Phase Noise Characterization of a Mode-Locked Quantum-Dot Coherent Optical Frequency Comb Source Laser

by

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Abstract

Modern coherent optical communication systems transmit 100 Gb/s dual-polarization quadrature phase shift keying (DP-QPSK) signals on each of the 100 channels in a dense wavelength division multiplexing (DWDM) system. Each of these channels require an individual optical carrier frequency to be generated from a discrete high-performance tunable laser. These arrays of discrete lasers are an expensive component in the transmitter of an optical communication channel. Thus, recent research has investigated an alternative laser source - optical frequency combs (OFCs). OFCs generate numerous spectral modes, referred to as comb lines, from a single photonic device. Each of these comb lines can be used as an optical carrier frequency for a channel in a DWDM system. Thus, the implementation of an OFC as a laser source for coherent optical communications minimizes the number of laser modules in a DWDM system leading to a drastic reduction in system cost and complexity. Additionally, OFCs exhibit an intrinsic coherence among each comb line across the spectrum. This coherence implies that the phase noise exhibited by each comb line in the OFC is related which allows for effective methods of phase noise compensation. Consequently, OFCs are heavily under research and development.

In this research, a prototype 25 GHz quantum-dot coherent optical frequency comb source laser is characterized in both the time and frequency domains. The output of the comb source laser is extensively observed and the optical and electrical spectra, spectral flatness, relative intensity noise, phase noise, linewidth and timing jitter are measured. These preliminary results led to the development of a coherent phase noise detection scheme capable of recovering time domain phase noise trajectories for pairs of comb lines. The detection scheme allowed the coherence exhibited by the comb source to be quantified using a correlation coefficient which compared recovered

phase noise trajectories for pairs of comb lines. Additionally, the statistical properties of the recovered phase noise trajectories are explored which enables experimental quantification of the relative contribution of amplified spontaneous emission (ASE) noise and timing jitter for each mode. Experimental results are shown to align with theoretical descriptions of phase noise in mode-locked lasers.

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List of Abbreviations

ADC	Analog-to-Digital
AOM	Acousto-optic Modulator
ASE	Amplified Spontaneous Emission
ASIC	Application Specific Integrated Circuit
AWG	Array-Waveguide Gratings
BPD	Balanced Photodiode
BPF	Bandpass Filter
CCS	Coherent Comb Source
CW	Continuous Wave
CWDM	Coarse Wavelength Division Multiplexing
DAC	Digital-to-Analog
DFB	Distributed Feedback Laser
DP	Dual Polarization
DSH	Delayed Self-Heterodyne
DSP	Digital Signal Processing
DUT	Device Under Test
DWDM	Dense Wavelength Division Multiplexing
EAM	Electro-absorption modulator
ECL	External Cavity Laser
EDFA	Erbium-Doped Fiber Amplifier
ETSO	Equivalent Time Sampling Oscilloscope
FFT	Fast Fourier Transform
FM	Frequency Modulation
FOE	Frequency Offset Estimation
FSR	Free Spectral Range
FWS	Finisar Waveshaper
GUI	Graphical User Interface

HRS	High Resolution Spectrometer
IQ	In-phase Quadrature
ITU	International Telecommunication Union
LASER	Light Amplification by Stimulated Emission of Radiation
LCA	Lightwave Component Analyzer
LCoS	Liquid Crystal on Silicon
LO	Local Oscillator
LPF	Low Pass Filter
LSA	Lightwave System Analyzer
LSRL	Lightwave Systems Research Laboratory
MLL	Mode-Locked Laser
MSA	Microwave Spectrum Analyzer
MSE	Mean Squared Error
MZM	Mach-Zehnder Modulator
OA	Optical Attenuator
O/E	Optical-to-Electrical
OFC	Optical Frequency Comb
OSA	Optical Spectrum Analyzer
PAM	Pulse Amplitude Modulation
PBS	Polarization Beam Splitter
PD	Photodiode
PDM	Polarization Division Multiplexing
PM	Phase Modulation
PNT	Phase Noise Trajectory
PSD	Power Spectral Density
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RBW	Resolution Bandwidth
RF	Radio Frequency
RIN	Relative Intensity Noise
RMS	Root Mean Squared

RNG	Random Number Generator
RTSO	Real-Time Sampling Oscilloscope
SNR	Signal-to-Noise Ratio
SOP	State of Polarization
SSB	Single-Sideband
SSE	Source Spontaneous Emission
TEC	Thermo-Electric Cooler
TIA	Trans-impedance Amplifier
TLS	Tunable Laser Source
UPS	Uninterruptable Power Source
VCSEL	Vertical-Cavity Surface-Emitting Laser
VISA	Virtual Instrument Software Architecture
VNA	Vector Network Analyzer
WDM	Wavelength Division Multiplexing

Chapter 1 - Introduction

1.1 – Introduction and Motivation

As we progress through the 21st century, the world has shifted into a digital era wherein information technologies influence nearly every aspect of daily life. The technological advancements made throughout this digital era have drastically changed the way the world functions. Society is now overflowing with increasingly powerful electronic devices which continue to push technological limits. As a result, there is an overwhelming and constant flow of data being transported around the globe. Life has become dominated by the need to transmit and receive data. A primary reason for this shift into the digital era is the ease and speed at which people are now able to access information. This has consequentially embedded high-bandwidth functionality deep within the roots of modern society. As a result, there is a never-ending growth in online traffic which is an increasingly challenging problem falling primarily on the shoulders of the telecommunications industry. Thus, the telecom industry is constantly under pressure to innovate and accommodate this ever-increasing demand for high-speed global connectivity.

It was only in the early 1990's when commercial global connectivity, which is now known as the *world wide web*, began to take form [1]. This infrastructure formed the beginnings of the modern internet which has since become a basic human necessity. As such, internet traffic has been steadily rising with current research forecasting monthly data traffic to increase by more than 120% in 2021 [2].

Figure 1.1: Expected Monthly Internet Traffic Growth (2016 – 2021)

This explosive increase in internet traffic has been driven by the expansion of cloud-based storage and services, social media services, video streaming, and machine-to-machine applications (e.g., Internet of Things). Furthermore, current and future internet-based applications such as artificial intelligence and augmented reality will continue to transform how people interact with each other, with data, and with their surroundings for many years to come [3]. Virtually all internet traffic is enabled by optical communications wherein binary data is transformed into optical pulses and propagated over optical fibers within a global communications network [4].

Although optical communication has firmly established itself as the backbone of the modern internet and enabler of the digital era, the technology surrounding optical communication is constantly under development to increase transmission rates towards the Shannon channel capacity. With no end to the aforementioned data increase in sight,

innovations are a constant and necessary industry priority. Optical communication systems can be separated into two distinct realms; non-coherent and coherent optical communications [5]. Though similar in their over-arching functionality of transmitting data optically, the methodology and techniques employed by non-coherent and coherent optical communication vary significantly.

1.2 – Non-Coherent and Coherent Optical Communication

Non-coherent optical communication is governed by a reception technique known as *direct detection*. At the transmitter of a non-coherent optical communication channel, digital data is embedded in the intensity of an optical signal using a technique known as intensity modulation or on-off keying. Optical carrier fields have their intensity modulated to correspond to a binary value, and a direct detection receiver is employed to recover the transmitted data. A direct detection receiver involves a high-speed photodiode which produces a photocurrent proportional to the received optical signal intensity. In this way, the recovered photocurrent encompasses the digital data embedded upon the transmitted optical carrier.

Non-coherent modulation can be accomplished using direct modulation or external modulators [6]. Direct modulation involves varying the drive current of a laser, which translates to a proportional increase or decrease in the optical output power of the signal. Thus, binary data can be transmitted by turning the laser off (or very close to off) and on at a high rate. Alternatively, external modulators, such as the Mach-Zehnder modulator (MZM), or Electro-absorption modulator (EAM) may be employed. In this modulation

scheme, the drive current of the laser is held constant producing a continuous wave (CW) optical output which passes through an external modulator. The external modulator is controlled by drive voltages proportional to the data being transmitted. Thus, varying the modulator drive voltages allows the modulator to vary the intensity of the optical carrier field allowing for transmission of the corresponding binary data. The modulated signal is then typically optically amplified using an erbium-doped fiber amplifier (EDFA) before it is launched into the channel.

In both described modulation techniques, the transmitted signal is received using direct detection which employs a single photodiode that produces a photocurrent. This resulting photocurrent is input to a trans-impedance amplifier (TIA), which changes the time-varying current to a time-varying voltage (proportional to the transmitted binary data). This voltage is then low pass filtered (LPF) to reduce out-of-band noise. In the absence of linear fiber impairments, such as attenuation, chromatic dispersion, and polarization mode dispersion as well as non-linear fiber impairments, the recovered voltage will yield an ideal representation of the transmitted data [7]. An example of a non-coherent transmitter and receiver can be seen in Figure 1.2 for the case where an external modulator is employed [8].

Figure 1.2: (a) Non-Coherent Transmitter (b) Non-Coherent Receiver

The major benefit of non-coherent optical communications is simplicity, as the alternative of coherent communications requires a much more extravagant transmitter and receiver. However, along with this inherent simplicity comes performance limitations. One such limitation is that the speed of the accompanying electronics (modulators, DACs) are directly proportional to the attainable transmission rates. Thus, increasing the net data rate of a non-coherent system typically requires an increase in the operating speed of the electronics being employed (although alternative multi-level modulation, e.g., Pulse Amplitude Modulation (PAM) could also be used). Additionally, signals are received using direct detection which implies that all phase information is lost, a characteristic which limits the spectral efficiency. However, recent research has demonstrated the use of Kramers-Kronig detectors, a form of direct detection which can reconstruct the phase of a

signal from its amplitude [9]. Kramers-Kronig detectors look to be bridging the gap between coherent and non-coherent technologies. Nevertheless, non-coherent optical communication dominated the telecommunications industry until the relatively recent resurgence of coherent optical communications [10].

Coherent optical communications differs due to its ability to preserve the amplitude and phase of a transmitted signal, at the cost of significant added complexity. This preservation allows for more spectrally efficient modulation formats to be employed, such as quadrature phase shift keying (QPSK), and M-ary quadrature amplitude modulation (QAM). Higher order modulation formats allow the net data rate of a system to be increased without necessarily increasing the speeds of the electronics or symbol rate [11]. This is done through IQ modulation which manipulates the amplitude and phase of a carrier signal to symbols which map to constellation points in the complex plane, each representative of unique bit sequences. Figure 1.3 (a-d) illustrates the constellation diagrams of BPSK, QPSK, 16-QAM, and 32-QAM, respectively.

Figure 1.3: Constellation Diagrams (a) BPSK, (b) QPSK, (c) 16-QAM, (d) 32-QAM

In each case, 2^N constellation points allow each transmitted symbol to represent a total of N bits. Thus as N increases, the number of *bits per symbol* increases. Therefore, net data rates can be increased without necessarily increasing the modulation or symbol rate, but by increasing the complexity of the modulation format. The relationship between bits per symbol and symbol rate are defined in Table 1.1 for various modulation formats achieving the same net bit rate.

Table 1.1: Properties of Complex Modulation Formats

Modulation Format	Bits Per Symbol	Symbol Rate
BPSK	1	1 x bit rate
QPSK	2	1/2 x bit rate
8-PSK	3	1/3 x bit rate
16-QAM	4	1/4 x bit rate
32-QAM	5	1/5 x bit rate

Table 1.1 demonstrates that for a particular bit rate, increasing the complexity of the modulation format allows for a decrease in the required symbol rate. However, as N is increased, the system performance must operate with higher confidence levels, as the margin for error consequently decreases, as demonstrated by the constellation diagrams of Figure 1.3.

Complex modulation is achieved using IQ modulators which split a continuous-wave optical carrier into orthogonal in-phase and quadrature components. The in-phase and quadrature components are separately modulated using individual MZMs and subsequently

recombined. Thus, IQ modulators can be realized with two MZMs [12]. Employing IQ modulation exploits signal orthogonality allowing data to be embedded in the phase of an optical carrier, enabling the complex modulation formats illustrated in Figure 1.3. Therefore, a prerequisite of coherent communications is the ability to preserve both the amplitude and phase information of a received optical signal. This makes coherent optical communication inherently more complex than its non-coherent counterpart.

The polarization diverse coherent receiver involves polarization beam splitters (PBS), a local oscillator (LO), 90 degree optical hybrids, balanced photodiodes (BPD), analog-to-digital converters (ADCs), and an application specific integrated circuit (ASIC). All of these components together form a coherent receiver enabling recovery of the amplitude and phase of the transmitted signal. Recovery of the amplitude and phase of the transmitted signal allows the transmitted symbols, and thus the transmitted binary data to be recovered [5].

In coherent (and less commonly non-coherent) optical communications, current telecommunication standards employ a technique known as polarization division multiplexing (PDM). PDM allows orthogonal optical signals of the same frequency to be simultaneously transmitted through an optical fiber, effectively doubling the information carrying capacity of a single carrier frequency [13]. This is accomplished by having optical carriers in both the 'X' and 'Y' polarizations of the fiber. Although the optical fiber will not maintain polarization of each carrier, original polarizations can be recovered through various digital signal processing techniques. An example of the dual polarization (DP) coherent transmitter and receiver can be seen in Figure. 1.4.

Figure 1.4: (a) Coherent Transmitter (b) Coherent Receiver

For simplicity, a single polarization coherent transmitter is shown. However, to generate a dual polarization carrier, the optical carrier frequency from the laser is split and delayed imposing a phase shift of $\frac{\pi}{2}$, and the IQ modulator, shown in Figure 1.4 (a) is duplicated. This allows for the generation of two orthogonal optical carriers modulated by individual

data sequences. The coherent receiver visualized in Figure 1.4 (b) is equipped for dual-polarization signal recovery, commonly referred to as a polarization diverse coherent receiver. This is accomplished by splitting the received optical signal, and the local oscillator signal (necessary for heterodyne detection) into orthogonal components using PBSs. This allows signals in both the ‘X’ and ‘Y’ polarizations to be recovered independently.

Although coherent optical systems are inherently more complex than their non-coherent counterparts, these systems have been well-refined and are widely deployed in the telecommunications industry today, enabling terabit-per-second data rates. The coherent receiver in particular plays an important role in the work completed throughout this thesis, and as such will be discussed in much further detail throughout subsequent sections.

1.3 – Wavelength Division Multiplexing

Although the technology surrounding the transmission of data on a single optical frequency in both the coherent and non-coherent domain is continually being updated, one of the most ground-breaking innovations in the history of optical communications occurred with the development of wavelength division multiplexing (WDM), a technology which increased the information carrying capacity of a single optical fiber by more than 1000% [14].

Prior to WDM systems, a single modulated optical carrier was transmitted along a single optical fiber. Therefore, increasing the capacity of a communication link pre-WDM involved laying more fiber. This solution proved incredibly costly, and led to an excess of dark fiber being laid. This approach was inefficient and led to the innovation of wavelength

division multiplexing. WDM was first demonstrated in the early 1980's when, at the time, the telecommunication industry was desperate for a drastic increase in bandwidth [15].

WDM technology involves increasing the throughput of an optical fiber by allowing a single fiber to transmit multiple carrier frequencies simultaneously through its core, as opposed to the traditional single carrier per fiber approach. This is realized through the implementation of optical multiplexers, or array-waveguide gratings (AWG). AWGs perform the function of combining multiple optical carrier frequencies from multiple fibers onto a single optical fiber which can then be transmitted throughout an optical network [16]. Additionally, the reverse functionality of de-multiplexing is achieved with a second AWG at the receiver, which takes the input of a single optical fiber and separates individual carrier frequencies onto separate fibers, each of which can be separately detected and digitally processed. A realization of a typical WDM configuration is shown below in Figure 1.5, where T_x is an optical transmitter and R_x is an optical receiver.

Figure 1.5: Typical WDM System

The implementation of a WDM system only required complexity to be added at the transmitter and receiver of a system, while previously deployed single mode fiber could still be utilized. This made WDM a very practical and cost-effective solution for increasing network bandwidth. WDM technology allowed the net throughput of a single optical fiber to scale proportionally to the number of optical carriers multiplexed onto it, drastically outperforming the previous single carrier per fiber solution.

WDM technology is typically categorized between coarse WDM (CWDM), and dense WDM (DWDM) [17]. CWDM is defined as a WDM system employing less than eight active wavelengths per fiber which implies that operating wavelengths are relatively far apart. CWDM is designed for short-reach applications. Contrastingly, DWDM employs much more than eight active wavelengths per fiber implying that operating wavelengths are much more *densely* packed. DWDM has become the modern approach for long-haul optical communication.

In a DWDM system, the operational wavelengths, or carrier frequencies, reside in the optical C-Band which spans 1530-1565 nm (195.9 – 191.6 THz). This region is dictated by the gain spectrum of the EDFAs necessary to compensate for the fiber attenuation which occurs in long-haul optical communication systems. The C-Band is typically divided into channels, each with a given bandwidth, standardized by the International Telecommunication Union (ITU) [18]. Thus, the number of available channels throughout the C-Band is governed by the desired channel spacing. Modern DWDM optical networks employ a 50 GHz (~0.04 nm) channel spacing, yielding a total of 100 channels. Thus, a DWDM system requires 100 modulated optical carriers to be generated and subsequently multiplexed onto a single optical fiber simultaneously.

1.4 – Lasers for Fiber Optic Communications

The implementation of a DWDM system relies on the ability to generate the necessary optical carrier frequencies. It is these optical carrier frequencies which are modulated to enable the transmission of data throughout an optical network. Optical carrier frequencies are typically generated using laser diodes (Light Amplification by Stimulated Emission of Radiation). Lasers are semiconductor based devices which are made by forming a double heterostructure [19]. A double heterostructure is composed of an intrinsic semiconductor layer inserted between a p-doped and n-doped semiconductor layer. Application of a current across the P-I-N device allows for the recombination of electron-hole pairs inside the intrinsic region, resulting in the stimulated emission of photons. The photons generated begin to resonate within the intrinsic region, which acts as an optical resonant cavity. Light is then emitted from the ends of the laser which can be coupled to a fiber for use in fiber optic communication. Throughout the light generation process modal discrimination is employed to ensure the optical output signal is resonating at a single frequency. Lasers which are cable of varying their output frequency are referred to as *tunable* laser sources (TLS). This tunability is necessary to generate the required optical carrier frequencies for all channels in a DWDM system.

There are many different types of modern tunable laser sources capable of generating optical carrier frequencies, including distributed feedback lasers (DFB), external cavity lasers (ECL), and vertical cavity surface-emitting lasers (VCSEL) [20]. Each of these photonic devices employ various methods of modal discrimination to ensure the optical output at resonates at a single discrete frequency.

Prior to coherent systems, the tunable laser sources used for optical communications exhibited optical linewidths on the order of a few megahertz which implied a significant amount of phase noise was present in the optical output (the concept of phase noise and optical laser linewidth is an integral part of the work completed in this thesis and will be discussed in much further detail throughout subsequent sections). Since non-coherent optical communication employs direct-detection, no data is encoded in the phase of the optical carrier, and as such the larger laser linewidth and larger phase noise is not detrimental to system performance. However, with coherent technology, larger linewidths (and consequently larger phase noise) becomes particularly problematic as the spectrally efficient complex modulation formats require data to be directly encoded in the phase of the optical carrier frequency. Thus, the large phase noise exhibited by megahertz linewidth lasers is not suitable for coherent optical communications. This led to a refinement in laser technology which now allows the TLSs employed in coherent optical communication systems to exhibit optical laser linewidths on the order of 100 kHz [21].

Since all modern optical communication networks employ DWDM, it is necessary to have many discrete tunable laser sources capable of generating a carrier frequency for each channel across the C-Band. Therefore, to implement DWDM system with a 50 GHz channel spacing, there exists a need for up to 100 discrete TLSs capable of generating carrier frequencies for all 100 channels. These high performance tunable laser sources are a necessity for the success of coherent optical communication. However, recent research has begun to explore the development of alternative laser devices capable of generating numerous optical carrier frequencies, commonly referred to as *optical frequency combs* (OFC) [22].

1.5 – Optical Frequency Combs

The output of an OFC is composed of equally spaced spectral modes referred to as *comb lines*. Comb lines are all separated by a specific frequency, referred to as the free spectral range (FSR) of the laser. Thus, if the FSR of the OFC matches the channel spacing in a DWDM system, each comb line can be used as an optical carrier frequency. Therefore, employing OFCs at the optical transmitter would significantly lessen the number of lasers required for a DWDM system.

Practically, the output of an OFC can be optical filtered using an AWG (similar to the AWG used for de-multiplexing at the receiver of a WDM system) to isolate each comb line. This would allow each comb line to be individually fed to an IQ modulator. Following modulation, the multiplexing of carriers would remain the same, using an AWG, with the resulting signal fed through the network. This optimization in the front-half of a DWDM system is visualized in Figure 1.6.

Figure 1.6: DWDM Coherent Optical Transmitter (a) Discrete Laser Array (b) Optical Frequency Comb

Figure 1.6 shows the front half of a WDM system with N channels configured with a discrete laser array in (a) and an optical frequency comb in (b). Each channel requires an

optical carrier to be generated at the corresponding channel frequency. Traditionally, this is accomplished with N different lasers, each generating a single optical carrier for each channel, as shown in Figure 1.6 (a). However, an optical frequency comb generates numerous comb lines, each of which can be selectively filtered and used as the necessary optical carrier frequency for a channel of a DWDM system, as shown in Figure 1.6 (b). This minimizes the number of laser modules in a DWDM system which would lead to a drastic reduction in system cost and complexity [23].

A second benefit of implementing OFCs as a laser source in a DWDM system (as opposed to the alternative discrete laser approach) surrounds the notion of *spectral coherence*. This coherence occurs due to the nature of the light generation process which employs a specific mechanism (such as passive/active mode-locking, or four-wave mixing) to ensure that all modes are phase locked. This allows all comb lines across the spectrum of an OFC to be *coherent* [24]. The particular mechanism to accomplish this coherence varies based on the technique used to generate the OFC. Nonetheless, all OFCs will be coherent which implies that the phase noises exhibited by the comb lines in the OFC are related, allowing for various phase noise compensation techniques to be implemented. By mitigating the phase noise present in the OFC through effective compensation, system complexity can be reduced by means of simplified digital signal processing at the receiver.

A third potential benefit of the OFC is a reduction in complexity at the receiver of a DWDM system. Figure 1.4 (b) showed that a polarization diverse receiver requires a local oscillator for down-conversion of the transmitted signal in each channel, the details of which will be investigated much more thoroughly in subsequent sections. However, similar to the optical transmitter, the receiver of the DWDM system requires N local oscillators, for each of the

N optical carrier frequencies. At the receiver, this is accomplished with a second array of discrete TLSs. Alternatively, an OFC could be employed to generate all N required local oscillator signals required for coherent detection.

These potential innovations have made optical frequency combs the focus of a significant amount of developmental research as they possess the ability to revolutionize coherent optical communication in terms of cost, efficiency and complexity. Recent literature has shown numerous methods of generating optical frequency combs, such as mode-locked lasers, Kerr micro-resonators, dispersive parametric mixing in highly non-linear fibers, and cascaded phase or Mach-Zehnder modulators, with each presenting their own set of advantages and disadvantages [23] [24] [25] [26] [27] [28]. However, throughout all methods of OFC generation there exist common issues, such as spectral flatness among comb lines, relative intensity noise, phase noise (and thus optical laser linewidth) and spectral tunability. Nevertheless, there have been many impressive demonstrations of terabit-per-second communication employing various optical frequency combs [23] [24] [25] [26] [27] [28].

The Lightwave System Research Laboratory (LSRL) has access to a prototype 25 GHz Coherent Comb Source (CCS) System, which falls into the category of a quantum-dot mode-locked laser. The research presented throughout this thesis provides an extensive characterization of the CCS system output in both the time and frequency domains. Preliminary characteristics of the CCS system, such as the optical and electrical spectra, relative intensity noise, phase noise, and optical laser linewidth are investigated. The research is furthered by observing the coherence present within the CCS system through the implementation of a time-domain phase noise recovery technique which allows for a

quantification of the correlation properties between pairs of comb lines [29]. Lastly, the intrinsic characteristics of the CCS system phase noise are further investigated through a novel phase noise trajectory analysis which examines the individual contributions of amplified spontaneous emission (ASE) noise and timing jitter to the overall phase noise. The characterization provides a significant amount of experimental insight regarding the benefits and limitations of the CCS system as a source for coherent optical communications. The observations made throughout this research highlight some fundamental issues with the CCS system which provides direction regarding areas of further development and refinement as the prototype moves towards practical implementation.

Chapter 2 – The Coherent Comb Source

2.1 - Introduction to the Coherent Comb Source System

The Lightwave Systems Research Laboratory at Queen's University has been provided with a prototype Coherent Comb Source system produced by the National Research Council of Canada, in collaboration with Ciena. Currently, only a handful of these devices have been produced for research purposes. The CCS system is a quantum-dot mode-locked laser (MLL). Multiple models have been produced with various free spectral ranges dictated by the length of laser cavity. The CCS system available at the LSRL has a FSR of 25 GHz.

2.2 – System Basics

The prototype CCS system is packaged without direct access to the laser chip. Due to this packaging, the CCS system is controlled through various methods that involve interfacing with this device (as opposed to providing direct current and temperature control to a laser chip). The front panel of the CCS system is shown in Figure 2.1 [30].

Figure 2.1: Front Panel of the CCS System

The CCS system is powered by a 120V source. Once turned on, the device takes 30 minutes to warm-up before the output is stable, as per the supplier's recommendation. There are two major parameters that contribute to the optical output of the device: the laser diode injection current, and the temperature setting of the thermo-electric cooler (TEC) upon which the laser diode is mounted. The laser diode injection current of the CCS system can be varied to directly impact the output power. Additionally, TEC temperature can be varied to adjust the spectral location of the optical output. A thermistor is used to actively measure the TEC temperature settings by means of a temperature dependent resistance. Real-time temperature feedback is acquired through measurements of the resulting thermistor resistance. This forms a closed control loop which allows for the temperature of the laser submount to be monitored and controlled. For simplicity, the user need only adjust for the desired thermistor temperature or resistance and the internal control circuit will make the necessary changes to the TEC driver, and thus the submount temperature. Control of these parameters can be accomplished through three methods of interfacing with the CCS device: standalone operation, a HyperTerminal command line, or a graphical user interface (GUI). Stand-alone device operation involves interacting with the two push buttons and LCD screen on the front of the device, shown in Figure 2.1. This control method is simple but cannot exploit the full functionality of the device, as the use of push buttons limits the possible control commands. The other two control methods are host PC-based and involve connecting the CCS system to a PC through a USB/RS232 connection, available at the rear of the system. The first PC-based mode of operation is through a HyperTerminal command line, wherein real-time device feedback regarding laser diode injection current, optical output power, and thermistor temperature and resistance are provided through constantly

updating lines of text, shown in Figure 2.2 [30]. These parameters can also be adjusted using a dedicated instruction set provided with CCS system, all of which are entered through the command line interface.

```

Use_USB_adaptor - HyperTerminal
File Edit View Call Transfer Help

Coherent Comb Source System - NRC
Laser diode, Tap, and Wavelength locker: #12
Analog hardware: #2; uC hardware: #0
uC firmware: V1.2 - Apr. 11, 2014

In-FLASH parameters:
f1=2.4934 V, Measured_VREFp
f2=13.31 k-Ohm, Default_R
f3=272.43 mA, Default_I
f4= 3.80 mW, Default_P
f5=300.00 mA, Max_Iset
f6= 4.00 mW, Max_POWset
f7=0.9867, I_Calib_Factor
f8=0.9300, POW_Calib_Factor
f9=0.0020, dS_for_Iadj_norm_SS
f10=0.0020, dP_for_Iadj_norm_SS
f11=0.0020, dS_for_Radj_norm_SS
f12=0.0020, dP_for_Radj_norm_SS
f13=2.10 k-Ohm, R_for_I1
f14=0.10075 mA, Ibias
f15=62.64 /nm, LOCKING SLOPE
St/Ac: 0.00/ 0.01mW ; Rf/Er: 0.1/ -0.1µA ; St/Ac:13.31/ 5.55k0 (~18.58/39.52°C) ; 20.00mA
Warm up time left: 30:00
St/Ac: 0.00/ 0.01mW ; Rf/Er: 0.1/ -0.1µA ; St/Ac:13.31/ 8.65k0 (~18.58/28.74°C) ; 40.00mA
Warm up time left: 29:59
St/Ac: 0.00/ 0.00mW ; Rf/Er: 0.0/ -0.1µA ; St/Ac:13.31/13.37k0 (~18.58/18.59°C) ; 60.00mA
Warm up time left: 29:58
St/Ac: 0.00/ 0.30mW ; Rf/Er: 5.0/ 5.1µA ; St/Ac:13.31/13.30k0 (~18.58/18.61°C) ; 80.00mA
Warm up time left: 29:57
St/Ac: 0.00/ 0.73mW ; Rf/Er: 12.8/ -5.6µA ; St/Ac:13.31/13.31k0 (~18.58/18.58°C) ; 100.00mA
Warm up time left: 29:56

Connected 0:04:56 VT1003 9600 8-N-1 SCROLL CAPS NUM Capture Print echo

```

Figure 2.2: HyperTerminal Command Line

The most effective PC-based mode of operation is through the custom GUI designed specifically to monitor and control the CCS system. When using the GUI, feedback regarding system parameters is provided in real-time through a command window and plotted in a separate window, which is a valuable tool for monitoring system behaviour. Commands can then be entered through a command window corresponding to the same instruction set mentioned previously. Figure 2.3 (a) shows the command window used for CCS command inputs, and Figure 2.3 (b) shows the updating plot of the corresponding device parameters [30].

(a)

(b)

Figure 2.3: (a) GUI Command Window (b) GUI Updating Plot

The custom GUI was the control method used throughout the duration of this research as it provided the most functionality and easily allowed tracking of device behaviour. Since there is no direct interaction with the laser diode chip, there is a significant amount of control circuitry present within the CCS system. Figure 2.4 provides an overview of the control circuitry when connected to a host PC. Once warmed-up, the output of the CCS system can be controlled in two distinct operational modes, automatic or manual [30].

Figure 2.4: Internal Electronics of the CCS System

Automatic mode employs a wavelength locking algorithm, which automatically locks and updates the laser diode injection current and TEC setting (which is sensed by the thermistor). Therefore, automatic mode does not allow the user to adjust the laser current or submount temperature. Instead, control of these parameters is performed using a dedicated command “wl+/-”, where “wl+” corresponds to increasing the location of the

spectrum by a single FSR while “wl-” does the opposite. In this way, the locking algorithm automatically adjusts the laser diode injection current and submount temperature to predefined settings which enables spectral tunability. Thus, automatic mode allows for control of the location of the CCS spectrum without direct access to the laser current or TEC settings. It should also be noted that the locking algorithm used during automatic operation is able to hold the optical output power of the CCS system constant by optimizing device settings. Unfortunately, over the course of this research the locking algorithm in the device became unstable, which meant the CCS system was controlled using the alternative manual operational mode.

Manual mode gives the user complete autonomy in controlling the laser current, as well as the submount temperature (through commands which are input to adjust the value of the thermistor temperature/resistance). Although more care must be taken while experimentally manipulating parameters to achieve the desired spectral settings, similar spectral tunability to automatic mode is attainable. A limitation of manual mode is that the locking algorithm is turned off making the CCS system unable to maintain a constant power while varying other device parameters.

The majority of the results obtained in this research correspond to the CCS system operating at the settings shown in Table 2.1, which will be referred to as normal operation. Using the same CCS device settings improves the consistency and repeatability of measurements.

Table 2.1: Normal Operation Settings of the CCS System

<i>Optical Output Power</i>	11.2 mW
<i>Thermistor Resistance</i>	9 k Ω
<i>Thermistor Temperature</i>	27 °C
<i>Laser Diode Current</i>	350 mA

The maximum allowable injection current of the CCS system is 480 mA. In the absence of the locking algorithm, changing the laser diode current directly impacts the optical output power of the CCS system.

A major safety concern when turning off the CCS system is that the user ensures to wind-down the injection current to the laser diode before removing power from the system. If this step is not taken prior to turning off the system the device may be damaged beyond repair. Therefore, this extra step must be taken each time the CCS system is turned off. This is achieved through the command “ld off”, which winds-down the laser diode injection current to 0 mA and notifies the user when it is safe to turn off the device. As an additional precautionary step an uninterruptable power source (UPS) was also employed as a back-up power supply to prevent against unexpected power outages.

2.3 – Optical Spectrum

A characterization of the CCS system’s optical spectrum is critical to fully understanding the output of the device. The optical output of the CCS system was observed extensively

using an optical spectrum analyzer (OSA), namely the Agilent 83453B High Resolution Spectrometer (HRS) [31]. Optical spectrum analysis relies on the ability to resolve an optical input signal into its spectral components and detect the optical power of each component. An optical input is incident on a medium (typically a diffraction grating) which separates the light into all contributing frequency components. A photodiode is then mechanically swept across this region of diffracted light and the optical power of each spectral segment (corresponding to a particular frequency resolution) is converted into a proportional electrical current. The OSA then typically performs internal signal processing which allows users to observe optical power as a function of frequency.

One of the most fundamental operational limitations of optical spectrum analysis is the notion of resolution bandwidth (RBW). Resolution bandwidth refers to the spectral resolution incident on the swept photodiode at a given point in time. Thus, a large resolution bandwidth yields higher detected powers, but the signal can be observed less precisely, corresponding to lower spectral resolution. Conversely, a small resolution bandwidth yields lower detected powers, however the signal can be observed more precisely, corresponding to higher spectral resolution. The HRS improves upon fundamental optical spectrum analysis through the use of a heterodyne receiver employing a local oscillator and calibrated O/E conversion which has the ability increase the spectral resolution obtained in measurements.

The HRS has three different resolution settings for observing an optical input signal: High Resolution, Normal Resolution and Wide Resolution. High Resolution mode, as the name implies, yields the highest resolution possible from the HRS, boasting the ability to resolve spectral components as little as 1 MHz apart. However, the trade-off with the High

Resolution measurement mode is a lengthy measurement time. Normal Resolution mode is recommended for most measurements when using the HRS, as it provides a good balance between spectral resolution, (a maximum of 15 MHz), and a manageable measurement time. However, due to the nature of the CCS optical spectrum, the HRS is restricted to use in Wide Resolution mode because of how much bandwidth the CCS spectrum covers. In Wide Resolution mode the quickest measurement time possible from the HRS is employed, trading-off a decrease in spectral-resolution (a maximum of 20 MHz). However, it should be noted that even the Wide Resolution mode of the HRS outperforms traditional OSAs. The optical spectrum of the CCS system at the settings described in Table 2.1 can be seen below in Figure 2.5, observed using the HRS in Wide Resolution mode.

Figure 2.5: Optical Spectrum of the CCS System

The optical spectrum of the CCS system under normal operation exhibits approximately 70 comb lines spanning 193.5 – 195.3 THz. The free spectral range (FSR) of the CCS system is 25 GHz, or approximately 0.2 nm. When observing the entirety of the optical spectrum, the majority of comb lines fall within a 4 dB range of spectral flatness, however if a subset of comb lines is considered (approximately 10) a spectral flatness of nearly 1 dB can be observed. The extinction ratio of the optical spectrum is approximately 40 dB. As mentioned in section 2.2, the properties of the optical spectrum can be changed by adjusting the submount temperature, as well as the laser diode injection current. This tunability allows the optical spectrum to cover the majority of the C-Band. Figure 2.6 shows the CCS optical spectrum operating at two different settings demonstrating the attainable spectral coverage. These operational settings are described as minimum and maximum settings corresponding to the minimum and maximum CCS system spectral locations. The values of the corresponding CCS system parameters are shown in Table 2.2.

Table 2.2: Operational Settings of the CCS System

	Normal	Minimum	Maximum
<i>Optical Output Power</i>	11.2 mW	18 mW	8.5 mW
<i>Thermistor Resistance</i>	9 kΩ	23 kΩ	3.2 kΩ
<i>Thermistor Temperature</i>	27 °C	7.2°C	52.7°C
<i>Laser Diode Current</i>	350 mA	350 mA	350 mA

Figure 2.6 demonstrates the spectral tunability of the CCS system. By adjusting the operational settings of the CCS system it has the ability to cover a range from 192.5 – 196 THz, spanning nearly the entire C-Band. However, it is important to note that at any given time the CCS system can only cover approximately 1.6 THz. Spectral tunability is an important characteristic for industry level applications wherein the optical frequency comb source would need to be capable of fulfilling DWDM requirements across the entirety of the C-Band.

Figure 2.6: Optical Spectrum of the CCS System Obtained at Various Operational Settings

2.4 – Relative Intensity Noise

The amplitude of each comb line within the output of the CCS system's spectrum exhibits a quantifiable fluctuation known as relative intensity noise (RIN), which is defined as the optical power fluctuations in the output of a laser [32]. RIN is a common phenomenon affecting photonic devices and is often used as an indication of device quality. RIN, defined in Equation 2.1, is measured in units of Hz^{-1} :

$$RIN = \frac{\langle \Delta P(t)^2 \rangle}{P_{opt}^2} \quad (2.1)$$

Where $\langle \Delta P(t)^2 \rangle$ is the time-averaged optical intensity fluctuations at a specified frequency in a 1 Hz bandwidth, and P_{opt} is the average optical power. RIN is predominantly caused by stimulated emission. Measuring RIN is an intricate process as there are numerous contributions that must be carefully considered to obtain accurate measurements. The HP 71400C Lightwave Signal Analyzer (LSA) is an instrument with a built-in personality capable of measuring the RIN of a device under test (DUT) [33]. The optical signal power P_{opt} is detected using a calibrated opto-electronic converter, which translates the optical power fluctuations into electrical power fluctuations that are then measured directly by the LSA. From this signal, the LSA measures a total noise, N_{Total} , which is the sum of three contributions:

$$N_{Total} = N_{Intensity} + N_{Shot} + N_{Thermal} \quad (2.2)$$

The terms contributing to the total noise are intensity noise, $N_{intensity}$, shot noise, N_{shot} , and thermal noise, $N_{Thermal}$. N_{shot} occurs due to the uncertain arrival time of photons at the photodetector and can be measured using the average photocurrent incident on the LSA. $N_{Thermal}$ is produced by the amplifier in the detector as well as the accompanying electronics after photodetection and can be measured under a dark condition. $N_{Intensity}$ is the desired RIN term corresponding the laser under test and can be attained by subtracting the shot and thermal noise contributions from the total measured noise. The LSA can attain these measurements by performing two sweeps of the incident optical signal. The first sweep leaves the optical signal unblocked, which allows for measured values corresponding to N_{Total} and N_{shot} . The second sweep blocks the optical input measuring the noise in the LSA, corresponding to $N_{Thermal}$. This allows for the isolation of $N_{Intensity}$. The RIN of the comb lines across the spectrum of the CCS system was measured using the LSA in combination with a Finisar Waveshaper (FWS). The FWS is a tunable optical filter with a spectral resolution of 1 GHz allowing for the isolation of individual comb lines across the CCS spectrum [34]. The experimental setup for the RIN measurement is shown in Figure 2.7.

Figure 2.7: Experimental Setup for Measuring RIN

The output signal from the CCS system is incident on the FWS which acts as a wavelength selective filter capable of isolating a single comb line. An individual comb line is then

incident on the LSA which internally performs the RIN measurement. Figure 2.8 shows the results of the RIN measurements superimposed on the corresponding comb lines of the CCS optical spectrum for reference.

Figure 2.8: RIN Measurements of Comb Lines across the CCS spectrum

The RIN values correspond to an average value taken over a RIN spectrum of 1 GHz. The results in Figure 2.8 demonstrate that the RIN is relatively stable across the spectrum with an average value of -117 dB/Hz, and a standard deviation of 0.7 dB/Hz. The consistency of these results implies that the amplitude variation of each individual comb line across the CCS spectrum is very similar as there is no discernable trend in RIN variation. For comparison, the RIN of a low-linewidth tunable laser source (TLS), namely the Agilent 81604B, was also measured [35]. Experimental results demonstrated that the Agilent TLS had a RIN of -147.74 dB/Hz in a 1 GHz measurement bandwidth. This result illustrates that the performance of the CCS system is not quite on par with commercially available

TLSs that would typically be used in a DWDM system [36]. Therefore, these RIN results demonstrate that the amplitude stability of individual comb lines is a characteristic that may require further development in this quantum-dot optical frequency comb source laser.

2.5 – Electrical Spectrum

The electrical spectrum of the CCS system was also observed and characterized using a Microwave Spectrum Analyzer (MSA). The experimental setup involves the full CCS output being incident on a fast photodiode which performs an optical to electrical (O/E) conversion. The photodiode is then directly connected to the MSA, as shown in Figure 2.9. The photodiode in the experimental setup had a bandwidth of 40 GHz, and required a 3V DC bias for proper operation. The MSA used in these measurements was an Agilent N9030A PXA Signal Analyzer [37].

Figure 2.9: Experimental Setup for Characterizing the Electrical Spectrum

Microwave spectrum analysis is typically done through a heterodyne mixing process which measures beat signals created by a local oscillator swept in frequency. The intermediate frequencies created from this process are then amplified and envelope detected. This allows the MSA to visualize electrical power of an input signal as a function of frequency. The RF spectrum of the CCS system as measured by the MSA is shown in Figure 2.10.

Figure 2.10: RF Spectrum of the CCS System

As expected, the electrical spectrum of CCS system contains spectral content surrounding 25.0187 GHz. The photodiode is a square-law device which detects the ~25 GHz beat signal from the CCS system output. The 3 dB radio frequency (RF) linewidth of the spectrum is 10 kHz. The spectral broadening demonstrated in Figure 2.10 arises due to a phenomenon known as phase noise.

Phase noise is a naturally occurring stochastic process that can be found in many areas of communications. In optical communications, phase noise occurs during the light generation process in a laser diode. Phase noise is defined as the short-term phase fluctuations that occur in an output signal [38] [39] [40] [41]. In the frequency domain, this phase noise process manifests itself as the broadening in the side bands of a signal from

the ideal discrete Dirac function. This effect can be seen below in Figure 2.11, which shows the spectrum of an ideal CW signal at a frequency f_0 in (a), and the spectrum of the same signal in the presence of phase noise in (b). In Figure 2.11 (a), the power spectral density (PSD) of a CW signal is an ideal Dirac function with all spectral content focused at f_0 . However, in Figure 2.11 (b) the same CW signal with phase noise shows that there is spectral content surrounding the ideal Dirac function which is visualized through the spectral broadening of the signal.

Figure 2.11:(a) Spectrum of CW Signal without Phase Noise (b) Spectrum of CW Signal with Phase Noise

In the time domain, this phase noise appears as a shift in the periodicity of the signal, resulting in a time domain phase noise representation referred to as timing jitter. Equation 2.3 provides a mathematical representation of an ideal noise-free CW signal, $v_{ideal}(t)$, where A is the peak amplitude, f_0 , is the carrier frequency, and t is time, corresponding to the spectrum of Figure 2.11 (a). However, a realistic signal also contains a stochastic phase noise process, $\varphi(t)$, which is present in $v_{real}(t)$ of Equation 2.4 corresponding to the spectrum of Figure 2.11 (b).

$$v_{ideal}(t) = Ae^{j(2\pi f_0 t)} \quad (2.3)$$

$$v_{real}(t) = Ae^{j(2\pi f_0 t + \varphi(t))} \quad (2.4)$$

Figure 2.12 visualizes the effect of the phase fluctuations, $\varphi(t)$, in the time domain, corresponding to $v_{ideal}(t)$ and $v_{real}(t)$. When observing Figure 2.12 (a), $v_{ideal}(t)$ is shown to be perfectly periodic, corresponding to the spectrum of Figure 2.11 (a). However, in Figure 2.12 (b), $v_{real}(t)$ is shown to be dynamically changing as function of time due to the phase fluctuations present in the signal, corresponding to the spectrum of Figure 2.11 (b). This is illustrated through the leading and lagging version of the original $v_{ideal}(t)$ signal. Thus, the phase fluctuations manifest as time domain fluctuations in the periodic signal, commonly referred to as timing jitter. If these phase fluctuations in a signal become too large it can become extremely problematic in a high-speed optical communication channel.

Figure 2.12: (a) Time Domain CW Signal without Phase Noise (b) Time Domain CW Signal with Phase Noise

The magnitude of the phase noise present in the CCS system was characterized using a built-in measurement personality available on the MSA. This technique is commonly used as it is simple to implement, requiring only a direct connection from the DUT to the MSA. The phase noise measurement is accomplished through an analysis of the single side band (SSB) power spectral density. SSB phase noise is denoted as $L(f)$, defined as the SSB power due to phase fluctuations in a 1 Hz bandwidth at a specified offset frequency, measured in units of dBc/Hz. A generic phase noise spectrum can be seen in Figure 2.13, which provides an overview of each contribution in a typical $L(f)$ curve. These four contributions are Flicker Frequency Modulation (FM) Noise, proportional to $\frac{1}{f^3}$, White FM Noise, proportional to $\frac{1}{f^2}$, Flicker Phase Modulation (PM), proportional to $\frac{1}{f}$, and White Noise [42].

Figure 2.13: Single Side Band Phase Noise Spectrum

Obtaining the phase noise spectrum, $L(f)$, of a signal allows other terms of interest, such as timing jitter, to be recovered. Timing jitter is related to phase noise spectrum through the relationship shown in Equation 2.5, which involves an integration over the entire spectrum, corresponding to the sum of terms A_{12} , A_{23} , A_{34} , and A_{45} in Figure 2.13.

$$Timing\ jitter = \frac{1}{2\pi f} \sqrt{2 \int_0^\infty L(f) df} \quad (2.5)$$

The phase noise measurement personality built into the MSA performs this calculation internally providing an estimation of the timing jitter, which is measured in seconds. The SSB spectrum measured for the CCS system can be seen in Figure 2.14 for a bandwidth of 1 GHz relative to the carrier. From this data, the built in phase noise measurement personality of the MSA calculated the corresponding timing jitter to be 5.25 ps.

Figure 2.14: Phase Noise Spectrum of the CCS System

2.6 – Optical Linewidth

Another quantifiable parameter that is directly affected by the magnitude of the phase noise present in a device is the optical laser linewidth [43]. The optical laser linewidth refers to the width of a laser output's optical spectrum. A single-mode laser produces a CW signal implying that it would have a single optical laser linewidth. However, since the CCS system is a multi-mode laser the optical laser linewidth of each distinct mode or comb line can be quantified individually. This implies that the optical laser linewidth of each comb line across the CCS system may vary. The spectral broadening due to phase noise, illustrated in Figure 2.12 (b), follows a Lorentzian profile. Lower linewidths correspond to less phase noise present in a device and is characteristic of high performance lasers.

Therefore, optical laser linewidth is another parameter that can be used as an indicator of device quality.

A common method of measuring the optical laser linewidth is through the delayed self-heterodyne (DSH) technique [44] [45]. The DSH technique involves a delay line interferometer, an acousto-optic modulator (AOM), a polarization controller (PC), an O/E conversion and a MSA. The FWS was used once again to perform optical filtering allowing the isolation of individual comb lines across the CCS spectrum. The experimental setup for the DSH measurements is shown in Figure 2.15.

Figure 2.15: Experimental Setup for the Delayed Self-Heterodyne Technique

The CCS system has a comb line isolated using a FWS, which is then split into two arms using a 50/50 optical splitter. One arm of the DSH setup proceeds through a polarization controller and acousto-optic modulator before recombination using a second 50/50 combiner. The purpose of the PC is to align the states of polarization of the two signals at the O/E in order to maximize the signal-to-noise ratio (SNR). The purpose of the AOM is to shift the intermediate frequency of the detected spectra to be greater than 0 Hz which allows both side bands of the detected signal to be observed by the MSA. However, it should be noted that a variation of the experiment can be performed without the use of the AOM and is referred to as the delayed self-homodyne method, which leaves the resulting spectra at baseband. The second arm of the DSH setup is delayed through a length of fiber

which decorrelates the signals in the two arms prior to recombination. This decorrelation process is critical to the success of the measurement technique, as it allows for a beat signal to be detected at the receiver. To ensure the signals are properly decorrelated the light must travel further than the coherence length, $L_{coherence}$, of the laser through the length of delay fiber. This relationship, defined in Equation 2.6, is governed by c , the speed of light, n , the refractive index of the medium and $\Delta\nu$, the optical laser linewidth [46].

$$L_{coherence} = \frac{c}{\pi n \Delta\nu} \quad (2.6)$$

For the measurements of the CCS system, it was assumed that the optical laser linewidth would not be less than 100 kHz, which implies a coherence length on the order of ~1 km. The LSRL has 25 km fiber spools readily available which were implemented in the DSH experimental setup. This ensured that the two arms in the measurement were properly decorrelated upon recombination prior to the O/E conversion. Figure 2.16 shows the resulting RF spectrum of the DSH measurement when observing the minimum linewidth comb line. The spectrum is centred at 100 MHz, which is due to the spectral shift produced by the AOM. The full width half maximum (FWHM) of the spectrum is equivalent to twice the optical laser linewidth, corresponding to 3 dB of attenuation [44]. Thus, the optical laser linewidth in Figure 2.16 is 2 MHz, taken as the single-sided spectral width 3 dB from the maximum amplitude. The DSH measurement was performed for all comb lines generated by the CCS system, with the results shown in Figure 2.17.

Figure 2.16: DSH Measurement Spectrum of Minimum Linewidth Comb Line

Figure 2.17: Results of the DSH Measurements on the CCS system

The results of this DSH measurement demonstrate that the optical laser linewidth varies from 2 – 12 MHz across the CCS spectrum. Additionally, the linewidths follow a distinctly quadratic trend. A second order polynomial has been superimposed on the data to emphasize this behaviour. These results indicate that the phase noise present in each comb line is not constant across the output of the CCS system. This observed quadratic trend in optical laser linewidth follows an expected phase noise scaling law present in MLLs, which has made the phase noise in MLLs the subject of a significant amount of recent research.

2.7 – Phase Noise in Mode-Locked Lasers

Phase noise is an intrinsic property governing the overall performance of semiconductor lasers which implies there is a breadth of literature discussing and quantifying its effects on various optical communication components and systems. More specifically, there has been a significant amount of research directly surrounding the phase noise properties of MLLs. As mentioned previously, recent literature has investigated quantum-dot optical frequency comb source devices with results consistently demonstrating a quadratic trend in the optical laser linewidth among comb lines.

In [47] the properties of the phase noise in semiconductor passively MLLs are characterized extensively, and mathematical time and frequency domain representations of the optical output signal are derived. [47] extends the derivation to attain mathematical representations of the phase noise present in each distinct mode of a MLL. The CCS system can be used to experimentally observe many of the relationships presented in [47].

Equation 2.7 describes the complex electric field of a semiconductor passively MLL with N modes, where E_n is the real amplitude, f_n is the frequency, $\theta_n(t)$ is the time-varying

phase noise process, and ϕ_n is the static phase - all of which correspond to the n^{th} longitudinal mode.

$$E(t) = \sum_{n=1}^N E_n e^{j(2\pi f_n t + \theta_n(t) + \phi_n)} \quad (2.7)$$

Under the mode-locked regime all modes of $E(t)$ will be equally separated and phase-locked. This relationship is described in Equation 2.8, where n_c is the minimum linewidth mode with a corresponding phase noise $\theta_c(t)$, and $\Delta\theta(t)$ is the phase noise difference between two adjacent spectral modes, separately defined in Equation 2.9.

$$\theta_n(t) = \theta_c(t) + (n - n_c)\Delta\theta(t) \quad (2.8)$$

$$\Delta\theta(t) = \theta_{n+1}(t) - \theta_n(t) \quad (2.9)$$

Equation 2.8 illustrates that the phase noise present on the n^{th} comb line has two distinct contributions. The first contribution, $\theta_c(t)$, occurs due to ASE noise. The second contribution, $\Delta\theta(t)$, occurs from the fluctuations due to timing jitter which increase as a function of modal separation, $(n - n_c)$. The fluctuations due to timing jitter are expressed as the difference between two adjacent spectral modes of the MLL, and is consistent across the spectrum as shown in Equation 2.9. Both the fluctuations due to ASE noise, $\theta_c(t)$, and the fluctuations due to timing jitter, $\Delta\theta(t)$, are time varying. Equation 2.8 implies that the quadratic trend in optical linewidth, observed in Figure 2.17, occurs from the scaling due

to modal separation, $n - n_c$, which directly impacts the relative contribution of the timing jitter in a MLL. Practically this means that as modes move further away from the distinct minimum linewidth mode (comb line), the magnitude of the phase noise will increase, which is observable through a parameter such as the optical laser linewidth. Moreover, the relative contributions of ASE noise and the timing jitter to the overall phase noise of a particular mode can be observed through an analysis of the corresponding variance of each term in Equation 2.8.

The variances of $\theta_c(t)$ and $\Delta\theta(t)$ both increase linearly with time and therefore represent random walk processes. However, for large observations of time they can be modelled as a Gaussian walk with zero mean and a distinct variance. By observing time domain representations of the phase noise trajectory corresponding to a given mode a significant amount of statistical information can be attained. A statistical analysis stems from the ability to model phase noise as a Wiener process, $W(t)$ [48]. By observing the difference in phase noise values of a particular n^{th} mode at times separated by a delay, τ , a zero-mean Gaussian random process $W_n(t)$ will be attained, shown in Equation 2.10.

$$\theta_n(t) - \theta_n(t - \tau) = W_n(t) \quad (2.10)$$

The variance of $W_n(t)$ can be used to observe different properties of the mode under observation, such as optical laser linewidth, as well as the relative contributions of ASE noise and timing jitter of Equation 2.8.

This can be experimentally accomplished on the CCS system using a phase noise trajectory (PNT) analysis. The PNT analysis is to be performed on recovered time domain phase noise traces corresponding to individual modes (comb lines). Figure 2.18 provides a simulated phase noise trajectory for illustrative purposes (details regarding these simulations are discussed in significant detail throughout Chapter 3).

Figure 2.18: Example of a Time Domain Phase Noise Trace

The time domain phase noise trajectory can be divided into a number of blocks of a given length. The initial and final value of each block can then be subtracted, referred to as phase noise differences, which will yield values of the desired zero-mean Gaussian process, $W_n(t)$, described in Equation 2.10. Performing this PNT analysis on a single phase noise trace will yield a range of phase differences with a variance, σ^2 , shown in Equation 2.11,

such that $\Delta\nu$ is the optical laser linewidth, T is the sampling period, and M is the block size.

$$\sigma^2 = 2\pi\Delta\nu TM \quad (2.11)$$

This PNT analysis can be performed on each comb line across the CCS system's spectrum allowing the variance of each comb line to be quantified to provide statistical insight into its phase noise properties. However, this requires an experimental setup which allows for the recovery of these time domain phase noise traces. Additionally, to properly isolate each contribution to the phase noise found in Equation 2.8 the experimental setup must be capable of recovering phase noise trajectories of two comb lines simultaneously. The PNT analysis is a feasible experimental approach which will allow for a more practical observation of the mathematical representations derived in [47]. Additionally, since the variance recovered is a function of optical laser linewidth it implies that the PNT analysis can also be used as an alternative method for estimating the optical laser linewidth of each comb line of the CCS system.

2.8 – Experimental Insights

The primary purpose of Chapter 2 is to provide a comprehensive overview of the CCS system utilized in this thesis, while motivating the simulated and experimental procedures explored in Chapters 3 and 4. The work presented in this thesis is an extensive characterization of the phase noise present in the 25 GHz CCS system, which is achieved

through the experimental implementation of a coherent detection schematic allowing for the simultaneous recovery of phase noise trajectories of two different comb lines.

The ability to observe the characteristics of the phase noise present across the CCS spectrum relies on the ability to accurately recover these time domain phase noise traces, and as such there has been a significant effort put-forth in optimizing the necessary experimental setup. The experimental setup is a heterodyne based coherent detection scheme employing two TLSs, two 90° optical hybrids, four balanced photodiodes, two polarization controllers and a real-time sampling oscilloscope.

One of the major advantages of the CCS system, and optical frequency combs in general, is the notion of a coherent spectrum. The concept of coherence was briefly introduced in 2.7, and demonstrated through Equation 2.8, which illustrates how the phase noise present on all comb lines exhibit a mathematical dependence. Although phase noise is randomly time-varying, a coherent spectrum maintains that this variation should be similar among all spectral modes. This coherence implies that the phase noise present in the CCS system can be compensated to allow for the comb lines to be implementable as a source for coherent optical communications.

Current DWDM optical communication systems generate each carrier frequency from individual discrete lasers implying that there is no phase relationship between carriers. However, the implementation of an OFC implies that the phase between carriers would be locked. Thus, along with the benefits of reducing the number of lasers at the transmitter of a DWDM coherent optical system, the coherence naturally present in optical frequency combs has the potential of reducing the complexity in signal processing at the receiver and

could possibly lead to many other innovations which exploit the intrinsic coherence among comb lines.

Although coherence has been theoretically described in [47], only a few methods have been implemented to experimentally quantify its presence across the spectrum of an OFC. All previous coherence measurements observe the output of the comb source in the frequency domain. This is accomplished through an analysis of the RF beat tone linewidth across the spectrum of an optical frequency comb [49]. Other experiments have attempted to quantify coherence through an optical and RF linewidth ratio as a function of mode number [50]. In both cases, the results showed that the term used to quantify coherence decreased as the spacing between observed spectral modes increased, indicating that coherence decreases with an increase in modal separation. Although this notion of coherence has been well-explored in the frequency domain, it has not yet been observed in the time domain.

The first major component of this thesis surrounds a novel time domain coherence quantification measurement of the phase noise present among the comb lines of the CCS system. This is accomplished through an analysis of the correlation properties between the phase noise trajectories recovered for pairs of comb lines across the spectrum, using a heterodyne coherent detection schematic. The experimental setup allows phase noise trajectories of two comb lines to be simultaneously recovered and analyzed offline in MATLAB. Since the traces are recovered simultaneously the fluctuations as a function of time should be synchronized. This allows for mathematical tools, such as the Pearson correlation coefficient, to be implemented as a measure of this synchronization. The Pearson correlation coefficient is based on covariance which measures how similarly two data sets vary together. By strategically comparing pairs of comb lines across the spectrum

of the CCS system, the correlation properties between the recovered phase noise traces can be used to quantify coherence. Although the results presented in thesis surround the coherence of the quantum-dot optical frequency comb source laser, the technique can be used to assess other OFCs and photonic devices.

The second major component of this thesis surrounds the characterization of the phase noise present in the CCS system through the novel phase noise trajectory analysis introduced in 2.7. As mentioned previously the PNT analysis can be used to estimate the optical laser linewidth of each comb line. Secondly, using the coherent detection schematic employed in this thesis, the relative contributions of the phase noise present in each comb line due to ASE noise and timing jitter can be experimentally isolated and characterized.

In both cases, simulations in MATLAB were completed to first verify the practicality of the experimental concepts, which are extensively outlined in Chapter 3. By first exploring the above ideas in simulation, idealized results are attained which act as a reference when observing experimental results. Additionally, a simulation allows the accompanying signal processing steps to be developed which ensures correctness prior to experimental implementation. Lastly, any idealities that may be present in the components used in the experimental setup can be modelled to more fully understand their impact on the recovered phase noise trajectories.

Chapter 3 – Simulated Experimental Setup

3.1 – Introduction to Simulation

Prior to implementing the experimental coherent detection scheme an extensive simulation of the intended experimental setup was performed using a model created within MATLAB. The primary purpose of the simulations was to ensure experimental validity as well as aid in the refinement of the digital signal processing techniques necessary to perform the characterizations introduced at the end of Chapter 2. The simulation relies heavily on original work with signals that were generated using a third-party tool, namely *Optilux* [51]. Optilux is a MATLAB implemented open source collection of software that allows for the design, simulation and analysis of optical communication systems. The Optilux toolbox was used to generate phase noise traces, such as the trace presented in Figure 2.18 of section 2.7, which can then be used for analysis. The generation of these phase noise traces is performed using the function `LaserSource`, which is part of the Optilux suite. `LaserSource`'s intended use is to create optical channels in a WDM system, allowing for the specification of channel peak power, wavelength, WDM channel spacing as well as various other parameters. However, to emulate CCS system behaviour, `LaserSource` was used to generate an optical field corresponding to a single wavelength, modelling the behaviour of a single comb line. In the absence of modulation the resulting generated signal contains solely phase noise. By varying the input parameters of `LaserSource` the

characteristics of the resulting phase noise trace can be controlled to more closely model the intrinsic properties of the CCS system.

3.2 – Simulated Phase Noise Traces

To simulate phase noise, LaserSource implements a Brownian bridge noise process, $B(t)$. A Brownian bridge is a continuous-time stochastic process which follows Brownian motion, however the initial and final values of the observed period are forced to zero [52]. The Brownian motion is modelled using a Wiener process (which was briefly discussed in 2.7), which is zero-mean Gaussian distributed probability density function with a given variance. Thus, the Brownian bridge can be defined as follows:

$$B(t) = W(t) \forall t \in [0, T] \text{ and } W(0) = W(T) = 0 \quad (3.1)$$

The results of the DSH measurements in section 2.6 demonstrated that the optical laser linewidths of the comb lines produced by the CCS system are not constant, implying that the magnitude of the phase noise present in each comb line varies. The Optilux function LaserSource has the ability to model this effect through changing the optical laser linewidth being used to generate the simulated phase noise traces, which can be changed as a function parameter. More specifically, this corresponds to an adjustment to the variance of the Wiener process used to create the Brownian Bridge. Figure 3.1 shows the results of three phase noise traces generated using LaserSource for an optical laser linewidth of 100 kHz. The traces are all zero mean, however they have been offset for visual clarity.

Figure 3.1: Simulated Phase Noise Traces for a 100 kHz Linewidth

Each trace in Figure 3.1 is 2 μs in duration. Three different traces are presented to demonstrate the ability of LaserSource to generate numerous random phase noise traces. This is accomplished through the MATLAB random number generator (RNG), which controls pseudo-random number generation. The RNG chooses a set of random numbers based on a seed. The seed of an RNG can be set to attain repeatable results. However, to attain a pseudo-random sequence of numbers, different seeds must be chosen which corresponds to selecting different sets of random numbers. Figure 3.2 shows the results of two randomly generated phase noise traces corresponding to optical laser linewidths of 2 MHz and 12 MHz which are the respective minimum and maximum linewidth comb lines observed in the DSH measurements in section 2.6.

Figure 3.2: Simulated Phase Noise Traces for a 1.6 and 25 MHz Linewidth

Figure 3.2 clearly visualizes the effect optical laser linewidth has on the magnitude of phase noise traces, demonstrating how larger optical linewidths correspond to larger phase fluctuations. The magnitude of these fluctuations can be quantified through the variance of each phase noise trace. However, since each simulated phase noise trace is inherently pseudo-random it is more accurate to consider the average variance of a large number of traces. The average variance of 1000 phase noise traces was calculated for linewidths of 2 MHz and 12 MHz, which led to variances of 1.201 rad and 9.032 rad respectively. These results can be extended by performing the same calculation for linewidths from 100 kHz to 30 MHz. The simulation considers linewidths larger than 12 MHz for completeness. Once again, each optical linewidth had 1000 pseudo-randomly generated phase noise traces of duration 2 μ s from which the variance of each individual trace and the resulting average

variance was calculated. Figure 3.3 presents the results of this measurement with average variance as a function of optical laser linewidth. The linear trajectory depicted in Figure 3.3 quantifies the direct proportionality of phase noise to optical laser linewidth. At larger optical linewidths (greater than 10 MHz) the calculated average variance begins to fluctuate, but still distinctly follows the expected linear trend. These minor variations occur from randomness presented in each trace generated through the simulation process as well as the increased magnitude of the phase fluctuations at larger optical linewidths.

Figure 3.3: Simulated Average Variance versus Optical Laser Linewidth

In any case, the ability to observe the properties of the phase noise physically present within the CCS system relies on a working experimental setup capable of properly recovering the types of phase noise trajectories visualized in Figures 3.1 and 3.2. As mentioned previously, this can be accomplished through the implementation of a heterodyne coherent

receiver. Thus, the capabilities and limitations of this coherent receiver should be modelled and analyzed prior to physical implementation. The following sections outline the basis of coherent detection as well as provide an illustrative walk-through of the model created to simulate coherent detection in MATLAB.

3.3 – Mathematical Foundations of a Coherent Receiver

The experimental setup used in Chapter 4 is an adaption of the coherent receiver currently deployed in modern optical communication systems. Coherent detection allows for the recovery of experimental phase noise traces corresponding to each comb line of the CCS system. Figure 3.4 presents a typical dual-polarization coherent receiver used widely throughout industrial telecommunication networks, which was briefly discussed in Chapter 1 [5].

Figure 3.4: Typical Coherent Receiver

Modern coherent optical systems employ dual-polarization signals which allows modulated optical carrier frequencies to propagate in both the X and Y polarizations of a single optical fiber. This transmission scheme doubles the throughput of each channel in a DWDM system. Dual polarization leads to the necessity of the polarization beam splitters at the input to a coherent receiver to separate orthogonal components of the input signal E_S and local oscillator signal E_{LO} prior to incidence on the 90° optical hybrids. In this research, PBSs are not necessary as signals arrive separately from isolated sources. The 90° optical hybrids perform an elegant mixing functionality which produces orthogonal variations of the down-converted signals which can then be detected using balanced receivers. The balanced receivers generate proportional photocurrents that maintain the integrity of phase information which can then be sampled using ADCs. The recovered digital signals are then subject to various DSP techniques enabling the recovery of the transmitted data. In modern optical networks, this DSP is completed in real-time through the implementation of an application specific integrated circuit (ASIC). However, in this research signals are sampled using a real-time sampling oscilloscope and processed offline in MATLAB.

A simulation of this coherent receiver was performed by modelling a single channel coherent setup, shown in Figure 3.5. It should be noted that the true path combinations within the 90° optical hybrids have been visualized to more clearly illustrate the functionality of the device [5].

Figure 3.5: Single Channel Coherent Detection Scheme

In Figure 3.5, $E_S(t)$, represents the complex electric field of the CW signal under observation, which experimentally corresponds to a single optical comb line of the CCS system:

$$E_S = A_S e^{j(\omega_s t + \varphi_s(t))} \quad (3.2)$$

where A_S is the amplitude of the signal, ω_s is the radian frequency of the signal, and $\varphi_s(t)$ is the signal phase noise process. It should be noted that in practical applications $E_S(t)$ would have an additional phase term corresponding to modulation. The second input signal, E_{LO} , is the local oscillator (LO) used to perform signal down-conversion:

$$E_{LO} = A_{LO} e^{j(\omega_{LO} t + \varphi_{LO}(t))} \quad (3.3)$$

Where A_{LO} is the amplitude of the LO, ω_{LO} is the radian frequency of the LO, and $\varphi_{LO}(t)$ is the LO phase noise process.

In the single channel coherent receiver each signal, E_S and E_{LO} , separately enter the 90° optical hybrid. The optical hybrid initially performs an internal 50/50 split of both signals. One arm of the LO signal is subject to a phase shift of $\frac{\pi}{2}$. The arms are then recombined and then 50/50 split once more, as shown in Figure 3.5, yielding four distinct outputs E_1 , E_2 , E_3 , and E_4 :

$$E_1 = \frac{1}{2}(E_S + E_{LO}) \quad (3.4)$$

$$E_2 = \frac{1}{2}(E_S - E_{LO}) \quad (3.5)$$

$$E_3 = \frac{1}{2}(E_S + jE_{LO}) \quad (3.6)$$

$$E_4 = \frac{1}{2}(E_S - jE_{LO}) \quad (3.7)$$

E_{1-4} are then incident on two balanced receivers. A single balanced receiver is composed two separate photodiodes. Thus, the setup in Figure 3.5 yields four photocurrents, I_{1-4} , corresponding to the individual photodetection of E_{1-4} . One balanced receiver takes the difference between I_1 and I_2 to yield the in-phase component of the signal, $I_{Inphase}$, while the other balanced receiver takes the difference between I_3 and I_4 to yield the quadrature component of the signal, $I_{Quadrature}$:

$$I_{Inphase} = I_1 - I_2 = R\sqrt{P_S P_{LO}} \cos(\omega_{IF}t + \varphi_s(t) - \varphi_{LO}(t)) \quad (3.8)$$

$$I_{Quadrature} = I_3 - I_4 = R\sqrt{P_s P_{LO}} \sin(\omega_{IF}t + \varphi_s(t) - \varphi_{LO}(t)) \quad (3.9)$$

R is the responsivity of the photodiode, P_s is the power of the input signal E_s , P_{LO} is the power of the local oscillator E_{LO} , and ω_{IF} is the intermediate frequency. The in-phase component, $I_{Inphase}$, is represented by a cosine and $I_{Quadrature}$ is represented by a sine which demonstrates the orthogonality achieved from the $\frac{\pi}{2}$ phase shift performed in one arm of the LO within the 90° optical hybrid.

Responsivity is a parameter that quantifies the efficiency of a photodiode in converting optical power to electrical current, measured in units of A/W. The input signal power, P_s , and the local oscillator power P_{LO} are time-averaged and proportional to the amplitudes A_s and A_{LO} :

$$P_s = \frac{|A_s|^2}{2} \quad (3.10)$$

$$P_{LO} = \frac{|A_{LO}|^2}{2} \quad (3.11)$$

It should be noted that in most communication systems the amplitude A_s would be time-varying corresponding to modulated data and transmission effects. However, in the case of the CCS system a comb line is being directly observed in the absence of modulation thereby implying a constant amplitude. The intermediate frequency, ω_{IF} , is a term that describes the frequency mismatch between the input signal frequency and the local oscillator frequency:

$$\omega_{IF} = |\omega_s - \omega_{LO}| = 2\pi f_{IF} \quad (3.12)$$

If the signal frequency and the local oscillator frequency are identically matched then $\omega_{IF} = 0$, and the coherent receiver is operating at baseband under homodyne conditions. Though ideal, homodyne operation is difficult to implement as it requires the use of a phase-locked loop which adds complexity to the optical receiver [5]. Alternatively, if the signal frequency and the local oscillator are not matched then $\omega_{IF} > 0$ and the coherent receiver is operating in the heterodyne condition. In modern coherent optical communications, the coherent receiver operates under heterodyne conditions and the residual intermediate frequency component is removed through DSP implementing various frequency offset estimation (FOE) and removal algorithms.

Since the detection of the in-phase and quadrature photocurrents yields orthogonal cosine and sine representations of the down-converted beat signal, they can both be used to create the following complex expression:

$$I_{Complex} = I_{Inphase} + jI_{Quadrature} = R\sqrt{P_s P_{LO}} e^{j(\omega_{IF}t + \varphi_s(t) - \varphi_{LO}(t))} \quad (3.13)$$

Equation 3.13 can be easily computed offline in MATLAB allowing for access to the phase information, $(\omega_{IF}t + \varphi_s(t) - \varphi_{LO}(t))$, from which the intermediate frequency can be estimated and removed to isolate the desired phase noise terms. In practice, the local oscillator used for coherent detection is a stable low-linewidth laser (on the order of 100

kHz) with minimal phase noise. Minimal phase noise from the LO allows the system to maintain the integrity of the transmitted data. However, results in section 2.6 showed that the minimum linewidth comb line from the CCS system is 2 MHz. This implies that for the intended purposes of this experimental setup the recovered phase noise traces will be dominated by the phase noise of the desired signal, $\varphi_s(t)$, corresponding to an individual comb line. This concept is verified by the simulation results presented in Figure 3.3, which illustrated that the variance of the phase noise trajectory in a 100 kHz linewidth laser was 0.1 rad, while for a 2 MHz linewidth laser was 1.8 rad. Therefore, the phase noise fluctuations of the larger linewidth signal are a factor of 18 larger than the low-linewidth local oscillator. This ensures that the phase noise of the larger linewidth signal will dominate the recovered phase trajectory.

3.4 – Simulating a Coherent Receiver

A simulation of the coherent receiver mathematically described in section 3.3 is performed to verify its capability of recovering the desired phase noise traces. This simulation was used as an idealistic reference for the expected behaviour of the physical experimental setup implemented in Chapter 4.

The simulation begins with the generation of two random phase noise traces to be processed by the modelled coherent receiver. Figure 3.6 presents a black phase noise trace generated from a 2 MHz linewidth laser modelling the phase noise present in a comb line of the CCS system denoted as $\varphi_{CL}(t)$, while the red phase noise trace was generated from a 100 kHz linewidth laser modelling the phase noise present in a typical LO denoted as

$\varphi_{LO}(t)$. These phase noise traces were then each placed on a simulated optical carrier frequency representing the CCS system comb line and a local oscillator.

Figure 3.6: Simulated Phase Noise Traces (2 MHz Comb Line, 100 kHz LO)

Modern coherent optical systems operate in the C-Band which spans 191.6 - 196 THz, which corresponds to the range of possible carrier frequencies. However, simulating signals at these frequencies is not practically feasible. A signal propagating at a frequency of 194 THz maintains a periodicity of 5.155 fs. Thus, simulating a signal at this frequency for a meaningful length of time would require an impractically large number of samples. To compensate for this limitation the low-pass equivalent of an optical signal is used, which removes the notion of a carrier frequency dependency in the signal models.

In this model, carrier frequencies were still included but these frequencies were drastically reduced to become practically implementable in MATLAB. This allows the concept of

FOE to be more fully explored. The generated phase noise traces in Figure 3.6 are 2 μs in duration implying that a carrier frequency with a periodicity of less than 2 μs should be chosen, which corresponds to frequencies greater than 0.5 MHz. For illustrative purposes, the comb line was assigned a carrier frequency, f_{CL} , of 20 MHz, while the local oscillator was assigned a frequency, f_{LO} , of 30 MHz. They are offset by 10 MHz to simulate the imperfect frequency matching present in heterodyne detection. The simulated comb line, $E_{CL}(t)$, and LO signal, $E_{LO}(t)$ are shown in Figure 3.7 in the absence of phase noise. This corresponds to real parts of Equation 3.14 and 3.15 respectively:

$$E_{CL}(t) = e^{j2\pi f_{CL}t} \quad (3.14)$$

$$E_{LO}(t) = e^{j2\pi f_{LO}t} \quad (3.15)$$

Figure 3.7: Real Part of the Simulated $E_{CL}(t)$ and $E_{LO}(t)$ (without Phase Noise)

The phase noise processes of Figure 3.6 can then be included in the comb line and LO signals, as per Equations 3.16 and 3.17 respectively.

$$E_{CL}(t) = e^{j(2\pi f_{CL} + \varphi_{CL}(t))} \quad (3.16)$$

$$E_{LO}(t) = e^{j(2\pi f_{LO} + \varphi_{LO}(t))} \quad (3.17)$$

A visualization of the comb line and LO signals with the phase noise processes $\varphi_{CL}(t)$ (linewidth of 2 MHz) and $\varphi_{LO}(t)$ (linewidth of 100 kHz) added can be seen in Figure 3.8, corresponding to the real parts of Equations 3.16 and 3.17 respectively.

Figure 3.8: Real Part of the Simulated $E_{CL}(t)$ and $E_{LO}(t)$ (with Phase Noise)

The simulated signals in Figure 3.8 demonstrate that the impact of $\varphi_{CL}(t)$ on $E_{CL}(t)$ is much more visually noticeable than $\varphi_{LO}(t)$ on $E_{LO}(t)$. Using these simulated signals, the functionality of the coherent receiver in Figure 3.5 was mathematically implemented using MATLAB. For clarity, Figure 3.9 visualizes the signal processing steps used to model behaviour of the 90° optical hybrid as well as the balanced receivers through the mathematical manipulation of the input signals $E_{CL}(t)$ and $E_{LO}(t)$. It should be noted that non-idealities such as imperfect splits, dual line uniformity and responsivity can easily be observed by manipulating parameters within this model. The effect of these non-idealities is explored in subsequent sections to more fully understand their impact on the recovered phase noise traces.

Figure 3.9: Mathematical Outline of a Coherent Receiver

Applying the signal processing steps outlined in Figure 3.9 results in two output currents from the balanced receivers, $I_{Inphase}(t)$, and $I_{Quadrature}(t)$, which are shown in Figure 3.10. Both signals are detected at an intermediate frequency, f_{IF} , of 10 MHz corresponding to the difference between f_{CL} and f_{LO} . Additionally, the signals are slightly offset in time due to the $\frac{\pi}{2}$ phase shift performed within the 90° optical hybrid, which mathematically

corresponds to orthogonal cosine and sine components of the detected beat signals as discussed previously:

$$I_{Inphase}(t) = R\sqrt{P_s P_{LO}} \cos(2\pi f_{IF}t + \varphi_{CL}(t) - \varphi_{LO}(t)) \quad (3.18)$$

$$I_{Quadrature}(t) = R\sqrt{P_s P_{LO}} \sin(2\pi f_{IF}t + \varphi_{CL}(t) - \varphi_{LO}(t)) \quad (3.19)$$

The responsivity of the photodiode was set to be 1 A/W, the amplitude of the input signals E_{CL} and E_{LO} were normalized to 1, and the optical splits within the 90° optical hybrid were idealized to be exactly 50/50. Thus, the amplitude of $I_{Inphase}(t)$, and $I_{Quadrature}(t)$, are 0.25, such that $R\sqrt{P_s P_{LO}} = 0.25$.

Figure 3.10: Recovered In-Phase and Quadrature Signals from the Balanced Receiver

Once $I_{Inphase}(t)$, and $I_{Quadrature}(t)$ have been recovered they can be combined to form $I_{Complex}(t)$:

$$I_{Complex}(t) = I_{Inphase}(t) + jI_{Quadrature}(t) = R\sqrt{P_s P_{LO}} e^{j(2\pi f_{IF}t + \phi_{CL}(t) - \phi_{LO}(t))} \quad (3.20)$$

This allows access to the phase information of $I_{Complex}(t)$ which can be attained through the MATLAB function *angle*. Mathematically, the function *angle* calculates the arctangent of the ratio of complex component to the real component in a complex expression:

$$angle(I_{Complex}(t)) = arctan\left(\frac{I_{Quadrature}(t)}{I_{Inphase}(t)}\right) \quad (3.21)$$

Performing this operation on the recovered traces in Figure 3.10 yields the following signal, shown in Figure 3.11, corresponding to the wrapped phase content of the signal in Equation 3.20.

Figure 3.11: Wrapped Phase of $I_{Complex}(t)$

Phase wrapping is a signal processing consequence that constrains the instantaneous phase of a signal between the interval of its principal value, $\pm\pi$. However, since signal processes are generally continuous functions of time there exists a need to *unwrap* the signal, which means to perform an operation that removes the discontinuities created by the process of phase wrapping. Unwrapping can be accomplished in MATLAB through use of the function *unwrap*, which corrects phase trajectories by adding multiples of $\pm 2\pi$ where there are discontinuities greater than a set threshold, nominally set to π . The unwrapped phase of $I_{Complex}(t)$ is shown in Figure 3.12.

Figure 3.12: Unwrapped Phase of $I_{\text{complex}}(t)$

Unwrapping the phase of $I_{\text{complex}}(t)$ reveals the presence of the residual frequency offset. Since ω_{IF} is a function of time the unwrapped phase follows a distinct linear trend. By estimating and removing this linear frequency offset the desired phase noise processes can be isolated. In this example, Figure 3.12 corresponds to $2\pi f_{IF}t + \varphi_s(t) + \varphi_{LO}(t)$, implying a residual frequency of f_{IF} . By adding $-2\pi f_{IF}t$ to the signal in Figure 3.12 the linear frequency offset can easily be removed. In practice, estimating and removing this frequency offset is not as straightforward as described. Frequency offset estimation in modern coherent optical communication systems is an intricate and critical portion of signal processing at the receiver. This had led to the development of many sophisticated algorithms that have optimized this estimation and removal process, which are explored in Chapter 4. Figure 3.13 shows the remaining phase trajectory, $\varphi_{CL}(t) + \varphi_{LO}(t)$, after

removing the frequency offset. The original phase noise processes $\varphi_{CL}(t)$ and $\varphi_{LO}(t)$ are also shown for comparison.

Figure 3.13: Phase of $I_{\text{complex}}(t)$ (Unwrapped and Frequency Offset Compensated)

The recovered phase is the difference of the phase noise processes from the original comb line, $\varphi_{CL}(t)$ and the local oscillator $\varphi_{LO}(t)$ as it is impossible to separate these contributions at the receiver. This emphasizes the importance of using a low-linewidth laser source as the LO to minimize any additional phase noise. Since the magnitude of the phase fluctuations from the low-linewidth LO are much smaller than the larger linewidth comb line, there is a nearly negligible effect on the recovered phase noise trajectory, as it is dominated by $\varphi_{CL}(t)$. The similarity of the recovered phase noise trajectory to the original $\varphi_{CL}(t)$ can be quantified using the mean squared error (MSE). Even though the model is idealistic, the MSE will not be zero due to the recovered phase noise trace being the

difference of $\varphi_{CL}(t)$ and $\varphi_{LO}(t)$. The MSE was calculated to be 0.0725 rad^2 , which will be used as the standard for ideal coherent receiver performance.

3.5 – Simulating Experimental Non-Idealities

The model developed in 3.4 observed an idealistic coherent receiver, however an experimental implementation of this coherent setup will introduce several predictable non-idealities. These non-idealities occur due to the physical components in the experimental setup which can be modelled to observe their impact on the recovered phase noise trajectories. The major non-idealities to consider are the dual-line uniformity, splitting ratios, the $\pi/2$ phase shift of 90° optical hybrids and the responsivity of the balanced receivers.

3.5.1 – Dual Line Uniformity

The dual-line uniformity is a performance metric of the 90° optical hybrid which corresponds to how similar the insertion losses of paired channels are, measured in dB [53]. In a single 90° optical hybrid there are four different values of dual line uniformity corresponding to the in-phase and quadrature components of the input signal and the LO. In practice, the insertion losses of all arms within the 90° optical hybrids are usually quite similar (indicative of a high degree of dual-line uniformity) which implies that the relative optical power of $E_{CL}(t)$ and $E_{LO}(t)$ will be similar in each of the four optical hybrid outputs. However, for demonstrative purposes many values of dual line uniformity were tested within the model to assess the impact on the recovered phase noise trajectories. This is modelled by adjusting the relative amplitudes of $E_{CL}(t)$ and $E_{LO}(t)$ within $E_{1-4}(t)$ of

Equations 3.4-3.7 corresponding to various values of dual-line uniformity. Equations 3.22-3.29 highlight the relationship between each of the four dual line uniformity values in a 90° hybrid with respect to the four outputs detected by the BPDs:

$$E_1 = \frac{1}{2}(E_{CL\ I1} + E_{LO\ I1}) \quad (3.22)$$

$$E_2 = \frac{1}{2}(E_{CL\ I2} - E_{LO\ I2}) \quad (3.23)$$

$$E_3 = \frac{1}{2}(E_{CL\ Q3} + jE_{LO\ Q3}) \quad (3.24)$$

$$E_4 = \frac{1}{2}(E_{CL\ Q4} - jE_{LO\ Q4}) \quad (3.25)$$

Where:

$$DLU_{CL\ I} = 20 \log_{10}(E_{CL\ I1}) - 20 \log_{10}(E_{CL\ I2}) \quad (3.26)$$

$$DLU_{LO\ I} = 20 \log_{10}(E_{LO\ I1}) - 20 \log_{10}(E_{LO\ I2}) \quad (3.27)$$

$$DLU_{CL\ Q} = 20 \log_{10}(E_{CL\ Q3}) - 20 \log_{10}(E_{CL\ Q4}) \quad (3.28)$$

$$DLU_{LO\ Q} = 20 \log_{10}(E_{LO\ Q3}) - 20 \log_{10}(E_{LO\ Q4}) \quad (3.29)$$

The impact of the dual line uniformity was quantified by calculating the MSE between the recovered phase noise traces and the original phase noise traces after adjusting the input powers of $E_{CL}(t)$ and $E_{LO}(t)$ at the output of the 90° optical hybrid. Though in practice each of the four dual line uniformities would be different, a single dual line uniformity was used for simplicity (corresponding to Equations 3.26 – 3.29), and was swept from 0 – 10 dB. Figure 3.14 shows the results of the dual-line uniformity versus the resulting MSE.

Figure 3.14: Dual Line Uniformity versus MSE (Simulation)

Results demonstrate that the MSE follows a quadratically increasing trend as the dual-line uniformity is increased. For values of dual line uniformity less than approximately 2 dB, the effect on the recovered phase noise trajectories is negligible.

3.5.2 – Splitting Ratios

A second non-ideality that may occur within the 90° optical hybrid is imperfect splitting and recombination of the input signals. The 90° optical hybrid performs a total of four 50/50 optical splits. This first two occur directly at the input of the device, splitting $E_{CL}(t)$ and $E_{LO}(t)$, while the second two splits yield $E_{1-4}(t)$ of Equations 3.4-3.7. In the simulation of section 3.4 perfect splitting ratios of 50/50 were used which can be easily

implemented in MATLAB. Although most practical optical splitting components are refined to be close to ideal, the consequence of any imperfections was explored in simulation to fully understand impact of improper splitting.

To model this effect, the splitting ratio was varied for all four splits simultaneously from 0.3 to 0.7 (which corresponds to 30/70 and 70/30 splits respectively, depending on the arm of the split being observed). This was implemented by changing the splitting factor governing the components in Equations 3.4-3.7 accordingly. Once again, the recovered phase noise trace was compared with the original phase noise trace using the MSE. Results of this characterization are shown in Figure 3.15 which yield a quadratic trend as the splitting ratio is varied away from the ideal at 0.5, which corresponds to a 50/50 split. These results demonstrate that if experimental splitting ratios found within the coherent receiver are approximately 50/50 the recovered phase noise traces will remain nearly ideal. An important caveat is that these results correspond to varying the splitting ratios *identically* when in practice this will not be the case. However, results of this simulation provide a generalized demonstration of how optical splits near 50/50 ensure minimum impairments on the recovered phase noise traces.

Figure 3.15: Splitting Ratio versus MSE (Simulation)

3.5.3 – Phase Shift

Another potential non-ideality in the 90° optical hybrid is the accuracy of the phase shift occurring in the one arm of the LO signal. This phase shift is critical in recovering orthogonal in-phase and quadrature components of the desired beat signal. In section 3.4 the split was modelled to be precisely $\frac{\pi}{2}$, however in practice this will not be the case. The simulation was used to assess the effect of an improper phase shift on the recovered phase noise trajectory. Once again, this was done by varying the phase shift in the model and then comparing the recovered phase noise trace with the original phase noise trace using the MSE. The results are shown in Figure 3.16, which demonstrate the MSE is minimized as the phase shift approaches the ideal $\frac{\pi}{2}$, as expected. These results demonstrate that if the

phase shift is at least 85% of $\frac{\pi}{2}$, the effect of the non-ideality will not have a significant impact on the recovered phase noise traces.

Figure 3.16: Phase Shift versus MSE (Simulation)

3.5.4 – Responsivity

The final non-ideality of the coherent receiver considered is the responsivity of the photodiode. Responsivity, as mentioned previously, is a parameter that quantifies the efficiency of a photodiode's optical to electrical conversion process [54]. In the model of 3.4, the responsivity of the photodiode was chosen to be 1 A/W, however in practical applications this value often falls closer to 0.5 - 0.7 A/W. The responsivity itself will not directly impair the recovered phase noise traces since the coherent receiver is being employed strictly as a phase detector. However, a mismatch in responsivity between the

balanced receivers in the experimental setup could have adverse effects on the recovered phase noise traces. For demonstrative purposes, it is assumed that the responsivity of the two photodiodes within a single balanced receiver are matched while the mismatch in responsivity between the two balanced receivers of the coherent detection setup is assessed. The responsivity mismatch is varied from 0 A/W to 0.5 A/W. Recovered phase noise traces were compared with the original phase noise trace using the MSE with the results shown in Figure 3.17.

Figure 3.17: Responsivity Mismatch versus MSE (Simulation)

Figure 3.17 illustrates that as the mismatch in responsivity between balanced receivers increases, the effect on the recovered phase noise traces increases quadratically. However, these results also demonstrate that if the responsivities of the balanced receivers are within ~ 0.1 A/W of each other there will be minimal effects on the recovered phase noise trace.

3.6 – Correlation Coefficient

The coherent receiver model in section 3.3 - 3.5 demonstrated that it is capable of correctly recovering time domain phase noise traces. Additionally, it also demonstrated that by choosing proper system components, the recovered phase noise traces will not be adversely affected by component non-idealities. Being able to correctly recover time domain phase noise traces from each comb line of the CCS system allows for a significant amount of novel device characterization.

One of the major parameters of interest is a quantification of the coherence within the CCS system. As mentioned in section 2.8, the CCS system's ability to generate a coherent spectrum is very advantageous from an implementation perspective. Using the CCS system to generate the required carrier frequencies for DWDM transmission can lead to drastic reductions in system complexity as the phase of all comb lines fluctuate together rather than independently. The similarity of these fluctuations can be quantified by comparing time-aligned phase noise evolutions of two different comb lines. Experimental implementation implies that the coherent setup explored in sections 3.3 - 3.5 must be doubled to allow for the simultaneous recovery of phase noise traces corresponding to two comb different lines (similar to the modern dual polarization coherent receiver of Figure 3.4 except that two LO signals are required).

After two phase noise traces corresponding to two different comb lines, $\varphi_{CL1}(t)$ and $\varphi_{CL2}(t)$, are simultaneously recovered their coherence can be quantified mathematically using the Pearson correlation coefficient [55]:

$$\rho(\varphi_{CL1}(t), \varphi_{CL2}(t)) = \frac{1}{N-1} \sum_{i=1}^N \left(\frac{\varphi_{CL1}(i) - \mu_{CL1}}{\sigma_{CL1}} \right) \left(\frac{\varphi_{CL2}(i) - \mu_{CL2}}{\sigma_{CL2}} \right) \quad (3.30)$$

where μ_{CL1} , μ_{CL2} , σ_{CL1} , and σ_{CL2} correspond to the mean and variance of comb lines 1 and 2 respectively, and N corresponds to the number of samples in traces $\varphi_{CL1}(t)$ and $\varphi_{CL2}(t)$. The Pearson correlation coefficient is a mathematical tool used to quantify linear correlation between two sets of data. The coefficient has boundaries of ± 1 , where $+1$ corresponds to full positive correlation, -1 corresponds to full negative correlation and 0 corresponds to no correlation. Therefore, by calculating the correlation coefficient between time-aligned phase noise traces their linear correlation can be quantified. This correlation coefficient provides mathematical insight into the coherence of the device by quantifying how similarly two data sets vary over time. Experimentally, these data sets correspond to the phase noise trajectories recovered from two different comb lines. To ensure the validity of this correlation coefficient calculation an additional preliminary step must be taken to ensure the recovered phase noise trajectories are in fact time-aligned. This is accomplished by performing a cross correlation to ensure the peak of the cross correlation function is located at a lag of 0 .

The functionality of the Pearson correlation coefficient can be explored by calculating its value resulting between randomly generated phase noise traces. Figure 3.18 presents three randomly generated phase noise traces. The black trace, $\varphi_1(t)$, and red trace, $\varphi_2(t)$ were simulated with a laser linewidth of 1 MHz and the blue trace, $\varphi_3(t)$, corresponds to a laser linewidth of 10 MHz.

Figure 3.18: Simulated Phase Noise Traces

The correlation coefficient was calculated between all three phase noise traces, $\varphi_1(t)$, $\varphi_2(t)$, and $\varphi_3(t)$, with the results summarized in Table 3.1.

Table 3.1: Correlation Coefficients between Simulated Phase Noise Traces

$\rho(X, Y)$	Correlation Coefficient
$\varphi_1(t), \varphi_2(t)$	0.300
$\varphi_1(t), \varphi_3(t)$	-0.479
$\varphi_2(t), \varphi_3(t)$	-0.525

Since the phase noise traces are generated randomly there is a no distinctive correlation between them as illustrated by the relatively low correlation coefficients. $\rho(\varphi_1(t), \varphi_2(t))$ was found to be 0.3 which implies that although the phase noise traces are independently and randomly generated there are some similarities in their evolution in time. Conversely, $\rho(\varphi_1(t), \varphi_3(t))$, and $\rho(\varphi_1(t), \varphi_2(t))$ are both negative which demonstrates that the phase noise traces were evolving oppositely in time.

An important caveat of the Pearson correlation coefficient is that it will never exceed the boundaries of ± 1 . This means that it will be blind to phase noise traces of different magnitudes as it strictly observes the normalized covariance between data sets. This is illustrated through the results in Table 3.1, as the calculated correlation coefficients provide no indication that the phase noise fluctuations of $\varphi_3(t)$ are 10 times larger than that of $\varphi_1(t)$ and $\varphi_2(t)$. This implies that an additional metric will be needed to quantify this effect when observing experimental results.

In the case of the CCS system it is hypothesized that the simultaneously recovered phase noise traces from two comb lines will yield higher correlation coefficients due to the phase noise properties of mode-locked lasers discussed previously in section 2.7. This effect can be idealistically modelled by setting the seed of the RNG used to generate repeatable phase noise traces but varying the simulated optical laser linewidths. This is observed in Figure 3.19, which shows two identical generated phase noise trajectories scaling proportional to linewidths of 1 MHz and 10 MHz.

Figure 3.19: Identical Phase Noise Traces Scaled (1 MHz and 10 MHz Linewidth)

The phase noise traces shown in Figure 3.19 are identical in terms of their respective time domain evolution, however the magnitude of the fluctuations is proportional to the different laser linewidths. This is emphasized by a correlation coefficient of 1 between the two traces. Similar behaviour is expected when practically observing the phase noise trajectories recovered for comb lines within the CCS system, subject to the variation in linewidths.

3.7 – PNT Analysis – Optical Linewidth (Simulation)

The PNT Analysis was briefly introduced in section 2.7 as a method for decomposing recovered phase noise trajectories to observe the relative contributions of ASE noise and fluctuations due to timing jitter occurring in a MLL. This is accomplished by segmenting a recovered phase noise trace into a set of blocks, each with M samples. The difference

between the initial and final values of each block is calculated which yields a set of recovered phase noise differences. Mathematically these recovered phase noise differences will be zero-mean Gaussian distributed (as per the Wiener process phase noise model observed in section 2.7) with a variance that will provide insight regarding the properties of the comb line under observation. One parameter that can be extracted from the variance is the optical laser linewidth, which is explored using the simulated phase noise traces to observe the accuracy of the technique. Figure 3.20 provides an example of a simulated phase noise trace generated from a laser with a linewidth of 10 MHz which will be subject to the PNT analysis and linewidth estimation.

Figure 3.20: Simulated Phase Noise Trace for PNT Analysis

This phase noise trace in Figure 3.20 has 2×10^6 samples. The $2 \mu\text{s}$ trace corresponds to a sampling rate of $\sim 1 \text{ TSa/s}$ (which is not practically implementable outside of an offline simulation environment). The phase noise trace was divided into blocks of 250 samples and the PNT analysis was executed. The resulting phase difference values from the initial and final values of each block can be seen in the Gaussian distribution of Figure 3.21 from which the variance can be extracted. A normalized Gaussian curve is superimposed on the distribution as a reference.

Figure 3.21: Gaussian Distribution of Phase Differences observed in the PNT Analysis

The variance of the distribution in Figure 3.21 is 0.0303 rad^2 . Using this variance, the optical laser linewidth can be inferred from the relationship presented in Equation 2.11 of section 2.7. This yields an estimated optical laser linewidth of 9.87 MHz, which is within 1.28 % of the true 10 MHz value.

An important parameter governing the accuracy of the PNT analysis is the choice of block size. In the above example, a predetermined optimal block size of 250 samples was employed. This optimal value was determined through an extension of the analysis shown above which observed the results of the PNT analysis for various block sizes. A simulation was performed wherein the block size used to segment the phase noise trace was varied from 50 to 10000 samples. Additionally, the PNT analysis was performed on 1000 phase noise traces instead of just a single trace to increase the accuracy of the results. Since phase noise is an inherently random process, a single trace may not yield an accurate representation of the statistical expectation. Therefore, the PNT analysis described above yielded 1000 variances which produced 1000 optical laser linewidth estimates per block size. The mean of all optical laser linewidth estimations was then calculated for each unique block size, with the results shown in Figure 3.22.

Figure 3.22: PNT Linewidth Estimation vs. Block Size

The mean optical laser linewidth estimation for a block size of 250 samples is 9.98 MHz which is within 0.19% of the true 10 MHz value. The results in Figure 3.22 demonstrate that block sizes between 200 and 500 samples yield nearly perfect optical laser linewidth estimations. Therefore, a block size of 250 samples was chosen to be the standard used throughout the PNT analysis.

3.8 – Simulation Conclusions

Chapter 3 provided a detailed simulation verifying the techniques which will be implemented experimentally in Chapter 4. The simulation observed the use of a coherent receiver as a phase detector and demonstrated its ability to recover the time-varying phase noise traces of an input signal. A detailed overview of the MATLAB model was provided

and the necessary signal processing steps for proper phase recovery were outlined. The results of the simulation demonstrated that near ideal phase recovery is possible using the proposed heterodyne coherent detection scheme. Additionally, many non-idealities that may be present within the experimental setup were modelled to quantify their potential impact on system performance. The analysis verified that commercially available components will not have a significant impact on the quality of the recovered phase noise traces subject to the non-idealities studied.

A preliminary overview of the coherence quantification using the Pearson correlation coefficient was also discussed. This technique will be applied to the experimentally recovered phase noise trajectories in Chapter 4 and the resulting correlation properties will provide insight regarding the level of coherence found within the CCS system. The results presented in Chapter 3 demonstrated how the Pearson correlation coefficient is capable of mathematically quantifying the similarity of two phase noise traces concurrently evolving in time. However, an important limitation highlighted by this preliminary exploration of the technique was the inability of the correlation coefficient to discriminate between two phase noise trajectories of different magnitudes.

Lastly an introduction to the PNT analysis was provided which outlined the details of the signal processing steps required. This simulation validated the theoretical foundation of the PNT analysis as demonstrated by the Gaussian distribution of phase noise differences expected from the derivations of section 2.7. The results also demonstrated the technique's effectiveness as a method for estimating optical laser linewidth. The PNT analysis will be used further in Chapter 4 to isolate and quantify the relative contributions of the phase noise in each mode of the CCS system.

Chapter 4 – Experimental

Implementation and Results

4.1 – Experimental Overview

The experimental characterization of the phase noise present within the CCS system relies on the ability to correctly recover time domain phase noise traces from individual comb lines. Chapter 3 introduced the fundamentals of a coherent detection scheme and demonstrated its ability to be implemented as a phase detector. This was accomplished through simulations performed using a MATLAB generated model. Chapter 3 provided the foundation for the experimental implementation of the coherent detection scheme as well as general methodology required to characterize the phase noise properties of the CCS system.

Chapter 4 presents the details of the experimental coherent detection scheme employed. The characteristics of each individual component within the experimental setup are discussed. This is followed by a full walkthrough of the phase noise recovery process, demonstrated using two comb lines selected as an illustrative example. This walk-through establishes the capabilities of the coherent detection scheme while allowing all experimental steps and signal processing procedures to be properly established. After the phase noise recovery process is outlined, a systematic experiment is introduced which allows obtaining the correlation properties between pairs of comb lines across the CCS spectrum. A PNT analysis is also performed on the recovered phase noise traces. The

results of the PNT analysis provide an estimate of optical laser linewidth for comb lines across the spectrum of the CCS system which are then compared with the DSH linewidth results of section 2.6. Additionally, the PNT analysis allows the relative contributions of the measured phase noise present within each comb line exhibited by the CCS system to be quantified based on the mathematical outline provided in [47], the results of which provide insight regarding the correlation properties observed.

4.2 – Experimental Setup

The physical implementation of the heterodyne coherent detection scheme is shown below in Figure 4.1. The setup has two channels, as the proposed CCS system characterization requires the simultaneous recovery of phase noise traces for two different comb lines.

Figure 4.1: Experimental Setup for Coherent Phase Noise Recovery

In Figure 4.1, BPF represents a bandpass filter, EDFA is an erbium doped fiber amplifier, FWS is the Finisar Waveshaper, TLS are the tunable laser sources, BPD are the balanced photodiodes and RTSO is the real-time sampling oscilloscope.

During operation, the CCS system has a portion of its spectrum optically pre-filtered using the BPF to minimize out-of-band noise as well as obey the power limitations of the cascading components. The bandpass filtered spectrum is then incident on an EDFA which is used to boost the amplitude of all comb lines. The amplified CCS spectrum is then incident on a FWS which is used to selectively filter two individual comb lines which also removes any added noise from the EDFA. Thus, the FWS has two distinct outputs, each of which is propagating a single discrete comb line. Each comb line then passes through a polarization controller prior to incidence on separate 90° optical hybrids. The polarization controllers are used to maximize the SNR of the detected beat signals by aligning the state of polarization (SOP) of the comb line to that of the TLS, which is the second input to each 90° optical hybrid. Each TLS output is closely matched in frequency to the corresponding comb line. Each 90° optical hybrid then performs an internal mixing process between the comb line and TLS output, which was outlined in detail throughout Chapter 3. The outputs of the 90° optical hybrids are then detected by four BPDs, yielding a total of four output photocurrents corresponding to the in-phase and quadrature components of the two comb lines. The photocurrents are then sampled using a RTSO, yielding digital representations of the in-phase and quadrature signals of detected heterodyne beat signals from each comb line. The detected traces are then saved and processed offline in MATLAB which allows for the recovery of the desired phase noise trajectories.

Each component present in the experimental setup of Figure 4.1 plays a pivotal role in the ability to properly recover the desired phase noise trajectories. Therefore, a much more

detailed description of each component is provided to more fully illustrate its respective role in the data collection process.

4.2.1 – Bandpass Filter

The bandpass filter employed in the experimental setup is the manually controlled Yenista XTM-50 tunable bandpass optical filter [56]. The Yenista BPF can be tuned over a wavelength range of 1450 - 1650 nm, spanning the entirety of the optical C-Band. Additionally, the bandwidth of the Yenista BPF can be adjusted from 6.25 - 625 GHz. The flatness of the passband is within 0.2 dB, and the roll-off of the filter is 500 dB/nm. Figure 4.2 provides a visualization of the Yenista BPF spectral profile while operating at the minimum and maximum bandwidth settings. This was accomplished by using the Yenista BPF to filter broadband noise which was then detected by the HRS. The spectral profiles are centred at 193.445 THz, corresponding to the centre of the optical C-Band.

Figure 4.2: Normalized Yenista BPF Spectral Profile at Two Bandwidth Settings

The two spectrums shown in Figure 4.2 have been normalized. In the experimental setup, the Yenista BPF is used to optically filter a portion of the CCS spectrum. The resulting optical spectrum obtained after the output signal from the CCS system passes through the Yenista BPF operating at the maximum bandwidth setting is shown in Figure 4.3, as detected by the HRS.

Figure 4.3: Yenista BPF Filtered CCS Spectrum (Maximum Bandwidth)

Figure 4.3 demonstrates that the Yenista BPF is capable of filtering approximately 30 comb lines. Pre-filtering of the CCS spectrum is critical prior to EDFA amplification to allow the selected comb lines to reach detectable power levels while ensuring the relative power levels of the CCS system optical signal obey experimental power limitations. The output power of the CCS spectrum prior to incidence on the BPF is on the order of 11.2 mW. However, this power is spread over nearly 70 discrete comb lines. Thus, the power of an individual comb line is significantly lower. Additionally, each component cascaded throughout the experimental setup yields insertion loss which further reduces the optical power of individual comb lines. Thus, optical amplification of the CCS spectrum is necessary to allow individual comb lines to reach detectable power levels. However, the necessity of this amplification introduces the issue of power restrictions in other components, such as the FWS. If the CCS system output is not optically pre-filtered and

the entirety of the CCS spectrum is amplified prior to incidence on the FWS, the optical power of the input signal to the FWS would exceed power limitations. Thus, a necessary step prior to signal amplification is optical pre-filtering.

4.2.2 – Erbium-Doped Fiber Amplifier

The EDFA employed in the experimental setup is the Pritel Low Noise High Power Optical Fiber Amplifier [57]. This EDFA has three operational modes, namely Pre-Amp, Power-Amp and Tandem. In all operational modes, the pump wavelength is 976 nm which is typical of EDFAs. The gain region of the amplifier is 1530 - 1565 nm corresponding to the optical C-Band. The Pre-Amp pump current ranges from 0 - 300 mA, while the Power-Amp pump current ranges from 0 - 550 mA. However, to maximize the possible gain from the EDFA, tandem operation should be utilized. Tandem operation refers to passing the input signal through both the Pre-Amp and Power Amp stages of the EDFA. In tandem mode, the Pre-Amp pump current should be set to the maximum value of 300 mA which then allows the desired signal gain to be refined by solely varying the pump current of the Power-Amp. Figure 4.4 shows the output power of an amplified optical signal as a function of Power-Amp pump current while operating in Tandem. The input signal was a CW signal generated from a TLS. The Pre-Amp pump current was maximized at 300 mA for all measurements, while the Power-Amp pump current was varied. The input and output power of the CW signal before and after amplification was measured using an optical power meter.

Figure 4.4: EDFA Output Power vs. Pump Current (Tandem Operation)

Figure 4.4 observes three different scenarios, a -10 dBm signal, a -3 dBm signal and a 0 dBm signal. In all cases the response is similar with the increase in output power exponentially decaying as the Power-Amp pump current approaches its maximal value. This behaviour is typical of EDFAs as the gain is known to saturate for large pump currents. In Figure 4.4, the maximum gain attained was 32 dB, however the results demonstrate that the gain of an EDFA is signal dependent. Thus, a lower power test signal will incur a larger gain, however all input signals will saturate around a similar maximum attainable output power. Overall, the performance demonstrated by the EDFA is typical and more than suitable for the required amplification of the pre-filtered CCS spectrum.

4.2.3 – The Finisar Waveshaper

The Finisar WaveShaper 4000s is a programmable optical filter which was briefly introduced in sections 2.4 and 2.6 as it was used for both the RIN and DSH measurements, respectively. However, the RIN and DSH measurements required only the basic functionality of the FWS, as it was only used to filter a single optical comb line. This implementation did not require any customization or automation. Thus, the principle of operation and functionality of the FWS 4000s is more formally introduced and discussed in this section as it plays a critical role in phase noise recovery process.

The FWS works on a principal similar to optical spectrum analysis, which was introduced in section 2.3. An optical input signal is incident on a cylindrical mirror which is then reflected between a diffraction grating and a Liquid Crystal on Silicon (LCoS) Optical Processor which together accomplish optical filtering. A visualization of the FWS operating principle is shown in Figure 4.5 [34]. The LCoS Optical Processor provides the FWS with the ability to control attenuation and phase as a function of frequency. The operational range of the FWS 4000s is 191.25 – 196.275 THz, spanning the entirety of the C-Band. The pass band bandwidth can be varied from 10 GHz – 5 THz, specified within a frequency resolution of 1 GHz. The FWS 4000s boasts a total of four output ports which allows the device to select pre-specified portions of the optical spectrum to be separated and propagated individually. This functionality is necessary for the success of the experimental setup in Figure 4.1, as the FWS must select two different comb lines and transmit them individually on separate outputs.

Figure 4.5: FWS Theory of Operation

There are two main ways to control the FWS. In both cases, the FWS must be connected to a host PC which has the necessary Finisar WaveManager software installed. The first FWS control method is through the WaveManager control suite GUI, shown in Figure 4.6.

Figure 4.6: Finisar WaveManager Application Suite

Using this GUI, the filter profile of the FWS can be easily adjusted by specifying bandwidth, centre frequency, and attenuation. This was the control method used to select individual comb lines across the CCS spectrum for the RIN and DSH measurements in 2.4 and 2.6, respectively.

The second control method involves generating custom filter profiles and uploading them to the FWS through virtual instrument software architecture (VISA) based MATLAB control. This type of control method, though more complex than the GUI alternative, allows for the full functionality of the FWS to be realized.

To create a custom filter profile, attenuation, phase and output port must be defined for all frequencies of the FWS, which can be easily done using MATLAB. Once the filter profile has been created within MATLAB it can be converted into an appropriate file and uploaded

to the FWS using the FWS command set. This led to the generation of a MATLAB script capable of easily selecting multiple comb lines from the CCS output. This is accomplished by generating filter profiles with adjustments made to the relative attenuation across the FWS frequency range corresponding to the desired output comb lines. Figure 4.7 provides an example of the types of filter profiles generated to select comb lines. The filter profiles are visualized by plotting the attenuation profile superimposed on a subset of comb lines. The optical spectrum has been normalized to 0 dBm for clarity. The passband for a filtered comb line was set to 20 GHz, centred at the corresponding peak frequency of the selected comb line. Once again, it should be noted that both filtered comb lines correspond to two unique outputs from different ports which are then separately incident on the two arms of the experimental setup in Figure 4.1.

Figure 4.7: FWS Profile Examples

Figure 4.7 presents a subset of the CCS spectrum to aid in the description of the filtering process. Two filter profiles are shown as examples which demonstrate the capability of the FWS in selecting desired comb lines. *Filter Profile 1* selects adjacent comb lines while *Filter Profile 2* selects two comb lines separated by 15 modes. In all cases, the attenuation for a 20 GHz region containing a desired comb line is set to 0 dB, while attenuation is maximized for the remainder of the spectrum. Thus, only comb lines within the created passbands of the filter profile are allowed to propagate. Although Figure 4.7 only presents two examples, the MATLAB script can be adjusted to generate a filter profile capable of isolating any two comb lines across the spectrum of the CCS system.

4.2.4 – 90° Optical Hybrids

The next major components of the coherent detection schematic are the two 90° optical hybrids. The optical hybrids are passive optical devices with two inputs and four outputs used to create orthogonal representations of heterodyne beat signals attained through an internal mixing process. The details of this mixing process were extensively observed throughout sections 3.3 and 3.4. The optical hybrids employed in the experimental setup of Figure 4.1 are the Kylia COH24X [53]. The operational wavelengths are 1520 - 1625 nm. Relevant device specifications of the two hybrids are summarized in Table 4.1, which include the phase shift and dual line uniformity of the two inputs, corresponding the signal under test (comb line) and the LO (TLS).

Table 4.1: Kylia COH24X Device Specifications

		Dual Line Uniformity (dB)	
	Phase Shift (deg)	<i>In-Phase</i>	<i>Quadrature</i>
Hybrid 1	86.1	0.43	0.53
Hybrid 2	86.7	0.55	0.91

Theoretically, the intended phase shift in an optical hybrid is 90 degrees. However, the practical phase shifts in these devices are both greater than 77 degrees. This is more than the required minimum phase shift determined in the simulation of section 3.5 to deem the impact on the recovered phase noise traces negligible.

The dual line uniformity presented is the maximum mismatch in insertion losses over the entirety of the operational wavelength range between paired channels. The channels correspond to the in-phase and quadrature components of the signal under test and LO. The practical dual line uniformity of the Kyla COH24X is less than 1 dB. This is lower than the 2 dB limit determined in simulation of 3.5 which ensures that the effect of the dual line uniformity will not impact the recovered phase noise traces.

4.2.5 – Balanced Receivers

The experimental setup shown in Figure 4.1 demonstrates that a total of four BPDs are necessary to accomplish coherent phase noise recovery. The BPDs employed are the General Photonics BPD-002 [58]. The functionality of the BPD was discussed extensively throughout sections 3.3 and 3.4. Each BPD has two optical inputs which are incident on two separate photodiodes internal to a single BPD. The BPD then performs an internal subtraction of the two detected photocurrents yielding a single RF output. For proper operation, each BPD must be biased at ± 12 V. The bandwidth of the BPDs are specified to be 200 MHz. To verify this, the bandwidth of each BPD was measured using an Agilent N4373A Lightwave Component Analyzer (LCA) which relies on the Agilent N5230A Vector Network Analyzer (VNA) [59] [60].

Vector Network Analysis is a common measurement technique used to characterize an electrical device's frequency response. An electrical signal generator is used to generate a CW input signal which is swept in frequency and transmitted through a device under test (DUT). The electrical output of the DUT is then connected to the VNA which performs the frequency response measurement based on the relationship between the known input and

relative measured output. However, since the BPD is an optical to electrical device, the input signal must be generated optically. The LCA accomplishes this through the sinusoidal modulation of the optical signal power, which can be detected by an O/E device, and analyzed with the VNA. By varying the frequency of this sinusoidal modulation, the frequency response of an O/E device can be characterized.

A single BPD requires two inputs which implies that the test signal, generated from the LCA, must be split. This is accomplished using a typical 50/50 optical splitter. It should be noted that the delay from the split is assumed to be identical. The experimental setup used to perform the frequency response characterization is shown in Figure 4.8. An optical attenuator (OA) is included to control the power of the generated test signal prior to splitting. It is important to note that the LCA and VNA, though separate devices, are connected and work intimately to perform a calibrated frequency response characterization.

Figure 4.8: BPD Characterization Experimental Setup

The frequency response of the four balanced photodiodes are shown in Figure 4.9. The results demonstrate that the performance of the BPD's is as expected, with a 3 dB bandwidth of 200 MHz.

Figure 4.9: Frequency Responses of the BPDs (Logarithmic)

An important observation made during this measurement is the notion of BPD saturation. The measured frequency responses in Figure 4.9 correspond to a -8 dBm optical test signal incident on both inputs of the BPD. It became apparent that if the input power of each arm exceeds approximately -8 dBm (with a slight variation between BPDs), the BPD becomes saturated. However, for optical input powers below this -8 dBm saturation threshold, the frequency response remains the same in shape (shown in Figure 4.9) but varies proportionally in magnitude relative to the input power. Thus, it is important to ensure that

the optical power of the signals input to the BPD are below the -8 dBm saturation threshold when implementing the experimental setup of Figure 4.1.

4.2.6 – Real Time Sampling Oscilloscope

The device responsible for digitally sampling the photocurrents from the four BPDs is the real-time sampling oscilloscope (RTSO). The RTSO employed in the coherent detection setup of Figure 4.1 is the Tektronix DPO4104 Digital Phosphor Oscilloscope. The Tektronix DPO4104 RTSO has a bandwidth of 1 GHz and a sampling rate of 5 GSa/s [61]. Additionally, this RTSO has a total of four input channels which allows for the measurement of all four photocurrents produced by the four BPDs to be captured by a single device simultaneously.

A real-time sampling oscilloscope, as the name implies, performs the sampling of an analog input signal in real-time which eliminates the need for an external trigger required of alternative equivalent-time sampling oscilloscopes (ETSO). Real-time sampling oscilloscopes sample an analog input signal sequentially to form a digital representation [62]. The architecture of RTSOs is inherently simpler than the ETSOs due to this sequential sampling, however the required speed of the accompanying electronics often limits instrument bandwidth. The coherent detection scheme in Figure 4.1, requires the heterodyne beat signals to be less than 200 MHz (BPD bandwidth) which implies that the bandwidth of 1 GHz provided by the Tektronix DPO4104 is more than adequate for the proposed comb line phase noise recovery setup.

The Tektronix DPO4104 RTSO can be controlled using two different methods. The first control method involves interacting with the push buttons present on the front-panel of the

RTSO. The second control method involves interfacing with the RTSO via a host PC which can be accomplished through VISA based commands executable using MATLAB.

A combination of both control methods was used throughout the data collection process. Physical interaction with the push buttons on the front panel of the RTSO was used to properly visualize the detected signals, with adjustments made to trace length, number of samples, and scale. Once these RTSO settings were manually applied, a MATLAB based VISA control script was used to automate the data collection process which involved continually capturing and saving sampled traces. Combining both available control methods allows for the automated recovery of numerous signal traces which drastically increases experimental efficiency.

4.2.7 – Tunable Laser Sources

The tunable laser sources employed as local oscillators in the experimental setup of Figure 4.1 are the Agilent 81600B Tunable Laser Modules. The Agilent 81600B is an ECL implying that the mode discrimination used to adjust the wavelength of the optical output is accomplished outside of the gain medium of the laser diode [63]. The Agilent 81600B modules have two different outputs. The first is a low source spontaneous emission (SSE) output with a maximum output power of -2 dBm while the second is a high power out with a maximum output power of 7 dBm. Both outputs of the TLS modules are tunable over 1455 nm - 1640 nm, however the low SSE out has a wavelength resolution of 0.1 pm, while the high-power output has a wavelength resolution of 1 pm. Thus, the low SSE output of the Agilent 81600B was used as the TLS in the experimental setup due to its superior spectral resolution.

The RIN of the TLS module is specified to be -145 dB/Hz over a 6 GHz bandwidth for both outputs. The nominal optical laser linewidth of the TLS modules is specified to be 100 kHz for both outputs. The low optical linewidth of the Agilent 81600B implies that the phase noise generated from the TLS will not impact the recovered phase noise traces of the observed CCS comb lines as was demonstrated by the simulation performed within Chapter 3. The optical spectrum of one low SSE TLS output for an optical power of -8 dBm is shown in Figure 4.10, as well as the minimum linewidth comb line, both measured by the HRS. Both optical spectra have been normalized and superimposed for comparison.

Figure 4.10: TLS and Comb Line Optical Spectrum

Figure 4.10 visually demonstrates that the optical spectrum of the TLS is narrower than the minimum linewidth comb line, emphasizing the low phase noise of the TLS in comparison to the CCS comb lines. Additionally, it is important to note that the comb line observed in

Figure 4.10 is the minimum linewidth mode, which implies all other comb lines yield a broader optical spectrum.

To confirm the specified linewidth of the TLS modules the DSH measurement was performed on the low SSE outputs using the experimental setup introduced in section 2.6. The results of this characterization yielded measured optical laser linewidths for the low SSE outputs of approximately 100 kHz, as expected. The DSH spectrum for the TLS device is shown below in Figure 4.11 for a MSA resolution bandwidth of 600 kHz. The spectrum is centred at 100 MHz due to the frequency shift performed by the AOM in the DSH experimental setup. Additionally, the spectrum has been normalized for clarity. Figure 4.11 shows that the DSH spectrum of the TLS is much narrower than the DSH spectrum of the minimum linewidth comb line in Figure 2.16 of section 2.6.

Figure 4.11: Tunable Laser Source DSH Spectrum

4.3 – Experimental Phase Noise Recovery Procedure

The experimental procedure for recovering phase noise trajectories of two comb lines simultaneously is presented through a visualized walk through in a similar manner to the simulation of sections 3.3 and 3.4. The optical spectrum of the CCS system is first pre-filtered using the BPF. The bandwidth of the BPF is maximized (625 GHz) which truncates the observed optical spectrum to approximately 30 comb lines. A visualization of the optically pre-filtered CCS spectrum was shown in Figure 4.3. However, the centre frequency of the BPF can be adjusted to select different portions of the CCS spectrum. The pre-filtered CCS spectrum is then amplified using the EDFA. The pump current of the EDFA pre-amp is set to 300 mA, while the pump current of the power amp is adjusted to attain an optical power of -5 dBm for each comb line. This places the pump current of the power amp at a nominal value of 160 mA. The FWS is then utilized to filter and isolate two comb lines of interest, controlled using a MATLAB script to generate the desired filter profile.

For illustrative purposes, two comb lines of the CCS spectrum are chosen to be observed and the entirety of the phase noise recovery process is demonstrated. In this example, the minimum linewidth comb line at 194.0573 THz (2 MHz linewidth) is chosen and referred to as *comb line 1* while a larger linewidth comb line at 194.4344 THz (3 MHz linewidth) is also chosen and referred to as *comb line 2*. The two comb lines are separated by 15 modes. Figure 4.12 shows the FWS filter profile generated to isolate comb lines 1 and 2, superimposed on the normalized optically pre-filtered CCS spectrum.

Figure 4.12: FWS Filter Profile on CCS Spectrum (Comb Lines 1 & 2)

Each FWS filtered comb line is output on a different port, implying two different outputs each propagating an individual comb line. To demonstrate the effectiveness of the FWS in its ability to isolate comb lines, both outputs of the FWS (each propagating comb lines 1 and 2, respectively) were combined using a 50/50 optical coupler and measured using the HRS. The resulting normalized spectrum is depicted in Figure 4.13. This visualizes the ability of the FWS to isolate individual comb lines across the CCS spectrum.

Figure 4.13: Optical Spectrum of Comb Lines 1 & 2 – (50/50 Coupled Outputs)

The next experimental step involves using the two TLSs to generate CW output signals that closely match the respective comb lines in terms of both power and frequency. Power matching is easily accomplished by comparing each comb line with the respective TLS using an optical power meter. Frequency matching is little more intricate, involving a 50/50 optical coupler to combine the output signal of the TLS with the filtered comb line which can then be measured using the HRS. This technique allows for fine adjustment of the TLS frequency to closely match that of the comb line. Figure 4.14 visualizes the described experimental setup used to perform frequency matching. Although the FWS has two outputs, frequency matching between a comb line and TLS must be done individually, as indicated by a single FWS output leading towards the HRS in Figure 4.14.

Figure 4.14: Frequency Matching Setup

The HRS is used to provide a visualization of the combined comb line and TLS output to allow the frequency of the TLS to be precisely adjusted. An intentional frequency mismatch is left between the comb line and TLS output (~ 100 MHz), as this was experimentally shown to optimize the recovered phase noise trajectories. Practically, the frequency difference between the TLS and comb line should not exceed 200 MHz due to the bandwidth limitations of the BPDs. Thus, the frequency difference must be monitored as the frequency of both the CCS spectrum and TLS may drift over time, which may correspond to a relative increase or decrease in frequency difference depending on orientation of the spectral shift. Figure 4.15 visualizes the optical spectrum measured by the HRS while performing the frequency matching procedure for *comb line 1*. A frequency difference of approximately 100 MHz exists between the two peaks detected in Figure 4.15. Though the relative power of the comb line and TLS are matched, the spectrum has been normalized for visual clarity.

Figure 4.15: Optical Spectrum while Matching TLS to Comb Line 1

After the frequency matching process has been performed for both comb lines 1 and 2, each comb line and TLS output are input to individual 90 degree optical hybrids. The TLS outputs are directly incident on the 90 degree optical hybrids while each comb line first passes through a polarization controller to align the comb line SOP to that of the TLS thereby maximizing the amplitude of the detected heterodyne beat signals. The outputs of the optical hybrids are detected by the array of BPDs. The resulting photocurrents are sampled using all four channels of the RTSO. The RTSO is used to capture signal traces which are saved and processed offline in MATLAB. The in-phase and quadrature signals for comb lines 1 and 2 are shown below in Figure 4.16, as detected by the RTSO.

Figure 4.16: Recovered In-Phase and Quadrature Signals of Comb Lines 1 & 2

The traces shown in Figure 4.16 have been subject to normalization and mean-removal. The traces are normalized to a peak-to-peak swing of 1 V. For both comb lines 1 and 2 the quadrature signal lags the in-phase signal due to the $\frac{\pi}{2}$ phase shift performed within the optical hybrids. Each recovered signal trace is 2 μs in duration, however the results shown in Figure 4.16 show a 0.2 μs subset of the recovered traces to more clearly visualize the periodicity of the signals. The RTSO has a sampling rate of 5 GS/s implying that each 2 μs trace contains a total of 10000 samples. After the signals are captured and saved by the RTSO they are imported into MATLAB to perform the remainder of the required digital signal processing steps, previously outlined in sections 3.3 and 3.4.

The first signal processing step is to use the recovered in-phase and quadrature components of comb lines 1 and 2 to reconstruct the respective complex signals of the form shown in Equation 4.1.

$$I_{Complex}(t) = I_{Inphase}(t) + jI_{Quadrature}(t) \quad (4.1)$$

This creates two complex signal representations corresponding to comb lines 1 and 2. The phase information of each complex signal can then be accessed using the MATLAB *angle* function. The resulting phase trajectories are then unwrapped using the MATLAB *unwrap* function which removes the discontinuities in the phase of the signal due to phase wrapping. The unwrapped phase trajectories of comb lines 1 and 2 are shown below in Figure 4.17.

Figure 4.17: Unwrapped Experimental Phase Trajectories of Comb Lines 1 & 2

The resulting phase trajectories in Figure 4.17 follow a distinct linear trend, as is expected from the time dependency of the residual intermediate frequency, f_{IF} . In the simulation example explored throughout sections 3.3 and 3.4, the frequency offset was pre-determined which allowed it to be easily compensated to correctly recover the phase noise trajectories. However, practically estimating and removing a residual frequency offset in a real signal is more complex. This is demonstrated through an observation of the frequency content in the recovered in-phase and quadrature components of comb lines 1 and 2, shown in Figure 4.18. It should be noted that the spectral content of the in-phase and quadrature components corresponding to a single comb line are identical, as expected.

Figure 4.18: Frequency Content of Recovered IQ Signals of Comb Lines 1 & 2

The frequency content of the in-phase and quadrature signals was attained by performing a fast Fourier transform (FFT) on the recovered traces in MATLAB. The amplitudes of the recovered spectrums have been normalized. The frequency content of comb line 1 surrounds -113.5 MHz, while the frequency content of comb line 2 surrounds -130.5 MHz. In both cases, visualizing the frequency content of the beat signals demonstrates how estimating and removing the residual frequency offset can be challenging due to the spread of spectral content. Simply detecting the peak of the signal spectrum may not necessarily yield an optimal estimate of the intermediate frequency which could compromise the integrity of the recovered phase noise trajectories. As such, many algorithms have been developed to provide an optimal frequency offset estimate, f_{IF} . One such method is the Hoffman frequency offset estimation algorithm [64].

The Hoffman algorithm was designed to be a hardware-efficient frequency offset estimation technique performed on digital symbols recovered by a receiver for a communications channel, implemented through an ASIC. As such, the mathematical process of the algorithm is outlined in terms of received symbols. However, the algorithm can easily be adapted for use with unmodulated signals. The premise of the Hoffmann algorithm surrounds calculating the instantaneous frequency of each received symbol. In this case, *symbols* can be treated as the *recovered samples* of the complex signals corresponding to comb lines 1 and 2 (which are the unwrapped phase trajectories of Figure 4.18). The instantaneous frequency, or the beat frequency of the heterodyne beat signals f_{IF} , can be calculated for each sample using Equations 4.2 and 4.3:

$$\omega_{IF}(n)T = \left(\left(\psi(n) - \psi(n-1) + \frac{\pi}{4} \right) \bmod \frac{\pi}{2} \right) - \frac{\pi}{4} \quad (4.2)$$

$$f_{IF}(n) = \frac{\omega_{IF}(n)T}{2\pi T} \quad (4.3)$$

Where $\psi(n)$ corresponds to the total phase of the complex signal, $I_{Complex}(t)$ recovered for each individual comb line and T is the *symbol* period – or in this case *sample* period. A manipulation of the phase difference is present in Equation 4.2 to remove any modulation present on signal, which in particular corresponds to QPSK. This phase adjustment brings the values to surround 0 radians effectively removing any QPSK modulation that would have been present in the recovered signal. In the case of measuring the phase noise of comb lines within the CCS system there is no modulation present on the recovered signals making this mathematical manipulation redundant. Nevertheless, it is left in Equation 4.2 for full completeness of the Hoffman algorithm definition. The radian frequency can then be isolated and converted to Hertz using the relationship in Equation 4.3. This leads to a range of instantaneous frequencies which can be easily visualized using a histogram. Performing this process on the recovered complex signals of comb lines 1 and 2 yields the following distributions of instantaneous frequencies, shown in Figure 4.19. By taking the mean of these distributions, a precise frequency offset estimation can be attained and used for compensation.

Figure 4.19: Histogram of Instantaneous Frequencies for Comb Lines 1 & 2

The mean intermediate frequencies of the two distributions are -113.4059 MHz and -129.6571 MHz for comb lines 1 and 2, respectively. Using these two intermediate frequency offset estimations, the contributions can be removed by adding $-2\pi f_{IF}t$ to the respective unwrapped phase trajectories visualized in Figure 4.17.

After performing this frequency offset compensation, the resulting phase trajectories will represent the desired phase noise processes of each comb line for the $2\ \mu\text{s}$ period. The resulting phase noise trajectories of comb lines 1 and 2 are shown in Figure 4.20.

Figure 4.20: Experimentally Recovered Phase Noise Traces for Comb Lines 1 & 2

Figure 4.20 demonstrates the capability of the experimental setup in recovering phase noise trajectories of two different comb lines simultaneously. The recovered traces resemble those which were generated and observed in the simulation of section 3.4. The phase noise trace corresponding to comb line 1 (2 MHz linewidth) has a variance of $1.225\ \text{rad}^2$, while the comb line 2 (3 MHz linewidth) has a variance of $3.064\ \text{rad}^2$. These experimental results agree with the simulated results observing the relationship between optical laser linewidth and variance explored in section 3.2. Figure 4.20 also visually demonstrates how the recovered phase noise traces of comb lines 1 and 2 evolve similarly over time, alluding to

the coherence present within the output of the CCS system. Another feature of the phase noise traces in Figure 4.20 is that the phase noise trajectory of comb line 2 is very close to being a scaled version of the phase noise trajectory of comb line 1. Thus, the recovered results shown in Figure 4.20 provide a preliminary visual demonstration of the scaling law present in MLLs which was introduced in section 2.7. The similarity in time evolution between these two phase noise traces can be quantified using the Pearson correlation coefficient. The resulting correlation coefficient between the phase noise traces of Figure 4.20 is 0.85. This strong correlation emphasizes how similarly the recovered phase noise trajectories of comb lines 1 and 2 are evolving in time. A measurable correlation is an expected result based on the mathematical description of the phase noise in MLLs presented in section 2.7. Thus, this experimental result quantitatively highlights the intrinsic coherence present within the CCS system.

To ensure the validity of the correlation coefficient, a preliminary step must be taken to verify that the recovered phase noise traces are properly aligned in time. This is accomplished by observing the cross-correlation between the two recovered phase noise trajectories which highlights any time displacement between the two sets of data. Thus, any time misalignment could then be compensated by shifting the recovered phase noise trajectories in time accordingly. However, experimental results demonstrated that the recovered phase noise trajectories were nominally time aligned.

An additional step can also be taken to optimize the recovered phase noise trajectories. Although the frequency offset estimation and compensation process is intended to completely remove the residual intermediate frequency, there is a potential that some of this content may remain in the recovered phase noise trajectory. This non-ideality is a

consequence of practically implementing a coherent receiver, as well as the limitations of the accompanying digital signal processing. Figure 4.17 visually demonstrated that a residual frequency offset corresponds to the recovered phase noise traces varying about a linear offset as a function of time. Since it is known that the phase noise processes of comb line 1 and 2 should mathematically evolve similarly as a function of time, the Pearson correlation coefficient can be utilized as parameter to optimize the slopes of the recovered phase noise traces. Thus, the slope of the phase noise trajectory can be finely adjusted to maximize the correlation coefficient, since that is the behaviour that is mathematically known and expected. Practically, this is implemented by holding one of the recovered phase noise traces constant while varying the linear offset of the other phase noise trace. This process is visualized in Figure 4.21, demonstrating how the linear offset of the phase trajectory can be varied to maximize the correlation coefficient. This is accomplished using an optimization process implemented in MATLAB which exploits the *fminsearch* function. The *fminsearch* function is designed to minimize the value of an input function. In this case, the goal is to maximize the correlation coefficient between the two recovered phase noise trajectories. Thus, the implementation of *fminsearch* must be used to minimize the negative Pearson correlation coefficient. To accomplish this, a small linear offset is added to one of the two phase noise traces and the resulting correlation coefficient is calculated. This process is repeated by incrementally varying the linear offset until a distinct maximum correlation coefficient value is obtained.

Figure 4.21: Demonstration of the Correlation Coefficient Optimization Process

After performing the correlation coefficient optimization process on the recovered phase noise trajectories corresponding to comb lines 1 and 2, the maximum correlation coefficient attained was 0.89 which is an increase of 0.04. Although the improvement in correlation coefficient in this example is modest, this process outlines an important procedural step taken to ensure optimum results are consistently attained from the experimental procedure. Visually, there is no discernable change in the optimized versus un-optimized phase noise traces in this example. This optimization process is based on the mathematical model of phase noise in MLLs presented in [47] and maintains the integrity of the results by minimizing the effect of any non-idealities occurring due to the practical limitations of experimental implementation.

The results presented throughout section 4.3 demonstrate the ability of the coherent detection scheme to recover time domain phase noise traces of two comb lines simultaneously. The resulting phase noise traces of Figure 4.20 maintain similar characteristics to the simulated results of sections 3.3 and 3.4, which provides validation regarding the accuracy of the experimental phase noise recovery procedure. Additionally, the necessary signal processing steps were outlined and visually demonstrated which included the implementation of a more precise frequency offset estimation algorithm. Furthermore, an optimization procedure was introduced to minimize the effect of any residual frequency offset present in the recovered phase noise trajectories post compensation. The correlation between the two recovered phase noise traces was also quantified, the results of which displayed a high degree of linear correlation. Thus, the proposed experimental setup has been experimentally validated and can be used to provide a more thorough characterization of the phase noise properties across the CCS spectrum.

4.4 – Correlation Properties

The coherence of the CCS system can be quantified by observing the correlation properties between phase noise trajectories recovered for pairs of comb lines across the spectrum. However, to obtain a meaningful characterization, the simultaneous phase noise recovery process must be performed systematically. Thus, a structured phase noise recovery process was designed to allow for a comprehensive analysis of the correlation properties between comb lines exhibited by the CCS system.

The experimental procedure involves the notion of recovering time domain phase noise traces corresponding to a *reference* comb line and a *test* comb line. The *reference* comb

line was chosen to be the minimum linewidth mode exhibited by the CCS spectrum (chosen from the DSH results) and is held constant throughout the entirety of the data collection procedure. Thus, one arm of the experimental setup in Figure 4.1 is dedicated to solely measuring the *reference* comb line. The *test* comb line is varied using the filtering capabilities of the FWS to select different comb lines across the spectrum of the CCS system, one at a time. Thus, the input to the second arm of the experimental setup will continually change to detect different comb lines across the CCS spectrum. In this way, phase noise traces can be systematically recovered for pairs of comb lines across the CCS spectrum allowing for a comprehensive characterization of the correlation properties of comb lines relative to a pre-determined *reference* comb line.

The minimum linewidth *reference* comb line has an optical laser linewidth of 2 MHz at 194.0573 THz, while the *test* comb line corresponds to any other comb lines in the spectrum, subject to the bandwidth limitations (625 GHz) of the BPF used for optical pre-filtering. However, the area of interest is the low linewidth region surrounding the *reference* comb line. Figure 4.22 shows the optical spectrum of the CCS system, highlighting the reference comb line as well as the low linewidth region of comb lines spanning a total of 30 modes.

Figure 4.22: Characterization Region of the CCS Optical Spectrum

Figure 4.22 shows the entirety of the CCS optical spectrum in black, while the minimum linewidth *reference* comb line and the low-linewidth region of *test* comb lines are shown in blue and red respectively. Therefore, the *test* comb line will be varied to observe comb lines within the low-linewidth red region of Figure 4.22, while the blue *reference* comb line is held constant.

The reason for limiting the characterization to a portion of the spectrum is two-fold. The first is because the linewidths of comb lines beyond the highlighted region rapidly increase. Since the CCS system is intended for implementation as a laser source for coherent DWDM transmission, minimum linewidth comb lines are of more practical interest. Secondly, the experimental setup is limited by the bandwidth of the bandpass filter used to optically pre-filter the CCS spectrum prior to optical amplification. As mentioned previously, the

maximum bandwidth of the Yenista BPF is 625 GHz, which corresponds to approximately 25 comb lines available for selection at single point in time. However, to maximize the availability of test comb lines the BPF can be tuned to have the *reference* comb line on either edge of the passband by simply shifting the centre frequency of the BPF. This allows access to entirety of the region shown in red. Contrastingly, comb lines beyond this region are not able to be compared with the chosen *reference* comb line using the outlined experimental setup. Additionally, to ensure there is no attenuation of the *reference* and *test* comb lines from the roll-off of the BPF, they are not placed at the maximal edges of the BPF passband. As a result, the *reference* and maximum *test* comb line were kept a few modes away from either edge of the BPF passband which further reduces the number of selectable comb lines to approximately 15 on either side of the *reference* comb line.

The experimentally observed *test* comb lines of the low-linewidth region are denoted with integer values, relative to n_c , where n_c represents the minimum linewidth mode from Equation 2.8 of section 2.7. For clarity, the *reference* comb line will be assigned a value of $n_c = 0$. The remainder of the comb lines surrounding the *reference* comb line are assigned integer values of n based on their relative location to the reference comb line. For example, the comb line directly to the right of the *reference* comb line n_c (corresponding to an increase in frequency) will be referred to as $n = 1$, and the comb line two modes to the right referred to as $n = 2$. Comb lines to the left of the *reference* comb line (corresponding to a decrease in frequency) will be given descending integer values based on their relation to the reference comb line, with the comb line directly to the left of the *reference* comb line referred to as $n = -1$.

The experimental setup was used to recover phase noise traces for a subset of test comb lines across the low-linewidth region of the CCS spectrum observed in Figure 4.22. The modes evaluated correspond to $n = \pm 1, \pm 2, \pm 5, \pm 10, \pm 15$. For each pair of comb lines, a total of 30 in-phase and quadrature traces were recovered from each *test* and *reference* comb lines resulting in 30 simultaneously recovered pairs of phase noise traces for each *test* comb line (value of n). The correlation coefficients for each of the 30 pairs of recovered phase noise traces were then calculated, resulting in a total of 30 correlation coefficients. The means of all 30 correlation coefficients obtained for each *test* comb line were calculated with results shown below in Figure 4.23.

Figure 4.23: Mean Correlation Coefficient vs. Comb Line

The *reference* comb line, corresponding to $n = n_c = 0$, was assigned a correlation coefficient of 1. The remainder of the correlation coefficients are visualized according to

the respective *test* comb line, where each value of n corresponds to phase noise traces being compared between the n^{th} comb line and the *reference* comb line.

The results in Figure 4.23 demonstrate that as modal separation, $|n|$, increases, the mean correlation coefficient tends to decrease. This decrease in mean correlation coefficient is symmetrical about the *reference* comb line, following a negative quadratic trend emphasized with a 2nd order polynomial fit.

The results of Figure 4.23 provide a significant amount of insight regarding the spectral properties of the CCS system. One of the most important characteristics to observe is the extremely high mean correlation coefficient for the spectral modes nearest to the *reference* comb line. For $|n| < 5$ the mean correlation coefficient is greater than 0.95 implying that the phase noise present in these modes of the CCS system are varying nearly identically as a function of time. This result indicates that the benefits of a coherent spectrum would likely be exploitable in a DWDM system for a small subset of neighbouring modes. Contrastingly, the mean correlation coefficient drops below 0.95 for modes such that $|n| > 5$. This is an important finding as it implies that the phase noise fluctuations, though still similar for comb lines such that $|n| > 5$, is quantifiably decreasing as a function of modal separation. In a more general sense, this result shows that although the CCS system is capable of generating a coherent spectrum of more than 70 comb lines, the relative coherence levels are not constant across the entirety of the spectrum. In section 2.7 it was shown that the phase noise of a particular comb line is the summation of ASE noise and timing jitter which scales as a function of modal separation, $|n|$. Thus, since this decrease in correlation is observed to be a function of modal separation it alludes to the notion that

this effect is caused by the relative increase in timing jitter fluctuations which scale as a function of modal separation.

An important note regarding these results is that the correlation properties shown in Figure 4.23 are not bound to the *reference* comb line chosen throughout the experimental characterization. If a different comb line is chosen to be the *reference* (any other mode), there will be the same quadratic decrease in the correlation properties of surrounding comb lines. The observed correlation remains true for any group of comb lines across the CCS spectrum. Thus, by strategically choosing groups of comb lines across the CCS spectrum, the correlation properties and in turn the coherence could still be strategically exploited for implementation in a coherent DWDM system. However, it is important to keep in mind that the area observed in Figure 4.23 is the low-linewidth region of comb lines exhibited by the CCS system. Thus, as mentioned previously, though strong correlation may be present between subsets of adjacent modes across the entirety of the CCS spectrum, the optical linewidths of the comb lines outside the low-linewidth region may be too large for practical implementation in a coherent communication system.

Another important note regarding the results of Figure 4.23 is that the *mean* correlation coefficient is observed. This is calculated from a set of 30 total correlation coefficients for each *test* comb line. When observing all 30 correlation coefficients, some notable variation is present. Therefore, observing a single pair of recovered phase noise traces for each *test* comb line may not provide an accurate characterization of device behaviour. Taking the mean of many traces allows the recovered correlation coefficient to stabilize allowing for a more confident and repeatable quantification. Figure 4.24 shows the average correlation coefficient as a function of the number of recovered correlation coefficients used in the

calculation. This visualizes the impact averaging has on the final results shown in Figure 4.23.

Figure 4.24: Mean Correlation Coefficient as a Function of the Number of Values Averaged

Figure 4.24 shows results for test comb lines corresponding to $n = 1, 2$, and 5 . In all cases, the correlation coefficient begins to stabilize as more values are considered. The volatility in the mean correlation coefficient when considering less than 15 results stresses the need to average numerous correlation coefficients for each mode. This observation led to the decision to recover 30 phase noise traces for each *test* comb line.

The experimental setup in Figure 4.1 was automated to capture the in-phase and quadrature traces for the corresponding *reference* and *test* comb line every 30 seconds. The traces were captured every 30 seconds due to the limitations in the trace saving and writing speed of

the RTSO, otherwise the capture rate would have been increased. Recovering data over a predetermined periodicity allows for quantities such as the correlation coefficients to be viewed as a function of time for each *test* comb line. Figure 4.25 observes the correlation coefficient as a function of time for $n = 1$, and 5.

Figure 4.25: Correlation Coefficient Variation over Time

The results in Figure 4.25 highlight the variation in recovered correlation coefficient over time, further emphasizing the necessity of calculating the mean of all recovered values. An important observation from the results of Figure 4.25 is that the variation in correlation coefficient as a function of time is much smaller for $n = 1$, than for $n = 5$. This effect can be quantified through the variance of the recovered correlation coefficients for each *test* comb line. The variance for $n = 1$ and $n = 5$ were found to be 0.049 and 0.126,

respectively. This result implies that as the modal separation $|n|$ increases, the variation in the recovered correlation coefficients also increases. Thus, the variance among the set of correlation coefficients for each *test* comb line is calculated with results shown below in Figure 4.26.

Figure 4.26: Variance in Recovered Correlation Coefficients

The results demonstrate that as $|n|$ increases, the variance over the range of recovered correlation coefficients also increases. This increase is quadratic in nature, emphasized by a 2nd order polynomial fit. Thus, not only does the mean correlation coefficient decrease with modal separation, but there is also measurable decrease in the consistency of the recovered correlation coefficients as a function of increasing modal separation.

The results in Figure 4.26 also show that the variance for modes such that $|n| < 5$ is minimal, implying that all recovered correlation coefficients for each of these *test* comb lines are very similar. However, for modes such that $|n| > 5$, the variance begins to scale rather quickly. These results appear to align with those observed in Figure 4.23 which presented the mean correlation coefficient versus *test* comb line. Observing the variance in the recovered correlation coefficients further emphasizes the notion that there is a practical limit to the coherence demonstrated by the CCS system.

The quantification of the variation in the recovered correlation coefficients for each *test* comb line is an intriguing result as it alludes to varying levels of coherence within the CCS system depending on the modes under observation. The correlation characterization demonstrates that in order to fully exploit the coherence in the CCS system (and more generally a MLL), subsets of neighboring modes need be considered to ensure maximal similarity between the phase noise present among comb lines. The contributions to the phase noise present on individual comb lines is observed in detail throughout section 4.5 using the PNT analysis which provides further insight towards the correlation observations made throughout section 4.4.

One of the properties of the phase noise trajectories recovered which is not quantified through the correlation coefficient is the scaling between a pair of simultaneously recovered phase noise traces, which was observed in Figure 4.20 of section 4.3. The increasing magnitude of phase noise across modes of the CCS spectrum was initially observed in the results of the DSH measurement. The DSH results illustrated that the optical laser linewidth of each comb line across the CCS system spectrum scales quadratically as a function of mode number. However, as discussed in section 3.6, the

correlation coefficient is bound to values between -1 and 1, which implies that this scaling phenomenon between phase noise traces of the *reference* and *test* comb lines will not be captured. Thus, a different parameter is necessary to quantify the changing magnitude between the recovered phase noise traces for the *reference* and *test* comb lines. This led to implementation of the normalized root mean squared (RMS) phase noise difference calculation.

The phase noise difference refers to taking the point-wise difference between the recovered phase noise traces of the *test*, and *reference* comb lines, shown in Equation 4.4.

$$\theta_{difference}(n) = \theta_{test}(n) - \theta_{reference}(n) \quad (4.4)$$

Where, $\theta_{reference}$ refers to the phase noise trajectory of the *reference* comb line while θ_{test} refers to the phase noise trajectory of the *test* comb line. This yields a set of phase differences from which an RMS value can be calculated. Thus, for each pair of recovered phase noise traces for a *reference* and *test* comb line, a corresponding RMS phase difference value was calculated. Equation 4.5 presents the RMS calculation, where M is the number of samples in the recovered phase difference array, $\theta_{difference}$.

$$RMS \text{ Phase Difference} = \sqrt{\frac{\sum_{n=1}^M \theta_{difference}(n)^2}{M}} \quad (4.5)$$

The calculated RMS phase difference value is then normalized with respect to the *reference* comb line to provide an appropriate scale. This allows for a more meaningful

characterization of the relative increase in phase noise fluctuations of the *test* comb lines. Thus, for each pair of recovered phase noise traces, the RMS phase difference is divided by the RMS value of the *reference* phase noise trajectory, as shown in Equation 4.6.

$$Normalized\ RMS\ Phase\ Difference = \frac{\sqrt{\frac{\sum_{n=1}^M \theta_{difference}(n)^2}{M}}}{\sqrt{\frac{\sum_{n=1}^M \theta_{reference}(n)^2}{M}}} \quad (4.6)$$

It should be noted that once again, for each *test* comb line, a total of 30 phase noise trajectory pairs were recovered, allowing for the calculation of 30 normalized RMS phase difference values. The mean of the resulting normalized RMS phase difference values are shown in Figure 4.27 for each *test* comb line. The *reference* comb line at $n = 0$ was assigned a value of 0.

Figure 4.27: Mean Normalized RMS Phase Difference

As expected from the results of the DSH measurements, the mean RMS phase difference increases quadratically as $|n|$ increases. Once again, the data has been fit with a second order polynomial to emphasize this behaviour. The normalized RMS phase difference provides context to how large the phase fluctuations are relative to the reference comb line. This quadratic behaviour is expected since the *reference* comb line represents the minimum linewidth mode and thus the smallest phase noise fluctuations. The results of the normalized RMS phase difference further validate how the magnitude of the recovered phase noise present on comb lines increases proportionally to modal separation, $|n|$. Once again, this can be attributed to the relative contribution of timing jitter fluctuations which grows as a function of modal separation, as defined in Equation 2.8 of section 2.7.

In the scenario where the *reference* comb line is not chosen to be the minimum linewidth mode, the trend would still be quadratic however the vertex would not be centred and would take on a value below 0, due to the nature of the calculation presented in Equation 4.4.

Although the mean of the normalized RMS phase difference provides the best overall representation of the varying magnitude in the recovered phase noise traces it is also important to quantify the variation among all 30 recovered normalized RMS phase difference values for each *test* comb line. Figure 4.28 shows the variance of the 30 normalized RMS phase difference values for each *test* comb line.

Figure 4.28: Variance in Normalized RMS Phase Differences

Figure 4.28 shows that the variance among the recovered RMS phase difference values also scale quadratically as a function of mode number, emphasized with a second order

polynomial fit. The results in Figure 4.28 further demonstrate that the consistency in the normalized RMS phase difference values decreases as $|n|$ increases, which agrees with the results of the variance in recovered correlation coefficients, illustrated in Figure 4.26.

A final important concept to consider is how the mean normalized RMS phase difference varies with respect to the mean correlation coefficient for each *test* comb line observed. This is visualized in Figure 4.29, with each data point corresponding to a comb line labelled with its respective value of n .

Figure 4.29: Normalized RMS Phase Difference versus Correlation Coefficient

Figure 4.29 demonstrates that as $|n|$ increases, the correlation coefficient decreases while the normalized RMS phase difference increases. Additionally, this relationship between correlation coefficient and RMS phase difference is linear. This provides evidence that the correlation between the phase noise of comb lines is directly influenced by the relative

difference in magnitude between the phase noise fluctuations of the *reference* and *test* comb lines. Therefore, the phenomenon responsible for this relative increase in the magnitude of the phase noise trajectories is also responsible for the decrease in correlation. Thus, the contribution causing this relative increase in phase noise as a function of modal separation needs to be explored, namely the fluctuations due to timing jitter.

In section 2.7 the phase noise of a particular comb line, $\theta_n(t)$, was mathematically described as the summation of two distinct contributions, ASE noise and timing jitter. The closed form expressions of Equations 2.8 showed that while the ASE noise contribution stays constant, the fluctuations due to timing jitter scale as a function of modal separation, $n - n_c$. This scaling was observed in the results of both the correlation coefficients and the normalized RMS phase difference values. Therefore, this behaviour can be further explored by quantifying the relative contributions of both the ASE noise and timing jitter using the PNT analysis. This will provide insight regarding the increase in phase noise magnitude and decrease in correlation observed across the CCS spectrum throughout section 4.4.

4.5 – PNT Analysis – Optical Laser Linewidth

A PNT analysis was first introduced in section 3.7, with the results demonstrating that the method can accurately estimate optical laser linewidth from recovered phase noise trajectories. Thus, the PNT analysis is used to provide a secondary method of estimating the optical laser linewidth for the low-linewidth region of comb lines across the CCS system spectrum.

In section 4.4, trace data was collected for a *reference* and *test* comb line simultaneously, with a total of 30 traces collected for each pair of comb lines. However, in estimating optical laser linewidth the notion of simultaneously recovering phase noise traces is unnecessary. This is because the measurement corresponds to an individual comb line, and thus individual phase noise trajectories corresponding to that particular comb line. Nevertheless, the same data used for the results in section 4.4 was utilized throughout this characterization to ensure consistency across all results presented throughout this research. For all 30 phase noise trajectories recovered from each *test* comb line, a PNT analysis was performed. The PNT analysis involves segmenting each recovered phase noise trace into blocks of M samples and taking the phase difference between the final and initial values of each block. The resulting collection of phase differences for each of the 30 recovered traces per comb line were then combined and a corresponding variance of these phase noise differences was calculated. The phase difference data of each *test* comb line is zero-mean Gaussian distributed and can be visualized using a histogram. Figure 4.30 shows an example of phase difference histograms from the PNT analysis for $n = 0$ and $n = 15$ where $M = 250$.

Figure 4.30: Histograms of the Recovered Phase Noise Differences ($n = 0$, $n = 15$)

Gaussian densities have been superimposed on both distributions. The results demonstrate a broadening in the recovered distribution of phase noise differences obtained from the PNT analysis for increasing values of $|n|$. This broadening mathematically corresponds to an increase in variance and consequently an increase in optical laser linewidth. The variance of the phase noise difference data for each *test* comb line observed was calculated and the corresponding optical linewidth was extracted using Equation 2.11 of section 2.7.

The resulting linewidths from the PNT analysis are shown in Figure 4.31, along with the results of the DSH measurement for comparison.

Figure 4.31: PNT and DSH Linewidth Results of CCS System

The linewidth results of the PNT analysis follow the same trend as the DSH measurements, with the measured linewidths increasing quadratically from a distinct minimum linewidth comb line. Both data sets have been fit with a second order polynomial to emphasize the quadratic behaviour. For the region shown in Figure 4.31 the DSH technique measured linewidths of CCS system comb lines between 2 - 6 MHz, while the PNT analysis yielded linewidths between 1 – 1.8 MHz. The attained PNT analysis linewidth estimates are slightly lower than the DSH measurements. This discrepancy occurs as a result of the DSH technique typically overestimating the optical laser linewidth due to an inherent frequency

drift occurring throughout the measurement [65]. Since the DSH technique relies on a low speed measurement combined with a large time delay, the envelope of any frequency drift will be measured, resulting in an optical laser linewidth which is often slightly larger than the true value. However, the similarity of both results demonstrates the experimental validity of the PNT analysis as an accurate method for the measuring optical laser linewidth.

An important consideration regarding the results of the PNT analysis is the amount of data required for an accurate estimation of optical linewidth. In this characterization, each phase noise trajectory trace had a total of 10000 samples. Since the optimal block size needed for the PNT analysis is 250 samples (as was pre-determined through the simulation performed in section 3.7), a PNT analysis performed on single phase noise trace recovered using the outlined experimental procedure will only yield a total of 40 phase difference values. Although a variance and in turn the corresponding optical laser linewidth can be calculated from the phase differences from a single phase noise trace, the accuracy and consistency significantly improves by considering more phase noise traces which inherently leads to more phase differences resulting from the PNT analysis. Figure 4.32 observes the effect that the number of traces considered in the PNT analysis has on the estimation of linewidth.

Figure 4.32: PNT Linewidth Results for Varying Number of Traces

Figure 4.32 observed three different PNT analyses, wherein 1, 10, and 30 phase noise traces were considered corresponding to 40, 400 and 1200 phase difference values used respectively. In each case the results demonstrate a similar quadratic trend, however considering more phase noise traces increases the consistency in the recovered optical laser linewidths. The variability seen in Figure 4.32 occurs due to the stochastic nature of phase noise. Thus, to minimize the impact of this inherent randomness one should increase the amount of data used in the PNT analysis. Nevertheless, the results in this section demonstrate the validity of the PNT analysis as a method for measuring the optical laser linewidth of an optical device, such as the CCS system.

4.6 – PNT Analysis – Phase Noise Contributions

The main purpose for the development and implementation of the PNT analysis is to quantify the relative contributions to the phase noise present in each mode throughout CCS system. As discussed previously, [47] demonstrated that the phase noise present in a single mode (or comb line) of a MLL is composed of two different contributions. The first contribution is due to laser ASE noise, denoted as $\theta_c(t)$, while the second contribution is due to the timing jitter fluctuations, denoted as $\Delta\theta(t)$, and is proportional to the modal separation, $n - n_c$. The summation of these two terms yields a time varying output phase noise term, $\theta_n(t)$, which is enumerated in Equation 2.8 of section 2.7.

The isolation of these phase noise contributions can be done using the data collected from the process outlined in section 4.4. The ASE noise, $\theta_c(t)$, corresponds to the phase noise of the minimum linewidth mode n_c , which experimentally refers to the *reference* comb line, such that $n_c = 0$. As mentioned previously, this comb line is constantly measured in one arm of the experimental setup in Figure 4.1. The second arm of the experimental setup is used to measure a *test* comb line which is varied across the CCS spectrum. Each test comb line corresponds to an integer value of n , with its value denoting its location relative to the minimum linewidth reference comb line, n_c . The recovered *test* phase noise traces correspond to $\theta_n(t)$. Since the recovered *reference* and *test* phase noise traces are captured simultaneously, the point-wise difference can be taken offline in MATLAB, as shown in Equation 4.6.

$$\theta_n(t) - \theta_c(t) = (n - n_c)\Delta\theta(t) \quad (4.6)$$

This results in the isolation of the scaled fluctuations due to timing jitter, $(n - n_c)\Delta\theta(t)$. The scaled fluctuations due to timing jitter can then be normalized by the known modal separation $n - n_c$, relative to the reference comb line. Therefore, for each pair of recovered phase noise trajectories for *reference* and *test* comb lines a total of four different phase noise trajectories can be observed, $\theta_n(t)$, $\theta_c(t)$, $(n - n_c)\Delta\theta(t)$ and $\Delta\theta(t)$. The relative magnitudes of each of these phase noise trajectories can be visualized by observing their time domain evolution. Figure 4.33 shows an example of recovered phase noise trajectories for the *reference* comb line, $\theta_c(t)$, and *test* comb line, $\theta_n(t)$, corresponding to $n = 15$, as well as the scaled and normalized fluctuations due to timing jitter, $15\Delta\theta(t)$ and $\Delta\theta(t)$.

Figure 4.33: Relative Phase Noise Contributions ($n = 0$ and $n = 15$)

The results in Figure 4.33 show that the fluctuations due to timing jitter are small in comparison to the contribution from ASE noise, $\theta_c(t)$. However, as the modal separation increases the fluctuations due to timing jitter scale proportionally which increases their relative impact on the overall recovered phase noise trajectory, $\theta_n(t)$, as is evidenced by the magnitude of the trace corresponding to $15\Delta\theta(t)$. Once the trajectories have been isolated, the relative contributions of the ASE noise and timing jitter of each *test* comb line can be quantified using the PNT analysis to more fully understand how each contribution impacts the overall phase noise of a comb line, $\theta_n(t)$. More specifically, a corresponding variance for each contribution can be attained.

The experimental setup and data collection process outlined in section 4.4 allows for the collection of 30 simultaneous phase noise traces for both the *reference* and each *test* comb lines. The point-wise difference of each pair of recovered phase noise trajectories was taken allowing for the recovery of $(n - n_c)\Delta\theta(t)$ and subsequently $\Delta\theta(t)$. This allowed the PNT analysis to be performed on each of the four phase noise trajectories, $\theta_c(t)$, $\theta_n(t)$, $(n - n_c)\Delta\theta(t)$ and $\Delta\theta(t)$, for each pair of *reference* and *test* comb lines. A block size of $M = 250$ was utilized to attain the phase differences of the PNT analysis. The phase differences from the 30 captured traces were then combined which allowed the variance of each contribution to be quantified. It should be noted that the inherent correctness of this characterization rests on the ability to correctly identify the minimum linewidth mode, corresponding to $\theta_c(t)$ where $n_c = 0$. Therefore, much care must be taken to ensure the minimum linewidth mode has been properly identified, otherwise the point-wise difference performed in Equation 4.6 will not properly represent the terms of interest. This

identification can be accomplished by actively comparing the results of the DSH and PNT analysis linewidth estimates.

A histogram of the resulting phase differences for $\theta_c(t)$ and $\theta_n(t)$ attained from the PNT analysis for the case where the *test* comb line corresponds to $n = 15$ was presented in Figure 4.30. The variances of the phase difference distributions in Figure 4.30 were used to estimate the linewidth of each comb line for $\theta_c(t)$ and $\theta_n(t)$, when $n = 15$. These two distributions showed that the variance of $\theta_n(t)$ is larger than that of $\theta_c(t)$, with the notable broadening occurring due to increasing values of $|n|$. For the same scenario where $n = 15$, the resulting phase difference histograms for $15\Delta\theta(t)$ and $\Delta\theta(t)$ are shown in Figure 4.34 for comparison.

Figure 4.34: Histograms of PNT Analysis for Timing Jitter

The results in Figure 4.34 easily visualize the contribution of timing jitter scaling proportional to modal separation through the variances recovered from the phase noise difference values. When comparing the distributions of $15\Delta\theta(t)$ and $\Delta\theta(t)$ in Figure 4.34 with the phase difference distributions of $\theta_c(t)$, and $\theta_n(t)$ in Figure 4.30, the resulting variances follow the expected trend illustrated by the time domain plots of Figure 4.33. The variance of the histogram for $\Delta\theta(t)$ in Figure 4.34 is small compared to that for

$\theta_c(t)$ in Figure 4.30. However, as the modal separation increases, the fluctuations due to timing jitter scale proportionally making the relative impact of timing jitter, $15\Delta\theta(t)$, comparable to $\theta_c(t)$. This can be more impactfully observed through the relative linewidths extracted from the recovered variances (extracted using equation 2.11 of section 2.7), quantified in Table 4.2.

Table 4.2: Linewidths Corresponding to Recovered PNT Analysis Variance ($n = 15$)

	$\theta_{15}(t)$	$\theta_c(t)$	$\Delta\theta(t)$	$15\Delta\theta(t)$
Linewidth (MHz)	1.841 MHz	1.120 MHz	0.003 MHz	0.692 MHz

The results in Table 4.2 provide a clear comparison of magnitude for the four phase noise trajectories, $\theta_c(t)$, $\theta_n(t)$, $\Delta\theta(t)$ and $(n - n_c)\Delta\theta(t)$ when $n = 15$. The PNT analysis was performed on all the recovered phase noise trajectories for each pair of *test* and *reference* comb lines. This allowed for a quantification of the relative contributions of ASE noise and timing jitter fluctuations for each mode throughout the low-linewidth region of the CCS optical spectrum. Thus, for each *test* comb line a corresponding variance was obtained using the PNT analysis for $\theta_n(t)$, $\theta_c(t)$, $(n - n_c)\Delta\theta(t)$, and $\Delta\theta(t)$, the results of which are shown in Figure 4.35.

Figure 4.35: Variance of the Contributions to $\theta_n(t)$

The largest variances shown in Figure 4.35 are due to $\theta_n(t)$ which correspond to the observed *test* comb lines, composed of the summation of $\theta_c(t)$ and $(n - n_c)\Delta\theta(t)$. The results of $\theta_n(t)$ scale quadratically from a minimum value of 0.35 rad^2 . The mean variance of the minimum linewidth *reference* comb line, $\theta_c(t)$, is 0.35 rad^2 and is approximately constant for each observed test comb line. This quantifies the relative contribution of ASE noise to the overall phase noise of each comb line.

The fluctuations due to timing jitter between adjacent modes, $\Delta\theta(t)$, is minimal and nearly constant across the observed CCS spectrum with an average value of approximately 0.018

rad². However, as the modal separation increases between the *test* comb line and the minimum linewidth *reference* comb line the impact of $\Delta\theta(t)$ begins to scale as a function of $n - n_c$. This is evident by observing the quadratic trend in the variance calculated for $(n - n_c)\Delta\theta(t)$.

The results in Figure 4.35 show that for modes such that $|n - n_c|$ is small, the overall phase noise is dominated solely by ASE noise, $\theta_c(t)$. However, as the modal separation between two observed comb lines increases, the relative contribution of the timing jitter, $\Delta\theta(t)$, begins to scale proportionally. Thus, for certain values of $|n|$, the impact of both the ASE noise and timing jitter become comparable. Additionally, the results of Figure 4.35 show that the scaled increase in timing jitter fluctuations is solely responsible for the observed increase in the magnitude of the recovered phase noise traces for increasing values of $|n|$, since the relative contribution of ASE noise is approximately constant.

One issue with the results shown in Figure 4.35 is how the variance of the timing jitter, $\Delta\theta(t)$, increases uncharacteristically for $|n| = 1$, which corresponds *test* comb lines adjacent to the *reference* comb line. For comb lines, such that $|n| > 1$, the variance of the timing jitter maintains a near constant variance of approximately 0.018 rad². However, for $n = \pm 1$, the measured variance is approximately 0.08 rad², which is a factor of 4 greater than the observed typical value. Although this increase is small in comparison to the relative contribution of ASE noise, the behaviour is unexpected and warrants further consideration.

To investigate the observed phenomenon, the *reference* comb line was changed from the minimum linewidth comb line to an arbitrarily chosen comb line ($n = 5$, *relative to the*

minimum linewidth comb line) across the spectrum. The PNT analysis was then performed for a few *test* comb lines, which included neighbouring modes as well as modes for where $n > 1$. The results from performing the PNT analysis demonstrated that modes adjacent to the chosen *reference* comb line still displayed a notable increase in the measured variance of the timing jitter fluctuations, $\Delta\theta(t)$. The mathematical derivations of $\Delta\theta(t)$ in [47] showed that the timing jitter fluctuations between adjacent modes is expected to be constant. Therefore, this relative increase in variance observed for $\Delta\theta(t)$ is likely a consequence of an imperfection in the experimental setup, although the cause of this measured behaviour remains unknown.

In the closed form expressions derived in [47], a caveat was that the time-varying phase noise of the minimum linewidth mode $\theta_c(t)$ and timing jitter $\Delta\theta(t)$ should be uncorrelated. This implies that ASE noise and timing jitter are two statistically independent and uncorrelated phase noise processes. However, if a measurable correlation between $\theta_c(t)$ and $\Delta\theta(t)$ exists, it implies a shift in the minimum linewidth mode across the spectrum. With respect to the experimental process this would practically imply an incorrect identification of the minimum linewidth mode, $\theta_c(t)$. Thus, to further validate the results attained, the correlation coefficient between $\theta_c(t)$ and $\Delta\theta(t)$ was calculated for the phase noise traces recovered for all pairs of *reference* and *test* comb lines across the CCS spectrum. The average correlation coefficient was found to be 0.069 (over all 30 traces for each of the 10 modes observed), which confirms that there is essentially no notable correlation between $\theta_c(t)$ and $\Delta\theta(t)$ thereby confirming a correct identification of the minimum linewidth comb line.

However, section 4.4 experimentally observed that there is a measurable correlation between the phase noise of the *reference* comb line $\theta_c(t)$ and *test* comb lines across the spectrum $\theta_n(t)$ which was visualized in Figure 4.23. The results demonstrated that the recovered correlation coefficient between the phase noise of comb line pairs decreased quadratically as a function of modal separation. Additionally, it was shown in Figure 4.35 that the relative contribution of the timing jitter, $(n - n_c)\Delta\theta(t)$, in the overall phase noise of the comb line also increases as a function of modal separation. Since results also demonstrated that the timing jitter, $\Delta\theta(t)$, is uncorrelated with the ASE noise, $\theta_c(t)$, it implies that the measured decrease in correlation across the CCS spectrum can be attributed to the relative increase in the timing jitter present in the overall phase noise of an individual comb line. Thus, the strong correlation present in the subset of comb lines surrounding the minimum linewidth *reference* comb line occurs because the relative contribution of the ASE noise dominates the relative contribution of the timing jitter in the overall phase noise. However, the results in Figure 4.35 have shown that as $|n|$ increases, the relative contribution of the uncorrelated timing jitter in the overall phase noise of a comb line consequently increases. Since this timing jitter is uncorrelated with the ASE noise $\theta_c(t)$, the overall correlation coefficient between pairs of comb lines decreases as function of modal separation, as experimentally observed. Therefore, the results attained throughout section 4.6 experimentally validate the mathematically derived phase noise expressions for phase noise in MLLs using the CCS system.

4.7 – Experimental Issue – Amplitude Variation

One unresolved issue with the recovered data surrounds the in-phase and quadrature photocurrents obtained from the experimental setup in Figure 4.1, which form the foundation of the recovered phase noise trajectories for the *reference* and *test* comb lines. The issue surrounds the variation noticed in the peak-to-peak amplitude of the sampled photocurrents for in-phase and quadrature components of each *test* and *reference* comb line. The experiment involved recovering 30 in-phase and quadrature photocurrent traces for each pair of comb lines. In each trace, the maximum and minimum values were calculated allowing for a quantification of the maximum peak-to-peak voltage swing across each IQ trace. This process was performed for all 30 traces captured for each *test* comb line. For illustrative purposes, the 30 peak-to-peak amplitude results are shown for the in-phase and quadrature components of the minimum linewidth *reference* comb line, $n_c = 0$, and the *test* comb line where $n = 15$ in Figure 4.36.

Figure 4.36: Peak-to-Peak Amplitude Variation versus Trace Number

The results show that the peak-to-peak amplitude swing of the in-phase and quadrature signals of both comb lines fluctuate significantly from trace to trace. However, an important observation is that the in-phase and quadrature components of the *reference* and *test* comb lines fluctuate together. Since this variation is happening to both the in-phase and quadrature components simultaneously, there is no detrimental effect on the recovered phase noise traces. Nevertheless, this observation is unexpected which insisted it be thoroughly investigated. Insight regarding this amplitude variation was attained by observing the change in the intermediate beat frequency, f_{IF} , as a function of trace number. For each recovered phase noise trace, the frequency offset is estimated and removed using the Hoffman algorithm which was thoroughly discussed in section 4.3. Thus, for each pair

of *reference* and *test* comb lines, 30 traces were recovered, implying 30 different frequency offset estimates. The resulting frequency offset estimates for each of the 30 recovered traces for the *reference* comb line and *test* comb line where $n = 15$ are shown in Figure 4.37.

Figure 4.37: Intermediate Frequency versus Trace Number ($n = 15$)

Observing the intermediate frequency variation as a function of trace number reveals that it follows the same trend as the amplitude variation shown in Figure 4.36, for the observed *reference* and *test* comb line. This result demonstrates that the observed amplitude variation is intimately related to the frequency difference between each comb line and the respective TLS output which implies the cause of the amplitude variation may stem from some non-

idealities present in the frequency response of the BPDs. This led to a thorough investigation of the frequency response for each BPD.

The amplitude variation observed within Figure 4.36 is on the order of 0.8 V, which would translate to more than a 2 dB variation in the measured magnitude response of the BPDs. However, the results in Figure 4.9 demonstrated that the measured frequency response of each BPD did not exhibit fluctuations of this magnitude. To highlight this, the frequency responses of the four BPDs are shown on a linear frequency scale in Figure 4.38.

Figure 4.38: Frequency Responses of the BPDs (Linear)

Throughout the data collection process, the intermediate frequency was monitored to maintain a value near 100 MHz, as is evidenced by the recovered frequency offset estimations in Figure 4.37. In each of the four BPD frequency responses of Figure 4.38

there is a slight gradual decrease in the amplitude of the magnitude response between 80 – 170 MHz, wherein most intermediate frequencies fall. However, this gradual decrease is less than 0.5 dB implying that based on the observations made, the amplitude variation observed in Figure 4.36 cannot be correctly attributed to the frequency responses of the BPDs.

Another potential source for the observed amplitude variation in Figure 4.36 is a variation in the SOP of the comb lines over time. The experimental setup of Figure 4.1 saves in-phase and quadrature traces every 30 seconds for a total of 30 traces. However, it is assumed that the SOPs of the observed comb lines and TLS signal are matched and constant at the inputs to the 90° optical hybrid. If a variation in either signal is present it would result in the observed amplitude variation of the detected in-phase and quadrature beat signals. To observe this potential variation in the comb line SOP, the *reference* comb line was passed through a polarization controller and was then incident on a polarization beam splitter (PBS). A PBS is a passive optical device which splits an input signal into orthogonal *X* and *Y* components. The polarization of the *reference* comb line was aligned using a PC to be completely in either the *X* or *Y* polarization of the PBS. This was accomplished by measuring the relative power of both PBS outputs. Once the SOP of the *reference* comb line was aligned with a single output of the PBS, both CW outputs were measured with photodiodes (PD) and sampled using the RTSO with traces collected every 30 seconds for a total of 30 traces, mimicking the behaviour of the performed experiment. The experimental setup employed can be seen in Figure 4.39.

Figure 4.39: Experimental Setup to Measure Polarization Fluctuations over Time

The mean value of each recovered trace was calculated yielding the average DC voltage of the *reference* comb line as a function of time. Results showed that the voltage over all 30 traces for the output of the PBS transmitting the comb line was approximately constant at 1.6 mV, while the other orthogonal PBS output remained at 0 V. Since the voltage of the *reference* comb line was shown to be constant it implies that the SOP does not measurably fluctuate over time. Therefore, the amplitude variation observed in Figure 4.36 cannot be attributed to a fluctuation in the SOP of the measured comb lines.

4.8 – Experimental Implementation Summary

Chapter 4 began by providing a detailed overview of the experimental setup used to perform the phase noise recovery process. Each component within the experimental setup was individually discussed allowing for relevant device specifications to be introduced and characterized. The entirety of the experimental setup was then observed and a systematic process outlined for the recovery of time domain phase noise traces for two comb lines simultaneously. This phase noise recovery process was described through an illustrative

walkthrough which aimed to provide a comprehensive description of each step required in the experimental procedure.

After the experimental phase noise recovery procedure was established, a systematic data collection method was outlined which allowed the correlation properties of comb lines across a subset of the CCS spectrum to be quantified. The data collected also provided insight regarding the changing magnitude of the recovered phase noise traces through a normalized RMS phase difference calculation. Together, the experimental results established that the observed decrease in correlation across the spectrum was intimately related to the increasing magnitudes of the phase noise across the spectrum of the CCS system for increasing values of n . This led to the implementation of the PNT analysis.

The PNT analysis allowed the relative contributions of the ASE noise and timing jitter to the overall phase noise of each comb line to be quantified. Results demonstrated that the ASE noise is constant across the CCS spectrum and dominates the overall phase noise if the modal separation between a pair of comb lines is relatively small. However, as the modal separation between a pair of comb lines increases, the relative contribution of the timing jitter also increases, resulting in an increase in the overall magnitude of the observed comb line's phase noise. Additionally, it was shown that the ASE noise and timing jitter fluctuations are uncorrelated. Thus, the PNT analysis allowed the relative contributions to the phase noise present on each comb line to be quantified as well as provided an explanation for measured decrease in correlation across the CCS spectrum. Furthermore, the results of the PNT analysis were also used to provide an estimate of the optical linewidth for each of the observed comb lines across the CCS spectrum. The results of this

linewidth characterization aligned with previous results of the DSH measurements performed in section 2.6.

Lastly, two unresolved issues in the recovered experimental data were discussed. The first issue was a relative increase in the variance of the timing jitter in modes adjacent to the minimum linewidth *reference* comb line. The second issue surrounded an inherent variation found within the recovered amplitudes of the in-phase and quadrature signals sampled by the RTSO through the recovered photocurrent traces. In both cases, the observed non-idealities were discussed and possible causes investigated. However, the source of these two issues was not found. Although these non-idealities are present in the results presented it should be noted that they do not affect conclusions drawn throughout Chapter 4.

Chapter 5 – Conclusions and Future

Work

5.1 – Conclusions

The research performed throughout this thesis provides an in-depth characterization of a prototype 25 GHz quantum-dot coherent comb source system. The output of the CCS system was extensively observed in both the time and frequency domains. Visualizing the optical spectrum verified the 25 GHz FSR and quantified the tunability of the laser source through an observation of the spectral range. The CCS system was shown capable of covering the entire optical C-Band which is a necessity for practical implementation as a laser source in a DWDM system. Observations of the optical spectrum also revealed that the spectral flatness of the entire CCS spectrum is within a range of 4 dB, and the comb lines yield an extinction ratio of approximately 40 dB. The electrical spectrum of the CCS system was also measured demonstrating a Lorentzian profile surrounding 25.0187 GHz. Furthermore, the phase noise of the CCS spectrum was quantified which led to a measured timing jitter of 5.25 ps. Modern tunable lasers are documented to exhibit jitter on the order of femtoseconds, which implies that the timing jitter, and thus the phase noise present in the CCS system is currently a major device limitation.

The next stage of measurements involved characterizing individual comb lines across the CCS spectrum. The RIN of each comb line across the CCS spectrum was shown to be relatively consistent with a mean value of -117 dB/Hz in a 1 GHz bandwidth. High

performance tunable lasers are shown to exhibit much less RIN, on the order of -140 dB/Hz in a 1 GHz bandwidth. Therefore, this measurement indicates that the amplitude stability of the individual comb lines exhibited by the CCS system is another property which is not yet on par with modern tunable lasers.

The optical laser linewidth of each individual comb line was also investigated using the DSH measurement technique. DSH results demonstrated that the linewidths of the comb lines across the CCS spectrum varied from 2 – 12 MHz, increasing quadratically from a distinct minimum linewidth comb line. This observation experimentally demonstrated that the linewidths, and thus the phase noise, are not constant for all comb lines across the spectrum, as expected. As a result, this led to a much more detailed analysis of the phase noise present within each comb line of the CCS system.

The next stage of CCS system characterization involved the design, simulation and implementation of an experimental setup capable of recovering time domain phase noise trajectories for two comb lines. This was accomplished by employing a coherent receiver as a phase detector. The experimental setup was first modelled in MATLAB to verify experimental validity. The model acted as a reference for ideal experimental performance, and allowed for an investigation of the impact component non-idealities would have on the recovered phase noise trajectories. Additionally, this model allowed much of the required signal processing steps for time domain phase noise recovery to be developed and tested prior to experimental implementation. Furthermore, simulating system behaviour allowed for preliminary characterization techniques, such as the Pearson correlation coefficient and the PNT analysis to be thoroughly explored and refined. Modelling the experimental setup

was a time-consuming but critical preliminary step taken throughout this work and was constantly referred to throughout the experimental process.

Practically implementing the experimental setup allowed simultaneous phase noise trajectories for pairs of comb lines across the CCS spectrum to be recovered. A systematic recovery procedure was employed which involved recovering phase noise traces for a *reference* comb line and *test* comb line. The *reference* comb line was chosen to be the minimum linewidth comb line and was held constant throughout the experimental phase noise recovery procedure. Contrastingly, the *test* comb line was varied across the spectrum. This procedure allowed for a meaningful spectral characterization to be achieved. Phase noise trajectories were simultaneously recovered for pairs of *reference* and *test* comb lines within the low-linewidth region of the CCS spectrum. This enabled a comparison of time-dependent properties, such as the correlation coefficient. Correlation coefficients were calculated for each of the observed *reference* and *test* comb line pairs. Results demonstrate that for *test* comb lines close to the *reference* comb line (less than ± 5 modes), the correlation was on the order of 0.95 implying a nearly identical evolution in time. However, as the modal separation is increased (greater than ± 5 modes), the correlation coefficient begins to decrease quadratically. This decrease in measured correlation coefficient implies that the phase noise between modes evolves less similarly as a function of time. Additionally, the quadratic nature of the decrease in correlation coefficient mirrors the trend of the increase in linewidths observed by the DSH measurements. The DSH linewidth measurements provided preliminary insight regarding the variation in magnitude of the phase noise across the comb lines of the CCS spectrum. This behaviour was further explored through a normalized RMS phase difference calculation which quantified the

magnitude of the phase difference between the recovered phase noise trajectories of the *test* and *reference* comb lines. Results of this calculation demonstrated that the magnitude of the phase noise fluctuations increase as a function of modal separation from the minimum linewidth *reference* mode, as expected from the preliminary DSH results. The attained results agree with current literature which mathematically describes the behaviour of phase noise within a MLL.

The recovered phase noise trajectories were then further investigated using a PNT analysis to experimentally observe the relative contributions of the ASE noise and timing jitter to the overall phase noise present within individual comb lines of the CCS system. Using the expressions in [47], the coherent phase noise recovery scheme was strategically employed to allow for the isolation of each phase noise contribution. Results of the PNT analysis demonstrated that contribution of ASE noise for a particular comb line throughout the CCS system is constant and dominates the overall phase noise of a comb line for *test* comb lines surrounding the minimum linewidth *reference* mode (less than ± 5 modes). However, as modal separation between the *reference* comb line and *test* comb line increases (greater than ± 5 modes), the relative contribution of the timing jitter increases proportionally and begins to yield a larger impact on the overall phase noise present in a single mode. This result demonstrates that the observed increase in the magnitude of the phase noise (as evidenced by the RMS difference results) across the CCS spectrum is due to the relative increase in timing jitter occurring as a function of mode number. Additionally, the correlation between the phase noise of the ASE noise contribution and the timing jitter was experimentally shown to be approximately 0 for all observed comb line pairs. This result implies that the decrease in the correlation coefficient for pairs of comb lines with

increasing modal separation across the CCS system can be attributed to the relative increase in timing jitter fluctuations. Lastly, the PNT analysis was used to provide an additional method for estimating the linewidth and was performed for a subset of comb lines across the CCS spectrum. The results of the PNT linewidth characterization agreed with those of the DSH measurement confirming the validity of the PNT analysis as a viable method for linewidth estimation.

One of the fundamental takeaways from the research completed throughout this thesis is that the phase noise present on each comb line across the spectrum of the CCS system is impractically large for use in a DWDM system. This was determined through various linewidth measurements which demonstrated that the minimum linewidth comb line was on the order of 1-2 MHz (DSH and PNT), whereas modern discrete tunable laser sources currently employed in modern coherent optical communication exhibit laser linewidths on the order of 100 kHz. These preliminary linewidth measurements drove the focus of the research towards obtaining a better fundamental understanding of the CCS system phase noise characteristics. A thorough analysis of these phase noise characteristics, such as the work performed throughout this thesis, provides insight regarding the potential of implementing phase noise compensation techniques (due to the measured intrinsic coherence) which would allow the CCS system to be used as a source for coherent optical communications.

A second important insight attained throughout this research surrounds the correlation properties across the spectrum of the CCS system. A proposed benefit of implementing an optical frequency comb (such as the CCS system) is the notion of a coherent spectrum which enables a potential reduction in the digital signal processing complexity at the

receiver of an optical communication channel. The results attained throughout this research have demonstrated that the correlation, and thereby the intrinsic coherence, of the CCS system measurably decreases as a function of modal separation. This result aligns with the theoretical description of MLL phase locking behaviour. However, results did experimentally observe a strong linear correlation in time domain phase noise trajectories for small subsets of comb lines (less than 10) across the CCS spectrum. Thus, to maximally benefit from the spectral coherence of the CCS system, individual sections of the CCS spectrum must be considered as opposed to the entirety of the spectrum.

In conclusion, the phase noise characteristics exhibited by the 25 GHz quantum-dot CCS system were extensively observed in both the time and frequency domains. The results demonstrated that although the magnitude of the phase noise present in the CCS system is currently too large to be practically implemented in a DWDM system, the measured intrinsic coherence present among all comb lines emphasizes the potential for effective methods of phase noise compensation. Additionally, all observed results were extensively compared with the theoretical expectations of MLLs to provide a better fundamental understanding of CCS system behaviour.

5.2 – Future Work

For the CCS system to become practically implementable as a source for coherent optical communication there exists the need to reduce the phase noise exhibited by individual comb lines. This leads to the possibility of a significant amount of further experimental work surrounding CCS system phase noise reduction. There have been numerous results demonstrated in recent literature which accomplish phase noise reduction in MLLs

(emphasizing that the magnitude of phase noise present in MLLs is a common issue) [66] [67] [68] [69]. However, some of the common approaches are shown to be inefficient or impractical for system level integration. As such, there exists the possibility of optimizing this phase noise reduction to be as efficient as possible from an implementation perspective.

One feasible approach which has been used to accomplish the simultaneous phase noise reduction of all modes is a feed-forward heterodyne scheme [66] [67] [68] [69]. This technique involves filtering a single optical comb line to be mixed with a local oscillator, the output of which is used to drive a MZM operating on the MLL output. Due to the coherence of the OFC, this approach has been demonstrated capable of reducing the phase noise of all comb lines exhibited simultaneously. Experimental results have shown comb line linewidths on the order of a few MHz reduced to the sub 100 kHz region. Thus, experimental implementation of the feed-forward heterodyne scheme is a very practical extension of the work that has been completed through this research. This feed-forward heterodyne scheme could be used as a reference for an effective method of phase noise compensation while other novel techniques are developed and explored. Any novel phase noise compensation techniques could then be tested and actively compared against the performance of the feed-forward heterodyne scheme through a quantification of the resulting comb line linewidths post phase noise compensation. An extensive investigation of possible phase noise compensation techniques would allow for an optimal method to be determined. The chosen technique could then be actively used to reduce the phase noise present on comb lines exhibited by the CCS system allowing the system to become practically implementable as a source for coherent optical communication. Thus, the CCS

system could then be used in a transmission experiment capacity. The performance of the CCS system in transmission experiments could then be compared to discrete lasers in terms of net average data rate, optical signal-to-noise, and bit error rates. Transmission experiments could test various complex modulation formats at the transmitter, as well as different digital signal processing algorithms at the receiver. Additionally, the performance of the CCS system could then be compared with similar photonic devices currently under investigation throughout the research community. Furthering research in this capacity will lead to a quantification of the performance benefits of implementing an optical frequency comb as a source for coherent optical communication. Additionally, the correlation properties of the phase-noise reduced CCS spectrum could be re-characterized which would observe the impact of phase noise reduction on the overall coherence present within CCS system. Overall, the work presented throughout this thesis has paved the way for a significant amount of further research and experimental work surrounding this CCS system.

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