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MACROMOLECULAR TUMBLING AND WOBBLING IN LARGE-AMPLITUDE OSCILLATORY SHEAR FLOW

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ABSTRACT

For a suspension of rigid dumbbells, in any simple shear flow, we recently solved for the diffusion equation for the orientation distribution function by a power series expansion in the shear rate magnitude. In this paper, we focus specifically on large-amplitude oscillatory shear flow (LAOS), for which we extend the orientation distribution function to the 6th power of the shear rate amplitude. We arrive at the Fourier solution for each harmonic contribution to the total orientation distribution function, separating each harmonic into its coefficients in and out-of-phase with $\cos n\omega t$, ψ'_n and ψ''_n , respectively. We plot, for the first time, the evolving normalized alternant macromolecular orientation. Moreover, to deepen our understanding of the macromolecular motions, we distinguish and study the two types of possible rotations, tumbling and wobbling.

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1. INTRODUCTION

Oscillatory shear flow is considered the most common method to characterize the rheological properties of viscoelastic fluids [1,2,3,4]. In this method, the strain rate applied to the fluid varies cyclically. In our work, we are interested in sinusoidal fluid displacement, and thus cosinusoidal shear rate [5]:

$$\dot{\gamma}(t) \equiv \frac{\partial v_x}{\partial y} = \dot{\gamma}^0 \cos \omega t \quad (1)$$

Table I and Table II define the symbols used herein, respectively, dimensional and dimensionless. Eq. (1) employs Cartesian coordinates, where x is the flow direction, y , the velocity gradient direction, and z , the vorticity direction (see Figure 3 of [6]). v_x is the velocity in the flow direction, such that $v_y = v_z = 0$. Eq. (1) can be nondimensionalized using the fluid relaxation time, λ :

$$\lambda \dot{\gamma}(t) = \lambda \dot{\gamma}^0 \cos \lambda \omega (t / \lambda) \quad (2)$$

in which two dimensionless groups appear, $\lambda \dot{\gamma}^0$, the Weissenberg number and $\lambda \omega$, the Deborah number.

We devote this paper to exploring the motions of the macromolecules in oscillatory shear flow. Specifically, we choose the suspension of rigid dumbbells (see Figure 1), as the simplest relevant model. By relevant, we at least mean that this model can predict the higher harmonics of the shear stress observed in the laboratory. Furthermore, at least qualitatively, the suspension of rigid dumbbells correctly predicts the behaviors of polymeric liquids and melts in the relevant limits of large-amplitude oscillatory shear flow (LAOS): steady shear flow ($\lambda \omega \rightarrow 0$), and in small-amplitude oscillatory shear flow ($\lambda \dot{\gamma}^0 \rightarrow 0$) (Sections 6. and 7. of [7]).

In fact, the rigid dumbbell model relies entirely on macromolecular orientation to explain the viscoelastic responses of polymeric fluids. It takes into consideration the orientability of the polymer in flow fields and disregards less important molecular stretching and bending motions, that appear to be of minor importance to the rheological properties [8].

Orientation is specified using two spherical coordinates, the polar angles, θ and ϕ , defined in Figure 1. The latitudinal coordinate, θ , is measured from the positive z direction, and ϕ , the longitudinal, from the positive x . The fraction of dumbbells that at any time t , are in the orientation range θ, ϕ to $\theta + d\theta$, $\phi + d\phi$ is then given by $\psi(\theta, \phi, t) d\theta d\phi$, where $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. This fraction of dumbbells is obtained by solving the diffusion equation (Eq. (15) of [8]; Eq. (14.4-1) of [9]; see [10], [11], [12], [13], [14], [15] to explore diffusion equation and application; see also Table I. of [16], Table I. of [17], Table 3 of [18], Table I. of [19]) in the following power series expansion in $\lambda \dot{\gamma}^0$ [20]:

$$\psi(\theta, \phi, t) = \frac{1}{4\pi} \left[1 + (6\lambda \dot{\gamma}^0) \psi_1(\theta, \phi, t) + (6\lambda \dot{\gamma}^0)^2 \psi_2(\theta, \phi, t) + (6\lambda \dot{\gamma}^0)^3 \psi_3(\theta, \phi, t) + \dots \right] \quad (3)$$

To be clear, Eq. (3) pertains to the periodic orientation distribution at which the fluid arrives some time after startup of oscillatory shear flow. We thus call this periodic fluid response *alternant*.

Our previous work arrived at the solution for the orientation distribution function, for any simple shear flow, up to and including the sixth power of the shear rate magnitude: ψ_1 , ψ_2 , ψ_3 (Eqs. (35), (46), and (55) of [20]; Eqs. (34), (45), and (54) of [8], Eqs. (22) through (24) of [21]), ψ_4 (Eq. (35) of [10]; Eq. (80) of [22]), ψ_5 and ψ_6 (Eqs. (81) and (82) of [22]). In this paper, we use these general results as a starting point to derive the Fourier solution for the special case of large-amplitude oscillatory shear flow [23].

Our work here is not the first finding about LAOS solution. In fact, the Fourier solution of the orientation distribution function for LAOS has been previously derived, up to and including the fourth power of the shear rate amplitude (see Eqs. (36), (44), (52) and (61) of [10]) and decomposed into its first six harmonics, zeroth through the fifth, each separated into components in and out-of-phase with the shear rate (see Eqs. (11) through (17), and Eqs. (19) through (20) of [17] or Eqs. (12) through (20) of [23]). This prior analysis of the molecular dynamics included the three most important regimes: nonlinear steady shear flow $(\lambda\omega, \lambda\dot{\gamma}^0) = (0, 1)$, nonlinear viscoelasticity $(\lambda\omega, \lambda\dot{\gamma}^0) = (1, 1)$, and linear viscoelasticity $(\lambda\omega, \lambda\dot{\gamma}^0) = (1, 0)$.

Here, we will correct and then extend the previous solution by completing ψ_4 , and then calculating the next ψ_5 and ψ_6 terms. We present our result for ψ as a function of its zeroth through sixth harmonics (Eq. (43) below), and explore its complex parts, both in and out-of-phase with the shear rate. In this paper, we focus our work on the nonlinear viscoelastic regime, particularly $(\lambda\omega, \lambda\dot{\gamma}^0) = (1, 1)$. We plot, for the first time, the *normalized* alternant macromolecular orientation induced by the oscillatory shear deformation that is predicted by each harmonic, and by the sum of all harmonics. By *normalized*, we mean that, within a single cycle, the probability distribution at each ωt shares a common color scheme, where the maximum orientation probability is red and the minimum is blue.

To deepen our understanding of the macromolecular motions caused by the oscillatory shear flow, we study and distinguish the two types of rotations, *tumbling* and *wobbling* (see Figure 1, Figure 2 and Figure 3). Following Section 3. of [24], by *wobbling*, we mean motions that widen the orientation distribution function along the vorticity direction. By *tumbling*, we mean motions that extend the orientation distribution function along the flow direction. For wobbling, we can thus study the normalized alternant macromolecular orientation either along the velocity gradient direction (top view) or along the flow direction (front view). For tumbling, we study the normalized alternant macromolecular orientation along the vorticity direction (side view).

2. METHOD

As mentioned earlier, the Fourier solution of the orientation distribution function for LAOS has been previously derived, up to and including the fourth power of the shear rate amplitude [10]. Here, we arrive at the complete Fourier solution of the orientation distribution function up to and including the sixth power of the shear rate amplitude. For this, we first complete the Fourier

solution of the fourth coefficient of the power series expansion of Eq. (3), after which we then arrive at the solutions for the fifth and sixth coefficients.

We thus start from the general solutions, for any simple shear flow, Eqs. (80) through (82) of [22]:

$$\begin{aligned} \psi_4(\theta, \phi, t) = & A_{0,2}P_2^0c_0 + A_{2,2}P_2^2c_2 + A_{0,4}P_4^0c_0 + A_{2,4}P_4^2c_2 + A_{4,4}P_4^4c_4 + A_{0,6}P_6^0c_0 \\ & + A_{2,6}P_6^2c_2 + A_{4,6}P_6^4c_4 + A_{6,6}P_6^6c_6 + A_{0,8}P_8^0c_0 + A_{4,8}P_8^4c_4 + A_{8,8}P_8^8c_8 \end{aligned} \quad (4)$$

$$\begin{aligned} \psi_5(\theta, \phi, t) = & A_{2,2}P_2^2s_2 + A_{2,4}P_4^2s_2 + A_{4,4}P_4^4s_4 + A_{2,6}P_6^2s_2 + A_{4,6}P_6^4s_4 + A_{6,6}P_6^6s_6 + A_{2,8}P_8^2s_2 \\ & + A_{4,8}P_8^4s_4 + A_{6,8}P_8^6s_6 + A_{8,8}P_8^8s_8 + A_{2,10}P_{10}^2s_2 + A_{6,10}P_{10}^6s_6 + A_{10,10}P_{10}^{10}s_{10} \end{aligned} \quad (5)$$

and:

$$\begin{aligned} \psi_6(\theta, \phi, t) = & A_{0,2}P_2^0c_0 + A_{2,2}P_2^2c_2 + A_{0,4}P_4^0c_0 + A_{2,4}P_4^2c_2 + A_{4,4}P_4^4c_4 + A_{0,6}P_6^0c_0 + A_{2,6}P_6^2c_2 \\ & + A_{4,6}P_6^4c_4 + A_{6,6}P_6^6c_6 + A_{0,8}P_8^0c_0 + A_{2,8}P_8^2c_2 + A_{4,8}P_8^4c_4 + A_{6,8}P_8^6c_6 + A_{8,8}P_8^8c_8 + A_{0,10}P_{10}^0c_0 \\ & + A_{2,10}P_{10}^2c_2 + A_{4,10}P_{10}^4c_4 + A_{6,10}P_{10}^6c_6 + A_{8,10}P_{10}^8c_8 + A_{10,10}P_{10}^{10}c_{10} + A_{0,12}P_{12}^0c_0 + A_{4,12}P_{12}^4c_4 \\ & + A_{8,12}P_{12}^8c_8 + A_{12,12}P_{12}^{12}c_{12} \end{aligned} \quad (6)$$

where P_n^m 's are the associated Legendre polynomials, and the coefficients, $A_{i,j}$'s, are functions of Bird-Armstrong paren functions, and are found in Subsections 7.1, 7.2 and 7.3 of [22].

Our Legendre polynomials are functions of $C \equiv \cos\theta$ and are generated by [25,26]:

$$P_n^m(C) \equiv \frac{(-1)^m}{2^n n!} (1-C^2)^{m/2} \frac{d^{n+m}}{dC^{n+m}} (C^2-1)^n \quad (7)$$

where the sign convention is dictated by the factor $(-1)^m$, called Condon-Shortley phase [27]. Some of the Legendre polynomials that we need are already evaluated (Table 3 of [10]). Using Eq. (7), we evaluate the new Legendre polynomials needed in this work (see Table III).

The paren functions are defined as follows (see Eq. (10) of [28] or Eqs. (28)-(30) of [29]):

$$(a;t) \equiv \frac{1}{\lambda} \int_{-\infty}^t e^{-a(t-t')/\lambda} f(t') dt' \quad (8)$$

$$(a,b;t) \equiv \frac{1}{\lambda^2} \int_{-\infty}^t \int_{-\infty}^{t'} e^{-a(t-t')/\lambda} e^{-b(t-t'')/\lambda} f(t') f(t'') dt'' dt' \quad (9)$$

$$(a,b,c;t) \equiv \frac{1}{\lambda^3} \int_{-\infty}^t \int_{-\infty}^{t'} \int_{-\infty}^{t''} e^{-a(t-t')/\lambda} e^{-b(t-t'')/\lambda} e^{-c(t-t''')/\lambda} f(t') f(t'') f(t''') dt''' dt'' dt' \quad (10)$$

and so on, where a , b and c are dimensionless.

To find the LAOS solution, we must evaluate the contributing paren functions for the specific case where $f(t) = \cos\omega t$. The contributing fourth order paren functions is derived in previous work (Eq. (82) of [10]). The fifth and sixth order paren functions, not studied before, are derived below.

Following the method of [8], we must first take the products of the real parts of complex variables. If u, v, w, n, x, y are complex, then:

$$\Re\{u\}\Re\{v\}\Re\{w\}\Re\{n\}\Re\{x\}$$

$$\equiv \frac{1}{16}\Re\left\{\begin{aligned} &uvwnx + u^*vwnx + uv^*w^*nx + uv^*wn^*x + uv^*wnx + \\ &uvw^*n^*x + uvw^*nx + uvwn^*x + uvwnx^* + u^*vwnx^* + \\ &uv^*w^*nx^* + uv^*wn^*x^* + uv^*wnx^* + uvw^*n^*x^* + uvw^*nx^* + uvwn^*x^* \end{aligned}\right\} \quad (11)$$

and:

$$\Re\{u\}\Re\{v\}\Re\{w\}\Re\{n\}\Re\{x\}\Re\{y\}$$

$$\equiv \frac{1}{32}\Re\left\{\begin{aligned} &uvwnxy + u^*vwnxy + uv^*w^*nxy + uv^*wn^*xy + uv^*wnxy + \\ &uvw^*n^*xy + uvw^*nxy + uvwn^*xy + uvwnx^*y + u^*vwnx^*y + \\ &uv^*w^*nx^*y + uv^*wn^*x^*y + uv^*wnx^*y + uvw^*n^*x^*y + uvw^*nx^*y + uvwn^*x^*y \\ &+ uvwnxy^* + u^*vwnxy^* + uv^*w^*nxy^* + uv^*wn^*xy^* + uv^*wnxy^* + \\ &uvw^*n^*xy^* + uvw^*nxy^* + uvwn^*xy^* + uvwnx^*y^* + u^*vwnx^*y^* + \\ &uv^*w^*nx^*y^* + uv^*wn^*x^*y^* + uv^*wnx^*y^* + uvw^*n^*x^*y^* + uvw^*nx^*y^* + uvwn^*x^*y^* \end{aligned}\right\} \quad (12)$$

where $\Re$ indicates “real part of” and the asterisks indicate “complex conjugate of”.

Following the pattern of Eqs. (8) through (10), we find:

$$(a, b, c, d, e; t) = \frac{1}{\lambda^5} \int_{-\infty}^t \int_{-\infty}^{t'} \int_{-\infty}^{t''} \int_{-\infty}^{t'''} \int_{-\infty}^{t'''} \begin{aligned} &e^{-a(t-t')/\lambda} e^{-b(t-t'')/\lambda} e^{-c(t-t''')/\lambda} \\ &\times e^{-d(t-t''')/\lambda} e^{-e(t-t''')/\lambda} \\ &\times \cos \omega t' \cos \omega t'' \cos \omega t''' \\ &\times \cos \omega t'''' \cos \omega t'''' \end{aligned} dt'''' dt'''' dt'' dt'' dt' \quad (13)$$

$$= \frac{1}{\lambda^5} \int_{-\infty}^t \int_{-\infty}^{t'} \int_{-\infty}^{t''} \int_{-\infty}^{t'''} \int_{-\infty}^{t'''} \begin{aligned} &e^{-a(t-t')/\lambda} e^{-b(t-t'')/\lambda} e^{-c(t-t''')/\lambda} \\ &\times e^{-d(t-t''')/\lambda} e^{-e(t-t''')/\lambda} \\ &\times \Re\{e^{i\omega t'}\} \Re\{e^{i\omega t''}\} \Re\{e^{i\omega t'''}\} \\ &\times \Re\{e^{i\omega t''''}\} \Re\{e^{i\omega t''''}\} \end{aligned} dt'''' dt'''' dt'' dt'' dt'$$

and:

$$(a, b, c, d, e, f; t) = \frac{1}{\lambda^6} \int_{-\infty}^t \int_{-\infty}^{t'} \int_{-\infty}^{t''} \int_{-\infty}^{t'''} \int_{-\infty}^{t''''} \int_{-\infty}^{t''''} \begin{aligned} &e^{-a(t-t')/\lambda} e^{-b(t-t'')/\lambda} e^{-c(t-t''')/\lambda} \\ &\times e^{-d(t-t''')/\lambda} e^{-e(t-t''')/\lambda} e^{-f(t-t''')/\lambda} \\ &\times \cos \omega t' \cos \omega t'' \cos \omega t''' \\ &\times \cos \omega t'''' \cos \omega t'''' \cos \omega t'''' \end{aligned} dt'''' dt'''' dt'''' dt'' dt'' dt' \quad (14)$$

$$= \frac{1}{\lambda^6} \int_{-\infty}^t \int_{-\infty}^{t'} \int_{-\infty}^{t''} \int_{-\infty}^{t'''} \int_{-\infty}^{t''''} \int_{-\infty}^{t''''} \begin{aligned} &e^{-a(t-t')/\lambda} e^{-b(t-t'')/\lambda} e^{-c(t-t''')/\lambda} \\ &\times e^{-d(t-t''')/\lambda} e^{-e(t-t''')/\lambda} e^{-f(t-t''')/\lambda} \\ &\times \Re\{e^{i\omega t'}\} \Re\{e^{i\omega t''}\} \Re\{e^{i\omega t'''}\} \\ &\times \Re\{e^{i\omega t''''}\} \Re\{e^{i\omega t''''}\} \Re\{e^{i\omega t''''}\} \end{aligned} dt'''' dt'''' dt'''' dt'' dt'' dt'$$

Following the method of Eqs. (60) through (71) of [8] we finally get, for the fifth order paren function:

$$(a, b, c, d, e; t) = \frac{1}{16}(p_5 + p_3 + p_1) \quad (15)$$

where:

$$p_5 = \Re \left\{ \frac{e^{5i\omega t}}{(a+b+c+d+e+5\lambda i\omega)(b+c+d+e+4\lambda i\omega)(c+d+e+3\lambda i\omega)(d+e+2\lambda i\omega)(e+\lambda i\omega)} \right\} \quad (16)$$

$$\begin{aligned} p_3 = & \Re \left\{ \frac{e^{3i\omega t}}{(a+b+c+d+e+3\lambda i\omega)(b+c+d+e+4\lambda i\omega)(c+d+e+3\lambda i\omega)(d+e+2\lambda i\omega)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{3i\omega t}}{(a+b+c+d+e+3\lambda i\omega)(b+c+d+e+2\lambda i\omega)(c+d+e+3\lambda i\omega)(d+e+2\lambda i\omega)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{3i\omega t}}{(a+b+c+d+e+3\lambda i\omega)(b+c+d+e+2\lambda i\omega)(c+d+e+\lambda i\omega)(d+e+2\lambda i\omega)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{3i\omega t}}{(a+b+c+d+e+3\lambda i\omega)(b+c+d+e+2\lambda i\omega)(c+d+e+\lambda i\omega)(d+e)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{3i\omega t}}{(a+b+c+d+e+3\lambda i\omega)(b+c+d+e+2\lambda i\omega)(c+d+e+\lambda i\omega)(d+e)(e-\lambda i\omega)} \right\} \end{aligned} \quad (17)$$

$$\begin{aligned} p_1 = & \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e)(c+d+e+\lambda i\omega)(d+e+2\lambda i\omega)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e)(c+d+e+\lambda i\omega)(d+e)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e)(c+d+e-\lambda i\omega)(d+e)(e+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e+2\lambda i\omega)(c+d+e+\lambda i\omega)(d+e)(e-\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{-i\omega t}}{(a+b+c+d+e-\lambda i\omega)(b+c+d+e-2\lambda i\omega)(c+d+e-\lambda i\omega)(d+e)(e-\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{-i\omega t}}{(a+b+c+d+e-\lambda i\omega)(b+c+d+e-2\lambda i\omega)(c+d+e-\lambda i\omega)(d+e-2\lambda i\omega)(e-\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e)(c+d+e+\lambda i\omega)(d+e)(e-\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{-i\omega t}}{(a+b+c+d+e-\lambda i\omega)(b+c+d+e-2\lambda i\omega)(c+d+e-3\lambda i\omega)(d+e-2\lambda i\omega)(e-\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e)(c+d+e-\lambda i\omega)(d+e)(e-\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{i\omega t}}{(a+b+c+d+e+\lambda i\omega)(b+c+d+e)(c+d+e-\lambda i\omega)(d+e-2\lambda i\omega)(e-\lambda i\omega)} \right\} \end{aligned} \quad (18)$$

and for the sixth:

$$(a, b, c, d, e, f; t) = \frac{1}{32} (p_6 + p_4 + p_2 + p_0) \quad (19)$$

where:

$$p_6 = \Re \left\{ \frac{e^{6i\omega t}}{(a+b+c+d+e+f+6\lambda i\omega)(b+c+d+e+f+5\lambda i\omega)(c+d+e+f+4\lambda i\omega)(d+e+f+3\lambda i\omega)(e+f+2\lambda i\omega)(f+\lambda i\omega)} \right\} \quad (20)$$

$$\begin{aligned} p_4 = & \Re \left\{ \frac{e^{4i\omega t}}{(a+b+c+d+e+f+4\lambda i\omega)(b+c+d+e+f+5\lambda i\omega)(c+d+e+f+4\lambda i\omega)(d+e+f+3\lambda i\omega)(e+f+2\lambda i\omega)(f+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{4i\omega t}}{(a+b+c+d+e+f+4\lambda i\omega)(b+c+d+e+f+3\lambda i\omega)(c+d+e+f+4\lambda i\omega)(d+e+f+3\lambda i\omega)(e+f+2\lambda i\omega)(f+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{4i\omega t}}{(a+b+c+d+e+f+4\lambda i\omega)(b+c+d+e+f+3\lambda i\omega)(c+d+e+f+2\lambda i\omega)(d+e+f+3\lambda i\omega)(e+f+2\lambda i\omega)(f+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{4i\omega t}}{(a+b+c+d+e+f+4\lambda i\omega)(b+c+d+e+f+3\lambda i\omega)(c+d+e+f+2\lambda i\omega)(d+e+f+\lambda i\omega)(e+f+2\lambda i\omega)(f+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{4i\omega t}}{(a+b+c+d+e+f+4\lambda i\omega)(b+c+d+e+f+3\lambda i\omega)(c+d+e+f+2\lambda i\omega)(d+e+f+\lambda i\omega)(e+f)(f+\lambda i\omega)} \right\} \\ & + \Re \left\{ \frac{e^{4i\omega t}}{(a+b+c+d+e+f+4\lambda i\omega)(b+c+d+e+f+3\lambda i\omega)(c+d+e+f+2\lambda i\omega)(d+e+f+\lambda i\omega)(e+f)(f-\lambda i\omega)} \right\} \end{aligned} \quad (21)$$

We solve Eqs. (15) and (19) for each value of a, b, c, d, e , and f and we give the code used to find each of these contributing paren functions in our Supplementary Material.

We next substitute the associated values of the Legendre polynomials and all the contributing paren functions into Eqs. (4), (5), and (6), to get the Fourier solution of ψ_4, ψ_5, ψ_6 respectively. We then, in turn, substitute these results for ψ_4, ψ_5, ψ_6 , and previous results for ψ_1, ψ_2, ψ_3 into Eq. (3) to find the Fourier solution of the orientation distribution function, up to and including the sixth power of the shear rate amplitude (see Eq. (43) below).

3. RESULTS

We arrive at the Fourier solution for each harmonic contribution to the total orientation distribution function, separating each harmonic into its coefficients in and out-of-phase with $\cos n\omega t$, ψ'_n and ψ''_n , respectively.

3.1. Zeroth Harmonic

The updated Fourier coefficient of the zeroth harmonic of the orientation distribution function for a rigid dumbbell suspension in large-amplitude oscillatory shear flow is given by:

$$\psi'_0 = \frac{1}{4\pi} \left[1 + (6\lambda\dot{\gamma}^0)^2 A'_2 + (6\lambda\dot{\gamma}^0)^4 A'_4 + (6\lambda\dot{\gamma}^0)^6 A'_6 \right] \quad (24)$$

where A'_i 's are the in-phase contributions of each ψ_i to the zeroth harmonic, and are defined as:

$$A'_i = \sum_{j=0, \text{even}}^{2i} \frac{a'_{i,j}}{b'_{i,j}} (C-1)^{\frac{j}{2}} (C+1)^{\frac{j}{2}} c_j \quad (25)$$

where $c_j = \cos j\phi$ and where j is only even. Values of $a'_{i,j}$ and $b'_{i,j}$ are given in Eqs. (44) through (73) in Subsection 6.1.

Figure 4 illustrates the normalized alternant macromolecular orientation distribution induced by the oscillatory shear deformation predicted by Eq. (24), in the nonlinear viscoelastic regime. The out-of-phase component is zero, thus the zeroth harmonic includes only a part in-phase with $\cos(0 \omega t)$. To study the zeroth harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 4 shown in Figure 5 and Figure 6. From Figure 5, we see that, over a complete alternant cycle, the zeroth harmonic orientation distribution extends along the flow direction, which means the zeroth harmonic contributes to the total tumbling. From Figure 6 we see that, over a complete alternant cycle, the zeroth harmonic orientation distribution widens along the vorticity direction, which means that the zeroth harmonic contributes to the total wobbling.

We next compare the contributions of the zeroth harmonic at each ωt . From Figure 4, we see that for the zeroth harmonic, the orientation distribution does not vary over a complete alternant cycle. This shows that the zeroth harmonic contributes to the total tumbling and wobbling equally at all ωt . This finding is verified by the movie shown in Figure 28. By comparing the values shown on the colorbar of Figure 4 through Figure 6 to the values of all other harmonics, we

show that the zeroth harmonic contributes the most to the total orientation distribution. However, the orientation distribution is marshmallow shaped, which illustrates the importance of including higher harmonics in our calculations, that will later sculpt the total orientation distribution to its final peanut or spherical shapes (see Figure 25), and thus allows correct predictions of orientations.

3.2. First Harmonic

The updated Fourier coefficients of the first harmonics are given by:

$$\psi'_1 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0) A'_1 + (6\lambda\dot{\gamma}^0)^3 A'_3 + (6\lambda\dot{\gamma}^0)^5 A'_5 \right] \quad (26)$$

and:

$$\psi''_1 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0) A''_1 + (6\lambda\dot{\gamma}^0)^3 A''_3 + (6\lambda\dot{\gamma}^0)^5 A''_5 \right] \quad (27)$$

where the A'_i 's are the contributions of each ψ_i that are in-phase with the first harmonic, and the A''_i 's, out-of-phase:

$$A'_i = \sum_{j=0, \text{even}}^{2i} \frac{a'_{ij}}{b'_{ij}} \lambda \omega (C-1)^{\frac{j}{2}} (C+1)^{\frac{j}{2}} s_j \quad (28)$$

and:

$$A''_i = \sum_{j=0, \text{even}}^{2i} \frac{a''_{ij}}{b''_{ij}} \lambda \omega (C-1)^{\frac{j}{2}} (C+1)^{\frac{j}{2}} s_j \quad (29)$$

where $s_j = \sin j\phi$ and where j is only even. Values of $a'_{i,j}$, $b'_{i,j}$, $a''_{i,j}$ and $b''_{i,j}$ are given in Eqs. (74) through (109) in Subsection 6.2 below.

Figure 7A illustrates the *complex* orientation distribution associated with the polymer motions arising at the test frequency, ω , induced by the oscillatory shear deformation and predicted by Eqs. (26) and (27). By *complex*, we here mean the sum of the components in and out-of-phase with $\cos\omega t$. Figure 7B and Figure 7C show the orientation distributions associated with the first harmonic components in and out-of-phase with $\cos\omega t$, respectively. To study the first harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 7 shown in Figure 8 and Figure 9. From Figure 8, we see that, over a complete alternant cycle, the first harmonic orientation distribution extends along the flow direction, which means the first harmonic contributes to the total tumbling. From Figure 9 we see that, over a complete alternant cycle, the first harmonic orientation distribution widens along the vorticity direction, which means that the first harmonic contributes to the total wobbling.

From Figure 7A, we see that for the first harmonic, the complex orientation distribution varies over a complete alternant cycle, with the orientations at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$ contributing most and orientations at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, least. This means that the total orientation distribution is sculpted by the first harmonic at

$\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$, and the marshmallows formed by the zeroth harmonic will deform. However, the total orientation distribution will stay almost the same at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$. This finding is verified by the movie shown in Figure 29. By comparing the values shown on the colorbar of Figure 7 to the values of all other harmonics, we show that the first harmonic contributes less to the total orientation distribution than the zeroth.

We also study the contributions of the components in and out-of-phase with $\cos \omega t$.

Comparing Figure 7A to Figure 7B and Figure 7C, we see that at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$, there is constructive interference between the components in and out-of-phase with $\cos \omega t$, explaining the high contributions of the complex orientation distribution. However, at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, the interference is destructive, explaining the low contributions. This destructive interference arises because:

$$\psi(\theta, \phi, t) = -\psi(\pi - \theta, \theta + \pi, \omega t) \quad (30)$$

There are also parts of the cycle where only the in-phase component contributes, $\omega t = 0, \pi$, and where only the out-of-phase component contributes, $\omega t = \frac{1}{2}\pi, \frac{3}{2}\pi$.

3.3. Second Harmonic

The updated Fourier coefficients of the second harmonic are given by:

$$\psi'_2 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^2 A'_2 + (6\lambda\dot{\gamma}^0)^4 A'_4 + (6\lambda\dot{\gamma}^0)^6 A'_6 \right] \quad (31)$$

and:

$$\psi''_2 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^2 A''_2 + (6\lambda\dot{\gamma}^0)^4 A''_4 + (6\lambda\dot{\gamma}^0)^6 A''_6 \right] \quad (32)$$

where A'_i 's are the in-phase contributions of each ψ_i to the second harmonic and A''_i 's, out-of-phase, defined by:

$$A'_i = \sum_{j=0, \text{even}}^{2i} \frac{a'_{i,j}}{b'_{i,j}} (C-1)^{\frac{j}{2}} (C+1)^{\frac{j}{2}} c_j \quad (33)$$

and:

$$A''_i = \sum_{j=0, \text{even}}^{2i} \frac{a''_{i,j}}{b''_{i,j}} (C-1)^{\frac{j}{2}} (C+1)^{\frac{j}{2}} c_j \quad (34)$$

Corresponding values of $a'_{i,j}$, $b'_{i,j}$, $a''_{i,j}$ and $b''_{i,j}$ are given in Eqs.(110) through (181) of Subsection 6.3 below.

Figure 10A illustrates the complex orientation distribution associated with the polymer motions arising at twice the test frequency, 2ω , induced by the oscillatory shear deformation, and predicted by Eqs. (31) and (32). Figure 10B and Figure 10C show the orientation distributions associated with the second

harmonic components in and out-of-phase with $\cos 2\omega t$, respectively. To study the second harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 10 shown in Figure 11 and Figure 12. From Figure 11 we see that, over a complete alternant cycle, the second harmonic orientation distribution extends along the flow direction, which means the second harmonic contributes to the total tumbling. From Figure 12 we see that, over a complete alternant cycle, the second harmonic orientation distribution widens along the vorticity direction, which means that the second harmonic contributes to the total wobbling.

From Figure 10A we see that for the second harmonic, the complex orientation distribution varies over a complete alternant cycle, with the orientations at $\omega t = \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$ contributing most and orientations at $\omega t = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$, least. This finding is verified by the movie shown in Figure 30. By comparing the values shown on the colorbar of Figure 10 to the values of all other harmonics, we show that the second harmonic contributes less to the total orientation distribution than the zeroth and the first.

Comparing Figure 10A to Figure 10B and Figure 10C, we see that at $\omega t = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$, only the component in-phase with $\cos 2\omega t$ contributes to the complex orientation distribution. However, at $\omega t = \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$, only the component out-of-phase with $\cos 2\omega t$ contributes.

3.4. Third Harmonic

The updated Fourier coefficients of the third harmonic are given by:

$$\psi'_3 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^3 A'_3 + (6\lambda\dot{\gamma}^0)^5 A'_5 \right] \quad (35)$$

and:

$$\psi''_3 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^3 A''_3 + (6\lambda\dot{\gamma}^0)^5 A''_5 \right] \quad (36)$$

where A'_i 's are the in-phase contributions of each ψ_i to the third harmonic and A''_i 's are the out-of-phase, defined by Eqs. (28) and (29), respectively.

Corresponding a'_j, b'_j, a''_j and b''_j are found in Eqs. (182) through (213) of Subsection 6.4 below.

Figure 13A illustrates the complex orientation distribution associated with the polymer motions arising at triple the test frequency, 3ω , induced by the oscillatory shear deformation, and predicted by Eqs. (35) and (36). Figure 13B and Figure 13C show the orientation distributions associated with the third harmonic components in and out-of-phase with $\cos 3\omega t$, respectively. To study the third harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 13 shown in Figure 14 and Figure 15. From Figure 14 we see that the orientation distribution extends along the flow direction, which means that third harmonic contributes to the total tumbling. From Figure 15 we see that the orientation distribution

widens along the vorticity direction, which means that the third harmonic contributes to the total wobbling.

From Figure 13A, we see that for the third harmonic, the complex orientation distribution varies over a complete alternant cycle, with the orientations at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$ contributing most and orientations at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, least. This finding is verified by the movie shown in Figure 31. By comparing the values shown on the colorbar of Figure 13 to the values of all other harmonics, we show that the third harmonic contributes less to the total orientation distribution than the zeroth, the first, and the second.

Comparing Figure 13A to Figure 13B and Figure 13C, we see that at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$, there is constructive interference between the components in and out-of-phase with $\cos 3\omega t$. However, at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, the interference is destructive. There are also parts of the cycle where only the in-phase component contributes, $\omega t = 0, \pi$, and where only the out-of-phase component contributes, $\omega t = \frac{1}{2}\pi, \frac{3}{2}\pi$.

3.5. Fourth Harmonic

The updated Fourier coefficients of the fourth harmonic are given by:

$$\psi'_4 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^4 A'_4 + (6\lambda\dot{\gamma}^0)^6 A'_6 \right] \quad (37)$$

and:

$$\psi''_4 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^4 A''_4 + (6\lambda\dot{\gamma}^0)^6 A''_6 \right] \quad (38)$$

where A'_i 's are the in-phase contributions of each ψ_i to the fourth harmonic and A''_i 's are the out-of-phase, defined by Eqs. (33) and (34), respectively.

Corresponding a'_j, b'_j, a''_j and b''_j are found in Eqs. (214) through (273) of Subsection 6.5 below.

Figure 16A illustrates the complex orientation distribution associated with the polymer motions arising at quadruple the test frequency, 4ω , induced by oscillatory shear and predicted by Eqs. (37) and (38). Figure 16B and Figure 16C show the orientation distributions associated with the fourth harmonic components in and out-of-phase with $\cos 4\omega t$, respectively. Figure 16 corrects Figure 7 of [23]. To study the fourth harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 16 shown in Figure 17 and Figure 18. From Figure 17 we see that, over a complete alternant cycle, the fourth harmonic orientation distribution extends along the flow direction, which means that the fourth harmonic contributes to the total tumbling. From Figure 18 we see that, over a complete alternant cycle, the fourth harmonic orientation distribution widens along the vorticity direction, which means that the fourth harmonic contributes to the total wobbling.

From Figure 16A we see that for the fourth harmonic, the complex orientation distribution appear to not vary over a complete alternant cycle. However, from the movie shown in Figure 32, we see that the orientation distribution does vary for ωt values not shown in Figure 16A. By comparing the values shown on the colorbar of Figure 16 to the values of all other harmonics, we show that the

fourth harmonic contributes less to the total orientation distribution than the zeroth, the first, the second, and the third.

Comparing Figure 16A to Figure 16B and Figure 16C, we see that only the component in-phase with $\cos 4\omega t$ contributes to the complex orientation distribution. However, the component out-of-phase with $\cos 4\omega t$ is zero.

3.6. Fifth Harmonic

The updated Fourier coefficients of the fifth harmonic are given by:

$$\psi'_5 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^5 A'_5 \right] \quad (39)$$

and:

$$\psi''_5 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^5 A''_5 \right] \quad (40)$$

where A'_i 's are the in-phase contributions of each ψ_i to the fifth harmonic and A''_i 's are the out-of-phase, defined by Eqs. (28) and (29), respectively.

Corresponding a'_j , b'_j , a''_j and b''_j are found in Eqs. (274) through (293) of Subsection 6.6 below.

Figure 19A illustrates the complex orientation distribution associated with the polymer motions arising at quintuple the test frequency, 5ω , that are induced by oscillatory shear deformation and predicted by Eqs. (39) and (40). Figure 19B and Figure 19C show the orientation distributions associated with the fifth harmonic components in and out-of-phase with $\cos 5\omega t$, respectively. To study the fifth harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 19 shown in Figure 20 and Figure 21. From Figure 20 we see that the orientation distribution extends along the flow direction, which means that fifth harmonic contributes to the total tumbling. From Figure 21 we see that the orientation distribution widens along the vorticity direction, which means that the third harmonic contributes to the total wobbling.

From Figure 19A we see that for the fifth harmonic, the complex orientation distribution varies over a complete alternant cycle, with the orientations at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$ contributing most and orientations at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, least. This finding is verified by the movie shown in Figure 33. By comparing the values shown on the colorbar of Figure 19 to the values of all other harmonics, we show that the fifth harmonic contributes less to the total orientation distribution than the zeroth, the first, the second, the third, and the fourth.

Comparing Figure 19A to Figure 19B and Figure 19C, we see that at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$, there is constructive interference between the components in and out-of-phase with $\cos 5\omega t$. However, at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, the interference is destructive. There are also parts of the cycle where only the in-phase component contributes, $\omega t = 0, \pi$, and where only the out-of-phase component contributes, $\omega t = \frac{1}{2}\pi, \frac{3}{2}\pi$.

3.7. Sixth Harmonic

The updated Fourier coefficients of the sixth harmonic are given by:

$$\psi'_6 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^6 A'_6 \right] \quad (41)$$

and:

$$\psi''_6 = \frac{1}{4\pi} \left[(6\lambda\dot{\gamma}^0)^6 A''_6 \right] \quad (42)$$

where A'_i 's are the in-phase contributions of each ψ_i to the fourth harmonic and A''_i 's are the out-of-phase, defined by Eqs. (33) and (34), respectively.

Corresponding a'_j , b'_j , a''_j and b''_j are found in Eqs. (294) through (321) of Subsection 6.7 below.

Figure 22A illustrates the complex orientation distribution associated with the polymer motions arising at sextuple the test frequency, 6ω , that are induced by oscillatory shear deformation and predicted by Eqs. (41) and (42). Figure 22B and Figure 22C show the orientation distributions associated with the sixth harmonic components in and out-of-phase with $\cos 6\omega t$, respectively. To study the sixth harmonic contribution to tumbling and wobbling of the total orientation distribution, we plot the side and top views of Figure 22 shown in Figure 23 and Figure 24. From Figure 23 we see that, over a complete alternant cycle, the sixth harmonic orientation distribution extends along the flow direction, which means that the sixth harmonic contributes to the total tumbling. From Figure 24 we see that, over a complete alternant cycle, the sixth harmonic orientation distribution widens along the vorticity direction, which means that the second harmonic contributes to the total wobbling.

From Figure 22A we see that for the sixth harmonic, the complex orientation distribution varies over a complete alternant cycle, with the orientations at $\omega t = \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$ contributing most and orientations at $\omega t = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$, least. This finding is verified by the movie shown in Figure 34. By comparing the values shown on the colorbar of Figure 22 to the values of all other harmonics, we show that the sixth harmonic contributes less to the total orientation distribution than the zeroth, the first, the second, the third, the fourth, and the fifth.

Comparing Figure 22A to Figure 22B and Figure 22C, we see that at $\omega t = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$, only the component in-phase with $\cos 6\omega t$ contributes to the complex orientation distribution. However, at $\omega t = \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$, only the component out-of-phase with $\cos 6\omega t$ contributes.

3.8. Total: Sum of First Seven Harmonics, Zeroth through Sixth

We next write our result for the new updated Fourier solution of the orientation distribution function, up to and including the sixth power of the shear rate amplitude, for a rigid dumbbell suspension in large-amplitude oscillatory shear flow as follows:

$$\psi(\theta, \phi, t) \equiv \sum_{n=0}^6 [\psi'_n \cos n\omega t + \psi''_n \sin n\omega t] \quad (43)$$

where ψ'_n and ψ''_n represent the Fourier coefficients in and out-of-phase with $\cos n\omega t$, found earlier. This new solution updates all the first four harmonics, and adds fifth and sixth harmonics.

Figure 25 illustrates the normalized alternant macromolecular orientation distribution induced by oscillatory shear deformation that is predicted by Eq. (43), in the nonlinear viscoelastic regime. As seen from this Figure 25, the total orientation distribution varies over a complete alternant cycle. Generally, molecules are forced to orient into a peanut shape, with higher probabilities along the direction of the flow, than the other two axes. The fact that the orientation distribution widens into nearly spherical at the end of the half cycle, after $\omega t = \pi$, illustrates the viscoelastic nature of the polymeric fluid. Once you change the shearing direction, for just a moment, the molecules are not subjected to shear and their orientations nearly randomize. This finding is verified by the movie shown in Figure 35.

To study the tumbling and wobbling rotations of the molecules predicted by the total orientation distribution function, we plot the side and top views of Figure 25 shown in Figure 26 and Figure 27. From Figure 26 we see that the orientation distribution extends along the flow direction, which means that molecules are likely to tumble. From Figure 27 we see that the orientation distribution widens along the vorticity direction, which means that molecules are likely to wobble.

4. CONCLUSION

We focus specifically on a suspension of rigid dumbbells in large-amplitude oscillatory shear flow (LAOS) at high shear rate amplitude (Weissenberg number) and high frequency (Deborah number). We extend the solution for the diffusion equation for the orientation distribution function to the 6th power of the shear rate amplitude. We arrive at the Fourier solution for each harmonic contribution to the total orientation distribution function, separating each harmonic into its coefficients in and out-of-phase with $\cos n\omega t$, ψ'_n and ψ''_n , respectively.

To deepen our understanding of the macromolecular motions, we distinguish and study the two types of possible rotations, tumbling and wobbling. We first investigate the contribution of each orientation harmonic to tumbling or wobbling. We find that the zeroth harmonic contributes the most, and the sixth, the least.

Further, we study the contributions of the components in and out-of-phase with $\cos n\omega t$ to the complex orientation distribution. We find that for odd harmonics, at $\omega t = \frac{1}{4}\pi, \frac{5}{4}\pi$, the components in and out-of-phase with $\cos n\omega t$ interfere constructively. However, at $\omega t = \frac{3}{4}\pi, \frac{7}{4}\pi$, the interference is destructive.

For the zeroth and the fourth harmonics, we find that only the component in-phase with $\cos n\omega t$ contributes to the complex orientation distribution. For the second and the sixth harmonics, we find that at $\omega t = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$, only the component in-phase with $\cos n\omega t$ contributes to the complex orientation

distribution. However, at $\omega t = \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$, only the component out-of-phase with $\cos n\omega t$ contributes. So for even harmonics, the contributions of the n and the $n+4$ harmonics resemble one another.

Finally, we study tumbling and wobbling rotations of molecules predicted by the total orientation distribution function, we see that the total orientation distribution extends along the flow direction and also widens along the vorticity direction, which means that molecules are likely to both tumble and wobble. We arrive at these conclusions by the use of three-dimensional imagery, and specifically, by the use of color-enhanced isometric, side and top views of the orientation distribution functions frozen at particular positions in time. We then enhance these findings with video-graphic imagery over a complete alternant cycle for most values of ωt .

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6. APPENDIX A: $a'_{i,j}$, $b'_{i,j}$, $a''_{i,j}$, $b''_{i,j}$

6.1. Zeroth Harmonic

$$a'_{2,0} = 15C^4 - 30C^2 + 7 \quad (44)$$

$$b'_{2,0} = 1920(1+W) \quad (45)$$

$$a'_{2,2} = -40 \quad (46)$$

$$b'_{2,2} = 1920(1+W) \quad (47)$$

$$a'_{2,4} = -15 \quad (48)$$

$$b'_{2,4} = 1920(1+W) \quad (49)$$

$$a'_{4,0} = - \left(\begin{aligned} &7541100C^8W^4 - 6879600C^6W^4 - 115156125C^8W^3 - 5798520C^4W^4 \\ &+ 726400500C^6W^3 + 2071440 * C^2W^4 - 2469092850C^8W^2 - 935716110C^4W^3 \\ &+ 614124W^4 + 8341422600C^6W^2 + 152855220C^2W^3 - 2925208125C^8W \\ &- 7400656620C^4W^2 + 45215107W^3 + 9923392500C^6W - 346583160C^2W^2 \\ &- 578812500C^8 - 2177484750C^4W + 678370894W^2 + 2315250000C^6 \\ &- 13714375500C^2W + 1877925000C^4 + 3914351075W - 8386350000C^2 + 2153427500 \end{aligned} \right) \quad (50)$$

$$b'_{4,0} = 206438400(9W+25)(9W+100)(W+49)(4W+1)(W+1)^2 \quad (51)$$

$$a'_{4,2} = -560 \left(\begin{aligned} &1890C^4W^4 - 27540C^2W^4 + 928725C^4W^3 - 16398W^4 - 2060010C^2W^3 \\ &+ 12242160C^4W^2 - 1094779W^3 - 19463760C^2W^2 + 15042825C^4W \\ &- 16195968W^2 - 23794290C^2W + 3727500C^4 - 51647655W \\ &- 6363000C^2 - 25028500 \end{aligned} \right) \quad (52)$$

$$b'_{4,2} = 206438400(9W+25)(9W+100)(W+49)(4W+1)(W+1)^2 \quad (53)$$

$$a'_{4,4} = 420 \left(\begin{aligned} &23940C^4W^4 + 7560C^2W^4 - 365575C^4W^3 + 29988W^4 + 1363950C^2W^3 \\ &- 7838390C^4W^2 + 2534657W^3 + 12022140C^2W^2 - 9286375C^4W \\ &+ 32391450W^2 + 14340750C^2W - 1837500C^4 + 55948065W \\ &+ 3675000 * C^2 + 20310500 \end{aligned} \right) \quad (54)$$

$$b'_{4,4} = 206438400(9W+25)(9W+100)(W+49)(4W+1)(W+1)^2 \quad (55)$$

$$a'_{4,6} = 58800(W+1)(18W^3 + 8827W^2 + 107765W + 35500) \quad (56)$$

$$b'_{4,6} = 206438400(9W+25)(9W+100)(W+49)(4W+1)(W+1)^2 \quad (57)$$

$$a'_{4,8} = -3675(9W+100)(19W-525)(W+1)(4W+1) \quad (58)$$

$$b'_{4,8} = 206438400(9W+25)(9W+100)(W+49)(4W+1)(W+1)^2 \quad (59)$$

$$\begin{aligned}
& \left(28003178246523480000C^{12}W^{15} - 85841022226783824000C^{10}W^{15} + 18134110514328378570000C^{12}W^{14} \right. \\
& + 70915485068141313600C^8W^{15} - 54575694872932778344800C^{10}W^{14} + 1897604426804486400C^6W^{15} \\
& + 3236419938740709363142500C^{12}W^{13} + 44243199360676709593200C^8W^{14} - 15130465199560751040C^4W^{15} \\
& - 9435692264430638446039800C^{10}W^{13} + 1439672780875357828800C^6W^{14} + 2092218500921028480C^2W^{15} \\
& + 212073423124311898691416875C^{12}W^{12} + 6909856149333805486919100C^8W^{13} - 9220763037996309880080C^4W^{14} \\
& - 172259155656447552W^{15} - 580809031569130493520641250C^{10}W^{12} + 1224770510619359235932400C^6W^{13} \\
& + 1223521569353328204960C^2W^{14} + 6081723344558465735064058125C^{12}W^{11} + 316501172037418430741003325C^8W^{12} \\
& - 1884159956275545371494500C^4W^{13} - 118771267113762478704W^{14} - 14360123625710744338541434350C^{10}W^{11} \\
& + 222237721725845130748743300C^6W^{12} + 214525473066727138859400C^2W^{13} + 81418388612560746771603954375C^{12}W^{10} \\
& + 1335714619182910964929406475C^8W^{11} - 156935744216823453444100395C^4W^{12} - 28570683986122463984700W^{13} \\
& - 119650025451698222026469373450C^{10}W^{10} + 13500151580273371959053007900C^6W^{11} + 12758254206218867132662590C^2W^{12} \\
& + 384178801667666341586420795625C^{12}W^9 - 212773364578460103738098954175C^8W^{10} - 5305722254191958687577087885C^4W^{11} \\
& - 3293164601182324745135901W^{12} + 881325464913899458911856247850C^{10}W^9 + 364224489240225894867664377300C^6W^{10} \\
& + 78409352035084920600349170C^2W^{11} - 210730336618578882699425689375C^{12}W^8 - 5393673518933922765976623215025C^8W^9 \\
& - 58265509338112239054057787335C^4W^{10} - 204356906703755087314459563W^{11} + 24731168308120790191474698749850C^{10}W^8 \\
& + 4660076666054790277112533617900C^6W^9 - 15309195147486162039758599530C^2W^{10} - 31837540716168086395995859985625C^{12}W^7 \\
& - 5754677718407638461267326442425C^8W^8 + 708561216122906054984221916055C^4W^9 - 7020095950647612342423546273W^{10} \\
& + 192312882500704968001566803786550C^{10}W^7 + 26311614268678040754011560300300C^6W^8 - 534358489249987221477357838710C^2W^9 \\
& - 13545620743680436370283346251875C^{12}W^6 - 312109488137268958873311183383775C^8W^7 + 23798249676051520646206498014175C^4W^8 \\
& - 139693639479673141663169748591W^9 + 702926134813018736199103483609650C^{10}W^6 + 8280961653001136709897566178900C^6W^7 \\
& - 7356254765320843099162596844550C^2W^8 - 272966486624431900052085698990625C^{12}W^5 - 847254853027148107121790303017925C^8W^6 \\
& + 249212405297424590961242982566745C^4W^7 - 1758468467106614611653600996535W^8 + 1350759610859275574710595356263750C^{10}W^5 \\
& - 612521530174178748063351560849700C^6W^6 - 45368911211293078190792789154090C^2W^7 - 290275438407705676960234200000000C^{12}W^4 \\
& - 1082592620003092217116747815540375C^8W^5 + 1284481991645519122504807780513635C^4W^6 - 16257620887020272418839895059969W^7 \\
& + 1431531304701227012445618456000000C^{10}W^4 - 2721961902112972105213754851223500C^6W^5 - 44273410103961175106217006943470C^2W^6 \\
& - 164872932249397121834021437500000C^{12}W^3 - 512203604938265177015489611200000C^8W^4 + 3270225788203526113098103646590625C^4W^5 \\
& - 113978758735541084139328097354587W^6 + 835512595499231411685548625000000C^{10}W^3 - 49289090246292016542084446784000000C^6W^4 \\
& + 839805475394446484671585935275750C^2W^5 - 47697526278816459420375000000000C^{12}W^2 + 1185166069146903467025777375000000C^8W^3 \\
& + 39222431755070344345388375960000000C^4W^4 - 526639453126336011128521574487625W^5 + 25549710160089293283105000000000C^{10}W^2 \\
& - 43597618351081283822886052500000000C^6W^3 + 35746740940440075979443853480000000C^2W^4 - 6486284080054290074062500000000C^{12}W \\
& + 1808011264446578462541270000000000C^8W^2 + 2007120662480016041815594237500000C^4W^3 - 1322813338480416641768388680000000W^4 \\
& + 37022243164755005532375000000000C^{10}W - 1909087573891771142713844000000000C^6W^2 + 5402652368705817349000701825000000C^2W^3 \\
& - 318134899212755625000000000000C^{12} + 491270020703044239281625000000000C^8W + 201395085701446455505687000000000C^4W^2 \\
& - 1655926805226176669327980697500000W^3 + 1908809395276533750000000000000C^{10} - 382569397919529081411150000000000C^6W \\
& + 360996769416222324480650000000000C^2W^2 + 3965735474240965425000000000000C^8 - 130457069291661088413937500000000C^4W \\
& - 1010521768393642703483573400000000W^2 - 2749652774714908062000000000000C^6 + 1065920812717242438131175000000000C^2W \\
& - 22938602553578967255000000000000C^4 - 282888021908598248383762500000000W + 104868608870535179670000000000000C^2 \\
& - 27030100540114323729000000000000
\end{aligned}$$

(60)

$$\begin{aligned}
b'_{6,0} = & 170039181312000(1+W)^3(49+W)^2(25+9W)^2(36+W)(3025+9W) \\
& \times (49+9W)(100+81W)(144+W)(1+4W)^2(100+9W)^2(49+4W)(1+9W)
\end{aligned}$$

(61)

$$\begin{aligned}
& \left(\begin{aligned}
& 20714908642243200C^8W^{15} - 30009281141548800C^6W^{15} + 12440693739416065200C^8W^{14} \\
& -2416787553922560C^4W^{15} - 17466378459706209600C^6W^{14} + 1998304551048960C^2W^{15} \\
& +2156747881785649614000C^8W^{13} - 2007738239913594720C^4W^{14} - 1479192778175616W^{15} \\
& -2804207432991763251600C^6W^{13} + 967138686504285120C^2W^{14} + 137757339800337882153375C^8W^{12} \\
& -675209394778433402160C^4W^{13} - 1008850129909754832W^{14} - 14512956596869582632700C^6W^{12} \\
& +84101868984655463760C^2W^{13} + 3677730351269905347780825C^8W^{11} - 93035595714085981586070C^4W^{12} \\
& -23982437960342085760W^{13} - 1558423829826380896690500C^6W^{11} - 12159382533785871820380C^2W^{12} \\
& +35878770992914950915027675C^8W^{10} - 5422710414893454298530810C^4W^{11} - 27434111006374633485873W^{12} \\
& +72643983366394903780667700C^6W^{10} - 2046663856279367628280740C^2W^{11} - 193208187305004589597277475C^8W^9 \\
& -149664468485394328865659470C^4W^{10} - 1757154858996218497089399W^1 + 2431545834842447119663337100C^6W^9 \\
& -106365332458860143881001580C^2W^{10} - 712261005430969067927156275C^8W^8 - 1968618351646082000574755010C^4W^9 \\
& -68133663699917012323435989W^{10} + 31188987247079847500423621100C^6W^8 - 2709819657431738527118081940C^2W^9 \\
& -59864557032264895784565909525C^8W^7 - 10588218731785937990623523970C^4W^8 - 1641303036245022382876264083W^9 \\
& +204121217431520887435847147700C^6W^7 - 38142763613049890320947720180C^2W^8 - 22898757400797979770147159975C^8W^6 \\
& +8823218995370202964072183890C^4W^7 - 24396102912952731455331783875W^8 + 712540712205862236069745333500C^6W^6 \\
& -310319802918889926464304133740C^2W^7 - 455004972663975628288088951025C^8W^5 + 342219465950444799273392044230C^4W^6 \\
& -217260452716397746358825619717W^7 + 137520564405826424941491006100C^6W^5 - 1456902056339202390654727018980C^2W^6 \\
& -495402505440522630758583930000C^8W^4 + 1393091377957366108392074626650C^6W^5 - 116126487585534194594922517431W^6 \\
& +1501413071905676948826839220000C^6W^4 - 3837497006711978897793689616300C^2W^5 - 295662278297269735307032500000C^8W^3 \\
& +2439756672056120266593221520000C^4W^4 - 3750957660051095733498590729425W^5 + 918150411861573619094100000000C^6W^3 \\
& -5571130326445408306878219924000C^2W^4 - 92064777534829673273700000000C^8W^2 + 2125397423781527998495743000000C^4W^3 \\
& -7097969695529434746910562358000W^4 + 297806965139869893955800000000C^6W^2 - 4414130092368467938417809600000C^2W^3 \\
& -13523472299511531472500000000C^8W + 929179664087130658834800000000C^4W^2 - 7415071360164634711987253700000W^3 \\
& +45552324117796443990000000000C^6W - 1835952975400033375162200000000C^2W^2 - 703358095217085000000000000C^8 \\
& +18889515328974201922500000000C^4W - 4032640474749759224405900000000W^2 + 243735708866958000000000000C^6 \\
& -36395916317196510097800000000C^2W + 1381530643091260200000000000C^4 - 1043424735875333089708500000000W \\
& -2638343211991779600000000000C^2 - 9480814540470573300000000000
\end{aligned} \right)
\end{aligned}$$

(62)

$$\begin{aligned}
b'_{6,2} &= 89181388800(1+W)^3(49+W)^2(25+9W)^2(36+W)(3025+9W) \\
&\times (49+9W)(200+81W)(144+W)(1+4W)^2(100+9W)^2(49+4W)(1+9W)
\end{aligned}$$

(63)

$$\begin{aligned}
& \left(\begin{aligned}
& 117495852782616000C^8W^{15} - 163491859606521600C^6W^{15} + 76087176983195994000C^8W^{14} \\
& -134179600154705280C^4W^{15} - 102096145832877241920C^6W^{14} + 48726720913086720C^2W^{15} \\
& +13579384358352626698500C^8W^{13} - 80817829392088686240C^4W^{14} + 26402272643984832W^{15} \\
& -17085547249676734363920C^6W^{13} + 26386449743394578880C^2W^{14} + 889818558563546428075875C^8W^{12} \\
& -1576778423216600847080C^4W^{13} + 14892727207939603344W^{14} - 979762785576364178959500C^6W^{12} \\
& +3606157309245022433520C^2W^{13} + 25517720326818737349919125C^8W^{12} - 1261151191168791436516590C^4W^{12} \\
& +2257738851149960783364W^{13} - 19533943529596440973222740C^6W^{11} + 45081129593627156116980C^2W^{12} \\
& +341615616556198937503233375C^8W^{10} - 45572798883725885695121730C^4W^{11} + 54774001012572231277347W^{12} \\
& +9240307569763722632863620C^6W^{10} - 15563833036240434171098580C^2W^{11} + 1611939027976222412250716625C^8W^9 \\
& -771730606649037475220150790C^4W^{10} - 9870997614710456256697899W^{11} + 5436246778239250242578974140C^6W^9 \\
& -891200478885553851377972220C^2W^{10} - 8841832305674638434940920375C^8W^8 - 4466343702874317310454413530C^4W^9 \\
& -716204277740855257485454305W^{10} + 80448347818525340609727998940C^6W^8 - 20708781332056516717658192580C^2W^9 \\
& -133584086920984977885297314625C^8W^7 + 30918322428309368553982616310C^4W^8 - 20643872279821640291444705007W^9 \\
& +539138727930826436951321263620C^6W^7 - 251017374361819731724682362020C^2W^8 - 568347723511218236518671382875C^8W^6 \\
& +561717692531793998767268006970C^4W^7 - 311014214953760792821491356407W^8 + 1863838969352944397738660938860C^6W^6 \\
& -1720270515268080668568324966780C^2W^7 - 1145313929892721258959800135625C^8W^5 + 3043037836423360277806641857310C^4W^6 \\
& -2592231452086467267994100722305W^7 + 3510712842601874963664211870500C^6W^5 - 6792179880547218320363668441620C^2W^6 \\
& -1217938902409953889343640000000C^8W^4 + 8047771634797656544868331280650C^6W^5 - 12175134823536200876905171421979W^6 \\
& +3715125956511584138014982400000C^6W^4 - 15504498043691155287673194462300C^2W^5 - 691774540906561350352537500000C^8W^3 \\
& +11257458466843925303063470080000C^4W^4 - 32455533255248284566825838750985W^5 + 2193764837762743977378150000000C^6W^3 \\
& -20249153723882260527765285120000C^2W^4 - 200129480890138990575000000000C^8W^2 + 8410610947990527217446855000000C^4W^3 \\
& -491097423139499306383521600000W^4 + 686063635180114661820000000000C^6W^2 - 14808212075198531163788250000000C^2W^3 \\
& -27215177958269748562500000000C^8W + 3215526324427149356089200000000C^4W^2 - 4157481199370085393356319900000W^3 \\
& +10179139224120678945000000000C^6W - 5754267937089596920951200000000C^2W^2 - 1334831744948625000000000000C^8 \\
& +565600784199415084785000000000C^4W - 18828023374950195241638040000000W^2 + 5339326979794500000000000000C^6 \\
& -1065942219233101406070000000000C^2W + 3502116896563593000000000000C^4 - 4149015904033506999538500000000W \\
& -7213712369749023600000000000C^2 - 33319584183430272420000000000
\end{aligned} \right)
\end{aligned}$$

(64)

$$b'_{6,4} = 475634073600(1+W)^3(49+W)^2(25+9W)^2(36+W)(3025+9W) \quad (65)$$

$$\begin{aligned} & \times (49+9W)(100+81W)(144+W)(1+4W)^2(100+9W)^2(49+4W)(1+9W) \\ & \left(\begin{aligned} & 2969293352269651989120C^4W^{14} - 624149590694982992640C^2W^{14} + 1780290766155798330865200C^4W^{13} \\ & - 1941116324549416085376W^{14} - 790718781976282593594720C^2W^{13} + 307369860871408798915267200C^4W^{12} \\ & - 1110704901272771437707984W^{13} - 236741303534608760643018720C^2W^{12} \\ & + 19438889239752846385359412575C^4W^{11} - 185875765531932410417562912W^{12} \\ & - 15752954109143683180213961310C^2W^{11} + 507730236729586510938852077370C^4W^{10} \\ & - 11162931305501024715756970593W^{11} - 277760417237508351637680490260C^2W^{10} \\ & + 4635164575734864849320596738785C^4W^9 - 144269874153261111760170377958W^{10} \\ & + 3384139342423545028938562154910C^2W^9 - 32329797271285835726259421879620C^4W^8 \\ & + 7467289301316416246959459281633W^9 + 169794008709466206977108627302440C^2W^8 \\ & - 988631436671666937900165036917295C^4W^7 + 301391903572326285463920751075356W^8 \\ & + 2244975700962798090113546257553790C^2W^7 - 7592407191226554658481493049916070C^4W^6 \\ & + 4544608484955484664205301006495249W^7 + 14000182238264826172481347290723180C^2W^6 \\ & - 25230874484642845566896340134099265C^4W^5 + 34029326765771065127524326424704890W^6 \\ & + 43213148321066338784287708271847330C^2W^5 - 39989941301415780513189099556038000C^4W^4 \\ & + 132003108864054547860569281147397695W^5 + 67322504669588642935989676189476000C^2W^4 \\ & - 31021492613504980699790707144500000C^4W^3 + 264509986456679646926731803890530000W^4 \\ & + 53070856425231493030163423757000000C^2W^3 - 11359000229929297883742031920000000C^4W^2 \\ & + 263647902486409389066822619294700000W^3 + 20262863417266009988335092240000000C^2W^2 \\ & - 1837646525001861189592168500000000C^4W + 124311093460817015436629797600000000W^2 \\ & + 3415278546251945327875977000000000C^2W - 10081997234272987046100000000000C^4 \\ & + 25735798251149964000120751500000000W + 19256104244039074057800000000000C^2 \\ & + 1834511814333925722487800000000000 \end{aligned} \right) \end{aligned} \quad (66)$$

$$b'_{6,6} = 8522226173214720(25+9W)^2(1+W)^2(1+4W)^2(49+W)^2(49+9W) \quad (67)$$

$$\begin{aligned} & \times (49+4W)(3025+9W)(144+W)(100+9W)^2(100+81W)(36+W+36)(1+9W) \\ & \left(\begin{aligned} & 6216711787440C^4W^{12} - 2298120945120C^2W^{12} + 3948931094807160C^4W^{11} \\ & - 16144760930256W^{12} - 1334899690378128C^2W^{11} + 669584871456132555C^4W^{10} \\ & - 9330064267974408W^{11} - 189221946896163798C^2W^{10} + 3874768325932739470C^4W^9 \\ & - 1548297111117227805W^{10} - 6501515615805672828C^2W^9 + 861701359296030883245C^4W^8 \\ & - 8797525810377658778W^9 + 112153278314670137766C^2W^8 + 6873604953843773764080C^4W^7 \\ & - 1478974948821453347307W^8 + 8049126588655169522880C^2W^7 - 11968747538316934125915C^4W^6 \\ & + 13028039883360913359920W^7 + 121350143962183634826102C^2W^6 - 419453548528533797103210C^4W^5 \\ & + 631196900798567676803213W^6 + 812278198455606831713076C^2W^5 - 1732897743129964282427325C^4W^4 \\ & + 6290527606876375397805766W^5 + 2515915133115553961725050C^2W^4 - 2686912937628637463707500C^4W^3 \\ & + 24680797970524972810216155W^4 + 3677931056907177755895000C^2W^3 - 1718993869501486866750000C^4W^2 \\ & + 43615505467694140910856500W^3 + 2503803790695690661500000C^2W^2 - 388969226218690425000000C^4W \\ & + 34532243964631195229650000W^2 + 693779885867473650000000C^2W - 2542536657045000000000C^4 \\ & + 10647879698597233855000000W + 5085073314090000000000C^2 + 859759186968990000000000 \end{aligned} \right) \end{aligned} \quad (68)$$

$$b'_{6,8} = 2264924160(1+W)^2(49+W)^2(25+9W)^2(36+W)(3025+9W) \quad (69)$$

$$\begin{aligned} & \times (49+9W)(100+81W)(144+W)(1+4W)(100+9W)(49+4W)(1+9W) \\ & a'_{6,10} = 77 \left(\begin{aligned} & 6508512W^8 + 3484118124W^7 + 444691677573W^6 + 11242866487322W^5 \\ & + 27216003084525W^4 - 1675937916851396W^3 - 14094592005765200W^2 \\ & - 19761578815160000W - 5260494960000000 \end{aligned} \right) \quad (70)$$

$$b'_{6,10} = 12582912(25+9W)(1+W)(1+4W)(49+W)(49+9W) \quad (71)$$

$$\begin{aligned} & \times (49+4W)(3025+9W)(144+W)(100+9W)(100+81W)(36+W) \\ & a'_{6,12} = 77(1899W^3 + 785194W^2 + 5986155W - 84892500) \quad (72) \end{aligned}$$

$$b'_{6,12} = 301989888(1+W)(49+W)(25+9W)(36+W)(3025+9W)(49+9W) \quad (73)$$

6.2. First Harmonic

$$a'_{1,2} = -1 \quad (74)$$

$$b'_{1,2} = 4(1+W) \quad (75)$$

$$a'_{1,2} = -1 \quad (76)$$

$$b'_{1,2} = 4(1+W) \quad (77)$$

$$a'_{3,2} = \begin{pmatrix} 790020C^4W^4 + 589680C^2W^4 - 12063975C^4W^3 - 93348W^4 \\ +64225350C^2W^3 - 258666870C^4W^2 - 5822239W^3 \\ +524550420C^2W^2 - 306450375C^4W + 17534962W^2 \\ +626289750C^2W - 60637500C^4 + 543422225W \\ +165375000C^2 + 330382500 \end{pmatrix} \quad (78)$$

$$b'_{3,2} = 506880(25+9W)(49+W)(100+9W)(1+4W)(1+W)^2 \quad (79)$$

$$a'_{3,4} = 5(81W^2 + 1097W + 400) \quad (80)$$

$$b'_{3,4} = 256(25+9W)(100+9W)(1+W)(1+4W) \quad (81)$$

$$a'_{3,6} = -7(19W - 525) \quad (82)$$

$$b'_{3,6} = 6144(25+9W)(1+W)(49+W) \quad (83)$$

$$a''_{3,2} = - \begin{pmatrix} 124740C^4W^4 - 554040C^2W^4 + 16302825C^4W^3 - 342252W^4 \\ -37393650C^2W^3 + 184224810C^4W^2 - 22463751W^3 \\ -337600260C^2W^2 + 209049225C^4W - 311551462W^2 \\ -481885650C^2W + 41002500C^4 - 1135347335W \\ -181125000C^2 - 656141500 \end{pmatrix} \quad (84)$$

$$b''_{3,2} = 506880(25+9W)(49+W)(100+9W)(1+4W)(1+W)^2 \quad (85)$$

$$a''_{3,4} = (153W^2 + 1922W + 1125) \quad (86)$$

$$b''_{3,4} = 128(25+9W)(100+9W)(1+W)(1+4W) \quad (87)$$

$$a''_{3,6} = 7(3W + 355) \quad (88)$$

$$b''_{3,6} = 6144(25+9W)(1+W)(49+W) \quad (89)$$

$$\begin{aligned}
& \left(\begin{aligned}
& 117495852782616000C^8W^{15} - 182647581576768000C^6W^{15} + 76087176983195994000C^8W^{14} \\
& + 15717411620830080C^4W^{15} - 114736930964121412800C^6W^{14} + 19698399928811520C^2W^{15} \\
& + 13579384358352626698500C^8W^{13} + 9020270044764488160C^4W^{14} - 1463089860783936W^{15} \\
& - 19412546636160095140800C^6W^{13} + 12040562592055448640C^2W^{14} \\
& + 889818558563546428075875C^8W^{12} - 84277686974625247320C^4W^{13} - 855609489058271472W^{14} \\
& - 1140982251118728024793500C^6W^{12} + 2485170936702352170720C^2W^{13} + 25517720326818737349919125C^8W^{11} \\
& - 255664287377697449294190C^4W^{13} - 15001781333375621580W^{13} - 24692502140701347749876100C^6W^{11} \\
& + 210568695263935692136740C^2W^{12} + 34161561655619893750323375C^8W^{10} - 20956237471658240155667970C^4W^{11} \\
& - 8921856088264942050813W^{12} - 76741115792396244407498700C^6W^{10} + 7365758850593589129058620C^2W^{11} \\
& + 1611939027976222412250716625C^8W^9 - 671911366361515415702880390C^4W^{10} - 54831714709849594825419W^{11} \\
& + 4693496597605241499355109100C^6W^9 + 92195953722874573529981940C^2W^{10} - 8841832305674638434940920375C^8W^8 \\
& - 10393702056755381622257822970C^4W^9 + 10705730872367945482348671W^{10} + 77630784156286166430355229100C^6W^8 \\
& - 617317442654422998944815380C^2W^9 - 133584086920984977885297314625C^8W^7 - 83339221953018405912786769290C^4W^8 \\
& + 373677265209781273760390097W^9 + 538838579165396029113188013300C^6W^7 - 28140031179567970764510768660C^2W^8 \\
& - 568347723511218236518671382875C^8W^6 - 334194954085705953749220891270C^4W^7 + 5144234101622967202211606185W^8 \\
& + 1889435964646189932009662475900C^6W^6 - 316822307261228044567617605580C^2W^7 \\
& - 1145313929892721258959800135625C^8W^5 - 480507680184993584414925784290C^4W^6 + 31726511336568586147407544863W^7 \\
& + 3577621772412438093175148662500C^6W^5 - 1765517883347606342589789198660C^2W^6 - 1217938902409953889343640000000C^8W^4 \\
& + 549390667508683639701728199450C^4W^5 + 30960426646126695878473431429W^6 + 3787415309832098601724956000000C^6W^4 \\
& - 5161018917343705392215240019900C^2W^5 - 691774540906561350352537500000C^8W^3 + 2354677953361426777905404640000C^4W^4 \\
& - 587276556219892646623486668025W^5 + 2229598170629212816380150000000C^6W^3 - 7985690629776758510643780000000C^2W^4 \\
& - 200129480890138990575000000000C^8W^2 + 2561113155005969030932815000000C^4W^3 - 2499772091216836349261436000000W^4 \\
& + 69321702820389224310000000000C^6W^2 - 6585240345220873729113210000000C^2W^3 - 27215177958269748562500000000C^8W \\
& + 119895444832305799055600000000C^4W^2 - 3778078579514557586713777500000W^3 + 10223322471569880225000000000C^6W \\
& - 2806124381514067227616800000000C^2W^2 - 1334831744948625000000000000C^8 + 23965141232494194610500000000C^4W \\
& - 25244529329805756814550000000000W^2 + 5339326979794500000000000000C^6 - 562941730163580602310000000000C^2W \\
& + 16432293341307690000000000000C^4 - 745399169732337369322500000000W - 41252729904459612000000000000C^2 \\
& - 733346915178567690000000000000
\end{aligned} \right) \quad (90)
\end{aligned}$$

$$\begin{aligned}
b'_{5,2} &= 7431782400(100 + 81W)(36 + W)(3025 + 9W)(49 + 4W)(144 + W) \\
&\quad \times (49 + 9W)(1 + 9W)(25 + 9W)^2(100 + 9W)^2(49 + W)^2(1 + 4W)^2(1 + W)^3 \quad (91)
\end{aligned}$$

$$\begin{aligned}
& \left(\begin{aligned}
& 1261555903440C^4W^{12} - 217491166368C^2W^{12} + 243982501108920C^4W^{11} \\
& + 36095472720W^{12} - 19070237347056C^2W^{11} + 9405184828490325C^4W^{10} \\
& + 1122827028856W^{11} + 3231622172128518C^2W^{10} + 103441907195102385C^4W^9 \\
& + 1291231921439709W^{10} + 226760147875359630C^2W^9 - 904047893400510810C^4W^8 \\
& + 76632955609386825W^9 + 5279224428914534532C^2W^8 - 2856257683237331430C^4W^7 \\
& + 2436744871578846870W^8 + 59355959961211681068C^2W^7 - 229170521902944025215C^4W^6 \\
& + 41730198858761352522W^7 + 350655208927595649294C^2W^6 - 782879451036482275875C^4W^5 \\
& + 375633079542676009513W^6 + 1075556468937739403958C^2W^5 - 1265845900225321987740C^4W^4 \\
& + 1757019111853100992597W^5 + 1711681683837485034024C^2W^4 - 998930387006837334000C^4W^3 \\
& + 4094643859838782069188W^4 + 1399422157895289242400C^2W^3 - 371987585083938600000C^4W^2 \\
& + 4550349247474620701200W^3 + 558970442211698160000C^2W^2 - 61006109832360000000C^4W \\
& + 2339901565071262360000W^2 + 9765440930649600000C^2W - 3376190160000000000C^4 \\
& + 562600393729464000000W + 5625946368000000000C^2 + 40732722979200000000
\end{aligned} \right) \quad (92)
\end{aligned}$$

$$\begin{aligned}
b'_{5,4} &= 884736(1 + 9W)(49 + 9W)(144 + W)(49 + 4W)(49 + W) \\
&\quad \times (36 + W)(100 + 81W)(1 + 4W)^2(100 + 9W)^2(1 + W)^2(25 + 9W)^2 \quad (93)
\end{aligned}$$

$$a'_{5,6} = - \left(\begin{aligned} &4444948928019600C^4W^{12} - 1643156475760800C^2W^{12} + 2823485732787119400C^4W^{11} \\ &- 2884882410806256W^{12} - 954453278620361520C^2W^{11} + 478753183091134776825C^4W^{10} \\ &- 1765831251135853944W^{11} - 135293692030757115570C^2W^{10} + 27704593525846908721050C^4W^9 \\ &- 333948813402666260007W^{10} - 4648583665301056072020C^2W^9 + 616116471896662081520175C^4W^8 \\ &- 2371552230227370700678W^9 + 80189593994989148502690C^2W^8 + 4914627541998298241317200C^4W^7 \\ &- 51880041087720368956913W^8 + 5755125510888446208859200C^2W^7 - 8557654489896607900029225C^4W^6 \\ &+ 2297894371774932481391088W^7 + 86765352932961298900662930C^2W^6 \\ &- 299909287197901664928795150C^4W^5 + 195784648652834689959274871W^6 + 580778911895758884674849340C^2W^5 \\ &- 1239021886337924461935537375C^4W^4 + 2136233833524047617234150034W^5 \\ &+ 1798879320177621082633410750C^2W^4 - 1921142750404475786550862500C^4W^3 + 8967636872541759738974314305W^4 \\ &+ 2629720705688632095464925000C^2W^3 - 1229080616693563109726250000C^4W^2 + 17204784685385937696871041500W^3 \\ &+ 1790219710347418822972500000C^2W^2 - 278112996746363653875000000C^4W + 14602001003462985218144150000W^2 \\ &+ 496052618395243659750000000C^2W - 18179137097871750000000000C^4 + 4739383381545291101805000000W \\ &+ 36358274195743500000000000C^2 + 395372879169573690000000000 \end{aligned} \right) \quad (94)$$

$$b'_{5,6} = 20242759680(W+1)^2(9W+25)^2(4W+1)(W+49)^2(9W+100) \times (9W+49)(4W+49)(9W+3025)(9W+1)(W+144)(81W+100)(W+36) \quad (95)$$

$$a'_{5,8} = 35 \left(\begin{aligned} &1373436W^7 + 244674999W^6 + 6439094550W^5 + 2081962267W^4 - 1350564943376W^3 \\ &- 10912431086416W^2 - 15517666057600W - 4287225600000 \end{aligned} \right) \quad (96)$$

$$b'_{5,8} = 196608(W+1)(9W+25)(4W+1)(W+49)(9W+100)(9W+49)(4W+49)(W+144)(81W+100)(W+36) \quad (97)$$

$$a'_{5,10} = 77(1899W^3 + 785194W^2 + 5986155W - 84892500) \quad (98)$$

$$b'_{5,10} = 145728(W+1)(9W+25)(W+49)(9W+49)(4W+49)(9W+3025)(W+36) \quad (99)$$

$$a''_{5,2} = \left(\begin{aligned} &7239080977128000C^8W^{15} - 9517113452448000C^6W^{15} + 4273854069181158000C^8W^{14} \\ &- 643923846357120C^4W^{15} - 5381418553037150400C^6W^{14} - 264974952049920C^2W^{15} \\ &+ 714735150286027909500C^8W^{13} - 612890069326547040C^4W^{14} - 979848059585472W^{15} \\ &- 795778110785098047600C^6W^{13} - 280712894979706560C^2W^{14} + 41902462293862963456125C^8W^{12} \\ &- 229108664751536054520C^4W^{13} - 657824660749048464W^{14} - 29903684079528799539300C^6W^{12} \\ &- 102851757986448124080C^2W^{13} + 868158885030311822668875C^8W^{11} - 32927347746804487260690C^4W^{12} \\ &- 151877986167802745700W^{13} + 624453805288051529055300C^6W^{11} - 17253069182577308721060C^2W^{12} \\ &- 10220847479629638634141375C^8W^{10} - 1922803783484279876679870C^4W^{11} - 995159971899106563533013W^{11} \\ &+ 1417303259485112163869651700C^6W^9 - 59683658972919932277219060C^2W^{10} - 47727779862577358282667625C^8W^8 \\ &- 609947427166399208179748070C^4W^9 - 35370142477593163733063583W^{10} + 16262061076603063675858482900C^6W^8 \\ &- 137967229897661477237277580C^2W^9 - 33419231793223098620135097375C^8W^7 - 2186742986862272332629798390C^4W^8 \\ &- 774058841069494592918713041W^9 + 101052456636184437960126161100C^6W^7 - 18356825940966699564146362860C^2W^8 \\ &- 117156479965568634777974605125C^8W^6 + 18124140844492634491136159430C^4W^7 - 10653157440398525704496654345W^8 \\ &+ 344281346006778192277313338500C^6W^6 - 145441324326534544320814296180C^2W^7 - 218217620024845019030833140375C^8W^5 \\ &+ 204661036935910091759326179810C^4W^6 - 92771011678280673083776597119W^7 + 659433151362492575561095627500C^6W^5 \\ &- 684728177322327892493080800060C^2W^6 - 223190904645053577285946200000C^8W^4 + 753490569280485403987992480150C^4W^5 \\ &- 510328095369966812708034013797W^6 + 725469391700305076368227600000C^6W^4 - 1861783374008637759857133504660C^2W^5 \\ &- 124122200270685850560442500000C^8W^3 + 1298463914338512314575340928000C^4W^4 - 1745222379147721153542814653655W^5 \\ &+ 454195046238544048353930000000C^6W^3 - 2868788168654414714033481590400C^2W^4 - 35502244151382693531000000000C^8W^2 \\ &+ 1141087653118242464377986600000C^4W^3 - 3538516363376843576843610484800W^4 + 152918663046696788976000000000C^6W^2 \\ &- 2489158049576230648367322480000C^2W^3 - 4797217951359294037500000000C^8W + 510594085267843701238800000000C^4W^2 \\ &- 4006233822774531011209691460000W^3 + 24219038203483891950000000000C^6W - 1178241160380328219586880000000C^2W^2 \\ &- 234452698405695000000000000C^8 + 107897091399867210399000000000C^4W - 2395676557238473202519000000000W^2 \\ &+ 1326125483062380000000000000C^6 - 277108223694291871333200000000C^2W + 850651429432293720000000000C^4 \\ &- 68424700105646755627900000000W - 2469802852738677456000000000C^2 - 68303104084584535320000000000 \end{aligned} \right) \quad (100)$$

$$b''_{5,2} = 2477260800(W+36)(81W+100)(W+144)(9W+1)(9W+3025) \times (4W+49)(9W+49)(9W+100)^2(W+49)^2(4W+1)^2(9W+25)^2(W+1)^3 \quad (101)$$

$$a_{5,4}'' = - \left(\begin{aligned} &498194051040C^4W^{12} + 64283103360C^2W^{12} + 85736597083200C^4W^{11} + 136065902112W^{12} \\ &+ 39481928146080C^2W^{11} + 1489818042992790C^4W^{10} + 48021804268128W^{11} + 6228638782383960C^2W^{10} \\ &- 88579686395385675C^4W^9 + 5934783144166074W^{10} + 310118797443296790C^2W^9 - 3583970577552736035C^4W^8 \\ &+ 319416765281284581W^9 + 6824236876524060930C^2W^8 - 51352971683929760250C^4W^7 + 8629129123991561961W^8 \\ &+ 77648305291368987780C^2W^7 - 347839425480555261960C^4W^6 + 126693054164566378134W^7 \\ &+ 478729145987623509420C^2W^6 - 1148881344615166118775C^4W^5 + 1029752645442633405908W^6 \\ &+ 1579952020938348867150C^2W^5 - 1905973066218489095835C^4W^4 + 4569461599725731661977W^5 \\ &+ 2784890444074912722330C^2W^4 - 1596173148411944518500C^4W^3 + 10635444563953187937945W^4 \\ &+ 2567127315916472302200C^2W^3 - 646249001527093650000C^4W^2 + 12503901860998462355180W^3 \\ &+ 116280643676773820000C^2W^2 - 113723417436615000000C^4W + 7260871192296051398000W^2 \\ &+ 221609142249138000000C^2W - 657540261000000000C^4 + 1940153088154264200000W \\ &+ 13390450264800000000C^2 + 183004027179120000000 \end{aligned} \right) \quad (102)$$

$$b_{5,4}'' = 2211840(W+1)^2(9W+25)^2(4W+1)^2(W+49)(9W+100)^2 \times (9W+49)(4W+49)(9W+1)(W+144)(81W+100)(W+36) \quad (103)$$

$$a_{5,4}'' = - \left(\begin{aligned} &821578237880400C^4W^{12} - 172359070884000C^2W^{12} + 474892897490411400C^4W^{11} - 485605812324528W^{12} \\ &- 81669302856020880C^2W^{11} + 75234946981096027125C^4W^{10} - 262296974287712952W^{11} - 5119861586307695370C^2W^{10} \\ &+ 3818892631983499120050C^4W^9 - 35533104013812492171W^{10} + 972286435868037636060C^2W^9 \\ &+ 50258329452842975939475C^4W^8 - 299033919527076641934W^9 + 100987204826793138372090C^2W^8 \\ &- 791451186301550041066800C^4W^7 + 174872032597424852093331W^8 + 3204990825730457360145120C^2W^7 \\ &- 2526235617899895634061925C^4W^6 + 9229588174656106293646224W^7 + 43304409171261784110180330C^2W^6 \\ &- 218351028337450940773992150C^4W^5 + 182646448422517148277748763W^6 + 288408821423204042406974700C^2W^5 \\ &- 734376167635049709730035075C^4W^4 + 1635628085787515816752243562W^5 + 930990363031608020032126950C^2W^4 \\ &- 106121124338156666346972500C^4W^3 + 6868870859691881100827092605W^4 + 1478434184622396806669865000C^2W^3 \\ &- 658425695544887913893250000C^4W^2 + 13972327880481727574510509100W^3 + 1143579922994120978866500000C^2W^2 \\ &- 147243166130235950775000000C^4W + 12849208015177204685458430000W^2 + 377661795483001179150000000C^2W \\ &- 9579067392004110000000000C^4 + 4695016644070451041833000000W + 29735087277315420000000000C^2 \\ &+ 461513628320597067600000000 \end{aligned} \right) \quad (104)$$

$$b_{5,6}'' = 20242759680(W+1)^2(9W+25)^2(4W+1)(W+49)^2(9W+100) \times (9W+49)(4W+49)(9W+3025)(9W+1)(W+144)(81W+100)(W+36) \quad (105)$$

$$a_{5,8}'' = 7 \left(\begin{aligned} &1627128W^7 + 255206538W^6 + 892311939W^5 - 315791363446W^4 \\ &- 694650116167W^3 - 45993583485592W^2 - 71271808465200W \\ &- 25049152800000 \end{aligned} \right) \quad (106)$$

$$b_{5,8}'' = 294912(W+1)(9W+25)(4W+1)(W+49)(9W+100)(9W+49)(4W+49)(W+144)(81W+100)(W+36) \quad (107)$$

$$a_{5,10}'' = 77(117W^3 + 41686W^2 - 35211W - 14910700) \quad (108)$$

$$b_{5,10}'' = 1048576(W+1)(9W+25)(W^2+49)(9W^2+49)(9W+3025)(W+36) \quad (109)$$

6.3. Second Harmonic

$$a_{2,0}' = 15(4W+1)(3W-5)C^4 + (-216W^2 + 78W + 150)C^2 + 36W^2 + 25W - 35 \quad (110)$$

$$b_{2,0}' = 384(9W+25)(W+1)(4W+1) \quad (111)$$

$$a_{2,2}' = (2W-1) \quad (112)$$

$$b_{2,2}' = 48(W+1)(4W+1) \quad (113)$$

$$a_{2,4}' = 5(3W-5) \quad (114)$$

$$b_{2,4}' = 128(9W+25)(W+1) \quad (115)$$

$$a_{2,0}'' = (20W+5)C^4 - (21W+15)C^2 + 3W + 4 \quad (116)$$

$$b_{2,0}'' = 16(9W+25)(W+1)(4W+1) \quad (117)$$

$$a_{2,2}'' = -1 \quad (118)$$

$$b_{2,2}'' = 16(W+1)(4W+1) \quad (119)$$

$$a_{2,4}'' = -5 \quad (120)$$

$$b_{2,4}'' = 16(9W + 25)(W + 1) \quad (121)$$

$$a_{4,0}' = \left(\begin{aligned} &1190427840C^8W^{10} + 529079040C^6W^{10} - 222775343280C^8W^9 - 1171532160C^4W^{10} \\ &+ 710215128000C^6W^9 - 861643008C^2W^{10} - 11909158603020C^6W^8 - 468629310240C^4W^9 \\ &+ 313668288W^{10} + 31828063775760C^6W^8 - 63426079296C^2W^9 - 189453854992185C^8W^7 \\ &- 17681883831720C^4W^8 + 38161400400W^9 + 458141137961220C^6W^7 - 4446792912816C^2W^8 \\ &- 1049654564017605C^8W^6 - 167429490055830C^4W^7 + 1795014661716W^8 + 2395653098770260C^6W^6 \\ &- 152660428819452C^2W^7 - 553106272750290C^8W^5 + 182558231311170C^4W^6 + 39974401955655W^7 \\ &+ 924453790049160C^6W^5 - 2126181211220844C^2W^6 + 5866426318162410C^8W^4 \\ &+ 9099873586111380C^4W^5 + 446607552068379W^6 - 15052055474239560C^6W^4 - 13276112053490808C^2W^5 \\ &+ 8782384083850755C^8W^3 + 30117956521322460C^4W^4 + 2534787392652590W^5 - 24054779004118380C^6W^3 \\ &- 34087781183473032C^2W^4 + 384785333370375C^8W^2 + 24898363815444690C^4W^3 + 6837470916274442W^4 \\ &- 12278502565465500C^6W^2 - 30616265405454444C^2W^3 + 648851822325000C^8W + 6945416130770250C^4W^2 \\ &+ 768632558556355W^3 - 2374546022100000C^6W - 7977098681166300C^2W^2 + 35735883750000C^8 \\ &+ 151698709470000C^4W + 2596482203371175W^2 - 142943535000000C^6 + 1619472531180000C^2W \\ &- 115943089500000C^4 - 303037705579000W + 517773249000000C^2 - 132952613850000 \end{aligned} \right) \quad (122)$$

$$b_{4,0}' = 4423680(W + 1)^2(4W + 1)^2(9W + 25)(9W + 1)(W + 36)(W + 49)(9W + 100)(4W + 49)(9W + 49) \quad (123)$$

$$a_{4,2}' = \left(\begin{aligned} &1488034800C^4W^9 - 2295825120C^2W^9 + 168840511560C^4W^8 - 960845328W^9 \\ &- 203933201040C^2W^8 + 3695917686615C^4W^7 - 81893404728W^8 - 4268313859230C^2W^7 \\ &+ 23321868389505C^4W^6 - 2252346522633W^7 - 30181363124490C^2W^6 + 12368365924320C^4W^5 \\ &- 27748616423319W^6 - 56455910318880C^2W^5 - 90673912535640C^4W^4 - 163487866890240W^5 \\ &+ 34317186760800C^2W^4 - 144257274630735C^4W^3 - 441113615870888W^4 + 145442158717230C^2W^3 \\ &- 74827131370425C^4W^2 - 465866329189199W^3 + 102673566494730C^2W^2 - 14713804770000C^4W \\ &- 147978262075665W^2 + 23417574516000C^2W - 894972750000C^4 + 13614093262000W \\ &+ 1527756300000C^2 + 6009342850000 \end{aligned} \right) \quad (124)$$

$$b_{4,2}' = 276480(W + 1)^2(4W + 1)^2(9W + 25)(9W + 1)(W + 49)(9W + 100)(4W + 49)(9W + 49)(81W + 100) \quad (125)$$

$$a_{4,4}' = - \left(\begin{aligned} &535692528C^4W^{10} + 714256704C^2W^{10} - 99721478808C^4W^9 + 1709114256W^{10} \\ &+ 140151935448C^2W^9 - 5457955081257C^4W^8 + 212360674320W^9 \\ &+ 5519971059498C^2W^8 - 90505945200069C^4W^7 + 9537969068631W^8 \\ &+ 74633900566554C^2W^7 - 554970209170230C^4W^6 + 194157327218211W^7 \\ &+ 403169891040252C^2W^6 - 693296694899298C^4W^5 + 1920059466073290W^6 \\ &+ 377360225343948C^2W^5 + 2505934754258859C^4W^4 + 8940208496814998W^5 \\ &- 2456609119170654C^2W^4 + 6584714881480575C^4W^3 + 16315286099500523W^4 \\ &- 6564826844985150C^2W^3 + 4964457548452500C^4W^2 + 4324057342884471W^3 \\ &- 5914976220825000C^2W^2 + 1188565229250000C^4W - 11570618316138700W^2 \\ &- 2009028346500000C^2W + 79413075000000C^4 - 7331730743470000W \\ &- 158826150000000C^2 - 877779189000000 \end{aligned} \right) \quad (126)$$

$$b_{4,4}' = 73728(W + 1)^2(4W + 1)(9W + 25)^2(9W + 1)(W + 36)(W + 49)(9W + 100)(4W + 49)(9W + 49)(81W + 100) \quad (127)$$

$$a_{4,6}' = -7 \left(\begin{aligned} &393660W^6 + 44130987W^5 + 917535735W^4 + 4903765383W^3 \\ &- 3760560385W^2 - 20801774000W - 8523550000 \end{aligned} \right) \quad (128)$$

$$b_{4,6}' = 18432(W + 1)(4W + 1)(9W + 25)(W + 49)(9W + 100)(4W + 49)(9W + 49)(81W + 100) \quad (129)$$

$$a_{4,8}' = 7(9W^3 - 1934W^2 - 38563W + 102900) \quad (130)$$

$$b_{4,8}' = 32768(W + 1)(9W + 25)(W + 36)(W + 49)(9W + 49) \quad (131)$$

$$a''_{4,0} = - \left(\begin{aligned} &8332994880C^8W^9 - 12565627200C^6W^9 + 199892674800C^8W^8 + 4709223360C^4W^9 \\ &- 28592313120C^6W^8 - 212891328C^2W^9 - 2856302087940C^8W^7 - 253526837760C^4W^8 \\ &- 1679616W^9 + 1766655075820C^6W^7 + 104327598240C^2W^8 - 113144393295675C^8W^6 \\ &- 18877708123380C^4W^7 - 2196179568W^8 + 374487924351240C^6W^6 \\ &+ 4752134834004C^2W^7 - 996136203133260C^8W^5 - 303917900090670C^4W^6 \\ &- 14930003592W^7 + 2887719025723560C^6W^5 + 50212270126800C^2W^6 \\ &- 2918957724481050C^8W^4 - 1868555165872320C^4W^5 + 3119480990289W^6 \\ &+ 8512802198873280C^6W^4 + 11075735177976C^2W^5 - 3058874545378680C^8W^3 \\ &- 3445715226319380C^4W^4 + 68169632407532W^5 + 9982864922507820C^6W^3 \\ &- 2232658069591920C^2W^4 - 1211007942303075C^8W^2 + 580356772850340C^4W^3 \\ &+ 541576438008926W^4 + 4897861470948600C^6W^2 - 8895831938168652C^2W^3 \\ &- 195602044050000C^8W + 2767750066737810C^4W^2 + 1762957347281076W^3 \\ &+ 954970056180000C^6W - 10235665592769120C^2W^2 - 10550594250000C^8 \\ &+ 1273724402832000C^4W + 2293200459759353W^2 + 58084992000000C^6 \\ &- 4436780299476000C^2W + 166222910700000C^4 + 1109491152635600W \\ &- 58575084120000C^2 + 154880130510000 \end{aligned} \right) \quad (132)$$

$$b''_{4,0} = 1105920(W+1)^2(4W+1)^2(9W+25)^2(9W+1)(W+36)(W+49)(9W+100)(4W+49)(9W+49) \quad (133)$$

$$a''_{4,2} = \left(\begin{aligned} &1025090640C^4W^8 + 8304651360C^2W^8 - 758412146520C^4W^7 - 244699056W^8 \\ &+ 1085472310320C^2W^7 - 22772743353675C^4W^6 - 25928063064W^7 \\ &+ 26225800770150C^2W^6 - 196390482033060C^4W^5 + 231491704437W^6 \\ &+ 221516549557560C^2W^5 - 527712096010470C^4W^4 + 17750828017548W^5 \\ &+ 698650404487020C^2W^4 - 581607755095920C^4W^3 + 190329377598874W^4 \\ &+ 904436648507520C^2W^3 - 279006467031495C^4W^2 + 711124850484016W^3 \\ &+ 500966001061470C^2W^2 - 54010997029500C^4W + 930175409071545W^2 \\ &+ 105485564154600C^2W - 3277365000000C^4 + 434986329435700W \\ &+ 6673723560000C^2 + 57260776720000 \end{aligned} \right) \quad (134)$$

$$b''_{4,2} = 552960(W+1)^2(4W+1)^2(9W+25)(9W+1)(W+49)(9W+100)(4W+49)(9W+49)(81W+100) \quad (135)$$

$$a''_{4,4} = \left(\begin{aligned} &1249949232C^4W^9 - 163683828C^2W^9 + 31214561112C^4W^8 + 171714492W^9 \\ &+ 26606108151C^2W^8 - 399231791469C^4W^7 + 24241092183W^8 \\ &+ 1585436613201C^2W^7 - 17400795877584C^4W^6 + 1241904561966W^7 \\ &+ 26686616715738C^2W^6 - 166022896925718C^4W^5 + 28735348547394W^6 \\ &+ 199835852035302C^2W^5 - 580807601687628C^4W^4 + 322346221935452W^5 \\ &+ 686490045351411C^2W^4 - 854177008140645C^4W^3 + 1757214691063303W^4 \\ &+ 1134404522614125C^2W^3 - 534565188454500C^4W^2 + 4354887366453090W^3 \\ &+ 905807861353500C^2W^2 - 119959739550000C^4W + 4612242329318120W^2 \\ &+ 307639666950000C^2W - 7815255000000C^4 + 1905043316764000W \\ &+ 24454185000000C^2 + 212837125200000 \end{aligned} \right) \quad (136)$$

$$b''_{4,4} = 6144(W+1)^2(4W+1)(9W+25)^2(9W+1)(W+36)(W+49)(9W+100)(4W+49)(9W+49)(81W+100) \quad (137)$$

$$a''_{4,6} = -7(271188W^5 - 201007251W^4 - 5751047538W^3 - 44049177319W^2 - 77408447900W - 31213000000) \quad (138)$$

$$b''_{4,6} = 36864(W+1)(4W+1)(9W+25)(W+49)(9W+100)(4W+49)(9W+49)(81W+100) \quad (139)$$

$$a''_{4,8} = -79(3W^2 - 237W - 30380) \quad (140)$$

$$b''_{4,8} = 8192(W+1)(9W+25)(W+36)(W+49)(9W+49) \quad (141)$$

$$a'_{6,0} = a_{6,0}^{(1)} + a_{6,0}^{(2)} \quad (142)$$

$$\begin{pmatrix} 38575348012520561806147584000C^{12}W^{29} - 114574020069766953708394905600C^{10}W^{29} \\ +37347656746566166261003911168000C^{12}W^{28} + 96592925312931166042309263360C^8W^{29} \\ -10920683769874807710071193600000C^{10}W^{28} - 10554189827267657782190407680C^6W^{29} \\ +13467080955320655845104170952704000C^{12}W^{27} + 90211495031374778476470785802240C^8W^{28} \\ -14180428198936893959889223680C^4W^{29} - 38556013732991009015682060118425600C^{10}W^{27} \\ -8845323694665821227437627801600C^6W^{28} + 4620899159982753374426628096C^2W^{29} \\ +2437669101384064414939633138626048000C^{12}W^{26} + 30136694451457562223819412864942080C^8W^{27} \\ -13394612839402067285776056975360C^4W^{28} - 480534389462975772408938496W^{29} \\ -6754414585021033597382449701209088000C^{10}W^{26} - 1185290975429586933927049546629120C^6W^{27} \\ +4413496027148828018192610754560C^2W^{28} + 244063938216529575593782453864808064000C^{12}W^{25} \\ +4727165519422982569787202356225003520C^8W^{26} - 5336600644112847605703014964019200C^4W^{27} \\ -49712688327950132388005675008W^{28} - 636570990406881616303386121593266841600C^{10}W^{25} \\ +437070869869653255493196147631144960C^6W^{26} + 1715562582857287489008526670659584C^2W^{27} \\ +13507039803420930064037619235820431968000C^{12}W^{24} + 343057552312077591767861213476233907200C^8W^{25} \\ -1144094130598492735227876964354928640C^4W^{26} - 214565535687271065870753243906048W^{27} \\ -30297097239595888187473531635706122816000C^{10}W^{24} + 156590742095664127717486353846453964800C^6W^{25} \\ +359013880054528504572359934825357312C^2W^{26} + 325543644961001923325060141694983303064000C^{12}W^{23} \\ +3297213076026378378287548724136479596800C^8W^{24} - 14198341038981996402352552261738977920C^4W^{25} \\ -52007094448103485888653638613159936W^{26} - 254386099965664151606621060450372633673600C^{10}W^{23} \\ +21197139148657149257135030046808381608960C^6W^{24} + 45094549246384190519038809847345846272C^2W^{25} \\ -6523285147402381315572822016046187932982000C^{12}W^{22} - 1374278773717366465483084147457930663705280C^8W^{23} \\ -10167599650146794036951358397650228299520C^4W^{24} - 8026505220206734023436261333454269440W^{25} \\ +54408415117138439404410158287748608689876000C^{10}W^{22} + 1561497031699981427694803906428471677530880C^6W^{23} \\ +3607272535620551692333739921721134175744C^2W^{24} - 809217799225484637294465683993430185052328500C^{12}W^{21} \\ -119571032233227331279486426314615783341026320C^8W^{22} - 360464200947912186792871693155729260130240C^4W^{23} \\ -848145487046270528997128547888293101824W^{24} + 3596736465656091660978378119460807054037931400C^{10}W^{21} \\ +68044325834478186919471176181020548128536000C^6W^{22} + 192550680820943259844215394431004753687680C^2W^{23} \\ -31738450625943866217429039016495582746379588875C^{12}W^{20} - 528440748378688161875786245260454385572830460C^8W^{21} \\ +1559768626725755107320317677638417522274800C^4W^{22} - 64379892843848605765553561816199022208832W^{23} \\ +122730465777030134615015795485687598335068239250C^{10}W^{20} + 1691266847932106249167337366867395669648467120C^6W^{21} \\ +7307734053716053607489963530103672191504416C^2W^{22} - 756042080825960846808167294083825523265189063375C^{12}W^{19} \\ -147413217731536450723858236242393783230794712365C^8W^{20} + 81013801614637901759193818311764305528335540C^4W^{21} \\ -3627241195720188046484132962266553100147952W^{22} + 2721260581153638926902237518819332766909285929850C^{10}W^{19} \\ +1494185526507250343213027069534746044908233180C^6W^{20} + 221219341130126584849004123102337549641896008C^2W^{21} \\ -12252396302150874415811981651359616830584508311750C^{12}W^{18} - 2753790820906999241841689471061253481226746888985C^8W^{19} \\ +42648308837598685961929834590727572391443177195C^4W^{20} - 154949065538166978265810127311560894048078564W^{21} \\ +4187470406627225237706695969371146593331963402500C^{10}W^{18} - 479800221094263909256362189086616353188886164020C^6W^{19} \\ +6249484845551488716172916638766522710018354098C^2W^{20} - 139977931469357839519273332449600212977730742397750C^{12}W^{17} \\ -34612106560836838699492216345973704388744504806890C^8W^{18} + 1282274120038633223291218555724513032367374986255C^4W^{19} \\ -5084023512024071257523046781661733041629332435W^{20} + 456819730146657471250848191627837014292916851138100C^{10}W^{17} \\ -21437301258684055407646290320397929325278344782600C^6W^{18} + 173761176248399409580062494433816502202842765914C^2W^{19} \\ -113347977444332874364120841349548331778904745638625C^{12}W^{16} - 276132013582742633007610925622214504447230167121930C^8W^{17} \\ +25838587197276949464429431384222269000223413510C^4W^{18} - 129085811903182183449156239284641026201413264839W^{19} \\ +3519509200164946793568033734938854997416352393821750C^{10}W^{16} - 43520784942786385242943725270971704433566862279080C^6W^{17} \\ +4268870939687185358803136736660747747764372864580C^2W^{18} - 6392344322368319890976855229976116235180566246667125C^{12}W^{15} \\ -1028201392444270785497068688963408421619396680836295C^8W^{16} + 367685551104105553265103917797825398980651355518550C^4W^{17} \\ -2546712448606569991922338070763820472016688753302W^{18} + 18533601776600110570918335727570867208088626725180350C^{10}W^{15} \\ -5671530045247619242039035248461208182808819687530860C^6W^{16} + 83019985959057630588636535077456304646063065783476C^2W^{17} \\ -23776473104039485560476248802888110322758738548768000C^{12}W^{14} + 4372586836867751256787147298813314871436042926927325C^8W^{15} \end{pmatrix} \quad (143)$$

(144)

$$\begin{aligned}
& \left(17550633713868415694274953827514919071190836250000000000C^6W^4 + 10284682403160033597665412837649397694934346720625000000C^2W^3 \right. \\
& - 70086235856634998153294122044251806919335306499696875000W^8 - 54163967945104148991667362165099598641984375000000000C^{10}W^3 \\
& + 864384351153188159871006783145765628906250000000000C^{12}W^2 - 35233864678318719329539797221568198214326562500000000C^8W^3 \\
& - 1649260296700402117131015517183700361882604968750000000C^4W^4 - 2732598545305390411372537057681854750917365835937500000W^5 \\
& - 487642289122848668818235796008487257343750000000000C^{10}W^2 + 36305191696552581062632777660309108735366875000000000C^6W^3 \\
& + 2208550760245535385657367605387713015590572187500000000C^2W^4 + 4064614969267344474932312853177187500000000000000C^{12}W \\
& - 5416115109563207846240474832112792310156250000000000C^8W^2 - 227199522532122195925308713125829262550814062500000000C^4W^3 \\
& - 6197812886496580380283681900484431673687548812500000000W^4 - 23735372908254728815058451051151125000000000000000C^{10}W \\
& + 43599457658093730315539266562945951716875000000000000C^6W^2 + 179946261653006012616677789018611437155765625000000000C^2W^3 \\
& + 7868595848171831260437605906250000000000000000000C^{12} - 38056489346766598897597761251840287500000000000000C^8W \\
& - 11911380524567439355226521908114894608906250000000000C^4W^2 - 58337602280452498799498466709656390003443437500000000W^3 \\
& - 4721157508903098756262563543750000000000000000000C^{10} + 27377624758205632316447928632493825000000000000000C^6W \\
& - 209052687356233379906467051743921387984375000000000000C^2W^2 - 980865971158094590127638541962500000000000000000C^8 \\
& + 520841591335345012665278604366772125000000000000000C^4W + 41008157013559337947583111912269919211562500000000000W^2 \\
& + 6800859151440241980300176478510000000000000000000C^6 - 507937573446941341498881323066739525000000000000000C^2W \\
& + 5673523818436776333610900999177500000000000000000C^4 + 12585839744372600898005616720514500750000000000000W \\
& \left. - 2593769820300066074510675184853500000000000000000C^2 + 668549528555095958188810797855450000000000000000 \right)
\end{aligned}$$

(145)

$$\begin{aligned}
b'_{6,0} = & 475634073600(16W+1)(9W+1)^2(36W+25)(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49)(81W+100) \\
& \times (4W+1)^3(36W+3025)(9W+25)^3(9W+3025)(W+49)^2(W+36)^2(W+169)(W+1)^3(9W+49)^2(W+9)(81W+3025)
\end{aligned}$$

$$a'_{6,2} = a^{(1)}_{6,2} + a^{(2)}_{6,2} \quad (146)$$

$$= \left(\begin{array}{l} 51369796018483263979860787200C^8W^{27} - 70092565709951747515473100800C^6W^{27} \\ +36220731024844775475362915942400C^8W^{26} + 7953725807402635293999759360C^4W^{27} \\ -47835147670403141270951401881600C^6W^{26} + 1356314404586666424155504640C^2W^{27} \\ +8738957588786184725507995433164800C^8W^{25} + 3482274603901826409795338895360C^4W^{26} \\ -2342212979906252288789839872W^{27} - 10738108850121994871177236618444800C^6W^{25} \\ +609515169713231211307125964800C^2W^{26} + 958321518845404095692362635345100800C^8W^{24} \\ -285593332933744884862527037685760C^4W^{25} - 2028454414396303843902416486400W^{26} \\ -987656185356147780528075356993126400C^6W^{24} - 47283181267172706247590413107200C^2W^{25} \\ +49262632221518123674627190399666342400C^8W^{23} - 283963425674080601052224499271680000C^4W^{24} \\ -699162376695810161473739310243840W^{25} - 23722643391535451689527670805471385600C^6W^{23} \\ -70030604201383807598839565541703680C^2W^{24} + 652163701848827919233698571856240787200C^8W^{22} \\ -47163912337392572847937476673751823360C^4W^{23} - 131263892742800273609878903052550144W^{24} \\ +2538665862212293474374584603155143705600C^6W^{22} - 16376707744652925017221189972441282560C^2W^{23} \\ -59000964300123345319686879200401335276000C^8W^{21} - 3709299950119329623206464379656869383680C^4W^{22} \\ -15317780928749266873823450826715989504W^{23} + 250068977875250192545830508802472809884800C^6W^{21} \\ -192922874501826797513965095764937559040C^2W^{22} - 386760937245350050127077630378527133154400C^8W^{20} \\ -162379563948141541872071533474834294827840C^4W^{21} - 1195670843194288369194720516123596835072W^{22} \\ +108064939501379866982706422896026451270400C^6W^{20} - 136805127397782789270702919952835493971840C^2W^{21} \\ -118598606353786334905630878686527977091906650C^8W^{19} - 4027906273800521628144245278303133215006080C^4W^{20} \\ -65417819583910956704003243334267761248224W^{21} + 284642121506988171241228686112226848816373400C^6W^{19} \\ -6323802704324534699316613730933631597617280C^2W^{20} - 2265774589650382028515929366473240398541770125C^8W^{18} \\ -4565473969072243724856635277483530944018700C^4W^{19} - 2589269895184605465175227688105332960656544W^{20} \\ +4986829604169403993655107462419218146788682500C^6W^{18} - 199781277279889167441786007483872032814029640C^2W^{19} \\ -28947191636192785036963491214018044699514174575C^8W^{17} + 330185982235389670164907446682416179167861650C^4W^{18} \\ -75783478994037721255074904672027061530508010W^{19} + 59920043176105475727588363445796713679056656300C^6W^{17} \\ -4447264334987991032328742561152722592842024700C^2W^{18} - 251541084696607794955050432973537875329868683775C^8W^{16} \\ +21196926307794989207845094266187396141629718790C^4W^{17} - 1665250274368199177504797944730426079968326285W^{18} \\ +494186326579295345817757198096276604023060865100C^6W^{16} - 71166132184909767257142154336039061633779462740C^2W^{17} \\ -1462811390423399732712492832782059454715703349675C^8W^{15} + 382161154510842453750125222786292210878745336390C^4W^{16} \\ -2774957822964267275533359151533609257474305903W^{17} + 2709675951598582214707470828459277173226668504300C^6W^{15} \\ -828882389184649258370917224920051597202750698100C^2W^{16} - 5314141512893639237613033410071577495756339021550C^8W^{14} \\ +407776027219957663518912231737069753496029971310C^4W^{15} - 352778624463596079230341756479794180488444352415W^{16} \\ +8832086072931104740279813743256559094511607993400C^6W^{14} - 7070842752770874595910527225904845845152172525300C^2W^{15} \\ -8990905268558602246818659324264502132243426294150C^8W^{13} + 28883953495301317822543170645811608555053437633100C^4W^{14} \\ -3430674396570543258262434504016518709785293821755W^{15} + 8415811949157185162721431704074321006259956036600C^6W^{13} \\ -44205046192940369322964056857089215863442539235720C^2W^{14} + 13910978208463280088725396044752354216190174342050C^8W^{12} \\ +140510023590373106933545471530208737385277321564060C^4W^{13} - 25514987722033438229115931212440497210846311110926W^{14} \\ -6470266099185222776317480761955773645343808842600C^6W^{12} - 201515015622038085984812865281827962584965781592840C^2W^{13} \\ +124698575475403000214357146500610086324559110678000C^8W^{11} + 472587788252934357360277923678100815799378303546380C^4W^{12} \\ -144676852973982486257747658186209808987030367956486W^{13} - 356371220279975342915232569845022324411908456534800C^6W^{11} \\ -662379071468623661948915490471793361654153038137960C^2W^{12} + 356556076680837126031957857784438268867866887656775C^8W^{10} \\ +1089026229073344875359585096523877713167755833527440C^4W^{11} - 621495026202895441353301438365135438404750436653278W^{12} \\ -946894684146587532280444078315875107742954163533900C^6W^{10} - 1539811963400044424731088099486627079387961721526960C^2W^{11} \\ +60475908488085984663994455213253953609141065575525C^8W^9 + 1672883875946647764512014281054730704416581220333130C^4W^{10} \\ -2002591670059740768252009436178057216735953679929536W^{11} - 1588887528885201126620800652093591468133532661676900C^6W^9 \\ -2447650019498707088236914542154082462037196438287820C^2W^{10} + 674976509631157893884508223546950610432425642238125C^8W^8 \\ +1596688803978198400539857854270640866525540718845950C^4W^9 - 4774728405938795565951805684078876430157411651693561W^{10} \\ -1798622467108627133717108561948684362728909066382500C^6W^8 - 2480175230953390054857482400897896905073200399593060C^2W^9 \\ +51121342292792750261379476372138369923775371953125C^4W^7 + 725847262627716453302999136585197644758145020588750C^6W^8 \\ -8282406402785978142583131069035704783508564624659995W^9 - 1401587940018677007307110108994629569392227376312500C^4W^7 \\ -1277850957555359483123426546376358171121420186950500C^2W^8 + 264103880168776326116614464991799233912330926562500C^6W^6 \\ -206621465723483219248333555606688362283862880031250C^4W^7 - 10246809278490396148834128110838305634375177435397875W^8 \end{array} \right) \quad (147)$$

$$\begin{aligned}
& \left(\begin{aligned}
& -751982397685394402411051922912882971548422618750000C^6W^6 + 193846664582547267262506100736795611465202111587500C^2W^7 \\
& + 92244939239452170439140780695633779437374062500000C^8W^5 - 523530250439468283030243087863453059992478378125000C^4W^6 \\
& - 8830812345776147031867045748862256027752996533996875W^7 - 274094669459985313727816896560762009013211250000000C^6W^5 \\
& + 800697675144522202405096383752058831612711746250000C^2W^6 + 21347519575142548480424308957254577257843750000000C^8W^4 \\
& - 3497429336675584363793043281181854693728956250000000C^4W^5 - 5137195207359244113033495071976537276537005955937500W^6 \\
& - 66174860050415728316537092302368107459125000000000C^6W^4 + 581030641196172567487522585379220600949992750000000C^2W^5 \\
& + 3163637929332311184087541970044973676562500000000C^8W^3 - 127126506205111541718276177115792722760187500000000C^4W^4 \\
& - 1918522730721903865438594335075474300354307937500000W^5 - 1018541103168254111513558002679533340625000000000C^2W^3 \\
& + 221582637601581788724974176928418264927975000000000C^2W^4 + 283333240227902874214802470215565781250000000000C^8W^2 \\
& - 27153920172993352244066775189440934065625000000000C^4W^3 - 415564868822289382468870719629559381877706250000000W^4 \\
& - 94109132368651159010943818547281437500000000000C^6W^2 + 4887286330402460820033198108821790228375000000000C^2W^3 \\
& + 1373991573914833273229300824237500000000000000C^8W - 332998257476218159412567949521516906250000000000C^4W^2 \\
& - 37558942124032721225434660341543269568437500000000W^3 - 467590565023888856511783046995000000000000000C^6W \\
& + 613201165495217598654171502246750612500000000000C^2W^2 + 2725950649463002242829721250000000000000000C^8 \\
& - 212390093725764571365869420511750000000000000000C^4W + 2366589137304242755521932087049635531250000000000W^2 \\
& - 944627663207807252650399500000000000000000000C^6 + 3983686249853750274813965840469000000000000000C^2W \\
& - 5354291618162613879887569050000000000000000000C^4 + 71441062107894445430228029682917500000000000000W \\
& + 1022522302812986387357943690000000000000000000C^2 + 367440607135650440315516843250000000000000000
\end{aligned} \right)
\end{aligned} \tag{148}$$

$$\begin{aligned}
b'_{6,2} &= 59454259200(16W+1)(9W+1)^2(36W+25)(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49) \\
&\times (81W+100)^2(4W+1)^3(36W+3025)(9W+25)^2(9W+3025)(W+49)^2(W+36)(W+1)^3(9W+49)^2(W+9)(81W+3025)
\end{aligned} \tag{149}$$

$$a'_{6,4} = a^{(1)}_{6,4} + a^{(2)}_{6,4} \quad (150)$$

$$= - \left(\begin{aligned} &156230159450708275314897715200C^8W^{29} - 204159592983549999597024706560C^6W^{29} \\ &+ 151411829023792899097267853721600C^8W^{28} - 117770934184693524068724375552C^4W^{29} \\ &- 19166827826607627384710025052160C^8W^{28} + 42237197420234813145876529152C^2W^{29} \\ &+ 54690563195525538779202594950860800C^8W^{27} - 112479692789635496026171866611712C^4W^{28} \\ &+ 3378912139874165460827111424W^{29} - 66267874555264751970240339450347520C^6W^{27} \\ &+ 35827597014308540020853118074880C^2W^{28} + 9926222624583182775036234417461299200C^8W^{26} \\ &- 44786709498425245323965177973645312C^4W^{27} + 1286113897835252193225717448704W^{28} \\ &- 11217491911296334811901465685084446720C^6W^{26} + 11170603100691346516613891522740224C^2W^{27} \\ &+ 998165739627719307535758045241237574400C^8W^{25} - 9685954854072276002800285735791120384C^4W^{26} \\ &- 711343967281983720891053765701632W^{27} - 984674419153000857782669420414614932480C^6W^{25} \\ &+ 1462430407701010398496568257739710464C^2W^{26} + 55674289460030484810437330663086480396800C^8W^{24} \\ &- 1250846957867435195113108934726302305792C^4W^{25} - 552068959244705969066012663294588928W^{26} \\ &- 37087477999648757149823513507990521313280C^6W^{24} + 12467627833016894123003518070519657472C^2W^{25} \\ &+ 1372068388744154818584072337378012917150000C^8W^{23} - 100087694563127844234470035041052145058816C^4W^{24} \\ &- 153474703782841809115894847472072977664W^{25} + 806810758926422771274868813544230541808960C^6W^{23} \\ &- 21211604879051300396079015060108117792768C^2W^{24} - 25134603719360673416090646540856647842544600C^8W^{22} \\ &- 4928169951111282954076648835417404902435552C^4W^{23} - 24072913901079211529871234379348004519424W^{24} \\ &+ 161482538013664311429546698866268862458542560C^6W^{22} - 3100340081305854925400085921105792985831104C^2W^{23} \\ &- 3303664861670384519266427468618409027166204275C^8W^{21} - 134335488218446997928509280979639879490218128C^4W^{22} \\ &- 2427548424747012603694436450252190764770512W^{23} + 8736431136689940715523775085171945520930023500C^6W^{21} \\ &- 2353595470441510067443311310334078574667497C^2W^{22} - 131760897815782485237243329569619658791307426375C^8W^{20} \\ &- 653336790098131022887524195487475387002455282C^4W^{21} - 167936083138119327549416192981850048475590744W^{22} \\ &+ 278848645707785187216777685259086939802049769500C^6W^{20} - 11436585087132630896598197426276630239834802484C^2W^{21} \\ &- 3187722456020915139350911903729566368258086794450C^8W^{19} + 99262861619269782816704572987791106514716158870C^4W^{20} \\ &- 8290191251959706321858546976066971726650247091W^{21} + 5978077760213140090785856089319698285693113100360C^6W^{19} \\ &- 383275385148833899630312800970147033868070049380C^2W^{20} - 52605484813835616833241634182493184188128682280850C^8W^{18} \\ &+ 4399794437574818159014539985767624838751293916052C^4W^{19} - 299751172975841561666646379053553203828131380743W^{20} \\ &+ 90145686099866313499173992006480247755022122133960C^6W^{18} - 9203769914830786117732475515807320348306858994872C^2W^{19} \\ &- 615021232758194717923806496132055650665699877699425C^8W^{17} + 103899213056732042932170364960609895515399192749972C^4W^{18} \\ &- 8080976665065307395390802159913020835268554673234W^{19} + 97056093090422772140857832326337467646911128770820C^6W^{17} \\ &- 16195492950919898856694980906445779434978527184440C^2W^{18} - 5136727435652738659290104445680657895926792962400325C^8W^{16} \\ &+ 1626986139982285823177675158484001050640762903283962C^4W^{17} - 164379450432436255118489744332024998427697971787154W^{18} \\ &+ 7403925807840957169246249623691756831409147375968020C^6W^{16} - 2116298213633882274480564894169212804451593710124924C^2W^{17} \\ &- 30272211518895175265454341776730847937394118786594900C^8W^{15} + 17974319026019208990285165750001743383745558757026594C^4W^{16} \\ &- 2541559085062310447402033174591345322058151647549473W^{17} + 38437056885233220160705438962192410625727246191827280C^6W^{15} \\ &- 20666701366333068202170527324105993010493260817230444C^2W^{16} - 120688384803477722158449498357394715998727192659197300C^8W^{14} \\ &+ 143293883443931533300946393518819431445873787722800392C^4W^{15} - 29975790796427048046677004174754140519338210646022277W^{16} \\ &+ 118558136730303159986428459744352098596784995792736080C^6W^{14} - 150765978100128464493863927217412588130018399937688752C^2W^{15} \\ &- 285697135898476089209000122570936733084357586906541125C^8W^{13} + 828268622757201588676208123410293592408569097185362376C^4W^{14} \\ &- 269750132816726257276714417670489738170188273267639636W^{15} + 68622549830202445774167474889102976327170834587464340C^6W^{13} \\ &- 815067354197853000234643116971682134761826258615411632C^2W^{14} - 149814475487552890477355885653849710198976319464469025C^8W^{12} \\ &+ 344234891835012736048505045770285457005921793486653602C^4W^{13} - 1846703684285560548280985974175194431520536043747765236W^{14} \\ &- 1246908231839111536641135820533182070186738539952091260C^6W^{12} - 3206474443083438160856816639669262405537285513409714156C^2W^{13} \\ &+ 1535472409684840356678562873037806160262536826712443150C^8W^{11} + 10037818716285148743075974717966399447814463383917458938C^4W^{12} \\ &- 9558436272274304397309174350345834481330241912034070533W^{13} - 6724230268332666086696323180813576043529544259196646200C^6W^{11} \\ &- 8848545076422753201764467064704064730051151683223771644C^2W^{12} + 6208040304141780011083608493515732114297506068846572750C^8W^{10} \\ &+ 19369213947013461425539479950870930066635265341091115732C^4W^{11} - 37042059562351859752250553946168838140312829575868378721W^{12} \\ &- 19196389934144260023971669854984580956720072938390934200C^6W^{10} - 15692843775459363714318818585280033611886385074714112056C^2W^{11} \end{aligned} \right) \quad (151)$$

$$\begin{aligned}
& \left(\begin{aligned}
& 12863138969765713044915705920480818807643006433654965625C^8W^9 + 20525662044631097491627851957783087499673774732270883540C^4W^{10} \\
& -105985703148081149386080620643970486940834482048373353074W^{11} - 35771554727872025760631549963889486332659409831593862500C^6W^9 \\
& -12479693177604444383962236344923178964243077609373157560C^2W^{10} + 17018846945854288806514855347484970322474082564307578125C^8W^8 \\
& -1457998296168672721024474005894932300367526032351302250C^4W^9 - 219487536141294773148713067576207616664292522017149783602W^{10} \\
& -45888876679175510044574440842636639497282366573950312500C^6W^8 + 15103552919343877579301675992890034523376966349344841500C^2W^9 \\
& +15116904965974837996620395794747840019139924991453125000C^8W^7 - 41650637214813483406003364119043031419971742670159131250C^4W^8 \\
& -319267344415562542062068509733786751856491182514717253575W^9 - 41086975064316761558746137572850317917363781854612500000C^6W^7 \\
& +60846770609288998571532852506353946129711987002308487500C^2W^8 + 9079235668785927525457105966472650676574674880468750000C^8W^6 \\
& -68544973357695267412351157761917106961552610850841250000C^4W^7 - 309635848334302643463841817923347707945064243221596381875W^8 \\
& -2554798742939806305457754047141279735518032001875000000C^6W^6 + 910233742282257714631110512518346343228310488169875000000C^2W^7 \\
& +3638903733716404331997279981318863335825929375000000000C^8W^5 - 60634062969399166743739239745428122666163059863187500000C^4W^6 \\
& -17739668362119184096951734707832735913186764911178875000W^7 - 10823724832647077147726909660989382693156653500000000000C^6W^5 \\
& +81091406556766464611692546521550310270256268642125000000C^2W^4 + 947805634998253203333827996830514418834117187500000000C^8W^4 \\
& -32972572045884299525321269020040249298658889350000000000C^4W^5 - 32784453823147937291010928881405715739637973371131250000W^6 \\
& -3022625743226119189446454266555400252858068750000000000C^6W^4 + 46195265847430150211999061078950281272928972100000000000C^2W^5 \\
& +15455199439392792275539599225575736286406250000000000C^8W^3 - 11208514922081626123549271074014846046757556875000000000C^4W^4 \\
& +28888929592185388011429681435086255222756786655000000000W^5 - 52985680082452571530653791936179442609625000000000000C^6W^3 \\
& +16807379961120290119376740970171546637760531250000000000C^2W^4 + 1489526923871654255617968776531576695312500000000000C^8W^2 \\
& -232634296096448887256215628194266797485162500000000000C^4W^3 + 24018729756197273602267115275796478076527841187500000000W^4 \\
& -5444197817537819888708314844395945181250000000000000C^6W^2 + 376782211472862878721265057712300987149575000000000000C^2W^3 \\
& +76272135234213261154487064495356250000000000000000C^8W - 27883133605808304297329621644933294288125000000000000C^4W^2 \\
& +8053844921192183815912327270343879307624862500000000000W^3 - 29349179591730925733731846121854500000000000000000C^6W \\
& +48813662884622296953544757563623547578750000000000000C^2W^2 + 157371916963436625208752118125000000000000000000C^8 \\
& -172331018132805733072502973218438500000000000000000C^4W + 138665938849551710016133209862837274539312500000000000W^2 \\
& -629487667853746500835008472500000000000000000000C^6 + 326041793693265989291227156365630300000000000000000C^2W \\
& -41288713092710138274194572096500000000000000000000C^4 + 1185701009686040842501906028605634505000000000000000W \\
& +85047104127266012357552198071800000000000000000000C^2 + 392826051314268042527190442518210000000000000000000
\end{aligned} \right) \quad (152)
\end{aligned}$$

$$\begin{aligned}
b'_{6,4} &= 63417876480(16W+1)(9W+1)^2(36W+25)(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49)(81W+100)^2 \\
&\quad \times (4W+1)^2(36W+3025)(9W+25)^3(9W+3025)(W+49)^2(W+36)^2(W+169)(W+1)^3(9W+49)^2(W+9)(81W+3025)
\end{aligned} \quad (153)$$

$$\begin{aligned}
& \left(613616005020964904861302283304960C^4W^{24} - 324963168853921069966668799672320C^2W^{24} \right. \\
& + 431824105964290173015338662150471680C^4W^{23} - 223823890785778037735724393431040W^{24} \\
& - 277972675875394573464618040053596160C^2W^{23} + 103799494199229182257030135925239572480C^4W^{22} \\
& - 157936117507472554055035923141033984W^{23} - 82407302930396608294687565493216890880C^2W^{22} \\
& + 11305770682339377238197290346279187453440C^4W^{21} - 39192595909348521952540758407627151360W^{22} \\
& - 10368263998541037465008508934671493155840C^2W^{21} + 573016989376973065208129239388603205864960C^4W^{20} \\
& - 4459653611380888174789734753477987273216W^{21} - 583620751864126685054451426127544129817600C^2W^{20} \\
& + 7005804191462122405005617869466680288102080C^4W^{19} - 207423817148002809706070120529371355271680W^{20} \\
& - 8191567289110718061461325552441900941808000C^2W^{19} - 71452970531481943133243332174479290634288600C^4W^{18} \\
& + 4494756767697245583836546587548028075903680W^{19} + 769299153747241405024811280096654049745348400C^2W^{18} \\
& - 45229065495784764341510223001871356203972922050C^4W^{17} + 1102497867220676436702534267387867842725890600W^{18} \\
& + 51249523052636440502981058768816227389274489700C^2W^{17} - 1354829644953646398175052492011662724214773559775C^4W^{16} \\
& + 65646109527822079930627669382918831917233504030W^{17} + 1578578301739166902968722288249726653716873337950C^2W^{16} \\
& - 25203162096634696949234681292993917963766847950825C^4W^{15} + 2236320035136609668177336221069776619566574862625W^{16} \\
& + 30036405657460571805193335215109274314983230904850C^2W^{15} - 310943902149699024773840990335816618648367079176160C^4W^{14} \\
& + 4992510617863610375793488205940318613927872740215W^{15} + 378772329009848413817335367928499983640784701321920C^2W^{14} \\
& - 2571608758819407553522785676308238182754061478827280C^4W^{13} + 766768275835649708921468497559743610625596429857440W^{14} \\
& + 3203046675178374994055159140809059022521435627652960C^2W^{13} - 13851522056510121462291329687466632978515299857699730C^4W^{12} \\
& + 8255831300997064723065427151776755278460273376745904W^{13} + 17637648361416087084583024621592655722095243687594980C^2W^{12} \\
& - 4361564462563006116721190576994731922802472081603190C^4W^{11} + 62477472101387547220280039953764781957764278488051310W^{12} \\
& + 56711000341757944982687904795279191118635268773500140C^2W^{11} - 42573151930903759138991792969801186584904211160339260C^4W^{10} \\
& + 328731579283943090197717232254144224212205371558064586W^{11} + 56432616594439690576219865236995419069798386545895800C^2W^{10} \\
& + 24146085566904516577914344902844607428640898104041770C^4W^9 + 1170169959735758705612573311400131539222504461460246980W^{10} \\
& - 326103990519080038090649400813939236981980846827318900C^2W^9 + 1178646405881575106401835840292361649434009524461633625C^4W^8 \\
& + 2656582653710117947825608858602486529913437036508214410W^9 - 1623149691220136621922387461687327624787609787407743250C^2W^8 \\
& + 256210107752583072690037555466006318348984246463734375C^4W^7 + 3252599439929800352020826738155270305718899926660257625W^8 \\
& - 3611363930638676120366985970711816246089644212615668750C^2W^7 + 3271517923996130732625725497716982941866357146564687500C^4W^6 \\
& + 280022520358562521670630772912006618404804416476634375W^7 - 4753043993796203025316784712250430422919791258899375000C^2W^6 \\
& + 2580632011399268865378093090658474539539748975187500000C^4W^5 - 5537238163841208930233386787679753629271774920345312500W^6 \\
& - 3904134389676177710449603872743610110722628013375000000C^2W^5 + 1250528720063889039835712281311539708598901556250000000C^4W^4 \\
& - 8717669187173905633830335755903913176409227645812500000W^5 - 1993667667009143019500577393815804644828636912500000000C^2W^4 \\
& + 358177192922207559274798098496253523481077187500000000C^4W^3 - 6548862874423166803742891503945914156892707043750000000W^4 \\
& - 606977923362411881298457113544784558969994375000000000C^2W^3 + 56352703729131783108107915215558856549343750000000000C^4W^2 \\
& - 26844463567834287567974191911204985697619328125000000000W^3 - 100953776989989422017284871114431183487687500000000000C^2W^2 \\
& + 426736631693620174763869331740709152500000000000000C^4W - 5855149447982728354575705965681024903016562500000000000C^2W \\
& - 795230103938193657071973484264685655000000000000000C^4W + 1172220541522377369010453393755750000000000000000C^2W \\
& - 6029315209490789999575662981069785557500000000000000W - 2238881882235164947474072338613500000000000000000C^2 \\
& - 213296272797560860143770047348385000000000000000000
\end{aligned}$$

(154)

$$\begin{aligned}
b'_{6,6} &= 17044452346429440(16W+1)(9W+1)(36W+25)(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49) \\
&\times (81W+100)^2(4W+1)^2(36W+3025)(9W+25)^2(9W+3025)(W+49)^2(W+36)(W+1)^2(9W+49)^2(W+9)(81W+3025)
\end{aligned}$$

(155)

$$\left(\begin{aligned} &295286994265938554880C^4W^{21} - 93529817188306329600C^2W^{21} + 277676580258915850172160C^4W^{20} \\ &-528844309187594932224W^{21} - 80633975068669449208320C^2W^{20} + 95292561771457328689931520C^4W^{19} \\ &-476542799686564511378688W^{20} - 24249895663560111099244800C^2W^{19} + 15941162572962734578349118240C^4W^{18} \\ &-166878100543439211935920128W^{19} - 3150132186715975701846650880C^2W^{18} + 1401293125997612952485182401600C^4W^{17} \\ &-29753327421235780313644150560W^{18} - 122482587040589872951726410000C^2W^{17} + 60435159110133454940472147743775C^4W^{16} \\ &-2904342275091099773615375401200W^{17} + 1321288377539192479877036708450C^2W^{16} + 455178631517496333509164595643150C^4W^{15} \\ &-146906146280286603918382856150025W^{16} + 1905747540325982495129513699860500C^2W^{15} - 78480485279366104635252107206976325C^4W^{14} \\ &-1961164472891544918659757445222770W^{15} + 108646147737174492987701050477312650C^2W^{14} \\ &-4169086846064360627570054407065349800C^4W^{13} + 196863274948742117443682006160934035W^{14} \\ &+3537167927315547773100742564879710000C^2W^{13} - 108110181192316330753948468703159552850C^4W^{12} \\ &+13316314639184210835491871996035972280W^{13} + 71524116758932432577235940782026873700C^2W^{12} \\ &-1699301447864817261523325286542643900780C^4W^{11} + 414053899952885321974983907309094314590W^{12} \\ &+916693642111540056115990352456811900600C^2W^{11} - 16845489516331908939066326970839536110210C^4W^{10} \\ &+7742044978132042945641978646936423400404W^{11} + 7273378911950403906180257648170565556420C^2W^{10} \\ &-102689836391569375866312189438415304452920C^4W^9 + 92183615412807834843800472086841730963118W^{10} \\ &+31071399820296423269736706711731021952800C^2W^9 - 34394535223693402323342746242599593349165C^4W^8 \\ &+700465466151116431322053531519781458575768W^9 + 14436216233807763740047150415548302836730C^2W^8 \\ &-339270444524213034402652848931714476577650C^4W^7 + 3264711188347642258061305698875944929198155W^8 \\ &-575548383076606424190471711486862177939500C^2W^7 + 1657013482664297526974945401704299336454375C^4W^6 \\ &+816614089298680022712762608771376590177870W^7 - 3151519840476345459135617742961565488428750C^2W^6 \\ &+6728115017333171212334114002242204183750000C^4W^5 + 4233993301309666406777057619535166082969375W^6 \\ &-8237082250227824992388413029067156903500000C^2W^5 + 10739204210731542316341661908027622912500000C^4W^4 \\ &-32614173799445614419111288556455410707850000W^5 - 12160135680491292888600379475976175425000000C^2W^4 \\ &+8684205813317856661609712585350207500000000C^4W^3 - 92009072564625519100185981498568840187500000W^4 \\ &-10351827423533686317964304415887655000000000C^2W^3 + 3525186532086742013691493869012937500000000C^4W^2 \\ &-107276461968673216176041581919154316500000000W^3 - 4857239600980503817928579234377875000000000C^2W^2 \\ &+6285529357539921073785810109500000000000C^4W - 608409852017536924182071401666595625000000000C^2W^2 \\ &-108805711029014475002494924110000000000000C^2W + 3670482028301728867842615000000000000000C^4 \\ &-15537429964845671691013082740170000000000000W - 7340964056603457735685230000000000000000C^2 \\ &-1241174098982176806412495530000000000000000 \end{aligned} \right) \quad (156)$$

$$b'_{6,8} = 4529848320(9W+1)(36W+25)(W+16)(W+144)(4W+49)(9W+100)(16W+49)(4W+1)(36W+3025) \quad (157)$$

$$\times (9W+25)^2(9W+3025)(W+49)^2(W+36)^2(W+169)(W+1)^2(9W+49)^2(W+9)(81W+3025)(81W+100) \quad (158)$$

$$a'_{6,10} = -77 \left(\begin{aligned} &2402351405568W^{13} + 1488979200417696W^{12} + 276202265233512672W^{11} + 17795790141673964562W^{10} \\ &+114162650968064172651W^9 - 27794524915553191333812W^8 - 1196183065857057047524390W^7 \\ &-22015137814823473469109546W^6 - 196784839249029283528827501W^5 \\ &-700811872822907502696877900W^4 + 520983838211944435940730000W^3 + 7340697633939861849399000000W^2 \\ &+10449672862315788208800000000W+305688040729449840000000000 \end{aligned} \right)$$

$$b'_{6,10} = 75497472(W+16)(W+144)(4W+49)(9W+100)(16W+49)(4W+1)(36W+3025) \quad (159)$$

$$\times (9W+25)(9W+3025)(W+49)(W+36)(W+1)(9W+49)(W+9)(81W+3025)(81W+100) \quad (160)$$

$$a'_{6,12} = -1001 \left(\begin{aligned} &37179W^6 + 21318363W^5 + 2267946186W^4 - 52556050058W^3 \\ &-2650566192705W^2 - 26755660094625W + 90136734187500 \end{aligned} \right) \quad (161)$$

$$a_{6,0}'' = a_{6,0}^{(1)} + a_{6,0}^{(2)} \quad (162)$$

$$a_{6,0}^{(1)} = - \left(\begin{aligned} &43870003622082207544246272000C^{12}W^{28} - 138670521396140091976895692800C^{10}W^{28} \\ &+ 48492123578521910287433072640000C^{12}W^{27} + 128710750547296547829118402560C^8W^{28} \\ &- 149745962985071011005028751769600C^{10}W^{27} - 22294290828728906692039802880C^6W^{28} \\ &+ 19450337002943089561605963067392000C^{12}W^{26} + 138521895992885712407919480668160C^8W^{27} \\ &- 16696670395733307766194831360C^4W^{28} - 59032941233188080282870826186444800C^{10}W^{26} \\ &- 26708552074066046807901776117760C^6W^{27} + 4785789863251958913127415808C^2W^{28} \\ &+ 3985587100362506568316550488765440000C^{12}W^{25} + 53318692469252657152574170722201600C^8W^{26} \\ &- 15417437512312881461885876305920C^4W^{27} - 140429651265105410852388864W^{28} \\ &- 11893875364557480219214290641453721600C^{10}W^{25} - 9200680388004189404529165192929280C^6W^{26} \\ &+ 4574857698958045735382250553344C^2W^{27} + 478270743553211693195519372863971072000C^{12}W^{24} \\ &+ 10315402560078154063181207130063697920C^8W^{25} - 6463682172545900138586104538931200C^4W^{26} \\ &- 134173391606591544006805684224W^{27} - 139954111904185290047812424679995806800C^{10}W^{24} \\ &- 1283544391339461001934517765816913920C^6W^{25} + 1815984531526006565152146871824384C^2W^{26} \\ &+ 36248480210200399821936478012069247040000C^{12}W^{23} + 1140597126267967470335645584353688899840C^8W^{24} \\ &- 1541479288930056192091586358778490880C^4W^{25} - 52056741602668021541446214504448W^{26} \\ &- 103333100008113022436875180842264568281600C^{10}W^{23} - 4818949329552053039214838299457359360C^6W^{24} \\ &+ 393132287023051605501200710404814848C^2W^{25} + 1821911868719715274595186284077594179472000C^{12}W^{22} \\ &+ 76402903356746650539057543955493247406080C^8W^{23} - 226086591705235010334719772630279114240C^4W^{24} \\ &- 10862819746812674488957138200668160W^{25} - 4998611820290436764075326053962082943234800C^{10}W^{22} \\ &+ 7372151498463844488210086946571278259200C^6W^{23} + 51500972014134834172536964344469904640C^2W^{24} \\ &+ 62647294971912985365013298691067010458140000C^{12}W^{21} + 3134033785320969272857614126367480608875280C^8W^{22} \\ &- 21317665783551715632456255874501178519040C^4W^{23} - 1357519406588207187407405695794592512W^{24} \\ &- 161856964407139742828432589942319950322902600C^{10}W^{21} + 1120053212717363090512636696596302444750880C^6W^{22} \\ &+ 4301418171412308440828243848387333842432C^2W^{23} + 1494562963276466785904833306300567536350757000C^{12}W^{20} \\ &+ 72626395911833486323581470390704557415881240C^8W^{21} - 1334807377696037785316273908517855479776480C^4W^{22} \\ &- 107069794827212539457400820764371475456W^{23} - 3483764236212113879993944863192651025241293425C^{10}W^{20} \\ &+ 73890625424587569565801211184481911867015600C^6W^{21} + 235400969199300981539105893278210019878096C^2W^{22} \\ &+ 24497563893736356633073080716123888288284008750C^{12}W^{19} + 451655100250611927493968127397924576972618895C^8W^{20} \\ &- 56911425729665302020618879882249726252802800C^4W^{21} - 5466902591987066390951259277512360747696W^{22} \\ &- 46293895712136823400968645676022979360719619725C^{10}W^{19} + 2974435302665156247440346446363793215211484590C^6W^{20} \\ &+ 8501572874640897642796578973932727129802136C^2W^{21} + 259484648894439602415467404744614792889013328000C^{12}W^{18} \\ &- 28146019520912346008229616021959462016635956175C^8W^{19} - 1675288793817999576701730800249575198179262770C^4W^{20} \\ &- 181728855379447859540577685931324707256712W^{21} - 241715070718720226702229877826144602684667148450C^{10}W^{18} \\ &+ 80113724321632823361573993510525951831413236230C^6W^{19} + 196686108971162096576804825913919677447632547C^2W^{20} \\ &+ 1330298926705149849561646023842246290565217547500C^{12}W^{17} - 1091170403799649284633225566053917024455386247010C^8W^{18} \\ &- 33867039434060321617515271929392495443698773160C^4W^{19} - 3868048674982460152421239497930145614504645W^{20} \\ &+ 3823666647883238324982671085564671668485822365850C^{10}W^{17} + 1496612748775217985680361622792271857975651281180C^6W^{18} \\ &+ 2494725668788017482828005700419331434620095655C^2W^{19} - 6562420528033027766582366901058132420204558692000C^{12}W^{16} \\ &- 21271305333534652884274831424541055381053287721210C^8W^{17} - 449026287486538019762788956723113989775822885940C^4W^{18} \\ &- 51541612022194594730568575164431904033232343W^{19} + 104726655609755938233872688078825977948464359212925C^{10}W^{16} \\ &+ 19545413713215196740192447095229252448864595446260C^6W^{17} - 553708286285517318673135897277338016699694138C^2W^{18} \\ &- 192956243835061277926829428084359204629576831658750C^{12}W^{15} - 270646624515917226980886548183183235198349793223675C^8W^{16} \\ &- 3188561657265681466645349272274139222166442168280C^4W^{17} - 55709861899342656687653370946601039620218666W^{18} \\ &+ 1276984566275786370464482142240613018778340976807225C^{10}W^{15} + 174499565141670836579737794406164722921194629578730C^6W^{16} \\ &- 682542826794322161977494385732808745435055128014C^2W^{17} - 1882190781783910825951001830328399986183290366897000C^{12}W^{14} \\ &- 2399545054755352351548562987512972411535437986521845C^8W^{15} + 6529848703850311416122422362074836101995402311650C^4W^{16} \\ &- 13433727154862624304095405711557945173161456346W^{17} + 9992972550293759891591861401973572035206170355560800C^{10}W^{14} \\ &+ 970291390120591886086854924935653060679038339258770C^6W^{15} - 14273474368163783026636852620426194770976596855239C^2W^{16} \\ &- 11243666452591556584436376577618493228540845536915000C^{12}W^{13} - 15113336921603789344726485714196901805748531884218040C^8W^{14} \\ &+ 453112450111501419104435338440477500316163839613680C^4W^{15} - 420610565252758166515059400193067702292428882407W^{16} \\ &+ 5428341647769903526717519448127901167525271219934100C^{10}W^{13} + 1984699522316419553819608589489793484706848887504960C^6W^{14} \end{aligned} \right)$$

$$\begin{aligned}
& \left(\begin{aligned}
& -164969970573415761076428384207611680428613288212843C^2W^{15} - 45509324198132017441089165075549212551796573830947000C^{12}W^{12} \\
& -67539014982252604169138744428684471316448193342465380C^8W^{13} + 5776380229206210169109722209451851844288308614926440C^4W^{14} \\
& -8876156405394755436857139943489368682501025141085W^{15} + 210204266091445430107101727527932848530758734684810425C^{10}W^{12} \\
& -16409610008723883044908584112236825767991350057583000C^6W^{13} - 1193974628334263476168801230981542404981330911839520C^2W^{14} \\
& -128844342594641290353665235152662335198416450572608750C^{12}W^{11} - 210660688090500241702052091224483302030486710786828855C^8W^{12} \\
& +43451990810301973434943733093578418812230772313799840C^4W^{13} - 125221492030872931989266472961906385231657801440968W^{14} \\
& +585855254518561128547184803237369174416299087780740725C^{10}W^{11} - 176996277918379607312603238795167628897880210339065630C^6W^{12} \\
& -5020028362245568992174877303971391274404305322181052C^2W^{13} - 257875084381108237350685658940022417559008768346082000C^{12}W^{10} \\
& -441856018061359248457165559291317152868394719153486665C^8W^{11} + 218366535651100519728314514310284213324157829001090930C^4W^{12} \\
& -1238448999367934173634954615440054525732593289682564W^{13} + 1177765356608682748471200942924909640632095576098346550C^{10}W^{10} \\
& -86621983025408348846007563752594005351147433975194550C^6W^{11} - 5305101833262983723255100537552043963482351182243451C^2W^{12} \\
& +75445518868010924224647004342473807623303064191028600C^4W^{11} - 8821775661106360188867691949487643741740195974237859W^{12} \\
& +1704329864902475684301514591748300170268958025734437850C^{10}W^9 - 2659409457877144820879516571448929561288180981231125940C^6W^{10} \\
& +72947721786468856880937092171420696278386810026497569C^2W^{11} - 364893083575595969577018149822645557605042455147550000C^{12}W^8 \\
& -308037836201907487023970817863729373158064166723080250C^8W^9 + 1790792937381745195205302300997976576207869210441342220C^4W^{10} \\
& -4571460716003728522638006628092563679833449086009633W^{11} + 1765783485880692430620300587644779565648307022358006875C^{10}W^8 \\
& -549362499305436898707597583977857768828336301057325580C^6W^9 + 510021069422432382718488208193490600260529572458327998C^2W^{10} \\
& -25476380159554891934286493125979551470440758345781250C^{12}W^7 + 276088153234567867755690532820268709999595860573831875C^8W^8 \\
& +2871867807120687374084003263870652427050068441988817960C^4W^9 - 172243716824059248243195009110286282494760147061585410W^{10} \\
& +1297877422362040159612876630934837808390083213615109375C^{10}W^7 - 780501640428270444873575840317359770232305412621192250C^6W^8 \\
& +1759665604383647389327023419317116358075683392770744050C^2W^9 - 122790385419690802811649799572746388138154036078125000C^{12}W^6 \\
& +688311659213889075918892967625717376596853156609078125C^8W^7 + 3007691910167285081816677012332073658042357386917630750C^4W^8 \\
& -46873453040633843652338492884394150912125136913736602W^9 + 667357920452716762393263704227270197893399521785937500C^{10}W^6 \\
& -7667879155650386117008595934790739406988116681185081250C^6W^7 + 3787059572861910397356718671171506166829197524428461375C^2W^8 \\
& -40190951949741797086112201158690278736626290625000000C^{12}W^5 + 633769072410457679714529910417447799254019633039062500C^8W^6 \\
& +1906642419962095974184924148936105140933120885393800000C^4W^7 - 912022763696091917808990184512222573505838475512459025W^8 \\
& +235271689823953889211081771689785704757024692187500000C^{10}W^5 - 5189393635407983922647249787735952793782684028443125000C^6W^6 \\
& +5377629252110850041606839584454307107591755983591671875C^2W^7 - 87470957116617895346391831360236592541456250000000C^{12}W^4 \\
& +337936512540521363132715181523646955005556487812500000C^8W^5 + 543519865528387326789119586689118002421679681117500000C^4W^6 \\
& -125330974396319094600380181991299127269112773618156875W^7 + 55303159023137217113807509100231564670998156250000000C^{10}W^4 \\
& -2389875874194538887910951935643595172220784798625000000C^6W^5 + 5131048600237621082627191688537814627359746822917187500C^2W^6 \\
& -1225879069720694706463059180044070558890625000000000C^{12}W^3 + 112192401943812299416068131970003456477195093750000000C^8W^4 \\
& -137654159476239041731804794176770361208675901500000000C^4W^5 - 1199353832022460487017086906221048417057899314914437500W^6 \\
& +8334587867953801329201665479847307581629687500000000C^10W^3 - 732037473542992037494417830274144314144902887500000000C^6W^4 \\
& +32947627267414066969042879398565422607912729704375000000C^2W^5 - 104651839882182843648948114889438176562500000000000C^{12}W^2 \\
& +23068488045425581493687705568645973030551562500000000C^8W^3 - 187159000517305189212505935702854056434651825000000000C^4W^4 \\
& -785159296324730193256617135795795017945868986887500000W^5 + 75781813942792242112720658425611758296875000000000C^{10}W^2 \\
& -143532503634571990553367161685884935014620625000000000C^6W^3 + 14045993791442241004833988442196291957703201312500000000C^2W^4 \\
& -488071317103570125940269214834875000000000000000C^{12}W + 278293521425876008213912143933590484328125000000000C^8W^2 \\
& -73722435708563434793111927438828026172925000000000000C^4W^3 - 343011739843014497053771271264415475551773266250000000W^4 \\
& +3720771851333298937352385723631462500000000000000C^{10}W - 16891446504067803639724149167931059346562500000000000C^6W^2 \\
& +384674618933118220950656502315110340785220937500000000C^2W^3 - 93944518751177952089318336250000000000000000C^{12} \\
& +174487097767969256841125195584395375000000000000000C^8W - 14649447555480158809738295003915650274250000000000000C^4W^2 \\
& -96202299214205682960886616249717074866405937500000000W^3 + 7452500936187253570075470768750000000000000000C^{10} \\
& -1058478160332678859950301415624345550000000000000000C^6W + 63634512689674530520148010773856646243593750000000000C^2W^2 \\
& +4241639505177018209106239723325000000000000000000C^8 - 14523393154118864073666408152303490000000000000000C^4W \\
& -16238608372907019238402570556817808526868750000000000W^2 - 265311080394404839547109888831300000000000000000C^6 \\
& +565270796935965097037926411876639732500000000000000C^2W - 5554729709984871177390510581700000000000000000000C^4 \\
& -146495149664006731556463482074177282500000000000000W + 2011292108584968772622304636340950000000000000000C^2 \\
& -5267526936026863976783765671567500000000000000000
\end{aligned} \right)
\end{aligned}$$

(164)

$$\begin{aligned}
b''_{6,0} &= 59454259200(16W+1)(36W+25)(9W+1)^2(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49)(81W+100) \\
&\quad \times (4W+1)^3(36W+3025)(W+36)^2(W+9)(W+49)^2(W+1)^3(9W+49)^2(9W+3025)(9W+25)^3(81W+3025)(W+169)
\end{aligned}$$

(165)

$$a_{6,2}'' = a_{6,2}^{(1)} + a_{6,2}^{(2)}$$

(166)

$$\begin{aligned} & \left(\begin{aligned} & 534879491774513047831078502400C^8W^{26} - 78382377848460229331479756800C^8W^{26} \\ & +402575324769416773636129249689600C^8W^{25} + 196726023452515288620592005120C^4W^{26} \\ & -581213586551682183225313001472000C^6W^{25} + 20849865657511877539560161280C^2W^{26} \\ & +107445830493956056496303796949708800C^8W^{24} + 136587263999846185050666420142080C^4W^{25} \\ & -3093268227764998689241694208W^{26} - 151135848287208737677489969767628800C^6W^{24} \\ & +17784400823464521423727133982720C^2W^{25} + 14084162654784078027887660163169843200C^8W^{23} \\ & +29926735417298341057072176218972160C^4W^{24} - 2349814192632995911151360016384W^{25} \\ & -18970357663910929859029203102865612800C^6W^{23} + 6205140797463135211686357707735040C^2W^{24} \\ & +1043526358856134398482877720165584083200C^8W^{22} + 2509428610653326965288154717674352640C^4W^{23} \\ & -665796421179639354558224911306752W^{24} - 1302896470683504877650954233369778816000C^6W^{22} \\ & +1109963433680452147054118870679183360C^2W^{23} + 47225751096061533667522039062641100748800C^8W^{21} \\ & +21809046878365201615322104763734341120C^4W^{22} - 94374568087250245637397880040957952W^{23} \\ & -51098951035295416964409878042895254630400C^6W^{21} + 110626734595659191692140982589758356480C^2W^{22} \\ & +1352666714014478127345650038568188087059600C^8W^{20} - 10543405041492684380093843345956295992320C^4W^{21} \\ & -7366824508774302249572434890868492032W^{22} - 1059312112942015937486973550030551936014400C^6W^{20} \\ & +6334975276014623668417774077653015377920C^2W^{21} + 2404289913105754881787893855079481605557400C^8W^{19} \\ & -837029009408300215407422532374963405536800C^4W^{20} - 31233160537702692026627774073159179264W^{21} \\ & -3742708572739767695489068154364933202216800C^6W^{19} + 200136470832487080464397994497927458462400C^2W^{20} \\ & +221944193830020286697584668868247875898934375C^8W^{18} - 32756838980348620039907986832598594604506000C^4W^{19} \\ & -5211396108794225778096403714970511280368W^{20} + 420209090811056181897366889161379139979754500C^6W^{18} \\ & +2308992727882602050384132370083770714917600C^2W^{19} - 420656344217078712051508071084875363980962375C^8W^{17} \\ & -785577901517635922787990798499657571167364550C^4W^{18} + 133969977744770471493197560662002846866680W^{19} \\ & +13198922560200125746569361691100406998159715500C^6W^{17} - 70757632481440658523689333420081035687849500C^2W^{18} \\ & -43160700903703714432536399093376381406874646275C^8W^{16} - 12273958490791095680134100854022370921554289450C^4W^{17} \\ & +9833914877798369803098915161114047787913975W^{18} + 217741600592458030884970760655964393416326226300C^6W^{16} \\ & -3797111711087233811352620797499570175026577300C^2W^{17} - 646954041601802420020346822269861631454346608225C^8W^{15} \\ & -125514194532622378674215122621006728109666150770C^4W^{16} + 263294407650417999141001240221785093149895385W^{17} \\ & +2346185447563519726878675040808659315874658256500C^6W^{15} - 86677591597274940509778739358627727921389749380C^2W^{16} \\ & -5574631734681160546378685389940778065602476194550C^8W^{14} - 787885062398836223117241057558363165355503203430C^4W^{15} \\ & +4020041134326996387602688673110144457848815613W^{16} + 17560662574780424981006645011245742032300739681800C^6W^{14} \\ & -1240631786461233815721483495619243707524658052620C^2W^{15} - 31799128260238295707753907072148370089532385194950C^8W^{13} \\ & -2114064428141689519976383210615876770486702000660C^4W^{14} + 35198547835838506427444280140309297071328604639W^{15} \\ & +93526823639749116649742692366313018775269275183800C^6W^{13} - 1218885170007404878176725614014377653999145142040C^2W^{14} \\ & -125554826854162475200894176026130091657843432768950C^8W^{12} + 9664557061803064075477010887687324299521322176860C^4W^{13} \\ & +109516670376169007523188101204683405570815150362W^{14} + 358230157662913456279080289185059592904297228755000C^6W^{12} \\ & -85113892355167900990898174520742352231479993212360C^2W^{13} - 349726469859270985482564187196204341791782736163050C^8W^{11} \\ & +127143584694452587834214274575002806449663901910780C^4W^{12} - 1312240265292599059612820689561261992511077880198W^{13} \\ & +991704754783059485871707157116227422277658688395400C^6W^{11} - 428628277762647033345674951889217657947939148391880C^2W^{12} \\ & -693529717473392053691625436755867503099772336784725C^8W^{10} + 642849383104169343041229569292162985956377547024420C^4W^{11} \\ & -20939921667180584012550837435693638448826133828278W^{12} + 1988221414886458435342426707521060799747281763906900C^6W^{10} \\ & -1563815282081654133667409650074058311523963343486520C^2W^{11} - 982680835322488018226767554374839622298792037882275C^8W^9 \\ & +1974795299793963209745916635081421459833011889740850C^4W^{10} - 151185324590254964448389009078060898343164082864266W^{11} \\ & +2885080919441443186466185973745934821181746549918300C^6W^9 - 4128629119951538572250042833776949096872332496146700C^2W^{10} \\ & -993772099745545346937076834707107459941719508206375C^8W^8 + 4031027660816617408477209190492213054083233411563950C^4W^9 \\ & -685566124272953206336434959489903089050844666736037W^{10} + 3018257992756987512698990687769088373061940806391500C^6W^8 \\ & -7850052216761521489452574749880422519090207994755300C^2W^9 - 712798562592435292994764693328317487781374776528125C^8W^7 \\ & +5635120911392123658730892940731199863072408190957750C^4W^8 - 2089088864308331618589564240374644631794511688160515W^9 \\ & +2256885838549294510487796146558008735910097742462500C^6W^7 - 10671417197868339116859280234416661568866771313435700C^2W^8 \\ & -358230473313052868881297030696800890347256719687500C^8W^6 + 5447687234788502956452488343461982268081106457731250C^4W^7 \\ & -4354416969421997089844655798876680459198298949604775W^8 + 1188333856660765016466710091142569920649496117500000C^6W^6 \\ & -10270644236514277016278509780740836327292742977377500C^2W^7 - 6216876565127661625146332440821627298834182483808125W^7 \\ & +430691271968078075167159434408380446468949250000000C^8W^5 - 6903612828544522808783015676998761887349787304000000C^2W^6 \end{aligned} \right) \end{aligned}$$

(167)

$$a_{6,2}^{(2)} = \left(\begin{aligned} &-28609211761902896465004625995965952680756250000000C^8W^4 + 1654928751422945423639721044839163920988272125000000C^4W^5 \\ &-6044645687700811414316436495888400050798315438937500W^6 + 10408704399735275155650291834878227895595000000000C^6W^4 \\ &-3178093076721792998345631948625707785612009550000000C^2W^5 - 4254574210516899467526307937126843736562500000000C^8W^3 \\ &+502158409343481410091249806697795004722737500000000C^4W^4 - 3954452572627895545307768061685476285986777412500000W^5 \\ &+1607914855432186654065855802559058182625000000000C^6W^3 - 97432940407876208766770590650795202906446000000000C^2W^4 \\ &-38295410940875639050656276025189659375000000000C^8W^2 + 97672953408423606756214535721883424625625000000000C^4W^3 \\ &-1704393793833586362003781458740869087569951250000000W^4 + 149172726173582189486649073772934225000000000000C^6W^2 \\ &-19103515359221672357370898804698023149575000000000C^2W^3 - 186629391505561186335642273996750000000000000C^8W \\ &+1142009672947033189887459721529432156250000000000C^4W^2 - 466907834873594614958729462377759887355562500000000W^3 \\ &+743860942508483536682121279988500000000000000C^6W - 2248480273589084987906286718250795430000000000000C^2W^2 \\ &-3718150103733531747975366750000000000000000C^8 + 7132855617261858346017868130418000000000000000C^4W \\ &-76429206899094568774222260792937083318750000000000W^2 + 1507056432786918283664324700000000000000000C^6 \\ &-141486384701113027436916077297121000000000000000C^2W + 179141160115976348607889502700000000000000000C^4 \\ &-666860774577658959094401647228224500000000000000W - 359141957943742209174216201000000000000000000C^2 \\ &-232536149068346886461967812350000000000000000 \end{aligned} \right) \quad (168)$$

$$b_{6,2}'' = 59454259200(16W+1)(36W+25)(9W+1)^2(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49) \quad (169)$$

$$\times (81W+100)^2(4W+1)^3(36W+3025)(W+36)(W+9)(W+49)^2(W+1)^3(9W+49)^2(9W+3025)(9W+25)^2(81W+3025)$$

$$a_{6,4}^{(i)} = a_{6,4}^{(1)} + a_{6,4}^{(2)} \quad (170)$$

$$\begin{aligned} & 88836757334716470277098700800C^6W^{28} - 131157928733430205188947312640C^6W^{28} + 9828401606622839473343313100800C^8W^{27} \\ & - 52927313381223265305866797056C^4W^{28} - 138743133597742008438708611973120C^2W^{27} + 31854986047002407461117231104C^2W^{28} \\ & + 39483591735889504039287322064793600C^8W^{26} - 48895861016501983751570951897088C^4W^{27} + 1797456616764403475911016448W^{28} \\ & - 5387982692309785195866363351336960C^6W^{26} + 30140638665279311972985641238528C^2W^{27} + 8109568822807461148735207816902297600C^8W^{25} \\ & - 20102285992530966537473981975089152C^4W^{26} + 1805868370875820156171064967168W^{27} - 10700770385668974299231286211524080640C^6W^{25} \\ & + 11561349595422436733861273496932352C^2W^{26} + 976434831240458079854534304317229446400C^8W^{24} \\ & - 4683010500305690352102921478779104256C^4W^{25} + 736405590770489342313549135175680W^{26} \\ & - 1238313357894165784250667882417222842880C^6W^{24} + 2371022142559585355978309805215708160C^2W^{25} \\ & + 74354594776728724352446532830520845574400C^8W^{23} - 674301524709893969544380805937479533568C^4W^{24} \\ & + 161356353001719497653799881936806912W^{25} - 8933515507139334244828268329560508972160C^6W^{23} \\ & + 28720549310916669900567754244344244736C^2W^{24} + 3761403905988742249521981787079671119637200C^8W^{22} \\ & - 62941814944018948165089403530261132418944C^4W^{23} + 20875349924078720195690678551394946816W^{24} \\ & - 4164806525973511982103977893760211992053200C^6W^{22} + 21262980909292257338507322224348735408256C^2W^{23} \\ & + 130475201013425897988259400112834763846504200C^8W^{21} - 3938435093764249232483067824665503524042064C^4W^{22} \\ & + 1610237659112213973171146666486444552832W^{23} - 126188624582106129392744642352067577510737380C^6W^{21} \\ & + 936113968352140445906127139206668020273296C^2W^{22} + 3150489437811271230372755841958108096294006875C^8W^{20} \\ & - 169532442092662679814708077288916152393105532C^4W^{21} + 64900978533176588124942029136154058995776W^{22} \\ & - 2354766294743752051433226092604376902100027920C^6W^{20} + 19524692746791146380252293963066884060174436C^2W^{21} \\ & + 52556351933554471339141882755412765600578508500C^8W^{19} - 5104234652058321465375795467059169942150855370C^4W^{20} \\ & - 40887778197693786504844018400129597853132W^{21} - 19356719953510878313225037546248658513659391400C^6W^{19} \\ & + 274404032974602285436763207658897454573575048C^2W^{20} + 573561235762192468639218725709301484920817383950C^8W^{18} \\ & - 107783440197905416242384217259295080904352096560C^4W^{19} - 166396866176206907112242717755251863332437945W^{20} \\ & + 249143827217374263699701084105879271866808618240C^6W^{18} - 31473028889084716183783952593313023065940791480C^2W^{19} \\ & + 3199176639873479334241197028714760349386894507700C^8W^{17} - 156354722010877935993343754645020343568764911364C^4W^{18} \\ & - 10665386915078359463660385261575832723281412420W^{19} + 10739925339540365961573596345787954362386871801220C^6W^{17} \\ & - 111972325587464450828851315682991922337092870704C^2W^{18} - 10762948412472376436985477172215792511847911109475C^8W^{16} \\ & - 14368519034523800177052295026290722281756709040172C^4W^{17} - 390295339023582218278491563840980171541810951898W^{18} \\ & + 181035830087966340359305360203311794413939372802560C^6W^{16} - 24364365226225510825493994927901995934015093641828C^2W^{17} \\ & - 404451707982963563109039139830418772297442503061600C^8W^{15} - 54830406609001286940630555007580433942084604745718C^4W^{16} \\ & - 9716671684097404788246566210652778424599937379048W^{17} + 194318259681328201530466810659480697849132627740240C^6W^{15} \\ & - 367186444009662911861672792803518193387338868337832C^2W^{16} - 4192714015704331726590592491668303290520744446347900C^8W^{14} \\ & + 522980268184400768340847592580148738602529391943696C^4W^{15} - 174269491256810535858128463923407222608657902733375W^{16} \\ & + 14614195838454530865691113323698822219178330002050480C^6W^{14} - 4025512848044078821348620702133649092989291064708880C^2W^{15} \\ & - 26425723017031596216713519022581372930623252017908400C^8W^{13} + 10485032138454527609098549046344787385425787106191448C^4W^{14} \\ & - 2309539131426038532829143863868068052842199701980272W^{15} + 79631337672953858325770707545765299423447204964652260C^6W^{13} \\ & - 3277555723372631227357883018741937754651036807041296C^2W^{14} - 113659116878438327725118120966388045256084362845478075C^8W^{12} \\ & + 90560544943777519346844237763929922026419480560720684C^4W^{13} - 22892529036905760275281115664851830864166949611663356W^{14} \\ & + 318735595430988418943778863881746643792856380501186800C^6W^{12} - 200073092023829329786891559841446886319159136023112996C^2W^{13} \\ & - 346268325029869074637615483631417248842263656275530700C^8W^{11} + 500112649100186840616183791996388726414002852440244394C^4W^{12} \\ & - 170549391765063730601235749618199805192287985662459772W^{13} + 942398678257347895906242861421548502861097343974859480C^6W^{11} \\ & - 918113926841319287275334893035002043307739422777672536C^2W^{12} - 757740821100880137859897676327346921342467968263455250C^8W^{10} \\ & + 191627570363282862413762188467184326582900244856094672C^4W^{11} - 955013854051472321789025551492507442555775396257658311W^{12} \\ & + 2060534668116964605795165148345557859165783830719615520C^6W^{10} - 3162660925411952158962154605931227075945824366032839096C^2W^{11} \\ & + 5232947207990111930209402663253767767613416136940444700C^4W^{10} - 4004798640752749143835647315374655172865628968840497188W^{11} \\ & + 3323906486874979610024750614905063928405256953526101500C^6W^9 - 8145270768894762125678737090844693892442504752506949072C^2W^{10} \\ & - 1195072257929427815688915424826294522350108452768367500C^8W^9 - 1353498148587702979257869559601660424535774628813828125C^6W^8 \\ & + 10266263397580712888473478226167066372059733145077653500C^4W^9 - 12493381836414269969081943224953016463889968530425413210W^{10} \\ & + 3931811735223034193484699053508469985387809469709300000C^6W^8 - 1558839578047883262013153487174396820653809704379435100C^2W^9 \\ & - 1089754870023050740797379470457284705228055016315625000C^8W^7 + 14451310419463395800994255692941828800995669759116593750C^4W^8 \\ & - 28728837945267731024632387043073229880729669237572720080W^9 + 3373017185509706048383541997705945400602103679850000000C^6W^7 \\ & - 21982018526564600691201435258680803212590460323702235000C^2W^8 - 613121316957983488851408304669979046523850064843750000C^8W^6 \end{aligned} \quad (171)$$

$$\begin{aligned}
& \left(+14483775761593401064981457865907365198977698226231250000C^4W^7 - 48131478065040108195520758990457889015272361213170526625W^8 \right. \\
& +2061415297512558005061821896103172616483293046500000000C^6W^6 - 22572345954681744085823256996845548255523483330418000000C^2W^7 \\
& -235082312007958273915649630110719023156090812500000000C^5W^5 + 10191506490546994707923689116203297493928015440562500000C^4W^6 \\
& -57913142444028197641205532545694813135894212673152025000W^7 + 873335963314409220905964338261025396711082350000000000C^6W^5 \\
& -16593654610900741406797694455843882368081626055050000000C^2W^6 - 59419670688480048044012441219493851042179687500000000C^8W^4 \\
& +49268408644833236925781249741609396240206877500000000C^4W^5 - 49165817017473326856650228414754996458772922259688750000W^6 \\
& +24645067454564939512797768168511944647070000000000000C^6W^4 - 8517896575157164004774115475665354510296555350000000000C^2W^5 \\
& -9495226723218868606182542374774928256562500000000000C^6W^3 + 1585343138124934841539490451678722130020070625000000000C^4W^4 \\
& -28782627701380189129048114510572516286080559922500000000W^5 + 438186392724242463738932829812064699420000000000000C^6W^3 \\
& -294364736917111825688165885758944850512373250000000000C^2W^4 - 90281096925843987300113228906755664062500000000000C^8W^2 \\
& +32425702896448643823623990988650750481112500000000000C^4W^3 - 11259649808109842023142272007221038909297335237500000000W^4 \\
& +456189567267732568560810610678280385000000000000000C^6W^2 - 64964510544626771447355445719117649854720000000000000C^2W^3 \\
& -45810301562194435922377666557112500000000000000000C^8W + 39245217347504483809180540537854117543750000000000000C^4W^2 \\
& -281285566942489717492191667680417345366122500000000000W^3 + 2485228931587563564116722851334800000000000000000C^6W \\
& -8485103398938442713370571820331170829500000000000000C^2W^2 - 9394451875117795208931833625000000000000000000C^8 \\
& +24722824518634156997282837400906450000000000000000C^4W - 41888517517956003432686009860299781582125000000000000W^2 \\
& +53718516932618527010983962000000000000000000000C^6 - 578811634868779363574811767664662400000000000000000C^2W \\
& +6033179692591103900572573949100000000000000000000C^4 - 331046778105424817598709618394683437000000000000000W \\
& \left. -1553417161292345439615500567592000000000000000000C^2 - 1048815006580011039963392378279700000000000000000 \right)
\end{aligned} \tag{172}$$

$$\begin{aligned}
b''_{6,4} = & 3963617280(16W+1)(36W+25)(9W+1)^2(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49)(81W+100)^2 \\
& \times (4W+1)^2(36W+3025)(W+36)^2(W+9)(W+49)^2(W+1)^3(9W+49)^2(9W+3025)(9W+25)^3(81W+3025)(W+169)
\end{aligned} \tag{173}$$

$$\begin{aligned}
& \left(6389175008447141713257667862200320C^4W^{23} - 844478287563531484232580299489280C^2W^{23} \right. \\
& +4800095591168871224354763299952230400C^4W^{22} - 1570401463582363017298197999255552W^{23} \\
& -1378847220453012708272782584601231360C^2W^{22} + 12769124276531838128356874101216768258560C^4W^{21} \\
& -1158918742061214885542687871465455616W^{22} - 591779011855342851123001180190739671040C^2W^{21} \\
& +166496475882221912549224050099466547232000C^4W^{20} - 318480432474309444973446460599284149248W^{21} \\
& -97658427172910210576732349001006378007040C^2W^{20} + 12237882465044832762495718274196611481793920C^4W^{19} \\
& -45296241320663998345507386956675920798464W^{20} - 8026999112505814349913934204962403699649280C^2W^{19} \\
& +547393180653711201354081196028475696508749600C^4W^{18} - 3723354439739455515793502506934762884567680W^{19} \\
& -367467461951325883008401032900203623506228800C^2W^{18} + 15407876996838465667908100638846721938835143300C^4W^{17} \\
& -182340251614945952575249657112797905126793440W^{18} - 9493693375348304292919627608461733305661640200C^2W^{17} \\
& +266009157069398789418268272584061177640855163625C^4W^{16} - 5067221704669513162608123117687876986578406460W^{17} \\
& -111692255266416021242913865922808842925523276050C^2W^{16} + 228306460001882529288847620216652573643711560825C^4W^{15} \\
& -52479426579315235882037441063624633806599502455W^{16} + 883524978267403765673617712595663731951571787950C^2W^{15} \\
& -8236151676697761023895758112291947071577749392000C^4W^{14} + 1459818371074944928619935359761770134616376335545W^{15} \\
& +58535915212544431001571916484307987859651047765600C^2W^{14} - 505242688004386930516152664508461508189258798747220C^4W^{13} \\
& +71716534267871356697019490266733496054213513758560W^{14} + 1115399082481720467594139541445803541674931948851880C^2W^{13} \\
& -7037082812439691921248915935516786187215940748807650C^4W^{12} + 1497542865719445464877399618919357258724538982173612W^{13} \\
& +12605473297554216119089340870851936933444801294686660C^2W^{12} - 56814423876311240254588770975848070044514054821253810C^4W^{11} \\
& +19452548292975558688423675789933471278506789660490846W^{12} + 94385761986247682520134645569413383979361220896149540C^2W^{11} \\
& -299761512702577253769518074530873588884189966128678500C^4W^{10} + 170741095990257954857158713385501792098918485832465358W^{11} \\
& +483960268406678511858925303455893106729397932147864840C^2W^{10} - 1069462975595311678936171389379753772887681028821886520C^4W^9 \\
& +1040187838343802602514971733319857842967698340816337244W^{10} + 1716344428382740978561528711055089989152278148096701680C^2W^9 \\
& -2603698456560899568470293884454572597223957954930434975C^4W^8 + 442717312598546863118049153066460784924282756701633800W^9 \\
& +4213676721987662810971532368267300591585317205704801150C^2W^8 - 4316111788136723112278039013415001107949629795579569375C^4W^7 \\
& +13121765879430746351836455782412703482242153597486525825W^8 + 712840819501596572265517726595435347841793275661958750C^2W^7 \\
& -482123069284976077177877667459856881634355380056062500C^4W^6 + 26846494812840465256640965600254875397401359211541270625W^7 \\
& +8224357686878521240685051472305141829363089150513625000C^2W^6 - 3557637541302849329642719099052447543262902997637500000C^4W^5 \\
& +3739768465438647162718357557707504068663713111373437500W^6 + 6347115374742878833427817877041830653481712235425000000C^2W^5 \\
& -1677276413381533550253935152456520834440510473750000000C^4W^4 + 34759499014541508585256222712699803033286869850912500000W^5 \\
& +3165682948182357004785380189516182740244973557500000000C^2W^4 - 478681096578821500290360993589375815076140187500000000C^4W^3 \\
& +20906873042983091095268572192908164687649031336250000000W^4 + 961300640241711919217003516904739380559221375000000000C^2W^3 \\
& -7582579529385666512104737969942289208818125000000000C^4W^2 + 775584903137279506676126285121009186442530681250000000W^3 \\
& +1606946659859896691478595761563551604391250000000000C^2W^2 - 57870423814487227868694416892220240650000000000000C^4W \\
& +1638647057324446233001392098815968834548568750000000000W^2 + 1272509664011388203136011574936917548500000000000000C^2W \\
& -15988887872634832427915832586665000000000000000000C^4 + 1705284745822900186567228800477023825700000000000000W \\
& \left. +3595815216331320616575178134466800000000000000000C^2 + 6216067372698291143735917972796349000000000000000 \right)
\end{aligned} \tag{174}$$

$$b_{6,6}'' = 17044452346429440(16W+1)(36W+25)(9W+1)(W+16)(W+144)(4W+49)^2(9W+100)^2(16W+49) \quad (175)$$

$$\times (81W+100)^2(4W+1)^2(36W+3025)(W+36)(W+9)(W+49)^2(W+1)^2(9W+49)^2(9W+25)^2(81W+3025) \quad (176)$$

$$\left(\begin{aligned} &335816581714204631040C^4W^{20} - 141964901089393536000C^2W^{20} + 361858137238379605824000C^4W^{19} \\ &- 566968169375570156544W^{20} - 137941458311484237896640C^2W^{19} + 138742002575179605178536960C^4W^{18} \\ &- 531934715427386823550656W^{19} - 48658216151506248003934800C^2W^{18} + 26561041474307839278438859200C^4W^{17} \\ &- 198042888192578873702372880W^{18} - 8514918476740590897126250260C^2W^{17} + 2888004247217148677077093233600C^4W^{16} \\ &- 38899173157198938571545775620W^{17} - 821488564856394495601285696875C^2W^{16} + 190526796791192177716557556649250C^4W^{15} \\ &- 4485253599503022662012073020175W^{16} - 44500896599800362488390079784350C^2W^{15} \\ &+ 791846038245497627336484179562450C^4W^{14} - 318581744441079840444766160794800W^{15} \\ &- 1216541472540178916916024041599875C^2W^{14} + 210383597112368418071385227916741000C^4W^{13} \\ &- 14146713165321732410975080692440145W^{14} - 3545111402330549487096317649577200C^2W^{13} \\ &+ 3502067547186841191809814934108338600C^4W^{12} - 383415791040076626345959978863627080W^{13} \\ &+ 869607509053409198661509201928314250C^2W^{12} + 32472156419184338099647283657707429500C^4W^{11} \\ &- 5523529287544155110320099717756439430W^{12} + 30776019214584697455308671819887039900C^2W^{11} \\ &+ 61789546554828583086455000766432067260C^4W^{10} - 5516122447763100786765086402209273920W^{11} \\ &+ 554561340206495579778931453891277903250C^2W^{10} - 2260500429652481610483743323917966519000C^4W^9 \\ &- 1372352183391679919386118753706089341534W^{10} + 6219637296697590239068064520522149270340C^2W^9 \\ &- 29338681704822802984967136063669494207160C^4W^8 + 27733761778811420790263486618774908490316W^9 \\ &+ 45857169912029739701978251332645671995425C^2W^8 - 17877173813353665276561011404282379383950C^4W^7 \\ &+ 287931933155620899432964347790147536729765W^8 + 224990236176693003731501872553603196348210C^2W^7 \\ &- 627467982040695794125124059218050307162750C^4W^6 + 1822271301315849756704676702461321742031760W^7 \\ &+ 730807215645230185175642223041084339054625C^2W^6 - 1316714516145775690602174739299756279600000C^4W^5 \\ &+ 7302406098879182503062815939353833096457875W^6 + 1550464509556139641019668790543314760850000C^2W^5 \\ &- 1639534318104725156627079786760335795000000C^4W^4 + 18523448553832557260409081178430455158900000W^5 \\ &+ 210198206684583558866160616389510407500000C^2W^4 - 117024461769239459602664591079285300000000C^4W^3 \\ &+ 291165861741862266505048541941478171357500000W^4 + 1755215473130179639215109750786162500000000C^2W^3 \\ &- 443646204603820424932422526604775000000000C^4W^2 + 27230520111857385468355966478449419000000000W^3 \\ &+ 839779244125108593198427237010962500000000C^2W^2 - 7634544885137736217338092406000000000000C^4W \\ &+ 14173800889944520936548671606609737500000000W^2 + 19553074706649337383013532433000000000000C^2W \\ &- 438225160355349047646966000000000000000C^4 + 359070303784301908781358975291000000000000W \\ &+ 1347024939723740257837857000000000000000C^2 + 30678216275760178544872853400000000000000 \end{aligned} \right)$$

$$b_{6,8}'' = 566231040(36W+25)(9W+1)(W+16)(W+144)(4W+49)(9W+100)(16W+49)(4W+1)(36W+3025) \quad (177)$$

$$\times (W+36)^2(W+9)(W+49)^2(W+1)^2(9W+49)^2(9W+3025)(9W+25)^2(81W+3025)(W+169)(81W+100) \quad (178)$$

$$a_{6,10}'' = 77 \left(\begin{aligned} &25014086067456W^{12} + 16693143023165904W^{11} + 3543891309498347352W^{10} + 317618446022856449613W^9 \\ &+ 13159235407216581730008W^8 + 259983824772329481211262W^7 + 1564523216843999598518404W^6 \\ &- 28383597064685668199755779W^5 - 583103090454876341677842220W^4 - 4051780237173890973624722000W^3 \\ &- 11813991470845199218921800000W^2 - 13377685812639586210080000000W - 4169532638355749280000000000 \end{aligned} \right)$$

$$b_{6,10}'' = 75497472(W+16)(W+144)(4W+49)(9W+100)(16W+49)(4W+1)(36W+3025) \quad (179)$$

$$\times (W+36)(W+9)(W+49)(W+1)(9W+49)(9W+3025)(9W+25)(81W+3025)(81W+100) \quad (180)$$

$$a_{6,12}'' = 1001(21141W^5 + 15022440W^4 + 2137253534W^3 + 41116948380W^2 - 775063275W - 5380789837500) \quad (181)$$

$$b_{6,12}'' = 37748736(W+36)(W+9)(W+49)(W+1)(9W+49)(9W+3025)(9W+25)(81W+3025)(W+169) \quad (182)$$

6.4. Third Harmonic

$$a_{3,2}' = \left(\begin{aligned} &1871100C^4W^3 - 1574640C^2W^3 - 779625C^4W^2 - 137052W^3 \\ &- 2506950C^2W^2 - 473550C^4W - 1494225W^2 + 733140C^2W \\ &- 40425C^4 - 2237078W + 110250C^2 + 220255 \end{aligned} \right) \quad (183)$$

$$b_{3,2}' = 101376(4W+1)(W+1)(9W+49)(9W+25)(9W+1) \quad (184)$$

$$a_{3,4}' = -5(1647W^2 + 791W - 400) \quad (185)$$

$$b_{3,4}' = 768(4W+1)(W+1)(81W+100)(9W+25) \quad (186)$$

$$a_{3,6}' = -75(9W-7) \quad (187)$$

$$b_{3,6}' = 6144(W+1)(9W+49)(9W+25) \quad (188)$$

$$a_{3,2}'' = \begin{pmatrix} 374220C^4W^3 - 379080C^2W^3 - 2817045C^4W^2 - 85860W^3 \\ +1089450C^2W^2 - 1055670C^4W - 732933W^2 \\ +3385980C^2W - 82005C^4 - 434950W + 362250C^2 + 1312283 \end{pmatrix} \quad (188)$$

$$b_{3,2}'' = 101376(4W+1)(W+1)(9W+49)(9W+25)(9W+1) \quad (189)$$

$$a_{3,4}'' = -5(81W^2 - 220W - 225) \quad (190)$$

$$b_{3,4}'' = 128(4W+1)(W+1)(81W+100)(9W+25) \quad (191)$$

$$a_{3,6}'' = -35(9W-71) \quad (192)$$

$$b_{3,6}'' = 6144(W+1)(9W+49)(9W+25) \quad (193)$$

$$a_{5,2}' = - \left(\begin{aligned} &7375494155420727705600C^8W^{18} - 9841766114110820843520C^6W^{18} + 426364826687910190080000C^8W^{17} \\ &+ 3486309314630825410560C^4W^{18} - 407732634387442471403520C^6W^{17} - 498517203302586777600C^2W^{18} \\ &+ 4579153074410976971078400C^8W^{16} + 65325247907633938022400C^4W^{17} + 9705044069244665856W^{18} \\ &+ 9600075206289977057187840C^6W^{16} - 39729016905862605373440C^2W^{17} - 264935435146279446735648000C^8W^{15} \\ &- 9079106494685262953863680C^4W^{16} + 783767050854100623360W^{17} + 975250939527137647445702400C^6W^{15} \\ &- 2926354686166056304558080C^2W^{16} - 10531377005418841084857036000C^8W^{14} - 451887364258377509358247680C^4W^{15} \\ &+ 29888030074228163921664W^{16} + 27175760115528196481564404800C^6W^{14} - 184028796625754285329294080C^2W^{15} \\ &- 174296718041284109298167910000C^8W^{13} - 7969504609068587118244771200C^4W^{14} - 107335413750599174799360W^{15} \\ &+ 400454799456467937458211051600C^6W^{13} - 6907384312150354787227982400C^2W^{14} - 1593056677652394436055715805125C^8W^{12} \\ &- 45997986613254547599379688400C^4W^{13} - 70945438201512824800812960W^{14} + 3502497087288241458133405785300C^6W^{12} \\ &- 151191877355040189082373485200C^2W^{13} - 8483055004184039126064286492500C^8W^{11} + 41898645073318686082222721650C^4W^{12} \\ &- 329393039824228383529776000W^{13} + 18664237300930562740742813610000C^6W^{11} - 2029065323602711686755102081100C^2W^{12} \\ &- 26051144734361075784116618533500C^8W^{10} + 8726683886067990567738562907400C^4W^{11} - 75515048285360850002821509045W^{12} \\ &+ 60042845351651882978979563497200C^6W^{10} - 17275056797131963370329066959600C^2W^{11} - 42392182722684746812221552345000C^8W^9 \\ &+ 6234108618369609367413418109400C^4W^{10} - 1021980989126893568557015718100W^{11} + 111082754754222474631786260019200C^6W^9 \\ &- 94633858404876705389718406074000C^2W^{10} - 22873705407627019305462165952350C^8W^8 + 235563889486788658070977985094000C^4W^9 \\ &- 8700729155319114235439217110460W^{10} + 98518275447962166957422179564920C^6W^8 - 334066700083214008874565134337600C^2W^9 \\ &- 4777853656881870499552818309000W^9 - 6931425967598670487757231215680C^6W^7 - 754212183694656531652392415446600C^2W^8 \\ &+ 31889349297969770711918693880000C^8W^7 + 506898289219329662473263283367340C^4W^8 + 62808229524064536981582066746100C^6W^6 \\ &+ 626891828309764696027227907022400C^2W^7 - 170340203307689964292618125762686W^8 - 101479199540109955687418772209040C^6W^6 \\ &- 106919776438496605940438647444160C^2W^7 + 45079150592365952139525648123000C^8W^5 + 440573867991050326072827395914680C^4W^6 \\ &- 392110485535287660614842261954560W^7 - 97508256061768301167536086514000C^6W^5 - 919978571943187272470857156056720C^2W^6 \\ &+ 16678813300881541924761010546875C^8W^4 + 167455443016359441550273317614880C^4W^5 - 572234879347482310927188852783244W^6 \\ &- 44207068224196263281356012387500C^6W^4 - 449824896217640202789136126255920C^2W^5 + 3397078849895200699817785312500C^8W^3 \\ &+ 26508226012934217667717305611250C^4W^4 - 511603227998618981182379504578840W^5 - 10541925244600545418305401250000C^6W^3 \\ &- 106500887297844908039285876023500C^2W^4 + 377974019849022489404531250000C^8W^2 - 2901781760112069678971362725000C^4W^3 \\ &- 26467606762012994239572279859125W^4 - 1316238961670757319852125000000C^6W^2 - 4750101030184572204526856850000C^2W^3 \\ &+ 212706395379597020343750000000C^8W - 1597764576859766494461712500000C^4W^2 - 71348258197691041930340636487500W^3 \\ &- 80386677017525583337500000000C^6W + 2327221411749347359503915000000C^2W^2 + 467191110732018750000000000C^8 \\ &- 176244843466835541735750000000C^4W - 7772022338742047276655248750000W^2 - 1868764442928075000000000000C^6 \\ &+ 355919944077491966770500000000C^2W - 575130266945769150000000000C^4 - 31284681755176272803625000000W \\ &+ 1443845546656086420000000000C^2 + 2566714203124986915000000000 \end{aligned} \right) \quad (194)$$

$$b_{5,2}' = 424673280(9W+1)^2(9W+25)^2(9W+49)^2(81W+100)(W+1)^2(4W+1)^2 \times (9W+100)(36W+25)(16W+49)(4W+49)(W+49)(W+36)(W+16)(W+9)(81W+3025)(16W+1) \quad (195)$$

$$a'_{5,4} = \left(\begin{aligned} &110770175238782976C^4W^{16} - 14307258895663104C^2W^{16} + 1955848904001054720C^4W^{15} + 4995480368627712W^{16} \\ &+ 3851914453308665856C^2W^{15} - 231019516749453122880C^4W^{14} + 1394963237614159872W^{15} + 351842222082402737280C^2W^{14} \\ &- 10700324833047652721520C^4W^{13} + 121013798552614077120W^{14} + 11382616338268522566240C^2W^{13} - 193255067033064707112540C^4W^{12} \\ &+ 4955689831941278470800W^{13} + 186947014654398167993880C^2W^{12} - 1808045567774599897341045C^4W^{11} + 112798273591823118309540W^{12} \\ &+ 1751757701698414349036370C^2W^{11} - 9463449720640124701483110C^4W^{10} + 1540062308779229081984715W^{11} \\ &+ 9827646640680141309931020C^2W^{10} - 28591350932593837376295195C^4W^9 + 13065742529722884733053690W^{10} \\ &+ 33720217401125642250527790C^2W^9 - 49129344818685691696240020C^4W^8 + 69621105351509049537907685W^9 \\ &+ 70892334501342639184454520C^2W^8 - 4220618691982208233933995C^4W^7 + 232040104988856366387112460W^8 \\ &+ 88542151242424306336217070C^2W^7 - 4070106278996441025463926C^4W^6 + 478514408052402525601211925W^7 \\ &+ 57938913918497981187758604C^2W^6 + 25310191297642581997387035C^4W^5 + 598763775030443958456679978W^6 \\ &+ 7447104191714524778086674C^2W^5 + 22989071089256595971539500C^4W^4 + 438882220810584403201135003W^5 \\ &- 15225976652990210865412200C^2W^4 + 8996183090465714193750000C^4W^3 + 173432100766457002501039500W^4 \\ &- 10007800246774751931300000C^2W^3 + 1695497897295262305000000C^4W^2 + 28104937877390579484230000W^3 \\ &- 237169596372237728000000C^2W^2 + 14222542646232900000000C^4W - 1468277838517821079000000W^2 \\ &- 223187256364262400000000C^2W + 413583294600000000000C^4 - 820448247199606200000000W \\ &- 689178430080000000000C^2 - 4989758564952000000000 \end{aligned} \right) \quad (196)$$

$$b'_{5,4} = 1769472(9W+1)(9W+25)^2(9W+49)(81W+100)^2(W+1)^2(4W+1)^2 \times (9W+100)(36W+25)(16W+49)(4W+49)(W+49)(W+36)(W+16)(W+9)(16W+1) \quad (197)$$

$$a'_{5,6} = +7 \left(\begin{aligned} &17438751061924008960C^4W^{15} - 3887531857282664448C^2W^{15} + 1000717417562246429760C^4W^{14} \\ &- 1186865755017956352W^{15} + 31016328794557501824C^2W^{14} + 10402238895599229410160C^4W^{13} \\ &+ 95382870847342111296W^{14} + 22337471813562827599392C^2W^{13} - 630875026488567322802340C^4W^{12} \\ &+ 26670933726253926232176W^{13} + 1110191215055908307071464C^2W^{11} - 24633857035202101918217415C^4W^{11} \\ &+ 1767769893576405733702716W^{12} + 25764771881210968167223662C^2W^{11} - 401643375337982409538929165C^4W^{10} \\ &+ 57998015546802264390715641W^{11} + 345517979965275258188733114C^2W^{10} - 3595269928762951774531582815C^4W^9 \\ &+ 1099353625688754207753254451W^{10} + 2862145801997098694689754142C^2W^9 - 18514227172462267161143293065C^4W^8 \\ &- 53571253759385861047515460065C^4W^7 + 92325529066812910045711702391W^8 + 47158408809485666715361026402C^2W^7 \\ &+ 12755046398952147804731299041W^9 + 14818503019609145597363742834C^2W^8 - 76601079413786132947558345815C^4W^6 \\ &+ 415387547321914939842207331551W^7 + 87547617388177707129255472014C^2W^6 - 18904625614401860939534396325C^4W^5 \\ &+ 1136742756027800748973623580521W^6 + 79750756537448661883807295850C^2W^5 + 87357617829205806536219390625C^4W^4 \\ &+ 1800635237928993035403028220443W^5 + 1067683759930771451056338750C^2W^4 + 112584033589472176511717437500C^4W^3 \\ &+ 1467434884880749176133858418625W^4 - 65575728939727393891378875000C^2W^3 + 54528714178995279567093750000C^4W^2 \\ &+ 391391055692367430933812877500W^3 - 55056483781954851483787500000C^2W^2 + 10516761121167196520625000000C^4W \\ &- 175077329804437136778546250000W^2 - 1676996032356111592125000000C^2W + 63626979842551125000000000C^4 \\ &- 113047492937153382910575000000W - 127253959685102250000000000C^2 - 1383805077093507915000000000 \end{aligned} \right) \quad (198)$$

$$b'_{5,6} = 8097103872(9W+1)(9W+25)^2(9W+49)^2(81W+100)(W+1)^2(4W+1) \times (9W+100)(36W+25)(16W+49)(4W+49)(W+49)(W+36)(W+16)(W+9)(81W+3025) \quad (199)$$

$$a'_{5,8} = -7 \left(\begin{aligned} &348940224W^9 + 4022030700W^8 - 757070111997W^7 - 29124820174135W^6 \\ &- 420150601643703W^5 - 2719689496939305W^4 - 7113799342723024W^3 \\ &- 4168103200391760W^2 + 5604505975824000W + 3151110816000000 \end{aligned} \right) \quad (200)$$

$$b'_{5,8} = 1179648(9W+25)(9W+49)(81W+100)(W+1)(4W+1) \times (9W+100)(16W+49)(4W+49)(W+49)(W+36)(W+16)(W+9) \quad (201)$$

$$a'_{5,10} = -77(16767W^4 + 57825W^3 - 12648767W^2 - 210251685W + 254677500) \quad (202)$$

$$b'_{5,10} = 97152(9W+25)(9W+49)(W+1)(W+49)(W+36)(W+9)(81W+3025) \quad (203)$$

$$\begin{aligned}
& (106891219643778662400C^8W^{18} - 101060789481390735360C^6W^{18} - 11314468590411145420800C^8W^{17} \\
& + 53985464466554880000C^4W^{18} + 26824285587371907317760C^6W^{17} - 50715487761723555840C^2W^{18} \\
& - 1388826231777817784582400C^8W^{16} - 7227421357209986580480C^4W^{17} - 9100406867219251200W^{18} \\
& + 2659485075298566276787200C^6W^{16} - 8388985273929478471680C^2W^{17} - 5439190281932980066003200C^8W^{15} \\
& - 571305584692008829232640C^4W^{16} - 1822930920491812134912W^{17} + 95535745946358548393203200C^6W^{15} \\
& - 736127390655669349463040C^2W^{16} - 1059311972601247454444124000C^8W^{14} - 6336397781666474873863680C^4W^{15} \\
& - 152914954957852326416640W^{16} + 1680609394741528859877964800C^6W^{14} - 36903669546308014870479360C^2W^{15} \\
& - 10573589044117052888938542000C^8W^{13} + 398077816100125763656958400C^4W^{14} - 7218538047552078666908928W^{15} \\
& + 13873507423490323949845831200C^6W^{13} - 1094177399917008592673671680C^2W^{14} - 37179205887120579442594058625C^8W^{12} \\
& + 14629605309286071555155292000C^4W^{13} - 215088747680049561151140960W^{14} + 4914033752033593840937308500C^6W^{12} \\
& - 19801939972695439026876962400C^2W^{13} + 260279839977953642867651053500C^8W^{11} \\
& + 220917352036270849037469099450C^4W^{12} - 4280873607549147060137562480W^{13} \\
& - 941991859587758955718329342000C^6W^{11} - 223196987831454832822164974700C^2W^{12} \\
& + 3442373132268172716547090888500C^8W^{10} + 1795269019831697347648871063400C^4W^{11} \\
& - 58485886953137204950867089825W^{21} - 8730255453797139309985658288400C^6W^{10} \\
& - 155458352515019777438637393200C^2W^{11} + 16431327301442644078726746123000C^8W^9 \\
& + 797413059688058553325760851000C^4W^{10} - 554309628145075828638294789060W^{11} \\
& - 39448253504861888297497286349600C^6W^9 - 6299600706219086967177299869200C^2W^{10} \\
& + 42416293624549891670240230001850C^8W^8 + 16522866211850138632452889966800C^4W^9 \\
& - 3640243270220344805370663924300W^{10} - 103810944099778124432025724959240C^6W^8 \\
& - 11572497184530942559347321607200C^2W^9 + 64165242834494731067881461364800C^8W^7 \\
& - 1457242212788902069891271864100C^4W^8 - 1638265085419103775538130617480W^9 \\
& - 167443926916516310339786411372160C^6W^7 + 10277560704249551926727695230840C^2W^8 \\
& + 5826393251027168309715202729200C^8W^6 - 77790627865993346890383152315520C^4W^7 \\
& - 49486758242384555818561201503750W^8 - 168306338834875652639511827250000C^6W^6 \\
& + 99456502335264128731615201860480C^2W^7 + 31779324669514969215015971362200C^8W^5 \\
& - 157689050874719506404766188878760C^4W^6 - 96940954219928295479774842331328W^7 \\
& - 104911400832095022331785726038400C^6W^5 + 217645355959670223891795054072240C^2W^6 \\
& + 10294043938282516375608939669375C^8W^4 - 150503449182783464975712255137520C^4W^5 \\
& - 115565588124829252645701741647260W^6 - 39531168069178713496042325827500C^6W^4 \\
& + 242062650920242858015993266063360C^2W^5 + 1949585992801391143878395062500C^8W^3 \\
& - 79743792707175024465503961313350C^4W^4 - 71862716131047533480684554518312W^5 \\
& - 8605021747664102785464053250000C^6W^3 + 149643978487504947288489244831380C^2W^4 \\
& + 207879020488989472309706250000C^8W^2 - 24333310777254842380934278425000C^4W^3 \\
& - 9064002408034531279496070076065W^4 - 1023990395303421829979325000000C^6W^2 \\
& + 511170231606707285131638736990000C^2W^3 + 11395848645232751266875000000C^8W \\
& - 4036659395677073168241010500000C^4W^2 + 14311373538205849071749473862500W^3 \\
& - 60866191428117134377500000000C^6W + 9236772698672719220817497400000C^2W^2 \\
& + 24617533325979750000000000C^8 - 317466612634250215158750000000C^4W \\
& + 7685026510424492186210527050000W^2 - 1392431757215499000000000000C^6 \\
& + 799853069803072526936100000000C^2W - 8931840009039084060000000000C^4 \\
& + 1333912227270850991202075000000W + 25932929953756113288000000000C^2 \\
& + 71718259288813762086000000000)
\end{aligned}$$

(204)

$$\begin{aligned}
b_{5,2}'' &= 141557760(9W+1)^2(9W+25)^2(9W+49)^2(81W+100)(W+1)^2(4W+1)^2(9W+100) \\
&\quad \times (W+36)(W+16)(36W+25)(4W+49)(W+49)(W+9)(16W+49)(16W+1)(81W+3025)
\end{aligned}$$

(205)

$$\begin{aligned}
& \left(\begin{aligned}
& 133297443127296C^4W^{16} + 977514582933504C^2W^{16} - 235559826776107008C^4W^{15} \\
& +122506977312768W^{16} + 237714459263447040C^2W^{15} - 16699733263678530240C^4W^{14} \\
& +199885657760741376W^{15} + 13803714618065737344C^2W^{14} - 436606235648316956880C^4W^{13} \\
& +12921351345303170112W^{14} + 336829722590569463136C^2W^{13} - 4971082166815169884500C^4W^{12} \\
& +435277713395859038064W^{13} + 3771706074515666935704C^2W^{12} - 16157970895394748446775C^4W^{11} \\
& +8505294276926431376316W^{12} + 13219145296345038893346C^2W^{11} + 162009987174874407217005C^4W^{10} \\
& +100760747413994795355669W^{11} - 116650230476369356708326C^2W^{10} + 1824283810986817624562685C^4W^9 \\
& +726653068881670674096921W^{10} - 1483138708089675483852054C^2W^9 + 7961760273241757590381445C^4W^8 \\
& +305638027779052916681849W^9 - 7178202284933896884351446C^2W^8 + 19182267607814867301600015C^4W^7 \\
& +637873789312512897328401W^8 - 19321458854023043228674674C^2W^7 + 27758186776061055973611519C^4W^6 \\
& +847415809826030047023459W^7 - 31522106584064036379954690C^2W^6 + 24685661642779121636835463C^4W^5 \\
& -2533424355371009518827997W^6 - 31888158879166461165773794C^2W^5 + 13353483120293565158027475C^4W^4 \\
& -55255498670140056440475717W^5 - 19726718531234544382692090C^2W^4 + 4214681979790360542712500C^4W^3 \\
& -55017407778355426648306521W^4 - 7075570675361136233103000C^2W^3 + 715154847306775615250000C^4W^2 \\
& -28989307206090013959764700W^3 - 1323528742411469121500000C^2W^2 + 56917967270400075000000C^4W \\
& -7968282291866808213750000W^2 - 111930618761622570000000C^2W + 1610973639450000000000C^4 \\
& -1019248962837981393000000W - 3280660314876000000000C^2 - 4483598665888440000000
\end{aligned} \right) \quad (206)
\end{aligned}$$

$$b''_{5,4} = 294912(9W+1)(9W+25)^2(9W+49)(81W+100)^2(W+1)^2(4W+1)^2 \quad (207)$$

$$\begin{aligned}
& \times (9W+100)(W+36)(W+16)(36W+25)(4W+49)(W+49)(W+9)(16W+49)(16W+1) \\
& \left(\begin{aligned}
& 252735522636579840C^4W^{15} + 9190382641330176C^2W^{15} - 26859194651434242240C^4W^{14} \\
& +203720148549485568W^{15} + 24443900398594385280C^2W^{14} - 3272400597832748450640C^4W^{13} \\
& +51136269608268023616W^{14} + 1992557248171035470496C^2W^{13} - 127217616404384964512340C^4W^{12} \\
& +4674254965120440325296W^{13} + 64008861807267882184680C^2W^{12} - 2450600685675631681135515C^4W^{11} \\
& +215247614222830961251116W^{12} + 952122640560643764126486C^2W^{11} - 23955871734419886818650665C^4W^{10} \\
& +5606855376131531354545701W^{11} + 4795035062495160260850930C^2W^{10} - 77635607987513237452162515C^4W^9 \\
& +85718664351905956279235991W^{10} - 52938180272548758101251194C^2W^9 + 649508008200905498847351435C^4W^8 \\
& +747893070870187317843698541W^9 - 1052385976833024095124561270C^2W^8 + 7868020478457268377677292435C^4W^7 \\
& +3084327180949924074975517771W^8 - 7886716699389071686741532934C^2W^7 + 35484978824439315791464529685C^4W^6 \\
& -215914344112360181160897709W^7 - 32887112143064948571195191370C^2W^6 + 84860755789198673296620788895C^4W^5 \\
& -81880322935503604111457478059W^6 - 82444829905024697146385808030C^2W^5 + 113965265531461984177958974125C^4W^4 \\
& -370306925348368782087847444897W^5 - 126201972750059345162667608250C^2W^4 + 85149228205987410051374287500C^4W^3 \\
& -806937361283096319479132780435W^4 - 115788974987652711379409775000C^2W^3 + 3318428519214212131498750000C^4W^2 \\
& -937287152186927300465582592500W^3 - 59729982487075798595197500000C^2W^2 + 5797307133004241980125000000C^4W \\
& -571166639804573515124514050000W^2 - 14734816686238923524250000000C^2W + 335267358720143850000000000C^4 \\
& -164290487428167903943875000000W - 1040728054706039700000000000C^2 - 16152976991220897366000000000
\end{aligned} \right) \quad (208)
\end{aligned}$$

$$b''_{5,6} = 2699034624(9W+1)(9W+25)^2(9W+49)^2(81W+100)(W+1)^2(4W+1) \quad (209)$$

$$\times (9W+100)(W+36)(W+16)(36W+25)(4W+49)(W+49)(W+9)(16W+49)(81W+3025) \quad (210)$$

$$a''_{5,8} = -7 \left(\begin{aligned} & 1259712W^9 - 2233853316W^8 - 144141339861W^7 - 3212557306782W^6 - 25324094813764W^5 \\ & +47527460481162W^4 + 1623155420258313W^3 + 7016749146062936W^2 + 10020838600995600W + 3682225461600000 \end{aligned} \right) \quad (211)$$

$$b''_{5,8} = 589824(9W+25)(9W+49)(81W+100)(W+1)(4W+1) \quad (212)$$

$$\times (9W+100)(W+36)(W+16)(4W+49)(W+9)(16W+49) \quad (213)$$

$$a''_{5,10} = -77(29W^4 - 116793W^3 - 3988889W^2 - 25259427W + 402588900) \quad (212)$$

$$b''_{5,10} = 2097152(9W+25)(9W+49)(W+1)(W+36)(W+49)(W+9)(81W+3025) \quad (213)$$

6.5. Fourth Harmonic

$$\begin{aligned}
& \left(\begin{aligned}
& 529079040C^8W^6 - 1175731200C^6W^6 - 11518491600C^8W^5 + 755827200C^4W^6 \\
& +21599161920C^6W^5 - 100776960C^2W^6 - 7184819880C^8W^4 - 13105709280C^4W^5 \\
& -8398080W^6 + 30837633120C^6W^4 + 3183105600C^2W^5 + 2748546675C^8W^3 \\
& -30684668400C^4W^4 - 245655504W^5 + 6560395380C^6W^3 + 9819312480C^2W^4 \\
& +1682796465C^8W^2 - 9392322990C^4W^3 - 743233320W^4 - 3801554820C^6W^2 \\
& +144616500C^2W^3 + 211930425C^8W + 17587186950C^4W^2 + 587665683W^3 \\
& -732305700C^6W - 18024382020C^2W^2 + 7441875C^8 + 742281270C^4W \\
& +2846791825W^2 - 29767500C^6 - 2305596900C^2W - 24144750C^4 + 701143321W \\
& +107824500C^2 - 27686925
\end{aligned} \right) \quad (214)
\end{aligned}$$

$$b'_{4,0} = 3538944(9W+1)(9W+49)(W+1)(4W+1)(36W+25)(9W+25)(16W+1)(W+9) \quad (215)$$

$$a'_{4,2} = -5 \begin{pmatrix} 7618738176C^4W^7 - 6530347008C^2W^7 - 63313125120C^4W^6 \\ -1088391168W^7 + 25728357888C^2W^6 - 327486821256C^4W^5 \\ -13932414720W^6 + 250355687760C^2W^5 - 333580480254C^4W^4 \\ -33788654280W^5 + 494701391628C^2W^4 - 76702008915C^4W^3 \\ +24827789682W^4 + 284773919238C^2W^3 + 19819867704C^4W^2 \\ +138357284733W^3 - 6235010496C^2W^2 + 4384214625C^4W \\ +130507820648W^2 - 6373696770C^2W + 182647500C^4 \\ +32843067985W - 311787000C^2 - 1226396500 \end{pmatrix} \quad (216)$$

$$b'_{4,2} = 221184(9W+1)(9W+49)(16W+49)(W+1)(4W+1)(36W+25)(9W+25)(16W+1)(81W+100) \quad (217)$$

$$a'_{4,4} = -5 \begin{pmatrix} 59521392C^4W^6 - 39680928C^2W^6 - 1226067192C^4W^5 - 31177872W^6 \\ +480090240C^2W^5 - 2331453537C^4W^4 - 289611288W^5 + 2413749618C^2W^4 \\ -542964303C^4W^3 + 1004799735W^4 + 2926247310C^2W^3 + 604992465C^4W^2 \\ +8686733913W^3 + 750785070C^2W^2 + 219751875C^4W + 7290427897W^2 \\ -247143750C^2W + 16537500C^4 + 1146396955W - 33075000C^2 - 182794500 \end{pmatrix} \quad (218)$$

$$b'_{4,4} = 294912(9W+1)(9W+49)(W+1)(4W+1)(36W+25)(9W+25)(81W+100)(W+9) \quad (219)$$

$$a'_{4,6} = 35(69984W^4 - 642330W^3 - 2459565W^2 - 847591W + 347900) \quad (220)$$

$$b'_{4,6} = 73728(9W+49)(16W+49)(W+1)(4W+1)(9W+25)(81W+100) \quad (221)$$

$$a'_{4,8} = 35(9W^2 - 206W + 105) \quad (222)$$

$$b'_{4,8} = 31072(9W+49)(W+1)(9W+25)(W+9) \quad (223)$$

$$a''_{4,0} = - \begin{pmatrix} 529079040C^8W^5 - 1116944640C^6W^5 - 1230843600C^8W^4 + 711970560C^4W^5 \\ +1205532720C^6W^4 - 123731712C^2W^5 - 1855246680C^8W^3 - 228776400C^4W^4 \\ -373248W^5 + 4253917500C^6W^3 + 231161040C^2W^4 - 608433525C^8W^2 \\ -4017211020C^4W^3 - 66756960W^4 + 2449235880C^6W^2 + 2206980324C^2W^3 \\ -66242610C^8W - 3963042450C^4W^2 - 333781884W^3 + 330307740C^6W \\ +2793982560C^2W^2 - 2197125C^8 + 321780960C^4W - 421006145W^2 + 12096000C^6 \\ -332604012C^2W + 34615350C^4 + 6685282W - 121980600C^2 + 32253255 \end{pmatrix} \quad (224)$$

$$b''_{4,0} = 442368(9W+1)(9W+49)(W+1)(4W+1)(36W+25)(9W+25)(16W+1)(W+9) \quad (225)$$

$$a''_{4,2} = \begin{pmatrix} 226445829120C^4W^6 - 172328601600C^2W^6 + 523778451840C^4W^5 \\ -11549039616W^6 - 715172094720C^2W^5 - 580121797500C^4W^4 \\ -188755712640W^5 - 576768149640C^2W^4 - 1425851652015C^4W^3 \\ -769372670268W^4 + 902864519550C^2W^3 - 714255565170C^4W^2 \\ -1155019406175W^3 + 1148793795540C^2W^2 - 92728647375C^4W \\ -638085258466W^2 + 177101485470C^2W - 3344250000C^4 \\ +2464973665W + 6809922000C^2 + 58429364000 \end{pmatrix} \quad (226)$$

$$b''_{4,2} = 221184(9W+1)(9W+49)(16W+49)(W+1)(4W+1)(36W+25)(9W+25)(16W+1)(81W+100) \quad (227)$$

$$a''_{4,4} = 5 \begin{pmatrix} 19840464C^4W^5 - 11573604C^2W^5 - 22902264C^4W^4 - 5380020W^5 \\ -25816077C^2W^4 - 125123859C^4W^3 - 77910093W^4 + 36558459C^2W^3 \\ -100887066C^4W^2 - 242490006W^3 + 105598017C^2W^2 - 24346875C^4W \\ -14038877W^2 + 57080625C^2W - 1627500C^4 + 139315976W + 5092500C^2 \\ +44322600 \end{pmatrix} \quad (228)$$

$$b''_{4,4} = 12288(9W+1)(9W+49)(W+1)(4W+1)(36W+25)(9W+25)(81W+100)(W+9) \quad (229)$$

$$a''_{4,6} = -35(416016W^3 + 601137W^2 - 1640639W - 1274000) \quad (230)$$

$$b''_{4,6} = 73728(9W+49)(16W+49)(W+1)(4W+1)(9W+25)(81W+100) \quad (231)$$

$$a''_{4,8} = -35(9W - 31) \quad (232)$$

$$b''_{4,8} = 16384(9W+49)(W+1)(9W+25)(W+9) \quad (233)$$

$$a'_{6,0} = a'^{(1)}_{6,0} + a'^{(2)}_{6,0} \quad (234)$$

$$\begin{aligned} & 7779902120172214145777664000000C^{12}W^{27} - 20826199521691773251774054400000C^{10}W^{27} \\ & -1058522917171455914161471488000000C^{12}W^{26} + 20858842467650537786651443200000C^8W^{27} \\ & +3551011090709932010059805491200000C^{10}W^{26} - 10808442106346479326068736000000C^6W^{27} \\ & -205294351551386829633856307095142400C^{12}W^{25} - 4077293863685283835655749632000000C^8W^{26} \\ & +3411965065689912097898496000000C^4W^{27} + 639553259697056773992316460901335040C^{10}W^{25} \\ & +1561302836223052745354615193600000C^6W^{26} - 382388795516955979992268800000C^2W^{27} \\ & -12957239340826304733595019698038374400C^{12}W^{24} - 684552583321799244471805557992325120C^8W^{25} \\ & +91059741434767132055096524800000C^4W^{26} - 33679229957455472492544000000W^{27} \\ & +39176050408137348139764464164208640000C^{10}W^{24} + 247471122714911199310952226462105600C^6W^{25} \\ & -58416217523784808744290877440000C^2W^{26} - 439357436121910996916888507911934668800C^{12}W^{23} \\ & -391866485033433508122608342698557440C^8W^{24} + 12630106498064176142409971426918400C^4W^{25} \\ & -10144836858136615538109972480000W^{26} + 1275171101086911765151873345218167316480C^{10}W^{23} \\ & +10755918186379211651039206972572303360C^6W^{24} - 8571988167059391590444301098680320C^2W^{25} \\ & -8661805776743409715313622632442108364800C^{12}W^{22} - 1125728002176794869922182068882558750720C^8W^{23} \\ & +31194283952077051031830613620490240C^4W^{24} - 130964352517269112972060600883200W^{25} \\ & +23197492277413668076977640217428911820800C^{10}W^{22} + 85041836305693171032594017302201221120C^6W^{23} \\ & -807830970490561927051640933410406400C^2W^{24} - 85760594630827487870297019275506375123200C^{12}W^{21} \\ & -14702310906477904373344008050837101639680C^8W^{22} + 253962342307337290898506224427536445440C^4W^{23} \\ & -101844783349674470890446952935456768W^{24} + 172253063115770700716995992156072097297920C^{10}W^{21} \\ & -8952474108750528746108080841377541529600C^6W^{22} - 43718365775439826210285819032174059520C^2W^{23} \\ & +176790681502390949887588601281633735276800C^{12}W^{20} + 7177091118244808867763007554488385153280C^8W^{21} \\ & +10728698230861020766251296238260095764480C^4W^{22} - 5415544194679364386087799277069201408W^{23} \\ & -2156438530788983532932568975648284517849600C^{10}W^{20} - 399976488672726634011512486936204090158080C^6W^{21} \\ & -1402520292415545690503179857972030566400C^2W^{22} + 18305782373040840230154277771348036524140400C^{12}W^{19} \\ & +643957422395316222899133390258004752142080C^8W^{20} + 273746797799116246368845852032317393281280C^4W^{21} \\ & -20828809088422905586017449290497918976W^{22} - 76549881442341451862341114353363171761022240C^{10}W^{19} \\ & -8550947213681720300179864862174021825326080C^6W^{20} - 26106256797075943119315152389225792350720C^2W^{21} \\ & +278924832843964145732178195110870354726441400C^{12}W^{18} + 124541589774452720771750396433969146075243280C^8W^{19} \\ & +4430109870978675136495733728358752649598720C^4W^{20} - 5944721822323533055500181669444973250816W^{21} \\ & -1048228169521161321661935820309608154910569200C^{10}W^{18} - 110570713554012869450668197505072920092337600C^6W^{19} \\ & -210438442945167694357125906312643421452800C^2W^{20} + 2399750904828258735005938863942177798791030625C^{12}W^{17} \\ & +1377977931774752144168520345947459418504370440C^8W^{18} + 44286937500902096170601778703555095190719120C^4W^{19} \\ & -127379602741564996611469362943995577261824W^{20} - 8759369800705258520615679932570930151712244550C^{10}W^{17} \\ & -885379823657416359709190021344086181754261280C^6W^{18} + 2134329406766101716170294996324921337158880C^2W^{19} \\ & +13368590454838518905998041930322731291957454500C^{12}W^{16} + 9811246049763447104449258461117690487131231255C^8W^{17} \\ & +218311735541107476399577710579276441744924360C^4W^{18} - 2060025807647492956776712090525399932462864W^{19} \\ & -48846418550323395452158832440870400954200863000C^{10}W^{16} - 3867650714364352734369520960583458633282719540C^6W^{17} \\ & +86224707436304982821222335099113185674870800C^2W^{18} + 50109953237139383891922480533514254158016920650C^{12}W^{15} \\ & +45844616762130852682364569588302028843096760700C^8W^{16} - 561589376786234451304248649627187274385895265C^4W^{17} \\ & -25192029804823996704087596006905231243917192W^{18} - 187213516919728710743052986638305184493504695740C^{10}W^{15} \\ & -1315131026100707833040729639456423918185064400C^6W^{16} + 1257554862704935094498236604437308829740565530C^2W^{17} \\ & +127395244088630734596985002002949282796962003900C^{12}W^{14} + 136423044635723615021183180986034771857333560870C^8W^{15} \\ & -18154647348596379219659444988129503601924132900C^4W^{16} - 232773876157636586947854070093374044873390007W^{17} \\ & -497641845303028388734030165705924210811128557000C^{10}W^{14} + 94177000557386280561169619391239333882547723000C^6W^{15} \\ & +11270524452292679868622956067813952141543677800C^2W^{16} + 216814068927577582640680077127999937041322945175C^{12}W^{13} \\ & +226792258101167290814468350163885209761557407140C^8W^{14} - 149327034754237458836287110445693741022632763850C^4W^{15} \\ & -1619652067745983666106607114283967852611882620W^{16} - 913800725947133026453564535429310225592453713930C^{10}W^{13} \\ & +630591844460026048897929235561747206692169002640C^6W^{14} + 68554744848410285384126914656354827733851714820C^2W^{15} \\ & +23517519953251467574609653446960097399985801800C^{12}W^{12} + 62148042046299421862823195651072083722049823345C^8W^{13} \\ & -724004816857594577573396935886852867970270314940C^4W^{14} - 8433349116814995486100849859971342710348782310W^{15} \end{aligned} \quad (235)$$

$$\begin{aligned}
& \left(\begin{aligned}
& -1134545334667951324362904630280628899376896746800C^{10}W^{12} + 2261553145416935051054794468420102760802048122580C^6W^{13} \\
& + 292455052668233817660991965288021954139135721400C^2W^{14} + 135831339398765499232366372670369699604807152700C^{12}W^{11} \\
& - 623638593710825837469095456970634019298842970120C^8W^{12} - 2332274603180718399920566996285721009447668725015C^4W^{13} \\
& - 32527209587995901465608826510812310476091949092W^{14} - 893524305162634254818992889802244155359121990120C^{10}W^{11} \\
& + 5232322443899324137045787656207454920835201647200C^6W^{12} + 882753909134351597995748588241970390271371269270C^2W^{13} \\
& - 6268524249171265850690631225759181639615714800C^{12}W^{10} - 1585128092568175293920524586394254942901745050780C^8W^{11} \\
& - 5176782345004047421211969120823118801366287844680C^4W^{12} - 91385953221832598945619251328617966225945546577W^{13} \\
& - 347177794341690727176871270410533146012642430400C^{10}W^{10} + 8251296174578889993704073767210405077475947292880C^6W^{11} \\
& + 1871062771963310407128575808141341753480920424400C^2W^{12} - 79911525364235218708196339100320273347775309025C^{12}W^9 \\
& - 2084975054758476349950252518815290715741999700880C^8W^{10} - 797438547594323963866677409714048965416367601980C^4W^{11} \\
& - 181462494411633094214405333288894654024497838584W^{12} + 75255639237503827897212805705068359978729534790C^{10}W^9 \\
& + 9050164059026167111663284005561080211831773361280C^6W^{10} + 2705525437080628655964005737989083898410747181720C^2W^{11} \\
& - 69269021738522805439828389732349039483524561900C^{12}W^8 - 1749991764040976493326737046696571938755711109655C^8W^9 \\
& - 8410817017338934842832633422827734983361317739920C^4W^{10} - 238815052202911939378872507635710500221353142244W^{11} \\
& + 18137401264057528887946427114689416256797680200C^{10}W^8 + 6924079739162166465544850590839065006078111752500C^6W^9 \\
& + 2472025751383976553303196070973307910055482356800C^2W^{10} - 32545506170789027888115488713849113335914461750C^{12}W^7 \\
& - 985764769787012117088948468692249633764746202740C^8W^8 - 5830745351140881476951917202270957315042381617695C^4W^9 \\
& - 171018053374149255563516091219797672059644132336W^{10} + 111639239615479767924815115576823990863421676100C^{10}W^7 \\
& + 3651086278799216304385202022674289806500182004080C^6W^8 + 1075251184969070991200820509073018590974202448870C^2W^9 \\
& - 9586875459649943295157041198697947603341962500C^{11}W^6 - 372152990166291302441123869671360621989915735130C^7W^7 \\
& - 2389210603432335465657474665002168210012200327060C^4W^8 + 12326908121737002861217035715728590523609617079W^9 \\
& + 38703513978595870069190546696683249800387675000C^{10}W^6 + 1286152300881418167763219305885841366162221410040C^6W^7 \\
& - 32642232542616239878336332964938718346889727800C^{12}W^8 - 1833251153469228314386419306017741785218984375C^{12}W^5 \\
& - 90432429358816152160390510001978651931499609500C^8W^8 - 364243587064972288426393557384818522374301844810C^4W^7 \\
& + 16343506930127696419231016045214319777581788532W^8 + 8338135123676374955462101378536540256415906250C^{10}W^5 \\
& + 283934875849822253615859940101120556343118262800C^6W^6 - 754496545542228079810397224193702976378909654780C^2W^7 \\
& - 227861239214755941254772929549194011750000000C^{12}W^4 - 12918078853445077151568498169529540361807740625C^8W^5 \\
& + 129462922816651581122876611941807768896050781700C^4W^6 + 174316333876160425262031410460305115225403021722W^7 \\
& + 113443128350452397845415104247493879750000000C^{10}W^4 + 33730409489939858926249841768665757054411587500C^6W^5 \\
& - 471134509345947426362681281263889850587251457800C^2W^6 - 18052322100056586733559175883408345312500000C^{12}W^3 \\
& - 865444836893823158673517156250100604890000000C^8W^4 + 76218708168982698986107449865155727345882584375C^4W^5 \\
& + 97857125158103943115301225180004945497729848860W^6 + 96233487122568437573342405537981851875000000C^{10}W^3 \\
& + 871690617354190375765238048988385383096000000C^6W^4 - 155661115013248201735188106955726027117081793750C^2W^5 \\
& - 87084606946748255134070723412187500000000C^{12}W^2 + 3503626791839549272621956187466543812500000C^8W^3 \\
& + 1551969694183070163583126353684322067814000000C^4W^4 + 32643015907172879420755281372236960712740520625W^5 \\
& + 488243441075211295430802660781312500000000C^{10}W^2 - 265742657138532717866618375038572483750000000C^6W^3 \\
& - 2902358184410064495503383208945941061076000000C^2W^4 - 2304124786171703335218042508593750000000C^{12}W \\
& + 3911244442391307527338220929744087500000000C^8W^2 + 1497904077507096791681382196700019361312500000C^4W^3 \\
& + 6456618919861990862069093273742387034861200000W^4 + 13435106758312458387959111691562500000000C^{10}W \\
& - 3207594710547651820610533722034401000000000C^6W^2 - 2924309914621385089694876134493477290125000000C^2W^3 \\
& - 254894585298731171377959375000000000000C^{12} + 200074485699304852573674260153437500000000C^8W \\
& + 64519118533071121009144225331269147500000000C^4W^2 + 705403242239576901810237644483560998987500000W^3 \\
& + 152936751179238702826775625000000000000C^{10} - 141481537295904257898320857261125000000000C^6W \\
& - 132300564863884751412754293976873515000000000C^2W^2 + 317740839377419692299202637500000000000C^8 \\
& + 6506978944578916727435095391259375000000000C^4W + 34967131934896332421561408223121880500000000W^2 \\
& - 2203064188999106569582175730000000000000C^6 - 607073642441716084233229501014375000000000C^2W \\
& - 18378761964485831984486235825000000000000C^4 + 241662955627607802935281397748562500000000W \\
& + 84022345976678525251398613050000000000000C^2 - 2165693322173942203397508253500000000000
\end{aligned} \right) \quad (236)
\end{aligned}$$

$$\begin{aligned}
b'_{6,0} &= 6794772480(25W+1)(16W+1)^2(9W+4)(36W+25)^2(25W+49)(9W+1)^2(W+16)(9W+100)(144W+3025)(16W+49) \\
&\times (4W+49)(4W+1)^2(81W+100)(9W+49)^2(W+9)^2(81W+3025)(9W+25)^2(4W+169)(W+36)(9W+121)(W+49)(W+1)^2
\end{aligned} \quad (237)$$

$$a'_{6,2} = a'^{(1)}_{6,2} + a'^{(2)}_{6,2} \quad (238)$$

$$a'^{(1)}_{6,2} = \begin{aligned} & 775596395980245348686757888000000C^8W^{28} - 775596395980245348686757888000000C^6W^{28} \\ & -139129210461218563362546379653120000C^8W^{27} + 399549658535277906899238912000000C^4W^{28} \\ & +297914455703166684189303106437120000C^6W^{27} - 339113575731622425183387648000000C^2W^{28} \\ & -23264103546607168214222008956538060800C^8W^{26} - 89076228343816063926842299514880000C^4W^{27} \\ & -60436082803655481715851264000000W^{28} + 42659989619380431090321812662045900800C^6W^{26} \\ & -69838365141449860649381768724480000C^2W^{27} - 1325299011762344960899547489377355563008C^8W^{25} \\ & -11541334877967016973363688840049459200C^4W^{26} - 15496454852335252976921990922240000W^{27} \\ & +2321314218573897816822946233750134980608C^6W^{25} - 8073247989421458143302236035560243200C^2W^{26} \\ & -39482489614628875887782711609266024022016C^8W^{24} - 424802716302333678740759278264560648192C^4W^{25} \\ & -1735206021274791232302799377766809600W^{26} + 66332562711673576802476441250803889405952C^6W^{24} \\ & -577144204510964229451765255930833272832C^2W^{25} - 646284097808268623713693498666945805623296C^8W^{23} \\ & -355167567833975599035429109213051944960C^4W^{24} - 114485467614913758164052011068566798336W^{25} \\ & +1001685421214548688016446673448197692190720C^6W^{23} - 26474981538422643230317562840295464435712C^2W^{24} \\ & -4256813634318212584047290998608973029956352C^8W^{22} + 45311583699461789865511435246158905143296C^4W^{23} \\ & -5011722588369208750787317192788028424192W^{24} + 4096206321950632197631246006638206514723840C^6W^{22} \\ & -811133001820439074092547284813482712772608C^2W^{23} + 47084992354086864268435271167333196246685696C^8W^{21} \\ & +16856648892456257991686263246052209226497536C^4W^{22} - 155439202828556185597820741141895056953344W^{23} \\ & -141927669269027389574323940125313782329798144C^6W^{21} - 17131974469901288600421025598236589414974464C^2W^{22} \\ & +1474181822420867518781484372578925868197791712C^8W^{20} + 330873786090851311235087764749990651364895232C^4W^{21} \\ & -3529860989119123308562375843610722960399104W^{22} - 3362552173571487064902767669421821154321234816C^6W^{20} \\ & -254322070754847165199624009801718299542391296C^2W^{21} + 18040520175705575237067925963269019936394127888C^8W^{19} \\ & +4139697851811679748696723081269926254116213056C^4W^{20} - 60059461869822911148149187919858372124379648W^{21} \\ & -39420977854623848598962674054148581601767892304C^6W^{19} - 2665846767912152594635380488889872275407368064C^2W^{20} \\ & +137956490827279639643940623183113107110223727941C^8W^{18} + 34797736555767614316775815716786574472335225744C^4W^{19} \\ & -775900607832576421952326761335850155822038560W^{20} - 301709043815136685273127141446402965471634786356C^6W^{18} \\ & -19384734496462494167834831947837026245804529264C^2W^{19} + 728388600761720715280359583834156099800025753515C^8W^{17} \\ & +196609345434614046191618539156976680385956033998C^4W^{18} - 7668532964180567419892537653829925423002822112W^{19} \\ & -1627353251434052869153508234047735060975742928780C^6W^{17} - 91030743651332573525935863053545406969521807380C^2W^{18} \\ & +2758434982740040783910696069599208737138955463855C^8W^{16} + 699916710279575187869251297131926952671415120210C^4W^{17} \\ & -58175879087075650417706746874741745862138220619W^{18} - 6390086546050277796046864900515053857552490285820C^6W^{16} \\ & -196129125180240291982651304825960811406705622700C^2W^{17} + 7615117083768234625928468860120566865337203061973C^8W^{15} \\ & +1108023989680316078518100165304304911733906193610C^4W^{16} - 338697945198158173935138848927234846565613583685W^{17} \\ & -18550785277959679693068164900289076682781227525508C^6W^{15} + 636584160474704182601983128033678216561787250340C^2W^{16} \\ & +1538773880227693557111208306975782281490650425846C^8W^{14} - 2797135429796952765770789551439748275642976641058C^4W^{15} \\ & -1507808882016071308491515811094698109293504120225W^{16} - 40053233941675100432401036412604705983729571901752C^6W^{14} \\ & +7458942387811926107424445252798789783991970380412C^2W^{15} + 22615784335942506360462012931853306179423670407086C^8W^{13} \\ & -23092877944552248930674165474160716421488709479740C^4W^{14} - 509281725082015516216974874570707952064253094699W^{15} \\ & -64224626763890950678399160706795498932389170280440C^6W^{13} + 33801164394812994851281578760722546065186596930632C^2W^{14} \\ & +23686976440122556307806697426721457462770337040382C^8W^{12} - 73917408474051711097026773192692676275847273130316C^4W^{13} \\ & -12865240436390587168884165043878249314879998473418W^{14} - 75742476372126489063168986802211043958032017440120C^6W^{12} \\ & +97298582124160477864619092515628503076000863017928C^2W^{13} + 16814814537196536385605160336706154679736696174394C^8W^{11} \\ & -148121411808897916465973650039377035863845560363436C^4W^{12} - 23647663754206372131237729915198122158281609392946W^{13} \\ & -64236464437412823888237405939240335236490903614296C^6W^{11} + 195151223128891252510476194807344983790468766965704C^2W^{12} \\ & +6947036218500336580079409588945651973128044446593C^8W^{10} - 203550105176030968645814781804134952978117843628212C^4W^{11} \\ & -2972644177342027630985075332881117080130895114146W^{12} - 37274684631291548745090339244142038619759406902244C^6W^{10} \\ & +280505354728598026935279946892939731121739729301896C^2W^{11} + 363433432352661210313147383161718676381760600727C^8W^9 \\ & -197325629564774463122645719037283358091003745599946C^4W^{10} - 20834448379229857467049326511058392485313846578102W^{11} \\ & -12896730269008990512336418397443049059564122794556C^6W^9 + 290427107926869611031385967931495696738029731970364C^2W^{10} \end{aligned} \quad (239)$$

$$\begin{aligned}
& \left(\begin{aligned}
& -1471632957973911525183803936816204344580207489661C^8W^8 - 135628572114794633417175004067631455242482882723174C^4W^9 \\
& +2819250902927425265796183212533002900892797982065W^{10} - 1029037809046573538249331786961837180304790929484C^6W^8 \\
& +214882122891261159196643139088892547918229502485284C^2W^9 - 984382724670777994880574585245847784127843004975C^4W^7 \\
& -65431193912526244023601720492346079083310341937918C^4W^8 + 25499723627163117431802294945573175084793373734247W^9 \\
& +1350305337432053038323193117507593943215426542700C^6W^7 + 111500769767792484936993495513191521662395949357780C^2W^8 \\
& -339804889381867437078302405204428234721847367500C^8W^6 - 21579742324707160950018424511880725781258018133530C^4W^7 \\
& +31606478988434720501796234862395016003566369373299W^8 + 731417163670468597907365483151843098719471450000C^6W^6 \\
& +39349596223780200876881198786155823765994818283180C^2W^7 - 71917433875978829412770849036081197893439500000C^8W^5 \\
& -4622081705589151754497220074437321558027558153000C^4W^6 + 22342343915837043478024008621414974989436217608625W^7 \\
& +187687344280331566102836772987719265512774000000C^6W^5 + 9002846526366350060400089389629719939284464354000C^2W^6 \\
& -9615014582548786791787464230905584896250000000C^8W^4 - 576684478712482734295871247710857927839310600000C^4W^5 \\
& +10148089910321188770462791981226119523569916244500W^6 + 28026503254106192101236838880623180277400000000C^6W^4 \\
& +1225917140827814249022437011145661440676399600000C^2W^5 - 804095167361332467290826647397681967500000000C^8W^3 \\
& -28493776353006821890423378827564641273260000000C^4W^4 + 2989652181789910673131984817752062702348504500000W^5 \\
& +251475646453033917237999051796234710000000000C^6W^3 + 77804704235059766824367900063297304207480000000C^2W^4 \\
& -40353069814429677538856163178695750000000000C^8W^2 + 1827944202282736219474400327382040371000000000C^4W^3 \\
& +549869482376814067657747103265412411760790000000W^4 + 132290517661171396527719058639453000000000000C^6W^2 \\
& -1350476502084394301678774724813938346000000000C^2W^3 - 1101365873241300228412848551400000000000000C^4W \\
& +318670193337502176597004257457493100000000000C^4W^2 + 575909710737279132343380869635312194725000000000W^3 \\
& +373007450321120069827757819280000000000000C^6W - 515628873859029233900974579638235800000000000C^2W^2 \\
& -12461488683259438824364440000000000000000C^8 + 1519920642177155435092186322088000000000000C^4W \\
& +2776702641578085834718301538146039850000000000W^2 + 4318297888949976012116112000000000000000C^6 \\
& -2753155969004719881596835193968000000000000C^2W + 24476761683029092022343172800000000000000C^4 \\
& +1813988142891307586392097374620000000000000W - 46743876700022234850648854400000000000000C^2 \\
& -1679728489762973441442362712000000000000000
\end{aligned} \right) \quad (240)
\end{aligned}$$

$$\begin{aligned}
b'_{6,2} = & 169869312(25W+1)(16W+1)^2(9W+4)(36W+25)^2((25W+49)(9W+1)^2((W+16)(9W+100)(25W+144)(144W+3025) \\
& \times (16W+49)^2(4W+49)(4W+1)^2(81W+100)^2(9W+49)^2((W+9)(81W+3025)(9W+25)^2(W+36)(9W+121)(W+49)(W+1)^2
\end{aligned} \quad (241)$$

$$a'_{6,4} = a'^{(1)}_{6,4} + a'^{(2)}_{6,4} \quad (242)$$

$$a'^{(1)}_{6,4} = \begin{aligned} &196928772416859170564997120000000C^8W^{28} - 206017792682252670744920064000000C^6W^{28} \\ &- 25428737718802041010862623948800000C^8W^{27} + 59308152438830314305355776000000C^4W^{28} \\ &+ 42744471758700991530907124367360000C^6W^{27} - 45414498565131159148167168000000C^2W^{28} \\ &- 538093507764062926859857406779392000C^8W^{26} - 3222735042212174531404584714240000C^4W^{27} \\ &- 30545928135788545337720832000000W^{28} + 7766095691098154024146407342794342400C^6W^{26} \\ &- 15966748920265183354506305863680000C^2W^{27} - 364181711439577199491059769057707601920C^8W^{25} \\ &+ 89576459705696572249592839579238400C^4W^{26} - 9906857042695606632263671480320000W^{27} \\ &+ 496171359249327437510750852500258996224C^6W^{25} - 2429277966643182478409409389966131200C^2W^{26} \\ &- 13429505953443445416907641493769559398400C^8W^{24} + 94561996389259456901339465863621238784C^4W^{25} \\ &- 1400813187493116768943376322817228800W^{26} + 16991750927518928835206209449555836952576C^6W^{24} \\ &- 200419574962309831697039031145966682112C^2W^{25} - 298533151748460419936501559161785211024640C^8W^{23} \\ &+ 8446201884865047983444115458391835471872C^4W^{24} - 115572299860015367360150435642127986688W^{25} \\ &+ 32867002622238930033315938301744712471552C^6W^{23} - 10107302298272530188699844613944702771200C^2W^{24} \\ &- 3764813780882906680012178771461667070988800C^8W^{22} + 369752680326625615218647949935547266302464C^4W^{23} \\ &- 6253465170833084441662447279797532809216W^{24} + 2665319733786800491999917110053221797849088C^6W^{22} \\ &- 332452763129883414724672101526623719402496C^2W^{23} - 12032723332367367889662191693410275105095760C^8W^{21} \\ &+ 9718462329680528253212967638983952744813568C^4W^{22} - 235952221725446812882890200021435561489664W^{23} \\ &- 34223411964715725872794408904415994672833216C^6W^{21} - 7332097682779696193096480891842102957774848C^2W^{22} \\ &+ 479981044652318598679828141455407422019953800C^8W^{20} + 162185239055383712197870490729932068960660000C^4W^{21} \\ &- 6435077322158039636867077269063417343480320W^{22} - 1393076383737583357232572690695107033074008864C^6W^{20} \\ &- 107246900449657292807267888733012027685747392C^2W^{21} + 10303147107889383660156852132550806829085682645C^8W^{19} \\ &+ 1665347013006244975853751083219824678060934448C^4W^{20} - 129597476335257506879069538046680964614446160W^{21} \\ &- 22112819038143278092494382052501726926195366740C^6W^{19} - 943931293606597486900295763178235131103295264C^2W^{20} \\ &+ 112778430566393415994387943678474192983741829400C^8W^{18} + 7942022226650543908079704137060085373162275902C^4W^{19} \\ &- 1949364859209363732520232518045309261842331512W^{20} - 221689733932551757055956803750594407927803405920C^6W^{18} \\ &- 2379991242381205282452681063918811039115431380C^2W^{19} + 806426161589457176573019740862308741892900626850C^8W^{17} \\ &- 39332042597121830401064409954802599112086676848C^4W^{18} - 21960745837216050114799087146887370207623665131W^{19} \\ &- 1546802385437171834071044072977698611252234919560C^6W^{17} + 58987188963574824046761885044569138245439646112C^2W^{18} \\ &+ 4016870659113995925836496373973916841029949846500C^8W^{16} - 1025891941620660403538534237491874333547382675380C^4W^{17} \\ &- 183989723022861921798852633688263318656154406632W^{18} - 7781439227952248929384432842447120674702442180240C^6W^{16} \\ &+ 973523602109376817241902343364761478780291916920C^2W^{17} + 14251951202305302045265924565743074249462966469395C^8W^{15} \\ &- 9158551981539076931717875139796831090682594789800C^4W^{16} - 1120517770764805680335222804889533018457784731102W^{17} \\ &- 28651934973745602506206985222391628819561626979404C^6W^{15} + 8225821319840628568348479843368526701359763764080C^2W^{16} \\ &+ 36172478718162386843705335267598292533412220152900C^8W^{14} - 50519780523003021846320297491846005147445338162894C^4W^{15} \\ &- 4671571071237500641425947311386840653612636064540W^{16} - 77624066623722630117952061118487081249661538356496C^6W^{14} \end{aligned} \quad (243)$$

$$\begin{aligned}
& \left(\begin{aligned}
& +46223648708203767110288906222737434143694741286452C^2W^{15} + 65010167157862771786081203695944543938718264993740C^8W^{13} \\
& -191158597483869650310415064582603268021644192351592C^4W^{14} - 10807817375419840487773142406654278246570831648557W^{15} \\
& -154415792592146801180246550449916776594820577039152C^6W^{13} + 184794777956787669602195236519691190343311106017520C^2W^{14} \\
& +80039396815360516678858078068046472937182405830800C^8W^{12} - 514400633609030795343290420500521083155864368808504C^4W^{13} \\
& +6317104727797110093092596483481260403680138606916W^{14} - 223076939855242821763538953924005518817330884436288C^6W^{12} \\
& +539102540948712471419200890365928540806951309004496C^2W^{13} + 61219340235386893314113681419586839003201877564235C^8W^{11} \\
& -998736392216465531093761391959591348487568516789408C^4W^{12} + 161108428026605013610926273338854924088748546890444W^{13} \\
& -227721578000406693704558025756645241304988204079404C^6W^{11} + 1158758495104145456116300437470223273214600025673408C^2W^{12} \\
& +17490829622968132437277845899944848462755645323200C^8W^{10} - 1404871018281076318415156027173160046035112875507070C^4W^{11} \\
& +687463410493312806016325898711317088726763231305360W^{12} - 153253390235198113702416814913032569977136072950016C^6W^{10} \\
& +1837358192978764287498593314073743715455730959450452C^2W^{11} - 18123218894780540115435996942194995182203390432670C^8W^9 \\
& -1427130265952400825101741401274973357321426158990208C^4W^{10} + 1729850934074653300193959346262197560704616994408715W^{11} \\
& -52752352376573235977881931154731642624396218513800C^6W^9 + 2138242973610256205538116325691284587832143629265664C^2W^{10} \\
& -26339261808392115581800130939352165125585202219900C^8W^8 - 1035164544265506920882408309793853755684859786976052C^4W^9 \\
& +2922039720300348884174686482278395953018951268665792W^{10} + 9764762263142555273669870864501745198654395243760C^6W^8 \\
& +1805044532735376527035709152541759288341725312334200C^2W^9 - 16827069702520184332556111850242344101518594881875C^8W^7 \\
& -523157548602080880374379802314504415090608504915432C^4W^8 + 3427964067020997053053992105146113062330252634012386W^9 \\
& +22218877468322886950877703076171125935184720903500C^6W^7 + 108287468226850848054014695121317873778735359999728C^2W^8 \\
& -655025972428725801191491360765739606229740937500C^8W^6 - 174500880514738553787743375273148102793201479107250C^4W^7 \\
& +2809586468109542645046773012140205673725951946021252W^8 + 12616344118204894481403627122638384397009254950000C^6W^6 \\
& +445966408322115697656639539265345006877380319750860C^2W^7 - 1646591204308357313378926183627426999707637500000C^8W^5 \\
& -32891624601187684717963014002927964586784241945000C^4W^6 + 1595628908837790104935414450816336408025398941035757W^7 \\
& +4000492693539622359439733360144076723750870000000C^6W^5 + 118376394123045651156434366098094546968599333606000C^2W^6 \\
& -267873039161318986254223352422712907378750000000C^8W^4 - 918335473689334921678348267900404149586465000000C^4W^5 \\
& +615073258649290959323634741232278007817213979721700W^6 + 77039036843433049003186249743348825684300000000C^6W^4 \\
& +17460175819488118861970826818634395541533897200000C^2W^5 - 27491145755501991031354901047779183937500000000C^8W^3 \\
& +1115933804424065585952986782276899918764300000000C^4W^4 + 154931641562879833287709164802540005722433127940000W^5 \\
& +8963686018819708156847311978436327655000000000C^6W^3 + 608975532063193805605214110039056760547320000000C^2W^4 \\
& -1685853079537732663559184632117418750000000000C^8W^2 + 26410834782237326684574294804983375797500000000C^4W^3 \\
& +23718806293429459519742668381567429366811714000000W^4 + 602773083557036466709311653480311500000000000C^6W^2 \\
& -219211509568059951215295937874446329402000000000C^2W^3 - 5533531673241376933946273956500000000000000C^8W \\
& +26209561066049677640752008283039375500000000000C^4W^2 + 1848694685834761847950815676842786608504100000000W^3 \\
& +21136643278648037980011288824400000000000000C^6W - 35364250811701400279253876862878760200000000000C^2W^2 \\
& -734096405660345773568523000000000000000000C^8 + 117834817911833746481234376299400000000000000C^4W \\
& +2509899934288449287039267161813947021000000000W^2 + 29363856226413830942740920000000000000000C^6 \\
& -205089700205191104588078353908656000000000000C^2W + 1926004109280939394714615608000000000000000C^4 \\
& -4775021279489888215199606538011592000000000000W - 396721185433310844816570020160000000000000C^2 \\
& -183242472916276637913558223915200000000000000
\end{aligned} \right)
\end{aligned}$$

(244)

$$\begin{aligned}
b'_{6,4} = & 905969664(25W+1)(16W+1)(9W+4)(36W+25)^2(25W+49)(9W+1)^2(W+16)(9W+100)(25W+144)(144W+3025)(4W+49) \\
& \times (4W+1)^2(81W+100)^2(9W+49)^2(W+9)^2(81W+3025)(9W+25)^2(4W+169)(W+36)(9W+121)(W+49)(W+1)^2(16W+49)
\end{aligned}
\tag{245}$$

(246)

$$\begin{aligned}
& (12867439161645397742832026910720000C^4W^{24} - 8867223105798680438479257600000000C^2W^{24} \\
& - 2319890860755432830514596145109401600C^4W^{23} + 540019112605203037563112079360000W^{24} \\
& + 1946157408486042167662132608368640000C^2W^{23} - 383855822667007230786225701083965554688C^4W^{22} \\
& + 1851278746891662656712550464041779200W^{23} + 345252697988888973942729751536127180800C^2W^{22} \\
& - 2163827596405606955513998295557726437376C^4W^{21} + 232680025679284939514295291060256505856W^{22} \\
& + 21203171133796694345793515871504990142464C^2W^{21} - 635324437453620390130513334795968887727104C^4W^{20} \\
& + 15495997678661161693494748968310275932160W^{21} + 689744015752225107734096847815264371531776C^2W^{20} \\
& - 10141674888097283078589871846554355657992768C^4W^{19} + 630489429834878526658970181907052081267712W^{20} \\
& + 12846187429266095508788235849236314104637056C^2W^{19} - 61309669365044086330868191421347961261321328C^4W^{18} \\
& + 16754053825374734543570380809475780549233600W^{19} + 119938932415723551843822268809989633763387744C^2W^{18} \\
& + 838482106717124123403113467149488387761551204C^4W^{17} + 298528140629809734081370299844021133256988944W^{18} \\
& - 170946558766900952109328087351751484419516520C^2W^{17} + 23705895737454661847183991129279960121710490455C^4W^{16} \\
& + 3526940289858149947064259644184701516308400772W^{17} - 20610873807270483983813128742558763856977894510C^2W^{16} \\
& + 277637403460628943635686850674867024641411212535C^4W^{15} + 25284292683807064800726732986249821935621793335W^{16} \\
& - 29531537934770557941512529687382452211060078030C^2W^{15} + 2032786345495885796571633326340762296205930889320C^4W^{14} \\
& + 61845853099019640106218075806906485922234235W^{15} - 2411185848087466797378771061152190121699917426320C^2W^{14} \\
& + 10193084461616865888192487025058807113753918015940C^4W^{13} - 780558965340649194140411541840365092367456761560W^{14} \\
& - 13073690413488322752235121977409231782288035908040C^2W^{13} + 36175060070410525028073680206150479060793394120898C^4W^{12} \\
& - 1048286271875793983207822095955950530257386505340W^{13} - 49549123204911542654566328434825715242870699955460C^2W^{12} \\
& + 91814876078494038720628806640917877543463138236146C^4W^{11} - 66618767996977076759014070960108210303503277083646W^{12} \\
& - 134137775399047788129584922545783399210237006130084C^2W^{11} + 165945216012335794474299446819546530925040765747204C^4W^{10} \\
& - 271641426891044719311098228257892951827103800866190W^{11} - 261161555936601328138960037790256598434408507656456C^2W^{10} \\
& + 209260940142367796547135417473814351326322402804768C^4W^9 - 762317212321094345853877932140852627344057658507132W^{10} \\
& - 363728290199106475786443438224113000250316876180576C^2W^9 + 175116063185156703601712148584892381309169203108543C^4W^8 \\
& - 1505414103845673216803902563015414467648894629604800W^9 - 354731110707062432456673237134356728759190971738174C^2W^8 \\
& + 84336305302306255703930546486974168033919319311151C^4W^7 - 2092315966807982602922422019519343758657679168389729W^8 \\
& - 229852223638741360154412202517489343560329341486270C^2W^7 + 815438031364184193357724796819927966108577526700C^4W^6 \\
& - 2027942316750341661988566613450992350820899682136657W^7 - 85181448601088616370929066681319259951567687429400C^2W^6 \\
& - 16835020379607139551002454966498872111342577300000C^4W^5 - 1336628342257832394933311165680033138808643337973780W^6 \\
& - 5865226723582657333794715226624999583087444120000C^2W^5 - 113213369797357108828988354374574783825059980000000C^4W^4 \\
& - 572093733150258884123027809122443732001453083476000W^5 + 98931891690737290620992446472118902640787800000000C^2W^4 \\
& - 3416919680969929954988821410872649896133000000000C^4W^3 - 144506433434086454458613524774155806524759521200000W^4 \\
& + 4587516903195742053855398811507461006807400000000C^2W^3 - 53260142574133240965568450636043192145000000000C^4W^2 \\
& - 16088619944899061301355470409406186260430580000000W^3 + 865902050883365593429164635849716597620000000000C^2W^2 \\
& - 39591919031275851947376379860022488000000000000C^4W + 763945297901874022728202954368052498038000000000W^2 \\
& + 71386583885801786025514725950600208000000000000C^2W - 107174449510617359452384310286240000000000000C^4 \\
& + 331315132605828863903989237961235950400000000000W + 204697772090072223769058042387520000000000000C^2 \\
& + 19501373512937699292743040432899520000000000000
\end{aligned}$$

$$\begin{aligned}
b'_{6,6} = & 1704445234642944(16W+1)(9W+4)(36W+25)(25W+49)(9W+1)(W+16)(9W+100)(25W+144)(144W+3025) \\
& \times (16W+49)^2(4W+49)(4W+1)^2(81W+100)^2(9W+49)^2(W+9)(81W+3025)(9W+25)^2(W+36)(9W+121)(W+49)(W+1)^2
\end{aligned} \tag{247}$$

$$\begin{aligned}
& \left(\begin{aligned}
& 14888419878954885120000C^4W^{21} - 2290526135223828480000C^2W^{21} - 1964730110484470488473600C^4W^{20} \\
& -1492312482039767040000W^{21} + 1065011373873299574374400C^2W^{20} - 401284055037377658462421248C^4W^{19} \\
& +1936949348998726209331200W^{20} + 160540410640181538769684992C^2W^{19} - 26389142460889817714352149760C^4W^{18} \\
& +409860470510866645950134016W^{19} + 9219890423044988596273250304C^2W^{18} - 939284837006849574248999657952C^4W^{17} \\
& +35599335697990217264837702400W^{18} + 268615285664216421374499933504C^2W^{17} - 19828607765809418006684202789840C^4W^{16} \\
& +1738832924796850027758652024608W^{17} + 3456408092685001552225062682560C^2W^{16} - 225633578547792881592592052989545C^4W^{15} \\
& +52775074214841148376021609820336W^{16} - 26966968855281865574293155642990C^2W^{15} \\
& -211154373754890225528842557991550C^4W^{14} + 1013757698214471716542817954653455W^{15} \\
& -2005795273627094628685578165394020C^2W^{14} + 37564293528095688380765658036295035C^4W^{13} \\
& +11368231365043021619291435901623250W^{14} - 41917917930447073611276762336743670C^2W^{13} \\
& +673293946250092415769116742869880300C^4W^{12} + 37955703448827583854319654242163155W^{13} \\
& -530657941905928101965691259969608600C^2W^{12} + 6508293690957053411376437977388490030C^4W^{11} \\
& -968002774091629177419554976027175860W^{12} - 4583038994267282360356455749445728220C^2W^{11} \\
& +40512922863098652545234028802338066300C^4W^{10} - 18789213778343117277854002777427953890W^{11} \\
& -28016558481794596398536717889170963640C^2W^{10} + 169077649624499562538411432790331220638C^4W^9 \\
& -176864832871593634975577037799437426980W^{10} - 122425877826442083564322196583480154812C^2W^9 \\
& +472113485785922366652204901163881268160C^4W^8 - 1055264387031906997150138209970260145266W^9 \\
& -380729951222710779733315584267070071264C^2W^8 + 85174910292555416819208502982244642667C^4W^7 \\
& -4229752533315330196724174104113862693440W^8 - 828558957888892529833074265802317932054C^2W^7 \\
& +898835156211079736507256380907197477490C^4W^6 - 11497874038297528085850958261107558403453W^7 \\
& -1218326211099356390284302400025276044740C^2W^6 + 361564924313236640763962569278580050375C^4W^5 \\
& -20883427371767154831249184108114549673406W^6 - 1122817776594888050923163861134262436750C^2W^5 \\
& -276649572267359188748641631721668287500C^4W^4 - 24365662249402437269345118540709552682625W^5 \\
& -520564445683194677821724306435868225000C^2W^4 - 410722074474793205516190709899738750000C^4W^3 \\
& -16789196601730315112878317174736344507500W^4 + 26620892868824424572073623726197500000C^2W^3 \\
& -190574337717396215547032444000625000000C^4W^2 - 5409364099289734492963551162011088750000W^3 \\
& +145159951854709966779877570625250000000C^2W^2 - 35401934289381547387726125900000000000C^4W \\
& +171130002451140999243738490381575000000W^2 + 5299166475166542056520569820000000000C^2W \\
& -2097418301886702210195780000000000000C^4 + 510317664028965340804455714900000000000W \\
& +4194836603773404420391560000000000000C^2 + 70924234227552960366428316000000000000
\end{aligned} \right)
\end{aligned}$$

(248)

$$b'_{6,8} = 452984832(36W+25)(25W+49)(9W+1)(W+16)(9W+100)(25W+144)(144W+3025)(4W+49)(4W+1) \quad (249)$$

$$\times(9W+49)^2(W+9)^2(81W+3025)(9W+25)^2(4W+169)(W+36)(9W+121)(W+49)(W+1)^2(81W+100)(16W+49) \quad (250)$$

$$\begin{aligned}
& \left(\begin{aligned}
& 17631936921600W^{13} - 3479369430057984W^{12} - 468779072131704384W^{11} \\
& -21257782429691034540W^{10} - 451337251789817958663W^9 - 3481265658459768201936W^8 \\
& +38513881690408788404078W^7 + 1124201838064982202361128W^6 + 10767315903720498457241169W^5 \\
& +51012844957022980603686332W^4 + 115605414989264562919633200W^3 + 89682091490174472059400000W^2 \\
& -25950638671831776672000000W-24455043258355987200000000
\end{aligned} \right)
\end{aligned}$$

$$b'_{6,10} = 37748736(W+16)(9W+100)(25W+144)(144W+3025)(4W+49)(4W+1)(9W+49) \quad (251)$$

$$\times(W+9)(81W+3025)(9W+25)(W+36)(9W+121)(W+49)(W+1)(81W+100)(16W+49) \quad (252)$$

$$a'_{6,12} = 1001 \left(\begin{aligned} & 8748W^6 - 1956285W^5 - 88234296W^4 - 1031036150W^3 \\ & +13334326648W^2 + 316813054635W - 240364624500 \end{aligned} \right) \quad (253)$$

$$b'_{6,12} = 100663296(9W+49)(W+9)(81W+3025)(9W+25)(4W+169)(W+36)(9W+121)(W+49)(W+1) \quad (253)$$

$$a''_{6,0} = a''_{6,0}^{(1)} + a''_{6,0}^{(2)} \quad (254)$$

$$\begin{aligned} & \left(\begin{aligned} & 12966503533620356909629440000000C^{12}W^{26} - 36774998868044812250618265600000C^{10}W^{26} \\ & + 1144614097423073543124418560000000C^{12}W^{25} + 37908434491613025267194265600000C^8W^{26} \\ & - 3025584355484054369642899046400000C^{10}W^{25} - 17226810980571952964645683200000C^6W^{26} \\ & + 23623770070449454660367865348096000C^{12}W^{24} + 2808569221474849397806706196480000C^8W^{25} \\ & + 3376184482068444446352998400000C^4W^{26} - 32529289609967473962152509809623040C^{10}W^{24} \\ & - 1114597084668860993164667781120000C^6W^{25} - 253138273780228785942036480000C^2W^{26} \\ & - 1358472523495984927909815729881088000C^{12}W^{23} - 9494394228510482022865744374005760C^8W^{24} \\ & + 212269255047170055225150013440000C^4W^{25} + 3825615095167378029281280000W^{26} \\ & + 6168077701349999558935190310158008320C^{10}W^3 + 207863345438441963716451442360320C^6W^{24} \\ & - 27007828335571365159594098688000C^2W^{25} - 105476840984653666264671853060606464000C^{12}W^{22} \\ & - 8793889401674229016365634418063769600C^8W^{23} - 5311452294726535848047829975040C^4W^{24} \\ & + 719551520421824666479362048000W^{25} + 387437181893028933850858901796887915520C^{10}W^{22} \\ & + 4320542428739755315570397692489236480C^6W^{23} - 2558891936232969906604607956058112C^2W^{24} \\ & - 3438291662190602143575716595027532032000C^{12}W^{21} - 467540796822922540460796021255305733120C^8W^{22} \\ & - 106265810513089291616207719647805440C^4W^{23} + 89038276054132882745494753247232W^{24} \\ & + 11889466892923026476418531425400531306240C^{10}W^{21} + 180726837227236799310513024786095216640C^6W^{22} \\ & - 246565710939198143056819078983647232C^2W^{23} - 67891510165125719092158130024391879520000C^{12}W^{20} \\ & - 12990310726390719598181498936025236486400C^8W^{21} + 20063050391858198446438104807331153920C^4W^{22} \\ & + 7083757797513861666132803028713472W^{23} + 227502361143005130109469404735291298436480C^{10}W^{20} \\ & + 3467369815744153405937237217241161408000C^6W^{21} - 16015038605538334287754948486173656064C^2W^{22} \\ & - 878552658645981231593678947800978359232000C^{12}W^{19} - 223170307438513458915722678953228062468480C^8W^{20} \\ & + 1703484431873504003136142860554671787520C^4W^{21} + 348635106742980585140042295297567744W^{22} \\ & + 2889757495205679148294990649968493246331360C^{10}W^{19} + 1835345326874495445692769651316997772800C^6W^{20} \\ & - 658936212281148563363863487755588011264C^2W^{21} - 7526298716016980974891289582811973573302000C^{12}W^{18} \\ & - 2433516400737945208773011702048361960794400C^8W^{19} + 62430724374325765773791126067083443438080C^4W^{20} \\ & + 10601178485861150913600882107398681344W^{21} + 24503795599945326062714692483334622136142940C^{10}W^{18} \\ & - 630214062855920900552985116986520668376640C^6W^{19} - 17879943378900816984982849198161958555776C^2W^{20} \\ & - 40873123434740419106798287889777095166733000C^{12}W^{17} - 15574560887098779930033219906258159782177940C^8W^{18} \\ & + 1373104503830443791516908009130971449663680C^4W^{19} + 189006210075161104947190924946269439616W^{20} \\ & + 132456637454999930919630786814322549780881755C^{10}W^{17} - 17267841541139206099514032378587205539246600C^6W^{18} \\ & - 331979584478831630279714905733919380324256C^2W^{19} - 116269705631204800736820485601931707896870625C^{12}W^{16} \\ & - 32830329827551623192063342196960013581054725C^8W^{17} + 20028329714726553255940525924121695493036040C^4W^{18} \\ & + 1335999233067475436687505703265623073376W^{19} + 374174046590031124656780047550115627386261450C^{10}W^{16} \\ & - 222648315718460157537031028025304720011799290C^6W^{17} - 4305771191037764599788730910795341758720468C^2W^{18} \\ & + 54409031082971757985083632188497888553940625C^{12}W^{15} + 348280937060635123718073518873325721384551825C^8W^{16} \\ & + 201849647864990086399880790109147844674878390C^4W^{17} - 21739334890810726286373332281081808208132W^{18} \\ & - 226238900247418741146434882401747057698420600C^{10}W^{15} - 1793597384903595118376724722089925422176955860C^6W^{16} \\ & - 39165494412134837646328202820826792297367553C^2W^{17} + 1940649448897038224491029973024119175458592125C^{12}W^{14} \\ & + 3744309322109323321296290064955900461012530625C^8W^{15} + 1427340574533729393113401811985508894065480225C^4W^{16} \\ & - 757474176027378803433641562138996818334537W^{17} - 6790630146886167534880528431948165053127786510C^{10}W^{14} \\ & - 9645572281472076324263444067021862086558973800C^6W^{15} - 245720745447985227089383383720438277816864950C^2W^{16} \\ & + 8517672170426286626997905897193314202278802375C^{12}W^{13} + 18265631042085135986315829777866772230507950935C^8W^{14} \\ & + 7079192974640365414894628204995728284931951075C^4W^{15} - 11102882716855743060999247531011747259867665W^{16} \\ & - 30431886063711445451029052943965463960402306045C^{10}W^{13} - 35249730609924816929559639000052578871913084820C^6W^{14} \\ & - 1003232011516931906722880083561568974042523040C^2W^{15} + 21010925279579169121659452290554365059623935250C^{12}W^{12} \\ & + 54678112891887177865479643256181068574548351850C^8W^{13} + 24238727383845951544326930020983532790258731115C^4W^{14} \\ & - 103141533526073680013719484738509229362383285W^{15} - 77898766284844818376549172776696664362166912820C^{10}W^{12} \\ & - 8702745524757340898788600333477967966209815730C^6W^{13} - 2129230903749021991783784174060656723547624478C^2W^{14} \\ & + 3376086529771075186841062570575096461915848250C^{12}W^{11} + 107617977470601312548127926153952949265050484670C^8W^{12} \end{aligned} \right) \quad (255)$$

$$\begin{aligned}
& \left(+54762166673686006331375657225906438587876786515C^4W^{13} - 663791314484979516450835812767356492989469567W^{14} \right. \\
& -131514564374732385657306985349099492976033150840C^{10}W^{11} - 140045617357307481213707316194435631109656066840C^6W^{12} \\
& +1721021932999945476992912961503198048999617167C^2W^{13} + 37037314821435831887804077938424145351847146250C^{12}W^{10} \\
& +141926812722198710505707116227654293162565469650C^8W^{11} + 7044082220891868673733713312363845382844566030C^4W^{12} \\
& -3057630037353937905428274559765865403263717732W^{13} - 153407525305021042038567632335192868624640797280C^{10}W^{10} \\
& -12956504000853514981020177651387402236791543200C^6W^{11} + 28912209644223204755649763820744387837574654924C^2W^{12} \\
& +28255578865910675076304183701797264333507415750C^{12}W^9 + 124164217039210258460254953844426374670794001530C^8W^{10} \\
& +10744322111881422262574573410167765292795802430C^4W^{11} - 10211122941489942984471152344435611625811842654W^{12} \\
& -125848714405306075966976322873421148029480564335C^{10}W^9 - 25002207981980195481862489569259696583188249200C^6W^{10} \\
& +104206845534404142177887162112706347735849120024C^2W^{11} + 15072293677855166427630580361052147023691373875C^{12}W^8 \\
& +67803706755190784472878897702614918673132651775C^8W^9 - 141594328247061401089675282699470128901223148130C^4W^{10} \\
& -24785954179513125565547342825467139486405620554W^{11} - 72879417340240116594868331433362670647701332990C^{10}W^8 \\
& +101329900853409179745539758203922337364366623490C^6W^9 + 218127137499317114802738359511206161228942646736C^2W^{10} \\
& +5608336740916370714348726335911774209047531125C^{12}W^7 + 17343401403174158642525913660845522167835806365C^8W^8 \\
& -286422754480323664937514508739882184418028854980C^4W^9 - 43517345854676192526525240214861290796363822626W^{10} \\
& -29619696773506817162334762634377272543681226480C^{10}W^7 + 148898542665476009998336987097446558274206861500C^6W^8 \\
& +297501766606578570179981781739473203543491065741C^2W^9 + 1443905749009119856984327305342941379479825625C^{12}W^6 \\
& -394506799700930121279591893566578697061116275C^8W^7 - 298352781127074967451033496202913019682217153715C^4W^8 \\
& -54624863804342577196898466533122152967412694561W^9 - 8333805438556513174543823708654596243749828750C^{10}W^6 \\
& +109524262943079378200628805226530940934861754440C^6W^7 + 270138914034387326224463697929671769119128674178C^2W^8 \\
& +253798660804112160654210008329129575981796875C^{12}W^5 - 52279446189658480470020562427501640982572525C^8W^6 \\
& -18888796603237608732524501848371539541890716065C^4W^7 - 48108019063452480498498313035808729339236668253W^8 \\
& -1590955914789489712733388962682760932249459375C^{10}W^5 + 49557098006752320680827092229707923595609642060C^6W^6 \\
& +161180911671518525845914334431514407057059925288C^2W^7 + 29855678736785024200305743635741875942187500C^{12}W^4 \\
& -2057137924552725418468025146964869582396012500C^8W^5 - 73691109611413925376845418850264932638647841025C^4W^6 \\
& -28896070545401090085504512497793058724449570273W^7 - 200934237473784902938926686912857716496875000C^{10}W^4 \\
& +14277847389886105187160475708663354114142606250C^6W^5 + 60576562512627080331448728932687657590569659010C^2W^6 \\
& +2279345866093365984374463250449546093750000C^{12}W^3 - 429142508357182633023669938212942410206437500C^8W^4 \\
& -1692184913667318361388297979300383769228183125C^4W^5 - 11306119272789928969448192515623087212058993195W^6 \\
& -16252657658708883008774351817130574625000000C^{10}W^3 + 2594887562956704891809992344556315987324950000C^6W^4 \\
& +12976511534918239532111449051413582191465833125C^2W^5 + 107098344738232437087577187186203125000000C^{12}W^2 \\
& -50136076562517069060336258886827933656250000C^8W^3 - 2017021845689100374216523450980795325391687500C^4W^4 \\
& -2627146239419750023757363700224704598861445250W^5 - 799060609182532674751459880191593750000000C^{10}W^2 \\
& +288706453944930456398052207256526388000000000C^6W^3 + 1091907052874574227749491559839950554382325000C^2W^4 \\
& +2786004675351805156694210390625000000000C^{12}W - 3227841405173425230335705670342973125000000C^8W^2 \\
& -73479576283581741566698542144722709618750000C^4W^3 - 267610212648798636700520442143738091997662500W^4 \\
& -21523721343137665700164814702812500000000C^{10}W + 186858610863403644311864247288598455000000000C^6W^2 \\
& -111631017345990622947738466409955402375000000C^2W^3 + 30432302802454794975483750000000000000C^{12} \\
& -1059623574333311973789411696309375000000000C^8W + 7054077324669343863376461776374606875000000C^4W^2 \\
& +17535330445887599508164518079441797861250000W^3 - 241415644191359040170893125000000000000C^{10} \\
& +634536974658634772984128756569375000000000C^6W - 32539159089483882362121855969360555750000000C^2W^2 \\
& -1374032881495632720798911475000000000000C^8 + 721516894901370383371615088048250000000000C^4W \\
& +7189214755978057917057286274663731125000000W^2 + 8594463245688527358183021990000000000000C^6 \\
& -2525088757921287754455482136822562500000000C^2W + 1799394139936790144927279100000000000000C^4 \\
& +620260723581912586619095247837662500000000W - 6515361543845055952777144918500000000000C^2 \\
& +17063579319814914080932185525000000000000
\end{aligned}$$

(256)

$$\begin{aligned}
b_{6,0}'' &= 424673280(25W+1)(16W+1)^2(9W+4)(9W+1)^2(25W+49)(36W+25)^2(81W+100)(4W+49)(4W+1)^2(16W+49) \\
&\times(9W+100)(144W+3025)(W+16)(9W+49)^2(W+36)(W+9)^2(9W+25)^2(4W+169)(9W+121)(W+49)(W+1)^2(81W+3025)
\end{aligned}$$

(257)

$$a''_{6,2} = a''_{6,2}^{(1)} + a''_{6,2}^{(2)} \quad (258)$$

$$(259)$$

$$a''_{6,2}^{(1)} = - \left(\begin{aligned} &55665199669832192213039185920000000C^8W^{27} - 77236474433032765973389639680000000C^6W^{27} \\ &+ 432318453727670530711466619699200000C^8W^{26} + 27460878287810664965603328000000000C^4W^{27} \\ &- 4778476562726065775043267054796800000C^6W^{26} - 2956051920465834094110965760000000C^2W^{27} \\ &+ 3510165840889250729541860219158528000C^8W^{25} + 991906534962629361809521901568000000C^4W^{26} \\ &+ 116605266242701027801300992000000W^{27} + 125901485257896559676471778437234688000C^6W^{25} \\ &- 252729498473859662126651120025600000C^2W^{26} - 8427298646582885348665383635593905438720C^8W^{24} \\ &- 125235772860273404965742724710400000000C^4W^{25} + 12317442875240876478145883013120000W^{26} \\ &+ 22628692748713249709886369642125005946880C^6W^{24} - 20003795764814997742680058792771584000C^2W^{25} \\ &- 470128295068281290539406879702105600102400C^8W^{23} - 11488627658712680996826646319189085388800C^4W^{24} \\ &+ 25242568486580527328457226898636800W^{25} + 1049057829153496383630614230046952338472960C^6W^{23} \\ &- 1921384701772882394891540821308974039040C^2W^{24} - 12872736724775057172686150935064979383543040C^8W^{22} \\ &- 421760034328741895867613674427321815162880C^4W^{23} - 38878469710480938246296617056867581952W^{24} \\ &+ 27194432843531452741683091242574470206745600C^6W^{22} - 122544820159094878350555670909510583992320C^2W^{23} \\ &- 21898903398907300390429874421689567478643200C^8W^{21} - 8380404803935701581458403363917619583198720C^4W^{22} \\ &- 3908892762943952134789162243458999521280W^{23} + 454921104646250813365985345276512523171450880C^6W^{21} \\ &- 481845045307259977333392037774720019850240C^2W^{22} - 2484664935975105868652179084870557928418926560C^8W^{20} \\ &- 86466070198515431574033572488012556945495040C^4W^{21} - 188924665968241022647280189219958606691584W^{22} \\ &+ 5185592399840085188525196285629149472145874560C^6W^{20} - 124186196557290252661240171970812458756464640C^2W^{21} \\ &- 192176998392314193483487776825922468562158400C^8W^{19} - 101683620939202204962476779010911621774094400C^4W^{20} \\ &- 5852314362988552021245042941331321907438080W^{21} + 4108103602352956166207117466930499615186143360C^6W^{19} \\ &- 221083387345133148974747930662833900451877760C^2W^{20} - 99899018698867150097717167353451750840627951605C^8W^{18} \\ &+ 1028795962330258631205305799114960093729156480C^4W^{19} - 126309004952107317248486480087206473196851936W^{20} \\ &+ 224579963422500169912245022800392628851933389260C^6W^{18} - 28155293627613204864812717713983627971969435520C^2W^{19} \\ &- 32001891136797355164182901652846569563662652875C^8W^{17} + 163472906058672504358479465589892675042387998170C^4W^{18} \\ &- 1974953084228877191462919132740864358914306880W^{19} + 802890982860639678784650926595702187458017504100C^6W^{17} \\ &- 26229732112372613576083541689308573278810552260C^2W^{18} - 370476951261620188039714618410484972361776436775C^8W^{16} \\ &+ 1408149357616095273682698544903547132164524445590C^4W^{17} - 22877956472760534404321390670562814857618298733W^{18} \\ &+ 1473737073206744791600291829269308575474896727700C^6W^{16} - 1812164231530865709330175253809847693347226920620C^2W^{17} \\ &+ 1878583934740018696766556100192531408124820904875C^8W^{15} + 8019705194451027226191933422403050137394612386350C^4W^{16} \\ &- 199097516149473012406445457072324025283606744835W^{17} - 1565829913662589736178713746108351803703118076900C^6W^{15} \\ &- 9353975682533140233168887746316920040721709487100C^2W^{16} + 11928366908371454675608833666397634729295297141570C^8W^{14} \\ &+ 31963517304457953783028017348709965576543494183850C^4W^{15} - 1313124828576643173379841149567460164141721725215W^{16} \\ &- 1993480240285683520774105122344884099623942255880C^6W^{14} - 36157938073906159843641122411101428106342670841300C^2W^{15} \\ &+ 35701963362703616721375457774251840631681775221650C^8W^{13} + 90869111621300481436397516275506585423314159109900C^4W^{14} \\ &- 6598122273456319192594514455428132367516341642525W^{15} - 69365491105294117295209484557515176322031796459160C^6W^{13} \\ &- 104384128337638606586349528095684145417365717489160C^2W^{14} + 69654141456954998396445876124155663103001301775890C^8W^{12} \\ &+ 184286742303682575189679727355908001782643644066780C^4W^{13} - 25326441825241691920615064804195501572690205643838W^{14} \\ &- 148705246577525130969436913910863956456675673539800C^6W^{12} - 223001007012142798923691946961755822820657713069880C^2W^{13} \\ &+ 95496560155609243565279565656165082873934602676950C^8W^{11} + 261825609860737188139815108209313784449898335765020C^4W^{12} \\ &- 743034564796602347411078232282312898520544480670W^{13} - 220979881459030302515277144910280018575679363831880C^6W^{11} \\ &- 345695464775076227049705105147313834166306644352760C^2W^{12} + 9440766560707079795637028829641877752413733921535C^8W^{10} \\ &+ 247194129347332396108139607245780254584420226649940C^4W^{11} - 166367137717501458185164457853368464331475496237406W^{12} \\ &- 236041143753846180275841291541506206545392608291460C^6W^{10} - 373024744133052008970592952653528364567434952837160C^2W^{11} \\ &+ 67791954903791030134557406220270432086657348646025C^8W^9 + 12973168242180188212703768487889704723752747752850C^4W^{10} \\ &- 283182925894246618489068409119436848946216638480570W^{11} - 183067122616983270169492390779088751807602499163660C^6W^9 \\ &- 251766035926705068428408531046165394649163854206740C^2W^{10} + 35259393253483980739152915221654053343631795935205C^8W^8 \\ &- 3084921660160120439273072677431843793438293434130C^4W^9 - 363918297504465524114845582469381653285147755886329W^{10} \\ &- 102748645171761317785794871055296651228946918356860C^6W^8 - 62792071563191794765145705439379873420515002687580C^2W^9 \\ &+ 13147569264150179336919227129884382698939325079375C^8W^7 - 64142214213637548485860692424433100272759108411370C^4W^8 \end{aligned} \right)$$

$$\begin{aligned}
& \left(\begin{aligned}
& -349155837477113788238507171526352645188874579255695W^9 - 41206382503151215999076632566897869928356986727700C^6W^7 \\
& +5843435114579469512805833244777559085539298475060C^2W^8 + 3456516474469535293046217927748565628729834322500C^8W^6 \\
& -54211188415880223760602214902401951413724802610590C^4W^7 - 245726997760424941773322270609085667703873623987507W^8 \\
& -11571800858455541483466544942324015882346449440000C^6W^6 + 72030492578555937868664197037321359303077033433020C^2W^7 \\
& +626456930450796684408495896467386982306852500000C^8W^5 - 24810472598773707951209908209763011303105809919000C^4W^6 \\
& -123314737285993403184435586002924944606533426466265W^7 - 2217739382391081500546425794022961951910504000000C^6W^5 \\
& +37942758094512110269251029120377893786125908980000C^2W^6 + 76128918857956386674025416420753577744450000000C^8W^4 \\
& -70357756595153007909797052252578975617173561800000C^4W^5 - 42021420592539981011544010421963821068334010215500W^6 \\
& -281726102016937300737661252981790891401800000000C^6W^4 + 11658595074533900829827898745512550416865574400000C^2W^5 \\
& +5987529602542228544279613598986816637500000000C^8W^3 - 1255849121507925378407486487865346983970380000000C^4W^4 \\
& -8782364298440225906345620261356367793063932300000W^5 - 22918023879739620658228396751757289110000000000C^6W^3 \\
& +2195777830065206278039499807979763921300200000000C^2W^4 + 288480371235767373687678788486294250000000000C^8W^2 \\
& -138062602018380602740672283998156251787000000000C^4W^3 - 795004682704029043842485100327227506213910000000W^4 \\
& -1132268990317688169956317419615042000000000000C^6W^2 + 2518957031582395422714260939655092991060000000000C^2W^3 \\
& +765779698903958115891692616900000000000000C^8W - 8890565912147288602026512408207756700000000000C^4W^2 \\
& +68983361002673025785029173066336730625500000000W^3 - 306185954351353891034695783308000000000000C^6W \\
& +16835374528673068351561402831727952000000000000C^2W^2 + 8498628808533786852515124000000000000000C^8 \\
& -301780609240007352005454876255360000000000000C^4W + 23705929050037756348653206562828110850000000000W^2 \\
& -344470041779867036266131360000000000000000C^6 + 58988983449561604930929644053848000000000000C^2W \\
& -409465508836517368246604577600000000000000C^4 + 197541159601050179973124881831324000000000000W \\
& +8208959038714107638267798880000000000000000C^2 + 5315111978705071740484497856800000000000000
\end{aligned} \right)
\end{aligned}$$

(260)

$$\begin{aligned}
b_{6,2}'' &= 424673280(25W+1)(16W+1)^2(9W+4)(9W+1)^2(25W+49)(36W+25)^2(81W+100)^2(4W+49)(25W+144)(4W+1)^2 \\
& (16W+49)^2(9W+100)(144W+3025)(W+16)(9W+49)^2(W+36)(W+9)(9W+25)^2(9W+121)(W+49)(W+1)^2(81W+3025)
\end{aligned}$$

(261)

$$a''_{6,4} = a''^{(1)}_{6,4} + a''^{(2)}_{6,4} \quad (262)$$

$$(263)$$

$$a''^{(1)}_{6,4} = - \left(\begin{aligned} &328214620694765284274995200000000C^8W^{27} - 389817980271321229938917376000000C^6W^{27} \\ &+ 31248250377855610420919549952000000C^8W^{26} + 66469198708534284144082944000000C^4W^{27} \\ &- 32147166918725407190931253493760000C^6W^{26} + 3325500119549136990633984000000C^2W^{27} \\ &+ 801011560844442680493165724999680000C^8W^{25} + 111046945980803700487516323840000C^4W^{26} \\ &+ 1858024825777892052172800000000W^{27} - 141324125131947300095599839520358400C^6W^{25} \\ &- 504984020843795535435403100160000C^2W^{26} - 30047779945035666969867507619747123200C^8W^{24} \\ &- 507965307840170630293252793558630400C^4W^{25} - 533162727510261572672252800000W^{26} \\ &+ 93190144599834626566706778450260566016C^8W^{24} - 312693555164767953444611428132454400C^2W^{25} \\ &- 2904269832849764427786456220583156275200C^8W^{23} - 26472009426315005202868721706843193344C^4W^{24} \\ &- 6132641541746106535450532770304000W^{25} + 5802162125222087603829648190495643498496C^6W^{23} \\ &- 4015542571284405339048304491371372544C^2W^{24} - 10576944058520871538170759121360692758400C^8W^{22} \\ &- 15502699390470945181124721524819470336C^4W^{23} - 8866141725017483864389396444019650560W^{24} \\ &+ 187843321838618190063054524706608096372736C^6W^{22} - 2629907870611788993444169025054949341184C^2W^{23} \\ &- 2339628631186252989256617498492102727161600C^8W^{21} + 44587482401347427441487077787839044059648C^4W^{22} \\ &- 646330650943321065022432306287744909312W^{23} + 3911871784145834452620648926995379344228608C^6W^{21} \\ &- 106853624931346742251079988803434625052672C^2W^{22} - 34731221756457076063996685680480046332388400C^8W^{20} \\ &+ 2031630820534613703356720815544183073652992C^4W^{21} - 29772366235325909963719468349751380733696W^{22} \\ &+ 55977917666495001950369883380799856517039520C^6W^{20} - 2935209862612198204900217535310912367495424C^2W^{21} \\ &- 356106954932394870163185295822451834994058200C^8W^{19} + 49513164010821262770619699887016401075361536C^4W^{20} \\ &- 946019081948580189073737352455594555658752W^{21} + 561317230116743908122120200419774701383598936C^6W^{19} \\ &- 57099786361680919061920341446703027568728864C^2W^{20} - 2503014918485675282546997612462115507535679525C^8W^{18} \\ &+ 783946860916220759933612945502090720288013944C^4W^{19} - 21705925482508571220789131491221246705565008W^{20} \\ &+ 3896877309503128425388228707233528573641844628C^6W^{18} - 808080983779706180482616528369050590567064920C^2W^{19} \\ &- 11377958764388364591830650186811204079382995375C^8W^{17} + 85922238157242965549821236753469688505476290C^4W^{18} \\ &- 369391708213242827203119573480480767620475040W^{19} + 17562503504616037013857026016645203836164828140C^6W^{17} \\ &- 8447310280918012001680627820332736390700622380C^2W^{18} - 2585436257819608998690913719534832954169824875C^8W^{16} \\ &+ 66807810334293491481397006276071702825984477270C^4W^{17} - 4738706277845810181648552455272391188664093061W^{18} \\ &+ 3847106354691868461946666585262793495891379180C^6W^{16} - 65691867050538196077827867795275725326125967412C^2W^{17} \\ &+ 39443158775966821300978747970269787435469702375C^8W^{15} + 370178374729243268097232144914747899901186530910C^4W^{16} \\ &- 46252246344697000736262806017589302425808308831W^{17} - 80494399537423044351608035114643938797514784340C^6W^{15} \\ &- 380195020690598067668639660058448337569228016340C^2W^{16} + 566523989912572987998025231792413823985118376950C^8W^{14} \\ &+ 1442615368367378595711408430220065819256126610690C^4W^{15} - 344941910599154055550619712804684862450438165515W^{16} \\ &- 102592679294830739232134840064916416927307176056C^6W^{14} - 1626758143172691797234492449952982684672192283620C^2W^{15} \\ &+ 2353802055345252834360492943360049232668669505450C^8W^{13} + 3787748247391402044794335999892560230955795236324C^4W^{14} \\ &- 1965873270045928352609409458430949440419610647985W^{15} - 4378792169691906846713800991935299857646906174216C^6W^{13} \\ &- 5060312029991364071965860973038005237284334824376C^2W^{14} + 5960624585815448590371042243754875942367055228250C^8W^{12} \\ &+ 5826324830317766373384784002643346415596874803356C^4W^{13} - 8531472842343374721837359406855129169975438391850W^{14} \\ &- 11670390068526218886903484378097101825596299941896C^6W^{12} - 11021879024414518014335231022698323192912500109256C^2W^{13} \\ &+ 10324295746088068017962653276088947972135199215350C^8W^{11} + 1439773561185444614762218609830697730291051846652C^4W^{12} \\ &- 27992129588526849857434901949611308785564181943158W^{13} - 21621096055427761081981893048469013558885318893008C^6W^{11} \end{aligned} \right)$$

$$\begin{aligned}
& \left(\begin{aligned}
& -15196673847175958260366989064748351529776039609928C^2W^{12} + 12704363198636009083795908446233696302872846438775C^8W^{10} \\
& -16156786316255042686165870504641894460678022905092C^4W^{11} - 68660782830109980324828704796738565930994033564934W^{12} \\
& -28866034387149404893958250285833704971216692022300C^6W^{10} - 7937164205818920436506628117017398670094771967136C^2W^{11} \\
& +11256861639873893094526544540819078266187609282325C^8W^9 - 42711635921578581024222595957009872676068811713366C^4W^{10} \\
& -123831755042355046565962696817479402572327910370178W^{11} - 28118809012674353993956278865062412802987022650916C^6W^9 \\
& +15551833843926710505147258403257662393262031947684C^2W^{10} + 7192960962790506045045790420824364603605682631025C^8W^8 \\
& -60336313819066596732006907121543591341432311847314C^4W^9 - 160104412620617126629919245547924381190031859954217W^{10} \\
& -19985726245330129392193933188477605157040281141828C^6W^8 + 42958066814365139520747767939456810581103247550140C^2W^9 \\
& +3292957686090560110513500346377536503919040604875C^8W^7 - 55672818977733192460531150740408051101763768238650C^4W^8 \\
& -142043342588504279624082363350569657504572842562075W^9 - 10270567901452302111125671663301045100261271293300C^6W^7 \\
& +5340379102425828924679548200035721749255949924220C^2W^8 + 1065758549447955481421311809062322189762557925000C^8W^6 \\
& -35312866858214488343284339666988101193815710505110C^4W^7 - 78379973380987359221440189623134413749892391163759W^8 \\
& -374916873858398087150726489780522723859353160000C^6W^6 + 41442567410827597371180683167776915256866674893212C^2W^7 \\
& +238983900244537285637245048005786419108572500000C^8W^5 - 15580810762235588456804596629005531056177039206000C^6W^6 \\
& -17871751313557432743316737785664842456916675480669W^7 - 94652953798263892937422775936079541077206000000C^6W^5 \\
& +21325805877632969222238946836781907645626922515200C^2W^6 + 36090909042039885426573596510616775753500000000C^8W^4 \\
& -4739903707865091755519233365934919443166668200000C^4W^5 + 7898953300719400756811951926990859224568228082600W^6 \\
& -15934658173638963161647028930597005424640000000C^6W^4 + 7315239635776358611100519862075445653174340240000C^2W^5 \\
& +3524609469936828472991462088562732087500000000C^8W^3 - 966675443214240500809251534551971258739720000000C^4W^4 \\
& +817250390112561505190176585453591581208038060000W^5 - 1705904759954599571802525683485712589000000000C^6W^3 \\
& +1641605468964241465446080481145192663640704000000C^2W^4 + 20902644670500299317982136773757750000000000C^8W^2 \\
& -125482784727885974878465370964891137103000000000C^4W^3 + 3133096136142106998572576524739945375727116000000W^4 \\
& -108805403236684848068518420189651200000000000C^6W^2 + 232414258589585152915659996985906474191600000000C^2W^3 \\
& +670758157925487584201558509200000000000000C^8W - 9538542202487903888657553283829293800000000000C^4W^2 \\
& +655785939574624948588315532924673362131500000000W^3 - 3687679506833921846344234626720000000000000C^6W \\
& +1951036784555683334634087982029362736000000000C^2W^2 + 876450320710698095293932000000000000000C^8 \\
& -37267743931237910628794371588704000000000000C^4W + 76160872474105026233808555894009096300000000000W^2 \\
& -501164006368452730131628800000000000000000C^6 + 85922316322646324640319086602284800000000000C^2W \\
& -5628622453729309761467124384000000000000000C^4 + 444207009618346644521027479224028800000000000W \\
& +1449252162138631313926997614080000000000000C^2 + 9784863035148791043390249593280000000000000
\end{aligned} \right)
\end{aligned}$$

(264)

$$\begin{aligned}
b_{6,4}'' &= 56623104(25W+1)(16W+1)(9W+4)(9W+1)^2(25W+49)(36W+25)^2(81W+100)^2(4W+49)(25W+144)(4W+1)^2(9W+100) \\
&\quad \times (144W+3025)(W+16)(9W+49)^2(W+36)(W+9)^2(9W+25)^2(4W+169)(9W+121)(W+49)(W+1)^2(81W+3025)(16W+49)
\end{aligned}$$

(265)

$$\begin{aligned}
& \left(\begin{aligned}
& 184701366299451646766901386280960000C^4W^{23} - 142004892582760985292164872273920000C^2W^{23} \\
& +14176936006536153791954225313821491200C^4W^{22} - 12378951114920887998540066324480000W^{23} \\
& -12270715751466512966321959060006502400C^2W^{22} + 103566554256123768601198631530367778816C^4W^{21} \\
& -409537155531353831988885096077721600W^{22} - 143509740622524811373887279056269279232C^2W^{21} \\
& -28058763815714107163013230103626807930880C^4W^{20} + 161496352341611802510292028215151788032W^{21} \\
& +24166546764468135126019567754560227237888C^2W^{20} - 1534459466090860158709429309742715596233152C^4W^{19} \\
& +19311972987553975433212799777805509185536W^{20} + 1484937442657199866596513709605880009170816C^2W^{19} \\
& -41314794145930248452908128720581047448674320C^4W^{18} + 1051263286004349682664339854824907605019200W^{19} \\
& +43630163962564838929898439423725364121473312C^2W^{18} - 688856928669769663585628836470844181726061924C^4W^{17} \\
& +34706655331423007522043472850030785917233904W^{18} + 792705062624836303243713135599713776182076936C^2W^{17} \\
& -7612069147537563274712053641391295150500352475C^4W^{16} + 766004579032272659694074545141840308168907356W^{17} \\
& +958267878124017208640008187739167420247041510C^2W^{16} - 56741444578466841475368891833409378277320691035C^4W^{15} \\
& +11839281994053570362031554017084474538952959733W^{16} + 78842776600592475667751721574333744260970803750C^2W^{15} \\
& -278706409308421536639729011713614618673802810400C^4W^{14} + 131262478675317803975809466878170417546825322485W^{15} \\
& +438329716386875028401597773145833411453709082240C^2W^{14} - 799485357366967463043870313804012547089794349140C^4W^{13} \\
& +1055807284564741316794329360492865315790514856800W^{14} + 1564313272805654846117864415853238699931782062440C^2W^{13} \\
& -457514553550666169841152777131213575154690738650C^4W^{12} + 6175476149681815928272935439779236205914645081580W^{13} \\
& +2873767864091417815009920474305345541812914120020C^2W^{12} + 6781209973919512021271291054781746687216724517014C^4W^{11} \\
& +26112556421106174091028165956517864786293968369030W^{12} - 2124202442341503486180546974986016103647649217548C^2W^{11} \\
& +3350372274737875335243288265604084679793409862780C^4W^{10} + 78497985646405243390205395565260256391000921865718W^{11} \\
& -30193987236761077321501343304093624644404198396728C^2W^{10} + 86943167081731217820607452125190394897703743862352C^4W^9 \\
& +161506233975423249232665263637748406658117722408764W^{10} - 96092110683111623433941898654857380037125653142176C^2W^9 \\
& +146666005760428352807864857547978465070724442926845C^4W^8 + 205629037984940210434245799428807170524026371904080W^9 \\
& -181231620219013902375195119011978518630905924989002C^2W^8 + 169245423754719598125837837710548316031721451709069C^4W^7 \\
& +99013773143154216413104111579917495708523376440141W^8 - 228674555976239528986164238364308495435241832274986C^2W^7 \\
& +134904764604997637177238162675205006216165297425900C^4W^6 - 153468959063087591574881781840644167701113882066851W^7 \\
& -198759281771813027635625469988806192951687664266840C^2W^6 + 73509052205513020919498757117438266023944908508000C^4W^5 \\
& -35725294463054861574913394674013727904902346212308W^6 - 118462556009025728596444998314325863291808626280000C^2W^5 \\
& +26649766477174863394326793633606780912356058800000C^4W^4 - 348985009123824200273921579424145354101859931341600W^5 \\
& -47054380482607066780394432971509100933452184800000C^2W^4 + 6128511314374740016881921556186740001474740000000C^4W^3 \\
& -198335900346325841725793909416985034411349693520000W^4 - 11794689451170571267763345642313049927727400000000C^2W^3 \\
& +825124018939484623518604888330184264814000000000C^4W^2 - 68097534062087500520237553824846200266352588000000W^3 \\
& -17063622646035649132482549841105655887320000000000C^2W^2 + 56523938222389877053942315091716612800000000000C^4W \\
& -13584172182104639213048462663061350367589200000000W^2 - 12303314107219377727403743730196799200000000000C^2W \\
& +146184117692661325055230469363808000000000000C^4 - 1408578869629036590657624740675649144960000000000W \\
& -328760248350292170658302000865536000000000000C^2 - 56832615978955804742728392894138048000000000000
\end{aligned} \right)
\end{aligned}
\tag{266}$$

$$\begin{aligned}
b''_{6,6} = & 852222617321472(16W+1)(9W+4)(9W+1)(25W+49)(36W+25)(81W+100)^2(4W+49)(25W+144)(4W+1)^2 \\
& \times (16W+49)^2(9W+100)(144W+3025)(W+16)(9W+49)^2(W+36)(W+9)(9W+25)^2(9W+121)(W+49)(W+1)^2(81W+3025)
\end{aligned}
\tag{267}$$

$$\begin{aligned}
& \left(\begin{aligned}
& 24814033131591475200000C^4W^{20} - 6012631104962549760000C^2W^{20} + 2292064581712903455744000C^4W^{19} \\
& -9853311291297454080000W^{20} - 320725248790816838707200C^2W^{19} + 53984668474013815791601920C^4W^{18} \\
& -1308626298829016965785600W^{19} + 24528880350460701387146304C^2W^{18} - 2431502321996043498052730880C^4W^{17} \\
& -37620080648581809466150848W^{18} + 3198903351038100079223035824C^2W^{17} - 212832059158403917969305872160C^4W^{16} \\
& +3420046206583355507216987376W^{17} + 155231661880155914112588383052C^2W^{16} - 7385740085315061448272204253200C^4W^{15} \\
& +357113391810042790605170576412W^{16} + 4428054388566849595197452134545C^2W^{15} - 155312569569830214313565916954975C^4W^{14} \\
& +15861296523804656129025433560045W^{15} + 8338748282777451095714502502040C^2W^{14} - 2163580754315265125245467398053575C^4W^{13} \\
& +430666610508664875652593370982535W^{14} + 1082622578400542156039192624486355C^2W^{13} \\
& -20326934669270891293102798947519900C^4W^{12} + 785633651582691116559454648748430W^{13} \\
& +9744124725604761323866016572186740C^2W^{12} - 125120302422971717410810461685871100C^4W^{11} \\
& +99820209012710642517186863434249200W^{12} + 58638874355305142463478038282334650C^2W^{11} \\
& -443699896270829541005346957967507650C^4W^{10} + 892397493529835105390976323842619310W^{11} \\
& +203834072844067261094985925296300960C^2W^{10} - 307341308939877787152611931084094050C^4W^9 \\
& +5560904904492374455587625669929348290W^{10} + 79808966758521266247423808744820310C^2W^9 \\
& +5306642264614367436147858769726464180C^4W^8 + 23291415790385429543798789440255572000W^9 \\
& -3207955213203763972897522970109565884C^2W^8 + 29294141142507775399086799111062508980C^4W^7 \\
& +59023199256961018307695639915279718208W^8 - 1848137327718385133764182865349317259C^2W^7 \\
& +80098989123916585922170389945939468585C^4W^6 + 52815084519737519198118093527007379069W^7 \\
& -56419864273719915638343199022631230712C^2W^6 + 131958904646266166728605277574603499825C^4W^5 \\
& -182323619279717114244457988314598433297W^6 - 108524957490963486771208681308317037225C^2W^5 \\
& +134475658359137739331490637193479120000C^4W^4 - 765334552521538908051347827907800680630W^5 \\
& -135133148615191590611474103395480962500C^2W^4 + 82346471732103759602033529094427250000C^4W^3 \\
& -1333980064252340267606316790962437210500W^4 - 106306672649297118873836265345021750000C^2W^3 \\
& +28012640895449474850495208148850000000C^4W^2 - 1285137176664087009046839687442826400000W^3 \\
& -49096475026575962818542749198775000000C^2W^2 + 4515277723135119955845951120000000000C^4W \\
& -687870121640403120850587988758465000000W^2 - 11240040952333750291259277060000000000C^2W \\
& +25041437734591374151255200000000000C^4 - 183651545653783484752949468148000000000W \\
& -76972853698499443305020400000000000C^2 - 1753040930043438773992734480000000000
\end{aligned} \right)
\end{aligned}$$

(268)

$$\begin{aligned}
b''_{6,8} &= 28311552(9W+1)(25W+49)(36W+25)(4W+49)(25W+144)(4W+1)(9W+100)(144W+3025)(W+16) \\
&\quad \times (9W+49)^2(W+36)(W+9)^2(9W+25)^2(4W+169)(9W+121)(W+49)(W+1)^2(81W+3025)(81W+100)(16W+49)
\end{aligned}$$

(269)

$$\begin{aligned}
a''_{6,10} &= -77 \left(\begin{aligned}
& 253091761228800W^{12} + 15113171718223488W^{11} - 145576655162307828W^{10} - 37866325354423896069W^9 \\
& -1446999385664188928736W^8 - 27425242258389583222926W^7 - 288611686067564250374144W^6 \\
& -1594661675965530915659493W^5 - 2917904684458540400059092W^4 + 11952824957073242155450000W^3 \\
& +64822627102393953975336000W^2 + 92925765483875910432000000W + 3335626110684599424000000
\end{aligned} \right)
\end{aligned}$$

(270)

$$\begin{aligned}
b''_{6,10} &= 18874368(4W+49)(25W+144)(4W+1)(9W+100)(144W+3025)(W+16)(9W+49) \\
&\quad \times (W+36)(W+9)(9W+25)(9W+121)(W+49)(W+1)(81W+3025)(81W+100)(16W+49)
\end{aligned}$$

(271)

$$\begin{aligned}
a''_{6,12} &= -5005 \left(\begin{aligned}
& 916W^5 + 2061W^4 - 4435049W^3 \\
& -126037441W^2 - 950705927W + 5739509160
\end{aligned} \right)
\end{aligned}$$

(272)

$$b''_{6,12} = 6291456(81W+3025)(W+1)(W+49)(9W+121)(4W+169)(9W+25)(W+9)(W+36)(9W+49)$$

(273)

6.6. Fifth Harmonic

$$a'_{5,2} = - \left(\begin{aligned} &1484939687616000000C^8W^{11} - 2561038837862400000C^6W^{11} - 2518439377608000000C^8W^{10} \\ &+ 1056814219238400000C^4W^{11} - 2425520532556800000C^6W^{10} + 5877312307200000C^2W^{11} \\ &- 71725359703168155600C^8W^9 - 2032835053318272000C^4W^{10} + 13407618700800000W^{11} \\ &+ 95405053811168787840C^6W^9 + 4462781369836032000C^2W^{10} - 236691058701821031000C^8W^8 \\ &- 80390812971434908320C^4W^9 + 416740844608243200W^{10} + 438029349667307521200C^6W^8 \\ &+ 50829426713917992960C^2W^9 - 303970541370423837525C^8W^7 - 335832431883239571840C^4W^8 \\ &+ 5918525052891129072W^9 + 807094709072839004580C^6W^7 + 199419019799905301328C^2W^8 \\ &- 152154545457135184425C^8W^6 - 486751721575668351390C^4W^7 + 40088820697073065944W^8 \\ &+ 717756344791880127300C^6W^6 + 277392880382859750564C^2W^7 - 597706336915414650C^8W^5 \\ &- 47336213184650889030C^4W^6 + 133719269630312082987W^7 + 285315657945821221320C^6W^5 \\ &- 132202622509796746908C^2W^6 + 24525385232674753350C^8W^4 + 452888464696942082340C^4W^5 \\ &+ 221422573140833027655W^6 + 20400555800788039800C^6W^4 - 666290473132440752568C^2W^5 \\ &+ 7885298271191192775C^8W^3 + 407445000038980658580C^4W^4 + 164452447626282117190W^5 \\ &- 14588155563287353740C^6W^3 - 556901855670889528488C^2W^4 + 962516234784102075C^8W^2 \\ &+ 111859096224972901770C^4W^3 + 24884580509266207318W^4 - 2906514951748008300C^6W^2 \\ &- 159422754326695592076C^2W^3 + 49061162737125000C^8W + 4201249436937514290C^4W^2 \\ &- 35587529114749448889W^3 - 178754896826100000C^6W - 17166675161738929932C^2W^2 \\ &+ 864808386750000C^8 - 252163514282744400C^4W - 18923599513237424117W^2 \\ &- 3459233547000000C^6 + 202627352646321120C^2W - 10646124613740000C^4 \\ &- 2996292924146714360W + 26726744350152000C^2 + 47511947857374000 \end{aligned} \right) \quad (274)$$

$$b'_{5,2} = 84934656(16W+49)(W+9)(9W+121)(25W+1)(16W+1)(36W+25) \quad (275)$$

$$\times (W+1)(4W+1)(9W+49)(25W+49)(9W+4)(9W+25)(81W+100)(9W+1) \quad (276)$$

$$a'_{5,4} = - \left(\begin{aligned} &1964205936000000C^4W^{10} - 1166619283200000C^2W^{10} + 11338527569040000C^4W^9 \\ &- 362230185600000W^{10} - 11314606582944000C^2W^9 - 32696993878913700C^4W^8 \\ &- 8471876763312000W^9 - 21906918396117720C^2W^8 - 315548078151399345C^4W^7 \\ &- 69142142405375460W^8 + 102191841612125370C^2W^7 - 713812934129326920C^4W^6 \\ &- 243942888588412545W^7 + 520275706406275680C^2W^6 - 672844614017144310C^4W^5 \\ &- 318529885512147624W^6 + 860787172093204860C^2W^5 - 266561025162334560C^4W^4 \\ &+ 191133183604701610W^5 + 597385437972372000C^2W^4 - 20896153530513345C^4W^3 \\ &+ 843253239091001568W^4 + 140057973620585370C^2W^3 + 10603420157538180C^4W^2 \\ &+ 646038190884574575W^3 - 3482684963871960C^2W^2 + 1769246568180000C^4W \\ &+ 173613131986953316W^2 - 2431870619673600C^2W + 67523803200000C^4 \\ &+ 14113100662688160W - 112518927360000C^2 - 814654459584000 \end{aligned} \right)$$

$$b'_{5,4} = 1769472(16W+49)(25W+144)(W+9)(16W+1)(36W+25) \quad (277)$$

$$\times (W+1)(4W+1)(9W+49)(25W+49)(9W+4)(9W+25)(81W+100)(9W+1) \quad (278)$$

$$a'_{5,6} = 35 \left(\begin{aligned} &561762897696000C^4W^8 - 271397140185600C^2W^8 - 1259996035137840C^4W^7 \\ &- 313495279153920W^8 - 1098450201996576C^2W^7 - 26472044921442120C^4W^6 \\ &- 7374338885021616W^7 + 6585960814001712C^2W^6 - 75003192009300555C^4W^5 \\ &- 44385762295180008W^6 + 47229373734401430C^2W^5 - 72698038597023840C^4W^4 \\ &- 15784727825591595W^5 + 100743107105317248C^2W^4 - 14165589335747850C^4W^3 \\ &+ 345024278717027808W^4 + 85269533105810196C^2W^3 + 11098555483595580C^4W^2 \\ &+ 689454967152509366W^3 + 20275732505868840C^2W^2 + 3969339527278125C^4W \\ &+ 350048719220374300W^2 - 3968782483916250C^2W + 294446665012500C^4 \\ &+ 31859044923689165W - 588893330025000C^2 - 6403836721243500 \end{aligned} \right)$$

$$b'_{5,6} = 8097103872(16W+49)(W+9)(9W+121)(36W+25) \quad (279)$$

$$\times (W+1)(4W+1)(9W+49)(25W+49)(9W+25)(81W+100)(9W+1) \quad (280)$$

$$a'_{5,8} = -35 \left(\begin{aligned} &5346000W^5 + 13365405W^4 - 149226782W^3 \\ &- 417436275W^2 - 131170888W + 43747200 \end{aligned} \right) \quad (281)$$

$$b'_{5,8} = 393216(16W+49)(25W+144)(W+9)(W+1)(4W+1)(9W+25)(81W+100) \quad (282)$$

$$a'_{5,10} = -2695(9W^2 - 86W + 33) \quad (283)$$

$$b'_{5,10} = 2097152(W+9)(9W+121)(W+1)(9W+49)(9W+25) \quad (283)$$

$$a''_{5,2} = \left(\begin{aligned} &42426848217600000C^8W^{11} - 77139724032000000C^6W^{11} - 1882167601406400000C^8W^{10} \\ &+ 28284565478400000C^4W^{11} + 2859002926214400000C^6W^{10} + 5142648268800000C^2W^{11} \\ &- 11650683798853490160C^8W^9 - 1445513511173760000C^4W^{10} + 1285662067200000W^{11} \\ &+ 23266274459625691200C^6W^9 + 368724026154240000C^2W^{10} - 12280927401393706440C^8W^8 \\ &- 11176694715882759840C^4W^9 + 38812697957952000W^{10} + 5424502307854226560C^6W^8 \\ &+ 1585389019495648320C^2W^9 + 38878056618272684505C^8W^7 - 10808153217383099760C^4W^8 \\ &+ 478634802819037200W^9 + 14101771853742734700C^6W^7 - 13536727139938866048C^2W^8 \\ &+ 103557316067878410045C^8W^6 + 91668432871485641430C^4W^7 + 2144332827394321464W^8 \\ &- 120154786479514990980C^6W^6 - 112600968462375701076C^2W^7 + 95184668300414123970C^8W^5 \\ &+ 290061790236496165230C^4W^6 - 225702669983954247W^7 - 191252727100703457000C^6W^5 \\ &- 306907327233030691908C^2W^6 + 41471030780752841730C^8W^4 + 314807260527129113100C^4W^5 \\ &- 26794128143588769891W^8 - 122556012913183275720C^6W^4 - 360025846729076371560C^2W^5 \\ &+ 8975472784423676685C^8W^3 + 136400877904586303820C^4W^4 - 75906648242392481150W^5 \\ &- 36069390881472708900C^6W^3 - 163276462901136841800C^2W^4 + 941653150292134665C^8W^2 \\ &- 1417752005854118130C^4W^3 - 90108247284134221630W^4 - 4537602890474359860C^6W^2 \\ &- 792914335472742372C^2W^3 + 44713384860375000C^8W - 12092821730505673290C^4W^2 \\ &- 32281116220892757139W^3 - 238555394613900000C^6W + 12230270334315730956C^2W^2 \\ &+ 759483959850000C^8 - 1162389536245336560C^4W - 13363743500557547543W^2 \\ &- 4295838947400000C^6 + 2481084760360166688C^2W - 2755925799556000C^4 \\ &- 733766186647962664W + 80006571193348800C^2 + 221260460267523600 \end{aligned} \right) \quad (284)$$

$$b''_{5,2} = 28311552(81W+100)(9W+49)(36W+25)(9W+25)(9W+4)(25W+1) \quad (285)$$

$$\times (W+1)(25W+49)(9W+1)(16W+49)(9W+121)(4W+1)(W+9)(16W+1) \quad (286)$$

$$a''_{5,4} = \left(\begin{aligned} &107138505600000C^4W^{10} - 71425670400000C^2W^{10} - 218682255568000C^4W^9 \\ &- 35712835200000W^{10} + 759497370912000C^2W^9 - 29781049405210740C^4W^8 \\ &- 551392633584000W^9 + 17141356518887160C^2W^8 - 102059778832994745C^4W^7 \\ &+ 87804447150780W^8 + 92723879823045030C^2W^7 - 10203046788711125C^4W^6 \\ &+ 37350507839205411W^7 + 210552900105537990C^2W^6 + 78031649522340990C^4W^5 \\ &+ 228716037852626079W^6 + 178937276188893180C^2W^5 + 208395090481516470C^4W^4 \\ &+ 579792732108383174W^5 - 43035234896620260C^2W^4 + 139760280017466255C^4W^3 \\ &+ 661108000114343982W^4 - 143352668760320250C^2W^3 + 38617097064913395C^4W^2 \\ &+ 272073932033954139W^3 - 62014795188340890C^2W^2 + 4021354516063500C^4W \\ &- 5559847988657641W^2 - 7613939101161960C^2W + 131508052200000C^4 \\ &- 22583452658781924W - 267809005296000C^2 - 3660080543582400 \end{aligned} \right)$$

$$b''_{5,4} = 884736(25W+144)(81W+100)(9W+49)(36W+25)(9W+25) \quad (287)$$

$$\times (9W+4)(W+1)(25W+49)(9W+1)(16W+49)(4W+1)(W+9)(16W+1) \quad (288)$$

$$a''_{5,6} = 35 \left(\begin{aligned} &48151105516800C^4W^8 - 26264239372800C^2W^8 - 2162446828871472C^4W^7 \\ &- 45232856697600W^8 + 753452124508512C^2W^7 - 12042177102922824C^4W^6 \\ &- 394912397587248W^7 + 8377681374094032C^2W^6 - 7247611975851819C^4W^5 \\ &+ 10848265404337080W^6 + 27132307410360438C^2W^5 + 48668646036849696C^4W^4 \\ &+ 132697621732046229W^5 + 24841970940181824C^2W^4 + 91271957550677046C^4W^3 \\ &+ 416184517311795360W^4 - 27458219074185900C^2W^3 + 55775608145061948C^4W^2 \\ &+ 402822028669343926W^3 - 58892236205372856C^2W^2 + 12119968673938125C^4W \\ &- 82762193470251460W^2 - 27847313740496250C^2W + 775758616132500C^4 \\ &- 174089917176297587W - 2408089348665000C^2 - 37375577285438700 \end{aligned} \right) \quad (289)$$

$$b''_{5,6} = 97103872(81W+100)(9W+49)(36W+25)(9W+25) \quad (290)$$

$$\times (W+1)(25W+49)(9W+1)(16W+49)(9W+121)(4W+1)(W+9) \quad (291)$$

$$a''_{5,8} = -35 \left(\begin{aligned} &291600W^5 - 6906159W^4 - 59354813W^3 \\ &- 62538761W^2 + 117455493W + 85201200 \end{aligned} \right) \quad (292)$$

$$b''_{5,8} = 196608(25W+144)(81W+100)(9W+49)(9W+25)(W+1)(16W+49)(4W+1)(W+9) \quad (293)$$

$$a''_{5,10} = -77(27W^2 - 1410W + 3043) \quad (294)$$

$$b''_{5,10} = 2097152(9W+49)(9W+25)(W+1)(9W+121)(W+9) \quad (295)$$

6.7. Sixth Harmonic

$$\begin{aligned}
 a'_{6,0} = - & \left(52109469576342097200000C^{12}W^{16} - 1683536709389513932800000C^{10}W^{16} - 46492422585744110976000000C^{12}W^{15} \right. \\
 & + 1913109897033538560000000C^8W^{16} + 138301278766116663859200000C^{10}W^{15} - 850271065348239360000000C^6W^{16} \\
 & - 376442781452876257029630720C^{12}W^{14} - 155143822552694325826560000C^8W^{15} + 91100471287311360000000C^4W^{16} \\
 & + 1338568489676275102464468480C^{10}W^{14} + 82581386542537013268480000C^6W^{15} + 7288037702984908800000C^2W^{16} \\
 & + 202965062130318561376343040C^{12}W^{13} - 167379428469599974580486400C^8W^{14} - 21130334481703027368960000C^4W^{15} \\
 & + 1214672950497484800000W^{16} + 1627285614819839102694424320C^{10}W^{13} + 897542798187428423149593600C^6W^{14} \\
 & + 1820233753844148099072000C^2W^{15} + 10260190365735588178177236960C^{12}W^{12} - 2923654668976340991355749120C^8W^{13} \\
 & - 189011887636072784803257600C^4W^{14} + 63680557643638944768000W^{15} - 23316071896638228530747675520C^{10}W^{12} \\
 & + 465172932439724679869031936C^6W^{13} + 2303954183218189646845440C^2W^{14} + 41262075002978091263911749840C^{12}W^{11} \\
 & + 30037360044369662904028687200C^8W^{12} + 987668124716507953368153600C^4W^{13} + 2060228321225580121401600W^{14} \\
 & - 121743006218285449255731784080C^{10}W^{11} - 33314330263273644371132924928C^6W^{12} - 390720217111012974712755456C^2W^{13} \\
 & + 7787822546322943352515617685C^{12}W^{10} + 183929204116488253061978892000C^8W^{11} + 20506091203622642724225828000C^4W^{12} \\
 & + 2755936902564170706936064W^{13} - 277652674264865082826601289870C^{10}W^{10} - 194588543250835337639770654752C^6W^{11} \\
 & - 4533933239061042955606558848C^2W^{12} + 79756236832260837395267135415C^{12}W^9 + 456557923782661219633821251475C^8W^{10} \\
 & + 111035391196354632890606071440C^4W^{11} + 162195914144092422366220320W^{12} - 354664512686904968274276356730C^{10}W^9 \\
 & - 502191487117144038689542135908C^6W^{10} - 21625520844692834179872931152C^2W^{11} + 43441045788897102323350257885C^{12}W^8 \\
 & + 585755194343601699131367981825C^8W^9 + 291461825898179208353296282635C^4W^{10} + 256754045726840650656215232W^{11} \\
 & - 265615900103951360054984963070C^{10}W^8 - 647362944908003210086347089292C^6W^9 - 49019060782082473467833074158C^2W^{10} \\
 & + 9368282104378052721862311315C^{12}W^7 + 369218687746056939135283574475C^8W^8 + 374391364123203131686767130665C^4W^9 \\
 & - 1689175955657523303925944147W^{10} - 111574084879270356942010942530C^{10}W^7 - 314951646255925722050203862148C^6W^8 \\
 & - 33117186485378882319156032922C^2W^9 - 1948256063848585086756725565C^{12}W^6 + 47033965707781620734423338725C^8W^7 \\
 & + 116125691843795791549093876035C^4W^8 - 10335567159187632347258284353W^9 - 20628386260747056273532389570C^{10}W^6 \\
 & + 177563646053875512792926973828C^6W^7 + 81243543243781491957950768802C^2W^8 - 1611931483484859267258963435C^{12}W^5 \\
 & - 78779941064948284724875783275C^8W^6 - 274247786267324176922018347635C^4W^7 - 25532244663583438493033526027W^8 \\
 & + 1406693959129422888734009970C^{10}W^5 + 301697641531581832717361658372C^6W^6 + 203169936447278743573198497438C^2W^7 \\
 & - 380957819601574875939023745C^{12}W^4 - 45855266584165967535751385325C^8W^5 - 340710702033120519349114098915C^4W^6 \\
 & - 34043524695499180873861785573W^7 + 1258478871716288383958654550C^{10}W^4 + 141764488103962817266034395548C^6W^5 \\
 & + 183477648899653153471966680222C^2W^6 - 43356127830773746339526175C^{12}W^3 - 9197473597172436927465930375C^8W^4 \\
 & - 152680641924498601622848090485C^4W^5 - 25338238491254916778508215637W^6 + 197118147267991817628349050C^{10}W^3 \\
 & + 29026422504865610696384220372C^6W^4 + 76468194582340790072490389778C^2W^5 - 2499476439039869521732500C^{12}W^2 \\
 & - 608046256537389188143968105C^8W^3 - 28872591394899915350695885215C^4W^4 - 10114196457401466797136505043W^5 \\
 & + 13185445470094310777595000C^{10}W^2 + 2381581919703798999781542732C^6W^3 + 13031246447522370360333155382C^2W^4 \\
 & - 68998963742852393250000C^{12}W - 7953078372151693483313100C^8W^2 - 982480147711898235496527585C^4W^3 \\
 & - 1779023534967609181095139257W^4 + 394695863190229639500000C^{10}W + 25344778599476946085850640C^6W^2 \\
 & - 542257709169493105544539686C^2W^3 - 716147825067675000000C^{12} + 398479020937859430450000C^8W \\
 & + 371739190708603993902855060C^4W^2 + 89998561093646820829929673W^3 + 4296886950406050000000C^{10} \\
 & - 2819270567173080742680000C^6W - 490117784945453831706456840C^2W^2 + 8927196738551091000000C^8 \\
 & + 8691809529469661409570000C^4W + 85281340115182549119363148W^2 - 61896945577997930400000C^6 \\
 & - 26089807683832517542260000C^2W - 51636681072991614600000C^4 + 7286144317267684594446000W \\
 & \left. + 236067864124148456400000C^2 - 60846979570820122680000 \right)
 \end{aligned} \tag{294}$$

$$\begin{aligned}
 b'_{6,0} = & 8153726976(9W+121)(9W+25)(9W+169)(W+1)(W+9)(9W+49)(4W+1)(81W+100) \\
 & \times (16W+49)(9W+1)(W+4)(36W+25)(25W+49)(9W+4)(16W+1)(36W+49)(25W+1)(36W+1)(81W+25)
 \end{aligned} \tag{295}$$

$$\begin{aligned}
& \left(54113679944662947840000000C^8W^{17} - 98388508990296268800000000C^6W^{17} - 2291356928140791956121600000C^8W^{16} \right. \\
& + 36075786629775298560000000C^4W^{17} + 3359317731990101425152000000C^6W^{16} + 6559233932686417920000000C^2W^{17} \\
& - 659909167332612307962445440000C^8W^{15} - 1839078212492109605990400000C^4W^{16} + 1639808483171604480000000W^{17} \\
& + 112590221363552318764250880000C^6W^{15} + 615016620948461711155200000C^2W^{16} - 541366700811919577663489245440C^8W^{14} \\
& - 55009145709448243366306176000C^4W^{15} + 7513369657576779709440000W^{16} + 1088943540633062662411892044800C^6W^{14} \\
& + 9924443626077931416254208000C^2W^{15} - 1531841767094336864297626756800C^8W^{13} - 484869506549531287473174735360C^4W^{14} \\
& + 1480323674298217383705024000W^{15} + 4527248350258911369705193002240C^6W^{13} + 19665626324259547881334471680C^2W^{14} \\
& + 2072507909834235763345721126400C^8W^{12} - 1336842032018768984600468341632C^4W^{13} + 14609286401509136191709018880W^{14} \\
& + 5513818694336817894075790114560C^6W^{12} - 818204229807022491726893591808C^2W^{13} + 24497749365482554190876633890950C^8W^{11} \\
& + 5125810670346344040683662837824C^4W^{12} + 59058741650904700982544568896W^{13} - 22736217078265207323748173581160C^6W^{11} \\
& - 919493724743276248854398990720C^2W^{12} + 67836292031364546140626322018025C^8W^{10} + 49893273466332503345103675161844C^4W^{11} \\
& - 166937855686160535520072099776W^{12} - 10833129512592378467575795955140C^6W^{10} - 47197238091366795232052913989832C^2W^{11} \\
& + 96381198969688026267774185231100C^8W^9 + 162812848083305715683954806657158C^4W^{10} - 3106028305312771863540047557194W^{11} \\
& - 207232542577688659330893315543360C^6W^9 - 137205254459154864808569502889316C^2W^{10} + 7777777502356760038796072084675C^8W^8 \\
& + 270973102238614688894664577415640C^4W^9 - 16183850882461523465260703374791W^{10} - 218386267144343327312330383940940C^6W^8 \\
& - 229572534500041005219855263606496C^2W^9 + 35372249815621055630654901251250C^8W^7 + 218387815031546558393922391475682C^4W^8 \\
& - 46355995946698101086024012650164W^9 - 131069560037750338698590174694360C^6W^7 - 197355528695288001292524874180332C^2W^8 \\
& + 7961290942106988412723104098175C^8W^6 + 37831955977299880472558092512636C^4W^7 - 80288777365574400131683709751021W^8 \\
& - 41931226123564718848483750694940C^6W^6 - 39748207684838093746709086230840C^2W^7 + 295160177296695575652839528400C^8W^5 \\
& - 58592735305475245241254197278934C^4W^6 - 84729518190792418889865897140254W^7 - 5641477723809095018514144107760C^6W^5 \\
& + 64059516969419012813003963775492C^2W^6 - 221188469718771379681470371835C^8W^4 - 39381521041600372118889453401808C^4W^5 \\
& - 52771021202831601333829524382481W^6 + 135149821932176856479398551660C^6W^4 + 50059288174934897381516333273136C^2W^5 \\
& - 41097730071804111641074500900C^8W^3 - 9539872912705640492374327688370C^4W^4 - 17886055595735190597963295319104W^5 \\
& + 101845875031558197652957244400C^6W^3 + 1400789911672208550206971869196C^2W^4 - 2889428667419556248658210000C^8W^2 \\
& - 903303064435654679314516090680C^4W^3 - 2399411753268996318887272020811W^4 + 8817007981825575502384680000C^6W^2 \\
& + 1760665678413573992682209217840C^2W^3 - 88366064395348670931000000C^8W - 16715030522054360913198348000C^4W^2 \\
& + 305818149467152378287861141820W^3 + 293290260211434614436000000C^6W + 45299519247125093286516264000C^2W^2 \\
& - 972643765957340400000000C^8 + 738442580319644457826800000C^4W + 158592159917410410715397342000W^2 \\
& + 3370516659760219200000000C^6 - 1142151851320598837426400000C^2W + 19104595178747712480000000C^4 \\
& + 15998723930091542055985800000W - 36484517560114895040000000C^2 - 131106121073935927920000000
\end{aligned}
\tag{296}$$

$$\begin{aligned}
b'_{6,2} &= 1019215872(25W+1)(81W+25)(36W+1)(9W+4)(16W+1)(36W+49)(25W+49)(9W+1)(36W+25) \\
&\times (W+4)(16W+49)(324W+3025)(4W+1)(25W+144)(81W+100)(W+9)(W+1)(9W+121)(9W+49)(9W+25)
\end{aligned}
\tag{297}$$

$$\begin{aligned}
& \left(\begin{aligned}
& 361871316502375680000000C^8W^{16} - 556725102311347200000000C^6W^{16} - 3021207771393681363200000C^8W^{15} \\
& -151834118812185600000000C^4W^{16} + 39131348297788188288000000C^6W^{15} + 145085935753866240000000C^2W^{16} \\
& -446549064191601797914488000C^6W^{14} - 16180612448442292569600000C^4W^{15} + 64951261936323840000000W^{16} \\
& +720410025540471057681120000C^6W^{14} + 4538465730879354086400000C^2W^{15} - 1352419025326794866109548880C^8W^{13} \\
& -6881867824256722821904000C^4W^{14} + 2951538433607370182400000W^{15} + 3924076312128697207791739200C^6W^{13} \\
& -63447105408718894650144000C^2W^{14} + 7974559642096732559964829740C^8W^{12} + 3253334097770154534282723360C^4W^{13} \\
& +39234549036173448515976000W^{14} + 1866921911165177755542617040C^6W^{12} - 2817621707288652585673410240C^2W^{13} \\
& +69473464669396673159315306625C^8W^{11} + 40559449596436032371214908040C^4W^{12} - 102451382279784573446345040W^{13} \\
& -6517681145956880236821518980C^6W^{11} - 31938036254755367349787627824C^2W^{12} + 217200582565005119628176864235C^8W^{10} \\
& +195680235716708656320619264710C^4W^{11} - 7376151511735315570314389844W^{12} - 310270226756749268904336699180C^6W^{10} \\
& -176314426031063614705039915524C^2W^{11} + 360865827915182062302348624105C^8W^9 + 438715285127992557657386116290C^4W^{10} \\
& -78224043059589746342929737663W^{11} - 708320699381346028706198145060C^6W^9 - 522658961720378719588820278188C^2W^{10} \\
& +339168289462577946686107647855C^8W^8 + 31680004038379975769097113910C^4W^9 - 414367449347626270004271705813W^{10} \\
& -939724448638446986075866928700C^6W^8 - 762711412034880389967526825380C^2W^9 + 170848593243001558497857979735C^8W^7 \\
& -553707536641219490775768719910C^4W^8 - 1266645677261395414242958489431W^9 - 746821611430038292178253470940C^6W^7 \\
& -16862332197950421498674361532C^2W^8 + 31374378560867316841149908625C^8W^6 - 1462076751644287691827195633590C^4W^7 \\
& -2288990697562792110510626598225W^8 - 340536153928974747669591809220C^6W^6 + 1091445768797469076404316856484C^2W^7 \\
& -9783931634505140305988679885C^8W^5 - 1430391198556282342357399048410C^4W^6 - 2294335132708798734843107393129W^7 \\
& -74900934316573346698856467020C^6W^5 + 1612642671432852685802326379964C^2W^6 - 6440504096593165724382712455C^8W^4 \\
& -740032937838327415912121570190C^4W^5 - 918868600492462596576123877807W^6 - 1090707793928383678093139940C^6W^4 \\
& +1000687115038187990098828226868C^2W^5 - 1375036808938971113036801700C^8W^3 - 201311407587572458386895704810C^4W^4 \\
& +322159949445416074688674495603W^5 + 2554647250602976930322182800C^6W^3 + 304692632483905828737231370140C^2W^4 \\
& -136964791935234586052730000C^8W^2 - 2317546980509927328749098200C^4W^3 + 430037046327070517431891237561W^4 \\
& +404494252061819904812520000C^6W^2 + 46413393415671366403403547792C^2W^3 - 62412441494466093030000000C^8W \\
& -48548114154312725525647200C^4W^2 + 141428140147082687638935533660W^3 + 22494842931625193052000000C^6W \\
& +2926844230254583358010661440C^2W^2 - 10312528680974520000000C^8 + 91649691628592623234800000C^4W + \\
& 20252440474819566254244278128W^2 + 41250114723898080000000C^6 - 79416151276658655288480000C^2W \\
& +2705635453802272992000000C^4 + 1043297909586373929952296000W - 5573108070800388518400000C^2 \\
& -25741758751998701892480000
\end{aligned} \right)
\end{aligned}
\tag{298}$$

$$\begin{aligned}
b'_{6,4} &= 5435817984(25W+1)(81W+25)(9W+4)(16W+1)(36W+49)(25W+49)(9W+1)(36W+25)(W+4) \\
&\times (16W+49)(4W+1)(25W+144)(81W+100)(W+9)(W+1)(9W+121)(9W+49)(9W+169)(9W+25)
\end{aligned}
\tag{299}$$

$$\begin{aligned}
& \left(\begin{aligned}
& 532009795097898937036800000C^4W^{14} - 485717371787545766092800000C^2W^{14} - 22727359926249817053371904000C^4W^{13} \\
& -257447747089045932134400000W^{14} + 13156117265736828684278784000C^2W^{13} - 640235599344463926810411630720C^4W^{12} \\
& -5474058649208199897619968000W^{13} + 576322377703994410603757349120C^2W^{12} - 5080860357086395645829189102880C^4W^{11} \\
& +66301639501506810772315455360W^{12} + 6108631229479227029615492458560C^2W^{11} - 13133403329654882424718750671720C^4W^{10} \\
& +2940370703160731567065769116896W^{11} + 24397120213984171639723388193360C^2W^{10} + 25431346853498432694086549603370C^4W^9 \\
& +3443682365366238042146937829272W^{10} + 21165220167244678119196989561420C^2W^9 + 231563829060682486666542439726905C^4W^8 \\
& +196343147059643203076776727491146W^9 - 139434189940137456963267022912050C^2W^8 + 579200459197887727300803099194820C^4W^7 \\
& +591603349887439203265479855315225W^8 - 538248797150733947337645879330120C^2W^7 + 724423388732611610222916932622030C^4W^6 \\
& +860793282174860650711363930856580W^7 - 898307725993387707658401745298460C^2W^6 + 479132926433091833940477167382270C^4W^5 \\
& +221329342279832144849163561411214W^6 - 810108601942862405536602187015260C^2W^5 + 151243586877304716961844040837705C^4W^4 \\
& -993839717119563225973432611876962W^5 - 377839693128123901399215812213970C^2W^4 + 10535242020352535544647745081420C^4W^3 \\
& -1313129960093918026791007180506711W^4 - 67778373694020204131753792088600C^2W^3 - 4560089077637899202692160989200C^4W^2 \\
& -647836653826115100362600960036660W^3 + 2656439922387348557564733972000C^2W^2 - 742030723916844606212938200000C^4W \\
& -126491867660237102052771346745360W^2 + 1202187011465808534341553840000C^2W - 2788392377936071845892800000C^4 \\
& -6612716092705516262864895192000W + 53256882594942353394144000000C^2 + 507373553221497171248054400000
\end{aligned} \right)
\end{aligned}
\tag{300}$$

$$\begin{aligned}
b'_{6,6} &= 340889046928588(9W+4)(16W+1)(36W+49)(25W+49)(9W+1)(36W+25)(W+4) \\
&\times (16W+49)(324W+3025)(4W+1)(25W+144)(81W+100)(W+9)(W+1)(9W+121)(9W+49)(9W+25)
\end{aligned}
\tag{301}$$

$$a'_{6,8} = -7 \left(\begin{aligned} &328302992160000C^4W^{11} - 151524457920000C^2W^{11} - 28137258607114800C^4W^{10} \\ &-452277548640000W^{11} + 8819504456637600C^2W^{10} - 343207495562910264C^4W^9 \\ &-467097028930800W^{10} + 186585336750307968C^2W^9 - 427410110258184339C^4W^8 \\ &+498189012322537656W^9 + 1185819954071837238C^2W^8 + 8665111696139889705C^4W^7 \\ &+9266893805766050181W^8 + 1786898580918874830C^2W^7 + 44518784744572319610C^4W^6 \\ &+59541144823406139345W^7 - 11099676218630335380C^2W^6 + 86538141874470158730C^4W^5 \\ &+135323802885616707690W^6 - 55641050246573191620C^2W^5 + 71376710665058070129C^4W^4 \\ &-70750468852109387190W^5 - 99848305122658323858C^2W^4 + 15898866313268092869C^4W^3 \\ &-728888960205169948311W^4 - 77382213273442850058C^2W^3 - 6752842039294991640C^4W^2 \\ &-984645858715469172371W^3 - 18945250637178816720C^2W^2 - 2665080755727390000C^4W \\ &-434608112429561181400W^2 + 2329513228945980000C^2W - 200437875237600000C^4 \\ &-32869263753208794800W + 400875750475200000C^2 + 677781003714720000 \end{aligned} \right) \quad (302)$$

$$b'_{6,8} = 905969664(25W+49)(9W+1)(36W+25)(W+4)(16W+49)(4W+1) \quad (303)$$

$$\times (25W+144)(81W+100)(W+9)(W+1)(9W+121)(9W+49)(9W+169)(9W+25)$$

$$a'_{6,10} = 385 \left(\begin{aligned} &566870400W^7 - 29110737096W^6 - 445637908710W^5 \\ &-827361045303W^4 + 5787403568960W^3 + 15148196768589W^2 \\ &+4642023863860W - 1299020184000 \end{aligned} \right) \quad (304)$$

$$b'_{6,10} = 25165824(16W+49)(324W+3025)(4W+1)(25W+144) \quad (305)$$

$$\times (81W+100)(W+9)(W+1)(9W+121)(9W+49)(9W+25)$$

$$a'_{6,12} = 1001(81W^3 - 8325W^2 + 48259W - 15015) \quad (306)$$

$$b'_{6,12} = 201326592(W+9)(W+1)(9W+121)(9W+49)(9W+169)(9W+25) \quad (307)$$

$$\begin{aligned}
& \left(\begin{aligned}
& 1042189391526841958400000C^{12}W^{15} - 3276883952204589619200000C^{10}W^{15} - 7872711530129462016000000C^{12}W^{14} \\
& + 371963224266092288000000C^8W^{15} + 16644085627492665619200000C^{10}W^{14} - 1782532554855058944000000C^6W^{15} \\
& - 185339992744778437387261440C^{12}W^{13} - 1544232903677956990208000C^8W^{14} + 303234425856336384000000C^4W^{15} \\
& + 54128523350761241384454720C^{10}W^{13} + 12403218608035002865152000C^6W^{14} - 6116745929290905600000C^2W^{15} \\
& - 951844150446551538988331520C^{12}W^{12} - 645448733324259938611070400C^8W^{13} - 7045387102492916673024000C^4W^{14} \\
& + 477192944838297600000W^{15} + 3253006042975506684242985840C^{10}W^{12} + 435135586179561514044900480C^6W^{13} \\
& + 1286444415208032871372800C^2W^{14} - 1907253803855129996592406080C^{12}W^{11} - 4339115164801463987040967440C^8W^{12} \\
& - 168270639881667971707117440C^4W^{13} + 16683081489502147123200W^{14} + 8552213798373683964943276860C^{10}W^{11} \\
& + 3082131537247491250096481760C^6W^{12} + 22736359174755008435427648C^2W^{13} - 48851664139607778387708320C^{12}W^{10} \\
& - 12387438959286542633658483300C^8W^{11} - 1173355032106069107972873120C^4W^{12} + 678814944982416636662592W^{13} \\
& + 9442624771215118024165496565C^{10}W^{10} + 8549562567471183382773826296C^6W^{11} + 126120904465842058810565808C^2W^{12} \\
& + 4476545227796159600063351370C^{12}W^9 - 12816384899059482992158075875C^8W^{10} - 2870397059593854524603492520C^4W^{11} \\
& + 11941065526590750406830768W^{12} - 2611642174953699581834474565C^{10}W^9 + 2883619678083172972013631834C^6W^{10} \\
& - 17832218518281800392261812C^2W^{11} + 8513165459868362505679046580C^{12}W^8 + 14159256934864101696407348325C^8W^9 \\
& + 3212487708747017108167081170C^4W^{10} + 104277359673724210458514380W^{11} - 19762016255023840743034016235C^{10}W^8 \\
& - 40168973916378654566575675914C^6W^9 - 2856093015565580708788533423C^2W^{10} + 7452406510423505908342335270C^{12}W^7 \\
& + 54278712481980591915796595025C^8W^8 + 34523015767407707653552703580C^4W^9 + 500788331141176883738585745W^{10} \\
& - 24387607234721104182209296665C^{10}W^7 - 104790849792144369493021147206C^6W^8 - 12702605084184165168875553297C^2W^9 \\
& + 3680069797872284286006416880C^{12}W^6 + 63413581335642468359257391325C^8W^7 + 83450607471509924358133358670C^4W^8 \\
& + 1387844998900493321758208085W^9 - 15349730785861546000659587085C^{10}W^6 - 121224850276486264516033857234C^6W^7 \\
& - 26332751777554734132681995823C^2W^8 + 1072551037709423151056046870C^{12}W^5 + 97739617848745750796957246280C^4W^7 \\
& + 2168303827620520176920241165W^8 - 5485035056301123681887544015C^{10}W^5 - 70351823497545099797596496586C^6W^6 \\
& - 27664651879660837991935065717C^2W^7 + 184034108692628660563477380C^{12}W^4 + 10031134044703570272394957575C^8W^5 \\
& + 55042445497169758579499462670C^4W^6 + 1589302492743279946618996245W^7 - 1111127669637418198035162585C^{10}W^4 \\
& - 17193013005771261991255633374C^6W^5 - 11410330548801276474014816073C^2W^6 + 18035018080287337153034010C^{12}W^3 \\
& + 686672974014099938872229475C^8W^4 + 9556159385584383382627561980C^4W^5 - 127386133586039561560724745W^6 \\
& - 122471234358481643743366335C^{10}W^3 + 99831870873988077377934654C^6W^4 + 2782545402212415742236552333C^2W^5 \\
& + 960956481996966153699000C^{12}W^2 - 165043073123565514553193525C^8W^3 - 3123122292159269893234076430C^4W^4 \\
& - 1081037120893827956586968825W^5 - 7043292366434710772116500C^{10}W^2 + 36764539464297751505364012075C^8W^6 \\
& + 758755509021197524703899746C^6W^3 + 4179626609525920081975321467C^2W^4 + 25375021105540005900000C^{12}W \\
& - 22628627757419291708185260C^8W^2 - 1565315512248802764049949880C^4W^3 - 772287967930696899158500025W^4 \\
& - 195194160176955127650000C^{10}W + 113971412577757849201883544C^6W^2 + 1464554668071090424944934845C^2W^3 \\
& + 256506360520410000000C^{12} - 908829673194349005750000C^8W - 211588750190116070390928960C^4W^2 \\
& - 255430872322258465811060877W^3 - 2034832811246235000000C^{10} + 5199541822249850076540000C^6W \\
& + 206875529588026061126405244C^2W^2 - 11581383635529107400000C^8 + 718570996717529976048000C^4W \\
& - 39841718643201535828316108W^2 + 72440607011841628560000C^6 + 1156088951212713998886000C^2W \\
& + 151666485764497704000000C^4 - 1155094798222516629791600W - 549163725116301935640000C^2 \\
& + 143824693687787460600000
\end{aligned} \right) \quad (308)
\end{aligned}$$

$$\begin{aligned}
b''_{6,0} &= 1019215872(25W+1)(81W+25)(36W+1)(9W+4)(16W+1)(36W+49)(25W+49)(9W+1) \\
&\quad \times (36W+25)(W+4)(16W+49)(4W+1)(81W+100)(W+9)(9W+121)(9W+25)(W+1)(9W+49)(9W+169)
\end{aligned} \quad (309)$$

$$\begin{aligned}
& (2354947182776998656000000000C^8W^{16} - 408494513252304138240000000C^6W^{16} + 2785829833670165428608000000C^8W^{15} \\
& +1679285354062727273600000000C^4W^{16} - 6060621865997133692006400000C^6W^{15} + 2620223078930288640000000C^2W^{16} \\
& -17378710098015594997136160000C^8W^{14} + 1927783719871108133529600000C^4W^{15} + 2451036489396710400000000W^{16} \\
& +5905554635874433145122944000C^6W^{14} + 882912754105504167628800000C^2W^{15} - 425023642062256771304922412800C^8W^{13} \\
& -19977356175207634697618880000C^4W^{14} + 121566753038639503088640000W^{15} + 595157116875976106037369584640C^6W^{13} \\
& +23884481769640739364894336000C^2W^{14} - 2677460892656353182247055251680C^8W^{12} - 464069490514665143702729103360C^4W^{13} \\
& +2753264240009232383706796800W^{14} + 4668415994800580991846160268160C^6W^{12} + 270895328659709430532520785920C^2W^{13} \\
& -8126345952330367807392374023920C^8W^{11} - 3304083255604550341325373958848C^4W^{12} + 35667152339610472936441461504W^{13} \\
& +17516441464532961353645081999040C^6W^{11} + 1601286866797993636380298026624C^2W^{12} - 12517191625627451392289411864805C^8W^{10} \\
& -11548893501703506773159368036416C^4W^{11} + 284293832603204278741157527968W^{12} + 36534409419054621993834443359860C^6W^{10} \\
& +4920514900529079415174074358272C^2W^{11} - 7150449602415738457699510204170C^8W^9 - 18605628321421001004553347785070C^4W^{10} \\
& +1438198274077895729239421379696W^{11} + 41480052937386406614073857844440C^6W^9 + 4822274947293902611975337251092C^2W^{10} \\
& +5820029234238592901113827640245C^8W^8 + 1638008482202344839108704707476C^4W^9 + 4634351791765210087002671101515W^{10} \\
& +18347527208365875626263658324460C^6W^8 - 16956669705160945363256212720776C^2W^9 + 12610445093285994867315826241280C^8W^7 \\
& +57790101330490859399434167945198C^4W^8 + 9199448221933092447601031666214W^9 - 10866962771951081922336205038960C^6W^7 \\
& -67324438836424126165571871100020C^2W^8 + 9108331137609596062414324032645C^8W^6 + 93912166815253391135247082009168C^4W^7 \\
& +9903850004088187643261836669797W^8 - 18223895484773739385700375209140C^6W^6 - 102663564852066192398105960813904C^2W^7 \\
& +3432604812616375265099152629630C^8W^5 + 66119686440329158130920689924206C^4W^6 + 2368805255827180865264176385392W^7 \\
& -944339567839335907198082174760C^6W^5 - 78620636207650735523666007262932C^2W^6 + 711272926620915656741527301595C^8W^4 \\
& +20812757441588290845627592300900C^4W^5 - 7090384004279014375634868479211W^6 - 2340714040654081626599062799340C^6W^4 \\
& -29127948633114282338700392417352C^2W^5 + 79182998149548892480925869980C^8W^3 + 1814153083369344929949960011314C^4W^4 \\
& -8913135633723001396561167126130W^5 - 288021049026278736445756214400C^6W^3 - 3851028027159041098403717962764C^2W^4 \\
& +457514293363873305313002000C^8W^2 - 419824136396543841531037117768C^4W^3 - 4791114268328605863562969989269W^4 \\
& -17602333482661828093618488000C^6W^2 + 305893000334248679706011847840C^2W^3 + 127095694128677145148200000C^8W \\
& -89391968410879895931096096800C^4W^2 - 1356243531381196794725857922676W^3 - 50514196904014598647200000C^6W \\
& +145315867844092578205596072000C^2W^2 + 1326669475840465680000000C^8 - 4432719057239952259870320000C^4W \\
& -199076716946586594529587871600W^2 - 537731321212606752000000C^6 - 834729371247481052999040000C^2W \\
& -63919180872733187232000000C^4 - 7892950345136849386187640000W + 128145088230598676160000000C^2 \\
& +829709942825302607376000000
\end{aligned}$$

(310)

$$\begin{aligned}
b''_{6,2} = & 339738624(25W+1)(81W+25)(36W+1)(9W+4)(16W+1)(36W+49)(25W+49)(9W+1)(36W+25) \\
& \times (W+4)(16W+49)(324W+3025)(4W+1)(25W+144)(81W+100)(W+9)(9W+121)(9W+25)(W+1)(9W+49)
\end{aligned}$$

(311)

$$\begin{aligned}
& (1206237721674585600000000C^8W^{15} - 176296282398593280000000C^6W^{15} - 219751196788778054400000C^8W^{14} \\
& +19026127233872640000000C^4W^{15} - 513139598082348652800000C^6W^{14} + 24712096292271360000000C^2W^{15} \\
& -26693758202673012947496000C^8W^{13} - 1611773884469112940800000C^4W^{14} + 8399524150632960000000W^{15} \\
& +2996538040230113908248000C^6W^{13} + 1354364133150624825600000C^2W^{14} - 232985648922202987874022960C^8W^{12} \\
& -33668443545853265891784000C^4W^{13} + 480455244377394048000000W^{14} + 342964462076369765530224480C^6W^{12} \\
& +22047887106223766157384000C^2W^{13} - 848837911421983581490211820C^8W^{11} - 192121606439387805075981120C^4W^{12} \\
& +10926497002458391832016000W^{13} + 165762490077274509647506300C^6W^{11} + 139336121480221752690169440C^2W^{12} \\
& -1304464982969947975080129225C^8W^{10} + 38426898547695767121266940C^4W^{11} + 126693320579638820651427120W^{12} \\
& +4222671779756837102861670120C^6W^{10} + 147169704274326869965738020C^2W^{11} + 228676038109668874878122970C^8W^9 \\
& +443576189304776661303825110C^4W^{10} + 804425898848728438943546256W^{11} + 5466233455426251190018288260C^6W^9 \\
& -2671174365693257141565238944C^2W^{10} + 3963331449401379776860324185C^8W^8 + 19114450366524626278007065440C^4W^9 \\
& +271900217372405801629824747W^{10} + 1560302545497852319687283760C^6W^8 - 15778630504488680399255975892C^2W^9 \\
& +6427898149468627638264718260C^8W^7 + 38818848721696354630678833810C^4W^8 + 3483674676399908125102808790W^9 \\
& -5545858355148655974981515580C^6W^7 - 41197170755932102504834997112C^2W^8 + 5215652284958241630193835145C^8W^6 \\
& +4261359135832220372997855540C^4W^7 - 6841436518782466040536489827W^8 - 8684780839001795421795547800C^6W^6 \\
& -57937639344703778479870899044C^2W^7 + 2432638486697199380463713850C^8W^5 + 24377580812933987538891475610C^4W^6 \\
& -34655069864110972676424334856W^7 - 5982506578822270876967330820C^6W^5 - 43629012888945576224065441200C^2W^6 \\
& +668760977904117842515626855C^8W^4 + 4775427284747670935338024840C^4W^5 - 59158884342860745586430957851W^6 \\
& -2216744926831431939604606560C^6W^4 - 14819250580947043743216381516C^2W^5 + 106200098316300942105892740C^8W^3 \\
& -2172691796067169285055427490C^4W^4 - 50970159706825133314598296882W^5 - 443100643518598889684636160C^6W^3 \\
& -36585765178630771529102488C^2W^4 + 9184564470895810422426000C^8W^2 - 1443280664950841326940628760C^4W^3 \\
& -219504494343425259737585805W^4 - 44812390698003007848384000C^6W^2 + 1282015238565440722140054432C^2W^3 \\
& +388447787545494276600000C^8W - 286809977534975157028125920C^6W^2 - 404841749911459630523757708W^3 \\
& -209099651916337234400000C^6W + 337722434093139582760850304C^2W^2 + 6156152652489840000000C^8 \\
& -19195905249978170030800000C^4W - 120415780388570361652558384W^2 - 35201563103266560000000C^6 \\
& +36937757590472145897280000C^2W - 395352231947293420800000C^4 + 60075060185298744195198400W \\
& +101794903045295560960000C^2 + 6872849390851475391360000
\end{aligned}$$

(312)

$$b''_{6,4} = 113246208(25W+1)(81W+25)(9W+4)(16W+1)(36W+49)(25W+49)(9W+1)(36W+25)(W+4) \quad (313)$$

$$\times (16W+49)(4W+1)(25W+144)(81W+100)(W+9)(9W+121)(9W+25)(W+1)(9W+49)(9W+169) \quad (314)$$

$$\left(\begin{aligned} &6945683436000347233536000000C^4W^{13} - 5826213001068768498585600000C^2W^{13} + 79550798874432865848887040000C^4W^{12} \\ &- 2049695335645079551948800000W^{13} - 108015498615734411446144512000C^2W^{12} - 542665345813880330941890091200C^4W^{11} \\ &- 84687433514288362211662080000W^{12} - 88648447925900120045678444160C^2W^{11} - 12333136241582043152983992418320C^4W^{10} \\ &- 1305283814679415171746735817920W^{11} + 9476859377277004026248513195040C^2W^{10} - 74314692920441672233657734916260C^4W^9 \\ &- 8949492673228227475043944527888W^{10} + 75273691262787116102435247825480C^2W^9 - 211433118306564709446200503649235C^4W^8 \\ &- 18714070343075267879007502427748W^9 + 262188356261796509073944307819750C^2W^8 - 287953169950706032809552940368120C^4W^7 \\ &+ 1000560344444067139829825828813W^8 + 488481056588419676854132037933520C^2W^7 - 97820772250258342129043528242650C^4W^6 \\ &+ 729332287043115061211673506583528W^7 + 467548152837449227973224818526260C^2W^6 + 214893835528117645650545468752320C^4W^5 \\ &+ 1922144673400616887318083143602790W^6 + 107851904722015465271517839252160C^2W^5 + 293296060117974607456227744531285C^4W^4 \\ &+ 2537356330948528618636194200477888W^5 - 214939042618113417654364006653930C^2W^4 + 153538170417150408316750544668260C^4W^3 \\ &+ 1651840499290671180371779533055413W^4 - 208206562479961352483430615969000C^2W^3 + 36911419195590811126741232743920C^4W^2 \\ &+ 37601609514535011513840656182852W^3 - 67944290226679573941566308065120C^2W^2 + 3601048174998801560987045400000C^4W \\ &- 74989123733128402121443512458128W^2 - 7592576276444421458567348208000C^2W + 114099586630247714133052800000C^4 \\ &- 40562436768819142900841377089600W - 256603857035209557297177600000C^2 - 4435897751866381076275063680000 \end{aligned} \right) \quad (315)$$

$$b''_{6,6} = 3408890469285888(9W+4)(16W+1)(36W+49)(25W+49)(9W+1)(36W+25)(W+4) \quad (316)$$

$$\times (16W+49)(324W+3025)(4W+1)(25W+144)(81W+100)(W+9)(9W+121)(9W+25)(W+1)(9W+49) \quad (317)$$

$$\left(\begin{aligned} &656605984320000C^4W^{10} - 271481320440000C^2W^{10} - 2651695161429600C^4W^9 \\ &- 591111195480000W^{10} - 1742078992074300C^2W^9 - 140334404558304528C^4W^8 \\ &- 21010357938005100W^9 + 33530371782286401C^2W^8 - 953701260785499798C^4W^7 \\ &- 192560249325004803W^8 + 460857296017447461C^2W^7 - 2311844550476689800C^4W^6 \\ &+ 344732707367700687W^7 + 2261825300468003490C^2W^6 - 614916889631761860C^4W^5 \\ &+ 1186982972679375090W^6 + 5154926124621803310C^2W^5 + 6101289921518957880C^4W^4 \\ &+ 56367647795119161450W^5 + 4523722130119964805C^2W^4 + 9639174337511382378C^4W^3 \\ &+ 105751013648293561065W^4 - 1767279931269909111C^2W^3 + 5436935907769265328C^4W^2 \\ &+ 68316937811328678563W^3 - 5358385074786712056C^2W^2 + 1139654379803940000C^4W \\ &- 15822143944621449272W^2 - 2554177748875830000C^2W + 71791867667520000C^4 \\ &- 26416383158922689680W - 220675225811040000C^2 - 5025832933369248000 \end{aligned} \right) \quad (318)$$

$$b''_{6,8} = 113246208(25W+49)(9W+1)(36W+25)(W+4)(16W+49)(4W+1) \quad (319)$$

$$\times (25W+144)(81W+100)(W+9)(9W+121)(9W+25)(W+1)(9W+49)(9W+169) \quad (320)$$

$$a''_{6,10} = -385 \left(\begin{aligned} &7400808000W^6 + 20867765940W^5 - 951360644511W^4 \\ &- 5733913962454W^3 - 5780532438679W^2 + 7312426327404W \\ &+ 5315523998400 \end{aligned} \right) \quad (321)$$

$$b''_{6,10} = 25165824(16W+49)(324W+3025)(4W+1)(25W+144) \quad (322)$$

$$\times (81W+100)(W+9)(9W+121)(9W+25)(W+1)(9W+49) \quad (323)$$

$$a''_{6,12} = -1001(81W^2 - 1710W + 2689) \quad (324)$$

$$b''_{6,12} = 12582912(9W+169)(9W+49)(W+1)(9W+25)(9W+121)(W+9) \quad (325)$$

Table I: Dimensional Variables.

Variable	Dimensions	Symbol
Cartesian coordinate, distance from stationary plate	L	y
Cartesian coordinate, flow direction	L	x
Cartesian coordinate, transverse to flow direction	L	z
Frequency of oscillation	t^{-1}	ω
Relaxation time of fluid	t	λ
Shear rate	t^{-1}	$\dot{\gamma}(t)$
Shear rate magnitude	t^{-1}	$\dot{\gamma}^0$
Time	t	t
Velocity, i th component	L/t	v_i

Legend: M = mass; L = length; t = time; T = temperature

Table II: Dimensionless Variables and Groups.

Abbreviation for $\cos\theta$ in spherical coordinates	C
Abbreviation for $\sin j\phi$ in spherical coordinates	s_j
Abbreviation for $\cos j\phi$ in spherical coordinates	c_j
Associated Legendre polynomials	P_n^m
Azimuthal angle	ϕ
Component in-phase with $\cos n\omega t$	ψ'_n
Component out-of-phase with $\cos n\omega t$	ψ''_n
Orientation distribution expansion, i th term	$\psi_i(\theta, \phi, t)$
Orientation distribution function	$\psi(\theta, \phi, t)$
Paren functions	$(a, b, \dots; t)$
Polar angle	θ
Time dependent part of shear rate for velocity profile, assumed linear in y	$f(t)$

Table III: The associated Legendre polynomials. P_n^m in terms of $C \equiv \cos\theta$.

$P_6^0(C) = \frac{1}{16}(231C^6 - 315C^4 + 105C^2 - 5)$
$P_6^4(C) = \frac{945}{2}(C-1)^2(C+1)^2(11C^2 - 1)$
$P_8^0(C) = \frac{6435}{128}C^8 - \frac{3003}{32}C^6 + \frac{3465}{64}C^4 - \frac{315}{32}C^2 + \frac{35}{128}$
$P_8^2(C) = -\frac{315}{16}(C-1)(C+1)(143C^6 - 143C^4 + 33C^2 - 1)$
$P_8^4(C) = \frac{10395}{8}(C-1)^2(C+1)^2(65C^4 - 26C^2 + 1)$
$P_8^6(C) = -\frac{135135}{2}(C-1)^3(C+1)^3(15C^2 - 1)$
$P_8^8(C) = 2027025(1 - C^2)^4$
$P_{10}^0(C) = \frac{46189}{256}C^{10} - \frac{109395}{256}C^8 + \frac{45045}{128}C^6 - \frac{15015}{128}C^4 + \frac{3465}{256}C^2 - \frac{63}{256}$
$P_{10}^2(C) = -\frac{495}{128}(C-1)(C+1)(4199C^8 - 6188C^6 + 2730C^4 - 364C^2 + 7)$
$P_{10}^4(C) = \frac{45045}{16}(C-1)^2(C+1)^2(323C^6 - 255C^4 + 45C^2 - 1)$
$P_{10}^6(C) = -\frac{675675}{8}(C-1)^3(C+1)^3(323C^4 - 102C^2 + 3)$
$P_{10}^8(C) = \frac{34459425}{2}(C-1)^4(C+1)^4(19C^2 - 1)$
$P_{10}^{10}(C) = 654729075(1 - C^2)^5$
$P_{12}^0(C) = \frac{676039}{1024}C^{12} - \frac{969969}{512}C^{10} + \frac{2078505}{1024}C^8 - \frac{255255}{256}C^6$ $+ \frac{225225}{1024}C^4 - \frac{9009}{512}C^2 + \frac{231}{1024}$
$P_{12}^4(C) = \frac{135135}{128}(C-1)^2(C+1)^2(7429C^8 - 9044C^6 + 3230C^4 - 340C^2 + 5)$
$P_{12}^8(C) = \frac{654729075}{8}(C-1)^4(C+1)^4(161C^4 - 42C^2 + 1)$
$P_{12}^{12}(C) = 316234143225(1 - C^2)^6$

Figure 1: The dumbbell orientation is given in terms of the spherical coordinates θ and ϕ . The dumbbell consists of identical beads (of identical mass and of identical diameter) connected by one thin, absolutely rigid (dimensionless, massless) rod.

Figure 2: Tumbling motion of rigid dumbbell beads in response to large-amplitude oscillatory shear flow.

Figure 3: Wobbling motion of rigid dumbbell beads in response to large-amplitude oscillatory shear flow.

Figure 4: Zeroth harmonic of ψ , ψ'_0 , from Eq. (24), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_0 , ψ_2 , ψ_4 , and ψ_6 contribute.

Figure 5: Zeroth harmonic of ψ , ψ'_0 , from Eq. (24), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_0 , ψ_2 , ψ_4 , and ψ_6 contribute. Side view.

Figure 6: Zeroth harmonic of ψ , ψ'_0 , from Eq. (24), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_0 , ψ_2 , ψ_4 , and ψ_6 contribute. Top view.

Figure 7: Complex first harmonic of ψ (A), $\psi'_1 \cos \omega t + \psi''_1 \sin \omega t$ with Eqs. (26) and (27), and first harmonic components, in-phase (B) and out-of-phase (C) with the shear rate, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_1 , ψ_3 , and ψ_5 contribute.

Figure 8: Complex first harmonic of ψ (A), $\psi'_1 \cos \omega t + \psi''_1 \sin \omega t$ with Eqs. (26) and (27), and first harmonic components, in-phase (B) and out-of-phase (C) with the shear rate, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_1 , ψ_3 , and ψ_5 contribute. Side view.

Figure 9: Complex first harmonic of ψ (A), $\psi'_1 \cos \omega t + \psi''_1 \sin \omega t$ with Eqs. (26) and (27), and first harmonic components, in-phase (B) and out-of-phase (C) with the shear rate, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_1 , ψ_3 , and ψ_5 contribute. Top view.

Figure 10: Complex second harmonic of ψ (A), $\psi'_2 \cos \omega t + \psi''_2 \sin \omega t$ with Eqs. (31) and (32), and second harmonic components, in-phase (B) and out-of-phase (C) with $\cos 2\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_2 , ψ_4 , and ψ_6 contribute.

Figure 11: Complex second harmonic of ψ (A), $\psi'_2 \cos \omega t + \psi''_2 \sin \omega t$ with Eqs. (31) and (32), and second harmonic components, in-phase (B) and out-of-phase (C) with $\cos 2\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_2 , ψ_4 , and ψ_6 contribute. Side view.

Figure 12: Complex second harmonic of ψ (A), $\psi'_2 \cos \omega t + \psi''_2 \sin \omega t$ with Eqs. (31) and (32), and second harmonic components, in-phase (B) and out-of-phase (C) with $\cos 2\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_2 , ψ_4 , and ψ_6 contribute. Top view.

Figure 13: Complex third harmonic of ψ (A), $\psi'_3 \cos \omega t + \psi''_3 \sin \omega t$ with Eqs. (35) and (36), and third harmonic components, in-phase (B) and out-of-phase (C) with $\cos 3\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_3 , and ψ_5 contribute.

Figure 14: Complex third harmonic of ψ (A), $\psi'_3 \cos \omega t + \psi''_3 \sin \omega t$ with Eqs. (35) and (36), and third harmonic components, in-phase (B) and out-of-phase (C) with $\cos 3\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_3 , and ψ_5 contribute. Side view.

Figure 15: Complex third harmonic of ψ (A), $\psi'_3 \cos \omega t + \psi''_3 \sin \omega t$ with Eqs. (35) and (36), and third harmonic components, in-phase (B) and out-of-phase (C) with $\cos 3\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_3 , and ψ_5 contribute. Top view.

Figure 16: Complex fourth harmonic of ψ (A), $\psi'_4 \cos \omega t + \psi''_4 \sin \omega t$ with Eqs. (37) and (38), and fourth harmonic components, in-phase (B) and out-of-phase (C) with $\cos 4\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_4 , and ψ_6 contribute.

Figure 17: Complex fourth harmonic of ψ (A), $\psi'_4 \cos \omega t + \psi''_4 \sin \omega t$ with Eqs. (37) and (38), and fourth harmonic components, in-phase (B) and out-of-phase (C) with $\cos 4\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_4 , and ψ_6 contribute. Side view.

Figure 18: Complex fourth harmonic of ψ (A), $\psi'_4 \cos \omega t + \psi''_4 \sin \omega t$ with Eqs. (37) and (38), and fourth harmonic components, in-phase (B) and out-of-phase (C) with $\cos 4\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_4 , and ψ_6 contribute. Top view.

Figure 19: Complex fifth harmonic of ψ (A), $\psi'_5 \cos \omega t + \psi''_5 \sin \omega t$ with Eqs. (39) and (40), and third harmonic components, in-phase (B) and out-of-phase (C) with $\cos 5\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_5 contribute.

Figure 20: Complex fifth harmonic of ψ (A), $\psi'_5 \cos \omega t + \psi''_5 \sin \omega t$ with Eqs. (39) and (40), and third harmonic components, in-phase (B) and out-of-phase (C) with $\cos 5\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_5 contribute. Side view.

Figure 21: Complex fifth harmonic of ψ (A), $\psi'_5 \cos \omega t + \psi''_5 \sin \omega t$ with Eqs. (39) and (40), and third harmonic components, in-phase (B) and out-of-phase (C) with $\cos 5\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime, where only ψ_5 contribute. Top view.

Figure 22: Complex sixth harmonic of ψ (A), $\psi'_6 \cos \omega t + \psi''_6 \sin \omega t$ with Eqs. (41) and (42), and sixth harmonic components, in-phase (B) and out-of-phase (C) with $\cos 6\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_6 contribute.

Figure 23: Complex sixth harmonic of ψ (A), $\psi'_6 \cos \omega t + \psi''_6 \sin \omega t$ with Eqs. (41) and (42), and sixth harmonic components, in-phase (B) and out-of-phase (C) with $\cos 6\omega t$, over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\gamma^0 = 1$, nonlinear viscoelastic regime, where only ψ_6 contribute. Side view.

Figure 25: Sum of first seven harmonics of ψ , $\sum_{n=0}^6 [\psi'_n \cos n\omega t + \psi''_n \sin n\omega t]$, Eq. (43), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime.

Figure 26: Sum of first seven harmonics of ψ , $\sum_{n=0}^6 [\psi'_n \cos n\omega t + \psi''_n \sin n\omega t]$, Eq. (43), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime. Side view.

Figure 27: Sum of first seven harmonics of ψ , $\sum_{n=0}^6 [\psi'_n \cos n\omega t + \psi''_n \sin n\omega t]$, Eq. (43), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime. Top view.

Figure 28: Zeroth harmonic of ψ , ψ'_0 , from Eq. (24), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, where only ψ_0 , ψ_2 , ψ_4 , and ψ_6 contribute, nonlinear viscoelastic regime. Multimedia view.

Figure 29: Complex first harmonic of ψ (A), $\psi'_1 \cos \omega t + \psi''_1 \sin \omega t$ with Eqs. (26) and (27), where only ψ_1 , ψ_3 , and ψ_5 contribute, nonlinear viscoelastic regime. Multimedia view.

Figure 30: Complex second harmonic of ψ (A), $\psi'_2 \cos \omega t + \psi''_2 \sin \omega t$ with Eqs. (31) and (32), where only ψ_2 , ψ_4 , and ψ_6 contribute, nonlinear viscoelastic regime. Multimedia view.

Figure 31: Complex third harmonic of ψ (A), $\psi'_3 \cos \omega t + \psi''_3 \sin \omega t$ with Eqs. (35) and (36), where only ψ_3 , and ψ_5 contribute, nonlinear viscoelastic regime. Multimedia view.

Figure 32: Complex fourth harmonic of ψ (A), $\psi'_4 \cos \omega t + \psi''_4 \sin \omega t$ with Eqs. (37) and (38), where only ψ_4 , and ψ_6 contributes, nonlinear viscoelastic regime. Multimedia view.

Figure 33: Complex fifth harmonic of ψ (A), $\psi'_5 \cos \omega t + \psi''_5 \sin \omega t$ with Eqs. (39) and (40), where only ψ_5 contributes, nonlinear viscoelastic regime. Multimedia view.

Figure 34: Complex sixth harmonic of ψ (A), $\psi'_6 \cos \omega t + \psi''_6 \sin \omega t$ with Eqs. (41) and (42), where only ψ_6 contribute, nonlinear viscoelastic regime. Multimedia view.

Figure 35: Sum of first seven harmonics of ψ , $\sum_{n=0}^6 [\psi'_n \cos n\omega t + \psi''_n \sin n\omega t]$, Eq. (43), over a complete alternant cycle, $\omega t = 0$ to $\omega t = 2\pi$, with $\lambda\omega = 1$ and $\lambda\dot{\gamma}^0 = 1$, nonlinear viscoelastic regime. Multimedia view.

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Eq.(62), $\int_{-\infty}^0 \int_{-\infty}^{s'}$ should be $\int_0^{\infty} \int_{s'}^{\infty}$; In Eqs. (88) through (91), " $+(\lambda\dot{\gamma}^0)^2$ " should be " $-(\lambda\dot{\gamma}^0)^2$ "; In captions to Figs. 7 and 8, "Coefficients" should be "Minus coefficients".

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