Optimum ML Detection for DF Cooperative Diversity Networks in the Presence of Interference

by

Tian Lu

A thesis submitted to the
Department of Electrical and Computer Engineering
in conformity with the requirements for
the degree of Master of Applied Science

Queen's University
Kingston, Ontario, Canada
July 2014

Abstract

Unlike the existing detectors for decode-and-forward (DF) relaying in the ideal case without interference, this thesis considers a more practical scenario where arbitrary interference is present. We consider a DF cooperative diversity network consisting of one source, multiple relays, one destination and multiple interferers affecting the relays as well as the destination. Under this scenario, for the first time in the literature, we develop the exact closed-form rules for optimum maximum-likelihood (ML) detectors for DF systems employing any one- or two-dimensional modulations in the presence of interference. In particular, we derive the optimum ML detection rules for two transmission schemes: simultaneous transmission and orthogonal transmission. In each transmission scheme, we derive the detection rules by considering two different cases of the instantaneous channel state information (CSI): the CSI of the second-hop is known or unknown at the relays. Numerical results demonstrate that the developed optimum detectors significantly outperform the conventional detector which simply ignores the existence of interference.

Acknowledgments

First of all, I would like to appreciate my supervisor, Dr. Kim for his help, guidance and support. All his encouragement and motivation helped me to get where I am today. Without his involvement, this work would not have been accomplished. Over the two years, I have benefited and learnt a lot from his creative ideas, exceptional foresight, and hard-working attitude. It is totally my honor to be his student. I also would like to thank Dr. Chan for his valuable comments.

Secondly, my sincere gratitude to the thesis defense committee members: Dr. Yousefi, Dr. Liu and Dr. Alajaji. Thank you so much for organizing and attending my defense. Your comments and suggestions are truly appreciated.

Thirdly, I would like to appreciate Peng Liu for his help along the way. He worked as a role model in the lab, and helped every member whenever someone is in need. As a PHD student, he devoted so much time to share his insightful thoughts with all the Master students in the lab. All of those times that he has gone the extra miles for us, we did notice.

Most importantly, I would like to express my deepest gratitude to my great parents for their unconditional love, support and encouragement throughout my life. The success and accomplishments I achieved today totally reflect their wisdom and hard-working attitude. I am grateful to them forever.

Last but not least, I am grateful to my partner, Jason Hua. I am glad we are classmates/lab partners/life partners for six years and now we are becoming co-works in the same company. Hold our hands, endure the cold winds, and walk until the journey ends.

List of Abbreviations

AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
BER	Bit-Error Rate
BPSK	Binary Phase-Shift Keying
BFSK	Binary Frequency-Shift Keying
CDF	Cumulative Distribution Function
CF	Compress-and-Forward
C-MRC	Cooperative Maximum Ratio Combining
CSCG	Circularly-Symmetric Complex Gaussian
CSI	Channel State Information
DF	Decode-and-Forward
DL	Direct Link
LAR	Link-adaptive Regeneration

i.i.d.	Independent and Identically Distributed
INR	Interference-to-Noise Ratio
LF	Likelihood Function
LLF	Log-Likelihood Function
LOS	Line-of-Sight
λ-MRC	λ Maximum Ratio Combining
M-PSK	M-ary Phase-Shift Keying
M-PAM	M-ary Pulse Amplitude Modulation
M-QAM	M-ary Quadrature Amplitude Modulation
MAP	Maximum A Posteriori Probability
MED	Minimum Euclidean Distance
MIMO	Multiple-Input-Multiple-Output
MISO	Multiple-Input-Single-Output
ML	Maximum-Likelihood
MRC	Maximum Ratio Combining
OT	Orthogonal Transmission
PAM	Pulse Amplitude Modulation
PDF	Probability Density Function

PL	Piecewise Linear
PSK	Phase-Shift Keying
QAM	Quadrature Amplitude Modulation
SER	Symbol Error Rate
SIMO	Single-Input-Multiple-Output
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
ST	Simultaneous Transmission

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Chapter 1

Introduction

1.1 An Overview of Cooperative Communication

In the conventional point-to-point wireless communications, the continuous communication between the transmitting device to the receiving device is not guaranteed as the channel links may suffer high level multipath fading. Then the cooperative communication with the assistance of relay nodes was proposed as a solution to broaden the coverage, mitigate fading impairment, and increase the data rate at a low cost. In a cooperative wireless network, each mobile device can function as a source terminal transmitting its own signal, or a relay terminal to assist other device to forward signals to the destination. In particular, relay transmission helps combat heavy path loss and provides additional diversity without requiring the multiple antennas at the source and destination. Then, two most well-known relay protocols were developed in [14] for cooperative diversity networks: amplify-and-forward (AF) protocol, and decode-and-forward (DF) protocol. In the AF protocol, the relay node receives a noisy version of the signal from a source terminal, and then amplifies and forwards the received signal to the destination for decoding. In the DF protocol, the relay decodes the received signal first, and then retransmits the detected signal to the destination. Through each protocol, multiple independent replicas of the transmitted signals due to cooperation among users result in spatial diversity, which can significantly improve the network performance and robustness. Then we will introduce the measurements we take to evaluate the performance of the cooperative communication.

1.1.1 Performance Measures and Detection Rules

To evaluate the performance of the cooperative communications, we introduce some most general performance measures and decision rules here.

Error Probability

In wireless transmission, error occurs when some bits of a data stream going over the channel have been altered due to noise, interference and distortion. The symbol-error-rate (SER) and the bit-error-rate (BER) are the two most commonly used error probability measures in digital communication. The instantaneous BER or SER can be computed in terms of the instantaneous signal-to-noise ratio (SNR) γ as follows [16, 18]:

$$P_e = \alpha Q(\sqrt{\beta\gamma}), \tag{1.1}$$

where α and β are the constants depending on the modulations. In (1.1), the SNR γ is a function of the channel coefficient h which is a random variable. Due to the randomness from the channel coefficient, it would be more desirable to find the average BER/SER, where we take expectation of the instantaneous BER/SER with respect to γ, as shown as follows [9]:

$$\mathbb{E}[P_e] = \alpha \int_0^\infty Q(\sqrt{\beta x}) f_\gamma(x) dx, \tag{1.2}$$

where $f_\gamma(x)$ is the probability density function (PDF) of γ.

Diversity Gain

Providing multiple independent replicas of the same information yields diversity gains. In addition to the information from the direct link, each parallel relay also provides an independent copy of the same information to assist the decision-making in the destination. The diversity gains improve the performance of the system because as we supply more copies of the information to the receiver, the probability that all the signal components will fade simultaneously is reduced considerably. Consider a network with L independent branches, the average error probability in (1.2), at high SNR, can be approximated as [23]

$$\mathbb{E}[P_e] \approx \binom{2L-1}{L} \frac{1}{(a\gamma)^L}, \tag{1.3}$$

where a is a modulation-dependent constant, and the SNR exponent L represents the diversity gain. As we can see from (1.3), increasing the diversity gain L would dramatically decrease the error probability.

Spectral Efficiency

Spectral efficiency is a measure of how efficiently a frequency spectrum is utilized, which can be computed as [23]

$$\rho = \frac{R}{W}, \quad \text{(bps/Hz)} \tag{1.4}$$

where R is the information bit rate and W is the bandwidth.

Detection Rules

As signal transmitted over a channel, it may be corrupted by noise or interference.

Thus, we wish to design a signal detector at the receiver that makes a decision on the transmitted signal such that the probability of a correct decision is maximized. We consider a received signal given by

$$\mathbf{r} = \mathbf{s}_m + \mathbf{n}, \quad m = 1, \cdots, M \tag{1.5}$$

where $\mathbf{r}$, $\mathbf{s}_m$ and $\mathbf{n}$ are $N \times 1$ vectors. We first introduce maximum a posteriori probability (MAP) decision rule, which is based on selecting the signal corresponding to the maximum of the set of posterior probabilities $P(S_m|r)$, as given by [18, 22]

$$\hat{m} = \arg \max_{m=1,\cdots,M} \Pr(\mathbf{s}_m|\mathbf{r}), \tag{1.6}$$

where $\Pr(\mathbf{s}_m|\mathbf{r})$ is the posterior probability. Applying Bayes' rule, (1.6) can be expressed as

$$\hat{m} = \arg \max_{m=1,\cdots,M} p(\mathbf{r}|\mathbf{s}_m) \Pr(\mathbf{s}_m), \tag{1.7}$$

where $p(\mathbf{r}|\mathbf{s}_m)$ is called the likelihood function. If we assume the transmitted signals are equiprobable with $\Pr(\mathbf{s}_m) = \frac{1}{M}$, then the MAP rule becomes maximum-likelihood (ML) decision rule given as follows [18]:

$$\hat{m} = \arg \max_{m=1,\cdots,M} p(\mathbf{r}|\mathbf{s}_m). \tag{1.8}$$

In other words, the ML decision is based on determining the maximum of the likelihood function, $p(\mathbf{r}|\mathbf{s}_m)$, over the M signals. To this end, we consider an additive white Gaussian noise (AWGN) channel, and the corresponding likelihood function is given by [18]

$$p(\mathbf{r}|\mathbf{s}_m) = \frac{1}{(\pi\sigma^2)^N} \exp\left[-\frac{||\mathbf{r}-\mathbf{s}_m||^2}{\sigma^2}\right]. \tag{1.9}$$

After substituting (1.9) into (1.8), we can obtain [18]

$$\begin{aligned} \arg \max_{m=1,\cdots,M} p(\mathbf{r}|\mathbf{s}_m) &= \arg \max_{m=1,\cdots,M} \log p(\mathbf{r}|\mathbf{s}_m) \\ &= \arg \max_{m=1,\cdots,M} \left(-N\log(\pi\sigma^2) - \frac{||\mathbf{r}-\mathbf{s}_m||^2}{\sigma^2}\right) \\ &= \arg \min_{m=1,\cdots,M} ||\mathbf{r}-\mathbf{s}_m||. \end{aligned} \tag{1.10}$$

The final expression in (1.10) is called the minimum Euclidean distance (MED) decision rule, which is based on finding the signal $\mathbf{s}_m$ that is closest in distance to the received signal vector $\mathbf{r}$. In summary, the MAP decision rule is equivalent as the ML decision rule when the transmitted signals are equally probable. Then, for an AWGN channel only, the ML decision rule is equivalent as the MED decision rule. In particular, if the transmitted signals are equiprobable, and the channel is AWGN, MAP, ML and MED are basically the same optimum decision rules.

1.2 Literature Review

In recent years, cooperative diversity has been studied as a promising technique to broaden the coverage range, increase the channel capacity, and mitigate fading impairments, emerging as a reliable potential candidate for wireless communications. There are two most well-known relaying strategies developed for cooperative diversity networks: amplify-and-forward protocol, and decode-and-forward protocol [14,21]. Extensive research has been done on these protocols with respect to various aspects:

performance analysis [20], diversity gain [5,28] and optimum/sub-optimum detections [24, 25], etc. Due to the ease of combining with channel codes and incorporating into network protocols, we will focus on DF relaying strategy in this thesis.

The majority of the previous works regarding the detection, however, have been intensively focused on a relatively ideal scenario, where interference was completely ignored. That was because, in the presence of interference, the system model becomes onerously complicated, and the mathematical analysis becomes more challenging. In the literature, there are optimum and many sub-optimum schemes for the interference-free environment [4,5,20,24,25]. The maximum-likelihood (ML) detection rule for DF cooperative systems was first presented in [15] for binary phase shift keying (BPSK) constellation. Then, in [4, 5], both coherent and noncoherent ML detections for DF systems with binary frequency shift keying (BFSK) were studied, and the piecewise linear (PL) combiner was introduced as a useful alternative for ML detector. In [3], an ML detector for the DF protocol was developed for arbitrary constellations. Since the ML detector has the exponential detection complexity and non-linear characteristic, various sub-optimum detectors with lower complexity but degraded performance were proposed: the λ maximum-ratio-combining (λ-MRC) [20], the cooperative maximum-ratio-combining (C-MRC) decoder [24], and the link-adaptive regeneration (LAR) detection rule [25]. However, it is questionable whether ignoring interference would be a realistic assumption for practical wireless networks.

Due to the broadcast nature of wireless signal transmissions, interference always exists over a wide range of frequency bands in almost all practical wireless communication systems. For example, interference may come from other authorized users of the same spectrum, or from other frequency channels injecting energy into the channel of interest. In addition, in many of the emerging wireless systems, such as ad-hoc and sensor networks, mesh networks, cognitive networks, femtocells, heterogeneous networks and cellular networks with multihop coverage extensions, interference typically exists and may affect the system performance. Thus, interference is a major and unavoidable issue in practical wireless transmissions. Although the issue of interference has been considered in some recent works, most of them were not addressing the detection problem. For example, in [29], the outage performance for a DF network was analyzed, assuming the destination is noise-ignorant and corrupted by co-channel interference while the relay is not affected by any interference. The authors of [10, 11] have analyzed the outage performance and error probability of AF and DF relaying systems with co-channel interference. The interference alignment technique was performed in [2, 26], where interference can be canceled within the multiple-input multiple-output (MIMO) systems. Even the remaining very few works on detection are restricted to the AF network with interference affecting only the relays such as in [19]. To the best of our knowledge, the problem of performing optimum ML detection in the presence of interference under the DF framework has never been studied.

1.3 Contributions of Thesis

In this thesis, we derive the optimum ML detectors in closed-form for a DF system in the presence of interference. Specifically, we consider a general system model, where a source transmits information to a destination via multiple intermediate relays in the presence of multiple interferers affecting the relays as well as the destination. We adopt M-pulse amplitude modulation (PAM) and M-quadrature amplitude modulation (QAM) as the modulation schemes for the desired signal and the interference. We begin by developing detection rules in two transmission schemes: simultaneous transmission and orthogonal transmission. In particular, we consider system-wise optimum DF scenario: the ML detection is employed both at the relays and at the destination to avoid any performance degradation in the DF cooperative system. Also, it is assumed that only partial channel state information (CSI) for the interferers is available at the relays and the destination. For each transmission scheme, we derive the exact optimum detection rules in closed-form by considering two different cases: the instantaneous CSI of the second hop is known or unknown at the relays. Numerical results demonstrate that the derived detectors significantly outperform the conventional scheme which completely ignores the interference.

1.4 Organization of Thesis

The rest of this thesis is organized as follows: In Chapter 2, we introduce the system model and describe the transmission schemes. In Chapter 3, we derive the detection rules for two transmission schemes. Numerical results are presented in Chapter 4 and conclusions are drawn in Chapter 5.

Notation: We use $A := B$ to denote that A by definition, equals B. Also, $\boldsymbol{n}_1 \sim \mathcal{CN}(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$ indicates that $\boldsymbol{n}_1$ is a circularly symmetric complex Gaussian (CSCG) random vector with mean vector $\boldsymbol{\mu}_1$ and covariance matrix $\boldsymbol{\Sigma}_1$. Similarly, $\boldsymbol{n}_2 \sim \mathcal{N}(\boldsymbol{\mu}_2, \boldsymbol{\Sigma}_2)$ indicates that $\boldsymbol{n}_2$ is a real Gaussian random vector with mean vector $\boldsymbol{\mu}_2$ and covariance matrix $\boldsymbol{\Sigma}_2$. For random variable X, $f(x)$ denotes its probability density function (pdf). Moreover, $(\cdot)^H$ denotes the operator of conjugate transpose,

$(\cdot)^*$ denotes the operator of conjugate, $(\cdot)^T$ denotes the operator of transpose and $\mathbb{E}[\cdot]$ is the expectation operator. The operators $\Re[\cdot], \Im[\cdot], \det(\cdot)$ and $\log(\cdot)$ denote the real part, imaginary part, matrix determinant, and logarithm, respectively. In addition, $\boldsymbol{I}_{N_k}$ denotes an $N_k \times N_k$ diagonal matrix with diagonal elements being one. Finally, for a row vector $\boldsymbol{a} = [a_1, \cdots, a_n]$, Diag($\boldsymbol{a}$) denotes the diagonal matrix with $[a_1, \cdots, a_n]$ on its main diagonal matrix and zero elsewhere.

Chapter 2

System Model

Consider a cooperative diversity communication system with one source, L interferers, K relays, and a destination, where each node is working in the half-duplex mode, as illustrated in Fig. 2.1. The k-th relay node has N_k ($N_k \geq 1$) antennas, whereas other terminals including the source, the destination, and the interferers have only one antenna. Let h_{sd} represent the fading coefficient of the direct-path channel from the source to the destination, $\boldsymbol{h}_{sk} = [h_{sk,1}, \cdots, h_{sk,N_k}]^T, k = 1, \ldots, K$, denote the first-hop channel from the source to the k-th relay with N_k antennas, and $\boldsymbol{h}_{kd} = [h_{kd,1}, \cdots, h_{kd,N_k}]^T, k = 1, \ldots, K$, denote the second-hop channel from the k-th relay to the destination. Let $\boldsymbol{g}_{jk} \sim \mathcal{CN}(\mathbf{0}, \Omega_{jk}\boldsymbol{I}_{N_k})$ denote the complex channel coefficient from the j-th interferer to the k-th relay for $j = 1, \cdots, L; k = 1, \cdots, K$. Also, let $g_{jD} \sim \mathcal{CN}(0, \Omega_{jD})$ denote the complex channel coefficient from the j-th interferer to the destination. The fading coefficients associated with different wireless channels are independent with one another.

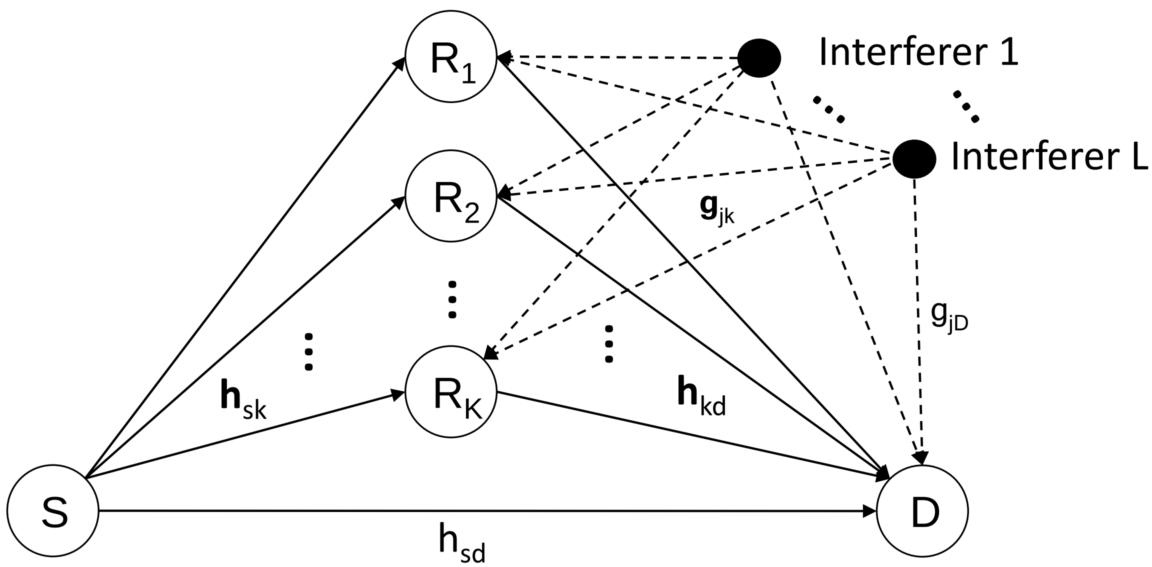

Figure 2.1: Multi-branch decode-and-forward cooperative diversity network with multiple interferers and AWGN at both the relays and the destination.

It is assumed that for the coherent detection of the desired signal x_0, the instantaneous CSI h_{sd}, $\boldsymbol{h}_{sk}$, and $\boldsymbol{h}_{kd}$ are known at the destination for $k = 1, \ldots, K$. As for the relays, we assume that the instantaneous CSI of the first-hop, $\boldsymbol{h}_{sk}$, is known at the relays, and we consider two possibilities for the instantaneous CSI of the second-hop: i) $\boldsymbol{h}_{kd}$ is known at the relays, or ii) $\boldsymbol{h}_{kd}$ is not known at the relays. For CSI signaling of $h_{sd}, \boldsymbol{h}_{kd}$, and $\boldsymbol{h}_{sk}$ to the destination, the source and each relay are required to transmit pilot symbols [12]. It is assumed that we have $h_{sd} \sim \mathcal{CN}(0, \vartheta_{sd}), \boldsymbol{h}_{sk} \sim \mathcal{CN}(0, \vartheta_{sk}\boldsymbol{I}_{N_k})$, and $\boldsymbol{h}_{kd} \sim \mathcal{CN}(0, \vartheta_{kd}\boldsymbol{I}_{N_k})$. For CSI of the interferers, however, we assume that the relays and the destination only have the *partial* CSI, i.e., the channel variance Ω_{jk} and Ω_{jD}.

Throughout the thesis, we only consider the uncoded system. Let $x_0 \in S_0$ denote the desired signal transmitted from the source with $\mathbb{E}[|x_0|^2] = 1$, where S_0 is the signal constellation. Let $x_j^{(t)} \in S_j,\ j = 1, \cdots, L$ denote the interference signal transmitted from the j-th interferer with $\mathbb{E}[|x_j^{(t)}|^2] = 1$ in time slot t, where S_j denotes its constellation. For M-PAM, S_j is given by $S_j^{PAM}(M) = \{\pm d_0, \pm 3d_0, \cdots, \pm(M-1)d_0\},\ j = 0, 1, \cdots, L$. Similarly, for M-QAM or $I \times J$-QAM ($M = I \times J$), the inphase component $\Re[x]$ and the quadrature component $\Im[x]$ of each element of S_j are selected independently from $S_j^{PAM}(I)$ and $S_j^{PAM}(J)$ respectively, for $j = 0, 1, \cdots, L$. In the constellations, $2d_0$ is the Euclidean distance between two adjacent signal points given as follows:

$$d_0 := \begin{cases} \sqrt{\dfrac{3}{M^2 - 1}}, & \text{for } M\text{-PAM;} \\ \sqrt{\dfrac{3}{I^2 + J^2 - 2}}, & \text{for } I \times J\text{-QAM.} \end{cases} \tag{2.1}$$

In this thesis, we will consider two different transmission schemes for the cooperative diversity networks: simultaneous transmission and orthogonal transmission.

2.1 Simultaneous Transmission

In simultaneous transmission, the transmission is completed within two time slots. In the first time slot, the source broadcasts its desired signal $\sqrt{E_0}x_0$ to all the relays and the destination. Meanwhile, all the interferers also transmit their own signals $\sqrt{E_j^I}x_j^{(1)}$, which are overheard by all the relays as well as the destination. In the first time slot, the signal $y_{sd}^{(1)}$ received by the destination is given by

$$y_{sd}^{(1)} = \sqrt{E_0}h_{sd}x_0 + \sum_{j=1}^{L}\sqrt{E_j^I}g_{jD}x_j^{(1)} + n_{sd}, \tag{2.2}$$

where n_{sd} is the additive white Gaussian noise (AWGN) at the destination with $n_{sd} \sim \mathcal{CN}(0, \sigma_d^2)$. In the first time slot, the signal vector $\boldsymbol{y}_{sk}^{(1)} = [y_{sk,1}^{(1)}, \cdots, y_{sk,N_k}^{(1)}]^T$ received by each relay with N_k antennas is given by

$$\boldsymbol{y}_{sk}^{(1)} = \sqrt{E_0}\boldsymbol{h}_{sk}x_0 + \sum_{j=1}^{L}\sqrt{E_j^I}\boldsymbol{g}_{jk}x_j^{(1)} + \boldsymbol{n}_k,\ k = 1, \cdots, K \tag{2.3}$$

where $\boldsymbol{n}_k = [n_{k,1}, \ldots, n_{k,N_k}]^T$ is the AWGN at each relay with $\boldsymbol{n}_k \sim \mathcal{CN}(0, \sigma_k^2\boldsymbol{I}_{N_k})$. Note that the interferers affect the relays as well as the destination.

In the second time slot, each relay node assists the communication from the source to the destination using the DF protocol, where each relay detects the transmitted symbol by ML detection and retransmits the detected signal $\hat{x}_{0k}$ to the destination. Meanwhile, all the interferers also transmit their own signal $\sqrt{E_j^I}x_j^{(2)}$ in the second time slot. Hence, the signal $y_{rd}^{(2)}$ received by the destination in the second time slot is given by

$$y_{rd}^{(2)} = \sum_{k=1}^{K}\sqrt{\frac{E_k^R}{N_k}}\boldsymbol{w}_k\boldsymbol{h}_{kd}\hat{x}_{0k} + \sum_{j=1}^{L}\sqrt{E_j^I}g_{jD}x_j^{(2)} + n_{rd}, \tag{2.4}$$

where E_k^R is the total transmission energy for the k-th relay with N_k antennas and n_{rd} is the AWGN at the destination with $n_{rd} \sim \mathcal{CN}(0, \sigma_d^2)$. Note that the received signal $y_{rd}^{(2)}$ contains only the detected symbol $\hat{x}_{0k}$ from the relay, not the original symbol x_0 transmitted from the source. The vector $\boldsymbol{w}_k$ in (2.4) denotes a weighting vector of dimension $N_k \times 1$, which is designed based on the available CSI of the second-hop. The determination of the optimum weighting vector will be discussed later.

2.2 Orthogonal Transmission

In orthogonal transmission, each relay is designed to transmit in different time slots to yield the best diversity performance. In the first time slot, the transmissions of the source and the interferers are exactly the same as in the simultaneous transmission: the source broadcasts its desired signal x_0 to all the relays as well as the destination, while all the interferers also transmit their own signals and they affect every relay node as well as the destination node. Therefore, the signal $y_{sd}^{(1)}$ received from the direct link, and the signal $\boldsymbol{y}_{sk}^{(1)}$ received from k-th relay are exactly the same as in (2.2) and (2.3), respectively. In contrast to the simultaneous transmission, however, from the second time slot, each relay transmits data in an orthogonal fashion. Specifically, in time slot $k+1, k = 1, \cdots, K$, only the k-th relay detects the transmitted symbol by ML detection and retransmits the detected signal $\hat{x}_{0k}$ to the destination. Meanwhile, all the interferers also transmit their own $(k+1)$-th signal $\sqrt{E_j^I} x_j^{(k+1)}$ and it is received by the destination. Thus, the signal $y_{kd}^{(k+1)}$, $k = 1, \cdots, K$, received by the destination in the $(k+1)$-th time slot is given as

$$y_{kd}^{(k+1)} = \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} + \sum_{j=1}^{L} \sqrt{E_j^I} g_{jD} x_j^{(k+1)} + n_{kd}, \quad k = 1, \cdots, K. \tag{2.5}$$

Similarly, the weighting vector $\boldsymbol{w}_k$ is designed based on the available CSI of the second-hop, which will be discussed later. By deriving the optimum ML detectors for the two transmission schemes, we will determine their corresponding advantages and disadvantages later.

Chapter 3

Exact Optimum ML Detection in Closed-Form

In this chapter, we study the optimum ML detection in the presence of interference for the two transmission schemes: simultaneous transmission and orthogonal transmission. In order to achieve truly optimum end-to-end performance in a DF network, we require all the terminals including the relays and the destination to perform the optimum ML detection. Thus, the end-to-end optimum detection consists of three key steps: i) ML detection at each relay, ii) forwarding the ML-detected signals from the relays to the destination, and iii) the ML detection at the destination. Since the first two steps are the same for the two transmission schemes, we present them in the first two subsections. Specifically, we begin this chapter by demonstrating the exact ML detection performed at each relay. Subsequently, the analysis of forwarding the detected signals from the relays to the destination is provided, where the transition probability is derived. In the last two subsections, we finally study the ML detections at the destination for the simultaneous transmission and the orthogonal transmission, respectively.

3.1 Exact ML Detection at Each Relay

In this section, we study the exact ML detection rule at each relay considering the interference. Note that each relay detects its own signal *independently*. Thus, we only consider a single relay in our analysis. In the following lemma, we first derive the ML detection rule in the presence of interference.

Lemma 1: With the partial CSI of the interferers, the optimum ML detector at the k-th relay is given by

$$\hat{x}_{0k} = \arg\max_{x_0\in S_0} \sum_{x_1^{(1)}\in S_1} \cdots \sum_{x_L^{(1)}\in S_L} \frac{1}{(\sum_{j=1}^{L} E_j^I \Omega_{jk}|x_j^{(1)}|^2+\sigma_k^2)^{N_k}} \exp\left(-\frac{\|\boldsymbol{y}_{sk}^{(1)}-\sqrt{E_0}\boldsymbol{h}_{sk}x_0\|^2}{\sum_{j=1}^{L} E_j^I \Omega_{jk}|x_j^{(1)}|^2+\sigma_k^2}\right). \tag{3.1}$$

Proof. See Appendix A.1. □

Note that (3.1) is truly optimum and valid for any type of M-PAM and M-QAM modulation. The term $\sum_{j=1}^{L} E_j^I \Omega_{jk}|x_j^{(1)}|^2$, which appears in the denominators inside and outside of the exponent in (3.1), stands for the interference. This interference term varies depending on the modulation of the interference signal, rather than being fixed. The detected signal $\hat{x}_{0k}$ obtained by (3.1) is used as the transmitted signal at the k-th relay to assist the formation of the received signals of $y_{rd}^{(2)}$ for the simultaneous transmission and $y_{kd}^{(k+1)}$ for the orthogonal transmission at the destination.

In order to proceed to the ultimate destination detection, we have to do further performance analysis utilizing the relay's ML detector of (3.1). However, due to the complicated expression of (3.1) including the L summations of the interference signals, it is difficult to directly use (3.1). In the following lemma, we derive a much simpler, yet mathematically equivalent, ML detection form.

Lemma 2: (3.1) is equivalent to the following:

$$\hat{x}_{0k} = \arg \min_{x_0 \in S_0} \left| x_0 - \frac{\boldsymbol{h}_{sk}^H \boldsymbol{y}_{sk}^{(1)}}{\sqrt{E_0} \|\boldsymbol{h}_{sk}\|^2} \right|^2. \tag{3.2}$$

Proof. See Appendix B. □

Note that, even with the variation in the interference signal and modulation, the equivalence is always hold between equations of (3.1) and (3.2). More specifically, the equivalence here means that the ML estimates, $\hat{x}_{0k}$, obtained by (3.1) and (3.2) are mathematically the same, where no inequalities or no approximations were applied. The disclosure of the equivalence is unanticipated as the equality is always true despite of the number of interferers and the variation in the denominators inside and outside of the exponents in (3.1).

3.2 Transition Probability

Using the result obtained by (3.2), we now derive the *instantaneous* signal-to-interference-plus-noise-ratio (SINR) of the ML detector at the relays. Firstly, by substituting (2.3) into (3.2), we can derive a *soft* estimate $\check{x}_{0k}$ of x_0, which is in the form of

$$\check{x}_{0k} = x_0 + \mathcal{I}_{0k} + \eta_{0k}, \tag{3.3}$$

where the interference component $\mathcal{I}_{0k}$ and the additive noise component η_{0k} are given by

$$\mathcal{I}_{0k} = \frac{\boldsymbol{h}_{sk}^H \sum_{j=1}^{L} \sqrt{E_j^I} \boldsymbol{g}_{jk} x_j^{(1)}}{\sqrt{E_0} \|\boldsymbol{h}_{sk}\|^2}, \tag{3.4}$$

$$\eta_{0k} = \frac{\boldsymbol{h}_{sk}^H \boldsymbol{n}_k}{\sqrt{E_0} \|\boldsymbol{h}_{sk}\|^2}. \tag{3.5}$$

Note that, given the instantaneous CSI $\boldsymbol{h}_{sk}$, the noise component η_{0k} is CSCG. In addition, given $\boldsymbol{h}_{sk}$ and $\{\boldsymbol{g}_{jk}\}_{j=1,k=1}^{L,\ K}$, the interference component $\mathcal{I}_{0k}$ is also CSCG when it is conditioned on the interfering signals $\{x_j^{(1)}\}_{j=1}^{L}$. As a result, conditioned on $\boldsymbol{h}_{sk}$, $\{\boldsymbol{g}_{jk}\}_{j=1,k=1}^{L,\ K}$, and $\{x_j^{(1)}\}_{j=1}^{L}$, the interference plus noise component $\mathcal{N}_{0k} := \mathcal{I}_{0k} + \eta_{0k}$ is CSCG with $\mathcal{N}_{0k} \sim \mathcal{CN}(0, \Omega_k)$, where

$$\Omega_k = \frac{\sum_{j=1}^{L} E_j^I \Omega_{jk} |x_j^{(1)}|^2 + \sigma_k^2}{E_0 \|\boldsymbol{h}_{sk}\|^2}. \tag{3.6}$$

Therefore, the instantaneous SINR, $\gamma_{SINR,k}$, given $\boldsymbol{h}_{sk}$, $\{\boldsymbol{g}_{jk}\}_{j=1,k=1}^{L,\ K}$, and $\{x_j^{(1)}\}_{j=1}^{L}$, are obtained by

$$\begin{aligned} \gamma_{SINR,k} &= \frac{\mathbb{E}[|x_0|^2]}{\Omega_k} \\ &= \frac{E_0 \|\boldsymbol{h}_{sk}\|^2}{\sum_{j=1}^{L} E_j^I \Omega_{jk} |x_j^{(1)}|^2 + \sigma_k^2}. \end{aligned} \tag{3.7}$$

The next step is to analyze the transition probability. Considering the potentially undetected errors at the relay, the transition probability is the probability that the relay detects symbol $\hat{x}_0$ when x_0 was in fact transmitted by the source. Firstly, considering the case of M-PAM, it is possible to derive the transition probability as follows:

$$\mathrm{Pr}_{M\text{-PAM}}(\hat{x_0}|x_0) = \Psi(x_0, \hat{x}_0, \gamma, M, d_0)$$
$$= \begin{cases} Q(\sqrt{2\gamma}(|x_0 - \hat{x}_0| - 1)d_0), & \text{if } \hat{x_0} = \pm(M-1)d_0, \\ Q(\sqrt{2\gamma}(|x_0 - \hat{x}_0| - 1)d_0) - Q(\sqrt{2\gamma}(|x_0 - \hat{x}_0| + 1)d_0), & \\ \qquad \text{if } \hat{x}_0 \epsilon \{\pm(M-3)d_0, \pm(M-5)d_0, \ldots, \pm 1\}, \end{cases} \tag{3.8}$$

where γ is the instantaneous signal-to-noise-ratio (SNR) and d_0 is given in (2.1).

Proof. See Appendix C. □

As we discussed earlier in this section, the interference plus noise component of (3.3) has a Gaussian distribution, from which we derived our instantaneous SINR. Thus, this SINR, $\gamma_{SINR,k}$ has a mathematically equivalent meaning of SNR. Consequently, we substitute $\gamma_{SINR,k}$ into γ in (3.8).

Since the demodulation of an arbitrary $I \times J$-QAM signal is equivalent to the demodulation of two independent PAM signals, I-ary PAM of the in-phase signal $\Re[x_0]$ and J-ary PAM of the quadrature signal $\Im[x_0]$. Thus, using the transition probability of M-ary PAM, we can obtain the transition probability of $I \times J$-QAM accordingly. For instance, the transition probability for the in-phase component is given in the same form of (3.8) with M replaced by I, x_0 replaced by $\Re[x_0]$, and $\hat{x_0}$ replaced by $\Re[\hat{x_0}]$. In summary, for $I \times J$-QAM, the transition probability can be obtained by

$$\mathrm{Pr}_{I\times J\text{-QAM}}(\hat{x}_0|x_0) = \Psi(\Re[x_0], \Re[\hat{x}_0], \gamma, I, d_0) \times \Psi(\Im[x_0], \Im[\hat{x}_0], \gamma, J, d_0). \tag{3.9}$$

With the derivation of the optimum detection at each relay and the transition probability, we will investigate the ML detections at the destination for two transmission schemes in the following two sections.

3.3 Exact ML Detection at the Destination for Simultaneous Transmission

In this section, for simultaneous transmission scheme, we study the ML detection rule at the destination in the presence of interference. Let $\hat{x}_d^{ST}$ denote the ML estimate of x_0 at the destination based on the two received signals: $y_{sd}^{(1)}$ in (2.2), which comes directly from the source, and $y_{rd}^{(2)}$ in (2.4), which comes from the relays.

Theorem 1: For simultaneous transmission, the ML detection rule at the destination is given in closed-form as follows:

$$\hat{x}_d^{ST} = \arg \max_{x_0 \in S_0} \sum_{\substack{x_j^{(1)} \in S_j \\ j=1,\cdots,L}} \sum_{\substack{x_j^{(2)} \in S_j \\ j=1,\cdots,L}} \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} \frac{1}{\Upsilon} \times \Lambda(\gamma_{SINR,k})$$
$$\times \exp\left\{ -\frac{1}{\Upsilon} \left(\sum_{j=1}^{L} E_j^I \Omega_{jD} \left| \left[x_j^{(2)}, -x_j^{(1)} \right] \begin{bmatrix} y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0 \\ y_{rd}^{(2)} - \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} \end{bmatrix} \right|^2 \right.\right.$$
$$\left.\left. + \sigma_d^2 \left(\left| y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0 \right|^2 + \left| y_{rd}^{(2)} - \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} \right|^2 \right) \right) \right\}, \tag{3.10}$$

where

$$\begin{aligned}\Upsilon &:= \sigma_d^6 + \sigma_d^2 \left(\sum_{j=1}^{L} E_j^I \Omega_{jD} (|x_j^{(1)}|^2 + |x_j^{(2)}|^2) \right) \\ &\quad + \sum_{j \neq i} E_j^I E_i^I \Omega_{jD} \Omega_{iD} \left(|x_j^{(1)}|^2 |x_j^{(2)}|^2 - x_j^{(1)} (x_j^{(2)})^* (x_i^{(1)})^* x_i^{(2)} \right),\end{aligned} \tag{3.11}$$

$$\Lambda(\gamma_{SINR,k}) := \begin{cases} \prod_{k=1}^{K} \Psi(x_0, \hat{x}_{0k}, \gamma_{SINR,k}, M, d_0), & \text{if } x_0, \hat{x_{0k}} \in M\text{-PAM}; \\ \prod_{k=1}^{K} \Psi(\Re[x_0], \Re[\hat{x}_{0k}], \gamma_{SINR,k}, I, d_0) \times \Psi(\Im[x_0], \Im[\hat{x}_{0k}], \gamma_{SINR,k}, J, d_0), \\ \qquad\qquad \text{if } x_0, \hat{x_{0k}} \in I \times J\text{-QAM}, \end{cases} \tag{3.12}$$

and $\Psi(\cdot)$ is given in (3.8).

Proof. See Appendix D. □

Note that, the ML detection rule in (3.10) is truly optimum and closed-form. The derived detector can be applied for the most practical scenarios of multiple interferers and multiple relays with multiple antennas (including the single antenna case) and the most general rectangular QAM modulations.

Depending on different CSI conditions, the weighting vector $\boldsymbol{w}_k$ in (3.10) can be determined accordingly. If the instantaneous CSI from all the relays to the destination is *unknown* at the relays, the weighting vector is simply set as $\boldsymbol{w}_k = \mathbf{1}_k = [1, \cdots, 1]$. However, if the relays have the knowledge of the CSI from all the relays to the destination, we can enhance the performance of the ML detector by employing $\boldsymbol{w}_k = \sqrt{N_k} \frac{\boldsymbol{h}_{kd}^H}{\|\boldsymbol{h}_{kd}\|}$. Given $\{\hat{x}_{0k}\}_{k=1}^{K}$, this weighting vector is optimum in the sense that the SINR or SNR is maximized. Because the channel from the relays to the destination is a multiple-input-single-output (MISO) channel, the weighting vector is also optimum in the ML sense. With the optimum $\boldsymbol{w}_k$, we are able to allow all the signal components to be constructively added at the destination.

We now consider an important special case where the interferers are far away from the destination, i.e., the channel gains from the interferers to the destination are zero. This is the case that the interference only affects the relays. For the described model, the ML detection at the destination is given in the following corollary:

Corollary 1: For simultaneous transmission, the ML detection at the destination with interference affecting only the relays is given in closed-form as follows:

$$\hat{x}_d^{ST} = \arg \max_{x_0 \in S_0} \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2}\left|y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0\right|^2\right) \\ \times \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2}\left|y_{rd}^{(2)} - \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k}\right|^2\right) \times \sum_{\substack{x_j^{(1)} \in S_j \\ j=1,\cdots,L}} \Lambda(\gamma_{SINR,k}), \tag{3.13}$$

where $\Lambda(\gamma_{SINR,k})$ is the transition probability given by (3.8) and (3.12). In the above equation, we set $g_{jD} = 0$ in $y_{sd}^{(1)}$ given in (2.2), and $g_{jk} \neq 0$ in $y_{rd}^{(2)}$ given in (2.4).

Proof. See Appendix E. □

Another special case is that the interferers are far away from the relays and they affect only the destination. We obtain the following corollary by setting the channel gains from the interferers to each relay to be zero.

Corollary 2: For simultaneous transmission, the ML detection at the destination with interference affecting only the destination is given in closed-form as follows:

$$\begin{aligned}\hat{x}_d^{ST} = & \arg \max_{x_0 \in S_0} \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2} \left| y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0 \right|^2 \right) \\ & \times \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2} \left| y_{rd}^{(2)} - \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} \right|^2 \right) \times \Lambda(\gamma_{SNR,k}),\end{aligned} \tag{3.14}$$

where we set $g_{jD} \neq 0$ in $y_{sd}^{(1)}$ given in (2.2), and $g_{jk} = 0$ in $y_{rd}^{(2)}$ given in (2.4). Note that the transition probability here, $\Lambda(\gamma_{SNR,k})$, is in terms of the SNR instead of the SINR since all the relays are interference free. ■

A final special, yet important, scenario is the case of no-interference, where the interferers are far away from the destination as well as all the relays. We obtain the following corollary by setting the channel gains from the interferers to the destination and from the interferers to each relay to be zero.

Corollary 3: For simultaneous transmission, the ML detection at the destination without interference is given in the same form of (3.14) with $g_{jD} = 0$ in $y_{sd}^{(1)}$ given in (2.2), and $g_{jk} = 0$ in $y_{rd}^{(2)}$ given in (2.4). ■

Note that even this special result of Corollary 3 has not been explicitly reported in the literature to the best of our knowledge, although some numerical results were available in [3]. Thus, our result in Theorem 1 can be considered as a very general one compared to the results in the literature.

3.4 Exact ML Detection at the Destination for Orthogonal Transmission

For the orthogonal transmission, we study the ML detection at the destination considering interference. Let $\hat{x}_d^{OT}$ denote the ML estimate of x_0 at the destination based on the $(K+1)$ received signals: $y_{sd}^{(1)}$ in (2.2), which comes from the direct link, and $y_{1d}^{(2)}, y_{2d}^{(3)}, \cdots, y_{Kd}^{(K+1)}$ in (2.5), which come from the relays in (K) time slots.

Theorem 2: For orthogonal transmission, the ML detection at the destination is given in closed-form:

$$\hat{x}_d^{OT} = \arg \max_{x_0 \in S_0} \Bigg\{ \sum_{\substack{x_j^{(t)} \in S_j \\ j=1,\cdots,L \\ t=1,\cdots,K+1}} \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} \Lambda(\gamma_{SINR,k}) \times \frac{\exp\left(-(\boldsymbol{Y}^{OT} - \boldsymbol{\mu}^{OT})^H \left[\boldsymbol{\xi}\right]^{-1} (\boldsymbol{Y}^{OT} - \boldsymbol{\mu}^{OT}) \right)}{\det(\boldsymbol{\xi})} \Bigg\}, \tag{3.15}$$

where vectors $\boldsymbol{Y}^{OT}$ and $\boldsymbol{\mu}^{OT}$ are defined as follows:

$$\boldsymbol{\mu}^{OT} = \begin{bmatrix} \sqrt{E_0} h_{sd} x_0 \\ \sqrt{\frac{E_1^R}{N_1}} \boldsymbol{w}_1 \boldsymbol{h}_{1d} \hat{x}_{01} \\ \sqrt{\frac{E_2^R}{N_2}} \boldsymbol{w}_2 \boldsymbol{h}_{2d} \hat{x}_{02} \\ \vdots \\ \sqrt{\frac{E_k^R}{N_K}} \boldsymbol{w}_K \boldsymbol{h}_{Kd} \hat{x}_{0K} \end{bmatrix}; \quad \boldsymbol{Y}^{OT} = \begin{bmatrix} y_{sd}^{(1)} \\ y_{1d}^{(2)} \\ y_{2d}^{(3)} \\ \vdots \\ y_{Kd}^{(K+1)} \end{bmatrix}. \tag{3.16}$$

In (3.15) above,

$$\boldsymbol{\xi} := \sum_{j=1}^{L} E_j^I \Omega_{jD} \{\boldsymbol{x}_j^{(t)}\}_{t=1}^{K+1} \{(\boldsymbol{x}_j^{(t)})^H\}_{t=1}^{K+1} + \text{Diag}\left([\sigma_d^2, \sigma_d^2, \cdots, \sigma_d^2] \right), \tag{3.17}$$

where

$$\boldsymbol{x}_j^{(t)} := \left[\ x_j^{(1)}, \ x_j^{(2)}, \ \cdots, \ x_j^{(K+1)} \ \right]^T. \tag{3.18}$$

Proof. See Appendix F. □

Note that the above ML detection rule is truly optimum, and it can be applied to most practical scenarios of multiple interferers, and multiple relays with multiple antennas. Similarly to the previous section, we now consider three special cases in the following:

Corollary 4: For orthogonal transmission, the ML detection at the destination with interference affecting only the relays is given in closed-form as follows:

$$\begin{aligned}\hat{x}_d^{OT} = \arg \max_{x_0 \in S_0} & \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2}\left|y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0\right|^2\right) \times \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} \\ & \times \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2}\sum_{k=1}^{K}\left|y_{kd}^{(k+1)} - \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k}\right|^2\right) \times \sum_{\substack{x_j^{(1)} \in S_j \\ j=1,\cdots,L}} \Lambda(\gamma_{SINR,k}),\end{aligned} \tag{3.19}$$

where we set $g_{jD} = 0$ in $y_{sd}^{(1)}$ given in (2.2), and $g_{jk} \neq 0$ in $y_{kd}^{(k+1)}$ given in (2.5). ■

Corollary 5: For orthogonal transmission, the ML detection at the destination with interference affecting only the destination is given in closed-form as follows:

$$\begin{aligned}\hat{x}_d^{OT} = \arg \max_{x_0 \in S_0} & \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2}\left|y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0\right|^2\right) \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} \Lambda(\gamma_{SNR,k}) \\ & \times \frac{1}{\sigma_d^2} \exp\left(-\frac{1}{\sigma_d^2}\sum_{k=1}^{K}\left|y_{kd}^{(k+1)} - \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k}\right|^2\right),\end{aligned} \tag{3.20}$$

where we set $g_{jD} \neq 0$ in $y_{sd}^{(1)}$ given in (2.2), and $g_{jk} = 0$ in $y_{kd}^{(k+1)}$ given in (2.5). ■

Corollary 6: For orthogonal transmission, the ML detection at the destination without interference is given in the same form of (3.20) with $g_{jD} = 0$ in $y_{sd}^{(1)}$ given in

(2.2), and $g_{jk} = 0$ in $y_{kd}^{(k+1)}$ given in (2.5). ■

Again, even this special result of Corollary 6 has not been explicitly reported in the literature. Therefore, the result of Theorem 2 is a very general one.

Comparisons between the two transmission schemes are provided in Table 3.1. Under the same M-ary modulation, the simultaneous transmission achieves higher spectrum efficiency compared to the orthogonal transmission. The orthogonal transmission, however, taking advantage of the orthogonal channels, obtains better diversity order compared to simultaneous transmission when the second-hop CSI is unknown at the relays. With the additional condition of knowing the second-hop CSI at the relays, both transmission schemes have the same diversity order. With both schemes, one can improve the performance by increasing the diversity orders and constructively adding all the signal components at the destination. The derivations of diversity orders are given in Appendix G.

Table 3.1: Comparisons between Two Transmission Schemes

	Simultaneous Transmission (ST)	Orthogonal Transmission (OT)
Spectral efficiency (bps/Hz)	$\frac{\log_2 M}{2}$	$\frac{\log_2 M}{K+1}$
Diversity order (D_{ML}) when second-hop CSI is unknown at the relays	2	$K+1$
Diversity order (D_{ML}) when second-hop CSI is known at the relays	$1+\sum_{k=1}^{K} N_k$	$1+\sum_{k=1}^{K} N_k$

From Table 3.1, we can conclude that simultaneous transmission is better than orthogonal transmission when the second-hop CSI is known at the relays, because simultaneous transmission achieves higher spectral efficiency and the same diversity order. On the other hand, when the second-hop CSI is unknown at the relays, orthogonal transmission can be better than simultaneous transmission in high SNR, because the orthogonal transmission achieves higher diversity order, which would lead to a performance advantage in high SNR. This can be confirmed numerically in Fig. 4.3 in Chapter 4.

Chapter 4

Numerical Results

4.1 Numerical Results

In this chapter, we present simulation results of ML performance for the two transmission schemes under different CSI assumptions. Let $\mathcal{D}_{k,j}$ denote the distance from the j-th interferer to the k-th relay, $\mathcal{D}_{d,j}$ denote the distance from the j-th interferer to the destination, and $\mathcal{D}_{d,k}$ denote distance from the k-th relay to the destination for $j = 1, \ldots, L; k = 1, \ldots, K$. Assuming the path loss exponent to be four, we set $\Omega_{jk} = (\frac{\mathcal{D}_{k,j}}{\mathcal{D}_0})^{-4}$ and $\Omega_{jD} = (\frac{\mathcal{D}_{d,j}}{\mathcal{D}_0})^{-4}$, where $\mathcal{D}_0$ is the reference distance. We assume constellations with equiprobable symbols, i.e., $\Pr(x_j^{(t)}) = 1/M_j$. We set $\sigma_d^2 = \sigma_k^2 = \sigma^2$ and $E_k^R = E_0$. The SNR in the first hop is defined as $SNR_1 := \frac{E_0}{\sigma^2}$ and the interference-to-noise ratio (INR) is defined as $INR_1 := \frac{E_j^I}{\sigma^2}$. In the second hop, the SNR per relay is defined as $SNR_2 := \frac{E_k^R}{\sigma^2}$; and thus, we have $SNR_1 = SNR_2$, which will be simply denoted by SNR in the following. By varying the value of this SNR, we can plot the average SER for the proposed ML schemes with uniform power allocation as well as for the conventional ML scheme. In this paper, the conventional

ML scheme indicates the existing ML detector which was developed by not considering any interference: interference was completely ignored at the relays as well as the destination, given in Corollary 3 for simultaneous transmission and Corollary 4 for orthogonal transmission[1]. But we will run the simulation under the same interference environment for all the schemes. Firstly, we investigate the performance of our developed ML detectors by presenting the simulated SER results for various scenarios. Moreover, we compare the performance between the two transmission schemes considering different CSI conditions.

First of all, we compare the performance of our derived ML detectors with the conventional scheme where interference was completely ignored. Note that the simulation was running under the same multi-interference environment for our derived ML detectors and the conventional ML detectors. In the following simulations, we assume $N_1 = N_2 = \cdots = N_K = N$ and $M_0 = M_1 = \cdots = M_L = M$. It is also assumed that we employ the symmetric networks with $\mathcal{D}_{d,j} = \mathcal{D}_0, \mathcal{D}_{d,k} = \mathcal{D}_0$, and $\mathcal{D}_{k,j} = \mathcal{D}_0$ for $j = 1, \cdots, L; k = 1, \cdots, K$. In Fig. 4.1, the performance of the derived ML detectors is presented with different antenna numbers, $N = 1, \cdots, 5$, while with $K = 2, L = 1$, and $M = 4$, for the simultaneous transmission with the second-hop CSI knowing at the relays. When $N = 1$, the simultaneous transmission scheme with the second-hop CSI at the relays are equivalent to the scheme without the second-hop CSI at the relays, both of which significantly outperform the conventional scheme. As the number of antennas increases, the performance difference gets bigger. The poor performance of the conventional scheme can be seen from the error floor exhibited in medium and high SNR range. That is because the conventional scheme simply

[1]Recall that the explicit expression of Corollary 3 and 4 have not been reported in the literature, although some numerical results for Corollary 3 were presented in [3]

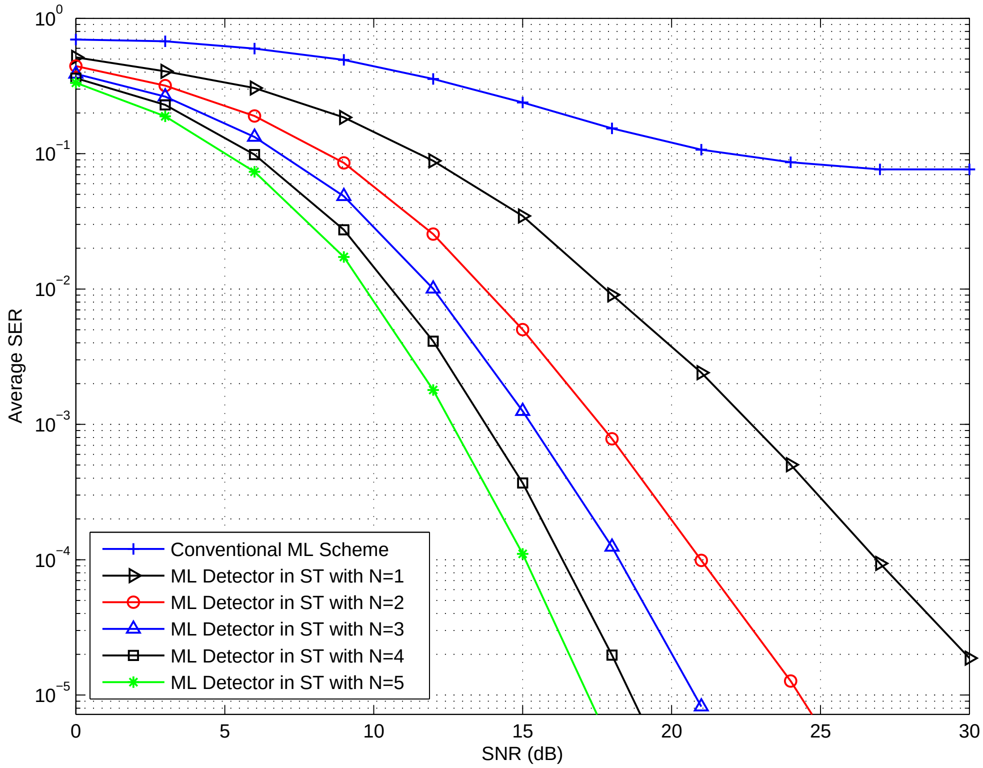

Figure 4.1: Average SER of Simultaneous Transmission (ST) with CSI $\boldsymbol{h}_{kd}$ at the relays versus the SNR γ for a 2-relay network with the direct link in Rayleigh fading, for $x_0 \in$ 4-QAM and $x_j^{(t)} \in$ 4-QAM.

ignores the interference even through interference is the dominating factor in high SNR.

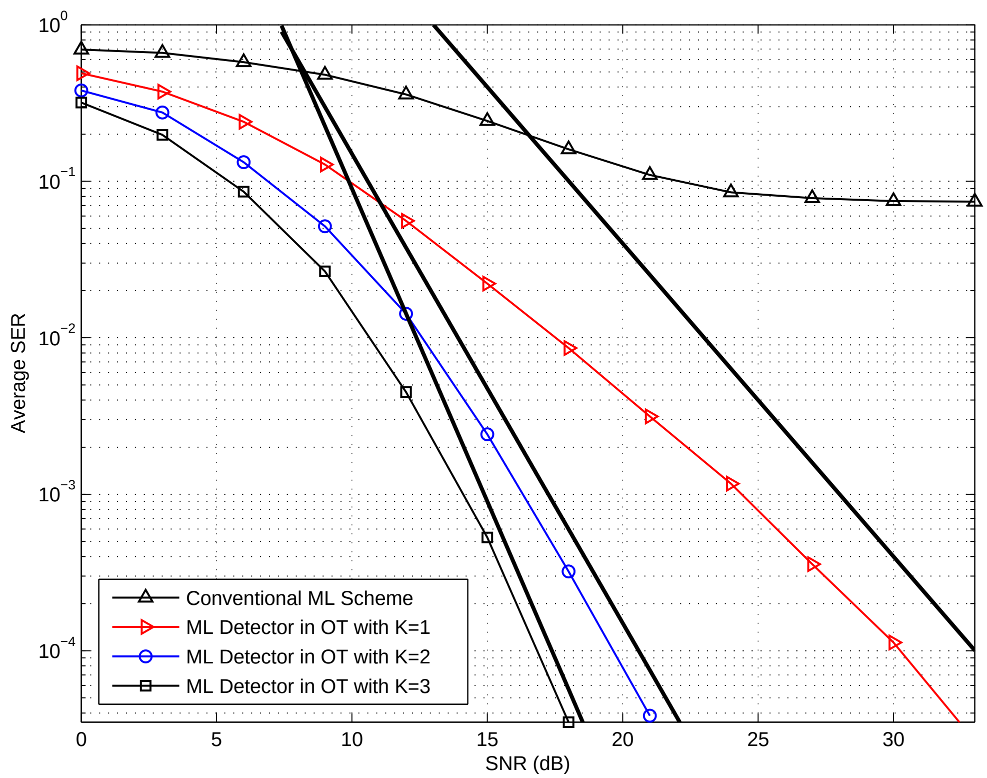

Figure 4.2: Average SER of Orthogonal Transmission (OT) without CSI $\boldsymbol{h}_{kd}$ at the relays versus the SNR γ for a 2-antenna network with the direct link in Rayleigh fading, for $x_0 \in$ 4-QAM and $x_j^{(t)} \in$ 4-QAM. The solid lines of $\frac{c_1}{(SNR)^2}$, $\frac{c_2}{(SNR)^3}$ and $\frac{c_3}{(SNR)^4}$ were drawn to show the diversity order.

Fig. 4.2 demonstrates the effects when we vary the number of relays for $K = 1, 2, 3$, in the orthogonal system without knowing the second-hop CSI at the relays, while with $N = 2, L = 1$, and $M = 4$. From the result, one can see that the developed ML detector significantly outperforms the conventional scheme. As the number of relays increases, the diversity order increases, which yields better performance. In the figure, the lines of $\frac{c_1}{(SNR)^2}, \frac{c_2}{(SNR)^3}$ and $\frac{c_3}{(SNR)^4}$, respectively, were drawn to show that each of them is parallel to each curve in high SNR range of ML detector with different number of relays, where c_1, c_2 and c_3 are positive constants. The parallelism further confirms the diversity order of $K + 1$ for orthogonal transmission without the second-hop CSI at the relays, as shown in Table 3.1.

Now, we compare the performance of two transmission schemes. First of all, we need to guarantee that our comparison is carried on under the same spectrum efficiency. If, for instance, we have 2 relays ($K = 2$) in both systems, $M^{ST} = 4$ and $M^{OT} = 8$ could be one of the candidates to achieve the equivalence of the equation above, according to Table 3.1. All the comparisons below are under the setups with the same spectral efficiency.

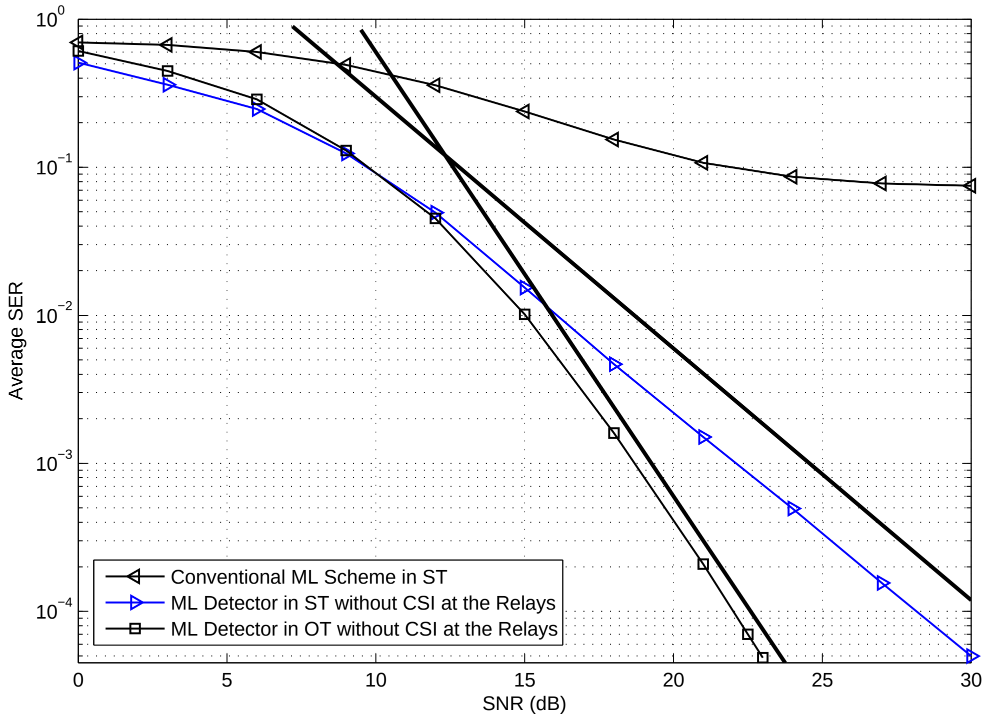

Figure 4.3: Average SER of Simultaneous Transmission (ST) without CSI $\boldsymbol{h}_{kd}$ at the relays, Orthogonal Transmission (OT) without CSI $\boldsymbol{h}_{kd}$ at the relays versus the SNR γ for a 2-relay, 2-antenna network with the direct link in Rayleigh fading, for $x_0^{ST} \in$ 4-QAM, $x_0^{OT} \in$ 8-QAM, and $x_j^{(t)} \in$ 4-QAM. The solid lines of $\frac{c_1}{(SNR)^2}$ and $\frac{c_2}{(SNR)^3}$ were drawn to show the diversity order.

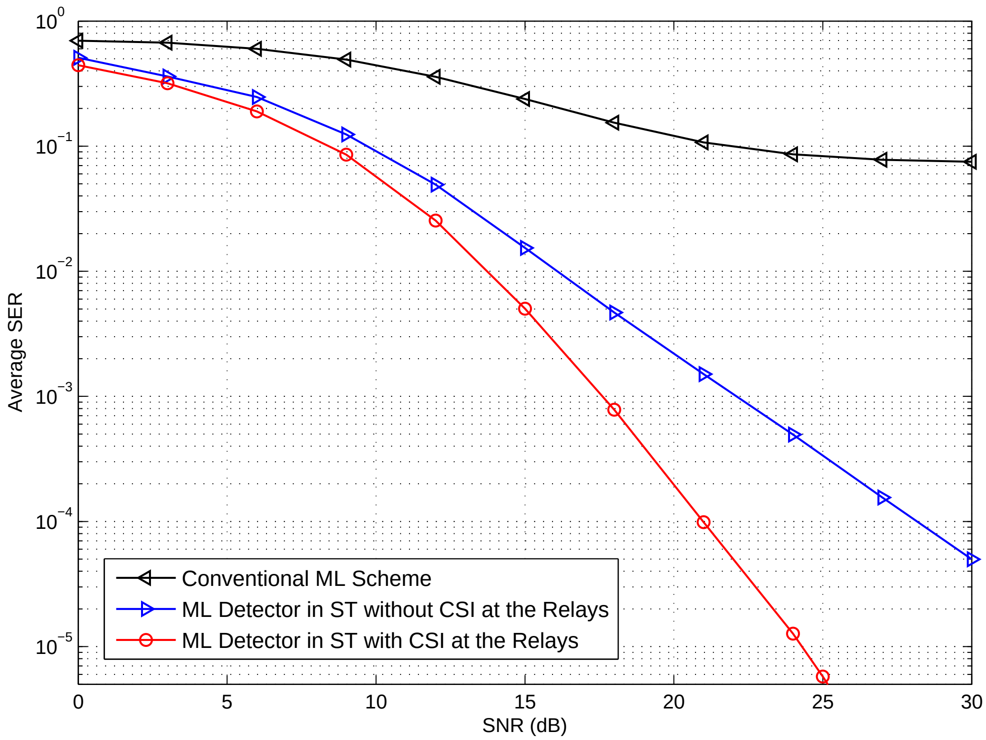

Figure 4.4: Average SER of Simultaneous Transmission (ST) with and without CSI $\boldsymbol{h}_{kd}$ at the relays versus the SNR γ for a 2-relay, 2-antenna network with the direct link in Rayleigh fading, for $x_0 \in$ 4-QAM and $x_j^{(t)} \in$ 4-QAM.

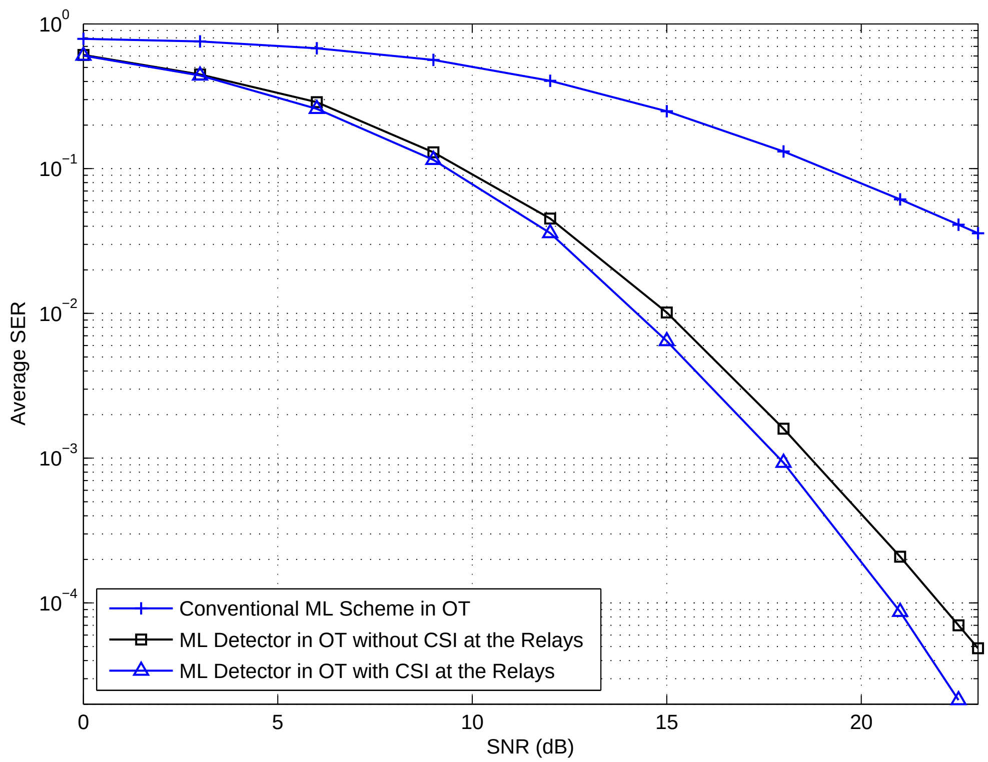

Figure 4.5: Average SER of Orthogonal Transmission (OT) with and without CSI $\boldsymbol{h}_{kd}$ at the relays versus the SNR γ for a 2-relay, 2-antenna network with the direct link in Rayleigh fading, for $x_0 \in$ 8-QAM and $x_j^{(t)} \in$ 4-QAM.

In Fig. 4.3, we compared 3 schemes under the same spectral efficiency: conventional ML scheme, simultaneous transmission scheme without CSI at the relays, and the orthogonal transmission scheme without CSI at the relays. Both of the proposed ML detectors significantly outperform the conventional scheme. As can be seen in the figure, the curve of the ML detector in orthogonal transmission yields a better performance than the one in simultaneous transmission in the high SNR range (e.g., greater than 10 dB). That is because, with the orthogonal transmission, increasing the number of relays will increase the diversity order. Whereas in the simultaneous transmission, increasing the number of relays will not change the diversity order. According to Table 3.1, the orthogonal transmission without CSI at the relays obtains a diversity order of $K + 1 = 3$, whereas the ST without CSI at the relays obtains a diversity order of 2. The lines of $\frac{c_1}{(SNR)^2}$ and $\frac{c_2}{(SNR)^3}$ were drawn to show the parallelism for the diversity order.

Fig. 4.4 shows the ML performance for the simultaneous transmission with and without second-hop CSI at the relays. As can be seen in the figure, the ML scheme with the additional knowledge of $\boldsymbol{h}_{kd}$ outperforms the ML scheme without the knowledge of $\boldsymbol{h}_{kd}$. That is because the optimum designed weight along with the second-hop CSI can greatly increase the diversity order to yield a better performance. Similarly, Fig. 4.5 shows the ML performance for orthogonal transmission with and without second-hop CSI at the relays. Simulation result shows that, knowing $\boldsymbol{h}_{kd}$ at the relays, the ML detector yields a better performance due to the constructively added signal components. In addition, with the second-hop CSI, the antennas form the effective orthogonal channels, which contributes to the diversity order. When we increase the number of antennas for each relay, the performance difference gets even bigger between these two schemes.

Chapter 5

Conclusions

5.1 Conclusion

In this thesis, we have derived the optimum ML detectors for decode-and-forward cooperative networks in the presence of interference where only partial CSI of the interferers is known. A key feature of the detection rules is their relative generality and practicality, so that they can be applied to the most general systems of multiple interferers and multiple relays with multiple antennas (including the single case). In addition, the derived detectors are applicable for the most practical scenarios with various signal and interferer modulation/constellation, such as M-QAM, M-PAM. We have considered two transmission schemes: simultaneous transmission and orthogonal transmission. In each scheme, we began by deriving the ML detection at the relays, and further developed the detection algorithm at the destination. We have proved that, depending on the setup, one scheme can yield a better performance than the other. Thus, a transmission scheme should be chosen carefully according to different conditions. As a special case, the ML detectors for the interference free case have been obtained, whose expressions had not been reported in the literature to the best of our knowledge. Numerical results showed that the developed ML detectors significantly and extensively outperforms the conventional scheme where the interference was completely ignored. Throughout our analysis, no approximation or inequality was applied. Therefore, we can claim that, by employing the ML rule, the developed detectors guarantee the true optimality when considering the interference, and no other detector would achieve better performance.

5.2 Future Work

I believe the work presented in this thesis has potential to be extended further in different ways. First of all, despite the fact that we are the first one in the literature developing ML detectors for DF systems in the presence of interference, the detectors were derived based on an important yet reasonable assumption: we do not know the interference channel, and only partial CSI for the interferers is available at the relays and the destination. Thus, possible detectors can be developed by reducing the current assumption. More specifically, the CSI of the interferers is completely unknown at the relays and the destination, and channel estimation might be required to further assist the development of detectors. Similarly, the noncoherent detection was not considered in the thesis, thus possible detectors can be derived under noncoherent detection where the destination does not have the access of the instantaneous CSI h_{sd}, $\boldsymbol{h}_{sk}$ and $\boldsymbol{h}_{kd}$.

In addition, the detectors for AF protocol in the presence of interference may be developed in the following steps. In [19], the authors proposed a detector for AF network with interference affecting only the single-antenna relays in orthogonal transmission. As an extension to the existing work, we may consider detection rules for the interference affecting relays as well as the destination under AF protocol. Then, it would be very useful to explore the multi-antenna relay model. In that case, the channel distribution would be changed from exponential distribution to chi-square distribution, which would definitely add difficulty in the mathematical performance analysis. Later on, the simultaneous transmission and orthogonal transmission should be considered to compare the spectral efficiency and diversity gain trade-offs.

Thirdly, despite the achievement of strong performance, the optimum ML detectors considering interference come with high computational complexity. It is therefore possible to develop suboptimum detectors or extending the existing suboptimum detection rules to reduce the computational complexity, yet causing small performance loss under interference environment. We have already provided a solution by extending the existing C-MRC detector into a detection rule which applies to the multi-antenna relay system in the presence of interference, provided in (G.7). Similar work could be done to extend the LAR detection rule or λ-MRC detection rule to consider interference. We could even develop new detection algorithms to reduce computational complexity while considering interference.

Moreover, for the current system, we have uniform power on every relay with the same modulation. For future improvement, we may consider an optimum relay power allocation scheme or modulation optimization problem. Alternatively, relay selection could be another way to improve the performance and to efficiently locate the transmission power. We can either create our own relay selection scheme based on maximizing the first-hop SINR or adopt some suitable existing relay selection schemes. There is also possibility to extend our system model to MIMO by employing multiple antennas on the source node and the destination node. Under MIMO system, more transmission approaches should be considered: all the antennas on the source node transmit the same desired signal or they can transmit completely different signals. Then beamforming techniques can be employed at the first-hop and second-hop to enhance the overall performance.

Finally, an emerging hot topic called *Energy Harvesting* or *Simultaneous Wireless Information and Power Transfer* (SWIPT) can be incorporated into our developed detectors. Energy harvesting is an emerging solution for prolonging the lifetime of a battery or an energy-constrained mobile device in wireless networks. As the radio-frequency signals can carry information and energy, the relay nodes can take advantage of this fact to process the information and harvest energy from the received signals to become self-sustain. Most of the existing works on energy harvesting do not consider interference [1, 7, 8, 13, 17, 27, 30]. However, it is quite interesting to consider interference, as the interferers' signals can help the relays to scavenge even more energies. In other words, under energy harvesting, the interference is not a completely bad factor, as the relay transmission can be powered by the interferer's signals. In that case, the trade-offs in the performance analysis might be shown once we increase the number of interferers. On the other hand, most of the existing works on energy harvesting are restricted to analysis on power allocation, outage probability, capacity and throughput [1, 7, 8, 13, 17, 27, 30]. It would be a great angle to study the detection rules by employing energy harvesting.

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Appendix A

Proofs

A.1 Proof for Lemma 1

We start by assuming that the complete instantaneous CSI from all terminals including L interferers were available at each relay. For the time being, it is assumed that we are interested in detecting all signals including the interference signals. Then the optimum ML detection at the relay is given by

$$(\hat{x}_0, \hat{x}_1^{(1)}, \cdots, \hat{x}_L^{(1)}) = \arg \max_{x_0 \in S_0, x_1^{(1)} \in S_1, \cdots, x_L^{(1)} \in S_L} \prod_{k=1}^{K} \mathcal{M}_k, \tag{A.1}$$

where

$$\mathcal{M}_k = \frac{1}{\sigma_k^2} \exp\left(- \frac{\left\| \boldsymbol{y}_{sk}^{(1)} - \sqrt{E_0} \boldsymbol{h}_{sk} x_0 - \sum_{j=1}^{L} \sqrt{E_j^I} \boldsymbol{g}_{jk} x_j^{(1)} \right\|^2}{\sigma_k^2} \right). \tag{A.2}$$

As the instantaneous CSI of $\boldsymbol{g}_{j1}$ is unknown, we now take expectation of $\mathcal{M}_k$ with respect to $\boldsymbol{g}_{j1}$. Then we have

$$\begin{aligned}\mathbb{E}_{\boldsymbol{g}_{1k}}[\mathcal{M}_k] =& \frac{1}{\sigma_k^2}\exp\left(-\frac{\left\|\boldsymbol{\Phi}_{1k}\right\|^2}{\sigma_k^2}\right)\\ &\times\mathbb{E}_{\boldsymbol{g}_{1k}}\left[\exp\left(-\frac{\|\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\|^2-2\Re\{(\boldsymbol{\Phi}_{1k})^*\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\}}{\sigma_k^2}\right)\right]\\ =& \frac{1}{\sigma_k^2}\exp\left(-\frac{\left\|\boldsymbol{\Phi}_{1k}\right\|^2}{\sigma_k^2}\right)\\ &\times\mathbb{E}_{\Re[\boldsymbol{g}_{1k}]}\left[\exp\left(-\frac{\|\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\|^2\Re[\boldsymbol{g}_{1k}]^2-2\Re\{(\boldsymbol{\Phi}_{1k})^*\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\}\Re[\boldsymbol{g}_{1k}]}{\sigma_k^2}\right)\right]\\ &\times\mathbb{E}_{\Im[\boldsymbol{g}_{1k}]}\left[\exp\left(-\frac{\|\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\|^2\Im[\boldsymbol{g}_{1k}]^2-2\Im\{(\boldsymbol{\Phi}_{1k})^*\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\}\Im[\boldsymbol{g}_{1k}]}{\sigma_k^2}\right)\right],\end{aligned} \tag{A.3}$$

where

$$\boldsymbol{\Phi}_{1k} = \boldsymbol{y}_{sk}^{(1)} - \sqrt{E_0}\boldsymbol{h}_{sk}x_0 - \sum_{j=1,j\neq 1}^{L}\sqrt{E_j^I}\boldsymbol{g}_{jk}x_j^{(1)}. \tag{A.4}$$

Since $\Re[\boldsymbol{g}_{1k}] \sim \mathcal{N}(0,\Omega_{1k}/2)$ and $\Im[\boldsymbol{g}_{1k}] \sim \mathcal{N}(0,\Omega_{1k}/2)$, we can apply the following property in (A.3), if $W \sim \mathcal{N}(0,\xi/2)$. The property is given as

$$\mathbb{E}_w[\exp(2bW - aW^2)] = \frac{\exp(\frac{b^2\xi}{1+a\xi})}{\sqrt{1+a\xi}}. \tag{A.5}$$

After some manipulations, we can obtain

$$\begin{aligned}\mathbb{E}_{\boldsymbol{g}_{1k}}[\mathcal{M}_k] &= \frac{\exp\left(-\frac{\left\|\boldsymbol{\Phi}_{1k}\right\|^2}{\sigma_k^2}\right)}{\sigma_k^2\left(1+\frac{\|\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\|^2}{\sigma_k^2}\Omega_{1k}\right)}\exp\left(\frac{\left(\Re\{(\boldsymbol{\Phi}_{1k})^*\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\}\right)^2\Omega_{1k}}{\sigma_k^4+\sigma_k^2\|\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\|^2\Omega_{1k}}\right)\\ &\quad\times\exp\left(\frac{\left(\Im\{(\boldsymbol{\Phi}_{1k})^*\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\}\right)^2\Omega_{1k}}{\sigma_k^4+\sigma_k^2\|\sqrt{E_1^I}\boldsymbol{g}_{1k}x_1^{(1)}\|^2\Omega_{1k}}\right)\\ &= \frac{1}{\sigma_k^2+E_1^I\Omega_{1k}|x_1^{(1)}|^2}\exp\left(-\frac{\|\boldsymbol{\Phi}_{1k}\|^2}{\sigma_k^2+E_1^I\Omega_{1k}|x_1^{(1)}|^2}\right).\end{aligned} \tag{A.6}$$

Based on the same approach, we keep taking expectations with respect to $\boldsymbol{g}_{2k},\cdots,\boldsymbol{g}_{Lk}$ for each relay k, and we will get

$$\mathbb{E}_{\boldsymbol{g}_{1k}}\cdots\mathbb{E}_{\boldsymbol{g}_{1L}}[\mathcal{M}_k] = \frac{\exp\left(\frac{\|\boldsymbol{y}_{sk}^{(1)}-\sqrt{E_0}\boldsymbol{h}_{sk}x_0\|^2}{\sum_{j=1}^{L}E_j^I\Omega_{jk}|x_j^{(1)}|^2+\sigma_k^2}\right)}{\sum_{j=1}^{L}E_j^I\Omega_{jk}|x_j^{(1)}|^2+\sigma_k^2}. \tag{A.7}$$

After substituting (A.7) into (A.1), we would achieve the optimum ML detection at each relay with partial CSI information for detecting all the signals. By re-considering (A.1), we are interested in detecting our desired signal, x_0. Thus, we take expectation of (A.1) with respect to $\{x_j^{(t)}\}_{j=1}^{L}$, and the detection rule becomes (3.1).

B Proof for Lemma 2

Let $a(\{x_j^{(1)}\}_{j=1}^{L})=\sum_{j=1}^{L}E_j^I\Omega_{jk}|x_j^{(1)}|^2+\sigma_k^2$ and $Y_0=\|\boldsymbol{y}_{sk}^{(1)}-\sqrt{E_0}\boldsymbol{h}_{sk}x_0\|^2=E_0\|\boldsymbol{h}_{sk}\|^2\left|x_0-\frac{\boldsymbol{h}_{sk}^H\boldsymbol{y}_{sk}^{(1)}}{\sqrt{E_0}\|\boldsymbol{h}_{sk}\|^2}\right|^2$, where $Y_0\geq 0$. For M-QAM or M-PAM, the defined $a(\{x_j^{(1)}\}_{j=1}^{L})$ is a positive variable. Now, we re-write (3.1) as

$$\hat{x}_0 = \arg \max_{Y_0}\left\{\zeta\left(Y_0, a(\{x_j^{(1)}\}_{j=1}^{L})\right)\right\}, \tag{B.1}$$

where

$$\zeta\left(Y_0, a(\{x_j^{(1)}\}_{j=1}^{L})\right) = \sum_{x_1^{(1)}\in S_1} \cdots \sum_{x_L^{(1)}\in S_L} \frac{1}{\left(a(\{x_j^{(1)}\}_{j=1}^{L})\right)^{N_k}} \exp\left(\frac{-Y_0}{a(\{x_j^{(1)}\}_{j=1}^{L})}\right). \tag{B.2}$$

Clearly, $\zeta(Y_0, a(\{x_j^{(1)}\}_{j=1}^{L}))$ is a *monotonic decreasing function* of $-Y_0$ for any positive $a(\{x_j^{(1)}\}_{j=1}^{L})$. The realization of monotonicity can be easily seen from its definition, whether $a(\{x_j^{(1)}\}_{j=1}^{L})$ is a constant or variable. Then it can be seen that

$$\begin{aligned}\arg\max_{Y_0}\{\text{monotonic decreasing function of } (-Y_0)\} &= \arg\max_{Y_0}\{-Y_0\} \\ &= \arg\min_{Y_0}\{Y_0\}.\end{aligned} \tag{B.3}$$

C Proof for Transition Probability

Since the demodulation of an arbitrary $I \times J$-QAM signal is equivalent to the demodulation of two independent PAM signals, I-ary PAM of the in-phase signal and J-ary PAM of the quadrature signal. Thus, by achieving the transition probability of I-ary PAM, we can obtain the transition probability of $I \times J$-QAM accordingly. Let us define a received signal $y = \sqrt{E_0}x + n$, where n is the AWGN with $n \sim \mathcal{CN}(0, \sigma^2)$. For I-PAM, let the original signal x and the detected signal $\hat{x}$ selected from the set $S_j^{PAM}(I) = \{\pm d, \pm 3d, \cdots, \pm(I-1)d\} = \sqrt{\frac{3}{I^2-1}}\{\pm 1, \pm 3, \cdots, \pm(I-1)\}, j = 0, 1, \cdots, L$, where $2d$ is the Euclidean distance between two adjacent signal points defined in [6]. An error occurs when the detected signal $\hat{x}$ resides in the wrong decision region of I-PAM signal constellation. There are two cases for an error to occur: the wrong decision region belongs to the end regions of the signal constellation ($\hat{x} = \pm(I-1)d$), and the wrong decision region belongs to the middle regions of the signal constellation ($\hat{x} \in \{\pm d, \pm 3d, \cdots, \pm(I-3)d\}$). If for the case of $\hat{x} \in \{\pm d, \pm 3d, \cdots, \pm(I-3)d\}$, the transition probability that the relay detects $\hat{x}$ when in fact x was transmitted by the source, can be obtained as follows:

$$\begin{aligned}
\Pr(\hat{x}|x) &= \Pr(\sqrt{E_0}d(\hat{x}-1) \leq \Re[y] \leq \sqrt{E_0}d(\hat{x}+1)|x) \\
&= \Pr\left(\sqrt{\frac{3E_0}{I^2-1}}(\hat{x}-x-1) \leq \Re[n] \leq \sqrt{\frac{3E_0}{I^2-1}}(\hat{x}-x+1)\right) \\
&= Q\left(\sqrt{\frac{6E_0}{(I^2-1)\sigma^2}}(|x-\hat{x}|-1)) - Q(\sqrt{\frac{6E_0}{(I^2-1)\sigma^2}}(|x-\hat{x}|+1)\right).
\end{aligned} \tag{C.1}$$

If for the case of $\hat{x} = \pm(I-1)d$, the transition probability can be obtained as follows:

$$\begin{aligned}
\Pr(\hat{x} = (I-1)d|x) &= \Pr(\Re[y] > \sqrt{E_0}d(\hat{x}-1)|x) \\
&= Q\left(\sqrt{\frac{6E_0}{(I^2-1)\sigma^2}}(|x-\hat{x}|-1)\right); \\
\Pr(\hat{x} = -(I-1)d|x) &= \Pr(\Re[y] < \sqrt{E_0}d(\hat{x}+1)|x) \\
&= Q\left(\sqrt{\frac{6E_0}{(I^2-1)\sigma^2}}(|\hat{x}-x|-1)\right).
\end{aligned} \tag{C.2}$$

If we substitute d, which is defined in (2.1), into (C.1) and (C.2), we can obtain (3.8).

D Proof for Theorem 1

Since the detection at the destination is based on two *dependent* received signals of $y_{sd}^{(1)}$ and $y_{rd}^{(2)}$, we need to employ the ML joint detection rule. Firstly, we define the following vectors based on (2.2) and (2.4),

$$\begin{aligned} \boldsymbol{\mu}^{ST} &= \begin{bmatrix} \sqrt{E_0} h_{sd} x_0 \\ \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} \end{bmatrix}; \\ \boldsymbol{Y}^{ST} &= \begin{bmatrix} y_{sd}^{(1)} \\ y_{rd}^{(2)} \end{bmatrix} = \begin{bmatrix} \sqrt{E_0} h_{sd} x_0 + \sum_{j=1}^{L} \sqrt{E_j^I} g_{jD} x_j^{(1)} + n_{sd} \\ \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} + \sum_{j=1}^{L} \sqrt{E_j^I} g_{jD} x_j^{(2)} + n_{rd} \end{bmatrix}, \end{aligned} \tag{D.1}$$

where $g_{jD} \sim \mathcal{CN}(0, \Omega_{jD})$, $\boldsymbol{Y}^{ST} \sim \mathcal{CN}(\boldsymbol{\mu}^{ST}, \boldsymbol{\Sigma})$, and $\boldsymbol{\Sigma}$ is the covariance of the dependent signals of $y_{sd}^{(1)}$ and $y_{rd}^{(2)}$, derived as

$$\begin{aligned} \boldsymbol{\Sigma} &= \mathbb{E}\{(\boldsymbol{Y}^{ST} - \boldsymbol{\mu}^{ST})(\boldsymbol{Y}^{ST} - \boldsymbol{\mu}^{ST})^H\} \\ &= \begin{bmatrix} \sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(1)}|^2 + \sigma_d^2, & \sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(1)} (x_j^{(2)})^* \\ \sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(2)} (x_j^{(1)})^*, & \sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(2)}|^2 + \sigma_d^2 \end{bmatrix}. \end{aligned} \tag{D.2}$$

We denote $\det(\boldsymbol{\Sigma})$ as the determinant of the covariance and can find

$$\begin{aligned} \det(\boldsymbol{\Sigma}) = &\left(\sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(1)}|^2 + \sigma_d^2 \right) \left(\sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(2)}|^2 + \sigma_d^2 \right) \\ &- \left(\sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(1)} (x_j^{(2)})^* \right) \left(\sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(2)} (x_j^{(1)})^* \right). \end{aligned} \tag{D.3}$$

After some manipulations, we can get the following:

$$\begin{aligned}\det(\mathbf{\Sigma}) =&\sigma_d^6 + \sigma_d^2 \left(\sum_{j=1}^{L} E_j^I \Omega_{jD} (|x_j^{(1)}|^2 + |x_j^{(2)}|^2) \right) \\ &+ \sum_{j \neq i} E_j^I E_i^I \Omega_{jD} \Omega_{iD} \left(|x_j^{(1)}|^2 |x_j^{(2)}|^2 - x_j^{(1)} (x_j^{(2)})^* (x_i^{(1)})^* x_i^{(2)} \right).\end{aligned} \tag{D.4}$$

With the partial knowledge of CSI at the destination, we need to take expectations with respect to $\boldsymbol{g}_{1k}, \ldots, \boldsymbol{g}_{Lk}$, which are the similar steps we have derived in Appendix A.1. Due to the space limitation, we skip the steps for expectations with respect to $\{\boldsymbol{g}_{jk}\}_{j=1,k=1}^{L,\ K}$ as well as $\{x_j^{(t)}\}_{j=1}^{L}$. Finally, the optimum ML detection would be given by

$$\begin{aligned}\hat{x}_d^{ST} =& \arg \max_{x_0 \in S_0} \sum_{\substack{x_j^{(t)} \in S_j \\ t=1,2 \\ j=1,\cdots,L}} f(y_{sd}^{(1)}, y_{rd}^{(2)} | x_0, \{x_j^{(1)}, x_j^{(2)}\}_{j=1}^{L}, h_{sd}, \{\boldsymbol{h}_{sk}, \boldsymbol{h}_{kd}\}_{k=1}^{K}) \\ =& \arg \max_{x_0 \in S_0} \sum_{\substack{x_j^{(t)} \in S_j \\ t=1,2 \\ j=1,\cdots,L}} \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} f(y_{sd}^{(1)}, y_{rd}^{(2)} | x_0, \{\hat{x}_{0k}\}_{k=1}^{K}, \{x_j^{(1)}, x_j^{(2)}\}_{j=1}^{L}, h_{sd}, \{\boldsymbol{h}_{sk}, \boldsymbol{h}_{kd}\}_{k=1}^{K}) \\ & \times \prod_{k=1}^{K} \Pr(\hat{x}_{0k} | x_0, \{x_j^{(1)}\}_{j=1}^{L}, \boldsymbol{h}_{sk}),\end{aligned} \tag{D.5}$$

where $\Pr(\hat{x}_{0k} | x_0, \{x_j^{(1)}\}_{j=1}^{L}, \boldsymbol{h}_{sk})$ represents the transition probability. Now we denote

$$\Theta = f(y_{sd}^{(1)}, y_{rd}^{(2)} | x_0, \{\hat{x}_{0k}\}_{k=1}^{K}, \{x_j^{(1)}, x_j^{(2)}\}_{j=1}^{L}, h_{sd}, \{\boldsymbol{h}_{sk}, \boldsymbol{h}_{kd}\}_{k=1}^{K}).$$

We would like to pursue a close-form expression for Θ so that the ML detection rule will be easily determined. Then we have

$$\Theta = \frac{1}{\pi \cdot \det(\boldsymbol{\Sigma})} \times \exp\left(-\frac{(\boldsymbol{Y}^{ST} - \boldsymbol{\mu}^{ST})^H}{\det(\boldsymbol{\Sigma})} \times \begin{bmatrix} \sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(2)}|^2 + \sigma_d^2, & -\sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(1)} (x_j^{(2)})^* \\ -\sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(2)} (x_j^{(1)})^*, & \sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(1)}|^2 + \sigma_d^2 \end{bmatrix} (\boldsymbol{Y}^{ST} - \boldsymbol{\mu}^{ST}) \right), \tag{D.6}$$

where $\det(\boldsymbol{\Sigma})$ was derived in (D.4) and $(\boldsymbol{Y}^{ST} - \boldsymbol{\mu}^{ST})^H$ can be found in (D.1). Due to the complication of the three matrices multiplication and expansion, we partitioned all the terms into two groups: terms associated with the interference and terms containing the noise variance. By factoring out all the common terms in each group, we achieve a closed-form expression for Θ as

$$\Theta = \frac{1}{\pi \cdot \det(\boldsymbol{\Sigma})} \exp\left(-\frac{1}{\det(\boldsymbol{\Sigma})} \sum_{j=1}^{L} E_j^I \Omega_{jD} \left| \begin{bmatrix} x_j^{(2)} & -x_j^{(1)} \end{bmatrix} \begin{bmatrix} y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0 \\ y_{rd}^{(2)} - \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} \end{bmatrix} \right|^2 - \frac{\sigma_d^2}{\det(\boldsymbol{\Sigma})} \left(|y_{sd}^{(1)} - \sqrt{E_0} h_{sd} x_0|^2 + |y_{rd}^{(2)} - \sum_{k=1}^{K} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k}|^2 \right) \right). \tag{D.7}$$

After substituting (D.7) and (D.4) into (D.5), we obtain the ML detection rule of (3.10).

E Proof for Corollary 1

When the channel gains of the interferers to the destination are zero, the detection at the destination is based on two *independent* received signals of $y_{sd}^{(1)}$ and $y_{rd}^{(2)}$. Due to the independence, the ML detection rule has a major change in the likelihood function. Instead of (D.5), we have

$$\begin{aligned}
\hat{x}_d^{ST} &= \arg\max_{x_0 \in S_0} \sum_{\substack{x_j^{(1)} \in S_j \\ j=1,\cdots,L}} f(y_{sd}^{(1)}, y_{rd}^{(2)} | x_0, \{x_j^{(1)}\}_{j=1}^{L}, h_{sd}, \{\boldsymbol{h}_{sk}, \boldsymbol{h}_{kd}\}_{k=1}^{K}) \\
&= \arg\max_{x_0 \in S_0} f(y_{sd}^{(1)} | x_0, h_{sd}) \times \sum_{\substack{\hat{x}_{0k} \in S_0 \\ k=1,\cdots,K}} f(y_{rd}^{(2)} | \{\hat{x}_{0k}\}_{k=1}^{K}, \boldsymbol{h}_{kd}) \\
&\quad \times \sum_{x_1^{(1)} \in S_1} \cdots \sum_{x_L^{(1)} \in S_L} \prod_{k=1}^{K} \Pr(\hat{x}_{0k} | x_0, \{x_j^{(1)}\}_{j=1}^{L}, \boldsymbol{h}_{sk}).
\end{aligned} \tag{E.1}$$

By determining each likelihood function, we obtain the expression of (3.13).

F Proof for Theorem 2

From (3.16), we have $\boldsymbol{Y}^{OT} \sim \mathcal{CN}(\boldsymbol{\mu}^{OT}, \boldsymbol{\xi})$ and $\boldsymbol{\xi}$ is the covariance of the dependent signals of $y_{sd}^{(1)}$, $y_{1d}^{(2)}$, $\cdots$, $y_{Kd}^{(K+1)}$, derived as

$$
\begin{aligned}
\boldsymbol{\xi} &= \mathbb{E}\{(\boldsymbol{Y}^{OT} - \boldsymbol{\mu}^{OT})(\boldsymbol{Y}^{OT} - \boldsymbol{\mu}^{OT})^H\} \\
&= \begin{bmatrix}
\sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(1)}|^2 + \sigma_d^2, & \sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(1)} (x_j^{(2)})^*, & \cdots, & \sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(1)} (x_j^{(K+1)})^* \\
\sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(2)} (x_j^{(1)})^*, & \sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(2)}|^2 + \sigma_d^2, & \cdots, & \sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(2)} (x_j^{(K+1)})^* \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(K+1)} (x_j^{(1)})^*, & \sum_{j=1}^{L} E_j^I \Omega_{jD} x_j^{(K+1)} (x_j^{(2)})^*, \cdots, & & \sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(K+1)}|^2 + \sigma_d^2
\end{bmatrix}, \\
&= \sum_{j=1}^{L} E_j^I \Omega_{jD} \begin{bmatrix}
x_j^{(1)} (x_j^{(1)})^*, & x_j^{(1)} (x_j^{(2)})^*, & \cdots, & x_j^{(1)} (x_j^{(K+1)})^* \\
x_j^{(2)} (x_j^{(1)})^*, & x_j^{(2)} (x_j^{(2)})^*, & \cdots, & x_j^{(2)} (x_j^{(K+1)})^* \\
\vdots & \vdots & \ddots & \vdots \\
x_j^{(K+1)} (x_j^{(1)})^*, & x_j^{(K+1)} (x_j^{(2)})^*, & \cdots, & x_j^{(K+1)} (x_j^{(K+1)})^*
\end{bmatrix} + \mathrm{Diag}\left([\sigma_d^2, \sigma_d^2, \cdots, \sigma_d^2]\right), \\
&= \sum_{j=1}^{L} E_j^I \Omega_{jD} \begin{bmatrix} x_j^{(1)} \\ x_j^{(2)} \\ \vdots \\ x_j^{(K+1)} \end{bmatrix} \begin{bmatrix} (x_j^{(1)})^*, & (x_j^{(2)})^*, \cdots, & (x_j^{(K+1)})^* \end{bmatrix} + \mathrm{Diag}\left([\sigma_d^2, \sigma_d^2, \cdots, \sigma_d^2]\right).
\end{aligned} \tag{F.1}
$$

With $\boldsymbol{x}_j^{(t)}$ defined in (3.18), we can finally obtain the covariance, $\boldsymbol{\xi}$, in (3.17).

G Proof for Table I

In this appendix, we only present the diversity order analysis for the orthogonal transmission scheme. The diversity analysis for the simultaneous transmission is similar and is not presented due to the space constraint. Directly deriving D_{ML} is difficult because ML detectors are non-linear. Thus, our approach is to derive an upper and lower bound of the diversity gain. Specifically, we use the following bounds:

$$D_{C-MRC} \leq D_{ML} \leq D_{\max}, \tag{G.1}$$

where $D_{\max}$ is the maximum diversity gain that can be achieved for the system. Also, D_{C-MRC} is the diversity gain of the C-MRC detection scheme, which is known as a best suboptimum scheme [24].

For the two cases with and without second-hop CSI at the relays, we first determine $D_{\max}$. When the second-hop CSI is available at the relays, $D_{\max}$ is simply $1+\sum_{k=1}^{K} N_k$, which is the total number of physical (source-to-destination) channels in the systems. When the second-hop CSI is not available at the relays, substituting $\boldsymbol{w}_k = \mathbf{1}_k = [1, \cdots, 1]$ into (2.5), the received signal at the destination, which was relayed by the k-th relay, is given by:

$$y_{kd}^{(k+1)} = \sqrt{E_k^R}\bar{h}_{kd}\hat{x}_{0k} + \sum_{j=1}^{L} \sqrt{E_j^I} g_{jD} x_j^{(k+1)} + n_{kd},\ k = 1, \cdots, K, \tag{G.2}$$

where $\bar{h}_{kd} = \frac{1}{\sqrt{N_k}} \sum_{r=1}^{N_k} h_{kd,r}$. Since $\bar{h}_{kd} \sim \mathcal{CN}(0, \varpi_{kd})$ is Gaussian distributed, $y_{kd}^{(k+1)}, k = 1, \cdots, K$, of (G.2) represent K received signals through K relays where each relay equivalently has a single antenna. Including the direct channel of (2.2), therefore, the total number of equivalent channels that are independent is $K + 1$. Thus, when second-hop CSI is not available at the relays, $D_{\max}$ is $K + 1$.

We now derive D_{C-MRC} in the following. In the literature, the C-MRC scheme was developed only for single antenna relays for the interference-free case. Therefore, we first develop the C-MRC scheme for our system model: multiple-antenna relays in the presence of interference. To that end, the SINR for each hop must be determined. The SINR for the direct link is given by $\gamma_{sd} = \frac{E_0|h_{sd}|^2}{\sum_{j=1}^{L} E_j^I \Omega_{jD} |x_j^{(k+1)}|^2 + \sigma_d^2}$. Also, the SINR for the first-hop, $\gamma_{SINR,k}$, is given in (3.7), which has a chi-square distribution. The second-hop SINR is given as follows:

$$\gamma_{kd}^{OT} = \begin{cases} \frac{1}{\sum_{j=1}^{L} \frac{E_j^I \Omega_{jD}}{\sigma_d^2} |x_j^{(k+1)}|^2 + 1} \times \frac{E_k^R |\bar{h}_{kd}|^2}{\sigma_d^2}, & \text{if second-hop CSI is unknown at relays;} \\ \frac{1}{\sum_{j=1}^{L} \frac{E_j^I \Omega_{jD}}{\sigma_d^2} |x_j^{(k+1)}|^2 + 1} \times \frac{E_k^R \|\boldsymbol{h}_{kd}\|^2}{\sigma_d^2}, & \text{if second-hop CSI is known at relays.} \end{cases} \tag{G.3}$$

Note that, without the CSI at the relays, γ_{kd}^{OT} has an exponential distribution. With the CSI at the relays, however, γ_{kd}^{OT} has a chi-square distribution. Then, the two-hop source-to-relay and relay-to-destination channel has end-to-end bit error probability (BER) given by:

$$P_{eq}^b(\gamma_{SINR,k}, \gamma_{kd}^{OT}) = [1 - P^b(\gamma_{SINR,k})] P^b(\gamma_{kd}^{OT}) + [1 - P^b(\gamma_{kd}^{OT})] P^b(\gamma_{SINR,k}). \tag{G.4}$$

For each of the two-hop channels through relays, one can think of an equivalent one-hop channel whose SINR is given by:

$$\gamma_{eq,k}^{OT} := \frac{1}{\alpha} \{Q^{-1}[P_{eq}^b(\gamma_{SINR,k}, \gamma_{kd}^{OT})]\}^2, \ k = 1, \cdots, K, \tag{G.5}$$

where α is a constant depending on the constellation, and $P_{eq}^b(\gamma_{SINR,k}, \gamma_{kd}^{OT})$ is given in (G.4). By defining $\gamma_{\min}^{OT} := \min\{\gamma_{kd}^{OT}, \gamma_{SINR,k}\}$, it can be shown that the equivalent one-hop SINR can be bounded as follows [24]:

$$\gamma_{\min}^{OT} - \frac{3.24}{\alpha} < \gamma_{eq,k}^{OT} \leq \gamma_{\min}^{OT}. \tag{G.6}$$

Considering the interference, it can be shown that C-MRC detector is given as follows:

$$\hat{x}_{d,C-MRC}^{OT} = \arg\min_{x_0 \in S_0} \left| \beta_{sd}^{OT} y_{sd}^{(1)} + \sum_{k=1}^{K} \beta_{kd}^{OT} y_{kd}^{(k+1)} - \left(\beta_{sd}^{OT} \sqrt{E_0} h_{sd} + \sum_{k=1}^{K} \beta_{kd}^{OT} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \right) x_0 \right|^2, \tag{G.7}$$

where the weights β_{sd}^{OT} and β_{kd}^{OT} are given as

$$\beta_{sd}^{OT} = \frac{\sqrt{E_0} h_{sd}^*}{\sum_{j=1}^{L} E_j^I \Omega_{jD} + \sigma_d^2}, \tag{G.8}$$

$$\beta_{kd}^{OT} = \frac{\gamma_{eq,k}^{OT}}{\gamma_{kd}^{OT}} \frac{\sqrt{\frac{E_k^R}{N_k}} (\boldsymbol{w}_k \boldsymbol{h}_{kd})^*}{\sum_{j=1}^{L} E_j^I \Omega_{jD} + \sigma_d^2}. \tag{G.9}$$

Note that the developed C-MRC detector is a generalized one in the sense that multiple antennas at the relays and the presense of interference are considered. With the above detector, we first analyze their error performance for single-relay BPSK case. For the extensions to the general constellations and multiple relays, the approach in [24] will be used.

Let the C-MRC output defined as:

$$\begin{aligned} y_d^{C-MRC} &= \beta_{sd}^{OT} y_{sd}^{(1)} + \beta_{kd}^{OT} y_{kd}^{(k+1)} \\ &= \beta_{sd}^{OT} \sqrt{E_0} h_{sd} x_0 + \beta_{kd}^{OT} \sqrt{\frac{E_k^R}{N_k}} \boldsymbol{w}_k \boldsymbol{h}_{kd} \hat{x}_{0k} \\ &+ \beta_{sd}^{OT} (\sum_{j=1}^{L} \sqrt{E_j^I} g_{jD} x_j^{(1)} + n_{sd}) + \beta_{kd}^{OT} (\sum_{j=1}^{L} \sqrt{E_j^I} g_{jD} x_j^{(k+1)} + n_{kd}) \end{aligned} \tag{G.10}$$

where the interference and noise components $\beta_{sd}^{OT}(\sum_{j=1}^{L}\sqrt{E_j^I}g_{jD}x_j^{(1)}+n_{sd})+\beta_{kd}^{OT}(\sum_{j=1}^{L}\sqrt{E_j^I}g_{jD}x_j^{(k+1)}+n_{kd}) \sim \mathcal{CN}(0,N_d)$ and the variance is given as $N_d = (|\beta_{sd}^{OT}|^2+|\beta_{kd}^{OT}|^2)(\sum_{j=1}^{L}E_j^I\Omega_{jD}+\sigma_d^2)$.

Let us define $P^{b,OT}$ as the BER for the C-MRC detector for the orthogonal transmission, and we can obtain:

$$\begin{aligned}
P^{b,OT} &= [1-Q(\sqrt{2\gamma_{SINR,k}})]Q\left(\frac{\beta_{sd}^{OT}\sqrt{E_0}h_{sd}+\beta_{kd}^{OT}\sqrt{\frac{E_k^R}{N_k}}\boldsymbol{w}_k\boldsymbol{h}_{kd}}{\sqrt{\frac{N_d}{2}}}\right)\\
&\quad + Q(\sqrt{2\gamma_{SINR,k}})Q\left(\frac{\beta_{sd}^{OT}\sqrt{E_0}h_{sd}-\beta_{kd}^{OT}\sqrt{\frac{E_k^R}{N_k}}\boldsymbol{w}_k\boldsymbol{h}_{kd}}{\sqrt{\frac{N_d}{2}}}\right),\\
&= [1-Q(\sqrt{2\gamma_{SINR,k}})]Q\left(\frac{\sqrt{2}\left(\rho_d\gamma_{sd}+\gamma_{eq,k}^{OT}\right)}{\sqrt{\left(\rho_d\gamma_{sd}+\frac{(\gamma_{eq,k}^{OT})^2}{\gamma_{kd}^{OT}}\right)}}\right)\\
&\quad + Q(\sqrt{2\gamma_{SINR,k}})Q\left(\frac{\sqrt{2}\left(\rho_d\gamma_{sd}-\gamma_{eq,k}^{OT}\right)}{\sqrt{\left(\rho_d\gamma_{sd}+\frac{(\gamma_{eq,k}^{OT})^2}{\gamma_{kd}^{OT}}\right)}}\right),
\end{aligned} \tag{G.11}$$

where $\rho_d = 1+\sum_{j=1}^{L}\frac{E_j^I\Omega_{jD}}{\sigma_d^2}$. Using (G.6), after some mathematical manipulations, we can obtain an upper-bound for $P^{b,OT}$ as follows:

$$P^{b,OT} \le \begin{cases} P_1+P_2, & \text{if } \gamma_{eq,k}^{OT} \le \gamma_{\min}^{OT} < \gamma_{sd};\\ P_1, & \text{if } \gamma_{sd} \le \gamma_{\min}^{OT}, \end{cases} \tag{G.12}$$

where

$$P_1 = Q\left(\sqrt{2(\rho_d\gamma_{sd} + \gamma_{eq,k}^{OT})}\right);$$
$$P_2 = Q(\sqrt{2\gamma_{SINR,k}})Q\left(\frac{\sqrt{2}\left(\rho_d\gamma_{sd} - \gamma_{eq,k}^{OT}\right)}{\sqrt{\rho_d\gamma_{sd} + \gamma_{eq,k}^{OT}}}\right). \tag{G.13}$$

From (G.12), we will prove that $\mathbb{E}[P_1 + P_2]$ achieves full diversity, which means that the C-MRC detector achieves full diversity. Applying Chernoff bounding and taking expectations over the instantaneous SINRs, we obtain

$$\mathbb{E}[P_1] \leq \begin{cases} \frac{1}{2}e^{1.62}\frac{1}{1+\rho_d\mu_{sd}}\left(\frac{1}{N_k(1+\rho_k\mu_{sk})^{N_k}} + \frac{1}{1+\rho_d\mu_{kd}}\right) \approx (k_1\bar{\gamma})^{-2}, \\ \qquad\qquad \text{if CSI is unknown at the relays;} \\ \frac{1}{2}e^{1.62}\frac{1}{1+\rho_d\mu_{sd}}\left(\frac{1}{N_k(1+\rho_k\mu_{sk})^{N_k}} + \frac{1}{N_k(1+\rho_d\mu_{kd})^{N_k}}\right) \approx (k_2\bar{\gamma})^{-(N_k+1)}, \\ \qquad\qquad \text{if CSI is known at the relays,} \end{cases} \tag{G.14}$$

where $\rho_k = 1+\sum_{j=1}^{L}\frac{E_j^I\Omega_{jk}}{\sigma_k^2}$, $\mu_{sd} = \frac{E_0\vartheta_{sd}}{\sigma_d^2}$, $\mu_{sk} = \frac{E_0\vartheta_{sk}}{\sigma_k^2}$, and $\mu_{kd} = \frac{E_k^R\vartheta_{kd}}{\sigma_d^2}$. Also, k_1 and k_2 are the constants which depend on the variance of interference plus noise, and $\bar{\gamma} = \frac{E_0}{\sigma_d^2}$ denotes the average SNR. Similarly, we take expectations over the instantaneous SINRs to obtain the following:

$$\begin{aligned} E[P_2] &= \int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\infty}\int_{\gamma_3=0}^{\infty} P_2 f_{\rho_d\gamma_{sd}}(\gamma_1) f_{\gamma_{SINR,k}}(\gamma_2) f_{\gamma_{kd}^{OT}}(\gamma_3)d\gamma_1 d\gamma_2 d\gamma_3, \\ &\leq \mathcal{A} + \mathcal{B}, \end{aligned} \tag{G.15}$$

where

$$\mathcal{A} = \int_{\gamma_2=0}^{\infty}\int_{\gamma_3=0}^{\infty}\int_{\gamma_1=\gamma_{min}}^{\infty} \frac{e^{-\gamma_2}}{4}\exp\left(-\frac{(\gamma_1-\gamma_{min})^2}{\gamma_1+\gamma_{min}}\right) f_{\rho_d\gamma_{sd}}(\gamma_1) f_{\gamma_{SINR,k}}(\gamma_2) f_{\gamma_{kd}^{OT}}(\gamma_3) d\gamma_1 d\gamma_2 d\gamma_3,$$
$$\mathcal{B} = \int_{\gamma_2=0}^{\infty}\int_{\gamma_3=0}^{\infty}\int_{\gamma_1=0}^{\gamma_{min}} \frac{e^{-\gamma_2}}{2} f_{\rho_d\gamma_{sd}}(\gamma_1) f_{\gamma_{SINR,k}}(\gamma_2) f_{\gamma_{kd}^{OT}}(\gamma_3) d\gamma_1 d\gamma_2 d\gamma_3, \tag{G.16}$$

where $\gamma_{min} = \min(\gamma_2, \gamma_3)$. In the following, we derive an upper-bound of $\mathbb{E}[P_2]$ for the two CSI assumptions.

Orthogonal Transmission with Second-hop CSI at the Relays

Note that γ_{sd} has an exponential distribution, and both $\gamma_{SINR,k}$ and γ_{kd}^{OT} have chi-square distributions with degree of freedom N_k. Let us define $\lambda_{sd} = \rho_d\mu_{sd}, \lambda_{sk} = \rho_k\mu_{sk}$, and $\lambda_{kd} = \rho_d\mu_{kd}$. Then we can rewrite $\mathcal{A}$ as follows:

$$\mathcal{A} = \frac{1}{4}\frac{1}{\lambda_{sd}(\lambda_{sk})^{N_k}(\lambda_{kd})^{N_k}\Gamma^2(N_k)} \int_{\gamma_2=0}^{\infty}\int_{\gamma_3=0}^{\infty} \gamma_2^{N_k-1}\gamma_3^{N_k-1}$$
$$\times \exp\left(-\gamma_2(1+\frac{1}{\lambda_{sk}})\right)\exp(-\frac{\gamma_3}{\lambda_{kd}}) \times f_A(\gamma_{min}) d\gamma_1 d\gamma_2 d\gamma_3, \tag{G.17}$$

where we can find an upper bound for $f_A(\gamma_{min})$

$$f_A(\gamma_{min}) = \int_{\gamma_1=\gamma_{min}}^{\infty} \exp(-\frac{\gamma_1}{\lambda_{sd}}) \exp\left(-\frac{(\gamma_1-\gamma_{min})^2}{\gamma_1+\gamma_{min}}\right) d\gamma_1$$
$$\leq (1+\sqrt{\pi\gamma_{min}})\exp(-\frac{\gamma_{min}}{\lambda_{sd}}). \tag{G.18}$$

After some mathematical manipulations and integrations, we can obtain that $\mathbb{E}[P_2] \leq \mathcal{A} + \mathcal{B}$, where

$$\begin{aligned}\mathcal{A} \leq & \frac{1}{4\lambda_{sd}(\lambda_{sk})^{N_k}\Gamma^2(N_k)} \times \Bigg\{ \left(\frac{\lambda_{sd}}{\lambda_{sd}+\lambda_{kd}}\right)^{N_k} \frac{(\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}})^{N_k}\Gamma(2N_k)}{N_k(1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}})^{2N_k}} \\ & \times {}_2F_1\left(1, 2N_k, N_k+1, \frac{\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}}}{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}}}\right) \\ & + {}_2F_1\left(1, 2N_k, N_k+1, \frac{\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}}}{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}}}\right) \\ & + \frac{\pi}{2}\left(\frac{\lambda_{sd}}{\lambda_{sd}+\lambda_{kd}}\right)^{N_k} \frac{(\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}})^{N_k}\Gamma(2N_k+1)}{N_k(1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}})^{2N_k+1}} \\ & \times {}_2F_1\left(1, 2N_k+1, N_k+1, \frac{\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}}}{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}}}\right) \\ & + \frac{\pi}{2} \frac{\Gamma(2N_k+1)}{\lambda_{kd}^{N_k}(N_k+1)(1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}})^{2N_k+1}} \\ & \times {}_2F_1\left(1, 2N_k+1, N_k+2, \frac{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}}}{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}}}\right) \Bigg\}, \end{aligned} \tag{G.19}$$

$$\begin{aligned}\mathcal{B} = & \frac{1}{2\lambda_{sk}^{N_k}\Gamma^2(N_k)} \Bigg\{ \frac{\lambda_{sk}^{N_k}\Gamma^2(N_k)}{(1+\lambda_{sk})^{N_k}} - \left(\frac{\lambda_{sd}}{\lambda_{sd}+\lambda_{kd}}\right)^{N_k} \frac{(\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}})^{N_k}\Gamma(2N_k)}{N_k(1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}})^{2N_k}} \\ & \times {}_2F_1\left(1, 2N_k, N_k+1, \frac{\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}}}{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}}}\right) \\ & - \frac{\Gamma(2N_k)}{\lambda_{kd}^{N_k}(N_k)(1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}})^{2N_k}} \\ & \times {}_2F_1\left(1, 2N_k, N_k+1, \frac{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{kd}}}{1+\frac{1}{\lambda_{sd}}+\frac{1}{\lambda_{sk}}+\frac{1}{\lambda_{kd}}}\right) \Bigg\}, \end{aligned} \tag{G.20}$$

where $\Gamma(\cdot)$ is the Gamma function defined as $\Gamma(x) := \int_0^\infty s^{x-1}e^{-s}ds$. Also, ${}_2F_1(a,b;c;z)$ is the hyper-geometric function defined as ${}_2F_1(a,b;c;z) := \sum_{n=0}^{a} \frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}$. Thus, $\mathbb{E}[P_2] \leq \mathcal{A}+\mathcal{B} \approx (k_3\bar{\gamma})^{-(N_k+1)}$ and $P^{b,OT} = \mathbb{E}[P_1+P_2] \approx (k_4\bar{\gamma})^{-(N_k+1)}$ has full diversity order of $N_k + 1$.

We have proved that, with the second-hop CSI at the relays, $D_{C-MRC} = D_{\max} = N_k + 1$ in (G.1). Thus, we can conclude that $D_{C-MRC} = D_{ML} = D_{\max} = N_k + 1$ for the single-relay BPSK case. Note that the diversity analysis of D_{C-MRC} for BPSK readily generalizes to higher order constellations through the concepts of worst-case Euclidean distance among constellation points, as in [24]. Furthermore, the diversity analysis of D_{C-MRC} for a single-relay network readily generalizes to the case of multi-branch network due to the independence among different branches [24]. Overall, it can be shown that the ML detector achieves full diversity order of $1 + \sum_{k=1}^{K} N_k$ for K relays, regardless of the underlying constellations.

Orthogonal Transmission without Second-hop CSI at the Relays

Using the same approach above, we can determine the diversity analysis when the second-hop CSI is not available at the relays. Recall that in (G.14), we have shown that $\mathbb{E}[P_1]$ has a diversity order of 2 when single-relay BPSK is considered. Now, the $\gamma_{SINR,k}$ has a chi-square distribution with the degree of freedom N_k; and both γ_{sd} and γ_{kd}^{OT} have the exponential distributions. After some mathematical manipulations, we can obtain the following:

$$\mathbb{E}[P_2] \leq (k_5\bar{\gamma})^{-(N_k+1)}, \tag{G.21}$$

where k_5 is a constant. Therefore, we have $P^{b,OT} = \mathbb{E}[P_1 + P_2] \approx (k_2\bar{\gamma})^{-2} + (k_5\bar{\gamma})^{-(N_k+1)}$, which gives diversity order of 2. For the single-relay and BPSK case, using the result of $D_{\max} = K + 1 = 2$, we can conclude that $D_{\max} = D_{ML} = D_{C-MRC} = 2$. Finally, extension to any general constellations and K relays is done following the same approach as in [24]. Overall, for K relays with any general constellations, our ML detector without second-hop CSI at the relays achieves full diversity order of $K + 1$.