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SLIP HEATING IN DIE DROOL WITH VISCOUS DISSIPATION

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ABSTRACT

Plastics can build-up on the die lip during extrusion. This phenomenon is called *die drool* and can be costly for plastics producers, requiring periodic shutdowns for die cleaning. Die drool has been attributed to the cohesive fracture of the melt into a *drool layer* and a *bulk layer*. The bulk layer slips on the drool layer after fracture, resulting in heating at the slip interface, called *slip heating*. The heat generated through slip heating can contribute to polymer thermal degradation, and to the die drool degradation. The impact of slip heating on die temperature rise has been investigated by neglecting viscous dissipation [Gilbert and Giacomin, *CJChE*, accepted (2014)]. This work considers viscous dissipation and its importance to slip heating. We find that viscous dissipation and slip heating contribute equally to the melt temperature rise, and we conclude with a worked example showing the importance of these two heating sources during polymer processing. We also develop two sufficient conditions for the accurate use of our results, $Pé \ll 1$ and $\partial\Theta/\partial\zeta \ll \partial^2\Theta/\partial\zeta^2$.

Keywords: slip heating, die drool, viscous dissipation, drool layer, plastics extrusion, die lip buildup

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I. Steady Velocity Profiles in Drool and Bulk Layers

The y – component of the equation of motion in terms of shear stress for Cartesian coordinates (Eq. (B.5-2) of [1]) reduces to:

$$\frac{dp}{dy} = -\frac{d\tau_{xy}}{dx} \quad (1)$$

for pressure-driven flow between closely spaced parallel plates (see Figure 1).

For a Newtonian melt with temperature independent viscosity, Eq. (1) becomes:

$$\frac{dp}{dy} = \mu \frac{d^2 v_y}{dx^2} \quad (2)$$

When Eq. (2) is integrated over the length of the die, L , we obtain:

$$\frac{d^2 v_y}{dx^2} = \frac{\Delta p}{L\mu} \quad (3)$$

Substituting the dimensionless groups:

$$\varphi \equiv \frac{-v_y \mu L}{\Delta p b^2} \quad (4)$$

and:

$$\xi \equiv \frac{x}{b} \quad (5)$$

into Eq. (3) yields:

$$\frac{d^2 \varphi}{d\xi^2} = -1 \quad (6)$$

which applies for both the bulk and drool layers.

For the drool layer velocity, Eq. (6) is subject to the no slip boundary condition at the wall:

$$\varphi_d(-d/b) \equiv \varphi_d(-a) = 0 \quad (7)$$

and to the slipping interface condition:

$$\left. \frac{d\varphi_d}{d\xi} \right|_{\xi=0} = \sigma_c \equiv \frac{-\tau_c L}{b\Delta p} \quad (8)$$

We find the drool velocity profile:

$$\varphi_d(\xi) = \frac{a^2 - \xi^2}{2} + \sigma_c(\xi + a) \quad (9)$$

Using the other boundary condition at the slipping interface:

$$\varphi_b(0) = \varphi_d(0) + \varphi_s = \frac{a^2}{2} + \sigma_c a + \varphi_s \quad (10)$$

and the mid-plane condition:

$$\left. \frac{d\varphi_b}{d\xi} \right|_{\xi=1} = 0 \quad (11)$$

we find the bulk layer velocity profile:

$$\varphi_b(\xi) = \frac{a^2 - \xi^2}{2} + \xi + \sigma_c a + \varphi_s \quad (12)$$

that will be used in Eq. (23) to determine the impact of viscous dissipation on the melt temperature rise.

II. Results: Steady Temperature Rise from Viscous Dissipation and Slip Heating

The dimensional equation of energy in terms of heat flux in Cartesian coordinates (see Eq. B.8-1 [1]) reduces to:

$$\rho \hat{C}_p v_y \frac{\partial T}{\partial y} = -\frac{\partial q_x}{\partial x} - (\tau : \nabla v) \quad (13)$$

for time-steady temperature rise where conductive heating in the flow direction is negligible. Using Eq. A.7-1 (C) of [1] to replace $(\tau : \nabla v)$ in Eq. (13) yields:

$$\rho \hat{C}_p v_y \frac{\partial T}{\partial y} = -\frac{\partial q_x}{\partial x} - \tau_{yx} \frac{\partial v_y}{\partial x} \quad (14)$$

and for a Newtonian fluid, Eq. (14) becomes:

$$\rho \hat{C}_p v_y \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial x^2} + \mu \left(\frac{\partial v_y}{\partial x} \right)^2 \quad (15)$$

Eq. (15) can be rewritten in dimensionless form:

$$-\text{Pé} \frac{\partial \Theta}{\partial \zeta} = \frac{\partial^2 \Theta}{\partial \zeta^2} + \text{Br} \left(\frac{\partial \varphi}{\partial \zeta} \right)^2 \quad (16)$$

where:

$$\Theta \equiv \frac{T - T_0}{T_0} \quad (17)$$

$$\zeta \equiv \frac{y}{L} \quad (18)$$

$$\text{Br} \equiv \frac{b^4}{\mu k T_0} \left(\frac{\Delta p}{L} \right)^2 \quad (19)$$

and:

$$\text{Pé} \equiv \frac{b^4 \Delta p}{\alpha L^2 \mu} \quad (20)$$

Convective heating can be neglected when:

$$\text{Pé} \ll 1 \quad (21)$$

which is a sufficient condition for the accurate use of our results. A second sufficient condition is satisfied when the temperature gradient along the die falls well below the second derivative of the temperature through the melt:

$$\frac{\partial \Theta}{\partial \zeta} \ll \frac{\partial^2 \Theta}{\partial \zeta^2} \quad (22)$$

When convective heating is small, Eq. (16) reduces to:

$$\frac{d^2 \Theta}{d\zeta^2} = -\text{Br} \left(\frac{\partial \varphi}{\partial \zeta} \right)^2 \quad (23)$$

which is used below to develop the temperature profiles for the drool and bulk layers.

We solve for the drool and bulk temperature profiles subject to an isothermal die wall:

$$\Theta_d(-a) = \Theta_w \quad (24)$$

an adiabatic mid-plane:

$$\left. \frac{d\Theta_b}{d\zeta} \right|_{\zeta=1} = 0 \quad (25)$$

temperature continuity at the slipping interface:

$$\Theta_d(0) = \Theta_b(0) \quad (26)$$

and constant heat generation at the slipping interface:

$$K \left. \frac{d\Theta_d}{d\zeta} \right|_{\zeta=0} - \left. \frac{d\Theta_b}{d\zeta} \right|_{\zeta=0} = F \quad (27)$$

where the dimensionless interfacial heat flux is:

$$F \equiv \frac{q_i b}{k_b T_0} \quad (28)$$

and the thermal conductivity ratio is:

$$K \equiv \frac{k_d}{k_b} \quad (29)$$

The latter allows for different thermal properties of the drool and bulk layers, as in the case of thermal degradation. When there is no degradation or the thermal properties of the drool and bulk layers do not differ, $K = 1$. By integrating Eq. (23) and then applying the above boundary conditions, we find the drool temperature profile:

$$\Theta_d(\zeta) = \text{Br} \left(\frac{a^4 - \zeta^4}{12} + \sigma_c \frac{\zeta^3 - a^3}{3} + \sigma_c^2 \frac{a^2 - \zeta^2}{2} \right) + \left(\frac{F}{K} + \frac{\text{Br}}{3K} \right) (\zeta - a) + \Theta_w \quad (30)$$

and the bulk temperature profile:

$$\Theta_b(\zeta) = \text{Br} \left(\frac{a^4 - \zeta^4}{12} + \frac{\sigma_c a^3 + \zeta^3 + \zeta}{3} + \frac{\sigma_c^2 a^2 - \zeta^2}{2} \right) + \left(\frac{F}{K} + \frac{\text{Br}}{3K} \right) a + \Theta_w \quad (31)$$

These can be used to determine the true temperature of the polymer melt in systems with cohesive slip. Because σ_c is always unity (see section V of [2,3]), we get:

$$\Theta_d(\zeta) = \text{Br} \left(\frac{a^4 - \zeta^4}{12} + \frac{a^3 + \zeta^3}{3} + \frac{a^2 - \zeta^2}{2} \right) + \left(\frac{F}{K} + \frac{\text{Br}}{3K} \right) (\zeta + a) + \Theta_w \quad (32)$$

and:

$$\Theta_b(\zeta) = \text{Br} \left(\frac{a^4 - \zeta^4}{12} + \frac{a^3 + \zeta^3 + \zeta}{3} + \frac{a^2 - \zeta^2}{2} \right) + \left(\frac{F}{K} + \frac{\text{Br}}{3K} \right) a + \Theta_w \quad (33)$$

The maximum melt temperature occurs at the mid-plane, $\zeta = 1$:

$$\Theta_{\max} \equiv \Theta_b(1) = \text{Br} \left(\frac{a^4 + 1}{12} + \frac{a^3}{3} + \frac{a^2}{2} \right) + \left(\frac{F}{K} + \frac{\text{Br}}{3K} \right) a + \Theta_w \quad (34)$$

It is the goal of the plastics engineer to prevent the maximum temperature rise from exceeding the polymer degradation point. Eq. (33) is a main result of this work, and the following worked example demonstrates its usefulness.

III. Worked Example: Roles of Slip Heating and Viscous Dissipation in Temperature Rise

A plastics engineer must determine the temperature rise in her sheet extrusion die from slip heating. She is also interested in the importance of slip heating. Her system specifications are: $T_0 = T_w = 423 \text{ K}$, $h = 0.3 \text{ mm}$, $\tau_c = 0.12 \text{ MPa}$, $u_s = 200 \text{ mm/s}$, $k_d = k_b = 0.1 \text{ W/m-K}$, $\tilde{C}_p = 1.6 \text{ J/g-K}$, $\rho = 0.90 \text{ g/mL}$, $\mu = 600 \text{ Pa-s}$, $L = 40 \text{ mm}$, $\Delta P = -35 \text{ MPa}$, and $w = 1.0 \text{ m}$.

She first determines the drool layer thickness (see Eq. 18 of [4]):

$$d \equiv \frac{h}{2} + \frac{\tau_c L}{\Delta p} = 12.9 \text{ } \mu\text{m} \quad (35)$$

which corresponds to a bulk layer thickness:

$$b \equiv \frac{h}{2} - d = 0.137 \text{ mm} \quad (36)$$

Applying these values and the system parameters, she calculates the heat generation at the interface from slip heating to be:

$$q_i = u_s \tau_c = 24.0 \text{ kW/m}^2 \quad (37)$$

and then its dimensionless value:

$$F \equiv \frac{q_i b}{k T_0} = 7.78 \times 10^{-2} \quad (38)$$

and the dimensionless drool thickness:

$$a \equiv \frac{d}{b} = 9.38 \times 10^{-2} \quad (39)$$

She then determines the maximum melt temperature in the presence of both slip heating and viscous dissipation, using Eq. (34), to be:

$$\Theta_{\max} = 8.56 \times 10^{-3} \quad (40)$$

which corresponds to a temperature rise from both slip heating (SH) and viscous dissipation (VD) of:

$$\Delta T_{SHVD} = 3.62 \text{ K} \quad (41)$$

By setting $\text{Br} = 0$ in Eq. (34), she obtains an expression for temperature rise from slip heating alone:

$$\Theta_{\max} = aF \quad (42)$$

which matches Eq. (44) of [2]. Using Eq. (42) she gets:

$$\Theta_{\max} = aF = 7.29 \times 10^{-3} \quad (43)$$

which corresponds to a temperature rise of:

$$\Delta T_{SH} = 3.09 \text{ K} \quad (44)$$

Using Eqs. (44) and (41), she next constructs the ratio:

$$\frac{\Delta T_{SH}}{\Delta T_{SHVD}} = 0.851 \quad (45)$$

from which she learns that slip heating is the primary contribution to overall heating.

Because her system may require adjustment, she decides to investigate the relative role of slip heating and viscous dissipation for other system specifications. She thus sets $\Theta_w = 0$ and divides Eq. (34) by the expression in Eq. (42) to develop the ratio of temperature rise from slip heating to the temperature rise from both slip heating and viscous dissipation:

$$\Phi = \frac{\Delta T_{SH}}{\Delta T_{SHVD}} = \left(\frac{\Delta p}{\mu u_s \Xi L} \left(\frac{h^3}{96} - \frac{h^2}{48} \Xi + \frac{h}{24} \Xi^2 + \frac{\Xi^3}{4} + \frac{\Xi^4}{6h + 12\Xi} - \frac{\Xi^3}{3K} \right) + \frac{1}{K} \right)^{-1} \quad (46)$$

where:

$$\Xi \equiv \frac{\tau_c L}{\Delta p} \quad (47)$$

is a characteristic length. Eq. (46) is a main result of this work. Using Eq. (46) with $K = 1$ she generates Figure 2 showing the fraction of total heating from slip heating as a function of slit width for different die pressure drops. Asymptotes arise at small gaps where the drool layer disappears and slip heating no longer occurs. When the drool layer exists, small die gaps, $h \leq 0.1$ mm, have negligible amounts of viscous heating, and as the gap increases viscous heating becomes the primary heating source. The plastics engineer should consider both slip heating and viscous dissipation in thermal analyses of drooling dies except for very large die gaps, $h \geq 5$ mm, where slip heating can be neglected, and very small die gaps, $h \leq 0.1$ mm, where viscous dissipation can be neglected.

She notes that increasing viscosity or slip speed amplifies the contribution of slip heating to total melt heating. Increasing die length or polymer cohesive fracture strength increases the importance of viscous dissipation for small gaps but expands the region of slip heating dominance to larger gaps. In other words, increasing these parameters shifts the curves in Figure 2 down and right. As die pressure drop increases, slip heating matters more at low gaps but the threshold where viscous dissipation becomes the major contributor is reduced to smaller gaps.

IV. Conclusion

Slip heating and viscous dissipation must both be considered in temperature analysis of drooling systems. Eq. (46) is a main result of this work and should be used to determine when one or the other can be neglected. When $\Phi \approx 0$, slip heating may be excluded. When $\Phi \approx 1$, viscous dissipation should be neglected, and Eqs. (43) and (44) of [2] become appropriate choices for thermal analysis. However, when $0 \ll \Phi \ll 1$, neither heating source may be neglected, and Eqs. (32) and (33) of this work should be used. Eq. (34) is the other main result, and it is used to determine the maximum temperature rise in the polymer. Plastics processors must ensure that this temperature rise falls below the degradation point of the polymer melt.

Figure 1: Velocity profiles and concomitant steady temperature profiles for the drool and bulk layers, where $u_s \equiv u_y^- - u_y^+$.

Figure 2: Slip heating to total heating ratio, Φ , as a function gap at three pressure drops, where all other properties are held constant,
 $\Phi(\Delta p, h, \mu, u_s, L, \tau_c) = \Phi(\Delta p, h, 600 \text{ Pa}\cdot\text{s}, 200 \text{ mm/s}, 40 \text{ mm}, 0.12 \text{ MPa})$.

V. REFERENCES

- [1] R.B. Bird, W.E. Stewart and E.N. Lightfoot, *Transport Phenomena*, Revised 2nd Ed., Wiley & Sons, New York (2007).
- [2] P.H. Gilbert and A.J. Giacomini, "Slip Heating in Die Drool," *Canadian Journal of Chemical Engineering*, (Accepted: May 2014).
- [3] P.H. Gilbert and A.J. Giacomini, "Slip Heating in Die Drool," PRG Report No. 001, Queen's University, Canada (March 2014).
- [4] A.M. Schmalzer and A.J. Giacomini, "Die drool theory," *Journal of Polymer Engineering*, **33**, 1-18 (2013).