Material Interface Control in Multi-Material Topology Optimization using Pseudo-Cost Domain Method

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SUMMARY

The recent drive for producing lightweight and high performance designs on reduced timelines has promoted the need for computational design generation tools such as Multi-Material Topology Optimization (MMTO). However, MMTO has drawn some industry skepticism as it assumes different material elements to be perfectly fused together. To address this concern, in this paper, a novel pseudo-cost domain (PCD) method is proposed which mathematically determines individual material interfaces in MMTO solutions. The proposed methodology employs a user defined joint cost model to weigh the distinct material interfaces relative to each other. An innovative approach to tailor the MMTO design considering the relative cost of each material interface is presented. The proposed methodology can consider any number of materials and their respective interfaces, and it is defined in such a way that increasing the number of materials has minimal effect on computational time. The methodology is formulated in a smooth and differentiable manner and the sensitivity expressions required by gradient-based optimization solvers are presented. A series of example problems are provided to demonstrate the efficacy of the proposed methodology.

KEY WORDS: multi-material topology optimization; joint cost; material interface; dissimilar materials; manufacturability; pseudo-cost domain

This article has been accepted for publication and undergone full peer review but has not been through the copyediting, typesetting, pagination and proofreading process which may lead to differences between this version and the Version of Record. Please cite this article as doi: 10.1002/nme.6545

1 | INTRODUCTION

In recent years with the widespread industry adoption of Topology Optimization (TO), the structural design process has been transformed. This is due to the significant benefits provided by TO in the design process. These benefits include generation of clean slate designs, unconventional and unintuitive solutions, as well as rapid generation of design alternatives. TO has been integrated into the structural design process for many industries. In the automotive sector Yang et al.¹ of Ford Motor Company demonstrated the benefits of using TO in automotive applications by studying a truck frame, a deck lid, and a space frame structure in 1995. Similar studies have been performed on automotive chassis, doors, and engine cradles.^{2,3} In the aerospace sector, TO studies have been performed on aircraft pylons, engine brackets, fuselage, wings, and landing gear systems.⁴ Zhu et al.⁵ discussed the application of TO for designing various system and component-level aerospace structures.

Since its inception in 1988, topology optimization has seen significant advances.⁶ Bendsøe and Sigmund⁷ have provided a comprehensive summary of the field in their monograph. Similar reviews have been published by Eschenauer and Olhoff⁸, Sigmund⁹, and Deaton and Grandhi¹⁰. Researchers have proposed several different methods for performing TO, with four methods that received significant consideration. They are (a) density-based methods e.g. solid isotropic material with penalization (SIMP); (b) boundary variation methods e.g. level-set and phase-field methods; (c) hard-kill methods e.g. evolutionary structural optimization (ESO) and its later version bi-directional ESO (BESO); and (d) nongradient methods e.g. genetic algorithms (GAs). Sigmund and Maute⁹ presented a comparative review between the above mentioned methods and advised that density-based methods are superior compared to their counterparts due to higher computational efficiency while still solving for good local optima. As a result, density-based methods received significant attention in both the research community and industry.

Since this work will require a large number of design variables, computational efficiency is one of the most important factors when choosing the TO framework to build upon. Thus, density-based TO has been selected for the proposed method. The density-based method utilizing the SIMP interpolation scheme is typically used with a fixed finite element (FE) mesh where the density of each element is independently optimized between full material existence (solid) and elimination (void). The design variables are termed “pseudo-densities” and can take any real value between unity and some infinitesimal value. The elastic modulus of intermediate densities, or pseudo-densities between 0 and 1, is penalized to encourage convergence to a discrete solid-void structure. For a two-phase material, the interpolated mass of each element $W^{(1,2)}$ is calculated as:

$$W^{(1,2)}(\rho^1) = (\rho^1)^q W^1 + (1 - (\rho^1)^q) W^2 \quad (1)$$

where ρ^1 is the design vector. $\rho_e^1 = (0, 1]$ for all elements e , where $e = 1, \dots, n$ with n being total number of elements. W^1 and W^2 are the material weights of the two material phases. q is the penalty factor; although $q > 1$ can be used to penalize intermediate densities, many papers and our numerical examples shows that $q = 1$ is sufficient as the amount of the elements with

intermediate densities is small in most cases.¹¹ Therefore $q = 1$ is used by default for weights in this article.

Similarly, the interpolated elastic modulus of each element $E^{(1,2)}$ is calculated as:

$$E^{(1,2)}(\rho^1) = (\rho^1)^p E^1 + (1 - (\rho^1)^p) E^2 \quad (2)$$

E^1, E^2 are the original elastic moduli of the two materials; p is the penalty factor, which penalizes the intermediate densities. Single material topology optimization (SMTO) uses $E^2 = 0$, $W^2 = 0$ to represent the void.

Bendsøe and Sigmund⁷ expanded the above method to a SIMP-like multi-material topology optimization (MMTO) scheme that interpolates between two material phases and one void phase. Thus, it determines optimum location and distributions of multiple material types. MMTO broadens the capabilities of the SMTO by allowing for optimal material selection and material existence to be determined simultaneously. Hvejsel and Lung¹² generalized the MMTO method by defining a general interpolation scheme for any number of material phases. For an M phase material with one void and m solid phases, where $m = M - 1$, the SIMP formulation can be extended to:

$$W^{(1,\dots,M)}(\rho^1, \dots, \rho^{M-1}) = \sum_{i=1}^M \alpha_i W^{(M-i+1)} \quad (3)$$

$$\text{where } \alpha_i = \left[1 - (\rho^i - \rho^i \delta_{iM})^q \right] \prod_{t=1}^{i-1} (\rho^t)^q$$

$$\delta_{iM} = \begin{cases} 1, & i = M \\ 0, & i \neq M \end{cases}$$

where ρ^j is the j^{th} nominal density; $W^{(j)}$ is the original weight of the element filled with the j^{th} material, which is obtained by the multiplication of the element volume and the physical density of the j^{th} material (not the nominal density ρ^j); $W^{(1,\dots,M)}$ is the interpolated element weight.

$$E^{(1,\dots,M)}(\rho^1, \dots, \rho^{M-1}) = \sum_{i=1}^M \beta_i E^{(M-i+1)} \quad (4)$$

$$\text{where } \beta_i = \left[1 - (\rho^i - \rho^i \delta_{iM})^p \right] \prod_{t=1}^{i-1} (\rho^t)^p$$

$$\delta_{iM} = \begin{cases} 1, & i = M \\ 0, & i \neq M \end{cases}$$

where $E^{(j)}$ is the original elastic modulus of the j^{th} material; $E^{(1,\dots,M)}$ is the interpolated elastic modulus. For TO in this form, material E^M will always represent a void material. Therefore, single-material interpolation in the form presented in (2) is realized by setting $M = 2$, and recognizing that $E^2 = 0$.

With MMTO, the added design freedom always yields TO results of similar or superior performance compared to the standard SMTO using SIMP. Several researchers verified this by comparing the performance of SMTO and MMTO for industry problems.^{3,13–16} For the MMTO method described above, $M-1$ design vectors are required to solve an M phase MMTO problem, which includes one void phase; or m design vectors for an m material MMTO problem. The size of each design vector is equal to the total number of finite elements in the model, n . Thus, the total number of design variables in an m material MMTO are equal to the product of m design vectors and n finite elements. In order to reduce the number of design variables, Zuo and Saitou¹⁷ presented an ordered SIMP interpolation method; whereas Yin and Ananthasuresh¹⁸ presented a gradually formed continuous peak function for material interpolation, both of which required only single set of design variables. Other MMTO methods include the moving morphable component method proposed by Zhang et al.^{19,20} and the multi-component topology optimization method proposed by Zhou et al.^{21,22}

Apart from density-based method, advances have been made in other methods to solve MMTO problems as well. In the boundary variation method, Wang and Wang proposed a multi-phase level-set model requiring m level-set functions to represent a structure of $n=2^m$ material phases.²³ Later, the authors presented a level-set based variational approach for the design of multi-material structures.²⁴ Other researchers including Liu et al.²⁵ Wang et al.²⁶ Ghasemi et al.²⁷ Yulin and Xiaoming²⁸ Cui et al.²⁹ have proposed level-set based MMTO methods.

Wang and Zhou³⁰ proposed MMTO with a phase-field method using Van der Waals–Cahn–Hilliard theory to define a dynamic process of phase transition. Later the authors presented a phase-field method for MMTO using a generalized Cahn–Hilliard model.³¹ Tavakoli and Mohseni³² provided a 115-line MATLAB implementation for an alternating active-phase algorithm for MMTO through division of a multi-phase topology optimization problem into a series of binary phase topology optimization sub-problems. Other phase-field based MMTO methods include the methods proposed by Blank et al.³³ and Wallin et al.³⁴ Huang and Xie³⁵ and Ghabraie³⁶ presented the bi-directional evolutionary TO algorithms to solve MMTO problems.

Even though MMTO shows promise, it has faced some industry skepticism as most MMTO methods consider the interfaces between different materials to be perfectly bonded. Florea et al.³⁷ highlighted this issue by comparing the numerical optimum design obtained using MMTO and the equivalent manufacturable design. They concluded that manufacturable design considering joints provided significantly lower stiffness compared to that of its numerically described value. To describe the true structural performance and manufacturability of a multi-material design, joints between components must be explicitly defined. This necessitates the need to effectively determine material interfaces between dissimilar materials.

Clausen et al.³⁸ proposed a SIMP-based method for including coated structures and prescribed material interface properties into TO. It includes the normalized norm of the spatial gradient of the design field into the material interpolation function, enforcing coating material at interfaces by attributing particular properties. Chu et al.³⁹ extended the method to handle TO of the multi-material structure with a graded interface. The method utilized a two-step filtering approach with density smoothing and projection scheme to describe the multi-material structure and identify the material interface.

Woischwill and Kim⁴⁰ and Florea et al.⁴¹ proposed a method for performing both multi-material and multi-joint topology optimization using a decomposition approach. The proposed method was limited to two material MMTO and performed MMTO and joint optimization sequentially, where the process of material and joint optimization occurred in two independent

loops. Florea et al.⁴² later proposed a novel methodology for performing simultaneous multi-material and multi-joint topology optimization, but was still restricted to only two materials.

Almost all previously mentioned methods can only determine material interface for two material MMTO problem. Chu et al.³⁹'s method can determine individual material interfaces for more than two material MMTO problem; however it did not consider joint cost during the process of determining material interface. Also, Chu et al.³⁹ calculated spatial gradients using nodal densities which must use the shape function matrix information which limits the use of this method to small scale problems as commercial FEA software like OptiStruct and Ansys do not provide shape function information.

The objective of this paper is to propose a novel, effective and efficient numerical method that can consider joint cost while determining individual material interfaces and which is easily scalable for complex, practical 3D problems.

All methods currently available in the literature determine material interface by calculating the spatial gradients of design vectors (ρ) and then subsequently utilizing different smoothing and projection functions, which is fundamentally different from the method proposed in this paper. Even though the spatial gradient calculations in proposed method increase linearly with the number of materials considered in MMTO, similar to Chu et al.³⁹'s method, this method provides user with the flexibility to reduce the number of spatial gradient calculations significantly by forgoing meager design freedom hence reducing the computational time.

A novel total joint cost constraint has been developed using the proposed material interface detection method for MMTO to provide users with the ability to tailor the MMTO result as per the design requirements.

The remainder of this paper is organized as follows: in Section 2, the approach to determine the material interface using a novel pseudo-cost domain (PCD) method is presented. The standard minimum compliance MMTO problem statement is reformulated and posed such that it includes a total material interface cost constraint. In Section 3, the sensitivity expressions required by gradient-based optimization solvers are derived. In Section 4, the proposed methodologies for considering the relative material interface cost are demonstrated using several two-dimensional (2D) geometries and load cases discretized using regular meshes. This does not imply any lack of extensibility of the method to more complex domains and discretization but for simplicity of implementation; results produced using 3D domains and load cases will be discussed in future work. Finally, in Section 5, conclusions and discussions surrounding the proposed methodologies are presented and potential future research directions are discussed.

2 | PSEUDO-COST DOMAIN METHOD TO DETERMINE MATERIAL INTERFACE

This section presents a method that determines the weighted material interface between m materials in an m material MMTO using novel PCD method. First, an overview of general MMTO and a standard problem statement are provided. Next, the proposed methodology for determining the weighted material interface using the PCD approach is presented. The procedure is divided into four steps which involve assigning relative joint cost to each material interface, representing the relative joint cost model in the PCD, calculating the weighted centroid at each location, and finally computing the spatial gradient of the weighted centroid.

2.1 | Multi-material topology optimization

In the current approach, the multi-phase SIMP technique described in (3) and (4) is used to perform MMTO. In these problems, the objective is to minimize the total compliance of the domain and the constraint is normalized mass fraction. The optimization problem is formulated as:

$$\begin{aligned}
 & \underset{\rho^1, \rho^2, \dots, \rho^{m-1}, \rho^m}{\text{minimize}} && C(\rho^j) = f^T \underline{u} = \underline{u}^T \underline{K} \underline{u} \\
 & \text{subject to} && g_1(\rho^j) = \frac{\sum_e W_e}{\sum_e W_e^1} \leq \bar{W} \\
 & && \underline{K} \underline{u} = \underline{f} \\
 & && \forall \text{ element } e, \rho_e^j \in (0, 1], j = 1, \dots, m
 \end{aligned} \tag{5}$$

where C is the total compliance; $\underline{f}$ is the external force vector; $\underline{u}$ is the displacement vector; $\underline{K}$ is the global stiffness matrix; W_e is the interpolated element weight; W_e^1 is the element weight when it is composed entirely of material 1; $\bar{W}$ is the weight fraction limit; ρ^j includes all design vectors, and ρ_e represents the design variables relevant to the e^{th} design cell.

In this work, three material MMTO is studied using isotropic steel, aluminum, and titanium with properties shown in Table 1.

TABLE 1 Properties of materials considered in the current study

	<i>Steel</i>	<i>Titanium</i>	<i>Aluminum</i>	<i>Magnesium</i>
<i>Elastic Modulus, E [GPa]</i>	210	110	70	45
<i>Poisson's Ratio, ν [-]</i>	0.32	0.36	0.35	0.30
<i>Physical Density, ρ [10^3 kg/m³]</i>	7.85	4.50	2.70	1.75
<i>Specific Stiffness, E/ρ [10^6 m²/s²]</i>	26.75	24.44	25.93	25.71

For m material MMTO, m design vectors are used; for three material MMTO, three design vectors are considered, ρ^1, ρ^2 , and ρ^3 . Here ρ^1 can be considered as the ‘material existence’ design vector as its value is used to determine if the element is void or full density. ρ^2, ρ^3 can be considered as ‘material selection’ design vectors as they are used to select material between steel, aluminum, and titanium. Figure 1 displays the method of interpreting the MMTO results based on the design variable values.

FIGURE 1 Process to interpret materials in an multi-material topology optimization (MMTO) solution from different design variable values; here $\rho_{crit}^1 = \rho_{crit}^2 = \rho_{crit}^3 = 0.5$

Figure 2 shows an example of three material MMTO result obtained by the problem statement shown in (5) using steel, aluminum, and titanium.

FIGURE 2 (a) Dimension and load case for three point loaded beam, discretized using four node bilinear quadrilateral elements of side length and thickness of 1 mm (b) MMTO result considering steel, aluminum, and titanium with $\bar{W} = 0.65$

2.2 | Relative material joint cost model definition

The first step in determining the weighted material interface using the PCD method is to assign the relative material interface costs. These costs are decided by the designer based on the requirements and limitations of their design process. Some of the factors which would be considered when determining these costs include material joining costs, material joining capabilities, joint properties, and joint reliability.

For an m material MMTO, there are $\frac{m!}{2(m-2)!}$ relative joint costs to be considered. The relative joint costs assigned can be visualized by drawing an $m-1$ -dimension geometry where each vertex represents a material and the lengths of each side represents the relative joint cost of the respective materials. This model is called a relative joint cost model or JCM. Considering a three material MMTO formulation considering steel, titanium, and aluminum there can be three types of material interfaces, steel-aluminum (ST-AL), steel-titanium (ST-TI), and titanium-aluminum (TI-AL). A JCM for this formulation with the relative joint cost ratios of 3:2:1 is shown below.

FIGURE 3 Example of joint cost model (JCM) for a three material MMTO formulation considering steel (ST), titanium (TI), and aluminum (AL) with relative material interface joint cost ratios of 3:2:1

As mentioned earlier, designer must select the relative joint cost ratios carefully considering individual joint preferences for the final MMTO design. Different joint cost ratios would provide

distinct final MMTO results. The MMTO problem using the JCM shown in Figure 3 would favor TI-AL interface twice as much as the ST-TI interface and trice as much as the ST-AL interface.

2.3 | Representation of relative material joint cost model in the pseudo-cost domain

The next step for determining the weighted material interface after deciding the JCM is to represent the JCM in the PCD (ξ). The PCD is an $m-1$ -dimension domain which is used to represent the JCM. The origin of the domain can be selected arbitrarily with the lone criteria that no JCM vertex should lie on the origin of the PCD to avoid numerical inconsistency. The scale of the JCM does not matter as long as the joint cost ratios selected in step 1 are kept consistent.

An example of the PCD for three material MMTO with the JCM from Figure 3 is shown in Figure 4. It should be noted that the vertices of the JCM can be located in any quadrant, but in the current study all are located in the first quadrant. A subroutine can be added within the MMTO solver to select the JCM dimension and determine its vertex coordinates in PCD based on the user provided relative material joint cost ratios so that the user does not have to explicitly provide the coordinate information.

To determine the coordinates for a 1-D JCM in PCD, one vertex is initialized first to a particular coordinate and the coordinates of the rest of the vertices are determined relative to the first vertex with distance equal to the user defined interface joint cost values. Similarly, for a 2-D JCM one vertex is initialized first to a particular set of coordinates. The coordinates of second vertex are determined in a predefined direction (e.g. along x-axis) with distance from the first vertex equivalent to the user defined interface joint cost between the first and second material. Finally, the third vertex is determined by using simple distance formulas as values of two vertices and three sides of the 2-D JCM are now established.

It is important to note that each point in the design domain has an individual and identical PCD. Figure 5 demonstrates the method of this PCD. It can be observed that each location in the design domain possess an individual PCD and all PCDs are identical (with an identical JCM). Table 2 details the PCD dimension and visualization of the JCM for 1–5 material MMTO.

FIGURE 4 (a) Example of a JCM represented in a pseudo-cost domain (PCD) (b) Demonstrates the process of determining vertex coordinates for a JCM in a PCD. Here ξ_i is the

i^{th} coordinate system in the PCD $i = 1, \dots, m-1$. ξ_i^j j^{th} material in the i^{th} coordinate system in the PCD. $j = 1, \dots, m$

It should be noted that Figure 4(a) is not drawn to scale. Also, the exact relative material interface joint cost ratio of 3:2:1 is not used as it cannot be represented using a 2-D JCM instead 3.01:2.04:1.00 is used. If exact relative material interface joint cost ratios of 3:2:1 needs to be used a 1-D JCM shown in Figure 6 should be utilized. A 2-D JCM is selected in this study just to provide better understanding of the proposed method.

FIGURE 5 Physical interpretation of the PCD. x and y are coordinates of the physical domain. Each point in the physical domain possess an individual and identical PCD (with an identical JCM)

It should be noted that the JCM and PCD dimensions mentioned in Table 2 are suggested for general case. There can be specific scenarios where the user can select lower dimensions of JCM and PCD based on the relative material interface costs selected. For e.g. the JCM shown in Figure 3 can be presented in 1-D PCD as well shown below

FIGURE 6 Special case of PCD representing three material MMTO using 1-D PCD

One of the steps in PCD method requires spatial gradients calculation of the coordinates of weighted centroid discussed in section 2.5 of this paper. The number of spatial gradient calculations are equal to the dimension of JCM. Thus, an MMTO job considering a 2-D JCM would require two spatial gradient calculations per iteration whereas an MMTO job considering a 1-D JCM would require only one spatial gradient calculation per iteration.

Using a lower dimension JCM and PCD with the same relative joint cost ratios would not affect the final MMTO results; however, it would reduce the computational time as it would require fewer spatial gradient calculations. But, for the sake of generalization, in this paper, 2-D PCD is used for three material MMTO.

TABLE 2 Dimension and shape of different JCMs and PCDs based on the number of materials considered in MMTO

Number of Materials	Pseudo-Cost Domain Dimension	Joint Cost Model Example
Two	1D	
Three	2D	
Four	3D	
		Cannot be visualized
m	$(m-1)$ D	$\begin{pmatrix} \xi_1^1, \dots, \xi_{m-1}^1 \end{pmatrix}$ $\vdots$ $\begin{pmatrix} \xi_1^m, \dots, \xi_{m-1}^m \end{pmatrix}$

2.4 | Weighted centroid calculation

Once the PCD has been developed, the next step is to determine the weighted centroid (P) for each individual PCDs. The coordinates of the weighted centroid $(\phi^1, \phi^2, \dots, \phi^{m-1})$ are obtained by interpolating the vertices of the JCM in the PCD using material selection design vectors, $\rho^2, \dots, \rho^{m-1}$. Thus, P is independent of material existence design vector, ρ^1 . The weighted centroid is usually located close to the vertices of the JCM, where each vertex represents a particular material. The proximity of weighted centroid to a vertex in JCM depends on the type of material used at that location in the design domain. The equation for determining the weighted centroid is shown below

$$\phi_s^{(1, \dots, m)}(\rho^2, \dots, \rho^{m-1}) = \sum_{i=1}^m \gamma_i \xi_s^{(m-i+1)} \quad (6)$$

$$\text{where } \gamma_i = \left[1 - (\rho^{i+1} - \rho^{i+1} \delta_{im})^r \right] \prod_{t=1}^{i-1} (\rho^{t+1})^r$$

$$s = 1, \dots, m-1 \quad \delta_{im} = \begin{cases} 1, & i = m \\ 0, & i \neq m \end{cases}$$

where m is the number of materials used in the MMTO; $\xi_s^{(j)}$ is the s^{th} coordinate of j^{th} material in the PCD; $\phi_s^{(1, \dots, m)}$ is the interpolated s^{th} coordinate of weighted centroid in the PCD for each element, r is the penalty factor used to steer the material selection design variables $(\rho^2, \dots, \rho^{m-1})$ to discrete 0-1 values. Different values of r provide different levels of discreteness in determined material interface at each iteration. The recommended range of r value is from 1 to 4 as values higher than that result in poor MMTO convergence performance. In this study $r=2$ is used as it provided the best results and convergence performance.

It should be noted that the coordinates of weighted centroid are interpolated between m materials and not M material phases, like in the case of elastic modulus interpolation. The void elements in the design domain are interpreted as elements with material m i.e. material with lowest elastic modulus in PCD. The void-material interface determined by this process is later removed by using the material existence design vector (ρ^1) shown in (11). This approach is utilized to keep the current weighted centroid calculation step independent of material existence design vector (ρ^1) ; effectively determine material interfaces at intermediate densities and to increase computational efficiency.

For $m=3$ the following equation is obtained from the above recursive equation.

$$\phi_1^{(1,2,3)}(\rho^2, \rho^3) = \left\{ (\rho^2)^r \left[(\rho^3)^r \xi_1^1 + (1 - (\rho^3)^r) \xi_1^2 \right] + (1 - (\rho^2)^r) \xi_1^3 \right\} \quad (7)$$

$$\phi_2^{(1,2,3)}(\rho^2, \rho^3) = \left\{ (\rho^2)^r \left[(\rho^3)^r \xi_2^1 + (1 - (\rho^3)^r) \xi_2^2 \right] + (1 - (\rho^2)^r) \xi_2^3 \right\} \quad (8)$$

Figure 7 demonstrates the physical interpretation of the weighted centroid. The location of the weighted centroid can only be either inside or on the JCM, in the PCD. For a converged solution, the weighted centroid will most likely be located on either of the vertices of the JCM.

FIGURE 7 Physical interpretation of the weighted centroid in the PCD

2.5 | Spatial gradient magnitude calculation

Once the location of a weighted centroid in the PCD has been determined, the spatial gradient of the coordinates of this weighted centroid must be determined. It should be noted that a weighted centroid is defined in the PCD and its spatial gradient is measured with respect to the change in position within the design domain. The Sobel kernel is used to determine the gradient of the coordinates of a weighted centroid. The Sobel kernel has been extensively used in image processing for edge detection using a discrete convolution operator. This step provides the weighted material interface for the MMTO result.

$$\underline{\psi} = \|\underline{G}^d \underline{\phi}_s\| \quad (9)$$

$$\underline{\psi} = \sqrt{\sum_d^D \sum_s^{m-1} (\underline{G}^d \underline{\phi}_s)^2} \quad (10)$$

$$d = 1, 2, 3 \quad s = 1, \dots, m-1$$

$\underline{\psi}$ is the vector containing the sum of the gradient magnitude of the centroid coordinates for each element. $\underline{G}^d$ is an $n \times n$ matrix of gradient estimation coefficients, where n is the same length as $\underline{\phi}_s$. A $\underline{G}^d$ matrix is needed for each of the d spatial directions, which will be x , y , and z ($d = 1, 2$, and 3 , respectively). Each row of $\underline{G}_{ei}^d$ is filled such that if the i^{th} element is a neighbor of the e^{th} element, it has the appropriate Sobel coefficient based on their relative position. If they do not neighbor, the entry is set to 0. Ultimately, the operation inside the norm of (9) implies that to find the gradient in direction d for element e , all entries in the e^{th} row of $\underline{G}_{ei}^d$ must be multiplied by

their corresponding ϕ_s entries, and then a summation must be carried out over the i^{th} index. This is the standard multiplication of an $n \times n$ matrix with an $n \times 1$ vector and is analogous to performing a weighted sum over all neighboring elements.

Once the gradient magnitude ψ is obtained, the next step is to develop a total joint cost constraint. With the current approach both material-material interfaces and some void-material interfaces are determined for an MMTO result. Thus, to remove the void-material interfaces and increase robustness of the process, the gradient magnitude vector is multiplied by the penalized material existence design vector. (11) shows the final total joint cost which is used as a constraint in optimization.

$$\eta = \left\{ \left(\rho^1 \right)^h \right\}^T \psi \quad (11)$$

where h is the penalty factor used to discretize material existence design variable (ρ^1) to 0-1 values. This penalization helps remove the void-material interfaces from gradient magnitude vector effectively at each iteration. The recommended range of h value is from 1 to 4 as values higher than that would result in poor MMTO convergence performance. In this study $h=2$ is used as it provided the best results and convergence performance.

FIGURE 8 (a) An MMTO result discussed in section 2.1 and shown in Figure 2 (b) Weighted material interface obtained through usage of the PCD method by considering the JCM shown in Figure 3

The material interface determined by this method is presented in Figure 8(b) for the three material MMTO result shown in Figure 2. It displays the plot of the normalized vector obtained by the product of spatial gradient magnitude of each element with its respective penalized material existence design variable. The JCM shown in Figure 3 was considered. It can be observed from the figure that all individual material interfaces are identified and weighted accurately by this method, with ST-AL being the most expensive and TI-AL being the least.

2.6 | Revised optimization problem statement

Using the additional total joint cost information defined by (11), the standard minimum compliance MMTO problem statement can be revised to consider the relative material joining costs. Integration of this additional information into optimization allows designers to explore

different cost-effective design ideas and flexibility to tailor the MMTO design considering material joints.

The total joint cost equation can be implemented into the standard optimization problem statement by two distinct ways: (i) it can be implemented as an inequality constraint as shown in (12) or, (ii) it can be considered in the objective function by using a weighted multi-objective function considering compliance and total joint cost (13).

$$\begin{aligned}
 & \underset{\rho^1, \rho^2, \dots, \rho^{m-1}, \rho^m}{\text{minimize}} & C(\rho^j) &= \tilde{f}^T \tilde{u} = \tilde{u}^T \tilde{K} \tilde{u} \\
 & \text{subject to} & g_1(\rho^j) &= \frac{\sum_e W_e}{\sum_e W_e^{-1}} \leq \bar{W} \\
 & & g_2(\rho^j) &= \frac{\eta}{\eta^{ref}} \leq \bar{\eta} \\
 & & \tilde{K} \tilde{u} &= \tilde{f} \\
 & & \forall \text{ element } e, \rho_e^j &\in (0,1], j=1, \dots, m
 \end{aligned} \tag{12}$$

The value of total joint cost η cannot be predicted beforehand as it depends on the load case, number of materials, and mass fraction constraint. Thus, to effectively constraint the total joint cost, an initial unconstrained MMTO is performed and the total joint cost of the result is noted, η^{ref} . All subsequent MMTO iterations are constrained relative to the unconstrained MMTO total joint cost as a benchmark. This is logical as an unconstrained result would generate the most optimum design as it will have maximum design freedom.

$$\begin{aligned}
 & \underset{\rho^1, \rho^2, \dots, \rho^{m-1}, \rho^m}{\text{minimize}} & J(\rho^j) &= \gamma \frac{C}{C_0} + (1-\gamma) \frac{\eta}{\eta_0} \\
 & \text{subject to} & g_1(\rho^j) &= \frac{\sum_e W_e}{\sum_e W_e^{-1}} \leq \bar{W} \\
 & & \tilde{K} \tilde{u} &= \tilde{f} \\
 & & \forall \text{ element } e, \rho_e^j &\in (0,1], j=1, \dots, m \\
 & & \gamma &\in [0,1]
 \end{aligned} \tag{13}$$

where J is the linear combination of the two objective functions, i.e., compliance and total joint cost. γ is the biobjective weighting factor and can be any value between 0 and 1. C_0 , and η_0 are normalization factors which scale each of the objective functions such that they have similar magnitudes.

3 | SENSITIVITY ANALYSIS

In the current approach, the MMTO problem statement (12) and (13) are solved using a gradient-based optimizer (method of moving asymptotes). Thus, it is necessary to determine the sensitivities for the objective function and constraints. With implementation of the total joint cost

constraint, the sensitivity calculations of compliance and mass fraction are not altered. These sensitivity expressions are widely available and shown below.

Compliance sensitivities:

$$\frac{\partial C}{\partial \rho^j} = -\frac{\varepsilon}{E} \frac{\partial E}{\partial \rho^j} \quad (14)$$

$$\frac{\partial E^{(1,\dots,M)}}{\partial \rho^j} = \frac{\partial \left(\sum_{i=1}^M \beta_i E^{M-i+1} \right)}{\partial \rho^j} = \sum_{i=1}^M \left(\frac{\partial \beta_i}{\partial \rho^j} \right) E^{M-i+1} \quad (15)$$

$$\text{where } \beta_i = \left[1 - \left(\rho^i - \rho^i \delta_{iM} \right)^p \right] \prod_{t=1}^{i-1} \left(\rho^t \right)^p$$

$$\delta_{iM} = \begin{cases} 1, & i = M \\ 0, & i \neq M \end{cases}$$

$$\text{Note that } \frac{\partial \beta_i}{\partial \rho^j} = 0 \text{ when } i > j$$

(14) presents the compliance sensitivities, where ε is the element strain energy vector. (15) provides the sensitivity of the interpolated elastic modulus (shown in (4)) with respect to the pseudo-density, ρ^j with $j=1,\dots,m$. Here M is the number of phases; $m+1$ (number of materials, m , and the void phase)

The elastic modulus sensitivities for three material MMTO are described below. Three materials ($m=3$) necessitate three design vectors ρ^1, ρ^2 , and ρ^3 for MMTO

$$\frac{\partial E}{\partial \rho^1} = p \left(\rho^1 \right)^{p-1} \left\{ \left(\rho^2 \right)^p \left(\rho^3 \right)^p \left(E^1 - E^2 \right) + \left(\rho^2 \right)^p \left(E^2 - E^3 \right) + E^3 \right\} \quad (16)$$

$$\frac{\partial E}{\partial \rho^2} = p \left(\rho^1 \right)^p \left(\rho^2 \right)^{p-1} \left\{ \left(\rho^3 \right)^p \left(E^1 - E^2 \right) + E^2 - E^3 \right\} \quad (17)$$

$$\frac{\partial E}{\partial \rho^3} = p \left(\rho^1 \right)^p \left(\rho^2 \right)^p \left(\rho^3 \right)^{p-1} \left\{ \left(E^1 + E^2 \right) \right\} \quad (18)$$

Mass fraction sensitivities:

$$\frac{\partial g_1}{\partial \rho^j} = \frac{1}{\sum_e W_e^1} \frac{\partial W}{\partial \rho^j} \quad (19)$$

$$\frac{\partial W^{(1,\dots,M)}}{\partial \rho^j} = \frac{\partial \left(\sum_{i=1}^M \alpha_i W^{M-i+1} \right)}{\partial \rho^j} = \sum_{i=1}^M \left(\frac{\partial \alpha_i}{\partial \rho^j} \right) W^{M-i+1} \quad (20)$$

$$\text{where } \alpha_i = \left[1 - (\rho^i - \rho^i \delta_{iM}) \right] \prod_{t=1}^{i-1} (\rho^t)^q$$

$$\delta_{iM} = \begin{cases} 1, & i = M \\ 0, & i \neq M \end{cases}$$

$$\text{Note that } \frac{\partial \alpha_i}{\partial \rho^j} = 0 \text{ when } i > j$$

(19) presents the mass fraction constraint sensitivities. (20) displays the interpolated mass (shown in (3)) sensitivity with respect to pseudo-densities.

The mass sensitivities for three material MMTO are described below. As mentioned in section 1, the penalty factor $q=1$ is selected for mass interpolation

$$\frac{\partial W}{\partial \rho^1} = q (\rho^1)^{q-1} \left\{ (\rho^2)^q (\rho^3)^q (W^1 - W^2) + (\rho^2)^q (W^2 - W^3) + W^3 \right\} \quad (21)$$

$$\frac{\partial W}{\partial \rho^2} = q (\rho^1)^q (\rho^2)^{q-1} \left\{ (\rho^3)^q (W^1 - W^2) + W^2 - W^3 \right\} \quad (22)$$

$$\frac{\partial W}{\partial \rho^3} = q (\rho^1)^q (\rho^2)^q (\rho^3)^{q-1} \left\{ (W^1 + W^2) \right\} \quad (23)$$

Total joint cost constraint sensitivities:

The sensitivity of the total joint cost constraint shown in (11) is presented below:

$$\frac{\partial \eta}{\partial \rho^j} = \frac{\partial \left\{ (\rho^1)^h \right\}^T}{\partial \rho^j} \psi + \left\{ (\rho^1)^h \right\}^T \frac{\partial \psi}{\partial \rho^j} \quad (24)$$

Here, the spatial gradient vector $\underline{\psi}$ is dependent only on material selection design variables. It is independent of ρ_j^1 (material existence design variable). Thus, (24) can be divided into two parts shown below:

$$\frac{\partial \eta}{\partial \rho_j^1} = \begin{cases} h \left\{ \left(\rho^1 \right)^{h-1} \right\}^T \underline{\psi} & , j=1 \\ \left\{ \left(\rho^1 \right)^h \right\}^T \frac{\partial \underline{\psi}}{\partial \rho_j^1} & , j>1 \end{cases} \quad (25)$$

Here, $\frac{\partial \underline{\psi}}{\partial \rho_j^1}$ is the sensitivity of the spatial gradient vector of the coordinates of the weighted centroid. Its sensitivities are shown below:

$$\frac{\partial \underline{\psi}}{\partial \rho_j^1} = \sum_s^{m-1} \left(\frac{\sum_d^D \left(G^d \underline{\phi}_s \right) \left(G^d \frac{d \underline{\phi}_s}{d \rho_j^1} \right)}{\| G^d \underline{\phi}_s \|} \right) \quad (26)$$

In (26), $\frac{d \underline{\phi}_s}{d \rho_j^1}$ is the sensitivity of the s^{th} coordinate of the weighted centroid in the PCD. Its recursive equation is presented in (6) and its sensitivity equation is shown below:

$$\frac{\partial \underline{\phi}_s^{(1, \dots, m)}}{\partial \rho_j^1} = \frac{\partial \left(\sum_{i=1}^m \gamma_i \xi_s^{(m-i+1)} \right)}{\partial \rho_j^1} = \sum_{i=1}^m \left(\frac{\partial \gamma_i}{\partial \rho_j^1} \right) \xi_s^{(m-i+1)} \quad (27)$$

$$\text{where } \gamma_i = \left[1 - \left(\rho^{i+1} - \rho^{i+1} \delta_{im} \right)^r \right] \prod_{t=1}^{i-1} \left(\rho^{t+1} \right)^r$$

$$s = 1, \dots, m \quad \delta_{im} = \begin{cases} 1, & i = m \\ 0, & i \neq m \end{cases}$$

$$\text{Note that } \frac{\partial \gamma_i}{\partial \rho_j^1} = 0 \text{ when } i > j$$

The total joint cost sensitivities for three material MMTO are described below. Three materials ($m=3$) necessitate three design vectors ρ^1, ρ^2 , and ρ^3 and a 2D PCD with coordinate system ξ_1, ξ_2 . The weighted centroid for each element will also have two coordinates, ϕ_1 and ϕ_2 .

$$\frac{\partial \eta}{\partial \rho^1} = h \left\{ \left(\rho^1 \right)^{h-1} \right\}^T \left(\sum_d^D \left(\left(G^d \phi_1 \right)^2 + \left(G^d \phi_2 \right)^2 \right) \right)^{1/2} \quad (28)$$

$$\frac{\partial \eta}{\partial \rho^2} = \left\{ \left(\rho^1 \right)^h \right\}^T \left(\frac{\sum_d^D \left(\left(G^d \phi_1 \right)^T \left(G^d \frac{\partial \phi_1}{\partial \rho^2} \right) + \left(G^d \phi_2 \right)^T \left(G^d \frac{\partial \phi_2}{\partial \rho^2} \right) \right)}{\sum_d^D \|G^d \phi_1\| + \sum_d^D \|G^d \phi_2\|} \right) \quad (29)$$

$$\frac{\partial \eta}{\partial \rho^3} = \left\{ \left(\rho^1 \right)^h \right\}^T \left(\frac{\sum_d^D \left(\left(G^d \phi_1 \right)^T \left(G^d \frac{\partial \phi_1}{\partial \rho^3} \right) + \left(G^d \phi_2 \right)^T \left(G^d \frac{\partial \phi_2}{\partial \rho^3} \right) \right)}{\sum_d^D \|G^d \phi_1\| + \sum_d^D \|G^d \phi_2\|} \right) \quad (30)$$

$$d = 1, 2$$

The sensitivities of ϕ_1 and ϕ_2 with respect to ρ^2 and ρ^3 i.e. $\frac{\partial \phi_1}{\partial \rho^2}, \frac{\partial \phi_1}{\partial \rho^3}, \frac{\partial \phi_2}{\partial \rho^2}, \frac{\partial \phi_2}{\partial \rho^3}$ are shown below:

$$\begin{aligned} \frac{\partial \phi_1^{(1,2,3)}}{\partial \rho^2} &= \frac{r}{\rho^2} \left\{ \left(\rho^2 \right)^r \left[\left(\rho^3 \right)^r \xi_1^1 + \left(1 - \left(\rho^3 \right)^r \right) \xi_1^2 - \xi_1^3 \right] \right\} \\ \frac{\partial \phi_1^{(1,2,3)}}{\partial \rho^3} &= \frac{r}{\rho^3} \left\{ \left(\rho^2 \right)^r \left[\left(\rho^3 \right)^r \xi_1^1 - \left(\rho^3 \right)^r \xi_1^2 \right] \right\} \end{aligned} \quad (31)$$

$$\begin{aligned} \frac{\partial \phi_2^{(1,2,3)}}{\partial \rho^2} &= \frac{r}{\rho^2} \left\{ \left(\rho^2 \right)^r \left[\left(\rho^3 \right)^r \xi_2^1 + \left(1 - \left(\rho^3 \right)^r \right) \xi_2^2 - \xi_2^3 \right] \right\} \\ \frac{\partial \phi_2^{(1,2,3)}}{\partial \rho^3} &= \frac{r}{\rho^3} \left\{ \left(\rho^2 \right)^r \left[\left(\rho^3 \right)^r \xi_2^1 - \left(\rho^3 \right)^r \xi_2^2 \right] \right\} \end{aligned} \quad (32)$$

3.1 | Filtering

Following the objective and constraint sensitivity calculations, the sensitivities are filtered using the mesh independent sensitivity filter presented by Sigmund and Petersson⁴³. This process is performed to prevent the formation of checkerboards in the final TO results. The equation used to modify the sensitivities is shown below:

$$\left(\frac{\partial \Phi}{\partial \rho_e^j}\right)^* = \frac{1}{\rho_e^j \sum_{k=1}^N H_k} \sum_{k=1}^N \left(H_k \rho_k^j \frac{\partial \Phi}{\partial \rho_k^j} \right) \quad (33)$$

$$H_k = \max_k \{v - \text{distance}(k, e), 0\}$$

where $\frac{\partial \Phi}{\partial \rho_e^j}$ is the objective or constraint function sensitivity for ρ^j design variable of e^{th}

element. The sensitivity of the e^{th} element, $\frac{\partial \Phi}{\partial \rho_e^j}$, is modified to $\left(\frac{\partial \Phi}{\partial \rho_e^j}\right)^*$, which is a weighted sum of the sensitivities of neighboring elements lying within a circle of a given radius, v . The sensitivity of e^{th} element itself is not considered. The $\text{distance}(k, e)$ is the center-to-center distance between the e^{th} element and its k^{th} neighbor. N is the number of neighbours.

4 | NUMERICAL IMPLEMENTATION AND RESULTS

4.1 | Numerical implementation

The MATLAB 88-line TO code presented by Andreassen et al.⁴⁴ was used as a base to develop the presented work. The modified code utilized a density-based MMTO approach using a modified SIMP scheme described in (4). In the current study, only 2D geometries are studied but the mathematical methodology presented in the earlier sections has been developed independent of the spatial dimensionality or FE discretization scheme. The authors will address 3D geometries and load cases in future works. The method of moving asymptotes (MMA) is used as the optimizer in the current approach to solve the optimization problem. Convergence of the optimization was reached when each of the objective functions had stabilised to a maximum of 1% variation over five sequential iterations. A SIMP penalty factor of $p=3$ is used for the current study.

Four fundamental 2D load cases are considered in this work: (a) Three point loaded beam (b) Full Messerschmitt-Bölkow-Blohm (MBB) beam (c) Half MBB beam (d) 2D Mitchell Structure. Four node bilinear quadrilateral elements are used, all with side lengths and thicknesses of 1 mm. The geometric dimensions and applied loads for each unique load case are denoted in Figure 9. The properties of materials used for the study are detailed in Table 1.

FIGURE 9 Dimensions, loads, and constraints for the four example models presented in this work: (a) Three point loaded beam (b) Full Messerschmitt-Bolkow-Blohm (MBB) beam (c) Half MBB beam (d) 2D Mitchell Structure

4.2 | Numerical verification of sensitivity expressions

The analytical sensitivities derived in Section 4 were verified by comparing them with the sensitivities obtained through the finite difference method (FDM). Even though FDM provides accurate sensitivities, analytical methods are used to calculate sensitivities due to their computational benefits. The analytical sensitivities only require one evaluation of the structural finite element model (FEM) problem per iteration, whereas with sensitivities obtained through FDM, one FEM evaluation is required for every design variable at each optimization iteration, resulting in large computational expense, even for modestly sized problems. Thus, it is necessary to accurately and efficiently determine analytical sensitivities used in TO. To perform the comparison between analytical and FDM sensitivities, a $50 \times 25\text{ mm}$ simply supported beam load case was solved and stopped after 20 iterations. At this point, FDM sensitivities were computed for every design variable.

Figure 10(a) Shows the sampling locations for the data presented in Table A1. The sampling locations were chosen at random. (b) Shows the interpreted MMTO result. (c) Shows the material existence design variable (ρ^1), (d) and (e) show the material selection design variables (ρ^2, ρ^3 respectively). Table shows the ρ^1, ρ^2 , and ρ^3 sensitivities comparison. In all cases, the sensitivities show excellent agreement.

FIGURE 10 Load case and geometry used for the sensitivity verification process. (a) Shows the TO load case of the three point loaded beam along with the spatial locations of the design variables used for sensitivity verification through comparison of FDM and analytical sensitivities (b) Shows the intermediate MMTO result with the materials binned using the method shown in Figure 1. (c), (d) and (e) are the respective intermediate ρ^1, ρ^2 and ρ^3 values

4.3 | Material interface control analysis

In this section, different aspects of material interface control are analyzed considering three point loaded beam model shown in Figure 9(a).

Problem statement study:

As noted in section 2.6, the material interface control can be implemented in TO by either using it as an inequality constraint (12) or as an objective function (13). In this section both methods are studied to compare their respective performance. Pareto frontiers of relative compliance versus relative total joint cost are generated to illustrate the trade-off between the two competing

functions i.e. compliance and total joint cost. This is accomplished by varying the total joint cost constraint limit for (12) and weight factor for (13).

The extremities of the Pareto frontier correspond to the examples of maximal interface cost obtained by standard MMTO and optimum result with no consideration for interface cost obtained by standard SMTO. The model load case image is displayed at the bottom left-hand corner and JCM and image color bar at the top right corner of each Pareto frontier.

The Pareto frontier is obtained by progressively restricting the design freedom by limiting the allowable total joint cost. Values of compliance and total joint cost are reported as relative compliance and relative total joint cost by normalizing the raw quantities at each constraint limit by the raw quantities obtained using unconstrained MMTO.

Figure 11 and Figure 12 illustrate the Pareto frontier obtained using the JCM showed in Figure 3. The expected trade-off between compliance and total joint cost is seen; as total joint cost or weight factor is progressively reduced, the MMTO design freedom is increasingly constrained resulting in designs with increased compliance. The relative compliance increases in a somewhat exponential fashion and ultimately results in selection of a single-material steel result.

It can also be observed in Figure 11 and Figure 12 that MMTO is progressively replacing the steel-aluminum interface with steel-titanium interface as the total joint cost constraint gets increasingly restricted in case of (12) or minimizing total joint cost is increasing preferred in case of (13). This is expected as the JCM used in the study prefers the steel-titanium interface over steel-aluminum interface.

FIGURE 11 Pareto frontier of a three point loaded beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm, considering total joint cost as an inequality constraint (problem statement (12))

Comparing Figure 11 and Figure 12 it can be noticed that both approaches perform similarly and provide designer comparable control to tailor the TO results based on the total joint cost. Even though either approach can be used to implement total joint cost in TO, problem statement (12)

is selected for further exploration in this paper as it provides better control over the final total joint cost obtained in the MMT0 result compared to (13). If a particular final total joint cost value is required to be attained, it is easier to achieve using (12) rather than (13). The final total joint cost value cannot be predicted using (13).

FIGURE 12 Pareto frontier of a three point loaded beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm considering total joint cost as an objective function (problem statement (13)). The biobjective weighting factor (γ) value is displayed over each result

Material interface control performance study:

Since problem statement (12) is selected for further exploration, in this section its convergence performance is studied. The CPU times for each problem from Figure 11 are shown in Table 3. The differences in computational expense are caused by differences in the number of iterations since there is one FE analysis, sensitivity analysis and optimization per iteration. The problem was solved on a Windows desktop PC with an Intel i7-6800 K CPU clocked at 3.40 GHz and 65536 MB of DDR4 RAM clocked at 2133 MHz. All problems were running in parallel with one core assigned per problem.

The convergence histories of results shown in Figure 11 with total joint cost constraint limits of $\bar{\eta} = 0.8, 0.6, 0.4$ and 0.2 are displayed in Figure 13. Here the compliance and percent constraint violation values for mass fraction and total joint cost are displayed for every iteration.

Convergence History Plots for Compliance, Mass Fraction and Total Joint Cost Constraint ($\bar{\eta} = 0.8$)

Convergence History Plots for Compliance, Mass Fraction and Total Joint Cost Constraint ($\bar{\eta} = 0.6$)

Convergence History Plots for Compliance, Mass Fraction and Total Joint Cost Constraint ($\bar{\eta} = 0.4$)

Convergence History Plots for Compliance, Mass Fraction and Total Joint Cost Constraint ($\bar{\eta} = 0.2$)

FIGURE 13 Convergence histories of results shown in Figure 11 with total joint cost constraint $\bar{\eta} = 0.8, 0.6, 0.4$ and 0.2 . The left plot shows the objective (compliance) convergence as well as

the mass fraction constraint violation. The right plot shows the total joint cost constraint violation

Both the compliance and mass fraction constraint violation plots (shown in left) for each problem exhibit relatively smooth convergence behavior with no oscillations whereas the total joint cost constraint violation convergence plot (shown in right) does show some oscillations. This is expected as total joint cost constraint is a global constraint which can alter significantly with a slight material change in the design space. This behavior is noticeable for stricter total joint cost constraint problems.

TABLE 3 CPU time for three point loaded beam examples shown in Figure 11

Relative Total Joint Cost [%]	No. of Iterations	Total CPU Time [hh:mm:ss]
100	96	3:08:21
90	66	2:28:04
80	80	2:51:36
70	56	2:12:50
60	65	2:24:16
50	74	2:41:43
40	65	2:25:40
30	117	4:19:41
20	98	4:10:54
10	131	4:17:01
0	119	1:16:10

It can also be observed that the mass fraction and total joint cost constraint violation for every problem converges to a zero percent violation. Thus, both the mass and total joint cost constraint are fully utilized for each and every problem. This explains why some of the results in Figure 11 obtain steel parts enclosed by thin aluminum or titanium structures as the optimizer utilizes all the allowable mass and joint cost for minor improvement in stiffness. This issue can be mitigated by tightening the total joint cost constraint values.

Material interface control in two and four material MMTO:

The material interface control methodology explained in this paper can be used for m different materials. The generalized equations of PCD method and material interface control considering m -materials are presented in section 2 and 3. In this section, material interface control considering two and four materials are studied.

Figure 14 shows the Pareto frontier of a three point loaded beam model with a mass fraction constraint of 65% considering steel and aluminum materials. Since only two materials are considered for MMTO, any joint cost except zero can be considered. A linear JCM with its vertices coordinates in PCD equal to one and two ($\xi_1^1 = 1$ and $\xi_1^2 = 2$) is used which results in the cost of joining steel and aluminum to unity. The constraint performs as expected, as it progressively reduces the material interface with the reduction in allowable total joint cost.

FIGURE 14 Pareto frontier of a three point loaded beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm, considering steel and aluminum for MMTO with joint cost of unity

Figure 16 shows the Pareto frontier of a three point loaded beam model with a mass fraction constraint of 65% considering steel, titanium, aluminum, and magnesium materials. A 3D regular tetrahedron with coordinates shown in Figure 15 is used as a JCM for the study. Each material interface is assigned identical joint cost, which in this case is unity. Thus, the material interface constraint would limit the overall amount of material interfaces in the MMTO design irrespective of the type of materials involved in the interface. The constraint performs as expected, as it progressively reduces the material interface with the reduction in allowable total joint cost.

FIGURE 15 3D regular tetrahedron joint cost model (JCM) for a four material MMTO formulation considering steel (ST), titanium (TI), aluminum (AL) and magnesium (MG) with identical material interface joint cost ratios of 1:1:1:1

FIGURE 16 Pareto frontier of a three point loaded beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm, considering steel, titanium, aluminum and magnesium for MMTO with joint cost of unity for every material interface

Even though the methodology mentioned in this paper can be used for m different materials, to effectively demonstrate the capabilities of PCD method with different JCMs three material MMTO is studied in the subsequent sections.

4.4 | Material interface control detail study

In this section, simultaneous minimization of structural compliance and restriction of total joint cost using the problem statement (12) is demonstrated using the four load cases detailed in Figure 10 and three materials. Two different types of JCMs are studied as shown in Table 4. It can be noted that JCM 1 has steel-aluminum as the most expensive interface, whereas in JCM 2 the steel-titanium interface is most expensive. The titanium-aluminum interface is maintained as the cheapest interface in both studies as a control. These relative joint costs were chosen arbitrarily to demonstrate method capabilities. Two different JCMs are studied to demonstrate the impact of the JCM in the current methodology. This can be observed by comparing the Pareto frontiers of individual JCM.

The following examples illustrate the influence of variation in loading, geometry, mass fraction, and JCM relative to the geometry. In each case study, a higher than normal mass fraction constraint is used with all examples featuring mass fraction constraint above 50% compared to typical practical lightweighting TO problems having constraint ranging from 10% to 30%. This is done in order to obtain sufficient material interface to effectively demonstrate the

capabilities of the presented methodology. Due to the use of higher mass fractions, the difference in compliance values of SMTO and MMTO designs is modest.

TABLE 4 Types of JCMs used in the current study

4.5 | Three point loaded beam

The following example demonstrates the influence of adding a total joint cost constraint to the standard MMTO problem for a three point loaded beam. Figure 11 illustrated the Pareto frontier obtained using the JCM 1. As stated previously for results shown in Figure 11, MMTO is progressively replacing the steel-aluminum interface with steel-titanium interface as the total joint cost constraint gets increasingly restricted. This is expected as JCM 1 is used in the study which selectively prefers the steel-titanium interface over steel-aluminum.

Figure 17 illustrates a nearly identical configuration to that of Figure 11, but with the JCM 2. Thus, both share identical load case and mass fraction values, with the only difference as weighing of individual material interfaces. Comparing the Pareto frontiers in Figure 11 and Figure 17, it can be observed that both use different approaches to mitigate the tighter total joint cost constraint. While the former replaces the expensive interface with the relatively less expensive interface, the latter is modifying the perimeter of the steel loadpath to reduce the total joint cost. Also, it can be observed that for the total joint cost constraint of 20%, the JCM 1 provides an MMTO result utilizing all three materials whereas JCM 2 provides result consisting only of steel and aluminum.

It is also noted that the trade-off between compliance and total joint cost for the Figure 11 is more ordered compared to Figure 17. It is important to note that the severity of the trade-off is highly dependent on the geometric configuration, optimization constraints, and JCM. This observation is further illustrated by the subsequent examples in this section.

This example shows that the presented PCD method is effectively able to determine and weigh individual material interfaces based on the selected JCM and accordingly alter the MMTO design to restrict the use of expensive material interface when implemented as a total joint cost constraint.

FIGURE 17 Pareto frontier of a three point loaded beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm, considering JCM 2, with steel-titanium interface as most expensive

4.6 | Full Messerschmitt-Bölkow-Blohm (MBB) beam

The following two examples illustrate the influence of total joint cost constraint to a standard MMTO problem for a full MBB beam. Figure 18 shows the Pareto frontier considering JCM 1. It should be noted that unconstrained MMTO provides designs primarily composed of steel and aluminum. As seen in Figure 18, as total joint cost is restricted, in order to reduce the now expensive steel-aluminum interface, titanium utilization is increased. When total joint cost is further restricted voids are introduced at material interface between bottom steel section and center aluminum section to decrease the total joint cost. This can be noticed in problems with joint cost constraint of 60%, 40% and 20% where this void region is gradually increasing between steel and aluminum as the total joint cost is restricted more and more. Thus, MMTO is naturally placing voids at strategic locations providing designer with the specific material joining locations.

Similar trend is observed in Figure 19 which displays the Pareto frontier of the same full MBB load case considering JCM 2. It can be observed in Figure 19 that rather than eliminating the costly steel-titanium interface, MMTO reduces the interface surface area in order to minimize total joint cost. The compliance values increase considerably for JCM1 and JCM 2 results, resultant from the loss of the main steel loadpath, as a consequence of total joint cost reduction.

This example presents an additional benefit of using the presented PCD method as a total joint cost constraint, as it naturally provides precise joint locations by strategically placing voids at material interfaces.

FIGURE 18 Pareto frontier of a full Messerschmitt-Bölkow-Blohm (MBB) beam with a mass fraction of $\bar{W} = 0.60$, filter radius $r = 3$ mm, considering JCM 1, with steel-aluminum interface as most expensive

FIGURE 19 Pareto frontier of a full Messerschmitt-Bölkow-Blohm (MBB) beam with a mass fraction of $\bar{W} = 0.60$, filter radius $r = 3$ mm, considering JCM 2, with steel-titanium interface as most expensive

4.7 | Half Messerschmitt-Bölkow-Blohm (MBB) beam

The following two examples illustrate the influence of the total joint cost constraint on a standard MMTO problem for a half MBB beam. This particular example is used to demonstrate the functionality of the proposed methodology to alter the unconstrained MMTO design when a designer wants to restrict excessive and redundant utilization of a material. Figure 20 shows the Pareto frontier for a half MBB beam considering JCM 1. It can be observed in the unconstrained MMTO result that it uses a section of titanium on the lower steel loadpath, which a designer may consider impractical for implementation. The proposed methodology is suitable for this case as it takes material joining costs into account, discouraging usage of some materials.

JCM 2 is more suitable for this problem as it restricts the steel-titanium interface, which can be seen in Figure 21. In the Pareto frontier shown in Figure 20, usage of JCM 1 results in progressive replacement of the steel-aluminum interface with steel-titanium, whereas the Pareto frontier shown in Figure 21, which utilized JCM 2, the steel-titanium interface is progressively reduced without being entirely replaced. It can be noted that at intermediate total joint cost constraint values, the titanium section used in the bottom segment is reduced and a more manufacturable design is obtained.

FIGURE 20 Pareto frontier of a half Messerschmitt-Bölkow-Blohm (MBB) beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm, considering JCM 1, with steel-aluminum interface as most expensive

FIGURE 21 Pareto frontier of a half Messerschmitt-Bölkow-Blohm (MBB) beam with a mass fraction of $\bar{W} = 0.65$, filter radius $r = 3$ mm, considering JCM 2, with steel-titanium interface as most expensive

This example also displays an added benefit of using the presented PCD method as a total joint cost constraint, as it allows the designer to alter the final MMTO design as desired.

4.8 | 2D Mitchell Structure

Finally, the following two examples illustrate the influence of the total joint cost constraint on a standard MMTO problem for a 2D Mitchell structure. Unlike previous examples the 2D Mitchell structure provided comparable designs between JCM 1 and 2, till strict joint cost constraint, which can be noted in Figure 22 and Figure 23. Comparing both Pareto frontiers, it can be noted that both follow the similar trend of reducing both steel-titanium and titanium-aluminum interfaces when total joint cost is increasingly constrained. The Pareto frontier shown in Figure 22, which uses JCM 1, removes all steel-aluminum interfaces and replaces them with steel-titanium interfaces whereas the Pareto frontier in Figure 23 retain certain amount of steel-aluminum interfaces. Also, it can be observed that the central steel section in the latter result takes on a circular shape to minimize surface area of the steel-titanium interface. Significant difference in MMTO results using JCM 1 and JCM 2 is obtained when the total joint cost constraint is restricted to 20%; with former providing three material solution whereas the latter providing only two material solution.

FIGURE 22 Pareto frontier of a 2D Mitchell structure with a mass fraction of $\bar{W} = 0.50$, filter radius $r = 3$ mm, considering JCM 1, with steel-aluminum interface as most expensive

FIGURE 23 Pareto frontier of a 2D Mitchell structure with a mass fraction of $\bar{W} = 0.50$, filter radius $r = 3$ mm, considering JCM 2, with steel-titanium interface as most expensive

5 | CONCLUSIONS AND DISCUSSIONS

In this work, a novel PCD method that determined individual material interfaces in an MMTO result was demonstrated. The pivotal benefit of this method is the ability to weigh the individual material interfaces with respect to each other by means of a user defined JCM. Based on the developed method, a total joint cost constraint was created to consider relative material interface cost within density-based MMTO. The methodology was formulated in a smooth and differentiable manner, permitting calculation of total joint cost constraint sensitivity expressions which are required in gradient-based optimizers (MMA in this case). The methodology provides designers with a tool for selection of the most effective multi-material design given particular structural and manufacturing cost requirements.

The proposed methodology uses the spatial gradient of the weighted centroid, which is defined in the PCD to determine individual material interfaces. The major contribution of this work is the development of a novel methodology for determination of individual material interfaces for an MMTO result. The presented implementation can be used for any number of materials. The total joint cost constraint and its sensitivities with respect to TO design variables were presented for m materials. The methodology for MMTO considering the total joint cost constraint was illustrated using several numerical examples, each demonstrated the validity, generality, and effectiveness of the approach. Four different 2D models were used to thoroughly convey that the proposed methodology works for a range of problems and to provide insight into its operation.

All four models illustrated the impact of a JCM on final MMTO results. The three point loaded beam model showed how MMTO takes different approaches to meet the total joint cost constraint when different JCMs are used. The full MBB beam model displayed the capability of the proposed method to determine optimum locations to join two materials. The half MBB beam study shows the use of the tool for simplification of MMTO results by removing small material sections which provide low stiffness but add significant additional joining costs. The 2D Mitchell structure showed the ability of the method to preserve the optimum MMTO result for different JCMs by making subtle changes to interface selection.

A disadvantage to the total joint cost constraint implementation into MMTO is that the total joint cost can make abrupt changes during the optimization procedure. This is generally caused by the conflicting nature of compliance optimization and total joint cost constraint satisfaction. This somewhat unstable behavior can make optimization convergence challenging. Improving the stability of this objective function to increase reliability of convergence remains a challenge to be addressed in future work.

The results shown in this work achieve the desired goal of joint cost consideration of different materials during an MMTO study; however, it should be noted that other methods presented in the literature have been shown to determine individual material interfaces and implement joint material properties at the interfaces. The approach these works take can be generally summarized as taking the spatial gradient of the material selection design vector to determine material

interfaces, is fundamentally different from the proposed method in this paper. The results from those studies are generally restricted to two materials and it is difficult to extend them to complex 3D geometries. The results generated through the presented methodology are more extensible to generalized design domains and boundary conditions.

ACKNOWLEDGEMENTS

This research was funded by the Natural Sciences and Engineering Research Council of Canada (NSERC) and General Motors of Canada. Technical advice and direction were gratefully received from Derrick Chow, Ralph Brown and Jonathan Rossiter at General Motors.

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APPENDIX A

TABLE A1 Sensitivity verification for the total joint cost constraint function. Analytical sensitivities are compared with sensitivities calculated using central difference method with step size of 1E-3

Element ID (<i>e</i>)	FDM $\frac{\Delta \psi}{\Delta \rho^1}$	Analytical $\frac{\partial \psi}{\partial \rho^1}$	$\frac{\Delta \psi / \Delta \rho^1}{\partial \psi / \partial \rho^1} \times 100\%$	FDM $\frac{\Delta \psi}{\Delta \rho^2}$	Analytical $\frac{\partial \psi}{\partial \rho^2}$	$\frac{\Delta \psi / \Delta \rho^2}{\partial \psi / \partial \rho^2} \times 100\%$	FDM $\frac{\Delta \psi}{\Delta \rho^3}$	Analytical $\frac{\partial \psi}{\partial \rho^1}$	$\frac{\Delta \psi / \Delta \rho^3}{\partial \psi / \partial \rho^1} \times 100\%$
69	0.0000	0.0000	100.00%	0.0000	0.0000	100.00%	0.0000	0.0000	100.00%
112	1.0769	1.0769	100.00%	0.4342	0.4342	100.00%	0.8877	0.8877	100.00%
135	0.4613	0.4613	100.00%	0.0975	0.0975	100.00%	-0.5673	-0.5673	100.00%
246	0.1452	0.1452	100.00%	-0.0362	-0.0362	100.00%	-0.1866	-0.1866	100.00%
269	0.0066	0.0066	100.00%	-0.0165	-0.0165	100.00%	0.0046	0.0046	100.00%
354	0.7137	0.7137	100.00%	0.0474	0.0474	100.00%	-0.4476	-0.4476	100.00%
371	1.6330	1.6330	100.00%	0.0307	0.0307	100.00%	0.0955	0.0955	100.00%
417	0.0122	0.0122	100.00%	-0.0544	-0.0544	100.00%	0.0039	0.0039	100.00%
456	1.0481	1.0481	100.00%	0.0140	0.0140	100.00%	-0.6351	-0.6351	100.00%
511	0.0007	0.0007	100.00%	-0.0355	-0.0355	100.00%	0.0011	0.0011	100.00%
583	0.0014	0.0014	100.00%	0.0609	0.0609	100.00%	-0.0157	-0.0157	100.00%
686	0.0000	0.0000	100.00%	-0.0037	-0.0037	100.00%	0.0001	0.0001	100.00%
772	0.0891	0.0891	100.00%	0.5478	0.5478	100.00%	0.8107	0.8107	100.00%
823	0.0000	0.0000	100.00%	0.3008	0.3008	100.00%	0.4460	0.4460	100.00%
906	0.0059	0.0059	100.00%	0.0777	0.0777	100.00%	-0.2456	-0.2456	100.00%
923	0.0133	0.0133	100.00%	0.5615	0.5615	100.00%	0.8324	0.8324	100.00%
1011	0.0460	0.0460	100.00%	0.1148	0.1148	100.00%	-0.3889	-0.3889	100.00%
1066	1.5109	1.5109	100.00%	0.1312	0.1312	100.00%	0.3828	0.3828	100.00%
1141	0.0000	0.0000	100.00%	0.0413	0.0413	100.00%	0.0613	0.0613	100.00%
1185	0.0255	0.0255	100.00%	0.2255	0.2255	100.00%	-0.7214	-0.7214	100.00%