Natural Convective Heat Transfer from Horizontal Isothermal Surface of Polygons of Octagonal and Hexagonal Shapes

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Abstract

Heat transfer often occurs effectively from horizontal elements of relatively complex shapes in natural convective cooling of electronic and electrical devices used in industrial applications. The effect of complex surface shapes on laminar natural convective heat transfer from horizontal isothermal polygons of hexagonal and octagonal flat surfaces facing upward and downward of different aspect ratios has been numerically investigated. The polygons surface is embedded in a large surrounding plane adiabatic surface, where the adiabatic surface is in the same plane as the surface of the heated element. For the Boussinesq approach used in this work, the density of the fluid varies with temperature, which causes the buoyancy force, while other fluid properties are assumed constants. The numerical solution of the full three dimensional form of governing equations is obtained by using the finite volume method based CFD code, FLUENT14.5. The solution parameters include surface shape, dimensionless surface width, different characteristic length, the Rayleigh number, and the Prandtl number. These parameters are considered as follows; Prandtl number is 0.7, the Rayleigh numbers are between 10^3 and 10^8 , and for various surface shapes the width-to-height ratios are between 0 and 1. The effect of different characteristic length has been investigated in defining the Nusselt and Rayleigh numbers for such complex shapes. The effect of these parameters on the mean Nusselt number has been studied, and correlation equations for the mean heat transfer rate have been derived.

Introduction

The phenomenon of convective heat transfer, which naturally occurs in elements with relatively complex geometrical shapes, have found numerous applications in the cooling of electronic components and devices. Consequently, the study of natural convective heat transfer has attracted significant attention, in particular, for simple two-dimensional shapes, which could be used to in deriving generalized equations pertinent to those practical cooling applications.

The literature is abundant with studies of natural convective heat transfer from upward and downward two-dimensional facing horizontal plates, albeit being of relatively simple shapes and in various situations. Insights from these prior studies, offered numerous empirical and analytical correlations, which have been reviewed in many textbooks, e.g., see [1-6]. In most simple configurations, the analytical expression for the mean Nusselt number is $Nu = C Ra^n$, where Ra is the Rayleigh number and the coefficient C and exponent n are constants. The values of C and n can be fitted to model convective heat transfer between bodies of various geometries and orientations and are generally affected by Pr , the characteristic length, geometry and orientation of the heating element and the range of the Rayleigh number, etc.

A significant body of work on the theoretical studies of natural convective heat transfer between horizontally placed surfaces can be found, e.g., see [7-11]. While the earlier experimental studies of the same phenomenon exist, e.g., see [12-22]. A verity of situations has been considered, in the literature, for natural convective heat transfer between three-dimensional bodies, e.g., see [1-5]. The aforementioned studies analyzed a

variety of inclination angles in relation to the vector of gravity. In general, a unique, yet different relationship between Nu and Ra for natural convective heat transfer can be found for specific shapes and aspect ratios. However, introducing the concept of the characteristic length, allows the derivation of a single relationship between Nu and Ra , which can accommodate a variety of these situations, albeit being chosen somewhat arbitrarily as discussed by Yovanovich and Jafarpur [23]. If the definition of Nu and Ra incorporates the ratio of the surface area of the element to its perimeter, then a unique relationship between the Nusselt and Rayleigh numbers arise at a given Prandtl number. In such situations, the variation of Nusselt and Rayleigh numbers is effectively identical for all element shapes, e.g., see [1-5]. It is found that if the length scale based on A/P where A and P are the surface area of the element and its perimeter, respectively, then the variation of Nu and Ra for a given Pr is effectively the same for all horizontal element shapes. Yovanovich *et al*, e.g., see [23-26], showed that utilizing the characteristic length h to describe the body gravity function reveals the sensitivity of the area mean Nusselt number on the element orientation and geometry. However, the authors found that defining the characteristic length as $\sqrt{A}$, is a superior choice for the length scale as it renders the body-gravity function insensitive to the orientation and geometry of the said element.

Although countless analytical correlation equations are presented in the literature for natural convective heat transfer, they do not account for the shrinking element sizes. In particular, as Ra and the size of the element's surface area becomes smaller, it becomes increasingly imperative to consider the effects of thermal diffusion and three-dimensional flow near the edges of the elements. The inadequacy of the generalized empirical

formulas, available in the literature, becomes fully manifested, when three-dimensional flow exists, and the size of the surface area of the elements becomes smaller. Consequently, utilizing these formulas underestimate the rate of heat transfer. For these situations, the numerical solution of the full three-dimensional governing equations is warranted. For example, numerical solutions for complex narrow plane surfaces with different aspect ratios and inclination angles in a variety of flow regions have been discussed in [25-35], where some empirical correlations have been developed.

The work presented in references [36, 37] considered the numerical study of natural convective heat transfer from horizontal surfaces of complex shapes, while not including polygons. Furthermore, reference [38] considered the numerical study of natural convective heat transfer from vertical surfaces of hexagonal and octagonal shapes. Finally, the work presented in [39] considered a brief overview of many previous works of Oosthuizen and Kalendar for natural convective heat transfer from horizontal and near horizontal surfaces in laminar to turbulent flow with different shapes and surface boundary conditions. As a consequence, the focus of this work is on horizontal surfaces of polygons (octagonal and hexagonal shapes) facing upward and downward, depicted in Fig. 2, that are relatively narrow, i.e., exhibiting a small width, w , to height, h , ratios. The present study is a continuous of a series of studies of natural convective heat transfer of complex shapes [36-39]. However, the main target of the work presented therein is to find correlations for the heat transfer rate produced under these circumstances, while completely accounting for three-dimensional flow near the edge of relatively narrow heated elements. Specific and careful attention is given to the adequate numerical modeling of narrow heated surface of a variety of shapes and sizes. A detailed

comparison with other scenarios is performed allowing the present results to be validated for an extensive range of different plane surface shapes and geometries, exceeding those considered in the past studies.

The outcome of the study presented herein, can be effectively utilized in deriving generalized equations pertinent to those practical cooling applications, albeit analyzing surfaces which are not identical to the ones present in aforementioned applications. Furthermore, numerical investigate of the natural convective heat transfer between isothermal heated plates is also presented. The plates considered are those with horizontally facing plane surfaces exhibiting various aspect ratios. In general a large horizontal adiabatic surface is used to house the plates within the same plane as the heated elements. The flow edifice considered is shown in Fig. 1. The various shapes of the heated elements analyzed are: square, octagonal, hexagonal and diamonds shape of different aspect ratio as shown in Fig. 2. It is assumed that the element exhibits a higher temperature than the surrounding fluid. Both upward and downward facing elements are considered as shown in Fig. 3. Furthermore, for comparative purposes results have also been obtained for a wide range of heated square element. A relatively extensive array of Rayleigh numbers, element geometries, dimensionless element sizes, and different length scales are considered. In this work, the Prandtl number has been restricted to 0.7, approximating the value of air thus being relevant to the many applications motivating this study.

Fig. 1 Flow conditions modeled in the present study.

Fig. 2 Surface shapes and geometries modelled: (a) Equilateral Hexagon ($L=w$), (b) Unequal Hexagon ($L>w>0$), (c) Rhombus ($w=0$), (d) Equilateral Octagon ($L=w, h=d$), (e) Unequal Octagon ($L>w>0, h=d$), (f) Rhombus ($w=0, h=d$), (g) Unequal Octagon ($L>w>0, h\neq d$), (h) Unequal Octagon ($w=0, h\neq d$), (i) Square ($h=L$). Reprinted from Kalendar et al. [38], with permission from Heat Transfer-Asian Research.

Fig. 3 Upward and downward facing element orientations.

Numerical Solution Procedure

The assumed flow in this work has been steady and laminar. The Boussinesq approach used in this work, the density of the fluid varies with temperature, which causes the buoyancy force, while other fluid properties are assumed constants. The full three-dimensional form of the governing equations, which being written in terms of dimensionless variables has been solved numerically. These governing equations and its dimensionless variables are being described in our previous work of Kalendar et al. [38].

The numerical simulation domain used in modeling the various shapes is as shown in Fig. 4. The boundary conditions imposed on the numerical solutions for the different surface shapes shown in Fig. 2, are in terms of a set of dimensionless variables.

The following dimensionless variables were defined:

$$\begin{aligned}
X &= \frac{x}{h}, Y = \frac{y}{h}, Z = \frac{z}{h}, U_X = \frac{u_x}{u_r}, u_r = \frac{\alpha}{h} \sqrt{Ra Pr} \\
U_Y &= \frac{u_y}{u_r}, U_Z = \frac{u_z}{u_r}, P = \frac{(p - p_F)h}{\mu u_r}, \theta = \frac{T - T_F}{T_H - T_F}
\end{aligned} \tag{1}$$

where θ is the dimensionless temperature. The x -coordinate is measured in the direction normal to the heated element surface, the y -coordinate and the z -coordinate are measured in the horizontal direction in the plane of the heated element surface (see Fig. 4).

In terms of these dimensionless variables, the governing equations are:

$$\frac{\partial U_X}{\partial X} + \frac{\partial U_Y}{\partial Y} + \frac{\partial U_Z}{\partial Z} = 0 \tag{2}$$

$$U_X \frac{\partial U_X}{\partial X} + U_Y \frac{\partial U_X}{\partial Y} + U_Z \frac{\partial U_X}{\partial Z} = \sqrt{\frac{Pr}{Ra}} \left(-\frac{\partial P}{\partial X} + \frac{\partial^2 U_X}{\partial X^2} + \frac{\partial^2 U_X}{\partial Y^2} + \frac{\partial^2 U_X}{\partial Z^2} \right) \tag{3}$$

$$U_X \frac{\partial U_Y}{\partial X} + U_Y \frac{\partial U_Y}{\partial Y} + U_Z \frac{\partial U_Y}{\partial Z} = \sqrt{\frac{Pr}{Ra}} \left(-\frac{\partial P}{\partial Y} + \frac{\partial^2 U_Y}{\partial X^2} + \frac{\partial^2 U_Y}{\partial Y^2} + \frac{\partial^2 U_Y}{\partial Z^2} \right) + \theta \tag{4}$$

$$U_X \frac{\partial U_Z}{\partial X} + U_Y \frac{\partial U_Z}{\partial Y} + U_Z \frac{\partial U_Z}{\partial Z} = \sqrt{\frac{Pr}{Ra}} \left(-\frac{\partial P}{\partial Z} + \frac{\partial^2 U_Z}{\partial X^2} + \frac{\partial^2 U_Z}{\partial Y^2} + \frac{\partial^2 U_Z}{\partial Z^2} \right) \tag{5}$$

$$U_X \frac{\partial \theta}{\partial X} + U_Y \frac{\partial \theta}{\partial Y} + U_Z \frac{\partial \theta}{\partial Z} = \frac{1}{\sqrt{Ra Pr}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right) \tag{6}$$

The assumed boundary conditions are as follows; for heated element surface, the dimensionless velocity components are nulled (set to zero), while the dimensionless temperature is set to unity. In the case of the adiabatic base, the imposed boundary conditions are set to zero for both dimensionless velocities perpendicular to all surfaces and dimensionless temperature gradient perpendicular to the surface. On the outer planes

of the numerical solution, both dimensionless temperature and velocity components are set to zero in the plane of the surface. Furthermore, both dimensionless temperature and pressure components at the surface are also set to zero, albeit on the upper outer plane of the numerical solution domain as shown in Eq. (7).

$$\text{CDLEJMKP} : U_X = 0, U_Y = 0, U_Z = 0, \theta = 1$$

$$\text{FNST except for CDLEJMKP} : U_X = 0, U_Y = 0, U_Z = 0, \frac{\partial \theta}{\partial X} = 0$$

$$\text{AHTF} : U_Y = 0, U_Z = 0, \theta = 0$$

$$\text{ABNF} : U_X = 0, U_Y = 0, \theta = 0 \tag{7}$$

$$\text{ABRH} : U_X = 0, U_Z = 0, \theta = 0$$

$$\text{NBRS} : U_Y = 0, U_Z = 0, \theta = 0$$

$$\text{HRST} : P = 0$$

Fig. 4 Numerical solution domain for upward facing heated element

The mean Nusselt number has been expressed in terms of mean heat transfer rate from the heated surface as follows:

$$Nu = \frac{\bar{q}' z}{k(T_H - T_F)} \quad (8)$$

where the Nu is the mean Nusselt numbers for the heated element surface based on temperature differences, and z is the characteristic lengths used in this study of w or m or $\sqrt{A}$ and T_F is the undisturbed fluid temperature. The commercial finite volume method modelling software FLUENT[®] 14.5, has been used for solving the dimensionless governing equations while imposing the aforementioned boundary conditions discussed above. In the present study, hexahedral cells were created with a fine mesh near the heated surface wall and the heated surface edges using GAMBIT. A non-uniform grid distribution was used in the planes perpendicular and parallel to the main flow direction. Close to the heated surface, the number of grid points or control volumes was increased to enhance the resolution and accuracy.

A fine mesh was rendered near the walls and the edges of the heated surface using hexahedral cells. To improve fidelity of the numerical simulations, detailed grid and convergence criterion were independently tested. The total grid of 10^6 nodes was found to be sufficient as further refinement did not change the predicted mean mean Nusselt number results. In fact the actual number of nodes utilized was found to depend on the Rayleigh number and size of the heated surface. Consequently, the heat transfer predictions in this work are within 1% independence of both the number of grid points and of the convergence-criterion. Furthermore, the effect of locating the outer surfaces of the solution domain (i.e., surfaces THRS, ABRH, AHTF, BRSN, and ABNF in Fig. 4) from the heated surface was minimized to ensure that the heat transfer results were independent to within 1%. The average Nusselt number was stable within 1% for residual values below 10^{-4} . In particular, each equation for mass and momentums have been iterated until the residuals fall below 10^{-4} , while the energy equation has been iterated

until the residual falls below 10^{-6} . The pressure-based solver utilized a second-order upwind scheme for convective terms in the mass, momentums and energy equations. In the case of pressure discretization, the Presto scheme has been employed while the SIMPLE-algorithm has been used for pressure-velocity coupling discretization.

Table 1 Correlation equations available in literatures for natural convective heat transfer from a horizontal upward facing surface of finite shape with a constant wall temperature boundary condition in laminar flow region.

No.	Reference	Geometry	Fluid/Pr number	Range	Correlation Equation
1	Fishenden & Saunders [12]	Square $\mathcal{L} = w$	Gasses & Liquids	$10^5 < Ra < 10^7$	$Nu = 0.54 Ra^{0.25}$
2	Al-Arabi & Saker [14]	Rectangle	Air	$10^4 < Ra < 10^7$	$Nu = 0.54 Ra^{0.25}$
3	Yousef et al. [17]	Rectangle	Air	$2 \times 10^5 < Ra < 10^9$	$Nu = 0.7 Ra^{0.25}$
4	Al-Arabi & El-Riedy [13]	Square & Rectangle $\mathcal{L} = w$	Air	$2 \times 10^5 < Ra_L < 4 \times 10^7$	$Nu = 0.7 Ra^{0.25}$
5	Churchill [15]	$\mathcal{L} = A/P$	Gasses & Liquids	$0 < Ra < 10^9$	$Nu = \frac{0.766 Ra^{1/5}}{[1 + (0.322/F(Pr))^{11/20}]^{4/11}}$
6	Rohsenow et al. [6]	Many Shapes $\mathcal{L} = A/P$	Air	$1 < Ra < 10^{10}$	$Nu_1 = \frac{0.831 \times C_1 \times Ra^{0.25}}{1.4}$ $Nu = \frac{1.4}{\ln[1 + 1.4/Nu_1]}$
7	Lee et al [25]	Square and Rectangle $\mathcal{L} = \sqrt{A}$	Gasses & Liquids	$10^0 < Ra < 10^{11}$	$Nu = Nu^0 + F(Pr)G_L Ra^{0.25}$ $Pr = \frac{0.67}{(1 + (0.5/F(Pr))^{9/16})^{4/9}}$ $G_L = 2^{1/8} \left(\frac{l}{W} \right)^{1/8}$
8	Martorell et al. [21]	Rectangle & Square $\mathcal{L} = w$ $L/w = 2.3-27.8$	Air	$2.9 \times 10^2 < Ra < 3.3 \times 10^5$	$Nu = 1.2 \times Ra^{0.175}$
9	Lewandowski et al. [10]	Rectangle	Air	$10^5 < Ra < 10^8$	$Nu = 1.228 \times Ra^{0.2}$

10	Rohsenow et al. [6]	$\mathcal{L} = A/P$	Air	$10^3 < Ra < 10^{10}$	$Nu^T = \frac{0.527 Ra^{1/5}}{[1 + (1.9/F(Pr))^{9/10}]^{2/9}}$ $Nu = \frac{1.4}{\ln[1 + 1.4/Nu^T]}$
11	Lloyd and Moran [8]	Square $\mathcal{L} = w$	Air	$3 \times 10^6 < Ra < 4 \times 10^7$	$Nu = 0.622 \times Ra^{0.25}$
12	Yovanovich [26]	Many Shapes $\mathcal{L} = \sqrt{A}$	Gasses & Liquids	$10^0 < Ra < 10^{11}$	$Nu = 0.638 \times Ra^{0.2}$
13	Yovanovich [26]	Many Shapes $\mathcal{L} = \sqrt{A}$	Gasses & Liquids	$10^0 < Ra < 10^{11}$	$Nu = Nu^0 + F(Pr)G_L Ra^{0.25}$ $Pr = \frac{0.67}{(1 + (0.5/F(Pr))^{9/16})^{4/9}}$ $G_L = 2^{1/8} \left(\frac{l}{W} \right)^{1/8}$

Table 2 Correlation equations available in literatures for natural convective heat transfer from a horizontal downward facing surface of finite shape with a constant wall temperature boundary condition in laminar flow region.

No.	Reference	Geometry	Fluid/Pr number	Range	Correlation Equation
14	Fishenden & Saunders [12]	Square $\mathcal{L} = w$	Gasses & Liquids	$10^5 < Ra < 10^7$	$Nu = 0.25 Ra^{0.25}$
15	Bosworth [13]	Square $\mathcal{L} = w$	Air	$Ra < 10^7$	$Nu = 0.35 Ra^{0.25}$
16	Churchill [15]	Square $\mathcal{L} = A/P$	Air	$10^3 < Ra \times \psi(Pr) < 10^{10}$	$Nu = 0.6(Ra_L \psi(Pr))^{1/5}$ $\psi(Pr) = \left[(1 + (0.492)^{9/16})^{-16/9} \right]$
17	Rohsenow et al. [6]	Many Shapes $\mathcal{L} = A/P$	Air	$10^3 < Ra < 10^{10}$	$Nu^T = \frac{0.527 Ra^{1/5}}{[1 + (1.9/F(Pr))^{9/10}]^{2/9}}$ $Nu = \frac{2.5}{\ln[1 + 2.5/Nu^T]}$
18	Fujii and Imura [16]	Square $\mathcal{L} = w$	Pr = 0.7	$10^5 < Ra < 10^{11}$	$Nu = 0.58 Ra^{0.2}$
19	Yovanovich [26]	Square and Rectangle $\mathcal{L} = \sqrt{A}$	Gasses & Liquids	$10^0 < Ra < 10^{11}$	$Nu = Nu^0 + F(Pr)G_L Ra^{0.25}$ $Pr = \frac{0.67}{(1 + (0.5/F(Pr))^{9/16})^{4/9}}$ $G_L = \frac{2^{1/8}}{2} \left(\frac{l}{W} \right)^{1/8}$

Results

The numerical solutions of the governing equations have the following parameters: the Rayleigh number, Ra , the dimensionless aspect ratio of the heated surface, $W = w/h$, the characteristic length scale, $4A/P_t$, the Prandtl number, Pr , and the shapes of the heated element surfaces and its orientation. Results obtained for $Pr = 0.7$, $10^3 \leq Ra \leq 10^8$, octagonal shapes for $0 \leq W \leq 0.6$, hexagonal shapes for $0 \leq W \leq 0.577$ and square shape for W equal 1 have been considered. The aspect ratio of $W = 0.577$ for hexagonal surface shape and $W = 0.4142$ for octagonal surface shape were selected because these values give the shapes with identical sides and when the aspect ratio of $W = 0$, these shapes become a rhombus (diamond) as shown in Fig. 2.

Natural convective heat transfer rates from horizontal upward facing and downward facing element surfaces of different simple shapes with constant wall temperature boundary condition have been investigated experimentally and theoretically by a number of researchers. Lists of selected correlation equations obtained from these studies are shown in Tables 1 and 2. In order to validate our numerical results, the numerical mean Nusselt numbers are compared with the mean Nusselt numbers given by these empirical correlation equations, for different Ra , and for the case of heated element surface facing upward and downward respectively as shown in Figure 5. A square element surface of aspect ratio, W , equals to 1 is selected for a comparison with wide uses empirical correlation equations available in literatures for this surface shape. The results given in this figure show that the numerical mean Nusselt number plotted with the mean Nusselt number given by the empirical correlation equations of Fishenden and Saunders [12] and Rohsenow et al. [6] are in a good agreement.

Fig. 5 Numerical versus empirical mean Nusselt numbers with different Ra based on $w=1$ for: (a) square upward facing surface and (b) square downward facing surface.

The empirical correlation equations available in literatures are based on different characteristic length scales as shown in tables 1 and 2. In order to select the best characteristic length scale for horizontal surfaces of octagonal, hexagonal and diamond shapes, the upward facing octagonal heated surface element of equal sides has been selected, as a sample to be presented here. Figures 6 and 7 show the effect of different characteristic length scales on the mean Nusselt number, for a horizontal upward facing heated element surface of octagonal shape of equal sides. These figures show that, the numerical results based on the characteristic length of $(4A/P_l)$ and the results obtained from the empirical correlation equations given by [12, 6] using the characteristic length scale of $\sqrt{A}$, for Rayleigh numbers and Nusselt numbers are in a good agreement and well predict the mean Nusselt number compared to other length scales such as w and (A/P_l) using correlation equations given in Table 1 and 2. Yovanovich and Jafarpur [23-

26] based on their results found that the characteristic length scale $\sqrt{A}$ was superior to other length scales for two and three dimensional body shapes considered in their studies. While Oosthuizen and Kalendar [39] introduce, in their results, the characteristic length scale $(4A/P_t)$ for different surface shapes. Based on their results they found that this new characteristic length scale well predicts the results for two-dimensional different surface shapes, this characteristic length has not been used as a comparison with other length scales in previous study of Yovanovich and Jafarpur [23, 24]. As a result, we select the characteristic length scale of $(4A/P_t)$ for Rayleigh numbers and Nusselt numbers as continuous of previous work of Oosthuizen and Kalendar [39] for all different shapes considered in this study.

Figures 7, 8 and 9 show the comparison between the numerical mean Nusselt numbers and the mean Nusselt numbers obtained from the empirical correlation equations, of Fishenden and Saunders [12] (No.1 in Table 1) and Rohsenow et al. [6] (No. 6 in Table 1) for the upward facing; octagonal $h=d$, hexagonal $h=d$ and octagonal $h \neq d$ surface heated elements respectively. These comparison are taken with different values of Ra based on $(4A/P_t)$ as a characteristic length scale and with different aspect ratio W . The following general observations have been seen from figure 7; at lower values of $Ra < 10^4$ and for all considered values of W , the numerical mean Nusselt numbers are higher than those obtained from the correlation equations of Fishenden and Saunders [12] and Rohsenow et al. [6]. A good agreement, between the numerical mean Nusselt numbers and those obtained from the empirical equation of Fishenden and Saunders [12], has been obtained when Ra between 10^4 and 10^5 . At higher values of $Ra > 10^5$; the numerical mean Nusselt numbers have higher values than the mean Nusselt numbers obtained from the

empirical correlation equation of Rohsenow et al. [6] and lower values than the mean Nusselt numbers obtained from Fishenden and Saunders [12] for all considered values of W . Figure 8, for hexagonal $h=d$, shows that at the lowest values of Ra , the numerical mean Nusselt numbers have higher values than those obtained from the empirical equations of Fishenden and Saunders [12] and Rohsenow et al. [6], while at the highest values of Ra , the numerical mean Nusselt numbers have a lower value than those obtained from the empirical equation of Fishenden and Saunders [12]. For all other values of Ra there are a good agreement between the numerical mean Nusselt numbers and those obtained from the empirical equation of Fishenden and Saunders [12] for all considered values of W . Figure 9 shows that at the lower values of $Ra < 2 \times 10^3$, the numerical mean Nusselt number has a higher value than that obtained from the empirical equation of Fishenden and Saunders [12], while at the higher value of $Ra > 2 \times 10^5$, the numerical mean Nusselt number has a lower value than that obtained from the empirical equation of Fishenden and Saunders [12]. For all other intermediate values of Ra there are a good agreement between the numerical mean Nusselt numbers and those obtained from the empirical equation of Fishenden and Saunders [12] for all values of W considered in this study. The results in these figures show that the percentage of the mean Nusselt number are higher at lower values of Rayleigh numbers and dimensionless width than there counterparts at higher values of Rayleigh numbers and dimensionless width. This is due to edge effects near the surface edges and its corresponding corners in a region of temperature flow differences which cause three dimensional flows near the edges and corners. It is worth noting that the edge effects is more pronounced at lower

values of Rayleigh numbers due to thicker boundary layer than that at higher values of Rayleigh numbers.

Fig. 6 Numerical versus empirical mean Nusselt numbers with different Ra based on $\sqrt{A}$ for upward facing octagon surface ($h=d$, different W).

Fig. 7 Numerical versus empirical mean Nusselt numbers with different Ra based on m for upward facing octagon surface ($h=d$, different W).

Fig. 8 Numerical versus empirical mean Nusselt numbers with different Ra based on m for upward facing hexagonal surface of $h=d$ for different W .

Fig. 9 Numerical versus empirical mean Nusselt numbers with different Ra based on m for upward facing octagon surface ($h \neq d$, different W).

Figures 10, 11 and 12 show the numerical mean Nusselt numbers verses the mean Nusselt numbers obtained from the empirical correlation equations of Rohsenow et al. [6] (No. 17 in table 2) and Fujii and Imura [16] (No.18 in table 2), for the downward facing; octagonal $h=d$, hexagonal $h=d$ and octagonal $h \neq d$ surface heated elements respectively. These comparisons have been taken with different values of Ra based on m as a characteristic length scale and with different aspect ratio, W . The following general observations have been seen from figure 10; at lower values of $Ra < 10^5$ and for all values of W considered in this study, the numerical mean Nusselt numbers are higher than those obtained from the correlation equations of Rohsenow et al. [6] and Fujii and Imura [16]. The numerical mean Nusselt numbers and those obtained from the empirical equation of Rohsenow et al. [6] are in a good agreement when Ra is between 10^5 and 8×10^6 . While at higher values of $Ra > 8 \times 10^5$; the numerical mean Nusselt numbers have lower values than the mean Nusselt numbers obtained from the empirical correlation equation of Rohsenow et al. [6] and higher values than the mean Nusselt numbers obtained from the empirical correlation equation of Fujii and Imura [16] for all values of W considered. Figure 11, for hexagonal $h=d$, shows that at the lowest values of Ra , the numerical mean Nusselt numbers have higher values than those obtained from the empirical equations of Rohsenow et al. [6] and Fujii and Imura [16], while at the highest values of Ra , the numerical mean Nusselt numbers have a lower values than those obtained from the empirical equation of Rohsenow et al. [6]. For all other values of Ra there is a good agreement between the numerical mean Nusselt numbers and those obtained from the empirical equation of Rohsenow et al. [6] for all values of W considered in this study. Figure 12 shows that at the lower values of $Ra < 2 \times 10^5$, the numerical mean Nusselt

number has a higher value than that obtained from the empirical equation of Rohsenow et.al. [6], while at the higher value of $Ra > 8 \times 10^6$, the numerical mean Nusselt number has a lower value than that obtained from the empirical equation of Rohsenow et al. [6]. For all other intermediate values of $2 \times 10^5 < Ra < 8 \times 10^6$ there is a good agreement between the numerical mean Nusselt numbers and those obtained from the empirical equation of Rohsenow et al. [6], for all values of W considered in this study.

Fig. 10 Numerical versus empirical mean Nusselt numbers with different Ra based on m for downward facing octagon surface ($h=d$, different W).

Fig. 11 Numerical versus empirical mean Nusselt numbers with different Ra based on m for downward facing hexagonal surface of $h=d$ for different W .

Fig. 12 Numerical versus empirical mean Nusselt numbers with different Ra based on m for downward facing octagon surface ($h \neq d$, different W).

Figure 13 shows the variations of the numerical mean Nusselt number with various values of Ra based on h as a characteristic length scale and with different values of W . This comparison between the numerical mean Nusselt numbers and Ra have been carried out for hexagonal and octagonal surface shapes facing upward and facing downward as in (Fig. 13(b, d)) and (Fig. 13(a, c)) respectively. This characteristic length was selected to show the effect of the dimensionless width W of octagonal and hexagonal surface shapes on the heat transfer rates. It will be seen that the mean Nusselt number tends to increase with decreasing W , this effect increases for higher values of Ra and lower values of W .

The local Nusselt number distributions shown in Figs. 14 and 15 illustrate the changes in the three-dimensional flow patterns over the heated surfaces of hexagonal and octagonal shapes facing upward and facing downward for different aspect ratios and for Ra equal to 10^5 . Although the previous figure show the flow symmetry, this symmetry is not utilized in this study for the sole purpose of simplifying the 3D mesh structure using Gambit and to avoid the skew of mesh structure near the sharp corners resulting in convergence problems. These figures show how the aspect ratio have an effect on the heat transfer rate over the heated surfaces of hexagonal and octagonal shapes which show zones of high and low local Nusselt number distributions. The three dimensional flow pattern near the edges of the heated surface of octagonal and hexagonal surfaces changes significantly near the edges which increase the heat transfer rate. The pressure on the facing up orientation surfaces is negative which cause an inward flow toward the center of the heated surface, which creates near the edges a higher heat transfer rate. While the pressure on the facing down orientation surfaces is positive which cause an outward flow toward the edges of the heated surfaces.

Fig. 13 Comparison between numerical mean Nusselt numbers with Rayleigh numbers based on h for different aspect ratios of (a) Octagonal facing up surface, (b) Hexagonal facing up surface, (c) Octagonal facing down surface and (d) Hexagonal facing down surface

Fig. 14 The distributions of local Nusselt number over the upward facing surfaces of a hexagon (left), octagon ($h=d$) (middle) and octagon ($h \neq d$) (right) at various W for $Ra=10^5$

Fig. 15 The distributions of local Nusselt number over the downward facing surfaces of a hexagon (left), octagon ($h=d$) (middle) and octagon ($h \neq d$) (right) at various W for $Ra=10^5$

In attempting to correlate the results for the cases of horizontal octagonal and hexagonal surfaces of different aspect ratios, the mean Nusselt number for a surface of dimensionless size, $m=4A/P_t$, is given by an equation of the form:

$$Nu = \text{function} (Ra, Pr, m) \quad (9)$$

Therefore for constant Prandtl number;

$$Nu = \text{function} (Ra, m) \quad (10)$$

For hexagonal surface,

$$\begin{aligned}
 A &= 2 \left[\frac{2w + 2L \cos 60}{2} \right] (L \sin 60) \\
 &= (2w + 0.577h)(0.5h) \\
 P_t &= 4L + 2w = 2w + 2.308h
 \end{aligned} \tag{11}$$

Hence in this case,

$$\begin{aligned}
 m &= \frac{4A}{P_t} = \frac{2 \left(\frac{2w}{h} + 0.577 \right) h}{2 \frac{w}{h} + 2.308} \\
 \therefore W &= \frac{w}{h} \\
 \therefore m &= \frac{(4W + 1.154)h}{(2W + 2.308)}
 \end{aligned} \tag{12}$$

From which it follows that,

$$M = \frac{m}{h} = \frac{(4W + 1.154)}{(2W + 2.308)} \tag{13}$$

For octagonal surface of $h=d$,

$$\begin{aligned}
 A &= 2 \left[\frac{2w + 2L \cos 45}{2} \right] (L \sin 45) + w(w + 2L \cos 45) \\
 &= \frac{h^2 - w^2 + 2wh}{2} \\
 P_t &= 4L + 4w = 1.1715w + 2.8285h
 \end{aligned} \tag{14}$$

Hence in this case,

$$\begin{aligned}
 m &= \frac{4A}{P_t} = \frac{\frac{h^2 - w^2 + 2wh}{2}}{1.1715w + 2.8285h} \\
 W &= \frac{w}{h} \\
 \therefore m &= \frac{2(1 - W^2 + 2W)}{2.8285 + 1.1715W}
 \end{aligned} \tag{15}$$

From which it follows that,

$$M = \frac{m}{h} = \frac{2(1 - W^2 + 2W)}{2.8285 + 1.1715W} \quad (16)$$

For octagonal surface of $h \neq d$,

$$\begin{aligned} A &= 2 \left[\frac{2w + 2L \cos 45}{2} \right] (L \sin 45) + (w + 2L \cos 45) L \\ &= 0.4142h^2 + wh \end{aligned} \quad (17)$$

$$P_t = 6L + 2w = 2w + 2.4852h$$

Hence in this case,

$$\begin{aligned} m &= \frac{4A}{P_t} = \frac{4h(0.4142h + w)}{2.4852h + 2w} \\ W &= \frac{w}{h} \\ \therefore m &= \frac{h(1.6568 + 4W)}{2.4852 + 2W} \end{aligned} \quad (18)$$

From which it follows that,

$$M = \frac{m}{h} = \frac{(1.6568 + 4W)}{2.4852 + 2W} \quad (19)$$

The mean Nusselt numbers and Rayleigh numbers based on m are given by,

$$Nu_m = Nu M, \quad Ra_m = Ra M^3 \quad (20)$$

For the present numerical results of all surface shapes considered in this study it has been found that the results can be characterized by a similar form of Eq. (10), for the cases of hexagon and octagon surfaces of different W , where $0.577 \geq W \geq 0$ for the hexagonal surface, $0.6 \geq W \geq 0$ for the octagonal surface when $h=d$, $0.4142 \geq W \geq 0$ for the octagonal surface when $h \neq d$, $10^3 \geq Ra \geq 10^7$ for upward facing heated surface and $10^3 \geq Ra \geq 10^8$ for downward facing heated surface.

For upward facing heated surfaces, i.e., by:

$$Nu_u^T = 0.463 Ra_m^{0.25}$$

$$Nu_{uemp} = \frac{2.5}{\ln \left[1 + \frac{2.5}{Nu_u^T} \right]} \quad (21)$$

For downward facing heated surfaces, i.e., by:

$$Nu_d^T = 0.5 Ra_m^{0.2}$$

$$Nu_{demp} = \frac{2.5}{\ln \left[1 + \frac{2.5}{Nu_d^T} \right]} \quad (22)$$

Comparisons of the mean Nusselt numbers given by the correlation equations Eqs. (21) and (22), with the present numerical results based on the characteristic length m for upward and downward facing surfaces of all surface shapes considered in this study and for different Rayleigh numbers between 10^3 and 10^8 are shown in Figs. 16 and 17. These figures show that for all aspect ratios the mean Nusselt number are effectively the same and the results are correlated by a simple form of an equation.

The results of the present correlation equations are in a good agreement with the numerical results of all the surface shapes considered in this study. The present correlation equations have the same form of the correlation equations given by Rohsenow et al. [6]. The differences of the mean Nusselt numbers between the present correlation equation and the numerical results for the upward facing heated surfaces are between 5.3% and 0.04%. While the differences of the mean Nusselt numbers between the present correlation equation and the numerical results for the downward facing heated surfaces are between 13% and 0.07%. The highest differences for the downward facing surfaces occur at lowest values of Rayleigh number considered in this study.

Fig. 16 Comparison of the numerical Nu_{uemp} with Ra_m based on the characteristic length m for isothermal upward facing with various W of all considered surface shapes.

Fig. 17 Comparison of the numerical Nu_{demp} with Ra_m based on the characteristic length m for isothermal downward facing with various W of all considered surface shapes.

Conclusions

The present numerical results for the natural convective heat transfer from horizontal upward and downward facing hexagonal, octagonal and diamond shapes of different aspect ratios indicate that for all the surface shapes considered, using the characteristic length scales of m and $\sqrt{A}$ expressed in terms of Nusselt and Rayleigh numbers are superior compared to other length scales. It was shown that, the heat transfer rate increases as the dimensionless width of the heated surface decreases. In particular, a

higher heat transfer rate effect occurs at lower values of the dimensionless width and higher values of Ra for all cases considered in this study. This study indicated that the edges of the surface shapes considered have a great effect on the flow disruption over the surface and the heat transfer rates. This effect varies differently between upward facing and downward facing surface. The results presented in this study show that if the results for all the surface shapes considered are expressed in terms of Nusselt and Rayleigh numbers based on the characteristic length, m , then the variations of Nu_m with Ra_m for all the surface shapes considered could be described in simple equation form, Eq. (10). Correlation equations for estimating the mean Nusselt numbers based on the characteristic length, m , for all the surface shapes with different aspect ratios considered in this study are well correlated by Eqs. (21) and (22) for isothermal upward and downward heated surfaces respectively.

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Nomenclature

A	Surface area of the heated element (m^2)
C	Parameter in equation for heat transfer rate
d	Total width of the heated element surface (m)
g	Gravitational acceleration (ms^{-2})
h	Vertical height of the heated element surface (m)
k	Thermal conductivity of fluid ($W\ m^{-1}\ K^{-1}$)
L	Side length of the heated element surface (m)
m	Characteristic length scale ($4A/P_t$)

Nu	Mean Nusselt number based on the characteristic length and temperature differences from the heated surfaces given by empirical correlation equations
$Nu_{\sqrt{A}}$	Mean Nusselt number based on the characteristic length $\sqrt{A}$ and $(T_H - T_F)$ from the heated surfaces
Nu_m	Mean Nusselt number based on the characteristic length $(m=4A/P_t)$ and $(T_H - T_F)$ from the heated surfaces
Nu_w	Mean Nusselt number based on the characteristic length w and $(T_H - T_F)$ from the heated surfaces
Nu_u^T	Mean Nusselt number based on m and $(T_H - T_F)$ from upward facing heated surfaces given by correlation equation
Nu_d^T	Mean Nusselt number based on m and $(T_H - T_F)$ from downward facing heated surfaces given by correlation equation
Nu_{uemp}	Mean Nusselt number for upward facing heated surfaces given by correlation equation
Nu_{demp}	Mean Nusselt number for downward heated surfaces given by correlation equation
P	Dimensionless pressure
p	Pressure (Pa)
P_t	Total perimeter of the heated element surface (m)
Pr	Prandtl number
Ra_w	Rayleigh number based on selected characteristic length w and $(T_H - T_F)$
Ra_m	Rayleigh number based on selected characteristic length $(4A/P_t)$ and $(T_H - T_F)$
$Ra_{\sqrt{A}}$	Rayleigh number based on selected characteristic length $(\sqrt{A})$ and $(T_H - T_F)$
T	Temperature (K)
T_F	Undisturbed fluid Temperature of fluid (K)
T_H	Temperature of heated element surface (K)
U_X	Dimensionless velocity component in X direction
u_x	Velocity component in x direction (ms^{-1})
u_r	Reference velocity (ms^{-1})

U_Y	Dimensionless velocity component in Y direction
u_y	Velocity component in y direction (ms^{-1})
U_Z	Dimensionless velocity component in Z direction
u_z	Velocity component in z direction (ms^{-1})
w	Vertical and horizontal side width of heated surface (m)
W	Dimensionless aspect ratio of heated element surface, w/h
X	Dimensionless coordinate normal to heated element surface
x	Coordinate normal to heated element surface (m)
Y	Dimensionless horizontal coordinate in plane of heated element surface
y	Horizontal coordinate in plane of heated element surface (m)
Z	Dimensionless horizontal coordinate in plane of heated element surface
z	Horizontal coordinate in plane of heated element surface (m)
α	Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
β	Bulk expansion coefficient (K^{-1})
ν	Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
θ	Dimensionless temperature

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