ESSAYS ON NETWORKS
AND MACROECONOMICS

by

STEVEN W. KIVINEN

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Abstract

This thesis consists of three essays related by the themes of networks and macroeconomics. The first essay examines the role of networks in workers’ search for employment. Unemployed workers gain employment by sending resumes directly to employers, and indirectly through employed friends. I find that the amount of search effort a worker undertakes is related to how many employed friends it has, and whether networks are substitutes or complements in search. When search costs are linear I find that complementary networks cause those with the most friends to search, while substitutable networks cause search effort to be independent of an individual’s network position. Finally, I examine the role of aggregate links on aggregate matching.

The second essay examines the impact of networks on aggregate labour market variables, such as unemployment and unfilled vacancies. It is concluded that networks lead to wage heterogeneity, increased unemployment volatility, and higher autocorrelation of vacancy rates. The conclusions follow analytically using an approximation method, and quantitative magnitudes are discussed using numerical simulations. Finally, issues of network formation and multiple equilibria are addressed.

The final essay analyzes the role of imitation on firm pricing decisions. A generalization of Calvo pricing is developed. It is shown that price dynamics are much different in a model with imitation. I demonstrate analytically that sticky inflation
can arise. I partially characterize equilibrium prices and inflation. I finish the chapter by generalizing the network effects.
Co-authorship

Chapter 4 is coauthored with Bill Dorval at Queen’s University.
Dedication

To my family for their support and patience.
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All errors are my own.
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<tr>
<td>$\mu_R$</td>
<td>Random search intensity.</td>
</tr>
<tr>
<td>$\mu_N$</td>
<td>Network search intensity per employed friend.</td>
</tr>
<tr>
<td>$L$</td>
<td>Labour force (set of workers).</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of social ties.</td>
</tr>
<tr>
<td>$U$</td>
<td>Set of unemployed workers.</td>
</tr>
<tr>
<td>$U(d^E)$</td>
<td>Set of unemployed workers with $d^E$ employed friends.</td>
</tr>
<tr>
<td>$U(d)$</td>
<td>Set of unemployed workers with $d$ friends.</td>
</tr>
<tr>
<td>$u$</td>
<td>Unemployment rate.</td>
</tr>
<tr>
<td>$u(d^E)$</td>
<td>Unemployment rate amongst workers with $d^E$ employed friends.</td>
</tr>
<tr>
<td>$u(d)$</td>
<td>Unemployment rate amongst workers with $d$ friends.</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of employed workers.</td>
</tr>
<tr>
<td>$E(d)$</td>
<td>Set of employed workers with $d$ friends.</td>
</tr>
<tr>
<td>$d$</td>
<td>Number of friends of unemployed worker.</td>
</tr>
<tr>
<td>$d^E$</td>
<td>Number of employed friends of unemployed worker.</td>
</tr>
<tr>
<td>$d^U$</td>
<td>Number of unemployed friends of unemployed worker $i$.</td>
</tr>
<tr>
<td>$D$</td>
<td>Number of friends of employed worker.</td>
</tr>
<tr>
<td>$D^E$</td>
<td>Number of employed friends of employed worker.</td>
</tr>
<tr>
<td>$D^U$</td>
<td>Number of unemployed friends of employed worker.</td>
</tr>
<tr>
<td>$V$</td>
<td>The set of vacancies.</td>
</tr>
<tr>
<td>$V_w$</td>
<td>The set of waiting vacancies.</td>
</tr>
<tr>
<td>$v$</td>
<td>Vacancy rate; number of vacancies per worker.</td>
</tr>
<tr>
<td>$v_w$</td>
<td>Waiting vacancy rate; number of waiting vacancies per worker.</td>
</tr>
<tr>
<td>$\bar{d}^E$</td>
<td>Average number of employed friends amongst unemployed workers.</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>Average number of friends amongst unemployed workers.</td>
</tr>
<tr>
<td>$m(u, v)$</td>
<td>Matching function; the number of matches per unit labour.</td>
</tr>
<tr>
<td>$m_u(u, v)$</td>
<td>Number of matches per unemployed worker.</td>
</tr>
<tr>
<td>$m_v(u, v)$</td>
<td>Number of matches per vacancy.</td>
</tr>
<tr>
<td>$m_u(u, v, d^E)$</td>
<td>Number of matches per unemployed worker with $d^E$ employed friends.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Probability that an application sent through random search is considered.</td>
</tr>
<tr>
<td>$\mu_V$</td>
<td>The rate at which processing vacancies become waiting vacancies.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Cost of search effort.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\Upsilon$</td>
<td>Match surplus.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Separation rate.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Productivity.</td>
</tr>
<tr>
<td>$c$</td>
<td>Vacancy cost rate.</td>
</tr>
<tr>
<td>$r$</td>
<td>Rate of return.</td>
</tr>
<tr>
<td>$W_U(d)$</td>
<td>Value function for unemployed worker with $d$ friends.</td>
</tr>
<tr>
<td>$W_W(d)$</td>
<td>Value function for employed worker with $d$ friends.</td>
</tr>
<tr>
<td>$W_V$</td>
<td>Value function for vacancy.</td>
</tr>
<tr>
<td>$W_J(d)$</td>
<td>Value function for job filled with a worker with $d$ friends.</td>
</tr>
<tr>
<td>$w(d)$</td>
<td>Wage of worker with $d$ friends.</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>Average wage.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Worker's bargaining power parameter.</td>
</tr>
<tr>
<td>$\varepsilon_{x,\tau-b}(\mu_N)$</td>
<td>Elasticity of $x$ with respect to $p - b$ given $\mu_N$.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Rate of link accumulation of workers of the same employment status.</td>
</tr>
<tr>
<td>$\hat{h}$</td>
<td>Homophily parameter.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Rate of link destruction.</td>
</tr>
<tr>
<td>$\text{supp}(x)$</td>
<td>Support of the distribution of $x$.</td>
</tr>
<tr>
<td>$\phi(x)$</td>
<td>Set of absorbing states of $x$.</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Period $t$ consumption.</td>
</tr>
<tr>
<td>$N_t$</td>
<td>Period $t$ hours worked.</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution parameter.</td>
</tr>
<tr>
<td>$B_t$</td>
<td>Period $t$ bonds.</td>
</tr>
<tr>
<td>$P_t(i)$</td>
<td>Period $t$ price of firm $i$'s good.</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>Period $t$ price of one-period bonds.</td>
</tr>
<tr>
<td>$W_t$</td>
<td>Period $t$ wage.</td>
</tr>
<tr>
<td>$A$</td>
<td>Total factor productivity.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Elasticity of output with respect to labour.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Risk aversion parameter.</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Leisure preference parameter.</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Period $t$ expectations operator.</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>Probability that a firm may change its price.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Probability that a firm optimizes, given that it may change its price.</td>
</tr>
<tr>
<td>$\nu_j(x)$</td>
<td>Probability that a firm with price $x$ changes its price after $j$ periods.</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Period $t$ log-linear price deviation from steady-state.</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>Period $t$ log-linear inflation deviation from steady-state.</td>
</tr>
<tr>
<td>$mc$</td>
<td>Marginal cost of production.</td>
</tr>
</tbody>
</table>
\( f(x) \) Probability distribution for degrees.
\( F(x) \) Cumulative distribution for degrees.
\( f^E(x) \) Probability distribution for \( E \)-degrees.
\( F^E(x) \) Cumulative distribution for \( E \)-degrees.
\( f^U(x) \) Probability distribution for \( U \)-degrees.
\( F^U(x) \) Cumulative distribution for \( U \)-degrees.
\( h(x) \) Probability distribution for prices.
\( H(x) \) Cumulative distribution for prices.
\( b \) Unemployment benefit.
\( \theta \) Labour market tightness; vacancies per unemployed worker.
\( \Phi_t(x) \) A firm’s cost function producing output \( x \) at time \( t \).
\( Y_t(i) \) Output of firm \( i \) at time \( t \).
\( Y_{t+k,t} \) Output of firm at time \( t + k \) that last reset its price at \( t \).
\( \chi \) Degree set.
\( \chi^E \) \( E \)-degree set.
\( M(u, v, \chi^E) \) Number of matches, as a function of \( \chi^E \).
Chapter 1

Introduction

1.1 Networks, Unemployment, and Inflation

Most markets are not centralized and anonymous. Nor are markets completely separated from society and geography. Consumers, workers, and firms have complex interactions (referred to as a network) that impact economic decisions and aggregate economic outcomes. For instance, workers often rely on family and friends for information about job postings. Likewise, firms are more likely to interact with firms in the same industry and the same geographic location than other firms. This thesis is concerned with the role networks play as a mechanism for information diffusion in the labour market and goods market.

In this thesis I present three articles. Chapter two studies the role of social networks in the job search process. Chapter three builds on chapter two by examining the influence of social networks on the dynamics of unemployment, and other labour market variables. Chapter four looks at firms that imitate each other and how that behaviour affects the dynamics of aggregate prices and inflation. Chapters two and
three are related in that both examine the labour market.

Chapter two begins with the premise that social networks are useful for matching unemployed workers with unfilled vacancies. My environment naturally relates the chance of getting a job at any given point in time to the social structure in which the labour market is embedded.

Several questions arise. Exactly how is the probability of finding a job dependent upon the social structure? Does having access to networks affect the behaviour of job searchers? I answer these questions using both statistical and game theoretical techniques to build a model of search in the labour market.

There are several interesting findings. First, those with more friends have a better chance of becoming employed, ceteris paribus. Second, heterogeneity in search effort depends on whether networks complement search effort or are a substitute for search effort. Finally, when there is no congestion then networks with more links lead to more matches. However, when congestion is present then giving all workers more links can lead to no change in the number of matches.

Chapter three looks at a special case of the environment in chapter two and examines the effect of networks on important labour market variables. How do networks affect wages, unemployment, and vacancy creation?

There are several results. First, those hired through networks receive higher wages on average than those who do not apply through networks, but for reasons different than those found in the previous literature. In particular, those with more friends receive higher wages on average because their outside option is better and those with more friends tend to get hired through networks. This result informs discussion about the wage premium paid to those who are hired through referral1.

1See Montgomery (1991) for discussion on the referral wage premium.
Second, changes in productivity lead to larger changes in equilibrium unemployment when there are networks than without (given that the matching rate remains constant). Finally, the presence of networks introduces interesting dynamics into vacancy creation. Namely, the number of vacancies exhibits a larger autocorrelation in the presence of network effects.

The last two results suggest a solution to an ongoing discussion in macroeconomics. The unemployment volatility puzzle roughly states that search models under-predict unemployment volatility. A related short-coming is that search models under-predict the autocorrelation of vacancy rates.

To see if network effects are large enough to increase fluctuations in unemployment, I undertake numerical simulations. I follow a routine calibration strategy, choosing network effects to be within realistic values. The simulations find that realistic parameter values can increase unemployment fluctuations by a significant magnitude when network effects are present compared to the model without network effects. Furthermore, there are small increases in the autocorrelation of vacancy rates.

Two caveats arise. First, only a model with network congestion can generate significant differences in volatility over the baseline model. Second, a trade-off is identified between network effects and a realistic vacancy-unemployment relationship. The Beveridge curve in the U.S. is observed to be downward sloping in the short run, but for large network effects the Beveridge curve is upward sloping in my model. I run a simulation with large network effects that produces a positively sloped Beveridge curve, and a simulation with small network effects that produces a negatively sloped (but nearly flat) Beveridge curve. I find that the latter simulation can only increase unemployment volatility by roughly 50%, whereas matching the data
requires volatility be roughly 20 times larger.

Finally, I analyze labour markets with non-fixed networks. I find that non-congested networks that form through a process called preferential attachment can generate multiple equilibria. Furthermore, when workers tend to gain friends with the same employment status then changes in productivity can lead to jumps between equilibria.

Chapter four departs from analyzing the labour market. Here I am motivated by one question: what happens to prices when firms imitate in a monopolistically competitive framework? The topic is related to networks in that a firm may only view the outcomes of a small number of other firms; firms have local information.

The environment is a generalization of a common sticky price model: Calvo pricing. I characterize the equilibrium and find that, for certain parameter values, sticky inflation will arise. Sticky inflation is the tendency of inflation to not respond immediately to changes in the fundamentals of the economy. This model is one of only a few New Keynesian models to have both sticky prices and sticky inflation.

Despite restricting attention to transitory dynamics, he price dynamics are too complex to solve analytically for the general case. First, I provide a specific example in which the optimal price does not depend on the aggregate price distribution. This case is easy to solve and I demonstrate how inflation stickiness varies with the level of imitation. Second, I examine the general case using a log-linear approximation around a zero inflation steady-state.
1.2 Organization of Thesis

Chapter two contains the essay “Search, Matching, and Social Networks.” Chapter three presents “Unemployment and Social Networks.” Chapter four presents “Imitation and Price Dynamics,” which is based on a joint work with Bill Dorval. Chapter 3 has its own appendices containing proofs and algorithms. Chapter five concludes, and contains suggestions for future work. I conclude this chapter with a review of the relevant literature.

1.3 Literature Review

There is a common theme to chapter two and three: the evidence on, and theories of, the role of social networks in the labour market. I summarize this literature first. Other literatures I summarize are the unemployment volatility puzzle literature, the sticky inflation literature, and the imitation literature. Finally, I discuss the relevant network theory literature.

1.3.1 Social Networks and the Labour Market

The propagation of job information through a network depends on the number of connections, the strength of the connections, and other properties of the network. Granovetter (1973) theorizes on the role of different types of ties on matching unemployed workers to vacancies. Ties are said to be transitive if the following holds true: A and B are friends and B and C are friends, therefore A and C are friends. Transitivity is often expressed probabilistically. The hypothesis goes that weak ties (acquaintances) tend to be less transitive than strong ties (close friends or relatives).
Therefore, weak ties provide novel information about vacancies and should be more useful for job search\textsuperscript{2}. Recent work by Goyal and van der Leij (2011) challenges the importance of weak ties in producing novel information.

Granovetter (1974) is a large study on the empirics of job search through social networks. Classifying ties by frequency of interaction\textsuperscript{3}, more than 50 percent of respondents answer that they came to be informed about their current positions through a tie of some kind. Weak ties appear to be the most important. A host of other studies reach similar conclusions (Myers and Schultz (1951), Rees and Schultz (1970), Corcoran et. al. (1980)). Jackson (2008) discuss the empirical evidence thoroughly.

Boorman (1975) is the earliest known work by an economist in this area, which builds a model to analyze the implications of Granovetter (1973) on the labour market. Montgomery (1991) finds that heterogeneity in ability and social ties can lead to equilibrium wage dispersion when agent’s tend to be friends with the same type\textsuperscript{4}.

Calvo-Armengol and Jackson (2004) investigates the role of social networks on inequality in the labour market. When an agent’s number of job offers depends on the number of employed friends he has then the model produces negative duration dependence. That is, agents who have been unemployed longer have a lower chance of gaining employment. Furthermore, path-connected agents’ employment statuses are correlated. This means that initial conditions matter for employment prospects, as well as location in the network.

Bramoulle and Saint Paul (2011) perform a similar analysis of negative duration

\textsuperscript{2}This is known as the \textit{strength of weak ties hypothesis}.
\textsuperscript{3}Weekly interaction between two workers denotes a strong tie, monthly interaction a weak tie, yearly interaction a very weak tie, and any other interaction gives a non-existent tie.
\textsuperscript{4}This is a type of \textit{homophily}. 
dependence, but allowing the social network to evolve over time. They demonstrate that when agents of the same employment status tend to connect to one another then changes in the social network lead duration dependence. In fact, they find that the magnitude of the duration dependence is small in the absence of an evolving social network.

There have been several attempts to embed networks into a standard search model. Every attempt involves providing a microfoundation for a matching function\(^5\) that includes a network effect. Fontaine (2007) derives a matching function from an urn-ball foundation, but limits his analysis to a small and simple set of networks\(^6\). Calvo-Armengol and Zenou (2005) analyze the impact of social networks on matching in a full-fledged search model and find that increasing the number of total links in the network can decrease the amount of matching. The results rely on the fact that time is discrete. Galenianos (2013) and Galenianos (2014) both analyze network effects in search models. The former addresses the role networks play in learning about match quality. The latter examines the cyclical properties of matching efficiency. To solve the model, both papers assume that the network (and each neighbourhood) is a continuum of nodes.

### 1.3.2 The Unemployment Volatility Puzzle

The main theoretical framework to analyze frictional unemployment is search theory. Diamond (1982), Mortensen and Pissarides (1994), and Pissarides (2000) cover the baseline models used by macroeconomists.

---

\(^5\)See Pissarides and Petrongolo (2001) for more on matching functions.

\(^6\)Only networks that can be partitioned into complete sub-networks are considered.
Andolfatto (1996) uses search in the labour market to address problems with standard real business cycle models. He notices that search models predict unemployment volatility to be too small compared to observed U.S. unemployment volatility. Shimer (2005) calibrates a standard search model (as in Pissarides (2000)) to U.S. data. He finds that (i) predicted unemployment volatility (relative to an H-P filtered trend) is two orders of magnitude smaller than the observed U.S. unemployment volatility over the period 1951-2003, and (ii) there is very little propagation of productivity shocks. Cardullo (2010) provides a survey of the so-called unemployment volatility puzzle\textsuperscript{7}. Costain and Reiter (2007) do a similar exercise as Shimer (2005), but allow unemployment benefits to vary.

Proposed solutions to the unemployment volatility puzzle have been varied. As in Cardullo (2010), I separate these into three categories: sticky wages, new calibration strategies, and microfoundations. The first set of proposals argues that unemployment is not volatile because wages are too flexible in the model. Namely, Nash bargaining is an improper solution concept for determining wages. Various exogenous sticky wage models have been proposed: Shimer (2004), Farmer and Hollenhorst (2006), Gertler and Trigari (2006). All can generate increases in unemployment and vacancy volatility.

A major criticism of such models is that wages or wage stickiness is determined exogenously. Hall (2005) overcomes the criticism by having wages determined by a social norm. Rudanko (2007) proposes a model of long-term wage contracts, where the contracts are a solution to the risk preferences of workers for income smoothing.

\textsuperscript{7}The first finding (i) is often referred to as the unemployment volatility puzzle, while both (i) and (ii) are collectively referred to as the Shimer puzzle.
A second major criticism of all the proposed sticky wage models is given by Pissarides (2009). While average wages tend to be sticky, the wages of new hires are quite flexible. However, it is the rigidity of these wages that drive the increases in predicted unemployment volatility. Therefore, sticky wages likely explain only a small fraction of the discrepancy in unemployment volatility.

The second approach to solving the unemployment volatility puzzle is to challenge the calibration strategy. Hagedorn and Manovskii (2008) point out that Shimer (2005) makes two important missteps in his calibration strategy. First, the bargaining power parameters is chosen to satisfy the Hosios (1990) condition; bargaining power is set in such a way so as to maximize welfare. In principle, there is no reason to believe that welfare is maximized in search environments. Second, the utility from being unemployed is set at the upper bound of estimates for average unemployment benefits, which is 40% of the utility when employed. Of course, unemployment benefits are not the only source of utility (or disutility) from being unemployed.

To solve the problem, Hagedorn and Manovskii (2008) “back out” the parameters required to generate observed unemployment volatility. The result worth mentioning is that the unemployment utility is over 99% of the utility from being employed. The strongest criticism against this approach is that the gap in utility between employment and unemployment is too small to be justified.

The final set of proposals are quite heterogenous, but can be lumped under the “microfoundations” category. These authors believe that Pissarides (2000) suffers from being too simple to explain the macroeconomy. The work of Chapter 3 falls into this category.
Silva and Toledo (2007) add turnover costs to the standard model. The environment lowers the firm’s profit margin, amplifying unemployment. Costain and Reiter (2007) include technological change that depends on match specificity. Barnichon (2014) includes a demand side of the market. These second two papers endogenize productivity changes.

The other failure of standard search models has to do with propagation of productivity shocks. The lack of propagation is revealed in the data in two ways. First, the predicted contemporaneous correlation between tightness (vacancy-unemployment ratio) is 1 whereas the observed correlation is 0.4. Second, the predicted autocorrelation of vacancies is somewhat lower than the observed autocorrelation. The most widely cited solution to this problem is to include a small number of lags in vacancy creation. Fujita and Ramey (2007) takes this approach and succeed in matching the autocorrelations of interest.

1.3.3 Sticky Inflation and New Keynesian Models

Sticky prices are a standard aspect of any New Keynesian economic model. Early examples include Taylor (1979), and Calvo (1983). Both have individual firms changing prices independently. The first has firms deterministically changing prices every fixed number of periods, while the latter has firms changing prices with a constant probability. Criticism of the simplistic environment led to models of state-dependent pricing, where the idea is that the probability that a firm changes its price depends on how costly maintaining its current price is.

There are several dimensions on which sticky price models fail empirically. The important one for this thesis is that basic sticky price models do not produce sticky
inflation. Sticky price models produce quick adjustments in inflation when there is an exogenous change in a parameter. Christiano, Eichenbaum, and Evans (1997) demonstrate that exogenous changes in monetary policy do not affect prices immediately but rather four to six quarters down the road. The result appears to be robust qualitatively.

There have been few attempts to rectify sticky price models. Altig, Christiano, Eichenbaum, and Linde (2005) introduce rigidities on capital usage to incite rigidities in inflation. Mankiw and Reis (2006) produce a model of sticky information in which information about the state of the world moves slowly through the population.

1.3.4 Imitation, Diffusion, and Competition

The results of chapter 4 contribute to the literature on industrial organization and imitation. Vega-Redondo (1997) presents an evolutionary game theoretical model of Cournot competition. The environment has agents imitating the successful strategies of other agents. An agent observes the strategies and payoffs of all players in the previous period and selects the strategy with the highest corresponding payoff. The main result is that the stochastically stable equilibrium has all agents playing the Walrasian (or perfectly competitive) output level.

Bergin and Bernhardt (2009) extend the environment of Vega-Redondo (1997) to include long memories. Agents view a finite history of strategies and corresponding payoffs. The main result is that when memory is sufficiently long, the stochastically stable equilibrium has all players producing the monopolist’s output level.

Selten and Apesteguia (2005) and Selten and Ostmann (2001) examine Cournot

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8The theoretical exception is when the probability that any firm changes its price is zero.
competition on a network: firms compete with neighbours. Simple topologies, such as a circle network, are examined and the solution concept of “imitation equilibrium” is proposed in the latter article. The former article tests the latter with experiments.

A related literature on imitation in the adoption of technologies is related to this thesis. Bass (1969) develops a model to explain the adoption of particular products. The model has agents choosing a new product stochastically: by adopting the product on one’s own (innovation), adopting the product because someone else has adopted the product (imitation). The result is that the rate at which the proportion of agents choosing the new product changes varies non-monotonically over time.

Griliches (1960) provides a similar analysis of the adoption of hybrid corn seeds. Within a given U.S. state, the proportion of farmers using the hybrid seed evolves as predicted in the model from Bass (1969).

1.3.5 Network Theory

The theory of graphs is the primary mathematical framework for analyzing social networks. An introduction to graph theory in general is provided by West (2001). The theory of random graphs is developed by Erdos and Renyi (1959) and Erdos and Renyi (1960). A random network is a fixed set of nodes and a probability on each edge\(^8\). More sophisticated models address shortcomings in basic random graphs (Albert and Barabasi (1999), Bollobas (1985), Jackson and Rogers (2007), Watts and Strogatz (1998)).

Though random graph models can be useful, they become intractable with small increases in complexity. A popular technique for approximating random graphs or

\(^8\)An edge is a pair of nodes, and is present with probability \(p\) and absent with probability \(1 - p\).
fixed graphs with randomly changing nodes is a mean-field approximation.

To give a concrete example, consider the three-dimensional Ising model. Particles (nodes) exist in \( \mathbb{Z}^3 \) and are connected to their immediate neighbours. For instance, a particle in position \((i, j, k)\) is connected to six other particles in positions \((i + 1, j, k)\), \((i - 1, j, k)\), \((i, j + 1, k)\), \((i, j - 1, k)\), \((i, j, k + 1)\), and \((i, j, k - 1)\). The probability that a particle takes on a particular state (e.g. solid, liquid, or gas) depends on the state of its neighbouring particles, and external factors. Baxter (1982) explains that this model is unsolvable and must be approximated. A mean-field approximation considers a representative particle that has a probability distribution over states, has its six neighbours take on a probability distribution over states, and solves for the case where these probability distributions over states are equal. The approximation is considered good for the \( n \)-dimensional Ising model when \( n > 3 \). McComb (2004), Vega-Redondo (2007), and Baxter (1982) provide introductions to the Ising model and mean-field approximations.

Mean-field approximations are now used widely outside of statistical physics. Applications to random graph models include Albert and Barabasi (1999), and Jackson and Rogers (2007). Durlauf (1999) provides a simple economic model of discrete choice with social interaction similar to the Ising model. An applications to the labour market can be found in Bramoulle and Saint Paul (2011).
Chapter 2

Search, Matching, and Social Networks

2.1 Introduction

The use of social networks is pervasive in the matching of workers and firms. Petrongolo and Pissarides (2001) declare, “The matching function summarizes a trading technology between agents who place advertisements, read newspapers and magazines, go to employment agencies, and mobilize local networks\(^1\) that eventually bring them together into productive matches.” Nonetheless, there are few attempts to explicitly incorporate local networks into standard labour search models\(^2\). This paper models social networks in the labour market and thereby provides microfoundations for the matching function.

First, I derive a network-augmented matching function from first principles. I use

\(^1\)Emphasis my own.

\(^2\)There are a few exceptions discussed in the Chapter 1 literature review.
the “urn-ball” approach common to the literature, and consider the limiting case of a large number of searchers in continuous time. The result is a matching function that is a simple function of the network topology.

There are well-known shortcomings to the “urn-ball” approach\(^3\). Next I use a telephone-line queuing process from Stevens (2007). The basic idea is that there is a period of time between when vacancies are created until they are ready to be filled. The result is a network-augmented matching function with desirable properties.

Third, I look at the impact of networks on search effort. The result depends on whether networks are complementary to search effort or substitutes for search effort. I find that complementary network effects lead to those with more employed friends tending to search more intensely, while substitutable network effects lead to an individual’s search effort that is independent of their number of friends.

Finally, I look at the consequences of adding “congestion” to the network. As unemployed agents compete for the attention of common friends networks become less useful. The main result is that the aggregate number of links matter very little for aggregate matching. However, network effects still matter even in the aggregate.

### 2.2 Model

In this section I describe the role networks play in job search, and formal ways to describe networks. Finally, the matching function is derived and its properties are described.

\(^3\)The number of matches with zero vacancies is positive.
2.2.1 Networks and Search

Consider the following scenario. An unemployed agent \( i \) has \( d_i^E \) employed friends and is filling out applications. The agent engages in random search by applying directly to firms. The agent also engages in network search by having employed friends fill out applications on his behalf.\(^3\)

If \( \mu_R \) random search applications are filled out per period and each employed friend fills out \( \mu_N \) applications per period, total applications from the worker per period are \( \mu_R + \mu_N d_i^E \). Thus, the number of applications, and consequently the rate at which an unemployed worker receives job offers depends on a property of the network.

The following example illustrates the interaction. Figure 2.1 depicts a society. Agents B and C are unemployed. However, agent B has access to an employed friend. Thus, there is a total of \( \mu_R + \mu_N \) applications from B and a total of \( \mu_R \) from C per period.

To abstract from the problem that the distribution of firm size and vacancies over firms matter, I suppose that all vacancies are individual firms and that employed agents randomly communicate with these firms. The details will be described when discussing the nature of vacancies.

I also abstract from the issue that the number of unemployed friends of employed friends may matter. If B were employed instead of A one could imagine that the network application rate would fall due to competition between A and C for B’s vacancy. In the model it is assumed that employed agents send out applications randomly and independently (as opposed to the same vacancy). Thus, B would send A’s application to a vacancy, and C’s to a (with probability 1) different vacancy. This

\(^3\)Random and network search are often referred to as formal and informal search, respectively.
assumption is relaxed in Section 2.4.

These two assumptions greatly improve the tractability of the model while maintaining important features. I will now provide a series of formal definitions needed for the following analysis.

**Definition 2.1 (Network)**

A network is a pair \((L, T)\), where \(L\) is the set of nodes and \(T \subset L \times L\) is the set of ties.

A typical element of \(L\) is \(i\), and an element of \(T\) is \((i, j)\).\(^4\) The network can be interpreted as a set of workers and corresponding friendships. I use \(L\) to denote the labour force, which is taken as parametric throughout the analysis. The set \(T\) is fixed, though this assumption is relaxed in Chapter 3.

Agents are either employed or unemployed. Thus, the labour force \(L\) is partitioned into an employed set \(E\) and unemployed set \(U\). The partition of \(L\) induces a partition of the set of ties \(T\).

**Definition 2.2 (Tie Types)**

The set \(T\) is partitioned as follows:

\[(i)\] \(T_{UU} = \{(i, j) \in T | i \in U, j \in U\}\)

\[(ii)\] \(T_{EE} = \{(i, j) \in T | i \in E, j \in E\}\)

\[(iii)\] \(T_{UE} = \{(i, j) \in T | i \in U, j \in E\}\)

Notice that \(T_{UU} \subset U \times U\), \(T_{EE} \subset E \times E\), and \(T_{UE} \subset U \times E\). The partitioning of ties into types is important for describing nodes. The following definition allows one to characterize nodes by network location.

\(^4\)There is no distinction between \((i, j)\) and \((j, i)\). This is referred to as an *undirected* network.
Definition 2.3 (Neighbourhood and Degree)

(i) The neighbourhood of an unemployed agent $i$ is the set $n_i = \{ j | (i, j) \in T_{UE} \cup T_{UU} \}$. An employed agent’s neighbourhood is denoted $N_i = \{ j | (i, j) \in T_{EU} \cup T_{EE} \}$.

(ii) The degree of an unemployed agent $i$ is $d_i = |n_i|$. An employed agent $i$ has degree $D_i = |N_i|$.

I adopt the convention that neighbourhoods and degree in lowercase (uppercase) will refer to unemployed (employed) agents. Like ties, neighbourhoods and degrees can be distinguished by employment status.

Definition 2.4 (Neighbourhood Types and Degree Types)

(i) An unemployed agent $i$ has an $S$-neighbourhood denoted $n_i^S = \{ j | (i, j) \in T_{US} \}$, where $S \in \{ E, U \}$. Similarly an employed agent $i$ has an $S$-neighbourhood denoted $N_i^S = \{ j | (i, j) \in T_{ES} \}$.

(ii) An unemployed agent $i$ has an $S$-degree denoted $d_i^S = |n_i^S|$, where $S \in \{ E, U \}$. Similarly an employed agent $i$ has an $S$-degree denoted $D_i^S = |N_i^S|$.

A tie’s type is determined by the status of the agents at each end. Notice that $d_i^E + d_i^U = d_i$ for every $i \in U$ and $D_i^E + D_i^U = D_i$ for every $i \in E$. Similarly, $n_i^E \cup n_i^U = n_i$ and $n_i^E \cap n_i^U = \emptyset$ for all $i \in U$. It is the set $T_{UE}$, and thus $d_i^E$, that is important for job search.

Most of the results in this paper rely on the size of individual neighbourhoods being small relative to the network as a whole (i.e. the ratio of an individual’s degree to the population being close to 0). To ensure this, I assume that $N_i$ and $n_k$ are finite.

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5 The function $| \cdot | : X \rightarrow \mathbb{Z}_+$ denotes the cardinality of the set $X$.
6 Due to the network being undirected, $T_{UE} = T_{EU}$. 
for all $i \in E$ and $k \in U$, and $|L| \rightarrow \infty$.

**Assumption 2.1**

*Neighbourhoods are finite: $|n_k| < +\infty$ and $|N_i| < +\infty$ for all $k \in U$ and $i \in E$.*

Up until this point I have described the individuals (nodes), the bilateral relationships (ties), and employment status. The following definition provides a description of a network’s aggregate properties.

**Definition 2.5 (Degree Distributions and Degree Sets)**

(i) The *degree distribution*, $f : \mathbb{Z}^+ \rightarrow [0, 1]$, of a network gives, for each degree $d \in \mathbb{Z}^+$, the fraction of agents with that degree.

(ii) The *S-degree distribution* of a network gives, for each $S$-degree $d^S$, the fraction of agents with that $S$-degree. It is denoted $f^S(d^S)$, where $S \in \{U, E\}$.

(iii) The degree set of a network is the set of pairs $\{(f(d), d)\}_{d \in \mathbb{Z}^+} \equiv \chi$. The $S$-degree set is the pair $\{(f^S(d^S), d^S)\}_{d^S \in \mathbb{Z}^+} \equiv \chi^S$.

Every network can be described by its degree distribution, or, equivalently, degree set. However, the degree distribution does not uniquely identify the network\(^8\). Furthermore, $E$-degree distributions and $U$-degree distributions are pinned down by the degree distribution and the distribution of unemployment. In this chapter, both are taken as fixed.

There is one more important property of the network. An agent is described by an employment status and neighbourhood characteristics. Therefore, a neighbour can be described by an employment status and neighbourhood characteristics. The reason

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\(^7\)I also impose this property on the set of vacancies $V$.

\(^8\)The networks $L_1$ and $L_2$ with the same degree distribution will be isomorphic.
that it is important is that, as will be described in subsequent sections, neighbours with many employed friends are more likely to find jobs and be employed. This affects one’s neighbourhood composition and job prospects.

The following definition describes the number of neighbours and $S$ - neighbourhoods that have $k$ employed friends.

**Definition 2.6 ($k$-neighbours)**

(i) (Unemployed) worker $i$’s $k$-neighbourhood is the subset of worker $i$’s neighbourhood whose members have $k$ employed friends, and is denoted $n_i(k) = \{ j | j \in n_i, d_j^E = k \}$.

(ii) (Unemployed) worker $i$’s $k_S$-neighbourhood is the subset of worker $i$’s $S$-neighbourhood whose members have $k$ employed friends, and is denoted $n_i^S(k) = \{ j | j \in n_i^S, d_j^E = k \}$, where $S \in \{ U, E \}$.

(iii) (Unemployed) worker $i$’s $k$-degree is the number of $k$-neighbours that worker $i$ has, and is denoted $d_i(k) = |n_i(k)|$.

(iv) (Unemployed) worker $i$’s $k_S$-degree is the number of $k_S$-neighbours that worker $i$ has, and is denoted $d_i^S(k) = |n_i^S(k)|$ where $S \in \{ U, E \}$.

The definition of $k$-neighbours demonstrates how complex network analysis can become. The partitioning of $L$ based on employment status and neighbourhood (which itself depends on employment status and degree) can be carried further by identifying a neighbour with his $k$-neighbourhood, etc.

### 2.2.2 Urn-Ball Matching

A useful approach to describing the total number of matches in a given period is with a *matching function*. Normally, matches are exogenously given as a function of the
total number of searchers on both sides of the market. Furthermore, it is standard to assume that the function has several desirable properties, such as constant returns to scale.

There is a large literature that derives the matching function from first principles, albeit without network effects. One common approach is referred to as the urn-ball method. I utilize the urn-ball method to provide foundations for the matching function.

Consider a set of urns, $V$, and a set of agents $U$. Each agent possesses $\mu_R > 0$ balls and places each ball in a particular urn with probability $\frac{1}{|V|}$. Thus, agents pick the urn in which to drop a ball randomly with replacement.

Now suppose that each urn belongs to a different firm. Once every ball is placed in some urn the firm draws a ball from its urn at random. Some firms receive no applications and thus draw zero balls. Therefore, a firm draws a ball at random conditional on its urn containing at least one ball. If a firm draws a ball belonging to worker $i$ then the worker gets the job. In the case of worker $i$ getting several balls drawn he chooses the job at random.

The above environment describes a common urn-ball process. Our process is slightly different. Each employed agent also places balls in urns. An employed agent randomly draws an urn with replacement $\mu_N$ times for each of his employed friends. Let $u = \frac{|U|}{|L|}$ and $v = \frac{|V|}{|L|}$ be the aggregate unemployment rate and vacancy rate respectively. The following definition allows us to discuss matching.

**Definition 2.7 (Matches)**

(i) The matching function gives the number of matches for unemployed workers per worker as a function of the unemployment rate ($u$), vacancy rate ($v$), and period
length ($\Delta t$), and is denoted by $m(u, v, \Delta t)$.

(ii) The $d^E$-matching function gives the number of matches for unemployed workers with $d^E$ employed friends per worker with $d^E$ employed friends as a function of unemployment, vacancies, and period length, and is denoted by $m(u, v, \Delta t, d^E)$.

(iii) The matching rate is the number of matches per worker per unit time, and is denoted $\frac{m(u, v, \Delta t)}{\Delta t}$.

(iv) The $d^E$-matching rate is the number of matches per worker with $d^E$ employed friends per unit time, and is denoted $\frac{m(u, v, \Delta t, d^E)}{\Delta t}$.

I will often suppress notation by having $m(u, v, \Delta t, d^E) \equiv m(\Delta t, d^E)$, $\lim_{\Delta t \to 0} \frac{m(u, v, \Delta t)}{\Delta t} \equiv m(u, v)$, and $\lim_{\Delta t \to 0} \frac{m(u, v, \Delta t, d^E)}{\Delta t} \equiv m(u, v, d^E)$. Furthermore define

$$m_u(u, v) \equiv \frac{m(u, v)}{u}$$

$$m_u(u, v, d^E) \equiv \frac{m(u, v, d^E)}{u(d^E)}$$

$$m_v(u, v) \equiv \frac{m(u, v)}{v}$$

The following proposition provides a matching function with network effects in a continuous time environment. It augments results common in the literature, as described in Petrongolo and Pissarides (2001), by having an agent’s search intensity be a linear function of his number of employed contacts. Let $\bar{d}^E = \frac{\sum_{d_E = 0}^{d_E = \max} f_E(d^E) u(d^E) d^E}{u}$, which is the average $E$-degree amongst unemployed agents.

**Proposition 2.1**

Let the number of matches per worker over a time interval $\Delta t$ be given by $m(u, v, \Delta t)$, and suppose that $|V|, |U| \to +\infty$ where $\frac{|V|}{|U|} = \theta < +\infty$ and

$$\bar{d}^E = \frac{\sum_{d_E = 0}^{d_E = \max} f_E(d^E) u(d^E) d^E}{u}$$,
\[ \frac{|U(d^E)|}{|U|} = \alpha(d^E) < +\infty \text{ for all } d^E. \]

If:

(i) Each unemployed agent searches with intensity \( \mu_R \),

(ii) Each employed agent searches with intensity \( \mu_N \) on behalf of each unemployed friend, and

(iii) When an unemployed agent sends out an application there is a probability \( 1 - \xi \) that the application is destroyed,

then when \( \Delta t \to 0 \) the aggregate matching rate is

\[ m(u, v) = (\mu_R \xi + \mu_N d^E)u \]

Proof:

When workers apply randomly and independently to vacancies we have: \# of matches = \# of vacancies \times the probability a vacancy gets filled. Rewrite this as

\[ M(u, v, \chi^E) = |V| \times p \]

where \( M(u, v, \chi^E) \) is the total number of matches as a function of the unemployment rate, vacancy rate, and the \( E \)-degree set. Whether a particular vacancy gets filled or not is a binary random variable and we can write \( p = 1 - q \) where \( q \) = the probability that all unemployed workers apply elsewhere.

If \( \kappa(d^E) = \mu_R \xi + \mu_N d^E \) is the “search intensity” of a worker with \( d^E \) employed
friends then (because workers randomly and independently apply)

\[
q = \prod_{d^E=0}^{d^E_{\text{max}}} q(d^E) = \prod_{d^E=0}^{d^E_{\text{max}}} (1 - \frac{1}{|V|})^{\kappa(d^E)|U(d^E)|}
\]

where \(|U(d^E)|\) is the number of unemployed workers with \(d^E\) employed friends and 
\(q(d^E) \equiv 1 - \frac{1}{|V|})^{\kappa(d^E)|U(d^E)|}\) is the probability that all unemployed workers with \(d^E\) friends do not apply to the vacancy. Therefore,

\[
\frac{M(u, v, \chi^E)}{|L|} = v(1 - \prod_{d^E=0}^{d^E_{\text{max}}} (1 - \frac{1}{v|L|})^{\kappa(d^E)u(d^E)|L|})
\]

where \(|L|\) is the size of the labour force.

Taking \(|L| \to +\infty\) for all \(d^E\), and letting \(v, \frac{v}{u} \equiv \theta\), and \(\frac{u(d^E)}{u} \equiv \alpha(d^E)\) be constant and finite for all \(d^E\) we get

\[
m(u, v, \chi^E) = v(1 - \prod_{d^E=0}^{d^E_{\text{max}}} e^{-\kappa(d^E)\alpha(d^E)\theta}})
\]

which can be rewritten as

\[
m(u, v, \chi^E) = v(1 - e^{-\bar{\kappa}})
\]

where \(\bar{\kappa}\) is the average over unemployed workers and \(m(u, v, \chi^E)\) is matches per worker.

To find the continuous time version, let search intensity be \(\kappa(d^E)\Delta t\) and look for matches per worker per unit time:

\[
\frac{m(u, v, \chi^E)}{\Delta t} = \frac{v}{\Delta t}(1 - e^{-\bar{\kappa}\Delta t})
\]
Applying L’Hopital’s rule gives

\[
\lim_{\Delta t \to 0} \frac{m(u, v, \chi^E)}{\Delta t} = \tilde{\kappa} u
\]

The assumption (iii) adds uncertainty to the application process. The reason for the assumption will become clear in the next section. The uncertainty is only applied to the random search process (as opposed to the network search process). The justification is that unemployed agents send applications to a firm not knowing whether a vacancy is available. In contrast, employed agents know whether the firm is ready to hire. The results do not depend on the probability \(\xi\) only applying to random search.

The matching function has several desirable properties. First, it is linear in \(u\). As is evident in Petrongolo and Pissarides (2001), linearity is a standard property of urn-ball matching functions in continuous time. The second desirable property is the linearity in \(\bar{d}^E\), which makes the function easy to work with.

Our result is new to the literature. Although several papers examine matching functions with networks, all are non-linear functions of the network topology\(^9\), restrict the set of possible network topologies, or are only partially microfounded. The matching function linear in \(d^E\) derived in Proposition 2.1 only relies on the number of nodes being infinite and the neighbourhoods being finite.

\(^9\)The topology of a network with \(n\) nodes refers to the specific pattern of ties.
2.2.3 Telephone Line Queuing Process

There are well-established shortcomings with the urn-ball matching function. In continuous time the function does not depend on the number of vacancies.\textsuperscript{10} Here I follow Stephens (2007) and add a telephone-line queuing process to endogenize $\xi$.

To illustrate the idea suppose that vacancies come in two types: processing and waiting. Processing vacancies are those vacancies that have been created but are unready to be filled. The justification for this is that time and effort is involved in between the decision to create a vacancy and the interview process. Waiting vacancies are those vacancies that are ready to be filled. Let $V_w \subset V$ be the set of waiting vacancies and $v_w = \frac{|V_w|}{|L|}$.

Random search occurs according to a telephone-line queuing process. Workers call a vacancy (uniform) randomly. If the vacancy is a processing vacancy then it does not pick up the telephone. If the vacancy is a waiting vacancy then it picks up the phone and a match is created. Thus, the probability that a phone call reaches a waiting vacancy is $\frac{v_w}{v}$. The matching function can be rewritten as $(\mu_R \frac{v_w}{v} + \mu_N \bar{d}E)u$.

Network search has an added benefit. Namely, employed workers know that a vacancy is waiting. Thus, all network applications reach a waiting vacancy\textsuperscript{11}.

To pin down $\frac{v_w}{v}$ one must discuss the determinants of $v_w$. If processing vacancies become waiting vacancies at rate $\mu_V$ then the inflow of waiting vacancies is $\mu_V (v - v_w)$. Furthermore, vacancies are being filled at the matching rate $m(u, v)$. Therefore, waiting vacancies evolve according to

\textsuperscript{10}It is possible to include vacancies but then the function fails to satisfy $m(0, v) = m(u, 0) = 0$.

\textsuperscript{11}As mentioned earlier, the results do not rely on the assumption. The difference to Proposition 1 is a matching function of $(\mu_R + \mu_N \bar{d}E)\xi u$. 
\[ \dot{v}_w = \mu_V (v - v_w) - \mu_R u \frac{v_w}{v} - \mu_N \bar{d}^E u \]  

(2.1)

where \( \bar{d}^E \) is the mean \( E \)-degree. The first term on the righthand side says that the stock of waiting vacancies (per worker) is increasing as processing vacancies are turned into waiting vacancies at rate \( \mu_V \). The second two terms describe the loss of waiting vacancies as they are matched with workers. Note that the term as written can be negative. I assume that, in steady-state, the network effect is small enough so as not to fill all vacancies.

### 2.2.4 Properties of the Matching Function

Here I highlight an important property of our network matching function. To highlight the role of the networks mechanism in matching I provide a preliminary result. The following proposition states a key property of the matching function when all unemployed agents have the same \( E \)-degree.

**Proposition 2.2 (Steady-State Matching Function)**

*Suppose that \( \dot{v}_w = 0 \) and \( \mu_N > 0 \). If \( d_i^E = d^E > 0 \) for all \( i \) then the matching function \( m(u, v) \) exhibits decreasing returns to scale in \( (u, v) \).*

**Proof:**

First I derive the aggregate matching function as a function of the network. Setting \( \dot{v}_w = 0 \) yields a steady-state fraction of waiting vacancies \( v_w = \frac{\nu v - \mu_N \bar{d}^E u}{\mu_R u + v} \). I restrict attention to values where the fraction is non-negative.

The total number of matches (over a small interval) is \( m(u, v, \bar{d}^E) = \mu_R u \frac{v_w}{v} + \)
Notice the decomposition into a random search and network search component. Rearranging gives the aggregate matching function as

\[
m(u, v, d^E) = \mu_R \frac{v_w^*}{v} + \mu_N d^E u
\]

(2.2)

\[
= \frac{\mu_R u (\mu_V v - \mu_N d^E u)}{\mu_R u + \mu_V v} + \mu_N d^E u
\]

(2.3)

\[
= \frac{\mu_V v u}{\mu_R u + \mu_V v} (\mu_R + \mu_N d^E)
\]

(2.4)

\[
= \frac{\mu_V v (\mu_R + \mu_N d^E)}{\mu_R + \mu_V \theta}
\]

(2.5)

Let \( \lambda > 1 \) and the all agents have \( d^E \).

\[
m(\lambda v, \lambda u) = \frac{\lambda v \lambda u \mu_V (\mu_R + \mu_N d^E)}{\lambda \mu_R u + \lambda \mu_V v}
\]

\[
= \frac{\lambda v u \mu_V (\mu_R + \mu_N d^E)}{\mu_R u + \mu_V v}
\]

It is clear that a constant \( d^E \) yields a constant returns to scale matching function. However, \( d^E \) must decrease with unemployment. The result follows.

Employed agents act as intermediaries in the search process. Doubling the number of searchers leads to less than double the number of applications being filled out. The assumption that all unemployed agents have the same number of employed friends ensures that the high \( E \)-degree employed agents are not the agents that become unemployed. Though it is a sufficient condition it is not necessary.

Decreasing returns to scale is empirically justifiable. Petrongolo and Pissarides (2001) provide a detailed survey of the estimation of matching functions. While
the aggregate U.S. matching function is not statistically significantly different from constant returns to scale, the point estimate is gives decreasing returns to scale.

2.3 Networks and Search Effort

Here the impact of network effects on search effort is examined. First I analyze the case where network effects are complementary, and then I analyze the case where they are perfectly substitutable. Let $\mu_{Ri}$ be the search effort exerted by unemployed worker $i$. Generally, networks and search effort are perfectly substitutable if

$$\frac{\partial^2 m(u,v,d^E_i)}{\partial \mu_{Ri} \partial d^E_i} = 0$$

and complementary if

$$\frac{\partial^2 m(u,v,d^E_i)}{\partial \mu_{Ri} \partial d^E_i} > 0.$$ Two specific cases are examined.

2.3.1 Complementary Networks

To examine search effort one needs a model with costs and benefits. In this section I present a very stylized search model. Suppose that unemployed workers choose $\mu_{Ri} \geq 0$ at a cost of $\mu_{Ri} \rho$, where $\rho \geq 1$ is a constant. The benefit to becoming employed is that the worker splits a surplus with the employer equal to $\beta \Upsilon$, where $\Upsilon$ is the surplus and $\beta$ is the worker’s share. Suppose $\Upsilon$ is constant and sufficiently large.\(^\text{12}\)

The interaction between search effort and networks takes the form of $m(u,v,d^E) = \frac{\mu_{V} \mu_{R} (1+\mu_{N} d^E_i) u}{(\mu_{R} + \mu_{V} v)}$. With endogenous effort an agent $i$’s strategy will depend on his number of employed friends $d^E_i$. In particular, an agent’s matching rate is given by:

\(^{12}\text{If } \Upsilon \text{ is small enough then agents will reject the match.}
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\[ m_i = \frac{\mu_R u + \mu_N d^E_i m(u, v, \bar{d}^E)}{\sum_{j \in U} \mu_R^j (1 + \mu_N d^E_j)} \] (2.6)

Notice the matching rate depends on aggregate variables \( u, v, \) and \( \bar{d}^E, \) as well as the individual \( d^E_i. \) Let \( b \) be the utility from being unemployed. The agent’s optimization problem is given by

\[ \max_{\mu_R} b - \frac{\mu_R^E}{\bar{q}} + m_i \beta \Upsilon \] (2.7)

The solution concept we use is Nash Equilibrium.

**Definition 2.6: Nash Equilibrium**

A Nash Equilibrium \( \{\mu_{R_i}^E\}_{i \in U} \) has \( \mu_{R_i}^E \) as a solution to (2.11) for all \( i \in U. \)

The solution concept is rather straightforward. The next proposition describes equilibrium.

**Proposition 2.3**

Let \( d_{max}^E \) be the largest degree in the support of \( f^E(.) \) and \( \beta \Upsilon > 1, \) where \( \Upsilon \) is the surplus from a match and \( \beta \) is the worker’s share of the surplus as a wage. In a Nash Equilibrium:

(i) For \( \xi > 1, \) \( \frac{\partial \mu_{R_i}^E}{\partial d_i^E} > 0. \)

(ii) For \( d_i^E < d_{max}^E, \) \( \lim_{\varrho \to 1+} \mu_R(d^E) = 0. \)

(iii) For \( d_i^E = d_{max}^E, \) \( \lim_{\varrho \to 1+} \mu_R(d^E) = \frac{\mu_{R_i}^E}{u} ((1 + \mu_N d_{max}^E) \beta \Upsilon - 1). \)
Proof:

Let $\rho = \varrho - 1$ and $K = \sum_{j \in U} \mu_{Rj}(1 + \mu_N d_j^E)$. The first order condition to a given individual’s optimization problem is

$$0 = -\mu_{Ri}^\rho + \frac{(1 + \mu_N d_i^E)m(u, v, \bar{d}_i^E)\beta \Upsilon}{K}$$

(2.8)

Rearranging gives

$$\mu_{Ri} = \left(\frac{(1 + \mu_N d_i^E)m(u, v, \bar{d}_i^E)\beta \Upsilon}{K}\right)^{\frac{1}{\rho}}$$

(2.9)

However, $K$ depends on the value of each individual’s $\mu_{Ri}$. Solving for this gives

$$K = (m(u, v, \bar{d}_i^E)\beta \Upsilon)^{\frac{1}{1+\rho}} \left(\sum_{j \in U} (1 + \mu_N d_j^E)^{\frac{1+\rho}{\rho}}\right)^{\frac{\rho}{1+\rho}}$$

(2.10)

This gives the result of

$$\mu_{Ri}^* = \left(\frac{(m(u, v, \bar{d}_i^E)\beta \Upsilon)^{\frac{1}{1+\rho}}}{(\sum_{j \in U} (1 + \mu_N d_j^E)^{\frac{1+\rho}{\rho}} \frac{\rho}{1+\rho})}\right)^{\frac{1}{\rho}}$$

(2.11)

The results follow. It is clear that for any $\rho > 0$ individual search effort is increasing in $d_i^E$ as individuals do not affect the aggregate variables, showing (i).

Results (ii) and (iii) are demonstrated by taking the limit of $\rho \to 0^+$ (equivalent to $\xi \to 1^+$). We see that the first term stay finite. The second term is a fraction. The denominator is a $L_{\frac{1+\rho}{\rho}}$ norm and goes to $(1 + \mu_N d_{max}^E)$ as $\rho$ goes to 0. Thus, the
fraction goes to zero if \( 1 + \mu_N d_i^E < 1 + \mu_N d_{max}^E \), or \( d_i^E < d_{max}^E \), and 1 if \( d_i^E = d_{max}^E \).

Thus, \( \mu_{Ri} \) depends on \( m(u, v, \bar{d}^E) \) which depends on \( \mu_{Ri} \). The positive solution to the fixed point (there is a 0 solution as well) gives the result \( (ii) \).

The first part of the proposition says that an individual’s search effort is increasing in one’s degree. In other words, when employed friends complement search effort then those with more employed friends tend to exert more search effort.

The second part says that as search costs approach linearity, only those with the most friends undertake any search effort. The intuition is that those with the most friends undertake so much search effort that the marginal benefit to search effort for everyone else goes to zero.

### 2.3.2 Importance of Complementarity

Here we compare the previous result with networks that are substitutable. Our main result is that search effort is the same, independent of an individual’s \( E \)-neighbourhood size.

Suppose that the aggregate matching function takes the following form:

\[
m(u, v, \bar{d}^E) = \frac{\mu_V vu(\mu_R + \mu_N \bar{d}^E)}{\mu_R u + \mu_V v} \quad (2.12)
\]

An individual’s matching function is given by:

\[
m_i = \frac{(\mu_{Ri} + \mu_N d_i^E) m(u, v, \bar{d}^E)}{(\sum_{j \in U} \mu_{Rj} + \mu_N d_j^E)} \quad (2.13)
\]
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Notice that substitutable network have $\frac{\partial^2 m_i}{\partial \mu_R \partial \mu_N} = 0$, unlike complementary networks. The next result says that search effort is independent of the number of employed friends.

**Proposition 2.4**

*In a Nash Equilibrium with $m_i$ as given in (2.17)*

(i) $\mu_{Ri}^* = \mu_R^*$ for all $i \in U$.

(ii) $\mu_R^* = \mu_V \theta (\beta \Upsilon - 1)$

**Proof:**

The first-order condition is given by

$$0 = -\mu_R^\rho + \frac{m(u, v, d^E) \beta \Upsilon}{(\mu_R + \mu_N d^E)} \Rightarrow \mu_{Ri}^* = \left(\frac{m(u, v, d^E) \beta \Upsilon}{(\mu_R + \mu_N d^E)}\right)^{\frac{1}{\rho}}$$

As the optimal effort for $i$ is independent of $d^E_i$ the strategy is the same for every agent. That demonstrates (i).

Solving for $\mu_R^*$ involves setting $\rho = 0$ in equation (2.10) and solving for $\mu_R$. The result (ii) follows.

One draws several conclusions from Proposition 2.3 and Proposition 2.4. First, for search effort to depend on networks there must be some complementarity. Writing applications must be more efficient for agents with many employed friends. Second, changes in the number of vacancies $v$ affects search effort differently under complementary networks. In particular, complementary networks have (in the limit of $\rho \to 0^+$)
the majority of agents not responding to changes in vacancies or unemployment.

The second point has important implications for search effort over the business cycle. Research\textsuperscript{13} finds that search effort varies little over the business cycle. Search models with endogenous search effort predict that unemployment should be close to acyclical, because low returns to search during recessions should lead to large variations in search effort or non-participation in the labour force. My results provide a rationale for why aggregate search effort is relatively constant: only those very few agents with the most friends exert significant search effort when search costs are approximately linear.

\section*{2.4 Extensions and Robustness}

\subsection*{2.4.1 Non-Constant Surplus}

It is assumed in the previous section that $\beta \Upsilon$ is constant, and independent of an individual’s number of links. This assumption is a good one if the firm and the individual both have no way to identify the number of links that an individual has, and matching is a non-repeated game.

In reality, and in the dynamic model of Chapter 3, an agent with more links has a better outside option and therefore should be able to negotiate a higher wage. Let $\Upsilon(d_i^E)$ be individual $i$’s surplus as a function of $i$’s number of employed friends. For now we assume that $\Upsilon'(d_i^E) \geq 0$. Then the results of Proposition 2.3 hold. To see why consider that the marginal benefit to search \( \frac{(1+d_i^E)\beta \Upsilon(d_i^E)}{(\mu_R+\mu_N d_i^F)} \) is still increasing in $d_i^E$.

What about the case of substitutable networks? It is straightforward to show

\textsuperscript{13}See Shimer (2004).
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that the search effort begins to behave a lot like the complementary case. To see how notice that the marginal benefit to search $\frac{\beta \Upsilon(d^E_i)}{(\mu_R + \mu_N d^E_i)}$ depends positively on $d^E_i$ whereas before it was independent of it.

2.4.2 Congestion and Aggregate Links

An important assumption that was made is that an employed friend of $i$, which I will call $j$, sends out $\mu_N$ applications on $i$’s behalf, independent of the number of unemployed friends $j$ has. Therefore, $j$ sends out applications at rate $\mu_N D_U^j$. If $j$’s friends must compete for $j$’s time then I call this congestion.

Though the result is less elegant, it is possible to respecify the model and derive a matching function with congestion. Suppose that each employed agent picks an unemployed friend in a uniformly random fashion and only sends out one application on that friend’s behalf. The rate at which a given employed friend sends off an application on $i$’s behalf becomes $\mu_N \frac{1}{D_U^j}$. Therefore, the total network search for $i \in U$ is

$$\Gamma_i \equiv \sum_{j \in n^E_i} \left( \frac{\mu_N}{D_U^j} \right) \quad (2.16)$$

The network effect now depends on indirect connections. Namely, an agent has the highest prospects for employment when he has many employed friends, each of which has no other unemployed friends.

Let $\bar{\Gamma}$ be the average $\Gamma_i$ for unemployed workers. The following proposition is the counterpart to Proposition 2.1 with congestion.
Proposition 2.5

Let the number of matches per worker over a time interval $\Delta t$ be given by $m(u,v,\Delta t)$, and suppose that $|V|, |U| \to +\infty$ where $\frac{|V|}{|U|} = \theta < +\infty$ and $\frac{|U(d^E)|}{|U|} = \alpha(d^E) < +\infty$ for all $d^E$.

If:

(i) Each unemployed agent searches with intensity $\mu_R$,

(ii) Each employed agent $j$ searches with intensity $\frac{\mu_N}{d^E_j}$ on behalf of each unemployed friend, and

(iii) When an unemployed agent sends out an application there is a probability $1 - \xi$ that the application is destroyed,

then when $\Delta t \to 0$ the aggregate matching rate is

$$m(u,v) = (\mu_R \xi + \mu_N \Gamma)u$$

The proof is sufficiently similar to the proof of Proposition 2.1 that it is omitted here.

The matching function with congestion resembles the matching function without congestion. However, they differ in one important fashion: the impact of the aggregate number of links. Without congestion, doubling every agent’s number of friends of each status\textsuperscript{14} doubles the network effect. With congestion doubling everyone’s number of friends has ambiguous results.

\textsuperscript{14}Doubling $D^E_j$, $D^U_j$, $d^E_i$, and $d^U_i$ for $i \in U$ for all $j \in E$ and $i \in U$. This can be thought of as doubling each agent’s neighbourhood size while keeping the composition of employed and unemployed neighbours unchanged.
The following corollary formalizes the idea. If we multiply each agent’s number of employed friends and unemployed friends by some constant then the network effects react differently with and without congestion.

**Corollary 2.5**

Consider the aggregate network effects with and without congestion, $\mu_N d^E$ and $\mu_N \Gamma$. Let $\gamma \geq 1$ be the amount by which each agent’s $U$-degree and $E$-degree is multiplied.

(i) Without congestion the new aggregate network effect is $\gamma \mu_N d^E$.

(ii) With congestion the change in aggregate network effect is ambiguous.

**Proof:**

Part (i) is straightforward as $\frac{\mu_N \sum_{i \in U} (\gamma d^E_i)}{u} = \gamma \mu_N d^E$. Part (ii) takes a bit more. Notice that although the size of each $n^E_i$ grows with $\gamma$ the composition matters. Let $n^E_i(\gamma) = n^E_i \cup \hat{n}^E_i(\gamma)$. Then the network effect with congestion is:

$$\frac{\mu_N}{u} \left( \sum_{i \in U} \left( \sum_{j \in n^E_i(\gamma)} \frac{1}{\gamma D^U_j} + \sum_{j \in \hat{n}^E_i(\gamma)} \frac{1}{\gamma D^U_j} \right) \right)$$

$$= \frac{\mu_N \bar{\Gamma}}{\gamma} + \frac{\mu_N \bar{\Gamma}(\gamma)}{\gamma}$$

It is easy to see that the change in the network effect may increase, decrease, or stay the same depending on the new neighbours’ values of $D^U_j$. 

\qed
2.5 Discussion

The contribution of this chapter is to derive a network matching function with desirable properties and analyze the effect of networks on search effort undertaken by unemployed workers. The former is done with an urn-ball foundation and a telephone-line queuing process foundation. Search effort is analyzed in a static framework taking into consideration the distinction between complementarity search effort and substitutable search effort.

The matching function results provide us with the tools to pursue further research. For instance, one can use the matching function from Proposition 2.2 to analyze equilibrium in a search model of the type from Pissarides (2000). I explore this avenue in Chapter 3.

The results on search behaviour are relevant to the fluctuation of search behaviour over the business cycle (or lack thereof). It also opens the door to further analysis on endogenous network effects.
2.6 Figures

Figure 2.1: Example Society

Description: An example of a networked labour market.
Chapter 3

Unemployment and Social Networks

3.1 Introduction

Debates on the the causes and nature of unemployment continue to this day. Can network effects inform us on the nature of unemployment, vacancy creation, and wages? This chapter investigates the role networks play in labour market equilibrium and addresses several failures of standard search models.

First, search models of the labour market do a poor job of explaining the short-run volatility of unemployment (and other variables) in the post-war era. In fact, observed unemployment rates are two orders of magnitude more volatile than a benchmark search model predicts.

Second, vacancy rates observed over the post-war era exhibit a higher autocorrelation than search models predict. Another way to say this is that search models lack a sufficient propagation mechanism for productivity shocks.
I develop a model of unemployment and social networks by building on the results from the previous chapter. I utilize a matching function that takes social networks into account. My model predicts that, when the network structure is fixed, the equilibrium unemployment rates and matching rates have a larger response (in absolute value) to productivity shocks than in baseline search models. Vacancies also exhibit more persistence outside the steady-state.

When examining unemployment volatility, the results rely on variables being in steady-state and are qualitative in nature. To check the robustness of the analytical results several numerical simulations are provided. I find that unemployment rates, matching rates, and vacancy rates all exhibit higher volatility. Labour market tightness does not. The results rely on networks being prone to congestion. Furthermore, a tradeoff is identified between network induced volatility and a realistic Beveridge curve.

Finally, I examine networks that evolve over time. I find that multiple equilibria can arise when networks do not exhibit congestion and links tend to form for those who already have many links. In addition, when workers tend to link with those of the same employment status changes in productivity can lead to jumps between equilibria.

### 3.2 Search Model with Fixed Networks

Here I present a search model and solve a notion of steady-state network effects. The analysis of unemployment volatility is done using a mean-field approximation. The approach is follows Bramoulle and Saint Paul (2010) closely.

The network can be described using an $|L| \times |L|$ adjacency matrix $G$. A typical
element of $G$, denoted $G_{ij}$, takes on a value of 1 if $(i, j) \in T$ and 0 otherwise. Let the distribution of unemployment be described by the $|L|$-dimensional vector $s$, where $s_i = 1$ if $i$ is employed and 0 otherwise.

Let time be discrete and $\lambda$ be the probability at which agents lose their jobs. At the individual level, variables evolve stochastically. For a given individual, the transition probabilities are $\Pr(s'_i = 1|s_i = 0) = m(u, v, d^E_i)$, and $\Pr(s'_i = 0|s_i = 1) = \lambda$. One can rewrite an unemployed individual $i$’s $E$-degree as:

$$
\begin{align*}
  d^E_i &= \sum_{j \in L} s_j G_{ij} \\
  &\text{(3.1)}
\end{align*}
$$

Up until this point I have defined a Markov process in which individual $E$-degrees and employment status evolve stochastically. While the dynamics of this model are generally very complex, I present an approximation model in the next section.

### 3.2.1 Mean-Field Approximation

The goal of the model is to analyze the labour market dynamics. The dynamics of the model presented thus far can get very complicated. For instance, the evolution of $S$-degrees are stochastic as workers lose and gain jobs randomly\(^1\). I call the model of the previous section the true model.

To overcome these issues, I use the method employed by Bramoulle and Saint Paul (2010) and apply an approximation model called a mean-field approximation\(^2\). Let the mean-field approximation be in continuous time and allow $d^E$ to take on any value in

---

\(^1\)Even if the means of aggregate variables move deterministically, the individual variables are stochastic.

\(^2\)Calvo-Armengol and Zenou (2005) overcomes these issues by randomly drawing the set of links every period. Galenianos (2013) and Galenianos (2014) overcome these issues by having neighbourhoods be infinitely large.
The mean field approximation (i) imposes the same distribution of employment over neighbours for all workers, (ii) has the aggregate distribution of employment evolve deterministically, and (iii) has individual distributions be consistent with the aggregate distribution. Another way to think of (i) is as having a representative agent for each \( d \), because each agent with degree \( d \) has the same \( E \)-degree.

In continuous time, \( m(u,v) \) is the matching rate and \( \lambda \) is the separation rate. The following equations describe the mean-field approximation for all \( d \) and \( k \):

\[
\begin{align*}
\dot{d}^E(d, d(k), k) &= m_u(u, v, k)(d(k) - d^E(d, d(k), k)) - \lambda d^E(d, d(k), k) \\
\dot{d}^U(d, d(k), k) &= -m_u(u, v, k)d^U(d, d(k), k) + \lambda (d(k) - d^U(d, d(k), k)) \\
\dot{D}^E(D, D(k), k) &= m_u(u, v, k)(D(k) - D^E(D, D(k), k)) - \lambda D^E(D, D(k), k) \\
\dot{D}^U(D, D(k), k) &= -m_u(u, v, k)D^U(D, D(k), k) + \lambda (D(k) - D^U(D, D(k), k))
\end{align*}
\]

where \( m_u(u, v, k) \) is the matching rate of an unemployed agent with \( k \) employed friends and \( \lambda \) is the separation rate. Equations (3.2) through (3.5) depend on \( k \), which is the number of employed friend a given neighbour has, and \( d(k) \), the number of total
friends with $k$ employed friend the agent has. The variable $k$ matters because agents with more employed friends will become employed faster. For an agent with degree $d$ the $E$-degree is given by $d^E(d) = \sum_k d^E(d, d(k), k)$. Furthermore, if an agent has a total of $d$ friends then $d = \sum_k d(k)$ by definition.

Notice that $d^U(d, d(k), k) = d(k) - d^E(d, d(k), k)$. The first term of equation (3.2) describes the number of $U$-neighbours that gain employment whereas the second term of equation (3.2) describes the number of $E$-neighbours that lose jobs. Therefore, there is no net loss or gain in total ties.

The steady-state for equation (3.2) is $d^{E*}(d, d(k), k) = \frac{m_u(u,v,k)}{\lambda + m_u(u,v,k)} d(k)$. However, $k$ is in steady-state and determined by the agent’s degree. Therefore, the steady-state $E$-degree for the worker with degree $d$ is given by $d^{E*}(d) = \sum_k d^{E*}(d, d(k), k)$ or:

$$
d^{E*}(d) = \left( \sum_{\alpha=0}^{d_{\text{max}}} \left( \frac{d\left(d^{E*}(\alpha)\right)}{d} \right) \frac{m_u(u,v,d^{E*}(\alpha))}{\lambda + m_u(u,v,d^{E*}(\alpha))} \right) d
$$

where $d^{E*}(\alpha)$ represents the steady-state $E$-degree of a neighbour with $\alpha$ friends. The fraction $\frac{d(d^{E*}(\alpha))}{d} \in [0, 1]$ is the number of friends with degree $\alpha$ that the representative person with $d$ friends has, and is (in steady-state) a fixed property of the structure of the network. This is because $E$-degrees are determined by $d$ in steady-state. Normally, a mean-field approximation would impose $\frac{d(d^{E*}(\alpha))}{d}$ to be the average proportion of friends with degree $\alpha$ conditional on degree $d$. While the level and distribution of unemployment depend on this fraction, the results on productivity presented in this section are either independent of the fraction or impose $f(d) = 1$ for some $d > 0$. I make the following assumption for the remainder of this section.

**Assumption 3.1**

*The fraction of neighbours with degree $\alpha$ is the same as the fraction of the population.*
with degree $\alpha$: \[ \frac{d(d^E(\alpha))}{d} = f(\alpha) \] for all $d$ and $\alpha$.

This assumption imposes that the degree distribution of neighbours for a typical worker with degree $d$ is the population distribution. There are a few implications of this assumption. First, degree homophily is the tendency of those with similar degrees to be connected. Assumption 3.1 rules this out. Second, the unemployment rate amongst neighbours is the same as the unemployment rate in the population. The main results of the paper do not rely on Assumption 3.1.

The mean-field approximation is considered a good one if (3.6) averaged across $d$ for unemployed workers is close to (3.1) averaged over all unemployed individuals $i$. Mean-field approximations are known to be good approximations to random network models in many circumstances. Bramoulle and Saint Paul (2010) apply the approximation to a labour market model with search frictions. Jackson and Rogers (2007) apply the approximation to a network formation model. For general discussions, see Vega-Redondo (2007) and McComb (2004) for physical applications and general discussion. The following analysis is done with a mean-field approximation$^3$.

When the population is large, aggregate variables move deterministically due to the law of large numbers. The unemployment rate for those with $d^E$ employed friends evolves according to

\[ \dot{u}(d^E) = \lambda(1 - u(d^E)) - u(d^E)m_u(d^E) \] (3.7)

The overall unemployment evolves according to $\dot{u} = \sum_{k=0}^{+\infty} f^E(k)\dot{u}(k)$.

$^3$Simulations not presented here show that our analysis approximates the true model well. Bramoulle and Saint Paul (2010) use a similar model and come to the same conclusion.
CHAPTER 3. UNEMPLOYMENT AND SOCIAL NETWORKS

The following proposition uses the steady-state exogenous effort matching function from chapter 2. The result describes the properties of the matching function when network effect are endogenized. In particular, the mean-field approximation is assumed to be in steady-state.

Proposition 3.1 (Steady-State Matching Function)
Suppose that unemployment and networks are in steady-state, \( \frac{u(d)}{u} \) is constant for all \( d \), and \( 0 < \mu_N < \bar{\mu}_N \).

(i) The matching function \( m(u, v) \) exhibits decreasing returns to scale in \( (u, v) \).

(ii) \( m(u, v) \) is strictly increasing in \( d^{E} \) and \( v \).

(iii) There exists a \( \hat{u} \in [0, 1] \) such that the matching function is increasing in \( u \) on \( [0, \hat{u}) \) and decreasing on \( (\hat{u}, 1] \).

Proof:

The proof has two steps: solve the matching function when \( \dot{u} = \dot{d}^{E} = 0 \), and show \( (i) - (iii) \) are true.

Step 1: Imposing \( \dot{u}(d) = \dot{d}^{E}(d) = 0 \) gives steady-state \( u(d) \) and \( d^{E}(d) \).

\[
\begin{align*}
u^*(d) &= \frac{\lambda}{\lambda + m_u(u, v, d^{E*}(d))} \quad (3.8) \\
d^{E*}(d) &= \frac{m_u(u, v, d^{E*}(d))}{\lambda + m_u(u, v, d^{E*}(d))} \quad (3.9) \\
\Rightarrow d^{E*}(d) &= (1 - u^*)d \quad (3.10)
\end{align*}
\]

The last step comes from equation (3.6) and Assumption 3.1.
Step 2: To verify the properties, one must first substitute $d^E$ into the matching function. Note that $m(u, v) = \sum_k f^E(k)\alpha(k)m(u, v, k)$ where $\alpha(k) \equiv \frac{u(k)}{u}$. Therefore, with $\alpha(k)$ constant for all $d$ we need only show that each $m(u, v, d^E*(d))$ is decreasing returns to scale in $(u, v)$.

$$m(u, v, d^E(d)) = \frac{\mu_V v}{\mu_R + \mu_V \theta}(\mu_R + \mu_N d^E(d))$$

(3.11)

$$= \frac{\mu_V v}{\mu_R + \mu_V \theta}(\mu_R + \mu_N(1 - u)d)$$

(3.12)

To show $(i)$ let $\gamma > 1$.

$$m(\gamma u, \gamma v, d^E*(d)) = \frac{\gamma \mu_V v}{\mu_R + \mu_V \theta}(\mu_R + \mu_N(1 - \gamma u)d)$$

(3.13)

$$< \frac{\gamma \mu_V v}{\mu_R + \mu_V \theta}(\mu_R + \mu_N(1 - u)d)$$

(3.14)

$$= \gamma m(u, v, d^E*(d))$$

(3.15)

To show $(ii)$ one need only differentiate $m(u, v)$ with respect to $v$ and $d$ respectively. The proof is sufficiently trivial to be left to the reader.

To show $(iii)$ one need only take the derivative of $m(u, v)$ with respect to $u$. Equivalently, we look at each component of the matching function.

$$\frac{\partial m(u, v, d^E*(d))}{\partial u} = \frac{m(u, v, d^E*(d))\mu_V \theta}{(\mu_R + \mu_V \theta)u} - \frac{v\mu_N d}{(\mu_R + \mu_V \theta)}$$

(3.16)

$$= \frac{\theta}{\mu_R + \theta}((\theta \mu_R + (\theta(1 - u) - u)\mu_N d)$$

(3.17)
This term will be positive or negative depending on the size of the parameters, $\theta$, and $(1 - u)$. The cutoff $\hat{u}$ is

$$0 = \theta \mu_R + \theta \mu_N d(k) - (1 + \theta)\hat{u}\mu_N d(k)$$

$$\Leftrightarrow 0 = \hat{u}^2 + v\hat{u} - \frac{v(\mu_R + \mu_N d(k))}{\mu_N d(k)}$$

The quadratic equation always has a positive real root and a negative real root. The (positive) solution is

$$\hat{u} = \frac{v}{2}(-1 + [1 + \frac{(\mu_R + \mu_N d(k))}{\mu_N d(k)v}]^{\frac{1}{2}})$$

Therefore, if $u > \hat{u}$ then matches decrease in $u$. \hfill \square

The result characterizes the matching function when network effects are in equilibrium. The key result is that the matching function exhibits decreasing returns to scale. When the number of vacancies and unemployed double, the non-steady state matching function exhibits constant returns to scale. However, the number of intermediaries (employed agents) decreases. There are more workers searching, but each with less intensity.

The number of matches increases with $v$ and $\bar{d}^E$. It is conceivable that the network effect is big enough to clear the market, which justifies the bound on $\mu_N \bar{d}^E$. Finally, increases in the unemployment rate always increases matches at low unemployment rates. However, it is possible for unemployment to decrease the number of matches at
high levels of unemployment. The properties derived are common to the literature.\textsuperscript{4}

\subsection*{3.2.2 Equilibrium}

A matching function has been derived, and now a full-fledged search and matching model may be analyzed. Here I augment Pissarides (2000) by incorporating the network effect. An equilibrium definition is given and results on volatility, persistence, and wages are stated.

One can imagine a scenario in which wages are set through bargaining and workers are heterogeneous. A non-cooperative bargaining solution can be complex and obscure the role of networks. To maintain the focus on changes in social networks (as opposed to wage setting) I determine wages with Nash bargaining as in Pissarides (2000). Furthermore, the analysis assumes that both firms and workers can observe a worker’s current network position.\textsuperscript{5}

Unless otherwise mentioned, I assume that $S$- neighbourhood, and unemployment are in steady-state. This assumption implies that each unemployed worker of degree $d_i$ will have the same neighbourhood composition, which makes the analysis much simpler.

Let $r$ be the (common) rate of return. $r$ can be interpreted as the rate at which a worker or firm increases its utility or profit over time. Let $b$ be unemployment benefits, which represent the instantaneous utility from being unemployed. Let $\tau$ be productivity, which is the rate at which an employed worker produces the good. We will denote the vacancy-unemployment ratio as $\theta = \frac{v}{u}$ and call it labour market

\textsuperscript{4}Galenianos (2013) and Galenianos (2014) provide a matching function with the above properties when workers have a continuum of friends. Calvo-Armengol and Zenou (2005), with the exception of point (ii), derives the results with random regular networks.

\textsuperscript{5}This includes neighbourhood composition and degree.
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tightness. It is easier to describe a firm’s vacancy creation decision, and resulting equilibrium, in terms of $\theta$ instead of $v$. Finally, let $c > 0$ be a parameter that describes the cost incurred by the firm from maintaining a vacancy over time. I follow Pissarides (2000) and I assume vacancy costs are proportional to productivity.

Until this point we have described the unemployment rate of those with a particular $d^E$-degree. In the equilibrium described below, there will be a one-to-one relationship between a worker’s degree $d$ and his $E$-degree $d^E$. Therefore, we use $u(d)$ to denote the unemployment rate of workers with degree $d$. Unemployment depends on the total number of friends an individual has as well as the number of employed friends.

Let $W_U(d)$ be the flow utility of being unemployed with degree $d$. $W_W(d)$ the flow utility of working with degree $d$. The flow utility of a vacancy is $W_V$. Similarly, filled jobs have flow utility depending on the type of worker hired, denoted $W_J(d)$. Because wages can be conditioned on a worker’s degree value functions are too. The value of vacancies, $W_V$, does not depend on any individual worker’s degree because of the uncertainty in the matching process.

The value functions\(^6\) associated with employment status and vacancy status are:

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\(^6\)These are valid assuming $d^E_i$, and $u$ are in steady-state. Otherwise the value functions have additional terms related to the changes in state-variables.
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\[ rW_U(d) = b + m_u(d)(W_W(d) - W_U(d)) \] (3.20)

\[ rW_W(d) = w(d) + \lambda(W_U(d) - W_W(d)) \] (3.21)

\[ rW_f(d) = \tau - w(d) + \lambda(-W_f(d)) \] (3.22)

\[ rW_V = -c\tau + \sum_{i \in U} m_v(d_i)(W_f(d_i) - W_V) \] (3.23)

for all \( d \) where \( c > 0, \tau > 0, \) and \( b > 0 \).

The value functions are the utility for workers and firms, depending on whether they are in a match a match or not. The value functions differ from the baseline Pissarides (2000) model in an important respect: utility depends on degree \( d \). This is because the future probability of becoming employed (potentially) depends on \( d \). This means wages will depend on degree, which means the value of a job depends on \( d \). The utility from an unfilled vacancy, denoted \( W_V \), does not depend on \( d \) because it is unknown which worker will fill the vacancy.

The steady-state conditions and Nash bargaining allow us to derive expressions for unemployment, wages, and \( S \)-neighbourhoods. To complete the model one must determine the vacancy rate \( v \). Each vacancy is a firm and profit maximization involves deciding between creating a vacancy or not. The final condition is the free-entry condition, \( W_V = 0 \). The free entry condition is similar to a zero-profit condition; firms earn exactly zero profits from creating a vacancy. If vacancies earn positive profits then vacancies would continue to enter the market until \( \theta \) becomes infinite in equilibrium. If vacancies earn negative profits then no firm is willing to create a vacancy and \( \theta = 0 \) in equilibrium.
The following definition is the equilibrium concept for fixed networks.

**Definition 3.1 (Fixed Network Equilibrium)**

Given a network \((L,T)\), a Fixed Network Equilibrium (FNE) satisfies:

(i) **Steady-State Unemployment:** 
   \[ \dot{u}(d) = 0 \quad \forall d \]

(ii) **Steady-State Neighbourhood:** 
   \[ \dot{d}_i^e = \dot{d}_u^e = \dot{D}_j^u = 0, \quad \forall i \in U, j \in E \]

(iii) **Nash-Bargaining:** 
   \[ w(d) = \arg\max (W(d) - U(d))^\beta (W_J(d) - W_V)^{1-\beta} \]

(iv) **Free-Entry:** 
   \[ W_V = 0 \]

Conditions \((i)\), \((iii)\), and \((iv)\) are similar to Pissarides (2000). The main difference is that wages, and thus value functions, depend on an agent’s degree.\(^7\) Condition \((ii)\) is a steady-state conditions on \(S\)-neighbourhoods.

### 3.2.3 Results on Volatility and Persistence

A FNE can explain some labour market statistics due to the decreasing returns to scale matching function. First, changes in productivity, \(p\), lead to changes in unemployment rate, \(u\). The changes in \(u\) affect the \(E\)-neighbourhoods of unemployed workers. This leads to a lower matching rate and lower steady-state unemployment. There is a feedback effect as changes in \(u\) are reinforced by less network matching.

Second, the vacancy rate \(v\) is a jump variable. Thus, changes in \(\tau\) lead to jumps in \(v\). However, now \(u\) has a larger effect on \(v\). One can see this by examining equilibrium \(\theta\). In a baseline search model \(\theta\) is constant for any given set of parameters; exogenous changes in \(\tau\) lead to a one-time jump in \(\theta\). In the model presented here, exogenous changes in

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\(^7\)Technically, they depend on \(d^E\). Conditions \((i)\) and \((ii)\) imply that \(d^{E*}_i\) is entirely determined by \(d_i\).
changes in $\tau$ lead to an initial jump in $\theta$ followed by a gradual change towards a steady-state.

When looking at the effect of $\tau$ on $u$ one must look at the direct effect and the indirect effect (through $\theta$). The indirect effect is assessed by examining the elasticity of $\theta$ with respect to $\tau$. The next result states that the equilibrium responses in $\theta$ and $u$ to changes in productivity are larger with network search than with no network search.

**Proposition 3.2 (Volatility)**

Let $\varepsilon(\mu_N)_{x,(\tau-b)}$ be elasticity of $x$ with respect to $\tau - b$ for $\mu_N$. Assume that $d_i = d > 0$ for all $i$, $\mu_R + \mu_N\bar{d}E$ is constant, and $\frac{\mu_N}{\mu_R}$ is constant. Then there exists a $\bar{\mu}_N > 0$ such that if $0 < \mu_N < \bar{\mu}_N$ then in FNE:

(i) $|\varepsilon(\mu_N)_{u,(\tau-b)}| > |\varepsilon(0)_{u,(\tau-b)}|$

(ii) $\varepsilon(\mu_N)_{\theta,(\tau-b)}$ changes ambiguously with $\mu_N$

The proof is in the appendix. The result demonstrates that equilibrium unemployment has a larger response to productivity shocks, and should be more volatile. The feedback effect of losing intermediaries (employed workers) between firms and the unemployed magnifies the response of each variable to exogenous changes in $\tau$. The impact of productivity changes on $\theta$ is ambiguous. On one hand, an increase in productivity shortens the average waiting time for a vacancy to get filled which increases the value of a vacancy. On the other hand, increases in match efficiency increase wages, decreasing the value of a vacancy. Which effect is stronger depends on the parameters of the model. Granted the results are limited to the steady-state, if transitory dynamics are of little consequence then the result is a good approximation. I examine non-steady state dynamics in section 3.3.
The reason for the restriction of $d_i = d > 0$ is that the proportion of unemployed agents with a given degree is not constant, but a function of $\tau$. The reason for the upper bound $\bar{\mu}_N$ is that a sufficiently large $\mu_N$ can lead the model to become unstable.

The next proposition looks at the non-equilibrium dynamics of vacancy creation. Essentially, if Nash bargaining wage determination, and free-entry ((iii) and (iv) of FNE definition) remain then $\theta$ is a function of past $p$ and $v$ exhibits more persistence. Let $\theta(\mu_N)$ be the tightness that satisfies (iii) and (iv) at $\mu_N \geq 0$.

**Proposition 3.3 (Persistence)**

Suppose (iii)-(iv) of FNE continue to hold, and at $t_0$ $d^E \neq 0$ and $\dot{u} \neq 0$.

$\dot{\theta}(\mu_N) \neq 0$ for $t > t_0$ implies $\mu_N > 0$.

The proof is in the appendix. The result is significant because it means that tightness is need not be completely determined by $\tau$, but also depends on previous $\theta$. This adds some persistence to the model that is absent when $\mu_N = 0$; the labour market tightness in the original model is not a function of flow variables. Now changes in $d^E$ and $u$ have an impact on $\theta$ by steadily increasing (decreasing) the matching efficiency when $\tau$ increases (decreases).

### 3.2.4 Wages

Equilibrium average wages behave like the equilibrium wages in standard search models. Due to heterogeneity in the matching rate our model produces a distribution of wages. Let $w_R$ be the average wage of those hired through random search and $w_N$ be the average wage of those hired through network search. The following result states that the wages are higher on average for those hired through their network.
Proposition 3.4

In a fixed network equilibrium, \( w_N \geq w_R \). For regular networks \( w_N = w_R \).

The proof is in the appendix and involves demonstrating that the distribution of wages conditional on being hired through a contact first-order stochastically dominates the distribution of wages conditional on being matched through random search. The idea is that those with larger degree get higher wages, and those with larger degrees tend to get hired through their network.

3.3 Numerical Simulations

3.3.1 Calibration

I follow the calibration strategy of Shimer (2005) closely\(^8\). The addition of U.S. data beyond 2003 changes the aggregate statistics very little. Productivity follows a discretized Ornstein-Uhlenbeck process to match the volatility present in U.S. data, though the state space is smaller than Shimer (2005) due to computational limitations. The unemployment utility, \( b = 0.4 \), is chosen within the range of unemployment benefits as a percentage of mean income.

The bargaining power, \( \beta = 0.72 \), is chosen to satisfy the Hosios Condition in a standard search model. Unlike Shimer (2005), the matching function used here has a non-constant elasticity of vacancies. Nonetheless, the chosen bargaining weight is close to the average elasticity of vacancies.

The separation rate, \( \lambda = 0.09 \), is chosen to be close to that observed in U.S.\(^{8}\) Hagedorn and Manovskii (2008) take issue with the calibration strategy. These criticisms do not change the main message that networks can explain some, but not all, of the unemployment volatility puzzle.
data. The parameters left are used to target relevant means, varying the relationship between $\mu_R$ and $\mu_N$ across simulations. Table 3.1 summarizes the choices across the three simulations.

I target mean unemployment at 0.056, and mean matching rate at 1.5. Because I measure vacancies as an index, we have the freedom to target $\theta$ to 1.25 without loss of generality. I set $d_i = d = 100$ for every agent. Finally, we keep $\mu_N d$ and $\mu_R$ within an empirically reasonable range across our second two simulations. The small network effect has matching through networks on average accounting for approximately $\frac{3}{11}$ of matches, whereas the large network effect account for $\frac{3}{5}$ of all matches on average.

Because the data is filtered, the network effect will be filtered out if it operates at a quarterly frequency. Therefore, we run simulations at a rate of 15 times more frequent than a quarter. Intuitively this means that unemployed workers talk to employed friends roughly five times per month, allowing feedback effects to work between quarters. The data is then aggregated by capturing every 15th observation.

### 3.3.2 Numerical Results

The results of the no network effect model reproduce Shimer (2005) relatively accurately, despite using a different matching function and different productivity process specification. Table 3.2 shows the standard deviation from an HP filtered trend with smoothing parameter $10^5$ (applied post-aggregation) and autocorrelation across models for no network effect, large network effect, and small network effect. The statistics are averaged across 1000 model simulations each, with standard errors in brackets.

Unemployment and matching rates are impacted most by network effects. Unemployment and matching rate volatility with large network effects is about 3.2 times
the volatility with no network effects. The small network effect gives the volatilities of unemployment and matching rates as being roughly 1.6 times the respective volatilities without network effects.

Vacancies are affected somewhat, roughly doubling in volatility and exhibiting a small increases in autocorrelation with large network effects. Vacancies are barely affected (if at all) with small network effects. Surprisingly, the volatility of tightness appears to be unaffected by network effects.

Adding network effects disconnects the unemployment-vacancy relationship. Table 3.3 shows how each variable is contemporaneously correlated with vacancies. The correlation is high and negative in the data, low and negative with small network effect, and positive with large network effects.

The unemployment-vacancy relationship is referred to as the Beveridge curve. The positively sloped Beveridge curve from my simulations suggests a bound on network effect. Much of the correlation can be regained by setting low network effects and artificially increasing the volatility of $\theta$. The fact that networks affect match efficiency alone, and not $\theta$, leads to results similar (but not identical) to a stochastic separation rate.

### 3.4 Implications of Non-Fixed Networks

Observed social networks evolve over time. Ties break and new connections are formed while employment statuses change. Here I present a coevolutionary model of networks and unemployment. Our main result is that multiple equilibria can arise when the rate of link formation depends on the number of agents available to link to. For instance, one accumulates links to employed agents faster when the unemployment
rate is low. These multiple equilibria can exhibit tipping points\(^9\).

Before continuing, I will discuss a few concepts in the formation of networks. Links are accumulated in two important ways. First, agents meet randomly or in ways that do not depend on network structure. Second, agents often find friends through current friends, which suggests that the rate of an agent’s link accumulation depends positively on the number of links he currently has. Preferential attachment (PA) is the idea that agents with more links gain more friends per period. In my model, PA is necessary for the existence of multiple equilibria.

There also exist biases in the network formation process that depend on an agent’s employment status. The tendency of agents with the same employment status to be linked is called economic homophily.\(^10\) In the presence of preferential attachment it is important to distinguish between the different manners in which homophily might present itself. First-Order Economic Homophily (FH) is the idea that agents of a particular employment status are more likely to accumulate links with agents of the same employment status. For instance, if \(i\) is employed then \(i\) is more likely to meet \(j\) if \(j\) is also employed.

Second-Order Economic Homophily (SH) is the idea that an employed friend of a particular status is more likely to refer a friend of the same status. For instance, independent of agent \(i\)’s status, if \((i, j) \in T\) and \(j\) is employed then \(i\) is more likely to meet \(k\) through \(j\) if \(k\) is also employed. This means that employed friends are more useful for meeting new employed friends.

The results in this section rely on PA and SH. To make things clear we assume

\(^9\)A previous version of the paper analyzed unemployment volatility in an evolving network. The results of the previous section are largely robust to changes in the set \(T\). However, when matching rates and/or separation rates are sufficiently high the model can exhibit instabilities and qualitatively counterfactual results.

\(^10\)I borrow this terminology from Bramouille and Saint Paul (2010).
that there is pure SH in the sense that only employed friends are useful for gaining employed friends, though the assumption is not required.

### 3.4.1 Mean-Field Approximation

Much like the previous section, I apply a mean-field approximation. The true model is described by equation (3.1). However, in this section $G$ is not fixed. Links are destroyed according to $Pr(G'_{ij} = 0|G_{ij} = 1) = \delta$. Link creation, described by $Pr(G'_{ij} = 1|G_{ij})$, is more complicated and depends on employment status.

The mean-field approximation is described by the appropriate laws of motion. The specific laws of motion for workers’ $S$-neighbourhoods are different than before. An unemployed worker $i$ has $d^E$ employed friends, which evolves according to:

\[
\dot{d}^E(d^E, d^U, k) = -\delta d^E(d^E, d^U, k) + \hat{h}\rho g(d^E)(1 - u(k)) \\
+ m_u(u, v, k)d^U(d^E, d^U, k) - \lambda d^E(d^E, d^U, k)
\]

Notice the similarities with the fixed network model. The final two terms, written as $m_u(u, v, k)d^U(d^E, d^U, k) - \lambda d^E(d^E, d^U, k)$, describe the current friends (with $k$ employed friends) who lose and gain employment. The other two terms, $-\delta d^E(d^E, d^U, k) + \hat{h}\rho g(d^E)(1 - u(k))$, describe the attaching and severing of links.

The term $\delta$ describes the rate at which links become detached. Because the mean-field approximation is a deterministic approximation of a stochastic process, $\delta$ can also be thought of as the probability that a link is destroyed. Thus, $-\delta d^E(d^E, d^U, k)$
is (approximately) the total number of links destroyed over a unit interval.

To understand link formation first examine $g(d^E)(1 - u)$. The second term, $(1 - u(k))$ says that the more agents with $k$ employed friends that are employed, the faster the rate at which others befriend them. Simply put, the more employed agents that are in the economy, the faster one connects to them.

The term $g(d^E)$ says that the rate of employed friend accumulation depends on the number of employed workers that an agent is connected to\textsuperscript{11}. When $g(x) = x$ then there is perfect preferential attachment and the model is analogous to “Model B” of Barabasi and Albert (1999). If $g(x) = 1$ then link formation is purely random, and the model is related to Erdos and Renyi (1959). It is the cases in between, referred to as imperfect preferential attachment, that can yield interesting multiple equilibria. For the purposes of this chapter, it is assumed that $g$ is strictly increasing. The other mean-field equations are:

\begin{align*}
\dot{d}^U(d^E, d^U, k) &= -\delta d^U(d^E, d^U, k) + \rho g(d^U)u(k) \tag{3.25} \\
&\quad - m_u(u, v, k)d^U(d^E, d^U, k) + \lambda d^E(d^E, d^U, k)
\end{align*}

\begin{align*}
\dot{D}^E(D^E, D^U, k) &= -\delta D^E(D^E, D^U, k) + \rho g(D^E)(1 - u(k)) \tag{3.26} \\
&\quad + m_u(u, v, k)D^U(D^E, D^U, k) - \lambda D^E(D^E, D^U, k)
\end{align*}

\textsuperscript{11}Notice $g$ is not a function of total links. This is because of second-order homophily.
\[
\dot{D}^U(D^E, D^U, k) = -\delta D^U(D^E, D^U, k) + \hat{h}\rho g(D^U)u(k) + m_u(u, v, k)D^U(D^E, D^U, k) + \lambda D^E(D^E, D^U, k)
\] (3.27)

The parameter \(\hat{h}\) captures FH. Namely, the creation of links is penalized if the workers are of different employment statuses. \(\rho\) captures the rate of link formation common to all individuals. The non-linearity of the link formation process can lead to multiple equilibria. Figure 3.1 shows an example of a law of motion in one dimension. Notice that there are three absorbing states.

### 3.4.2 Equilibrium

Because the set of links, \(T\), can change over time one needs a more general equilibrium definition than a FNE. Here we define an equilibrium to accommodate the evolving nature of the network. Let \(F_t\) and \(F_t^S\) for \(S \in \{U,E\}\) be the cumulative distribution distributions corresponding to \(f_t\) and \(f_t^S\), respectively.

**Definition 3.2: Network Equilibrium**

*Given a set of nodes \(L\), a network equilibrium (NE) satisfies:*

(i) **Steady-State Unemployment:** \(\dot{u}(d) = 0 \forall d\)

(ii) **Steady-State E-Degree Distributions:** \(F_t^S(d^E) = F_t^S(d^E) \forall t, S \in \{U, E\}\)

(iii) **Steady-State U-Degree Distributions:** \(F_t^S(d^U) = F_t^S(d^U) \forall t, S \in \{U, E\}\)

(iv) **Nash-Bargaining:** \(w(d^E) = \arg\max(WW(d^E) - W_U(d^E))^\beta(W_U(d^E) - W_V)^{1-\beta}.

(v) **Free-Entry:** \(W_V = 0\)
Let the set of NE be denoted \( \Gamma(L) \). A network equilibrium is a fixed network equilibrium with an endogenous distribution of total links. This is highlighted by conditions \((ii)\) and \((iii)\) requiring the degree distribution be in steady-state.

### 3.4.3 Network Equilibrium with \( \hat{h} = 1 \)

The special case of no first-order economic homophily \( \hat{h} = 1 \) leads to a conveniently navigable model. The following proposition states some of the basic properties of the equilibrium.

**Proposition 3.5 (Properties of Network Equilibrium)**

Let \( \hat{h} = 1 \) and \( \text{supp}(x) \) be the support of distribution \( f(x) \). Let \( \phi(x) \) be the set of absorbing states of \( x \). Then for any NE:

1. \( \text{supp}^* (d^E) \subseteq \phi(d^E) \)
2. \( \text{supp}^* (d^E) = \text{supp}^* (D^E) \) and \( \text{supp}^* (d^U) = \text{supp}^* (D^U) \).
3. There exists a function \( g(d^E) \) and parameters \( (\delta, \rho) \) such that \( |\phi(d^E)| = 3 \).

The proof is in the appendix.

The proposition makes several important points. First, \((i)\) limits the set of equilibrium distributions. The set of distributions is restricted to the set of absorbing states. Second, \((ii)\) says that the type degree distributions are independent of one’s employment status. Once an individual has a steady-state number of employed (and unemployed) friends he maintains that number, even if his employment status changes. Finally, \((iii)\) says that there exist parameters that lead to multiple absorbing states (and thus multiple equilibria) for \( d_i^E(k) \).

Therefore, the model is very tractable and multiple equilibria can arise. Point \((ii)\) means that a large set \( \phi(x) \) can lead to a large set of NE. To keep the analysis clear a
refinement of the set of NE is presented. Namely, I concentrate on networks in which individuals have the same number of links.

**Definition 3.3: Regular Equilibria**

The set of degenerate steady-state distributions are referred to as regular steady-states. The equilibria with regular steady-states are referred to as regular equilibria. The set of regular equilibria is denoted by $\Gamma_r(L) \subseteq \Gamma(L)$.

Notice that when $|\phi(dE)| = 1$ then $\Gamma_r(L) = \Gamma(L)$. The refinement is not useful for settings with a unique absorbing state. However, the refinement is very useful for dealing with multiple absorbing states.

The following proposition discusses equilibrium Beveridge curves.

**Proposition 3.6**

Consider $(dE(v), dU(v), u(v))$ associated with a regular steady-state (given $v \in \mathbb{R}_+$) at $h = 1$. If there exists another regular steady-state network $(dE(v)', dU(v)')$ where $dE(v') > dE(v)$ and $dU(v') > dU(v)$, then there exists another steady-state unemployment rate $u(v)'$ where $u(v)' < u(v)$.

The proof is in the appendix. The idea is that when there are multiple steady-state $dE^*$ then there are potentially many steady-state $u^*$ as well. The Beveridge curve is an equilibrium relationship between $v$ and $u$, and in regular equilibrium is described by

$$u^* = \frac{\lambda}{\lambda + \frac{\mu v}{\mu ru + \mu v} (\mu_R + \mu_N dE^*)}$$

It is easy to see that multiple $d^*$ may lead to multiple $u^*$. The proof is slightly more involved, because $dE^*$ depends on $u$. 
The result says that at any (non-equilibrium) vacancy rate there may be multiple unemployment steady-states. This is equivalent to saying there are several equilibrium Beveridge curves. The following example combined with Figure 3.2 and Figure 3.3 drives the point home. The numerical example illustrates how a change in $\lambda$ can reduce the set of equilibria from three to one. In particular, a high unemployment equilibrium can disappear temporarily, and lead to a large and persistent change in unemployment.

### 3.4.4 Example

Consider the function $g(x) = 1 + (x - 1)^{\frac{3}{2}}$, $\delta = 1$, and $\rho = 1$. Notice that $g(x)$ is strictly increasing, strictly concave for $x > 1$, and strictly convex for $x \in (0, 1)$. When the parameters (including productivity) are calibrated to have $u$ close to 0 then there are obviously two stable steady states (and a third unstable steady-state). However, when the parameters are chosen to set $u$ close to 1, then there is only one equilibrium.

Figure 3.2 shows $\dot{d}^E$ under the two scenarios. Figure 3.3 shows two Beveridge curves associated with the larger and smaller $d^{E*}$ respectively.

### 3.4.5 Network Equilibrium with $\hat{h} < 1$

The benefit of working with no homophily is the tractability of the model. The drawback is that (as Bramoulle and Saint Paul (2010) point out) there is zero duration dependence from link destruction. As both duration dependence and homophily are observed in the data, it is good to know whether the results of the previous section extend to $\hat{h} < 1$.

To see how homophily affects the equilibrium suppose only one equilibrium exists.
It is easy to see from the laws of motion (the system of equations (3.23) – (3.26)) that the absorbing value \( d_{E}^{*} \) is smaller than the absorbing value \( D_{E}^{*} \). In fact, an equilibrium distribution is on \( I \equiv [d_{E}^{*}, D_{E}^{*}] \).

Now suppose that there are multiple absorbing states \( d_{E}^{*} < d_{M}^{*} < d_{H}^{*} \) and \( D_{L}^{*} < D_{M}^{*} < D_{H}^{*} \). There are two associated intervals \( I_{H} \equiv [d_{H}^{*}, D_{H}^{*}] \) and \( I_{L} \equiv [d_{L}^{*}, D_{L}^{*}] \). Any equilibrium distribution is on a subset of these two intervals.

The next result says that multiple equilibria exist when \( I_{H} \) and \( I_{L} \) do not overlap. If the intervals overlap then agents end up in the intersection asymptotically. Furthermore, the intervals will not overlap if there is not “too much” homophily.

**Proposition 3.7**

Suppose the laws of motion are such that when \( \hat{h} = 1 \) there are three absorbing states for \( d_{E} \) and \( D_{E} \) each (two stable and one unstable) and the correspondence of absorbing states is continuous. Let \( I_{L} \) and \( I_{H} \) be the relevant intervals. The following are equivalent:

(i) There are multiple equilibria.

(ii) \( I_{L} \cap I_{H} = \emptyset \)

(iii) \( \hat{h} \in (\hat{h}_{L}, \hat{h}_{H}) \) where \( 0 < \hat{h}_{L} < 1 < \hat{h}_{H} < +\infty \)

The proof is in the appendix. The result is a robustness check on the assumption that \( \hat{h} = 1 \). The result says that \( \hat{h} \) is not too far from 1, when the set of absorbing states for \( d_{E} \) and \( D_{E} \) are continuous, if and only if there are multiple equilibria. A way to interpret this is that the rate of link accumulation cannot be “too dependent” on employment status. The second point (ii) is a technical point that says that the supports of different equilibrium distributions must not overlap.

When \( \hat{h} > 1 \) we say that network formation exhibits *heterophily*, though such an
assumption produces the counterfactual prediction of positive duration dependence. Another implication of the result is that \( \hat{h} = 0 \) implies a unique equilibrium.

### 3.5 Discussion

The role of network effects in labour market has yet to be fully explored. I contribute to the literature by developing models with fixed networks and evaluate network effects on the volatility and persistence of important labour market variables. I characterize the equilibrium and find that the existence of network effects increases the volatility of unemployment, vacancies, tightness, and matching rates. Networks also increase the propagation of exogenous changes in productivity.

Our results motivate the further investigation of network effects in search and matching models of the labour market. First, I rely on Nash bargaining by imposing an information structure that is unrealistic. How can bargaining with asymmetric information affect the model’s predictions? Second, the result of including network effects together with other mechanisms for increasing volatility, such as rigid wages or endogenous productivity, has yet to be explored. I leave this for future research.
### 3.6 Tables

#### Table 3.1: Calibrated Parameters for Different Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Network Effect</th>
<th>Large Effect</th>
<th>Small Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.09</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>$c$</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.72</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>$b$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\mu_R$</td>
<td>0.165</td>
<td>0.165</td>
<td>0.165</td>
</tr>
<tr>
<td>$\mu_{Rd}$</td>
<td>0</td>
<td>1</td>
<td>0.125</td>
</tr>
<tr>
<td>$\mu_{R}$</td>
<td>110</td>
<td>54</td>
<td>98</td>
</tr>
</tbody>
</table>

#### Table 3.2: Standard Deviations for Different Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$m$</th>
<th>$p$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No NE</td>
<td>0.010</td>
<td>0.010</td>
<td>0.017</td>
<td>0.013</td>
<td>0.017</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.012)</td>
<td>(0.003)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>Large NE</td>
<td>0.032</td>
<td>0.021</td>
<td>0.017</td>
<td>0.035</td>
<td>0.017</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Small NE</td>
<td>0.016</td>
<td>0.012</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

#### Table 3.3: Autocorrelation of $v$ and contemporaneous correlations.

<table>
<thead>
<tr>
<th></th>
<th>No Network</th>
<th>Large Network</th>
<th>Small Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$ Autocorrelation</td>
<td>0.84</td>
<td>0.88</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>corr($u,v$)</td>
<td>-0.75</td>
<td>0.56</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>
3.7 Figures

Description: An example of the mean-field approximation having multiple steady-states.
Figure 3.2: Changes in Equilibrium

Description: A change in productivity can change the number of equilibria.

Figure 3.3: Beveridge Curves

Description: An example of multiple equilibrium Beveridge curves.
Chapter 4

Imitation and Price Dynamics

4.1 Introduction

Confucius declares\(^1\), “Man has three ways of acting wisely. First, on meditation: that is the noblest. Second, on imitation: that is the easiest. Thirdly, on experience: that is the bitterest.” Standard macroeconomic theory assumes that firms do the first and optimize profits. In this chapter, a model is developed in which firms may imitate the successful strategies of other firms\(^2\).

The main result is that imitation and imperfect information lead to sticky prices and sticky inflation. Firms that imitate only change prices when other firms have a better history of pricing. The probability of such an event is generally less than one. The result is that the probability of changing one’s price is higher if more firms are playing more profitable prices.

The result has empirical implications. The theoretical impulse response functions

\(^1\)Translation by Muller (1990).

\(^2\)Here we do not analyze learning from experience. Presumably this model can incorporate learning by extending the memory of a firm.
closer match those estimated with U.S data by Christiano, Eichenbaum, and Evans (1997). My model predicts that the rate at which aggregate prices change depends on how many firms have changed prices in the past, implying that the rate is non-linear over time. The result is an “S”-shaped impulse response function that resembles diffusion processes.

This chapter is organized as follows. First I present the New Keynesian model of Galí (2008). The model incorporates the pricing dynamics from Calvo (1983), in which every firm changes its price with some probability $\theta$ and maintains its current price otherwise. So-called Calvo pricing is a standard way of incorporating sticky prices in a monopolistically competitive framework. Second, I augment Galí’s model by adding imitation. I solve for the price dynamics under two different methods of price aggregation. Third, I characterize the equilibrium price and how it relates to imitation. Finally, I relate the price dynamics to the properties of a firm’s network.

4.2 Model

In this section I introduce the basic New Keynesian model described in Galí (2008) and use it as a benchmark model, which is a dynamic version of Dixit and Stiglitz (1977). The presentation includes an exposition of Calvo pricing, where I highlight my innovation.

4.2.1 Households

The households in this paper behave exactly as in Galí (2008). More specifically, there is a representative infinitely lived household that maximizes the following discrete time
problem:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

s.t.

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + W_tN_t + T_t$$

where $C_t$ is a consumption index:

$$C_t = \left( \int_0^1 C_t(i)^{\epsilon - 1} \right)^{\frac{1}{\epsilon}}$$

where $C_t(i)$ is the amount of good $i$ consumed by the household at time $t$ and produced by firm $i$. It is assumed that these goods are indexed by the interval $[0, 1]$. Therefore, given this set up, the households have preferences for variety. $P_t(i)$ is the price of good $i$, and $B_t$ is the number of one-period bonds purchased (at price $Q_t$). Also, $\lim_{T \rightarrow \infty} E_t\{B_T\} \geq 0$ must be satisfied for all $t$ (No-Ponzi scheme condition).

It can be shown that the budget constraint can be written as:

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t$$

where $P_t$ is an aggregate price index characterized by the following equation:

---

$^3$Each firm produces a different good, and each good is substitutable.

$^4$See Appendix 3.1 of Galí (2008)
The demand equation for good $i$ is:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$  \hspace{1cm} (4.6)$$

Finally, assume that the household has the following preferences over labour and consumption:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$  \hspace{1cm} (4.7)$$

The first order conditions are:

$$-\frac{U_{n,t}}{U_{c,t}} = C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t}$$  \hspace{1cm} (4.8)$$

$$Q_t = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$  \hspace{1cm} (4.9)$$

### 4.2.2 Firms

While the households are identical to Galí (2008), firms are different in an important respect. Here we introduce the imitation mechanism in an easily accessible framework.

For simplicity I assume that there is no capital, and that each firm produces
according to the following production function:

\[ Y_t(i) = A_t N_t(i)^{1-\alpha} \] (4.10)

where \( Y_t(i) \) is the output of firm \( i \) at time \( t \), \( N_t(i) \) is the labour input, and \( A_t \) is productivity/technology common to all firms. All firms face the same demand function, (4.6), and take the aggregate consumption index, (4.3), and the aggregate price index, (4.5), as given.

Now consider a discrete time environment in which a unit interval of firms each set the price of their good, \( P_t(i) \). With some probability \( \vartheta \in [0, 1] \) firm \( i \) cannot update its price while with probability \( 1 - \vartheta \) the firm may change its price. This exogenous pricing probability is Calvo pricing. Notice that \( \vartheta = 0 \) is the special case of flexible prices.

My environment differs from Calvo pricing in that a firm that is allowed to change its price might not optimize. Assume that with probability \( \gamma \in [0, 1] \) firm \( i \) is be able to choose the price to maximize its profit while with probability \( 1 - \gamma \) it is matched with another firm and might imitate its price. In other words, firms randomly do one of three things: optimize prices, imitate prices, or leave prices unchanged. I assume that a firm will only imitate the other firm if the latter has a better pricing history (in terms of previous period profits). Therefore, my environment is a generalization of Calvo pricing.

For the time being I will introduce some notation without solving the optimization problem. Let \( P_t^*(\vartheta, \gamma) \) be the optimal price for firms re-optimizing at time \( t \) with fixed parameters \( \vartheta \) and \( \gamma \), \( P_t(\vartheta, \gamma) \) be the aggregate price at \( t \), and \( f_t(x) \) be the number of firms with price \( x \) at \( t \).
For example, suppose that at $t = 0$ all firms are playing the optimal price $P_0^*(\vartheta, \gamma)$, which means $h_0(P_0^*(\vartheta, \gamma)) = 1$. When $\vartheta = 0$ and $\gamma = 1$ (flexible prices) then this will always be the case. Furthermore, given that all firms have the same production function and information set they will all play the same optimal price every period $h_t(P_t^*(0,1)) = 1$, for all $t$.

Note that Calvo pricing is a special case in which $\gamma = 1$ and $\vartheta \in [0, 1]$. When $\gamma = 1$ it can be shown that the aggregate price dynamics are described by\(^5\):

$$P_t(\vartheta, 1) = [(1 - \vartheta)P_t^*(\vartheta, 1)^{1-\epsilon} + \vartheta P_{t-1}(\vartheta, 1)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \quad (4.11)$$

Every firm that is re-optimizing is choosing the same price, $P_t^*(\vartheta, 1)$, while the mass of firms that do not re-optimize correspond to the distribution of prices from the previous period.

Re-arranging gives us a formula in terms of the inflation rate between period $t - 1$ and $t$, $\Pi_t$:

$$\left(\frac{P_t(\vartheta, 1)}{P_{t-1}(\vartheta, 1)}\right)^{1-\epsilon} = \Pi_t^{1-\epsilon} = \vartheta + (1 - \vartheta) \left(\frac{P_t^*(\vartheta, 1)}{P_{t-1}(\vartheta, 1)}\right)^{1-\epsilon} \quad (4.12)$$

In the model with $\gamma < 1$, aggregate prices also depends on the probability of being matched with a firm that has a better pricing history. In general, this probability will depend on the cumulative distribution of prices at a given point in time, denoted $H_t(x)$. The model is discussed in the subsequent section.

\(^5\)See Galí (2008)
4.3 Imitation

Consider the simple scenario in which the parameters of the model are constant (non-stochastic), every firm plays the same initial price \( f_0(P_0) = 1 \), and the long run optimal price is larger than the initial price \( \lim_{t \to +\infty} P_t^* > P_0 \). This simple environment is easy to analyze and clearly demonstrates our mechanism at work. The analysis presented in this chapter is restricted to these transitory dynamics, though symmetric results hold for the case of \( \lim_{t \to +\infty} P_t^* < P_0 \).

Proposition 4.1 describes the conditional probability of a price change for a given firm.

**Proposition 4.1**

Let \( \gamma < 1 \) and \( \nu_j(P_t(i)) \) be the probability that a firm that last set its price at \( t \) to price \( P_t(i) \) will change its price at \( t + j \) for \( j \geq 1 \).

\( (i) \) \( \nu_j(P_t(i)) = (1 - \vartheta)(\gamma + (1 - \gamma)(1 - H_{t+j-1}(P_t(i)))) \) for \( t \geq 1 \).

\( (ii) \) \( \nu_{j+1}(P_t(i)) > \nu_j(P_t(i)) \)

\( (iii) \) \( \nu_1(P_t(i)) = (1 - \vartheta)\gamma \) and \( \lim_{j \to +\infty} \nu_j(P_t(i)) = 1 - \theta \)

The proof of \( (i) \) follows directly from the definitions, and \( (ii) \) and \( (iii) \) follow almost immediately from \( (i) \). Therefore, the proof is omitted.

The proposition discusses the probability of changing one’s price \( j \) periods after the last price change. The probability varies as the distribution of prices varies. The probability of changing one’s price increases over time, in contrast to Calvo pricing under which the probability of changing the price remains constant.

Before examining the price dynamics in the New Keynesian model I look at how
average prices evolve. Such an exercise will give insight into the mechanism of imitation. Proposition 4.2 describes the evolution of average prices for an arbitrary sequence of optimal prices.

**Proposition 4.2**

Consider the sequence of optimal prices \( \{ P_s^*(\vartheta, \gamma) \} \) where \( P_t^*(\vartheta, \gamma) \) is (weakly) increasing in \( t \) and all firms play \( P_0(\vartheta, \gamma) \) at \( t = 0 \). Average prices evolve according to

\[
\bar{P}_t = (1 - \vartheta)(\gamma P_t^*(\vartheta, \gamma) + 2(1 - \vartheta)(1 - \gamma) \sum_{s=0}^{t-1} h_{t-1}(P_s^*(\vartheta, \gamma)) H_{t-1}(P_s^*(\vartheta, \gamma)) P_s^*(\vartheta, \gamma)) \\
- (1 - \gamma) \sum_{s=0}^{t-1} h_{t-1}(P_s(\vartheta, \gamma))^2 P_s^*(\vartheta, \gamma)) + \vartheta \bar{P}_{t-1}
\]

**Proof:**

Consider the law of motion for the probability distribution function of firm prices \( h_t(P_s) \) (and corresponding cumulative distribution function \( H_t(P_s) \)) where \( p_s \) is an optimal price at period \( s \leq t \):

\[
h_t(P_s) - h_{t-1}(P_s) = (1 - \vartheta)(1 - \gamma)(H_{t-1}(P_s) - h_{t-1}(P_s)) h_{t-1}(P_s) \\
- (1 - \vartheta)\gamma h_t(P_s) - (1 - \vartheta)(1 - \gamma)(1 - H_{t-1}(P_s)) h_{t-1}(P_s)
\]

Some rearranging yields the following recursive equation:

\[
h_t(P_s) = \vartheta h_{t-1}(P_s) + 2(1 - \vartheta)(1 - \gamma) H_{t-1}(P_s) h_{t-1}(P_s)
\]
Given the above result we can substitute in the distribution in the following equation:

\[
\bar{P}_t = \sum_{s=0}^{t} h_t(P_s)P_s 
\]

\[
= (1 - \vartheta) \gamma P_t + \sum_{s=0}^{t-1} h_t(P_s)P_s 
\]

\[
= (1 - \vartheta) \gamma P_t + 2(1 - \vartheta)(1 - \gamma) \sum_{s=0}^{t-1} H_{t-1}(P_s)h_{t-1}(P_s)P_s 
\]

\[
- (1 - \vartheta)(1 - \gamma) \sum_{s=0}^{t-1} h_{t-1}(P_s)^2P_s + \vartheta \bar{P}_{t-1} 
\]

The result follows.

\[ \square \]

Notice that \( \gamma = 1 \) reduces to Calvo pricing and \( \gamma = 0 \) leads to price changes only within currently existing prices. The result says that current prices depend on the past prices and distributions, whereas Calvo pricing has current prices only depend on the previous period’s average price. The difference provides a source for sticky inflation.

Proposition 4.2 describes the evolution of prices in a recursive way. The problem of explicitly solving the distribution of prices is intractable for most sequences of optimal prices. The next result is for the special case in which there is one optimal price that is different from the original. All firms begin at the original price and move towards the new price \( P^* \).
Corollary 4.2

Suppose that $P^*_t(\vartheta, \gamma) = P^* = P^*_0(\vartheta, \gamma) \equiv P^*_0$ for all $t > 0$. Then prices change according to:

$$\bar{P}_t - \bar{P}_{t-1} = (1 - \vartheta)\gamma P^*_t + (1 - \gamma)h_{t-1}(P^*)h_{t-1}(P^*_t)P^* + (1 - h_{t-1})P_0 - \bar{P}_{t-1}$$

where $h_t(P^*) = \frac{1 - e^{-(1 - \vartheta)t}}{1 + \frac{1}{\gamma}e^{-(1 - \vartheta)t}}$

Though the above result does not apply to the full model of Galí (2008), it highlights the role of imitation. Namely, Figure 4.1 illustrates the evolution of prices for various parameters. Furthermore, the distribution of prices is solved exactly as in Bass (1969). Notice that the S-shaped curve exhibits sticky inflation as the inflation rate does not peak initially.

4.4 Price Dynamics

In this section, I analyze aggregate price dynamics in the New Keynesian framework of Galí (2008) with imitation. The results in this section differ from those in Proposition 4.2 in two ways. First, New Keynesian models do not use average prices, but use the price index in (4.5) (and (4.17)). This makes solving a monopolistic competition model much easier. Second, we express the prices in terms of a log-linear approximation around a steady-state inflation rate of zero.

As discussed in Section 4.2, prices are aggregated with the following index:
CHAPTER 4. IMITATION AND PRICE DYNAMICS

\begin{align}
P_t \equiv \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \tag{4.17}
\end{align}

It is straightforward to demonstrate that the index is equivalent to:

\begin{align}
P_t \equiv \left[ \vartheta(P_t - 1)^{1-\epsilon} + (1 - \vartheta)(1 - \gamma) \sum_{i=0}^{t-1} h_{t-1}(P^*_i)H_{t-1}(P^*_i)(P^*_i)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{4.18}
\end{align}

Let \( \pi_t \) be logged change in aggregate prices (inflation) at time \( t \), \( p_t \) be logged aggregate prices at time \( t \), and \( p^*_t \) be the optimal logged price for a firm at time \( t \). The following proposition describes the evolution of logged aggregate prices.

**Proposition 4.3**

The evolution of optimal prices toward a long-run steady-state is described by the following equation:

\[
\pi_t = (1 - \vartheta) \left[ \gamma(p^*_t - p_{t-1}) + (1 - \gamma) \sum_{i=0}^{t-1} (p^*_i - p_{i-1}) \right]
\]

**Proof:**

Rewrite the price index as \( \Pi_t^{1-\epsilon} = F(X_0, X_1, ..., X_t) \) where \( \Pi_t = \frac{P_t}{P_{t-1}} \) and \( X_t = \frac{P^*_t}{P_{t-1}} \). Deriving the log-linear approximation around the steady-state is a widely used
technique. It is easy to demonstrate that the log-linear approximation around inflation rate $z$ is

\[(1 - \epsilon)z^{1-\epsilon}\pi_t = (1 - \epsilon)\gamma (1 - \vartheta)X_t^{1-\epsilon}(p_t^* - p_{t-1})\]

\[-(1 - \gamma) \sum_{i=0}^{t-1} (1 - \epsilon)h_{t-1}(P_i^*)H_{t-1}(P_i^*)X_t^{1-\epsilon}(p_i^* - p_{t-1})\]

The distributions are not differentiable (in the equilibrium of interest) and are left undifferentiated. Evaluating at the zero-inflation steady-state involves setting $z = 1$ also implies $X_i = 1$ for all $i$ and $h_{t-1}(P_i^*) = H_{t-1}(P_i^*) = 1$ for all $i$.

The result follows.

\[\Box\]

The proposition provides a few insights. First, past prices enter the equation directly. How the sequence of optimal prices compares to $t - 1$ aggregate prices influences current inflation. Second, the evolution of inflation does not depend on the distribution of prices directly (only through the aggregate prices). This is entirely due the the log-linear approximation and should make future numerical work much simpler.

Finally, though past inflation does not enter the equation directly for arbitrary $t$, something can be said about the inflation over time. The following corollary states the inflation equation for the first two periods.

**Corollary 4.3**

$\pi_t$ for $t \in \{1, 2\}$ are described by:
\[ \pi_1 = (1 - \vartheta)\gamma(p_1^* - p_0) \]  
\[ \pi_2 = (1 - \vartheta)\gamma(p_2^* - p_1) + \frac{(1 - \gamma)(1 - 2\gamma(1 - \vartheta))}{\gamma} \pi_1 \] 

The proof follows from direct calculation and is omitted. A clear implication of the result is that if there is “enough” imitation then second period inflation depends positively on past inflation. In particular, if \( \gamma < \frac{1}{2(1 - \vartheta)} \) then the relationship will be positive.

4.5 Equilibrium

To discuss the full effect of imitation we must solve for the monopolistic competition equilibrium. While the equilibrium is not solved explicitly here, it is characterized as a fixed point. I begin examining the firm’s optimization problem, and comparing my approach to Galí (2008).

In the Calvo pricing case, a typical firm faces the following pricing problem:

\[ \max_{P_t^*} \sum_{k=t}^{\infty} \vartheta^k Q_{t,t+k}(P_t^*Y_{t+k,t} - \Psi_t(Y_{t+k,t})) \] 

where \( Q_{t,t+k} \equiv \beta^k(C_{t+k}/C_t)^{-\sigma} P_t/P_{t+k} \) is the stochastic discount factor for nominal payoffs, \( \Psi_t(\cdot) \) is the cost function, \( \vartheta \) is the probability for a firm of not being able to re-optimize next period, \( P_t^* \) is the price set by a firm at time \( t \) and \( P_t \) is the price.
level (aggregate price index) at time $t$. $Y_{t+k,t}$ is the output in period $t+k$ of a firm that last reset its price in period $t$.

Every firm adjusting its price in period $t$ will choose the same price, $P_t(i) = P_t^* \forall i$, so indexation is dropped. The first order condition is:

$$\sum_{k=t}^{\infty} \vartheta^k Q_{t,t+k}(P_k^* - M\psi) = 0 \quad (4.22)$$

where $\psi \equiv \Psi'(Y_{j,t+k})$ is the marginal cost in nominal term and $M \equiv \frac{\epsilon}{1-\epsilon}$. Let $MC_{t+k}$ be the real marginal cost at time $t+k$. Galí (2008) demonstrates that the log-linearization of the first-order condition can be written as:

$$p_t^* = m + (1 - \beta \vartheta) \sum_{k=t}^{\infty} (\vartheta \beta)^k (mc_{t+k,t} + p_{t+k}) \quad (4.23)$$

With imitation the pricing problem of the firms is slightly different from Calvo’s. Firms that can optimize must take into account the impact of their price on the probability of imitating into a better price in the future. Recall $\nu_t(P_t(i))$ as defined in Section 4.3. Those firms are now facing the following maximization problem:

$$\max_{P_t} \sum_{k=t}^{\infty} \prod_{l=0}^{k} (1 - \nu_t(P_t^*)) Q_{t,t+k}(P_t^*Y_{t+k,t} - \Psi_t(Y_{t+k,t})) \quad (4.24)$$

where $\prod_{l=0}^{k} (1 - \nu_t(P_t(i)))$ is the probability that a firm with price $P_t(i)$ will not be able to change its price at $t+k$. 
In general, the first order conditions will be different under imitation and difficult to solve. However, we are interested in a special case. We care about the optimization when all firms begin at the same initial price and move towards a long-run steady-state. In this case the distribution of prices will be discrete in equilibrium.

The key to our next proposition is that, in the special case of transitory dynamics, the firm can ignore the impact of their prices on the probability of imitating in the future. The idea is that when optimal prices increase in average prices then a firm will never find it optimal to “jump” a future optimal price.

Proposition 4.4

Suppose that $\frac{\partial P^*_t}{\partial P_t} > 0$. The optimal price characterized by

$$P^*_t = \frac{\epsilon}{\epsilon - 1} \sum_{j=0}^{+\infty} \frac{\Pi_{k=0}^j(1 - \nu_k(P^*_t)))Q_{t,t+j}Y_{t+j}P^{\epsilon-1}_{t+j}}{\sum_{h=0}^{+\infty} \Pi_{l=0}^h(1 - \nu_l(P^*_t)))Q_{t,t+h}Y_{t+h}P^{\epsilon-1}_{t+h}}$$

Proof:

The proof involves demonstrating that the first order condition of the maximization problem is essentially unchanged (except for the probabilities). It must be shown that the influence of a firm’s price on the probability of imitating is ignored because it will never increase profits.

Consider the sequence of optimal prices $\{P^*_i\}_{i=0,1,...}$ and suppose that the optimal price increases in the average price. Or

$$\frac{\partial P^*_t}{\partial P_t} \geq 0$$
∀t. Then for \( P_0 < P^*_\infty \) the sequence of prices is always increasing, or \( P^*_t < P^*_t+1 \) as average prices continue to increase.

The probability of a firm imitating is unchanged for \( p^*_t \in [P^*_t-1, P^*_t+1] \). If \( P^*_t \) is outside this interval then either \( p_t \) is not monotonic in \( t \) or \( P^*_t+1 \) (or \( P^*_t-1 \)) cannot be optimal, which is a contradiction.

Given that this is true the \( P^*_t \) is solved for, and it is increasing in \( P_t \).

The result essentially says that the optimal price under imitation is the optimal price under Calvo pricing, with the probabilities changing over time according to the probability of imitating. The significance of the result is that the firm’s decision is not affected by its ability to change the probability of imitation.

### 4.5.1 Imitation and Networks

An implicit assumption in the analysis thus far is that firms who imitate only have information about a small number of firms (one in this case). For sticky inflation to be significant then firms must be limited to having local information about the history of prices and profits. Conversely, if a firm can observe all strategies and histories in the population then imitation is equivalent to (one period lagged) optimization.

One way to view matching in this model is as a random network with a particular degree distribution being generated every period. In this section I generalize the matching process to allow each firm to observe the history of \( d \) other firms. This is a random \( d \)-regular network, which is a special case of the Configuration Model. See Newman (2010) and Jackson (2008) for an exposition of the Configuration Model.

Allowing firms to observe \( d \) other firms’ information gives a new probability of
Proposition 4.5

Let $\gamma < 1$ and $\nu_j(P_t(i))$ be the probability that a firm that last set its price at $t$ to price $P_t(i)$ will change its price at $t + j$ for $j \geq 1$.

(i) $\nu_j(P_t(i)) = (1 - \vartheta)(\gamma + (1 - \gamma)(1 - H_{t+j-1}(P_t)^d))$ for $t \geq 1$.

(ii) $\frac{\partial \nu_j(P_t(i))}{\partial d} \geq 0$

(iii) $\lim_{d \to +\infty} \nu_j(P_t(i)) = 1 - \vartheta$

Proposition 4.5 states that the probability of a firm changing its price is increasing in the number of firms it matches with. Furthermore, as that number gets large the probability of changing one’s price approaches the Calvo pricing probability. The only deviation from Calvo pricing is that imitating firms optimize with a one-period lag.

4.6 Discussion

I have identified a mechanism that generates sticky inflation by generalizing a commonly used sticky price model. I demonstrate the performance of the mechanism analytically and numerical.

The model leaves open several avenues for future research. First, the model is not stochastic. Future models should allow changes in the parameters (such as money supply or productivity) to vary stochastically. The effects of imitation on the steady-state distribution of prices is left for future research.
Second, the model has no role for menu costs. Namely, a firm may decide whether to imitate versus optimize based on the expected value of observing the true state of the world. A state-dependant pricing model with imitation is also left for future research.
CHAPTER 4. IMITATION AND PRICE DYNAMICS

4.7 Figures

Figure 4.1: Prices with Imitation

Description: An example of average prices with and without imitation.
Chapter 5

Conclusion

In conclusion, this thesis explored the importance of networks effects in the macroeconomy. Chapter 2 discusses the role of social networks on the probability of acquiring a job, as well as the incentive to exert effort in job search. Chapter 3 discusses the unemployment volatility puzzle and the shifting of the Beveridge curve. The importance of social networks for these issues is assessed. Both chapters conclude that networks are theoretically important. A numerical simulation in Chapter 3 suggests that network effects cannot wholly explain the unemployment volatility puzzle.

Chapter 4 investigates the role of imitation on inflation. It is argued that imitation is a mechanism for sticky inflation. The result relies on firms having information on a finite number of firms, which can be interpreted as a network effect.

Future work will involve testing empirically the theoretical conclusions in Chapter 2 and Chapter 3. Namely whether networks can explain shifts in the Beveridge curve and whether networks are important for search effort.

The results of Chapter 4 can be built upon along several dimensions. First, numerical simulations may be used to generalize the results of the chapter. Second,
the inclusion of monetary policy shocks (or productivity shocks) is of interest. However, expanding the analysis to a stochastic environment produces many technical issues that must be overcome. Third, instead of generalizing Calvo pricing I plan to generalize state-dependant pricing with imitation.

Finally, empirical work can be done along two paths. The first is in the spirit of Christiano, Eichenbaum, and Evans (1998). Can imitation generate impulse response functions in line with the data? The second is to measure the behaviour of firms directly. Do firms imitate enough to explain sticky inflation?

All of these questions are left for future research.
Bibliography


Appendix A

Mathematical Appendix

This appendix contains proofs omitted from Chapter 3, and details on the numerical simulations from Chapter 3.

A.1 Proofs

A.1.1 Proof of Proposition 3.2

The proof has two steps. First, the equilibrium, described Definition 3.1, is shown to be reducible to two equations. Second, given these equations (displayed as (A.4) and (A.5) below) I can calculate $\epsilon_{u,\tau-b}$ and $\epsilon_{\theta,\tau-b}$. The proof is done for $\frac{\mu_C}{\mu_R} = 1$, though the results are not sensitive to the restriction.

Step 1: The FNE conditions establish several equations. Substituting the steady-state $E$-degree, denoted $d^{E*}$, into the matching function reduces the equilibrium conditions to the following equations. Note that in equilibrium each agent with degree $d$ will have the same $d^{E*}$. I will suppress the * notation. The remaining conditions are
APPENDIX A. MATHEMATICAL APPENDIX

\[ 0 = \lambda(1 - u) - \frac{\theta}{1 + \theta}(\mu_R + \mu_N(1 - u)d)u \]  \hspace{1cm} (A.1)

\[ 0 = (1 - \beta)(W_W(d) - W_U(d)) - \beta(W_J(d) - W_V) \]  \hspace{1cm} (A.2)

\[ 0 = \tau - w(d) + \frac{\tau c(r + \lambda)(1 + \theta)}{\mu_R + \mu_V(1 - u)d} \]  \hspace{1cm} (A.3)

The equation (A.1) is the steady-state imposed on (3.6), with \( d^{Ex} \) substituted in, (A.2) is the (rearranged) first-order condition from Nash bargaining given by (iii) in Definition 3.1, and (A.3) is the result of imposing the free-entry condition (stated as (iv) from Definition 3.1) on the equations (3.19)-(3.22).

The above equation are analogous to the equations of Pissarides (2000). Equation (A.1) imposes steady-state unemployment, equation (A.2) is the first-order condition for Nash bargaining, and equation (A.3) is the job-creation equation. Notice the equations that depend on \( d \).

\[ 0 = \lambda(1 - u) - \frac{\theta}{\mu_R + \theta}(\mu_R + \mu_N(1 - u)d)u \]  \hspace{1cm} (A.4)

\[ \text{1In the case of a general degree distribution with} \ n \ \text{types, there is a system of} \ 2n + 1 \ \text{equations, because (A.3) just depends on the average wage.} \]
\[
\frac{(1 - \beta)(\tau - b)}{c} = (r + \lambda)\frac{(\mu_R + \theta)}{\mu_R + \mu_N(1 - u)d} + \beta \theta \quad (A.5)
\]

**Step 2:** Let \( \eta_\theta \equiv \frac{\partial m_u}{\partial \theta} \frac{\theta}{m_u} \) and \( \eta_u \equiv \frac{\partial m_u}{\partial u} \frac{u}{m_u} \) be elasticities. Notice that \( \frac{\mu_u}{\mu_R} \) constant implies that \( \eta_\theta \) does not vary with \( \mu_N \) or \( \mu_R \).

Taking the total derivative of the system one can solve for the elasticities. Let \( \eta_x \) be the elasticity of \( m_u \) with respect to \( x \).

\[
\begin{align*}
\epsilon_{u,\tau-b} &= \frac{-(1 - \beta)(\tau - b)\eta_\theta m_u}{c\theta[(m_u + \lambda)(r + \lambda)(1 - \eta_\theta) + \beta) + \eta_u m_u((r + \lambda)(1 - 2\eta_\theta) + \beta)]} \quad (A.6) \\
\epsilon_{\theta,\tau-b} &= \frac{(\lambda + m_u(1 + \eta_\theta))}{\eta_\theta m_u} |\epsilon_{u,\tau-b}| \quad (A.7)
\end{align*}
\]

Increasing \( \mu_N \) from zero to a positive number increases \( \eta_u \) from zero to a positive number, keeping \( (\mu_R + \mu_N(1 - u)d) \) constant. In this case the absolute value of \( (A.6) \) increases as proposed.

The absolute value of \( (A.7) \) changes in an ambiguous way.

\[\Box\]

### A.1.2 Proof of Proposition 3.3

The proof involves keeping the job creation equation and wage equation from the previous proof and taking the time derivative of \( \theta \). The idea is that the network effect, which shows up in the matching function, will create persistence in changes in \( \tau \). Totally differentiating the job creation equation gives
\[ \left( \frac{(r + \lambda)}{\mu_R + \mu_N d^E} + \beta \right) \frac{\partial \theta}{\partial t} = \frac{(r + \lambda)(\mu_R + \theta)}{(\mu_R + \mu_N d^E)^2} \frac{\partial \mu_N d^E}{\partial t} + \Xi \]  

(A.8)

The \( \Xi \) is included because the non-steady-state decision problem depends on the change in \( d \) and \( u \). If the network effect is zero or the system is in steady-state then \( \Xi = 0 \), and when network effects are present its value is irrelevant. It is clear that \( \frac{\partial \theta}{\partial t} \neq 0 \) requires a network effect \( \mu_N > 0 \). Being outside of the steady-state requires that \( E[\mu_N \frac{\partial d^E}{\partial t}] \) is positive.

\[ \square \]

### A.1.3 Proof of Proposition 3.4

To show that the average wage is higher for referred agents I (Step 1) show that wages are higher for those of higher \( E \)-degree, and (Step 2) show the degree distribution conditional on being hired through network search first order stochastically dominates (FOSD) the degree distribution conditional on being hired through random search.

**Step 1:** A property of Nash bargaining is that those with a higher outside option get a higher share of the surplus. In FNE, the outside option of a worker of \( E \)-degree \( d_i^E \) is described by the value functions

\[ rW_U(d_i^E) = b + m_u(d^E)(W_W(d_i^E) - W_U(d_i^E)) \]

and

\[ rW_W(d_i^E) = w(d_i^E) + \lambda(W_U(d_i^E) - W_W(d_i^E)) \]
Furthermore, in FNE all workers of the same $d$ have degree $d^E_i$ (i.e. a one to one mapping). The matching rate $m_u(d^E_i)$ is higher for those of higher degree and thus the wage is higher for those of higher degree.

**Step 2:** Now it must be demonstrated that the conditional distribution of a network-hired person, denoted $p(d|N)$, first-order stochastically dominates that of a randomly hired person, $p(d|R)$. The trick is to find the probability of being hired through a channel $H \in \{R, N\}$ given degree, denoted $p(H|d)$. Knowledge of the degree distribution gives $p(d)$ and $p(H) = \sum_d p(H|d)p(d)$. We apply Bayes’ rule to obtain $p(d|H)$ and can verify $p(d|N)$ first-order stochastically dominates $p(d|R)$.

Recall that the matching function for workers with degree $d$ (or number of matches per unemployed worker with degree $d$) can be decomposed into $\mu_R \xi$ and $\mu_N (1-u)d$. Because the population is large these give the number of workers of degree $d$ matched randomly and through the network respectively. The telephone-line queuing process endogenizes $\xi$ making it depend only on aggregate variables (independent of $d$), and thus taken as a constant.

The expressions are:

$$p(R|d) = \frac{\mu_R \xi}{\mu_R \xi + \mu_N (1-u)d} \quad (A.9)$$

$$p(N|d) = \frac{\mu_N (1-u)d}{\mu_R \xi + \mu_N (1-u)d} \quad (A.10)$$

Applying Bayes’ rule gives

$$p(d|N) = \frac{f(d)(1-u(d))\frac{\mu_N (1-u)d}{\mu_R \xi + \mu_N (1-u)d}}{\sum_k f(k)(1-u(k))\frac{\mu_N (1-u)k}{\mu_R \xi + \mu_N (1-u)k}}$$
and
\[ p(d|R) = \frac{f(d)(1 - u(d))\mu_R \xi \mu_N(1-u)d}{\sum_k f(k)(1 - u(k))\mu_R \xi \mu_N(1-u)k} \]

To show that the top FOSD the bottom we calculate the distribution of each, denoted \( P(x|H) \), by summing over \( d \) from 0 to \( x \geq 0 \).

\[
P(x|R) = \sum_{d=0}^{x} p(d|R) \quad (A.11)
\]

\[
P(x|N) = \sum_{d=0}^{x} p(d|N) \quad (A.12)
\]

It is easy to verify that \( (A.11) \geq (A.12) \) for all \( x > 0 \) and therefore \( (A.12) \) FOSD \( (A.11) \). Because \( w(d) \) is weakly increasing in degree and by definition of FOSD it must be the case the the conditional average is weakly larger. Furthermore, \( (A.11) = (A.12) \) when \( d_i = d \) for all \( i \).

\[\square\]

### A.1.4 Proof of Proposition 3.5

The set of absorbing states of \( d^E_i \) require equations (3.24)-(3.27) to be 0. By definition of equilibrium the distribution of \( E \)-degree distribution must not be changing. If \((ii)\) holds (and I show it does in a moment) it is clear that any distribution with \( supp(d^E) \cap \phi(d^E)^C \) cannot be an equilibrium as all agents are moving toward the same absorbing state.

If \((ii)\) did not hold then which steady-state a worker gravitates to depends on employment status.
To prove (ii) examine equations (3.24)-(3.27) while imposing $\hat{h} = 1$.

$$\dot{d}^E(d^E, d^U, k) = -\delta d^E(d^E, d^U, k) + \rho g(d^E)(1 - u(k))$$  \hspace{1cm} (A.13)

$$\quad + m_u(u, v, k)d^U(d^E, d^U, k) - \lambda d^E(d^E, d^U, k)$$

$$\dot{d}^U(d^E, d^U, k) = -\delta d^U(d^E, d^U, k) + \rho g(d^U)u(k)$$  \hspace{1cm} (A.14)

$$\quad - m_u(u, v, k)d^U(d^E, d^U, k) + \lambda d^E(d^E, d^U, k)$$

$$\dot{D}^E(D^E, D^U, k) = -\delta D^E(D^E, D^U, k) + \rho g(D^E)(1 - u(k))$$  \hspace{1cm} (A.15)

$$\quad + m_u(u, v, k)D^U(D^E, D^U, k) - \lambda D^E(D^E, D^U, k)$$

$$\dot{D}^U(D^E, D^U, k) = -\delta D^U(D^E, D^U, k) + \rho g(D^U)u(k)$$  \hspace{1cm} (A.16)

$$\quad - m_u(u, v, k)D^U(D^E, D^U, k) + \lambda D^E(D^E, D^U, k)$$

It is clear that (A.13)-(A.14) and (A.15)-(A.16) are identical systems. Therefore, the solutions must be equal implying the set of absorbing states is equal.

Finally, (iii) is proven by the example in the text. \qed
A.1.5 Proof of Proposition 3.6

In a regular steady-state, unemployment and networks are described by:

\[ 0 = -\lambda (1 - u) + \frac{\mu v \rho (\mu_R + \mu_N d^E)}{\mu_R u + \mu_N v} \]  
(A.17)

\[ 0 = -\delta d^E + \rho g d^E (1 - u) + \frac{\mu v \rho (\mu_R + \mu_N d^E)}{\mu_R u + \mu_N v} d^U - \lambda d^E \]  
(A.18)

\[ 0 = -\delta d^U + \rho g d^U u - \frac{\mu v \rho (\mu_R + \mu_N d^E)}{\mu_R u + \mu_N v} d^U + \lambda d^E \]  
(A.19)

It is not difficult to collapse these into two equation relating \( u \) to \( d^E \) and \( d^U \):

\[ u = \frac{\lambda (\mu_R u + \mu_N v)}{\lambda (\mu_R u + \mu_N v) + (\mu_R + \mu_N d^E)/(\mu_R u + \mu_N v)} \]  
(A.20)

\[ u = \frac{\delta (d^E + d^U) - \rho g (d^U)}{\rho (g(d^E) - g(d^U))} \]  
(A.21)

The equation (A.20) is the usual steady-state unemployment equation, expressed as a fixed point. Equation (A.21) is the result of manipulating in the mean-field approximation equations for \( d^E \) and \( d^U \). Note that the \( D^E \) and \( D^U \) equations are
irrelevant for the case of $h = 1$.

These equations can be collapsed to a final equation:

$$u = \frac{\mu_v v}{\mu_R \lambda} \left( \frac{\rho (g(d^E) - g(d^U)) (\mu_R + \mu_N d^E)}{\rho g(d^E) - \delta (d^E + d^U)} - \lambda \right)$$  \hspace{1cm} (A.22)

Equation (A.22) gives a relationship between steady-state $u$ and steady-state $d^E$ and $d^U$, taking $v$ as exogenous. It is easy to verify that the equation is positive if $\lambda$ is sufficiently small and $0 < u \rho (g(d^E) - g(d^U))$ and small.

It is easy to verify that $\frac{\partial u}{\partial d^E} < 0$ when $\rho g'(d^E) > \delta$, which is permitted. Also, $\frac{\partial u}{\partial d^U} < 0$.

Therefore, $u$ and $(d^E, d^U)$ are negatively related in steady-state, and a larger steady-state $(d^E, d^U)$ gives a smaller steady-state $u$ (keeping $v$ constant).

A.1.6  Proof of Proposition 3.7

$(i) \Rightarrow (ii)$

Suppose $I_L \cap I_H \neq \emptyset$. Then all workers must eventually enter this interval. Those employed with $D_i^E > D^E*$ will, with probability 1, become unemployed and enter the interval while those unemployed with $d_i^E < d^E*$ will, with probability 1, become employed and enter the interval. Once $d_i^E, D_j^E \in I_L \cap I_H$ for all $i$ and $j$ then there is a steady-state distribution that is unique.

$(ii) \Rightarrow (i)$

Suppose $I_L \cap I_H = \emptyset$. Then all agents will end up in these intervals, with probability 1. Once in the intervals they will never leave. For instance, even if unemployed
Agents will end up in disconnected intervals and the steady-state distributions within each interval will exist, but the number of agents in each intervals depends on initial conditions. Therefore, multiple equilibria exist corresponding to how many agents begin in each basin of attraction.

$$(iii) \Leftrightarrow (ii)$$

It is straightforward to verify that when $h = 1$ then $I_L \cap I_H = \emptyset$. Similarly, when $h = 0$ or $h \to +\infty$ we have $I_L \cap I_H \neq \emptyset$. Because the correspondence of steady-states is continuous in $h$ at $h = 1$ the result follows.

For each step the same logic applies to the distributions of $d^U_i$ and $D^U_i$.

\[\square\]

### A.2 Numerical Simulations

#### A.2.1 Discussion

Chapter 3 presents numerical results. In this appendix, I describe the differences between my code and the “standard” code used for simulating search models.


Unemployment and $d^E$ follow their respective laws of motion, given an initial value. To ensure that the network effect is not filtered out by the H-P filter, I generate data 15 times more frequent than quarterly. One can interpret this as having friends
communicate roughly every 6 days.

Solving for $\theta$ is done using equation (A.5). This involves solving a non-linear system of equations; for every $p$ there is a corresponding $\theta$, all else equal. My model and Shimer (2005) differ in an important respect: in my model, all else is not equal. Namely, Shimer (2005) need only solve this system once, whereas I must solve the system each time that $d^E$ changes. Therefore, my model is more computationally intensive.

Once the simulations are complete, one needs to de-trend the data using an H-P filter. I use code from Ivorski to de-trend the simulation data. Standard deviations are calculated as deviation from this trend.

Each simulation has a burn period of 500 units of time (where a unit is roughly 6 days). Each simulation is run 1000 times to generate empirical distributions of the moments of interest.

**A.2.2 Code**

The following is the code for one MATLAB simulation of the model. The values of $\mu_V$, $\mu_R$, and $\mu_N$ are left empty.

**Main Code**

The following is the code from the simulation.

```octave
div=15;
n=500 + 212*div;
h=80;
delta=0.0003;
```
\begin{verbatim}
omega = 4 / div;
R = poissrnd(omega, n, 1);
u = zeros(n, 1);
v = zeros(n, 1);
m = zeros(n, 1);
d = zeros(n, 1);
a = 0.72;
lambda = 0.09 / div;
c = 0.2;
r = 0.012 / div;
b = 0.4 / div;
muR =;
muV =;
muN =
wh = zeros(n, 1);
theta = zeros(n, 1);
p = zeros(n, 1);
y = zeros(n, 1);
k = 1;
deg = 100;
d(1) = 94;
u(1) = 0.05;
theta(1) = 1;
s1 = h + 1;
\end{verbatim}
while k < n+1

d(k+1) = d(k) + ((\mu_R + \mu_N d(k)/(\text{deg-d}(k))) \cdot (\text{deg-d}(k)) \cdot \mu_V \cdot \theta(k)

(\mu_R + \mu_V \cdot \theta(k)) - \lambda \cdot d(k));

c1 = d(k)/(\text{deg-d}(k));

x0 = ones(2*h+1,1);

f = @(w)gun3(w,c1);

[w,fval] = fsolve(f,x0);

if R(k) > 0

j = 1;

x = zeros(R(k),1);

s = zeros(R(k),1);

x(1) = p(k);

z = rand(R(k),1);

while j < R(k)+1

if z(j) < 0.5*(1-x(j)/(h*delta))

s(1) = s1;

x(j+1) = x(j) + delta;

x1 = x(j+1);

s(j+1) = s(j) - 1;

s1 = s(j+1);

else

s(1) = s1;

x(j+1) = x(j) - delta;

x1 = x(j+1);

end

end
\[ s(j+1) = s(j) + 1; \]
\[ s1 = s(j+1); \]
end
\[ wh(j) = s1; \]
\[ j = j + 1; \]
end
\[ p(k+1) = x1; \]
\[ \theta(k+1) = w(s1); \]
else
\[ p(k+1) = p(k); \]
\[ \theta(k+1) = \theta(k); \]
end
\[ u(k+1) = u(k) + \lambda(1-u(k)) - u(k)(\mu_R + \mu_N \cdot d(k)/(\deg-d(k))) \cdot \\
(\mu_V \cdot \theta(k))/(\mu_R + \mu_V \cdot \theta(k)) \cdot \text{div}; \]
\[ \text{if } u(k+1) < 0; \]
\[ u(k+1) = 0; \]
\[ \text{else} \]
\[ u(k+1) = u(k+1); \]
end
\[ k = k + 1; \text{ end} \]
\[ l = 2; \]
while \( l < n + 1 \)
\[ v(l) = u(l) \cdot \theta(l); \]
\[ \text{if } v(l) < 0 \]
v(l)=0;
else
v(l)=v(l);
end

m(l)=(muR+muN*d(l)/(deg-d(l)))*muV*theta(l)/((muR+muV*theta(l))*div);
if m(l)<0
m(l)=0;
else
m(l) = m(l);
end
l=l+1;
end
q=1;
lu = zeros(212,1);
ld = zeros(212,1);
ltheta = zeros(212,1);
lp = zeros(212,1);
lm = zeros(212,1);
lv = zeros(212,1);
lagy=zeros(212,1);
lagu=zeros(212,1);
lagtheta=zeros(212,1);
lagm=zeros(212,1);
lagp=zeros(212,1);
lagd=zeros(212,1);
while q<213
lu(q)=u(q*div+n-212*div);
lagu(q) = u(q*div+n-213*div);
ld(q)=d(q*div+n-212*div);
lagd(q) = d(q*div+n-213*div);
ltheta(q)=theta(q*div+n-212*div);
lagtheta(q) = theta(q*div+n-213*div);
lp(q)=p(q*div+n-212*div);
lagp(q) = p(q*div+n-213*div);
lm(q)=div*m(q*div+n-212*div);
lagm(q) = div*m(q*div+n-213*div);
lv(q)=u(q*div+n-213*div)*theta(q*div+n-212*div);
lagv(q) = u(q*div+n-214*div)*theta(q*div+n-213*div);
q=q+1;
end
para = 10^-5;
@(s)HP(lu,w);
devu=log(lu)-log(HP(lu,para));
stdu= sqrt(sum(devu.^2));
@(s)HP(lv,w);
devv=log(lv)-log(HP(lv,para));
stdv= sqrt(sum(devv.^2));
@(s)HP(lagv,w);
devlagv=lagv-HP(lagv,para);
stdlagv= sqrt(sum(devlagv.^2));
@(s)HP(ltheta,w);
devtheta=log(ltheta)-log(HP(ltheta,para));
stdtheta= sqrt(sum(devtheta.^2));
@(s)HP(lp,w);
devp=lp-HP(lp,para);
stdp= sqrt(sum(devp.^2));
@(s)HP(ld,w);
devd=log(ld)-log(HP(ld,para));
stdd= sqrt(sum(devd.^2));
@(s)HP(lm,w);
devm=log(lm)-log(HP(lm,para));
stdm= sqrt(sum(devm.^2));

Function Code
The following is the function code for gun3. The HP function code is from Ivorski
and is omitted.

function F = gun3(w,c1)
div=15;
a=0.72;
r=0.012/div;
lambda=0.09/div;
omega1=3.9/div;
\[
\begin{align*}
\mu_R &=; \\
\mu_V &=; \\
\mu_N &=; \\
b &= 0.4/\text{div}; \\
h &= 80; \\
delta &= 0.0003; \\
c &= 0.2; \\
g &= 1; \\
I &= \text{eye}(2\cdot h+2); \\
\text{Il} &= I(2:2\cdot h+2,1:2\cdot h+1); \\
\text{Ih} &= I(1:2\cdot h+1,2:2\cdot h+2); \\
\text{prod} &= -\delta h: \delta: \delta h; \\
pr &= -\text{transpose}(\text{prod}); \\
D &= \text{diag}(pr); \\
F &= (r+\lambda+\omega_1)(\mu_Vw + \mu_R\text{ones}(2\cdot h+1,1)) \\
     / (\mu_V(\mu_R+\mu_Nc_1)) \\
     + a\cdot w - (1-a)\cdot (1-b)\cdot \exp(g\cdot pr)/c - \\
(\omega_1/2)\cdot ((\text{eye}(2\cdot h+1)-D/(h\cdot \delta))\cdot \\
(\text{Il}*(\mu_Vw+\mu_R\text{ones}(2\cdot h+1,1)) + \\
(\text{eye}(2\cdot h+1)+D/(h\cdot \delta))\cdot \\
(\text{Ih}*(\mu_Vw+\mu_R\text{ones}(2\cdot h+1,1)))/(\mu_V(\mu_R+\mu_Nc_1));
\end{align*}
\]