Liquidity in the Housing Market: Credit Standards, Volatility Clustering and Asymmetric Price Adjustment

by

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Abstract

This dissertation studies various economic issues related to the housing market. First, I investigate how trading frictions in a housing market affect lenders' decisions. In Chapter 2, by employing a dynamic house search model with long-term mortgage debt, I find that a more liquid housing market reduces not only the possibility of a borrower to default but also the lender's cost upon default. These benefits induce lenders to require lower credit requirements and issue larger loans. Second, Chapter 3 is an empirical study of volatility clustering (ARCH/GARCH effects), *i.e.* periods of swings followed by periods of relative calm, in home price of Canadian cities. I find that most Canadian major cities exhibit ARCH/GARCH effects in house price return and several cities also show TGARCH effects, *i.e.* price volatility is asymmetric in response to positive and negative shocks. Furthermore, I analyze the determinants of house price volatility and document asymmetric price adjustment in Canada. Finally, in Chapter 4, I develop a static house search model with indebted sellers to study the asymmetric adjustment of house prices. The analysis shows that positive changes of the average house price are larger and more likely to reflect the underlying shocks than negative changes due to the equity effect, *i.e.* sellers with less house equity tend to post higher asking prices. This result is consistent with empirical findings that the house price exhibits downward rigidity.

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Chapter 1

Introduction

Since the recent U.S. sub-prime mortgage crisis, much academic and policy work has focused on the role of credit market conditions in the housing market boom-bust cycle. It has been commonly argued that easy credit market terms, including low down-payments and high mortgage approval rates, helped generate large swings in the housing market.[1] Most of the literature takes changes in credit terms as exogenously given and ignores the interaction between the credit market and the housing market. In my view, in addition to the common view that lenient credit standards may cause more active trading in housing markets, causality may also run the other way. That is, more active trading in the housing market may in turn encourage more relaxed credit standards, which will further encourage housing demand, and so on.

In Chapter 2, I study explicitly how housing market liquidity - the ease with which houses are bought and sold - affects lending standards by incorporating long-term mortgages into two dynamic housing models: one with search frictions and the other

[1]See, Campbell, Davis, Gallin, and Martin (2009), Dell'Ariccia, Igan, and Laeven (2012), Duca, Muellbauer, and Murphy (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2010), Glaeser, Gottlieb, and Gyourko (2010), Himmelberg, Mayer, and Sinai (2005), Khandani, Lo, and Merton (2013), Mayer and Sinai (2009), Ortalo-Magné and Rady (2006).

without. In both models, house prices and down-payment ratios[2] are endogenously determined in equilibrium. With search frictions, I show that the sales rate - the probability of selling - affects both the rate at which a financially distressed owner can sell his/her house to avoid default, and how long a foreclosed house stays in the mortgage company's inventory. Thus, market liquidity affects both the expected default rate of a mortgage and the mortgage company's expected loss upon default. Together, these determine the size of a loan that the mortgage company is willing to issue at origination. Without search frictions, the down-payment ratio rises upon a temporary positive housing demand shock because the mortgage company's expected loss upon default would otherwise rise at a higher rate than its expected revenue. By contrast, when the economy with search frictions experiences such a shock, the down-payment ratio falls because the mortgage company benefits from the reductions in the expected default rate and the expected loss upon default.

In Chapter 3, I examine the properties of house price volatility in Canada at both national and city levels. First, Autoregressive Conditional Heteroskedastic (ARCH) and Generalized Autoregressive Conditional Heteroskedastic (GARCH) models are employed to analyze possible time variation of house price volatility. Most major Canadian cities (9 out of 11), as well as the nation as a whole, exhibit volatility clustering (ARCH/GARCH effects)[3] during 1981 - 2014. Among these cities with ARCH/GARCH effects, asymmetric volatility (TGARCH effect), of either sign[4], is observed in Toronto, Vancouver, Winnipeg and Canada.

[2]The down-payment ratio refers to the ratio of down-payments to the purchase price, i.e. one minus the loan to value ratio (LTV) at origination of a mortgage.

[3]Volatility clustering refers to the observation, as noted by Mandelbrot (1963), that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes."

[4]Positive (negative) asymmetric volatility refers the phenomenon that volatility is higher (lower) in down markets than in up markets.

I also examine the dynamic interaction between house price volatility and key macroeconomic variables for the nation as a whole. On one hand, VAR, Granger causality and variance decomposition (VDC) analyses demonstrate that house price volatility is significantly affected by house price appreciation, house sales growth rate and population growth rate. On the other hand, volatility affects GDP growth rate, house price appreciation rate, sales growth rate and volatility itself. Finally, I find that positive changes in house prices are more frequent and larger than negative changes in most of the studied cities over the sample period.

Chapter 4 studies theoretically the observed house price downward rigidity[5] in a housing model with search frictions and sellers with heterogeneous mortgage debt. Without frictions, when the housing market experiences a shock of any type, the adjustment of house prices is instantaneous, and the direction of the shock has no role to play. However, if the search and matching frictions are present, the price adjustment pattern may be different. Search frictions make it possible for sellers to trade off the asking price and the sales rate. Sellers determine asking prices based on individual mortgage position and a reference price. The latter reflects the fundamental economic conditions. Heterogeneity of sellers leads to dispersion of asking prices. A negative shock to the reference price makes sellers less willing to lower their asking prices because of falling equity, while a positive shock gives sellers more incentives to sell faster because of rising equity. This is the so called "equity effect". Analytical results show that positive changes of the average house price are larger and more likely to reflect underlying shocks than negative changes.

[5]For empirical evidence of house price downward rigidity, see Engelhardt (2003), Genesove and Mayer (2001) and Tsai (2012).

Chapter 2

Housing Liquidity and Lending Standards

2.1 Introduction

In this chapter, I study how housing market liquidity affects lending standards by incorporating long-term mortgages into two dynamic housing models: one with search frictions and the other without. In both models, house prices and down-payment ratios are endogenously determined in equilibrium. With search frictions, I show that market liquidity affects both the expected default rate of a mortgage and the mortgage company's expected loss upon default, which altogether affect the volume of a loan that the mortgage company is willing to issue at origination. Moreover, combined with heterogeneity in seller's mortgage debt, trading frictions in the housing market naturally induce non-degenerate house price distribution in the equilibrium for identical houses. I also analyze the dynamics of these two models in response to local income shocks and show that housing liquidity is an important factor that can generate the negative co-movement between the house price and the down-payment ratio.

In the U.S., the overall credit constraint was eased and lenders began favoring mortgages with lower down-payment ratios and higher risks in the early part of last decade, the same period that was accompanied with booming housing markets.[1] Figure 2.2 illustrates that down-payment ratios[2] for first-time home buyers, who are most likely to be affected by down-payment constraints, were negatively correlated with house prices. The effects of collateral constraint, which directly affects the down-payment ratio (or Loan-To-Value ratio) in an economy with housing, have been studied extensively through Iacoviello (2005) and Iacoviello and Neri (2010). However, in most of such literature, the exact form of the collateral constraint is set exogenously instead of being derived based on optimal decisions of agents.

This chapter studies a factor that may endogenously determine credits available to borrowers. It is commonly accepted that houses are illiquid because they are bought and sold in decentralized markets, in which search and matching play important roles. Thus, it is natural to ask the following questions: (1) How do lending standards (the ease with which buyers are able to obtain funding from lenders)[3] depend on liquidity

[1]Gerardi, Lehnert, Sherland, and Willen (2008) show that many subprime loans in that period were characterized by high leverage at origination and non-traditional amortization schedules. Duca, Muellbauer, and Murphy (2011) construct a series for loan to value ratio (LTV) faced by first-time home buyers and show that the overall LTV ratio increased from 2000 to 2005. Barlevy and Fisher (2011) use data compiled for over 200 U.S. cities between 2000 and 2008 to find that interest only (IO) mortgages were used sparingly in cities in which an elastic housing supply kept house prices in check, but were common in cities with an inelastic supply in which house prices rose sharply and then crashed. Dell'Ariccia, Igan, and Laeven (2012) document and show that lending standards (denying rates) declined more in areas that experienced larger credit booms (more applicants) and greater price appreciation. Mian and Sufi (2009) find that regions with high latent demand from 2001 to 2005 experienced large relative decreases in denial rates, increases in mortgages originated, and increases in house price appreciation, despite the fact that the same regions experienced significantly negative relative income and employment growth over this time period.

[2]This series consistently measures down-payment ratios on conventional mortgages, which corresponds to the Freddie Mac home price series that is based on homes bought with conforming conventional mortgages.

[3]Lending standards may include the level of mortgage approval rates, down-payment ratios, document requirements, mortgage rate, etc. This chapter focuses on down-payment ratios. Many believe that low down-payment mortgages are more risky because they are more likely to be underwater in down housing markets.

(the ease with which houses are traded) in housing markets? (2) How important is the presence of trading frictions in generating observed negative co-movement of house prices and down-payment ratios?

To answer these questions, I develop a dynamic housing model with a frictional housing market and long-term mortgages. House prices, entry of new buyers, construction and mortgage terms are all endogenously determined. I also analyze a counterpart model with a frictionless housing market.

In the model, the housing market is characterized by competitive search. In particular, agents make a trade-off between the house price and the matching probability. House buyers take on long term collateral loans issued by a representative mortgage company to finance their purchases and use labor income to make down-payments at origination. Mortgages suffer from lack of commitment in that borrowers can default on their debt obligation at any time. In each period, borrowers may experience an adverse shock at an exogenous rate. In the search economy, the shock induces borrowers to fall into financial distress. A distressed borrower must decide whether to sell his/her house and pay off mortgage debt or to default on the loan.[4] When choosing to sell, he/she may not be able to find a buyer to trade with due to search frictions. Upon a failing sale, the distressed borrower is forced to default and his/her house is foreclosed. In the non-search economy, the adverse shock is a default shock, *i.e.* borrowers who experience such a shock will default on their mortgage contracts

[4] In the event of financial distress, it is always in owners' best interest to sell before going into foreclosure if they have positive equity. In an economy with perfect liquid housing markets, distressed owners with positive equity would never default because they can immediately sell their houses and pay off mortgage debt. However, according to reports from RealtyTrac, less than 50% of homeowners who went into foreclosure had negative equity. Time-consuming search and matching is responsible for this observation. The appraisal value of a house is not equal to its true liquidation value in decentralized housing markets. Appraisal value is typically estimated based on the most recent sale prices of houses with similar characteristics, whereas liquidation value takes not only the price of a house but also the probability of selling into consideration.

right away.[5]

After calibrating the model to U.S. city-level data, I have found the following results: First, in the search economy, market liquidity plays an important role in setting lending standards through its effects on the expected default rate of a mortgage and the mortgage company's expected loss upon default. Intuitively, in a more liquid housing market, sellers get to match with buyers more easily and can sell their houses more quickly. A financially distressed borrower is more likely to be able to sell his/her house to avoid falling into the foreclosure process. Thus, the expected default rate of a mortgage is lower. Similarly, since the mortgage company can sell foreclosed houses more quickly, the expected carrying cost of a foreclosed house is lower as well. In summary, higher housing market liquidity reduces both the owner's exposure to default risk and the mortgage company's expected loss on a foreclosed house. Given these two improved factors, the mortgage company is willing to offer mortgages with more relaxed terms, which means lower down-payment ratios in the search economy. Using coefficients of the matching function as proxies to measure trading frictions in the housing market, I find that both the expected default rate and the down-payment ratio are lower in economies with more efficient housing markets. In the non-search economy, housing liquidity is irrelevant and loan terms depend only on the exogenous default rate.

Second, in the search model, the probability of selling (trading price) is approximately flat for sellers with low LTV ratios, but decreases (increases) for sellers with median to high LTV ratios. This is consistent with Genesove and Mayer (1997, 2001),

[5] In the steady state of the non-search economy, all borrowers have positive equity on their houses. Since there is no transaction cost of selling a house, a borrower will sell the house to pay off mortgage debt and never default if he/she suffers the same type of shocks as in the search economy. Thus, to make the two models comparable, I assume that borrowers default directly upon receiving an adverse shock in the non-search economy.

who find evidence that sellers with high leverage post higher asking prices, wait longer on the market, and sell at higher prices.

Third, responses of the down-payment ratio to housing demand shocks in these two models are qualitatively different. In both economies, the demand in the housing market is determined endogenously by the value of living in a city. New buyers move to a city whenever the expected value of doing so exceeds their next best alternative. An unexpected increase in the value of living in a city attracts more buyers to that city. In the search economy, the average house price exhibits short-term momentum (*i.e.,* serially correlated growth rates), whereas the average down-payment ratio falls initially and gradually returns to its long-run trend thereafter.

The positive housing demand shock results in an immediate increase in market tightness. It takes time for buyers to find a house due to search frictions, and for new housing construction to respond. Since not all newly entering buyers are matched, and entry is persistent due to the persistence of income shock, demand gets accumulated in the market over time, generating future increases in both market tightness and sales rates in the city.

With a more liquid housing market, the mortgage company becomes more willing to offer larger loans such that the down-payment ratio falls. Several factors account for this result. First, the expected default rate of a mortgage falls since distressed borrowers have a better chance to find a buyer. Second, because the carrying cost of foreclosed houses falls, the mortgage company's expected loss upon default drops. Moreover, persistent house price appreciation accelerates owners' housing equity accumulation. Higher housing equity leads distressed sellers to choose submarkets with higher market tightness and sales rates. This further decreases the expected default rate of a mortgage. Finally, proceeds from foreclosure sales become higher as the resale value of foreclosed houses becomes higher than before. With all these improved factors, both borrowers' exposure to default and the mortgage company's expected loss upon default fall. Thus, the mortgage company has a strong incentive to issue larger loans. The increase in loan size is so high that the down-payment ratio falls although the house price rises.

As time goes on, house prices and market tightness continue to rise before eventually fall. The later a mortgage is originated after the shock, the more periods before its maturity will be with falling house prices and lower sales rates. Both the expected default rate and the expected loss upon default rise over time. This induces the loan volume and the down-payment ratio to monotonically return to their long-run trends after initial deviations.

In the non-search economy, the average house prices, loan volume and down-payment ratio all jump initially and gradually settle into long-run trend thereafter. The initial increase in the loan volume is because resale value of foreclosed houses becomes higher than before. Since the expected default rate of a mortgage does not change in the non-search economy, the mortgage company's expected loss upon a default rises at a higher rate than its expected revenue if the down-payment ratio does not increase. This breaks the mortgage company's zero profit condition. Hence, in contrast to its fall in the search economy, the down-payment ratio rises after the shock.

2.1.1 Related literature

This chapter contributes to the literature on the interaction between housing market liquidity and lending standards, such as Hedlund (2014), Ungerer (2012) and Guren and McQuade (2013). Hedlund (2014) focuses on matching unconditional business cycle moments in the data. The model matches U.S. data well and highlights the feedback mechanism in the housing market: trading frictions tighten endogenous credit constraints, and credit constraints magnify trading frictions in the real estate market. Households in Hedlund (2014)'s model are heterogeneous in income streams, credit records and asset portfolios (with respect to mortgage debt and savings). To address the two-sided heterogeneous matching problem in housing markets, Hedlund (2014) introduces real estate firms as middle men to bridge transactions between sellers and buyers. Sales in the sequential selling and buying markets occur in a competitive search environment. The value of this setup is to make the model tractable and the computation is no more difficult than that of Krusell and Smith (1998), because of the block recursive property (see Menzio and Shi (2010)) of the equilibrium. The model, however, hinders the direct impact of fundamental market tightness - the ratio of total buyers to total sellers - on the economy. In general, when the housing market becomes tighter, two forces affect the probability of selling. On one hand, because higher shadow price[6] increases (decreases) their house equity (gain from purchases), sellers (buyers) might shift their destination submarkets to those with higher (lower) matching probabilities. On the other hand, sellers (buyers) can sell (buy) the houses more quickly (slowly) just because the market becomes "hot". In Hedlund (2014),

[6]In Hedlund (2014), the shadow price is the competitive price at which real estate firms purchase new housing from construction firms.

however, only the former factor takes effect. The existence of real estate agents accounts for this result. Free entries of real estate agents into sequential buying and selling markets offset the direct effect of fundamental market tightness on the economy. By contrast, in my model, buyers and sellers trade directly such that both forces take effects, and the change in fundamental market tightness is the major force.

Ungerer (2012) uses a macroeconomic general equilibrium model with credit-constrained consumers and housing market search frictions to study the phenomenon that the volume of housing sales jumps quite high in response to a temporary moderate fall in the policy interest rate. Housing market search frictions are incorporated into a new Keynesian macroeconomic model as in Iacoviello (2005). Borrowers' credit constraint is determined by the liquidation value of collateralized houses, which does not only relate to the transaction price, but also to how quickly houses can sell. The latter matters because the faster that houses sell, the lower carrying costs that lenders must assume. A temporarily low interest rate makes home purchase relatively valuable, which attracts more potential buyers and allows sellers to find buyers more quickly. In turn, higher housing sales rate increases the housing liquidation value and allows lenders to threaten foreclosure more effectively. Ex-ante, this raises the willingness of lenders to provide credit. Because homeowners are not allowed to default and mortgages are modeled as one-period loans, the model cannot study the impact of higher sales rate on borrowers' default rates, which is the main issue discussed in this chapter.

Guren and McQuade (2013) studies how foreclosures can exacerbate a housing bust and delay the housing market's recovery in a random match environment. While the effects of foreclosure on price-default spirals during a downturn in frictional housing market are well analyzed, their model do not model mortgage explicitly and hence is silent about how liquidity in housing market affects credit standards directly.

The stochastic general equilibrium models with housing markets have been extensively analyzed in recent literature, including Iacoviello and Pavan (2013), Favilukis, Ludvigson, and Van Nieuwerburgh (2010), Kiyotaki, Michaelides, and Nikolov (2011), and Ríos-Rull and Sánchez-Marcos (2008) among others. These papers generally integrate frictions in financial markets into a GE model to study the role of the housing market in business cycles. They usually treat mortgages as short-term contracts and do not allow borrower to default.

Since the recent sub-prime mortgage crisis, there is a large literature focusing on household default. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) introduces an important equilibrium concept to study household default in general equilibrium models. In their framework, different loans made to different types of borrowers are traded in distinct markets. Applying this method, Mitman (2011), Hintermaier and Koeniger (2011), and Jeske, Krueger, and Mitman (2010) investigate borrower default in the mortgage industry. However, in these papers, mortgages are defined as one-period loans and households need to refinance every period. Chatterjee and Eyigungor (2012), Corbae and Quintin (2011), and Garriga and Schlagenhauf (2009) relax this restriction and use heterogenous agents models with long-term mortgage loans to study foreclosures in last decade. This chapter extends the literature by studying household default with long-term mortgages and illiquid housing markets.

Housing markets are naturally decentralized and several authors have argued that search and matching may play important roles in housing markets. Ríos-Rull and

Sánchez-Marcos (2008) finds positive co-movement of housing prices and sales volume. Krainer (2008) documents the fact that prices and sales are negatively correlated with average time on the market. Moreover, Caplin and Leahy (2011) finds that there is a significant negative correlation between vacancies and housing price appreciation.

Along with the research in frictional labor markets, random search is used to model trading frictions in housing markets in the early literature, see Krainer (2001), Novy-Marx (2009), Burnside, Eichenbaum, and Rebelo (2011), and Caplin and Leahy (2011) among others. In a random search framework, agents get matched at exogenously given rates and cannot control the matching probability by their activities. However, as documented by Merlo and Ortalo-Magné (2004) and Merlo, Ortalo-Magné, and Rust (2008), house sellers usually tend to lower their asking prices to attract more buyers, which is broadly consistent with directed search theory. Therefore, this chapter models housing using a directed search framework, as in Diaz and Jerez (2013), Albrecht, Gautier, and Vroman (2010), and Head, Lloyd-Ellis, and Sun (2014). My model structure in this chapter is based on Head, Lloyd-Ellis, and Sun (2014), who develop a dynamic search model of the housing market to generate dynamics qualitatively and quantitatively (partially) consistent with empirical observations.

The remainder of this chapter is organized as follows. The search model structure is developed in section 2.2. Section 2.3 presents agents' problems and the recursive equilibrium for the search model. Section 2.4 introduces the non-search model. Section 2.5 presents the calibration for the search economy. Section 2.6 discusses steady-state statistics of the search model, and Section 2.7 discusses dynamic implications of a positive income shock for both types of economies. Section 2.8 concludes the chapter. The last section lists tables and figures mentioned in this chapter. Appendix A.1 lists laws of motion of agents' measures in these two economies. Finally, Appendix A.2 presents the computational algorithm for the search model.

2.2 Model Environment

Time is infinite and discrete. Each time period is indexed by t. The economy consists of a single housing market, considered a city, and the "rest of the word", where households decide whether to enter the city at the beginning of each period. The aggregate economy is populated by measure Q_t of *ex ante* identical households, which grows exogenously at net rate μ.

Each household lives infinitely and inelastically supplies one unit of labor every period. In period t, the unit of labor supplied earns income y_t, which is in terms of consumption goods and follows a stationary stochastic process in log-levels. A household's period-by-period utility from consumption c_t and housing z_t is given by

$$\mathcal{U}_t = u(c_t) + z_t, \tag{2.1}$$

where $z_t = z^H > 0$ if the household owns a house and enjoys living in it, and $z_t = 0$ otherwise. The function $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable, with the boundary properties: $u(0) = 0$, $u(\infty) = 0$ and $u'(0)$ is sufficiently large. All agents discount future with factor β. Consumption goods are non-storable and there is no technology for households to save across periods.[7]

At the beginning of each period, a measure μQ_t of new households arrive in the economy. Each of these households has a best alternative to entering the city, denoted as ε. The variable ε is independently and identically distributed across new households according to a stationary distribution function $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$. There is a critical alternative value, ε_t^c , at which a new household is indifferent whether entering the city or not. Upon entering the city, households are randomly and permanently separated into two types, those who derive utility from owning a house, *i.e.,* buyers, and those who do not, *i.e.,* perpetual renters. The critical value is

$$\varepsilon_t^c = \psi U_t + (1 - \psi) W_t^p, \tag{2.2}$$

where ψ is the fraction of buyers and is exogenously given. The respective values U_t and W_t^p are of being a buyer and a perpetual renter in the city. In other words, the population inflow depends on the value of living in the city, either as a buyer or as a perpetual renter.

Houses are indivisible and identical. Each household can own at most one house. Purchased houses are occupied immediately to provide housing services and owner premiums. Houses depreciate over time, regardless of whether they are occupied or not. Any depreciation is assumed to be offset by maintenance. Let m denote the cost

[7]This assumption is used to maintain tractability. With this assumption, buyers in the housing market are homogenous at the onset of each period because they earn identical labor income and do not have any savings from the previous period. As a result, the matching problem in this model only involves heterogeneity on the seller's side. This significantly simplifies the analysis. If house traders are heterogeneous on both sides, although one may be able to characterize the steady state under certain conditions (see Shimer and Smith (2000), Smith (2006), and Eeckhout and Kircher (2010)), the dynamics of the model in response to shocks are intractable. Note that even if households are allowed to save, the channels through which housing market liquidity affects lending standards still work as long as borrowers may default due to a failing sale.

of maintenance incurred by the owner to offset depreciation every period.

In the city, there are a large number of competitive construction firms that build new housing units. Houses constructed in period t become available at the beginning of period $t+1$. Each new house requires one unit of land, which can be purchased in a competitive market at unit price $q_t = \mathcal{Q}(N_t)$ and incurs a construction cost $k_t = \mathcal{K}(N_t)$, where N_t denotes the amount of new houses built in period t and available at $t+1$. Maintenance costs must be paid for unsold houses in inventory to offset depreciation.

Non-owner households in the city also need dwellings. They receive housing services from landlords. Because of the model's complexity, I assume that landlords sell housing services competitively at a price equal to a fixed fraction of the average concurrent house trading price, *i.e.,* $R_t = \varsigma P_t$. Rental housing services are not counted into the city's housing stock.

At the end of each period, perpetual renters may experience a relocation shock with probability $\pi_p \in (0, 1)$ that induces them to leave the city. On receiving this shock, perpetual renters move out immediately and receive a continuation value, L. Otherwise they remain as perpetual renters into the next period. Similarly, with probability $\pi_h \in (0, 1)$, owners and buyers receive shocks that cause them to leave the city at the end of each period. Upon receiving this shock, owners and buyers move out immediately and receive the continuation value, L. Owners who has left the city, namely, relocated owners, have vacant houses that they may want to sell. In addition, relocated owners with mortgage debt must also decide whether to continue their loan contracts.

The housing market is characterized by competitive search. There are a variety of submarkets, each indexed by (θ, p), where θ denotes the market tightness (*i.e.,* the ratio of the measure of buyers $[B]$ to measure of sellers $[S]$) and p denotes the trading price upon a match. Sellers and buyers take (θ, p) across all submarkets as given and decide which submarket to enter and search for a transaction. Respective matching probabilities of a buyer, $\gamma(\theta)$, and a seller, $\rho(\theta)$, are computed according to the function, $\mathcal{M}(B, S)$. Buyers and sellers can enter any submarket at no cost. This free-entry condition determines the tradeoff between the price and the matching probability across submarkets. Intuitively, higher price submarkets have lower levels of tightness so that a buyer who is willing to pay a higher price gets compensated by enjoying a higher chance to trade with a seller.

The distribution of sellers in the housing market is non-degenerate. There are three types of sellers: construction firms, the mortgage company, and household sellers. Household sellers are consisted of staying owners and relocated owners. Construction firms sell houses they build, the mortgage company sells houses seized from defaulting borrowers, and household sellers sell houses they own. Moreover, household sellers are heterogeneous in their mortgage debt because they may be at different mortgage repayment stage when they decide to sell.

Households must obtain mortgages to help finance a house purchase, because they cannot save and the periodic labor income is not sufficient to pay the full purchase price in any active submarket. The mortgage sector is perfectly competitive and is represented by one mortgage company that issues long-term mortgage contracts to borrowers. The mortgage company is owned by risk-neutral investors who consume all profits and losses *ex post*.[8] To finance its loans, the mortgage company trades

[8] An alternative method is to assume all *ex post* profits and losses are transferred in a lump sum to households. This assumption complicates the computation but not significantly changing the qualitative results.

one-period risk-free bonds at an exogenous interest rate i in the international bond market. In addition, the mortgage company incurs service cost (premium) ϕ on the opportunity cost of funds loaned to buyers.

Mortgage contracts are standard fixed-rate mortgages with finite maturity T.[9] Let m_0 and r_m represent the size of a loan at origination and the mortgage rate, respectively. For a contract $(m_0; r_m, T)$, the constant periodic payment $x(\cdot)$ is given by

$$x(m_0) = \frac{r_m}{1 - (1 + r_m)^{-T}} m_0. \tag{2.3}$$

Given the number of completed payments $n \in [0, T - 1]$, the principle balance d is

$$d(m_0, n + 1) = (1 + r_m)d(m_0, r_m, n) - x(m_0, r_m) \tag{2.4}$$

and $d(m_0, 0) = m_0$. Since both $x(\cdot)$ and $d(\cdot)$ are unrelated to t after origination, I use (m_0, n) to represent the state of a mortgage hereafter. A borrower can terminate his/her mortgage contract at any time. A termination is a foreclosure if the borrower cannot pay off his/her outstanding mortgage balance. To focus on the relationship between the volume of credit available to borrowers and housing market liquidity, I assume that the mortgage rate r_m is exogenously given. Nevertheless, m_0 is endogenously determined such that the equilibrium profit on the contract is zero.

At the beginning of each period, staying owners with mortgage debt may experience an adverse shock, such as accidents or unexpected illness, with probability π_d. A distressed staying owner must terminate his/her current mortgage contract within the same period, either by paying off the outstanding debt or by defaulting. In the

[9] Conventional U.S. mortgages typically have a fixed 30-year term and about 70% of these mortgages have fixed rates, even though this percentage has decreased in recent years.

event of default, a borrower's mortgage balance is set to zero and a foreclosure flag is placed on his/her credit record, *i.e.,* $f = 1$. The mortgage company repossesses the borrower's house, puts it in real-estate-owned (REO) inventory, and sets it up for sale in the decentralized housing market from the following period. Upon a successful sale, the mortgage company loses a fraction $\chi \in (0, 1)$ of the revenue as costs, *e.g.,* legal fees. If the value of a house in REO inventory is higher than the outstanding mortgage balance, the remaining amount goes to the borrower. As a penalty for default, buyers with $f = 1$ lose access to the mortgage market and are thus excluded from the housing market. At the beginning of the next period, the foreclosure flag stays on a buyer's record with probability $\pi_f \in (0, 1)$.[10]

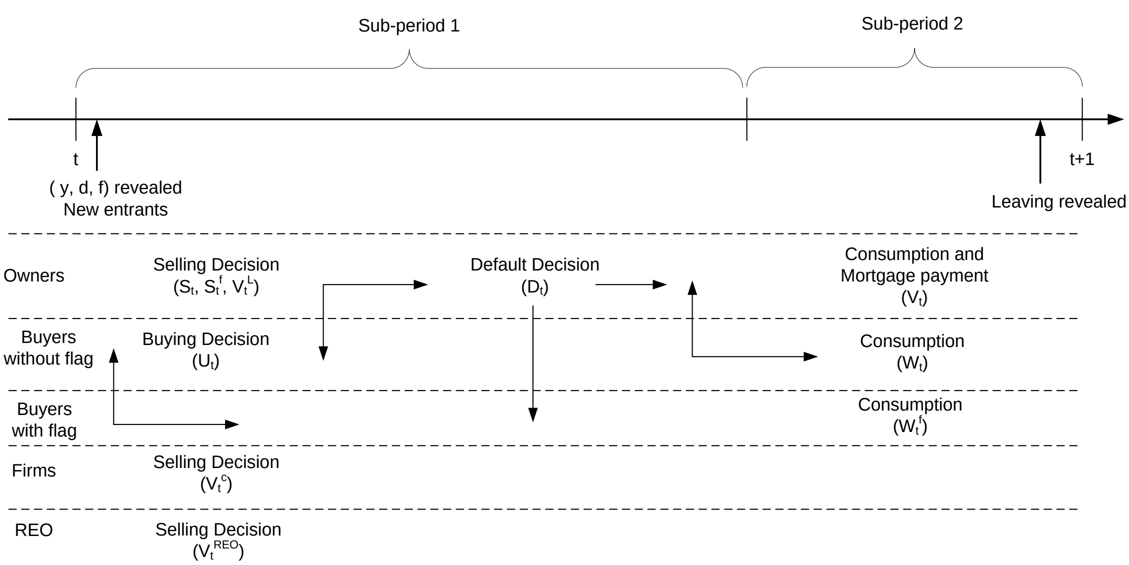

Figure 2.1: Time Line

Timing. Figure 2.1 illustrates the timing of the search model. Each period consists of two sub-periods. At the beginning of sub-period 1, new households with $\varepsilon \leq \varepsilon_t^c$

[10]According to the policies of Fannie Mae and Freddie Mac, foreclosure filings stay on a borrower's credit record for a finite number of years.

enter the city. Income shocks, financial distress shocks and shocks on the foreclosure flag are all revealed. Then the housing market opens. In the market, construction firms sell newly built houses, and the mortgage company sells foreclosed houses in its inventory. Owners and buyers without a foreclosure flag decide whether to enter the housing market. If so, they determine which submarket (p, θ) to search and trade in. The mortgage sector is active after the housing market closes. Borrowers decide whether to default on their current mortgage contracts. New owners take on mortgages to finance their purchases. In sub-period 2, households receive income, make payments (maintenance, down payments, mortgage payments or rents), and consume whatever is left. At the end of the period, relocation shocks are revealed for all households and those who experience such a shock leave the city immediately.

2.3 Equilibrium in the Search Economy

2.3.1 Household decisions

I now describe agents' values and decisions, starting from sub-period 2 and going backward to sub-period 1.

Decisions in sub-period 2

Let W_t^p denote the value of being a perpetual renter, who remains a renter (and never chooses to buy a house) until he/she leaves the city. It follows that

$$W_t^p = u(c_t) + \pi_p \beta L + (1 - \pi_p)\beta E_t W_{t+1}^p \qquad (2.5)$$

$$\text{s.t.} \qquad c_t = y_t - R_t.$$

That is, with probability π_p, a perpetual renter leaves the city and receives continuation value L, and otherwise continues into the next period as a renter.

Buyers with foreclosure flags on their credit records have no access to mortgage credit and are excluded from the housing market. They remain renting until moving out of the city or the foreclosure flag disappears from their credit records. Let $W_t^f(a)$ be the value of such a buyer with asset balance a in sub-period 2. The balance a represents the buyer's intra-period asset holding. This balance is positive if (i) the buyer defaulted on his/her previous mortgage in the preceding sub-period 1; and (ii) the value of a foreclosed house in the REO inventory more than covers his/her outstanding mortgage balance. Otherwise, $a = 0$. Thus,

$$\begin{aligned} W_t^f(a) &= u(c_t) + \pi_h \beta L + (1-\pi_h)\beta E_t\Big[\pi_f W_{t+1}^f(0) + (1-\pi_f)U_{t+1}\Big] \\ &\text{s.t.} \qquad c_t = y_t + a - R_t. \end{aligned} \tag{2.6}$$

Conditional on staying in the city, with probability π_f the foreclosure flag stays on this household's record and it moves into the next period with expected value $W_{t+1}^f(0)$. With probability $1 - \pi_f$, the foreclosure flag is lifted and this household will continue with value U_{t+1} as a buyer searching for a house in sub-period 1 of period $t + 1$.

Buyers without a foreclosure flag at the beginning of sub-period 2 are either previous staying sellers who successfully sold their houses or buyers who failed to purchase a house in sub-period 1. The former type of sellers may hold positive intra-period asset a, which came from sale proceeds net of outstanding mortgage debt. These buyers will enter the housing market next period if they do not experience a relocation shock at the end of current period. The value of such a buyer is given by

$$\begin{aligned} W_t(a) &= u(c_t) + \pi_h \beta L + (1-\pi_h)\beta E_t U_{t+1} \\ &\text{s.t.} \qquad c_t = y_t + a - R_t. \end{aligned} \tag{2.7}$$

In sub-period 2 of period t, a staying owner who holds a mortgage originated before period t has principle balance $d(m_0, n \in [0, T-1])$. Let $V_t(m_0, n)$ denote the value of such an owner. It follows that, for $n \in [0, T-2]$,

$$\begin{aligned} V_t(m_0, n) &= u(c_t) + z^H + \beta E_t \Big\{ \pi_h V^L_{t+1}(m_0, n+1) + (1-\pi_h) \\ &\quad \times \Big[\pi_d S^f_{t+1}(m_0, n+1) + (1-\pi_d) S_{t+1}(m_0, n+1) \Big] \Big\} \\ &\text{s.t.} \qquad c_t = y_t - x(m_0) - m, \end{aligned} \tag{2.8}$$

where $V^L_{t+1}(\cdot)$, $S^f_{t+1}(\cdot)$ and $S_{t+1}(\cdot)$ are the owner's respective values of exiting the city, being hit by a distress shock, and staying in the city without a distress shock. For $n = T-1$, we have

$$\begin{aligned} V_t(m_0, T-1) &= u(c_t) + z^H + \beta E_t \Big[\pi_h \bar{V}^L_{t+1} + (1-\pi_h) \bar{S}_{t+1} \Big] \\ &\text{s.t.} \qquad c_t = y_t - x(m_0) - m, \end{aligned} \tag{2.9}$$

where $\bar{V}^L_{t+1}$ and $\bar{S}_{t+1}$ denote respective values of relocated and staying owners without mortgage debt at the beginning of the following sub-period.

A new owner who has just purchased a house in the preceding sub-period 1 takes the loan issued by the mortgage company, m_0, and pays the difference between the purchasing price and m_0, *i.e.,* the down payment. The periodic mortgage payment starts from the next period. Let $V_t^0(p, m_0)$ denote the value of such a new owner. It follows that

$$\begin{aligned} V_t^o(p_t, m_0) &= u(c_t) + z^H + \beta E_t \Big\{ \pi_h V_{t+1}^L(m_0, 0) + (1 - \pi_h) \\ & \quad \times \Big[\pi_d S_{t+1}^f(m_0, 0) + (1 - \pi_d) S_{t+1}(m_0, 0) \Big] \Big\} \\ & \quad \text{s.t.} \qquad c_t = y_t - (p_t - m_0) - m. \end{aligned} \tag{2.10}$$

Finally, owners without mortgage debt do not suffer from distress shocks. They stay in the city until experiencing a relocation shock. The value of such an owner is given by

$$\begin{aligned} \bar{V}_t &= u(c_t) + z^H + \beta E_t \Big[\pi_h \bar{V}_{t+1}^L + (1 - \pi_h) \bar{S}_{t+1} \Big] \\ & \quad \text{s.t.} \qquad c_t = y_t - m. \end{aligned} \tag{2.11}$$

Decisions in sub-period 1

Buyers without a foreclosure flag search for a house with market value U_t:

$$U_t = \max_{(p^b \geq 0, \theta^b)} \Big\{ \gamma(\theta^b) V_t^o(p^b, m_0) + \Big(1 - \gamma(\theta^b)\Big) W(0) \Big\}. \tag{2.12}$$

Buyers choose to enter the submarkets that provide the maximum expected payoff to search for a trade. In the submarket (p^b, θ^b), with probability $\gamma(p^b, \theta^b)$, a buyer can purchase a house at price p^b and receive value $V_t^o(p^b, m_0)$ in the following sub-period 2. Otherwise, he/she remains as a buyer without a foreclosure flag and receives value $W_t(0)$. Because they begin making mortgage payments from the next period following their purchases and do not save over time, new borrowers are identical to the mortgage company. In other words, all buyers obtain the same loan $m_{0,t}$ in period t, regardless of the house prices they paid.

A staying owner who holds mortgage debt (m_0, n) and does not experience a distress shock has the value $S_t(m_0, n)$:

$$
\begin{aligned}
S_t(m_0, n) = \max_{(p^s \geq 0, \theta^s)} & \Big\{ \rho(p^s, \theta^s) W_t\Big(\max\left\{p - d(m_0, n), 0\right\}\Big) + \Big(1 - \rho(p^s, \theta^s)\Big) \\
& \times \max_{D_t \in \{0,1\}} \Big[(1 - D_t) V_t(m_0, n) + D_t W_t^f \Big(\max\left\{\beta V_{t+1}^{REO} - d(m_0, n), 0\right\}\Big)\Big]\Big\},
\end{aligned}
\tag{2.13}
$$

where V_{t+1}^{REO} is the value of a vacant house in the mortgage company's REO inventory. The owner decides whether to put his/her house up for sale, and chooses a single submarket (p^s, θ^s) to enter if he/she does. After a successful sale, he/she pays off outstanding debt by sale proceeds, takes the profit $a = \max\{p^s - d(m_0, n), 0\}$ and receives the value $W_t(a)$ as a buyer without a foreclosure flag in the following sub-period 2. If he/she did not enter the housing market, or entered the market but failed to sell, the owner then decides whether to continue current mortgage contract. $V_t(\cdot)$ and $W_t^f(\cdot)$ are the owner's values upon default and no-default, respectively.

In the event of financial distress, a staying owner with mortgage debt must terminate his/her contract within the same period. If the house is sold, the owner receives proceeds of the sale net of debt and becomes a buyer without a foreclosure flag. Otherwise, the owner is forced to default and a foreclosure flag is put on his/her credit record. If the value of a house in REO inventory more than covering the outstanding balance, the difference is sent to the owner.[11] Thus, the value of a distressed owner

[11] In the current setup, distressed owners can use proceeds from sales, but not labor income, to pay off outstanding mortgage debt. Relaxing this constraint would complicate the model and does not change the results qualitatively.

with (m_0, n) is:

$$\begin{aligned} S_t^f(m_0,n) &= \max_{(p^{sd}\geq 0,\theta^{sd})} \Big\{\rho(p^{sd},\theta^{sd})W_t\Big(\max\big\{p^{sd}-d(m_0,n),0\big\}\Big) \\ &\quad +\Big(1-\rho(p^{sd},\theta^{sd})\Big)W_t^f\Big(\max\big\{\beta E_t V_{t+1}^{REO}-d(m_0,n),0\big\}\Big)\Big\}. \end{aligned} \tag{2.14}$$

A staying owner without mortgage debt decides whether to sell his/her house and receives value $\bar{S}_t$:

$$\bar{S}_t = \max_{(p^{sw}\geq 0,\theta^{sw})} \Big\{\rho(p^{sw},\theta^{sw})W_t(p) + \Big(1-\rho(p^{sw},\theta^{sw})\Big)\bar{V}_t\Big\}. \tag{2.15}$$

Similar to staying owners who are not hit by distress shocks, a relocated owner with mortgage debt makes selling and default decisions. Upon a successful sale, the owner pays off outstanding debt with sale proceeds, takes the profit and receives continuation value L thereafter. Note that the labor income and the rent in the rest of world, y_t^L and R_t^L, may differ from y_t and R_t. Let $V_t^L(m_0,n)$ denote the value of such a relocated owner:

$$\begin{aligned} V_t^L(m_0,n) &= \max_{(p^L\geq 0,\theta^L)} \Bigg\{\rho(p^L,\theta^L)\Big[u\Big(\max\big\{p^L-d(m_0,n),0\big\}+y_t^L-R_t^L\Big)+\beta L\Big] \\ &\quad +\Big(1-\rho(p^L,\theta^L)\Big)\max_{D_t^L}\Big\{(1-D_t^L)\Big[u\Big(y-R_t^L-x(m_0)-m\Big) \\ &\quad +\beta E_t V_{t+1}^L(m_0,n+1)\Big] \\ &\quad +D_t^L\Big[u\Big(\max\big\{\beta E_t V_{t+1}^{REO}-d(m_0,n),0\big\}+y_t^L-R_t^L\Big)+\beta L\Big]\Big\}\Bigg\}. \end{aligned} \tag{2.16}$$

A relocated owner who does not have debt receives value $\bar{V}_t^L$ as

$$\begin{aligned}\bar{V}_t^L &= \max_{(p^{Lw}\geq 0,\theta^{Lw}} \Big\{\rho(p^{Lw},\theta^{Lw})\Big[u(p^{LW}+y_t^L-R_t^L)+\beta L\Big] \\ &\quad +\Big(1-\rho(p^{Lw},\theta^{Lw})\Big)\Big[u(y_t^L-R_t^L-m)+\beta E_t\bar{V}_{t+1}^L\Big]\Big\}. \qquad (2.17)\end{aligned}$$

Finally, let V_t^c and V_t^{REO} respectively denote the values of a vacant house in a construction firm's inventory and in the mortgage company's REO inventory. Then we have

$$V_t^c = \max_{(p^c\geq 0,\theta^c)} \Big\{\rho(p^c,\theta^c)p^c+\Big(1-\rho(p^c,\theta^c)\Big)\Big[-m+\beta E_t V_{t+1}^c\Big]\Big\}, \qquad (2.18)$$

$$\begin{aligned}V_t^{REO} &= \max_{(p^{REO}\geq 0,\theta^{REO}} \Big\{\rho(p^{REO},\theta^{REO})(1-\chi)p^{REO} \\ &\quad +\Big(1-\rho(p^{REO},\theta^{REO})\Big)\Big[-m+\beta E_t V_{t+1}^{REO}\Big]\Big\}. \qquad (2.19)\end{aligned}$$

Note that the mortgage company loses a fraction χ of the sale price as the foreclosure discount.

2.3.2 The mortgage company

No arbitrage implies that the mortgage company earns zero profit on each loan contract. In particular, the expected return net of expected foreclosure costs should be exactly equal to the opportunity cost of funds, which is the interest rate i of the international bonds plus the servicing premium ϕ. In this model, households cannot save over time. Moreover, the first regular repayment of all new mortgages starts from the period after the one when the mortgage is initiated. In other words, all new borrowers are identical to the mortgage company at the point of initiating the loan.

Therefore, the representative mortgage company issues the same loan $m_{0,t}$ to all new borrowers in period t, regardless of the house price they pay.

Let $\iota = (m_0; r_m)$ denote the mortgage held by a resident owner. Let $P_t^{\iota}(m_0, n)$ be the mortgage's present value at the beginning of sub-period 2 of period t after $n \geq 0$ payments have been made up to period $t-1$. Correspondingly, $P_t^{L\iota}(m_0, n)$ is the present value of the mortgage held by an owner that has relocated elsewhere. Note that here $n \geq 1$ for a relocated owner because one repayment has been made by the beginning of the first sub-period 2 after the household relocated. It follows that for all $n \in \{0, 1, \cdots, T-1\}$,

$$
\begin{aligned}
& P_t^{\iota}(m_0, n) \\
= \;& x(m_0)\mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \\
& \times E_t \left\{ \begin{array}{l}
\pi_h \left\{ \begin{array}{l} \rho(\theta_{t+1}^{L*}) \min\left[p_{t+1}^{L*}, d(m_0, n+1)\right] \\ +(1-\rho(\theta_{t+1}^{L*})) \left\{ \begin{array}{l} D_{t+1}^{L*}(m_0, n) \min\left[\beta V_{t+2}^{REO}, d(m_0, n+1)\right] \\ +\ (1 - D_{t+1}^{L*}(m_0, n)) P_{t+1}^{L\iota}(m_0, n+1)] \end{array} \right\} \end{array} \right\} \\
+(1-\pi_h) \\
\times \left\{ \begin{array}{l}
\pi_d \left\{ \begin{array}{l} \rho(\theta_{t+1}^{sd*}) \min\left[p_{t+1}^{sd*}, d(m_0, n+1)\right] \\ +\ (1-\rho(\theta_{t+1}^{sd*})) \min\left[\beta V_{t+2}^{REO}, d(m_0, n+1)\right] \end{array} \right\} \\
+(1-\pi_d) \times \left\{ \begin{array}{l} \rho(\theta_{t+1}^{s*}) \min\left[p_{t+1}^{s*}, d(m_0, n+1)\right] \\ +(1-\rho(\theta_{t+1}^{s*})) \\ \times \left\{ \begin{array}{l} D_{t+1}^{*}(m_0, n) \min\left[\beta V_{t+2}^{REO}, d(m_0, n+1)\right] \\ +\ (1 - D_{t+1}^{*}(m_0, n)) P_{t+1}^{\iota}(m_0, n+1) \end{array} \right\} \end{array} \right\}
\end{array} \right\}
\end{array} \right\}
\end{aligned} \tag{2.20}
$$

and for all $n \in \{1, \cdots, T-1\}$,

$$
\begin{aligned}
P_t^{L\iota}(m_0, n) \;=\;& x(m_0) + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \\
& \times E_t \left\{ \begin{array}{l} \rho(\theta_{t+1}^{L*}) \min\left[p_{t+1}^{L*}, d(m_0, n+1)\right] \\ +(1-\rho(\theta_{t+1}^{L*})) \left\{ \begin{array}{l} D_{t+1}^{L*} \min\left[\beta E_t V_{t+2}^{REO}, d(m_0, n+1)\right] \\ +(1 - D_{t+1}^{L*}) P_{t+1}^{L\iota}(m_0, n+1) \end{array} \right\} \end{array} \right\}
\end{aligned} \tag{2.21}
$$

where $p^{sd*}_{t+1},\ \theta^{sd*}_{t+1},\ p^{s*}_{t+1},\ \theta^{s*}_{t+1},\ p^{L*}_{t+1},\ \theta^{L*}_{t+1},\ D^{*}_{t+1},\ D^{L*}_{t+1}$ are policies that households will follow in period $t+1$ given mortgage balance $(m_0, n+1)$. Moreover, $I_{\{n\neq 0\}}$ and $I_{\{n\neq T-1\}}$ are index functions such that

$$\begin{aligned} \mathbb{I}_{\{n\neq 0\}} &= \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{otherwise} \end{cases} \\ \mathbb{I}_{\{n\neq T-1\}} &= \begin{cases} 0, & \text{if } n = T-1 \\ 1, & \text{otherwise} \end{cases} \quad . \end{aligned}$$

The first index function $I_{\{n\neq 0\}}$ is such that a borrower starts making regular repayments from the period next to origination. The second one $I_{\{n\neq T-1\}}$ indicates that the mortgage matures after the current repayment is made. The present value $P^{\iota}_t(m_0, n)$ is equal to the period-t repayment $x(m_0)$ plus the discounted expected value of period $t+1$. The latter is affected by whether the borrower is hit by the moving shock or the financial distress shock, and whether the borrower decides to sell the house or to default given the particular situation she is in.[12] If the borrower sells the house in period $t+1$, the amount that the mortgage company will receive is the minimum of the sale proceeds and the outstanding debt $d(m_0, n+1)$. The value $P^{L\iota}_t(m_0, n)$ is determined in a similar way except that a relocated borrower does not experience any moving or distress shock.

Finally, the mortgage company's zero-profit condition is given by

$$P^{\iota}_t(m_0, 0) - m_0 = 0. \tag{2.22}$$

[12] To compute the present value of a mortgage contract ι at origination, we first compute the value for $n = T-1$, and then use backward induction to obtain $P^{\iota}_t(\cdot,\cdot)$ for $n \in [0, T-2]$.

Thus the problem solved by the mortgage company is to find the value of $m_{0,t}$ that satisfies (2.22) in period t.[13]

2.3.3 The recursive equilibrium

Definition. A recursive equilibrium is a collection of value functions, $\{U_t,\ S_t(\cdot),\ S_t^f(\cdot),\ \bar{S}_t,\ V_t^L(\cdot)_t,\ \bar{V}_t,\ V_t^c,\ V_t^{REO},\ W_t^p,\ W_t(\cdot),\ W_t^f(\cdot),\ V_t(\cdot),\ S_t(\cdot)\ V_t^o(\cdot),\ \bar{V}_t\}$ and associated policy functions $\{p_t^{s*}(\cdot),\ D_t^*(\cdot),\ p_t^{sw*},\ p_t^{sL*}(\cdot),\ D_t^{L*}(\cdot),\ p_t^{sLw*}\}$, together with the entry value cutoff ε_t^c, a measure of active submarkets indexed by $(p, \theta(p))$, mortgage contracts $m_{0,t}$, rents R_t, the measure of buyers participate in all active submarkets B_t^{sum}, and the *per capita* laws of motion of perpetual renters, F_t, buyers, B_t, B_t^R, owners, Φ_t, Φ_t^L, the construction firm's inventory, Φ_t^c, and the REO inventory, Φ_t^{REO}. These functions and values satisfy the following:

1. New households enter the city optimally;

2. Household optimization: households' value and policy functions solve their corresponding problems;

3. REO optimization: REO value and policy functions solve the mortgage company's REO problems;

4. Firm optimization: firm's value and policy functions solve the construction firm's problems:

$$k_t(N_t) + q_t(N_t) = \beta E_t V_{t+1}^c; \tag{2.23}$$

[13] Without loss of generality, if there are multiple values of $m_{0,t}$ that satisfy (2.22), we choose the highest one as the solution.

5. Buyers freely enter any active submarket:

$$U_t = \gamma(\theta)V_t^o(p^b, m_{0,t}) + \Big(1 - \gamma(\theta)\Big)W_t(0);$$

6. The mortgage company earns zero profits on each mortgage contract $(m_{0,t}, r_m)$ at origination;

7. Landlords sell housing services competitively at price $R_t = \varsigma P_t$;

8. All buyers without a foreclosure flag search for a house: $B_t = B_t^{sum}$

Corresponding *per capita* laws of motion are listed in Appendix A.2.1.

The non-degenerate distribution of sellers is illustrated in equations (A.4) - (A.7). In general, this distribution should serve as a state variable in agents' problems. In this model, however, agents' decisions are independent of the distribution of sellers, although the distribution affect aggregate statistics, *i.e.* the equilibrium is block recursive as in Shi (2009) and Menzio and Shi (2010).

Competitive search and free entry of buyers assure this. Because competitive search fixes the terms of trade before matching takes place, the optimal trade-off between selling price and matching probability implies that different sellers make their own submarket choices. The sellers who sell at a particular submarket care about only the matching rate and the trading price there, but neither the participants in that submarket nor the distribution of sellers over other submarkets. Free entry of homogenous buyers renders the measure of sellers in each submarket irrelevant because the tightness - the ratio of buyers to sellers - adjusts until buyers get the market utility U_t, which does not depend on the distribution of sellers.[14]

Each active submarket can be indexed by the trading price p only. Recall the problem of buyers without a foreclosure flag in (2.12). Free entry of buyers implies that all active submarkets (p, θ) have to offer buyers the same payoff U_t:

$$U_t = \gamma(\theta)V_t^o(p, m_0) + \Big(1 - \gamma(\theta)\Big)W_t(0),$$

Thus, we can rewrite market tightness, θ, as a function of p:

$$\theta = \gamma^{-1}\left(\frac{U_t - W_t(0)}{V_t^o(p, m_{0,t}) - W_t(0)}\right). \tag{2.24}$$

Finally, to compute B_t^{sum}, I use the inverse functions of $\rho(\theta)$ and measures of all types of sellers to derive measures of buyers in all active submarkets and then take summation, as illustrated in the equation (A.8).

2.4 The Non-Search Economy

When houses are perfectly liquid, new buyers and staying buyers[15] can purchase a house immediately and move in. Sellers can sell houses without delay. Neither construction firms nor the mortgage company holds any inventory. At the beginning of sub-period 1, with probability, π_d^n, a staying owner with mortgage debt may experience a default shock that induce him/her to default on the loan. For mortgage holders who do not experience a default shock, negative housing equity is the only

[14]The distribution of sellers does *indirectly* affect the market utility U_t through its impact on the population inflow.

[15]Staying buyers consist of those owners who sold their houses during the preceding period, and those previous default borrowers whose foreclosure flags disappear in current period.

factor that may trigger defaults.[16]

In sub-period 2, households' problems are identical to those in the search model. In sub-period 1, agents face different problems, here I list them with superscript n. The equation numbers are corresponding to those in the search economy. Buyers without foreclosure flags purchase houses at competitive price p_t in the unique housing market and have the value U_t^n.

$$U_t^n = V_t^o(p_t, m_0). \tag{n.2.12}$$

A staying owner who holds mortgage debt and does not experiences a default shock decides whether to sell his/her house. If he/she chooses to sell, $H_t^s = 0$, the owner pays off outstanding debt by sale proceeds, takes the profits and thereby becomes a buyer without a foreclosure flag. If he/she did not choose to sell, $H_t^s = 1$, the owner needs to decide whether to continue current mortgage contract.

$$\begin{aligned} S_t^n(m_0, n) = & \max_{H_t^s \in \{0,1\}, D_t^n \in \{0,1\}} \Big\{ (1 - H_t^s) W_t \Big(\max\{p_t - d(m_0, n), 0\} \Big) \\ & + H_t^s \Big[(1 - D_t^n) V_t(m_0, n) + D_t^n W_t^f \Big(\max \big\{ \beta V_{t+1}^{nREO} - d(m_0, n), 0 \big\} \Big) \Big] \Big\}. \end{aligned} \tag{n.2.13}$$

In the non-search economy, a mortgage holder who experiences a default shock must terminate his/her contract by defaulting and has the value $S_t^{nf}(m_0, n)$:

$$S_t^{nf}(m_0, n) = W_t^f \Big(\max \big\{ \beta E_t V_{t+1}^{nREO} - d(m_0, n), 0 \big\} \Big). \tag{n.2.14}$$

Staying owners without mortgage debt just decide whether to sell their houses and

[16]Not all owners with negative equity will default. They must take the cost of default into consideration.

have the value $\bar{S}_t^n$:

$$\bar{S}_t^n = \max_{H_t^{sw}\in\{0,1\}} \Big\{(1-H_t^{sw})W_t(p_t) + H_t^{sw}\bar{V}_t\Big\}. \tag{n.2.15}$$

A relocated owner with/without mortgage debt makes selling and default decisions and receives value $V_t^{nL}(m_0,n)$ and $\bar{V}_t^{Ln}$, respectively.

$$\begin{aligned} V_t^{nL}(m_0,n) = &\max_{H_t^{sL}\in\{0,1\}, D^nL_t\in\{0,1\}} \Big\{(1-H_t^{sL})\Big[u\Big(\max\big\{p_t - d(m_0,n),0\big\} + y_t^L - R_t^L\Big) + \beta L\Big] \\ &+ H_t^{sL}\Big\{(1-D_t^{nL})\Big[u\Big(y_t^L - R_t^L - x(m_0) - m\Big) + \beta E_t V_{t+1}^{nL}(m_0,n+1)\Big] \\ &+ D_t^{nL}\Big[u\Big(\max\big\{\beta E_t V_{t+1}^{nREO} - d(m_0,n),0\big\} + y_t^L - R_t^L\Big) + \beta L\Big]\Big\}\Big\}, \end{aligned} \tag{n.2.16}$$

$$\begin{aligned} \bar{V}_t^{Ln} = &\max_{H^{sLw}\in\{0,1\}} \Big\{(1-H_t^{sLw})\Big[u(p_t + y_t^L - R_t^L) + \beta L\Big] \\ &+ H_t^{sLw}\Big[u(y_t^L - R_t^L - m) + \beta E_t \bar{V}_{t+1}^{nL}\Big]\Big\}. \end{aligned} \tag{n.2.17}$$

The values of a vacant house in a construction firm's inventory, V_t^{nc}, and in the mortgage company's REO inventory, V_t^{nREO} are

$$V_t^{nc} = p_t \tag{n.2.18}$$

$$V_t^{nREO} = (1-\chi)p_t. \tag{n.2.19}$$

For a mortgage contract $\iota = (m_0; r_m)$, let $P_t^\iota(m_0,n)$ be its present value at the beginning of sub-period 2 of period t after n payments have been made. Moreover, $P_t^{L\iota}(m_0,n)$ is for the mortgage ι held by a relocated owner. It follows that for all

$n \in \{0, 1, \cdots, T-1\}$,

$$
\begin{aligned}
& P_t^{\iota}(m_0, n) \\
& = x(m_0)\mathbb{I}_{\{n \neq 0\}} + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \\
& \times E_t \left\{ \begin{array}{l}
\pi_h \left\{ \begin{array}{l}
(1 - H_{t+1}^{sL*}) \min\left[p_{t+1}, d(m_0, n+1)\right] \\
+H_{t+1}^{sL*} \left\{ \begin{array}{l}
D_{t+1}^{nL*}(m_0, n) \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\
+(1 - D_{t+1}^{nL*}(m_0, n)) P_{t+1}^{L\iota}(m_0, n+1)
\end{array} \right\}
\end{array} \right\} \\
+(1-\pi_h) \left\{ \begin{array}{l}
\pi_d^n \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\
+(1 - \pi_d^n) \\
\times \left\{ \begin{array}{l}
(1 - H_{t+1}^{s*}) \min\left[p_{t+1}, d(m_0, n+1)\right] \\
+H_{t+1}^{s*} \left\{ \begin{array}{l}
D_{t+1}^{n*}(m_0, n) \\
\times \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\
+(1 - D_{t+1}^{n*}(m_0, n)) \\
\times P_{t+1}^{\iota}(m_0, n+1)
\end{array} \right\}
\end{array} \right\}
\end{array} \right\}
\end{array} \right\}
\end{aligned}
\quad \text{(n.2.20)}
$$

and for all $n \in \{1, \cdots, T-1\}$,

$$
P_t^{L\iota}(m_0, n) = x(m_0) + \frac{\mathbb{I}_{\{n \neq T-1\}}}{1+i+\phi} \times E_t \left\{ \begin{array}{l}
(1 - H_{t+1}^{sL*}) \min\left[p_{t+1}, d(m_0, n+1)\right] \\
+H_{t+1}^{sL*} \left\{ \begin{array}{l}
[D_{t+1}^{nL*} \min\left[\beta V_{t+2}^{nREO}, d(m_0, n+1)\right] \\
+(1 - D_{t+1}^{nL*}) P_{t+1}^{L\iota}(m_0, n+1)
\end{array} \right\}
\end{array} \right\} \quad \text{(n.2.21)}
$$

where, H_{t+1}^{sL*}, H_{t+1}^{s*}, D_{t+1}^{n*}, D_{t+1}^{nL*} are policies that households will follow in period $t+1$

with mortgage balance $(m_0; n+1)$.

The definition of the recursive equilibrium is similar to that in the search model except the housing market clearing condition:

$$B_t = S_t^{sum},$$

where B_t is the demand of houses and S_t^{sum} is the total supply of houses:

$$S_t^{sum} = \sum_{n=0}^{T} \Big(1 - H_t^s(\cdot)\Big)\Phi_t(m_{0,t-n}, n) + \sum_{n=0}^{T} \Big(1 - H_t^{sL}(\cdot)\Big)\Phi_t^L(m_{0,t-n}, n) + \Phi_t^c + \Phi_t^{REO}.$$

Corresponding *per capita* laws of motion are listed in Appendix A.2.2.

2.5 Calibration

The search model is calibrated to match selected facts of the U.S. economy. Table 2.1 gives parameter values for the baseline search model. Above the line, numbers are set to match indicated targets directly. Values below the line are set jointly such that specified steady-state values generated by the search model match given targets.

To start, the period-by-period utility on consumption is set to be

$$u(c_t) = \log(c_t).$$

The matching function takes the Cobb-Douglas form:

$$M(B, S) = \min\left\{\varpi B^{\eta} S^{1-\eta}, S\right\},$$

where B is the measure of buyers and S is the measure of sellers. Then the probability that a buyer matches with a seller is

$$\gamma(\theta) = \frac{M(\cdot)}{B} = M\Big(1, \frac{1}{\theta}\Big) = \min\big\{\varpi\theta^{\eta-1}, 1\big\}.$$

And the probability that a seller matches with a buyer is

$$\rho(\theta) = \frac{M(\cdot)}{S} = M(\theta, 1) = \theta\gamma(\theta).$$

The unit construction cost is defined as

$$k_t = \frac{1}{\kappa}(N_t)^{\frac{1}{\zeta}}, \tag{2.25}$$

where N_t is new houses built in period t and available at $t+1$. The price of land is in the form of

$$q_t = \bar{q}(N_t)^{\frac{1}{\xi}}, \tag{2.26}$$

where ξ represents the elasticity of new land supply with respect to its prices. Since any depreciation is offset by maintenance, the total housing stock in the city evolves as $H_{t+1} = H_t + N_t$.

β is set to reflect an annual interest rate of 4%. μ is chosen to match the annual population growth during the 1990s. Income in the steady-state is normalized to one unit of consumption good. Thus, present values and prices are all measured relative to steady-state *per capita* income.

I set π_p to match the annual fraction of renters that move between counties, which is, on average, approximately 12%, according to the Census Bureau. Similarly, π_h is set to match the annual fraction of home-owners who move between counties (3.2%). The continuous value of leaving the city, L, is equal to the steady-state value of being a perpetual renter, V_{ss}^{p}.

The value of the elasticity of new construction with respect to the price of housing, ζ, equals the median elasticity for the 45 cities studied by Green, Malpezzi, and Mayo (2005), which they estimate to be $\zeta = 5$. Saiz (2010) studies the relationship between house prices and the stock of houses based on a long difference estimation between 1970 and 2000 for 95 U.S. cities. His supply elasticity estimates vary from 0.60 to 5.45 with a population-weighted average of 1.75 (2.5 unweighted). I set $\xi = 1.75$ in the baseline search model.

There are three components of the mortgage rate: $r_m = i + \phi + \varrho$. The annual yield on international bonds, i, is set at 4%. ϕ is service cost and ϱ represents a moderate risk premium. Without a positive ϱ, the present value of any mortgage contract will be less than the loan issued at origination because of the positive default probability. ϕ and ϱ are determined jointly in calibration.

I target a waiting period of 5 years after a foreclosure before a borrower can take out a new mortgage. This time frame is consistent with the policies of Fannie Mae and Freddie Mac, which guarantee most U.S. mortgages. In the model, the waiting period is stochastic and depends on the probability that a foreclosure flag stays on a borrower's credit record. I set $\pi_f = 0.80$ to give an expected duration of 5 years.

The steady-state unit price of land $\bar{q}$ is set so that the relative share of land in the house price is 30% (see Davis and Palumbo (2008), and Saiz (2010))) in the steady state. The maintenance cost m is chosen to be 2.5% of the steady state house price according to Harding, Rosenthal, and Sirmansa (2007). The average house price of a house is 3.2 times annual income or 12.8 times quarterly income.

I define a period to equal one year instead of one quarter.[17] This is because house prices are too high for no-saving buyers to make a typical 15-20% down-payments with only one quarter's worth of labor income.[18]

For the rent-to-price ratio, I use data from the Lincoln Institute of Land Policy. They estimate annual rents for owner-occupied units based on data from the Census Bureau on annual rents paid to rental units. They divide these estimated rents by the average self-reported value of owner-occupied units to obtain the rent to price ratio. Prior to the recent housing boom this number was fairly stable and hovered around 5%.

I choose remaining 8 parameters such that 8 steady-state statistics match their average counterparts in the U.S. data. The value of ψ is calibrated so that the ownership rate in the city $\sum \Phi_{ss}/(\sum \Phi_{ss} + B_{ss} + B_{ss}^R + F_{ss})$ equals 66%. The Census Bureau reports the ownership rate among households whose head is between 35 and 44 is roughly 66.7%. According to Federal Housing Finance Board data, the average contract rate on conventional, fixed rate mortgages between 1995 and 2004 is 7.2%. The average down-payment ratio is targeted to 20%. I target the annual default rate at 1.6%, which is near the average annual foreclosure rate among all mortgages during the 1990s in the Mortgage Bankers Association's National Delinquency survey.

The loss severity rate is the present value of all losses on a given loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are

[17] One year can be arguably too long for search frictions to take effect on sales in the housing markets. In the model, I assume that subperiod 1 takes one quarter while subperiod 2 takes three quarters. This implies that sellers and buyers can only search for a trade in the first quarter of every year. This assumption is arbitrary for normal buyers and sellers except those who are in financial distress and stop making payments. Delinquent mortgages quickly fall into the foreclosure process.

[18] If a period equals one quarter, borrowers can only afford down-payments that are less than 7.9% of the average price.

caused both by transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a data set of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. Based on these numbers I choose parameters so that in the event of default and on average

$$\frac{\min\left\{\beta V^{REO}, d\right\}}{d} = 0.54,$$

that is, the average loss severity rate is 46%.

Pennington-Cross (2010) examines sub-prime mortgage data and finds that 50% of delinquent loans with loan-to-value ratios between 80% and 90% end up being repossessed, compared to 55% of delinquent loans with loan-to-value ratios between 90% and 100%, and 59% of delinquent loans with loan-to-value ratios above 100%. In addition, Phillips and Vanderhoff (2004) find that 30% of defaulted conventional fixed rate loans and 50% of defaulted conventional adjustable rate loans transition to REO and Ambrose and Capone (1996, 1998) report that 32% to 38% of defaulted FHA loans transition to foreclosure. Based on these numbers, I choose parameters such that in the event of financial distress, the average probability of successful sale is 0.665, *i.e.* 33.5% delinquent loans end up being repossessed.

Evidence available from the American Housing Survey (AHS) suggests that prior to 2003 the ratio of the original loan size to yearly income is around 2.72 on average. Therefore I choose parameters such that $m_{0,ss}/y_{ss} = 2.72$.

In addition, the dynamics of the model depend crucially on two elasticities: the elasticity of $G(\cdot)$, evaluated at ε^c, $\alpha_p = \varepsilon^c G'(\varepsilon^c)/G(\varepsilon^c)$ and the elasticity of the matching function with respect to the number of buyers, η. These two parameters are calibrated jointly by using estimates of the relative standard deviations of population growth and sales growth in response to income shocks as in Head, Lloyd-Ellis, and Sun (2014).

The parameters of the non-search economy are identical to those in Table 1, except for π_d^n, z^H and ψ, which are adjusted such that the steady-state statistics match the relevant targets again.

2.6 The Steady State

I now analyze the steady-state implications for the search economy. In the steady state, household sellers' selling and default decisions vary by type and by mortgage debt. All owners have positive housing equity. Staying owners who do not experience a financial distress shock do not enter the housing market, no matter what their mortgage balances are. Relocated owners that have a LTV ratio over 75% default upon unsuccessful sales in the period right after moving out of the city. Other relocated owners continue to make payments until successful sales or until their mortgages mature. All distressed owners search for a trade in the housing market.

The main idea of this chapter is that the default rate is determined by the severity of illiquidity in the housing market. In cities where houses can be sold at higher rates, distressed borrowers are less likely to default on their mortgage contracts. This encourages the mortgage company to issue larger loans and ask for lower downpayments, whereas in cities where the housing market is sluggish, loan size at origination is smaller and the default rate is higher. To test this hypothesis, I conduct two comparative statics by changing parameter values of the matching function: the coefficient ϖ and the elasticity of the measure of matches with respect to the measure of buyers, η.

In both experiments, all other parameters remain identical to the baseline model. The results may be different if the parameter set is re-calibrated to match all targets. The point here is to emphasize how search frictions affect lending standard. Thus, keeping the same parameter set is reasonable. The results are depicted in Figure 2.3.

The top two panels describe values of the down-payment ratio, ν, and the probability of a mortgage ending in foreclosure as functions of ϖ, respectively. The probability of a period t originated mortgage ending in foreclosure, $\Pi_{d,t}$, is computed as:

$$\begin{aligned} \Pi_{d,t} &= \sum_{i=1}^{T}(1-\pi_h)^i(1-\pi_d)^{i-1}\pi_d\Big[1-\rho\Big(\theta\big(p^{sd*}_{t+i}(i-1)\big)\Big)\Big] \\ &+ \sum_{i=1}^{T}(1-\pi_h)^{i-1}\pi_h * D^{L*}_{t+i}(i-1)\Big[1-\rho\Big(\theta\big(p^{sL*}_{t+i}(i-1)\big)\Big)\Big], \end{aligned} \tag{2.27}$$

where $p^{sd*}_{t+i}(i-1)$ and $p^{sL*}_{t+i}(i-1)$ are the borrower's optimal trading prices at period $t+i$ if he/she is in and not in the city, respectively. The first item is the summation of probabilities of default if the owner stays in the city. The second item is the summation of probabilities of default if the owner leaves the city.

Both ν and Π_d are monotonically decreasing in ϖ. The higher the value of ϖ, the less friction there is in the housing market such that agents can find trade partners more easily. In a more liquid housing market, the expected default rate is lower because distressed borrowers can liquidate their collateral more quickly. The loan size at origination is larger because of lower expected default rate and a faster rate at which houses in REO inventory sell.

The two bottom panels in Figure 2.3 show similar results for different values of

η. Intuitively, buyer's trading surplus from housing transactions is increasing in η.[19] A higher η implies a higher value of being a buyer, which, in turn, increases the value of living in the city. More households are attracted to the city and the market is tighter in the steady-state. The probability of selling is higher and the expected default rate is lower, which leads to a lower down-payment ratio. Analytically, the seller's matching rate, $\rho(\theta(\cdot)) = \varpi(\theta)^{\eta}$, increases in η if $\theta > 1$, *i.e.*, $\partial\rho(\cdot)/\partial\eta|_{\theta>1} > 0$. For $\eta \in [0.1,\ 0.4]$, the computation result shows that buyer to seller ratios θ in all active submarkets are greater than one. Thus, higher η results in higher probabilities of selling, lower default rates and larger loans at origination.

Figure 2.4 depicts the probabilities of selling for distressed owners and relocated owners with different LTV ratios. For each LTV ratio, distressed sellers choose submarkets with higher matching rates compared to relocated sellers. Two reasons account for this result. First, distressed sellers are forced to default if they fail to sell their houses, whereas relocated sellers are free to make default decisions upon failing sales. Second, relocated households receive continuation value L, which does not depend on their credit records. By contrast, distressed sellers who has defaulted continue staying in the city, and are punished by being excluded from mortgage markets in the future. In summary, the cost of failing to sell is much higher for distressed sellers than for relocated sellers. Thus, distressed sellers are more likely to enter submarkets in which their houses can be sold more quickly.

For both types of sellers, the probability of selling is approximately flat for sellers with low LTV ratios, and decreases for those with median to high LTV ratios. Intuitively, prospective sellers' loss upon failing sales is decreasing in the LTV ratio. Sellers with high LTV ratios are less eager to sell and are more likely to bet for higher

[19] See (Head, Lloyd-Ellis, and Sun, 2014) and (Moen, 1997)

prices. As LTV ratio decreases, sellers become more anxious to sell and are more likely to ask lower prices in exchange for higher probabilities of selling. There is a threshold LTV ratio such that for sellers with LTV ratios lower than it, it is optimal to sell their houses at the price with highest sales rate, i.e. in the most liquid sub-market. In addition, the marginal decrease in the probability of selling is higher for both types of sellers with higher LTV ratios.

2.7 Equilibrium Dynamics

2.7.1 Qualitative implications

In this section, I describe the qualitative implications of the equilibrium dynamics for both economies. To begin with, I assume that the process followed by the log of income, $\log y_t$, is a simple AR(1) process with persistence parameter 0.92 and innovation standard deviation 0.01.

The implied impulse response functions following a positive shock to local income are depicted in Figures 2.5-2.9. The four panels of Figure 2.5 depict the impulse responses for local income, population growth rates, house prices, and construction rates. Red-dotted lines are the dynamics of variables in the non-search economy. In both economies, the positive shock to local income induces immediate entry and the population growth rate rises. Although the responses of city population growth rates in the two economies are qualitatively similar[20], the initial entry is much more rapid in the search economy. There are two possible reasons accounting for this. First, households value a house not only because it provides dwelling but also because

[20]In the non-search economy, the population growth rate jumps above 2% and falls monotonically thereafter.

of its liquidation value in the future. After the income shock, because the house price appreciation is persistent and market tightness remains high for some time, the house's liquidation value rises. The value of living in the city increases and more households move there. In the non-search economy, there is no such liquidation effect. Second, as observed in the bottom-left panel in Figure 2.5, the house price jumps much higher in the non-search economy than it does in the search economy. High house price reduces both buyers' payoffs and the value of living in the city. The higher the price jumps, the more offset the positive effect of higher local income is. Thus, the increase in the value of living in the city is lower in the non-search economy, which results in a lower change in the population growth rate.

The responses of house prices and construction rates differ qualitatively across two economies. The search model generates serial correlation in both of these two variables. By contrast, the non-search economy does not generate such dynamic in either prices or construction rates. The force that generates serial correlation in both house price appreciation and the construction rate is the change in housing market liquidity. To conceptualize this, first consider Figure 2.6, which depicts responses of market tightness, average matching probabilities of buyers, average matching probabilities of sellers, and default rates among all mortgages.

Initially, an increase in the value of living in the city generates an immediate increase in search activity as more people enter the market and begin searching for a house. Ignoring, for now, any responses in the measures of houses for sale, the overall ratio of buyers to sellers (market tightness) rises, which reduces the average rate at which buyers find houses through the matching process. Because the price of a house reflects its future resale value (because owners may need to sell the house if they are in financial distress or leave the city), the increase in tightness, which raises the per-period probability of selling, increases overall transaction prices. Because of search frictions, not all newly entering buyers are matched with sellers immediately. Because entry is persistent due to the persistence of the positive income shock, unmatched buyers get built up in the market after the shock (as depicted in the upper-left panel of Figure 2.6), which generates further increases in both overall tightness and the rate (on average across submarkets) at which houses can be sold. The additional increase in tightness also further decreases the rate at which a buyer can find a house, as depicted in the upper-right panel of Figure 2.6. As houses become more liquid over time, their value and transaction prices increase further as well, which results in persistent appreciation of house prices in response to the relative income shock.

On average, the probability of selling rises initially because of the immediate increase in search activity as more buyers enter and begin searching in the market. As houses become more liquid, the average rate of selling continues to increase for a period of time. Distressed sellers have a better chance of selling their houses to avoid falling into default. This results in a decrease in the overall default rate, as shown in the bottom-right panel of Figure 2.6. Later, because the number of vacant houses in the market rises as the number of owners increases and new construction rises, and because entry slows, the population growth rate and the probability of selling back to their long-run trend.

The responses of down-payment ratios in the two economies are also qualitatively different, as shown in Figure 2.7. In the search model, the loan volume jumps, the down-payment ratio falls in the period with the shock. Several forces contribute to this result. First, as houses become more liquid over time, the expected default rate remains low for a while. Thus, borrowers' exposure to risk falls for the entire period of new mortgages. Second, because the house price remains high for an extended period, proceeds from foreclosure sales remains above its long-run trend level, which implies that the houses in REO inventory have a higher value V_t^{REO}. Third, higher sales rate reduces the time it takes to sell houses in the REO inventory, which decreases the carrying costs of the mortgage company and further increases the value of V_t^{REO}. Finally, borrowers who originate mortgages before the shock have higher net equity on their houses than their counterparts in the steady state. Higher equity induces distressed sellers to choose submarkets with higher sales rates. This further decreases the overall default rate. Reductions in the expected default rate of a mortgage and the expected loss upon default give the mortgage company strong incentive to issue larger loans. The increase in loan volume is so high that the down-payment ratio falls while house price rises immediately following the shock.

Because mortgages are long-term contracts, and because both market tightness and house prices follow hump-shaped evolutionary paths, the later the loan is originated, the less the mortgage company benefits from lower default rates and the fewer proceeds it receives from foreclosed house sales. The loan volume and the down-payment ratio monotonically return to their long-run trends after initial changes.[21]

In the non-search economy, by contrast, both loan volume and down-payment ratio jump up initially and gradually return to long-run trends thereafter. The reason why larger loans are incurred is that proceeds from foreclosure sales are higher than proceeds in the steady state for a long time after the shock. Since the expected default rate of a mortgage does not change, the mortgage company's expected loss

[21] The change in absolute volume of a loan at origination is more persistent in the search model than in the non-search model. In the latter, house price falls much faster and market tightness plays no role.

upon default rises at a higher rate than its expected revenue if the down-payment ratio does not increase. This breaks the mortgage company's zero profit condition. Hence, the down-payment ratio jumps right after the shock.[22] Because the house price falls with time, the increase in the expected loss in new mortgages also falls, which causes the down-payment ratio to return to its long-run trend.

The four panels of Figure 2.8 depict dynamic paths of probabilities of selling for distressed sellers with different mortgage balances over time. The upper-right panel depicts the probability of selling for a distressed seller who originated a mortgage contract one period before the shock. In the period of shock, the distressed sellers who had become owners a few periods earlier have higher housing equity compared to their counterparts in the steady state due to the jump in house prices. These distressed sellers prefer to selling their houses quickly and their matching probabilities jump higher than average. The effect of higher housing equity on the probability of selling is not significant for owners with low to medium levels of mortgage debt; thus, the increases in their sales rates are mostly caused by the overall rise in market tightness. This is consistent with the findings above that submarket choices of borrowers with high leverages are the most sensitive to equity values.

The impulse functions of the buyer's market utility, the measure of buyers, the loss severity rate of REO houses, and the value of REO inventory for the search economy are depicted in Figure 2.9. The first panel depicts the evolution of the buyer's market utility. Recall that the market utility is buyer's expected payoff before entering the housing market. Initially, with the higher value of living in the city and the moderate fall in the rate at which buyers can find a house, market utility jumps above the

[22]This result does not critically depend on the assumptions of no saving and exogenously given default rate.

steady state level. Over time, because local income decreases persistently after the initial jump and because more buyers search in the housing market, market utility decreases monotonously back to its steady state level. Note that market utility reverts to its steady state level much faster than other variables. This is because the negative effects of higher house prices and lower matching rates dominate the positive effects of higher income (compared with the steady state). The loss severity rate initially falls because all mortgages used to compute this rate are originated before the shock and because house prices increase with the shock. As time goes on, because more mortgages are originated after the shock and with higher principles, the loss severity rate rises over time.

The impulse responses of the economies to a negative local income shock are depicted in figures 2.10 - 2.14. The dynamics are nearly symmetric to those in response to a positive income shock. One exception is that immediately after the shock, some non-distressed owners have negative equity and some of them choose to default on their loans.

2.7.2 Quantitative implications

Using the process constructed for income, I generate sample paths for key variables by simulation, and use these to construct annualized time series. Moments from these series, along with the corresponding moments for the U.S. economy,[23] are presented in Table 2.2. The table contains the standard deviations of house price appreciation, housing stock growth, population growth and the down-payment ratio relative to that

[23]The moments for the U.S. economy are taken from Head, Lloyd-Ellis, and Sun (2014). The researchers reported the figures by using data compiled for the metropolitan statistical areas of 106 cities in the U.S. from 1981 to 2008. Due to the lack of city-level data on the average LTV ratio for the U.S. economy, empirical moments of down-payment ratios are not reported.

of local per capita income growth, and the correlations of these variables with local income growth. The first column in the table reports the results for the numbers for the U.S. economy. The second column reports the results for the search model and the third column reports the results for the non-search economy.

The baseline model generates price volatility relative to income at approximately 109% of what is observed in the data. By contrast, the volatility generated by the non-search model is much higher. Housing stock growth, population growth and the down-payment ratios in the data are all less volatile than local income growth in both models. Compared with moments in the non-search model, population growth (σ_{pop}/σ_y) is more volatile and down-payment ratio (σ_ν/σ_y) is less volatile in the search model. Consistent with empirical evidence, the down-payment ratio ($\sigma_{\nu,p}$) is negatively correlated with the average house price in the search model. However, in the non-search economy, the correlation between the down-payment ratio and the house price is positive and significant. Despite of abstracting from many factors, this analysis illustrates that trading frictions in the housing market is indeed an important factor to explain the observed comovement between the housing market and the credit market.

2.8 Conclusion

This chapter studies how housing liquidity affects lending standards by incorporating long-term mortgage contracts into a dynamic search model. Two key features of the model are: (1) mortgages are long-term loans with finite maturity and borrowers are allowed to default, and (2) the housing market is characterized by competitive search with homogenous buyers and heterogeneous sellers. A parallel model with no search is also developed to make comparisons.

I find that housing liquidity affects lending standards by influencing the expected default rate of a mortgage and the mortgage company's expected loss upon default. In addition, consistent with empirical evidence, sellers with higher LTV ratios are more aggressive, ask for higher prices and sell more slowly. Moreover, the model also provides a theory for explaining the stylized fact that the down-payment ratio and the house price are negatively correlated.

A policy that affects the liquidity in the housing market may also change credit market conditions, which in turn may amplify or dampen the policy's effect on housing markets and other macro variables. For example, a decrease in the property tax makes home purchase more valuable. This attracts more potential buyers and makes houses more liquid. The reductions in both the expected default rate and the expected carrying cost induce more lenient lending standards, which in turn encourage more housing demand, and so on. Thus, in addition to evaluate a policy's direct effects on the housing market, we also need to take the feedback from the credit market into consideration because of the liquidity channel examined in this chapter.

2.9 Tables and Figures

Table 2.1: Calibration Parameter Values

Parameter	Value	Target	Data
		Parameters determined independently	
β	0.96	Annual interest rate	4.0%
π_p	0.120	Annual mobility of renters	12%
π_h	0.032	Annual mobility of owners	3.2%
ξ	1.75	Median price-elasticity of land supply	1.75
i	0.040	International bond annual yield	4.0%
T	30	Fixed rate mortgage maturity (years)	30
μ	0.012	Annual population growth rate	1.2%
π_f	0.80	Average duration (years) of foreclosure flag	5
$\bar{q}$	0.96	Average land price-income ratio	30%
m	0.08	Residential housing gross depreciation rate	2.5%
ζ	5	Median price elasticity of new construction	5
R/P	0.05	Average rent to price ratio	5%
		Parameters determined jointly	
χ	0.440	Loss severity rate	46%
ϕ	0.0246	Average down-payment ratio	20%
ϱ	0.0074	Average annual FRM-yield	7.20%
ψ	0.570	Fraction of households that rent	33.3%
π_d	0.060	Annual foreclosure rate	1.6%
z^H	0.3280	Average loan-to-income ratio at origination	2.72
ϖ	0.56	Average fraction of delinquent loans repossessed	33.5%
κ	0.137	Average price of a house	3.2
η	0.1880	Relative volatility of sales growth	1.32
α_p	6.200	Relative volatility of population growth	0.17

Table 2.2: Volatilities and co-movements (models and U.S. cities)

Moments	US cities	Baseline	No search
σ_p/σ_y	1.60	1.74	2.54
σ_h/σ_y	0.11	0.11*	0.11*
σ_{pop}/σ_y	0.17	0.17*	0.09
$\sigma_{p,y}$	0.76	0.98	0.99
$\sigma_{h,y}$	0.54	0.16	0.23
$\sigma_{pop,y}$	0.76	0.48	0.19
σ_ν/σ_y		0.21	0.65
$\sigma_{\nu,y}$		-0.60	0.93
$\sigma_{\nu,p}$		-0.65	0.94

Note: value marked with a * is calibrated targets. Moments for US cities are cited from empirical results of Head, Lloyd-Ellis, and Sun (2014).

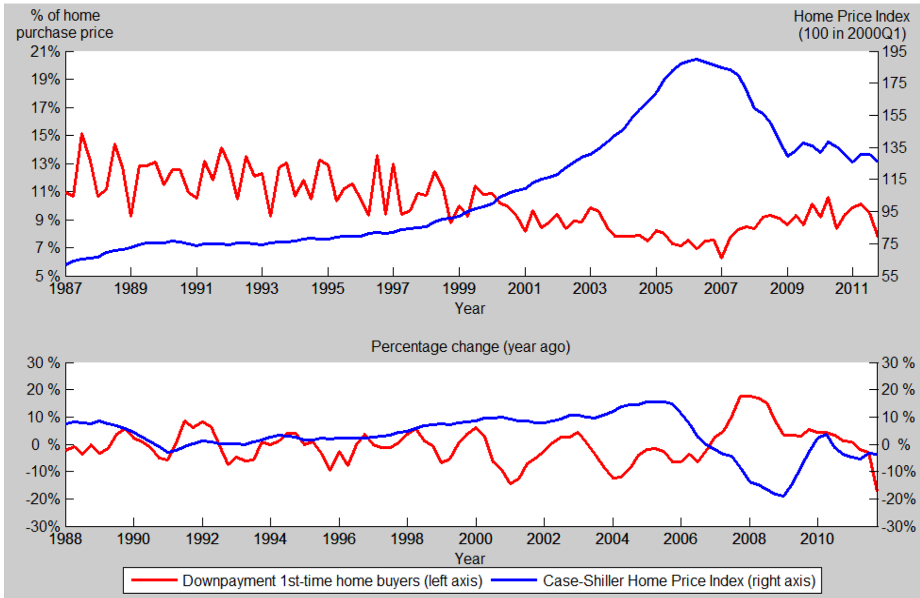

Figure 2.2: Values and percentage changes (from one year earlier) in average first-time home buyers down-payment ratios and S&P/Case-Shiller U.S. National Home Price Index. Source: American Housing Survey (AHS) 2007, 2009, 2011 national data.

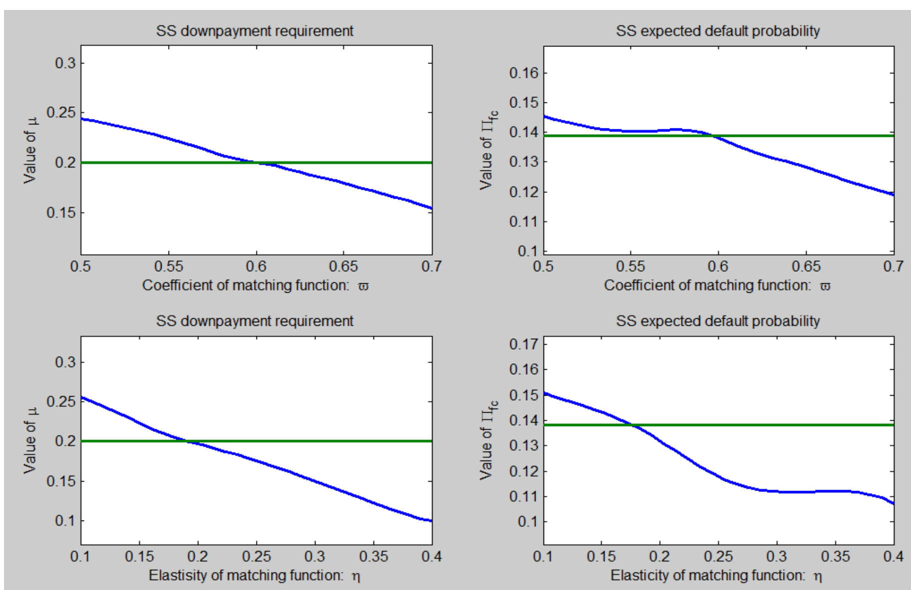

Figure 2.3: Down-payment ratio and default rate for different parameters of $M(B, S)$.

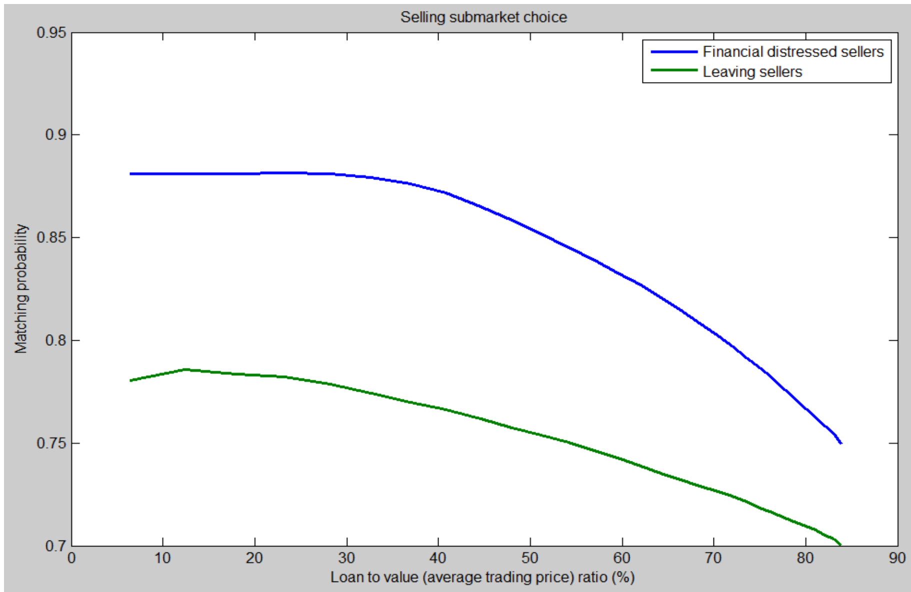

Figure 2.4: Submarket choices of staying and relocated sellers

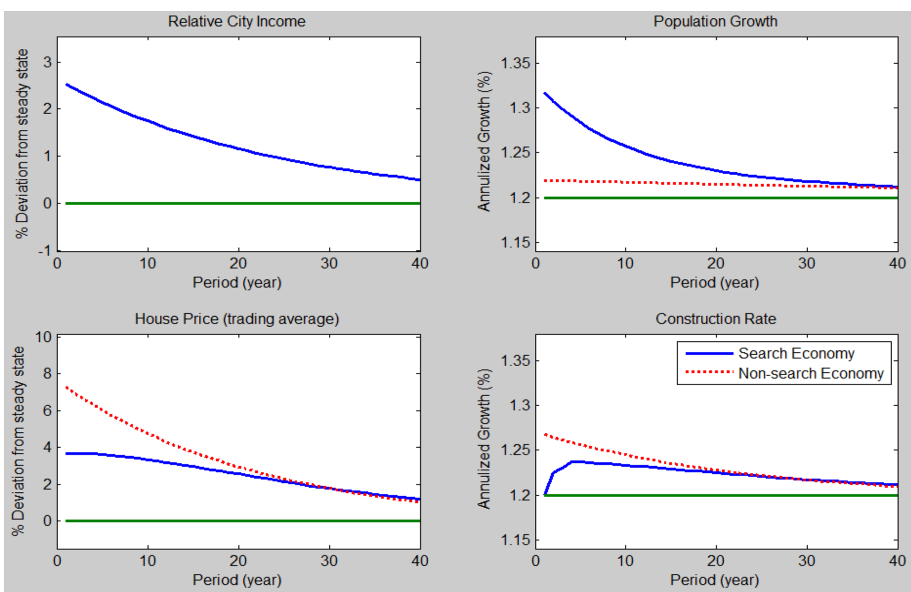

Figure 2.5: Impulse responses with a positive income shock

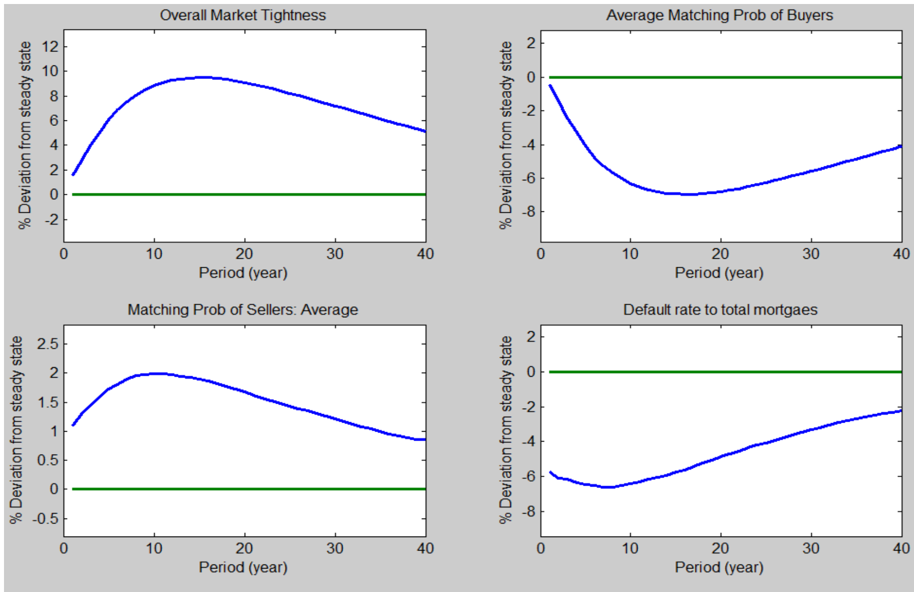

Figure 2.6: Impulse responses with a positive income shock

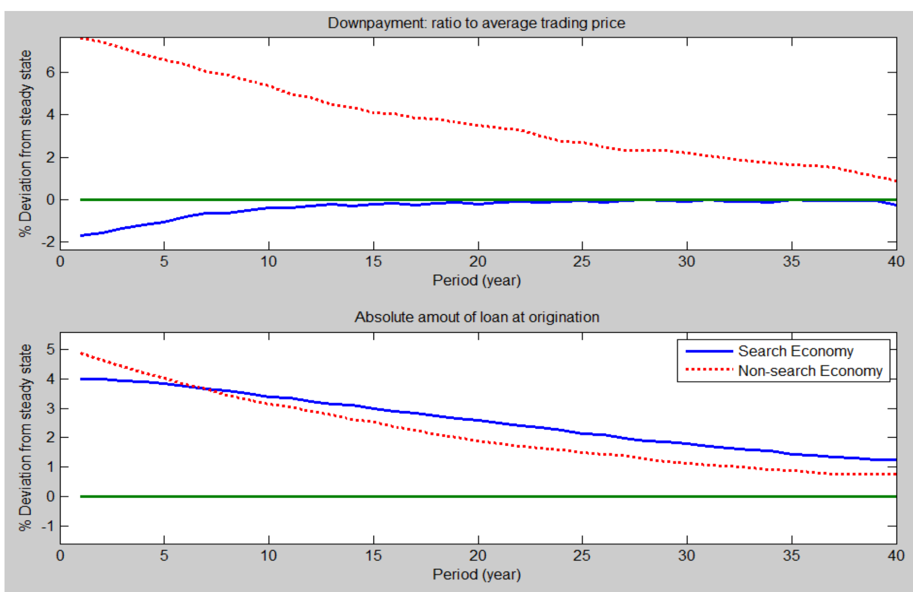

Figure 2.7: Impulse responses with a positive income shock

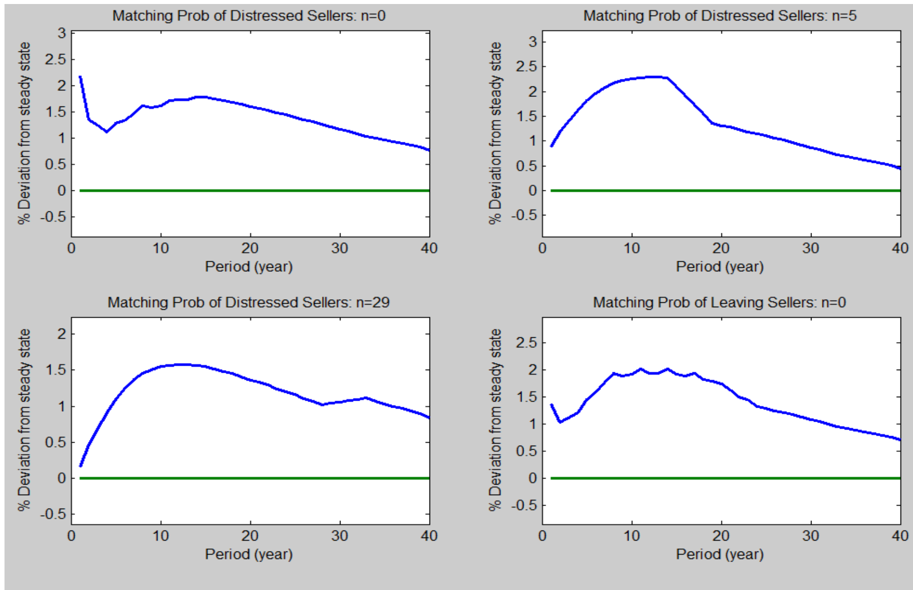

Figure 2.8: Impulse responses with a positive income shock

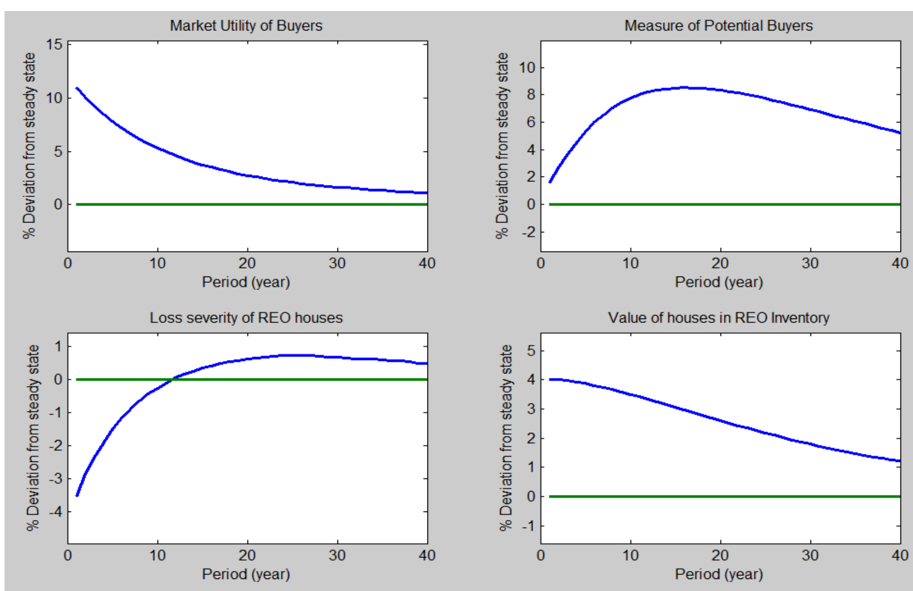

Figure 2.9: Impulse responses with a positive income shock

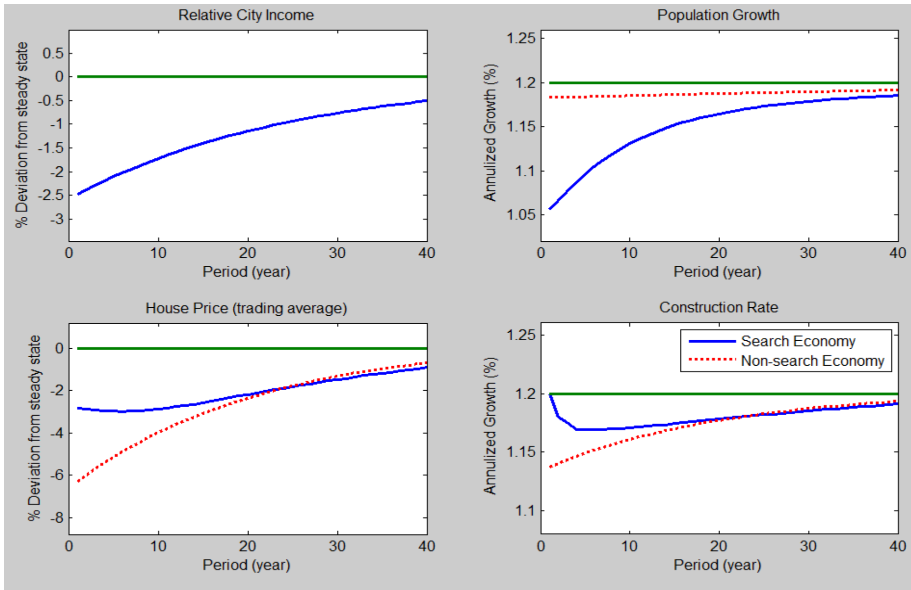

Figure 2.10: Impulse responses with a negative income shock

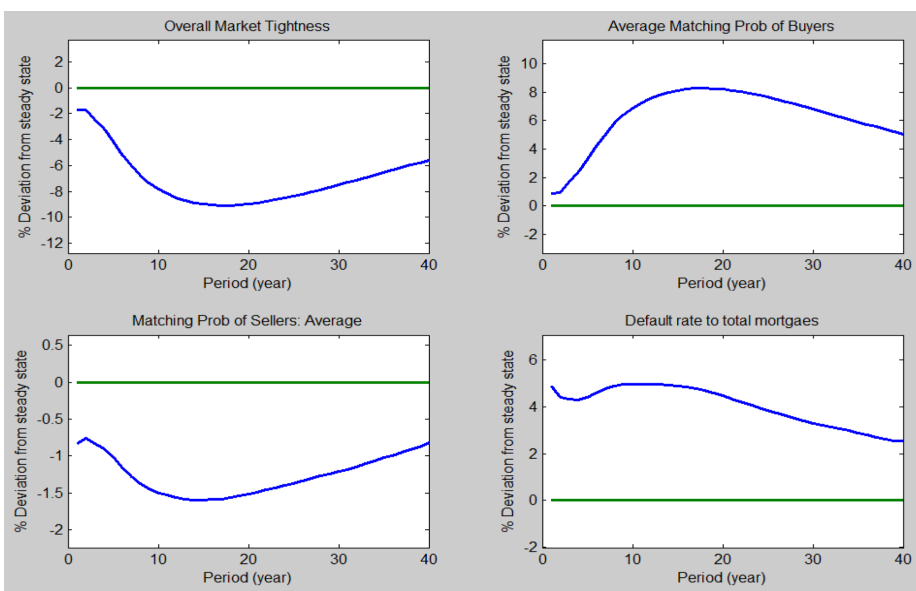

Figure 2.11: Impulse responses with a negative income shock

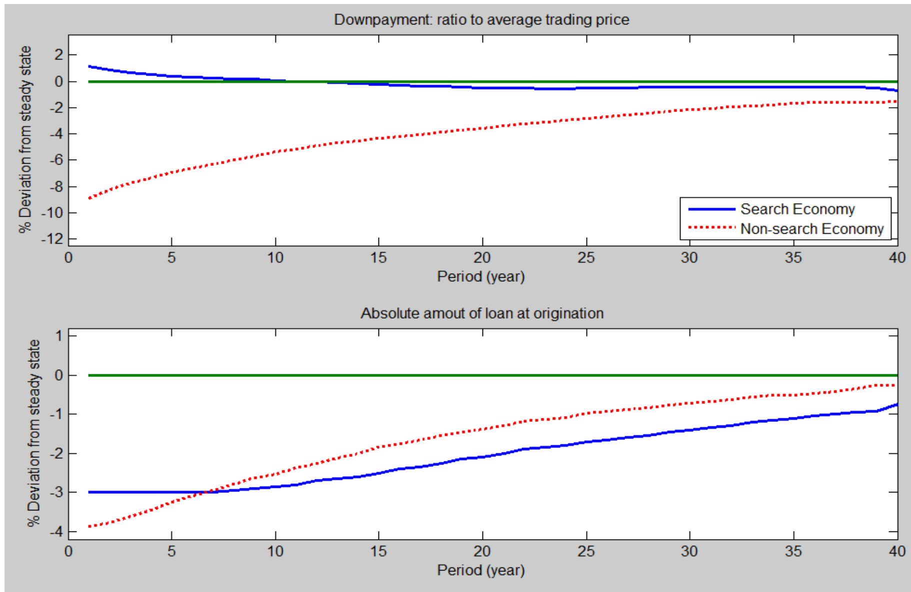

Figure 2.12: Impulse responses with a negative income shock

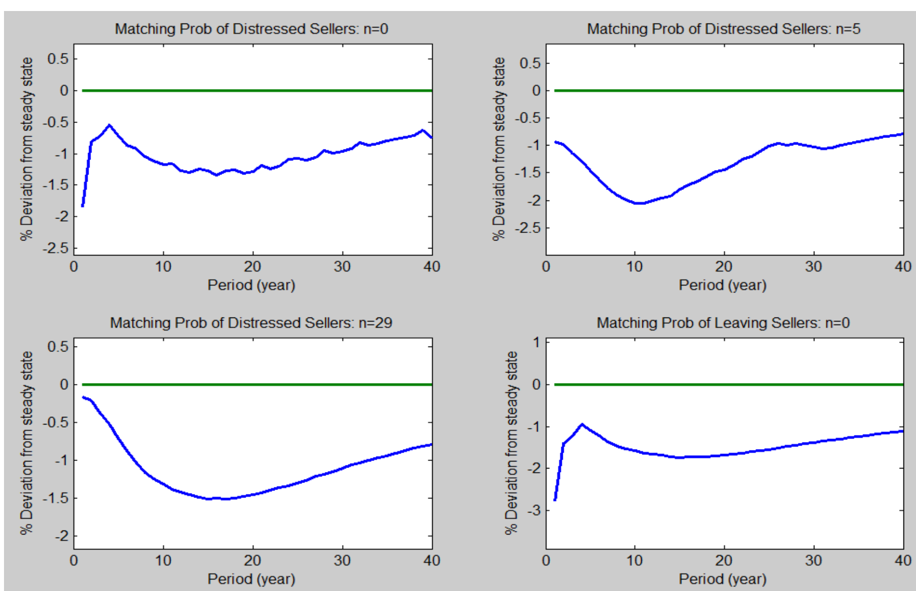

Figure 2.13: Impulse responses with a negative income shock

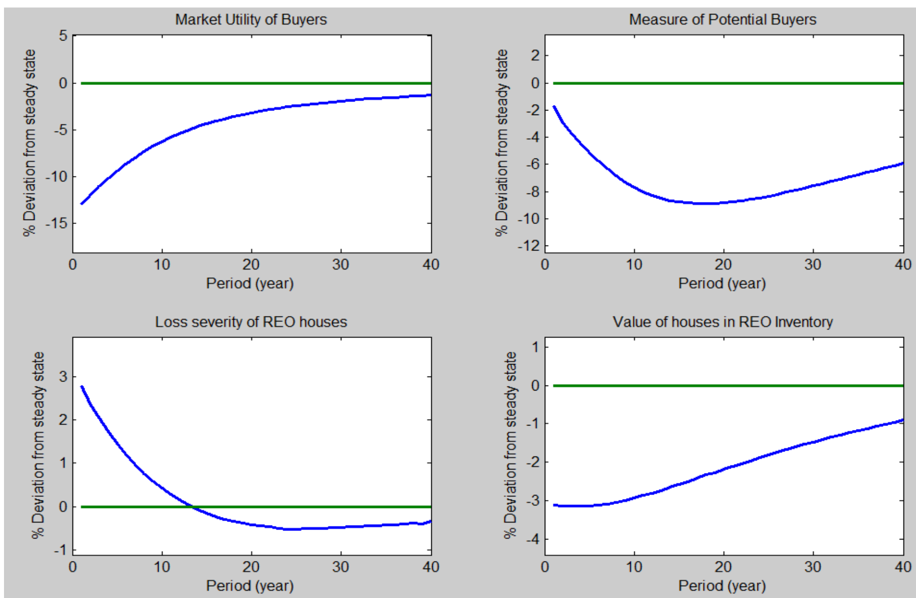

Figure 2.14: Impulse responses with a negative income shock

Chapter 3

Exploring House Price Volatility in Major Canadian Cities

3.1 Introduction

In this chapter, I examine the properties of house price volatility in Canada at both national and city levels. By employing full Lagrange Multiplier (LM) tests on well specified Autoregressive-moving-average (ARMA) models of house prices, I find that most major Canadian cities (9 out of 11), as well as the nation as a whole, exhibit house price volatility clustering during 1981 - 2014. Among these cities with Generalized Autoregressive Conditional Heteroskedastic (GARCH) effects, asymmetric volatilities (TGARCH effect), of either sign, are observed in Toronto, Vancouver, Winnipeg and Canada. For cities with ARCH effects, the conditional variance GARCH process often leads to much larger variance during volatile periods than the unconditional variance.

I also examine the dynamic interaction between house price volatility and key macroeconomic variables for the nation as a whole. On one hand, VAR, Granger causality and variance decomposition (VDC) analyses demonstrate that house price volatility is significantly affected by house price appreciation, house sales growth rate and population growth rate. On the other hand, volatility affects GDP growth rate, house price appreciation rate, sales growth rate and volatility itself. Finally, I find that positive changes in house prices are more frequent and larger than negative changes in most of the studied cities over the sample period.

The housing market plays an important role in an economy. In Canada, the housing sector accounts for nearly one-third of the fixed capital stock and over one-fifth of household expenditures in 2010. A house is not only providing dwelling but also making a significant contribution to the total asset of many households. The total value of individual and multiple dwelling has been estimated to be $1.73 trillion in 2010.[1] Given significance of the housing sector, the housing market has drawn a lot of attention from investors and researchers. There is a large body of literature on the determinants of house prices, such as Case and Shiller (1989), Iacoviello (2005) and Glaeser, Gyourko, Morales, and Nathanson (2013). Most of existing studies focus mainly on the return of house prices (the first moment), while little in the literature on price volatility (the second moment) has been developed.

House is a special asset that has dual role of providing dwelling and investment. Similar with many other financial assets, the degree of price volatility is used to assess the investment risk. For example, extensive studies have found that house price volatility is a determinant of borrower pre-payment and mortgage default (see Foster and Van Order (1984), Crawford and Rosenblatt (1995) and LaCour-Little, Marschoun, and Maxam (2002)). In this respect, the volatility of house price is one of the most important variables for the house investment decision, which in turn generates significant effects on the whole economy.

[1]Statistics Canada (CANSIM tables), in 2007 CAD.

To investigate the volatility of house price, it is important to test if there is volatility clustering and analyze its pattern if it does exist. As highlighted by Miles (2008) and Wong, Yiu, Tse, and Chau (2006), failure to incorporate volatility clustering may lead to inaccurate modeling results and sub-optimal allocation. For Canada, Hossain and Latif (2009) identify evidence of time varying house price volatility (GARCH effect) and find that house price volatility is affected significantly by gross domestic product (GDP) growth rate, house price appreciation rate and inflation. Lee (2009) shows that volatility clustering is found in many Australian capital cities and the response of price volatility to shocks is often asymmetric. Miles (2008) finds that volatility clustering effects are evident in over half of the states in the U.S. and the signs and magnitudes vary widely. His additional results also demonstrate the evidence of shock's asymmetric effect on price volatility. Interestingly, the states that are in areas not traditionally associated with strong house markets, Michigan for example, have positive TARCH effects, *i.e.* a negative shock raises volatility. By contrast, the states with strong house markets, New Jersey for instant, exhibit negative TARCH effects (where a negative shock reduces volatility).

In recent empirical literature, economists have documented the downward rigidity of house price, which is the phenomenon that prospective sellers are reluctant to reduce asking prices in down house markets. By examining the real estate trading data of central Boston during the 1990s, Genesove and Mayer (2001) confirm the presence of the "disposition effect"[2] in real estate markets. Tsai and Chen (2009) demonstrates the presence of a price defensive effect in the U.K. house market. The results in their paper indicate that the volatilities between house prices moving up and

[2]The disposition effect describes the tendency of investors to sell shares with prices that increased while retaining assets that dropped in value (Shefrin and Statman (1985)).

down are asymmetric. That is, when bad news occurs, the variance decreases. Tsai (2012) finds that house price volatility is indeed asymmetrically adjusted to money supply, *i.e.* house prices tend to over-react in upturn and under-react in downturn to monetary shocks. Dufrénot and Malik (2012) study the relationship between the business cycle and house prices and discovers that during a recession, house prices were not informative. This observed inefficiency may appear in the form of house price rigidity.

The remainder of this chapter is structured as follows. Section 3.2 provides data source and show the pattern of Canadian house price paths over the studied period. Section 3.3 - 3.12 sequentially conduct unit root test, ARMA model specification, GARCH effect test, GARCH-M test, Augmented-GARCH test, Asymmetric GARCH test, conditional and unconditional variance comparison, VAR Granger causality test and VDC, and asymmetric adjustment analyses. Section 3.13 concludes. The last section lists tables and figures mentioned in this chapter.

3.2 Data Source

This chapter uses quarterly data from 1980.Q2 to 2014.Q1 for Canada and its major cities. The following time series were used: gross domestic production growth rate ($GDPG_t$), Inflation rate ($CPIG_t$), population growth rate ($POPG_t$), change in mortgage rate ($MRTC_t$), average house price appreciation across Canada (HPG_t) and change in house sale volume (HSG_t).[3] $GDPG_t$, $CPIG_t$, $POPG_t$ and $MRTC_t$ are compiled by using the corresponding time series of GDP_t, CPI_t, POP_t, and MRT_t from CANSIM tables (Statistics Canada). Average house price HP_t and total sales

[3]These growth and change rates are defined as log difference of the original level series.

volume HS_t were collected from Multiple Listing Service (MSL) of Canadian Real Estates Association (CREA).[4]

Figure 3.1 and Figure 3.2 present Canadian national house price series as well as its changes. From 2000 to 2006, the Canadian housing market experienced a long booming period and the aggregate price almost doubled. Following the U.S. sub-prime crisis in 2007, people realized that the rapid growth in house prices in previous years may be a bubble and the price suffered a most marginal decline during 2007 - 2009. Thereafter, not like the U.S. housing market, Canadian housing market recovered much faster due to Canada's better economic performance during and after the crisis. The house price went back to its level before crisis in 2010 and soared after 2013. In addition, we can observe from these two graphs that a fierce housing market fluctuation is illustrated if the slope of price only slightly changed. For example, the increase trend of price sequence after 2002 is represented by the abrupt slope in Figure 3.1.

As discussed in Miller and Peng (2006), city (MSA) level data rather than national data are more appropriate for analyzing the metropolitan house price volatility. Hence, unlike Lin and Fuerst (2013) who use provincial level data, the following analysis utilizes quarterly price return over 1980:Q2 to 2014:Q1 in Canadian cities. The quarterly unit sales, HS_t, are used as the measure of the activity in house markets and the 11 most active cities are chosen for this study. The city list includes: Calgary, Edmonton, Fraser Valley, Hamilton-Burlington, London-St. Thomas, Mississauga, Ottawa, Toronto, Vancouver, Windsor-Essex and Winnipeg. Table 3.1 provides the summary statistics of prices appreciation in these eleven cities. On average, Toronto

[4]Note that only HP_t, HS_t and CPI_t are used in analyses on Canadian cities. Other variables are required in the section that uses a VAR model to study the determinants of volatility at national level.

has the biggest housing market that represents nearly 20% of the overall house trades and is followed by Vancouver (8%), Calgary (6%) and Edmonton (4%) during the studied period. The paths of house prices in these cities are depicted in Figure 3.3. Most of these series exhibit upward long-run trends after 2000, particularly in Vancouver, Toronto, Calgary and Fraser Valley. The house price in Windsor-Essex, however, kept decreasing after 2005. There are good reasons to believe that this depreciation was caused by the downfall of Detroit, the U.S. city close to Windsor.

3.3 Unit root tests

Unit root tests were performed for examining the stationary of city price appreciation series. Both Augmented Dickey-Fuller and Phillips-Perron unit root tests have been conducted and the results are reported in Table 3.2.

All time series of house prices reject the hypothesis of containing a unit root at 1% level. We can safely say these variables are stationary, *i.e.* their corresponding level variables are difference stationary.

3.4 ARMA models for return

In this section, I estimate the following Autoregressive-moving-average (ARMA) models of house price return for each city as well as Canada as a whole.

$$HPG_t = \alpha + \sum_{i=1}^{p} \beta_i HPG_{t-i} + \sum_{j=1}^{q} \iota_j \varepsilon_{t-j} + \varepsilon_t, \tag{3.1}$$

where β_i and ι_j are AR and MA coefficients, respectively. There are different criteria that could be used to choose the optimal lag length, such as AIC, BIC, maximum likelihood, etc. Because ARCH models assume that the residuals are uncorrelated[5], I adopt a conservative strategy of adding enough AR and MA lags such that there is no remaining autocorrelation in residuals according to the results of a Lagrange Multiplier (LM) test for autocorrelation. The estimation results are reported in Table 3.3.

The estimated residuals of the ARMA models for the nation and the cities are plotted in Figures 3.4, 3.5 and 3.6. The residual plots of these ARMA models seem to demonstrate the clustering of variance: strong (weak) variations are more likely to be followed by strong (weak) variations and *vice versa.* In other words, variances of house price return do not remain constant rather they are time variant. But we still need to take a full Lagrange Multiplier (LM) test to see if there is indeed ARCH effect in each series.

3.5 Full LM tests for ARCH effects

With the clustering of variance, the use of ARCH/GARCH is most appropriate to estimate volatility series. The process can be expressed as:

$$HPG_t = \alpha + \sum_{i=1}^{K} \beta_i HPG_{t-i} + \sum_{j=1}^{N} \iota_i \varepsilon_{t-j} + \varepsilon_t \tag{3.2}$$

$$\varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{P} \delta_i \varepsilon_{t-1}^2 + \sum_{j=1}^{Q} \eta_j \sigma_{t-j}^2, \tag{3.3}$$

[5] There is autocorrelation among the squared residuals.

where ε_t is a white noise and *omega*$_0$ is the constant term in variance equation. σ_t^2 is the heteroskedastic conditional variance, which is correlated to the lagged-error terms and conditional variance. δ_i and η_j are the coefficients of P ARCH terms and Q GARCH terms respectively.

Prior to employing a GARCH model, it is important to perform a formal LM test. For the best fitted ARMA models of each city, a full LM test for ARCH effects is conducted following Engle (1982):

$$\varepsilon_t^2 = \omega_0 + \delta_1 \varepsilon_{t-1}^2 + \delta_2 \varepsilon_{t-2}^2 + ... + \delta_q \varepsilon_{t-q}^2, \tag{3.4}$$

where ε_t^2 is the squared residual from the ARMA model, and LM test is performed by $LM = T \cdot R^2$, where T is the sample size and R^2 is derived from regression of equation (3.4). The results of LM tests are listed in Table 3.4.

As displayed, ARCH effects are significant in most Canadian cities, except Calgary and Hamilton-Burlington. Canada as a whole also exhibits clustering volatility. There is little pattern to which cities have ARCH and which do not. In terms of conditional variability, those cities with ARCH effects, on average, have larger variances than those cities without ARCH effects.

3.6 Testing for GARCH Effects

For those cities that the null hypothesis of a constant variance is rejected, a GARCH(P, Q) model is estimated. Despite GARCH(1,1) is the most frequently used specification in the literature, I select the optimal lag length in the variance equation by using the Schwarz criterion (Schwarz, 1978). Table 3.5 shows the estimation results.

Here I report the summation of significant ARCH and GARCH terms because they are better for describing the persistence of volatility. Most series have both significant ARCH and GARCH terms. Vancouver, however, has only a significant GARCH term.

3.7 Testing for GARCH-M Effects

This section conducts an analysis to determine whether the conditional variance directly affects house price return in the mean equation.[6]

$$HPG_t = \alpha + \sum_{i=1}^{K} \beta_i HPG_{t-i} + \sum_{j=1}^{N} \iota_j \varepsilon_{t-j} + \lambda \sigma_t^2 + \varepsilon_t. \tag{3.5}$$

The test results are depicted in Table 3.6. London-St. Thomas and Canada as a whole have negative GARCH-M effects, *i.e.* house prices get a decrease in mean returns from an increase in the conditional variance.

3.8 Testing for Augmented-GARCH effects

Next, I test the effect of returns on its variance by exploring the augmented conditional variance equation (3.6):

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{P} \delta_i \varepsilon_{t-1}^2 + \sum_{j=1}^{Q} \eta_j \sigma_{t-j}^2 + \zeta HPG_t. \tag{3.6}$$

[6] Previous studies have found that some markets exhibit a positive GARCH in mean effects, while others have a negative effect, see Dolde and Tirtiroglu (1997).

Here, returns are added to equation (3.3) for those cities exhibiting conditional variance.[7] The results in Table 3.7 indicate that house price growth affects volatility significantly for the whole Canada and for three cites: London-St. Thomas, Mississauga and Vancouver. There is no theoretical restriction on the expected sign of the returns parameter. In Mississauga and Canada, higher returns increase the conditional variance, while the effect is opposite in London-St. Thomas and Vancouver.

3.9 Testing for Asymmetric GARCH effects

Note that in the above ARCH/GARCH equation (3.3), σ_t^2 is symmetrically correlated to the lagged-error terms regardless of whether they are positive or negative. In this section, I test if the house price return series have asymmetric volatilities by employing both GJR-GARCH model and EGARCH model.

3.9.1 GJR-GARCH effects

The threshold generalized autoregressive conditional heteroskedasticity (TGARCH) model is proposed by Glosten, Jagannathan, and Runkle (1993). The rationale behind the model goes back to the finding of *leverage effects* (Black, 1976): the increase in equity price volatility is higher in response to negative than to positive shocks. The parameter of interest in a TGARCH model is the coefficient on shocks to house price returns. While the effect has been found to typically be positive on other assets in the literature, there have been no theoretical restrictions on the sign of the threshold coefficient in the housing market. The real estate market may exhibit

[7] This is a standard practice in many GARCH studies. For example, studies of inflation and inflation uncertainty add the inflation rate - the dependent variable - to the variance equation to see if higher inflation results in inflation uncertainty, as measured by the GARCH process.

different statistical properties, according to Case and Shiller (1989). The TGARCH variance equation is:

$$\sigma_t^2 = \omega_0 + \sum_{i=1}^{P} \delta_i \varepsilon_{t-1}^2 + \sum_{j=1}^{Q} \eta_j \sigma_{t-j}^2 + \lambda \varepsilon_{t-1}^2 D_{t-1}, \tag{3.7}$$

where $D_{t-1} = 1$ if $\varepsilon_{t-1} \geq 0$; otherwise $D_{t-1} = 0$.[8] This setting enables positive lagged-error to explain the asymmetric behavior of the conditional variance. The GJR-GARCH model estimation results are listed in Table 3.8.

There are two cities, London-St. Thoms and Mississauga, where a sensible GJR-GARCH model could not be estimated due to a lack of convergence in the estimates. The results show that two cities with the most active house markets, Toronto and Vancouver, exhibit significant negative TARCH effects, *i.e.* positive shocks increase volatility, while negative shocks decrease volatility.[9] This finding is consistent with Miles (2008), who found positive TARCH effects are announced in U.S. areas not traditionally associated with strong house markets while negative TARCH effects are observed in higher-priced (more active) house markets. In other cities, we cannot reject the hypothesis that house price volatility responses symmetrically to positive and negative return shocks. Because of their significant shares in the total trading volume in Canada, the TGARCH effects in Toronto and Vancouver house prices lead the national house price to exhibit negative TARCH effect as well.

[8] It is common to have $D_{t-1} = 1$ if $\varepsilon_{t-1} < 0$ and $D_{t-1} = 0$ otherwise. In Stata, however, the dummy is defined on positive lagged-errors, following the form of GJR-GARCH (Glosten, Jagannathan, Runkle 1993).

[9] If the house return is defined in levels rather than in log difference, the threshold coefficient is much more significant at 1% for Canada, Toronto and Vancouver.

3.9.2 EGARCH effects

I also conduct asymmetric GARCH test by employing EGARCH models, Nelson (1991). The EGARCH model has several theoretical superiority than the GARCH model. It can be used to investigate asymmetries and the conditional variance is always positive (Asteriou and Hall, 2007). Michayluk, Wilson, and Zurbruegg (2006) and Lee (2008) also demonstrate the asymmetric volatility in real estate markets. Moreover, Engle and Ng (1993) and Stevenson (2002) have found evidence that EGARCH models offer more intuitively appealing results and perform surprisingly well in stock and real estate markets. The model is given by:

$$\log \sigma_t^2 = \omega_0 + \sum_{i=1}^{P} \delta_i g(z_{t-i}) + \sum_{j=1}^{Q} \eta_j \log \sigma_{t-j}^2, \tag{3.8}$$

where

$$\begin{aligned} g(z_t) &= \theta z_t + \gamma(|z_t| - \sqrt{2/\pi}) \\ z_t &= \frac{\varepsilon_t}{\sigma_t}. \end{aligned}$$

The interested parameter is θ. A significant positive θ indicates an asymmetric effect such that positive (negative) shocks increase (decrease) conditional volatility. Table 3.9 depicts the estimation results.

Consistent with the estimation from GJR-GARCH model, Canada and Toronto show negative TARCH effects. Vancouver, however, does not have a significant TARCH effect in its EGARCH model. Interestingly, an EGARCH(1,1) model fits Winnipeg best and the series exhibits a significant positive TARCH effect.

3.10 Comparing unconditional variances with GARCH variances

In this section, for each of the cities exhibiting GARCH effects, the variance of the house price over the sample is computed and compared with the highest value of the GARCH series. The results are listed in Table 3.10.

As displayed, the conditional variance GARCH process often leads to much larger variance during volatile periods than the unconditional variance. Thus, if the GARCH is greater than the unconditional variance, the probability of large losses is much larger than what standard mean-variance analysis will suggest. Thus, value-at-risk models should take this into consideration.

3.11 Var model estimation at national level series

To examine the dynamic interactions between house price volatility and key macroeconomic variables, following Hossain and Latif (2009) and Miller and Peng (2006), I use a VAR estimation model developed by Sims (1982). Due to limitation of space, here I focus on national level data, while city level data can also be tested following the same procedure. Note that the series are all in log difference terms.

Table 3.11 summarizes autocorrelation of the variables under study and their pairwise correlations. The table shows high magnitude of positive first-order autocorrelations for all variables except changes in the mortgage rate and the house sales volume. The volatility of house price appreciation exhibits very high positive autocorrelations for all four orders. The population growth rate and the change in inflation demonstrate a high positive fourth-order autocorrelation.

3.11.1 VAR model

The following VAR model has been estimated.

$$\begin{pmatrix} Y_t \\ VOLTY_t \end{pmatrix} = \begin{pmatrix} A_k \\ a_k \end{pmatrix} + \begin{pmatrix} \sum_{k=1}^{p} B_k Y_{t-k}^{+} \\ \sum_{k=1}^{p} b_k Y_{t-k}^{+} \end{pmatrix} + \begin{pmatrix} \sum_{k=1}^{p} C_k Y_{t-k}^{-} \\ \sum_{k=1}^{p} c_k Y_{t-k}^{-} \end{pmatrix} + \begin{pmatrix} \sum_{k=1}^{p} D_k \text{VOLTY}_{t-k} \\ \sum_{k=1}^{p} d_k \text{VOLTY}_{t-k} \end{pmatrix} + \begin{pmatrix} U_t \\ u_t \end{pmatrix}, \tag{3.9}$$

where Y_t is a vector of $GDPG_t$, $CPIG_t$, $POPG_t$, $MRTC_t$, HPG_t, and HSG_t; Y_t^+ (Y_t^-) is a vector of positive (negative) values of Y_t or 0; A_k, B_k, C_k and D_k are the vector of coefficients and U_k is a vector of error terms that is orthogonal to the space spanned by the explanatory variables. $VOLTY_t$ is the estimated quarterly volatility series by fitting the unpredictable components of house price appreciation rate from the ARMA model to GARCH model mentioned above. The lower-case coefficients are the corresponding coefficients in the equation in which $VOLTY_t$ is the dependent variable. The Schwarz information criterion suggests that the lag length for VAR is one. I use the ordering of variables as following: $MRTC^+$, $CPIG^-$, HSG^-, HSG^+, HPG^-, HPG^+, $VOLTY$, $GDPG^-$, $CPIG^+$, $POPG$, $MRTC^-$ and $GDPG^+$. Note that variables that are expected to have predictive power for other variables are put first. Table 3.12 reports the estimated coefficients of the equation for house price volatility.

The adjusted R^2 is 0.9155. This indicates that the lagged values of the variables may explain a large proportion of the house price volatility. The coefficients of positive mortgage rate change, positive and negative house price change, volatility and population growth are significant, *i.e.* fluctuations in these macroeconomic variables can result in house price volatility change.

3.11.2 Granger causality test

In this section, I use Granger causality test to investigate the factors that determine house price appreciation volatility and also to identify the effect of volatility on other variables. Table 3.13 reports the test results.

According to the results in Table 3.13, the volatility of house price appreciation is significantly affected by price appreciation (both positive and negative), mortgage rate (positive) and change in sales volume (positive). In addition, population growth has effect on price appreciation volatility at 10% significant level. On the other hand, volatility affects house price appreciation (both positive and negative), sale growth rate (negative), GDP growth rate (positive) and inflation (both positive and negative).

3.11.3 Variance decomposition (VDC) analysis

The VDC analysis illustrates how much of the forecast error variance of house price volatility can be explained by exogenous shocks to other variables. The VDC technique decomposes the total variance of the volatility in each of the future periods and determines how much of this variation each macroeconomic variable can explain. Before estimating VDC, the innovations are orthogonalized by the Choleski decomposition method in which the endogenous variables are specified following the aforementioned ordering. This is equivalent to imposing a recursive VAR model. The VDC results of one standard deviation (SD) shock to the variables are presented in Table 3.14 up to 10 quarters.

The VDC results indicates that the disturbance originating from the positive house price appreciation explains 40.22% of the variation in house price volatility after four quarters. Even after 10 quarters it explains 44.21% of the variation in house price volatility. Current volatility accounts for 37.65% after 4 quarters and 27.58% after 10 quarters of the total variation in house price volatility. Thus, the positive house price appreciation and current volatility appear to be the most important variables that cause changes in house price volatility. These two variables account nearly 80% of the variation in house price volatility after four quarters. The high percentage of variance explained by volatility is a reflection of the aforementioned residual clustering. Consistent with the defensive behaviour of house price, the negative house price appreciation does not account much for changes in volatility.

3.12 Price adjustment

Finally, I report in Table 3.15 the statistics for house price adjustment in the Canadian cities studied above. Interestingly, positive price changes are more frequent than negative changes in all studied cities. In all cities except Vancouver, the magnitude of price change is on average higher for positive change than for negative change.

3.13 Conclusion

In this chapter, I analyze the volatility clustering of the house price in eleven Canadian cities. The nation and nine cities exhibit non-constant variance during 1981 - 2014. Conditional variance directly affects house price returns in London-St. Thomas as well as Canada. The price return itself affects volatility for the whole Canada and for three cites: London-St. Thomas, Mississauga and Vancouver. In Mississauga and Canada, higher returns increase the conditional variance, while the effect is opposite in London-St. Thomas and Vancouver. The asymmetric volatility of price return is tested by employing GJR-GARCH model and EGARCH model. Canada and Toronto show negative TGARCH effects in both GJR-GARCH and EGARCH tests. Vancouver exhibits positive TGARCH effect in GJR-GARCH test but no such effect in EGARCH test. By contrast, the positive TGARCH effect is demonstrated for Winnipeg in EGARCH test, but not in GJR-GARCH test. For all cities with ARCH effects, the conditional variance GARCH process often leads to much larger variance during volatile periods than the unconditional variance.

I also examine the dynamic interaction between house price volatility and key macroeconomic variables. VAR, Granger causality and variance decomposition (VDC) analyses demonstrate that house price volatility is significantly affected by house price appreciation, house sales growth rate and population growth rate. In the other direction, volatility affects GDP growth rate, house price appreciation rate, sales growth rate and volatility itself. Finally, I examine the statistics for house price adjustment and find that positive price changes are more frequent and larger than negative changes in most of the cities studied.

3.14 Tables and Figures

Table 3.1: Descriptive Statistics

City	Mean	Maximum	Minimum	Std.dev.	Skewness	Kurtosis
Canada	0.0056	0.0713	-0.0858	0.0269	-0.4782	4.4767
Calgary	0.0039	0.1237	-0.0884	0.0309	0.6326	5.5522
Edmonton	0.0033	0.1198	-0.0558	0.0281	1.0144	5.5815
Fraser Valley	0.0037	0.1173	-0.1317	0.0397	-0.9127	5.7239
Hamil-Burling	0.0075	0.0768	-0.0987	0.0264	-0.4123	4.5915
London-Thomas	0.0040	0.1075	-0.0941	0.0280	-0.0700	5.3283
Mississauga	0.0066	0.2687	-0.2355	0.0473	0.3588	14.0447
Ottawa	0.0061	0.0638	-0.0526	0.0206	0.1539	3.5225
Toronto	0.0071	0.0990	-0.0840	0.0315	0.2612	3.9998
Vancover	0.0054	0.1163	-0.1347	0.0429	-0.8326	4.1670
WindsorEssex	0.0019	0.0926	-0.1370	0.0334	-0.6526	6.3827
Winnipeg	0.0054	0.1101	-0.0706	0.0266	0.0560	4.8163

Table 3.2: Unit Root Test

City	ADF test	PP test	City	ADF test	PP test
Calgary	-5.587 (0.000)***	-6.218 (0.000)***	Ottawa	-6.785 (0.000)***	-10.478 (0.000)***
Edmonton	-4.348 (0.000)***	-6.447 (0.000)***	Toronto	-5.160 (0.000)***	-7.854 (0.000)***
Fraser Valley	-6.499 (0.000)***	-7.234 (0.000)***	Vancover	-6.088 (0.000)***	-7.786 (0.000)***
Hamil-Burling	-5.536 (0.000)***	-8.294 (0.000)***	WindsorEssex	-6.280 (0.000)***	-11.890 (0.000)***
London-Thomas	-6.439 (0.000)***	-12.274 (0.000)***	Winnipeg	-6.082 (0.000)***	-12.628 (0.000)***
Mississauga	-7.882 (0.000)***	-13.417 (0.000)***			

Note: A constant is included in the unit-root test, i.e. trend is assumed in the original non-difference series. MacKinnon approximate p-values are reported in parentheses. One lagged difference term is included. *, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively.

Table 3.3: ARMA model specification

City	AR lags	MA lags
Canada	1	0
Calgary	1	0
Edmonton	3	0
Fraser Valley	1	0
Hamil-Burling	5	0
London-Thomas	2	1
Mississauga	1	0
Ottawa	1	0
Toronto	1	0
Vancover	1	0
Windsor-Essex	1	3
Winnipeg	2	1

Table 3.4: LM test for ARCH effects

City	LM test	p-value
Canada	14.490	0.002***
Calgary	5.927	0.115
Edmonton	12.125	0.007***
Fraser Valley	14.190	0.002***
Hamil-Burling	3.128	0.372
London-Thomas	9.647	0.022**
Mississauga	27.038	0.000***
Ottawa	6.378	0.095*
Toronto	16.576	0.001***
Vancover	8.624	0.035**
WindsorEssex	34.862	0.000***
Winnipeg	19.084	0.000***

Notes: LM test with three lags were performed, Lag(4) and Lag(5) are also computed for comparison. *, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively.

Table 3.5: GARCH estimation: Canadian cities

City/Coef.	Model	$\sum_{i=1}^{P} \delta_i$	$\sum_{j=1}^{Q} \eta_j$
Canada	GARCH(1,1)	0.2867***	0.6616***
Edmonton	GARCH(1,1)	0.4953***	0.4464***
Fraser Valley	GARCH(1,1)	0.4523**	0.3903*
London-Thomas	GARCH(1,3)	0.0751***	0.9410***
Mississauga	GARCH(1,2)	0.5146***	-0.3059**
Ottawa	GARCH(1,4)	0.3940***	0.0748**
Toronto	GARCH(1,1)	0.4308***	0.5487***
Vancover	GARCH(1,1)	0.1804	0.6692***
WindsorEssex	GARCH(1,1)	0.1765***	0.7722***
Winnipeg	GARCH(2,2)	0.1624***	0.7902***

The optimal lag length in the variance equation is selected by using the Schwarz criterion.The summation of significant ARCH/GARCH term's coefficients is reported. Note: the corresponding ARMA model coefficients are not reported in this table. *, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively.

Table 3.6: GARCH-M estimation: Canadian cities

City	Model	Est. γ
Canada	GARCH-M(1,1)	-15.8440**
Edmonton	GARCH-M(1,1)	-1.2752
Fraser Valley	GARCH-M(1,1)	-5.0571
London-Thomas	GARCH-M(1,1)	-11.1540*
Mississauga	N/A	N/A
Ottawa	GARCH-M(1,1)	-0.9894
Toronto	GARCH-M(1,1)	-4.6592
Vancover	GARCH-M(1,1)	-0.3904
WindsorEssex	GARCH-M(1,1)	-1.5922
Winnipeg	GARCH-M(2,1)	5.7415

*, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively. "N/A" represents that a sensible GARCH-M model could not be estimated due to a lack of convergence in the estimates.

Table 3.7: Augmented GARCH estimation: Canadian cities

City	Model	Estimated ζ
Canada	A-GARCH(1,1)	0.0014*
Edmonton	A-GARCH(1,1)	0.0020
Fraser Valley	A-GARCH(1,1)	0.0007
London-Thomas	A-GARCH(1,1)	-0.0019**
Mississauga	A-GARCH(1,1)	0.0079***
Ottawa	A-GARCH(1,1)	0.0012
Toronto	A-GARCH(1,1)	0.0020
Vancover	A-GARCH(1,1)	-0.0047*
WindsorEssex	A-GARCH(1,1)	0.0000
Winnipeg	A-GARCH(2,2)	-0.0011

*, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively.

Table 3.8: GJR-GARCH estimation: Canadian cities

City	Model	Estimated λ
Canada	GJR-GARCH(1,2)	0.2425*
Edmonton	GJR-GARCH(1,1)	-0.1740
Fraser Valley	GJR-GARCH(1,1)	0.4056
London-Thomas	N/A	N/A
Mississauga	N/A	N/A
Ottawa	GJR-GARCH(1,1)	0.5475
Toronto	GJR-GARCH(1,1)	0.3867*
Vancover	GJR-GARCH(1,1)	0.2493*
WindsorEssex	GJR-GARCH(1,1)	0.0095
Winnipeg	GJR-GARCH(1,2)	-0.2548

Note: The optimal lag length in the variance equation is selected by using the Schwarz criterion. *, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively. "N/A" represents that a sensible GJR-GARCH model could not be estimated due to a lack of convergence in the estimates.

Table 3.9: EGARCH estimation: Canadian cities

City	Model	Estimated θ
Canada	EGARCH(1,3)	0.1445***
Edmonton	EGARCH(1,2)	-0.1136
Fraser Valley	EGARCH(1,1)	0.0669
London-Thomas	EGARCH(1,1)	-0.0257
Mississauga	EGARCH(1,3)	-0.0101
Ottawa	EGARCH(1,1)	0.1742
Toronto	EGARCH(1,1)	0.1648*
Vancover	EGARCH(1,1)	0.0165
WindsorEssex	EGARCH(1,1)	-0.0401
Winnipeg	EGARCH(1,1)	-0.1091*

Note: The optimal lag length in the variance equation is selected by using the Schwarz criterion. *, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively.

Table 3.10: Unconditional variance and GARCH variance, $\times 10^{-3}$

City	Variance	GARCH
Canada	0.7223	4.4986
Edmonton	0.7887	4.5173
Fraser Valley	1.5767	9.756
London-Thomas	0.786	1.8291
Mississauga	2.241	37.6324
Ottawa	0.4235	1.6518
Toronto	0.9896	5.8299
Vancover	1.8439	4.5288
WindsorEssex	1.114	6.9358
Winnipeg	0.7072	3.2451

The first column shows the unconditional variance of the price index for each city and the second shows the highest value that the GARCH series, attained over the sample.

Table 3.11: Autocorrelation and pairwise correlation

Lags/Variables	$GDPG_t$	$CPIG_t$	$POPG_t$	$MRTC_t$	HPG_t	HSG_t	$VOLTY_t$
Autocorrelation							
Lag 1	0.3870	0.4411	0.1974	-0.1234	0.3407	-0.098	0.8859
Lag 2	0.0929	0.1333	-0.2727	-0.3228	0.0468	-0.3026	0.7546
Lag 3	0.0388	0.2321	0.1510	-0.0381	0.1105	0.1540	0.6596
Lag 4	0.0161	0.3549	0.8398	0.2033	-0.0451	-0.3063	0.5818
Pairwise Correlations							
$GDPG_t$	1						
$CPIG_t$	-0.2632	1					
$POPG_t$	0.0242	-0.0939	1				
$MRTC_t$	0.2113	-0.4492	0.3282	1			
HPG_t	0.2574	-0.2645	0.0193	0.0106	1		
HSG_t	0.0593	0.0606	-0.0426	-0.3079	0.4447	1	
$VOLTY_t$	-0.2215	0.2973	0.3828	0.0390	-0.1517	-0.0263	1

Table 3.12: Determinants of house price volatility, VAR volatility equation

Variables	Coef	Std. Err	z	P>z	[95% Conf.	Interval]
$MRTC^+_{t-1}$	-0.0085	0.0035	-2.4500	0.0140	-0.0153	-0.0017
$CPIG^-_{t-1}$	0.0035	0.0110	0.3200	0.7510	-0.0180	0.0250
HSG^-_{t-1}	-0.0002	0.0004	-0.5600	0.5750	-0.0009	0.0005
HSG^+_{t-1}	0.0004	0.0003	1.4000	0.1630	-0.0002	0.0010
$RHPG^-_{t-1}$	-0.0120	0.0013	-9.3700	0.0000	-0.0145	-0.0095
$RHPG^+_{t-1}$	0.0102	0.0010	10.3800	0.0000	0.0083	0.0121
$VOLTY_{t-1}$	0.7615	0.0325	23.4200	0.0000	0.6978	0.8252
$GDPG^-_{t-1}$	-0.0007	0.0031	-0.2100	0.8320	-0.0068	0.0055
$CPIG^+_{t-1}$	-0.0045	0.0034	-1.3100	0.1910	-0.0111	0.0022
$POPG_{t-1}$	0.0317	0.0185	1.7100	0.0870	-0.0046	0.0679
$MRTC^-_{t-1}$	-0.0026	0.0035	-0.7400	0.4580	-0.0095	0.0043
$GDPG^+_{t-1}$	0.0012	0.0024	0.5100	0.6080	-0.0035	0.0060
$Cons$	-0.0002	0.0001	-2.4900	0.0130	-0.0003	0.0000

Table 3.13: Granger Causality Test

Hypothesis	F-stat	P-val	Hypothesis	F-stat	P-val
$GDPG^{+}$ →volty	0.2628	0.608	volty→ $GDPG^{+}$	7.1725***	0.007
$GDPG^{-}$ →volty	0.0449	0.832	volty→ $GDPG^{-}$	0.7805	0.377
$POPG$ →volty	2.9345*	0.087	volty→ $POPG$	0.3898	0.532
$CPIG^{+}$ →volty	1.7136	0.191	volty→ $CPIG^{+}$	6.7218**	0.010
$CPIG^{-}$ →volty	0.1007	0.751	volty→ $CPIG^{-}$	3.6580*	0.056
$MRTC^{+}$ →volty	6.0268**	0.014	volty→ $MRTC^{+}$	0.0310	0.860
$MRTC^{-}$ →volty	0.5516	0.458	volty→ $MRTC^{-}$	0.4578	0.499
HPG^{+} →volty	107.84***	0.000	volty→ HPG^{+}	5.6109**	0.018
HPG^{-} →volty	87.711***	0.000	volty→ HPG^{-}	11.302***	0.001
HSG^{+} →volty	1.949**	0.163	volty→ HSG^{+}	0.6408	0.423
HSG^{-} →volty	0.3147	0.575	volty→ HSG^{-}	9.1029***	0.003

*, ** and *** indicate the parameter estimate is significant at 10%, 5% and 1% level, respectively.

Table 3.14: Variance decomposition of house price volatility (percentage)

Variable	1	2	3	4	5	6	7	8	9	10
$MRTC^{+}$	1.34	1.03	0.71	0.53	0.44	0.39	0.36	0.35	0.33	0.33
$CPIG^{-}$	1.84	0.93	0.94	0.75	0.62	0.55	0.51	0.49	0.47	0.46
HSG^{-}	0.60	2.17	1.91	1.64	1.47	1.37	1.30	1.26	1.22	1.20
HSG^{+}	0.00	6.02	5.74	5.20	4.72	4.33	4.05	3.85	3.71	3.62
HPG^{-}	0.49	5.93	5.87	5.47	5.42	5.56	5.73	5.87	5.97	6.03
HPG^{+}	0.27	23.73	35.59	40.22	42.08	42.95	43.46	43.80	44.04	44.21
$VOLTY$	95.45	59.12	44.69	37.65	33.69	31.30	29.77	28.75	28.06	27.58
$GDGP^{-}$	0.00	0.01	1.36	3.59	5.56	7.01	7.98	8.61	9.03	9.31
$CPIG^{+}$	0.00	0.18	0.28	0.23	0.19	0.17	0.16	0.16	0.16	0.16
$POPG$	0.00	0.70	2.70	4.42	5.45	6.01	6.33	6.53	6.66	6.76
$MRTC^{-}$	0.00	0.12	0.17	0.19	0.19	0.17	0.16	0.15	0.15	0.14
$GDPG^{+}$	0.00	0.07	0.05	0.11	0.16	0.18	0.19	0.20	0.20	0.20

Table 3.15: Price asymmetric adjustment: Canadian cities

City/Stat.	Pos. Change	Mean of Pos.	Mean of Neg
Canada	61.53%	0.0212	-0.0193
Calgary	54.96%	0.0238	-0.0203
Edmonton	52.67%	0.0219	-0.0174
Fraser Valley	56.49%	0.0282	-0.0281
Hamil-Burling	65.65%	0.0219	-0.0199
London-Thomas	59.54%	0.0204	-0.0201
Mississauga	55.73%	0.0326	-0.0262
Ottawa	64.89%	0.0172	-0.0143
Toronto	64.07%	0.0252	-0.0212
Vancover	59.54%	0.0316	-0.0332
WindsorEssex	53.44%	0.0238	-0.0233
Winnipeg	59.54%	0.0216	-0.0184

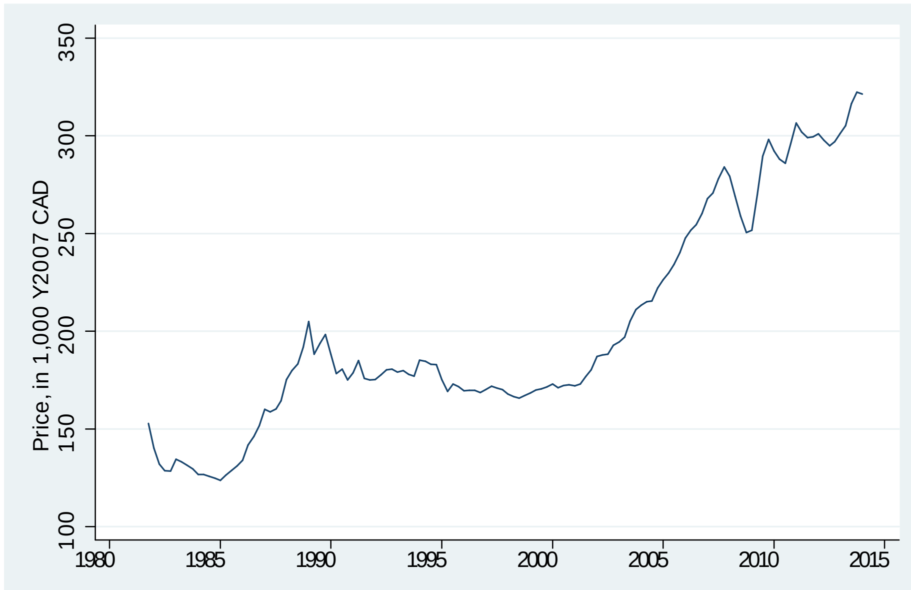

Figure 3.1: Aggregate house price in Canada, in $1,000 Y2007 CAD.

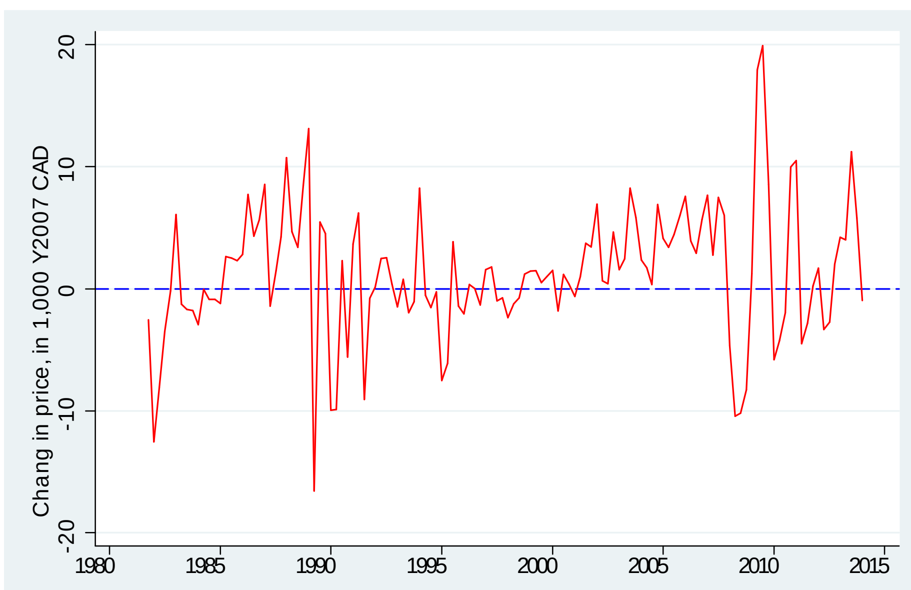

Figure 3.2: House price change in Canada, in $1,000 Y2007 CAD.

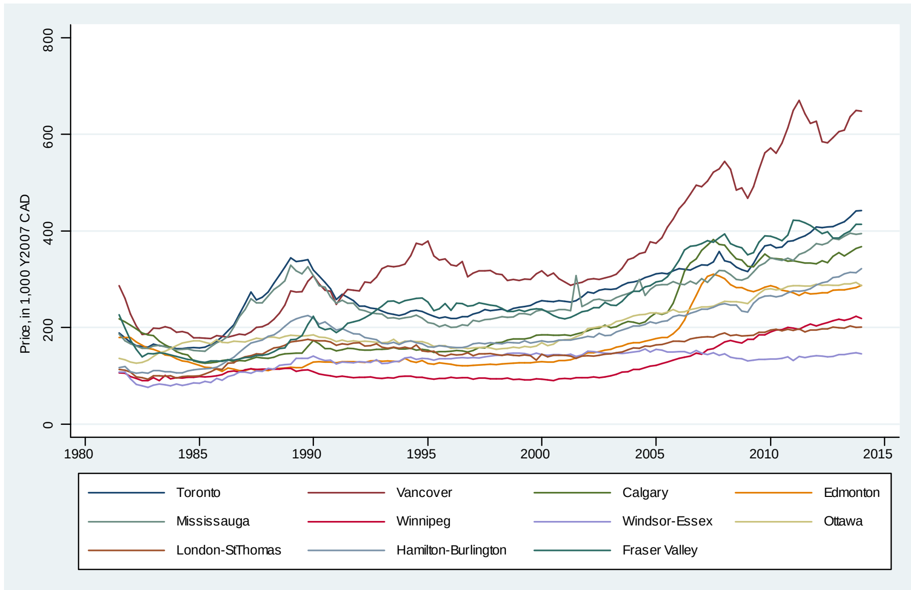

Figure 3.3: Price of major Canadian cities, in $1,000 Y2007 CAD.

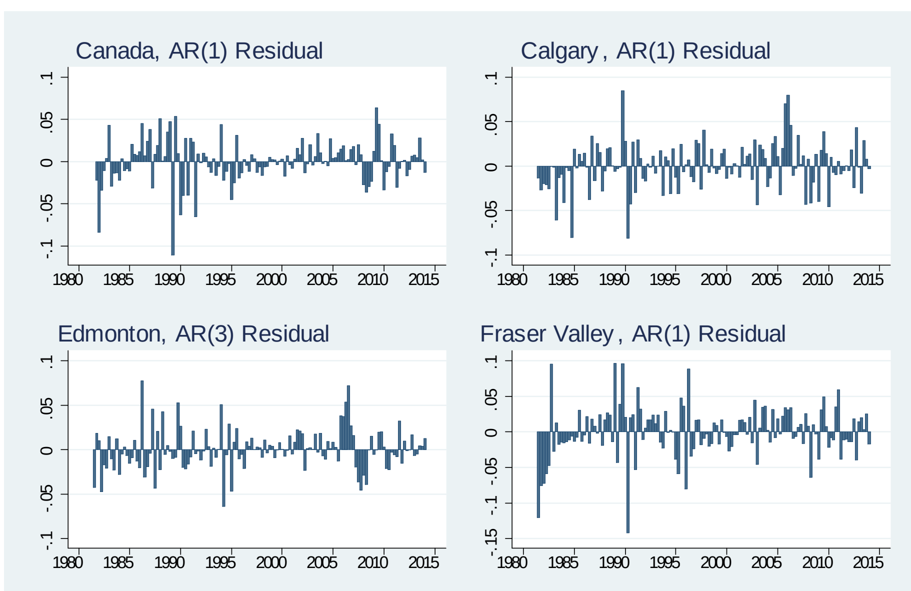

Figure 3.4: HPG_t ARMA Residual Clustering

Figure 3.5: HPG_t ARMA Residual Clustering

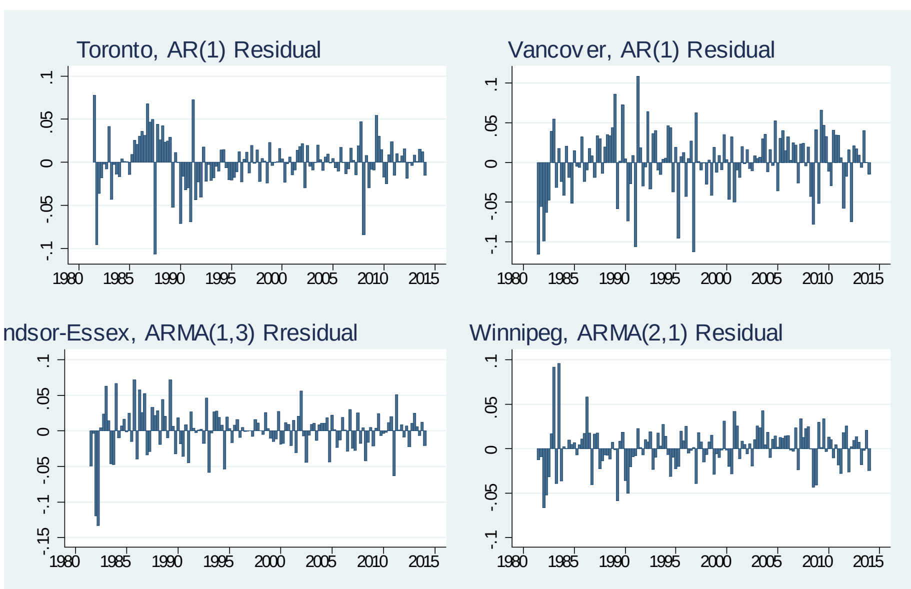

Figure 3.6: HPG_t ARMA Residual Clustering

Chapter 4

Asymmetric House Price Adjustment and Search With Indebted Sellers

4.1 Introduction

In this chapter, I develop a static model of house search with indebted sellers to illustrate asymmetric house price adjustment. Heterogeneity of sellers leads to dispersion of asking prices. A negative shock to the reference price[1] of houses makes sellers less willing to lower their asking prices because of falling house equity, while a positive shock gives sellers more incentive to sell faster because of rising equity. I name this the "equity effect" in determining sellers' asking prices. Numerical analyses demonstrate that positive (average) house price changes are larger and more likely to reflect the underlying shocks than negative price changes in the model.

In many markets, prices rise faster than they fall, *i.e.* displaying the so called "rocket and feather" phenomenon. This pattern of asymmetric price adjustment has

[1] To simplify analysis, I abstract from modeling fundamental factors, such as household income, construction cost, *etc.* Instead, I assume a reference price to reflect the fundamental economic factors. It can be interpreted as the construction cost or the buying price for real estate agents, see Hedlund (2014).

been reported in a broad range of product markets[2]. Despite the abundant empirical evidence on asymmetric price adjustment, there is little theoretical work plausibly explaining this phenomenon.

The first attempt to explain why prices adjust asymmetrically to cost shocks is tacit collusion. Based on Tirole (1988), Borenstein, Cameron, and Gilbert (1997) argue that because sellers take the old output price as a natural focal point, prices are downward sticky when input costs drop. Tacit collusion, however, is not likely to happen in more competitive markets, according to Peltzman (2001).

Several recent theoretical papers, such as Bayer and Ke (2011), Yang and Ye (2008), Tappata (2009), Lewis and Marvel (2011), Lewis (2011) and Cabral and Fishman (2012), attempt to explain asymmetric price adjustment in search models. Common in these models, consumer search causes equilibrium price dispersion and some heterogeneity of consumers (in terms of their search costs) causes different search behaviors upon positive and negative cost shocks. These two factors can generate the "rocket and feather" phenomenon.

Another type of models that has the potential to explain the asymmetric price adjustment phenomenon is based on the prospect theory, which was initially proposed by Kahneman and Tversky (1979). Shefrin and Statman (1985) use the "disposition effect" to describe the tendency of investors to sell shares with prices that increased while retaining assets that dropped in value. Dobrynskaya (2008) proposes a behavioral model, in which trader's utility is characterized by loss aversion, to explain

[2]Karrenbrock (1991), Bacon (1991), Duffy-Deno (1996), Borenstein, Cameron, and Gilbert (1997), Eckert (2002), and Galeotti, Lanza, and Manera (2003) study the gasoline markets across developed economies. Hannan and Berger (1991) and Neumark and Sharpe (1992) find asymmetric adjustment in bank's mortgage rates. In a study over 77 consumer and 165 producer goods, Peltzman (2001) significantly broadens the evidence for this asymmetrical price behavior. In addition, Engelhardt (2003) and Genesove and Mayer (2001) have demonstrated that the downward price rigidity also exists in the housing market.

several stylized facts of asymmetric price rigidity.

In a perfect competitive housing market, price adjustment is instantaneous and symmetric in response to positive and negative shocks. However, if search and matching frictions present, the adjustment pattern may be different. When buyers' search is directed by sellers' asking prices, it is possible for sellers to trade off the asking price and the sales rate. Depending on their equity levels, sellers' trade-off strategies may be asymmetric in response to positive and negative shocks. This can lead to the observed defensive behavior of the house price.

In the following economy with search frictions and indebted sellers, what generates asymmetric price adjustment is the following "equity effect". Define a seller's loss upon failing sale[3] to be the difference between her/his values upon successful and failing sales. A seller's loss upon failing sale is increasing in her/his equity on the house. Sellers with low house equity are less eager to sell and more likely to bet for higher prices (at lender's cost of loss). By contrast, with higher house equity, sellers are more anxious to sell and more likely to ask lower prices in exchange for a better chance to sell in order to avoid large loss upon failing sale.

Consider a scenario in which the housing market suffers a negative shock that induces a decrease on the reference house price. Sellers' house equity deteriorates and their losses upon failing sales decrease. Due to the equity effect, sellers become reluctant to reduce their asking prices at the same pace as the reference price. Thus, average house price adjustment is small and slow. By contrast, with a positive shock that induces the reference price to rise, sellers have larger house equity and their losses upon failing sales increase. Due to the equity effect, sellers become eager to sell and the average probability of selling rises. The magnitude of the price change,

[3] "Failing sale" here refers to the case that a seller fails to find a buyer to trade with.

however, is ambiguous. On one hand, sellers would not increase their asking price too much at the cost of sales rate. On the other hand, if the positive shock also incurs more demand such that the sales rates for all asking prices become higher than before, sellers are able to ask higher prices without large decrease in the probability of selling. If the latter effect is large enough, the average trading price may rise at a even higher rate than the reference price does. Thus, the average house price responds quickly and often largely to positive demand shocks.[4]

The equity effect driven asymmetric house price adjustment may be a good candidate to explain the empirical evidence of informativeness of the house price. Dufrénot and Malik (2012) analyze the relationship between house price development and the business cycle. Employing a time-varying transition probability Markov switching framework, they provide empirical evidence that house price growth may be a useful leading indicator for turning-point detection in the U.S., U.K., and Spain. However, they locate evidence of an asymmetric significance of the information obtained from house price development in business cycle states. They find that during a recession, house prices are not informative. In a recession (at least at the start of the recession), the house price adjusts slowly. The change is less and cannot deliver much information about the underlying economic environment changes. In a booming period, by contrast, the housing market is more liquid and the change in price can clearly reflect the shocks.

In Section 4.2, I describe the model environment. Section 4.3 studies agent problems. In Section 4.4, I conduct numerical analyses to demonstrate how asymmetric house price adjustment appears in the model due to the equity effect. Section 4.5

[4] In the short run, shocks to housing market are more likely from the demand side rather than from the supply side.

concludes. The last section lists figures mentioned in this chapter.

4.2 Model environment

This section presents a static house search model, which is a simplified version of the model in Chapter 2. The model is used to demonstrate the aforementioned equity effect on the house price in response to shocks to the reference price. The environment is as follows.

It is a one-period closed economy. Before entering the housing market, the measure of sellers is S. Sellers are different in their mortgage debt[5], $d \in D$. The measure of sellers on D follows distribution $g(d)$, where $g(d)$ is the probability distribution function (PDF).

Houses are identical and indivisible. Each household can hold at most one unit of house. The housing market is characterized by directed search, *i.e.* sellers post asking prices to attract buyers. The directed search mechanism implies a trade-off between the asking price and the probability of selling: a seller who asks a lower price gets compensated by enjoying a higher chance of sale. Abstracting from modeling buyers' behavior, I assume the following function $m : p \to [0, \omega]$ that maps a seller's asking price to the probability of selling:

$$m(p; P) = \begin{cases} \omega & \texttt{if } p \leq (1-\kappa_s)P \\ \omega\left(1 - \left(1 - \frac{P-p}{\kappa_s P}\right)^{\upsilon}\right) & \texttt{if } (1-\kappa_s)P < p \leq P \\ 0 & \texttt{if } p > P \end{cases} \tag{4.1}$$

where κ_s is the maximum selling discount and P is the reference price. $\omega \in (0, 1]$

[5] In my view, mortgage debt is an important source that causes sellers' distinction in their reservation values, which are used in typical search literature on housing markets.

and $\upsilon > 0$ are parameters. The search friction in the housing market is akin to an endogenous adjustment cost. If sellers ask a price equal to the reference price minus the adjustment cost $\kappa_s P$, their houses sells with probability ω. Otherwise, if they post a higher price, $m(p; P)$ is less than ω. Sellers cannot get in trades with $p > P$, *i.e.* $m(p; P) = 0$. It is easy to check that $\partial m(\cdot)/\partial P > 0$. This reflects an implicit assumption: positive (negative) shocks that cause the reference price P to rise (fall) also incur more (less) demand such that $m(p; P)$ shifts to the up (down).

After the housing market closes, there are two types of sellers: (i) matched sellers with values $V_{SM}(p, d)$; (ii) unmatched sellers with values $V_{SU}(d)$. Functions $V_{SM}(p, d)$ and $V_{SU}(d)$ take the following linear forms:

$$\begin{aligned} V_{SM}(p, d) &= V_B + p - d \\ V_{SU}(d) &= \underline{V} \end{aligned}$$

I assume that a seller receive $\underline{V}$ upon a failing sale no matter the level of his/her mortgage debt d.

4.3 Agent problems

The timing is as follows. At the beginning of the period, a shock to the reference price may hit the economy and is observed by all sellers. Then the housing market opens. Sellers optimally post asking prices to attract buyers for a trade. After the market closes, Sellers receive values according to their status.

A seller with mortgage debt d solves the following problem:

$$\begin{aligned} W(d;P) &= \max_p \Big\{ m(p;P)V_{SM}(p,d) + \Big(1 - m(p;P)\Big)V_{SU}(d) \Big\} \\ &= \max_p \Big\{ m(p;P)\Big(V_{SM}(p,d) - V_{SU}(d)\Big) + V_{SU}(d) \Big\}. \end{aligned}$$

Then sellers' policy function $f : d \to p$ can be derived according to the first order condition:

$$m'(p;P)\Big(V_{SM}(p,d) - V_{SU}(d)\Big) + m(p;P)\frac{\partial V_{SM}(p,d)}{\partial p} = 0$$

4.4 Numerical analyses

In this section, I conduct numerical analysis to demonstrate how the asymmetric house price adjustment takes effect due to the equity effect. The parameters $(\omega, \upsilon, \kappa_s, P, V_B, \underline{V})$ are set as the following:

$\omega = 0.85$	$\upsilon = 2.78$	$\kappa_s = 0.2$	$P = 3.2$	$V_B = 5.0$	$\underline{V} = 0$

The coefficient of $m(p;P)$, ω, is less than one. This implies that sellers cannot sell houses for sure even when they set asking price at the lowest level. The maximum selling discount, κ_s, is set to be 0.2.[6] The value of a seller who failed to sell, $\underline{V}$, is normalized to zero and other parameters are set to be consistent with the steady state values in Chapter 2.

[6]Garriga and Schlagenhauf (2009) use data from the American Housing Survey to find that capital gains shocks fall in a range of 21.9%. Moreover, RealtyTrac's information reveals that the average pre-foreclosure discount is about 20%.

4.4.1 Mortgage debt distributions

First, I examine how the mortgage debt distribution $g(d)$ affects the average trading price of houses. The maximum of debt is set to be 80% of the reference price.[7] I parameterize the loan balance distribution $g(d)$ as a beta distribution with parameters b_a and b_b scaled to have support on $D = (0, 0.8P]$. This flexible distribution allows me to value the equity effect among different distributions of loan balance by changing the values of b_a and b_b. Figures 4.1 and 4.2 depict the beta distributions for $b_a = 1$ and $b_a = 2$ respectively with various values of b_b. For each value of b_a, the higher the value of the second shape parameter b_b, the more skew to the left the transformed beta distribution, *i.e.* more sellers have higher mortgage debt.

Note the following. First, I do not include the non-zero probability density mass point for $d = 0$ in order to emphasize the effect of household mortgage debt on the response of the average price to shocks. Second, the beta distribution is transformed to skew to the left because refinancing, prepayment and default can all terminate a mortgage before its maturity in the real world.

Figures 4.3 and 4.4 illustrate the average asking price and the average probability of selling as functions of b_b. Larger values of b_b represent economies in which more sellers have higher mortgage debt and less house equity. The average price is monotonically increasing in the value of b_b, while the average probability is monotonically decreasing in b_b. This is caused by the equity effect: sellers with less house equity (higher debt) are more willing to ask for higher prices and sell houses at lower rates.

[7] This reflects the fact that the down-payment is at least 20% of the purchase price for conventional loans.

4.4.2 Reference price shocks

In this section, I demonstrate how the average trading price responds to shocks to the reference price P. The beta distribution used in following analysis is $(b_a = 2, b_b = 4.75)$. Figure 4.5 displays the asking price and the probability of selling as functions of a seller's mortgage debt.

At the beginning of the period, a shock hits the reference price P. As mentioned before, the shock also shifts the probability function $m(p; P)$ to the same direction as the change in P. At the same time, higher (lower) value of P increases (decreases) all sellers' house equity by the same amount. Figure 4.6 depicts how the average trading price respond to shocks on P. The green line is for positive shocks and the blue line is for negative shocks. Note that negative shocks lead to a fall in the average price but the percentage changes in this graph are at absolute values. With positive shock to the reference price P, the average price rises at almost the same magnitude as P. By contrast, when P decreases, the average price falls at a significantly lower pace. In other words, the price exhibits a pattern of defensive characteristics, *i.e.* downward rigidity. In addition, the difference between these two curves is increasing in the magnitude of shocks.

This result is also caused by the equity effect. With positive reference price shocks, sellers have higher house equity and would like to sell houses more quickly. Combined with the upward change in the probability of selling, the house price adjusts quickly and fully reflects the underlining shocks. By contrast, with negative price shocks, sellers' equity deteriorates and they become reluctant to sell because they are better off betting for higher prices at the risk of lenders.

4.5 Conclusion

In this chapter, I develop a static house search model to demonstrate the roles that search frictions and households' mortgage position play in the asymmetric house price adjustment. The search friction makes it possible for sellers to trade off between the asking price and the sales rate. Indebted sellers generate dispersion of asking prices such that an equity effect takes place to generate the defensive characteristics of the average house price.

4.6 Figures

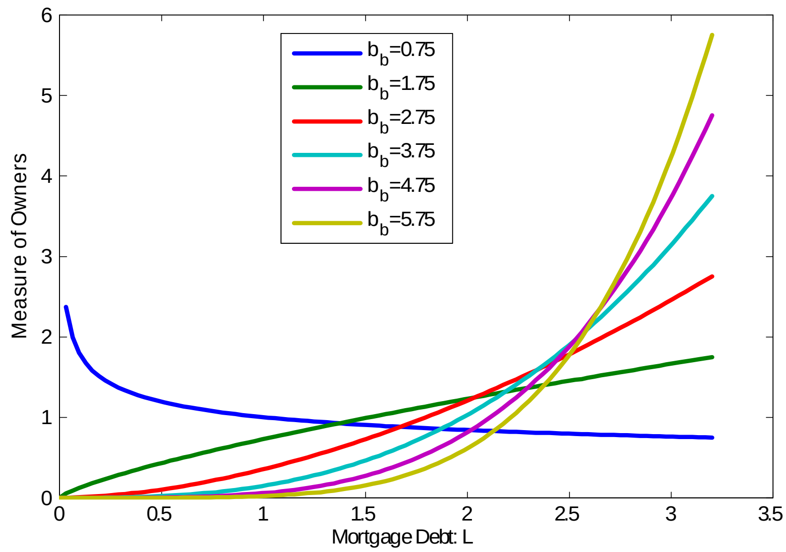

Figure 4.1: Distribution of mortgage debt: transformed Beta distribution with $b_a = 1$ and various b_b.

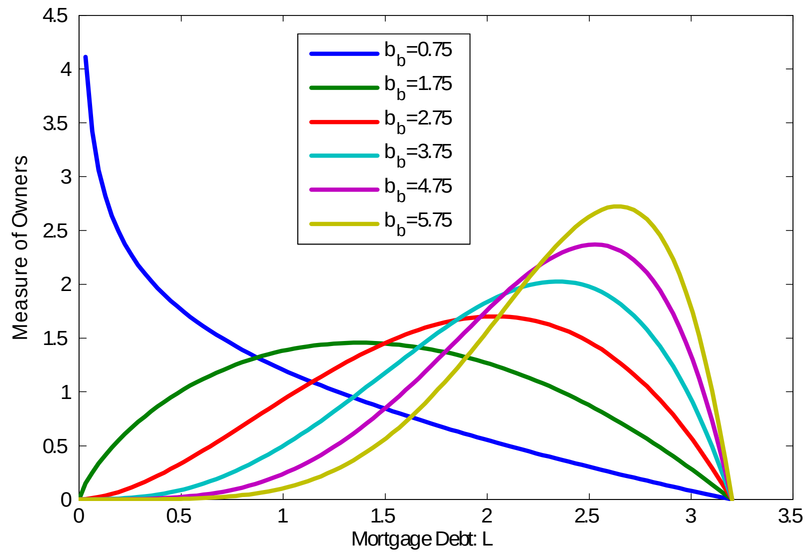

Figure 4.2: Distribution of mortgage debt: transformed Beta distribution with $b_a = 2$ and various b_b.

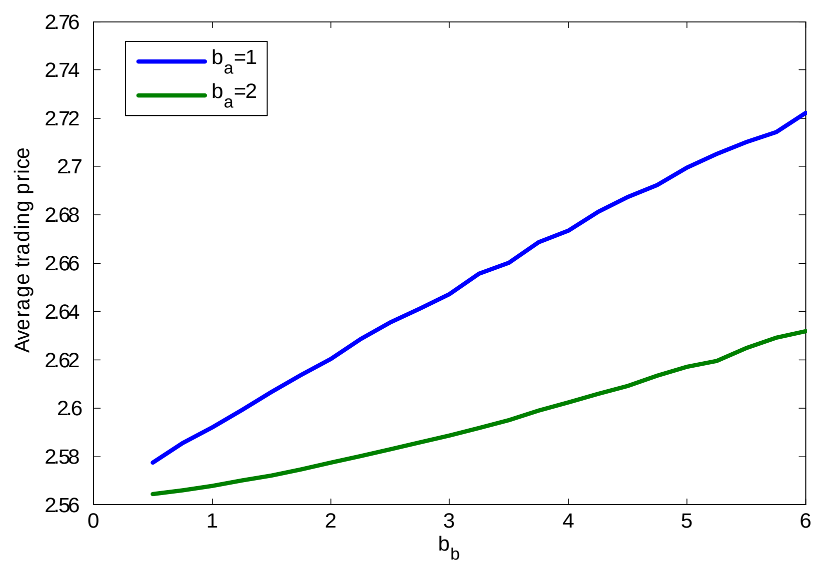

Figure 4.3: Average asking price for loan distributions (Beta $b_a \in \{1, 2\}$, $b_b \in [0.5, 6]$).

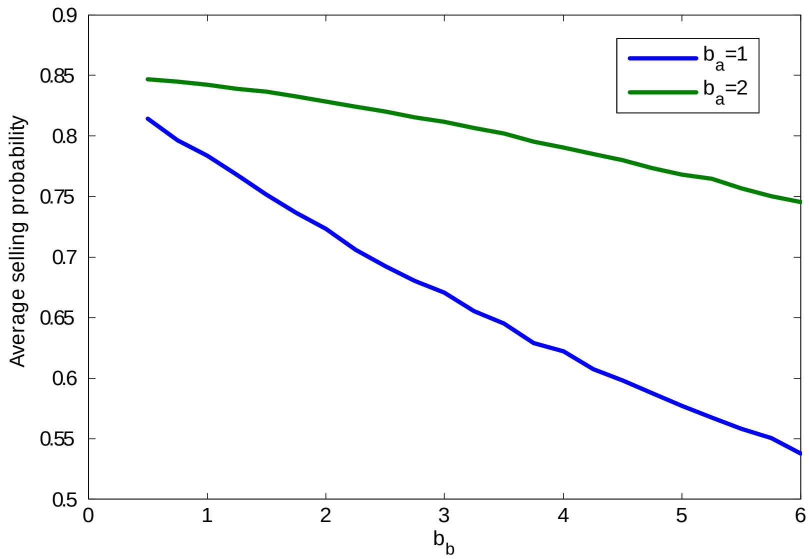

Figure 4.4: The average probability of selling for loan distributions (Beta $b_a \in \{1, 2\}$, $b_b \in [0.5, 6]$).

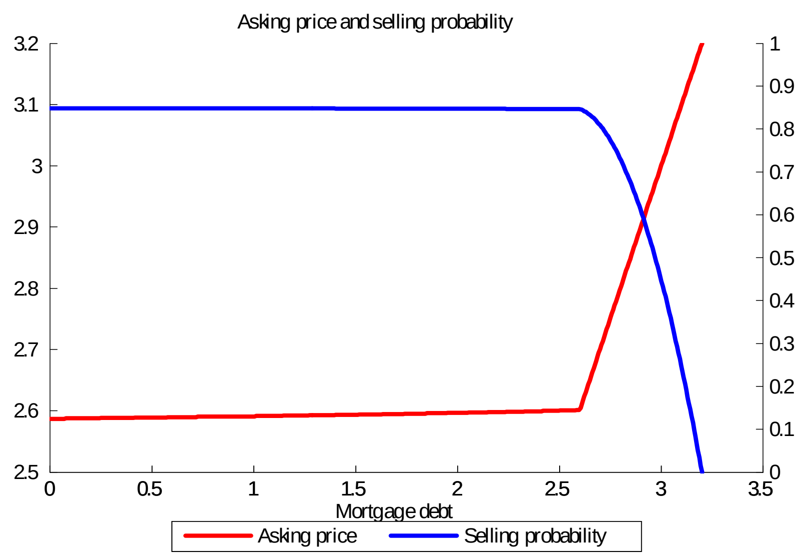

Figure 4.5: The asking price and the probability of selling as functions of seller's mortgage debt. The left vertical axis is for asking price, while the right axis is for the probability of selling.

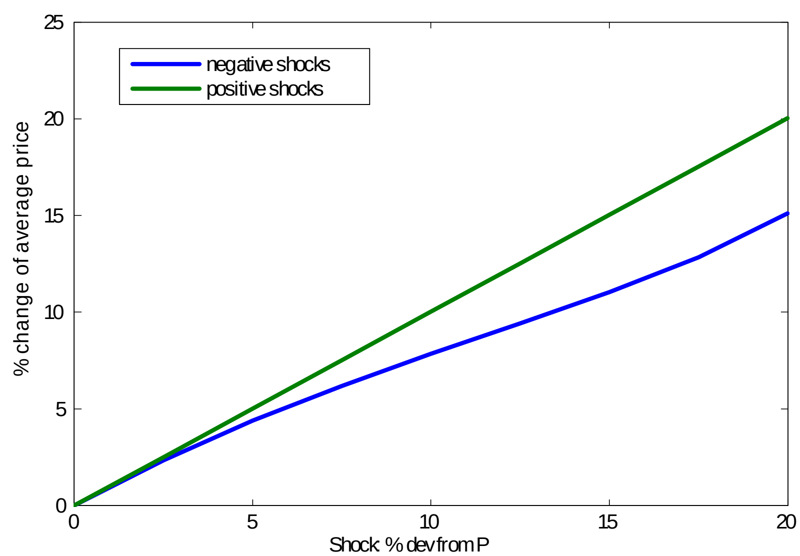

Figure 4.6: The response of average trading price to shocks on the reference price P. The percentage changes of the average price to negative shocks are negative and are shown as absolute values in the graph.

Chapter 5

Conclusion

5.1 Conclusion

This thesis studies how search and matching frictions can explain empirical observations in housing markets. Chapter 2 studies how the liquidity of house trades affects lender's behavior by incorporating mortgage contracts into a dynamic house searching model. I find that housing liquidity affects lending standards by influencing the expected default rate of a mortgage and the mortgage company's expected loss upon default. The model also provides a theory for explaining the stylized fact that the down-payment ratio and the house price are negatively correlated. In addition, a note-worthy feature of the housing search model is that it generates price dispersion for identical houses. It is unusual in the literature so far to study equilibrium house price dispersion in an economy with frictional markets. In summary, Chapter 2 confirms that the interaction between housing market liquidity and lending standards can be much more complex than one may have thought. In particular, it shows that when evaluating the effect of a policy on housing markets, we need to take not only its direct effect but also the feedback from the credit market into consideration.

As is usually the case with models of heterogeneous agents, numerical solution of the model described in Chapter 2 is very challenging. The state vector is of large dimension because of search frictions and endogenously determined terms of multi-period mortgages. For example, to determine terms of a mortgage contract, the mortgage company takes not only past mortgages but also expected future mortgages into consideration because both of them will affect borrowers' behavior, which in turn affects the payoff of the loan in the future. Thanks to recent advances in technology, there are ways to alleviate the computation burden. For instant, taking advantage of the CUDA GPU[1] computing technology, I can dramatically shorten the computation time .

The search model can be extended to study other housing issues. For example, by modeling the mortgage company's problem more sophisticatedly, we can use it to study the supply of different mortgage products in decentralized housing markets. Moreover, by relaxing the assumption of homogenous buyers, the model might be further extended to study households' dynamic choices among various loan contracts.

In Chapter 3, I investigate the volatility of home prices in Canadian cities and document the following observations. First, most major Canadian cities exhibit house price volatility clustering during 1981 - 2014. Second, asymmetric volatility of price return is observed in Toronto, Vancouver, Winnipeg and Canada as a whole. Moreover, VAR, Granger causality and variance decomposition (VDC) analyses demonstrate that house price volatility is significantly affected by house price appreciation, house sales growth rate and population growth rate. On the other hand, volatility

[1] CUDA stands for Compute Unified Device Architecture, it is a parallel computing platform and programming model created by NVIDIA and implemented by the graphics processing units (GPUs) that they produce.

affects GDP growth rate, house price appreciation rate, sales growth rate and volatility itself. Finally, I document the statistics for house price adjustment and find that positive changes in house prices are more frequent and larger than negative changes in most of the studied cities.

These findings are important for guidance in the development of theoretical models of housing markets, particular for those used for policy analysis. Models that fail to reconcile the aforementioned volatility patterns may lead to inferior results. For instant, the conditional variance GARCH process often leads to much larger variance during volatile periods than the unconditional variance. Without awareness of this, a house price stabilizing policy may underreact in booming periods or overreact in recession.

Chapter 4 illustrates that trading frictions may be an important factor accounting for the observed asymmetric adjustment in house prices. The search friction makes it possible for sellers to trade off between the asking price and the sales rate. This leads to the downward rigidity of the house price because the equity effect takes place for sellers with mortgage debt.

A dynamic analysis can be achieved by embedding the static model into the search framework in Chapter 2. A dynamic model would allow us (i) to simulate the model to analyze the asymmetric adjustment pattern in comparison with real data; (ii) to study the informativeness of price change in business cycles; (iii) to tackle the empirical observation of volatility patterns that discussed in chapter 3. This, however, is a substantial undertaking that I leave for future work.

Bibliography

ALBRECHT, J., P. GAUTIER, AND S. VROMAN (2010): "Directed search in the housing market," manuscript, Georgetown University.

AMBROSE, B., AND C. CAPONE (1996): "Do Lenders Discriminate in Processing Defaults," *Cityscape*, 2(1), 89–98.

AMBROSE, B., AND C. CAPONE (1998): "Modeling the Conditional Probability of Foreclosure in the Context of Single-family Mortgage Default Resolutions," *Real Estate Economics*, 26(3), 391–429.

ASTERIOU, D., AND S. G. HALL (2007): *Applied Econometrics: A Modern Approach Using Eviews and Microfit Revised Edition*. Palgrave Macmillan.

BACON, R. W. (1991): "Rockets and feathers: the asymmetric speed of adjustment of UK retail gasoline prices to cost changes.," *Energy Economics*, 13(3), 211–218.

BARLEVY, G., AND J. FISHER (2011): "Mortgage Choices and Housing Speculation," manuscript, Federal Reserve Bank of Chicago.

BAYER, R. C., AND C. KE (2011): "Are rockets and feathers caused by search or informational frictions?," Max Planck Institute for Tax Law and Public Finance Working Paper 2011-12.

Black, F. (1976): "Studies of Stock Market Volatility Changes," in *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economics Statistics Section*, pp. 177–181.

Borenstein, S., A. C. Cameron, and R. Gilbert (1997): "Do gasoline prices respond asymmetrically to crude oil price changes?," *Quarterly Journal of Economics*, 112(1), 305–339.

Burnside, C., M. Eichenbaum, and S. Rebelo (2011): "Understanding Booms and Busts in Housing Markets," No. w16734. National Bureau of Economic Research.

Cabral, L., and A. Fishman (2012): "Business as usual: A consumer search theory of sticky prices and asymmetric price adjustment," *International Journal of Industrial Organization*, 30(4), 371–376.

Campbell, S., M. Davis, J. Gallin, and R. Martin (2009): "What Moves Housing Markets: A Variance Decomposition of the Rent-Price Ratio," *Journal of Urban Economics*, 66, 90–102.

Caplin, A., and J. Leahy (2011): "Trading Frictions and House Price Dynamics," *Journal of Money, Credit and Banking*, 43, 283–303.

Case, K. E., and R. Shiller (1989): "The Efficiency of the Market for Single Family Homes," *American Economic Review*, 79(1), 125–37.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Rios-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75(6), 1525–1589.

Chatterjee, S., and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102(6), 2674–2699.

Corbae, D., and E. Quintin (2011): "Mortgage Innovation and the Foreclosure Boom," manuscript, University of Wisconsin.

Crawford, G., and E. Rosenblatt (1995): "Efficient Mortgage Default Option Exercise: Evidence from Loan Loss Severity," *Journal of Real Estate Research*, 10, 543–555.

Davis, M. A., and M. Palumbo (2008): "The Price of Residential Land in Large U.S. Cities," *Journal of Urban Economics*, 63, 352–384.

Dell'Ariccia, G., D. Igan, and L. Laeven (2012): "Credit Booms and Lending Standards: Evidence from the Subprime Mortgage Market," *Journal of Money, Credit and Banking*, 44(2-3), 367–384.

Diaz, A., and B. Jerez (2013): "House Prices, Sales, and Time on the Market: A Search-Theoretic Framework," *International Economic Review*, 54(3), 837–872.

Dobrynskaya, V. (2008): "Reference-dependent preferences, loss aversion and asymmetric price rigidity," Center for Advanced Studies Working Paper 13.

Dolde, W., and D. Tirtiroglu (1997): "Temporal and Spatial Information Diffusion in Real Estate Price Changes and Variances," *Real Estate Economics*, 25, 539C565.

Duca, J., J. Muellbauer, and A. . Murphy (2011): "House Prices and Credit Constraints: Making Sense of the U.S. Experience," *Economic Journal*, 121, 533–551.

Duffy-Deno, K. T. (1996): "Retail price asymmetries in local gasoline markets," *Energy Economics*, 18(1-2), 81–92.

Dufrénot, G., and S. Malik (2012): "The changing role of house price dynamics over the business cycle," *Economic Modelling*, 29(5), 1960–1967.

Eckert, A. (2002): "Retail price cycles and response asymmetry," *Canadian Journal of Economics*, 35(1), 52–77.

Eeckhout, J., and P. Kircher (2010): "Sorting and Decentralized Price Competition," *Econometrica*, 78(2), 539–574.

Engelhardt, G. (2003): "Nominal loss aversion, housing equity constraints, and household mobility: evidence from the United States," *Journal of Urban Economics*, 53, 171–195.

Engle, R. F. (1982): "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50(4), 987–1008.

Engle, R. F., and V. K. Ng (1993): "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48(5), 1749–1778.

Favilukis, J., S. Ludvigson, and S. Van Nieuwerburgh (2010): "Macroeconomic Effects of Housing Wealth, Housing Finance and Limited Risk Sharing in General Equilibrium," No. w15988. National Bureau of Economic Research.

Foster, C., and R. Van Order (1984): "FHA Terminations: A Prelude to Rational Mortgage Pricing," *Journal of the American Real Estate and Urban Economics Association*, pp. 273–291.

Galeotti, M., A. Lanza, and M. Manera (2003): “Rockets and feathers revisited: an international comparison on European gasoline markets,” *Energy Economics*, 25(2), 175–190.

Garriga, C., and D. Schlagenhauf (2009): “Home Equity, Foreclosures, and Bailouts,” Manuscript, Federal Reserve Bank of St. Louis.

Genesove, D., and C. Mayer (1997): “Equity and Time to Sell in the Real Estate Market,” *The American Economic Review*, 87(3), 255–269.

——— (2001): “Loss Aversion and Seller Behavior: Evidence from the Housing Market,” *The Quarterly Journal of Economics*, 116(4), 1233–1260.

Gerardi, K., A. Lehnert, S. Sherland, and P. Willen (2008): “Making Sense of the Subprime Crisis,” *Brookings Papers on Economic Activity*, 2008(2), 69–159.

Glaeser, E., J. Gottlieb, and J. Gyourko (2010): “Can Cheap Credit Explain the Housing Boom?,” No. w16230. National Bureau of Economic Research.

Glaeser, E. L., J. Gyourko, E. Morales, and C. G. Nathanson (2013): “Housing Dynamics,” No. w12787. National Bureau of Economic Research.

Glosten, L., R. Jagannathan, and D. Runkle (1993): “On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks,” *Journal of Finance*, 68, 1779–1801.

Green, R., S. Malpezzi, and S. Mayo (2005): “Methropolitan-specific Estimates of the Price Elasticity of Supply of Housing, and Their Sources,” *American Economic Review*, 95, 334–339.

Guren, A., and T. McQuade (2013): "How Do Foreclosures Exacerbate Housing Downturns," mimeo.

Hannan, T. H., and A. N. Berger (1991): "The rigidity of prices: evidence from the banking industry," *American Economic Review*, 81(4), 938–945.

Harding, J. P., S. S. Rosenthal, and C. Sirmansa (2007): "Depreciation of housing capital, maintenance, and house price inflationation: Estimates from a repeat sales Model," *Journal of Urban Economics*, 61(2), 193–217.

Hayre, L. S., and M. Saraf (2008): "A Loss Severity Model for Residential Mortgages," *Journal of Fixed Income*, pp. 5–31.

Head, A., H. Lloyd-Ellis, and H. Sun (2014): "Search, Liquidity and the Dynamics of House Prices and Construction," *American Economic Review*, 104(4), 1172–1210.

Hedlund, A. (2014): "The Dynamics of Frictional Housing and Mortgage Markets," University of Pennsylvania Working paper.

Himmelberg, C., C. Mayer, and T. Sinai (2005): "Assessing High House Prices: Bubbles, Fundamentals and Misperceptions," *Journal of Economic Perspectives*, 19(4), 67–92.

Hintermaier, T., and W. Koeniger (2011): "Debt Portfolios," No. 5653. Discussion paper series//Forschungsinstitut zur Zukunft der Arbeit.

Hossain, B., and E. Latif (2009): "Determinants of housing price volatility in Canada: a dynamic analysis," *Applied Economics*, 41(27), 3521–3531.

Iacoviello, M. (2005): "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95(3), 739–764.

Iacoviello, M., and S. Neri (2010): "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2(2), 125–64.

Iacoviello, M., and M. Pavan (2013): "Housing and Debt Over the Life Cycle and Over the Business Cycle," *Journal of Monetary Economics*, 60(2), 221–238.

Jeske, K., D. Krueger, and K. Mitman (2010): "Housing and the Macroeconomy: The Role of Implicit Guarantees for Government Sponsored Enterprises," No. 2005-15. Federal Reserve Bank of Atlanta.

Kahneman, D., and A. Tversky (1979): "Prospect theory: an analysis of decision under risk," *Econometrica*, 46, 171–185.

Karrenbrock, J. D. (1991): "The behavior of retail gasoline prices: symmetric or not?," *Federal Reserve Bank of St. Louis Review*, pp. 19–29.

Khandani, A. E., A. Lo, and R. Merton (2013): "Systemic risk and the refinancing ratchet effect," *Journal of Financial Economics*, 108(1), 29–45.

Kiyotaki, N., A. Michaelides, and K. Nikolov (2011): "Winners and Losers in Housing Markets," *Journal of Money, Credit and Banking*, 43(2-3), 255–296.

Krainer, J. (2001): "A Theory of Liquidity in Residential Real Estate Markets," *Journal of Urban Economics*, 49, 32–63.

——— (2008): "Falling house prices and rising time on the market," FRBSF Economic Letter. Federal Reserve Bank of San Francisco (March).

Krusell, P., and A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

LaCour-Little, M., M. Marschoun, and C. Maxam (2002): "Improving Parametric Mortgage Prepayment Models with Non-Parametric Kernel Regression," *Journal of Real Estate Research*, 24, 229–328.

Lee, C. (2009): "Housing Price Volatility and its Determinants," *International Journal of Housing Markets and Analysis*, 2(3), 293–308.

Lee, C. L. (2008): "Volatility Spillover in Australian Commercial Property," *Pacific Rim Property Research Journal*.

Lewis, M., and H. Marvel (2011): "When Do Consumers Search?," *Journal of Industrial Economics*, 59(3), 457–483.

Lewis, M. S. (2011): "Asymmetric price adjustment and consumer search: An examination of the retail gasoline market," *Journal of Economics & Management Strategy*, 20(2), 409–449.

Lin, P., and F. Fuerst (2013): "Volatility clustering, risk-return relationship and asymmetric adjustment in Canadian housing markets," *Journal of Real Estate Portfolio Management*, Forthcoming.

Mandelbrot, B. B. (1963): "The Variation of Certain Speculative Prices," *The Journal of Business*, 36(4), 394–419.

Mayer, C., and T. Sinai (2009): *U.S. House Price Dynamics and Behavioral Finance*chap. 5. Federal Reserve Bank of Boston.

Menzio, G., and S. Shi (2010): "Block Recursive Equilibria for Stochastic Models of Search on the Job," *Journal of Economic Theory*, 145(10), 1453–1494.

Merlo, A., and F. Ortalo-Magné (2004): "Bargaining Over Residential Real Estate: Evidence From England," *Journal of Urban Economics*, 56, 192–216.

Merlo, A., F. Ortalo-Magné, and J. Rust (2008): "The Home Selling Problem: Theory and Evidence," Cowles Foundation Summer Conference.

Mian, A., and A. Sufi (2009): "House prices, home equity-based borrowing, and the U.S. household leverage crisis," No. w15283. National Bureau of Economic Research.

Michayluk, D., P. J. Wilson, and R. Zurbruegg (2006): "Asymmetric Volatility, Correlation and Returns Dynamics Between the U.S. and U.K. Securitized Real Estate Markets," *Real Estate Economics*, 34(1), 109–131.

Miles, W. (2008): "Volatility Clustering in U.S. Home Prices," *Journal of Real Estate Research*, 30(1), 73–90.

Miller, N., and L. Peng (2006): "Exploring Metropolitan House Price Volatility," *Journal of Real Estate Finance and Economics*, 33(5), 5–18.

Mitman, K. (2011): "Macroeconomic Effects of Bankruptcy and Foreclosure Policies," unpublished paper, Penn Institute For Economic Research.

Moen, E. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385–411.

Nelson, D. B. (1991): "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, 59(2), 347–370.

Neumark, D., and S. A. Sharpe (1992): "Market structure and the nature of price rigidity: evidence from the market for consumer deposits," *Quarterly Journal of Economics*, 107(2), 657–680.

Novy-Marx, R. (2009): "Hot and Cold Markets," *Real Estate Economics*, 37(1), 1–22.

Ortalo-Magné, F., and S. Rady (2006): "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints," *Review of Economic Studies*, 73(2), 459C485.

Peltzman, S. (2001): "Prices Rise Faster Than They Fall," *Journal of Political Economy*, 108, 466–502.

Pennington-Cross, A. (2010): "The Duration of Foreclosures in the Subprime Mortgage Market: A Competing Risks Model with Mixing," *Journal of Real Estate Financial Economics*, 40, 109–129.

Phillips, R., and J. Vanderhoff (2004): "The Conditional Probability of Foreclosure: An Empirical Analysis of Conventional Mortgage Loan Defaults," *Real Estate Economics*, 32(4), 571–588.

Ríos-Rull, J.-V., and V. Sánchez-Marcos (2008): "Aggregate Shocks and the Volatility of House Prices," mimeo.

Saiz, A. (2010): "The Geeographic Determinants of Housing Supply," *The Quarterly Journal of Economics*, 125(3), 1253–1296.

Schwarz, G. E. (1978): "Estimating the dimension of a model," *Annals of Statistics*, 6(2), 461–464.

Shefrin, H., and M. Statman (1985): "The disposition to sell winners too early and ride losers too long: theory and evidence," *Journal of Finance*, 40, 777–791.

Shi, S. (2009): "Directed search for equilibrium wage-tenure contracts," *Econometrica*, 77(2), 561–584.

Shimer, R., and L. Smith (2000): "Assortative Matching and Search," *Econometrica*, 68, 343–369.

Sims, R. . (1982): "Policy analysis with econometric models," *Brookings Paper on Economic Activity*, pp. 107–152.

Smith, L. (2006): "The Marriage Model With search frictions," *Journal of Political Economy*, 114, 1124–1144.

Stevenson, S. (2002): "An Examination of Volatility Spillovers in REIT Returns," *Journal of Real Estate Portfolio Management*, 8(3), 229–238.

Tappata, M. (2009): "Rockets and feathers: Understanding asymmetric pricing," *RAND Journal of Economics*, 40(4), 673–687.

Tirole, J. (1988): *The Theory of Industrial Organization*. Cambridge: MA: MIT Press.

TSAI, I. (2012): "The asymmetric impacts of monetary policy on housing prices: A viewpoint housing price rigidity," *Economic Modelling*, 31, 405–413.

TSAI, I., AND M. CHEN (2009): "The asymmetric volatility of house price in the UK," *Property Management*, 27, 80–90.

UNGERER, C. (2012): "Monetary Policy, Hot Housing Markets and Household Debt," Working paper, London School of Econonomics.

WONG, S. K., C. Y. YIU, M. K. S. TSE, AND K. W. CHAU (2006): "Do the Forward Sales of Real Estate Stabilize Spot Prices?," *Journal of Real Estate Finance and Economics*, 32(3), 289–304.

YANG, H., AND L. YE (2008): "Search with learning: understanding asymmetric price adjustments," *RAND Journal of Economics*, 39(2), 574–564.

Appendix A

Housing Liquidity and Lending Standard

A.1 Laws of motion

A.1.1 The search economy

The evolutions of household distributions and agents' stocks in the search economy are described below. The measure of perpetual renters evolves according to

$$(1+\mu)F_t = (1-\pi_p)F_{t-1} + (1-\psi)G(\varepsilon_t^c)\mu. \tag{A.1}$$

Let B_t denote the measure of buyers without a foreclosure flag at the beginning of sub-period 1 of period t. It follows that

$$\begin{aligned}(1+\mu)B_t &= \psi G(\varepsilon_t^c)\mu + (1-\pi_f)B_t^R + (1-\pi_h) \\ &\times \Big\{B_{t-1} - \sum_{n=0}^{T-1}\rho\Big(\theta\big(p_{t-1}^L(\cdot)\big)\Big)\Phi_{t-1}^L(m_{0,t-1-n}, n) - \rho\Big(\theta\big(p_{t-1}^{Lw}(\cdot)\big)\Big)\Phi_{t-1}^{Lw} \\ &\quad -\rho\big(\theta(p_{t-1}^c)\big)\Phi^c - \rho\big(\theta(p_{t-1}^{REO})\big)\Phi^{REO}\Big\}.\end{aligned} \tag{A.2}$$

The first item is the inflow of population and the second is the measure of buyers whose previous foreclosure flags disappear this period. The items in the big bracket compute the measure of buyers from last period who were not matched with sellers and did not leave the city. Note that staying sellers who successfully sold houses in last period became buyers without a foreclosure flag in this period if they are still in the city.

The measure of potential buyers with a flag at the beginning of sub-period 1, B_t^R, evolves according to

$$\begin{aligned}(1+\mu)B_t^R &= (1-\pi_h)\Big\{\pi_f B_{t-1}^R + (1-\pi_d) \\ &\quad \times \sum_{n=0}^{T-1}\Big[1-\rho\Big(\theta\big(p_{t-1}^s(\cdot)\big)\Big)\Big]D_{t-1}(\cdot)\Phi_{t-1}(m_{0,t-1-n},n) \\ &\quad +\pi_d \sum_{n=0}^{T-1}\Big[1-\rho\Big(\theta\big(p_{t-1}^{sd}(\cdot)\big)\Big)\Big]\Phi_{t-1}(m_{0,t-1-n},n)\Big\}. \qquad \text{(A.3)}\end{aligned}$$

In the big bracket, the first item is the measure of buyers whose previous foreclosure flags stay in records. The other two items are the measures of staying owners who defaulted last period. Note owners without mortgage debt, $n = T$, are not subject to distress shocks.

$\Phi_t(m_0, n)$ is the measure of owners in the city who have made n payments at the beginning of sub-period 1 of period t. For every n, the corresponding loan volume at origination is $m_{0,t-n}$. $\Phi(\cdot)$ evolves as

$$\begin{aligned}(1+\mu)\Phi_t(m_0,n) &= (1-\pi_h)(1-\pi_d)\Big[1-\rho\Big(\theta\big(p_{t-1}^s(\cdot)\big)\Big)\Big] \\ &\quad \times\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-n},n-1)\end{aligned}$$

$$
\begin{aligned}
(1+\mu)\Phi_t(m_0,0) \;&=\; (1-\pi_h)\Big\{(1-\pi_d)\sum_{n=0}^{T-1}\rho\Big(\theta\big(p^s_{t-1}(\cdot)\big)\Big)\Phi_{t-1}(m_{0,t-1-n},n) \\
&+\pi_d\sum_{n=0}^{T-1}\rho\Big(\theta\big(p^{sd}_{t-1}(\cdot)\big)\Big)\Phi_{t-1}(m_{0,t-1-n},n) \\
&+\rho\big(\theta(p^c_{t-1})\big)\Phi^c+\rho\big(\theta(p^{REO}_{t-1})\big)\Phi^{REO}\Big\} \\
(1+\mu)\Phi_t(\emptyset) \;&=\; (1-\pi_h)\Big\{(1-\pi_d)\Big[1-\rho\Big(\theta\big(p^s_{t-1}(\cdot)\big)\Big)\Big] \\
&\times\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-T},T-1) \\
&+\Big(1-\rho\big(\theta(p^{sw}_{t-1})\big)\Big)\Phi_{t-1}(\emptyset)\Big\}. \qquad \text{(A.4)}
\end{aligned}
$$

If a staying owner with balance (m_0, n) in period t experienced no shock, chose not to sell and did not default, he/she is counted in the measure of staying owners with balance $(m_0, n+1)$ in period $t+1$. $\Phi_t(m_0, 0)$ is the measure of owners who purchased houses last period, staying in the city and are not hit by financial distress shocks. Note that the measure of new owners equals the measure of sellers getting in matched in every period. Staying owners who made last payment are counted into the measure of staying owners without mortgage, $\Phi_t(\emptyset)$.

Similarly, $\Phi^L_t(m_0, n)$ is the measure of relocated owners who have made n payments at the beginning of sub-period 1 of period t. For every n, the corresponding loan volume at origination is $m_{0,t-n}$. It follows that

$$
\begin{aligned}
(1+\mu)\Phi_t^L(m_0,n) &= \Big[1-\rho\Big(\theta\big(p_{t-1}^{sL}(\cdot)\big)\Big)\Big]\Big(1-D_{t-1}^L(\cdot)\Big)\Phi_{t-1}^L(m_{0,t-n},n-1) \\
&\quad +\pi_h(1-\pi_d)\Big[1-\rho\Big(\theta\big(p_{t-1}^{s}(\cdot)\big)\Big)\Big]\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-n},n) \\
(1+\mu)\Phi_t^L(m_0,0) &= \pi_h\Big\{(1-\pi_d)\sum_{n=0}^{T-1}\rho\Big(\theta\big(p_{t-1}^{s}(\cdot)\big)\Big)\Phi_{t-1}(m_{0,t-1-n},n) \\
&\quad +\pi_d\sum_{n=0}^{T-1}\rho\Big(\theta\big(p_{t-1}^{sd}(\cdot)\big)\Big)\Phi_{t-1}(m_{0,t-1-n},n) \\
&\quad +\rho\big(\theta(p_{t-1}^{c})\big)\Phi^c+\rho\big(\theta(p_{t-1}^{REO})\big)\Phi^{REO}\Big\} \\
(1+\mu)\Phi_t^L(\emptyset) &= \pi_h\Big\{\Big(1-\rho\big(\theta(p_{t-1}^{sw})\big)\Big)\Phi_{t-1}(\emptyset)+(1-\pi_d)\Big[1-\rho\Big(\theta\big(p_{t-1}^{s}(\cdot)\big)\Big)\Big] \\
&\quad \times\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-T},T-1)\Big\} \\
&\quad +\Big[1-\rho\Big(\theta\big(p_{t-1}^{sL}(\cdot)\big)\Big)\Big]\Big(1-D_{t-1}^L(\cdot)\Big)\Phi_{t-1}^L(m_{0,t-T},T-1) \\
&\quad +\Big(1-\rho\big(\theta(p_{t-1}^{sLw})\big)\Big)\Phi_{t-1}^L(\emptyset). \qquad \text{(A.5)}
\end{aligned}
$$

Relocated owners with (m_0,n) hold $(m_0,n+1)$ next period if they fails to find a buyer and choose to continue mortgage contracts. With probability π_h, a staying owner with (m_0,n) becomes a relocated owner with $(m_0,n+1)$ in the following period.

Let Φ_t^c be the stock of houses in construction firms' inventory at the beginning of sub-period 1 of period t, it evolves as

$$(1+\mu)\Phi_t^c=\Big(1-\rho\big(\theta(p_{t-1}^c)\big)\Big)\Phi_{t-1}^c+N_t, \qquad \text{(A.6)}$$

where N_t is new houses that are built last period and available this period.

The stock of houses in REO inventory at the beginning of sub-period 1, Φ_t^{REO}, changes over time following

$$
\begin{aligned}
(1+\mu)\Phi_t^{REO} \quad = \quad & \Big(1-\rho\big(\theta(p_{t-1}^{REO})\big)\Big)\Phi_{t-1}^{REO} \\
& +\pi_d \sum_{n=0}^{T-1} \Big[1-\rho\Big(\theta\big(p_{t-1}^{sd}(\cdot)\big)\Big)\Big]\Phi_{t-1}(m_{0,t-1-n},n) \\
& +(1-\pi_d)\sum_{n=0}^{T-1} \Big[1-\rho\Big(\theta\big(p_{t-1}^{s}(\cdot)\big)\Big)\Big]D_{t-1}(\cdot)\Phi_{t-1}(m_{0,t-1-n},n) \\
& +\sum_{n=1}^{T-1} \Big[1-\rho\Big(\theta\big(p_{t-1}^{sL}(\cdot)\big)\Big)\Big]D_{t-1}^{L}(\cdot)\Phi_{t-1}^{L}(m_{0,t-1-n},n). \qquad \text{(A.7)}
\end{aligned}
$$

Finally, the total measure of buyers in housing market, B_t^{sum}, can be derived as:

$$
\begin{aligned}
B_t^{sum} \quad = \quad & (1-\pi_d)\sum_{n=0}^{T-1}\rho^{-1}\Big(\theta\big(p_t^{s}(\cdot)\big)\Big)\Phi_t(m_{0,t-n},n)+\pi_d\sum_{n=0}^{T-1}\rho^{-1}\Big(\theta\big(p_t^{sd}(\cdot)\big)\Big)\Phi_t(m_{0,t-n},n) \\
& +\sum_{n=0}^{T-1}\rho^{-1}\Big(\theta\big(p_t^{sL}(\cdot)\big)\Big)\Phi_t^{L}(m_{0,t-n},n)+\rho^{-1}\big(\theta(p_t^{sw})\big)\Phi_t(\emptyset)+\rho^{-1}\big(\theta(p_t^{sLw})\big)\Phi_t^{L}(\emptyset) \\
& +\rho^{-1}\big(\theta(p_t^{c})\big)\Phi_t^{c}+\rho^{-1}\big(\theta(p_t^{REO})\big)\Phi_t^{REO}. \qquad \text{(A.8)}
\end{aligned}
$$

A.1.2 The non-search economy

In the non-search model, we have the following laws of motion:

$$
(1+\mu)F_t=(1-\pi_p)F_{t-1}+(1-\psi)G(\varepsilon_t^c)\mu \qquad \text{(n.A.1)}
$$

$$
\begin{aligned}
(1+\mu)B_t = & \psi G(\varepsilon_t^c)\mu+(1-\pi_f)B_t^R \\
& +(1-\pi_h)\sum_{n=0}^{T}\Big(1-H_t^s(\cdot)\Big)\Phi_{t-1}(m_{0,t-1-n},n) \qquad \text{(n.A.2)}
\end{aligned}
$$

$$
\begin{aligned}
(1+\mu)B_t^R =&(1-\pi_h)\Big\{\pi_f B_{t-1}^R + \pi_d^n \sum_{n=0}^{T-1} \Phi_{t-1}(m_{0,t-1-n}, n) \\
&+ (1-\pi_d^n)\sum_{n=0}^{T-1} H_t^s(\cdot) D_{t-1}(\cdot)\Phi_{t-1}(m_{0,t-1-n}, n)\Big\} \qquad \text{(n.A.3)}
\end{aligned}
$$

$$
\begin{aligned}
(1+\mu)\Phi_t(m_0, n) =&(1-\pi_h)(1-\pi_d^n)H_t^s(\cdot)\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-n}, n-1) \\
(1+\mu)\Phi_t(m_0, 0) =&(1-\pi_h)(1-\pi_d^n)B_{t-1} \\
(1+\mu)\Phi_t(\emptyset) =&(1-\pi_h)\Big\{(1-\pi_d^n)H_t^s(\cdot)\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-T}, T-1) \\
&+ H_t^s(\cdot)\Phi_{t-1}(\emptyset)\Big\} \qquad \text{(n.A.4)}
\end{aligned}
$$

$$
\begin{aligned}
(1+\mu)\Phi_t^L(m_0, n) =&H_t^{sL}(\cdot)\Big(1-D_{t-1}^{sL}(\cdot))\Big)\Phi(m_{0,t-n}, n-1) \\
&+ \pi_h(1-\pi_d^n)H_t^s(\cdot)\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-n}, n) \\
(1+\mu)\Phi_t^L(m_0, 0) =&\pi_h(1-\pi_d^n)B_{t-1} \\
(1+\mu)\Phi_t^L(\emptyset) =&\pi_h\Big\{(1-\pi_d^n)H_t^s(\cdot)\Big(1-D_{t-1}(\cdot)\Big)\Phi_{t-1}(m_{0,t-T}, T-1) \\
&+ H_t^s(\cdot)\Phi_{t-1}(\emptyset)\Big\} + H_t^{sL}(\cdot)\Big(1-D_{t-1}^L(\cdot)\Big)\Phi_{t-1}^L(m_{0,t-T}, T-1) \\
&+ H_t^{sLw}(\cdot)\Phi_{t-1}^L(\emptyset) \qquad \text{(n.A.5)}
\end{aligned}
$$

$$
(1+\mu)\Phi_t^c = N_t \qquad \text{(n.A.6)}
$$

$$
\begin{aligned}
(1+\mu)\Phi_t^{REO} =& \pi_d^n \sum_{n=0}^{T-1} \Phi_{t-1}(m_{0,t-1-n}, n) + \sum_{n=1}^{T-1} H_t^{sL}(\cdot) D_{t-1}^L(\cdot) \Phi_{t-1}^L(m_{0,t-1-n}, n) \\
&+ (1-\pi_d^n) \sum_{n=0}^{T-1} H_t^s(\cdot) D_{t-1}(\cdot) \Phi_{t-1}(m_{0,t-1-n}, n). \qquad \text{(n.A.7)}
\end{aligned}
$$

A.2 Computation detail for the economy with search

The model is numerically solved according to the following algorithms.

A.2.1 Steady state

Loop 1: Make an initial guess of buyer's market utility U_{ss}.

1. Loop 2: Make an initial guess for the loan at origination, $m_{0,ss}$ and compute periodic payment $x(m_{0,ss})$ and balance path $\{d(m_{0,ss}, n)\}_{n=0}^{T-1}$

   (a) Set $\underline{p} = m_{0,ss}$ and make an initial guess for $\underline{V}^o = V^o(\underline{p}, m_{0,ss})$. This implies

   $$V^o(p; m_{0,ss}) = \underline{V}^o + u\Big(y_{ss} - (p - m_{0,ss}) - m\Big) - u(y_{ss} - m).$$

   We can find $\bar{p}$ such that $V^o(\bar{p}; m_{0,ss}) = U_{ss}$, *i.e.* the matching probability for buyers is one in that submarket. Given house price space $P = [\underline{p}, \bar{p}]$ and intra-period asset space $A = [0, \bar{p}]$, tightness in each submarkets, $\theta(p)$, value functions, $W(a)$, $\bar{V}_{ss}^L$, V_{ss}^c, V_{ss}^{REO}, $\bar{V}_{ss}$, $\bar{S}_{ss}$, $V^L(m_{0,ss}, n)$, $V(m_{0,ss}, n)$, $S(m_{0,ss}, n)$, $S^f(m_{0,ss}, n)$ and corresponding policy functions can be found by using (2.5) - (2.9), (2.11), and (2.12) - (2.19). Then we update $\underline{V}^{o\prime} = V^{o\prime}(\underline{p}, m_{0,ss})$ by (2.10). If $|\underline{V}^{o\prime} - \underline{V}^o| < \epsilon$, go to step (b). Otherwise, set $\underline{V}^o = \underline{V}^{o\prime}$ and repeat this step;

(b) Using household policy functions derived above to find the present value of the mortgage contract, $P^{\iota}(m_{0,ss}; r_m)$. If $P^{\iota}(m_{0,ss}; r_m) > (<)0$, guess a new $m'_{0,ss} > (<) m_{0,ss}$ and go back to (a). Otherwise, go to next step;

2. Using equations (A.1) - (A.7) to find stationary distributions B_{ss}, B^R_{ss}, Φ_{ss}, Φ^L_{ss}, Φ^c_{ss}, Φ^{REO}_{ss}. Solve the total measure of buyers in all submarket, B^{sum}_{ss}, using (A.8). If $|B^{sum}_{ss} - B_{ss}| < \epsilon$, steady state is found. Otherwise, update u_{ss} and go back to Loop 1.

A.2.2 Dynamic paths

The economy is on steady state initially and hit by an income shock at $t = 1$.

Loop 1: Let TT be a large integer and assume that the economy goes back to steady state before period TT;

1. Loop 2: Make an initial guess of path of buyer's market value, $\{U_t(t)\}_{t=1}^{TT}$.

   (a) Start from last period TT, for each t

   i. Loop 3: Make an initial guess of $m_{0,t}$ and compute periodic payment $x(m_{0,t})$ and balance path $\{d(m_{0,t}, n)\}_{n=0}^{T-1}$

   A. Set $\underline{p}_t = m_{0,t}$ and make an initial guess for $\underline{V}^o_t = V^o_t(\underline{p}_t, m_{0,t})$. This implies

$$V^o_t(p_t, m_{0,t}) = \underline{V}^o_t + u\Big(y_t - (p_t - m_{0,t}) - m\Big) - u(y_t - m)$$

Then we can find $\bar{p}_t$ such that $V^o_t(\bar{p}_t; m_{0,t}) = U_t(t)$. Given house price space $P_t = [\underline{p}_t, \bar{p}_t]$ and intra-period asset space $A_t = [0, \bar{p}_t]$

and next period value functions, tightness in each submarkets, $\theta_t(p)$, value functions, $W_t(a)$, $\bar{V}_t^L$, V_t^c, V_t^{REO}, $\bar{V}_t$, $\bar{S}_t$, $V^L(m_{0,t}, n)$, $V(m_{0,t}, n)$, $S(m_{0,t}, n)$, $S^f(m_{0,t}, n)$ and corresponding policy functions can be found by using (2.5) - (2.9), (2.11), and (2.12) - (2.19). Then we update $\underline{V}_t^{o\prime} = V_t^{o\prime}(\underline{p}_t, m_{0,t})$ by (2.10). If $|\underline{V}_t^{o\prime} - \underline{V}_t^o| < \epsilon$, go to step (ii). Otherwise, set $\underline{V}_t^o = \underline{V}_t^{o\prime}$ and repeat this step;

B. Using household policy functions derived above to compute the present value of the mortgage contract, $P^\iota(m_{0,t}; r_m)$. If $P^\iota(m_{0,t}; r_m) >$ $(<)0$, guess a new $m'_{0,t} > (<) m_{0,t}$ and go back to (i). Otherwise, go to next step;

(b) Value and policy functions for household owners who have mortgages that originated before $t = 1$ can be solved by using $\left\{\theta_t, W_t, \bar{V}_t^L, V_t^c, V_t^{REO}, \bar{V}_t, \bar{S}_t\right\}_{t=1}^{TT}$ derived above;

(c) Using equation (A.1) - (A.7) to find the evolution paths of distributions B_t, B_t^R, Φ_t, Φ_t^L, Φ_t^c, Φ_t^{REO}. Solve the total measure of buyers in all submarket, B_t^{sum}, using (A.8). If $\max|B_t^{sum} - B_t| < \epsilon$, dynamic paths are found and go to step 2. Otherwise, update $\left\{U_t\right\}_{t=1}^{TT}$ and go back to step (a).

2. If the economy does not converge back to the initial stationary steady state at $t = TT$, increase TT and go back to Loop 1. Otherwise, the computation is done.