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Substructuring Verification of a Rear Fuselage Mounted Twin-Engine Aircraft

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Abstract

Dynamic substructuring allows for the reduction of large complex structures into substructures to increase computational efficiency and to isolate the local dynamic behaviors of concern. However, errors such as truncation, continuity and rigid body mode errors still limit the applicability of this method experimentally. Additionally, the feasibility of implementing substructuring techniques on finite element models of multi-component fuselage structures has yet to be shown in the literature. The objective of this paper is twofold: first to introduce a feasible substructuring methodology that mitigates the experimental and multi-component limitations with current methods; and secondly, to investigate the modal properties and applicability of substructuring analysis on a rear fuselage mounted twin-engine aircraft. This configuration is not well understood in the literature despite having been shown to have increased interior cabin noise and vibration levels. Experimental validation of the computational model was first performed. A substructuring analysis of the validated computational model produced natural frequencies of the local and global modes that agreed within 6.47% on average, and pseudo-orthogonality terms greater than 0.89 for all modes considered. This methodology proved to be useful for generating an accurate representation of local modes within a global structure for the aircraft configuration studied. This will allow for future work to more thoroughly investigate the local modes using innovative design methods with confidence that the local modes will correlate with the global modes.

Keywords: substructuring, orthogonality, modal analysis, aft fuselage

mounted engines

Nomenclature

$[I]$	=	Identity Matrix
$[M]$	=	Mass Matrix [kg]
$[\Gamma]$	=	Transformation Matrix
$\{\phi\}$	=	UMM Modal Shape Vector
$[\Phi]$	=	UMM Modal Shape Matrix
Subscripts		
A	=	analytical modes
A _{active}	=	experimentally active components of the analytical modal shape matrix
A _{deleted}	=	experimentally inactive components of the analytical modal shape matrix
g	=	global modes
l	=	local modes
X	=	experimental modes

1. Introduction

1.1. Dynamic Substructuring

Dynamic substructuring (DS) was introduced into the literature by Hurty [1] to provide a method of reducing the structural modes of a global system to local component modes. This methodology proved a viable means of analyzing the dynamic behavior of large, complex structures that are difficult to represent either analytically or experimentally. Using this technique, the analysis becomes more efficient and allows for an increased level of detail to be investigated in the component of interest. It can also allow for multiple research groups to collaborate on a global system, generating much faster results. For these reasons, DS research was furthered by Przemieniecki [2] and Hurty [3] into what eventually became a branch of substructuring analysis known as component-mode

synthesis (CMS). Three of the most common and wide-spread CMS methods were introduced by Craig and Bampton [4], MacNeal [5], and Rubin [6].

15 The Craig-Bampton method [4] is widely used in the aerospace industry. This method reduces the system by modeling only the adjoining degrees of freedom (DOFs) between the two substructures. The limitation to this method is the means of experimental validation of local subsystems. Experimental data taken at contact points is prone to error due to coupling effects between the
20 two systems [7]. Additionally, when dealing with large scale subsystems with several interacting systems, it becomes infeasible to manually define all adjoining DOFs. To mitigate the experimental issues, Allen and Mayes [8] introduced the Modal Constraint for Fixture and Subsystem (MCFS) approach. This method avoids the use of physical coordinates by connecting the substructures with
25 modal coordinates. This approach allows for the experimental substructure to be connected with finite element substructures, and has been shown to be compatible with substructures with multiple connections [9].

A review of substructuring techniques was performed by de Klerk et al. [10]. The authors highlight that despite being introduced over four decades ago at
30 the time of the publication, DS research is still ongoing. Specifically, the authors highlighted the difficulties in experimental substructuring that include truncation errors, continuity errors, rigid body mode errors, and experimental errors, amongst others. These errors are inherent to experimental modal analysis, and a well-executed experiment can only attempt to minimize them. The authors
35 suggest the use of the system equivalent reduction expansion process (SEREP) method [11] in attempting to mitigate continuity errors.

The SEREP method was introduced by O’Callahan et al. in 1989 to mitigate continuity errors present in common model reduction techniques such as Guyan and Dynamic reduction [11]. The SEREP method uses a transformation matrix
40 containing all available information, both the active and deleted sets of data. The SEREP method is the only method able to provide an exact means of mapping between the computational and experimental DOFs. The benefit of SEREP over alternative model reduction methods is well-established. O’Callahan and

Li [12], O’Callahan [13], Friswell and Inman [14], Friswell [15], and Aglietti et
 45 al. [16] have all demonstrated the ability of the SEREP method to overcome
 truncation errors when compared to other model reduction techniques.

Recent work performed by Rohe and Allen [17], Manzato et al. [18], and
 D’Ambrogio et al. [19] attempted to mitigate some of the issues with experi-
 mental substructuring. Rohe and Allen proposed a model validation test in an
 50 attempt to validate the substructuring model with experimental data. How-
 ever, the data used in this work was purely computational, with random error
 applied to simulate experimental results. The work performed by Manzato and
 D’Ambrogio focused on mitigating the experimental errors in Frequency Based
 Substructuring (FBS). FBS is a branch of substructuring that uses the frequency
 55 response functions (FRFs) of the substructure in an attempt to correlate the two
 sets of data. The authors attempted to mitigate the effect of hard to measure
 DOF locations by optimally selecting a subset of internal DOFs. The limitation
 of experimental FBS is that this methodology is more prone to measurement
 errors, as these errors get magnified during the matrix inversion process inherent
 60 to this method. The CMS method uses the modal parameters of the structure
 that are less affected by random measurement errors [20]. Despite recent efforts,
 experimental substructuring methods still contain significant issues making it
 difficult to apply in practice.

This work introduces a novel methodology to experimentally validate a CMS
 65 substructuring analysis. This method will address the truncation, continuity
 and rigid body mode errors prevalent in experimental dynamic substructuring
 identified by de Klerk et al. [10] as follows. The truncation errors will be min-
 imized by comparing two full-sized computational modal vectors that contain
 orders of magnitude more DOFs than the experimental modal vectors used in
 70 traditional experimental DS methods. The continuity errors will be minimized
 by using the SEREP method to perform the initial experimental validation of
 the FE model to accurately represent the entire computational model in a re-
 duced subset of DOFs. Additionally, continuity will be ensured by comparing
 the same DOFs for all analyses. The rigid body mode errors will be eliminated

75 by performing the experimental data collection with free-free boundary conditions to include the complete set of modal vectors for validation. Minimization of the above errors will be accomplished by introducing a new methodology that has two main steps. First, the finite element model of the local subsections will be validated by experimental data collected with free-free boundary conditions.
 80 Second, modified boundary conditions will be applied to the validated computational local subsystems, and the modal properties will be compared with the computational global system. Well established methods of model validation such as the pseudo-orthogonality check (POC) will be implemented throughout the analysis to prove orthogonality, first between experimental and computational local modes, then between computational local and global modes.
 85 This methodology improves upon the MCFS method by eliminating the rigid body mode error and the non-physical complex modes observed due to the fixed experimental boundary conditions [9]. Additionally, this work provides a more thorough comparison of the substructuring analysis by including the POC to
 90 prove the orthogonality between local and global modes, as opposed to examining only FRFs and damping ratios as performed in previous work [8, 9]. This method overcomes the limitations of the Craig-Bampton method by comparing feasible experimental DOFs and providing a feasible method of substructuring a system with multiple connection points. This methodology also bypasses the
 95 necessity to model all adjoining contact areas within the global system, as only the local substructures of interest are needed.

1.2. Fuselage Mounted Twin-Engine Aircraft

Extensive noise and vibration research has been performed at the NASA Langley Research Center investigating interior cabin noise. Early work per-
 100 formed on single engine light aircraft showed that cabin noise is a function of two sources: the mechanical (structure-borne) source and the acoustic source [21, 22, 23, 24]. It was also shown that the two sources can be treated independently, and the total interior noise can be calculated as a logarithmic sum of the noise from the two sources [22, 25, 26]. From this point, research shifted to

105 focus on the two sources in isolation. Modal analysis techniques were used to extract the modal properties of the fuselage structure, while acoustic measurements were used to better understand the acoustic response of the fuselage to the airborne source.

Throughout these years of analysis, there was minimal work performed on rear fuselage engine mounted configurations. Of the work that has been done, 110 it has been demonstrated that structure-borne engine excitations are of greater importance due to the reduced transmission path from the engine to the fuselage [27, 28]. Wing mounted engines have been shown to have a reduced structure-borne component due to the added damping provided by the fuel stored in the wings [25, 29]. Due to the proximity of the engines to the fuselage, more discrete frequency bands were noticed in the fuselage response [28]. This indicates the increased importance that the turbine rotational frequency and the blade passing frequency of the engine has on the response of the fuselage for this configuration.

120 To determine the effect of noise and vibration on passenger comfort levels, NASA proposed the Ride Comfort Model [30]. The noise and vibration stimuli are treated as two separate sources in this model. Each passenger has a subjective psycho-physical discomfort level associated with the noise and vibration level independently. Once the passenger has assessed their discomfort level to the noise and vibration stimuli using separate estimates, the two stimuli 125 can be combined to give an overall representation of passenger discomfort. It was shown that tonal noise sources increased passenger annoyance beyond what was observed for a broadband noise source at an equivalent sound pressure level [31, 32, 33]. It has also been shown that the perceived discomfort level was more significantly influenced by vibration stimulus than noise stimulus 130 [34]. Considering fuselage mounted engines contribute both increased tonal noise, and increased vibration levels over wing mounted engines, the ride comfort model would suggest that fuselage mounted engines would lead to a decreased passenger comfort level.

135 Recently, research has trended towards investigating the use of advanced

design techniques. Krog et al. [35] was amongst the first to implement topology optimization in aircraft design. They implemented topology optimization on an Airbus A350 leading edge rib to minimize the compliance of the structure subject to a specified reduction in mass. This allowed for the stiffness of the rib to be maximized while simultaneously reducing the mass therefore providing substantial fuel savings without sacrificing structural integrity. Remouchamps et al. [36] expanded upon this work by performing a bi-level optimization scheme including both geometric and topology optimization on an Airbus A350 engine pylon. Martins and Lambe [37] performed a survey on the use of multidisciplinary design optimization (MDO) in the aerospace industry. Zhu et al. [38] performed a survey on the recent advances of topology optimization techniques applied to aircraft design. The implementation of topology optimization to dynamic response design was discussed. The authors mention the potential of topology optimization as a method of shifting the natural frequencies of an aerospace structure outside of the frequency range of concern. Warwick et al. [39] was the first to successfully accomplish this on a pre-stiffened aircraft bulkhead. The limitation of this work and of all topology optimization work on a rear fuselage mounted twin-engine aircraft to date is the assumption that the analysis performed on a local aircraft component will be consistent when the component is assembled inside the fuselage. The computational demand of topology optimization necessitates the analysis to be performed on a local aircraft component. Therefore, there exists a need for a substructuring analysis of a rear fuselage mounted-twin engine aircraft to analyze the validity of this assumption allowing for advanced tools like topology optimization to be used with confidence in future work.

The objective of this research is to introduce a novel hybrid substructuring verification technique to verify the modal analysis results of a CMS substructuring analysis on a rear fuselage mounted twin-engine aircraft. The hybrid methodology contains a combination of both experimental and computational substructuring techniques, allowing for the truncation, continuity and rigid body mode errors prevalent in experimental dynamic substructuring to be overcome

using computational analysis, while still allowing for experimental data to be used validate the computational data. The limited data available on this configuration adds to the relevance of this research. This investigation will provide
 170 information on the dynamic properties of this configuration that are currently not well understood. Verification of the substructuring analysis will allow for more computationally efficient research to be performed on the local components of this configuration with confidence that the results will be physically significant to the assembly. Innovative design methodologies such as topology
 175 optimization can then be applied to this configuration in future work with the confidence established by the error bounds presented herein. This will eventually lead to the reduction of the vibration levels of fuselage mounted twin-engine aircraft, and a more comfortable enjoyable ride for the passengers.

2. Methodology

180 2.1. Experimental Validation

The rear fuselage mounted twin-engine aircraft assembly consists of three main components: the bulkhead, and the front and rear engine support frames (FESF and RESF). These three components were modeled computationally using the commercial finite element modeling software, ANSYS [40]. Experimental
 185 modal analysis was performed to validate the computational models for the bulkhead and the FESF. The experimental data for the RESF was collected by Chamberlain and Mechefske [41] and the computational model was validated by Warwick et al. [42]. The following subsection will cover the model validation procedure in depth.

190 2.2. Model Validation Techniques

The validation of a finite element model with experimental modal analysis results is a well-studied methodology. In the text, “*Modal Testing: Theory and Practice*” D.J. Ewins explains this methodology in detail [43]. The parameters

of correlation are the natural frequencies (eigenvalues) and mode shapes (eigen-
 195 vectors) of the two data sets. The methodology can be summarized as follows,
 with the methods performed in sequence of least to most rigorous.

2.2.1. Determination of Correlated Mode Pairs (CMPs)

The first step is to identify which modes of the computational model were
 excited experimentally. This is done by qualitative comparison of the computa-
 200 tional mode shapes with the experimental mode shapes. The quantity of CMPs
 is limited to the frequency bandwidth under consideration, and the experimen-
 tal testing methodology. Only modes within the experimental frequency band-
 width can be identified. In addition, depending on the experimental suspension
 method, the boundary conditions may not be perfectly free-free as intended.
 205 It is therefore not recommended to correlate any modes less than five times
 the first rigid body mode, as there may be contamination of rigid body modes
 into the flexible modes. Once the CMPs have confidently been determined, the
 quantitative analysis can commence.

2.2.2. Natural Frequency Comparison

210 The experimental natural frequencies are plotted against the computational
 natural frequencies, and a linear regression is performed to determine the quality
 of the correlation between the two data sets. Perfect correlation would result in
 a slope of one, however, in practice, a slope of 0.9 is deemed acceptable due to
 the limitations of experimental data collection.

215 Alternatively, the natural frequencies of each CMP can be directly com-
 pared. Depending upon the application, the target percentage difference be-
 tween the computational and experimental modes may vary. An application
 that is primarily concerned with the frequency value may have tighter spec-
 ifications, while an application more concerned with the local distribution of
 220 displacements within specific mode shapes may place less importance on the
 correlation between the natural frequency values.

2.2.3. Modal Assurance Criterion (MAC)

The MAC is a quantitative comparison of the correlation between the two sets of mode shapes. The equation is given in Eq. (1).

$$MAC = \frac{|\{\phi\}_X^T \{\phi\}_A|^2}{(\{\phi\}_X^T \{\phi\}_X)(\{\phi\}_A^T \{\phi\}_A)} \quad (1)$$

where $\{\phi\}_X$ is a vector of the experimental unit modal mass (UMM) normalized mode shapes, and $\{\phi\}_A$ is a vector of the analytical UMM normalized mode shapes. The MAC matrix provides a measure of the least squares deviation of the modal vectors from a perfect correction. This parameter quantifies the degree of correlation between two sets of modal vectors. However, it provides a measure of correlation and not proof of correlation. For more information on the proper use of the MAC matrix see Allemang [44].

2.2.4. Pseudo-Orthogonality Check (POC)

To prove correlation, a pseudo-orthogonality check (POC) must be performed to prove that the experimental eigenvectors are orthogonal to the mass matrix and the analytical eigenvectors. This relationship is given in Eq. (2).

$$[\Phi]_A^T [M] [\Phi]_X = [I] \quad (2)$$

where $[\Phi]_A$ is a matrix of the analytical UMM normalized mode shape vectors, and $[\Phi]_X$ is a matrix of the experimental UMM normalized mode shape vectors. This relationship is regarded as the most rigorous test of correlation between experimental and computational data, as the experimental eigenvectors are used along with the analytical eigenvectors and mass matrix to prove a relationship that can be derived from the equation of motion for free vibration.

However, performing this check comes with inherent complications. The most prominent complication arises from the intrinsic difference in size of the experimental and computational eigenvectors. The experimental eigenvectors contain one term for each experimentally measured DOF. The computational eigenvector contains one term for each computational DOF. There are 6 DOFs

for each node in the finite element mesh. Consequently, the mass matrix is an $n \times n$ matrix where n is the number of DOFs in the computational model. For Eq. 2 to be mathematically valid, both the computational eigenvectors and mass matrix needs to be reduced to match the number and location of the experimental DOFs. Precision may be lost as not all reduction methods are exact. Additionally, when a large amount of mass is reduced to a measurement location that is highly flexible, it may cause errors in the system. These errors are due to the measurement location being weighted heavily in the orthogonality calculation, but may represent only a subsidiary motion of the overall modal vector [44]. This added error is unavoidable and must be considered when assessing the results of the POC.

There are several matrix reduction techniques commonly used to reduce the computational DOFs down to the experimental DOFs. The original method is the Guyan reduction scheme [45]. The limitation of this reduction scheme is that the transformation matrix only involves the stiffness matrix, and is therefore only exact for static analyses.

The system equivalent reduction expansion process (SEREP) method was formulated to overcome this issue [11]. It uses a combination of the active and deleted degrees of freedom to create a transformation matrix as defined in Eq. 3 [46].

$$[\Gamma] = \begin{bmatrix} [\Phi_{Active}] \\ [\Phi_{Deleted}] \end{bmatrix} [\Phi_{Active}]^\dagger \quad (3)$$

where $[\Gamma]$ is the reduction/expansion or transformation matrix, $[\Phi_{Active}]$ contains the components of the analytical mode shape matrix that contain the experimental DOFs, and $[\Phi_{Deleted}]$ contains the components of the analytical mode shape matrix that do not contain the experimental DOFs and therefore need to be removed from the system in the reduction process. The SEREP provides an exact means of mapping between the analytical DOFs and the experimentally active DOFs.

It is important to consider all aspects of the correlation to validate the

275 computational model. The qualitative correlation is just as important as the
 MAC and POC, as the assessment of CMPs sets the framework for more in
 depth quantitative analyses. Likewise, it is important to consider both the
 MAC and POC results, as both have their advantages and disadvantages. The
 overall quality of correlation between the two data sets is only understood when
 280 all correlation techniques are considered together.

2.3. Substructuring Verification

Upon validation of all three components, fixed boundary conditions were
 implemented on the outer surface area of the frames of each component to
 replicate the interaction of each component with the skin of the fuselage. A
 285 modal analysis was performed on the local components with fixed boundary
 conditions. A modal analysis was performed on the assembled fuselage with
 free-free boundary conditions. Finally, the individual modal analysis results of
 each local component were compared to the modal analysis results of the global
 assembled structure. The same methodology used for validation of the computa-
 290 tional model described in the previous subsection was introduced as a method
 to verify the substructuring analysis. The advantage to this methodology is
 the consistency between the experimental local DOFs, the computational local
 DOFs, and the global DOFs. This consistency allows for rigorous mathematical
 correlation checks to be performed at all stages of the CMS verification pro-
 295 cess. The rigor of this methodology allows for errors common to experimental
 substructuring methods to be avoided.

3. Setup

3.1. Fuselage Design

A half-scale mock-up of a twin-engine rear fuselage aircraft was developed
 300 based off a generalized 13 passenger business jet. The mock-up was half-scaled
 due to experimental limitations. As a result, the half-scaled model was used
 for all experimental testing and computational modelling in this work. The

mock-up contains 12 frames, 36 stringers, a front and rear engine support frame (FESF and RESF), a front and rear bulkhead, and the skin. The FESF and RESF protrude through the fuselage skin and form the engine mount location, as indicated by the rectangle in Fig. 1. The front bulkhead separates the passenger cabin from the unpressurized aft section of the aircraft. The primary components of focus are the two engine support frames and the front bulkhead, as the engine provides the source of vibration and the main transmission path to the passenger cabin is from the FESF and RESF to the front bulkhead. Four of the twelve frames are attached to the front/rear bulkheads, and the FESF and RESF. The frames provide a method of riveting these components to the skin. From this point forward, this mock-up will be denoted as the global assembly.

Figure 1: Side and isometric views of the half-scale mock-up of a generalized fuselage mounted twin-engine aircraft. Dimensions in meters.

The bulkhead consists of a series of stiffeners, seven vertical, and two horizontal. The stiffeners are each in the shape of an L-bracket 30 mm x 30 mm x 1.6 mm made from 6061-T6 Aluminum. The base of the bulkhead is formed from 5052 Aluminum 1.63mm thick with a diameter of 1,340 mm. There are 36 semi-circular cut-outs with a diameter of 28 mm to allow for the stringers to pass through the frame unobstructed. The mass of the bulkhead is 8.79 kg. The relevant dimensions of the bulkhead are given in Fig. 2.

The FESF and the RESF are both made from 6061-T6 aluminum 1.63mm thick, the FESF with a diameter of 1,140 mm, and the RESF with a diameter of 926 mm. The RESF and FESF contain two engine support beams that protrude

Figure 2: Front view of the bulkhead (dimensions in meters).

through the fuselage skin, and form the engine mounts. They had dimensions
 190.5mm X 480mm X 12.7mm and are made from 6061-T6 aluminum. There
 are only 30 semi-circular cut-outs for these components, as the stringers did
 not run through the engine support frames to ensure structural integrity. The
 masses of the FESF and RESF are 8.45 kg and 8.2 kg respectively. The FESF
 and RESF are displayed in Fig. 3.

A summary of the material properties for 6061-T6 and 5052 Aluminum are
 given in Table 1, along with the material properties of 7075-T6 Aluminum used
 for the skin. 5052 Aluminum was used for the frames and stringers of the
 assembly.

Figure 3: Front view of the FESF (left) and the RESF (right) (dimensions in meters).

Table 1: Material properties for the various grades of aluminum used to manufacture the assembly.

Material	Young's Modulus, GPa	Poisson's Ratio	Density, kg m^{-3}
5052	70.3	0.33	2700
6061-T6	68.9	0.33	2703
7075-T6	71.3	0.33	2804

3.2. Experimental Equipment

335 A PCB 086C02 modal impact hammer fitted with a soft vinyl tip was used to excite the test subject. Due to the inherent modal density of the components, a low frequency bandwidth was desired to increase the resolution of the measurement to adequate levels and not to oversaturate the analyzer input with undesired frequency content. For this reason, the soft tip was applied to the impact hammer, resulting in a bandwidth of 0 Hz to 75 Hz. Three PCB 352C22
340 accelerometers were roved around the components, with added masses equivalent to the accelerometer mass of 0.474 g used at all unmeasured DOF locations to ensure consistent mass distribution throughout the duration of the test. The accelerometers and dummy masses were attached to the components via petro
345 waX.

Both the hammer and the accelerometers were connected as channel inputs into a LDS Dactron Photon 2 DAQ unit with 24 bit A/D resolution and a

built-in signal conditioning amplifier. All data acquisition and Fast Fourier Transform (FFT) analysis was conducted via this unit. Force and response time
350 domain histories along with Frequency Response Function (FRF) and coherence data were extracted from the DAQ unit using the associated RT Pro software. Subsequently, ME Scope VES 4000 Modal Analysis software was used for post-processing and modal parameter extraction.

To avoid coupling effects inherent to fixed boundary conditions, free-free
355 boundary conditions were approximated by freely suspending the components from a steel test structure with soft elastic bands connected to the component via carabiners. The components were suspended perpendicular to the primary direction of motion to minimize the added damping and stiffness effects of the connections. Figure 4 illustrates the experimental test set-up of the FESF.

Figure 4: Experimental test set-up of the FESF.

360 3.3. Experimental Procedure

A similar experimental procedure was followed as presented in the work of Chamberlain and Mechefske [41]. The key difference was that three accelerometers were used to decrease the measurement time. As a result, the roving accelerometer testing methodology was used to properly extract one full column of the FRF matrix for data analysis. In addition, there was a decrease in
 365 the number of measurement DOFs, as the bulkhead had 36 DOFs and the FESF had 46 DOFs. The orientation of the measurement DOFs for the bulkhead and the FESF are displayed in Fig. 5. The orientation of the DOFs was chosen based off the preliminary computational results. Degree of freedom locations
 370 were chosen to avoid nodal points of each mode, while ensuring an even distribution of DOFs around the geometry. The driving point contains both input and output DOFs, and is located at DOF eleven and seven for the bulkhead and FESF respectively. The driving point was carefully selected such that the location was modally active for as many modes as possible. Due to Maxwell's
 375 Reciprocity Theorem [43], if the location of the excitation is at the node of a mode, that mode will not appear on the FRFs and therefore will not be extracted experimentally. Reciprocity was verified by exchanging excitation and response locations. This ensured that the linearity assumption was valid.

The remaining details of the experimental testing methodology can be seen in
 380 Chamberlain and Mechefske [41], as consistency between the three components was required. Validity checks were performed on all tests in accordance with ISO 7626-5 [47].

3.4. Computational Procedure

3.4.1. Model Validation of the Local Components

385 The manufacturing drawings displayed in Figs. 2 and 3 along with the material properties given in Table 1 were used to generate the geometries in Solid Edge. The geometries were imported into ANSYS and used to generate a finite element mesh using SHELL181 elements. These elements contain four nodes with six DOFs per node, and are suitable for moderately thin structures

Figure 5: Experimental degree of freedom locations for the bulkhead (left) and the FESF (right).

[48]. Shell elements were proven to be the most efficient element type for the RESF by Warwick et al. [42]. Point masses of 0.474 g were added to the nodes corresponding to the location of experimental DOF locations shown in Fig. 5.

The Lanczos method was used to perform modal analysis on the components. A mesh convergence test was performed to verify the mesh quality. Additionally, the mesh skewness was considered to aid in eliminating poor quality elements on a local scale. Mesh skewness is a metric that quantitatively evaluates how close to perpendicular the sides of a quadrilateral shaped element are, and how close to 60° the sides of a triangular shaped element are. All elements with a mesh skewness less than 0.8 are considered “good” [40]. A summary of the mesh properties of the two components is given in rows one and two of Table 2.

The effect of non-linearities present in full-scale aircraft was investigated by Kerschen et al. [49]. The authors performed both a linear and non-linear modal analysis, and found non-linear modes involving wing bending, wing torsion, tail bending and landing gear modes. A modal assurance criterion (MAC) matrix was generated between the linear and non-linear modes, and produced values greater than 0.999 between the mode shapes. The relative error between the natural frequencies was 0.2%. Since Kerschen et al. [49] showed that non-linear

effects of a full-sized aircraft had negligible impact on the modal properties of the aircraft, linearity was assumed in this work. This analysis was performed using
 410 free-free boundary conditions without pre-stress to mimic the experimental set-up. The components were meshed uniformly, with no discontinuities. Therefore, both the bolt and rivet joints located on the FESF and the bulkhead respectively were taken to be perfectly bound.

3.4.2. Local Components with Boundary Conditions

415 Upon validation of the computational model, the local components used for the substructuring analysis were analyzed. The mesh from the validated computational model was used in this analysis to ensure consistency. Fixed boundary conditions were applied to the outer surface of the frames to simulate the frame to skin interaction of the assembled model. For the bulkhead, there
 420 were 36 faces of the frames that were made rigid, and for the FESF and the FESF, there were 28 faces of the frames along with the outer nodes of the engine support beam that were made rigid. Additionally, the added point masses were eliminated as they would not be present in application. The details of the finite element mesh for the three components are presented in Table 2.

3.4.3. Global Assembly

425 The global fuselage assembly was created by computationally modelling the remaining nine frames, thirty-six stringers, rear bulkhead and fuselage skin. 3D four-node shell elements were used for all components in the global assembly. The mesh convergence test and skewness considerations were implemented
 430 to mesh the new components of the assembly such as the stringers, skin, rear bulkhead and remaining frames. The local component models were then added. The local FE models were bonded to the skin of the global assembly with bound contacts. Bound contacts assume that the contact points between the frames of the local components and the skin of the assembly are perfectly bound, i.e.
 435 there is zero relative displacement between the two surfaces. The contacts were placed between the exterior frames and the interior of the fuselage skin. This

represents the location where the components are riveted to the aircraft fuselage in practice. Free-free boundary conditions were used to analyze the modal properties of the assembled fuselage. The mesh sizing of each component remained
440 consistent from the local component analysis to the assembly. Table 2 displays the properties of the finite element mesh of the assembly.

Table 2: Mesh properties for the six geometries analyzed.

	No. Nodes	Average Skewness	% Quad4 Elements	% Tri3 Elements
Bulkhead	36131	0.037	99.8%	0.2%
FESF	35651	0.070	99.7%	0.3%
Bulkhead Fixed	32368	0.052	99.6%	0.4%
FESF Fixed	54628	0.074	99.6%	0.4%
RESF Fixed	44702	0.054	99.6%	0.4%
Assembly	360062	0.043	99.7%	0.3%

4. Results

4.1. Model Validation

4.1.1. Experimental Validity Checks

445 Experimental validity checks were performed to investigate the free-free boundary condition assumption and the effect of accelerometer added mass. The boundary conditions were investigated by doubling the stiffness of the elastic supports. The added stiffness of the boundary conditions resulted in an upward shift of the natural frequencies, as expected. Therefore, the elastic
450 bands with the lowest feasible stiffness were used, and the lowest flexural mode was not observed to contain rigid body contamination for the bulkhead local mode.

Consistent mass distribution was ensured by applying dummy masses equivalent to the 0.474 g mass of the accelerometer at all unmeasured DOF locations.

The added mass of the accelerometers was accounted for by performing a modal test with a dummy mass at the measurement DOF locations. This had the effect of doubling the added mass at the measurement DOFs. The FRFs with and without the added mass were compared and consistent results were observed. This showed that the added mass effect of the accelerometers was negligible.

For more details of the validity checks, please refer to Chamberlain and Mechefske [41], as the same experimental procedure was followed in this work.

4.1.2. Bulkhead

Upon performing modal analysis of the bulkhead both computationally and experimentally, frequency response functions were generated. Figure 6 shows the driving point FRF magnitude for the bulkhead. Qualitative correlation was noticed, with a small shift in natural frequencies. Modal parameter extraction was performed on the experimental data set to provide a quantitative data for comparison. Extraction of the modal parameters of the experimental data set produced five correlated mode pairs with the computational data. The frequencies of the correlated modes were compared and a linear regression of the two data sets produced a slope of 0.94. The mode shapes were then quantitatively compared by generating a MAC and POC between the two sets of mass normalized eigenvectors. The computational model was reduced to the size of the experimental DOFs by use of the SEREP. The POC produced two modes greater than 0.9, and all five modes greater than 0.80 as shown in Fig. 7.

4.1.3. Front Engine Support Frame

Similarly, the front engine support frame produced a driving point frequency response function with qualitative correlation. Modal parameter extraction was then performed, producing seven correlated mode pairs between the computational and experimental data. A slightly reduced bandwidth was used for the FESF, as rigid body contamination was seen for frequencies below 15 Hz. Due to the high modal density at low frequencies, this was expected, and the framework established by Ewins [43] was used to eliminate modes with rigid body

Figure 6: Experimental and computational FRFs of the bulkhead.

contamination from the CMPs. It is important to note that high modal density for aircraft components is well established in the literature, specifically by Unruh et al. and Kerschen et al. [24, 49]. Upon quantitative comparison of the CMPs, the linear regression of natural frequencies produced a slope of 0.948, and the POC produced four modes over 0.94 and all seven modes over 0.87 as shown in Fig. 8.

4.2. Substructuring Verification

Upon experimental validation, the fixed boundary conditions were applied to the local computational models. This produced an alternative set of modal parameters. This data along with the global assembly modal data was used to perform the substructuring verification. Correlated mode pairs were generated between the local and global systems. Figure 9 shows the first CMP for the bulkhead. A bandwidth of 0-150 Hz was used for this analysis, as the compo-

Figure 7: Pseudo-orthogonality check between the experimental and computational model of the bulkhead with free-free boundary conditions.

nents were less modally dense with the implemented boundary conditions. This produced ten CMPs for the bulkhead and the FESF, and five CMPs for the RESF. It is important to note that all modes within the frequency bandwidth
 500 studied could form a CMP, unlike the previous subsection where certain modes were not excited due to the limitations of experimental modal analysis. Table 3 shows the natural frequencies of the CMPs for each of the three components studied.

As seen in Table 3, the average frequency discrepancy for the three compo-
 505 nents was 11.77%, 3.92%, and 0.97% for the bulkhead, FESF, and RESF respectively. The average discrepancy between all modes investigated was 6.47%.

Figure 8: Linear regression of the computational and experimental natural frequencies (left) and the POC between the computational and experimental modes and the computational mass matrix (right) for the FESF.

Figure 9: Correlated mode pair between the local system of the bulkhead (left) and the global system (right). The colours indicate the total displacement magnitude.

When a linear regression was performed between the computational and experimental natural frequencies, slopes of 0.87, 0.945, and 0.989 were generated for the bulkhead, FESF and RESF respectively. The MAC and POC results are summarized in Table 4. The MAC and the POC for the FESF is shown in Fig. 10. Note that all modes studied produced a POC value greater than 0.891, and all but one mode produced a POC value greater than 0.919. This indicates the accuracy of this substructuring methodology, and allows for future work to investigate the local modes more thoroughly with confidence that they

Table 3: CMP frequencies for the three components.

Mode	Bulkhead			FESF			RESF		
	Local, Hz	Global, Hz	% Diff	Local, Hz	Global, Hz	% Diff	Local, Hz	Global, Hz	% Diff
1	21.126	20.565	2.66%	25.932	25.692	0.93%	39.713	39.579	0.34%
2	41.101	35.274	14.18%	47.071	46.711	0.76%	72.854	72.419	0.60%
3	58.684	56.313	4.04%	64.170	63.195	1.52%	100.06	98.046	2.01%
4	72.888	59.096	18.92%	85.419	85.158	0.31%	134.87	134.63	0.18%
5	75.164	70.048	6.81%	99.516	97.512	2.01%	150.07	147.45	1.75%
6	107.88	88.979	17.52%	107.89	99.974	7.34%			
7	108.44	94.298	13.04%	130.30	114.04	12.48%			
8	140.05	122.82	12.30%	132.89	127.48	4.07%			
9	141.20	121.05	14.27%	141.48	139.51	1.39%			
10	149.48	128.56	14.00%	154.10	141.21	8.36%			
Average			11.77%			3.92%			0.97%

will correlate well with the global modes of the fuselage.

Table 4: MAC and POC values for the three components.

Mode	Bulkhead		FESF		RESF	
	MAC	POC	MAC	POC	MAC	POC
1	0.990	0.997	1.000	1.000	0.999	1.000
2	0.966	0.982	0.998	0.999	0.999	1.000
3	0.984	0.997	0.997	0.999	1.000	1.000
4	0.909	0.952	0.996	0.998	0.996	1.000
5	0.947	0.982	0.982	0.994	0.991	0.999
6	0.776	0.891	0.938	0.966		
7	0.789	0.928	0.966	0.990		
8	0.760	0.919	0.948	0.985		
9	0.673	0.930	0.982	0.997		
10	0.733	0.982	0.797	1.000		

The results indicate the importance of including the POC as a proof of correlation, as modes producing MAC values less than 0.8 produced a POC value greater than 0.89 in all cases. Therefore, modes that would be disregarded as having poor correlation by use of the MAC, are indeed orthogonal with respect

Figure 10: MAC matrix between the local and global modes of the FESF (left) and the POC matrix between the local and global modes of the FESF (right).

to the mass matrix. This suggests that the rigor of this correlation analysis was necessary to generate a complete understanding of the correlation between the modal properties of the local and global structures.

5. Discussion

5.1. Linearly Dependent Modes

Inherent to this analysis was the generation of linearly dependent modes. This can be seen as non-zero off diagonal terms in both the MAC and POC matrices. For example, in Fig. 10, component mode 6 and assembly mode 8 produce a MAC value of 0.259 and a POC value of 0.348. At first, this may be thought of as an erroneous result, as both the MAC and the POC should produce the identity matrix for linearly independent mode shapes. However, upon further inspection, the assumption of linearly independent mode shapes is not always valid. To further investigate this issue, the mode shapes for component mode 6 and assembly mode 8 are given in Fig. 11. It is clear that there are inherent similarities between the two modes. The locations of maximum displacement and the modally inactive regions are largely consistent between both mode shapes. Although the similarities are not strong enough to denote these as CMPs, there will be an inherent linear dependence between these two modes. Therefore, the POC will always be non-zero for these two modes.

Figure 11: Local mode 6 (left) and global mode 8 (right) for the FESF. Note that the skin and several frames and stringers were removed for illustrative purposes only.

To gain a deeper mathematical understanding, an orthogonality check was performed between the local modes and the global modes separately. This relationship is given in Eq. (4) and (5).

$$[\Phi]_l^T [M] [\Phi]_l = [I] \quad (4)$$

$$[\Phi]_g^T [M] [\Phi]_g = [I] \quad (5)$$

where the subscript l indicates the local modes and the subscript g indicates the global modes of the fuselage. This resulted in off-diagonal terms of 0.186 and 0.102 for the local orthogonality check and the global orthogonality check respectively. This indicates that there is an inherent linear dependence between mode 6 and mode 8 on both the local and global scale. This result gets magnified when the POC is performed between the two sets of mode shapes.

5.2. Modal Coupling

The results of this study generate substantial insight into the modal properties of a rear fuselage mounted twin-engine aircraft. The results indicate that the local and global natural frequencies matched within 6.47% on average in the frequency bandwidth studied. Additionally, the POC was greater than 0.89

for all modes studied. This shows that the modal properties of the critical fuselage components contain a high level of consistency when they are analyzed in isolation and when they are analyzed in an assembled fuselage.

Analyzing the modal results of the assembled fuselage in greater detail showed a high level of modal coupling at frequencies greater than 80 Hz. Reviewing the local and global bulkhead data in Tab. 3, it is evident that three out of four modes less than 80 Hz contain discrepancies less than 7%, whereas all modes greater than 80 Hz have discrepancies greater than 12%. This was due to the modal coupling observed at frequencies greater than 80 Hz. Modal coupling introduced vibrations of multiple components of the fuselage for a single mode shape. For example, vibration of the skin and stiffeners of the fuselage had a major impact on bulkhead modes, generating the larger discrepancies for the bulkhead than the other two components studied. Therefore, it is important to understand at what frequency modal coupling begins to occur, as that limits the accuracy of the substructuring method above that frequency.

5.3. Passenger Comfort

This work identifies the critical aircraft components pertaining to passenger comfort. It is seen in this work that the bulkhead and FESF are both twice as modally dense as the RESF in the frequency bandwidth studied. Therefore, both the bulkhead and FESF are equally likely to generate resonance for an excitation frequency within the 0 Hz to 150 Hz bandwidth. Considering that the bulkhead provides a direct transmission path for engine induced noise and vibration to propagate into the passenger cabin, the authors suggest that the bulkhead should be the component of focus for improving passenger comfort levels moving forward.

The results discussed in the previous subsections provide significant insight regarding increasing passenger comfort for a rear fuselage mounted twin-engine aircraft. Firstly, the confidence bounds generated in this work for the local analysis of individual aircraft components sets the framework the accurate implementation of topology optimization, as performed by Warwick et al. [39].

The work of Warwick et al. [39] was able to generate a new aircraft bulkhead design through the implementation of topology optimization that improved passenger comfort by shifting the natural frequencies away from the engine excitation frequency. Without the understanding of the relationship between local and global modes, the advanced methodology introduced by Warwick et al. [39] would not be of use as it is too computationally demanding to be performed on the assembled aircraft. Therefore, the results of this work provides the ability to investigate new methods of increasing passenger comfort for a rear fuselage mounted twin-engine aircraft not previously feasible.

5.4. Minimizing Truncation Error

The hybrid substructuring methodology introduced in this work was the first to use both experimental and computational data to verify a substructuring analysis. Substructuring methods typically use experimental data to validate the computational model. As seen Sec. 3.3 of this work, experimental data collection is often limited to 20-100 degrees of freedom. With tri-axial accelerometers, the number of degrees of freedom may be increased to 200-500. The second stage of this substructuring methodology compares the local and global computational models using only validated computational data. From Tab 2. it is seen that the bulkhead had the minimum number of nodes for any local model containing 32,368 nodes (194 208 DOFs). Therefore, this methodology was able to increase the number of DOFs by three orders of magnitude from experimental substructuring methods common in the literature.

The accuracy of the computational models depended on mesh density, and therefore the number of DOFs in each model. Consequently, the mesh convergence tests used to determine appropriate mesh density prior to computational analysis prove that truncation errors present in experimental substructuring methods were overcome. The number of DOFs used to overcome truncation error was equivalent to the number of DOFs required to satisfy the mesh convergence criteria.

6. Conclusion

A method of experimental component-mode synthesis dynamic substructuring was introduced to mitigate truncation, continuity and rigid body mode error prevalent in this field. The method overcame truncation errors by using modal vectors with orders of magnitude more DOFs than commonly used in the literature. The method overcame continuity errors by using the SEREP method to reduce the modal vectors without sacrificing accuracy, and maintaining continuity between measurement DOFs for all analysis sets. The method overcame rigid body mode errors by performing experimental modal analysis with free-free boundary conditions to ensure that the complete set of modal vectors within the specified bandwidth were available for analysis.

The method first validated the computational models of local component modes of a global assembly. The validated computational local models were then used to verify a dynamic substructuring analysis, by implementing the SEREP method to reduce the computational data sets to the size of the experimental DOFs without sacrificing accuracy. Several correlation techniques commonly used in experimental modal analysis were implemented to prove correlation between the local and global modes of the substructure.

The substructuring analysis was performed on a fuselage mounted twin-engine aircraft. This configuration was previously not well understood in the literature despite being at an increased risk for engine induced structural vibrations. The substructuring method was introduced to this configuration, focusing on the local modes of three main components: the bulkhead, the front engine support frame, and the rear engine support frame. The results indicated that the local and global natural frequencies matched within 6.47% on average between the three components studied in the frequency bandwidth of 0-150 Hz. A pseudo-orthogonality check between the local and global mode shapes produced values greater than 0.89 for all 15 modes studied between the three components. The discrepancy between the modal assurance criterion values and the pseudo-orthogonality check values highlights the importance of proving the or-

thogonality relationship between the two sets of modes by inclusion of the mass matrix, as additional modes proved to be orthogonal. Linear dependence was noted between several modes; qualitatively by inspection of maximum displacement locations on the component, and quantitatively by comparison of local
 645 and global orthogonality checks with the pseudo-orthogonality check.

The method was able to overcome the truncation and multi-component limitations of other well-established methods in the literature. The DOF locations were suitable for experimental testing, as they were orientated to provide the
 650 most accurate experimental results and to avoid the coupling and complex mode shape results noticed in the literature for similar methods that place DOFs at the boundary. Computationally, the method provided a substantially reduced modeling time, as only the three local substructures of interest were modeled. Despite the reduced modeling time, the local results compared well with the
 655 global system. The method proved to be easily adaptable into finite element analysis, which allows for a wide range of users to implement this methodology. These results show promise for future research investigating detailed models and innovative design methodologies for this configuration with confidence that the local component results will correlate to the global assembly within the precision
 660 presented in the work herein.

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670 **References**

- [1] W. C. Hurty, Vibrations of structural systems by component mode synthesis, Transactions of the American Society of Civil Engineers 126 (1960) 157–175.
- [2] J. S. Przemieniecki, Matrix structural analysis of substructures, AIAA Journal 1 (1963) 138–147.
- [3] W. C. Hurty, Dynamic analysis of structural systems using component modes, AIAA journal 3 (1965) 678–685.
- [4] R. Craig, M. Bampton, Coupling of substructures for dynamic analyses, AIAA journal 6 (1968) 1313–1319.
- [5] R. H. MacNeal, A hybrid method of component mode synthesis, Computers & Structures 1 (1971) 581–601.
- [6] S. Rubin, Improved component-mode representation for structural dynamic analysis, AIAA journal 13 (1975) 995–1006.
- [7] P. Avitabile, Modal testing: a practitioner’s guide, John Wiley & Sons, 2017.
- [8] R. L. Mayes, M. S. Allen, Comparison of FRF and Modal Methods for Combining Experimental and Analytical Substructures., Technical Report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2007.
- [9] R. L. Mayes, P. Hunter, T. W. Simmermacher, M. S. Allen, Combining Experimental and Analytical Substructures with Multiple Connections., Technical Report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2007.
- [10] D. d. Klerk, D. J. Rixen, S. Voormeeren, General framework for dynamic substructuring: history, review and classification of techniques, AIAA journal 46 (2008) 1169–1181.

- [11] J. C. O'CALLAHAN, System equivalent reduction expansion process, in: Proc. of the 7th Inter. Modal Analysis Conf., 1989.
- [12] J. O'Callahan, P. Li, A non smoothing serep process for modal expansion, in: PROCEEDINGS-SPIE THE INTERNATIONAL SOCIETY FOR OPTICAL ENGINEERING, SPIE INTERNATIONAL SOCIETY FOR OPTICAL, pp. 232–232.
- [13] J. O'Callahan, Modal vector correlation techniques, in: Proceedings of the 16th International Modal Analysis Conference, volume 3243, p. 197.
- [14] M. I. Friswell, D. J. Inman, Reduced-order models of structures with viscoelastic components, AIAA journal 37 (1999) 1318–1325.
- [15] M. Friswell, J. Penny, S. Garvey, Model reduction for structures with damping and gyroscopic effects, in: Proceedings of the international seminar on modal analysis, volume 3, KU Leuven; 1998, pp. 1151–1158.
- [16] G. Aglietti, S. Walker, A. Kiley, On the use of serep for satellite fem validation, Engineering Computations 29 (2012) 580–595.
- [17] D. P. Rohe, M. S. Allen, Investigation of the effectiveness of using an experiment to validate experimental substructure models, Mechanical Systems and Signal Processing 43 (2014) 192–216.
- [18] S. Manzato, C. Napoli, G. Coppotelli, A. Fregolent, W. D'Ambrogio, B. Peeters, Experimental coupling and decoupling of engineering structures using frequency-based substructuring, in: Dynamics of Coupled Structures, Volume 4, Springer, 2016, pp. 447–461.
- [19] W. D'Ambrogio, A. Fregolent, Replacement of unobservable coupling dofs in substructure decoupling, Mechanical Systems and Signal Processing 95 (2017) 380–396.
- [20] M. Imregun, Structural modification and coupling dynamic analysis using measured frf data, in: Proc. 5th Int. Modal Analysis Conf., April, volume 1136.

- [21] S. Jha, J. Catherines, Sources and characteristics of interior noise in general aviation aircraft, *The Journal of the Acoustical Society of America* 59 (1976) S64–S64.
- [22] S. Jha, J. Catherines, Interior noise studies for general aviation types of aircraft, part i: Field studies, *Journal of Sound and Vibration* 58 (1978) 375–390.
- [23] S. Jha, J. Catherines, Interior noise studies for general aviation types of aircraft, part ii: Laboratory studies, *Journal of Sound Vibration* 58 (1978) 391–406.
- [24] J. F. Unruh, D. C. Scheidt, Engine induced structural-borne noise in a general aviation aircraft, *SAE Technical Paper Series* (1979).
- [25] J. Unruh, S. Dobosz, Fuselage structural-acoustic modeling for structure-borne interior noise transmission, *Journal of vibration, acoustics, stress, and reliability in design* 110 (1988) 226–233.
- [26] S. Snyder, C. Hansen, The design of systems to control actively periodic sound transmission into enclosed spaces, part ii: mechanisms and trends, *Journal of sound and vibration* 170 (1994) 451–472.
- [27] J. Van Dyke, J. Schendel, C. Gunderson, M. Ballard, Cabin noise reduction in the dc-9, in: *Proceedings of the AIAA Commercial Aircraft Design Meeting*.
- [28] J. F. Wilby, Interior noise of general aviation aircraft, *SAE transactions* (1982) 3133–3144.
- [29] J. F. Unruh, Aircraft propeller induced structure-borne noise (1989).
- [30] J. D. Leatherwood, Combined effect of noise and vibration on passenger acceptance (1984).
- [31] J. D. Leatherwood, Annoyance response to simulated advanced turboprop aircraft interior noise containing tonal beats (1987).

- [32] D. Osborne, Passenger comfort—an overview, *Applied Ergonomics* 9 (1978) 131–136.
- [33] L. Richards, On the psychology of passenger comfort, human factors in transport research edited by Dj Osborne, Ja Levis 2 (1980).
- 755 [34] K. P. Shepherd, J. D. Leatherwood, S. A. Clevenson, Effect of low-frequency tones and turbulent-boundary-layer noise on annoyance (1983).
- [35] L. Krog, A. Tucker, G. Rollema, Application of topology, sizing and shape optimization methods to optimal design of aircraft components, in: *Proc. 3rd Altair UK HyperWorks Users Conference*.
- 760 [36] A. Remouchamps, M. Bruyneel, C. Fleury, S. Grihon, Application of a bi-level scheme including topology optimization to the design of an aircraft pylon, *Structural and Multidisciplinary Optimization* 44 (2011) 739–750.
- [37] J. R. Martins, A. B. Lambe, Multidisciplinary design optimization: a survey of architectures, *AIAA journal* 51 (2013) 2049–2075.
- 765 [38] J.-H. Zhu, W.-H. Zhang, L. Xia, Topology optimization in aircraft and aerospace structures design, *Archives of Computational Methods in Engineering* 23 (2016) 595–622.
- [39] B. T. Warwick, C. K. Mechefske, I. Y. Kim, Topology optimization of a pre-stiffened aircraft bulkhead, *Structural and Multidisciplinary Optimization* (2019) 1–19.
- 770 [40] Ansys release 18.2 documentation, 2017.
- [41] D. A. Chamberlain, C. K. Mechefske, Experimental modal analysis of a half-scale model twin-engine aircraft rear fuselage engine mount support frame, in: *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V008T12A008–V008T12A008.
- 775

- [42] B. T. Warwick, C. K. Mechefske, I. Y. Kim, Computational modal analysis of a twin-engine rear fuselage mounted aircraft support frame, in: ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, American Society of Mechanical Engineers, pp. V008T12A007–V008T12A007.
- [43] D. J. Ewins, Modal testing: theory and practice, volume 15, Research studies press Letchworth, 1984.
- [44] R. J. Allemang, The modal assurance criterion—twenty years of use and abuse, Sound and vibration 37 (2003) 14–23.
- [45] R. J. Guyan, Reduction of stiffness and mass matrices, AIAA journal 3 (1965) 380–380.
- [46] P. Avitabile, Study of modal vector correlation using various techniques for model reduction, in: Proc. of the 10th IMAC, pp. 572–583.
- [47] I. 7626-5, Vibration and Shock - experimental determination of mechanical mobility. Part 5: measurement using impact excitation with an exciter which is not attached to the structure.
- [48] ANSYSInc., Element reference, 2009, pp. 1173–1188.
- [49] G. Kerschen, M. Peeters, J.-C. Golinval, C. Stéphan, Nonlinear modal analysis of a full-scale aircraft, Journal of Aircraft 50 (2013) 1409–1419.