Abstract

With the focus of the automotive industry on decreasing vehicle weight and improving fuel efficiency, aluminum is being used for structural components in automobiles. Given the high strain rates associated with vehicle impact, it is necessary to understand the rate sensitivity of any potential alloy (e.g., AA5754) in order to accurately predict deformation behaviour. Furthermore, the magnitude and strain path associated with the residual strains remaining after forming of the component also play a major role in how the material will behave.

It has been found that AA5754 sheet exhibits negative rate sensitivity up to a strain rate of 0.1/s, and positive strain rate sensitivity at strain rates between 0.1/s and 1500/s. Increasing the strain rate also has the effect of increasing the yield stress as well as the ductility. When a strain path change is involved between the prestrain stage and subsequent uniaxial loading, it has the effect of reducing the rate sensitivity of the material as well as reducing the overall flow stress. A rate-sensitive adaptation of the Voce material model was successfully implemented in LS-DYNA and used to predict the response of AA5754 sheet in bending for applied strain rates of 0.001/s and 0.1/s.
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Contents

Abstract ........................................................................................................................................... ii
Acknowledgements ...................................................................................................................... iii
List of Tables ............................................................................................................................... ix
List of Figures .............................................................................................................................. xi
Nomenclature ............................................................................................................................. xxii

Chapter 1: Introduction .................................................................................................................. 1

1.1 Background ............................................................................................................................. 1
1.2 Motivation for the present work ............................................................................................. 5
1.3 Objectives ................................................................................................................................ 6
1.4 Outline .................................................................................................................................... 7

Chapter 2: Literature review ......................................................................................................... 8

2.1 Material behaviour at high strain rates .................................................................................. 9
  2.1.1 General behaviour ............................................................................................................ 9
  2.1.2 Rate sensitivity of Al-Mg alloys (5000 series) ............................................................... 10
  2.1.3 The PLC effect and negative rate sensitivity ................................................................. 12
  2.1.4 Diffuse necking ............................................................................................................. 13
  2.1.5 Elongation to failure ..................................................................................................... 16
Chapter 3: Rate sensitivity of AA5754 ................................................................. 40

3.1 Introduction ........................................................................................................ 40

3.2 Preliminary testing ............................................................................................. 41

3.2.1 Evaluation of subsize sample geometry ....................................................... 41

3.2.2 Preliminary evaluation of material models ..................................................... 46

3.3 Experimental methods – uniaxial tensile testing ............................................. 49

3.3.1 Uniaxial tensile testing at strain rates of 0.001/s and 0.1/s ............................ 49

3.3.2 Uniaxial tensile testing at strain rates of 500/s and 1500/s ............................ 50

3.4 Experimental results – uniaxial tensile testing ................................................. 51

3.4.1 Results for uniaxial tensile tests at strain rates of 0.001/s and 0.1/s .......... 51

3.4.2 Results for uniaxial tensile tests at strain rates of 500/s and 1500/s .......... 54

3.4.3 Comparison of uniaxial tensile tests for all strain rates ............................. 57

3.5 Discussion .......................................................................................................... 62
Chapter 4: Effects of prestrain on the rate sensitivity of AA5754

4.1 Introduction ................................................................. 67
4.2 Preliminary Testing ...................................................... 68
4.3 Experimental methods – uniaxial tensile testing following prestrain .................................................. 69
   4.3.1 Applying plane strain prestrain ........................................ 70
   4.3.2 Test matrix: uniaxial tensile testing of prestrained samples ........................................ 73
   4.3.3 Uniaxial tensile testing of prestrained samples at strain rates of 0.001/s and 0.1/s ........................................ 75
   4.3.4 Uniaxial tensile testing of prestrained samples at strain rates of 500/s and 1500/s ........................................ 76
4.4 Experimental results – uniaxial tensile testing following prestrain ........................................ 77
   4.4.1 Uniaxial tensile testing of prestrained samples at strain rates of 0.001/s and 0.1/s ........................................ 77
   4.4.1.1 Results for transverse prestrain ........................................ 78
   4.4.1.2 Results for longitudinal prestrain ........................................ 87
   4.4.1.3 Comparison between results for transverse and longitudinal prestrain ........................................ 95
   4.4.2 Uniaxial tensile testing of prestrained samples at strain rates of 500/s and 1500/s ........................................ 102
4.5 Discussion ................................................................. 105
   4.5.1 Effect of prestrain on uniaxial flow stress ........................................ 105
   4.5.2 The effect of prestrain on rate sensitivity ........................................ 112
   4.5.3 Prestrained samples at strain rates of 500/s and 1500/s ........................................ 118

Chapter 5: Effects of prestrain and strain rate on deformation of AA5754 in bending

5.1 Introduction ................................................................. 120
5.2 Preliminary work .......................................................... 121
   5.2.1 Development of the bend test fixture ........................................ 121
5.3 Experimental methods - bending ........................................ 127
5.3.1 Test matrix-bending .................................................................129
5.3.2 Bend testing at applied strain rates of 0.001/s and 0.1/s .................130
5.3.3 Testing at an applied strain rate of 80/s .........................................130
5.4 Experimental results - bending .......................................................132
5.4.1 Bending results for applied strain rates of 0.001/s and 0.1/s ..........132
5.4.1.1 Bending results comparison between applied strain rates of 0.001/s and
        0.1/s .....................................................................................................134
5.4.1.2 Bending results comparison between prestrain orientations ............139
5.4.2 Results for applied strain rate of 80/s ..........................................142
5.4.3 Overall rate comparison ...............................................................147
5.5 Discussion .....................................................................................149
5.5.1 Bending results and flow stress ....................................................150
5.5.2 Impact dynamics ..........................................................................157

Chapter 6: Implementation and validation of a constitutive model for prestrained
AA5754 .................................................................................................160

6.1 Introduction ...................................................................................160
6.2 Preliminary work ............................................................................161
6.3 Development of a rate-dependent Voce material model ......................165
6.3.1 Rate-dependent Voce material model .........................................165
6.3.2 Implementation of rate-sensitive Voce material model in LS-DYNA ....170
6.3.3 Determining material constants ....................................................177
6.4 FEM simulation .............................................................................188
6.5 Test matrix: FEM simulation of bending ...........................................191
6.6 Numerical predictions ....................................................................192
6.6.1 Load comparison between experimental test and FEM simulation ....194
6.6.2 Deformation comparison between experimental test results and FEM
        simulation ..........................................................................................198
6.6.3 Comparison of energy absorption between experimental test and FEM
        simulation ..........................................................................................200
6.7 Discussion ........................................................................................................... 202
  6.7.1 Voce-MA material model ...........................................................................202
  6.7.2 Discussion of FEM model predictions .........................................................206
  6.7.3 Strain rate during bending ..........................................................................209
  6.7.4 Necessity of a rate-dependent model .........................................................211
  6.7.5 Necessity of a model incorporating prestrain .............................................213

Chapter 7: Conclusions and Recommendations .............................................. 214

  7.1 Conclusions ......................................................................................................214
  7.2 Recommendations ............................................................................................218

References .........................................................................................................219

Appendix A .........................................................................................................226
Appendix B .........................................................................................................232
Appendix C .........................................................................................................237
Appendix D .........................................................................................................240
Appendix E .........................................................................................................243
Appendix F .........................................................................................................247
Appendix G .........................................................................................................249
List of Tables

Table 3-1: Experimental test matrix for uniaxial tensile tests at strain rates between 0.001/s and 1500/s. ........................................................................................................... 49

Table 3-2: Hopkinson bar test parameters ........................................................................... 51

Table 4-1: Average Young’s modulus and yield stress values for 3mm AA5754 in the transverse and longitudinal material directions. ......................................................... 68

Table 4-2: Average prestrain values for the six prestrain groupings. ......................... 73

Table 4-3: Experimental test matrix for uniaxial tensile tests of samples prestrained in the transverse direction at strain rates between 0.001/s and 1500/s. .......... 74

Table 4-4: Experimental test matrix for uniaxial tensile tests of samples prestrained in the longitudinal direction at strain rates of 0.001/s and 0.1/s. .................... 75

Table 4-5: Hopkinson bar test parameters. ........................................................................ 76

Table 5-1: Experimental bend test matrix for applied strain rates between 0.001/s and 80/s. ............................................................................................................................. 129

Table 6-1: Equations used to calculate the multiplicative rate parameter (m). .......... 182

Table 6-2: Equations used to calculate the additive rate parameter (m’). .................... 183

Table 6-3: Test matrix for FEM simulations ...................................................................... 191

Table B1: Statistical summary for 1.5% transverse prestrain at 0.001/s……………….233
Table B2: Determining the characteristic Voce fit (for true plastic strain)…………… 234

Table B3: Yield stresses for uniaxial tensile test repeats in longitudinal and transverse directions…………………………………………………………………. 235
List of Figures

Figure 1-1: Strain rate sensitivity comparison between AA5754 and three different types of steel, modified from [4].................................................................2

Figure 1-2: Structural framework of a Chevrolet SSR with hydroformed siderails [6]........................................................................................................3

Figure 1-3: Sequential collapse of a square tube during a high-rate axial crush..........4

Figure 1-4: The loading stages of a hydroformed component subjected to axial crush.................................................................................................6

Figure 2-1: True stress-plastic strain curves for AA5083-H116 at different strain rates. [14] ........................................................................................................11

Figure 2-2: Yield stress versus strain rate for Al-Mg alloys with varying Mg content [16]........................................................................................................12

Figure 2-3: Uniform elongation versus strain rate for Al-Mg alloys with varying Mg content [16] ..........................................................................................16

Figure 2-4: Total elongation versus strain rate for Al-Mg alloys with varying Mg content [16] ..........................................................................................17

Figure 2-5: a) Undulations [27], b) shear bands [29] and c) cracking [27] on the outer surface of an AA5754 sheet sample in bending. .........................18

Figure 2-6: Illustration of a) isotropic and b) kinematic hardening schemes. ..............22

Figure 2-7: Two test methods for determining the rate sensitivity factor (m) [3] ..........29
Figure 2-8: Tensile split Hopkinson bar apparatus.................................................................32

Figure 2-9: Bend test configuration from Tam and Calladine fitted with strain
    gauges, and final deformed shape [52]. .................................................................34

Figure 2-10: Illustration of the hydroforming process.......................................................35

Figure 2-11: Thickness variation and the strain state after hydroforming.......................35

Figure 2-12: Two different types of responses following prestrain with a path change.
    [55]. ..............................................................................................................37

Figure 3-1: a) ASTM-E8 tensile sample geometry, b) subsize tensile sample
    geometry. Dimensions are in mm. ..........................................................................42

Figure 3-2: Flow stress curves of true stress and true strain for samples with ASTM-
    E8 standard tensile geometry and subsize tensile geometry. a) strain rate
    of 0.001/s, b) strain rate of 0.1/s. Note that the true stress has not been
    corrected past the point of diffuse necking. .....................................................43

Figure 3-3: The yield region of true stress and true strain curves for samples with
    ASTM-E8 standard tensile geometry and subsize tensile geometry. a) strain rate
    of 0.001/s, b) strain rate of 0.1/s. ......................................................................44

Figure 3-4: Stress-strain curves for AA5182 samples with a) width-to-thickness ratio
    of 35.7, b) width-to-thickness ratio of 6 [68]. ..................................................46

Figure 3-5: Material model curve fits to true stress versus plastic strain test data at
    strain rates of a) 0.001/s and b) 1500/s. ...........................................................48

Figure 3-6: Integrated aluminum bars used to grip the Hopkinson bar specimens. .......50

Figure 3-7: Voce curve fit to the true stress versus plastic strain data up to the point
    of diffuse necking for specimens tested in uniaxial tension at strain rates
    of a) 0.001/s and b) 0.1/s..............................................................52

Figure 3-8: Characteristic Voce curve for uniaxial tensile tests at strain rates of
    0.001/s and 0.1/s. .........................................................................................53

Figure 3-9: Voce curve fit to the true stress versus true plastic strain data for
    specimens tested in uniaxial tension at strain rates of a) 500/s and b)
    1500/s. ...........................................................................................................55

Figure 3-10: Engineering stress-strain curve generated from the first loading pulse of
    a tensile split Hopkinson Bar test at 500/s. .....................................................56
Figure 3-11: Characteristic Voce curve for uniaxial tensile strain rates of 500/s and 1500/s. .......................................................... 56

Figure 3-12: Flow stress curves of true stress versus true plastic strain in uniaxial tension for a range of strain rates between 0.001/s and 1500/s. ................. 58

Figure 3-13: Yield stress from raw data points and the Voce curve fits from Figure 3-12. ............................................................................................................. 58

Figure 3-14: An example of the Voce curve fit for a sample tested at 1500/s. .......... 59

Figure 3-15: Normalized Voce curve fits for strain rates between 0.001/s and 1500/s. 59

Figure 3-16: Maximum stress at the onset of diffuse necking for strain rates of 0.001/s and 0.1/s.......................................................... 60

Figure 3-17: True plastic strain at the onset of diffuse necking for strain rates of 0.001/s and 0.1/s. ......................................................................................... 61

Figure 3-18: True plastic strain at failure for strain rates of 0.001/s and 0.1/s. ........ 61

Figure 3-19: Serrations typical of the Portevin Le-Chatelier effect, at different strain rates for AA5754.......................................................................................... 64

Figure 4-1: True stress versus true strain for 3mm AA5754 sheet tested under uniaxial tension in the transverse and longitudinal material directions. ...... 69

Figure 4-2: Wide-grip configuration used for the application of plane strain prestrain............................................................................................. 71

Figure 4-3: Blank geometries used for plane strain prestrain in the a) transverse and b) longitudinal directions. The location of the subsize samples is also shown. All dimensions are in mm.......................................................... 72

Figure 4-4: Longitudinal blank with grid pattern used for measuring the achieved level of prestrain. ............................................................ 73

Figure 4-5: A comparison between strain rates of 0.001/s and 0.1/s for uniaxial tensile samples subjected to transverse prestrains ranging from 2% to 12%. ............................................................................................................. 80

Figure 4-6: Samples subjected to transverse prestrains of different magnitudes, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s. .......... 81
Figure 4-7: Hardening rate of samples subjected to different magnitudes of transverse prestrain, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s. ................................................................. 82

Figure 4-8: Yield stress values from individual samples for all 6 transverse prestrain groupings for strain rates of 0.001/s and 0.1/s. ................................................................. 83

Figure 4-9: Percentage change in the overall hardening rate between a strain rate of 0.001/s and 0.1/s (Equation (28)), for samples subject to transverse prestrain................................................................. 84

Figure 4-10: Maximum stress at the onset of diffuse necking for varying amounts of prestrain in the transverse direction. ................................................................. 85

Figure 4-11: True equivalent plastic strain at the onset of diffuse necking for varying amounts of prestrain in the transverse direction. ................................................................. 86

Figure 4-12: Nominal equivalent plastic strain at failure for varying amounts of prestrain in the transverse direction. ................................................................. 86

Figure 4-13: A comparison between strain rates of 0.001/s and 0.1/s for samples subjected to longitudinal prestrain with magnitudes from 2% to 12%, tested in uniaxial tension................................................................. 88

Figure 4-14: Samples subjected to longitudinal prestrains of different magnitudes, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s. ................................................................. 89

Figure 4-15: Hardening rate for samples subjected to longitudinal prestrains of different magnitudes, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s. ................................................................. 90

Figure 4-16: Yield stress values from individual tests for all 6 longitudinal prestrain groupings for strain rates of 0.001/s and 0.1/s. ................................................................. 92

Figure 4-17: Percentage change in the overall hardening rate between a strain rate of 0.001/s and 0.1/s, for longitudinal prestrain. ................................................................. 92

Figure 4-18: Maximum stress at the onset of diffuse necking for varying amounts of prestrain in the longitudinal direction. ................................................................. 93

Figure 4-19: True equivalent plastic strain at the onset of diffuse necking for varying amounts of prestrain in the longitudinal direction. ................................................................. 94

Figure 4-20: Nominal equivalent plastic strain at failure for varying amounts of prestrain in the longitudinal direction. ................................................................. 94
Figure 4-21: A comparison between strain rate and prestrain orientation for samples subjected to plane strain prestrains between 2% and 12%, tested in uniaxial tension.

Figure 4-22: Effect of prestrain orientation on yield stress values obtained from individual samples at strain rates of a) 0.001/s and b) 0.1/s.

Figure 4-23: Maximum stress at the onset of diffuse necking for varying amounts of prestrain in the longitudinal direction (L) and transverse direction (T). a) strain rate of 0.001/s, b) strain rate of 0.1/s.

Figure 4-24: True equivalent plastic strain at the onset of diffuse necking for varying amounts of prestrain in the longitudinal and transverse directions. a) strain rate of 0.001/s, b) strain rate of 0.1/s.

Figure 4-25: True equivalent plastic strain at failure for varying amounts of prestrain in the longitudinal and transverse directions. a) strain rate of 0.001/s, b) strain rate of 0.1/s.

Figure 4-26: Voce curve fit to the true stress versus true plastic strain data for specimens prestrained to 12% in the transverse direction and then tested in uniaxial tension at strain rates of a) 500/s and b) 1500/s.

Figure 4-27: Characteristic Voce curve for specimens prestrained to 12% in the transverse direction and then tested in uniaxial tension at strain rates between 0.001/s and 1500/s.

Figure 4-28: The flow stress curve for loading involving prestrain without a path change. a) 6% prestrain in uniaxial tension was followed either immediately or after a 2 week delay by uniaxial tensile loading. b) Results following a 2 week delay are compared for uniaxial prestrain and 6.4% equivalent plane strain prestrain.

Figure 4-29: Comparison of the saturation stress between transverse and longitudinal prestrain orientation for strain rates of a) 0.001/s and b) 0.1/s.

Figure 4-30: Comparison of the hardening rate for samples prestrained to 12% in the transverse and longitudinal directions at a strain rate of 0.001/s.

Figure 4-31: The effect of prestrain magnitude ε_p on the critical strain for serrated flow ε_c in AA6082 as shown by An et al. [23].

Figure 4-32: Percentage change in the saturation stress between samples prestrained in the transverse direction and the monotonic saturation stress.
Figure 4-33: Multiplicative rate parameter, m, for samples with a) transverse and b) longitudinal prestrain. ................................................................. 117

Figure 4-34: Experimental true stress versus true equivalent plastic strain for 12% transverse prestrain, tested in uniaxial tension at strain rates of 500/s and 1500/s. ................................................................................................. 119

Figure 5-1: Typical axial crush deformation of a hollow tube with a square cross-section. .............................................................................. 121

Figure 5-2: Assembled bend test fixture......................................................................................... 123

Figure 5-3: Dimensioned drawing for the end plates of the bend test fixture. Dimensions are in inches. ................................................................. 124

Figure 5-4: Dimensioned drawing for the inserts of the bend test fixture. Dimensions are in inches. ........................................................................ 124

Figure 5-5: Bend test fixture in place in the Instron. ................................................................. 125

Figure 5-6: Bend test fixture in place in the drop tower .......................................................... 126

Figure 5-7: a) Free bends developed during an axial crush. b) Free bends developed in the bend test fixture. c) Obstructed bend. .................... 127

Figure 5-8: a) Transverse and b) longitudinal prestrain blank geometries used for bending samples. Dimensions are in millimetres. ......................... 128

Figure 5-9: a) Bend sample geometry (mm), b) loading orientation, c) deformed orientation. ........................................................................................................ 128

Figure 5-10: Drop tower test apparatus at the University of Waterloo [46]. ......................... 131

Figure 5-11: Typical response of a sample in bending at low applied strain rates. ......... 133

Figure 5-12: Matching peak loads at 1mm for 3 repeated tests, and then averaging to produce a characteristic bending curve................................................. 134

Figure 5-13: Comparison between the characteristic bending curves for samples with no prestrain at applied strain rates of 0.001/s and 0.1/s. ..................... 135

Figure 5-14: Comparison between the characteristic bending curves for samples with transverse prestrain at applied strain rates of 0.001/s and 0.1/s........... 136

Figure 5-15: Comparison between the characteristic bending curves for samples with longitudinal prestrain at applied strain rates of 0.001/s and 0.1/s........ 138
Figure 5-16: Comparison between peak loads achieved during bending between transverse and longitudinal prestrain orientations at an applied strain rate of 0.001/s........................................................................................................139

Figure 5-17: Comparison between peak loads achieved during bending between transverse and longitudinal prestrain orientations at an applied strain rate of 0.1/s........................................................................................................140

Figure 5-18: Comparison between minimum loads achieved during bending for transverse and longitudinal prestrain at an applied strain rate of 0.001/s........................................................................................................141

Figure 5-19: Comparison between minimum loads achieved during bending for transverse and longitudinal prestrain at an applied strain rate of 0.1/s........................................................................................................141

Figure 5-20: Final position of a completed drop tower bend test........................................143

Figure 5-21: Comparison between bending curves following prestrain in the transverse and longitudinal directions at an applied strain rate of 80/s........................................................................................................145

Figure 5-22: Comparison between peak loads achieved during bending following prestrain in the transverse and longitudinal orientations at an applied strain rate of 80/s........................................................................................................146

Figure 5-23: Comparison between minimum loads achieved during bending following prestrain in the transverse and longitudinal orientations at an applied strain rate of 80/s........................................................................................................146

Figure 5-24: Comparison between peak loads achieved during bending between applied strain rates of 0.001/s, 0.1/s and 80/s for samples prestrained in the a) transverse and b) longitudinal directions..................................................148

Figure 5-25: Comparison between minimum loads achieved during bending between applied strain rates of 0.001/s, 0.1/s and 80/s for samples prestrained in the a) transverse and b) longitudinal directions..................................................149

Figure 5-26: Correlation between the reduced modulus and the corresponding peak load at buckling ..........................................................................................................................152

Figure 5-27: Correlation between the yield stress and the peak load at buckling for transverse and longitudinal prestrains, at strain rates of a) 0.001/s and b) 0.1/s ..........................................................................................................................153

Figure 5-28: Development of surface roughness in the high strain region, for a sample tested in bending..................................................................................................................155
Figure 5-29: Correlation between the minimum load during bending, and the strain increment necessary to initiate diffuse necking in uniaxial tension. ........ 156

Figure 5-30: Comparison between the energy absorbed and the prestrain magnitude at an applied strain rate of 0.001/s with transverse prestrain......................... 156

Figure 5-31: Amount of displacement applied between the start of loading, and the peak load for samples with no prestrain at strain rates of 0.001/s, 0.1/s and 80/s. ..................................................................................................... 159

Figure 6-1: The variation of the Johnson-Cook rate parameter (C) with strain rate, for samples with no prestrain................................................................. 163

Figure 6-2: The best fit predictions using a Voce-Johnson-Cook material model (Equation (32)) as compared with experimental results with no prestrain. 163

Figure 6-3: The variation of the Voce-Johnson-Cook rate parameter (C) with equivalent prestrain................................................................. 164

Figure 6-4: Effects of the multiplicative rate sensitivity parameter (m) at a strain rate of 0.1/s on a baseline Voce fit obtained at 0.001/s. ........................................ 166

Figure 6-5: Effects of the additive rate sensitivity parameter (m') at a strain rate of 0.1/s on a baseline Voce fit obtained at 0.001/s. ........................................ 167

Figure 6-6: The best fit predictions using a Voce-MA material model (Equation (33)) as compared with experimental results with no prestrain. .................. 169

Figure 6-7: The variation of the Voce-MA rate parameters m and m' with strain rate, for no prestrain and 12% prestrain................................................................. 170

Figure 6-8: Flow chart for the Voce-MA rate-sensitive material model.................. 172

Figure 6-9: Plastic return method shown in the plane of deviatoric stress............. 173

Figure 6-10: Flow chart showing the steps taken to determine the input parameters for the Voce-MA constitutive material model........................................ 178

Figure 6-11: Window created in the ANSYS GUI to input the prestrain magnitude. .... 179

Figure 6-12: The variation of the Voce parameter A with prestrain, for a strain rate of 0.001/s. ....................................................................................................... 180

Figure 6-13: The variation of the Voce parameter B with prestrain, for a strain rate of 0.001/s. ....................................................................................................... 181
Figure 6-14: The variation of the Voce parameter $n$ with prestrain, for a strain rate of 0.001/s. .................................................................................................................. 181

Figure 6-15: The variation of the Voce-MA multiplicative rate parameter ($m$) with prestrain, at a strain rate of 0.1/s, using a base strain rate of 0.001/s............ 184

Figure 6-16: The variation of the Voce-MA additive rate parameter $m'$ with prestrain, at a strain rate of 0.1/s, using a base strain rate of 0.001/s....................... 184

Figure 6-17: The variation of the Voce-MA rate parameter $m$ with prestrain, at a strain rate of 1500/s, using a base strain rate of 0.001/s. ......................... 185

Figure 6-18: The variation of the Voce-MA rate parameter $m'$ with prestrain, at a strain rate of 1500/s, using a base strain rate of 0.001/s. ...................... 185

Figure 6-19: The variation of the Voce-MA multiplicative rate parameter ($m$) with strain rate with no prestrain.............................................................. 186

Figure 6-20: The variation of the Voce-MA additive rate parameter ($m'$) with strain rate with no prestrain. .......................................................... 186

Figure 6-21: The stress-strain curve for 5.7% prestrain at a strain rate of 0.1/s, as calculated by LS-DYNA using the material constants determined from the ANSYS macro compared with experimental uniaxial test data......... 188

Figure 6-22: Geometry of the model used for the bend simulation. Dimensions are in mm. ............................................................................................................ 190

Figure 6-23: Loading conditions for the bend simulation. .................................................. 190

Figure 6-24: Lateral offset used to initiate buckling of the sample. ......................... 191

Figure 6-25: Comparison between FEM simulation and experimental test results for different magnitudes of transverse prestrain at applied strain rates of 0.001/s and 0.1/s. ................................................................. 193

Figure 6-26: Comparison of peak loads between individual experimental bend tests and FEM simulation................................................................. 195

Figure 6-27: Comparison of FEM simulation peak loads between strain rates of 0.001/s and 0.1/s. ................................................................. 196

Figure 6-28: Comparison of minimum loads between individual experimental bend tests and FEM simulation................................................................. 197
Figure 6-29: Comparison of FEM simulation minimum loads between strain rates of 0.001/s and 0.1/s. ................................................................. 197

Figure 6-30: Typical deformation of the sample in bending. a) partial bend, b) full bend................................................................. 198

Figure 6-31: Dimensions used to compare sample deformation. ................................................. 199

Figure 6-32: Comparison between the deformed shape from the test and simulation. a) width, b) height......................................................... 200

Figure 6-33: Comparison between the amount of energy absorbed during the simulation and test. ................................................................. 201

Figure 6-34: A representative load versus displacement curve for a uniaxial tensile test at 0.001/s for a sample with 12% transverse prestrain.......................... 204

Figure 6-35: Comparison between the Voce curve fit, and the load versus displacement curve for a uniaxial tensile test at 0.001/s for a sample with 12% transverse prestrain. ................................................................. 204

Figure 6-36: True equivalent strain for a) no prestrain, b) 12% transverse prestrain at an applied strain rate of 0.1/s, where the elements with strains above the corresponding failure strains are shown in red. ................................................. 205

Figure 6-37: Relationship between the peak load at buckling and the reduced modulus for prestrained samples in bending. ........................................... 207

Figure 6-38: Relationship between the peak and minimum loads and the number of nodes in the FEM simulation. ......................................................... 208

Figure 6-39: Relationship between the energy absorbed and the number of nodes in the FEM simulation................................................................. 208

Figure 6-40: Deformed shape of the sample at the peak load for a sample with no prestrain at an applied strain rate of 0.001/s. ................................. 209

Figure 6-41: Plastic strain rate versus displacement comparison between elements at two different locations within a sample with no prestrain at an applied strain rate of 0.1/s................................................................. 210

Figure 6-42: Comparison between load curves from an FEM bend simulation at an applied strain rate of 0.1/s, using the Voce-MA material model, and a quasi-static, rate-insensitive model............................................ 211
Figure 6-43: Comparison between load curves from an FEM bend simulation at an applied strain rate of 0.1/s, using the Voce-MA material model, and a rate-insensitive model .......................................................... 212

Figure 6-44: Comparison of load versus displacement during bending between simulations at an applied strain rate of 0.001/s for a sample with no prestrain, and a sample with 12% transverse prestrain ........................................... 213

Figure A1: An illustration of the different values used to determine the true plastic strain ........................................................................................................ 227

Figure A2: An illustration of the plastic adjustment used to determine the true plastic strain ........................................................................................................ 227

Figure A3: Determining the limits of strain data used from Hopkinson bar results… 229

Figure A4: Method for determining the Voce curve fit for results at 1500/s ………… 230

Figure A5: Voce curve fits for 1.5% transverse prestrain at 0.001/s plotted against a) true plastic strain, b) equivalent true plastic strain ................................. 231

Figure C1: Prestrain blank grid pattern a) before prestrain and b) after prestrain. Examples of horizontal and vertical pairs of dots are shown……………… 238

Figure C2: Spreadsheet used to calculate the applied major and minor prestrain a) prior to prestrain and b) post prestrain .............................................................. 239

Figure D1: As-received 3mm AA5754 sheet .................................................... 241

Figure D2: Void nucleation of a notched sample subject to plane strain prestrain in the transverse direction followed by tensile loading to failure at 1500/s ................................................................................................. 242

Figure D3: Void nucleation of a sample subject to plane strain prestrain in the transverse direction followed by a 180° bend .............................................. 242

Figure G1: A comparison between strain rates of 0.001/s and 0.1/s for samples subjected to transverse prestrain with magnitudes from 2% to 12%……… 249

Figure G2: A comparison between strain rates of 0.001/s and 0.1/s for samples subjected to longitudinal prestrain with magnitudes from 2% to 12%…. 250
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 \ldots a_6$</td>
<td>Voce-MA material constants, Equation (34)</td>
</tr>
<tr>
<td>$A$</td>
<td>yield stress, Equation (9)</td>
</tr>
<tr>
<td>$B$</td>
<td>saturation stress, Equation (9)</td>
</tr>
<tr>
<td>$C(\dot{\varepsilon})$</td>
<td>rate-sensitive rate parameter, Equation (32)</td>
</tr>
<tr>
<td>$c_2, c_3, c_4, c_5$</td>
<td>curve fitting constants, Equations (14) and (15)</td>
</tr>
<tr>
<td>$C_{ijkl}$</td>
<td>elastic stiffness matrix</td>
</tr>
<tr>
<td>$A, B, C, D$</td>
<td>curve fitting constants, Equations (13) and (25)</td>
</tr>
<tr>
<td>$\delta e$</td>
<td>plastic strain increment</td>
</tr>
<tr>
<td>$e$</td>
<td>engineering strain</td>
</tr>
<tr>
<td>$E_r$</td>
<td>reduced modulus</td>
</tr>
<tr>
<td>$E_t$</td>
<td>tangent modulus</td>
</tr>
<tr>
<td>$f(\varepsilon)$</td>
<td>strain hardening rule</td>
</tr>
<tr>
<td>$f$</td>
<td>porosity, Equation (8)</td>
</tr>
</tbody>
</table>
yield function, Equation (10)
shear modulus
hardening rate
moment of inertia
power law constant, Equation (2)
column effective length factor, Equation (30)
length, Equation (30)
multiplicative rate sensitivity parameter, Equation (19)
additive rate sensitivity parameter, Equation (20)
rate-sensitive multiplicative rate parameter
rate-sensitive additive rate parameter
hardening exponent, Equation (2)
constant, Equation (9)
buckling load
\[ R = \frac{\varepsilon_w}{\varepsilon_r}, \] ratio between strain in the width direction and the strain in the thickness direction in a uniaxial tensile test
time
scale factor, Equation (38)
percent change in overall hardening
true strain
effective plastic strain rate
equivalent prestrain
base engineering strain rate
\( \varepsilon_1 \)  
major principal strain

\( \varepsilon_2, \varepsilon_3 \)  
minor principal strain

\( \varepsilon_{eq} \)  
von Mises equivalent strain

\( \varepsilon_{kl} \)  
deviatoric strain tensor

\( \varepsilon_p \)  
effective plastic strain

\( \dot{\varepsilon}_p \)  
effective plastic strain rate

\( \varepsilon_{p\text{max}} \)  
strain at the onset of diffuse necking, Equation (27)

\( \varepsilon_{p\text{min}} \)  
strain at yield, Equation (27)

\( \dot{\varepsilon}_T \)  
effective plastic true strain rate

\( \Theta_{\text{overall}} \)  
overall hardening rate

\( \sigma \)  
true stress

\( \sigma_f \)  
flow stress

\( \sigma_{\text{dev}} \)  
deviatoric stress tensor

\( \sigma_{\text{max}} \)  
stress and strain at the onset of diffuse necking, Equation (27)

\( \sigma_{\text{min}} \)  
stress and strain at yield, Equation (27)
Chapter 1: Introduction

1.1 Background

Automobile manufacturers are continuously looking for ways to reduce vehicle weight, thereby reducing fuel consumption and emissions, and improving handling and safety. Two ways of accomplishing this goal are to optimize the vehicle design so that unnecessary material is removed and to find applications where steel could be replaced with a less dense material such as aluminum. Aluminum alloys have already been used for door panels, hoods, engines, suspension components and interior trim [1], but the new focus is to replace steel structural components with aluminum alloy components, as the higher strength-to-weight ratio of aluminum results in significant weight savings. High-end automobiles have already begun to incorporate increased amounts of aluminum into their vehicles, but there is still an untapped market for this material in high volume production vehicles [2].

In addition to weight reduction, there are other advantages of using aluminum, such as superior corrosion resistance, the ability of 90% of the aluminum used in
automobiles to be recovered and recycled, and the potential for increased safety during a crash situation.

The disadvantages of aluminum alloys are that they are susceptible to microstructural damage, and it is not known how much damage is incurred during forming or how it affects the so-called crashworthiness of the component in service. Aluminum alloy sheet generally exhibits a lower level of formability than steel because of its low R ratio, which means the material is more susceptible to thinning [3]. The performance of aluminum alloys under the high deformation rates associated with crash have not been widely investigated, and it has even been shown that some alloys may exhibit decreased strength under high strain rates [4]. This negative rate sensitivity with increasing strain rates can be seen for AA5754 in Figure 1-1, whereas the three steels all show positive rate sensitivity.

![Image: Engineering Stress - Strain for All Materials]

Figure 1-1: Strain rate sensitivity comparison between AA5754 and three different types of steel, modified from [4].
Aluminum alloys are more ductile than steel, and therefore, have the ability to decrease the peak initial impact force felt by the occupant during a crash [5]. One component that is designed to absorb impact forces during crash is the longitudinal siderail (see Figure 1-2 [6]), which deforms in a sequential collapse, as shown in Figure 1-3. This component must maintain structural integrity while undergoing extremely large strains, and any reduction of ductility due to previous manufacturing processes, or within the material itself due to strain rate effects, must be understood if the effectiveness during crash is not to be compromised. One of the major unknowns in incorporating aluminum alloy structural components into automobiles is the behaviour of these formed members in a crash situation. In order to accurately predict the peak load, energy absorption and deformed shape, the material behaviour under high strain rates, large strains and complex stress states must be rigorously tested and understood.

Figure 1-2: Structural framework of a Chevrolet SSR with hydroformed siderails [6].
In addition to new materials, automobile manufacturers are also looking at innovative methods for forming components. Tube hydroforming is the process by which pre-bent tubes are formed circumferentially by first placing them in a die and then filling them with fluid under pressure. This method can produce long components of greater quality since they can be made as a continuous piece instead of a series of joined sections. Furthermore, they can be made with an improved tolerance in less time and with less wasted material [7].

As a result of such forming processes, components are placed into the automobile in a prestrained state, with a non-uniform thickness distribution. Therefore, it is not only important to understand the constitutive behaviour of aluminum alloys under high strain rates, but it is also necessary to understand how prestrain magnitude and orientation affects the residual ductility and high-rate properties of the material.

While Europe favours 6000 series alloys for structural and sheet components, North America is focusing on AA5754-O because of its superior formability and weldability [5]. As a result of the relatively recent interest in using AA5754 for high-
volume automotive applications, its behaviour under high-rate axial crush situations has yet to be fully explored [8].

1.2 Motivation for the present work

Finite element method (FEM) computer simulations are an integral step in the automotive design process, and require accurate material properties to properly represent the behaviour of the component or system of interest. The primary objective of this research was to explore the response of AA5754 sheet to a range of strain rates and prestrains in order to implement a constitutive material model that would accurately represent post-forming behaviour during a crash situation.

The load cases considered in this thesis were loosely based on a hydroformed S-rail with a square cross-section undergoing high-rate axial crush. During the hydroforming process, a state of plane strain is created along the sides of the square cross-section in the transverse material direction as seen in Figure 1-4 a). The prestrained component is then loaded in the longitudinal material direction at a higher strain rate and deforms in a series of bends, as illustrated in Figure 1-4 b).

Specifically, this thesis examines high-rate loading in uniaxial tension at strain rates between 0.001/s and 1500/s for samples with plane strain prestrain in the longitudinal and transverse material directions. The key outcome from the experimental data is the effect of prestrain with a path change on the subsequent flow stress and on the rate sensitivity of AA5754 sheet.

Following the experimental testing program, a rate-dependent material model was implemented within the explicit dynamic FEM software LS-DYNA, and then used to
predict the behaviour of prestrained AA5754 under high-rate bending. This material model was validated through experimental tests that investigated the effect of prestrain and strain rate on the bending behaviour of AA5754 sheet.

Figure 1-4: The loading stages of a hydroformed component subjected to axial crush.

1.3 Objectives

The primary objectives of this thesis are to:

- Determine the rate sensitivity of as-received AA5754 sheet over a range of strain rates between 0.001/s and 1500/s;
- Determine the effects of plane strain prestrain in the transverse and longitudinal material directions on the subsequent stress-strain curves obtained at strain rates between 0.001/s and 1500/s;
- Quantify the effects of plane strain prestrain in the transverse and longitudinal material directions on bending of AA5754 sheet at strain rates of 0.001/s, 0.1/s and 80/s;
- Implement a rate-sensitive constitutive material model, and use it to predict the load-deflection response and deformed shape of prestrained AA5754 sheet samples subject to bending at applied strain rates of 0.001/s and 0.1/s.
1.4 Outline

This thesis is divided into the seven chapters outlined below.

- Chapter 1 explains the motivation and objectives for this research.
- Chapter 2 provides background applicable to this thesis, noting relevant studies from the literature.
- Chapter 3 looks at the behaviour of AA5754 sheet subject to monotonic loading under uniaxial tension at strain rates between 0.001/s and 1500/s.
- Chapter 4 focuses on the behaviour of AA5754 sheet subject to plane strain prestrain in the transverse and longitudinal material orientations, followed by loading in uniaxial tension in the longitudinal direction at strain rates between 0.001/s and 1500/s.
- Chapter 5 examines the behaviour of AA5754 sheet subject to plane strain prestrain in the transverse and longitudinal material directions, followed by bending at applied strain rates of 0.001/s, 0.1/s and 80/s.
- Chapter 6 outlines the implementation of a constitutive material model to predict the deformation of prestrained AA5754 sheet in bending at strain rates of 0.001/s and 0.1/s.
- Chapter 7 draws overall conclusions as to the rate sensitivity of AA5754 and the effect that prestrain has on this behaviour, and proposes future work.
Chapter 2: Literature review

One of the major unknowns in incorporating aluminum alloy structural components into automobiles is the behaviour of these formed members in a crash situation. In order to accurately predict the peak load, energy absorption and deformed shape, the material behaviour under high strain rates, large strains and complex stress states must be known. While much research has been performed on the strain rate sensitivity of various aluminum alloys, very little is known about the effects of prestrain on the rate sensitivity of AA5754.

Quasi-static loading is considered to have strain rates on the order of $10^{-3}$/s, intermediate rates are between 1/s and 10/s, high strain rates are considered to be on the order of $10^2$/s to $10^3$/s, and very high strain rates are $5\times10^3$/s and greater [9, 10]. A typical automobile crash can produce strain rates on the order of 100/s, although in an axial crush situation, the folds may produce local deformation rates that are even higher.
This chapter presents a review of the following topics:

- Material behaviour at high strain rates;
- Constitutive material models;
- Test methods for different strain rates;
- Prestrain effects on high-rate properties.

2.1 Material behaviour at high strain rates

2.1.1 General behaviour

Metals typically have high-rate properties that are different from their quasi-static properties; specifically, the flow stress, work hardening behaviour, yield stress, uniform elongation and elongation to failure may all vary depending on the strain rate during loading [9].

Automotive steel sheet typically exhibits positive strain rate sensitivity, resulting in a higher degree of strain hardening with increased strain rate, as seen in Figure 1-1. Accounting for positive strain rate sensitivity is not critical when simulating automotive impacts, provided that material failure does not occur, as models ignoring rate effects will result in conservative predictions of deformation behaviour. Aluminum alloys exhibit much lower rate sensitivity than steel [9] and, as a result, their quasi-static material properties are often used in computer simulations. However, certain materials such as aluminum-magnesium (Al-Mg) alloys can exhibit negative rate sensitivity [11], and
simulations that do not account for their inherent decrease in strength will result in an overestimate of the energy absorbed during impact.

2.1.2 Rate sensitivity of Al-Mg alloys (5000 series)

In general, rate sensitivity refers to the response of a material’s flow stress and hardening rate to changes in strain rate, although the yield stress, uniform elongation and ductility may all be described in terms of their rate sensitivity as well.

Al-Mg alloys can exhibit both negative and positive rate sensitivity depending upon the specific alloy, the range of strain rates of interest and the temperature. For AA5754, Kang et al. [12] found a negative strain rate sensitivity as the strain rate increased from 1x10^{-5} to 1x10^{-3}/s, while testing performed by the University of Waterloo showed a positive rate sensitivity at high strain rates of 500/s, 1000/s and 1500/s [13]. The Aluminum Association [5] has shown a 25% increase in the flow stress of AA5754 at a strain rate of 70/s when compared with quasi-static results. There have been no studies looking at whether there is a transition from negative to positive rate sensitivity for AA5754 over the entire range of strain rates between quasi-static and high rates, although this change-over has been observed in other 5000 series alloys. For example, Figure 2-1 depicts negative rate sensitivity for AA5083-H116 at strain rates below 3.95/s and a positive rate sensitivity above 3.95/s [14], while Wagenhofer et al. [15] reported that such a transition occurs at 0.1/s for AA5086. However, not all Al-Mg alloys experience this transition, as AA5056-O exhibits only negative rate sensitivity between quasi-static and high rates [10].
While Young’s modulus is not rate-sensitive for aluminum alloys, the yield strength may show an increase with increasing strain rate. McGregor et al. [17] found a 25% increase in the yield strength of AA5754 between quasi-static rates and a strain rate of 70/s, while United States Steel [4] has reported a slight decrease in the yield stress at a strain rate of 10/s when compared to quasi-static behaviour. In Figure 2-2, the data from Mukai et al. [16] demonstrates that the yield stress of various Al-Mg alloys remains relatively constant for strain rates up to approximately 100/s, after which the yield stress rises rapidly with increasing strain rate. No reason for this trend, however, is given.

It is clear that there is no consistent behaviour with respect to strain rate sensitivity of the 5000 series alloys over the range of strain rates from quasi-static to high-rate, and the research literature for AA5754 is mostly limited to quasi-static behaviour. Hence, there is a need for testing of AA5754 from quasi-static up to high strain rates.
2.1.3 The PLC effect and negative rate sensitivity

It has been well documented that the Portevin-Le Chatelier (PLC) effect is the mechanism largely responsible for the negative rate sensitivity observed in 5000 series alloys at certain temperatures and strain rates [18-21]. The occurrence of the PLC effect is clearly identified by the jerky nature of the material’s stress-strain curve, and the macroscopic banding that develops on the material’s surface. When a mobile dislocation is waiting at an obstacle, it may be pinned by diffusing solute atoms, corresponding to Mg atoms in the case of AA5754. An increased force is required to break the dislocation free of the solute atoms, and the successive pinning and unpinning events result in serrated flow. If the applied strain rate is increased, the solute atoms are not able to diffuse fast enough to pin dislocations before they can overcome obstacles. As a result, the magnitude of serrated flow and the stress needed for dislocation motion are both

Figure 2-2: Yield stress versus strain rate for Al-Mg alloys with varying Mg content [16].
reduced, and a negative rate sensitivity ensues. At large strain rates, the PLC effect disappears altogether, signifying the end of the region of negative rate sensitivity and the start of positive rate sensitivity. The PLC effect can also disappear at certain temperatures. For example, AA5754 shows negative rate sensitivity at room temperature but a very slight positive rate sensitivity at 223K [22].

At a certain value of plastic strain, called the critical strain, the PLC effect is initiated, and the stress-strain behaviour transitions from being homogeneous to inhomogeneous. The critical strain is different for each material, and is dependent upon the strain rate, temperature and level of prestrain. Van den Beukel [21] showed that this critical strain increases with increasing strain rate and decreasing temperature, while more recently An et al. [23] have found that the onset of the PLC effect is also delayed by prestrain with a path change. It has been reported in one series of tests that the critical strain for AA5754 at room temperature and a strain rate of 6x10^{-4}/s is 1.3% [24].

The PLC bands that are evident on the surface of the sample are a result of the serrated flow of the material. These deformation bands are regions of locally increased strain that nucleate and propagate along the sample in the direction of loading. In automotive applications, this reduction in the surface quality is a significant issue when a smooth finish is required for certain components such as painted exterior panels [12].

2.1.4 Diffuse necking

During tensile loading, diffuse necking (or plastic instability) is the condition where deformation continues under a decreasing load; its onset is identified by the point of maximum load on a load-displacement curve, or by the point of maximum stress on an
engineering stress-strain curve. Diffuse necking occurs when the rate of thinning becomes greater than the rate of strain hardening, and deformation becomes unstable. The process is initiated at a material inhomogeneity or specimen defect and this condition marks the end of uniform elongation.

The onset of diffuse necking in uniaxial tension occurs when the hardening rate and the flow stress are equal, as predicted by the Considère criterion:

\[
\frac{d\sigma}{d\varepsilon} = \sigma. \quad (1)
\]

For many materials, the simplest model for representing the stress-strain relationship is the power law hardening curve (Holloman [25]) given by:

\[
\sigma = K \varepsilon^n \tag{2}
\]

where: 
- \(K\) = constant
- \(n\) = hardening exponent
- \(\varepsilon\) = total strain.

The derivative of Equation (2) with respect to the strain gives the hardening rate,

\[
\frac{d\sigma}{d\varepsilon} = nK \varepsilon^{n-1}. \quad (3)
\]

Using Equation (1) for the stress at diffuse necking,

\[
\frac{d\sigma}{d\varepsilon} = \sigma = nK \varepsilon^{n-1}. \quad (4)
\]
By substituting Equation (2) into Equation (4) and simplifying, it is found that when the curve of the flow stress is fit to a power law hardening model, the total strain at diffuse necking is equal to the hardening exponent $n$,

$$K\varepsilon^n = \frac{nK\varepsilon^n}{\varepsilon}$$

$$\varepsilon = n.$$

Equation (5) indicates that a material with a lower hardening rate will not be able to undergo the same degree of plastic strain before the onset of diffuse necking in comparison to a material with a larger hardening rate.

Mukai et al. [16] determined that for Al-Mg alloys the relationship between uniform elongation and strain rate varies greatly depending on the percentage of Mg present, as can be seen in Figure 2-3. With a low Mg content, the uniform elongation remains constant for all strain rates, but with an increased Mg content, the elongation is rate-dependent, and increases with increasing strain rates of at least $1/s$. Kang et al. [24] showed that the strain at diffuse necking for AA5754 is less than predicted using the Considère condition (Equation (1)) because the local strains within the PLC bands are higher than the average sample strain.
2.1.5 Elongation to failure

As loading continues in uniaxial tension under plastic instability, the deformation with eventually become localized and a macroscopic neck will form. Materials with a positive strain rate sensitivity experience additional hardening within the neck region as the strain rate increases locally, which tends to stabilize the neck [26]. Elongation can then continue well past the point of plastic instability. For materials with negative rate sensitivity the instability is compounded as the flow stress decreases within the neck, where the local strain rate is higher. The triaxial state of stress in the neck can also cause void nucleation at second phase particles or inclusions which may eventually lead to ductile failure.

Figure 2-3: Uniform elongation versus strain rate for Al-Mg alloys with varying Mg content [16].
In AA5754, past the point of diffuse necking, the formation and growth of shear bands becomes more prevalent. Shear bands are regions of intense dislocation motion which result from resolved shear loads, and when PLC bands are also present, they can interact with each other [12, 24]. That is, when a traveling PLC band is superimposed with a shear band, a localized neck can form and initiate premature failure.

In general, the elongation to failure of 5000 series aluminum increases with increasing strain rates, and this increase can be significant at high strain rates [10]. However, other alloys show a reduction in the elongation to failure over a certain range of strain rates. Mukai et al. [16] discovered that alloys with 2 and 3% Mg exhibit a decrease in ductility between $10^{-4}$ and $10^{-3}$ while an alloy with 5% Mg showed the same trend between $10^{-4}$ and 10/s, as is evident in Figure 2-4.

![Figure 2-4: Total elongation versus strain rate for Al-Mg alloys with varying Mg content [16].](image-url)
2.1.6 Failure in uniaxial tension and bending in AA5754

Tensile failure in AA5754 is typically caused by the localization of a shear band rather than damage caused by the nucleation and coalescence of voids. Localization is initiated at the site of a shear band superimposed with a traveling PLC band. Kang et al. [24] have demonstrated that very little damage is present near the fracture surface or within the shear bands, indicating that damage is not a precursor to failure in AA5754.

Failure in bending can occur either on the inside of the bend by cracking or buckling, or on the outside of the bend due to cracking associated with shear bands. Unlike tensile deformation, no neck is formed prior to failure in bending. As loading progresses, the outer surface of the bend will develop surface undulations, where the valleys form local geometric stress concentrations. Cracks may eventually grow from these surface defects through localized shear in the orientation of maximum shear stress, as illustrated in Figure 2-5 [27, 29].

![Figure 2-5: a) Undulations [27], b) shear bands [29] and c) cracking [27] on the outer surface of an AA5754 sheet sample in bending.](image)
2.2 Material models

2.2.1 Implementation of an elastic-plastic material model

In order for a material model to predict post-yield behaviour using a standard continuum mechanics approach, the following four components must be defined:

1) Yield function;
2) Hardening rule;
3) Hardening scheme;
4) Flow rule.

Yield Function

The yield function defines the boundary between fully elastic and elastic-plastic behaviour, and this boundary evolves with material hardening. Two of the more common yield functions are the Tresca criterion:

\[ \hat{\tau} \geq \frac{\sigma_y}{2} \]  \hspace{1cm} (6)

and the von Mises criterion:

\[ \frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 \right] \geq \sigma_y^2 \]  \hspace{1cm} (7)

where: \( \hat{\tau} = \) maximum shear stress
\( \sigma_y = \) yield stress
\( \sigma_1, \sigma_2, \sigma_3 = \) principal stresses.
While both of these yield functions are commonly used for ductile materials, the discontinuous boundary of the Tresca criterion can cause computational problems when implemented in an elastic-plastic material model.

When a material susceptible to void nucleation and damage is being modeled, the evolving porosity and the resulting softening can be accounted for using the Gurson yield criterion [28]:

$$\Phi = \left( \frac{\sigma_e}{\bar{\sigma}} \right)^2 + 2f \cosh \left( \frac{\sigma_e}{2\bar{\sigma}} \right) - 1 - f^2 = 0$$  \hspace{1cm} (8)

where: $\bar{\sigma}$ = flow stress of the matrix material
$f$ = porosity
$\sigma_e = \sqrt[3]{\frac{3}{2} \sigma_{ij} \sigma_{ij}}$
$\sigma_{ij}$ = deviatoric component of stress.

The Gurson yield criterion may also be used to model the growth of shear bands and subsequent failure for load cases such as bending [29].

**Hardening Rule**

The hardening rule provides the relationship between the flow stress and the effective plastic strain and may incorporate effects of temperature and strain rate. A basic model for describing the strain hardening that accompanies plastic deformation is the power-law hardening (Holloman [25]) equation:
\[ \sigma = K \varepsilon^n \] \hspace{1cm} (2)

where:
- \( K \) = constant
- \( n \) = hardening exponent
- \( \varepsilon \) = total strain.

When considering aluminum alloys, the characteristic decrease in hardening rate at higher strains is better captured by the Voce equation [30]:

\[ \sigma_f = B - (B - A) e^{-n\varepsilon_p} \] \hspace{1cm} (9)

where:
- \( A \) = yield stress
- \( B \) = saturation stress
- \( n \) = constant
- \( \varepsilon_p \) = effective plastic strain.

The parameter \( B \) represents the asymptote of maximum stress to which the curve approaches, also called the saturation stress.

**Hardening Scheme**

The hardening scheme defines the change in the yield surface as the material hardens. Isotropic hardening dilates the yield surface equally. As a result, hardening in tension results in the same hardening in compression. Kinematic hardening translates the yield surface, so that its size remains the same; hence, hardening in tension produces a reduction in the yield stress in compression. The concepts of isotropic and kinematic hardening are illustrated in Figure 2-6.
Flow Rule

The flow rule describes the direction of the plastic strain increment. For metals, the rule of normality is typically employed, where the plastic strain increment is directed along the outward normal of the yield surface such that:

\[
 d\varepsilon_y^p = \lambda \frac{df}{d\sigma_y} 
\]

(10)

where: 
- \(\varepsilon_y^p\) = plastic strain
- \(\lambda\) = scalar
- \(f\) = yield function
- \(\sigma_y\) = stress.

2.2.2 Rate-dependent constitutive equations

There are numerous constitutive models that attempt to capture strain-rate effects, such as elasto-plastic [31, 32], elasto-viscoplastic [33], and viscoplastic [34] models. They range from very simple approximations such as Cowper-Symonds [35] which
relates the yield stress under static conditions to the yield stress under dynamic conditions, to the more complex Steinburg and Lund [36] model which incorporates equivalent plastic strain, pressure, and internal energy. The Cowper-Symonds relationship is:

$$\frac{\sigma_y^d}{\sigma_y^s} = 1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{1/p}$$

(11)

where: $\sigma_y^d$ and $\sigma_y^s$ = dynamic and static yield stress
$\dot{\varepsilon}$ = strain rate
$D$ and $p$ = material constants.

Equation (11) provides a very simple method for incorporating strain rate effects into existing material models such as power-law plasticity and piecewise-linear in explicit FEM codes such as LS-DYNA [37].

Another simple model that can be used for fcc materials is:

$$\sigma = c' \varepsilon^n \dot{\varepsilon}^m$$

(12)

where: $c'$ = constant
$n$ = hardening exponent
$m$ = rate sensitivity parameter,

which is based on power law hardening and incorporates the term $m$ to account for the strain rate sensitivity [3].
Elasto-plastic models have proven to be the most widely incorporated into explicit FEM codes, with the most commonly accepted models being those proposed by Johnson-Cook [31] and Zerilli-Armstrong [32].

**Johnson-Cook**

The Johnson-Cook model [31] is an elasto-plastic model based on a curve fitting technique and is derived from experimental test results, such that:

\[
\sigma_f = \left( A + B \varepsilon_p^* \right) \left( 1 + C \ln \dot{\varepsilon}_p^* \right) \left( 1 - T^* \right)
\]

where:
- \( \sigma_f \) = flow stress
- \( A, B, C, n, m \) = input constants
- \( \varepsilon_p \) = equivalent plastic strain
- \( \dot{\varepsilon}_p^* \) = dimensionless plastic strain rate and
- \( T^* = \left( T - T_{room} \right) / \left( T_{melt} - T_{room} \right) \).

Equation (13) is considered valid for a range of strain rates from quasi-static up to \( 10^3 / s \) [32], although it has been modified by some researchers to account for higher strain rates [38, 39].

The terms in the first bracket of Equation (13) represent a power-law hardening relation for the flow stress. This baseline flow stress is then factored by the terms in the second and third brackets to account for strain rate effects and thermal softening, respectively.

While the Johnson-Cook material model has benefits, including its ease of use and the minimal testing required for determining the parameters, it lacks the physical basis for application to a wide variety of situations. Furthermore, its use is restricted to
the range of testing rates for which the model was fit. There are also no terms to account for the interaction between strain and temperature or strain rate, and there is no way of differentiating between the strain-rate sensitivity of face-centred cubic versus body-centred cubic materials. It is also widely known that grain size has a significant effect on plastic response, due to smaller grains being more resistant to dislocation motion [40, 41], yet grain size effects are not considered in the Johnson-Cook model.

**Zerilli-Armstrong**

The Zerilli-Armstrong model [32] is derived from the theories of thermal activation and dislocation motion, and incorporates effects of strain hardening, strain rate, thermal softening, grain size, and the crystal structure of the material.

For face centred cubic materials (fcc):

\[
\sigma = \Delta \sigma_G' + c_2 \varepsilon_p^{1/2} \exp\left(-c_3 T + c_4 T \ln \dot{\varepsilon}_p^*\right) + k l^{-1/2} \tag{14}
\]

and for body centred cubic materials (bcc):

\[
\sigma = \Delta \sigma_G' + c_2 \varepsilon_p^n + c_1 \exp\left(-c_3 T + c_4 T \ln \dot{\varepsilon}_p^*\right) + k l^{-1/2} \tag{15}
\]

where: $\Delta \sigma_G'$ = factored yield stress
$\varepsilon_p$ = equivalent plastic strain
$\dot{\varepsilon}_p^*$ = dimensionless plastic strain rate
$n$, $c_1$, $c_2$, $c_3$, $c_4$ and $c_5$ are material constants
$T$ = temperature
$k$ = microstructural stress intensity
$l$ =average grain diameter.
In Equations (14) and (15), the term $\Delta \sigma' G$ takes into account the influence of solute atoms and the original dislocation density on the yield stress. The term $k l^{-1/2}$ is the Hall-Petch relationship [40, 41] which accounts for the effect of grain size on the flow stress. The difference in behaviour between fcc and bcc materials is captured by coupling or uncoupling the plastic strain with the thermal and strain-rate terms.

The Zerilli-Armstrong model is applicable to a wide range of fcc materials, but underestimates the strength of bcc materials, and is only valid for very high strain rates ($10^4$-s$^{-1}$-$10^6$-s$^{-1}$) and relatively low temperatures (less than one half of the melting temperature). Nevertheless, the Zerilli-Armstrong model improves upon the major concerns associated with the Johnson-Cook model, although more effort is required to determine the various material constants.

### 2.2.3 Models that can represent a change from negative to positive rate sensitivity

The rate-dependent material models presented in Section 2.2.2 are applicable only to materials exhibiting positive rate sensitivity. While some researchers have focused specifically on developing models to represent materials with a negative rate sensitivity [42], they are designed only for this response and are not able to capture transitions to positive rate sensitivity. Other models that can represent both positive and negative rate sensitivity [43, 44], are very much theoretical and have not been tested for different materials or over a range of strain rates. No models have emerged as being commonly used or widely accepted. In LS-DYNA, the yield stress and tangent modulus can be input as functions of the strain rate as a way of capturing changes in rate sensitivity [37].
One straightforward method for generating a material model that can capture transitions between negative and positive rate sensitivity, is to introduce a rate-dependent, rate parameter into an existing material model. For example, the term from the Johnson-Cook model [31] that incorporates strain rate:

\[ C \ln(\dot{\varepsilon}_p^*) \]  

(16)

can be combined with any hardening rule \( f(\varepsilon) \). The constant \( C \) can then be made to vary with strain rate in order to produce a model that captures the transition between negative and positive rate sensitivity, such as:

\[ \sigma_f = f(\varepsilon) C(\dot{\varepsilon}_p) \ln(\dot{\varepsilon}_p^*) \].  

(17)

Other rate terms that could be used are from the Zerilli-Armstrong model [32]:

\[ c_1 \exp\left(-c_4 \ln(\dot{\varepsilon}_p^*)\right), \]  

(18)

an exponential term [45]:

\[ \sigma = f(\varepsilon) \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^m, \]  

(19)

or additive term [45]:
\[ \sigma = f(\dot{\varepsilon}) + m' \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \]  

(20)

where:  
\( f(\dot{\varepsilon}) \) = hardening rule  
\( m \) = multiplicative rate parameter  
\( m' \) = additive rate parameter  
\( \dot{\varepsilon}_0 \) = base strain rate  
\( \dot{\varepsilon} \) = effective strain rate.

These rate-dependent equations provide a practical method for predicting the flow stress based on curve fitting techniques and experimental stress-strain curves.

### 2.3 Test methods for different strain rates

Figure 2-7 depicts two methods used to determine the strain-rate sensitivity parameter (\( m \)) in Equation (12). The first method compares stress-strain curves for two specimens at two different strain rates, and the second method compares the change in stress resulting from a jump in strain rate during the testing of one specimen [3], such that:

\[
m = \left( \frac{\Delta \sigma}{\sigma} \right) \frac{1}{\ln(\dot{\varepsilon}_2 - \dot{\varepsilon}_1)}. \]

(21)

The second method has the advantage that many changes in strain rate can be performed on one sample, but the difficulty is that the testing machine must be able to respond quickly to changes in strain rate and some machines are not capable of altering the strain rate during a test.
Tests to determine strain rate sensitivity can be performed in tension, compression, shear or bending. Testing in bending provides a stable method for achieving large strains, although stress-strain relations are difficult to determine from a bend test. Torsion tests can provide very large shear strains as they eliminate the plastic instability associated with necking in tensile specimens. The test method employed depends upon the strain rate required, and can be divided into low (up to 0.1/s), intermediate (between 1/s and 10/s) and high (100/s and greater) strain rates.

2.3.1 Tensile testing at low strain rates

Tensile testing at low strain rates, on the order of 0.001/s, is typically performed using standard tensile testing machines and strains are measured with an extensometer. When serrated flow is present, the loading rate is controlled by the cross-head velocity, and the strain rate reported is the nominal engineering strain rate. Oscillations in the data are eliminated, and the results show minimal variation, which makes this testing method ideal for detailed comparisons involving subtle trends in strain rate sensitivity.
2.3.2 Tensile testing at intermediate strain rates

Tensile testing at intermediate strain rates, on the order of 1/s, is most often performed with a servo-hydraulic tensile testing machine and strains are measured using an extensometer. One must be careful in interpreting these results, as the strain rate may not be constant throughout the duration of the test due to the acceleration of the cross-head from its resting position at the start of the test.

At slightly higher strain rates, on the order of 100/s, a drop tower apparatus can be used in conjunction with a tensile fixture to produce constitutive data. Strains are monitored with strain gauges or laser velocity systems [46]. Dynamic oscillations are present which can mask subtle rate effects, and the strain rate slows down as the sample absorbs energy. As a result, the drop tower is typically used to validate material models, and to determine absorbed energy and deformation modes of full-scale components. A sled test may also be used for the same purpose.

2.3.3 High-rate tensile testing

High strain rate testing in tension introduces many challenges, such as the measurement of strain, because extensometers and strain gauges typically used to measure strains under quasi-static rates cannot be employed. The most popular high-rate tensile test method is the tensile split Hopkinson bar.

The Hopkinson Bar test apparatus is one of the most popular methods for producing material data under high strain rates. It has developed over the last century through the work of Hopkinson [47], Kolsky [48], and Lindholm [49]. Samples can be loaded in torsion, tension or in compression and can produce strain rates up to $1.5\times 10^4$/s
Stress-strain data can only be extracted up to the point of necking, since the sample is not deforming uniformly beyond that point.

A typical Hopkinson bar apparatus, such as the one at the University of Waterloo, is presented in Figure 2-8. The gas gun is pressurized and the striker bar is fired along the incident bar towards the end cap with a high velocity. The striker makes contact with the end cap and creates a tensile pulse, or stress wave, in the incident bar. This pulse travels down the bar and loads the sample in tension as some of the wave is transmitted and some is reflected. The detected strain in the transmitter bar and the strain in the reflected bar are measured to determine the strain in the sample according to:

\[
\sigma_s = E_b \frac{A_b}{A_s} \varepsilon_T
\]  

\[
\dot{\varepsilon}_s = -2 \frac{C_0}{L_s} \varepsilon_R
\]  

\[
\varepsilon_s = -2 \frac{C_0}{L_s} \int_0^t \varepsilon_R dt
\]

where:
- \( \sigma_s \) = sample stress
- \( \dot{\varepsilon}_s \) = sample strain rate
- \( A_s \) = sample area
- \( L_s \) = sample length
- \( \varepsilon_s \) = sample strain
- \( C_0 \) = speed of sound
- \( E_b \) = Young’s modulus of the bars
- \( A_b \) = area of each bar
- \( \varepsilon_R \) = reflected strain
- \( \varepsilon_T \) = transmitted strain.
It is critical to choose an appropriate specimen geometry, as dynamic equilibrium must exist in the sample during the test. The gauge length must be short enough so that the pressure in the specimen is constant, while being long enough so that a state of uniaxial tension is reached at the centre. Parameters such as the striker material, the gas gun pressure, and the length of the striker bar can all be varied to obtain the required strain rate, strain magnitude and test duration.
2.3.4 High-Rate Bending

High-rate bending is a more complex process than high-rate tension because the strain rate and the amount of strain are not constant through the thickness of the specimen. Davies and Magee [51] have shown that the dynamic factor (ratio between the flow stress at high-rate versus low rate) for aluminum alloy samples tested separately in tension and bending are similar. This correspondence suggests that the high-rate collapse of a structure in bending can be predicted using high-rate tensile properties, although this link was not confirmed.

High-rate bend tests are not typically employed to obtain material data due to the difficulties in accurately measuring stresses and strains. Tam and Calladine [52] performed high-rate bend tests using the setup shown in Figure 2-9 with overall load and strains being recorded. Two identical samples with an initial pre-bend at the centre are clamped back to back, and can be loaded either quasi-statically or under high-rates using a drop tower. While this setup cannot be used to measure material constants, as stress-strain curves cannot easily be obtained, it can provide a means for evaluating constitutive models developed from associated tensile data. Information such as the amount of energy absorbed, the peak load and the average load are all useful when studying automotive impacts.
2.4 Prestrain effects on high-rate properties

Forming operations introduce thickness changes, residual stresses, and work hardening into components and it is important to understand the effects that these changes will have on the deformation and failure behaviour during impact. Lee et al. [53] looked at the effects of forming in crash simulations, and found that the residual stresses have almost no effect on the predicted behaviour during crash. The strain hardening of the material after the forming process, however, did have a significant influence on crash behaviour. That is, omitting the work hardened state of the material resulted in significant variations between predicted and experimental force-displacement curves.

2.4.1 Hydroforming

During the hydroforming process a tubular section is placed into a die and then filled with fluid. The fluid is pressurized and the tube expands to fill the die, creating the
required cross-section, as illustrated in Figure 2-10. When end-feeding is not used, in the case of a square cross-section, the thickness profile changes, as depicted in Figure 2-11, and thinning occurs at the corners. A state of plane strain in the transverse direction is developed across the sides of the tube because the ends are not able to contract in the longitudinal direction.

Figure 2-10: Illustration of the hydroforming process.

Figure 2-11: Thickness variation and the strain state after hydroforming.
2.4.2 Prestrain effects on flow stress and failure

The effect of prestrain involving a path change on the subsequent flow stress behaviour of a metal falls into two main categories, as described by Fernandes et al. [54] and illustrated in Figure 2-12 [55]. In the first case, the initial yield (upon reloading) is higher than the monotonic curve, the hardening rate is reduced, and the flow stress approaches the monotonic curve from above. In the second case, the initial yield is lower than the monotonic curve, the hardening rate is increased, and the flow stress approaches the monotonic curve from below. One factor that determines which response will occur is the development and reorientation of dislocation substructure with applied strain. Alloys that undergo dynamic strain aging, such as AA5754, as well as materials undergoing load reversal (compression to tension or vice versa) [54] tend to exhibit the second type of behaviour. It has been discussed by Wilson et al. [56] that the reduced hardening rate exhibited by the first case is a result of the annihilation or rearrangement of dislocation barriers developed during prestrain. The solute atoms present in materials that undergo dynamic strain aging (eg. Mg atoms in AA5754) cause internal stresses which act in the direction opposite to the applied stress during prestrain. With a change in strain path, the internal stresses in the new direction are less than in the monotonic case, and this results in a decrease in the flow stress, but an increased hardening rate as the new internal stresses develop [57].
The majority of research investigating prestrain effects on aluminum has considered commercial purity aluminum (AA1050) [58] as well as 2000 [55, 59] and 6000 [57, 60] series alloys. Much of the work focuses on formability with the intention of predicting forming limit diagrams or the limits of uniform elongation. Most studies consist of two-stage tensile tests with changes in orientation due to the difficulty in obtaining uniform prestrain in large specimens, although some tests consider biaxial stretching since it is a load path typical of sheet forming [59]. Laukonis et al. [55] demonstrated that, for AA2036-T4, the residual uniform flow capability in uniaxial tension increases with biaxial prestrain, while Lloyd and Sang [61] found that the capability remained unchanged with uniaxial tensile prestrain at an orientation of 90° with respect to the subsequent loading. It was also shown that for AA1100 and 3003,
prestrains of at least 10% in uniaxial tension followed by a 90° change in orientation caused almost immediate diffuse necking in a subsequent tensile test.

An [57] performed sequential compression tests with plane strain prestrain followed by uniaxial compression using AA6082. The initial microstructure, the angle of change in strain path, and the prestrain magnitude all had significant effects on the hardening rate. It was determined that the largest increase in hardening rate came with the largest prestrain magnitude, and when the material was rotated through the largest angle (e.g. transverse to longitudinal sheet directions), but with no path change. The change between prestrain in plane strain to uniaxial compression did not cause a significant increase in the hardening rate.

When looking at the effects of prestrain on bending, Lloyd et al. [62] concluded that with increasing prestrain, the bend angle corresponding to the maximum load during a cantilever bend test decreases, resulting in lower bendability. Sarkar et al. [27] determined that the iron content in AA5754 sheet also has a large effect on the bendability. Prestrain in AA5754 sheet with an iron content of 0.3% caused damage and voids at the Fe-containing particles which promote accelerated strain localization during bending.

The mechanism responsible for the behaviour of a prestrained material is the response of the dislocation substructure to changes in loading orientation or load path. In commercially pure aluminum, a well-defined dislocation cell structure is developed during prestrain, which is then annihilated and reformed during the next stage of loading [63]. In alloys with a high solute content, the dislocation distribution is more
homogeneous following prestrain, and the behaviour during the second stage of loading is determined by the reorientation of internal stresses [56].

Just as with the rate sensitivity of aluminum alloys, the effects of prestrain are not easily predicted. The behaviour following prestrain depends upon the microstructure of the alloy, the prestrain magnitude and the path change involved, and two-stage testing must be performed in order to determine the response of the specific alloy of interest.

2.4.3 Prestrain effects on rate sensitivity

There has been very little work examining the effects of prestrain on the rate sensitivity of aluminum alloys, and no literature has been found dealing specifically with AA5754. Lee and Lin [64, 65] showed that for 304L stainless steel bars, prestrain has an affect on the strain rate sensitivity. Compressive prestrain was followed by high-rate loading in compression; an increase in the strain rate sensitivity with increasing prestrain was found. These results were accurately predicted using a Zerilli-Armstrong material model by determining separate sets of parameters for the prestrained and non-prestrained samples. The model parameters themselves did not account for the effects of prestrain. Bate [58] found that with a path change of 63° between AA1050 tensile specimens, the rate sensitivity of the material was eliminated. This was attributed to the disruption of the dislocation cell structure following prestrain being rate-insensitive. Kocks and Brown [66], however, showed no effects of prestrain, with a change in orientation, on the rate sensitivity of pure aluminum tested in compression.
Chapter 3:  Rate sensitivity of AA5754

3.1  Introduction

This chapter looks at the rate sensitivity of 3mm AA5754 sheet over a range of strain rates between 0.001/s and 1500/s* (see note below). Monotonic loading in uniaxial tension was applied in the longitudinal material direction using two test apparatus: a servo-hydraulic Instron for strain rates of 0.001/s and 0.1/s, and a tensile split Hopkinson Bar for strain rates of 500/s and 1500/s.

The AA5754 sheet was provided in the O-temper condition, and had a chemical composition of 0.2% Si, 0.34% Fe, 0.022% Cu, 0.27% Mn, 2.76% Mg, 0.018% Cr, 0.012% Zn, 0.012% Ti, with the balance being Al [67].

* Unless otherwise noted, the strain rates referred to in this thesis represent engineering strain rates, calculated by taking the loading velocity divided by the original gauge length of the sample. The engineering strain rate remains constant during loading, as opposed to the true strain rate, which decreases as the sample gauge length increases.
3.2 Preliminary testing

3.2.1 Evaluation of subsize sample geometry

In order to obtain a wide range of strain rates, two different test apparatus were used. A consistent sample geometry was needed, as the Hopkinson bar apparatus required an unconventional subsize geometry for the reasons discussed in Section 2.3.3. It was necessary to ensure that the subsize samples produced the same results as the standard ASTM-E8 tensile geometry when tested in the Instron. Robinson et al. [68] noted that specimen geometry can influence the behaviour of materials exhibiting the Portevin-LeChatelier effect when loaded in tension.

The ASTM-E8 [69] and subsize geometries shown in Figure 3-1 were loaded under uniaxial tension in the longitudinal direction at strain rates of 0.001/s and 0.1/s. Displacement control of the cross-head was employed, and 25.4mm and 10mm extensometers were used to record the change in gauge length of the ASTM-E8 and subsize samples, respectively.

All dimensions are ±0.02mm

a) ASTM-E8 tensile sample geometry
Figure 3-1:  a) ASTM-E8 tensile sample geometry, b) subsize tensile sample geometry. Dimensions are in mm.

Figure 3-2 shows the true stress-strain curves for the ASTM-E8 and subsize samples at strain rates of 0.001/s and 0.1/s. The neck formed outside the extensometer gauge region for the ASTM-E8 samples; therefore, the curves end at the onset of diffuse necking, which corresponds to the maximum load. The neck formed inside the extensometer gauge region for the subsize samples; therefore, the curves continue until failure, although the true stress has not been corrected past the point of diffuse necking. For the range of strains from approximately 5% up to diffuse necking, the specimen geometry has no effect on the flow stress. At strains below 5%, the subsize sample shows higher stresses due to the absence of a yield plateau, as shown in Figure 3-3. The ASTM-E8 sample shows a clear yield plateau, whereas the subsize sample exhibits a gradual transition from elastic to plastic behaviour. The difference in stresses gradually decreases until approximately 5% strain, at which point the two curves converge.
Figure 3-2: Flow stress curves of true stress and true strain for samples with ASTM-E8 standard tensile geometry and subsize tensile geometry. a) strain rate of 0.001/s, b) strain rate of 0.1/s. Note that the true stress has not been corrected past the point of diffuse necking.
Figure 3-3: The yield region of true stress and true strain curves for samples with ASTM-E8 standard tensile geometry and subsize tensile geometry. a) strain rate of 0.001/s, b) strain rate of 0.1/s.
The differences are due to the specimen geometry, specifically the width-to-thickness ratio of the cross-section, as shown by Robinson et al. [68]. Figure 3-4 shows that as the width-to-thickness ratio of the gauge region decreased from 35.7 to 6 that the length of the yield plateau decreased as well. This is the same as what was observed between the ASTM-E8 sample with a width-to-thickness ratio of 4.2, and the subsize sample with a ratio of 0.58.

Robinson et al. [68] has shown that a thinner specimen approaches more of a plane stress condition and the deformation mode is a series of local necks through the thickness of the sample. These local necks correspond to the serrations on the stress-strain curve, and are a manifestation of the Portevin-Le Chatelier effect discussed in Section 2.1.3. A thicker specimen deviates away from plane stress towards a plane strain condition, and deformation proceeds through successive shear band formation. These variations in deformation modes affect the orientation and propagation of the PLC bands, resulting in differences in the yield plateau and magnitude of the serrations.

The difference in the deformation mode between the two geometries extends to the mode of failure as well. The failure of a sample in plane stress is more dependent on the orientation and location of the through-thickness necks originating from one of the PLC bands, whereas the failure of a plane strain sample is more dependent on the state of stress.
In the current study, both the ASTM-E8 and the subsize samples fall somewhere between plane stress and plane strain, and will exhibit deformation modes between the two extremes discussed above. Even though the geometry does have an affect on the recorded flow stress following the initial yield, both geometries produce similar overall work hardening rates. Since this research focused on only qualitative comparisons of ductility, the differences in elongation to failure and failure mode are not of great concern. Furthermore, only the region between yielding and diffuse necking is used to characterize the material’s plastic flow behaviour, therefore, it was decided that the subsize geometry would be used for all subsequent testing in this thesis.

3.2.2 Preliminary evaluation of material models

The following three material models were evaluated based on their fit with true stress versus plastic strain data for AA5754 at strain rates of 0.001/s and 1500/s:
Voce

\[ \sigma_f = B - (B - A)e^{-n \varepsilon_f} \quad (9) \]

Johnson-Cook

\[ \sigma_f = \left( A + B \bar{\varepsilon}_p^n \right) \left( 1 + C \ln \dot{\varepsilon} \right) \quad (13) \]

Zerilli-Armstrong

\[ \sigma_f = A + B \bar{\varepsilon}_p^{0.5} e^{(-C + D \ln \dot{\varepsilon})} \quad (25) \]

Figure 3-5 a) compares the best fits for the three material models against the test data for a strain rate of 0.001/s (test results will be presented in Section 3.4.1). Both the Johnson-Cook \( (R^2 = 0.991) \) and Zerilli-Armstrong \( (R^2 = 0.987) \) models overpredict the flow stresses at the start and end of the plastic curve, while underpredicting the flow stress in the middle of the curve. The Voce model predicts the experimental results well for all regions of the plastic curve and has the highest \( R^2 \) value of 0.998.

Figure 3-5 b) compares the best fits for the three material models against the test data for a strain rate of 1500/s. All three material models are equally as effective at predicting the experimental data \( (R^2 = 0.993) \). The Voce model overpredicts the first 1% of the plastic curve, while the Johnson-Cook and Zerilli-Armstrong models capture the start and end of the curve well, but underpredict the middle portion of the curve.

The Voce curve is the best fit for the AA5754 data over the entire range of strain rates. However, the Voce model would have to be adapted to be able to incorporate rate effects.
Figure 3-5: Material model curve fits to true stress versus plastic strain test data at strain rates of a) 0.001/s and b) 1500/s.
3.3 Experimental methods – uniaxial tensile testing

A series of samples were machined from the 3mm AA5754 flat sheet using the subsize sample geometry of Figure 3-1 b), and then loaded under uniaxial tension in the longitudinal material direction, following the test matrix shown in Table 3-1. The holes in the grip region were only necessary for samples tested in the Hopkinson Bar, but they were also included in the samples tested in the Instron, as they enabled the samples to be gripped during machining.

Table 3-1: Experimental test matrix for uniaxial tensile tests at strain rates between 0.001/s and 1500/s.

<table>
<thead>
<tr>
<th>Strain rate</th>
<th>Apparatus</th>
<th>Repeats</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>Instron</td>
<td>5</td>
</tr>
<tr>
<td>0.1/s</td>
<td>Instron</td>
<td>6</td>
</tr>
<tr>
<td>500/s</td>
<td>Hopkinson Bar</td>
<td>3</td>
</tr>
<tr>
<td>1500/s</td>
<td>Hopkinson Bar</td>
<td>7</td>
</tr>
</tbody>
</table>

3.3.1 Uniaxial tensile testing at strain rates of 0.001/s and 0.1/s

The testing of specimens at strain rates of 0.001/s and 0.1/s was performed using a servo-hydraulic Instron testing apparatus. The subsize samples were loaded in the longitudinal material direction under uniaxial tension. A constant rate of cross-head displacement was applied, which produced a constant engineering strain rate in the gauge region. The samples were not loaded using strain-rate control due to the serrated flow that occurs in AA5754 at strain rates of 0.001/s and 0.1/s.
3.3.2 Uniaxial tensile testing at strain rates of 500/s and 1500/s

The tensile split Hopkinson Bar apparatus from the University of Waterloo was employed for testing at high strain rates of 500/s and 1500/s, following the setup discussed in Section 2.3.3. A set of aluminum bars was machined for mounting the samples, similar to the bars shown in Figure 3-6. The slots could accommodate material with a thickness up to 3mm, and the screws were tightened to grip the specimen. Integrated bars made from the same material as the specimen greatly reduce the distortion of stress waves as they pass from the incident bar into and through the sample. Specific test parameters are listed in Table 3-2 with the gas gun pressure providing the required strain rates. The striker length of 0.5m provided a single pulse long enough to significantly deform the sample, but was not enough to take the samples up to the point of diffuse necking, or up to the point of failure. Momentum trapping was used to “capture” the stress wave and prevent the samples from being loaded repeatedly. Strain gauges were mounted on the incident and transmitted bars and the voltage output was recorded through a high-speed digital oscilloscope.

Figure 3-6: Integrated aluminum bars used to grip the Hopkinson bar specimens.
Table 3-2: Hopkinson bar test parameters

<table>
<thead>
<tr>
<th>Strain Rate</th>
<th>Gas Gun Pressure (kPa)</th>
<th>Striker Material</th>
<th>Striker Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500/s</td>
<td>42.75</td>
<td>Aluminum</td>
<td>0.5</td>
</tr>
<tr>
<td>1500/s</td>
<td>222.01</td>
<td>Aluminum</td>
<td>0.5</td>
</tr>
</tbody>
</table>

3.4 Experimental results – uniaxial tensile testing

The stress-strain curves up to the point of diffuse necking was determined from the raw data for each sample, and then fit to the following form of the Voce material model:

\[ \sigma_f = B - (B - A)e^{-n\varepsilon_p}. \]  

One characteristic curve of true plastic strain versus true stress was produced for each of the four strain rates. The results are grouped into the following sub-sections.

3.4.1 Results for uniaxial tensile tests at strain rates of 0.001/s and 0.1/s

Raw data points of load and gauge displacement were analyzed using the method of Appendix A. At a quasi-static strain rate of 0.001/s, AA5754 was found to have a yield stress of 95.3 MPa and a Young’s modulus of 69.7 GPa. For each specimen, a Voce curve, Equation (9), was fit to the true stress versus plastic strain data, up to the point of diffuse necking as shown in Figure 3-7.
Figure 3-7: Voce curve fit to the true stress versus plastic strain data up to the point of diffuse necking for specimens tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s.
Curves falling outside the 95% confidence interval were identified using the statistical analysis outlined in Appendix B, and an average of the remaining curves was taken to produce a characteristic Voce curve for each strain rate, as plotted in Figure 3-8. The maximum strain value of the curves corresponds to the sample with the smallest strain at diffuse necking, and is not a measure of the strain at diffuse necking across all samples. The individual curves were truncated at this value, so that the characteristic curve was based on data points over a common range of strains.

![Figure 3-8: Characteristic Voce curve for uniaxial tensile tests at strain rates of 0.001/s and 0.1/s.](image-url)
3.4.2 Results for uniaxial tensile tests at strain rates of 500/s and 1500/s

The raw strain gauge voltage and time data collected by the oscilloscope was converted to stress and strain data through the use of calibration files and a program called CSHB provided by the University of Waterloo. The calibration files, which are specific to the incident and transmitter bars employed, provide the correlation between strain and strain gauge voltage. The program CSHB determines the stress and strain values from conventional Hopkinson bar theory based on the difference in interface velocities at either ends of the sample. The resulting stress-strain data was then analyzed using the method of Appendix A to obtain a Voce curve fit of true stress versus true plastic strain, as presented in Figure 3-9.

It is important to note that only the first loading pulse was recorded, and the samples were only loaded up to a characteristic strain dependent upon the length of the stress wave. Failure did not occur, and it was not known whether diffuse necking was initiated as the decrease in stress following the peak load may have been a local oscillation, as illustrated in Figure 3-10. The curves in Figure 3-9 are terminated at the point of maximum load for the data that was available and not at the point of diffuse necking as was the case for the data collected at strain rates of 0.001/s and 0.1/s.

Curves falling outside the 95% confidence interval were identified using the statistical analysis outlined in Appendix B, and an average of the remaining curves was taken to produce a characteristic Voce curve for each strain rate, as shown in Figure 3-11.
a) Strain rate of 500/s

b) Strain rate of 1500/s

Figure 3-9: Voce curve fit to the true stress versus true plastic strain data for specimens tested in uniaxial tension at strain rates of a) 500/s and b) 1500/s.
Figure 3-10: Engineering stress-strain curve generated from the first loading pulse of a tensile split Hopkinson Bar test at 500/s.

Figure 3-11: Characteristic Voce curve for uniaxial tensile strain rates of 500/s and 1500/s.
3.4.3 Comparison of uniaxial tensile tests for all strain rates

The characteristic flow stress curves for the four strain rates considered between 0.001/s and 1500/s are plotted in Figure 3-12. The strain rate sensitivity can be described in terms of two characteristics: the yield stress and the hardening rate. An increase in the yield stress and an increased hardening rate are both indications of positive strain rate sensitivity. Figure 3-13 shows that the yield stress remains constant at strain rates between 0.001/s and 0.1/s and then increases at strain rates of 500/s and 1500/s. The yield stress from the Voce curve fits at 500/s and 1500/s are higher than the raw data because the Voce fit overpredicts the stresses at low values of strain, as is evident in Figure 3-14. Curves of the flow stress, normalized to 100MPa using $\sigma - \sigma_y$ (the difference between the flow stress and the yield stress), are depicted in Figure 3-15, which provides a qualitative comparison of the hardening rates. The hardening rate decreases between strain rates of 0.001/s and 0.1/s but increases slightly between 500/s and 1500/s. It is difficult to compare the hardening rate between 0.1/s and 500/s due to differences in the overall shape of the curve.
Figure 3-12: Flow stress curves of true stress versus true plastic strain in uniaxial tension for a range of strain rates between 0.001/s and 1500/s.

Figure 3-13: Yield stress from raw data points and the Voce curve fits from Figure 3-12.
Figure 3-14: An example of the Voce curve fit for a sample tested at 1500/s.

Figure 3-15: Normalized Voce curve fits for strain rates between 0.001/s and 1500/s.
Figures 3-16 and 3-17 indicate that the maximum true stress and strain at diffuse necking both decrease with increasing strain rate. The true plastic strain at failure, however, increases with increasing strain rate, as can be seen in Figure 3-18. In these three figures, the data points for 500/s and 1500/s were not included because tests at these strain rates were not necessarily taken to the point of diffuse necking.

Figure 3-16: Maximum stress at the onset of diffuse necking for strain rates of 0.001/s and 0.1/s.
Figure 3-17: True plastic strain at the onset of diffuse necking for strain rates of 0.001/s and 0.1/s.

Figure 3-18: True plastic strain at failure for strain rates of 0.001/s and 0.1/s.
3.5 Discussion

Negative strain rate sensitivity is typical of materials that exhibit dynamic strain aging, or the Portevin-Le Chatelier (PLC) effect. Figure 3-19 a) exhibits the serrated flow for AA5754 which is characteristic of the PLC effect. Figure 3-19 b) and c) show that the magnitude of the serrations decreases with increasing strain rate, and have completely disappeared by a strain rate of 1/s. (The sampling rate for these tests is not equivalent, but there were sufficient points to identify serrations if they existed). The PLC effect occurs when a mobile dislocation is waiting at an obstacle, and it is pinned by diffusing solute atoms, corresponding to Mg atoms in the case of AA5754 (see Section 2.1.3). An increased force is required to break free of the solute atoms, and the successive pinning and unpinning events result in serrated flow. If the applied strain rate is increased, the solute atoms are not able to diffuse fast enough to pin dislocations before they can overcome obstacles. In this case, there is no serrated flow, and the stress needed for dislocation motion is reduced. This results in negative rate sensitivity for strain rates with decreasing PLC effect, such as between 0.001/s and 0.1/s. The suppression of the PLC effect marks the end of the region of negative rate sensitivity, and the start of positive rate sensitivity. The positive rate sensitivity that was observed between 0.1/s and 1500/s is a result of dislocations not being able to overcome obstacles in the short time period that is present at higher strain rates. Further testing would need to be performed at strain rates between 0.1/s and 500/s to determine exactly where the transition between negative and positive rate sensitivity occurs, but preliminary testing has shown that this threshold is around a strain rate of 1/s.
a) strain rate of 0.001/s

Start of serrated flow

b) strain rate of 0.1/s

Start of serrated flow
c) strain rate of $1/s$

Figure 3-19: Serrations typical of the Portevin Le-Chatelier effect, at different strain rates for AA5754.

Mukai et al. [16] have shown that the 0.2% proof stress (yield stress) for Al-Mg alloys is rate-independent up to strain rates of 100/s and then becomes highly rate-dependent, as shown in Figure 2-2. This behaviour is evident in Figure 3-13, where the yield stress is constant between 0.001/s and 0.1/s, and then increases between 0.1/s and 1500/s. At lower strain rates, the flow stress exhibits rate sensitivity due to the PLC effect, which only initiates after a critical strain. This critical strain is evident in Figure 3-19 a) where the serrations do not appear until approximately 3% true plastic strain. At initial yield, the mechanism causing rate dependency, the PLC effect, may not have initiated; as a result, the yield stress may not be rate-dependent. On the other hand, at higher strain rates, the PLC effect is no longer present, and the rate sensitivity of the flow
stress is dependent upon the rate of dislocation motion. Since the mechanism driving rate dependency is present at the onset of loading, this results in the yield stress being rate-sensitive.

The presence of the PLC effect and the resulting negative rate sensitivity between 0.001/s and 0.1/s explains why the true stresses and strains at diffuse necking decrease with increasing strain rate, as shown in Figures 3-16 and 3-17. The onset of diffuse necking occurs when the hardening rate and the flow stress are equal, as predicted by the Considère criterion:

\[ \frac{d\sigma}{d\varepsilon} = \sigma. \quad (1) \]

For many materials, the power law hardening curve given by:

\[ \sigma = K\varepsilon^n, \quad (2) \]

provides a reasonable representation of the flow stress. Section 2.1.4 shows the derivation of the equation for the total strain at diffuse necking, which is equal to the hardening exponent \( n \):

\[ \varepsilon = n. \quad (5) \]

Equation (5) means that a flow stress curve with a lower hardening rate is expected to have a lower strain at diffuse necking. When the data for 0.001/s and 0.1/s is fit to the power law hardening curve of Equation (2), the hardening exponents and the strain at diffuse necking are found to be 0.285 and 0.278, respectively. The experimental
values of 0.167 and 0.145 are much lower than the predicted values, due to the locally higher strains present in the PLC bands, thus lowering the overall sample strain observed at diffuse necking [24]. The effect of a decreased hardening rate was demonstrated in Figures 3-16 and 3-17, where the true stress and strain values at diffuse necking are seen to decrease as the strain rate increases from 0.001/s to 0.1/s.

Even though the level of strain at diffuse necking is higher at a lower strain rate of 0.001/s, the failure strain increases with increasing strain rates, as shown in Figure 3-18. When the PLC effect is present, the failure strain is reduced due to the interaction between PLC bands and shear bands [24], thus causing premature failure. At a strain rate of 0.1/s, the PLC effect is not as severe as it is at 0.001/s, so premature failure is less likely.
Chapter 4: Effects of prestrain on the rate sensitivity of AA5754

4.1 Introduction

This chapter looks at the influence of prestrain with a path change on the behaviour and rate sensitivity of 3mm AA5754 sheet in uniaxial tension for strain rates between 0.001/s and 1500/s. Prestrain paths consisted of plane strain in either the longitudinal (rolling) or transverse material direction at a strain rate of 0.001/s. Prestrains were followed by loading under uniaxial tension in the longitudinal material direction using two test apparatus: a servo-hydraulic Instron for strain rates of 0.001/s and 0.1/s, and a tensile split Hopkinson Bar for strain rates of 500/s and 1500/s.
4.2 Preliminary Testing

Initially, the properties of the AA5754 sheet were determined in the transverse and longitudinal material directions so that the effect of prestrain orientation on the rate sensitivity of AA5754 could be more clearly understood. The ASTM-E8 uniaxial tensile sample [69] geometry was employed, and loaded at a strain rate of 0.001/s using the servo-hydraulic Instron. Four repeats in the transverse direction and six repeats in the longitudinal direction were performed.

Table 4-1 shows the average yield stress and Young’s modulus for the transverse and longitudinal directions. While both of the material constants are higher for the longitudinal direction, the t-test presented in Appendix B indicates that the Young’s modulii are not statistically different. Figure 4-1 compares the true stress versus plastic strain data for AA5754 in the transverse and longitudinal sheet directions, and shows that the transverse direction has a lower hardening rate. This trend is typical of rolled sheet, where the properties are often superior in the rolling direction. The maximum strain value of the curves corresponds to the smallest strain at diffuse necking between all of the repeats, and is not a measure of the strain at diffuse necking across all samples. The individual curves were truncated at this value so that the characteristic curve was based on data points over a common range of strains.

<table>
<thead>
<tr>
<th></th>
<th>Transverse</th>
<th>Longitudinal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>67.7</td>
<td>69.7</td>
</tr>
<tr>
<td>Yield stress (MPa)</td>
<td>93.2</td>
<td>95.5</td>
</tr>
</tbody>
</table>

Table 4-1: Average Young’s modulus and yield stress values for 3mm AA5754 in the transverse and longitudinal material directions.
Figure 4-1: True stress versus true strain for 3mm AA5754 sheet tested under uniaxial tension in the transverse and longitudinal material directions.

4.3 Experimental methods – uniaxial tensile testing following prestrain

Six different magnitudes of plane strain prestrain were applied in the transverse and longitudinal material directions according to the method outlined in Section 4.3.1. These are referred to as prestrain groupings of 2%, 3%, 6%, 8%, 10%, and 12% since there was some variability in the magnitude of prestrain between samples within the same group (see Section 4.3.1). Subsize tensile samples were machined from the prestrained blanks using the geometry previously shown in Figure 3-1, and subsequently tested under uniaxial tension in the longitudinal direction. Tests at strain rates of 0.001/s and 0.1/s were performed using a servo-hydraulic Instron, and tests at strain rates of 500/s and 1500/s were performed using a tensile split Hopkinson bar.
The time between prestraining and subsequent tensile testing was on the order of weeks due to the time needed for machining the subsize samples. It has been shown that the delay in two-stage testing can slightly reduce the flow stress [58], but it has also been reported by An [57] that the delay does not affect the trends in strain hardening.

4.3.1 Applying plane strain prestrain

Plane strain prestrain was applied to a notched rectangular blank using a servo-hydraulic Instron with the wide-grip configuration depicted in Figure 4-2. Prestrain was applied at a quasi-static strain rate of 0.001/s, and although the strain rate during prestraining affects the subsequent flow stress [58], this influence was not explored in this thesis. Two different blank geometries were used for prestrain in the transverse and longitudinal directions, as shown in Figure 4-3. The locations of the subsize uniaxial samples are also indicated. Notches were machined out of each side of the blank to reduce the chance of premature failure at the grip-to-sample interface. The blank height was chosen to minimize the gauge length while still having a plane strain region large enough to machine out the subsize tensile samples. A small gauge length coupled with a large width maximized the size of the plane strain region.

A grid pattern of dots with a centre-to-centre spacing of 2.2mm was applied to each blank in the plane strain region, as illustrated in Figure 4-4, to enable determination of the actual level of prestrain achieved. Image analysis software (ImagePro) was employed to determine the initial and final spacing of the centre of each dot, and the engineering strain between each row of dots was then calculated, as outlined in Appendix C. This method determined the percentage strain to within ±0.38% (a 95% confidence
interval, sample size of 144), and a large part of this error is due to the degradation of the ink dots as they deform.

There was significant variability between the target prestrain, and the prestrain that was achieved due to the nature of the wide-grip mechanism and the jerky flow of the AA5754 sheet. As the blank was loaded, there was a delay before the grip faces engaged and fully clamped the sample. During this delay, the blank could pull out of the grips, resulting in an increase in the effective gauge length. Because of this uncertainty, the target prestrain was controlled with applied load and not applied displacement. However, variability in the prestrain could not be completely eliminated due to the jerky flow of the material. Table 4-2 shows the average prestrain achieved for each grouping (including prestrain in both the transverse and longitudinal sheet directions) as well as the standard deviation and range. The variability within a grouping could be up to 8% of the average prestrain; therefore, the only way to compare sets of subsequent stress-strain data was to convert prestrain into equivalent strain.

Figure 4-2: Wide-grip configuration used for the application of plane strain prestrain.
Figure 4-3: Blank geometries used for plane strain prestrain in the a) transverse and b) longitudinal directions. The location of the subsize samples is also shown. All dimensions are in mm.

All dimensions are ±0.5mm
Figure 4-4: Longitudinal blank with grid pattern used for measuring the achieved level of prestrain.

Table 4-2: Average prestrain values for the six prestrain groupings.

<table>
<thead>
<tr>
<th>Prestrain Grouping</th>
<th>Average Prestrain (%)</th>
<th>Range</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.87</td>
<td>1.66-2.16</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>2.93</td>
<td>2.47-3.22</td>
<td>0.23</td>
</tr>
<tr>
<td>6</td>
<td>5.96</td>
<td>5.67-6.66</td>
<td>0.28</td>
</tr>
<tr>
<td>8</td>
<td>8.15</td>
<td>7.81-8.63</td>
<td>0.24</td>
</tr>
<tr>
<td>10</td>
<td>10.14</td>
<td>9.6-10.51</td>
<td>0.28</td>
</tr>
<tr>
<td>12</td>
<td>12.12</td>
<td>11.74-13.03</td>
<td>0.30</td>
</tr>
</tbody>
</table>

4.3.2 Test matrix: uniaxial tensile testing of prestrained samples

The test matrices for the uniaxial tensile testing of samples prestrained in the transverse and longitudinal orientations are shown in Tables 4-3 and 4-4 respectively. Uniaxial tensile tests at strain rates of 0.001/s and 0.1/s were performed for both transverse and longitudinal prestrain, at prestrain magnitudes of 2, 3, 6, 8, 10 and 12%.
The number of repeats was determined to be the number necessary to produce a statistically significant characteristic curve of flow stress (using the method of Appendix B). Typically 5 or 6 repeats were necessary for each test. Uniaxial tensile tests at strain rates of 500/s and 1500/s were only performed with 12% transverse prestrain.

Table 4-3: Experimental test matrix for uniaxial tensile tests of samples prestrained in the transverse direction at strain rates between 0.001/s and 1500/s.

<table>
<thead>
<tr>
<th>Strain rate</th>
<th>Apparatus</th>
<th>Prestrain Grouping</th>
<th>Repeats</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>Instron</td>
<td>2%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12%</td>
<td>5</td>
</tr>
<tr>
<td>0.1/s</td>
<td>Instron</td>
<td>2%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8%</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12%</td>
<td>4</td>
</tr>
<tr>
<td>500/s</td>
<td>Hopkinson Bar</td>
<td>12%</td>
<td>2</td>
</tr>
<tr>
<td>1500/s</td>
<td>Hopkinson Bar</td>
<td>12%</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 4-4: Experimental test matrix for uniaxial tensile tests of samples prestrained in the longitudinal direction at strain rates of 0.001/s and 0.1/s.

<table>
<thead>
<tr>
<th>Strain rate</th>
<th>Apparatus</th>
<th>Prestrain Grouping</th>
<th>Repeats</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>Instron</td>
<td>2%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12%</td>
<td>6</td>
</tr>
<tr>
<td>0.1/s</td>
<td>Instron</td>
<td>2%</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12%</td>
<td>5</td>
</tr>
</tbody>
</table>

4.3.3 Uniaxial tensile testing of prestrained samples at strain rates of 0.001/s and 0.1/s

The testing of specimens at strain rates of 0.001/s and 0.1/s was performed using a servo-hydraulic Instron testing apparatus. The prestrained subsize samples were loaded in the longitudinal material direction under uniaxial tension, with the strain rate being controlled through the displacement of the cross-head. A 10mm extensometer was fastened to the sample using a combination of elastics and clips to prevent slipping on the very small gauge cross-section.
4.3.4 Uniaxial tensile testing of prestrained samples at strain rates of 500/s and 1500/s

The tensile split Hopkinson bar apparatus from the University of Waterloo was employed for testing at high strain rates of 500/s and 1500/s, following the setup discussed in Section 2.3.3. A set of aluminum bars with integrated slots was machined for gripping the samples. The slots could accommodate up to a 3mm thick specimen, while shims were used within the slot to grip prestrained specimens with thicknesses as low as 2.6mm. Specific test parameters are shown in Table 4-5, with the gas gun pressure providing the required strain rates. A striker length of 0.5m provided a single pulse long enough to significantly deform the sample, but not enough to take the prestrained samples up to the point of diffuse necking or failure on the first pulse. Data was only recorded for the first pulse, even though the samples were broken by subsequent pulses. Strain gauges were mounted on the incident and transmitted bars and the voltage output was recorded through a high-speed digital oscilloscope.

<table>
<thead>
<tr>
<th>Strain Rate</th>
<th>Gas Gun Pressure (kPa)</th>
<th>Striker Material</th>
<th>Striker Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500/s</td>
<td>42.75</td>
<td>Aluminum</td>
<td>0.5</td>
</tr>
<tr>
<td>1500/s</td>
<td>222.01</td>
<td>Aluminum</td>
<td>0.5</td>
</tr>
</tbody>
</table>
4.4 Experimental results – uniaxial tensile testing following prestrain

The true stress versus plastic strain data up to the point of diffuse necking was determined for each sample, and fit using the following form of the Voce material model:

\[ \sigma_f = B - (B - A)e^{-n\varepsilon_p}. \]  \hspace{1cm} (9)

A characteristic flow curve was produced for each of the four strain rates, at each prestrain orientation, and for each prestrain grouping. The flow stress was plotted against true equivalent plastic strain to take into account the differences in the amount of prestrain applied. The von Mises equivalent strain was determined using:

\[ \varepsilon_{eq} = \sqrt{\frac{2}{3}(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)} \]  \hspace{1cm} (26)

where: \( \varepsilon_1 = \) major strain  
\( \varepsilon_2 = \) minor strain  
\( \varepsilon_3 = -\varepsilon_1 - \varepsilon_2. \)

The flow stress was also plotted against true plastic strain as presented in Appendix D.

4.4.1 Uniaxial tensile testing of prestrained samples at strain rates of 0.001/s and 0.1/s

Raw data points of load and gauge displacement were analyzed using the method of Appendix A. For each specimen, a Voce curve (Equation (9)) was fit to the true stress
versus plastic strain data, and the true stress versus equivalent plastic strain data up to the point of diffuse necking. Curves with parameters or equivalent prestrain magnitudes falling outside the 95% confidence interval were identified based on the statistical analysis outlined in Appendix B. An average of the remaining curves was taken to produce a characteristic Voce curve for each combination of strain rate, prestrain magnitude and prestrain orientation. The maximum strain value of the curves corresponds to the smallest strain at diffuse necking between all of the repeats, and is not a measure of the strain at diffuse necking across all samples. The individual curves were truncated at this value so that the characteristic curve was based on data points over a common range of strains. As a baseline, Figure 3-8 in Section 3.4.1 compares the flow stress curves for strain rates of 0.001/s and 0.1/s for samples without prestrain.

4.4.1.1 Results for transverse prestrain

The flow stress curves for samples with transverse prestrain are compared for strain rates of 0.001/s and 0.1/s in Figure 4-5 for each of the 6 prestrain groupings, plotted against true equivalent plastic strain. For all cases except 8% transverse prestrain, a strain rate of 0.1/s exhibits lower flow stresses than 0.001/s. At prestrains of 2% and 3%, there is a clear decrease in the hardening rate between 0.001/s and 0.1/s, whereas prestrains of 6% and 12% show a decrease in the yield stress and an overall decrease in the flow stress between 0.001/s and 0.1/s. The curve for 10% prestrain at 0.001/s appears somewhat anomalous, as its hardening rate is higher than expected. Figure 4-6 compares the curves for the different prestrain groupings with the monotonic curve in order to illustrate the effect of prestrain with a path change on flow stress. It can be seen that
following plane strain prestrain in the transverse direction, the initial yield stress is lower than the monotonic curve, the hardening rate is increased, and the flow stress approaches the monotonic curve from below.

Figure 4-7 indicates that the hardening rate following prestrain is higher than for the monotonic case at strain rates of 0.001/s and 0.1/s. As loading progresses, the hardening rate approaches that of the monotonic curve.

![Graphs showing stress-strain behavior](image)

- a) 2% transverse prestrain
- b) 3% transverse prestrain
- c) 6% transverse prestrain
- d) 8% transverse prestrain
Figure 4-5: A comparison between strain rates of 0.001/s and 0.1/s for uniaxial tensile samples subjected to transverse prestrains ranging from 2% to 12%.

- e) 10% transverse prestrain
- f) 12% transverse prestrain

a) strain rate of 0.001/s
Figure 4-6: Samples subjected to transverse prestrains of different magnitudes, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s.
Figure 4-7: Hardening rate of samples subjected to different magnitudes of transverse prestrain, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s.

Figure 4-8 presents the yield stress for each individual sample plotted against its corresponding equivalent prestrain for strain rates of 0.001/s and 0.1/s. The yield stress appears to increase linearly with increasing prestrain between 2% and 10% prestrain, and there is no clear difference in the values between the two strain rates. The apparent decrease in the yield stress between 0.001/s and 0.1/s for 6% and 12% prestrain in Figure 4-5 is a result of comparing characteristic curves, and does not indicate any change in the rate sensitivity of the yield stress. The linear fits of Figure 4-8 have similar slopes, indicating that the rate sensitivity of the yield stress does not change with increasing prestrain.
Figure 4-8: Yield stress values from individual samples for all 6 transverse prestrain groupings for strain rates of 0.001/s and 0.1/s.

The overall hardening rate ($\Theta_{\text{overall}}$) between yield ($\sigma_{\text{min}}$ and $\epsilon_{\text{p(min)}}$) and the onset of diffuse necking ($\sigma_{\text{max}}$ and $\epsilon_{\text{p(max)}}$) was determined for each characteristic curve using:

$$\Theta_{\text{overall}} = \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})}{(\epsilon_{\text{p(max)}} - \epsilon_{\text{p(min)}})}.$$  \hspace{1cm} (27)

A common range of strains was considered for both strain rates within each prestrain grouping, determined by truncating the curves to match the one with the minimum strain at diffuse necking. The percent change in the overall hardening rate ($\Delta_{\Theta}$) between 0.001/s and 0.1/s was determined from:

83
\[
\Delta_\Theta = -\left( \frac{\Theta_{\text{overall} \mid 0.001/s} - \Theta_{\text{overall} \mid 0.1/s}}{\Theta_{\text{overall} \mid 0.001/s}} \right).
\]  

(28)

The percent change in the hardening rate for each prestrain grouping is plotted in Figure 4-9, where the negative values indicate that the hardening rate decreases as the strain rate is increased from 0.001/s to 0.1/s. The smaller prestrains of 0, 2 and 3% exhibit a larger decrease in the subsequent hardening rate, which is an indication of negative rate sensitivity. Prestrains of 6% and greater show a much smaller change in the hardening rate, which indicates a reduction in rate sensitivity with higher magnitudes of prestrain.

Figure 4-9: Percentage change in the overall hardening rate between a strain rate of 0.001/s and 0.1/s (Equation (28)), for samples subject to transverse prestrain.
In general, Figures 4-10 and 4-11 demonstrate that stresses and strains at diffuse necking are higher for a strain rate of 0.001/s, even though some of the prestrain levels do not show statistically significant differences. The strain at failure, however, is higher at a strain rate of 0.1/s for all prestrain values up to 10%, as can be seen in Figure 4-12. For a strain rate of 0.1/s, the failure strain remains relatively constant with increasing prestrains, whereas, the failure strain increases with increasing prestrains for a strain rate of 0.001/s. The strain at failure was taken as the nominal equivalent plastic strain at failure, which was calculated as the sum of the true equivalent prestrain and the uncorrected true equivalent strain at failure as measured by the extensometer.

Figure 4-10: Maximum stress at the onset of diffuse necking for varying amounts of prestrain in the transverse direction.
Figure 4-11: True equivalent plastic strain at the onset of diffuse necking for varying amounts of prestrain in the transverse direction.

Figure 4-12: Nominal equivalent plastic strain at failure for varying amounts of prestrain in the transverse direction.
4.4.1.2 Results for longitudinal prestrain

The flow stress curves for samples with longitudinal prestrain are compared for strain rates of 0.001/s and 0.1/s in Figure 4-13 for each of the 6 prestrain groupings, plotted against true equivalent plastic strain. In all cases, a strain rate of 0.1/s exhibits lower flow stresses than 0.001/s. At prestrains of 6, 8, and 12%, there is a clear decrease in the hardening rate between 0.001/s and 0.1/s, whereas, at prestrains of 2, 3 and 10% there is a decrease in the yield stress and an overall decrease in the flow stress between 0.001/s and 0.1/s. Figure 4-14 compares the curves for the different prestrain groupings with the monotonic curve in order to illustrate the effect that prestrain with a path change has on the flow stress. Following plane strain prestrain in the longitudinal direction, the initial yield stress is lower than the monotonic curve, the hardening rate increases, and the flow stress approaches the monotonic curve from below.

Figure 4-15 reveals that the hardening rate following prestrain is higher than for the monotonic case at strain rates of 0.001/s and 0.1/s. As loading progresses, the hardening rate approaches that of the monotonic curve.

![Graphs](image)

a) 2% longitudinal prestrain  
b) 3% longitudinal prestrain
Figure 4-13: A comparison between strain rates of 0.001/s and 0.1/s for samples subjected to longitudinal prestrain with magnitudes from 2% to 12%, tested in uniaxial tension.

c) 6% longitudinal prestrain  
d) 8% longitudinal prestrain

e) 10% longitudinal prestrain  
f) 12% longitudinal prestrain
Figure 4-14: Samples subjected to longitudinal prestrains of different magnitudes, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s.
Figure 4-15: Hardening rate for samples subjected to longitudinal prestrains of different magnitudes, tested in uniaxial tension at strain rates of a) 0.001/s and b) 0.1/s.
Figure 4-16 presents the yield stress for each individual sample plotted against its corresponding equivalent prestrain for strain rates of 0.001/s and 0.1/s. The yield stress appears to increase linearly with increasing prestrain between 2% and 10% prestrain, and there is no clear difference in the values between the two strain rates. The apparent decrease in the yield stress between 0.001/s and 0.1/s for 2, 3 and 10% prestrain in Figure 4-13 is a result of comparing characteristic curves, and does not indicate any change in the rate sensitivity of the yield stress. The linear fits of Figure 4-16 have similar slopes, indicating that the rate sensitivity of the yield stress does not change with increasing prestrain.

The percentage change in the hardening rate between 0.001/s and 0.1/s was determined using Equations (27) and (28), and is plotted against true equivalent prestrain in Figure 4-17. The negative values indicate that the hardening rate decreases as the strain rate is increased from 0.001/s to 0.1/s. Without prestrain there is a large reduction in overall hardening, which is an indication of negative rate sensitivity. Prestrains of 2% and greater produce much smaller changes in the hardening rate, which indicates a reduction in rate sensitivity when prestrain with a path change is applied.
Figure 4-16: Yield stress values from individual tests for all 6 longitudinal prestrain groupings for strain rates of 0.001/s and 0.1/s.

Figure 4-17: Percentage change in the overall hardening rate between a strain rate of 0.001/s and 0.1/s, for longitudinal prestrain.
In general, Figures 4-18 and 4-19 demonstrate that the stresses and strains at diffuse necking are higher for a strain rate of 0.001/s, even though some of the prestrain levels do not show statistically significant differences. The nominal equivalent plastic strain at failure, however, is higher at a strain rate of 0.1/s for all prestrain values, as can be seen in Figure 4-20. For a strain rate of 0.1/s, the failure strain exhibits only a slight increase with increasing levels of prestrain, whereas the failure strain increases significantly with increasing prestrains for a strain rate of 0.001/s.

![Graph](image)

Figure 4-18: Maximum stress at the onset of diffuse necking for varying amounts of prestrain in the longitudinal direction.
Figure 4-19: True equivalent plastic strain at the onset of diffuse necking for varying amounts of prestrain in the longitudinal direction.

Figure 4-20: Nominal equivalent plastic strain at failure for varying amounts of prestrain in the longitudinal direction.
4.4.1.3 Comparison between results for transverse and longitudinal prestrain

The flow stress curves for each prestrain grouping are shown in Figure 4-21 for the two prestrain orientations and the two strain rates, plotted against true equivalent plastic strain. Prestrain in the longitudinal direction causes higher subsequent flow stresses than prestrain in the transverse direction at the corresponding strain rate, in all cases except for 10% prestrain. (Again, as previously noted in Section 4.4.1.1 the curve at 0.001/s with 10% transverse prestrain appears to be an anomaly, as the hardening rate is much higher that the other three curves in Figure 4-21 e)). Overall, the data presented in Figure 4-21 indicates that the orientation of the prestrain has a significant effect on the flow stress during subsequent loading.

Figure 4-22 compares the yield stresses for each sample individually plotted against its corresponding equivalent prestrain for both prestrain orientations. The yield stress increases linearly with increasing prestrain and the samples with longitudinal prestrain have higher yield stresses than those with transverse prestrain, demonstrating that prestrain orientation has an effect on the yield stress during subsequent uniaxial loading.
a) 2% prestrain

b) 3% prestrain

c) 6% prestrain
d) 8% prestrain

e) 10% prestrain

f) 12% prestrain

Figure 4-21: A comparison between strain rate and prestrain orientation for samples subjected to plane strain prestrains between 2% and 12%, tested in uniaxial tension.
Figure 4-22: Effect of prestrain orientation on yield stress values obtained from individual samples at strain rates of a) 0.001/s and b) 0.1/s.
In general, Figures 4-23 and 4-24 demonstrate that stresses and strains at diffuse necking are higher for prestrain in the longitudinal direction, even though some of the prestrain levels do not show statistically significant differences. Figure 4-25 indicates no significant difference in the nominal equivalent plastic strain at failure between longitudinal and transverse prestrains. The failure strain shows a slight increase with increasing prestrains at a strain rate of 0.1/s, whereas the failure strain increases significantly with increasing prestrains for both orientations at a strain rate of 0.001/s.
Figure 4-23: Maximum stress at the onset of diffuse necking for varying amounts of prestrain in the longitudinal direction (L) and transverse direction (T). a) strain rate of 0.001/s, b) strain rate of 0.1/s.

a) strain rate of 0.001/s

b) strain rate of 0.1/s
Figure 4-24: True equivalent plastic strain at the onset of diffuse necking for varying amounts of prestrain in the longitudinal and transverse directions. a) strain rate of 0.001/s, b) strain rate of 0.1/s.
Figure 4-25: True equivalent plastic strain at failure for varying amounts of prestrain in the longitudinal and transverse directions. a) strain rate of 0.001/s, b) strain rate of 0.1/s.

4.4.2 Uniaxial tensile testing of prestrained samples at strain rates of 500/s and 1500/s

For the high-rate tensile split Hopkinson bar tests, the raw strain gauge voltage and time data was collected by a digital oscilloscope and then converted to stress and strain data through the use of calibration files and a program called CSHB provided by the University of Waterloo (as discussed in Section 3.4.2). The resulting stress-strain data was subsequently analyzed using the method outlined in Appendix A to obtain a Voce curve fit of true stress versus true plastic strain, as plotted in Figure 4-26. It is important to note that data was only recorded for the first loading pulse, which meant that the point of diffuse necking or failure was not necessarily reached. As a result, the curves
in Figure 4-26 terminate at a characteristic strain corresponding to the peak load, and not at the point of diffuse necking as was the case for the strain rates of 0.001/s and 0.1/s (Section 3.4.2).

Curves falling outside the 95% confidence interval were identified using the statistical analysis outlined in Appendix B, and an average of the remaining curves was taken to produce a characteristic Voce curve for each strain rate. Tests at 500/s and 1500/s were only performed with 12% transverse prestrain, and their characteristic curves are depicted in Figure 4-27 along with the results from 0.001/s and 0.1/s (with 12% transverse prestrain). The flow stress curve for 500/s is linear, and does not have the typical shape of the Voce curve fit that is exhibited for 0.001/s, 0.1/s and 1500/s.

![Graph showing true stress versus true plastic strain for a strain rate of 500/s.](image)

a) strain rate of 500/s
Figure 4-26: Voce curve fit to the true stress versus true plastic strain data for specimens prestrained to 12% in the transverse direction and then tested in uniaxial tension at strain rates of a) 500/s and b) 1500/s.

Figure 4-27: Characteristic Voce curve for specimens prestrained to 12% in the transverse direction and then tested in uniaxial tension at strain rates between 0.001/s and 1500/s.
4.5 Discussion

The following discussion focuses on two major topics: the effect of prestrain on the subsequent uniaxial flow stress of AA5754 sheet, and the effect of prestrain on the material’s rate sensitivity. A brief explanation is also presented as to why the high-rate results obtained at strain rates of 500/s and 1500/s are not used to draw conclusions within this section.

4.5.1 Effect of prestrain on uniaxial flow stress

There are three characteristics of prestrain that must be considered: the change in the loading condition (ie. plane strain versus uniaxial), the change in orientation and the prestrain magnitude. The effects of prestrain that have been observed in this thesis are primarily a result of the path change from plane strain to uniaxial tension, and not the fact that the loading was applied in two stages, that the direction of major loading changed between applied paths or that there was a delay between the two stages of loading. Figure 4-28 a) shows that when a sample is prestrained to 6% in uniaxial tension, followed immediately by loading in uniaxial tension in the same direction (no change in path or orientation), the prestrain has no effect on the flow stress upon reloading. That is, during second stage loading, the initial yield stress lies directly on the monotonic curve, and the flow stress continues to increase from thereon as if no prestrain had been applied. Figure 4-28 a) also shows that when there is a delay between application of prestrain and subsequent loading (a two-week delay in this case), there is a slight reduction in the yield stress, but the flow stress quickly returns to the monotonic curve (ie. within about 3% strain in Figure 4-28 a)). This reduced yield point is likely a result of a thermally-
activated recovery process in which dislocations move small atomic distances to form lower energy configurations. Recovery can occur in Al alloys at room temperature due to the relatively low melting temperature. Figure 4-28 b) illustrates, however, that thermal recovery only accounts for part of the initial reduction in the flow stress observed following plane strain prestrain, and that thermal recovery has no effect on the subsequent strain hardening [57]. Therefore, the overall behaviour observed in this chapter is primarily attributed to the path change between plane strain prestrain and subsequent uniaxial loading.

The effect of prestrain involving a path change on the flow stress, falls into two main categories as described by Fernandes et al. [54] and illustrated in Figure 2-12. In the first case, the initial yield stress is higher than the monotonic curve, the hardening rate is reduced, and the flow stress approaches the monotonic curve from above. In the second case, the initial yield stress is lower than the monotonic curve, the hardening rate is increased, and the flow stress approaches the monotonic curve from below. Alloys that undergo dynamic strain aging, such as AA5754, tend to exhibit the second type of behaviour. It has been discussed by Wilson et al. [56] that the reduced hardening rate exhibited by the first case is a result of the annihilation or rearrangement of the dislocation barriers developed during prestrain. The solute atoms present in materials that undergo dynamic strain aging cause internal stresses which act in the direction opposite to the applied stress during prestrain. With a change in strain path, the internal stresses in the new direction are less than in the monotonic case, which results in a decrease in the flow stress, but an increased hardening rate as the new internal stresses develop [57]. This response is evident in Figures 4-6 and 4-14. Since the same response
is exhibited for both transverse and longitudinal prestrains, it appears that the path change from plane strain to uniaxial tension, rather than the change in orientation, causes this transient behaviour.

Figure 4-28: The flow stress curve for loading involving prestrain without a path change. a) 6% prestrain in uniaxial tension was followed either immediately or after a 2 week delay by uniaxial tensile loading. b) Results following a 2 week delay are compared for uniaxial prestrain and 6.4% equivalent plane strain prestrain.
The severity of the prestrain is not only a function of the magnitude of the prestrain, but also depends on the change in orientation between the two stages of loading. All samples were subject to the path change from plane strain to uniaxial tension, but those subjected to prestrain in the transverse direction also contained a 90° change in major strain orientation that can be considered a more severe path change.

Since prestrain with a path change has the effect of reducing the flow stress in AA5754, a more severe prestrain path such as plane strain in the transverse direction should produce consistently lower flow stresses than prestrain in the longitudinal direction. Figure 4-6 shows that with a 90° change in orientation, the flow stress of the prestrained samples approach a value below that of the monotonic curve, whereas without the orientation change, the saturation stresses approach the monotonic curve (see Figure 4-14). It is not uncommon for a second stage of loading to exhibit this “permanent softening” effect [70], where the flow stresses remain below the monotonic curve. Permanent softening is evident in Figure 4-29, where the saturation stress (as determined by the parameter B from the Voce fit for the characteristic curves) has been plotted for monotonic loading and for prestrained samples in the two orientations. At strain rates of 0.001/s and 0.1/s, the saturation stress following longitudinal prestrain is much closer to the monotonic curve than for the case of transverse prestrain. At 0.001/s, the longitudinal prestrain samples fall just below the monotonic line, whereas at 0.1/s, they actually surpass the monotonic saturation stress. This result suggests that by introducing a 90° change in orientation, and making the prestrain more “severe”, a permanent softening of the flow stress occurs.
Figure 4-29: Comparison of the saturation stress between transverse and longitudinal prestrain orientation for strain rates of a) 0.001/s and b) 0.1/s.
It is evident in Figure 4-21 that the flow stress curves following longitudinal prestrain are consistently higher than those following transverse prestrain. One reason is that transverse prestrain causes permanent softening of the flow stress curve, but the anisotropy of the material may also have an effect. Figure 4-1 illustrates that the flow stress for samples loaded in uniaxial tension in the longitudinal direction is higher than in the transverse direction. The higher stress reached during prestrain in the longitudinal direction undoubtedly contributes to the higher initial yield stress upon reloading, as shown in Figure 4-22.

The overall shapes of the flow stress curves for prestrained samples can be described in terms of the change in hardening rate as loading progresses. Figure 4-30 provides a comparison between the hardening rates for samples prestrained to 12% in the transverse and longitudinal directions at a strain rate of 0.001/s. The samples with transverse prestrain have a higher hardening rate initially, because the stress rises from a lower yield stress. As loading progresses, the hardening rate for the samples with longitudinal prestrain becomes higher, as the flow stress continues to rise to approach the monotonic curve, whereas the samples with transverse prestrain approach a lower value. These characteristics are typical of the prestrain orientations in general, irrespective of the amount of prestrain or the strain rate.

Diffuse necking is predicted to occur when the true stress and the hardening rates are equal, as discussed in Section 2.1.4. Figure 4-30 indicates that at the point of diffuse necking (which corresponds to the maximum strain on the graph), samples with longitudinal prestrain have a higher hardening rate; this elevated rate explains the larger
strains and corresponding stresses for the longitudinal prestrain orientation, shown previously in Figures 4-11 and 4-19.

![Graph showing hardening rate vs. true equivalent plastic strain](image)

**Figure 4-30:** Comparison of the hardening rate for samples prestrained to 12% in the transverse and longitudinal directions at a strain rate of 0.001/s.

It is clear that prestrain with a path change reduces the flow stress of AA5754 sheet during subsequent loading, and a larger change in orientation between the two loading stages produces a permanent softening of the flow stress curve. Furthermore, the combined effect of prestrain path and orientation change results in the stresses and strains at diffuse necking being higher for samples prestrained in the longitudinal direction.
4.5.2 The effect of prestrain on rate sensitivity

It has been discussed in Section 2.1.3 that it is the presence of the PLC effect that causes the rate sensitivity of AA5754; therefore, to understand the effect of prestrain on rate sensitivity, one must determine the effect of prestrain on the PLC phenomenon.

Strain rate sensitivity can be described in terms of two flow curve characteristics: the yield stress and the hardening rate, where an increase in the yield stress and the hardening rate with an increase in applied strain rate indicates positive rate sensitivity.

Figures 4-8 and 4-16 indicate that the yield stress increases with increasing prestrain, and the values of the yield stress are similar between 0.001/s and 0.1/s. This correspondence is expected, as the yield stress is not rate-sensitive until higher strain rates are reached (see Figure 2-2). The yield stress remains rate-insensitive with increasing prestrains in either the transverse or longitudinal directions.

Figure 4-9 illustrates that the percentage change in the overall hardening rate between 0.001/s and 0.1/s is reduced with increasing prestrain after a transverse prestrain of at least 6%. Figure 4-17 depicts the same trend with longitudinal prestrains of at least 2%, which shows that overall, there is a reduction in the rate sensitivity of AA5754 with increasing prestrain involving a path change.

This reduction in rate sensitivity may be tied to the fact that the PLC effect is initiated after a critical strain, which varies with the amount of prestrain [23]. The critical strain increases with increasing prestrain when a prestrain of at least 2.5 - 4% is applied, as shown in Figure 4-31 for AA6082. The critical strain also depends on the change in orientation between the two loading stages, where a larger change results in higher critical strains. The micromechanisms responsible for this delay in critical strain are not
known. The observed behaviour implies that as the amount of prestrain increases, less of the subsequent deformation is occurring in conjunction with the PLC effect. Since the PLC effect causes negative rate sensitivity, it follows that as the amount of prestrain increases, negative rate sensitivity decreases.

The delay in the onset of the PLC effect with prestrain also helps to explain the reason why uniaxial tensile samples tested at 0.001/s show a larger degree of permanent softening than those tested at 0.1/s. This trend is evident in Figure 4-32 where the saturation stresses for 0.001/s drop well below the monotonic curve, whereas they are only slightly lower for 0.1/s. It was previously discussed in Section 2.1.3 that the increased forces required to break pinned dislocations away from solute atoms (eg. Mg) is the primary cause of the increase in flow stress. Hence, if the onset of the PLC effect is delayed, less of an increase in the flow stress is expected, which accounts for the large decrease in the saturation stress at 0.001/s with applied prestrain when compared to either the monotonic curve or testing at 0.1/s. At a strain rate of 0.1/s, the PLC effect is not as severe because the Mg solute atoms are not able to diffuse fast enough to pin mobile dislocations; therefore, any delay in the PLC effect would not have as large of an effect as it would at 0.001/s.
Figure 4-31: The effect of prestrain magnitude $\varepsilon_p$ on the critical strain for serrated flow $\varepsilon_c$ in AA6082 as shown by An et al. [23].

Figure 4-32: Percentage change in the saturation stress between samples prestrained in the transverse direction and the monotonic saturation stress.
The delay in the onset of the PLC effect with increasing prestrain also explains the changes in rate sensitivity of the failure strain. In Figure 4-25 b), the strain rate of 0.1/s produces relatively constant values of the failure strain with increasing prestrain, presumably because the PLC effect is not significant at this strain rate. Therefore, the delay in the PLC effect, caused by the prestrain, does not have a large effect on the failure strain. For tests conducted at 0.001/s, the PLC effect causes premature failure resulting in lower failure strains. However, as the prestrain increases, the PLC effect and premature failure are both delayed, causing the failure strain to increase with increasing prestrain, as shown in Figure 4-25 a). Figure 4-12 and Figure 4-20 clearly illustrate that the failure strains become similar as the level of prestrain increases, which is another indication of prestrain reducing the rate sensitivity of AA5754.

One way to quantify the rate sensitivity of AA5754 as a function of prestrain is to fit the stress-strain data to a rate-sensitive material model and compare the magnitudes of the rate sensitivity parameters. A rate-sensitive Voce material model (described in Section 6.3.1) is defined by:

$$
\sigma_f = \left( B - (B - A) e^{-n\varepsilon_p} \right) \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m + m' \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)
$$

(29)

where: A, B, n are Voce parameters
ε_p = true plastic strain
\dot{\varepsilon} = engineering strain rate
\dot{\varepsilon}_0 = base engineering strain rate
m, m' = multiplicative and additive rate sensitivity parameters, respectively.

The multiplicative rate sensitivity parameter (m) is a measure of the rate sensitivity of the hardening rate as it affects the shape of the curve. The additive rate sensitivity parameter...
(m') provides a vertical shift in the flow stress; however, since it also accounts for any
shift caused by the parameter m, it can not be taken strictly as a measure of the rate
sensitivity of the flow stress (see Section 6.3.1 for further details).

The constant \( \dot{\varepsilon}_0 \) in Equation (29) represents the base strain rate of 0.001/s, against
which the rate sensitivity parameters m and m' are determined. The Voce parameters A,
B and n are determined for a strain rate of 0.001/s, and used to calculate the m and m'
parameters that produce the best fit for the stress-strain data at 0.1/s. Only the
statistically significant samples were considered (ie. the same samples used to produce
the characteristic Voce curves, based on the method of Appendix B). The multiplicative
rate sensitivity parameter (m) is plotted against prestrain for both prestrain orientations in
Figure 4-33.

Figure 4-33 a) illustrates that the magnitude of the multiplicative rate parameter
decreases significantly with increasing transverse prestrain of at least 6%, while Figure
4-33 b) shows that for any magnitude of longitudinal prestrain, the rate sensitivity is
minimal. Hence, it can be concluded that prestrain in either orientation has the effect of
reducing the rate sensitivity of the hardening rate. It appears that a larger amount of
prestrain is required in the transverse direction to reduce the rate sensitivity, in
comparison to the longitudinal direction, although the reason for this difference is
unknown.
Figure 4-33: Multiplicative rate parameter, m, for samples with a) transverse and b) longitudinal prestrain.
One other factor to consider is that prestrain increases dislocation density. With a finite number of Mg solute atoms available, there is a limit to the PLC effect. As prestrain increases, there may be more interactions between dislocations than between solute atoms and dislocations. If the PLC effect that causes the rate sensitivity of AA5754 is diminished, then a decrease in the rate sensitivity with increasing prestrain is expected.

It has been shown that prestrain with a path change causes a reduction in the rate sensitivity of the hardening rate as well as the failure strain. Since the rate sensitivity of the yield stress is minimal, the overall rate sensitivity of AA5754 is reduced with increasing prestrain. While the orientation of the prestrain was found to have a significant effect on the flow stress curves during subsequent uniaxial loading (as shown in Section 4.5.1), it does not appear to have a significant effect on rate sensitivity.

### 4.5.3 Prestrained samples at strain rates of 500/s and 1500/s

The characteristic curve of flow stress for a prestrained sample tested at a strain rate of 500/s does not have the typical shape of the other experimental curves presented in Figure 4-27. This irregularity is caused by the fact that strain gauge data for the samples tested in the Hopkinson Bar was only recorded for the first stress wave pulse. As a result, the stress-strain curves are only plotted for a small range of strains. Figure 4-34 shows the portion of the experimental true stress versus true equivalent strain data obtained from the Hopkinson Bar tests, and subsequently used to fit the characteristic flow stress curves at 500/s and 1500/s. Even though the range of data in Figure 4-34 is at a constant strain rate, there still appears to be dynamic effects present. Therefore, when
only a small portion of the data is used to fit the curve, the resulting shape may not be indicative of the overall shape of the curve. This issue is compounded when large amounts of prestrain are applied because the initial hardening rate during subsequent uniaxial loading is very high. In the case of 500/s, this may have been the only portion of the curve that was captured. Because of this uncertainty surrounding the high-rate data, it was not used to draw any conclusions for this section of the thesis. However, data obtained at 1500/s is employed for the constitutive modeling work outlined in Section 6.3.

Figure 4-34: Experimental true stress versus true equivalent plastic strain for 12% transverse prestrain, tested in uniaxial tension at strain rates of 500/s and 1500/s.
Chapter 5: Effects of prestrain and strain rate on deformation of AA5754 in bending

5.1 Introduction

This chapter examines the effects of strain rate and prestrain on the deformation behaviour of 3mm AA5754 sheet in bending, a type of loading that is seen during an axial crush scenario. This work also provides experimental data for validation of the constitutive material model implemented in Section 6.3.

Figure 5-1 shows the typical accordion style deformation of a hollow tube with a square cross-section under a high-rate axial crush. The structure deforms in a series of bends, which are constrained at the edges as they transition to the bends of the adjacent faces. While the loading is more complex than a simple free bend, the current study focuses on the pure bending of prestrained rectangular samples at applied strain rates of
0.001/s, 0.1/s and 80/s. Prestrain was applied in plane strain in either the transverse or longitudinal material direction, at magnitudes of 0, 3%, 6% and 12%. A specially designed test fixture allowed for a servo-hydraulic Instron to be employed for bending at nominal rates of 0.001/s and 0.1/s, and a drop tower to be used at 80/s.

![Image](image.jpg)

Figure 5-1: Typical axial crush deformation of a hollow tube with a square cross-section.

### 5.2 Preliminary work

#### 5.2.1 Development of the bend test fixture

A test fixture was designed to enable simple rectangular sheet samples to be loaded in bending using either an Instron or a drop tower. The fixture was designed to consistently produce bends at the sample midpoint, and to allow for unobstructed development of the bend. The assembled test fixture is shown in Figure 5-2, and the dimensions of the components are shown in Figure 5-3 and Figure 5-4. The assembly consisted of rectangular inserts screwed into the two circular end plates. The top plate was free to slide down the four bolts (8mm diameter, 50mm non-threaded, 30mm
threaded), and the 8 nuts allowed the position of the bottom plate to be adjusted. The sample was placed between the two inserts, and the position of the bottom plate was raised to provide a slightly compressive load. The nuts were finger tightened so the fixture gripped the sample in place until loading was applied.

When employed in the Instron, the larger, flat end plates allowed the fixture to sit on a lower cylinder while the upper cylinder applied load to the top plate as shown in Figure 5-5. When used in the drop tower, the fixture sat on the stand, and the striker was free to impact the top plate as shown in Figure 5-6. The preload applied by the test fixture was necessary in this case to stabilize the sample before it was hit by the striker. Another option was to have a free-standing sample, but this setup would require the ends of the sample to be machined within a very tight tolerance, in addition to the possibility that the sample could be hit slightly off-centre, or could fall out of the fixture before being loaded. The fixture that was implemented allowed the plates to be centred so that loads were applied without any offset in order to avoid unsymmetrical bending and instability. Even if the loading was slightly off-centre, the 4 bolts ensured that the plates compressed parallel to each other and that bending occurred at the centre of the sample. The inserts were included so that the bend would be allowed to develop in either direction without obstruction, just as it would in the axial crush situation depicted in Figure 5-7 a).
Figure 5-2: Assembled bend test fixture.
Figure 5-3: Dimensioned drawing for the end plates of the bend test fixture. Dimensions are in inches.

All dimensions are ± 0.05”

Figure 5-4: Dimensioned drawing for the inserts of the bend test fixture. Dimensions are in inches.

All dimensions are ± 0.05”
Figure 5-5: Bend test fixture in place in the Instron.
Figure 5-6: Bend test fixture in place in the drop tower.
5.3 Experimental methods - bending

Prestrain was applied using the method discussed in Section 4.3.1 in either the transverse or longitudinal material directions employing the geometries depicted in Figure 5-8. Rectangular samples 25.4mm x 19.1mm were machined from the prestrained sheet blanks, and subsequent loading was applied in compression in the longitudinal direction, producing a bend about the transverse axis, as illustrated in Figure 5-9. Applied strain rates of 0.001/s, 0.1/s and 80/s were employed, which refers to the engineering strain rate of the ends of the sample relative to the initial sample length, and not the varying strain rate occurring within the sample itself. For example, the drop tower impacted the sample at a velocity of 2050mm/s, and with a sample length of 25.4mm, this corresponds to an applied strain rate of 80.7/s. Locally, the maximum strain rate within the sample is higher than the applied strain rate, and the strain rate distribution within the sample changes as the bend progresses.
Figure 5-8: a) Transverse and b) longitudinal prestrain blank geometries used for bending samples. Dimensions are in millimetres.

Figure 5-9: a) Bend sample geometry (mm), b) loading orientation, c) deformed orientation.
5.3.1 Test matrix-bending

Samples were pretrained to 3%, 6% and 12% in the transverse and longitudinal orientations. Three repeats of each combination of strain rate, prestrain magnitude and prestrain orientation were performed, as summarized in Table 5-1.

Table 5-1: Experimental bend test matrix for applied strain rates between 0.001/s and 80/s.

<table>
<thead>
<tr>
<th>Applied strain rate</th>
<th>Apparatus</th>
<th>Prestrain Grouping</th>
<th>Prestrain Orientation</th>
<th>Repeats</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>Instron</td>
<td>0%</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>Transverse</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>Transverse</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12%</td>
<td>Transverse</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>Longitudinal</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>Longitudinal</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12%</td>
<td>Longitudinal</td>
<td>3</td>
</tr>
<tr>
<td>0.1/s</td>
<td>Instron</td>
<td>0%</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3%</td>
<td>Transverse</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6%</td>
<td>Transverse</td>
<td>3</td>
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5.3.2 Bend testing at applied strain rates of 0.001/s and 0.1/s

The bend testing of specimens at applied strain rates of 0.001/s and 0.1/s, was performed using a servo-hydraulic Instron testing apparatus and the bend test fixture presented in Figure 5-2. The prestrained rectangular sheet samples were loaded in the longitudinal material direction under compression to produce a bend around the transverse axis. The applied strain rate was controlled through the displacement of the cross-head, and loading continued until a maximum compressive load of 20kN was reached.

5.3.3 Testing at an applied strain rate of 80/s

The drop tower apparatus from the University of Waterloo and the bend test fixture shown in Figure 5-2 were used for bend testing at an applied strain rate of 80/s. The drop tower apparatus is pictured in Figure 5-10 [46], where a striker mass guided by a track was released from a specified height to impact the test fixture and load the sample in compression. A height of 0.215m provided the required impact velocity of 2.05m/s, and a mass of 47.5kg generated sufficient energy to deform the sample. The striker was instrumented with a load cell in order to measure the impact load over time. The corresponding displacements were determined based on the measured velocity-time relation. A firm, thin piece of rubber was placed on top of the bend test fixture, and between the support and the bottom of the bend test fixture to provide some damping to the system.
Figure 5-10: Drop tower test apparatus at the University of Waterloo [46].
5.4 Experimental results - bending

Plots of load versus sample end displacement were created for all bend tests. In order to compare results between samples, the displacement scale was individually offset to match the peak load at a displacement of 1mm. Other methods such as matching the displacement at the midpoint between maximum and minimum load, and matching the spike in the load at the end of the test were evaluated, but did not prove to be as effective in consistently matching the curves.

5.4.1 Bending results for applied strain rates of 0.001/s and 0.1/s

Data points of load and cross-head displacement were recorded by the Instron, and when plotted, produced the typical shape of the load versus displacement curve for bending, as presented in Figure 5-11. Initially, the load increases as the sample is strictly loaded in axial compression. The peak value corresponds to the buckling load, and at this point, inelastic buckling occurs as the sample has already yielded. The mode of loading transitions from direct compression to bending, and as a result, the load decreases as the bend becomes more severe. The minimum load is reached just prior to the two ends of the sample coming into contact, at which time the load rises as the sample is now loaded through its thickness.
A characteristic curve representing an average between the 3 repeats of each test was determined from the following steps:

1. The displacement scale was offset to place the peak load at a displacement of 1mm.

2. The forces were determined for displacement increments between 1mm and 18mm.

3. An average of the force values between the 3 repeats at specific displacement values was taken to produce the characteristic curve.

Figure 5-12 shows the curves for three individual tests with no prestrain at an applied strain rate of 0.001/s, with the peaks matched at a displacement of 1mm. The closeness of the curves indicates that the test is very repeatable, with the most notable
difference being a slight variation in peak value. Surface defects, material inhomogeneity, loading eccentricity and the length of the sample are all factors that can contribute to the differences in the peak values, but for the bend tests performed in this thesis, these effects are not considered significant.

![Displacement vs Load](image)

Figure 5.12: Matching peak loads at 1mm for 3 repeated tests, and then averaging to produce a characteristic bending curve.

5.4.1.1 Bending results comparison between applied strain rates of 0.001/s and 0.1/s

Figure 5.13 compares the characteristic curves between applied strain rates of 0.001/s and 0.1/s for samples with no prestrain. Both applied strain rates have the same overall shape, they both converge to a similar minimum load, and the only noticeable, albeit slight, difference is in the peak load.

Figure 5.14 compares the characteristic curves between applied strain rates of 0.001/s and 0.1/s for samples with 3%, 6% and 12% transverse prestrain. For each
prestrain magnitude the overall shape of the bending curve is independent of the strain rate. The only distinguishable, yet small, differences are between the peak and the minimum load values.

Figure 5-13: Comparison between the characteristic bending curves for samples with no prestrain at applied strain rates of 0.001/s and 0.1/s.

a) 3% transverse prestrain
Figure 5-14: Comparison between the characteristic bending curves for samples with transverse prestrain at applied strain rates of 0.001/s and 0.1/s.
Figure 5-15 compares the characteristic curves between applied strain rates of 0.001/s and 0.1/s for samples with 3%, 6% and 12% longitudinal prestrain. Again, the only distinguishable differences between the strain rates are the peak and the minimum load values, which are both higher for 0.001/s.

![Diagram showing characteristic curves for different strain rates.](image)

a) 3% longitudinal prestrain
Figure 5-15: Comparison between the characteristic bending curves for samples with longitudinal prestrain at applied strain rates of 0.001/s and 0.1/s.
5.4.1.2 Bending results comparison between prestrain orientations

Figure 5-16 compares the peak loads for transverse and longitudinal prestrain at an applied strain rate of 0.001/s and Figure 5-17 provides the same comparison at an applied strain rate of 0.1/s. For both orientations and strain rates, the peak load increases with increasing prestrain. At 0.001/s, the longitudinal direction produces slightly higher peak loads, while at 0.1/s the transverse direction produces slightly higher peak loads, although neither of these results are statistically significant.

Figure 5-16: Comparison between peak loads achieved during bending between transverse and longitudinal prestrain orientations at an applied strain rate of 0.001/s.
Figure 5-17: Comparison between peak loads achieved during bending between transverse and longitudinal prestrain orientations at an applied strain rate of 0.1/s.

Figure 5-18 compares the minimum loads for transverse and longitudinal prestrain at an applied strain rate of 0.001/s and Figure 5-19 presents the same comparison at an applied strain rate of 0.1/s. For both orientations and strain rates, the minimum load decreases with increasing prestrain. At 0.001/s, only the data points at 12% prestrain show a difference between the two orientations, with the longitudinal direction producing higher minimum loads. At 0.1/s, the minimum load does not vary significantly with prestrain orientation.
Figure 5-18: Comparison between minimum loads achieved during bending for transverse and longitudinal prestrain at an applied strain rate of 0.001/s.

Figure 5-19: Comparison between minimum loads achieved during bending for transverse and longitudinal prestrain at an applied strain rate of 0.1/s.
5.4.2 Results for applied strain rate of 80/s

Figure 5-20 depicts the final resting position of the drop tower bend test with the sample in its fully deformed state. Results from the drop tower were output in the form of load versus displacement. Comparisons between the prestrain orientations are made for each sample at each prestrain magnitude, as shown in Figure 5-21. In this case, individual sample results were used, instead of a characteristic curve, because of the oscillations resulting from the impact dynamics caused by the rigid mass striking the bend test fixture. The displacement scale was offset for each test to match the peak loads at a displacement of 2mm. There is some variation between repeats of the sample test reflected mainly in the differences in peak loads.

Figure 5-22 compares the peak loads between the two prestrain orientations and it can be seen that the peak load increases with increasing prestrain, and is independent of prestrain orientation.

Figure 5-23 compares the minimum load between the two prestrain orientations. Because of the oscillations present, an average of the load for displacements between 15 and 18mm was taken. The minimum load decreases with increasing prestrain, with the longitudinal orientation producing slightly higher values.
Figure 5-20: Final position of a completed drop tower bend test.
a) no prestrain

b) 3% prestrain
Figure 5-21: Comparison between bending curves following prestrain in the transverse and longitudinal directions at an applied strain rate of 80/s.
Figure 5-22: Comparison between peak loads achieved during bending following prestrain in the transverse and longitudinal orientations at an applied strain rate of 80/s.

Figure 5-23: Comparison between minimum loads achieved during bending following prestrain in the transverse and longitudinal orientations at an applied strain rate of 80/s.
5.4.3 Overall rate comparison

Figure 5-24 shows the peak load versus equivalent prestrain at applied strain rates of 0.001/s, 0.1/s and 80/s, for individual samples prestrained in the transverse and longitudinal directions. It can be seen that the peak load at 80/s is a minimum of 25% higher than the peak loads at 0.001/s and 0.1/s for all prestrains.

Figure 5-25 shows the minimum load versus equivalent prestrain at applied strain rates of 0.001/s, 0.1/s and 80/s, for individual samples prestrained in the transverse and longitudinal directions. The minimum load decreases with increasing prestrain and with increasing strain rate for all cases.
Figure 5-24: Comparison between peak loads achieved during bending between applied strain rates of 0.001/s, 0.1/s and 80/s for samples prestrained in the a) transverse and b) longitudinal directions.
Figure 5-25: Comparison between minimum loads achieved during bending between applied strain rates of 0.001/s, 0.1/s and 80/s for samples prestrained in the a) transverse and b) longitudinal directions.

5.5 Discussion

In the following section the bend test results are discussed by considering the flow stress levels of the corresponding stress-strain curves at the appropriate prestrain orientation and strain rate, as well as the impact dynamics of the drop tower testing. Comparisons between different load cases will be based on the peak loads and minimum loads, as they provide a way of directly comparing the bending behaviour of the material.
5.5.1 Bending results and flow stress

The deformation behaviour of prestrained AA5754 sheet in bending can be better understood by looking at the different characteristics of the stress-strain curves for different prestrain orientations, magnitudes, and strain rates. The peak loads plotted in Figure 5-24 correspond to the critical buckling loads, $P_{cr}$, which can be predicted for intermediate length compression members using the classical Euler formula with a reduced modulus (reduced modulus theory) defined by:

$$P_{cr} = \frac{\pi^2 E \gamma I}{(KL)^2}$$  \hspace{1cm} (30)

where: $K = \text{column effective length factor (K=1 for two pinned ends)}$

$I = \text{unsupported length}$

$E_\gamma = \text{reduced modulus}$

$I = \text{moment of inertia}.$

The reduced modulus is given by:

$$E_\gamma = \frac{4EE_t}{\left(\sqrt{E} + \sqrt{E_t}\right)^2}$$  \hspace{1cm} (31)

where: $E_t = \text{tangent modulus}.$

Although the tangent modulus varies along the stress-strain curve, for simplicity, it can be taken as the modulus just after yield. Equation (30) shows that there is a positive linear relationship between the peak load and the reduced tangent modulus of the
corresponding stress-strain curve. However, when the reduced modulii obtained from the characteristic Voce fit of the flow stress curves (Section 4.4) are plotted against the experimental peak loads, as presented in Figure 5-26, the increase in peak load with increasing reduced modulus is not evident. One reason for the absence of this trend may be that each data point was determined from the average characteristic modulus (based on a statistical average of up to five Voce curve fits) and the average of three experimental peak loads. The data processing method as well as averaging both parameters may have overshadowed the trend, as it will be demonstrated through FEM simulation predictions, in Chapter 6, that peak load does increase with increasing reduced modulus.

Figure 5-27 indicates that the peak load at buckling increases with increasing yield stress for transverse and longitudinal prestrains at strain rates of 0.001/s and 0.1/s. Since inelastic buckling takes place after the yield stress is reached, a higher yield stress will allow higher loads to develop before instability occurs. It has been shown in Figure 4-8 (Section 4.4) that higher prestrain magnitudes result in higher yield stresses, which explains the increase in peak load with increasing prestrain from Figure 5-24.
Figure 5-26: Correlation between the reduced modulus and the corresponding peak load at buckling.
Figure 5-27: Correlation between the yield stress and the peak load at buckling for transverse and longitudinal prestrains, at strain rates of a) 0.001/s and b) 0.1/s.

In Figure 5-25 the minimum load during bending is seen to decrease with increasing prestrain, presumably because, as the bend progresses, the strains in the sample increase and the load drops once strain localization starts. Figure 5-28 illustrates that even with the naked eye, surface roughness develops in the high strain region on the outer surface of the bend, producing the undulations depicted in Figure 2-5. These undulations are initiation sites for strain localization, which causes an overall load drop to occur earlier for prestrained samples. Similarly, in a tensile test, the point at which the load starts to drop indicates the onset of diffuse necking, and this occurs after a smaller applied strain for prestrained samples. By the end of the bend test, a larger proportion of
the highly prestrained sample will contain shear bands, resulting in a lower overall minimum load than for samples with smaller magnitudes of prestrain.

In tension, diffuse necking is often followed by the formation of shear bands and ultimately, their interaction with PLC bands may initiate failure. In bending, the surface undulations provide initiation sites for shear bands, which decreases the load carrying capacity of the sample. Even though the failure mechanism of AA5754 sheet in tension and bending are significantly different, they both involve shear localization, and the onset of diffuse necking in a tensile test can be used as a predictor for the onset of shear localization in a bend test. Figure 5-29 demonstrates that there is indeed a correlation between the minimum load at the end of a bend test and the onset of diffuse necking in a tensile test. The larger the increment of strain needed to initiate diffuse necking (corresponding to smaller magnitudes of prestrain), the larger the minimum load.

This correlation between the minimum load (or in other words, the load carrying capability of the sample at the end of the bend test) and increasing prestrain can also be looked at in terms of the amount of energy absorbed by the sample during bending. Energy was calculated as the area under the load-displacement curve between displacements of 1mm (corresponding to the peak load) and 16mm. Figure 5-30 shows that the energy absorbed between 1mm and 16mm decreases with increasing magnitudes of prestrain. Since energy absorption is dependent upon the load carrying capacity of the sample, then a decreasing level of energy absorption is also an indication of decreasing overall load with increased prestrain.
Figure 5-28: Development of surface roughness in the high strain region, for a sample tested in bending.

Region of increased surface roughness.

a) applied strain rate of 0.001/s
Figure 5-29: Correlation between the minimum load during bending, and the strain increment necessary to initiate diffuse necking in uniaxial tension.

Figure 5-30: Comparison between the energy absorbed and the prestrain magnitude at an applied strain rate of 0.001/s with transverse prestrain.
There are no clear trends evident when comparing the prestrain orientation. Figure 5-16 shows that the peak loads are higher for longitudinal prestrain, whereas Figure 5-17 illustrates that they are higher for transverse prestrain. Figure 5-18 shows that the minimum loads are higher for longitudinal prestrain, while Figure 5-19 indicates no consistent trend. No consistent trends are observed at an applied strain rate of 80/s (see Figure 5-21). Based on the results from Section 5.4, differences between the two prestrain orientations are subtle. Moreover, when only three repeats are performed, and the strain rates within the sample are not constant, small behavioural differences are difficult to discern.

5.5.2 Impact dynamics

The comparison between peak loads for different strain rates from Figure 5-24 indicates that there is a relatively large increase between the intermediate applied strain rate of 80/s and the slower applied strain rates of 0.001/s and 0.1/s. This increase is much larger than would be expected based on the yield stress of the stress-strain curve at 80/s. One of the possible reasons for such a large increase is the dynamic buckling that occurs at higher strain rates. Vaughn et al. [71] state that when dynamic buckling occurs, the lateral inertia of the sample provides additional resistance to buckling and, as a result, the peak load can increase significantly. This delay in buckling allows for larger axial compressive strains to occur prior to instability, and the applied displacement required to reach the peak load is a relative measure of these compressive strains. For samples with no prestrain, a displacement of 2.8mm was applied before the peak load was reached at a strain rate of 80/s, whereas it only took 1.4mm before buckling occurred at 0.001/s, as
can be seen in Figure 5-31. Hence, more axial compressive strain occurs at 80/s prior to buckling. The 0.001/s and 0.1/s bend samples undergo the same displacement prior to buckling because the strain rates are not high enough to produce significant lateral inertial effects.

Because of the impact dynamics that occur during buckling, it is not possible to identify whether the rate sensitivity of the peak load between 0.1/s and 80/s is positive or negative. Since lateral inertia effects do not exist at the slower rates, it follows that there is a modest negative rate sensitivity between the peak loads observed at 0.001/s and 0.1/s.

When comparing the minimum loads plotted in Figure 5-25, applied strain rates of 0.001/s and 0.1/s exhibit the negative rate sensitivity that is expected, while the applied strain rate of 80/s produces minimum loads that are lower than expected; that is, strain rates greater than 0.1/s showed a positive rate effect, as discussed in Chapter 3. Again, this unexpected behaviour can be explained by the influence of impact dynamics on buckling, which produces larger axial compressive strains prior to buckling at higher strain rates. Higher levels of axial compressive strain mean that a smaller increment of bending is needed to initiate shear localization. Since more of the deformation occurs while shear bands are present, the larger axial compressive strains at 80/s will result in a lower minimum load.

Because of the impact dynamics that occur during buckling at 80/s, it is not possible to identify whether minimum load is rate-sensitive between applied rates of 0.1/s and 80/s. However, Figure 5-25 does reveal a negative rate sensitivity for minimum load between 0.001/s and 0.1/s.
Another factor that may influence the bending results observed at 80/s is the fact that the applied velocity is not constant throughout the test. In the drop tower, the striker mass deforms the sample, energy is absorbed, and the velocity of the striker decreases. However, the nominal strain rate is only expected to drop to approximately 60/s based on the initial velocity and the energy absorbed, which should not result in a significant difference when comparing the 80/s behaviour with that observed at 0.001/s and 0.1/s.

![Displacement applied prior to peak load](image)

Figure 5-31: Amount of displacement applied between the start of loading, and the peak load for samples with no prestrain at strain rates of 0.001/s, 0.1/s and 80/s.
Chapter 6: Implementation and validation of a constitutive model for prestrained AA5754

6.1 Introduction

This chapter looks at the development of a rate-sensitive constitutive model for 3mm AA5754 sheet prestrained in the transverse material direction, and its implementation as a user-defined material model in LS-DYNA. The material model was validated through comparison with bend tests performed at applied strain rates of 0.001/s and 0.1/s. FEM simulations were not performed for an applied strain rate of 80/s because the predicted behaviour could not be used for validation of the material model due to the additional complexity involved when modeling dynamic buckling. Simulations were also not performed for longitudinal prestrain as characteristic stress-strain curves were not available at a strain rate of 1500/s. It was necessary to have experimental data for three different strain rates in order to determine the appropriate material constants.

In this chapter, all experimental results used to defined material constants, or used for curve fitting purposes, refer to the curves of flow stress versus true plastic strain
presented in Appendix G. The term $\dot{\varepsilon}$ refers to the effective true plastic strain rate for all constitutive equations in the chapter, but when these equations are used for curve-fitting purposes the effective engineering plastic strain rate is employed (see footnote in Section 3.1). When implemented in LS-DYNA, a conversion is performed between the true strain rates calculated in the FEM simulation and the engineering strain rate required for the material model (Equation (42)).

6.2 Preliminary work

As discussed in Section 2.2, there are many rate-dependent material models, combined with a variety of hardening schemes, that can be adopted to represent the rate sensitivity for a wide range of materials. The Voce hardening curve has proved to be the best fit for AA5754 over a range of prestrains and strain rates; therefore, it seems appropriate to consider a rate-dependent model with Voce hardening. The major challenge is that AA5754 exhibits a transition from negative to positive rate sensitivity over the range of strain rates of interest, and there are few models available that can represent this behaviour. The most commonly used rate-sensitive models, such as Johnson-Cook [31] and Zerilli Armstrong [32], are not able to model this transition. Models that capture the PLC effect typically do so by considering dislocation interaction on a microscopic scale and are not straightforward or practical for large-scale modeling. Ho and Krempl [43] capture changes in rate sensitivity using viscoplasticity theory based on overstress, but again, this is a rather complex model, and the material constants would be difficult to define accurately. Another option to capture the transition between
negative and positive rate sensitivity is to start with a hardening equation, and then introduce a rate sensitivity parameter that varies with strain rate.

As a preliminary step, the Voce hardening rule was adapted to include the rate dependency of the Johnson-Cook model, as well as a rate-sensitive strain rate parameter \( C(\dot{\varepsilon}) \) to obtain a Voce-Johnson-Cook expression as:

\[
\sigma_f = \left( B - (B - A) e^{-n\varepsilon_p} \right) \left[ 1 + C(\dot{\varepsilon}) \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right] \tag{32}
\]

where: \( B, A \) and \( n \) are Voce parameters
\( C(\dot{\varepsilon}) \) = rate-sensitive rate parameter
\( \dot{\varepsilon} \) = effective plastic strain rate
\( \dot{\varepsilon}_0 \) = base strain rate (0.001/s in this thesis).

The Voce-Johnson-Cook model specifies a quadratic relationship between the rate sensitivity parameter \( C \) and the natural logarithm of strain rate (see Figure 6-1), as fit to the experimental results for samples with no prestrain. This curve reflects the transition between negative rate sensitivity and positive rate sensitivity. Predictions made with this rate sensitivity factor, however, do not capture the proper response for 500/s and 1500/s at lower strains, as shown in Figure 6-2. Figure 6-3 plots the rate sensitivity parameter \( C \) against prestrain for a strain rate of 0.1/s. The rate sensitivity parameter is relatively constant (other than at 0% prestrain), which is not expected, as a reduction in the rate sensitivity has been observed with increasing prestrain. Hence, it was concluded that a rate-dependent Johnson-Cook rate sensitivity parameter \( C \) still does not capture the effects that are important for the AA5754 alloy under study. Rather, a model that can
capture changes in rate sensitivity and hardening rate, while also dealing with the large range of yield stresses, is necessary.

Figure 6-1: The variation of the Johnson-Cook rate parameter (C) with strain rate, for samples with no prestrain.

Figure 6-2: The best fit predictions using a Voce-Johnson-Cook material model (Equation (32)) as compared with experimental results with no prestrain.
One other consideration was whether or not a damage-sensitive model was suitable for the current study. Preliminary testing, summarized in Appendix D, showed that no appreciable damage occurred up to the point of diffuse necking in uniaxial tensile samples, and only a small amount of damage was evident once failure started. The material model implemented in this thesis only considers strains up to the point of diffuse necking, and does not consider any type of failure modeling. As a result, damage was not included in the material model.

Figure 6-3: The variation of the Voce-Johnson-Cook rate parameter (C) with equivalent prestrain.

Figure 6-3: The variation of the Voce-Johnson-Cook rate parameter (C) with equivalent prestrain.
6.3 Development of a rate-dependent Voce material model

6.3.1 Rate-dependent Voce material model

Wagoner [45] adapted the Voce hardening model to incorporate strain rate effects by using simple multiplicative or additive rules. The material model developed in this thesis makes use of both rules as well as incorporating rate dependency into the rate sensitivity parameters. Strain rate effects can be modeled by combining either a multiplicative term or an additive term with the strain hardening rule \( f(\varepsilon) \), such that:

Multiplicative: \[
\sigma_f = f(\varepsilon) \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m
\] (19)

Additive: \[
\sigma_f = f(\varepsilon) + m' \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)
\] (20)

where: 
- \( m \) = multiplicative rate sensitivity parameter
- \( m' \) = additive rate sensitivity parameter
- \( \dot{\varepsilon}_0 \) = base strain rate (0.001/s in this thesis).

The effect of the multiplicative term is primarily to change the hardening rate of the curve, but it has the secondary effect of offsetting the flow stress, as shown in Figure 6-4. Figure 6-5 demonstrates that the additive term shifts the curve up or down to reflect changes in the yield stress. In the present work, both terms were necessary as there are significant changes in the hardening rates with different strain rates, and the strain rates
of 500/s and 1500/s caused large increases in the yield stress. The Voce hardening rule can be combined with both the multiplicative (m) and additive (m') rate sensitivity terms, where both m and m' are functions of the strain rate to obtain:

Voce-MA

\[
\sigma_f = \left( B - (B - A) e^{-n\varepsilon_p} \right) \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^m + \left( m' \ln \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \right),
\]  

(33)

where: 
\( m(\dot{\varepsilon}) \) = rate-sensitive multiplicative rate parameter 
\( m'(\dot{\varepsilon}) \) = rate-sensitive additive rate parameter 
\( \dot{\varepsilon}_0 \) = base strain rate (0.001/s in this thesis).

Equation (33) will be referred to as the Voce-MA material model.

Figure 6-4: Effects of the multiplicative rate sensitivity parameter (m) at a strain rate of 0.1/s on a baseline Voce fit obtained at 0.001/s.
The Voce parameters A, B and n were determined for the base strain rate ($\dot{\varepsilon}_0$) of 0.001/s, and the m and m' parameters were determined by fitting the results for 0.1/s, 500/s and 1500/s. Figure 6-6 depicts the best fit curves for samples with no prestrain using the Voce-MA model (Equation (33)). The curve fits for 500/s and 1500/s, using the Voce-MA model, are much better than those employing the Voce-Johnson-Cook model in Figure 6-2. The rate sensitivity parameters m and m' are plotted against strain rate for samples with no prestrain and for 12% transverse prestrain in Figure 6-7. The negative rate sensitivity between 0.001/s and 0.1/s is reflected by a decrease in the multiplicative rate parameter and the positive rate sensitivity above 0.1/s is indicated by an increase in the multiplicative rate parameter. The additive rate parameter compensates for the shift in the flow stress caused by the multiplicative rate parameter.
Since the multiplicative and additive rate parameters both follow a quadratic relationship with the natural logarithm of strain rate, Equation (33) was rewritten to incorporate the strain rate dependency of the rate parameters as:

\[
\sigma_f = \left[ B - (B - A) e^{-\alpha_p} \right] \left( \frac{\hat{\varepsilon}}{\dot{\varepsilon}_0} \right)^{\alpha_1 (\ln \hat{\varepsilon})^2 + \alpha_2 (\ln \hat{\varepsilon}) + \alpha_3} + \left( a_4 (\ln \dot{\varepsilon})^2 + a_5 (\ln \dot{\varepsilon}) + a_6 \right) \ln \left( \frac{\hat{\varepsilon}}{\dot{\varepsilon}_0} \right) \tag{34}
\]

where: \( m = a_4 (\ln \dot{\varepsilon})^2 + a_5 (\ln \dot{\varepsilon}) + a_6 \)

\[ m' = a_4 (\ln \dot{\varepsilon})^2 + a_5 (\ln \dot{\varepsilon}) + a_6 \]

\( a_1 \ldots a_6 \) = fitting constants.

This constitutive material model (Equation (34)) was implemented into LS-DYNA, where the rate-dependent rate sensitivity parameters (\( m \) and \( m' \)) are able to account for the transition from negative to positive rate sensitivity with increasing strain rate.
Figure 6-6: The best fit predictions using a Voce-MA material model (Equation (33)) as compared with experimental results with no prestrain.

a) multiplicative rate parameter (m)
6.3.2 Implementation of rate-sensitive Voce material model in LS-DYNA

The rate-sensitive Voce material model was implemented in the explicit FEM software LS-DYNA through a user-defined material model (UMAT). This implementation allowed for a dynamic model to be run in LS-DYNA, with the UMAT updating the stress-strain curve for each element in response to changes in the local strain rate during the simulation.

Figure 6-7: The variation of the Voce-MA rate parameters $m$ and $m'$ with strain rate, for no prestrain and 12% prestrain.
The UMAT algorithm follows the flow chart shown in Figure 6-8, with the three key parts being:

- green boxes: updating of the yield surface, stresses and plastic strain
- blue boxes: calculation of the strain rate.
- orange boxes: calculation of the flow stress

The UMAT code is presented in Appendix E.

Yield Surface

This section of the code was written by W. B. Lievers (Queen’s University), as a modification to a UMAT originally written by Dr. M. J. Worswick (now at the University of Waterloo). It involves the numerical implementation of the plastic return method illustrated in Figure 6-9.

The strain increment corresponding to the timestep for the current cycle was used to calculate the deviatoric trial stresses $\sigma_{ij}^T$ according to:

$$\sigma_{ij}^T = \sigma_{ij}^* + C_{ijkl}^* d\varepsilon_{kl}^*$$

(35)

where: $\sigma_{ij}^*$ = deviatoric stress tensor
$d\varepsilon_{kl}^*$ = increment of deviatoric strain
$C_{ijkl}$ = elastic stiffness matrix.
Figure 6-8: Flow chart for the Voce-MA rate-sensitive material model.
Figure 6-9: Plastic return method shown in the plane of deviatoric stress.

The updated stresses are determined according to the following criteria based on the von Mises yield function:

\[
\frac{3}{2} \sigma'_{ij} \sigma'_{ij} \leq \bar{\sigma}^2
\]  

(36)

where: \( \bar{\sigma} = \) flow stress.

If Equation (36) is satisfied, the stress state is within the elastic region, meaning there is no plastic strain increment and the updated stresses for the following timestep are:

\[
\sigma_{ij}^{n+1} = \sigma_{ij}^T.
\]  

(37)
If Equation (36) is not satisfied, the stresses are returned to the yield surface along the radial direction (plastic strains are oriented normal to the yield surface), and the updated stresses for the following timestep are:

$$\sigma_{ij}^{n+1} = \alpha \sigma_{ij}^T + \frac{\sigma_{kk}^n}{3},$$  \hspace{1cm} (38)

where $\alpha = \frac{\bar{\sigma}}{\sqrt{\frac{3}{2} \sigma_{ij}^T \sigma_{ij}^T}}$ is a scale factor applied to the deviatoric trial stress $\sigma_{ij}^T$.

The plastic strain increment (dep) is determined as:

$$dep = \frac{(1 - \alpha) \sqrt{\frac{3}{2} \sigma_{ij}^T \sigma_{ij}^T}}{(3 * G) + h} \hspace{1cm} (39)$$

where: $G$ = shear modulus

$$h = \frac{d \bar{\sigma}}{d \bar{\varepsilon}^p}, \text{ hardening rate.}$$

The plastic strain increment (Equation (39)) is then used to update the total plastic strain as:

$$\varepsilon_{p}^{n+1} = \varepsilon_{p}^n + dep \hspace{1cm} (40)$$
An isotropic hardening scheme was adopted, which means that the yield surface expands radially with increasing strains. Considering that there is a path change between prestrain and subsequent bending, which produces a transition from tension to compression on the inner surface of the bend, a kinematic hardening model may be more appropriate. Kinematic hardening is used when loading is reversed, and produces a shift in the yield surface as shown in Figure 2-6. It has been shown [29] that for metals an isotropic-kinematic hardening scheme is more appropriate; however, this would require additional parameters and additional tests to be performed. Therefore, as a first step, an isotropic von Mises yield surface was employed because all of the necessary material parameters could be determined from the uniaxial tensile test data.

**Strain Rate**

The true plastic strain rate is calculated as:

$$\dot{\varepsilon}_T = \frac{\sum_{n=1}^{n=1000} de_{pn}}{\sum_{n=1}^{n=1000} dt_n}$$  \hspace{1cm} (41)

which is the total true plastic strain increment over 1000 steps divided by the total time increment over 1000 steps. Taking the strain rate over an increment of 1000 steps produced much smoother model predictions as the strain rate changes by orders of magnitude when calculated based on the plastic strain increment for each timestep (due to the explicit dynamic nature of the FEM software). One disadvantage of averaging out the
strain rate is that there is a delay before the model responds to changes in strain rate, but the benefits of the increased stability of the calculations outweighs this downside.

If the plastic increment is zero, (ie. when the stresses remain fully elastic), the strain rate is set to the base strain rate of $0.001/s$.

**Flow Stress**

The flow stress ($\sigma_f$) is calculated as:

$$
\sigma_f = \left( B - (B - A) e^{-n e_p} \right) \left( \frac{\dot{e}_f}{\dot{e}_0} \right)^{\frac{a_1 (\ln \dot{e})^2 + a_2 (\ln \dot{e}) + a_3}{e + a_4 (\ln \dot{e})^2 + a_5 (\ln \dot{e}) + a_6 \ln \left( \frac{\dot{e}}{\dot{e}_0} \right)}}
$$

(34)

based on the material constants that are defined in the LS-DYNA input deck, the plastic strain $e_p$, and the engineering strain rate $\dot{e}$. The engineering strain rate ($\dot{e}$) is calculated from the true strain rate ($\dot{e}_T$) using:

$$
\dot{e} = \dot{e}_T \left( e^{\dot{e}_T} \right).
$$

(42)

Equation (42) is based on the engineering strain rate because the material constants are fit from uniaxial tensile test data collected for one specific strain rate. The engineering strain rate is calculated using the velocity of the applied loading and the gauge length of the sample, and it stays constant throughout the test. The true strain rate, however, does not remain constant because it is based on the instantaneous gauge length, which
increases throughout the test. Hence, Equation (42) is needed to convert the true strain rate, employed in LS-DYNA, to the engineering strain rate used in calculating the flow stress.

6.3.3 Determining material constants

In this section, all experimental results used to define material constants, or used for curve fitting purposes, refer to the curves of flow stress versus true plastic strain presented in Appendix G.

Figure 6-10 provides a flow chart indicating how the material constants needed for the Voce-MA material model were calculated. The amount of transverse prestrain remained constant for all elements throughout the FEM simulation; therefore, it was input as part of the preprocessing stage. All material model constants were calculated for this specific prestrain orientation and magnitude, which was input through the ANSYS graphical user interface (GUI), as shown in Figure 6-11. The material constants were calculated using a macro written in ANSYS (see Appendix F). The calculated constants were output by ANSYS and then manually entered into the LS-DYNA input deck as material constants.
Figure 6-10: Flow chart showing the steps taken to determine the input parameters for the Voce-MA constitutive material model.

1. **Preprocessor input**
   - Transverse Prestrain
   - **A, B, n**
     - Voce parameters for a specific transverse prestrain
   - **m(0.001/s)**
   - **m(0.1/s)**
   - **m(1500/s)**
     - Determine the multiplicative rate sensitivity parameters $m$, for the specific transverse prestrain, at strain rates of 0.001/s, 0.1/s and 1500/s
   - **m'(0.001/s)**
   - **m'(0.1/s)**
   - **m'(1500/s)**
     - Determine the additive rate sensitivity parameters $m'$, for the specific transverse prestrain, at strain rates of 0.001/s, 0.1/s and 1500/s

2. **UMAT input**
   - **A, B, n, a₁, a₂, a₃, a₄, a₅, a₆**

   $\sigma = (B - (B - A)e^{-n_{e_p}}) \left( \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \left( a_1 (\ln \dot{\varepsilon})^2 + a_2 (\ln \dot{\varepsilon}) + a_3 \right) + \left( a_4 (\ln \dot{\varepsilon})^2 + a_5 (\ln \dot{\varepsilon}) + a_6 \right) \left( \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)$
The Voce parameters $A$, $B$ and $n$ were calculated for the specific magnitude of transverse prestrain based on fitting curves to the characteristic stress-strain data presented in Appendix G at the base strain rate ($\dot{\varepsilon}_0$) of 0.001/s. Figures 6-12 through 6-14 plot the Voce parameters $A$, $B$ and $n$ against equivalent prestrain, with the best fits corresponding to second-order polynomials for parameters $A$ and $B$, and a third-order for the parameter $n$. The best-fit equations for these parameters are given by:

\begin{align*}
A &= -4333.2x^2 + 1264.9x + 105.6 \\
B &= 5190.3x^2 - 790.4x + 280.4 \\
n &= -38067x^3 + 7539.4x^2 - 113.0x + 14.5.
\end{align*}

The parameter $A$ is the yield stress of the Voce curve fit, and as expected, it increases with increasing prestrain, as was previously shown in Figure 4-8. The linear fit employed in Figure 4-8 was used to illustrate the relative difference between the yield
stresses at the two strain rates, even though the quadratic relation shown in Figure 6-12 is more appropriate.

The parameter \( n \) is a general representation of the overall hardening of the Voce curve, and as expected, it increases with increasing prestrain. A cubic relation was needed to capture the flat portions of the curve at low and high values of prestrain, as is evident in Figure 6-14.

The curve fits used to define the Voce parameters (Equation (33)) were compared with the overall shape of curve fits for longitudinal prestrain to ensure that they were consistent. Both prestrain orientations were best-fit using a quadratic relation for parameters \( A \) and \( B \), and a cubic relation for \( n \).

Figure 6-12: The variation of the Voce parameter \( A \) with prestrain, for a strain rate of 0.001/s.
Figure 6-13: The variation of the Voce parameter $B$ with prestrain, for a strain rate of 0.001/s.

Figure 6-14: The variation of the Voce parameter $n$ with prestrain, for a strain rate of 0.001/s.
**Constants a₁…a₆**

The constants a₁…a₆ which define the quadratic relationship between the rate sensitivity parameters m and m' and strain rate were calculated by following the steps outlined below.

1. The multiplicative rate parameter m was calculated at strain rates of 0.001/s, 0.1/s and 1500/s for the specified equivalent prestrain (ε₀), using the curve fits presented in Table 6-1.

   **Table 6-1: Equations used to calculate the multiplicative rate parameter (m).**

<table>
<thead>
<tr>
<th>Strain rate</th>
<th>Equation</th>
<th>Figure reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>m (0.001) = 0</td>
<td>This is the base strain rate</td>
</tr>
<tr>
<td>0.1/s</td>
<td>m (0.1) = 0.136*ε₀ - 0.018</td>
<td>Figure 6-15</td>
</tr>
<tr>
<td>1500/s</td>
<td>m (1500) = 0.222*ε₀ + 0.004</td>
<td>Figure 6-17</td>
</tr>
</tbody>
</table>

2. The additive rate parameter m' was calculated at strain rates of 0.001/s, 0.1/s and 1500/s for the specific equivalent prestrain (ε₀), using the curve fits given in Table 6-2.
Table 6-2: Equations used to calculate the additive rate parameter (m')

<table>
<thead>
<tr>
<th>Strain rate</th>
<th>Equation</th>
<th>Figure reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>( m'(0.001) = 0 )</td>
<td>This is the base strain rate</td>
</tr>
<tr>
<td>0.1/s</td>
<td>( m'(0.1) = -20.330 \varepsilon_0 + 2.230 )</td>
<td>Figure 6-16</td>
</tr>
<tr>
<td>1500/s</td>
<td>( m'(1500) = -69.559 \varepsilon_0 + 1.146 )</td>
<td>Figure 6-18</td>
</tr>
</tbody>
</table>

3. A quadratic curve fit was performed on the values of \( m(0.001) \), \( m(0.1) \) and \( m(1500) \) versus ln of strain rate (ln \( \varepsilon \)), so that \( m \) could be calculated for any strain rate. The outcome is shown in Figure 6-19 with no prestrain. (The equation is different for each prestrain magnitude). The curve fit was performed by solving the set of three equations and three unknowns (a\(_1\), a\(_2\), and a\(_3\)), according to the macro provided in Appendix F. The constants a\(_1\), a\(_2\), and a\(_3\) are input into the material model for the specific prestrain.

4. A quadratic curve fit was performed on the values of \( m'(0.001) \), \( m'(0.1) \) and \( m'(1500) \) versus ln of strain rate (ln \( \dot{\varepsilon} \)), so that \( m' \) could be calculated for any strain rate. This result is depicted in Figure 6-20 with no prestrain. (The equation is different for each prestrain magnitude). The curve fit was performed by solving the set of three equations and three unknowns (a\(_4\), a\(_5\), and a\(_6\)), following the macro included in Appendix F. The constants a\(_4\), a\(_5\), and a\(_6\) are input into the material model for the specific prestrain.
Figure 6-15: The variation of the Voce-MA multiplicative rate parameter \( m \) with prestrain, at a strain rate of 0.1/s, using a base strain rate of 0.001/s.

\[
y = 0.136x - 0.018
\]

Figure 6-16: The variation of the Voce-MA additive rate parameter \( m' \) with prestrain, at a strain rate of 0.1/s, using a base strain rate of 0.001/s.

\[
y = -20.330x + 2.230
\]
Figure 6-17: The variation of the Voce-MA rate parameter $m$ with prestrain, at a strain rate of 1500/s, using a base strain rate of 0.001/s.

Figure 6-18: The variation of the Voce-MA rate parameter $m'$ with prestrain, at a strain rate of 1500/s, using a base strain rate of 0.001/s.
\[ y = 0.0004372x^2 + 0.0001181x - 0.0200461 \]

Figure 6-19: The variation of the Voce-MA multiplicative rate parameter (m) with strain rate with no prestrain.

\[ y = -0.041987x^2 + 0.097610x + 2.677766 \]

Figure 6-20: The variation of the Voce-MA additive rate parameter (m') with strain rate with no prestrain.
Figures 6-15 and 6-16 are plots of the rate sensitivity parameters for a strain rate of 0.1/s, as compared to a base rate of 0.001/s. These two figures demonstrate that both rate sensitivity parameters vary linearly with prestrain, with the multiplicative parameter \( m \) increasing and the additive parameter \( (m') \) decreasing with increasing prestrain. These trends are expected since the rate sensitivity decreases \( (m \text{ and } m' \text{ approach zero}) \) with increasing prestrain. Figures 6-17 and 6-18 present plots of the rate sensitivity parameters for a strain rate of 1500/s, as compared to a base rate of 0.001/s. While only two data points were collected, a linear relationship was fit, following the approach used for 0.1/s (Figures 6-15 and 6-16).

Figure 6-19 shows the relationship between the multiplicative rate sensitivity parameter \( (m) \) and strain rate, indicating the transition between negative to positive rate sensitivity with increasing strain rate. The additive rate sensitivity parameter \( (m') \) plotted in Figure 6-20, compensates for the offset in the flow stress caused by the parameter \( (m) \).

Figure 6-21 presents the stress-strain curve for 5.7% prestrain at a strain rate of 0.1/s that was calculated in LS-DYNA using the material constants determined from the ANSYS macro (Appendix F). Since diffuse necking and failure were not modeled, the stress-strain curve in the material model continues to a strain of 200% at the saturation stress. The difference between the stress-strain curves from the material model and the test up to the point of diffuse necking can be attributed to the curve fitting techniques used to determine the material constants.
Figure 6-21: The stress-strain curve for 5.7% prestrain at a strain rate of 0.1/s, as calculated by LS-DYNA using the material constants determined from the ANSYS macro compared with experimental uniaxial test data.

6.4 FEM simulation

The explicit dynamic FEM software LS-DYNA was used to simulate bending of a 25.4mm x 19.1mm x 3mm (thick) rectangular sample as it deformed in the bend test fixture described in Section 5.2.1.

The FEM model consisted of two loading plates and the 25.4mm x 19.1mm rectangular sample, as illustrated in Figure 6-22. The thickness of the sample changed for each model due to the different magnitudes of prestrain considered. A horizontal plane of symmetry through the middle of the specimen was employed to reduce the size of the model, as can be seen in Figure 6-23. Loads were applied by defining a displacement versus time curve for the bottom surface of the plate in the vertical (Y)
direction. Buckling was initiated by applying a lateral offset of 0.26mm to the middle of the sample geometry, as indicated in Figure 6-24. This lateral offset was necessary because computer simulations utilize ideal materials; hence, without a mechanism for initiating buckling, the sample would simply deform in axial compression and buckling would never occur. The magnitude of the offset was determined through calibration with the maximum load of 8.915kN for a sample without prestrain deformed at a strain rate of 0.001/s. This level of offset was then used for all models as it represented the buckling load for this particular combination of material, length of sample and end conditions. It also provided another way of evaluating the material model, as the geometry was not recalibrated each time.

The loading plates were modeled using a linear elastic material model with a Young’s modulus of 200GPa, a density of 7850kg/m³, and a Poisson’s ratio of 0.3 to represent a mild steel. Contact was defined as automatic single surface contact between all surfaces of the model including the plates and the ends of the sample.

Loading was prescribed by applying a displacement of 12.7mm to the loading plate over a time of 1000 seconds, which simulated an applied strain rate of 0.001/s. The time was reduced to 10 seconds for an applied strain rate of 0.1/s.

Mass scaling was employed to decrease simulation times by specifying the timestep size to be 0.01s and 0.0001s for applied strain rates of 0.001/s and 0.1/s, respectively. In order to achieve the specified timestep, LS-DYNA uniformly increased the mass of each part within the model. This increase in the mass was small enough to avoid any additional dynamic effects.
Figure 6-22: Geometry of the model used for the bend simulation. Dimensions are in mm.

Figure 6-23: Loading conditions for the bend simulation.
6.5 Test matrix: FEM simulation of bending

Computer simulations were performed for strain rates of 0.001/s and 0.1/s, with prestrain magnitudes of 0, 3%, 6% and 12% in the transverse direction, following the test matrix in Table 6-3. The values of the transverse prestrain and the thickness were taken as the average of the three experimental repeats presented in Section 5.4.

Table 6-3: Test matrix for FEM simulations

<table>
<thead>
<tr>
<th>Nominal Strain Rate</th>
<th>Transverse Prestrain (%)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001/s</td>
<td>0</td>
<td>2.91</td>
</tr>
<tr>
<td>0.1/s</td>
<td>0</td>
<td>2.91</td>
</tr>
<tr>
<td>0.001/s</td>
<td>3.26</td>
<td>2.87</td>
</tr>
<tr>
<td>0.1/s</td>
<td>3.35</td>
<td>2.86</td>
</tr>
<tr>
<td>0.001/s</td>
<td>6.31</td>
<td>2.78</td>
</tr>
<tr>
<td>0.1/s</td>
<td>6.31</td>
<td>2.78</td>
</tr>
<tr>
<td>0.001/s</td>
<td>12.78</td>
<td>2.64</td>
</tr>
<tr>
<td>0.1/s</td>
<td>12.95</td>
<td>2.63</td>
</tr>
</tbody>
</table>
6.6 Numerical predictions

Output from the FEM simulations was in the form of the total reaction force along the plane of symmetry and the displacement of the loading plate. The displacement values were multiplied by 2 to account for the symmetry of the model. Figure 6-25 compares the FEM simulation predictions and the experimental results for different magnitudes of transverse prestrain at applied strain rates of 0.001/s and 0.1/s. The characteristic bend curve from the experimental test was plotted, which was an average of three repeats. The geometry of the model was calibrated to match the peak load for a sample with no prestrain at 0.001/s, and all cases employed this same geometry. Therefore, a comparison of the predicted versus experimental buckling load provides a way of evaluating the effectiveness of the material model. Qualitatively, the overall shapes of the curves match, as do the peak loads and the minimum loads.

a) no prestrain, applied strain rate of 0.001/s  b) no prestrain, applied strain rate of 0.1/s
c) 3% prestrain, applied strain rate of 0.001/s  


d) 3% prestrain, applied strain rate of 0.1/s


e) 6% prestrain, applied strain rate of 0.001/s  


f) 6% prestrain, applied strain rate of 0.1/s


g) 12% prestrain, applied strain rate of 0.001/s  


h) 12% prestrain, applied strain rate of 0.1/s

Figure 6-25: Comparison between FEM simulation and experimental test results for different magnitudes of transverse prestrain at applied strain rates of 0.001/s and 0.1/s.
6.6.1 Load comparison between experimental test and FEM simulation

Figure 6-26 compares peak load values between individual tests and the FEM simulation at applied strain rates of 0.001/s and 0.1/s. The numerical predictions follow the trend of increasing peak load with increasing prestrain that is evident in the test results. The FEM model predicts higher peak loads for an applied strain rate of 0.001/s, as shown in Figure 6-27 (maximum difference of 3.0%), whereas the tests did not exhibit any difference in peak loads between applied rates of 0.001/s and 0.1/s (Figure 5-24a)).

Figure 6-28 compares minimum load values between individual tests and the FEM simulation at applied strain rates of 0.001/s and 0.1/s. The minimum load was taken as an average of the load values between displacements of 15mm and 17mm due to oscillations in the data. For all prestrain magnitudes, except for one (6% prestrain, 0.001/s), the minimum loads predicted by the simulation were higher than the test results for both strain rates (maximum difference of 12.1%). The simulation predictions follow the trend of decreasing minimum loads with increasing prestrain that can be seen in the experimental results (see Figure 5-25). The FEM model exhibits no dependence of the minimum loads on strain rate, as illustrated in Figure 6-29.
Figure 6-26: Comparison of peak loads between individual experimental bend tests and FEM simulation.

a) applied strain rate of 0.001/s

b) applied strain rate of 0.1/s
Figure 6-27: Comparison of FEM simulation peak loads between strain rates of 0.001/s and 0.1/s.

a) applied strain rate of 0.001/s
b) applied strain rate of 0.1/s

Figure 6-28: Comparison of minimum loads between individual experimental bend tests and FEM simulation.

Figure 6-29: Comparison of FEM simulation minimum loads between strain rates of 0.001/s and 0.1/s.
6.6.2 Deformation comparison between experimental test results and FEM simulation

Comparisons between the experimental test results and FEM model predictions related to overall sample deformation are illustrated in Figure 6-30. The deformed shapes compare well, with the FEM simulation showing the same rotation of the sample ends and the flaring out of the sample sides in the bend region. Dimensions of the height and width of the sample after deformation, as defined in Figure 6-31, are compared in Figure 6-32 for applied strain rates of 0.001/s and 0.1/s. The FEM predictions and test results both exhibit no dependence of the deformed dimensions on strain rate. The simulation predicts dimensions that are larger than those measured in the tests, but the trend of decreasing dimensions with increasing prestrain is captured.

Figure 6-30: Typical deformation of the sample in bending. a) partial bend, b) full bend.
Figure 6-31: Dimensions used to compare sample deformation.

a) width of deformed sample
6.6.3 Comparison of energy absorption between experimental test and FEM simulation

The amount of energy absorbed during deformation is a critical measurement when designing a structural component for automotive crash. Figure 6-33 reveals that, in general, the FEM model predicts more energy absorption than what was actually determined from the corresponding experimental test. This comparison was based on taking the area under the load versus displacement curve between the peak load and a 15mm incremental end displacement. The simulation closely predicts the amount of energy absorbed when no prestrain was applied, but the predictions deviate from the experiments with increasing prestrain (6.6% difference at 12.95% prestrain and 0.1/s).

Figure 6-32: Comparison between the deformed shape from the test and simulation. a) width, b) height.
Both the FEM model and the test data exhibit a trend of decreasing energy absorption with increasing prestrain.

Figure 6-33: Comparison between the amount of energy absorbed during the simulation and test.
6.7 Discussion

The discussion will focus on the following topics with regard to the effectiveness of the computer simulation in predicting the response of prestrained AA5754 sheet in bending:

- the rate-sensitive Voce-MA material model;
- the overall FEM simulation;
- the changes in strain rate throughout the simulation;
- the necessity of including a rate-dependent material model;
- the necessity of including the effects of prestrain in the material model.

6.7.1 Voce-MA material model

The rate-dependent Voce-MA material model was based on strains up to the point of diffuse necking in a uniaxial tensile test, and did not consider the reduction in load carrying capacity of the material past this point, or macroscopic failure. The drop in load between the onset of diffuse necking and failure in a uniaxial tensile test is highlighted in Figure 6-34. The Voce hardening curve was extrapolated past the strain at diffuse necking, as strains up to 80% were seen during bending. The sustained rate of hardening in the fit curve means that, at some point, the Voce-MA model overpredicts the load carrying capacity of the material in uniaxial tension, as illustrated in Figure 6-35. In bending, diffuse necking is suppressed but the material model is still not able to predict the overall drop in load caused by the development of the systematic pattern of shear
bands at the outer surface of the bend, as was seen in Figure 2-5. As a result, after a certain point during the bend simulation, the FEM model predicts larger loads than what was observed during experiments (see Figure 6-28). It is also the reason why the peak loads are more accurate than the minimum loads (3.0% versus 12.1%, see Section 6.6.1). This overprediction of load carrying capacity also results in the FEM model showing a greater level of energy absorption (see Figure 6-33). The difference in energy absorption between the bending test and simulation becomes larger as the amount of prestrain increases because more of the sample has failed, as indicated by the red regions in Figure 6-36. The failure strains are 24.1% for 0% prestrain, and 16.6% for 12% transverse prestrain.

Not accounting for the reduction in load carrying capacity due to shear localization is likely the largest factor in the FEM models overpredicting energy absorption during bending. The implementation of a failure model, however, is very complex and would require more knowledge about the failure modes and mechanisms of AA5754 sheet in bending. Another reason for the FEM models overpredicting energy absorption during bending is the possibility of void damage being present in the regions of high strain (see Appendix D).
Figure 6-34: A representative load versus displacement curve for a uniaxial tensile test at 0.001/s for a sample with 12% transverse prestrain.

Figure 6-35: Comparison between the Voce curve fit, and the load versus displacement curve for a uniaxial tensile test at 0.001/s for a sample with 12% transverse prestrain.
Figure 6-36: True equivalent strain for a) no prestrain, b) 12% transverse prestrain at an applied strain rate of 0.1/s, where the elements with strains above the corresponding failure strains are shown in red.
Two other factors that may contribute to the inaccuracies in the material model are the errors associated with predicting the material constants based on curve fitting techniques (see Figure 6-21), and the use of isotropic hardening rather than kinematic hardening. Isotropic hardening overestimates the flow stress in compression; (at the inner side of the bend) as a result, the model would appear stiffer when compared with the experimental results.

6.7.2 Discussion of FEM model predictions

In Section 5.5, it was stated that, according to Equation (30) for inelastic buckling, the peak load is dependent upon the reduced modulus. This trend is evident in Figure 6-37 as the peak load increases with an increasing reduced modulus. Data was only taken from results of prestrained samples, as the reduced modulus differs significantly between monotonic and prestrained cases.

Overall, Figures 6-26, 6-28, 6-32 and 6-33 show that the FEM model predicts higher peak loads, minimum loads, absorbed energy and final sample dimensions than measured from experiments. One reason for these discrepancies is that in a FEM model, the continuum of the sample is represented by a finite number of nodes (or degrees of freedom). Such approximations create models that are stiffer than real components. The more nodes used in meshing the sample, the closer the predicted values will be to the test results. This improvement is demonstrated in Figure 6-38 and Figure 6-39, where the peak loads, minimum loads and absorbed energy decrease as the number of nodes used to mesh the geometry increases. The solution eventually converges to a lower bound with a sufficient number of nodes. Three elements were used through the thickness of the mesh.
sensitivity sample in this study, with element sizes of 1.8, 1.2 and 0.8 producing a corresponding increase in the number of nodes. In general, a finer mesh would have produced more accurate predictions, but the simulation run time would also have increased significantly.

![Graph showing the relationship between peak load at buckling and reduced modulus for prestrained samples in bending.](image)

Figure 6-37: Relationship between the peak load at buckling and the reduced modulus for prestrained samples in bending.
Figure 6-38: Relationship between the peak and minimum loads and the number of nodes in the FEM simulation.

Figure 6-39: Relationship between the energy absorbed and the number of nodes in the FEM simulation.

It is not known whether or not the lateral offset used to initiate buckling changed the deformation at the bend site, and contributed to the difference between the width and height of the deformed samples in Figure 6-32. The offset of 0.26mm was relatively
small compared to the overall size of the sample, and the fact that the entire sample yields prior to buckling allows the initial geometric imperfection to develop a plastic hinge with an independent geometry. Figure 6-40 illustrates that by the time the peak load occurs, there is no evidence of the initial geometric offset.

Figure 6-40: Deformed shape of the sample at the peak load for a sample with no prestrain at an applied strain rate of 0.001/s.

Since the computer simulation did not include the bend assembly, any friction between the end plate and the vertical bolts was not captured in the predicted results. However, the increase in the force due to friction would be negligible when compared to the force due to bending.

6.7.3 Strain rate during bending

Simulating a bending experiment provides a good test of the validity of the rate-sensitive material model as the strain rate is not constant throughout the simulation, and neither is it constant from one element to another.

The true plastic strain rate throughout the simulation at two different locations is plotted in Figure 6-41. An element near the end of the sample exhibits an increase in strain rate up to the point of buckling, followed by an immediate drop as bending
initiates. The strain rate then drops to zero as the ends of the sample undergo rigid body rotation and do not experience any additional strain. An element located at the centre of the sample on the outer surface of the bend sees an increase in strain rate as bending initiates, and then a gradual decrease as the bend progresses.

When the peak load is reached, the strain rate for all of the elements in the sample are of the same order of magnitude as the applied strain rate, which implies that the rate sensitivity should be evident when looking at the peak loads, as seen in Figure 6-27. When looking at the minimum loads near the end of the simulation, the strain rates within the sample vary from zero to the applied strain rate. Hence, the minimum load may not show rate effects as clearly because the “average” strain rate within the sample will be much lower than the applied strain rate. This “smearing out” effect is evident in Figure 6-29, where a reduction in the minimum load is expected between 0.001/s and 0.1/s, but not seen.

![Graph showing plastic strain rate versus displacement comparison between elements at two different locations within a sample with no prestrain at an applied strain rate of 0.1/s.](image)

**Figure 6-41:** Plastic strain rate versus displacement comparison between elements at two different locations within a sample with no prestrain at an applied strain rate of 0.1/s.
6.7.4 Necessity of a rate-dependent model

In order to evaluate the necessity of considering the rate sensitivity of AA5754, a simulation was performed at an applied strain rate of 0.1/s using a quasi-static piecewise linear material model (rate-insensitive) to represent the stress-strain curve for a strain rate of 0.001/s. Rate-insensitive material models, with constants derived from a quasi-static tensile test, are commonly used in FEM simulations. Figure 6-42 compares the predicted load curve for an FEM simulation with a rate-insensitive quasi-static material model, versus a rate-sensitive Voce-MA material model. The rate-insensitive model with quasi-static properties overpredicts the peak load by 2.3%. The amount of energy absorbed during deformation is overpredicted by 1.5% due to the larger peak load.

Figure 6-42: Comparison between load curves from an FEM bend simulation at an applied strain rate of 0.1/s, using the Voce-MA material model, and a quasi-static, rate-insensitive model.
The need for a material model that can adapt to changes in strain rate throughout the FEM simulation was evaluated by performing a simulation using a piecewise linear material model (rate-insensitive) to represent the stress-strain curve for a strain rate of 0.1/s. This setup evaluates the effect of not considering differences in strain rate within each element of the sample and how these rates change as deformation progresses. Figure 6-43 compares the predictions from this simulation with those obtained using the rate-sensitive Voce-MA material model. The rate-insensitive model underpredicts the amount of energy absorbed during deformation by 1.9% when compared with the rate-sensitive model, as a result of the lower load predictions during the later stages of the bend test. Where rate sensitivity is considered, the elements away from the bend experience very low strain rates (Figure 6-41) and, as a result, have a higher flow stress than if all of the elements are simply assumed to have the properties for a strain rate of 0.1/s.

![Comparison between load curves from an FEM bend simulation at an applied strain rate of 0.1/s, using the Voce-MA material model, and a rate-insensitive model.](image)

Figure 6-43: Comparison between load curves from an FEM bend simulation at an applied strain rate of 0.1/s, using the Voce-MA material model, and a rate-insensitive model.
6.7.5 Necessity of a model incorporating prestrain

As was demonstrated in Figure 6-33, the amount of energy absorbed during bending decreases with increasing prestrain. At an applied strain rate of 0.001/s, the bending simulation predicted that 40.4J of energy would be absorbed for a sample with no prestrain and 35.1J for a sample with 12% prestrain, over the same displacement. This is a reduction of 13% in energy absorption that would not be accounted for if prestrain effects were ignored in the material model. Figure 6-44 illustrates the substantial difference in predicted load versus displacement curves for a non-prestrained sample versus a sample prestrained at 12% in the transverse direction.

There is also an 11% decrease in the predicted height of the deformed geometry between a sample with no prestrain and one with 12% transverse prestrain. Hence, if an accordion style crush was modeled, the omission of prestrain effects would cause an incorrect prediction of contact events between the folds.

![Graph showing load versus displacement comparison](image)

Figure 6-44: Comparison of load versus displacement during bending between simulations at an applied strain rate of 0.001/s for a sample with no prestrain, and a sample with 12% transverse prestrain.
Chapter 7: Conclusions and Recommendations

7.1 Conclusions

The rate sensitivity of 3mm AA5754 sheet loaded in uniaxial tension in the longitudinal material direction was determined over a range of strain rates between 0.001/s and 1500/s. The effect of strain rate on the stresses and strains at the onset of diffuse necking, as well as on the failure strain were also examined. The following conclusions were made:

- AA5754 exhibits a region of negative rate sensitivity between 0.001/s and 0.1/s, and a region of positive rate sensitivity between 0.1/s and 1500/s.
- The strains at diffuse necking decrease between strain rates of 0.001/s and 0.1/s due to the lower hardening rate. The observed strains at diffuse necking are much lower than those predicted using the Considère criterion due to the increased strain within the PLC bands.
• The failure strain increases between strain rates of 0.001/s and 0.1/s due to the corresponding reduction in the severity of the PLC effect, which causes premature failure.

AA5754 sheet was subjected to plane strain prestrain in either the transverse or longitudinal material orientation, then tested in uniaxial tension in the longitudinal direction at strain rates between 0.001/s and 1500/s. The following conclusions were made:

• The rate sensitivity of AA5754 is reduced with increasing magnitudes of prestrain, a trend that can be explained by a delay in the onset of the PLC effect.

• Prestrain involving a path change from plane strain to uniaxial tension alters the flow stress curve during subsequent loading, such that it exhibits a lower yield stress, a higher hardening rate, and it approaches the monotonic curve from below.

• Prestrain in the transverse direction results in a permanent softening of the flow stress during subsequent loading.

• The strains at diffuse necking are larger for samples pretrained in the longitudinal direction due to the higher hardening rate.

• Prestrain in the longitudinal direction produces a higher yield stress upon subsequent loading, in comparison to the transverse result, possibly due to the anisotropy of AA5754.
The rate sensitivity of the failure strain decreases with increasing prestrain in either orientation due to a delay in the onset of the PLC effect.

Small rectangular samples of AA5754 sheet were tested in bending at strain rates of 0.001/s, 0.1/s and 80/s, following plane strain prestrain in either the transverse or longitudinal material directions. The following conclusions were made:

- The negative rate sensitivity that exists for AA5754 between strain rates of 0.001/s and 0.1/s is exhibited during bending.

- In uniaxial tension, prestrain in the longitudinal direction produced larger yield stresses and larger strains at diffuse necking when compared with prestrain in the transverse direction, but this did not translate into a similarly consistent trend during bending.

- At applied strain rates of 80/s, lateral inertial effects increase the peak load, and decrease the minimum load during bending.

- The peak load during bending increases with increasing prestrain and increasing yield stress.

- The minimum load decreases with increasing prestrain.
A rate-dependent Voce material model was implemented in LS-DYNA, and validated by simulating bend tests at strain rates of 0.001/s and 0.1/s, for transverse prestrains of 0, 3, 6, and 12%. It was concluded that:

- The rate-sensitive Voce-MA material model implemented in LS-DYNA was effective in predicting the peak loads and the overall behaviour of a rectangular sheet sample in bending.
- The amount of energy absorption during bending was predicted to within 6.6% for applied strain rates of 0.001/s and 0.1/s, and transverse prestrains of 0, 3, 6 and 12%.
- Not taking into account the reduction in load carrying capability associated with the development of shear bands resulted in an overprediction of the energy absorption and the deformed sample dimensions.
- It is imperative to include prestrain effects in a sheet bending FEM simulation to accurately predict energy absorption.
- It is necessary to consider the rate sensitivity of the material, but not necessarily a rate-sensitive material model, for the AA5754 alloy.
7.2 Recommendations

It is recommended that future research investigates the following areas:

- While the number of test repeats performed was enough to distinguish between trends in rate sensitivity, more repeats would be needed to better understand the subtle trends associated with changes in the prestrain orientation.
- More repeats of tests at high strain rates of 500/s and 1500/s for prestrained samples are needed to be able to determine the effects of prestrain when AA5754 exhibits positive rate sensitivity.
- More testing needs to be performed in the intermediate strain rates (1/s to 10/s) in order to determine when the transition from negative to positive strain rate sensitivity occurs.
- It would be interesting to look at more levels of prestrain below 6% to determine what amount of prestrain is necessary to produce a reduction in rate sensitivity.
- In order to ultimately perform an FEM simulation of a hydroformed S-rail under axial crush, the Voce-MA material model would have to be adapted to incorporate varying prestrain amounts, as the strain distribution in a hydroformed tube is not uniform.
- The necessity of a failure model has also been discussed in relation to the accuracy of absorbed energy predictions; hence, it would be desirable to include a failure criterion in the material model.
References


visor.uwaterloo.ca/worswick/smf/HighRate/HR_overview_website_files/frame.htm.


Appendix A

Determining the true stress versus plastic strain curves from raw test data

This section outlines the procedure for converting the load displacement data obtained from the Instron and the stress-strain data from the Hopkinson bar into true stress versus plastic strain curves and true stress versus equivalent plastic strain curves.

A 1.1: Strain rates of 0.001/s and 0.1/s, without prestrain

For strain rates of 0.001/s and 0.1/s, the raw data was output from the Instron in the form of load and displacement points, and the strains were shifted so the curve passed through the origin. True stresses and strain were calculated, and the true stress versus plastic strain curve covering the range of strains from the onset of yield to the onset of diffuse necking was determined. The onset of yield was determined by employing the 0.2% offset method [72] with a Young’s modulus of 70 GPa. The onset of localization was taken as the strain corresponding to the maximum stress. The method for determining true plastic strain ($\varepsilon_{pTRUE}$) is illustrated in Figure A1 such that:

$$\varepsilon_{pTRUE} = \varepsilon_{TRUE} - P - \left( \frac{\sigma_{TRUE}}{E_A} \right)$$

(A1)

where: $E_A$ = apparent modulus

P = plastic adjustment.
The apparent modulus ($E_A$) was taken to be the slope of the linear portion of the true stress-strain curve. The plastic adjustment ($P$) took into account the amount of strain that occurred at the yield plateau as shown in Figure A2, and was determined from:

$$P = \varepsilon_{\text{true}} \text{ (at start of plastic strain)} - \frac{\sigma_{\text{true}} \text{ (at onset of yield)}}{E_A}.$$  (A2)

Figure A1: An illustration of the different values used to determine the true plastic strain.

Figure A2: An illustration of the plastic adjustment used to determine the true plastic strain.
A 1.2 Strain rates of 500/s and 1500/s, without prestrain

For strain rates of 500/s and 1500/s, the raw data was output from the Hopkinson bar in the form of voltage and time, which was then converted to engineering stress and strain through the use of a postprocessor provided by C. Salisbury [73]. True stresses and strains were calculated, and the strains were shifted so the curve passed through the origin. The range of the data points used to perform the Voce curve fit started at the maximum of the following two criteria:

- True strain at the start of the constant strain rate;
- True strain at the end of the yield plateau.

All specimens tested at a strain rate of 500/s used data starting at the end of the yield plateau, and all specimens tested at 1500/s used data starting at the constant strain rate.

The end of the true plastic stress-strain curve was taken as the minimum of the following two criteria:

- True strain at the point of maximum stress;
- True strain at the end of the constant strain rate.

All specimens tested at 500/s and 1500/s used the true strain level at the point of maximum stress. These limits are illustrated in Figure A3.
Figure A3: Determining the limits of strain data used from Hopkinson bar results.

For specimens tested at 500/s, the start of the plastic curve occurred at the end of the yield plateau, and the true plastic strains ($\varepsilon_{p\text{TRUE}}$) were determined using Equations A1 and A2.

For the specimens tested at 1500/s, a Voce fit was performed on the data points starting at the point where the strain rate became constant. The Voce curve was then extrapolated back to the point at the end of the yield plateau, to obtain the plastic stress-strain curve, as illustrated in Figure A4.
A 1.3: Strain rates of 0.001/s, 0.1/s, 500/s and 1500/s with prestrain

The stress-strain curves in this section were determined using the sample geometry following prestrain. A true plastic curve and an effective plastic curve were generated for specimens that had been prestrained, as shown in Figure A5. The true plastic curves for strain rates of 0.001/s and 0.1/s were produced by applying the method of Section A1.1 and for strain rates of 500/s and 1500/s using the method of Section A1.2.

The effective plastic curve takes into account the equivalent amount of strain that had been applied during prestrain. The true plastic curve was shifted by the equivalent von Mises strain ($\varepsilon_{eq}$) calculated from:

$$
\varepsilon_{eq} = \sqrt{\frac{2}{3}(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)}.
$$

(A3)
The major and minor principal strains ($\varepsilon_1$ and $\varepsilon_2$ respectively) were measured following prestraining, and $\varepsilon_3$ was calculated by assuming that:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0. \quad (A4)$$

![Diagram](image)

<table>
<thead>
<tr>
<th>True plastic strain</th>
<th>A</th>
<th>A'</th>
<th>B</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>True equivalent plastic strain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>137.626</td>
<td>268.18</td>
<td>16.403</td>
<td></td>
<td></td>
</tr>
<tr>
<td>87.065</td>
<td>268.18</td>
<td>16.403</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure A5: Voce curve fits for 1.5% transverse prestrain at 0.001/s plotted against a) true plastic strain, b) equivalent true plastic strain.

The Voce curve fits (Equation (9)) for the flow stress plotted against true plastic strain and equivalent true plastic strain (Figure A5) result in the same values for the parameters B and n. The initial yield parameter, however, changes with the different horizontal axes. The parameter A is used for flow curves plotted against true plastic strain, and A' is used with equivalent true plastic strain.
Appendix B

Statistical analysis used to identify significant test results

B 1.1: Statistical significance

This section outlines the procedure for determining which of the test repeats would be used to represent the characteristic stress-strain curve for a given set of testing conditions.

For each sample, the Voce parameters $A$ or $A'$, $B$ and $n$ were recorded, as well as the true equivalent prestrain (if applicable). The $A$ parameter is specifically for curves plotted against plastic strain, and $A'$ parameter is for curves plotted against the true equivalent plastic strain. The $t$ distribution was used to determine the 95% confidence interval for each parameter according to:

$$s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$  \hspace{1cm} (B1)

with confidence interval limits:

$$\bar{x} \pm t \frac{s}{\sqrt{n}}$$  \hspace{1cm} (B2)

where: $s = \text{sample standard deviation}$
$n = \text{sample size}$
$\bar{x} = \text{sample mean}$
$t = t$ value corresponding to the level of confidence desired.
Any parameter falling outside of this range was highlighted, and any sample with at least one of the 5 parameters outside of the 95% confidence interval was removed from the subsequent calculations. The Voce curve fit was based on the average of the remaining samples, which were samples 462, 496 and 486 in this case, summarized in Table B1.

Table B1: Statistical summary for 1.5% transverse prestrain at 0.001/s.

<table>
<thead>
<tr>
<th>number of samples</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>t value</td>
<td>2.776</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sample number</th>
<th>A</th>
<th>B</th>
<th>n</th>
<th>A'</th>
<th>true equivalent prestrain</th>
</tr>
</thead>
<tbody>
<tr>
<td>460</td>
<td>137.12</td>
<td>272.89</td>
<td>15.53</td>
<td>91.51</td>
<td>0.01864</td>
</tr>
<tr>
<td>462</td>
<td>137.63</td>
<td>268.18</td>
<td>16.40</td>
<td>87.07</td>
<td>0.01996</td>
</tr>
<tr>
<td>495</td>
<td>120.63</td>
<td>251.12</td>
<td>16.09</td>
<td>77.60</td>
<td>0.01771</td>
</tr>
<tr>
<td>496</td>
<td>134.33</td>
<td>272.08</td>
<td>15.38</td>
<td>83.19</td>
<td>0.02129</td>
</tr>
<tr>
<td>486</td>
<td>134.10</td>
<td>267.70</td>
<td>16.23</td>
<td>78.04</td>
<td>0.02159</td>
</tr>
</tbody>
</table>

| average       | 132.76  | 266.39  | 15.93   | 83.48 | 0.01984                    |
| standard deviation | 6.97  | 8.84   | 0.45    | 5.95  | 0.00167                    |
| lower         | 124.11  | 255.42  | 15.37   | 76.09 | 0.01777                    |
| upper         | 141.41  | 277.37  | 16.48   | 90.87 | 0.02191                    |

Remove 495 and 460

Once the statistically significant samples had been identified, the Voce parameters were used to predict the flow stress at 0.005 increments of strain for each sample, as presented in Table B2. The maximum strain is limited by the sample that has the lowest strain at diffuse necking. An average of these flow stresses was taken, and the average was fit to the Voce material model. The final column shows the parameters and predicted stress values for the characteristic Voce curve, corresponding to the specific strain rate,
and amount and orientation of prestrain (1.5% transverse prestrain at 0.001/s plotted against plastic strain in this case).

Table B2: Determining the characteristic Voce fit (for true plastic strain).

<table>
<thead>
<tr>
<th>462</th>
<th>496</th>
<th>486</th>
<th>average</th>
<th>characteristic Voce fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>137.63</td>
<td>134.33</td>
<td>134.10</td>
<td>135.35</td>
</tr>
<tr>
<td>B</td>
<td>268.18</td>
<td>272.08</td>
<td>267.70</td>
<td>269.28</td>
</tr>
<tr>
<td>n</td>
<td>16.40</td>
<td>15.38</td>
<td>16.23</td>
<td>16.00</td>
</tr>
</tbody>
</table>

| 0    | 137.63 | 134.33 | 134.10 | 135.35 |
| 0.005| 147.91 | 144.52 | 144.51 | 145.65 |
| 0.01 | 157.38 | 153.97 | 154.11 | 155.15 |
| 0.015| 166.10 | 162.71 | 162.96 | 163.92 |
| 0.02 | 174.14 | 170.81 | 171.12 | 172.02 |
| 0.025| 181.54 | 178.30 | 178.65 | 179.50 |
| 0.03 | 188.37 | 185.25 | 185.59 | 186.40 |
| 0.035| 194.65 | 191.67 | 191.99 | 192.77 |
| 0.04 | 200.44 | 197.63 | 197.89 | 198.65 |
| 0.045| 205.77 | 203.14 | 203.33 | 204.08 |
| 0.05 | 210.69 | 208.24 | 208.34 | 209.09 |
| 0.055| 215.22 | 212.97 | 212.97 | 213.72 |
| 0.06 | 219.39 | 217.34 | 217.24 | 217.99 |
| 0.065| 223.23 | 221.40 | 221.17 | 221.93 |
| 0.07 | 226.77 | 225.15 | 224.79 | 225.57 |
| 0.075| 230.03 | 228.62 | 228.14 | 228.93 |
| 0.08 | 233.03 | 231.84 | 231.22 | 232.03 |
| 0.085| 235.80 | 234.82 | 234.06 | 234.89 |
| 0.09 | 238.35 | 237.58 | 236.68 | 237.54 |
| 0.095| 240.70 | 240.13 | 239.10 | 239.98 |
| 0.1  | 242.86 | 242.30 | 241.33 | 242.23 |
| 0.105| 244.86 | 244.69 | 243.39 | 244.31 |
| 0.11 | 246.69 | 246.71 | 245.28 | 246.23 |
| 0.115| 248.38 | 248.59 | 247.03 | 248.00 |
| 0.12 | 249.94 | 250.33 | 248.64 | 249.64 |
| 0.125| 251.38 | 251.94 | 250.12 | 251.15 |
| 0.13 | 252.70 | 253.43 | 251.49 | 252.54 |
| 0.1319| 253.17 | 253.96 | 251.97 | 253.03 |
**B 1.2: T-test**

This section presents an example of the t-test used to determine whether two sample means are statistically different.

The values of the yield stress for the six repeats in the longitudinal direction, and the four repeats in the transverse direction are summarized in Table B3. The means and the standard deviations are also shown.

Table B3: Yield stresses for uniaxial tensile test repeats in longitudinal and transverse directions.

<table>
<thead>
<tr>
<th></th>
<th>Longitudinal (MPa)</th>
<th>Transverse (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.7</td>
<td>92.2</td>
<td></td>
</tr>
<tr>
<td>95.2</td>
<td>92.2</td>
<td></td>
</tr>
<tr>
<td>95.0</td>
<td>95.2</td>
<td></td>
</tr>
<tr>
<td>95.4</td>
<td>93.0</td>
<td></td>
</tr>
<tr>
<td>94.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>94.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average:</td>
<td>95.5</td>
<td>93.2</td>
</tr>
<tr>
<td>Standard deviation:</td>
<td>1.127</td>
<td>1.418</td>
</tr>
</tbody>
</table>

The pooled estimate of the common standard deviation is calculated as follows:

\[ s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = 1.548 \]  \hspace{1cm} (B3)

where: \( n_1 \) and \( n_2 \) = sizes of samples 1 and 2
\( s_1 \) and \( s_2 \) = standard deviations of samples 1 and 2.

The value of \( t \) is equal to,

\[ t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = 2.319 \] \hspace{1cm} (B4)

where: \( \bar{x}_1 \) and \( \bar{x}_2 \) = means of samples 1 and 2.
For 0.05 level of significance, and \( n_1 + n_2 - 2 = 8 \) degrees of freedom, the critical values of \( t \) are \( t_c = \pm 2.306 \).

If \(-2.306 < t < 2.306\), then it can be concluded that the yield stresses are statistically the same, and if \( t \) falls outside of the range, then the yield stresses are statistically different. Since \( t = 2.319 \) falls outside of the range, the yield stress in the longitudinal direction is different than the yield stress in the transverse direction.
Appendix C

Determining the magnitude of the applied plane strain prestrain

This section outlines the steps employed for calculating the major and minor strains resulting from the application of plane strain prestrain. References are made to Figure C1, which depicts the grid pattern and defines the strain-related parameters, and Figure C2, which shows the spreadsheet used for the calculations.

1. A grid pattern of dots was applied to the unloaded blank, at a spacing of 2.2mm as shown in Figure C1.
2. This base grid was scanned, and the 25 dots in the centre of the sample were cropped and imported into the software Image Pro.
3. The horizontal positions of the centres of the 25 dots were recorded in the column marked “X position”, and the vertical positions of the centres were recorded in the column marked “Y position” in Figure C2 a).
4. The distance, $d_1$, between the 20 vertical pairs of dots, and the distance, $d_2$, between the 20 horizontal pairs of dots was determine as shown in Figures C1 and C2 a), and the average of these 20 measurements was taken ($d_{1avg}$ and $d_{2avg}$).
5. The blank was prestrained, and steps 2 through 4 were repeated for the same 25 dots where $d_1'$ and $d_2'$ are the distances between vertical and horizontal pairs of dots and $d_{1avg}'$ and $d_{2avg}'$ are the corresponding averages as shown in Figure C2 b).
6. The following formulae were used to calculate the overall major \((e_1)\) and minor \((e_2)\) engineering strain,

\[
e_1 = \frac{d_{1\text{ avg}} - d_{1\text{ avg}'}}{d_{1\text{ avg}}}, \quad e_2 = \frac{d_{2\text{ avg}} - d_{2\text{ avg}'}}{d_{2\text{ avg}}}. \tag{C1}
\]

Figure C1: Prestrain blank grid pattern a) before prestrain and b) after prestrain. Examples of horizontal and vertical pairs of dots are shown.

a) Base grid

b) Prestrained grid
Object  | X position | $d_2$ | $d_{2\text{avg}}$ | Object  | Y position | $d_1$ | $d_{1\text{avg}}$
---|---|---|---|---|---|---|---
PS100 base dots IP:
1  | 31.924  |  |  | 1  | 30.150  |  |  
6  | 31.192  |  |  | 2  | 30.501  |  |  
13 | 30.904  |  |  | 3  | 31.082  |  |  
17 | 30.686  |  |  | 4  | 31.360  |  |  
21 | 30.840  |  |  | 5  | 32.645  |  |  
2  | 106.940 | 75.016 | 6  | 105.317 | 75.168 |  
8  | 106.206 | 75.015 | 8  | 105.476 | 74.974 |  
11 | 106.349 | 75.445 | 9  | 106.481 | 75.399 |  
19 | 105.836 | 75.149 | 7  | 106.390 | 75.030 |  
22 | 105.934 | 75.094 | 10 | 107.473 | 74.827 |  
3  | 181.519 | 74.579 | 13 | 180.704 | 75.387 |  
9  | 180.826 | 74.620 | 11 | 180.667 | 75.191 |  
14 | 180.986 | 74.637 | 14 | 181.556 | 75.075 |  
20 | 180.351 | 74.515 | 15 | 182.113 | 75.723 |  
23 | 180.270 | 74.336 | 12 | 182.442 | 74.969 |  
4  | 255.806 | 74.287 | 17 | 256.078 | 75.374 |  
7  | 255.061 | 74.235 | 19 | 256.315 | 75.648 |  
15 | 255.224 | 74.238 | 20 | 256.964 | 75.408 |  
16 | 254.681 | 74.330 | 16 | 256.769 | 74.656 |  
25 | 254.741 | 74.471 | 18 | 257.581 | 75.140 |  
5  | 330.806 | 75.000 | 21 | 330.469 | 74.390 |  
10 | 329.765 | 74.704 | 22 | 330.818 | 74.503 |  
12 | 330.010 | 74.786 | 23 | 331.368 | 74.404 |  
18 | 329.370 | 74.689 | 25 | 332.013 | 75.243 |  
24 | 329.958 | 75.217 | 24 | 332.416 | 74.834 |  
3  | 261.988 | 74.265 | 17 | 263.527 | 74.196 |  
7  | 260.986 | 73.986 | 19 | 264.281 | 74.169 |  
14 | 261.095 | 74.169 | 20 | 264.816 | 74.196 |  
16 | 260.192 | 74.165 | 16 | 264.729 | 74.704 |  
25 | 260.228 | 74.173 | 18 | 265.667 | 74.345 |  
5  | 336.482 | 74.494 | 21 | 340.292 | 74.765 |  
9  | 335.472 | 74.486 | 22 | 340.791 | 76.510 |  
15 | 335.284 | 74.189 | 23 | 341.631 | 76.815 |  
18 | 334.737 | 74.545 | 25 | 342.179 | 77.450 |  
24 | 335.082 | 74.854 | 24 | 342.545 | 76.878 |  

| Object  | X position | $d_2'$ | $d_{2\text{avg}}'$ | Object  | Y position | $d_1'$ | $d_{1\text{avg}}'$
---|---|---|---|---|---|---|---
PS100 base dots IP:
1  | 38.424  |  |  | 1  | 31.772  |  |  
6  | 38.079  |  |  | 2  | 32.066  |  |  
11 | 37.606  |  |  | 3  | 33.076  |  |  
17 | 36.986  | 4  | 32.813 | 19 | 36.578  | 5  | 34.894 |  
21 | 36.578  | 5  | 33.849 | 2  | 113.527 | 75.104 | 6  | 108.832 | 77.060 |  
10 | 112.181 | 74.102 | 10 | 109.519 | 77.453 |  
12 | 112.528 | 74.922 | 8  | 110.000 | 76.924 |  
19 | 111.467 | 74.481 | 7  | 110.028 | 77.215 |  
22 | 111.585 | 75.007 | 9  | 110.960 | 77.110 |  
4  | 187.723 | 74.196 | 11 | 186.653 | 77.821 |  
8  | 187.000 | 74.819 | 12 | 186.417 | 76.898 |  
13 | 186.925 | 74.397 | 13 | 187.188 | 77.188 |  
20 | 186.027 | 74.561 | 14 | 188.100 | 78.072 |  
23 | 186.055 | 74.470 | 15 | 188.321 | 77.362 |  
3  | 261.988 | 74.265 | 17 | 263.527 | 74.196 |  
7  | 260.986 | 73.986 | 19 | 264.281 | 74.169 |  
14 | 261.095 | 74.169 | 20 | 264.816 | 74.704 |  
16 | 260.192 | 74.165 | 16 | 264.729 | 74.704 |  
25 | 260.228 | 74.173 | 18 | 265.667 | 74.345 |  
5  | 336.482 | 74.494 | 21 | 340.292 | 74.765 |  
9  | 335.472 | 74.486 | 22 | 340.791 | 76.510 |  
15 | 335.284 | 74.189 | 23 | 341.631 | 76.815 |  
18 | 334.737 | 74.545 | 25 | 342.179 | 77.450 |  
24 | 335.082 | 74.854 | 24 | 342.545 | 76.878 |  

minor strain -0.33  major strain 2.83

b) calculations after prestrain is applied

Figure C2: Spreadsheet used to calculate the applied major and minor prestrain a) prior to prestrain and b) post prestrain.
Appendix D

Determining the susceptibility of AA5754 sheet to damage

Extensive evaluation of this particular batch of 3mm AA5754 sheet was performed in order to determine whether or not it was necessary to include the effects of voids and damage in the constitutive material model. Samples were prestrained in plane strain in the transverse direction, then tested under quasi-static bending, high-rate uniaxial and high-rate triaxial (notched sample producing elevated triaxiality) loading to failure. These load cases were chosen as they would maximize the potential for damage to occur.

The samples were mounted in bakelite, polished and then viewed in through-thickness planes under an optical microscope. The evaluation of the degree of damage present was strictly qualitative, involving the visual detection of voids. Select samples were also viewed under the scanning electron microscope (SEM) but no more information was gathered than what could be observed with the optical microscope.

No damage was seen in the as-received 3mm flat sheet material, as is evident in Figure D1. In general, there was no detectable damage for any of the load cases, in regions of uniform deformation, which indicates that damage does not need to be included in a constitutive material model for this particular batch of 3mm AA5754 sheet. Damage was only observed in two situations: adjacent to a neck or failure surface, and near the outer surface of a tight-radius bend. Figure D2 shows isolated damage near the failure surface of a notched sample (increased triaxiality) tested at a strain rate of 1500/s.
Figure D3 depicts void nucleation and linking of particles that is widespread throughout the entire outer region of the bend.

The minimal, isolated damage that was observed in these tests indicate that damage does not need to be incorporated into the constitutive material model for AA5754 behaviour up to the point of strain localization. Void nucleation, growth, and coalescence should be incorporated into models seeking to predict extremely large strains (e.g. tight-radius bending) up to the point of final fracture, which is beyond the scope of this thesis.

Figure D1: As-received 3mm AA5754 sheet. (Rolling direction is horizontal; through-thickness direction is vertical).
Figure D2: Void nucleation of a notched sample subject to plane strain prestrain in the transverse direction followed by tensile loading to failure at 1500/s.

Figure D3: Void nucleation of a sample subject to plane strain prestrain in the transverse direction followed by a 180° bend.
Appendix E

Voce-MA rate-sensitive material model

This section presents the Voce-MA rate-sensitive material model used in the FEM computer simulation of the bend test.

```fortran
subroutine umat49 (cm, eps, sig, hisv, dt1, capa, etype, time, temp)

material constants in array cm:
1 k bulk modulus
2 g shear modulus
3 ys yield stress
4 itype hardening rule type
5 etan tangent modulus
6 A Voce constant
7 B Voce constant
8 n Voce constant
9 e0dot base strain rate
10 a1 first constant for multiplicative rate parameter m
11 a2 second constant for multiplicative rate parameter m
12 a3 third constant for multiplicative rate parameter m
13 a4 first constant for additive rate parameter m'
14 a5 second constant for additive rate parameter m'
15 a6 third constant for additive rate parameter m'

dimension cm(*), eps(*), sig(*), hisv(*)
character(*) etype

if (dt1.gt.0.) then
  d1=eps(1)/dt1
  d2=eps(2)/dt1
```
d3 = \frac{\text{eps}(3)}{\text{dt}1} \\
d4 = \frac{\text{eps}(4)}{\text{dt}1} \\
d5 = \frac{\text{eps}(5)}{\text{dt}1} \\
d6 = \frac{\text{eps}(6)}{\text{dt}1}

\text{else}
\begin{align*}
    d1 &= 0. \\
    d2 &= 0. \\
    d3 &= 0. \\
    d4 &= 0. \\
    d5 &= 0. \\
    d6 &= 0. \\
\end{align*}
\text{end if}

\begin{align*}
\text{calc} & \text{ulate deviatoric components of strain rate} \\
\text{---} & \text{---} \\
\text{dd} &= -(d1+d2+d3) \\
\text{davg} &= \text{dd}/3.0 \\
\text{d1d} &= d1 + \text{davg} \\
\text{d2d} &= d2 + \text{davg} \\
\text{d3d} &= d3 + \text{davg} \\
\end{align*}

\begin{align*}
\text{calc} & \text{ulate stress increments (dsig1-dsig6)} \\
\text{---} & \text{---} \\
\text{dsig1} &= 2.0*\text{cm(2)}*d1d*\text{dt}1 \\
\text{dsig2} &= 2.0*\text{cm(2)}*d2d*\text{dt}1 \\
\text{dsig3} &= 2.0*\text{cm(2)}*d3d*\text{dt}1 \\
\text{dsig4} &= \text{cm(2)}*d4*\text{dt}1 \\
\text{dsig5} &= \text{cm(2)}*d5*\text{dt}1 \\
\text{dsig6} &= \text{cm(2)}*d6*\text{dt}1 \\
\end{align*}

\begin{align*}
\text{calc} & \text{ulate trial stresses} \\
\text{---} & \text{---} \\
\text{pold} &= -(\text{sig}(1)+\text{sig}(2)+\text{sig}(3))/3.0 \\
\text{trial1} &= \text{pold} + \text{sig}(1) + \text{dsig1} \\
\text{trial2} &= \text{pold} + \text{sig}(2) + \text{dsig2} \\
\text{trial3} &= \text{pold} + \text{sig}(3) + \text{dsig3} \\
\text{trial4} &= \text{sig}(4) + \text{dsig4} \\
\text{trial5} &= \text{sig}(5) + \text{dsig5} \\
\text{trial6} &= \text{sig}(6) + \text{dsig6} \\
\text{dpres} &= \text{cm}(1)*\text{dt}1*\text{dd} \\
\text{ptrial} &= \text{pold} + \text{dpres} \\
\end{align*}

\begin{align*}
\text{calc} & \text{ulate flow stress} \\
\text{---} & \text{---} \\
\text{call sigflow(cm,eps,hisv,sigf,hard,dt1)} \\
\end{align*}

\begin{align*}
\text{von Mises stuff} \\
\text{---} & \text{---} \\
\text{aj2} &= 0.5*(\text{trial1}**2 + \text{trial2}**2 + \text{trial3}**2) + \\
& \text{trial4}**2 + \text{trial5}**2 + \text{trial6}**2 \\
\text{sj2} &= \text{sqrt}(3.0*\text{aj2}) \\
\end{align*}
sj2 = amax1(1.e-30, sj2)
c
if(sj2.gt.1.e-30) then
   scale = sigf/sj2
else
   scale = 1.0
end if
c
write(*,*) dt1, sigf, sj2, scale, hisv(1)
c
if(scale.lt.1.0) then
   strial1 = scale*strial1
   strial2 = scale*strial2
   strial3 = scale*strial3
   strial4 = scale*strial4
   strial5 = scale*strial5
   strial6 = scale*strial6
   dep = (1.0-scale)*sj2/(3.0*cm(2)+hard)
else
   dep = 0.0
end if
c
-------------------------------------------------------------------------------------------------
c     update stresses and plastic strain
c     -----------------------------------------------------------------
sig(1) = strial1-ptrial
sig(2) = strial2-ptrial
sig(3) = strial3-ptrial
sig(4) = strial4
sig(5) = strial5
sig(6) = strial6
hisv(1) = hisv(1)+dep
write(*,*) dep
c
-------------------------------------------------------------------------------------------------

c Author: D Wowk
c Date:   June 4, 2008
c
c update other history variables
c     -----------------------------------------------------------------
if(dt1.gt.0.0) then
   einc = dep
   epaccum = einc+hisv(3)
dtaccum = dt1+hisv(4)
count = hisv(5)+1
if(count.eq.1000) then
   count = 0
   hisv(2) = epaccum/dtaccum
   epaccum = 0
dtaccum = 0
end if
hisv(3) = epaccum
hisv(4) = dtaccum
hisv(5) = count
write(*,*) hisv(2), rate, hisv(3), hisv(4), hisv(5)
else
hisv(2) = 0.001
end if

c
return
end

c SIGFLOW

c Author: W. Brent Lievers
c Date: May 6, 2008
c
Code to implement different hardening behaviours. Must calculate the
flow stress (sigf) as well as the hardening rate (hard). As above,
based on code in the Gurson implementation UMAT46.
c
itype = 1: linear hardening
c

subroutine sigflow(cm,eps,hisv,sigf,hard,dt1)
c
dimension cm(*),hisv(*),eps(*)
itype= nint(cm(4))
-----------------------------------------------------------------
Author: D Wowk
Date: June 4, 2008
-----------------------------------------------------------------
linear hardening
---------------------------------
if(itype.eq.1)then
  sigf= cm(3)+hisv(1)*cm(5)
  hard= cm(5)
else if(itype.eq.2)then
  voce= cm(7)-(cm(7)-cm(6))*EXP(-hisv(1)*cm(8))
  sigf= voce
  hard= cm(8)*(cm(7)-cm(6))*(EXP(-hisv(1)*cm(8)))
if(hisv(2).gt.0.0000099)then
  rate=hisv(2)
  rate=hisv(2)*exp(hisv(1))
  em=(rate/cm(9))**(cm(10)*((LOG(rate))**2)+cm(11)
  addrp=(cm(13)*((LOG(rate))**2)+cm(14)*LOG(rate)
  ! +cm(15))*LOG(rate/cm(9)))
  sigf= (sigf*em)+addrp
  hard= hard*em
c
write(*,*) hisv(2), rate, sigf, dep, dt1
end if

c
return
end

SIGFLOW
Appendix F

Determining the material constants for the Voce-MA material model

This section presents the macro used to calculate the material constants which are used as input for the rate-sensitive Voce-MA material model.

*ASK,trans,amount of equivalent prestrain in the transverse direction,0
!calculate the Voce parameters using curve fits

*SET,n,-38067*(trans**3)+7539.4*(trans**2)-(112.95*trans)+14.539
*SET,B,5190.3*(trans**2)-(790.42*trans)+280.41
*SET,A,-4333.2*(trans**2)+(1264.9*trans)+105.6

!Calculate m and m' for the specific prestrain value at 0.1/s and 1500/s using linear fits

*SET,m01,(0.1361*trans)-0.018
*SET,m01p,(-20.33*trans)+2.2304
*SET,m1500,(0.2223*trans)+0.0042
*SET,m1500p,(-69.559*trans)+1.146

!Perform a quadratic curve fit for m and m' using 3 points, m at 0.001/s (0), at 0.1/s and at 1500/s

*SET,x1,log(0.001)
*SET,x2,log(0.1)
*SET,x3,log(1500)
*SET,x4,x2-x1
*SET,x5,x2**2-x1**2
*SET,x6,x3-x1
*SET,x7,x3**2-x1**2

!Perform the fit for m

*SET,a2,(m01*x7-m1500*x5)/(x4*x7-x6*x5)
*set,a1,(m01-a2*x4)/x5
*SET,a3,-a1*(x1**2)-(a2*x1)
!
!Perform the fit for m'
!

*SET,a5,(m01p*x7-m1500p*x5)/(x4*x7-x6*x5)
*set,a4,(m01p-a5*x4)/x5
*SET,a6,-a4*(x1**2)-(a5*x1)
!
!
/EOF
Appendix G

True stress versus plastic strain curves for prestrained samples under uniaxial tension

Figure G1: A comparison between strain rates of 0.001/s and 0.1/s for samples subjected to transverse prestrain with magnitudes from 2% to 12%.
Figure G2: A comparison between strain rates of 0.001/s and 0.1/s for samples subjected to longitudinal prestrain with magnitudes from 2% to 12%.