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**REVIEW OF NONLINEAR OSCILLATORY SHEAR FLOW
NOTATIONS AND PRESENTATIONS: POLYMERIC LIQUIDS**

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ABSTRACT

We review notations for, along with ways of presenting, the shear stress responses to large-amplitude oscillatory shear flow (LAOS). We find that the Fourier loss and storage viscosities to be the simplest primal notations for interpreting LAOS. The relative intensities provide the best evidence for oscillatory shear entering the large-amplitude regime. Deviation from linear viscoelastic can be observed through the distortions of Lissajous loops. We explore these loops in their elastic and viscous projections. The centerpiece of our review is Table II, which summarizes experimental measurements for polymer melts and solutions, targeting 21st century publications. Our review also provides conversion formulae to go from any of many notations to the Fourier loss and storage viscosities. Table II also defines which part of the LAOS measurement field has yet to be ploughed and shines light on which notations have been used, and for which purposes, to interpret nonlinearities.

Keywords: Large-amplitude oscillatory shear flow; LAOS; medium-amplitude oscillatory shear flow; MAOS; stress decomposition; Fourier-transform rheology; FTR

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I EVOLUTION OF NOTATIONS IN LAOS

For the past half-century, the large-amplitude oscillatory shear (LAOS) experiment has been exploited to elucidate nonlinear viscoelastic behaviors of complex fluids or soft matters [1]. Since the earliest implementations of LAOS [2,3,4], several notations arose to report nonlinear behaviors. In this work, only measurable quantities involving the shear stress response in LAOS are considered. Table 1 of [5] summarizes normal stress difference measurements in LAOS along with notations.

LAOS is performed by subjecting fluid to a cosinusoidal shear rate, $\dot{\gamma} = \dot{\gamma}_0 \cos \omega t$, (see Fig. 1 of [6]). If the experimental condition falls within the large-amplitude regime (cross-hatching in Fig. 14 of [7]), the shear stress response contains significant higher harmonics, which are all odd (Eq. (21) of [8]; Eq. (66) of [9]):

$$\tau_{yx} = - \underbrace{\sum_{n=1, \text{odd}}^{\infty} \dot{\gamma}^0 \eta'_n \cos n\omega t}_{\text{viscous } (\sigma'' \equiv \sigma''_1 + \sigma''_3 + \dots)} - \underbrace{\sum_{n=1, \text{odd}}^{\infty} \dot{\gamma}^0 \eta''_n \sin n\omega t}_{\text{elastic } (\sigma' \equiv \sigma'_1 + \sigma'_3 + \dots)} \quad (1)$$

For linear viscoelasticity (LVE), only $\eta' \equiv \lim_{\dot{\gamma}^0 \rightarrow 0} \eta'_1$ and $\eta'' \equiv \lim_{\dot{\gamma}^0 \rightarrow 0} \eta''_1$ appear in Eq. (1).

The higher harmonics were first observed as wave distortion in the time domain (see Figure 1a and also Figure 2 of [10]), and provoked the study of nonlinear oscillatory shear. All of the loss and storage viscosities in Eq. (1), η'_n and η''_n , are readily obtained from a measured stress wave using the discrete Fourier transform [1]. We call calculation Fourier transform rheology (FTR) [11,12] because we transform $\tau_{yx}(t)$ in the time domain to its Fourier coefficients in the frequency domain (see Fig. 2 of [13]).

Another way of observing the nonlinear shear stress response is through its phase difference with the cosinusoidal shear rate wave:

$$\tau_{yx} = - \sum_{n=1, \text{odd}}^{\infty} \tau_n \cos(n\omega t + \delta_n) \quad (2)$$

where δ_n is the phase angle, and where $\eta'_n = (\dot{\gamma}^0)^{-1} \tau_n \cos \phi_n$ and $\eta''_n = -(\dot{\gamma}^0)^{-1} \tau_n \sin \phi_n$. Eq. (1) can also be written in the form of power series in the shear rate amplitude:

$$\tau_{yx} = - \sum_{m=1, \text{odd}}^{\infty} \gamma_0^m \sum_{n=1, \text{odd}}^m [G''_{nm} \cos n\omega t + G'_{nm} \sin n\omega t] \quad (3)$$

thus $\eta'_n = \omega^{-1} \sum_{m=n}^{\infty} \gamma_0^{m-1} G''_{nm}$ and $\eta''_n = \omega^{-1} \sum_{m=n}^{\infty} \gamma_0^{m-1} G'_{nm}$. When harmonics higher than the third are negligible, i.e. $\tau_{yx} = O(\gamma_0^5)$, the shear stress exhibits weak nonlinear behaviors, and we call this regime medium-amplitude oscillatory shear (MAOS) (see Eq. (15)ff. of [14]). Regardless, Eq. (1) is the simplest and the most primal in LAOS. Furthermore, whereas the coefficients in Eq. (1), (η'_n, η''_n) , are functions of both ω and $\dot{\gamma}^0$, those in Eq. (3), (G''_{nm}, G'_{nm}) , are just functions of ω . The alternative to Eq. (3) is the power series expansion in $\dot{\gamma}^0$ (Eq. (157) of [15]):

$$\tau_{yx} = - \sum_{m=1, \text{odd}}^{\infty} (\dot{\gamma}^0)^m \sum_{n=1, \text{odd}}^m [\eta'_{nm} \cos n\omega t + \eta''_{nm} \sin n\omega t] \quad (4)$$

where $\eta'_n = \sum_{m=n}^{\infty} (\dot{\gamma}^0)^{m-1} \eta'_{nm}$ and $\eta''_n = \sum_{m=1}^{\infty} (\dot{\gamma}^0)^{m-1} \eta''_{nm}$. However, whereas $(G''_{nm}, G'_{nm}) [=] (G''_{11}, G'_{11})$, the dimensions of the Fourier coefficients in Eq. (4) are complicated as $(\eta'_{nm}, \eta''_{nm}) [=] (\eta'_{11}, \eta''_{11}) / (\dot{\gamma}^0)^{m-1}$.

In this review, all notation conversions will be referred to the one given in Eq. (1), i.e. η'_n and η''_n . For instance, we define the loss and storage Fourier moduli as $G''_n \equiv \omega \eta''_n$ and $G'_n \equiv \omega \eta'_n$. For linear viscoelasticity (LVE), $G'' \equiv \lim_{\dot{\gamma}^0 \rightarrow 0} G''_1$ and $G' \equiv \lim_{\dot{\gamma}^0 \rightarrow 0} G'_1$. Also, to recast Eq. (1) in the form of loss and storage intensities, we set $I''_n \equiv \dot{\gamma}^0 \eta''_n$ and $I'_n \equiv \dot{\gamma}^0 \eta'_n$, so that $I_n \equiv \sqrt{I'^2_n + I''^2_n}$ (see Fig. 2 of [13]).

The concept of intrinsic quantities is widely used in LAOS by simply taking each targeted quantity, x , in its limit of vanishing strain amplitude, i.e. $x_0 = \lim_{\gamma_0 \rightarrow 0} x$. For example, recently, this concept was applied to the complex moduli (Eq. (6) of [16]):

$$G'_{n,0} \equiv \lim_{\gamma_0 \rightarrow 0} G'_n; G''_{n,0} \equiv \lim_{\gamma_0 \rightarrow 0} G''_n; G^*_{n,0} \equiv \lim_{\gamma_0 \rightarrow 0} G^*_n \equiv \lim_{\gamma_0 \rightarrow 0} \sqrt{G'^2_n + G''^2_n} \quad (5), (6), (7)$$

These provide insight information of the remainder, which is a sole function of test frequency. Quantities made intrinsic are denoted by the subscript 0.

Of immense relevance to LAOS is the n th harmonic *relative intensity* with respect to the first, $I_{n/1} \equiv I(n\omega)/I(\omega)$. In practice, advanced rheometers, such as the MCR 702 (Anton-Parr), now report $I_{n/1}$. One can also extract I_n graphically from $\tau_{yx}(\gamma)$ or $\tau_{yx}(\dot{\gamma})$ loops [1,17]. In our opinion, $I_{n/1}$, where $n > 3$, are the best quantities for determining unequivocally the limits of linear viscoelasticity in oscillatory shear flow (see Figure 1b).

Hyun and Wilhelm [18] proposed the nonlinear material function, $Q_3 \equiv I_{3/1}/\gamma_0^2$ for identifying the $I_{3/1} \propto \gamma_0^2$ regime. Called MAOS, this regime presents as a plateau in $Q_3(\gamma_0)$ curves (see Fig. 4 of [18]). Therefore, Q_3 is a practical nonlinear parameter that normally used to identify MAOS. The nonlinear material function, Q_3 , are further generalized for n th order, $Q_n \equiv I_{n/1}/\gamma_0^{n-1}$ where $n = 3, 5, 7, \dots$ (Eq. (7) of [16]).

When the strain amplitude is considered *moderate*, the corresponding plateau value determines $Q_{3,0}(\omega) \equiv \lim_{\gamma_0 \rightarrow 0} Q_3$. Furthermore, $Q_{3,0}(\omega)$ curves are constructed to depict MAOS responses. The Figure 2 of [19] illustrates how to construct a master curve of $Q_{3,0}(\omega)$. This review reports progress in nonlinear oscillatory shear measurements and notations, both MAOS and LAOS.

Colloids are elastic liquids. Insightfully, Cho *et al.* [20] show that the sine terms in the shear stress response, Eq. (1), represent *elastic* contributions, and the cosine terms,

viscous (see also Appendix B of [21]). They thus proposed the following decomposition (Eq. (3) of [21]):

$$\tau_{yx} = - \underbrace{\sum_{n=1, \text{odd}}^{\infty} G''_{Tn} \gamma_0^n \cos^n \omega t}_{\text{viscous } (\sigma'' \equiv \sigma''_1 + \sigma''_3 + \dots)} - \underbrace{\sum_{n=1, \text{odd}}^{\infty} G'_{Tn} \gamma_0^n \sin^n \omega t}_{\text{elastic } (\sigma' \equiv \sigma'_1 + \sigma'_3 + \dots)} \quad (8)$$

Both σ' and σ'' are single-valued functions that form *intra-cycles* when also plotted against $\tau_{yx}(\gamma)$ and $\tau_{yx}(\dot{\gamma})$ loops respectively (see Fig. 2 of [22]). Comparing the intra-cycles reveals the colloid's proximity to purely elastic or viscous behaviors. These two single-valued functions can be rewritten in terms of the loss and storage viscosities [see Eq. (1)].

Ewoldt *et al.* [23,24] advanced the decomposition by proposing:

$$\tau_{yx} = - \underbrace{\sum_{n=1, \text{odd}}^{\infty} \dot{\gamma}^0 v_n T_n(\cos \omega t)}_{\text{viscous } (\sigma'' \equiv \sigma''_1 + \sigma''_3 + \dots)} - \underbrace{\sum_{n=1, \text{odd}}^{\infty} \gamma_0 e_n T_n(\sin \omega t)}_{\text{elastic } (\sigma' \equiv \sigma'_1 + \sigma'_3 + \dots)} \quad (9)$$

where $v_n = \eta'_n$ and $e_n = \omega \eta''_n (-1)^{(n-1)/2}$, and where $T_n(x)$ are the Tschebycheff polynomials of the first kind. When orders exceeding the third are negligible, the stress is in the MAOS regime (see Eq. (15)ff. of [14]). Hence, v_n and e_n are also practical measurable notations to identify MAOS.

To capture nonlinearities, such as those exhibited by wormlike micellar solutions, the following four material functions are often reported: (i) large-rate, (ii) minimum-rate dynamic viscosities (Eqs. (8) and (9) of [23]):

$$\eta'_L \equiv \left. \frac{\tau_{yx}}{\dot{\gamma}} \right|_{\dot{\gamma}=\pm \dot{\gamma}^0} = \sum_{n=1, \text{odd}}^{\infty} \eta'_n; \quad \eta'_M \equiv \left. \frac{d\tau_{yx}}{d\dot{\gamma}} \right|_{\dot{\gamma}=0} = \sum_{n=1, \text{odd}}^{\infty} n \eta'_n (-1)^{(n-1)/2} \quad (10), (11)$$

(iii) large-rate and (iv) minimum-rate Fourier moduli (Eqs. (6) and (7) of [23]):

$$G'_L \equiv \left. \frac{\tau_{yx}}{\gamma} \right|_{\gamma=\pm \gamma_0} = \omega \sum_{n=1, \text{odd}}^{\infty} \eta''_n (-1)^{(n-1)/2}; \quad G'_M \equiv \left. \frac{d\tau_{yx}}{d\gamma} \right|_{\gamma=0} = \omega \sum_{n=1, \text{odd}}^{\infty} n \eta''_n \quad (12), (13)$$

Whereas G'_M and η'_M are slopes at the ordinates of $\tau_{yx}(\gamma)$ and $\tau_{yx}(\dot{\gamma})$ loops respectively, G'_L and η'_L are slopes of the secants connecting the origin to the intra-cycle extremities, $\sigma'(\gamma)|_{\gamma_0}$ and $\sigma''(\dot{\gamma})|_{\dot{\gamma}^0}$, respectively. Fig. 3 of [24] depict η'_L, η'_M, G'_L and G'_M on $\tau_{yx}(\gamma)$ and $\tau_{yx}(\dot{\gamma})$ loops, as well as their corresponding intra-cycles.

Ewoldt *et al.* [23] quantified distortions of $\tau_{yx}(\gamma)$ and $\tau_{yx}(\dot{\gamma})$ loops, respectively, with the following *shear-thickening ratio* and *strain-stiffening ratio*:

$$T \equiv (\eta'_L - \eta'_M) / \eta'_L, \quad S \equiv (G'_L - G'_M) / G'_L \quad (14), (15)$$

Thus, absent higher harmonics, $\eta'_L = \eta'_M$ and $G'_L = G'_M$, so that $T = S = 0$. Of course, $T = S = 0$ can also obtain with higher harmonics. For colloids, $T = S = 0$ might mean either small-amplitude or where higher harmonics are absent from the nonlinear

response, such as when molten mozzarella behaves as a Lodge rubber-like liquid (see Fig. 9 of [25];[26]). Continuing with the *ontology*, $T > 0$ signifies *shear-thickening* behavior, and $T < 0$, *shear-thinning*, and $S > 0$ signifies *strain-stiffening* behavior, and $S < 0$, *strain-softening* (see discussion after Fig. 2 of [23]; see also pp. 58–59 of [27] for strain-induced behaviors). The notation given in Eqs. (9)–(15) can be readily obtained from the raw stress signal by use of the software developed by MIT [28]. Table I summarizes all notations in this review.

II DEPICTION OF NONLINEARITIES

Lissajous loops are the most fundamental and powerful way of observing fluid nonlinearity through their distortions from ellipticity. These loops involve three quantities: τ_{yx} , $\dot{\gamma}$ and γ . Three-dimensional loops can thus be constructed having τ_{yx} as height above the $\dot{\gamma}\gamma$ -plane (see Figure 1c and also Fig. 3 of [29]). Projection of these 3D-loops onto the $\tau_{yx}\dot{\gamma}$ -plane, compares the liquid to the purely viscous fluids (see Fig. 7 of [30]), and the projections onto the $\tau_{yx}\gamma$ -plane, to the purely elastic solid. In these ways, these 3D-loops and their projections deepen our physical insight. The corresponding intra-cycles can also be included on the projections. The $\tau_{yx}(\gamma)$ loop area, $\oint \tau_{yx} d\gamma$, is the viscous dissipation per unit volume per cycle, and governs the average temperature rise in the deforming colloids (see Eq. (177)ff. of [15]). In the limit of low frequency, the $\tau_{yx}(\dot{\gamma})$ loop collapses to the curve for steady shear behavior. Thus, the smaller the $\tau_{yx}(\dot{\gamma})$ loop area, $\oint \tau_{yx} d\dot{\gamma}$, the closer we get to the purely viscous curve, $\tau_{yx}(\dot{\gamma}) = \sigma''(\dot{\gamma})$.

Test conditions can be mapped onto the $\lambda\omega$ - $\lambda\dot{\gamma}^0$ plane (see figure on p. 133 of [31]), where λ is the longest colloid relaxation time. Called a Pipkin map, this can be used to indicate the LAOS regime, which is bounded by the ordinate (steady shear flow: $\lambda\omega = 0$) and, at least, by $\lambda\dot{\gamma}^0 = 1$ (compare Figure 2 with Fig. 1 of [32]). The Lissajous loops can be miniaturized and mapped into cells on the $\lambda\omega$ - $\lambda\dot{\gamma}^0$ plane to create the *Ewoldt grid* (see §Ewoldt Grids of [33]). This clever combination is recently widely used. Now widely used [34,35,36,37], these clever combinations of Lissajous loops with the Pipkin map offer a phenomenal view of how colloids reveal their nonlinearity in oscillatory shear flow.

III INTERPRETATION OF NONLINEARITIES

Table II classifies chronologically the literature on shear stress measurements and reporting notations in nonlinear oscillatory shear of polymeric liquids. Dealy and coworkers [4,49] used Lissajous loop distortions to elucidate nonlinear behaviors of polymer melts in LAOS.

From Table II, we see the huge impact that $I_{n/1}$ have had on the field of nonlinear oscillatory shear flow, since its use on atactic polypropylene [11]. $I_{n/1}$ were then

successfully used to probe morphology and dispersion of polymer blends [38,63,80]. Of course, the problem with using $I_{n/1}$ is that information is lost. For instance, η'_3 and η''_3 from whence $I_{3/1}$ comes, cannot be deduced from $I_{3/1}$. Many articles thus opt for the Ewoldt grids [36,37,39, 40,41] over $I_{n/1}$.

For yield-stress fluids (YSF), some construct two plots, each with two curves: (i) $\sigma'_{\max}(\gamma|_{\sigma'_{\max}})$ with $\sigma_{\max}(\gamma|_{\sigma_{\max}})$, and (ii) $\sigma''_{\max}(\dot{\gamma}|_{\sigma''_{\max}})$ with $\sigma_{\max}(\dot{\gamma}|_{\sigma_{\max}})$ (see Fig. 10 of [22]). Called *stress bifurcation*, this technique is used to determine YSF solid-liquid transitions. Whereas, departures of the $\sigma'_{\max}(\gamma|_{\sigma'_{\max}})$ curve from $\sigma_{\max}(\gamma|_{\sigma_{\max}})$ on Plot (i) indicate the start of yielding, departures of $\sigma''_{\max}(\dot{\gamma}|_{\sigma''_{\max}})$ from $\sigma_{\max}(\dot{\gamma}|_{\sigma_{\max}})$ on Plot (ii) indicate the end of yielding. We find this a remarkable use for LAOS.

The shear-thickening ratio is plotted against strain or shear rate amplitudes to interpret intra-cycle behaviors. Our Table II indicates that, filled polymers, the popularity of $T(\gamma_0)$ is increasing, and specifically, to explain network stability [41,42]. That is, in the filled system, the liquid matrix percolates through the network, so long as $\partial S(\gamma_0)/\partial \gamma_0 \geq 0$ or $\partial T(\gamma_0)/\partial \gamma_0 \geq 0$, otherwise, the network ruptures.

A host of experimental errors in LAOS are manifested by even harmonics in the measured shear stress. For instance, if the measured time series, $\tau_{yx}(t)$, is collected too soon after startup, then even harmonics appear in the frequency domain. Further, even harmonics will also appear in the frequency domain whenever an aperiodic $\tau_{yx}(t)$ is collected. On a $\tau_{yx}(\dot{\gamma})$ loop, these even harmonics reveal themselves as two-fold asymmetry, that is, as $\tau_{yx}(\dot{\gamma}) \neq -\tau_{yx}(-\dot{\gamma})$. One must never misinterpret even harmonics in the measured shear stress as material properties, that is, as $I_{2/1}$ or $I_{4/1}$ or the like [43].

When the shear stress response to LAOS is aperiodic [44,45], slip can be to blame (see Figure 4 of [67]). For this, strange attractors are sometimes uncovered in the *phase plane*, that is, where $\tau_{yx}(t)$ is plotted against its own time derivative [46,47]. To suppress slip, the plate of the shear fixture can be roughened [40]. Furthermore, some report that the measurements of G'_M or η'_M are unaffected by slip.

IV CONCLUSION

This paper reviews notations for along with ways of presenting the shear stress responses to large-amplitude oscillatory shear flow (LAOS). We find the Fourier loss and storage viscosities, η'_n and η''_n , to be the simplest primal notations for LAOS. The relative intensities, $I_{n/1}$, provide the best evidence for oscillatory shear entering the

large-amplitude regime. When Q_3 versus γ_0 is constant, at $Q_{3,0}$, this nonlinear behaviour is called medium-amplitude oscillatory shear (MAOS).

Deviation from linear viscoelastic can be observed through the distortions of Lissajous loops. These loops in elastic projections, $\tau_{yx}(\gamma)$, provide the strain-stiffening ratio, S , which elucidate strain-induced behaviours. By contrast, in viscous projections, $\tau_{yx}(\dot{\gamma})$, Lissajous loops provide the shear-thickening ratio, T , which elucidate shear-rate-induced behaviours. Graphs of these strain-induced and shear-rate-induced behaviors reveal the convexities of purely elastic or purely viscous contributions, $\sigma'(\gamma)$ and $\sigma''(\dot{\gamma})$ curves, on $\tau_{yx}(\gamma)$ and $\tau_{yx}(\dot{\gamma})$ loops.

The centerpiece of our review, Table II, summarizes experimental measurements for polymer melts and solutions, targeting 21st century publications. Our review also provides conversion formulae to go from any of many notations to the Fourier loss and storage viscosities, η'_n and η''_n . Table II also defines which part of the LAOS measurement field has yet to be ploughed. Table II also shines light on which notations have been used, and for which purposes, to interpret the nonlinearities.

Although we have delimited our review to the shear stress response, we admit that a like review of the normal stress difference responses to LAOS would be at least as interesting. We leave this for another day.

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VI APPENDICES

a (τ_n, ϕ_n) to (η'_n, η''_n)

To convert from (τ_n, ϕ_n) to (η'_n, η''_n) , we use the identity:

$$\tau_n \cos(n\omega t + \phi_n) = \tau_n \cos \phi_n \cos n\omega t - \tau_n \sin \phi_n \sin n\omega t \quad (16)$$

which, when compared with Eq. (1), gives:

$$\tau_n \cos \phi_n = \dot{\gamma}_0 \eta'_n; -\tau_n \sin \phi_n = \dot{\gamma}_0 \eta''_n \quad (17), (18)$$

So that:

$$\eta'_n = \dot{\gamma}_0^{-1} \tau_n \cos \phi_n; \eta''_n = -\dot{\gamma}_0^{-1} \tau_n \sin \phi_n \quad (19), (20)$$

which relate (η'_n, η''_n) to (τ_n, ϕ_n) .

b (G''_{nm}, G'_{nm}) to (η'_n, η''_n)

Expanding Eq. (3) gives:

$$\begin{aligned} \tau_{yx} = & -\gamma_0 [G''_{11} \cos 1\omega t + G'_{11} \sin 1\omega t] \\ & -\gamma_0^3 [G''_{13} \cos 1\omega t + G'_{13} \sin 1\omega t + G''_{33} \cos 3\omega t + G'_{33} \sin 3\omega t] \\ & -\gamma_0^5 [G''_{15} \cos 1\omega t + G'_{15} \sin 1\omega t + G''_{35} \cos 3\omega t + G'_{35} \sin 3\omega t + G''_{55} \cos 5\omega t + G'_{55} \sin 5\omega t] \end{aligned} \quad (21)$$

By comparing this with Eq. (1), we find:

$$\eta'_n = \omega^{-1} \sum_{m=n}^{\infty} \gamma_0^{m-1} G''_{nm}; \quad \eta''_n = \omega^{-1} \sum_{m=n}^{\infty} \gamma_0^{m-1} G'_{nm} \quad (22), (23)$$

which relate (η'_n, η''_n) to (G''_{nm}, G'_{nm}) .

c $(\eta'_{nm}, \eta''_{nm})$ to (η'_n, η''_n)

Expanding Eq. (4) gives:

$$\begin{aligned} \tau_{yx} = & -(\dot{\gamma}^0)^1 [\eta'_{11} \cos 1\omega t + \eta''_{11} \sin 1\omega t] \\ & -(\dot{\gamma}^0)^3 [\eta'_{13} \cos 1\omega t + \eta''_{13} \sin 1\omega t + \eta'_{33} \cos 3\omega t + \eta''_{33} \sin 3\omega t] \\ & -(\dot{\gamma}^0)^5 [\eta'_{15} \cos 1\omega t + \eta''_{15} \sin 1\omega t + \eta'_{35} \cos 3\omega t + \eta''_{35} \sin 3\omega t + \eta'_{55} \cos 5\omega t + \eta''_{55} \sin 5\omega t] \end{aligned} \quad (24)$$

By comparing this with Eq. (1), we find:

$$\eta'_n = \sum_{m=n}^{\infty} (\dot{\gamma}^0)^{m-1} \eta'_{nm}; \quad \eta''_n = \sum_{m=n}^{\infty} (\dot{\gamma}^0)^{m-1} \eta''_{nm} \quad (25), (26)$$

which relates (η'_n, η''_n) to $(\eta'_{nm}, \eta''_{nm})$. Substituting $\eta'_n \equiv \omega^{-1} G''_n$ and $\eta''_n \equiv \omega^{-1} G'_n$ into Eqs. (25) and (26) gives:

$$G''_n = \omega \sum_{m=n}^{\infty} (\dot{\gamma}^0)^{m-1} \eta'_{nm}; \quad G'_n = \omega \sum_{m=n}^{\infty} (\dot{\gamma}^0)^{m-1} \eta''_{nm} \quad (27), (28)$$

which correct Eqs. (172) and (173) of [15], and relate $(G''_n, G'_n) = (\eta'_{nm}, \eta''_{nm})$.

d (η'_n, η''_n) to (η'_M, η'_L) and (G'_M, G'_L)

Substituting (Eq. (10) of [7]):

$$G''_n = \omega \eta'_n; \quad G'_n = \omega \eta''_n \quad (29), (30)$$

into (Eqs. (3))ff. of [23]):

$$\eta'_M = \frac{1}{\omega} \sum_{n=1, \text{odd}}^{\infty} n G''_n (-1)^{(n-1)/2}; \quad \eta'_L = \frac{1}{\omega} \sum_{n=1, \text{odd}}^{\infty} G''_n \quad (31), (32)$$

$$G'_M = \sum_{n=1, \text{odd}}^{\infty} n G'_n ; G'_L = \sum_{n=1, \text{odd}}^{\infty} G'_n (-1)^{(n-1)/2} \quad (33), (34)$$

we find:

$$\eta'_M = \sum_{n=1, \text{odd}}^{\infty} n \eta'_n (-1)^{(n-1)/2} ; \eta'_L = \sum_{n=1, \text{odd}}^{\infty} \eta'_n \quad (35), (36)$$

$$G'_M = \omega \sum_{n=1, \text{odd}}^{\infty} n \eta''_n ; G'_L = \omega \sum_{n=1, \text{odd}}^{\infty} \eta''_n (-1)^{(n-1)/2} \quad (37), (38)$$

which relate (η'_n, η''_n) to (η'_M, η'_L) , and (η'_n, η''_n) to (G'_M, G'_L) .

Table I: Notation

Name	Symbol [Unit]
Dynamic viscosities, large-rate and minimum-rate	$\eta'_L, \eta'_M [M/Lt]$
Elastic and viscous Tschebycheff coefficients, n th order	$e_n [M/Lt^2], v_n [M/Lt]$
Fourier moduli, large-rate and minimum-rate	$G'_L, G'_M [M/Lt^2]$
Fourier moduli, loss and storage, n th harmonic	$G''_n, G'_n [M/Lt^2]$
Fourier moduli, loss and storage, n th harmonic, m th order of γ_0	$G''_{nm}, G'_{nm} [M/Lt^2]$
Fourier modulus, Complex, n th harmonic	$G_n^* \equiv G'_n + iG''_n [M/Lt^2]$
Fourier viscosities, loss and storage, n th harmonic	$\eta'_n, \eta''_n [M/Lt]$
Fourier viscosities, loss and storage, n th harmonic, m th order of $\dot{\gamma}^0$	$\eta'_{nm}, \eta''_{nm} [M/Lt^m]$
Fourier viscosity, Complex, n th harmonic	$\eta_n^* \equiv \eta'_n - i\eta''_n [M/Lt]$
Intensity moduli, loss and storage, n th harmonic	$I'_n, I''_n [M/Lt^2]$
Intensity moduli, n th harmonic	$I_n = \sqrt{I'^2_n, I''^2_n} [M/Lt^2]$
Intensity moduli, relative, n th harmonic	$I_{n/1} \equiv I(n\omega)/I(\omega) [M/Lt^2]$
Longest relaxation time	$\lambda [t]$
Nonlinear coefficient and its intrinsic value, n th order	$Q_n \equiv I_{n/1}/\gamma_0^{n-1}, Q_{n,0} [M/Lt^2]$
Phase angle, n th harmonic	$\delta_n [\emptyset]$
Shear rate and its amplitude	$\dot{\gamma}, \dot{\gamma}^0 [t^{-1}]$
Shear stress	$\tau_{yx} [M/Lt^2]$
Shear stress, amplitude of n th harmonic	$\tau_n [M/Lt^2]$
Shear stress, viscous and elastic contributions	$\sigma'' \equiv \Sigma \sigma''_n, \sigma' \equiv \Sigma \sigma'_n [M/Lt^2]$
Strain and its amplitude	$\gamma, \gamma_0 [\emptyset]$
Strain-stiffening and shear-thickening ratios	$S, T [\emptyset]$
Taylor moduli, loss and storage, n th order	$G''_{Tn}, G'_{Tn} [M/Lt^2]$
Test frequency	$\omega [t^{-1}]$

Legend: $M \equiv$ mass; $L \equiv$ length; $t \equiv$ time; $T \equiv$ temperature; $\emptyset \equiv$ dimensionless

Table II: Literature on shear stress measurements and reporting notations in nonlinear oscillatory shear of polymeric liquids.

First Author (year) [Ref.]		(Polym ₁ /Polym ₂) _{filler} [Solvent] Temperature°C	Reporting Notations
1.	MacDonald (1969) [2]	PS[Aroclor® 1242] 25 ; PAM[glycerol(aq)] 25 ; PIB[Primol® 355] 25 ; Phenoxy 212	$\eta'_1/\eta'(\gamma_0), G'_1/G'(\gamma_0)$
2.	Onogi (1970) [10]	PS[DEP] ; PS _c [DEP] 40-220	$\tau_{yx}(t), G'_1(\omega), G''_1(\omega)$
3.	Matsumoto (1973) [48]	PS _{Poly(S-co-DVB)} [DEP] 10-70	$G'_1(\gamma_0), G''_1(\gamma_0), G'_3(\gamma_0), G''_3(\gamma_0)$
4.	Tee (1975) [49]	HDPE 190 ; LDPE 170 ; PS 190	$\tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma}), \tau_{yx} /\dot{\gamma}^0(\omega)$
5.	Tsang (1981) [50]	br-PE ; PE 170	$\tau_{yx}(\dot{\gamma})$
6.	MacSporran (1982) [51]	PAM[glycerol(aq)] 20	$\eta'_1(\gamma_0), G'_1(\gamma_0), G'_1(\dot{\gamma}_0)$
7.	Pearson (1982) [52]	PS, PS[TCP] 180	$G'_1/G'(\gamma), G'_1/G''(\gamma)$
8.	Liu (1983) [53]	PIB[decalin] 24	$\tau_{yx}(t), \tau_{yx}(\gamma)$
9.	MacSporran (1984) [54]	PAM[glycerol(aq)] 20	$G'_3(\omega), G''_3(\omega), \delta_3(\omega), \eta'_3(\gamma_0), G'_3(\gamma_0), \eta'_5(\gamma_0), G'_5(\gamma_0)$
10.	Hatzikiriakos (1991) [44]	HDPE 180	$\tau_{yx}(t)$
11.	Adrian (1991) (1992) [46]	PU 195, 205	$\tau_{yx}(t), \tau_{yx}(\dot{\gamma}), \dot{\tau}_{yx}(\tau_{yx}), \tau_n (\omega)$
12.	Adrain (1994) [47]	LLDPE 190	$\tau_{yx}(t), \tau_{yx}(\dot{\gamma}), \dot{\tau}_{yx}(\tau_{yx}), \tau_n (\omega)$
13.	Raghavan (1995) [55]	Mineral Oil _{A®R805} 25	$G'_1(\gamma_0), G''_1(\gamma_0)$
14.	Reimers (1996) [56]	PS[DEP] 20	$G'_1(\omega, \gamma_0), G''_1(\omega, \gamma_0), \delta_1(\omega, \gamma_0), \dots, \delta_5(\omega, \gamma_0), \tau_3(\omega, \gamma_0), \tau_5(\omega, \gamma_0), G'_1(\gamma_0), \dots, G'_5(\gamma_0), G''_1(\gamma_0), \dots, G''_5(\gamma_0)$
15.	Raghavan (1997) [57]	PPG _{A®D150} 25	$G'_1(\gamma_0), G''_1(\gamma_0), G'_1(\omega), G''_1(\omega), \eta^*_1 (\omega), \eta^*_1 (\dot{\gamma}_0)$
16.	Reimers (1998) [58]	PS[TCP] 20	$\tau_{yx}(t), G'_1(\omega, \gamma_0), \delta_3(\omega, \gamma_0), G'_1(\gamma_0), \dots, G'_5(\gamma_0), G''_1(\gamma_0), \dots, G''_5(\gamma_0)$
17.	Wilhelm (1998) [11]	at-PP 25	$\tau_{yx}(t), I_{n/1}(\omega)$
18.	Yosick (1998) [59]	LDPE 25	$\tau_{yx}(\dot{\gamma})$
19.	Wilhelm (1999) [60]	Oppanol® B 100 ; Oppanol® B 200 79.85	$\tau_{yx}(t), I_{n/1}(\omega), I_{3/1}(\gamma_0)$
20.	Yziquel (1999) [61]	PO _{A®200} ; PO _{A®300} ; PO _{A®R974} ; PPG _{A®200} ; PPG _{A®R974} 25	$G'_1(\gamma_0), G''_1(\gamma_0), \tau_{yx}(\gamma)$
21.	Chopra (2000)	SMA/PMMA 210-225	$\tau_{yx}(t), G'_1(\omega), G''_1(\omega), \tau_{yx}(\dot{\gamma})$

	[62]		
22.	Wilhelm (2000) [63]	Oppanol® B 100 25	$\tau_{yx}(t), I_{n/1}(\omega), I_{3/1}(\gamma_0)$
23.	Kallus (2001) [13]	PS ; PBA ; PS/PBA ; PS/PBA(aq) ; PS/PBA[NaCl] 25	$\tau_{yx}(t), I_{n/1}(\omega), I_{3/1}(\gamma_0), \dots, I_{7/1}(\gamma_0)$
24.	Debbaut (2002) [64]	HDPE 170	$ G_1^* (\gamma_0), I_n(n), I_n(\omega), \tau_{yx}(\gamma), \tau_{yx}(t)$
25.	Hyun (2002) [27]	PVA(aq) ; PVA _{Borax} (aq) 25	$G_1'/G'(\gamma_0), G_1''/G''(\gamma_0)$
26.	Sternstein (2002) [65]	PVAc _{SiO₂} 90	$G_1'(\gamma_0), G_1''(\gamma_0), G_1''/G_1'(\gamma_0)$
27.	Clemeur (2003) [66]	LDPE 180	$\tau_{yx}(t), \tau_{yx}(\gamma), \tau_{yx}(\omega)$
28.	Hyun (2003) [67]	PVA/Borax 25	$G_1'(\gamma_0), G_1''(\gamma_0), \tau_{yx}(t), \tau_{yx}(\gamma), I_{3/1}(\gamma_0), \dots, I_{7/1}(\gamma_0)$
29.	Neidhöfer (2003) [68]	PS[DOP] 20.85-81.85	$I_{n/1}(\omega), I_{3/1}(\gamma_0), \Phi_3(\gamma^0) \equiv \delta_3 - 3\delta_1, \dots, \Phi_7(\gamma^0) \equiv \delta_7 - 7\delta_1$
30.	Fleury (2004) [69]	PE ; <i>br</i> -PE 140, 150, 160, 170	$I_{n/1}(\omega), I_{3/1}(\gamma_0), I_{5/1}(\gamma_0), I_{3/1}(\omega), I_{5/1}(\omega)$
31.	Neidhöfer (2004) [70]	PS[DOP] ; <i>br</i> -PS[DOP] 21.85-77.35	$I_{3/1}(\omega), \Phi_3(\omega) \equiv \delta_3 - 3\delta_1$
32.	Narumi (2005) [71]	SO _{Melamine} ; SO _{PMMA} ; SO _{Melamine/PMMA} 20	$\tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma})$
33.	Paquien (2005) [72]	PDMS _{A@200} 25	$\tau_{yx}(\dot{\gamma}), G_1'(\gamma_0), G_1''(\gamma_0)$
34.	Zhu (2005) [73]	PBD _{SiO₂} 40	$G_1'(\gamma_0), G_1''(\gamma_0), G_1'(t), G_1''(t)$
35.	Hyun (2006) [74]	PEO-PPO-PEO(aq) 22-26	$G_1'(\gamma_0), G_1''(\gamma_0), \tau_{yx}(t), I_{n/1}(\omega), I_{3/1}(\gamma_0), I_{5/1}(\gamma_0), I_{7/1}(\gamma_0)$
36.	Sugimoto (2006) [75]	<i>br</i> -PP 180	$\tau_{yx}(t)$
37.	Guido (2007) [76]	PIB/PDMS 35	$I_{3/1}(\dot{\gamma}^0)$
38.	Hyun (2007) [77]	PP 180 ; <i>br</i> -PP 180 ; PLA/Epoxy 180 ; PS 170-240 ; HDPE 220	$I_{3/1}(\gamma_0)$
39.	Vittorias (2007) [78]	PE 120-180	$I_{n/1}(n), I_{3/1}(\gamma_0), \Phi_3(\gamma_0) \equiv \delta_3 - 3\delta_1, \delta(G^*), \Phi_{3,0}(G^*), \Phi_{3,0}(\omega), I_{3/1}(G''/G')$
40.	Xu (2007) [79]	PDMS _{CaCO₃} -40-50	$G_1'(\gamma_0), G_1''(\gamma_0)$
41.	Carotenuto (2008) [80]	PIB ; PDMS ; PIB/PDMS 23, 30	$I_{3/1}(\gamma_0), I_{5/1}(\gamma_0), I_3'/\gamma_0(\gamma_0), I_3''/\gamma_0(\gamma_0), I_5'/\gamma_0(\gamma_0), I_5''/\gamma_0(\gamma_0), I_{2/1}(\gamma_0)$
42.	Filipe (2008) [43]	PE 180	$\tau_{yx}(t), I_{2/1}(\gamma_0), I_{3/1}(\gamma_0)$
43.	Stadler (2008) [81]	mLLDPE ; LDPE 110, 150	$\tau_{yx}(\dot{\gamma}), G_1'/ G^* (\gamma_0), G_3'/ G^* (\gamma_0)$
44.	Anderson (2009) [82]	PEO _{SiO₂} 75	$G_1'(\gamma_0), G_1''(\gamma_0)$
45.	Hyun (2009) [18]	PS 147, 160, 190 ; <i>comb</i> -PS 170, 180, 190, 200	$I_{3/1}(\gamma_0), Q_3(\gamma_0), Q_{3,0}(\gamma_0)$

46.	Li (2009) [83]	PBD ; PBD[oligo-] ; PI ; PDMS 24	$\tau_{yx}(\gamma), \tau_{yx}(t), \tau_{yx}(\gamma, t), \tau_{yx} (\gamma_0), G'_1(t), G''_1(t), I_{n/1}(n)$
47.	Brader (2010) [84]	KCl(aq) _{Latex Beads} 15, 18, 20	$G'_1(\gamma_0), G''_1(\gamma_0), \tau(\gamma), I_{n/1}(\omega), \delta_1(\gamma_0), \delta_3(\gamma_0), \delta_5(\gamma_0)$
48.	Rogers (2010) [85]	star-PBD[squalene] 20	$\tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma}), e_0(n), e_2(n), e_n(n), v_0(n), v_2(n), v_n(n), e_n(t), v_n(t), G'_1(t), G''_1(t)$
49.	Cho (2010) [21]	PEO(aq) 20	$\sigma'(\gamma), \sigma'/G'(\gamma_0 G'/G^*), \sigma''(\dot{\gamma}), \sigma''/G''(\gamma_0 G'/G^*)$
50.	Giacomin (2011) [15]	HDPE _C 160	$I_n(\omega), \tau_{yx}(\dot{\gamma})$
51.	Rogers (2011) [86]	star-PBD[squalene] 20	$\tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma}), I_{3/1}(\gamma_0), I_{5/1}(\gamma_0), I_{7/1}(\gamma_0)$
52.	Sun (2011) [87]	PEG _{Laponite} [NaCl] 25	$G'_1(\gamma_0), G''_1(\gamma_0), \tau_{yx}(\gamma), \tau_{yx}(t), I_n(\omega), I_{n/1}(\gamma_0), G'_M(\gamma_0), G'_L(\gamma_0), \eta'_M(\gamma_0), \eta'_L(\gamma_0)$
53.	Vittorias (2011) [88]	HDPE 190	$I_{3/1}(\gamma_0), Q_{3,0}(\gamma_0), Q_{3,0}(\omega)$
54.	Hyun (2012) [89]	PP _{C₂₀A} ; PP/PS _{C₂₀A} ; PP _{CaCO₃} 180, 185	$I_{n/1}(\omega), G'(\gamma_0), G''(\gamma_0), I_{3/1}(\gamma_0)$
55.	Wode (2012) [90]	PS-b-P4VN ; PS-b-P4VN _{MWCNT} 155	$G'_1(\gamma_0), G''_1(\gamma_0), G''_1/G'_1(\gamma_0), G'_1(t), G''_1(t), G''_1/G'_1(t)$
56.	Abbasi (2013) [91]	LDPE 130	$I_{3/1}(\gamma_0), Q_{3,0}(\gamma_0)$
57.	Ewoldt (2013) [92]	PVA/Borax(aq) 25	$G'_1(\gamma_0), G''_1(\gamma_0), e_1(\gamma_0), e_3(\gamma_0), v_1(\gamma_0), v_3(\gamma_0)$
58.	Lim (2013) [39]	PCL/MWCNT ; PCL/OMMT ; PCL/PCC 130	$\tau_{yx}(t), \tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\gamma), I_{n/1}(n), I_{3/1}(\gamma_0), Q_{3,0}(\gamma_0)$
59.	Ahirwal (2014) [93]	i-PP ; LLDPE ; LDPE 190	$\delta_{1,0}(G_{1,0}^*), I_{n/1}(n), I_{3/1}(\gamma_0), I_{5/1}(\gamma_0), e_{3/1}(\gamma_0), v_{3/1}(\gamma_0)$
60.	Bharadwaj (2014) [32]	cis-1,4 PI 25	$I'_1(\gamma_0), I'_3(\gamma_0), I''_1(\gamma_0), I''_3(\gamma_0), G'_{13}/\gamma_0^3(\omega), G''_{13}/\gamma_0^3(\omega)$
61.	de Souza Mendez (2014) [94]	PAM 25	$ \tau_{yx} (\gamma_0), G'_1(\gamma_0), G''_1(\gamma_0)$
62.	Giacomin (2014) [95]	cis-1,4 PI 52.8	$I_{3/1}(\dot{\gamma}^0), I_{5/1}(\dot{\gamma}^0)$
63.	Hoyle (2014) [96]	HDPE 155 ; LDPE 150	$I_{3/1}/\omega(\dot{\gamma}^0), I'_{3/1}(\dot{\gamma}^0), I''_{3/1}(\dot{\gamma}^0)$
64.	Kim (2014) [97]	PEGDA/SDS(aq)/PDMS 50	$G'(\gamma_0), G''(\gamma_0), \tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma}), I_{3/1}(\gamma_0), G'_M(\gamma_0), \eta'_L(\dot{\gamma}_0)$
65.	Kim (2014) [98]	(C ₁₄ D ₃₀ /C ₁₄ H ₃₀) _{SiO₂} 28, 29	$G'_1(\gamma_0), G''_1(\gamma_0), \tau_{yx}(\gamma_0, \dot{\gamma}_0), \tau_{yx}(\gamma_0), \tau_{yx}(\dot{\gamma}_0), \eta^* (\dot{\gamma})$
66.	Shamim (2014) [99]	PBD[oligo-] -30, 25 ; PS[DEP] -40	$\tau_{yx}(t), \tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma}), \sigma'(\gamma), e_n(\gamma_0), e_n(\omega), I_{3/1}(\gamma_0^2)$
67.	Ilyin (2015) [100]	PIB ; PI _{SiO₂} 20	$\tau_{yx}(t), G'_1(\gamma_0), G''_1(\gamma_0), G'_M(\gamma_0), G'_L(\gamma_0), \tau_{yx}(\gamma), S(\gamma_0), T(\gamma_0)$
68.	Khandavalli (2015) [101]	PEO _{A@200} ; PPG _{A@200} 20	$G'_1(\gamma_0), G''_1(\gamma_0), \tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma}), G'_L(\gamma_0),$

			$G'_M(\gamma_0), \eta'_L(\gamma_0), \eta'_M(\gamma_0), S(\gamma_0), T(\gamma_0)$
69.	Blackwell (2016) [14]	Carbopol® 940 25	$\sigma'_1(\gamma_0) - G' \sin \omega t, \sigma''_1(\gamma_0) - G'' \sin \omega t, \sigma'_3(\gamma_0), \sigma''_3(\gamma_0), I_n(n)$
70.	Cziep (2016) [19]	PS ; PI ; PpMS ; P2VP ; PEO ; HDPE $T_g + 60$	$Q_{3,0}(\omega)$
71.	Nie (2016) [16]	OBC 135-180	$ G_{11}^* (\gamma_0), \delta_1(\gamma_0), G_{33}^* (I_{3/1}), \delta_3(I_{3/1}), G_{55}^* (I_{5/1}), Q_{3,0}(\omega), Q_{5,0}(\omega), G'_{33,0}(\omega), G''_{33,0}(\omega), \delta_{1,0}(G_{11,0}^*), \delta_{3,0}(G_{33,0}^*), G_{55}^* (\omega)$
72.	Song (2016) [102]	PI ; star-PI 25	$Q_{3,0}(\omega)$
73.	Andablo-Reyes (2017) [103]	UHMWPE 160	$\tau_{yx}(t)$
74.	Goudoulas (2017) [35]	PAM(aq) 20, 25	$\tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma}), G'_1(\gamma_0), G''_1(\gamma_0), G'_M(\gamma_0), G'_L(\gamma_0), v_{3/1}(\gamma_0), v_{5/1}(\gamma_0), e_{3/1}(\gamma_0), e_{5/1}(\gamma_0)$
75.	Kádár (2017) [104]	EBA ; EBA _{GNP} ; EBA _{C/GNP} 160	$I_{3/1}(\gamma_0), G'_1(t), G''_1(t), Q_{3,0}(t)$
76.	Ma (2017) [105]	PDMS _{A®200} ; PDMS _{A®R974} 25	$G'_1(\gamma_0), G''_1(\gamma_0), \eta'_1 (\gamma_0)$
77.	Yang (2017) [40]	PDMS/8CB 38	$\tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma}), G'_M(\tau_{yx}), G'_L(\tau_{yx}), \eta'_M(\tau_{yx}), \eta'_L(\tau_{yx})$
78.	Aliabadian (2018) [42]	HPAM _{A®R816} 35	$G'_1(\gamma_0), G''_1(\gamma_0), \eta'_L(\gamma_0), \eta'_M(\gamma_0), S(\gamma_0), T(\gamma_0), \tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma})$
79.	Du (2018) [41]	PDMS _{GO} ; PDMS _{rGO} ; PDMS _{rGO-POSS} 20	$\tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma}), G'_M(\gamma_0), G'_L(\gamma_0), \eta'_M(\gamma_0), \eta'_L(\gamma_0), S(\gamma), T(\gamma), I_{3/1}(\gamma_0), \dots, I_{7/1}(\gamma_0)$
80.	Goudoulas (2018) [36]	PEO(aq) 25	$G'_1(\gamma_0), G''_1(\gamma_0), G'_M(\gamma_0), G'_L(\gamma_0), v_3/v_1(\gamma_0), v_5/v_1(\gamma_0), e_3/e_1(\gamma_0), e_5/e_1(\gamma_0), \tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma})$
81.	Kamkar (2018) [37]	PVDF _{CNT} ; PVDF _{MnO₂NW} ; PVDF _{CNT/MnO₂NW} 240	$S(\gamma_0), T(\gamma_0), \tau_{yx}(\gamma), \sigma'(\gamma), \tau_{yx}(\dot{\gamma}), \sigma''(\dot{\gamma})$
82.	Singh (2018) [106]	PVA/Borax Gel 25	$G'_1(\gamma_0), G'_3(\gamma_0), G''_1(\gamma_0), G''_3(\gamma_0), G'_1(\omega), e_1(\omega), e_3(\omega), v_1(\omega), v_3(\omega)$
83.	Yang (2018) [22]	PDMS _{8CB} 38 ; PEB _{SiO₂} 160 ; Carbopol® 940 25 ; Laponite® 25	$\sigma'(\gamma), \sigma'(\dot{\gamma}), \tau_{yx}(\gamma), \tau_{yx}(\dot{\gamma})$

Legend: 8CB $\equiv$ 4-octyl-4-biphenylcarbonitrile; A® $\equiv$ Aerosil®; C $\equiv$ carbon black; C® $\equiv$ Closite®; CNT $\equiv$ carbon nanotube; DEP $\equiv$ diethyl phthalate; DOP $\equiv$ dioctyl phthalate; DVB $\equiv$ divinyl benzene; EBA $\equiv$ poly(ethylene-butyl acrylate); GNP $\equiv$ graphite nanoplatelets; GO $\equiv$ graphene oxide; rGO $\equiv$ reduced graphene oxide; HDPE $\equiv$ high-density polyethylene; HPAM $\equiv$ hydrolyzed polyacrylamide; LDPE $\equiv$ low density polyethylene; LLDPE $\equiv$ linear low density polyethylene; mLLDPE $\equiv$ metallocene catalyzed linear low density polyethylene; MWCNT $\equiv$ multiwall carbon nanotube; OBC $\equiv$ olefin multiblock copolymers;

OMMT $\equiv$ organomodified montmorillonite; P2VP $\equiv$ poly(2-vinylpyridine); P4VN $\equiv$ poly(4-vinylpyridine); PAM $\equiv$ polyacrylamide; PBA $\equiv$ polybutylacrylate; PBA $\equiv$ polybutylacrylate ;PBD $\equiv$ polybutadiene; PCC $\equiv$ precipitated calcium carbonate; PCL $\equiv$ polycaprolactone; PDMS $\equiv$ polydimethylsiloxane; PE $\equiv$ polyethylene; PEB $\equiv$ poly(ethylene-co- α -butene); PEO $\equiv$ polyethylene oxide; PEG $\equiv$ polyethylene glycol; PEGDA $\equiv$ polyethylene glycol diacrylate; PI $\equiv$ polyisoprene; PIB $\equiv$ polyisobutylene; PLA $\equiv$ polylactic acid; PMMA $\equiv$ polymethyl methacrylate; PO $\equiv$ paraffin oil; POSS $\equiv$ polyhedral oligomeric silsesquioxane; PP $\equiv$ polypropylene; PPG $\equiv$ polypropylene glycol; PpMS $\equiv$ poly(*p*-methylstyrene); PPO $\equiv$ polypropylene oxide; PS $\equiv$ polystyrene; PU $\equiv$ polyurethane; PVA $\equiv$ polyvinyl alcohol; PVAc $\equiv$ polyvinyl acetate; PVDF $\equiv$ polyvinylidene fluoride; S $\equiv$ styrene; SDS $\equiv$ sodium dodecyl sulfate; SMA $\equiv$ poly(styrene-co-maleic anhydride); SO $\equiv$ silicone oil; TCP $\equiv$ tricresyl phosphate; UHMWPE $\equiv$ ultrahigh molecular weight polyethylene.

Figure 1: Typical nonlinear depictions: (a) $\tau_{yx}(t)$, (b) $I_n(n)$ and (c) $\tau_{yx}(\dot{\gamma}, \gamma)$ with projections of $\tau_{yx}(\gamma)$ and $\tau_{yx}(\dot{\gamma})$, along with corresponding intra-cycles, $\sigma'(\gamma)$ and $\sigma''(\dot{\gamma})$. **Black**, **blue** and **red** are SAOS, MAOS and LAOS responses.

Figure 2: Pipkin map showing **Newtonian** region ($\lambda\dot{\gamma}^0 < 1 \cap \lambda\omega < 1$), **steady-shear** flow ($\lambda\omega < 1$), **SAOS** ($\lambda\dot{\gamma}^0 < 1$) and **LAOS** ($\lambda\dot{\gamma}^0 > 1 \cap \lambda\omega > 1$). The 3D Lissajous plots show typical stress responses in all four regimes.

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