EXPERIMENTAL AND NUMERICAL INVESTIGATIONS INTO
OPTIMAL PARTIAL CONCRETE FILLING OF
FRP AND STEEL TUBULAR POLES

By

Jeffrey Richard Mitchell

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ABSTRACT

Glass fibre-reinforced polymer (GFRP) tubular poles can be superior to conventional poles, in that they are lighter in weight and more durable. Thin-walled tubular poles, however, tend to fail in flexure by local buckling, before fully utilizing the high tensile strength of GFRP. Increasing the wall thickness would solve this problem, but at a significant material cost. A simple and economical solution is to partially fill the tube with concrete. The aim of this study is to establish the optimal length of concrete filling that is required to achieve the highest moment capacity at a minimum dead weight in cantilevered GFRP and steel poles.

The study comprises experimental and numerical phases. Six 3660 mm long and 220 mm in diameter GFRP tubes of 4.15 mm wall thickness as well as four 1855 mm long and 114 mm in diameter steel tubes of 3 mm wall thickness, were filled with concrete of varying lengths, ranging from zero to a 100% of the span. The tubes were tested to failure in cantilever bending. The completely filled tubes achieved nearly double the strength of the hollow ones. Furthermore, it was found that the optimal ratio of concrete filling length was 0.34 and 0.46 of the span, for the GFRP and steel tubes, respectively. This is defined as the minimum filling length required to achieve the capacity of the completely filled tube.

Numerical models have been developed to predict the behaviour of partially concrete-filled GFRP and steel tubes as well as the optimal filling ratio. The models incorporate other models developed for hollow and completely filled tubes and account for the slight non-linearity of multi-layer GFRP tubes, concrete, and steel plasticity. An
important feature of the models is their ability to account for ovalization and local buckling of the hollow part of thin tube. The models were successfully validated and used in a parametric study to investigate the effects of key parameters, namely diameter-to-thickness (D/t) ratio, GFRP laminate structure and steel yield strength. It was shown that the optimal filling ratio increases as D/t ratio is reduced or as more GFRP fibres become oriented longitudinally. However, it was unaffected by the steel yield strength.
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NOTATION

\( A_c(i) \) Net area of concrete within the strip \( i \)
\( A(i) \) Cross-sectional area of the GFRP tube within the strip \( i \)
\( A_s(i) \) Cross-sectional area of the steel tube within the strip \( i \)
\( B(i) \) Width of half the strip \( i \)
\( c \) Neutral axis depth within a beam cross-section
\( CC(i) \) Compressive force in the concrete within strip \( i \)
\( CCC \) Total compressive force in the concrete
\( CF(i) \) Compressive force in the GFRP tube within strip \( i \)
\( CFF \) Total compressive force in the GFRP tube
\( D \) Diameter of the GFRP tube taken to the tube mid-thickness (in the flexure numerical model)
\( D \) Depth of beam specimens, equal to the outer diameter of the specimen, or
\( D_h(x_h) \) Width of a hollow steel tube after ovalization at a distance \( x_h \) measured from the free end
\( D_O \) Outer diameter of the PCFFT and PCFST specimens
\( D_s(x_h) \) Depth of hollow steel tube after ovalization at a distance \( x_h \) measured from the free end
\( D_{xx} \) Longitudinal bending stiffness of GFRP tube
\( D_{yy} \) Circumferential stiffness of GFRP tube
\( E \) Elastic modulus of steel tube
\( E_{co} \) Tangent modulus of concrete
\( ERR \) Tolerance value of force equilibrium for analysis
\( E_{sec} \) Secant modulus of concrete
\( E_x \) Elastic modulus of the GFRP tube in the longitudinal direction
\( E_y \) Elastic modulus of GFRP tube in the hoop direction
\( f'_c \) Unconfined concrete uniaxial compressive strength
\( f_c \) Concrete compressive stress at strain \( \varepsilon_c \)
\( f_c(i) \) Stress in strip \( i \) for concrete
\( f_{cr} \) Cracking strength of concrete in tension
\( f_{i(i)} \) Stress in strip \( i \) for GFRP tube
\( f_u \) Ultimate strength of steel, GFRP, or CFRP rebar or steel wire
\( f_y \) Yield strength of steel tube
\( h(i) \) Distance to centroid of a general strip \( i \) measured from the mid-thickness of the GFRP or steel tube in compression
\( h_i \) Height of each strip within the cross-section of the PCFFT
\( i \) Identification number of a general strip of height \( h_i \) in the cross-section of an RCFFT
\( I(x_h) \) Moment of inertia of a hollow GFRP tube after ovalization at a distance \( x_h \) measured from the free end
\( I_o \) Initial moment of inertia of hollow GFRP tube
\( K_N \) Normalizing factor for hollow steel tubes
\( L \) Total length of PCFFT or PCFST
\( L(i) \) Length of part of the perimeter of the GFRP or steel tube on one side of the cross-section in strip \( i \)
\( L_h \) Length of hollow portion of GFRP or steel tube
\( M \) Total bending moment in the CFFT cross-section
\( M(x_h) \) Moment in a hollow GFRP tube at a distance \( x_h \) measured from the free end
\( MCC \) Internal moment of the compression forces of the concrete
\( MCF \) Internal moment of the compression forces of the GFRP tube
\( M_{full} \) Moment capacity of a CFFT or CFST
\( M_h \) Moment applied to the base of a hollow GFRP tube
\( M_{hollow-B} \) Moment capacity of a hollow GFRP or steel tube governed by buckling
\( M_{hollow-S} \) Moment capacity of a hollow GFRP or steel tube governed by material fracture
\( M_o \) Moment at the base of a hollow GFRP tube due to ovalization
\( M_s(x_h) \) Moment in a hollow steel tube at a distance \( x_h \) measured from the free end
\( MTC \) Internal moment of the tension forces of the concrete
\( MTF \) Internal moment of the tension forces of the GFRP tube
\( M_{alt} \) Moment capacity of a hollow GFRP tube governed by material fracture
Notation

\(n\) Number of strip layers within a CFFT or CFST section for analysis
\(n_f\) Number of longitudinal segments in the filled portion of a PCFFT or PCFST
\(n_h\) Number of longitudinal segments in the hollow portion of a PCFFT or PCFST
\(P\) Failure load of PCFFT or PCFST
\(P_{full}\) Failure load of CFFT or CFST
\(P_{full}\) Maximum load of concrete-filled GFRP or steel tube
\(r\) Radius of GFRP or steel tube
\(r\) Factor in Popovics concrete model, defined as \(E_{co}/(E_{co} - E_{sec})\)
\(t\) Structural thickness of the GFRP or steel tube
\(TC(i)\) Tension force in the concrete in layer \(i\)
\(TCC\) Total tension force in the concrete
\(TF(i)\) Tension force in the GFRP tube in layer \(i\)
\(TFF\) Total tension force in the GFRP tube
\(x\) Distance measured from the free end to any point along the length of a PCFFT or PCFST
\(x_f\) General position along the concrete-filled portion of the tube, measured from the free end
\(x_h\) General position along the hollow portion of the tube, measured from the free end
\(x_{if}\) Length of longitudinal segment in concrete-filled GFRP or steel tube
\(x_{ih}\) Length of longitudinal segment in hollow GFRP or steel tube
\(x_{opt}\) Optimal concrete filling height for PCFTTs or PCFSTs
\(xx(i)\) Horizontal distance from the vertical axis to the steel tube with a strip \(i\)
\(xx_1(i)\) Horizontal distance from the vertical axis to the lower bound of the steel tube with a strip \(i\)
\(xx_2(i)\) Horizontal distance from the vertical axis to the upper bound of the steel tube with a strip \(i\)
\(Y\) Distance from fixed end (or face of concrete filling) to the location of local buckling failure in a PCFFT
\begin{align*}
y(i) & \quad \text{Distance from neutral axis to mid-height of strip } i \\
y(x) & \quad \text{Deflection at point } x \text{ along the beam} \\
y(x_h) & \quad \text{Depth to neutral axis in a hollow GFRP after ovalization at a distance } x_h \text{ measured from the free end} \\
yy(i) & \quad \text{Vertical distance from the horizontal axis to the steel tube with a strip } i \\
yy_1(i) & \quad \text{Vertical distance from the horizontal axis to the lower bound of the steel tube with a strip } i \\
yy_2(i) & \quad \text{Vertical distance from the horizontal axis to the upper bound of the steel tube with a strip } i \\
\alpha_1 & \quad \text{Factor accounting for bond characteristics in tension concrete calculations} \\
\alpha_2 & \quad \text{Factor accounting for nature of loading in tension concrete calculations} \\
\varepsilon & \quad \text{Strain at extreme top and bottom surfaces of a hollow steel tube} \\
\varepsilon_b & \quad \text{Longitudinal strain at bottom surface of concrete-filled GFRP or steel tube} \\
\varepsilon_{bu} & \quad \text{Ultimate strain at bottom surface of concrete-filled GFRP or steel tube} \\
\varepsilon_c & \quad \text{Strain in the concrete corresponding to general stress } f_c, \text{ or} \\
\varepsilon'_c & \quad \text{Strain in concrete corresponding to the unconfined concrete strength } f'_c \\
\varepsilon_{cr} & \quad \text{Cracking strain of concrete in tension, corresponding to } f_{cr} \\
\varepsilon_{crit} & \quad \text{Critical buckling strain in a hollow steel tube} \\
\varepsilon_t & \quad \text{Longitudinal strain at top fibre of GFRP tube} \\
\varepsilon_{tu} & \quad \text{Ultimate strain at top surface of concrete-filled GFRP or steel tube} \\
\theta & \quad \text{Fibre angle of an angle ply within an FRP tube} \\
\mu & \quad \text{Poisson’s ratio for steel} \\
\mu_{xy} & \quad \text{Poisson’s ratio for GFRP tubes} \\
\zeta(x_h) & \quad \text{Ovalization ratio at distance } x_h \text{ from free end of hollow GFRP or steel tube} \\
\zeta_o & \quad \text{Maximum ovalization ratio at midspan of hollow GFRP or steel tube} \\
\sigma_1 & \quad \text{Longitudinal stress in hollow GFRP tube due to ovalization} \\
\sigma_2 & \quad \text{Longitudinal stress in hollow GFRP tube due to bending} \\
\sigma_{cr}(x_h) & \quad \text{Critical buckling stress in a hollow GFRP tube at a distance } x_h \text{ measured from the free end} \\
\sigma_T & \quad \text{Total longitudinal stress in hollow GFRP tube due to bending and ovalization}
\end{align*}
\( \varphi_1(i) \)  Angle in radians between vertical centreline of the section and the radius defining the beginning of the arc \( L(i) \) in the cross-section of a PCFFT or PCFST

\( \varphi_2(i) \)  Angle in radians between vertical centreline of the section and the radius defining the end of the arc \( L(i) \) in the cross-section of a PCFFT or PCFST

\( \psi(x_h) \)  Curvature of the cross-section of a beam specimen in flexure at a distance \( x_h \) from the free end of the tube

\( \Psi_N \)  Normalizing factor for hollow steel tubes

\( \psi_s(x_h) \)  Curvature in a hollow steel tube at a distance \( x_h \) measured from the free end

\( \Omega \)  Dimensionless parameter used for hollow GFRP tubes
CHAPTER 1: INTRODUCTION

1.1 Introduction

Fibre-reinforced polymers (FRPs) are increasingly used in structural engineering applications. FRPs are a composite material consisting of glass, carbon, or aramid fibres embedded in a resin matrix. The fibres provide the tensile strength and stiffness, while the matrix binds the fibres together and distributes stresses throughout the fibres. There has been numerous structural applications of FRPs, including reinforcement bars in new concrete structures, FRP sheets and strips to rehabilitate or strengthen existing structures, and FRP tubes filled with concrete in new construction to provide both permanent formwork and reinforcement. FRPs are generally lightweight, have high tensile strength, durable (i.e. corrosion-resistant), which makes them an attractive alternative to traditional construction materials, particularly with regard to reducing long-term maintenance costs.

FRP materials have also been used to fabricate poles for transmission line supports and have the potential for other types of poles, including highway sign-supporting structures, bridge poles, and telecommunication towers. A typical FRP pole consists of a hollow, thin-walled FRP tube. Traditionally, poles have been fabricated using wood, steel, or concrete.

1.2 Research Significance

Hollow FRP poles, like all thin-walled structures, are vulnerable to premature failure due to local buckling. Local buckling is an undesirable failure mode because the full capacity of the FRP material is not utilized. Furthermore, the local buckling load is
difficult to predict since it largely depends on geometric imperfections, which are difficult to measure or observe. Increasing the wall thickness of the tube mitigates the local buckling problem, but the high cost of FRP makes this an expensive solution. The present thesis suggests that a more economical remedy is to fill a portion of the tube with concrete. A number of studies [Fardis and Khalil (1981), Seible (1996), Fam and Rizkalla (2002)] have presented the benefits of completely concrete-filled FRP tube (CFFT) systems. The concrete core supports the tube and prevents inward local buckling, while the tube confines and protects the concrete from environmental exposure and provides flexural reinforcement at the most efficient location. It also acts as permanent formwork, which reduces construction time and cost. In light of all the benefits of CFFTs, there are some drawbacks in some situations that need to be addressed. In pure flexure, Fam (2000) has shown that the concrete core after cracking provides little stiffness or strength (both are largely governed by the FRP tube), but mainly supports the tube against local buckling. At the same time it significantly increases the deadweight of the member. This is particularly significant in utility poles that are loaded in cantilever bending.

Figure 1-1(a) illustrates the problem under consideration. A point load applied to the tip of a pole will create a bending moment that varies linearly from a maximum value at the base to zero at the tip. Because the bending moment reduces along the pole’s length, filling the entire tube with concrete is unnecessary. A complete filling adds undesired deadweight and cost. A more effective solution is to use concrete in a portion of the tube near the base only. The concrete will strengthen the tube and prevent inward local buckling in the region of maximum moment, while the remaining hollow portion,
where the bending moment is lower, allows the FRP’s lightweight benefits to be maintained. The primary objective of this study is to determine the optimal height of concrete filling, $x_{opt}$. This study defines the optimal concrete filling height as the minimum amount of concrete required to achieve the maximum bending capacity. If less than the optimal amount of concrete is used, the full capacity of the tube material will not be utilized. If more than the optimal amount of concrete is used, the pole will have extraneous deadweight that does not contribute to bending strength. Previous studies by Fam (2000) and Ibrahim (2000) have presented the performance of completely concrete-filled FRP tubes and completely hollow FRP tubular poles, respectively. The present study adds the benefits of concrete-filling to a lightweight hollow FRP tubular pole in cantilever bending to devise a pole that has the same flexural strength as a completely concrete-filled tube, but does not sacrifice much of the lightweight benefit of a hollow tube.

1.3 Hypothesized Design Approach

Figure 1-1(b) summarizes the hypothesis of this research problem. The optimal filling amount is hypothesized to be the height of concrete that will allow the tube to fail by rupture of the FRP, likely in tension, but possibly in compression, near the base (location A in Figure 1-1(a)), while simultaneously failing by local buckling (if the wall is too thin) at a short distance from the face of the concrete (location B in Figure 1-1(a)), or by FRP fracture, likely in compression (if the wall is thick) at the concrete face (location C in Figure 1-1(a)). Whether the tube will fail by local buckling or fracture in the hollow part depends on the laminate structure and diameter-to-thickness (D/t) ratio.
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Introduction

The following section develops simple expressions for the optimal length of concrete filling ($x_{opt}$) of a partially concrete-filled FRP tube (PCFFT) and a partially concrete-filled steel tube (PCFST), based on the problem illustrated in Figure 1-1. An experimental program and an numerical parametric study will validate these expressions in Chapter 5.

Knowing the moment capacities of the concrete-filled and hollow cross-sections, $M_{full}$ and $M_{Hollow-B}$ (based on buckling), an expression for $x_{opt}$ is derived from Figure 1-2(a) as follows:

$$x_{opt} = \left( 1 - \frac{M_{Hollow-B}}{M_{full}} \right) L - Y$$

(1-1)

For FRP tubes, expressions for $M_{Hollow-B}$, $M_{full}$, and $Y$ are presented in Chapter 2. The distance ($Y$) (location of local buckling from concrete face) dependent on the hollow length as $Y = \lambda L_h$, and $L_h = L - x_{opt}$.

When the hollow tube is governed by material strength, its moment capacity is $M_{Hollow-S}$, and $x_{opt}$ is established from Figure 1-2(b) as follows:

$$x_{opt} = \left( 1 - \frac{M_{Hollow-S}}{M_{full}} \right) L$$

(1-2)

In the absence of a discrete expression for the location ($Y$) where buckling will occur, it is assumed that the tube will buckle at $Y = D$ from the face of the concrete filling, where $D$ is the diameter. This is the case for steel tubes studied in this thesis. As with a hollow GFRP tube, a hollow steel tube is governed by a failure mode that ranges from local buckling before yielding to the full plastic moment capacity. This depends on the class of the section, which is a function of $(D/t)$ ratio and yield strength. Details regarding the moment capacity of a hollow steel tube are presented in Chapter 2. The
concrete-filled steel tube (CFST) will always fail by reaching the full plastic moment capacity near the base. Various methods for calculating the capacity of a CFST are also discussed in Chapter 2.

1.4 Objectives

The principal objectives of this research program are to investigate the flexural performance of PCFFTs and PCFSTs. Specific objectives addressed in this study include:

1. Establish the optimum length of partial concrete filling
2. Observe the variation in failure load as the amount of concrete filling in FRP and steel tubes is gradually increased from zero to 100%
3. Develop an numerical model to predict the moment-curvature and load-deflection behaviour of PCFFTs and PCFSTs.
4. Evaluate the following specific parameters on the flexural behaviour of PCFFTs and PCFSTs
   a) (D/t) ratio (for both PCFFTs and PCFSTs)
   b) FRP tube laminate structure for PCFFTs
   c) Yield strength of steel tube for PCFSTs
5. Use the results of the parametric study to verify the hypothesized design approach proposed in Section 1-3 of this chapter.

1.5 Scope

The scope of the study consists of an experimental investigation and an numerical model to study the behaviour of PCFFTs and PCFSTs. The purpose of the experimental
program was to observe the effects of varying the amount of concrete filling in a PCFFT or PCFST on flexural behaviour, ranging from completely hollow to completely concrete-filled. Six prismatic PCFFTs and four PCFSTs were tested in cantilever bending.

An numerical model was developed to predict the moment-curvature and load-deflection responses of PCFFTs and PCFSTs. The model builds on other models developed by Fam (2000), Ibrahim (2000), and Brazier (1927). For the concrete-filled portion, the model assumes strain compatibility and uses a layer-by-layer approach to perform a cracked section analysis that accounts for the material nonlinearities of concrete, GFRP, or steel. The model for the hollow portion of the tube accounts for ovalization of the cross-section during bending. The model was validated by the results of the experimental program. This was followed by a parametric study conducted to evaluate the effects of changing the (D/t) ratio and laminate structure for GFRP tubes, and changing the (D/t) ratio and yield strength for the steel tubes. The goal was to observe the effects each parameter has on the optimal concrete-filling length.

1.6 Thesis Outline

The contents of the thesis are briefly outlined below:

Chapter 2: presents a review of literature related to previous research on GFRP and steel monopoles, CFFTs, CFSTs, various models, and design standards governing the behaviour of FRP monopoles and hollow and concrete-filled steel tubes.
Chapter 3: provides a detailed description of the experimental program, including design and fabrication of test specimens, instrumentation, test setup, and procedures. Ancillary tests conducted to determine the properties of constituent materials are also explained.

Chapter 4: provides the results of the experimental investigations into the flexural behaviour of PCFFTs and PCFSTs, including failure modes and the effect of changing the amount of concrete filling on behaviour and strength, as well as the results of the ancillary material tests.

Chapter 5: presents the proposed numerical model for flexure of PCFFTs and PCFSTs, verification of the model using experimental results, and parametric studies.

Chapter 6: presents conclusions of the study and recommendations for future research into PCFFTs and PCFSTs.

References.
Figure 1-1: (a) Problem schematic (b) Effect of concrete fill length on moments
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Concrete filling

(a) When hollow part fails by local buckling

Concrete filling

(b) When hollow part fails by strength limit

Figure 1-2: Design parameters for partially filled tubes
CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter presents the historical development of GFRP tubular poles, as well as the manufacturing process of GFRP tubes. Previous research related to buckling prevention of GFRP and steel hollow tubes is discussed, and various design standards are introduced. The first part of this chapter discusses the shortcomings of poles fabricated using conventional materials, followed by the development and the manufacturing process of GFRP tubes, as well as advantages and disadvantages of GFRP as a material for poles. The next section examines previous studies on the behaviour of hollow tubular sections. The third section details studies conducted on preventing buckling of GFRP tubular poles. Finally, research on buckling prevention of steel tubes is investigated.

Traditional poles are fabricated using wood, steel, or concrete, but these conventional materials have their shortcomings. Wooden poles have durability issues [Vanderbilt and Criswell, 1988] and are vulnerable to woodpecker damage [Harness and Walters, 2004]. Chemical additives can increase wooden poles service life, however the environmental impact is a concern. Steel poles are capable of resisting high loads, but corrosion is an issue that requires costly treatment, typically by painting or galvanizing. However, galvanized steel poles are still vulnerable to corrosion in storage yards, and at or below the ground line [Zamanzadeh et al, 2006]. Concrete poles are very heavy, which results in high transportation and installation costs. Maintenance to prevent corrosion of steel reinforcement, particularly due to de-icing chemicals, is a concern [Charour and Soudki, 2006].
2.2 Fabrication of GFRP Tubes

The following section briefly describes three methods of fabricating FRP tubular members: pultrusion, filament-winding, and centrifugal casting. The growing use of FRP materials is attributed to a number of factors including advances made with materials, product engineering, and process technology. Inherent advantages of FRP material include its light weight, high strength, corrosion resistance, and non-conductivity.

2.2.1 Pultrusion

Pultrusion is suitable for members of a uniform cross-section, such as tubes, rods, beams, and channels. Pultrusion involves pulling resin-impregnated fibres through a heated die that is formed to the desired cross-sectional geometry (Figure 2-1). Pultrusion is the recommended method when complex cross-sectional geometry is desired. Fibres are typically in the longitudinal direction, although it is possible to pultrude multidirectional fibres. After the fibres and resin are cured and hardened in the die members are cut at the desired length.

2.2.2 Filament-winding

Filament-winding is typically used to produce hollow tubular shapes. It involves wrapping continuous resin-impregnated fibres around a rotating mandrel (Figure 2-2). A major advantage of filament-winding is that the fibres can be oriented in any configuration to produce the desired mechanical properties specified by a designer. This
can be controlled through the relative rotational velocity of the mandrel as well as the linear velocity of the fibre feeder.

### 2.2.3 Centrifugal Casting

Centrifugal casting is also used for fabricating hollow tubular members. It is a similar process to that of spun-cast concrete. Resin and fibres are loaded into a mould that rotates at high speeds (Figure 2-3). Centrifugal forces push the fibres to the interior surface of the mould, while the resin is distributed throughout the reinforcing fibres. Heat is often applied to accelerate the curing process.

### 2.3 Design and Testing Guidelines of GFRP Poles


#### 2.3.1 AASHTO Design Guidelines

This section outlines the AASHTO (2001) method for designing poles for flexure. The determination of wind loads for a pole of any material (i.e. FRP, steel, wood), as well as flexural resistance of FRP poles will be discussed. Other considerations need to be made for other geometric shapes, as well as for other types of loading, such as axial compression. However, only flexure is discussed here.
2.3.1.1 Design Wind Load

A pole is subjected to four types of loads: dead, wind, ice, and fatigue loads. Dead load consists of the member self-weight, as well as the weight of all attachments. Since flexural loads are applied primarily by wind, only wind load determination will be detailed here. The same method for calculating wind load is used for poles of any material.

Wind pressure, cross-sectional area and shape, and gust effects govern the wind load acting on a pole. The wind pressure is given as:

\[
P_z = 0.613K_zGV^2I_rC_d
\]  

(2-1)

where \(K_z\) is the height and exposure factor, \(G\) is the gust factor, \(V\) is the basic wind velocity, \(I_r\) is the wind importance factor, and \(C_d\) is the drag coefficient. The \(K_z\) is a dimensionless coefficient that corrects the magnitude of the wind pressure, referenced to a height above the ground of 10 m for the variation of wind speed with height. Wind varies with height due to frictional drag associated with different types of terrain. The gust factor corrects the effective velocity pressure for the dynamic interaction of the structure with the gustiness of the wind. The basic wind velocity, \(V\), is the given wind velocity for a particular geographic region. The importance factor allows 50-year mean recurrence interval wind speeds to the adjusted to represent 10-, 25-, and 100-year mean recurrence interval. The drag coefficient accounts for the effect of the cross-sectional geometry on wind pressure.
2.3.1.2 Flexural Design

This section discusses the AASHTO method for designing FRP poles. Thin-walled tubular members are vulnerable to local buckling. AASHTO defines the critical buckling stress in terms of the critical bending stress. Johnson (1985) found that the critical bending stress is about 30% higher than the buckling stress in axial compression. For round tubular members, AASHTO gives the critical bending stress $F_b$ as:

$$F_b = \frac{0.75E_1K_1}{n_a\left(\frac{D}{t}\right)^{\frac{1}{2}}\mu\frac{1}{2}} \leq \frac{F_{bu}}{n_a}$$  \hspace{1cm} (2-2)

where $E_1$ is the elastic modulus in the longitudinal direction, $K_1$ is the orthotropy factor, $n_a$ is the safety factor, $D$ is the tube diameter, $t$ is the tube wall thickness, $\mu$ is Poisson’s ratio, and $F_{bu}$ is the ultimate bending strength of the FRP material. For bending strength, $n_a$ is given as 2.5. The $K_1$ factor is given as:

$$K_1 = 1.414 \left[1 + \nu_{12}\left(\frac{E_2}{E_1}\right)^{\frac{1}{2}}\left(\frac{E_2}{E_1}\right)^{\frac{1}{2}}\left(\frac{G}{E_1}\right)^{\frac{1}{2}}\right]$$  \hspace{1cm} (2-3)

where $E_2$ is the elastic modulus in the circumferential direction, $\nu_{12}$ is Poisson’s ratio in the longitudinal direction, and $G$ is the in-plane shear modulus. Poisson’s ratio, $\mu$, is given as:

$$\mu = 1 - \nu_{12}^2\left(\frac{E_2}{E_1}\right)$$  \hspace{1cm} (2-4)

The allowable tensile stress, $F_t$, in bending is given as:

$$F_t = \frac{F_{tu}}{n_a}$$  \hspace{1cm} (2-5)

where $F_{tu}$ is the ultimate tensile strength of the FRP material.
2.3.2 Laboratory Testing of FRP Lighting Poles

The ANSI C136.20-1990 standard provides details on manufacturing considerations, and laboratory testing procedures, strength requirements, and deflection limits of FRP poles. More information regarding manufacturing, design, and handling considerations is also found in ASCE’s “Recommended Practice for Fiber-Reinforced Polymer Products for Overhead Utility Line Structures” (ASCE, 2003).

In order for the manufacturer to fabricate and test an appropriate pole, the user must specify a number of details, including: (a) the type of pole mounting – either anchor base or direct embedment; (b) length, weight, effective projected area (EPA), and support arm attachment details; (c) weight and EPA of the luminaries; (d) maximum wind velocity for installation location; (e) operating and maintenance practices, and any equipment that will be used to service the pole, plus the subsequent additional load applied to the pole; (f) applicable local or state codes, if they differ from the national standards; and finally, (g) details on access openings, if any; or any other custom features the user may desire.

The surface of the pole shaft should resist the environment in which it is installed. It should also be able to resist twice the maximum bending moment applied by specified wind loads. When handholes are specified, the shaft should resist the moment when the handholes are on the compression side. The lateral deflection of the tip should not exceed 10% of the pole height above the ground line when subjected to specified wind loads. Also, there should be less than 1% permanent deflection for the specified wind load. During testing, the load should be applied for 5 minutes at 25° C ± 1.5° C, with deflection measurements being taken 5 minutes after the load is released.
To determine whether the pole will satisfy strength and deflection requirements, the wind force and subsequent base moment are calculated by the AASHTO method presented in Section 0. For the deflection requirements, the design force is calculated as:

$$F_d = \frac{MB}{(L - L_1)}$$

(2-6)

where $F_d$ is the force, $MB$ is the base moment, $L$ is the overall pole length, and $L_1$ is the length from the top of the pole to the applied force. Deflection requirements are checked when the force $F_d$ is applied. For strength, twice the force $F_d$ is applied to the pole to check for failure.

### 2.4 General Behaviour of Hollow Tubes in Flexure

Brazier (1927) conducted the first study into the buckling and ovalizing behaviour of elastic tubular members of circular cross-section. Brazier found that a tube cross-section becomes increasingly ovular as the applied longitudinal curvature is increased (Figure 2-4). In original studies by St. Venant, it was thought that the bending moment would increase linearly with curvature until the tube material failed. But Brazier showed that as the cross-section ovalizes, the moment-curvature response will flatten and eventually reaches a maximum, before decreasing. The point immediately following the maximum moment is the point of instability when the tube will buckle. Brazier introduced 2nd-order terms to derive an expression for the critical bending moment, as well as the maximum amount of ovalization at the point of buckling. Brazier assumed that the tube was infinitely long, the radial displacements were small relative to the radius, and that the tube was inextensional in the circumferential direction. The critical
moment was determined by minimizing the total strain energy along the length of the tube, with respect to change in applied curvature. The critical moment is given as:

\[ M_{cr} = \frac{2\sqrt{2}}{9} \frac{E\pi rt^2}{\sqrt{1-\nu^2}} \]  

(2-7)

where \( E \) is the elastic modulus of the tube, \( r \) is the original radius of the tube, \( t \) is the wall thickness, and \( \nu \) is Poisson’s ratio. The maximum amount of radial displacement at the point of instability is given as:

\[ \omega = \frac{2}{9} r \]  

(2-8)

where \( \omega \) is the change in radius.

Reissner (1961, 1962) conducted a more general investigation into cylindrical members of arbitrary cross-section. When a special case of a circular cross-section was considered, the critical moment approached that derived by Brazier (Equation 2-7). Reissner developed a formulation to relate the normalized curvature to the normalized bending moment. The relationship was an expansion of \( n \)-terms. When terms up to only 2\(^{nd}\)-order were considered, as was prescribed by Brazier, the critical moment and curvature approached those of Brazier. The behaviour up to the critical point also agreed well with that of the numerical solution that included all \( n \) terms. Both Reissner and Karamanos (2002) showed that although Brazier’s 2\(^{nd}\)-order relationship between ovalization and curvature was accurate up to the point of instability, it failed to model post-buckling behaviour very well.
Keward (1978) reworked Brazier’s critical moment expression to account for orthotropic material properties of composite (FRP) tubes, and derived the following expression:

\[
M_{cr} = \frac{4\sqrt{2}}{3\sqrt{3}} \sqrt{\frac{D_{22}}{E_x t}} \pi r t
\]  

(2-9)

where \(D_{22}\) is the circumferential bending stiffness, \(E_x\) is the elastic modulus in the longitudinal direction, \(r\) is the radius, and \(t\) is the wall thickness of the FRP tube.

Li (1996) studied the static and dynamic instability of circular hollow tubes. He adopted Brazier’s assumptions and the 2\textsuperscript{nd}-order relationship between ovalization and curvature to produce the following static bending critical moment:

\[
M_{cr} = \frac{2\sqrt{2}}{9} \pi r t^2 \sqrt{\frac{E_1 E_2}{(1 - \nu_{12} \nu_{21})}}
\]  

(2-10)

where \(E_1\) is the longitudinal elastic modulus, \(E_2\) is the circumferential bending modulus, and \(\nu_{12}\) and \(\nu_{21}\) are Poisson’s ratio in the longitudinal and circumferential directions. Li’s prediction matched that of Keward. When studying dynamic instability, Li found that the critical moment under cyclic loading was approximately 30% less than the static case and the radial displacement at the point of instability was doubled.

Ibrahim (2000) completed a study on the behaviour of filament-wound completely hollow GFRP poles. He developed a rational and a numerical model, each of which was validated through experimental testing. The ultimate goal of the study was to develop optimum configurations for GFRP poles, with respect to laminate structure and cross-sectional geometry. The current study differs from Ibrahim’s work in that Ibrahim studied completely hollow GFRP tubes only, whereas the present study adds a small
amount of concrete near the base of the hollow GFRP tube to improve performance. The testing evaluated the effects of wall thickness, fibre orientation, and layer sequence on the behaviour and ultimate load carrying capacity of the poles. Ibrahim tested twelve scaled specimens and twelve full-scale specimens in cantilever bending. Each specimen was attached to a concrete block to create the fixed end support for the cantilever. For the scaled specimens, a 430 mm high concrete plug was embedded into the concrete block and the FRP tube was placed on top of the plug. The full-scale specimens were embedded into a concrete base and a 1000 mm long, and 25 mm thick, tapered GFRP sleeve was inserted into the tube within the concrete base to stabilize the tube within the base, where stresses are high. The scaled specimens were 2500 mm long, with a free length of 2070 mm. Each tube was tapered with a base diameter of 100 mm and a tip diameter of 75 mm. The wall thickness varied from 0.88 mm to 2.2 mm, and the longitudinal fibre angles varied from 25° to 35°, in 5° increments. Load was applied at a distance of 1940 mm from the base, where the lateral deflection was measured. The full-scale poles were 6250 mm long and similarly embedded into a concrete base. The base diameter of 416 mm was tapered to 305 mm at the tip. Longitudinal fibres were angled at 5°, 10°, and 20°. Load was applied at 600 mm from the tip, in adherence with ANSI standards.

Ibrahim’s experimental program indicated that GFRP tubular poles were an adequate replacement for poles of traditional materials. The GFRP poles were able to achieve similar ultimate loads with a significant weight reduction. The scaled poles had a capacity-to-weight ratio of 0.145 kN/kg, which was 52% higher than wooden poles of the same capacity. The lateral deflection ranged from 11% to 18% of the pole’s free length.
The capacity-to-weight ratio for the full-scale poles was 0.47 kN/kg, which was 500% higher than wooden poles of equivalent capacity. The lateral deflection did not exceed 13% of the free height. The best capacity-to-weight ratios were achieved when the tube included both circumferential and longitudinal fibres. When considering tubes with an equal number of layers, the addition of circumferential fibres reduced the stiffness by 20% when only longitudinal fibres were used, although the capacity was 22% higher.

Three failure modes were observed during testing: local buckling on the compression side, diagonal fracture parallel to the fibre direction on the compression side due to in-plane shear stress, and fractured fibres on the tension side due to transverse fibre pull-out. Local buckling was observed for poles with a radius-to-thickness (r/t) ratio of r/t = 57. Diagonal fracture was observed when r/t = 40, and tension fraction occurred when r/t = 23. Ten of the full-scale poles failed by local buckling. The load-deflection relationship was linear up to around 70% of the ultimate load. The behaviour then became non-linear at higher loads due to progressive failure of the resin. All specimens experienced ovalization of the cross-section, which was highly non-linear.

An numerical model was developed that accounted for the non-linear ovalization of the cross-section. Ibrahim derived the following relationship between the maximum ovalization ratio and bending moment:

\[
M_o = 2.85 \pi r \sqrt{\pi E_x D_{yy} t \left( l + \frac{\pi^4}{12 \Omega^2} \right) \left( 1 - \frac{3 \xi_o}{\pi} \right) \sqrt{\xi_o}} \tag{2-11}
\]

where \(M_o\) is the base moment, \(E_x\) is the elastic modulus in the longitudinal direction, \(r\) is the initial tube radius, \(D_{yy}\) is the circumferential bending stiffness of the FRP tube, \(t\) is the
wall thickness, $\Omega$ is equal to $(L/r)\sqrt{12D_{yy}/r^2E_t}$, and $\xi_o$ is the maximum ovalization ratio at the midpoint of the hollow length, defined as the change in radius divided by the original radius. Ovalization is assumed to have a sinusoidal distribution along the hollow length. The ovalization at any distance $x$ from the free end is given as $\xi(x) = \xi_o\sin(\pi x/L)$.

The subsequent moment of inertia at any point in the hollow length is $I(x) = I_o(1 - 1.5\xi_o\sin(\pi x/L))$, where $I_o$ is the initial moment of inertia of the undeformed circular cross-section. Initially, the distance from the neutral axis to the extreme surfaces, $y$, is equal to the radius, $r$, but after ovalization begins, it becomes $y = r(1-\xi_o)$. The hollow tube model also assumes that the circumferential strains are zero, since they are small relative to the longitudinal strains; and that ovalization is prevented at the base and at the tip.

Ibrahim also presented the relationship between ovalization and stress. The total stress in a hollow GFRP tube is the summation of two stresses: stress created by the ovalizing of an initially straight tube, and stress created by the applied bending moment. The total stress is given as:

$$\sigma_T = \sigma_1 + \sigma_2$$

(2-12)

where $\sigma_1$ is the stress created by ovalization, which is equal to:

$$\sigma_1 = \frac{\pi^2E_rr^2}{4L^2} \left( \xi_o\sin\left(\frac{\pi X_h}{L_h}\right) \right)$$

(2-13)

The bending stress, $\sigma_2$, is equal to:
where $L_h$ is the length of the hollow FRP tube, and $X_h$ is the distance from free end along the hollow length.

The model agreed well with the test results, as the average experimental-to-theoretical strength ratio was 0.85. Ibrahim also developed a finite-element model using ANSYS, a commercial finite-element software package. The model accounted for the non-linear pole behaviour and included a strength check based on the Tsai-Wu failure criterion. The model agreed very well with the experimental results, with an average experimental-to-theoretical ratio of 0.99.

Ibrahim’s parametric study revealed that the optimal layer sequencing of circumferential-to-total layers was 2/8, with the longitudinal fibres as close to 0° as possible. The optimal diameter for 6.1 m poles was 300 mm to 350 mm. For 12.2 m poles, the optimal diameter was 450 mm to 525 mm, and for the 18.3 m poles it was between 600 mm and 700 mm.

Ibrahim and Polyzois (1999) presented a simple method for determining the critical load for a FRP pole. The critical load $P_{cr}$ is given as:

$$P_{cr} = \frac{M_{cr}}{(L-Y)}$$

(2-15)
where $M_{cr}$ is the critical moment, $L$ is the length of the pole, and $Y$ is the location above the base where the tube will buckle. Ibrahim modified Brazier’s equation to account for FRP’s orthotropic properties. $M_{cr}$ and $Y$ are given as:

$$M_{cr} = \frac{2\sqrt{2}}{9} \pi E_x r t^2 \left[ 2.1 \frac{E_y}{E_x} - 0.84 \left( \frac{E_y}{E_x} \right)^2 \right]$$

$$Y = \left( 0.13 - 0.04 \frac{E_y}{E_x} \right) \sqrt{\frac{r}{l}} L_h$$

Ibrahim and Polyzois validated this prediction with a finite element parametric study. The parametric study found that the critical load was greatly affected by the laminate structure. The critical load decreased as the fibre angle, relative to the longitudinal axis, increased, but the addition of circumferential fibres increased the buckling load.

### 2.5 Preventing Buckling of GFRP Tubes

This section discusses research related to buckling prevention of hollow GFRP tubular members. The merits of filling a GFRP tube with concrete – either the complete cross-section, or with a void – are presented. Another study investigated the benefits of increasing the diameter of the GFRP tube near the base.

#### 2.5.1 Behaviour of Concrete-Filled GFRP Tubes

Concrete-filled FRP tubes (CFFT) have been used in practice for bridge piers, piles, and beams. The CFFT system was developed to combine the concrete and FRP tube in a way that utilizes each material effectively. The concrete provides stability to the
tube to prevent inward local buckling, while the tube confines and reinforces the concrete, and also acts as permanent formwork. A number of researchers have studied the benefits of filling hollow GFRP tubular members with concrete. Fardis and Khalil (1981), Seible (1996), and Fam and Rizkalla (2002), to name a few, have conducted studies on the benefits of filling GFRP tubes with concrete.

Fam and Rizkalla conducted an in-depth investigation into the behaviour of CFFTs. The experimental program included 20 beams with a span ranging from 1.07 m to 10.4 m, and a diameter of 89 mm to 942 mm. Nine types of GFRP tubes were used to study the effects of laminate structure and wall thickness on the behaviour. Various cross-sectional configurations were tested as well, including completely-filled tubes and with tubes a concentric and eccentric circular voids. The study showed that concrete filling would sufficiently support the GFRP and prevent local buckling. Strength and stiffness were both increased. Tubes that were initially weaker or less stiff experienced the greatest benefit of concrete-filling. When beams were tested with a circular void, the strength was slightly reduced by about 9%, although the strength-to-weight ratio was about 35% higher than a completely filled cross-section.

Fam and Son (2007) presented an equation for the moment capacity of a concrete-filled FRP tube. The formula is based on past experimental results and the finite element results of the parametric study of the same problem. Various D/t ratios and laminate structures are accounted for by the reinforcement index, $\omega$, which is presented as follows:

$$\omega = 100 \frac{4t f_{ut}}{D f_c}$$  \hspace{1cm} (2-18)
where \((4t/D)\) is the reinforcement ratio, which is the ratio of the cross-sectional areas of the FRP tube and the concrete filling. \(D\) is the diameter and \(t\) is the wall thickness, \(f_{ut}\) is the ultimate tensile strength of the FRP tube, and \(f_c'\) is the compressive strength of the concrete filling. When the filled moment capacity \(M_{Full}\) is normalized with respect to diameter and concrete strength, the normalized moment, \(\overline{M}_{Full}\), is given as

\[
\overline{M}_{Full} = M_{Full} / (D^4 f_c')
\]

When the normalized moment is plotted against the reinforcement index, a clear trend emerges from the empirical data that is represented by:

\[
\overline{M}_{Full} = 0.0045 \omega^{0.815}
\]  \hspace{1cm} (2-19)

when Equation 2-18 is substituted into Equation 2-19, the normalized moment is rearranged to reveal the filled moment capacity, \(M_{full}\), as:

\[
M_{Full} = 0.0045 D^4 f_c' \left(100 \frac{4t}{f_{ut}} \right)^{0.815}
\]  \hspace{1cm} (2-20)

Qasrawi (2007) studied partially concrete-filled GFRP tubular poles, except the partial filling for that study was relative to the cross-section, not the length. Qasrawi tested seven spun-cast concrete-filled GFRP tubes (SCFTs), plus two control specimens. An experimental program and numerical parametric study were conducted to study the effect of concrete wall thickness, presence of steel rebar, and FRP laminate structure on behaviour. Qasrawi found that SCFTs had similar strength to conventional prestressed concrete poles, although the SCFT stiffness was less. There was a minimum concrete wall thickness required to achieve the same capacity as a completely filled cross-section, and to prevent inward spalling of the partial concrete-filling. For the experimental study, the minimum inner diameter-to-outer diameter \((D_i/D_o)\) ratio was 0.6. The parametric
study found that the \((D_l/D_o)\) ratio had a significant effect on cracking moment, but an insignificant effect on post-cracking stiffness. It was also found that increasing the percentage of longitudinal fibres increased the strength and stiffness. As either the tube’s wall thickness or the percentage of longitudinal fibres was increased, the minimum required concrete wall thickness increased.

### 2.5.2 Buckling Prevention of FRP Tubes by Increasing Inertia Near Base

Desai and Yuan (2006) developed a numerical model to study the bending and buckling performance of a stepped FRP pole (Figure 2-5). Both GFRP and CFRP poles were considered. The pole includes a length at the base of moment of inertia \(I_B\), and a top portion of \(I_T\), where \(I_B\) is greater than \(I_T\). The base length, \(a\), is referred to as the “rigidity length” by Desai and Yuan (Figure 2-5). A small tapered section connects the two lengths. The wall thickness was constant along the length. The purpose of the study was to investigate the bending and buckling performance for various lengths, \(I_T/I_B\) and \(a/L\) ratios, and material. Two types of load were applied to the pole: a unit compressive load was put on the pole tip, and the ASCE design wind load was applied transversely. The pole length varied from 3.66 m to 12.2 m, and the applied wind load ranged from 2.8 kN to 3.4 kN, respectively. The rigidity length ratio varied from 0.2 to 0.6, while \(I_T/I_B\) varied from 0.2 to 0.5. The wall thickness was constant at 12.7 mm. The geometry was the same for both GFRP and CFRP poles.

The study found that once the pole length exceeded 9.1 m, they became vulnerable to local buckling. They also found that the rigidity length ratio had little effect when the pole length was greater than 9.1 m, but it did factor significantly for shorter
poles. The buckling load of the CFRP poles was around 175% higher than for the GFRP. The $I_T/I_B$ ratio contributed significantly to the pole buckling strength, particularly for higher $I_T/I_B$ ratios. The most abrupt change occurred when $I_T/I_B$ was increased from 0.4 to 0.5. The pole length-deflection distribution illustrates that for lower $I_T/I_B$, the rigidity length has a greater effect on the deflection than for higher $I_T/I_B$. Naturally, longer poles deflected more. Analysis of the bending stress revealed a stress concentration at the step for poles over 9.1 m with small rigidity length ratios. But for a rigidity length ratio of 0.6, the stress was linear to the base of the pole.

2.6 Preventing Buckling in Steel Tubular Poles and Other Retrofit Methods

The following sections will discuss measures that have been taken to prevent buckling of steel tubular members. As with the CFFTs discussed earlier, steel tubes experienced significant benefits from concrete-filling. Methods for predicting the capacity of CFSTs are presented as well. Partially concrete-filled steel tubes are discussed, as are steel-concrete sectional composite poles, and externally bonded CFRP sheets for strengthening.

2.6.1 Concrete-Filled Steel Tubes (CFST)

A number of studies have been conducted to predict the moment capacity of a CFST. Elchalakani (2001) conducted an in-depth study of the behaviour of CFSTs. An expression was derived to predict the ultimate capacity of a CFST and an experimental program to validate the expression was conducted. The experimental program consisted of twelve specimens; six were concrete-filled, and six were left hollow. Each specimen
was 1500 mm long. The (D/t) ratio ranged from 12 to 110. The average yield strength of the tubes was 419 MPa, and the average ultimate strength was 523 MPa. The testing apparatus was designed to induce a uniform bending moment along the middle portion of the tube. Each tube was mounted on two hinged supports, one at each end. The apparatus was able to apply a bending moment without significant axial or shear forces.

In general, the concrete-filled members were stronger and more ductile than the hollow members. The hollow tubes experienced kinking and buckling after large rotations, while the CFSTs did not experience kinking. Filling the tube with concrete increased the capacity by 3% to 37%. Tubes with the greatest (D/t) ratio experienced the most significant benefits, in both strength and ductility, since hollow slender tubes will buckle at very low loads. These results agree well with studies by Lu and Kennedy (1994) and Zhao and Grzebieta (1999) who reported strength increases of 10-30% and 15-35%, respectively.

Elchalakani used a simplified rigid plastic approach to predict the ultimate moment capacity of a CFST. Since no slippage was observed during the experimental program, perfect bond was assumed for the theoretical derivation. Figure 2-6 illustrates this model. The ultimate moment capacity, $M_{ult}$, is determined by summing the moments in the steel tube and concrete core, given as:

$$M_{ult} = M_{cc} + M_{st} + M_{sc}$$  \hspace{1cm} (2-21)

where $M_{cc}$ is the moment created by the concrete in compression, $M_{st}$ is the moment created by the steel in tension, and $M_{sc}$ is the moment created by the steel in compression. Equilibrium dictates that:

$$M_{cc} = \frac{2}{3} f_c r_f^3 \cos \gamma_o$$ \hspace{1cm} (2-22)
\[ M_{st} = M_{sc} = 2 f_y r_m^2 t \cos \gamma_o \] (2-23)

where \( \gamma_o \) is the angular location of the plastic neutral axis, which is given as:

\[
\gamma_o = \frac{\pi}{4} \left( \frac{f_c r_i^2}{f_y r_m t} \right) - 2 + \frac{1}{2} \left( \frac{f_c r_i^2}{f_y r_m t} \right) \] (2-24)

where \( f_c \) is the compressive strength of the concrete, \( r_i \) is the inner radius of the steel tube, \( f_y \) is the steel yield strength, \( r_m \) is the average radius of the steel tube, and \( t \) is the wall thickness.

The author generally found strong agreement between the experimental and predicted values. It should be noted that for the three slender specimens, \( f_y \) was substituted with \( f_u \) since the members underwent significant strain hardening. After this adjustment was made, the author’s predicted capacity was, on average, 7% lower than the observed capacity, as opposed to 12% without the substitution. The author compared the experimental results to predictions offered in various design standards, including AISC-LRFD (American Institute of Steel Construction, 1993), AIJ (Architectural Institute of Japan, 1987), CIDECT (1995), and EC4 (Eurocode4, 1992). All the design standards underestimated the capacity observed in the experiments. The author made the same steel strength substitution for all design standards to find the following differences (the difference without the substitution is given in parentheses): 14% (24%) for AISC, 10% (23%) for AIJ, 5% (14%) for CIDECT, and 3% (10%) for EC4.

Han (2004, 2006) conducted two comprehensive investigations into the behaviour of concrete-filled circular, square, and rectangular hollow structural sections that were
filled with normal concrete. Han’s (2004) study included 16 square or rectangular hollow sections tested in bending. The width-to-depth ratio ranged from 1 to 2, the (D/t) ratio varied from 20 to 50, the length was 1100 mm, $f_c'$ values included 27 to 40 MPa, $f_y$ varied from 294 to 330 MPa, and the shear span-to-depth ratio varied from 1.67 to 2.1. In the Han (2006) study, 36 circular or square hollow structural sections were filled with self-consolidating concrete. The steel yield strength ranged from 235 to 282 MPa, (D/t) ratio varied between 47 and 105, the shear span-to-depth ratios ranged from 1.25 to 6, and the 28-day concrete strength included 50, 60 and 80 MPa.

During the (2004) study, Han formulated a simplified design method to predict the ultimate capacity of a CFST in flexure. Results of the formula were compared to experimental results in both studies, as well as to the values predicted by various design standards. The method was based on the composite member reaching the maximum moment when the extreme compression surface achieved a strain of 0.01, which is the strain at which Han found the moment began to stabilize in the experimental program. This strain, which may have been slightly conservative, was selected with practical design considerations in mind. Han’s design approach is as follows:

$$M_u = \gamma_m W_{scm} f_{scy} \tag{2-25}$$

where $M_u$ is the moment capacity of a CFST, $\gamma_m$ is the flexural strength index, $W_{scm}$ is the section modulus of the CFST, and $f_{scy}$ is the nominal yielding strength of the composite section. For a circular section, $\gamma_m$ is given as:

$$\gamma_m = 1.1 + 0.48 \ln(\xi + 0.1) \tag{2-26}$$

where $\xi$ is the confinement factor, which is equal to $(A_s f_{sy} / A_c f_{ck})$, where $A_s$ is the area of the steel cross-section, $f_{sy}$ is the yield strength of the steel, $A_c$ is the area of concrete in the
cross-section, and $f_{ck}$ is the characteristic concrete strength ($0.67 f_{cu}$), where $f_{cu}$ is the ultimate concrete strength. For a circular section, $W_{scm}$ is given as:

$$W_{scm} = \left(\frac{\pi D^3}{32}\right)$$  \hspace{1cm} (2-27)

where $D$ is the outer diameter of the steel tube. For a circular cross-section, $f_{scy}$ is given as:

$$f_{scy} = (1.14 + 1.02 \xi) f_{ck}$$  \hspace{1cm} (2-28)

Han (2006) found that for CFSTs with circular cross-section, his formulation predicted a moment capacity that was typically 20% lower than the observed ultimate moment from the experimental program. Han compared the results to the moment capacities predicted by various design standards: AIJ (Architectural Institute of Japan, 1997), AISC-LRFD (American Institute of Steel Construction, 1999), BS5400 (British Standard Institute, 1979), and EC4 (Eurocode4, 1994). Han found that AIJ and AISC-LRFD predicted moments about 37% less than observed, BS5400 gave moments 28% lower, and EC4 predicted 20% lower moments. For square sections, Han’s model predicted only 6% less than observed.

\subsection*{2.6.2 Composite Concrete-Steel Monopole (Partially Filled)}

Fouad (2006) developed a composite steel-concrete monopole when a utility company required poles that could support higher loads while having smaller ground line dimensions. The pole featured a thin-walled hollow steel tube that was filled with concrete at the base. The purpose of the concrete was to provide stability to the thin-walled tube, which was vulnerable to local buckling. The concrete also increased the size of the compression zone, which could resist larger loads. Concrete was added to the base
only because stresses were highest at the base of a cantilevered pole, and designers did not want to add unnecessary weight. The concrete filling can include additional reinforcement if desired. The primary advantages of this monopole were increased load, longer spans, and smaller dimensions at ground level. Fouad’s study differs from the present study in that Fouad made no attempt to optimize the amount of concrete filling. The amount of concrete filling was the same for all of Fouad’s tests.

Seven 12-sided poles were tested. Poles were typically circular or multi-sided polygons. Five of the poles were composite poles, and the remaining two poles were completely hollow control specimens. Each composite pole was 27 m long and made of two sections: a 10 m base filled with ready-mix concrete, and an overlapping 18 m hollow portion. The same hollow section was used for all seven specimens. Additional steel reinforcement was added to three of the composite poles. The members were tested horizontally with 3.35 m of the base embedded into concrete, leaving a free length of 24 m. Load was applied 609.6 mm from the tip. Three different 28-day concrete strengths were used: 39.6 MPa, 40 MPa, and 51 MPa. The wall thickness of the filled portion was 4.75 mm with a base diameter of 655 mm. The top of the filled portion had a diameter of 554 mm. The hollow section was 9.5 mm thick, and the base of the hollow portion had a diameter of 554 mm and a tip diameter of 400 mm.

Tests showed that the composite poles had a significantly higher strength and stiffness when compared to the hollow control specimens (Figure 2-7). The composite pole with additional reinforcement (Test 7) was 250% stronger than its hollow counterpart (Test 6) with an increased stiffness. The authors also found that the concrete strength and the presence of additional reinforcement had a strong influence on the
outcome. In 2005, 24.4 m free-length partially concrete-filled composite monopoles were installed in Florida.

2.6.3 Steel-Concrete Sectional Composite Pole

A sectional composite pole (Sharpless, 2004) was developed. It combined multiple materials not in the cross-section, but along the length of the pole, with various sections comprised of different materials. The sectional pole consisted of a spun-cast concrete section at the base, with one or more hollow steel tubular sections on top. The different sections were slip-spliced together. This configuration utilized the strengths of each material. Steel tubular sections were light, but they could be expensive and vulnerable to corrosion, particularly below grade, as well as buckling. On the other hand, concrete sections were durable and relatively inexpensive, but their weight made transportation difficult and expensive. When combined in the aforementioned configuration, the concrete base section would resist corrosion better than steel, and since the concrete section was relatively small, transportation and installation were easier. Furthermore, the lightweight steel section was safe from corrosion and easy to install, thus long-term maintenance was not an issue. Buckling in the steel was avoided because it was away from the region of maximum moment.

The sectional composite pole has been used in a number of projects. The first application involved the lengthening of existing spun concrete poles in Austin, Texas, in 1995 (Sharpless, 2004). Most other subsequent applications involved installation in environmentally sensitive areas. The small, relatively lightweight spun concrete bases lengths were easier to install and less disruptive to the environment. The sectional
composites have been successful on very large-scale projects as well. Two 65.5 m tall poles spanned 609.6 m, a span that previously required six wooden poles (Sharpless, 2004). On a different project, the costs of using sectional composites, all-tubular steel, or all-spun concrete poles were compared. The sectional composites were 11% and 23% less expensive than spun-cast concrete and steel poles, respectively. The best use of the sectional composite poles was in wetlands, or other areas where steel was vulnerable to corrosion near ground level; and in areas where the multi-stage erection process could simplify installation.

2.6.4 Strengthening Steel Monopoles with Externally Bonded CFRP

Lanier (2007) investigated strengthening techniques for steel monopole communication towers. Increased demands for wireless technology require the installation of additional equipment, and subsequently additional loads. Existing strengthening methods for steel monopole towers include welding steel plates to the surface. This solution is undesirable due to the poor fatigue performance of welds, and the high cost associated with welding. The proposed solution used externally bonded CFRP sheets or strips to increase the strength and stiffness of the pole. CFRP strips are lightweight and relatively easy to install. The experimental program consisted of three poles loaded in cantilever bending by a point load applied to the free tip. Each pole was 6096 mm long with a tapered 12-sided cross-section. The base and tip width were 457 mm and the tip width was 330 mm, respectively. Each unstrengthened pole was initially loaded to 60% of its yield strength to serve as a control for the strengthened members. After the unstrengthened pole was unloaded and CFRP was applied, the strengthened
pole was loaded to the same midspan deflection achieved by the unstrengthened pole. Following this stage, the pole was loaded to failure. Three different strengthening materials were used: Test 1 pole was strengthened with high-modulus CFRP sheets; Test 2 pole was strengthened with high-modulus CFRP strips; and Test 3 pole used intermediate-modulus CFRP strips. Stiffeners were added to the tube base to reduce stresses. CFRP was applied in various configurations (Figure 2-8). Test 1 included multiple layers of various lengths of CFRP sheets applied to the three upper and three lower surfaces, as well as transverse sheets for the lower half of the pole’s length. There were seven continuous transverse sheets near the base, then six transverse sheets spaced at 200 mm, followed by two transverse sheets at 305 mm. Test 2 and Test 3 included layers of CFRP strips bonded to the three upper and lower surfaces, without any transverse layers. When each strengthened member was loaded to each respective midspan deflection, the average load increase was about 30%, and the stiffness was generally greatly increased. The high-modulus strips reported the highest stiffness increase. The failure mode for Test 1 was simultaneous tensile rupture of the CFRP and buckling of the steel tube, at 200 mm from the base of the pole. The ultimate failure mode of Test 2 was buckling of the steel tube. Prior to buckling, the compression CFRP strips had crushed at a strain of 0.15%, and the tensile CFRP simultaneously ruptured and delaminated at 0.18% strain. All CFRP sheets either ruptured or delaminated prior to buckling. The ultimate failure mode of Test 3 was also local buckling. Prior to buckling, the tensile strips delaminated and ruptured 200 mm from the base, followed by crushing of the CFRP strips in compression at 200 mm from the base. The pole buckled shortly after the crushing of the compressive strips.
The study concluded that high- and intermediate-modulus CFRP will significantly increase the strength and stiffness of a steel monopole, particularly when the design loads are within the pole’s elastic range. The high-modulus sheets provided the greatest strength increase, but the least stiffness increase. The high-modulus strips provided the greatest increase in stiffness, but the lowest increase in strength. The intermediate-modulus strips provided a good compromise between the advantages and disadvantages of the high-modulus sheets and strips.

2.7 Design Standards for CFSTs in Pure Bending

This section presents design formulae from various design standards, including CSA S16-01 (2001), AIJ (AIJ, 1987), AISC-LRFD (AISC-LRFD, 1993), CIDECT (CIDECT, 1995).

2.7.1 Canadian Steel Standards, CAN/CSA S16-01

The Canadian design standard (CSA S16-01) offers two methods to predict the capacity of a CFST in flexure. Figure 2-9 from Bruneau and Marson (2004) presents the CSA S16-01 model. The composite section moment resistance, $M_{rc}$, is given as:

$$M_{rc} = C_r e + C'_r e'$$

(2-29)

where $C_r$, $e$, $C'_r$, and $e'$ are given as:

$$C_r = \phi F_y \beta \frac{D_t}{2}$$

(2-30)

$$e = b_i \left[ \frac{1}{2\pi - \beta} + \frac{1}{\beta} \right]$$

(2-31)
\[
C_r = \phi f_c \left[ \frac{\beta D^2}{8} - \frac{b_c}{2} \left( \frac{D}{2} - a \right) \right] 
\]

\[
e' = b_c \left[ \frac{1}{2\pi - \beta} + \frac{b_c^2}{1.5\beta D^2 - 6b_c(0.5D - a)} \right] 
\]

where \( F_y \) is the steel yield strength; \( \beta \) is the angle, in radians, from the centroid to the neutral axis; \( D \) is the tube’s diameter; \( t \) is the tube’s wall thickness; \( f_c' \) is the concrete compressive strength, \( \phi \) and \( \phi_c \) are safety factors for steel and concrete, respectively. For the actual capacity, the safety factors are given a value of unity. The parameters \( a, b_c, \) and \( \beta \) are given as:

\[
a = \frac{b_c}{2} \tan \left( \frac{\beta}{4} \right) 
\]

\[
b_c = D \sin \left( \frac{\beta}{2} \right) 
\]

\[
\beta = \frac{\phi A_s F_y + 0.25\phi_c D^2 f_c' \left[ \sin \left( \frac{\beta}{2} \right) - \sin^2 \left( \frac{\beta}{2} \right) \tan \left( \frac{\beta}{4} \right) \right]}{0.125\phi_c D^2 f_c' + \phi Dt F_y} 
\]

where \( A_s \) is the cross-sectional area of the steel tube. A second method is also given. This method is slightly more conservative than the above method, but it provides a closed-form solution that is much simpler for a designer to use. The alternative moment capacity, \( M_{rc} \), is given as:

\[
M_{rc} = \left( Z - 2th_n^2 \right) \phi F_y + \left[ \frac{2}{3} (0.5D - t)^3 - (0.5D - t) h_n^3 \right] \phi f_c' 
\]

where \( h_n \) is given as:

\[
h_n = \frac{\phi_c A_{ce} f_c'}{2D\phi_c f_c' \left( 2\phi F_y - \phi_c f_c' \right)} 
\]
where \( Z \) is the plastic modulus of the steel tube alone, and \( A_c \) is the area of the inner concrete filling.

### 2.7.2 American Steel Standard – AISC-LRFD

The commentary of the code recommends using a simplified equation for the ultimate strength of a composite section. This equation is for a general cross-section. Many of the parameters are set equal to zero in the case of a concrete-filled tube (CFT):

\[
M_n = Z f_y + \frac{1}{3} \left( h_2 + 2 C_r \right) A_r f_y + \left( \frac{h_2}{2} - \frac{A_w f_y}{1.7 f_y h_1} \right) A_w f_y \tag{2-39}
\]

where \( M_n \) is the ultimate moment of composite cross section; \( A_w \) is the web area of the encased steel (\( A_w = 0 \) for CFT); \( A_r \) is the area of reinforcing steel (\( A_r = 0 \) for CFT); \( Z \) is the plastic section modulus of the hollow steel tube; \( C_r \) is the average distance to the internal reinforcement (\( C_r = 0 \) for CFT); \( h_1 \) is the width of the member perpendicular to the plane of bending; \( h_2 \) is the width of the member parallel to the plane of bending; \( f_c \) is the concrete cylinder strength; \( f_{yr} \) is the yield strength of reinforcement steel (\( f_{yr} = 0 \) for CFT); and \( f_y \) is the yield strength of the steel tube.

### 2.7.3 Japanese Design Standard – AIJ

\[
M_u = s M_u + e M_u \tag{2-40}
\]

\[
e M_u = \frac{1}{12} (r_0 f_c) d^3 \sin^3 \theta \tag{2-41}
\]

\[
s M_u = s Z_p f_y \tag{2-42}
\]
where $c_{Mu}$ is the ultimate moment contribution of concrete, $s_{Mu}$ is the ultimate moment contribution of steel, $f_c$ is the concrete strength, $d$ is the outer diameter of the steel tube, $\theta$ is the angular location of the plastic neutral axis, $s_{Zp}$ is the plastic modulus of the steel section, $f_y$ is the yield strength of the steel.

### 2.7.4 CIDECT

The International Committee for Research and Technical Support for Hollow Section Structures (CIDECT, 1995) provides the composite section moment capacity as:

$$M_{pl,rd} = m_0 \frac{d^3 - (d - 2t)^3}{6} f_{yd}$$

(2-43)

where $m_0$ is a factor; $f_{yd}$ is the design yield strength of the steel; $t$ is the wall thickness, and $d$ is the outside diameter of the steel tube.

### 2.8 Design Standards for Hollow Steel Tubes

The following section provides design equations from various standards for the design of hollow steel tubular members. Only guidelines for pure bending are presented here.

#### 2.8.1 API Code

The American Petroleum Institute API RP 2A-LRFD-93 code provides guidelines offshore platforms. The guidelines for the design of hollow cylindrical members are valid for $t \geq 6$ mm, $D/t \leq 300$, and $f_y \leq 414$ MPa. For pure bending, API provides a stress limit, $f_b$, for various $(D/t)$ ratios:
Chapter 2  Literature Review

\[ f_b = \left( \frac{Z}{S} \right) F_y \quad \text{for} \quad \frac{D}{t} \leq \frac{10340}{F_y} \quad (2-44) \]

\[ f_b = \left[ 1.13 - 2.58 \left( \frac{F_y D}{E t} \right) \right] \left( \frac{Z}{S} \right) F_y \quad \text{for} \quad \frac{10340}{F_y} < \frac{D}{t} \leq \frac{20680}{F_y} \quad (2-45) \]

\[ f_b = \left[ 0.94 - 0.76 \left( \frac{F_y D}{E t} \right) \right] \left( \frac{Z}{S} \right) F_y \quad \text{for} \quad \frac{20680}{F_y} < \frac{D}{t} \leq 300 \quad (2-46) \]

where \( S \) is the elastic section modulus, and \( Z \) is the plastic section modulus.

### 2.8.2 Canadian Steel Design Standard CSA-S16-01

The type of failure of the hollow tube is based on the class of the section, which depends on the (D/t) ratio and yield strength. Class 1 and Class 2 sections are considered ‘compact’ sections that are capable of achieving the full plastic moment. Class 3 sections are capable of achieving the yield moment, but not the plastic moment. Class 4 sections will buckle prior to achieving the yield moment. For Class 1 and Class 2 sections, the moment capacity, \( M \) is given as:

\[ M = Z_x F_y \quad \text{for} \quad \frac{D}{t} \leq \frac{18000}{F_y} \quad (2-47) \]

where \( Z_x \) is the plastic section modulus in the longitudinal direction. For Class 3 sections, the moment capacity is given as:

\[ M = S_x F_y \quad \text{for} \quad \frac{D}{t} \leq \frac{66000}{F_y} \quad (2-48) \]

where \( S_x \) is the section modulus in the longitudinal direction.
Class 4 sections are governed by the cold-formed steel design standard CSA S136-01, which are discussed in the following section.

### 2.8.3 Canadian Steel Standard for Cold-formed Steel Members CSA S136-01

CSA S136-01 provides buckling stress limits for cold-formed steel pipes, as well as slender steel sections. The buckling stress, $F_c$, limits are given for various $(D/t)$ ratios:

$$ F_c = 1.25 F_y $$

for $\frac{D}{t} \leq 0.07 \frac{E}{F_y}$ (2-49)

$$ F_c = \left[ 0.965 + 0.02 \left( \frac{E}{F_y} \right) \right] F_y $$

for $0.07 \frac{E}{F_y} < \frac{D}{t} \leq 0.319 \frac{E}{F_y}$ (2-50)

$$ F_c = 0.328 \left( \frac{E}{F_y} \right) $$

for $0.319 \frac{E}{F_y} < \frac{D}{t} \leq 0.441 \frac{E}{F_y}$ (2-51)

where $E$ is the elastic modulus of the steel tube, and $F_y$ is the tube yield strength.

### 2.8.4 LRFD Specification for Structural Steel Buildings (1999)

The LRFD specification provides a critical bending moment for various $(D/t)$ ratios. The specification provides a bending moment for compact and non-compact sections, but not for slender sections. The critical moments, $M$, are given as:

$$ M = Z F_y $$

for $\frac{D}{t} \leq 0.07 \frac{E}{F_y}$ (2-52)
\[
M = \left\{ \left[ 0.021 \frac{E}{(D/t)} \right] + F_y \right\} S \quad \text{for } 0.07 \frac{E}{F_y} \leq \frac{D}{t} \leq 0.31 \frac{E}{F_y} \tag{2-53}
\]

where \( Z \) is the plastic section modulus of the steel tube, \( S \) is the section modulus of the steel tube, and \( F_y \) is the tube yield strength.
Figure 2-1: Pultrusion process for fabricating FRP sections (Warner, 2000)

Figure 2-2: Filament-winding process for fabricating FRP tubes (Warner, 2000)
Figure 2-3: Centrifugal casting process for fabricating FRP tubes (Warner, 2000)

Figure 2-4: Ovalization behaviour of a hollow tube (Karamanos, 2001)
Chapter 2  Literature Review

Figure 2-5: FRP poles with various rigidity lengths (Desai and Yuan, 2006)

Figure 2-6: Model for predicting capacity of a CFST (Elchalakani, 2001)
Figure 2-7: Load-deflection response of partially concrete-filled steel tube (Test 7) and hollow steel tube (Test 6) (Fouad, 2006)
Figure 2-8: Details for strengthening steel poles with externally bonded CFRP (Lanier, 2007)
Figure 2-9: CSA S16-01 model for predicting moment capacity of CFST (Bruneau and Marson, 2004)
CHAPTER 3: EXPERIMENTAL PROGRAM

3.1 Introduction

The experimental program investigated the flexural behaviour of circular, partially concrete-filled FRP tubes (PCFFTs) and partially concrete-filled steel tubes (PCFSTs). The objective was to determine the optimal height of concrete filling, which was defined in Chapter 1. Six PCFFT specimens, and four PCFST specimens were tested. The geometry and laminate structure of the six GFRP tubes used in the PCFFT specimens was the same. Also, similar steel tubes were used in all PCFST specimens. Each test specimen was filled with a unique amount of concrete such that the cantilevered tubes ranged from completely hollow to completely filled within their test spans. This chapter describes the materials used to fabricate the test specimens, the fabrication process, the experimental procedure, instrumentation, and coupon tests for the GFRP and steel tubes, as well as and concrete cylinder testing.

3.2 Materials used for Test Specimens

The PCFFT specimens were fabricated using filament-wound GFRP prismatic tubes, while the PCFST specimens were fabricated using prismatic steel tubes. Mechanical properties of the GFRP tubes were established through tension tests of coupons, manufacturer data, and predictions using Classical Lamination Theory (CLT). Mechanical properties of the steel tubes were obtained by tension tests of steel coupons. Mechanical properties of the concrete core were determined by compressive tests of cylinders.
3.2.1 GFRP Tubes

Six filament-wound GFRP tubes were provided by Ameron International (Burkburnett, Texas, USA). The tubes shared the same geometry and laminate structure. Each tube was 3660 mm long, with a 220 mm outer diameter and 4.15 mm structural wall thickness. A thin liner was bonded to the inner surface of the tubes to facilitate the filament-winding process. This liner was neglected in the tube’s structural thickness because it does not contribute to the structural performance of the tube. The structural wall thickness consisted of nine asymmetric [+5/-88] continuously wound layers, plus one [+10] roving layer. The tube had a fibre-volume fraction of 51%, which corresponds to a fibre-weight fraction of 68%. The ratio of hoop fibres [-88] to axial fibres [+5] was approximately 2:1. Details regarding layer thicknesses and stacking sequence are provided in Table 3-1 and mechanical properties of the GFRP tubes are provided in Table 3-2. The mechanical properties determined by coupon testing, in terms of stress-strain curves, are detailed in Chapter 4. These mechanical properties are compared to those provided by the manufacturer, and those predicted by CLT.

3.2.2 Steel Tubes

Four steel tubes were used to fabricate the PCFST specimens. Each tube was 1855 mm long, with an outer diameter of 114.3 mm and a 3 mm wall thickness. Mechanical properties of the steel were obtained by longitudinal coupon testing and are presented in Table 3-3. The results of the coupon tests, in terms of stress-strain curves, are detailed in Chapter 4.
3.2.3 Concrete

The same concrete mix was used for all test specimens and was provided by Lafarge Canada. Three standard 102 mm by 204 mm (4” by 8”) cylinders were poured for each specimen and tested in compression to determine the concrete strength at the time of testing. Results of the compression tests are summarized in Table 3-4. For each specimen, the value given in Table 3-4 is the average compressive strength of three cylinders. Although all specimens used concrete from the same mix, differences in the compressive strength exist since the specimens were tested over a period of several months. The low compressive strength for PCFFT5 in Table 3-4 is not a concern since Fam (2000) has shown that in flexure, the strength of the concrete filling has little effect on the performance of a concrete-filled tube.

3.3 Description of Test Specimens

The following section provides details of the test specimens. Six PCFFT and four PCFST specimens were fabricated for cantilever bending tests.

3.3.1 PCFFT Specimens

Six specimens were fabricated by filling the GFRP tubes with various amounts of concrete. Table 3-5 presents the geometry of each PCFFT specimen. Each tube was originally 3660 mm long, with a 220 mm outer diameter, and a 4.15 mm structural wall thickness. The set of six tubes included two control specimens, one hollow (0% concrete within test span) and one completely filled (100%). The amount of filling in the remaining tubes included 13%, 30%, 51%, and 72% within the test span. To achieve a
successful clamped end for the cantilever fixed support, a 702 mm long portion of all the six tubes was filled with concrete. The aforementioned concrete fill percentages are with respect to the portion of the tube beyond the 702 mm base (i.e. the test span).

### 3.3.2 PCFST Specimens

The four PCFST specimens were fabricated similar to the PCFFTs. Each steel tube was 1855 mm long, with a 114.3 mm outer diameter (D), and a 3 mm wall thickness (t). The (D/t) ratio of the tube was 38.1. Canadian Standard Association CSA S16-01 classifies the section as compact (i.e. the plastic moment will be achieved before buckling). The amounts of concrete filling for the steel tubes within the test span were 0%, 15%, 33%, and 100%, beyond the fixed base. The concrete-filled part of the tube for clamping within the fixed end was 394 mm. Table 3-6 presents a summary of the PCFSTs.

### 3.4 Fabrication of Specimen

All specimens were fabricated on the same day from the same batch of concrete. The inner surface of the tubes was cleaned to enhance the bond with concrete. The tubes were propped vertically and secured to a scaffolding (Figure 3-1(a)) to allow filling from above. The tubes were mounted on sheets of plywood and the edge of each tube at its base was sealed with silicon to prevent any leakage. Concrete was poured manually using the standard 150x300 mm cylinder moulds (Figure 3-1(b)). It is envisioned that in real life practice, concrete filling could be achieved by pumping through a small access hole on the side. The estimated volume of concrete required for each of the partially-
Chapter 3  Experimental Program

filled tubes was converted to a number of cylinder moulds. The final height of the concrete fill was measured after the concrete had hardened.

3.5 Experimental Setup and Loading

The following sections detail the testing apparatus and method of loading for the experimental program. The application of load and the assembly of the fixed support are explained. Details regarding the type and location of instrumentation for data collection are provided.

3.5.1 Experimental Setup for PCFFTs

Typically, monopoles are loaded laterally, mainly by wind pressure. As an approximation of this effect, each specimen was loaded by a single point load at the free end in cantilever bending (Figure 3-2). Although a uniformly distributed load on the pole would more accurately reflect a wind loading, a single point load was used because it does accurately reflect a number of practical circumstances. Poles are not loaded only by wind acting directly on the pole. In the case of a sign-supporting structure, the sign applies a point load where it is connected to the pole. A similar scenario exists in the case of a pole supporting overhead power lines. Using a single point load also had some advantages for the experimental program in that a uniformly distributed load is difficult to apply to a long circular beam in the laboratory. The fixed support was provided by gripping the 702 mm long concrete-filled end portion (Section 3.3.1) of the tube. The tube was placed between two steel channel sections lined with quick-set plaster that was moulded to conform to the curved surfaces of the tube. The channels were clamped
together to a rigid steel base using threaded rods. Load was applied at a distance of 2665 mm from the face of the fixed base using a manually operated hydraulic jack with 250 mm stroke (Figure 3-3 (b)). The load cell was positioned directly below the hydraulic jack and fitted with a spherical cap to ensure that the load was always transferred vertically to the load cell, in the case that the jack was slightly tilted due to end rotation. To avoid ovalization and local crushing of the GFRP tube at the loading point, a vertical steel stiffener plate was inserted inside the tube, and a steel ring was placed around the tube above the jack for bracing. A lubricated steel pin was used between the loading end of the jack and the tube to facilitate free rotation (Figure 3-3 (a)). In the event that the stroke of the hydraulic jack was fully expended before failure of the member, temporary screw-type mechanical jacks were used to support the deformed specimen while the hydraulic jack was reloaded. Short HSS sections were then inserted beneath the hydraulic jack to allow for further loading. Additionally, to maximize the stroke, a second loading jack with 150 mm stroke, axially aligned with the first jack, was used when required (Figure 3-3 (c)).

3.5.2 Experimental Setup for PCFSTs

The steel tube specimens were tested in a similar fashion to the GFRP specimens (Figure 3-4). The same gripping method was used to create the fixed end, and the loaded end arrangement was also similar to the PCFFTs. The tested length of the steel tubes was 1398 mm, from the face of the fixed end to the loading point. Ovalization was also prevented at the load point by using a vertical steel plate inserted inside the tube at the end.
3.5.3 Instrumentation

Each set of test specimens, PCFFTs and PCFSTs, was instrumented in a similar fashion (Figure 3-5). Strain was measured using electrical resistance foil strain gauges, manufactured by Showa Measuring Instruments Co. Ltd, Tokyo, Japan. The strain gauges were 5 mm long, 120 ohm resistance of type N11-FA-5-120-11. The gauges were attached to the specimens using Vishay’s M-Bond 200 adhesive system. A grinder was used to prepare the surface of the steel tubes to ensure a strong bond. Longitudinal strains were measured at the fixed end, at both the compression and tension sides. Hoop strains were also measured at the fixed end on the compression side. Longitudinal strains were also measured at the end face of the concrete filling, where applicable. For tubes with partial concrete-filling, longitudinal and hoop strains were measured on the compression side at one diameter distance from the face of the concrete fill, as this was the anticipated location of local buckling. Longitudinal compressive and tensile strains at the base were also measured using 100 mm displacement type strain gauge transducers (PI gauges). Displacements were generally measured by linear potentiometers (LPs) directly above the load at the free end, and at the face of the fixed base to measure any slight movements of the support. Load was measured using a 111 kN capacity Strain Sert Universal flat Load Cell, Model FL25U-3DGKT. All readings from strain gauges, PI gauges, LPs, and the load cell were recorded during the test by Vishay’s Micro Measurements System 5000 and the data acquisition program Strain Smart.
3.6 Ancillary Tests

The following sections discuss the procedures for determining mechanical properties of the materials used for testing. Tension tests were performed on three GFRP coupons and three steel coupons. Three concrete cylinders were tested in compression following each bending test.

3.6.1 GFRP Coupons

Three coupons were cut from one of the GFRP tubes in the longitudinal direction. The coupon ends were wrapped with unidirectional GFRP sheets to eliminate gripping problems related to the curvature of the coupons. The clear distance between the grips was 25 mm. It was desired to minimize this length to reduce the fibre discontinuity effects associated with the helical orientation of the fibres. Mandal (2004) showed that the length of GFRP coupons played a significant role in the outcome of coupon tests. Longer coupons were more likely to fail prematurely, while shorter coupons approached the true material strength of the full-scale tubes. Figure 3-6(a) illustrates that as the coupon length increases, the bandwidth of fibres that contribute to the tensile strength decreases. One strain gauge, of the same type described for the beam tests, was installed on each side of each coupon (Figure 3-6(b)). The average strain for each coupon was used to determine the stress-strain relationship. The coupons were tested in tension using a Riehle loading machine (Figure 3-6 (c)) in accordance with ASTM D3039 (ASTM, 1995). A photo of a GFRP coupon before testing is shown in Figure 3-6(d). The stress in the coupon was determined by dividing the force by the coupon’s cross-sectional area. Results of the GFRP coupon tests are detailed in Chapter 4.
3.6.2 Steel Coupons

Three steel coupons were cut from a section of the same steel tube. The coupon geometry was scaled down by a factor of 0.75 from the standard ASTM A370–03a (ASTM, 2003) coupon (Figure 3-7). The coupons were scaled due to the short length that was available from the steel tubes from which the coupons were cut. Results of the steel coupon tests are detailed in Chapter 4.

3.6.3 Concrete Testing

Standard compression tests were performed in accordance with ASTM C39 (ASTM, 1996) on three 102 x 204 mm concrete cylinders after each cantilever bending test, excluding the hollow members. All cylinders were poured from the same concrete mix as used for the beam tests.
### Table 3-1: Laminate structure and stacking sequence for GFRP tubes

<table>
<thead>
<tr>
<th>Layer</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle (°)</td>
<td>-88</td>
<td>+10</td>
<td>-88</td>
<td>+5</td>
<td>-88</td>
<td>+5</td>
<td>-88</td>
<td>+5</td>
<td>-88</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>0.33</td>
<td>0.022</td>
<td>0.36</td>
<td>0.38</td>
<td>0.95</td>
<td>0.54</td>
<td>0.54</td>
<td>0.49</td>
<td>0.54</td>
</tr>
</tbody>
</table>

![Diagram of laminate structure and stacking sequence for GFRP tubes.](image)

**Structural thickness, t = 4.15 mm**

### Table 3-2: Mechanical properties of GFRP tubes

<table>
<thead>
<tr>
<th>GFRP Tube</th>
<th>Longitudinal Tension</th>
<th>Circumferential Tension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial modulus (GPa)</td>
<td>Strain at 1st ply failure</td>
</tr>
<tr>
<td>Manufacturer Data</td>
<td>20.6</td>
<td>NA</td>
</tr>
<tr>
<td>Coupon Test</td>
<td>18.3</td>
<td>0.005</td>
</tr>
<tr>
<td>Laminate Theory</td>
<td>17.8</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

* premature failure (discussed in Chapter 4)

### Table 3-3: Mechanical properties of steel tube based on coupon tests

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, $E$ (GPa)</td>
<td>200</td>
</tr>
<tr>
<td>Yield strength, $f_y$ (MPa)</td>
<td>300</td>
</tr>
<tr>
<td>Yield strain</td>
<td>0.0037</td>
</tr>
<tr>
<td>Ultimate strength, $f_u$ (MPa)</td>
<td>365</td>
</tr>
<tr>
<td>Ultimate strain</td>
<td>0.213</td>
</tr>
</tbody>
</table>
Table 3-4: Mechanical properties of concrete

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Compressive Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GFRP/PCFFT Specimens</strong></td>
<td></td>
</tr>
<tr>
<td>PCFFT1</td>
<td>NA</td>
</tr>
<tr>
<td>PCFFT2</td>
<td>44.7</td>
</tr>
<tr>
<td>PCFFT3</td>
<td>40.5</td>
</tr>
<tr>
<td>PCFFT4</td>
<td>43.2</td>
</tr>
<tr>
<td>PCFFT5</td>
<td>35.8</td>
</tr>
<tr>
<td>PCFFT6</td>
<td>43.7</td>
</tr>
<tr>
<td><strong>Steel/PCFST Specimens</strong></td>
<td></td>
</tr>
<tr>
<td>PCFST1</td>
<td>NA</td>
</tr>
<tr>
<td>PCFST2</td>
<td>41.4</td>
</tr>
<tr>
<td>PCFST3</td>
<td>40.8</td>
</tr>
<tr>
<td>PCFST4</td>
<td>42.6</td>
</tr>
</tbody>
</table>

Table 3-5: Details of GFRP tube/PCFFT test specimens

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>Outer diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Test span (L) (mm)</th>
<th>Concrete filling length (x) beyond base (mm)</th>
<th>Filling length percentage beyond base</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCFFT1</td>
<td>220</td>
<td>4.15</td>
<td>2665</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PCFFT2</td>
<td>220</td>
<td>4.15</td>
<td>2665</td>
<td>350</td>
<td>13%</td>
</tr>
<tr>
<td>PCFFT3</td>
<td>220</td>
<td>4.15</td>
<td>2665</td>
<td>793</td>
<td>30%</td>
</tr>
<tr>
<td>PCFFT4</td>
<td>220</td>
<td>4.15</td>
<td>2665</td>
<td>1346</td>
<td>51%</td>
</tr>
<tr>
<td>PCFFT5</td>
<td>220</td>
<td>4.15</td>
<td>2665</td>
<td>1912</td>
<td>72%</td>
</tr>
<tr>
<td>PCFFT6</td>
<td>220</td>
<td>4.15</td>
<td>2665</td>
<td>2665</td>
<td>100%</td>
</tr>
</tbody>
</table>

![Diagram of GFRP tube/PCFFT test specimen](image)
Table 3-6: Details of steel tube/PCFST test specimens

<table>
<thead>
<tr>
<th>Specimen number</th>
<th>Outer diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Test span (L) (mm)</th>
<th>Concrete length (x) above base (mm)</th>
<th>Filling length percentage above base</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCFST1</td>
<td>114.3</td>
<td>3</td>
<td>1398</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PCFST2</td>
<td>114.3</td>
<td>3</td>
<td>1398</td>
<td>217</td>
<td>16%</td>
</tr>
<tr>
<td>PCFST3</td>
<td>114.3</td>
<td>3</td>
<td>1398</td>
<td>478</td>
<td>34%</td>
</tr>
<tr>
<td>PCFST4</td>
<td>114.3</td>
<td>3</td>
<td>1398</td>
<td>1398</td>
<td>100%</td>
</tr>
</tbody>
</table>

![Diagram](image)
Figure 3-1: Fabrication of specimens

(a) Shoring system

(b) Concrete casting
Figure 3-2: Experimental setup for PCFFTs
(a) Connection between GFRP tube and loading jack

(b) Single jack loading

(c) Dual jack loading

Figure 3-3: Loading jack details
Figure 3-4: Experimental setup for PCFSTs
Chapter 3  Experimental Program

Concrete filling

D

L

P

Table:

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain gauge (SG)</td>
<td>Axial, circumferential strain</td>
</tr>
<tr>
<td>100 mm PI gauge (PI)</td>
<td>Average axial strain</td>
</tr>
<tr>
<td>Linear potentiometer (LP)</td>
<td>Deflections at load point and base</td>
</tr>
</tbody>
</table>

Note: Instrument orientation same for PCFFT and PCFST

Figure 3-5: Instrumentation details
(a) Short vs. long coupons  (b) Details of instrumentation  (c) Testing of coupons

(d) Photo of GFRP coupons before testing

Figure 3-6: Details of GFRP coupons
Figure 3-7: Details of steel coupons
CHAPTER 4: EXPERIMENTAL RESULTS AND DISCUSSION

4.1 Introduction

This chapter presents the results of the ancillary material tests and the ten cantilever specimens tested in bending. The ancillary tests included standard compression tests for concrete cylinders and tensile tests on GFRP coupons. The cantilever bending tests included six PCFFT specimens with GFRP tubes, and four PCFST specimens with mild steel tubes. The tubes were filled with various amounts of concrete and tested in bending. The primary objective of the experimental program was to determine the optimal height of concrete filling for the GFRP and steel tubes. The optimal height of filling is defined as the minimum height of concrete required to achieve the highest bending capacity. In theory, for thin tubes, this should be the height at which the tube fails by rupture of the GFRP at the base, while simultaneously failing by local buckling at a small distance away from the face of the concrete filling. For the steel tube tested in this study, the optimal concrete height should correspond to reaching the full plastic moment capacity at the base, while reaching the yielding capacity of the hollow part (or local buckling), simultaneously.

4.2 Ancillary Test Results

The following sections discuss the results of the ancillary tests used to determine the material properties of the GFRP tubes and steel tubes used in the experimental program.
4.2.1 GFRP Coupon Tension Tests

Three longitudinal GFRP coupons cut from the tube were tested in tension to determine the material properties of the tube. The 190x25 mm coupons were tested according to ASTM Standard D 3039/D 3039M-95a.

Figure 4-1 presents the stress-strain behaviour of the three tested coupons. Each coupon exhibited a somewhat bilinear response. The change in slope at a strain around 0.005 indicated that the circumferential fibres had failed. The average ultimate strength of the three tested coupons was 219 MPa, with a standard deviation of 14.5 MPa. Due to instrumentation malfunction, a projected visual trend line was used to complete the stress-strain curves. Figure 4-2 shows the failed coupons.

4.2.2 Steel Coupon Tension Tests

Three steel coupons were tested in tension according to ASTM Standard A370-03a to determine the stress-strain relationship. Figure 4-3 presents a photo of the coupons after testing. During the tension test, load and stroke were recorded, but strain was accidentally not recorded. Figure 4-4 shows the stress-strain curves based on stroke measurements, which does not produce accurate strain. Therefore, the strains had to be corrected. Assuming an elastic modulus of 200 000 MPa, the proportional limit strain value was established (Point (a) in Figure 4-5). An average ultimate strain value was determined by measuring the length of the failed coupons to determine the change in length, which was then divided by the original length. The original tested length was the length between the grip tabs, not the entire coupon length. The average ultimate strain (Point (b) in Figure 4-5) was 0.213. Figure 4-5 shows the corrected final curve.
strains between the proportional limit strain and the ultimate strain were determined by linear interpolation. Figure 4-6 illustrates how the strain was linearly interpolated. The stress-strain curve based on stroke was first adjusted to satisfy initial slope of 200 GPa before the interpolation. Using the 0.2% offset method, the yield strain and stress were determined as 0.0037 and 300 MPa, respectively, from the final curve. The proportional limit strain and stress were 0.00135 and 270 MPa, respectively. The average maximum stress was 365 MPa.

4.3 Results of Cantilever Bending Tests

The following sections present the results of the experimental program. Results of bending tests on six PCFFTs and four PCFSTs are included. Load-deflection and load-strain behaviour is discussed for each specimen. The failure mode of each specimen is also presented and discussed, and the optimal concrete filling length ratio is established from the experimental results.

4.3.1 GFRP Tube PCFFT Tests

Six GFRP tubes were tested in cantilever bending, by the method described in Chapter 3. Each tube was filled with a unique amount of concrete, including 0%, 13%, 30%, 51%, 72%, and 100% of the free length. The objective was to determine the optimal filling length, which was estimated by correlating the failure loads to the concrete filling lengths. Load-deflection and load-strain relationships were measured for all specimens, and are presented in Figures 4-7 to 4-24, along with photos of each specimen’s failure mode.
4.3.1.1 Load-Deflection Behaviour and Failure Modes

This section discusses the load-deflection plots of each PCFFT. The tip deflection was corrected to account for rotation at the base, as suggested by the ANSI standard for Fibre-Reinforced Plastic (FRP) Lighting Poles (ANSI C136.20-1990). For each specimen, deflection was measured at the face of the fixed support to measure the rotation of the base. The effects of this rotation on total deflection were subtracted from the measured tip deflection. Each specimen, except for hollow PCFFT1, experienced a decrease in stiffness following cracking of the concrete. Figure 4-25 presents the load-deflection curves for the completely hollow PCFFT1 and completely filled PCFFT6 specimens. The benefits of concrete filling are obvious, as the failure load nearly doubled when the tube was filled with concrete. The stiffness also nearly doubled, and the ultimate deflection increased by about 30%. The concrete filling also changed the failure mode from buckling combined with crushing on the compression side of the hollow tube, to rupture of the GFRP tube in tension in the concrete-filled tube. The hollow tube buckled at a distance (1.5D) from the fixed support, where D is the diameter.

Figure 4-26 shows the load-deflection plot of all specimens. It is evident that PCFFT4 and PCFFT5 achieved the same failure load as PCFFT6, also with comparable stiffness. Both specimens also failed by rupture of the GFRP tube in tension, which indicates that complete filling of the tube with concrete is unnecessary. On the other hand, PCFFT2 failed prematurely by local buckling at a distance (D) from the face of the concrete filling, but at a higher load than PCFFT1. It demonstrated that filling only 15% of the tube’s height with concrete increased the failure load by about 50%. PCFFT3 also failed by buckling in the hollow length at about 0.33D from the face of the concrete
filling, but at a load significantly higher than PCFFT1. In fact, it almost achieved the capacity of PCFFT6. This indicates that the optimal filling height is slightly above 30%. PCFFT3 achieved the greatest amount of deflection, nearly span/4 (L/4), although its behaviour, similar to all the specimens, remained fairly linear-elastic to failure. Figures 4-12 and 4-15 show the local buckling failure of specimens PCFFT2 and PCFFT3, while Figures 4-18, 4-21, and 4-24 show the tension failure of specimens PCFFT4, PCFFT5, and PCFFT6, respectively.

It should also be noted that the location of local buckling moved closer to the face of the concrete filling as the length of the hollow part reduced, although local buckling is affected by a number of parameters, including imperfections, which were not measured for these experiments.

### 4.3.1.2 Load-Strain Behaviour

Looking at the load-tensile strain plots of each PCFFT specimen at the base, it is difficult to observe a true failure strain since the plots are incomplete due to strain gauge premature failure. Strain gauges often fail prematurely by detaching, either fully or partially, from the surface of the GFRP tube. Micro-cracking in the resin disables the strain gauge by causing areas of the strain gauge to detach from the surface of the tube. Also, the outermost layer of the tubes used in this study was continuously wound in the circumferential direction. Bending causes these fibres to separate, which increases the likelihood of the strain gauge detaching from the surface. The only tensile strain gauge to survive was from PCFFT1, which had no concrete filling and failed in compression. A projected visual trend line was used to complete the load-strain plots in this chapter.
Longitudinal compressive strain at the base in partially filled tubes was generally between 0.01 and 0.015. This far exceeds the 0.0035 crushing strain of concrete, which suggests partial confinement of concrete, even though loading is mainly in bending.

Circumferential strains at the base on the compression side were minimal tensile strains as shown in Figure 4-27. Figure 4-28 presents the variation of the neutral axis depth at the base with the applied load. PCFFT1 was completely hollow, so the neutral axis depth remained near mid-depth of the cross-section, but it reduced slightly due to ovalization of the cross-section. PCFFT5 and PCFFT6 behaved similarly because each tube failed by tensile rupture of the GFRP. PCFFT3, even though it failed prematurely by local buckling, behaved similarly to PCFFT5 because PCFFT3 nearly achieved the totally filled tube capacity. The neutral axis depth of PCFFT5 and PCFFT6 that failed by tensile rupture of the tube ranged from 60 to 80 mm, with an average of 70 mm, which is about (D/3). PCFFT4 was excluded from this discussion because a meaningful load versus neutral axis depth curve could not be constructed due to the premature failure of both strain gauges (Figure 4-17) at very low loads.

4.3.1.3 Optimal Filling Length Ratio for PCFFT Specimens

Figure 4-29 presents the relationship between concrete filling length ratio and failure load. The figure clearly illustrates that the failure load gradually increases with concrete filling until reaching a plateau equivalent to the totally concrete-filled tube capacity. The point at which the plateau is reached is the optimal filling length. By linear fitting of the data points of the two trends of “flexural tension failure” points and
“local buckling failure” points, respectively, an intersection point is established, which indicates that the optimal filling length is predicted as 34% of the tube’s length.

4.3.2 Steel Tube PCFST Tests

Four steel tubes were tested in cantilever bending, as discussed in Chapter 3. Each tube was filled by a unique amount of concrete, including 0%, 15%, 33%, and 100% of the free length. The objective was to determine the optimal filling length. This section discusses the results of the PCFST tests. Load-deflection and load-strain behaviour is discussed, as well as failure modes and the optimal filling length ratio.

4.3.2.1 Load-Deflection Behaviour and Failure Modes

Load-deflection and load-strain plots, along with photos of failure modes, are shown in Figures 4-30 to 4-41. Figure 4-42 presents a comparison of the load-deflection response of all PCFST specimens. The benefit of completely filling the tube with concrete is obvious, as the ultimate load was increased by almost 80%, and ductility increased significantly. The completely filled tube, PCFST4, yielded excessively and reached near a plastic plateau, but the steel never fractured in tension. The test was stopped when all available stroke from the loading jack was utilized. The test for the completely hollow tube, PCFST1, was stopped after the tube had buckled and the load had begun to decrease. It is evident that adding concrete will increase the ultimate load. No distinct cracking point was observed. This occurred because the steel tube was stiffer than the concrete, while the concrete was stiffer than the GFRP tube ($E_s = 200$ GPa,
\( E_{\text{concrete}} = 28.5 \text{ GPa} \quad E_{\text{GFRP}} = 17.76 \text{ GPa} \). It should be noted that the initial stiffness of all specimens is almost similar. Stiffness began to reduce when the steel tube started to yield. Figure 4-42 clearly shows that the yielding load increased as more concrete was added. PCFST2 underwent inward buckling (Figure 4-35), similar to PCFST1 (Figure 4-32), but at a higher load. The maximum load reached was 10.76 kN, which is approximately 30% higher than the 8.29 kN achieved by PCFST1. The buckling occurred very close to the face of concrete filling, well within one diameter from the face of the concrete fill in both specimens. PCFST3 and PCFST4 experienced outward buckling near the fixed end after excessive yielding, but neither tube ruptured (Figures 4-38 and 4-41).

### 4.3.2.2 Load-Strain Behaviour

Axial strain was measured at the base of each tube on both the compression and tension sides. Figure 4-43 presents the load-strain relationships at the base for all tested specimens. Not all of the curves are complete due to instrumentation malfunction. The load-strain relationship is similar to that of the load-deflection. Greater strains were achieved when there was more concrete filling. It is clear that all specimens yielded, and even PCFST1 and PCFST2 yielded prior to buckling. Each specimen yielded in tension before yielding in compression, although PCFST1 yielded on both sides almost simultaneously. PCFST1 yielded in tension at 6.46 kN, while PCFST4 yielded in tension at 9.43 kN, an increase of about 45%.

Figure 4-44 shows the load-axial strain behaviour at the face of the concrete filling in specimens PCFST2 and PCFST3, whereas Figure 4-45 shows at the load-axial
strain behaviour at distance D from the concrete face in specimens PCFST1, PCFST2, and PCFST3. The figures indicate that PCFST1 and PCFST2 yielded prior to buckling, whereas PCFST3 had just begun to yield when it reached its ultimate load. In both figures, PCFST2 underwent significant plastic deformations, while PCFST3 was only beginning to become plastic. Figure 4-46 presents the load-strain behaviour in the circumferential direction, on the compression side, at the base. Hoop strains at the base are in tension and are created by Poisson’s ratio effect. The figure shows that all tubes yielded in the circumferential direction as well. The high strains suggest some confinement effect of the concrete core, which help enhance the moment capacity and ductility. Much higher hoop strains were achieved in the steel tubes than in the GFRP ones. This occurred because of the higher Poisson’s ratio for the steel tubes (0.3) than for GFRP tubes (0.087) used in this study. Figure 4-47 illustrates the load-hoop strain behaviour at distance (D) from the face of concrete filling for PCFST1, PCFST2, and PCFST3. PCFST1 and PCFST2 yielded prior to the ultimate load, while PCFST3 did not yield.

Figure 4-48 illustrates the relationship between load and neutral axis depth at the base. The neutral axis depth for hollow PCFST1 remained very close to the tube’s radius, while the neutral axis depth for PCFST2, PCFST3, and PCFST4 gradually reduced as the load was increased. All three specimens with concrete filling behaved almost similarly, with a neutral axis depth being about (0.35D) at ultimate.
4.3.2.3 Optimal Filling Ratio Length for PCFST Specimens

Only PCFST1 reached a maximum load that began to drop during testing. All other specimens were halted when the load appeared to be approaching a plateau, due to stroke limitations. Figure 4-49 presents the ultimate load versus concrete fill length ratio. The trend suggests that the optimal filling ratio is about of 0.46L.
Figure 4-1: Results of GFRP coupon tests

Figure 4-2: GFRP coupons after testing
Figure 4-3: Steel coupons after failure

Figure 4-4: Stress-strain curves for steel coupons from stroke measurements
Figure 4-5: Final adjusted steel stress-strain curve

Figure 4-6: Method for adjusting steel stress-strain curve
Figure 4-7: Load-deflection behaviour of PCFFT1 (0% full)

Figure 4-8: Load-strain behaviour of PCFFT1 at base (0% full)
Figure 4-9: Failure mode of PCFFT1 (0% full)

Figure 4-10: Load-deflection behaviour of PCFFT2 (13% full)
Figure 4-11: Load-strain behaviour of PCFFT2 at base (13% full)

Figure 4-12: Failure mode of PCFFT2 (13% full)
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Figure 4-13: Load-deflection behaviour of PCFFT3 (30% full)

Figure 4-14: Load-strain behaviour of PCFFT3 at base (30% full)
Figure 4-15: Failure mode of PCFFT3 (30% full)

Figure 4-16: Load-deflection behaviour of PCFFT4 (51% full)
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Figure 4-17: Load-strain behaviour of PCFFT4 at base (51% full)

Figure 4-18: Failure mode of PCFFT4 (51% full)
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Figure 4-19: Load-deflection behaviour of PCFFT5 (72% full)

Figure 4-20: Load-strain behaviour of PCFFT5 at base (72% full)
Figure 4-21: Failure mode of PCFFT5 (72% full)

Figure 4-22: Load-deflection behaviour of PCFFT6 (100% full)
Figure 4-23: Load-strain behaviour of PCFFT6 at base (100% full)

Figure 4-24: Failure mode of PCFFT6 (100% full)
Figure 4-25: Load-deflection behaviour of PCFFT1 (0%) and PCFFT6 (100%)

Figure 4-26: Load-deflection behaviour of all six specimens
Figure 4-27: Load-hoop strain behaviour of all six specimens at base

Figure 4-28: Variation of neutral axis depth at base for all specimens, excluding PCFFT4
Figure 4-29: Optimal concrete filling length ratio for PCFFTs

Figure 4-30: Load-deflection behaviour of PCFST1 (0% full)
Figure 4-31: Load-strain behaviour of PCFST1 (0% full) at base

Figure 4-32: Failure mode of PCFST1 (0% full)
Figure 4-33: Load-deflection behaviour of PCFST2 (15% full)

Figure 4-34: Load-strain behaviour of PCFST2 (15% full) at base
Figure 4-35: Failure mode of PCFST2 (15% full)

Figure 4-36: Load-deflection behaviour of PCFST3 (34% full)
Figure 4-37: Load-strain behaviour of PCFST3 (34% full) at base

Figure 4-38: Failure mode for PCFST3 (34% full)
Figure 4-39: Load-deflection behaviour for PCFST4 (100% full)

Figure 4-40: Load-strain behaviour of PCFST4 (100% full) at base
Figure 4-41: Failure mode of PCFST4 (100% full)

Figure 4-42: Load-deflection behaviour of PCFST1 to PCFST4
Figure 4-43: Load-strain behaviour of PCFST1 to PCFST4 at the base

Figure 4-44: Load-axial strain behaviour of PCFST1, PCFST2, and PCFST3 at distance (D) from face of concrete filling
Figure 4-45: Load-axial strain behaviour of PCFST2 and PCFST3 at the face of concrete filling

Figure 4-46: Load-hoop strain behaviour of PCFST1 to PCFST4 at base on compression side
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Figure 4-47: Load-hoop strain behaviour of PCFST1, PCFST2, and PCFST3 at distance (D) from concrete filling face

Figure 4-48: Variation of neutral axis depth at base for PCFST1 to PCFST4
Figure 4-49: Optimal concrete filling length ratio for PCFSTs
CHAPTER 5: NUMERICAL MODELLING

5.1 Introduction

A numerical model was developed to predict the behaviour of partially concrete-filled FRP tubes (PCFFTs) and partially concrete-filled steel tubes (PCFSTs). The PCFFT model incorporated other models developed by Fam (2000) and Ibrahim (2000). The Fam (2000) model dealt with completely concrete-filled GFRP tubular members, while Ibrahim (2000) investigated hollow GFRP tubular poles. Parts of each model were combined to predict the behaviour of PCFFTs. For the PCFSTs, a similar model to Fam (2000) was developed for the concrete-filled portion, accounting for the steel tube, while the hollow tube behaviour was based on Brazier’s (1927) equations for steel tubes. The models were based on strain compatibility, internal force equilibrium, and material constitutive relationships, which accounted for the non-linearity of multi-layer GFRP tubes, concrete, and steel plasticity. An important feature of the model was the ability to account for ovalization and local buckling of the hollow part of the tube when the wall was thin. GFRP tube properties were derived using Classical Lamination Theory (CLT) and the Ultimate Laminate Failure (CLT-ULF) approach, which accounted for the progressive failure of individual layers within the laminate thickness. Once the model was validated by the results of the experimental program and other finite element studies from literature, a parametric study was conducted to investigate the effects of key parameters on the behaviour. For the GFRP tubes, the diameter-to-thickness (D/t) ratio and laminate structure were studied, while for the steel members, the (D/t) ratio and yield stress were studied. The equations from some design standards were also assessed.
5.2 Numerical Model for Partially Filled Tubes

The following sections present the numerical models that were developed to predict the behaviour of PCFFT s and PCFSTs. The modelling procedure is described and then validated by the results of the experimental program and a finite element model from literature. For background information regarding models used for completely concrete-filled FRP or steel tubes, or completely hollow FRP or steel tubes, see Appendix A.

5.2.1 Model Procedure

Detailed procedures for obtaining the load-deflection behaviour of a partially concrete-filled GFRP or steel tube are described in the following sections. The following three sections detail the procedures for determining the complete moment-curvature responses for a concrete-filled GFRP or steel tube, a hollow GFRP tube, and a hollow steel tube. The last section explains the load-deflection procedure for a partially concrete-filled tube. The load-deflection procedure is the same when either a GFRP or steel tube is used.

5.2.2 Moment-Curvature Response of Concrete-Filled Tubes

A detailed explanation of the model is given in Appendix A. Figure 5-1 provides a flow chart for the following procedure for determining the moment-curvature response of a concrete-filled GFRP or steel tube (Figure A-6 in Appendix A). The procedure is the same for each material; the only difference is the material properties. For each material, the moment-curvature response is complete when the tube material ruptures.
F1. Specify the tube outer diameter $D_o$, structural wall thickness $t$, and material properties of GFRP or steel tube, and concrete.

F2. Assume a strain value at the top surface of the tube, $\varepsilon_t$.

F3. Assume a neutral axis depth $c$.

F4. For each strip in the cross-section, calculate the area of concrete, $A_c(i)$, and the area of the tube, either GFRP or steel, $A_f(i)$.

F5. Calculate the strain in each layer, $\varepsilon(i)$, and the corresponding stress, $f_t(i)$ and $f_c(i)$, in the tube and concrete, respectively.

F6. Calculate compressive, $CF(i)$ and $CC(i)$, and tensile, $TF(i)$ and $TC(i)$, force in the tube and concrete, respectively.

F7. Check equilibrium by summing the forces. If equilibrium is not satisfied, return to Step F3 and assume a new neutral axis depth. Repeat until equilibrium is satisfied.

F8. Check whether the ultimate strain of the tube, $\varepsilon_{tu}$ or $\varepsilon_{bu}$, is exceeded in compression or tension, respectively. If either ultimate strain is exceeded, the member is failed.

F9. Determine the moments of the forces in each layer about the neutral axis. The summation of all the moments is the total moment in the member for the strain applied in Step F2.

F10. Compute the curvature as $\psi = \varepsilon_t/c$.

F11. Return to Step F2 and assume a new strain. Repeat this process until the ultimate strain is exceeded and the complete moment-curvature response is developed.
5.2.3 Moment-Curvature Response of Hollow GFRP Tubes

A detailed explanation of the model is given in Appendix A. Since the amount of ovalization varies along the length of the hollow part of the tube (Figure 5-2 (a)), each section, effectively has a different moment-curvature response. In this case, when a bending moment is applied to the base of the pole and distributed linearly to the tip (Figure 5-2 (b)), the curvature at any section along the length of the tube can be determined from the appropriate \( M - \Psi \) curve (Figure 5-2 (c)). In this section, the moment-curvature responses were established all the way until material failure in tension or compression. A more detailed check of local buckling failure criteria is conducted in Section 5.2.3.4 when the load-deflection curve is constructed in the event that local buckling occurs before material failure. Figure 5-3 provides a flow chart for the following procedure for determining the moment-curvature response of a hollow GFRP tube:

HF1. Enter diameter, \( D_o \), tube wall thickness, \( t \), material properties, hollow length, \( L_h \), initial moment of inertia, \( I_o \) (Equation A-36 in Appendix A), and the ultimate moment based on material strength, \( M_{ulh} \) (Equation A-41 in Appendix A).

HF2. Divide the hollow length into \( n_h \) segments in the longitudinal direction of equal length \( x_{ih} \). The distance from the free end of the hollow tube, \( x_h \), will be equal to \( L_h \) when \( n = n_h \).

HF3. Apply a bending moment, \( M_h \), to the base of the hollow tube. The moment, \( M_h(x_h) \), at each segment along the length is given as \( M_h(x_h) = M_h*(x_h / L) \).
HF4. Assume a maximum ovalization ratio, $\xi_o$, at mid-length, $L_h/2$. Calculate $M_o$ (Equation 5-32) and check equilibrium between $M_o$ and $M_h(x_h)$. If $M_o \neq M_h(x_h)$ assume another $\xi_o$ until equilibrium is satisfied. If $M_h(x_h) > M_{ult}$, the tube has experienced material failure.

HF5. When equilibrium is satisfied, determine ovalization ratio, $\xi(x_h)$, at all sections of the hollow length from $\xi_o$ using Equation A-34 in Appendix A.

HF6. Calculate moment of inertia, $I(x_h)$, using Equation A-32 in Appendix A.

HF7. The curvature, $\psi(x_h)$, needs to be determined at all $n_h$ segments along the length of the hollow tube in order to establish the $M(x_h) - \psi(x_h)$ at each distance, $x_h$, along the length. The curvature at any segment at $x_h$ distance from the free end is calculated as $\psi(x_h) = M(x_h) / E_s I(x_h)$.

HF8. Return to Step HF3, apply a higher moment $M_h$ to the base, and repeat until $M_{ult}$ is exceeded at all $n_f$ segments and the complete moment-curvature response (up to material fracture) is developed at all $n_f$ segments.

5.2.4 Moment-Curvature Response of Hollow Steel Tube

A detailed explanation of the model is given in Appendix A. Figure 5-4 provides a flow chart for the following procedure for determining the complete moment-curvature response of a hollow steel tube until material failure. Local buckling is checked later when establishing the load-deflection curve in Section 5.2.5. Figure 5-2 presented in Section 5.2.3 for hollow GFRP tubes is also applicable to hollow steel tubes.
HS1. Input diameter $D_o$, thickness $t$, hollow length $L_h$, rupture strain of steel $\varepsilon_{ult}$, and stress-strain function.

HS2. Divide the hollow length into $n_h$ segments in the longitudinal direction of equal length $x_{ih}$. The distance from the free end of the hollow tube, $x_h$, will be equal to $L_h$ when $n = n_h$.

HS3. Starting from the first section ($n = 1$), up to mid-span section ($n = L_h/2$), complete steps HS4 to HS13, then move to the next section, $n$, and repeat the process until the complete moment-curvature response is established for each section, $n$.

HS4. Apply strain, $\varepsilon$, to the top surface of the tube.

HS5. Calculate the new depth and width of the deformed cross-section, $D_v(x_h)$ and $D_h(x_h)$, respectively. For the first strain, these will be equal to $D$, since the cross-section is not yet deformed. The neutral axis depth, $c$, will always be half of $D_v(x_h)$, due to symmetry.

HS6. Check if the strain $\varepsilon$ is greater than the rupture strain ($\varepsilon \geq \varepsilon_{ult}$). If yes, the tube is failed by rupture after excessive yielding.

HS7. Divide $D_v(x_h)$ into $n_L$ layers.

HS8. Calculate strain, $\varepsilon(i,x_h)$, and tube area, $A_s(i,x_h)$, in all layers.

HS9. Calculate stress and force in each layer. Determine the moment from each layer about the section’s neutral axis.

HS10. Sum the moments, and multiply by 4 (since only one quarter of the cross-section is considered, due to symmetry). This is the total bending moment, $M_h(x_h)$, in the cross-section for the strain applied in Step HS4.
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HS11. Determine the curvature, $\psi_h(x_h)$, created by the strain $\varepsilon$ applied in Step HS4 as

$$\psi_h(x_h) = \frac{2 \varepsilon}{D_v(x_h)}.$$  

For the cross-section under consideration, $n$, establish a point on its moment-curvature response ($M_h(x_h) - \psi_h(x_h)$).

HS12. Calculate the amount of ovalization created by the curvature in Step HS11 using Equation A-43 in Appendix A (this is $\xi_\alpha$). Determine the ovalization at the section, $n$, under consideration, $\xi(x_h)$, using Equation A-34 in Appendix A.

HS13. Return to Step HS4 and apply a new strain. Repeat until the cross-section is failed by rupture of the steel tube.

HS14. When the tube is failed at cross-section, $n$, return to Step HS3 and repeat for the next section, $n + 1$.

5.2.5 Load-Deflection Response of Partially Filled Tube

The deflection of the partially filled cantilevered tube is determined by using the moment-area theorem (Hibbler, 2002), with which the curvature along the length of the tube is integrated. For a partially filled tube, a bending moment is applied to the fixed end of the cantilever and is linearly distributed along the length. Based on the applied moment anywhere along the length, the corresponding curvature can be calculated from the moment-curvature responses developed in Sections 5.2.2, 5.2.3, and 5.2.4 for a concrete-filled, hollow GFRP, and hollow steel sections, respectively. Figure 5-5 illustrates schematics of the development of the load-deflection curve, and the procedure is summarized as follows:
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1. Divide the hollow length, $L_h$, into $n_h$ sections, where each section, $i_h$, has equal length $x_{ih}$. The distance from the tip to any section is $x_h = x_{ih}i_h$.

2. Apply a moment, $M$, to the base of the pole. Since the bending moment diagram is linear, the moment in the hollow portion, from the tip to the face of the concrete, is $M(x) = M*(x_h/L)$, where $L$ is the total length of the partially concrete-filled tube.

3. For the moment $M(x)$ at each section of the hollow tube, determine the corresponding curvature, $\psi_h(x)$, from the specific moment-curvature response of that section, determined in Section 5.2.3 for GFRP, or Section 5.2.4 for steel.

4. For GFRP, at every section along the hollow length, determine the applied stress from Equation A-38 in Appendix A and check if the applied stress has exceeded the critical stress from Equation A-42 in Appendix A at that section. If the tube has not buckled, check for material failure of the tube by comparing $M(x = L_h)$ at the end of the hollow length ($x = L_h$) to $M_{ult}$ calculated using Equation A-41 in Appendix A. For steel, check whether the critical strain in Equation A-61 in Appendix A is exceeded at any section along the hollow length.

5. Divide the concrete-filled length into $n_f$ sections, where each section, $i_f$, has equal length $x_{if}$. The distance from the tip to any concrete-filled section is $x_f = L_h + x_{if}i_f$. The bending moment at any section along the concrete-filled portion is $M(x) = M*(x_f/L)$.

6. Determine the curvature, $\psi_f(x)$, at each section of the concrete-filled portion for the moment $M(x)$ using the moment-curvature response developed in Section 5.2.2.
7. Check whether the moment at the base, \( M(x = L) \), is greater than the ultimate moment for the filled portion that was determined in Section 5.2.2. If the ultimate moment is exceeded, the member is failed.

8. Once the curvatures are known along the full length of the pole for the moment \( M \) specified in Step 2, calculate the deflection, \( y(x) \), using the moment-area method.

9. Return to Step 2 and apply a new moment. Continue until one failure criterion in Steps 4 or 7 is achieved either in the hollow or concrete-filled part.

5.3 Model Validation

The following sections discuss the model predictions that were compared to experimental results for the ten specimens in the experimental program. Figures 5-6 to 5-12 show the responses of the PCFFTs, and Figures 5-13 to 5-17 show the responses for the PCFSTs. There is generally good agreement between the predicted and observed results.

5.3.1 Validation for Specimens with GFRP Tubes

Figures 5-6 to 5-11 compare the load-deflection responses and load-strain (at the base) responses of the tested and modelled specimens. The average difference between the experimental and predicted ultimate load was 5.3%. The average largest difference in the deflection at a given load was 14%. Visual inspection indicates the experimental and predicted stiffness are almost similar to failure. The cracking point is slightly underestimated by the model, although it is still near the observed cracking point. It
should be noted that the model predictions agreed better with the experimental results when the GFRP tube was filled with larger amounts of concrete. The assumed sinusoidal distribution of ovalization in the hollow portion may be the cause for inaccuracy in specimens featuring small amounts of concrete filling.

5.3.1.1 Optimal Length of Concrete Filling

Figure 5-12 compares the ultimate load versus concrete-filling length ratio relationship from the experimental program and the model predictions. Extra points were established using the model, particularly at the transition region, in order to establish an accurate prediction of \( (x_{opt}/L) \) ratio. The model predicts an optimal filling length ratio of 0.34, which agrees well with the experimental ratio of 0.33.

5.3.2 Validation for Specimens with Steel Tubes

Figures 5-13 to 5-17 compare the predicted and observed responses of PCFSTs. The average difference between the experimental and predicted ultimate load was 5.5%. The average largest difference in the deflection at a given load was 34%. The largest discrepancy tended to occur in the region where the steel tube was transitioning between elastic and plastic behaviour. All specimens showed excessive yielding, but in some of the experimental responses, the maximum load was not quite reached due to stroke limitations. The trend of each experimental curve, however, is very similar to that of each predicted curve. Unlike the PCFFT's, the PCFST's lack a distinct cracking point. This is likely due to the steel tube's much higher elastic modulus compared to GFRP tubes. Each figure also shows the failure load predicted based on Canadian design
standard CSA S16-01 (2001). Design standards regarding steel pipe sections were discussed in detail in Chapter 2. The design standard underestimates the actual capacity, by around 30%, for the concrete-filled specimens in this study.

5.3.2.1 Optimal Length of Concrete Filling

Figure 5-17 presents the relationship between the concrete filling length ratio and the failure load. The plot compares the variation observed experimentally to that predicted by the model and that based on the Canadian design standard CSA S16-01 (2001). The experimental optimal filling length ratio was 0.46. The model and CSA S16-01 standard predicted an optimal filling ratio of 0.37 and 0.17, respectively. The prediction using the CSA S16-01 standard is much lower because the ultimate loads, especially of concrete-filled tubes, are underestimated.

5.4 Parametric Study

The following sections discuss the parametric study that was conducted using the model developed in this chapter. The discussion includes the parameters that were studied for PCFFTs and PCFSTs, and the results of the parametric study.

5.4.1 PCFFT Parametric Study

After the model was validated using the experimental results, a parametric study was conducted to investigate the effects of various (D/t) ratios and fibre orientations on performance. The (D/t) ratios included 125, 75, and 40. Angle-ply laminates with fibre
angles of ±10°, ±25°, ±35°, ±45°, and ±75° with respect to the longitudinal axis were used. Also, cross-ply laminates of completely longitudinal and circumferential fibres [0/90] were considered in various ratios, namely [1:3], [1:1], and [3:1]. The length of the modelled tube was 6 m; the outer diameter remained a constant, 300 mm, while the wall thickness was 2.4 mm, 4.0 mm, and 7.5 mm. Table 5-1 presents the circumferential bending stiffness of the tubes ($D_{yy}$), which is needed to analyze hollow tubes, and Figure 5-18 presents the predicted stress-strain curves of the different laminates. These values and curves were predicted using CLT-ULF.

Table 5-2 and Table 5-3 summarize the parametric study results for the angle-ply and cross-ply laminates, respectively. Figures 5-19 and 5-20 present load-deflection curves for some angle-ply and cross-ply PCFFT's, respectively. Figure 5-21 to 5-23 and Figures 5-24 to 5-26 present plots of the concrete fill length – to – total length ratio (x/L) versus failure load and (x/L) versus the normalized failure loads (P/P$_{full}$) for the angle-ply and cross-ply GFRP tubes, respectively. Each figure clearly illustrates that the failure load gradually increases as the (x/L) ratio increases until reaching a plateau. The point where the plateau is reached is the optimal filling ratio ($x_{opt}$/L). The figures also indicate that the ultimate load is highest when the fibre angles in the angle-ply laminates are nearest the longitudinal axis, or as the ratio of longitudinal fibres increase in cross-ply laminates.

### 5.4.1.1 Effects of Laminate Structure for Angle-ply Laminates

Figure 5-27 presents the relationship between ($x_{opt}$/L) ratio and fibre angle for different (D/t) ratios. In general, the ($x_{opt}$/L) ratio reduced as the fibre angle increased
further from the longitudinal axis, until a minimum was reached at ±45°. When the angle was increased to ±75°, the \( \frac{x_{\text{opt}}}{L} \) ratio increased again. A fibre angle near ±45° minimized \( \frac{x_{\text{opt}}}{L} \) because the fibres were contributing strongly in both the longitudinal (for flexural strength) and circumferential (for buckling resistance) directions. The ±10° fibres required the largest amount of concrete to achieve \( x_{\text{opt}} \) because the primarily longitudinal fibres tended to buckle at low load levels within the hollow portion, although they achieved the highest concrete-filled strength.

### 5.4.1.2 Effect of Laminate Structure for Cross-ply Laminates

Figure 5-28 presents the relationship between \( \frac{x_{\text{opt}}}{L} \) ratio and the longitudinal-to-circumferential fibre ratio for different \( (D/t) \) ratios. For \( (D/t) \) of 75 and 125, the \( \frac{x_{\text{opt}}}{L} \) ratio is reduced when there are more fibres in the circumferential direction, but for \( (D/t) \) of 40, this trend is reversed. When most of the fibres are in the longitudinal direction (3:1), there is little buckling resistance for the hollow portion of the large \( (D/t) \) tubes, while the concrete-filled capacity for (3:1) is much higher due to the longitudinal fibres. Therefore, a large amount of concrete is required due to the high vulnerability to buckling. For the small \( (D/t) \) tubes, the thick wall compensates for the effect of fibre ratio on buckling capacity.

### 5.4.1.3 Effects of \( (D/t) \) for Angle-ply Laminates

Figure 5-29 presents the relationship between optimal filling \( \frac{x_{\text{opt}}}{L} \) and \( (D/t) \) ratios for different fibre angles. As expected, minimizing \( (D/t) \) decreased the \( \frac{x_{\text{opt}}}{L} \)
ratio because thicker wall FRP tubes reduced the need for concrete. Generally, less concrete is required as the ratio of \( M_{\text{hollow}} / M_{\text{full}} \) increases.

### 5.4.1.4 Effects of (D/t) for Cross-ply Laminates

Figure 5-30 presents the relationship between \( x_{\text{opt}} / L \) and (D/t) ratios for cross-ply laminates. As was observed with the angle-ply laminates, \( x_{\text{opt}} / L \) decreases as (D/t) is made smaller, for all laminates.

### 5.4.2 PCFST Parametric Study

The parametric study for the PCFSTs investigated the effects of (D/t) ratio and yield strength of the tube. The (D/t) ratios were 30, 80, and 160, while the yield strengths were 300, 400, and 550 MPa. These conventional yield values were taken from Table 6-3 of the Handbook of Steel Construction (CISC, 2004), which provides mechanical properties of available steel products, to ensure practicality of the study. The length of the tubes was assumed 6 m, the diameter was 120 mm, and the wall thicknesses were 4.0 mm, 1.5 mm, and 0.75 mm. The model establishes predictions of optimal length of concrete filling based on two methods of analysis. The first method uses the developed model, and accounts for either material failure by excessive yielding of the steel tube, following the stress-strain curve until steel fracture, or local buckling through the critical buckling strain presented in Equation A-61 in Appendix A. The second method of analysis uses the Canadian design standards CSA S16-01 and CSA S136. Generally, the design standards tend to underestimate the tube’s ultimate capacity, particularly for
CFSTs. However, analysis based on the design standards is included because designers may feel compelled to adhere to design guidelines, even though they underestimate the true behaviour of the member.

Figure 5-31 presents the stress-strain curves assumed for the various yield strengths used in the parametric study, following the same expressions used to model the steel tested in this study. Figures 5-32 and 5-33 present the moment-curvature behaviour of the hollow steel tube with a (D/t) of 160 and 30, respectively, for each steel grade. The plots compare the predicted moment-curvature responses when ovalization and buckling have been ignored (i.e. strength of material governs) and when ovalization and local buckling are permitted. The effects of ovalization are obvious, as the ovalized section has a reduced capacity and ductility. As expected, the moment capacity increases with increasing (D/t) and yield strength. The plots also include the ultimate capacity predicted by the Canadian design standard CSA S16-01. It is noted that when (D/t) is 160 the predicted moment capacity for $f_y = 400$ MPa and 550 MPa is similar because of the large (D/t) ratio as the strength predicted by the standards is independent of yield strength. This indicates that for large (D/t), the design standard shows no advantage to using high-grade steel for hollow tubes.

The moment-curvature relationships for the completely filled tubes used in the parametric study are included in Figures 5-34 and 5-35 for the (D/t) of 160 and 30, respectively. Figure 5-36 shows the load-deflection response for various PCFSTs with different (D/t) ratios and yield strengths, and with different concrete fill ratios. Figures 5-37 to 5-39 present the variation of (x/L) with maximum load and the variation of (x/L) with normalized maximum load ($P/P_{\text{full}}$). As was observed with the PCFFTs, the failure
load of a PCFST will gradually increase when more concrete is added to the tube until the load reaches a plateau.

### 5.4.2.1 Effect of Yield Strength

Figures 5-40 to 5-42 present the \((x_{\text{opt}}/L)\) versus \(f_y\) relationship for each (D/t) ratio. Each figure includes the relationship based on the model, which could be governed either by material strength or local buckling, and the design standards. For all (D/t) ratios, the optimal filling ratio is less when established by the code provisions because the concrete-filled capacity usually underestimated. Generally, the optimal filling ratio is nearly constant for all \(f_y\). The main observation taken from these results is that the yield strength of the steel tube has a negligible effect on the optimal filling ratio. A tube with a higher yield strength will produce a pole with a higher strength for all amounts of concrete filling, but the optimal filling amount will not vary significantly.

### 5.4.2.2 Effect of (D/t) Ratio

Figure 5-43 presents the relationship between optimal concrete filling length ratio and (D/t) for all yield strengths. The three curves are nearly coincident, and each is fairly linear. The reason a thinner tube requires more concrete filling is that the hollow tube buckles at lower loads. As was observed with the PCFFTs, the optimal concrete filling reduces as the hollow tube capacity approaches the concrete-filled tube capacity.
5.5 Assessment of Design Expressions

The following sections compare the results of the parametric study to values predicted by the simple design expressions that were hypothesized in Chapter 1.

5.5.1 Design Expression Comparison for PCFFTs

Design expressions are presented in Equations 1-1 to 1-2 in Chapter 1 that predict the optimal concrete filling length, $x_{\text{opt}}$, for partially concrete-filled FRP tubes. In the equations, $x_{\text{opt}}$ is primarily based on the capacities of a hollow and a concrete-filled FRP tube (i.e. Equations 2-16 and 2-20, respectively). Therefore, the accuracy of the design expressions (Equations 1-1 and 1-2) is largely based on the accuracy of Equations 2-16 and 2-20. It should be noted that Equations 2-16 and 2-20 were obtained by fitting empirically derived and finite element data.

Figure 5-44 compares Equation 2-16 to the developed model’s capacity for a hollow GFRP tube, and Figure 5-45 compares Equation 2-20 to the developed model’s capacity for a concrete-filled GFRP tube. Generally, there is a good agreement, although the model seems to slightly underestimate the ultimate loads predicted by Equation 2-16 for hollow GFRP tubes. This overestimation by Equation 2-16 suggests that the design expressions may slightly underestimate the optimal filling ratio. Figures 5-46 to 5-48 compare the optimal filling length ratio for the PCFFTs, for each case of (D/t) obtained with the model and design expressions. In all figures, the prediction of the model exhibits a similar trend to the prediction of the design expressions. For (D/t) of 125, the average percentage difference was 12%; for (D/t) of 75, was 23%, and for (D/t) of 40, was 30%, largely due to the large outlier for ±45°. These large discrepancies occurred...
when section fracture, not buckling, governed, which means the model and design prediction used the same hollow capacity. The difference in totally filled capacity (between the model and Equation 2-20) for the $\pm 45^\circ$ and $\pm 75^\circ$ is 9% and 14%, respectively. The 9% difference for $\pm 45^\circ$ tube produced a 120% increase in optimal filling, while the $\pm 75^\circ$ tube had a 50% increase in optimal filling. This suggests that the design expressions are very sensitive to changes in the ratio of hollow-to-filled capacity.

5.5.2 Design Expression Comparison for PCFSTs

Due to the absence of an adequate prediction of the ultimate capacity of a concrete-filled steel tube, Equations 1-1 and 1-2 were not used for predictions based on the material failure of steel. The design expression was used only with respect to the Canadian design standards. Figures 5-49 to 5-51 compare the optimal filling length ratio versus yield strength for each (D/t) ratio. The figures include the optimal filling length ratios that are predicted using Equations 1-1 and 1-2 and by using the model (governed by CSA S16-01 failure criteria). Generally, the Equations 1-1 and 1-2 predict the optimal filling ratio quite accurately. The average difference between the model and the design expressions was 6%. The PCFSTs lack the large discrepancies that were found with the PCFFTss since the PCFST model and Equations 1-1 and 1-2 were based on the same failure criteria. The small differences are likely due to the design expression assumption that buckling failure will occur at a distance $D$ from the face of the concrete filling. In the model, and as was observed in the experimental program, buckling occurred closer to the face of the concrete filling, than a $D$ distance.
5.6 Assessment of Model Using FEA from Literature

Fam and Son (2007) used the finite element method through program ANSYS to address the same problem being studied in this thesis for FRP tubes. The parametric study conducted in this chapter to assess the model developed in this thesis used the same cases studied by Fam and Son (2007), to facilitate comparisons. Figure 5-52 shows the effect of laminate structure on \( \frac{x_{opt}}{L} \) ratios predicted by using the model and FEA by Fam and Son. For cross-ply laminates, the average difference between the developed model and the FEA model was 17%, or 0.06\( L \) of concrete filling. For angle-ply laminates, the average difference was 23%, or 0.09\( L \) of concrete filling.
Table 5-1: Circumferential bending stiffness $D_{yy}$ of GFRP tubes used in parametric study

<table>
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<th>Laminate</th>
<th>$+\theta/-\theta$</th>
<th>$[0/90]$</th>
<th>$D_{yy}$ (kN.m)</th>
<th>$D_{yy}$ (kN.m)</th>
<th>$D_{yy}$ (kN.m)</th>
<th>$D_{yy}$ (kN.m)</th>
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<td>t = 2.4 mm</td>
<td>t = 4 mm</td>
<td>t = 4 mm</td>
<td>t = 4 mm</td>
<td>t = 4 mm</td>
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<tr>
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<td>$25^\circ$</td>
<td>$35^\circ$</td>
<td>$45^\circ$</td>
<td>$75^\circ$</td>
<td>$3:1$</td>
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<td>0.011</td>
<td>0.014</td>
<td>0.019</td>
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Table 5-2: Summary of parametric study results for angle-ply GFRP laminates (optimal filling highlighted)

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<th>X/L</th>
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<th>$P_a/P_{full}$</th>
<th>X (mm)</th>
<th>X/L</th>
<th>$P_a$ (kN)</th>
<th>$P_a/P_{full}$</th>
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Table 5-3: Summary of parametric study results for cross-ply GFRP laminates (optimal filling ratio (x/L) highlighted)

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<th>D/t = 40</th>
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<td>Pu (kN)</td>
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6.1 Summary

A hollow tube loaded in flexure is vulnerable to premature failure by local buckling. Buckling is an undesirable failure mode because it fails to utilize the full material strength of the member. This may be the case for steel and GFRP tubular poles loaded in cantilever bending. The proposed solution to this problem is to fill a portion of the tube, near the base, with concrete. This is proposed in lieu of increasing the wall thickness of the tube, which is more costly. The partial concrete filling will support the tube against inward local buckling in the region of maximum moment, while the remaining hollow portion where the bending moment is reduced will allow for utilizing the lightweight benefits of FRP. This study evaluated the flexural performance of partially concrete-filled GFRP tubes (PCFFTs) and partially concrete-filled steel tubes (PCFSTs). The primary objective was to determine the optimal length of concrete filling that would permit the pole to simultaneously experience material failure of the tube at the base, and local buckling failure of the hollow part, just above the end of concrete fill. The simultaneous failure allows for optimum use of tube material and concrete, while maintaining a minimum deadweight. The experimental program consisted of six PCFFTs and four PCFSTs loaded in cantilever bending. Each specimen was filled with a unique amount of concrete.

An numerical model was developed to predict the complete flexural performance of PCFFTs and PCFSTs. For the concrete-filled portion, a strain compatibility model, which adopts cracked section analysis and equilibrium, uses a layer-by-layer approach,
and accounts for material nonlinearities of the GFRP laminate, steel tube, and concrete, in both compression and tension, has been used. For the hollow portion, the model accounts for the ovalization of the cross-section for both GFRP and steel tubes as well as material failure and local buckling failure. The model was verified by the experimental results and showed good agreement. The model was used to conduct a parametric study that investigated the effects of a number of parameters on the performance of PCFFTs and PCFSTs, including D/t ratio and laminate structure for PCFFTs, and D/t ratio and yield strength for PCFSTs. This study has only focused on prismatic tubes. Tapered tubes are beyond the scope of this thesis.

6.2 Conclusions

The following conclusions were drawn from the experimental and numerical investigations of PCFFTs:

1. There is an optimum length of partial concrete fill in cantilever-type FRP tubular poles. This length would result in achieving maximum flexural strength at a minimum dead weight. As the length of concrete fill increases, the failure load gradually increases until it reaches a plateau at the optimum length.

2. The optimum length of the partial concrete fill increases as the (D/t) ratio increases (i.e. in thinner-walled tubes).

3. For angle-ply laminates, the optimum length of the partial concrete fill reduces as the angle of fibers relative to the longitudinal axis in angle-ply laminates increases, until reaching a minimum at 45°, after which the optimal filling ratio will increase.
4. For cross-ply laminates, the optimal filling ratio increases when more fibres are in the longitudinal direction, and reduces when more fibres are in the circumferential direction.

5. When the optimum concrete fill length is provided, the pole would fail due to material failure of the tube within the concrete-filled part at the fixed base and due to failure of the hollow part, at or shortly above the end of the concrete fill, simultaneously. Failure of the hollow part of the tube may be due to local buckling or material failure, depending on \((D/t)\) ratio and laminate structure.

6. The experimental program revealed that the hollow portion will buckle at a variable distance from the face of the concrete filling, and that this distance will decrease as the length of concrete fill is increased.

7. PCFFTs may undergo large deflections; however, the behaviour remains linear-elastic to failure.

8. Although a PCFFT with the optimal concrete-filling length (or more) will achieve the same capacity as a completely filled tube, there is a loss in stiffness associated with partial removal of concrete.

9. Although laminates, either angle- or cross-ply, with primarily circumferential fibres require a smaller amount of concrete for the optimal filling, the strength and ductility are highest when the laminate consists of primarily longitudinal fibres.

10. A simple expression to predict the optimal concrete filling length for PCFFTs has been developed and is based on the moment capacities of both the concrete-filled and hollow sections. For \((D/t)\) of 125, the average percentage difference was 12%; for \((D/t)\) of 75, was 23%, and for \((D/t)\) of 40, was 30%.
The following conclusions were drawn from the experimental and numerical investigations of PCFSTs:

1. As was observed with PCFFTs, complete filling of the tube with concrete is unnecessary to achieve the maximum ultimate capacity.

2. The optimal filling length ratio increases as the (D/t) ratio is increased.

3. The steel yield strength has a negligible effect on the optimal filling length ratio. However, increasing the yield strength increased the flexural strength.

4. Three failure modes govern the behaviour of PCFSTs: tensile failure (excessive yielding) of the steel tube at the fixed end of the filled portion, local buckling in the hollow length, and material failure (excessive yielding) in the hollow length near the face of the concrete filling. These depend on (D/t) ratio.

5. In the experimental and numerical studies, the hollow tube tended to buckle very near the face of the concrete filling.

6. PCFSTs will undergo large deformations, and the behaviour can undergo large plastic deformations prior to failure, particularly for members that fail in tension. Local buckling failure can reduce the amount of plastic deformation that happens.

7. When the behaviour was limited by design standards, the optimal filling length ratio was significantly underestimated because design standards underestimated the capacity of the concrete-filled steel tube, much more so than the hollow tube.

8. A simple expression to predict the optimal concrete filling for PCFSTs differed from the model prediction by an average of 6%.
6.3 Recommendations for Future Work

More research is required to fully understand the behaviour and capabilities of PCFFTs and PCFSTs and to make them viable as practical products. This includes:

1. Evaluation of the system under combined axial compression and flexure to observe the effect axial compression has on optimal filling and to develop load-moment interaction diagrams. Also, the effect of wind pressure, instead of point loads, should be evaluated.

2. Evaluation of torsion performance, since sign-supporting structures are vulnerable to wind-induced torsion.

3. Dynamic tests should be undertaken to determine the effect the partially-filled concrete core has on the natural frequency of the system.

4. Since a tube with primarily longitudinal fibres in the filled portion provide the greatest strength and ductility, and a tube with primarily circumferential fibres in the hollow portion provides the greatest buckling resistance, a splice, or some other mechanism, should be developed to produce a PCFFT where the filled and hollow portions consist of a different laminate structure, one that will optimize each portion’s performance.

5. Evaluate different cross-sectional geometries, including effects of tapering the pole on optimal fill ratio.
REFERENCES


ASTM A370-03a. “Test methods and definitions for mechanical testing of steel products.”


References


APPENDIX A: NUMERICAL MODEL BACKGROUND

A.1 Introduction

Appendix A provides background information regarding moment-curvature models for concrete-filled GFRP and steel tubes, and completely hollow GFRP and steel tubes. The concrete-filled tube model was developed by Fam (2000), the hollow GFRP tube model was developed by Ibrahim (2000), and the hollow steel tube model was developed by Brazier (1927).

A.2 Model Description of Concrete-filled FRP or Steel Tubes

The model is based on the assumption that plane sections remain plane when subjected to bending. This assumption allows the strain to be distributed linearly through the cross-section. Strain compatibility (i.e. full bond) between the tube and the inner core was also assumed. Previous studies by Fam (2000), Mandal (2005), Cole (2006), and Qasrawi (2007) have adopted and validated these assumptions. The member is assumed to be cracked under a general bending moment, $M$. The depth to the neutral axis is given as $c$. A summary of the model, which was developed by Fam (2000) for concrete-filled FRP tubes, is given in the following sections.

A.2.1 Section Geometry

The circular geometry of the section is more complex than a typical rectangular cross-section. Stresses in the section vary over the section’s depth, and are also applied to a variable width. Reinforcement (FRP tube) is provided continuously around the
circumference of the section, as opposed to at a constant depth in a conventional steel-reinforced rectangular section. A layer-by-layer approach is adopted to integrate the stresses over the section. The geometry is idealized into discrete layers (Figure A-1). The concrete-filled tube model for either a GFRP tube or steel tube is structured the same. The only difference is the material properties. The average diameter of the section, \( D \), is given as:

\[
D = D_0 - t
\]

(A-1)

where \( D_0 \) is the outer diameter of the section, and \( t \) is the structural wall thickness of the tube. The diameter is divided into \( n \) number of layers of equal thickness, \( h_i \), given as:

\[
h_i = \frac{D}{n}
\]

(A-2)

For a given layer \( i \), the depth, \( h(i) \), to the mid-height of the layer is given by:

\[
h(i) = h(i-0.5)
\]

(A-3)

The length of the perimeter of the tube on one side of a given layer, \( L(i) \), is given as:

\[
L(i) = 0.5D[
\phi_2(i) - \phi_1(i)
]\]

(A-4)

where \( \phi_1(i) \) and \( \phi_2(i) \) are angles, in radians, between the vertical axis of the section and the two radii bounding the segment. These angles are determined as:

\[
\phi_1(i) = \cos^{-1}\left[\frac{0.5D - h(i) + 0.5h_i}{0.5D}\right]
\]

(A-5)

\[
\phi_2(i) = \cos^{-1}\left[\frac{0.5D - h(i) - 0.5h_i}{0.5D}\right]
\]

(A-6)

The area of the FRP or steel tube within a given strip is determined as:

\[
A_f(i) = 2L(i)t
\]

(A-7)

The total area of the concrete within each strip is given by:
\[ A_f(i) = 2B(i)h(i) - 0.5A_f(i) \] (A-8)

where \( B(i) \) is half the width of the strip, calculated as:

\[ B(i) = 0.5D \sin \varphi(i) \] (A-9)

where \( \varphi(i) \) is the angle, in radians, between the vertical axis of the section and the radius to the perimeter at mid-height of a given layer, calculated as:

\[ \varphi(i) = \cos^{-1} \left( \frac{0.5D - h(i)}{0.5D} \right) \] (A-10)

Equations A-7 and A-8 can then be used to calculate the areas of FRP (or steel) and concrete in each strip.

### A.2.2 Material Constitutive Properties

The following sections discuss the material constitutive properties for the GFRP and steel tubes, and for the concrete filling.

#### A.2.2.1 GFRP Tubes

Material properties of the GFRP tube can be derived from either coupon testing or from Classical Lamination Theory’s Ultimate Laminate Failure approach (CLT-ULF), which dictates that the laminate is not failed until all layers have failed. Details of classical lamination theory are found in Daniel and Ishai (1994). Figure A-2 compares the stress-strain curves obtained experimentally and theoretically by CLT-ULF. The coupon tests were discussed in detail in Chapter 4. The stress-stress curve from CLT-ULF was fitted by a second-order function. For the GFRP tube in compression (Figure A-3), the stress-strain curve from CLT-ULF was fitted by a linear function.
A.2.2.2 Steel Tubes

The stress-strain response for the steel tubes was determined by fitting a curve to the coupon test results (Figure A-4). The coupon results are detailed in Chapter 4. The steel stress-strain curve is represented by four separate equations, since there are four distinct regions of behaviour: the elastic region, the strain-hardening region, the plastic region, and the strain-softening region near rupture. The equations are given in Figure A-4, and the same curve is also used for compression.

A.2.2.3 Concrete

The nonlinear stress-strain behaviour of the concrete in compression was modeled by Popovics (1973) (Figure A-5), assuming extended strain softening (i.e. curve is not terminated at 0.003 strain) as suggested by Fam and Rizkalla (2002) to account for partial confinement in flexure. Popovics’ model for the unconfined compressive strength of concrete, $f_{cc}$, is given as:

$$ f_{c}(i) = \frac{f_{c} \left( \frac{\varepsilon_{c}(i)}{\varepsilon_{c}} \right)^{r}}{r - 1 + \left( \frac{\varepsilon_{c}(i)}{\varepsilon_{c}} \right)^{r}} $$  \hspace{1cm} (A-11)

where $f_{c}$ is the unconfined concrete strength, obtained from cylinder tests, $\varepsilon_{c}(i)$ is the corresponding strain in concrete, $\varepsilon_{c}$ is the strain corresponding to $f_{c}$, $r = E_{co}/(E_{co} - E_{sec})$. $E_{co}$ and $E_{sec}$ are the tangent and secant modulus of concrete, respectively, where $E_{co} = 5000 \sqrt{f_{c}'}$ and $E_{sec} = f_{c}'' / \varepsilon_{c}''$. 
Prior to cracking ($\varepsilon(i) < \varepsilon_{cr}$), the tensile stress in concrete is:

$$f_{ct}(i) = E_{ct} \varepsilon(i) \quad (A-12)$$

Concrete’s tension stiffening effects were accounted for by the model developed by Vecchio and Collins (Collins and Mitchell, 1997) (Figure A-5). Therefore, after cracking, the tensile stress in concrete is given as:

$$f_{ct}^{cr} = \frac{\alpha_1 \alpha_2 f_{cr}}{1 + \sqrt{500(\varepsilon(i) - \varepsilon_{cr})}} \quad (A-13)$$

where $f_{cr}$, the cracking strength of concrete, is calculated as $f_{cr} = 0.6 \sqrt{f_c}$ and $f_c = E_{cs} \varepsilon_{cr}$ (CSA A23.3-94), where $\varepsilon_{cr}$ is the cracking strain. $\varepsilon(i)$ is the applied strain in the concrete. $\alpha_1$ and $\alpha_2$ account for bond characteristics and the nature of the loading, respectively. $\alpha_1 = 0.3$ for the GFRP tube used in this study (Fam, 2000), and $\alpha_2 = 1.0$ for monotonic loading.

A.2.3 Stresses Acting on Cross-section

The strains at the extreme top and bottom surfaces of the tube are $\varepsilon_t$ and $\varepsilon_b$, respectively. The strain in the extreme surfaces of the concrete is given as $\varepsilon_c$. At any particular load, the strain in a given layer is given by:

$$\varepsilon(i) = \frac{\varepsilon_t[D-h(i)] + \varepsilon_b h(i)}{D} \quad (A-14)$$

Figure A-6 illustrates the distribution of stresses and strains throughout the cross-section. The stresses are calculated from the strains using the materials stress-strain curves described in Section A.2.2.
A.2.3.1 Internal Forces and Moments

Internal forces in the cross-section are determined by integrating the stresses over the cross-sectional area. The compression force in the concrete, $CC(i)$, is determined by:

$$CC(i) = f_c(i)A_c(i)$$  \hspace{1cm} (A-15)

where $f_c(i)$ is given in Equation A-11, and $A_c(i)$ is given in Equation A-8. The compression force in the tube, $CF(i)$, is determined by:

$$CF(i) = f_{c_{frp}}(i) \text{ (or) } f_s(i)A_f(i)$$  \hspace{1cm} (A-16)

where $f_{c_{frp}}(i)$ and $f_s(i)$ are the stresses in FRP or steel tube, respectively, and are calculated from the equations given in Figures A-3 and A-4, respectively. $A_f(i)$ is given in Equation A-7. The tension force in concrete, $TC(i)$, is determined by:

$$TC(i) = f_{c_t}(i)A_c(i)$$  \hspace{1cm} (A-17)

where $f_{c_t}(i)$ is given in Equations A-12 and A-13, and $A_c(i)$ is given in Equation A-8. The tension force in the tube, $TF(i)$, is determined by:

$$TF(i) = f_{f_{frp}}(i)(\text{or} f_s(i))A_f(i)$$  \hspace{1cm} (A-18)

where $f_{f_{frp}}(i)$ and $f_s(i)$ are from the equations given in Figures A-2 and A-4.

The total compressive force in concrete and the tube, $CCC$ and $CFF$, respectively, are calculated by summing the forces in the layers located above the neutral axis. $CCC$ and $CFF$ are given as:

$$CCC = \sum_{i=1}^{i-\left[\frac{c-0.5i}{D}\right]} CC(i)$$  \hspace{1cm} (A-19)

$$CFF = \sum_{i=1}^{i-\left[\frac{c-0.5i}{D}\right]} CF(i)$$  \hspace{1cm} (A-20)
The total tensile force in the concrete and the tube, \( TCC \) and \( TFF \), respectively, are determined by summing the forces in all strips below the neutral axis. \( TCC \) and \( TFF \) are given as:

\[
\text{TCC} = \sum_{i=1}^{i=n} \text{TCC}(i) \tag{A-21}
\]

\[
\text{TFF} = \sum_{i=1}^{i=n} \text{TFF}(i) \tag{A-22}
\]

For equilibrium, the tensile and compressive forces must satisfy the following condition:

\[
\text{CCC} + \text{CFF} + \text{TCC} + \text{TFF} = 0 \tag{A-23}
\]

When equilibrium is satisfied, internal moments are calculated. The internal moment contribution of a particular layer is determined by multiplying the force in that layer by that particular layer’s distance to the neutral axis. The internal moments, in the concrete and tube, are calculated as:

\[
\text{MCC} = \sum_{i=1}^{i=n} [\text{CC}(i)y(i)] \tag{A-24}
\]

\[
\text{MCF} = \sum_{i=1}^{i=n} [\text{CF}(i)y(i)] \tag{A-25}
\]

\[
\text{MTC} = \sum_{i=1}^{i=n} [\text{TC}(i)y(i)] \tag{A-26}
\]

\[
\text{MTF} = \sum_{i=1}^{i=n} [\text{TF}(i)y(i)] \tag{A-27}
\]
where $MCC$ is the moment contribution of concrete in compression, $MCF$ is the moment contribution by the GFRP tube in compression, $MTC$ is the moment contribution of concrete is tension, and $MTF$ is the moment contribution of the GFRP tube in tension. The moment arm, $y(i)$, is determined as follows:

In compression: 
$$y(i) = c - 0.5t - h(i)$$  \hspace{1cm} (A-28)

In tension: 
$$y(i) = h(i) - c + 0.5t$$  \hspace{1cm} (A-29)

The total moment in the cross-section is the sum of the moments as follows:

$$\text{Moment} = MCC + MCF + MTC + MTF$$  \hspace{1cm} (A-30)

The curvature, $\psi$, which corresponds to the aforementioned moment, is given as:

$$\psi = \frac{\varepsilon}{c}$$  \hspace{1cm} (A-31)

### A.2.4 Failure Criteria

A number of different failure modes can govern the strength of partially filled tubes. Failure can occur in the concrete-filled portion of the tube, or in the hollow portion. This section is focused on concrete-filled tubes.

Failure in the concrete-filled FRP tube is usually governed by rupture of the tube in tension, unless the tube is very thick, in which case, crushing of the tube in compression may occur first. Fam (2000) has shown that concrete inside the tube does not fail as long as the tube is intact. For GFRP tubes, the ultimate strength can be determined by either coupon testing or CLT-ULF. In this study, the ultimate strength predicted by CLT-ULF was used, since the coupons failed prematurely at lower strains as discussed in Chapter 4. For the steel tubes, failure is governed by the steel tube. Coupon tests showed that the steel undergoes excessive yielding followed by strain softening,
then rupture. For the GFRP and steel tubes used in this study, once the strains corresponding to the ultimate strengths (rupture) of the materials were reached at the extreme tension side, analyses were terminated.

A.3 Model Description of Hollow GFRP or Steel Tubes

The follow sections present the model that was developed by Ibrahim (2000) for hollow GFRP tubes and by Brazier (1927) for hollow steel tubes. Modelling hollow tubes is slightly more complex than concrete-filled tubes because cross-sectional ovalization must be accounted for.

A.3.1 Hollow GFRP Tube

A hollow tubular member in flexure undergoes a cross-sectional distortion called ovalization. As the curvature of the beam is increased, the cross-section gradually changes from circular to ovular (Figure A-7). The following expressions are adopted from the model by Ibrahim (2000) for hollow FRP tubes fixed at one end and loaded at the free end, which is a braced cross-section. The relationship between ovalization and bending moment in a cantilever FRP tube is given as follows:

\[
M_o = 2.85 \pi r \sqrt{\pi E_x D_{yy} f \left(1 + \frac{\pi^4}{12\Omega^4}\right) \left(1 - \frac{3}{\Omega}\right)} \sqrt{\xi_o}
\]  

(A-32)

where:

\[
\Omega = \left(\frac{L_o}{r}\right) \left(\frac{12 D_{yy}}{r^2 E_x f}\right)
\]  

(A-33)
where $M_o$ is the base moment, $E_x$ is the elastic modulus in the longitudinal direction, $r$ is the initial outer radius of tube, $D_{yy}$ is the circumferential bending stiffness of the FRP tube, $t$ is the wall thickness, $\xi_o$ is the maximum ovalization ratio at the midpoint of the hollow length, and $L_h$ is the length of the hollow part of the tube. The variable $\Omega$ in Equation A-33 has no physical significance and is merely used to simplify Equation A-32. The ovalization ratio $\xi_o$ is defined as the change in radius divided by the original radius, $r$ (Figure A-7). The maximum ovalization ratio is calculated based on the applied bending moment $M_o = P*L_h$, where $P$ is the lateral load applied, and then iteration is used Equation A-32 to establish $\xi_o$. Ovalization is assumed to have a sinusoidal distribution along the hollow length, $L_h$. Note that ovalization is equal to zero at both ends because the base is fixed and the tip is assumed braced with a stiffener. The ovalization, $\xi(x_h)$, at any distance $x_h$ from the free end is given as:

$$\xi(x_h) = \xi_o \sin \left( \frac{\pi x_h}{L_h} \right)$$  \hspace{1cm} (A-34)

where $\xi_o$ is the maximum ovalization ratio at midspan of the hollow part of the tube and $L_h$ is the length of the hollow part of the tube. The subsequent moment of inertia at any distance $x_h$ in the hollow length is:

$$I(x_h) = I_o \left( 1 - 1.5\xi_o \sin \left( \frac{\pi x_h}{L_h} \right) \right)$$  \hspace{1cm} (A-35)

where $I_o$ is the initial moment of inertia of the undeformed circular cross-section, which is given as:

$$I_o = \pi r^4 t$$  \hspace{1cm} (A-36)
where \( r \) and \( t \) are the outer radius and wall thickness, respectively. Initially, the distance from the neutral axis to the extreme surfaces of the cross-section, \( y(x_h) \), is equal to the radius, \( r \), but after ovalization begins it becomes:

\[
y(x_h) = r(1-\xi(x_h)) \tag{A-37}
\]

where \( M(x_h) \) is the applied moment at a distance \( x_h \) along the hollow tube, and \( E_x \) is the elastic modulus of the GFRP tube in the longitudinal direction.

The hollow tube model also assumes that the circumferential strains are zero, since they are small relative to the longitudinal strains.

The total longitudinal stress, \( \sigma_T \), in a hollow GFRP tube is the summation of two stresses (Ibrahim, 2000): a stress created by the ovalizing of an initially straight tube, and stress created by the applied bending moment. The total stress at a distance \( x_h \) is given as:

\[
\sigma_T(x_h) = \sigma_1(x_h) + \sigma_2(x_h) \tag{A-38}
\]

where \( \sigma_1(x_h) \) is the stress created by ovalization, which is equal to:

\[
\sigma_1(x_h) = \frac{\pi^2 E_x r^2}{4L_h^2} \left( \tilde{\varepsilon}_o \sin \left( \frac{\pi x_h}{L_h} \right) \right) \tag{A-39}
\]

The bending stress, \( \sigma_2(x_h) \), is equal to:

\[
\sigma_2(x_h) = 2.85 \sqrt{\frac{\pi D_{yy} E_x \left( 1 + \frac{\pi^4}{12 \Omega^2} \right)}{r^2 t}} \frac{\left( 1-\xi_o \sin \left( \frac{\pi x_h}{L_h} \right) \right)}{\sqrt{\xi_o}} \left[ 1 + \frac{3 \xi_o}{\pi} \left( 1-1.5 \xi_o \sin \left( \frac{\pi x_h}{L_h} \right) \right) \right] \left( \frac{x_h}{L_h} \right) \tag{A-40}
\]
where $E_x$ is the elastic modulus of the tube in the longitudinal direction, $D_{yy}$ is the circumferential bending stiffness of the tube, $r$ is the radius of the tube, $t$ is the wall thickness of the tube, $x_h$ is the distance along the tube from the free end, $L_h$ is the length of the hollow part of the tube, $\xi_o$ is the maximum ovalization ratio in the hollow part of the tube, and $\Omega$ is a dimensionless parameter from Equation A-33.

### A.3.1.1 Hollow GFRP Tube Failure Criteria

A hollow GFRP tube can have a material failure of the tube, in tension or compression, or local buckling. For material failure, this occurs when the stress at extreme fibres exceeds the ultimate stress $\sigma_{ult}$. For a hollow tube, the moment that corresponds to the ultimate stress is determined as:

$$M_{ult} = 0.79tD^2\varepsilon_{ult}$$

(A-41)

where $t$ is the structural wall thickness, $D$ is the outer diameter ($2r$), $E_x$ is the longitudinal modulus of elasticity, and $\varepsilon_{ult}$ is the ultimate strain. $\varepsilon_{ult}$ can be in either compression or tension, whichever is lower, and is established from Figures A-2 and A-3. It should be noted that Equation A-41 is the ultimate moment for a cross-section that is perfectly circular, and not ovalized, which means that Equation A-41 assumes that the material fracture will occur immediately at the fixed end of the pole (location of maximum moment). Due to ovalization, and its sinusoidal distribution along the length (Figure A-7), the moment of inertia and neutral axis depth vary along the length. Therefore, it could be possible for this failure mode (fracture of the tube material) to occur a small distance away from the base, at a slightly lower moment than that from Equation A-41. To prove
that the hollow GFRP tube would fail by material fracture at only the location of maximum moment, bending stresses in perfectly circular tubes were compared to tubes undergoing ovalization. The minimum thickness was determined that would allow the GFRP tube to fail simultaneously by local buckling at a distance $Y$ (Equation 2-17) from the base (Figure A-8(a)), and by material fracture at the base (Figure A-8(b)). The tubes investigated in Figure A-8 are the same tubes used in the parametric study in Section 5.3.1 of Chapter 5. For this minimum thickness the tube will experience the maximum amount of ovalization that will allow it to fail by material fracture. If a tube with this minimum thickness satisfies the assumption of Equation A-41 for ovalized tubes, then all tubes governed by material fracture (increased thickness) are safe. For the thicknesses in Figure A-8, Figures A-9 and A-10 compare the bending stress distribution along the length of ovalized GFRP tubes and perfectly circular GFRP tubes. The stress distribution for the perfectly circular poles is perfectly linear, while the stress distribution for the ovalized poles is somewhat non-linear, due to the ovalized cross-section. In all cases, the bending stress at the base is higher than at any other location which indicates that the ultimate stress of the material is reached at the base before being reached elsewhere along the length. Therefore, Equation A-41 is a safe assumption.

For local buckling failure, Tatting (1996) derived a formula for the critical buckling stress, $\sigma_{cr}(x)$, in an FRP tube. Ibrahim (2000) rearranged Tatting’s formula to put the critical stress in terms of the maximum ovalization ratio:

$$\sigma_{cr}(x_h) = \frac{2 \sqrt{D_{xx} E_t t}}{rt} \left( 1 - 3 \xi_o \sin \left( \frac{\pi x_h}{L_b} \right) \right) \quad (A-42)$$
where $E_y$ is the circumferential elastic modulus, $t$ is the wall thickness, $\zeta_o$ is the maximum ovalization ratio, $L_h$ is the length of hollow tube, $x_h$ is the distance of section $x$ from the free end of the hollow tube, $D_{xx}$ is the longitudinal bending stiffness. The tube fails by local buckling when the stress calculated by Equation A-38 exceeds the critical stress calculated from Equation A-42.

### A.3.2 Hollow Steel Tube

Like hollow GFRP tubes, hollow steel tubular sections in flexure undergo cross-sectional ovalization. Ovalization of elastic tubes was first studied by Brazier (1927). Brazier introduced 2\textsuperscript{nd} order effects to the analysis of tubes in flexure to derive the following relationship between the ovalization ratio and normalized curvature:

$$
\zeta_o = \left(\frac{\psi}{\Psi_N}\right)^2
$$

(A-43)

where $\zeta_o$ is the maximum ovalization ratio, $\psi$ is the applied curvature, and the normalizing factor, $\Psi_N$, is equal to:

$$
\Psi_N = \frac{t}{r^2\sqrt{1-\mu^2}}
$$

(A-44)

where $\mu$ is Poisson’s ratio for steel, $r$ and $t$ are the radius and thickness of the tube, respectively.

The model for the hollow steel tube is structured similarly to the concrete-filled tube model, in that it adopts a layer-by-layer approach. But since the absence of concrete allows the cross-section to ovalize, the model had to account for the changing geometry. Guarracino (2003) showed that the ovalized cross-section very closely resembles an
ellipse, so an elliptical geometry was applied to the deformed section. As with the 
hollow GFRP model, ovalization is assumed to have a sinusoidal distribution along the 
length of the tube, where $\xi_o$ is the maximum ovalization at midspan, and $\xi(x_h)$ is the 
ovalization anywhere (distance $x_h$ from the free end) along the length of the tube. Figure 
A-9 presents the geometry of the ovalized section. The following equations consider 
only one quarter of the ellipse. The average depth of the initial, undeformed circular 
section, $D$, is given as:

$$D = D_0 - t$$ (A-45)

where $D_0$ is the outer diameter of the section, and $t$ is the structural wall thickness of the 
tube. Once ovalization begins, the depth, $D_v$, and width, $D_h$, become:

$$D_v = D(1 - \xi(x_h))$$ (A-46)

$$D_h = D(1 + \xi(x_h))$$ (A-47)

This assumes that the vertical and horizontal ovalization ratios, $\xi(x_h)$, are equal. The 
depth is divided into $n$ number of layers of equal height, $h_i$, given as:

$$h_i = \frac{D_v}{n}$$ (A-48)

For a given layer $i$, the depth, $h(i)$, to the mid-height of the layer is given by:

$$h(i) = h(i) - 0.5$$ (A-49)

The length of the perimeter of the tube on one side of a given layer, $L(i)$, is given as:

$$L(i) = 0.5 \sqrt{xx(i)^2 + yy(i)^2} [\varphi_2(i) - \varphi_1(i)]$$ (A-50)

where $xx(i)$ is the horizontal distance from the $y$-axis axis to the tube, and $yy(i)$ is the 
vertical distance from the $x$-axis to the tube. $\varphi_1(i)$ and $\varphi_2(i)$ are angles, in radians, between
the $y$-axis of the section and the two radii bounding the segment. These are determined as:

$$x_{x(i)}=0.5D_h \sqrt{1-\left(\frac{yy(i)}{0.5D_v}\right)^2}$$  \hspace{1cm} (A-51)$$

$$yy(i)=0.5D_v-h(i)$$  \hspace{1cm} (A-52)$$

$$\phi_1(i)=\cos^{-1}\left[\frac{yy(i)+0.5hi}{\sqrt{x_{x(i)}^2+yy(i)^2}}\right]$$  \hspace{1cm} (A-53)$$

$$\phi_2(i)=\cos^{-1}\left[\frac{yy(i)}{\sqrt{x_{x(i)}^2+yy(i)^2}}\right]$$  \hspace{1cm} (A-54)$$

The values $x_{x_1(i)}$, $yy_1(i)$, $x_{x_2(i)}$, and $yy_2(i)$ are the horizontal ($x_x$) and vertical ($yy$) distances to the lower and upper bounds of the layer considered. These are given as:

$$x_{x_1(i)}=0.5D_h \sqrt{1-\left(\frac{yy_1(i)}{0.5D_v}\right)^2}$$  \hspace{1cm} (A-55)$$

$$yy_1(i)=0.5D_v-h(i)+0.5hi$$  \hspace{1cm} (A-56)$$

$$x_{x_2(i)}=0.5D_h \sqrt{1-\left(\frac{yy_2(i)}{0.5D_v}\right)^2}$$  \hspace{1cm} (A-57)$$

$$yy_2(i)=0.5D_v-h(i)-0.5hi$$  \hspace{1cm} (A-58)$$

The area of the steel tube within a given strip, $A_s(i)$ is determined as:

$$A_s(i)=L(i)t$$  \hspace{1cm} (A-59)$$

Strain in a given layer is calculated similarly to the concrete-filled model. The only difference is that the new ovalized section depth, $D_v$, must be considered:
where \( \varepsilon \) is the strain applied to the extreme top and bottom surface of the tube. Stresses, forces, and moments are calculated identically to the method presented in the concrete-filled model in Section A.2.3.1, excluding the concrete contribution. The total moment needs to be multiplied by a factor of 4 since Equation A-59 accounts for only one quarter of the tube cross-section.

**A.3.2.1 Failure Criteria**

Korol (1979) developed an expression for the critical buckling strain, \( \varepsilon_{\text{crit}} \), of a hollow steel tube as follows:

\[
\varepsilon_{\text{crit}} = \frac{4t}{3D} \sqrt{\frac{E_t}{E_s}}
\]  

(A-61)

where \( \varepsilon_{\text{crit}} \) is the critical buckling strain, \( t \) is the wall thickness, \( E_t \) is the tangent modulus of the material, and \( E_s \) is the secant modulus. The values \( E_t \) and \( E_s \) are calculated from the stress-strain curve for steel (Figure A-10).
Figure A-1: Schematic of idealized concrete-filled tube model
Figure A-2: Stress-strain behaviour of GFRP tube in axial tension

Figure A-3: Stress-strain behaviour of GFRP tube in axial compression
Figure A-4: Stress-strain behaviour of steel tube

Figure A-5: Stress-strain behaviour of concrete on compression and tension
Figure A-6: Stress and strain distributions on the cross-section of concrete-filled tubes

Figure A-7: Ovalization of GFRP cantilevered tube (adopted from Ibrahim, 2000)
Critical loads for hollow GFRP tubes governed by (a) local buckling failure and (b) material fracture failure.
Figure A-9: Variation of moment of inertia and neutral axis depth for a circular and an ovalized cross-section

Figure A-10: Variation of bending stress along length of circular and ovalized hollow GFRP tubes by fracture of the GFRP tube material
Figure A-11: Cross-section geometry of ovalized hollow steel tube
Figure A-12: Properties of steel stress-strain curve
APPENDIX B: NUMERICAL MODEL

B.1 PCFFT Numerical Model

External ConcreteFilled

!Filled Portion Variables
Real::Strain,M,C, Bot,L,LF,fpc,t,Xif
Real::MCFmax,PFmax,Mult,MA,MB, MD, MHcrit, Mbase
Real::S,Es,Fy,Is,Mn,My
REAL,DIMENSION(0:2500,1:2)::MCF

!Hollow Portion Variables
Real::D22, Mcr, Omega, SigmaCR
Real::Rout, Rin, XH, CurveXX, CurveX, SlopeXX, SlopeX !, DPFF

Real, Dimension(0:25001,0:1101,1:2)::CurvePFH, CurvePF, CurvePFF
Real, Dimension(0:25001,0:1101):::MF, MI, Dpf, Apf, Xbar
Real, Dimension(0:25001,0:101):::I, MH, CurveH, Zeta, EftH, Y, Ebolt, SigmaTot
Real, Dimension(0:25001)::PH, Zet, ZetaPFH
Real::Delta, X1, X2, LH, Xih, Xi, CHmax, Pmax, Xcrit, XXXXX, Etult, Ebult, Efrac, Ef y, A, B, CC,
D, Sigma1, Sigma2, OO
Real, Parameter::PI = 3.141592654

OPEN(1, File = 'MCFull- exp.CSV')
!OPEN(2, File = 'MCHollow- 1-3-40n.CSV')
!OPEN(3, File = 'CurvePF- 75-125-46n.CSV')
!OPEN(4, File = 'DeltaPF- 75-125-46n.CSV')
!OPEN(5, File = 'CurveCheck4.CSV')
!OPEN(6, File = 'Random.CSV')

!Hollow Tube Material Props
D22 = 0.0913*10**6
D11 = 0.1855*10**6
!EftH(0) = 17294.0
E2 = 17762.0 !MPa
E1 = E2
Ef y = 27500.0
!E1 = 13481.0
Ebult = 0.035
Etult = 0.016
If (Etult <= Ebult) Then
    Efrac = Etult
ElseIf (Etult > Ebult) Then
    Efrac = Ebult
EndIf

!Geometric Props
Rout = 220
t = 4.15
Rin = Rout - t
L = 2665.0 !mm - TOTAL LENGTH
LF = 0.0 !FILLED LENGTH
LH = L - LF
!LH = 2665.0

!Concrete Strengths
fpc = 40.0
fpch = 0.0

Io = 0.25*PI*((Rout**4)-(Rin**4))

Y = Rout
Omega = (LH/Rout)*((12.0*D22/((Rout**2)*E2*t))**0.25)
OO = 1 + ((PI**4)/(12.0*(Omega**4)))

!Critical Buckling Moment for Hollow Tube - FRP
Mcr =
(4.0*(2.0**0.5)/(3.0*(3.0**0.5)))*(((D22/(10**6))/(E2*1000.0*(t/1000.0)))**0.5)*E2*1000.0*PI* &
    (Rout/1000.0)*t/1000.0
!Mcr = 20.52

Mult = 0.79*t*((2*Rout)**2)*E2*Efrac

If (Mult > (Mcr*10**6)) Then
    MHerit = Mcr*10**6
ElseIf (Mult < Mcr*10**6) Then
    MHerit = Mult
EndIf
Mcrack = 0.0

!FILLED TUBE MOMENT-CURVATURE

! If (fpc <= 0.0) Then
! GoTo 84
! EndIf

J = 1
Strain = 0.00001
MCFmax = 0

MCF(0,1) = 0.0
MCF(0,2) = 0.0

!X = 110.0

Filled_MC:Do

Call FP(Test1,A,B,ConcreteFilled,Root,Value,M,C,Strain,Bot,fpc),X)

MCF(J,1) = C
MCF(J,2) = M

If (MCF(J,2) > MCFmax) Then
MCFmax = MCF(J,2)
EndIf

Write(1,10) J,Strain,Bot,MCF(J,2),MCF(J,1), Root
10 Format(I6.3, ',', f25.5, ',', f25.5, ',', f25.3, ',', f25.15, ',', f25.5)

!Print '(I10.3,f10.5,f10.5,f15.3,f15.9)', J,Strain,Bot,MCF(J,2),MCF(J,1)

If (Strain > Etult) Then
GoTo 84
EndIf

If (Bot > Ebult) Then
GoTo 84
EndIf

J = J + 1
LimitF = J
Strain = Strain + 0.00001

EndDo Filled_MC

!HOLLOW TUBE MOMENT-CURVATURE

84 Xi = LH/100.0

If (LH <= 0) Then
   GoTo 85
EndIf

GoTo 90

Do N = 1,100

   X = Xi*N
   Q = 1

   I(1,N) = Io
   EftH(1,N) = E1
   Y(1,N) = Rout

   ! StrainH = 0.0001
   ZetaO = 0.0

   MomentCurvature:Do

       PH(Q) = Q*500
       Sigma = -151167.0*(StrainH**2) + 13481.0*StrainH
       MH(Q,N) = PH(Q)*X
       MH(Q,N) = Sigma*I(Q,N)/Y(Q,N)
       CurveH(Q,N) = StrainH/Y(Q,N)
       EftH(Q,N) = Sigma/StrainH
       MD = MH(Q,N)

       ! If (StrainH >= 0.035) Then
       !       GoTo 85
       ! EndIf
!Write(6,60) Q, StrainH, Sigma, MH(Q,1), CurveH(Q,1), I(Q,1), EftH(Q,1)
!60 Format(I6.3,"",f10.6, ",", f25.12, ",", f25.3, ",", f25.15, ",", f25.15, ",", f25.15)
   If (MH(Q,N) >= 1.05*Mult) Then !3.2*Mcr*10**6) Then
   !3.5*Mcr*10**6) Then
       GoTo 87
   ElseIf (MH(Q,100) >=1.05*Mult) Then
       GoTo 85
   EndIf

! Find maximum ovalization, ZetaO

   Call
   FPZETA(MH(Q,N),EftH(Q,N),D22,Omega,Rout,ZetaO,t,CalcZetaO)

   Zet(Q) = ZetaO
   Zeta(Q,N) = Zet(Q)*sin(PI*X/LH)

   Ebot(Q,N) = MH(Q,N)*Y(Q,N)/(EftH(Q,N)*I(Q,N))
   CurveH(Q,N) = Ebot(Q,N)/Y(Q,N)

   Q = Q + 1
   LimitQ = Q
!StrainH = StrainH + 0.0001

   Y(Q,N) = 0.5*(2*Rout*(1.0-Zeta(Q-1,N)))
   I(Q,N) = Io*(1.0-1.5*Zeta(Q-1,N)*sin(PI*X/LH))
   !EftH(Q,N) = E1
   !EftH(Q,N) = ABS(-302334.0*Ebot(Q-1,N) + 13481.0)
   Omega =
   (LH/Rout)*((12.0*D22/((Rout**2)*EftH(Q,N)*t))**0.25)

!Print*, N, Q,MH(Q-1,N),I(Q,N), EftH(Q,N)

   !Eft(Q) = -157474.0*Ebot + 14365.0
   !Print '(f6.3,f6.3,f12.5,f10.13,f8.15,f8.5)',
   Etop,Ebot,M,Curve,ZetaO,I(ZetaO)

   EndDo MomentCurvature

87 GoTo 89
89 EndDo

85    Do V = 1, LimitQ
        Write(2,20) V, Ebot(V,100),MH(V,100)/1000000,CurveH(V,100),Zeta(V,50),MH(V,50)/1000000,CurveH(V,50)
        20 Format(I6.0, "", f25.12, "", f25.3, "", f25.15, "", f25.15, "", f25.15,
        ",", f25.15)
        EndDo

!GoTo 90
!HOLLOW PORTION CURVATURE

!85     V = 1
        II = 1
        Xih = LH/100.0
        Xif = LF/1000.0

        XXXX = 0.9

        !Hollow Portion Curvature

                N = 0
                K = 1
                MI(Ppf,100) = 0
        Ppf = 1
        PartiallyFilledCurve: Do

                If (LH <= 0) Then
                        GoTo 45
                EndIf

                Do N = 1,100
                        K = 1
                        XH = Xih*N
                        MI(Ppf,N) = MCF(Ppf,2)*(XH/L)
                        MA = MI(Ppf,N)
                        !MI = Ppf*1000.0*(XH/L)

        !Print*, MA/1000000.0, Mult/1000000.0, MCFmax/1000000.0
CurvePFH(Ppf,N,1) = XH

Do While (MI(Ppf,N) > MH(K,N))
    MB = MH(K,N)
    If (MI(Ppf,N) > MH(K,N)) Then
        K = K + 1
    EndIf
EndDo

CurvePFH(Ppf,N,2) = ((MI(Ppf,N)-MH(K-1,N))*(CurveH(K+1,N)-CurveH(K-1,N)))/(MH(K+1,N)-MH(K-1,N)) &
& + CurveH(K-1,N)

ZetaPFH(Ppf) = ((MI(Ppf,N)-MH(K-1,N))*(Zet(K+1)-Zet(K-1)))/(MH(K+1,N)-MH(K-1,N)) &
& + Zet(K-1)

A = 2.85*(PI*D22*E1*OO/((Rout**2)*t))**0.5
B = (1-ZetaPFH(Ppf)*sin(PI*XH/LH))*(ZetaPFH(Ppf))**0.5
CC = 1+(3.0*ZetaPFH(Ppf)/PI)
D = 1-1.5*ZetaPFH(Ppf)*sin(PI*XH/LH)

Sigma1 = (((PI**2)*E1*(Rout**2)/(4.0*LH**2)))*ZetaPFH(Ppf)*sin(PI*XH/LH)
Sigma2 = (A*B/(CC*D))*(XH/LH)
SigmaTot(Ppf,N) = Sigma1+Sigma2

SigmaCR = ((2.0*(D11*Efy*t)**0.5)/(Rout*t))*(1-3.0*ZetaPFH(Ppf)*sin(PI*XH/LH))

!Print*, A,B
!Print*, Sigma1,Sigma2,SigmaCR,XH/LH,ZetaPFH(Ppf)

If (SigmaTot(Ppf,N) >= SigmaCR) Then
    Mbase = 2.85*PI*Rout*((PI*E1*D22*t*OO)**0.5)*&
&+(1-
(3.0*ZetaPFH(Ppf)/PI))*(ZetaPFH(Ppf))**0.5
    Print","buckling fail",
    (Mbase/(LH*1000)),XH/LH,SigmaTot(Ppf,N),SigmaCR
    LimitJ = Ppf-1
    GoTo 88
EndIf
If (MI(Ppf,100) >= Mult) Then
    Print*, "Fracture", (MI(Ppf,100)/(LH*1000)),
Mult/1000000
    LimitJ = Ppf-1
    GoTo 88
EndIf

!Print*, Ppf,N,MA, Mcr

! If (MI(Ppf,100) >= (Mcr*10**6)/XXXX) Then
!    Print*, "buckling", MF(Ppf-1,100+1000)/(L*1000), Mcr,
Mult/1000000
!    LimitJ = Ppf-1
!    GoTo 88
! ElseIf (MI(Ppf,100) >= Mult) Then
!    Print*, "fracture",MI(Ppf,100)/(LH*1000), Mcr,
Mult/1000000
!    LimitJ = Ppf-1
!    GoTo 88
! EndIf

!DeltaH(Ppf,N) = 0.5*CurvePFH(Ppf,N,2)*LH*(2.0/3.0)*LH

!CurveH(J,N,2) = ((Curve(J,N)-Curve(J-1,N))/(M(J,N)-M(J-1,N)))*MF
!Delta(J,N) = 0.5*CurvePPF(J,N,2)*L*(2.0/3.0)*L

EndDo

45 Filled_Curve: Do W = 1,1000

II = 1

XF = LH + Real(Xif*W)
MF(Ppf,100+W) = MCF(Ppf,2)*(XF/L)
MFFF = MF(Ppf,100+W)

CurvePFF(Ppf,100+W,1) = XF

If (XF >= LH) Then
    Do While (MF(Ppf,100+W) > MCF(II,2))
        If (MF(Ppf,100+W) > MCF(II,2)) Then
            II = II + 1
        EndIf
EndIf
CurvePPF(Ppf,100+W,2) = ((MF(Ppf,100+W)-MCF(II-1,2))*(MCF(II+1,1)-MCF(II-1,1)) / (MCF(II+1,2)-MCF(II-1,2))) + MCF(II-1,1)

! CurveX = CurvePPF(Ppf,100+W,2)
! CurveXX = CurvePPF(Ppf-1,100+W,2)

! SlopePPF(Ppf,100+W,2) = SlopePPF(Ppf,100+W-1,2)+0.5*(CurveX+CurveXX)*Xif

! SlopeX = SlopePPF(Ppf,100+W,2)
! SlopeXX = SlopePPF(Ppf-1,100+W,2)

! DPFF(Ppf,100+W,2) = DPFF(Ppf,100+W-1,2) + 0.5*(SlopeX+SlopeXX)*Xif

If (XF >= LH) Then
   If (MF(Ppf,100+1000) > (MCFmax - 0.01*10**6)) Then
      LimitJ = Ppf-1
      Print*, "tension failure",
      MF(Ppf,100+1000)/(L*1000), Mult/1000000.0, MCFmax/1000000.0
      GoTo 88
   EndIf
EndIf

!If (Ppf >= 951) Then
!   Write(5,500), Ppf,W,Mf(Ppf,W),MCF(II+1,2),MCF(II,2),MCF(II-1,2),MCF(II+1,1),MCF(II,1),MCF(II-1,1)
!   !EndIf
!

!DeltaPF(Ppf,W) = (0.5*CurvePFH(Ppf,N,2)*LH*(2.0/3.0)*LH) + LF*CurvePPF(Ppf,1,2)*(LH+0.5*LF) + &
!   & 0.5*(CurvePPF(Ppf,W,2)-CurvePPF(Ppf,1,2))*LF*(LH+(2.0/3.0)*LF)
!
!Print*, DeltaPF(Ppf,W)
!Print
'(I6.3,I6.3,f15.5,f15.5,f10.3,f15.9)',N+LimitH,P,MCFmax,MI,CurvePF(N+LimitH,P,1),&
& CurvePF(N+LimitH,P,2)
EndDo Filled_Curve

! Do N = 1,100
!  XXXX = Int((Xcrit/LH)*N)
!
!If (XH <= Xcrit) Then
!  If (XH <= 5165.0) Then
!    If (MI(Ppf,N) >= Mcr*10**6) Then
!      MCC = MI(Ppf,XXXX)
!      If (MI(Ppf,XXXX) >= MHcrit) Then
!        Print*,"buckling failure",
MF(Ppf,N),MCF(Ppf,2),Ppf,N
!
EndIf
!
EndDo
!
Ppf = Ppf + 1
LimitPpf = Ppf

EndDo PartiallyFilledCurve

Z = 1
P = 1

!88 GoTo 90

!888 Do Z = 1,1100
88 Do Z = 1,1100
  Do P = 1,LimitJ
    CurvePF(P,Z,2) = CurvePFH(P,Z,2)+CurvePFF(P,Z,2)
    !DeltaPF(P,Z) = (0.5*CurvePF(P,Z,2)*LH*(2.0/3.0)*LH) +
    !                  & 0.5*(CurvePFF(P,Z,2)*LF*(LH+0.5*LF)+ &
    !                  & 0.5*(CurvePFF(P,Z,2)-
    CurvePFF(P,101,2)*LF*(LH+0.5*LF)+ &
    CurvePFF(P,Z,2))*LF*(LH+(2.0/3.0)*LF)
  EndDo
EndDo
Appendix B

!Determine Panel Areas and Centroids

\[ Z = 0 \]
\[ P = 0 \]
\[ Apf(0,0) = 0.0 \]
\[ Xbar(0,0) = 0.0 \]
\[ Dpf(0,0) = 0.0 \]

\[
\text{Do } P = 1, \text{LimitJ} \\
\quad \text{Do } Z = 1, 1100 \\
\hspace{1cm} \text{If } (Z \leq 100) \text{ Then} \\
\hspace{2cm} \text{Apf}(P,Z) = 0.5 \times (\text{CurvePF}(P,Z,2)+\text{CurvePF}(P,Z-1,2)) \times (\text{CurvePF}(P,Z,1)-\text{CurvePF}(P,Z-1,1)) \text{ !panel area} \\
\hspace{2cm} \text{Xbar}(P,Z) = 0.5 \times (\text{CurvePF}(P,Z,1)+\text{CurvePF}(P,Z-1,1)) \text{ !panel centroid} \\
\hspace{2cm} \text{Dpf}(P,Z) = \text{Dpf}(P,Z-1) + \text{Apf}(P,Z) \times \text{Xbar}(P,Z) \text{ !dflxn} \\
\hspace{1cm} \text{ElseIf } (Z < 1100) \text{ Then} \\
\hspace{2cm} \text{Apf}(P,Z) = 0.5 \times (\text{CurvePF}(P,Z,2)+\text{CurvePF}(P,Z-1,2)) \times (\text{CurvePF}(P,Z,1)-\text{CurvePF}(P,Z-1,1)) \text{ !panel area} \\
\hspace{2cm} \text{Xbar}(P,Z) = 0.5 \times (\text{CurvePF}(P,Z,1)+\text{CurvePF}(P,Z-1,1)) \text{ !panel centroid} \\
\hspace{2cm} \text{Dpf}(P,Z) = \text{Dpf}(P,Z-1) + \text{Apf}(P,Z) \times \text{Xbar}(P,Z) \text{ !dflxn} \\
\hspace{1cm} \text{EndIf} \\
\quad \text{EndDo} \\
\text{EndDo} \\
\]

\[
\text{Do } P = 1, \text{LimitJ} \\
\quad \text{Write}(4,400), \text{Dpf}(P,1099), \text{MF}(P,1099)/(L \times 1000) \\
\quad 400 \text{ Format}(f25.15","",f25.15) \\
\text{EndDo} \\
\]

\[
\text{Do } P = 1, \text{LimitJ} \\
\quad \text{DeltaPF}(P,1100) = (0.5 \times \text{CurvePF}(P,100,2) \times LH \times (2.0/3.0) \times LH) + \\
\quad \text{CurvePF}(P,101,2) \times LF \times (LH+0.5 \times LF)+ & \\
\quad \& 0.333 \times (\text{CurvePF}(P,1100,2)- \\
\quad \text{CurvePF}(P,101,2) \times LF \times (LH+(3.0/4.0) \times LF) \\
\quad \text{!EndDo} \\
\]

\[ Z = 1 \]
P = 1

Do Z = 1,1100
  ! Write(3,300),
  LimitPpf, MF(LimitPpf,Z), MI(LimitPpf,Z), MI(LimitPpf,90), CurvePFH(LimitPpf,Z,1),
  CurvePFH(LimitPpf,Z,2), &
  !       & CurvePFF(LimitPpf,Z,1),
  CurvePFF(LimitPpf,Z,2)
  Write(3,300), CurveP( LimitJ,Z,1), CurveP( LimitJ,Z,2) !,
  SlopePFF(LimitJ,Z,2) !, CurvePF(440,Z,1), CurvePF(440,Z,2), CurvePF(441,Z,1),
  CurvePF(441,Z,2)
  ! EndDo

! Do Q = 1,LimitJ
! Write(4,400),
Q, MI(Q,INT(XXXX*100)), MF(Q,1100)/(L*1000), DeltaPF(Q,1100), CurvePF(Q,1100,2)
, Mct !, DPFF(Q,1100,2) !DeltaH(Q,1000)
  ! 400 Format(I10.3, ",",f25.13, ",",f25.13, ",",f25.13, ",",f25.13, ",",f25.13,
  ",",f25.3, ",",f25.5)
! EndDo

90 End

SubRoutine FP(Test1,A,B,F,Xnew,FXN,ZZ,C,eee,Bot,fc)

Real::A,B,FA,FB,Xold,FXO,Xnew,FXN,Tol1,Tol2,Ntol,Unit,U,Test,Test2,Size,
NMax,Mom,C,ZZ,Bot,fc
Logical:::Test1
Real,External:::ConcreteFilled

A = 30
B = 110
Ntol = 10000
Tol1 = 1.E-8
Tol2 = 0
Xnew = 0
Xold = 0
FXN = 0
Value = 0
U = 0
E = eee

Unit = 1
1      Unit = 0.5*Unit
       U = 1.0+Unit

IF (U>1.0) GoTo 1

! Protect against unreasonable tolerance
Tol2 = Tol2+Unit

! Initialization
Call ConcreteFilled(Test1,A,Value,Mom,C,E,Bot,fc)
FA = Value
SFA = Sign(1.0,FA)
Call ConcreteFilled(Test1,B,Value,Mom,C,E,Bot,fc)
FB = Value
Top = A*FB-B*FA
Diff = FB-FA
Xold = Top/Diff

If (Xold < 0) Then
    Xold = -Xold
EndIf

!If (Xold < A) Then
!    Xold = A
!EndIf

Call ConcreteFilled(Test1,Xold,Value,Mom,C,E,Bot,fc)
FXO = Value
Test = SFA*FXO

IF (Test>0) Then
    A = Xold
    FA = FXO
Else
    B = Xold
    FB = FXO
EndIf

! Begin iteration
Do K = 1,Ntol

    Top = A*FB-B*FA
    Diff = FB-FA
    Xnew = Top/Diff

    If (Xnew < 0) Then
        Xnew = -Xnew
    EndIf

    ! If (Xnew < A) Then
    !     Xnew = A
    ! EndIf

    Xdiff = ABS(Xnew-Xold)
    Xold = Xnew
    RERR = Xdiff/ABS(Xnew)

    ! Check Relative error criterion
    If (RERR.LE.Tol2) Return
        Call ConcreteFilled(Test1,Xnew,Value,Mom,C,E,Bot,fc)
        FXN = Value
        ZZ = ABS(Mom)
        Size = Abs(Fxn)
    ! Check size of function value
    If (Size.LE.Tol1) Return
        SFxn = Sign(1.0,Fxn)
        Test2 = SFxn*FXO
        Test = SFA*FXN
    ! Update EndPoints
    If (Test>0) Then
        A = Xnew
        FA = FXN
        If (Test2>0) Then
            FB = FB/2
        EndIf
    Else
        B = Xnew
        FB = FXN
        If (Test2>0) Then
            FA = FA/2

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End If
End If

5  FXO = FXN
6  Continue
!Print*, A,B,Xnew
EndDo

End SubRoutine FP

Function ConcreteFilled(AA,FF,EE,BB,CA,GG,Bot,HH)

IMPLICIT NONE

REAL, INTENT(IN) :: FF,GG,HH
REAL, INTENT(OUT) :: EE,BB,CA,Bot
LOGICAL, INTENT(OUT) :: AA
    REAL, DIMENSION(0:300000) :: H,E,FC,FT,CC,CF,TC,TF,AF,AC,B,PT,PC,Y,AP,ESC,EST,MO,YT,YC,X
    REAL, DIMENSION(0:300000) :: Q,Q1,Q2
    REAL :: MCC,MCF,MTC,MTF,N,MOMENT,MPTF,MPCF,Mcon,Mfrp,OD,Strain,ConcreteFilled
    REAL :: FPC,ST,T,ECP,ALPHA1,ALPHA2,ETU,EBU
    REAL :: SOFT,EFC,EFT,EFCSOFTWARE,EFTSOFTWARE,ERROR,ECF,ETF,EFCU,EFTU,D,HI
    REAL :: EC,ESEC,R,FCR,ECR,ECT,C,EB,ET,AFRP,ACON,CCC,MAXCONCSTRESS
    REAL :: CFF,TCC,TF,TFF,ERR,PTF,PCF,CURVATURE,NN,EB1
    INTEGER :: I
    LOGICAL :: FAIL

!  MCC to Defl are defined in various thesis stuff (see Dr. Fam about it)
!  CURVold etc are for post-failure finding deflections and other post-failure stuff
!these are variables that remember if the
FAIL = .FALSE.

!  Geometry and Material Properties

OD = 220.0
ST = 4.15
T = ST
!FCP=40.0
FPC = HH

ECP=0.0035
ALPHA1=0.3
ALPHA2=1.0
N=1000
Etu=0.02
Ebu=0.035
!Soft = 0.00493
Efc = 17290.0
!Eft = 17762.0
!Efc = 17291.0
!Efc = 26687.5
!Eft = 17762.0
!Efc = Eft !coupon1
!Eft = Efc !coupon1
!Efecsoft=8418.0
!Eftsoft = 8190.0
!Soft = 0.00493

!GENERAL CONCRETE GEOMETRIC PROPERTIES

!Ktr=(53*(15.9/200)*2*3.2*(FCP/1000)**0.25)/2 !confining effect for Ld
D=OD-T !D at tube mid-thickness
hi=D/N !ie. 219mm / 500 = <0.5 it's the size of each layer
EC=5000.0*(FPC)**0.5 !damn thing for concrete model => concrete initial modulus?
ESEC=FPC/ECP !ditto => secant modulus of concrete?
R=EC/(EC-ESEC) !ditto => stiffness amplification?
FCR=0.6*(FPC)**0.5 !ditto => concrete cracking stress?
ECR=FCR/EC !ditto => concrete strain at cracking stress?

Moment = 0
Curvature = 0

AFRP = 0
ACON = 0
CCC = 0
CFF = 0
TCC = 0
TFF = 0
MCC = 0
MCF = 0
MTC = 0
MTF = 0
MaxConcStress = 0
ERR = 0
EE = 0
Bot = 0
!Eft = 0

Do I = 1,N

    CC(I) = 0
    CF(I) = 0
    TC(I) = 0
    TF(I) = 0
    YC(I) = 0
    YT(I) = 0
    E(I) = 0
    FC(I) = 0
    AF(I) = 0
    AC(I) = 0
    FC(I) = 0
    FT(I) = 0
    X(I) = 0
    H(I) = 0

EndDo

I = 0

ET = GG
!ET = 0.0001
C = FF

Eb = ((D/(C-0.5*T)) * Et - Et)
Bot = Eb

! If (Bot > EBU) Then
!     AA = .True.
! EndIF

!Eft = -302334.0*Bot + 13481.0
!Eft = -110342.0*Bot + 14249.0
!Eft = 17762.0
!Eft = -362634.0*Bot + 13629.0
!Efc = 17290.0
!Efc = Eft

!Eb = -((D-C+0.5*T)/(C-0.5*T))*ET
!Eb1 = -(((1/H(I))/H(I))*D*ET)
!C = (ET/(ET+EB1))*D+0.5*T

If (ABS(EB) > ABS(EBU)) Then
!AA = .True.
!EndIF

DO I=2,(N-1) !from 2 to 999
H(I)=(I-0.5)*hi !stores the test elevation of each layer (ie. midheight)
!E(I) = ((C-0.5*T-H(I))/(C-0.5*T))*ET !calculates the strain of each layer (E(I) is +ve comp, -ve tens.)
E(I) = (ET*(H(I)-D)+EB*H(I))/D
Q(I)=ACOS((0.5*D-H(I))/(0.5*D))
!Q1(I)=22/7/2-ATAN((0.5*D-H(I)+0.5*hi)/(0.5*D)**2-(0.5*D-H(I)+0.5*hi)**2)**0.5
!Q2(I)=22/7/2-ATAN((0.5*D-H(I)-0.5*hi)/(0.5*D)**2-(0.5*D-H(I)-0.5*hi)**2)**0.5
Q1(I) = ACOS((0.5*D-H(I)+0.5*hi)/(0.5*D))
Q2(I) = ACOS((0.5*D-H(I)-0.5*hi)/(0.5*D))
AF(I)=T*D*(Q2(I)-Q1(I)) !area of frp tube for that particular layer
B(I)=0.5*D*SIN(Q(I))
AC(I)=2*B(I)*hi-0.5*AF(I) !area of concrete for that particular layer
If (AC(I)<0) Then
AC(I) = 0
EndIf
AFRP=AF(I)+AFRP !summed total frp tube
ACON=AC(I)+ACON !summed total concrete core
ENDDO

I = 0

! Concrete and FRP in Compression

!If (FPC <= 0) Then
! GoTo 100
! EndIf
DO I = 2, (INT((C-0.5*T)/hi))

H(I)=(I-0.5)*hi  !stores the test elevation of each layer (ie. midheight)

!E(I)= ((C-0.5*T-H(I))/(C-0.5*T))*ET
E(I) = (ET*(H(I)-D)+EB*H(I))/D
X(I) = ABS(E(I))/ECP

!IF (E(I)>0) THEN

FC(I)=FPC*X(I)*R/(R-1+(X(I))**R) !stress in concrete layer from model, based on E(I):strain at each layer

!    ! IF (MaxConcStress < FC(I)) THEN !concrete elasto-plastic:
!        MaxConcStress=FC(I)
!        ! stress increases until peak
!    ELSE
!        FC(I)=MaxConcStress
!        ! then maintains this peak stress forever
!    ENDIF
!If you want no confinement, remove the preceding three lines and concrete will soften after peak

YC(I)=C-0.5*T-H(I)  !calcs moment arm to current layer

!If (AC(I)>0) Then
CC(I)=FC(I)*AC(I) !concrete force is layer stress * layer concrete area
!EndIf

CCC=CCC+CC(I) !CCC=total force
MCC=MCC+(CC(I)*YC(I)) !MCC=moment from concrete in compression

!END IF

ENDDO

100  I = 0

! FRP in Compression

DO I = 2, (INT((C-0.5*T)/hi))
\[ H(I) = (I - 0.5) \times h_i \]
\[ E(I) = ((C - 0.5 \times T - H(I)) / (C - 0.5 \times T)) \times E_T \]
\[ E(I) = (E_T \times (H(I) - D) + E_B \times H(I)) / D \]
\[ !CF(I) = 0 \]

\[ !IF (E(I) > 0) THEN \]
\[ !IF (ABS(E(I)) < ABS(E_TU)) THEN \]
\[ CF(I) = ABS(E(I)) \times E_{fc} \times AF(I) \]
\[ CF(I) = (-151167.0 \times ABS(E(I))^2 + 13481.0 \times ABS(E(I))) \times AF(I) \]
\[ !IF (E(I) < \text{Soft}) THEN \]
\[ CF(I) = E(I) \times E_{fc} \times AF(I) \]
\[ CF(I) = \text{layer force = layer strain} \times \text{tube comp mod} \times \text{area of tube in layer} \]
\[ !ELSE \]
\[ CF(I) = \text{Soft} \times E_{fc} \times AF(I) + (E(I) - \text{Soft}) \times E_{fcsoft} \times AF(I) \]
\[ !ENDIF \]
\[ !ENDIF \]
\[ !ENDIF \]
\[ YC(I) = C - 0.5 \times T - H(I) \]
\[ CFF = CFF + CF(I) \]
\[ MCF = MCF + (CF(I) \times YC(I)) \]
\[ ENDDO \]

\[ I = 0 \]

\[ !Concrete in Tension \]

\[ !IF (FPC <= 0) \text{Then} \]
\[ !GoTo 200 \]
\[ !EndIf \]

\[ DO \]
\[ I = (INT((C - 0.5 \times T) / h_i)), (N - 1) \]

\[ H(I) = (I - 0.5) \times h_i \]
\[ E(I) = ((H(I) - C + 0.5 \times T) / (D - C + 0.5 \times T)) \times E_B \]
\[ E(I) = (E_T \times (H(I) - D) + E_B \times H(I)) / D \]
\[ !TC(I) = 0 \]

\[ !IF (E(I) < 0) THEN \]
\[ IF (E(I) < \text{ECR}) THEN \]
FT(I)=EC*ABS(E(I)) !tensile stress in layer based on layer strain and initial concrete modulus (-ve!)
ELSEIF (ABS(E(I))> ECR) THEN !if layer strain is less than cracking strain go to 60
FT(I)=ALPHA1*ALPHA2*FCR/(1+(500*(ABS(E(I))-ECR))**0.5) !tension stiffening if strain>cracking strain (-Ve)
ENDIF

TC(I)=FT(I)*AC(I) !force in layer is stress*area of concrete in layer (-ve)
TCC=TCC+TC(I) !total force (adding a -ve number, so more -ve)
YT(I)=H(I)-C+0.5*T !moment arm
MTC=MTC+(TC(I)*YT(I)) !total moment (+ve)
!ENDIF
ENDDO

200 I = 0

! FRP in Tension

DO I = (INT((C-0.5*T)/hi)) , (N-1)
H(I) = (I-0.5)*hi
E(I) = ((H(I)-C+0.5*T)/(D-C+0.5*T))*EB
E(I) = (ET*(H(I)-D)+EB*H(I))/D
TF(I) = 0

!IF (E(I)<0) THEN
!IF(ABS(E(I))<ABS(Ebu)) THEN  !for strain in 1st part of bilinear stuff
!  TF(I)=Soft*Eft*AF(I)+(E(I)-Soft)*Eftsoft*AF(I)
! ELSE
!  TF(I)=ABS(E(I))*Eft*AF(I)  !tension force in layer=layer strain*area of tube in layer (-ve)
  TF(I)=(-151167.0*ABS(E(I))**2 + 13481.0*ABS(E(I)))*AF(I)
ENDIF
!ENDIF

13481.0*ABS(E(I)))*AF(I)
!
!ENDIF

!IF (ABS(E(I))> ABS(EbU)) THEN
!  AA = .TRUE.
!  EXIT
!ENDIF
!tens force in layer in 2nd bilinear
YT(I)=H(I)-C+0.5*T  !moment arm  
TFF=TFF+TF(I)  !total tube tension force (-ve)  
MTF=MTF+(TF(I)*YT(I))  !total moment

!ENDIF  
ENDDO

ERR= CCC+CFF-TCC-TFF  !sums forces  
EE = ERR

!Print*,CCC,CFF,TCC,TFF

!IF(ABS(ERR)< error) THEN  
!checks if forces in equilibrium, if not, move NA and start over  
!  GOTO 64

  MOMENT=MCC+MCF+MTC+MTF  !sums moment  
  CURVATURE=ABS(ET)/(C-T)  !calcs curvature based on current top concrete strain

    BB = Moment  
    CA = Curvature

!Print*, C, Moment, ET, EB !,ERR,CCC,CFF,TCC,TFF !  

!ENDIF

!220 Print*, C, ET, Err, Moment, Curvature

END Function ConcreteFilled
B.2 PCFST Numerical Model

External Concrete Filled

! Filled Portion Variables
Real::Strain,M,C,Bot,L,LF,fpc,t,Xif
Real::MCFmax,PFmax,Mult,MA,MB,MD,MHcrit
Real::Es,Fy,Is,Mn,Mp,Zx
REAL,DIMENSION(0:4000,1:2)::MCF
Real, Dimension(0:3000,0:51)::Fs,MH,CurveH,I,Ys,StrainH,Etan,Esec,a,b,Straincrit,
Sigma,StrainPFH,ZetaPFH,IPFH

! Hollow Portion Variables
Real::Mcr,Mbase,ZetaO
Real::Rout,Rin,XH,CurveXX,CurveX,SlopeXX,SlopeX,Kn,MomH,CH, CurveMid,Elog
Real,Dimension(0:3001,0:1051,1:2)::CurvePFH,CurvePF,CurvePFF
Real,Dimension(0:3001,0:1051)::Dpf,Apf,Xbar
Real,Dimension(0:3001,0:1051)::MF,MI,DeltaPF,H,DeltaH
Real,Dimension(0:5001,0:1051)::Zeta
Real,Dimension(0:5001)::Zet
Real::Delta,X1,X2,LI,CHmax,Max,Max,XXX,CC,CCC,CCCC,C101,
MHmax,CurveBase,MM,Mf,CHmid,MBB
Real,Parameter::PI = 3.141592654
Logical::Failure

! OPEN(1,File = 'MCFull-Steeln.CSV')
OPEN(2,File = 'MCHollow-Steel-0new.CSV')
! OPEN(3,File = 'CurvePF-Steel-0n.CSV')
! OPEN(4,File = 'DeltaPF-Steel-36.5n.CSV')
! OPEN(5,File = 'Stresses.CSV')
! OPEN(6,File = 'ellipse.CSV')

! Geometric Props
Rout = 57.15
! Rout = 60.0
L = 3.0
Rin = Rout - t
L = 1398.0  !mm - TOTAL LENGTH
L = 1398.0
LF = 0.0  !FILLED LENGTH
LH = L - LF

! Steel Tube Properties
Es = 200000.0  !Initial elastic modulus
Ey = 0.001345   !C1 (L)
Ey = 0.0013   !yield strain
Ey = 0.00155   !fy = 350
Ey = 0.0018  !fy = 400
\begin{verbatim}

!Ey = 0.0022  !fy = 480
!Ey = 0.00255 !fy = 550

fy = 300.0

Elog = 0.0359 !for fy = 300
!Elog = 0.0364 !for fy = 400
!Elog = 0.03715 !for fy = 550

!Fy = Es*Ey    !yield stress
S = PI*((Rout**4)-(Rin**4))/(4.0*Rout) !section modulus
Is = S*Rout   !moment of inertia
Zx = (4.0/3.0)*((Rout**3)-(Rin**3))  !plastic modulus
Y = Rout     !neutral axis depth

!Critical Moment of Hollow Tube
!Mn = (0.021*Es/(2.0*Rout/t)+Fy)*S    !Buckling
!Yielding
If ((2.0*Rout/t) <= 0.07*Es/Fy) Then
  Mcrit = Mp
ElseIf ((0.07*Es/Fy) < (2.0*Rout/t) <= (0.31*Es/Fy)) Then
  Mcrit = Mn
ElseIf ((2.0*Rout/t) > (0.31*Es/Fy)) Then
  Print*, "Slender"
  GoTo 90
EndIf

!Mn = 12.62*10**6
!Mn = 12.0*10**6
Mcrit = 12.76*10**6
!My = Mcrit
!Concrete Strengths
fpc = 40.0
fpch = 0.0

!Critical Moments
MHcrit = 2.62*10**6
MFcrit = 4.43*10**6
Mp = Zx*Fy 13.20*10**6
!My = 16.15*10**6
!GoTo 84

!FILLED TUBE MOMENT-CURVATURE

\end{verbatim}
J = 1
Strain = 0.0001
MCFmax = 0

MCF(0,1) = 0.0
MCF(0,2) = 0.0

!X = 110.0

Filled_MC:Do

Call FP(Test1,A,B,ConcreteFilled,Root,Value,M,C,Strain,Bot,fpc),X)

MCF(J,1) = C
MCF(J,2) = M

If (MCF(J,2) > MCFmax) Then
    MCFmax = MCF(J,2)
EndIf

If (MCF(J,2) < MCFmax) Then
    GoTo 84
EndIf

Write(1,10) J,-Strain,Bot,MCF(J,2)/1000000,MCF(J,1), Root,MCF(J,2)/(L*1000)

!Print '(I10.3,f10.5,f10.5,f15.3,f15.9)', J,Strain,Bot,MCF(J,2),MCF(J,1)

If (Strain > 0.213) Then
    GoTo 84
EndIf

If (Bot > 0.213) Then
    GoTo 84
EndIf

J = J + 1
LimitF = J
Strain = Strain + 0.0001
EndDo Filled_MC

!HOLLOW TUBE MOMENT-CURVATURE

84  \( Xi = LH/50.0 \)

\( U = 0.3 \)
\( Kn = \frac{1}{((\text{Rout}**2)*((1.0-U**2)**0.5))} \)
\( \text{MomH} = 0.0 \)
\( MHmax = 0.0 \)
\( MHmid = 0.0 \)
\( CurveMid = 0.0 \)

!Print*, MCFmax, MCFMax/(L*1000)
!GoTo 90

!GoTo 90

If (LH <= 0) Then
    GoTo 86
EndIf

Do N = 1,50

!N = 1
!X = Xi*N
!Q = 1

!I(1,N) = Is
!Ys(1,N) = Rout
!StrainH(1,N) = 0.0001
!Zeta(Q) = 0.0

!If (N > 99) Then
!   GoTo 19
!EndIf

MomentCurvature:Do
    ZetaO = Zeta(Q,N)
Call HollowSteel(Test1,CH,MomH,StrainH(Q,N),Bot,ZetaO,Q)

CurveH(Q,N) = CH
MH(Q,N) = 2.0*MomH

If (MHmax < MH(Q,50)) Then
  MHmax = MH(Q,50)
Else
  GoTo 87
EndIf

If (MHmid < MH(Q,25)) Then
  MHmid = MH(Q,25)
  CurveMid = CurveH(Q,25)
EndIf

!Write(2,20) Q, CurveH(Q,N),MH(Q,N)/1000000.0,StrainH(Q,N),Bot,ZetaO
!20 Format(I6.0, ',', f25.12, ',', f25.12, ',', f25.15, ',', f25.15, ',', f25.15)

! If (MH(Q,N) >= 11.45*10**6) Then
!   Print*, CH, 2.0*MomH, Bot
!   GoTo 87
!   ElseIf (MH(Q,100) >= 11.45*10**6) Then
!     GoTo 85
!   EndIf

Q = Q + 1
LimitQ = Q

!Zet(Q) = (CH/Kn)**2
Zeta(Q,N) = ((CH/Kn)**2)*sin(PI*X/LH)

StrainH(Q,N) = StrainH(Q-1,N) + 0.0001

If (StrainH(Q,N) >= 0.213) Then
  GoTo 87
ElseIf (StrainH(Q,50) >= 0.213) Then
  GoTo 85
EndIf
Appendix B

EndDo MomentCurvature

87 GoTo 89

89 EndDo

85  Do V = 1, LimitQ
     Write(2,20) V, CurveH(V,50),MH(V,50)/1000000.0,-
StrainH(V,50),Bot,Zeta(V,25),CurveH(V,25),MH(V,25)/1000000.0, StrainH(V,25)
     20 Format(I6.0, ",", f25.12, ",", f25.12, ",", f25.15, ",", f25.15, ",", f25.15,
",", f25.15, ",", f25.15, ",", f25.15)
     EndDo

!  Do V = 1, LimitQ
!     Write(5,50)
!     V,StrainH(V,50),Esec(V,50),Etan(V,50),a(V,50),b(V,50),Straincrit(V,50)
!     50 Format(I6.0, ",", f25.12, ",", f25.12, ",", f25.15, ",", f25.15, ",", f25.15,
",", f25.15, ",", f25.15, ",", f25.15)
!     EndDo

GoTo 90

!HOLLOW PORTION CURVATURE

86  V = 1
    II = 1
     Xih = LH/50.0
     Xif = LF/1000.0

     If (LH > 0) Then
        Xcrit = LH - 2*Rout
        XXXX = Xcrit/LH
     ElseIf (LH <= 0) Then
        XXXX = 1
     EndIf

!Hollow Portion Curvature

     N = 0
     K = 1

     Ppf = 1
Appendix B

! Mbase = 250000.0
! CurveBase = 0.000001

PartiallyFilledCurve: Do

If (LH <= 0) Then
  !Mbase = 1000.0
  GoTo 45
EndIf

Do N = 1, 50

18   K = 1

   XH = Xih*N

   CurvePFH(Ppf,N,2) = CurveBase*(XH/L)
   !MI(Ppf,N) = MHmax*(XH/L)
   !MI(Ppf,N) = 1.5*MCF(Ppf,2)*(XH/L)
   MA = CurvePFH(Ppf,N,2) !
   !MA = MI(Ppf,N)

   CurvePFH(Ppf,N,1) = XH

   !   Do While (MI(Ppf,N) > MH(K,N))
   !     MBB = MH(K-1,N)
   !     MB = MH(K,N)
   !     If (MI(Ppf,N) > MH(K,N)) Then
   !       K = K + 1
   !     EndIf
   !   EndDo

   !Print*, K, MI, MH(K,N)
   !   CurvePFH(Ppf,N,2) = ((MI(Ppf,N)-MH(K-1,N))*(CurveH(K+1,N)-
   CurveH(K-1,N)))/(MH(K+1,N)-MH(K-1,N)) &
   !   + CurveH(K-1,N)
\[
\text{MI}(Ppf,N) = ((\text{CurvePFH}(Ppf,N,2)-\text{CurveH}(K-1,N))*(\text{MH}(K+1,N) - \text{MH}(K-1,N))/((\text{CurveH}(K+1,N)-\text{CurveH}(K-1,N))) & \\
& + \text{MH}(K-1,N))
\]

\[
\text{StrainPFH}(Ppf,N) = ((\text{MI}(Ppf,N)-\text{MH}(K-1,N))*(\text{StrainH}(K+1,N)-\text{StrainH}(K-1,N)))/(\text{MH}(K+1,N)-\text{MH}(K-1,N)) & \\
& + \text{StrainH}(K-1,N)
\]

If (StrainPFH(Ppf,N) < Ey) Then
\[
\text{Es}(Ppf,N) = \text{Es} \\
\text{Etan}(Ppf,N) = \text{Es}
\]
ElseIf (StrainPFH(Ppf,N) < Elog) Then
\[
\text{Esec}(Ppf,N) = (28.831*\log(\text{StrainPFH}(Ppf,N))+459.44)/\text{StrainPFH}(Ppf,N) !fy=300 \\
\text{Etan}(Ppf,N) = 28.831/\text{StrainPFH}(Ppf,N) !fy=300 \\
\text{Esec}(Ppf,N) = (30.315*\log(\text{StrainPFH}(Ppf,N)) + 566.15)/\text{StrainPFH}(Ppf,N) !fy=400 \\
\text{Etan}(Ppf,N) = 30.315/\text{StrainPFH}(Ppf,N) !fy=400 \\
\text{Esec}(Ppf,N) = (32.747*\log(\text{StrainPFH}(Ppf,N)) + 724.24)/\text{StrainPFH}(Ppf,N) !fy=550 \\
\text{Etan}(Ppf,N) = 32.747/\text{StrainPFH}(Ppf,N) !fy=550
\]
EndIf

\[
\text{Straincrit}(Ppf,N) = (4.0*t/(3.0*2.0*Rout))*(\text{Etan}(Ppf,N)/\text{Esec}(Ppf,N))**0.5
\]

Print*, Ppf, XH, MI(Ppf,N), StrainPFH(Ppf,N), Straincrit(Ppf,N)

\[
\text{ZetaPFH}(Ppf,N) = (\text{CurvePFH}(Ppf,N,2)/\text{Kn})*\sin(\pi*XH/LH) \\
\text{a}(Ppf,N) = \text{Rout}*(1.0+\text{ZetaPFH}(Ppf,N)) \\
\text{b}(Ppf,N) = \text{Rout}*(1.0-\text{ZetaPFH}(Ppf,N)) \\
\text{IPFH}(Ppf,N) = 0.25*\pi*((\text{a}(Ppf,N)*\text{b}(Ppf,N)**3.0)-((\text{a}(Ppf,N)-t)*t*(\text{b}(Ppf,N)-t)**3.0)) \\
\text{Sigma}(Ppf,N) = \text{MI}(Ppf,N)*\text{b}(Ppf,N)/\text{IPFH}(Ppf,N)
\]

If ((StrainPFH(Ppf,N)) >= Straincrit(Ppf,N)) Then
\[
\text{LimitJ} = \text{Ppf} \\
\text{Failure} = .\text{True.}
\]
Print*, "critical strain", XH, Straincrit(Ppf,N), StrainPFH(Ppf,N), MI(Ppf,N)
G o T o  8 8
EndIf

MM = MI(Ppf,N)
!MM = CurvePFH(Ppf,N,2)

!If (N > 49) Then
!If (MI(Ppf,50) >= Mp) Then
  If (MI(Ppf,50) >= MHmax-0.5*10**6) Then
    Print*, "plastic failure", Ppf
    LimitJ = Ppf-1
    GoTo 88
!ElseIf (CurvePFH(Ppf,25,2) >= CurveMid) Then
!   ElseIf (MI(Ppf,50) >= MHmid) Then
!     If (MI(Ppf,50) >= MHmax-0.01*10**6) Then
!       Print*, "buckling"
!     LimitJ = Ppf-1
!     GoTo 88
!ElseIf (MI(Ppf,100) >= Mult) Then
!ElseIf (MI(Ppf,100) >= My) Then
!   Print*, "fracture"
!   LimitJ = Ppf-1
!   GoTo 88
EndIf
!EndIf

EndDo

Mface = MI(Ppf,50)
Mbase = (L/LH)*Mface

If (LF <= 0.0) Then
  GoTo 888
EndIf

45 Filled_Curve: Do W = 1,1000
II = 1

XF = LH + Real(Xif*W)
MF(Ppf,50+W) = Mbase*(XF/L)

!MF(Ppf,50+W) = MCF(Ppf,2)*(XF/L)
MFFF = MF(Ppf,50+W)

CurvePFF(Ppf,50+W,1) = XF

If (XF > LH) Then
   Do While (MF(Ppf,50+W) > MCF(II,2))
      If (MF(Ppf,50+W) > MCF(II,2)) Then
         II = II + 1
      EndIf
      If (II >= 3900) Then
         If (MF(Ppf,50+W) > MCF(II-1,2)) Then
            print*, "timeout", MF(Ppf-1,1050), Mface
            GoTo 88
         EndIf
      EndIf
   EndDo
EndIf

CurvePFF(Ppf,50+W,2) = ((MF(Ppf,50+W)-MCF(II-1,2))*(MCF(II+1,1)-MCF(II-1,1)) / (MCF(II+1,2)-MCF(II-1,2))) &
                         & + MCF(II-1,1)

If (XF >= LH) Then
   If (MF(Ppf,50+1000) > (MCFmax - 0.01*10**6)) Then
      !If (MF(Ppf,50+1000) > MFcrit) Then
         LimitJ  = Ppf
         Print*, "tension failure"
         GoTo 88
      EndIf
   EndIf
EndIf

!If (Ppf >= 951) Then
   !    Write(5,500), Ppf,W,Mf(Ppf,W),MCF(II+1,2),MCF(II,2),MCF(II-1,2),MCF(II+1,1),MCF(II,1),MCF(II-1,1)
   !EndIf
\[ \Delta \text{PF}(P_{pf},W) = (0.5 \times \text{CurvePFH}(P_{pf},N,2) \times LH \times (2.0/3.0) \times LH) + \]
\[ LF \times \text{CurvePFF}(P_{pf},1,2) \times (LH + 0.5 \times LF) + \]
\[ 0.5 \times (\text{CurvePFF}(P_{pf},W,2) - \text{CurvePFF}(P_{pf},1,2)) \times LF \times (LH + (2.0/3.0) \times LF) \]

\text{Print*}, \Delta \text{PF}(P_{pf},W)

\text{Print}
'I6.3,I6.3,f15.5,f15.5,f10.3,f15.9)',N+\text{LimitH},P,MCF\text{max},MI,\text{CurvePF}(N+\text{LimitH},P,1),&
\& \text{CurvePF}(N+\text{LimitH},P,2)

\text{EndDo Filled_Curve}

! Do N = 1,100
! XXXX = Int((Xcrit/LH)*N)
!
! If (XH <= Xcrit) Then
! If (XH <= 5165.0) Then
! If (MI(P_{pf},N) >= Mcr*10**6) Then
! MCC = MI(P_{pf},XXXX)
! If (MI(P_{pf},XXXX) >= MHcrit) Then
! Print*,"buckling failure",
MF(P_{pf},N),MCF(P_{pf},2),P_{pf},N
! GoTo 88
! EndIf
! EndIf
! EndDo

888 P_{pf} = P_{pf} + 1
LimitP_{pf} = P_{pf}

! If (Failure >= .TRUE.) Then
! GoTo 88
! EndIf
!
! If (LH <= 0) Then
! Mbase = Mbase + 250000.0
! EndIf
CurveBase = CurveBase + 0.000001

\text{EndDo PartiallyFilledCurve}
!88 !Do JJ = 1,100
! Write(6,60) JJ, MI(LimitJ,JJ), Xih*Real(JJ) !CurvePFH(LimitJ,JJ,1)
! 60 Format (I6.0, ",",f12.3,"",f15.3)
!EndDo

88 Z = 1
P = 1
print*, "88"

!Write(6,60) Ppf,MI(Ppf,50)

!Distribute Curvature Along Beam Length

CurvePF(0,0,1) = 0.0
CurvePF(0,0,2) = 0.0

Do Z = 1,11050
  Do P = 1,LimitJ
    CurvePF(P,Z,2) = CurvePFH(P,Z,2)+CurvePFF(P,Z,2)
  EndDo
EndDo

!Determine Panel Areas and Centroids

Z = 0
P = 0
Apf(0,0) = 0.0
Xbar(0,0) = 0.0
Dpf(0,0) = 0.0
!PFdelta = 0.0

Do P = 1,LimitJ
  Do Z = 1,11050
    If (Z <= 50) Then
Appendix B

Apf(P,Z) = (CurvePF(P,Z,2)+CurvePF(P,Z-1,2))*(CurvePF(P,Z,1)-CurvePF(P,Z-1,1)) !panel area

Apf(P,Z) = 0.5*(CurvePF(P,Z,2)+CurvePF(P,Z-1,2))*(CurvePF(P,Z,1)-CurvePF(P,Z-1,1)) !panel area

Xbar(P,Z) = 0.5*(CurvePF(P,Z,1)+CurvePF(P,Z-1,1)) !panel centroid


ElseIf (Z < 1050) Then

Apf(P,Z) = (CurvePF(P,Z,2)+CurvePF(P,Z-1,2))*(CurvePF(P,Z,1)-CurvePF(P,Z-1,1)) !panel area

Xbar(P,Z) = 0.5*(CurvePF(P,Z,1)+CurvePF(P,Z-1,1)) !panel centroid


EndIf

EndDo

EndDo

Do P = 1,LimitJ
Write(4,400), Dpf(P,1049), MF(P,1049)/(L*1000)
400 Format(f25.15","",f25.15)
EndDo

!Do Z = 1,1100
!Write(6,600), Z, Apf(LimitJ,Z),Xbar(LimitJ,Z),Dpf(LimitJ,Z)
!600 Format(I6.0","",f25.15","",f25.15,"",f25.15)
!EndDo

!Do P = 1,LimitJ
!
DeltaPF(P,1100) = (0.5*CurvePF(P,100,2)*LH*(2.0/3.0)*LH) +
CurvePF(P,101,2)*LF*(LH+0.5*LF)+ &
    & 0.333*(CurvePF(P,1100,2)-
CurvePF(P,101,2))*LF*(LH+(3.0/4.0)*LF)
!
If (P <= 1007) Then
!
DeltaPF(P,1100) = 0.67*CurvePF(P,1100,2)*LF**2
!
ElseIf (P > 1007) Then
!
DeltaPF(P,1100) = 0.67*CurvePF(P,Jy,2)*(Xy)**2 + 0.5*(LF-
Xy)*CurvePF(P,Jy,2)*(Xy+LF)+ &
    & 0.67*(CurvePF(P,1100,2)-CurvePF(P,Jy,2))*(LF-Xy)* (0.25*Xy+0.75*LF)
!
EndIf

!EndDo
Z = 1
P = 1

Do Z = 1, 1050
  Write(3,300), LimitPpf, MF(LimitPpf,Z), MI(LimitPpf,Z), MI(LimitPpf,90), CurvePFH(LimitPpf,Z,1), CurvePFH(LimitPpf,Z,2), & CurvePFF(LimitPpf,Z,1), CurvePFF(LimitPpf,Z,2)
  Write(3,300), CurvePF(LimitJ,Z,1), CurvePF(LimitJ,Z,2), MI(LimitJ,Z), MF(LimitJ,Z), SlopePFF(LimitJ,Z,2), CurvePF(440,Z,1), CurvePF(440,Z,2), CurvePF(441,Z,1), CurvePF(441,Z,2)
EndDo

! Do Q = 1, LimitJ
!  Write(4, 400), Q, MI(Q, INT(XXXX*100)), MF(Q,1100)/(L*1000), DeltaPF(Q,1100), CurvePF(Q,1100,2) !, Mcrit !, DPFF(Q,1100,2) !DeltaH(Q,1000)
!  EndDo

!90
90 End
SubRoutine FP(Test1,A,B,X,new,FXN,ZZ,C,eee,Bot,fc)

Real::A,B,FA,FB,Xold,FXO,Xnew,FXN,Tol1,Tol2,Ntol,Unit,U,Test,Test2,Size, NMax,Mom,C,ZZ,Bot,fc
Logical::Test1
Real,External::ConcreteFilled

A = 30
B = 110
Ntol = 10000
Tol1 = 1.E-8
Tol2 = 0
Xnew = 0
Xold = 0
FXN = 0
Value = 0
U = 0

E = eee

  Unit = 1
  1 Unit = 0.5*Unit
  U = 1.0+Unit

IF (U>1.0) GoTo 1

! Protect against unreasonable tolerance
  Tol2 = Tol2+Unit

! Initialization
! Call ConcreteFilled(Test1,A,Value,Mom,C,E,Bot,fc)
FA = Value
SFA = Sign(1.0,FA)
Call ConcreteFilled(Test1,B,Value,Mom,C,E,Bot,fc)
FB = Value
Top = A*FB-B*FA
Diff = FB-FA
Xold = Top/Diff

If (Xold < 0) Then
  Xold = -Xold
EndIf

!If (Xold < A) Then
See Appendix B
SFxn = Sign(1.0,Fxn)  
Test2 = SFxn*FXO  
Test = SFA*FXN

! Update EndPoints  
If (Test>0) Then  
  A = Xnew  
  FA = FXN  
  If (Test2>0) Then  
    FB = FB/2  
  EndIf  
Else  
  B = Xnew  
  FB = FXN  
  If (Test2>0) Then  
    FA = FA/2  
  EndIf  
EndIF

5   FXO = FXN  
6   Continue  
!Print*, A,B,Xnew  
EndDo

End SubRoutine FP

Function ConcreteFilled(AA,FF,EE,BB,CA,GG,Bot,HH)

IMPLICIT NONE

REAL, INTENT(IN) :: FF,GG,HH
REAL, INTENT(OUT) :: EE,BB,CA,Bot
LOGICAL, INTENT(OUT) :: AA

REAL, DIMENSION(0:300000) :: H,E,FC,FT,CC,CF,TC,TF,AF,AC,B,PT,PC,Y,AP,ESC,EST,MO,YT,YC,X
REAL, DIMENSION(0:300000) :: Q,Q1,Q2
REAL :: MCC,MCF,MTC,MTF,N,MOMENT,MPTF,MPCF,Mcon,Mfrp,OD,Strain,ConcreteFilled
REAL :: FPC,ST,T,ECP,ALPHA1,ALPHA2,ETU,EBU
REAL :: SOFT,EFC,EFT,EFCSOFTWARE,EFTSOFTWARE,ERROR,EFC,ETF,EFCU,EFTU,D,HI
REAL ::
EC,ESEC,R,FCR,ECR,ECT,C,EB,ET,AFRP,ACON,CCC,MAXCONCSTRESS,MaxSteelStress
REAL :: CFF,TCC,TFF,ERR,PTF,PCF,CURVATURE,NN,EB1, CurveYield,Ey
INTEGER :: I
LOGICAL :: FAIL
! MCC to Defl are defined in various thesis stuff (see Dr. Fam about it)
! CURVold etc are for post-failure finding deflections and other post-failure stuff
!these are variables that remember if the
FAIL = .FALSE.

! Geometry and Material Properties

OD = 114.3
ST = 3.0
T = ST
!FCP=40.0

FPC = HH

ECP=0.0035
ALPHA1=0.25
ALPHA2=1.0
N=1000
Etu=0.215
Ebu=0.215

Ey = 0.0013 !fy = 300
!Ey = 0.00155 !fy = 350
!Ey = 0.0018 !fy = 400
!Ey = 0.0022 !fy = 480
!Ey = 0.00255 !fy = 550

!Soft = 0.00493
!Efc = 17294.0
!Eft = 17762.0
!Efc = 17291.0
!Efc = 26483.0
!Eft = 26483.0
!Efc = 20600.0 !coupon1
!Eft = Efc !coupon1
!Efcsoft=8418.0
!Eftsoft = 8190.0
!Soft = 0.00493
GENERAL CONCRETE GEOMETRIC PROPERTIES

\[ \text{Ktr} = \left(53\times(15.9/200)\times2\times3.2\times(FCP/1000)^{0.25}\right)/2 \]

Confining effect for Ld

\[ D = OD - T \]

D at tube mid-thickness

\[ hi = D/N \]

I.e. 219mm / 500 = <0.5 it's the size of each layer

\[ EC = 5000.0\times(FPC)^{0.5} \]

Damn thing for concrete model => concrete initial modulus?

\[ ESEC = FPC/EC \]

Ditto => secant modulus of concrete?

\[ R = EC/(EC-ESEC) \]

Ditto => stiffness amplification?

\[ FCR = 0.6\times(FPC)^{0.5} \]

Ditto => concrete cracking stress?

\[ ECR = FCR/EC \]

Ditto => concrete strain at cracking stress?

Moment = 0
Curvature = 0

AFRP = 0
ACON = 0
CCC = 0
CFF = 0
TCC = 0
TF = 0
MCC = 0
MCF = 0
MTC = 0
MTF = 0
MaxConcStress = 0
MaxSteelStress = 0
ERR = 0
EE = 0
Bot = 0
!Eft = 0

Do I = 1, N

CC(I) = 0
CF(I) = 0
TC(I) = 0
TF(I) = 0
YC(I) = 0
YT(I) = 0
E(I) = 0
FC(I) = 0
AF(I) = 0
AC(I) = 0
FC(I) = 0
FT(I) = 0
X(I) = 0
H(I) = 0
EndDo

I = 0
ET = GG
!ET = 0.0001
C = FF

Eb = ((D/(C-0.5*T)) * Et - Et)
Bot = Eb

! If (Bot > EBU) Then
!  AA = .True.
! EndIF

!Eb = -((D-C+0.5*T)/(C-0.5*T))*ET
!Eb1 = -(((1/H(I))/H(I))*D*ET)
!C = (ET/(ET+EB1))*D+0.5*T

!If (ABS(EB) > ABS(EBU)) Then
!  AA = .True.
!EndIF

DO I=2,(N-1)  !from 2 to 999
H(I)=(I-0.5)*hi  !stores the test elevation of each layer (ie. midheight)
!E(I)= ((C-0.5*T-H(I))/(C-0.5*T))*ET  !calculates the strain of each layer (E(I) is +ve comp, -ve tens.)
E(I) = (ET*(H(I)-D)+EB*H(I))/D
Q(I)=ACOS((0.5*D-H(I))/(0.5*D))
!Q1(I)=22/7/2-ATAN((0.5*D-H(I)+0.5*hi)/(0.5*D)**2-(0.5*D-H(I)+0.5*hi)**2)*0.5)
!Q2(I)=22/7/2-ATAN((0.5*D-H(I)-0.5*hi)/(0.5*D)**2-(0.5*D-H(I)-0.5*hi)**2)*0.5)
Q1(I) = ACOS((0.5*D-H(I)+0.5*hi)/(0.5*D))
Q2(I) = ACOS((0.5*D-H(I)-0.5*hi)/(0.5*D))
AF(I)=T*D*(Q2(I)-Q1(I)) !area of frp tube for that particular layer
B(I)=0.5*D*SIN(Q(I))
AC(I)=2*B(I)*hi-0.5*AF(I) !area of concrete for that particular layer
If (AC(I)<0) Then
   AC(I) = 0
EndIf
AFRP=AF(I)+AFRP !summed total frp tube
ACON=AC(I)+ACON !summed total concrete core
ENDDO

I = 0

! Concrete and FRP in Compression

!If (FPC <= 0) Then
! GoTo 100
!EndIf

DO I = 2 , (INT((C-0.5*T)/hi))

H(I)=(I-0.5)*hi  !stores the test elevation of each layer (ie. midheight)
!E(I)= ((C-0.5*T-H(I))/(C-0.5*T))*ET
E(I) = (ET*(H(I)-D)+EB*H(I))/D
X(I) = ABS(E(I))/ECP

!IF (E(I)>0) THEN
FC(I)=FPC*X(I)*R/(R-1+(X(I)**R)) !stress in concrete layer from model, based on E(I):strain at each layer

! IF (MaxConcStress < FC(I)) THEN !concrete elasto-plastic:
! MaxConcStress=FC(I)
! stress increases until peak
! ELSE
! FC(I)=MaxConcStress
! then maintains this peak stress forever
! ENDF

!If you want no confinement, remove the preceding three lines and concrete will soften after peak
YC(I)=C-0.5*T-H(I)  !calcs moment arm to current layer
!If (AC(I)>0) Then
    CC(I)=FC(I)*AC(I) !concrete force is layer stress * layer concrete area
!EndIf

CCC=CCC+CC(I) !CCC=total force
MCC=MCC+(CC(I)*YC(I)) !MCC=moment from concrete in compression

!END IF

ENDDO

100 I = 0

! FRP in Compression

DO I = 2 , (INT((C-0.5*T)/hi))

    H(I)=(I-0.5)*hi
    !E(I)= ((C-0.5*T-H(I))/(C-0.5*T))*ET
    E(I) = (ET*(H(I)-D)+EB*H(I))/D

    !CF(I) = 0

    !IF (E(I)>0) THEN
        !IF (ABS(E(I))< ABS(ETU)) THEN

        ! FRP STRESS-STRAIN PROPS
        ! CF(I) = ABS(E(I))*Efc*AF(I)

        ! STEEL STRESS-STRAIN CURVES
        ! fy = 300 MPa
If (E(I) <= Ey) Then
    CF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (E(I) <= 0.0359) Then
    CF(I) = (28.831*log(ABS(E(I)))+459.44)*AF(I)
ElseIf (E(I) < 0.1423) Then
    CF(I) = (28.381*Log(0.0359) + 459.44)*AF(I)  ! +
    28.23*ABS(E(I)))*AF(I)
ElseIf (E(I) <= 0.214345) Then
    CF(I) = (-689954*(ABS(E(I))**3)+331248*(ABS(E(I))**2)-
    53036*ABS(E(I))+3196.6)*AF(I)
EndIf

!fy = 350 MPa

! If (E(I) <= Ey) Then
!    CF(I) = 200000.0*ABS(E(I))*AF(I)
! ElseIf (E(I) <= 0.03615) Then
!    CF(I) = (29.439*log(ABS(E(I)))+513.17)*AF(I)
! ElseIf (E(I) < 0.1425) Then
!    CF(I) = (29.439*Log(0.03615) + 513.17)*AF(I)  ! +
!    28.23*ABS(E(I)))*AF(I)
! ElseIF (E(I) <= 0.214345) Then
!    CF(I) = (-689954*(ABS(E(I))**3)+331765*(ABS(E(I))**2)-
!    53202*ABS(E(I))+3259.9)*AF(I)
! EndIf

!fy = 400 MPa

! If (E(I) <= Ey) Then
!    CF(I) = 200000.0*ABS(E(I))*AF(I)
! ElseIf (E(I) <= 0.0364) Then
!    CF(I) = (30.315*log(ABS(E(I)))+566.15)*AF(I)
! ElseIf (E(I) < 0.1427) Then
!    CF(I) = (30.315*Log(0.0364) + 566.15)*AF(I)  ! +
!    28.23*ABS(E(I)))*AF(I)
! ElseIF (E(I) <= 0.214345) Then
!    CF(I) = (-689954*(ABS(E(I))**3)+332283*(ABS(E(I))**2)-
!    53368*ABS(E(I))+3323.2)*AF(I)
! EndIf

!fy = 480 MPa

! If (E(I) <= Ey) Then
!    CF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (E(I) <= 0.0368) Then
!   CF(I) = (31.642*log(ABS(E(I)))+650.6)*AF(I)
ElseIf (E(I) < 0.1431) Then
!   CF(I) = (31.642*log(0.0368)+650.6)*AF(I) + 28.23*ABS(E(I))*AF(I)
ElseIf (E(I) <= 0.2143) Then
!   CF(I) = (-689954*(ABS(E(I))**3)+333111*(ABS(E(I))**2)-53634*ABS(E(I))+3424.6)*AF(I)
EndIf

fy = 550 MPa

If (E(I) <= Ey) Then
!   CF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (E(I) <= 0.03715) Then
!   CF(I) = (32.747*log(ABS(E(I)))+724.24)*AF(I)
ElseIf (E(I) < 0.1436) Then
!   CF(I) = (32.747*log(0.03715)+724.24)*AF(I) + 28.23*ABS(E(I))*AF(I)
ElseIf (E(I) <= 0.214345) Then
!   CF(I) = (-689954*(ABS(E(I))**3)+333835*(ABS(E(I))**2)-53867*ABS(E(I))+3513.4)*AF(I)
EndIf

YC(I)=C-0.5*T-H(I) !moment arm
CFF=CFF+CF(I) !total force
MCF=MCF+(CF(I)*YC(I)) !total moment

ENDDO

I = 0

Concrete in Tension

If (FPC <= 0) Then
!   GoTo 200
!EndIf

DO  I = (INT((C-0.5*T)/hi)) , (N-1)

H(I) = (I-0.5)*hi
E(I) = ((H(I)-C+0.5*T)/(D-C+0.5*T))*EB
E(I) = (ET*(H(I)-D)+EB*H(I))/D
TC(I) = 0

!IF (E(I)<0) THEN
  !IF (E(I)<ECR) THEN
    FT(I)=EC*ABS(E(I)) !tensile stress in layer based on layer strain and initial concrete modulus (-ve!)
  ELSEIF (ABS(E(I))> ECR) THEN   !if layer strain is less than cracking strain go to 60
    FT(I)=ALPHA1*ALPHA2*FCR/(1+(500*(ABS(E(I))-ECR))**0.5) !tension stiffening if strain>cracking strain (-Ve)
  ENDIF
ENDIF

TC(I)=FT(I)*AC(I)  !force in layer is stress*area of concrete in layer (-ve)
TCC=TCC+TC(I)  !total force (adding a -ve number, so more -ve)
YT(I)=H(I)-C+0.5*T  !moment arm
MTC=MTC+(TC(I)*YT(I)) !total moment (+ve)
!ENDIF
ENDDO

200  I = 0

! FRP in Tension
DO I = (INT((C-0.5*T)/hi)) , (N-1)
  H(I) = (I-0.5)*hi
  !E(I) = ((H(I)-C+0.5*T)/(D-C+0.5*T))*EB
  E(I) = (ET*(H(I)-D)+EB*H(I))/D
  !TF(I) = 0

  !IF (E(I)<0) THEN
    !IF(ABS(E(I))<ABS(Ebu)) THEN
      ! IF(ABS(E(I)) > Soft) THEN  !for strain in 1st part of bilinear stuff
        ! TF(I)=Soft*Eft*AF(I)+(E(I)-Soft)*Eftsoft*AF(I)
      ELSE
        ! FRP STRESS-STRAIN PROPS
      ENDIF
    ENDIF
  ENDIF
ENDDO
TF(I)=ABS(E(I))*Eft*AF(I) !tension force in layer=layer strain*area of tube in layer (-ve)

! STEEL STRESS-STRAIN CURVES

! fy = 300 MPa
If (E(I) <= Ey) Then  
   TF(I) = 200000.0*ABS(E(I))*AF(I) 
ElseIf (E(I) <= 0.0359) Then 
   TF(I) = (28.831*log(ABS(E(I)))+459.44)*AF(I) 
ElseIf (E(I) < 0.1423) Then 
   TF(I) = (28.381*Log(0.0359) + 459.44)*AF(I) ! + 28.23*ABS(E(I)))*AF(I) 
ElseIf (E(I) <= 0.214345) Then 
   TF(I) = (-689954*(ABS(E(I))**3)+331248*(ABS(E(I))**2)-53036*ABS(E(I))+3196.6)*AF(I) 
EndIf

! fy = 350 MPa
If (E(I) <= Ey) Then  
   TF(I) = 200000.0*ABS(E(I))*AF(I) 
ElseIf (E(I) <= 0.03615) Then 
   TF(I) = (29.439*log(ABS(E(I)))+513.17)*AF(I) 
ElseIf (E(I) < 0.1425) Then 
   TF(I) = (29.439*Log(0.03615) + 513.17)*AF(I) ! + 28.23*ABS(E(I)))*AF(I) 
ElseIf (E(I) <= 0.214345) Then 
   TF(I) = (-689954*(ABS(E(I))**3)+331765*(ABS(E(I))**2)-53202*ABS(E(I))+3259.9)*AF(I) 
EndIf

! fy = 400 MPa
If (E(I) <= Ey) Then  
   TF(I) = 200000.0*ABS(E(I))*AF(I) 
ElseIf (E(I) <= 0.0364) Then 
   TF(I) = (30.315*log(ABS(E(I)))+566.15)*AF(I) 
ElseIf (E(I) < 0.1427) Then
TF(I) = (30.315*Log(0.0364) + 566.15)*AF(I)  ! +
28.23*ABS(E(I))*AF(I)
!important If (E(I) <= 0.214345) Then
!   TF(I) = (-689954*(ABS(E(I))**3)+332283*(ABS(E(I))**2) -
53368*ABS(E(I))+3323.2)*AF(I)
!   EndIf
!
fy = 480 MPa
!
If (E(I) <= Ey) Then
!   TF(I) = 200000.0*ABS(E(I))*AF(I)
! ElseIf (E(I) <= 0.0368) Then
!   TF(I) = (31.642*Log(ABS(E(I)))+650.6)*AF(I)
! ElseIf (E(I) <= 0.1431) Then
!   TF(I) = (31.642*Log(0.0368) + 650.6)*AF(I)  ! +
28.23*ABS(E(I))*AF(I)
!   EndIf
!
fy = 550 MPa
!
If (E(I) <= 0.1431) Then
!   TF(I) = 200000.0*ABS(E(I))*AF(I)
! ElseIf (E(I) <= 0.03715) Then
!   TF(I) = (32.747*Log(ABS(E(I)))+724.24)*AF(I)
! ElseIf (E(I) <= 0.1436) Then
!   TF(I) = (32.747*Log(0.03715) + 724.24)*AF(I)  ! +
28.23*ABS(E(I))*AF(I)
!   EndIf
!
YT(I)=H(I)-C+0.5*T  !moment arm
TFF=TFF+TF(I) !total tube tension force (-ve)
MTF=MTF+(TF(I)*YT(I))  !total moment
!
!ENDIF
ENDDO
ERR = CCC+CFF-TCC-TFF  !sums forces
EE = ERR

!Print*, CCC, CFF, TCC, TFF

!IF(ABS(Err)< error) THEN !checks if forces in equilibrium, if not, move NA
and start over

! GOTO 64

MOMENT = MCC + MCF + MTC + MTF  !sums moment
CURVATURE = ABS(ET)/C

!CURVATURE = ABS(ET)/(C-T)  !calcs curvature based on current top
concrete strain

BB = Moment
CA = Curvature

!Print*, C, Moment, ET, EB !, ERR, CCC, CFF, TCC, TFF
!

!ENDIF

!220 Print*, C, ET, Err, Moment, Curvature

END Function ConcreteFilled

Function HollowSteel(AA, CA, BB, GG, Bot, ZO, QQ)

IMPLICIT NONE

REAL, INTENT(IN) :: GG, ZO
REAL, INTENT(OUT) :: BB, CA, Bot
LOGICAL, INTENT(OUT) :: AA
Integer, Intent(IN):: QQ
REAL, DIMENSION(0:300000) :: H, E, FT, CC, CF, TC, TF, AF, AC, B, PT, PC, Y, AP, ESC, EST, MO, YT, YC, X
REAL, DIMENSION(0:300000) :: Q, Q1, Q2, xx, xx1, xx2, yy, yy1, yy2, CS, TS
REAL :: MCC, MCF, MTC, MTF, N, MOMENT, MPTF, MPCF, Mcon, Mfrp, OD, Strain, HollowSteel
REAL :: FPC, ST, T, ECP, ALPHA1, ALPHA2, ETU, EBU, AFRP1, ACON1
REAL :: SOFT, EFC, EFT, EFCSOFT, EFTSOFT, ERROR, ECF, ETF, EFCU, EFTU, D, HI
REAL :: EC,ESEC,R,FCR,ECR,ECT,C,EB,ET,AFRP,ACON,CCC,MAXCONCSTRESS,MaxSteelStress
    REAL :: CFF,TCC,TFF,ERR,PTF,PCF,CURVATURE,NN,EB1,Ey,Oval,Dv,Dh
    INTEGER :: I,j
    LOGICAL :: FAIL
    ! MCC to Defl are defined in various thesis stuff (see Dr. Fam about it)
    ! CURVold etc are for post-failure finding deflections and other post-failure stuff
    ! these are variables that remember if the
    FAIL = .FALSE.

    ! Geometry and Material Properties

    OD = 114.3
    !OD = 120.0
    ST = 3.0
    T = ST
    !FCP=40.0

    FPC = 0.0

    N=1000
    Etu=0.215
    Ebu=0.215
    Ey = 0.0013   !fy = 300
    !Ey = 0.00155 !fy = 350
    !Ey = 0.0018  !fy = 400
    !Ey = 0.0022   !fy = 480
    !Ey = 0.00255 !fy = 550

    !GENERAL CONCRETE GEOMETRIC PROPERTIES

    Oval = ZO

        D = OD-T   !D at tube mid-thickness
    ! hi = D*(1.0-Oval)/N   !ie. 219mm / 500 = <0.5 it's the size of each layer

    Moment = 0
    Curvature = 0
AFRP = 0
ACON = 0
AFRP1 = 0
ACON1 = 0
CCC = 0
CFF = 0
TCC = 0
TFF = 0
MCC = 0
MCF = 0
MTC = 0
MTF = 0
MaxConcStress = 0
MaxSteelStress = 0
ERR = 0
Bot = 0
!Eft = 0

Do I = 1,N

    CC(I) = 0
    CF(I) = 0
    TC(I) = 0
    TF(I) = 0
    YC(I) = 0
    YT(I) = 0
    E(I) = 0
    FC(I) = 0
    AF(I) = 0
    AC(I) = 0
    FC(I) = 0
    FT(I) = 0
    X(I) = 0
    H(I) = 0

EndDo

I = 0

ET = GG
!ET = 0.0001

Dv = D*(1.0-Oval)
Dh = D*(1.0+Oval)
hi = Dv/N
C = 0.5*Dv

Eb = (Dv/C) * Et - Et
Bot = Eb

!Write(6,60) QQ,Dv,Dh,C,Oval,hi,Et,Eb
!60 Format(I6.0","f10.5","f10.5","f10.5","f10.5","f10.5","f10.5","f10.5")

DO I=1,(0.5*N)+1 !from 2 to 499
  H(I)=(I-0.5)*hi !stores the test elevation of each layer (ie. midheight)
  E(I) = (ET*(H(I)-Dv)+EB*H(I))/Dv
  yy(I) = 0.5*Dv-h(I)
  yy1(I) = 0.5*Dv-h(I)+0.5*hi
  yy2(I) = 0.5*Dv-h(I)-0.5*hi
  xx(I) = 0.5*Dh*(1.0-((yy(I)**2)/(0.5*Dv)**2))**0.5
  xx1(I) = 0.5*Dh*(1.0-((yy1(I)**2)/(0.5*Dv)**2))**0.5
  xx2(I) = 0.5*Dh*(1.0-((yy2(I)**2)/(0.5*Dv)**2))**0.5
  Q(I) = ACOS((0.5*Dv-H(I))/((xx(I)**2)+(yy(I)**2))**0.5)
  Q1(I) = ACOS(((0.5*Dv-H(I)+0.5*hi))/((xx1(I)**2)+(yy1(I)**2))**0.5)
  Q2(I) = ACOS(((0.5*Dv-H(I)-0.5*hi))/((xx2(I)**2)+(yy2(I)**2))**0.5)
  !AF(I)=T*D*(Q2(I)-Q1(I)) !area of frp tube for that particular layer
  AF(I) = 2.0*(((xx(I)**2)+(yy(I)**2))**0.5)*(Q2(I)-Q1(I))
  B(I) = (((xx(I)**2)+(yy(I)**2))**0.5)*sin(Q(I))
  !AC(I)=2.0*B(I)*hi-0.5*AF(I) !area of concrete for that particular layer
  !If (AC(I)<0) Then
  !  AC(I) = 0
  !EndIf
  AFRP=AF(I)+AFRP !summed total frp tube
  !ACON=AC(I)+ACON !summed total concrete core

! If (QQ > 59) Then
!  If (QQ < 61) Then
!    Write(6,60) QQ, I, H(I),yy(I),xx(I), E(I),Dv
!    60 Format(16.0,","f10.5",","f10.5",","f10.5",","f10.5",","f10.5")
!  EndIf
! EndIf
ENDDO

!print*, I, AFRP, ACON,Dv,Dh
I = 0

!Do i = (0.5*N)+1, N
!   AF(I) = AF(N-I)
!   AC(I) = AC(N-I)
!   E(I) = -E(N-I)
!   H(I) = (I-0.5)*hi
!   AFRP1 = AFRP + AF(I)
!   ACON1 = ACON + AC(I)

!Write(6,60) I,AF(I),AC(I),AF(N-I),AC(N-I)
!60 Format(I6.0,,E10.5,,E10.5,,E10.5,,E10.5)

!EndDo

I = 0

If (QQ > 19) Then
   If (QQ < 21) Then
      ! Do I = 1,(0.5*N)+1
      !   Write(6,60) E(I), H(I)
      !   60 Format(f10.6,,f10.6)
      !   EndDo
   EndIf
EndIf
!    Concrete and FRP in Compression

!If (FPC <= 0) Then
!   GoTo 100
!EndIf

100  I = 0

! Steel in Compression

DO I = 1, (0.5*N)+1 !(INT((C-0.5*T)/hi))
H(I)=(I-0.5)*hi
E(I)= ((C-0.5*T-H(I))/(C-0.5*T))*ET
E(I) = (ET*(H(I)-Dv)+EB*H(I))/Dv

!fy = 300

If (ABS(E(I)) <= Ey) Then
    CF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.0359) Then
    CF(I) = (28.381*log(ABS(E(I)))+459.44)*AF(I)
ElseIf (ABS(E(I)) < 0.1423) Then
    CF(I) = (28.381*Log(0.0359) + 459.44)*AF(I) + 100.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
    CF(I) = (-689954.0*(ABS(E(I))**3)+331248.0*(ABS(E(I))**2)-53036.0*ABS(E(I))+3196.6)*AF(I)
EndIf

!fy = 350 MPa

If (ABS(E(I)) <= Ey) Then
    CF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.03615) Then
    CF(I) = (29.439*log(ABS(E(I)))+513.17)*AF(I)
ElseIf (ABS(E(I)) < 0.1425) Then
    CF(I) = (29.439*Log(0.03615) + 513.17)*AF(I) + 28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
    CF(I) = (-689954*(ABS(E(I))**3)+331765*(ABS(E(I))**2)-53202*ABS(E(I))+3259.9)*AF(I)
EndIf

!fy = 400 MPa

If (ABS(E(I)) <= Ey) Then
    CF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.0364) Then
    CF(I) = (30.315*log(ABS(E(I)))+566.15)*AF(I)
ElseIf (ABS(E(I)) < 0.1427) Then
    CF(I) = (30.315*Log(0.0364) + 566.15)*AF(I) + 28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
    CF(I) = (-689954*(ABS(E(I))**3)+332283*(ABS(E(I))**2)-53368*ABS(E(I))+3323.2)*AF(I)
EndIf
If $|E(I)| \leq Ey$ Then
\[ CF(I) = 200000.0 \times |E(I)| \times AF(I) \]
ElseIf $|E(I)| \leq 0.0368$ Then
\[ CF(I) = (31.642 \times \log |E(I)| + 650.6) \times AF(I) \]
ElseIf $|E(I)| < 0.1431$ Then
\[ CF(I) = (31.642 \times \log(0.0368) + 650.6) \times AF(I) + 28.23 \times |E(I)| \times AF(I) \]
ElseIf $|E(I)| \leq 0.214345$ Then
\[ CF(I) = (-689954 \times |E(I)|^3 + 333111 \times |E(I)|^2 - 53634 \times |E(I)| + 3424.6) \times AF(I) \]
EndIf

If $|E(I)| \leq Ey$ Then
\[ CF(I) = 200000.0 \times |E(I)| \times AF(I) \]
ElseIf $|E(I)| \leq 0.03715$ Then
\[ CF(I) = (32.747 \times \log |E(I)| + 724.24) \times AF(I) \]
ElseIf $|E(I)| < 0.1436$ Then
\[ CF(I) = (32.747 \times \log(0.03715) + 724.24) \times AF(I) + 28.23 \times |E(I)| \times AF(I) \]
ElseIf $|E(I)| \leq 0.214345$ Then
\[ CF(I) = (-689954 \times |E(I)|^3 + 333835 \times |E(I)|^2 - 53867 \times |E(I)| + 3513.4) \times AF(I) \]
EndIf

If $I < 2$ Then
Write(6,60) I,E(I),CS(I)
60 Format (I6.0,,f10.6,,f10.6)
EndIf

\[ YC(I) = C - H(I) + 0.5 \times T \] ! moment arm
\[ CFF = CFF + CF(I) \] ! total force
\[ MCF = MCF + (CF(I) \times YC(I)) \] ! total moment

ENDDO

I = 0

GoTo 999

Steel in Tension
DO I = (0.5*N)+1, N-1 !(INT((C-0.5*T)/hi)) , (N-1)
   H(I) = (I-0.5)*hi
   !E(I) = ((H(I)-C+0.5*T)/(D-C+0.5*T))*EB
   E(I) = (ET*(H(I)-Dv)+EB*H(I))/Dv

! STEEL STRESS-STRAIN CURVES

!fy = 300
If (ABS(E(I)) <= Ey) Then
   TF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.0359) Then
   TF(I) = (28.831*log(ABS(E(I)))+459.44)*AF(I)
ElseIf (ABS(E(I)) < 0.1423) Then
   TF(I) = (28.381*Log(0.0359) + 459.44)*AF(I) ! +
   28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
   TF(I) = (-689954*(ABS(E(I))**3)+331248*(ABS(E(I))**2)-
   53036*ABS(E(I)))+3196.6)*AF(I)
EndIf

!fy = 350 MPa
If (ABS(E(I)) <= Ey) Then
   TF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.03615) Then
   TF(I) = (29.439*log(ABS(E(I)))+513.17)*AF(I)
ElseIf (ABS(E(I)) < 0.1425) Then
   TF(I) = (29.439*Log(0.03615) + 513.17)*AF(I)  ! +
   28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
   TF(I) = (-689954*(ABS(E(I))**3)+331765*(ABS(E(I))**2)-
   53202*ABS(E(I)))+3259.9)*AF(I)
EndIf

!fy = 400 MPa
If (ABS(E(I)) <= Ey) Then
   TF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.0364) Then
   TF(I) = (30.315*log(ABS(E(I)))+566.15)*AF(I)
ElseIf (ABS(E(I)) < 0.1427) Then
   TF(I) = (30.315*Log(0.0364) + 566.15)*AF(I)
EndIf
TF(I) = (30.315*Log(0.0364) + 566.15)*AF(I) 
28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
TF(I) = (-689954*(ABS(E(I))**3)+332283*(ABS(E(I))**2)-53368*ABS(E(I))+3323.2)*AF(I)
EndIf

fy = 480 MPa

If (ABS(E(I)) <= Ey) Then
TF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.0368) Then
TF(I) = (31.642*log(ABS(E(I)))+650.6)*AF(I)
ElseIf (ABS(E(I)) < 0.1431) Then
TF(I) = (31.642*Log(0.0368) + 650.6)*AF(I) 
28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
TF(I) = (-689954*(ABS(E(I))**3)+333111*(ABS(E(I))**2)-53634*ABS(E(I))+3424.6)*AF(I)
EndIf

fy = 550 MPa

If (ABS(E(I)) <= Ey) Then
TF(I) = 200000.0*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.03715) Then
TF(I) = (32.747*log(ABS(E(I)))+724.24)*AF(I)
ElseIf (ABS(E(I)) < 0.1436) Then
TF(I) = (32.747*Log(0.03715) + 724.24)*AF(I) 
28.23*ABS(E(I))*AF(I)
ElseIf (ABS(E(I)) <= 0.214345) Then
TF(I) = (-689954*(ABS(E(I))**3)+333835*(ABS(E(I))**2)-53867*ABS(E(I))+3513.4)*AF(I)
EndIf

YT(I)=H(I)-C+0.5*T 
moment arm
TFF=TFF+TF(I) 
total tube tension force (-ve)
MTF=MTF+(TF(I)*YT(I)) 
total moment

ENDIF
ENDDO

ERR= CCC+CFF-TCC-TFF 
sums forces
!Print*, CCC, CFF, TCC, TFF

!IF(ABS(ERR)< error) THEN !checks if forces in equilibrium, if not, move NA and start over
!     GOTO 64

999   MOMENT = MCF !+ MTF !sums moment
       CURVATURE = ABS(ET)/C
       !CURVATURE = ABS(ET)/(C-T) !calc curvature based on current top concrete strain

       BB = Moment
       CA = Curvature

!Print*, C, Moment, Curvature !, ET, EB !, ERR, CCC, CFF, TCC, TFF

!ENDIF

!220 Print*, C, ET, Err, Moment, Curvature

END Function HollowSteel

SubRoutine FPZeta(C, D, E, G, H, Xnew, J, F)

Real:: A, B, FA, FB, Xold, FXO, Xnew, FXN, Tol1, Tol2, Ntol, Unit, U, Test, Test2, Size, NMax, Value
Real:: C, D, E, G, H, J
Real, External:: CalcZetaO

!C = moment from previous
!D = Eft
!E = D22
!G = Omega
!H = Rout
!J = t

A = 0.0       !first bound on zeta
B = 0.1       !second bound on zeta
Ntol = 10000
Tol1 = 1.E-8
Tol2 = 0
Xnew = 0
Xold = 0
FXN = 0
Value = 0 ! Zeta value from CalcZeta function
U = 0

Unit = 1
1 Unit = 0.5*Unit
U = 1.0+Unit

IF (U>1.0) GoTo 1

! Protect against unreasonable tolerance
Tol2 = Tol2+Unit

! Initialization
Call CalcZetaO(A,Value,C,D,E,G,H,J)
FA = Value
SFA = Sign(1.0,FA)
Call CalcZetaO(B,Value,C,D,E,G,H,J)
FB = Value
Top = A*FB-B*FA
Diff = FB-FA
Xold = Top/Diff

If (Xold < 0) Then
  Xold = -Xold
EndIf

Call CalcZetaO(Xold,Value,C,D,E,G,H,J)
FXO = Value
Test = SFA*FXO

IF (Test>0) Then
  A = Xold
  FA = FXO
Else
  B = Xold
  FB = FXO
EndIf

! Begin interation

Do K = 1,Ntol
Top = A*FB-B*FA
Diff = FB-FA
Xnew = Top/Diff

If (Xnew < 0) Then
   Xnew = -Xnew
EndIf

Xdiff = ABS(Xnew-Xold)
Xold = Xnew
RERR = Xdiff/ABS(Xnew)

! Check Relative error criterion
If (RERR.LE.Tol2) Return
   Call CalcZetaO(Xnew,Value,C,D,E,G,H,J)
   FXN = Value
   Size = Abs(Fxn)
EndIf

! Check size of function value
If (Size.LE.Tol1) Return
   SFxn = Sign(1.0,Fxn)
   Test2 = SFxn*FXO
   Test = SFA*FXN
EndIf

! Update EndPoints
If (Test>0) Then
   A = Xnew
   FA = FXN
   If (Test2>0) Then
      FB = FB/2
   EndIf
Else
   B = Xnew
   FB = FXN
   If (Test2>0) Then
      FA = FA/2
   EndIf
EndIf

5   FXO = FXN
6   Continue
!Print*, A,B,Xnew
EndDo
End SubRoutine FPZeta
FUNCTION CALCZETAO(Z, Value, C, D, E, G, H, J)

IMPLICIT NONE

Real::CalcZetaO
REAL, PARAMETER :: PI = 3.141592654
!REAL :: ZETAOFINAL, ZERO, ZETAO
Real,Intent(In)::C,D,E,G,H,J,Z
Real,Intent(Out)::Value

!Z = ZetaO test value
!C = moment from previous
!D = Eft
!E = D22
!G = Omega
!H = Rout
!J = t
Value = 0

Value = 2.85*PI*H*(PI*(ABS(D))*E*J*(1.0+(PI**4/(12.0*G**4))))**0.5*(1.0-3.0*Z/PI) &
   & * Z**0.5 - C

ENDFUNCTION CALCZETAO