Obtaining a Quality Model for Manufacturing Systems and Establishing a Maintenance-Quality Link

by

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Abstract

This thesis describes the application of the stochastic-flow-modeling (SFM) approach to represent the quality behavior of a manufacturing system. Initially, a simple, one-product type SFM is discussed and then a more complex multiple-product manufacturing system is developed. This quality SFM-based model has aggregation by station, product, and operational shift. Subsequently, potential supervisory control architectures that could be used in conjunction with this quality-based SFM are discussed and developed. Distribution parameter fitting is explored using static and adaptive approaches and a comparison between these two approaches is given. Then, the accuracy of the SFM modeling technique is demonstrated using two simulation examples.

Effective equipment maintenance is essential for a manufacturing plant seeking to produce high quality products. The impact of equipment reliability and quality on throughput have been well established, but the relationship between maintenance and quality is not always clear nor direct. Therefore, after developing a SFM to represent the quality of a manufacturing system, the focus of this work shifts towards identifying correlations between maintenance and quality. This thesis describes a statistical modeling method that makes use of a Kalman filter to identify correlations between independent sets of maintenance and quality data. With such a method,
maintenance efforts can be better prioritized to satisfy both production and quality requirements. In addition, this method is used to compare results from the theoretical maintenance-quality model to data from an actual manufacturing system. Results of the analysis indicate the potential for this method to be applied to preventive, as well as reactive maintenance decisions, since ageing aspects of equipment are also considered in the model.
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List of Acronyms

CFM  Continuous Flow Model

CI   Confidence Interval

CTI  Continuous/Throughput Improve Maintenance

DC   Down Check Maintenance

DES  Discrete Event Systems

EMR  Emergency Maintenance/Repairs

FSM  Finite State Machine

FTQ  First Time Quality

IMC  Imperfect Maintenance Concept

MQ   Maintenance Quality

PFB  Preventive - Frequency Based Maintenance

RC   Running Check Maintenance

RME  Relative Mean Error
**SFM**  Stochastic Flow Model

**sFTQ**  Sliding First Time Quality

**SQI**  Station Quality Index

**SRP**  Scheduled Repair Maintenance

**sSQI**  Sliding Station Quality Index

**TPM**  Total Productive Maintenance
Chapter 1

Introduction

1.1 Motivation

Most manufacturing firms are large, complex systems characterized by several decision-making subsystems such as finance, personnel, marketing, and operations. Such firms may have a number of plants and warehouses that produce a large number of different products with a wide variety of machines and equipment. Moreover, these manufacturing systems are subject to discrete events such as construction of new facilities, purchase of new equipment and disposal of old, machine setups, failures and repairs, and the introduction of new products. These events may be deterministic or stochastic. Therefore, management must recognize and react to these events. Trying to model a manufacturing system while taking into account all of these modeling parameters is not an easy job. Due to the large size of these systems and the presence of these events, obtaining exact optimal policies to run these systems is nearly impossible both theoretically and computationally [41].

Furthermore, effective equipment maintenance is essential for a manufacturing
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A plant seeking to produce high quality products. The impact of equipment reliability and quality on throughput have been well established, however, the relationship between maintenance and quality is not always clear nor direct. Frequently, in manufacturing plants when product quality does not meet requirements, manufacturing engineers first check the product for any defects due to the process. In most cases, engineers are able to find the defective component of the product and replace it. However, sometimes poor quality is not simply due to defective components; rather, it is due to defects in the processing station. In this sense, ageing of equipment plays a crucial role in decreasing the quality of the overall product. With time, mechanical parts in an assembly line, such as conveyor belts and gears, become worn and therefore, an increased number of products are rejected due to lower product quality.

Ensuring that proper maintenance is done on plant equipment can help achieve target quality levels. Regular maintenance also helps to detect problems at an early stage and hence reduces the cost of repairs [15, 29, 36]. Effective maintenance includes monitoring machinery as well as trying to measure deterioration rates in order to predict failures. According to George Herbert’s 1640 poem [34], “the loss of the horseshoe nail caused the loss of the horseshoe, the horse, the rider, the battle, and eventually the kingdom.” It would have been worthwhile, then, to have spent as much as the kingdom is worth to maintain the nail in its proper place. Therefore, performing regular maintenance may be seen as an extra expense in the short-term, but in the long-term one can see the benefits of doing so [14, 34].

There are very few theoretical notions that describe the essential role of maintenance in manufacturing. However, empirical studies have indicated that there are
four main competitive factors of manufacturing: quality, delivery performance, flexibility and cost [25]. It can clearly be seen that quality is a factor that affects a manufacturer’s performance and therefore, it is an operating aspect that should be taken seriously by manufacturers.

1.2 Literature Review

1.2.1 Hybrid Modeling of Manufacturing Systems

In general, manufacturing system operations are modeled as discrete event systems (DES). This class of system representations is widely recognized in queuing systems [12]. This work complements such a DES model formulation by incorporating the real-time behavior of manufacturing systems. One such approach is the use of so-called timed-DES models [9]. A timed-DES model is a DES in which the occurrence of each event is marked by a time variable counter. Although the timed-DES formulation is easy to set up for simple systems, the complex, multi-scale nature of large manufacturing systems limits its applicability. In previous research [1, 28], it was found that the addition of continuous-time dynamics to DES models using a hybrid modeling approach provides greater flexibility in modeling. In this approach, each event can be characterized by specific dynamical properties, such as process delay and process dynamics. This formulation can be used to introduce real-time information such as product demand changes, personnel changes, and so on, along with all the dynamically relevant variables that may influence real-time decision making processes. Such models, which belong to a class of hybrid systems, allow the generic application of existing modeling platforms. Using this modeling approach, a realistic
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A manufacturing system was modeled and simulated [1].

One way to address the dynamical nature of manufacturing processes is to consider a hybrid systems model [11, 39, 40, 47]. Hybrid control systems are those that involve continuous-time dynamics and discrete-event behavior and require controllers that may also have mixed continuous and discrete dynamics [2, 10]. Such systems arise in engineering applications where time-varying processes (e.g., chemical processes) are interconnected with software and/or hardware (e.g., a switching mechanism that opens and closes valves). Models for hybrid systems are particularly useful for industrial problems that involve both logical-time constraints (where specifications may involve requirements on the relative ordering of events) as well as real-time constraints (e.g., safety constraints in plant start-up and shut-down). Hybrid systems models may also arise when systems integration issues are considered. In a number of applications, such as tar sand mining operations, process information arising from both discrete and continuous systems must be processed to provide effective control and/or optimization of system operations.

Recently, a class of models called continuous flow models (CFM) [30] have been proposed for the analysis of manufacturing systems. For each work station, the continuous flow models monitor the ability of a station to do work, its capacity and its product outflow. They provide an elegant way to develop event-based continuous-time models that are suitable for a large class of manufacturing systems. Successful applications of the CFM paradigm have led to new approaches for throughput optimization in manufacturing systems [38, 42], real-time scheduling [37] and optimal resource allocation [46], among others. Continuous flow models provide the modeling capabilities of the finite state machines (FSM) representation of DES while circumventing the
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discrete nature of DES models. This attribute is important when one considers the use of process models in optimization. The DES formulation requires intensive search algorithms to find suboptimal solutions. As highlighted in the literature, it is relatively easy to use CFM in the development of optimal algorithms. Stochastic flow models (SFM) have also been proposed to take into account the stochastic nature of process operations [13, 51, 52]. Recent developments have demonstrated that CFM can yield control policies that can be implemented in real-time [52]. In contrast to DES and CFM, the theory of hybrid systems remains an emerging area. Although it provides a much more flexible and general approach to modeling the dynamics of manufacturing systems, the current theory can only deal with relatively simple applications that are generally treated as benchmark applications. More research is required to develop a theory of hybrid systems that is widely applicable to complex manufacturing systems.

1.2.2 Maintenance Terminology

Due to increased competition in global operations, many manufacturers have made significant changes to the way they operate. These changes have lead to changes in other areas of the business such as maintenance, which has been recognized by many manufacturers as a key factor for enhanced performance and increased profitability [15, 26, 29, 36].

In general there are three types of maintenance: proactive maintenance, reactive maintenance and aggressive maintenance. Proactive maintenance can further be broken down to two types: preventive maintenance and predictive maintenance. Preventive maintenance is a periodic maintenance done on plant equipment after a specific
period of time or amount of machine use to ensure that normal operating conditions are being satisfied [21, 23]. This type of maintenance uses estimated probabilities to predict when a failure will occur. Some examples of preventive maintenance include lubrication of plant equipment, fine tuning, adjustments, equipment inspection and replacement of parts. Therefore, preventive maintenance helps to detect problems at an early stage, reduces machine breakdowns and extends the life of plant equipment. However, it increases downtime of plant equipment to allow for inspections to take place [43].

Predictive maintenance is a condition-based type of maintenance that is performed in response to detecting certain conditions in plant equipment [21, 45]. This type of maintenance uses diagnostic equipment to determine the physical condition of plant equipment such as temperature, noise, lubrication and corrosion [17]. Therefore, when one of these physical conditions goes beyond a certain threshold, predictive maintenance is performed to restore it to its original condition or at least as close as possible to its original condition. Thus, plant equipment is only serviced when it is defective [43].

Although preventive and predictive maintenance may appear to be similar, they differ significantly in the way they are performed. Preventive maintenance is usually done after a certain period of time, determined from historical data. As for predictive maintenance, it is done in response to an abnormal reading on a monitoring device [35]. Both types of maintenance contribute to reduce equipment breakdown and increase plant equipment life. Furthermore, preventive maintenance is normally performed periodically, in contrast to predictive maintenance [23, 33, 43].
Reactive maintenance is usually performed in response to a failure of plant equipment. During reactive maintenance, temporary repairs/adjustments may be done to minimize equipment downtime, while permanent repairs are postponed [20]. According to [45], maintenance manpower and the resources expended to keep the manufacturing plant running are minimized with reactive maintenance. Unfortunately, one cannot overlook the fact that reactive maintenance leads to unpredictable and fluctuating production capacity, as well as, an increase in the overall maintenance costs to repair tremendous failures [4, 20].

Finally, aggressive maintenance strategies, such as Total Productive Maintenance (TPM) do not only try to avoid machine failures, but also aim to enhance overall equipment operation. TPM was developed by Japanese manufacturing plants to improve various manufacturing aspects including product quality. It focuses on eliminating the ”six major losses” listed below [43]:

- equipment failure
- set-up and adjustment time
- idling and minor stoppages
- reduced speed
- defects in process
- reduced yield.
1.2.3 The Maintenance-Quality Link

The relationship between maintenance and quality has been an active topic of debate for some time. Some benchmarking studies have been published in [18] and [50] which indicated that there was no direct relationship between maintenance and quality enhancement. In general, however, it is currently well-established in the literature that performing maintenance on plant equipment has a significant effect on product quality. In [27], the author states that equipment reliability and maintenance have a significant effect on quality. Other studies such as [22, 24, 32, 49], indicate that implementing various maintenance strategies such as TPM improved manufacturers’ competitiveness and led to better overall results.

Table 1.1: Results of regression analysis of maintenance strategies on maintenance performance\textsuperscript{a} [43].

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Coefficient of Improvement of Product Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggressive Maintenance</td>
<td>$0.253^{d}$</td>
</tr>
<tr>
<td></td>
<td>(0.066)</td>
</tr>
<tr>
<td>Proactive Maintenance</td>
<td>$0.194^{c}$</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
</tr>
<tr>
<td>Reactive Maintenance</td>
<td>$-0.112^{b}$</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Standard errors are in parentheses, \textsuperscript{b}$p < 0.10$, \textsuperscript{c}$p < 0.01$, and \textsuperscript{d}$p < 0.001$.

In [43], the author attempts to prove that there is a relationship between maintenance and quality. The author does this by surveying several plants on their maintenance management policies. The maintenance manager and production manager at each plant were sent a survey and asked to report on the operating characteristics of their plants. Out of the 708 surveys sent to 354 plants, 125 plant managers and 162 maintenance managers responded. Performance was measured by asking the respondents to indicate the effect of maintenance on improvements in product quality
using a five-point Likert-type scale (1 = less than 20% of quality improvement was the result of maintenance efforts, 5 = more than 80% of quality improvement was due to maintenance).

Table 1.1 reports the results of regression with contribution to improvement in product quality as the dependent variable with regards to the different types of maintenance performed. In this table, for each type of maintenance, the coefficient of the dependent variable is given in the second column with its standard error given in brackets underneath. For aggressive maintenance, the results obtained show that the coefficient of improvement of product quality is a positive one, which means that aggressive maintenance has a positive relationship with product quality. Similarly, proactive maintenance has a significant, positive relationship with product quality, since its coefficient is also a positive one. On the other hand, for reactive maintenance, the coefficient of improvement of product quality is a slightly negative one, which indicates that reactive maintenance has a less significant, negative relationship with product quality. According to [43], these results are in agreement with published results. Whether a given type of maintenance has a positive impact, or not, it would appear that a relationship between maintenance and quality exists.

Additionally, the p-value is shown for each coefficient reported in Tab. 1.1. This value tells you the probability of the reported coefficient coming up in a random distribution. For example, there is only a 0.1% chance that the result obtained for the coefficient of improvement of product quality for aggressive maintenance came up in random distribution, i.e. one can say with a 99.9% probability of being correct that the independent variable (aggressive maintenance) is having some effect, assuming the model is specified correctly.
Furthermore, the authors state in [8] that a relationship between maintenance and quality does exist, but more adequate models are yet to be developed. Manufacturing plants are often successful in retrieving data about plant maintenance and product quality. Attempts to relate maintenance data to quality data through a mathematical model have failed. According to [8], there are three possible reasons for this:

- Only recently have manufacturers realized the importance of maintenance and its link to profitability.

- The relationship between maintenance and other functions in an organization is a complex one.

- Outputs of the maintenance department are difficult to define and therefore, linking input and output is even more difficult.

In [8], the authors manage to reveal the relationship between maintenance and quality and underline its importance. They propose a general framework by which the link between maintenance and quality can be obtained and incorporated into production. Figure 1.1 shows how maintenance, quality and production interact. Production has two outputs; a primary output and a secondary output, which is maintenance. From this figure, one can see that production affects quality and equipment conditions are affected by maintenance which, in turn, affects the quality of the overall product. The authors of [8] proposed the following ideas to relate maintenance and quality.
Models using Imperfect Maintenance Concepts

Preventive maintenance is a diagnostic type of maintenance performed to keep plant equipment in nearly perfect condition. Most existing preventive maintenance models assume that the condition of plant equipment goes back to its original condition when preventive maintenance is performed. However, this is not true. In reality, equipment condition deteriorates with time and does not return to its original condition no matter how good the maintenance program is. Therefore in [8], the authors propose another dimension to existing preventive maintenance models by adding the imperfect maintenance concept (IMC). IMC basically takes into account the deterioration of plant equipment conditions with respect to time, which in turn affects the scheduling of quality control inspections. Using this modeling idea, the authors of [5, 6, 7] are able to determine a relationship between preventive maintenance and quality-related costs.
Models using the Taguchi Approach

According to [8], Taguchi defines quality as “the loss incurred owing to deviation of product characteristics from their target values.” This deviation is measured using a quadratic loss function. The Taguchi approach performs preventive maintenance as soon as the deviation goes below a certain threshold, thereby reducing the divergence away from the target levels and improving quality [8, 44].

Previous research has confirmed that a relationship between maintenance and quality does exist, but unfortunately, few researchers have tried to find a mathematical formulation for it. The authors of [8] were able to determine how maintenance scheduling affects quality, but a direct maintenance-quality mathematical formulation was not obtained. Hence, more research is needed in this area of manufacturing systems’ quality and the effect of maintenance. In this thesis, an alternative way to identify correlations between independent maintenance and quality data sets in a manufacturing environment is proposed, thus, attempting to fill the gap that exists in this field of research.

1.3 Outline of Thesis

In this thesis, we exploit the flexibility of hybrid system models to develop a comprehensive understanding of manufacturing process quality. In addition, the design of a supervisory control architecture will be considered using modeling approaches such as SFM and CFM based representations that are tailored to the analysis of manufacturing systems. One of the objectives of this research is to study the dynamics of manufacturing systems using a SFM-based hybrid system modeling paradigm. In this formulation, key manufacturing events such as blockage, breakdown, working,
repair, etc... trigger changes in the dynamics of key states that monitor product flow, resource allocation and maintenance applications. In contrast to previous work where the dynamics of these states was primarily modeled using delays, the current project focuses on the use of SFM and CFM to model the state dynamics that indicate the quality level of the manufacturing system.

This modeling approach, which has been recently investigated [3] in the context of manufacturing systems, provides an effective platform to incorporate other common objectives in addition to quality such as throughput optimization and resource allocation subject to stochastic fluctuations. Furthermore, this work considers the validation of dynamical models of manufacturing systems using CFM/SFM-based hybrid models that are compared to quality data from a real operational manufacturing line.

Once a validated model is obtained, the research focuses on maintenance and its effects on quality. More specifically, we propose a mathematical formulation that can identify correlations between independent sets of maintenance and manufacturing quality data.

In chapter 4, some quality definitions are introduced. Quality is defined as the ability of a station to process parts while minimizing the number of parts that get rejected. Two methods for calculating production quality are proposed and their effects on maintenance are examined. The first method is based on the calculation of a production station’s quality over a certain time interval. This is then followed by an example using real manufacturing data to demonstrate how a station’s quality is affected by maintenance. A Kalman filter is used to estimate production quality to provide an alternative production quality calculation method and the results are then discussed.
In chapter 6, a novel quality definition that incorporates maintenance detection into the quality calculation is introduced and its results are discussed. This is followed by a brief analysis and discussion of modeling approaches for plant ageing and maintenance. A summary of the conclusions and future work is presented in chapter 8.
Chapter 2

Manufacturing Line Flow and Quality Models

2.1 System Model

2.1.1 Single-Product Type SFM Model

In this work, the development of a dynamic model of manufacturing system quality is based on a stochastic flow model (SFM) of each station for an assembly line of automotive engines. The SFM uses information about the probability of the occurrences of reject codes for each engine model type and each production shift to compute the number of rejects on the line for any given period of time. Figure 2.1 shows a single-station discrete SFM, where $u(t)$ (parts/sec) is the rate of parts entering the station while $v(t)$ (parts/sec) is the output rate which is equal to the production rate $\rho(t)$ at which the station is able to produce parts. The buffer size is given by $x_b(t)$ (parts).
CHAPTER 2. MANUFACTURING LINE FLOW AND QUALITY MODELS

Note that the station switches between two states, 0 (OFF) or 1 (ON), such that:

\[
\begin{aligned}
    s(t) = \begin{cases} 
        1 & \text{machine working} \\
        0 & \text{otherwise}
    \end{cases}
\end{aligned}
\]  \hspace{1cm} (2.1)

where \( s(t) \) denotes the operational state of the system.

The working and non-working states are triggered by random events that cause the station to change its current state. More elaborate logical statements may also be considered in practice.

The rate of parts entering the station, \( u(t) \), is determined by production planning and scheduling. The rate at which the station produces parts, \( v(t) \), is a more complex function of processing conditions. A simple rule-based operational constraint for the output rate:

\[
\begin{aligned}
    v(t) = \begin{cases} 
        0 & \text{if } s(t) = 0 \text{ or } x_b(t) = 0 \\
        \rho(t) & \text{otherwise}
    \end{cases}
\end{aligned}
\]  \hspace{1cm} (2.2)

where the buffer content \( x_b(t) \) is determined by the following ordinary differential
equation:

\[ \dot{x}_b(t) = u(t) - v(t) \quad (2.3) \]

At each station, the total number of rejects is calculated using the following ordinary differential equation:

\[ \dot{x}_r(t) = \Delta(t)\delta(t) \quad (2.4) \]

where \(x_r\) is the number of rejects (with initial conditions \(x_r(0) = 0\)), \(\delta(t)\) is a unit impulse.

Before defining the function \(\Delta(t)\), some additional definitions must be introduced. First, a parameter, called \(\lambda\), is introduced to represent the average rate of engines produced between the occurrence of two rejects. The estimation of this parameter is obtained using historical data with an algorithm presented in section 2.2. As shown in Figure 2.2, historical data fits an exponential distribution \(E(\lambda)\).

Following the definition of the reject occurrence distribution, a threshold \(th\) must be introduced to isolate the reject probabilities from the non-reject probabilities. The exponential distribution \(E(\lambda)\) has a probability density function \(f_{E(\lambda)}(t) = \lambda e^{-\lambda t}\).

From the definition of the density function, the integral of \(f_{E(\lambda)}(t)\) from the limits 0 to \(\infty\) is equal to 1 (100 %), but there is also \(\frac{100}{\lambda}\) % that corresponds to the reject rate and thus \((100 - \frac{100}{\lambda})\) % for the non-reject rate. The boundary between these two modes (reject and non-reject) is defined by the threshold \(th\). In other words, this threshold, deduced from the exponential distribution \(E(\lambda)\), is defined by \(\int_0^{th} f_{E(\lambda)}(t)dt = \frac{1}{\lambda}\), where the solution of this equation yields an expression for \(th\):

\[ th = -\frac{1}{\lambda} \ln \left( \frac{\lambda - 1}{\lambda} \right) \quad (2.5) \]
From the exponential distribution $E(\lambda)$, a random sample, called $t_\lambda$, is obtained for each engine exiting the work station. This sample is compared to the reject threshold $th$ defined above. If the sample $t_\lambda$ is under the threshold when the engine exits the station, then a reject flag is given to the engine. Finally, the function $\Delta(t)$ which generates rejects on the station is defined by:

$$
\Delta(t) = \begin{cases} 
1 & \text{if } v(t) \neq 0 \text{ and } t_\lambda \leq th \\
0 & \text{otherwise, i.e.: no reject}
\end{cases}
$$

(2.6)

This type of model, described above, can be used to provide high-level abstractions of discrete event systems. Rejects are generated by impulse functions using the value
of the sample $t_\lambda$ compared to the threshold. The same abstraction is done with the output flow modeling $v(t)$.

### 2.1.2 Five Station Manufacturing System

As the manufacturing system grows in size, it becomes more complex and modifications to the model are required. This section examines an example represented by a slightly more complex five-station system that is still simple enough to describe, yet represents a realistic production line configuration (Figure 2.3), where the modeling approach follows that of the previous section. Each station is indexed by $i$ with $i \in \{1, 2, ..., 5\}$ and the relation $v_i = u_{i+1}$ is used to link stations.

![Figure 2.3: Simple five-station SFM quality system](image)

However, this five-station model introduces a further complication with the introduction of a parallel station which measures quality (station $S_4$ in Figure 2.3). Thus, in Figure 2.3, stations $S_1$ and $S_5$ are defined as *processing stations*, stations $S_2$ and $S_3$ are *control stations* and finally, station $S_4$ is a *quality station* $Q_1$. Products can move from one station to the next (for example, from $S_1$ to $S_2$). But, in some instances, as at the exit of station $S_3$, two paths are possible. From $S_3$, engines can be directed...
to $S_4$ or station $S_5$. If the product has not triggered a quality reject before station $S_3$, this product goes to station $S_5$. If a reject has occurred, the product must be repaired, and is therefore sent to station $S_4$. In station $S_4$, the problem identified with the engine at station $S_3$ is repaired. The product then returns to station $S_2$ to be processed and diagnosed a second time. To complete the model, new definitions are required to generalize Equations (2.1) to (2.5).

A set $L_i$ is defined as the set of all stations linked with the input of station $i$, for example in Figure 2.3, $L_2 = \{S_1, S_4\}$. The input $u_i$ of the station $i$ can be defined as:

$$u_i(t) = \sum_{l \in L_i} b_{l,i}(t)v_l(t)$$

where $b_{l,i}$ is a Boolean function (with the property $\sum_i b_{l,i} = 1$) that determines in which line the product is produced from station $l$ to station $i$. In Figure 2.3, two choices are possible after station $S_3$. For example, if the product must be repaired then $b_{3,4}(t) = 1$ and $b_{3,5}(t) = 0$.

### 2.1.3 Complete Manufacturing System Model

The previous section provided definitions for the modeling of all controlling stations and associated variables in the system. However, the product type and the production shift were not taken into account, and therefore, further variables need to be defined to represent the dynamics of the system before defining an entire manufacturing line. The above five-station model is now extended to a generalized model of a complete manufacturing system.

A product is defined by a unique number $p$ (such as a unique serial number). This product identification number belongs to a product type $j$ defined by $j = J(p)$ where $J$ is a function that gives the engine model type $j$ of an engine that has a
unique product number $p$. A production shift is defined by a number $k$ and defined by $k = \mathcal{K}(t)$ where $\mathcal{K}$ is a function of time.

In this section, the model of the entire manufacturing line is developed for all possible configurations $(i, j, k)$, meaning for each station $i \in I = \{\text{stations in the manufacturing line having rejects}\}$, for each product type $j \in J = \{\text{product type}\}$ and each shift $k \in K = \{\text{production shift}\}$.

The same functions, as defined previously in Equations (2.1) to (2.5) can be defined for all configurations $(i, j, k) \in I \times J \times K$. Functions $s_{i,j,k}(t)$, $v_{i,j,k}(t)$, etc... are also defined. In this work, the SFM dynamics of the entire manufacturing line is defined as a hybrid stochastic system for each specific station $i \in I$, and the equations of the operating state $s_i$, the output rate $v_i$ and the derivative of the buffer content are given by:

\[
s_i(t) = s_{i,j,k}(t) = \begin{cases} 
1 & \text{if a product is made in station } i \\
0 & \text{otherwise} 
\end{cases} 
\tag{2.8}
\]

\[
v_i(t) = v_{i,j,k}(t) = \begin{cases} 
0 & \text{if } s_i(t) = 0 \text{ or } x_{b_i}(t) = 0 \\
\rho_{i,j,k}(t) & \text{otherwise} 
\end{cases} 
\tag{2.9}
\]

\[
\dot{x}_{b_i}(t) = u_i(t) - v_i(t) 
\tag{2.10}
\]

where the input $u_i$ is defined by Equation (2.7).
Before defining the ordinary differential equation of \( x_r(t) \), the reject function must be defined for each product as:

\[
\dot{R}_p(t) = \begin{cases} 
1 & \text{if } v_{i,j,k}(t) \neq 0 \text{ and } t_{\lambda_{i,j,k}} \leq t_{h_{i,j,k}} \\
0 & \text{otherwise}
\end{cases}
\]  

(2.11)

where the variable \( t_{\lambda_{i,j,k}} \) is a random variable with an exponential distribution \( \mathcal{E}(\lambda_{i,j,k}) \) having a parameter \( \lambda_{i,j,k} \) defined for station \( i \), product type \( j \), shift \( k \). The variable \( t_{h_{i,j,k}} \) is a threshold extracted from \( \mathcal{E}(\lambda_{i,j,k}) \) as explained in Equation (2.5).

The function of the total number of rejects at station \( i \) \( x_r(t) \) is given by:

\[
\dot{x}_r(t) = \dot{R}_p(t)\delta(t)
\]  

(2.12)

where \( x_r(t) \) is the number of rejects at station \( i \) (with initial conditions \( x_r(0) = 0 \)).

When product \( p \) exits the last station of the manufacturing line, a reject index function gives:

\[
\dot{X}_r(t) = \begin{cases} 
1 & \text{if } R_p(t) \neq 0, \\
0 & \text{otherwise.}
\end{cases}
\]  

(2.13)

2.2 Distribution Fitting

In the previous sections, \( \lambda_{i,j,k} \) parameters were simply stated without description of how these values may be obtained. In this section, the estimation of these parameters is explained.

2.2.1 Static approach

Assumption 2.2.1. At any given time \( t \), the set of all rejects and the corresponding production scheduling data provide a set \( R_{N_{i,j,k}} = \{ r_{i,j,k}^1, r_{i,j,k}^2, \ldots, r_{i,j,k}^{N_{i,j,k}} \} \) of \( N_{i,j,k} \)
An estimation of the average rate of rejects among all products can be obtained by maximizing the likelihood function. The likelihood function is the joint probability density function after substitution of the available observations:

\[
L(\lambda_{i,j,k}|R_{N_{i,j,k}}) = \prod_{m=1}^{N_{i,j,k}} \lambda_{i,j,k} e^{-\lambda_{i,j,k} r_{i,j,k}^{m}}.
\]  

(2.14)

Since the logarithm function is monotonic, to maximize the likelihood function or the logarithmic likelihood function is identical. The logarithmic likelihood function is given by:

\[
\Lambda(\lambda_{i,j,k}|R_{N_{i,j,k}}) = \ln(L(\lambda_{i,j,k}|R_{N_{i,j,k}})) = N_{i,j,k} \ln(\lambda_{i,j,k}) - \lambda_{i,j,k} \sum_{m=1}^{N_{i,j,k}} r_{i,j,k}^{m}.
\]  

(2.15)

To find an estimation \(\hat{\lambda}_{i,j,k}\) that maximizes the function \(\Lambda\), a necessary condition is that the gradient of \(\Lambda\) with respect to \(\lambda_{i,j,k}\) becomes zero, \(\frac{\partial \Lambda}{\partial \lambda_{i,j,k}} = 0\). The derivative of the previous equation with respect to \(\lambda_{i,j,k}\) yields:

\[
\frac{\partial \Lambda(\lambda_{i,j,k}|R_{N_{i,j,k}})}{\partial \lambda_{i,j,k}} = \frac{N_{i,j,k}}{\lambda_{i,j,k}} - \sum_{m=1}^{N_{i,j,k}} r_{i,j,k}^{m}.
\]  

(2.16)

The expected value is given by \(\mathbb{E}_{N_{i,j,k}}[r_{i,j,k}^{m}] = \frac{1}{N_{i,j,k}} \sum_{m=1}^{N_{i,j,k}} r_{i,j,k}^{m}\), so the estimation value is:

\[
\hat{\lambda}_{i,j,k} = \frac{1}{\mathbb{E}_{N_{i,j,k}}[r_{i,j,k}^{m}]}
\]  

(2.17)

In Equation (2.11), the variable \(t_{\lambda_{i,j,k}}\) is a random variable that belongs to the \(\lambda_{i,j,k}\) exponential distribution: \(t_{\lambda_{i,j,k}} \sim \mathcal{E}(\lambda_{i,j,k})\).
2.2.2 Adaptive Approach

An estimation that takes into account the most recent reject data cannot be performed adequately with the mean estimation scheme described above. For this reason, the mean estimation has a strong filtering behavior as the size of the sample data increases (see dashed plots on Figures 2.4 and 2.5). Filtering conceals non-stationary events or changes in the mean value that may be important in the assessment of a control strategy and the analysis of process dynamics. In this section, an adaptive estimation algorithm is described to avoid this filtering problem, which can track non-stationary events in rejected data measurements $r_{i,j,k}$ with an adaptation of the filtering smoothness. For more explanation about the proposed algorithm, readers can refer to [16].

Obviously, when the entire manufacturing line is taken into account, the parameter $\lambda$ depends on station $i$, engine $j$ and shift $k$. Formally, we denote the dependence of $\lambda$ on $i$, $j$ and $k$ as $\lambda(i,j,k) = \lambda_{i,j,k}$. The same notation is used for $r(i,j,k) = r_{i,j,k}$.

First, we introduce a constant $M \in \mathbb{N}^+$ which corresponds to the length of the data window ($M$ is the memory length given to the estimation, the greater $M$ is, the smoother the estimations are). Next, define $\bar{\gamma} \in (0,1]$ as the upper bound of step estimations $\gamma_n^*$ where $\bar{\gamma}$ is the greatest step that is possible to give to $\gamma_n^*$ estimations. We also define $\Delta \in [0, +\infty)$ as the acceptable error range for the iterates $\lambda_n$, $\alpha \in (0, 1]$ as the error probability for the standard normal distribution and $z_{1-\alpha}$ as the associated percentile. Then, $\mu \in (0, 1]$ is introduced as the $\lambda_n$ estimation weight. To begin the adaptive approach, we first set $n = 1$, and the initial condition of new observation gain $\gamma_1 \in (0, \bar{\gamma}]$. The approach is then described by the following six steps:
1. Generate the normalized iteration value

\[ \tilde{\lambda}_n(i, j, k) = \frac{\lambda_n(i, j, k)}{\sqrt{\gamma_n(i, j, k)}}. \]  

(2.18)

2. If the number of iterations \( n \) is an integer multiple of \( M \), perform steps 3 and 4, otherwise, set \( \gamma_{n+1}(i, j, k) = \gamma_n(i, j, k) \) and proceed directly to step 5.

3. Calculate the normalized sample mean

\[ \tilde{\lambda}^{\text{mean}}_n(i, j, k) = \frac{1}{M} \sum_{m=n-M+1}^{n} \tilde{\lambda}_m(i, j, k) \]  

(2.19)

and normalized sample variance

\[ \tilde{\kappa}_n = \frac{1}{M-1} \sum_{m=n-M+1}^{n} \left( \tilde{\lambda}_m(i, j, k) - \tilde{\lambda}^{\text{mean}}_n(i, j, k) \right)^2. \]  

(2.20)

4. Calculate the dynamic part of the step value at iteration \( n \)

\[ \hat{\gamma}^*_n = \min \left( \bar{\gamma}, \frac{\Delta^2}{z_{1-\alpha}^2 \tilde{\kappa}_n} \right), \]  

(2.21)

and update:

\[ \gamma_{n+1}(i, j, k) = (1 - \mu)\gamma_n(i, j, k) + \mu \hat{\gamma}^*_n. \]  

(2.22)

where \( \mu \) denotes the weight given to the \( \gamma_{n+1}(i, j, k) \) estimation, the closer \( \mu \) is to 0, the smoother the estimations are, conversely, the closer \( \mu \) is to 1, the greater the amount of importance given to \( (\hat{\gamma}^*_n) \) is.

5. Finally, calculate

\[ \lambda_{n+1}(i, j, k) = \lambda_n(i, j, k) + \gamma_n(i, j, k)r_n(i, j, k). \]  

(2.23)
6. Set \( n = n + 1 \) and repeat from step 1.

This algorithm is a step-by-step algorithm. The estimation of parameter \( \lambda_{i,j,k} \) is updated with the actual reject rate \( r_n(i,j,k) \) when a new reject occurs for the configuration \( (i,j,k) \).

In the above algorithm, the variable \( n \) (iteration number) is not taken as a global variable for every configuration \( (i,j,k) \). The variable \( n \) should be defined locally as \( n(i,j,k) \). For the remainder of this chapter, it is written as \( n \) to ease readability.

### 2.2.3 Estimation strategy

As mentioned above, the maximum likelihood approach provides a simple method to estimate \( \lambda \) (see section 2.2.1) that is based on the mean reject rate calculation. The main drawback of the method is that the parameter estimation dynamics are dampened as the amount of data increases and therefore, the adaptive approach is proposed as an alternative. By construction, this alternative estimation of \( \lambda \) cannot be implemented when the number of rejects is less than the constant \( M \). For this reason, both methods are combined. As a result, the likelihood estimation algorithm is utilized when the number of rejects is less than the window size \( M \) and the adaptive algorithm is used when the number of rejects is sufficiently large (greater than or equal to \( M \)). The resulting strategy provides a novel alternative to existing techniques that is suitable for the assessment of manufacturing quality of a system modeled by a SFM approach.
2.2.4 Comparison

This section provides comparison results between various approaches to estimate the parameter $\lambda_{i,j,k}$. Results are shown in Figures 2.4 and 2.5 for different parameter values of the adaptive approach.

Figure 2.4: $\lambda$ Estimation for a specific configuration as a function of $n_{i,j,k}$ - comparison with different values for the parameter $\mu$
Figures 2.4 and 2.5 show the $\lambda_{i,j,k}$ estimations as a function of the number of rejects using the approach described in section 2.2.3. These two figures show the estimation evolution for a specific 3-tuple $(i, j, k)$, the same evolution is observed with other 3-tuples.

In Figure 2.4, different estimation evolutions are compared for different values of the parameter $\mu$. This smoothing factor $\mu$ defined in Equation (2.22) gives less noisy estimations, and in Figure 2.4 several values are given to $\mu$ to show the estimation smoothness. As $\mu$ approaches 1, the estimations become smoother, and the opposite occurs when $\mu$ approaches 0. These estimations are compared to the static approach estimations and to a sliding window mean estimation. The sliding window approach
provides a better detection of changes in parameter values. It does that by calculation parameter estimates with the same weight $M$. The window moves with time and after $M$ rejects are detected, the parameters are estimated.

The estimations for the adaptive approach gives better results than the mean estimation approaches (see Figures 2.4 and 2.5). With only a window size equal to 10, the adaptive approach estimations are more filtered than with the sliding window mean estimation approach, which has a window size of 50. Furthermore, the estimations done using the adaptive approach are less smooth than the mean estimation using the static approach (mean estimation).

In Figure 2.5, the aim is to compare the estimations done with the adaptive approach, using different window lengths $M$, to estimations done with the static approach (mean estimations with or without a sliding window). Figure 2.5 shows that the smaller $M$ is, the faster the estimation is performed, conversely the opposite occurs when $M$ is bigger. For the same reasons given for Figure 2.4, the estimations done with the adaptive approach are better.

Several simulations indicated that the choice of $\mu$ and $M$ parameters are important. For this study the optimal choice for $\mu$ and $M$ are 0.9 and 10, respectively, which results in the optimal value of first time quality.

### 2.3 First Time Quality Index

The purpose of the model is to devise a control scheme that can respond to fluctuations in quality that occur in a manufacturing line. Quality in manufacturing lines is usually monitored by elaborate sensor-based and/or person-based fault detection controls. The counting of rejects provides an effective measure of quality.
Therefore, this measure of quality is the percentage of parts that exit the station without rejects. If \( N \) is the number of the last station of the entire manufacturing line, the measure called first time quality (FTQ) is given by:

\[
FTQ(t) = \frac{\int_0^t v_N(\tau)d\tau - \int_0^t \dot{X}_r(\tau)d\tau}{\int_0^t v_N(t)d\tau} \times 100
\]  

(2.24)

## 2.4 Model Computation

### 2.4.1 Distribution Parameter Estimation

The simulation of the entire manufacturing line and the calculation of the first time quality as given by Equation (4.1) are done using the estimation of the average reject time \( \lambda_{i,j,k} \). This estimation uses actual data from the assembly line and historical reject data. The \( \lambda_{i,j,k} \)-estimation is done off-line using a set of representative data from the engine assembly line. The adaptive estimation algorithm allows one to change \( \lambda_{i,j,k} \) in real-time to adjust for potential changes in line conditions or to incorporate data from new stations or new engines that were not represented in the past data.

As previously described, initial conditions and parameter values are required to use the estimation program. The choice of these constants can affect the performance of the estimation program. For the purpose of simulation and model validation, the following values were chosen: \( M = 20, \bar{\gamma} = 0.8, \Delta = 0.3, \alpha = 5\% \) (the value of the percentile \( z_{1-\alpha} \) is 1.96), \( \mu = 0.9 \) and \( \gamma_1 = 0.3 \). This initial set of parameters is assumed to be the same for every station, every product (here engine) type and every production shift. However, it is possible to tailor the initial conditions and the parameters of this algorithm to every configuration. In this way, the smoothness of the \( \lambda_{i,j,k} \) estimations can be altered to provide more or less dampening as required.
The amount of historical data needed to extract an accurate estimation of the \( \lambda \)-parameter is difficult to estimate because it depends on each configuration \((i, j, k)\). However, a minimum of two rejects in the same configuration are needed to begin the \( \lambda \)-parameter estimation algorithm for a specific configuration. Due to the adaptive component of the algorithm, a convergence is observed if the reject rate is stable, otherwise it will depend on the value of the initial parameters \((M, \tilde{\gamma}, \Delta, z_{1-\alpha}, \mu \) and \( \gamma_1 \) - see Figure 2.4 or 2.5). Sufficient conditions do not exist that exhibit the convergence of \( \lambda \)-parameters, due to the adaptive aspect of the algorithm.

2.4.2 Simulation Description

Once the distribution parameter is estimated for every reject code recorded in a manufacturing quality database, a simulation of the FTQ index can then be performed. The simulation of the assembly line can be decomposed into several sections. Each section is decomposed in three main parts: inline, control and quality loop stations (see Figure 2.3).

To start the simulation, engines are sent to the buffer before the first station of the line. From here, an engine moves through the assembly line from station to station. At each station, a random sample \( t_{\lambda_{i,j,k}} \) is compared to the threshold \( th_{i,j,k} \). The probability that the random sample belongs to the reject zone is equal to \( \frac{1}{\lambda_{i,j,k}} \). If the sample is associated to a reject, the engine is labeled with a reject flag otherwise no flag is created. When an engine exits the last station of the control line, without having generated a reject then the engine is sent to the next section or exits the entire manufacturing line. But, if the engine is labeled with a reject flag (the engine has had a reject in the current section), the engine is sent to the quality loop to be repaired.
When the engine is repaired, it re-enters the control line without any reject flag, but the engine is still labeled as an engine that has had a reject. This label cannot be changed since it affects FTQ.

When an engine exits the entire manufacturing line, it has no reject and the integral of $v_N$ is incremented by one unit. However, if the engine has been flagged as an engine that has had a reject, then the variable $X_r$ is incremented by one. The FTQ index is updated according to Equation (2.24).

## 2.5 Summary

In this chapter, a dynamic model of manufacturing system quality based on the SFM approach was introduced. Initially, a single-product type SFM model was discussed and then extended to a multi-station system. Static and adaptive parameter estimation approaches were also discussed and their performance was assessed. Finally, a quality measure, FTQ, was developed to estimate the quality of manufacturing systems.
Chapter 3

Results: FTQ Simulation

In this chapter, the accuracy of the SFM approach is examined. The SFM approach is initially used to estimate the FTQ for a single configuration and then used to estimate the FTQ of a system that incorporates some hybrid modeling aspects. For each estimation, two examples generated from real manufacturing data are discussed.

3.1 FTQ Estimation for single configuration

Before discussing the FTQ simulation results of the hybrid modeling aspects, we first examine the model’s accuracy using simulation results. Accuracy here is defined as the difference between the expected value and the simulated value. A process is simulated using typical distributional information. The ability of the FTQ estimation scheme to recover the known distribution parameters and FTQ results is examined. The SFM model for FTQ estimations is performed on a single configuration for N=83 stations. An exhaustive comparison is computationally prohibitive due to the large number of possible combinations of configurations (32 engine types, 3 shifts, 83 stations). From
CHAPTER 3. RESULTS: FTQ SIMULATION

Figure 3.1: Comparison for the three shifts of FTQ estimated by the SFM approach and the expected value for engine type 10

Figure 3.2: Comparison for the three shifts of FTQ estimated by the SFM approach and the expected value for engine type 5
this point of view, two cases are presented with two different quality levels. These two cases are illustrated respectively in Figures 3.1 and 3.2 along with results for the three corresponding production shifts.

In Figures 3.1 and 3.2, the FTQ simulation is compared to the expected value. Note that between the interval 0h to 1h30 the FTQ estimation equals 0 because no engines exit the line. After the instant 2h30 or 3h, the estimation convergence is visible. In Figure 3.2, the accuracy of the SFM model is striking and the bias with the expected value is quite small, especially with the second shift. The errors between the estimations and the expected values are smaller than 3%.

Expected values of all configurations can be used to give an idea of the global quality for each configuration. These expected values are given in Figure 3.3.

![Figure 3.3: FTQ expected value for all configurations](image-url)
3.2 FTQ Simulation with Hybrid Modeling Aspects

Hybrid control systems are those that involve continuous-time dynamics and discrete-event behavior and require controllers that may also have mixed continuous and discrete dynamics [2, 10]. Clearly, all the possible combinations of discrete states cannot be studied and illustrated in this work. The number of all switching combinations is equal to 9120 (96 * 95) for a single station. In this section, the number of stations is $N = 83$. Only two examples are discussed with 7 and 5 different discrete states for the first station corresponding to the first and second examples respectively.

The goal is to show with two examples the accuracy of the SFM approach while taking into account hybrid modeling aspects of the system, such as changes in product types and production shifts. In the following results, the estimation of the FTQ by the SFM model is compared to the FTQ index calculated from the historical data (processed engines and repaired engines stored in the database).

3.2.1 Example 1

For the first example, data from the first station of the historical data set are used as input to the simulation. The first engine enters the line at time instant $t_0 = 0$. The simulation duration is approximately 16 hours. Figure 3.4 illustrates the mode evolution on the first station where 7 discrete states, called modes, are defined. Each mode shows what product-type is being processed at the station and which shift is on duty.
Figure 3.4: Mode evolution for the first station

An initial first simulation of FTQ is illustrated in Figure 3.5. The results obtained are close to the actual value of FTQ. Since the calculation of FTQ is based on an average calculation, the initial FTQ values oscillate but convergence is observed after 6 hours. The relative mean error (RME) between the observed FTQ and the value calculated from the SFM is less than 2% after \( t = 6.5h \) and less than 1% after \( t = 10h \). Relative mean error here is calculated as:

\[
RME = \frac{FTQ_{sim} - FTQ_{data}}{FTQ_{data}}
\]  

The calculation of FTQ is based on statistical properties that give a random component to the FTQ estimation. If several estimations are performed (see Figures 3.5 or 3.7), the results will not be exactly the same, even if the parameters are the same in these different simulations. These differences are due to the statistical properties of the SFM simulator. Figure 3.5 shows different results of simulations where the actual FTQ value is accurately predicted by the SFM model. The estimation convergence is
Figure 3.5: Comparison of several FTQ estimations done by the SFM approach and FTQ calculated from the historical data seen after $t = 6h$. The largest errors are observed (Figure 3.5) before this convergence and do not exceed 10% between $t = 2.5h$ and $t = 6h$. These various results indicate good performance of the stochastic flow model, because after $t = 6.5h$ the relative mean error does not exceed 2% and still decreases with time.

### 3.2.2 Example 2

For the second example, a different day is picked from the data set. The simulation duration is 10 hours. Figure 3.6 illustrates the mode evolution on the first station where 5 modes are defined.
In this second example, the comparison between the estimation of the FTQ by the SFM model and the FTQ index calculated from the historical data is extended to the comparison with the expected value of the FTQ defined over all stations (expected quality intervals are plotted corresponding to a [99%, 101%]-confidence interval and a [95%, 105%]-confidence interval (CI). The results are illustrated in Figure 3.7.

Figure 3.7 shows the expected value of the FTQ, confidence intervals, the FTQ calculation from the historical data and 10 simulation results. In this example, the accuracy of the SFM algorithm is clearly visible because after the instant $t = 2.5h$ all estimations are included in the 5%-error confidence interval. Except for one simulation in which values are high, all simulations converge to the expected value after $t = 9h$. After the instant $t = 5h$, 50% of simulations are found within the 1%-error confidence interval. Therefore, the results in both examples demonstrate that the SFM model provides a very accurate prediction of the FTQ of the assembly line.
CHAPTER 3. RESULTS: FTQ SIMULATION

3.3 Summary

Some very encouraging results were obtained from the FTQ estimation simulation for both the single configuration and hybrid systems. In the case of the FTQ estimation for the single configuration, the estimation convergence is very obvious after about 3 hours of simulation and the accuracy of the SFM approach is evident. The results obtained from the FTQ simulation with hybrid modeling aspects incorporated also confirmed the accuracy of the SFM approach.

Figure 3.7: Comparison of several FTQ estimations done by the SFM approach and FTQ calculated from the historical data and FTQ expected value
Chapter 4

Quality Definitions

In the following chapters, we focus on the identification of the possible correlations between independent sets of maintenance and manufacturing quality data. We first propose some alternative quality definitions used in further developments in the remainder of this thesis. FTQ, introduced previously, will be re-introduced in the following section with slightly modified variables to ease comparison with other quality definitions in the following sections.

4.1 Basic Definitions

Definition 4.1.1. First Time Quality or FTQ refers to a quality index over the entire manufacturing line. More precisely, it is the ratio of parts going through the entire manufacturing line without any rejects, over the total number of parts. This ratio is calculated from an initial time instant $t_0 = 0$ to time $t$. 

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\[ FTQ(t) = \frac{\int_0^t v_n(s)ds - \int_0^t r(s)ds}{\int_0^t v_n(s)ds} \] (4.1)
\[ = 1 - \frac{\int_0^t r(s)ds}{\int_0^t v_n(s)ds} \] (4.2)

where \( FTQ(t) \) is the First Time Quality at time \( t \) using the output function \( v_n(s) \) and reject function \( r(s) \) over the interval \([0,t]\) and \( n \) is the number of the last station of the entire manufacturing line. \( v_n(t) = 1 \) if a part exits a station and 0 otherwise. If a reject is detected at any station, \( r(t) = 1 \), otherwise \( r(t) = 0 \).

**Definition 4.1.2.** Station Quality Index or \( SQI_i \) is the quality index of a particular station \( i \). \( SQI_i \) is the ratio of parts leaving station \( i \) without being rejected over the total number of parts leaving station \( i \). This ratio is calculated from an initial time index \( t_0 = 0 \) to time \( t \).

\[ SQI_i(t) = \frac{\int_0^t v_i(s)ds - \int_0^t r_i(s)ds}{\int_0^t v_i(s)ds} \] (4.3)
\[ = 1 - \frac{\int_0^t r_i(s)ds}{\int_0^t v_i(s)ds} \] (4.4)

where \( SQI_i(t) \) is the Station Quality Index of station \( i \) at time \( t \) using the output function \( v_i(s) \) and reject function \( r_i(s) \) over the interval \([0,t]\). \( v_i(t) = 1 \) if a part exits station \( i \) and 0 otherwise. If a reject is detected at station \( i \), \( r_i(t) = 1 \), otherwise \( r_i(t) = 0 \).

Therefore, to summarize the previous two definitions; FTQ is a global quality, based on the entire manufacturing line, whereas \( SQI_i \) is a local quality, referring to a specific station.
4.2 Sliding Window Approach

Equations (4.2) and (4.4) produce quality estimations that tend to become constant after a certain number of observation data is processed. For a better detection of changes in quality of the produced parts, a method that provides quality estimates with the same weight $N$ is proposed. The window moves with the current process-time. Therefore, after a certain number of parts $N$ is reached, FTQ is calculated using a more dynamic, sliding window approach, defined as sliding FTQ or sFTQ.

**Definition 4.2.1.** Sliding FTQ or sFTQ calculates the first time quality of the parts produced over a fixed window of size $N$, where $N$ is the number of parts. $N$ is used instead of time. Since time does not take into account production breaks, it would provide less accurate results. sFTQ is defined as follows:

$$sFTQ(m) = \frac{m - (m - N + 1) + 1 - \sum_{n=m-N+1}^{m} r(n)}{m - (m - N + 1) + 1}$$  \hspace{1cm} (4.5)$$

$$= \frac{N - \sum_{n=m-N+1}^{m} r(n)}{N}$$  \hspace{1cm} (4.6)$$

$$= 1 - \frac{\sum_{n=m-N+1}^{m} r(n)}{N}$$  \hspace{1cm} (4.7)$$

where $N$ is the fixed number of parts going through the station that is taken into account or the size of the sliding window, $m$ is the number of parts at time $t$ after an initial time $t_0 = 0$ and $\sum_{n=m-N+1}^{m} r(n)$ is the total number of parts exiting the
manufacturing line that have had at least one reject over the line. Thus, the FTQ is calculated using the sliding window approach only when \( m \geq N \).

Similarly, a sliding SQI or \( sSQI_i \) technique is adopted after \( N \) parts pass through station \( i \).

\[
sSQI_i(m) = 1 - \frac{\sum_{n=m-N+1}^{m} r_i(n)}{N} \tag{4.8}
\]

where \( N \) is the fixed number of parts taken into account or the size of the sliding window, \( m \) is the number of parts that passed through a particular station \( i \) after an initial time \( t_0 = 0 \) and \( \sum_{n=m-N+1}^{m} r_i(n) \) is the number of parts that are rejected at station \( i \) during production. Therefore as soon as \( m \geq N \), the SQI\(_i \) is calculated using the sliding window approach.

### 4.3 Instantaneous SQI

In the previous sections, FTQ, SQI\(_i \), sFTQ and \( sSQI_i \) were defined. All four of these quality measures can be calculated from the data set provided for this study. Furthermore, SQI\(_i \) can be seen as the mean of the instantaneous quality, \( q_i \). Given equation (4.4), the derivation of this observation is evident:

\[
Q_i(t) = SQI_i(t) = \frac{1}{t} \int_0^t q_i(s)ds \tag{4.9}
\]

\[
\dot{Q}_i(t) = -\frac{1}{t^2} \int_0^t q_i(s)ds + \frac{1}{t} \int_0^t q_i(s)ds
\]

\[
= -\frac{1}{t} Q_i(t) + \frac{1}{t} q_i(t) \tag{4.10}
\]
then,

\[ q_i(t) = t \dot{Q}_i(t) + Q_i(t) \]  \hspace{1cm} (4.11)

### 4.4 Classifying Different Maintenance Work Types

Different manufacturers have different names for various maintenance work types. These maintenance types range from the routine preventive-type maintenance to more urgent emergency maintenance. In this study, the data set contains six types of maintenance work types as shown in Tab.4.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Work Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTI</td>
<td>Continuous/Throughput Improve</td>
</tr>
<tr>
<td>DC</td>
<td>Down Check</td>
</tr>
<tr>
<td>EMR</td>
<td>Emergency Maintenance/Repairs</td>
</tr>
<tr>
<td>PFB</td>
<td>Preventive - Frequency Based</td>
</tr>
<tr>
<td>RC</td>
<td>Running Check</td>
</tr>
<tr>
<td>SRP</td>
<td>Scheduled Repair</td>
</tr>
</tbody>
</table>

The six maintenance work types are grouped together depending on the nature of the maintenance type. For example, CTI and RC are considered to be regular, routine maintenance and therefore, they are grouped together. Similarly, SRP is considered to be a preventive maintenance and therefore, SRP and PFB are grouped together. Finally, EMR and DC are not grouped together, because the first one is an emergency maintenance and the latter is merely a check. They are each studied separately.
4.5 Summary

In this chapter, the definitions of FTQ, SQI, sFTQ, sSQI, and instantaneous SQI were introduced. The sliding window approach was also introduced. It is used for a better detection of changes in quality of the produced parts. Finally, different maintenance work types were presented and divided into groups depending on the nature of the maintenance type.
Chapter 5

Correlating Maintenance and Quality

In this chapter, several attempts at identifying correlations between maintenance data and quality data using the definition of SQI and instantaneous SQI are studied. The results of these attempts, which are based on examples derived from real manufacturing data, are shown and discussed.

5.1 The Case for SQI

A closer look at EMR and DC maintenances in relation to the SQI revealed some interesting facts. After a DC maintenance is applied, there is a delay after a maintenance occurrence (represented by the vertical lines), before any improvement is seen in the SQI (Figure 5.1). This delay might be associated with some sort of production break. Surprisingly, an EMR maintenance can cause the quality to decrease (Figure 5.2). Furthermore, the duration of an EMR maintenance is usually quite
short. SQI plots were plotted for several stations to confirm these facts.

It can clearly be seen from Figure 5.1 that the first DC maintenance ($\approx 127$ days) takes just less than one day to show an improvement in SQI, while the second DC maintenance ($\approx 133$ days) takes nearly two days to show any improvement in SQI.

Figure 5.1: DC maintenance shows some improvement, but after some delay

Figure 5.2 shows how an EMR maintenance can sometimes lead to a decrease in quality, but at other times, leads to an increase in quality. Similar results were obtained for other stations.
Figure 5.2: EMR decreases and increases quality (before 140 days and after 170 days)

5.2 The Case for Instantaneous SQI

5.2.1 Chattering Problems

Equation (4.11) is an estimation of the instantaneous quality at a station \( i \) that cannot be estimated directly from the data set due to chattering problems as seen in Figure 5.3.

Therefore, a more robust observer should be implemented to reduce chattering problems. A Kalman filter is introduced to obtain estimates for the instantaneous quality at a specific station.
Figure 5.3: Chattering problems observed when equation (4.11) is estimated directly

### 5.2.2 Kalman Filter

Starting from the quality measurement using equations (4.9) and (4.11), the state-space representation is defined as:

\[ \dot{x}(t) = A(t)x(t) + w_x(t) \]  

(5.1)

where \( x(t) \) represents the state defined by \([Q(t), \dot{Q}(t), \ddot{Q}(t), q(t)]\), where \( q(t) = 1 - \frac{1}{\rho(t)} \), \( A(t) \) is the time-varying matrix and \( w_x(t) \) is the state noise.

The measurement equation is given by:

\[ y(t) = C x(t) + w_y(t) \]  

(5.2)

where \( y(t) \) represents the measurement, \( C \) the measurement matrix and \( w_y(t) \) the measurement noise.
Noises $w_x(t)$ and $w_y(t)$ are supposed to be uncorrelated, white and gaussian with covariance matrices $W_x$ and $W_y$, respectively.

Matrices $A$ and $C$ are defined by:

$$A(t) = \begin{pmatrix} -1/(t + \epsilon) & 0 & 0 & 1/(t + \epsilon) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & t & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

The calculation of the eigenvalues of $A$ gives the state-space representation as defined by equation (5.1). This is an unstable system. Three poles are zero and the last pole has a negative real part that depends on $1/(t + \epsilon)$, where $\epsilon$ is a constant. To satisfy the convergence of the instantaneous quality estimations, an observer is implemented. The basic principle of an observer is to close the loop using the measurement as input and to optimize the correlation between the measurements and the output estimations while minimizing noise effects. In this work, the observer is given by:

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + K(t)(y(t) - C\hat{x}(t))$$

$$\hat{y}(t) = C\hat{x}(t)$$

(5.3) (5.4)

where $\hat{x}$ corresponds to the state estimation, $\hat{y}$ the output estimation and $K$ is the observer gain matrix.

The dynamic equation of the error $\tilde{x}$ between the state $x$ and its estimation $\hat{x}$ can be calculated:

$$\dot{\tilde{x}}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$$

$$= (A(t) - K(t)C)\tilde{x}(t) + w_x(t) - K(t)w_y(t)$$

(5.5)
From Equation (5.5), the choice of matrix \( K \) gives the stability of the observer, if the real parts of \( (A(t) - K(t)C) \)-eigenvalues are negative. Moreover the choice of \( K \) can be realized by minimizing the covariance of the estimation error:

\[
\dot{P}(t) = \text{Cov}[\dot{x}(t)] \\
= (A(t) - K(t)C)P(t) + P(t)(A(t) - K(t)C)^T \\
+ W_x + K(t)W_y(t)K(t)^T \\
\tag{5.6}
\]

Initial values of \( P(t) \) can be estimated from historical data. To find the matrix \( K \) that minimizes the covariance error \( P \), the influence of \( K \) toward \( P \) is calculated:

\[
\frac{\partial \text{trace}(\dot{P}(t))}{\partial K(t)} = -P(t)C^T - P(t)C^T + 2K(t)W_y \\
\tag{5.7}
\]

If \( \frac{\partial \text{trace}(\dot{P}(t))}{\partial K(t)} \) is equal to 0, that gives the necessary condition to obtain the minimum value of \( P \), which turns out to be:

\[
K(t) = P(t)C^TW_y^{-1} \\
\tag{5.8}
\]

If the expression of \( K(t) \) given by Equation (5.8) is substituted into Equation (5.7), it gives:

\[
\dot{P}(t) = A(t)P(t) + P(t)A(t)^T - P(t)C^TW_y^{-1}CP(t) + W_x \\
\tag{5.9}
\]

which is a Riccati equation with the asymptotic convergence property for the eigenvalues. The greatest estimation error converges to 0. For more details see [19], [31] and [48].
5.2.3 Instantaneous SQI using a Kalman Filter

A Kalman filter is used to get an estimate of the instantaneous quality of a particular station. A schematic of this model for a single station can be seen below in Figure 5.4:

![Schematic of instantaneous SQI estimation technique](image)

where $u_i(t)$ is the number of products entering station $i$, $v_i(t)$ is the output from station $i$ going to station $i + 1$, $x_b(t)$ is the number of products in the buffer, $r_i$ is the number of rejects that take place at station $i$, $Q_i$ is the SQI at station $i$ and $\hat{q}_i$ is the estimated instantaneous SQI at station $i$ calculated using the Kalman filter.

The way the model in Figure 5.4 works is that it initially calculates the SQI ($Q_i$) as shown previously and then feeds it into the Kalman filter. The Kalman filter then uses this information to calculate an estimate of the instantaneous quality $\hat{q}_i$ at a particular station $i$. 
5.2.4 Incorporating Maintenance Data into Instantaneous SQI Plots

The next step is to incorporate maintenance data into the instantaneous SQI plots. The simulation is run for a period of 600 hours or 25 days. The simulation starts after 150 days from the beginning of the data (in the data set provided) and ends at 175 days. This area was chosen after several SQI plots proved that there were many fluctuations in the data in that region, which meant more analysis could be done, as compared to other areas, where there was less variation in the SQI.

The first segment of the manufacturing line is the main focus of this section. Instantaneous SQI plots were plotted for all the stations in the first segment, except for a couple of stations, since no reject data was available for these stations. Some stations resulted in instantaneous SQI plots that did not provide any useful information because there was either no maintenance data for that station within the simulated time period or very few variations in the instantaneous SQI were observed. On the other hand, there are two stations that provide useful information for the analysis of the maintenance data. Stations B and C are considered to be very useful, since they show a lot of fluctuations in the instantaneous SQI and have a variety of maintenance occurrences, including EMR. EMR is an emergency maintenance. Theoretically speaking, an emergency maintenance is expected to occur when there is a sharp decrease in quality and an immediate improvement is expected after it is done. With this focus in mind, stations B and C are analyzed further.

Since the instantaneous quality is a quality index, it cannot have a value greater than one. However, in the following figures, the reader will notice that the instantaneous quality is sometimes greater than 1. The reason for this is that these estimations
are not filtered and no thresholds are applied.

![Station B - Instantaneous SQI Results (EMR Maintenance)](image)

Figure 5.5: Zoom-in on section with EMR

Figure 5.5 is a zoom-in on the instantaneous quality plot obtained for station B. It is evident that as soon as the instantaneous quality decreases, an EMR maintenance takes place (solid, vertical line) and as soon as the maintenance ends (dashed, vertical line), the instantaneous quality increases. This is a really good indication that the theory stated above is valid. Unfortunately, this result does not hold for all stations.

An example where this theory does not apply is station C. A zoom-in (Figure 5.6) on the first EMR maintenance occurrence shows that a maintenance occurs slightly after a sharp decrease in quality reaches a minimum, which might be associated with some type of delay. However, what is really difficult to comprehend is why the previous drops in quality, which were more severe drops (such as at 320 hours of simulation time, for example), were not given any attention in terms of emergency maintenance.
Figure 5.6: Zoom-in on first EMR occurrence

Figure 5.7: Zoom-in on second EMR occurrence
The second EMR occurs later on after 490 hours of simulation time. The result of this maintenance is a decrease in quality and it is very difficult to justify why this would happen. This can clearly be seen in Figure 5.7.

5.3 Summary

We noticed that according to the definitions of SQI and instantaneous SQI, when a maintenance event is detected, a change in quality appears after some delay, which might be attributed to the time it takes for a station that has just been serviced to start up again and synchronize with the other stations. It was also interesting to note that according to these definitions of quality, maintenance events sometimes lead to a decrease in quality.
Chapter 6

A New Quality Definition

In this chapter, a new quality definition known as Maintenance Quality is defined. This definition of quality is then examined to correlate maintenance and quality data sets in an effective manner. Several examples of preventive and emergency maintenance are examined to test the accuracy of this mathematical formulation. The results of these examples are then discussed in detail.

6.1 Maintenance Quality...A New Quality Definition

Previous quality definitions and calculations do not provide any expected results associated with maintenance occurrence. No relationship between quality and maintenance occurrence was clearly exhibited in a global sense. Quality is simply plotted against maintenance occurrence and a relationship is sought by analyzing the plots and looking at what happens after maintenance takes place. Therefore, another more
accurate method for calculating quality has to be defined.

**Definition 6.1.1.** Maintenance Quality or MQ calculates the level of quality after a maintenance occurrence is detected. For a given time interval, when the first maintenance point is detected, MQ is calculated as $sSQI$ until the second maintenance point is detected, given that the number of parts that have gone through the station being examined has reached the size of the sliding window ($N$). If the size of $N$ is not reached, MQ waits until $N$ parts pass through the station and then calculates $sSQI$. It disregards the first condition which stops calculating $sSQI$ when the second maintenance point is reached. MQ is calculated as $sSQI$ only until the second maintenance point is detected, given that $N$ is reached within that time interval, or until $N$ is reached regardless of how many maintenance points are detected during that interval. After that, MQ is calculated as the SQI of the closest maintenance point after $sSQI$ is calculated and the next one after that. This is computed as follows:

$$MQ_i(t) = \begin{cases} 
0 & \text{if } t < N \\
\alpha(sSQI_i(M) + \beta SQI_i[M+,t](t)) & \text{if } t \geq M \\
sSQI_i(t) & \text{if } N \leq t < M 
\end{cases} \quad (6.1)$$

where

- $\alpha$ and $\beta$ are weights used to indicate how much confidence is given to $sSQI_i(t)$ and $SQI_i(t)$ used in defining $MQ_i(t)$.

- $\alpha = \frac{1}{t - M + 1} \quad (6.2)$
$\beta = t - M$  \hspace{1cm} (6.3)

Therefore, $m - M + 1$ values are used when calculating $MQ_i$.

$\bullet$

$$sSQI_i(m) = 1 - \frac{\sum_{n=m-N}^{m} r_i(n)}{N}$$ \hspace{1cm} (6.4)

where $N$ is the fixed number of parts taken into account or the size of the sliding window, $m$ is the number of parts that pass through a particular station $i$ after an initial time $t_0 = 0$ and $\sum_{n=m-N+1}^{m} r_i(n)$ is the number of parts that are rejected at station $i$ during production.

$\bullet$

$$SQI_i(t)_{[M^+,t]} = \frac{\sum_{i=M+1}^{t} p_i(t) - \sum_{i=M+1}^{t} r_i(t)}{\sum_{i=M+1}^{t} p_i(t)}$$  \hspace{1cm} (6.5)

$$= \frac{t - (M + 1) + 1 - \sum_{i=M+1}^{t} r_i(t)}{t - (M + 1) + 1}$$  \hspace{1cm} (6.6)

$$= \frac{t - M - \sum_{i=M+1}^{t} r_i(t)}{t - M}$$  \hspace{1cm} (6.7)

where $SQI_i(t)$ is the Station Quality Index of station $i$ at time $t$ using the output function $p_i(t)$ and reject function $r_i(t)$ over the interval $[M^+,t]$. $p_i(t)=1$ if a part exits station $i$ and 0 otherwise. If a reject is detected at station $i$, $r_i(t)=1$, otherwise $r_i(t)=0$. $M$ is the time at which a maintenance event is recorded.
CHAPTER 6. A NEW QUALITY DEFINITION

The final definition used to calculate $MQ_i(t)$ is:

$$MQ_i(t) = \begin{cases} 
0 & \text{if } t < N \\
\frac{1}{t-M_j+1}(sSQI_i(M_j) + (t - M_j)SQI_i[M_j^+,t]) & \text{if } t \geq M_j \\
sSQI_i(t) & \text{if } N \leq t < M_j
\end{cases}$$

(6.8)

where $M_j$ is the time at which the $j^{th}$ maintenance event is recorded.

6.2 MQ - Theoretical Results

Maintenance helps to keep plant equipment in good operating condition so that the products being produced satisfy a certain quality criteria such as the ISO/QS 9000 quality requirements. Therefore, one would expect an improvement in product quality after maintenance is performed on plant equipment.

![Figure 6.1: Expected results for $MQ_i$](image-url)
Figure 6.1 shows a theoretical view that explains how Equation (6.8) is used to calculate Maintenance Quality. By looking at the figure, one can see that for the first period from $t_0 = N$ until the first maintenance is scheduled at $t_1 = M_1$, the calculated Maintenance Quality just oscillates around a specific quality level. This is due to the fact that a sliding window approach is used to calculate this first segment. However, from $t_1$ onwards, the quality initially increases as expected due to maintenance and then tends to decrease before another maintenance event occurs at $t_2 = M_2$. This is due to the fact that the method used for calculating Maintenance Quality changes and relies on the definition of SQI.

6.3 Relating Maintenance Quality to Maintenance Occurrence

To observe the results of the new quality definition, five stations in four different sections of the manufacturing line are examined. Due to the size of the data set used, the time interval selected for this analysis is only between 100 and 200 days from the start time of the data set.

Two sets of results are obtained. The first set of results studies the relationship between preventive (scheduled maintenance) and its effect on Maintenance Quality, whereas the second set of results analyzes the effect of emergency maintenance (unscheduled maintenance) on Maintenance Quality.
6.3.1 Preventive Maintenance Results

The first set of results examines preventive maintenance done on the plant. Preventive maintenance is used to periodically maintain plant equipment to prevent it from reaching a point when it can no longer be fixed. As a result, breakdowns decrease and plant equipment does not depreciate as much [34].

Station D

Results obtained for the first station studied are now discussed. The Maintenance Quality is calculated over 100 days of operation and starts at the 100\textsuperscript{th} day of the data set. In the figures that follow, the dashed, vertical lines represent the time at which a preventive maintenance event takes place.

Figure 6.2: MQ results for station D.
By looking at Figure 6.2, it can be seen that the lowest the Maintenance Quality for station D is just below 0.995 (or 99.5%), which corresponds to a very good level of quality. One can see that after maintenance occurs, a slight delay with a low quality level, but then the quality level increases and gets close to 1 (or 100%). This delay may be due to temporarily shutting down the station for maintenance and starting it up again. The results obtained for station D do not closely follow the expected results. However, there is some resemblance, in the sense that, the first part of the plot is relatively constant since MQ is calculated using the sSQI definition. It can also be noted that from that point onwards, between every two maintenance points, the quality would increase, but unfortunately, it does not decrease before the next maintenance point is detected as hypothesized. Nevertheless, it is safe to say that there is a relationship between maintenance occurrence and increase in quality. This observation is based on the results obtained, which indicate that following the detection of a maintenance event, an initial decrease in quality is observed, followed by a significant improvement in quality.

Station E

Figure 6.3 shows the results obtained for station E, focusing on the time interval 24 – 44 days. We first notice that in Figure 6.3 the results closely resemble the hypothesized results. The quality initially increases after the second maintenance occurrence is detected until it reaches a peak and then decreases as time goes by and another maintenance point is about to be detected. Figure 6.3 is only a window of the results obtained for this station, but in this window of time, it can be seen that the quality never goes below 92%. However, there is a point in the plot of the overall
results for this station at which the quality goes down to about 25%. Furthermore, a
direct relationship between maintenance and quality can clearly be observed.

![Station E - Maintenance Quality Results (Preventive Maintenance)](image)

Figure 6.3: Zoom-in on MQ results for station E.

### 6.3.2 Emergency Maintenance Results

Emergency maintenance is also referred to as unscheduled maintenance. This usually
occurs when a station at the manufacturing plant suddenly breaks down and has to
be fixed immediately to avoid any loss in production. In Figs. 6.4, 6.5 and 6.6, the
solid, vertical line indicates the time at which an emergency maintenance event takes
place.
In Figure 6.4, an EMR maintenance point can be seen after 72 days mark and an increase in quality occurs after about 79 days of simulation. However, it is not clear whether the cause of this increase in quality is the preventive maintenance that took place or the EMR maintenance or both. Unfortunately, this was the only EMR maintenance point observed at this station for the time interval being examined. Better results were obtained for station E.
Station E

Several EMR maintenance points are observed at this station. In Figure 6.5, it is clear that when an EMR maintenance occurs after about 22 days of simulation time, the results are positive and an increase in quality is spotted. Just to make sure that this trend is not merely a coincidence, another EMR maintenance point at this station is examined.

Figure 6.5: Zoom-in on EMR’s effect on MQ for station E
Again, similar results are obtained when an EMR maintenance point is detected. Maintenance Quality increases and follows the theoretical trend described previously, where the quality first increases and then decreases. This result supports the hypothesis that there is a direct relationship between maintenance occurrence and increase in Maintenance Quality.

### 6.4 Summary

Through the definition of Maintenance Quality, we were able to come up with a novel mathematical formulation that correlates independent sets of maintenance and quality data. Several examples were used to demonstrate how the results obtained from this mathematical formulation matched the theoretical results.
Chapter 7

Ageing and Maintenance Modeling

Ideas

According to the results obtained using MQ, there is significant supporting evidence of a direct relationship exists between performing maintenance and improving quality. The next step is to provide some simple ideas on how ageing and maintenance of plant equipment can be modeled.

7.1 Ageing

Ageing can be defined as the time between two maintenances or the number of parts passing through a station between two maintenances. This study proposes to model the ageing based on the part number.
7.1.1 Linear ageing

In this part, it is assumed that the $\lambda$-parameters decrease as a linear function of the number of parts passing through a station:

$$\lambda_{i,j,k}^m(t) = \lambda_{i,j,k}^r(t) \left(1 - \frac{p_i(t)}{a} \frac{1}{100}\right)$$  \hspace{1cm} (7.1)

where $\lambda_{i,j,k}^m$ represents to the modified value of the $\lambda$-parameter according to the station $i$, the part type $j$ and the shift $k$ at time $t$, $\lambda_{i,j,k}^r$ the real value of the $\lambda$-parameter, $p_i$ the part passage number (all parts mixed), $a$ an ageing speed constant (every $a$ parts that pass through the station $i$, the $\lambda$-parameter decreases by 1%).

The modified parameter $\lambda^m$ is used in the model as the reject rate parameter.

7.2 Maintenance

This $\lambda$-parameter, called $\lambda^m$, decreases as the number of parts passing through a station increases, modeling the ageing of the manufacturing lines. It is obvious that if plant maintenance is neglected, production quality will decrease and the number of rejects will increase significantly. Now, three types of maintenance will be defined:

- cyclic maintenance; the period between two maintenances is the same (for example every 12 hours),
- low quality maintenance; the maintenance is done on a station when the station has too many rejects,
- reconfiguration maintenance; a specific maintenance is done for a specific part type to make a new configuration of stations.
7.2.1 Cyclic Maintenance

In this case of maintenance, stations are periodically inspected every given time period $T_m$. The following algorithm describes the approach:

**Cyclic Maintenance Algorithm**

- Initialization,

\[ t_{m_i} = t_{i,0} \]

for each time instant $t$

\[ \text{if } t - t_{m_i} \geq T_m \text{ then} \]

\[ p_i = 0 \]

\[ t_{m_i} = t \]

end if

end for

All stations are not required to have the same maintenance period. In this case, the greatest common factor of maintenance periods can be defined $p_m$ and for a specific station the maintenance period is defined by: $T_{m_i} = \alpha_i p_m$, with $\alpha_i \in \mathbb{N}$.

7.2.2 Low Quality Maintenance

Unlike other maintenance definitions, low quality maintenance is not based on a time period. The occurrence of a maintenance is stochastic. In fact, this type of maintenance is done when a station has a very low production quality.

This way, the number of rejects associated with each station is calculated and called $r_i$. $SQI_i$ gives a dynamical estimation of the quality for each station.
maintenance can be easily done for a given threshold by comparing the \( SQI \) value to the threshold \( th_i \). The algorithm is proposed hereafter.

According to this algorithm, the maintenance aim is to reset the \( \lambda \)-parameter to the real value, it gives: \( \lambda_{i,j,k}^m = \lambda_{i,j,k}^r \) as explained in Equation (7.1) when \( p_i \) equals zero. This action stops the ageing of the station by resetting \( r_i \) to zero.

**Low Quality Maintenance Algorithm**

- for each new part in station \( i \),

\[
p_i = p_i + 1
\]

if the part has a reject in the station then

\[
r_i = r_i + 1
\]

end if

\[
SQI_i(t) = \begin{cases} 
1 & \text{if } p_i = 0, \\
1 - \frac{r_i}{p_i} & \text{otherwise.}
\end{cases}
\]

if \( SQI_i(t) < th_i \) then

\[
p_i = 0
\]

\[
r_i = 0
\]

end if

end for

**7.2.3 Reconfiguration Maintenance**

The reconfiguration maintenance is a very specific maintenance, that can be done when a new part type enters the manufacturing line. In this case, the maintenance consists of reconfiguring or adapting the station to the new part type.
Reconfiguration Maintenance Algorithm

- for each new part in station $i$,

  if the part has a specific product type $j$

  \[
  t_{m_i} = t
  \]

  \[
  p_i = 0
  \]

  \[
  r_i = 0
  \]

end

end

7.3 Summary

Several maintenance models were discussed in this chapter, each having their own advantages and disadvantages. The ageing and maintenance modeling ideas in this chapter form the basis for the next step in this research project. The effect of ageing and maintenance on plant equipment will hopefully be incorporated into our model to form a complete manufacturing system model that will help improve the manufacturing quality of the goods produced by these systems.
Chapter 8

Conclusions

The modeling of manufacturing systems and obtaining optimal policies to run these systems is an area of great interest. Although many researchers have proposed modeling techniques such as discrete event systems (DES) and timed DES, the complex, multi-scale nature of large manufacturing systems limits the applicability of these techniques and therefore, hybrid modeling techniques are used to overcome these difficulties. One of the objectives of this research was to study the dynamics of manufacturing systems using a SFM-based hybrid system modeling paradigm. In contrast to previous work where the dynamics of these states was primarily modeled using delays, our project focused on using SFM to model the state dynamics that indicate the quality level of the manufacturing system.

This thesis demonstrates how the stochastic flow model (SFM) approach provides a suitable tool for the dynamic modeling of first-time quality (FTQ) for manufacturing systems. This modeling approach, which has been recently investigated [3] in the context of manufacturing systems, provides an effective platform to incorporate other common objectives in addition to quality such as throughput improvement and
resource allocation subject to stochastic fluctuations. Two examples from an automotive manufacturing environment were used to prove the accuracy and potential of using this modeling technique to determine the quality of manufacturing systems. In the first example, the relative mean error between the observed FTQ and the value calculated from the SFM was less than 2% after 6.5 hours of simulation and less than 1% after 10 hours of simulation. In the second example, the comparison between the estimation of the FTQ by the SFM model and the FTQ index calculated from the historical data was extended to include a comparison with the expected value of the FTQ defined over all stations. The accuracy of the SFM algorithm was evident in this example, with all the estimations being within the 5%-error confidence interval after 2.5 hours of simulation and 50% of simulations being within the 1%-error confidence interval after 5 hours of simulation. Therefore, both examples proved that the SFM is a very accurate predictor of the FTQ in an assembly line.

A novel adaptive estimation algorithm was developed for the estimation of model parameters. This technique uses a likelihood estimation algorithm when the number of rejects is less than the window size M. It uses an adaptive algorithm when the number of rejects is greater than or equal to M, thereby accounting for non-stationary behavior of the model parameters. This parameter estimation strategy provides a novel alternative to existing techniques and is suitable for assessing the manufacturing quality of a system that is modeled using the SFM approach.

The correlations between independent sets of maintenance and manufacturing quality data was analyzed. Several quality definitions were introduced and their results were discussed. Using the definitions of quality defined in sections 4.1 and 4.3, we obtained some interesting findings. We noticed that according to the definitions
of SQI and instantaneous SQI, when a maintenance event is detected, a change in quality appears after some delay, which might be attributed to the time it takes for a station that has just been serviced to start up again and synchronize with the other stations. It was also interesting to note that, according to these definitions of quality, maintenance events sometimes lead to a decrease in quality. For example, DC maintenance provided an improvement in SQI after a small delay. It was also found that EMR caused both a decrease and an increase in SQI. The results obtained from the instantaneous SQI plots agreed with those obtained from the initial SQI plots. Both sets of plots showed that an EMR maintenance can sometimes result in a decrease in quality.

Previous quality definitions (SQI and instantaneous SQI) and calculations did not provide any expected results associated with maintenance occurrence. No relationship between quality and maintenance occurrence was clearly exhibited in a global sense. Quality was simply plotted against maintenance occurrence and a relationship was sought by analyzing the plots and looking at the effect of maintenance. A more accurate method for calculating quality was proposed. The new definition of quality, Maintenance Quality, provided some very encouraging results. These results matched the theoretical results and proved that a direct relationship between maintenance occurrence and increase in quality was present. This was mainly due to the development of this novel definition that took into account maintenance occurrence and calculated the quality accordingly. The new maintenance definition provided a mathematical formulation that identifies the direct correlation existing between maintenance and quality.
The last step in this research project was to generate some ageing and maintenance modeling ideas that will be used in future work to create a complete model for the manufacturing line that performs maintenance on the plant in an optimum manner such that high quality products are produced while minimizing costs. Other future work includes modeling other automotive plants and applying these quality definitions to them. These plants will be modeled using historical data obtained from the plant such as downtime, average cycle time and production time. The model will then be verified by comparing the output from the model to historical data and see how well they match. After the model is verified, maintenance and quality data will be obtained and the definition of Maintenance Quality will be implemented. By doing so, we hope that we can help bridge the gap between maintenance and quality in the manufacturing industry.
Bibliography


