Numerical simulations of rough-wall turbulent boundary layers

by

Junlin Yuan

A thesis submitted to the
Department of Mechanical and Materials Engineering
in conformity with the requirements for
the degree of Doctor of Philosophy

Queen’s University
Kingston, Ontario, Canada
July, 2015

Copyright © Junlin Yuan, 2015
Abstract

At sufficiently high Reynolds number, all surfaces are rough, and roughness affects most flows in engineering and the natural sciences. Examples range from atmospheric boundary layers over buildings and canopies, to engineering surfaces with erosion, deposits, etc. To study the roughness effects, we take a high-resolution approach to capture the flow around individual roughness elements using direct and large-eddy simulations (DNS and LES); the goal is to elucidate phenomena which have been difficult to access using physical experiments, and to help develop engineering correlations and models. First, most experiments and turbulence models are based on a standardized type of roughness, sand-grain roughness, which can be described using a single length scale. The relationship between the geometry of an arbitrary surface and the canonical one must be known, to predict critical flow parameters such as the drag, using either experimental correlations or turbulence models. Using numerical experiments, we relate this length-scale to the roughness geometry, and propose a guideline for its prediction in the industrial setting. Next, to explain the dependence of drag on the topographical details, we examine the role of the wake of the roughness elements in the drag generation of a rough surface. The wake field is found to promote vertical momentum transfer and near-wall instability; it might provide a link between geometry details and the engineering modeling of roughness effects. Lastly, we focus on a more realistic flow scenario – the one with freestream accelerations – and study the
combined effects of roughness and acceleration, a phenomenon widely present in engineering flows over airfoils or complex landscapes. It is first shown, by comparing equilibrium accelerating flows obtained in the present study with the non-equilibrium flows in the literature, that the roughness and acceleration effects are interdependent and depend on the flow equilibrity. Then, using DNS data of a spatially developing flat-plate boundary layer, it is found that the effect coupling develops as the roughness affects the turbulence time scale and thus the flow susceptibility of the acceleration stabilization, while acceleration changes the wake velocity and ultimately the roughness destabilization level.
Co-Authorship

Chapter 2 of this thesis will be also referenced in the text as Yuan & Piomelli (2014a) and has been published as:


Chapter 3 of this thesis will be also referenced in the text as Yuan & Piomelli (2014c) and has been published as:


Chapter 4 of this thesis will be also referenced in the text as Yuan & Piomelli (2014b) and has been published as:


Chapter 5 has been submitted to Journal of Fluid Mechanics as:

Junlin Yuan and Ugo Piomelli, *Numerical simulation of a spatially developing accelerating boundary layer over roughness*
Acknowledgments

My sincere thanks go to my advisor, Professor Ugo Piomelli, who mentored me on both research and career, and taught me confidence in assertion and persuasion. I also wholeheartedly thank Professors Xiaohua Wu and Andrew Pollard for your guidance and help, and Professors Boegman, Rival, and Pharoah for making my thesis more convincing and complete.

I am grateful to IREQ, especially Anne-Marie, Jonathan, and Martin, who endowed me an industrial perspective and offered generous computational support. Appreciation is also given to Hartmut and Gang in HPCVL, for all your help to make my simulations possible.

Special thanks must be made to my generations of lab-mates in 6 years, especially Iftekhar, Matt, Carlo, Omid, Valerio, Wen, Rayhaneh, Amir, Ali, Pouya, Mojtaba; I am honoured to go through, together with you, those 10 versions of manuscript and 1001 APS rehearsals, and to all my friends who successfully pulled me away from becoming a workaholic. In addition, I must show my gratitude to Jane, Gabrielle, Kate, Chris, and other department staff for all the help that I could not even start listing.

Lastly, thank you my mom, my dad, and Antoine, for making me smile whenever I thought of you.
List of Tables

2.1 Statistics of rough surfaces. .................................................. 24
2.2 Spatial resolution in $x$- and $z$-directions for all cases. ............... 24
2.3 $k_s/\bar{k}$ and the start of fully rough regime ($k_{s,cri}^+$) for all surface types. ................................................................. 30
2.4 S2 surfaces with various stretching factors, $Str$. $Re_r = 1000$, $k/h = 0.067$. 34

3.1 Parameters for all cases. $k_c$: roughness crest; $\bar{k}$: mean roughness height. $n_j$ ranges from 128 to 256. $N_{x_i}$: grid points per sand-grain element. ............. 49

4.1 Summary of simulation parameters. Values of roughness Reynolds number $k^+$. ................................................................. 67
4.2 Boundary layer parameters in all cases. $H$ is the shape factor; $\theta$ is the momentum thickness. ......................................................... 71

5.1 Simulation parameters ($L$ is the length of the computational domain). . . . 109
List of Figures

1.1 (a) The schematics of a typical Francis turbine (top view, source: http://www.learnengineering.org/), and (b) roughness patches obtained at different locations of an existing Francis turbine (side view, indicated by the arrow in (a)) (Yuan et al., 2014). .................................................. 2

1.2 Regions of flow acceleration (FPG) and deceleration (APG) on an airfoil. .......................................................... 3

2.1 Visualization of (a)–(d) a single tile of four surfaces denoted by S1, S2, S3, S4, (e) a complete surface formed from S3 tiles, and (f) 1/4 of a complete sand-grain (SG) surface. Colouring is based on $y/k$. .................................................. 21

2.2 Autocorrelation function $R_{k'k'}(\circ)$ of surface height fluctuations, as a function of the separation $r$ normalized by mean height, and the parabola $R_{\text{para}}(r)$ (——) fitted to determine the Taylor microscale $\lambda$ of the surface height fluctuations. Case SGk3 is shown. .................................................. 23

2.3 Profiles of streamwise mean velocity in cases with $k = k_2$ and $Re_\tau = 1000$. + Smooth-wall experiment (Schultz & Flack [2013]), ——— smooth, © SG, ◯ S1, □ S2, △ S3, ▽ S4. .................................................. 26

2.4 (a)–(c) Reynolds normal stresses for cases with $k_2$ and $Re_\tau = 1000$, compared with the smooth case; --- $y = 5k$. (d)–(f) Normal components of the anisotropy tensor of the Reynolds stresses near the wall. + Smooth-wall experiment by Schultz & Flack [2013]. .................................................. 27
2.5 Reynolds shear stress for cases with $k_2$ and $Re_\tau = 1000$, compared with the smooth case. .......................................................... 28

2.6 Dependence of roughness function on the roughness Reynolds numbers based on (a) mean height and (b) equivalent sand-grain height. --- $\Delta U^+ = 1/\kappa \ln k_s^+ - 3.5$, ◀ SG , ○ S1, □ S2, △ S3, ▽ S4, × Nikuradse sand grains (Nikuradse, 1933). Lines are for visual aid only. .............. 29

2.7 Dependence of the roughness function on the effective slope on various surfaces with $\bar{k}/h = 0.04$ (clear symbols) and $\bar{k}/h = 0.067$ (filled symbols); $Re_\tau = 1000$. + Napoli et al. [2008], --- $ES = 0.35$. Current surfaces: ◀ SG, ○ S1, □ S2, △ S3, ▽ S4. ......................................................... 31

2.8 Actual values of $k_s$ and fitted correlations: (a) slope/shape method, (b) slope-rms method, and (c) moments method. ......................... 32

2.9 Artificial compressing of S2 to modify $ES$ while keeping the same mean height. One tile is shown. .................................................. 33

2.10 Variation of the roughness function as the original S2 surface is compressed by the stretching factors of 4, 8 and 16. ■ Current surfaces, + Napoli et al. [2008], --- $ES = 0.35$. ......................... 34

2.11 Wall-normal profiles of time- and space-averaged total drag distribution for all surfaces with the height $k_2$. Dashed lines indicate the location of respective zero-plane displacement. ........................................ 40

2.12 Convergence of the root-mean-square of drag averaged in time and from $n_r$ sandgrain roughness samples, $\langle f_d \rangle_{n_r}$, in two cases, (Yuan & Piomelli [2014b]) as $n_r$ increases. $\langle f_d \rangle$ is the value averaged using the largest number of samples (22311 samples). ................................................. 41
3.1 (a) Visualization of 1/8 of the surface R2; wall-normal profiles of (b) DA streamwise velocity (\(\langle \bar{u} \rangle_i\) and \(\langle \bar{u} \rangle\)) and (c) roughness geometry function. ................................................................. 49

3.2 (a) Streamwise velocity and (b) Reynolds stresses for cases SM (---), R1 (○), and R2 (△). + smooth-wall experiment [Schultz & Flack, 2013]. \(k_R\): top of the roughness sublayer. ......................................................... 50

3.3 Form-induced stresses for cases R1 (empty symbols) and R2 (filled symbols):
\[(a) \langle \tilde{u}^2 \rangle^+, (b) \nabla \langle \tilde{v}^2 \rangle^+, \triangle \langle \tilde{w}^2 \rangle^+, \text{ and } \diamond \langle \tilde{u} \tilde{v} \rangle^+. \] ................................................................. 51

3.4 Stress balance of case R2. Total stress from momentum balance, \(\triangle\)
total drag from momentum balance, \(\square\) total drag from \(\langle F_1 \rangle\) integral, \(\Box\) Reynolds shear stress, \(\diamond\) form-induced shear stress, + viscous stress due to DA mean shear. ................................................................. 52

3.5 TKE budgets for cases SM (line), R1 (empty symbol), and R2 (filled symbol).
All terms normalized by \(u^4_\tau/\nu\). \(\diamond\) \(k_c\), roughness crest. \(\diamond\) \(P_s\), \(\triangle\) \(P_w\), \(\diamond\) \(\epsilon\), \(\diamond\)
\(T_t\), \(\angle T_{\nu}\), \(\nabla T_p\). Note that the actual bottom-wall location varies. ................................. 55

3.6 Comparison of normal Reynolds stress budgets between cases R1 (empty symbols) and R2 (filled symbols): (a) streamwise, (b) wall-normal, and (c) spanwise components. \(\diamond P_s\), \(\triangle P_w\), \(\diamond\) \(\epsilon\), and \(\nabla\) \(\Pi\); \(\square\) \(T_t\) for case R1, \(\Box\) \(T_t\) for case R2 . All terms normalized by \(u^4_\tau/\nu\). ................................................................. 56

3.7 Comparison of normal form-induced stress budgets between cases R1 (empty symbols) and R2 (filled symbols): (a) streamwise, (b) wall-normal, and (c) spanwise components. Line: shear production (--- R1, --- R2), \(\nabla\) pressure work, \(\triangle P_w\), and \(\diamond\) viscous dissipation and diffusion. All terms normalized by \(u^4_\tau/\nu\). ................................................................. 58

4.1 Schematic of the simulation domain in a sink flow. From here on \(\eta\) is written as \(y\) for simplicity. ................................................................. 64

viii
4.2 Visualization of sand-grain roughness R3 for 1/8 of the domain using the iso-surface of \( \phi \) with value 0.5 (i.e., borders of roughness), coloured by the wall-normal location, with the colours purple and yellow representing the lower and higher ends, respectively.

4.3 Smooth wall simulations: \( \_\_\_\_ K = 0.45 \times 10^{-6}, \_\_\_\_ K = 2.5 \times 10^{-6} \). (a) Mean velocity; thin black dashed line: \( U^+ = 1/0.41 \log y^+ + 5 \); (b) Reynolds stresses. \( \_\_\_\_ \) Spalart DNS (Spalart, 1986), \( K = 2.5 \times 10^{-6} \); \( \_\_\_\_ \) Dixit & Ramesh (2010), \( K = 2.18 \times 10^{-6} \); \( \_\_\_\_ \) Jones & Launder (1972), \( K = 2.5 \times 10^{-6} \); + Jones et al. (2001), \( K = 0.54 \times 10^{-6} \). Superscript “+” indicates normalization in wall units.

4.4 Total shear stress and Reynolds shear stress in the smooth case with \( K = 1.5 \times 10^{-6} \). \( \_\_\_\_ \) current results; \( \_\_\_\_ \) Spalart DNS (Spalart, 1986) & Launder (1972).

4.5 Effects of roughness and acceleration on (a), (b) the Reynolds number and (c), (d) the friction coefficient. Lines connect cases with constant \( k/\delta \) in (a) and (c), and cases with constant \( k^+ \) in (b) and (d). Hollow symbols are data from current study: \( \_\_\_\_ \) smooth; \( \_\_\_\_ \) rough. Solid symbols are reference data:

4.6 Mean velocity profiles in inner scaling. (a) Effect of \( K \); (b) Effect of \( \bar{K} \). \( \_\_\_\_ \) Universal logarithmic law.

4.7 Dependence of roughness function \( \Delta U^+ \) (case K4R2 not shown) on \( k^+ \).

4.8 Roughness effects on the streamwise and wall-normal components of the Reynolds stress tensor, normalized by \( u_\tau \), in cases with (a) K3 and (b) K4.

4.9 Separate effects of \( K \) and \( \bar{K} \) on the normal Reynolds stresses and the Reynolds shear stress. \( \_\_\_\_ \) K1R1, \( \_\_\_\_ \) K3R1, \( \_\_\_\_ \) K3R3.
4.10 Normal components of the anisotropy tensor for rough cases K1R1 (--- ), K3R1 (---- ), and K3R3 (----- ) plotted in (a)–(c) outer scaling and (d)–(f) inner scaling. .................................................. 78

4.11 Energy budgets of cases K1R1 (--- ), K3R1 (---- ), K3R3 (----- ) in wall units. (a) Production and viscous dissipation, (b) turbulent diffusion and (c) viscous diffusion. .................................................. 79

4.12 Quadrant contributions from Q2 (--- ) and Q4 (---- ) events in (a) smooth cases K1R0 (○), K3R0 (□), K4R0 (▽) and (b) rough cases K1R1 (□), K3R1 (○), K3R3 (△). .................................................. 80

4.13 Quadrant contributions from Q2 and Q4 events versus the wall-normal location in outer scaling. Smooth cases: K1R0 (filled ○), K3R0 (filled □); rough cases: K1R1 (○), K3R1 (□), K3R3 (△). .................................................. 81

4.14 Contours of (a) streamwise, (b) wall-normal, and (c) spanwise time-averaged velocities in the (x, z) plane y = d for case K3R3, normalized by $u_\tau$ and $\delta_\nu$. The white contour lines denote the fluid-solid interface ($\phi = 0.5$). .................................................. 82

4.15 Root-mean-square of the spatial fluctuation of the time-averaged velocity for cases K3R0 and K3R3, normalized by the root-mean-square of the local turbulent fluctuation in the respective direction: --- streamwise, ---- wall-normal, --- spanwise. $k_{R3}$ is the roughness height for group R3. .................................................. 83

4.16 Isosurfaces of $u'^+ = -3$ (yellow) and $Q'^+ = 0.01$ (coloured by $(y - d)^+$) in cases (a) K1R1, (b) K3R1 and (c) K3R3. Rough surfaces are shown in white. .................................................. 84

4.17 Isosurfaces of components of $Q$: mean-flow component (white), turbulent component (purple), and the component corresponding to mean-flow and turbulence interaction (yellow). .................................................. 85

4.18 Two-point correlations of streamwise fluctuations, $R_{uu}$, centered on $y = k$. Contour levels: from 0.3 to 0.9 with increment 0.1. Cases (a) K1R1, (b) K3R1, and (c) K3R3 are shown. .................................................. 86
4.19 Integral length scale in (a) the streamwise and (b) spanwise directions in inner scaling. —— K1R0, --- K3R0, □ K1R1, ○ K3R1, △ K3R3. Black and red line colours denote K3 and K1 cases, respectively. 87

4.20 Instantaneous velocity vectors and contours of negative $\omega_z$ in case (a) K1R1, (b) K3R1, (c) K3R3. The convection velocity $U_c = 0.8U_\infty$ is subtracted from the velocity field. Lines indicate angles of hairpin packets. Contour level: $-5.7 \leq \omega_z\delta/U_\infty \leq -0.8$. Every other grid point in $x$ and $y$ directions is used for plotting velocity vectors. 89

4.21 Two-point correlations of streamwise fluctuations, $R_{uu}$, centred on $y = 0.3\delta$ in fully turbulent rough-wall cases: (a) K1R1, (b) K3R1, (c) K3R3, (d) K4R3. Contour levels: 0.3 to 0.9 with increment 0.1. 90

4.22 Integral length scale in (a) the streamwise and (b) spanwise directions in inner scaling. —— K1R0, --- K3R0, □ K1R1, ○ K3R1, △ K3R3. Black and red line colours denote K3 and K1 cases, respectively. 92

4.23 Contour lines of $|u'v'|_Q^2$ (black) and $|u'v'|_Q^4$ (white) induced by the hairpin structures (isosurfaces of $QX_o/U_{\infty,o} = 800$) in Case K3R3. Plane located at $(y - d)/\delta = 0.5$. Contour of negative $\omega_z\delta/U_\infty$ shows hairpin heads. Contour line levels: from $\sigma_{u'^v'}$ to $20\sigma_{u'^v'}$ with increment $\sigma_{u'^v'}$. 93

4.24 Averaged separation (inverse of density) of Q2 events detected with $H = 1$ at different near-wall elevations in smooth cases. *Chambers et al. (1983) experiment. 95

4.25 (a) Averaged separation and (b) intermittency for strong Q2 events with $H = 1$ in cases K1R1, K3R1, and K3R3, compared to corresponding smooth cases. Black and red line colours denote K3 and K1 cases, respectively. 96
4.26 Averaged separation of strong Q2 events in case K1R1, K3R1, and K3R3

in inner scaling: (a) $H = 1$, --- - : cases with $k^+ \approx 40$; (b) $H = 2$. Black

and red line colours denote K3 and K1 cases, respectively. The blue line

highlights the collapse of profiles corresponding to $k^+ \approx 40$.

5.1 Sketch of the configurations for the smooth (Piomelli & Yuan, 2013) and

rough cases.

5.2 Contours of (a) pressure gradient $(\partial P/\partial x)/(U_{\infty,o}^2/\delta^*_o)$ and (b) $K$. Thick and

thin lines represent the upper limit of the boundary layer and the streamlines,

respectively.

5.3 Contours of (a) $\langle u \rangle/U_{\infty,o}$ and (b) $\langle v \rangle/U_{\infty,o}$; --- $\delta$. Boundary layer inte-

gral parameters: (c) acceleration parameter $K$; (d) Reynolds number $Re^*_\delta$ =

$U_\infty \delta^*/\nu$, (e) friction coefficient, $C_f = 2u_\tau^2/U_{\infty}^2$ and (f) friction velocity,

$u_\tau/U_{\infty,o}$.

5.4 (a) Streamwise variation of the roughness Reynolds number; --- $k^+ = 60$.

(b) Streamwise variation of $2(u_\tau/U_{RS})^2$; --- the constant value (0.032) in

the fully rough region ($220 < x/\delta^*_o < 400$).

5.5 Profiles of mean streamwise velocity $U$ at various streamwise locations, nor-

malized in (a) wall units and (b) outer scaling; solid and dashed lines present

the smooth and rough cases, respectively. The thin dashed lines in (a) indi-
cate $U^+ = y^+$ and the universal logarithmic law $U^+ = \log y^+ / 0.40 + 5.0$; the

thin lines in (b) show the velocity profiles at the reference location $x/\delta^*_o = 0$.

5.6 Diagnostic function in (a) the smooth and (b) the rough cases. In (a), ---

$\Xi = y^+$ and $\Xi = 2.50$ (corresponding to $\kappa = 0.40$); $\Delta y^+ = 35$, $\bigcirc y/\delta = 0.2$.

In (b), --- $\Xi = 2.50$; $\Delta y/k = 2$, $\bigcirc y/\delta = 0.2$. 

xii
5.7 Variation of (a) local logarithmic slope and (b) local intercept. Symbols: ○ smooth case, △ rough case. Shaded regions indicate the streamwise regions where the logarithmic profile is not well defined.

5.8 Comparison of the roughness function between the Nikuradse sandgrain and the current numerical sandgrain (Yuan & Piomelli, 2014a,c), whose $k_{s∞} = 1.6 k$. The solid line shows the fitted profile for Nikuradse sandgrain in the fully rough regime (Nikuradse, 1933).

5.9 (a) Variation of the product of $\tilde{\kappa} \tilde{B}$ as a function of the intercept with and without the roughness correction; the arrow denotes the start of the quasi-laminarization. (b) An enlargement focusing on the rough-case data.

5.10 Horizontal contours of (a) $\tilde{u}/U_{∞,o}$ and (b) instantaneous $u'/U_{∞,o}$ at $y = d$ in a streamwise region centred at $x/\delta_o^* = 20$, and (c) contour of the time-averaged velocity, $\bar{u}/U_{∞,o}$, in a $xy$-plane, with the dashed line representing the contour line of $\bar{u} = 0$. The white and black arrows indicate, respectively, the recirculation between two close elements and the flow reattachment between two elements separated with a longer streamwise distance.

5.11 Profiles of the dispersive stresses, including (a) the streamwise normal component, and (b) the other two normal components and the shear component, normalized by local $u_τ$. —— $x/\delta_o^* = 50$, —— $x/\delta_o^* = 220$, --- $x/\delta_o^* = 300$, --- $x/\delta_o^* = 400$. Arrows indicate the direction of the flow acceleration.

5.12 Two-point correlations of $\tilde{u}$ with (a) streamwise, and (b) spanwise separations, at three $x$ locations ($x/\delta_o^* = 50, 220, 350$).

5.13 Profiles of normal Reynolds stresses, normalized by local $U_{∞}$: (a)-(c) smooth; (d)-(f) rough. —— $x/\delta_o^* = 50$; --- $x/\delta_o^* = 200$; --- $x/\delta_o^* = 300$. The arrow indicates the direction of flow acceleration.
5.14 Profiles of normal Reynolds stresses, normalized by local $u_\tau$: (a)-(c) smooth;
(d)-(f) rough. $x/\delta_o^* = 50$; $x/\delta_o^* = 200$; $x/\delta_o^* = 300$. The arrow indicates the direction of flow acceleration.

5.15 Profiles of Reynolds shear stress in (a) smooth and (b) rough cases, normalized by $u_\tau$. $x/\delta_o^* = 50$; $x/\delta_o^* = 200$; $x/\delta_o^* = 300$; $x/\delta_o^* = 400$.

5.16 The evolution of Reynolds-stress invariants at three wall-normal locations for $50 \leq x \leq 330$: (a) smooth and (b) rough cases. Symbols show the most upstream location. Arrows indicate the direction of increasing $x$.

5.17 Effects of FPG on budget terms of (a) $\langle u'^2 \rangle$, (b) $\langle v'^2 \rangle$, and (c) $\langle w'^2 \rangle$, normalized in wall units. $x/\delta_o^* = 50$, $x/\delta_o^* = 200$, $x/\delta_o^* = 300$. Arrows show the direction of flow acceleration.

5.18 Ratio between the time scales of the turbulence, the wake shear, and the mean shear in (a) smooth case and (b) rough case. $x/\delta_o^* = 50$, $x/\delta_o^* = 220$, $x/\delta_o^* = 300$. Thick lines: $S^*$; thin lines: $S^*_w$. Arrows indicate increase of $x$. 

xiv
## List of Symbols

### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>APG</td>
<td>adverse pressure gradient</td>
</tr>
<tr>
<td>DA</td>
<td>double-averaging</td>
</tr>
<tr>
<td>DNS</td>
<td>direct numerical simulation</td>
</tr>
<tr>
<td>ES</td>
<td>effective slope</td>
</tr>
<tr>
<td>FPG</td>
<td>favourable pressure gradient</td>
</tr>
<tr>
<td>IBM</td>
<td>immersed boundary method</td>
</tr>
<tr>
<td>LES</td>
<td>large-eddy simulation</td>
</tr>
<tr>
<td>MKE</td>
<td>mean kinetic energy</td>
</tr>
<tr>
<td>Q1–Q4</td>
<td>first to fourth quadrant of turbulent fluctuations</td>
</tr>
<tr>
<td>RHS</td>
<td>right-hand-side</td>
</tr>
<tr>
<td>RMS</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds-averaged Navier-Stokes</td>
</tr>
<tr>
<td>SG</td>
<td>sand-grain</td>
</tr>
<tr>
<td>SM</td>
<td>smooth</td>
</tr>
<tr>
<td>TKE</td>
<td>turbulent kinetic energy</td>
</tr>
<tr>
<td>WKE</td>
<td>wake kinetic energy</td>
</tr>
<tr>
<td>ZPG</td>
<td>zero pressure gradient</td>
</tr>
</tbody>
</table>
Roman symbols

\( B \)  
mean-velocity profile intercept in the logarithmic region

\( b_{ij} \)  
Reynolds stress anisotropy tensor

\( C_f \)  
friction coefficient

\( C_D \)  
drag coefficient of a single roughness element

\( \overline{C_D} \)  
mean drag coefficient of rough surface

\( d \)  
zero-plane displacement

\( f_d \)  
total drag

\( f_p, f_\nu \)  
form drag and viscous drag

\( F_i \)  
IBM body force in \( i \) direction

\( G_i \)  
sink-flow growth term in \( u_i \)-momentum equation

\( h \)  
channel half-height

\( H \)  
strength threshold of quadrant events

\( k \)  
certain quantification of roughness height in an average sense

\( k_s \)  
grain size of Nikuradse uniform sandgrain, or equivalent sand-grain height

\( k_{s,\infty} \)  
equivalent sandgrain height (fully rough regime)

\( k^+ \)  
roughness Reynolds number

\( k_{s,cri}^+ \)  
critical roughness Reynolds number as the start of fully rough regime

\( k_{\text{max}}, k_c \)  
roughness crest

\( k_u \)  
kurtosis of surface height statistics

\( k_R \)  
top of roughness sublayer

\( \overline{k} \)  
mean value of local roughness-height distribution

\( K \)  
acceleration parameter

\( \mathcal{K} \)  
TKE
$L_x, L_y, L_z$  
domain size

$N_x, N_z$  
horizontal resolution of a single roughness element

$n_i, n_j, n_k$  
grid size

$P$  
pressure

$P_s, P$  
shear production

$P_w$  
wake production

$Q$  
flow rate

$Re$  
Reynolds number

$s_k$  
skewness of surface height statistics

$\overline{S}$  
mean shear rate

$S^*$  
ratio between time scale of turbulence and that of mean shear rate

$S_w$  
wake shear rate

$S_w^*$  
ratio between time scale of turbulence and that of wake shear rate

$t$  
time

$T$  
total simulation time

$T_p$  
pressure transport

$T_t$  
turbulent transport

$T_v$  
viscous transport

$u_i$  
instantaneous velocity components

$u_\tau$  
friction velocity

$U_i$  
mean velocity components

$U_{i,\infty}$  
freestream velocity

$U_c$  
convection velocity

$U_{RS}$  
mean velocity at the top of roughness sublayer
$x_i$  
  direction

$X$  
  local distance from the sink (sink flow)

**Greek symbols**

$\alpha$  
  local slope of rough surface

$\gamma$  
  burst intermittency

$\Delta x, \Delta y, \Delta z$  
  grid spacing

$\Delta U^+$  
  roughness function

$\delta$  
  boundary layer thickness

$\delta^*$  
  displacement thickness

$\delta_{ij}$  
  Kronecker Delta

$\delta_\nu$  
  viscous length scale

$\epsilon$  
  viscous dissipation

$\eta$  
  Kolmogorov length scale, or the second-order invariant of Reynolds stress anisotropy tensor (Chapter 5)

$\theta$  
  momentum thickness

$\kappa$  
  von Kármán constant

$\lambda$  
  Taylor micro-scale of local roughness-height fluctuations

$\Lambda_s$  
  surface slope/density parameter

$\nu$  
  kinematic viscosity

$\nu_{sgs}$  
  subgrid-scale viscosity

$\xi$  
  third-order invariant of Reynolds stress anisotropy tensor

$\Xi$  
  diagnostic function of logarithmic region

$\Pi$  
  pressure work

$\rho$  
  density

$\sigma$  
  RMS of fluctuations
\( \tau \)  
total shear stress

\( \tau_{ij} \)  
subgrid-scale stress tensor

\( \tau_w \)  
wall shear stress

\( \phi \)  
fraction of a grid cell occupied by fluid

\( \Phi \)  
roughness geometric function

\( \omega \)  
turbulent vorticity

**Others symbols**

\( \overline{\cdot} \)  
non-dimensionalization using \( X_o \) and \( U_{\infty,o} \) in Chapter 4, time-average in Chapters 3 and 5

\( \langle \cdot \rangle_t \)  
time-average in Chapters 2 and 4, since \( \overline{\cdot} \) is used otherwise

\( \langle \cdot \rangle \)  
time- and superficial space-average in Chapters 2 and 4, superficial spatial average in other chapters

\( \langle \cdot \rangle_i \)  
intrinsic spatial average

\( \widetilde{\cdot} \)  
spatial variation of time-averaged quantity

\( \hat{\cdot} \)  
non-divergence-free quantity before pressure correction

\( \cdot' \)  
turbulent fluctuations

\( \cdot^+ \)  
non-dimensional quantity normalized using viscous length and velocity scales, \( \nu/u_\tau \) and \( u_\tau \)

\( \cdot_o \)  
quantity at the inlet of simulation domain
Table of Contents

Abstract i

Co-Authorship iii

Acknowledgments iv

List of Tables v

List of Figures vi

List of Symbols xv

Table of Contents xx

Chapter 1:

General introduction 1

1.1 Background and motivation 1

1.2 Literature review 4

1.3 Objectives and outline 12

Chapter 2:

Estimation and prediction of the roughness function on realistic surfaces 14

2.1 Abstract 14
2.2 Introduction ......................................................... 15
2.3 Problem formulation ........................................... 20
2.4 Results ............................................................. 26
2.5 Conclusions ......................................................... 35
Appendices ............................................................. 36

**Chapter 3:**

Roughness effects on the Reynolds stress budgets in near-wall turbulence ................................................. 43

3.1 Abstract ............................................................. 43
3.2 Introduction ......................................................... 44
3.3 Problem formulation ........................................... 46
3.4 Results ............................................................. 50
3.5 Conclusions ......................................................... 58

**Chapter 4:**

Sink-flow boundary layers over rough surfaces ......................... 60

4.1 Abstract ............................................................. 60
4.2 Introduction ......................................................... 61
4.3 Problem formulation ........................................... 64
4.4 Results ............................................................. 69
4.5 Conclusions and discussions ................................... 98

**Chapter 5:**

Numerical simulation of a spatially developing accelerating boundary layer over roughness .......................... 101

5.1 Abstract ............................................................. 101
5.2 Introduction ......................................................... 102
5.3 Problem formulation .................................................. 106
5.4 Results ................................................................. 110
5.5 Conclusions and discussions ........................................ 131

Chapter 6:

Discussion and future work .............................................. 134
6.1 Discussion ............................................................. 134
6.2 Future work ........................................................... 137

Bibliography ............................................................... 140
Chapter 1

General introduction

1.1 Background and motivation

The flow over rough surfaces has been the object of investigation since the early measurements of [Nikuradse (1933)]. Boundary layers over rough walls are significantly different than those over smooth walls. In the latter case, for instance, turbulence is never completely independent of Reynolds number. If the roughness is large enough, on the other hand, some quantities (most notably, the friction coefficient) become independent of Reynolds number.

Roughness plays an important role in many fields of study; in geophysical and environmental applications, for instance, most surfaces of interest are rough (hilly terrain, plant canopies, urban environments, etc.). In engineering, roughness is particularly important in electronic cooling, turbomachinery, and duct and pipe flows. The last two areas of application are of interest to the power industry. Roughness is especially important in penstocks (the ducts that convey water to the turbines) and in the turbines themselves.

This thesis is motivated by the significant effects of surface roughness on the hydraulic turbine performances. Figure 1.1(a) shows the top view of a Francis turbine, the most common type of water turbine in use. The flow comes into the spiral casing from the side; it is then directed by the stay vanes and the guide vanes and impact the runner blades at design
While having a universal law describing the impact of roughness based on only one parameter is very attractive for engineers, finding the equivalent sand grain for an arbitrary surface is not trivial and this poses a first real challenge when trying to address the impact of roughness. A lot of studies were performed to find a proper relation between geometrical characteristic of the surface and equivalent sand grain, but most were application specific as reported by Bons \[4\]. Acknowledging this, the IEC code \[5\] recommended to use a simple linear relation, $K_s = 5 \times Ra$, between sand grain and arithmetic roughness.

To gain an understanding of the problem, roughness samples were taken on various sites on real turbines (figure 1) and brought back to IREQ laboratory for digitalisation and measurement with an optical profilometer CHR150 from Microphotonics. Table 1 presents the arithmetic average value $Ra$ obtained for 4 different samples. Following the ASME standard \[9\], a Gaussian filter was used to separate between oscillations of the surface from roughness. The filter value is the maximal distance that a roughness element can have before being smoothed. For expected $Ra$ above 10 microns, the recommended cutoff length is 8 mm. One can note on table 1 the impact of the filter settings on the results meaning that even finding the proper arithmetic roughness is not trivial. This motivated to a great extent the need for high resolution simulation to characterise the behaviour of rough surface.

Table 1. Measured value (in μm) of arithmetic roughness in a ?? years old turbine

<table>
<thead>
<tr>
<th>Name (location)</th>
<th>Unfiltered 8 mm</th>
<th>2.5 mm</th>
<th>0.8 mm</th>
<th>0.2 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 (guide vanes)</td>
<td>293.3</td>
<td>109.4</td>
<td>47.2</td>
<td>22.5</td>
</tr>
<tr>
<td>S2 (runner)</td>
<td>74.1</td>
<td>56.4</td>
<td>33.8</td>
<td>17.5</td>
</tr>
<tr>
<td>S3 (stay vane)</td>
<td>968.5</td>
<td>212.3</td>
<td>83.3</td>
<td>36.0</td>
</tr>
<tr>
<td>S4 (runner)</td>
<td>75.6</td>
<td>45.0</td>
<td>31.5</td>
<td>16.2</td>
</tr>
</tbody>
</table>

Figure 1. Surfaces of an hydraulic distributor, while having a universal law describing the impact of roughness based on only one parameter is very attractive for engineers, finding the equivalent sand grain for an arbitrary surface is not trivial and this poses a first real challenge when trying to address the impact of roughness. A lot of studies were performed to find a proper relation between geometrical characteristic of the surface and equivalent sand grain, but most were application specific as reported by Bons \[4\]. Acknowledging this, the IEC code \[5\] recommended to use a simple linear relation, $K_s = 5 \times Ra$, between sand grain and arithmetic roughness.

To gain an understanding of the problem, roughness samples were taken on various sites on real turbines (figure 1) and brought back to IREQ laboratory for digitalisation and measurement with an optical profilometer CHR150 from Microphotonics. Table 1 presents the arithmetic average value $Ra$ obtained for 4 different samples. Following the ASME standard \[9\], a Gaussian filter was used to separate between oscillations of the surface from roughness. The filter value is the maximal distance that a roughness element can have before being smoothed. For expected $Ra$ above 10 microns, the recommended cutoff length is 8 mm. One can note on table 1 the impact of the filter settings on the results meaning that even finding the proper arithmetic roughness is not trivial. This motivated to a great extent the need for high resolution simulation to characterise the behaviour of rough surface.

Table 1. Measured value (in μm) of arithmetic roughness in a ?? years old turbine

<table>
<thead>
<tr>
<th>Name (location)</th>
<th>Unfiltered 8 mm</th>
<th>2.5 mm</th>
<th>0.8 mm</th>
<th>0.2 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 (guide vanes)</td>
<td>293.3</td>
<td>109.4</td>
<td>47.2</td>
<td>22.5</td>
</tr>
<tr>
<td>S2 (runner)</td>
<td>74.1</td>
<td>56.4</td>
<td>33.8</td>
<td>17.5</td>
</tr>
<tr>
<td>S3 (stay vane)</td>
<td>968.5</td>
<td>212.3</td>
<td>83.3</td>
<td>36.0</td>
</tr>
<tr>
<td>S4 (runner)</td>
<td>75.6</td>
<td>45.0</td>
<td>31.5</td>
<td>16.2</td>
</tr>
</tbody>
</table>

Figure 1. (a) The schematics of a typical Francis turbine (top view, source: [http://www.learnengineering.org/](http://www.learnengineering.org/)), and (b) roughness patches obtained at different locations of an existing Francis turbine (side view, indicated by the arrow in (a)) (Yuan et al., 2014).
angles; finally, the flow exits to the draft tube through the runner exist. Hydraulic turbines have long lifespans, sometimes over 100 years. Over this lifespan, surface degradation occurs and significant roughness contaminates the solid surfaces: corrosion, impacts from debris, cavitation, and cracks alter the initial smooth finish. Figure 1.1(b) gives a side view of the Francis turbine, with visualization of patches of the rough surfaces (S1 to S4) at different locations; it can be seen that these surfaces are different in characteristics, due to the variation of the mechanisms through which they are generated. These surface degradations increase with time, affecting the overall performance of equipment in a significant way: in a Francis turbine, roughness can result in a loss of efficiency of 1.25% (Grenier et al., 2007), a significant amount in light of the high efficiency of over 90% for a typical turbine. Methods for surface smoothing are available; the cost of smoothing penstocks and turbine surfaces is, however, significant, both because of direct costs and due to the shut-down time needed. The benefit can justify the investment, but the power industry needs an accurate way to evaluate losses related to surface roughness as a decision-making tool.

This thesis studies turbulent flows over roughened walls, as a first step to propose improvements for the applications of interest to the power industry. The results on the physics and modeling, however, would benefit much wider applications (e.g., aerospace and ship industries and environmental studies) beyond the power industry. These studies are performed using high-resolution simulation techniques (DNS and LES), at lower Reynolds numbers (sufficient, however, to achieve fully rough flows, i.e., the flows in which the roughness effects on the turbulence are fully developed). The study will not attempt to address
the geometric complexities that are encountered in hydro-electric plants; instead, some fluid-dynamical non-equilibrium processes, specifically, flow acceleration (favourable pressure gradient, FPG) as prominent features of flow over turbine blades (Figure 1.2), will be studied. Here, “equilibrium” describes the self-similarity of the flow quantities in the flow direction. The objective then simplifies to the study of flat-plate boundary layer flows over surface roughness with FPG.

1.2 Literature review

1.2.1 Roughness effects on turbulent flows

Experimental studies

Researchers studying the pressure drop in pipes were the first to recognize the importance of roughness in turbulent flows. Systematic experimental studies of turbulent flow in rough pipes were carried out by Nikuradse (1933) and Colebrook (1939), and resulted in the well-known Moody diagram. Since then, a substantial amount of work has been done to understand the dynamics of turbulent flows over rough-walls, both for engineering and atmospheric applications. Reviews by Raupach and co-workers (Raupach & Thom, 1981; Raupach et al., 1991) and Finnigan (2000) summarize the research on roughness in atmospheric applications, while those by Raupach et al. (1991) and Jiménez (2004) discuss engineering flows. A complete review of the vast body of experimental investigations is beyond the scope of this introduction, and this introduction is limited to the studies that are more relevant to the current work.

The roughness effects on the mean flow depend on how far the roughness elements extend into the flow above. When the roughness height, $k$ (determined as certain statistical quantification of the local height fluctuations), is much higher than the viscous length scale, $\delta_v = \nu/\tau$ (where $\nu$ is the kinematic viscosity, $\tau$ is the friction velocity), $k$ is the only
important length scale in the inner region; therefore, the logarithmic profile of the mean velocity $U$ in the overlap region does not depend on $\delta_{\nu}$, but $k$ alone,

$$U^+(y^+) = \frac{1}{\kappa} \log \left( \frac{y}{k} \right) + C,$$

where $y$ is the wall-normal location, the subscript “+” denotes the normalization using wall units (friction velocity, $u_\tau$, and $\delta_{\nu}$), $\kappa$ is the Kármán constant, and $C$ is the intercept of the logarithmic profile. This is called the “fully rough” regime, and corresponds to the constant-friction region in the Moody chart for sufficiently high Reynolds numbers. In analogy to a smooth-wall profile, the logarithmic profile of $U$ can also be expressed as

$$U^+(y^+) = \frac{1}{\kappa} \log y^+ + B - \Delta U^+(k^+),$$

where $B = 5.0$ is the universal logarithmic intercept for the smooth-wall flows, and the “roughness function”, $\Delta U^+$, depends on both $k^+$ and the surface texture, and quantifies the momentum deficit due to roughness. In the fully rough regime, $\Delta U^+$ is a logarithmic function of $k^+$. Here, $k^+$ is also called the “roughness Reynolds number”, quantifying how far the roughness extends into the turbulent boundary layer. To account for the variation of $\Delta U^+$ with the roughness type, usually a “universal measurement” is used to quantify the momentum deficit for an arbitrary surface: $k^+_s = k_s u_\tau / \nu$, the roughness Reynolds number based on the Nikuradse sandgrain roughness height [Nikuradse 1933], $k_s$, that produces the same $\Delta U^+$ in the fully rough regime. It follows that $k_s$ is a fixed parameter of a surface, characterizing its ability to increase frictional drag; $k_s$ is also called the “equivalent sandgrain height”.

For $k$ not significantly larger than $\delta_{\nu}$, the flow is in the “transitionally rough” regime. In this regime, $\Delta U^+$ is no longer a logarithmic function of $k^+$; for certain types of roughness (riblets, for example), $\Delta U^+$ takes negative values for a range of $k^+$ in this regime,
leading to reduced friction compared to the smooth-wall flow. For \( k \) of the order of \( \delta_\nu \),
the viscous effects become dominant close to the wall, and the flow behaves effectively as a
smooth-wall flow; this is called the “hydraulically smooth” regime. Note that the roughness
Reynolds numbers separating the three regimes vary with the roughness type; for example,
Langelandsvik et al. (2008) showed that the range of the transitionally rough regime is
\( 1.4 < k_s^+ < 18 \) for a commercial steel pipe, while for Nikuradse sandgrain, transitionally
rough regime applies to \( 5 < k_s^+ < 80. \)

Most of the investigations of roughness effects on turbulence have concentrated on canonical
flows: zero-pressure-gradient (ZPG) boundary layers, channels and pipes. Several forms
of roughness have been studied (sand-grain, rods, perforated plates, wire screens, protuber-
ances, and so on). Important features of the flow over a general roughness have been
established: the increase in wall-normal fluctuations (\( v' \)) near the wall (for example, see
Krogstad et al., 1992) and the decreased anisotropy of the Reynolds stresses (for example,
Shafi & Antonia, 1995) have been observed in many studies. In addition, when normalized
by wall units, the turbulent statistics are affected by roughness within the near-wall region
only (Schultz & Flack, 2007; Kunkel et al., 2007); this region is called the “roughness sub-
layer”. The thickness of this sublayer depends on the fluid-dynamical quantity examined,
but is of the order of \( 3 - 5k_s \) (Flack et al., 2007). Outside of the sublayer, roughness is
unable to affect the shape of the turbulent structures (Volino et al., 2007) due to a large
separation between the length scales of the roughness and the large turbulent structures.

It is important to observe that the roughness geometry plays a role on both the frictional
drag and the near-wall turbulence structures. For example, Volino et al. (2009) compared a
surface consisting of two-dimensional (2D) transverse square bars and a three-dimensional
(3D) woven roughness with the same height, and observed that the 2D geometry leads to
one order-of-magnitude higher \( k_s \) than the 3D geometry, due to the effects of the large
spanwise length scale of the 2D geometry. Flack & Schultz (2010) compared a wide range
of roughness geometries (sandpaper, semi-spheres, pyramids, etc.) and found that they give
drastically different $k_s/k$, where $k$ is the peak-to-trough height. For two different types of roughness (mesh and rods) that produce the same $k_s$, Krogstad & Antonia (1999) observed significant differences in the velocity spectra, the turbulence productions, and diffusions. In addition, Christensen and co-workers (Wu & Christensen, 2006; Johnson & Christensen, 2009; Mejia-Alvarez & Christensen, 2010) studied patches of realistic roughness replicated from a turbine blade and observed that both the large- and small-scale surface structures strongly affect the turbulence motions.

For practical applications, it is important to predict the wall friction over a given surface; this is because, for most engineering turbulence models, the roughness effects on the turbulence are not resolved, but modelled, requiring a single roughness input to represent the change of turbulence length and velocity scales at the wall. This input is usually $k_s$, and is not known a priori.; therefore, it needs to be determined either through experiments or numerical simulations (resolving the roughness surface) prior to the engineering simulations, or by predicting $k_s$ based on the surface details. For the latter approach, various correlations have been proposed from a wide range of studies to predict $k_s$ (summarized in the review by Flack & Schultz, 2010). However, it remains elusive how to select a correlation for an arbitrary surface.

**Numerical studies**

Over the last ten years the advances in computer power have made it possible to compute flows over rough walls using direct numerical simulations (DNS), in which all the scales of motion are resolved (including the Kolmogorov length) and no modelling is required. This type of numerical simulation captures the flow around the roughness elements, providing an accurate determination of the drag, flow statistics, and turbulent structures within and above the roughness sublayer. The main obstacle of DNS is the computational cost (see the discussion in Jiménez (2004)): to achieve fully rough flows one usually needs $k_s^+ > 80$; at the same time one needs to have a domain large enough to minimize contamination of the
entire boundary layer by the roughness ($\delta > 20k_s$, where $\delta$ is the boundary layer thickness). Meeting both criteria is expensive, and most of the numerical simulations that have been performed so far relax one (or both) of these requirements.

In many studies, the immersed-boundary methods (IBM) were used to model the presence of the roughness elements, especially in the case with complex roughness geometries where a body-fitted grid is hard to generate. In general, the IBM is based on a simple (usually Cartesian) grid structure, and uses localized forces to set velocities to zero on the solid-fluid interface immersed in the mesh. Reviews of this type of method can be found, for example, in Mittal & Iaccarino (2005).

Numerical experiments have been used to calculate mean-flow and turbulent statistics on different types of surface. For instance, Leonardi et al. (2003) performed DNS of channel flows in which square rods were used to roughen one of the walls. They captured the recirculation zones occurring immediately upstream and downstream of each roughness element and studied the relation between the drag and the roughness geometry. Orlandi & Leonardi (2008) carried out DNS of channels using square and circular protrusions to generate 3D roughness; they observed a close correlation between roughness function and $u'$ intensity at the roughness crest. Bhaganagar and co-workers (Bhaganagar et al., 2004, 2007) studied a channel flow with one wall roughened by 3D “egg-carton” roughness, in the transitionally rough regime. They observed that roughness increases the intensity of the pressure fluctuations and decreases their length scale; however, the pressure fluctuations give different statistics compared to those of the velocity fluctuations, for instances, in the thickness of the roughness sublayer and in the length scale. DNS was also carried out by Coceal et al. (2006) on regular arrays of cubical obstacles; significant dispersive stress – the stress arising due to the spatial heterogeneity of the rough surface – was observed within the arrays, while, above the arrays, the Reynolds stress dominates.

The ability of DNS to provide insights into the turbulent structures, both near the wall and in the outer layer, has also been demonstrated in many studies. For examples,
Sung (2007) and Lee et al. (2011) studied turbulent boundary layers with zero pressure gradient and found that roughness affects both the near-wall streaky structures and the outer-layer hairpin packets. However, major differences in the structure shape resulted only in the inner layer, indicated by the length and width of the low-speed streaks. Ikeda & Durbin (2007) also observed, from DNS of the flow in a channel roughened by square rods, that the 2D roughness, similar to a 3D one, generates 3D flow structures. Recently, Castillo et al. (2013) carried out DNS of a turbulent boundary layer over a turbine roughness geometry. Through the mode decomposition of the turbulent motions, it was observed that roughness redistributes TKE so that the energy is shifted from larger-scale structures to those with smaller scales; also, it decreases the characteristic length scales of turbulence.

Large-eddy simulations (LES), in which the momentum- and energy-transporting eddies are resolved and only the smaller, dissipative scales are modelled, were applied to rough-wall boundary layers for many years. LES is usually used to achieve the fully rough regime, where the large anisotropic turbulent eddies, of the same order-of-magnitude of the roughness length scale, are much larger than the Kolmogorov scale. But LES is less applied to the transitionally rough regime, where a well-resolved surface requires resolution closer to the Kolmogorov scale, and thus the LES subgrid-scale model becomes unnecessary. For this reason, LES has been often applied in atmospheric boundary-layer studies with high roughness Reynolds numbers and relatively low $\delta/k$. For instance, Xie et al. (2008) carried out LES of flows over urban-like obstacles with $\delta/k = 10$ (where $k$ is the obstacle height) and $k^+ \approx 400$. Kanda et al. (2004) studied flows over various types of building array with $\delta/k = 6$, and found that the array type has strong effects on the details of turbulent organized structures around the cubes, but not on the type of the structures.

Although numerical simulation has been mostly used to study roughness effects in canonical wall-bounded flows, such as flat-plate turbulent boundary layers with ZPG and steady channel flows, it also shows a strong potential to elucidate the physics underlying more complex scenarios, such as flows with acceleration, deceleration, curvature, rotation, or
transience. In addition, the role of roughness topography on the flows should be further studied numerically, extending from regular roughness (such as bars and blocks) to realistic surfaces with a wide spectrum of length scales; it is easier to numerically simulate a specific surface than to manufacture it physically for experiments.

### 1.2.2 Effects of freestream acceleration

Spatially accelerating flows result from pressure gradients or surface curvatures. They are present in a wide range of realistic scenarios, such as the leading edge of an airfoil and an atmospheric flow approaching complex landscape. Previous studies are mostly focused on flow acceleration over a smooth wall. For a turbulent boundary layer under strong accelerations, although the kinetic energy of the mean flow is increased as a result of acceleration, turbulence may become less vigorous and the flow may revert to a “quasi-laminar” state (a state in which the turbulence is present, but its effect on the mean-momentum transport is negligible compared to the pressure gradient), through a process called “relaminarization”.

The mechanisms of relaminarization on a smooth wall have been widely studied. Narasimha & Sreenivasan (1973) attributed the reversion to the domination of pressure forces over the slowly responding Reynolds stresses in an originally turbulent flow, which leads to the generation of a new laminar boundary layer stabilized by FPG. The turbulence, however, does not diminish in the region of strong acceleration, but an increasing fraction of it plays a passive role in the boundary-layer development (Lauder, 1964). Moreover, the turbulence production was found to exceed dissipation in the quasi-laminar region (Narasimha & Sreenivasan, 1973), thus the reversion is more than mere Reynolds number effect. Warnack & Fernholz (1998) observed that the Reynolds shear stress development in the outer layer lags behind that of the wall shear stress, \( \tau_w \), and that the turbulent anisotropy increases in the inner layer, with most of the TKE residing in the streamwise turbulent fluctuations, \( u' \). Such decrease of the wall-normal (\( v' \)) and spanwise fluctuations (\( w' \)) was later pointed
out by Bourassa & Thomas (2009) to enhance the stability of streaks and near-wall vortices (Jiménez & Pinelli, 1999; Schoppa & Hussain, 2002). However, there were not enough data to explain these variations of wall-normal and spanwise anisotropies in the inner layer. Very recently, the DNS of a similar type of flow carried out by Piomelli & Yuan (2013) confirmed that the decrease of Reynolds shear stress (in wall units) during relaminarization is due to the significant damping of $v'^{+}$ (with the “+” indicating normalization in wall units), instead of a decorrelation between $u'$ and $v'$. It was proposed that the mean-flow variation first damps the pressure fluctuations, which leads to weaker TKE redistribution among Reynolds normal stresses and reductions in $v'$ and $w'$, ultimately stabilizes the low-speed streaks.

A few studies have investigated the accelerating flows on rough walls. Coleman et al. (1977) measured the velocity field in a turbulent boundary layer over an arrangement of copper spheres, with a flexible upper wall of the wind tunnel, generating mild acceleration. The study focused on equilibrium boundary layers in the sense that the mean velocity and Reynolds stress profiles become self-similar and the pressure gradient parameter and the Reynolds number become a constant. FPG was found to make the surface “rouugher” by increasing $k^{+}$; therefore, the flow does not develop towards the transitionally rough or the effectively smooth state. The intensities of $v'$ and $w'$ decrease compared to the freestream, $U_{\infty}$, throughout the boundary layer while $u'$ peak is unaffected, leading to a more anisotropic turbulence.

Cal and co-workers (Cal et al., 2008, 2009) performed experimental studies of roughness in non-equilibrium boundary layers with mild accelerations achieved using a tilted bottom wall. Competing effects between the roughness and FPG were observed: while roughness makes the mean velocity ($U$) profiles less full and tends to intensify both $u'$ and $v'$ and the Reynolds shear stress, FPG makes the $U$ profiles fuller and slightly reduces $u'$ and $v'$ in the outer part of the boundary layer. However, in the inner part of the boundary layer ($y/\delta < 0.3$), FPG further increases the more intense turbulent fluctuations on the
rough wall versus the smooth case; this is different from the smooth-wall FPG effects on the turbulence, and does not agree with the observations of Coleman et al. (1977).

Experimental studies were also carried out by Tachie and co-workers Tachie and co-workers (Tachie et al., 2007; Tachie & Shah, 2008) with rib roughness in an open channel with converging side walls, or in a channel with converging top and bottom walls, generating widely varying accelerations from mild to strong levels. Contrary to the observations of Cal et al. (2009) and Coleman et al. (1977), their data showed that the friction coefficient, the mean velocity defect, and the Reynolds shear stress are largely independent of pressure gradient. This is possibly due to the limited streamwise distance for the FPG to exert its effects.

The above results indicate that the effects of roughness and FPG are interdependent, and appear affected by the state of the flow (equilibrium or not). Deeper understandings on the flow dynamics are required to explain how the two effects are related, and how they are manifested in different flow states.

1.3 Objectives and outline

The goals of this thesis include both the fundamental understanding of rough-wall flows and the performance analysis of the engineering correlations. The studies start from a practical motivation: is there a universal approach to accurately predict $k_s$ for various surfaces found inside a typical hydraulic turbine? Then, to establish the link between the geometry and the drag, we study the flow dynamics in the vicinity of roughness elements based on DNS data. Finally, the study is extended to accelerating boundary layers to elucidate the roughness effects in more realistic scenarios. The thesis is decomposed into the following parts.

- In Chapter 2, we use LES in a simple channel-flow configuration with replicated turbine-blade roughness imposed at the bottom wall, to obtain the actual flow quantities, including $k_s$, for these surfaces. Then, these data are used to examine the performance of the existing $k_s$ correlations. The implementation of the immersed
boundary method and the total-drag calculation applied throughout this thesis are discussed in the appendices.

- The dependence of the friction on the topographical details motivates us to study the flow dynamics inside the roughness sublayer in Chapter 3; this sublayer has been shown important for the layers above. The focus is on the effects on the wall-normal and spanwise turbulent fluctuations, since these components are important indicators of wall-normal momentum transfer and near-wall instability.

- Then we focus on rough-wall flows with FPG. We first study the sink flows in Chapter 4. A flow database with a wide range of FPG strengths and roughness heights is obtained from DNS and LES, to carry out a parametric study of the combined effects of roughness and acceleration on both flow statistics and coherent turbulent structures.

- Finally, a more complicated, spatially developing accelerating boundary is studied using DNS in Chapter 5. It is motivated by the coupling previously observed between FPG and roughness effects, as well as the different trends of the results between sink-flows and non-equilibrium FPG flows. The focus is on explaining the coupling and its dependence on flow equilibrity.
Chapter 2

Estimation and prediction of the roughness function on realistic surfaces

2.1 Abstract

Large-eddy simulations are carried out in turbulent open-channel flows to determine the roughness function and the equivalent sand-grain roughness height, $k_s$, over sand-grain roughness and different types of realistic roughness replicated from hydraulic turbine blades. A range of Reynolds numbers and mean roughness heights is chosen, leading to both transitionally and fully rough regimes. The start of the fully rough regime is shown to depend on the roughness type, and $k_s$ depends strongly on the surface topography. We then examine several existing correlations that predict $k_s$ in the fully rough regime based on the information of the surface geometry. In the cases where the surface slope is an important parameter, the moments of surface height statistics do not predict the roughness function, while the existing forms of slope-based correlations perform well. The range of applicability
of various correlations is shown to vary with the roughness topography, as the critical value of the effective slope, separating the waviness and roughness regimes, is shown to be higher for a realistic surface, compared to the value for the more regular types of roughness that were previously studied.

2.2 Introduction

2.2.1 Roughness effects

Experimental and numerical studies are usually carried out on surfaces with uniform distributions of elements of standard shapes, such as spheres, bars, cylinders and so on, although more realistic surfaces have been considered recently (Bons, 2002; Wu & Christensen, 2006; Flack & Schultz, 2010). Realistic roughness differs from regular (or “modelled”) roughness: it is characterized by a wider spectrum of wavelengths and random distribution of structures of each scale. Wu & Christensen (2006) compared different patches of turbine-blade roughness and observed significant dependence of the most intense ejection and sweep events on the surface topography. In later investigations (Wu & Christensen, 2007, 2010), they showed that outer-layer similarity also applies to realistic roughness; specifically, to the mean velocity, Reynolds normal and shear stresses, and the overall spatial structure of the outer-layer turbulence agree with the smooth-wall flow. Christensen and co-workers (Johnson & Christensen, 2009; Mejia-Alvarez & Christensen, 2010) compared a full surface and its lower-order representation and found that the larger surface scales predominantly determine the roughness effects on the turbulent statistics and the instantaneous turbulent events; however, the lower-order surface does not reproduce well the most intense ejection and sweep events in the roughness sublayer. Licari & Christensen (2011) conducted experiments to study turbine roughness reconstructed from a subset of the surfaces scales, and found that both the small and large scales contribute to the enhancement of turbulence.
Anderson & Meneveau (2011) showed that, as smaller scales of a fractal roughness are progressively included, the friction keeps increasing, indicating a difference in the friction on modelled roughness compared to a realistic one, which includes smaller scales. Recently, transitionally rough flows have been studied by Busse et al. (2013), who performed direct numerical simulations (DNS) in open-channel flows on several types of realistic surface; they also observed that drag depends on the three-dimensional topography of the surfaces and is not solely determined by the physical roughness height.

2.2.2 Equivalent sand-grain height

These results show that the topographical features of rough surfaces play a role in affecting the strong, energy-producing turbulence events near the wall, and thus affect the drag. Therefore, simulations of rough-wall flows should take into account detailed features of the surface. Common numerical approaches to include roughness effects include: (1) direct or large-eddy simulations (DNS or LES) with a body-fitted mesh or immersed boundary methods (IBM), which resolve the detailed surface but require high computational cost; (2) the discrete-element method (DEM), which does not resolve the surface but involves major modifications of the governing equations, and has been shown to work mainly for small-scale roughness structures and roughness peaks (other than valleys) (Bons et al. 2008), whereas a realistic roughness is often characterized by multiple scales and coexistence of peaks and valleys; and (3) rough-wall correction to the turbulence models for the Reynolds-Averaged Navier-Stokes (RANS) equations, which models roughness effects without physically resolving the surface or changing the basic equations.

In RANS solvers, the single parameter used to describe the roughness is the equivalent sand-grain height, $k_s$, which relates the drag of an arbitrary type of roughness to that of the uniform sand-grain roughness studied by Nikuradse (1933). What can be usually measured, however, is the mean height, $\bar{k}$. The drag depends on both $\bar{k}$ and the roughness texture; these two factors combine to determine $k_s$. For a given surface type, i.e., a set of surfaces
with the same texture, \( k_s \) depends on \( \bar{k} \) alone. To relate \( \bar{k} \) of an arbitrary surface to its \( k_s \), one uses the mean velocity profile in the logarithmic region assuming fully rough flows: over the Nikuradse sand grains, the mean velocity is

\[
U^+(y^+) = \frac{1}{\kappa} \ln \left( \frac{y}{k_s} \right) + 8.5, \tag{2.1}
\]

while for an arbitrary surface, it is

\[
U^+(y^+) = \frac{1}{\kappa} \ln \left( \frac{y}{\bar{k}} \right) + C, \tag{2.2}
\]

where \( C \) depends on the surface type. A superscript “+” indicates normalization using \( u_\tau \) and the viscous length-scale, \( \delta_\nu \). Equating the right-hand-sides of Equations (2.1) and (2.2) gives that \( k_s/\bar{k} \) is a constant dependent on the surface texture. Note that this linear relation between \( k_s \) and \( \bar{k} \) applies to the fully rough regime only.

### 2.2.3 Prediction of equivalent sand-grain height

Correlations have been proposed to predict the constant \( k_s/\bar{k} \) solely based on the geometric characteristics of the surface. Reviews are found in [Bons (2010)] and [Flack & Schultz (2010)]. A set of commonly used correlation is based on the surface slope. For instance, [Sigal & Damberg (1990)] and [van Rij et al. (2002)] proposed

\[
\ln \left( \frac{k_s}{\bar{k}} \right) = a \ln(\Lambda_s) + b, \tag{2.3}
\]

where

\[
\Lambda_s = \left( \frac{S}{S_f} \right) \left( \frac{S_f}{S_s} \right)^{-1.6}, \tag{2.4}
\]
and $S$, $S_f$, and $S_s$ are the reference surface area before adding roughness, the total frontal area of the roughness, and the total windward wetted surface, respectively; hence $\Lambda_s$ contains both slope and shape information for the surface. Reasonable collapse was obtained for gas-turbine roughness using this correlation [Bons 2002]. Note that other forms of $\Lambda_s$ have been proposed for regular 2D or 3D roughness [Dvorak 1969; Simpson 1973; Dirling 1973], but the form in Equation (2.4) is best adapted for realistic surface. The constants $a$ and $b$ are empirical coefficients. Another correlation of this type was proposed by Bons (2005); it is based on the root-mean-square of local surface slope, $\alpha_{\text{rms}}$:

$$\frac{k_s}{k} = a\alpha_{\text{rms}}^2 + b\alpha_{\text{rms}},$$

(2.5)

where $\alpha$ is the local streamwise slope angle, which can be easily obtained from a 1D surface trace.

A second class of correlations is based on the moments of height statistics, including the second ($k_{\text{rms}}$), the third (skewness, $s_k$), and sometimes higher-order moments (kurtosis, $k_u$); one of these correlations was proposed by Flack & Schultz (2010) and was shown to work well on several types of realistic surfaces (gravel, honed surfaces, etc.):

$$\frac{k_s}{k} = a\frac{k_{\text{rms}}}{k} (1 + s_k)^b.$$  

(2.6)

The “effective slope” ($ES$) of a surface is an additional parameter that can be used to categorize the roughness types with regards to the importance of surface slope. It is defined as

$$ES = \frac{1}{L} \int_L \left| \frac{dk(x)}{dx} \right| \, dx,$$

(2.7)

where $k(x)$ is the height array (obtained from a 1D trace of the surface), $x$ is the streamwise direction, and $L$ is the surface length in $x$. For modelled roughness constructed with closely packed pyramids (Schultz & Flack 2009) and with random sinusoidal waves (Napoli et al.,
it has been shown that the surface slope is an important parameter only when the
surface is not sufficiently steep: when $ES < 0.35$ ("waviness regime"), $\Delta U^+$ is sensitive to
$ES$, and does not scale with the roughness height. When $ES \geq 0.35$ ("roughness regime"),
on the other hand, the surface slope does not affect the drag, and $\Delta U^+$ can be predicted
using Equation (2.6) (Flack & Schultz 2010). The categorization using $ES$ is of industrial
interests, since $ES$ is easy to obtain, and indicates in which regime the surface falls; then
a correlation appropriate for this regime can be chosen to predict $k_s$. Therefore, a general
guideline for choosing the $k_s$-correlation for an arbitrary roughness is possible, if the critical
$ES$ value (separating the waviness and roughness regimes) is known for the given type of
roughness.

It is the purpose of this paper to (1) investigate the performance of existing fully-rough
$k_s$-correlations in the waviness regime using large-eddy simulations, and (2) modify the
surfaces artificially to obtain a wide range of $ES$ to study the dependence of the critical
$ES$ on the surface topography. To this end, we simulate open-channel flows over realistic
roughness replicated from hydraulic turbine blades; a wide range of roughness Reynolds
number is covered to determine the start of the fully rough regime. The problem set-up and
numerical techniques are first introduced in Section 2.3, then the mean flow and turbulent
statistics are studied in Section 2.4.1 to validate the simulations; the estimation method
for $k_s$ is discussed in Section 2.4.2, and the evaluation of the $k_s$-correlations is carried
out in Section 2.4.3; finally, the critical $ES$ is studied in Section 2.4.4. Conclusions and
recommendations for future work will follow in Section 2.5.
2.3 Problem formulation

2.3.1 Governing equations and numerical techniques

The incompressible flow of a Newtonian fluid is governed by the equations of conservation of mass and momentum:

\[ \frac{\partial \rho_i}{\partial x_i} = 0, \]
\[ \frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i u_j}{\partial x_i} = -\frac{\partial P}{\partial x_i} + \nu \nabla^2 u_i + \frac{\partial \tau_{ij}}{\partial x_i} + F_i. \]

\(x_1, x_2\) and \(x_3\) (or \(x, y\) and \(z\)) are, respectively, the streamwise, wall-normal and spanwise directions, and \(\overline{u}_i\) (or \(\overline{u}, \overline{v}\) and \(\overline{w}\)) are the filtered velocity components in those directions; \(P = p/\rho\) is the modified pressure, \(\rho\) the density and \(\nu\) the kinematic viscosity. The sub-grid stress \(\tau_{ij} = \overline{u_i u_j} - \overline{u_i} \overline{u_j}\) is modeled using the Lagrangian-averaged eddy-viscosity model [Meneveau et al., 1996], due to the capability of this model to capture spatial flow-heterogeneity by tracking the fluid particle-paths in time. The overbar denoting the filtering operation is hereafter omitted for simplicity. The simulations are performed using a well-validated code that solves (2.8) and (2.9) on a staggered grid using second-order, central differences for all terms, a second-order semi-implicit time advancement, and MPI (Message Passing Interface) parallelization [Keating et al., 2004].

An open-channel flow is simulated with no-slip and rigid free-slip boundary conditions applied to the bottom wall and to the top boundary, respectively. The free-slip condition has been used in the literature to simulate waveless open-channel flows with low Froude numbers, representing slow and tranquil flows [Pan & Banerjee, 1995]. Periodic conditions are used in both the \(x\)- and \(z\)-directions. An immersed-boundary method is used to impose no-slip boundary condition on the rough surfaces, which are well-resolved by the grid. It is based on the volume-of-fluid approach [Scotti, 2006]: the volume fraction occupied by the fluid, \(\phi(x, y, z)\), of each cell is calculated in pre-processing. In the current fractional-step
Figure 2.1: Visualization of (a)–(d) a single tile of four surfaces denoted by S1, S2, S3, S4, (e) a complete surface formed from S3 tiles, and (f) 1/4 of a complete sand-grain (SG) surface. Colouring is based on $y/k$.

framework, a body force, $F_i$ in the $x_i$ direction, is obtained from the velocity predicted without the forcing term, $\hat{u}_i'$, and the time step, $\Delta t$,

$$F_i(x, y, z) = -(1 - \phi(x, y, z)) \frac{\hat{u}_i'(x, y, z)}{\Delta t}.$$  \hfill (2.10)

By including $F_i$ into the right hand size of Equation (2.9) in the actual prediction step, the actual predicted velocity, $\hat{u}_i(x, y, z)$, at the boundary cell is reduced by $(1 - \phi)$. The eddy viscosity is reduced by $(1 - \phi)$ to account for the decrease of the sub-grid length scale. The forces are distributed in the boundary cells of roughness, and are zero inside the roughness. The numerical method is detailed in Appendix A. The force $F_1(x, y, z)$ is integrated throughout the domain to obtain the total drag on a surface, i.e., the sum of both pressure and viscous drag (Appendix B).
2.3.2 Surfaces

The various surface segments and a complete surface used in the present study are shown in Figure 2.1(a–e). The iso-surface of $\phi = 0.5$, representative of the solid surface, is coloured by the wall-normal location. Each surface is produced from a rectangular patch of roughness, called a “tile” (Figures 2.1(a–d)). For the current study of generic realistic roughness, it is assumed that the surface characteristics are homogeneous, i.e., there is no preferential tile direction; this assumption applies to a wide range of surfaces, but, under some circumstances, roughness generated by fluid flow can show preferential direction.

To produce the surface, first, the tile is scaled, preserving all length ratios, to match the target average height $k$. Then, it is duplicated in both $x$ and $z$ directions to achieve a horizontal domain of size at least $6h \times 3h$. Random rotation ($\alpha_{\text{rotation}} = N\pi/2$, where $N = 0, 1, 2,$ or 3) is applied to the tiles during the duplication, to achieve homogeneous surface characteristics. A linear smoothing function is applied to the interface between two patches to ensure a seamless match between tiles, and also surface periodicity in the $x$- and $z$-directions. An example of the final surface (from S3 tiles) is shown in Figure 2.1(e). For the sand-grain (SG) roughness, we use the roughness model proposed by Scotti (2006): the virtual sand-paper is constructed from randomly oriented and distributed ellipsoids of the same shape and size (with the three semi-axes of the ratio $1:1.5:2$); one quarter of a complete SG surface is shown in Figure 2.1(f).

To predict accurately the flow on a surface that represents homogeneous roughness, (1) each tile needs to be well resolved, and (2) the number of tiles should be sufficient to reproduce the random distribution and rotation of surface structures. To ensure that both conditions are met, we choose the cut-off length scale of the surface to be $o(1/10)$ of the Taylor microscale, $\lambda$, of the surface height fluctuations, since $\lambda$ is a good indication of the size of an equivalent “roughness element” in the context of realistic roughness. The calculation of $\lambda$ is similar to the determination of the Taylor microscale of turbulent fluctuations: first
Figure 2.2: Autocorrelation function $R_{k'k'}(\circ)$ of surface height fluctuations, as a function of the separation $r$ normalized by mean height, and the parabola $R_{\text{para}}(r)$ (—— ) fitted to determine the Taylor microscale $\lambda$ of the surface height fluctuations. Case SGk3 is shown.

The autocorrelation of the surface height fluctuations, $R_{k'k'}(r)$, is determined as a function of separation, $r$, in $x$ or $z$ (Figure 2.2); then a parabola, $R_{\text{para}}(r)$, is fitted to the $R_{k'k'}$ profile, \textit{i.e.},

$$R_{\text{para}}(r) = Ar^2 + 1,$$  \hspace{1cm} (2.11)

where the numerical factor $A$ is determined from equaling $(d^2R_{\text{para}}/dr^2)|_{r=0}$ to $(d^2R_{k'k'}/dr^2)|_{r=0}$.

Once the parabolic profile is known, $\lambda$ is obtained as the separation at which $R_{\text{para}} = 0$.

As will be shown in Table 2.2, the shape of the characteristic structures of the surface ($\sim o(\lambda)$) is resolved, and our computational power allows each domain to contain around $60^2$ samples of structure of the size $\lambda^2$—a sufficient sample pool to produce relatively random distribution and rotation of structures of this size.

The characteristic parameters of all surfaces are tabulated in Table 2.1. SG and S3 have the steepest surface slopes (high values of $ES$ and $\alpha_{\text{rms}}$, and low values of $\Lambda_s$). Surfaces SG and S4 give relatively high values of positive $s_k$, indicating the peaky nature of the surfaces, whereas S1, with negative $s_k$, is valley-dominated. For all surfaces, the kurtosis is close to 3, indicating that randomness is achieved.
Table 2.1: Statistics of rough surfaces.

<table>
<thead>
<tr>
<th>Surface type</th>
<th>(k_{\text{rms}}/k)</th>
<th>(s_k)</th>
<th>(k_u)</th>
<th>(ES)</th>
<th>(\Lambda_s)</th>
<th>(\alpha_{\text{rms}}) (in radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG</td>
<td>0.452</td>
<td>0.462</td>
<td>2.867</td>
<td>0.379</td>
<td>39</td>
<td>0.281</td>
</tr>
<tr>
<td>S1</td>
<td>0.199</td>
<td>-0.024</td>
<td>3.226</td>
<td>0.104</td>
<td>675</td>
<td>0.109</td>
</tr>
<tr>
<td>S2</td>
<td>0.378</td>
<td>0.135</td>
<td>2.561</td>
<td>0.085</td>
<td>1176</td>
<td>0.091</td>
</tr>
<tr>
<td>S3</td>
<td>0.419</td>
<td>0.198</td>
<td>2.021</td>
<td>0.164</td>
<td>288</td>
<td>0.147</td>
</tr>
<tr>
<td>S4</td>
<td>0.324</td>
<td>0.239</td>
<td>3.413</td>
<td>0.086</td>
<td>1158</td>
<td>0.101</td>
</tr>
</tbody>
</table>

Table 2.2: Spatial resolution in \(x\)- and \(z\)-directions for all cases.

<table>
<thead>
<tr>
<th>Surface type</th>
<th>Symbol</th>
<th>grid points per (\lambda)</th>
<th>total grid points ((n_i \times n_k))</th>
<th>(\Delta x^+)</th>
<th>(\Delta z^+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>□</td>
<td>–</td>
<td>128 \times 128 – 128 \times 192</td>
<td>18 – 47</td>
<td>9 – 10</td>
</tr>
<tr>
<td>SG</td>
<td>△</td>
<td>4 \times 9 – 6 \times 12</td>
<td>256 \times 256 – 512 \times 512</td>
<td>9 – 15</td>
<td>4 – 11</td>
</tr>
<tr>
<td>S1</td>
<td>□</td>
<td>9 \times 9 – 13 \times 13</td>
<td>384 \times 256 – 640 \times 384</td>
<td>4 – 17</td>
<td>4 – 17</td>
</tr>
<tr>
<td>S2</td>
<td>□</td>
<td>7 \times 7 – 9 \times 9</td>
<td>256 \times 128 – 512 \times 256</td>
<td>10 – 42</td>
<td>10 – 42</td>
</tr>
<tr>
<td>S3</td>
<td>△</td>
<td>6 \times 8 – 10 \times 13</td>
<td>384 \times 256 – 640 \times 384</td>
<td>3 – 21</td>
<td>3 – 16</td>
</tr>
<tr>
<td>S4</td>
<td>△</td>
<td>8 \times 8 – 11 \times 10</td>
<td>256 \times 128 – 384 \times 256</td>
<td>7 – 30</td>
<td>7 – 30</td>
</tr>
</tbody>
</table>

2.3.3 Parameters

The domain size in \(x\) and \(z\) ranges approximately from \(6h \times 3h\) to \(12h \times 12h\). This size is sufficient to accommodate the largest turbulent structures on a smooth-wall channel flow; on 3D regular roughness or realistic gas-turbine surfaces, roughness has been found not to affect the size of large-scale structures in the outer layer \cite{Volino2009, Wu2010}. To cover both transitionally rough and fully rough regimes, two Reynolds numbers and three average roughness heights are used: for \(Re = 400\), \(\bar{k} = 0.020h\ (k_1)\), \(0.040h\ (k_2)\), and \(0.067h\ (k_3)\); for \(Re = 1000\), \(\bar{k} = 0.040h\ (k_2)\), and \(0.067h\ (k_3)\). The maximum height \((k_{\text{max}}, \text{or the peak-to-valley height})\) varies from 0.03h to 0.14h. \(\bar{k}\), normalized by the momentum thickness, ranges between 0.2 and 0.68, slightly lower or similar to values used in other studies on turbine-blade roughness \cite{Bogard1998, Bons2008}. The spatial
resolutions are tabulated in Table 2.2. Note that the variation of $\Delta x^+$ and $\Delta z^+$ for each surface is the result of variations of both the Reynolds number and the roughness height. The grid sizes $\Delta x^+$ and $\Delta z^+$ range between 3 and 40, similar to the values used in previous rough-wall simulations using the same numerical techniques (Yuan & Piomelli, 2014b); the resolution in $y$ satisfies $\Delta y^+ < 1$ in the region $y \leq k_{\text{max}}$ and $\Delta y^+ < 25$ at the top boundary; the friction coefficient $C_f = 2(Re_\tau/Re_b)^2$ (where $Re_b$ is the Reynolds number based on the mean bulk velocity) in the smooth cases is within 0.2% of Dean’s correlation (Dean, 1978). The total number of grid points ranges from $128 \times 128$ to $640 \times 512$ in $x$ and $z$, and from 82 to 300 in $y$.

The equations are integrated in time for approximately $100h/u_\tau$ after a steady state is achieved. The angle brackets $\langle \cdot \rangle$ denote quantities that are averaged in time and over the homogeneous directions, $x$ and $z$. $U_i(y)$ is the velocity averaged in time and the total domain (including both fluid and solid domains),

$$U_i(y) = \langle u_i(x, y, z, t) \rangle.$$  

(2.12)

Note that the spatial average among the total domain is also called “superficial average”.

The turbulent fluctuation $u'_i$ are calculated by subtracting the time-averaged velocity from the total one:

$$u'_i(x, y, z, t) = u_i(x, y, z, t) - \langle u_i(x, y, z, t) \rangle_t,$$  

(2.13)

where $\langle \cdot \rangle_t$ denotes averaging over time only. Statistical convergence within 1% is achieved for the mean velocity and second moments statistics.
2.4 Results

In this section, the cases with $k_2$ and $Re_\tau = 1000$ are first used to investigate the effects of roughness geometry on the mean flow and the turbulent stresses (Section 2.4.1); then, the start of the fully rough regime is identified for each of the present surfaces, and $k_s/\bar{k}$ is determined (Section 2.4.2); $k_s$ is then used to evaluate existing $k_s$-correlations (Section 2.4.3), and the critical $ES$ is studied in Section 2.4.4.

2.4.1 Mean flow and turbulent stresses

The profiles of $U$ are shown in Figure 2.3. Note that the presented cases differ only in the value of $\bar{k}$. The zero-plane displacement, $d$, is subtracted from the wall-normal location to collapse the logarithmic regions of the smooth and rough cases. $d$ is defined as the wall-normal location where the drag appears to act (i.e., the centroid of the local drag profile, $Raupach et al., 1991$),

$$d = \frac{\langle \int_0^\infty F_1(x, y, z, t) y \, dy \rangle}{\langle \int_0^\infty F_1(x, y, z, t) \, dy \rangle}$$  (2.14)
Figure 2.4: (a)–(c) Reynolds normal stresses for cases with $k_2$ and $Re_\tau = 1000$, compared with the smooth case; $y = 5\overline{k}$. (d)–(f) Normal components of the anisotropy tensor of the Reynolds stresses near the wall. Smooth-wall experiment by Schultz & Flack (2013).

For all surfaces studied, $d \approx \overline{k}$. The smooth-wall profile in the current LES collapses with the experimental data obtained by Schultz & Flack (2013). In the logarithmic region, the rough-wall profiles display an offset from the smooth-wall profile, with the amount of the displacement, $\Delta U^+$, depending on the surface topography.

The normal components of the Reynolds stress tensor are shown in Figure 2.4(a–c), compared to the experimental results (Schultz & Flack 2013). In the rough-wall cases, the peaks of the normal stresses move slightly away from the wall due to the displacement effect of the roughness on the mean flow; the peak magnitude of $\langle u'^2 \rangle^+$ is significant damped, a phenomenon widely observed in experiments (Volino et al., 2009) and simulations (Busse et al., 2013; Scotti, 2006). Farther away from the wall, the profiles collapse, consistent with outer-layer similarity; the roughness sublayer, defined here as the layer in which roughness alters the profiles of the normal Reynolds stresses, is confined to the region $y \lesssim 5\overline{k}$ for all
Insights on the topographical effects can be obtained from the anisotropy of Reynolds stresses near the wall (Figure 2.4(d–f)). The normal components of the Reynolds stress anisotropy tensor are

$$b_{\alpha\alpha} = \frac{\langle u_\alpha u_\alpha \rangle}{\langle u_i u_i \rangle} - \frac{1}{3},$$

(2.15)

with no summation over greek indices. In the vicinity of a smooth wall, the vertical velocity fluctuation $v'$ is significantly damped ($\langle (v')^2 \rangle^+ (0) = 0$ and thus $b_{22}(0) = -1/3$), while almost all the TKE resides in $u'$ and $w'$. On a rough wall, however, $v'$ and $w'$ can be significant near the wall at $y \approx d$, especially for the SG and S3 surfaces, resulting in more isotropic distributions of TKE. The very different levels of near-wall augmentations of $v'$ and $w'$ reflect dramatic differences in the way various roughness types affect the flow.

The Reynolds shear stress profiles are compared in Figure 2.5. Despite the significant topographical effects in near-wall Reynolds stress anisotropy, the effect on the Reynolds shear stress is only a slight change of the shape in the roughness sublayer: different surfaces correspond to different distribution of the total drag.
2.4.2 Calculation of $k_s$

Since the linear relation between $k_s$ and $\bar{k}$ is valid only inside the fully rough regime, the beginning of this regime needs to be first identified. A range of $k^+$ is achieved with various Reynolds numbers and $\bar{k}/h$, and $\Delta U^+$ is obtained from the mean-velocity profiles and plotted in Figure 2.6(a). The start of the logarithmic relation between $\Delta U^+$ and $k^+$ is the start of the fully rough regime, where the viscous length scale becomes unimportant, and the mean velocity $U(y)^+$ scales with $y/k_s$ in the logarithmic region,

\[
\frac{1}{\kappa} \ln y^+ + 5.0 - \Delta U^+ = \frac{1}{\kappa} \ln \left( \frac{y}{k_s} \right) + 8.5. \tag{2.16}
\]

The logarithmic region is identified as the plateau region of $y^+(dU^+/dy^+)$. $k_s$ is calculated from Equation (2.16) once $\Delta U^+$ is obtained; its values are used to plot the roughness function in Figure 2.6(b); all curves asymptote to a single line in the fully rough regime. The critical roughness Reynolds number $k_{s,cri}^+$, marking the start of the fully rough regime,
Table 2.3: $k_s/\overline{k}$ and the start of fully rough regime ($k_{s,cri}^+$) for all surface types.

<table>
<thead>
<tr>
<th>Surfaces</th>
<th>SG</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{s,cri}^+$</td>
<td>$\approx 80$</td>
<td>$\approx 20$</td>
<td>$\approx 30$</td>
<td>$\approx 40$</td>
<td>$\approx 20$</td>
</tr>
<tr>
<td>$k_s/\overline{k}$</td>
<td>2.68</td>
<td>0.32</td>
<td>0.48</td>
<td>1.27</td>
<td>0.29</td>
</tr>
</tbody>
</table>

and the ratio $k_s/\overline{k}$ for all surfaces are tabulated in Table 2.3. $k_s/\overline{k}$ varies significantly (from 0.3 to 2.7), and the fully rough regime starts at a lower $k_{s,cri}^+$ for realistic surfaces (especially S1 and S4) than for SG, where it is established for $k_{s,cri}^+ \approx 80$. Earlier establishment of fully rough regime for realistic surfaces was also observed by Langelandsvik et al. (2008), who found $k_{s,cri}^+$ to be 18 on commercial steel pipe. The roughness function for Nikuradse sand grains is also compared in Figure 2.6(b). It shows around the same $k_{s,cri}^+$ as Scotti’s sand-grain model, SG, but its profile does not collapse with that of SG, indicating that the SG model does not reproduce precisely the uniform sand-grain surface. This is probably because the SG model was calibrated with uniform sand grains in the transitionally rough regime only (Scotti 2006).

The length scale $k_s$, however, does not collapse the roughness function for all surfaces in the transitionally rough regime; S1, S2 and S4 present a gradual increase of $\Delta U^+$, while SG and S3 show a much more sudden growth. These results are consistent with the phenomena previously observed that modelled roughness with large distinct elements shows a delayed and more sudden increase of $\Delta U^+$, while surfaces with a wider spectrum of scales are more likely to display a smoother increase of drag (Jiménez 2004), since different surface scales are gradually triggered one after another to contribute to the drag as the Reynolds number increases.
2.4.3 Evaluation of $k_s$ correlations

To evaluate the existing $k_s$ correlations, the fully rough data are used, namely, the $k_s/\kappa$ values obtained from the cases with $k/h = 0.067$ and $Re_\tau = 1000$ on all surfaces. Figure 2.7 shows the relation between the roughness function and $ES$ (filled symbols). $\Delta U^+$ does not correspond one-to-one to $ES$, due to the effects of other surfaces parameters, such as $k_{rms}$ and $s_k$; for example, S2k3 and S4k3 have the same $ES$ but differ by 30% in $\Delta U^+$. Nevertheless, a general dependence of $\Delta U^+$ on $ES$ is observed for all $k_3$ cases, indicating that the current surfaces are in the waviness regime. The cases with a lower average height, $k_2$, are also plotted for comparison (clear symbols), showing the dependence of $\Delta U^+$ on $k^+$. Also plotted are the results of numerical experiment by Napoli et al. (2008) on a different type of roughness (composite of sinusoidal waves) with a nearly constant average height; the dashed line indicates the critical value of $ES$ marking the end of the waviness regime; for their surfaces, this value is around 0.35. The current surfaces show the same behaviour.
in the waviness regime, but the range of ES is not large enough to investigate the critical ES for these surfaces.

The correlations of the forms specified in Equations \((2.3)\) (slope/shape method), \((2.5)\) (slope-rms method), and \((2.6)\) (moment method) are considered; for each correlation, the constants \(a\) and \(b\) are fitted with the five data points, and the resultant correlation is plotted in Figure 2.8. Figures 2.8(a) and 2.8(b) show the slope-based correlations; the forms of these two correlations are found to represent reasonably well the variation of the current data. Larger scatter is found when the moment method is used (Figure 2.8(c)). In particular, surfaces S3 and S4 have similar \(k_{\text{rms}}/K\) (within 22\%) and \(s_k\) (within 20\%) as shown in Table 2.1 but differ by a factor of 4 in \(k_s/K\); this is probably due to the fact that moments of height statistics do not contain slope information directly. Thus the moment-based correlation is not suitable for the surfaces whose slope is an important parameter, i.e., in the waviness regime.

### 2.4.4 Critical ES

To find out whether the range of the waviness regime is the same for all surfaces, higher values of ES need to be reached. We systematically vary ES by artificially compressing
Figure 2.9: Artificial compressing of S2 to modify ES while keeping the same mean height. One tile is shown.

the tiles of surface S2k3 in the x- and z-directions, while keeping the size in y unchanged (Figure 2.9). The stretching factor, $Str$, is defined as the ratio of the original horizontal dimension to its final value after compression. $Str$ is chosen to be 4, 8, and 16. Table 2.4 presents the spatial resolutions and resulting $ES$. For higher $Str$, more roughness patches are required to fill the flow domain; therefore, a lower surface resolution was used to limit the computational costs. However, such coarsening affects $k_{rms}/\bar{k}$ and $s_k$ by only 0.1% and 0.5%, respectively; it does, however, decrease ES by 9% for S2Str4 and by 25% for S2Str16. The grid coarsening is not expected to affect significantly the prediction of $k_s$, since the sensitivity of $\Delta U^+$ to ES tends to saturate at high values of ES (as will be shown), at which point $k_{rms}/\bar{k}$ and $s_k$ play the dominant role in determining $\Delta U^+$ (Flack & Schultz, 2010).

The effect of ES on the roughness function obtained from the stretched surfaces is shown in Figure 2.10. Note that, unlike in Figure 2.7, the data points here correspond to surfaces that differ only in the slope (but not in $k_{rms}$ or $s_k$). For surface S2, the saturation
Table 2.4: S2 surfaces with various stretching factors, $Str$. $Re_{r} = 1000$, $\overline{k}/h = 0.067$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Str</th>
<th>points per $\lambda$ (in $x, z$)</th>
<th>total grids ($n_i \times n_j \times n_k$)</th>
<th>ES</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2Str1</td>
<td>1</td>
<td>$9 \times 9$</td>
<td>$256 \times 213 \times 256$</td>
<td>0.09</td>
</tr>
<tr>
<td>S2Str4</td>
<td>4</td>
<td>$7 \times 7$</td>
<td>$512 \times 213 \times 256$</td>
<td>0.18</td>
</tr>
<tr>
<td>S2Str8</td>
<td>8</td>
<td>$5 \times 5$</td>
<td>$384 \times 213 \times 256$</td>
<td>0.35</td>
</tr>
<tr>
<td>S2Str16</td>
<td>16</td>
<td>$5 \times 5$</td>
<td>$640 \times 213 \times 384$</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Figure 2.10: Variation of the roughness function as the original S2 surface is compressed by the stretching factors of 4, 8 and 16. ■ Current surfaces, + Napoli et al. (2008), --- $ES = 0.35$. 

34
of the $ES$ effect occurs at a much higher $ES$ value than for the more regular roughness (pyramids) and the 2D waves previously studied \cite{Napoli2008, Flack2010}: the plateau of $k_s/\bar{k}$ is approached for $ES \approx 0.7$, compared to the critical value of 0.35 previously found \cite{Napoli2008}. This indicates that the topographical details significantly affect separation between the waviness and roughness regimes. The wider waviness regime observed for realistic roughness is linked to its multi-scale nature: surface structures of larger wavelengths contribute to the waviness of the surface; the stronger sheltering effects of the large-scale protuberances on those of smaller ones lead to a delay in the contribution of the small-scale structures to the increase of friction.

2.5 Conclusions

Four types of realistic roughness with different topographical characteristics and the sand-grain roughness are compared. Their effects on the mean flow and turbulent stresses are studied using large-eddy simulations in an open-channel flow configuration. The beginning of the fully rough regime is identified for each surface, and within this regime, $k_s$ is determined by matching the measured roughness function to that of the uniform sand grain of Nikuradse. The ratio between $k_s$ and the average roughness height, $\bar{k}$, is shown to depend on the roughness type.

The current surfaces are shown to be in the waviness regime defined by \cite{Schultz2009}. The resulting $k_s/\bar{k}$ is then used to evaluate the performance, in this regime, of three existing forms of $k_s$-correlations that predict $k_s$ based solely on the surface information; these include correlations based on slope/shape parameter, on slope-rms, and on moments of height statistics. The slope-based correlations yield collapse of data, while the moment-based correlation gives wider scatter, since the moments do not contain slope information and thus do not scale $k_s/\bar{k}$ in the cases where the surface slope is an important parameter.

The effective slope ($ES$) categorizes rough surfaces according to the importance of the
surface slope to the roughness function; thus it offers a guideline in choosing $k_s$-correlations for an arbitrary rough topography. Of crucial importance is the critical $ES$ that separates the waviness and roughness regimes. By artificially compressing a realistic surface and systematically increasing $ES$, we found that the critical $ES$ of a realistic surface can be much higher than the value of 0.35 found for pyramid roughness and 2D wave-composite. Future studies are required to study and model the dependence of the critical $ES$ on roughness topography.

Appendices

A Immersed boundary method

Immersed boundary methods (IBM) are widely used to handle moving or deforming bodies with complex surface geometries embedded in a flow. It does not require the Eulerian grid to be body-conforming, since the no-slip boundary condition is imposed on the boundary surface by spreading boundary forces to the Eulerian cells. The IBM was first introduced by Peskin (1972), who calculated the boundary force on the Lagrangian grid points as a singular function using Hooke’s law, and spread it on to the neighbouring Eulerian cells with regularized delta functions. With a similar approach, Goldstein et al. (1993) obtained the forcing function from a feed-back mechanism. These approaches, however, require some empirical parameters, and pose strict constraints on the time step or the deformation from immersed boundary. Direct formulations of the forcing function were introduced by Fadlun et al. (2000), who modified the discretized momentum equation so that the interpolated velocity at the interface equals the required value, giving sharp interfaces. However, the interpolation is easy to carry out only for simple and regular interface geometries. Balaras (2004) extended the approach to complex geometries.

In this thesis, to represent the complex roughness geometry while maintaining the simplicity of the Cartesian approach, we use an IBM based on the volume-of-fluid (VOF)
approach. This technique was first applied by Scotti (2006) to the study of roughness with DNS. The volume-of-fluid IBM method was first introduced by Hirt & Nichols (1981) to study the interface between different types of incompressible fluid. In this method, the volume fraction $\phi$ of a certain fluid in each cell is calculated from a conservation equation,

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{v}) = 0; \quad (2.17)$$

then, the amount of fluid transferred from the upstream cell to the downstream one is calculated from the product of $\phi$ and the flux boundary area. This method is simple and effective. It describes immersed interfaces in a piecewise-linear sense, and ensures conservation of mass for each type of fluid (with conservation of total $\phi$).

In this study, the interface is between steady surface roughness and a fluid; thus both the time-derivative term and the convection term in (2.17) equal zero. Instead, the volume fraction of each cell occupied by fluid, $\phi$, is calculated in pre-processing. Note that, due to the staggered grid arrangement, different volume fractions are used for the three velocity components and the subgrid-scale viscosity. A body force, $F_i$, is imposed on the right-hand side of the momentum equation to reduce the velocity proportionally to the solid volume in each cell.

In the current fractional-step framework, the prediction step is carried out twice. Take DNS as an example; in the first step, a preliminary predicted velocity, $\tilde{u}'_i$, is obtained without $F_i$,

$$\left[1 - \Delta t \frac{\partial}{\partial y} \left( \nu \frac{\partial}{\partial y} \right) \right] \tilde{u}'_i = u^n_i + \Delta t \left( \frac{3}{2} RHS^n - \frac{1}{2} RHS^{n-1} \right) + \frac{1}{2} \Delta t \frac{\partial}{\partial y} \left( \nu \frac{\partial u^n_i}{\partial y} \right), \quad (2.18)$$
where $RHS^n$ denotes the sum of convection, pressure, and the streamwise- and spanwise-viscous terms, at time step $n$, and $\Delta t$ is the time step size. Note that the implicit time-advancement is applied to the wall-normal viscous term only. Then $F_i$ is obtained as

$$F_i^n = -(1 - \phi) \frac{\tilde{u}_i'}{\Delta t}. \quad (2.19)$$

The second prediction step, in the same form as Equation (2.18), is then carried out with $F_i^n$ and $F_i^{n-1}$ as the source terms in $RHS^n$ and $RHS^{n-1}$, and the final predicted velocity, $\tilde{u}_i$, is obtained. Note that, for LES, the eddy viscosity must also be reduced by $(1 - \phi)$ to account for the decrease of sub-grid length scale. $\tilde{u}_i$ at the boundary cell is approximately reduced by $(1 - \phi)$; the reduction of $(1 - \phi)$ is exact, however, when explicit Euler time-advancement is used. Note that $F_i^n$ depends on $\Delta t$: for decreasing $\Delta t$, $F_i^n \Delta t$ tends towards the 0/0 limit, reaching singularity for $\Delta t = 0$. However, this singularity is not reached numerically, since $\Delta t$ is always positive. Yuan (2011) studied the behaviour of the fluid-solid interface with the present IBM under the same fractional-step numerical framework as this thesis, and showed that the decrease of $\Delta t$ leads to an asymptotic zero-velocity in the interface cells; effectively, the fluid-solid interface moves towards the fluid side by up to one grid size. Although, in principle, this dependency violates the definition of a “solid surface”, numerically the difference is negligible providing that a sufficiently fine spatial resolution (and thus small grid size) is available in the vicinity of the fluid-solid interface.

This approach is similar to the one used by Scotti (2006), who obtained $\tilde{u}_i$ by scaling directly $\tilde{u}_i'$ with the volume fraction without the second prediction step, i.e., $\tilde{u}_i = (1 - \phi) \tilde{u}_i'$, and used no explicit body-force term. The advantage of the present IBM, compared to the approach of Scotti (2006), is that the explicit forcing terms can be integrated directly to obtain the total drag, $f_d$ (described in Appendix B), bypassing the use of velocity and pressure distributions, which are highly complicated due to surface-shape randomness. The forces are distributed in the boundary cells, and are zero inside the roughness.
Compared to direct-forcing methods, this method is simple to implement and does not depend on the discretization scheme. It does, however, lead to a diffused interface whose exact location is determined only to the order of the grid size. In the application studied here, the roughness length scale is large compared to the grid size; thus, the grid-wise pixelation of the roughness surface does not systematically affect the structural statistics in the roughness sublayer.

B Roughness forcing and drag calculation

The sum of pressure and viscous drag on a rough surface at a given location and time, \( f_d \), is obtained from the streamwise body force imposed by the IBM, \( F_1 \), as

\[
f_d(x, z, t) = \int_0^\infty F_1(x, y, z, t) \, dy.
\]

(2.20)

To explain this approach, we consider the stress balance obtained as vertical integral of the averaged \( u \)-momentum equation,

\[
\int_0^\infty \frac{\partial \langle P \rangle}{\partial x} \, dy = \langle f_d \rangle + \nu \left. \frac{\partial \tilde{U}}{\partial y} \right|_0^\infty - \left. UV \right|_0^\infty - \left. (\tilde{u} \tilde{v}) \right|_0^\infty - \left. (u'v') \right|_0^\infty + \left. \nu_{sgs} \frac{\partial \tilde{U}}{\partial y} \right|_0^\infty,
\]

(2.21)

where \( \tilde{\cdot} \) denotes the spatial variation of a time-averaged quantity in the roughness sublayer (due to roughness wakes; for more details see Chapter 3), and \( \nu_{sgs} \) is the subgrid-scale viscosity obtained from the LES model. At \( y = 0 \), the velocity magnitude is zero; the wall-normal velocity-gradient is negligible (since, in the vicinity of the domain bottom, roughness occupies most of the physical space, and thus the fluid is nearly static). Similarly, at \( y = \infty \), \( V = \partial U / \partial y = \tilde{u}_i = u'_i = 0 \). Therefore, Equation (2.21) reduces to

\[
\frac{\partial \langle P \rangle}{\partial x} h = \langle f_d \rangle,
\]
Figure 2.11: Wall-normal profiles of time- and space-averaged total drag distribution for all surfaces with the height $k_2$. Dashed lines indicate the location of respective zero-plane displacement.

\[ \langle F_1 \rangle + y + \text{SG}_{k_2} \text{S}_{1k_2} \text{S}_{2k_2} \text{S}_{3k_2} \text{S}_{4k_2} y = d \]

\[ \langle f_d \rangle \] is the total drag of the surface. Note that \( \langle f_d \rangle \) is negative, because \( \langle F_1 \rangle(y) \) is in the opposite direction of the mean flow (according to Equation (2.19)) and thus is negative for all \( y \); however, the local \( \langle f_d \rangle_t(x,z) \) can be either positive or negative depending on the direction of local time-averaged velocity in the vicinity of roughness.

The body force \( F_1(x,y,z,t) \) is non-zero only within the interface cells. Figure 2.11 shows the wall-normal distribution of the time- and space-averaged body force, \( \langle F_1 \rangle(y) \) for all cases with the mean height $k_2$; both $F_1$ and $y$ are normalized using wall units; therefore, the sum of the area below the curve is unity, since the integration of the forcing gives the total drag. \( \langle F_1 \rangle(y) \) reaches zero at the lower boundary of the domain (since $u = 0$ at $y = 0$), and at the maximum height of the rough surface. The respective centroid of the force distribution, i.e. $d^+$, is compared with the profile; its value ranges from 40 to 50. Since $\bar{k}^+ = 40$ for these cases, $d \approx \bar{k}$.
Figure 2.12: Convergence of the root-mean-square of drag averaged in time and from \( n_r \) sandgrain roughness samples, \( \langle f_d \rangle_{n_r} \), in two cases, (Yuan & Piomelli [2014b]) as \( n_r \) increases. \( \langle f_d \rangle \) is the value averaged using the largest number of samples (22311 samples).

C Drag convergence with number of roughness elements

The distribution of \( f_d \) is affected by the randomness of the roughness distribution. For the numerical sandpaper model, this randomness is manifested by the sandgrain location and rotation angle that satisfy a uniform distribution; for surfaces S1–S4, the randomness is shown by the shape and distribution of the “equivalent” roughness elements defined as the surface structures of a characteristic size \( \lambda \). Since only a finite (and relatively small) number of roughness elements are included in the calculations, it is necessary to verify that the drag calculation was not affected by insufficient sample. We use the time-averaged \( f_d \) in cases K1R1 and K3R1 in studies of flows over the sandpaper model (Yuan & Piomelli [2014b]), which are among the cases with the largest number of roughness samples (22311 samples) in the thesis. We performed plane averages over small portions of the domain itself (and, thus, a smaller number of roughness elements). We then evaluated the root-mean-square of the plane-averaged force, for various subdomains, and compared it with that obtained from the entire domain. Figure 2.12 shows the root-mean-square (RMS) variation of the force averaged over a subset containing \( n_r \) roughness elements. As \( n_r \to \infty \), of course, the RMS approaches zero (the sample of roughness elements becomes adequate). For the domain
containing 1000-3000 roughness samples, the standard deviation of the force is around 1%-2% of the mean force. Given that a sample size of $o(1000)$ is used in each case in this study, the sampling is considered sufficient. In addition, the comparison between the two cases shows that the $f_d$ uncertainty increases slightly as the roughness Reynolds number $k^+$ grows from 7 to 27, approaching the lower limit of the fully rough regime (Figure 2.6(a)); the uncertainty corresponding to the case with $k^+ = 27$ is representative for cases with higher $k^+$, since the roughness wake is fully developed (in the fully rough regime), leading to an asymptotic spatial variation level of $\langle f_d \rangle_t$. 
Chapter 3

Roughness effects on the Reynolds stress budgets in near-wall turbulence

3.1 Abstract

The physics of the roughness sublayer are studied by direct numerical simulations of an open-channel flow with sandgrain roughness. A double-averaging approach is used to separate the spatial variations of the time-averaged quantities and the turbulent fluctuations. The spatial inhomogeneity of velocity and Reynolds stresses results in an additional production term for the turbulent kinetic energy, the “wake production”; it is the excess wake kinetic energy, generated from the work of mean flow against the form drag, that is not directly dissipated into heat, but instead converted into turbulence. The wake production promotes wall-normal turbulent fluctuations, and increases the pressure work which ultimately leads to more homogeneous turbulence in the roughness sublayer, and to the increase of Reynolds shear stress and the drag on the rough wall. In the fully rough regime, roughness directly
affects the generation of the wall-normal fluctuations, while, in the transitionally rough regime, the region affected by roughness is separated from the region of intense generation of these fluctuations. The budget of the wake kinetic energy and the connection between the wake and the turbulence suggest strong interactions between the roughness sublayer and the outer layer that are insensitive to the variation of the outer-layer conditions. Furthermore, the present results may have implications for the relationship between the roughness geometry and the flow dynamics in the region directly affected by roughness.

3.2 Introduction

Roughness plays an important role in many fields of study. A substantial amount of work has been carried out to understand the dynamics of turbulent flows over rough walls, both for engineering and atmospheric applications. Roughness effects on the flow are summarized by [Raupach et al. (1991)] and [Jiménez (2004)]. Roughness increases the drag on the wall due to the pressure drag, resulted from the wake created downstream of a roughness element. Many studies have shown that, far away from the wall, roughness does not affect the turbulent statistics, but it sets the velocity scale by increasing the friction coefficient.

Inside the roughness sublayer (defined as the region where roughness causes spatial variations of time-averaged turbulent statistics), the flow dynamics are significantly altered as the turbulent structures become more three-dimensional. In order to capture the interaction between the turbulence and the spatial variation of the time-averaged velocity field (wake), the space-time double-averaging approach can be used ([Raupach & Shaw, 1982]). Earlier studies of the flow over canopies ([Finnigan, 2000] [Raupach et al., 1991]) found that the wake leads to an additional source term in the budget of the turbulent kinetic energy (TKE), the “wake production”.

Roughness and sediments are different from canopies in the vertical distribution of the density of the roughness elements. Recent studies on flow over gravel beds ([Mignot et al.], 44
Dey & Das (2012) found that the effect of the wake on TKE is negligible compared to the production due to the shear of the mean velocity (shear production).

DNS and LES have been used to study the roughness effects on wall-bounded flows, e.g., Kanda et al. (2004), Coceal et al. (2006), Orlandi & Leonardi (2008), Leonardi & Castro (2010), and Lee et al. (2011). Their main focus has been on the flow above the roughness and the parameterization of the drag. The layer below the roughness crest, however, is also important; understanding this layer would provide insights to the roughness effects in the regions above. It was observed by Coceal et al. (2006) that the wake depends on the geometry of the roughness, and that, although smaller than the Reynolds stresses, the dispersive stresses (resulting from the wake component) are significant in the lower part of the roughness sublayer. However, the effects of roughness on the budgets of specific Reynolds normal stresses are not well studied. It is important to understand the roughness effects on the wall-normal and spanwise turbulent fluctuations, since these two components are more direct indicators (than TKE) of inner-outer-layer interactions, near-wall structure instability, and drag generation. To this end, we use direct numerical simulations of the turbulent flow over sandgrain roughness in the transitionally and fully rough regimes, to study the roughness effects on the budgets of the Reynolds stresses, and to investigate the energy transfer between the wake and the turbulence.
3.3 Problem formulation

3.3.1 Governing equations and numerical techniques

The incompressible flow of a Newtonian fluid is governed by the equations of conservation of mass and momentum,

\[
\frac{\partial u_i}{\partial x_i} = 0, \tag{3.1}
\]

\[
\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + F_i, \tag{3.2}
\]

where \(u_i\) and \(P\) are the velocities and pressure.

\(x_1, x_2\) and \(x_3\) (or \(x, y\) and \(z\)) are, respectively, the streamwise, wall-normal and spanwise directions, and \(u_i\) (or \(u, v\) and \(w\)) are the velocity components in those directions; \(P = p/\rho\) is the modified pressure, \(\rho\) the density and \(\nu\) the kinematic viscosity. We performed direct numerical simulations of (3.1) and (3.2) using a staggered grid and second-order, central differences for all terms, a second-order semi-implicit time advancement, and MPI (Message Passing Interface) parallelization (Keating et al., 2004). An immersed-boundary method (IBM) based on the volume-of-fluid approach (Appendix A in Chapter 2) is applied: a body force, \(F_i\) is used to impose the no-slip boundary condition on the rough surfaces; it is non-zero in the interface cell only.

3.3.2 Averaging approach

Following Mignot et al. (2009), a double-averaging (DA) approach is applied to decompose a flow quantity, \(\theta\), into space-time average, \(\langle \cdot \rangle_i\) (bar and brackets denote temporal and spatial averages, respectively, subscript \(i\) denotes intrinsic spatial average), the spatial disturbance of the temporal average, \(\tilde{\theta}\), and the turbulent fluctuation (denoted by the prime),

\[
\theta(x, y, z, t) = \langle \theta \rangle_i(y) + \tilde{\theta}(x, y, z) + \theta'(x, y, z, t). \tag{3.3}
\]
Two spatial averaging approaches are used (Nikora et al., 2007). The intrinsic spatial average, \( \langle \cdot \rangle_i \), is carried out in the \((x,z)\)-plane, in the fluid domain only

\[
\langle \theta \rangle_i = \frac{1}{A_f} \int_{A_f} \theta \, dA,
\]  

(3.4)

where \( A_f \) is the area of fluid domain, while the superficial average, \( \langle \cdot \rangle \), is carried out over the whole \((x,z)\)-plane

\[
\langle \theta \rangle = \frac{1}{A_{tot}} \int_{A_{tot}} \theta \, dA,
\]  

(3.5)

where \( A_{tot} \) is the total area, including both solid and fluid domains. Above the fluid layer occupied by roughness, these two spatial-average operations are equivalent.

Applying the intrinsic average to the time-averaged \( u \)-momentum equation for the open-channel flow, one obtains

\[
- \langle \frac{\partial P}{\partial x} \rangle_i + \nu \langle \nabla^2 \bar{u} \rangle_i - \langle \frac{\partial \bar{u} \bar{v}}{\partial y} \rangle_i - \langle \frac{\partial \bar{u}' \bar{v}'}{\partial y} \rangle_i = 0
\]  

(3.6)

Note that the IBM force \( F_i \) does not appear in (3.6) since it is zero inside the fluid domain. Equation (3.6) can be written as

\[
- \frac{\partial \langle P \rangle}{\partial x} + \nu \frac{\partial^2 \langle \bar{u} \rangle}{\partial y^2} - \frac{\partial \langle \bar{u} \bar{v} \rangle}{\partial y} - \frac{\partial \langle \bar{u}' \bar{v}' \rangle}{\partial y} + \nu (\nabla^2 \bar{u}) = 0,
\]  

(3.7)

where the last two terms on the left hand side are, respectively, the pressure drag, \( f_p \), and the viscous drags, \( f \nu \) (Raupach & Shaw, 1982); they arise due to the non-commutativity between the operators \( \partial/\partial x \) and \( \langle \cdot \rangle_i \) below the roughness crest. The sum of these two terms is the total drag. It can be shown that \( F_1 \), averaged in the interface cell in the \((x,z)\)-plane, equals the total drag, \( -\langle \partial \bar{P}/\partial x \rangle + \nu (\nabla^2 \bar{u}) \).
3.3.3 Simulation parameters

An open-channel flow of domain size $6h \times 1h \times 3h$ in $x$, $y$, and $z$ is simulated; this size is sufficient to accommodate the largest turbulent structures on a smooth-wall channel flow. No-slip and symmetric boundary conditions are applied to the bottom wall and to the top boundary, respectively. Periodic conditions are used in both the $x$- and $z$-directions. A constant pressure gradient is applied to drive the flow. A virtual sandgrain model (Scotti, 2006) is used to impose roughness at the bottom boundary: the bottom domain is separated into tiles of size around $2k_s \times 2k_s$; in each tile a roughness element is planted, which constitutes a randomly rotated ellipsoid with semiaxes equaling $k_s$, $1.4k_s$, and $2k_s$. Note that, for this virtual sandgrain surface, $k_s$ is known a priori since the model is calibrated, in a specific range of roughness Reynolds numbers, to give the same roughness function as the Nikuradse sandgrain with a height $k_s$; however, the shape of the elements is not exactly the same as those of Nikuradse. A fraction of the surface is visualized in Figure 3.1(a) as iso-surface of the fluid volume fraction equaling 0.5. The Reynolds number based on the half-channel height ($h$) and the friction velocity ($u_\tau$) is $Re_\tau = 1000$. Two roughness heights are used to yield a transitionally rough and a fully rough flow: a low-roughness case ($R1$, $k_s/h = 0.02$, $k_s^+/\kappa = 22$), and a high-roughness one ($R2$, $k_s/h = 0.07$, $k_s^+/\kappa = 72$). The roughness crests ($k_c$) for $R1$ and $R2$ are 3% and 9% of $h$, respectively. A smooth-wall case (case SM) is also included for comparison.

Table 3.1 tabulates the parameters and spatial resolutions. Uniform grids are used in $x$ and $z$, while the grid is refined near the wall in $y$ direction. For the rough cases, $\Delta y^+ < 1$ for $y \leq k_c$, resulting in more than 31 and 51 grid points in $y$ below the mean roughness height and the crest, respectively. For the smooth case, $\Delta y_{\text{min}}^+ = 0.3$, with 4 points below $y^+ = 1$. The total number of grid points varies between 70 and 130 millions. The grid sizes in $x$ and $z$ directions are less than or equal to 12 and 6 wall units, respectively, sufficient for DNS resolution.
Figure 3.1: (a) Visualization of 1/8 of the surface R2; wall-normal profiles of (b) DA streamwise velocity ($\langle u \rangle_i$ and $\langle \overline{u} \rangle$) and (c) roughness geometry function.

Table 3.1: Parameters for all cases. $k_c$: roughness crest; $\bar{k}$: mean roughness height. $n_j$ ranges from 128 to 256. $N_{x_i}$: grid points per sand-grain element.

<table>
<thead>
<tr>
<th>Case</th>
<th>$k_s^+$</th>
<th>$k_s/h$</th>
<th>$k_c/h$</th>
<th>$k/h$</th>
<th>$N_x \times N_z$</th>
<th>$n_i \times n_k$</th>
<th>$\Delta x^+ \times \Delta z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>512 $\times$ 512</td>
<td>12 $\times$ 6</td>
<td></td>
</tr>
<tr>
<td>R1</td>
<td>22</td>
<td>0.02</td>
<td>0.03</td>
<td>0.013</td>
<td>7 $\times$ 7</td>
<td>1024 $\times$ 512</td>
<td>6 $\times$ 6</td>
</tr>
<tr>
<td>R2</td>
<td>72</td>
<td>0.07</td>
<td>0.09</td>
<td>0.040</td>
<td>20 $\times$ 20</td>
<td>1024 $\times$ 512</td>
<td>6 $\times$ 6</td>
</tr>
</tbody>
</table>

The shape resolution of a roughness element is quantified by $N_{x_i}$, the number of grid points (in $x_i$) used to resolve the shape of each sandgrain element. A grid-convergence study (not shown) was carried out, and it was found that the present resolution is sufficient to obtain the Reynolds stresses and their budgets. For all cases, data are collected from a total simulation time of $T = 50h/u_\tau$ after the transient. A convergence study showed that decreasing the simulation time to $0.5T$ leads to errors of less than 1% for third- or lower order turbulent statistics.
3.4 Results

3.4.1 Turbulent statistics

The DA streamwise velocity is shown in Figure 3.1(b). Below the roughness crest, the velocity varies almost linearly, consistent with Model III of the velocity distribution proposed by Nikora et al. (2004) applied to rough-wall flows with relatively low submergence ($h/k_s$). The superficial average ($\langle \overline{u} \rangle = \Phi \langle \overline{u} \rangle$) is lower than the intrinsic one, since the roughness geometry function, $\Phi(y)$, defined as the fraction of space occupied by fluid in the horizontal plane at $y$, is less than one (Figure 3.1(c)). The distribution of $\Phi(y)$ is not affected by the variation of the spatial resolution. The zero-plane displacement ($d$), representing the effective elevation of the boundary layer due to roughness, is obtained as the location of the centroid of the wall-normal distribution of the time- and space-averaged drag force profile (Yuan & Piomelli 2014b); $d \approx 0.8k_s$ for both rough cases.

Figure 3.2(a) shows the DA velocity profiles in wall units. The roughness functions, $\Delta U^+$, obtained as the offsets of $\langle \overline{\tau} \rangle^+$ profiles in the logarithmic region from the universal logarithmic law, are 3.5 and 7.8 for cases R1 and R2, respectively, yielding $k_s^+ = 22$ and 72.
Figure 3.3: Form-induced stresses for cases R1 (empty symbols) and R2 (filled symbols): (a) $\langle \tilde{u}^2 \rangle^+$, (b) $\nabla \langle \tilde{v}^2 \rangle^+$, $\Delta \langle \tilde{w}^2 \rangle^+$, and $\diamond \langle \tilde{u} \tilde{v} \rangle^+$.

based on the Colebrook correlation (Colebrook, 1939); case R2 is in the fully rough regime, while case R1 is transitionally rough.

The Reynolds stresses in cases R1 and R2 are compared in Figure 3.2(b) with smooth-wall experimental data from channel-flow studies with the same Reynolds number (Schultz & Flack, 2013); agreement is obtained in the outer layer, supporting the wall similarity hypothesis. Case R1 preserves a high near-wall peak of the streamwise normal Reynolds stress, while the peak is significantly damped for case R2, due to the destruction of the buffer layer. The top of the roughness sublayer is denoted by $k_R$; its location will be discussed below. Inside the roughness sublayer, all normal Reynolds stresses in wall units are damped, compared to case SM. The magnitudes of the other two shear stresses are negligible compared to $\langle u'v' \rangle$, due to the randomness of the rough surface.

The form-induced (or dispersive) stresses are shown in Figure 3.3 for both rough cases; the wall-normal and spanwise components are smaller than the corresponding components of the Reynolds stresses, consistent with previous experimental rough-wall and canopy studies (Nikora et al., 2001; Mignot et al., 2009); the streamwise form-induced stress is comparable to the Reynolds stress, a characteristic of relatively low submergence (Manes et al., 2007). The peak of $|\langle \tilde{u} \tilde{v} \rangle^+|$ is around 0.15, consistent with previous DNS study by Coceal.
Figure 3.4: Stress balance of case R2. —— Total stress from momentum balance, $\triangle$ total drag from momentum balance, —— total drag from $\langle F_1 \rangle$ integral, □ Reynolds shear stress, ○ form-induced shear stress, + viscous stress due to DA mean shear.

The region with non-negligible $\langle u_i^2 \rangle$ is the roughness sublayer, whose thickness, $k_R$, is shown to be around $2k_s$. The magnitude of the form-induced stresses relative to the Reynolds stresses increases with $k_s^+$, since, in the roughness sublayer, $\langle u_i^2 \rangle^+$ decreases while $\langle u_i^2 \rangle^+$ increases with higher $k_s^+$, due to the lower submergence.

Figure 3.4 shows the stress balance for case R2; the mean pressure gradient is balanced by the sum of other stresses,

$$\tau^+ = \int_{y^+}^{h^+} (f_v^+ + f_p^+)dy^+ - \langle u'v' \rangle^+ + \frac{\partial \langle \pi \rangle^+}{\partial y^+} - \langle \bar{u} \bar{v} \rangle^+ = 1 - \frac{y^+}{h^+}. \quad (3.8)$$

The viscous stress is small, consistent with fully rough flow. The form-induced shear stress is also small compared to the Reynolds shear stress. The total drag, $(f_v^+ + f_p^+)$, calculated from momentum balance, agrees with the IBM force integral.
3.4.2 Budgets of Reynolds and dispersive stresses

In the turbulent flow over roughness, the total kinetic energy can be decomposed into three parts,

\[ \frac{1}{2} \langle u_i u_i \rangle_i = \frac{1}{2} \langle \bar{u}_i \rangle_i \langle \bar{u}_i \rangle_i + \frac{1}{2} \langle \tilde{u}_i \tilde{u}_i \rangle_i + \frac{1}{2} \langle u'_i u'_i \rangle_i, \]  

(3.9)

where the terms on the right hand side are, respectively, the mean-flow kinetic energy (MKE), the wake kinetic energy (WKE) and the TKE. The MKE is converted to small-scale TKE (and consequently heat) in two ways (Finnigan, 2000): first, the energy goes through the whole energy cascade; second, MKE first generates WKE of scale \( k_s \) due to the work of the mean flow against the total drag, and the WKE generates TKE of scales smaller than \( k_s \) through the energy cascade ("short-circuited" cascade (Raupach et al., 1991)). The interactions between WKE and TKE are also two-ways: WKE generates TKE of scales smaller than \( k_s \), while TKE is converted to WKE through the work of large-scale turbulent structures against the form drag.

The budget of Reynolds stresses and TKE were derived by Raupach & Shaw (1982) for flow over canopy and by Mignot et al. (2009) for flow over roughness. The budgets of the normal Reynolds stresses are (no summation over Greek index)

\[-2 \langle v'_a v'_a \rangle \frac{\partial \langle \bar{u}_a \rangle_i}{\partial y} - 2 \left( \langle \tilde{u}'_a \tilde{u}'_j \frac{\partial \tilde{u}_a}{\partial x_j} \rangle \right) - \frac{\partial}{\partial y} \langle \tilde{u}'_a \tilde{u}'_a \bar{v} \rangle - \frac{\partial}{\partial y} \langle u'_a u'_a v' \rangle \]
\[-2 \left( \langle \tilde{u}'_a \frac{\partial P}{\partial x_a} \rangle \right) + \nu \frac{\partial^2}{\partial y^2} \langle u'_a u'_a \rangle - \epsilon_{aa} = 0. \]  

(3.10)

Summing over the three components yields the budget of the TKE,

\[-\langle u'_i v' \rangle \frac{\partial \langle \bar{u}_i \rangle_i}{\partial y} - \left( \langle \tilde{u}'_i \tilde{u}'_j \frac{\partial \tilde{u}_i}{\partial x_j} \rangle \right) - \frac{\partial}{\partial y} \langle \tilde{u}'_i \tilde{u}'_i \bar{v} \rangle / 2 \]
\[-\frac{\partial}{\partial y} \langle u'_i u'_i v' \rangle / 2 - \frac{\partial}{\partial y} \langle P' v' \rangle + \nu \frac{\partial^2}{\partial y^2} \langle u'_i u'_i \rangle / 2 - \epsilon_k = 0, \]  

(3.11)
where the terms on the left hand side are, respectively, shear production ($P_s$), wake production ($P_w$), transport due to wake, turbulent transport ($T_t$), pressure transport ($T_p$), viscous transport ($T_\nu$), and viscous dissipation ($\epsilon$). $\epsilon$ is obtained as the residual of the sum of all other terms. In the normal-stress budget the pressure work, $\Pi_{\alpha\alpha}$, is also present.

The budget of WKE, $1/2\langle \tilde{u}_i \tilde{u}_i \rangle$, was derived by Raupach & Shaw (1982). Taking into account the vertically varying roughness geometry function, $\Phi(y)$, the budgets of the normal dispersive stresses ($\langle \tilde{u}_\alpha^2 \rangle$) read

$$
-2\langle \tilde{u}_\alpha \tilde{v} \rangle \frac{\partial \langle \tilde{u}_\alpha \rangle}{\partial y} + 2 \langle \tilde{u}_\alpha \tilde{u}_j \frac{\partial \tilde{u}_\alpha}{\partial x_j} \rangle - \frac{\partial}{\partial y} \langle \tilde{u}_\alpha \tilde{u}_\alpha \tilde{v} \rangle - 2 \frac{\partial}{\partial y} \langle \tilde{u}_\alpha \tilde{u}'_\alpha \tilde{v}' \rangle - 2 \frac{\partial}{\partial y} \langle \tilde{u}_\alpha \partial \tilde{P}/\partial x_\alpha \rangle + 2 \nu \langle \tilde{u}_\alpha \nabla^2 \tilde{u}_\alpha \rangle = 0 \tag{3.12}
$$

where the six terms on the left hand side are, respectively, shear production of WKE, wake production ($-P_w$), transport due to wake, transport due to turbulence, work done by pressure, and viscous diffusion and dissipation at the wake level.

The term $P_w$ appears with opposite signs in the budgets of TKE and WKE; it is the net transfer from WKE to TKE as the result of their interactions. Previous studies on flows above canopies found that $P_w$ is usually positive (WKE converts to TKE) with magnitudes comparable to $P_s$ (Raupach et al., 1991; López & García, 2001). In the experimental study of flow over gravel beds (Mignot et al., 2009), however, the magnitude of $P_w$ was found to be less than 5% of $P_s$.

The TKE budgets normalized by wall units are shown in Figure 3.5. Case SM has been validated with the smooth-wall channel-flow DNS results reported by Hoyas & Jiménez (2008) (not shown). Above the roughness sublayer ($y > k_R$), the R1 budget terms agree well with case SM, but in the roughness sublayer, they differ from the smooth case mainly in that, approaching the wall, both $\epsilon_k$ and $T_\nu$ become zero, indicating a quiet region without turbulence at the root of the roughness.
Figure 3.5: TKE budgets for cases SM (line), R1 (empty symbol), and R2 (filled symbol). All terms normalized by $u_r^4/\nu$. --- $k_c$, roughness crest. $\diamond P_s$, $\bigtriangleup P_w$, $\bigcirc \epsilon$, $\triangleright T_t$, $\triangleleft T_{\nu}$, $\nabla T_p$. Note that the actual bottom-wall location varies.

Figure 3.5 also shows that, for both rough cases, the location of the peak of the shear production is close to the roughness crest, connected to the shear layers formed near the crest [Mignot et al. 2009; Ikeda & Durbin 2007]. For case R2, $P_s$ peaks at $0.73k_c$ from the actual bottom wall (where $\Phi = 0$), while, for case R1, it peaks at $0.93k_c$. It was observed by Mignot et al. (2009) that, for flow over gravel beds, $P_s$ peaks around $0.9k_c$. $P_s$ decreases from its peak location to the virtual wall. Dissipation peaks slightly below the production peak; the transport terms are non-negligible in the roughness sublayer only. Both the turbulent diffusion and the viscous diffusion take energy from the region with high production (near $k_c$) and transport it towards the wall to balance dissipation. These observations are consistent with the literature [Finnigan 2000; Mignot et al. 2009; Hong et al. 2011]. Furthermore, the pressure transport removes TKE from near the roughness crest, and transports it to the lower part of the roughness sublayer; for case R2, the pressure transport is the most important source term in the lowest third of the sublayer, consistent
Figure 3.6: Comparison of normal Reynolds stress budgets between cases R1 (empty symbols) and R2 (filled symbols): (a) streamwise, (b) wall-normal, and (c) spanwise components. $\Diamond P_s$, $\triangle P_w$, $\circ \epsilon$, and $\triangledown \Pi$; $----- T_t$ for case R1, $--- T_t$ for case R2. All terms normalized by $u^+_L/\nu$.

with LES results in flow within forest canopy \cite{Dwyer1997}, while in the transitionally rough case R1, the viscous diffusion is the most important source term in the lowest part of the sublayer. Compared to $P_s$ and $\epsilon$, $P_w$ is much smaller, consistent with \cite{Mignot2009}. The sign and magnitude of $P_w$ are affected by the roughness height, which will be explained later.

Selected terms of the normal Reynolds stress budgets (in wall units) are shown in Figure 3.6. The wall-normal distance is normalized using $k_s$. As the flow approaches the fully rough regime from R1 to R2, $P_{s,uu}^+$ decreases significantly; so do the viscous dissipations of $\langle u'^2 \rangle$ and $\langle w'^2 \rangle$, while, below the roughness crest, $\Pi_{vv}^+$ increases. This indicates that roughness results in more energy being redistributed to $\langle v'^2 \rangle$, and that stronger wall-normal fluctuations distort the near-wall quasi-streamwise vortices, homogenizing the turbulence near the wall, consistent with the previous observations from DNS that roughness disturbs the near-wall low-speed streaks \cite{Lee2011}.

For case R1, $\Pi_{vv}^+$ exhibits two peaks; the inner peak is located slightly below the roughness crest, where the shear production peaks, while the outer peak is outside the roughness
sublayer, at \((y - d)^+ \approx 30\). The outer peak exceeds dissipation, with the excess energy transported into the region of the inner peak. As the roughness height increases from R1 to R2, the inner peak of \(\Pi_{vv}^+\) moves upwards and merges into the outer peak. Meanwhile, excess energy is gained around the roughness crest, and transported both towards the wall and towards the outer layer. This indicates that, in a transitionally rough flow, the roughness is somewhat isolated from the most important region for the generation of wall-normal turbulent fluctuations, while, in a fully rough flow, roughness directly affects its generation.

The wake production for \(\langle u'^2 \rangle\) is mostly negative below \(k_c\) for R1 and and negligible for R2; for the other two normal components, it is positive, with non-negligible peak magnitudes compared to the pressure work (17%\(\Pi_{vv}\) and 23%\(\Pi_{ww}\) in case R2). Such difference may be due to the fact that \(\langle u'^2 \rangle\) is mostly associated with turbulent structures with larger scale than the roughness length scale, converting significant amount of TKE to WKE as they work against the form drag; therefore, more \(\langle u'^2 \rangle\) is converted to WKE compared to the amount being converted from WKE, hence the negative \(P_{w,uu}\). On the other hand, \(\langle v'^2 \rangle\) and \(\langle w'^2 \rangle\) receive a more significant contribution from turbulent structures smaller than \(k_s\); thus \(\langle v'^2 \rangle\) and \(\langle w'^2 \rangle\) generate less WKE than the amount being converted from WKE through energy cascade. For \(v'\) and \(w'\) intensities, the wake production is important, despite a weaker wake contribution to the TKE budget.

Figure 3.7 shows the dispersive stress budgets. Two significant sources are present: the shear production and the pressure work. The shear production is due to the work of the mean flow against the form-induced stress; it contributes to \(\langle u'^2 \rangle\) only. The pressure work of WKE can be written as

\[
\langle \bar{u}_i \frac{\partial \bar{P}}{\partial x_i} \rangle = -\frac{\partial \langle \bar{P} \bar{v} \rangle}{\partial y} + \langle \bar{u} \rangle \left\langle \frac{\partial \bar{P}}{\partial x} \right\rangle_i,
\]

with the two terms on the right hand side being the pressure transport and the energy gained from the mean flow working against the form drag; the latter is shown contributing
Figure 3.7: Comparison of normal form-induced stress budgets between cases R1 (empty symbols) and R2 (filled symbols): (a) streamwise, (b) wall-normal, and (c) spanwise components. Line: shear production (--- R1, — R2), ▽ pressure work, Δ −P_w, and ○ viscous dissipation and diffusion. All terms normalized by u_*^4/ν.

significantly to ⟨˜v^2⟩ and is the only source of ⟨˜v^2⟩ and ⟨˜w^2⟩. The WKE is partly converted to TKE through wake production, partly dissipated into heat. As the roughness height increases from R1 to R2, a larger portion of ⟨˜v^2⟩ and ⟨˜w^2⟩ is converted to Reynolds stresses instead of being directly dissipated, due to the further separation between the roughness length scale and the viscous length scale.

3.5 Conclusions

Direct numerical simulations are carried out in open-channel flows over sand-grain roughness in the transitionally and the fully rough regimes. A double-averaging (DA) technique is applied to separate the spatial disturbance of the time-averaged field and the turbulent field. Significant spatial variations of time-averaged flow quantities are generated due to the three-dimensionality of the roughness geometry. The form-induced stresses are small compared to their Reynolds stress counterparts, but the magnitude of this difference becomes smaller as the roughness height increases.

The work of the form drag near k_c converts mean-flow energy to wake energy, which is
partly dissipated into heat and partly converted to turbulence through wake production. The intensified wall-normal turbulent fluctuations lead to higher pressure work near \( k_c \), and more homogeneous energy redistribution between the Reynolds stresses. In the fully rough regime, the wake dynamically affects the region of \( \langle v'^2 \rangle \) generation, while, in the transitionally rough regime, its effect is limited below \( k_c \).

Despite significant transfer from the wake to the turbulent component in the wall-normal and spanwise directions, for the streamwise component, the conversion is mostly from turbulence to wake energy, further contributing to a more even TKE redistribution; this is possibly due to the larger turbulent scales that are associated with this Reynolds stress component. The wake production of TKE, however, is negligible, because its values for the three normal Reynolds stresses tend to cancel out, and the magnitudes of total TKE production and dissipation are greater than the energy redistributed.

The current results shed some light on the role of the spatial inhomogeneity on the generation of drag in rough-wall boundary layers. The spatial inhomogeneity, being dependent on the roughness geometry, and in turn altering the energy redistribution and strongly affecting the Reynolds shear stress through its production, may provide the link between geometry detail and the dynamics in the roughness sublayer. Furthermore, the fact that the wake energy depends on the work of mean flow against the form drag as a source term suggests that the wake field is more persistent compared to turbulence in a non-equilibrium boundary layer: while the turbulence production relies on active fluctuations and the mean shear, the presence of the wake requires non-zero mean velocity only. The persistent wake indicates strong interactions between the roughness sublayer and the outer layer, insensitive to the variation of the outer-layer conditions.
Chapter 4

Sink-flow boundary layers over rough surfaces

4.1 Abstract

Turbulent sink flows over smooth or rough walls with sand-grain roughness are studied using large-eddy and direct numerical simulations. Mild and strong levels of acceleration are applied, yielding a wide range of Reynolds number \((Re_\theta = 372 - 2748\) based on the momentum thickness, \(\theta\)) and cases close to the reverse-transitional state. Flow acceleration and roughness are shown to exert opposite effects on boundary-layer integral parameters, on the Reynolds stresses, budgets of turbulent kinetic energy, and properties of turbulent structures in the vicinity of the rough surface; statistics exhibit similarity when plotted using inner scaling for cases with the same roughness Reynolds number, \(k^+\). Acceleration leads to a decrease of \(k^+\), while roughness increases it. For cases with higher \(k^+\), the low-speed streaks become destabilized, and turbulent structures near the wall are distributed more uniformly in the wall-parallel plane; they are less extended in the streamwise direction, but more densely distributed. Higher \(k^+\) also causes decorrelation of the outer-layer hairpin
packets with the near-wall structures, probably due to the direct impact of random roughness elements on the hairpin legs. Wall-similarity applies for the fully turbulent cases, in which the outer-layer turbulent statistics are affected by acceleration only. It is shown that being in the hydraulically-smooth regime is a necessary condition for reverse-transition, supporting the idea that relaminarization starts from the inner region, where roughness effects dominate.

4.2 Introduction

Turbulent boundary layers subject to a favourable pressure gradient (FPG) are found in many engineering applications, including airfoils, turbine blades or curved ducts; in many of these applications roughness is also important. A vast body of literature on smooth-wall FPG boundary layers exists. The review by Narasimha & Sreenivasan (1979) remains a landmark, and outlines many of the open questions and issues. Here, we only summarize the principal ones. It is known that, when the acceleration is sufficiently large, the flow may revert to a quasi-laminar state; the state between fully turbulent flow and fully relaminarized flow is called “reverse-transitional”. The acceleration can be characterized by the parameter \( K = (\nu/U_\infty^2)(dU_\infty/dx) \) (\( U_\infty \) is the freestream velocity). In sink flows (in which the acceleration acts for an infinite distance and \( K \) is not a function of the streamwise location, i.e., the accelerating flow is equilibrium), Spalart (1986) showed that turbulence cannot be sustained if \( K \) is higher than a critical value, \( K_{\text{crit}} \), between \( 2.5 \times 10^{-6} \) to \( 3.0 \times 10^{-6} \). In realistic spatially developing boundary layers, of course, the acceleration cannot act for infinite distances, and complete relaminarization occurs rarely. However, the state of the flow is still significantly altered by strong acceleration, and even the mean velocity profile is modified. Reviews of current knowledge can be found in several articles (Narasimha & Sreenivasan, 1973; Narasimha, 1985; Narasimha & Sreenivasan, 1979) and in more recent
studies (Bourassa & Thomas, 2009; Piomelli & Yuan, 2013; Piomelli et al., 2000). Ex-
perimental investigations of relaminarization due to flow acceleration started in the early
1960s. Among the major findings of these studies was the fact that, at least in the outer
region of the boundary layer, dissipation remains smaller than production. Narasimha &
Sreenivasan (1973) conjectured that, since the streamwise and wall-normal fluctuations do
not lose their correlation, relaminarization is due to pressure forces dominating over nearly
frozen Reynolds stresses. Recent simulations of accelerating flows over smooth, flat plates
(Piomelli & Yuan, 2013; Piomelli et al., 2000; De Prisco et al., 2007) show that, in the
region of maximum acceleration, frozen turbulence advected from upstream is still present,
but does not keep up with the freestream acceleration.

The sink flow is the simplest accelerating boundary layer and has been studied nu-
merically (Spalart, 1986) and experimentally (Jones & Launder, 1972; Jones et al., 2001;
McEligot & Eckelmann, 2006; Dixit & Ramesh, 2010); in this type of flows, $K$ and the
Reynolds number remain constant, resulting in statistical similarity in the flow direction.
Therefore, a sink flow is substantially less expensive computationally, compared to spatially
developing accelerating flows. In smooth-wall sink flows, typical responses of the turbu-
ulence include thickening of the viscous sublayer, damping of fluctuations especially in the
wall-normal component, lower bursting rate, and larger near-wall coherent structures.

More recent studies have investigated the interaction between roughness and pressure
gradients. Tachie et al. (2007) conducted experiments in flows over bar roughness under
FPG due to converging side-walls in an open channel. From the study of a wide range of
$k^+$ they concluded that, while in the hydraulically smooth regime the flow responses were
similar to those on a smooth wall, in the cases of fully rough flows no apparent FPG effect
was observed on drag characteristics, on the mean flow, or on turbulent quantities. However
it remained unclear whether this conclusion applied also to cases with FPG generated by
converging top walls. Tachie & Shah (2008) studied the same type of roughness with
an inclined bottom wall, using stronger acceleration. It was found that near-wall flows
were still governed by roughness effects, while, in the outer layer, acceleration decreased Reynolds stresses and increased the triple velocity correlations and transport of turbulent kinetic energy. Both these studies featured strongly non-equilibrium accelerating flows.

Cal et al. (2008, 2009) studied quasi-equilibrium boundary layer flows on a tilted plane with sand-grain roughness and mild acceleration; the acceleration was applied for a considerable distance for both the fully rough and transitionally rough flows. They observed a general increase of friction coefficient $C_f$ and decrease of Reynolds number based on momentum thickness, $Re_\theta$, as acceleration was imposed on the rough wall. Competing effects of $K$ and roughness were found on the mean flow, and on the Reynolds stresses in the outer layer; close to the wall, however, acceleration significantly intensified the fluctuations, an effect opposite to a smooth-wall flow. It is not clear whether these effects were due to the fact that the flow is spatially developing, or to the mild levels of acceleration used.

The interaction between strong acceleration and near-wall roughness effects was observed by Piomelli & Yuan (2013) in large-eddy simulations of strongly non-equilibrium boundary layer flows: under strong freestream acceleration, relaminarization was achieved over both the smooth wall and a rough surfaces with low roughness height. However, when the roughness was significant, the accelerating flow did not relaminize; instead, the generation and growth of streaks were disrupted, and the near-wall wall-normal and spanwise fluctuations did not decrease during acceleration. It was concluded that the inner layer plays a dominant role in the relaminarization process.

To further understand the interaction of strong FPG and roughness, it appears desirable to remove the effects of spatial development and investigate the simplest equilibrium accelerating flow, the sink flow. It is the objective of this paper to carry out a parametric study of both roughness and acceleration in the transitionally rough regime, with a wide range of $K$. The combination of acceleration and roughness leads mostly to fully turbulent flows, with another two cases close to the reverse-transitional state. We focus on the
4.3 Problem formulation

The incompressible sink-flow of a Newtonian fluid is governed by the equations of conservation of mass and momentum:

\[ \frac{\partial u_i}{\partial x_i} = 0, \]  
\[ \frac{\partial u_j}{\partial t} + \frac{\partial u_i u_j}{\partial x_i} = -\frac{\partial P}{\partial x_j} + \frac{1}{Re} \nabla^2 u_j + G_j + F_j. \]  

The equations have been made dimensionless using a reference velocity and length, \( U_\infty \) and \( X \) (which will be specified later). \( x_1, x_2 \) and \( x_3 \) (or \( x, y \) and \( z \)) are, respectively, the streamwise, wall-normal and spanwise directions, and \( u_j \) (or \( u, v \) and \( w \)) are the velocity components in those directions; \( P = p/\rho \) is the modified pressure, \( \rho \) the density and \( Re = U_\infty X/\nu \) the Reynolds number. The term \( F_j \) in Equation (4.2) is a body force imposed by the immersed boundary method used to impose non-slip boundary conditions on the rough surface, and will be explained later.
The mathematical treatment of the sink flow follows the approach proposed by Spalart (1986): the domain is transformed into similarity coordinates \((x, \eta, z)\), with \(\eta = yX_o/X\). The schematic of a sink flow on the \((x, \eta)\)-plane is shown in Figure 4.1. Here, \(X_o\) is a constant related to the mean flow-rate towards the sink, and \(X\) the distance from the sink.

If the boundary-layer thickness \(\delta\) is much smaller than \(X\), we can consider a domain centred around the position \(X_o\), and assume that the turbulence is spatially homogeneous since the statistical quantities vary with \(X \gg \delta\), and the turbulent fluctuations vary slowly compared with \(\delta\). The reference velocity and the reference length for the streamwise locations inside the domain can then be approximated by \(U_{\infty, o}\) (the free-stream velocity at \(x = X_o\)) and \(X_o\). Note that both reference quantities are independent of \(K\); in fact, \(K\) is the normalized viscosity, \(K = \nu/Q\), where \(Q = U_{\infty}(X)X = U_{\infty, o}X_o\) is a constant; thus \(K\) is determined by \(\nu\) alone. The effect of the acceleration is then included through growth terms that can be obtained from the transformation of the equations into the similarity coordinates, followed by a multiple-scale procedure to simplify the equations. The growth terms are given by Spalart (1986):

\[
G_1 = -\frac{\langle u \rangle}{X_o} (\langle u \rangle + 2u') + \frac{U_{\infty}^2}{X_o}, \quad G_2 = 0, \quad G_3 = -\frac{\langle u \rangle}{X_o} w',
\]  

(4.3)

where \(\langle \cdot \rangle\) denotes an appropriate average (in this case, carried out over time and the homogeneous flow directions), and \(u'\) and \(w'\) are fluctuations from the averages. With this transformation, only two parameters are present: the acceleration parameter \(K\) and the roughness height, \(\kappa = k/X_o\). For simplicity, \(\eta\) is replaced with \(y\) in the following.

In large-eddy simulations (LES), Equations (4.1) and (4.2) are solved for filtered quantities, and the divergence of the sub-grid stress tensor, \(\tau_{ij} = \overline{u_i u_j} - \overline{u_i} \overline{u_j}\), appears on the right-hand-side of the momentum equation. In the present study, \(\tau_{ij}\) is modelled using the Lagrangian Dynamic Eddy-Viscosity model (Meneveau et al., 1996), due to its capability of capturing flow heterogeneity by tracking the fluid particle-paths in time.

65
The simulations are performed using a well-validated code that solves the governing equations (4.1) and (4.2) on a staggered grid using second-order, central differences for all terms, a second-order accurate semi-implicit time advancement, and MPI parallelization (Keating et al., 2004). Periodic boundary conditions are used in the streamwise and spanwise directions (since the flow is assumed to be homogeneous on the scale $\delta \ll X$). A free-slip boundary condition is imposed on the top boundary.

On the bottom wall, an immersed boundary method based on volume-of-fluid method is used to impose the non-slip conditions on the roughness surface. We use the roughness model proposed by Scotti (2006): a virtual sand-paper is constructed from randomly oriented and distributed ellipsoids of the same shape and size (with the three semi-axes equaling $k$, $1.5k$, and $2k$); this model was found to give $k_s = k$ in the transitionally rough regime. Note that, this $k$ is not the mean height in Chapter 2 but around 1.5 times of the mean-height value. The volume fraction of each grid cell occupied by the fluid (volume-of-fluid, or $\phi$) is calculated in pre-processing. Visualization of the surface with $\phi = 0.5$ (representative of the shape of the rough wall) is shown in Figure 4.2. The implementation of the immersed boundary method is detailed previously in Chapter 2.

A total of 12 simulations were run, identified by $KnRn$, where $n = 1,\ldots,4$ denotes
Table 4.1: Summary of simulation parameters. Values of roughness Reynolds number $k^+$.  

<table>
<thead>
<tr>
<th></th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
<th>K4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R0 (smooth)</td>
<td>0.45 × 10^{-6}</td>
<td>0.80 × 10^{-6}</td>
<td>1.50 × 10^{-6}</td>
<td>2.50 × 10^{-6}</td>
</tr>
<tr>
<td>R1 ($k = 3.0 \times 10^{-4}$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>R2 ($k = 6.0 \times 10^{-4}$)</td>
<td>36.3</td>
<td>20.2</td>
<td>10.6</td>
<td>–</td>
</tr>
<tr>
<td>R3 ($k = 9.5 \times 10^{-4}$)</td>
<td>–</td>
<td>45.9</td>
<td>23.4</td>
<td>13.4</td>
</tr>
</tbody>
</table>

Varying acceleration parameters $K$, and $m = 0, \ldots, 3$ different roughness heights. Table 4.1 gives the values of $K$, $k$, and $k^+$ in all cases. $k^+$ values within a 25% range are considered constant; thus, we will consider cases K2R1, K3R2 and K4R3 as having approximately the same value of $k^+$. In all cases, $\delta/k \geq 25$, and $k^+$ is in the transitionally rough regime; $Re_\theta$ ranges between 372 and 2853. As will be shown later, only cases K4R0 and K4R2 are close to the reverse-transitional state.

Due to the wide range of Reynolds numbers, LES must be used for the cases with milder accelerations (K1 and K2), while direct simulations are used for cases K3 and K4. The domain size in all cases is $0.2X_o \times 0.06X_o \times 0.04X_o$, equivalent to approximately $9\delta \times 3\delta \times 2\delta$. The domain sizes in $x$ and $z$ directions are similar to those used by Spalart (1986), and the streamwise and spanwise two-point correlations of the streamwise turbulent fluctuations, calculated at $y \leq 0.5\delta$, fall below 0.1 at half the domain length or width.

Uniform grids are used in $x$ and $z$ directions, while stretching is applied in the $y$ direction outside of the roughness layer. In DNS, $\Delta x^+$ and $\Delta z^+$ are less than 15 and 4, respectively, and $\Delta y^+ < 0.5$ in the region below the top of the roughness elements ($y \lesssim 1.5k$). In LES, $\Delta x^+ = 17 - 30$, $\Delta z^+ = 6 - 9$, and $\Delta y^+ < 1$ in the region $y \lesssim 1.5k$. The stretching rate of the $y$ grid is less than 4% in all cases. Note that even the LES is highly resolved, with grid sizes only marginally worse than those of direct simulations of smooth-wall flows in the literature. Between 20 and 100 million total grid points were required. The roughness
geometry is resolved by at least 16 grid points in the \((x,z)\) plane, and by more than 30 grid points in the \(y\) direction; this surface resolution is similar to the one used in the work of Scotti (2006), where the current roughness model was validated. The simulations were initialized by a calculation at a lower value of \(K\), and integrated in time for \(15\delta/u_\tau\) units to achieve a statistically steady state. Data was then collected for 30 additional time units to calculate statistics.

In the following, the angle brackets \(\langle \cdot \rangle\) denote quantities that are averaged in time and over the homogeneous directions \(x\) and \(z\) (including both solid and fluid domains). \(U_i(y)\) is the time- and space-averaged velocity

\[
U_i(y) = \langle u_i(x,y,z,t) \rangle, \tag{4.4}
\]

whereas \(\langle \cdot \rangle_t\) denote averaging over time.

\[
\bar{u}_i(x,y,z) = \langle u_i(x,y,z,t) \rangle_t - U_i(y) \tag{4.5}
\]

is the deviation of the local time-averaged velocity from the time-and space-averaged one. Note that \(\bar{u}_i(x,y,z)\) is non-zero only in the vicinity of the roughness. Turbulent fluctuations \(u'_i\) are calculated by subtracting the time-averaged velocity from the total one:

\[
u'_i(x,y,z,t) = u_i(x,y,z,t) - \langle u_i(x,y,z,t) \rangle_t \tag{4.6}
\]

\[
= u_i(x,y,z,t) - [U_i(y) + \bar{u}_i(x,y,z)]. \tag{4.7}
\]

First and second moments of the velocity calculated using half the sample differed from those calculated using the full sample by less than 4%.
4.4 Results

In this section, first the smooth-wall sink flows are compared with previous results. Then, the flow statistics (Section 4.4.2) are studied to investigate the start of reverse-transition in the context of rough-wall sink flows. Next, we focus on the fully-turbulent cases to determine what causes the gradual change to the reverse-transition state, and what the effects are on the statistics. To explain the flow statistics, the mean-flow structures (Section 4.4.3) and turbulent structures in both near-wall region (Section 4.4.4) and the outer region (Section 4.4.5) are then studied among the fully-turbulent cases.

4.4.1 Smooth-wall sink flow

The mean velocity and turbulent fluctuations of the highest- and lowest-$K$ cases for smooth walls are compared in Figure 4.3 with experimental (Jones et al., 2001; Dixit & Ramesh, 2010) and other DNS results (Spalart, 1986). As $K$ increases, $U$ falls above the universal
The total shear stress and Reynolds shear stress for case K3R0 are compared with DNS and experimental results in Figure 4.4. Excellent agreement is obtained, indicating that, despite the difference in friction velocity, the current results capture the same overall logarithmic law, and the slope slightly decreases from 1/κ. Meanwhile, the inner peak of the streamwise Reynolds stress \( \langle u'^2 \rangle \) moves away from the wall due to the thickening of the viscous sublayer, and the wall-normal component \( \langle v'^2 \rangle \) is significantly reduced throughout the boundary layer. Good agreement is achieved with experimental results. Compared to DNS results obtained by Spalart (1986), the current simulation gives a 4% lower friction velocity, shown by the higher maximum \( U^+ \), which is closer to the experimental results, as well as 8% lower peak for \( \langle u'^2 \rangle^+ \). This is probably due to the finer grid size in the \((x, z)\)-plane in wall units (at least twice finer than the former DNS) used in the current study; a grid-refinement study has shown that a finer grid leads to lower friction velocity, higher peak of \( \langle u'^2 \rangle^+ \) and low peak of \( \langle v'^2 \rangle^+ \).
Table 4.2: Boundary layer parameters in all cases. $H$ is the shape factor; $\theta$ is the momentum thickness.

<table>
<thead>
<tr>
<th>Case</th>
<th>$k/\delta$ (%)</th>
<th>$Re_\theta = U_\infty \theta/\nu$</th>
<th>$C_f \times 10^3$</th>
<th>$H$</th>
<th>$\delta_{99.9}^+$</th>
<th>$\bar{\theta}_{99.9}$</th>
<th>$\theta \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1R0</td>
<td>0</td>
<td>2037</td>
<td>4.15</td>
<td>1.32</td>
<td>1752</td>
<td>0.017</td>
<td>9.2</td>
</tr>
<tr>
<td>K1R1</td>
<td>1.9</td>
<td>2748</td>
<td>5.93</td>
<td>1.37</td>
<td>1874</td>
<td>0.015</td>
<td>12.7</td>
</tr>
<tr>
<td>K2R0</td>
<td>0</td>
<td>1248</td>
<td>4.48</td>
<td>1.35</td>
<td>1116</td>
<td>0.019</td>
<td>10.0</td>
</tr>
<tr>
<td>K2R1</td>
<td>1.7</td>
<td>1510</td>
<td>5.83</td>
<td>1.34</td>
<td>1166</td>
<td>0.017</td>
<td>12.1</td>
</tr>
<tr>
<td>K2R2</td>
<td>3.1</td>
<td>2028</td>
<td>7.49</td>
<td>1.34</td>
<td>1472</td>
<td>0.019</td>
<td>16.2</td>
</tr>
<tr>
<td>K3R0</td>
<td>0</td>
<td>662</td>
<td>4.88</td>
<td>1.43</td>
<td>584</td>
<td>0.018</td>
<td>9.9</td>
</tr>
<tr>
<td>K3R1</td>
<td>1.5</td>
<td>749</td>
<td>5.64</td>
<td>1.36</td>
<td>687</td>
<td>0.019</td>
<td>11.0</td>
</tr>
<tr>
<td>K3R2</td>
<td>2.9</td>
<td>1016</td>
<td>6.87</td>
<td>1.33</td>
<td>808</td>
<td>0.020</td>
<td>15.2</td>
</tr>
<tr>
<td>K3R3</td>
<td>4</td>
<td>1266</td>
<td>8.62</td>
<td>1.31</td>
<td>1028</td>
<td>0.024</td>
<td>19.1</td>
</tr>
<tr>
<td>K4R0</td>
<td>0</td>
<td>372</td>
<td>4.84</td>
<td>1.58</td>
<td>337</td>
<td>0.017</td>
<td>9.3</td>
</tr>
<tr>
<td>K4R2</td>
<td>2.9</td>
<td>484</td>
<td>6.19</td>
<td>1.39</td>
<td>462</td>
<td>0.021</td>
<td>12.1</td>
</tr>
<tr>
<td>K4R3</td>
<td>4.2</td>
<td>696</td>
<td>7.87</td>
<td>1.31</td>
<td>574</td>
<td>0.023</td>
<td>17.4</td>
</tr>
</tbody>
</table>

acceleration effects as the DNS.

The grid requirements for smooth-wall cases are assumed to apply to rough cases also, since, in the transitionally rough regime, the roughness scales are of the same order of the scales of near-wall structures. Additionally, the domain size of the smooth cases is deemed sufficient for the rough cases, assuming that roughness does not increase the scale of the largest eddies, which occur far from the wall, where the roughness is not expected to play a significant role; this assumption is supported by results from various authors, e.g., Volino et al. (2007) and Wu & Christensen (2010), and will be validated a posteriori by examining the outer-layer structure size.

### 4.4.2 Flow statistics

Results of the boundary layer parameters are shown in Table 4.2. The Reynolds number based on momentum thickness, $Re_\theta$, and the friction coefficient

$$C_f = 2 \left( \frac{u_\tau}{U_\infty} \right)^2,$$

(4.8)
Figure 4.5: Effects of roughness and acceleration on (a), (b) the Reynolds number and (c), (d) the friction coefficient. Lines connect cases with constant $k/\delta$ in (a) and (c), and cases with constant $k^+$ in (b) and (d). Hollow symbols are data from current study: □ smooth; △ rough. Solid symbols are reference data: ♦ Dixit & Ramesh (2010); ● Jones & Launder (1972); ▼ Spalart (1986).
where
\[ u_\tau = \sqrt{-(f_d)/\rho}, \tag{4.9} \]
are shown in Figure 4.5. \( f_d \) is the sum of pressure and viscous drag on rough surfaces; its calculation is described in Chapter 2. Good agreement is obtained with available experimental (Jones & Launder 1972, Dixit & Ramesh 2010) and numerical (Spalart 1986) smooth-wall data. \( Re_\theta \) decreases with \( K \) and increases with roughness, as observed by Cal et al. (2008). \( C_f \) was found to increase with \( K \) by Cal et al. (2008), while results obtained by Tachie et al. (2007) showed negligible variation of \( C_f \) with \( K \). In the current study, however, an increase of \( K \) with a constant \( K \) decreases \( C_f \), while roughness increases \( C_f \), as was observed in all studies. Notice that \( u_\tau/U_\infty = (C_f/2)^{1/2} \) is only mildly affected by \( K \) [Figure 4.5(c)]; also, the viscous length scale is
\[ \overline{\delta}_\nu \equiv (\nu/u_\tau)/X_o = K(U_\infty/u_\tau). \tag{4.10} \]

Then, \( k^+ \), which can be written as
\[ k^+ = \frac{(k/X_o)(u_\tau/U_\infty)}{K}, \tag{4.11} \]
can be viewed as a relative measure of the effects of roughness and acceleration, since the numerator is essentially determined by \( k \). Following the constant-\( k^+ \) lines in Figure 4.5(b) and (d), it is found that, as \( K \) increases, \( Re_\theta \) decreases, dominated by the acceleration effect, and \( C_f \) increases, dominated by the roughness effect.

Figure 4.6 shows the mean velocity profiles in wall units for calculations with increasing acceleration [Figure 4.6(a)] and roughness height [Figure 4.6(b)]. The zero-plane displacement, \( d \) is defined as the wall-normal location where the drag appears to act (i.e., the centroid of the local drag profile). With the sand-grain model, \( d/k \) is around 0.8, insensitive to FPG; this value is the same as that obtained by Scotti (2006) in open-channel flow.
Figure 4.6: Mean velocity profiles in inner scaling. (a) Effect of $K$; (b) Effect of $\bar{k}$. Universal logarithmic law.

simulations, probably because the shape of the forcing distribution depends more on the type of roughness, compared to the effects of external conditions such as flow acceleration. Note that the various thicknesses are measured from the plane $y = d$.

In most cases (the main exception being the high-acceleration, low-roughness case, K4R2), we observe clearly a logarithmic layer, which acceleration displaces upwards and roughness downwards. Non-universal log-law constants have been used to describe equilibrium sink flows with similar or lower levels of acceleration compared to the current study (Dixit & Ramesh 2008, 2010), but in our calculations the von Kármán constant $\kappa$ (calculated by considering the plateau region of $y^+dU^+/dy^+$) was found to be within the accepted range around the universal value ($\kappa \simeq 0.4$) except in case K4R2. In this case, $k^+ \simeq 10$, close to the hydraulically smooth regime, and the pressure gradient is strong, leading to a case close to reverse transition. On the smooth wall, Dixit & Ramesh (2008) observed a gradual 18% increase of $\kappa$ as $K$ increases from $7.7 \times 10^{-7}$ to $2.9 \times 10^{-6}$; on the current R2 rough wall, the relatively sudden increase of $\kappa$ as $K$ approaches K4 indicates that flow reversion occurs only when the flow is nearly in the hydraulically smooth regime, where $\kappa$ approaches the smooth-wall value; for higher $k^+$, $\kappa$ is insensitive to the strengthening of
Figure 4.7: Dependence of roughness function $\Delta U^+$ (case K4R2 not shown) on $k^+$. Rough cases: $\bigcirc$ K1, $\square$ K2, $\triangle$ K3, $\triangledown$ K4; $+$ Colebrook (1939).

acceleration.

Since the slope of the logarithmic region is close to the universal value, the roughness function can be defined as the offset of the logarithmic profile from the universal logarithmic law,

$$\Delta U^+ \equiv U^+ - \frac{1}{\kappa} \log(y - d)^+ - B,$$

where $\kappa = 0.41$ and $B = 5.0$. From Figure 4.7 we observe that the sink-flow results collapse with those from experimental studies on equilibrium pipe flows. The robustness of the dependence of the roughness function on the roughness Reynolds number may deserve further attention.

The roughness-induced momentum deficit is not dominantly affected by acceleration nor roughness; instead, it is affected by the ratio between the strengths of the two. In non-equilibrium accelerating boundary layers, Tachie et al. (2007) found, based on the assumption of a universal log-law, that $\Delta U^+$ is affected by roughness height but not by acceleration. This does not necessarily contradict the current results, since, in their studies, $k^+$ is not significantly affected by $K$ either. The current work serves to clarify that both $K$
and $\bar{k}$ are important in determining the mean velocity deficit in developed FPG flows.

The roughness effects on the Reynolds stresses are presented in Figure 4.8 compared to the smooth case with the same $K$. It is known that, for low blockage ratio $k/\delta$, three-dimensional roughness does not affect outer-layer Reynolds stresses in a high-Reynolds-number boundary layer with zero-pressure-gradient (ZPG), when normalized with $u_\tau$. Here, it is found that wall similarity also applies to boundary layers that are subject to certain strength of acceleration, as shown by the K3 cases [Figure 4.8(a)] and cases with weaker $K$ (not shown); among the K3 cases, $Re_\theta = 625 - 1276$ and $k/\delta \leq 0.04$. The thickness of the roughness sublayer, defined as the layer where the statistics of the rough cases differ from the smooth one, is around $5k$, consistent with earlier observations of $3 - 5k_s$ for a variety of three-dimensional roughness. On the other hand, in the K4 cases [Figure 4.8(b)] the Reynolds stresses do not collapse; specifically, the increase of $\langle u'^2 \rangle$ due to roughness is less fast than that of $u_\tau^2$, while the increase of $\langle v'^2 \rangle$ is faster. In these cases, $k/\delta$ are similar to those in the K3 cases, but the Reynolds numbers are much lower ($Re_\theta = 372 - 696$). Case K4R0 and K4R2 are in the reverse-transitional state; a deviation is, therefore, expected from the outer-layer similarity hypothesis based on fully turbulent flows.

Figure 4.8: Roughness effects on the streamwise and wall-normal components of the Reynolds stress tensor, normalized by $u_\tau$, in cases with (a) K3 and (b) K4.
Figure 4.9: Separate effects of $K$ and $\overline{k}$ on the normal Reynolds stresses and the Reynolds shear stress.

In Figure 4.9, the Reynolds stresses are compared among three cases: cases K1R1 and K3R1 show the effects of $K$ increase only, while cases K3R1 and K3R3 show the effects of $\overline{k}$ increase only. K1R1 and K3R3 have the same $k^+ (\approx 40)$ while K3R1 corresponds to a lower $k^+ (\approx 10)$. Similar to the effects on smooth-wall flows, an increase of $K$ increases the streamwise component of the Reynolds normal stress in the near-wall region, and damps the wall-normal and spanwise components; the Reynolds shear stress also decreases near the wall. On the other hand, an increase of $\overline{k}$ decreases the streamwise component and increases both the other two normal components of Reynolds stresses, and results in a higher $\langle u'v' \rangle^+$. The effects of $K$ are visible throughout the lower half of the boundary layer (especially for $v'^+$ and $w'^+$), consistent with the observations by Cal et al. (2009) that the $u_\tau$ scaling does not absorb the FPG effects. However, the current $K$ effects appear to start from the inner
peak, while in their studies, the $K$ effects are significant throughout the boundary layer and appear to start from the outer layer. The roughness effect on $\langle u'^2 \rangle$ in the current study is consistent with their observations, showing a significant decrease within the region $y - d < 0.2\delta$, whereas its effects in $\langle v'^2 \rangle$, $\langle w'^2 \rangle$ and Reynolds shear stress are much more significant than in their studies, in which they were found to be minimal. Comparison between cases K1R1 and K3R3 shows that cases with the same $k^+$ give nearly identical Reynolds-stress profiles; the only difference is the slightly lower peak values in the case with higher $K$ and higher $\overline{k}$.

The Reynolds stress anisotropy tensor $b_{ij}$ is defined as

$$b_{ij} = \frac{\langle u'_i u'_j \rangle}{\langle u'_k u'_k \rangle} - \delta_{ij} \frac{3}{3}. \quad (4.13)$$
Figure 4.11: Energy budgets of cases K1R1 (solid), K3R1 (---), K3R3 (--) in wall units. (a) Production and viscous dissipation, (b) turbulent diffusion and (c) viscous diffusion.

Anisotropies for the same three rough cases are shown in Figure 4.10. The outer-layer anisotropy (in the region \((y-d) > 0.35\)) is dominantly determined by \(K\), as shown by the collapse of cases K3R1 and K3R3 in this region in Figure 4.10 (a)–(c); this is consistent with the outer-layer similarity valid for cases outside of the reverse-transitional state. In the wall region, the anisotropy profiles mostly collapse for cases with \(k^+ \approx 40\), as shown by cases K1R1 and K3R3 in the region \((y-d)^+ < 50\) in Figure 4.10 (d)–(f), suggesting a near-wall similarity based on the roughness Reynolds number.

The near-wall turbulent-kinetic-energy (TKE) budgets are compared in Figure 4.11 normalized by \(u_\tau\) and \(\delta_v\). Note that the effects of the growth terms have been included into energy production and mean-flow advection; the mean-flow advection is negligible for all three cases and thus is not presented. Cases K1R1 and K3R3 collapse well for all budget terms. The case with lower \(k^+\), however, shows a peak of production closer to the wall, at \((y-d)^+ \approx 10\); the energy generated at this vertical location is transported towards the wall through viscous and turbulent diffusion (Figure 4.11 (b) and (c)), and dissipated.
in the region \((y - d)^+ \lesssim 10\), where the viscous dissipation exhibits a peak (Figure 4.11 (a)). The higher-\(k^+\) cases show much weaker energy diffusion, especially viscous, due to the fact that these cases are much farther away from the hydraulically-smooth regime; as a result, the energy dissipated in the vicinity of roughness \(((y - d)^+ \lesssim 10)\) is mostly generated at the same wall-normal location (shown by the approximate equality of production and dissipation in this region in Figure 4.11 (a)).

Quadrant analysis is also carried out. Following Wallace et al. (1973), the averaged Reynolds shear stress is decomposed into contributions from four quadrants,

\[
\langle u'v' \rangle_Q(y, H) = \langle u'(x, y, z, t)v'(x, y, z, t)I_Q(x, y, z, t, H) \rangle,
\]

where \(I_Q\) is an indicator function defined as

\[
I_Q(x, y, z, t, H) = \begin{cases} 
1 & \text{if } |u'v'|_Q(x, y, z, t) \geq H\sigma_u(y)\sigma_v(y), \\
0 & \text{otherwise},
\end{cases}
\]
Figure 4.13: Quadrant contributions from Q2 and Q4 events versus the wall-normal location in outer scaling. Smooth cases: K1R0 (filled ○), K3R0 (filled □); rough cases: K1R1 (○), K3R1 (□), K3R3 (△).

and $\sigma_u(y)$, $\sigma_v(y)$ are the root-mean-square deviations of $u'$ and $v'$. $H$ is the strength threshold for the quadrant events to be considered. In all the cases, the contribution of the total sweeping (Q2) and ejecting (Q4) events, with $H = 0$, is $3 - 5$ times larger (in magnitude) than the contribution of the other two quadrants; therefore attention is given to Q2 and Q4. The wall-normal distributions of the quadrant contributions are compared for the smooth cases [Figure 4.12(a)] and three rough cases [Figure 4.12(b)]. The smooth case resembles non-accelerating flows [Kim et al., 1987]: sweeps are more significant in the near-wall region and ejections in the outer flows, respectively, with the equal contributions occurring around $y^+ \approx 15$. Increase of $K$ on both smooth and R1 rough walls shows that the main effect of $K$ is to promote the Q2 contributions and to decrease those of Q4 in the region $(y - d)^+ > 20$. Less difference in the near-wall region is observed. Profiles of K3R1 and K3R3 show that an increase of $\bar{k}$ tends to restore the contributions in the region $15 \lesssim (y - d)^+ \lesssim 50$; near the wall, Q2 contributions from the rough cases are generally 30% higher than on a smooth wall. The role of roughness in intensifying ejections close to the wall and sweeps in the outer layer was also observed in experiments [Grass, 1971].
Figure 4.14: Contours of (a) streamwise, (b) wall-normal, and (c) spanwise time-averaged velocities in the \((x, z)\) plane \(y = d\) for case K3R3, normalized by \(u_\tau\) and \(\delta_\nu\). The white contour lines denote the fluid-solid interface \((\phi = 0.5)\).

Krogstad et al. (1992). In Figure 4.13, the wall-normal location is normalized using \(\delta\). For \((y - d)/\delta \gtrsim 0.2\), the rough-wall profiles fall on the respective smooth-wall profile with the corresponding \(K\), indicating that the outer-layer similarity also applies to stress contribution of total Q2 and Q4 quadrant events.

### 4.4.3 Mean-flow structures

Roughness induces mean-flow heterogeneity near the wall, shown in Figure 4.14 by the horizontal contours of time-averaged velocity components at \(y = d\) in Case K3R3. Recall that such spatial variation is due to \(\tilde{u}_i\) components. The velocities are normalized by \(u_\tau\).

Comparing Figure 4.14 (a) and (b), two types of motions can be identified: the mean sweeping motions, which appear as long streaks of positive \(\langle u \rangle_t\) and negative \(\langle v \rangle_t\), and the nearly vertical ejection motions, \(i.e.,\) regions of locally high magnitude of positive \(\langle v \rangle_t\). The sweeping motions may be related to the mean-flow “channeling phenomenon” observed on pyramid roughness by Hong et al. (2011). In addition, Figure 4.14 (c) shows the mean
spanwise motions of the flow bypassing a single roughness element or a block of elements.

The magnitude of the mean-flow heterogeneity versus the wall-normal direction is studied by considering the RMS of the mean-flow variation, $\sigma_{\bar{u}_j}(y)$, at each $y$ location in Figure 4.15. For $y < k$, the variation of $\bar{u}_j$ is stronger than that of $u'_j$, while for $y > 3k$, a plateau of minimum $\sigma_{\bar{u}_j}/\sigma_{u'_j}$ is reached. The non-zero value of $\sigma_{\bar{u}_j}$ in the outer layer is due to the limited temporal sampling of the flow, as is shown by the non-zero $\sigma_{\bar{u}_j}/\sigma_{u'_j}$ in the smooth case K3R0. Given that the thickness of the roughness sublayer is $5k$ for current roughness, the layer of significant mean-flow heterogeneity ($y/k < 3$) is inside the roughness sublayer.

### 4.4.4 Near-wall turbulent structures

The turbulent vortices, shown by the isosurfaces of the second invariants of the full-velocity tensor

$$Q = -\frac{1}{2} \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j}$$

(4.16)
and the low-speed regions, shown by the isosurfaces of $u'^+ = -3$, are presented in Figure 4.16 for cases K1R1, K3R1 and K3R3. In all cases, the near-wall vortices closely correlate with the distribution of roughness elements. In case K3R1, two types of near-wall vortices are clearly identifiable: the roughness-scale vortices attached at the tip of the rough elements, and the vortices elongated in the streamwise direction, similar to the quasi-streamwise vortices over a smooth wall. The latter are mostly decorrelated from the roughness distribution; the low-speed regions are stable for a very long streamwise distance, usually from $700\delta_\nu$ to $1000\delta_\nu$, a feature similar to the low-speed streaks over a smooth wall. In the cases with $k^+ \approx 40$, the attached eddies extend into the layers of quasi-streamwise vortices; these quasi-streamwise vortices are more densely distributed compared to case K3R1; the low-speed regions are more limited in their streamwise lengths, usually $200 - 300\delta_\nu$, and are lifted up frequently upstream of relatively tall roughness elements. Similar to the experimental observations of coherent structures in a fully-rough channel flow with pyramid roughness by Talapatra & Katz (2012), the prevalent structures in the two higher-$k^+$ cases

Figure 4.16: Isosurfaces of $u'^+ = -3$ (yellow) and $Q^+ = 0.01$ (coloured by $(y-d)^+$) in cases (a) K1R1, (b) K3R1 and (c) K3R3. Rough surfaces are shown in white.
Figure 4.17: Isosurfaces of components of $Q$: mean-flow component (white), turbulent component (purple), and the component corresponding to mean-flow and turbulence interaction (yellow).

are densely-distributed low-laying spanwise and groove-parallel vortical structures of the scale of the roughness, as well as quasi-streamwise vortices. The near-wall structures do not completely match the experimental observations due to the lower roughness Reynolds number and the randomness of roughness rotation and location. In the region $(y-d)^+ \gtrsim 50$, hairpin vortices are observed in in cases K3R1 and K3R3; the hairpins in case K1R1 mostly appear above the region visualized, which is limited to the region $(y-d)/\delta < 0.05$, where the dominant structures are tilted, quasi-streamwise vortices.

To classify the near-wall vortices, $Q$ is separated into three components,

$$Q = -\frac{1}{2} \frac{\partial (\bar{u}_j + U_j)}{\partial x_i} \frac{\partial (\bar{u}_i + U_i)}{\partial x_j} - \frac{1}{2} \frac{\partial u'_j}{\partial x_i} \frac{\partial u'_i}{\partial x_j} - \frac{\partial (\bar{u}_j + U_j)}{\partial x_i} \frac{\partial u'_i}{\partial x_j},$$  \hspace{1cm} (4.17)

where the terms on the right-hand-side are, from left to right, the contribution from near-wall mean-flow heterogeneity, the contribution from turbulence, and the component due to the interaction between mean flow and turbulence. The isosurfaces of all components are plotted in Figure 4.17; the case K3R1 is chosen as an example due to the clear separation of
these components. It is found that the attached roughness-scale eddies are due to mean-flow separations: they are either mean-flow eddies, or turbulent vortices that exist in the regions of high mean-flow gradient due to roughness. These eddies contribute to the differences of flow structures between smooth- and rough-wall flows. The wall-normal extension of the attached eddies depends on the height of roughness. For case K3R1, these eddies are mostly restricted to the region $y^+ < 14$, where viscous effects are significant, whereas in K3R3 and K1R1, they extend into the layer that is important for turbulent production, and lead to more densely distributed quasi-streamwise vortices, probably because they augment streaks instability. Such roughness effect is consistent with earlier observations that the near-wall Reynolds-stress anisotropy and TKE budgets satisfy similarity for cases with the same $k^+$, since $k^+$ is the ratio between the physical roughness height and the thickness of the layer where viscosity is important.
To compare quantitatively the size and inclination of the near-wall turbulent structures, Figure 4.18 shows the two-point correlations of streamwise fluctuations, $R_{uu}$, in the $(x, y)$-plane centred on $y = k$, for the three cases. $R_{uu}$ at a reference wall-normal location, $y_{ref}$, is defined as

$$R_{uu}(\Delta x, \Delta y, y_{ref}) = \frac{\langle u'(x, y_{ref}, z) u'(x + \Delta x, y_{ref} + \Delta y, z) \rangle}{\sigma_w(y_{ref})^2},$$

(4.18)

where $\Delta x$ and $\Delta y$ are separations in the streamwise and the wall-normal directions. Following Volino et al. (2007), the inclination angle is obtained from the line fitted using the least-square method from points farthest upstream and downstream from the correlation peak on each of the contour levels from 0.4 to 0.9 with increment 0.1. A decrease of $k^+$ (from K1R1 and K3R3 to K3R1) leads to more elongated near-wall coherent structures in the streamwise direction, and the wall-normal dimension becomes more limited; as a results, case K3R1 exhibits a shallower inclination ($3^\circ$) compared to case K1R1 ($13^\circ$) and case K3R3 ($10^\circ$). This phenomenon can be explained by larger streamwise coherence due to less frequent lift-ups of low-speed streaks and a decrease of bursting.

To study the variation of large-scale structure size against the wall-normal location,
Figure 4.19 compares the integral length scales, defined as

\[ L_{\alpha \beta}(y) = \int_0^\infty R_{u_{\alpha} u_{\beta}}(y, \Delta r) d(\Delta r) \],

where \( R_{u_{\alpha} u_{\beta}} \) is the two-point spatial correlation in the \((x, z)\)-plane, defined as

\[ R_{u_{\alpha} u_{\beta}}(y, \Delta r) = \frac{\langle uu'_\alpha(y, r) uu'_\beta(y, r + \Delta r) \rangle}{\sigma_{u'_\alpha}(y) \sigma_{u'_\beta}(y)} \],

and \( r \) denotes the location in \( x \) (for \( L_{\alpha \beta, x} \)) or \( z \) (for \( L_{\alpha \beta, z} \)). To remove errors due to fluctuations in the two-point correlation caused by insufficient sample, the integration is carried out to the value of \( \Delta r \) at which the correlation coefficient first crosses 0.2. \( L_{11,x}^+ \) in both two smooth cases and case K3R1 presents a peak in the wall region due to the flattening and elongation of near-wall structures; it then decreases slightly. Qualitatively similar trends were observed in experimental smooth-wall sink-flow studies [Dixit & Ramesh 2010]. In the rough cases with higher \( k^+ \), \( L_{11,x}^+ \) is quite different: cases K1R1 and K3R3 almost collapse and produce a monotonically increasing \( L_{11,x}^+ \). The effects of \( K \) and \( \bar{k} \) on the spanwise integral length are absorbed into the viscous length scale.

4.4.5 Outer-layer turbulent structures

Hairpin vortices, either symmetric, asymmetric, or one-sided, are frequently observed in the outer layer (Figure 4.16 (b), (c)). The visualizations of velocity vectors in Figure 4.20 using the approach of Adrian et al. (2000) demonstrate the existence of the hairpin packets, shown by the shear layer separating the low-speed and high-speed region, and by the chains of negative-\( \omega_z \) regions, which can be interpreted as hairpin heads. In all three cases, the inclinations of the packets are around 12°–14° and the packet extensions are around 1\( \delta \), insensitive to either acceleration or roughness. The inclination of the local shear layer upstream of a single hairpin varies widely, often from 30° to 65°, and its value range is not
Figure 4.20: Instantaneous velocity vectors and contours of negative $\omega_z$ in case (a) K1R1, (b) K3R1, (c) K3R3. The convection velocity $U_c = 0.8U_\infty$ is subtracted from the velocity field. Lines indicate angles of hairpin packets. Contour level: $-5.7 \leq \omega_z\delta/U_\infty \leq -0.8$. Every other grid point in $x$ and $y$ directions is used for plotting velocity vectors.
Figure 4.21: Two-point correlations of streamwise fluctuations, $R_{uu}$, centred on $y = 0.3 \delta$ in fully turbulent rough-wall cases: (a) K1R1, (b) K3R1, (c) K3R3, (d) K4R3. Contour levels: 0.3 to 0.9 with increment 0.1.

significantly different from case to case; the average separation between hairpins in a packet varies from case to case in Figure 4.20 when normalized by $\delta$, but, under inner scaling, they are all around 100–150 wall units. These observations of hairpin structures and packets are consistent with experimental observations in ZPG turbulent boundary layers (Adrian et al., 2000); no significant change is caused by either roughness or acceleration.

The size and inclination of the packets is shown by the contour of two point correlation of $u'$ (Figure 4.21). The inclination angles, obtained using the same fitting approach in Figure 4.18, vary weakly in all three cases (Figure 4.21 (a)–(c)) and in a case with higher $K$ (K4R3, Figure 4.21 (d)), from $10^\circ$–$12^\circ$, within the $10^\circ$–$15^\circ$ range obtained in many former experimental and numerical studies of ZPG turbulent boundary layers and channel flows.
In smooth-wall sink flows, Dixit & Ramesh (2010) associated the decrease of hairpin-packet inclination angle to the reverse-transition; they obtained the inclination of overall near-wall structures from the location of maximum cross-correlation between wall-shear stress and \( u' \) in the region \( 0.1\delta < y < 2.5\delta \), and found that an increase in \( K \) from \( 0.77 \times 10^{-6} \) to \( 1.74 \times 10^{-6} \) weakly decreases the structure inclination by 20%. It is interesting, however, that here \( K \) is shown to exert no visible effect in transitionally-rough sink flows, even when \( K = 2.5 \times 10^{-6} \) (case K4R3), which, in the case of a smooth-wall, would lead to nearly relaminarized flow, and much smaller inclination angle. (Dixit & Ramesh, 2010)

On the role of roughness on outer-layer structure inclination, weak roughness effects were observed in a number of experimental studies, e.g., by Volino et al. (2007), where the inclinations on rough-wall ZPG boundary layers fall in the 10°–15° range, consistent with the current study. However, a significant role of roughness in increasing outer-layer structural inclination was observed in several experimental studies, e.g., Krogstad & Antonia (1994), and in DNS studies by Lee et al. (2011), on ZPG boundary layers; this may be due to the higher blockage ratio \( k/\delta \), since a necessary condition for the outer-layer similarity in rough-wall turbulence is a significant separation between the roughness length scale \( k \) and the boundary layer thickness (Jiménez, 2004).

The correlation contours in Figure 4.21 also show an increase of structure size as \( K \) increases. In case K3R1, the increase in the dimension of the eddies results in connections between the hairpin packets and the near-wall structures upstream (from \( x/\delta = -1.0 \) to \( -0.5 \) in Figure 4.21 (b)); this phenomenon, however, is not observed in case K3R3 (Figure 4.21 (c)). This indicates that, in case K3R3, the hairpins in the outer layer are decorrelated with the structures in the region directly affected by the randomly-distributed roughness; these structures include the legs of primary hairpins whose heads have already evolved into the outer layer, and any secondary hairpin they generate from the legs.

Figure 4.22 compares the integral length scales defined in Equation (4.19), normalized by \( \delta \), in the streamwise and spanwise directions. Both \( L_{11,x} \) and \( L_{11,z} \) in the outer layer
cluster into two branches that differ in acceleration level. This indicates that acceleration dominantly determines the scales of the largest structures in the outer layer: a stronger acceleration leads to a larger size of the outer-layer structures. This may be explained by the “pure-wall-flow” nature of a sink flow, in which the outer-region structures also scale with the viscous length scale. Increasing from K1 to K3 results in almost 100% increase of the outer-layer \( L_{11,x}/\delta \) (Figure 4.22 (a)), while, when plotted in wall units, it reduces to a 15% increase in \( L_{11,x}^+/\delta \) (Figure 4.19 (a)); if the cut-off value of \( R_{uu} \) for calculating the integral length is changed to 0.5, \( L_{11,x}^+ \) can be shown insensitive to \( K \) outside of the roughness sublayer. The roughness effect is negligible in the outer layer; but the roughness decreases the streamwise integral length scale in the roughness sublayer, and the amount of decrease is augmented by a higher \( k \). The decrease of size of large structures near the wall for the sand-grain (SG) roughness agrees with experimental observations on three-dimensional mesh roughness by Volino et al. (2007), while it is opposite to the increase of the streamwise extent of \( R_{uu} \) observed throughout the boundary layer over two-dimensional bars (Volino et al., 2009) and in the roughness sublayer above a type of realistic roughness (Wu & Christensen, 2007).
Figure 4.23: Contour lines of $u'v'|_{Q_2}$ (black) and $u'v'|_{Q_4}$ (white) induced by the hairpin structures (isosurfaces of $Q_{X_o}/U_{\infty,o} = 800$) in Case K3R3. Plane located at $(y-d)/\delta = 0.5$. Contour of negative $\omega_z/\delta/U_{\infty}$ shows hairpin heads. Contour line levels: from $\sigma_{u'}\sigma_{v'}$ to $20\sigma_{u'}\sigma_{v'}$ with increment $\sigma_{u'}\sigma_{v'}$.

The correlation between strong quadrant events and the turbulent structures in the outer layer is shown in Figure 4.23. Compared to Q4 events, the distribution of Q2 events relates much more closely to the hairpin heads: strong Q2 events with $H \geq 1$ are present below and immediately upstream of the hairpin heads. The Q4 events distribute widely outside of the region enclosed by the hairpins. For $H = 1$, the Q4 events occupy larger total area compared to Q2 events, but Q2 events reach much higher strength than Q2; as a result, Q2 events contribute more than Q4 events to the averaged Reynolds shear stress at this elevation.

The good correlation between the patches of strong Q2 events and the hairpin structures allows us to study the density of the outer-layer turbulent structures. Note that the notion of “structure density” in space is in analogy to the “bursting frequency” in time, which is widely used by experimentalist in analyzing turbulent structures. The quantification of a burst event includes the burst size, denoted by $A$, and the mean separation between two events, $r$. The burst intermittency (or “space ratio”), $\gamma$, defined as the ratio between the total event area and the total domain area, quantifies the strength of turbulent production.
Various detection techniques have been used in the literature to identify bursts; reviews of major methods can be found in Robinson (1991) and Bogard & Tiederman (1986). Bogard & Tiederman (1986) concluded that the $u'v'$ quadrant method (Wallace et al., 1973) is less prone to false detection. Here the quadrant technique is also employed; specifically, a burst event is considered to be a strong ejection corresponding to values of $H$ equal to 1, 2, and 4. The threshold value $H = 1$ was found to provide the highest percentage of ejections detected (Bogard & Tiederman, 1986), while higher $H$ are also used to study stronger events, although only a subset of them is detected.

The intermittency $\gamma$ is calculated by counting the grid cells with $|u'v'|_{Q2}(x, y, z)$ stronger than the threshold. To quantify $A$, first, for a $(x, z)$-plane, a masked field of relatively strong $Q2$ stress, $u'v'|_{Q2}$, is obtained as

$$u'v'|_{Q2}(x, y, z, H) = u'(x, y, z)v'(x, y, z)I_{Q2}(x, y, z, H). \quad (4.21)$$

Then the two-point auto-correlation, $R_{QQ, st}$, of the marked field is calculated as

$$R_{QQ}(\Delta x, \Delta z, y, H) = \frac{\langle u'v'|_{Q2}(x, y, z, H) u'v'|_{Q2}(x + \Delta x, y, z + \Delta z, H) \rangle}{\sigma_{Q2}^2}, \quad (4.22)$$

where $\sigma_{Q2}$ is the standard deviation of $u'v'|_{Q2}$ in the corresponding $(x, z)$-plane. $A$ is then obtained as the averaged area of coherent ejection events,

$$A(y, H) \equiv \iint_{R_{QQ} \geq 0.1} R_{QQ}(\Delta x, \Delta z, y, H) d\Delta x d\Delta z, \quad (4.23)$$

where the value 0.1 is used as it is effective in capturing the size variation. $r$ can then be defined as

$$r(y, H) \equiv \sqrt{A(y, H) / \gamma(y, H)}. \quad (4.24)$$

The intermittency and period ($T_b$) of bursts in smooth-wall sink flows were studied in
Figure 4.24: Averaged separation (inverse of density) of Q2 events detected with $H = 1$ at different near-wall elevations in smooth cases. *Chambers et al. (1983) experiment.

experiments by Chambers et al. (1983); detection was made when the local variance of wall-shear stress exceeded a critical value, with the threshold chosen by matching their detection with Blackwelder & Kaplan (1976) in their boundary-layer studies, where the detection is based on the local velocity variance that is higher than 1.2 times of the variance obtained from short-term average, at the elevation $y^+ = 15$. To validate the current approach, we compare our smooth-wall results with Chambers et al. (1983); the experimental results are treated using Taylor hypothesis with the assumption that, near the wall, the event convection velocity, $U_C$, varies linearly with $u_\tau$, 

$$r_{exp} = T_b U_C \sim T_b u_\tau,$$

where the subscript “exp” indicates data from experiments. The response of near-wall $r$ in current smooth-wall cases (K1R0, K3R0, K4R0) are compared with $r_{exp}$ in Figure 4.24 with $H = 1$. Here a low $H$ value is used to detect a high percentage of bursts. A range of near-wall locations from $y^+ = 5$ to 30 is considered. All data points are normalized with the corresponding value in case K1R0. All current data follow closely with the growth of $r_{exp},$
and it is not surprising to find that the most near-wall data best match the experimental results obtained with the signal of wall-shear stress. The relatively strong Q2 events at $y^+ = 5$ represent the kinks of the low-speed streaks, which have been shown to be frequently the early stage of secondary hairpin vortices (Zhou et al., 1999).

Figure 4.25 (a) shows the effects of roughness and acceleration on the average separation (i.e., the inverse of density) of the coherent turbulent structures: acceleration significantly decreases the density (or increases $r$) throughout the boundary layer. Roughness, on the other hand, causes more frequent appearance of structures close to the wall, but does not affect the outer layer ($(y-d)/\delta > 0.2$). The spatial intermittency of the events (Figure 4.25 (b)), as a product of event density and event size, is not significantly affected by acceleration, despite a slightly decrease in the outer layer; this is because, although acceleration decreases the number of coherent structures, the size of the structures becomes larger (shown by the increases of integral length scales in Figure 4.22). Roughness has no significant effect on $\gamma$ in the outer layer, consistent with experimental observations by Wu & Christensen (2007). Varying $H$ from 1 to 4, we found that the effects of the threshold value to the above outer-layer observations are minimum.
When the structure density is plotted in inner scales (Figure 4.26(a)), the cases collapse onto two profiles: the low-$k^+$ case collapse with the two smooth cases; the high-$k^+$ cases show much lower $r^+$ compared to the cases with low or no roughness in the region $5 < (y - d)^+ < 100$. This means that structure density also follows the $k^+$-based similarity; a higher $k^+$ leads to more frequent near-wall events with strength $|u'v'|_{Q2} > \sigma_u \sigma_v'$. This phenomenon was also observed earlier from near-wall structure visualization (Figure 4.16), where the number of quasi-streamwise vortices are distributed more densely in cases with higher $k^+$. However, for higher threshold values, such as $H = 2$ (Figure 4.26(b)) or $H = 4$ (not shown), the density in the near-wall region is the same on both smooth and rough walls. Thus, roughness increases the number of relatively weak turbulent structures, but not those strong enough to result in Q2 events with $|u'v'|_{Q2} > 2\sigma_u \sigma_v'$.
4.5 Conclusions and discussions

Large-eddy and direct numerical simulations are carried out for equilibrium sink flows over a smooth wall or sand-grain roughness. The acceleration parameter, $K$, and the normalized roughness height, $\bar{k}$, are parameters quantifying the acceleration and roughness effects. The roughness Reynolds number $k^+$ can be used as an indicator of the relative strength of these two effects.

Acceleration is found to decrease $Re_\theta$ and $C_f$, while roughness increases both. When $k^+$ is kept constant, $Re_\theta$ and $C_f$ are dominantly affected by $K$ and $\bar{k}$, respectively. Roughness tends to prevent the reverse-transitional state (characterized by a significant decrease of the log-law slope), which occurs only when the flow is close to the hydraulically smooth regime and subjects to a strong acceleration ($K4$). In the fully turbulent state, the roughness function depends on $k^+$ only, and agrees with experimental data obtained from equilibrium pipe flows with the same type of roughness.

Wall-similarity applies to the Reynolds stresses for cases far from the reverse-transitional state; when the flow is close to reverse-transition, $u_\tau$ does not absorb the effects of acceleration or roughness in a significant part of the lower boundary layer. In any case, a higher acceleration increases the peak of $\langle u'^2 \rangle^+$ and decreases the peaks of $\langle v'^2 \rangle^+$ and $\langle u'v' \rangle^+$, while a higher roughness exerts the opposite effects.

The statistics in the near-wall region exhibit similarity based on $k^+$: cases with higher $k^+$ show more homogeneous distribution of energy in three components of turbulent fluctuations and higher Reynolds shear stress $\langle u'v' \rangle^+$; the production of turbulent kinetic energy peaks farther away from the wall, and is mostly balanced by local viscous dissipation, with negligible viscous diffusion and weaker turbulent diffusion. The similarity also applies to turbulent structures: as $k^+$ increases, their average inclination becomes steeper, the streamwise size of near-wall turbulent structures decreases, and the relatively weaker Q2 events (with strength lower than $2\sigma_{u'}\sigma_{v'}$) appear more frequently. This is because $k^+$ is a ratio of
the physical roughness height to the thickness of the viscous layer; for high \( k^+ \), the hairpin packets near the wall are shorter and distributed more uniformly in the \((x, z)\)-plane, presumably due to the inability of an initial hairpin to form secondary hairpins that are aligned with itself in the streamwise direction, as the randomly-distributed roughness directly affects the evolution of hairpin legs, where the secondary hairpins are often formed. The higher structure density is also due to the fact that a higher \( k^+ \) augments the magnitudes of \( v' \) and \( w' \), both of which have been shown to play an important role in streak instability (Schoppa & Hussain 2002; Jiménez & Pinelli 1999). The role of \( K \) comes in by decreasing \( k^+ \), reducing the impact of roughness on hairpin development, and thus increasing the streamwise packet length; effectively, the average packet inclination angle decreases.

In the outer layer, wall-similarity applies to transitionally rough cases in the Reynolds stresses, and in the statistics of turbulent structures including average inclination and size of hairpin packets, as well as their density. The disappearance of roughness effects in the outer layer may be because that, as the hairpins develop into this layer, their spanwise size increase, and the resultant hairpin-merging phenomenon absorbs the roughness-scale spanwise misalignment among hairpins, removing the effects of roughness on decreasing streamwise extents of hairpin packets. For the fully turbulent cases, \( K \) affects the outer-layer flow mainly by increasing \( \delta_v/\delta \); as a result, the cases with stronger acceleration shows larger coherent structures compared to the boundary layer thickness, and less number of these structures in the domain.

It is interesting to observe that, unlike in a smooth-wall sink flow, a strong \( K \) does not necessarily lead to reverse transition in transitionally-rough sink flows; this is shown by the fact that an increase of \( K \) does not result in a decrease of the inclination angle of hairpin packets in the outer layer, which is considered the cause of reverse-transition in sink flows (Dixit & Ramesh 2010). Instead, reverse transition occurs only when the flow is in the hydraulically-smooth regime and, at the same time, is subject to a strong acceleration. This finding supports the idea that acceleration-induced relaminarization starts from the
near-wall region, and can be overruled by near-wall destabilization (Piomelli & Yuan, 2013).
Chapter 5

Numerical simulation of a spatially developing accelerating boundary layer over roughness

5.1 Abstract

Direct numerical simulation of an accelerating boundary layer over a rough wall has been carried out to investigate the coupling between the effects of roughness and strong freestream acceleration. While the favourable pressure gradient is sufficient to achieve quasi-laminarization on the smooth wall, on the rough wall the flow reversion is prevented, with higher friction coefficient, faster increase of turbulence intensity compared to the freestream velocity, and more isotropic turbulence near the wall. The logarithmic region of the mean-velocity profile presents an initial decrease in slope as in the smooth case, but soon recovers, as the fully rough regime is reached and a new overlap region is established. A strong coupling between
the roughness and acceleration effects develops as roughness leads to more responsive turbulence and prevents the strong acceleration from stabilizing the turbulence, and the acceleration intensifies the velocity scale of the wake field (i.e., the near-wall spatial heterogeneity of the time-averaged velocity distribution). The combined effect is a “rougher” surface as the flow accelerates. In addition, the link between the local values of the freestream and the near-wall velocity depends on the flow history; this explains the different flow responses observed in previous studies, in terms of friction coefficient, turbulent kinetic energy, and Reynolds stress anisotropy. This study elucidates the near-wall flow dynamics, which may be used to explain other non-canonical flows over rough walls.

5.2 Introduction

Spatially accelerating flows are present in a wide range of applications in engineering and the natural sciences, ranging from airfoils and turbine blades, to ducts and atmospheric flows on complex landscape. Although the kinetic energy of the mean flow increases as a result of the acceleration, turbulence may become less vigorous and the flow may revert to a quasi-laminar state, through a process called “relaminarization” or, more accurately, “quasi-laminarization”. Various parameters are used to quantify the acceleration level; one of the most widely used is the acceleration parameter, \( K = \left( \nu / U_\infty^2 \right) dU_\infty / dx \), where \( x \) is the streamwise direction, \( U_\infty \) is the freestream velocity in this direction, and \( \nu \) is the kinematic viscosity. In sink flows (the equilibrium accelerating boundary layer that occurs in a long converging duct with straight walls), Spalart (1986) showed that turbulence cannot be sustained if \( K \) is higher than a critical value of \( 2.5 - 3.0 \times 10^{-6} \).

The mechanisms of quasi-laminarization have been widely studied. Narasimha & Sreenivasan (1973) attributed the reversion to the dominance of pressure forces over the slowly responding Reynolds stresses in an originally turbulent flow, which leads to the generation
of a new laminar boundary layer stabilized by the favourable pressure gradient (FPG). Although the turbulence does not decrease in the region of strong acceleration, it plays an increasingly passive role in the boundary-layer development (Launder 1964). The turbulence production was found to exceed dissipation in the quasi-laminar region (Narasimha & Sreenivasan 1973), indicating that the reversion is not a pure Reynolds-number effect.

The quasi-laminarization has been found to be associated with a decrease of bursting frequency and stabilization of the near-wall turbulent structures. McEligot & Eckelmann (2006) observed that the burst frequency is very sensitive to the acceleration, decreasing as $K$ increases; the intensity of the wall-normal fluctuations also decreases, leading to weaker momentum transport.

Bourassa & Thomas (2009) related quasi-laminarization to the stabilizing effects of acceleration on near-wall streaky structures caused by the decrease of the wall-normal and spanwise fluctuations, which enhances the stability of streaks and near-wall vortices (Jiménez & Pinelli 1999; Schoppa & Hussain 2002). Furthermore, quadrant analysis (Wallace et al. 1973) showed that a weaker inner-outer-layer interaction results from a decrease of both sweeps (Quadrant 4, Q4, events) and ejections (Q2 events): Q4 events are eliminated and alias to third-quadrant (inward interaction) events due to the acceleration, and Q2 events become more intermittent, due to the increased spanwise separation of streaks. Additionally, Joshi et al. (2014) related quasi-laminarization to a decrease of Q2 events that is believed to be associated with sweeping events and local APG regions, which are eliminated by the strong favourable mean pressure gradients.

Recent numerical simulations by Piomelli & Yuan (2013) showed that the damping of wall-normal and spanwise turbulent fluctuations is due to the decrease of the magnitude of the pressure fluctuations, which is caused by a decrease of the source term of the Poisson equation for the rapid component of the pressure fluctuations. This lessens the redistribution of turbulent kinetic energy (TKE) into wall-normal and spanwise fluctuations, leading to quasi-one-dimensional (1D) turbulence in the quasi-laminarization zone. Intensifying the
wall-normal and spanwise fluctuations by introducing wall roughness, on the other hand, leads to strongly destabilized near-wall structures, and either a complete lack of quasi-laminarization or an early re-establishment of the equilibrium turbulent flow, depending on the roughness height. The results suggest that the near-wall region has an important effect on the quasi-laminarization process.

Roughness is known to play a major role in the dynamics of near-wall turbulence, and a substantial amount of work has been carried out to understand the dynamics of turbulent flows over rough walls, both for engineering and environmental applications, summarized in [Raupach et al. (1991) and Jiménez (2004)]. Roughness increases the drag on the wall due to the pressure drag, resulted from the wake created downstream of a roughness element. Many studies (e.g., Flack et al., 2005) have shown that, far away from the wall, roughness does not affect the turbulence statistics, but sets the velocity scale at the wall. In addition, near the rough wall the TKE is more homogeneously distributed in the three directions, and the turbulent structures are clearly altered.

Although most of the experimental and numerical work has concentrated on canonical pipe, channel flows, or boundary layers, several studies have addressed the interaction of roughness with the favourable pressure gradient. Coleman et al. (1977) measured the velocity field in a rough-wall accelerating boundary layer in a quasi-equilibrium state, in the sense that the velocity achieved self-similarity and the Reynolds number and accelerating parameter were constant, but the exact equilibrium behaviour (Rotta, 1962) was not satisfied. A favourable pressure gradient was found to make the surface “rougher” by increasing the friction velocity, \( u_\tau \); the increase of \( u_\tau \) was faster than that of \( U_\infty \), producing a higher friction coefficient, \( C_f = 2(u_\tau/U_\infty)^2 \). The turbulent kinetic energy decreased when normalized using \( U_\infty \), especially for its wall-normal and spanwise components, while the streamwise component was unchanged, leading to higher anisotropy. Tachie and co-workers (Tachie et al., 2007; Tachie & Shah, 2008) conducted experiments in flows over bar roughness under strong FPG achieved with converging side-walls or an inclined bottom wall in open-channel
flows. Contrary to the observation of Coleman et al. (1977), their data showed that $C_f$, the mean velocity defect, and the Reynolds stresses were largely independent of the pressure gradient. This was probably due to the limited streamwise distance for the FPG to exert its effects. Cal et al. (2008) and Cal et al. (2009) performed experimental studies in non-equilibrium (without self-similarity of flow quantities in the streamwise direction) boundary layers subjected to acceleration achieved using a tilted plane, with fairly mild acceleration ($K < 0.5 \times 10^{-6}$). FPG leads to an increase of $C_f$, and decrease of bulk Reynolds number.

In the outer layer, competing effects of FPG and roughness were found on the mean flow and the Reynolds stresses; however, close to the rough wall, both roughness and acceleration intensify the fluctuations significantly, with the Reynolds stress anisotropy unaffected by FPG; this was different from the observations of both Coleman et al. (1977) and Tachie & Shah (2008). Similar observations on the opposite effects of FPG and roughness and the higher $C_f$ were also made from large-eddy simulations (LES) of spatially developing flows over roughness by Piomelli & Yuan (2013).

The sink flow, on the other hand, exhibits a different behaviour. Yuan & Piomelli (2014b) showed that a stronger acceleration causes a decrease in $C_f$, instead of promoting it as in the non-equilibrium accelerating flows. The opposite effects of FPG and roughness on the drag and turbulent intensities combine to influence the flow in a way that depends on the roughness Reynolds number, $k_s^+$, where $k_s$ is the equivalent sandgrain height, defined as the mean height of the uniform sandgrain roughness (Nikuradse, 1933) that produces the same drag as the roughness in question, and $+$ denotes quantities normalized by the friction velocity, $u_\tau$, and $\nu$. They found that flow reversion is achieved only when the flow is close to the hydraulically smooth regime, i.e., when the FPG is strong enough to overcome the roughness effects resulting in a low value of $k_s^+$. In the outer layer, FPG effects always dominate those of roughness.

These results indicate that the effects of roughness and FPG are interdependent, and are affected by the state of the flow (equilibrium or not). Deeper understating on the
flow dynamics is required to explain how the two effects are related and manifested in different flow states. Moreover, the different results from previous studies of non-equilibrium flows with mild $K$ or a relatively short accelerating distance motivate the study of a non-equilibrium accelerating boundary with strong $K$ lasting for a sufficient distance for its effect to be significant. To this end, we carried out direct numerical simulations (DNS) of a spatially developing rough-wall boundary layer flow with freestream acceleration, compared to a smooth-wall LES taken from case LES3s in [Piomelli & Yuan (2013)]. The problem set-up and numerical techniques are introduced in Section 5.3. The boundary layer (Section 5.4.1) and mean velocity (Section 5.4.2) developments are then presented. Then the time-averaged wake fluctuation and its statistics are discussed in Section 5.4.3. Lastly, the turbulent statistics are shown in Section 5.4.4 and the causes of the differences in Reynolds stress anisotropy are studied in details in Section 5.4.5.

5.3 Problem formulation

The incompressible flow of a Newtonian fluid is governed by the equations of conservation of mass and momentum:

\[ \frac{\partial u_i}{\partial x_i} = 0, \]
\[ \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + F_i, \]

where $x_1$, $x_2$ and $x_3$ (or $x$, $y$ and $z$) are, respectively, the streamwise, wall-normal and spanwise directions, and $u_i$ (or $u$, $v$ and $w$) are the velocity components in those directions; $P = p/\rho$ is the modified pressure, and $\rho$ the density. For the LES, (5.1)–(5.2) are solved for filtered quantities, and the divergence of the sub-grid stress tensor, $\tau_{ij} = \bar{u}_i\bar{u}_j - \bar{u}_i\bar{u}_j$, appears on the right hand side of the momentum equation; $\tau_{ij}$ is modelled using a dynamic eddy-viscosity model [Germano et al., 1991; Lilly, 1992], in which the coefficient is adjusted.
using the Lagrangian-Averaging procedure \cite{Meneveau et al., 1996}. The numerical simulations are performed using a well-validated code that solves \eqref{eq:5.1} and \eqref{eq:5.2} on a staggered grid using second-order, central differences for all terms, a second-order semi-implicit time advancement, and MPI (Message Passing Interface) parallelization \cite{Keating et al., 2004}.

In the rough-wall studies, an immersed-boundary method (IBM) based on the volume-of-fluid approach (Chapter 2) is used to impose the no-slip boundary condition on the rough surfaces. A body force, \( F_i \) in the \( x_i \) direction, results from the IBM; it is non-zero in the interface cell only. In the smooth-wall case, \( F_i = 0 \).

For both cases, the acceleration is achieved by imposing a spatially varying streamwise velocity at the domain top, \( U_\infty(x) \); the vertical component, \( V_\infty(x) \), is calculated through mass balance. Note that, although \( U_\infty(x) \) differs from the velocity immediately at the top of the boundary layer, \( U(x, \delta) \), its streamwise gradient amounts to roughly the same \( K(x) \) distribution as the one obtained at \( y = \delta \) (as will be shown later); therefore, \( U_\infty(x) \) is here called the “freestream velocity”. A no-slip boundary condition is applied at the bottom wall (or on the rough surface through IBM); the turbulent inflow at \( x/\delta^*o = 0 \) (reference plane) is generated from a rescaling/recycling region \cite{Lund et al., 1998}. Periodic boundary conditions are applied in the spanwise direction \( z \), and the convective outflow \cite{Orlanski, 1976} is used at the domain outlet.

Figure 5.1 presents the sketch of the configuration. The long domain in \( x \) allows the boundary layer to evolve for a sufficient distance (around \( 300\delta^*_o \), where \( \delta^* \) is the displacement thickness, and the subscript “\( o \)” denotes the reference plane) for the acceleration effects to develop; the spanwise domain size is sufficient to enclose the largest turbulent eddies in the outer layer; moreover, \( L_y/\delta_{95} \geq 6 \) (where \( \delta_{95} \), or \( \delta \), is the boundary layer thickness), to avoid blockage effects. The domain size and \( \delta \) in the rescaling region are adjusted so that, at \( x = 0 \), the rough-wall boundary layer matches (within 2%) the local Reynolds number of the smooth case, \( Re_{\delta^*o} = 1240 \) based on \( \delta^*_o \) and \( U_\infty o \). The inlet Reynolds number and the \( K(x) \) distribution are the same as Case 2 in the experimental studies of
Figure 5.1: Sketch of the configurations for the smooth and rough cases.

Warnack & Fernholz (1998). Roughness starts at a location downstream of the recycling plane, while leaving sufficient distance before the reference plane for the boundary layer to become well-developed at $x/\delta_0^+ = 0$. The present study is focused mainly on the FPG region ($50 \lesssim x/\delta_0^+ \lesssim 350$).

The roughness is modelled as a dense distribution of randomly rotated ellipsoids (“sand grains”) with the same shape and size (Scotti, 2006); the semiaxes of the ellipsoid are $k$, $1.4k$, and $2k$, where $k$ is termed the “roughness height”. This roughness model has been calibrated so that $k$ approximates $k_s$ in the transitionally rough regime. The fully rough regime starts for $k^+ \approx 60$; in this regime, the equivalent sandgrain height is a fixed value (denoted by $k_{s\infty}$) and equals $1.6k$ (from the data in Chapter 2) for the present roughness.

As the freestream accelerates, $k^+$ increases from around 20 to 80, developing from the transitionally rough flow to a fully rough flow at $x/\delta_0^+ \approx 220$. $\delta$ is between 10 and 20 times larger than $k$, with the lower limit reached only within a narrow band in the FPG region; these values are within the common range used in numerical simulations in the literature (Lee et al., 2011; Bhaganagar et al., 2007; Ikeda & Durbin, 2007).

For the rough case, $10 \leq \Delta x^+ \leq 21$ and $5 \leq \Delta z^+ \leq 14$; the coarser limit is reached only
Table 5.1: Simulation parameters ($L$ is the length of the computational domain).

<table>
<thead>
<tr>
<th>Cases</th>
<th>$Re_{\delta^*,o}$</th>
<th>$k/\delta$</th>
<th>$k^+$</th>
<th>$(n_i, n_j, n_k)$</th>
<th>$(L_x, L_y, L_z)/\delta_o^*$</th>
<th>$\Delta x^+$</th>
<th>$\Delta y_{\min}^+$</th>
<th>$\Delta z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>1280</td>
<td>0.00–0.10</td>
<td>23–80</td>
<td>(1792, 156, 256)</td>
<td>(543, 27, 27)</td>
<td>23–42</td>
<td>0.3–0.8</td>
<td>8–23</td>
</tr>
<tr>
<td>Rough</td>
<td>1312</td>
<td>0.05–0.10</td>
<td>23–80</td>
<td>(4608, 270, 512)</td>
<td>(591, 27, 27)</td>
<td>10–18</td>
<td>0.2–0.7</td>
<td>5–14</td>
</tr>
</tbody>
</table>

at the end of the FPG region due to the increased friction velocity. The present grid sizes in plus units are comparable to the values used in existing DNS studies on rough-wall boundary layers and channel flows. For examples, Castillo et al. (2013) used $\Delta x^+ = 23$, $\Delta z^+ = 11$, and $\Delta y_{\min}^+ = 0.5$; in Ikeda & Durbin (2007), $\Delta x^+ = 12$, $\Delta z^+ = 11$, and $\Delta y_{\min}^+ = 0.4$; similar sizes were also used in Coceal et al. (2006), where $\Delta x^+$ and $\Delta z^+$ equal to 16 were considered sufficient. It was pointed out by Moin & Mahesh (1998) that a sufficient spacial resolution should capture most of the dissipation, which occurs for scales of $O(\eta)$, where $\eta = (\nu/\epsilon)^{1/4}$ ($\epsilon$ is the dissipation rate of TKE) is the Kolmogorov length scale. For a curved channel flow, scales less than $15\eta$ contributed to most of the dissipation (Moser & Moin, 1987). The present resolution gives $\Delta x/\eta \leq 10$ and $\Delta z/\eta \leq 7$. In addition, this resolution is capable to resolve the shape of roughness elements and the flow structures developed around them: from 5 to 8 grid points in $x$ and 10 points in $z$ are used to resolve each roughness element; 101 points in $y$ are used below the roughness crest.

The total simulation time is $85\delta_o^*/u_\tau$ and $145\delta_o^*/u_\tau$ for the rough and smooth cases, respectively, sufficient to achieve convergence for second and third-order turbulent statistics and Reynolds-stress budgets. A summary of simulation parameters is presented in Table 5.1.

For the smooth case, a flow quantity is decomposed into two components

$$\theta(x, y, z, t) = \langle \theta \rangle(x, y) + \theta'(x, y, z, t), \quad (5.3)$$
where $\langle \theta \rangle$, $\bar{\theta}$, and $\theta'$ are the spatial average, the temporal average, and the turbulent fluctuation, respectively. For the rough case, $\theta$ also includes the wake component $\tilde{\theta}$, the spatial variation of the time-averaged field due to the spatial heterogeneity of the roughness geometry,

$$
\theta(x, y, z, t) = \langle \bar{\theta} \rangle_i(x, y) + \tilde{\theta}(x, y, z) + \theta'(x, y, z, t).
$$

(5.4)

Note that, in the rough case, $\langle \theta \rangle_i$ is the intrinsic spatial average, averaging in the fluid domain only [Mignot et al., 2009; Nikora et al., 2007]; in contrast, the superficial average, $\langle \theta \rangle$, is obtained by averaging $\theta$ in the whole domain. Above the fluid layer occupied by roughness, these two spatial-average operations are equivalent. Also note that $\tilde{\theta}$ is significant in the roughness sublayer only. In the smooth case the spatial averaging is performed in the spanwise direction only, while in the rough case, the averaging is also carried out over $50k$ in the streamwise direction to compensate for the limited sampling of random roughness elements in the spanwise direction. The averaging length is large compared to the roughness scale, but small enough (between 1%-10%) compared to the acceleration length scale, $U_\infty(x)/(dU_\infty(x)/dx)$, in the FPG region.

5.4 Results

5.4.1 Boundary layer development

The contours of the pressure gradient ($\partial P/\partial x$, normalized by $U_{\infty, o}$ and $\delta_o$) and $K$ are presented in Figure 5.2. The free-stream variation leads to pressure gradients throughout the boundary layer. Figure 5.2(b) shows that the vertical variation of $K$ in the potential-flow region (the region between the top domain boundary and the upper limit of the boundary layer) is negligible; therefore, the prescribed $K(x)$ is considered directly imposed on the boundary layer. In addition, pressure fluctuations with small wavelengths are observed inside the boundary layer due to the pressure gradients formed as a result of the roughness;
Figure 5.2: Contours of (a) pressure gradient \( (\partial P/\partial x)/(U_{\infty,0}^2/\delta_o^* ) \) and (b) \( K \). Thick and thin lines represent the upper limit of the boundary layer and the streamlines, respectively.
Figure 5.3: Contours of (a) $\langle \bar{u} \rangle / U_{\infty, o}$ and (b) $\langle \bar{v} \rangle / U_{\infty, o}$; $\delta$. Boundary layer integral parameters: (c) acceleration parameter $K$; (d) Reynolds number $Re_{\delta^*} = U_{\infty} \delta^*/\nu$, (e) friction coefficient, $C_f = 2u_{\tau}^2 / U_{\infty}^2$ and (f) friction velocity, $u_{\tau} / U_{\infty, o}$.

these pressure fluctuations are not smoothed out in the spatial averaging process, due to the relatively limited roughness samples in the $z$-direction.

The contours of the streamwise and wall-normal mean velocities are shown in Figure 5.3(a–b). In both cases, the boundary layer becomes thinner in the FPG region, with irrotational flow entrained into the boundary layer. The streamwise development of $Re_{\delta^*}$ and $C_f$ are shown in Figures 5.3(d–e). The total drag, for the rough-wall case, is the sum of viscous and pressure components, and is calculated from the vertical integral of the IBM force (Yuan & Piomelli 2014b). Acceleration causes a decrease of the local Reynolds number, as the boundary layer thinning is faster than the increase of $U_{\infty}$. This is more significant in the smooth case; on the rough wall, the dip in Reynolds number is much less significant.

On the smooth wall, during the initial acceleration $C_f$ stays almost constant, as the wall
shear stress adapts to the acceleration; in the region of quasi-laminarization, $250 \lesssim x/\delta^* \lesssim 300$, however, $u_\tau$ remains almost constant and $C_f$ exhibits a dip. The rough case, on the other hand, presents a significant increase of $C_f$ in the acceleration region, indicating much faster increase of $u_\tau$ than $U_\infty$: the turbulent motions on the rough wall are not, in this case, attenuated by acceleration, but instead intensified.

To explain the variation of the rough-wall $u_\tau$, we consider the dependence of the form drag on the velocity relative to the roughness element. In the fully rough regime, the drag is mostly pressure drag, thus the drag force exerted by each roughness element can be expressed as

$$D \approx \frac{1}{2} \rho \mathcal{U}(x)^2 C_D A,$$  

where $C_D$ is the drag coefficient of the element, dependent on the element shape, $A$ is the bed area covered by the element, and $\mathcal{U}$ is the velocity outside the wake region, taken here as the velocity at the top of the roughness sublayer, $U_{RS}(x) = \langle \mathcal{U}(x) \rangle$.

For the flow above the roughness sublayer in an averaged sense, this amounts to a wall shear stress, $\tau_w = \rho u^2_\tau$, where
Figure 5.5: Profiles of mean streamwise velocity $U$ at various streamwise locations, normalized in (a) wall units and (b) outer scaling; solid and dashed lines present the smooth and rough cases, respectively. The thin dashed lines in (a) indicate $U^+ = y^+$ and the universal logarithmic law $U^+ = \log y^+/\theta + 5.0$; the thin lines in (b) show the velocity profiles at the reference location $x/\delta_o^* = 0$.

which, in the fully rough flows, is the density of the pressure drag per unit area,

$$
\tau_w(x) \approx \frac{\rho}{2} U_{RS}(x)^2 C_D,
$$

(5.6)

where $C_D$ is the drag coefficient averaged among all roughness elements. To verify this dependence of drag on $U_{RS}$, Figure 5.4 plots $2\tau_w(x)/\rho U_{RS}(x)^2$ compared to $k^*_w(x)$. The fully rough regime ($k^+ \gtrsim 60$) is achieved for $x/\delta_o^* \gtrsim 220$ (Figure 5.4(a)), where $2\tau_w/\rho U_{RS}^2$ becomes constant (Figure 5.4(b)). This indicates that the increase of drag in the FPG region is due to the near-wall acceleration, caused by the acceleration of $U_{\infty}$ and the thinning of the boundary layer.
Figure 5.6: Diagnostic function in (a) the smooth and (b) the rough cases. In (a), \( \Xi = y^+ \) and \( \Xi = 2.50 \) (corresponding to \( \kappa = 0.40 \)); \( \triangle y^+ = 35 \), \( \bigcirc y/\delta = 0.2 \). In (b), \( \Xi = 2.50 \); \( \triangle y/k = 2 \), \( \bigcirc y/\delta = 0.2 \).

5.4.2 Mean velocity

The mean streamwise velocity \( U = \langle \overline{u} \rangle \) is shown in Figure 5.5 for both cases. Here, \( d \) is the zero-plane displacement, representing the effective elevation of the boundary layer due to roughness; \( d \) is obtained as the location of the centroid of the wall-normal profile of the time- and space-averaged drag force (Scotti, 2006), and \( d \approx 0.8k \), not noticeably affected by the FPG. Acceleration leads to a decrease of the slope of the logarithmic layer (a well-known property of accelerating boundary layers), and, in the smooth case, to a laminar-like mean flow in the quasi-laminar region (\( x/\delta^* = 300 \) in Figure 5.5(a)), and a well-mixed region with weaker velocity gradient, \( \partial U/\partial y \), in the outer layer (Figure 5.5(b)). The universal logarithmic profile is restored after retransition, as shown at \( x/\delta^* = 400 \). Similarly, the rough-wall mean flow displays a decrease of logarithmic slope; however, Figure 5.5(b) shows an earlier restoration to its initial value, before \( x/\delta^* = 300 \).

The local values of the logarithmic slope and intercept are denoted as \( \tilde{\kappa}(x) \) and \( \tilde{B}(x) \) to distinguish them from the universal values for canonical boundary layer (taken here
as \( \kappa = 0.40 \) and \( B = 5.0 \). Following Spalart (1988), a diagnostic function \( \Xi(x, y) = (y - d)^+ \partial U^+(x, y)/\partial y^+ \) is used to obtain \( \tilde{\kappa}(x) \), with \( d = 0 \) in case of a smooth wall. At each \( x \) location, \( 1/\tilde{\kappa} \) is obtained as the local minimum of \( \Xi(y - d)^+ \) in the overlap region, i.e., the inflection point of \( U(y)^+ \). Figure 5.6 displays the variation of \( \Xi \) profiles. \( \tilde{B} \) is then obtained at the same wall-normal location from

\[
\tilde{B}(x) = U^+(x, y) \frac{1}{\tilde{\kappa}(x)} \log(y - d)^+.
\] (5.7)

The results are plotted in Figure 5.7. In the upstream zero-pressure-gradient (ZPG) regions, \( \tilde{\kappa} \) in the smooth case is 0.43, decreasing to 0.36 in the rough case; such trend and magnitude of the difference agree with the literature. Nagib & Chauhan (2008) (referred to as NC08) collected experimental data from various studies of smooth-wall ZPG boundary layers and demonstrated that the slope shows a scatter between around 0.35 and 0.45 for flows with low Reynolds numbers \( (Re_{\delta^*} < 4000) \); in addition, Leonardi & Castro (2010) (LC10) observed from DNS of channel flows that the slope decreases with the roughness.
function, from 0.41 to 0.36 for a type of cube roughness. Entering the FPG region, both cases exhibit a similar decrease of the slope for up to \( x/\delta_0^* \approx 200 \); this was explained by Nickels (2004) by the destruction of the constant-stress region, and consequently a decrease of the velocity scale in the logarithmic region. The Reynolds shear stress profiles, which highlight the destruction of the constant-stress region, will be shown later. In addition, the logarithmic region moves away from the wall in wall units (Figure 5.6): in the smooth case at \( x/\delta_0^* \approx 50 \), it extends between \( y^+ = 35 \) and \( y/\delta = 0.2 \), while it moves out of this range at \( x/\delta_0^* = 220 \). This is due to the thickening of the viscous sublayer, and thus the shortening (and ultimate disappearance) of the overlap region. The rough case resembles the smooth case in the upward shift of the log-law region, as is shown in Figure 5.6(b) for \( 50 < x/\delta_0^* < 170 \). This resemblance is due to the residual viscous sublayer, which is not completely destroyed by the roughness in the transitionally rough regime (Jiménez, 2004).

After \( x/\delta_0^* \approx 200 \), while the smooth-wall slope keeps decreasing throughout the rest of the acceleration region, the rough-wall slope returns to the canonical value (Figure 5.7(a)). This early recovery corresponds to the re-establishment of the logarithmic region between \( y/k = 2 \) and \( y/\delta = 0.2 \), as shown in Figure 5.6(b) for \( x/\delta_0^* = 300 \). This is likely because, as the viscous sublayer disappears, a new overlap region is established between the inner layer \( (y/\delta < 0.2) \) and the region outside the roughness sublayer \( (y/k > 2) \). In this \( x \)-region, \( 1/\tilde{\kappa} \) tends towards the canonical value due to the recovery of the constant-stress region. Note that during the slope recovery, within a narrow streamwise band, the profile of \( \Xi \) shows no local minimum, and the logarithmic region is not well defined; for example, this occurs in the smooth case at \( x/\delta_0^* = 330 \) (Figure 5.6(a)) and at \( x/\delta_0^* = 220 \) in the rough case (Figure 5.6(b)). This region is shaded in Figure 5.7.

In a canonical boundary layer, the higher drag on the rough wall can be quantified using the roughness function, \( \Delta U^+ \), which in ZPG boundary layers, channels and pipes is a function of \( k_s^* \) only; it can be used to relate the rough-wall intercept \( B_R \) to the smooth-wall
Figure 5.8: Comparison of the roughness function between the Nikuradse sandgrain and the current numerical sandgrain (Yuan & Piomelli, 2014a,c), whose $k_{\infty} = 1.6k$. The solid line shows the fitted profile for Nikuradse sandgrain in the fully rough regime (Nikuradse, 1933).

value $B_S$: 

$$\Delta U^+(x) = B_S(x) - B_R(x). \quad (5.8)$$

The values of $\Delta U^+$ for $k^+$ ranging from 20 to 100 for the present rough surface are collected from channel-flow simulations with this roughness model (Yuan & Piomelli, 2014a,c), and plotted in Figure 5.8. As mentioned earlier, $\Delta U^+$ matches the Nikuradse sandgrain value ($k_s = k$) in the transitionally rough regime for $k^+ \leq 40$, and it becomes a logarithmic function of $k^+$ (when the flows enters the fully rough regime) for $k^+ \geq 60$, where the Nikuradse sandgrain with $k_{\infty} = 1.6k$ gives the same values of $\Delta U^+$.

We use $\Delta U^+$ to correct the intercept in the non-equilibrium boundary layer, by interpolating the $\Delta U^+-k^+$ relation (Figure 5.8) using the local $k^+(x)$ shown in Figure 5.4(a). In Figure 5.7(b), the corrected rough-wall intercept, $B_R(x) + \Delta U^+(x)$, is compared with the smooth-wall value. Acceptable agreement is observed in the ZPG regions and, interestingly, in the first part of the acceleration region before the restoration of the canonical value ($50 < x/\delta^* < 220$). This indicates that the $\Delta U^+-k^+$ correlation obtained in channel...
flows with dynamic equilibrium may also apply to non-equilibrium accelerating flows. This observation also holds in sink flows (Yuan & Piomelli, 2014b) and wall jets (Banyassady & Piomelli, 2014). After the recovery of the rough case and before $x/\delta_o^* = 350$, $\Delta U^+$ fails to correct $B_R$ to $B_S$, since, in this region, the smooth case is in the quasi-laminar region, where $B_S$ is far from the canonical value.

For canonical wall bounded flows and accelerating or decelerating boundary layers over a smooth wall, Nagib & Chauhan (2008) showed that $\tilde{\kappa}\tilde{B}$ is a function of $\tilde{B}$, and that effectively only one variable is required to describe the logarithmic profile for a wide range of smooth-wall flows. Figure 5.9 compares the present data with the NC08 relation. As expected, the smooth-wall data matches it well; on the other hand, the uncorrected rough-wall data agrees, for $\tilde{B} \leq 0.7$ (in the ZPG region), with the modified fit obtained by Leonardi & Castro (2010) from experimental and numerical data of dynamically equilibrium channel and boundary layer flows over roughness. For large $\tilde{B}$ (in the FPG region), however, the present data deviate noticeably from the LC10 relation. When the intercept is corrected by $\Delta U^+$, however, the rough-wall data also match the NC08 relation in both dynamically
5.4.3 Wake fluctuations

The wake fluctuations result from the small-scale recirculation downstream of the roughness elements. Figure 5.10 compares the wake fluctuations (Figure 5.10(a)) with the turbulent equilibrium and non-equilibrium regions, as shown in Figure 5.9(b). This indicates a possible way to define of the roughness function applied for non-equilibrium boundary layers where the logarithmic slope varies: instead of the specific local values of smooth-wall intercept, the roughness function can be obtained by collapsing the $\tilde{\kappa}\tilde{B}$ values on the experimental correlation.

Figure 5.10: Horizontal contours of (a) $\bar{u}/U_{\infty,o}$ and (b) instantaneous $u'/U_{\infty,o}$ at $y = d$ in a streamwise region centred at $x/\delta_o^* = 20$, and (c) contour of the time-averaged velocity, $\bar{\nu}/U_{\infty,o}$, in a $xy$-plane, with the dashed line representing the contour line of $\bar{\nu} = 0$. The white and black arrows indicate, respectively, the recirculation between two close elements and the flow reattachment between two elements separated with a longer streamwise distance.
Figure 5.11: Profiles of the dispersive stresses, including (a) the streamwise normal component, and (b) the other two normal components and the shear component, normalized by local $u_r$. $x/\delta_o^* = 50$, $x/\delta_o^* = 220$, $x/\delta_o^* = 300$, $x/\delta_o^* = 400$. Arrows indicate the direction of the flow acceleration.

ones (Figure 5.10(b)) in the $xz$-plane at $y = d$. In this near-wall region, the turbulence is shown by the low-speed streaks, shortened by the presence of roughness. The wake fluctuations correlate closely with the roughness geometry. A low-speed region is formed immediately downstream of most of the roughness elements. When the streamwise distance between two tall elements is large enough, the flow reattaches before the next element, as shown by the time-averaged velocity (for example, at $x/\delta_o^* = 24.5$ in Figure 5.10(c)); this phenomenon leads to the short high-speed streaks (due to the wake component) in Figure 5.10(a). On the other hand, when two tall elements are close to each other in the streamwise direction (at $x/\delta_o^* = 21.5$ in Figure 5.10(c)), no reattachment occurs between them. Strong shear layers are present in the vicinity of the roughness crest in the roughness sublayer. These shear layers and recirculation regions were also observed by Chan-Braun et al. (2011) in their DNS study. The mean flow channeling phenomenon, similar to the streaks with high-magnitude positive $\bar{u}$, was also observed by Hong et al. (2011). In the near-wall region shown in the contours, the peak magnitude of $\bar{u}$ is of the same order as that of $u'$. 

121
Figure 5.12: Two-point correlations of $\tilde{u}$ with (a) streamwise, and (b) spanwise separations, at three $x$ locations ($x/\delta_0^* = 50, 220, 350$).

Figure 5.11 shows $y$-profiles of the dispersive stresses, $\langle \tilde{u}_i \tilde{u}_j \rangle$, normalized by the local $u_\tau$. Four representative streamwise locations are selected: two are in the upstream and downstream ZPG regions ($x/\delta_0^* = 0$ and 400), the others in the FPG region ($x/\delta_0^* = 200$ and 300).

The roughness sublayer is the region in which the dispersive stresses are significant, and its thickness is approximately $2k$. The streamwise-stress profiles, $\langle \tilde{u}^2 \rangle$, scaled by the local friction velocity, collapse slightly better than those of the other components (Figure 5.11(a)); since the pressure drag is generated by the wake fluctuations in the streamwise direction, the streamwise wake component is expected to be an appropriate velocity scale in the roughness layer. The other two components of the normal dispersive stress increase significantly as the flow accelerates (Figure 5.11(b)), possibly due to the decreasing $\delta/k$ that results from the thinning of the boundary layer in the acceleration region (shown in Figure 5.3(a–b)). Similar effects of $\delta/k$ on the dispersive stresses were also observed in channel-flow DNS by Yuan & Piomelli (2014c). This result is also consistent with the observation by Coceal et al. (2006) in DNS of flows over cube roughness with low submergence ($\delta/h = 4 - 6$, where $h$ is cube height): they found that the dispersive shear stress is approximately $0.15u_\tau^2$, more than twice the value observed by Mignot et al. (2009) in experiments of channel flows over
a gravel bed with high submergence ($\delta/k > 40$).

To study the average size of the wake structures, the two-point correlation of $\tilde{u}(x,y,z)$ is calculated, at each $(x,y)$ location,

$$R_{11}(x, r) = \frac{\langle \tilde{u}(x) \tilde{u}(x + r) \rangle}{\langle \tilde{u}^2 \rangle},$$  \hspace{1cm} (5.9)

where $r$ is the separation vector. The correlation is then integrated for $0.5 \leq y/k \leq 2$ to obtain its vertically averaged value in the roughness sublayer, $\langle R_{11} \rangle_y$. We considered only streamwise and spanwise separations, and the corresponding vertically averaged correlations are $\langle R_{11,1} \rangle_y$, and $\langle R_{11,3} \rangle_y$. The separation is normalized with the roughness height $k$. Figure [5.12] shows that neither the streamwise nor the spanwise correlation varies in $x$: the integral length scale of the wake fluctuations is not affected by the FPG, but only by the size of the roughness elements. The correlation length is much larger in $x$ than in $z$, due to the effect of the high-speed $\tilde{u}$ streaks discussed above (Figure 5.10(a)); in the spanwise direction, $\tilde{u}$ becomes uncorrelated for separations larger than $2k$, reflecting the lateral extent of both low-speed and high-speed regions of $\tilde{u}$.

5.4.4 Turbulence statistics

The normal Reynolds stresses in outer scaling are shown in Figure 5.13, at three streamwise locations that cover the FPG region. In the smooth case, the Reynolds stresses decrease significantly in the outer layer relative to $U_\infty$; while, on the rough wall, they adjust faster to $U_\infty$ in this region; the Reynolds-stress profiles change in shape, so no collapse is obtained. Near the wall, $\langle \tilde{u}^2 \rangle$ increases at the same rate as $U^2_\infty$ on both walls; the other two components increase faster than $U^2_\infty$ on the rough wall, while stay almost unchanged in magnitude (thus decrease under this scaling) on the smooth wall. This trends are consistent with the more homogeneous redistribution of TKE into its components in rough-wall turbulence, observed by several researchers [Schultz & Flack 2007; Shafi & Antonia 1995]. The fact
Figure 5.13: Profiles of normal Reynolds stresses, normalized by local $U_\infty$: (a)-(c) smooth; (d)-(f) rough. $x/\delta_0^* =$ 50; $x/\delta_0^* =$ 200; $x/\delta_0^* =$ 300. The arrow indicates the direction of flow acceleration.
that all components in the other layer are higher on the rough wall than on the smooth wall is also consistent with the results of Cal et al. (2009); they also observed that, different from the present results, the acceleration effect is absorbed by the normalization using $U_\infty$, due to the much weaker acceleration and to the fact that their accelerating boundary layers are closer to the self-similar state.

Figure 5.14 compares the normal Reynolds stresses, normalized by the local friction velocity. Similar to the what was observed by Cal et al. (2009), the normalized Reynolds stress in the outer layer decreases in the FPG region in both cases, showing that turbulence increases more slowly than the drag. In the rough case, this also means that the turbulence lags behind the wake field development.
The Reynolds shear stress, normalized by the local friction velocity, is shown in Figure 5.15. On the smooth wall, it decreases significantly throughout the acceleration region, while in the rough case, after an initial decrease, it starts to recover at $x/\delta^*_o \approx 220$; the recovery appears to originate from the near-wall region and spread outward. In both cases, a change of shape towards the sink-flow profile [Spalart, 1986; Yuan & Piomelli, 2014b] is observed, characterized by the sharper peak and the disappearance of a constant-stress region, as mentioned earlier. This shape change of the Reynolds shear stress can be explained by the impact of the pressure gradient on the near-wall total shear stress [Nickels, 2004]. At $x/\delta^*_o = 400$, the ZPG state is almost recovered in both cases.

### 5.4.5 Reynolds stress anisotropy

The streamwise evolution of the Reynolds-stress anisotropy tensor

$$ b_{ij} = \frac{\langle u'_i u'_j \rangle}{\langle u'_k u'_k \rangle} - \frac{1}{3} \delta_{ij}, $$

\( (5.10) \)
Figure 5.16: The evolution of Reynolds-stress invariants at three wall-normal locations for $50 \leq x \leq 330$: (a) smooth and (b) rough cases. Symbols show the most upstream location. Arrows indicate the direction of increasing $x$.

during acceleration is shown in Figure 5.16 in terms of its second and third invariants (Lumley, 1978)

$$\eta^2 = -\frac{1}{6} b_{ij} b_{ji}, \quad \xi^3 = \frac{1}{6} b_{ij} b_{jk} b_{ki}. \quad (5.11)$$

Three wall-normal locations are considered: one in the outer region ($y/\delta = 0.5$), one in the logarithmic region, $(y - d)^+ = 100$, and the last very near the wall, $(y - d)^+ = 15$ for the smooth case and $y/k = 2$ for the rough one. On the smooth wall, turbulence develops towards 1D turbulence in the logarithmic region and below, almost reaching the 1D state (the upper right corner of the triangle) near the wall. On the contrary, on the rough wall the anisotropy in the logarithmic region is unaffected, while near the roughness crest it shifts towards the isotropic state (the lowest vertex of the triangle). In both cases, the anisotropy in the outer layer is not noticeably affected by the acceleration.

To explain the difference in Reynolds stress anisotropy, we investigate the Reynolds stress budget. The budget of Reynolds stresses and TKE were derived by Raupach & Shaw (1982) for flow over canopy and by Mignot et al. (2009) for flow over roughness. The budgets
of the normal Reynolds stresses, \( \langle u_\alpha'^2 \rangle \), in a boundary layer are

\[
\begin{align*}
- \frac{\partial}{\partial x_j} \langle \bar{u}_j \rangle \langle u_\alpha'u_\alpha' \rangle - \frac{\partial}{\partial x_j} \langle u_\alpha'u_\alpha' \rangle \frac{\partial \langle \bar{u}_\alpha \bar{u}_\alpha \rangle}{\partial x_j} \\
- 2 \langle u_\alpha'u_\alpha' \rangle \frac{\partial \langle \bar{u}_\alpha \rangle}{\partial x_j} - 2 \left\langle u_\alpha'u_\alpha' \frac{\partial \bar{u}_\alpha}{\partial x_j} \right\rangle - 2 \left\langle u_\alpha' \frac{\partial P'}{\partial x_\alpha} \right\rangle \\
+ \nabla^2 \langle u_\alpha'u_\alpha' \rangle - \epsilon_{\alpha\alpha} = 0
\end{align*}
\]  

(5.12)

(no summation on Greek indices). Here, \( \epsilon_{\alpha\alpha} \) is the viscous dissipation rate of \( \langle u_\alpha'^2 \rangle \). The wake production, \( P_w \), appears with a positive sign in the budgets of the normal dispersive stresses, and represents the energy exchange between the turbulent and wake fields.

The wake fluctuations interact with turbulent fluctuations in two ways: first, the turbulent structures with scales larger than the wake scale work against the form drag, converting TKE to wake kinetic energy (WKE); at the same time, the WKE is converted through energy cascade to TKE associated with structures with scales smaller than the wake structures. The wake production represents the net conversion between TKE and WKE. \cite{Finnigan2000} observed, in DNS of a transitionally rough open-channel flow, that \( P_w \) is negative for the streamwise normal Reynolds stress, probably because this Reynolds stress component is mostly associated with larger-scale turbulent structures, converting TKE to WKE; on the other hand, \( P_w \) is positive for the wall-normal and spanwise components of Reynolds stress, since these two components are associated with smaller-scale turbulent structures, and more WKE is converted to TKE through energy cascade.

The effects of acceleration on the source terms of the three Reynolds stress components are shown in Figure 5.17. In the upstream ZPG region \( (x/\delta^*_o = 50) \), the Reynolds stress budgets are consistent with the picture above. As the flow accelerates, the shear production decreases, mostly due to the decrease of Reynolds shear stress compared to \( \tau_w \) (Figure 5.15(b)), i.e., turbulence adjusts to the acceleration more slowly than the drag. For the
same reason, the pressure work, $\Pi_{\alpha\alpha} = -2\left\langle u'_\alpha \left( \partial P'/\partial x_\alpha \right) \right\rangle$, decreases outside of the roughness sublayer. However, $\Pi_{22}$ adapts faster to the inner scaling below the roughness crest, $k_c$; this is presumably due to both the intensification of $v'$, as a result of turbulence-wake interactions, and an increase of $P'$ intensity, resulting from the addition of the wake-related terms (the second and fourth terms) in the source of the $P'$ Poisson equation,

$$\nabla^2 P' = -2 \frac{\partial \left\langle u' \right\rangle}{\partial x_j} \frac{\partial u'_i}{\partial x_i} - 2 \frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} - \frac{\partial^2}{\partial x_i \partial x_j} \left( u'_i u'_j - \left\langle u'_i u'_j \right\rangle \right) - \frac{\partial^2 \tilde{u}'_i \tilde{u}'_j}{\partial x_i \partial x_j}. \quad (5.13)$$

It is also worth noting that both the shear production and the pressure work peak around the roughness crest. The wake production becomes less important for $\langle u'^2 \rangle$, but more so for $\langle v'^2 \rangle$ and $\langle w'^2 \rangle$, probably because the further separation between the roughness length scale and the viscous length scale (as $k^+$ increases) leads to larger amount of WKE being converted to TKE through energy cascade from $k$ towards the Kolmogorov scale.

To explain the fact that the FPG does not increase the flow anisotropy in the roughness sublayer, we should consider the effects of both the persistent wake production (that does not vanish as the flow accelerates) and the near-wall pressure work, which remains significant, unlike the smooth case, in which it diminishes under strong FPG [Piomelli & Yuan, 2013].

Figure 5.17: Effects of FPG on budget terms of (a) $\langle u'^2 \rangle$, (b) $\langle v'^2 \rangle$, and (c) $\langle w'^2 \rangle$, normalized in wall units. $x/\delta^*_o = 50$, $x/\delta^*_o = 200$, $x/\delta^*_o = 300$. Arrows show the direction of flow acceleration.
Figure 5.18: Ratio between the time scales of the turbulence, the wake shear, and the mean shear in (a) smooth case and (b) rough case. \( x/\delta_o^* = 50, \quad x/\delta_o^* = 220, \quad x/\delta_o^* = 300 \). Thick lines: \( S^* \); thin lines: \( S_{w^*}^* \). Arrows indicate increase of \( x \).

Both these two effects contribute to more even redistribution of TKE among all three directions. The pressure work appears to be the leading factor here, due to its higher magnitude. However, the contribution of wake production may increase for flows with higher Reynolds number, as Figures 5.17(b) and 5.17(c) indicate, and by the observation of Yuan & Piomelli (2014c) that, for a higher \( k^+ \), \( \Pi^+ \) decreases, whereas \( P_{w^+}^+ \) is almost unchanged.

The difference in turbulence anisotropy can also be explained by the different importance of the mean shear compared to the shear of the turbulence (namely, by the ratio between the time scale of the turbulence and that of the mean shear),

\[ S^* = \frac{\mathcal{S}\mathcal{K}}{\epsilon}, \tag{5.14} \]

where \( \mathcal{S} = 2(\mathcal{S}_{ij}\mathcal{S}_{ij})^{1/2} \) is the mean strain rate, \( \mathcal{S}_{ij} = (\partial\langle u_i \rangle/\partial x_j + \partial\langle u_j \rangle/\partial x_i)/2 \) is the strain-rate tensor, \( \mathcal{K} \) is TKE, and \( \epsilon \) is the TKE dissipation rate. For homogeneous turbulence under a strong uniform mean shear, the eddies are distorted by the flow sufficiently rapidly that the turbulence is not significantly modified by the change in strain rate through the
production terms, but is modified by the stretching of the individual vortex elements; a
development towards 1D turbulence (with all TKE in \(u'\)) then follows \(\text{[Hunt] 1978}\). Lee
et al. \(\text{[1990]}\) compared a boundary layer with a homogeneous turbulent flow with uniform
shear, and concluded that these effects of strong mean shear also apply to a near-wall region
in a turbulent shear flow. Figure\(5.18\) compares \(S^*\) in the present cases. In the smooth case,
\(S^*\) peaks at the top of the viscous sublayer and asymptotes to a constant value of around
4.5 in the logarithmic region; this constant results from the fact that \(-\langle u'v'\rangle/K \approx 0.3,\)
the TKE production \(P \approx -\langle u'v'\rangle \partial U/\partial y\), and that \(P \approx \epsilon\). This peak value reaches 60 in
the quasi-laminarization region, significantly affecting the turbulent eddy shape and the
Reynolds stress anisotropy. In the rough case, The peak of \(S^*\) is located at the roughness
crest \((1.5k)\), and decreases with acceleration towards the value in the logarithmic region
\((4.5)\), as the logarithmic region reaches the top of the roughness sublayer when the fully
rough regime is achieved. The ratio between the time scale of turbulence and that of the
wake strain rate,
\[
S^*_w = S_w K/\epsilon
\]
(5.15)
(where the shear of the wake \(S_w(x,y)\) is approximated as \((\bar{u}^2)^{1/2}/k\)) is also plotted in
Figure\(5.18(b)\). \(S^*_w\) peaks inside the roughness sublayer, with values of the order of one.
Note that the non-zero \(S^*_w\) outside the roughness sublayer is due to the finite temporal
sampling. These observations indicate that neither the mean shear nor the wake strain is
sufficient to result in a significant change of Reynolds stress anisotropy in the rough case.

5.5 Conclusions and discussions

A direct numerical simulation is carried out on a flat-plate turbulent boundary layer over
roughness with strong non-equilibrium freestream acceleration, to investigate the effects
of the strong FPG on a rough-wall boundary layer. Such acceleration leads to quasi-
laminarization on a smooth wall. But roughness prevents the flow reversion. Instead,
the acceleration leads to higher $C_f$, faster increase of TKE compared to $U_\infty^2$, and more isotropic turbulence near the roughness crest.

It is shown that the roughness leads to a wake field that is persistent throughout acceleration. The intensity of the wake scales mostly with the friction velocity, and the integral length scale scales with the roughness length scale. The wake promotes the Reynolds stress isotropy in two ways. First, it increases $v'$ and $w'$ through a production mechanism transferring wake energy to turbulence. In addition, it gives rise to an augmentation of the pressure work term, and more even redistribution of TKE in the three directions. As a result, the wake field maintains the inner-outer-layer interactions and the turbulence production cycle through instability mechanisms (Jiménez & Pinelli, 1999; Bourassa & Thomas, 2009).

The mean velocity profile departs from the universal logarithmic profile for both cases during acceleration. In the transitionally rough regime, it varies in a way similar to the smooth case, including a decrease of $1/\tilde{\kappa}$ and upward shift of the logarithmic region, due to the residual viscous sublayer. After the fully rough regime is reached, $1/\tilde{\kappa}$ returns to the canonical value, associated with a newly established overlap region between the inner layer, $y/\delta < 0.2$, and the outer layer, $y/k > 2$. The $\Delta U^+ - k^+$ correlation, obtained from the dynamically equilibrium state, can be applied to the present non-equilibrium boundary layer to restore the smooth-wall relationship between the $\tilde{\kappa}$ and $\tilde{B}$. This may have indications in determining the rough-wall drag in turbulence models.

The effects of the acceleration and roughness are closely coupled. The acceleration leads to increase of the mean strain rate (most importantly, the mean shear rate, $\partial U/\partial y$), and tends to stabilize the near-wall turbulence by linearly stretching the turbulent eddies, which results in relaminarization on a smooth wall (Hunt, 1978; Lee et al., 1990; Piomelli & Yuan, 2013). As an opposite effect, acceleration in the freestream causes the increase of mean velocity at the top of the roughness sublayer, $U_{RS}$, which is shown to linearly promote the wake intensity in the fully rough regime. On the other hand, roughness reduces the time scale of the turbulence near the wall, resulting in more responsive turbulence

132
to the mean flow variations; this counteracts the FPG effect in linear stretching of the turbulent structures. As a result, the combined effects are a higher $\partial U/\partial y$, which impacts the turbulence mainly through a higher production, and an increase of wake intensity, which leads to higher pressure drag and higher transfer between the energy of the wake and that of the turbulence.

The difference in such coupling in different flow states (i.e., equilibrium or not) may now be explained by the difference in the acceleration history, which affects the variation of $U_{RS}$. In the present non-equilibrium boundary layer, $U_{RS}$ increases faster than the freestream due to the flow entrainment and a thinner boundary layer; this results in a “rougrher” flow, as well as higher $C_f$, higher turbulence intensity, and lower Reynolds stress anisotropy. Similar observations in a spatially developing flow by Cal et al. (2009) support this theory. Tachie & Shah (2008) also reported the increase of drag. In a sink flow, on the other hand, $U_{RS}$ decreases (relative to $\nu/k$) with $K$ due to the decrease of flow rate $Q$, as $K = \nu/Q$; thus $k^+$ decreases, and the flow tends towards the hydraulically smooth regime and laminarization (Yuan & Piomelli, 2014b). This study considers a scenario more realistic than canonical rough-wall boundary layers. Although different flow behaviours result from the added complexity under different freestream conditions, they point to the same underlying physics, which may also help explain the flows in other realistic rough-wall boundary layers, such as the ones with adverse pressure gradient and rotation.
Chapter 6

Discussion and future work

6.1 Discussion

A detailed conclusion section is provided in each of the preceding chapters. This chapter is focused on the implications of the most important findings of this thesis, by partially reiterating them in a more general perspective, to outline the directions for future work in this field.

The general objective of this thesis was to identity and quantify the roughness effects in representative flow scenarios inside hydraulic turbines. The most relevant roughness effects for engineering design and analysis are the increase of frictional drag and the change of pressure drag (due to the impacts on flow separations); the frictional drag is the focus in the first part of this thesis. Specifically, it is necessary to know the ability of a given rough surface in increasing the frictional drag, prior to the engineering simulations; this ability is often characterized by $k_s$ of the surface. Currently, there is no established industrial guideline to choose the most suitable $k_s$-correlations for a given surface, to predict $k_s$ based on the physical characteristics of the surface. In this study, the drag-modelling abilities of existing correlations are compared using the actual $k_s$ values obtained numerically for four different turbine-blade surfaces. For the present surfaces in the waviness regime (where
the local surface slope is an important parameter), results show that the correlations based on the parameters that quantify the local slope perform well, while the one based on the moments of surface height statistics is not as successful in this regime, since the moments do not directly contain slope information. In addition, these results lead to a possible general approach to choose $k_s$-correlations based on the regime type, which can be identified using a critical effective slope (ES) value, separating the roughness and waviness regimes; this critical value was found previously to be 0.35 for selected idealized surfaces. However, we show that this value does not apply generally to realistic surfaces; further studies are required to identify the dependence of the critical ES on the surface type.

Most of the studies on $k_s$-correlations have focused on the fully rough regime, since, in this regime, $\Delta U^+$ (or the increase of friction drag compared to a smooth wall) increases with roughness height $k$ through a known logarithmic law. In the transitionally rough regime, however, the increase of $\Delta U^+$ is more complicated, differing from surface to surface (Jiménez, 2004; Flack & Schultz, 2010); this adds to the difficulty of accurate drag prediction. Through a wide range of $k^+$ ranging from the transitionally to fully rough regimes, the present data are among the first numerical evidence that confirm the experimental observations of such variation in drag behaviour among different surfaces. Future study is required using resolved flow data like the present ones, to detail the underlaying reason for such surface dependence in the transitionally rough regime, and to investigate possible prediction approaches.

To explain the role of the surface topography on roughness effects, the roughness sublayer is studied in channel flows, and the wake component of the velocity fields is obtained from a double-averaging decomposition; the wake fields are the quantities in interest since they are related directly to the specific roughness geometry. From here on in the thesis, the numerical sandpaper model (Scotti, 2006) is used as a reasonably generic roughness. The wake field is found to increase the portion of the TKE in the wall-normal and spanwise directions, through two mechanisms: a more even TKE redistribution and a direct conversion from the
wake energy to these two Reynolds stress components. The increase of wall-normal turbulent fluctuations, $v'$, facilitates a higher vertical momentum transfer between the roughness sublayer and the outer layer, leading to an increase of drag. These results demonstrate the link between roughness geometry and the amount of frictional drag increase. They indicate that an effective $k_s$-correlation must be able to model the overall effects of the wake on the turbulence.

Another indication from the wake-field results is the wide presence of the roughness effects in more complex flows. It is shown that, unlike the TKE which depends on non-zero mean shear (i.e. shear production) as a main source, the wake kinetic energy relies on the non-zero mean velocity itself; therefore, the wake kinetic energy is likely to be more persistent than that of the turbulence in realistic scenarios. Since the roughness effects are due to the interaction between the wake and turbulence, the persistent wake field translates into a wide presence of roughness effects even in realistic, spatially developing flows, whenever the mean-velocity magnitude is non-negligible near the wall.

To identify the particular roughness effects in more realistic boundary layers, the studies of the combined effects of roughness and flow acceleration are carried out. Previous studies present mixed results, shown by various near-wall turbulence responses and different levels of flow sensitivity to FPG effects. To clarify the contribution of each factor to the combined effects, the present parametric study is carried out to include a wide range of strengths of both factors. The simplest accelerating flow—the sink flow—is simulated for this purpose. FPG and roughness are found to exert opposite effects on the mean flow and turbulence (shown by the mean momentum deficit, Reynolds stresses, etc.) on a continuous basis, with FPG leading to transition towards laminarization and roughness preventing it by promoting near-wall instability. The combined effect, either towards laminarization or the turbulent state, mostly depends on the roughness Reynolds number, $k^+$, as a comparison between the roughness and viscous effects. In the fully turbulent cases (yielded by a high $k^+$), the outer-layer similarity applies for both Reynolds stresses and turbulent structures, including
the density, average size, and inclination of the hairpin vortices.

Different from the highly idealized sink flows, realistic accelerating flows often show no self-similarity, and are characterized by spatially varying acceleration level and Reynolds number. To study the flow dynamics in this type of flows, we consider a flat-plate boundary layer under spatially developing freestream acceleration; strong acceleration levels and the fully rough regime are reached, to elicit clear flow responses from both FPG and roughness. Two separate mechanisms are found to produce a coupling between the FPG and roughness effects. First, roughness leads to higher responsiveness of the turbulence intensity to the increase of the mean shear, and thus the resistance to the eddy stretching that constitutes the FPG stabilization. At the same time, FPG leads to higher mean shear rate, increasing the TKE production, and accelerates the near-wall flow, intensifying the wake magnitude. In combination, the two effects strengthen the momentum transfer and instability near the wall, leading to higher drag; relaminarization is prevented even under strong acceleration. In comparison, the sink flow may be regarded as a particular type of general accelerating flow, where the equilibrium-flow state dictates, under a given kinematic viscosity, that a higher acceleration parameter is achieved through a reduction in flow rate; this leads to a decrease in the near-wall velocity and the wake intensity; ultimately, \( k^+ \) approaches the smooth-wall limit, and laminarization can be achieved under sufficiently strong accelerations, as the roughness effect is eliminated and, as a result, the FPG stabilization becomes active. The dependence of the near-wall velocity on the history of acceleration may explain the varying responses reported in the literature for different experimental set-ups.

6.2 Future work

The present studies take advantage of high-resolution numerical simulations, which provide near-wall flow data that are usually not available in experiments, and manipulate a variety of realistic surfaces that are otherwise difficult to reproduce physically. The results not only
have practical indications for the turbine-analysis purposes in the hydro-power industry, but also help understand the flow physics in real-life boundary layers over rough surfaces. Future efforts in various directions are required to translate fundamental understanding into engineering practices.

The first question is whether the $k_s$ prediction can be improved based on the understanding of the wake effects on the drag. A realistic surface consists of a wide spectrum of length scales and a vast selection of surface characteristics (including the moments of height statistics, slope parameters, etc.); to formulate an effective $k_s$-correlation, the parameters most directly associated with the drag need to be identified. It might be beneficial to evaluate the importance of a surface parameter using its level of correlation with the wake field, since it is the wake, not the geometry, that is directly related to the drag distribution. In addition, it might be practical to develop a wake-modelling approach, to predict the coverage of the roughness-scale separations downstream of roughness protrusions, and thus the magnitude of the pressure drag. This approach would apply to a wide range of realistic surfaces, including heterogeneous ones. This would be very beneficial, since the common $k_s$-correlations do not contain the information of the flow direction, and thus only apply for homogeneous roughness; in reality, however, natural roughness is usually characterized by preferred directions in the flow directions of air or water. Accurate prediction of the roughness parameter is not only attractive in engineering turbulent boundary layers, but also in environmental flows, such as the predictions of flows over canopies or cities.

Secondly, comparisons between the present high-resolution simulations and the engineering simulations, such as RANS, are required to evaluate the performances of the RANS turbulence models on realistic rough-wall flows. It is conjectured that, in strongly accelerating flows, a rough-wall RANS would compare more favourably to the DNS results than would a smooth-wall RANS, due to the faster adapting turbulence stress to the mean flow variations in the rough case. However, it would be useful to study whether the drag variation in the rough case, as a result of the near-wall velocity variation, is well modelled by
the RANS. Moreover, to complete the study of realistic rough-wall boundary layers, other factors should also be considered, including deceleration, curvature, and rotation. The current findings of underlying physics over roughness may also help explain the flows in these complex scenarios.
Bibliography


