

THREE ESSAYS ON THE SIZE AND CONTRIBUTION
OF INTANGIBLE INVESTMENT TO THE OVERALL
CAPITAL STOCK

by

NAZIM BELHOCINE

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Abstract

This thesis aims to contribute to a better understanding of the overall magnitude of intangible investment and the impact of this intangible investment on the behavior of the capital stock and on the value of capital goods.

I begin by constructing a data set to document firms' expenditures on an identifiable list of intangible items in Canada. I then examine the implications of treating intangible spending as the acquisition of final (investment) goods on estimates of GDP growth for Canada. I find that investment in intangible capital by 2002 is almost as large as the investment in physical capital. Furthermore, the growth in GDP and labor productivity may be underestimated by as much as 0.1 percentage point per year during this same period.

I proceed by measuring the size of the stock of the intangible capital in Canada using newly released data on the market value of all securities in the economy. The approach taken relies on a quantitative application of the q-theory of investment to generate the quantity of capital owned by firms. I find that the intangible capital stock accounted for approximately 30% of overall capital since 1994. Of this, the R&D reported by national accounts makes up only 23%. These results imply that official Canadian statistics failed to account for 26% of the value of the capital stock in their 2005 quarterly data collection.

Finally, I extend the q-theory of investment to model explicitly the decision of firms to invest in intangibles. I then use the model to measure the contribution of intangible goods to the overall capital stock in the U.S. The model departs from the one mentioned earlier in that it highlights the embodiment of intangible goods in tangibles and the role of relative price movements in the measurement of the contribution of each type of investment to the overall capital stock. I find that the growth in the overall capital stock from the late-80s until 2000 was driven mainly by an increase in the contribution of intangibles. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the movements in the price of intangibles rather than focusing only on their rising share in overall investment.

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Chapter 1

Introduction

In the last few years, there has been a growing perception among academics and policy-makers that a significant and increasing part of total business investment is directed towards intangible investment. Intangible investment is the expenditure on items which have a knowledge component, such as research and development, training, organizational change, marketing and software. To some researchers, this phenomenon is “what put the new in the new economy” (Nakamura, 1999), while others acknowledge that “although investment in intangible capital is not counted as capital investment in the national income and product accounts, they appear to be quantitatively important.” (Bernanke, 2005)

Unfortunately, the lack of systematic statistical information on intangible investment makes it difficult to directly substantiate this phenomenon, monitor its progress and assess its importance for growth. Moreover, the difficulty of defining intangibles, given their impalpable nature, contributes to an opacity of language and, consequently, to a lack of agreement on their precise size. Finally, researchers’ various goals in measuring and using intangibles have led to diverse approaches in evaluating their magnitude, sometimes with conflicting results.

The current state of omission and mismeasurement of intangible capital has several implications. First, because spending on intangibles is not treated as investment, aggregate savings and investment may be significantly understated in official statistics. Monetary policymakers could be misled by such an imprecise picture of the economy in setting interest rates. Second, resource allocation and investment decisions within firms and across firms in a given industry become more difficult. Third, fiscal policy can be affected in various ways such as in the design of a fair tax system. Finally, the lack of good information on intangibles will lead to opaqueness and volatility in capital markets given the increased difficulty of estimating the future cash flows that some investments will generate.

This thesis aims to fill a gap in the literature by measuring the size of intangible investment and intangible capital for Canada and the U.S. Chapter 2 takes a direct approach to measuring intangibles by documenting the expenditures of Canadian firms on an identified list of intangible inputs using various statistical sources from 1998 to 2005. This approach was pioneered by Nakamura (1999) and made more comprehensive by Corrado *et al.* (2005) for the U.S. Chapter 3 illustrates the use of the market value of securities to measure the stock of intangible capital in Canada from 1990 to 2005. This method was first used by Hall (2001) for the U.S. Chapter 4 brings these two different approaches together to measure the stock of intangible capital in the U.S. while removing a critical assumption that the price of intangibles is the same as the price of tangibles. The results are contrasted with the studies of Hall (2001) and McGrattan and Prescott (2007a). Chapter 5 concludes the thesis with a discussion on possible future avenues.

The System of National Accounts (SNA) is the ultimate conceptual framework that sets the international statistical standard for the measurement of economic activity. It consists of a set of internationally agreed concepts, definitions, classifications and accounting rules, which are approved by the United Nations Statistical Commission and published jointly by the United Nations, the Commission of the European Communities, the International Monetary Fund, the Organization for Economic Cooperation and Development, and the World Bank. Being the most comprehensive macroeconomic standard, the SNA also serves as the main reference point for statistical standards.

The most current version of the SNA is from 1993. The 1993 SNA has changed from its 1953 and 1968 versions over time to reflect the evolution of national economies and their increasing interdependence. However, the 1953 and 1968 versions paid very little attention to intangible investment. Most intangible items were accounted for as current costs (in case of purchase of service) or as labor and material costs (Young, 1998).

During the revision exercise leading up to the 1993 SNA, there was substantial discussion regarding the nature of investment and the possibility of including some intangibles. This debate was fueled by claims that a new era known as the knowledge economy started and that some firms' valuations had a large component related to assets that were intangibles. The result of the SNA revision was the creation of a category called "produced intangible assets" which encompassed software and major databases; entertainment, literary and artistic originals; and mineral exploration. However, the experts felt that other intangible producing activities such as R&D,

training, market research and advertising did not “lead to the creation of assets that can be easily identified, quantified and valued for balance sheet purposes. Such expenditures therefore continue to be classified as intermediate consumption, even though it is recognized that they are undertaken for the purpose of trying to secure future benefits.” (United Nations, 1998) Several countries implemented satellite accounts for R&D but these typically cover short periods of time and are not standardized.

The end of the 1990s marked an increased effort in setting measurement standards for intangible investment, partly under the leadership of the statisticians at the OECD (see OECD, 1998). Subsequently, the 1997 and 2002 versions of the North American Industrial Classification System were developed to better account for industries that depend heavily on intangible asset formation such as the cultural industries and the information and communication sector (see Statistics Canada, 2002.)

In 2003, the United Nations Statistical Commission called for an update of the SNA 1993 “to bring the accounts into line with the new economic environment, advances in methodological research and needs of users” (United Nations, 2008a). The 1993 SNA Rev.1, as it will be called, is scheduled to be finalized and adopted by 2009. Among the most notable updates is the treatment of R&D as investment under the umbrella of “intellectual property products”. This heading will merge the existing category “produced intangible assets” with R&D expenditures. Capitalizing R&D expenditures is a major shift in national income practices. Almost all OECD member countries will have implemented this change by 2014 (United Nations, 2008b).

While the road towards including intangible items in national income accounts

was slow, today there is clear momentum towards the reporting and inclusion of a more comprehensive list of intangible items. The difficulties now lie in how best to account for these items as opposed to ignoring them all together.

The lack of complete data collection on intangibles at national levels has led researchers to devise differing approaches to obtain a clearer picture of the magnitude of intangibles. The following paragraphs briefly outline these approaches. Note that the focus will be on those papers that look at the economy as a whole, as opposed to only one segment, and that attempt to account for the sum of all types of intangible capital goods¹. When relevant, the link between each chapter of the thesis and the approach being introduced will be explained. To classify these different approaches, it will be useful to adopt an asset perspective framework. This framework is based on the fact that, in equilibrium, the cost of production of an extra unit of asset (or capital good) must equal its market value which must also equal the expected present value of the future earnings it generates.

The first method used in the literature to evaluate intangible investment will be referred to as the direct approach. At the root of most investigations into the level of intangible investment lies a dissatisfaction with the practice of national income

¹There is a large literature in the field of Industrial Organization which aims at uncovering the size or the value of some of the components of the intangible capital stock. These papers typically use panel or survey data which cover short periods of time or just some portions of the economy. The focus of this research strand is on industry dynamics as exemplified by their focus on firms' entry, exit, mergers and the dynamic of life and growth of population of firms or plants. Two indicative studies are the papers of Atkeson and Kehoe (2006) for the U.S. and Baldwin and Gelatly (2006) for Canada. The first paper focuses on the measurement of organizational capital for a panel of plants in the late 1980s and covers two years. The second paper investigates the expenditures of a set of Canadian firms on organizational change using survey data.

accountants in treating expenses on intangibles as operating costs. Given that intangibles are assets, they should be capitalized because they are not entirely used up in the production of final output. In this way, they ought to be treated as investment instead of being expensed as intermediate consumption goods (Nakamura, 2003a and Corrado *et al.* 2005.) This practice by national accountants may have been reasonable when investment in such assets was a negligible portion of total investment. However, that doesn't seem to be the case anymore. If businesses are employing substantial resources to create these intangible assets, it should be possible to discern the impact of these investments on various aspects of the economy. If not, then perhaps the omission is unimportant (Nakamura, 2001a). Using different data sources, these authors construct a data set to document the expenditures of firms on an identified list of intangible inputs. Today, there is a fair agreement among national accountants as to what the core components of this list should consist of (see Vosselman, 1998).

In the chapter, *Treating Intangible Inputs as Investment Goods: the Impact on Canadian GDP*, I adopt this direct approach to evaluate the magnitude of intangible investment. After elaborating on Corrado *et al.* (2005) who argue against the current convention in national income accounts, I examine the implication of treating intangible spending as the acquisition of final (investment) goods on estimates of Canadian GDP growth. I find that investment in intangibles by 2002 was almost as large as the investment in physical capital. This finding is in line with similar results for the U.S. and the U.K. reported by Corrado *et al.* (2005) and Marrano and Haskel (2006), respectively. Furthermore, I find that the growth in GDP and labor productivity may be underestimated by as much as 0.1 percentage point per year during

this same period. In light of the current debates at various statistical agencies about capitalizing intangibles, this chapter confirms the need to consider such expenditures as investments and to collect this data as an integral part of the system of national income accounts.

Another group of approaches to evaluating intangible investment attempt to back out their value by making use of the aggregate implications derived from macroeconomic models. These approaches depart markedly from the direct approach in that they impose specific structures on the data.

One such method, which will be referred to as the “value approach”, is based upon using either the securities value of corporations or more directly, a reinterpretation of the National Income Account (NIA) data. The use of the stock market data to measure the size of the capital stock was first introduced by Martin Bailey (1981) and subsequently updated and amended by Robert Hall (2001) to measure the size of intangible capital. This approach relies on a quantitative application of the baseline q-theory of investment which is used to generate the quantity of capital owned by firms. From this inferred capital stock, the component which is recorded in national income accounts is subtracted to obtain the unrecorded capital stock.

In the chapter, *The Stock of Intangible Capital in Canada: Evidence from the Aggregate Value of Securities*, I use this method to investigate the size of the intangible capital stock in Canada using newly released data on the market value of all securities in the economy. My findings indicate that the intangible capital stock accounts for 29% of the overall capital in Canada since 1994. Of this, the R&D reported by the

national accounts makes up only 23%. These results imply that official Canadian statistics failed to account for 26% of the value of the capital stock in their 2005 quarterly data collection. In addition, I find that the behavior of intangible capital stock is similar to that in the U.S. and the U.K. as reported by Hall (2001) and Eliades and Weeken (2004), respectively. The stock of intangible capital increases continuously until the year 2000, at which point it falls and then flattens out.

McGrattan and Prescott (2005a) and (2007a) also take a value approach but unlike Hall (2001), they use a general equilibrium growth model to match certain aggregate time series data. Their approach is based upon reframing national accounts in order to correct for the wrongly expensed intangible investment and to uncover the value of intangibles investment of corporations using a standard real business cycle model. McGrattan and Prescott (2007a) elaborate on this approach. Their findings will be contrasted with the results of the second last chapter described below.

A third method that was proposed to measure intangibles is to use the fact that intangibles are assets and evaluate the present value of the future flow of income that they generate by using a variation of the user-cost of capital model. This final method which will be referred to as the “earnings approach”, was undertaken by Hall (2003) with little success in finding any trace of intangibles. The framework of the final chapter, discussed below, has the potential to resolve the apparent puzzle found by Hall (2003) of little intangibles when using the earnings approach. This will be discussed in the conclusion chapter of the thesis.

The problem with the research described above is the reliance on the assumption

that the price of tangibles, which is reported by national income accounts, is equal to the price of intangibles (most likely based on the premise that they are both capital goods). This assumption ought to be at odds with the facts. On the one hand, the investment deflator for physical capital constructed by national accounts exhibits a downward trend starting in the mid-1950s. On the other hand, given that the relative cost of the main input to intangible production -skilled labor- rose substantially in the 80s and 90s, the price of intangibles should inherit this rise. As a result, the price of intangibles does not behave like the price of physical investment goods but actually moves in the opposite direction: it is flat in the 70s, rises progressively up to the year 2000 and declines afterwards.

In the chapter, *The Embodiment of Intangible Investment Goods: a Q-Theory Approach*, I use the findings of the direct approach for the U.S. to uncover the size of intangible capital using the value approach while allowing the price of intangibles to be different than the price of tangibles. I do this by extending the q-theory of investment to account explicitly for the production of intangibles and use the model to measure the contribution of embodied intangible goods to the overall capital stock. The investment decision of the firm is modeled as a two stage optimization problem: 1) a static decision over the optimal combination of intangible and tangible investment goods to produce a final aggregate investment good and then 2) a dynamic decision over the amount of final aggregate investment to acquire. The model quantitatively implies that the downward trend in the aggregate investment deflator series reported by national accounts, which accounts only for the presence of tangible investment goods, has a significant downward bias in the 90s. The model also shows that the

growth in the overall capital stock from the late-80s until 2000 was driven mainly by an increase in the contribution of intangibles. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the movements in the price of intangibles rather than focusing only on their rising share in overall investment.

Chapter 2

Treating Intangible Inputs as Investment Goods: the Impact on Canadian GDP

2.1 Introduction

This chapter constructs a data set to document firms' expenditure on a list of intangible items. The approach adopted follows the work of Corrado *et al.* (2005) where intangible investment is mainly measured at cost. The constructed data set is then used to measure the impact of capitalizing intangible investment, as opposed to expensing it, on GDP growth. It is found that intangible investment averaged 9.6% of GDP for the period 1998 to 2004. This investment is as large as the investment in physical capital. This result is in line with similar findings for the U.S. and the U.K. Furthermore, the growth in GDP and labor productivity are found to be underestimated by as much as 0.1 percentage point per year during this same period. In light of current debates at various statistical agencies about capitalizing intangibles, this study confirms the need to consider such expenditures as investments and to collect this data as an integral part of the system of national income accounts.

Attempting to take a similar approach applied at various time points during the past 10 years, Baldwin *et al.* (2005) notice that there are no reliable data in Canada that would give a complete account of expenditures on intangible capital. The lack of reliable data is most likely due to the nature of past surveys: whatever relevant data these surveys collected was likely out of curiosity rather than key elements that would be required by the system of national accounts. Hence, the data in Canada appears to be sparse, often discontinuous and when the data turns out to be almost complete, it is not analyzed in a comprehensive way.

In this chapter, I show that the data are rich enough to offer an estimate of the size of intangible investment. The measurement approach used consists of summing up the expenditures and costs involved in producing items that are considered to be intangible. Relying on a specific identification of certain items offers a lower bound for the value of intangibles in two ways: first, it will unlikely exhaust all the components of intangible capital because some non-core items are hard to value (see Vosselman, 1998) and second, the market value of intangible investment goods, in the short-run, can be higher than the cost of their production if there are scarcity rents, for example. Above all, this approach is arguably more reliable because it is not based on backing out the value of intangibles using a tightly specified model as is done in other approaches to measuring intangibles (see in particular Hall, 2001). This latter procedure bears the risk of being contaminated by generating intangible quantities and values that are very sensitive to the assumptions adopted; hence, producing estimates that are quite imprecise.

Nakamura (1999) was the first to look at the expenditures of firms over time on

two important intangible items at the aggregate level, namely R&D and advertising. His paper points to the increasing share of these intangible items from 1953 until 1997 while tangible investment in plant and equipment was no higher in the 1990s than in the 1950s and 1960s. Nakamura recognizes that it is necessary to account for the spending of firms on other items such as executive time, software and on a wider range of creativity costs to obtain a much clearer picture of intangible investment. This is what Nakamura (2001a) and (2003a) sets out to do: to collect data on the expenditures of firms on R&D (broadly defined), software, advertising and the like. The author finds that direct and indirect empirical evidence points to an investment in intangibles by U.S. private firms of at least one trillion dollars annually. This amount roughly equals U.S. gross investment in nonresidential tangible assets.

Corrado *et al.* (2005) build on Nakamura's approach by adopting an intangible classification which comes close to that advocated by OECD statisticians (Vosselman, 1998). They also take an extra step by distinguishing those expenditures that generate long-lasting revenue flows from those whose returns are exhausted too quickly. Corrado *et al.* (2005) reach the same conclusion as Nakamura (2001a, 2003a); namely that, by 2000, intangible investment was as large as the investment in tangible capital in the U.S. Moreover, they estimate that measured productivity growth increases by .25 percentage points per year between 1995 and 2002 if intangibles are capitalized. In Corrado *et al.* (2006), the authors follow up on their 2005 paper by extending their data coverage in time and using the investment in intangibles to build a series for the intangible capital stock from 1950 until 2005. This series allows them to conduct a growth accounting exercise and study the impact of capitalizing intangibles on

income shares. One of their major findings is that after 1995, capital (both tangible and intangible) deepening surpasses TFP as the principal source of growth.

The work of Corrado *et al.* (2005) was followed by similar studies conducted for the U.K., France and Germany, Japan and Netherlands respectively, by Marrano and Haskel (2006), Hao *et al.* (2007), Fukao *et al.* (2007) and van Rooijen-Horsten *et al.* (2008). These studies reach similar findings in terms of the rise in intangible investment throughout the 1990s. In the U.K., intangible investment reached 10.8% of GDP in 2004, a similar level as tangible investment. It is interesting to note that other studies found lower intangible investment levels: 8.3% of GDP in Japan between 2000 and 2002 (includes both the private and public sector), 7.6% of GDP in France in 2004, 6.5% of GDP in Germany in 2004 and 7.5% of GDP in Netherlands between 2001 and 2004. To date, no such work has been performed for Canada.

The main contribution of this chapter is to quantify the extent to which aggregate investment data is understated in Canada. This will provide a sense of the size of the current omission and how accurate our actual picture of the economy is. The second contribution is to provide more evidence to policy-makers that supports the movement towards the capitalization of intangibles, which is being advocated by many statistical agencies and national accountants.

The rest of the chapter proceeds as follows. Section 2 describes the current treatment of intangibles by the national income accounts and develops a framework to discuss the reasons against such convention. Section 3 details the methodological approach adopted through a discussion of the data construction. Section 4 reports

the data sources and manipulations. Section 5 discusses the findings from the data collection and contrasts the results with similar studies conducted in the U.S. and the U.K. Section 6 investigates the impact of capitalizing intangibles on the growth of GDP. Finally, section 7 concludes with a discussion of future work.

2.2 National Accounts Convention and the Rationale Against It

This section develops a three-sector framework to discuss the current treatment of intangibles by national income accounts and the consequence for GDP of capitalizing intangible expenditures. This framework, borrowed from Corrado *et al.* (2006), also facilitates the discussion that will ensue of the rationale against the current national accounts convention².

There are three sectors in the economy: 1) an intangible investment good sector, 2) a tangible investment good sector and 3) a consumption good sector. All production functions are assumed to be homogeneous of degree one. We also assume that there is perfect competition. These assumptions are necessary to ensure that the compensation of various inputs add up to the value of output. The production function for intangible output, tangible output and consumption goods is given respectively by $\dot{A} = F^A(L_A, K_A, A_A)$, $I = F^K(L_K, K_K, A_K)$ and $C = F^C(L_C, K_C, A_C)$. Each sector i produces its own output using labor L_i , its accumulated stock of tangibles K_i and its accumulated stock of intangibles A_i ³. The two investment goods accumulate

²Using a different set of assumptions, Howitt (1996) and Nakamura (2003b) develop related frameworks where intangibles are treated as investment goods.

³For ease of exposition, there are no exogenous sources of TFP growth.

according to

$$\begin{cases} K' = (1 - \delta_K)K + I \\ A' = (1 - \delta_A)A + \dot{A} \end{cases} \quad (2.1)$$

where the prime exponent denotes next period's stock quantity. The revenues for each sector is given by:

$$\begin{cases} p^A \dot{A} = wL_A + r^K K_A + r^A A_A \\ p^I I = wL_K + r^K K_K + r^A A_K \\ p^C C = wL_C + r^K K_C + r^A A_C \end{cases} \quad (2.2)$$

The aggregate amount of labor, physical capital and intangible capital are defined respectively by $L = L_A + L_K + L_C$, $K = K_A + K_K + K_C$ and $A = A_A + A_K + A_C$.

In this framework where intangibles are treated as final capital goods, the NIA identity will be written as:

$$\underbrace{p^Q Q}_{\text{Value added approach}} = \underbrace{p^C C + p^I I + p^A \dot{A}}_{\text{Expenditure approach}} = \underbrace{wL + r^K K + r^A A}_{\text{Income approach}} \quad (2.3)$$

where $p^Q Q$ is defined as the total value added produced in this economy. The current treatment of intangibles by NIA views them as intermediate inputs. Therefore, NIA implicitly assumes the following identity:

$$\underbrace{p^Q Q - p^A \dot{A}}_{\text{Value added approach}} = \underbrace{p^C C + p^I I}_{\text{Expenditure approach}} = \underbrace{r^K K + wL}_{\text{Income approach}} . \quad (2.4)$$

An obvious consequence of the current practice by NIA is that GDP and income are undervalued. But why is there such a convention in the first place? The argument

often used, to paraphrase Griliches (1994), is that intangibles are “difficult-to-measure goods” for two main reasons⁴: the first lies in the fact that these goods are rarely exchanged on the market and are mainly produced in-house. As a result, there are no market transactions⁵. This observation has three major implications. The first is that p^A becomes a shadow price which needs to be calculated using a specific economic model. The second implication is that the quantity data \dot{A} is difficult to obtain or to separate in the magnitude $p^A \dot{A}$. Finally, it is difficult, from an accounting point of view, to verify the truthful and accurate reporting of $p^A \dot{A}$, given that \dot{A} is mainly produced inside firms.

The second reason intangibles are difficult to measure is related to the special attributes these goods carry, which are not found in physical investment goods or in consumption goods⁶. The first attribute is visibility: vintages of the stock A are difficult to observe. This means that the depreciation rate δ_K is difficult to obtain. The second attribute is appropriability. Intangibles create externalities which mean that the measured p^A and r^A may only reflect private benefits and costs. In other words, they do not accurately reflect the true value or real compensation they deserve. The third attribute is the rivalry characteristic of some of the intangible goods: the marginal cost to produce an extra unit tends to zero, which concretely implies that $p^A \approx 0$. The fourth attribute is the fact that intangibles often have the characteristic of a public good: the same quantities are available to all users at the same time. This

⁴Corrado *et al.* (2006) and Nakamura (2001b) have a similar discussion. The discussion provided here builds and expands on theirs.

⁵What accountants refer to as the existence of an arm’s length transaction.

⁶Although many of the following features could also characterize some forms of tangible capital. The emphasis here is on the disproportionate extent to which they are present in intangibles than in tangibles

means that $A \neq A_A + A_K + A_C$ but instead $A = A_A = A_K = A_C$. The fifth attribute is related to uncertainty in the outcome of the production of self-constructed goods. Indeed, there is always a positive probability that the production process leads to $\dot{A} = 0$. A sixth attribute is that intangibles include elements that are short-lived and some that are long-lived. Concretely, this means that $\delta_A \in [0, 1]$ (*i.e.*, for some of intangible investment goods, depreciation can be of 100%). A final attribute is the lag that can exist between the production of intangibles and their full exploitation.

Do these characteristics provide grounds for the convention of viewing them as expenses as opposed to investment goods? I argue that the answer is no: the issue of expensing versus capitalizing an input should depend on the type of input (capital versus intermediate input) not on: 1) the ease of measurement of the input or 2) the differences in the economic attributes of some inputs.

What distinguishes an intermediate input from a capital good? Capital formation is defined as the expenditure on inputs that will not be consumed by firms in the accounting period. Consumption by firms is the act of using up goods and services in the current period (United Nations (1998)). These “consumed” goods are known as intermediate inputs. Capital is then a produced good “that is used repeatedly or continuously in production over several accounting periods (more than one year)” (United Nations (1998)). A business expenditure that aims to acquire a capital good will be recorded by national income accounts as capital formation only if it is *identifiable* and if it involves the *acquisition* of a capital good from the market instead of being produced in-house.⁷ The requirement of identification is met whenever national

⁷Software expenditures are an exception. Since 2001, even when produced in-house, software is treated as capital expenditure (see Statistics Canada (2001)).

income accountants can classify the expenditure on the item under a well-defined category of products. On the other hand, the necessity to observe that the item was acquired from the marketplace ensures the existence of an accurate valuation of the good which is captured by the market price.

Given these two requirements and the definition of capital formation, all capital expenditures by firms which are either non-identifiable or are intended to produce a capital good in-house do not end up being recorded as capital formation. The convention in national income accounts is to treat this spending as intermediate consumption expenditure. Consequently, this practice lowers the value added of final produced output and understates the existing stock of capital in the economy.

Research and experimental development (R&D) expenditures offer a good illustration of the consequence of this convention. Even though national income accounts incorporate data on R&D spending, this expenditure is treated as an expense rather than an investment mainly because of the lack of a market price on the output of R&D activities⁸. Training expenses constitute a different example where no data is systematically collected by national income accounts since it is a difficult good to identify or classify.

Intangible investment is believed to have been small before the 1990s, so the issue of their capitalization was seen to be minor. However, both direct and indirect evidence points today for an important share of these expenditures in overall investment.

⁸Some R&D spending leads to the creation of a patent which will carry a price if commercialized. However, the market for patents is extremely thin: very few patents change hands. For example, Serrano (2006) documents that only about 20% of all U.S. patents issued to small innovators (*i.e.*, firms that were issued no more than five patents in a given year) are traded once or more.

Moreover, given the official definitions of capital goods and intermediate inputs, intangibles have all the characteristics of capital goods and hence should be collected by national income accounts as investment goods.

2.3 Data Collection Approach

This chapter uses a “direct” measurement approach which consists of summing up the expenses and costs involved in purchasing or producing items that are considered to be intangible. A consensus has emerged over time among national income accountants on what those items should be. Vosselman (1998) outlines the core components of intangible investment as: R&D, education and training, software, marketing, mineral exploration, licenses, brands, copyrights and patents. The supplementary categories of intangible investment are: organizational development, engineering and design, construction and use of databases, remuneration for innovative ideas and other human resource development (training excluded).

In the selection of items for which data will be collected, I follow the categorization of Corrado *et al.* (2005) of intangible investment. After identifying and listing the items that represent intangible goods, I investigate the sources that might supply data regarding the spending on such goods. The expenditures are then converted into investments by retaining the fraction which will be accumulated over time. Finally, I calculate the new real investment and add it to real GDP in order to calculate the resulting new growth rate.

There are four types of data approaches that are adopted to document the spending of firms on intangibles. The first data that is collected is “bought-in” expenditure data. These data consist of items that have a recorded transaction on the market. These data are typically available if a survey of purchases exists, for example, in the case of prepackaged software. A second approach to collecting data is a consequence of the non-existence of bought-in expenditure data. It consists of gathering the revenue estimates of knowledge-good providers. For example, the revenues of the advertising industry can be used as an approximation for firms’ expenditures on advertising. A third type of data that is used stems directly from activities of the firm, which is known as “own-account” spending or “self-constructed” goods. These data are hard to collect without a particular survey, for example, the R&D surveys. Finally, when all else fails, it is necessary to make an educated guess on the size of some expenditures given certain background information. For example, the own-account spending on organizational change and development is set as 20% of the wage of executives by Corrado *et al.* (2005).

The following is a detailed list of intangibles and their definitions:

1. **Innovative property**

- Scientific R&D: this item is mainly made of industrial R&D (science and engineering R&D) that usually leads to a *patent* or a *license* and can be split into:
 - R&D conducted in manufacturing, utilities, construction and agriculture.

- Spending for the discovery of new natural reserves through mineral exploration and other geophysical and geological explorations:
 - * R&D expenditures in mining, oil and gas extraction.
 - * Other geophysical and geological explorations.
- Non-scientific R&D
 - Information-sector industries: spending for the development of entertainment and artistic originals usually leading to a *copyright* or *license*.
 - Other new product development: this category encompasses new product development, design and research expenses that do not necessarily lead to a patent or copyright:
 - * Financial sector : new product development costs in the finance, insurance and real estate industries.
 - * Other service industries: estimates of R&D in the remaining services industries.

2. Computerized information

- Computerized databases: encompasses the expenditures on data processing activities (processing of data, data entry, data scanning, etc.) and database activities (on-line database publishing, on-line directory publishing, etc.).
- Software: comprises own-account spending on software (software developed inside the firm) and purchased software (either custom made software or general purpose software).

3. Economic competencies

- Brand equity: expenditures for the development of brands and trademarks made of two activities:
 - Purchased advertising: spending on the acquisition of advertising services.
 - Market and consumer research: either conducted inside the firm or purchased from the industry.
- Firms' investment in human capital: or employer-provided training is made of:
 - Direct firm expenses on training: in-house trainers, tuition payment, etc.
 - Indirect firm expenses: lost output from employees being trained *i.e.*, the opportunity cost measured as the wage of employees.
- Organizational structure: costs of organizational change and firm development composed of:
 - Purchased organizational change and development: typically from the “management consulting” industry.
 - Own account: estimated as the dollar value of senior executive time spent on developing business models and corporate cultures.

2.4 Data Sources

This section documents the data sources used for each intangible item. The North American Industrial Classification System (Statistics Canada (2007)) codes were relied upon as much as possible as they allow a very rich and well-defined description of specific items. Also, using these codes will make the data even more comparable across countries, given the international character of the NAICS.

2.4.1 Computerized Information

Computerized information is made of two items: software and computerized databases. I will begin with software. “Software refers in general to the encoded instructions executed by electronic devices, including computers, for performing operations and functions. This includes both systems software and user tools (operating systems, compilers, performance measurement and job accounting tools, etc.) and applications software (word processing, spreadsheets, payroll systems, etc.)” (Jackson (2001)). In 2001, Statistics Canada started producing software data, in conformity with the new SNA 1993 guidelines, and including it in GDP as part of non-residential investment. These data are available today from 1981 until 2007 under the CANSIM series label V3860272. Computer software expenditures are made of own-account spending on software (*i.e.*, developed in-house) and purchased software (either custom-made or pre-packaged). Note that the expenditure on software developed in-house for firms’ own-use is approximated by the wage bill of computer programmers and system analysts. Figure 2.1 shows the evolution of overall software

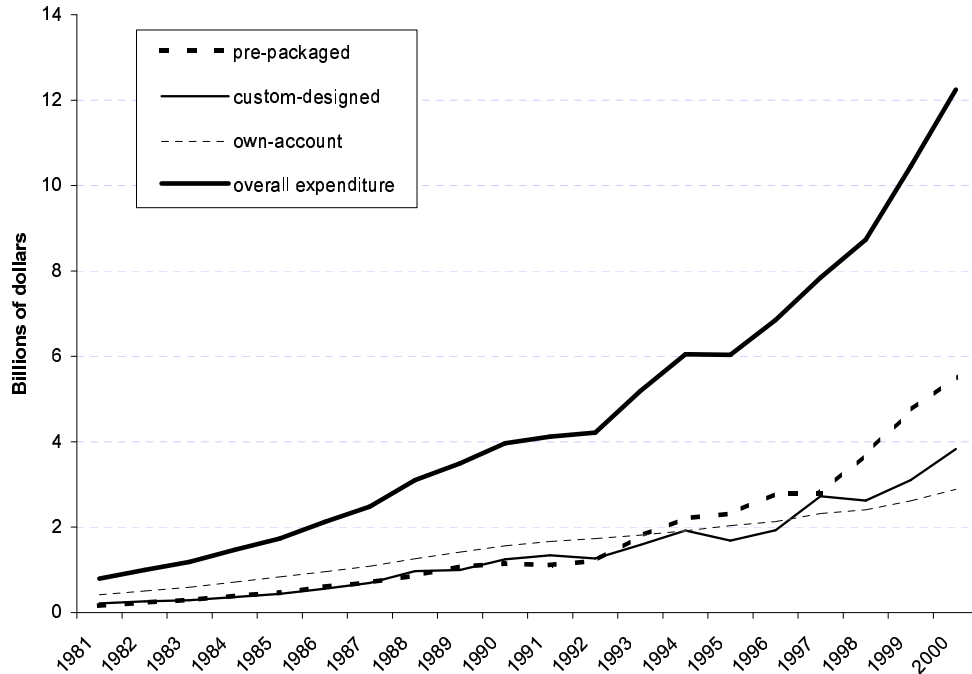


Figure 2.1: Software Investment (overall and by type). Source: Jackson (2001).

investment from 1981 to 2000 and the investment in each type of software during this same period. The expenditure on software rose in almost an exponential way. Expenditures on all types have experienced a similar rise although pre-packaged software dominates other types of software expenditures. In 2000, commercial software represented about 45% of business expenditures on software with own-account and custom-design software accounting for respectively 25% and 30% of total expenditures on software.

As for the second item, computerized databases, the “Annual Survey of Software Development and Computer Services” collects data on the revenues of firms involved in “data processing, hosting and related services” [NAICS 51821]. The activities covered by the sampled establishments “include specialized hosting activities, such as

web hosting, streaming services or application hosting, or may provide general time-share mainframe facilities to clients. Data processing establishments may provide complete processing and preparation of reports from data supplied by the customer; specialized services, such as automated data entry; or they may make data processing resources available to clients on an hourly or time-sharing basis.” (Statistics Canada (2007)). These data are available from 1997 to 2005 under the CANSIM series label V1929941. The own-account spending on computerized databases is likely included in the own-account spending on software given how this latter item is calculated (see former paragraph). As a result, there is no special provision made for the computerized databases developed in-house. Note that the R&D conducted in this sector is collected as part of the business enterprise research and development (BERD) tables under “information and cultural industries” [NAICS 51]. It is necessary to subtract R&D of data processing companies from their revenues to avoid double-counting. Unfortunately, the R&D in this sector is inseparable from the broader R&D of the “information and cultural industries”. Given there are 9 sectors within this industry, the R&D of the data processing companies was approximated as one-ninth of the industry’s overall R&D expenditure.

2.4.2 Innovative Property

It is possible to distinguish two broad categories of R&D: scientific and non-scientific.

The scientific component can be considered as industrial R&D, an activity which typically leads to a patent. This component is made up of two items. The first item

consists of R&D expenditures in manufacturing, utilities and construction and agriculture. These data are collected under the CANSIM vectors V29793132, V29793128, V29793131 and V29793121, respectively. These data are collected as part of the BERD tables and are available by industry group based on NAICS at an annual frequency from 1994 until 2007. Some data observations are missing in some series due to confidentiality agreements. Wherever this problem was encountered, two adjacent observations were used to make an extrapolation.

The second item consists of spending on the discovery of new natural reserves through mineral exploration and other geophysical and geological explorations. It represents R&D of the “mining, oil and gas extraction” industry, collected as part of BERD and given by the CANSIM vector V29793125, and of “other geophysical and geological exploration”. This latter item is approximated from the output of the surveying and mapping services collected under the “Annual Survey of Service Industries Surveying and Mapping”. The corresponding CANSIM vector is V1929009.

Non-scientific R&D is made up of two large categories: the information-sector industries and other new product development. The information-sector industries are basically the service-sector’s R&D which leads to a copyright. These industries are covered under the umbrella of “information and cultural industries” [NAICS 51]. It includes the publishing industries (including software publishers), motion picture and sound recording industries, broadcasting and telecommunications. This data is collected under the CANSIM vector V29793164. It is necessary to exclude the R&D of “software publishers” [NAICS 5112] as it has already been counted in software expenditures. Unfortunately, the R&D of this sector is inseparable from the broader

R&D of the “information and cultural industries”. As mentioned earlier, given there are 9 sectors within this industry, R&D of software publishers was approximated as one-ninth of the overall R&D expenditure of this industry.

The second component of the non-scientific R&D category “other new product development”, consists of the design and research expenses that do not necessarily lead to a patent or copyright. It includes the financial sector, which is made of the finance, insurance and real estate industries [NAICS 52, 53]. Those data are available as part of BERD from 1994 until 2007 under the CANSIM series label V29793165. It also includes all R&D conducted in the remaining service industries (wholesale and retail trade [NAICS 41, 44-45], transportation and warehousing [NAICS 48-49], architectural and engineering services [NAICS 5413], health care and social assistance [NAICS 62], etc.). These are published by Statistics Canada under the CANSIM label V29793160 (corrected here to omit the financial sector and the information-sector industries). To avoid double-counting with other categories, it is necessary to remove from this series the R&D conducted by the “computer systems design and related services” [NAICS 5415] and by the “management, scientific and technical consulting services” [NAICS 5416] since they are both part of this category of remaining service industries⁹.

⁹Corrado *et al.* (2005) have a separate category for R&D conducted in the social sciences and humanities (firms involved in conducting fundamental and experimental research in economics, sociology, and related fields). It is not possible to single out such category here since R&D specific to the social sciences and humanities [NAICS 54172] is inseparable from the broader category of “scientific research and development services” [NAICS 5417] which is part of the “other service industries”.

2.4.3 Economic Competencies

The category of economic competencies is made up of three items: brand equity, training and organizational change.

The cost of the development of a brand involves two activities: advertising and market and consumer research. Data on purchased advertising is approximated by the revenues of firms involved in “Advertising and Related Services” [NAICS 5418]. These establishments are primarily engaged in “creating mass-media advertising or public relations campaigns; creating and implementing indoor/outdoor display advertising campaigns, direct mail advertising campaigns and specialty advertising campaigns; placing advertising in media for advertisers or advertising agencies, etc.” (Statistics Canada (2007)). These data have been collected by Statistics Canada from 1997 to 2005 under the “Annual Survey of Advertising and Related Services” and are published under the series label V1927659.

As for the spending on market and consumer research, the data are taken from a 2004 survey of the Canadian market research industry by Datamonitor. The industry “consists of the provision of services involving the collection and analysis of information about consumers, businesses and markets” (Datamonitor (2004)). The report provides the revenues of this industry from 1999 until 2003 with projections as far as 2008. The report notes that 16% of the revenues came from the public sector in 2003. This ratio will be used throughout the period to remove the share of the public sector from the initial data set. The final number is doubled to account for intramural market research following the practice of Corrado *et al.* (2005).

Training costs consists of direct and indirect expenses. The direct expense is the cost of developing workforce skills (*i.e.* on-the-job training by in-house trainers, outside trainers, tuition reimbursement for job-related education, and outside training funds). The indirect expenses are made of the opportunity cost of the training activity (*i.e.*, the value of lost output) which is approximated by the wage and salary of employee time spent in formal and informal training. There are no training expenditure data collected by Statistics Canada which made the training cost data overall hard to obtain. As a result, some assumptions had to be made in order to put a dollar value on this activity. The direct firm expenses on training per employee are estimated by the Conference Board of Canada and published in “Learning and Development Outlook 2005” (pp.6) for the years 1996, 1998, 2000, 2002, 2004. These expenses are multiplied by the employment data produced by Statistics Canada under the “labor force survey” and given by the label series V2461119. Data for the missing years is extrapolated as an average of the two adjacent years. The indirect data on wage and salary costs of employee time are arbitrarily set at the same amount as the “direct firm expenses”.¹⁰

Finally, firms’ expenditure on organizational change is estimated by looking at two items. The first item is the portion that is purchased. To proxy these purchased new capabilities, we use the revenue of “management, scientific and technical consulting services” [NAICS 5416] which consists of “management consulting services” [NAICS 54161] and “scientific and technical consulting” [NAICS 54162, 54169]. These data are stored in CANSIM under the series label V1929084 from 1998 to 2005. The

¹⁰This seems to have been the case for the U.K. in 2004 from the work of by Marrano and Haskel (2006). Corrado *et al.* (2005) find that indirect costs are 4 times the direct costs between the years 1998-2000.

second item is the “own account” component estimated as 20% of the wage of senior executives by Corrado *et al.* (2005). This component was only available in the “census of population” data in real terms for the two years 1995 and 2000. These data are collected using the 1991 Standard Occupational Classification and the wage bill used here was the one of “senior management occupations”. The wage bill in 2000 was multiplied by 20% to obtain an estimate of own account organizational development. The ratio of purchased to own account expenditures in the year 2000 is applied to the amounts purchased in all other years in order to obtain the own account data for the remaining years.

2.5 Summary of Findings and Comparison with Other Countries

2.5.1 Findings for Canada

Table 2.1 details firms’ expenditures on each intangible item as a percentage of the GDP reported by national accounts¹¹.

Notice that the shares of each of the major three categories of intangibles in GDP from 1998 to 2004 are quite stable. The share of computerized information is the smallest and comprises about 1.1% of the reported GDP. The ratio of spending in the economic competencies category is second in size, with an average of 3.75%. Finally, the ratio of the innovative property is the largest, with an average of 4.6%.

Overall, intangible investment averaged 9.6% as a percent of GDP for the period

¹¹Appendix A reports the constructed data series on the spending in intangibles, by item.

Type of intangible investment	Spending as a % of Reported GDP							
	1998	1999	2000	2001	2002	2003	2004	2005
Computerized information	1.09	1.07	1.02	1.19	1.11	1.11	1.06	1.03
Computer Software	0.97	0.95	0.87	0.98	0.89	0.89	0.85	0.83
Computerized databases	0.12	0.12	0.15	0.21	0.22	0.22	0.21	0.20
Innovative property	4.30	4.35	4.49	4.72	4.78	4.80	4.83	4.97
Scientific R&D	1.65	1.68	1.79	1.97	1.91	1.86	1.87	1.90
Mineral exploration	1.04	1.03	1.00	1.08	1.10	1.06	1.09	1.11
Non-Scientific R&D	1.61	1.64	1.70	1.68	1.77	1.87	1.88	1.96
Copyright and license costs	0.03	0.03	0.03	0.05	0.05	0.09	0.10	0.11
New product dvpmt costs in financial industry	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03
R&D in remaining service industries	1.57	1.60	1.66	1.61	1.70	1.76	1.75	1.82
Economic competencies	3.99	4.07	3.98	3.83	3.90	3.73	3.84	3.79
Brand equity	0.56	0.54	0.54	0.54	0.52	0.48	0.48	0.50
Advertising expenditure	0.47	0.45	0.45	0.45	0.42	0.39	0.39	0.41
Market research	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
Firm specific human capital	2.38	2.40	2.36	2.15	2.23	2.13	2.26	2.16
Direct firm expenses	1.19	1.20	1.18	1.08	1.11	1.06	1.13	1.08
Wage and salary costs of employee time	1.19	1.20	1.18	1.08	1.11	1.06	1.13	1.08
Organizational structure	1.04	1.13	1.08	1.14	1.16	1.12	1.10	1.13
Purchased	0.66	0.71	0.68	0.72	0.73	0.70	0.69	0.71
Own account	0.39	0.42	0.40	0.42	0.43	0.41	0.41	0.42
Total	9.38	9.49	9.49	9.74	9.79	9.64	9.73	9.78

Table 2.1: Decomposition of Intangible Expenditures by Item

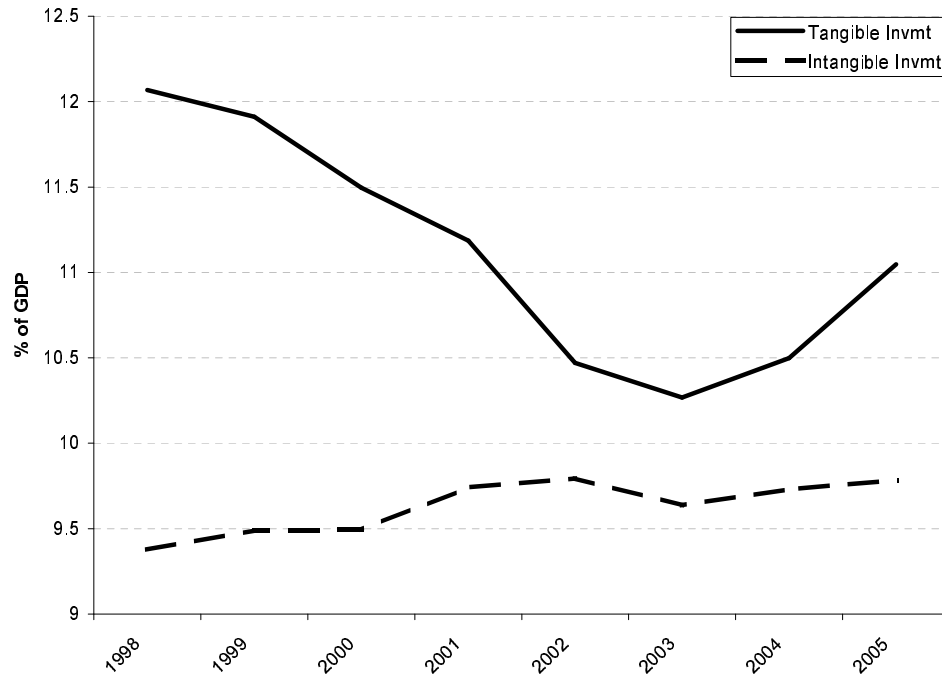


Figure 2.2: Tangible Versus Intangible Investment.

1998 to 2004. This intangible investment almost matched the investment in physical capital around the year 2002¹². Figure 2.2 shows the evolution of both tangible and intangible investment. Notice that as tangible investment declines until 2000, intangible investment continues to climb, reaching its peak at almost 10% of GDP. After 2001, both forms of investment flatten out, then pick up steam in 2003, albeit at different speeds.

¹²Tangible or physical investment is defined as business investment in non-residential structures and equipment as reported by Statistics Canada under the vector V647541. Note that software investment was removed from this vector given it is considered as an intangible investment.

2.5.2 Comparison with the U.S. and the U.K.

The findings for Canada are contrasted with the findings for the U.S., by Corrado *et al.* (2005) for the years 1998-2000, and with the findings for the U.K., by Marrano and Haskel (2006) in 2004. Table 2.2 shows each category of intangible investment and its share in GDP for each country. As mentioned earlier, the investment in intangibles in Canada almost matches the investment in physical capital in 2002. It is striking to find the same observation in the U.S. and the U.K. However, Canada's investment in intangibles is lower than the U.K. and the U.S., although not far from the level reported for the U.K. This result is mainly driven by a lower investment in economic competencies and in particular, in brand equity and intramural organizational change.

2.6 Impact of Including Intangibles on GDP

It is important to explore the consequence of the omission of intangibles not only on aggregate investment but on real GDP growth. Table 2.3 reports the growth rates of real GDP with and without intangible investment and the resulting discrepancy.

Overall intangible spending from 1998 to 2004 was deflated by the GDP deflator to obtain the real intangible spending of firms. The resulting series obtained is added to real GDP and the new growth rate of GDP is calculated and then compared to that obtained without intangibles. I find that real GDP growth in Canada is, on average, understated by 0.1 percentage point per year for the period considered with a standard deviation of 0.23 percentage points. The period from 1999 to 2001 experienced an under-estimation while the period from 2002 to 2003 were over-estimated.

Type of intangible investment	Spending as a % of GDP		
	US (98-2000)	UK (2004)	CAN (2000)
Computerized information	1.65%	1.70%	1.02%
Software: purchased		1.70%	0.87%
Computerized databases			0.15%
Innovative property	4.58%	3.23%	4.49%
Scientific R&D	1.98%	1.06%	1.79%
Mineral exploration	0.19%	0.04%	1.00%
Non-Scientific R&D	2.41%	2.13%	1.70%
Copyright and license costs	0.81%	0.21%	0.03%
New product devpmt costs in financial industry	0.79%	0.69%	0.01%
New architectural & engineering designs	0.73%	1.20%	1.66%
R&D in social science and humanities	0.08%	0.03%	
Economic competencies	6.91%	5.94%	3.98%
Brand equity	2.53%	1.59%	0.54%
Advertising expenditure	2.33%	1.20%	0.45%
Market research	0.20%	0.39%	0.09%
Firm specific human capital	1.25%	2.44%	2.36%
Direct firm expenses	0.24%	1.27%	1.18%
Wage and salary costs of employee time	1.01%	1.17%	1.18%
Organizational structure	3.13%	1.91%	1.08%
Purchased	0.87%	0.60%	0.68%
Own account	2.26%	1.31%	0.40%
Total	13.14%	10.87%	9.49%

Table 2.2: Comparison of Expenditures in Intangibles Across U.S., U.K. and Canada

Table 2.3: Impact of Including Intangibles on Real GDP Growth

Year	GDP Growth with Intangibles	Reported GDP Growth	Discrepancy
1998			
1999	5.87%	5.53%	0.34%
2000	5.26%	5.23%	0.03%
2001	2.03%	1.78%	0.25%
2002	2.87%	2.94%	-0.07%
2003	1.53%	1.82%	-0.29%
2004	3.44%	3.30%	0.14%

In comparison, Corrado *et al.* (2005) find that U.S. GDP growth is underestimated by about 0.25 percentage point per year during a similar period.

2.7 Conclusion

This chapter follows a direct approach to document firms' expenditures on an identified list of intangible inputs for which there is now wide agreement among national accountants. The implications of treating intangible spending as an acquisition of final (capital) goods on GDP growth for Canada were then examined. Intangible investment averaged 9.6% of GDP per year for the period 1998 to 2004 and was found to be almost as large as the investment in physical capital around the year 2002. This result is in line with similar findings for the U.S. and the U.K. However, Canada's investment in intangibles is slightly lower than the U.K. and quite a bit lower than the U.S. Finally, I find that the growth in GDP and labor productivity may be underestimated by as much as 0.1 percentage point per year during this same period. The discussion on the need to capitalize intangibles and the magnitude of the findings

demonstrate the necessity to report such expenditures as investments and to collect this data as an integral part of the system of national income accounts.

Chapter 3

The Stock of Intangible Capital in Canada: Evidence from the Aggregate Value of Securities

3.1 Introduction

Many firms devote resources to the production of capital goods that are not intended for commercialization but instead are kept in-house. Such capital goods include research and development expenditures, training expenses, brand equity and organizational change and development. For reasons explained below, this capital formation is not recorded in national income accounts. As a consequence, little is known about its overall size. Using newly released Canadian data, this chapter represents, to my knowledge, a first attempt to uncover the magnitude of this unrecorded capital stock in Canada.

The methodology followed is a quantitative application of the q-theory of investment. The market value of all firms in the economy is given by the aggregate value of their net financial liabilities. Assuming that investors price securities rationally, the value of these liabilities can then be used to infer the overall stock of installed

capital inside firms. From this inferred capital, I subtract that part which is recorded in national income accounts and back out the unrecorded capital stock. Baily (1981) pioneered this approach by empirically linking the value of securities to the stock of capital and Hall (2001) applied it using a q-theory of investment to infer the stock of capital in the U.S. economy. This chapter applies Hall's approach to Canadian data.

What does this unrecorded capital consist of? The two preceding chapters discussed how all capital expenditures by firms which are either non-identifiable or are intended to produce a capital good in-house do not end up being recorded as capital formation. The convention in national income accounts is to treat this spending as intermediate consumption expenditure. Consequently, such practice understates the existing stock of capital in the economy. Accountants distinguish two categories of capital goods: tangible and intangible. Tangible capital goods comprise a list of items that have a physical embodiment such as machines, tools and equipment. On the other hand, intangible capital goods are associated with items that have a knowledge or informational component such as patents, copyrights and brands. Since most intangible capital is created in-house with the two characteristics of being difficult to identify and generally not acquired or sold on the market, most unrecorded capital is often assimilated to, or completely identified with, intangible capital. This needs not be the case but it points to natural candidates that would compose the stock of the unrecorded capital that this chapter identifies. Indeed, the unrecorded capital could, for example, include mismeasurement of the tangible capital stock. Therefore, equating the unrecorded capital stock with intangible capital is only an approximation. However, for the sake of exposition, the terms "unrecorded capital" and "intangible

capital” will be used interchangeably throughout this text.

Notwithstanding, preliminary estimates point to an average investment level in intangibles in the U.S. of 6% of GDP in the early 1990s. By the late 1990s, investments in intangible capital by U.S. businesses were argued to have been as large as investment in traditional, tangible capital (Corrado *et al.*, 2005). The picture in Canada is less clear. Baldwin *et al.* (2005) note that there are no reliable data in Canada that would give a complete account of expenditures on intangible capital.

As detailed below, such a state of affairs can be remedied by taking the indirect approach of Hall (2001) to measuring the stock of capital. I find that the size of the intangible stock has been increasing from 1994 to 2001 and averaged 29% of the overall capital from 1994 to the middle of 2006. The nature of this stock is shown to consist of about 23% R&D capital with the rest made up of other intangible capital goods. This finding implies that official Canadian statistics failed to account for \$380 billion worth of capital stock in their 2005 quarterly data collection or about 26% of the inferred capital stock. The results obtained mirror qualitatively the findings of similar approaches conducted for the U.S. and the U.K. which document a substantial rise in the size of the intangible capital stock up to 2001.

The chapter is organized as follows: the next section reviews the related literature, section 3 introduces the model, section 4 describes the data, section 5 presents the empirical results, section 6 outlines the sensitivity analysis conducted to assess the robustness of the findings and the chapter concludes with a comparison of the results of Chapter 2 with the findings in this Chapter.

3.2 Relationship to Related Literature

There is a growing body of literature that attempts to evaluate the size of unrecorded capital investment at the aggregate level as discussed in the previous two chapters. At the root of most of these investigations lies a dissatisfaction with the practice of national income accounts in treating expenses on intangibles as intermediate inputs or operational costs. Given that intangibles are assets, they should be capitalized and treated as investment goods instead of being expensed as intermediate consumption goods.

Hall (2001) and McGrattan and Prescott (2005a) rely on the unmeasured levels of intangible capital to rationalize the rise in the stock market in the late 90s in the U.S. and in the U.K. Hall (2001) shows that the rise in the stock market coincides with an ever increasing accumulation of intangible capital. McGrattan and Prescott (2005a) are able to rationalize the size of intangible investment found in Corrado *et al.* (2005) while using the change in tax regulations to explain the different performance of the U.K. and the U.S. stock markets. McGrattan and Prescott (2005b) show that by explicitly accounting for intangible capital, one can explain the productivity paradox. In particular, they argue that GDP in national income accounts is undervalued because of the expensing of intangible investment which ultimately created a downward bias in the estimates of productivity in the early 90s.

The paper of Eliades and Wecken (2004) applies Hall's methodology to the U.K. These authors find no trace of intangible capital for the U.K. before 1990 but reach the same qualitative results as Hall (2001) for the late 90s.

To the best of our knowledge, no studies to date have attempted to give a macroeconomic account of the overall stock of intangible capital in Canada.

3.3 Methodology

3.3.1 A Quantitative Approach to the Q-Theory

The model is a standard neoclassical model of investment as developed in Hayashi (1982). It ultimately relates the value of securities to the value of installed capital which then allows to back out the unobserved quantity of installed capital.¹³

There is perfect competition in input and output markets. The production function is homogeneous of degree one in capital, k , and labor, l , and is denoted by $F(k, l)$. Firms buy capital from the market or produce it in-house. The problem of the firm is to choose the optimal level of labor and investment so as to maximize the net present value of future profits subject to the technology of investment accumulation, the starting level of capital and the non-feasibility of Ponzi schemes:

$$\max_{\{x_s, l_s\}} v_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{F(k_{s-1}, l_s) - w_s l_s - x_s - C(x_s, k_{s-1})\} \quad (3.1)$$

s.t.

$$k_{s-1} \quad (3.2)$$

$$k_s = (1 - \delta)k_{s-1} + x_s \quad (3.3)$$

¹³The empirical performance of the q-theory of investment appears to be decent but not more (see Caballero (1999) for a survey). The belief here is that past tests of the theory suffered from specification problems by not taking into account the investment of firms in intangibles (Hall (2004) pp.914-915 provides a related discussion.) Moreover, the exercise in this chapter is not intended to test the theory but instead to explore its quantitative implications.

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T v_{s+T} = 0 \quad (3.4)$$

where k stands for capital stock, x for investment, δ for the depreciation rate, l for labor, r for the real interest rate, w for the real wage rate and $C(\cdot)$ for the adjustment cost function. The value function v_t is the net present value at time t of future payout to securities' holders. Indeed, after the firm pays inputs their due, the left over income is paid to owners. Their ownership materializes through the possession of titles in the form of securities. Hence, v_t is also the value of the firm.

It is assumed that there is perfect substitutability between the recorded investment by national income accounts and the unrecorded investment *i.e.*,

$$x_s = x_s^{Recorded} + x_s^{Unrecorded}. \quad (3.5)$$

Note that the unrecorded investment consists of the sum of both in-house produced capital goods (for example, training expenses that goes into producing human capital inside the firm) and the externally acquired capital goods. The latter are nevertheless expensed because of the convention in national accounts of expensing all intangible capital goods (the purchase of a patent for example). Notice also that this approach is not intended to explain the reasons for the firm's choice to not commercialize the in-house produced capital good. As such, there is no inherent difference between a capital good bought from the market and a capital good produced in-house; perfect competition will ensure that they both have the same price.

The Lagrangian \mathcal{L} at time t is given by

$$\mathcal{L}_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \{F(k_{s-1}, l_s) - w_s l_s - x_s - C(x_s, k_{s-1}) - q_s [k_s - (1-\delta)k_{s-1} - x_s]\} \quad (3.6)$$

where q is the Lagrangian multiplier or the shadow price of an additional unit of capital. The first order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial x_s} : \quad q_s = 1 + C_x(x_s, k_{s-1}) \quad (3.7)$$

$$\frac{\partial \mathcal{L}_t}{\partial l_s} : \quad w_s = F_l(k_{s-1}, l_s) \quad (3.8)$$

$$\frac{\partial \mathcal{L}_t}{\partial k_s} : \quad q_s(1+r) = F_k(k_s, l_{s+1}) - C_k(x_{s+1}, k_s) + (1-\delta)q_{s+1} \quad (3.9)$$

$$\frac{\partial \mathcal{L}_t}{\partial q_s} : \quad k_s = (1-\delta)k_{s-1} + x_s. \quad (3.10)$$

Note that bubbles in the shadow price of capital are ruled out, *i.e.* $\lim_{T \rightarrow \infty} (1+r)^{-T} q_{s+T} = 0$. Equation 3.7 illustrates the equality of the lifetime return to increasing capital by one unit with its marginal cost given by the price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. Equation 3.8 states the usual equilibrium condition for the labor market whereby the real wage is equal to the marginal product of labor. Equation 3.9 shows the dynamic equilibrium equation of the q with its continuation value. Finally, Equation 3.10 recasts the investment technology constraint.

Following Hayashi (1982), the adjustment cost function is quadratic and displays constant returns to scale:

$$C(x_t, k_{t-1}) = \frac{\alpha}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1}. \quad (3.11)$$

where α is the adjustment cost parameter. Its exact meaning will be explained below. Assuming $s = t$ and substituting this cost function into the first order condition that described the equality of the q with the marginal cost of augmenting capital by one

unit, we obtain the following equation:

$$x_t = \frac{1}{\alpha}(q_t - 1)k_{t-1}. \quad (3.12)$$

This is known as the investment equation since it relates the behavior of investment to the shadow price of capital q_t . Investment is positive when the lifetime return to increasing capital by one unit exceeds its price. This equation has limited empirical use since q_t is by definition a shadow price and therefore, it is unobservable.

Hayashi (1982) showed that by combining all the first order conditions with the no-Ponzi scheme constraint, the value of the firm v_t would be equal to the value of the installed capital $k_t q_t$. This finding combined with the first order condition of the equality of the q with the marginal cost of increasing capital by one unit results in a recursive system of the form:

$$\begin{cases} v_t = k_t q_t \\ q_t = 1 + C_x(x_t, k_{t-1}) \Leftrightarrow x_t = \frac{1}{\alpha}(q_t - 1)k_{t-1} \end{cases} \quad (3.13)$$

This approach is introduced in Hall (2001). The system of two equations can be solved for the pair (k_t, q_t) at each t given v_t and k_{t-1} . To obtain an explicit solution, rewrite the investment equation as

$$x_t = \frac{1}{\alpha}\left(\frac{v_t}{k_t} - 1\right)k_{t-1}. \quad (3.14)$$

Substituting the investment term x_t by the capital accumulation expression and re-arranging, we obtain the following quadratic equation:

$$\alpha k_t^2 + [1 - \alpha(1 - \delta)]k_{t-1}k_t - v_t k_{t-1} = 0 \quad (3.15)$$

Hall (2001) shows that a unique solution exists for a general convex cost function with constant returns to scale. This equilibrium is stable and is therefore not sensitive to initial conditions in the long-run. The positive root expresses the law of motion of the capital stock¹⁴:

$$k_t = \frac{1}{2\alpha} \left([\alpha(1 - \delta) - 1]k_{t-1} + \sqrt{([1 - \alpha(1 - \delta)]k_{t-1})^2 + 4\alpha v_t k_{t-1}} \right). \quad (3.16)$$

All variables are observable and v_t is a sufficient statistic to back out the stock of capital in the economy. k_t is therefore the endogenous variable to be calculated at each point in time. Notice here that at every t , $k_t = k_t^{Recorded} + k_t^{Unrecorded}$. $k_t^{Recorded}$ is the observed capital stock constructed from national income accounts by cumulating the recorded investment $x_t^{Recorded}$ overtime.

3.3.2 Empirical Strategy

The parameters in the law of motion of the capital stock need to be specified. For the sake of comparison, the same parameters as those in Hall (2001) are used. These are also used by Eliades and Weeken (2004). Section 6 analyzes the impact of specifying different ranges of parameters on the implied stock of capital k_t and further discusses the rationale for the choice of certain parameter values.

In order to account for irreversibility in investment, it is assumed that the cost function is piece-wise quadratic :

$$C(x_t, k_{t-1}) = \begin{cases} \frac{\alpha^+}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t > 0 \\ \frac{\alpha^-}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t < 0 \end{cases} \quad (3.17)$$

¹⁴Since a capital stock is a positive quantity, the negative root is meaningless in this context.

where the adjustment-cost parameter α^+ (α^-) represents the time it takes for the capital stock to double (halve) when q doubles (halves). To see this, rewrite the investment equation as

$$\frac{x_t}{k_{t-1}} = \frac{1}{\alpha}(q_t - 1). \quad (3.18)$$

If q doubles permanently, say from one to two, it will initially cause the investment-capital ratio to increase by $\frac{1}{\alpha}$. For the investment-capital ratio to double, the increase in $\frac{1}{\alpha}$ must be repeated for α periods. By allowing the downward adjustment-cost parameters to be higher than the upward adjustment-cost parameter, this asymmetry in the investment decision will reflect irreversibility of investment.

Hall (2001) cites the work of Shapiro (1986) to justify the choice of a doubling time parameter of 8 quarters. He also sets the downward adjustment-cost parameter to ten times higher than the upward adjustment-cost parameter. The depreciation rate of 2.6% per quarter is used by national income accounts for physical capital. Finally, to start the iteration on the law of motion of capital, the value of the initial capital stock k_{t-1} needs to be set. We will assume that the economy is in a steady-state equilibrium at the pre-initial quarter, *i.e.* q_{t-1} takes its equilibrium value of 1. Since investment will be nil at this pre-initial quarter, the relationship $v_t = k_t q_t$ implies that $k_{t-1} = v_{t-1}$. Because the recursion was shown to be insensitive to initial condition, this equilibrium assumption is not going to affect the behavior of the system in the long-run. In fact, the derivative of the capital stock in 1994 with respect to the initial condition is only 0.1 and dies to 0 soon after. It will be shown in the sensitivity analysis section that this assumption is inconsequential after the recursion runs for

Table 3.4: Parameter values

Name	Parameter	Value	Rationale
Upward adjustment-cost	α^+	8	Shapiro (1986)
Downward adjustment-cost	α^-	80	Hall (2001)
Depreciation rate	δ	0.026	Hall (2001)
Initial capital stock	k_{t-1}	v_{t-1}	Assuming $q_{t-1} = 1$ at $s = t - 1$

some quarters¹⁵.

Table 3.4 summarizes the parameter values used and the rationale for the choice of each value.

3.4 Data Description

The Canadian system of national accounts is made of three main accounts: the national income and expenditure account (NIEA), the financial flow accounts (FFA) and the national balance sheet accounts (NBSA). The FFA reports the flow of assets and liabilities that occur during a period while the NBSA reports the evolution of the stock of assets and liabilities overtime. The NBSA can be viewed as an aggregate account statement that merges all balance sheets of firms, for all sectors of the economy. The financial liabilities that are reported can be divided into three major categories: shares, bonds and other liabilities (loans and mortgages, short-term paper, trade payables, life insurance and pensions).

Until recently, the data on the aggregate value of financial instruments given by

¹⁵The focus throughout the text is on the period post 1994 given that the recursion is shown to be invariant to the initial capital level after 4 years. The results from 1990 and 1994 are therefore not as precise (see Figure 3.10 in the sensitivity analysis section.)

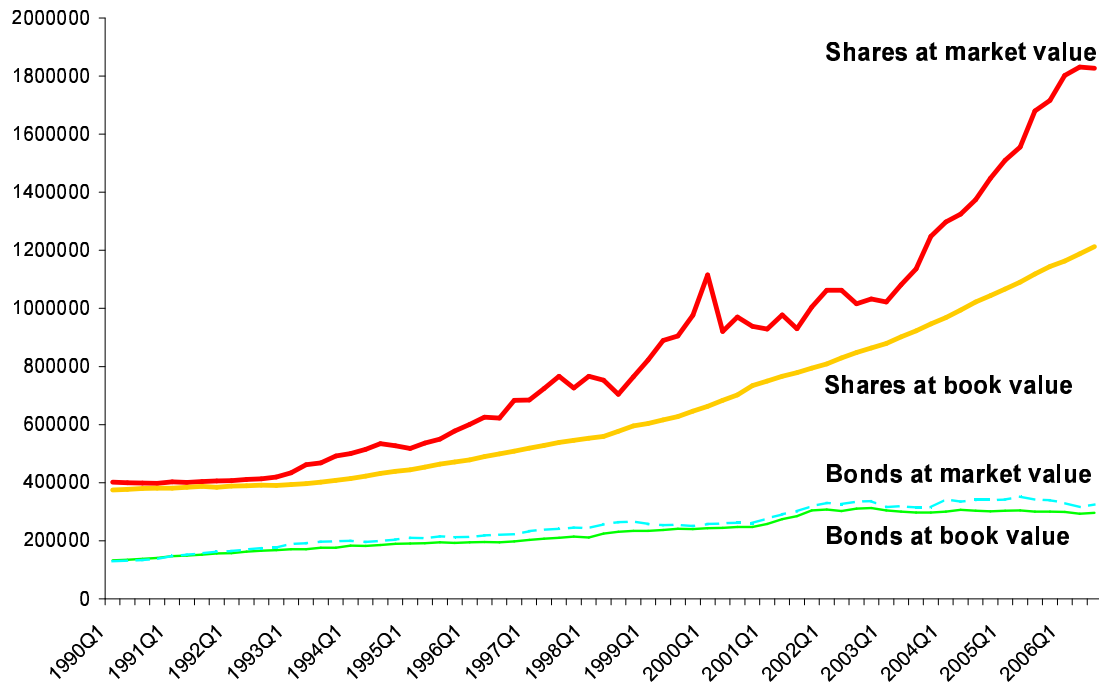


Figure 3.3: Shares and Bonds in Book and Market Value (in millions)

the NBSA was available only at book value. Since June 2004, Statistics Canada produces this data at market value. The data coverage starts in 1990 at quarterly frequency. This represents a major improvement in the reporting given that the discrepancy between the historical price of an asset and its current market value can be substantial, especially for equities and long-term financial assets. Figure 3.3 illustrates the difference in the size of shares and bonds when measured at book value versus at market value.

It is interesting to note that the balance sheet for the U.S. economy reports only equity at market value and leaves bonds at book value. Hall (2001) manipulates the stock of bonds at book value to obtain a series at market value. In the end, the two bonds' series evolve in the same way as those shown for Canada in Figure 3.3. This

cross-checking with an official statistical agency is reassuring and highlights that most of the variation in the market value of firms in the U.S. and in Canada is associated with equity.¹⁶

The market value of net financial claims (financial liabilities minus financial assets) is used as the measure of v_t since the value of the ownership claims is a reflection of the installed capital inside the firm. Indeed, v_t was defined as the present value of payouts to securities' holders. Assuming that investors are rational, it follows that the present value of payouts v_t will equal the value of securities on the market. Since for all t , $v_t = q_t k_t$, then the value of securities equals the value of the installed capital stock.

Notice that v_t includes all financial claims towards firms net of financial assets that firms hold against others. This represents a departure from the way the literature in the q-theory of investment interpreted v_t as covering only equity values or as consisting of equity plus bonds. This departure is mainly due to the availability of data.

Another way of understanding the equality of v_t with net financial claims is to rearrange the usual accounting balance sheet of a firm, like the one illustrated in Table 3.5¹⁷. A firm's balance sheet stresses the distinction between assets and liabilities.

The modified accounting framework shown in Table 3.6 uses the equality of assets and liabilities to recast the balance sheet into financial assets versus non-financial assets with the result that the two must be equal. As Table 3.6 indicates, net financial

¹⁶The financial instruments that constitute "other liabilities" were not displayed on the graph because their market value is reported to be similar to their book value. This is most likely the case given the majority of these financial instruments are not traded (for example, loans and mortgages).

¹⁷This exposition is borrowed from Hall (2001).

Table 3.5: Accounting Balance Sheet

Assets	Liabilities
Current Assets	Current Liabilities
Cash	Accounts payable
Accounts receivable	Notes payable
Marketable securities	Other payables
Inventory	
Long-Term Assets	Long-Term Liabilities
Land	Notes payable
Plant and Equipment	Bonds payable
Other Assets	Owner's Equity
Goodwill	Equity
Intangible assets	Retained earnings
=Total Assets	=Total Liabilities & owner's equity

liabilities serve as an estimate to the value of the firm's productive assets v_t , for all t . Note that this balance sheet approach is an identity (accounting convention) while the equality between assets and liabilities under the q-theory of investment is an equilibrium condition.

When conducting the data analysis, the focus will be on the non-farm, non-financial corporate sector. This sector is chosen because it is the most amenable to fit the perfectly competitive framework of this chapter. The removal of the farming sector aims to control for the presence in the overall capital stock of land, a capital input in fixed supply, which therefore earns rents. The choice of the corporate sector ensures that securities are continually priced to reflect accurately new information regarding the value of the capital stock. This would not be true for the installed capital of unincorporated businesses. Another reason to focus on this sector is dictated by the fact that the farming sector, the non-corporate sector and the financial sector

Table 3.6: Modified Accounting Framework

Financial claims	Nonfinancial assets
Equity outstanding	Plant and equipment
Debt outstanding	Land
Value of payables and other financial obligations	Inventories
	Intangibles
<i>Less</i> equity, debt, receivables, cash and other financial claims on others	<i>Less</i> nonfinancial assets claimed on others
=Net financial claims outstanding	=Net value of nonfinancial assets

suffer from data quality problems. The use of the non-farm, non-financial corporate sector is not restrictive given that this sector owns around 90% of the non-residential fixed capital stock in the economy.

In terms of the needs of this study, the NBSA data suffers from two limitations. First, the data starts in the first quarter of 1990, limiting the determination of the capital stock to the period post-1990. Statistics Canada plans to publish NBSA tables at market value starting in 1970 at yearly frequency. Their release was planned for September 2006 but has been postponed *sine die*.

The second limitation relates to the composition of the equity data. The valuation of equity at market value in the NBSA is made difficult by the existence of two types of shares: listed (quoted) and unlisted (unquoted). Only listed shares have a market value while unlisted shares are evaluated at book value (Statistics Canada (2004)). As a result, the reported value of corporate shares in the NBSA does not price all categories of equity present in the balance sheet of corporations at market

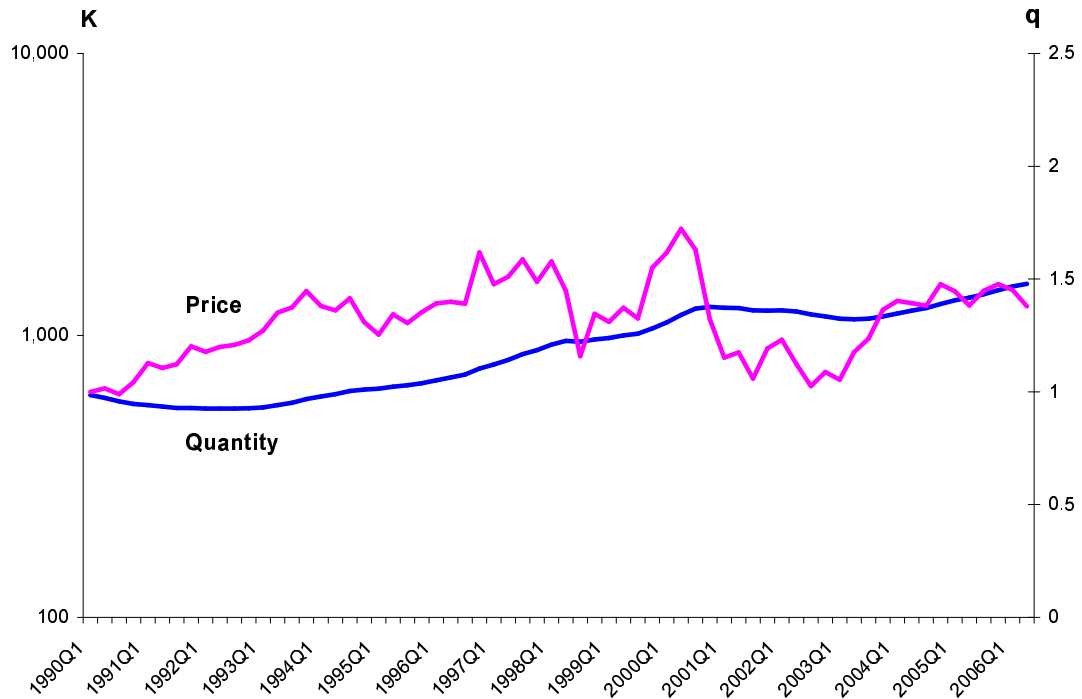


Figure 3.4: Quantity and Shadow Price of the Inferred Capital Stock

value. I report in Appendix B the ensuing data series and I explain how I convert the book value data into market value. A full description of data sources and data manipulations can also be found in Appendix B.

3.5 Results

Figure 3.4 shows the solution to the recursive system with the breakdown of the value of the installed capital v_t into a shadow price q_t , represented on the right axis, and an inferred stock of capital k_t , represented on the left axis in log scale. The figure shows a smoothly increasing inferred capital stock in the economy from 1990 to 2006. The shadow price of capital is constantly above one, a finding that is representative

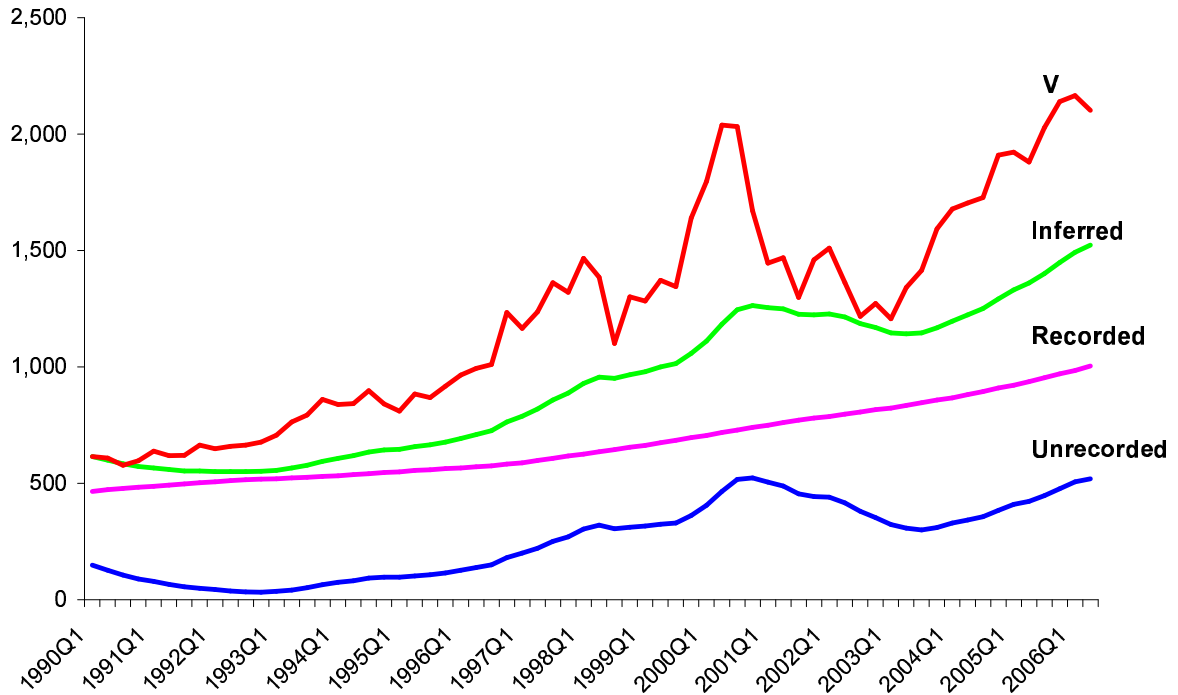


Figure 3.5: Decomposition of the Aggregate Value of Firms (in billions of 1997 dollars)

of many calculations of the q in the literature ¹⁸.

Figure 3.5 shows the breakdown of the components of the aggregate value of firms. The difference between the value of securities v and the inferred quantity of capital reflects the price q . The shadow price is the variable that absorbs all the volatility in securities values. The figure also uncovers the size of the unrecorded capital stock (intangibles) by subtracting the recorded capital stock in national income accounts from the inferred quantity of capital. Note that there is a smooth pattern of increase of the stock of intangibles from 1992 to 2001. After 2001, this stock falls to a level comparable to the one in 1998 and increases back after 2003.

The ratio of intangibles to the inferred capital stock averages 29% from 1994

¹⁸See Laitner and Stolyarov (2003) and the references therein.

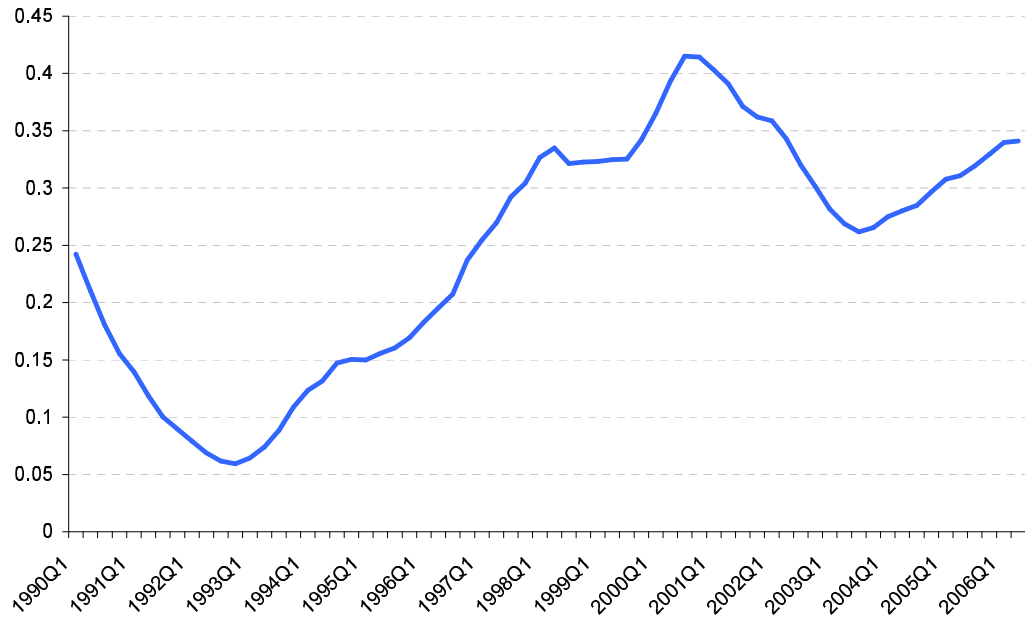


Figure 3.6: Ratio of Intangible Capital Stock to the Inferred Capital Stock

onwards with a standard deviation of 7.9%. The behavior of this ratio is depicted in Figure 3.6. The relative size of intangible capital increases with the rise of the value of securities in the late 90s. It grows from a proportion of 12% in 1994 to a proportion of 41% at the peak of the value of securities in the last quarter of 2000. With the fall in the value of securities in 2001, the relative stock of intangible capital falls to reach a proportion of 33% by the end of 2003. The recent rise in the value of securities is once again accompanied by a rise of the stock of intangible capital.

The coincidence in the rise of Canadian securities values with the accumulation of intangible capital reflects the slower pace at which the accumulation of the recorded capital proceeds relative to the pace at which the value of securities rise. The same process works in opposite direction in the case of a fall in the value of securities. The relative rise of intangible capital in the overall capital stock reflects the increasing

reliance of companies on knowledge capital. This is viewed as a consequence of the information-technology revolution that started in the 70s (Greenwood and Jovanovic (1999), Hobijn and Jovanovic (2001)).

The proportion of intangibles in the overall capital stock is smaller than the one found in Hall (2001) for the U.S. and bigger than the one found in Eliades and Weeken (2004) for the U.K. Both papers document a sharp rise in the proportion of intangibles accompanying a run up in the value of securities in the late 90s. Although Figure 3.5 shows a comparable rise for Canada, the sharpness of this increase is actually about half as large as that of the U.S. The share of intangible capital in the U.S. by the end of 2000 constituted half of the inferred stock of capital. It is important to note that an extension of the findings of Hall (2001) to the year 2005 shows a collapse in the relative size of intangibles to levels slightly below the ones found for Canada in 2005¹⁹. Both countries experienced the same qualitative behavior and are seemingly heading to a similar steady-state ratio.

Next, the nature of the unrecorded capital stock is explored. This information will also help estimate the size of the capital stock for which national income accounts do not collect any data.

As mentioned in the introduction, there are no available data from national income accounts on the investment of firms on different types of intangible capital goods. Statistics Canada collects only data on R&D expenditures. The stock of R&D is calculated using the perpetual inventory method and is compared with the size of

¹⁹These results can be obtained from the author upon request. No such exercise was conducted for the U.K.

the unrecorded capital. The procedure is detailed in Appendix B and the results are shown in Figure 3.7. This figure depicts the evolution of the R&D stock together with the evolution of the unrecorded capital stock. The ratio of the R&D stock to the unrecorded capital stock falls from a level of 43% in 1994 to a level of 12% by 2000. This proportion grows afterwards to reach an average of 20% between 2001 and 2005. This trend shows that the composition of the intangible capital stock shifted towards less R&D capital goods.

About 23% of the size of the unrecorded capital is made of R&D capital since 1994 on average. The rest would be made of the accumulated stock of capital that resulted from expenditures on training, organizational change, advertising and any expenditure which is intended to increase future production. Since only 23% of the 29% of unrecorded capital can be explained, we conclude that Statistics Canada misses about 26% of the overall productive capital stock in its data collection. This represented about \$380 billion in the last quarter of 2005.

3.6 Sensitivity Analysis

Three checks for robustness are conducted in this section. It is found that for a range of depreciation rates, starting values and adjustment costs parameters, the qualitative results pertaining to the behavior and to the relative size of intangible capital stock are unchanged. The size of the stock of intangibles does naturally vary depending on the range of investment adjustment cost and depreciation rates considered.

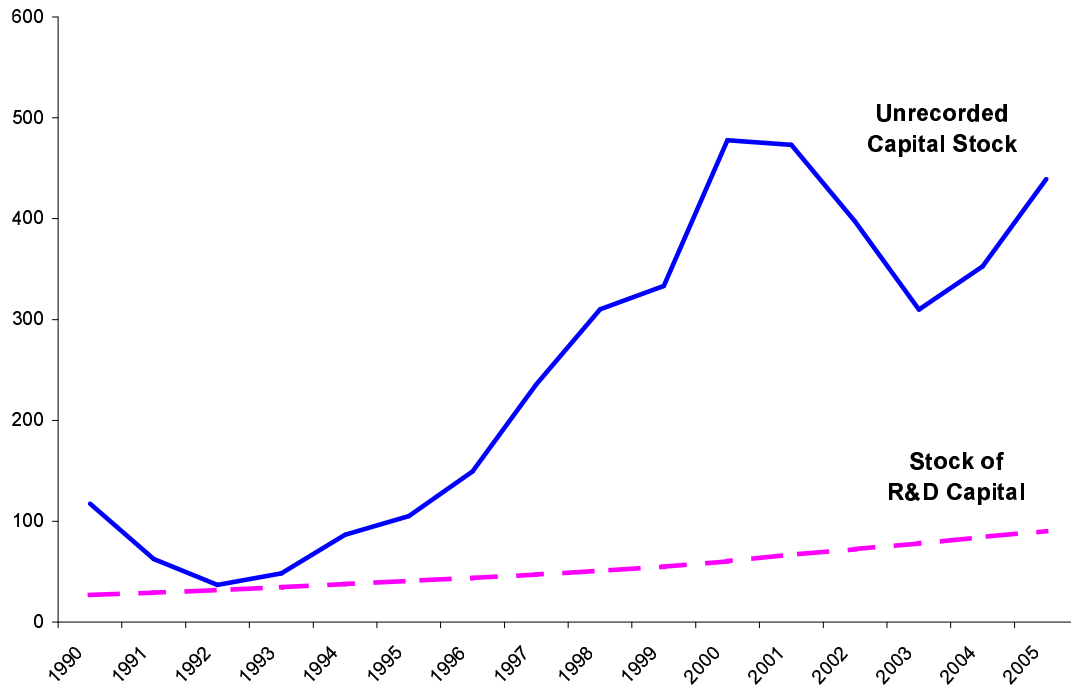


Figure 3.7: The Part of R&D in the Unrecorded Intangible Capital Stock (in billions)

3.6.1 Allowing for Different Adjustment Cost Parameters

First, the experiment of removing the asymmetry assumption in the adjustment cost parameters by allowing $\alpha^- = \alpha^+ = 8$ is conducted. The quantity of intangibles and the corresponding q in the case of symmetry are superimposed on the values obtained with the baseline model. This result reflects the absence of negative net investment *i.e.*, gross investment has always, at the very least, kept up with depreciation. In other words, no instances of decline in the firms' value have ever provoked discarding of capital. Therefore, the assumption on the relationship between the upward and the downward adjustment cost parameters is innocuous.

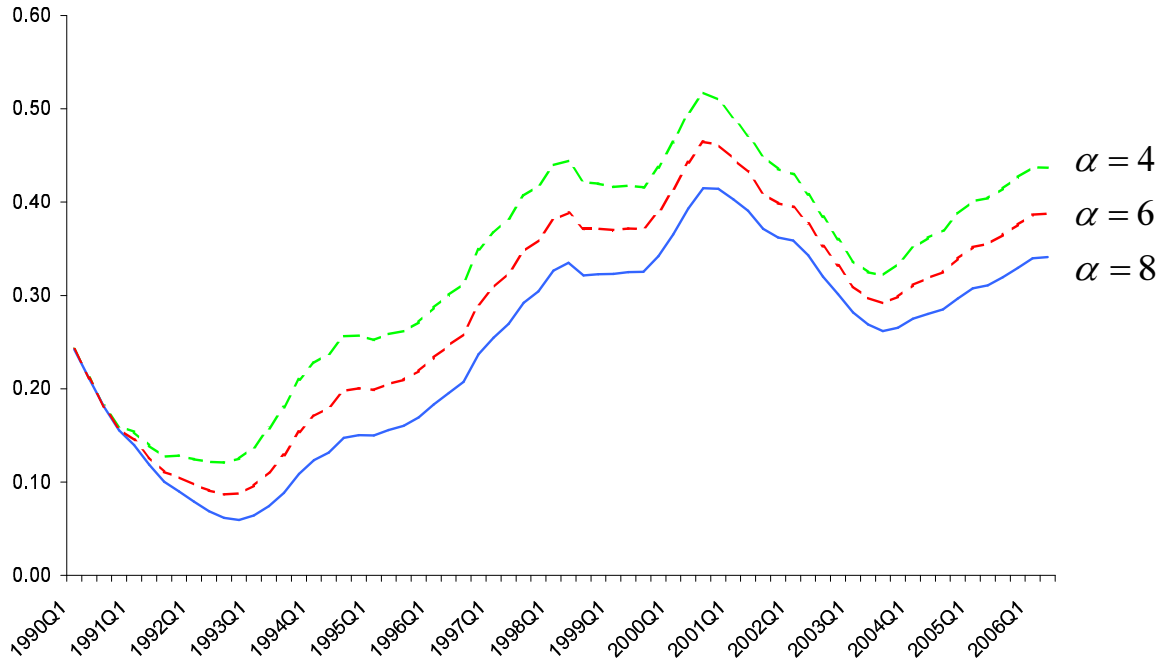


Figure 3.8: Sensitivity to Various Adjustment Costs of the Ratio of Intangible Capital to the Inferred Capital Stock

No studies were found for Canada that attempted to estimate the adjustment-cost parameter involved in increasing the amount of installed capital. In the baseline calibration exercise, the upward adjustment cost parameter was set to match the finding of Shapiro (1986). Hall (2004) followed essentially the procedure in Shapiro (1986), elaborating on the econometrics and using annual manufacturing industry data rather than aggregate data. He found even smaller adjustment cost parameters, in the range reported by Cooper and Haltiwanger (2006) for plant level data. Groth and Khan (2007) confirm the results of Hall (2004) when allowing for investment adjustment costs on top of the capital adjustment costs. Following these findings, lower adjustment costs were considered in the sensitivity analysis: adjustment costs of $\alpha^+ = 6$ and $\alpha^+ = 4$ corresponding respectively to an adjustment period of capital

following a shock of a year and half and a year. The implications on the implied ratio of intangibles to tangible capital levels are shown in Figure 3.8.

As the adjustment cost parameter decreases, the implied capital stock will increase given that the firm can now install capital at a lower cost. In particular, halving the adjustment time will increase the ratio of intangible capital stock to an average of 38% for the period considered. The high estimate that was assumed in setting $\alpha^+ = 8$ is viewed appropriate at this stage of the evidence and research on intangibles. Indeed, little is known about the exact size of the stock of intangible capital and its relative size in the overall capital stock, which warrants a conservative approach. Further research will provide more guidance on this matter.

3.6.2 Allowing for Different Depreciation Rates

There is no information on the depreciation rate of intangible capital in general. However, estimates of the depreciation rate for R&D capital are available. The Bureau of Economic Analysis (1994) estimates the depreciation rate of R&D to be around 11% while the Bureau of Labor Statistics (1989) uses a rate of 10%. Adams (1990) calculated an annual depreciation rate for basic research of between 9% to 13%, while Nadiri and Prucha (1996) estimated a depreciation rate of 12% for industrial R&D. No similar work was found documenting such rates for Canada. Given that R&D was found to compose only 23% of the overall intangible capital stock, the depreciation rate to use for the overall intangible capital is not obvious.

Nevertheless, the depreciation rate was allowed to vary between 11% and 9% in

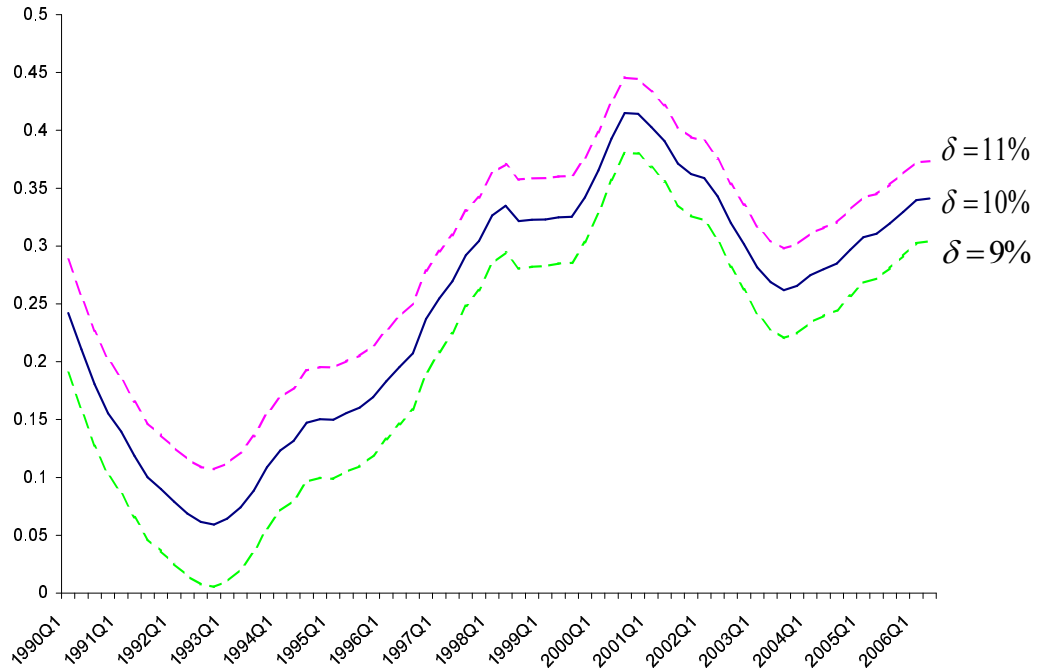


Figure 3.9: Sensitivity of the Ratio of Intangible Capital to the Overall Capital Stock to Various Depreciation Rates

accordance with the only evidence available that was just cited. With a depreciation rate of 11%, the ratio of intangible capital stock to the overall level of capital increases by 3 percentage points to a level of 32%. Increasing the depreciation rate lowers the inferred capital stock but lowers even more the physical capital stock which nets out to an increase in the intangible capital stock. With a depreciation rate of 9%, the intangible capital stock averages 25% of the overall capital stock. These bounds are shown in Figure 3.9.

The form of the theoretical law of motion of the capital stock from the model does not allow the use of a different depreciation rate for the physical capital stock and the intangible capital. In light of what the aforementioned statistical agencies use, the physical depreciation rate of 10% applied implicitly on intangibles is ultimately

a reasonable level to consider.

3.6.3 Allowing for Different Initial Values

In the baseline calibration, the initial capital stock k_{t-1} was set to equal the level that prevails when the system is in equilibrium at $t - 1$. This corresponds to v_{t-1} or a level equal to \$614 billion. Here, the initial capital stock is set to equal the recorded capital stock. In other words, we assume that at $t - 1$, the stock of intangibles is zero. The level of the recorded capital stock is now initially equal to \$466 billion. Hence, the initial starting value is lower than in the baseline model. Figure 3.10 shows the implied intangible level of capital for both scenarios. We can see that despite the sizeable difference in starting values, the implied level of intangibles converges to a common value by 1996, with a similar and almost equal values starting in 1994. Hence, the pre-1994 downward trend in intangible levels is not accurate and the estimates are afterwards robust to the choice of a starting value.

3.7 Conclusion

In this chapter, newly released Canadian data on the aggregate market value of all securities in the economy was used to calculate the size of the intangible capital stock. It was found that the stock of unrecorded capital was about 29% of the overall capital since 1994. The accumulation of intangible capital played a bigger role in the rise of the capital stock in the late 90s than the accumulation of physical capital. This relative rise coincided with the increase in the value of securities. Similar studies

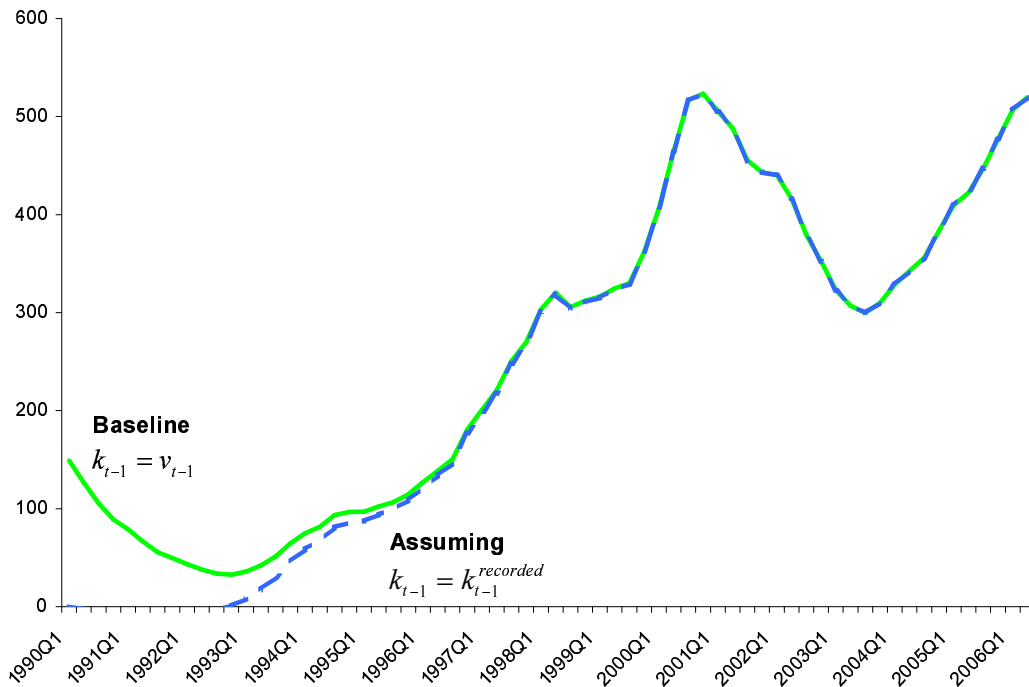


Figure 3.10: Sensitivity of the Intangible Capital Stock to Various Initial Values

conducted for the U.S. and the U.K. reached similar conclusions on the increasing prevalence of intangible capital. The nature of the intangible capital stock was shown to consist of 23% R&D capital. The composition of intangible capital shifted towards less R&D capital goods overtime. Since Statistics Canada collects data only on R&D capital, it misses about 26% of the overall capital stock in its data collection. This chapter provided a sense of how much national income accounts underestimated the stock of capital in times of a shift in the form of capital owned by businesses.

It is possible to compare the findings of this chapter with the previous one. Using the results of Chapter 2, the growth of the investment of firms in intangibles between 1998 and 2005 averaged 6.6% per year. On the other hand, backing out the investment in intangibles from the inferred stock of intangible capital documented in this chapter

shows that the similar growth rate is equal to 7.15%. The absolute numbers are hard to compare because of the extreme volatility which characterizes the investment in intangibles that is backed out from the value of securities. However, the average yearly stock of intangible capital implied by the results of Chapter 2 for the period considered is about \$365 billions²⁰. This number is comparable to the average inferred stock of intangibles per year of \$387 billions found in this chapter. This cross-checking of the findings is reassuring.

²⁰The procedure to calculate the stock of intangibles is based on the perpetual inventory method. This approach is similar to the one used to calculate the stock of R&D capital described in Appendix B. The depreciation rate used is also the same.

Chapter 4

The Embodiment of Intangible Investment Goods: a Q-Theory Approach

4.1 Introduction

Recent empirical findings point to a considerable increase in the share of intangible expenditures in overall investment since the early 80s. Still, intangible investment is rarely accounted for in investment models. This chapter extends the q-theory of investment to account for both the existence of intangible investment goods and the movement of their price over time. The model highlights the embodiment of intangible goods in tangibles and the role of relative price movements in the measurement of the contribution of each type of investment good to the overall capital stock. The model is used to: 1) derive the quantitative implications on the behavior of the capital stock and 2) to measure the contribution of embodied intangible investment goods to the overall capital stock in the U.S.

The increasing share of intangibles since 1950 is documented in a comprehensive study by Corrado *et al.* (2006), as discussed in Chapter 2. They find that the share

of intangible investment in overall investment since 1950 has steadily increased over time and that by 2000, investment in intangible capital was as large as investment in physical capital.

Other authors have attempted to measure the size of intangibles by taking indirect approaches, as discussed in Chapters 1 and 3. For example, Hall (2001) uses a q-theory model to infer the amount of intangible investment such that his model matches the movements in the stock market. McGrattan and Prescott (2007a) use a general equilibrium model to generate the intangible series which makes movements in hours worked, in producing both intangibles and tangibles, match some aggregate moments.

A key limitation to all these approaches is that none of them separately identify indices for the price and quantity of intangible investment goods. Corrado *et al.* (2006) and Hall (2001) simply assume that the price of intangibles is the same as the price of tangible investment goods. McGrattan and Prescott (2007a) guess a price series in order to infer an intangible investment series.

The distinction between the price and the quantity of intangibles is an important issue because it is not clear how much of the increase in the overall expenditure on intangibles is attributable to the *real* contribution of intangibles. Moreover, Hall's lack of distinction between the price of tangibles and intangibles effectively implies some unlikely movements in the stock of intangibles, including that they were negative for a decade after 1974 and extremely volatile in the last decade.

What is more, the substantial increase in the relative cost of the main input to intangibles production, skilled labor, suggests that the relative price of intangibles

also rose (Katz and Autor (1999) and Lemieux (2007) review the literature on the rise in the skill-premium). A closer look at the aforementioned list of intangible items reveals that they are mainly produced by a class of skilled workers made of university graduates and executives. The real average compensation of these two types of workers has been increasing since the early 1980s until it peaked in 2000 and declined somewhat. This compensation pattern should translate into a rise in the unit cost of intangibles up to the year 2000 and a subsequent fall.

In order to account for both the existence of intangible investment goods and the movement of their price over time, I develop and implement a generalized q-theory of investment. Previous authors have mainly assumed that intangible goods are disembodied and that the stock of intangibles can be conceptualized as separately evolving from the physical capital stock of firms. However, many intangible investments made by firms appear to be “embodied” in a composite stock made of tangible and intangible capital. One might think of this composite stock as combinations of “hardware” (e.g. office buildings, computers, machines) and “software” (organizational design, operating systems, blueprints) which have been brought together by past investments.

This is precisely how the investment process is modeled, with units of intangible and tangible goods being optimally combined as intermediates in the production of a final aggregate investment good. It is this final investment good that is accumulated overtime and used in output production. Changes in the share of expenditures on the two types of investment goods over time reflect exogenous technological change in the final investment good sector.

The price of intangible goods reflects the costs of producing them which, in turn, reflects the wages of skilled labor. I construct a price series for intangibles based upon the compensation received by university graduates and executives and estimates of their share in the production of intangibles (similar to those used by Corrado *et al.* (2006)).

Both the rise in the price of intangibles and their increasing share in overall investment results in an aggregate investment deflator whose behavior contrasts markedly with the (physical) investment deflator reported by national income accounts; while the investment deflator of national accounts exhibits a downward trend starting in the mid-1950s, the investment deflator constructed from the model to include intangible investment has a downward trend up to the mid-1980s and then rises up to 2000 and declines afterwards²¹. In other words, the behavior of the acquisition cost of capital is dramatically different when intangibles are accounted for.

The model successfully generates a smoothly-behaving series of capital stock with a market value that predominantly remains above and close to its acquisition cost; this reflects a Tobin's q that fluctuates closely around its equilibrium value, a desirable feature which is not observed in empirical measures of Tobin's q when capital exclusively consists of tangible investment. This result is a direct implication of: 1) the new trend observed in the acquisition cost of capital once intangibles are accounted for and 2) the increased size of capital stock once intangible investment goods are accounted for. These successful quantitative implications suggest that the mixed econometric success of the q -theory might be a consequence of the omission of

²¹The secular fall in the price of physical investment goods is a well documented fact. See in particular Greenwood *et al.* (1997) and Krusell (1998).

intangible investment²².

Finally, the model is used to measure the extent to which the composition of the stock of capital in the economy has shifted over time towards the inclusion of more intangible capital at the expense of tangible capital. The decomposition of the capital stock into its investment constituents shows that the rise in the overall capital stock from the mid-80s until the late 90s was driven mainly by an increase in intangible investment despite an increase in tangible investment. In particular, I find that once the price movements of the two investment goods are accounted for, the relative contribution of accumulated intangible investments to the overall capital stock was higher prior the 1990s and substantially lower during the 1990s than other authors have found.

Taken together, the findings in this chapter underscore the importance of intangible investment as a source of value for the firm and as a key component of any investment theory. Moreover, this chapter provides a consistent account of the compositional changes that have occurred in the last 25 years in the U.S. economy by bringing together the evidence on the rise of the skill premium, the evidence on the increasing importance of intangible investment and the evidence on the behavior of aggregate securities.

The chapter proceeds as follows. In section 2, the approach followed is contrasted with other approaches that try to account for the presence of intangibles. The most

²²These potential specification problems are mentioned in a related discussion by Hall (2004) pp. 914-915. See Chapter 5 for an elaboration on this point.

notable feature that distinguishes this work are: 1) the linkage of the unit cost of intangibles to the wage behavior of skilled workers and 2) the embodiment of intangibles in tangible investment goods. Section 3 outlines how the extension of the q-theory is constructed. It will also describe the approach used to disentangle the contribution of each type of investment good to the capital stock. Section 4 discusses how the parameters are calibrated and what data sources are used to document some of these choices. This section also details the technology by which skilled workers, identified as university graduates and executives, produce intangibles. Section 5 describes the findings with regards to the inferred capital stock, the behavior of Tobins' q and the changing composition of the capital stock over time. Section 6 compares the findings with the work and results of the two closest approaches in the literature, namely Hall (2001) and McGrattan and Prescott (2007a). Section 7 outlines the sensitivity analysis conducted to assess the robustness of the results. Section 8 concludes with a discussion of future research.

4.2 Contrast with Other Approaches

The baseline neoclassical model of investment, also known as the q-theory of investment, predicts that the decision of a firm to invest is a function of a trade-off between the benefit of increasing capital by one unit and the cost of acquiring and installing the extra unit. Hayashi (1982) showed the conditions under which the marginal benefit of increasing capital is identified with the net value of outstanding securities. Figure 4.11 shows the evolution of the aggregate value of net securities together with the acquisition cost of capital or its replacement cost. The q-theory

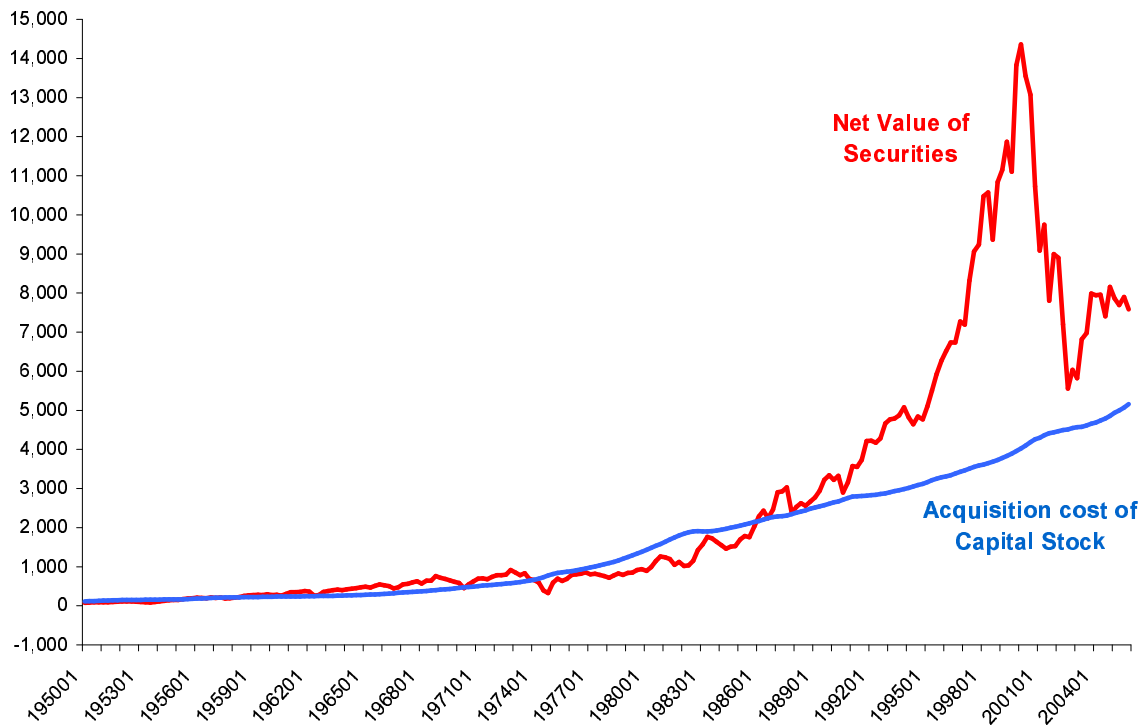


Figure 4.11: Net Value of Securities and Value of Capital Stock (in billions). Source: Hall (2001), NIPA and author's calculations

states that, from this graph, the firm can deduce the benefits and costs it faces in its decision to adjust capital by one unit: if the capital in place has a higher value than the capital that is not yet installed then the firm should take advantage of the arbitrage opportunity. Any discrepancy between the cost and benefit is due to capital adjustment costs which slow down the arbitrage process.

Another way of presenting this tradeoff is through Tobin's q , which is the ratio of the net value of securities to the acquisition cost of capital. Tobin's q reflects the incentive of the firm to adjust its capital stock: the firm should invest if the value is bigger than one, otherwise the firm should disinvest. Figure 4.12 illustrates the behavior of Tobin's q .

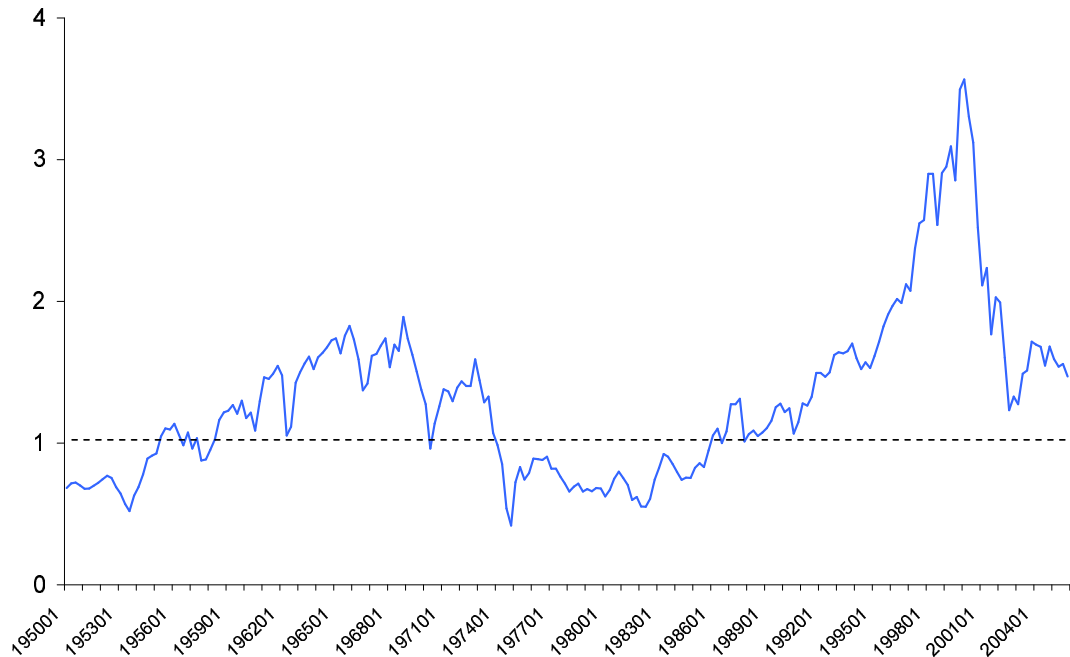


Figure 4.12: Behavior of Tobin's q . Source: Hall (2001), NIPA and author's calculations

Two aspects of this behavior are difficult to rationalize from the perspective of the q -theory. First, we observe a large discrepancy between the benefit and the cost of adjusting capital by one unit suggesting a very long adjustment period. In fact, the econometric implementation of the q -theory by Summers (1981) implied an adjustment period of 8 years²³. This number is viewed as unrealistic. Second, the period from the mid-1970s until the mid-1980s features an aggregate value of firms below the acquisition cost of capital. This is viewed as a puzzle because the firm's capital can be sold at a higher value than when kept inside the firm (see Baily, 1981). In other words, there are arbitrage opportunities that agents are not taking advantage of.

²³Other studies confirmed the long implied adjustment period. See Chirinko (1993) for a review of the literature. However, more recent work which uses natural experiments, exemplified by the paper of Cummins *et al.* (1994) and (1996) have found shorter adjustment periods.

Four broad research directions have been pursued to explain these anomalies in Tobin's q . The first line of attack was to study how financial markets do not accurately reflect the fundamental value of the firm. This literature developed different models but none of them seem to be immune from problems despite the strong theoretical and empirical evidence for the existence of bubbles (see LeRoy (2004) for a review of this literature). The second line of attack was to relax the restrictive assumptions that allowed the value of the installed capital to reflect the marginal value of increasing the capital stock; the reliance on perfect competition, constant returns to scale and quadratic adjustment costs might not be accurate. This literature had some success in improving on the q -theory but the improvements are sometimes viewed as marginal (see Caballero (1999) for a review of this literature). A third approach was to question the use of a representative firm at the aggregate level and to focus instead on modeling the heterogeneity observed at the plant level. This literature abandoned the macroeconomic approach to modeling investment (see the review of this literature in Bond and Van Reenen (2007)). Finally, one explanation that has received attention in the past few years is the existence of another category of investment that has not yet been accounted for in theoretical and empirical investigations. This category of investment goods consists of intangible goods. The argument is that if the net value of securities reflects, under rational valuation, the value of installed capital stock, then periods of marked departure of the net value of securities from the acquisition cost of tangible capital stock is evidence for the accumulation of intangible capital by firms.

This last possibility was proposed by Hall (2001). His paper uses the q -theory of

investment to generate the stock of capital in the economy. From this inferred capital, the component which is recorded in national income accounts as physical capital is subtracted and the residual is assimilated to intangible capital. One crucial assumption in Hall's work is the assumption that the intangible and tangible investment goods are perfect substitutes and that their price is equal. These two assumptions are violated, under the framework of this paper, as discussed in the introduction: the share of intangibles in the overall capital expenditures has been increasing overtime and the unit cost of intangibles behaves the opposite of the price of physical capital. Finally, one anomaly in Hall's findings is that the quantity of intangibles falls below zero for a decade, starting in the mid-70s.

Eliades and Weeken (2004) apply Hall's methodology to the U.K. These authors also find negative intangibles throughout the 70s, the 80s and early 90s. However, they reach the same qualitative results as Hall (2001) for the late 90s. In Chapter 3, I applied this approach to Canada for the period after 1990 and reaches the same qualitative conclusions as those found for the U.S. and the U.K.

I depart from Hall's approach by: 1) relaxing the assumption of equality between the price of intangibles and tangibles and 2) allowing some degree of substitutability between the two investment goods.

McGrattan and Prescott (2005a) make another attempt to use the unmeasured levels of intangible capital to rationalize the rise in the U.S. and U.K. stock markets in late 90s. The authors depart from Hall's work by taking a general equilibrium approach with no frictions aside from the existence of taxes. The authors rationalize

the size of intangible investment found in Corrado *et al.* (2005) in the 1990s while using the change in tax regulations to account for the differing performance between the U.K. and the U.S. stock markets. McGrattan and Prescott (2005b) show that by explicitly accounting for intangible investment in an otherwise standard real business cycle model, one can explain the low productivity levels in the early 90s. In particular, they argue that GDP in national income accounts is undervalued because intangible investment is expensed, which ultimately created a downward bias in the productivity estimates for the early 90s. Taking this work further, McGrattan and Prescott (2007a) extend the baseline real business cycle model to allow for the production of intangibles by the representative agent. The goal of their work is to reconcile the real business cycle model's prediction of a fall in hours worked after the 1990s with actual evidence of their increase. In particular, their model is calibrated to match some aggregate macroeconomic series and features two stocks of capital: a tangible capital stock and an intangible capital stock.

In this chapter, I calibrate the extended q-theory model to the aggregate value of securities and the extended model features only one capital stock index that embodies both tangible and intangible investment goods. Another difference with the work of McGrattan and Prescott is the way the price of intangibles is calculated: McGrattan and Prescott (2007b) state that they guessed the price of intangibles in their solution to derive a series of intangible investment. As mentioned above, I derived the price of intangibles from the behavior of the cost of its main inputs.

Finally, the proposed model is most comparable to the work of Hall (2000). In

Hall's paper, he focuses on the period from 1990 to 2000 and tries to link the behavior of university graduate wages with the formation of intangible capital. Hall does not however account for the changing structure of firms' investment as done in the extended model I propose. In addition, Hall assumes that the intangible and tangible capital stocks evolve over time separately while I develop a model with a homogeneous capital stock. Moreover, the inclusion of executives in the class of skilled labor are another conceptual and empirical difference between Hall's work and the model developed below. Finally, the long-term approach I adopt illuminates the pre-1990 events as well as the post-1990 events.

Although most studies find that the size of intangible capital is substantial, Hall (2003) and Bond and Cummins (2000) are exceptions. They both show, using different data, that the returns to physical capital exhaust all payments to capital and hence, nothing is left over to reward the services of intangible capital. This is held as evidence for the absence of a substantial intangible capital stock which is puzzling in light of the findings in the previously cited papers. In Chapter 5, I discuss how to reconcile the findings of Hall (2003) and Bond and Cummins (2000) with the results obtained here.

4.3 Including Intangibles in the Q-theory of Investment

The standard neoclassical model of investment as developed in Hayashi (1982) is extended to account for the production of intangibles. Once produced, intangible

investment goods are combined with tangible investment goods, which are bought from the market, to produce a final investment good that accumulates into a capital stock which is used in production. Ultimately, the model relates the value of securities to the value of the capital in place within firms. This key relationship allows the generation of a series for the capital stock and the construction of a series for Tobin's q .

There is perfect competition in input and output markets. The firm employs two types of labor, skilled and unskilled. An amount l^u of unskilled labor is used for the production of output only. It is paid w^u . Skilled labor is used for two tasks: the amount l^s is used for the production of output and the remainder, h^s , is used for the production of intangibles. Skilled labor is paid w^s . The production of intangibles is governed by the following technology:

$$x^I = \theta h^s \tag{4.1}$$

where θ is a productivity parameter. The existence of this function is motivated by the need to capture the link between the rise in the wage paid to skilled labor and the increase in the price of intangible investment goods. This function will allow the ratio of the intangible to tangible price to vary over time instead of being set equal to one as Hall (2001) does. Skilled labor consists of university graduates and executives. The price of intangible capital is denoted p^I and will be given, after optimization, by

$$p^I = \frac{w^s}{\theta}. \tag{4.2}$$

The construction of this unit cost will be discussed in the calibration section, once

the wage of skilled workers is specified as a function of the wage of university graduates and executives. The production of output proceeds according to $F(k_{t-1}, l_t^u, l_t^s)$ where $F(\cdot)$ is assumed to be homogeneous of degree one. The price of output is the numeraire.

The model departs from the baseline q-theory model by defining a composite investment good which is accumulated into a capital stock. At each period t , the firm combines intangible goods x_t^I with tangible investment goods x_t^T to produce a final investment good x_t according to the following constant elasticity of substitution (CES) function

$$x_t = \left(a_t (x_t^T)^\rho + (1 - a_t) (x_t^I)^\rho \right)^{\frac{1}{\rho}}. \quad (4.3)$$

The exponent ρ is the elasticity of substitution between the two intermediate inputs and is assumed to be less than or equal to 1 while a_t represents the income share of each input. This share is allowed to vary over time and is calibrated to capture the evidence on the increasing spending on intangibles relative to tangibles. This is important because the variation in the shares will account for the changing structure of the economy in the past 50 years towards the use of relatively more intangible capital. This CES functional form makes apparent that this changing structure is a biased technological change.

There is no evidence on the empirical value of ρ . Hall (2001) assumed that tangible and intangible investment are perfect substitutes so implicitly ρ in his model is equal to 1 and $a_t = 1 - a_t$. In the calibration section, the baseline model is specified with $\rho = 0$ and in the sensitivity analysis section, various values of ρ are considered.

Tangible investment goods are bought from the market at a price p^T which is taken as given by the firm. The aggregate investment good accumulates according to

$$k_t = (1 - \delta)k_{t-1} + x_t \quad (4.4)$$

where δ is the depreciation rate. The adjustment of the capital stock is subject to output losses modeled as a cost function assumed to be quadratic and homogeneous of degree one that is denoted by $C(x_t, k_{t-1})$.

At each period, firm's profit is given by

$$\hat{v}_t = F(k_{t-1}, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s - w_t^s h_t^s - p_t^T x_t^T - C(x_t, k_{t-1}). \quad (4.5)$$

The firm's problem is to choose the optimal level of labor and investment in order to maximize the net present value of future profits subject to: 1) the technology of production of intangibles and of the final investment good, 2) the capital accumulation equation, 3) the starting level of capital and 4) the transversality condition:

$$\max_{\{l_t^u, l_t^s, h_t^s, x_t^T, x_t\}} v_s = \sum_{t=s}^{\infty} \left(\frac{1}{1+r} \right)^{t-s} \hat{v}_t \quad (4.6)$$

s.t.

$$x_t^I = \theta_t h_t^s$$

$$x_t = \left(a_t (x_t^T)^\rho + (1 - a_t) (x_t^I)^\rho \right)^{\frac{1}{\rho}}$$

$$k_t = (1 - \delta)k_{t-1} + x_t$$

$$k_{t-1} \quad (4.7)$$

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T v_{s+T} = 0. \quad (4.8)$$

The value function v_s is the net present value at time s of future payout to securities' holders: after the firm pays inputs their due, the leftover income is paid to owners.

Their ownership materializes through the possession of titles in the form of securities. Hence, v_s is also the value of the firm at time s .

The model can be shown to be equivalent to a standard q-theory optimization problem through a two-stage optimization procedure. The only difference will lie in the interpretation of the price index of investment goods. The first stage is a static problem which consists in choosing x^T and x^I to minimize the expenditure on the production of x within each period. The second stage recasts the above dynamic problem in such a way that it is solved at the start.

The static problem can be written as

$$\min_{x^T, x^I} p^T x^T + p^I x^I \quad (4.9)$$

$$s.t. \quad \left(a(x^T)^\rho + (1-a)(x^I)^\rho \right)^{\frac{1}{\rho}} \leq x. \quad (4.10)$$

Replacing the optimal solutions x^{T*} and x^{I*} into the objective function leads to the minimum cost function:

$$p^T x^{T*} + p^I x^{I*} = x \left(\left(\frac{p^T}{a} \right)^{\frac{\rho}{\rho-1}} + \left(\frac{p^I}{1-a} \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \quad (4.11)$$

$$= x p^x \quad (4.12)$$

where p^x reflects the unit cost of an investment good or the price index of aggregate investment, x . The new dynamic problem of the firm can be written as:

$$\max_{\{l_s^u, l_s^s, x_s\}} v_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \hat{v}_s \quad (4.13)$$

$$\hat{v}_t = F(k_{t-1}^x, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s - p_t^x x_t - C(x_t, k_{t-1}) \quad (4.14)$$

s.t.

$$k_t = (1 - \delta)k_{t-1} + x_t$$

$$k_{t-1}$$

$$\lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T v_{s+T} = 0$$

The solution to this standard problem is detailed in Appendix C, section 4.1. The usual first-order condition on the equality of the lifetime return to increasing capital by one unit with its marginal cost is given by

$$\lambda_t = p_t^x + C_x(x_t, k_{t-1}) \quad (4.15)$$

where λ_t is the shadow price of a unit of installed capital. The right-hand side is the marginal cost given by the summation of the acquisition price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. In order to obtain sharper results with respect to the investment decision of the firm, the adjustment cost function is specified as quadratic and homogeneous of degree one, as is often done in the literature:

$$C(x_t, k_{t-1}) = \frac{\alpha}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1}. \quad (4.16)$$

Substituting this function into the first-order condition results in the following equation:

$$\frac{x_t}{k_{t-1}} = \frac{1}{\alpha} (\lambda_t - p_t^x). \quad (4.17)$$

This is known as the investment equation since it relates the behavior of investment to the difference between the value of capital in place λ_t and its acquisition cost p_t^x . Investment is positive when the lifetime return to increasing capital by one unit

exceeds its marginal cost and vice versa. To get around the fact that λ_t is by definition unobservable, the finding of Hayashi (1982) that

$$v_t = \lambda_t k_t \quad (4.18)$$

is used to obtain the following expression:

$$\frac{x_t}{k_{t-1}} = \frac{1}{\alpha} \left(\frac{v_t}{k_t} - p_t^x \right). \quad (4.19)$$

Finally, in order to implement the model quantitatively, this relationship is combined with the equation for the investment term x_t to obtain the following quadratic equation:

$$\alpha k_t^2 + ([p_t^x - \alpha(1 - \delta)]k_{t-1}) k_t - v_t k_{t-1} = 0 \quad (4.20)$$

Hall (2001) shows that a unique solution exists for a general convex cost function with constant returns to scale. This equilibrium is stable and is therefore not sensitive to initial conditions in the long-run. k_t is the endogenous variable to be solved for and generated at each point in time. The positive root expresses the law of motion of the capital stock:

$$k_t = \frac{-[p_t^x - \alpha(1 - \delta)]k_{t-1} + \sqrt{([p_t^x - \alpha(1 - \delta)]k_{t-1})^2 + 4\alpha v_t k_{t-1}}}{2\alpha}. \quad (4.21)$$

All variables are observable and the pair (v_t, p_t^x) is a sufficient statistic to generate the stock of capital in the economy. This pair of variables is assumed to be taken as given by the firm.

Once a series for the capital stock is obtained, the contribution of each type of investment to the overall capital stock can be recovered in the following way: the

capital accumulation equation $k_t = (1 - \delta)k_{t-1} + x_t$ is substituted forward to obtain:

$$k_{t+T} = (1 - \delta)^{T+1}k_{t-1} + \sum_{i=0}^T (1 - \delta)^i \{x_{t+T-i}\}. \quad (4.22)$$

Since the technology of production of the final investment good is homogeneous of degree one, the Euler theorem applied to x_t leads to:

$$\begin{aligned} x_t &= \frac{\partial x_t}{\partial x_t^T} x_t^T + \frac{\partial x_t}{\partial x_t^I} x_t^I \\ &= \left(\frac{p_t^T}{p_t^x} \right) x_t^T + \left(\frac{p_t^I}{p_t^x} \right) x_t^I. \end{aligned} \quad (4.23)$$

It is now possible to link the stock of capital to the investment over time in each type of investment good by substituting this last relationship in the capital equation:

$$k_{t+T} = (1 - \delta)^{T+1}k_{t-1} + \sum_{i=0}^T (1 - \delta)^i \left\{ \left(\frac{p_{t+T-i}^T}{p_{t+T-i}^x} \right) x_{t+T-i}^T + \left(\frac{p_{t+T-i}^I}{p_{t+T-i}^x} \right) x_{t+T-i}^I \right\}. \quad (4.24)$$

This equation will be used to disentangle the contribution of each type of investment into the overall capital stock. Each type of investment is weighted by its relative price which allows the capital stock to be written in efficiency terms. This is a direct consequence of the aggregation formulation that was assumed through the production function of the final investment good.

4.4 Calibration

4.4.1 Constant Parameters

The parameter values and paths of some exogenous parameters in the law of motion of capital need to be specified. For most of the parameters that are shared with

the model developed in Chapter 3, the values used are similar. For ease of exposition, the next paragraph repeats Chapter 3's discussion of the constant parameters .

The adjustment-cost parameter α represents the time it takes for the capital stock to double (halve) when λ doubles (halves). To see this, note that if λ doubles permanently, say from one to two, it will initially cause the investment-capital ratio to increase by $\frac{1}{\alpha}$. For the investment-capital ratio to double, the increase in $\frac{1}{\alpha}$ must be repeated for α periods. Hall (2001) cites the work of Shapiro (1986) to justify the choice of a doubling time parameter of 8 quarters. The depreciation rate of 2.6% per quarter is used by national income accounts for physical capital. To start the iteration on the law of motion of capital, the value of the initial capital stock k_{t-1} needs to be specified. We will assume that at the pre-initial quarter, the value of the firm reflects its quantity of installed capital *i.e.* $k_{t-1} = v_{t-1}$. This is similar to assuming that $\lambda_{t-1} = 1$. Since the recursion was shown to be insensitive to initial conditions, this assumption will not affect the behavior of the system in the long-run.

Given the desire to model intangibles as embodied in tangibles, we allow a certain degree of complementarity between the two types of investment goods. It will be enough to set ρ equal to zero as a baseline case. This will transform the CES function into a Cobb-Douglas function and as a result, the composite investment good will capture the embodiment of intangibles in tangible investment goods. The shares will then be represented by the exponents on each input which makes the final investment good an aggregator of two intermediate investment goods. Indeed, it will be a share-weighted function that is apparent to an index of investment. The weights represent the share of each intermediate investment good in the overall investment expenditure.

Table 4.7: Parameter values

Name	Parameter	Value	Rationale
Adjustment-cost	α	8	Shapiro (1986)
Depreciation rate	δ	0.026	Hall (2001)
Initial capital stock	k_{t-1}	v_{t-1}	Assuming $\lambda_{t-1} = 1$
Elasticity of substitution	ρ	0	Embodiment assumption

In fact, it is a Divisia index approach to combining two investment goods. Following this logic, x will then be viewed as an index of aggregate investment. In the sensitivity analysis section, ρ will be allowed to take a value bigger and lower than 0.

Assuming $\rho = 0$ implies that p^x in equation 4.11 takes the following form:

$$p_t^x = \left(\frac{p_t^T}{a_t} \right)^{a_t} \left(\frac{p_t^I}{1 - a_t} \right)^{1 - a_t} \quad (4.25)$$

Table 4.7 summarizes the parameter values used and the rationale for the choice of each value.

4.4.2 Varying Parameters

The market value of net financial claims (financial liabilities minus financial assets) is used as the measure of v_t since the value of ownership claims are a reflection of the installed capital inside the firm. Indeed, v_t was defined as the present value of payouts to securities' holders. Assuming that investors are rational, it follows that the present value of payouts v_t will equal the value of securities on the market. Since for all t , $v_t = \lambda_t k_t$, then the value of securities equals the value of the installed capital stock.

Notice that v_t includes all financial claims towards firms, net of financial assets.

These claims are made of equity, bonds and all other other liabilities (loans and mortgages, short-term paper, trade payables, life insurance and pensions). The definition of v_t represents a departure from most of the q-theory literature's interpretation of v_t . Traditionally, v_t covers only equity values or equity plus bonds. This departure is mainly due to the new types of data that are available for use.

Most of the data to measure v_t is taken from the national balance sheet account at market value from 1950Q1 to 2005Q4. Equity is reported at market value and all the other liabilities are at book value. These were converted by Hall (2001) into market value. Data analysis focuses on the non-farm, non-financial corporate sector. This sector was chosen because it best fits the perfectly competitive framework of this chapter. The removal of the farming sector aims to control for the presence of land in the overall capital stock, a capital input in fixed supply, which therefore earns rents. The choice of the corporate sector ensures that securities are continually priced to accurately reflect new information regarding the value of the capital stock. This would not be true for the installed capital of unincorporated businesses. Another reason to focus on this sector is dictated by the fact that the farming sector, the non-corporate sector and the financial sector suffer from data quality problems. The use of the non-farm, non-financial corporate sector is not restrictive given that this sector owns around 90% of the non-residential fixed capital stock in the economy.

A full description of data sources and data manipulations can be found in Appendix C, section 4.2.

The chapter takes the view that intangibles are being produced by a class of

skilled workers. This class is made of two broad categories of workers: on the one hand, scientists and middle managers that we categorize as university graduates and denote (h^{Univ}) who create raw intangibles and on the other hand, executives (h^{Exec}) who create organizational designs and structures. Raw intangibles and organizational designs are combined to produce intangibles following

$$x_t^I = \theta_t h_t^s = \theta_t (h_t^{Exec})^{\phi_t} (h_t^{Univ})^{1-\phi_t}. \quad (4.26)$$

Assuming perfect competition, the expression of the unit cost of an intangible capital good is then given by

$$p_t^I = \frac{1}{\theta_t} \left(\frac{w_t^{Exec}}{\phi_t} \right)^{\phi_t} \left(\frac{w_t^{Univ}}{1-\phi_t} \right)^{1-\phi_t} \quad (4.27)$$

where θ is output per worker and ϕ is allowed to vary to reflect the variable weight that characterized executives compensation overtime in the overall intangible production wage bill

$$\phi_t = \frac{w_t^{Exec} h_t^{Exec}}{w_t^{Univ} h_t^{Univ} + w_t^{Exec} h_t^{Exec}}. \quad (4.28)$$

This specification highlights the relationship between the labor market for high-skilled workers and the cost of intangibles. Since national income accounts do not collect information on the investment of firms in intangibles, and because the market for intangibles is extremely thin²⁴, little is known about their aggregate price. The view taken in this chapter is that the major manufacturers of intangible goods, as listed in Corrado *et al.* (2005), are university graduates and executives. Due to the

²⁴Some R&D spending leads to the creation of a patent which will carry a price if commercialized. However, the market for patents is extremely thin: very few patents change hands. For example, Serrano (2006) documents that only about 20% of all U.S. patents issued to small innovators (*i.e.*, firms that were issued no more than five patents in a given year) are traded once or more.

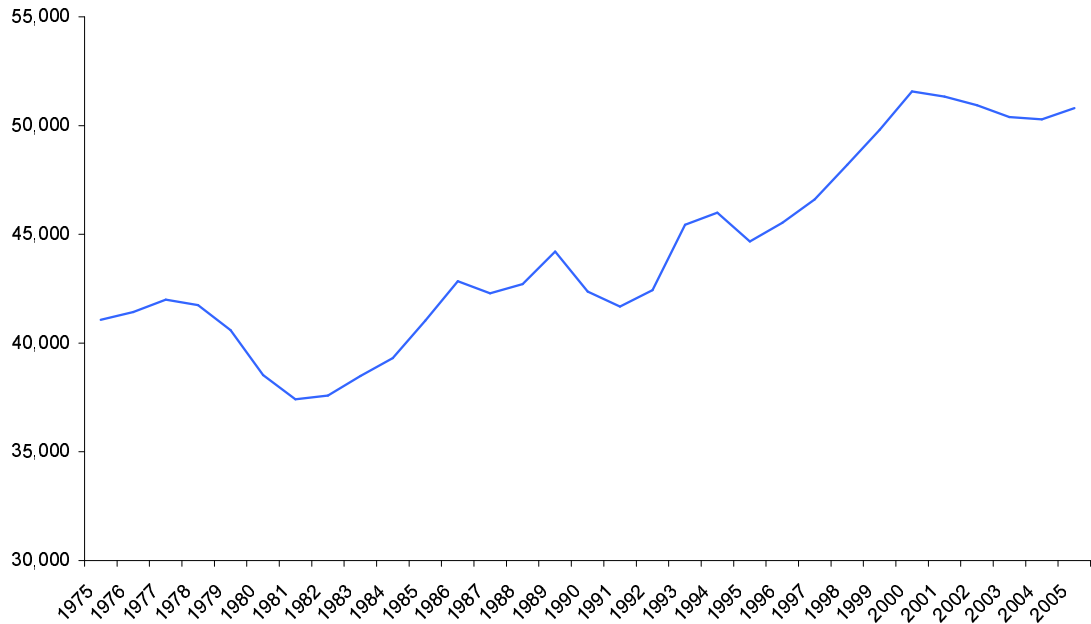


Figure 4.13: Real Mean Earnings of University Graduates. Source: BLS.

rapid increase in demand of intangibles as documented by Corrado *et al.* (2006), this category of workers experienced a widely documented increase in their wage premium starting in the early 80s (see Katz and Autor (1999) and Lemieux (2007) for an up-to-date review of this literature.) Figure 4.13 shows the evolution of the real wage of university graduates from 1970 to today. The upward trend observed up to 2000 and the subsequent fall is still preserved if this wage is taken as a ratio of non-university graduates.

Another category of workers which experienced an impressive rise in their wages is executives. Frydman and Saks (2007) report data from 1935 on CEO compensation. Figure 4.14 reproduces their findings. There is an upward trend starting in the early 80s. The rise and fall in the stock market around 2000 seems to have had an important effect on the compensation trend. This documented evolution in the

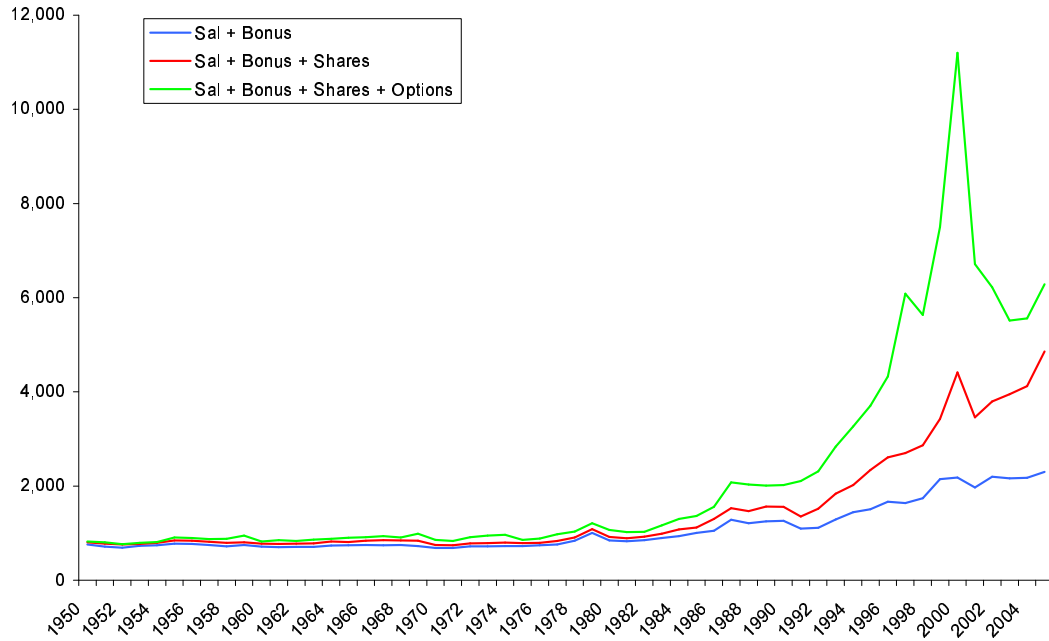


Figure 4.14: Real Mean Compensation of Executives (in thousands). Source: Frydman and Saks (2007)

labor market for high-skilled workers is important in valuing the competitive price of produced intangible goods and thereby accurately constructing an index for the price of aggregate investment.

Using the wage of executives and college graduates together with the shares of these inputs allows the construction of the intangible unit cost²⁵. The behavior of this price is depicted in Figure 4.15 in log scale. The price of intangibles p^I is rising at a constant rate from 1950 until mid-80s. Afterwards, its growth rate accelerates driven by the rise in the wage of CEOs and in their increasing share in the overall wage bill up to 2000. The growth rate of the intangible price falls afterwards.

²⁵Note that the calibration of all starting values of the varying parameters is detailed in Appendix C, section 4.3.

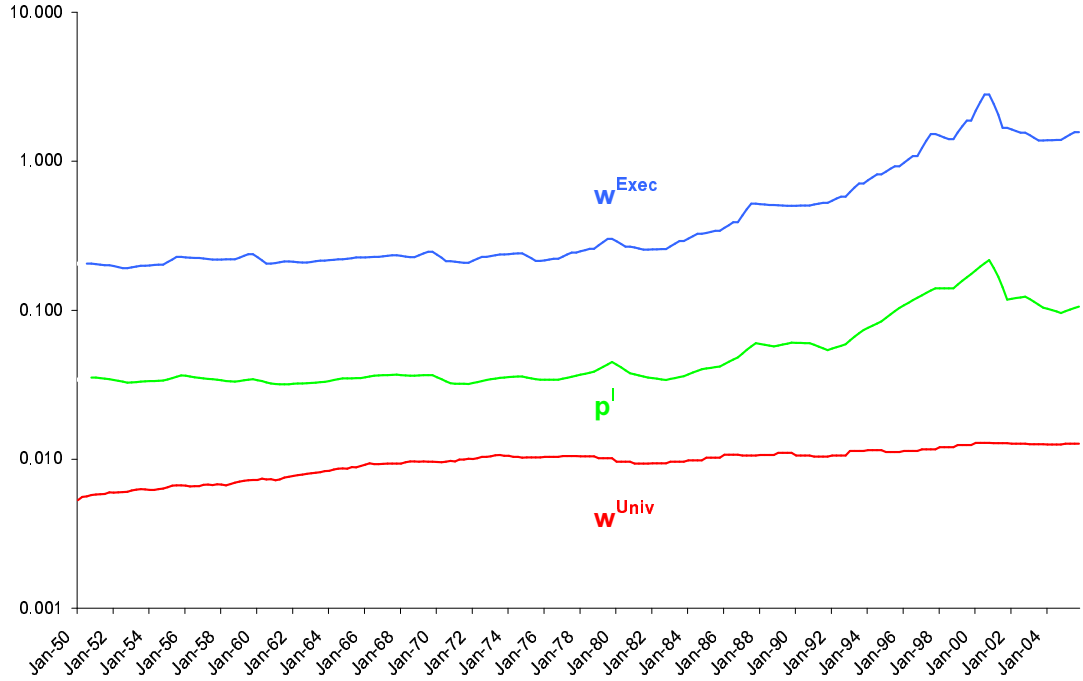


Figure 4.15: Real Wages and Price of Intangible Investment Goods.

It is possible to construct an aggregate price index that combines the price of intangible and tangible investment goods once the share of tangible expenditure in overall investment expenditures given by:

$$a_t = \frac{p_t^T x_t^T}{p_t^T x_t^T + p_t^I x_t^I} \quad (4.29)$$

is calibrated. Corrado *et al.* (2006) report the spending of U.S. firms on an identified list of intangible inputs as mentioned before. The National Income and Product Accounts (NIPA) recorded 20% of the reported expenditures in intangibles by Corrado *et al.* (2006). $p^T x^T$ is the private fixed investment as recorded by NIPA from which the included recorded intangibles are subtracted. These recorded intangibles consisted of software, mineral exploration, and architectural and design services. The time series behavior of this ratio captures the biased technological change which resulted in the

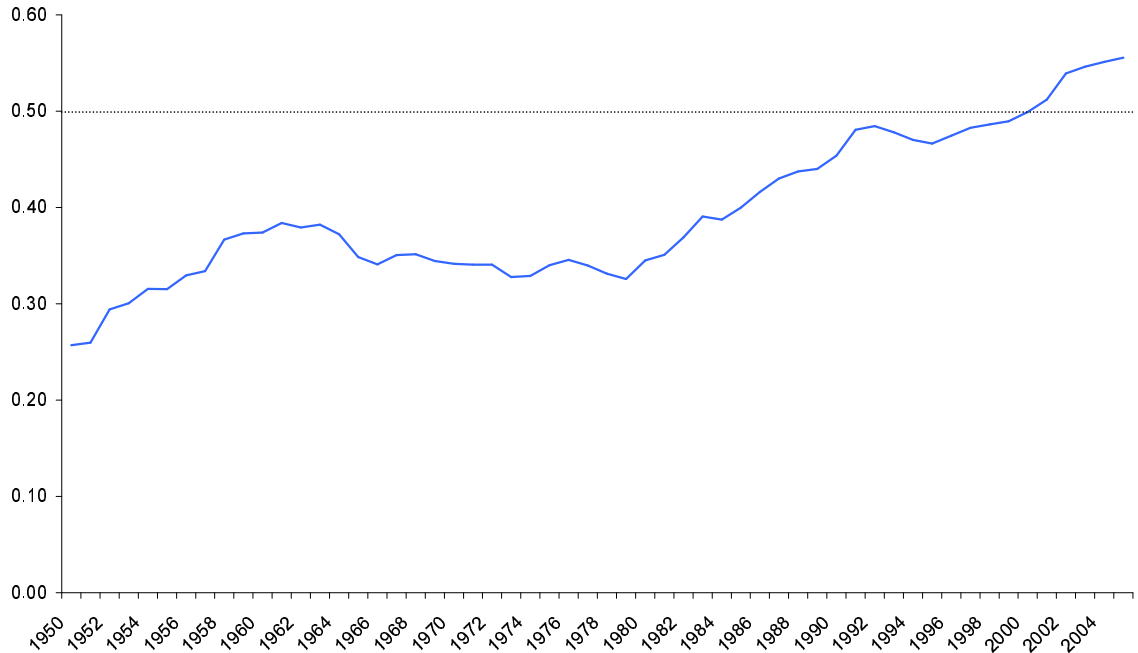


Figure 4.16: Share of Intangible Expenditure in Overall Investment Expenditure $1-a$. Source: Corrado *et al.* (2006) and author's calculations.

use of relatively more x^I . Figure 4.16 shows the behavior of a overtime.

As mentioned in the introduction, there is a striking upward trend beginning as early as the 1950s. After 2000, firms' investment in intangibles began to surpass the investment in tangible investment goods. For the purpose of this chapter, this is important in constructing an accurate measure of aggregate investment. Indeed, because the share of expenditure of firms in intangibles has been increasing overtime, the reported aggregate investment by national income accounts, which consists mainly of tangible investment, is only a partial reflection of the investment activity of firms.

It is now possible to use the behavior of the share series in the equation

$$p_t^x = \left(\frac{p_t^T}{a_t} \right)^{a_t} \left(\frac{p_t^I}{1-a_t} \right)^{1-a_t} \quad (4.30)$$

to derive the series on the aggregate investment price index p^x . This is shown in Figure 4.17 together with the price series of tangibles p^T . Both the rise in the price of intangibles and their increasing share in overall investment results in an aggregate investment deflator whose behavior contrasts markedly with the (physical) investment deflator reported by national income accounts; while the investment deflator of national accounts has a downward trend beginning in the mid-1950s, the investment deflator from the model has a downward trend until the mid-1980s and then rises until 2000 at which point it peaks and then falls²⁶. In other words, the behavior of the acquisition cost of capital is dramatically different when intangibles are accounted for. In particular, note that the price of intangibles from Figure 4.15 behaves opposite than that of the price of physical capital reported in Figure 4.17. This drives the unique shape of the aggregate price of investment, as illustrated in Figure 4.17, since it is a shared weighted index.

4.5 Quantitative Findings and Results

4.5.1 Behavior of the Stock of Capital

Figure 4.18 shows how the value of the firm v_t is contrasted to the generated capital stock k_t and to its acquisition cost $p_t^x k_t$.

The first notable feature is that the implied stock of capital smoothly increases until the peak of 2000 and then falls and flattens out. The second notable feature is the contrast between the series of the acquisition cost of capital with the one

²⁶The secular fall in the price of physical investment goods is a well documented fact. See in particular Greenwood *et al.* (1997) and Krusell (1998).

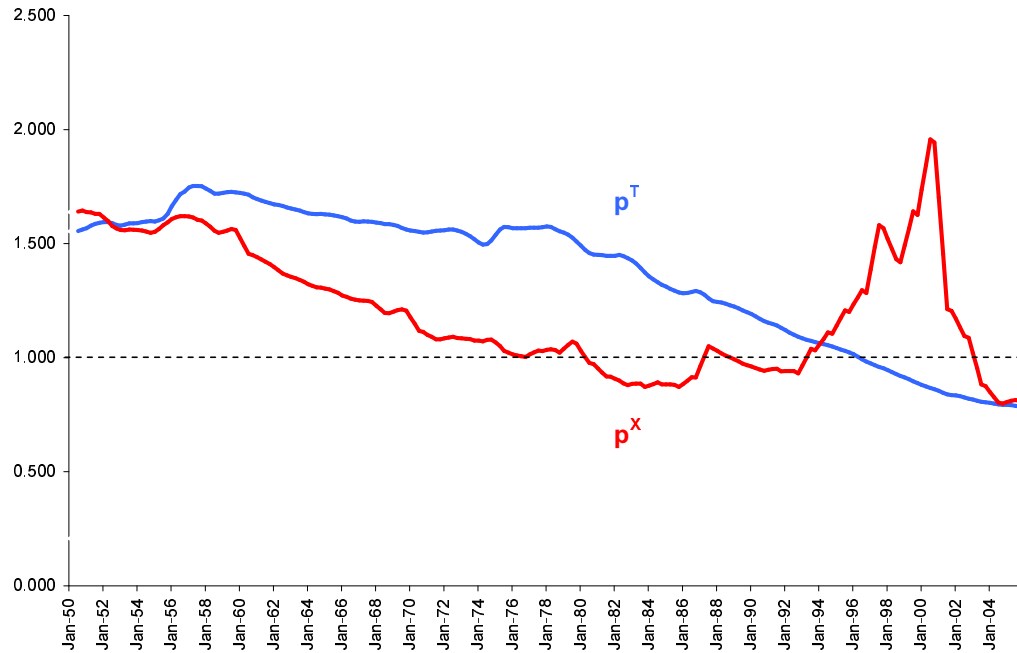


Figure 4.17: Real Price of Investment Goods. Source: NIPA and author's calculations.

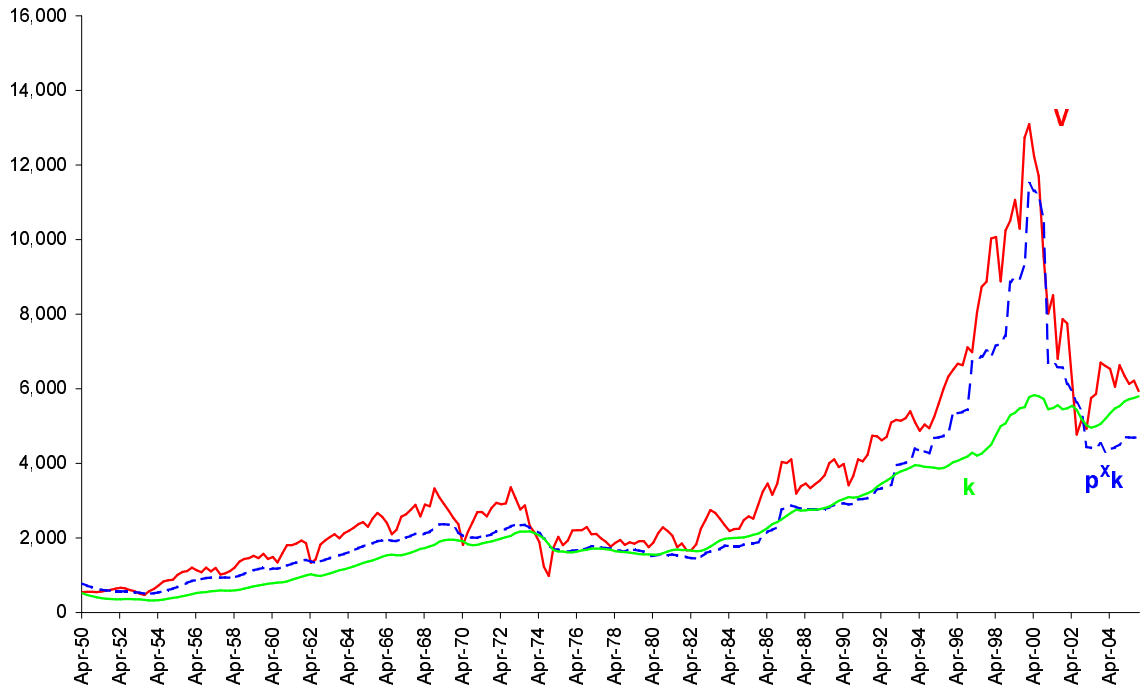


Figure 4.18: The Aggregate Value of Firms and the Inferred Capital Stock.

shown in Figure 4.11 which uses the national accounts investment deflator. The smoothness depicted from official statistics in Figure 4.11 is at odds with what Figure 4.18 depicts. The rise in the acquisition cost of capital as a result of the larger share of intangibles and their rising cost is at the source of this discrepancy. Note that allowing for irreversible investment doesn't affect the behavior of the capital stock series. Appendix C, section 4.4, extends the model to allow for irreversible investment and discusses why this addition is innocuous.

4.5.2 Contribution of Intangibles and Tangibles to Capital Stock

Figure 4.19 shows the decomposition of the contribution of the series of intangibles and tangible capital. Notice that the contribution of intangible investment goods has been substantial.

The rise in the overall capital stock from the mid-80s until the late 90s was driven mainly by an increase in intangible investment. After 2000, the contribution of intangibles falls consistently to reach a level similar to the one in 1990. These movements in the contribution of intangibles underscore the importance of this type of capital as a source of variation in the value for the firm and as a key component of any investment theory. The fall in the contribution of intangibles after mid-70s draws a parallel with the explanation of Greenwood and Jovanovic (1999) on the reasons for the fall in the value of securities in the early 70s. These authors argue that it is the arrival of the information technology at a time where the stock market incumbents of the day were not ready to implement it which depressed stock prices. The arrival

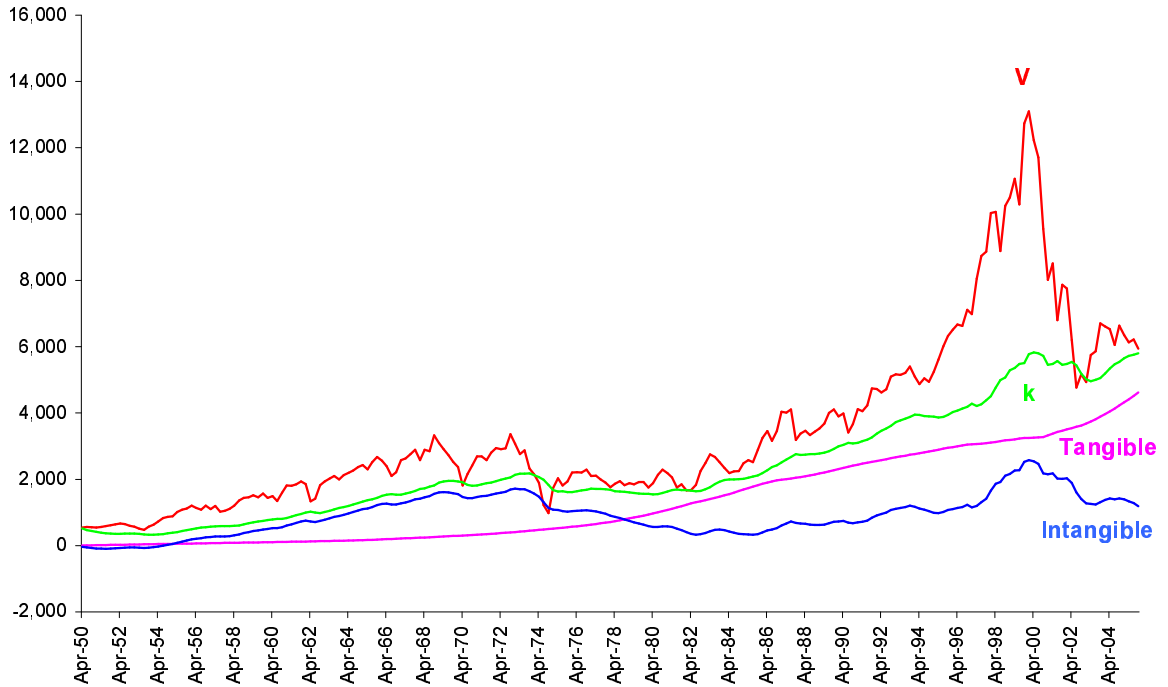


Figure 4.19: The Contributions to the Inferred Capital Stock

of this general purpose technology effectively destroyed the value of existing capital. This suggests that intangible capital experienced obsolescence and its value started to fall accordingly more rapidly than for the price of tangibles as suggested by Figure 4.17.

4.5.3 Implications for Tobin's q Series

Tobin's q is the ratio of the value of an additional unit of capital *in place* to the price of acquiring new capital.

$$q_t = \frac{\lambda_t}{p_t^x} \quad (4.31)$$

Since $v_t = \lambda_t k_t$, then

$$q_t = \frac{v_t}{p_t^x k_t} \tag{4.32}$$

The result from the extended q -theory shows that the behavior of Tobin's q is almost always positive. Figure 4.20 depicts the fact that the market value of the capital stock predominantly remains above and close to its acquisition cost; this reflects a Tobin's q that fluctuates closely around its equilibrium value, a desirable feature which is not observed in empirical measures of Tobin's q when capital is exclusively made of physical investment goods. In particular, Tobin's q does not fall below one for extended periods. Hall (2001) delivers a Tobin's q which is more volatile. His result stems from an absence of a rise and then a fall in the price of intangibles which would push up p_t^x before 2000 and lower it afterwards. This would offset the rise and then fall in the value of the firm and keep Tobin's q around its equilibrium value. The next section explores more thoroughly the differences in the findings with Hall (2001).

Note that other authors such as Laitner and Stolyarov (2003) deliver a theoretical Tobin's q that is positive and constant due to how they specify the production of intangibles in their model. In particular, their model is not one of adjustment costs but one of vintage capital goods and technological revolutions; it is not comparable to the setup of the model in this chapter.

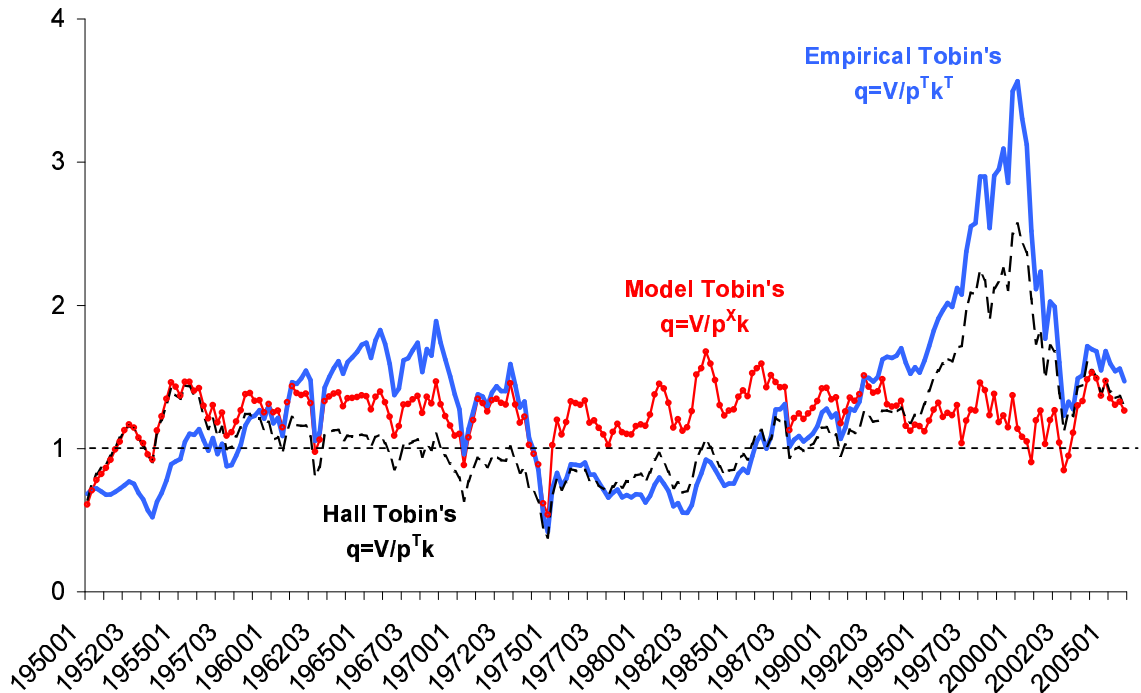


Figure 4.20: Behavior of Tobin's q

4.6 Comparison of Findings with Existing Literature

In this section, I will compare the model developed in this chapter with the models used both by Hall (2001) and by McGrattan and Prescott (2007a). The comparison will abstract from the presence of adjustment costs, taxes and difference in depreciation rates and will focus on the expression of the firm's value and the implications of assumptions made about the measurement of the size of the capital stock. The results of each paper will also be contrasted with the findings described above.

Recall that the forward substitution in the capital accumulation equation led to the general expression given by equation 4.24, reproduced here for convenience:

$$k_{t+N} = (1 - \delta)^{N+1} k_{N-1} + \sum_{i=0}^N \left((1 - \delta)^i x_{t+N-i} \right).$$

The first term tends to zero as N becomes large. The equation can then be rewritten as

$$k_{t+N} = \sum_{i=0}^N (1 - \delta)^i x_{t+N-i}. \quad (4.33)$$

This expression will be shown to differ in important ways across approaches.

4.6.1 Reframing Hall (2001)

As mentioned in the introduction, the model developed in this chapter differs from Hall (2001) by relaxing the assumption that $p^I = p^T$ and by allowing x^T and x^I to have some degree of substitutability. In the case of Hall (2001), the value of the firm is given by

$$V_t = p_t^T k_t = p_t^T (k_t^T + k_t^I) \quad (4.34)$$

where p^T is the (physical) investment price deflator from national accounts. Indeed, Hall assumes that there is one price of investment goods *i.e.*, $p^T = p^I$ and implicitly assumes that there are two equations for the capital stock at each point in time, one for each type of investment good:

$$\begin{cases} k_{t+N}^T = \sum_{i=0}^N (1 - \delta)^i x_{t+N-i}^T \\ k_{t+N}^I = \sum_{i=0}^N (1 - \delta)^i x_{t+N-i}^I \end{cases} \quad (4.35)$$

4.6.2 Reframing McGrattan and Prescott (2007a)

In McGrattan and Prescott (2007a), the model they use implies an expression for the value of the firm that can be written as:

$$V_t = p_t^T k_t^T + p_t^I k_t^I = k_t^T + p_t^I k_t^I \quad (4.36)$$

where p_t^T is the price of tangible investment goods (considered to be equal to the numeraire, assumed to be the consumption good) and p_t^I is the price of intangible capital goods. The price of intangibles is guessed as part of the solution to their general equilibrium model. Unfortunately, the authors do not report the series for p_t^I . The model of McGrattan and Prescott features two capital accumulation equations similar to Equation 4.35, as done by Hall as well.

4.6.3 Models' Comparison

In this chapter, the value of the firm without adjustment costs can be written as

$$V_t = p_t^x k_t = \left(\frac{p_t^T}{a_t} \right)^{a_t} \left(\frac{p_t^I}{1 - a_t} \right)^{1 - a_t} k_t. \quad (4.37)$$

Note that the two investment goods in this chapter are not accumulated because they are intermediate goods. However, in Hall (2001) and in McGrattan and Prescott (2007a), the two investment goods are final so they are separately accumulated over time. Moreover, the non-embodiment in Hall and in McGrattan and Prescott of investment goods results in two prices, one for each accumulated investment good while in this chapter, there is only one price for the accumulated investment good.

This one aggregate price deflator is attached to a single overall stock of capital, as it is commonly modeled when using an aggregate production function.

The comparison of the expressions for capital stock in each model will also reveal another consequence of the embodiment hypothesis. To see this, note that the embodiment of intangibles is reflected when equation 4.33 is expanded after using the Euler theorem on x_{t+N-i} :

$$\begin{aligned} k_{t+N} &= \sum_{i=0}^N (1-\delta)^i \left\{ \left(\frac{p_{t+N-i}^T}{p_{t+N-i}^x} \right) x_{t+N-i}^T + \left(\frac{p_{t+N-i}^I}{p_{t+N-i}^x} \right) x_{t+N-i}^I \right\} \\ &= \sum_{i=0}^N (1-\delta)^i \left(\frac{p_{t+N-i}^T}{p_{t+N-i}^x} \right) x_{t+N-i}^T + \sum_{i=0}^N (1-\delta)^i \left(\frac{p_{t+N-i}^I}{p_{t+N-i}^x} \right) x_{t+N-i}^I. \end{aligned} \quad (4.38)$$

Both parts of the right-hand-side of this last expression are similar to the corresponding equations for Hall (2001) and for McGrattan and Prescott (2007a) when we allow $p^T = p^I$. In this case,

$$k_{t+N} = k_{t+N}^T + k_{t+N}^I. \quad (4.39)$$

Such a capital stock exists in Hall (2001) by construction but not in McGrattan and Prescott (2007a) because this is not the capital stock which enters into the production of intangibles or in the production of final output. This is a major theoretical difference.

4.6.4 Comparison of Findings

Hall's implied capital stock series has a very pronounced bell-shape around 2000 as shown in Figure 4.21. Given that the acquisition cost of capital goods is identified to the tangible deflator which was falling throughout, his model's implication of a

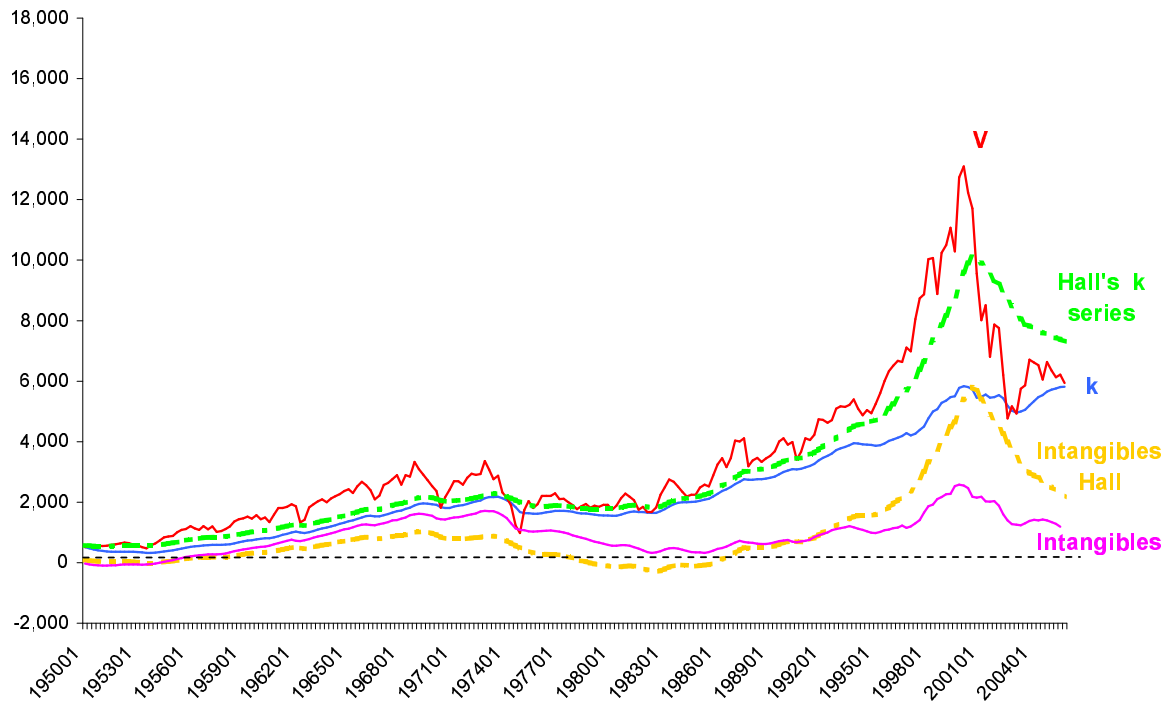


Figure 4.21: Comparison of findings with Hall (2001)

rise and then fall in the value of securities is for the firm to invest massively and then disinvest. Everything else the same, the secular fall in the price of new investment goods leads to more investment. This is not the case in the extended model presented here: the acquisition cost of capital becomes heavily skewed towards intangibles whose price begins to rise starting in the 80s. It then falls after 2000 causing the acquisition cost of capital to have a bell shape around 2000 (recall Figure 4.17). The implication is that the firm in this chapter has less incentive to accumulate capital before 2000 and to disinvest after 2000 once the price of aggregate investment falls. This explains why the extended model features a much less pronounced bell-shape around 2000.

The assumption of the equality between the prices of intangibles and tangibles

leads to Hall's finding that the quantity of intangible capital is negative from the mid-70s until the mid-80s. The model in this chapter does not feature such an anomaly. To see why, observe that the constructed price series of aggregate investment in Figure 4.17 falls faster than the tangible price series up until the mid-80s. Firm's incentive to invest in the extended model is therefore bigger than in Hall's model. As a result, the accumulation of capital is higher during this period than in Hall, which makes the implied contribution of intangible investment higher.

Comparing the extended model's results with the findings of McGrattan and Prescott (2007a) is not as straightforward because they use a general equilibrium framework and because they emphasize different series. In particular, the price of intangibles is not reported in their paper. In their technical appendix (McGrattan and Prescott, 2007b) the authors only mention that they guessed this price series. Furthermore, McGrattan and Prescott do not report a series for intangible investment alone, which makes it hard to compare the size of intangible investment directly. However, the authors report two series that include, in one way or another, intangible investment: 1) intangible investment as a share of total GDP (their Figure 6) and 2) a graph containing the series of overall investment and tangible investment (their Figure 12).

It is not possible to use the first series to make quantitative comparisons regarding the amount of intangible investment because McGrattan and Prescott make many corrections to GDP. However, this series can be used to compare the qualitative findings. In particular, McGrattan and Prescott find that intangibles are falling in the early 90s while in this chapter intangible investment was shown to rise during

that period. The behavior of intangible investment around 2000 is similar.

The second series is used to approximate the difference between the overall investment and the tangible investment, which is the intangible investment found in McGrattan and Prescott. Following this method of backing out intangible investment, it seems that intangible investment is quite small between 1990 and 1994. It then rises until 1999; This rise is about 20%. Afterwards, intangible investment falls to a very low level. The drop is about 40% between 2000 and 2003. On the other hand, the extended q-theory model results in a rise of intangible investment by 150% between 1990 and 1992. Then it falls by 150% between 1992 and 1994. After 1994, intangible investment increases until 1999 by 300% and falls afterwards by 350%. The bottom line of these comparisons is that the extended model reports more intangible investment growth before 2000 and more decline after 2000 than that in McGrattan and Prescott (2007a).

To sum up, the above discussion implies that the reported results in this paper lie somehow in between the findings of Hall and of McGrattan and Prescott.

4.7 Sensitivity Analysis

In this section, two experiments are conducted to assess the robustness of the results. The first involves specifying a different value for the elasticity of substitution than was assumed in the baseline model ($\rho = 0$). In order to explore the consequence of assuming less complementarity, *i.e.* when $\rho \in (0; 1]$, and more complementarity, *i.e.* when $\rho \in (-\infty; 0)$, between the two investment goods compared to the baseline

Periods	Baseline model	Rho = -0.5	Rho = 0.5	Using only skilled labor	Using only the "salary" component	Using only the "salary+shares" component
1950-1960	0.17	0.22	0.17	0.06	0.13	0.07
1960-1970	0.79	0.76	0.79	0.77	0.79	0.78
1970-1980	0.66	0.62	0.65	0.65	0.66	0.66
1980-1990	0.24	0.32	0.26	0.32	0.26	0.29
1990-1995	0.27	0.41	0.30	0.38	0.30	0.34
1995-2000	0.34	0.54	0.42	0.48	0.39	0.43
2000-2005	0.30	0.35	0.34	0.35	0.32	0.35

Table 4.8: Sensitivity of the Contribution of Intangibles Relative to the Overall Capital Stock

case, two values of ρ are used. These values are arbitrarily chosen. To explore the consequence of less complementarity, or equivalently, more substitution, ρ is set equal to 0.5. Then, to explore the consequence of more complementarity, or equivalently, less substitution, ρ is set equal to -0.5 . The impact of changing the value of ρ on the model will appear through the value of p^x , as shown in Equation 4.11, and ultimately on the generated stock of capital and the contribution of each type of investment good according to Equation 4.24.

The second experiment involves using different wage series data when calculating the price index of intangibles. Here again, I explore the consequence of first using only the real wage of university graduates and second, different portions of the components of executive compensation, on the price of the composite investment good. The impact of the experiments on the contribution of intangibles relative to the overall capital stock is summarized in Table 4.8.

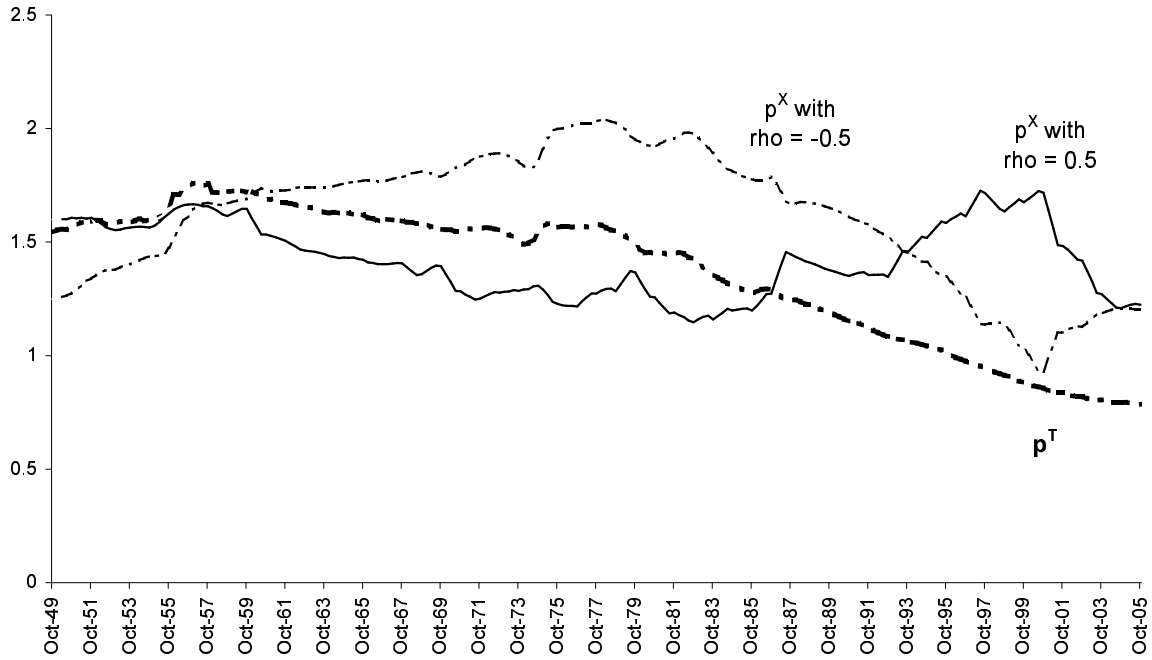


Figure 4.22: Impact of Different Values of the Elasticity of Substitution on the Price of Composite Investment Good

4.7.1 Allowing for Different Elasticity of Substitution Values

Figure 4.22 depicts the consequence of allowing for more and less complementarity between the two investment goods on the price of the composite investment good. By setting $\rho = 0.5$, the two investment goods are made less complementary to each other. This results in an investment price index which has an overall behavior that is similar to the price index from the baseline model but nevertheless, its rise and fall are less pronounced. On the other hand, allowing $\rho = -0.5$ leads p^x to behave almost opposite to its value with $\rho = 0.5$. By looking at Equation 4.11, it becomes obvious why this occurs: with more substitutability allowed, the income share of each investment good relative to its own price becomes the relevant variable. The opposite occurs when we allow for more complementarity. In any case, it is interesting to note

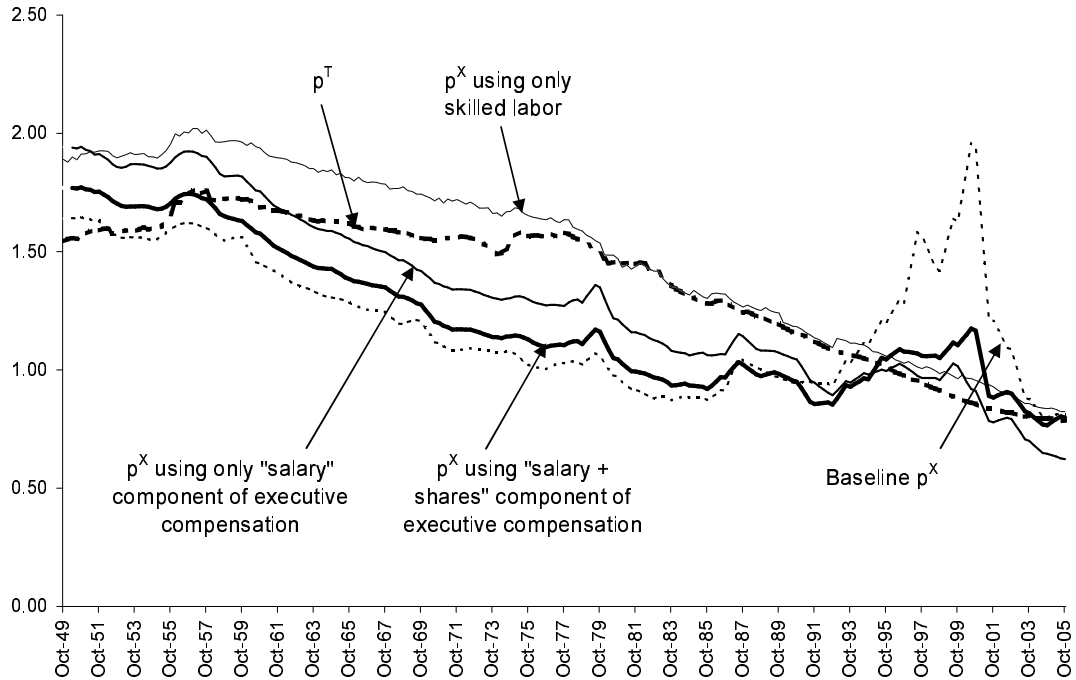


Figure 4.23: Impact of Different Wage Series on Price of Composite Investment Good

that the behavior of the generated capital and the contribution of each input are not significantly affected in either scenario. Table 4.8 shows the discrepancies in the relative contribution of intangibles for each value of ρ .

4.7.2 Allowing for Different Wage Series

In this exercise, I consider the impact of using first only the real wage of university graduates and second, only one component of the executive compensation at a time on the price of the composite investment good. Figure 4.23 reproduces the findings of the baseline model shown in Figure 4.17 and the results of each experiment²⁷.

²⁷Note that because the wage data series are being changed, the implied initial values had to be adjusted each time in accordance with the procedure described in Appendix C, Section 4.3 and mentioned in Footnote 27.

The first experiment involved using only the wage of skilled labor. Because labor productivity, θ , grows faster than the wage of skilled workers, the price of intangibles falls monotonically over time. However, given that this fall is not as steep as the fall in the price of tangibles before the 80s and after the mid-90s, I find that the price of the composite investment good is almost always higher than the price of tangibles. Note that the rise in the composite investment good price after the mid-90s relative to the reported tangible price series is due to the increasing share of intangibles in overall investment. This causes the relatively benign fall in the price of intangibles to reduce the fall in the price index of the composite investment good. This finding suggests that official statistics still under-estimated the price of capital goods by the late 90s even though we don't observe the same pattern as in the baseline case. As for the impact on the contribution of intangibles and tangibles to the overall capital stock, the qualitative findings remain the same as the baseline specification (see Table 4.8). However, quantitatively, the contributions track the ones found in Hall, admittedly with a different interpretation, as explained in the previous section. This happens because the composite price index series behaves almost similarly to the price of tangibles. That the quantitative findings are similar to Hall is not surprising: the complementarity assumption works through the price index. If the constructed price index resembles the behavior of the price of tangibles, then the behavior of the generated capital and the contribution of each input is almost the same.

The second experiment involves using the different components of the executive wage compensation which are shown in Figure 4.14. Figure 4.23 shows that as I eliminate each component of executive compensation at a time, the overall behavior of

the composite investment index is the same but the rise up to 2000 and the subsequent fall, are less pronounced. Note also that as I downplay the compensation of executives, and at the same time their share in wage compensation relative to skilled workers, the composite investment index price series gets closer to the price of tangibles. As a result, the implied stock of capital is closer to the one reported by Hall.

To summarize, the non-inclusion of executive compensation or the inclusion of different components of executive compensation alters the behavior of the composite investment price index but does not take away from the main message of the text: namely, that the reported index of the price of capital goods has a downward bias in the 90s due to the rise in the contribution of intangibles in overall investment. This exercise shows also that the derived capital series and the contribution of intangibles and tangibles to the overall capital stock is not qualitatively affected.

4.8 Conclusion

This chapter extended the q -theory of investment to account for intangible investment and used the model to measure the contribution of embodied intangible goods to the overall capital stock in the U.S. The chapter also explored the quantitative implications of the model on the behavior of the capital stock and Tobin's q .

The price trend of aggregate investment was shown to contrast markedly with the aggregate investment deflator series reported by national accounts because this deflator series does not account for the price effect of intangibles. In addition, the extended model successfully generated a smoothly-behaving series of capital stock

with a market value that predominantly remains above its book value; this reflects a Tobin's q that is mostly above one, a feature which is not observed in empirical measures of Tobin's q when capital is exclusively made of tangible investment. Furthermore, the model shows that the rise in the overall capital stock from the mid-80s until the late 90s was driven mainly by an increase in intangible investment. However, the contribution of intangibles fell consistently after 2000. These results underscore the importance of accounting for the movements in the price of intangibles rather than focusing only on their rising share in overall investment. These findings confirm the changing nature of the stock of capital in the economy and the quantitative importance of intangibles, underscoring the importance of intangibles as a source of value for the firm and as a key component of any investment theory.

Chapter 5

Conclusions and Future Work

In this thesis, I explore the evidence on the investment of firms on intangible capital, both for Canada and the U.S. In addition, I investigate the impact of this knowledge good investment on the overall capital stock and on the value of capital goods. Since direct and indirect evidence on intangible investment points to its rising share in overall investment, it is important to monitor the progress of this phenomenon and draw the implications on the growth of the capital stock and the value of capital goods.

This thesis contributes to the existing knowledge on intangible investment in several ways. Chapter 2 explores the evidence on the expenditures of Canadian firms on a set of identifiable items which are considered as intangible. The results point to a rising investment in intangibles, which by 2002 reached almost the same level as the investment in physical capital. This finding is similar to related studies conducted for the U.S. and the U.K.

In Chapter 3, the use of the aggregate value of securities as a proxy for the stock of installed capital in Canadian corporations points to a similar rise in the stock of intangibles. As a result, one-fourth of the value of the capital stock is unaccounted

for by Canadian statistics.

Finally, this thesis goes beyond the current practice in the literature, which consists of setting the price of intangible capital goods equal to the price of tangible capital goods, by constructing a price index of intangibles for the U.S. Chapter 4 documents that the time series behavior of these two prices behaves almost opposite to one another. This fact implies that the aggregate investment deflator reported by national accounts has a significant upward bias in the 70s and a downward bias in the 90s. As a result of this price movement of intangibles, the contribution of intangible investment to the overall capital stock points to a rising role before 2000 and a smaller role afterwards.

There are many promising avenues for future research. For Chapter 2, I plan to extend the period covered from 1980 to 1998. The data for this period is even sparser and requires better consolidation. The goal is to conduct a growth accounting exercise to measure the implication on TFP measurement of the omission of intangibles in the estimates of the overall capital stock. Related to this, a second research direction is to use the investment data on intangibles to calculate the stock of intangibles in the Canadian economy. This will involve making compromises regarding which appropriate depreciation rate and price of capital to use. As Chapter 4 shows, there is evidence that the behavior of the price of intangibles is drastically different from the behavior of the price of physical capital goods or of consumption goods. Finally, one dimension that was not studied and discussed in Chapter 2 is the implication of the capitalization of intangibles on the income side of the national income identity. The income shares of the inputs will be affected in a non-trivial way. With a longer data

set, it will be useful to examine how the pattern of these shares changed and draw conclusions regarding the winners and losers of the increasing share of intangibles in overall investment.

For Chapter 3, it would be desirable to extend the model to account for differences that are inherent in the two capital goods. For example, the model could be enriched by explicitly accounting for the non-rival nature of most intangible goods and allow for the existence of spillovers. The perfectly competitive structure and the non-existence of externalities that the model assumed did not allow for such possibilities. Another avenue that needs further exploration is the impact of the presence of rents once imperfect competition is introduced. Part of the value of the firm will then include a portion that is made up of rents. If this portion is large then the baseline model could be confusing some of the capital accumulation for rent accumulation with the consequence of underestimating the shadow price of capital. An analogous implication would be the overestimation of the quantity of capital. I plan to address this issue by extending the framework of this Chapter 3 to accommodate for the existence of rents and measure their economic importance. Finally, the future extension of the data for the period pre-1990 by Statistics Canada will be very valuable as it will open the door to computing a longer series of intangibles and explore the consequence of the existence and behavior of intangible capital on the measures of productivity performance of the Canadian economy.

Two main avenues are proposed for future work related to Chapter 4. First, the price of aggregate investment reflected two secular stylized facts: 1) the larger role for intangibles in production and 2) the rise in their cost of production as illustrated

by the growth in compensation of skilled labor and executives. The link between the labor market for high-skilled workers and capital markets was shown to be key in obtaining these results. This feature might provide a better understanding of the mixed performance of the econometric estimation of the investment equation. These regressions tend to produce low R^2 's and serially correlated residuals. Moreover, additional regressors, such as output and cash flow, also appear to be important factors in the investment decision, as they typically have statistically significant coefficients (Chirinko, 1993). These problems may be due to the omission of intangibles when valuing the true cost of new investment goods. In future work, I plan to use the data series on the acquisition cost of capital corrected for the inclusion of intangibles, as constructed above, to investigate whether the econometric estimation of the investment equation did indeed suffer from specification problems.

A second future extension to Chapter 4 is to explore the insight that intangibles are produced mainly by skilled workers, in order to address the puzzle found in Hall (2003). In that paper, Hall finds that the payment to intangibles is almost nil. He reaches this conclusion after comparing the income stream accrued from physical capital as predicted by the baseline user-cost of capital theory, with the income stream observed from earnings of firms. Hall finds that the present value of the future flow of income generated by intangibles is close to zero, suggesting that their size is unimportant. I suspect that the treatment of intangibles as expenditures (as opposed to capital goods) caused reported earnings to be undervalued. This omission could deliver the finding that once physical capital is paid for its services, there is nothing left to reward intangible capital. I plan to redo Hall's (2003) exercise

using earnings that are corrected for this mismeasurement and investigate whether Hall's findings hold for this correction.

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Appendix A: Breakdown of Spending in Intangibles by Item

Table 2.9 reports the constructed data series on the spending of firms in intangibles by items.

Type of intangible investment	Spending in Millions of Dollars							
	1998	1999	2000	2001	2002	2003	2004	2005
Computerized information	9962	10512	11016	13166	12818	13457	13675	14179
Computer Software	8867	9307	9419	10890	10227	10806	11000	11364
Computerized databases	1095	1205	1597	2276	2591	2651	2675	2815
Innovative property	39570	42951	48702	53552	55382	58407	63563	68491
Scientific R&D	15083	16468	19272	21779	21983	22589	24079	25073
Mineral exploration	9463	9802	11645	12197	12274	13225	14328	15596
Non-Scientific R&D	15025	16681	17785	19576	21125	22594	25156	27822
Copyright and license costs	251	263	310	566	623	1124	1346	1545
New product devpmt in the financial industry	134	109	142	173	211	235	318	354
R&D in remaining service industries	14640	16309	17333	18837	20291	21235	23492	25923
Economic competencies	36252	39792	42620	42198	44734	44975	49244	51664
Brand equity	5083	5324	5817	5928	5951	5857	6173	6867
Advertising expenditure	4342	4438	4796	4943	4894	4735	4984	5613
Market research	740	887	1021	985	1057	1123	1189	1254
Firm specific human capital	21800	23582	25365	23854	25660	25828	29151	29558
Direct firm expenses	10900	11791	12682	11927	12830	12914	14576	14779
Wage and salary costs of employee time	10900	11791	12682	11927	12830	12914	14576	14779
Organizational structure	9369	10885	11438	12415	13123	13289	13920	15239
Purchased	6023	6997	7353	7981	8436	8543	8948	9796
Own account	3346	3888	4085	4434	4687	4747	4972	5443
Total	85784	93254	102338	108916	112933	116839	126482	134335

Table 2.9: Breakdown of Spending in Intangibles by Item

Appendix B: Data Manipulations and Sources for Chapter 3

The procedure to insure that all shares of the non-farm non-financial corporate sector are in market value is as follows. First, I obtain the market value of all listed shares from the World Federations of Exchanges website (<http://www.world-exchanges.org/WFE/>). This series represents the market capitalization of the TSX and is available starting January 1995 under the heading “statistics/monthly”. I extend the series backward to 1990 using the S&P/TSX composite index obtained from CANSIM (series label V122620). Second, I take the level of shares at book value and market value of all corporations from the NBSA (respectively the labels V20682659 and V28368658). I obtain the value of unlisted shares at book value by subtracting the market capitalization of the TSX from the level of shares of all corporations at market value. I then obtain the value of listed shares at book value by subtracting from the shares of all corporations at book value the unlisted shares portion just calculated. Finally, I construct a price index by dividing the listed shares at market value by the listed shares at book value. I then use this price index to inflate the book value series of the non-farm non-financial corporate sector. I use it on both the book value series of liabilities and assets of this sector (given respectively by the labels V20682692 and V20682673). These two data series are reported in Table 2.1.

The nominal net market value of securities is obtained by subtracting the value of liabilities series from the value of assets. This amount is deflated by the investment deflator which is obtained by dividing the gross nominal private investment series (V498927) by the gross real private investment (V1992271). The resulting series is the variable v_t .

The series for the recorded capital stock is calculated by cumulating overtime the quarterly investments in fixed capital by the non-farm, non-financial corporate sector taken from the FFA (V34914) while removing at the same time the depreciated part at each quarter assuming a depreciation rate of 10%. This investment in fixed capital

Quarters	Market value of liabilities	Market value of financial assets	Quarters	Market value of liabilities	Market value of financial assets
1990Q1	1,069,347	461,118	1998Q2	2,761,116	1,356,233
1990Q2	1,057,895	452,767	1998Q3	2,224,033	1,104,446
1990Q3	1,006,034	432,616	1998Q4	2,633,052	1,311,218
1990Q4	1,019,384	435,703	1999Q1	2,601,037	1,305,822
1991Q1	1,062,623	462,979	1999Q2	2,809,568	1,427,221
1991Q2	1,060,513	460,003	1999Q3	2,770,789	1,413,242
1991Q3	1,060,993	458,014	1999Q4	3,343,751	1,703,599
1991Q4	1,101,445	472,715	2000Q1	3,691,009	1,891,020
1992Q1	1,091,813	479,557	2000Q2	4,256,924	2,181,759
1992Q2	1,114,047	491,099	2000Q3	4,220,281	2,161,963
1992Q3	1,117,999	493,858	2000Q4	3,504,809	1,807,549
1992Q4	1,157,615	513,798	2001Q1	3,031,529	1,560,048
1993Q1	1,227,042	548,535	2001Q2	3,167,348	1,640,818
1993Q2	1,323,151	587,364	2001Q3	2,781,554	1,439,411
1993Q3	1,392,012	628,004	2001Q4	3,151,825	1,646,010
1993Q4	1,531,017	693,552	2002Q1	3,286,813	1,705,871
1994Q1	1,528,409	699,204	2002Q2	3,030,176	1,591,654
1994Q2	1,546,083	705,558	2002Q3	2,701,386	1,419,164
1994Q3	1,656,469	764,850	2002Q4	2,831,656	1,497,154
1994Q4	1,572,813	736,239	2003Q1	2,648,691	1,392,792
1995Q1	1,520,312	707,575	2003Q2	2,940,932	1,545,696
1995Q2	1,672,590	787,231	2003Q3	3,120,966	1,654,865
1995Q3	1,640,113	776,379	2003Q4	3,563,604	1,935,844
1995Q4	1,735,100	828,036	2004Q1	3,823,860	2,102,784
1996Q1	1,814,899	867,267	2004Q2	4,126,673	2,300,326
1996Q2	1,914,776	923,999	2004Q3	4,153,015	2,317,611
1996Q3	1,961,333	954,634	2004Q4	4,450,058	2,487,784
1996Q4	2,347,897	1,132,167	2005Q1	4,574,016	2,597,895
1997Q1	2,265,592	1,111,234	2005Q2	4,697,921	2,687,315
1997Q2	2,463,546	1,221,725	2005Q3	5,053,895	2,910,910
1997Q3	2,730,379	1,367,555	2005Q4	5,254,254	3,042,614
1997Q4	2,582,517	1,258,609	2006Q1	5,342,077	3,109,073
1998Q1	2,876,877	1,398,559	2006Q2	5,341,878	3,104,318

Table 2.1: Financial Assets and Liabilities at Market Value

is deflated each quarter by a deflator obtained from dividing the nominal value of non-residential and equipment investment series (V498929) by the real value of non-residential and equipment investment (V1992273) both taken from the NIEA. The initial level of the capital stock is taken to be the one given in the last quarter of 1961 made of the the summation of the stock of machinery and equipment (V33216), the stock of non-residential capital (V33215), all taken from the NBSA of the non-farm, non-financial corporate sector.

Statistics Canada does not collect data on the R&D expenditure incurred by the non-farm, non-financial corporate sector. The closest data available for the needs of this study is the nominal R&D business enterprise expenditure (V617324) collected at yearly frequency. To calculate the R&D investment of the non-farm, non-financial corporate sector, the relative investment amounts of this sector in the overall business investment was computed and used to scale the R&D expenditures data accordingly. The relative investment of the non-farm, non-financial sector at each quarter was calculated by dividing the investment in fixed capital of the non-farm, non-financial corporate sector taken from the FFA (V34914) by the investment in fixed capital by all businesses (V498929) taken from the NIEA. Once the R&D portion of this sector was obtained, it was deflated by the same deflator used for the stock of recorded capital, at yearly basis. Finally, the stock of R&D capital was calculated using the perpetual inventory method²⁸:

$$R_t = I_t^{R\&D} + (1 - \delta)R_{t-1}$$

where R_t is the stock of R&D at time t , $I_t^{R\&D}$ is the investment flow in R&D at time t and δ is the depreciation rate set equal to 10%. The initial stock of R&D in 1963 was calculated using the formula obtained by backward recursive substitution of the above equation:

$$R_{1963} = \left(\frac{1 + g}{\delta + g} \right) I_{1963}^{R\&D}$$

where g is the historical average growth rate of R&D expenditures. It is assumed that preceding the initial observation, there was a long period of constant investment growth in R&D of g which is set equal to the average growth rate for the period 1963-2005. In any case, the stock of R&D obtained in 1990 is not sensitive to these initial starting point assumptions given the small magnitude of investment in R&D in the early 60s.

²⁸There is a long tradition, prominently described in Griliches (1979), of calculating the stock of R&D capital as a weighted sum of past expenditures in R&D.

Appendix C: Chapter 4

Appendices

4.1 Derivation of First-Order Conditions

The firm's problem is

$$\begin{aligned} \max_{\{l_t^u, l_t^s, x_t\}} v_t &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \hat{v}_s \\ \hat{v}_t &= F(k_{t-1}^x, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s - p_t^x x_t - C(x_t, k_{t-1}) \\ \text{s.t.} \\ k_t &= (1 - \delta)k_{t-1} + x_t \\ k_{t-1} \\ \lim_{T \rightarrow \infty} \left(\frac{1}{1+r} \right)^T \hat{v}_{s+T} &= 0. \end{aligned}$$

The Lagrangian \mathcal{L} at time s and the first order conditions are given by

$$\begin{aligned} \max_{\{l_t^u, l_t^s, x_t, k_t, \lambda_t\}} \mathcal{L}_s &= \sum_{t=s}^{\infty} \left(\frac{1}{1+r} \right)^{t-s} \{ F(k_{t-1}, l_t^u, l_t^s) - w_t^u l_t^u - w_t^s l_t^s \\ &\quad - p_t^x x_t - C(x_t, k_{t-1}) - \lambda_t [k_t - (1 - \delta)k_{t-1} - x_t] \} \end{aligned}$$

$$\frac{\partial \mathcal{L}_t}{\partial x_t} : \quad \lambda_t = p_t^x + C_x(x_t, k_{t-1})$$

$$\frac{\partial \mathcal{L}_t}{\partial l_t^u} : \quad w_t^u = F_{l^u}(k_{t-1}, l_t^u, l_t^s)$$

$$\frac{\partial \mathcal{L}_t}{\partial l_t^s} : \quad w_t^s = F_{l^s}(k_{t-1}, l_t^u, l_t^s)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial k_t} : \quad & \lambda_t(1+r) = F_k(k_{t-1}, l_t^u, l_t^s) - C_k(x_{t+1}, k_t) + (1-\delta)\lambda_{t+1} \\ \frac{\partial \mathcal{L}_t}{\partial \lambda_t} : \quad & k_t = (1-\delta)k_{t-1} + x_t \end{aligned}$$

where λ is the Lagrangian multiplier or the shadow price of an additional unit of capital. We assume that bubbles in the shadow price of capital are ruled out, *i.e.* $\lim_{T \rightarrow \infty} \left(\frac{1}{1+r}\right)^T q_{t+T} = 0$. The first equation illustrates the equality of the lifetime return to increasing capital by one unit with its marginal cost given by the price of a unit of capital plus the marginal adjustment cost of installing this unit of capital. This equation determines the optimal investment amount to be chosen by the firm. The second and third equation state the usual equilibrium condition for the labor market whereby the real wage is equal to the marginal product of labor. The next equation shows the dynamic equilibrium equation of λ with its continuation value. The last equation recasts the investment technology constraint.

4.2 Data Sources and Definitions

The deflator used is the quarterly CPI and the base year adopted is 1996. This series is obtained from the Bureau of Labor Statistics which reports the CPI on a monthly basis for all urban consumers from 1913 until today (Series ID: CUUR0000SA0). I take the average of three consecutive months to obtain the quarterly equivalent.

The data on output per worker are published by the Bureau of Labor Statistics under the heading “major sector productivity and costs index”. Because of data availability, I use the output per person of the business sector from 1950 until 1957 (Series ID: PRS84006163) and then the output per person of the non-financial corporate sector from 1957 until 1975 (Series ID: PRS88003163).

The data on wages of workers by educational attainment is collected annually by the Bureau of Labor Statistics in the Current Population Survey and reported every March for the whole economy. I use table A-3 entitled “Mean Earnings of Workers 18 Years and Over, by Educational Attainment, Race, Hispanic Origin, and Sex: 1975 to 2005” to obtain data on mean annual earnings and number of workers by educational attainment. There are 5 educational levels: not a high school graduate, high school graduate, some college/associate degree, bachelor’s degree and advanced degree. Earnings refer to the total income people receive for work performed as an employee during the income year. This includes wages, salary, armed forces pay,

commissions, tips, piece-rate payments, and cash bonuses earned, before deductions are made for items such as taxes, bonds, pensions, and union dues. The wage of university workers is calculated as follows. I multiply the mean annual earnings of workers with a bachelor's degree and advanced degree by their respective number of workers and divide the result by the total number of workers with a bachelor's and advanced degree. I extend the data from 1975 until 1950 using the growth of output per worker. The implicit assumption is that the mean earnings of university graduates grew at the same rate as productivity per worker from 1950 to 1975. This assumption has a strong empirical support during this period (see Lemieux (2007)).

The wage bill of skilled labor from 1975 to 2005 is obtained by multiplying the mean earnings of university graduates by the number of university graduates. I extend the data back to 1950 using the economy wide compensation of employees. This series is provided by the Bureau of Economic Analysis in their Table 1.10. entitled "gross domestic income by type of income", line 2. This extension relies on the same assumption as the one outlined in the previous paragraph.

The data on executive compensation and its composition are taken from Frydman and Saks (2007). This series is a major improvement on previous studies which collected data on executive compensation for short samples, with different sample designs and employed different methodologies to value compensation and its components (see for example Antle and Smith (1985), Hall and Liebman (1998) and Bebchuk and Grinstein (2005).) The work of Frydman and Saks (2007) is the first comprehensive panel dataset on executive compensation that spans the period 1936 to 2003. The sample follows the compensation of individual officers in the largest 50 publicly traded corporations ranked according to the value of sales in 1940, 1960 and 1990. This amounted to a total of 102 firms. Frydman and Saks discuss the representativeness of their sample in Appendix Section 3, and conclude that it is representative of the largest 300 publicly-traded corporations. They limit their analysis to the top three officers in order to maintain a consistent group of individuals over time, but the results are robust to including the 4th and 5th highest-paid executives.

The data on the number of chief executives is taken from the Occupational Employment Statistics (OES) Survey produced by the Bureau of Labor Statistics. The occupational title is "chief executive" with the occupational code 11-1011. This data is available from 1998 to 2005. From 1983 until 1997, the OES survey used a somewhat different classification system. The closest occupational definition for our purpose is the "management, business, and financial operations occupations" (Series ID: LNU02032202). I use the growth rate of this occupation to extend the data on the number of chief executives backward to 1983. Finally, I use the growth rate of the employment in the private sector to extend this data backward to 1950. This data are

produced by the Bureau of Economic Analysis under Table 6.5 A-D with the heading “full-time equivalent employees by industry”.

The wage bill of executives is calculated by multiplying the average CEO compensation reported by Frydman and Saks (2007) by the number of chief executives described in the previous paragraph.

The price of tangible investment goods is the national income and product accounts implicit deflator for fixed non-residential investment. This series is published by the Bureau of Economic Analysis under Table 7.1 “quantity and price indexes for GDP quarterly”, line 32.

The data on the real capital stock and the value of securities are taken from Hall (2001) and are extended to 2005. The data on intangible expenditures are taken from Corrado *et al.* (2006).

4.3 Calibration of Starting Values

The initial value of p^I is calibrated such that the firm’s investment condition given by equation 4.19 holds in the long-run. In other words,

$$\delta\alpha = \left(\frac{\bar{v}}{\bar{k}} - \bar{p}^x\right)$$

where the variables with a bar denote their sample average. Once the value p_0^I is found, θ_0 is deduced and made to grow at the rate of change of output per worker. Also, p_0^I is used to calculate the starting value of p^x .

The empirical starting value of the share of executives in overall wage bill is $\phi_0 = 0.47$. Corrado *et al.* (2006) calibrate this value to be $\phi_0 = 0.21$ on the basis that executives spend 20% of their time in activities that involve organizational change and design. I chose this starting value to be an average of the two values $\phi_0 = 0.33$. The results are not sensitive to departures from this initial value inside the set of values made of these two boundary values.

Finally, the starting value of a is taken from the constructed series on the share of tangible expenditures in overall investment expenditures. It is found that $a_0 = 0.74$.

4.4 Allowing for Irreversibility in Investment

The generated stock of capital is not sensitive to the existence of irreversibility in investment. It is possible to assume that the cost function is piece-wise quadratic :

$$C(x_t, k_{t-1}) = \begin{cases} \frac{\alpha^+}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t > 0 \\ \frac{\alpha^-}{2} \left(\frac{x_t}{k_{t-1}} \right)^2 k_{t-1} & \text{if } x_t < 0 \end{cases}$$

where the adjustment-cost parameter α^+ (α^-) has the same interpretation as in the model: it represents the time it takes for the capital stock to double (halve) when λ doubles (halves). By allowing the downward adjustment-cost parameter to be higher than the upward adjustment-cost parameter, this asymmetry in the investment decision will reflect irreversibility of investment. Setting $\alpha^+ = 8$ as in the main text and allowing α^- to be arbitrarily set at up to ten times higher than the upward adjustment-cost parameter, the generated capital stock will not be affected. The result behind this finding comes from the fact that gross investment is almost always above depreciation expenditures in the data. In other words, net investment is almost always positive so there is little evidence on irreversibility at the aggregate level. In addition, when net investment is negative, its magnitude is quite small.