NON-RANGE BASED COOPERATIVE LOCALIZATION FOR VANETS IN URBAN ENVIRONMENTS

by

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Abstract

Location-Based Services (LBS) and Intelligent Transportation Systems (ITS) demand positioning accuracy and availability requirements. In urban canyons, Global Navigation Satellite Systems (GNSS) suffer from signal blockage, severe multipath, and low Carrier-to-Noise (C/No) ratio which degrade positioning accuracy and availability. Therefore, applications solely relying on GNSS have limited performance. In this thesis, we present a novel unified Cooperative Positioning (CP) solution which enhances positioning accuracy and availability in urban canyons. The proposed system exploits the fact that vehicles have different positioning resources and is based on Angle Approximation (AA). AA requires no infrastructure or other aiding sensors, AA is distributed and addresses two core challenges (limited positioning accuracy and availability) in a unified solution. AA artificially generates the hindered pseudorange by sharing pseudoranges between vehicles using Dedicated Short Range Communication (DSRC). To enhance the performance of the AA technique, we propose the Absolute Sum of Double Differencing (ASODD) method which increases the probability of selecting the most accurate generated pseudorange. We also propose a vehicle selection method called Absolute Sum of Single Differencing (ASOSD). As the distance between vehicles decrease, the accuracy of the proposed system increases and hence ASOSD is utilized to increase the probability of selecting the nearest assisting
vehicle to the target vehicle. We have developed an Orbit Simulator to evaluate the performance of our system.

In addition, we employ the proposed cooperative system to assist the loose integration between the Inertial Navigation System (INS) and the GPS system (using Extended Kalman Filter) during partial GPS outages. Using raw data from inertial sensors and GPS receivers in real road trajectories, we implement the cooperative INS/GPS loose integration and show that our cooperative integrated system outperforms the non-cooperative integrated system. The performance metrics used are the 2D positioning Root-Mean-Square (RMS) error, the maximum 2D positioning error and the Positioning Accuracy Gain (PAG). Specifically, the PAG gain is around 88%, 80% and 60% when the number of blocked satellites is one, two and three respectively.
List of Publications


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## List of Abbreviations

- **2D**  
  Two-Dimensional

- **3D**  
  Three-Dimensional

- **A-GPS**  
  Assisted-GPS

- **A/D**  
  Analog-to-Digital

- **AA**  
  Angle Approximation Method

- **ASK**  
  Amplitude Shift Keying

- **ASODD**  
  Absolute Sum of Double Differencing

- **ASOSD**  
  Absolute Sum of Single Differencing

- **BDS**  
  BeiDou Navigation Satellite System

- **BER**  
  Bit Error Rate

- **C/A Code**  
  Course/Acquisition Code

- **CAV**  
  Candidate Assisting Vehicle

- **CC**  
  Control Channel
<table>
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<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CLC-EKF</td>
<td>Cooperative Loosely Coupled Extended Kalman Filter</td>
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<td>CP</td>
<td>Cooperative Positioning</td>
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<tr>
<td>CSMA/CA</td>
<td>Carrier Sense Multiple Access / Code Avoidance</td>
</tr>
<tr>
<td>CSMA/CD</td>
<td>Carrier Sense Multiple Access / Code Detection</td>
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<tr>
<td>DD</td>
<td>Double Differencing</td>
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<td>DGPS</td>
<td>Differential GPS</td>
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<td>DL</td>
<td>Down-Link</td>
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<tr>
<td>DLL</td>
<td>Delay Locked Loop</td>
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<tr>
<td>DoP</td>
<td>Dilution of Precision</td>
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<tr>
<td>DORIS</td>
<td>Doppler Orbitography and Radio-positioning Integrated by Satellite</td>
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<td>DoT</td>
<td>Department of Transportation</td>
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<tr>
<td>DR</td>
<td>Dead Reckoning</td>
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<tr>
<td>DSRC</td>
<td>Dedicated Short Range Communication</td>
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<td>DTG</td>
<td>Dynamically Tuned Gyroscopes</td>
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<tr>
<td>ECEF</td>
<td>Earth-Centered-Earth-Fixed coordinate system</td>
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<td>ENU</td>
<td>East-North-Up</td>
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<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
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<tr>
<td>FCC</td>
<td>Federal Communications Committee</td>
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<tr>
<td>FLL</td>
<td>Frequency Locked Loop</td>
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<tr>
<td>GBAS</td>
<td>Ground-based Augmentation System</td>
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<td>GLONASS</td>
<td>Global Navigation Satellite System</td>
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<td>GNSS</td>
<td>Global Navigation Satellite System</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>HRC</td>
<td>High Resolution Correlator</td>
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<td>HS-GPS</td>
<td>High Sensitivity GPS receivers</td>
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<tr>
<td>IFOG</td>
<td>Interferometer Fiber-optic Gyroscope</td>
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<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
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<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
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<tr>
<td>ITS</td>
<td>Intelligent Transportation System</td>
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<tr>
<td>KF</td>
<td>Kalman Filter</td>
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<tr>
<td>LBS</td>
<td>Location Based Services</td>
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<tr>
<td>LC</td>
<td>Loosely Coupled</td>
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<tr>
<td>LoS</td>
<td>Line of Sight</td>
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<tr>
<td>MAC</td>
<td>Medium Access Control</td>
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<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>MANET</td>
<td>Mobile Ad-Hoc Network</td>
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<tr>
<td>MCS</td>
<td>Monitor Control Station</td>
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<tr>
<td>MEMS</td>
<td>Microelectromechanical Systems</td>
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<tr>
<td>MEO</td>
<td>Medium Earth Orbit</td>
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<tr>
<td>NCO</td>
<td>Numerically Controlled Oscillator</td>
</tr>
<tr>
<td>NLC-EKF</td>
<td>Non Cooperative Loosely Coupled Extended Kalman Filter</td>
</tr>
<tr>
<td>Non-CP</td>
<td>Non Cooperative Positioning</td>
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<td>PAG</td>
<td>Positioning Accuracy Gain</td>
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<tr>
<td>PCF</td>
<td>Pre-Correlation Filter</td>
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<tr>
<td>PDD</td>
<td>Packet Delivery Delay</td>
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<tr>
<td>PDR</td>
<td>Packet Delivery Rate</td>
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<tr>
<td>PF</td>
<td>Particle Filter</td>
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<tr>
<td>PLE</td>
<td>Path-Loss Exponent</td>
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<tr>
<td>PRN</td>
<td>Pseudorange Number</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<td>RISS</td>
<td>Reduced Inertial Sensors System</td>
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<td>RLG</td>
<td>Ring Laser Gyroscopes</td>
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<td>RMS</td>
<td>Root-Mean-Square</td>
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RSS  Received Signal Strength
RSU  Road Side Unit
RTK  Real-Time Kinematic
RTT  Round Trip Time
SBAS Satellite-based Augmentation System
SD   Single Differencing
SDR  Software Defined Radio
SNR  Signal to Noise Ratio
STEM Sparse Topology and Energy Management
TC   Tightly Coupled
TDoA Time Difference of Arrival
ToA  Time of Arrival
U-TC Ultra-Tightly Coupled
UL   Up-Link
V2I  Vehicle to Infrastructure Communication
V2V  Vehicle to Vehicle Communication
VANET Vehicular Ad-Hoc Network
WHO World Health Organization
WSN

Wireless Sensor Network
Chapter 1

Introduction

1.1 Motivation

According to Transport Canada, the number of injuries due to traffic accidents in 2013 was 165,306 [1]. The World Health Organization (WHO) stated that the number of deaths due to traffic accidents reaches 1.24 million annually [2]. Aside from fatalities, traffic congestions cost Americans at least 124 billion dollars a year [3]. Intelligent Transportation Systems (ITS) aim at reducing traffic accidents and congestion. In addition, ITS systems enable many applications including entertainment and driver assistance applications. The recent developments in Vehicular Ad-Hoc Networks (VANETs) and Dedicated Short Range Communication (DSRC) enabled many of the ITS applications. Information about the position of the vehicles are used by many ITS applications and Location-Based Services (LBS). For example, in automated driving modes and safety critical applications, vehicles have to exchange their accurate positions. The required positioning accuracy and availability of the vehicles’ positions depends on the application.
1.2 Problem Statement

In urban environments, the positioning accuracy and availability of land vehicles is limited. Tall buildings block many signals from different Global Navigation Satellite Systems (GNSS). Buildings in urban areas also reflect GNSS signals causing multipath effect. The measured pseudoranges are also contaminated with errors due to uncompensated atmospheric delays and satellite clock biases. There are also many challenges on the receiver side that include:

- A limited number of vehicles employ advanced multipath and jamming detection and mitigation techniques.

- A limited number of vehicles are capable of decoding multi-constellation GNSS signals.

- A limited number of vehicles are capable of removing the uncompensated atmospheric errors using complex error models.

Accurate positioning of vehicles is required by most safety critical ITS applications. Positioning systems can be categorized as non-cooperative (conventional) and cooperative systems. Due to the harsh signal environment in urban areas, non-cooperative systems suffer from limited positioning accuracy [4]. Recently, Cooperative Positioning (CP) has been proposed as an ideal solution to the problem of limited positioning accuracy in urban environments. CP takes advantage of the fact that vehicles have different positioning resources and uses DSRC to exchange positioning information between vehicles and subsequently estimate accurate positions. Most of the proposed systems rely on ranging methods to estimate the distance between vehicles or between vehicles RSUs. Ranging methods introduce range errors to the estimated
1.3. THESIS OUTLINE AND CONTRIBUTIONS

distances [5]. These errors propagate to the final computed position and thus the performance of the range based CP systems are also limited.

1.3 Thesis Outline and Contributions

In chapter 2, the predominant positioning systems used in land vehicles are introduced. Global Positioning System (GPS) and Inertial Navigation System (INS) are briefly described, and the errors affecting their positioning performance are also presented. Furthermore, different configurations of Kalman Filters (KF) used to optimally fuse GPS and INS states are discussed. In addition, the paradigm of Vehicular Ad-hoc Networks (VANETs) and the Dedicated Short Range Communication standard (DSRC) which enable CP are both introduced. Finally, a literature review is conducted on the existing CP techniques for VANETs.

In chapter 3, we introduce a novel cooperative non-range based positioning system. The concept of Angle Approximation (AA) is introduced and numerical examples are used to present the variables affecting the performance of the proposed system. Then, the Absolute Sum of Double Differencing (ASODD) which is a proposed satellite selection method is explained and analytically derived. This method is used to increase the probability of selecting the most accurate pseudorange generated by the AA method. Vehicles are generally surrounded by many neighbors and therefore a vehicle selection criteria called Absolute Sum of Single Differencing (ASOSD) is introduced. ASOSD aids the proposed cooperative system in selecting the vehicle which is expected to provide the most accurate pseudoranges. Outdoor Experiments are used to test the viability of the proposed system. Finally, extensive simulations are conducted to test the effect of several parameters on the performance of the proposed
cooperative system. These parameters include the effect of the distance between vehicles, the standard deviation of the measured pseudorange error, the elevation mask of the satellites and the number of common visible satellites between the vehicles. Furthermore, we study the effect of the number of assisting vehicles and the standard deviation of the measured pseudorange error on the proposed assisting vehicle selection method. The main contributions of this chapter are as follows:

- A novel CP system is proposed that does not depend on estimating the distance between the vehicles. The AA method is distributed and does not require the installation of any Road Side Units (RSU). Our CP system has been accepted for publication [6].

- We propose a satellite selection method called ASODD. An analytical derivation proves that ASODD increases the probability of selecting the most accurate generated pseudorange.

- We propose a method for the selection of the assisting vehicle called ASOSD. Using analytical derivations, we show that ASOSD increases the probability of selecting the assisting vehicle that generates the most accurate pseudorange.

- The viability of the proposed cooperative system was tested using two NovAtel Receivers in an outdoor scenario.

- To measure the performance of our system we use extensive MATLAB simulations. The simulations were focused on: the effect of the distance between vehicles; the number of common satellites; the minimum satellite elevation; the standard deviation of the pseudorange error; and the number of assisting vehicles.
In chapter 4, we introduce a CP system that utilizes the exchange of pseudoranges from assisting vehicles to aid INS/GPS Loosely Coupled integration using Extended Kalman Filter (LC-EKF). The proposed system enhances the performance of the LC-EKF during partial GPS outages. First of all, we introduce the Reduced Inertial Sensor System (RISS) mechanization process. We then present the system and measurement model of the LC-EKF which is used to fuse the INS and the GPS states. Finally, the experimental setup, evaluation criteria, and results are presented. The main contributions of this chapter are as follows:

- We propose a CP system that uses AA and subsequently ASODD to enhance the positioning accuracy of RISS/GPS LC-EKF during partial GPS outages in urban environments.

- We perform real experiments by collecting road trajectories in Kingston. Then, the conventional RISS/INS LC-EKF is implemented and compared to the proposed cooperative RISS/INS LC-EKF in terms of 2D position RMS error, the maximum position error and the Positioning Accuracy Gain (PAG). The proposed system outperforms the existing LC-EKF during all simulated GPS partial outages. Specifically, the PAG gain is around 88%, 80% and 60% when the number of blocked satellites is one, two and three respectively.
Chapter 2

Background and Literature Review

2.1 Introduction

Global Positioning System and Inertial Navigation System are the two most utilized systems for the positioning of vehicles. This chapter introduces the principle of operation and sources of error of GPS and INS. Moreover, the predominant INS/GPS integration modes are briefly described. In VANETs, vehicles communicate with RSUs and with vehicles in their communication range using DSRC. In this chapter, VANETs, and DSRC are introduced followed by a review of the non-cooperative positioning and CP methods for VANETs.

2.2 Satellite Positioning Systems

Positioning can be either absolute or relative. In relative positioning, the position of the vehicle can estimated using sensors embedded in the vehicles’ platform. The estimated position in this case is relative to the initial position of the vehicle. On the other hand, absolute positioning refers to the positioning of receivers using the time of flight of signals from satellites with known locations. GPS, Global Navigation
Satellite System (GLONASS), BeiDou Navigation Satellite System (BDS), Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS) and Galileo positioning system (Galileo) are existing satellite positioning systems. Currently, GLONASS and GPS are the only satellite constellations that provide global coverage and are referred to as Global Navigation Satellite Systems (GNSS). The technical difference between the operation of the Russian (GLONASS) and the American (GPS) satellite systems are not within the scope of this research. Here, GPS is briefly introduced.

Global Positioning System is a US-based navigation system which provides users with Positioning, Navigation and Timing services. This system consists of three segments depicted in Figure 2.1.

![Global Positioning System Segments](image)
Space Segment
The GPS space segment consists of six orbits inclined at 55° from the equatorial plane. A minimum of 24 satellites are operational in order to guarantee providing positioning service to users all over the globe 95% of the time. Each orbit consist of four satellites orbiting the earth at a mean radius of 26,560 km which is at the Medium Earth Orbit (MEO). Each satellite covers an orbit in 12 hours. Satellites use Code Division Multiple Access (CDMA) to transmit at the same frequency simultaneously. To enhance the accuracy of positioning for a variety of applications, satellites transmits at L1 (1.57542 GHz), L2 (1.2276 GHz) and L5 (1.17645 GHz).

Control Segment
The GPS control segments consists of 16 monitoring sites and 12 command and control antennas used to analyze the performance of the GPS satellites and subsequently send control commands to the satellite. The monitoring sites receive and analyze messages from visible satellites, then send their conclusions to the Monitor Control Station (MCS). The MCS provides command and control of the GPS satellites. Using the information from the monitoring station, the MCS estimates an accurate position of the satellite and then sends this data to the satellites. Consequently, satellites broadcast their accurate position to users. Moreover, one of the functions of the MCS is to detect the failure of satellites and possibly reposition satellites to achieve optimal performance. For the MCS to communicate with satellites, ground antennas transmitting at the S-band are used. The number of ground antennas is four, and they co-exist with the monitoring stations.

User Segment
Land vehicles, marine vessels, and aerial vehicles are all examples of GPS users. GPS
receivers acquire and track signals from visible satellites and employ algorithms that use information embedded in the GPS signal to estimate the receivers’ positions. The accuracy an estimated position depends on the accuracy of the measured pseudoranges to the satellites. Moreover, positioning accuracy might also depend on the receivers’ hardware capabilities. For example, High Sensitivity GPS (HS-GPS) receivers are capable of tracking signals from satellites at lower Carrier-to-Noise Ratio ($C/N_0$) compared to standard GPS receivers. In addition, the accuracy of the estimated position can be a function of the employed state estimation filter. Shortly, the GPS sources of error will be briefly presented.

2.2.1 GPS Principle of Operation

To estimate the position of the GPS receiver, the distances between three satellites and the GPS receiver have to be estimated. However, since the clock bias of the receiver is unknown, the receiver has to estimate the distance to a fourth additional satellite. Moreover, the receiver has to know the position of each of the four satellites. Given four satellites and four distances, the latitude, longitude, altitude and the clock bias of the receiver can be estimated using Trilateration if the distances are perfect estimates. However, estimation techniques are normally used instead of Trilateration since the distances are erroneous. If the number of visible satellites is less than four, then it is not possible to compute a 3D position. Figure 2.2 depicts positioning a user using four satellites.
Satellite Position

The position of four satellites and the range to each of the satellites has to be determined in order to estimate the position of the receiver. The receiver estimates the position of the satellites using the navigation message. Each satellite periodically sends a navigation message which modulates the transmitted carrier signal (L1). This navigation message contains the ephemeris data which helps the receiver estimate the satellites’ positions. In addition, the satellite clock errors and some parameters used to estimate ionospheric errors by single frequency receivers are part of the navigation message. Furthermore, the message contains information about the health status of the satellite. The source of the navigation message is the MCS which monitors the satellites and then sends messages to each satellite using ground antennas.
Range Estimation

Now that the position of the satellite is estimated from the ephemeris data, the range to the satellite has to be also estimated by the receiver. There are two main observables that can be used to estimate the range between the receiver and the satellite. The GPS observables are pseudorange measurements, carrier phase measurements and Doppler measurements. Since carrier phase is out of the scope of this research, only pseudorange and Doppler measurements will be introduced.

Carrier Demodulation and Doppler Estimation

In order to estimate the range to the satellite, the GPS receiver has to first demodulate the received signal. This is performed using a mixer which multiplies the received L1 signal with a locally generated replica of the L1 signal. A Numerically Controlled Oscillator (NCO) is used to generate a clone of the received L1 signal. By demodulating the received signal, the carrier frequency is removed. Since the satellite is moving at a very high speed and the receiver could also be moving, the center frequency of the received signal is shifted due to Doppler effect. The Frequency Locked Loop (FLL) of the GPS receiver is used to estimate the Doppler shift in real time and use this estimate to control the NCO and hence correctly demodulate the received signal. This Doppler shift is also used by the receiver to estimate the velocity of the vehicle.

Pseudorange Estimation

Each satellite sends an L1 signal coded with a Pseudo-random Number (PRN). Using CDMA satellites can transmit at the same frequency all the time utilizing different codes. After the Doppler estimate and carrier demodulation step, the receiver has to estimate the time between the transmission of the signal from the satellite to the time the signal is received. By multiplying this time with the speed of light, the
range and then the position of the satellite can be estimated. Equation 2.1 shows the relation between transmitting time, receiving time, speed of light and the measured range denoted by $t_{TX}$, $t_{RX}$, $c$ and $\rho$ respectively. Here, the range is called pseudorange because the measured quantity is not merely a function of the transmission and reception time but is also a function of the misalignment of the receiver’s clock and the satellites’ clocks.

$$\rho = c \times (t_{TX} - t_{RX}) \quad (2.1)$$

The pseudorange is estimated by the Delay Locked Loop (DLL) which multiplies many shifted replicas of a locally generated PRN with the received PRN. The DLL searches for the shift which corresponds to the maximum correlation. The delay at which the maximum correlation occurs is known as the correlation peak. By computing the auto-correlation function between the received PRN and a generated replica, the DLL estimates the shift at which the correlation peak occurs and hence the difference between the transmission time and the reception time can be computed. Subsequently, using (2.1), the pseudorange to the satellite can be estimated.

The estimated pseudorange is an initial guess as it contains many some uncompensated errors. The pseudorange between the $i^{th}$ receiver and the $s^{th}$ satellite can be modeled by the following equation:

$$\rho_i^s = R_i^s + c(\delta t_{RX} - \delta t_{TX}) + Ion^s + Tro^s + \varepsilon_i^s \quad (2.2)$$

where

- $R_i^s$ is the true range from vehicle $i$ to satellite $s$. 
2.2. SATELLITE POSITIONING SYSTEMS

- $\delta t_{RX}$ is the clock bias of receiver $i$.
- $\delta t_{TX}$ is the clock bias of satellite $s$.
- $Ion^s$ is the error due to Ionospheric errors.
- $Tro^s$ is the error due to Ionospheric errors.
- $\varepsilon_i^s$ is the error due to multipath and receiver’s noise.

Assume the satellite clock bias and the ionospheric and tropospheric errors are correctly compensated. Furthermore, assume the error due to multipath and the receiver’s noise are negligible. Equation 2.2 can be approximated to the following equation:

$$\rho_i^s = R_i^s + c\delta t_{RX}$$

(2.3)

where

$$R_i^s = \sqrt{(x_i - x^s)^2 + (y_i - y^s)^2 + (z_i - z^s)^2}$$

(2.4)

In (2.4), the position of the $s^{th}$ satellite in the Earth-Centered-Earth-Fixed (ECEF) is denoted by $x^s$, $y^s$ and $z^s$. Also, the position of the $i^{th}$ receiver in the Earth-Centered-Earth-Fixed (ECEF) is denoted by $x_i$, $y_i$ and $z_i$. Since the receiver’s 3D position and clock bias are four unknowns, using pseudorange measurements to four satellite, the position of the receiver can be accurately estimated using the concept of Trilateration.
2.2. SATELLITE POSITIONING SYSTEMS

2.2.2 GPS Errors

Given that the receiver’s clock bias is estimated, the pseudorange measured by the GPS receiver should indicate the distance between the receiver and the satellite. However, due to many factors, the pseudorange measurements are contaminated with errors. Table 2.1 shows typical errors in pseudoranges due to different sources of error (not in dense urban environments). Estimating an accurate position relies heavily on the receiver’s ability to compensate for the errors in all the measured pseudoranges.

Atmospheric Delays

The two mediums that affect the propagation delay of electromagnetic waves transmitted from satellites are the ionospheric and the tropospheric layers. The effect of the ionospheric layer stretches from 50 to 1000 km above the surface of Earth [7]. The radiation of the sun causes photons to impinge atoms and molecules resulting in a layer of free electrons and ions. The level of ionization is a function of the solar activity across the ionospheric layer. By varying the ionization level, the refractive indexes of the ionospheric layers change. Radio waves transmitted from receivers on earth below 30 MHz are reflected by the Ionospheric layer. However, at the L-Band, the radio wave passes through the ionospheric layer but the propagation delay of the signal is affected. Hence, the amount of ionospheric delay depends on the frequency of the signal. The single frequency GPS receiver uses the parameters of the Klobuchar model in the navigation message to estimate the ionospheric delay [8]. Using Klobuchar model, single frequency receivers compensate for around 50% of the ionospheric delay [9]. On the other hand, Dual frequency receivers use two frequencies to accurately estimate the ionospheric delay at the expense of the complexity of the receiver. Another method to determine the ionospheric delay is by using a
reference receiver with a known position. If a user has access to the pseudoranges and the position of the reference receiver within its vicinity, the user can determine the ionospheric delay. This is due to the fact that ionospheric delays do not change rapidly over small distances (less than 50 km).

Contrary to the dispersive properties of the ionosphere, the troposphere is not a frequency selective medium. However, it is also a refractive medium. Radio waves traveling in free space encounter a delay in the troposphere due to the existence of water vapor and gases like Nitrogen and Oxygen. This layer stretches from 8 to 50 km which is right below the ionosphere. There are several models used to predict the tropospheric delay like Chao model [10] and Hopfield model [11]. The ionosphere and troposphere delays are both function in the elevation of the satellite relative to the receiver. Specifically, radio waves transmitted from high elevation satellites experience less delay compared to lower elevation satellites.

Clock Bias

Since time of arrival is used as a ranging method, the synchronization between the satellite clock, the receiver clock and the GPS system time is very critical. If the clocks are off by 1 microsecond, multiplying this quantity by the speed of light results in around 300 meters of error in the range between the satellite and the receiver. For this reason, GPS satellites use atomic clocks (cesium and rubidium oscillators) which have very small drifts but are very expensive. Although atomic clocks are very accurate, they still drift relative to the GPS time. This drift is monitored by the monitoring station and subsequently the MCS updates the navigation message with the satellite clock error parameters. Then, the ground antennas broadcast the new
navigation message to the satellites and hence users can compensate for each satellite’s clock drift using the clock error parameters.

Atomic clocks can not be used for commercial receivers due to their high cost. Hence, the user segment consists of cheap clocks with very large drifts and unknown initial clock bias. Fortunately, pseudoranges are measured relative to the receiver’s clock and therefore the measurements from all satellites are affected by the same clock bias. By assuming that the clock bias is unknown, using a fourth satellite, the position and the clock bias of the receiver can be estimated.

Multipath

The dominant source of position errors in urban environments produced by GNSS receivers for high accuracy applications is multipath [12]. In order to accurately measure the range between the satellite and the receiver, a Line of Sight (LoS) signal is expected. Receiving delayed versions of the same signal (with different amplitudes and phases) from the satellite due to the existence of reflectors increases the error in the estimated position. Many factors affect the significance of multipath on the errors introduced to the measured pseudoranges. These factors are:

**Type of Signal Modulation:** The autocorrelation function of the L1 C/A GPS signal is different from the E1 Galileo signal. Specifically, the autocorrelation function of the latter has three peaks compared to one peak for the L1 signal. In a multipath environment, more than one peak makes it more challenging for the DLL discriminator to lock onto the correct peak.

**Correlator Spacing:** The correlator spacing between early and late depends on the Pre-Correlation Filter (PCF) bandwidth. If the PCF bandwidth is high, then the correlator spacing is low and thus reduce the effect of multipath. The correlator
Discriminator Function: Different types of DLL discriminators have distinct resilience to multipath errors (each discriminator has a unique function). Some are resilient to short delay multipath, others are resilient to medium and long multipath delays. As a rule of thumb, the narrower the correlator spacing (controlled by the bandwidth of PCF), the less the DLL discriminator loop is affected by multipath.

Chip Duration: Multipath error significance is greatly dependent on the chipping rate. As the duration of the chip decreases, the effect of the multipath signal decreases, since only very short time-delayed multipath signals will confuse the DLL discriminator.

Environment: The amplitude of the multipath (relative to LoS), the phase and the number of significant multipath are all environment dependent. The multipath characteristics has different effects on the tracking loops.

Multipath errors can be mitigated using signal processing techniques like High Resolution Correlators (HRC) [13]. However, the first line of defense would be the choice and orientation of the antenna. It is worth noting that multipath effects in open sky environments like highways are negligible.

Receiver Noise
Modern receivers have many tracking channels which are used to track several satellites at the same time. Synchronization between different tracking channels ensures accurate pseudorange measurements. Errors due to front end circuitry, synchronization between tracking channels, sampling, quantization and noise received at the antenna can accumulate and result in an error known as receiver noise. The error due to the noise at the receiver is limited but can affect applications requiring highly
2.3. INERTIAL NAVIGATION SYSTEM (INS)

accurate positions.

**Ephemeris Data**

Errors due to ephemeris data occurs when the orbital parameters are not computed correctly by the control segment. Hence, the GPS receiver would estimate an inaccurate satellite position using the orbital parameters in the navigation message. Subsequently, the estimated range between the satellite and the receiver is also inaccurate due to the erroneous satellite position.

<table>
<thead>
<tr>
<th>Source</th>
<th>Function of</th>
<th>One-Sigma (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionosphere</td>
<td>Solar activity, Signal frequency and Satellite Elevation</td>
<td>4.0</td>
</tr>
<tr>
<td>Troposphere</td>
<td>Percentage of water vapor and gases and Satellite Elevation.</td>
<td>0.7</td>
</tr>
<tr>
<td>Satellite Clock</td>
<td>Atomic clock accuracy</td>
<td>2.1</td>
</tr>
<tr>
<td>Ephemeris Data</td>
<td>Ephemeris Model</td>
<td>2.1</td>
</tr>
<tr>
<td>Multipath</td>
<td>Environment</td>
<td>1.4</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>Receiver Software and Hardware</td>
<td>0.5</td>
</tr>
</tbody>
</table>

2.3 Inertial Navigation System (INS)

GNSS systems provide absolute positioning. In dense urban environments, satellites can be partially or completely blocked. Moreover, in indoor environments, GNSS signals are very weak and are not suitable for positioning (except for very high sensitivity receivers). In these situations, relative positioning can be effective since it does not rely on external signals for state estimation. Using Dead Reckoning (DR) techniques, data from several sensors are collected and the position, velocity and attitude of the platform are estimated. In this section, the principle of operation of the INS system is briefly introduced.
2.3. INERTIAL NAVIGATION SYSTEM (INS)

2.3.1 INS Principle of Operation

An INS system consists of an Inertial Measurement Unit (IMU), an Analog to Digital converter (A/D), a signal processing stage and a mechanization stage. These blocks are discussed below.

**Inertial Measurement Unit (IMU):** Accelerometers and gyroscopes are the building units of any IMU. The three axis gyroscope measures angular rates over three perpendicularly aligned (theoretically) axes. Interferometer Fiber-optic Gyroscope (IFOG), Ring Laser Gyroscopes (RLG), Dynamically Tuned Gyroscopes (DTG) and Microelectromechanical gyroscopes (MEMS) are all types of gyroscopes used in different applications ranging from submarine to land navigation. In addition, a three axis accelerometer measures the specific forces over three perpendicularly aligned (theoretically) axes. Gravimeters, Quartz resonators and mechanical floated instruments are some of the technologies used to implement accelerometers for various applications depending on the required accuracy. Numerous accelerometer and gyroscope technologies, their applications and accuracies are illustrated in [14]. In this research, MEMS-based sensors are used because of their very low cost and hence applicability for many commercial applications. However, raw data from standalone MEMS-based navigation sensors have very complex errors (very hard to model) and this will be discussed in the next section.

**Analog to Digital Converter (A/D):** In order to perform complex digital computations, an analog signal has to be sampled, quantized and then digitized. Some of these computations include noise filtering and mechanization. An A/D converter is used to convert the analog output of the accelerometers and gyroscopes to a digital output. This output can be stored inside registers in a microcontroller and several
2.3. INERTIAL NAVIGATION SYSTEM (INS)

computations can be performed. Two of the most important characteristics of A/D converters are the sampling frequency and the number of levels (quantization) used to represent the input to the A/D converter.

**Signal processing:** The raw data from accelerometers and gyroscopes are contaminated with high and low frequency noise. In the signal processing stage, the INS designer aims at removing deterministic errors from the system. This ensures that known errors in the raw data do not propagate to the states of the systems.

**Mechanization:** The input to the mechanization stage is the digitized raw data of the 3-axis accelerometers and the gyroscopes. The mechanization process performs several computations to estimate the position, velocity and attitude of the platform (given an initial state). The first step is to compute the attitude angles which are the pitch, raw and azimuth using the angular rates and specific forces. Using the rotation matrix from the last iteration, the specific forces are transformed from the body frame to the navigation frame. The transformed specific forces have to compensated for gravity using a known model. Moreover, it has to be compensated for the Coriolis effect (this effect is negligible for low speeds). Subsequently, an integration is computed to estimate the velocity of the platform in the East-North-Up (ENU) frame. In addition, a second integration is computed to estimate the position (geodetic) of the platform. Figure y shows the main steps of the mechanization process.

2.3.2 Inertial Sensor Errors

Sensors manufactured using different technologies have different error characteristics. It is very important to understand the error characteristics of the inertial sensors in order to be able to compensate for such errors. Errors in the raw data measured by
accelerometers and gyroscopes can either be deterministic or stochastic errors. It is much easier to compensate for deterministic errors compared to stochastic errors.

**Deterministic Errors:** These are systematic errors in the inertial sensors measurements that can be removed using laboratory calibration. Most inertial sensor manufacturers remove deterministic errors and provide users with calibrated raw data. Some of the common deterministic errors are bias offset, scale factor, quantization, axes non-orthogonality and misalignment errors. Bias offset is the existence of an output from the accelerometers and gyroscopes when the input to the system is zero. One of the characteristics of the systematic bias offset is that it does not depend on the current sensor measurements. The scale factor is the deviation of the true gradient that describes the relationship between the inputs and the outputs of the inertial sensors.

Quantization is another source of error and it is a deterministic error which is a function of the number of levels used to represent the input. As the number of the levels increase, the quantization error decreases but the memory requirements and hence the cost of the INS increases. Furthermore, non-orthogonality errors occur when sensors are not perfectly orthogonal due to manufacturing errors. Moreover, mounting imperfection results in the misalignment between the sensitive axes of the inertial sensors relative to the axes of the body frame. There are many methods employed by manufacturers to remove the deterministic errors. Some of these methods are discussed in [15].

**Stochastic Errors:** While most of the deterministic errors in the inertial sensor measurements can be removed by the manufacturers, random errors are very hard to compensate for. Random errors are usually modeled using different stochastic
processes depending on the characteristics of the error. Run-to-Run bias offset, bias drift, scale factor instability and white noise are the main stochastic errors contaminating the accelerometer and gyroscope measurements. Run-to-Run bias offset is the change in the value of the bias from one run to the other. This error can be modeled as a random constant process since the value of the run-to-run bias offset is constant per run. Moreover, the bias drift is the amount of drift that affects the bias offset which is random in nature. while scale factor is a deterministic error characterized by a gradient that defines the relationship between the input and the output of the inertial sensors, due to variations in the temperature, the scale factor value might randomly change.

Finally, white noise is a random process which is uniformly distributed over the entire spectrum. The samples of white noise are uncorrelated in time but at a certain instant can be modeled as a Gaussian white noise with zero mean and finite standard deviation. The source of white Gaussian noise is the intrinsic characteristics of the semiconductor material or due to the variations of the power source.

2.4 INS/GPS Integration

The solution computed using GPS has different characteristics compared to the solution computed by INS. The errors in the solution computed by GPS does not accumulate over time since at each time epoch new pseudoranges are used to estimate the current states. In other words, the estimated states using GNSS systems are time invariant with homogeneous accuracy [16]. However, single GPS receivers can not provide information about the orientation of the platform over the three perpendicular axes. In addition, GPS receivers are susceptible to signal blockage, low carrier to
noise ratio, multipath, low satellite visibility in urban areas and jamming. Moreover, the solution rate of most commercial GPS receivers is limited to 1 Hz.

The solution from an INS system depends on the previous state. Hence, INS systems accumulate errors over time. For example, just in 50 seconds, a standalone MEMS-based INS system could provide a position with more than 100 meters errors. Strategic and Tactical grade INS systems provide very good accuracy (but errors still slowly accumulate over time) at the expense of very high cost. In other words, INS short-term errors are small but they degrade exponentially (depending on the grade of the INS system) in an unbounded manner and thus external aiding sources are essential [17]. However, INS systems are not susceptible to signal blockage or jamming. They provide very high data rate (100-200 Hz) with information about the orientation (attitude) of the platform over the three axes.

<table>
<thead>
<tr>
<th>Features</th>
<th>GPS</th>
<th>INS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positioning Type</td>
<td>Absolute Positioning</td>
<td>Relative Positioning</td>
</tr>
<tr>
<td>Accuracy</td>
<td>Good in long-term</td>
<td>Good in short-term</td>
</tr>
<tr>
<td>Attitude</td>
<td>Not Available</td>
<td>Available</td>
</tr>
<tr>
<td>Sensitive to Jamming</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Sensitive to Gravity</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Sensitive to Previous State</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Solution Rate</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Functions Indoor</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of GPS and INS important features

Due to the complementary characteristics of the GPS and INS systems, integrating both systems has been extensively used in practice. Table 2.2 summarizes the characteristic of the solution offered by the commercial GPS receiver and an INS system. GPS solution can be used to aid INS by slowing down the accumulation
2.5. INS/GPS MODES OF INTEGRATION

There are several modes of integrating INS and GPS data. The difference between the modes of integration lies in the type of data that is integrated. As the type of integrated data approaches the raw INS and GPS data, the integration is considered deeper, centralized and more complex. According to [18], the most common modes of integration are Uncoupled, Loosely Coupled (LC), Tightly Coupled (TC) and Ultra-Tightly Coupled (U-TC) integrations.

2.5.1 Uncoupled

The uncoupled integration represents a system where INS and GPS operate independently and provide two solutions. GPS solution is used to reset INS position and velocity errors without estimating the errors due to sensor drifts. The object to be localized adopts the GPS solution whenever it is available. However, during GPS outages, the INS solution is used. Even though this integration is the least complex approach, uncompensated inertial sensor errors accumulate over time (azimuth errors grow exponentially leading to very poor accuracy during outages). Moreover, since the GPS standalone solution is used whenever it is available (INS ignored), abrupt
GPS outliers due to multipath or poor satellite geometry severely affects the accuracy of the navigation solution. Due to all the aforementioned reasons, uncoupled integration is rarely used in practice.

### 2.5.2 Loosely Coupled

In the LC approach, the INS and the GPS measurements are first processed independently. The IMU sends the raw INS measurements to the mechanization stage which computes the navigation solution. Also, the GPS raw measurements are processed using either a Weighted Least Squares (WLS) or a KF to obtain a navigation solution. At this point, two navigation solutions are available, using another filter, the difference between the solutions, the confidence of the designer in the GPS solution and the confidence of the designer in the INS solution are all used to estimate the INS state errors. These errors are then used to correct the INS estimate and compute the final navigation solution. In a closed loop feedback architecture, the estimated INS errors are used by the mechanization stage to remove the errors in the INS states and the sensor errors before computing future estimates. However, in an open loop architecture, the estimated INS errors are not sent back to the mechanization stage (ensuring the sensor errors are uncorrelated). The open loop architecture may only be used when high grade INS system is used or when the nominal trajectory is known. This is due to the exponential growth of INS errors in low-grade INS systems like MEMS-based sensors [19]. Figure 2.3 shows the block diagram of a closed loop LC integration.

Since INS and GPS measurements are processed independently, this is a decentralized integration. LC integration is robust because INS and GPS operate separately [16].
Thus, if one of the systems fail for any reason, the output of the other system can be used and the integrated solution can be ignored. However, when the number of visible satellites is less than four, INS solution is used in a standalone mode. In urban areas, limited satellites are visible and the LC filter will not use the available GPS measurements to aid the INS system if the number of visible satellites is less than four. Hence, the error in the navigation solution will grow exponentially during GPS partial outages. This is the main drawback of the LC integration.

2.5.3 Tightly Coupled

In the TC approach, the IMU sends the raw inertial sensor measurements to the mechanismization stage which computes the INS states. Subsequently, the GPS pseudoranges
and pseudorange-rates (GPS observables) are predicted using the INS states and the GPS ephemeris data (satellite position and velocity). Now, a filtering technique like KF or PF is used to estimate the errors in the INS states using the difference between the measured and predicted GPS observables. The confidence in the measured and predicted GPS observables are parameters used KF to estimate the error in the INS states. In a closed loop feedback architecture, the estimated INS errors are used by the mechanization stage to remove the errors in the INS states and the sensor errors before computing future estimates. Figure 2.4 shows the block diagram of the TC integration.

Since INS and GPS measurements are processed together, this is a centralized in-
2.5. INS/GPS MODES OF INTEGRATION

tegration. The main advantage of the TC approach is that even when the number of visible satellites is less than four, the pseudoranges and pseudoranges-rates of the visible satellites can still aid the INS system and hence limit the growth of error during partial GPS outages. However, the robustness of the TC filter is questionable because if one of the systems fail, there will be no solution. Moreover, the TC filter has two more states (receiver’s clock bias and drift) compared to the LC filter.

2.5.4 Ultra-Tightly Coupled

In this mode of integration, raw INS and GPS data are integrated. The raw measurements from the inertial sensors are used to aid the tracking loops of the GPS receiver. For example, this helps the receiver’s tracking loops in limiting the search domain for the accurate carrier Doppler shift and the code phase shift estimates and hence result in faster acquisition and tracking. Moreover, the GPS pseudoranges and pseudorange-rates are used to estimate the INS errors. This is the deepest and most complex form of integration that requires access to the firmware of the GPS receiver (which is not easily provided by the manufacturers). The receiver has to be a Software Defined Receiver (SDR) to allow others to implement a U-TC integration. However, most GPS receivers on the market use dedicated hardware architecture. Another issue with the U-TC integration is the propagation of undetected errors in the tracking loops. Finally, U-TC integration provides only one solution which is the integrated solution.
2.6 Vehicular Ad Hoc Networks (VANets)

The development of numerous applications for ITS systems has provided motivation for establishing a network of vehicles [20]. A vehicular network is a subclass of a Mobile Ad Hoc Network (MANet). Contrary to MANet, nodes in a VANet can move at very high speeds. This unique characteristic imposes a new constraint on vehicular networks which is fast and reliable communication between vehicles [21]. Current advancements in software, hardware and communication technologies enabled the implementation of such networks. VANETs enable applications leading to safer roads, efficient driving and passenger entertainment [22]. Vehicle collision warning, advanced driver assistance, platooning, Internet access and dissemination of road information along with many services are some of VANETs applications [24]. Dissemination of road information is an application requiring the existence of a communication link between the vehicles and a centralized entity (RSUs). On the hand, applications like vehicle collision warning rely on the frequent exchange of state information between vehicles. In this case, state information includes position, velocity, acceleration, attitude or inter-vehicular distance.

2.6.1 Dedicated Short Range Communication (DSRC)

A variety of wireless technologies are utilized for Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication. Dedicated Short Range Communication is one the technologies enabling wireless communication in VANets. The US Department of Transportation (DOT) reported that the utilization of DSRC for V2V communications addresses 82% of all crashes in the United States where drivers are not impaired. Consequently, thousands of lives and billions of dollars can be saved [25].
Nodes in VANet share a wireless medium where the bandwidth is limited. Some of the protocols designed for sharing wireless mediums are Carrier Sense Multiple Access with Collision Detection (CSMA/CD), Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) and Sparse Topology and Energy Management (STEM). The IEEE 802.11 wireless standard uses CSMA/CA for Medium Access Control (MAC). The Federal Communications Commission (FCC) has dedicated a bandwidth of 75 MHz in the band from 5.85 to 5.925 GHz for Vehicle-to-Vehicle communications. This band consists of 6 service channels. The bandwidth allocated to each service channel is 10 MHz. Also, a 5 MHz band is reserved for a control channel (CC). The supported data-rate ranges from 3 Mbps to 27 Mbps depending on the utilized modulation technique and channel coding rate [26]. The European Telecommunications Standards Institute (ETSI) has also assigned 30 MHz for the V2V communications in the 5.9 GHz band [27]. Moreover, DSRC in the US and Europe uses the IEEE 802.11p standard [28]. IEEE 802.11p is a modified version of IEEE 802.11 protocols allowing reliable inter-communication between high speed vehicles and communication between vehicles and RSUs (this is achieved by doubling the transmission time for every symbol compared to the transmission time of IEEE 802.11a) . The Japanese Association of Radio Industries and Businesses (ARIB) assigned the band 5.770-5.850 GHz for DSRC. The bandwidth per channel is 5 MHz with 7 Downlink (DL) and 7 Uplink channels (UL). The data-rate per channel is 1 Mbps using Amplitude Phase Shift Keying (ASK) and 4 Mbps using Quadrature Phase Shift Keying (QPSK) [29].

There are different classes of DSRC devices defined by the FCC. The DSRC devices are categorized mainly according to the maximum transmitted power by the device.
2.6. VEHICULAR AD HOC NETWORKS (VANETS)

The maximum range of communication between vehicles is a function of the transmitted power. Table 2.3 shows the four classes defined by the FCC. For each class the corresponding maximum transmitted power in dBm and communication range in meters are shown [30]. It is important to note that the range of communication between vehicles shown in Table 2.3 is based on LoS condition, where the path-loss exponent is constant and the received power is inversely proportional to the square root of the distance between the transmitter and the receiver (no shadowing effect) [31]. In urban areas, the LoS might be blocked and hence decrease the effective communication range.

**Table 2.3: FCC Device Classification**

<table>
<thead>
<tr>
<th>Device Class</th>
<th>Maximum Transmitted Power (dBm)</th>
<th>Communication Range (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>D</td>
<td>28.8</td>
<td>1000</td>
</tr>
</tbody>
</table>

**DSRC Performance**

In order to assess the performance of DSRC, two main metrics are used in the literature. Namely, Packet Delivery Rate (PDR) and Packet Delivery Delay (PDD). Applications have different PDR and PDD requirements and hence it is essential to investigate the performance of DSRC under various traffic and channel conditions. For example, V2V safety applications require a minimum message exchange rate of 10 packets per second [32].

Some studies analyzed the effect of the number of vehicles per km on PDR and PDD. The effect of dense traffic conditions wherein the number of vehicles could reach up to
100 per km is investigated in [33]. A PDR of 65% and PDD of 1 ms is achieved when the number of vehicles is 100 vehicle per km and the packet length is 200 bytes. The PDR value increases as the number of vehicles per km decrease, while the PDD value decrease under the same traffic conditions. Moreover, in [34] the effect of varying the communication range on PDD and PDR is studied. Results show a decrease in PDR and an increase in PDD as the transmission range increases.

VANETs operate in different environments including rural, suburban and urban areas. In dense urban environments, strong multipath signals due to many reflecting surfaces are dominant and lead to constructive and destructive interference. In [35], the effects of Signal-to-Noise ratio (SNR) and multipath on the received Bit-Error-Rate (BER) are examined. As SNR increases, BER decreases. In addition, doubling the power of multipath signals result in increasing the BER by two orders of magnitude.

2.6.2 Positioning for VANETs

To provide a certain service, many VANET applications require the state information of the vehicles. One of the important states is the position of the vehicle. In general, most applications have constrains on the accuracy of the vehicle’s position. The difference between the true and the estimated position is a measure of position accuracy. In addition to the accuracy of the vehicle’s position, the availability of the position data is also critical. Position availability is the percentage of time a vehicle has enough information to estimate its position. It is worth noting that the word position and location are interchangeable in the literature. However, in the context of this thesis, position refers to coordinates of the vehicle, while location identifies the place of the vehicle on the map (location of the vehicle is limited to roads defined
on the map).

In [22], vehicle provides positioning information all the time but with varying accuracies. The author divided the applications into three main categories depending on the location accuracy required by the application. The three categories are:

- **Applications based on Low-accurate Location:**
  
  Data dissemination and routing protocols are used by many applications requiring limited location accuracy which could reach up to 30 m [22]. However, as the position accuracy degrades, the performance of such protocols deteriorates. For example, data dissemination protocols are used in applications where RSUs broadcast information about road conditions to vehicles in their vicinity. Accordingly, traffic efficiency applications could suggest changing the driver’s route in the case of congestion. If the estimated location of the vehicle has very low accuracy, the received road information might be irrelevant and hence applications could provide incorrect suggestions.

- **Applications based on Moderate-accurate Location:**
  
  Platooning and Blind Crossing applications require moderate-accurate location which range from 1 m to 5 m. When some vehicles are driving to a common location, platooning application allows vehicles to follow one leading vehicle autonomously while maintaining a fixed inter-vehicular distance. If the location accuracy degrades, it is possible that the following vehicles might lose track of the leading vehicle. Moreover, the relative distance constraint could be violated if maintaining the distance is based on the estimated erroneous position (not by using accurate LoS sensors like ultrasonic sensors which are used in parking-assist applications). One of the interesting applications that require moderate
positioning accuracy is resource allocation for predictive video streaming [?].

- Applications based on **High-accurate Location**: Automated parking and safety applications require sub meter level accuracy. These systems would not tolerate inaccurate positions as it would result in jeopardizing the safety of passengers or passing pedestrians.

There are two main approaches used to enhance positioning of vehicles. Non-cooperative and CP positioning approaches. This research focuses on positioning of vehicles in very challenging environments like urban areas where most of the conventional positioning techniques fail to meet the requirements of ITS applications.

**Non Cooperative Positioning**

Non-cooperative positioning methods consist of two main classes. The first class contains all standalone aiding systems that assist GPS receivers. For instance, data from sensors are fused with the navigation solution from GPS receivers in order to enhance positioning accuracy and availability. In addition, map matching techniques are used to constrain the GPS estimated position to roads. Examples of standalone aiding systems are vision-based systems [36], inertial sensors [37], odometers [38] and digital maps [39]. The main focus of this research is not using standalone solutions to enhance vehicular positioning and this due to their cost and limited performance. The second class of non-cooperative methods utilize systems built specifically to assist GPS receivers in estimating the ionospheric and tropospheric errors. Since these atmospheric errors are the same within a defined geographical area, a reference GPS receiver with known position can be used to precisely compute and then propagate these errors to GPS receivers in their vicinity. Differential Global Positioning System
(DGPS), Satellite Based Augmentation System (SBAS) and Real Time Kinematic (RTK) and Assisted-GPS (A-GPS) are all examples of systems built specifically to aid GPS receivers in removing atmospheric errors [40].

Users receive the value of ionospheric and tropospheric propagation delays from DGPS stations and SBAS geostationary satellites. Hence, a more accurate estimate of pseudoranges to visible satellites can be achieved. In SBAS, a ground station computes the atmospheric errors and sends these estimates to the SBAS satellites. Thus, GPS receivers can also use the SBAS satellite position and range (not affected by atmospheric delays) to enhance positioning accuracy and availability. It is important to mention that the accuracy of the transmitted atmospheric errors increases as the GPS receivers are closer to the reference station. However, DGPS and SBAS are not able to compensate for multipath errors and receiver noise because they are not common between the GPS receivers and the reference station. Moreover, in dense urban areas many GPS signals are completely blocked and hence positioning accuracy and availability degrade. In addition, if GPS receivers are close to a jamming source, the broadcast errors from DGPS and SBAS are useless unless GPS receivers employ anti-jamming techniques. Finally, RTK is an aiding system that consists of a communication link between a reference station and a user. The positioning in RTK is based on carrier phase rather than code phase. Hence, the positioning accuracy could reach mm level. However, the availability of five visible satellites is required in order to solve the integer ambiguity (number of carrier cycles to the satellite). It is obvious that this solution cannot be deployed in dense urban areas due to the limited satellite visibility and the impracticality of computing the integer ambiguity each time a temporary satellite outage occurs [41].
When the $C/N_0$ of the signal transmitted from the satellite is low, the GPS receiver takes a long time to find the autocorrelation peak. A-GPS system is used to assist GPS receivers when the received signals are weak by decreasing satellite acquisition time and saving around 30 seconds to find a position. However, A-GPS performance is exactly the same as standalone GPS when more than four weak signals are available. Moreover, the main challenge for vehicles in urban areas is signal blockage, multipath and jamming. These problems can not be resolved by using GPS augmentation systems and thus CP methods are proposed.

**Cooperative Positioning**

One of the main problems in Wireless Sensor Networks (WSN) is estimating the position of the nodes. Positioning information of WSN nodes is very important as most WSN applications are location-based. Extensive research has been conducted in the area of WSN localization. Many of the proposed methods rely on CP techniques to enhance the positioning accuracy of the nodes. Some of the WSN positioning methods have been re-proposed in the domain of VANETs. However, the main difference between WSN and VANETs is the high mobility of the nodes in the network. Moreover, WSN nodes have very limited processing capabilities compared to nodes in VANETs. Some CP methods rely either on the availability of ranging information to other vehicles using V2V or the availability of ranging information to RSUs using V2I. For instance, knowing the position of RSUs (or the position of neighboring vehicles) and the range information, multilateration or trilateration can be used to estimate the position of the vehicle. Other CP methods do not use ranging techniques due to their limited performance.
2.6. VEHICULAR AD HOC NETWORKS (VANETS)

Ranging Based Techniques

In order to enhance the position of a vehicle, ranging methods are proposed to estimate the distance between the vehicle to be localized and a reference point with known position. This reference can be a RSU, an adjacent vehicle or a cellular tower. There are two main techniques used to estimate the range between a vehicle and another reference:

- Signal Strength-Based Ranging
- Time-Based Ranging

**Signal Strength-Based Ranging:** The concept behind Received Signal Strength (RSS) based ranging is that the strength of an RF signal decays as the signal propagates. The relation between received power at any receiving antenna location and distance between the transmitting and the receiving antenna is computed using (2.5) [42]. The transmitted power is denoted by \( P_t \) (power in the far-field at a reference power from the transmitting antenna), the distance between receiver \( j \) and transmitter \( i \) is noted by \( D_{ij} \) and the received power is denoted by \( P_r \).

\[
P_r(D_{ij}) = P_t - 10\alpha \log(D_{ij})dBm
\]  

(2.5)

The distance traveled by the RF signal is inversely proportional to the received signal power. In free space, the Pathloss Exponent (PLE) denoted by \( \alpha \) in Equation 2.5 is equal to 2 and thus the power received is inversely proportional to the square of the distance traveled by the RF signal. The PLE varies according to the medium through
which the RF signal is propagating. Assuming a constant $\alpha$ in a changing environment leads to high ranging errors. Moreover, multipath signals severely affect the performance of the RSS-based range due to the effect of destructive and constructive interference. Even though RSS based ranging is very simple and cheap, it is one of the most inaccurate ranging methods.

Some research has been directed towards RSS-based ranging in cellular networks [43]. However, the achieved accuracy is in the range of hundreds of meters and therefore is not suitable for CP. Other research directions in the field of WSN localization [44–46], assume that $\alpha$ is the same for all links and is non-changing over time and thus not suitable for VANETs dynamic environment where many obstacles exist.

The work in [47] and [48], attempts to enhance the positioning accuracy of vehicles in urban areas where multipath is a dominant source of error. Once a vehicle detects a multipath, it requests positions and relative distance from vehicles in its vicinity using CP. An optimization problem is then formulated incorporating information from the participating vehicles with accurate positions and their distances to the target vehicle. The distances are computed using RSS which result in significant relative distance errors.

In [49–51], only two Global Positioning System (GPS) satellites are visible and the distance from the vehicle to a Base-station with an accurate location is used to estimate the position of the vehicle. The limitations of the proposed solutions stem from using inaccurate ranging methods to estimate the distance between the vehicle to be localized and the Base-station.

**Time-Based Ranging:** The first type of Time-Based ranging is Time of Arrival. In order to estimate the distance between receivers $A$ and $B$, receiver $A$ sends a packet
with a time stamp denoted by $T_A$ which represents the transmission time to receiver $B$. Receiver $B$ receives the packet and adds another time stamp denoted by $T_B$ which represents the reception time at receiver $B$. Subsequently, receiver $B$ sends the packet to receiver $A$. The difference between the reception time and the transmission time multiplied by the speed of light is the distance between receivers $A$ and $B$. To achieve a meter level accuracy, the clock synchronization between receiver $A$ and $B$ has to be in terms of nanoseconds. In vehicular communications, DSRC uses IEEE802.11p which is based on IEEE802.11. The clock synchronization in IEEE802.11 protocols is in terms of microseconds [52]. This means that the error in the calculated distance will be in terms of thousands of meters.

Another Time-based ranging technique is Time Difference Of Arrival (TDOA). In this ranging method, two signals from two stations are transmitted and the vehicle to be localized processes the difference between the arrival time of both signals to identify the locus of the vehicle [53]. The most important constraint is that the two stations have to be synchronized to the nanosecond level. Vehicles using DSRC can not reach such level of synchronization. TOA and TDOA require very complex hardware to achieve the time synchronization between vehicles and therefore both methods are not practical for determining ranges between vehicles in VANETs.

A third Time-Based ranging approach is Round Time Trip (RTT) and is the most promising technique in terms of ranging accuracy. The distance is computed using the round time trip of the signal between two vehicles. Synchronization is not required between the two vehicles because the relative distance is computed relative to one clock. However, the processing and the queuing time need to be modeled in order to estimate the correct relative distance. Assuming the processing time and queuing
time are known, the relative distance between vehicles $i$ and $j$ denoted by $D_{ij}$ can be computed using:

$$D_{ij} = c \times \left( \frac{RTT - \tau_{\text{processing}} - \tau_{\text{queueing}}}{2} \right)$$ \hspace{1cm} (2.6)

Where $c$ is the speed of light, $RTT$ is the round trip time, $\tau_{\text{processing}}$ is the processing time at the receiver and $\tau_{\text{queueing}}$ is the queueing time at the receiver. Assuming that $\tau_{\text{processing}}$ and $\tau_{\text{queueing}}$ can be perfectly modeled is one of the RTT drawbacks. $\tau_{\text{processing}}$ depend on the type of the receiver, age and temperature and these effect change over time. Moreover, $\tau_{\text{queueing}}$ is affected by the number of vehicles measuring the distance to a certain vehicle. If only one vehicle needs to know its distance to another vehicle, $\tau_{\text{queueing}}$. As the number of vehicles increase, $\tau_{\text{queueing}}$ increases. In order to solve the problem of imperfect modeling, RTT is performed many times and the mean value is used to estimate the range. However, the accuracy of the estimated range is proportional to the number of RTTs and inversely proportional to the latency per range estimation. Moreover, the V2V bandwidth is limited and therefore packets drop rate increases as the number of vehicles and the number of RTTs per range increases. Table summarizes the relationship between the number of RTTs, the latency and the achieved range accuracy [54]. In [55], GPS signals are completely blocked and vehicles rely on cooperative positioning to enhance their position. An error free RTT ranging technique is used. Even though the proposed system provides promising results, in dense urban areas high bandwidth will be utilized to achieve sufficient RTT rounds to reach an error free range.

In [56], a localization framework for VANETs is proposed. Using TOA as a ranging method, the distance between vehicles is estimated. Moreover, a particle filter is used to fuse GNSS position, odometer reading and distance between vehicles to enhance
the position of vehicles. A map matching algorithm is used to enhance the vehicle’s position. The author assumes that the error in the estimated distance due to mis-synchronization between the receivers is in terms of tens of meters which should reflect a mis-synchronization in terms of nanoseconds. This is not a realistic assumption and hence the results are not practical. Moreover, in urban areas, multipath signals are dominant due to many reflectors and thus degrades the performance of TOA and TDOA. References [57–60] present different TOA and TDOA ranging approaches. Due to their synchronization requirements, these approaches are not practical for VANET positioning and will not be discussed further.

**Non Ranging-Based Techniques**

Range based techniques suffer from many limitations that prevent the usage of such techniques in CP. A detailed discussion of the limitations of the ranging-based techniques is presented in [5]. Non-Range based techniques do not rely on time or signal strength ranging techniques. References [61, 62] assume no GNSS coverage is available and propose a method by which vehicles can estimate their positions and also their lane. Two RSUs on the opposite sides of the road broadcast their position and road geometry information. Using the broadcasted information, the odometer measurements and the Carrier-Frequency-Offset (CFO) of the received signals from the RSUs, vehicles compute their position and lane. This approach is very expensive.

Table 2.4: Range Error and Latency as a function of RTT iterations.

<table>
<thead>
<tr>
<th>RTT Iterations</th>
<th>Ranging Error (m)</th>
<th>Latency (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>9.03</td>
<td>0.1</td>
</tr>
<tr>
<td>300</td>
<td>6.64</td>
<td>0.3</td>
</tr>
<tr>
<td>500</td>
<td>4.72</td>
<td>0.5</td>
</tr>
<tr>
<td>1000</td>
<td>1.70</td>
<td>1</td>
</tr>
</tbody>
</table>
since it requires RSUs storing information about the road geometry to be installed at each intersection.

Using only two GNSS satellites, vehicle to infrastructure communication and RSS of the DSRC packets, reference [63] proposes a CP positioning method. This method is based on intersecting the equation of the line representing the street with the TDOA hyperboloid from the two GNSS satellites to estimate the vehicles position. A Doppler shift filter is used to mitigate multipath. This method assumes an RSU is installed at each intersection and broadcasts its positions. Hence, it is an expensive solution. The filter used to mitigate multipath is based on observing Doppler shifts from the RSUs. However, for low vehicle speeds, Doppler shifts are not observed due to the high noise threshold [63].

In [64], pseudorange measurements are exchanged between two vehicles. Using Double Differencing (DD) and a tightly coupled particle filter is used to estimate the relative distance between vehicles. The concept of DD is close to DGPS since the common errors are removed from the pseudoranges of both vehicles. The real time response of the particle filter is questionable and the proposed method assumes full satellite coverage which is not suitable for VANETs in dense urban areas.

In [65], a system is proposed for estimating the relative distance between vehicles in urban areas when the number of visible satellites is at least four. Pseudoranges are exchanged between vehicles and a form of DD is applied to remove the common errors. A tight integration is adopted fusing the DD values to estimate the relative distance between vehicles. Even though, the proposed system eliminates the need for a DGPS infrastructure, it does not mention multipath. Multipath is dominant in dense urban areas and will certainly degrade the performance of the proposed system. In [66], a
system is proposed for enhancing the position of vehicles in urban areas when the number of visible satellites is at least four. Loose integration is adopted for fusing GPS position with range-rate estimated from the CFO of the received signals from adjacent vehicles. The range-rate estimation is based on observing Doppler shifts. In order to be able to observe the Doppler shifts, a minimum relative speed between vehicles should exist. In urban areas, the relative speeds between vehicles are very low and hence the proposed system is not effective.

Due to the aforementioned reasons, range based CP systems will not be able to provide the required positioning accuracy. In the next chapter we will propose a new Non-Range based CP system to overcome the limitations of ranging methods and provide the required positioning accuracy in urban environments.
Chapter 3

Non-Range Based Cooperative Positioning System

3.1 Introduction

Intelligent Transportations Systems and Location Based Services have different positioning accuracy and availability requirements [22]. Positioning accuracy refers to the difference between the true and the estimated positions. Positioning availability of a vehicle is the proportion of time the vehicle has enough information to estimate its position. Global Navigation Satellite Systems (GNSS) provide location information for numerous ITS applications. Computing the position of a vehicle requires visibility to at least 4 satellites [67]. Accordingly, when the number of visible satellites is less than four, the positioning availability decreases. In urban canyons, the number of visible satellites is very limited due to tall buildings and many other obstacles. Moreover, limited number of vehicles can decode multi-constellation GNSS signals. In other words, some vehicles are able to decode GPS and GLONASS signals while others are only able to decode GPS signals. In addition, not all receivers have the necessary hardware (array of antennas) or employ complex signal processing techniques to detect and mitigate jammed GNSS signals. Jamming signals can completely block
an entire GNSS constellation. All the aforementioned cases lead to limited satellite visibility in urban areas and hence decrease positioning availability.

Positioning accuracy depends on many factors like the number of visible satellites, the accuracy of pseudorange measurements, the geometry of visible satellites relative to the vehicle’s position and the type of GNSS observables used to estimate the vehicle’s position (Code or Carrier phase). Severe multipath can significantly decrease the accuracy of the pseudorange measurements. Multipath effect in dense urban canyons may cause an error up to 150 meters for C/A (Course/Acquisition) code measurements, and up to 15 meters for P-code [68]. Also, if a vehicle is able to decode GPS and GLONASS signals however GLONASS signals are jammed in a certain geographical area, the vehicle will not be able to use GLONASS satellites to improve its estimated position. All the aforementioned cases lead to either limited satellite visibility or poor accuracy of the pseudorange measurements and hence decrease the accuracy of the estimated position.

In the Section 2, a unified solution for limited positioning availability and accuracy in urban areas is presented. The proposed technique is called Angle Approximation (AA) and it relies on sharing pseudoranges and angle information between vehicles to re-generate pseudoranges that were originally hindered. To further enhance the performance of the AA technique, in Section 3 a method called Absolute Sum of Double Differencing (ASODD) is proposed that would assist the receiver in selecting the satellite which generates the least erroneous pseudorange. The ASODD method is also analytically derived in details. In Section 4, a method for vehicle selection is proposed to guide the receiver in selecting the most promising assisting vehicle. To test the viability of the proposed system, the experimental setup is presented and
results are discussed in Section 5. Finally, the developed Orbit Simulator (OS) is used to study the effect of the distance between vehicles, GPS errors, elevation mask, number of common satellites and number of assisting neighbours on the performance of the proposed system.

3.2 Angle Approximation

In this Section, the concept of Angle Approximation (AA) is presented. The developed Orbit Simulator (OS) is then introduced. The OS is used to provide numerical examples depicting the factors affecting the accuracy of the AA. Moreover, numerical examples are used to justify the need for a pseudorange selector.

3.2.1 Concept

Assume pseudoranges from satellites $k$, $m$ and $n$ are available to vehicles $i$ and $j$, while the pseudorange from satellite $f$ is only available to vehicle $i$. Figure 3.1 depicts only satellites $k$ and $f$. This occurs in urban canyons due to the existence of obstacles. Moreover, Blockage even for small distances between vehicles can also occur when the receiver of vehicle $j$ has the capability of only decoding GPS signals, while the receiver of vehicle $i$ is capable of decoding both GPS and GLONASS signals. Furthermore, vehicle $j$ might have the capability of decoding GPS/GLONASS but a jamming signal could lead to the blockage of GLONASS constellation (especially of vehicle $j$ does not employ complex anti-jamming algorithms) while vehicle $i$ has uses complex anti-jamming algorithms. Therefore, vehicle $i$ has access to more satellites than vehicle $j$. Furthermore, inaccurate pseudorange measurements occurs when vehicle $j$ has detected a severe multipath affecting the pseudorange from vehicle $j$ to satellite $f$. 
but not the pseudorange from vehicle \( i \) to satellite \( f \). Hence, vehicle \( j \) is either not capable of computing a 3D position due to the limited number of equations compared to the number of unknown states or is capable of computing a position with very poor accuracy.

Denote by \( d_{ij} \) the Euclidean distance in meters between the two vehicles \( i \) and \( j \). The pseudoranges between vehicle \( i \) and satellites \( k \) and \( f \) are denoted by \( \rho^k_i \) and \( \rho^f_i \), respectively. Likewise, the pseudoranges between vehicle \( j \) and satellites \( k \) and \( f \) are respectively denoted by \( \rho^k_j \) and \( \rho^f_j \). Global Positioning System (GPS) satellites orbit Earth at an approximate altitude of 20,200km. Other GNSS Systems like the Russian system, Global Navigation Satellite System (GLONASS) orbit Earth at around 19,100 km while the European system, Galileo, orbits Earth at an altitude close to 23,222km [40]. Since the distances between the satellites and the vehicles
are significantly larger than the distance between the vehicles, \( \rho_i^k \), \( \rho_j^k \) and also \( \rho_i^f \), \( \rho_j^f \) are almost parallel. Here we are considering relative distances between vehicles ranging from 0 to 100m, which is the communication range for class B DSRC systems in VANETs. For the maximum communication coverage of 100m between vehicles, the ratio between the distance to the satellite and the distance between vehicles is 202,000 to 1m. Due to this large ratio, [69] uses the approximation of parallel lines to compute the distance between vehicles.

If pseudoranges \( \rho_i^k \), \( \rho_j^k \) and pseudoranges \( \rho_i^f \), \( \rho_j^f \) are almost parallel then the angle between pseudoranges \( \rho_i^k \) and \( \rho_i^f \) denoted by \( \theta_{i;i}^k \) in Figure 3.1 is almost equal to the angle between pseudoranges \( \rho_j^k \) and \( \rho_j^f \) denoted by \( \theta_{j;i}^k \). Now let us assume that satellites \( k \) and \( f \) have direct LOS to vehicle \( i \). While vehicle \( j \) has a direct LOS to satellite \( k \) but the signal from satellite \( f \) to vehicle \( j \) is completely blocked, jammed or is effected by severe multipath. Here the goal is to generate the hindered pseudorange \( \rho_j^f \) using the proposed Angle Approximation Technique (AA). The following steps demonstrate how \( \rho_j^f \) is generated using AA:

1. Vehicle \( j \) receives periodic beacons containing visible satellites from vehicle \( i \) using DSRC and detects that the signal from satellite \( f \) is blocked or jammed. Or vehicle \( j \) detects that \( \rho_j^f \) is affected by severe multipath using one of the existing multipath detection method [70].

2. In the first case, vehicle \( j \) would request the position of satellite \( f \) (if it does not already have it) and the pseudorange measurements \( \rho_i^k \) and \( \rho_i^f \). In the second case, vehicle \( j \) has the position of satellite \( f \) and only requests the pseudorange measurements \( \rho_i^k \) and \( \rho_i^f \) from vehicle \( i \).

3. Now vehicle \( j \) is ready to execute the AA technique. First the angle \( \theta_{i;i}^k \) is
3.2. ANGLE APPROXIMATION

computed using the cosine rule in (3.1)

4. Under the assumption \( \theta_i^{k,f} \approx \theta_j^{k,f} \), vehicle \( j \) generates pseudorange \( \rho_j^f \) using (3.3). Denote by \( \rho_j^{f,k} \) the generated pseudorange using \( \rho_i^k \). Here, \( \rho_j^{f,k} \) is called an Artificial Candidate Pseudorange (ACP) for the hindered pseudorange \( \rho_j^f \).

\[
\theta_i^{k,f} = \cos^{-1} \left( \frac{(\rho_i^k)^2 + (\rho_i^f)^2 - (a)^2}{2\rho_i^k\rho_i^f} \right) \tag{3.1}
\]

where \( a \) is the Euclidean distance between satellites \( k \) and \( f \) and is computed by using (3.2).

\[
a = \sqrt{(x_k - x_f)^2 + (y_k - y_f)^2 + (z_k - z_f)^2} \tag{3.2}
\]

\[
\rho_j^{f,k} = \frac{\sin(\theta_1)}{\sin(\theta_2)} \rho_j^k \tag{3.3}
\]

where \( \theta_1 \) and \( \theta_2 \) depicted in Figure 3.1 are computed using (3.4) and (3.5)

\[
\theta_1 = 180 - \theta_2 - \theta_i^{k,f} \tag{3.4}
\]

\[
\theta_2 = \sin^{-1} \left( \frac{\sin(\theta_i^{k,f})}{a} \right) \rho_j^k \tag{3.5}
\]

It is important to mention that there is another method that can be used to determine the hindered pseudorange \( \rho_j^f \) without having to compute two angles (using the cosine rule instead of the since rule). Under the assumption \( \theta_i^{k,f} \approx \theta_j^{k,f} \), Equation 3.6 can be reduced to a quadratic form.

\[
\cos \theta_i^{k,f} \approx \cos \theta_j^{k,f} = \frac{(\rho_j^k)^2 + (\rho_j^{f,k})^2 - (a)^2}{2\rho_j^k\rho_j^{f,k}} \tag{3.6}
\]

\[
2\rho_j^k\rho_j^{f,k} \cos \theta_i^{k,f} - (\rho_j^k)^2 - (\rho_j^{f,k})^2 + (a)^2 \approx 0 \tag{3.7}
\]
3.2. ANGLE APPROXIMATION

\[-1(-\rho_j^{f,k})^2 + 2\rho_j^k\rho_j^{f,k} \cos \theta_i^{k,f} - (\rho_j^k)^2 + (a)^2 \approx 0\] (3.8)

\[(\rho_j^{f,k})^2 - 2\rho_j^k\rho_j^{f,k} \cos \theta_i^{k,f} + ((\rho_j^k)^2 - a^2) \approx 0\] (3.9)

Equation 3.9 can be represented by the following quadratic equation:

\[A(\rho_j^{f,k})^2 + B\rho_j^{f,k} + C \approx 0\] (3.10)

where A, B and C are equal to:

- \(A = 1\)
- \(B = -2\rho_j^k \cos \theta_i^{k,f}\)
- \(C = ((\rho_j^k)^2 - a^2)\)

The solution to (3.10) is:

\[\rho_j^{f,k} \approx -B \pm \sqrt{B^2 - 4AC} = \rho_j^k \cos \theta_i^{k,f} \pm \sqrt{(-\rho_j^k \cos \theta_i^{k,f})^2 - 4(((\rho_j^k)^2 - a^2))} \] / 2 (3.11)

Solving (3.10) for the positive root or using the sine rule in (3.3) results in computing \(\rho_j^{f,k}\).

In the previous example, the number of common satellites between vehicle \(i\) and \(j\) is three. Therefore, the AA technique can be executed several times using three angle approximations resulting in three different ACPs for the hindered pseudorange.

Assuming \(\theta_i^{k,f} \approx \theta_j^{k,f}\), AA generates the hindered pseudorange using satellite \(k\), such that \(\rho_j^{f,k}\) is expressed by 3.12. Here, \(\mu^{f,k}\) denotes the error due to the inaccuracy of the \(\theta_i^{k,f} \approx \theta_j^{k,f}\) assumption.

\[\rho_j^{f,k} = \rho_j^f + \mu^{f,k}\] (3.12)
3.2. ANGLE APPROXIMATION

Similarly, assuming $\theta^m_{ij} \approx \theta^n_{ij}$ and $\theta^m_{ij} \approx \theta^n_{ij}$ respectively generate $\rho^m_{ij}$ and $\rho^n_{ij}$. Now the most critical question becomes: which ACP should the receiver use to compute its position ? The ultimate goal is to use the ACP which is based on the most accurate angle approximation since it would result in the least error. Therefore, we have to understand the factors affecting the accuracy of the angle approximation assumption.

3.2.2 Numerical Examples

We developed a simulation model called Orbit Simulator (OS) using MATLAB to analyze the effect of different factors such as the distance between vehicles and the satellite geometry on the error in the ACP resulting from the angle approximation assumption. In the simulation model, points $i$ and $j$ are separated by a distance $d_{ij}$. Point $i$ is fixed at coordinates $(0,0,0)$ while point $j$ has coordinates $(x,y,0)$. Here points $i$ and $j$ represent the two vehicles.

Two satellites $k$ and $f$ are at an altitude of $20,200km$ relative to the $x$-$y$ plane (approximate altitude for GPS satellites). Satellite $k$ is fixed at an elevation angle of $60^\circ$ and an azimuth angle of $90^\circ$. Satellite $f$ is moving and is assigned values from an elevation of $60^\circ$ to $90^\circ$ and from an azimuth of $0^\circ$ to $360^\circ$. The elevation angle of satellites range from $0^\circ$ to $90^\circ$, however, we constrain the minimum elevation angle to $60^\circ$ since we are assuming an urban canyon environment where low elevation satellites are blocked. Satellite $k$ is visible to points $i$ and $j$ while satellite $f$ is only visible to point $i$. Using AA we assume $\theta^k_{ij} \approx \theta^k_{ij}$ and consequently generate $\rho^k_{ij}$. Then we compute the Range Error (RE) which is the absolute difference between the actual range from satellite $f$ to point $j$ namely $\rho^f_{ij}$ and the generated range $\rho^k_{ij}$. RE
is calculated for every position of satellite $f$. Here the term range is used rather than pseudorange since there are no inherent random errors in the true range or the generated range.

Figures 3.2 and 3.3 respectively show the RE when the distance between the points

![Graph showing range error (m) vs azimuth and elevation degrees.]

Figure 3.2: Range Error when vehicles are separated by 5m and aligned on the x-axis.

is set to 5m and 50m. The coordinates of point $j$ is (5, 0, 0) and (50, 0, 0) for Figures 2 and 3 respectively. Figure 3.4 shows the RE when the distance between the points is set to 50m, while the coordinates of point $j$ is (0, 50, 0). Analyzing Figure 3.2, 3.3 and 3.4 we deduce three important factors affecting the error in the generated range using AA:

1. As the distance between the vehicles decrease, the generated range has lower error for a given satellite and vehicle positions. This is intuitive because the angle approximation assumption becomes more accurate. Figures 3.2 and 3.3 respectively show a maximum RE of 2.54m and 24m for the same geometry
Figure 3.3: Range Error when vehicles are separated by 50m and aligned on the x-axis.

but for relative distances of 5 and 50m. Also the shape of the RE envelope is identical but amplified for the 50m scenario.

2. The satellite geometry relative to the vehicle’s geometry is the main contributor to the RE. This can be observed by comparing Figure 3.3 and 3.4 where the distance between the points is 50m for both scenarios, the only difference is the coordinates of point $j$.

3. Depending on the geometry of the satellites relative to the vehicles, the error in the generated range varies from 0 to 24m when the distance between the vehicles is 50m. Using the AA technique without knowing the geometry of the satellites relative to the vehicles leads to a high uncertainty in the error of the generated range.
3.3 Satellite Selection Criteria

In this Section, a method is proposed to assist the GNSS receiver in increasing the probability of selecting the satellite which will generate the most accurate ACP. The method is named Absolute Sum of Double Differencing (ASODD). A detailed analytical prove and analysis of the selection method is presented.
3.3. SATELLITE SELECTION CRITERIA

3.3.1 Absolute Sum of Double Differencing (ASODD)

ASODD is a selection method that takes as an input all the observed pseudoranges from the target vehicle and the assisting vehicle and all the Artificial Candidate Pseudoranges (ACPs) representing the hindered pseudorange. For every ACP, ASODD computes a positive indicator. To decide which ACP to select, ASODD selects the candidate with the minimum indicator value. In other words, we claim that the indicators computed by ASODD reflect the error in generated pseudoranges with a high probability.

Following the AA assumption, the angle between the two pseudoranges from satellites $k$ and $f$ to vehicle $i$ is approximately equal to the angle between the two pseudoranges from satellites $k$ and $f$ to vehicle $j$. Hence, the double differencing of the pseudoranges $k$ and $f$ from vehicle $i$ and $j$ tends to zero. The idea behind ASODD is to apply a sequence of mathematical operations to the ACPs which leads to the accumulation of the error due to AA. The absolute value of the sum of the double differencing of the pseudoranges indicates how far each generated ACP is from meeting the angle approximation assumption.

**ASODD Proof**

Assume satellites $k$, $m$ and $n$ are visible to the participating vehicle $i$ and the target vehicle $j$. While satellite $f$ is only visible to vehicle $i$. Using the assumptions $\theta^k_i = \theta^k_j$, $\theta^m_i = \theta^m_j$ and $\theta^n_i = \theta^n_j$, the AA technique generates three pseudoranges $\rho^f_{j,k}$, $\rho^f_{j,m}$ and $\rho^f_{j,n}$ with errors $\mu^f_{j,k}$, $\mu^f_{j,m}$ and $\mu^f_{j,n}$ respectively. Here we will derive the ASODD indicator for $\rho^f_{j,k}$ in details. Equation 3.13 depict the actual pseudorange.

$$\rho^f_j = R^f_j + \beta_j + \alpha^f + \varepsilon^f_j$$  (3.13)
where

- $R^f_j$ is the true range from vehicle $j$ to satellite $f$.

- $\beta_j$ is the clock bias of vehicle $j$ and the receiver’s noise (common errors to a single receiver).

- $\alpha^f$ is the clock bias of satellite $f$, the ionosphere and troposphere errors (common errors to a single satellite).

- $\varepsilon^f_j$ is the error due to multipath.

Our main aim is to attenuate the value $\rho^f_j$ in (3.12) and amplify the generated error denoted by $\mu^{f,k}$, such that the error in the candidate pseudorange is observable.

Denote by $ASODD^{f,k}$ the ASODD indicator for the ACP $\rho^{f,k}_j$. This quantity is calculated by taking the sum of the absolute value of the double differencing between the ACP and all the other observable pseudoranges from vehicles $i$ and $j$. $ASODD^{f,k}$ is given by

$$ASODD^{f,k} = \sum_{s=k,m,n} |\Delta \rho^s_i - \Delta \rho^s_{j,k}| \tag{3.14}$$

where $s$ is the number of common visible satellites between vehicles $i$ and $j$

$$ASODD^{f,k} = |\Delta \rho^k_i - \Delta \rho^k_{j,k}| + |\Delta \rho^m_i - \Delta \rho^m_{j,k}| + |\Delta \rho^m_i - \Delta \rho^m_{j,k}| \tag{3.15}$$
\[ \Delta \rho_i^{k_f} = \rho_i^k - \rho_i^f \]
\[ = (R_i^k + \beta_i + \alpha_i^k + \varepsilon_i^k) - (R_i^f + \beta_i + \alpha_i^f + \varepsilon_i^f) \]
\[ = (R_i^k - R_i^f) + (\beta_i - \beta_i) + (\alpha_i^k - \alpha_i^f) + (\varepsilon_i^k - \varepsilon_i^f) \]
\[ = (R_i^k - R_i^f) + (\alpha_i^k - \alpha_i^f) + (\varepsilon_i^k - \varepsilon_i^f) \]
\[ = \Delta R_i^{k_f} + \Delta \alpha_i^{k_f} + \Delta \varepsilon_i^{k_f} \quad (3.16) \]

Similarly, \( \Delta \rho_i^{m_f} \) is the difference between \( \rho_i^m \) and \( \rho_i^f \) and \( \Delta \rho_i^{n_f} \) is the difference between \( \rho_i^n \) and \( \rho_i^f \). This step removes the receiver’s clock bias from all the pseudoranges of vehicle \( i \). For vehicle \( j \) there is a slight difference due to the error in the ACP. \( \Delta \rho_j^{k_f,k} \) is the difference between \( \rho_j^k \) and \( \rho_j^{f,k} \), and is given by:

\[ \Delta \rho_j^{k_f,k} = \rho_j^k - \rho_j^{f,k} \]
\[ = (R_j^k + \beta_j + \alpha_j^k + \varepsilon_j^k) - (R_j^f + \beta_j + \alpha_j^f + \varepsilon_j^f + \mu_j^{f,k}) \]
\[ = (R_j^k - R_j^f) + (\beta_j - \beta_j) + (\alpha_j^k - \alpha_j^f) + (\varepsilon_j^k - \varepsilon_j^f) - \mu_j^{f,k} \]
\[ = (R_j^k - R_j^f) + (\alpha_j^k - \alpha_j^f) + (\varepsilon_j^k - \varepsilon_j^f) - \mu_j^{f,k} \]
\[ = \Delta R_j^{k_f} + \Delta \alpha_j^{k_f} + \Delta \varepsilon_j^{k_f} - \mu_j^{f,k} \quad (3.17) \]

Similarly, \( \Delta \rho_j^{m_f,k} \) and \( \Delta \rho_j^{n_f,k} \) is the difference between \( \rho_j^m \) and \( \rho_j^{f,k} \) and \( \rho_j^n \) and \( \rho_j^{f,k} \) respectively. This step removes the receivers clock bias and noise from all the pseudoranges of vehicle \( j \). The effect of the second step of the double differencing is removing
3.3. SATELLITE SELECTION CRITERIA

the satellites clock bias.

\[
|\Delta \rho_i^{kf} - \Delta \rho_j^{kf,k}| = |(\Delta R_i^{kf} + \Delta \alpha^{kf} + \Delta \varepsilon_i^{kf}) - (\Delta R_j^{kf} + \Delta \alpha^{kf} + \Delta \varepsilon_j^{kf} - \mu^{f,k})|
\]

\[
= |(\Delta R_i^{kf} - \Delta R_j^{kf}) + (\Delta \alpha^{kf} - \Delta \alpha^{kf}) + (\Delta \varepsilon_i^{kf} - \Delta \varepsilon_j^{kf}) + \mu^{f,k}|
\]

\[
= |(\Delta R_i^{kf} - \Delta R_j^{kf}) + (\Delta \varepsilon_i^{kf} - \Delta \varepsilon_j^{kf}) + \mu^{f,k}|
\]

\[
= |\Delta R_{ij}^{kf} + \Delta \varepsilon_{ij}^{kf} + \mu^{f,k}|
\] (3.18)

Likewise, the absolute double differencing in the ASODD indicator; \(|\Delta \rho_i^{mf} - \Delta \rho_j^{mf,k}|\) and \(|\Delta \rho_i^{nf} - \Delta \rho_j^{nf,k}|\) can be calculated. Therefore, (3.15) can be represented as:

\[
ASODD_{f,k}^{j} = |\Delta R_{ij}^{kf} + \Delta \varepsilon_{ij}^{kf} + \mu^{f,k}| + |\Delta R_{ij}^{mf} + \Delta \varepsilon_{ij}^{mf}|
\]

\[
+ |\Delta R_{ij}^{nf} + \Delta \varepsilon_{ij}^{nf} + \mu^{f,k}|
\] (3.19)

where:

- \(\Delta R_{ij}^{kf}, \Delta R_{ij}^{mf}\) and \(\Delta R_{ij}^{nf}\) are the double differencing terms for the difference in the true ranges.

- \(\Delta \varepsilon_{ij}^{kf}, \Delta \varepsilon_{ij}^{mf}\) and \(\Delta \varepsilon_{ij}^{nf}\) are the double differencing terms for the multipath error.

It is not removed because it is not common between the pseudoranges. Here we assume multipath is almost zero because pseudoranges affected by severe multipath are not used to generate hindered pseudoranges. Therefore, 3.19 can be approximated by:

\[
ASODD_{f,k}^{j} = |\Delta R_{ij}^{kf} + \mu^{f,k}| + |\Delta R_{ij}^{mf} + \mu^{f,k}|
\]

\[
+ |\Delta R_{ij}^{nf} + \mu^{f,k}|
\] (3.20)
3.3. SATELLITE SELECTION CRITERIA

Following the same steps, $ASODD^f,m$ and $ASODD^f,n$ can be derived. Equations 3.21 and 3.22 depict the final stage of the derivation.

\[
ASODD^f,m = |\Delta R_{ij}^{kf} + \mu^f,m| + |\Delta R_{ij}^{mf} + \mu^f,m| + |\Delta R_{ij}^{nf} + \mu^f,m| \tag{3.21}
\]

\[
ASODD^f,n = |\Delta R_{ij}^{kf} + \mu^f,n| + |\Delta R_{ij}^{mf} + \mu^f,n| + |\Delta R_{ij}^{nf} + \mu^f,n| \tag{3.22}
\]

Equations 3.20, 3.21 and 3.22 consist of the double differencing terms for the true ranges and the error in the generated pseudoranges. The error in the generated pseudoranges accumulates several times for each indicator depending on the number of common satellites between the target and the assisting vehicle. Consequently, ASODD increases the probability of observing the magnitude of the error for each ACP. Hence, the ASODD indicator can be used to select the least erroneous ACP.

Denote by $ACP_{sel}^f$ the selected ACP representing the hindered pseudorange $\rho^f_j$ in the case of three common satellites ($k$, $m$ and $n$). The ACP with the minimum ASODD indicator value is selected and this selection can be written in the following form:

\[
ACP_{sel}^f = \min(ASODD^f,k, ASODD^f,m, ASODD^f,n) \tag{3.23}
\]

ASODD Limitations

In some situations ASODD might not succeed in selecting the least erroneous ACP. This occurs when the double differencing terms for the true range are large enough
concealing the errors due to AA. The value of the double differencing term for the true ranges is a function of the elevation of the satellites and the distance between the target and the assisting vehicle. Specifically, the elevation of the satellites and the distance between vehicles are inversely proportional to the double differencing term for the true ranges of the ASODD indicator. In urban areas, satellite visibility is normally limited to high elevation satellites. Generally, when the double differencing term for the true ranges is large relative to the error due to AA, the error observability is limited.

Due to the aforementioned limitations, ASODD is more practical when the number of common satellites is less than four and no other information is available to inform the receiver about the accuracy of the position computed using each ACP. Therefore, the receiver will not be able to choose an ACP.

3.4 Multiple Neighbors

In urban areas, the target vehicle is normally surrounded by many assisting vehicles. Some of those vehicles can assist the target vehicle in either enhancing positioning availability or accuracy. In this section, we propose a method by which one assisting vehicle is selected from the pool of neighboring assisting vehicles. After vehicle selection, AA is applied to generate ACPs and then ASODD is used to select the final ACP. Here the final ACP refers to the pseudorange which will be used in the final position estimation.
3.4. MULTIPLE NEIGHBORS

3.4.1 Absolute Sum of Single Differencing (ASOSD)

Each assisting vehicle transmits its pseudoranges to the target vehicle. The target vehicle selects only one vehicle and then uses AA method to generate ACPs. Subsequently, the ASODD method selects one of the ACPs. Here we propose a method for vehicle selection. Assume we have three satellites $k, m$ and $n$ common to vehicles $A, B, C$ and a target vehicle $D$ while satellite $f$ is only available to the assisting vehicles. Figure 3.5 shows the block diagram of the proposed system when more than one assisting vehicle is used to enhance the position availability or accuracy of the target vehicle. The proposed method is called Absolute Sum of Single Differencing (ASOSD).
3.4. MULTIPLE NEIGHBORS

This method calculates a distance indicator for each assisting vehicle using the common pseudoranges between the assisting vehicle and the target vehicle. The accuracy of the AA method is affected by the distance between vehicles. The probability of generating ACPs with high errors from an assisting vehicle separated from the target vehicle by a large distance is higher than the probability of generating ACPs with high errors from an assisting vehicle at smaller distances. Hence, the distance between the candidate assisting vehicles and the target vehicle is the most important factor affecting the accuracy of the generated ACPs. The ASOSD method uses the difference between the common pseudoranges of the target and the Candidate Assisting Vehicles (CAV) to calculate a distance indicator for each vehicle. This method assumes that each CAV can estimate its GNSS receiver’s clock bias and the pseudoranges transmitted to the target vehicle are corrected for the error due to the clock bias. Equation 3.24 is used to calculate the ASOSD distance indicator for an assisting vehicle $v$ denoted by $ASOSD_v$. Assuming $S$ is the number of common satellites between the target and assisting vehicle.

$$
ASOSD_v = \sum_{s=1}^{S} |\rho_D^s - \rho_v^s - Clk_v|
$$

$$
= \sum_{s=1}^{S} |(R_D^s + \beta_D + \alpha_D^s + \varepsilon_D^s) - (R_v^s + \beta_v + \alpha_v^s + \varepsilon_v^s) - Clk_v| \tag{3.24}
$$

Removing the assisting vehicle’s clock bias denoted by $Clk_v$ in (3.24) is very important as it is not common to all assisting vehicles. However, the target’s clock bias does not have to be removed since the all ASOSD indicators are referenced to the target vehicle. In other words, the target vehicle’s clock bias equally affects the ASOSD indicator for each assisting vehicle. Assuming the assisting vehicle’s clock bias is
correctly estimated and hence $Clk_v \simeq \beta_v$. Moreover, all common errors between pseudoranges to the same satellites are removed. These common errors are due to the uncompensated satellite clock bias, ionospheric and tropospheric delays. Equation 3.25 shows the ASOSD indicator for assisting vehicle $v$ after removing the common errors and the assisting vehicle’s clock bias.

$$ASOSD_v = \sum_{s=1}^{S} |(R^*_D + \beta_D + \alpha^s + \varepsilon^s_D) - (R^*_v + \beta_v + \alpha^s + \varepsilon^s_v) - Clk_v|$$

$$= \sum_{s=1}^{S} |(R^*_D - R^*_v) + \beta_D + (\alpha^s - \alpha^s) + (\varepsilon^s_D - \varepsilon^s_v) + (\beta_v - Clk_v)|$$

$$= \sum_{s=1}^{S} |(R^*_D - R^*_v) + \beta_D + (\varepsilon^s_D - \varepsilon^s_v)|$$  (3.25)

The non common errors between the target vehicle $D$ and the assisting vehicle $v$ is denoted by $\varepsilon^s_v$. Assuming non common errors like multipath or receiver noise at the assisting vehicles are approaching zero. This occurs for example when assisting vehicles are capable of mitigating short delay multipath signals or are not affected by multipath at all. Hence, Equation 3.25 can be expressed as:

$$ASOSD_v = \sum_{s=1}^{S} |(R^*_D - R^*_v) + \beta_D + \varepsilon^s_D|$$  (3.26)

The effects of the target’s clock bias denoted by $\beta_D$ and the non common errors on the pseudoranges of the target vehicle denoted by $\varepsilon^s_D$ does not affect the accuracy of the ASOSD distance indicator because they are common to all assisting vehicles. Assuming the common satellites are denoted $k, m$ and $n$, the ASOSD indicator for
the assisting vehicle $v$ can be expressed as:

$$ASOSD_v = \sum_{s=1}^{S} |(R_D^s - R_{k}^s) + \beta_D + \varepsilon_D^s|$$

$$= |(R_D^s - R_n^s) + \beta_D + \varepsilon_D^s| + |(R_D^s - R_D^m) + \beta_D + \varepsilon_D^s|$$

$$+ |(R_D^s - R_v^s) + \beta_D + \varepsilon_D^s|$$ (3.27)

Using the ASOSD method reduces the complexity of the overall system and enables system scalability. When selecting one assisting vehicle based on the distance indicator, the AA and the ASODD selection method is applied only to one vehicle instead of all the assisting vehicles. Hence, a practical real time scalable system can be implemented.

**ASOSD Limitations**

As the effect of non common errors like multipath and noise at the assisting vehicle’s receiver increase, the performance of the ASOSD indicator degrades. In this case, the ASOSD does not represent the distance between the vehicles. However, we always assume that assisting vehicles have strong multipath mitigation capabilities. Even if an assisting vehicle can only detect but not mitigate multipath and an excess of assisting vehicles are available, we can simply discard pseudoranges from such vehicles and only apply ASOSD on assisting vehicles with multipath mitigation capabilities or vehicles that are not affected by multipath. Another limitation of the ASOSD method is that it is a function of the satellite geometry relative the target and assisting vehicle’s position. Therefore, even if the non common errors of the assisting vehicles are somehow removed or the assisting vehicles are not affected by multipath, the ASOSD might not indicate the correct distances especially if two assisting vehicles are around
the same distance from the target vehicle.

3.5 Experimental Results

To test the viability of the proposed system, two NovAtel GNSS receivers were positioned at the Royal Military College of Canada. The receivers were separated by 5, 15 and 20m. For each relative distance setup, pseudoranges were collected for a duration of 15 minutes. During the experiment, 12 satellites were visible, 8 GPS satellites and 4 GLONASS satellites. Our analysis was performed on the GPS satellites. Since the 8 GPS satellites were visible to receivers \( i \) and \( j \), we intentionally block one satellite at a time from receiver \( j \).

Then AA is applied to generate the blocked pseudorange given 7 angle approximations resulting in 7 generated pseudoranges. To measure the error in the generated pseudoranges we compute the absolute difference between the generated pseudoranges and the observed pseudorange. This procedure was conducted for each satellite visible to receiver \( j \). Table 3.1 shows the averaged satellites elevation and azimuth angles over the duration of both experiments (30 minutes). Tables 3.2 and 3.3 show

<table>
<thead>
<tr>
<th>PRN Number</th>
<th>Elevation (°)</th>
<th>Azimuth (°)</th>
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<tbody>
<tr>
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<td>62.9</td>
<td>87</td>
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<tr>
<td>11</td>
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<td>28</td>
<td>46.6</td>
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<tr>
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<td>72.4</td>
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</tr>
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<td>7</td>
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<tr>
<td>4</td>
<td>63.8</td>
<td>154.8</td>
</tr>
<tr>
<td>17</td>
<td>46.3</td>
<td>265.3</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>168.7</td>
</tr>
</tbody>
</table>
3.5. EXPERIMENTAL RESULTS

Table 3.2: Errors in pseudoranges using AA for a distance of 5m between receivers.

<table>
<thead>
<tr>
<th></th>
<th>$\rho_1^i$</th>
<th>$\rho_2^i$</th>
<th>$\rho_3^i$</th>
<th>$\rho_4^i$</th>
<th>$\rho_5^i$</th>
<th>$\rho_6^i$</th>
<th>$\rho_7^i$</th>
<th>$\rho_8^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1^j$</td>
<td>1.9</td>
<td>0.5</td>
<td>2.1</td>
<td>1.1</td>
<td>1.0</td>
<td>4.0</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\rho_2^j$</td>
<td>2</td>
<td>2.3</td>
<td>4</td>
<td>3.3</td>
<td>2.7</td>
<td>6.0</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>$\rho_3^j$</td>
<td>0.6</td>
<td>2.4</td>
<td>3.3</td>
<td>1.2</td>
<td>1.0</td>
<td>4.0</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>$\rho_4^j$</td>
<td>2.7</td>
<td>4.1</td>
<td>3.0</td>
<td>3.4</td>
<td>3.3</td>
<td>5.0</td>
<td>2.6</td>
<td></td>
</tr>
<tr>
<td>$\rho_5^j$</td>
<td>0.7</td>
<td>1.6</td>
<td>1.0</td>
<td>3.0</td>
<td>0.7</td>
<td>4.0</td>
<td>5.0</td>
<td></td>
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<tr>
<td>$\rho_6^j$</td>
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<td>3.0</td>
<td>2.0</td>
<td>4.5</td>
<td>2.0</td>
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<tr>
<td>$\rho_7^j$</td>
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<td>4.1</td>
<td>3.6</td>
<td>4.5</td>
<td>4.0</td>
<td>3.4</td>
<td></td>
<td>3.6</td>
</tr>
<tr>
<td>$\rho_8^j$</td>
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<td>0.4</td>
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Table 3.3: Errors in pseudoranges using AA for a distance of 20m between receivers.

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<tr>
<th></th>
<th>$\rho_1^i$</th>
<th>$\rho_2^i$</th>
<th>$\rho_3^i$</th>
<th>$\rho_4^i$</th>
<th>$\rho_5^i$</th>
<th>$\rho_6^i$</th>
<th>$\rho_7^i$</th>
<th>$\rho_8^i$</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_2^j$</td>
<td>3.1</td>
<td>4.0</td>
<td>2.5</td>
<td>6.8</td>
<td>0.5</td>
<td>8.2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>$\rho_3^j$</td>
<td>6.1</td>
<td>4.1</td>
<td>3.0</td>
<td>7.0</td>
<td>7.5</td>
<td>2.5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>$\rho_4^j$</td>
<td>2.0</td>
<td>3.1</td>
<td>3.2</td>
<td>1.0</td>
<td>0.7</td>
<td>13</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>$\rho_5^j$</td>
<td>5.0</td>
<td>2.1</td>
<td>6.8</td>
<td>0.8</td>
<td>4.2</td>
<td>3.9</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>$\rho_6^j$</td>
<td>8.0</td>
<td>1.0</td>
<td>8.3</td>
<td>0.5</td>
<td>3.2</td>
<td>7.2</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>$\rho_7^j$</td>
<td>4.1</td>
<td>6.0</td>
<td>2.0</td>
<td>10</td>
<td>4.1</td>
<td>5.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>$\rho_8^j$</td>
<td>9.8</td>
<td>5.5</td>
<td>10</td>
<td>3.5</td>
<td>4.5</td>
<td>6.5</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

the average of the absolute errors in the generated pseudoranges after applying AA when receivers are separated by 5 and 20m respectively. The number of averaged samples is 900 (15minutes at a rate of 1Hz). The error is measured in meters. The experiments were performed sequentially starting with 5m and then 20m separation between receivers $i$ and $j$.

For simplicity we refer to satellites with PRN numbers 1, 11, 28,30,7,4,17 and 8 as 1,2,3,4,5,6,7 and 8 respectively. For example, the number in the first row and second column of Table 3.2 represents the averaged absolute error when pseudorange denoted by $\rho_2^i$ (pseudorange from receiver $i$ to satellite 2) is used to generate the blocked pseudorange denoted by $\rho_1^j$. The first row and first column is empty because $\rho_1^j$ cannot
be generated using $p^1_j$ (this applies for all the diagonal elements).

Comparing tables 3.2 and 3.3, we find that the errors in Table 3.2 are generally lower than the errors in Table 3.3 due to the difference in the distance between receivers. As the distance between vehicles decrease, the errors in the generated pseudoranges by AA decreases. In tables 3.2 and 3.3, the bold errors refer to the minimum error in the generated pseudoranges. Since the pseudorange errors are not observable and are of high variance, applying ASODD selection method to decide which generated pseudorange should be used is essential. As it would increase the probability of selecting the least erroneous generated pseudorange. Theoretically, in tables 3.2 and 3.3 the errors in the generated pseudorange should be symmetrical. However, since the pseudorange error is computed by taking the absolute difference between the generated pseudorange and the observed pseudorange which is affected by noise, the generated pseudorange errors are not symmetrical.

Since the experiments were performed sequentially, errors in table 3.2 and 3.3 should not be compared directly. This is due to the fact that the satellites positions are not the same for both experiments. However, the general magnitude of the error can be compared since it is a function of the relative distance between the receivers.

In order to test the performance of ASODD, 6 GPS satellites were assumed visible to receivers $i$ and $j$ separated by $20m$. For 2 minutes the pseudorange from one satellite to receiver $j$ was blocked and the AA technique was used to generate 5 pseudoranges from each visible satellite common to both vehicles. ASODD was applied and the pseudorange with the least indicator was selected.

Figures 3.6 and 3.7 depict the pseudorange error and the position error in meters when satellite 1 and satellite 6 are blocked respectively. Least squares estimator was
3.5. EXPERIMENTAL RESULTS

Figure 3.6: ASODD succeeds in selecting the satellite generating the pseudorange with the minimum error (SAT-4) resulting in least position error.

applied to the observed pseudoranges and also to the generated pseudoranges. The absolute difference was used as the position error. Figure 3.6 shows pseudorange errors after applying AA using different satellites.

The ASODD method was able to select the satellite with the least generated pseudorange error and therefore the minimum position error was achieved. Instead of blocking satellite 1, in Figure 3.7 satellite 6 was blocked and the same procedure was conducted. ASODD was not able to select the satellite generating the least pseudorange error. However, the second best solution was selected.

In order to study the level of accuracy achieved by the AA technique and the ASODD method for different distances ranging from 0 to 100m (DSRC range) and for different pseudorange error standard deviation (whether common or non common errors), relying on experiments is not sufficient. During experiments only limited satellite geometry, relative distances between vehicles and pseudorange error standard deviation
3.6 Simulations

In this section, the Orbit Simulator environment is used to evaluate the effect of different variables on the performance of the proposed system. Figure 3.8 shows the block diagram depicting an abstract view of the simulation process. To setup the environment, some variables such as the number of common satellites between the target and assisting vehicles, the elevation mask of the satellites (minimum satellite elevation), the standard deviation of the common pseudoranges error and the number of
assisting vehicles have to be declared. When the effect of one variable is investigated, all other variables are set to a value that does not change during the simulation session. Depending on which variable is under investigation, a setup file is chosen using the setup file selector. Subsequently, the environment variables are used to generate 10,000 scenarios which correspond to one simulation session. For every scenario, the satellite positions are randomly selected above a certain elevation mask (determined by the setup file). After that, AA and then ASODD is performed for each assisting vehicle. The selected ACPs are then sent to the vehicle selection block so that the final ACP value is determined. In case only one assisting vehicle is simulated, the vehicle selection block is bypassed. Finally, the difference between the selected ACP and the actual range is computed and error analysis is performed.

First of all, the effect of the relative distance between the target and assisting vehicles
on the accuracy of the generated ACPs and the selected ACP using ASODD is presented. Secondly, the effect of multipath, receiver noise, troposphere delay, ionosphere delay and ephemeris data on the accuracy of the generated ACPs and the selected ACP using ASODD is presented. Moreover, the effect of the satellite geometry (represented in the elevation mask) on the accuracy of the proposed system is studied. Also, the effect of the number of common satellites on the performance of the AA and ASODD method is investigated. After studying the effects of different variables on the accuracy of the generated ACP from one neighbor, we investigate the effect of employing multiple assisting vehicles on the performance of the proposed system using the proposed selection methods. We also investigate the effect of multipath, receiver’s noise, ionosphere and troposphere errors on the ability of ASOSD method to correctly infer the nearest assisting vehicle.

3.6.1 Effect of Distance

In [71], the minimum elevation of the visible satellites was 75 degrees since all other satellites were masked due to tall buildings in downtown Calgary. In our simulations, four satellites with different elevation and azimuth angles were generated 10,000 times for each distance between vehicles (from 10 to 100m). In order to simulate all possible satellite geometry in urban environments relative to different vehicle positions, no limitation was set on the azimuth angle, however, the minimum elevation angle for the randomly generated satellites was set to 67.5 degrees. At each epoch, one satellite was blocked only from the target vehicle and the three common satellites were used to generate the blocked pseudorange using AA resulting in three ACPs. The ASODD indicator was used to select the final ACP. The number of assisting vehicles was set
3.6. SIMULATIONS

Figure 3.9: Cumulative Distribution Function Representing the range error at different distances between the target and the assisting vehicle to one vehicle and the standard deviation of the ranges to the satellites was set to zero.

Figure 3.9 depicts the Cumulative Distribution Function (CDF) of the range error at different distances. The Best ACP curve represents the RMSE of the most accurate generated ACP at a certain distance. The ASODD ACP represents the RMSE of the ACP selected by the ASODD method. The Best ACP curve is viewed as an upper bound on the performance of the proposed system. As the distance increases, the upper bound becomes less steep and hence higher probability of large range RMSE errors. Consequently, the pool of ACPs that are available for the ASODD method
Table 3.4: Mean and Standard Deviation of the most accurate generated ACP and the ACP selected by ASODD at different distances.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Best ACP $\mu$ (m)</th>
<th>$\sigma$ (m)</th>
<th>ASODD ACP $\mu$</th>
<th>$\sigma$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.86</td>
<td>0.80</td>
<td>1.41</td>
<td>1.09</td>
</tr>
<tr>
<td>20</td>
<td>1.69</td>
<td>1.58</td>
<td>2.75</td>
<td>2.15</td>
</tr>
<tr>
<td>30</td>
<td>2.57</td>
<td>2.42</td>
<td>4.21</td>
<td>3.28</td>
</tr>
<tr>
<td>60</td>
<td>5.09</td>
<td>4.81</td>
<td>8.28</td>
<td>6.46</td>
</tr>
<tr>
<td>80</td>
<td>6.86</td>
<td>6.43</td>
<td>11.31</td>
<td>8.76</td>
</tr>
</tbody>
</table>

The mean and standard deviation are proportional to the distance between vehicles. Specifically, the mean of the range error in the selected ACP is approximately limited to 14% of the distance between vehicles. Finally, Figure 3.10 shows the relation between the distance between vehicles (10-100m) and the percentage of time ASODD was able to successfully select the most accurate ACP. The average success rate of the ASODD is 53% and it does not depend on the distance between vehicles. This is due to the fact that the double differencing terms representing the true range in Equations 3.20, 3.21 and 3.22 proportionally increase with the increase in the distance between the vehicles. Moreover, the double differencing terms representing the error due to ACP in Equations 3.20, 3.21 and 3.22 proportionally increase with the increase in the distance between the vehicles. Therefore, increasing the distance does not affect the selectivity of the ASODD method. The probability of detecting the
3.6. SIMULATIONS

Figure 3.10: ASODD success rate as a function of the distance between vehicles

worse solution by the ASODD method is 69% and it is also not dependent on the distance between vehicles.

3.6.2 Effect of Pseudorange Error Standard Deviation

There are two main categories of errors that affect the measured pseudoranges. Common and Non Common errors. In a certain geographical area, pseudorange errors due to uncompensated ionospheric and tropospheric delays are the same for all vehicles and hence are called common errors. Moreover, the error due to the incorrect estimation of the satellite clock bias from the ephemeris data is also common to all GPS receivers regardless of the position of the receiver. On the other hand, Non Common errors include multipath and receiver noise. These errors are not the same for different vehicles in a specific geographical area. This is due to the fact that
3.6. SIMULATIONS

multipath is a function of very small position variations and the receiver noise is specific to the Noise Figure (NF) of the front end of the vehicle’s receiver. Here, we model Common and Non Common errors as two Gaussian random variables depicted in Equation 3.28.

\[ \rho^f_j = R^f_j + CE + NCE \]  

(3.28)

where:

- \( \rho^f_j \) : Measured pseudorange from vehicle \( j \) to satellite \( f \)
- \( R^f_j \) : True range from receiver \( j \) to satellite \( f \)
- \( CE \sim \mathcal{N}(0, \sigma_{CE}) \) : Common errors like ionosphere, troposphere and satellite clock bias are modeled using Gaussian random variables denoted by \( CE \) with a standard deviation denoted by \( \sigma_{CE} \)
- \( NCE \sim \mathcal{N}(0, \sigma_{NCE}) \) : Non Common errors like multipath and receiver noise are modeled using a Gaussian random variable denoted by \( NCE \) with a standard deviation denoted by \( \sigma_{NCE} \)

The number of assisting vehicles during this simulation was set to one vehicle. In addition, the number of common satellites was set to three. The effect of \( CE \) and \( NCE \) errors on the accuracy of the generated ACPs were studied independently. Also, the accuracy of the selected ACP by the ASODD method was investigated.

Figure 3.11 shows the effect of the standard deviation of the measured pseudorange \( CE \) and \( NCE \) on the RMSE of the generated ACPs and the ACP selected by the ASODD method. The distance between the target and assisting vehicle is 10m. For each curve, either the \( \sigma_{CE} \) is zero while \( \sigma_{NCE} \) is varying or vice versa. Regardless of
Figure 3.11: Effect of the standard deviation of measured pseudorange common and non common errors on the RMSE of the generated ACPs and the ACP selected by the ASODD method.

As the standard deviation of the measured pseudorange error increases, the RMSE of the most accurate generated ACP increases and the RMSE of the selected ACP increases. Moreover, the RMSE of the most accurate ACP pseudorange is not affected by the type of pseudorange error (whether \( CE \) or \( NCE \)). However, the RMSE of the selected ACP by the ASODD method when \( \sigma_{CE} \) is varying and \( \sigma_{NCE} \) is zero is less compared to the case when \( \sigma_{NCE} \) is varying and \( \sigma_{CE} \) is zero. The ASODD method removes the common errors and then decides which ACP has the least ASODD indicator. When the errors are not common to the pseudoranges of the target and assisting vehicle, ASODD does not remove these errors and hence the RMSE of the selected ACP increases relative to the common error case.
3.6. SIMULATIONS

Figure 3.12: Effect of the standard deviation of pseudorange common and non-common errors on the RMSE of the generated ACPs and the ACP selected by the ASODD method when the distance between vehicles is 10m and 50m.

Figure 3.12 depicts the RMSE of the ACPs as a function of the standard deviation of the common pseudorange errors received by both vehicles. The results shown are for a distance of 10m and 50m between vehicles. In general, as $\sigma_{CE}$ or $\sigma_{NCE}$ increase, the RMSE of the most accurate generated solution by AA increases and consequently the RMSE of the selected solution by ASODD increases. This is intuitive, since the standard deviation of the measured pseudoranges affects the accuracy of the angle approximation.

We also deduce from Figure 3.12 that the effect of the measured pseudoranges on the RMSE of the proposed system is more dominant on the vehicles separated by a distance of 10m relative to the vehicles separated by 50m. The RMSE achieved by
3.6. SIMULATIONS

ASODD at 10m varies between 1.4m for a standard deviation of 1m to 5.8m for a standard deviation of 7m. However, the RMSE achieved by ASODD at 50m only varies between 7m for a standard deviation of 1m to 8.5m for a standard deviation of 8m. This shows that the dominant source of error in our proposed system when the distance between the vehicles is relatively small is the standard deviation of measured pseudoranges. However, when the distance between vehicles is relatively large, the dominant source of error is the Angle Approximation. Therefore, selecting the participating vehicles depending on the standard deviation of the measured pseudorange error and the distance between the target vehicle and the assisting vehicle is very critical.

3.6.3 Effect of Satellite Elevation

In urban areas, most visible satellites are at high elevation due to the existence of tall buildings and obstacles. From Figures 3.3, 3.4 and 3.2, we conclude that the performance of the proposed system is highly affected by the satellite geometry relative to the position of the target and the assisting vehicles. Here, we investigate the effect of the satellite geometry on the performance of the AA and the ASODD selection method.

Only one assisting vehicle is used in the simulations and three satellites were common to both the target and the assisting vehicle. The distance between the target and the assisting vehicle was set to 10m. Different elevation masks were set and 10,000 scenarios were generated for every elevation mask. The elevation masks used were 45, 60, 72 and 84 degrees. For every elevation mask, the RMSE of the most accurate ACP generated by the AA method was computed. Moreover, the RMSE of the ACP
selected by the ASODD method was computed.

Figure 3.13 shows the RMSE of the most accurate generated ACP by AA and the ACP selected by the ASODD method. As the elevation mask increases, only high elevation satellites are visible. The RMSE of the generated ACPs decreases as the elevation of the satellites increase. Consequently, the RMSE of the selected ACP by the ASODD method decreases. However, the ASODD success rate which represents the number of times the ASODD is able to select the most accurate ACP is not affected by the elevation mask. Moreover, the number of times the ASODD method ignores the worse generated ACP from all the three generated ACPs is also not affected by the elevation mask.
3.6. SIMULATIONS

So far we have discussed the effect of the elevation mask on the RMSE of the proposed system. In urban areas, GPS L1 signals from satellites at high elevation angles are less affected by ionospheric and tropospheric effects compared to low elevation satellites. Moreover, signals from high elevation satellites are less susceptible to multipath. Therefore, using high elevation satellites to generate ACPs is preferred over low elevation satellites.

3.6.4 Effect of Number of Common Satellites

The number of generated ACPs by the AA method is directly proportional to the number of satellites visible to the target vehicle and the assisting vehicle. Here we investigate the effect of the number of common satellites on the most accurate generated

Figure 3.14: Effect of the Number of Common Satellites on the RMSE of the generated ACPs and the ACP selected by the ASODD method.
Figure 3.15: ASODD success rate as a function of the number of common satellites between the target and the assisting vehicle.

ACP. Moreover, we study the effect of increasing the number of common satellites on the accuracy of the ACP selected by the ASODD method.

The number of common satellites used were 1, 2, 3 and 4. For every number of common satellites used, 10,000 scenarios were generated and the AA method was applied to generate ACPs and ASODD was used to select one ACP. The distance between the target vehicle and the assisting vehicle was set to 10m and only one assisting vehicle was used in the simulations.

Figure 3.14 shows the effect of the number of common satellites on the RMSE of the most accurate ACP generated by the AA method and the ACP selected by the ASODD method. The RMSE of the most accurate ACP generated by the AA
increases as the number of common satellites increase. The accuracy of the ACP selected by the ASODD method increases as the number of common satellites increase. However, the percentage of increase is very low. This decrease in ASODD success rate in detecting the most accurate ACP as the number of common satellites increase is depicted in Figure 3.15. Therefore, the ASODD method acts as a bottleneck especially when the number of common satellites is larger than three. In the next chapter, we propose a new method of selection that relies on the residuals of the Least Square estimator. This method requires at least four common satellites between the target vehicle and the assisting vehicle.

3.6.5 Multiple Assisting Vehicles

There are many variables affecting the performance of the proposed cooperative system and one of the most important variables is the number of assisting vehicles. In the case where multiple assisting vehicles are available, the ASOSD method is used to infer the nearest assisting vehicle. This is due to the fact that the accuracy of the generated ACPs is inversely proportional to the distance between the target vehicle and the assisting vehicle. Here we will investigate the effect of increasing the number of assisting vehicles on the performance of the proposed cooperative system. Moreover, the effect of common and non common errors in the measured pseudoranges on the ability of the ASOSD method to detect the nearest assisting vehicle will be investigated.

Effect of the Number of Assisting Vehicles

First of all, the performance of the ASOSD should be assessed in comparison with a simpler method. This method relies on averaging the selected ACPs by the ASODD
method for each assisting vehicle. In other words, all the assisting vehicles are treated equally regardless of their distance from the target vehicle. The number of assisting vehicles was varied from 1 to 20 vehicles. The generation of the position of the assisting vehicles was random and spanned an area of 100\textit{m} around the target vehicle. To determine the RMSE of the generated ACP for a specific number of assisting vehicles, 10,000 scenarios were generated for a fixed number of assisting vehicles. The elevation mask was set to 67.5 degrees for all scenarios.

Figure 3.16 shows the performance of two vehicle selection methods as a function of the number of assisting vehicles. The RMSE of the averaged ACPs from all assisting vehicles decreases as the number of assisting vehicles increases from one to six vehicles. After that the RMSE of the averaged ACP seems to stay at around 7\textit{m}
regardless of the increase in the number of assisting vehicles. The gain from averaging the error in ACPs is reduced by the effect of increasing the number of assisting vehicles. These assisting vehicles can either be close or far away from the target vehicle. The proposed ASOSD method infers the nearest assisting vehicle and AA and ASODD is only applied to the selected assisting vehicle. Obviously, selecting the nearest assisting vehicle significantly reduces the RMSE of the final ACP as the number of assisting vehicles increase. Specifically, The RMSE of the final ACP selected by ASOSD drops from 8.5\(m\) to 2.7\(m\). The assisting vehicles are generated randomly (following a uniform distribution in 100\(m\) communication range) and therefore, as the number of assisting vehicles increase, the probability of generating a vehicle near the target vehicle increases.

Using ASOSD as a vehicle selection method does not only decrease the RMSE of the final ACP but also reduces the complexity of the system and enhances its scalability. In order to simply average the selected ACPs (by ASODD) from all assisting vehicles, the AA method has to be performed for each assisting vehicle. On the other hand, the ASOSD selects only one assisting vehicle and hence, AA and ASODD are executed once.

**Effect of the Pseudorange Error Standard Deviation**

There are two types of measured pseudorange errors, common errors like satellite clock bias and atmospheric errors and non common errors like multipath and receiver’s noise. Here we investigate the effect of both types of errors on the ASOSD ability to detect the nearest assisting vehicle. Moreover, averaging ACPs from assisting vehicles is compared to the ASOSD performance. The number of assisting vehicles is set to five and the elevation mask is set to 67.5 degrees. Pseudorange errors are
modeled as two Gaussian random variables representing the standard deviation of the common and non common errors. For every type of error, 10,000 scenarios are randomly generated and the RMSE of the final ACP is computed by averaging the selected ACPs from five assisting vehicles. Moreover, the ASOSD method is used to select the nearest assisting vehicle and the final ACP is also computed. Figure 3.17 shows the performance of the ASOSD method compared to averaging ACPs as a function of the measured pseudorange common errors. Common errors are simulated by varying the measured pseudoranges error from 1m to 8m. In order to simulate common pseudorange errors, the same generated random variable is added to each pair of pseudoranges (to the same satellite) of the assisting vehicle and the target vehicle. As the standard deviation of the common errors increases, the RMSE of the
Figure 3.18: Performance of the ASOSD method compared to averaging ACPs as a function of the measured pseudorange Non Common Errors (multipath and receiver’s noise).

ACP resulting from averaging ACPs from the five assisting vehicles and the RMSE of the ACP resulting from the assisting vehicle selected by ASOSD increases. However, the RMSE of the ACP generated from the vehicle selected by ASOSD is less compared to the ACP resulting from averaging. Since the errors are common to both pseudoranges of the target and assisting vehicles, the ASOSD removes the common errors and then infers which vehicle should be used to generate the ACPs. The RMSE of the final ACP increases as the standard deviation of the common errors increase because the generated ACPs contain larger errors. The ability of the ASOSD to infer the nearest assisting vehicle is not affected by the pseudorange common errors.

Figure 3.18 depicts the performance of the ASOSD method compared to averaging ACPs as a function of the measured pseudorange Non Common Errors. The RMSE
of the final ACP generated by averaging ACPs from assisting vehicles is around $7m$ and is not significantly affected by the increase in the pseudorange error standard deviation. Since the Non Common pseudorange error is modeled as a Gaussian random variable with mean zero and the errors in pseudoranges from the vehicles to different satellites are uncorrelated, the effect of averaging ACPs from the five assisting vehicles is on average, canceling the errors due to non common pseudorange errors. On the other hand, the RMSE of the ACP generated from the assisting vehicle selected by ASOSD increases as the pseudorange non common error standard deviation increases. ASOSD is not able to remove non common errors and hence the ASOSD distance indicator becomes less meaningful as the standard deviation of the non common pseudorange errors increase. Beyond a standard deviation of $5m$, the performance of the ASOSD becomes worse than averaging ACPs. Therefore, if the standard deviation of the pseudorange non common errors increase beyond a certain threshold, ASOSD should not be used.
Chapter 4

Enhancing the Performance of 3D-RISS/GPS
LC-EKF Integration in Urban Environments using Cooperative Positioning

4.1 Proposed Cooperative System

In an urban environment, vehicles are equipped with different positioning resources. Some expensive vehicles are capable of decoding many GNSS constellations while others might only track GPS satellites. Different vehicles employ INS systems that vary in grades. Some vehicles use advanced signal processing techniques to detect and mitigate jamming and short delayed multipath signals while other vehicles are only capable of detecting and mitigating medium and long delayed multipath signals and might not have the necessary hardware resources (array of antennas) to mitigate the effect of jamming signals. The need for utilizing resources from neighboring vehicles in a cooperative manner to assist in enhancing the positioning availability and accuracy of other vehicles is critical in dense urban areas due to the following reasons:

1. Limited number of visible satellites due to tall buildings and barriers.
2. Effect of multipath due to the existence of many reflectors.

3. Poor satellite geometry resulting in high Dilution of Precision (DoP).

4. Limited number of vehicles capable of decoding signals from different GNSS constellations

In this chapter, we propose a cooperative system based on the Angle Approximation method which can be used to enhance the performance of INS and GPS integration based on LC-EKF. INS/GPS Integration takes advantage of the complementary error characteristics of INS and GPS systems. In an LC configuration, INS system is used to interpolate positions during GPS partial or full outages. INS errors accumulate over time in an unbounded manner and therefore only short GPS outages can be interpolated effectively (with good accuracy). In order to design a navigation system which is commercially viable and affordable, MEMS-based sensors are used in vehicular navigation. The errors of MEMS-based sensors are very complex and thus can be used in standalone mode only for very short durations. On the other hand, GPS errors are bounded but the visibility of at least four satellites is essential for estimating a 3D position and velocity. Even though LC integration is very simple when it comes to hardware implementation, its main drawback is the low positioning accuracy when GPS outages are prolonged. This is due to the fact that only INS solution is used when the number of visible satellites is less than four. Moreover, under full GPS coverage conditions, positioning accuracy can still be very low due to poor satellite geometry, atmospheric errors, multipath, low carrier to noise ratio and many other factors.

Figure 4.1 depicts the proposed cooperative RISS/GPS LC-EKF (CLC-EKF) system which consists of three main components. The first component is the 3D Reduced
4.1. PROPOSED COOPERATIVE SYSTEM

Inertial Sensor System (RISS) mechanization process used to estimate the states of the vehicle. The second component is the GPS position estimation block which is used to compute the GPS position of the vehicle. When the number of visible satellites to the target vehicle is at least four, a LS algorithm is applied to the measured pseudoranges. However, if the number of visible satellites is less than four, the target vehicle finds an assisting vehicle and applies AA to generate all possible ACPs. Subsequently, ASODD is used to select one ACP which is then used to estimate the GPS position of the target vehicle. The last component of our proposed system is using LC-EKF for the fusion of the RISS and the GPS solution to estimate an optimal position. Since the AA and ASODD methods are covered in details in chapter 3, it will not be discussed here. In this chapter, we start by introducing the 3D-RISS mechanization process. We then describe in details the system and measurement...
models of the LC-EKF. In addition, the EKF filtering algorithm is briefly explained. Finally, the experimental setup, evaluation criteria and the results are presented.

4.2 The 3D Reduced Inertial Sensor System

A full IMU system consists of 3 gyroscopes monitoring angular rotations across the 3-axis of the vehicle. In addition, 3 accelerometers that measure the specific forces across the 3-axis of the vehicle. The full IMU is the most accurate mechanization process, however it is expensive and of high complexity. The 3D RISS [72, 73] is an attempt to reduce the cost and complexity of a full IMU system and thus yield a navigation system that is commercially affordable for land vehicles. The first 3D RISS system proposed in the literature consists of 1 gyroscope perpendicular to the horizontal plane and 2 accelerometers parallel to the horizontal plane (one parallel to the x-axis and the other parallel to the y-axis) along with speed information from the wheel rotation sensor (odometer). The second proposed partial IMU consists of 1 gyroscope perpendicular to the horizontal plane and 3 accelerometers. Two accelerometers are parallel to the horizontal plane (one parallel to the x-axis and the other is parallel to the y-axis) and the third accelerometer is parallel to the z-axis. Here we will discuss the first approach. The advantages of using the partial IMU (3D RISS) over full IMU are:

1. 3D RISS is cheaper and computationally less complex compared to full IMU system.

2. The elimination of two gyroscopes removes the effect of the complex stochastic errors of the bias and drift of the gyroscopes. These stochastic errors are very hard to model as they are a function of many variables.
3. Computing pitch and roll angles using accelerometers prevents the accumulation of error in the 3D RISS model. The full IMU system uses previous pitch and roll angles to compute the current attitude angles and thus results in the accumulation of errors.

4. In the 3D RISS system, integration is performed only once to the East, North and Up velocities to compute the latitude, longitude and altitude. This slows down the rate of growth of error. However, in the full IMU system, integration is performed twice. The first integration is used to compute the velocities and then position is computed by integrating velocities.

The process by which the RISS system computes the states of the land vehicle from the raw data of the sensors is called mechanization. Denote by \( f_x \) and \( f_y \) the transversal and forward specific forces from the accelerometers respectively. Moreover, the angular rotation rate from the vertical gyroscope and the speed from the odometer are denoted by \( \omega_z \) and \( v_{od} \) respectively. Gyroscope and accelerometer known biases are some of the errors that are compensated before the beginning of the mechanization process. The system states representing the latitude, longitude and altitude are denoted by \( \varphi \), \( \lambda \) and \( h \) respectively. In addition, the system states representing the East, North and Up velocity are denoted by \( v^e \), \( v^n \) and \( v_u \) respectively. The first step in the 3D RISS mechanization process is the computation of the attitude angles. The pitch and roll angles are computed using specific forces from the accelerometers and an accurate gravity model. Equations 4.1 and 4.2 are used to compute the pitch and roll angles denoted by \( p \) and \( r \). The integration of the speed from the odometer yields
the acceleration and is denoted by $a_{od}$

$$p = \sin^{-1}\left(\frac{f_y - a_{od}}{g}\right) \quad (4.1)$$

$$r = -\sin^{-1}\left(\frac{f_x - v_{od}\omega_z}{g\cos(p)}\right) \quad (4.2)$$

The azimuth angle denoted by $Az$ is computed using the previous azimuth angle and the current angular rotation rate. Two effects have to be removed from the current $\omega_z$ measurement. The stationary effect of Earth’s rotation denoted by $\omega_e$ on the output of the vertical gyroscope is removed. The angular rotation rate of Earth is around 15 degrees per hour. Moreover, the non-stationary effect (Coriolis effect) is also compensated. Equation 4.3 is used to compute $Az$. The current and previous azimuth angles are denoted by $Az_k$ and $Az_{k-1}$ respectively. The stationary effect is removed by the term $\omega_e \sin(\varphi)$ and the non stationary effect is removed by the term $\frac{v^e \tan(\varphi)}{R_N + h}$. Here, $R_N$ is the normal radius of curvature of the Earth’s ellipsoid.

$$Az_k = Az_{k-1} - \left(\omega_z - \omega_e \sin(\varphi) - \frac{v^e \tan(\varphi)}{R_N + h}\right) \quad (4.3)$$

The attitude angles $p$, $r$ and $Az$ are computed and the next stage of the mechanization is computing the East, North and Up velocities. This is performed by transforming the speed measurements from the body frame to the local frame using the transformation matrix. The current attitude angles are used to compute the transformation matrix. Using Equations 4.4, 4.5 and 4.6, $v^e$, $v^n$ and $v^u$ is computed.

$$v^e = v_{od} \sin(Az) \cos(p) \quad (4.4)$$
4.3. SYSTEM AND MEASUREMENT MODEL FOR 3D-RISS/GPS LC-EKF

\[ v^n = v_{od} \cos(Az) \cos(p) \]  
\[ v^\mu = v_{od} \sin(p) \]  

(4.5)  
(4.6)

The final stage of the mechanization process is the computation of the position of the platform. East, North and Up velocities are integrated to respectively compute the latitude, longitude and altitude. Equations 4.7, 4.8 and 4.9 yield states of the system representing its position. The sampling time is denoted by \( \Delta t \) and the Meridian radius of curvature of the Earth’s ellipsoid is denoted by \( R_M \). The main reason for the unbounded error of the INS system on the long run is the integration process. This is due to the fact that sensors’ biases and drifts propagate to the velocities through the attitude angles, and velocities are then integrated every epoch to compute the position of the vehicle.

\[ \varphi_k = \varphi_{k-1} + \frac{v^n_k + v^n_{k-1}}{2(R_N + h)} \Delta t \]  
\[ \lambda_k = \lambda_{k-1} + \frac{v^\mu_k + v^\mu_{k-1}}{2(R_M + h)} \Delta t \]  
\[ h_k = h_{k-1} + \frac{v^u_k + v^u_{k-1}}{2} \Delta t \]  

(4.7)  
(4.8)  
(4.9)

4.3 System and Measurement Model for 3D-RISS/GPS LC-EKF

In this section, the system and the measurement model of the 3D-RISS/GPS LC-EKF integration is introduced. The main reference for the system and measurement model is [74] unless stated otherwise.

Since a form of KF is used as a data fusion method, the system and the measurement model have to meet the requirements of the KF to achieve optimal state estimation. The three main requirements for KF to yield optimal state estimation are:
4.3. SYSTEM AND MEASUREMENT MODEL FOR 3D-RISS/GPS

1. The system and the measurement model are linear. Most practical models are
not linear and thus a form of linearization is applied before using KF.

2. The system and the measurement noise are uncorrelated and behave as two zero
mean Gaussian random variables with known auto-covariance functions.

3. The initial state of the system is a Gaussian random vector that is uncorrelated
with the system and the measurement model.

4.3.1 System Model

The discrete-time system model of the 3D-RISS/GPS integration can be expressed
using the following equation:

\[ x_k = \phi_{k,k-1}x_{k-1} + G_{k-1}w_{k-1} \]  \hspace{1cm} (4.10)

where

- \( x_k \) is the current state vector of the system
- \( x_{k-1} \) is the previous state vector of the system
- \( \phi_{k,k-1} \) is the discrete-time linear state transition matrix which models the de-
terministic relation between the previous and the current state vectors. Given
the dynamic coefficient matrix \( F \) of a continuous system, the linearized state
transition matrix is shown in Equation 4.11. where \( I \) is the identity matrix.

\[ \phi = (I + F\Delta t) \]  \hspace{1cm} (4.11)
4.3. SYSTEM AND MEASUREMENT MODEL FOR 3D-RISS/GPS

LC-EKF

- $G_{k-1}$ is the noise coupling matrix.

- $k$ is the measurement epoch.

- $w_{k-1}$ is the system noise. The expectation of $w_k$ denoted by $\mathbf{E}[w_k]$ and the covariance of the state vector denoted by $\mathbf{E}[w_i^T w_j]$ (where $i$ and $j$ are elements of the system noise vector) are given by Equations

\begin{equation}
\mathbf{E}[w_k] = 0
\end{equation}

\begin{equation}
\mathbf{E}[w_i^T w_j] = \begin{cases} 
  Q_k & i = j \\
  0 & i \neq j 
\end{cases}
\end{equation}

Equations 4.1 to 4.9 are the RISS mechanization equations. These equations are non-linear and in order to meet the KF linearity requirements, linearization of the RISS mechanization equations is essential. Taylor series expansion is applied to the system equations describing the rate of change of the states. Only the first order terms are considered in the transition matrix. Moreover, very small terms are ignored to reduce the complexity of the system. The process of linearization forces the states of the system to become errors in the states. Therefore, the RISS states of the system are given by vector $4.14$. The error in latitude, longitude and altitude is denoted by $\delta \varphi$, $\delta \lambda$ and $\delta h$ respectively. East, North and Up velocity errors are denoted by $\delta v^e$, $\delta v^n$ and $\delta v^u$ respectively. Moreover, $\delta Az$, $\delta a_{od}$ and $\delta b_z$ denote the error in azimuth, error in acceleration due to wheel rotation sensor measurement and gyroscope bias error respectively.

\begin{equation}
\mathbf{x}_k = [\delta \varphi, \delta \lambda, \delta h, \delta v^e, \delta v^n, \delta v^u, \delta Az, \delta a_{od}, \delta b_z]
\end{equation}
Error in Position Linearized Transition Equations

Error in Latitude
The rate of change of the latitude is given by:

\[ \dot{\varphi} = \frac{v_n}{(R_M + h)} \]  

(4.15)

After applying Taylor series, Equation 4.15 can be re-written as:

\[ \delta \dot{\varphi} = \frac{\delta v_n}{(R_M + h)} - \frac{v_n \delta h}{(R_M + h)^2} \]  

(4.16)

To reduce the complexity of the system, \( \frac{v_n \delta h}{(R_M + h)^2} \) is ignored due to the large value of the denominator. Therefore, the RISS equation describing the transition from the previous to the current error in latitude is given by:

\[ \delta \dot{\varphi} \approx \frac{\delta v_n}{(R_M + h)} \]  

(4.17)

Error in Longitude
The rate of change of the longitude is given by:

\[ \dot{\lambda} = \frac{v^e}{(R_N + h) \cos(\varphi)} \]  

(4.18)

After applying Taylor series, Equation 4.18 can be re-written as:

\[ \delta \dot{\lambda} = \frac{\delta v^e}{(R_N + h) \cos(\varphi)} + \frac{v^e \sin(\varphi) \delta \varphi}{(R_N + h) \cos^2(\varphi)} + \frac{v^e \delta h}{(R_N + h)^2 \cos^2(\varphi)} \]  

(4.19)
To reduce the complexity of the system, \( \frac{v^e \delta h}{(R_N + h)^2 \cos^2 \varphi} \) is ignored due to the large value of the denominator. Therefore, the RISS equation describing the transition from the previous to the current error in longitude is given by:

\[
\delta \lambda \approx \frac{\delta v^e}{(R_N + h) \cos \varphi} + \frac{v^e \sin \varphi \delta \varphi}{(R_N + h) \cos^2 \varphi} \quad (4.20)
\]

**Error in Altitude**

The rate of change of the altitude is given by:

\[
\dot{h} = v^a \quad (4.21)
\]

After applying Taylor series, Equation 4.22 is the RISS equation describing the transition from the previous to the current error in altitude.

\[
\delta \dot{h} = \delta v^a \quad (4.22)
\]

**Error in Velocities Linearized Transition Equations**

**Error in East Velocity**

The rate of change of East velocity is given by:

\[
\dot{v}^e = a_{od} \sin(Az) \cos(p) + v^n \dot{A}z \quad (4.23)
\]

After applying Taylor series and some approximations to reduce complexity, Equation 4.24 is the RISS equation describing the transition from the previous to the current
4.3. SYSTEM AND MEASUREMENT MODEL FOR 3D-RISS/GPS

LC-EKF

Error in East velocity.

\[
\delta \dot{v}_e = \sin(Az)\cos(p)\delta a_{od} + a_{od}\cos(Az)\cos(p)\delta Az \quad (4.24)
\]

Error in North Velocity

The rate of change of North velocity is given by:

\[
\dot{v}_n = a_{od}\cos(Az)\cos(p) + v_e \dot{Az} \quad (4.25)
\]

After applying Taylor series and some approximations to reduce system complexity, Equation 4.26 is the RISS equation describing the transition from the previous to the current error in North velocity.

\[
\delta \dot{v}_e = \cos(Az)\cos(p)\delta a_{od} + a_{od}\sin(Az)\cos(p)\delta Az \quad (4.26)
\]

Error in Up Velocity

The rate of change of Up velocity is given by:

\[
\dot{v}_u = a_{od}\sin(p) \quad (4.27)
\]

After applying Taylor series, Equation 4.28 is the RISS equation describing the transition from the previous to the current error in Up velocity.

\[
\delta \dot{v}_u = \sin(p)\delta a_{od} \quad (4.28)
\]
Error in Azimuth Linearized Transition Equation

The rate of change of the azimuth angle is given by:

\[ \dot{\hat{A}z} = - \left( (\omega_z - b_z) - \omega^e \sin(\varphi) - \frac{v^e \tan(\varphi)}{R_N + h} \right) \]  

(4.29)

After applying Taylor series and some approximations to reduce system complexity, Equation 4.30 is the RISS equation describing the transition from the previous to the current error in the azimuth angle.

\[ \delta \dot{\hat{A}z} \approx \delta b_z + \left( \omega^e \cos(\varphi) + \frac{v^e \sec^2(\varphi)}{R_N + h} \right) \delta \varphi + \frac{\tan \varphi}{R_N + h} \delta v^e \]  

(4.30)

Gyroscope and Odometer Error Models

There are many models used to compensate the stochastic errors of the wheel rotation sensor and the gyroscope bias. One of the predominant models is using first-order Gauss-Markov process. This model has a deterministic term and a stochastic term. The time constant (reciprocal of the autocorrelation time) \( \beta \) and the variance \( \sigma^2 \) are the two parameters characterizing stationary first-order Gauss-Markov processes.

\[ \dot{\delta b_z} = -\beta_z \delta b_z + \sqrt{2\beta_z \sigma_z^2} w(t) \]  

(4.31)

\[ \dot{\delta a_{od}} = -\beta_{od} \delta a_{od} + \sqrt{2\beta_{od} \sigma_{od}^2} w(t) \]  

(4.32)

Equations 4.31 and 4.32 show the first-order Gauss-Markov models for the wheel rotation sensor error and the gyroscope bias respectively. Here, \( \beta_z \) and \( \beta_{od} \) are the time constant of the wheel rotation sensor errors and the gyroscope bias respectively. Moreover, \( \sigma_z^2 \) and \( \sigma_{od}^2 \) are the variance of the wheel rotation sensor errors and the
gyroscope bias respectively. The time constant and the variance of the error depends on the characteristics of the gyroscope and the wheel rotation sensor.

### 4.3.2 Measurement Model

The discrete-time linear measurement model relating the system states to the error in the measurement is given by:

\[
\delta z_k = H \delta x_k + \eta_k \tag{4.33}
\]

where

- $\delta z_k$ is the error in the measurement vector.

- $H$ is the design matrix. Here the design matrix is not changing with time.

- $\eta_k$ is the measurement noise. The expectation of $\eta_k$ denoted by $E[\eta_k]$ and the covariance of the state vector denoted by $E[\eta_i^T \eta_j]$ (where $i$ and $j$ are elements of the measurement noise vector) are given by:

\[
E[\eta_k] = 0 \tag{4.34}
\]

\[
E[\eta_i^T \eta_j] = \begin{cases} 
  R_k & i = j \\
  0 & i \neq j
\end{cases} \tag{4.35}
\]

In the LC implementation of EKF, the integration is performed on the level of states of the system which makes LC implementation simpler than TC implementation. Here we only use the latitude, longitude and altitude estimated by the GPS to aid
the INS system. Velocities can also be used to aid the INS system. The measurement vector is given by:

$$\delta z = \begin{bmatrix} 
\delta \varphi_{GPS} - \delta \varphi_{INS} \\
\delta \lambda_{GPS} - \delta \lambda_{INS} \\
\delta h_{GPS} - \delta h_{INS}
\end{bmatrix}$$

Since only position information is used to aid the INS system in an LC integration, the design matrix relating the error states to the measurement error is given by:

$$H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
4.4 Extended Kalman Filter

There are many types of filters used to fuse INS and GPS solutions in order to produce an optimal estimation of the states of the system. The most common filters are KF and Particle Filter (PF) which are recursive Bayesian estimators. In this research, we have implemented a closed loop configuration of KF which is refereed to as Extended Kalman Filter (EKF). The main advantage of PF over KF is that the system and measurement noise are not constrained to Gaussian distributions and hence PF could provide more accurate solutions. Moreover, the system and measurement models are not limited to linear systems just like the case in KF. However, the main advantage of KF over PF is the low complexity of the filter and the real time response. The optimality of the KF solution is a function of:

- The validity of the linear model relative to the actual non-linear model.

Therefore, the measurement model can be expressed as:

\[
\delta z = \begin{bmatrix}
\delta \varphi_{GPS} - \delta \varphi_{INS} \\
\delta \lambda_{GPS} - \delta \lambda_{INS} \\
\delta h_{GPS} - \delta h_{INS}
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\delta \varphi \\
\delta \lambda \\
\delta h \\
\delta v^e \\
\delta v^n \\
\delta v^u \\
\delta A_z \\
\delta a_{od} \\
\delta b_z
\end{bmatrix} + \eta
\]
The validity of the assumption that the system and the measurement noise can be modeled as normally distributed random variables.

EKF is a closed loop implementation of the KF algorithm where error states computed by KF are fed back to the INS mechanization stage to predict a more accurate INS solution and to keep the system model in the linearity region. Using EKF provides a compromise between the optimality of the PF and the simplicity of the KF. Some critical parameters have to be initialized before the operation of KF. The initial system noise covariance matrix is denoted by $Q_0$ and the initial measurement covariance matrix is denoted by $R_0$. Moreover, the initial error states of the system are denoted by $\delta \hat{x}_0$ and the initial state covariance matrix is denoted by $P_0$. The state covariance matrix is an indication of the errors in the states estimated by the KF and this indicator varies each epoch depending on the accuracy of the predicted (INS solution) and the measured (GPS solution) states.

The KF algorithm consists of two main stages which are the prediction stage and the correction stage. In the prediction stage, the system transition matrix $\phi_{k,k-1}$ is used to predict the current error states at epoch $k$ denoted by $\delta \hat{x}_k^p$ from the previous corrected errors states denoted by $\delta \hat{x}_{k-1}^c$. Moreover, the prediction stage includes computing the predicted state covariance matrix denoted by $P_k^p$ using the knowledge of the previous corrected state covariance matrix denoted by $P_{k-1}^c$, the system transition matrix and the system noise covariance matrix. Equations 4.36 and 4.37 depict the prediction of the states and the states covariance matrix of the system.

\begin{align*}
\delta \hat{x}_k^p &= \phi_{k,k-1} \delta \hat{x}_{k-1}^c \quad (4.36) \\
P_k^p &= \phi_{k,k-1} P_{k-1}^c \phi_{k,k-1}^T + Q_0 \quad (4.37)
\end{align*}
In this EKF, the system noise covariance matrix does not vary over time and is specific to the error characteristics of the inertial sensors. The next stage of the KF is the correction stage. In this stage, the KF gain denoted by $K_k$ is computed as a function of the predicted state covariance matrix and the measurement noise covariance matrix which is denoted by $R_k$. As $P_k^p$ increases relative to $R_k$, the KF gain increases giving the measured error states higher weights relative to the predicted error states. On the other hand, if $R_k$ is large relative to $P_k^p$ which means that the measurement is not reliable, the KF gain decreases giving the measured error states lower weights relative to the predicted error states. The KF gain is computed using the following Equation:

$$K_k = P_k^p H_k^T \left( H_k P_k^p H_k^T + R_k \right)^{-1}$$

(4.38)

Here $H_k$ refers to the design matrix which relates the system states to the measurement states. The $R_k$ is propagated from the LS estimator to the KF at each epoch and represents the confidence in the $\delta \varphi$, $\delta \lambda$ and $\delta h$. The next step in the correction stage is to calculate the corrected states. The innovation vector depicted in Equation 4.39 is a quantity describing the difference between the measured error states denoted by $\delta z_k$ and the predicted error states. The corrected error states denoted by $\delta \mathbf{x}_k^c$ are computed using Equation 4.40. When $K_k$ approaches zero (measurement is unreliable), the corrected and predicted error states are equal. As the KF gain increases (measurement is reliable), a larger quantity of the innovation vector is used to correct the predicted error states. The final step of the KF correction stage is shown in Equation 4.41 and is used to compute the corrected state covariance matrix based on the calculated KF gain and the predicted state covariance matrix. The corrected
4.5. EXPERIMENTS AND RESULTS

states and the corrected states covariance matrix are propagated to the next epoch.

\[ v_k = \delta z_k - H_k \delta \hat{x}_k^p \]  
\[ \delta \hat{x}_k^c = \delta \hat{x}_k^p + K_k v_k \]  
\[ P_k^c = P_k^p + K_k H_k P_k^p \]

In an EKF, the state prediction step depicted in Equation 4.36 is not considered because of the closed loop implementation. In the closed loop implementation, the corrected error states are fed back to the INS mechanization stage and the previous error states are set to zero. Therefore, the predicted error states are always zero and the first step in the prediction stage is unnecessary.

4.5 Experiments and Results

In order to test the proposed cooperative system, a target and an assisting vehicle are employed in a real road trajectory in Kingston. In this section, we present the equipment used to conduct a real road trajectory in order to evaluate the proposed cooperative system. Secondly, the evaluation criteria which is used to test the performance of our system is introduced. In addition, the experimental setup for the real road trajectory is described. Finally, the results are presented and analyzed.

4.5.1 Equipment

There are two IMUs used in this experiment, a high tactical grade IMU from Novatel called IMU-CPT and a MEMS-based IMU called Crossbow IMU300CC. High grade IMUs are not used in commercial land vehicle applications due to their high cost.
### Table 4.1: IMU Characteristics of IMU-CPT [75] and Crossbow IMU300CC [76]

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>IMU-CPT</th>
<th>Crossbow IMU300CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>15.2x16.8x8.9 cm</td>
<td>7.62x9.53x8.13 cm</td>
</tr>
<tr>
<td>Gyro Technology</td>
<td>FOG</td>
<td>MEMS</td>
</tr>
<tr>
<td>Weight</td>
<td>2.36 Kg</td>
<td>0.59 Kg</td>
</tr>
<tr>
<td>Max Data-rate</td>
<td>100 Hz</td>
<td>200 Hz</td>
</tr>
<tr>
<td>Start-up Time</td>
<td>N/A</td>
<td>&lt;1s</td>
</tr>
</tbody>
</table>

**Accelerometer Characteristics**

<table>
<thead>
<tr>
<th>Range</th>
<th>±10g</th>
<th>±2g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>±50mg</td>
<td>&lt;±30mg</td>
</tr>
<tr>
<td>Bias Stability</td>
<td>±0.75mg</td>
<td>N/A</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>4000 ppm</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>Random Walk</td>
<td>N/A</td>
<td>&lt;0.15m/s/√hr</td>
</tr>
</tbody>
</table>

**Gyroscope Characteristics**

<table>
<thead>
<tr>
<th>Range</th>
<th>±375°/s</th>
<th>±100°/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bias</td>
<td>20°/hr</td>
<td>&lt;±2°/s</td>
</tr>
<tr>
<td>Bias Stability</td>
<td>±1°/hr</td>
<td>N/A</td>
</tr>
<tr>
<td>Scale Factor</td>
<td>1500 ppm</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>Random Walk</td>
<td>&lt;0.0667°/√hr</td>
<td>&lt;2.25°/√hr</td>
</tr>
</tbody>
</table>

However, here we use IMU-CPT to aid the GPS system in estimating the reference solution. On the other hand, MEMS-based IMUs can be used in commercial land vehicle applications because they are less expensive and light weight. The proposed system uses data from Crossbow IMU300CC. The characteristics of the high tactical grade and the MEMS-based IMUs are depicted in Table 4.1. It is important to mention that the IMU-CPT employs a full IMU mechanization process by utilizing 3-axis accelerometers and 3-axis gyroscopes, however, the RISS mechanization used in this research only utilizes the vertical gyroscope and the two horizontal accelerometers from the MEMS-based IMU along with the wheel rotation sensor readings to compute the states of the target vehicle.
The RISS mechanization process requires information about the speed of the vehicle. This is acquired by using a speed logger. CarChip [77] is a data logger which was connected to the OBDII interface of the target vehicle. This data logger records the speed of the vehicle and saves it on a flash memory. The memory can be accessed offline through the CarChip software installed on a laptop. The logged speeds are then synchronized with the GPS time tag using a software developed at the Navigation and Instrumentation Research Lab (located in the Royal Military College of Canada).

There are two types of GNSS receivers used in our experiments, the NovAtel SPAN-SE reference system and the NovAtel ProPak-G2plus GPS receiver. The SPAN-SE unit integrates GNSS signals with the high tactical grade IMU-CPT in a tightly coupled KF using L1 and L2 signals. The SPAN-SE is installed in the target vehicle with two antennas and the output from the SPAN-SE is used as a reference solution. The second type of receivers used is a GPS receiver (NovAtel ProPak-G2plus) capable of decoding only GPS signals.

The equipment installed on the target vehicle is:

1. Two L1 and L2 GNSS antennas (GPS-702-GG) offering combined GPS and GLONASS signal reception. Here, two antennas are used to aid SPAN-SE unit in accurately determining the heading of the target vehicle.

2. The SPAN-SE GNSS receiver which offers GPS standalone solution and an integrated IMU-CPT/GNSS solution.

3. A high tactical grade IMU called IMU-CPT offering full IMU 3D solution.

4. A Data logger (CarChip) that is connected to the OBDII interface collecting speed logs.
The equipment installed on the assisting vehicle are:

1. An L1 and L2 GNSS antenna (GPS-702-GG) offering combined GPS and GLONASS signal reception.

2. A NovAtel ProPak-G2plus GPS receiver. The pseudoranges from GLONASS satellites are not processed.

4.5.2 Evaluation Criteria

The purpose of this experiment is to investigate if the proposed cooperative 3D-RISS/GPS LC-EKF (CLC-EKF) is capable of outperforming the conventional non-cooperative 3D-RISS/GPS LC-EKF (NLC-EKF) during partial GPS outages in urban environments. If the target vehicle does not have an INS system installed, there will be no solution available during partial GPS outages. However, if a commercial INS system is installed, a solution is available but its accuracy degrades exponentially as the GPS outage duration prolongs. In order to evaluate the proposed cooperative system, we collect data from two vehicles in an open sky environment and then manually introduce GPS partial outages. The 2D RMS error of the position estimated by the CLC-EKF is compared to the 2D RMS error of the position estimated by the NLC-EKF. The 2D RMS value is computed using the following Equation:

$$RMS = \frac{\sum_{n=1}^{N} \sqrt{(E_n - \hat{E}_n)^2 + (N_n - \hat{N}_n)^2}}{N}$$

(4.42)

Where $E_n$ and $N_n$ are respectively the reference East and North of the target (convert latitude and longitude into East and North) vehicle for the nth pseudorange samples. Moreover, $\hat{E}_n$ and $\hat{N}_n$ are respectively the estimated East and North of the target using
the positioning systems under evaluation (CLC-EKF or NLC-EKF). The number of available pseudorange samples are denoted by \( N \). The second metric used to evaluate the proposed system is the Positioning Accuracy Gain (PAG) and is given by Equation 4.43. Where \( RMS_{CLC-EKF} \) is the 2D RMS position error of the CLC-EKF system and \( RMS_{NLC-EKF} \) is the 2D RMS position error of the NLC-EKF system. Finally, the third metric used to evaluate the performance of the proposed system is the maximum 2D position error.

\[
PAG = \frac{RMS_{NLC-EKF} - RMS_{CLC-EKF}}{RMS_{NLC-EKF}} \times 100 \tag{4.43}
\]

4.5.3 Experimental Setup

Figure 4.2: Road Trajectory estimated using NovAtel SPAN-SE unit
The road trajectory of the target vehicle and the assisting vehicle is shown in Figure 4.2. This trajectory is computed using the SPAN-SE unit installed on the target vehicle. On the map, points marked "A" and "B" are the start and end point of the trajectory respectively. Moreover, the blue crosses show the segments of the trajectory where partial GPS outage was introduced later in the offline phase. The speed of the target vehicle throughout the trajectory is shown in Figure 4.3. The average speed is 20.42 Km/hr, the maximum speed is 46.44 Km/hr and the speed standard deviation is 14.87 Km/hr. The average distance between the target and the assisting vehicle is 35 meters and the minimum distance is 15 meters. The trajectory was conducted in an open sky environment so we can easily block any visible satellite in the offline stage and test our system with different satellites’ geometries relative to the position of the vehicles. The number of visible GPS satellites to the target vehicle
is shown in Figure 4.4. In order to mimic the limited number of visible satellites in urban areas due to tall buildings, initially only four satellites were made visible to the target and the assisting vehicle in the outage segments of the trajectory. The number of common satellites is four and is denoted by $CS$. The four satellites with the highest elevation angles were used as the common visible satellites.

The next step is to reduce the number of common satellites by blocking one, two or three satellites from the four satellites which were initially visible to the target vehicle. Thus, mimicking the effect of vehicles in an urban area having with different positioning resources. The number of visible satellites is still four for the assisting vehicle. This might occur for example when the assisting vehicle is capable of decoding GPS and GLONASS while the target vehicle decodes only GPS signals. Now the conventional 3D-RISS/GPS integration using LC-EKF will rely only on RISS solution.
(positioning accuracy degrades exponentially during the outage) since GPS is partially blocked from the target vehicle and the number of visible satellites is less than four. Here, we apply the proposed cooperative system which applies AA and consequently generates a number of ACPs that is equal to $CS$. The next step is to select the most accurate ACP using the ASODD selection criteria if $CS$ is not equal one. The selected ACP or ACPs (depending on the number of blocked satellites) are then passed to the LS algorithm along with the measured pseudoranges to the visible satellites to compute the GPS position of the target vehicle. Finally, a LC-EKF is used to integrate the GPS solution with the 3D-RISS solution and a final estimate of the error states of the system are computed. These error states are used to correct the solution of the 3D-RISS system.

The duration of each of the three outage regions is 100 seconds. We know that the accuracy of the generated ACPs is a function of the satellite geometry relative to the target and assisting vehicle. In order to diversify the possible geometries and hence produce reliable results, the three outage regions were chosen such that a sharp change in the direction of the trajectory occurs during each outage segment. Moreover, another technique used to diversify the possible geometries is by blocking all possible combinations of visible satellites to the target vehicle and then averaging the horizontal RMS error.

### 4.5.4 Results

Figures 4.5, 4.7 and 4.9 depict the estimated trajectory on Google maps using the conventional NLC-EKF and the proposed CLC-EKF for the first, second and third outage segments respectively. Both NLC-EKF and CLC-EKF are compared to the
reference solution from the SPAN-SE unit. The highest elevation satellite was blocked for a duration of 100 seconds. The number of common satellites between the target and the assisting vehicles was three and hence three ACPs were generated using AA. Subsequently, ASODD was applied and one ACP was selected and then used to compute the GPS position using LS algorithm. Finally, LC-EKF was applied to the RISS and the GPS position to produce the CLC-EKF solution. On the other hand, the NLC-EKF relied only on the 3D-RISS output since a minimum of 4 satellites is required to compute a GPS position.

Figures 4.6, 4.8 and 4.10 depict the 2D position error in meters using the conventional NLC-EKF and the proposed CLC-EKF for the first, second and third outage segments respectively. These errors are for the same simulation setup (blocking the highest elevation satellite) that was used to estimate the trajectories in Figures 4.5, 4.7 and 4.9. It is clear that the NLC-EKF position errors accumulate over time during the outage segments of the trajectory. On the other hand, the CLC-EKF position errors are not accumulative and are significantly better than the conventional NLC-EKF.

Tables 4.4, 4.3 and 4.4 shows the RMS and the maximum 2D position error for the estimated position using NLC-EKF and CLC-EKF for the first, second and third outage segments respectively. The RMS and maximum 2D position error is computed for different number of blocked satellites between the target and the assisting vehicle during the outage segments. The CLC-EKF was applied for one , two and three blocked satellites. For a specific number of blocked satellites, there are many combination of visible satellites. The 2D position error for each combination is different due to the sensitivity of the AA to satellite geometry relative to the position
of both vehicles and also due to different accuracies of the measured pseudoranges by the target vehicle. In order to compute a realistic RMS error of the 2D position estimated by CLC-EKF, all possible combinations of satellites were considered in the RMS calculations for each number of blocked satellites setup. The RMS errors shown in Tables 4.4, 4.3 and 4.4 are a result of applying CLC-EKF to all possible satellite combinations for each number of blocked satellites.

First of all, we observe that the solution of the CLC-EKF is always better than the NLC-EKF in terms of the 2D RMS position error and the maximum position error regardless of the number of blocked satellites for all three outage segments. Moreover, as the number of blocked satellites decrease, the gain of the proposed cooperative system increases compared to the conventional non-cooperative system. The 2D RMS position error and the maximum error of the NLC-EKF is the higher for third outage segment compared to the first and the second outage segment. This error also affects the proposed system due to the uncompensated gyroscope errors. Correcting the gyroscope angular rate errors should result in lower heading errors and therefore lower 2D RMS position errors for the NLC-EKF and the CLC-EKF systems. Using the proposed cooperative system (CLC-EKF) results in higher positioning accuracy during partial GPS outages. The PAG gain is around 88%, 80% and 60% when the number of blocked satellites is one, two and three respectively.
4.5. EXPERIMENTS AND RESULTS

Figure 4.5: Estimated trajectory using NLC-EKF and CLC-EKF compared to the reference solution for the first outage when the highest elevation satellite is blocked.

Figure 4.6: Position Error of NLC-EKF and CLC-EKF for the first outage when the highest elevation satellite is blocked.
Table 4.2: Outage # 1

<table>
<thead>
<tr>
<th>No. of Blocked Satellites</th>
<th>NLC-EKF RMS (m)</th>
<th>NLC-EKF Max. Error (m)</th>
<th>CLC-EKF RMS (m)</th>
<th>CLC-EKF Max. Error (m)</th>
<th>PAG (%)</th>
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<td>9.04</td>
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<td>33</td>
<td>53.5</td>
<td>14.1</td>
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</table>

Figure 4.7: Estimated trajectory using NLC-EKF and CLC-EKF compared to the reference solution for the second outage when the highest elevation satellite is blocked
4.5. EXPERIMENTS AND RESULTS

Figure 4.8: Position Error of NLC-EKF and CLC-EKF compared to the reference solution for the second outage when the highest elevation satellite is blocked

<table>
<thead>
<tr>
<th>No. of Blocked Satellites</th>
<th>NLC-EKF RMS (m)</th>
<th>NLC-EKF Max. Error (m)</th>
<th>CLC-EKF RMS (m)</th>
<th>CLC-EKF Max. Error (m)</th>
<th>PAG (%)</th>
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Figure 4.9: Estimated trajectory using NLC-EKF and CLC-EKF compared to the reference solution for the third outage when the highest elevation satellite is blocked

Figure 4.10: Position Error of NLC-EKF and CLC-EKF for the third outage when the highest elevation satellite is blocked
### Table 4.4: Outage # 3

<table>
<thead>
<tr>
<th>No. of Blocked Satellites</th>
<th>NLC-EKF RMS (m)</th>
<th>NLC-EKF Max. Error (m)</th>
<th>CLC-EKF RMS (m)</th>
<th>CLC-EKF Max. Error (m)</th>
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5.1 Summary and Conclusion

ITS applications demand specific positioning availability and accuracy requirements. In urban areas, GNSS signals are hindered due to tall buildings, multipath effect, jamming and limited GNSS channels per receiver; this leads to limited positioning availability and accuracy. The two main positioning systems for land vehicles are non-cooperative positioning and CP systems. The conventional non-cooperative positioning systems have limited performance due to the harsh environment in urban areas. Therefore, recent research has been directed towards CP systems. The motivation behind CP is the varying positioning resources of vehicles. For example, some vehicles are capable of decoding GPS and GLONASS signals while other vehicles only decode GPS signals. Most CP methods employ ranging methods like RSS, ToA or RTT to determine the distance between vehicles. The error in the estimated distance is large, and it propagates to the final computed position. Due to the limitations of the ranging methods, recent proposed CP approaches have very limited performance. In this thesis, we propose a novel distributed non-range based CP system. In this
system, AA uses the exchanged pseudoranges from the assisting vehicle and the target vehicle to generate ACPs representing the hindered pseudorange. We then propose an ACP selection method called ASODD. Since ACPs have varying range accuracies depending on the position of the satellites relative to the vehicles, ASODD is used to select the most accurate ACP, which is later used to compute the position of the target vehicle. We developed an OS simulator on MATLAB environment and applied extensive simulations. Our goal was: to study the effect of the distance between vehicles; the standard deviation of the pseudorange error; the satellite elevation mask; and the number of common visible satellites.

As the distance between vehicles increase, the accuracy of the proposed cooperative system degrades. This occurs due to the invalidity of the angle approximation assumption as the distance between vehicles increase. Moreover, the distance between vehicles does not have an effect on the selectivity of the ASODD method. Also, as the standard deviation of the pseudorange error decrease, the accuracy of the generated ACPs increase. Here, the dominant source of error for small distances between vehicles is the measured pseudorange error, while the dominant source of error at large distances between vehicles is the angle approximation assumption. As the number of common visible satellites increase, the accuracy of the best ACP increases, however, the performance of the ASODD is not enhanced by increasing the number of common visible satellites. Finally, the positioning accuracy of the proposed system increases as the minimum satellite elevation increases.

In urban environments, target vehicles are surrounded by several neighboring vehicles. It is possible that more than one vehicle is able to assist the target vehicle. Each assisting vehicle provides ACPs with different accuracies. We proposed an assisting
vehicle selection method called ASOSD. Since the accuracy of the proposed system is a function of the distance between the target vehicle and the assisting vehicle, ASOSD is used as a distance indicator. Moreover, OS is used to investigate the effect of the number of assisting vehicles on the ASOSD accuracy compared to averaging all the available solutions. The RMS error of the range computed by averaging all the selected ACPs was almost constant as the number of vehicles increased. However, the RMS error of the range computed by the assisting vehicle that is selected by ASOSD decreased significantly as the number of assisting vehicles increased. Finally, simulations show that ASOSD outperforms the averaging method when common pseudoranges errors are introduced. Results of the simulations also show the resilience of the ASOSD to errors due to like ionospheric delays, tropospheric delays, and satellite clock biases. ASOSD removes these common errors before deciding which assisting vehicle is closest to the target vehicle. However, the accuracy of the proposed system is limited when the magnitude of the non-common errors like multipath increase (these errors can not be removed by single differencing).

Finally, we propose the CLC-EKF system which integrates RISS and GPS using EKF and assists GPS during partial outages. The proposed system is implemented and tested using road trajectories and compared to the conventional NLC-EKF system. The proposed CLC-EKF outperforms the conventional NLC-EKF system in terms of position RMS error and the maximum position error. Specifically, the PAG gain is around 88%, 80% and 60% when the number of blocked satellites during partial GPS outages is one, two and three respectively.
5.2. FUTURE WORK

5.2 Future Work

The research on CP for land vehicles in urban areas has been limited to range based systems. This thesis provides a unified non-range based CP system capable of increasing positioning availability and accuracy of land vehicles in harsh environments. The insights attained in this thesis open interesting future research directions that include:

- **Artificial Candidate Pseudorange Selection:**
  The proposed AA method generates ACPs with varying accuracies. We have seen that the RMS error of the selected ACP does not decrease as the accuracy of the most accurate ACP increase (when the number of common visible satellites increase). Therefore, the performance of the proposed system is capped by the selectively of the ASODD method. Investigating different selection methods with higher selectivity will significantly increase the accuracy of the final estimated position.

- **Assisting Vehicle Selection:**
  We proposed an assisting vehicle selection method called ASOSD. The selection was merely based on a distance indicator between the target and assisting vehicle. While the accuracy of the ACPs generated by AA depends on the distance between the vehicles, it is not the only factor influencing the accuracy of the generated ACPs. The measured pseudoranges by each assisting vehicle varies in terms of range accuracy (multipath effect). Therefore, the assisting vehicle selection method should also consider the accuracy of the measured pseudoranges.
5.2. FUTURE WORK

- **Advanced GNSS Simulator:**
  In this thesis, we developed an OS environment to investigate the effect of different parameters on the performance of the proposed CP system. In order to simulate the actual orbital paths of the different GNSS satellites, an advanced GNSS simulator like SPIRENT should be used. Trying to automate thousands of simulation scenarios in a GNSS simulator environment is challenging, however, more practical results can be acquired.

- **Enhancing LC-EKF performance during full GNSS coverage:**
  In this thesis, we proposed the CLC-EKF CP system which provides a higher positioning accuracy during partial GPS outages compared to the conventional NLC-EKF system. Under full GNSS coverage, the positioning accuracy provided by NLC-EKF is also limited due to atmospheric delays and multipath effects in urban areas. The proposed CP system could be used to enhance the positioning accuracy of the NLC-EKF during full GNSS coverage. However, the CP system should have the capability of determining whether the CP solution achieves better accuracy than the NLC-EKF solution or vice versa and accordingly select the final solution.

- **Enhancing TC-EKF performance using the proposed CP system:**
  The first advantage of LC-EKF compared to TC-EKF is the smaller filter size (less number of states). Moreover, LC-EKF provides RISS standalone, GPS standalone and the integrated RISS/GPS solutions. However, TC-EKF provides only RISS standalone and the integrated RISS/GPS solutions. Therefore, LC-EKF is more robust when RISS standalone solution fails as it still provides the GPS solution. On the other hand, TC-EKF provides higher positioning...
accuracy under partial GNSS coverage compared to LC-EKF. Both LC-EKF and TC-EKF systems provide similar accuracies under full GNSS coverage. The proposed CP system can be used to generate ACPs representing the hindered pseudoranges and then the selected ACP can be directly integrated with the TC-EKF filter, thus potentially increasing the accuracy of the TC-EKF system.
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