TRANSLATING MESSAGES FROM CURRICULUM STATEMENTS INTO CLASSROOM PRACTICE: COMMUNICATION IN GRADE 9 APPLIED MATHEMATICS

by

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ABSTRACT

This qualitative case study describes how two teachers translate communication messages from curriculum statements into classroom practice. These illuminative cases illustrate the perspectives and practices of two teachers who support the spirit of the Ontario Mathematics Curriculum by working to implement communication in the Foundations of Mathematics (MFM1P), Grade 9, Applied course (Ontario Ministry of Education, 2005). Data for this study were collected from individual interviews with teachers, classroom observations, and document analysis.

Grade 9 Applied mathematics teachers across Ontario indicate on surveys that they support communication in the mathematics classroom (Education Quality and Accountability Office, 2007; Suurtamm & Graves, in press). Despite evidence of support for this aspect of the curriculum, findings from this study point to a need for finer analysis of teachers’ perspectives and practices when it comes to communication in mathematics. The cases presented in this thesis illustrate different images, or meanings, associated with communication in mathematics. Furthermore, even in unique cases where the gap between curriculum developers and teachers images is minimal, the idealized vision of communication may not be realized in classroom practice since teachers may face challenges in implementation. The teachers report that despite additional challenges involved with implementing communication in Grade 9 Applied mathematics, teaching in this context can be a rewarding experience.

To minimize the gap between images of communication that are translated from curriculum statements into classroom practice, findings from this study indicate that curriculum developers must find ways to help teachers understand the rationale behind curriculum initiatives. Future research might explore ways to help familiarize teachers with the theory and research underlying communication in mathematics. Research might also examine the impact that these initiatives have on teachers’ perspectives and practices.
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CHAPTER 1:
INTRODUCTION

Research Context

This thesis describes how two teachers translate communication messages from curriculum statements into classroom practice, in the context of the *Foundations of Mathematics, Grade 9, Applied* course. The focus here is on communication messages since communication, through active student participation in the classroom, is viewed by those promoting mathematics curriculum reform as a fundamental process in teaching and learning. The importance is recognized in reform movements around the world. For instance, according to Sfard, “today, rather than speaking about ‘acquisition of knowledge,’ many people prefer to view learning as *becoming a participant in a certain discourse*” [emphasis in original] (2000, p. 160). This perspective is prominent in reform movements advocated by the National Council of Teachers of Mathematics (NCTM, 2000), who consider communication to be an essential process for both mathematics and mathematics education.

In Ontario, mathematics curriculum policy (Ontario Ministry of Education, 2005a) and resource documents (Consortium of Ontario School Boards, 2001, 2003, 2005, 2007; Ontario Ministry of Education, 2000b, 2003, 2004) support the communication focus that is posited in research literature. Ontario teachers are encouraged to promote active student participation in a “math-talk learning community” by encouraging active student communication in instruction and assessment (Hufferd-Ackels, Fusun, & Sherin, 2004, p. 87; Ontario Ministry of Education, 2005a). Even though communication is promoted in the curriculum, research suggests that the degree of implementation of curriculum initiatives varies among teachers (Remillard, 2005). Some teachers reject and subvert recommendations while other teachers “wholeheartedly” embrace them (p. 212). This research describes the perspectives, practices, and experiences of
two teachers who wholeheartedly embrace the communication emphasis that is posited in Ontario Mathematics Curriculum documents.

This study was conducted in the context of the *Foundations of Mathematics, Grade 9, Applied* course in Ontario. In this province, mathematics courses in Grade 9 and 10 are offered in two types, academic and applied:

> Academic courses develop students’ knowledge and skills through the study of theory and abstract problems. These courses focus on the essential concepts of a subject and explore related concepts as well. They incorporate practical applications as appropriate.

> Applied courses focus on the essential concepts of a subject, and develop students’ knowledge and skills through practical applications and concrete examples. Familiar situations are used to illustrate ideas, and students are given more opportunities to experience hands-on applications of the concepts and theories they study. (Ontario Ministry of Education, 2005a, p. 6)

The Ontario Mathematics Curriculum for Grade’s 9 and 10 also highlights the importance of communication in mathematics instruction and assessment. In this document, communication is one of seven mathematical process expectations that “are to be integrated into student learning in all areas of this course” (Ontario Ministry of Education, 2005a, p. 38). Moreover, in *The Achievement Chart for Mathematics*, communication is also one of four categories “of knowledge and skills within which the expectations for any given mathematics course are organized,” assessed and evaluated (p. 18).

The *Foundations of Mathematics, Grade 9, Applied* course provides a rich context for this study. In addition to encouraging communication in curriculum documents at this level, a number of resource documents are available to assist teachers in implementing the Grade 9 Applied curriculum. These include resources such as *Think Literacy* documents (Ontario Ministry of Education, 2003, 2004), *Targeted Implementation and Planning Supports (TIPS)* (Consortium of Ontario School Boards, 2003), *Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM)* (Consortium of Ontario School Boards, 2005, 2007), *The Ontario Curriculum Exemplars* (Ontario Ministry of Education, 2000b), and *Course Profiles* (Consortium of Ontario School Boards, 2001). Although some of these documents were developed prior to
2005, when the revised curriculum was introduced, they were designed to support the 1999
Ontario Mathematics Curriculum, which also emphasized the importance of communication in
mathematics instruction and assessment.

The *Foundations of Mathematics, Grade 9, Applied* context is rich, not only because
there are a variety of resources available to support curriculum implementation, but also because
the dynamics of the Grade 9 Applied class create potential challenges for teachers who work to
support communication in this context. Although teachers may face challenges in implementing
communication in any mathematics course, the Grade 9 Applied course is a rich area for research
because students in this context have demonstrated low levels of academic achievement in
mathematics (Education Quality and Accountability Office [EQAO], 2007a; King, Warren,
Boyer, & Chin, 2005). Furthermore, responses to the EQAO student questionnaire suggest that
these students do not enjoy mathematics. Only 28% of female students and 40% of male students
who responded to the 2006-2007 survey indicate that they “like mathematics” (EQAO, 2007a, p.
69). Consequently, research in a Grade 9 Applied context is valuable for exploring teachers’
experiences with engaging students who struggle and do not enjoy mathematics in group work
and math-talk in the classroom. Grade 9 Applied mathematics teachers across Ontario also
indicate on provincial surveys (EQAO, 2007b; Suurtamm & Graves, in press) that they support
communication in mathematics instruction and assessment. This qualitative case study research
describes how two teachers, who would also indicate their support for this mathematical process,
translate communication messages from curriculum statements into classroom practice in this
context.

**Overview of Chapter One**

This chapter consists of five parts. In the first part, the purpose of the study is described.
Next, key terms are defined. In the third part, the conceptual framework that guides all aspects of
this research is presented. Subsequently, the significance of the study is explained. Finally, I outline the thesis structure with a brief overview of the chapters in this thesis.

**Purpose**

The purpose of this study is to describe how messages concerning communication are translated from mathematics curriculum statements into classroom practice. Specifically, this thesis describes the perspectives, plans, and practices of two *Foundations of Mathematics, Grade 9, Applied*, Ontario teachers who are working to support this mathematical process in their classrooms. The focus here is not to describe how Ontario teachers in general have accepted the communication emphasis that is mandated in curriculum policy documents, but is, instead, to describe the experiences of two teachers who are interested and engaged in developing “a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants” (Hufferd-Ackles, et al., 2004, p. 82).

**Key Terms**

For the purpose of this research, it is important to acknowledge a distinction between three levels of curriculum. These levels are defined by Travers and Westbury (1989), in an analysis of mathematics curricula for the International Studies in Educational Achievement, as the intended, implemented, and attained curriculum. At the level of the educational system, there is the intended curriculum which is the “collection of intended outcomes, together with course outlines, official syllabi, and textbooks” (Travers & Westbury, p. 6). At the classroom level, there is the implemented curriculum. It is at the classroom level that the implemented curriculum content is “translated into reality by the teacher” (Travers & Westbury, p. 6). The third level, the attained curriculum, describes knowledge, attitudes, and skills that students actually acquire from the first two levels. This thesis focuses on how communication messages are “translated into reality” by two teachers who support this mathematical process in classroom practice.
**Conceptual Framework**

All researchers approach a study with some ideas and foci that come together to form a conceptual framework (Miles & Huberman, 1984). The conceptual framework for this research consists of two parts. The first part is based on the focus of this study on the translation of messages from intended to implemented curriculum. The second part focuses on communication in mathematics.

**Gap Between Images of Curriculum Messages**

Since the main purpose of this study is to describe how curriculum statements are translated from intended to implemented curriculum, the notion of a “gap between images” in the teacher-curriculum relationship is an important part of the conceptual framework (MacDonald & Walker, 1976, p. 45). This concept was introduced by MacDonald and Walker and is based on the idea that the image, or vision, of a curriculum initiative that is idealized by developers is often different from the vision that is implemented in professional practice. MacDonald and Walker also explain that there may be a gap between teachers and academic theorists’ images of a curriculum innovation:

Interpreters of the R, D and D [Research Development and Diffusion] model have widely assumed that the gap between the intents of developers and the realisations of practitioners is explicable in terms of a process of adulteration which begins at the point of entry into the schools and is a result of institutional conservatism compounded by ineptness and indifferences on the part of the innovating teachers. Plus, of course, the failures in communication alluded to earlier. Dissemination followed by disintegration. These interpreters are typically academic theorists who have taken the trouble to familiarize themselves with the concepts, values and practices of an innovation. They have a picture of the curriculum which they then use to evaluate school practice. They assume that the picture they have acquired is the same picture that the practitioner acquired in the course of induction or training for the innovation. It is our purpose in this chapter to contend that in many cases there is a significant discrepancy between these two pictures, a discrepancy that has nothing to do with communication. (MacDonald & Walker, p. 46)

The notion of a gap between images is a central feature of the curriculum negotiation model that was developed by MacDonald and Walker (1976), extended by Pitman (1981), and further extended in application to the *Targeted Implementation and Planning Supports (TIPS)*
resource by Jarvis (2006). Curriculum negotiation is defined in this thesis, as it is by Jarvis, as “a complex process which involves the development, mediation, and implementation of a given curricular product” (p. 7). This process is “affected by many factors such as timing, flexibility, accountability, system scale, existing beliefs and attitudes among participants, support and training mechanisms, funding, and communication” (Jarvis, p. 7). The negotiation models describe differences between the idealized and implemented visions of a curriculum in terms of differences in motivations of members involved in the negotiation process.

MacDonald and Walker’s original curriculum negotiation model is based on theories related to “processes of social invention, transmission and implementation,” and three case studies that demonstrate how the cases complement the theories (1976, p. 2). These researchers suggest that curriculum is only “part of the package, the rest consists of … all the many human purposes that people prosecute with whatever resources come to hand” (MacDonald & Walker, p. 45). Curriculum developers may assume that their views of policy documents are the same as those held by practitioners; however, through the negotiation process, different images emerge not because of poor communication from developers to practitioners but because the two do not share the same “educational values and have overlapping visions of curriculum excellence” (MacDonald & Walker, p. 44).

Pitman (1981) extended the negotiation model during his research concerning the dissemination of secondary school science courses in Victoria, Australia and Hong Kong. In his model, Pitman recognizes the importance of negotiations with mediators and students. He agrees with MacDonald and Walker that curriculum implementation depends on teachers’ interpretations of initiatives; however, he suggests that the negotiations are not as simple as in the original model. He argues that students play an integral role because teachers will distort curriculum ideas based on their assumptions of what is best for students. Jarvis (2006) extended the negotiation model further by exploring the development of the TIPS document in Ontario. Through his observations of case studies, he found that, in this curricular context, the negotiation process is
not linear as is suggested in the previous models, but is cyclical at particular stages in the process. Although extensions in MacDonald and Walker’s model acknowledge different members in the process and the cyclical nature of negotiations at different stages, the main feature of these models—the gap between images of curriculum—remains and is the primary focus in the proposed research.

**Images and Purposes of Communication in Mathematics**

All aspects of this research are guided by the general notion of a gap between images of curriculum initiatives. The specific focus here, however, is on images of mathematics communication. The framework that guides this aspect of the study is provided in Ontario Mathematics Curriculum policy documents (Ontario Ministry of Education, 2005a) and in the math-talk learning community model that is outlined by Hufferd-Ackles et al. (2004).

In this research, I explore the relationship between images of communication that are presented in curriculum policy and resource documents and those held by *Foundations of Mathematics, Grade 9, Applied* teachers. These documents present particular ideas about what communication should look like in mathematics instruction and assessment. In the Ontario Mathematics Curriculum for grades one through twelve, communication is highlighted as one of seven mathematical processes expectations that should be addressed throughout each course (Ontario Ministry of Education, 2005a, 2005b, 2007). According to the communication expectation, during each mathematics course in the curriculum, students should “communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions” (Ontario Ministry of Education, 2005a). In assessment, according to the Ontario *Achievement Chart for Mathematics*, students should demonstrate their ability to communicate mathematically for different purposes and audiences, using various mathematical forms with appropriate terminology and conventions (Ontario Ministry of Education, 2005a).

According to the math-talk learning community framework developed by Hufferd-Ackles et al. (2004), the key components of a classroom that supports communication, or math-talk learning, are identified in terms of questioning, explaining mathematical thinking, taking responsibility for learning, and the source(s) of mathematical ideas. These aspects of the math-talk model are outlined in a research synopsis on the Ontario Ministry of Education website:

*Questioning* [emphasis in original] in an effective Math-talk Learning Community features a shift away from the teacher as questioner to students and teacher as co-questioners. In this community, students are encouraged to ask questions of their peers in order to understand one another’s thinking. *Explaining mathematical thinking* is closely related to, and an obvious product of, good questioning. Students are increasingly afforded the opportunity to articulate their ideas and new learning to the teacher and to each other within a supportive environment. In a Math-talk Learning Community students are able to explain, defend and justify their mathematical thinking with confidence. In a more traditional classroom the key source of mathematical ideas was often the teacher, solving problems in a procedural manner for students to then imitate. Whereas, in this environment, students as well as the teacher are seen as important sources of mathematical ideas. The mathematical “talk” often features the negotiation of student understanding of a given concept, and the ideas of students are considered as valid and worthy of further exploration. In the Math-talk Learning Community students increasingly take responsibility for their own learning and for the evaluation of others and self…. (Ontario Ministry of Education, 2008b, p. 25)

Images of communication that are outlined in Ontario Mathematics Curriculum policy and resource documents reflect a particular epistemological perspective on the nature of mathematical knowledge, where mathematical knowledge is “the specific subject matter of the teaching and learning processes in mathematics instruction” (Steinbring, 2005, p. 7). There are two opposing images of the nature of mathematics which influence different ideas about the
purposes of communication in the classroom. These two views and the purposes of communication that they support are an important part of this conceptual framework.

One perspective on the nature of mathematical knowledge is based on Ernest’s (1991) absolutist image of mathematics. From this perspective, communication is important for helping students absorb, or discover, the rules of the subject. This image acknowledges that mathematics is an objective, consistent and absolute body of knowledge (Ernest, 1991). Even though this is a common view (Steinbring, 2005), the vision for communication articulated in mathematics curriculum reform documents stems from an opposing, fallibilist, image (Ernest, 1991) of the subject. From this perspective, the purpose of communication is to facilitate students’ construction of knowledge. This view acknowledges that mathematics content is neither absolute nor complete but emerges in “the context of social construction and individual interpretation processes” (Ernest, 1991; Steinbring, 2005, p. 7). Communication is important not only for constructing knowledge that is shared in the classroom community, but also for supporting each individual student’s construction of his or her own understandings of mathematics. The two opposing views of mathematics are an important part of the framework because they influence different perspectives on the purposes of communication: (1) for absorbing knowledge, (2) for constructing knowledge.

Summary: A Framework for Exploring the Translation of Communication Messages

In summary, since the central purpose of this research is to describe the translation of communication messages from curriculum statements into classroom practice, all aspects of this study are guided by the notion of a gap between images of curriculum (MacDonald & Walker, 1974, Pitman, 1981, & Jarvis, 2006). The specific focus here, however, is on images of communication that are described by the Ontario Ministry of Education (2005a) and by Hufferd-Ackles et al. (2004). Since different perspectives on the purposes of communication are influenced by opposing views of the nature of mathematics (Ernest, 1991), these two perspectives also comprise an important part of the framework.
Significance of the Study

This area of research is significant and findings from this particular study are valuable for members of the mathematics education community. The argument for research in this area is based on a description of the context of the study, and is therefore provided following the review of relevant literature in the next chapter of this thesis. I argue that although there is an emphasis on communication in literature and in curriculum resources, there are variations in how curriculum messages are implemented in classroom practice. Therefore, it is important to explore the perspectives and experiences of teachers who are working to support curriculum initiatives.

Findings from this study are valuable for anyone who is interested in curriculum, particularly mathematics curriculum, because they provide insight into how curriculum statements are translated into classroom practice by teachers who are committed to implementing new program ideas. The findings provide us with a sense of different meanings that are associated with communication in mathematics. Findings may also be valuable for mathematics teachers since they provide some examples of how communication messages can be implemented in classroom practice.

Thesis Structure

This thesis consists of six chapters. In the first chapter, I have outlined the purpose, key terms, conceptual framework, and the significance of findings. In Chapter Two, the study is contextualized in a review of relevant literature. I discuss research into the teacher-curriculum relationship and classroom application of curriculum messages. I also describe the increased emphasis on communication in mathematics curriculum reform and explanations for supporting communication in the mathematics classroom. Following the description of relevant literature, I discuss the communication emphasis in Ontario and outline the specific communication expectations in mathematics curriculum policy and resource documents. Since the research questions addressed in this study are based on concepts explained in the literature review, these
questions are outlined in the final section of Chapter Two. My approach to addressing the research questions is described in Chapter Three, with an explanation of my choices for methods and the chronological stages of the research. In Chapter Four I describe the perspectives, plans, and practices of the two teachers who participated in this research. Major themes that emerged from these two cases are consolidated in Chapter Five, in a discussion on the translation of communication messages from curriculum policy into classroom practice. I also discuss and illustrate my own images of mathematics communication. In Chapter Six I report on the main conclusions of this thesis and the implications for practice and research.
CHAPTER 2: RELEVANT LITERATURE AND RESEARCH QUESTIONS

The description of relevant literature presented in this chapter expands on the conceptual framework for this study. I contextualize the curriculum and communication focus with a description of literature that discusses variations in classroom application of curriculum messages. I also illustrate images of communication in mathematics curriculum reform, the Ontario Mathematics Curriculum, and in *Foundations of Mathematics, Grade 9, Applied* context. I summarize this description with an explanation of the importance of this research. Since the research questions addressed in this thesis are based on concepts defined in the literature review, I conclude this chapter with an outline of these questions.

**Variations in Classroom Application of Curriculum Messages**

There are considerable variations in the degree to which teachers implement ideas that are presented in curriculum materials. In a review of literature related to the “teacher-curriculum” relationship, Remillard found that there have been “instances of teachers rejecting and subverting recommendations in their guides as well as instances of teachers wholeheartedly embracing them” (2005, p. 212). In Canada, O’Shea (2003) acknowledges that there have been a limited number of studies conducted to describe how curriculum objectives are realized in classrooms; however, from the few that exist, it is apparent that “in Ontario far fewer changes [have] occurred in classrooms than an examination of the official curricula … suggest” (O’Shea, 2003, p. 886). His observation demonstrates the importance of research that describes how curriculum objectives are implemented in classroom practice.

A variety of factors influence how curriculum initiatives are translated into classroom practice. For example, as Remillard recognizes, “the teacher-curriculum relationship is intertwined with other teaching practices, is dependent on the particular teacher and curriculum,
and is situated in a specified context” (2005, p. 212). Moreover, Chavez (2003) demonstrates the potential impact of discrepancies between images of curriculum. In his study, Chavez described how mathematics teachers used curriculum and resource materials in intermediate level classrooms in the United States. He found that teachers views of curriculum “and the match, or lack of it, between their own views about mathematics and mathematics teaching and the philosophy of the textbook—whether it is explicit or not—were the primary factors that determined how the textbook was used” (Chavez, p. 157).

Of course it is possible to explore how teachers translate messages from curriculum statements into classroom practice for any school subject, but mathematics provides a rich context for study. This argument is clearly demonstrated by Remillard (2005), who suggests that mathematics education is important to explore because teachers have tended to rely more heavily on curriculum materials in this subject than in others. Moreover, at the present time, these materials represent curriculum reform efforts, which call for significant changes in mathematics instruction. Contemporary reform ideas, which have had an apparent influence in revised Ontario mathematics curriculum documents (Ontario Ministry of Education, 2005a, 2005b, 2006), are described next.

**Contextualizing the Communication Emphasis**

Images of communication and the importance of this mathematical process are emphasized in mathematics curriculum reform. Below I describe the images of communication in reform initiatives advocated by the National Council of Teachers of Mathematics (NCTM). I also explain epistemological reasons and learning theory dimensions of communication in the mathematics classroom. Finally, I describe research studies that focus on communication in mathematics.
Mathematics is a challenging subject, so it is no surprise that “teaching mathematics has been, and will likely always be, subject to change and improvement” (Sfard, 2000, p. 157). Improving the quality of teaching and learning is a primary objective of mathematics education reform efforts. As Sfard recognizes, since the exact shape of reform varies from one country to another, “it is difficult to speak about the reform movement in general” (p. 179). Therefore, the specific focus here is on the NCTM Standards movement.

The NCTM is a professional organization of mathematics teachers from the United States and Canada (Lindquist, 2003). The NCTM reform movement began with the 1989 publication of the Curriculum and Evaluation Standards for School Mathematics. This standards-based reform initiative, which encourages active student participation in learning, continues to appear in the recently published Principles and Standards for School Mathematics (NCTM, 2000). A central feature of these initiatives is an emphasis on five mathematical “Process Standards—Problem Solving, Reasoning and Proof, Communication, Connections, and Representation—[to] highlight the ways of acquiring and using content knowledge” (NCTM, 2000, p. 29).

Even though the NCTM Standards are widely accepted, there is controversy surrounding this movement. Concerns are apparent at websites such as Mathematically Correct (http://www.mathematicallycorrect.com). Through this website, some members of the American mathematics education community express concerns about the so-called “Fuzzy Math.” Members criticize reform, stating that the initiatives lack rigour and do not present the fundamentals of mathematics. Despite criticisms, the standards movement has had a significant influence in the mathematics education community, particularly in Canada (Lindquist, 2003). This is clear in revised Ontario Mathematics Curriculum documents (Ontario Ministry of Education, 2005a, 2005b, 2007), where several mathematical processes are explicitly expected to be incorporated in instruction and assessment.
Even though all of the process standards are important, the focus here is on communication because a common feature in most curriculum reform efforts is that they “consistently call for balanced and student-centered communication in which students take an active role in classroom discourse” (Webb, Nemer, & Ing, 2006, p. 64). The significance of this mathematical process is demonstrated in the extensive body of literature dedicated to its study (Alrø & Skovsmose, 2002; Elliot & Kenney, 1996; Hufferd-Ackles, Fusun & Sherin, 2004; Pimm, 1987; Reinhart, 2000; Steinbring, Bussi, & Sierpinski, 1998).

Communication is emphasized in NCTM reform initiatives and in Ontario Mathematics Curriculum documents. Both the NCTM and the Ontario Ministry of Education recognize that communication is “essential” for learning mathematics (Ontario Ministry of Education, 2005a, p. 16; NCTM, 2000, p. 60). According to the NCTM’s communication standard, which consists of four categories, students should: (1) use communication to organize their thinking; (2) communicate for different audiences; (3) “analyze and evaluate the mathematical thinking and strategies of others;” and, (4) “use the language of mathematics to express mathematical ideas precisely” (2000, p. 60). Communication messages presented in revised Ontario Mathematics Curriculum documents (Ontario Ministry of Education, 2005a, 2005b, 2007) are similar to the NCTM standard. Before I outline the specific communication expectations for Ontario teachers, I discuss the theoretical explanations for communication in mathematics.

**Explanations for Communication in Mathematics Education**

Calls for communication are grounded in epistemological views of the nature of mathematics as a body of knowledge, and in psychological theories about learning. In this section, I explain how epistemology and learning theories influence calls for communication.

**Epistemological reasons.** Different images of the nature of mathematics as a body of knowledge influence contemporary views of communication in the mathematics classroom. Ernest (1991, 1998) uses the terms absolutist and fallibilist to acknowledge two core images of
mathematics. Roulet illustrates the distinction between these images by raising the following question:

Do we live in a world that is governed by fixed mathematical rules which, over the centuries, we have discovered and recorded as mathematical theorems, or is mathematics a human construct that we project onto our world whenever we find patterns of events that appear to support our creations? (1998, p. 21)

From the former, absolutist perspective, mathematics axioms are accepted as true and proof preserves truth; therefore, results that are derived in mathematical proofs must be true (Ernest, 1991). Thus, according to Ernest, the absolutist image of the nature of mathematics is that mathematical knowledge is certain and absolutely true. This position has been accepted for centuries; however, there are flaws in the assumptions of this perspective. For example, Euclid’s Elements (300 BC), which was widely accepted in mathematics education as a representation of mathematical truths, was challenged in the nineteenth century when alternative forms of one of the postulates, the parallel postulate, were realized and resulted in the expansion of geometrical concepts. In 1930, mathematician, Kurt Gödel also challenged the absolutist view of mathematics as a complete system of truth when he demonstrated that it is impossible to prove all mathematical truths (Ernest, 1991). This is a significant problem for the absolutist image of mathematics because it demonstrates the impossibility of establishing absolute certainty.

Challenges to the absolutist perspective led to an opposing, fallibilist image of the nature of mathematics. In its negative form, this new image rejects absolutism, and in its positive form it presents an image of mathematical truth as flawed and open to revision (Ernest, 1991). Unlike the absolutist view, which encourages the development of mathematical knowledge by discovering the static rules of the subject, from a fallibilist perspective, mathematics is a growing and changing social construct. This image of the nature of mathematics supports a social constructivist view of mathematics knowledge. From this perspective, interactive communication in the mathematics classroom is encouraged because according to Ernest, “interactions with other persons … especially through negative feedback, provides the means for developing a fit between
an individual's subjective knowledge of mathematics, and the socially accepted, objective mathematics” (1991, p. 81). This relationship between objective and subjective knowledge is a critical feature of social constructivism because “subjective knowledge re-creates objective knowledge, without the latter being reducible to the former” (Ernest, 1991, p. 83). Thus, since mathematical knowledge continues to evolve, it is not absolute so communication in a math-talk community is necessary for generating and warranting new knowledge.

Calls for student communication in mathematics are grounded in social constructivist images of mathematical knowledge creation. Despite the fact that an epistemological position is not explicitly recognized in reform and in Ontario Mathematics Curriculum documents, the image of the nature of mathematics as an evolving social construct is held by leaders of curriculum reform initiatives. For example, Thomas Romberg, a leader of the NCTM initiatives states that “[t]he term that we did not use in writing up the Standards (but we certainly talked about) is what might best be called the social constructivist's notion of learning” (McLeod, Stake, Schappelle, Mellissinos, & Gierl, 1996). Roulet (1997) recognizes in his background research paper for the Ontario Ministry of Education, concerning revisions to the mathematics curriculum, that the social constructivist view is not only supported by the NCTM, but also by leaders in the Ontario Association for Mathematics Education [OAME] and Ontario Mathematics Coordinators Association [OMCA]. Clearly, even though writers of the NCTM Standards and the Ontario Mathematics Curriculum documents may not have completely adopted constructivist views, they have been influenced by them. Philosophical foundations for images of mathematics that influence pedagogy are important; however, calls for communication also stem from constructivist learning theories. The prominent ideas in these theories and parallels with social constructivist philosophy are described next.

Learning theory dimensions. From a philosophical perspective, mathematics communication is important because the nature of the subject as a body of knowledge is socially constructed. From a constructivist perspective in psychology, communication is important
because students learn mathematics through individual interpretations in a social context. In a broad sense, “constructivism in psychology is…based on the principle that knowledge is not passively received but actively built up by the cognizing subject” (Ernest, 1991, p. 103). A main feature of constructivism, which is apparent in calls for communication in mathematics curriculum reform, is:

The classroom needs to be seen as a ‘community of discourse engaged in activity, reflection, and conversation’…. The learners (rather than the teacher) are responsible for defending, proving, justifying, and communicating their ideas to the classroom community. Ideas are accepted as truth only insofar as they make sense to the community and thus rise to the level of ‘taken-as-shared.’ (Fosnot, 1996, p. 30)

The central idea that distinguishes constructivism from other theories of learning is that from this perspective, the process of obtaining knowledge is an “adaptive function” and not an “independent reality” (Fosnot, 1996, p. 3). Constructivist ideas emerged with the work of Jean Piaget and challenged the dominant perspective in the Western world because from this point of view, acquiring knowledge is no longer associated with obtaining an accurate view of the world but is based on each student’s previous understandings and experiences of the world (von Glasersfeld, 1996). Piaget’s work initiated discussions in the mathematics education community about communication in teaching and learning; however, over time there has been a shift from a focus on mathematical language to a focus on communication and discourse in mathematics (Sierpinska, 1998). According to Sierpinska, this change represents a shift in prominent theoretical perspectives. Originally the views of knowledge construction were mainly cognitive and focused on the internal processes of constructing knowledge; this changed however, when Vygotsky’s views, which took into account social and cultural aspects of learning, became popular.

Vygotsky acknowledged the social aspect of constructivism by focusing on the role of “the adult and the learners’ peers as they conversed, questioned, explained, and negotiated meaning” (Fosnot, 1996, p. 20). Vygotsky agreed with Piaget that knowledge is not static but is
based on the interactions that each learner has with the world; however, Vygotsky (1981) placed more emphasis on the importance of the social and cultural context. For Vygotsky:

[T]he notion of internalization applied only to the development of higher mental functions and hence the social or cultural line of development. In this account internalization is a process involved in the transformation of social phenomenon into psychological phenomenon. Consequently, Vygotsky saw social reality as playing a primary role in determining the nature of internal intrapsychological functioning (as quoted in Wertsch, 1985, p. 63).

Thus, in extending Piaget’s notion of constructivism, Vygotsky acknowledge this movement from social to internal thought.

Vygotsky’s ideas have influenced education and have been extended in more recent learning theories. For instance, Cobb, Boufi, McClain, and Whitenack (1997) suggest that while Vygotsky argues that there is a direct relationship between mathematics development and participation in discourse, he does not take into account individual differences in student thinking. Cobb et al. acknowledge that the relationship between social and psychological processes is indirect. From this perspective, active student participation in lessons through mathematics communication makes learning possible but does not guarantee it. Each individual student must participate in, and contribute to, discourse.

Cobb et al. explore the indirect relationship between social and psychological processes and recognize the importance of “reflective discourse” in the mathematics classroom (1997, p. 258). In this discourse, the teachers’ and students’ actions become objects of discussions. The teacher’s role is critical because learning in discourse is not guaranteed. According to Cobb et al., the teacher must use “considerable wisdom and judgment” to know when to initiate shifts in the direction of lessons that will support students’ mathematical development (p. 269). To illustrate reflective discourse and the teacher’s role, Cobb et al. describe classroom episodes of first grade students’ development of counting strategies. In one example, the class is trying to determine how many ways to get five monkeys into two trees. A shift in discourse occurred when the teacher asked students if there was a way to know that they had found all possible solutions. The
direction of the lesson changed, based on the contributions that students made in response to this question. This image of a mathematics classroom that supports student contributions through mathematics communication is representative of contemporary images of this mathematical process in teaching and learning.

Research Exploring Communication in Classroom Practice

Clearly, the epistemological notion of social constructivism and constructivist theories in psychology support active student participation through communication in the creation of both collective mathematics knowledge and individual student learning. With increased emphasis on constructivism in the mathematics education community, there has been more research exploring aspects of constructivist teaching methods, particularly those aspects related to communication. Some important studies that contribute to understanding constructivist visions of mathematics communication describe the importance of communication and what it can or should look like in classrooms (Fonzi & Smith, 1998; Craven, 2000; Hufferd-Ackles et al., 2004; Wilson & Lloyd, 2000) and discuss considerations for teachers (Frykholm, 2003; Franks & Jarvis, 2001; Keiser & Lambdin, 1996; Rousseau & Powell, 2005; Sfard, 2000).

Images of communication and benefits of this process are discussed by Whitin and Whitin (2002), who describe how communication can facilitate learning the concepts of prime and composite numbers. Findings from their analysis of a fourth grade mathematics lesson suggest that “talking was an effective way for children to clarify their thinking, discuss new possibilities, extend the thinking of others, and rehearse their ideas for writing” (p. 211). The value of communication, as it is outlined by the NCTM, is also recognized in an article by Toronto District School Board coordinator, Stewart Craven (2000), who describes an elementary classroom vignette and addresses questions about why students should learn to communicate in mathematics, and how teachers can foster communication in their classrooms. He argues that students should learn to make explanations and create arguments not only because curriculum
guidelines call for this process but because children will need these skills in their future occupations.

An important description of communication in the mathematics classroom is provided in Hufferd-Ackles et al.’s “math-talk learning community” framework (2004, p. 81). The purpose of this model is to guide teachers in implementing communication and to “facilitate researcher and teacher educator understanding of this process” (p. 81). Even though elementary school mathematics was the focus in Hufferd-Ackle et al.’s research, the model presented in this article is referenced in professional learning guides for Ontario teachers (Ontario Ministry of Education, 2008a), in a research synopsis on the Ontario Ministry of Education website (Ontario Ministry of Education, 2008b), and in an article produced by a partnership between The Literacy and Numeracy Secretariat and the Ontario Association of Deans of Education (Bruce, 2007).

The math-talk framework outlined by Hufferd-Ackles et al. (2004) describes key components of a classroom that supports communication. Math-talk learning is identified in terms of questioning, explaining mathematical thinking, taking responsibility for learning, and the source(s) of mathematical ideas in the classroom. The math-talk framework is based primarily on a case study of one elementary school teacher who began teaching a course in a traditional manner and worked towards implementing whole-class discourse in a math-talk community. The model consists of four levels, which represent “the development of the math-talk learning community” in the transition from traditional teacher-directed discussion to whole-class discourse (Hufferd-Ackles et al., p. 87). In the shift from the lowest level, Level 0, to the highest level, Level 3, of a math-talk community there is a “shift from teacher as questioner to students and teacher as questioners”; “students increasingly explain and articulate their math ideas”; there is a “shift from teacher as the source of all math ideas to students’ ideas also influencing direction of lesson”; and, “students increasingly take responsibility for learning and evaluation of others and self” (Hufferd-Ackles et al., 2004, p. 88). Since this model has been referenced as the preferred
communication framework in Ontario, it is an important part of the conceptual framework for this thesis.

In addition to describing images of communication and the importance of this mathematical process, research studies have explored considerations that teachers must take into account when they implement interactive communication in the mathematics classroom. Sfard (2000, 2001), for example, who views communication as equivalent to thinking, recognizes that given the nature of mathematics, there are potential problems with deciding how to relax the rules of the subject. She describes examples of classroom lessons that illustrate two different types of discourse. One type places less emphasis on the formal aspects of mathematics and the other places more emphasis on the rules. Sfard (2000) acknowledges a distinction between discourse that takes place in mathematics classrooms and the discourse of professional mathematicians. In the classroom, the rules of the subject must be relaxed but Sfard (2000) questions how much teachers can do this while maintaining the well-defined, coherent nature of the subject. She cautions that carelessly relaxing the rules can make the subject meaningless and impossible for students to learn.

There are a number of tensions that teachers may face when implementing communication. For example, time is an important factor to consider. Implementing math-talk communication in instruction and assessment may take more time than traditional teacher-directed instruction and assessment tasks that focus on right answers instead of explanations. This issue is raised by Keiser and Lambdin (1996) in a study conducted to determine why teachers were not covering curriculum units at a suggested pace. From teachers’ responses to a questionnaire and classroom observations, Keiser and Lambdin found that teachers needed more time to incorporate activities such as group work in class, to plan such activities outside of class time, and to assess student communication.

The teacher’s role in supporting a math-talk community can also cause tensions for teachers. For example, Frykholm (2003) explored how several teachers implemented mathematics
curriculum reform in elementary mathematics classrooms. Findings from this study suggest that when teachers encourage interactive student communication they must address a variety of perspectives and solutions, and make on the spot decisions. This may cause them to become uncomfortable and therefore avoid implementing communication activities. This finding is also recognized in a study conducted in Ontario by Franks and Jarvis (2001). These researchers explored communication in secondary mathematics and found that when teachers implement communication, “the potential for a multiplicity of meanings is high and, consequently, so is the potential for confusion and frustration” (p. 66). Data for Franks and Jarvis’s study were collected from discussions in working groups of mathematics educators, one of the author’s activities in exploring written communication with a small group of teachers over an extended period of time, and observations of how students communicated in an assignment. Even though implementing communication can lead to frustration, Franks and Jarvis recognize that overall, inclusive communication can be “motivating and liberating” for the teacher and students and that teachers should provide opportunities for students to engage in such communication (p. 66).

Wilson and Lloyd (2000) conducted a qualitative study with a purpose similar to that of this thesis—to describe mathematics teachers’ experiences and the challenges that they faced when implementing a high school curriculum that encouraged communication. The focus in this study was on the challenges that three teachers faced with respect to sharing authority with students in this type of classroom environment. An important finding from this study is that teachers conceptions of mathematics as a subject influenced how they implemented communication.

Even though the challenge of shared authority was not faced by teachers who participated in this research, the cases in this research demonstrate how teachers’ conceptions of mathematics are reflected in their perspectives and practices. Moreover, the focus here is not only on understanding challenges involved with implementing communication, but on describing the gap between images of communication presented in curriculum statements and those held by teachers.
in classroom practice. Findings from this research contribute to literature in the area of communication, particularly constructivist images of the purposes of communication, by describing the images of communication held by two teachers who teach the *Foundations of Mathematics, Grade 9, Applied* course in Ontario.

**Emphasis on Communication in the Ontario Mathematics Curriculum**


The communication emphasis is not new in Ontario. As early as the 1980’s, communication was highlighted in the introductory, “Process Components,” sections of Ontario Mathematics Curriculum documents (Ontario Ministry of Education, 1985). Despite the emphasis in these sections, communication was not an explicit expectation. It was possible to implement the guideline without reading the introductory sections of the documents. Roulet demonstrated this in a background research paper that was produced to initiate discussion concerning secondary curriculum reform in Ontario. He explained that although the Ministry emphasized the importance of communication in the introductory sections of curriculum guideline, this advice had little impact in the classroom since the curriculum could be implemented without reading these sections. Consequently, he recommended that to support changes in instructional
practices—such as increased communication in classroom practice—in addition to specifying content expectations, curriculum guidelines should include examples and descriptions of expected classroom interactions:

Language and communication play an important role in learning and the guideline tells us that students should be involved in oral and written presentations of explanations and meanings. Provincial reviews that report, ‘The most commonly used methods for teaching … mathematics are presentation of information to the class by chalkboard or overhead projector and assignment of individual work’\textsuperscript{125} show that the advice provided in the guidelines has had little impact. This lack of progress can be traced to the organization of the guideline, where courses are described by long lists of topics with few references back to the initial teaching suggestions. It is possible to employ the guideline without ever reading the introductory pages where the image of mathematics and its teaching and learning is set. A new guideline, to be effective in altering teaching practice along with specifying new content, will need to be supported by well developed examples and descriptions of the expected classroom interaction. (Roulet, 1997, Instruction and Assessment section, para. 3)

The expectation for communication became more explicit when the Ontario Mathematics Curriculum was revised in the 1990’s. In a comparison of the 1985 and 1999 Grade 9 curriculum documents, Roulet, Cooke and Lim (1999) explained that the format of revised policy documents made mathematical processes, such as communication, a compulsory expectation:

A look at the Process Components section (pp. 17-21) of the 1985 Guideline shows that the teaching approaches implied by the new curriculum are not really new. Our old document suggested an experiential approach, a focus on applications and mathematical modelling, and the use of problems for which students did not have ready-made solution procedures. It was suggested that students talk about their mathematics as they tested their thinking and reasoning. The significant difference between the old and new curricula is in format and the new implied compulsory use of teaching practices that actively involve students. In the 1985 Guideline the long lists of topics made no mention of the instructional ideas suggested in the introductory pages. Mathematics teaching did not have to change. To meet the expectations of the new curriculum, instruction must move beyond teacher presentation and student practice of isolated mathematical routines. (Is this a New Way of Teaching section, para. 1)

When the Ontario Mathematics Curriculum was revised in the late 1990’s, the importance of communication was highlighted throughout curriculum documents. Even though these documents did not contain an introductory section dedicated to describing the mathematical processes, the importance of communication was described in these sections of the guidelines.
The Ministry explained that communication expectations were embedded in each course in the curriculum:

The importance of communication in mathematics is a highlight of the elementary school curriculum and continues to be a highlight in secondary school. In all strands and all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their high school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group. (Ontario Ministry of Education and Training, 1999, p. 4)

Thus, in the late 1990’s, the importance of communication was not only recognized in curriculum documents, but this mathematical process became a compulsory teaching component of the guidelines. To encourage communication, course expectations included verbs such as “communicate,” “describe,” and “explain.”

The importance of communication in assessment was also emphasized in curriculum in the late 1990’s. Communication was one of four categories of knowledge and skills identified in The Achievement Chart for Mathematics. According to the guidelines, teachers were expected to “review the required curriculum expectations and link them to the categories to which they relate” (Ontario Ministry of Education and Training, 1999, p. 38). The four categories included: “Knowledge/Understanding, Thinking/Inquiring/Problem Solving, Communication, and Application” (Ontario Ministry of Education and Training, 1999, p. 37). Students were to be assessed on their level of achievement for each category. “The descriptions of the levels of achievement given in the chart [were to] be used to identify the level at which the student has achieved the expectations” (p. 38). According to the communication category of the achievement chart, students were to be assessed on their ability to communicate “reasoning orally, in writing, and graphically” and to use “mathematical language, symbols, visuals, and conventions” (p. 39). To achieve a Level 4 (the highest level), students were required to demonstrate their ability to communicate concisely and to justify their reasoning. They are also required to regularly use correct language, symbols, visuals, and conventions.
Curriculum Policy and Resource Documents

Communication continues to be a compulsory component of the most recently revised Ontario Mathematics Curriculum (Ontario Ministry of Education, 2005a, 2005b, 2007). The expectation is not only more heavily emphasized, it is also articulated more clearly and consistently throughout documents. In the introductory sections, the importance of communication is highlighted in a section titled The Mathematical Processes. This section of the guideline describes seven mathematical processes: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. Students are expected to “learn and apply” these processes as they “work to achieve the expectations outlined within the strands of [each] course” (Ontario Ministry of Education, 2005a, p. 12). The Ministry explains that process expectations should be addressed throughout mathematics courses rather than in connection with particular strands:

In the 1999 mathematics curriculum, expectations relating to the mathematical processes were embedded within individual strands. The need to highlight these process expectations arose from the recognition that students should be actively engaged in applying these processes throughout the course, rather than in connection with particular strands. (Ontario Ministry of Education, 2005a, p. 12)

The most recently revised Ontario Mathematics Curriculum was developed through a collaborative process and was informed by current research. Suurtamm and Graves describe the members involved in developing the curriculum and that the revised guideline reflects current thinking in mathematics education:

The current Ontario math curriculum for intermediate mathematics is aligned with curricula in other jurisdictions and reflects current thinking and research in mathematics education. It has evolved through a collaborative process involving practitioners, policy makers and mathematics education research, and the most recent revisions of the curriculum began with the writing of background research reports to inform the curriculum writers of current research. These revisions also included discussions with various stakeholders so that the mathematics education community, as well as other communities, were participants in the curriculum revisions. (Suurtamm & Graves, 2007, p. 2)
These documents clearly reflect current thinking about communication in mathematics. The Ontario Ministry of Education posits, for example, that communication is “an essential process in learning mathematics” (Ontario Ministry of Education, 2005a, p. 16).

Communication is one of seven mathematical process expectations that are highlighted at the beginning of each course in the revised Ontario Mathematics Curriculum. These processes are expected to be “integrated into student learning in all areas” of each course (Ontario Ministry of Education, 2005a, p. 38). According to the communication expectation for the Foundations of Mathematics, Grade 9, Applied course, students should demonstrate their ability to “communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions” (p. 38).

Communication continues to be one of four categories of The Achievement Chart for Mathematics in the revised mathematics curriculum. The four categories now include “Knowledge and Understanding,” “Thinking,” “Communication,” and “Application.” Communication is defined in the achievement chart as “[t]he conveying of meaning through various oral, written, and visual forms” (Ontario Ministry of Education, 2005a, p. 19). Students are to be assessed on their ability to express and organize their ideas and thinking using different forms; to communicate for different audiences using different forms; and to use appropriate conventions, vocabulary, and terminology in different forms. Communication is expected to be assessed according to four levels of achievement, where Level 3 represents the “‘provincial standard’ for achievement of expectations in a course” (Ontario Ministry of Education, p. 18). The highest level, Level 4, “identifies achievement that surpasses the standard. It should be noted that achievement at level 4 does not mean that the student has achieved expectations beyond those specified for a particular course” (p. 18).

A number of resource materials are available to support teachers with implementing the Ontario Mathematics Curriculum. These documents provide samples of what communication might look like in the classroom and in assessment (Consortium of Ontario School Boards, 2001,
2003, 2005, 2007; Ontario Ministry of Education, 2000b, 2003, 2004). Even though some of these documents were developed prior to the introduction of the revised Ontario Mathematics Curriculum, they support the contemporary emphasis on communication and are still available to support teachers in implementing the curriculum. Since the context of this research is the

*Foundations of Mathematics, Grade 9, Applied* course, I describe materials that are available for teachers of this course next, in a description of this context and the importance of research in this area.

**Foundations of Mathematics, Grade 9, Applied, MFM1P**

Even though communication is emphasized in all of the revised Ontario Mathematics Curriculum documents (Ontario Ministry of Education, 2005a, 2005b, 2007), this research was conducted in the context of the *Foundations of Mathematics, Grade 9, Applied* course. In the guidelines for this course, the importance of communication is highlighted in the introductory sections, under the heading *The Mathematical Processes* (Ontario Ministry of Education, 2005a, p. 12). Communication is also an explicit process expectation that should be “integrated into student learning in all areas” (Ontario Ministry of Education, 2005a, p. 38). The mathematical processes are interconnected; however, the communication process expectation states that by the end of the course, students will “communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions” (p. 38).

Not only is communication expected in mathematics instruction, it is one of four categories of knowledge and skills for which students should be assessed. Incorporating communication into assessment is an important feature of this curriculum because as Marzano, Pickering and McTighe (1993) recognize, not only are learning and assessment “intimately tied,” teachers tend to, consciously or unconsciously, focus their instruction on what will be tested (Marzano et al., p. 11). According to *The Achievement Chart for Mathematics*, students in the *Foundations of Mathematics, Grade 9, Applied* course should be assessed on their ability to
express and organize their thinking “using oral, visual, and written forms”; to “communicate for
different audiences … and purposes”; and to “use conventions, vocabulary, and terminology of
the discipline” (Ontario Ministry of Education, 2005a, p. 21).

The *Foundations of Mathematics, Grade 9, Applied* course provides a rich context for
study. Not only is communication a compulsory component of the curriculum guideline for this
course, but a number of resources are available to support teachers with implementing the
curriculum. Resources that support the spirit of the Ontario Mathematics Curriculum and
emphasize communication in mathematics include: *Think Literacy: Cross-Curricular
Approaches, Grades 7-12* (Ontario Ministry of Education, 2003), *Think Literacy: Cross-
Curricular Approaches, Grades 7-12 – Mathematics: Subject-Specific Examples, Grades 7-9*
(Ontario Ministry of Education, 2004), *Targeted Implementation and Planning Supports (TIPS)*
(Consortium of Ontario School Boards, 2003), and *Targeted Implementation and Planning
Supports for Revised Mathematics (TIPS4RM)* (Consortium of Ontario School Boards, 2005,
2007). Teachers may also refer to *Course profiles: Mathematics, grade 9 academic and applied*
(Consortium of Ontario School Boards, 2001) and *The Ontario curriculum exemplars: Grade 9

Suurtamm and Graves recognize that the “resource that appears most closely connected
to teachers’ classroom practice is the TIPS document” (2007, p. 4). TIPS (Consortium of Ontario
School Boards, 2003) and TIPS4RM (Consortium of Ontario School Boards, 2005, 2007) are
comprehensive resources that include detailed lesson planning templates and activities, posters
for the classroom and for teachers, and information about the mathematical processes.
Information about mathematical processes includes details on the role of students, instructional
strategies, and sample questions and feedback that can be used in the classroom. A vision of
communication in relation to the other mathematical processes in the Ontario Mathematics
Curriculum is also illustrated in the TIPS4RM resource.
The *Think Literacy* documents (Consortium of Ontario School Boards, 2003, 2004) outline reading, writing, and oral communication strategies that teachers can use to support communication in their mathematics classrooms. The descriptions of strategies include explanations of the purposes, payoffs, resources, and further support that are available for teachers. Teacher participants in this research used a variety of strategies from the *Think Literacy* documents, including Word Walls, Think/Pair/Share, Placemat, Four Corners, and the Anticipation Guide. A Word Wall is a “wall, chalkboard or bulletin board listing key words that will appear often in a new unit of study, printed on card stock and taped or pinned to the wall/board” (Consortium of Ontario School Boards, 2004, p. 22). For Think/Pair/Share, “students individually consider an issue or problem and then discuss their ideas with a partner” (2004, p. 96). The Placemat strategy is outlined in the *Think Literacy* document for mathematics:

> In this easy-to-use strategy, students are divided into small groups, gathered around a piece of chart paper. First students individually think about a question and write down their ideas on their own section of the chart paper. Then students share ideas to discover common elements, which can be written in the centre of the chart paper. (2004, p. 102)

For the Four Corners activity, “students individually consider an issue and move to an area in the room where they join others who share their ideas” (2004, p. 106). “An *Anticipation Guide* is a series of questions or statements related to the topic or point of view of a particular text. Students work silently to read and then agree or disagree with each statement” (2004, p. 10).

The *Foundations of Mathematics, Grade 9, Applied* course provides a rich arena for research not only because of the availability of a variety of resources to support curriculum implementation, but also because of the challenges that teachers may face in this context. Teachers may face challenges in implementing communication in any mathematics classroom. Grade 9 Applied teachers face additional challenges involved with the dynamics of this class. Students across Ontario have demonstrated low levels of academic achievement in this course (Education Quality and Accountability Office [EQAO], 2007a; King, Warren, Boyer, & Chin, 2005). Furthermore, their responses to the EQAO student questionnaire indicate that a large
percentage of Grade 9 Applied students across Ontario do not enjoy mathematics. Only 28% of female and 40% of male students who responded to the 2006-2007 survey indicated that they “like mathematics” (EQAO, 2007a, p. 69). Students in the *Foundations of Mathematics, Grade 9, Applied* course must participate in the provincial standardized EQAO test. Preparing for any standardized test may cause tensions for teachers. This is demonstrated in the results of Rousseau and Powell’s (2005) qualitative case study of two teachers implementing mathematics curriculum reform. Their findings demonstrate that teachers may feel the need to incorporate more teacher-directed instruction to cover the content of tests.

EQAO results provide valuable information about *Foundations of Mathematics, Grade 9, Applied* teachers and students across Ontario. In addition to providing information about student performance on the test, responses to the student questionnaire give us a sense of their attitudes toward mathematics and their perceptions on their performance. Specifically, findings from the 2006-2007 survey suggest that students in the *Foundations of Mathematics, Grade 9, Applied* course believe that they are not good at mathematics. Only 25% of female and 41% of male students indicated on this survey that they are good at mathematics (EQAO, 2007, p. 69). Responses to the EQAO teacher survey also indicate that teachers across the province support communication in the *Foundations of Mathematics, Grade 9, Applied* course. The majority of teachers (68%) who participated in the 2006-2007 survey responded with “often or very often” to an item that asked how often they engage students in activities related to the communication category of *The Achievement Chart for Mathematics*.

Findings from the *Curriculum Implementation in Intermediate Mathematics (CIIM)* project also demonstrate that teachers across Ontario support communication in their classrooms. The purpose of the CIIM project, which began in January 2006, was to “examine how this inquiry-oriented mathematics curriculum for Grades 7-10 in Ontario is implemented and understood by the multiple members involved” (Suurtamm & Graves, 2007, p. 1). Data for this project were collected from focus group interviews, a web-based questionnaire, and case studies.
Responses to the CIIM teacher questionnaire indicate that intermediate teachers support communication in mathematics. Of the Grade 9 Applied teachers who responded to the CIIM survey, 61% suggested that they expect students to explain their reasoning or justify solutions in most or every mathematics lesson (Suurtamm & Graves, in press, p. 10). Furthermore, almost all (91%) of the intermediate mathematics teachers who responded to the survey “agreed that it is somewhat or very important to provide opportunities for students to explain and provide reasons to support solutions” (Suurtamm & Graves, in press, p. 8). Clearly, data collected for the CIIM project and for the EQAO assessment of mathematics indicate that teachers across Ontario support communication in the *Foundations of Mathematics, Grade 9, Applied* course. This thesis describes how two teachers translate their visions of communication into mathematics classroom practice.

**Summary: Rationale for Research**

In summary, I argue for the significance of research that describes how teachers translate communication messages from curriculum statements into classroom practice. Since there are variations in classroom application of curriculum messages, it is important to explore how teachers implement initiatives. Mathematics is a rich forum for this study since curriculum materials reflect curriculum reform ideas, which call for particular approaches to teaching—including increased communication in the mathematics classroom. The images of communication presented in curriculum reform materials and in Ontario Mathematics Curriculum policy documents reflect social constructivist theories on learning and the nature of mathematics. Therefore, it is important to explore the extent to which teachers’ images of communication in mathematics reflect theories underlying calls for communication.

The *Foundations of Mathematics, Grade 9, Applied* course is a rich context for this research because communication is a compulsory component of the curriculum for this course and numerous resource materials are available to support teachers in implementation. It is also
interesting to explore how teachers engage students in a context where students struggle with learning mathematics and may not enjoy the subject. The context is also important because teachers across Ontario indicate on questionnaire items that they support communication. This study describes how two teachers, who would also indicate their support for this mathematical process, translate communication messages from curriculum statements into classroom practice in the *Foundations of Mathematics, Grade 9, Applied* course.

**Research Questions**

The specific questions addressed in this thesis are based on the general purpose of the research, the conceptual framework, and concepts that are discussed in the review of relevant literature. The general purpose of this study is to describe how communication messages are translated from curriculum statements into classroom practice by two teachers who work to support this mathematical process. The following more specific questions were addressed:

- What are teachers’ images of mathematics communication?
  - To what extent is there a gap between images of communication in curriculum statements and in teaching practice?
  - To what extent are the teachers reasons for implementing communication practical—communication is a fun way to learn the rules of mathematics—or based on theories underlying calls for communication—communication is important for each student’s learning and for constructing knowledge in a community?

- How do teachers implement communication in classroom practice?
  - What types of activities do they use in instruction and assessment?
  - How do they guide communication during instruction?
  - How do they incorporate communication in assessment?
  - What experiences and resources influence implementation?
• What enablers and barriers are faced when supporting communication in practice?
  – What challenges are faced when working to support mathematics communication?
  – What or who is helpful in supporting efforts to implement communication?
CHAPTER 3:

METHOD

This chapter reports on the procedures and methods that were employed in this study to address the research questions. In the first section, I outline the research design and demonstrate the appropriateness of a qualitative case study approach. Next, I describe my role as the researcher. In the third section, I explain the participant selection process. Subsequently, I explain how data were collected and analyzed. This chapter concludes with an account of the steps that I took to enhance the trustworthiness of my findings.

Research Design

For the purpose of this study, qualitative research methods—including semi-structured interviews with teachers, classroom observations, and the analysis of instructional and assessment materials—were employed to describe the perspectives, practices, and experiences of two teachers in the context of the Foundations of Mathematics, Grade 9, Applied course.

Appropriateness of a Qualitative Case Study Approach

Given the descriptive nature of this research, a qualitative approach is appropriate (McMillan & Schumacher, 2006). This study relays how two teachers translate communication messages from mathematics curriculum expectations into classroom practice. Qualitative data were valuable for illuminating each participant’s “experience…in his or her own words” (Patton, 2002, p. 47). Such data were also important for describing how curriculum initiatives are implemented in practice because, as Bodgan and Biklen recognize, “if you want to know about the process of change in a school and how the various school members experience change, qualitative methods will do a better job” (1998, p. 39).

A case study approach is essential for illuminating the “phenomenon of interest” (Patton, 2002, p. 25). In this thesis, the phenomena, “or ‘heart,’ of the study,” (Miles & Huberman, 1994,
were the teachers’ perspectives, practices, and experiences with implementing communication in the *Foundations of Mathematics, Grade 9, Applied* course. In each teacher’s case the heart of study was bounded by “a somewhat indeterminate boundary [that] defines the edge[s] of the case[s]: what will not be studied” (Miles & Huberman, 1994, p. 25). This boundary was set by the classroom context (e.g., classroom setting and experiences of students), as well as information, demands, and experiences of teachers in relation to the mathematics curriculum from outside the classroom (e.g., professional development workshops, expectations prescribed in curriculum and support documents).

Since this thesis describes only two cases, I do not claim that the findings are generalizeable. The rich descriptions of each case are valuable since they enhance our experiential knowledge of different images, practices, and experiences with implementing communication in the mathematics classroom (Stake, 2005). As Stake recognizes, this knowledge is important for contributing to our understanding of similar situations. Findings from this thesis provide insight into the gap between images of mathematics communication, different approaches to implementing communication in mathematics instruction and assessment, and challenges that are faced in classroom practice.

**Rationale for Multiple Data Sources**

Multiple qualitative data sources were essential for this research. Analysis of messages outlined in curriculum policy and resource documents provide a sense of the intended curriculum. Data collected from interviews, observations, teacher writings, and from the collection of materials that are used in instruction and assessment provide information about the implemented curriculum and the process by which communication statements are translated into classroom practice.

Interviews, observations, and document analysis were necessary for addressing the research questions. Specifically, these data sources were necessary for describing teachers’ images of communication, approaches to implementation, and the enablers and barriers that are
often faced in practice. Interviews were the primary source for data in this thesis. They were integral to this study because, as Patton recognizes, “we interview people to find out those things we cannot directly observe” (2002, p. 340). For this research, interviews provided insight into teachers’ images of communication, perspectives on implementation, and past or present experiences that could not be observed. During the interviews, I asked open-ended questions to encourage participants to respond in their own words (McMillan & Schumacher, 2006; Patton, 2002).

Interview data were triangulated with observational data because, as MacCallum found when he explored classroom practices related to supporting problem solving in mathematics classrooms, observations provided a “greater understanding of each participant’s world than would have been possible had there been an exclusive reliance on interviews” (1996, p. 38). Observations also “enabled an individualization of subsequent interviews with participants by referring to particular social and actual practice” (MacCallum, p. 38). Document data, obtained from teacher writings, instructional and assessment materials provided a context for the interview questions. These forms of data were necessary because documents “frequently reveal what people will not or cannot say…. These artifacts, often available to researchers for asking, provide a kind of operational definition of what teachers value” (Eisner, 1998, p. 184). Clearly, interviews were the primary source for data although it was important to triangulate data collected from this source with observation and document data. Furthermore, observation and analysis of instructional and assessment materials were important for providing insight into teachers’ practices and experiences with implementing communication in the mathematics classroom.

**My Role as the Researcher**

As the principal researcher, my role in this research was to collect and analyze the data. I conducted interviews, observed classrooms, got to know my participants, and interpreted the data. To minimize subjectivity, I continued to reflect on my own background and perspectives on the
research. This helped me to remain open to new themes that emerged. I also documented my own perspectives on mathematics communication. My images of this mathematical process are discussed in Chapter Five of this thesis. In the fifth chapter, I describe my perspectives and how they have changed through my experiences as a mathematics student, mathematics education researcher, and as the principal investigator of this study.

My decision to conduct research that explores the gap between images of communication in the intended and implemented mathematics curriculum emerged through my experiences as a mathematics student and my participation in a curriculum analysis project. In high school, I was a student during the double cohort in Ontario. At this time, the five year high school program in Ontario was replaced with a four year program and a revised mathematics curriculum was introduced. I was taught the older 1985 curriculum and graduated a year prior to the double cohort (Ontario Ministry of Education, 1985). Throughout high school, I took Academic-level courses and in many cases my mathematics classes were taught through a more teacher-centered communication. Some of my mathematics teachers shared their thoughts about the new curriculum expectations—particularly those related to assessment and the increased emphasis on communication. Their opinions on this emphasis were not always favourable. Since I was successful in a teacher-directed learning environment, I did not see the need for a more student-centered approach to learning.

After high school, I entered an undergraduate program where I majored in mathematics. In this program my classes were very teacher-directed. I did, however, spend more time talking to my peers and professors about mathematics outside of class time. After completing the second year of the undergraduate program, I participated in a curriculum analysis project. For this project, I analyzed messages contained in the revised Ontario Mathematics Curriculum documents. The focus of the analysis was on mathematical processes, particularly the communication process. The emphasis in these documents on student-centred communication was interesting for me because I did not have much experience with this as a mathematics
student. After analysing the messages in curriculum documents, I became interested in finding out how teachers across Ontario have embraced the new focus. This experience is what led me to pursue this area of research for my Master’s degree.

Although I am interested in mathematics communication, I did not have any assumptions or strong biases about this process or about teachers’ views. I was successful in a teacher-directed classroom but I was also open to developing a more theoretical and practical understanding of a student-centred approach to teaching mathematics. I entered the Master’s program immediately after completing the undergraduate program. Consequently, my experiences with being a mathematics student and with curriculum analysis research were the only experiences that would potentially influence my opinions. I did not develop any assumptions from teaching because aside from tutoring mathematics, I did not have classroom experience. My interviews in this study enabled me to probe my participants responses further because I had no assumptions about teaching. One potential bias did emerge as I conducted this research and became familiar with the literature concerning communication in mathematics. After developing a theoretical understanding of communication, I developed a greater appreciation for communication in the mathematics classroom. Since my favourable attitude toward the process may lead to this bias, I remained cognizant of this orientation to allow for new themes to emerge as I collected data.

**Participants**

Purposeful sampling was used to find participants who were knowledgeable and informative about the phenomenon of interest (McMillan & Schumacher, 2006). Since the degree of implementation of curriculum initiatives varies among teachers (Remillard, 2005), I was looking for teachers who are exemplary in their support of the emphasis on communication that is posited in the Ontario Mathematics Curriculum policy documents (Ontario Ministry of Education, 2005a). The teachers selected for this study were information-rich not only because their program exemplifies the communication focus in the curriculum, but also because they were able to
clearly articulate their curricular decisions and instructional actions, which helped me gain a more thorough understanding of their perspectives, practices and experiences.

Participants were located using the reputational case strategy; that is, by “recommendation of knowledgeable experts” (McMillan & Schumacher, 2006, p. 320). I asked for recommendations from experts in the Ontario mathematics education community—including members of the Ontario Association for Mathematics Education (OAME) and the Ontario Mathematics Coordinators Association (OMCA), as well as from mathematics coordinators and department heads.

During the summer months prior to data collection, an email was sent to the OMCA looking for names of potential participants. Shelly (all names used are pseudonyms), who was working for the Ministry of Education at the time, but would be teaching the *Foundations of Mathematics, Grade 9, Applied* course in September, responded to the email and expressed an interest in participating. Shelly’s name was also recommended to me by mathematics teachers whom I met at workshops and conferences, and by mathematics education researchers. Once I received approval from the Queen’s University Graduate Research Ethics Board to conduct my research, I sent Shelly electronic copies of the Letter of Information and Consent Form (See Appendix A) and applied to Shelly’s School Board to request permission to work with her. After receiving approval, Shelly’s principal was contacted by email to inform him of the study. In October, 2007, after receiving approval, I met informally with Shelly. She signed two copies of the Consent Form. I kept one signed copy and she kept a copy for herself. I also gave Shelly a hard copy of the Letter of Information.

After receiving approval from Shelly’s School Board, I asked for permission to involve a second participant. Once the Board approved, I contacted the Mathematics Coordinator to see if she had recommendations for potential participants. The Coordinator contacted heads of mathematics departments across the Board. One department head, Rita, sent me a message to recommend the “wonderful 9 Applied teacher” at her school (HEMA, Oct. 31). I sent Rita and
Helen electronic copies of the Letter of Information and Consent Form and we met informally in early November. At this meeting, Helen agreed to participate and signed a copy of the Consent Form. Then, I sent an email to Helen’s principal to notify him of her participation in the research.

Shelly and Helen provide rich and complementary cases for this thesis. Both teachers support the spirit of the curriculum by working to implement communication in their classrooms; however, these teachers come from very different backgrounds. Shelly has a strong mathematics background and is heavily involved in the mathematics education community. She is passionate about mathematics and has also been actively involved in the development of mathematics curriculum policy and resource documents in Ontario. Although Helen appreciates the value of mathematics as a subject, her passion is in helping students learn. For her, the subject that she teaches is not as important. She has a strong background in Special Education, with a specific focus on English and Mathematics. From her experiences in Special Education, Helen has developed strategies for helping students who do not find learning easy. She teaches mathematics because this is a subject that many students find difficult. Although Helen has not been as actively involved in developing curriculum resources in Ontario, she is a member of the Ontario Association for Mathematics Educators and presents at workshops and conferences. She has also been involved in a project with her school that is intended to help students who struggle in mathematics to be successful in this subject. Therefore, with their varying passions and professional backgrounds, Helen and Shelly provided two illuminative cases for this thesis.

Data Collection

Data were collected in three overlapping phases, beginning in November, 2007 and ending in March, 2008. Even though data collection and analysis are addressed in separate sections of this thesis, the two occurred simultaneously throughout all phases of the research so that I could “cycle back and forth between thinking about the existing data and generating strategies for collecting new—often better quality—data” (Miles & Huberman, 1984, p. 49).
To address the purpose and specific research questions in this thesis, data were collected in three phases from both cases simultaneously. Information about teachers’ images of communication was elicited during the first phase. The focus of the second phase was on implementation—including enablers and barriers that are faced in classroom practice. During each phase, individual semi-structured interviews that lasted approximately one hour were conducted with each teacher. The interviews were intended to elicit information about particular topics. For example, my first interview sought to gather information about the teacher’s background and perspectives on mathematics communication. Interview questions were developed in advance, based on the specific purposes of this study, and also emerged in the field. Questions emerged during interviews and informal discussions, classroom observations, and from analyzing documents (e.g., personal writings, lesson plans, and assessment tasks). The semi-structured interview format ensured that important questions were asked and provided the flexibility to explore other questions that arose (Patton, 2002). I decided the topics in advance but the “sequence and wording of the questions during the interview” (McMillan & Schumacher, 2006, p. 351). The three phases of data collection are outlined in Table 1. Sample questions for individual interviews are provided in Appendix B. Below I describe the purpose of each phase and how the data collection for each participant matched or differed from my intended method.
Table 1: Data Collected for Each Phase

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<tr>
<th>PHASE 1: BACKGROUND, IMAGES &amp; PLANS</th>
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<tr>
<td>Informal Meeting</td>
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<td>Document Analysis</td>
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<th>PHASE 2: PRACTICES AND EXPERIENCES</th>
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<td>Classroom Observations</td>
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<th>PHASE 3: SUMMARY &amp; PARTICIPANT CONFIRMATION</th>
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<td>Interviews</td>
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Phase 1: Background, Images and Plans

The purpose of the first phase was to elicit information about the teachers’ backgrounds, images of communication, and their plans for implementing this mathematical process in classroom practice. This phase also served to establish rapport with the teachers. Data were collected from interviews, document analysis, and from personal writings. I asked both teachers to prepare a personal statement to describe their past experiences with communication, and a concept map to illustrate their images of mathematics communication (see Appendix D for instructions for the writings). I planned to discuss these writings with participants in our first semi-structured interview. Since Helen was very busy, she was unable to complete the writings.
Therefore, based on my own interpretations of data collected from our first informal meeting and in our first interview in November, I constructed my own version of a concept map to represent her images (see Appendix F for the construction that was verified by Helen). We discussed my construction during our second interview.

Data collection for the first phase began with my first informal meeting with Shelly. While the main purpose of the meeting was to establish rapport, Shelly also described her class and her perspectives on communication in mathematics. She illustrated her images of communication in a concept map and discussed her perspectives in a personal statement (see Appendix E). We discussed her constructions during our first semi-structured interview. Since the interview took place in the classroom after Shelly taught her *Foundations of Mathematics, Grade 9, Applied* class, I observed her mathematics lesson for the day. Data collected from observations were used for the second phase of data collection. During our first interview, I learned about Shelly's background—including her experiences as a mathematics student and her considerable involvement in the Ontario mathematics education community. Shelly also reflected on how both of these experiences have influenced her teaching.

To establish rapport, I met informally with Helen. At this first meeting, Helen agreed to participate and shared some of her perspectives on teaching. Since Helen was unable to complete the personal statement and concept map, I prepared interview questions to elicit information related to the topics addressed in these writings. During our first interview, Helen shared information about her background and perspectives on mathematics communication. During our second interview, I asked her about her perspectives on communication even further in a discussion surrounding the concept map that I constructed to represent her vision of this mathematical process. My concept map, which was verified by Helen, is provided in Appendix F. Helen elaborated on particular points and expanded on the details that I included in my construction. Since data collection began late in the semester, and because our interview took place after Helen’s classes, I observed the three mathematics classes that she taught that day—
Foundations of Mathematics, Grade 9, Applied, Foundations of Mathematics, Grade 10 Applied, and the Grade 9 Locally Developed Course. These classes were observed for the purposes of the second phase of data collection.

To understand each participant’s plans for implementing communication in mathematics instruction and assessment, I asked each teacher for copies of materials that they used for instruction and assessment purposes. Although we talked about their plans for the implementation in the majority of our interviews, this was the primary topic of my second interview with Shelly and third interview with Helen.

**Phase 2: Practices and Experiences**

The purpose of the second phase of data collection was to explore teachers’ practices and experiences with implementing communication in mathematics instruction and assessment. This phase began with the classroom observations that took place during the first phase of data collection. Images of communication, presented in curriculum expectations (Ontario Ministry of Education, 2005a) and Hufferd-Ackles et al.’s (2004) framework guided the observations. I focused on how each teacher supported math-talk lessons and recorded the teacher’s actions in setting up lessons and responding to students. Since one main purpose of the second phase of the research was to have each teacher reflect on her actions in the classroom, during our first interview I asked questions related to specific instances that took place during the lessons.

After observing lessons and analyzing instructional and assessment tasks that call for communication, I conducted one individual interview with each participant. The main purpose of the interview in this phase was to elicit the teachers’ perspectives on how students performed with respect to communication on assessment tasks. We also discussed classroom practices. Interview questions concerning practices emerged primarily from classroom observations. Since the interviews took place after students completed their summative assessment tasks, the teachers’ experiences with summative assessments became an important focus of the interview.

Assessment tasks—including the teacher’s expectations for students, students’ actual
performance, and evaluation—were also discussed during the interview. After data were collected for the second phase, I analyzed all of the data that were collected up to that point and prepared a summary of my interpretations for each site (See Appendix G). This summary was used in the third and final phase of data collection.

**Phase 3: Summary and Participant Confirmation**

The purpose of this phase was to provide Shelly and Helen with an opportunity to respond to my interpretations of the data collected from their sites. For this purpose, I conducted a final informal semi-structured interview with each participant. Prior to the interview, I developed a summary of my findings for each case. During the interview, each teacher was asked to comment on the points that I included in the summary and to elaborate on my interpretations. I also asked the participants questions that emerged from the first two phases of the research but had not yet been addressed. I sent each teacher a copy of the summary and made sure that they were aware that could contact me at any time after the interview if they had any questions or comments. After this last set of interviews, the final analysis of data took place and conclusions were drawn.

**Data Analysis**

Throughout the research, I analyzed data that were collected from both cases simultaneously. I kept both hard and electronic copies of all data. At the end of each phase of collection, I analyzed the data to look for “blind spots” so that these could be addressed or probed further in subsequent interviews (Miles & Huberman, 1984, p. 49). I focused on predetermined themes that related to my conceptual framework and my specific research questions. When themes or issues emerged in one case, I explored them in the second case. For example, when one teacher indicated a particular challenge that involved implementing communication in the *Foundations of Mathematics, Grade 9, Applied* course, I raised the issue with the other
participant to elicit her views. Even though I analyzed data simultaneously across cases, I present each case individually in the fourth chapter of this thesis.

Throughout the research, data were analyzed in “three concurrent flows of activity: data reduction, data display, and conclusion drawing/verification” (Miles & Huberman, 1994, p. 12). Since analysis and collection “form an interactive and cyclical process” (Miles & Huberman, p. 12), these three flows took place throughout the entire collection and final analysis process. The details of these three flows with respect to this study are described next.

**Data Reduction**

The first flow, data reduction, is “the process of selecting, focusing, simplifying, abstracting, and transforming the data that appear in written-up field notes or transcriptions” (Miles & Huberman, 1994, p. 10). This process began with choices of the conceptual framework—gap between images of communication within the intended curriculum and those held by teachers—and the specific research questions, which stem from the framework and from the literature. The process continued throughout the phases of data collection as I organized, transcribed and coded data, generated themes, made summaries, and wrote memos and field notes.

Codes for this research were generated in advance and they also emerged in the field. Initially, I generated codes based on research questions and the conceptual framework. These included for example: “communication in instruction,” “communication in assessment,” “challenges,” and “images of mathematics.” Data categorized in the instruction and assessment categories were analyzed further into categories that were generated based on curriculum statements (Ontario Ministry of Education, 2005a), teacher resources (Consortium of Ontario School Boards, 2007; Ontario Ministry of Education, 2000b), and the literature (Hufferd-Ackles et al., 2004). For example, one sub-category based on Hufferd-Ackles et al.’s model concerned “developing a math-talk community.”
Codes also emerged in the field, since “the notes and transcripts of interviews and index cards on which ideas and comments have been written can be used more or less inductively to generate thematic categories” (Eisner, 1998, p. 189). One code that emerged in the field concerned “the Foundations of Mathematics, Grade 9, Applied” context. After observing lessons and talking to participants about their attempts to implement communication in this context, I found that in both cases, the dynamics of this class is unique and, therefore, an important part of this research. The data reduction flow of analysis continued as I collected and coded data. I developed themes and then made summaries of my interpretations. This process continued well into the process of writing this thesis.

Data Display

The second flow in analysis is “data display,” which is the process of presenting information in “an organized, compressed assembly of information that permits conclusion drawing and action” (Miles & Huberman, 1994, p. 11). Miles and Huberman recognize that a researcher should not rely on extended text (e.g., field notes) to display data because it is challenging for people to comprehend large amounts of information presented in this form, which can cause us to “jump to hasty, partial, unfound conclusions” (1994, p. 11). Therefore, data collected in this study were organized in a display that made information more accessible and easier for me to see what was happening (Miles & Huberman, 1994).

Data were displayed in matrices according to coded themes and participants’ pseudonyms. Direct quotes and summarizing phrases were labelled with the first letter of the participant’s pseudonym, the data source, and the date. For direct quotes, line numbers from interview transcripts were included in the code. For example, a direct quote from line 12 of the interview with Shelly that was conducted on November 26th is coded (SINT, Nov. 26, 12). For a description of the data coding scheme and a data log for each participant, see Appendix C.

The matrix format used for this thesis is similar to the form that Jarvis (2006) used to display data collected for his doctoral research. Jarvis also displayed data according to participant
and theme. The matrices that he used are displayed at a website that he created for his thesis (http://www.nipissingu.ca/faculty/danj/PHD/HOME.htm). Since the unit of analysis in this research is teachers’ perspectives, experiences, and practice, the analytical focus was on making comparisons among participants with respect to these categories.

**Conclusion Drawing and Verification**

The third flow of analysis is “conclusion drawing and verification” (Miles & Huberman, 1984, p. 22). Throughout the research, I documented my interpretations and verified conclusions by analyzing relationships and patterns in data, reviewing my findings with colleagues for “intersubjectivity consensus” (Miles & Huberman, 1994, p. 11), and checking my findings with participants. Miles and Huberman (1994) recognize that researchers often begin to make conclusions in the early stages of research; however, it is important that they stay open and be skeptical about these. To avoid predetermined conclusions, as mentioned previously in this chapter, I documented my assumptions and biases prior to collecting data.

**Enhancing Trustworthiness**

To enhance the trustworthiness of the findings from this research, I documented all of my thoughts carefully, triangulated perspectives, and reflected on my own orientation and role in collecting the data.

To enhance the validity of my findings, I triangulated multiple data sources and perspectives (McMillan & Schumacher, 2006; Patton 2002). I triangulated my interpretations of Shelly’s personal writings with her responses to interview questions. For Helen, I illustrated my interpretations of her images in a concept map and then she expanded on my interpretation during a follow-up interview. I also discussed my interpretations of findings from observations and document analysis with each teacher during follow-up in interviews. Finally, before making conclusions about the study, I discussed a summary of my findings from each participant in a final interview with them.
To enhance the quality of the data, I took detailed field notes and phrased the interview questions in participant language. I also include direct quotations in this thesis to illustrate each participant’s perspective. I took notes and tape recorded all interviews. Since technology sometimes fails, I used two tape recorders. The notes that I took during interviews were not detailed because I focused on giving the participant my full attention and did not want my note taking to be distracting. Instead I recorded key words in my notes and related them to my own interpretations at the time and points raised by the participant that I thought might be important to probe further. These basic notes were important for locating information on tapes. I took detailed notes during observations because they are “not simply important, they are crucial. They provide the reminders, the quotations, the details that make for credible description and convincing interpretation” (Eisner, 1998, p. 188).

To minimize subjectivity, which is a potential concern in qualitative research (Patton, 2002), I used critical reflexivity to critique myself and I recorded detailed field notes that described all steps taken during data collection and decisions that were made throughout the process. It is important to keep a detailed record because, as Miles and Huberman state, “we have to begin . . . by logging and then describing our procedures clearly enough so that others can understand them, reconstruct them, and subject them to scrutiny” (1994, p. 281). The data log that outlined the coding scheme and descriptions of the data that were collected from each participant is outlined in Appendix C. Not only did I record all of my thoughts and decisions throughout the process, but also prior to beginning the research I reflected on and documented my own assumptions and biases that might have influenced my interpretations. I describe these thoughts in the next section of this chapter.
CHAPTER 4:
TWO ILLUMINATIVE CASES

This chapter reports on the perspectives and practices of two teachers who support the spirit of the Ontario Mathematics Curriculum by working to translate communication messages from curriculum statements into classroom practice in the *Foundations of Mathematics, Grade 9, Applied* course. These two illuminative cases provide a sense of different images of communication in mathematics teaching practice.

Shelly and Helen, the two teachers who participated in this research, come from different backgrounds and experiences with the Ontario Mathematics Curriculum. Both teachers are involved in the process of implementing initiatives. Shelly has also been involved in the processes of planning and producing curriculum policy and ancillary resource documents. Additionally, she presents curriculum ideas—related to mathematical processes and developing a math-talk learning community—at workshops and conferences for teachers. These opportunities have significantly influenced her understanding of the innovations. Her case is a valuable part of this thesis since it provides insight into the perspectives, practices, and experiences of a teacher who translates images of communication from the world of curriculum developers into the world of professional practice. Helen’s perspectives and classroom practices have been more heavily influenced by pragmatic experiences in the world of professional practice. She has a strong background in Special Education, which has contributed to her success in helping students in the *Foundations of Mathematics, Grade 9, Applied* course to learn mathematics. Since students in this course often struggle with this subject, Helen’s case is an important part of this research. Her case is rich and complementary to Shelly’s.

Shelly and Helen are both passionate about helping the students in their classes learn mathematics. Helen, however, is passionate about teaching in general while Shelly is more passionate about teaching mathematics specifically. Helen is a teacher of struggling students. She
was invited to teach mathematics, primarily in non-academic courses, because of her ability to help students who struggle. For her, the subject that she teaches is not important—her passion lies in teaching. Shelly is also passionate about teaching but her passion is related more specifically to mathematics. She has always enjoyed mathematics. Before becoming a teacher, she volunteered in mathematics classrooms. She enjoyed the experience and now teaches a range of mathematics courses, including both academic and non-academic classes. Helen and Shelly’s passions are reflected in their perspectives on communication in mathematics. Helen describes communication in general terms, related to her opinions on what is best for her students. Shelly describes communication as it relates more specifically to teaching mathematics. Below I elaborate on Helen and Shelly’s perspectives on communication in mathematics.

Shelly’s and Helen’s cases are presented in separate sections below. Each section begins with a description of the teacher’s professional background. The focus in this description is on the teacher’s experiences with mathematics, teaching, and the curriculum. Next, I outline the teacher’s perspectives on the revised Ontario Mathematics Curriculum (Ontario Ministry of Education, 2005a), communication in mathematics, and on the nature of mathematics as a subject. Subsequently, I discuss the teacher’s practices and experiences with implementing communication in mathematics instruction and assessment in the context of the Foundations of Mathematics, Grade 9, Applied course. I conclude the presentation of each case with a summary and a brief discussion on the gap between images of communication.

**Shelly’s Case**

Shelly has been heavily involved in the curriculum revision process in Ontario. She is also an active participant in the mathematics education community in this province. Consequently, she has a rich understanding of how developers envision the revised Ontario Mathematics Curriculum. She has been involved in the mathematics education community since her second year of teaching, when she became a chapter representative on the Board of the
OAME. Through this experience, she “met all of the people who were involved in mathematics education and mathematics leadership in the province and … just started to get involved in projects” (SDOC, Nov. 18, 43). Her role as chapter representative led to her being part of the “writing team for the 1999 policy document, course profiles and both TIPS and TIPS4RM documents” (SDOC, Nov. 18, 41). Shelly has also participated in field testing for a number of years. She has tested ideas “for EQAO, including investigations … [and] for anybody who wants to try [ideas] out” (S_INT, Nov. 26, 105). If an idea is in line with something new, she likes to try it because it gives her “something to try and something to think about” (S_INT, Nov. 26, 105).

Shelly has also worked with the Ontario Ministry of Education. When data collection for this research began, she was returning to the classroom from a two-year secondment to the Ministry (S_INT, Nov. 26, 95). At the Ministry, she “had many opportunities to discuss the current research literature at a high level” (SDOC, Nov. 18, 45). This experience helped her put ideas, and research concerning curriculum, “into perspective so that [she] could get a better format … for what she was trying to do. It hasn’t changed significantly, what [she does]. It changes [her] understanding of what [she does] more than anything” (S_INT, Nov. 26, 131). Shelly’s participation in the mathematics education community and in the world of curriculum developers has had an impact in her classroom (SDOC, Nov. 18, 46).

Not only is Shelly innovative in supporting curriculum initiatives in her own classroom, she also plays a major role as the Head of the Mathematics Department at her school. She helps her colleagues understand the vision that is represented in the intended curriculum. She provides “support and leadership” in her department, helping her colleagues understand and embrace aspects of the revised curriculum—particularly those related to “communication” and “thinking,” which are part of The Achievement Chart for Mathematics in the revised Ontario Mathematics Curriculum documents (Ontario Ministry of Education, 2005a; S_INT, Nov. 26, 701).

Shelly’s passion for teaching mathematics and for participating actively in the mathematics education community stems from her appreciation of mathematics as a subject and
from a number of experiences that led to her becoming a teacher. She has always liked mathematics, and because of her experiences with a teacher who struggled with teaching this subject to her class in high school, Shelly learned to “figure it out on [her] own and then show other people how it worked” (S\textsubscript{INT}, Nov. 26, 18). Her brother was always in her class and he struggled with learning mathematics, so all throughout high school, she re-taught lessons to him. This experience made teaching mathematics more meaningful for Shelly and has influenced her perspectives on teaching (S\textsubscript{INT}, Nov. 26, 17).

After high school, Shelly went on to complete an undergraduate degree. She graduated with honours—majoring in mathematics and with English as a second teachable subject. After completing the degree, she was enrolled in a Chartered Accounting program but left when she became a mother. Shelly was a stay-at-home mom for 15 years during which she volunteered in mathematics classrooms. Eventually, “because [she] was in the school all the time, [she] accidentally got a job as a classroom assistant because they needed someone who could do some math” (S\textsubscript{INT}, Nov. 26, 32). After about a year and a half, she decided to become a teacher and went to a Faculty of Education to complete a Bachelor of Education degree (S\textsubscript{INT}, Nov. 26, 35).

Shelly’s perspectives and practices have also been influenced by her experiences with teaching mathematics to her own children. When she began teaching, her children were in high school. They were both identified as gifted learners because of their ability in mathematics. Since they “hated” the subject, Shelly knew that she had to teach them differently from the way they were being taught so she “used them as guinea pigs lots of times” in her lessons (S\textsubscript{INT}, Nov. 26, 38). Helping her own children influenced Shelly’s teaching because she learned to be more child-centered and to take on a more constructivist view (S\textsubscript{INT}, March 5, 42). “Working with them made [her] realize that if [she] was going to develop conceptual understanding, [she] needed to get them to think and do and play and all that kind of stuff” (S\textsubscript{INT}, Jan. 18, 44). Shelly’s child-centred views are demonstrated in her classroom practices, which are described below.
Before describing Shelly’s classroom practices, I illustrate her perspectives on communication in the mathematics curriculum. The description of Shelly’s perspectives and practices demonstrates how images of communication are translated from the intended curriculum into classroom practice by a teacher who is passionate about mathematics and has experienced curriculum both in the world of curriculum developers and in the world of professional practice.

 Perspectives on the Curriculum, Communication, and Mathematics

For Shelly, communication is an essential expectation and a fundamental part of The Achievement Chart for Mathematics in the Ontario Mathematics Curriculum (Ontario Ministry of Education, 2005a). Seven mathematical processes are defined in Ontario Mathematics Curriculum policy documents: “problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating” (Ontario Ministry of Education, 2005a, p. 12). Shelly embraces this emphasis in her classroom and sees communication as “overriding” (SINT, Nov. 26, 390). Her view is that “there’s five processes in the curriculum, under the umbrella of problem solving and held together by communicating because that’s how you find out if they’re [students] using the process” (SINT, Nov. 26, 397).

With respect to assessment, the Ontario Mathematics Curriculum documents describe four categories of achievement: “Knowledge and Understanding,” “Thinking,” “Communication,” and “Application” (Ontario Ministry of Education, 2005a, p. 20). Communication is an essential part of the Achievement Chart for Shelly. She suggested that even if she were no longer required to report on this category, it would still be important for her because it is something that students will “take away from high school” (SINT, Nov. 26, 83). She went on to explain that “in terms of thinking and communicating, they have to use those, so those two categories stand out for me” (SINT, Nov. 26, 83).

Shelly’s vision of communication has been influenced by revisions to the Ontario Mathematics Curriculum. She explained that “when the new … curriculum came out and then we
had four categories of the Achievement Chart, and one of them was communication, I thought long and hard about what communication was” (S\textsubscript{INT}, Nov. 26, 57). Shelly initially thought about communication in two ways: “Correct use of terminology, symbols and conventions (written and oral); and clear and concise explanations of reasoning, justification of solutions, and connections to context (written and oral)” (S\textsubscript{DOC}, Nov. 18, 7). More recently, she has also “tied communication to the problem solving processes as outlined in the revised curriculum … and math talk in the classroom (oral)” (S\textsubscript{DOC}, Nov. 18, 9), and to “reflecting on how [students] learn” (S\textsubscript{INT}, Nov. 26, 66).

Shelly illustrated her images of mathematics communication in a concept map. Her construction, which is shown in Appendix E, represents her images of communication in mathematics as a subject and communication in mathematics teaching and learning. For Shelly, “communication in mathematics is really about vocabulary and terminology” and is also connected “to the mathematical processes” (S\textsubscript{INT}, Nov. 26, 387). After reflecting on her concept map, Shelly decided that assessment is not only an essential part of communication in mathematics, but is also important for communication in mathematics teaching and learning.

… it [assessment] is critical. It’s the assessment that tells you what the kids know and that impacts your teaching. It’s the communication from the students that lets you know what they know and impacts on their learning, their evaluation …. So this is a really key piece that’s right smack in the middle. (S\textsubscript{INT}, Nov. 26, 474)

In constructing her concept map, Shelly focused mainly on what communication means for mathematics teaching and learning (S\textsubscript{INT}, Nov. 26, 189). In her opinion, literacy strategies, such as those encouraged in the \textit{Think Literacy} documents (Ontario Ministry of Education, 2003, 2004), are an important part of communication for mathematics teaching and learning. These strategies help her anticipate what she will get out of her lessons (S\textsubscript{INT}, Nov. 26, 189). For example, using an “Anticipation Guide” (Ontario Ministry of Education, 2004, p. 20) helps her plan lessons because it “lays out on the table what they [students] know… or what their prior knowledge is or what their misconceptions…. [T]hat’s sort of teaching and learning because it
tells me that I have more work to do” (S\textsc{int}, Nov. 26, 189). Similarly, in a “Graffiti” exercise (Ontario Ministry of Education, 2004, p. 66) for a lesson on percent, ratio, and rate, Shelly received feedback on students’ prior knowledge. Through the activity, she learned that her students did not have a “solid background” in this area so she had to “get at some of the basic ideas and help them [students] to summarize what those ideas were…. So, literacy strategies helped [her] that way” (S\textsc{int}, Nov. 26, 199).

Feedback is an integral part of mathematics communication for Shelly. Teacher-to-student, student-to-student, and student-to-teacher communication are all important but feedback is necessary in all of these forms of communication (S\textsc{int}, Nov. 26, 221). She suggested that for the “[w]hole communication in mathematics, feedback is key, and it’s not just giving me feedback on how they are doing, but listening to them discuss their ideas and giving each other feedback on how the investigation is going” (S\textsc{int}, Nov. 26, 225). Having students investigate in pairs and groups helps them “to clarify their thinking” (S\textsc{int}, Nov. 26, 221). Shelly explained that helping her brother learn mathematics influenced her perspective on having students bounce “ideas off each other to clarify their thoughts” (S\textsc{int}, Nov. 26, 208). When students do this, it reminds Shelly of when she was teaching her brother. She would discuss the mathematics with her brother and when he asked questions about how something worked Shelly had to describe the process and this helped clarify her thinking. This reminds Shelly that it is important to give students opportunities to communicate with each other. In Shelly’s opinion, this type of student-centred communication is tied to “developing conceptual understanding and whole class discussions” (S\textsc{int}, Nov. 26, 208).

Shelly’s vision of mathematics communication is grounded in her social constructivist image of “the nature of mathematical knowledge and the process of its development both in individuals and societies” (Roulet, 1998, p. 25). Despite facing some challenges with implementing her vision, Shelly demonstrates this view in practice. In the majority of the lessons that I observed, activities were open-ended and generated discussion among members of the
class. Furthermore, Shelly does not act as an authority in her classroom since lessons are often based on student contributions. Instead of acting as an authority by evaluating student responses, Shelly encourages her students to come to their own conclusions and to justify them.

Shelly’s images of the nature of mathematics are demonstrated in how she guides communication in her lessons. Many of her lessons revolve around open-ended tasks that generate discussion and then the contributions of the class are consolidated in a discussion of what members of the community have developed. In one lesson for example, students were asked to agree or disagree with four statements and to commit to their decision. Shelly did not explain right or wrong answers to the statements but instead incorporated a “Think/Pair/Share” strategy (Ontario Ministry of Education, 2004) that encouraged students to anticipate and justify their own answers. Shelly’s role in guiding this lesson is described as follows:

> From the front of the room, Shelly calls for her students’ attention. She says, “Today we will be working on a *Growing Dilemma* activity. You should begin by reading the following statements by yourself: (a) when you double the length of a square, the perimeter doubles; (b) when you double the length of a square, the area doubles; (c) when you double length of square, the diagonal doubles; (d) when you double the size of a cube, the volume doubles”. Students take a minute to read the statements. Then, Shelly tells the class, “You have to anticipate an answer to each statement and commit to it”. Students make their decision and then Shelly says, “Now, talk to your partner about why you agree or disagree—do not change your answers!” Then Shelly hands out square shapes to each pair of students and they begin to explore what happens in each of the four cases. Shelly walks around the classroom, visiting each pair. She asks questions and clarifies the task for students. She continues saying to pairs of students and to the entire class, “Remember, you must stick with your responses”. Shelly does not confirm or deny the students responses. (S_OBS, Nov. 22)

This classroom episode illustrates Shelly’s perspective on constructing knowledge in the classroom. She does not act as an authority in determining the correctness of student contributions. Instead, students come to their own conclusions. Shelly’s role consists mainly of clarifying the task, making sure that students are on task, and asking questions. During the task, students work in pairs with manipulatives while they construct their own understanding of the concept.
Shelly’s images of the purposes of communication in mathematics influence her commitment to implementing a math-talk learning community in her classroom. For her, communication is not merely a fun way to engage students in mathematics, but is necessary for constructing knowledge and understanding in the classroom. She demonstrated her commitment to continued support for math-talk learning after an unsuccessful attempt in one of her lessons (S_OBS, Nov. 26). Throughout the lesson, it was difficult for Shelly to maintain student engagement and she told her class that for the next lesson their desks would be placed in rows. After the class, Shelly shared her perspectives on the outcomes of this lesson. She shared some of her opinions on why students were not engaged and explained that since communication and interaction is so important, she would not leave her students in rows:

If you just leave them in rows … you lose all of this conversation, all of this feedback loop [referring to the concept map that she constructed], all of this communication and you lose it all. And it’s not worth the risk, right, because then its individuals that won’t be working or thinking…. (S_INT, Nov. 26, 258)

Shelly confirmed that she tends to support a vision of mathematics as a subject that is socially constructed rather than a set of rules, skills, or procedures that are absorbed. She expressed her opinion during our final interview, when I explained the different images of mathematics to her and asked where she would see herself. She confirmed that she would see mathematics as more of a fallible subject that is socially constructed. To her, knowledge is constructed in the classroom. She voiced her opinion that students should “construct their own understanding and their own knowledge. I don’t think that I can pour the knowledge into their brain. If it was absolute then it would be just one way to look at it… and that’s not me” (S_INT, March 5, 235).

Not only does Shelly tend to support a social constructivist perspective on the nature of mathematics, but she also acknowledges that with the focus on processes such as reasoning and proving, the contemporary mathematics curriculum is designed to encourage this vision of
mathematics. She explained, however, that time constraints may cause a teacher to support a more absolutist perspective that emphasises skill development:

You can’t implement the curriculum if you’re only thinking of doing this [absorbing knowledge], and in 9 Applied there is a lot about thinking, it’s about reasoning, it’s about proving. And if all you’re ever doing is drilling integers and fractions and solving equations, you’re never going to think about relationships …. Time is what you need the most of, and in order to get them to really think and reason and do all of that, you need to spend some time. But all the time you spend there you remove from skill development so it’s hard to balance the two things… and I am more apt to give up time there then with getting them to think …. [W]hen they come away from school, they need to be better at thinking than they do at solving equation, so I am perfectly happy and prepared to spend a good amount of time to get them thinking and analysing … I try really hard for a balance but I’m probably going to err on the side of thinking, higher order thinking than I am on the side of rote skills. (S\textsc{INT}, March 5, 459)

Shelly’s images of communication and the nature of mathematics are apparent in her practices which are illustrated below in a discussion of how she incorporates communication in her classroom.

\textbf{Practices and Experiences}

Shelly’s support for communication is clear in the physical arrangement of her classroom and in her teaching practices. Communication is defined and displayed in a word wall and in several posters on the walls of her classroom. One poster dedicated to communication is part of four posters that describe the \textit{Rubric Components for the Categories of the Achievement Chart}. This generic rubric is part of a \textit{C.A.R.E. Package} that was developed to support teachers in implementing the 1999 Grade 9 and 10 curriculum and is available on the OAME website. A Word Wall and smaller posters, titled “Communication Prompts” and “I am communicating when…” are also displayed in the classroom. More recently, Shelly has added posters of the mathematical processes, which are available on the Ontario Ministry of Education website. These posters are displayed on the wall of every mathematics classroom in her department (S\textsc{INT}, March 5). Making the processes available to students helps them understand what Shelly is looking for when she assesses communication. Furthermore, displaying the processes in every classroom is
important because it shows students that they must work on the processes in all of the mathematics courses.

Shelly provides opportunities for her students to employ and practice the mathematical processes. She discusses the processes with her students at the beginning of the school semester through a graffiti exercise (Appendix E). For this activity, students work in groups at stations. At each station, they use words, pictures, examples, and diagrams to represent one of the mathematical processes on a piece of chart paper. Then, the students move to a different station, where they “read what the previous group has already written, they can clarify, add an example, or add new information”. Each group of students rotates from station to station until they have had the opportunity to discuss each process. The charts that the class develop “represent the class’s collective prior knowledge of the processes”. Students are informed that they will work on the processes throughout the year and that they “will form the basis for much for their assessment in the Thinking and Communications categories”.

Shelly implemented the graffiti exercise with both the Foundations of Mathematics, Grade 9, Applied and Principles of Mathematics, Grade 9, Academic classes that she was teaching when this research was conducted. She voiced her opinion that the differences were “interesting. The processes, when I put them up …. The 9 Applied’s had a better sense of the math processes as they related to math than the Academic’s did” (SINT, Jan. 18, 836). Shelly elaborated on the differences in her students’ opinions:

They [the Applied class] understood that they were making connections that were related to math .... In my Academic class the connections were about the internet, ipods, text messaging … it was social connections. They never even thought about math because they were free. It was open .... And that … again it’s a level of confidence .... In the Applied class they knew it had to be connected to math and so they did not allow themselves to go anywhere. They went to the math. And… the only one they really misrepresented was reasoning and proving. And that’s probably the one that the Academics got the best. So there were two totally opposite reactions to the processes. It was really interesting. (SINT, Jan. 18, 842)

Shelly followed up on the graffiti exercise with an activity in which the students applied their understanding of the processes. She explained how she implemented this activity in the
Foundations of Mathematics, Grade 9, Applied class that she was teaching in her second semester:

…we started with the graffiti exercise about the processes…then the next day I did the, “Which Cylinder Has the Greater Volume? [Appendix E] I used the rice for that… and I did a four corners and they had to go to: “they both would be equal,” “the tall skinny one holds more,” “the short fat one holds more,” or “I really don’t know.” And…my class divided evenly, both classes divided evenly between short fat and they’re equal because they have the same amount of paper to wrap around the cylinder … and then we had the rice so they could see the answer and then we did it mathematically so that we could prove what we knew and then the kids related it back to the processes, “what processes did we use?” Because we had done the graffiti the day before … So they know what they are and they’re using the words, which is good. (SINT, March 5, 357)

Even though Shelly worked to support math-talk learning in her Foundations of Mathematics, Grade 9, Applied class, she voiced her opinion that she was having difficulty reaching a high-level math-talk community. She did however feel that she was more successful in the other classes that she was teaching at the time. She described the difference between her Foundations of Mathematics, Grade 9, Applied class and her other classes:

I’m working towards it but I don’t have it. I feel like I get close sometimes in 9 Academic. That the kids will share with each other and teach each other, and they can ask questions. They pose the ‘what if’ questions, not me. But I’m a long way from that in the Applied class. I pose the questions …. Data Management is a math-talk community but it’s a different course and it’s almost easy to have it set up that way. So I feel like I’m okay there, getting there in Academic and not close in Applied. Applied is a problem because they don’t have the confidence, and they don’t trust themselves to be the ones to ask questions. And sometimes they’re really not interested in asking questions. A lot of them don’t like math or have it in their head that they don’t like math …. (SINT, Nov. 26, 489)

When the semester was over, I asked Shelly if she managed to develop a math-talk learning community. She said that “it depends” (SINT, March 5, 715). When students were reviewing for the provincial standardized EQAO test, they worked together in groups, and they “were really talking about it and arguing about answers and explaining to each other how they did it” and she thought to herself, “they have come a long way”. However, Shelly still did not “think of them as a math-talk learning community” (SINT, March 5, 715). She finds that students do not trust themselves: they “argue and they would all agree that the answer would be ‘a’ and then they
would say, ‘is it a?’ So in a sense of being able to defend what they do…they didn’t make it, I don’t think” (SINT, March 5, 715).

Even though Shelly does not feel that she managed to reach a fully developed math-talk community in her classroom, I observed evidence of math-talk in her lessons. This is exemplified in the following description of how Shelly guides a whole-class discussion to review the concept of proportions:

Shelly stands at the front of the room and asks the class, “What is a proportion?” Two students raise their hands. Shelly takes a few steps toward one of the students and asks him to share his thoughts. The student provides a brief response and Shelly thanks him for his contribution. Then Shelly asks the second student to elaborate on the description that was provided by the first student. This student expands on the first student’s response. Shelly then follows up by restating the main ideas of the two contributions for the rest of the class. At the same time, she represents the answers on the board at the front of the room. As Shelly is writing, a student in the class asks, “How do we know this?” Shelly asks the class if someone can explain. Another student raises her hand to indicate that she would like to respond. (SOBS, Nov. 19)

This episode exemplifies math-talk learning since students are asking questions and explaining their own mathematical thinking as well as the thinking of their peers. Students are also taking responsibility for learning by asking questions and elaborating on the responses of their peers. This whole class discussion is also based on students ideas—an important aspect of the math-talk framework (Hufferd-Ackles et al., 2004). I observed this type of interaction in Shelly’s classroom on a number of occasions. Students tend to be comfortable with contributing to lessons. Shelly has noticed that a “positive side effect of encouraging communication and math talk in [her] classroom is that many students feel comfortable volunteering answers and strategies” (SDOC, Nov. 18).

Not only is math-talk heavily emphasized in Shelly’s mathematics lessons, but communication plays a major role in assessment. Communication is assessed through a variety of tasks—including journals, test questions, performance tasks, quizzes and projects (see Appendix B for samples). Shelly voiced her opinion on the importance of communication questions and using a variety of tasks in assessment:
I have always believed that a variety of assessments is critical to gathering evidence of how and what students have learned. Asking thinking and communication questions in a variety of assessments gives students an opportunity to show what they know. The variety also helps students with opportunities that match their learning style. (SDOC, Nov. 18, 75)

Communication questions that require students to demonstrate conceptual understanding play an important role on Shelly’s tests. When the Ontario Mathematics Curriculum was revised and communication became a category of The Achievement Chart for Mathematics, Shelly reflected on the types of questions that she asked on tests. Thinking about these questions helped her “uncover a few misunderstandings” that she had (SDOC, Nov. 18, 57). She described an experience with a Principles of Mathematics, Grade 10, Academic test that influenced the types of questions that she asks on tests:

I gave students three points on a grid and asked them to determine if the triangle formed by joining these points was an equilateral, isosceles, scalene or right angle triangle. Students would calculate lengths of sides and slopes to compare and use the results to determine the type of triangle. On the same test I posed this question: ‘If you were given four points on a grid, how would you determine that the quadrilateral formed with these points as the vertices would be a rhombus?’ To answer this question I expected students to outline the procedure they would use to answer this question. I assumed that if students could do the first question they would also be able to do the second. I was quite surprised that I had students who could do the first one but not the second (They had memorized problems like it without understanding the process). As well, I had students who could do the second question but not the first (these students understood the process but had very weak skills and couldn’t perform the operations correctly). I realized that the students who could do both questions had good procedural skills and understanding. (SDOC, Nov. 18, 57)

This experience helped Shelly to realize the importance of balancing questions that call for procedural and conceptual understanding on assessments. Students should demonstrate procedural knowledge but also be able to communicate their understanding. Shelly now includes both types of questions on her tests.

An example of a question that calls for communication and requires students to demonstrate both conceptual and procedural understanding was on an “Optimization” test that she used with her Foundations of Mathematics, Grade 9, Applied class (see Appendix E). The question consisted of two parts. In the first part, students are expected to draw a picture to
represent a situation and to “describe using words, charts or graphs how [they] would be able to determine the maximum area” without actually solving the problem. To get a Level 4 (the highest level) for communication, Shelly expected her students to tell her all of the steps that they went through in class when they were solving optimization problems (S\textsc{INT}, Nov. 28, 270). Shelly explained how student communication on this type of test question shows her the depth of her students’ understanding:

So this is, ‘how do you do it?’ This is doing it, and they’re different. The students who can...follow all the steps and do it, and follow the instructions, cannot generalize to write the whole story here.... that tells you whether they are operating at a very concrete level, or whether they’ve generalized and conceptualized. So when I’m looking for communication, communication starts to tell me the level of understanding, the depth of their understanding. If they can do this generically for any kind of optimization problem, and do it specifically then they have good procedural knowledge and ... understand the concept. If they can do only one, then they have only one concept without the other. So on a test, I try to have both kinds of questions—do they have good procedural knowledge and can they explain how things work? And that’s the other... that helps me to see if they have conceptual knowledge .... [I]t’s how much they can generalize or connect that tells me what level they’re at. And I don’t have a specific rubric for that because it’s not a performance task it’s just where they’re at and where they should be and if they can conceptualize this is the kind of answer I expect. (S\textsc{INT}, Nov. 28, 279)

Shelly uses rubrics to assess communication. “Communication is assessed on all tests, as a level where the generic rubric is used to assess use of terminology and form and as well certain questions are highlighted with a communication symbol (a C inside a circle) that are assessed for clear and concise explanations” (S\textsc{DOC}, Nov. 18, 19). For communication questions, Shelly expects her students to “explain, describe, justify, reflect, compare representations or make connections” (S\textsc{DOC}, Nov. 18, 19). Shelly finds that communication questions provide her with the opportunity to see how well her “students understand concepts or how they are thinking about them. This technique has really helped students to see the difference between right answers and proper communication since they are marked using different criteria” (S\textsc{DOC}, Nov. 18, 19).

Shelly develops rubrics for all tasks that are evaluated on communication (see Appendix E for samples). She described how she develops rubrics for these tasks:

I always write Level 3 first – this is what I expect students to do, this is what I want them to be able to do – that’s Level 3. Level 4 will make stronger connections or have more
representations or maybe solve the problem in more than one way or something like that. And I will show them that in the rubric so that students who are attacking some of these problems from different strategies can demonstrate that they can do it using more than one technique …. And, then, what’s missing? What’s missing from this, from the standard that would be a level 2? Where do you make your mistakes? What kinds of things, what omissions make it a level 2? And then what further omissions or further mistakes make it a level 1? …. So I usually make my standard first and build up for Level 4. Like what do they do really well that makes it above standard and then what are the pieces that they’re going to likely miss or omit, or slip up on that make it a Level 2 or Level 1. (SINT, Nov. 28, 135)

Shelly stresses that a clear rubric is important for grading communication. With a clear rubric she has very little or no difficulty assigning and justifying marks. She explained, “if your rubric is clear and your criteria [are] clear than you can say, ‘you got a three because those pieces were missing’, and there’s no argument, it’s when everything is vague that there is an argument” (SINT, Jan. 18, 528). Shelly explained how her first experience with using rubrics has helped her to see the importance of a clear rubric:

The very first rubric I made, I got exactly what I asked for but it was not what I wanted to mark… and it was the best lesson that I ever could have learned … the rubric has to speak to what it is I want to mark, and the kids will give me what I ask for so I have to show them what I want to mark, what I want to assess. And when you do that, they will rise. [They] can get a Level 4 because they know what it’s supposed to look like … how are they supposed to know that you value more than one solution if you don’t tell them. So I have rubrics that say, ‘For a Level 4, show me how to do this problem in more than one way’ and so then they will… why would they show you a second solution if you haven’t said, ‘well for a level 4, I need that?’ It’s like you’re not telling them what a level four looks like … and so even [with the rubric] sitting there, some students cannot physically figure out how to show you another way. But they know they should be striving for that … so to me, a good rubric gives as much information to the kids as it does for me. (SINT, Jan. 18, 590)

**Case Summary: Translating a Vision from the World of Developers into the World of Professional Practice**

Shelly is passionate about teaching mathematics and has a rich vision of the discipline. She participates actively in the world of curriculum developers and in the mathematics education community in Ontario. In addition to being a member of the world of developers, Shelly has had opportunities to “familiarize [herself] with the new curriculum as propounded by the developers in journal articles or at conferences specially held to acquaint the theorist with the concepts,
values and practices of the [curriculum] innovation[s]” (MacDonald & Walker, p. 46). These experiences have contributed to her understanding of the vision of communication that is presented in the Ontario Mathematics Curriculum.

Shelly’s images of communication in mathematics are strongly linked to those held by curriculum developers. She describes communication in terms of a math-talk learning community, as it relates to the other mathematical processes in the curriculum, and how it fits into the Achievement Chart for Mathematics. She uses a variety of strategies from ancillary support resources for teachers—including TIPS (2003), TIPS4RM (Consortium of Ontario School Boards, 2005, 2007) and Think Literacy documents (Ontario Ministry of Education, 2003, 2004).

Even though the gap between Shelly’s images and those presented in the Ontario Mathematics Curriculum documents is minimal, she faces challenges in implementing this vision in her Foundations of Mathematics, Grade 9, Applied class. Consequently, there is a gap between developers’ images and those that are actually implemented in classroom practice in this context. Specifically, even though she is strongly committed to implementing communication in mathematics instruction and assessment, she voiced her opinion that because of the dynamics of her Grade 9 Applied class, she was unable to develop a high-level math-talk learning community in this context.

**Helen’s Case**

Helen is passionate about teaching and helping students who struggle in mathematics. For her, the subject that she teaches is not important. She explained: “to tell you the truth, it doesn’t really matter what I teach, as long as I’m teaching. I enjoy teaching so it doesn’t matter” (HINT, Nov. 27, 95). Her perspectives and practices have been heavily influenced by her classroom experiences and her strong background in Special Education. She is successful in helping students who struggle with learning mathematics. Her success with these students led to her being asked to work in the school’s mathematics department.
Helen was educated in England. She graduated with a Bachelor of Arts degree and completed her teacher training in Special Education—focusing on English and Mathematics (H_{INT}, Nov. 27, 9). She began her teaching career by teaching art but could not continue because she was allergic to the materials, so she switched into Special Education (H_{INT}, Nov. 27, 9). She moved to Canada in the 1980’s and started teaching English and Mathematics to students in special education classes (H_{INT}, Nov. 27, 35). In the 1990’s, when Helen began teaching mathematics in Ontario, there were three different levels of courses—Basic, General, and Advanced. Helen first started teaching Basic Level courses. These courses were intended to “serve the needs of the students who may not participate in post-secondary education and [to] provide a good preparation for direct entry into employment” (Ontario Ministry of Education, 1985, p. 9). Then Helen taught Grade 9 and 10 General Level Mathematics in language intensive special education. General Level courses were “considered as appropriate preparation for employment, careers, or further education in certain programs in the colleges of applied arts and technology and other non-degree-granting post-secondary educational institutions” (Ontario Ministry of Education, 1985, p. 11).

When the Ontario curriculum was revised in the 1990’s, and students needed to complete an additional course in mathematics to graduate, Helen was asked to go into the mathematics department to help students in Applied and Essentials level courses so that they could graduate (H_{INT}, Nov. 27, 35). Helen explained that she was asked to work for the mathematics department because of her background in special education:

[Students] were used to my teaching and a lot of the teachers are not special education based… so they don’t have the background knowledge that I do, and so they were not getting through to them. So that’s the reason I went into the mathematics department, so that I could continue with the students so they would be successful …. (H_{INT}, March 3, 203)

When Helen merged into the mathematics department she needed to take Additional Qualification courses because some of the courses that she took in England were not recognized in Ontario. She completed a course in Special Education, specializing in behaviour, and a course
in mathematics (HINT, Dec. 14, 476). For her specialist course, Helen focused on behaviour. This was important for her because that was where she was “being placed a lot, with students that are having massive behaviour problems and could not go into regular classrooms” (HINT, March 3, 132). In the Additional Qualifications mathematics course that Helen took, one of the “big pushes” was for encouraging communication in the classroom (HINT, Dec. 6, 360). The impact that Helen’s experiences in this course had on her perspectives on implementing communication are described in the perspectives section to follow.

Even though Helen has not been heavily involved in the curriculum revision process in Ontario, she participates in the mathematics education community by attending workshops and conferences. She has attended a variety of workshops and has presented some of the strategies that she uses in her *Foundations of Mathematics, Grade 9, Applied* class at these workshops (HINT, Dec. 14, 514; HINT, Jan. 23, 168). She is also involved in a school-wide action research project that was developed to help reduce the Grade 9 failure rate at her school and to encourage students to select appropriate pathways in high school. Teachers in Helen’s Mathematics Department work as a team in implementing this initiative and together they present their experiences at conferences and workshops. For the project, students in the Locally Developed course are placed in one of five groups based on their level of conceptual understanding of mathematics. Students in Groups 1-4 work on developing basic mathematical conceptual understanding, while students in Group 5 work on completing the *Foundations of Mathematics, Grade 9, Applied* curriculum. Helen teaches Group 5, so when this research was conducted she was teaching the *Foundations of Mathematics, Grade 9, Applied* curriculum to students in the Locally Developed course. She also taught this curriculum to students who were enrolled in the Grade 9 Applied course. Therefore, her perspectives on teaching in these different contexts are described below. The focus here, however, is on implementing the curriculum in the *Foundations of Mathematics, Grade 9, Applied* class.
Helen is passionate about teaching. For her, the subject that she teaches is not important. Since she is successful in helping students who struggle with learning mathematics, she was asked to transfer into the Mathematics Department. Even though she is more passionate about teaching in general than about teaching mathematics specifically, she does value the importance of mathematics as a subject. She demonstrated this by suggesting that this subject “should be held in such respect as English ….You’ve got to remember what’s in our world …. You can’t get away from it ….You need math. You need a massive amount of mathematics in any career” (HINT, March 3, 381).

From Helen’s perspective, it is important for mathematics teachers to take courses in Special Education. She suggests that Special Education should be a component of the Bachelor of Education program because it is important for teachers to learn strategies for teaching students who may not learn mathematics in traditional, teacher-directed, classrooms. She explained that teachers are going to have to work with these students so they need a lot of introduction to this in their Bachelor of Education courses. The teachers need to “understand that you can’t just throw curriculum at them and just expect that they’re going to suck it up like sponges because they don’t” (HINT, March 3, 342).

Helen was taught mathematics in a teacher-directed, “lecture style,” form. After learning more about Special Education she realized the importance of teaching according to the learning styles of her students. Helen voiced her opinion that if one’s students are not auditory learners, which is the case for most students in the Foundations of Mathematics, Grade 9, Applied course, then they will not be successful in a teacher-directed classroom. Based on this recognition, Helen “built up strategies on how to work with the students who didn’t find learning easy” (HINT, Nov. 27, 9). These strategies, which support teaching mathematics in a student-centred environment, are described in the practices section later in this description of Helen’s case.

Before describing the strategies that Helen uses in classroom practice, I discuss her perspectives on communication in the mathematics curriculum and on mathematics as a subject.
Helen’s case demonstrates how communication messages are translated from the intended mathematics curriculum into classroom practice by a teacher whose perspectives and practices have been influenced less by the world of developers and more by classroom experiences and her strong background in Special Education.

**Perspectives on the Curriculum, Communication, and Mathematics**

From Helen’s perspective, teachers have always asked their students to communicate. With the most recently revised mathematics curriculum, the only difference is that this process is a separate expectation, which is just another way of looking at it. Helen explained that “we’ve always asked them to communicate to us. It just hasn’t been segmented into that little pocket… ‘this is communication’ … [W]hen we wrote our tests, there’s always been those word problems…. It just hasn’t been weighted in that fashion” (H_\text{INT}, Nov. 27, 189).

In Helen’s opinion, the contemporary curricular emphasis on communication is on the written form. However, she does not agree with this view and even though other people may not agree with her, in her classroom students can communicate information in any form that works best for them. Communicating through various forms is important in her classroom because English is a second language for almost every student, so they often struggle with the written language. Therefore, Helen said that “if I am unsure, but yet I know they understand more, I will talk to them and they will continue to communicate to me and I will give a communication mark based on that” (H_\text{INT}, Nov. 27, 225).

Helen’s images of communication in the mathematics classroom are heavily influenced by her perspectives on the students she is teaching. From her experiences with students in Applied courses, she finds that they often believe that they are not good at mathematics so the most important thing for her is to help her students to be successful and to change their attitude towards the subject (H_\text{INT}, Dec. 6, 583). Since Helen finds that her students are often not auditory learners, an important strategy for helping her students succeed is to teach according to their learning styles (H_\text{INT}, March 3, 321). Helen begins her semester by determining her students
learning styles. She finds that students in the Applied course are often not auditory learners so they will not succeed in a traditional, teacher-directed, classroom:

…one of the things that I do first is I find out what my learners are … and that’s your basis that you work from. And in Applied level you’ll have your kinaesthetic learners and you’ll have your visual learners - 99.9% are not auditory … and we just … carry on talking and talking and talking and these kids don’t learn that way …. The information just doesn’t get there. (HINT, March 3, 257)

So for Helen, it is important for students to participate actively in her classroom because her students are typically not auditory learners.

Helen voiced her opinion that the main premise behind her teaching is that students can do better and that they should have opportunities to improve and show her what they know. This belief influences her perspective on the importance of having students communicate through various forms:

… that’s the whole premise behind what I do with my students is that if they don’t like their mark and they’re willing to do something, they’ll work on it. And it’s the same with the communication. I know that they know more than sometimes what they put down and so I’ll say, ‘can you explain to me’ And so they can explain to me verbally and as long as they know what they’re supposed to be doing and then can explain to me, whether it’s in doodle drawings, I don’t care. (HINT, March 3, 227)

Thus, communicating through different forms is important in Helen’s classroom so that students can show her what they know.

Helen’s perspectives on communication in mathematics have also been influenced by her experiences in the Additional Qualifications course that she took. In the course, Helen analyzed the Ontario Mathematics Curriculum. She explained, “We literally just took the curriculum and cut it into pieces and then as groups, we discussed it. And then it all came apart when each of the groups presented. So that’s how we went through the curriculum” (HINT, Jan. 23, 618). Helen does not believe that her experiences with dissecting the curriculum changed her perspective. She understands the emphasis on certain ideas but suggests that it is not always possible with her students (HINT, Jan. 23, 626). Helen voiced her opinion that some curriculum ideas “would be nice, but that isn’t the clientele, that isn’t what we’re dealing with” (HINT, March 3, 626). She
explained that the Additional Qualifications course gave her some ideas for “things that I’m going
to do and that I’ve used and worked and other’s, nah, it’s not going to work” (HINT, March 3,
626). With respect to the emphasis on communication in these courses, although the push was
clear, it did not change her view of the importance—it only re-emphasized it (HINT, Jan. 23, 523).

Even though dissecting the curriculum has not necessarily changed Helen’s perspective
she appreciates the increased emphasis on communication because she can now give her students
credit for showing the processes that they use in solving problems:

How I was taught, the answer is it and that was your mark … and when I actually started
putting marks for processes it was … ‘oh, I like this’. Even though their answer is wrong,
I can give marks and this is nice. This is better than saying ‘no, it’s wrong. You did
nothing right.’ This isn’t really good for the kids self-esteem because if you go through
the process and you go, ‘oh, you got the idea … you made that mistake’ and you circle it.
But at least they got certain marks, and I like that. I can give them something. I can say
‘yes, you know up to here but you just went a bit wrong there’. (HINT, Jan. 23, 638)

This demonstrates that Helen acknowledges the importance of having students communicate
more than just solutions to problems. Her view is clearly influenced by her perspective that many
of her students are not confident with their abilities in mathematics. Helen also finds that having
students teach each other is important for building confidence. She explained that it “boosts their
egos” which is important because “it makes them feel special that they’ve actually accomplished
this and they can help somebody else that hasn’t got to that stage (HINT, March 3, 255).

Several aspects of Helen’s images of mathematics communication are illustrated in a
concept map (See Appendix F). I constructed this map to describe my sense of Helen’s
perspectives. She reacted to and expanded on this construction during one of our interviews.
Helen’s vision of communication is quite broad. Her images reflect her perspectives on teaching
in general and her opinions on what is best for her students. For Helen, it is difficult to define
what communication means in her mathematics classroom because this concept is “massive”. She
explained for example that her students are always communicating to her:

Whether the student’s communicating within their group, or the class, or myself, they’re
actually communicating verbally, or with their quizzes, it’s all communication, it’s all the
same. They just may speak to me differently to when they’re speaking to each other.…
Helen’s images of communication also include non-verbal communication of attitudes. In her opinion, students communicate their attitudes all the time. They often enter her class with a negative attitude toward mathematics so her goal is to change this. She explained that over time her students communicate a more positive attitude toward the subject. She sees this in “the way they sit, the way they question…. They’re more eager to answer, and even without prompting them to answer, where at the beginning maybe a couple of the people would want to answer” (HINT, Dec. 6, 599).

Helen talked about her images of student-student, teacher-student, and student-teacher communication. For Helen, student-student communication is important because her students are not auditory learners so they need to interact and contribute to lessons. Student-student interaction is also important for preparing students to communicate to the teacher. She explained that since English is a second language for many of her students, she provides them with opportunities to communicate with each other before sharing information with her:

I’ve even had students talking with other students and then giving me the information in English when they’re talking Cantonese. And they do really well on just arithmetic but as soon as that written language is in there and it’s gone into a word problem, they have to communicate with each other and then they give me the information afterwards. And that’s okay. It’s still doing the same job. (HINT, Nov. 27, 234)

For Helen, student-teacher communication is important because when she listens to her students she receives feedback on their attitudes and their understanding of the mathematics. Her perspective on the importance of having her students communicate so that they can show their knowledge is demonstrated in her reasons for assessing various forms of communication. She explained that she wants to give each student a mark that they deserve because they may know something and just have difficulty communicating in writing. So, if they can “demonstrate that they know it [the math], [she will] give them the mark because they’ve demonstrated they know what they’re supposed to do” (HINT, March 3, 361). Therefore, by considering different forms of
communication, Helen attempts to get better answers from her students. In terms of teacher-student communication, Helen considers it important for showing her students “where we are coming from so that is why we need to work on the communication, on ‘look, this is where it’s from, this is why we do certain things’” (HINT, Nov. 27, 204).

Helen links communication in mathematics to thinking and to showing the process taken in problem solving. She voiced her opinion that sometimes the steps in solving problems are “very intricate, that’s why it’s called communication, that’s why it’s called thinking, inquiring, problem solving, because you have to see the process” (HINT, Dec. 6, 851). Helen described an experience in England when she was writing an exam and since the wording of the question was unclear, she did not understand the question the way that it was intended. Consequently, she received no marks for her explanation because the final answer was wrong. After describing this experience, Helen expressed her opinion on the importance of giving students marks for showing their thinking:

That isn’t what you were taught to do over there. It was literally, you had the right answer or you had the wrong answer and I had the wrong answer …. That is one thing that had to change, that there has to be some thought. Like nowadays I would have got a mark. It would have been, ‘wow, you did actually think about this.’ (HINT, Dec. 6, 964)

Helen also expressed her opinion that student-centred communication is important for getting her students to think during lessons. Even though Helen likes to have some control in the direction of her lesson by making sure that students are “on the right path,” she does not mind if they do “a little diversion” (HINT, March 3, 426). She explained that when this happens, “[i]t means that they’re thinking, and if it is a little left field, we’ll talk about that, and then we’ll swing around and come back to the initial problems, and that’s okay because that’s just communication” (HINT, March 3, 426). Helen uses strategies, such as those from the Think Literacy document for mathematics (Ontario Ministry of Education, 2004), for encouraging thinking during discussions. For example she encourages her students to agree or disagree with statements. This is part of the “Anticipation Guide” strategy from that is found in the Think
Helen explained that she uses this type of strategy because “more than anything, it’s just to get them going. And then for them to realize that they’re not just going to sit there and do nothing. They have to think” (HINT, Jan. 23, 182).

Helen demonstrates some aspects of a social constructivist view but her images of the nature of mathematics are mixed. Her tendency to maintain some control over the direction of her lessons so that students are “on the right path” reveals her inclination towards a less socially constructed image of the nature of the subject. I explained the different views of mathematics to Helen during our final interview and asked where she would see herself. She voiced her opinion that mathematics is not absolute:

> It’s not so structured as in it’s absorbing rules because first thing is, that’s memorization and they don’t have to do that. Whatever job you’re in … they’re not going to say, ‘I’m sorry, you cannot do that job because you haven’t memorized those rules.’ It’s there, it’s available …. My husband is an engineer; he has … manuals and manuals of what … things that have changed, the rules that have changed …. It’s not something that you have to completely memorize. (HINT, March 3, 393)

Helen went on to explain that in practice she does not take math as “straight rules” but gives marks based on how students interpret things. Her statement reflects her perspective on mathematics as a school subject rather than mathematics as a discipline. This demonstrates that for her, mathematics is the content that she teaches rather than a larger way of thinking.

Even though Helen voiced her opinion on the importance of a more fallible image of the nature of mathematics, she faces constraints that lead her to teach in a less socially constructed manner. For example, she explained that “it would be nice if we could just investigate everything in a perfect land” (HINT, March 3, 444). She went on to state that, “I try to do as much investigation as possible but restraints, because to do an investigation, it’s a complete class and sometimes you just don’t have that” (HINT, Jan. 23, 452). She shared an example of how she put a lot of work into developing a rich investigation related to the Pythagorean Theorem but it took the entire class and the students did not accomplish much because they spent a lot of time getting
organized with cutting out squares that they were using for the activity. Helen has not tried the activity again because even though she thinks it is an important activity, she did not accomplish much. She suggests that it’s “neat to do it” but does not always work (H_{INT}, March 3, 465). Thus, for Helen, communication is a motivational device rather than an essential method of constructing knowledge in the classroom.

For Helen, time is a constraint, particularly in her *Foundations of Mathematics, Grade 9, Applied* class. She explained that this is because “[y]ou’ve got to finish curriculum and we’ve got EQAO [provincial standardized mathematics test] and EQAO is based on the entire curriculum….” (H_{INT}, March 3, 470). Helen’s students also influence how she supports knowledge construction in her classroom. Students hold a more absolutist image of mathematics, where the teacher or the textbook is the authority. They like to have ‘right’ answers. For example, they “cannot tolerate wrong answers in the back of the book” (H_{INT}, Dec. 6, 462). When answers are wrong, the students get frustrated and since Helen wants to help students succeed, she makes sure that the answers are correct before assigning homework. This is important for the students because when they come up with an answer that is different from the one that is in the back of the book they assume that they cannot do any more questions and do not try anymore.

Helen’s images of communication and the nature of mathematics are illustrated in her practices and, therefore, are also demonstrated in the discussion below of how she incorporates communication in her classroom. For the most part, mathematical knowledge in her classroom is discovered under her guidance rather than collectively constructed in the community. Even though Helen recognizes that mathematics subject matter is not completely absolute, she does not see communication as being essential for constructing new knowledge.

*Practices and Experiences*

Helen works to establish a classroom community that supports communication right from the beginning of the semester. Her support for this mathematical process is displayed in the physical arrangement of her classroom. Students’ desks are arranged so that they can sit in groups...
of up to four students. At the end of the semester, when the students’ desks were placed in rows for the EQAO test, Helen explained that this was foreign for her students, and that it is important for the desks to be arranged in a manner that encourages communication:

This is so foreign to have just lines of desks … it’s always set up … the classroom, so they do work together. You think sometimes they’re maybe just chit chatting but they are not. Math is in there …. Not everybody does it, but I like it. Not everybody has their classroom set like I do either. They still have it like this. But that’s old. That’s old school, and we’re told that they’ve got to do more communication. They’ve got to do more things together. (HINT, Jan. 23, 143)

Helen’s support for mathematics communication is also evident in a large word wall that is displayed at the back of her classroom. The word wall is bright—with fluorescent colours in the background to attract attention (HINT, Dec. 6, 416). Word Walls are now emphasized in the recently released *Think Literacy* document for mathematics (Ontario Ministry of Education, 2004, p. 22), but Helen has always used them. She explained how her reasons for using word walls stem from her background in special education:

In special education, we’ve always been doing this type of thing because we’ve realized that they [students] actually need things around. A lot of mathematics teachers, a lot of teachers, they just want to keep it a secret, not realizing that these kids just physically cannot memorize this type of stuff …. [A]nd it isn’t in the curriculum where they have to memorize...so why are we hiding it? And so, I’ve always done the word walls, I’ve always had them up in special education and so this has just grown from it, so it’s not just the words …. [T]here’s the parabola on there. They’ve got to remember the axis of symmetry, they’ve got to remember the zeros. We did the optical illusions. I’ve got the algebraic rules, solving algebraic equations, it’s all there. Tables and graphs. (HINT, Dec. 6, 437)

So for Helen, the word wall is important for making the mathematics available to her students because, in her opinion, many of them are not able to memorize.

Helen establishes her expectations for communication in the *Foundations of Mathematics, Grade 9, Applied* class right form the beginning of the semester. Students are expected to communicate with her and with each other frequently. She encourages student contributions by posing questions for the class. If they do not respond, she asks particular students to share their thoughts. She explained that it does not take students long to become comfortable with this:
It doesn’t usually take long for them to realize my expectations, and I think they get used to it. They also get used to actually getting up out of their seats. It might be a little foreign at the beginning but when I say, ‘oh, you agree or disagree with me?’ and so I will say, ‘all triangles have three sides’ and they either agree or disagree. But then they’ve got to come up with a reason why they agree or disagree … and you put them on the spot, and they get used to that. (H_{INT}, Jan. 23, 156)

Helen does not assign students to groups but lets them “mingle into their own” because they do not always work well together (H_{INT}, Jan. 23, 577). Helen explained that it does not take long to establish a community in her classroom: “within a week, that’s it, they’re gelled together and it doesn’t change from that” (H_{INT}, Jan. 23, 771). In Helen’s opinion, having students mingle into their own groups is particularly important with students in Applied level courses because they are together in the majority of their classes, so they already know each other and who they get along with. These students will not be productive in certain groups. This was a challenge that she faced in her *Foundations of Mathematics, Grade 9, Applied* class: “with the 9 Applied … I have such an array of age groups that it’s just kind of, they’re not gelling together because they really don’t have other things in common” (H_{INT}, Dec. 14, 325). She also cautions that “you have to watch that they do not gel and make a really bad group because sometimes that happens” (H_{INT}, Jan. 23, 777).

The interactions that take place during a mathematics lesson in Helen’s class depend on the purpose of the lesson. Helen described some activities that she uses to encourage communication on a review day:

What you find on a review day … there’s a lot of information that they will be getting concerned if there’s a test coming up, and I will get feedback. I will have them working in groups—we will have manipulatives out … I like to do a jeopardy game with them and I get a lot more feedback, especially when there’s treats involved. I will do something like a multiple choice for the answers and they get candies if they get the right answer. Or I will have them write on the boards and anybody that has the right answer, they all get candies. (H_{INT}, Nov. 27, 299)

Discourse in Helen’s class tends to be heavily guided. This was demonstrated, for example, during a lesson that I observed on graphing stories. Her role in guiding this lesson is described as follows:
Helen stands beside her desk at the front of her room. She asks the class, “what would my graph look like if my desk was a sensor and I moved?” At the same time, Helen draws the x and y-axes of a graph on the board and represents the sensor at the origin. Then she asks the class “was I walking toward or away from the sensor?” A student responds with a short answer and Helen begins to describe another situation. Helen pretends that she is driving a car and says to her students, “Say you are in your car and the desk is a stop sign. If I walk slowly, like this, and then speed up, what would the graph look like?” Helen continues to ask her students what would happen in different situations. At the same time, she illustrates what will happen on the board. (H_OBS, Nov. 27)

This description demonstrates Helen’s attempt to make mathematics interesting by connecting it to real life situations. She also displays information on the board for the visual learners in her class. The actual discussion during the lesson is heavily guided—students ask questions when they do not understand and they respond to the questions that Helen asks.

When Helen does not need to write on the front board or on the overhead, she often talks to the class from her desk at the front of the room. In the lessons that I observed, Helen began by talking to students from her desk and then, during whole-class discussions, she stood at the front of the room and wrote on the front board or on the overhead projector. When I told Helen that I noticed that she often talks to students from her desk, she explained that she does this because she prefers to “be at their level” (H_INT, Dec. 14, 295). In her opinion, with these students, “if you stand over them, they sometimes perceive that as that you feel as though you’re bigger than them.” So Helen likes to “keep to one level” because of “the way that it’s perceived by the students” (H_INT, Dec. 14, 297).

Helen has developed teaching strategies based on her Special Education background and her opinions on what is best for her students. For her, two of the most important strategies are to find out who her learners are and to rephrase things so that her students will understand (H_INT, Dec. 6, 273). Since her students are typically visual and kinaesthetic learners, she gives them manipulatives and visuals such as algebra tiles. Helen explained that “by actually using the algebra tiles, it does make sense and they’ve got something to play with, and they’ve got something to look at” (H_INT, Dec. 6, 264). Helen also makes sure that the information is displayed...
on the overhead at the front of the room. She likes using the overhead rather than the front board because she can see her students and make sure that they are on task.

Helen explained that it is important for teachers to be able to rephrase things and to try different directions so that students can understand concepts (HINT, Dec. 6, 243). She suggested that teachers need to use “very simple, basic language” (HINT, Dec. 6, 273). She also voiced her opinion that “yes, we do have to have the language for mathematics, and when we are talking about monomials … I go through what other things I want” (HINT, Dec. 6, 273). She exemplified this by describing how she introduces monomials to her students. First, she talks about the prefix—mono. Then she moves on to binomials and trinomials—connecting the prefixes to concepts that the students understand. For example, they talk about a bicycle and a tripod or a tricycle. She needs to repeat herself and make sure that students see the relevance in the concepts. She explained:

I’ve said it three times and three different ways …. I don’t say, ‘oh, this is a monomial, they’re all part of the family of polynomials ….’ I break it up into things that they can think about. Like I said the tripod, the tricycle …. When they were little, they had a three wheel bike and we know what that that’s what a tricycle is. (HINT, Dec. 6, 279)

Clearly Helen has developed strategies for implementing communication in mathematics instruction. Many of these strategies have been influenced more heavily by her special education background than by revisions to the Ontario Mathematics Curriculum. Helen also assesses communication through a variety of tasks, including the summative performance tasks that students complete at the end of the semester. Even though a communication mark is not shown on the students’ report cards, the levels that students receive for communication on these tasks contribute to 10% of the final mark that is reported.

Revisions to the Ontario Mathematics Curriculum have not influenced Helen’s opinion on what communication looks like in assessment. In her opinion, the mathematics on tests and assignments has not changed. However, she explained that because of this emphasis, the tests are arranged differently according to categories of The Achievement Chart for Mathematics:
Sometimes we will start with the same test but then …. This part, this question, I’m specifically going to just mark it on communication. This one, thinking, inquiring, problem solving …. You’ve literally just rearranged your test. Very similar, if not identical to what it was. It’s just been separated into it’s little components. (H_{INT}, Dec. 6, 717)

An example of the type of communication question that Helen includes on a *Foundations of Mathematics, Grade 9, Applied* test on slope is, “Explain in detail ALL the ways that you know of to find slope of a linear relationship”. Helen explained how she grades this type of question with respect to communication:

How I did the communication. I also took into consideration how they’d actually done on the test, which sometimes is better than just a specific communication question. Because the word ‘ALL,’ even though it’s in capitals, they don’t, they just think of a couple of things and then they don’t think about what encompasses all, and so that’s why I will do that but then I will take into consideration what they’ve communicated through the actual test, to tell me what do they know, what don’t they know? (H_{INT}, Dec. 14, 5)

When Helen assigns a grade level for communication on a test, she focuses on what the student has communicated to her. Helen described how she decides on an appropriate grade level for a student. The question that she used as an example describes a person who says that $y^2 + 4$ can be factored. Another person, Trevor says it cannot. The question asks, “Who is correct?” (H_{INT}, Dec. 6, 5).

So Level 1 is, well they’ve got an idea but it’s not fully correct. Level 2, they know something but it’s not really telling me…. Like they might come up with this, $(y – 2)(y + 2)$ but tell me…that it is correct, $y^2 + 4$, that’s what this works out to be and not think, ‘oh my goodness, (-2 ) x 2, oh, that’s -4…. If they go that step, we’re into about [Level] 3. If they can actually do that step…that’s Level 4. They know what they’re doing, they’ve understood. (H_{INT}, Dec. 6, 71)

So Helen grades communication on the basis of understanding—she gives a level for how much they have communicated that they know.

Helen also works to support collaborative communication in the final summative performance tasks. She gives students 15 minutes to look over a word problem individually and then they have 15 minutes to talk to each other. The types of problems are “very thinking, inquiring, problem solving, communication” (H_{INT}, Dec. 14, 435). These problems are found in the student workbook that accompanies the mathematics textbook that Helen uses in her
Foundations of Mathematics, Grade 9, Applied class. For example, Helen uses the “Getting on Track” task from this book (Brosseau, Brosseau & Etienne, 2006, p. 17). This task consists of several questions related to a race track. One question that encourages communication is, “How much longer is one lap around the track if you run in the outside lane instead of the inside lane? Describe how the race officials might make up for this difference” (p. 18). Since students have worked on these activities during previous lessons, Helen tweaks the numbers for the final summative assessments. Students often do not realize that they have seen the question before.

In Helen’s opinion, oral communication is important in the summative task for providing students with opportunities to seek clarification. For the summative task, students are allowed to communicate orally but they are not allowed to write anything down. Helen voiced her opinion that the oral communication is important because “if there is a problem, lets say of a composite figure and they need to find the area, if they need clarification…they can get clarification to make sure that they understand” (HINT, Dec. 14, 378). Early in the semester, Helen explained that she did not expect that her students would be able to communicate for very long on the summative task. She was concerned that they would not stay on task. However, after they completed the task, she voiced her opinion that they were very cooperative.

Case summary: Translating a Vision Influenced by Experiences in the World of Professional Practice

Helen is passionate about teaching. She teaches mathematics, particularly in Applied-level and Locally Developed courses, because of her ability to help students who struggle with learning. She describes communication in terms of what she believes is best for her students. For example, many of her students are not auditory learners so they must participate actively in the classroom. English is also a second language for many of her students so they need to communicate with each other before communicating with her. In Helen’s opinion, communication through a variety forms is important for her students because many of them struggle with written language.
Throughout her teaching career, Helen has tried a number of strategies in the classroom and has identified those that are successful with her students. She has also developed strategies from professional activities, such as workshops and an Additional Qualifications mathematics course. Helen’s expectations of her students impact her images of communication in mathematics since they influence her decisions to accept ideas that she believes will work with her students and she rejects ideas that she believes will not work. For example, the Additional Qualifications course in mathematics gave her “some ideas of what [she wants] to do and things that [she] certainly will never do” (HINT, Jan. 23, 629). She explained that some of the ideas that are promoted “would be nice, but that isn’t the clientele, that isn’t what we are dealing with” (HINT, Jan. 23, 628). For her, communication is important for helping students learn mathematics but is not essential for constructing knowledge and understanding. Her perspectives and teaching practices reflect a more practical rather than theoretical vision of communication in mathematics.
CHAPTER 5:

INTERPRETATION AND DISCUSSION OF THEMES

In this chapter I discuss themes that emerged as I conducted this research. In the first section, I describe different images of communication in mathematics. I contrast images held by Shelly and Helen in the world of professional practice. Then I compare their images with those held by members of the world of developers of the Ontario Mathematics Curriculum. I also discuss my own images of mathematics communication. In the second section, I draw out similarities and differences in Helen and Shelly’s classroom practices and experiences with translating communication messages into the mathematics classroom. This includes a discussion of their experiences with teaching students in the Foundations of Mathematics, Grade 9, Applied course and a description of how they work to overcome challenges involved with supporting communication.

A Gap Between Images of Communication in Mathematics

In this section I contrast Helen and Shelly’s images of communication in mathematics. I describe how different sources have influenced their images. I also discuss their commitments to implementing communication in mathematics teaching practice. Next, I compare their images with those presented in the Ontario Mathematics Curriculum. This section concludes with a description of my own images of mathematics communication.

Images in the World of Professional Practice

Even though there are similarities in their perspectives and practices, Helen and Shelly have different visions of communication in mathematics teaching practice. Furthermore, their images have been influenced to different degrees by revisions to the Ontario Mathematics Curriculum. Because of her involvement in the world of developers, Shelly’s images have been more heavily influenced by the curriculum revision process. Therefore, her vision is more strongly linked to that represented in curriculum documents (Ontario Ministry of Education,
Helen’s perspectives and teaching practices have been more heavily influenced by her experiences in professional practice. The influence of her experiences, particularly in Special Education and teaching students who struggle with mathematics, is reflected in her images of communication in mathematics teaching practice.

Since Shelly has “been working on mathematical communication since 1999 when the new curriculum was implemented” (S\text{DOC}, Nov. 18, 6), she has had the opportunity to reflect on the meaning of this mathematical process. She initially thought about communication in two ways: “Correct use of terminology, symbols and conventions (written and oral); and clear and concise explanations of reasoning, justification of solutions and connections to context (written and oral)”.

When the Ontario Mathematics Curriculum was revised, she began to link communication to the “problem solving processes as outlined in the revised curriculum (reasoning and proving, reflecting, selecting tools and computational strategies, connecting and representing) and math talk in the classroom (oral)” (S\text{DOC}, Nov. 18, 7).

Helen’s images of communication are less specifically related to mathematics than Shelly’s. Although she acknowledges that mathematical form and word problems are important in mathematics, she describes communication in more general terms by explaining that “whether [a] student is communicating within their group, or the class, or myself, they’re actually communicating verbally, or with their quizzes. It’s all communication; it’s all the same …. It’s just a very big, communication is just massive” (H\text{INT}, Dec. 6, 549). Her vision has not been significantly influenced by revisions to the Ontario Mathematics Curriculum. From her perspective, “we’ve always asked [students] to communicate to us. It just hasn’t been segmented into that little pocket - ‘this is thinking, inquiring, problem solving and this is communication…’” (H\text{INT}, Nov. 27, 189). She also explained that “there have always been those word problems in there; it just hasn’t been weighted in that fashion, it’s just another way of looking at [it]” (H\text{INT}, Nov. 27, 191).
Shelly’s understanding of the Ontario Mathematics Curriculum and the contemporary emphasis on communication in mathematics teaching practice have been influenced by the opportunities that she has had to work in the world of curriculum developers. She described experiences that have impacted her classroom practices:

I have had the good fortune to be involved as part of the writing team for the 1999 policy document, course profiles and both TIPS and TIPS4RM documents. As well I have just returned from a two-year secondment to the Ministry of Education. As a result I have the good fortune to field test new ideas, implement the new curriculum with a deeper understanding than many other teachers and I have had many opportunities to discuss the current research literature at a high level. All of this has had an impact in my classroom. (SDOC, Nov. 18, 41)

Shelly explained that working with the Ministry contributed to her understanding of the rationale behind curriculum initiatives. The experience also helped her “put ideas and research into perspective so that [she] could…get a better format for what [she] was trying to do” in the classroom (SINT, Nov. 26, 131). She voiced her opinion that this experience has not “changed significantly” what she does in the classroom but has changed her “understanding of what [she does] more than anything” (SINT, Nov. 26, 132). Before working with the Ministry, Shelly found that she was “sort of haphazardly trying things” in the classroom (SINT, Nov. 26, 130). She would get ideas from workshops or by talking to people, and “get excited about doing something,” but “would just do it but not understand where it all comes from and… how you can use that kind of way to develop the kids’ ideas” (SINT, Nov. 26, 128). While Shelly now understands the rationale behind curriculum changes and the emphasis on communication, Helen sees the new emphasis on this mathematical process as, “somebody has come up with a new idea and said, ‘oh, well we should have 20% on communication. We should go about whatever…and it’s just a different way of putting the numbers down” (HINT, Dec. 6, 681).

Even though revisions to the curriculum have not significantly influenced Helen’s perspectives and teaching practices, she has acquired ideas for implementing the curriculum from Additional Qualifications courses that she took just after the Ontario Mathematics Curriculum was revised in 1999. In the courses, Helen was introduced to strategies and activities that support
curriculum initiatives. In particular, she learned about strategies in the *Think Literacy* document for mathematics (Ontario Ministry of Education, 2004), which support curriculum initiatives. Helen also changed her practices by placing the desks in her class in groups rather than in rows. She explained that “previously all of these desks were in rows” (HINT, Dec. 14, 361). In her opinion, the majority of mathematics teachers place their desks in rows, “unless [the teacher has] had a course and really want[s] [their] kids to work together, they’re not like this. The desks are not like this. They are all in lines, just like they always have been. It has not changed” (HINT, Dec. 14, 363).

Clearly, participating in an Additional Qualifications course that supports curriculum initiatives has influenced Helen’s perspectives and teaching practices. However, her beliefs about what is best for her students have a more significant impact on her perspectives and teaching practices. Her expectations of students influence her decisions to accept particular ideas that are promoted in these courses and to reject others. She voiced her opinion that the courses gave her “some ideas of what [she wants] to do and things that [she] certainly will never do” (HINT, Jan. 23, 629). She explained that some of the ideas that are promoted “would be nice, but that isn’t the clientele, that isn’t what we are dealing with” (HINT, Jan. 23, 628).

Helen’s perspectives and practices have been most heavily influenced by her Special Education background. From Special Education courses, she learned to teach according to the needs, as she perceives them, of her students. She explained that “when [she] got into all the information about Special Education and how to teach the special needs, [she] realized how it really wasn’t Special Education, how it was really good teaching” (HINT, Nov. 27, 11). Through Special Education, she learned to teach according to the learning styles of her students. In her perspective, since the majority of her students are not auditory learners, they are not successful in a teacher directed classroom. Therefore, in her opinion, “if you just do lecture style, you know by the statistics that you are not hitting as many students as you would hope, because they just don’t learn in that way” (HINT, Dec. 14, 583).
Helen recognizes the importance of communication but she also acknowledges that it is not always possible for her students to interact in the classroom. Helen explained that although she knows from Special Education that communication is important, it is not always possible with particular groups of students:

I can see in a class of 33 kids in front of you, where it’s… it just doesn’t happen. And, like it’s really nice to have your desks like this and let the kids converse. But… [t]he dynamics of a class of 33 Grade 9’s. It doesn’t always work. And it’s nothing to do with your teaching. It’s the dynamics of the class. And there are always those outside things that you have no control over…. The students don’t work together so you go back into your files, which… it’s not what we really want. We want them to work in groups, because we know that when they come out of school they [need to] be able to work together to be a team player. So if we’ve had them set in rows and rows and rows like I was taught, how are you a team player? Other than when you go to play football. How are you a team player? And what if you hate sports? Do you never learn to be a team player just because you don’t play a sport? (HINT, Dec. 6, 886)

This statement demonstrates that her rationale for encouraging communication is based on her perspective that students must learn to be team players. Furthermore, this statement reflects Helen’s vision of the nature of mathematics as a subject. Although she considers communication to be an important part of mathematics instruction, her vision is more practical and not strongly based on theories underlying calls for communication—that communication is important for each student’s learning and for constructing knowledge in a community.

Shelly, on the other hand, is more committed to implementing communication in her classroom. She demonstrated her commitment after a very negative experience in her classroom when students were not engaged in the lesson. Even though she decided to begin the next lesson with students in rows, she would still have them work together because, in her opinion, “[i]f you just leave [students] in rows…. you lose all of this conversation…all of this communication…… it’s not worth the risk, because then it’s individuals that won’t be working or thinking or doing stuff” (SINT, Nov. 26, 258). Shelly also explained that having students work together is important because it helps them develop conceptual understanding:

I like the kids to share and learn together, and think together because I know that it helps them when they’re bouncing ideas off of each other to clarify their thoughts. Reminded me of when I was re-teaching what I knew to my brother that I learned it better, just
because I had to talk about it and share and when he would ask a question about, “I don’t get how this works” even in me having to describe what I was thinking about how it worked, clarified my thinking so I try to give students opportunities to do that. And I... and that sort of tied with developing conceptual understanding and whole class discussions. (S_{INT}, Nov. 26, 207)

Shelly’s commitment to implementing communication and her acknowledgement of the importance of conversation for developing conceptual understanding demonstrate that her images of communication in mathematics teaching practice are based more heavily on theories underlying calls for communication. In particular, her vision of the importance of communication is based on her understanding of the importance of communication for constructing knowledge and understanding.

Despite differences in Helen and Shelly’s images and levels of commitment to implementing communication in their classrooms, there are some similarities in their practices. For instance, both teachers use activities that encourage student engagement in the classroom. Helen incorporates open-ended questions in her lessons and uses strategies from the *Think Literacy* (Ontario Ministry of Education, 2004) documents for mathematics to engage students in lessons. Even though she likes to maintain control over the direction of whole-class discussions, she has voiced her opinion that it is important for her students to contribute to lessons because they must “realize that they’re not just going to sit there and do nothing. They have to think” (H_{INT}, Jan. 23, 182). Shelly also uses strategies from the *Think Literacy* documents to engage students in lessons. However, her purposes for using these activities are somewhat different. While Helen’s purposes are more motivational, Shelly emphasizes the importance of these activities for pedagogical or learning purposes. Shelly explained that she uses “many literacy strategies to engage [students], including graffiti, four-corners, think/pair/share and investigations that allow them to develop their own understanding of concepts” (S_{DOC}, Nov. 18, 13).

Both Helen and Shelly also consider communication important for understanding student thinking and the processes that they use for solving problems, but there are subtle differences in their perspectives. Helen focuses on communication for management and motivational purposes,
while Shelly focuses on students’ learning. Helen demonstrated her appreciation for the increased emphasis on communication for understanding student thinking in her description of an experience that she had as a mathematics student in England. She explained that although she thought about the question correctly, since her final answer was incorrect, she would receive no credit for showing how she thought about the problem:

> It was literally, you had the right answer or you had the wrong answer and I had the wrong answer. So there wouldn’t have been any thought. It wouldn’t have even been read… That is one thing that had to change—that there has to be some thought. Like nowadays I would have got a mark. It would have been “wow, you did actually think about this. (HINT, Dec. 6, 964)

Helen’s motivational focus is demonstrated in her statement that giving students credit for communicating their thinking is “better than saying no, it’s wrong, you did nothing right” (HINT, Jan. 23, 639). She explained that not giving credit for communicating the process is not “really good for the kids’ self-esteem … if you go through the process and you go, ‘you got the idea…. you just made that mistake’ and you circle it … at least they got certain marks, and I like that” (HINT, Jan. 23, 639).

Shelly also likes students to explain the processes that are used in solving problems. Her focus, however, is on students’ learning. When students communicate processes to her, it shows her if they have conceptual understanding. Shelly’s perspective was demonstrated in an experience that she had with a Grade 10 test. On the test, she asked students “to determine if the triangle formed by joining these points was an equilateral, isosceles, scalene or right angle triangle. Students would calculate lengths of sides and slopes to compare and use the results to determine the type of triangle” (SDOC, Nov. 18, 60). On the same test she asked, “If you were given four points on a grid, how would you determine that the quadrilateral formed with these points as the vertices would be a rhombus?” (SDOC, Nov. 18, 62). Shelly expected that students would outline the procedure that they used in the first question. She found that there were students that “could do the first [question] but not the second (They had memorized problems like it without understanding the process)” (SDOC, Nov. 18, 67). She also had “students who could do
the second question but not the first (these students understood the process but had very weak skills and couldn’t perform the operations correctly)” (SDOC, Nov. 18, 68). From this experience, Shelly learned about the importance of balancing questions on tests that require procedural understanding and problems that require students to describe the processes that they use to show that they have conceptual understanding. This demonstrates Shelly’s view of the importance of communication problems for understanding her students’ learning.

In this section, I have demonstrated that despite some similarities in practice, Shelly and Helen have different images of communication in mathematics. The description in this section gives us a sense of different images and commitments that teachers may have in professional practice. I expand on this discussion in the next section by examining how Shelly and Helen’s images compare with those presented in the Ontario Mathematics Curriculum.

Images in the World of Developers

Helen and Shelly both support aspects of mathematics communication that are highlighted in Ontario Mathematics Curriculum documents. In particular, both teachers would agree that “[c]ommunication is the process of expressing mathematical ideas and understandings orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words” (Ontario Ministry of Education, 2005a, p. 16). Both teachers would also agree that in their classrooms, “[s]tudents communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, or the whole class” (Ontario Ministry of Education, 2005, p. 16). Helen’s support for this definition is clear in her explanation that “whether the student is communicating within their group, or the class, or myself, they’re actually communicating verbally, or with their quizzes, it’s all communication” (HINT, Dec. 6, 549). Shelly’s support for communication for a variety of audiences and through different forms is demonstrated in her practices and in her construction of a concept map that illustrates her perspective on mathematics communication (see Appendix E).
Helen and Shelly would also agree with the statement in the Ontario Mathematics Curriculum that “[e]ffective classroom communication requires a supportive and respectful environment that makes all members of the class comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher” (Ontario Ministry of Education, 2005a, p. 16). Both teachers develop communities in their classrooms that encourage students to be comfortable with talking and sharing. Helen, for example, voiced her opinion that this is essential in her classroom:

I want them to talk. I want them to share their ideas. I don’t want them to be scared of opening their mouths and somebody laughing at them. Because how can we expect them to be productive if they do not open their mouths because they’re concerned, ‘well I might say something foolish and somebody laughs at me’? Because that’s the biggest step that they can make is to actually open their mouths and speak because if they don’t do that then we haven’t taught them very well…. I want them to know that it’s a safe place, and that there isn’t going to be any giggling if they say something. If there is, I address it, because… if I left it, they’d just assume ‘well that is fine, the kids must be right, I must be stupid or something.’ (HINT, Dec. 6, 881)

Shelly also explained that she makes sure that her students feel comfortable with making contributions in the classroom because if they do not “feel comfortable, they won’t get involved in a whole class discussion,” which she considers important for learning (SINT, Nov. 26, 207). Shelly also stated that a “positive side effect of encouraging communication and math talk in [her] classroom is that many students feel comfortable volunteering answers and strategies” (SDOC, Nov. 18, 34).

According to the Ontario Mathematics Curriculum, “[t]eachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate” (Ontario Ministry of Education, 2005a, p. 16). Several examples of opportunities are outlined in the documents. One suggestion is to “ensure that students are exposed to and use new mathematical vocabulary as it is introduced (e.g., by means of a word wall; by providing opportunities to read, question, and discuss)” (Ontario Ministry of Education, 2005a, p. 16). In this research, both teachers use word walls to make sure students are exposed to vocabulary. Another strategy that both Helen and Shelly find helpful is to use “funny names” to help students remember
vocabulary. Shelly explained that “funny names” are helpful but that it is also important to use proper terminology:

Before OSS [Ontario Secondary School Diploma] I used to use funny names and mnemonic devices all the time to help students remember procedures because I didn’t think that they would be able to use and understand proper terminology. As a result when they moved from my class to the next grade they often had to learn new words for certain operations often their new teacher’s pet terms. Now, since we are all stressing proper terminology, I have realized that students certainly can handle it and we are much more consistent. Distribution is distribution in everyone’s class; I will still use some funny expressions to help them remember certain procedures but I always use proper terminology, and my students are very capable of using proper terminology as well. (SDOC, Nov. 18, 48)

Helen also voiced her opinion that she must use “very simple, basic language” and that she must relate concepts to everyday life so that her students see relevance. She also uses funny names with her students but acknowledges that “yes, we do have to have the language for the mathematics” (HINT, Dec. 6, 273).

When comparing Shelly’s images of mathematics communication to those represented in curriculum and resource documents, it is clear that her vision is aligned with that of members of the world of developers. She understands the Hufferd-Ackles et al. (2004) math-talk framework and works to support a math-talk learning community in her classroom. Her perspectives and practices are also based upon theories underlying calls for communication. Despite her support for the communication process, Shelly faces challenges in implementing a math-talk learning community with students in her *Foundations of Mathematics, Grade 9, Applied* class. The main challenge that she faced with the class that she was teaching when this research was conducted was with low student confidence. Because of this challenge, which is discussed in more detail in a subsequent section below, Shelly was not able to implement a completely developed math-talk learning community (SINT, Nov. 26, 355).

Even though Helen’s images of communication are similar to those presented in curriculum documents, her vision is of a practical nature and is based more heavily on her classroom experiences. She interprets communication messages based on her expectations of
what works best with her students. For example, in her classroom, it is important for students to communicate “for different audiences…and purposes…in oral, visual, and written forms” (Ontario Ministry of Education, 2005a, p. 21) because English is a second language for many of them. Since her students struggle with the written form of communication, they have opportunities to communicate in a form that works best for them (HINT, Nov. 27, 234). Since Helen’s images of communication are influenced heavily by classroom experiences, her reasons for implementing communication are more practical—communication is a way to engage students in learning the rules of mathematics—rather than based on theories underlying calls for communication.

The discussion in this section demonstrates that when we compare images of communication in mathematics held by two members of the world of professional practice with those presented in curriculum documents, there are similarities but there is also a potential “gap between images” of this mathematical process (MacDonald & Walker, 1976, p. 45). The participants in this research have different motivations for encouraging communication. Both teachers work to implement the same curriculum and use activities from resources that support the philosophy of the curriculum but Shelly emphasis the importance of communication in connection with student learning, while Helen focuses on the importance of communication for motivating her students.

**My Images of Mathematics Communication**

In this section, I describe my own images of mathematics communication and how they have changed over time. Originally, my experiences as a mathematics student, tutor, and teaching assistant influenced my perspectives on mathematics communication. My vision of this mathematical process evolved through my involvement in curriculum analysis research and experiences in the Master of Education program. The most significant influence on my images of mathematics communication comes from experiences with conducting this research. Reflecting
on this concept and exploring what this looks like in theory and in practice have contributed to my understanding of this mathematical process.

Before becoming a student in the Master of Education program, my perspectives on mathematics communication were influenced by my experiences as a mathematics student and tutor, and by my involvement in a curriculum analysis research project. In high school, my perspectives on mathematics communication were more traditional. The emphasis was on mathematical language and form of solutions. As a student, I was expected to use correct mathematical notation, to underline answers, and show the correct steps in solving problems. My vision of the nature of mathematics was also quite absolutist—the teacher and the textbook were the authorities of mathematical knowledge.

My views of communication and the nature of mathematics changed when I completed an undergraduate program with a major in pure mathematics. In this program, I began to value the importance of discussing mathematics and explaining thinking for contributing to my own learning. Discussing mathematics with my peers and professors, explaining my mathematical thinking, and writing mathematical proofs helped me to clarify my thoughts and provided me with a deeper understanding of mathematical concepts. In high school, I found that I often practiced solving exercises and was able to describe steps taken in solving problems, but I did not always have a deep conceptual understanding. In university, when I was expected to provide elaborate explanations of my reasoning, I became better at explaining why things work rather than just how they work.

Through my experiences as a teaching assistant for undergraduate courses in discrete mathematics and linear algebra, I began to understand the importance of communication in assessment. As a teaching assistant, I graded assignments and found that students often provided unclear explanations and that they faced difficulties with communicating mathematically. In some cases, this made it difficult for me to see what the students actually understood. For example, some students would provide in-depth explanations. This made it easy for me to understand their
thinking and to assign marks. Other students would come to conclusions that appeared to be appropriate but they would leave out information, which made it difficult for me to know if they actually knew what they were doing or if they omitted details because they could not make connections. After facing this type of challenge as a teaching assistant, I began to value the importance of a student’s ability to provide clear and concise explanations in mathematics.

Tutoring undergraduate mathematics students at a math-help centre and working with a student in the Grade 9 Applied program also influenced my perspectives on communication. Through these experiences, I saw that communication is important for helping students clarify their thoughts, for helping me understand their thinking, and for developing conceptual understanding. Oftentimes, when I worked on problems with students, I would ask them to think about what they knew about the problem. For example, I would ask students to talk about what they thought the problem was asking and the information that they thought might be helpful for solving the problem. When they reflected on the problem and what they knew, they often made connections that they did not make when they were thinking on their own. One difference between tutoring undergraduate students and tutoring the student enrolled in the Grade 9 Applied course was that the Grade 9 Applied student was less confident in her ability to do mathematics. Consequently, she required more guidance and support. Without the support, she was more willing to give up.

My views of the nature of mathematics as a subject and the importance of communication for constructing knowledge changed when I was in the undergraduate program. I began to see that mathematics is not a completely absolute subject. I noticed that mathematics textbooks, teachers, professors, and researchers did not always have the answers and that mathematics was continuing to change. I learned about the flaws in Euclid’s Elements (300 BC) and about Kurt Gödel, who showed that it is impossible to prove all mathematical truths and therefore establishing absolute certainty. Even though I began to understand that mathematics as a subject is not absolute, I still did not see communication as an essential part of mathematics instruction. I was both comfortable
and successful in a teacher-directed classroom environment so I did not see the importance of student-centred communication. I did, however, value the importance of talking about mathematics and constructing understanding among my peers because this became an important part of the undergraduate mathematics experience.

After completing my second year in the undergraduate program, I participated in a curriculum analysis project in which I analyzed messages presented in Ontario Mathematics Curriculum documents (Ontario Ministry of Education, 2000b, Ontario Ministry of Education and Training, 1997, 1999). Through the analysis, I noticed that there was a significant push for communication in mathematics instruction and assessment. Since my classroom experiences as a student were primarily in teacher-directed environments, I became interested in how teachers have embraced this emphasis in high school mathematics classrooms. Consequently, I decided to explore this further in the Master of Education program. In this program, I learned more about theories underlying calls for communication and research related to communication. I have also had various opportunities to converse with mathematics educators and researchers, to attend workshops and conferences that focus on mathematics communication, and, most importantly, to conduct research for this thesis. Through these experiences, I have developed a more theoretical and a practical understanding of what mathematics communication is and why it is important.

I now see communication as important for constructing knowledge in a community and for developing conceptual understanding. After observing the practices of teachers who are working to support communication, I now understand the importance of a student-centred classroom environment. Reflecting on the meaning and value of mathematics communication in this research has helped me to see that different people hold different images of mathematics communication. Consequently, it is important for me to illustrate my own images of this mathematical process. My vision is represented in a concept map in Appendix H. This concept map represents my vision of mathematics communication in terms of what it means for mathematics teaching and learning and for mathematics as a subject.
Translating Images of Communication into Mathematics Classroom Practice

Since findings from this research provide insight into the dynamics of a *Foundations of Mathematics, Grade 9, Applied* class, this section begins with a description of the challenges and rewards involved with teaching in this context. Following this description, I highlight similarities and differences in Shelly and Helen’s approaches to implementing communication in mathematics instruction and assessment. I also explain how these teachers handle the challenges involved with implementing communication in their classrooms.

*Foundations of Mathematics, Grade 9, Applied*

Findings from this research give us a sense of what it is like to teach and work to support communication in the *Foundations of Mathematics, Grade 9 Applied* course. In particular, findings demonstrate that this is both a challenging and rewarding course to teach.

**A challenging context.** Data collected across Ontario suggests that students in the *Foundations of Mathematics, Grade 9 Applied* course have demonstrated low levels of academic achievement in mathematics. This was demonstrated by King et al. who found that, in 2003-2004, “failure rates [were] notably high in the Grade 9 Applied course” (2005, p. 38). This is also demonstrated in results of the EQAO provincial standardized assessment of achievement for students in Grade 9 mathematics. Even though results from 2006-2007 indicate that “over the past five years, the percentage of students taking applied mathematics who attained a Level 3 [the provincial standard] or above has increased by 14 percentage points,” only 35% of these students are at or above the standard. Similarly, teachers in this study found that students in these classes struggle with mathematics.

In both cases, teachers faced challenges with student confidence and attitudes towards mathematics. Findings from the EQAO student questionnaire also indicate that students in the *Foundations of Mathematics, Grade 9, Applied* course demonstrate low confidence in mathematics in comparison to students in the *Foundations of Mathematics, Grade 9, Academic* course. Findings from the 2006-2007 survey suggest that students in the Grade 9 Applied course
do not believe that they are not good at mathematics. Only 25% of female and 41% of male students indicated on this survey that they are good at mathematics (EQAO, 2007, p. 69). Closer to half of the students in the Grade 9 Academic course indicated that they believe that they are good at mathematics (43% of female students and 57% of male students). Findings from the EQAO student questionnaire also suggest that a large percentage of Grade 9 Applied students across Ontario do not enjoy mathematics. Only 28% of female and 40% of male students who responded to the 2006-2007 survey indicated that they “like mathematics” (EQAO, 2007a, p. 69), whereas closer to half of the students in the Academic course indicated that they like mathematics (47% of females and 57% of males).

The class dynamics in the *Foundations of Mathematics, Grade 9, Applied* course influenced the implementation of communication in this context. Shelly, for example, described how the dynamics of her Grade 9 Applied class prevented her from developing a high-level math-talk learning community:

> Applied is a problem because they don’t have the confidence, and they don’t trust themselves to be the ones to ask the questions. And sometimes they’re really not interested in asking questions. A lot of them don’t like math or have it in their head they don’t like math. And so they’re not curious about how things work or what if you did it that way or what if you tried that. And so it’s really hard to get a math-talk learning community where you’re the one posing and answering the questions. I can get answers but I’m still posing. (S_INT, Nov. 26, 497)

In Shelly’s opinion, she was able to develop a math-talk community in the other classes that she was teaching at the time the research was conducted. In the semester after this research was conducted, Shelly was teaching two *Foundations of Mathematics, Grade 9, Applied* classes. She explained that she thought that she might get much closer in developing a math-talk community in these classes, but that “[i]t probably still [wouldn’t] be perfect because of the nature of the students and their insecurities about math” (S_INT, March 5, 347-349).

Since the students in Shelly and Helen’s Grade 9 Applied classes have low self-confidence, getting the “right” answer is important to them. Shelly explained how her students had low self-confidence:
… but this particular class has no self-confidence, and you can tell: ‘is this right? Is this right? Am I doing this right? I don’t know what I’m doing. Is this what I have to do?’ and then they will change their mind because someone will say, ‘you have to do this.’ And they will say, ‘I told you that was wrong!’… so, getting the right answer is important to them but they have no confidence in their ability to do that. And I’ve been working on it but it’s not really changing very fast. They still are struggling with that. (SINT, Nov. 26, 349)

Similarly, Helen explained that “this is the clientele” that she is working with… with the Applied [student]…they want to know, ‘am I doing it right?’… and they need that because they are uncertain” (HINT, Nov. 27, 143).

An additional challenge that Helen and Shelly faced in their Foundations of Mathematics, Grade 9, Applied classes was caused by having older students in the class. From Shelly’s perspective, having three Grade 10 students in her Grade 9 Applied course had a significant influence on the classroom dynamics. Shelly described the challenge:

So it was having three students that were a presence in the classroom… they influenced because they were older and street smart so they had a presence in the classroom exhibiting behaviours and attitudes that I didn’t want in my classroom. So that was a challenge. (SINT, March 5, 64)

Shelly explained that these students “really inhibited [her] 9’s - they had a bigger impact than [she] realized” (SINT, March 5, 93). She noticed the impact of the Grade 10 students after she started teaching two different sections of the Foundations of Mathematics, Grade 9 Applied course with only Grade 9 students in the second semester.

Helen also faced challenges with having older students in her Grade 9 Applied class. She explained that the students in this class were “not gelling together because they really [did not] have other things in common” (HINT, Dec. 14, 325):

The main difference I saw with this 9 Applied class was the ages… I even had a student from Grade 12 in it… massive age difference - maturity, everything. All the way through 9 to 12 in there and they had their pockets in the classroom. It’s just really interesting. So they did their own grouping, just these were the pockets and that’s what they did and they never ever moved from them. (HINT, Jan. 23, 762)

Shelly also faced challenges in finding students who would work together. She explained that “in an Applied class, it is more challenging to match up partners and groups” (SINT, March 5,
She found that there are some students “who have strong language difficulties and so you can’t match them with anybody because they won’t talk” (SINT, March 5, 393). For Shelly, one problem in finding students who would work together is that students in her Grade 9 Applied classes are often “looked after by Special Education,” by “ESL [English as a Second Language]” because “they have language difficulties,” or are “being monitored by the Student Success team” (SINT, March 5, 392-407). Like Shelly, the majority of Helen’s students were either Special Education or ESL (HINT, Dec. 6, 308).

Since students in the Applied course often believe that they cannot succeed in mathematics, both Shelly and Helen stress the importance of helping them develop positive attitudes towards the subject. Shelly, for example, described how the background knowledge of her students was not very strong and since they believe that they are not good at mathematics, it is important to set them up for success so that they are motivated to try harder and overcome this belief:

Most of them have taken Applied because they believe they can’t do math. Somewhere along the line they’ve had a bad experience. Almost all of my students in this semester, almost all of them failed Grade 8 math so they don’t believe they can do it and when I gave them back their first test, a whole bunch of students have said, ‘oh miss can I take this home and show my mother’. ‘Of course you can have it home, I want your mom to sign it’. ‘This is the best I’ve ever done….’ We worked hard to make sure they knew it…. So there’s all kinds of reasons why that might happen but when you set up your class, which I try to do so that they really are successful and they understand and they have strategies…. Because we work really hard at doing that and they’re successful. They start to believe….Success motives kids to try harder. (SINT, March 5, 179-208)

Helen also finds that students in her Grade 9 Applied class think that they “suck at math” (HINT, Jan. 23, 324). She voiced her opinion that if they were “strong students, they wouldn’t be in [her] classroom…. There’s something making it so they can’t succeed in the academic classroom” (HINT, Jan. 23, 323). She explained that the majority of students in her Applied courses “get frustrated and they’ll tell you right from day one, ‘I suck at math.’ You’ve got to build their confidence” (HINT, Jan. 23, 324).
A rewarding experience. Even though both teachers who participated in this study described challenges involved with teaching students who struggle with mathematics, they also explained how rewarding the experience can be. Shelly, for example, explained that she has to work harder to teach the Applied course but that it is a rewarding experience for her:

I don’t have to work nearly as hard in 9 Academic as I do in Applied. It’s just how much they need you. And the impact that you can have if they can be successful… if you care about math and you care about them and they can be successful you have a huge impact. One of the teachers in my department said I think I’d like to try Applied because she just hears me telling some of the stories…. Applied is good because the kids need you so much. And you make a difference in their life more so than…. I mean a good Academic student is going to do well no matter who their teacher… not an Applied student. (SINT, March 5, 536-542)

Helen is passionate about teaching in general. She stated that “it doesn’t really matter what I teach, as long as I’m teaching. I enjoy teaching so it doesn’t matter” (HINT, Nov. 27, 95). She teaches mathematics in Applied-level courses because she has a reputation for helping these students succeed (HINT, Nov. 27, 88). She “believe[s] that they can do better and with more practice and more assistance, and maybe working it in a different way…. then maybe the next time they will get it or the time after that” (HINT, Nov. 27, 113). Helen demonstrates her enthusiasm for helping students who struggle with mathematics by mainly teaching College, Locally Developed, and Applied-level courses. These courses are designed for students headed for college or direct entry into the workplace, rather than university. Helen teaches these courses because she is successful in helping these students graduate.

It is important for teachers to try to understand and connect with students in the Grade 9 Applied course. The relationships that teachers develop can also be rewarding. Helen described the importance of connecting with students by explaining that the “main thing is, you’ve got to know their names. And if you don’t know their names they think that you don’t care…. [T]hey’re very very sensitive to those things…. They have to realize that you’re real, that you go and do your shopping and they see you and they’re, wow you’re shopping” (HINT, March 3, 298). Helen explained that “if you are direct with them and you work with them and they know you as a
person, you’re more likely to get something back, where if you are just the person that lectures at the front of the classroom, with this type of student, it doesn’t work as well (H\textsubscript{INT}, Dec. 14, 596).

Helen described the importance of understanding that what works with one class may not work with another. She also explained that if you know your students, they are more likely to tell you what they do not understand:

[Y]ou always get the feedback from the students. If they know you and you know them, you’re going to get that feedback. They’re going to say to you, “miss I didn’t understand this”. So you’re going to have to find different ways. So when I write a lesson for the class I’ve always got room for change because what may work with one class doesn’t necessarily work with another class. And I need to know that these are individuals… that these are different groups and those groupings work differently. (H\textsubscript{INT}, March 3, 333)

Connecting with students is important not only in the classroom but also makes teaching the *Foundations of Mathematics, Grade 9, Applied* course a rewarding experience. Shelly, for example, described the impact that a teacher who helps these students succeed can have. Furthermore, she explained that when she walks into the classroom, she explained that if you connect with them and help them to be successful, they will remember that:

When I walk in [the classroom]…. I get a good warm reception. That’s the one nice thing about Applied… If you connect with them, then… they drop in…. They are really true blue. When you get to know them… or if you help them to be successful when they didn’t think they could they really remember that. So, Applied kids are really great that way, more so than Academic…. I have students who want to be in my class because they know they’re going to get good instruction and they have a good chance of being successful so they’ll do that. Two years later, they hardly talk to you in the mall…. Applied kids scream to me at the mall…. Truly it’s different. It really sums up the difference how they relate to their teachers. (S\textsubscript{INT}, March 5, 525)

The relationships that Helen developed with her students were also clear as I spent time in her classroom. The majority of our meetings took place in her classroom and on a number of occasions students, including students who were not in her class at the time but she taught in previous years, stopped by to talk to her about mathematics or to share their successes in mathematics.

Now that I have described some of the challenges and rewards involved with teaching the *Foundations of Mathematics, Grade 9, Applied* course, I will describe similarities and differences
Communication in Mathematics Instruction

Helen and Shelly work to support student-centred communication in their classrooms. Both teachers also find that strategies and activities from resource materials, such as TIPS (Consortium of Ontario School Boards, 2003), TIPS4RM (Consortium of Ontario School Boards, 2005, 2007) and Think Literacy (Ontario Ministry of Education, 2003, 2004), are helpful for supporting this mathematical process (SINT, Nov. 26, 189; SINT, March 5, 60; HINT, Jan. 23, 656).

Helen explained that there are “a lot of good things out there that help. It’s just getting time. Once you’re in the classroom that’s it,” but the TIPS and Think Literacy documents are helpful because they “always [have] some neat little things to do” and “different ideas for group work” (HINT, Jan. 23, 664).

One strategy from the Think Literacy document for mathematics (Ontario Ministry of Education, 2004, p. 10) that both Shelly and Helen like to use in their classrooms is the Anticipation Guide strategy. “An Anticipation Guide is a series of questions or statements related to the topic or point of view of a particular text. Students work silently to read and then agree or disagree with each statement” (Ontario Ministry of Education, 2004, p. 10). Helen described an example of how she uses this strategy to engage students in her lessons:

I will say, ‘oh you agree or disagree with me?’ And so I’ll say, ‘okay, all triangles have three sides.’ And so they either agree with me or disagree. But then, they’ve got to come up with a reason why they agree or disagree… [A]nd you put them on the spot, and they get used to that. You’ll always have somebody sitting on the fence and I’ll say nu un, you’re not sitting on the fence you have to chose. (HINT, Jan. 23, 158)

Shelly also likes to use the Anticipation Guide strategy in her classroom. She explained how useful this strategy is for eliciting information about her students’ prior knowledge:

Using literacy strategy helps me, because when I choose one I have to think about what I’m getting out of it. So if I have an anticipation guide for the kids then, and I’m asking them for questions that I’m getting them to anticipate the answer to. Then I have to do
something in my lesson that addresses each of those four questions so that at the end of it they’ve got… they’ve agreed and disagreed appropriately. So it forces me to plan better and it lays out on the table what they know in terms, or what their prior knowledge. (S\textsubscript{INT}, Nov. 26, 189)

Messages in the \textit{Think Literacy} document for mathematics (Ontario Ministry of Education, 2004) also acknowledge that one purpose of the Anticipation Guide strategy is to “Help students to activate their prior knowledge and experience and think about the ideas they will be reading” (Ontario Ministry of Education, 2004, p. 10).

Helen and Shelly take somewhat different approaches to guiding communication in their classrooms. Both teachers encourage student contributions by asking questions but there is a difference in the types of questions that they ask. Furthermore, students in their classes are willing to ask questions. During lessons, Shelly often stands and walks around the room. When students work together, she visits each pair or group of students. She explained that she does this because if she does not, the students will stop when they do not understand something and they will not ask for help:

They will quit until their questions are answered…. So suppose they’re doing an optimization problem and it’s a 3-sided figure and they forget how to calculate the length. They will wait. So sometimes they will ask their partner and get it or they will wait. When I go around and say ‘oh, you’re not working’ [they will say] ‘but, I don’t know how to do this.’ And then I will show them how to do it and they will work again. But they’re not… where in some classes they’ll say miss, miss, I have a question. My Academic class, I have a question, I have a question because they want to keep going. These guys will use it as an excuse to chill out for 10 minutes. (S\textsubscript{INT}, March 5, 154)

When Shelly walks around the room, she also gets feedback on her students thinking, which helps her plan the direction that she will take in the lesson. For her it is important to listen “to [students] discuss their ideas and giving each other feedback on how the investigation is going” (S\textsubscript{INT}, Nov. 26, 226).

During lessons, Helen spends more time guiding lessons from her desk at the front of the room. In the lessons that I observed, Helen began by talking to students from her desk and then during whole-class discussions, she stood at the front of the room and wrote on the front board or on the overhead projector. When I told Helen that I noticed that she often talks to students from
her desk, she explained that she does this because she prefers to “be at their level” (HINT, Dec. 14, 295). In her opinion, with these students, “if you stand over them, they sometimes perceive that as that you feel as though you’re bigger than them.” So Helen likes to “keep to one level” because of “the way that it’s perceived by the students” (HINT, Dec. 14, 297).

Helen writes everything down because her students tend to be visual learners. She explained that the overhead projector is important because when she uses it, she can write and at the same time maintain eye contact with her students:

I also believe in using the overhead rather than the board because you can actually deal with them. As soon as you start turning around, because of their attention span, they can often get into mischief. So, I try desperately to use the overhead as much as possible so I have eye contact at all times with them. (HINT, Dec. 6, 267)

Shelly also writes things down for her students during instruction. The main difference in how these teachers guide instruction in their classrooms is that Shelly prefers to move around the room, while Helen prefers to be at the same level as her students when she is not writing.

Differences in Shelly and Helen’s roles in guiding instruction during whole-class discussions are also demonstrated in the descriptions of their practices presented in Chapter Four of this thesis. During whole-class discussions, both teachers tend to talk to students from the front of the room. Even though Helen supports “diversion” in her lessons, the description of her practices demonstrates that her lessons are more heavily guided—students respond to the questions that she asks and they ask her questions when they do not understand.

Helen guides lessons heavily based on her expectations of her students. She explained that “[y]ou have to be guiding them all the time and...if they are not attentive it means that you have gone too quickly or they haven’t been listening when they should have been and you’ve got to direct your attention to them” (HINT, March 3, 296). While Shelly’s lessons are less directed and based more on student contributions, Shelly acknowledges that because the students in her Grade 9 Applied class have no confidence she needs to set them up for success so lessons are more heavily guided than she would like them to be:
They have no confidence so if they do… if I let them go take the investigation wherever it would take them. And I will if they’re going in an interesting or a direction that will give them some good results then I like that but if they’re heading and they’re calculating everything all backwards and they’re going to get all the wrong answers then I don’t let them continue. I try to ask some probing questions to say ‘no, what are you counting here? You’re counting sides, okay, how many sides would there be in the square? Let’s count them….’ [B]ecause they won’t take a risk if they’re wrong so I’ve got to set them up for success and I want them to do it right so that they know what they’re doing and they don’t get discouraged so… some of these investigations are very guided. I would prefer them to be less guided but this particular class has no self confidence. (SINT, Nov. 26, 349)

Clearly, even though lessons in Shelly’s classes are based more on student contributions than Helen’s, both teachers acknowledge that the students in their Grade 9 Applied classes need a lot of guidance.

The description in this section illustrates some similarities and differences in activities and strategies that Helen and Shelly use to guide instruction in their classrooms. Next, I will compare the activities and strategies that they use for implementing communication in mathematics assessment.

**Communication in Mathematics Assessment**

Shelly and Helen assess student communication through a variety of tasks—including quizzes, tests, journals, and performance tasks. Even though they use similar types of activities, there are differences in how they assess this mathematical process. One similarity among the assessment practices of these two teachers is how communication is graded on tests. In particular, both teachers have adopted the curriculum document’s direction to grade communication by assigning students a level out of four. They also highlight test questions that will be marked for communication. Shelly explained what this looks like on her tests: “Certain questions are highlighted with a communication symbol (a C inside a circle) that are assessed for clear and concise explanations. For these questions students are asked to explain, describe, justify, reflect, compare representations or make connections” (SDOC, Nov. 18, 19).

In both Shelly and Helen’s cases, students’ responses to communication questions contribute to the overall mark that they receive for communication on the test.
Both teachers also take a holistic look at a test before assigning an overall mark for communication. Shelly explained that when she marks tests, she gives feedback on communication questions first. For example, if a student provides an excellent explanation, she will make a note on the test. She also gives feedback on their use of terminology, conventions, and form. After making notes on the test, she takes a holistic look at her feedback before assigning a level for communication (SINT, Nov. 28, 363). For many of the tests that Helen marks, she also looks at tests holistically before assigning a final level for communication. She explained that when she does this, she will assign a mark based on “what they have done – lets say they’ve communicated to me that they know 70%, 75%. I’m going give them 3 out of 4 because that’s what they communicated to me that that’s how much they know” (HINT, Dec. 6, 84).

Shelly and Helen hold different perspectives on the value of rubrics for assessing communication. Shelly believes in using rubrics to assess communication, while Helen does not. For Shelly, rubrics are important because with a good rubric, it is easy to assign a level for communication. Shelly also makes sure that her students have a copy of the rubric that she will use to assess a task that is graded with respect to communication because a “good rubric gives as much information to the kids as it does to [the teacher]” (SINT, Jan. 18, 556). Helen, on the other hand, explained that the reason she does not like rubrics is “because you put a level on it then you have to put a number on it, so why not just put a number on it?” (HINT, Dec. 14, 42).

Shelly finds that with a clear rubric it is easy to assess communication because she can circle, on the rubric, to show her students what needs work. Helen does not find rubrics helpful because, in her opinion, she can “circle all the inappropriate things” directly on tests, (HINT, Dec. 14, 42). Helen also does not “think [rubrics are] very helpful to the students either because if these students could actually work towards that perfect 4, they would not be in with [her]” (HINT, Dec. 14, 46). In Shelly’s opinion, rubrics are actually important for helping students reach a Level 4 because rubrics show them what they should aim for to reach this level. Shelly explained that a
rubric is important for showing students her students what to aim for and that when she develops levels for a rubric she needs to be consistent in her expectations:

And you say, I want to mark communication, I have to have a rubric on there and I want to tell them what it is that I want to see - clear, concise explanations, do I want form… the narrative link to the mathematics…? That kind of thing you have to tell them…. It’s that whole age old Level 4 should be something that they don’t know what they’re aiming for… so nobody can get a Level 4…. When it first came out, people thought that level 4 was above grade level. No, it’s, your expectation. Your Level 3 is what you reasonably expect kids to be able to do, Level 4 is above that but it’s not above grade level. I do not expect them to do grade 10 work in grade 9.… [M]y Level 4’s often will connect to other situations, have more than one solution, more creativity, complexity… depending on the task. So Level 4 is above expectation, not above grade. (S_INT, Jan. 18, 572)

The differences between Shelly and Helen’s perspectives on the value of rubrics for assessing communication are clear. Their opinions reflect their visions of the nature of mathematics. In particular, Helen holds a more absolute vision of mathematics so she assesses the correctness of the processes in solving problems. Shelly, on the other hand, values multiple approaches to problems. Consequently, rubrics are essential for assigning communication marks because proper communication does not depend on knowledge. For example, while Shelly acknowledges that “communication questions provide [her] with an opportunity to see how well students understand concepts or how they are thinking about them,” “good thinking and communicating doesn’t depend on content” (S_DOC, Nov. 18, 24).

Now that I have described the Foundations of Mathematics, Grade 9, Applied context and compared Helen and Shelly’s strategies for incorporating communication in instruction and assessment, I explain challenges that they face in implementing communication and their perspectives, on and strategies for overcoming, these challenges.

**Overcoming Challenges**

The discussion of relevant literature that is presented in Chapter Two of this thesis demonstrates that teachers may face challenges when implementing communication in the mathematics classroom. Data collected for this study gives us a sense of challenges related to implementing communication in any mathematics classroom and also, more specifically, to
implementing this mathematical process in the Foundations of Mathematics, Grade 9, Applied course. Challenges that emerged in this research included: (1) classroom dynamics; (2) selecting appropriate activities; (3) developing rubrics; (4) time; and consistency among teaching staff.

**Class dynamics.** The impact of the classroom dynamics on a teacher’s attempt to implement student-centred communication is demonstrated in the above description of the challenges involved with teaching the *Foundations of Mathematics, Grade 9 Applied* course. In both cases, because of the characteristics of students in these contexts, it was difficult for the teachers to encourage aspects of communication that they considered important. Shelly, for example, was unable to reach a fully-developed math-talk learning community in her classroom. Helen, on the other hand, found that her students did not always work well together, which made it difficult for her to encourage group work. Based on their experiences, both teachers developed strategies that they found helpful for working to overcome these challenges.

Even though Shelly values the importance of developing a math-talk learning community (Hufferd-Ackles et al., 2004) in her classroom, she was unable to implement a fully developed community with students in her Grade 9 Applied class because of the dynamics of this class. In particular, she found that the Applied class is a problem because students are not confident, they believe that they do not like math, and they are not interested in asking questions (S\textsubscript{INT}, Nov. 26, 497). The major influence in her first semester Grade 9 Applied class was having three older students in the class. In the second semester, when the students in her class were all the same grade, Shelly was more successful in working towards a math-talk learning community. She found that although some of her students were still “struggling with the mathematics,” they were willing to “try and engage in the activities” (S\textsubscript{INT}, March 5, 85).

Shelly was more successful in developing a math-talk learning community with students in her second semester *Foundations of Mathematics, Grade 9, Applied* classes. The students in this class were more confident. Shelly explained that having a SMART Board in her classroom for the second semester helped because students “get a chance to go up and use the SMART
Board and show what they know on the SMART Board” (S\textsubscript{INT}, March 5, 85). Shelly voiced her opinion that the problem with the group of students in her first semester class may have been that they “didn’t have confidence” but a major difference, which helped her develop a community in the second semester was having a SMART Board (S\textsubscript{INT}, March 5, 133). She explained how having a SMART Board influenced the type of community that she was able to develop in her classroom:

It could be that that particular group [first semester Grade 9 Applied class] didn’t have confidence. I also have… there’s one big difference. Now when we go up we can talk and draw on the SMART Board, it’s more fun and maybe that’s the difference too. Like it maybe isn’t just about confidence but it’s less risky to go up and draw a picture with someone else. So without the SMART Board maybe there is more risk. But it’s different…. which is good because it’s positive. (S\textsubscript{INT}, March 5, 133)

Even though Helen values group work in the mathematics classroom, she found that students in her class did not work well together. A number of students in her class were of different ages and therefore had different interests and levels of maturity, so they developed “pockets” in the classroom (H\textsubscript{INT}, Jan. 23, 762). Helen described one strategy that she uses instead.

They can’t work together. They just can’t do that. So what I’ve normally done instead of doing that. I usually get my computer out and there’s a website…they have lesson plans but they also have little games and… I put this unit in and it would come out with questions doing usually silly things, so it keeps them amused. Really interesting graphics and there’s four answers and you have to chose the right one… that can work and I give candies out. That will work but as soon as you try to group them and I’ve tried all different groups with them, it does not work. (H\textsubscript{INT}, Dec. 14, 341)

Helen also explained that she likes to play a jeopardy game with her students because she gets “a lot more feedback, especially when there’s treats involved” (H\textsubscript{INT}, Nov. 27, 299). She uses “multiple choice for the answers and they get candies if they get the right answer. Or I’ll have them write on the boards and anybody that has the right answer, they all get candies” (H\textsubscript{INT}, Nov. 27, 301). Her strategies for encouraging communication demonstrate her focus on communication for motivation and management rather than on learning.
**Selecting appropriate activities.** On several occasions, Shelly discussed the significance of selecting appropriate activities for encouraging communication in the classroom. The importance of selecting appropriate tasks is also demonstrated by “[academic task researchers” (Henningsen & Stein, 1997, p. 525). According to Henningsen and Stein, a mathematical task is “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical concept, idea, or skill” (p. 528). These researchers demonstrate the importance of selecting appropriate activities by illustrating how the task that is selected to be the topic of whole-class or small group communication can affect the quality of student communication in the classroom. These researchers found that when teachers chose tasks that were inappropriate with respect to clarity and specificity of expectations, students’ did not engage in high-level thinking.

From Shelly’s experiences, she has also found that it is important to select appropriate activities and to make sure that students are not only engaged in them but that they have a sense of why she has selected the task. Shelly explained the importance of selecting appropriate activities and how difficult it can be for teachers:

> So you don’t do activities to do activities. You do activities to pull out the important ideas in math that you want to present. Activities are important if they can do that for you, and if they’re not then they’re not worth doing. It’s not about fun - although you want them to have fun while they’re doing stuff. It’s about connecting the activity that you’re doing to the math that you want them to learn and that’s hard. That’s hard for a lot of teachers to do. And I’ve got 20 years experience so it’s hard. (SINT, March 5, 280)

This statement demonstrates Shelly’s focus on encouraging communication activities to promote learning. Even though Shelly has faced challenges in selecting appropriate activities, she does not find it difficult anymore. She explained that now she sees them “everywhere” (SINT, March 5, 317). In her earlier years of teaching, she would find activities that she thought were great but she “didn’t really know why [she] did them” (SINT, March 5, 317).

Shelly emphasizes that it is important for teachers to think about, “where is the math” when they select activities (SINT, March 5, 321). To help teachers in her department select appropriate activities, Shelly encourages teachers to discuss and analyze activities. She described,
for example, how she brought the whole department together to analyze tasks when the Grade 9 curriculum was revised:

[When] they first introduced the changed Grade 9 curriculum we brought in our whole department and got all these activities and we sat down to analyze what we had done. We said, ‘what was the point of some of the activities?’ So we went… and our line became ‘Where’s the math?’ So we thought, ‘well this activity was really good because it’s got at these ideas’. Then we did this one but it didn’t get at any new ideas and we spent a whole day on it - let’s pull that one out. That’s how we decided… and at first—and it was as a group—we said ‘Where’s the math?’ ‘What’s the math that was important? Is it new math? Is it new learning? Is it worth doing?’ Because the activity was fun but then it has got to be worth doing or why are you doing it? And we recognized that we had way too many activities as a group, because we just… it was like a new way to do business. Oh, this will be really fun for the kids, this will really help the kids… So we did a lot of that. Wait a minute, where’s the math? That’s my first question, if I’m going to do this, where’s the math? Or, here’s the math, can I think of something that will help the kids to see that in a way that’s more fun than me just telling them about it. So you can go about it from both ends. (SINT, March 5, 317)

From spending time in the mathematics department office, I observed teachers talking about activities on a number of occasions. Since teachers may face challenges in selecting appropriate activities, it may be helpful to share experiences with colleagues and analyze activities together.

**Developing rubrics.** A second challenge that Shelly discussed involved developing rubrics. Although she acknowledge the significance of using rubrics to assess communication and thinking categories of *The Achievement Chart for Mathematics*, she explained that it can be challenging for teachers to develop because they must think about what to expect from students. Shelly explained that she thinks the reason many teachers do not like rubrics is because “you can’t just average” marks (SINT, Jan. 18, 219). She explained that thinking about “what the kids have demonstrated” in terms of thinking and communication can be challenging, whereas with the application category of the Achievement Chart a teacher can just average marks:

That’s the problem. I think that’s why a lot of people hate the thinking and the communicating category because it has to reflect what the kid’s doing and that’s why people hate rubrics, right, because you can’t just average them [marks] out and say to heck with it. You have to really think about what the kid’s have demonstrated and why they’ve demonstrated a 2 here and a 4 there and how is this one different than that one? And how your collections of 3’s and 4’s have to be put together. It’s not easy to do. If you look at assessment for application, you’ve got all these test marks, you’ve got quizzes that are marked, you’ve got assignments that are marked. You can average them. Their most consistent is their average. (SINT, Jan. 18, 219)
When I asked Shelly if she found it challenging to develop rubrics she explained that she “did at first but not anymore” (S\textsc{int}, Nov. 28, 154). She shared some of her strategies for developing rubrics. In particular, she emphasized the importance of being clear. With a clear rubric it is easy to assess communication:

The rubrics that I’m writing now are getting clearer as to what’s there. And so when you have a really clear rubric you can see what you expect and what the kids have to do. Like for instance if part of the rubric is showing that you can do the problem in more than one way and they don’t do that then it’s easy. (S\textsc{int}, Nov. 28, 154)

When the criteria are clear, Shelly finds it easy to assign marks. “It’s when everything’s vague that there’s an argument, where it looks subjective. So a rubric, a good rubric is not subjective” (S\textsc{int}, Jan. 18, 530).

Although Shelly sometimes has trouble developing “good rubrics,” she is “getting better” (S\textsc{int}, Jan. 18, 533). She explained that students will give a teacher exactly what he or she asks for. To illustrate this, Shelly described a cartoon. In this cartoon, there was an “elementary school kid. A teacher asked him to write the biggest number that he could on the board…. And he took the chalk and he made a big number one - from the top to the bottom.” Shelly explained that she had a similar experience the first time she developed a rubric:

I use that when I do assessment because the very first rubric I made, I got exactly what I asked for but it wasn’t what I wanted to mark. It was like that big number one. The kids gave me what I asked them to give me but it wasn’t what I wanted to mark and it was the best lesson that I ever could have learned because then I started, the rubric has to speak to what it is I want to mark. (S\textsc{int}, Jan. 18, 533)

Shelly’s experiences taught her the importance of being clear about criteria in rubrics. She explained that it takes time to learn to develop good rubrics. It takes “practice and getting [information] and then trying to mark it and saying this rubric doesn’t help me. This isn’t what I wanted” (S\textsc{int}, Jan. 18, 565).

Shelly described how she develops rubrics. She explained that “you learn” from your experiences. When she develops criteria for her rubrics, she thinks about what she wants to see. She explained that she develops Level 3 first, since this is what she expects that the average
student should be able to attain, and that Level 4 is something superior to that but not above grade
level:

I want to tell them what it is I want to see - clear, concise explanations… the narrative link to the mathematics and the narrative links, that kind of thing. You have to tell them…. Your Level 3 is what you reasonably expect kids to be able to do. Level 4 is above that but it’s not above grade level. I do not expect them to do Grade 10 work in Grade 9…. Then Level 4 is that they have connected it to something. So, my Level 4’s will often connect to other situations, have more than one solution, more creativity, complexity, depending on what the task is. So, Level 4 is above expectation, not above grade. So, I want, I expect everybody to get a Level 3 because that’s what the course is saying - you should be able to get a Level 3. Level 4 is this. Pieces missing, you dropped into Level 2. More pieces missing, more errors, you dropped into Level 1. So when I make a rubric I start with Level 3. (SINT, Jan. 18, 572)

Not only is it important for being clear when developing rubrics but it is also necessary to be consistent with criteria across levels of achievement. She cautions that, especially for deciding on criteria for a Level 4, a teacher needs to use the same criteria but that there has to be something superior about the criteria to make it a Level 4:

I have to be careful when I’m doing level 4 because the other thing that happens in Level 4 is people add more criteria. You can’t—you have to assess on the same criteria all the way across. You can’t assess Level 3 for this and this and this and for level 4 all of a sudden mark beautiful organization and you haven’t had organization in any of the other levels. But if you want to value organization, it has to be here and there has to be something about the organization that is superior. You cannot all of a sudden add new details…. That’s a common mistake that people make—that Level 4 is all of a sudden assessing something different… So, it’s finding what you value in what you want to mark…. Like what is it that a kid could do that would show me that they really get this—more than I expect or better than could be expected? And that’s making connections, that’s depth of understanding, it’s creativity. it’s a novel way to put things together maybe, it’s lots of solutions, it’s representing it four or five different ways. All of those kinds… your processes come in there. (SINT, Jan. 18, 572)

Shelly went on to explain that after she decides on the criteria that she will value across the levels, then “usually the difference between [levels] are errors and omissions” (SINT, Jan. 18, 572).

Time. One challenge that emerged in this research concerned time constraints. This challenge is also discussed in relevant research literature. For example, Keiser and Lambdin (1996) conducted a study to determine why Grade 7 and 8 teachers in the United States were not covering curriculum units at the suggested pace. From teachers responses to questionnaire items
and classroom observations, these researchers found that it can take more time for teachers to incorporate activities such as group work in the classroom, to assess student communication, and to employ engaging activities. Furthermore, with respect to time as a constraint for covering curriculum content, Rousseau and Powell (2005) found that when teachers are under pressure to cover content, particularly for standardized tests, they may rely on more teacher-directed instruction.

Helen also described how time can be a constraint in her classroom; in particular, time for finding resources, time for investigations, and time to cover content. With respect to having time to find resources, Helen explained that “there are a lot of good things out there that help, it’s just getting time. Once you are in the classroom, that’s it…. There [are] a lot of [resources] out there, it’s literally getting time to go through and dissect” (HINT, Jan. 23, 656). Helen also explained that she would like to do more investigations in her class because her students learn though investigations. She finds it difficult to spend a lot of time on investigations though, because she needs to cover curriculum:

I want to be able to do a lot of investigation and that is the push now, is, that’s the way our students learn. The Applied level especially, learn through investigation, learn…. But it’s the time that it takes them to investigate when you can just give them a formula like that… And this is where the formula came through that this is a third. But it’s something nice to do. It isn’t something essential to do. And when you’re stuck with time, and the same… and that was how you do investigations, investigating (inaudible)…. Can you afford to spend that time? But yet, if they don’t know the nitty-gritty of Pythagoras…. And I do, I try to do those investigations but the investigation will take the entire class where you can just say “a² + b² = c².” End of it. (HINT, Jan. 23, 703)

Helen explained that it can be very difficult with “EQAO, the exam, and the summatives,” which are all based on the curriculum. So if she wants to spend extra time on an investigation of “surface areas and volumes” she may lose time on another area. She explained that “it’s difficult because it’s [the curriculum] so full. If you have a strong class, if, when, okay. But I do not get them very often” (HINT, Jan. 23, 720).

Shelly also faces time constraints in her classroom but she is still heavily committed to implementing activities that encourage students to communicate and reason. In her opinion,
mathematics is connected more to thinking and reasoning than to the particular content that is taught. Shelly discussed time as a constraint to covering curriculum content. She explained that you are not implementing the curriculum properly if you focus on teaching students the rules. The curriculum is more about thinking and reasoning:

You can’t implement the curriculum if you’re only thinking of doing this [absorbing rules], and in 9 Applied there is a lot of stuff about thinking, it’s about reasoning, it’s about proving. And if all you’re ever doing is drilling integers and fractions and solving equations, you’re never going to think about relationships, you’re never going to think about optimization….. Time is what you need the most of, and in order to get them to really think and reason and do all of that, you need to spend some time. But all the time you spend there you remove from skill development, so it’s hard to balance the two things because actually getting them to be able to solve equations well too, takes time. And I’m more apt to give up time there than I am with getting them to think…. When they come away from school, they need to be better at thinking than they do at solving equations….. So it’s a trade off. You don’t have time to do everything really well. I try really hard for a balance but I’m probably going to err on the side of thinking, higher order thinking than I am on the side of rote skills. (SINT, March 5, 459)

Clearly time can be a constraint for teachers and they must make decisions about how they will spend class time. In this study, teachers’ decisions about where to spend time tended to be based on their beliefs about what is important for their students and their images of mathematics. Helen acknowledges that her students learn best through investigations but since she must cover curriculum content, she cannot spend as much time on them as she wants. She stated her opinion that investigations are something that are “nice to do” but not “essential to do” (HINT, Jan. 23, 703). Shelly, on the other hand, considers it more important to spend time on developing her students thinking and reasoning skills because they will forget the mathematics content but will need to be able to think and reason in the future.

This research demonstrates that some of the challenges that teachers who work to support communication in their mathematics classroom face include time constraints, developing rubrics, selecting appropriate activities, and classroom dynamics. In this section, I have described Shelly and Helen’s perspectives on these challenges and their strategies for overcoming these in classroom practice.
Summary: Communication from Curriculum Statements into Classroom Practice

In this chapter I have contrasted Helen and Shelly’s images of communication in mathematics. I discussed differences between the images of communication held by these teachers in the world of professional practice and those images presented, by members of the world of developers, in the Ontario Mathematics Curriculum. I also described how Shelly and Helen translate communication messages from curriculum statements into classroom practice in the *Foundations of Mathematics, Grade 9, Applied* course. The discussion in this chapter demonstrates that despite some similarities, different meanings may be associated with communication in mathematics. Furthermore, even in unique cases where the gap between curriculum developers and teachers images is minimal, the idealized vision of communication may not be realized in classroom practice since teachers may face challenges in implementation.
CHAPTER 6:
CONCLUSIONS: IMPLICATIONS FOR PRACTICE AND RESEARCH

This chapter reports on the main conclusions of this thesis and the implications for practice and research. In the first section, I provide a summary of the study. Next, I describe limitations and implications for practice and research. I conclude with a statement of conclusions that may be drawn from this work.

Summary

This thesis describes how two teachers translate communication messages from curriculum statements into classroom practice in the *Foundations of Mathematics, Grade 9, Applied, MFM1P* course. The conceptual framework for this study is based on the notion of a gap between images of curriculum initiatives held by curriculum developers and by teachers in the world of professional practice (MacDonald & Walker, 1976; Jarvis, 2006; Pitman, 1981). The framework for mathematics communication stems from curriculum statements (Ontario Ministry of Education, 2005a) and from a math-talk learning community model (Hufferd-Ackles et al., 2004) that is widely referenced in Ontario Mathematics Curriculum resources. Since there are different reasons for communication, which are influenced by differing images of mathematics (Ernest, 1991), these reasons are in important part of this framework. The reasons are: (a) for better absorbing absolute knowledge, and (b) for student social construction of knowledge.

Three specific questions were addressed in this research: (a) what are the teachers’ images of communication in mathematics? (b) how do they implement communication in classroom practice? (c) what enablers and barriers do they face in classroom practice? To address these questions, data were collected in three phases. The main focus in the first phase was to explore teachers’ images of mathematics communication. Interviews were conducted and documents were collected. The second phase served to elicit information about the teachers’
practices and experiences, including enablers and barriers, in implementing communication in classroom practice. Data for this phase were collected from classroom observations, analysis of lesson planning and assessment materials, and an individual interview with each teacher. The purpose of the third phase was to verify my interpretations of findings with participants. I created a summary of my interpretations for each case and discussed these with each participant during a semi-structured interview.

The findings from this research demonstrate different faces of communication in mathematics. In Chapter Four of this thesis, I highlighted two teachers’ images of communication and how they translate these images into classroom practice. One teacher, Shelly, is passionate about mathematics as a subject. She has also been actively involved in the Ontario Mathematics Curriculum revision process. Consequently, she has a rich understanding of the philosophy, research and theory underlying curriculum initiatives. Her background and experiences have influenced her images of communication and her classroom practices. Her images of communication are heavily tied to those presented in the Ontario Mathematics Curriculum. Although she works to translate her vision into classroom practices, she faces challenges with implementing a math-talk learning community in the Foundations of Mathematics, Grade 9, Applied course. Helen, a second teacher who participated in this research, is passionate about teaching and helping students who struggle to succeed in mathematics. She has a strong Special Education background and teaches mathematics in Applied-level courses because of her ability to help these students learn. Her perspectives and practices have been heavily influenced by her professional experiences—including experiences in the classroom, special education, and professional activities such as workshops, conferences, and Additional Qualifications courses.

In Chapter Five of this thesis I contrasted differences in images and implementation of communication in mathematics. Shelly’s images of communication are strongly tied to those presented in curriculum documents and support materials from the Ontario Ministry of Education. She describes communication in terms of mathematical processes and the categories of The
Achievement Chart for Mathematics. Her vision also reflects epistemological and cognitive theories underlying calls for communication. Helen describes communication in more general terms than Shelly. For her, communication is a “massive” concept. Her images reflect her perspectives on what is best for her students and are based more heavily on practical experiences than on theories underlying calls for communication in the mathematics curriculum. The discussion in Chapter Five demonstrates that even in unique cases, such as that with Shelly, where the gap between curriculum developers and teachers images of communication is minimal, the idealized vision of communication may not be realized in classroom practice since teachers may face challenges in implementation. The teachers report that despite additional challenges involved with implementing communication in Grade 9 Applied mathematics, teaching in this context can be a rewarding experience.

Since this thesis describes how two teachers translate communication messages from curriculum statements into classroom practice, the findings do not demonstrate how teachers across Ontario have embraced the contemporary emphasis on this mathematical process. The cases do, however, illustrate different images of communication that emerge among supporters of this mathematical process. The findings give us a sense of different images presented by members of the world of curriculum developers in curriculum documents, and held by teachers in the world of professional practice. My images and Shelly’s images also represents those of members of the world of academia since we can both be considered “academic theorist[s] who [have] taken the trouble to familiarize [ourselves] with the new curriculum as propounded by the developers in journal articles or at conferences specially held to acquaint the theorists with the concepts, values and practices of the innovation” (MacDonald & Walker, 1976, p. 46). The different images presented in this thesis demonstrate that although various members of the mathematics education community support communication in mathematics, there are various meanings attributed to this mathematical process. Consequently, we cannot simply ask teachers if they support
communication in the mathematics classroom. We must probe further to understand the meanings that they attribute to this mathematical process in teaching practice.

**Limitations**

There are two main limitations of this research: (a) limitations to the generalizibility of findings, and (b) limitations in data collection. First, given the case study nature of this thesis, findings may not be generalizeable. I have examined two cases which are likely to be unique in their details and, consequently, their images of communication in mathematics are not necessarily the same as those held by mathematics teachers across Ontario. The findings are important, however, since they provide us with experiential knowledge of teachers’ perspectives and practices. This knowledge can help us understand similar situations (Stake, 2005). Furthermore, the findings illustrate the relationship between images of mathematics communication represented in the curriculum that are intended by developers and images held by teachers in the world of professional practice.

Teachers across Ontario indicate on surveys (EQAO, 2007b, Surttamm & Graves, in press) that they support communication in their mathematics classrooms. Similarly, participants in this research would indicate support for this mathematical process. Although we cannot generalize these two teachers’ images to the teachers across the province, we can likely generalize the result that among those teachers who claim to be supportive of the curriculum statements on communication, there is a range in what this support actually means. Their statements of support are based on different images of communication in mathematics.

A second limitation of this research concerns gaining access to time on site during data collection. Since I faced challenges in securing participants, I began collecting data later than anticipated. Consequently, there was some overlap between phases of collection and I was not able to observe as many *Foundations of Mathematics, Grade 9, Applied* lessons as I would have liked. Furthermore, while Shelly felt that my presence in the classroom did not influence her
students’ behaviour, Helen suggested that when I was in the classroom, her students’ behaviour was not typical.

Even though I was not able to observe as many *Foundations of Mathematics, Grade 9, Applied* lessons as I would have liked and my presence influenced the behaviour of Helen’s students, these limitations to data collection do not impact the findings of this study. First, since I was not able to observe as many *Foundations of Mathematics, Grade 9, Applied* lessons as I hoped, I attended other lessons that the teachers were teaching in the fall term. This provided me with a better sense of their practices and roles in guiding communication. This also allowed for more opportunities to spend time with my participants and to have more informal conversations. My influence in the classroom did not impact findings since the main focus in observations was not on the students, but was on the teachers’ practices—including classroom activities and the teachers’ role in guiding lessons—and their images of mathematics and mathematics communication.

**Implications for Practice and Research**

This thesis has implications for curriculum leaders. The findings may also be valuable for teachers since they provide some examples of how communication messages can be implemented in classroom practice. This research is important for curriculum leaders since the two cases illustrate how different meanings associated with communication in mathematics may be translated into classroom practice. Below I describe some considerations that emerged through this research for teachers and curriculum leaders.

**Considerations for Teachers**

This thesis demonstrates the importance of understanding the rationale underlying curriculum initiatives. Shelly explained, for example, the importance of her experiences with becoming familiar with the research literature:
When I was at the Ministry, the one thing that I didn’t get a chance to do when I was teaching that I had a chance to do there was get caught up on the research literature—the latest literature on research. Up until then I would do a little bit in the summer. I had a sense of why doing things differently was important but I didn’t understand the research and I didn’t know the names. I didn’t know…why the change in the curriculum was so dependent on research. And…what was the research telling us about how kids learn—in particular, they learn mathematics—that influence the changes in the curriculum. I was unaware of all of that….As a teacher in the field you don’t know why some of the changes are being made and lots of times teachers don’t agree with them because they think that’s just somebody’s idea of doing it better….Going to the Ministry helped me to see that there was a solid basis and some good rationale for changing the curriculum. And that’s a nice thing to have as a teacher. (SINT, Nov. 26, 109)

Opportunities to engage in the study of background literature have helped Shelly move towards a math-talk learning community. She understands what this community should look like and why it is important in classroom practice. Shelly’s case demonstrates the importance of trying to understand the rationale behind increased emphasis on communication in the classroom.

Both cases in this research demonstrate challenges that are faced when implementing communication in classroom practice. Teachers must keep in mind that given the importance of increased communication in the classroom, they must remain persistent in overcoming challenges. Teachers may face time constraints in the classroom. They may also find it challenging to select appropriate tasks and to assess activities involving communication. The cases described in this thesis also demonstrate that the dynamics of a *Foundations of Mathematics, Grade 9, Applied* class may create additional challenges for teachers. However, both teachers who participated in this study suggest that teaching in this context can be a very rewarding experience.

**Considerations for Curriculum Leaders**

An implication of this thesis for users of survey data is that we really do not know what teachers are saying when they claim agreement with communication statements. Grade 9 Applied mathematics teachers across Ontario indicate on surveys that they support communication in the mathematics classroom (Education Quality and Accountability Office, 2007b; Suurtamm & Graves, in press). For example, on the EQAO teacher questionnaire, 68% of the *Foundations of*
Mathematics, Grade 9, Applied teachers indicated that they engage students in activities that support the communication category of The Achievement Chart for Mathematics (EQAO, 2007b, p. 10). Furthermore, teachers’ responses to a questionnaire that was distributed for the Curriculum Implementation in Intermediate Mathematics (CIIM) project suggest that intermediate mathematics teachers across Ontario recognize the importance of having students explain and actively participate by communicating in mathematics lessons (Suurtamm & Graves, in press). Of the Grade 9 Applied teachers who responded to the CIIM survey, 61% suggested that they expect students to explain their reasoning or justify solutions in most or every mathematics lesson (Suurtamm & Graves, p. 11). Almost all of the intermediate mathematics teachers who responded to the survey (92%) “agreed that it is somewhat or very important to provide opportunities for students to explain and provide reasons to support solutions” (Suurtamm & Graves, p. 8). Despite evidence of support for this aspect of the curriculum, findings from this study point to a need for finer analysis of teachers’ perspectives and practices when it comes to communication in mathematics. We need to ask detailed questions on surveys, and perhaps even visit classrooms and talk to teachers, to understand what this actually looks like in classroom practice.

To minimize the gap between teachers’ and developers’ images of communication, it is important for curriculum developers to find ways to increase teachers’ contact with the philosophy, theory, and research literature underlying calls for communication, as was the case with Shelly. Since teachers may not have a lot of time available for reading literature, developers must find clear and concise ways to introduce teachers to the literature. Leaders must present important aspects of research without oversimplifying the details. The Connecting Research and Practice in Mathematics series, which includes an article that summarizes the math-talk learning community framework (Bruce, 2007) is an example of a resource that may be helpful for teachers. Curriculum developers may also consider including summaries of background philosophy and theory in curriculum documents. This suggestion was also made by Roulet (1997) in his background research paper for curriculum renewal for secondary schools in Ontario. Roulet
recommended that “[w]hatever vision of mathematics is to guide the new curriculum, it must be clearly and consistently articulated. Extensive in-service work with teachers and communication with parents may be required if the image adopted does not match those presently popular” (Images of Mathematics section, para. 6).

Leaders must remember that there are a number of potential challenges that teachers face when they work to support the contemporary vision of communication in their classrooms. If teachers understand the importance of curriculum revisions and increased emphasis on communication, they may be more persistent in overcoming challenges that are faced in classroom practice. Therefore, curriculum developers may want to explore ways to increase teacher contact with the philosophy, theory, and research underlying curriculum initiatives.

**Recommendations for Future Research**

Several recommendations for future research emerged as I collected data for this thesis and discussed the findings with members of the mathematics education community. Recommendations are displayed in Table 2 and are arranged according to the three levels of curriculum—intended, implemented, and attained (Travers & Westbury, 1989).
Table 2: Recommendations for Future Research

<table>
<thead>
<tr>
<th>Intended Curriculum</th>
<th>Implemented Curriculum</th>
<th>Attained Curriculum</th>
</tr>
</thead>
<tbody>
<tr>
<td>− Explore images of communication in the world of developers</td>
<td>− Describe support for communication in the implemented curriculum in Ontario and conduct a finer analysis of what teachers mean when they report agreement with communication in the curriculum</td>
<td>− Explore student interactions in a math-talk classroom</td>
</tr>
<tr>
<td>− Describe influences of teacher reflection on images of communication</td>
<td>− Explore ways to bring background theory and research to teachers and examine the impact of these programs on images and practice</td>
<td>− Describe the development of skills, knowledge and/or attitudes in a math-talk learning community</td>
</tr>
</tbody>
</table>

First, at the level of the educational system, where the intended curriculum is developed, research that explores images of communication held by members of the “world of developers” (Pitman, 1981, p. 254) would be valuable for both teachers and curriculum leaders. Teachers would benefit from understanding the perspectives of leaders in the mathematics education community. Leaders themselves would benefit from making their images more explicit and in the process reflecting on these.

At the level of the implemented curriculum, where the intended curriculum is translated into classroom practice, several areas of research would be valuable. First, this research focused on the perspectives and practices of teachers who were selected because they support the contemporary emphasis on communication in mathematics. Research that explores how teachers across Ontario have embraced the contemporary emphasis on this mathematical process would be valuable. As findings from this study demonstrate, it is important to go beyond asking teachers if they support communication, but to instead conduct a finer analysis to get a sense of what this
means to teachers across the province. Given the importance of helping teachers understand the rationale behind curriculum initiatives, an important area of research would explore approaches to bringing background theory and research to teachers. This research may also examine the impact of these programs on images and practice. It would also be interesting to describe how teachers’ perspectives on mathematics communication change as they reflect on the meaning of this process and/or become familiar with the rationale underlying curriculum initiatives. For example, it might be interesting to describe how the images of the teachers who participated in this study have changed. Research that explores ways to increase the chances that teachers will change their images would also be valuable.

At the level of the attained curriculum, where students acquire knowledge, skills, and attitudes from the intended and implemented curriculum, future research that focuses on the students in a math-talk classroom would be important. Of course, research into this area already exists. For example, with the Hufferd-Ackles et al. (2004) math-talk framework and an Ontario study conducted by Kotsopoulos that focuses on the “nature of peer communication in mathematics” and “the relationship to learning and knowing” (2007, p. 10). Further research into student interactions that take place in a math-talk classroom would be valuable. It might also be interesting to explore how students develop mathematics skills, knowledge and/or attitudes in a math-talk learning community.

**Conclusions**

The different images presented in this thesis demonstrate that although various members of the mathematics education community support communication in mathematics, there are various meanings attributed to this mathematical process. Teachers translate these meanings into classroom practice. Consequently, we cannot simply ask teachers if they support communication. We need to probe further to understand the meanings that are attributed to this mathematical process in classroom practice. To minimize the gap between images of communication that are
translated from curriculum statements into classroom practices, developers must find clear and concise ways to communicate the rationale behind curriculum initiatives.
REFERENCES


APPENDIX A:

LETTER OF INFORMATION AND CONSENT FORM

Letter of Information

Title: Translating Mathematical Process Messages Concerning Communication from Curriculum Statements to Teaching Practice: Three Case Studies of Exemplary Grade 9 Applied Teachers

I am writing to request your participation in a research study aimed at understanding teachers’ efforts to translate mathematical process statements, particularly those concerning communication, into Grade 9 Applied mathematics instruction and assessment. I am currently a graduate student in the Faculty of Education, Queen’s University, working towards completion of my Master’s degree in education.

The ultimate goal of this research is to describe the translation of communication messages from curriculum statements to practice by documenting the thoughts, plans, and experiences of teachers who are working to establish a classroom math-talk learning community. Although not all mathematics lessons are appropriate for encouraging such a community, in a classroom that supports mathematical communication, or math-talk, students make an active contribution to lessons. For example, they may work in groups, participate in whole class discussions, question and elaborate on explanations from other students and/or the teacher. Their ideas provide a basis for lessons, and they share responsibility for their own learning and for the learning of their peers.

For the purposes of this research, the proposed method of study requires that I conduct three or four interviews with you, observe one or two math-talk lessons, and collect materials that you use for planning these lessons. I also intend to collect a sample assignment that will be graded in this course that you think calls for rich communication from students. This research will take place over the course of one semester, beginning early in the fall term, 2007 and ending in January 2008.

All interviews will be conducted at a time and location that is convenient for you. Each interview will last approximately one hour and will be audio-taped. The taped interviews will be transcribed, and the tape will be destroyed five years after the research has ended. Observations will also take place at times that are convenient for you. During observations I will take notes to make a written record of how you guide instruction in a math-talk community. All notes will be written up and maintained as a computer file.

I will also request that you prepare a brief personal writing, using as much space as you wish, in which you reflect upon your experiences with communication in mathematics. You will also be asked to prepare a concept map to illustrate your own personal image of mathematical communication. I will collect these writings prior to the first interview, in which they will be discussed.

None of the data collected for this study will contain your name, or the identity of your place of work. The place of work will be identified using general terms and a pseudonym only. I will take the necessary measures to ensure confidentiality. Only myself and my supervisor will have access to transcript data. The data will be used for research purposes and will not be used to evaluate you.
in any way. Data will be stored in a safe place and confidentiality will be protected to the furthest extent possible.
I do not foresee risks in your participation in this research. You are not obligated to answer any questions you find objectionable or that you are uncomfortable with, and you are assured that no information collected will be reported to anyone who is in authority over you. Participation is voluntary and you are free to withdraw from the study without any reasons at any point, and you may request removal of all or part of your data.

This research may result in publications of various types, including journal articles, professional publications, and conference presentations. Your name will not be attached to any form of the data that you provide, neither will your name or the identity of your place of work be known to anyone tabulating or analyzing the data, nor will these appear in any publication created as a result of this research. A pseudonym will replace your name on all data that you provide to protect your identity. If the data are made available to other researchers for secondary analysis, your identity will never be disclosed.

This research has been cleared by the Queen’s University General Research Ethics Board and by your school board. If you agree to participate, please sign the attached consent form and return it to me at your earliest convenience.

If you have any questions about this research, please contact me, Jill Lazarus, 613-533-6000 ext. 75952 (jilllazarus@gmail.com) or my supervisor, Dr. Geoffrey Roulet, 613-533-6000 ext. 74935 (rouletg@educ.queensu.ca). For questions, concerns or complaints about the research ethics of this study, contact the Dean of the Faculty of Education, Dr. Rosa Bruno-Jofré, 613-533-6210 (brunojor@educ.queensu.ca) or the General Research Ethics Board, 613-533-6081 (Chair.GREB@queensu.ca).

Sincerely,

Jill Lazarus
Title: Translating Mathematical Process Messages Concerning Communication from Curriculum Statements to Teaching Practice: Three Case Studies of Exemplary Grade 9 Applied Teachers

- I have read and retained a copy of the letter of information concerning the study Translating Mathematical Process Messages Concerning Communication from Curriculum Statements to Teaching Practice: Three Case Studies of Exemplary Grade 9 Applied Teachers and agree to participate in the study. All questions have been answered to my satisfaction. I am aware of the purpose and procedures of this study.

- I understand that the purpose of the study is to describe the translation of communication messages from curriculum statements to practice by documenting the thoughts, plans, and experiences of teachers who are working to establish a classroom math-talk learning community.

- I am aware that my participation will take the form of four interviews and two or three observations over the course of one school semester, beginning early in the fall semester, 2007 and ending in January 2008. I have been informed that interviews will be approximately one hour in length and will be audio-taped. I understand that I am expected to prepare two writings, one personal reflection and one concept map, and submit samples of instructional and assessment materials to the researcher.

- I understand that there are no known risks, discomforts or inconveniences associated with participation in the research study. I understand that confidentiality will be protected by appropriate storage of and access to data and by the use of pseudonyms.

- I have been notified that participation is voluntary and that I may withdraw at any point during the study and request removal of all or part of my data, without consequences.

- I am aware that I can contact the researcher, Jill Lazarus at 613-533-6000 ext. 75952 (jilllazarus@gmail.com) or her supervisor Dr. Geoffrey Roulet, 613-533-6000 ext. 74935 (rouletg@educ.queensu.ca) if I have any questions about this project. I am also aware that for questions, concerns or complaints about the research ethics of this study, I can contact the Dean of the Faculty of Education, Dr. Rosa Bruno-Jofré, 613-533-6210 (brunojor@educ.queensu.ca) or the General Research Ethics Board, 613-533-6081 (Chair.GREB@queensu.ca).

Please sign one copy of this Consent Form and return to Jill Lazarus. Retain the second copy for your records.

I HAVE READ AND UNDERSTOOD THIS CONSENT FORM AND I AGREE TO PARTICIPATE IN THE STUDY.

Participant’s name (Please print): ________________________________________________________________

Participant’s signature: ________________________________________________________________

Date: ______________________________________

Please write your e-mail or postal address at the bottom of this sheet if you wish to receive a copy of the results of this study.
APPENDIX B:
INTERVIEW TOPICS AND SAMPLE QUESTIONS

Phase 1 Interviews: Backgrounds, Images and Plans
Prior to the first interview, Shelly constructed a personal statement to describe her experiences and a concept map to illustrate her images. These constructions were the topic of our first interview.
Sample interview questions included:
- After looking at your personal statement, I am wondering if there is anything that you would like to add to what you have here?
- I noticed that you said “abc” in your statement. Can you please elaborate on this experience?
- Can you please explain to me how you have organized your concept map and why you have done it this way?
- Here (pointing at map) you have connected this (term) and this (term). Can you please explain to me why you have done this? How are these two things connected?

Helen was unable to create a personal statement and concept map. Instead, I elicited information about her background and images of communication during our first informal meeting and through two semi-structured interviews. After the first interview, I constructed a concept map to illustrate her image of mathematics communication. We discussed this map during our second interview.
Sample questions for the first two interviews included:
- Can you please talk about your past experiences with communication in mathematics and how they might influence how you teach now?
- Communication is explicitly recognized in Ontario Mathematics Curriculum documents. Can you please describe what communication means to you? If you were too look at communication in relationship to the other mathematical processes outlined in curriculum documents—a visual picture—how would you describe how they fit together?
- You mentioned that because I was here today, students did not behave as they typically do. I am wondering, what would you typically see in terms of communication if I was not here? What would you like to see?
After our last interview, I tried to draw a picture of how you describe communication. I brought it so that you can make any changes that you see appropriate (I explained my construction and probed for elaboration where necessary).

The purpose of the final interview in the first phase was to discuss the teachers’ plans for implementation and materials that they use to support communication in instruction and assessment.

Sample interview questions included:

- I found this activity interesting. Can you please elaborate on what you might expect to see in terms of communication when you implement this in the classroom?
  - After looking at this task, I have noticed that . . . . Can you please elaborate on this?
- Can you please describe, focusing on mathematics communication expectations, what you would like to see in student solutions to these types of assignment questions?

**Phase 2 Interview: Practices and Experiences**

Interview questions were based on data collected from observations and from the analysis of instructional and assessment materials. Sample questions included:

- In the lesson that I observed last time we met I noticed that you [did something]. What was it that you hoped would happen? How do you feel about what actually did happen?
- Now that you have had a chance to grade this assessment task, how do you feel students did with respect to communication?
  - What were some common problems, if any, that students had with respect to communicating mathematically in this assignment? What did they do well on?
  - Were there any parts of this assignment that were difficult to grade? If so, what were they, and what were the difficulties?
- I found this question on the assignment interesting [Or, we discussed this part in our last interview]. How did students do on this part? Was this what you expected?

**Phase 3 Interview: Summary and Participant Confirmation**

During the interview, I expanded on points that I included in the summary and asked the participant to expand, where appropriate.

Sample interview questions included:
While I was summarizing my notes from our previous interviews, I noticed “abc.” I am still unclear about “xyz.” Can you please elaborate on this for me?

Now that you have had an opportunity to look at the summary, is there anything that you would like to explain, or that you do not agree with?
APPENDIX C:
DATA CODING SCHEME

Event/Item Codes

In text, specific references are coded: \((N_{ABC}, \text{date}, ##)\)
In data log, references are coded \(N_{ABC} \text{(date)}\)

N  Participants initial
    S = Shelly Baker
    H = Helen Smith

ABC  Data Type
    INT = Interview
    DOC = Document
    OBS = Observation
    INF = Informal Discussion
    EMA = E-mail

Date  Date data was collected.

##  line number on interview transcripts
<table>
<thead>
<tr>
<th>Code/Date</th>
<th>Event</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEMA(June 19)</td>
<td></td>
<td>Shelly responded to e-mail to OMCA. Board Approval received Oct. 9th.</td>
</tr>
<tr>
<td>SINF(Oct. 17)</td>
<td>Phone conversation</td>
<td>Scheduled first informal meeting.</td>
</tr>
<tr>
<td>SINF(Oct. 20)</td>
<td>Math meeting</td>
<td>Met informally with Shelly. She described her Grade 9 Applied class. She also received hard copies of the Letter of Information and Consent Form for the study and signed one copy of the Consent Form.</td>
</tr>
<tr>
<td>SDOC(Nov. 18)</td>
<td>Materials received via e-mail</td>
<td>Received completed Concept Map, Personal Statement and Instructional/Assessment Materials (Instructions were sent to Shelly on Oct. 23).</td>
</tr>
<tr>
<td>SOBS(Nov. 19)</td>
<td>9 Applied lesson</td>
<td>Observed Grade 9 Applied lesson on Proportions.</td>
</tr>
<tr>
<td>SINF(Nov. 19)</td>
<td>Conversations</td>
<td>Made notes on informal conversations that I had with Shelly between classes.</td>
</tr>
<tr>
<td>SDOC(Nov. 22)</td>
<td>9 Applied, 9 Academic, and Data Management lessons</td>
<td>Observed all lessons that Shelly taught. This was a snow day but most students showed up for the 9 Applied class, which takes place in the afternoon.</td>
</tr>
<tr>
<td>SINF(Nov. 22)</td>
<td>Conversations</td>
<td>Made notes on informal conversations that I had with Shelly between classes.</td>
</tr>
<tr>
<td>SINF(Nov. 24)</td>
<td>Math meeting</td>
<td>Talked only briefly, primarily about her thoughts about participating in the research and my influence in the classroom.</td>
</tr>
<tr>
<td>SOBS(Nov. 26)</td>
<td>9 Applied lesson</td>
<td>Observed scaling exercises. This was the most challenging lesson of the semester - students were not engaged.</td>
</tr>
<tr>
<td>SINF(Nov. 26)</td>
<td>Conversations</td>
<td>Made notes on informal conversations that I had with Shelly between classes. Shelly shared her thoughts about how the lesson went and why it might have happened the way it did.</td>
</tr>
<tr>
<td>SINF(Nov. 26)</td>
<td>Interview 1: 9 Applied lesson</td>
<td>Talked about responses to Personal Statement, Concept Map and Shelly’s background. Since the lesson occurred so differently from what was planned, Shelly shared her perspectives on the lesson.</td>
</tr>
<tr>
<td>SDOC(Nov. 26)</td>
<td>Final version of concept map</td>
<td>During our first interview, Shelly modified her construction of the Concept Map.</td>
</tr>
<tr>
<td>SOBS(Nov. 28)</td>
<td>9 Applied lesson</td>
<td>Elastic Meter and Percent Activity. This lesson took place two days after the lesson that students were not engaged in. This lesson began with students desks placed in rows.</td>
</tr>
<tr>
<td>SINF(Nov. 28)</td>
<td>Conversations</td>
<td>Made notes on informal conversations that I had with Shelly between classes.</td>
</tr>
<tr>
<td>Date</td>
<td>Event Type</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Nov. 28</td>
<td>SINT</td>
<td>Interview 2: Took place after 9 Applied lesson. Discussion focused mainly on plans for implementing communication. Also followed up on questions that emerged during previous meetings.</td>
</tr>
<tr>
<td>Dec. 9</td>
<td>SEMA</td>
<td>E-mail. E-mail from Shelly to arrange for our next meeting. Shelly suggested that her week did not go as planned. She also described some of her plans for using a motion detector the next day and writing stories about graphs. Because of the change in the lesson plans, we decide to postpone an interview.</td>
</tr>
<tr>
<td>Dec. 10</td>
<td>SINF</td>
<td>Phone Conversation. Discussed data collection schedule. Shelly described events and activities that she implemented recently. She also described some activities that she planned to implement before the end of the semester.</td>
</tr>
<tr>
<td>Dec. 19</td>
<td>SDOC</td>
<td>Materials received via e-mail. Sent me more activities that she planned to use. She also described some of her plans for the activities.</td>
</tr>
<tr>
<td>Jan. 18</td>
<td>SINT</td>
<td>Interview 3: After school. Interview took place following the 9 Applied class. During the class, students were writing the EQAO test. During the interview, Shelly described how students performed on summative assessments and how she assesses communication. I also asked questions that emerged during previous meetings.</td>
</tr>
<tr>
<td>Jan. 18</td>
<td>SINF</td>
<td>Conversations. Shelly shared her perspectives on students’ performance on the summative assessments and described some examples of high and low levels of student communication on tasks. Shelly also talked about the EQAO test.</td>
</tr>
<tr>
<td>March 5</td>
<td>SINT</td>
<td>Interview 4: After school. Discussed my interpretations of the data collected from Shelly.</td>
</tr>
</tbody>
</table>
## Data Log: Helen Smith

<table>
<thead>
<tr>
<th>Code/Date</th>
<th>Event</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEMA (Oct. 31)</td>
<td>Received e-mail from Helen's Mathematics Department Head. She recommended Helen, the school’s “wonderful 9 Applied” teacher for this research. Our first informal meeting was scheduled via email.</td>
<td></td>
</tr>
<tr>
<td>HINF (Nov. 9)</td>
<td>Met informally with Helen. The mathematics department head joined us. Purpose of the meeting was to establish rapport. Helen signed a copy of the Consent form.</td>
<td></td>
</tr>
<tr>
<td>HDOC (Nov. 9)</td>
<td>Helen gave me a copy of the workbook that she uses for her Grade 9 Applied class.</td>
<td></td>
</tr>
<tr>
<td>HDOC (Nov. 9)</td>
<td>Helen gave me a sample test that she planned to give to her class.</td>
<td></td>
</tr>
<tr>
<td>HINF (Nov. 9)</td>
<td>Arranged for our next meeting. Helen shared some of her perspectives on teaching mathematics.</td>
<td></td>
</tr>
<tr>
<td>HODOC (Nov. 27)</td>
<td>Observed all classes that Helen taught that day. First observation of the 9 Applied lesson related to graphs. Students did a quiz and worked through questions from the workbook. Observed 9 Essentials lesson. Helen taught the same lesson to the Applied class because her Essentials class is working to complete the Applied credit.</td>
<td></td>
</tr>
<tr>
<td>HINF (Nov. 27)</td>
<td>Made notes on informal conversations that I had with Helen between classes.</td>
<td></td>
</tr>
<tr>
<td>HINT (Nov. 27)</td>
<td>Discussed Helen’s background and ideas about communication in mathematics. I used this information to construct a concept map of her images of mathematics communication.</td>
<td></td>
</tr>
<tr>
<td>HDOC (Dec. 6)</td>
<td>Concept map that I developed to represent Helen’s image of mathematics communication.</td>
<td></td>
</tr>
<tr>
<td>HINT (Dec. 6)</td>
<td>Since a teacher candidate was teaching the Grade 9 lessons, I could not observe but did have an interview with Helen to follow up on her perspectives on communication and her background.</td>
<td></td>
</tr>
<tr>
<td>HDOC (Dec. 6)</td>
<td>Helen gave me a copy of the test that she planed to give her class (the test she actually gave her class was similar but not the same)</td>
<td></td>
</tr>
<tr>
<td>HODOC (Dec. 14)</td>
<td>Observed 9 Essentials lesson, which was the same as the Applied lesson.</td>
<td></td>
</tr>
<tr>
<td>HINF (Dec. 14)</td>
<td>Made notes on informal conversations that I had with Helen between classes.</td>
<td></td>
</tr>
<tr>
<td>HINT (Dec. 14)</td>
<td>Discussed instructional and assessment practices. Also asked questions that emerged from analysing data that was collected during previous meetings.</td>
<td></td>
</tr>
<tr>
<td>Event</td>
<td>Date</td>
<td>Description</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Interview 4</td>
<td>Jan. 23</td>
<td>After school: talked mainly about summative tasks and asked HM some questions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>that emerged from data that I collected during previous meetings.</td>
</tr>
<tr>
<td>Conversations</td>
<td>Jan. 23</td>
<td>Made notes on informal conversations that I had with Helen before and after</td>
</tr>
<tr>
<td></td>
<td></td>
<td>the interview.</td>
</tr>
<tr>
<td>Course Outline</td>
<td>Jan. 23</td>
<td>Helen gave me a copy of the course outline (with course expectations and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>marking scheme) that she gives to her 9 Applied class.</td>
</tr>
<tr>
<td>Interview 5</td>
<td>March 3</td>
<td>Discussed my interpretations of the data collected from Helen.</td>
</tr>
</tbody>
</table>
APPENDIX D:

INSTRUCTIONS FOR PERSONAL STATEMENT AND CONCEPT MAP

Personal Statement and Concept Map

The instructions provided on the following page are for constructing a personal statement and concept map related to communication in mathematics. These tools will help me to learn more about your experiences with, and images of, communication in mathematics.

Data for this study will be stored electronically and thus it would be of assistance if I could have an electronic copy of these materials. If possible, please type your responses for the personal statement. If you are comfortable with SMART Ideas software, you may like to use this tool for constructing the concept map. Of course, it is not necessary that you use computer software. If you would like, feel free to use pencil and a blank piece of paper to construct the concept map so that you can make changes if you see them as required during the process.

Since I would like to look at your responses prior to our first interview, it would be helpful if you could please return these materials to me at least one week before we meet to discuss them.
Personal Statement

Before our first interview, I would like to learn more about your experiences with mathematics communication and how you think these might have influenced your current thoughts about this mathematical process. To do this, I will ask you to write a brief reflection on your previous experiences. This reflection can be as long as you wish. I am hoping for approximately one page but if you would like to write more this will also be appreciated. I would like to explore your responses further during our first interview.

Please respond to the following questions:

1. What experiences with mathematics communication have you had as a mathematics student or teacher (e.g., in the classroom, professional development, etc.)?
2. How do you think your experiences have influenced your current ideas about this mathematical process?

Concept Map for Mathematics Communication

Before our first interview, I would also like to understand more about your images of mathematics communication and what it looks like, both for mathematics in general and for mathematics teaching in particular. To do this, I will ask you to construct a concept map.

Constructing the Map

To begin, please place the main title “Mathematics Communication” in the center of the page. Then represent this process with respect to the two categories, “Communication in Mathematics” and “Communication in Mathematics Teaching and Learning.” For example, in the middle of the page, you may have:

- The “Communication in Mathematics” node represents terms which are associated with what communication means for mathematics as a subject.
- The “Communication in Mathematics Teaching and Learning” node would include any terms related to what communication looks like in ‘math-talk’ lessons and in communication-rich assessment activities.

Now, please generate names of associated concepts and place these terms as nodes on the map to show where they fit. Then connect related concepts using lines to show the relationships between terms. Since I am interested in understanding relationships between labels it may help, if you can, to label the connections between different nodes in the map. I would like to explore how you have organized this map in our first interview.
APPENDIX E:
DOCUMENTS COLLECTED FROM SHELLY

Shelly’s Construction of a Concept Map

Shelly’s Modification of her Concept Map
The Mathematical Processes Graffiti Activity

- Write each Math process on 5 different pieces of chart paper and post on the walls spaced out around the room. (Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing)

- Place students into 5 groups (use count off, or coloured cards, or stickers) and provide each group with one or two coloured markers

- **Instructions**: each of the 5 groups starts at a different one of the five chart locations; they have $1\frac{1}{2}$ minutes to write down everything the group knows about the word. They can use words, pictures, examples, and diagrams. When their time is up (Use a stop watch or timer) they move clockwise around the room to the next station. At this station they read what the previous group has already written, they can clarify, add an example, or add new information but they may not repeat what has already been written. After $1\frac{1}{2}$ minutes they move to the next station, and so on until each group has been to all 5 stations.

- **Debrief**: These charts represent the class’s collective prior knowledge of the processes; it’s not likely that they will have come up with the mathematical definitions but there should be some analogies that you can draw on. Summarize the students’ understanding of the meaning of the mathematical processes and share with the class the additional meaning of the words in a math context. (Show and post the processes posters)

- **Follow-up**: Inform students that we will be working on these processes throughout the year and that they will form the basis for much of their assessment in the Thinking and Communications categories

- Try to refer to the processes at appropriate times during the year but especially in the first two units.
Which Cylinder Has the Greater Volume?
(Source TIPS4RM Grade 9 Applied – Introductory Unit)

Paper can be folded in two different ways to form cylinders. In this activity, students examine how this affects the volume of the cylinders.

Prediction
Students predict how the volumes will compare and estimate the difference in volume. Do you think that the volumes will be equal or will one be bigger? If so, estimate how much bigger the volume of one cylinder will be compared to the other.

(Example: The volumes of both cylinders will be the same because we are using the same piece of paper.)

Materials
2 pieces of paper of equal size (8½ x 11), ruler, tape

Solutions

Case 1

Case 2

<table>
<thead>
<tr>
<th>Length</th>
<th>Radius</th>
<th>Height</th>
<th>New Length</th>
<th>New Width</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.9 cm</td>
<td>4.4 cm</td>
<td>21.6 cm</td>
<td>21.6 cm</td>
<td>27.9 cm</td>
<td>1338 cm³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Radius</th>
<th>New Length</th>
<th>New Width</th>
<th>Height</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 cm</td>
<td>21.6 cm</td>
<td>27.9 cm</td>
<td>27.9 cm</td>
<td>1036 cm³</td>
</tr>
</tbody>
</table>

Note: The paper is turned to create a new length and width.

Conclusion
Students make a conclusion based on their observations and measurements.

(Example: The volumes of the cylinders are different! The volume of one [1] is nearly 1.3 times as much as the other [2]).
In a banquet room, there are small square tables that seat one person on each side. The tables are pushed together to create larger rectangular tables.

a. Consider all possible arrangements of 12 square tables. Sketch each arrangement on grid paper.

b. Which arrangement seats the most people? Explain.

c. Which arrangement seats the fewest people? Explain.

d. Explain why the minimum perimeter might not be preferred for this situation.
Questions from  
Test#2: Optimization

Name __________________    Date_______________

Part A: Knowledge /27  Communication Level ______

1. Nikki is building a four sided rectangular dog pen with 24m of fence. She is trying to determine the maximum area that her fence will enclose. She created the following chart but left out some of her calculations.

<table>
<thead>
<tr>
<th>Perimeter (m)</th>
<th>Width, w (m)</th>
<th>Length, l (m)</th>
<th>Area, A, (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>24</td>
<td>4</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a. Calculate the length and area for the missing data. Show all calculations.

   [4]

   b. What is the largest area that Nikki can make for her dog pen? Explain how you know.

   [2]

2. A building supply store donated 30 m of fence to a daycare centre so that they could create a rectangular play area. The day care will build the play area next to their centre so they will only need to fence in three sides.

   a. Draw a picture of the situation and show one example of an area that would work. Calculate the length and the area for your example.

   [3]

   b. Describe using words, charts or graphs how you would be able to determine the maximum area for the rectangular play area. (You do not have to solve this problem)

   [4]
Geometry Performance Task: Original Work

Choose one of the following options: (to be submitted as part of the geometry project)

Option 1:
Using the geometry learned in this unit to design a geometry creation of your own. Include your picture and all geometric shapes and properties that you used. Give your picture an appropriate title. Show the whole geometric shape but colour/darken the parts of your shapes that are part of the picture. You can draw, use technology or use paper folding.

You must have at least 8 different geometric shapes/properties. If you satisfy these conditions accurately to create a picture, you will earn a level 3. For a level 4 or 4+, accurately add more geometric shapes and/or more creativity.

You must record the shapes or properties that you have used with your design, and submit this with your geometry report.

Option 2:
Research to find out how geometry is used in everyday life. Your report must include geometry studied in this unit. Be specific about the geometry and how it is being used. Use words, pictures, and/or numbers to explain what you found out.

You could look at occupations, sports or architecture. Your report must include connections to at least 8 different geometric shapes/properties. If you satisfy these conditions accurately to write your report, you will earn a level 3. For a level 4 or 4+, accurately make more connections to geometry.

Assessment: Thinking and Communication

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>-uses creative process with limited accuracy and effectiveness</td>
<td>-uses creative process with some accuracy and effectiveness</td>
<td>-uses creative process with considerable accuracy and effectiveness</td>
<td>-uses creative process with a high degree of accuracy and effectiveness</td>
</tr>
<tr>
<td>-describes some connections of geometry to the picture or report correctly with major errors and/or omissions using little mathematical terminology</td>
<td>-describes some connections of geometry to the picture or report correctly with minor errors or omissions using some mathematical terminology</td>
<td>- describes most aspects connections of geometry to the picture or report clearly and correctly using appropriate mathematical terminology</td>
<td>-describes all aspects of connections of geometry to the picture or report correctly in detail using appropriate mathematical terminology</td>
</tr>
</tbody>
</table>
Performance Task: The Bicycle Trip

Mary and Carolyn set out for a bicycle trip. The distance-time graph shows their progress as they reach their destination.

On a separate piece of paper write a story that describes their trip.

Details you should include:
- times they were together/apart, stopped, or going faster/slower
- possible events explaining the different sections of the graphs
- references to time and distance, as well as your calculations of speeds in a narrative style
- comparisons and contrasts

Write a creative story as you use the information in the graph.

Assessment: Communication

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>describes some aspects of the graph with errors and/or omissions using little mathematical terminology</td>
<td>describes some aspects of the graph correctly with minor errors or omissions using some mathematical terminology</td>
<td>describes most aspects of the graph clearly and correctly using appropriate mathematical terminology</td>
<td>describes all aspects of the graph clearly and correctly in detail using appropriate mathematical terminology</td>
</tr>
</tbody>
</table>
APPENDIX F:

DOCUMENTS COLLECTED FROM HELEN

My Representation of Helen’s Images of Mathematics Communication - Verified by Helen
APPENDIX G:

PARTICIPANT SUMMARIES

Participant Summary for Shelly

Background
- Got involved in second year of teaching—met people who are involved in mathematics education and mathematics leadership in the province (also became chapter representative for Y4MA on OAME, involved with TIPS, Course Profiles, field testing, policy writing and working for the Ministry).
- Now acts as support and leadership in the department and helps teachers embrace aspects of the new curriculum (thinking and communication category).

Experiences/Resources that Influenced Teaching
- Re-teaching math to brother in school influenced thoughts about teaching (e.g., re-teaching helps to clarify thinking). Also used children (who were in high school) as ‘guinea pigs’ because they hated math so needed to be taught differently than they were learning it.
- Being involved in the education community (TIPS, Ministry, etc.) has impacted teaching. Working at the Ministry helped see that there is a solid basis and good rationale for changing the curriculum.
- Helpful resources include the internet for ideas, other teachers, and various teacher resources such as TIPS, VanDeWalle, various textbooks and activities from magazines (e.g., Mathematics Teacher).

Students
- Three tenors.
- Students are not confident and do not like to risk being wrong. Need to be set up for success or they get discouraged. Investigations are more guided than would like them to be.
- In their heads they do not like math.
- In the Applied classroom if they get tired they will just quit writing. They also like to have their questions answered.

Thoughts about:

(1) Curriculum
- Follows curriculum and finds materials to support it. Would use textbook more if had one that was aligned with curriculum.
- Achievement Chart: When new curriculum came out, thought long and hard about the four categories of the Achievement Chart and what they meant. Communicating and Thinking categories stand out because those are the skills that students take away from high school.
- Mathematical Processes: Sees problem solving and communication as bigger than the other processes (5 processes in curriculum under umbrella of problem solving and held together by communication.

(2) Mathematics
- Need to balance conceptual and procedural understanding of math. Important for….
- Social construction—bouncing ideas off each other helps to clarify thoughts and develop conceptual understanding.

(3) Communication
- Thought about communication differently when new curriculum came out. Communication is an important part of the problem solving processes and of assessment.
- Communication has two parts: (1) form, convention, proper notation & terminology; (2) clear and concise explanations of thinking and justifying thinking. Have since added their reflecting on how they learn.
- Feedback is an important part of communication.
- Consolidation is also key.

Plans/Practices/Experiences

(1) Communication in Instruction
- Literacy strategies are important. Uses anticipation guides, band show, four corners, graffiti, think-pair-share (students like this).
- Worked towards math-talk community. Close in 9 Academic and there in Data Management but not in Applied. Applied students do not have confidence to be the ones who ask questions—do not ask “what if” questions.
- Develops a classroom environment that supports communication early in the school year. Begins with an open-ended task. Also talks to students about what the mathematical processes are.

(2) Communication in Assessment
- Uses journals, tests, performance tasks, quizzes, projects (geometry task—assessment as learning)
- Rubrics to assess communication (entire department does this)—important for rubrics to be clear and to ask for exactly what you want to get. Criteria for the different levels of communication are outlined for students on tasks that are graded with respect to communication (performance tasks, projects). When creating rubrics, writes level three first—what expects students to do. A good rubric gives as much information to the teacher as to the kids.
- Students also receive a level for communication on tests. Communication questions on tests are marked “C” and students get lots of feedback on the test and receive an overall level for communication.
- Open to differentiating assessment.
(3) Challenges

- Developing rubrics can be challenging—need to be clear about what is expected (department struggles with thinking category but not communication)
- Overall, assessing communication is not very challenging, only challenge is sometimes deciding between Level 1 or 2 when there are omissions.
- Difficult for some of the students in the class to work together—not a lot of choice for partners.
- Applied students do not have confidence and have it in their head that they do not like math—not curious to ask “why” questions.
- Can be difficult for students going from a math-talk classroom to a traditional classroom where they are not allowed to talk. When students come from a traditional classroom into a math-talk classroom (e.g., Academic class) takes time for students to realize that the answer is not the most important thing.
- Timing: More flexibility in Applied course because less material. Likely because of timing that teachers rely on textbooks and rely more heavily on procedural knowledge (this year ran out of time—teacher candidate in and other school activities cut into class time).
Participant Summary for Helen

Students
- Essentials course is learning Applied material and will get Applied credit. Nowhere behind Applied and in end better mark (smaller class).
- Predominantly boys.
- Do not have to memorize—not in the curriculum for these students. Students cannot memorize—their knowledge is very fragmented. Do not put information in long term memory, everything is today.
- They are uncertain and have no confidence in their answers—need feedback. Often say they “suck at math” and accept this—give up easily.
- Need to see why we do certain things (relevance).
- Students from all over the world. English is not their first language. Many students are language impaired/learning disabled. Essentials class is 100% special ed. Applied is 50% special ed. and 50% ESL—50% are also not Grade 9 which makes it a unique class (real applied class is quiet).
- Can’t tolerate answers in back of book wrong. Should not communicate about wrong answers because they can remember it wrong.

Background

General background information:
- Enjoys teaching—doesn’t matter what.
- Educated in UK (Art, Special Education—English and Mathematics). Moved to Canada approx. around 1980 and started teaching soon after.
- Taught art (but could not teach anymore because of health), then special education. Started teaching math around 1990’s and switched into math department when curriculum changed to help Grade 11 students pass.
- Took courses around 2,000 at OISE and at York (behaviour and math AQ courses).
- Presents at workshops (manipulatives).

Experiences with communication:
- As a student, was taught ‘lecture style.’ The answer was the mark, not marks for process.
- Courses at OISE pushed communication—more group work. Tribes was being pushed and this was part of one of the AQ courses.

Experiences/Resources that Influence Teaching
- Special education huge (just good teaching).
- From teaching English learned to “chunk” tasks for students.
- Specialist in behaviour helped to look past the little things.
- AQ course materials were useful because related to curriculum. Experiences with group work in courses also influenced (although sometimes difficult with Applied classes).
Thoughts about:

(1) Teaching (strategies)
- Need to teach according to learning styles of students—most students do not learn auditory (so lecture style does not work), more visual and kinaesthetic (special education background influenced this). Finds out learning style with a game and then teaches according to needs of students. Gets to know her students (including interests) and lets her students get to know her.
- Believes that students can do better so pushes them to rewrite quizzes and come in for extra help, and do homework.
- Always writes things down and believes in using the overhead. Maintains eye contact as much as possible. Also need to rephrase things for these students.
- It is important to change your teaching and not stick to ‘the time that you were taught’ because they are changing.

(2) Curriculum
- Always asked students to communicate it just wasn’t weighted this way/segmented—just another way of putting down the numbers.
- Have tried to isolate the written language.
- When curriculum was changing more emphasis on communication/groups—previously all desks were in rows.
- If students see that math is everywhere they are more susceptible to the curriculum.

(3) Mathematics
- Math is a set of rules, etc. and communication is a fun way to use the rules (interpretation). Also practical because they need to learn to communicate for college and jobs in the future.

(4) Communication
- Communication is everything (e.g., nonverbal, quizzes, oral, writing)
- Communication shows the process (and these students have to see the process) and why we do certain things.
- There have always been word problems.
- Even by writing a test, students communicate what they know and don’t know and what need to work on.
- Students can communicate orally or in writing. Need to rephrase things for these students and keep language simple and in everyday terms so they can remember, but tie in the language of math.
- Important for these students to communicate because they come from very diverse backgrounds.
- Uses word walls because math is not a secret.
- Should not communicate about something that is wrong. If one student gets an answer 10 and another 2, they are more likely to have an argument then constructive discussion.
- Important to learn to communicate for future (college, jobs).
Plans/Practices/Experiences

(1) Communication in Instruction
- Classroom is set up in groups to encourage students to work together. Creates a comfortable environment so students are not afraid to share answers. Lets students mingle into their own groups.
- Have used four corners, placemats for review, agree/disagree, debates, work stations but these classes do not always work well.

(2) Communication in Assessment
- Lots of quizzes.
- Focusing on communication just changes what you look for on a test.
- Students get a level on communication questions, marked “C” but often have to look at entire test to see what they communicated.
- 9 Applied students are lazy when it comes to communication problems.
- Does not penalize students for spelling or grammar. If something is unclear, will talk to them for clarification.
- Students talk to each other in different language and then share answer in English.
- Uses journals
- Does not use rubrics. Models communication on the board and circles inappropriate things. They don’t do well on word problems.

(3) Challenges
- Difficult to communicate knowledge when knowledge is fragmented. These students do not put information in long term memory.
- Can be very distractible. Period of day can be a problem (after lunch can be hyper).
- Communication worked well with adults but sometimes not so well with Applied level students. Students do not work well together. Some classes do, others do not.
- Communication gets students on EQAO
- Students get frustrated when answers in back of book are wrong.
- Sometimes students do not work well together and you have to put them back in rows. Nothing to do with your teaching, can be something that happened outside of the classroom.
- Time is an issue: would like to do investigations because that is how these students learn but it takes more time.
- Second major issue is that these students are not all Grade 9.

(4) Enablers/Support/Resources
- Uses McGraw-Hill textbook and workbook (likes structure and lots of examples)
- OAME website was helpful for Grade 11 course
- TIPS and Think Literacy resources very helpful.
- Regent prep. Good for multiple choice questions. (jeopardy game).
- Support from colleagues.
- AQ courses—used material from them because based on curriculum.
APPENDIX H:
MY IMAGES OF MATHEMATICS COMMUNICATION