A Bayesian Approach to Convergence Detection in Underground Excavations using LiDAR

by

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Abstract

This thesis deals with the subject of convergence detection in underground excavations — typically mining operations. Traditional methods for convergence monitoring involve operose surveying procedures that produce measurements in low density along a drift. The aim of this thesis is to introduce a novel convergence monitoring solution which extracts typical convergence features from point cloud scans in a drift. These features are extracted by convergence indicators. These indicators are amalgamated using a Bayesian statistical approach to build an inference about whether or not convergence is occurring. This algorithm was tested on simulated as well as actual mining convergence drift scans.
Acknowledgments

I would like to thank my thesis supervisor Josh Marshall for helping me with both vision and direction with respect to my thesis. I would also like to thank my friends and family for reminding me to have fun once in a while.
## Nomenclature

<table>
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<tr>
<td>2D/3D</td>
<td>2-dimensional and 3-dimensional</td>
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<tr>
<td>$P(,+,+,C)$</td>
<td>Probability of high convergence</td>
</tr>
<tr>
<td>$P(,+,C)$</td>
<td>Probability of low convergence</td>
</tr>
<tr>
<td>$P(,\neg,C)$</td>
<td>Probability of no convergence</td>
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<td>uGPS</td>
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Chapter 1

Introduction

1.1 Motivation

Mining operations have a great number of complex human interactions occurring within them that require synchronizing countless operations in such a way as to slowly extract the mineral wealth of a deposit. Any event within the mine that affects flow of traffic or occupies the time and resources of production crews has a rippling effect through the operation, causing far field impacts on the mining operation as a whole. In most cases, performing work in a mine, whether it is as benign as surveying or as imposing as drilling, causes that drift to no longer be easily traversable. Activities such as monitoring for underground convergence are no exception. This is especially troubling in active working headings.

Important though it is, convergence monitoring involves survey crews meticulously measuring and documenting aspects of mining drifts. These drifts can stretch thousands of kilometres in some mines [21] making the task incredibly cumbersome, not only for survey crews, but in terms of interruptions they pose to normal mining operations as well.
1.2 Objective

The objective of this thesis is to propose a novel algorithm for performing convergence detection with stationary or mobile LiDAR scanners. The scope of this thesis covers the definition of the desired data sets, and how these data sets are manipulated to probabilistically infer convergence. This probabilistic inferencing of the data was developed and tested on both simulated and real convergence data to determine if the algorithm responds as expected to convergence measurements made with mobile LiDAR scanner.

1.3 Thesis Outline

Chapter 2 begins by exploring what convergence in underground mines actually is. Firstly the governing equations of convergence are discussed in Section 2.1.1. Next, common manifestations of underground convergence are explored and put into context with real geologic structures. Finally, common mitigation techniques are also discussed.

Section 2.2 explores the different convergence monitoring technologies available; both current and antiquated. This section is broken up into both physical measurement methods — i.e., with surveyors and measuring sticks — to laser-based change detection methods with point cloud drift reconstructions.

Chapter 2 is completed with Section 2.3 where an amalgamation of research on probabilistic estimation is performed detailing aspects of Bayesian networks and fuzzy logic controllers. These methods will be used in the development of the thesis algorithm.

This algorithm is first seen in Chapter 3. Firstly, the desired data structure for the
input to the algorithm is defined. This is followed by the introduction of the concept of convergence indicators in Section 3.2. The derivation of each of these indicators from the data is outlined explicitly in Sections 3.2.1 through 3.2.5. Pseudo-code is used to give the reader a more complete understanding of how the computer handles the data. Chapter 3 ends with a practical application of the Bayesian theorem and fuzzy logic that is explored in Section 2.3. These methods are used to turn uncertain sensory measurements of the drift into probabilities of convergence.

Chapter 4 serves as a preamble for Chapter 5. Chapter 4 outlines the structure of the experimental data as well as the methods by which they were collected. Here, one learns about the uGPS rapid mapper and how it generates point cloud data. Photos and details of the experimental set up are shown in Section 4.2. The limitations of this data are also explained in Section 4.3. This chapter precedes Chapter 5 since it lays out how the data was structured for the experiment. The result is that the algorithm detailed in Chapter 3 was actually built to handle this structure of data. This is reasonable since this structure and the ideal one outlined in Section 3.1.1 are almost exactly the same.

After the development of the algorithm in Chapter 3 and the explanations in Chapter 4, Chapter 5 chronicles the three steps to validating the convergence monitoring algorithm on convergence data. Firstly, the simulated convergence shapes are introduced and justified given the typical modes of convergence outlined in Section 2.1. Next, these shapes are applied to individual cross sections followed by entire sections of actual drift scans. The chapter concludes with a case study on a mine with convergence in Section 5.4. This was done by comparing surveyed convergence data with data collected as laid out in Section 4.2. The accuracy of the convergence
monitoring algorithm is investigated for each event and reasons for inaccuracies in the field tests are explored.

Chapter 6 concludes the thesis and offers extrapolation into the future applications of the successful implementation of laser scan-based convergence monitoring technology.
Chapter 2

Background

2.1 Convergence Theory

Convergence in a mining environment refers to an elastoplastic deformation of rock, under a constant pressure, over a period of time. In geology, the study of this kind of solid material flow is known as rheology [8].

Convergence is caused by a supporting rock mass slowly straining over time due to naturally occurring stress fields present in the host rock. This stress field is the result of the combination of superimcumbant loading and lateral stresses in the host rock. Generally, in-situ stress magnitudes will increase linearly with mining depth, making deeper excavations more prone to convergence.

Mineral extraction enterprises can routinely schedule mines to operate for several decades, expecting excavations such as shafts and access drifts to remain open for long periods of time. This permanent infrastructure is also more prone to the effects of convergence.

Independent of these external properties, the intrinsic qualities of the supporting rock also greatly affect how convergence will be manifested within the rock mass.
2.1. CONVERGENCE THEORY

The following sections begin by discussing common models that describe the relationships between time, field stress, rock properties and convergence strain. Common manifestations of convergence on drift shape will also be discussed. The convergence background section ends by describing the negative effects of convergence on mining operations and common practices for its mitigation.

2.1.1 Convergence models

In order to understand the convergence models, one should first consider the conditions by which convergence is brought about. Initially there would be a deposit of solid rock under a certain depth of host rock. This host rock will have a pre-mining stress field present but no strain can occur because there is no vacancy for the rock to flow into. Now consider the introduction of a cylindrical void in the deposit analogous to a mining excavation — possibly a drift. As soon as the void is made, the perimeter of the drift experiences elastic deformation due to the sudden loss of confining pressure. This, immediate strain response is linear and elastic in nature and can be described with the Hookean solid model as in

$$\epsilon_H = \frac{\sigma}{E}$$  \hspace{1cm} (2.1)

where $\epsilon_H$ is the Hookean elastic strain, $\sigma$ is the applied stress and $E$ is the modulus of elasticity.

This is the first type of strain which causes convergence. Given the immediate nature of its occurrence, this strain is largely disregarded when studying drift convergence since it is difficult to measure and generally does not change over time. Non-reversible, time dependent strain is described as a “plastic flow” of material and
2.1. CONVERGENCE THEORY

Figure 2.1: Burger solid creep chart showing how the rock deformation changes over time. [16]

falls under the area of study known as rheology.

A limitation of the Hookean model is that it does not consider the time dependent stress response of rock. The Society of Mining Metallurgy and Exploration defines convergence as “the propensity for strain to increase under a constant load,” [9]. In a laboratory setting, it is possible to administer a constant load to a rock sample and study the time dependent strain of the rock. The resultant deformation is known as creep. Creep, as a result of the application of a constant stress, tends to follow a characteristic curve know as the creep curve, shown in Figure 2.1.

This creep curve is derived empirically to form the Burger solid model, wherein the strain relation is described in terms of three other solid model constitutive equations: the Hookean model Equation (2.1), the Kelvin model Equation (2.2) and the Maxwell
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solid model Equation (2.3). The Hookean elastic region occurs instantaneously after
the stress is applied, so it defines the initial offset on the strain axis of Figure 2.1.
The primary creep region of the curve shows a steady decrease in the rate of creep
modelled by the Kelvin solid model Equation (2.2). In the secondary or steady state
creep region, the linear relation of strain with respect to time can be modelled with
the Maxwell solid model,

\[ \epsilon_k = \frac{\sigma}{E} \left( 1 - e^{-\frac{Et}{\eta_k}} \right) \]  \hspace{1cm} (2.2)

\[ \epsilon_M = \frac{\sigma}{E} + \frac{\sigma t}{\eta_M} \]  \hspace{1cm} (2.3)

where \( \eta_M \) and \( \eta_k \) are each material viscosity coefficients, \( E \) is the Modulus of Elasticity,
\( \sigma \) is the constant applied stress and \( t \) is time.

Tertiary creep represents the accelerated formation and expansion of micro-cracks
within the rock material [27]. This acceleration of creep is more chaotic and difficult
to model. It also represents the failure of the structural integrity of the material.

By measuring creep rates in underground excavations, one can begin to determine
where on the creep curve the supporting walls lie. During the majority of the mine
life, the supporting geology will experience steady state creep. It is when the creep
begins to accelerate that one should call the structural integrity of the excavation into
question. As in the creep models, acceleration of creep means that serious mitigation
techniques need to be enacted to avoid structural failure of the excavations.

This creep curve is only valid for isotropic rock samples with no discontinuities
or faulting throughout. The model becomes inherently less accurate as it is used to
2.1. CONVERGENCE THEORY

describe larger and larger geologic systems. This is chiefly due to the unpredictable composition of naturally occurring rock masses. This empirical creep analogue can, however, be used to derive numerical models on which estimations of rates and magnitudes of rheological flow can be based, [12, 13].

In a perfect rock mass, under a uniform stress field, this creep will occur in areas of higher stress. For circular drifts in uniaxial, pre-mining stress field, one can use the hole-in-plate stress model to show where convergence would be most likely to occur. The hole-in-plate model expressed as:

\[
\begin{align*}
\sigma_r &= \frac{\sigma}{2} \left[ (1 - \frac{a^2}{r^2}) + (1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4}) \times \cos(2\theta) \right] \\
\sigma_t &= \frac{\sigma}{2} \left[ (1 + \frac{a^2}{r^2}) - (1 + 3 \frac{a^4}{r^4}) \times \cos(2\theta) \right]
\end{align*}
\] (2.4) (2.5)

where \( \sigma \) is the in-situ stress field, \( \sigma_r \) and \( \sigma_t \) are the radial and tangential stress components respectively, \( r \) is the distance away from drift centre and \( a \) is the actual drift radius [28].

A diagram, including the uniaxial, vertical stress field implied by this model, is shown in Figure 2.2. Notice the higher concentration of stress on the walls and the development of tensile stress on the back and floor.

Since convergence rate is proportional to applied stress, the areas subjected to greater compressive stress will converge faster. Rock also tends to be approximately 10 times weaker in tension than in compression [29]. Considering this, the sections in Figure 2.2 under tension are likely to fail due to tensile loading. This type of failure is known as ‘spalling’ and occurs frequently in the back of unsupported ground.

Again, this model is only valid for uniaxial ground stress surrounding a perfectly
circular drift in isotropic rock. For other drift shapes, such as rectangles which are wider than they are tall, different mechanisms of convergence prevail. Likewise, for different pre-mining stress conditions, different mechanics of convergence can occur. Finally, the addition of geologic structures such as faulting and foliation can change how convergence is expressed.

Consider the simple geologic structures in Figure 2.3. Figure 2.3(a) shows micro-crack propagation in a homogeneous solid to a point where the entire excavation begins failing along the exposed perimeter. This arises when the pre-mining stress field is hydrostatic — equal in all directions.

The buckling failure in Figure 2.3(b) occurs through re-distribution of the pre-mining stress field, similar to the redistribution of the hole-in-plate model in Figure 2.2. In a uniaxial stress field, this rectangular drift introduces a void above the floor and below the back. Due to this excavation, the layers of rock no longer have a confining stress to hold the foliations together. This causes drift convergence.
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(a) Complete shear failure, (b) buckling failure, (c) tensile splitting shearing and sliding.

Figure 2.3: Convergence manifestations in underground excavations [2].

The rheologic response in Figure 2.3(c) shows layers in the rock mass being compressed by the host rock. The mining excavation, once again, removes the confining pressure of these layers, allowing them to flow into the excavation. This lateral deformation is due to the lateral strain of the foliations in proportion to the Poisson ratio as they are being acted upon by vertical, uniaxial compressive stress. This type of pre-mining stress field orientation also causes tensile spalling in the back or the drift. Here the rock layers are competent enough to not fail in buckling, unlike Figure 2.3(b).

In general, elastoplastic flow as a result of in-situ stress causes the drift cross section to change. It is this overall change in profile that the convergence algorithm proposed in Chapter 3 is designed to detect.

2.1.2 Support and Mitigation Techniques

Supporting an excavation with expected rheologic flow is difficult. In addition to traditional rock bolt, screen and shotcrete support, a structure which yields to the
Figure 2.4: A drift being supported by an arch system which uses sliding plates to yield to convergence movement [9].

movement of the host rock is also important to incorporate [9]. Metal archways such as in Figure 2.4(a) provide resistance while yielding to some convergence movement using the sliding plate configurations shown in 2.4(b). In cases of advanced fracturing, such as in highly foliated geology or during prolific micro-cracking, resin grouts can be pumped into the rock mass in an attempt to re-adhere the fracturing host rock. For critical long term excavations, steel reinforced concrete linings or cast steel segments can be installed. These measures are very expensive and are usually done for shafts or areas where water in-flow is also a concern.
2.2 Convergence Monitoring Technology

Convergence can be a problem with as little as 1% change in drift diameter [2] and has the capacity to close entire drifts, where deformations on the order of a couple of meters can occur [9]. Convergence left un-mitigated will eventually fail catastrophically as outlined in Section 2.1.1. In order to have enough time to enact an appropriate mitigation strategy, one must measure and track the movement of ground early on, when the excavation is first made. After a certain magnitude of convergence, one might need to rehabilitate the excavation and if the rate of convergence begins to increase extra support methods may be required.

2.2.1 Physical Measurement Methods

One antiquated way to measure convergence magnitude is through the use of telltales. These are strings, hung exactly from the back to the floor so that there is no excess length. The length of string which begins to coil on the floor as the back sags serves as a measurable quantity for vertical convergence. This method neglects horizontal convergence measurement entirely.

A similar, but more modern approach to convergence monitoring involves measuring drift widths and heights using laser range devices such as disto meters [26]. Known points are marked on the walls at regular intervals along the drift in a pattern similar to the ones shown in Figure 2.5. The distances between the points are measured and recorded over time. As the distances begin to change, one can begin to make inferences about the magnitude and rate of convergence.

Some measurement solutions involve installing infrastructure into the host rock to track creep: a borehole extensometer is one such tool as shown in Figure 2.6. For this,
2.2. CONVERGENCE MONITORING TECHNOLOGY

(a) Triangular

(b) Star

Figure 2.5: Standard measurement patterns for convergence monitoring [7].

A borehole is drilled and a bolt is installed, having one end anchored at the foot of the borehole and one end anchored at the face of the rock with a steel plate. During convergence, the exposed face of the drift begins to creep into the open excavation while the foot of the hole remains relatively stationary relative to the face anchor. These measurement methods have the added advantage of being able to be taken autonomously by connecting their measurement outputs to a server. Unfortunately the installation and material costs are relatively high because not only are the sensors more expensive and single use, the drilling and installation takes a great deal of time and makes use of consumable resources to complete.

2.2.2 Laser Scanners in Convergence Monitoring

Each of the above methods only measure convergence at isolated points in the drift. Density of data is severely limited by the amount of time it takes to establish each individual measurement, so convergence is inferred for a few meters along a drift based on few and limited points taken at singular cross sections. Laser scanners can
2.2. CONVERGENCE MONITORING TECHNOLOGY

Figure 2.6: Diagram of a Multi Point Borehole Extensometer (MPBX) [7].

take thousands of measurements of the drift walls at a time, thus greatly increasing the density of measurement data. Unfortunately, these large volume point clouds of data have their own issues when one goes to extract convergence data from them.

Convergence monitoring with point clouds requires that two scans be taken at a known location in a mine at different times. These two point clouds then need to be aligned so that data from the initial scan is overlapping or coincident with the second scan. This data association is often done with computational methods such as iterative closest point matching (ICP) where the sum of square distances from points
2.3 Probabilistic Estimation Review

The use of probabilistic estimation is widely applied in computer sciences and robotics as a means of taking uncertain measurements from one’s environment and determining an optimum course of action despite the uncertain nature of input data. This type of computational reasoning has apparent advantages within the context of convergence monitoring when using the LiDAR scan data discussed in Section 2.2.2.
These point clouds collect vast quantities of data about the surrounding drift but poor data association methods make determining whether and where convergence has occurred more difficult. Probabilistic inferencing could be applied to drift scans to reduce uncertainty in convergence estimation.

The following section outlines basic concepts of fuzzy logic. The purpose is to build a working familiarity with the concepts in order to use them with the Bayesian theories presented in Section 2.3.2. Together, fuzzy logic and statistical amalgamation form the base on which convergence inference is built. This inferencing is used with the indicators presented in Chapter 3 to detect convergence using LiDAR data.

### 2.3.1 Fuzzy Inference Background

Fuzzy logic is a field of probability that deals with partial membership of a variable within a particular system state [22]. This is in contrast to Boolean logic where a variable must totally exist within a particular state, as in

$$
\mu_A(x) = \begin{cases} 
0 & \text{if } x \in A \\
1 & \text{if } x \notin A 
\end{cases}
$$

(2.6)

where a function $\mu$ is either 1 or 0 if variable $x$ does or does not belong to the set $A$.

In Boolean logic, a lamp can either be assigned a state of on or off. In fuzzy logic, the lamp state can belong to any proportion of the available states as in

$$
\mu_A : X \to [0, 1]
$$

(2.7)

where $X$ is a universal set, specified for each individual problem. In the lamp example,
2.3. PROBABILISTIC ESTIMATION REVIEW

$X$ could be voltage which affects the state of lamp brightness $\mu$ of the set $A$. The output $\mu_A$ would then be the percentage of total brightness.

Fuzzy logic has been applied in a great number of fields ranging from motor control [38] to artificial intelligence and machine learning [4]. Fuzzy logic can be used as a way of incorporating qualitative, human descriptors into the language of system control or computer processing, in spite of the traditional need for quantitative measurement.

Consider a control system, at a restaurant, which controls a valve to automatically fill a glass with sparkling water. To reduce fill time, one would like the glass to fill as quickly as possible. Filling the glass very quickly causes bubbles to form on top of the liquid level, so as the glass becomes more full the rate of flow needs to be reduced in order to keep the bubbles from flowing over the side of the glass. Traditionally, this is a problem for a proportional controller as in Equation (2.8):

$$Q = K_p(H - L)$$

(2.8)

where $Q$ is the flow rate, $K_p$ is the controller gain, $H$ is the glass height and $L$ is the liquid level.

Using qualitative measurements a person could reason that a nearly empty glass should be filled quickly at first, then the flow should be reduced to medium about half way up the glass, followed by low flow when the glass is nearly full. These qualitative descriptors of state are referred to as linguistic variables in fuzzy logic [37]. These variables have no numeric value, like measurements of height or flow, but they each correspond to states in the control system.

In a fuzzy controller these linguistic variables are used within sets. A form of
measurement for a state, such as liquid height, is then assigned a compatibility value within these sets. Compatibility refers to how strongly a variable fits into the definition of a set and is a numerical value from 0 to 1 [37]. Assigning the compatibility value is usually done with a membership function as in Figure 2.7.

Figure 2.7: An example of a membership function which assigns input values a compatibility value for any proportion of 2 out of the 3 states.

Here the $x$-axis could be flow rate and the blue, yellow and red lines represent the qualitative variables low, medium and high flow respectively. A flow of 12 Lpm could be considered a high flow. In the membership function (MF) the flow of 12 Lpm is 100 % compatible with the state high flow. The vector form for this is $\langle 0, 0, 1 \rangle$. 
According to the MF, a flow of 8 Lpm would be considered between high and medium flow. The compatibility measurement for this is 50 % medium flow and 50 % high flow as seen in Figure 2.7 and can also be written as a vector, $\langle 0, 0.5, 0.5 \rangle$. Likewise, the qualitative variable describing the liquid level in the glass could be empty, mid level and full, with the error metric $(H - L)$ from Equation (2.8) as the variable on the $x$-axis which determines how the state of the glass is categorized.

These fuzzy variables are now in a state where they can be mapped, from input to output, and made into a control system. For the sparkling water control unit, represented in Figure 2.8, the input would be fill level: either empty, mid level or full. The fill level compatibility vector would be derived from height measurements of the liquid in the glass, which are passed into a membership function. An output vector of $\langle 0, 0.25, 0.75 \rangle$ from the MF would signify that the glass is mostly full. After the sensor input is made into a fuzzy variable, through a process known as encoding [4], the sensor state information is manipulated to bring about the desired results. According to our logic, a mostly full glass should prescribe a lower flow rate. In this simple model, the input vector could simply be inverted and plugged back into the MF in Figure 2.7 in order to solve for flow. Indeed a vector of $\langle 0.75, 0.25, 0 \rangle$ would yield a mostly low level of flow at approximately 4 Lpm. The compatibility vector is then fed into a MF for the valve and the flow is determined. This is called decoding.

This system eradicates the need for governing constitutive equations between the quantities. This control method instead chooses to describe linguistic variables. Some systems cannot be described by conventional metrics, and are instead classified with one or more linguistic variables. Concepts such as friendliness, attractiveness or fear are good examples. When judging apparent friendliness of a person, one will
consider many factors simultaneously before forming an opinion. By virtue of using compatibility values and linguistic variables in lieu of constitutive equations, one could use fuzzy logic to describe and subsequently react to such approximate, non-measurable qualities as friendliness.

Another non-measurable quantity could be how sure one feels they are walking in the right direction in a forest. One could be 0 % compliant with the category ‘lost’ when they see a well marked path, or be 40 % lost as they walk towards where they might have remembered the parking lot to be. In this example, there is no binary point where one goes from 0 % lost to 100 % lost: likewise, there is no single variable which influences this feeling from surely located to definitely lost. The process of feeling lost comes about from a series of clues or inputs that suggest that one might not be in the right place. Approximate reasoning using fuzzy variables is well suited to amalgamating evidence based on uncertain measurements.
2.3. PROBABILISTIC ESTIMATION REVIEW

In Section 2.3.2, a statistical method for amalgamating evidence is introduced. This, along with fuzzy variables, will eventually make up the convergence inference algorithm proposed in Chapter 3.

2.3.2 Statistics and Bayesian Network Background

The constitutive mathematical component of a Bayesian network is the Bayesian theorem:

\[
P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}.
\]

The expression \( P(A|B) \) is the conditional probability that situation A will occur if B is known to occupy a certain state and is read as ‘probability of A given B’.

This rule is an extension of the definition of conditional probability:

\[
P(A|B) = \frac{P(A, B)}{P(B)},
\]

where the numerator has the joint probability \( P(A, B) \) — indicating the probability that both A and B will occur — instead of the conditional probability.

The chain rule in Equation (2.11) relates these types of probabilities as follows:

\[
P(A_n, A_{n-1}, \ldots, A_1) = P(A_n|A_{n-1}, \ldots, A_1) \times P(A_{n-1}, \ldots, A_1)
\]

where the probability of \( A_x \), where \( x = [1, n] \), occurring together equals the probabilities of \( A_x \), where \( x = [1, n-1] \), occurring together times the probability of \( A_n \) occurring given the other \( n - 1 \) events have also happened.

It may help to think of each event, A, as the height of a person on a ladder at discrete intervals \( n \in \mathbb{N} \) where \( n \) is the rung number on the ladder. According to the
chain rule, the probability that a person has climbed \( n \) rungs and \( n - 1 \) rungs and \( n - 2 \) rungs, etc. is the product of the probability that this person has climbed \( n - 1 \) rungs, \( n - 2 \) rungs, \( n - 3 \) rungs, etc. times the probability of climbing the \( n^{th} \) rung given that person previously climbed rung \( n - 1, n - 2, n - 3, \) etc.

In this case, it is true to say that once a person climbs up to a rung, say \( n - 1 \), that reaching the next rung \( n \) is no longer related to the other \( n - 2 \) rungs below. That is, the probability of reaching a state \( n \) depends only on the previous state \( n - 1 \). This is known as a first order Markov assumption [10] and condenses the chain rule into the joint probability function as follows:

\[
P(A_n, A_{n-1}, \ldots, A_1) = P(A_n|A_{n-1}, \ldots, A_1) \times P(A_{n-1}|A_{n-2}, \ldots, A_1) \times \ldots \times P(A_1).
\] (2.12)

Bringing Equation (2.12) into the definition of conditional probability, Equation (2.10) yields:

\[
P(A, B) = P(A|B) \times P(B)
P(A, B) = P(B|A) \times P(A).
\] (2.13)

From here, one should substitute Equation (2.13) into Equation (2.10) to produce the Bayes theorem Equation (2.9).

As will be discussed later in this section, the Bayes theorem provides a means by which one can calculate joint probabilities using conditional probabilities. This is necessary when the independent marginals of the joint probabilities are not stochastically independent. In statistics, two events are independent if the probability of one
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does not affect the probability of the other. Mathematically, this is written as:

\[ P(\alpha \land \beta) = P(\alpha)P(\beta) \]  \hspace{1cm} (2.14)

where \( P(\alpha \land \beta) \) is also known as the joint probability of the marginals \( P(\alpha) \) and \( P(\beta) \).

When two events are independent, calculating the joint probabilities in Equation (2.10) is as simple as multiplying the probabilities of both marginals. In a Bayesian network, this is infrequently the case.

A Bayesian network, like the one shown in Figure 2.9, is a graphical way to display causality between statistical events and can be used to visually discern independence [30]. The circles in Figure 2.9 represent statistical events and are called nodes. The arrows connecting the nodes are called edges, and imply that the node on the arrow side is caused by the node on the other side of the edge. This standard diagram for depicting a Bayesian network is known as a Directed Acyclic Graph (DAG). A DAG must have directed edges, implying causation, while not having any cyclical edge connections (acyclic) [17].

This simple DAG serves as a representation for an example scenario, adapted from a tutorial [23] to suit the convergence detection algorithm development. Figure 2.9 depicts the causality structure for a hypothetical diagnostic tool in detecting convergence. The presence or absence of convergence (\( C \) or \( \neg C \)) causes the two tests \( T_1 \) and \( T_2 \) to either show a confirmation or negation of the presence of convergence.

In this example, each test is identical and so they share the same false positive
Figure 2.9: A directed acyclic graph (DAG) where the nodes are statistical events and the arrows imply causality between them.

and false negative rate as in:

\[ P(T_n|C) = 0.9 \implies P(\neg T_n|C) = 0.1 \]  
\[ P(T_n|\neg C) = 0.2 \implies P(\neg T_n|\neg C) = 0.8. \]  

Note that the implication in Equation (2.15) stems from the law of total probability as in:

\[ 1 = \sum_m \sum_n P(A_m, B_n), \]  

where the summation of all possible states of all variables must equal 1. Note that this law also implies that summing the total probability of one variable, leads to the
2.3. PROBABILISTIC ESTIMATION REVIEW

total probability associated with the other variable, as in:

\[ P(A) = \sum_n P(A, B_n). \]  
(2.17)

When applied to the conditional probability law Equation (2.10), this equation becomes:

\[ P(A) = \sum_n P(A|B_n) \times P(B_n). \]  
(2.18)

Let us propose an example question of the Bayesian network in Figure 2.9. The question is, what is the probability of convergence if test 1 is positive? What if test 1 and test 2 are positive? This is referred to as calculating the posterior probability and can be written in terms of conditional probabilities as \( P(C|T_1) \) and \( P(C|T_1, T_2) \) respectively.

The first can be solved using the Bayes theorem Equation (2.9) as follows:

\[ P(C|T_1) = \frac{P(T_1|C) \times P(C)}{P(T_1)} \]  
(2.19)

where \( P(C) \) is known as the prior likelihood of convergence and \( P(T_1) \) is the marginal likelihood of a positive test result, calculated using total probability Equation (2.18), as in:

\[ P(T_1) = \sum_n P(T_1|C_n) \times P(C_n) \]  
(2.20)

\[ P(T_1) = P(T_1|C) \times P(C) + P(T_1|\neg C) \times P(\neg C). \]
When this is substituted back into Equation (2.19) it yields:

\[
P(C|T_1) = \frac{P(T_1|C) \times P(C)}{P(T_1|C) \times P(C) + P(T_1|\neg C) \times P(\neg C)}.
\]  

(2.21)

Notice how only known values, such as the false negative rates and prior likelihoods from Figure 2.9, remain in the Bayes theorem. The resultant probability of an area which is 15% predisposed to convergence after receiving one test with the accuracy values in 2.9 is 44.26%. This is the likelihood that convergence has not been falsely diagnosed. This small value is due to the improbability that convergence would exist in the first place (85% certain to be convergence free) and the failure rates associated with the testing method.

Now, if both tests yielded positive results, what would be the posterior probability of convergence? Calculating the joint conditional probability from these first principles can become daunting. As more convergence detection tests are added the operation becomes increasingly operose. How does one calculate conditional probabilities with large numbers of conditions? One way to render the calculation more computationally intuitive is through the use of Joint Probability Permutation Tables (JPTs) as in Figure 2.10.
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Figure 2.10: A joint probability permutation table for the Bayesian network example depicted in Figure 2.9.

This JPT shows all possible joint probability state permutations. The probabilities themselves are calculated using Equation (2.22). This equation uses the conditional probabilities derived in a similar manner as Equation (2.21), in conjunction with the chain rule Equation (2.12) to calculate the joint probabilities for each world. This process is laid out in Appendix B.1 but the entire process simplifies into a more compact equation as in:

\[
P(\omega_k) = P(T_1|C) \cdot P(T_2|C) \cdot P(C_k), \quad \text{if} \quad \omega_k \models T_1, T_2, C
\]

\[
P(\omega_k) = P(\neg T_1|C) \cdot P(T_2|C) \cdot P(C_k), \quad \text{if} \quad \omega_k \models \neg T_1, T_2, C
\]

\[
P(\omega_k) = P(T_1|\neg C) \cdot P(T_2|\neg C) \cdot P(\neg C_k), \quad \text{if} \quad \omega_k \models T_1, \neg T_2, \neg C
\]

\[
P(\omega_k) = P(\neg T_1|\neg C) \cdot P(\neg T_2|\neg C) \cdot P(\neg C_k), \quad \text{if} \quad \omega_k \models \neg T_1, \neg T_2, \neg C
\]

This equation considers the relative reliabilities of the convergence tests as well as the prior likelihood of convergence. The variable \(\omega_k\) refers to the world permutations in Figure 2.10 and each reliability or prior value used to calculate that world must match the states of that variable in that world. For example, world 1 is modelled by \(T_1, T_2\).
and $C$ as positive ($\omega_1 \models T_1, T_2, C$) so world 1 is calculated with the corresponding equation. Contrastingly, world 8 is modelled by $T_1, T_2$ and $C$ as negative ($\omega_8 \models \neg T_1, \neg T_2, \neg C$) and would use its corresponding variable states.

Once the JPT is populated with joint probabilities, it is possible to update these probabilities with new evidence. Returning to the example problem, the goal is to calculate $P(C|T_1, T_2)$ from the joint probabilities in Figure 2.10. The conditional probability $P(C|T_1, T_2)$ asks what is the probability of the presence of convergence given that test 1 and 2 are positive results. On the JPT, this can be queried by setting the probabilities of all worlds which do not satisfy this state equal to zero. This is because when the evidence, 2 positive test results, is added it makes the other world permutations impossible — i.e., there is now no probability of having a negative test result.

This leaves world 1 and world 5 untouched as in the intermediates shown in Figure 2.11(a). Now the remaining probabilities are multiplied by a constant so that each remaining possibility is equally scaled until the remaining total probability is equal to 1. This constant, $M$, is calculated as follows:

$$M = \frac{1}{\sum_{k}(\text{intermediate}_k)}$$  \hspace{1cm} (2.23)

Consistant with the law of total probability Equation (2.17), to calculate the probability of convergence, one simply sums the worlds in which convergence is positive.

This example is true for Boolean variables in the Bayesian network. Using the framework set out above, one can now apply this to fuzzy variables in lieu of Boolean
2.3. PROBABILISTIC ESTIMATION REVIEW

(a) Boolean update  

(b) Fuzzy variable update

Figure 2.11: Intermediate steps in updating the joint probabilities, given additional evidence.

In one example, test 1 is 80% compatible with a positive result and test 2 is 70% compatible. One still calculates the JPT in Figure 2.10 as before. This populates the table with relative certainties of the test results with respect to the existence of convergence. The fuzzy estimations are incorporated during the probability update phase of the JPT, as in Figure 2.11(b), and is referred to as soft evidence. This method of probability update is known as the all things considered model since it considers prior likelihoods and additional soft evidence [10].

The soft evidence is incorporated with:

\[
P'(\omega_k) = \begin{cases} 
\frac{q}{P(\beta)} P(\omega_n), & \text{if } \omega \models \beta \\ 
\frac{1-q}{P(\neg \beta)} P(\omega_n), & \text{if } \omega \models \neg \beta.
\end{cases}
\]  

(2.24)

Variable \( q \) is the compatibility value derived from membership functions like that in Figure 2.7 and \( \beta \) is a place-holder for the variable being considered; in this case, \( \beta \) is test 1 or test 2. When dealing with test 1, the top equation in Equation (2.24) applies to those worlds in Figure 2.10 where test 1 is positive while the second line
deals with those worlds where test 1 is negative. The expression $P(\beta)$ is the total probability that $\beta$ is either a positive or negative result. It is calculated with:

$$P(\beta) = \sum_k P'(\omega_k) \quad \forall \omega_k \models \beta$$

$$P(\neg \beta) = \sum_k P'(\omega_k) \quad \forall \omega_k \models \neg \beta.$$  \hspace{1cm} (2.25)

The resultant posterior probabilities, $P'(\omega_k)$ from test 1, are shown in the ‘Test 1’ column of Figure 2.11(b). This column then becomes $P(\omega_k)$ and the updating operation is performed again with the compatibility values of test 2. The results of this are in the ‘Test 2’ column of Figure 2.11(b).

What Equation (2.24) does is take existing world probabilities, $P(\omega)$, and multiplies them by a scaling value, $\frac{q}{P(\beta)}$, to achieve new probabilities for those worlds denoted $P'(\omega)$. The scaling factor scales the probabilities of $P'(\omega)$ such that the new sum of worlds where $\omega \models \beta$ is true, will equal the compatibility value ‘$q$’, as in:

$$q = \sum_k P'(\omega_k) \forall \omega_k \models \beta$$

$$1 - q = \sum_k P'(\omega_k) \forall \omega_k \models \neg \beta.$$  \hspace{1cm} (2.26)

Essentially, the higher the truth value for a particular state of indicator $\beta$, the higher the probabilities associated with worlds which contain that state of $\beta$. Conversely, worlds which show $\beta$ will sum to occupy an amount proportional to $1 - q$.

This concludes the background theory required to probabilistically amalgamate multiple uncertain measurements. This will be used heavily in Chapter 3 and specifically in Section 3.3.
Chapter 3

Proposed Method for Convergence Detection

This chapter contains the technical details describing the inner workings of the convergence detection system. Firstly, the general structure of the input data is described. Next the concept of ‘convergence indicators’ and a description of how they relate to the method of data collection are introduced. Following this is a detailed explanation of how the indicators are calculated and amalgamated, using the theories developed in Section 2.3, to give a probability of convergence.

3.1 Introduction of Method

The determination of drift convergence involves two distinct steps: 1) indicators are derived from drift cross sectional data; and 2) these indicators are amalgamated into a combined score or probability which indicates whether or not convergence may be prevalent.

The algorithm differs from current LiDAR scan change-detection algorithms where one attempts to directly compare two point clouds acquired at different times. What is different about this new approach is that it instead employs more macroscopic observations to infer whether or not convergence is occurring. This has at least two
3.1. INTRODUCTION OF METHOD

significant advantages: 1) it may produce fewer false positives (because singular errors will not be enough to suggest the presence of convergence); and, 2) the method is inherently robust to uncertainty in the acquired data meaning it does not require a highly-accurate, survey-grade positioning and/or expensive high-accuracy LiDAR system.

3.1.1 Data Structure

These macroscopic changes are extracted from cross sections of scanned mining drifts. First, 3-D point cloud data is gridded into a solid drift shell. This feature is standard in most mining software packages and allows section views to be sliced at any distinguishable area along a drift. Point cloud gridding generates a compete surface using point cloud points. This process is effectively 3-D interpolation and can be done with one of many methods; nearest neighbour, inverse distance weighting, triangulation and Kriging to name a few [20]. The solid drift shell now allows sections to be cut in spite of the distribution of point cloud points.

The data passing through the section is a 2-D line representing the drift perimeter where the section was taken. From here range measurements are defined from a single, arbitrary point within the perimeter of the drift to the drift edge. This transforms the definition of the drift perimeter from Cartesian coordinates to a set of polar ones. This polar coordinate form of data is more advantageous over its Cartesian counterpart because it not only defines the drift shape but also generates a signal plot of range versus angle around the drift. The advantages are explored later in Section 3.2.3 when signal analysis provides interesting ways to infer convergence.

In Section 3.2, the polar range data is assumed to be sampled at a half degree
resolution from the defined reference point. The sweep angle of the sensor is also assumed to cover 360 degrees of rotation, making 720 sample points overall. This defined data structure was chosen because it could easily be modified to fit the sensor output from the uGPS Rapid Mapper™. This is important because in Section 5.4 uGPS data, collected from a real mine, is passed through the convergence algorithm to detect convergence. The details of the uGPS scanner can be found in Section 4.1.

3.2 Convergence Indicators

This section exposes how the indicators are developed from the aforementioned cross-sectional range data. There are five indicators that are developed, in detail, in Sections 3.2.1 through 3.2.5.

3.2.1 Cross Sectional Area

In converging ground conditions it is expected that the cross sectional area of the drift is going to change. It is likely that the rock will begin swelling into the drift and the void cross sectional area will decrease over time.

The data is set up such that the range measurements $R(\theta)$, are sampled every 0.5 degrees $R(d\theta)$, over some sweep angle, $\theta$. To calculate an area from these range values one needs to integrate over the sweep angle of the scanner as in:

$$A = \int_{0}^{2\pi} R(\theta)d\theta. \quad (3.1)$$

The discrete nature of the range values, necessitates the change of the integral to
3.2. CONVERGENCE INDICATORS

a finite sum as in:

\[
A = 0.5 \times \frac{\pi}{180} \sum_{i=1}^{720} R(\theta_i). \tag{3.2}
\]

During data acquisition, it is possible that some range values would equal 0 or be marked as infinitely long. These values, along with any values that appear to be outliers, are filtered out in the area calculation algorithm. Outliers are defined as any point that is greater than a certain distance from all its neighbouring points. This distance was determined by trial and error and is calculated using the cosine law:

\[
d = \sqrt{R_i^2 + R_{i+1}^2 - 2R_iR_{i+1}\cos(d\theta)}, \tag{3.3}
\]

where \(R_i\) is the range value associated with the point in question and \(R_{i+1}\) is the range value of the neighbouring point being compared to. This is shown in Figure 3.1. The resulting distance threshold filter is shown in Algorithm 1. Tagged points are then ignored when calculating the summation in Equation (3.2).

![Figure 3.1: Shows how the perimeter length D is calculated using the cosine law and the available measurements from the scanner.](image)

This raises yet another problem with the area calculation. If some scan cross sections are using all 720 range values and some are using less, then the area values,
3.2. CONVERGENCE INDICATORS

Algorithm 1 Implementation of a filter which tags points which are beyond a certain ‘distance_threshold’ from their neighbours.

**Input:** $R_i, R_{i+1}$  
**Output:** $G_{\text{TAGGED}}$

1: function **Distance Filter**($R_i, R_{i+1}$) 
2: $C \leftarrow 0$  \Comment*{Initialize a counter} 
3: $D = \sqrt{R_i^2 + R_{i+1}^2 + 2(R_i)(R_{i+1})\cos(d\theta)}$  \Comment*{Compute $D$ with cosine law} 
4: if $D > \text{distance\_threshold}$ then 
5: $G_{\text{TAGGED}, C} = i$ 
6: end if 
7: end function

$A$, will not be directly comparable. That is, if the same cross sectional area were computed with 720 points and then with 700 points, the magnitude of $A$ would be different.

To get around this, the area is divided by the number of points used in the calculation (i.e., the number of non-tagged points) and then multiplied by 720, so that each area value is comparable to the true cross sectional area. A pseudo code implementation is shown in Algorithm 2.

### 3.2.2 Perimeter

Calculating the perimeter is similar to calculating the the area of the drift. Firstly, the data must be filtered, with anomalous points being tagged, as stated in Section 3.2.1. If left unfiltered, anomalous points could render the perimeter incorrect by an order of magnitude or more. Next, the perimeter is calculated by using the cosine law (3.3) applied between neighbouring points; $R_i$ and $R_{i+1}$ in Figure 3.1. These distances are then added together and form the perimeter metric, as in:

$$P = \sum_{i}^{720} \sqrt{R_i^2 + R_{i+1}^2 - 2R_iR_{i+1}\cos(d\theta)}.$$  \hspace{1cm} (3.4)
3.2. CONVERGENCE INDICATORS

Algorithm 2 Implementation of the area calculation algorithm with filtered points.
Input: $R$
Output: $A$

1: function AREA CALCULATION($R, G_{\text{TAGGED}}$) \Comment{Initialize area variable}
2: $A \leftarrow 0$
3: $C \leftarrow 0$ \Comment{Initialize counting variables}
4: $C_{\text{total}} \leftarrow 0$
5: for $i = 1 \rightarrow 720$ do
6: \hspace{1em} if $i = \text{Any}(G_{\text{TAGGED}})$ then \Comment{Count number of successive, un-TAGGED points}
7: \hspace{2em} $C = C + 1$
8: \hspace{2em} $C_{\text{total}} = C_{\text{total}} + 1$ \Comment{Count number of total un-TAGGED points}
9: \hspace{1em} else if $i \neq \text{Any}(G_{\text{TAGGED}})$ then \Comment{Sum area with un-TAGGED points}
10: \hspace{2em} $A = A + R_i \times 0.5 \times \frac{\pi}{180} \times C$
11: \hspace{1em} $C \leftarrow 0$
12: \hspace{1em} end if
13: end for
14: $A = \frac{A \times 720}{720 - C_{\text{total}}}$ \Comment{Account for number of un-used range values}
15: end function

The process of filtering out the tagged points $G_{\text{TAGGED}}$ is shown in Algorithm 1.

The pseudo code in Algorithm 3 shows how the tagged values are omitted from the perimeter calculation process.

Algorithm 3 Implementation of the perimeter calculation with filtered points.
Input: $R, G_{\text{TAGGED}}$
Output: $P$

1: function PERIMETER CALCULATION($R, G_{\text{TAGGED}}$) \Comment{Initialize perimeter variable}
2: $P \leftarrow 0$
3: $C \leftarrow 0$ \Comment{Initialize counting variable}
4: for $i = 1 \rightarrow 720$ do
5: \hspace{1em} if $i = \text{Any}(G_{\text{TAGGED}})$ then \Comment{Count number of successive, un-TAGGED points}
6: \hspace{2em} $C = C + 1$
7: \hspace{1em} else if $i \neq \text{Any}(G_{\text{TAGGED}})$ then \Comment{Sum perimeter}
8: \hspace{2em} $P = P + \sqrt{R_i^2 + R_{i+1}^2 + \cos(d\theta \times C)}$
9: \hspace{2em} $C \leftarrow 0$
10: \hspace{1em} end if
11: end for
12: end function
3.2.3 Spectrum Analysis

The spectrum analysis takes range values and calculates the Fast Fourier Transform (FFT) of the data. The idea is that the macroscopic changes in drift shape will be captured as amplitude spikes in the lower frequency domain of the Fourier transform. If one thinks of a complete circular scan of the drift as a single period, it makes sense to consider the swelling of the walls as an addition of a range modulation that occurs twice in a single period. For example, in Figure 3.2 the ranges on the right and left of the centre are shorter, while the ranges on the top and bottom are elongated. This particular modulation constitutes a radius change of $0.25 \times \cos(2\theta)$. By taking the FFT of the difference in radii which make up these drifts, one can see a spike in the 2-Hz column of the FFT chart.

In order to take the FFT of the range data, one must condition the data to be symmetrical. What this means is that the range values cannot simply be put through a FFT function as they were taken. If the scan was taken away from the geometrical centre of the drift, this would induce frequency domain noise in the resultant data. Consider the perfectly round drift shown in Figure 3.2. Range data taken from the geometrical centre would appear to be a constant value, because the range is never changing. If the range values were taken from near the base of the drift, the range data would look like the blue line in Figure 3.3.

Notice that by not taking the range values with respect to the geometric centre, the perfectly circular drift would now have an artificially induced 1-Hz sine wave in the signal data.

Even when one takes the scan ranges from the perfect geometric centre of the drift,
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Figure 3.2: An oval drift superimposed on a circular one. Used to illustrate how shape affects range signal data.

there is still another aspect to consider before taking the Fourier transform. The constant two-metre radius constitutes what is known as DC bias in the Fourier transform. This manifests itself as very low frequency noise (near 0 Hz) in the frequency domain. To reduce this unwanted signal noise, the average value of the cross-sectional radii is computed and subtracted from all the range values. To illustrate this point, Figure 3.4 shows the Fourier transform of the perfectly circular drift range data both with and without the aforementioned filtering steps.

This final manipulation of the range data allows the FFT to be taken. What follows is a practical explanation of how these steps were done.
Figure 3.3: Radial distance scan data from a perfectly circular drift. Black line represents ranges from the origin while the blue line represents ranges originating lower in the drift.

In order to calculate the geometric centre of the data, one must first build a grid of fixed resolution representing the scan data so that differences in scan resolution around the drift do not affect where the geometric centre will be placed. This fixed resolution grid is known as an occupancy grid [15], and can be generated with the help of a variety of open source packages. The geometric centre is calculated with a
3.2. CONVERGENCE INDICATORS

Figure 3.4: A Fast Fourier Transform (FFT) of the circular drift. In blue is the FFT resulting when the drift centre and average radius are accounted for. The red shows what noise this process has eliminated.

The standard centre of mass formula, as in:

\[
\begin{align*}
\bar{x} &= \frac{1}{M_t} \sum_i m_i x_i \\
\bar{y} &= \frac{1}{M_t} \sum_i m_i y_i,
\end{align*}
\]  

(3.5)

where \( M_t \) is the number of unoccupied blocks, \( m_i \) is the mass of each unoccupied block (set equal to 1) while \( x_i \) and \( y_i \) are the distances of that block from the origin.
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in the $x$ and $y$ directions respectively.

With the geometric centre of the cross section evaluated, it is now possible to calculate new range values from the centre of mass to the previously projected points. For the sake of clarity, let the initial range values be $R_{\text{init}}$ with Cartesian coordinates $x$ and $y$ while the range values with respect to the geometric centre shall be referred to as $R_C$. The range values $R_C$ will point from the centre of mass of the drift to the $x$ and $y$ points laid out from the $R_{\text{init}}$ range data.

The $R_C$ values are evaluated as:

$$x_C = x - \bar{x}$$

$$y_C = y - \bar{y}$$

$$R_C = \sqrt{x_C^2 + y_C^2}$$

Next the average $R_C$ range is evaluated and subtracted from all $R_C$ values. This final signal data is fed into a FFT algorithm and the frequency amplitude information is generated.

3.2.4 Principal Component Analysis

Principal Component Analysis (PCA) is a tool that exposes the co-variance structure of data. The PCA process outputs which direction in a set of data has the largest co-variance, followed by a linearly independent direction with next largest co-variance, and so on. In a two dimensional data set, like in the $x$ and $y$ cross sectional scan data, PCA will return two axes. The first axis, with the largest co-variance, will be the principal component. The second axis will be along the direction of least co-variance and will always be perpendicular (linearly independent) of the principal axis [18,35].
3.2. CONVERGENCE INDICATORS

Put in the context of convergence detection, principal co-variance directions show in what direction the drift is expanding and in which direction is it converging. The correlation between co-variance and drift width exists because a set of data with points far from one another will inherently have a larger co-variance. Consider Figure 3.5, with the lines indicating the principal component of each oval. The principal component output will always point to the direction of greatest co-variance, which in this case is the widest portion of the drift.

Computing the principal component direction requires determining the eigenvalue of the $x$ and $y$ scan co-variance matrix. The the co-variance matrix is:

$$M = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}. \tag{3.9}$$

For this matrix, the variance $\sigma_x^2$ and co-variance $\sigma_{xy}$ are calculated as in Equation (3.10), where variable $n$ is the number of samples and $\bar{x}$ and $\bar{y}$ are the average values of $x$ and $y$. As discussed in previous sections, the average $x$ and $y$ values for the drift will not represent the actual geometric centre of the drift because of differences of the scanner point density around the circumference of the drift. In this case, $\bar{x}$ and $\bar{y}$ are replaced by $x_C$ and $y_C$, the centre of mass coordinates computed in Section 3.2.3, namely:

$$\sigma_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - x_C)^2$$

$$\sigma_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - x_C)(y_i - y_C). \tag{3.10}$$
Figure 3.5: This figure shows the result of two PCA analysis.
The eigenvalue of matrix $M$ is defined as the determinant:

$$\det(M - \lambda I) = |M - \lambda I| = 0,$$

(3.11)

where $\lambda$ is the desired eigenvalue matrix and $I$ is the identity matrix.

Additionally, computing the principal component direction requires determining the eigenvector of the co-variance matrix $M$. An eigenvector $x$ of matrix $M$ must satisfy:

$$Ax = \lambda x.$$  

(3.12)

There exist recursive estimation models that calculate eigenvectors and their associated eigenvalues, so these scripts are not new for this work [3, 31, 36].

Convergence with PCA is measured as a ratio of the largest eigenvalue magnitude over the smallest eigenvalue magnitude. This metric can be described as the aspect ratio of the drift, since it is a ratio of two perpendicular measurements which describe the widest and narrowest sections of the drift.

By using all the scan points for this calculation, it is expected that a few, outlying points will have little influence over the PCA output. The theory is that a change in the overall shape of the drift will have a greater effect on the PCA aspect ratio than any small group of anomalous data points. Consider the example in Figure 3.5(b), where one point has been moved out of position from 3.5(a). Notice how the overall direction of the PCA vector is unaffected, while the magnitude of the primary component changes by only 5 %.
3.2.5 Phase Angle

Phase angle is a term used when discussing the misalignment of periodic signals. In the context of this convergence monitoring algorithm, phase disruption refers to angular misalignment of cross sectional scans. This indicator is not used directly in the Bayesian amalgamation to determine convergence, however it is a natural by-product of the principal component analysis. The phase angle was included because it could be used after the convergence detection algorithm to determine in what direction the drift is experiencing convergence.

To illustrate how phase shifts correlate to directions of convergence in a drift, consider the cross sections presented in Figure 3.6. Figure 3.6(a) is the cross section of two drifts. Each drift only differs from the other by a rotation of $\frac{\pi}{4}$ radians. Figure 3.6(b) shows the range data with respect to scan angle. It can be seen that a rotational change in the drift cross sections will tend to manifest itself as a phase change in the scan data. This type of change could occur in a drift as a result of uneven loading in the rock mass caused by introducing new excavations.

The direction of the principal component can be calculated by taking the arctangent of the principal eigenvector—the one with the largest associated eigenvalue. With this, successive scans can compare the direction of their principal component vectors.

3.3 Bayesian Amalgamation

In an idealized scenario, each of the four indicators would perfectly identify convergence. In other words, in a perfect world it would be possible to determine the convergence magnitude with high accuracy, with the indicator measurements never
Figure 3.6: Demonstration of how a rotational drift change will manifest itself as a phase change in the frequency domain of the drift signal.
deviating from the actual convergence of the drift. In practice, this is not the case. Sometimes the indicators are confronted with a drift scan that causes some anomalous spike in value. These errors can be caused by any number of factors. Errors such as a laser beam refracting away from the range sensor or being absorbed by the material would, for example, cause false (large) range values to be measured. Conversely, dirt on the sensor or dust clouds surrounding the area could cause artificially small laser range values. If one were to always trust a single indicator, one would be allowing the occasional miscalculation of that value to whole-heartedly influence one’s perception of drift convergence, even if the value makes no practical sense. For these reasons, a Bayesian Network (BN) is used to amalgamate information from the multiple indicators in order to derive an estimate of the probability of convergence.

The following sections first outline how this BN is set up, followed by an explanation for how these BN probabilities were updated using the principles outlined in Section 2.3. The chart in Figure 3.7 serves to depict a high-level overview of how the available data is amalgamated to produce the final probability of convergence.

3.3.1 Initialization

In this convergence detection algorithm, convergence is inferred from multiple computed indicators. Figure 3.8 shows this implicative relationship in the form of a Directed Acyclic Graph (DAG) between convergence and the aforementioned indicators. Similar to the convergence example in Section 2.3.2, the indicators are caused by convergence occurring. The change in indicator values serve as the tests for either confirming or negating the probability of convergence. As before, determining convergence from these test is just a form of conditional probability; \( P(C|T_1, T_2, T_3, T_4) \).
3.3. BAYESIAN AMALGAMATION

Figure 3.7: A flow chart depicting how the raw convergence indicators are amalgamated into a probability of convergence.
In this specific instance of the algorithm, each node is given three states: 1) high; 2) low; and, 3) negligible. For example, the convergence node will indicate either high, low or negligible convergence while the indicator nodes will show measurements consistent with either high, low or negligible convergence. These measurements are derived from laser scans of underground cross sections as described in Section 3.2, which are then passed into membership functions (MF) such as the one in Figure 2.7 on page 19.

Firstly, before measuring the drift, the reliability of each indicator is considered and assigned a value from zero to one. These values are comparable to false negative rates in 2.3.2. Having three possible states, however, necessitates a slightly larger number of conditional statements. These are summarized in Figure 3.9.

The symbols in the figure are; ++, + and ¬ for high, low and negligible convergence respectively.
In practice, these values may be assigned by studying acquired data, using the experience of a qualified practitioner of the system or through supervised learning algorithms which evolve the probabilities based on known data [5]. These are simply preliminary numbers for the sake of explaining the BN amalgamation.

According to the law of total probability (2.16), the sum of every conditional probability of any one indicator state must be one, since that represents all the possible permutations of that state; as in:

\[ 1 = \sum_n P(+ +|C_n) \]  

(3.13)

where \( C_n \) can be \(+ +\), \(+\), \(+\), or \(\neg C\).
Note that the probabilities for the convergence indicators reading negligible convergence are equivalent to those reading positive convergence, since errors favouring positive or negative test results are not expected, as in:

\[
P(+_{SA} | ++C) = P(+_{SA} + C) = P(+_{SA} | -C) \\
P(+_{SA} | +C) = P(+_{SA} + C) = P(+_{SA} | -C)
\]

These reliability values are then used to calculate the probabilities of different permutations in the Joint Probability Permutation Table (JPT). Recall from Section 2.3.2 that each permutation is known as a world and has a probability of existing given the state and associated reliability of each variable in that world. These joint probabilities serve to weight the reliability of the indicators against themselves. The example JPT, Figure 2.10, has 3 variables with 2 possible states, totalling 8 permutations. This relationship can be summarized as:

\[
n_{\text{worlds}} = (n_{\text{states}})^{n_{\text{variables}}}
\]

The conditional probability \( P(C|T_1, T_2, T_3, T_4) \) has 5 variables which can occupy 3 different states. Therefore, the JPT must have 243 worlds. This very large JPT can be found in Appendix A.2 on page 99.

### 3.3.2 Indicator Amalgamation

The derivation of the Joint Probability Table in Appendix A.2 is independent of the actual convergence monitoring measurements. This means that it can be modified
3.3. BAYESIAN AMALGAMATION

and generated in advance as a look-up table.

Returning to the workflow presented in Figure 3.7, once the reliability of each indicator is considered and added to the Joint Probability Table, these probabilities are updated with compliance values from the membership function.

The membership functions (MF) are calibrated based on experimental data for each indicator. As mentioned in Section 2.3.1, the purpose of the MF is to assign an indicator value into banks corresponding to linguistic variables representing high, low or negligible convergence. The membership function also assigns the probability that that indicator value is a member of that MF bank. In this way, the membership function weighs the certainty of any one indicator’s claim to represent one of the three states of convergence.

As in Section 2.3.2, the MF outputs are then used to update the JPT in Appendix A.2 using a three state implementation of equation (2.24), as in:

\[
P' (\omega_k) = \begin{cases} 
\frac{q_{++} + \beta}{P(++)} P(\omega_k), & \text{if } \omega_k \models ++ \\
\frac{q_{+\beta}}{P(+\beta)} P(\omega_k), & \text{if } \omega_k \models + \beta \\
\frac{q_{\beta}}{P(-\beta)} P(\omega_k), & \text{if } \omega_k \models - \beta 
\end{cases} \quad (3.16)
\]

where \(\beta\) can be any one of the indicators.

The intended outcome from applying such a method is to balance evidence and determine what level of convergence is actually occurring, despite general uncertainty with respect to the measurements and the implied uncertainty of the linguistic variables.

The percent likelihood of either of these levels of convergence is determined from the joint probability lookup table by summing those worlds where convergence is
either high, low or negligible, as in:

\[ P(C|\beta) = \sum_k \omega_k \quad \omega_k \models \beta \quad (3.17) \]

where \( C \) and \( \beta \) can be in any one of the three available states and \( \beta \) can incorporate all four indicator states.

This probability update holds true for the three-state example for as long as the sum of \( q \)-values in Equation (3.16) equals 1, as in:

\[ q_{++\beta} + q_{+\beta} + q_{-\beta} = 1. \quad (3.18) \]

This renders the system “sum normal”, meaning that it covers all possible outcomes of the system. As a result, the sum of all the worlds, \( \omega \), will still equal 1 after all the manipulations [17].
Chapter 4

Experiment Data Acquisition

In order to test the proposed method outlined in Chapter 3, both simulated and field data were collected to run through the probabilistic convergence algorithm. The simulated data was designed to resemble the output from the sensors used in the experimental field data. This was done so that only one instantiation of the algorithm was needed to serve as a tool for processing both data sets.

This chapter introduces the format of the collected field data and measurement procedures to ensure reproducibility of the experiments. Lastly, the limitations of the field data are examined.

4.1 The uGPS Rapid Mapper™

The uGPS Rapid Mapper™ was provided by Peck Tech Consulting for use in this experiment. The device, as seen in Figure 4.1, consists of two SICK LMS111 lasers and a 3-D inertial measuring unit (IMU) with triaxial gyroscopes.

This scanner generates LiDAR scan maps of underground excavations on a mobile platform in spite of the limitations associated with mapping without access to global
position reference points of some kind (like satellite-based GPS, not available in underground environments). The uGPS was designed to be mounted to a vehicle and driven around a mine to generate point cloud maps with a single pass. It estimates its position in small time-step increments - about 10Hz - through double integration of the IMU. This estimation is then corrected by matching scan results from the horizontally mounted SICK laser scanner near the bottom of the uGPS. This motion estimation is based on the work of N. J. Lavingne and J. Marshall [1,25].

By estimating and correcting with the IMU and horizontal laser scanner respectively, the uGPS is able to localize the position of its vertical laser scanner relative to the starting point. This starting point can also be tied into mine coordinates with the use of RFID tags.

The vertical SICK laser captures range measurements from the SICK to the drift perimeter. Using the estimation of its current position, these range measurements allow for the projection of a planar point cloud representing the cross section of the drift. Moving along the drift, more of these rings are measured and positioned in space using the uGPS motion estimate. The result is a point cloud made up of concentric rings based on polar range measurements, like the drift shown in Figure 4.2.

This data is very similar in form to the data proposed in Section 3.1.1. The only difference is that the SICK LMS 111 has a sweep angle of 270°, projecting 541 points on the drift walls and back.
4.2. DATA COLLECTION AND USE

4.2 Data Collection and Use

4.2.1 Details of Collection

The experiment was set up in an underground block caving mine known to have significant convergence issues. The anonymous mine site was not willing to have its name published with the data but was very generous in providing an actual testing ground for the convergence algorithm. This mine site was already using the triangular measurement method with a disto meter to track convergence, as shown in Figure 2.5 on page 14. The idea was to collate the industry standard measurements from surveyors with the output from the convergence algorithm. These surveyed results are discussed at greater length in Section 5.4.
The experiment was set up by mounting the uGPS on the front of an underground mining vehicle via a custom made hitch, depicted in Figure 4.3. The unit was mounted on the front to minimize the interference of vehicle generated dust with the scanner. The hitch was wrapped in a thick layer of duct tape to help dampen vibration transfer within the receiver. This also acted as a stabilizer so that the mount did not tend to move in the receiver.

The uGPS IMU and gyroscopes were calibrated every time that the hitch was mounted. This was done by parking the vehicle and scanner on a poured bed of concrete, underground, in the same place each time. The calibration process was automatic and consisted of taking average IMU readings over 10 seconds and using that number to zero the sensors for that day.

Driving down the drifts was done similarly each time. The same path was followed.
Figure 4.3: The underground surveyor vehicle with the front mounted uGPS unit.
from beginning to end, always in the same direction and at a consistent speed of 3-5 kph.

4.2.2 Details of Use

The uGPS produces point clouds as its default output — $X, Y$ and $Z$ of points in space. In order to be more useful for the convergence monitoring algorithm, the raw cross sectional range data were needed from each drift scan. Additionally, the positions and orientations of these scans needed to be extracted as well, since this is not part of the $X, Y, Z$ point cloud data.

As mentioned, the position and orientation was handled by an established algorithm written by J. Lavigne for the uGPS Rapid Mapper, as outlined in his thesis [24]. In this algorithm, the raw sensor data from the uGPS Rapid Mapper is used to generate an initial map of the drift. This map takes the form of an occupancy grid. The occupancy grid is essentially a plan view of what the drift looks like to the horizontally mounted SICK laser. The raw sensor data from a subsequent scan can then be projected into the drift occupancy grid. The horizontal laser data from the second scan can match its expected location in the original scan using the occupancy grid and its expected range values based on its estimated location. This is similar to the implementation of the fast polar scan matching algorithm presented in [11].

Doing this localization method with the uGPS sensor data means that one is able to extract the position and orientation of the vertical laser scanner. The range values from the vertical scanner are what is needed as inputs to the convergence monitoring algorithm. Knowing the scanner position allows one to compare the cross sections of subsequent scans.
4.3 Limitations of Data

The data was collected consistently from test to test and was applied to the same base map each time as outlined in Section 4.2.2 over 5 months of data collection.

In spite of this, the scan data was still subject to error which limits how the data should be interpreted. The main concern relates to how well the scans were localized with respect to one-another. This type of error can be summarized as being composed of three main types:

- IMU drift
- rotational alignment of scans
- perpendicular association error

The IMU drift is normally caused by cumulative error after double integrating acceleration values in order to form a position estimation. This type of error is different from normal, Gaussian noise because the integration tends to stack errors from one position to another. Looking at Figure 4.4, one possible explanation for why this very straight drift appears bowed is that IMU drift is somehow not being optimally corrected. Errors like this tend to make data association between drift scans less accurate.

The rotational alignment between scan slices of the same drift can have a noticeable effect on the convergence indicators. This type of error is shown in plan view in Figure 4.5(a). This occurs when a cross section from the baseline scan is matched to a cross section from a subsequent scan which is close in proximity but, due to differences in the driven path, do not have aligned orientations.
4.3. LIMITATIONS OF DATA

Figure 4.4: The point cloud of a perfectly straight drift, and another pass of the same drift displaying effects which could be attributed to IMU drift.

Despite being in the same location, the cross sections differ slightly from scan A to scan B as in Figure 4.5(b). This distortion resembles an overall change in drift cross section, despite only being a change of perspective. This can cause false positive readings from the convergence algorithm.

This problem was addressed by using the known orientation values of each scan.
4.3. LIMITATIONS OF DATA

Figure 4.5: How rotational error affects cross section shape.

cross section and projecting the range vectors of scan B onto the plane formed by scan A, as follows:

\[
\overline{R}_i \parallel \overline{N}_A = \frac{\overline{N}_A \times (\overline{R}_i \times \overline{N}_A)}{|\overline{N}_A|^2}
\]

where \( \overline{N}_A \) is the normal of scan plane A and \( \overline{R}_i \) are the 541 range vectors from scan B. Note that this operation is done for each range vector in scan B where \( i = 1 \rightarrow 541 \).

Perpendicular association error, or out of plane error, is the most difficult source of error to address. This type of error is impossible to avoid and results from scan planes, from one scan pass to another, not being perfectly coincident along the vehicle direction of travel as shown in Figure 4.6. Here the initial pass, in blue, has an out of plane error ‘D’ from the second scan pass in grey.

Comparing scan planes that are not perfectly coincident introduces some uncertainty. They do not scan the exact same cross sections, so there could be missing features in between the scan sections or possibly an expected change of drift cross
4.3. LIMITATIONS OF DATA

Figure 4.6: Two point cloud segments showing the out of plane ring error, D.

section from one scan to another that might be misinterpreted as convergence due to the misalignment.

This type of error is minimized by using two methods. With one, collecting data at a slow rate of speed makes the cross section spacing smaller and more dense, generating rings which are less than a few centimetres apart. This should capture all but the smallest features on the drift walls. In another, the convergence indicator values are linearly interpolated between scan slices, lowering the uncertainty between rings even further.
Chapter 5

Experimental Validation

The convergence detection algorithm was tested using three types of data sets. Firstly, drift shapes approximating convergence were simulated and run though the algorithm. Next, the simulated convergence shapes were applied to real drift cross sectional data and evaluated. Lastly, laser data from an underground mine experiencing convergence was examined.

The simulated convergence shapes are known as convergence modes and are introduced first. The first cross-sectional simulations are intended as a validation for whether or not overall cross sectional shape change could be detected and used by the convergence monitoring algorithm to detect drift change. Real drift data was then modulated by these shapes in order to determine if the irregular shapes of mining excavations interfere in any way with the ability to measure these convergence modes.

The section concludes with a comparison between surveyed convergence estimates and the output of the convergence monitoring system.
5.1 Simulated Convergence Modes

Four convergence mode shapes — concentric, oval, arch drift and toe spalling — have been developed in accordance with the theories presented in Section 2.1 and presented in Figure 5.1. Figure 2.3 shows 3 common cross sectional manifestations of convergence. Shapes shown in Figures 5.1(a) to 5.1(c) were designed to match these typical convergence modes. The concentric convergence mode resembles uniform shear failure while the oval convergence mode, as it is shown, mimics buckling drift failure. The arch drift convergence mode resembles plane sliding failure.

Additionally, the arch drift mode models the shape of a more typical mining drift. The arched back shape is common, for strength, but can succumb to spalling causing it to recede over time. The walls also tend to swell under high, tangential, compressive loads while the floor remains graded — a usual requirement for accessible drifts.

The toe spalling convergence mode does not follow any of the previously discussed convergence mechanisms but it does tend to occur underground [7]. It is especially prevalent in mines where blasting above a previously filled stope or drift is the primary method of mining. During the development of the first cut, the back and walls begin to degrade as a result of blasting damage and tensile spalling from underground stress fields. When this cut, or stope, is filled and a new cut is driven on top, the previous fractured and degraded walls now make up the base of the walls and pillars of the new cut. Stresses generating convergence would be able to plastically deform the drift inward — near the back — but the already degraded rock — near the toe — would be more likely to fragment given its loading history [8]. This combined with the constant attrition from an LHD bucket during the mucking process of any mining operation can cause the toe of the walls to become undercut.
5.2 Preliminary Simulation

The convergence monitoring algorithm was tested on the four simulated convergence modes shown in Figure 5.1. The convergence indicator responses were recorded for incremental drift convergence from zero to 0.5 metres of diameter change. One of these plots, shown in Figure 5.2, demonstrates the convergence response using the oval convergence mode. Initially, the oval was stretched horizontally then progressively compressed vertically through small incremental change. Over 100 increments, the oval was made to change into 0.5 metres of convergence where the drift is compressed.

Figure 5.1: The shape of the four simulated convergence modes.
horizontally and stretched vertically.

The resultant indicator values are shown on the $Y$-axis of Figure 5.2 and the level of convergence on the $X$-axis. Notice how the 2 Hz spectral amplitude values and cross sectional area change values are highly correlated to the amount of drift convergence. Phase angle also shifts 90 degrees when the converging drift changes from converged vertically to converged horizontally - an effective convergence change of 90 degrees. Some of the less consistent metrics appear to be perimeter change and principal component analysis which have anomalous spike values as the drift changes.

Examining the toe spalling indicator response in Figure 5.3 one can see a similar correlation with respect to convergence magnitude and the resultant indicator values.

In contrast to these observations, concentric convergence indicator responses in Figure 5.4 show that cross sectional area change is the most correlated indicator to this type with no other closely related responses.

Finally, arch drift convergence shows strong correlation with all of the convergence indicators as shown in Figure 5.5.

What one can take away from this is that macroscopic changes in drift cross section can be inferred from the convergence indicators. It should be noted that the indicators are not always going to be consistently reliable. Some will be better at detecting certain modes of convergence than others. Their relative reliability can be estimated through testing and factored in to the Bayesian amalgamation of the indicators to form convergence estimations. These prior probabilities can also be changed depending on what type of convergence one is expecting to see. Modifying the prior probabilities in this way gives the convergence algorithm great versatility based on human expert opinion and context.
Figure 5.2: Indicator response from simulated oval convergence.
Figure 5.3: Indicator response from simulated toe spalling convergence.
Figure 5.4: Indicator response from simulated concentric convergence.
Figure 5.5: Indicator response from simulated arch drift convergence.
5.3 Advanced Simulation

5.3.1 Two Dimensional

Using the convergence modes previously described, artificial convergence could be added to actual drift cross sections. This cross section was randomly selected from a drift scan taken at the volunteer mine site. Section 5.2 endeavoured to show how convergence indicators could be used to detect cross-sectional changes. This section aims to show that cross-sectional changes can also be extracted from strange drift shapes in order to be used to detect convergence and are not necessarily relegated to purely simulated data.

All four convergence modes shown in Figure 5.1 were applied to random mining cross sections as shown in Figure 5.6. The resulting convergence indicator responses for concentric, arch and toe spalling convergence modes are shown in Appendix A.1. Figure 5.7 displays how the oval convergence mode affected the convergence indicators. Through inspection, it can be observed that modulating the real drift cross section yielded even closer correlations between indicator response and convergence magnitude than in the simulated data. Turning to the indicator responses shown in Appendix A.1, one can see that this is true for two of the three remaining convergence modes, with concentric convergence being the only deviation from this.

The increased reliability as compared to the purely simulated data could be possible because of extended precision errors when generating and subsequently imitating drift scanning on the simulated cross sections. This likely comes about because the drift walls are generated from their exact centre, while the drift range values are also obtained, through ray casting [34], from a calculated centre. It could be that some error in the ray casting algorithm is causing the scan simulator to miss the drift walls
and register erroneous values. This idea is strengthened when one observes that drift data not derived from an exact centre — such as the measured cross sections or the arch convergence mode — are not subject to the same indicator discrepancies.

The exception to this is the real drift data subjected to the concentric convergence mode in Figure A.1. Similar to its simulated counterpart in Figure 5.4, there is not a strong correlation between concentric convergence and any of the indicators excepting cross sectional area change - which one expects to show a strong correlation, given the prescribed mechanism of convergence.

5.3.2 Three Dimensional

Moving on from tests on singular cross sections, similar tests were run on point cloud data from a mining drift. Here, all four convergence modes were imposed on sections of a drift and run through the convergence algorithm. The results are shown in Figure 5.8 where the red regions represent a greater likelihood of convergence and the blue show no convergence at all. The four convergence modes are: oval, arch drift, toe spalling and concentric from left to right. The real convergence distribution can be seen in Figure 5.9. The red regions indicate a convergence magnitude of 0.25 metres in the direction of the convergence mode.

One can see at first glance that the convergence algorithm was able to detect the converging regions. This shows that changing the cross section shape while subjecting that cross section to the convergence modes was not enough to render the algorithm unable to differentiate convergence. The range values for the converged drift were also modulated by Gaussian noise with a standard deviation of 10 cm. The convergence algorithm was still able to extract convergence despite Gaussian noise well above the
Figure 5.6: Actual mining drift cross sections with three convergence modes applied with a magnitude of 0.5 metres.

quoted noise for the uGPS SICK laser of ±3 cm [33].
Figure 5.7: The convergence indicator response for oval convergence applied to a real drift cross section.
Figure 5.8: A convergence probability map showing the results of the convergence algorithm on simulated data.
Figure 5.9: A convergence probability map showing the actual imposed convergence on the drift.
5.4 Test on Field Data

The field data was acquired at an anonymous mine site over a period of 6 months in a region of the mine which was expected to show convergence. This mine site uses block caving to extract the mineral wealth. An example of this mining method is depicted in Figure 5.10.

Figure 5.10: A diagram of a typical block caving operation [14]

In the north-west section of Figure 5.10, one can see the upper holes drilled to generate a bell for caving material to flow into. This bell will lead to a drawbell drift near its base where material accumulates. On the north-east section, the drawbell drift is on the left. To either side of the drawbell drift the cross-cutting panel drifts permit access to the accumulating material from the bell. The drawpoints are excavations cut from the panel drift to the drawbell drift where load-haul-dump machines
can actually interact with the material. It is in these panel drifts where convergence is most expected during block caving operations.

5.4.1 Convergence Survey Data

Convergence was expected to take place approximately 15 metres back from the ‘caving line’. In Figure 5.10, the hypothetical converging region would exist where the solid, non-caving, host rock above the excavations in the north-east section actually meets the lower rock mass which makes up the drift walls. This intersection marks the area where the overhung host rock is actually being supported by the rock mass below. This concentration of stress is known as abutment loading [7] and tends to induce the kind of plastic deformation one associates with convergence.

The map shown in Figure 5.11 shows a plan view of the drift of interest in the test mine. The green arrow indicates the approximate direction of propagation of the caving line.

The charts in Figure 5.12 show the displacement measurements taken by the mine survey team in that drift. The measurements were done with the triangle survey pattern, as in Figure 2.5, on the pillars between drawpoints. The pillars shown on the map are reflected on the convergence charts.
5.4. TEST ON FIELD DATA

Figure 5.11: A plan view of the mining drift showing the direction of mining and the labels for pillars.

Due to the time consuming nature of this manual survey method, the data density is not ideal and sometimes external factors mean that the cross section measurement could not be taken for certain pillars on certain days. These gaps in the data adversely affected how it could be interpreted.

Looking at Figure 5.12(a), the data shows a reduction in drift height at certain pillars over the six month period. Interestingly, these jumps in measurement mostly occur from left to right in Figure 5.11, apparently following the direction of the caving advance. Due to the irregular measurement periods for convergence, these convergence results must be taken loosely and non definitively. One gap, for example, in vertical convergence data between October - November at the 705 pillar make determining when vertical convergence occurred almost impossible. It might have occurred anywhere between October 20 and November 31. This, on its own, makes the data difficult to interpret but when one attempts to compare the 705 pillar to its neighbours, such as the 703 or 707 pillars, inferring a definitive progression of convergence becomes almost impossible. Pillar 707 could have converged any time between mid October and mid November while pillar 703 has no data available after the month of September.
5.4. TEST ON FIELD DATA

Figure 5.12: Drift measurements taken over 6 months at the mine production level.
5.4. TEST ON FIELD DATA

The drift width measurement data in Figure 5.12(b) also show loose evidence of this sequential loading pattern. Over the first 3 months there appears to be very little motion, with the exception of the massive data inconsistency in the 707 pillar. Survey notes indicate a ‘change of survey markers’ after October, possibly due to sloughing of the walls.

From early November onward the ‘705’, ‘707’ and ‘709’ pillars began showing horizontal convergence. When one compares the dates from 5.12(a) to 5.12(b), one can see some evidence of rheologic hysteresis as the horizontal convergence lags behind the application abutment loading by 2 months for the ‘707’ pillar and 3 months for the ‘709’ pillar. Again, this is difficult to say with confidence because huge data gaps, such as from November to January at the 705 pillar, fail to show exactly when convergence was occurring.

5.4.2 Convergence Algorithm

The convergence monitoring algorithm was run on data sets for each month from August 25th to December 1st while the uGPS unit was on loan to the mine site. The August 25th data was used as the baseline for comparison with the other four data sets. The four data sets were used to produce three plots each, which can be found in Appendix A.3. Firstly, a colour heat map representing convergence likelihood was produced. Next, two plots showing how the various indicators responded were produced.

In the convergence heat maps, such as the one in Figure 5.13, the blue areas represent low convergence probability while the red areas represent high convergence probability. The black regions represent areas which had a diameter change greater
than 1 metre. Areas with this level of diameter change were excluded from the convergence algorithm since changes of that magnitude were not expected in the real context of convergence. Changes of this magnitude were usually expected in the drawpoints, where the drift widens in Figure 5.13. This is because in an active mine, muck piles are constantly being removed and re-generated throughout the test interval. However, certain pillars between drawpoints are also being coloured black, indicating that they are being excluded from the convergence algorithm due to a diameter change greater than 1 metre. Looking at Figures A.5 through A.7 on page 105, one can begin to see why the drifts are being coloured black in these unexpected areas.

In Figure A.5 the grey drift is the base scan from August 25th to which the September scan is being compared. Notice that the alignment of the two scans apparently differs by almost 50% of the drift diameter towards the right side. Additionally, one can see that the edges which mark the beginnings of drawpoints are not well aligned. Due to the high variability in diameters between drawpoint cross sections, the data becomes too distorted to use.

This September scan represents the data with the most blacked out pillar sections. The October scan shown in Figure 5.13 represents a near perfect match, showing all the pillars and displaying the amount of convergence one would expect only 2 months into the experiment. Unfortunately, as time proceeded, the scan matches did not become any better. Figures A.6 and A.7 each show a tremendous amount of bowing at levels greater than 100% of the drift diameter. The perpendicular alignment, as a result, is just not accurate, rendering the comparison with the convergence algorithm inaccurate. The December scan data shown in Figure A.7 is also missing pillars ‘703’,
Figure 5.13: A convergence probability map from the October drift scan. The grey drift is the reference scan from August.

‘705’, ‘707’ and ‘709’ all together. These are the only drifts, according to Figure 5.12, which show any convergence by December. Therefore, the data is not suitable to confirm or deny the accuracy of the convergence algorithm on real world convergence data.

The misalignment of drift cross sections along the drift is described as perpendicular association error in Section 4.3 and is the most difficult type of data association error for the algorithm to compensate for. The existing control for this type of error involves interpolating between the two nearest reference scan matches to calculate convergence indicator values which correspond to a cross section on the new scan.
This system does not work if the scans are misaligned by dozens of scan cross sections.

The localization algorithm, outlined in Section 4.2.2, is supposed to match the location of a cross section in the new scan to an approximate location in the reference scan using horizontal laser range data. The strength of this system is such that it is adept at discerning motion in drifts where there are a lot of corners and features. In this instance, the areas of concern with poor localization are the areas around drawpoints, which have very distinct features for the scan matching algorithm to derive position estimations from.

This discrepancy in the strength of the localization algorithm and the actual output of the localization algorithm is unexpected and could indicate a problem in the data collection process. Broken down, it is likely a problem in one of two areas. It is possible that the set-up of the uGPS device was non-ideal, causing interference with the inertial measuring unit. In Figure 4.3(b) one can see that the uGPS is mounted to the vehicle receiver on the end of a 1 metre hitch. On a small vehicle with the hitch acting as a lever, slight road irregularities could cause the uGPS to bounce more aggressively, causing artificially high, initial IMU estimations used for positioning. These huge acceleration values would then overshadow the corrections made by the scan matching process. A second error could be in the localization algorithm itself. This localization algorithm was provided by N. J. Lavigne from Peck Tech as a compiled binary file, so it is not possible to investigate possible code problems. Any suggestion towards a root cause would be speculation. Suffice it to say that it is conceivable that the software exhibits localization problems.
Chapter 6

Conclusions and Discussions

6.1 Contributions

The convergence algorithm outlined in Chapter 3 is the primary contribution of this thesis. It suggests a novel way to approach convergence detection using 3-D point cloud data. Using 3-D point cloud data to detect convergence is, in itself, a more efficient way of determining convergence. It allows the collection of thousands of points rather than using only a few distinct points at sparse cross sections. It also allows the application of statistical techniques to acquire a more reliable picture of convergence. It also takes less manual effort to acquire the data, saving cost and improving efficiency in a mine environment.

The problem with existing point cloud methods is that such techniques are reliant upon the registration of two point clouds, which is difficult. One way to do this is by ICP registration but as convergence progresses, the scans become different by their very nature and are thus difficult to match. The detection of convergence then relies on point to point measurements between scans. Unfortunately, small accuracy issues in the scan matching technique tend to induce alignment errors which drown out the
actual evidence of convergence.

The idea of using overall excavation shape change, as proposed herein, allows one to apply statistical techniques for interpreting convergence in a robust way without allowing oneself to accept a single form of evidence as evidence for convergence.

Additionally, the idea of taking scans and measuring range values from the centre of excavations to the edge of a scan cross section shows promise when looking to extract physical manifestations of convergence. Using range-angle data to define drift shape allows one to use signal analysis techniques to extract drift attributes as though they were signal frequency components.

Finally, the collection of extensive point cloud scans from a converging mine constitutes a relatively unique data set when paired with the survey measurements of the converging drift. Perhaps revisiting the data with different localization techniques could improve the readability of the scan data.

6.2 Future Work

It is beyond the scope of this thesis to propose a scan association method for approximately aligning the new and reference scans. The data provided in Section 5.4 also exhibited too much bowing — as in the example shown in Figure 4.4 on page 62 — to be properly associated with ICP point cloud matching techniques. Future work must therefore be performed to investigate why the uGPS did not perform as expected.

It would also benefit this thesis if one could re-visit a converging mine site and re-evaluate the method by which scan data are collected. This would permit avoidance of the types of data association errors seen in Section 5.4.2 and to give an opportunity to fill in survey gaps present in Figure 5.12 on page 82. It would be interesting to
assemble stationary scan data, render point cloud solids and slice cross sections in order to extract convergence, as outlined in Section 3.1.1. One could then compare the algorithm to existing point cloud change detection techniques.

After future acquisition of more data, it would be useful to train the Bayesian network using actual convergence data and collected point cloud cross sections. This would replace the initial estimated BN reliability values of Figure 3.9 on page 51 with more statistically validated estimations.

6.3 Speculation and Final Thoughts

This thesis investigates a novel method of convergence detection in an underground mine. Once the technique can be proven to work on real underground data, the possibility for further exciting advancements based on this technology will follow.

For example, deformations can be used to back-calculate certain attributes of the surrounding rock mass. If the accuracy of the system in real circumstances was proven to be very small, it may be possible to monitor hard rock mines for areas which are experiencing minuscule convergence as a precursor to rock bursts. Turning to other ground control applications, field stress, for example, could conceivably be back calculated using measured strain and simple stress models. In addition to this, the phase angle attribute discussed in Section 3.2.5 could be put to use in determining the primary direction of in-situ stress. This combination of using laser scanners to compute the direction and magnitude of mining stresses would theoretically save incredible effort when compared to conventional stress field delineation techniques.

By applying this convergence algorithm to systems like the uGPS Rapid Mapper™, one could conceivably render convergence detection to be as simple and fast as
driving through a drift with a scanner. At this point, it is almost trivial to imagine the system as autonomous, being driven around by a computer controlled machine. Automation would allow this convergence monitoring technology to be utilized in areas where humans cannot go. Perhaps this means measuring the elastic convergence during the formation of new excavations or measuring civil infrastructure sites such as pipelines or hydro-electric dam tunnels. Extrapolating farther into the future, as mines become deeper and automation begins to become the only means by which earth-bound mineral excavation is even possible, statistically based convergence monitoring algorithms for self localizing scanners will be the best way to monitor the movement of permanent excavations underground.
Bibliography


[23] Knowitvideos. Bayesian Networks Tutorial, 2011. Available at: https://www.youtube.com/watch?v=VC\{\_\}kaOKfjzI\{\&\}list=PLQ0wNwW2kpURmDZc3b\{\_\}HsEkuQ1mX7ITfv\{\&\}index=22.


Appendix A

Additional Figures

A.1 Real Drift Cross Sections

Output from the convergence algorithm from incremental addition of convergence on a real mining cross section. This uses the same convergence modes as before and the cross sections can be seen in Figure 5.6.
A.1. REAL DRIFT CROSS SECTIONS

Figure A.1: The convergence indicator response for concentric convergence applied to a real drift cross section.

Figure A.2: The convergence indicator response for arch convergence applied to a real drift cross section.
Figure A.3: The convergence indicator response for toe spalling convergence applied to a real drift cross section.
A.2 Joint Probability Table

This joint probability table (JPT) contains all the permutations of the 5 variables considered in the Bayes network in Figure 3.8 with the three possible states for each variable - high, low and negligible.
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Figure A.4: The joint probability table used in the final convergence monitoring algorithm.
A.3. CONVERGENCE ALGORITHM EXPERIMENTAL VALIDATION

Figure A.5: September convergence probability map. The grey scan is the reference scan in August.

A.3 Convergence Algorithm Experimental Validation

These figures are the convergence algorithm results from the scans taken in the converging mine drift over a 6 month period. Red represents a higher probability of convergence while blue represents no change.
Figure A.6: October convergence probability map.
Figure A.7: October convergence probability map.
Appendix B

Calculations

B.1 Joint Probability Table Calculation Derivation

The purpose of this derivation is to use conditional probabilities, known from Figure 2.9, to calculate joint probabilities for a joint probability permutation table (JPT) like those in Figure 2.10.

Using the simplified example in Chapter 2, the chain rule (2.11) allows one to write the joint probability as follows:

\[ P(C, T_1, T_2) = [P(C|T_1, T_2)] \cdot [P(T_1|T_2)] \cdot P(T_2) \]  (B.1)

The point of the whole operation in Section 2.3.2 is to calculate the first element \( P(C|T_1, T_2) \) using the joint probability tables. The next steps will expand the 3 elements using basic statistical principles and show how terms cancel.

Firstly, look at a conditional probability equation derived in Chapter 2:

\[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \]  (B.2)
This equation further develops into Equation (B.3), as in:

\[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\neg A) \cdot P(\neg A)} \]  

(B.3)

Now define:

\[ P'(A|B) = P(B|A) \cdot P(A) \]  

(B.4)

and

\[ P'(\neg A|B) = P(B|\neg A) \cdot P(\neg A) \]  

(B.5)

In (B.2) the denominator \( P(B) \) is equal to \( P(B|A) \cdot P(A) + P(B|\neg A) \cdot P(\neg A) \) as in (B.3). This is simply the sum of the \( A \) and \( \neg A \) permutations of the numerator. Next substitute in (B.4) into (B.3) to yield:

\[ P(A|B) = \frac{P'(A|B)}{P'(A|B) + P'(\neg A|B)} \]  

(B.7)

With this, one can manipulate the Bayes theorem without having to sum \( P(B) \) with every permutation of \( A \). Now, assign the denominator of Equation (B.7) a variable, as in:

\[ \eta = \left[ P'(A|B) + P'(\neg A|B) \right]^{-1} \]  

(B.8)

Now, back in the context of the convergence example, one must calculate the
B.1. JOINT PROBABILITY TABLE CALCULATION DERIVATION

conditional probability in element 1 of Equation (B.1) - as in:

\[ P(C|T_1, T_2) \]  
(B.9)

This can be done by multiplying 2 conditional probabilities together, as in:

\[ P(C|T_1, T_2) = \frac{P(T_1|C) \cdot P(T_2|C) \cdot P(C)}{P(T_1, T_2)} \]  
(B.10)

The denominator can be calculated using the chain rule, as in:

\[ P(T_1, T_2) = P(T_1|T_2) \cdot P(T_2) \]  
(B.11)

through the law of total probability,

\[ P(T_1, T_2) = P(T_1|T_2, C) \cdot P(C|T_2) + P(T_1|T_2, \neg C) \cdot P(\neg C|T_2) \cdot P(T_2) \]  
(B.12)

simplifying with conditional independence of tests,

\[ P(T_1, T_2) = P(T_1|C) \cdot P(C) + P(T_1|\neg C) \cdot P(\neg C) \cdot P(T_2) \]  
(B.13)

now expanding \( P(T_2) \),

\[ P(T_1, T_2) = P(T_1|C) \cdot P(C) + P(T_1|\neg C) \cdot P(\neg C) \cdot P(T_2|C) \cdot P(C) + P(T_2|\neg C) \cdot P(\neg C) \]  
(B.14)
and substituting back into (B.10) yielding:

\[
P(C|T_1, T_2) = \frac{P(T_1|C) \cdot P(T_2|C) \cdot P(C)}{P(T_1|C) \cdot P(C) + P(T_1|\neg C) \cdot P(\neg C) \cdot P(T_2|C) \cdot P(C) + P(T_2|\neg C) \cdot P(\neg C)}
\]  

(B.15)

As mentioned in Section 2.3.2, the process grows incredibly complicated as the number of ‘tests’ increases. Now, taking the same calculation of \( P(C|T_1, T_2) \) using the principles in (B.7):

\[
P'(C|T_1, T_2) = P(T_1|C) \cdot P(T_2|C) \cdot P(C)
\]  

(B.16)

which leads to:

\[
P(C|T_1, T_2) = \eta \times P'(C|T_1, T_2)
\]  

(B.17)

\[
P(C|T_1, T_2) = \frac{P'(C|T_1, T_2)}{P'(C|T_1, T_2) + P'(\neg C|T_1, T_2)}
\]  

(B.18)

In summary, element 1 of (B.1) is expanded from (B.17) and element 2 - \( P(T_1|T_2) \) - from Equation (B.11), (B.12) and (B.13). Substituting back into Equation (B.1) yields:

\[
P(C, T_1, T_2) = \left[ \frac{P(T_1|C) \cdot P(T_2|C) \cdot P(C)}{P(T_1|C) \cdot P(T_2|C) \cdot P(C) + P(T_1|\neg C) \cdot P(T_2|\neg C) \cdot P(\neg C)} \right] \cdots
\]

\[
\cdots \times [P(T_1|C) \cdot P(C|T_2) + P(T_1|\neg C) \cdot P(\neg C|T_2)] \cdots
\]

\[
\cdots \times P(T_2)
\]  

(B.19)
B.1. JOINT PROBABILITY TABLE CALCULATION DERIVATION

Applying the Bayes rule to $P(C|T_2)$ and $P(\neg C|T_2)$ in element 2:

$$P(C, T_1, T_2) = \left[ \frac{P(T_1|C) \cdot P(T_2|C) \cdot P(C)}{P(T_1|C) \cdot P(T_2|C) \cdot P(C) + P(T_1|\neg C) \cdot P(T_2|\neg C) \cdot P(\neg C)} \right] \cdots$$

$$\cdots \times \left[ P(T_1|C) \cdot \left( \frac{P(T_2|C) \cdot P(C)}{P(T_2)} \right) + P(T_1|\neg C) \cdot \left( \frac{P(T_2|\neg C) \cdot P(\neg C)}{P(T_2)} \right) \right] \cdots$$

$$\cdots \times P(T_2)$$

(B.20)

Now factor out the $\frac{1}{P(T_2)}$ in element 2 and cancel it with element 3:

$$P(C, T_1, T_2) = \left[ \frac{P(T_1|C) \cdot P(T_2|C) \cdot P(C)}{P(T_1|C) \cdot P(T_2|C) \cdot P(C) + P(T_1|\neg C) \cdot P(T_2|\neg C) \cdot P(\neg C)} \right] \cdots$$

$$\cdots \times [P(T_1|C) \cdot P(T_2|C) \cdot P(C) + P(T_1|\neg C) \cdot P(T_2|\neg C) \cdot P(\neg C)]$$

(B.21)

Finally, cancel the denominator in element 1 with element 2 to yield:

$$P(C, T_1, T_2) = P(T_1|C) \cdot P(T_2|C) \cdot P(C)$$

(B.22)

This is a much simpler way to calculate joint probabilities. Using joint probability tables as in Section 2.3.2, calculation of likelihood of some event given multiple types of evidence could be made computer automated.