THE EFFECT OF PURPOSEFUL MATHEMATICS DISCOURSE IN THE CLASSROOM ON STUDENTS’ MATHEMATICS LANGUAGE IN THE CONTEXT OF PROBLEM SOLVING

by

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A thesis submitted to the Faculty of Education
in conformity with the requirements for
the degree of Master of Education

Queen’s University
Kingston, Ontario, Canada
September, 2016

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Abstract

This study examines how one secondary school teacher’s use of purposeful oral mathematics language impacted her students’ language use and overall communication in written solutions while working with word problems in a grade nine academic mathematics class.

Mathematics is often described as a distinct language. As with all languages, students must develop a sense for oral language before developing social practices such as listening, respecting others ideas, and writing. Effective writing is often seen by students that have strong oral language skills.

Classroom observations, teacher and student interviews, and collected student work served as evidence to demonstrate the nature of both the teacher’s and the students’ use of oral mathematical language in the classroom, as well as the effect the discourse and language use had on students’ individual written solutions while working on word problems. Inductive coding for themes revealed that the teacher’s purposeful use of oral mathematical language had a positive impact on students’ written solutions. The teacher’s development of a mathematical discourse community created a space for the students to explore mathematical language and concepts that facilitated a deeper level of conceptual understanding of the learned material. The teacher’s oral language appeared to transfer into students written work albeit not with the same complexity of use of the teacher’s oral expression of the mathematical register. Students that learn mathematical language and concepts better appear to have a growth mindset, feel they have ownership over their learning, use reorganizational strategies, and help develop a discourse community.
Acknowledgements

The completion of this thesis would not have been possible without the support of my family, friends, and the caring Queen’s University community. I want to offer my thanks to the participating school board, administrators, teacher, and students for their enthusiasm and support.

To my supervisor, Dr. Jamie Pyper, thank you for all your support throughout my thesis. Your continued patience and encouragement will always be remembered. Thank you for pushing me to think more deeply about my research questions and my analysis. To my committee member, Dr. Azza Sharkawy, thank you for challenging my thinking throughout the writing process. You helped me to clarify my thinking and direction and for that I am grateful. To my external examiner, Dr. Peter Taylor, thank you for your interest in my work, your conversation invoking questions, and your passion for understanding. To my friend and colleague, Dr. Stefanie Syer, thank you for reminding me that life is busy and to take a break. You offered me advice and feedback whenever I felt lost in academia. Your friendship and support helped me to stay the course and rethink issues and problems that arose throughout my research.

To my daughter, Elis, thank you for growing up in the Education Library. Your patience for Mommy’s “just five more minutes” will be forever appreciated. To my parents, thank you for all the support over the years and help with childcare. Dan, thank you for being such a constant in my life since the day I met you.
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<th>Full Form</th>
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<tbody>
<tr>
<td>ZPD</td>
<td>Zone of Proximal Development</td>
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<td>OAME</td>
<td>Ontario Association of Mathematics Educators</td>
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Chapter 1

Introduction

“… words create tightly knit webs of connections …”

(Kim, Ferrini-Mundy, Sfard, 2012, p. 87)

Mathematics is a language in and of its own (Moschkovich, 2010). As with all languages, students must develop a sense for oral language before developing both the social (e.g., listening and respecting others ideas) and written practices (Goodman & Goodman, 1976; Elley, 1991). Effective writing is often observed with students that have strong oral language skills (Shanahan, 2006; Cox et al., 1991). The development of oral language skills translates to improved conceptual understanding and written mathematical problem solving through a number of instructional modeling methods such as think-alouds, cooperative learning strategies, and collaborative tasks (Wachira et al., 2013; Kabasakalian, 2007). The opportunity for these methods may allow students to gain practice, comfort, and confidence in using the language of the mathematics classroom (i.e., a mathematics register; Schleppegrell, 2007; Halliday, 1978). This confidence is gained through the oral use of the language of mathematics. From the enculturation of the discourse community, students can demonstrate use of language and terminology that indicate the students understand the concepts that the language represents. Once oral language skills are developed many students are capable of transferring this same skill set into their written work (Swain, Kinnear, & Steinman, 2010). Oral language skills help in the development of written language, acquisition of academic vocabulary, and engagement of social practices (Swain, et al., 2010). The overall aim of this research was to inquire if the oral language adopted by the students from the teacher appeared within students’ written mathematics
solutions. Furthermore, I sought to explore if written language skills can be linked to students’ oral use within the mathematics discourse community of a classroom.

**Rationale**

Students in my mathematics classes often seem to struggle with problem solving. I questioned if they struggled with reading the structure of mathematics problems, if they didn’t understand the language used in word problems, or if they didn’t understand or have cultural experience with the word problems or inquiry problems they were given. From my experience, I believe that it is a mixture of these concepts that are the root of issues for students when trying to solve word problems in mathematics. When I began to look over the literature on these topics, I noticed that a great deal of research has been done looking at second language learning and how it affects students in mathematics courses in the United States. Through this research it can be seen that bilingual students who are allowed to move between languages are able to make stronger connections to their mathematics than unilingual students of mathematics (Adams, 2010; Moschkovich, 2010). Bilingual and second language research stemmed further research into how cultural and social practices affect student learning of mathematics and problems. A variety of researchers have noted that the culture we are raised in deeply impacts the way we learn mathematics (Le Roux, 2008). Cultures that are more mathematically literate, or that teach problem solving within practical situations develop students that are stronger at problem solving (Le Roux, 2008). Sociocultural studies also have been conducted on the culture of mathematics. Mathematics not only acts as a language that is to be learned but is its own culture as well (Gutierrez, Sengupta-Irving, & Dieckmann, 2010; Moschkovich, 2010). Students that are able to navigate the language use and social practices better understand the taught concepts and are stronger problem solvers (Moschkovich, 2004). Research has also been done to better
understand how reading structures of mathematics problems affect student learning and their problem solving skills with word problems in mathematics. Students that have strong literacy skills, and understand the language used in problems, also understand the problems being asked and are able to solve the word problems presented (Draper, 2002).

An assumption made by the majority of the research conducted in problem solving in mathematics is that if students understand the use of mathematical language in word problems, they understand the problems, and are able to solve the problem in some fashion (Draper, 2002). To extend the research conducted into the use of mathematical language by students in the classroom, research has demonstrated the use of oral mathematical language in paired work facilitates written problem solving (Kieran, 2002). Teachers use of language in the classroom has been seen to change based on the audience within the classroom. Teachers of students in private schools or high achieving classes, tend to use rigorous oral mathematical language whereas teachers of students moving into community college or work tend to use less academic or less rigorous mathematical language in the classroom (Atweh & Cooper, 1995). It was also noted in the same study that students were stronger at problem solving in the classes where the teacher was more rigorous with mathematical language use (Atweh & Cooper, 1995). There seems to be a gap in the literature that explicitly examines whether teachers’ use of rich oral mathematical language facilitates students’ written use of mathematical language and problem solving with word problems. This research aimed to illustrate if, and the extent to which, teachers’ use of rich oral mathematical language in the classroom translates into students’ improved written solutions of written word problems. To show this I first looked at the teachers’ use of rich oral mathematical language in the classroom and examined if students then adopted this language use in their own speech patterns. This allowed me to compare students’ language patterns against the
teachers, as well as against their written work. The goal was to see if students adopted the oral mathematical language used in class from their teacher, and then to see if this language transferred to students’ solutions of their written problems in class.

**Research Questions**

The value that teachers place on the development of students’ oral discourse within a mathematics class may predict the strength of the foundation of the students’ knowledge base and students’ ability to problem solve effectively in mathematics. As students work collaboratively to share ideas and strategies they may learn to be better problem solvers not only within their mathematics class, but throughout their secondary experience and in their daily lives (Mullins, Rummel & Spada, 2011; Lin & Liu, 2012).

Thus the main research question was:

How does the use of purposeful oral mathematics language in the classroom affect students’ mathematics discourse during problem solving and appear in their written work?

To further explore this question, three supporting questions were developed as follows:

- What is the nature of the teacher’s oral discourse in the classroom?
- What similarities and differences exist between the teacher’s and students’ oral mathematical discourse?
- To what extent do students’ use of oral mathematics language in class appear in students’ individual written solutions to problems?

**Conceptual Frameworks**

The context of this research was discourse (oral, written, gestural, and visual/symbolic) within a secondary school mathematics classroom. Emphasis in this research was placed on oral and written forms of discourse. Sociocultural Theory was paired with a Sociolinguistic Approach
and Social Constructivism to account for the important role which social interactions play in language development and mathematics proficiency issues. A description of each theory and its importance to this research follows.

**Sociocultural Theory**

Sociocultural Theory was primarily developed from the work of Lev Vygotsky. The essence of the theory is that learning is a social process and the building or development of knowledge and understanding occurs through social interaction and an associated discourse (oral, written, gestural, and visual/symbolic). Cognitive development occurs through meaningful exchanges of information (Piaget, 1969; John-Steiner, et al, 1994; Swain, et al., 2010).

Vygotsky’s zone of proximal development (ZPD) describes a zone in which children or learners collaborate and where knowledge acquisition may be increased by working and communicating with an expert (teacher or peer) (Vygotsky, 1978; Kozulin, Gindis, Ageyev, & Miller, 2003; Swain, et al., 2010). Vygotsky (1978) explains the zone of proximal development as “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance and collaboration with more capable peers” (Vygotsky, 1978, p. 86). Vygotsky (1978) later explains, “what the child is able to do in collaboration today, he will be able to do independently tomorrow” (Vygotsky, 1978, p.211). Children learn from observation, interaction, and participation with others (Sahlberg, 2010; Rogoff, 2008). Reflection on the experience (the learning activity) solidifies student learning.

Students learn mathematics, in particular, from understanding the concepts and then applying them (Cobb, 1995; Sfard, 2001; Schleppegrell, 2007). Mathematical understanding is developed through interaction with the teacher and other students (Cobb, 1996). According to
Vygotsky (1978), students learn practices within a given field, such as mathematics, when they are given the opportunity to work a little ahead and teach a little behind their own ability range. Pairing students with peers at different conceptual levels will improve both students’ level of understanding (Vygotsky, 1978). Using Vygotsky’s zone of proximal development in her case study of a mathematics student and her tutor, Moschkovich (2004) found that both social and written practices within the field of mathematics were learned through pairing or grouping students that utilized the zone of proximal development. The tutor in Moschkovich’s study brings the student’s mathematical knowledge forward to a higher level of understanding but also moves the student’s social and written practices toward a more acceptable level within the field of mathematics.

As children mature society looks for them to be able to transform their speech to written form. Vygotsky (1978) states:

> Writing should be meaningful for children, that an intrinsic need should be aroused in them, and that writing should be incorporated into a task that is necessary and relevant for life. Only then can we be certain that it will develop not as a matter of hand and finger habits but as a really new and complex form of speech.

Vygotsky (1978, p. 117-118)

There needs to be a reason for a child to want to learn to read or write within a certain style or format, not simply out of routine or to give habitual answers. Reading and writing in mathematics education is unique and specific as there is cross-over into different registers and writing styles (e.g., technical, organizational, or narrative; Moschkovich, 2010). Sociocultural Theory explains that the learner would have a mentor or guide who is an expert in the field who can teach both the academic knowledge or content, in addition to the social and written practices
that students must appropriate before becoming competent members in a given field of study (Gee, 1990; John-Steiner, et al, 1994; Cobb, 1995; Moschkovich, 2004; Swain, et al., 2010).

**Sociolinguistic Theory**

In Gee’s introduction to *Sociolinguistic Literacies* (1990), which is based on a sociolinguistic approach, he opened his discussion with a similar argument that there are different types of literacies the same way there are different types and registers of discourse. The sociolinguistic approach is the study of the uses of language, based on the cultural norms and practices, which are developed around the language and field of study themselves (Gee, 1990). Geé argued that these literacies are learned through the distinctive communities within which we all have grown up. He discussed the differences of social communities affecting students’ learning with an example of two girls telling stories during sharing time in their kindergarten class. One girl had told her story in a manner that adhered to the rules of sharing time, which was to emphasize order of importance. The other girl told her story in a chronological method. These two children were from different communities, white and black respectively, and heard different versions of storytelling at home. Each child told her story in the way that her family tended to tell stories. The black girl, Gee explains, is ahead in her creative writing form of literacies comparatively to the other students in her class, while the white girl is ahead in her sharing time literacies.

Each different community and social group masters a home-based Discourse that integrates words, actions, interactions, values, feelings, attitudes and thinking in specific and distinctive ways. Each such home-base Discourse is connected to a particular social group’s way of being in the world, their ‘form of life’, their very identity, who they take themselves to be.
Gee (1990) defines discourse as “combinations of sayings, doings, thinkings, feelings, and valuings” (Gee, 1990, p. xv). Gee used discourse as an inclusive term to incorporate all types of discourse. The ‘home-based discourse’ Gee (1990) mentioned is also what develops our language skills, either advancing or handicapping us when compared with the academic achievement of our peers. Home–based discourse is the language and culture of conversation developed at home (Gee, 1990). Our language skills can either handicap or benefit us within content areas such as mathematics since the mastery of academic vocabulary demonstrates the mastery of mathematical concepts. Students whose parents could support their children, through practice and discussion, to develop the discourse of mathematics are often more successful in mathematics classes. In the same way, students who attended additional help sessions or peer support sessions are more successful in mathematics classes since they have a support system to develop their discourse and mathematical understanding. Supported students are more successful within their mathematics classes because they are more comfortable with their mathematical discourse and this comfort grants a higher ability to take risks (Turner, et al., 2003; Morrone, Harkness, D'Ambrosio, Caulfield, 2004). Students gain a stronger knowledge base and sense of mathematical discourse community when learning concepts in the classroom. A common understanding of the language of mathematics will help each student grasp new mathematical concepts in class as well as induct students into the mathematical discourse community. The concept of multiple types of literacies and discourses exists in research literature but is an area of research requiring further inquiry for how these literacies affect students’ ability to develop common understanding of the language of mathematics with their teachers and peers.
All language learning can be deconstructed into the structure of language and meaning (Gee, 1990). For example, the language we use in the mathematics register is in part taken from the everyday language, words that we use every day to communicate and interact with others (Moschkovich, 2010) and the professional or academic language (Moschkovich, 2010; Schleppegrell, 2007). The compilation of the mathematics register helps us to define and understand concepts, as well as, describe mathematical meaning to one another. Approaches that focus on the development of language during the learning of mathematics that will in turn help students to develop their conceptual understanding of mathematics (Schleppegrell, 2007; Halliday, 1978; Veel, 1999).

**Social Constructivism**

Understanding the social nature of learning mathematics is enhanced with social constructivism. Social Constructivism (Charmaz, 2012; Cobb, 1996) is a hybrid theory of Constructivism (von Glasersfeld, 1984) and Social Cognitive Theory (Bandura, 1986) and enhances the ideas of activity, interaction, and discourse in mathematics. The main focus of Social Constructivism is theoretical pragmatism. Social Constructivism focuses on the production and reproduction of both school and academic practices demonstrating an understanding of social order within discourses, and social and academic interactions (Cobb, 1996). Constructivists (von Glasersfeld, 1984) believe that learning occurs through the construction of knowledge from ones’ own reality and ones’ interaction with objects based around a learning activity (von Glasersfeld, 2001). However, Constructivism does not account for the process of learning that occurs through social practices (Cobb, 1996). Social Constructivists believe that the construction of knowledge and one’s own ‘reality’ is through interaction with materials and activities, but maintains that knowledge and reality can be
constructed in more practical ways and is dependent upon the interaction with others as a means to gain both conceptual and social understanding of the practices within a given field of study.

Social Constructivists believe that reality is constructed by participation and interaction with others in the learning activity. Knowledge is a product of society and is culturally constructed through interaction with others (Ernest, 1999; Driver, 1989). Culturally constructed knowledge allows the individual the ability to create meaning from interactions with the environment and other participants within that environment. The environment where knowledge is constructed is dependent on cultural norms (i.e. is a library a place for study or for socializing within a given community? Is the mathematics classroom an environment in which we talk about how terminology is connected to meaning and actions?). The production of knowledge involves active cognizing by the individual participating in the learning activity (Von Glasersfled, 2001; Jonassen, 1991; Driver, 1989). A learner will gain the knowledge not by participating in the activity alone, but by thinking about the meaning behind the activity while participating in it and trying to predict the world around them from the observations during the activity. The learning activity is adaptive to the dynamic of the social grouping as well as the individual (Cobb, 1995; Jonassen, 1991; Driver, 1989). The more exploration of the activity a student engages in, the more knowledge a student has the potential of gaining. The acquired knowledge from the learning task is subjective and rich, in both sociocultural and individual processes (Ernest, 1999; Jonassen, 1991; Driver, 1989). Learning is seen as a social process that takes place through the engagement of social activities (Ernest, 1999; Driver, 1989).

**Social Cognitive Theory**

Social Cognitive Theory is a learning theory where individual knowledge is acquired through directly observing others within a social context. These contexts might be within a
social interaction, a life experience, or an outside media influence (Bandura, 1963; 1986; 1997). Students learn by watching others and, in particular, paying attention to the actions they perform in a given context (Bandura, 1963). For example, when a question says ‘determine the value of \(x\)’, students will solve an equation for ‘\(x\)’ based on the methods they observed in class.

According to Bandura (1963), social learning cannot occur unless four requirements are met: attention, retention, replication, and motivation. Cobb (1995) uses components of social cognitive theory throughout his social constructivist work. Cobb (1995) uses activity as a means to heighten students’ mental state and build behaviours within mathematics. Heightening a student’s mental state could be referred to as developing their critical thinking skills whereas the development of behaviours and rituals within mathematics could be seen as developing process skills for problem solving. Social cognitive theory assumes that knowledge is acquired through observation, mental readiness, and recognition that learning is not equivalent to behaviour modification. Bandura’s (1963) work sheds light onto how students might learn through activity in a variety of ways while Cobb’s (1995; 1997) work demonstrates that knowledge is gained through observation and participation of activity within the mathematics classroom.

Sociocultural Theory, a Sociolinguistic Approach, and Social Constructivism recognize the importance of the different roles that social interaction play in understanding mathematical concepts and how and when they are best applied for learning. The underdevelopment of mathematical language, and its meaning, can hinder students’ levels of success of learning mathematical concepts. Social interaction and activity allows students to build new concepts by activating prior knowledge through language use and play (Swain, et al., 2010). A learning activity provides a base for guided interactions with the teacher to introduce language and its meaning. It also provides interaction with peers to play with the language and incorporate
students’ shared lived experiences, and activate prior knowledge. The construction of knowledge within a mathematical discourse community allows for the development of a shared reality through a common use of language and mathematical practices.
Chapter 2

Literature Review

This review of literature moves thematically through impactful articles. To begin I examine mathematical language and reflect on the types of communication that exist in mathematics classrooms as ways of learning. I then review the literature on writing strategies, mathematical discourse communities, mathematics classroom culture, the mathematics register, and student-focused teaching and learning.

A few definitions are required to proceed. The term language is often used in two manners. One is the language we speak to communicate with others in our daily life, and the other manner is that of words and phrases we use to speak or write that has more specific uses, such as the language of instruction. The distinction is made clear because of the context in which they are used. The mathematics register is a set of meanings of words and phrases that belong to or are highly utilized by the subject area of mathematics. The literature will present three levels of language within the mathematics register: everyday, classroom, and academic/professional. Within each level of language more technical and subject specific terminology is utilized in mathematical discourse. Discourse is communication through oral, written, symbolic, and/or gestural notation. A mathematical discourse community is a community of people that are able to communicate their ideas, information, and arguments to one another through a common use of language to better understand how to solve and create problems.

A language, and its uses, is embedded in the culture itself. The language and culture of mathematics is rich in social practices and meanings that must be navigated to understand the underlying mathematical concepts (Moschkovich, 2010; Halliday, 1978; Forman & Ansell,
When learning the language of mathematics it can be separated into four components; oral, written, symbolic, and gestural practices (Moschkovich, 2010; Sfard, & Cole, 2003; Lemke, 1990; 1992). These practices all form mathematical discourse. In language learning we must learn to speak, read, and write (Harris, 1989; Bartholomae, 1986). From an understanding of applied linguistics, learning a language first comes from developing oral language skills, then to the written word. “Each subject area has its own ways of using language to construct knowledge, and students need to be able to use language effectively to participate in those ways of knowing” (Schleppegrell, 2007, pg. 140). In the context of learning mathematics, teachers’ oral language skills and the importance they place on these skills, impact how students use oral language within mathematics (Huang, Normandia, & Greer, 2005).

To learn to speak mathematically, students must participate in oral mathematical discourse and social interactions with teachers and their peers (O’Halloran, 2000; Moschkovich, 2002; Kieran, 2002). This is also referred to as engaging in the mathematics register (Temple & Doerr, 2012; Gutierrez et al., 2010; Gee, 1996; Halliday, 1978). Even passive participants in oral mathematical discourse have shown improvements in the use of oral language and social practices simply from being present in a mathematical discourse community (Moschkovich, 2010; Van Dijk, 1993).

If students try to use and learn the academic vocabulary orally, they can better structure questions and participate in activities in the classroom (Moschkovich, 2002; 2010). Students who can articulate what they do and do not understand are more likely to ask questions, get answers, and feel more successful (Bifuh-Ambe, 2009). Oral command of any language allows for deeper understanding of what one is learning and discussing (Sleep & Eskelson, 2012; Schleppegrell, 2007; Gee, 1996; Rogoff, 1990; Brown, Collins, & Duguid, 1989). When
students talk with their teachers and peers using academic language, they navigate how to use each new term correctly and where else it can be applied. This is similar to when English as a second language students learn new English words; they need to practice using the word in a sentence, see what phrases that word is commonly used in, look at idioms that use the word, and if that word has a double or triple meaning (Katsarou, 2012). Mathematics vocabulary in English speaking classrooms, often use vocabulary from ‘everyday language’ in the mathematics register, which introduces multiple meanings to common words (Temple & Doerr, 2012; Moschkovich, 2010; Halliday, 1978). The use of academically appropriate oral language in the mathematics classroom promotes student acclimatization to the mathematics register and become more versed in conceptual understanding of mathematics (Sleep & Eskelson, 2012; Schleppegrell, 2007; Huang et al., 2005).

Temple and Doerr (2012) address the idea of content area discourse as it relates to literacy in mathematics. Literacy, defined by Temple and Doerr (2012), is inclusive of oral language. Temple and Doerr (2012) observed a lesson by a grade ten teacher to examine the interactional strategies the teacher used to develop oral skills using the mathematical register. They found that whether the teacher was explicit or implicit in teaching the mathematical register, students would learn the register if the teacher taught it through the use of interactional strategies of questioning and “feedback” [asking follow up questions for clarification] (Temple & Doerr, 2012). By using mathematical language with a purpose, to answer the teacher’s questions and understand the problem set, students were practicing what Temple and Doerr (2012) refer to as “funneling” and “focusing” their knowledge and understanding through their oral discussions within the mathematics register (Temple & Doerr, 2012). Funneling and focusing could also be understood as the renegotiation of meaning through reorganizational strategies for learning. The students
demonstrated higher levels of conceptual understanding, when they participated in oral discussions with their peers and the teacher. The teachers’ continuous feedback enabled the students to reformulate and recognize the learned concepts under the teachers’ guidance (Temple & Doerr, 2012).

Mathematics learning also depends on the written word both for learning and the demonstration of learning. This dependency on writing comes from the use of symbols to represent phrases and sentences. Written communication in mathematics is important as it allows others to read and understand your work, which creates further discussion and generation of ideas.

Writing in mathematics is different from writing in other content areas due to the nature of math content as well as to the use of symbols as representations of concepts (Moschkovich, 2010; O’Halloran, 2005; Pimm, 1987). Students’ ability to use the symbols, academic language, and the evidence-based structure of their work, indicates a clear understanding of both the mathematical concepts, as well as the purpose behind each representation of symbols, words, and reasoning (Dove, 2009). Writing as a means of learning often prepares students for the demonstration of learning as they are learning to write in the subject specific style of discourse. Bangert-Drowns, Hurley, and Wilkins (2004) completed a meta-analysis on 48 school based writing-to-learn initiatives and found that in approximately 75% of the studies there was evidence of small positive changes in students’ learning and achievement. Most of the writing was content based or task specific and students were completing a form of writing in a structured manner so that they could demonstrate their learning of classroom material that was presented in a similar structure. Bangert-Drowns, Hurley, and Wilkins (2004) concluded that when students
were asked to write on a specific task that was relevant to the content they were learning, this reflective writing affected their learning and achievement levels positively.

The way in which a teacher uses writing within the classroom may help to develop the mathematical discourse community (Adams, 2010; Schleppegrell, 2007; Groves & Doig, 2004). In Adams’ (2010) study she examined the practices of twelve math and science teachers and their classes, noting that two distinct beliefs appeared in regard to literacy development within a mathematics classroom. The first was that literacy strategies in writing could be used as a method of review or rehearsal for students. The rehearsal method gave students practice with the language and how it was used in class but the bulk of the writing would be geared toward review and opportunity for this would present itself toward the end of the unit of study (Adams, 2010). The other method of literacy use was as reorganization. Reorganization of ideas through writing was used throughout the unit and provided students with more opportunity to write and opportunity to receive feedback (Adams, 2010). Both methods of writing improved student learning, but it was noted that students in the classes that practiced reorganizational writing had a deeper understanding of the vocabulary as well as the taught mathematical concepts (Adams, 2010). Reorganizational writing allows students the opportunity to “play” with new vocabulary so that they better understand how and when it is most appropriately used. In Adams’ (2010) study it can be noted that teachers that used reorganizational writing also gave more feedback to students regarding students’ use of language within mathematics, giving the students more opportunity to correct themselves in the practices of mathematical discourse prior to assessment or evaluation (Adams, 2010).

Mathematical concepts are learned in part through the cultural importance placed on mathematics and its relevance to the world outside of formalized education (Schleppegrell,
2007). Cultures that appreciate a given type of literacy, such as mathematical literacy, will place more emphasis on its role in school and there will be more support at home, school, and the community as a whole (Gee, 1990). The mathematics that we learn in secondary school is dependent upon both the purpose we are learning it for, and where we are learning it both geographically and culturally (Le Roux, 2008). In Le Roux’s (2008) study, she examined the use of real-world problems in mathematics in South African universities. Le Roux found that for South African students to successfully learn mathematics they needed the problems to be accessible and have relevance to their daily lives (Le Roux, 2008). Le Roux (2008) noted that real world problems often had many types of text within them for which not all students had previous experience. This created an issue of accessibility to the language used in the problems as well as cultural references within the problems (i.e. “A milk crate is moved 3m to the left” is a simple sentence to understand as long as you know what a milk crate is, and have some cultural reference to it). Also, in these real-world problems it was observed that the most dominant written language used was the academic language in the mathematical register. The concern with the use of mathematical language in a real-world or application problem, was that the problem then became too contrived and lost its meaning. The real-world problem becomes a textbook problem with the same inaccessible language and culturally devoid context. The authenticity of the problem itself is muddled by the problems being heavily based in written mathematical language. The problems that were presented in the South African program were similar to those used in the researcher’s university in Cape Town and were used across South Africa in a variety of universities. The problems that were used for this study were used in many other universities and represented typical written problems and language in the program. Le Roux (2008) noted that the problems did not develop interest from the students since the written work was focused
on academic language of the mathematical register and less on cultural relevance. The problems were too mathematical and less business oriented even though the problems were written using a local example, this did not seem to help to grant accessibility. It seemed that there were too many forms of written language (mathematical, business, literary, etc) for students to understand the problems to their full extent (Le Roux, 2008).

To understand the problem, s/he needs to be able to interpret the use of the mathematics register. Secondly, on the productive level, the interpreting students’ needs to be able to use the mathematics register appropriately in order to produce the correct solution. Thirdly, the presence of the school mathematical word problem genre sends a message to the interpreting students about the true value of the context, as many aspects of the business are not relevant when solving this mathematics problem. Yet, those aspects of the business context that are relevant require that the student buy in to the ‘profit-business discourse’. Lastly, the student has to have a conceptual understanding of the mathematical concepts of rate of change.

(Le Roux, 2008, p. 321)

Where the learner comes from, culturally, geographically, and economically, has much to do with how the learner will interpret the information provided. For example, when reading a short story about a teenager taking care of their grandparent with Alzheimer’s, each person will connect to that story differently depending on if they have been affected by ailing parents or grandparents, or live in a culture where children and grandchildren are expected to care for their elders. The reader’s gender might impact how they read that story through the implied duty culturally instilled upon a gender to take care of their elders. The reading of mathematics word problems is similar in that there are biases and cultural references that we must be sensitive to as
not to detract from the purpose of the problem nor cast stereotypes or gender roles upon the readers. The South African example illustrates that written problems may require many levels of language in the mathematics register, discourse, and prior knowledge to create accessibility to understanding the problem. Often, there is an expectation at the university level that students have previously developed a minimum level of discourse in a variety of registers. The use of unfamiliar and inaccessible language makes the real-world problems often disengaging for the students, and in Le Roux’s study (2008), the barrier created by the use of inaccessible language indicated that students were less likely to move into fields where they would use mathematics. In the analysis of the real world problem Le Roux (2008) also notes that often students were asked to transfer their results to “practical terms” or to ‘everyday language’ in the mathematics register. This process of transference might limit students from seeing the relevance in adopting the academic mathematical language, which will in turn limit their access into mathematics as a discipline itself (Le Roux, 2008).

Mathematics culture is based on human interaction (Le Roux, 2008; Moschkovich, 2010). The way in which we communicate or interact with one another while learning mathematics affects our conceptual understanding of mathematics (Moschkovich, 2010). Students and teachers interact each day in the classroom and this allows for the development of oral and written language skills.

…reform curricula portray mathematics as a human activity with applications to real life. Learning mathematics involves collaborating, communicating and participating in mathematics practices such as modeling, making conjectures, and proving.

(Le Roux, 2008, p. 307)
In Le Roux’s (2008) study she illustrates how word problems have to be relevant and accessible to the readers via level of literacy. Literacy is considered as both the type of language used within a field or subject area and as a level of vocabulary used within written works. Le Roux’s example above can also be used to indicate that word problems need to be free of cultural bias and conform to the local socio-cultural practices. Many South African communities act in a collaborative nature and Le Roux’s (2008) examination of real-world problems might suggest that providing problems that utilize students ability to collaborate with each other would benefit the students’ culture, creating a discourse community, where the mathematics is being taught as well as to develop stronger and deeper conceptual connections between mathematics and other fields of study. Learning mathematics, as Le Roux mentions, involves collaboration but often the learning and work is performed independently at the senior secondary and university levels. Le Roux (2008) suggests making a change toward the problems becoming more collaborative in nature to improve the learning process as well as to create more accessibility to the problem sets through collaboration and discussion (Le Roux, 2008).

Within a given field of study, students are often asked to know, or develop capability with the mathematical language and discourse practices throughout the duration of the course. In classrooms, there exist three distinct types of discourse alongside the mathematics register: everyday, classroom, academic/professional (Gutierrez et al., 2010; Sfard, & Cole, 2003). The ‘classroom language’ using the mathematics register is a hybrid between students’ everyday language and the language they use in academic settings such as in the high level math classes or their post-secondary classes (Gutierrez et al., 2010; Sfard, & Cole, 2003). ‘Academic language’ refers to the language used that is specific to a field such as mathematics, where professional language for a teacher might focus on learning strategies and mathematical process vocabulary
Teachers and other professionals will demonstrate the professional and academic language (and discourse) and have the ability to have students discuss the concepts and work using ‘everyday language’ (and discourse) which provide a simpler and more concise ways to express ideas in ‘academic language’ in the mathematics register (Gutierrez et al., 2010; Moschkovich, 2010). Moschkovich (2002) noted, in her case study of a tutor and student, the student needed to develop the practices and procedures within mathematical discourse to be able to successfully use and demonstrate understanding of the taught mathematical concepts that she had learned in her summer mathematics program. The student’s familiarity with the mathematics register gave her the ability to express her work and ideas within her program as well as the ability to ask pertinent questions for what she was learning (Moschkovich, 2002). The tutor shared his expertise with the student and taught the mathematical discourse through social interaction by talking through the problems as he worked through them (Moschkovich, 2002).

When learning mathematics a number of linguistic challenges face the learner. In Schleppegrell’s (2007) synthesis of research articles, she found a number of linguistics challenges to learning the range of language in the mathematical register (everyday, classroom, and academic/professional). Schleppegrell (2007) argues that there are two main features that impact our learning of registers: multiple semiotic systems, and grammatical patterns. Multiple semiotic systems include four main sub features: mathematics symbol notation, oral language, written language, graphs and visual displays. For example, the equation

$$3x - 4 = 8$$

must be understood in symbol, written, and oral language for students to be able to interpret and understand that the question is asking to solve for the value of ‘x’; that question is asking for
students to determine the value that is tripled, and less four that equals a value of eight. Multiple semiotic systems require students to be thinking within multiple representations of the same mathematical sentence at once. Grammatical patterns include five sub features: technical vocabulary, dense noun phrases, being and having verbs, conjunctions with technical meanings, and implicit logical relationships. For example, the following question is from the Nelson Functions 11 textbook used in Ontario,

For a fundraising event, a local charity organization expects to receive $15 000 from corporate sponsorship, plus $30 from each person who attends the event. a) Use function notation to express the total income from the event as a function of the number of people who attend. b) Suggest a reasonable domain and range for the function in part (a). Explain your reasoning. c) The organizers want to know how many tickets they need to sell to reach their fundraising goal. Create a function to express the number of people as a function of expected income. State the domain of this new function.

(Kirkpatrick, et al., 2008)

This question is full of technical vocabulary such as ‘function notation’, ‘domain and range’, and ‘function of expected income’. There are dense noun phrases ‘local charity organization expects to receive’ and ‘each person who attends’. Answering each part of the question requires having and being verbs. The use of “plus” is a conjunction with technical meaning. Implicit logical reasoning exists when part c of the question says, “The organizers want to know how many tickets they need to sell to reach their fundraising goal,” the goal of the problem was implied but not stated. The student then has to use implicit logical reasoning to determine the fundraising goal to which the question is referring.
Schleppegrell (2007) further states that teachers must focus on these features of the mathematics register to engage and support student learning. The explicit teaching of these features of the mathematical register will allow students to construct knowledge and make deeper connections to the terminology that they are being asked to learn and use within the field of mathematics (Schleppegrell, 2007). These features are learned when students talk to one another about mathematics problems (Schleppegrell, 2007). Multiple semiotic systems and grammatical patterns are features that connect mathematics to the language that it is learned in, and studies have shown that English language learners do score lower on word problems than fluent language learners (Schleppegrell, 2007).

In Sherin’s (2002) study she observed a teacher in his classroom throughout the course of a year. Sherin (2002) found that the teacher struggled to balance the creation of a discourse community while still ensuring the integrity of the course curriculum. In the study, students were given learning activities or problems to discuss and create arguments. Through these tasks students were able to interact with their peers and the teacher. The discussion and interaction produced through group and paired activities, allowed for students to see a variety of problem solving strategies, deepening students understanding and learning (Sherin, 2002; Cobb, 1996). The teacher in Sherin’s (2002) study struggled with creating a balance of time between creating a discourse community within the classroom and completing the curriculum expectations of the course. This teacher’s concern about completing the curriculum are important, however, further research in the field reveals that the development of discourse communities in mathematics classrooms do improve students understanding of mathematical concepts (Sleep & Eskelson, 2012; Moschkovich, 2004; 2010; Cobb, 1996) and hence increase the ability to complete a curriculum.
It was also noted in Sherin’s (2002) study, as the year progressed, a shift in teacher centered discourse occurred. During observations later in the year, there was more focus on mathematical concepts and less teacher participation to guide the discourse (Sherin, 2002; Groves & Doig, 2004). This shift demonstrates a learning curve for the students and teacher, about learning and participating as a member of a mathematical discourse community. The focus in the discourse community at the end of the year was on the deeper meaning behind the mathematics whereas the focus in the discussions at the beginning of the year was on process and behaviours based with the teacher as a central guiding figure. By the end of the year-long study the students were more independent and collaborative in their classroom discourse (Sherin, 2002).

In Khisty and Chval’s (2002) study on pedagogical discourse, the classrooms of two distinctly different teachers were examined. Two teachers taught bilingual students (Spanish and English), however one teacher was bilingual the other was not. Both teachers used a predominantly oral mode as their delivery mechanism for activities and reflection of learning in their mathematics classrooms. The bilingual nature of the students and the teacher is important as it illustrated the bilingual teacher’s ability to create accessibility to the mathematics through her students’ first language and create age appropriate discourse in two instructional languages. The first teacher, Ms. M, introduced vocabulary and scaffolded previously learned vocabulary and concepts within the students’ first language. Ms. M used oral language and visual representations as a means to renegotiate meanings and connections to the mathematical concepts in the activities. Ms. M accessed students’ prior knowledge in their first language and utilized mathematics register across two languages, which created accessibility for her students. The second teacher, Ms. T, delivered conceptual knowledge through narrative activities (i.e. acting
The two teachers had striking differences in their students’ achievement levels. Ms. M’s students made deeper grade level connections to concepts through Ms. M’s constant effort and emphasis on correct language use in both the language of instruction and students’ first language. The results of the study confirmed that students are able to develop their own meaning and connections to the work while having a means to access teacher feedback through continuous participation in the mathematical discourse when activities embed discourse, and promote social interaction and collaboration (Khisty & Chval, 2002).

The use of language by a teacher indicates its level of importance in a student’s education (Wachira et al., 2013; Khisty & Chval, 2002). Bandura’s (1963) social cognitive theory tells us that children learn by watching and doing; if students are exposed to teachers using academic vocabulary and mathematical process language, then they too will utilize this language. In Khisty and Chval’s (2002) study two teachers’ instructional strategies attempted to link discourse to problem solving. The teacher [Ms. M] that embedded academic language and connected mathematical concepts into the other subject areas and first languages (mainly Spanish) created a level of importance for the students to use mathematical language in each learning activity. Prioritizing mathematical language use by the teacher, encouraged students to use the same terminology to communicate with the teacher and other students, which created a common language (Khisty & Chval, 2002). In Wachira, Pourdavood, and Skitzki’s (2013) study of one teacher that engaged mathematics students through the promotion of mathematical language and the use of discourse communities, it was established that students developed stronger mathematics conceptual understanding, better use of mathematics vocabulary, were able to ask more meaningful questions, and shifted their perception of learning mathematics from a negative to positive disposition. This study used four explicit teaching strategies to develop a mathematics
discourse community: establishing expectations, mathematics language, mathematics community, and establishing formal discourse. The teacher prioritized the fourth strategy and students knew that their end goal of the course was to use formal discourse within the classroom (Wachira et al., 2013).

Students that form partnerships with other students offer useful strategies or explanations that help them make connections and make deeper meaning out of the content (Swain et al., 2010; Groves & Doig, 2004; Kieran, 2002; Cobb, Boufi, McClain, & Whiteneck, 1997; Cobb, 1996). Kieran (2002) researched the effect of partnered discourse in partnered and individual problem solving. She used same sex partners at the age of thirteen after they had completed a seven-week course on pre-algebra. The partners were given a problem set to complete together, afterward were given a problem set with similar problems to complete individually. It was noted that four of the six pairs did not have mutually beneficial partnerships. Mutually beneficial partnerships refer to pairs of students that worked together to solve the problems. Partners listened to one another and shared ideas equally. In the non-mutually beneficial partnerships there was a pattern in the discourse of the ‘strong’ student making a decision about the solution prior to being to work with their partner, while the ‘weaker’ student was finishing to read and understand the problem. The students that were stronger at problem solving in each partnership explained their steps as self-justifications and self-talk instead of maintaining a discourse with their partner. In the two groups that did have mutually beneficial partnerships the students confirmed that they both understood the problem in the same way and spent more time working to create a mutual understanding of justifications of each step they took toward the solution before continuing. The students that formed the mutually beneficial partnerships worked together to develop a solution in both oral and written formats. After their partnered work, the
same students were asked to complete a similar question set. Each of the students that were in the mutually beneficial partnerships completed their problem sets with a similar level of success to that of their partner. The students that were in non-mutually beneficial partnerships (i.e. students that did not participate in shared mathematical discourse) did not complete the individual problem set in a similar level of success to that of their partner (one performed notably higher than the other). The experience of using the mathematical discourse deepens student understanding even within a partnership of like ability (Kieran, 2002). It also was noted that if the partners had too great a gap in their ability ranges and became non-mutually beneficial partners they did not share the mathematical discourse and the stronger student did the work to complete the task (Kieran, 2002).

Progressive use of mathematical discourse in the classroom setting often enables students to clarify meanings and uses of vocabulary, symbols, and meanings themselves which in turn affect students’ conceptual understanding positively (Groves & Doig, 2004; Kieran, 2002; Cobb, et al., 1997; Cobb, 1996). From the research presented above, we can see that the use of mathematical discourse in the classroom enables students to make stronger connections to conceptual understanding (Groves & Doig, 2004; Kieran, 2002; Cobb, et al., 1997; Cobb, 1996). Research has suggested that students’ rich use of oral discourse facilitates the development of written discourse in a number of content areas including that of mathematics (Moschkovich, 2010). Students with strong oral discourse are often found to possess strong written discourses. From Moschkovich’s (2004) work on situational mathematical discourse it was seen that when a student understands the expected mathematical discourse, they are able to understand problems posed, as well as present stronger solutions to those problems. Kieran’s (2002) and Huang et al. (2005) work suggests that discourse in small group settings promotes stronger problem solving
ability. There exist gaps in research that would better connect the classroom use of mathematical discourse communities to the level of ability of students’ independent problem solving skills.

Mathematical discourse communities can be defined as communities in which “students are willing to engage in investigation and discourse in the creation of an atmosphere of trust and mutual respect” (Silver & Smith, 1996, pg. 22). From this definition it is evident that not all classrooms maintain mathematical discourse communities, as they might not have cultivated the classroom norms and social practices required. The work of Kieran (2002) and Huang et al. (2005) revealed that there were mathematical discourse communities within the classrooms of study, but they were not the focus of the research at the time. From the literature, we have seen that the students that engage in oral discourse with peers present stronger solutions to problems (Kieran, 2002). The work of Vygotsky on the Zone of Proximal Development highlights that students learn, in general, by working a little ahead of their own level and teaching a little behind their own level of understanding (Vygotsky, 1978; Kozulin et al, 2003; Swain, et al., 2010). The ZPD can be applied to the use of rich oral mathematical language in the classroom between students and teachers. The gap in the literature this research aims to fill is to demonstrate the effect of the teacher’s use of oral mathematical language in the classroom on students’ written work specifically when problem solving. There appears to be a lack of research that directly connects the rigor of the teacher’s mathematical language use to the students’ language use and skills in problem solving. Further research is required to determine the effects of group mathematical discourse during problem solving on students’ individual ability to problem solve. If exposed to oral and written discourse within a mathematical discourse community, students will have seen strategies and tool selection methods different from their own, which might broaden their range of problem solving tools and overall skill set (Moschkovich, 2004; 2010).
Mathematics could be taught as an applied language as it requires learning oral, written, and social practices of the field. The first step in learning all languages is to begin to comprehend it orally. As we learn oral content we question and are questioned and this begins our ability to negotiate meaning within our learned language. Many studies demonstrate the importance of students’ ability to communicate in mathematics in written formats as it allows them to communicate with others. First we must learn to communicate orally before we are able to communicate in writing. Research has shown that using writing strategies to reorganize student thinking positively impacts students understanding of mathematics concepts and improves achievement levels. Writing can also be used to develop discourse communities within mathematics classrooms. Writing-to-learn programs are most impactful when students get timely feedback with specific targets to develop. Learning a language is similar to learning a cultural way of life. Culture impacts our understanding of language through value, exposure, accessibility, and interaction. To understand the culture of mathematics allows for the development of understanding behind the social practices of the language of mathematics.

The language of mathematics utilizes levels of the mathematics register; everyday language, classroom language, and academic/professional language. When learning the language of mathematics, students move across the mathematics register acquiring higher levels of mathematics language through the development of understanding of social practices in mathematics culture. The linguistic challenges of learning mathematics can be overcome through student interactions with peers and experts.

The development of mathematical discourse communities within the classroom allow for students to engage in deeper understanding of mathematics concepts. A discourse community creates a shift away from teacher centered learning to student centered learning that moves
toward collaborative learning environments that foster independent learning. Teacher priority on
the learning of mathematical language impacts students’ level of importance of language learning
in mathematics. The development of mutually beneficial student partnerships allows for deeper
understanding of mathematical concepts, strategies, and language through the development of
mutual understanding of problems and justifications. Student collaboration leads to improved
student written work.
Chapter 3

Methodology

This chapter describes the method and procedures used to answer this study’s research questions. The research methodology, which is outlined, provides the rationale for using a constructivist grounded theory framework with a particular case. The description of this study’s procedure explains the process of participant selection, data collection, and data analysis. Trustworthiness is also discussed through a thick description of the actions that occurred during data collection and analysis. Trustworthiness is important to this research as it develops confidence in the researcher and the results of the study.

Research Methodology

The methodology for this study used constructivist grounded theory (Charmaz, 2012), which incorporates principles of grounded theory (Strauss & Corbin, 1990) and social constructivism (Cobb, 1996; Charmaz, 2012). Grounded theory and social constructivism are both strongly rooted in theoretical pragmatism. This means that the theory developed from research within these two frameworks are intended to improve practice within the field research is conducted. To clarify, the research conducted under these frameworks was intended to improve practice in a meaningful way with results that educators can see in their daily practice. Grounded theory grounds researchers’ work in data and involves the development of theory as part of the process. It is also imperative that the generated theory be both practical and applicable to the context of the research. Grounded theory was defined as the “discovery of theory from data” (Glaser & Strauss, 1967, p. 1). In Strauss and Corbin’s (1990) articulation of Grounded theory they defined three key elements to their theory: 1) theoretical sensitive coding, 2)
theoretical sampling, and 3) constant comparison. However, this definition has been more recently defined as a systematic methodology that involves the discovery of theory through the careful analysis of data (Corbin & Strauss, 1990; 1997; 1998; 2007; Glaser, 1978; 1998; McMillan & Schumaker, 2010; Oktay, 2012; Patton 2002; Strauss & Corbin, 1990). Grounded theory was guided by a central question “What theory emerges from systematic comparative analysis and is grounded in fieldwork so as to explain what has been and is observed?” (Patton, 2002, p. 133).

Grounded theory has continued to evolve as information, technology, and society has changed (Oktay, 2012; Strauss & Corbin, 1990). For example, grounded theory allowed for a structured approach to observe processes that were occurring within a mathematic classroom to determine if there existed connections between how students and the teacher interact with one another, while also considering the impact of those interactions on students’ written work when problem solving. Grounded theory provided flexibility in the research process, so that the collection of data, interview questions, and analysis, could be adapted as new themes emerged after the first data collection point (Strauss & Corbin, 1990). Using Grounded theory enables researchers to look at both the local and global view of the effects of discourse (e.g., a mathematical discourse) in relation to student written work through problem solving (Strauss & Corbin, 1990). The local view of the effects of mathematical discourse could later be defined based on how the use of discourse affects the students’ ability to gain deeper understanding of mathematical concepts within the class at hand. The global view of the effects of mathematical discourse were defined as how the use of discourse impacts long term understanding of mathematical concepts and students written work when problem solving. Grounded theory permitted the researcher the opportunity to look at the local or immediate impact of a study (i.e.
how did the use of discourse impact students written work when problem solving in class today?). Meanwhile, it gave the researcher the flexibility to adapt indicators in order to give clues to the long term effects of the use of the discourse community on students communication skills in written work when problem solving.

The flexibility of the research process using a grounded theory approach was complimented by the adaptable nature of social constructivism. Of constructivist grounded theory, Charmaz (2000, p. 510) says:

…first-hand knowledge of empirical worlds, takes middle ground between postmodern and positivism, and offers accessible methods for taking qualitative research into the 21st century. Constructivism assumes the relativism of multiple social realities, recognizes the mutual creation of knowledge by the viewer and the viewed, and aims toward interpretive understanding of subjects’ meanings.

In this interpretation, Charmaz (2000) defined constructivism as social constructivism. She believed that social constructivism offerd an opportunity to better understand the interplay between knowledge and social constructs, while still insisting on observing and recording these interactions as a means to better understand how and why these constructs existed, and how these interactions best aided in the development of these constructs.

Social constructivism (Cobb, 1996) evolved over the last forty years such that it was methodologically similar to grounded theory. Constructivism (Von Glasersfeld, 1984) in its original conception would not have been appropriate to this research, as it would not allow the researcher to have observed and analyzed simultaneously. Social constructivism allowed for adaptation to observations, and interview results, which means that the researcher had the ability to ‘re-focus’ interview questions after data collection much like that in grounded theory.
Constructivist grounded theory incorporates the methodology and structure of Strauss and Corbin’s (1990) methodology while also allowing for flexibility and practicality.

A particular case was chosen for this research as it allowed the researcher to examine a teacher and her students in her naturalistic classroom environment. The research question itself was developed from questions that were inspired from the researcher’s own classroom and were further driven by observations that occurred throughout her own experiences as a teacher. The research questions were inspired by personal classroom observations and as such seemed most appropriate to be answered from careful observations of oral transactions between a teacher and their students. The main purpose of this research was to examine how oral language from the classroom teacher transfers to students’ oral language in class and in their written work. To examine this transfer, a particular case is most appropriate because the teacher’s and students’ oral language use can be analyzed with the mathematical registers in mind, a journey of learning can be traced through classroom transcripts, and written evidence can be collected to articulate student use of learned mathematical language. Focusing on a particular case allowed the researcher to investigate nuances of language transfer within the classroom community.

**Procedure**

This research was based on observations and qualitative data coming from the social interaction that occurred during regularly scheduled classroom activities and the school day. There were four sets of data collected: 1) teacher interviews, 2) classroom observations, 3) students’ work, and 4) student interviews. All names of students, teachers, educational assistance, and administrators’ were changed during transcription. All recorded material will be destroyed five years after the research has been completed. The following section will detail participant selection, data collection, data analysis, and trustworthiness.
Participant Selection

Participants included one grade nine mathematics teacher and her grade nine academic mathematics students (N=16). The teacher was purposefully sampled as she was known to the researcher through her long history of involvement with the Ontario Association of Mathematics Educators (OAME) demonstrating an involvement in professional development and continual learning for herself and her students. The classroom teacher has an engineering degree and a bachelor of education degree. She has twenty years of experience teaching secondary school mathematics. The teacher has also participated in a variety of professional development programs within her school board and province. All students in her class were invited to participate in the research cycle.

Data Collection

The data collection was completed in mid spring of 2014. There were four main sources of data collected: the teacher interviews, classroom observations, student work, and the student interviews.

Teacher Interviews

The teacher participated in two interviews that were audio-recorded and transcribed. Semi structured interviews were approximately forty to sixty minutes in length. Interviews were conducted with the teacher two weeks before observations began and again directly after all observations were completed. These interviews aimed to explore how the teacher perceived the impact of her language use in her class. The teacher was asked her opinion of the importance of mathematical discourse at each grade level and pathway (i.e. academic, applied, locally developed, or university or college preparation). The teacher was asked her education level, mathematical background, interest in professional development, and hopes from participation in
the research. The teacher was asked a number of questions about her beliefs in the relationship between mathematics and language, the purpose of problem solving, the use of problem solving within the classroom, and her opinion of how and what is mathematical literacy. In the final interview the teacher was asked about what students learned and how she thought this learning might impact the teacher’s instructional strategies and the students’ learning strategies.

**Classroom Observations**

The audio recording captured the oral discourse that occurred in the classroom to enable the researcher to transcribe the conversations that occurred during the teacher’s lessons. The researcher observed one grade nine academic mathematics class for seventy-six minutes for three consecutive days. The lessons were taught on the topic of linear relationships with a focus on linearity and analytic linear relations. The researcher took field notes as well as placed audio recording devices around the room. Two video cameras were running throughout the class periods to ensure that all participation could be heard and appropriate individuals identified. The classroom observations recorded the teacher’s lessons, questions, and details of the mathematical register used. Students’ voices were audio recorded to capture the flow of the group conversations and group problem solving that emerged during the learning activities. The observations also recorded the student and teacher transactions as a whole group, and student’s peer conversations when working in small groups.

**Student Work**

Documentation of students’ problem solving was collected a few days before and after the classroom observations. The documentation collected were the solutions of two mathematical problems, prepared by the researcher, to compliment the lessons before and after the observation period. Both problems covered curriculum content but varied slightly in their conceptual
complexity. The problem given before the observations was labeled Task One, and the problem given after the classroom observations was labeled Task Two. The word problems used had supporting questions to prompt the students’ language use from their previous lessons. The prompts were focused on probing students’ language use and vocabulary knowledge. This allowed a comparison to be made between how the same student used mathematical language when problem solving prior to the introduction of new mathematical language by the teacher and following exposure to the teacher’s language use during lessons for several days. Task One and Task Two were collected by the researcher and student solutions were copied. The names of the teacher and students removed prior to copying. All anonymous hard copies were digitalized for further analysis.

**Student Interviews**

Nine students from class were selected and agreed to participate in a reflective interview each lasting approximately fifteen to twenty minutes. The students were selected upon the teacher’s recommendation for their consistent achievement at a particular performance level throughout the course. Interviewed students represented achievement levels two, three, and four. Students were asked questions regarding their beliefs in learning mathematics, the importance of mathematical language, and their understanding of how important mathematical language was to their teacher. Students were given time to reflect on their submitted Task One. These questions intended to establish their understanding of teacher expectations, opinion on language learning within mathematics, and understanding of language development within mathematics. The interviews were audio-recorded and transcribed.
Data Analysis

Upon completion of observations and interviews, audio recordings were transcribed. The video was used to ensure accuracy of what was said during class time and confirm the participant speaking. The transcribed work was reviewed, coded, and categorized as in the three-step coding process (Strauss & Corbin, 1990). Concepts and themes were developed from the categorized data.

Teacher Interviews

The teacher interviews transcripts were analyzed, resulting in three themes. The interview transcripts were reviewed through open coding to find words, phrases, or passages where the teacher indicated literacy and language learning strategies, identification of student language acquisition in class and indicators of personal pedagogy of mathematics education and applied linguistics role within mathematics. Axial codes were then developed to describe the purpose each word, phrases, or passage served in the conversation. The axial codes were then developed, in context, to determine possible themes.

An example of the coding process from the teacher interviews is in Table 3-1. For example, when the teacher said “when I am approaching an area of math that I find is vocabulary driven, I will make sure that students are aware of the terms. Now, I am pretty picky about precision in mathematical vocabulary,” this was coded as “Importance – math is vocab driven” and “Importance – Precision.” These two axial codes then developed into two themes, Importance of Language and Language Acquisition Through Collaborative Problem Solving. The theme Importance of Language came from the teacher repeatedly saying that she does something in her practice to ensure students are learning mathematical language, and to become strong communicators. The theme Language Acquisition Through Collaborative Problem
Solving was coded here from the teachers' statements “When I am approaching …”, “I find…”, “I will …”, and “I am pretty picky …” that indicated that she used her strategies because she appeared to have believed that students learned mathematical language if they were explicitly taught it.

Table 3-1

<table>
<thead>
<tr>
<th>Open codes</th>
<th>Axial codes / categories</th>
<th>Selective codes / themes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phrases</strong></td>
<td><strong>Words</strong></td>
<td></td>
</tr>
<tr>
<td>when I am approaching an area of math that I find is vocabulary driven, I will make sure that students are aware of the terms. Now, I am pretty picky about precision in mathematical vocabulary</td>
<td>Math Vocab driven Aware Terms Precision</td>
<td>Importance – math is vocab driven (1) (2)</td>
</tr>
<tr>
<td>we often will go through and define terms to make sure their understanding is pretty clear about those terms and exactly what they mean before we start a unit of study encourages students to be mathematically precise</td>
<td>Define terms Clear terms Exact Encourage Precise</td>
<td>Importance – clarity (1) Importance – precision (1)</td>
</tr>
</tbody>
</table>

Note: Excerpt from Teacher Interviews

Three themes emerged from the teacher interview data; Importance of Language, Language Acquisition Through Collaborative Problem Solving, and Problem Solving Inquiry.

These themes are presented with their associated categories from axial coding.
**Importance of Language.** Figure 3-1 shows the seven categories that come from the axial coding phase that supported the theme, *Importance of Language*, which is the importance the teacher holds regarding students’ mathematical language learning.

*Figure 3-1. Demonstration of Axial Codes to Selective Code, Importance of Language, for Teacher Interviews*
Language Acquisition Through Collaborative Problem Solving. Figure 3-2 shows the axial codes that support the theme Language Acquisition Through Collaborative Problem Solving.

Figure 3-2. Demonstration of Axial Codes to Selective Code, Language Acquisition Through Collaborative Problem Solving, for Teacher Interviews
**Problem Solving Inquiry.** In Figure 3-3, six distinct categories were determined from the interviews with the teacher that support the theme of the teacher’s pedagogy in mathematics for problem solving.

*Figure 3-3. Demonstration of Axial Codes to Selective Code, Problem Solving Inquiry, for Teacher Interviews*

**Classroom Observations**

The focus on the classroom observations was on the teacher’s and the students’ mathematical conversations. Teacher and student conversations and interactions that related to mathematical topics were transcribed and analyzed. The transcriptions were read and reviewed by the researcher and compared against the audio and video to ensure that each student participant was accredited for their statements, prior to having them reviewed by the thesis supervisor. After transcription was completed, the documents were reviewed per day, and
examined for language use, discussions of mathematical language, and indicators of language acquisition. The transcripts were coded using the same process of open, axial, and selective coding to develop themes from words, phrases, or passages where students were using mathematical language, strategies, reasoning or were attempting to use mathematical language.

An example of the coding process from the classroom observations is in Table 3-2, Joe [student] stated “Partial does not pass through the origin” which included the mathematical terms partial [partial variation] and origin [the centre (0,0) of the Cartesian plane]. These terms were used by Joe when giving a description of what he understood the term and concept ‘partial variation’ to mean, after having learned it in the last class. Due to the nature of the response the phrase was categorized as “exploring use of vocabulary” and “playing with terminology,” since Joe was attempting to use the learned mathematical terminology to make it part of his vocabulary. From these two categories, “exploring use of vocabulary” and “playing with terminology”, this statement contributed to the theme *Development of a Common Language* since Joe’s larger focus was on developing his language and communication skills within mathematics rather than producing mathematical thinking around the terminology to solve a problem. Table 3-3 presents an example of coding from the second day of classroom observation.

Table 3-2

<table>
<thead>
<tr>
<th>Phrases</th>
<th>Words</th>
<th>Axial Coding/ Categories</th>
<th>Selective Coding/ Themes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partial does not pass through the origin</td>
<td>Partial</td>
<td>Exploring use of vocabulary (2)</td>
<td>Development of a Common Language (2)</td>
</tr>
<tr>
<td>(Joe pg 3)</td>
<td>Origin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y is partially on the x</td>
<td>B value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partially on the b value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Joe pg 3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y – intercept (Joe pg 3)</td>
<td>Y-intercept</td>
<td>Concept (3)</td>
<td>Questioning/response (4)</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-------------</td>
<td>-------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3) Mathematical Thinking Within Context</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4) Questioning and Directing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slope of a line (Vanessa pg 4)</th>
<th>Slope</th>
<th>No understanding, asking for help (3) (1)</th>
<th>(1) Student Driven Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>As more years go by, the value of the house increases (Steve pg 4)</td>
<td>Trend/data trend</td>
<td>Explanation (3)</td>
<td>Activating prior knowledge (1)</td>
</tr>
</tbody>
</table>

**Note: Excerpt from Day 1**

<table>
<thead>
<tr>
<th>Table 3-3</th>
<th>Axial Coding/ Categories</th>
<th>Selective Coding/ Themes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phrases</td>
<td>Words</td>
<td></td>
</tr>
<tr>
<td>the place describing what the [finger pointing vertically]…. the 2 and the 3, and that’s the difference in distance…. Then we look at the difference between the 1 and the 2 which is the difference in time [teacher had labeled the x axis as time and the y axis as distance], and then that gives us a slope right there and it works out to be the slope because it is the differences between the rise and the run that is the slope. (jenn pg 3)</td>
<td>difference in distance</td>
<td>Definition (1)</td>
</tr>
<tr>
<td>rise over run</td>
<td></td>
<td>(1) Student Driven Learning</td>
</tr>
<tr>
<td>slope</td>
<td>Rehearsal (1)</td>
<td>Review (1)</td>
</tr>
<tr>
<td>slope equals rise divided by run.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(jenn pg 3) mathematical equations? (Ryan pg 3) | | Question/Response (4) | Misconceptions (3) | Questioning and Directing |
Five themes emerged from the classroom observation data; Student Driven Learning, Development of a Common Language, Mathematical Thinking Within Context, Questioning and Directing, and Nature of Classroom Discourse. These themes are presented now.
**Student Driven Learning.** In Figure 3-4, six distinct categories were determined from the axial coding phase. These categories demonstrated the theme *Student Driven Learning* through student statements and actions that reflected that students were attempting to gain an understanding of mathematical terminology or concepts in class.

*Figure 3-4. Demonstration of Axial Codes to Selective Code, Student Driven Learning, for Classroom Observations*
**Development of a Common Language.** In Figure 3-5, all categories that suggested similar types of actions or student responses were selectively coded to the theme of *Development of a Common Language.*

*Figure 3-5. Demonstration of Axial Codes to Selective Code, Development of a Common Language, for Classroom Observations*
**Mathematical Thinking Within Context.** In Figure 3-6, all categories with similar types of student responses were selectively coded to the theme of *Mathematical Thinking Within Context.* The theme was developed from the five categories that emerged during axial coding.

*Figure 3-6. Demonstration of Axial Codes to Selective Code, Mathematical Thinking, for Classroom Observations*
**Questioning and Directing.** In Figure 3-7, five distinct categories were determined from axial coding. These categories contributed to the theme *Questioning and Directing* through student statements that reflected that teacher had directed the learning of the classroom and students were responding to the teacher in order to understand mathematical terminology or concepts in class.

*Figure 3-7. Demonstration of Axial Codes to Selective Code, Questioning and Directing, for Classroom Observations*
The Nature of Classroom Discourse. In Figure 3-8, five distinct categories were determined from axial coding. These categories contributed to the theme Nature of Classroom Discourse through student statements and willingness to collaborate.

Figure 3-8. Demonstration of Axial Codes to Selective Code, Nature of Classroom Discourse, for Classroom Observations

After coding all five themes from the classroom observations, the frequency of each theme and the combination of themes during the three days of observation was determined. This was to see if one theme or combination of themes played a more dominant role in student language acquisition than the other themes. Student frequency of speech was recorded in a frequency chart. Whether a complete statement was one word or three sentences, it received a tally. The students’ and teacher’s frequency of speech throughout class was recorded, as well as, their frequency of asking questions, and frequency of one and two word statements (interactions).
**Student Work**

Student written work was also collected before and after the classroom observations. Student work was analyzed in two ways; improvement of mathematics register use, and improvement of frequency of mathematical language use. A rubric was developed based on the work of Schleppegrell (2007) to explore the type of language used by students before and after the three-day observation period where students learned course material and the mathematical language that accompanied it (see Appendix A). The rubric levels were developed from the conceptualization of language use as it pertained to the mathematical register and its various ways of being expressed, simplified everyday, everyday, classroom, and academic. Simplified everyday language has been added to the continuum as it accounts for English language that was used by students who might not be fluent in English (i.e. students with below grade level literacy skills or that are new to the instructional language, etc.) and a further simplifying of the language so students understand the flow of instruction. Appendix A shows the rubric used to assess student work. *Technical vocabulary* was defined as mathematical language in the classroom or academic register that described a mathematical concept or that had a unique meaning in the context of mathematics. A *dense noun phrase* was a phrase in student writing that contained a noun with more than one determiner. A determiner comes before the adjective and noun in a noun phrase (e.g. Half of the people, ‘Half of the’ is the determiner). *Conjunctions with technical meanings* were sentences that used ‘and’, ‘but’, ‘while’, ‘although’, and ‘because’ to connect a claim with an explanation. *Implicit logical relations* were the use of one of the five strategies to demonstrate student reasoning and decision making: *addition and replacement, comparison and contract, exemplification and restatement, cause and condition, and time and place* (“Clearer Writing”, n.d.).
Level 1 was defined as the use of simplified everyday language to describe student thinking or decision making within the task. A Level 1 assessment would mean that a student would not have used *technical vocabulary*, no *dense noun phrases*, no use of *having and being verbs*, no *conjunctions with technical meaning*, and no *implicit logical relations*. An assessment of Level 2, 3, and 4 would occur as ranging from having some use of *technical vocabulary* to using appropriate everyday language (Level 2), then a mix of everyday and academic language (Level 3), to then regularly using academic language (Level 4). Students were assessed as Level 2, 3, and 4 as their written work demonstrated sparse to *dense noun phrases*, some to many *being and having verbs*, inappropriate to consistent and accurate *conjunctions with technical meaning*, and inappropriate to consistent and accurate *implicit logical relations*. The before and after observation written tasks were compared for each student to determine a change in level of mathematical language use in written work over the course of one week of instruction.

The written student documentation, Task One and Task Two, was collected and analyzed through comparisons of each students’ work before and after the teacher had taught lessons on the material involved in the sample problems. The collected student work was discussed with the teacher; however, the teacher did not use the student work for feedback or assessment to ensure that opinions of the students’ work did not impact the research process. These discussions were noted, but not recorded, as they were informal discussions at the end of each observed period (or class). The goal for these conversations was to discuss student thinking and language use.

Students’ change in assessment from Task One to Task Two was recorded in an effort to determine the frequency of level change among students. Students’ frequency of mathematical language use for classroom and academic registers were recorded for both Task One and Task Two; the change in frequency was recorded for each student. The two measures from the
students’ submitted written work, the change in the level rubric score and the change in frequency of classroom and academic mathematical language used, were compared to frequency of student speech during the three days in the observed classroom. This comparison was designed to determine a connection between frequency of speech in class and improvement in use of mathematical language after a week of instruction.

Task One and Task Two were compared to the collected transcripts from the classroom observations. Each artifact was examined to determine if there was evidence that students’ incorporated learned mathematical language from the teacher’s oral discourse into their written work through problem solving. The data was recorded in tables that listed students and frequency of speech, improvement of frequency of mathematical language use, and improvement in register use. Student improvement levels in their written work were compared to the quantity of which they spoke. Rankings of each category were created and compared to determine if students that spoke frequently also experienced the most improvement in their written tasks.

**Student Interviews**

The interviews were individually coded for themes. The coding for the student interviews was again a three-step process of open, axial, and selective coding to develop three main themes. Responses were selected, phrases and words were extracted, and categories were placed on the words and phrases. For example, when Martin said “I think its important because if you use slang for mathematics … sometimes it can cause confusion” he explained in his own words why he believed learning the proper terminology was important to him. This was categorized as ‘Important for clarity when speaking with others’ which was then listed with all other categories that indicated students’ felt that learning mathematical language was important to them individually. Categories that related to each other were grouped and common themes developed.
See Table 3-4 for an example of the theme *Importance of Language from the Student Perspective*.

<table>
<thead>
<tr>
<th>Open</th>
<th>Axial Codes/ Categories</th>
<th>Selective Codes/ Themes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phrases</td>
<td>Words</td>
<td></td>
</tr>
<tr>
<td>the y intercept</td>
<td>y intercept</td>
<td>Connection to theoretical language (2)</td>
</tr>
<tr>
<td>Compare</td>
<td></td>
<td>Analysis of Problem (2)</td>
</tr>
<tr>
<td>This is the type of analysis that you do ...like just recently I got a phone plan</td>
<td>analysis</td>
<td>Personal connection (1)</td>
</tr>
<tr>
<td>I think it's important because... if you use slang for mathematics or uses non official vocabulary... some times it can cause confusion or ... if you always use the academic terms...it can also used in your head for you and other people, its just like learning it the right way so that you don't have to go back and change your mindset, like if a teacher tells you this is how you learn it.</td>
<td>Important for clarity when speaking with others (1)</td>
<td>Clarifies if you understand concepts (1)</td>
</tr>
<tr>
<td>like showing all your work, and writing like a final statement as well, ............... you kind of put it into a chronological order and finish with the statement t</td>
<td>Showing work Final statement Chronologic al order</td>
<td>Process can be clarified through understanding and use of terminology (1)</td>
</tr>
<tr>
<td>I think it’d be important just so that you are not solving when you should be evaluating, or so that everything is clear</td>
<td>Solving vs evaluating Wrong</td>
<td>Importance because vocabulary</td>
</tr>
</tbody>
</table>

55
Note: Excerpt from Student Interviews

The student interviews contributed three themes: *Importance of Language From the Students Perspective, Students Engaged in Language Acquisition, and Untangling Expectations.*

The *Importance of Language From the Students Perspective* demonstrated that students believed that learning mathematical language was important for their own understanding of mathematics.

The theme *Students Engaged in Language Acquisition* was evident when students would use mathematical language incorrectly during the student interviews, self-correct their mathematical language, or discuss how other students and themselves acclimate mathematical language in the classroom setting. The theme *Untangling Expectations* was evident when students explained how their reasons for learning mathematical language were different from how they understood their teacher’s reasons for the students to be learning mathematical language.
**Importance of Language From the Student Perspective.** Five distinct categories, which were understanding terminology deepens conceptual knowledge, connections to real world scenarios, clarifies work to self and peers, creates clear mathematical communicators (succinct), and can fully describe your thinking and process to others were grouped to reflect *Importance of Language from Student Perspective*. See Figure 3-9 to view how the theme *Importance of Language from the Student Perspective* was supported by these five categories.

![Diagram showing the relationships between the five categories of language importance.](image)

*Figure 3-9. Demonstration of Axial Codes to Selective Code, Student Importance of Learning Mathematical Language, for Student Interviews*
**Students Engaged in Language Acquisition.** In Figure 3-10, five distinct categories were determined from axial coding. These categories contributed to the theme *Students Engaged in Language Acquisition* through student statements that reflected student observation of language acquisition form teacher to student or between students.

*Figure 3-10. Demonstration of Axial Codes to Selective Code, Students Engaged in Language Acquisition, for Student Interviews*
**Untangling Expectations.** In Figure 3-11, four distinct categories were determined from axial coding. These categories contributed to the theme *Untangling Expectations* through student statements that reflected what students’ believed motivated the teacher to place emphasis or importance on developing mathematical language in class.

*Figure 3-11. Demonstration of Axial Codes to Selective Code, Untangling Expectations, for Student Interviews*

**Cross Data Analysis**

After all coding was completed and the themes were developed, themes from the different data sources were compared and contrasted. The teacher interview themes were compared to the themes developed from the classroom observations and the student interviews to look for commonalities in ways of thinking about mathematical language and its importance. This
process was seeking universal themes across data sources. From the classroom observations, students’ frequency of in-class speech was compared to their change in mathematical language use on two measures from their submitted written work, Task One and Task Two. The purpose of this comparison is to determine possible trends of students’ language acquisition from pattern in class speech frequency and written mathematical language use. The teacher’s and students’ questions during the classroom observations were noted and compared through similarities and differences then reviewed to determine their nature. The teacher’s and students’ oral register use were also compared to determine the nature of the classroom discourse. The purpose of understanding the nature of the teacher’s questions during class aimed to answer the research question itself. The student interview themes were compared to those of the teacher’s interviews. This comparison examined common understanding of mathematical language, and communication in classroom discourse.

**Trustworthiness**

Patton (2002a) said trustworthiness is promoted by “enhancing the quality and credibility of qualitative analysis” through addressing three main concerns in inquiry: 1) rigorous techniques and methods, 2) researcher credibility, and 3) philosophical belief in qualitative methods. Trustworthiness was enhanced during this study by using multiple data sources, developing quality data, and minimizing subjectivity. During the study rigorous methods were utilized to collect and analyze the data. A multi-method strategy was used to collect data from four different sources: teacher interview transcripts, classroom observation transcripts, students’ work, and student interview transcripts. Multi-method strategies enhance the quality of qualitative research by allowing “triangulation in data collection and analysis” (McMillian & Schumacher, 2006, p. 330). Detailed transcripts were used to ensure the correct interpretation of contextual statements.
during interviews and classroom observations. The researcher used questioning methods to confirm understanding when interview participants left ambiguity in responses, giving clarity and reducing bias in interpretation of meaning when transcribing the same statements. The recorded observations, and then transcripts, allowed for quotations from the teacher and students both in and out of the classroom as support for themes that were developed. Participant language allows for “verbatim accounts” (McMillian & Schumacher, 2006, p. 330), which provide accuracy in developing understanding of the participants experience within a study. Audio recording were taken to collect data for each interview and the three days of classroom observations. Mechanically recorded data allowed for the verbatim transcripts to be written, giving the ability to take contextual quotes from the interviews and observations. During the analysis all frequency counts from the classroom observations were compared to belief statements from both the teacher and student interviews, as well as compared against the submitted written work. These processes enhanced the rigor of the data collection and analysis as well as created a degree of quality of data.

Patton (2002) states:

By triangulating with multiple data sources, observers, methods, and/or theories, researchers can make substantial strides in overcoming the skepticism that greets singular methods, lone analysts, and single-perspective interpretations (p. 556).

Triangulation was achieved by comparing multiple data sources (teacher interviews to classroom observations, student interviews, and collected student written tasks), comparing the teacher’s and students’ perspectives of the importance of language learning in mathematics class, and through member checking.
Patton (2002) described researcher credibility as being “dependent on training, experience, track record, status, and presentation of self” (p. 584). During the data collection and analysis, the researcher used critical reflexivity to minimize subjectivity. Throughout the data collection, the researcher allotted time to question herself under the three reflexivity screens: participants, audience, and myself (McMillian & Schumacher, 2006, p. 331). These screens provided trust to be built between the researcher and participants so that participants did not feel judged (McMillian & Schumacher, 2006, p. 330). The researcher utilized her own experiences in learning to help clarify questions to students when conducting student interviews. McMillian & Schumacher (2006) stated “researchers use … personal experiences and abilities of engagement, balancing the analytical and creative through empathetic understanding and profound respect for participants perspectives” (p. 330).

Patton (2002) argues that philosophical belief in qualitative methods is a concern to be addressed within qualitative research. The researcher’s belief in qualitative methods was addressed prior to beginning the research through discussions with her supervisor and committee member as to the best methods of data collect and analysis for the research, and discussing the purpose behind the research for the researcher. The collaboration with these others minimized subjectivity through the data collection and analysis processes.
Chapter 4

Results

In this chapter, results will be presented from the data of teacher interviews, class observations, student work, and student interviews. Themes emerged during the teacher interviews when she discussed her class and her beliefs around teaching mathematics. The analysis of the data uncovered a link between the oral mathematical discourse of the students and their teacher, and between students’ written work during individual problem solving.

Teacher Interviews

The themes resulting from the analysis of the teacher interview data were: (1) Importance of Language, (2) Language Acquisition Through Collaborative Problem Solving, and (3) Problem Solving Inquiry. Importance of Language was defined as the teacher’s belief that language development in mathematics is facilitated by the teacher. Such language development subsequently allowed for precision in the students’ presentation of their work (to the class or their peers), granted students confidence in their work, allowed students to take ownership of the mathematical language and written work, and improved language allowed for deeper levels of collaboration among students. Language Acquisition Through Collaborative Problem Solving was the teacher’s acknowledgement that students’ explored and experimented with mathematical terminology (classroom and academic registers), by using, applying, and re-organizing terminology individually and collaboratively to allow for student built connections between prior knowledge, concepts in mathematics, and the world beyond the classroom. Language Acquisition Through Collaborative Problem Solving included the teacher’s recognition of student development of confident language use, active attempts at language use during class, and that
language development in mathematics was enriched by problem solving. The third theme, *Problem Solving Inquiry*, was the teacher’s belief that delivering the mathematics course curriculum through the process of problem solving was a means to create opportunities for conceptual discovery, building daily vocabulary from everyday language through teacher modeled language, and that language must be embedded within the context of problem solving. The teacher believed in peer collaboration that allowed student experimentation with both conceptual understanding and language development within small groups prior to large group interactions.

**Importance of Language**

*Importance of Language* was a theme that described the wide variety of reasons that the teacher explained it was important to purposefully teach mathematical language to her students. The teacher argued learning mathematical language was important because she believed that students could express themselves, and their articulation of their thoughts in the classroom facilitated a common understanding amongst their peers.

Students in mathematics to have the ability use the correct language, and to use it correctly in the correct context and understand it’s meaning. So, I find that if they are actively taught the correct terminology then it becomes part of their mathematics learning, then they use it in their writing and in their speaking and how they actually approach their math work.

(Teacher, Interview)

The teacher stated that students want to use the correct mathematical language to express themselves, understand concepts, and to understand the mathematical meanings behind academic vocabulary. The teacher believed that when students were purposefully taught mathematical
language that they will use it in both their oral and written language and this language use will change how they approach their mathematics work. From the teacher’s statement, it was seen that she believed language acquisition within mathematics impacts student understanding of mathematical concepts.

The teacher stated that by keeping mathematical language in the forefront of her teaching that students will build academic language, as well as make connections to the mathematical concepts represented by the terminology.

I always want to use the correct mathematical terminology and not water down how I am presenting vocabulary. So in my own statements to the class I try to keep the vocabulary in the forefront, using correct terms and referring to it in multiple ways [representations]. I keep going back to slope and keep repeating that it’s the rate of change of the dependent versus independent variable, I’m trying to build the concept every time that I use the word.

(Teacher, Interview)

Through the teacher’s use of appropriate academic mathematical terminology she indicated her belief that students were constructing mathematical knowledge when she gave her example of consistently going back to the first principles of slope. Repetition of appropriate language from her, and then repetition in language use from her students ensured both mathematical language acquisition and conceptual understanding.

The teacher stated that mathematical language was important within her class. She argued that language needed to be represented in a variety of ways for students to learn it.

I have always felt like language is important, and I think for me, it is stressing that putting things up in different ways in the classroom [word walls, anchor charts, etc].
Like putting stuff up and watching kids refer to it is important. Having it in front of them and not just saying it orally. Because for some kids, orally is just not enough, they have to see it visually… so having examples on the wall…or …definitions on the wall … I think is important.

(Teacher, Interview)

The teacher used visual cues within her classroom as a means to help students of a variety of learning styles so that they were able to use the visual references as reminders of vocabulary and concepts. The teacher’s efforts in creating her word walls and anchor charts signaled that she believed learning mathematical language was important to use her time in preparation for her class to develop these material as aids.

The teacher stated that she too is not an oral learner and that she also needed to learn using a variety of techniques.

I’m not necessarily an oral learner, I have to see things written out. For kids who need the visual cue to remember, I think it is important. I just have to keep reminding myself students aren’t all oral learners, they don’t just take in what they are hearing

(Teacher, Interview)

The teacher explained that she used a variety of techniques in teaching mathematical language to include students with different learning styles. She described a personal connection in her learning experience that helped to remind her that ‘students aren’t all oral learners’ and that some needed visual cues. The teacher was facilitating the development of mathematical language when she created multiple representations of the terminology and concepts.

The teacher appeared to believe that mathematical language gave students clarity and precision in what they say to each other but it also indicated importance from the teacher. When
the teacher used proper mathematical language it indicated to students that the term in use was important, as was the concept it represented.

I think that they [problem solving and mathematical language] are connected but I would have a very difficult time explaining how they are connected. Kids need the proper,… well all students….when I am learning I need the proper name. So naming something the most concise way to understand something is really important. I always think that naming something is critical in your understanding, whether it is a concept or an idea. That part of it really helps in your understanding because in a name we form a generalization of our understanding. Then we attach the understanding to that name. Sometimes vocabulary helps that as it gives you precision when you are trying to describe something. It is why we call you by your first name and not ‘Hey you tall person with the glasses, an green eyes, and a pony tail.’ We call you by your first name, because that embodies exactly who you are.

(Teacher, Interview)

From the statement “Kids need the proper… well all students….when I am learning I need the proper name”, the teacher indicated that she believed it was important for students’ conceptual understanding to understand and use the proper terminology, when learning any subject. She stated that “naming something is critical in your understanding, whether it is a concept or an idea” by this she was saying that the use of correct terminology gives precision and depth to one’s understanding of both concepts and ideas. The teacher said, “…in a name we form a generalization of our understanding. Then we attach the understanding to that name.” This meant that when the correct terminology was learned that surrounds a concept, meaning and importance is attached to the term. This leads to a common understanding of both mathematical terminology
and concepts. The teacher used an analogy to emphasize her point that a name gives precision and clarity when speaking to others. Precision in mathematics allows students to be articulate in their thoughts and process of problem solving. The teacher said she facilitated mathematical language development by connecting the proper name to the concept in a variety of ways.

The teacher discussed the importance of understanding terminology as it connected complex mathematical concepts to easily definable terms.

In grade nine kids do link those two things [problem solving and mathematical language]. Hopefully when they hear the word factoring they know that in the big scheme we are breaking things down. They may not think specifically about… you know these are the eight different factoring strategies we teach them. But they think about it in the wider concept. Again, it is helping students have a broader perspective around concepts and the individual skills that we are trying to teach them to back up those concepts. Because it is easier to be caught up in the skill and not for kids to miss the overall connection.

(Teacher, Interview)

The teacher described students’ cognitive ability to connect problem solving and mathematical language. She stated that she believed that grade nines “do link those two things” and that this was the beginning of a process of making connections to mathematical language. When the teachers stated “when they hear the word factoring they know that in the big scheme we are breaking things down,” she was hopeful that students would develop clear enduring understandings of the terminology so that when students hear the same terminology in future math classes that they will be able to connect the concept and understand that the underlying definition and processes remain the same regardless of new complexities. The teacher further
stated, “it is helping students have a broader perspective around concepts and the individual skills that we are trying to teach them to back up those concepts.” This meant that the teacher believed that students’ conceptual understanding was extremely important to the long-term understanding of mathematics and their ability to solve evolving problems. The teacher appeared to believe that students understanding of mathematical language was rooted in students enduring understanding of concepts in mathematics.

The theme of *Importance of Language* was seen through the teacher’s determination to develop students’ communication skills so that students use standardized language with one another, understand and explain solutions and mathematical thinking to one another, and to develop connections between mathematical terminology and the concepts they represent. The teacher emphasized mathematical language learning by using academic language in class, explained new terms as they arose, and dedicated time to the development of materials that aid in language learning for a variety of styles of learners. Through these actions the teacher demonstrated that mathematical language was developed from teacher questioning and directing of student learning.

**Language Acquisition Through Collaborative Problem Solving**

*Language Acquisition Through Collaborative Problem Solving* was demonstrated from teacher statements of times when she saw active language acquisition occurring within her daily class (both orally and written) and across the semester.

The teacher suggested that language acquisition was occurring and was allowing for meaning to be made from everyday language into classroom and academic language. She regarded this language acquisition as a continuum for students and saw progress across the semester and over student’s high school careers as they increase their mathematical vocabulary.
She suggested that students learn the mathematical register through its various types of language, everyday, classroom, and academic, but that the process of language acquisition will be different and unique for each student since they will all start at different locations along mathematical language learning continuum.

It [the development of oral and written language] does vary from student to student. Some students want to constantly talk about the ideas in math and then others, they want the written, they need to be able to see it written down in front of them, they aren’t necessarily comfortable just discussing problems orally, they need to write it down.

(Teacher, Interview)

The teacher stated that students come to her class with all different skill sets for communication in mathematics. When she said “Some students want to constantly talk about the ideas in math and then others, they want the written” she was indicating that students have all different communication and learning styles. She explained that when students come to class she was aware that some were more comfortable writing things down before they speak in class, while others were comfortable without wait time to collect their thoughts.

The teacher answered a question by the researcher that asked if she believed that there existed a discourse community in her grade nine mathematics class.

Well, I would have to say no in my grade nine math class, but in my grade 12 class, I would say yes. We have gotten to the stage where I feel we can talk about ideas and kids feel free to express either what they know about a topic or what they don’t know about something. I find that at the grade nine level they either don’t have the confidence or the vocabulary or the skills. So that’s where I notice the big difference
in them [students] going up the food chain in math. We try to build that and with some classes you definitely achieve that but it is a process. Sometimes you don’t realize, some of those social and collaborative skills that sometimes grade nines honestly just don’t have.

(Teacher, Interview)

The teacher explained that she did not believe that her grade nine students demonstrated characteristics of a discourse community. She described that her grade twelve class did have this skill set, which was evident by the way the students employ think aloud strategies about problems with one another. Grade nine students, the teacher explained, were still developing their confidence with the language use and stating their opinions in front of their peers. The teacher stated that the development of a discourse community was an end goal in the overall program but it was not always seen in the junior classes. She explained that the grade nine students were still developing the social skills necessary to be able to collaborate effectively. The teacher’s statement illustrated her belief in language acquisition as a process that can take years for students. When the teacher said that “We have gotten to the stage where I feel we can talk about ideas and kids feel free to express either what they know about a topic or what they don’t know about something” she was describing language acquisition as a collaborative process among peers at the senior level. She suggested that most grade nine students will develop the skill set to collaborate with their peers effectively to learn mathematical language and concepts as they mature and grow older. From the teacher’s statements regarding the grade nine class, students were learning to collaborate with each other and were learning how to build knowledge from one another’s mathematical discoveries.
*Language Acquisition Through Collaborative Problem Solving* was demonstrated by the teacher’s statement that students become accustomed to the new terminology and explore its uses in class until they use the terminology correctly each time.

Sometimes they were trying to utilize the vocabulary. They seem to have a handle on…there are some concepts that they seem to have a handle on. I think they have a handle on independent versus dependent variable.

(Teacher, Interview)

The teacher explained that in her class, it was noted that students had a deep understanding of independent and dependent variables as they used this language correctly in class on a regular basis. She stated that students were actively trying to use the mathematical language being taught in class and that she was able to track a consistent use of the terms ‘independent’ and ‘dependent variables.’

The teacher actively saw students acquire new mathematical language, apply it incorrectly, and correct themselves, demonstrating full integration of new terms into students’ vocabulary.

I find that they are [students] self-correcting as well, and I had that last week where a student said …oh I actually meant an expression but I used the word equation.

(Teacher, Interview)

The teacher stated that students were “self-correcting as well” meaning that students used mathematical terminology actively in class as well as used it incorrectly and then self-corrected. When the teacher saw this occur it signaled to her that the students understood the terminology and corrected an error using newly acquired terms.
The teacher commented on students’ need to collaborate to learn both the mathematical language and concepts. She stated her perception that students’ believed that mathematics was a topic that you learn by working alone. She argued that students must discuss mathematics together in order to learn.

They view math as an isolated process where they just need to work on it by themselves. …those [oral and written communication] are the most important skills. It [collaboration] gives them [students] an opportunity to talk and explain their thought process and to listen to someone else. Because so often we are okay with letting kids work it out on their own. Kids often struggle in silence but then other kids get the impression that mathematics is a very isolated process, ‘that really if I can’t figure it out on my own, I’m not really doing math.’

(Teacher, Interview)

The teacher believed that students within her class first express their thinking orally. The teacher stated that “often students are better orally than they are in written work” (Teacher, Interview). This statement reflected that students were able to explain their work orally better than when they were asked to explain it in writing. The teacher described the importance of discourse in student learning when she said, “gives them [students] an opportunity to talk and explain their thought process and to listen to someone else.” From this statement, it can be seen that the teacher created a safe learning environment. She stated that students need to learn to talk about mathematics, as well as, to listen to their peers understanding of mathematics. She suggested that listening to each other allowed students to learn different strategies and to learn contextual use of mathematics terminology. When students work with each other they have an opportunity to build their conceptual understanding with one another. She suggested that discussing
mathematics also gave students an opportunity to voice misconceptions regarding mathematical terminology or concepts. She stated that mathematics teachers often have students work in silence on their own as she stated “often we are okay with letting kids work it out on their own.” She continued to say that students “struggle in silence” when they were not sure of how to start a problem. She suggested that if students were given time to talk and discuss the problem before starting it, that more students would better understand the objective to the problem. She stated “kids get the impression that mathematics is a very isolated process,” which meant that when mathematics teachers had their students struggle working on their own, without any collaboration, that teacher was telling their students that mathematics was independent work and that it was to be completed on your own without the help of others.

The teacher demonstrated a belief in language acquisition when she commented on how students within her class acquire language differently from one another, actively attempt to use mathematical language, and give one another constructive feedback through peer and self-correction. These opportunities to acquire language occur during the usual collaborative problem solving experiences. She demonstrated that she believed that her students were learning language orally before they are learning to write language. From this, she suggested it was important to have students talking and listening to one another so that they can clear up misconceptions at the oral stage of learning mathematical language.

Problem Solving Inquiry

Problem Solving Inquiry was demonstrated by the teacher’s statements that indicated her core pedagogical beliefs of how mathematics was most effectively taught by her. The teacher stated that mathematics should centrally be taught through the solving of rich problems. Learning through problem solving provides opportunity for collaboration and developing
collaborative learning skills, developing and increasing mathematical vocabulary, accountability, and maintaining engagement throughout class. She suggested that problem solving is the most important mathematical process as it develops critical thinking.

My approach in the classroom is that problem solving is obviously a really important mathematical process. Having them [students] see math as a problem…. I think that the other thing is just thinking about math as problems that we can access that aren’t…[solved]

(Teacher, Interview)

The teacher explained that she teaches the grade nine curricula through problem solving. Her goal was that her students viewed the field of mathematics as a set of problems to be discovered and solved. The teacher hoped that students thought of mathematics as problems that can be accessed. When she said, “I think that the other thing is just thinking about math as problems that we can access that aren’t…[solved]” she indicated that mathematics should be approached as exploratory in nature, giving students a more natural curiosity. She argued that when mathematics was taught in this manner that students think more deeply about the mathematics that they have learned.

The teacher continued to explain that mathematics teachers often discuss problem solving to reflect the process of solving a word problem.

In mathematics we sometimes describe problem solving as things that aren’t really problems. That we know if we apply a certain number of algorithmic steps that we will get to the answer. That’s not really a problem, a problem is something that when we look at it we don’t necessarily know how to solve it.

(Teacher, Interview)
This distinction between problem solving and word problems to solve was important because each style of questioning develops different skill sets in students. Problem solving promotes critical thinking skills, while solving a word problem promotes the development in efficiency of algorithms. The teacher stated that problem solving occurs when students were given a task where they had to develop a solution to a problem in a situation or scenario, this situation might have no correct answer and might employ a variety of strategies and tools. She explained that true problem solving does not occur when answering questions that require routines or algorithms.

The teacher used problem solving as a way to engage her students. She suggested that if problems were developed without a readily available solution that students have to work toward linking concepts across their curriculum and skill set.

And so, it’s trying to have students link together various concepts and ideas, so I try in terms of problems solving, to incorporate it into the lesson, in terms of…. this is a problem that you aren’t necessarily going to look at and say…..oh I know the algorithm for solving that…because again that is not really a problem.

(Teacher, Interview)

Here the teacher said, “it’s trying to have students link together various concepts and ideas” which meant that she was using problem solving as a strategy to have students discover connections between concepts across different aspects of the mathematics curriculum. She implied that students have to work together in this process as students won’t be able to solve the problem easily “you aren’t necessarily going to look at it and say… oh I know the algorithm for solving that”, students need input from others to develop a solution. The teacher indicated that by working with other people students
developed their oral communication skills and their mathematical vocabulary because students were constantly negotiating terms during meaningful conversation.

The teacher used problems in a variety of ways within her lesson. There were times she used a problem to engage the students at the start of a lesson and generate discussion around their learning for the day.

And I use it often to start my lessons, as the introduction to a lesson to put it up as an intriguing question that is going to play into what we are going to be doing that day. And also as a jumping off point for where we are going the next day. So I might leave them, as part of their homework, something that pushes them in their own mathematical thinking in terms of a problem.

(Teacher, Interview)

When the teacher stated, “put it [the problem] up as an intriguing question” she used this strategy as a way to engage students in discussion. She indicated a belief that when students were engaged in mathematical discussions it built communication skills and terminology around the concepts that were connected to the problem.

The teacher indicated her view that teaching mathematics was teaching problem solving. She was careful in her definition of a problem as her intent was that learning to problem solve meant that students learned how to think mathematically. Problem solving was seen at the start, middle, or end of a lesson so long as true problem solving was heavily integrated into the classroom.

Every single day I try to embed problem solving. Because I don’t want students to feel like problem solving is the add-on that we do at the end, after we’ve learned the concepts.
The teacher suggested that she believed that problem solving created the reason to learn the concepts instead of viewing problem solving as an application after the conceptual learning has occurred.

She explained that problems allowed for concept discovery and in turn the generation of mathematical thinking and language. She suggested that teaching under this philosophy also built conceptual knowledge and mathematical vocabulary through collaboration.

I think they do pick up when somebody uses the correct vocabulary in class, I do think they pick up on that and start to … quote each other…so if one student is using the correct vocabulary … or uses a correct term. For instance… I think when a student uses an appreciate rate or a depreciate rate correctly, it then helps the other students use it correctly in the same context. I think that the partnered conversations really help with that, and help them see that the language is important.

The teacher indicated that using partnered conversations through collaboratively problem solving allowed students time to develop correct language use. She suggested that students take more risks when they can work on problems together to work through their thinking together, discuss strategy, and utilize mathematical language. She suggested that group work exemplified the reasons why using mathematical language created a common language amongst students, i.e. a discourse community, was important for their overall understanding in class. This was the teacher’s strategy to create a norm of risk taking in classroom discourse.
The teacher communicated a strong set of beliefs in teaching mathematics, which defined the theme, *Problem Solving Inquiry*. She indicated a belief that contextual problem solving was the premise for learning mathematics. She explained that through her definition of problem solving students saw the interconnectivity that exists within mathematics. The teacher suggested that when teaching through problem solving that students were engaged in their learning process, they learn from one another during classroom collaborations, and they learn the embedded mathematical language.

The teacher interviews produced three themes: (1) *Importance of Language*, (2) *Language Acquisition Through Collaborative Problem Solving*, and (3) *Problem Solving Inquiry*. These three themes show an integration of thinking between *Language Acquisition Through Collaborative Problem Solving* and *Problem Solving Inquiry* as the teacher demonstrated a belief that mathematical language was transferred to students throughout the semester through mathematical problem solving. She indicated that mathematical language was directly linked to the concepts learned within mathematics class and, as such, hold deep importance to her. Although the teacher did not believe that her grade nine students experienced a mathematical discourse community, the teacher indicated that her class had or were developing several characteristics of a discourse community.

**Classroom Observations**

Classroom observations were conducted over a three-day period. Five themes were developed from the coding process: (1) *Student Driven Learning*, (2) *Development of a Common Language*, (3) *Mathematical Thinking within Context*, (4) *Questioning and Directing*, and (5) *Nature of Classroom Discourse*. Analysis was also conducted on the breakdown of students’ oral transactions during class that were coded as combinations of the first four themes, and this
gave evidence to the fifth theme, the role of the nature of the classroom discourse in student learning.

*Student Driven Learning* is about students leading the class discussion through their own questions or comments. Students’ self direction of their learning was exemplified when they used and explored mathematical terminology that had been introduced through class, reading, a review of prior knowledge, or the reorganization and application of new concepts. Students’ demonstrated self-direction of their own learning when they asked for help involving or with mathematical concepts. *Development of a Common Language* described students’ exploration of mathematical terminology through its use in class while working with the whole class or in small groups. Students demonstrated *Development of a Common Language* through an increase in confidence in mathematical language use during class, the connecting of terms and concepts, and the connecting of explanation to algorithms when problems solving. The linking of mathematical language to ideas within mathematics, cross-curricularly, and to the world beyond the classroom demonstrated students’ acquisition of new terminology. *Mathematical Thinking within Context* was the strategy use, explanation of, description of, and creativity in mathematical problem solving by students and/or the teacher. *Questioning and Directing* was demonstrated through the teacher’s use of questioning and discourse to facilitate the direction of student responses during the lesson. The teacher lead the class through a mathematical concept, solved a problem, and focused on extracting explanations of ‘why’ from students when they were explaining their work in class.

*Student Driven Learning*

*Student Driven Learning* existed when students directed portions of the lesson based on their responses to teacher questioning or problem posing, students asked questions about
mathematical concepts or vocabulary, or students discussed problems together in small group settings without the teacher.

*Student Driven Learning* was evident when Vanessa explained that she did not understand part of the homework. She sought clarification from her teacher and peers on the concept of y-intercept.

I didn’t really get the part where is was…..where it was the first part of every problem…. I think it was the intersects? Like the intersects… I just didn’t get that.

(Vanessa, Classroom Observations)

This showed one way a student may direct the classroom learning. Vanessa expressed difficulty understanding homework through the misuse of the term ‘intercept’ as she was unsure of how to determine the y-intercept in her homework from the night before. This granted her control over the next few transactions of speech in class as she sought clarification.

Martin looked for clarification of the concept of slope when he attempted to reorganize the way he described slope allowing him to express it in his own words. “Is it true to say the slope determines if you have a positive relationship or a negative?” (Martin, Classroom Observations). His question also attempted to create a connection between prior knowledge from the week before in class. Martin gained control of his learning by directing the class conversation around helping him understand the concept of slope more fully.

Fred discussed an in-class task with a partner. He used his language to work through defining and applying his knowledge to an unfamiliar context. He sought clarification and/or confirmation on his thinking from a peer.

the first one is linear…… it is increasing by the same amount each time. [long pause] the second one is increasing by negative seven, negative seven, negative
seven, and then by negative fourteen, negative fourteen, negative fourteen. So it is linear because of this.

(Fred, Classroom Observations)

Fred used Student Driven Learning to guide his peer conversation to help him better understand the difference between linear and non-linear patterns. When Fred said, “it is linear because of this”, he opened the conversation to his peer group in effort to discuss the first differences that he found. He was facilitating his learning through peer discussion.

Each of these examples illustrated different ways in which students directed their learning during the lesson. Student Driven Learning allowed students to pose questions and ask for help when they did not understand the mathematical concepts or language used in class. Student Driven Learning was evident when students reviewed terminology from previous classes or asked for help from the teacher or their peers. Students directed their own learning when they developed questions on the terminology or the content. Students had many conversations in class with small groups giving opportunity for peer lead conversations. Student Driven Learning enabled students to discuss their misconceptions of terminology (i.e. slope and rate of change).

Development of a Common Language

Development of a Common Language emerged throughout the three days of observations when students adapted new terminology from class into their mathematical vocabulary. Students re-organized and applied terminology to new and familiar situations. Students used terminology in the scenario of the problem, which signaled that they understood the contextual meaning. Students were heard exploring the pronunciation, use, and application of mathematical terminology within class. Some students tried to understand new mathematical terms and attempted to create or apply that terminology to new situations as they arose. Further evidence of
Development of a Common Language appeared when students stated their personal opinion on the solution of a problem, self-correction when speaking, and asking for help with terminology.

In the following transaction, three students discussed a question assigned to them in class. In this discussion the students peer-corrected each other and applied a new term.

Fred: Hey, is there a model for number two?
Joe: Yes, there is.
Vanessa: Oh… model… is like an equation…. Why don’t they just say equation?

(Fred, Joe, and Vanessa, Classroom Observations)

Prior to Fred using the word ‘model’, the teacher said ‘model’ three times during class. After Fred said ‘model’ within his peer group Vanessa also began to understand the term ‘model’ in reference to mathematics. Both Fred and Vanessa demonstrated a variety of language acquisition within this oral transaction. Without stating the word, Joe indicated clear understanding of the term ‘model’ as it applied to mathematics, when he was able to answer Fred’s question and continued to offer his mathematical model when the teacher called for it during the whole class discussion of the questions.

In this next transaction the teacher gauged students’ prior knowledge on rates of change and slope. As she referred to a question posed on the board, students discussed the meaning of slope.

Teacher: What is your understanding of slope right now?
Martin: It’s a rate.
Teacher: It’s a rate. Rate of what?
Martin: It’s a rate of increase or decrease.
Teacher: It’s a rate of increasing or decreasing.
Martin: If the slope is like… slope determines how the line increases in a linear equation.

(Martin and Teacher, Classroom Observations)

Martin addressed the question by using terminology that he already had a clear understanding (i.e. rate of increase or decrease). Once Martin had associated rate of change to slope, he then re-organized his understanding of slope to create meaning of the word slope and connected it to linear equations. Martin demonstrated *Development of a Common Language* as actively acquired new language when he reorganized the meaning of slope and associated it with what he already understood of rate of change.

Here, students were given time to work in small groups. The intention was to discuss and work through several problems to determine linearity. The four students below debated their response to one of the questions that was about non-linearity. As they discussed the problem they used the term ‘linear’, ‘not linear’, and ‘partial’, all in an exploratory manner trying to gain command over the new terminology.

Quinton: Yes. It is **not a linear** equation.

Fred: It becomes a **linear** relationship. It starts going up by 4. Is there a special name for that? Like it goes...down but then it ..[arm motion to flatten out]. Could it be ….**partial or something**?

Vanessa: That’s what I said.

Ella: You said it didn’t go all the way through? So **how can that be a partial relation**? No you **can’t call it partial** because this is y =mx+b. This is partial, one can tell this is not….

(Quinton, Fred, Vanessa, and Ella, Classroom Observations)
Quinton described the table of values as “not a linear equation”. This demonstrated that he has connected representations of numeric tables and algebraic equations. Quinton had not come to use the term ‘non-linear’, nor had the teacher used the term ‘non-linear’ at that point in class. Fred mentioned that the graph became linear after a certain point in the table and asked if anyone knew a term they could use to describe it. Fred offers the term ‘partial’. Vanessa engaged in the misconception around the term ‘partial’. Ella questioned her peers, and then explained that the term ‘partial’ refers to ‘partial variation’ not that an equation is ‘partially linear’ (i.e. an equation that would be represented by two equations with at least one being linear). Ella tried to explain a partial variation would be represented by the slope-intercept form of a linear equation, y = mx+b, but the group had not created a full model to compare this against and the argument faded. In this transaction, Fred explored his pronunciation of the term ‘partial’ in different phrases so that he could attempt to apply it to new situations. Ella attempted to peer-correct using key characteristics of the term ‘partial variation’. Fred brought up the term ‘partial’ again in a whole class discussion.

During the next discussion, students reviewed a few questions with the teacher as a class. They discussed the characteristics of a table of values and whether or not this table demonstrated linearity. The students noticed a trend to the graph and wanted to call it linear but it doesn’t satisfy the definition of linearity. Quinton and Fred both observed that the first differences in the table of values had two sets of constant values, the first half of the table produced one first difference and the bottom half of the table produced a second consistent first difference.

Teacher: Quinton, what do you think now? He’s convinced now.
Quinton: …[long pause] it seems like it is not linear but its weird
Teacher: Yeah, it’s …like… weird… It’s non-linear.
Fred: Can it be partially linear? It’s linear after this point [points to point on the board]

Teacher: so how do you know from a graph when something is linear?

Joe: It’s a straight line

Teacher: Yes, it’s a straight line. Does this graph make a straight line?

Joe: Nope!

Fred: But … So I don’t know what it is but it is a straight line for part of it…can it be partially linear?

Teacher: It looks like its linear for some of the graph but not for all of the graph.

Fred: After zero it is linear but not before that its like …really weird.

(Quinton, Joe, Fred, and Teacher, Classroom Observations)

Quinton stated that he saw that the table would not be linear but his answer was not spoken confidently. His desk partner, Fred, asked if the relation could be called ‘partially linear’. When Fred created this new term he demonstrated a form of language acquisition as he attempted to apply the new terminology (i.e. partial of partial variation) to describe a piecewise linear function as ‘partially linear’ as it graphed two line segments that connected at one point. This provided an opportunity for clarification of misconceptions of the term linear. Joe peer-corrected with his reply to the teacher’s question “how do you know from a graph when something is linear?” as he stated that anything described as linear produces a straight line. This gave rise to another clarifying question from Fred when he asked “it is a straight line for part of it….can it be partially linear?” to which the teacher replied that the entire graph had to be linear, not just one section of it. After this transaction Fred and Quinton both understood the term linear more deeply as it applied to linear equations, tables, and graphs. Development of a
*Common Language* was demonstrated by Fred when he continued to ask for clarification on the term ‘partially linear’ which allowed him to better understand the definition of the term ‘linear’ and when and how to apply the word ‘partial’ within mathematics.

Fred, Quinton, Martin, Vanessa, Joe, and Ella all presented forms of *Development of a Common Language* that were heard during the class observations. Fred, Quinton, and Vanessa best demonstrated acquiring mathematical terminology through peer discussions, while Martin used his class time to build connections between mathematical concepts being learned in class. Joe and Ella both demonstrated the importance of peer-correction to the process of language acquisition. Martin was heard re-organizing the term ‘slope’ to make it connect to his prior knowledge of the term ‘rate of change’. Fred used the term ‘model’ in class when working with his peers to discuss the algebraic equation that he and his peers had developed. Fred attempted to apply terminology to new situations when he tried to expand the use of the word ‘partial’. When attempting to use his new term ‘partial’, Fred used the small group discussion as a method to test the vocabulary and think of how to place the word in the sentence to make it most meaningful before he attempted its use in a whole class discussion. *Development of a Common Language* was present within the observed classes, and was connected to the context of what students were learning.
Mathematical Thinking within Context

Mathematical Thinking within Context meant that students demonstrated an oral statement of mathematics, their work, or thinking related to problems discussed in class. Students described an understanding of learned mathematical concepts, an integration of mathematical language as it applied to solving problems presented in class, and the use of knowledge and application of various strategies while problem solving.

The following excerpt from the transcript started with a question that asked for clarification of the meaning of slope, then discussed slope as a rate of increase in the context of a homework question relating the slope to the appreciation rate of a house.

Vanessa: Yeah, I didn’t get the idea of the slope of the line at all.

Teacher: Okay so lets take up number eight. It gave you this equation

\[ y = 7500x + 125000, \]

it said it described the relationship between \( x \) years and \( y \) is the value of the house. What can you tell me about that equation?

Steve: As more years go by, the value of the house increases.

Teacher: Yes, and how do you know that?

Steve: Well cause depending on what \( x \) is and the number of years times the 7500 is higher than it was so the cost of the house is getting larger as the years pass by.

Teacher: Right, so if you put in the number one, you’ll get a certain number, and if you put in a value like two, you’ll get a bigger number, if you put in a number like three, you’ll get a bigger number. So a value of house is going up…. What do you call it when the value of something increases…like the value of a house…we talked about this last
there was a term…we talked about cars going down in value…and other investments that went up in value. So a good investment is something that goes up and a bad investment is something that goes down in value over time.

Jenn: Is the one going down called depreciation?

Teacher: If the one going down is called depreciation, …

Gina: Appreciation

Teacher: Yes, appreciation, and it doesn’t mean that yes I appreciate you as a person, it this context when something is appreciating in value it is going up. House generally do go up in value, generally. Steve made a good point that if I put in big numbers for x that the value is going to go up.

(Students and Teacher, Classroom Observation)

*Mathematical Thinking within Context* was represented in this transaction as the students connected language from everyday to their classroom and academic registers. For example, Steve responded to the teacher’s question “What can you tell me about that equation?” with a description of the trend of the scenario “As more years go by, the value of the house increases”. Steve indicated that he understood the general idea that if there is a positive slope that the house prices will increase over time.

Another example of where students pulled language from their everyday experience was in the description of the type of investment rate being either an appreciation rate or a depreciation rate. This created a connection of the word meaning as students had previously used the term in class, notice when the teacher said “we talked about this last week…there was a term…we talked about cars going down in value” to prompt students into recall. It was also seen that
students explained their reasoning behind their statements and responses, for instance, when the teacher asked Steve to explain his statement about the trend of the equation “Well cause depending on what x is and the number of years times the 7500 is higher than it was so the cost of the house is getting larger as the years pass by.” Steve explained how he thought about the trend of the equation in his own words granting other students the opportunity to be exposed to his mathematical thinking. There was a discussion of trends (data management) and slope (linear relations) as mathematical concepts throughout this transaction, which demonstrated that students were discussing mathematical concepts while learning the language of mathematics.

**Questioning and Directing**

*Questioning and Directing* means that the teacher was explicitly directing student learning during class time. This was seen when the teacher introduced a problem, redirected the class when working in small groupings, made corrections to student thinking in whole class discussions, or discussed misconceptions often through questioning techniques.

The teacher directed student learning to encourage students to think about a curriculum topic. She started by having students review their work from the day before to guide the class through what they had already learned on the topic.

First of all let’s talk about some of the key ideas that I asked you to look at yesterday.

Just to consolidate some stuff around slope before we’re going to talk about how we’re actually going to use it today.

*(Teacher, Classroom Observations)*

The teacher explicitly stated that she wanted students to “talk about some key ideas” in review so that they, as a team, could clear up any misconceptions that had developed from
the previous class and/or from their homework. She wanted to clarify misconceptions before students move onto new learning.

The teacher demonstrated *Questioning and Directing* when she introduced learning goals as she asked students to focus on one concept in their learning over the course of the lesson. She described how they were going to use the concept of slope to develop their own equations based on only two points within a Cartesian plane.

That’s our learning focus today. Our learning goal today is that we’ll be able to determine the slope between two points and, we’re hopefully going to be able graph a line using the slope and the y-intercept, and we’re going to be able to explain the slope relationship between the dependent and independent variable. So today is all about slope.

(Teacher, Classroom Observations)

The teacher related students’ previous learning to new strategies that students’ were then able to use based around the concept of slope. She then re-iterated that they, as a class, were going to also look to further develop students’ understanding of slope as it related to independent and dependent variables. The teacher used the learning goal itself as another mechanism to create more questions and inquiry for students.

The teacher continued to direct student learning by questioning students on what they currently understood about the idea of slope. This meant that students were expected to discuss slope as a group and students were not able to change the overall topic at this time during class.

Okay, so lets hear about some ideas about slope. Some of you already know about it, some of you wrote some [ideas] down last night. What are some of your ideas about slope? What do you already know about slope?
The teacher urged students to recall their homework and discuss the concept of slope as a means to generate discussion.

*Questioning and Directing* was clearly stated as the teacher discussed prior learning and referenced the learning goal for the day. She directed the learning, which in turn, directed many of the student questions that appear from that point in class. As mentioned above *Questioning and Directing* also appeared as class redirection.

Okay. I am going to interject with you because some of you are confused.

So Jenn had a great explanation for me, she said start with the point P, and then she said to do what?

This interjection allowed a number of students in the class who were working in small groupings to stop, reflect, and discuss their work with the teacher as a class. Students then returned to their problem solving in small groups. Although the *Questioning and Directing* only lasted for a few sentences it was impactful as students re-directed their own work after this point.

In the following transaction the teacher demonstrated direction of student learning by questioning student understanding to reveal a misconception. Fred, Martin, and the teacher discussed a solution to a problem posed in class regarding linearity. The table of values that were discussed represented a non-linear relation that was quadratic. Students were able to see a pattern in the first difference, which led to a few students developing a misconception that any pattern in the first difference implied that the table of values represented a linear relation.

Fred: So x is the one on the bottom. X is increasing equally.

Teacher: Okay, yes, you are right but what about y?
Fred: It doesn’t actually matter about y because … [interrupted]

Martin: Y is the only thing that matters here!

Teacher: Okay, so, I have one opinion that says it doesn’t actually matter about the y value,

Martin: It’s all about y. In every question we’ve looked at it has been all about y.

Teacher: Okay. How would you test that?

Martin: I would need to graph it.

Teacher: But in the other ones we looked at the y’s did matter.

Fred: I don’t think it [y values] matters. Can I draw something? [draws a graph of the table of values but changed the scale on the y axis so that the graph appears to be increasing by a constant amount along the y axis]

Teacher: Here is my question. When you draw a scale on a graph, you’ve got this one jumping up by this one [point to the consistent scale on the x-axis], and then this one jumping up by this one [point to the inconsistent scale on the y-axis], is that a problem?

Fred: Yeah, but it’s linear still.

Teacher: I don’t think so.

Fred: Oh it’s on.

Teacher: First of all, when you are drawing a graph, the scale has to go up by the same amount. It makes a difference.

Fred: Are you saying that I can’t actually graph these?

Teacher: Well, let’s graph these. I want everyone to get out a piece of graph paper and plot these ones. I want you to graph that relationship.
The teacher asked several students for their responses and noticed that a few were adamant that the table of values represented a linear relation. She began questioning students about the changes in the x value, and the y value, and asked which changes were the same and which impacted their understanding of the values being linear. Martin noticed that in all the problems that only the y-values had given results to explain linearity so far when he said, “In every question we’ve looked at it has been all about y.” When Martin said this, Fred disagreed, and drew a graph where he mislabeled the scale of the y-axis. When Fred mislabeled the scale the teacher took back the conversation and directed the class to observe the error in the labeling of the scale. The teacher took the opportunity to have all the students build a graph and a scale so that she could clarify this misconception that if one adjusted the scale to be inconsistent that the graph was linear or that certain pattern in the first different (i.e. the step pattern) produced a linear relation.

Questioning and Directing was demonstrated throughout the observational period in a variety of ways. The teacher demonstrated the explicit direction of student learning in the class through the discussion of learning goals as she explained drew out students learning from the day before and new applications they would learn by the end of that day. The teacher re-directed student learning or corrected student thinking and work when students became confused or began to build misconceptions when working in their small groups demonstrated Questioning and Directing. The teacher’s questioning style to elicit student thinking allowed misconceptions of the development of a scale to be discussed and minimize its negative impact on student learning of slope demonstrated teacher control of student learning. The students’ learning was
impacted by the teacher’s ability to direct learning during the observed lesson when the teacher
clarified mathematical terminology use and conceptual understanding for students.

**Examining the combination of themes within the classroom**

The four themes: (1) Student Driven Learning, (2) Development of a Common Language, (3) Mathematical Thinking within Context, and (4) Questioning and Directing were evident during classroom observations. There were three main combinations of themes that produced points of interest to the researcher when examining the nature of students’ oral transactions in the classroom observations. The combination of Questioning and Directing and Development of a Common Language demonstrated that teacher questioning and direction of the classroom discourse allowed for language acquisition. The second combination of Student Driven Learning and Development of a Common Language demonstrated that student self-direction of the learning environment minimized language acquisition. The third combination of Development of a Common Language and Mathematical Thinking within Context demonstrated that language acquisition occurred most when there was a strong and clear purpose to learning language (i.e. language acquisition occurred when there was a context of which to learn language in). See Figure 4-1 for frequency of occurrence of each theme and combinations of themes throughout the three-day observation period.

Figure 4-1 represents all students’ oral interactions during the three-day observational period. A student interaction was considered to be the turn a student took in class to speak, regardless of the length of the interaction. A student’s interaction could be anything from one word to three sentences. The chart was divided by percentage of interactions that were coded for each theme or combination of themes. For example, *Student Driven Learning* represented by ‘1’ on the chart was coded as twelve percent of all students’ oral interactions over the three days.
These interactions were not coded in combination with any other themes, only as *Student Driven Learning*. The combination of the themes *Student Driven Learning* and *Development of a Common Language* are represented by ‘1 2’ on the chart which accounted for zero percent of all student’s oral interactions. The combination of themes *Questioning and Directing* and *Mathematical Thinking within Context* was represented by ‘3 4’ and accounted for eleven percent of all students’ oral interactions in class. The first four individual themes were labeled ‘1’, ‘2’, ‘3’, and ‘4’, and represent *Student Driven Learning, Development of a Common Language, Mathematical Thinking within Context, and Questioning and Directing*, respectively.

![Frequency of Thematic Codes in Classroom Observations](image)

*Figure 4-1. The Frequency of Thematic Codes in Classroom Observations. (1) Student Driven Learning, (2) Development of a Common Language, (3) Mathematical Thinking within Context, and (4) Questioning and Directing.*
Figure 4-2 represents the same data as above. The labels were modified to allow for ease when referencing the chart during analysis. For example, *Student Driven Learning* that was previously represented by ‘1’, was re-labeled ‘A’ in Figure 4-1. The chart coding was labeled alphabetically clockwise in the same order as Figure 4-8. The first four individual themes that were labeled ‘1’, ‘2’, ‘3’, and ‘4’, were labeled ‘A’, ‘B’, ‘C’, and ‘D’, respectively.

![Frequency of Thematic Codes in Classroom Observations](image)

*Figure 4-2. The Frequency of Thematic Codes in Classroom Observations with alphabetical labeling.*

Presenting the frequencies of the themes and the nature of their co-occurrence offered an opportunity to explore the percentage of students’ oral interactions during the observational period. For example, all instances of *Questioning and Directing* (D) and *Development of a*
*Common Language* (B) accounted for fifty-one percent of all students’ oral interactions within the classroom observations (Figure 4-2, B+D+F+G+J+L). The combination of these themes indicated that when the teacher gave clear direction to the purpose and use of mathematical language within the class, students demonstrated language acquisition of the mathematical terminology being taught. Notice that when the two main themes, *Questioning and Directing* (D) and *Development of a Common Language* (B), were combined only with each other, it accounted for three percent of all students’ oral interactions in the classroom observations, see ‘F’ in Figure 4-2. This meant that *Questioning and Directing* was most effective in producing active language acquisition, and the *Development of a Common Language* when the teacher and students were discussing the language in the context of mathematical thinking and problem solving.

Combining all instances of *Development of a Common Language* (B) and *Student Driven Learning* (A) accounted for thirty-three percent of all students’ oral interactions (Figure 4-2, A+B+M+O). This combination of themes meant that mathematical language acquisition occurred less than one third of class time when students controlled the class discussion whether in small group discussions or whole class. Notice that when the two themes, *Development of a Common Language* (B) and *Student Driven Learning* (A), were combined only with each other it accounts for zero percent of all students’ oral transactions in the classroom observations, seen as ‘N’ in Figure 4-2, meaning the class time that was utilized by students to acquire mathematical language other than when the teacher controlled the students’ learning time (through *Questioning and Directing*).

When all instances of *Development of a Common Language* (B) and *Mathematical Thinking within Context* (C) were combined, they represented seventy-two percent of all students’ oral interactions (Figure 4-2, B+C+F+G+H+I+J+L+M+O). This combination of
themes meant that when mathematical terminology was given in context, the terminology was heard discussed, used, and learned for the majority of class time when students spoke. Notice that when these two themes were combined only with one another it accounted for twelve percent of all students’ oral interactions within the classroom observations, see ‘L’ in Figure 4-2. Students learn mathematical language best when they learn it contextually. From the classroom observations of this study, students spent seventy-two percent of their class time speaking, actively acquiring mathematical language while problem solving or discussing mathematics. Students acquired mathematical language best when the teacher directed student learning and maintained student focus in class, whereas student control of learning minimized students’ acquisition of mathematical language.

The Nature of Classroom Discourse

A fifth theme was developed during the analytic process. This theme was not developed during the coding process and was not explicit during the observations. The Nature of Classroom Discourse emerged as a theme from a comparison of the other themes to one another. During classroom observations, student frequency of speech was recorded. Whether a complete statement was one word or three sentences, it received a tally. Frequency totals can be seen in Table 4-1. Of the sixteen students thirteen spoke during class. This indicates that not all students in class spoke during the observation period and non-oral data collection (i.e. student written work) was required to determine if non-speakers acquired mathematical language across the observation period. Of the thirteen speakers in class, one out of six times that students spoke were questions to the teacher or peers, and one out of five times that students spoke was recorded as less than three spoken words. Student questions during the classroom observations were reviewed to determine their nature.
Table 4-1

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of times spoke</th>
<th>Number of one to two word responses</th>
<th>Number of Questions Asked</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher</td>
<td>304</td>
<td>16</td>
<td>240</td>
</tr>
<tr>
<td>Fred</td>
<td>110</td>
<td>21</td>
<td>40</td>
</tr>
<tr>
<td>Martin</td>
<td>69</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>Ryan</td>
<td>37</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>Joe</td>
<td>26</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Ella</td>
<td>23</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Quinton</td>
<td>21</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Vanessa</td>
<td>19</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Jenn</td>
<td>18</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Lynn</td>
<td>13</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Steve</td>
<td>11</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lauren</td>
<td>10</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Ben</td>
<td>6</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Neil</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>671</td>
<td>86</td>
<td>314</td>
</tr>
</tbody>
</table>

Note: Frequency count of classroom speech per student. Includes total number of times student spoke in class over a three-day observational period, the frequency of which those phrases were less than three words long, and the frequency of which those phrases were questions. Ordered high to low in the variable ‘Number of times spoke’.

Students’ questions, when asked in front of the entire class were most often for clarification, either to clarify the meaning of notation or a concept. Jenn, Martin, and Fred
illustrated clarifying questions during whole class discussions. During a discussion on slope that related slope and rate through a problem Jenn asked, “Why is it ‘hr’ beside the three dollars but just ‘h’ for the speed?” This question showed a question of notational clarification. During the same lesson on slope when Jenn posed her question, Martin asked a clarifying question, “Is it true to say the slope determines if you have a positive relationship or negative?” He sought clarification in the conceptual meaning of slope so that he could re-organize his own definition of the word and internalize the term into his vocabulary. During a lesson on determining the slope from two points on a line, Fred asked, “Why was the slope down three and not up?” This question showed that Fred needed to clarify his conceptual understanding of slope so that he could better understand slope through multiple representations. Fred’s question needed clarification as this was the first negative slope discussed in class during the observation period and Fred’s question demonstrated that he didn’t understand when the change in y was positive or negative affected the graph of a line that had a negative slope. Jenn, Martin, and Fred all demonstrated the clarification nature of student questions during whole class discussions.

When students asked questions within small peer groups, the nature of their questions was more diverse. Students’ questions sought clarification not only on notation and terminology but also on peer reasoning, strategy, and understanding of concept. During these transactions students were asked to work in small groups or pairs and to determine if three questions were linear or non-linear. Vanessa begins to talk to her peers with a question.

Vanessa: But how do we know?

Ryan: Because it is going up by a set amount. This one is going up by a set amount. [said with a condescending tone].

Vanessa: I know that, thank you. [slightly irritated tone]
Ryan: No two is not linear [second question on page]. It is not linear because it won’t make a straight line when you graph it. [said with irritation]

(Vanessa and Ryan, Classroom Observations)

Vanessa’s question, “How do we know [its linear]?” opened the discussion between her and Ryan. The question demonstrated her willingness to talk out the problem and collaborate. Vanessa’s question appeared to seek a reason for linearity. Ryan responded to the question by giving characteristics that signaled linearity that were present in the table of values that they were given. Vanessa asked a question to open the discourse, Ryan responded to the question demonstrating his conceptual understanding of linearity. Even though Ryan and Vanessa did not collaborate well, the nature of her question gave room to create collaborative discourse between them.

Lynn and Ella discuss the same problem. They also start their transaction with a question to open up discussion.

Lynn: What is a linear relation?

Ella: It goes like this. [unsure and draws a picture]

Lynn: Okay so. Show that these are linear relationships. Well, I think that we just explained it.

Ella: [Giggles]

Lynn: Yes, it is a linear relationship. [Long pause]

Lynn: Well, we know this is linear because both the x and y ... because it is decreasing or increasing by the same amount.

Ella: The next one isn’t linear, because it isn’t going up by the same amount.
Lynn: They aren’t linear because they aren’t appreciating or depreciating by the same amount?

Ella: I want to say that they aren’t linear but I can’t describe exactly why or exactly what they are.

(Lynn and Ella, Classroom Observations)

Lynn’s question, “What is a linear relation?” opened the discussion between Ella and Lynn and allowed them to determine characteristics that made a table of values linear. Ella’s response was to draw a picture, Lynn accepted the pictorial representation as true, and then re-stated the question again. This process created mutually beneficial transactions as the two students set up an unspoken set of norms that were based in discussion and collaboration. When the students discussed the second problem, Ella stated that the table of values was not linear and Lynn asked, “They aren’t linear because they aren’t appreciating or depreciating by the same amount?” which was a question to connect the group lesson into their peer discussion. Lynn explored the use of appreciation and depreciation with Ella, sought clarification for their reasoning as to why the table of values was not linear, and sought confirmation on her answer from Ella.

Ben and Fred also worked together on the same problem set where they asked questions of each other back and forth. Ben wanted to look for other strategies to determine linearity and asked, “Could we graph it?” Fred responded, “We can’t put it on the graph because if we graph it then it won’t be a straight line.” This question and response illustrated that Ben attempted to open up a discussion of strategies to solve the problem at hand. Fred’s response essentially answered Ben’s questions to say that the table was not linear. Fred’s response to Ben’s question also elicited a possible misconception that you can only graph linear relations. Without the
teacher’s participation in this transaction the misconception was not addressed at this point in the
lesson.

The nature of students’ questions when working in small groups or pairs was more broad
as it included the same types of clarification, as well as, exploration of meaning and
pronunciation of terminology, confirming solutions, comparisons of arguments and solutions,
exploration of mathematical concepts, and collaboration of ideas and strategies. This means that
students expand their understanding of mathematical concepts and language when they have
time to discuss them with their peers.

There were a number of times that students were the first to say academic mathematical
language during the observed lessons. This type of instance occurred eleven times over the
course of three days. The nature of the teacher response was recorded as questioning looking for
meaning or clarification, which means that the teacher was asking students to elaborate on their
responses to her. An example of this can be seen in Figure 4-10 where the teacher repeated the
terminology used by Jenn, “Direct variation and partial variation”, and then posed a question,
“what’s the difference?” and asked Jenn, or another student, to clarify the difference between the
term ‘partial variation’ and ‘direct variation’. The teacher used confirmational praise after Joe’s
detailed response describing direct and partial variation. She then repeated what he said and
chunked his response to break down the difference, emphasizing the conceptual importance of
the term. She continued to question on the difference between the two terms using a variety of
representations (i.e. graphical, numeric, and algebraic).
<table>
<thead>
<tr>
<th>Name</th>
<th>Statement</th>
<th>Student Key Terms/Teacher’s Nature of Comment or Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jenn</td>
<td>We learned that there are different types of linear relationships. There is direct variation and partial variation.</td>
<td>Linear relationships</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Direct variation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Partial Variation</td>
</tr>
<tr>
<td>Teacher</td>
<td>Direct variation and partial variation, what’s the difference?</td>
<td>Questioning – looking for meaning</td>
</tr>
<tr>
<td>Joe</td>
<td>So on a direct variation the line passes through the origin and the equation would be represented by $y=mx$, whereas on a partial variation the line does not pass through an origin and the line will be of the form $y = mx+b$ so there is a $b$ value.[hand gestures accompanied]</td>
<td>Direct variation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Line</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Passes through the origin</td>
</tr>
<tr>
<td></td>
<td></td>
<td>equation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Represented</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Line</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Whereas</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Partial</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Does not pass through the origin</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B$ value</td>
</tr>
<tr>
<td>Teacher</td>
<td>Well said, well summarized. So, we have a relationship $y=mx$, where $m$ is our slope. We’ve talked about slope but we haven’t talked about how to get that number. We will get closer to that today. So he [Joe] said that it goes through the origin. All the ones in the exercise went through the origin like this [draws a sketch of $y=4x$ on the board] and what is it called when the line looks like this?</td>
<td>Confirmation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Praise</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Repeating</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Questioning – looking for connection between representations (graphic and word)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Questioning - Looking for clarification</td>
</tr>
</tbody>
</table>

*Figure 4-1. Coding of the Nature of Teacher’s Questioning or Response when students said academic language prior to the teacher during classroom observation. Terms included Partial Variation, Direct Variation, Line, Linear, and $b$-value.*
The teacher’s responses were also noted as ‘connections’, when she helped a student to make a connection between two mathematical concepts being discussed. She often ‘confirmed’ student responses, ‘praised’ their work, or ‘corrected’ them. When the teacher made a connection between mathematical concepts, confirmed or corrected a student’s work, she repeated what the student said and re-phrased the student’s response. In one of eleven occurrences that the students were the first to say academic mathematical language, the only term not addressed in any way by the teacher was ‘equation’. However, this term was already well known to the class as stated in the pre-observational teacher interview.

The teacher used questioning in eight out of ten times that she spoke during class. The nature of her questioning included: review, activating prior knowledge, searching for student misconceptions, seeking clarification and explanations of student thinking, seeking terminology use and examples, reorganizing word patterns to create meaning, and the correction of notation use.

The teacher started class by asking students to describe what they learned last class. The teacher’s first question was to review and activate prior knowledge.

Teacher: What did you learn on last class? Talk to me. What did you learn from that exercise?

Jenn: We learned that there are different types of linear relationships. There is direct variation and partial variation.

Teacher: Direct variation and partial variation, what’s the difference?

(Teacher and Jenn, Classroom Observations)

After Jenn replied, the teacher posed another question, “What’s the difference?” Students were expected to give a more thorough description in their next response as the teacher was
attempting to have students to explain their understanding of the terminology and its concept.

In the next transaction the teacher asked a student to clarify their meaning, and explain their thinking. When the student didn’t reveal all his thinking, the teacher asked more questions in effort to extract student thinking.

   Teacher: How do you know it’s linear?
   Steve: It’s decreasing by two
   Teacher: Why doesn’t that matter?
   Steve: Because it’s the same all the way down.

   (Teacher and Steve, Classroom Observations)

Steve’s response to the teacher’s question was correct but very brief. The teacher’s second question attempted to have Steve explain his thinking. The teacher’s question appeared to want Steve to connect his previous response to the idea of first differences. Steve’s final response, “Because it’s the same all the way down” implied a logical connection to the idea of rate and first differences without Steve explicitly saying the term ‘rate’ or ‘first differences’.

   In the following transaction, the teacher was talking with the class. She asked the students to explain their thinking and to convince her that each table of values was linear or not.

   Teacher: Now, what else? You haven’t really convinced me that this is a linear relationship yet. You said it was linear and you know this…and you made some keen observations…. What else? You said it was linear, what else did you find? Yep.
   Joe: We can turn it into an equation.
Teacher: We can turn it into an equation. So before Joe says his equation that his
group determined, who else got an equation for this. Hands up. How
many of you got an equation that links x and y together? Two groups?
Three groups? [ 7 groups in total] So we are going to listen to these
groups, so what was your equation?

Ryan: y = -5 subtract 2x

Teacher: So he got y = -5 -2x. Is that the same one that you guys got Joe?

Joe: Yep!

Teacher: Okay. So the question is …. First of all….is that written in the form
y = mx+b?

Students: Yes. But… differently

Teacher: Okay so can we change the order so it looks more like y = mx+b? what
would it be?

Fred: -2x-5 [whispering to self]

Teacher: y =-2x-5. Now, does this equation always work for all the points I gave
you. It has to always work. Did you make sure and check for all the
points?

Martin: For that equation? No, I didn’t check for that equation.

Teacher: We’ll check. If we put 4 in there, will we get negative 13? Does it work?

If we put 4 in for x, do we get -13 out for y?

(Teacher, Joe, Ryan, and Martin, Classroom Observations)

When the teacher asked the students to convince her that the table of values represented a
linear relationship, Joe replied. “We can turn it into an equation.” This meant that the
teacher was looking for the solution to be represented in a different way. Joe offered the algebraic representation (i.e. an equation). The teacher asked the class to determine how many groups of students had developed an equation, then proceeded to have the students that had developed an equation explain how they created their equation to the rest of the class. The teacher’s questions enabled students to discuss the problem and its notation freely in class. The questions created discussion topics of which students could argue. The last question in the transaction, “Does it work? If we put 4 in for x, do we get -13 out for y?” was verification in nature. She asked students to check their work and tell the class if they had the same or different response.

Thus the nature of the classroom discourse was inquisitive and friendly. The teacher focused on the students’ answers and pressed for further explanations. Each transaction between the teacher and students employed discourse moves such as clarifying notation, conceptual understanding, and mathematical language acquisition. The teacher used questioning to activate prior knowledge, draw out student misconceptions, clarify and explain student thinking, reorganize word meaning, and correct notation use.

**Summary of Classroom Observations**

The classroom observations revealed five themes in total: (1) *Student Driven Learning*, (2) *Development of a Common Language*, (3) *Mathematical Thinking within Context*, (4) *Questioning and Directing*, and (5) the *Nature of the Classroom Discourse*. *Student Driven Learning* demonstrated that students directed portions of their learning for both mathematical language and concepts whether students discussed problems together in small group settings without the teacher or as a whole class. *Development of a Common Language* expresses students’ demonstration of characteristics of language acquisition such as the exploration, use,
and application of new mathematical terminology. Development of a Common Language encompasses students’ use of mathematical language confidently when discussing problems and when students peer and self-corrected terminology use. Questioning and Directing explained how the teacher directed student learning and how the teacher maintained students’ focus on their learning, discussed misconceptions with the whole class, and corrected student thinking and terminology use. From the classroom observations, it was seen that students were less likely to acquire new mathematical terminology when they had control of their learning in contrast to when the teacher had control of learning in the classroom. Mathematical Thinking within Context was evident throughout the classroom observations from oral statements of students’ mathematics work, problem solving strategies, or integration of mathematical language. From the classroom observations, it was seen that students acquired mathematical language through the application and use of contextual problem solving (i.e. Mathematical Thinking within Context). Students that spoke in class, asked questions one out of six times that they spoke. When the questions were directed at their peers they were richer in question type and more collaborative in nature. The nature of the classroom discourse was relaxed and jovial when the students worked together. Students pushed one another with questions about conceptual understanding of the mathematics they learned during the lesson.

Student Work

Student written work was collected before and after the classroom observations as a means to compare students’ use of classroom and academic mathematical language in their written work and to compare language use for students that actively and passively engaged in speaking during the observed classes. The most common range of improvement in language use appeared to be an improvement of one rubric level. The second most frequent improvement was a change of two
rubric levels. The maximum change of levels in language use was three rubric levels, and the minimum change in level of improvement in language use was a decline in one level. No themes emerged during this analysis, but the results from this data supported the themes from classroom observations and interviews.

Analysis indicated some notable changes. The three non-speakers (Leonard, Heather, and June) within the classroom observations had improvements in language use of two, two, and one rubric level, respectively. Ten of the thirteen students that spoke during class make improvements in language use over the week of observations. See Table 4-2 and Table 4-3 for the data on rubric level changes.

Table 4-2

<table>
<thead>
<tr>
<th>Name</th>
<th>Rubric Level from Task 1 Before Lessons</th>
<th>Rubric Level From Task 2 After Lessons</th>
<th>Change in Rubric Level After One Week of Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quinton</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Fred</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Martin</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Joe</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>June</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Leonard</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Ella</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Jenn</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Lynn</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Lauren</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Ben</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Neil</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Heather</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Ryan</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Vanessa</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Steve</td>
<td>2</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Note: List of Students’ Level on Assessment Rubric for Student Written Work. Includes Rubric Level for Task 1, Task 2, and the Change in Rubric Level.
Table 4-3

<table>
<thead>
<tr>
<th>Level change in language use in written work over the observation period</th>
<th>Frequency</th>
<th>Names</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement of one level</td>
<td>7</td>
<td>Ella, Jenn, Lynn, Lauren, Ben, Neil, Heather</td>
</tr>
<tr>
<td>Improvement of two levels</td>
<td>5</td>
<td>Fred, Martin, Joe, Leonard, June</td>
</tr>
<tr>
<td>No Change</td>
<td>2</td>
<td>Steve and Vanessa</td>
</tr>
<tr>
<td>Reduction of one level</td>
<td>1</td>
<td>Ryan</td>
</tr>
<tr>
<td>Improvement of three levels</td>
<td>1</td>
<td>Quinton</td>
</tr>
</tbody>
</table>

*Note: Frequency distribution of students’ written work change in level of language use over observation period.*

From the students observed, one of sixteen (Ryan) showed a reduction in his mathematical language use within his written work, two of sixteen (Steve and Vanessa) showed no change in their mathematical language use in their written work, and thirteen of sixteen showed at least one level improvement of mathematical language use in their written work. This demonstrated that most students, whether they spoke in class or not, made an improvement in language use from one week of instruction where language acquisition was not the sole focus of class, however, the largest improvements were seen by students that spoke more frequently in class.

A frequency count of the mathematical language use was conducted on the submitted student written work for both before and after the observation period see Table 4-4. This frequency count was completed for instances of academic language and classroom language. The statistics from these counts are as follows: the mean increase of mathematical language was 5.7 words or phrases per student. For non-speakers the mean was an increase was six words or phrases per student, for speakers (removing non-speakers) the mean was an increase of 5.6 words or phrases per student. Twelve out of the sixteen students in the class had an increase of three or more words or phrases in their written work, ten out of sixteen students in the class had
an increase of five or more words or phrases in their written work, six out of sixteen students in the class had an increase of seven or more words or phrases in their written work, and thirteen out of sixteen students experienced some level of improvement in language use across their written work versus no improvement or a reduction in mathematical language use. See Table 4-4 to see students listed in order of highest to lowest increase in mathematical language use in student written work from before and after the observational period.

For the six students who spoke less than twenty times during the classroom observations the mean frequency of speech was ten interactions per student. Each of these students had a change in mathematical language use ranging from an improvement of one to a deterioration of one rubric level, and a change in frequency of mathematical language use in written work of a range from an increase of five words per student to a decrease of one word per student. The mean improvement in frequency of mathematical language use in written work was three words or phrases per students. This increase is seven words or phrases per student more than students that spoke less frequently during the classroom observations. Therefore, students who spoke more than twenty times in class, had an average of fifty interactions, improved their frequency of mathematical language use in their written work by an average of ten words or phrases per student.

Five of the sixteen students’ (Leonard, Neil, June, Vanessa, and Ryan) results indicated improved level of mathematical language use in written work was not related to the frequency of speech in class. It appeared Leonard, Neil, and June improved their mathematical language use in their written work (from their level score and their frequency). Vanessa made no improvement between the first and last written task. Ryan’s use of mathematical language appeared to decrease in his written work from the first to the last written task.
Table 4-4

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of times spoke in class</th>
<th>Increases in Mathematical Language Use in Student Written Work Across Before and After Observational Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe</td>
<td>26</td>
<td>18</td>
</tr>
<tr>
<td>Martin</td>
<td>69</td>
<td>12</td>
</tr>
<tr>
<td>Quinton</td>
<td>21</td>
<td>10</td>
</tr>
<tr>
<td>Leonard</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Neil</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Ella</td>
<td>23</td>
<td>7</td>
</tr>
<tr>
<td>Fred</td>
<td>110</td>
<td>6</td>
</tr>
<tr>
<td>Lynn</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>Ben</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>June</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Heather</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Jenn</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>Lauren</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Vanessa</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>Steve</td>
<td>11</td>
<td>-1</td>
</tr>
<tr>
<td>Ryan</td>
<td>37</td>
<td>-2</td>
</tr>
</tbody>
</table>

*Note: Increases in Mathematical Language Use in Student Written Work Across Before and After Observational Tasks*

Student written work as a change in frequency of mathematical words or phrases, and change in written level of mathematical language use was compared to the frequency of which students spoke in class. It was noted that students that were non-speakers during the observed lessons did not have the lowest change in improvement of language use levels, nor the lowest change in frequency of mathematical words or phrases in their written work.

Thirteen out of sixteen of students demonstrated improvement in their written language use. There was a tendency that students who spoke more often in class had greater improvement in their mathematical language use in written work (See Table 4-4). Further analysis was conducted by ranking students from highest to lowest across three categories: frequency at which
a student spoke during class observations, the change in level score of mathematical language use in written work, and the change in frequency of mathematical words or phrases in the classroom or academic register in student written work. Students were considered to have a ‘consistent ranking’ if their name appeared within eight ranked positions on each list.

Students were considered to have ‘inconsistent rankings’ if their name appeared further than eight ranked positions from itself on one of the three categories, see Table 4-5 and Table 4-6. A distance of no more than eight ranked positions was chosen because it is half the amount of participants and as such allows the opportunity for non-speakers to demonstrate an ‘inconsistent ranking’ if they demonstrate more improvement than half of their class on the list. This will be clarified with some examples. Once students were ranked from highest to lowest, it was observed that only eleven of sixteen students (Martin, Quinton, Joe, Fred, Ella, Ben, Jenn, Lynn, Lauren, Heather, and Steve) were consistently placed across all three categories, meaning that students scored similarly in comparison to their peers in each category. For example, Martin was ranked as the second most frequent speaker in class, second highest improvement in language use level, and second highest in change in frequency of mathematical words or phrases. Martin was ranked similarly across all three categories (frequency at which a student spoke during class observations, the change in level score of mathematical language use in written work, and the change in frequency of mathematical words or phrases in the classroom or academic language in the mathematics register in student written work) in comparison to his peers, see Table 4-5 and Table 4-6. Heather was ranked in the bottom three for frequency of speech in class, fourth lowest improvement in language use level, and seventh lowest in change in frequency of mathematical words or phrases. Heather was consistently ranked within five places on each list. The other five students (Leonard, Neil, June, Vanessa, and Ryan) were present with consistent rankings across
two of the three categories, which meant that these five students had inconsistent rankings across the three categories.

Eleven of the sixteen (Martin, Quinton, Joe, Fred, Ella, Ben, Jenn, Lynn, Lauren, Heather, and Steve) students’ improved level of mathematical language use in written work could be anticipated to some extent by their frequency of speech in class. Five out of sixteen (Leonard, Neil, June, Vanessa, and Ryan) students’ improved level of mathematical language use in written work could not be anticipated by their frequency of speech in class since they did not present consistency in all three measured categories.

Frequency of speech was able to anticipate improvements in written work for Martin but not for Leonard who did not speak in class during the observation period but was ranked in the top five students for improved mathematical language in his written work over the course of the observation period. The reverse occurred with Ryan as he spoke thirty-seven times in class but made no overall change in his use of mathematical language in his submitted written work and had a negative change in his frequency of words or phrases in classroom or academic language within his submitted written work. Although Ryan, Vanessa, and Steve spoke in class, their written work was not reflective of improved mathematical language. See Table 4-6 for the rankings showing students in order of highest to lowest across the three categories. In Table 4-6 a rank of one in the left column meant students spoke frequently in class and had a high level of improvement in mathematical language use in their written work. A rank of eleven in the left column means students did not speak often in class and did not demonstrate a high level of improvement in mathematical language use in their written work. A rank of one in the right column means that the student showed a high level of improvement in mathematical language use in their written work but did not speak often in class, and a rank of five in the right column
means that the student demonstrated no improvement in mathematical language use in their written work but did speak in moderately high frequency in class.

Table 4-5

<table>
<thead>
<tr>
<th>Total Number of Times Speaking in observed classes</th>
<th>CHANGE in LEVEL</th>
<th>CHANGE in Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fred</td>
<td>Quinton</td>
<td>Joe</td>
</tr>
<tr>
<td>Martin</td>
<td>Fred</td>
<td>Martin</td>
</tr>
<tr>
<td>Ryan</td>
<td>Martin</td>
<td>Quinton</td>
</tr>
<tr>
<td>Joe</td>
<td>Joe</td>
<td>Leonard</td>
</tr>
<tr>
<td>Ella</td>
<td>June</td>
<td>Neil</td>
</tr>
<tr>
<td>Quinton</td>
<td>Leonard</td>
<td>Ella</td>
</tr>
<tr>
<td>Vanessa</td>
<td>Ella</td>
<td>Fred</td>
</tr>
<tr>
<td>Jenn</td>
<td>Jenn</td>
<td>Lynn</td>
</tr>
<tr>
<td>Lynn</td>
<td>Lynn</td>
<td>Ben</td>
</tr>
<tr>
<td>Steve</td>
<td>Lauren</td>
<td>Heather</td>
</tr>
<tr>
<td>Lauren</td>
<td>Ben</td>
<td>June</td>
</tr>
<tr>
<td>Ben</td>
<td>Neil</td>
<td>Jenn</td>
</tr>
<tr>
<td>Neil</td>
<td>Heather</td>
<td>Lauren</td>
</tr>
<tr>
<td>June</td>
<td>Ryan</td>
<td>Vanessa</td>
</tr>
<tr>
<td>Heather</td>
<td>Vanessa</td>
<td>Steve</td>
</tr>
<tr>
<td>Leonard</td>
<td>Steve</td>
<td>Ryan</td>
</tr>
</tbody>
</table>

*Note: Names of each participant are in place of the participants score, from highest to lowest, for each of the three categories described.*

Table 4-6

<table>
<thead>
<tr>
<th>Ranks</th>
<th>Present Across All Three Categories Within 8 Rankings</th>
<th>Present Across Two Categories Within 8 Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Martin</td>
<td>Leonard</td>
</tr>
<tr>
<td>2</td>
<td>Quinton</td>
<td>Neil</td>
</tr>
<tr>
<td>3</td>
<td>Joe</td>
<td>June</td>
</tr>
<tr>
<td>4</td>
<td>Fred</td>
<td>Vanessa</td>
</tr>
<tr>
<td>5</td>
<td>Ella</td>
<td>Ryan</td>
</tr>
<tr>
<td>6</td>
<td>Ben</td>
<td></td>
</tr>
</tbody>
</table>
Student Interviews

Student interviews were inductively analyzed, through open, axial, and selective coding. The three themes that resulted from this process were: (1) *Importance of Learning Mathematical Language from the Student Perspective*, (2) *Students Engaged in Language Acquisition*, and (3) *Untangling Expectations*. *Importance of Learning Mathematical Language from the Student Perspective* expressed student motivation for learning mathematical language (both oral and written). Students’ stated that learning mathematical language was important to them so that they were able to read peer work, become more succinct communicators, and be able to describe their thinking to others. Students stated that they felt that learning mathematical language would allow them to relate mathematics to the world around them, and to obtain a deeper understanding of the concepts that mathematics terminology represents. *Students Engaged in Language Acquisition* described students’ recognition and belief in the process of mathematical language acquisition, which supported student exploration of terminology through experimentation, using, applying, and re-organizing terminology to integrate it into student vocabulary, make linguistic connections to real world scenarios, and the development of confidence in mathematical language use. *Untangling Expectations* were students’ reasoning that their teacher required, believed in, or discussed, learning mathematical language so that students could complete their...
course work, to do well on assessments [particularly summative tasks], and ensure that others were able to follow your work in a coherent way.

**Importance of Learning Mathematical Language from the Student Perspective**

Captured in *Importance of Learning Mathematical Language from the Student Perspective* students expressed a desire to be clear in their meaning to other students, the teacher, and to themselves. They wanted to understand the similarities and differences in how to use mathematical terminology in and outside of class, and they wanted to be able to make meaningful connections between their work in class and in other classes.

*Importance of Learning Mathematical Language from the Student Perspective* was observed throughout the nine student interviews as students explained what mathematical language was, described its importance to their learning process, and discussed communication skills in mathematics. Students described that understanding mathematical terminology helped clarify their meaning to their peers and allows them to use more precision in their meaning when giving answers or writing solutions. Students explained that understanding the terminology used in class ensured that they understood tasks given in class as they were derived from the same mathematical language. Students rationalized that understanding mathematical language was an indicator for learning mathematics, and granted access to understanding problems in class and outside of class.

Ella discussed how using simplified everyday language and everyday language could negatively impact student communication skills within the classroom and in student work.

I think a lot of time when you talk about mathematics you… if you use more commonly used words then you can be misinterpreted, but if you use mathematics vocabulary than it might be more clear and to the point what you are talking about.
Ella stated that by using simplified everyday language, one’s peers and teacher could misinterpret what students were saying. She explained that she believed it was important to be clear in student meaning and the best way to do this was to use the mathematical language with the most appropriate terminology to express student thinking. Ella continued to explain that often they used a variety of language in the mathematics register that were adapted from everyday language and yet had a significantly different meaning in mathematics than it does in the world outside of mathematics.

It is also …like… a lot of processes that you do in math aren’t the same things as what you do everyday, so you need your own set of words to describe these processes that are very different, that you might do on a regular basis that you might do outside of the math class.

Ella stated that clarity existed within strong communication, which to her was the development and use of mathematical vocabulary. She stated an understanding that there were often words that were used within mathematics class that had different meanings outside of mathematics and that these differences needed to be precisely understood to ensure that there were no misconceptions or misinterpretations.

June explained how understanding mathematical language was important as it signaled if she understood the mathematical concepts the terms represented.

If I am not understanding a problem, it usually means that usually means that I am not understanding the whole concepts that we’ve been working with in class, …
because if I don’t know how to use these concepts in class then the vocabulary doesn’t really mean anything.

(June, Student Interview)

June commented that the lack of understanding (and completing) a problem occurred when she didn’t understand the overall concept, and when she did not understand a concept it indicated to her that she did not understand the terminology used in class.

Martin expressed the importance of understanding and use of mathematical language as a gage of students’ individual learning.

… key phrases that you might need to understand the definition of them, because on a test or something, you might understand like 90% of the question, but you say oh, like, what is that one thing, what is the angle bisector mean? So I know about angles and triangles but what does the angle bisector mean? That’s why you should always do like a self-check, do I understand what’s happening in class?

(Martin, Student Interview)

He stated that understanding academic language could allow for student understanding of problems or written questions, and that understanding mathematical vocabulary could essentially “unlock” a problem for a student.

Ella, June, and Martin exemplified student articulations during the interview process of why students believed that learning mathematical language was important for them. All interviewed students explained that learning mathematical language was important to them so that they could be strong communicators, understand underlying mathematical concepts, and relate their learning to others.
Students Engaged in Language Acquisition

*Students Engaged in Language Acquisition* meant that students reflected on a developing understanding of mathematical language during the observation period. *Students Engaged in Language Acquisition* appeared in students’ comments on long-term development of mathematical language over the semester or observational period as they attempted to correct their use of mathematical terminology from written work or oral discussion, and their description of the process of language acquisition as they experienced it during class.

Quinton commented on his own mathematical language acquisition during the observational period. He explained his acquisition of the term slope and mentioned that moving forward he would understand the various terms related to slope.

We are always prepared for what were going to be doing … Like for example, before going into this unit I would not have know what the slope of two different points but going into the next test I would know what that meant and the other terminology or vocabulary that will be used along side with slope.

(Quinton, Student Interview)

Quinton observed that with each new unit of study he was faced with new mathematical language. He stated that by the end of the unit that he, and his peers, were well prepared for their summative assessments that would use this terminology.

Ella reflected on her written work during her interview and noticed that she had a better understanding of the mathematical language and could improve her written work.

…where it [the problem] says that the values represents the base or the sum mathematically and I think that now I could use the b or like the y – axis [y-intercept] so it starts on the y-axis [y-intercept], to represent there, instead of the base, ….and I
could possibly identify the slope of the line to see if one is steeper than the other as oppose to carrying out the full equation with the numbers plugged into it.

(Ella, Student Interview)

Ella demonstrated a belief in active language acquisition as she looked through her before observations written work (Task One) and attempted to describe the mathematical vocabulary that she would use if she were to complete the work after the learning that occurred during the observation period. Ella also demonstrated that she was still engaged in the process of actively acquiring mathematical language as she had not fully integrated the term “y-intercept” into her vocabulary although she demonstrated clear understanding of the concept of a y-intercept as she connected it to the problems scenario and the terminology utilized within the problem.

Lauren discussed her observation of the teacher’s use of language and that she believed students would acquire the mathematical language from the teacher’s consistent use of the language in class.

She [teacher] uses it [mathematical language] a lot, and it [mathematical language] kinda becomes learned to say it like that [the way the teacher does]. So like if she says it a lot in class, then we’re going to pick up on that… if she asks a question based on work that we just did. Like another problem just like what she just did, …then you could hear us…sort of starting to use those words.

(Lauren, Student Interview)

Lauren gave an example of when students were observed “picking up” the teacher language as they worked in small groups on a similar type of problem to that of which the teacher might have just worked on in class. Lauren stated that students were then heard using the language in the small groups.

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Quinton, Ella, and Lauren described a variety of ways that they have observed mathematical language being acquired during the observational period. The students discussed their personal language acquisition that occurred as well as their observations of peer language acquisition.

**Untangling Expectations**

*Untangling Expectations* indicates students’ distinctions between reasons they believe learning mathematical language was important and the reasons the teacher believed were important. Students indicated that they believed the teacher’s emphasis of the importance of learning mathematical language was based more in written work than in oral usage. Students indicated they believed the teacher thought that using and understanding mathematical language was extremely important. However, when asked to give evidence of how they knew this understanding was important to the teacher, students predominately discussed written feedback regarding submitted written work and usually for summative purposes.

Neil described his experience learning mathematical vocabulary. He indicated that time in class was mostly spent working on problem solving and little time was spent explicitly on mathematical language development.

I don’t think that [enough time is spent on learning vocabulary]…because… once we learn a new word we’ll get an example of that, we’ll just move on to using that language but not really using it.

She’ll just give us a problem, and we’ll just try to process that language as we’re using it… [I think we spend] More [time] on our written work than in the class because we don’t lose a mark during class…if we use the wrong language but on a test…if we use different language then we get marks taken off.
Neil stated that he perceived feedback on mathematical terminology use occurred on summative written work and not during oral work in class time. Although, Neil stated that he felt that there was an expectation that students were to use appropriate mathematical language both in class and in their written work.

Quinton stated that the teacher placed great importance on learning mathematical language so that students were able to communicate a strong solution in their written work. He described the teacher’s process of giving written feedback to students on their summative work.

She [the teacher] would often stresses that if you don’t use the correct terminology or you don’t show your full step by step process, she would comment on that process, and she would say ‘bad form’. Meaning that you should think more of how you would demonstrate you process and that you should use more precise vocabulary and terms.

Quinton had an understanding that mathematical language use was very important to the teacher and that the development and use of this language was going to help him develop strong solutions with higher levels of precision. He only commented on written feedback on written summatives as his evidence of why he believed the teacher placed importance on students learning mathematical language.

Ryan stated that the teacher had a high expectation of student language development and the use of mathematical language in solutions. He did not specify if this expectation was for oral or written work.
I know that she extremely does [place emphasis on learning mathematical language]. She is always hard on me. She always says that you have to use certain words. When I think about it after she’s done, it does make more sense to use the words that she’s told me, it gives me a more detailed answer that’s more really…realistic and easier to understand.

(Ryan, Student Interview)

He emphasized that the teacher’s feedback did help him to gain further insight on the importance of learning mathematical vocabulary as it allowed him to be more succinct and clear in his meaning when describing his solution to others.

Neil, Quinton, and Ryan expressed Untangling Expectation was based on clear communication of solutions to their peers in writing more than strong oral communication. Each of the three students stated that feedback was given to them on their written work.

Student interviews produced three themes: (1) Importance of Language from the Student Perspective, (2) Students Engaged in Language Acquisition, and (3) Untangling Expectations. Eight out of nine interviewed students stated that learning mathematical language was important to them to communicate with others effectively, and all interviewed students described ways in which learning mathematical language benefited them as learners of mathematics. Students expressed an understanding that their teacher placed learning and the use of mathematical language in their work as a high priority. Both the students and the teacher interviews indicated that the students and the teacher observed active language acquisition within the classroom among students both from the teacher to student, and between students.
Summary of Results

In this chapter, results were presented from a three-day observational period that included data collected from teacher interviews, classroom observations, student work, and student interviews. Themes were developed from the data sources. From the teacher interviews three themes resulted: (1) Importance of Language, (2) Language Acquisition Through Collaborative Problem Solving, and (3) Problem Solving Inquiry. From the classroom observations five themes were developed: (1) Student Driven Learning, (2) Development of a Common Language, (3) Mathematical Thinking within Context, (4) Questioning and Directing, and (5) the Nature of Classroom Discourse. Lastly, from student interviews three themes were identified: (1) Importance of Learning Language from the Student Perspective, (2) Students Engaged in Language Acquisition, and (3) Untangling Expectations. From the teacher interview, student interviews, and from the classroom observations all participants believe in the value of mathematical language acquisition. Students and teachers both had essentially the same motivations for learning mathematical language in class: to communicate well with other students, ensure clear expression of reasoning and logic, to understand subtleties in the use of mathematical terminology in and outside of mathematics class, and to make meaningful connections across curriculum and community. From the student interviews it was evident that students also felt that the teacher’s motivations for students to learn mathematical language were more task oriented such as being successful on summative written work, to be able to complete their homework, and to be able to communicate their work in a standardized manner.

Students’ submitted written work from before and after observations illustrated improvement in both the written level of mathematical language use and the frequency of mathematical language use in written work over the period of one week’s instruction for thirteen
out of sixteen students. Mathematical language acquisition appeared to occur in the context of classroom discussions of mathematical problems (i.e. seventy-two percent of all students’ oral transactions in class were coded inclusively as Mathematical Thinking within Context and/or Development of a Common Language in all combinations). Students acquired mathematical language most when the teacher was able to focus student interactions (i.e. fifty-one percent of all students’ oral transactions in class were coded as Questioning and Directing and/or Development of a Common Language in combinations excluding Student Driven Learning).

Considering all available themes, an overarching theme emerged as the Importance of Learning Language. Both the teacher and the students wanted to be able to communicate effectively with others. The Development of a Common Language (i.e. language acquisition) through oral discussions more than in written work emerged as a common theme when student learning was rooted in problem solving (or inquiry based activities) and was facilitated by the teacher through Questioning and Directing. Teacher Questioning and Directing was pivotal to the creation of the theme the Nature of Classroom Discourse -- the opportunities for students to orally clarify their conceptual understanding of mathematics. These four themes were areas that contributed more to the results of the study than other themes presented. A thorough discussion of these overarching themes and other results from this chapter will be presented in the next chapter.
Chapter 5

Discussion

In this chapter, the question, ‘How does the use of purposeful oral mathematics language in the classroom affect students mathematics discourse during problem solving and appear in their written work?’, will be answered using the results presented in the previous chapter supported by current mathematics and linguistics research. To most effectively answer the research question, three other questions were also defined: What is the nature of the teachers’ oral discourse in the classroom?, What similarities and differences exist between teachers’ and students’ oral mathematical discourse?, To what extent do students’ use of oral mathematics language in class appear in students’ individual written solutions of problems?

From observations taken during three days of class, teacher and student interviews, and from student written work, this study has determined that the use of purposeful oral mathematics language in the classroom positively impacted students’ discourse during problem solving and appeared in their written work. There existed a transfer of mathematical terminology from the teacher’s oral language to the students’ oral language, which was seen in submitted student written work. Overall students improved their use of mathematical language over the course of one week of instruction. By exploring the nature of the teacher’s oral discourse in the class, it was determined that the teacher’s inquiry and discussion-based classroom practice created a mathematics discourse community which allowed students to feel comfortable to ask questions and speak during the observation period. The similarities and differences between the teacher’s and students’ oral discourse demonstrated that as the teacher spoke in a higher academic language of the mathematics register than the students, students acquired mathematical language.
In particular, students acquired mathematical language when the teacher controlled the discourse and could determine students’ misconceptions and openly discuss them during the classroom lesson time. Students’ oral language was acquired from the teacher’s use of oral language in class. Once students understood mathematical terminology and concepts orally they used these terms and concepts in their written work when prompted.

**The Nature of the Teacher’s Oral Discourse**

To answer research question one about the nature of the teacher’s oral discourse, three themes were important: teacher *Questioning* to keep students on task, *Developing a Common Language* developed a classroom discourse, and that the teacher’s belief in *Language Acquisition Through Collaborative Problem Solving* developed the central idea that mathematics, for her, is taught through problem solving.

The nature of the teacher’s oral discourse was flexible. She moved across the mathematical register using simplified everyday language to academic language. *Questioning and Directing* and *Development of a Common Language* was evident as she moved between language levels of the register in her discourse to ensure conceptual understanding and mathematical language learning. For example, consider the transaction between Steve, the teacher, and Jenn in a whole class discussion taking up a homework problem after Vanessa asked for clarification on the concept of slope. The teacher asked Steve to explain his earlier response further, when he did, she then simplified his everyday language further breaking the language down into an algorithm.

Steve: Well cause depending on what x is and the number of years times the 7500 is higher than it was so the cost of the house is getting larger as the years pass by.
Teacher: Right, so if you put in the number one, you’ll get a certain number, and if you put in a value like two, you’ll get a bigger number, if you put in a number like three, you’ll get a bigger number. So a value of house is going up…. What do you call it when the value of something increases…like the value of a house…..we talked about this last week….there was a term…we talked about cars going down in value…..and other investments that went up in value. So a good investment is something that goes up and a bad investment is something that goes down in value over time.

Jenn: Is the one going down called depreciation?

(Steve, Jenn, and Teacher, Classroom Observations)

The teacher simplified Steve’s everyday language to break it down to the conceptual understanding of the algorithm involved when she stated, “if you put in the number one, you’ll get a certain number, and if you put in a value like two, you’ll get a bigger number.” She simplified the term ‘increase’ to “a value of house is going up” to ensure student conceptual understanding. Using everyday language helped students understand the mathematical concepts, the teacher simplified the language further to ensure that all students in the classroom understood both the conceptual meaning and build their mathematical vocabulary. Vygotsky’s (1978) theory of Zone of Proximal Development asserts that students learn and acquired new language when they communicate and collaborate with an expert (peer or teacher). Having an expert (teacher) is valuable because mathematics classes use language structures differently than the way that language is used outside of the field (Schleppegrell, 2007, and Pimm, 1987).

Halliday (1978) described the mathematics register as,
…a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a ‘mathematics register’, in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes.

(p. 195-196)

Halliday (1978) suggested that learning mathematics draws on ‘everyday’ language and uses it to develop understanding of new terminology and concepts that have academic language that is to be acquired by students. Hence, students learn the mathematics register from their teacher. Since the levels of language in the register that exist within the mathematics classroom are everyday, a combination of everyday and academic (classroom), and academic language (Moschkovich, 2007; Schleppegrell, 2007), then the teacher’s flexibility in language use demonstrated mathematical language appropriate for students’ cognitive understanding and provided examples of new, slightly more advanced mathematical language within the mathematics register.

In another transaction, the teacher discussed slope as a rate, and as unit rate. She used three levels of register, classroom, everyday, and simplified everyday, to ensure that students gained conceptual understanding as well as acquired mathematical language with the goal of speaking in the classroom or academic language of the mathematics register.

Teacher: What do you call that? So she said, it’s the relationship of by how much y increases as x increases by one. Do you know what we call that?

Jenn: A rate?
Teacher: It is absolutely a rate. Can you give me an example with actual numbers?

Jenn: Every hour this person makes three dollars per hour.

Teacher: [Writes and says] $3/hr. Sometimes we use this [/] because as mathematicians we are notoriously lazy so if there is a way to write something symbolically then we do it. So three dollars per one hour.[pointing and emphasizing each word]. Give me another example of a rate like that [points to the board].

Vanessa: Sixty kilometers per hour.

Teacher: Perfect, sometimes we call these unit rates because it’s out of one or has a one on the bottom. Anyone else want to discuss their understanding of unit rates right now?

(Jenn, Vanessa, and Teacher, Classroom Observations)

Questioning and Directing and Development of a Common Language appears through the teacher’s activation of students’ prior understanding of the term ‘increase’ and applying it to a mathematical scenario. She used this term ‘increase’ in her description of a mathematical relationship between the variables ‘x’ and ‘y’. The teacher had students make connections to their prior knowledge of rates and unit rates, which deepened their conceptual understanding as they were prompted to re-organize their prior knowledge to incorporate the newly acquired mathematical language. The teacher used simplified everyday language when she said, “it’s out of one or has a one on the bottom” when she referred to unit rates to ensure that all students understood that unit rates were the rate of change with respect to any unit with a value of one. The teacher modeled flexibility with the mathematical register, which enabled her students to understand multiple representations of the same concepts.
In this last transaction, we saw the teacher use a mixture of classroom and academic terminology appropriate for the grade nine academic mathematics class. The students and teacher discussed slope as a rate when the rate of change was changing, leading to the terminology and concept of acceleration and deceleration.

Teacher: Negative slope, partial variation…. Is there another expression that we connect with speed or the change in speed… so we use delta to be the change in… so we have the change in …speed…[with students] over time. What do we call that in real life?... When your speed is changing?

Lauren: Acceleration.

Teacher: Acceleration… Well … In this case, what is it actually…

Students: Deceleration.

Teacher: Wow, great! Deceleration. So if you were going to describe this you would say… Oh my goodness this car is actually…doing what over time?

Ryan: Decelerating.

Teacher: Is it decelerating, how I decelerate? Where I slam on my breaks, then I slam on breaks again... Then I slam on my breaks again...is it doing that...?

Fred: No no its really … peaceful…like a steady rate…no its not.

Ryan: Is slow…no…it’s a steady rate…

Teacher: Again.

Ryan: It’s a steady rate of deceleration

Teacher: It’s a steady rate of deceleration. Good. So now…what you are going to be doing for me…
The teacher described the ‘the change in speed’ in the ‘classroom’ language, then simplified the language to “speed is changing” when she asked, “what do we call that in real life?” Students connected their prior knowledge of everyday language to volunteer the terms ‘acceleration’ and ‘deceleration’ to describe and instance when speed changed. The terms ‘acceleration’ and ‘deceleration’ were developed as the academic language in the mathematics register in context. The teacher then discussed a deceleration curve with students and continued to correct students in effort to develop their mathematics register, for example, the smooth curve as “its really … peaceful…like a steady rate” to “a steady rate of deceleration.”

**Similarities and Differences Between Students’ and Teacher’s Discourse**

To answer research question number two, the similarities and differences between the students’ and teacher’s discourse in the classroom are presented separately.

**Similarities**

The similarities of the students’ and teacher’s discourse in the classroom illustrated the norms of communication between the students and teacher within the observed lessons. Classroom observations demonstrated similarities in the nature of both student and teacher questions (the teacher theme of *Questioning and Directing* and the student theme of *Student Driven Learning*). The teacher’s and the students’ beliefs in language acquisition (via the teacher theme of *Language Acquisition Through Collaborative Problem Solving* and the student theme of *Students Engaged in Language Acquisition*) were demonstrated through the teacher’s use of questioning during the lessons. The teacher’s and the students’ *Importance of Language Learning* was seen through student and teacher questioning and negotiation of meaning.
Both the teacher and students asked clarifying questions throughout class. The teacher asked students to clarify their thinking, provide examples, and explain their meaning more fully, whereas the students asked the teacher to clarify the meaning of concepts, terminology, and notation. Both used clarification questions as a method to negotiate meaning of mathematical concepts and terminology. This negotiation helped the teacher and students to develop clear expectations for their communication in solutions and with others (Cazden & Beck, 2002; Yackel & Cobb, 1996; Cobb, Yackel & Wood, 1992).

From Figure 4-1, it was noticed that Student Driven Learning accounted for approximately twelve percent of all students’ oral interactions, and teacher driven Questioning and Directing accounted for approximately thirteen percent of students’ oral interactions during the lessons. The student-directed time mostly occurred when students were in small groups discussing an assigned problem or task. The teacher-directed time involved students answering the teacher’s questions in whole class discussions. The teacher and students directed similar percentages of students’ oral interactions during the observation period but each theme’s percentage of student interactions reflected different uses of class time and components of the learning process. In Ames’s (1992) article on student motivations she suggested that students would improve their learning and interest through the ability to direct some of their learning through mathematical discourse in the classroom. During her study two classes were observed, one where the teacher engaged in traditional teacher-centered practices and another that engaged in mathematical discourse through the use of inquiry based activities. Students in the class that practiced mathematical discourse learned and applied more strategies due to students’ in-depth conversations around problems before beginning them (Ames, 1992). Gee (1996) states that when students develop an understanding of language it also grants them power over their own
learning. With this power students develop the ability to ask questions and discuss what they are learning.

**Differences**

The differences between the students’ and teacher’s discourse in the classroom demonstrated strategies of how students acquired mathematical language within the observed lessons. The themes of *Importance of Language* from the teacher interviews and *Importance of Learning Language from the Student Perspective* during the student interviews demonstrated differences in the importance of the development of students’ oral mathematics register during the lessons. The themes of *Questioning and Directing* and *Student Driven Learning* from the classroom observations illustrated the difference between how the teacher and students use questioning differently within class and small group interactions for students to acquire new mathematical language and concepts. The themes of *Language Acquisition Through Collaborative Problem Solving* and *Mathematical Thinking in Context* from the teacher interviews and classroom observations demonstrated that the nature of the teacher’s oral discourse and problem solving approach allowed for open discussion of problems that clarified misconceptions early in student learning whereas the nature of students’ oral discourse when problems solving could consolidate misconceptions.

The differences between the teacher’s and students’ oral discourse in the mathematics classroom were the level of mathematics register used, the nature of questions, and the control of learning. The teacher used ‘classroom language’ most frequently in class where students’ predominately used the ‘everyday language’. However, the student written work from before and after the observation period reflected a transition from simplified everyday to ‘everyday’ and/or ‘classroom language’, which indicated that most students were able to use the classroom
language but chose not to during the classroom observations. The teacher’s and the students’ oral and written mathematical registers appeared to have a gap of one full mathematical register level (i.e. if the teacher speaks and writes using academic language and the students speak and write using classroom language). Orally, many students’ oral language use was somewhere between everyday and classroom, whereas the teacher’s oral language use was between the classroom and academic.

The nature of the teacher’s and students’ questions during the observations were also different. The teacher asked questions eight out of ten times that she spoke, while the students asked questions two out of ten times that they spoke. When the teacher asked questions in front of the whole class, the questions often sought explanation of reasoning and strategy whereas the students’ questions were clarification based on notation use and concept. When the teacher asked questions in small groups she sought clarification on notation use, written organization, structure of the solution, and conceptual understanding. Martino and Maher (1999) explained in their ten-year longitudinal study that an experienced teacher’s questions were a powerful tool that engaged students and improved overall students’ conceptual understanding. In Mueller, Yankelewitz, and Maher’s (2014) study, they found that teacher moves (such as questioning) exposed student thinking to their peers, brought out multiple students’ ideas giving way for other students to extend those ideas, and encouraged student justifications of thinking and reasoning. Mueller, Yankelewitz, and Maher (2014) also noted that the teacher moves developed the classroom norms that produced students that listened to one another, shared ideas, and promoted justification of student thinking. When the students asked questions in small groups to one another (without the teacher present), the questions were more in depth and often asked each other to explain their meaning, thinking, and process. The student questions in small groups
were developed around answering the question ‘why?’. In class discussions, the teacher’s questions generated interest in the learned topic that students then carried over to their small group discussions. Sahin and Kulm (2008) found that teacher questioning was represented in three formats: probing, guiding, and factual. According to Sahin and Kulm (2008) probing questions enable students to extend their conceptual understanding and allow for the connections to be made between prior learning and other representations. In these studies there were no specific examples as to the teacher moves that were utilized but it was suggested that further research be done in this field to compile a list of best practices that allow teachers to develop standards and norms that lead to mathematical discourse communities within the classroom.

Another notable difference between the teacher’s and students’ oral discourse was the learning that occurred for students when there was teacher facilitation of learning versus students’ own control or direction of their learning. Teacher direction of learning developed language acquisition more than during the times that students’ directed their learning. When the teacher asked questions of the whole group there was more accountability to answer the teacher’s questions than to answer peers’ questions. In large group settings, the teacher spent time introducing the language as it referred to the mathematical concept and continued to use the mathematical language throughout the discussion. In the whole class discussions, the students voiced their misconceptions orally and the teacher corrected the misconception before it became consolidated in student thought or writing. During large group discussions the teacher modeled the method of gradual release of responsibility (Fisher & Frey, 2013; Pearson & Gallagher, 1983). Within small group discussions that occurred, students often explored and experimented with language use and concept rules. When students had control of their learning they were more often working in small group settings, this meant that students were more likely to develop a
misconception in their thinking or definition and continued to build on the misconception. Kieran (2002) explained in her study of student partnerships that partnered learning was beneficial for both partners when students were appropriately paired. Kieran (2002) noted gains in students’ individual achievement after mutually beneficial partnerships. In this study, there were instances of student control where students worked through their misconceptions by returning to their definitions of terminology or concepts. This suggested that there were fewer mutually beneficial partnerships developed in the observed class when students worked with the desk partners than in Kieran’s (2002) study when they were assigned similarly leveled students.

The last difference to be discussed is one of the students’ and teacher’s perceptions of the importance of learning mathematical language. The theme of Untangling Expectations was evident when interviewed students discussed their perceptions of how the teacher felt it was important to learn language in the mathematics class. Students described their perception of the teacher’s importance of learning mathematical language as understanding and completing practice work, success on summative assessments, and communicating written solutions in a standardized way. The teacher theme of Importance of Language explained the teacher’s fortitude to develop strong communication skills for students to communicate with one another in a standardized way, explanations of students’ mathematical thinking, and connections between mathematical language and concepts. Student perceptions of what the teacher deemed as important about learning mathematical language were predominately driven around written assessments, whereas the teacher’s motivations were to create depth in meaning and conceptual understanding through language use. The students’ motives better aligned with the teacher’s articulated motives for learning mathematical language. This was further emphasized when interviewed students were asked “Do you feel your teacher stresses your mathematical language
use in class and/or in your written work?” and eight out of nine students interviewed responded that the teacher stressed the importance of language use and commented on examples of feedback in written work. Two of those eight students said that the teacher did stress the importance of mathematical language use orally and she gave examples through oral feedback, and one out of nine students said they were unsure if the teacher stressed the importance of language use.

The Extent of Students’ Use of Oral Mathematical Language In Written Work

To answer research question three, the themes Importance of Learning Language from the Student Perspective, Students Engaged in Language Acquisition, and Untangling Expectations from the student interviews and the collected student work were valuable. Students’ use of oral mathematical language allowed students to acquire mathematical language and apply it in their written work. The students experienced oral language acquisition and a change in the level of mathematical language used in the submitted written work.

Students’ use of acquired mathematical language appeared in the students second collected written task. From the collected work, there were thirteen of sixteen students that demonstrated improvement in mathematical language use. Students’ used an average of 5.6 more words or phrases per student, in the mathematical register of classroom or academic language. Students’ learned mathematical language from their teacher’s oral use of discourse during the lessons, applied mathematical language use in small groups during the lessons, and utilized mathematical language in their written work to explain their thinking, make claims, and describe their reasoning. Quinton was one example of this transfer of mathematical language. Before the classroom observations Quinton submitted Figure 5-1. After one week of instruction, a second problem task was given and Quinton submitted Figure 5-2. Quinton improved his solution
overall and his final statement within the solution. He used more mathematical language in his work and explained his solution more fully.

Figure 5-1. Quinton’s Submitted Written Task 1

Figure 5-2. Quinton’s Submitted Written Task 2

During the classroom observations Quinton spoke twenty one times in class. During a transaction described earlier Quinton responded to the teacher “Yes. It is not a linear equation” then later on in a whole group discussion he stated, “…it seems like it is not linear but its weird” then the teacher used the term ‘non-linear’ and discussed characteristics of linearity. On the third day during whole class discussion he said, “You can get “b” by itself, by taking the value you get from the slope times the “x” and moving it to the other side of the equation and adding the values together.” Quinton increased his use of mathematical language over the three days of lessons.
The last quote illustrated that Quinton has increased his duration of speech and was using more classroom and academic language such as the term ‘slope’.

During the teacher interviews, the teacher described her consistent use of mathematical language and development of multiple representations helped students understand mathematical language orally because the language was the concept. When she said, “I try to keep the vocabulary in the forefront, using correct terms and referring to it in multiple ways [representations]”, she indicated that she developed language throughout her course in a variety of ways so students were able to understand a representation of the mathematical language used. When she stated, “I’m trying to build the concept every time that I use the word”, she meant mathematical language was directly linked to the concepts they represent and that students must learn the mathematical language to come to a deep understanding of the concept. As the teacher builds the language use, students, like Quinton, begin to speak more frequently in whole class discussions to clarify their own misconceptions and used more classroom and academic language.

The Importance of Learning Language from the Student Perspective emphasized students’ appreciation that learning the mathematical language was important to solve problems, understand concepts, and to communicate clearly with others. Students indicated that they would acquire the same discourse of the mathematical register that the teacher spoke in class. For example, Lauren said, “She [teacher] uses it [mathematical language] a lot, and it [mathematical language] kinda becomes learned to say it like that [the way the teacher does].” When the teacher used mathematical language, the students acquired it. Lauren continued to explain that the students practiced oral mathematics language with each other. She stated, “she asks a question based on work that we just did. Like another problem just like what she just did,… then
you could hear us…sort of starting to use those words.” Lauren indicated that students acquired the language from the teacher orally, then practiced with other students orally when working in small groups. Oral language was transferred from the teacher to the students through classroom discourse in large and small group settings (Kieran, 2002; Moschkovich, 2004).

The teacher and the students described why they felt learning mathematical language was important to them. The teacher explained that her goal was for students to develop precision in how they discuss mathematics and to communicate clearly with others in a standardized way (e.g Importance of Language). The students described that they also wanted to learn mathematical language to communicate effectively with the teacher and peers, be able to complete assigned problems, and to understand the learned concepts (e.g Importance of Learning Language From the Student Perspective). The students’ explained in their interviews that they thought the teacher expected clear communication in their written solutions so that they could complete the practice and be successful on their assessments (e.g Untangling Expectations). During classroom observations, the teacher explained to students when words or phrases were of importance and when and how students would see these terms again in assessments. When the teacher placed emphasis on mathematical language students understood this emphasis, asked questions around these points, and utilized these words in their written work. Mendez, Sherin, and Louis (2007) found that when a teacher placed emphasis on discourse and student thinking, students participated more and developed better understanding of concepts learned in mathematics. Moschkovich (2002) explained that teacher placing emphasis on learning mathematical language through discourse improved student learning and allowed for an oral to written transfer of mathematical language. Khisty and Chaval (2002) described that when teachers talk, students listen, and learn from their talk. Khisty and Chaval (2002) suggested that teachers that believe
that their talk in class plays an important role in student learning develop strong classroom discourse practices where students master mathematical language and learn to control the direction of the discourse through the presentation of their own ideas.

Once students acquired language orally they could then transfer this language to written work (Moschkovich, 2004; Chamot, 2005; Chamot, & O’Malley, 1987). The student written work gave evidence that students acquired classroom and academic mathematical language over a one-week period. Most students’ written work after the observation period used more mathematical language, had better developed arguments, and used better overall communication. Students learned the mathematical register from their teacher orally and transfer it to their written work improving the quality of their mathematical language when solving problems.

From answering the three supporting research questions, What is the nature of the teachers’ oral discourse in the classroom?, What similarities and differences exist between teachers’ and students’ oral mathematical discourse?, To what extent does students’ use of oral mathematics language in class appear in students’ individual written solutions of problems? three overarching themes emerged, the Importance of Language, the Language Acquisition Through Collaborative Problem Solving, and the Development of a Common Language. These three themes through the use of Grounded Theory lead to the discovery in this research that students learned mathematical language and problem solving best when they had a growth mindset, and participated in the discourse community. This will be further explained when answering the overarching research question ‘How does the use of purposeful oral mathematics language in the classroom affect students mathematics discourse during problem solving and appear in their written work?’
Answering The Overarching Research Question

Purposeful oral mathematics language used by the teacher, affected students’ mathematics discourse during problem solving and the same classroom and academic mathematical language appeared in their written work. This was evident in the analysis of student interviews, the frequency counts of spoken word and written word, and rankings of consistent speech, and improved mathematical language use in written work. Students demonstrated greater improvements in mathematical language use when they employed re-organizational strategies with new terminology. Students showed greater improvements in mathematical language use when they appeared to have a growth mindset to their learning. A consistent and collaborative mathematical discourse community supported student gains in mathematical language use. These gains will be explored through discussions on: the development of a discourse community, the frequency of speech and reorganizational strategies, and growth mindset.

The Development of a Discourse Community

The themes that emerged from the data described the nature of oral discourse not only for the teacher but also for the students. From these interactions, it was seen that the teacher developed a discourse community for students to discuss problems, questions, and mathematical concepts and terminology. The teacher moved across all levels of language of the mathematics register (simplified everyday to academic language use) to safeguard against misconceptions and build students’ conceptual knowledge and mathematical language. The teacher’s development of a mathematical discourse community is evident in several themes. The themes Problem Solving Inquiry and Acquisition Through Collaborative Problem Solving were evidence of the teacher’s class being based in collaborative problem solving as a learning community. From the teacher and student interviews, the themes Importance of Language, Language Acquisition Through
Collaborative Problem Solving, and Importance of Learning Language from the Student Perspective emerged when there was evidence of the teacher’s register use being emulated by the students. From the classroom observations, the themes Questioning and Directing, Student Driven Learning, Mathematical Thinking in Context, and Development of a Common Language are evidence of the development of collaborative discussions, discourse, and the nature of teacher questioning.

The teacher’s oral discourse was not intimidating as she constantly worked to simplify language use to ensure students’ conceptual understanding. This had an effect of producing an environment that was conducive to student collaboration and open dialogue. The teacher stated during her interviews that she did not believe that she had created a discourse community with her students until students were seniors in her high school’s mathematics program but the nature of her discourse during student transactions indicated that she had built a discourse community within her grade nine academic mathematics class. Literature has characterized a mathematical discourse community by mode of arguments (precision, brevity, and logical coherence), generalizing and summarizing, making claims (detailed descriptions of the application of said claims), visualizing, developing relationships, and talking and writing about imagined concepts (Moschkovich, 2007; Schleppegrell, 2007; O’Halloran, 2005). The grade nine class exhibited precision and logical coherence in their arguments both orally and in their second written task, worked toward summarizing and generalizing their learning both orally and in writing, and made claims in their mathematics learning and explained their thinking behind their claims. The teacher and students engaged in the use of multiple representations of mathematical language and concepts that helped students to develop relationships between meanings. Students exhibited that
they talked and wrote about imagined problems throughout the observed classes. The class exhibited all the characteristics of a mathematics discourse community.

The themes *Importance of Language, Language Acquisition Through Collaborative Problem Solving*, and *Importance of Learning Language from the Student Perspective* were evident from interviews when the teacher and the students explained that the teacher’s register use was emulated by students. When the teacher used the ‘classroom language’ as her predominant level of register, she was indicating to students the acceptable form of discourse in the mathematics classroom and the field of mathematics. The teacher used ‘academic language’ to introduce and discuss concepts but maintained the ‘classroom language’ for much of the class time during observations. Students used mostly ‘everyday’ and ‘classroom language’ as their mathematical register to discuss concepts, problems, and ideas. The students consistently spoke one level of register below the teacher during the study. The teacher stated “I try to keep the vocabulary in the forefront…I keep going back to slope and keep repeating that it’s the rate of change of the dependent versus independent variable”, suggesting that she places emphasis on the mathematical language across its registers to make the language more accessible to students so they will acquire the new language being used. Student statements during the interviews, “So like if she says it a lot in class, then we’re going to pick up on that” (Lauren, Student Interviews), suggested that the terminology that the teacher used in class would be adapted by the students.

From Vygotsky’s (1978) theory of Zone of Proximal Development (ZPD) we know that students must learn in a higher level of the mathematical register that they are expected to utilize in discourse. To promote students’ mathematical language use to the ‘classroom’ and ‘academic’ range the teacher must speak predominantly using ‘academic language’ in the mathematics register. Schleppegrell (2007) stated that for students to learn mathematical language and
concepts a teacher’s language must have a higher lexical density, more relational processes, more long noun phrases, higher register of mathematics, and meaningful technical language.

The themes Problem Solving Inquiry and Language Acquisition Through Collaborative Problem Solving were illustrated by the teacher when she discussed her problem solving based class during the interviews. The teacher taught through problem solving. She described the problems as mathematics inquiries. Inquiry based learning improves student engagement, communication skills, collaboration skills, and develops students’ mathematics vocabulary (Savery, 2015; Lieberman & Hoody, 1998). During the teacher interview, the teacher stated that by teaching through problem solving, students were more collaborative; they discussed problems with their peers, learned and utilized the mathematical language, and understood concepts more deeply. During the classroom observations, it was seen that students worked collaboratively in small groups and as a whole class when assigned a problem, students were engaged in classroom discussions, and students used newly acquired mathematical language when prompted. The teacher’s strategy to teach all concepts through problem solving created a need to learn the terminology and fundamental concepts behind the terms so that the students could continue to communicate with each other. Students recognized this need and commented on learning mathematical language as important for this reason during student interviews. Gerber, Cavallo, and Marek (2001) stated that students in inquiry-based classes outperform non inquiry-based classrooms in reasoning ability. Students’ in inquiry-based classrooms spend more time learning through experience and the discussing the experience with their peers (Gerber, Cavallo, & Marek, 2001; Resnick, 1987). The teacher’s approach to learning mathematics through problem solving provided students with conceptual inquiries that promoted student discourse and developed the mathematics discourse community within the teacher’s classroom. Martino and
Maher (1999) stated that when students become engaged with a problem task that students’ move through a variety of problem solving strategies including the re-organization of knowledge and problem solving strategies. The teacher engaged students in the process of solving mathematical problems through maintaining a problem based learning environment.

The themes Questioning and Directing and Development of a Common Language were apparent when the teacher worked to develop connections between prior knowledge and introduced mathematical language. As seen in earlier sections of the classroom observation transcripts, the teacher regularly questioned students to activate prior knowledge that enabled students to connect previously learned or known terminology to newly learned concepts or terminology. For example, this occurred when the teacher drew out the terms ‘appreciation rate’, ‘depreciation rate’, ‘acceleration’, and ‘deceleration’. The teacher worked to connect the concept of slope to everyday experiences as was seen through the discussion on appreciation rates, speed, acceleration, rates, and unit rates. The nature of the teacher’s oral discourse allowed students to build relationships between prior knowledge and new concepts as the teacher structured the oral discourse to give students opportunities to re-organize conceptual and mathematical language meaning. Gutierrez et al. (2010) argued that students who participated in oral discourse communities that used multiple representations of language and concepts allowed students to develop meaning from their prior knowledge and build contextual meaning. Adams (2010) argued that students who were enabled to re-organize their mathematical language were able to develop deeper connections to the language and concepts. In Mason’s (1996) study, she documented that students build new concepts through ‘renegotiating and sharing meanings’ through class or peer argumentative exchanges. Mason (1996) further suggested that collaborative discourse enabled fifth grade students to master scientific discursive practices. The
teacher’s oral discourse enabled students to renegotiate their understanding of both mathematical language and concepts.

*Questioning and Directing* was evident from the teacher’s nature of questioning students’ thinking. Her nature of questioning created deeper conceptual understanding and more meaningful understanding of mathematical language. The teacher asked questions eight out of ten times that she spoke. The nature of the questions she asked reviewed or activated prior knowledge. She asked questions when searching for student misconceptions, clarification in student explanations, and clarification in student thinking and mathematical process. She utilized questioning when she expected terminology use, sought description of examples, or corrected student notation use. The teacher’s questions kept students on task and focused on learning mathematics concepts and terminology. The nature of the teacher’s questions developed the discourse community in her class as they sparked discussions and debates, pushed students to think about their thinking, and enabled students to feel empowered to ask questions of their own. Students that participated most frequently in the mathematical discourse community of the observed class had greater improved level of mathematical language use in their written work and used explanations and justifications in their problem summaries. In Martino and Maher’s (1999) study of grade three and four students, the teacher’s careful posing of questions and listening created a discourse community that promoted students’ explanation and justification of work and thinking.

From the nature of the teacher’s discourse, it was seen that the teacher facilitated student learning through questioning. Students were on task throughout class discussions as the teacher questioned their thinking when students veered toward misconceptions. The teacher’s flexible use of mathematical language across its levels of the register enabled students to hear and
understand multiple representations and strategies from their peers during whole class
discussions. The teacher’s inquiry based classroom gave students opportunities to collaborate,
discuss strategies, talk through their own strategies, and discuss misconceptions with peers. This
gave students confidence to speak in class and to ask clarifying questions when they needed. In
Kramarski and Mevarech’s (2003) study on enhancing mathematical reasoning through
collaboration students’ fluency and flexibility of speech were measured across two groups,
significant differences were found in students’ fluency and flexibility of language use in their
grade eight mathematics class from the start to the end of the study. Students were explicitly
taught to use mathematical discourse during the study with emphasis on improving collaboration
and oral communication (Kramarski & Mevarech, 2003).

The teacher’s questioning nature in her oral discourse deepened students’ understanding of
mathematical language and concepts. As the teacher asked questions regarding the problems or
concepts being discussed and students spoke of misconceptions that they had. This allowed the
teacher to clarify misconceptions while they were still part of students’ oral language before they
were consolidated in writing.

The teacher’s oral discourse built a mathematics discourse community, which allowed
students to hear one another’s strategies and built their knowledge from one another, thus
acquiring mathematical language that was meaningful to them. The development of a discourse
community allowed passive participants (non-speakers) to actively acquire mathematical
language as well, which was evident through students’ written work over the observation period.
The nature of the teacher’s oral discourse enabled many students to speak in class. Students that
spoke more frequently in class showed greater levels of improvement in their use of
mathematical language in written work. When students spoke in class they learned to collaborate
with one another to renegotiate meaning of learned terminology. Students that spoke more frequently in class gained more classroom and academic language than those that did not speak frequently. The nature of the teacher’s oral discourse positively impacted students written expression when problem solving.

**Frequency of Speech and Re-organizational Strategies**

Students explicitly stated that they could not solve problems in mathematics if they did not understand the language, for example, if the terminology was too difficult or unknown to them. Martin best described this point when he answered why he thought learning mathematical language was important.

...key phrases that you might need to understand the definition of them, because on a test or something, you might understand like 90% of the question, but you say oh, like, what is that one thing, what is the angle bisector mean? So I know about angles and triangles but what does the angle bisector mean? That’s why you should always do like a self-check, do I understand what’s happening in class?

(Martin, Student Interview)

Martin’s statement indicated his appreciation that if a student didn’t understand the one key term in the problem, this acted like a barrier to comprehending the problem in full and demonstrating their knowledge on the topic. Martin’s explanation about angles, triangles, and bisectors shows that he sincerely felt that learning mathematical language allowed him to read, comprehend, and solve problems effectively.

For students, the *Importance of Learning Language From the Student Perspective* was valuable for problem solving, conceptual understanding, and to communicate clearly with others. Students explained that without learning mathematical language they were unable to answer...
problems because they didn’t understand what they were being asked. For example, when students didn’t understand the language used in a problem they realized they had not understood the concept that the term represented. Students elaborated that understanding mathematical language allowed them to be more succinct, to explain their work to the teacher and their peers, and clarify meaning through commonly understood and used terms. Students indicated that their oral learning of mathematical language was impactful in their written communication - most students explained they used oral discourse to learn but described their communication skills in mathematics as written skills. Improved written communication skills when solving problems resulted from students learning purposeful oral mathematical language.

The greatest frequency speakers spoke more than twenty times during the observed three days and the mean frequency of speech was fifty interactions per student. These students had an improvement of mathematical language use of one to three rubric levels, and an improvement in frequency of mathematical language use in written work of between six and eighteen words (and phrases). The mean improvement in frequency of mathematical language use in written work was ten words or phrases per students. In Murray and Lang’s (1997) study on participation in the college classroom, they discovered that high levels of participation improved both the learning of the course content as well as students ability to problem solve.

Often, students that were able to re-organize word meaning or developed flexibility in understanding terminology and concepts across a variety of representations had greater levels of improvement of mathematical language use in their written work. Students that most notably used re-organization strategies asked questions during class until they could re-state terminology in a meaningful way. They summarized both terminology and conceptual understanding to the teacher and their peers, explored language use until they felt they understood its meaning well
enough to use it in class discussions, and had an improvement in frequency of mathematical language use in written work from six to eighteen words or phrases per student, which gave a mean improvement in frequency of twelve words or phrases per student. These students had the greatest improvement of mathematical language use. Adams (2010) noted a similar impact with students that utilized a reorganization method developed a deep and meaningful connection to the mathematical concepts (Adams, 2010). While Adams’ study used writing as the medium to re-organize student thinking, this study used oral discourse along with writing during problem solving tasks to re-organize understanding of mathematical language and concepts. In this study, students who orally re-organized their understanding of language in class made much greater levels of improvement of mathematical language use in their written work than those that did not.

During Rubin’s (1975) study on the second language learners, she determined seven distinct characteristics of good language learners. Good language learners, according to Rubin, are willing and accurate guessers, driven to communicate, uninhibited, pattern finders, practitioners, speech monitors, and attendants of meaning (Rubin, 1975). Rubin (1975) found that students described as being successful language learners practiced pronouncing words and making sentences. These students adapted new vocabulary into their classroom speech more quickly than students that did not practice.

**A Growth Mindset**

It was discovered that students and the teacher had a number of commonalities in their desire to learn and teach in their mathematics class. From both the student and the teacher interviews all students indicated the reasons for learning the mathematical language were the same as the reasons that the teacher had indicated. This shows that setting a norm in the
classroom is important so that students and teachers are reminded of what both the teachers and the students value in their learning process.

While thirteen of the sixteen participants did demonstrate at least one level of growth in their level of language in written work use over the observation period, it became apparent through observations and during interviews that the students that were ready to listen and interact with the teacher and other students, benefited the most from these interactions and from class time. The students that appeared to be ready to learn had a number of characteristics in that they listened to their teacher and to others, they maintained a positive outlook on their learning, and they continued to ask questions or seek help from the teacher or a peer. The three students that did not seem to exhibit these traits came across as though they did not listen well, maintained a negative outlook, did not seek help when given an opportunity to improve, and appeared less engaged in the process of learning than their classmates. These three students that appeared to have some characteristics that Dweck (2010) referred to a ‘fixed mindset’ did not demonstrate growth in their mathematical language skills over the observational period, and in some cases regressed over the same learning period. The three students (Ryan, Steve, and Vanessa) that appeared to have a fixed mindset had the lowest change in levels of improvement in language use among the class with two (Steve and Vanessa) scoring no change in language use over the observation period and one student (Ryan) regressing his language use over the observation period. Students that had the appearance of a fixed mindset had the lowest change in improvement of language use levels across all measures.

In this study, students with the appearance of a growth mindset made greater improvements in their written work in comparison to students that appeared to have a fixed mindset. For example, students that appeared to have a growth mindset improved their
mathematical language use in written work between one and three levels, and had a mean improvement in frequency of mathematical language use in written work of 7.6 words or phrases per student. This is evident for thirteen out of sixteen students that appeared to be open minded to learning, and were willing to engage in the learning process, and discussed their work with others had more gains in mathematical language use in their written work over the observation period. Alternatively students with the appearance of a fixed mindset stagnated or decreased in their level of mathematical language use in written work, and had a mean decrease in frequency of mathematical language use in written work of one word or phrase per student.

Students with the appearance of a growth mindset made more gains in mathematical language use than students with the appearance of a fixed mindset. Growth mindsets were evident during student interviews when five of nine students interviewed demonstrated a willingness to use newly acquired mathematical language. Each of these students had improvements in written level of mathematical language use ranging from one to three rubric levels, and improvements in frequency of mathematical language use in written work ranging from three to twelve words per student. The mean improvement in frequency of mathematical language use in written work was eight words or phrases per student. Four students appeared to be unwilling to use newly acquired mathematical language. Ben and Lauren seemed unsure of how to apply the new mathematical language where as Steve and Ryan (students that appeared to have fixed mindsets) seemed confident that their answers were correct and there was no possibility for improvement in their solution or language use. Ben and Lauren both had improvements in written level of mathematical language use of one, and improvements in frequency of mathematical language use in written work of five and two words per students. The mean improvement in frequency of mathematical language use in written work for these four
students was one word or phrase per student. Steve had a reduction in his written work by one level of mathematical language use, and decreased in frequency of mathematical language use in written work by one word or phrase. Ryan had no change in his written level of mathematical language use but did have a decrease in his frequency of mathematical language use in written work. Students with a growth mindset experienced more overall growth in learning mathematical language than students with a fixed mindset. Research on student mindset has recently gained momentum since students with growth mindsets out performed students with fixed mindsets in a number of studies (Yeager & Dweck, 2012; Dweck, 2010).

During the interview each student was asked to reflect on their submitted written work and describe anything that they would change if they were asked to complete the task again. Based on their response they were coded as ‘willing to use their newly acquired vocabulary’ by stating “I would change …” Or “I could …” and continuing their statement to refer to something they used or learned in class over the observation period. Students were coded as ‘unwilling to use their newly acquired vocabulary’ by stating “I don’t think so” or “I don’t think I could.” Students’ names, frequency of speech in class, change in written level of mathematical language use, change in frequency of written mathematical language use, and students’ willingness to use newly acquired vocabulary were placed in a table. The table was sorted for highest to lowest frequency of improvement mathematical language use in written work.
Table 5-1

<table>
<thead>
<tr>
<th>Interviewed</th>
<th>Frequency of speech in class</th>
<th>Change in written level of mathematical language use</th>
<th>Change in frequency of improvement of mathematical language use in written work</th>
<th>Believe Learning Mathematical Language is important</th>
<th>Students willingness to use newly acquired vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Martin</td>
<td>69</td>
<td>2</td>
<td>12</td>
<td>Yes</td>
<td>“I could...”</td>
</tr>
<tr>
<td>Quinton</td>
<td>21</td>
<td>3</td>
<td>10</td>
<td>Yes</td>
<td>“I would show...”</td>
</tr>
<tr>
<td>Neil</td>
<td>4</td>
<td>1</td>
<td>8</td>
<td>No</td>
<td>“The only thing is... this question here...”</td>
</tr>
<tr>
<td>Ella</td>
<td>23</td>
<td>1</td>
<td>7</td>
<td>Yes</td>
<td>“I think that now I could use...”</td>
</tr>
<tr>
<td>Ben</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>Yes</td>
<td>“I don’t think that I could”</td>
</tr>
<tr>
<td>June</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>Yes</td>
<td>“I might change...”</td>
</tr>
<tr>
<td>Lauren</td>
<td>10</td>
<td>1</td>
<td>2</td>
<td>Yes</td>
<td>“I don’t think so.”</td>
</tr>
<tr>
<td>Steve</td>
<td>11</td>
<td>-1</td>
<td>-1</td>
<td>Yes</td>
<td>“Honestly I feel like I did really good on this. I didn’t ...like...I knew exactly what I was doing for the whole problem.”</td>
</tr>
<tr>
<td>Ryan</td>
<td>37</td>
<td>0</td>
<td>-2</td>
<td>Yes</td>
<td>“Not really. It’s kinda obvious.”</td>
</tr>
</tbody>
</table>

Note: Student willingness to use newly acquired mathematical language compared to students change in frequency of improvement of mathematical language use in written work.

All students that were coded as ‘unwilling to use newly acquired vocabulary’ were in the bottom five of the nine interviewed listed in Table 4-9. All students that were coded as ‘unwilling to use newly acquired vocabulary’ were in the bottom eight scores of change in frequency of mathematical language use in written work out of the entire class, see Table 4-7. Lauren, Steve, and Ryan show up in the bottom four when compared to the rest of the class. Steve and Ryan
consistently scored in the bottom two for improvements in their written work for both their class and their interview group, demonstrating a consistently low change in achievement across all indicators. Steve and Ryan had similar types of statements in the interview when asked to review their submitted before observation written work and take a few moments to reflect if there was anything that they would change about how they answered the problem now, after a week of instruction on the topic. Steve’s statement “Honestly I feel like I did really good on this. I didn’t …like…I knew exactly what I was doing for the whole problem” demonstrated that he felt his work was correct, and he didn’t need to change anything because he had understood and implemented a correct solution. Ryan’s statement “Not really. It’s kinda obvious” also indicated that Ryan believed that his work was well done and that he had the correct answer. When Ryan said, “It’s kinda obvious” he implied that there was nothing challenging about the problem and that he had also understood and implemented the correction solution. It is unclear if Steve and Ryan don’t see value in changing their solution or if they didn’t make a connection to an opportunity to implement any newly acquired mathematical vocabulary. Both Steve and Ryan had mathematical errors in their solutions.

The purposeful use of oral mathematics language affected students’ mathematics discourse during problem solving and appeared in their written work. Student interviews revealed that students believed that learning mathematical language was important for them to be able to solve problems, understand concepts, and communicate clearly with others. The frequency counts of spoken word and improved mathematical language use in written work indicated that students that spoke more frequently in class had more opportunity to clarify their understanding and acquire language compared to those who spoke less, but that students present within a classroom that discusses mathematical strategies, concepts and terminology will improve their written
language use as well. Students had greater improvements in mathematical language use when they re-organized new terminology until it was meaningful to them. Students who appeared to have a growth mindset during the observation period also demonstrated greater improvements in mathematical language use. These improvements in mathematical language also indicate a potential mathematics mindset that means students with a growth mindset re-examine a problem from multiple perspectives and collaborate with other students to determine solutions (Dweck, 2010; Duckworth, Peterson, Matthews, & Kelly, 2007).

Summary

The teacher’s use of purposeful oral mathematics language in the classroom improved students’ oral and written use of mathematics language. The nature of the teacher’s oral discourse created a discourse community in which students spoke in class and clarified misconceptions allowing them to acquire mathematical language orally. Student’s that spoke frequently in class acquired more mathematical language and this appeared in their written task. The similarities between the teacher’s and the students’ oral discourse were the development of the norms of communication, negotiation of the meaning of terms, accountability for the time that each person or group directed class time, and the use of questioning in the classroom. The differences between the teacher’s and students’ oral discourse were the gap in register use, and the nature of questioning and directing of learning time during class. The differences in the teacher’s and the students’ oral discourse created a Zone of Proximal Development where students were exposed to new mathematical language and concepts. Students learned mathematical language from their teacher who used more advanced language. Students acquired mathematical language from the teacher and their peers when the teacher questioned student thinking and process to prompt further explanation of their ideas. Students appeared to acquire
more mathematical language when the teacher controlled student learning by asking questions to expose their thinking and address misconceptions. Student’s oral mathematics language from class noticeably appeared in their submitted written work. Thirteen of sixteen students improved their mathematical language use when learning mathematical language in their classroom lessons, including students that did not speak in class. The teacher commented, during interviews, that she noticed the transfer of mathematical language and knowledge that occurred over the course as students self-corrected during class. The students valued the learning of mathematical language in order to understand the problems and solve them. Students noticed the transfer of oral mathematical language from the teacher to students and commented that if the teacher used new mathematical language in class, the students would be heard using it shortly after. The mathematical language that students learned in class appeared in their written work. The teacher’s use of purposeful oral mathematical language also improved students’ written solutions when problem solving as the students could explain their thinking more fully because they had access to more mathematical language.

Limitations

There were several points during the data collection and analysis that had potential for limitations. One such point was that students were not asked if they received additional help or tutoring outside of class; this could potentially impact students understanding of the mathematical concepts and language used both when speaking and writing. The implication that this might have on the data collected for non-speakers that showed written improvement is that the non-speakers may have been speaking to peers or parents or teachers for help outside of class time. This would mean that these three students that were labeled as non-speakers during classroom observations could have been practicing their mathematical language outside of class.
time. These students would create skewed data suggesting that non-speakers or passive participants are learning the mathematical language as it appears in their written form without orally practicing it, while in reality the students could be practice the mathematical language with an expert at home.

Another point of limitation might be the rapport that the classroom teacher had with students. Some students were more comfortable approaching the teacher and coming in for help during breaks in the schedule. This would allow those students an opportunity to improve their language skills and potentially having further gains in mathematical language use both in oral and written communication.

Often teachers live and work in the same community, as did the classroom teacher in this study. As a community member often students know the teacher outside of the school community and have previously developed relationships that allow for students to be more comfortable learning in the teachers classroom. There were two students in the classroom who were related to the teacher. One student demonstrated high levels of language acquisition and an overall positive attitude toward learning where the other student was more negative toward learning and showed lower levels of language acquisition. While the students’ prior relationship was not noticeable in the classroom environment, one student commented that he/she felt the teacher was harder on him/her due to the family tie but the other student that was also related made no mention of feeling this way.

A third possible limitation is that students often do not hand in their best work when they know it is for formative versus summative purposes. The written work that was collected before and after the classroom observations may not have included students’ best work because it was not for a summative purpose. However, both pieces of work were not summative and students
were given the same message for Task One and Task Two of the written work, that it was formative and would not count toward their mark. The teacher mentioned during her interviews and in informal discussions that students often would show their best work on their summatives because they counted toward their grade. It is this researchers’ belief that the growth that was seen across the learning time period is most important to this study versus seeing that all students can produce academic mathematical language by the end of a learning period.

Implications for Future Research

From this research there are four main questions that arose that give way to new distinct research. The first limitation gave way to wonder if all students that speak more, acquire written language faster (more readily) than their passive learning counter parts. There is currently a lot of research in the area of the introverted learner and one might wonder if being introverted when expected to learn through collaborative tasks might be seen as a limitation in itself. This also suggests the question, are introversive and passive learners comparable.

A second possible area of research to explore could be into types of teacher facilitation and their impact on student learning and mathematic confidence. Although this seems as though it is seeking best practices or high yield strategies for learning mathematics, the intent of this research would be to better understand how teachers are successfully facilitating discourse communities, inquiry based learning, and student self-directed learning within the mathematics classroom and beyond.

A third area of interest from this study would be to discover what experiences cultivate a sense of a fixed mindset in some students to their learning while others are more willing to learn and continually try new things. This question is important to understand why so many students apparently enter secondary mathematics programs with a fear of mathematics or a lesser
expectation of achievement in mathematics. Understanding how fixed mindsets develop could help unpack how students view their mindset with different life priorities (i.e. a student might have a growth mindset when playing sports but does not apply this mindset when in their mathematics class or vice versa). Some students may have growth mindsets in various areas of their lives but when in specific classes or situations they exhibit more of a fixed mindset. Could this shift in mindsets be connected with learning priorities? For example, students that know that they would like to become engineers might place a higher priority on learning mathematics in their secondary school program than students who want to become chefs. Any professions students select will have varying degrees of mathematics and numeracy within them, but perhaps the students wanting to become chefs might have a growth mindset in their grade ten foods and nutrition class versus their grade ten mathematics course. Within this study, three students appeared to have a fixed mindset toward the mathematics class but they also did not seem to be lacking confidence in the subject area. What allows those students to decide their work is ‘good enough’ in their minds whereas other students drive for a complete understanding and ‘perfection’ of achievement?

A last point for possible further research stems from understanding growth mindset and its connection to language learning. Much research has been conducted on second language and bilingual learners in mathematics looking at high-yield strategies but little has been conducted to examine the connection between mindset and content area learning in a second language. This raised the question ‘Do students that are learning the instructional language (e.g. ESL, or ELL, etc) develop a growth mindset more readily than first language learners?’
References


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# Appendix A: Rubric for Student Written Work

<table>
<thead>
<tr>
<th>Name:</th>
<th>Level 1 (simplified everyday)</th>
<th>Level 2 (everyday)</th>
<th>Level 3 (classroom)</th>
<th>Level 4 (academic)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grammatical Patterns</strong></td>
<td>Written Language</td>
<td>Written Language</td>
<td>Written Language</td>
<td>Written Language</td>
</tr>
<tr>
<td>Technical Vocabulary</td>
<td>Uses overly simplified language to describe concepts in mathematics Eg. Goes up Increase</td>
<td>Uses appropriate everyday language to describe concepts in mathematics Eg. Amount of increase Positive change Increase over time</td>
<td>Uses a mix of appropriate everyday language and some terminology specific to mathematics to describe concepts in mathematics Eg. Increasing Slope Rate/amount of increase</td>
<td>Uses mathematics terminology to appropriately describe learned concepts in mathematics and applies language with precision. Eg. Positive Slope Rate of increase</td>
</tr>
<tr>
<td>Dense Noun Phrases</td>
<td>None</td>
<td>Few Eg. “If I were to choose one of the plans, I would choose the Gold Standard Plan. The first reason being that this plan is cheaper and the cost per minute is $0.02…”</td>
<td>Some Eg. “If I were to choose one of the plans, I would choose the Gold Standard Plan. The first reason being that this plan is cheaper and the cost per minute is $0.02…”</td>
<td>Dense Eg. “If I were to choose between the two plans, I would choose the Premium Plus Plan because, although, the Gold Standard Plan does seem appealing at first with 400 minutes for a flat fee, as the rate is double the rate of the Premium Plan. The Premium Plan has the best rate per minute and will have more than twice the amount of minutes available when the costs are the same for both plans.”</td>
</tr>
<tr>
<td>Being and Having Verbs</td>
<td>None</td>
<td>Few Eg. “I would choose the Gold Standard Plan, if you are going to use more than 2000 minutes per month than the Premium Plan is better but I will not.”</td>
<td>Some Eg. “If I were to choose one of the plans, I would choose the Gold Standard Plan. The first reason being that this plan is cheaper and the cost per minute is $0.02…”</td>
<td>Dense Eg. “If I were to choose between the two plans, I would choose the Premium Plus Plan because, although, the Gold Standard Plan does seem appealing at first with 400 minutes for a flat fee, as the rate is double the rate of the Premium Plan. The Premium Plan has the best rate per minute and will have more than twice the amount of minutes available when the costs are the same for both plans.”</td>
</tr>
<tr>
<td>Grammatical Patterns</td>
<td>Written Language</td>
<td>Written Language</td>
<td>Written Language</td>
<td>Written Language</td>
</tr>
<tr>
<td>----------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Conjunctions with Technical Meaning ('and', 'but', 'while', 'although', 'because')</td>
<td>Used - without technical meaning Eg. “I would choose the Premium Plan because it is $55.”</td>
<td>Inappropriate - Meaning not made Eg. “I would choose the Premium Plan because it is cheaper.”</td>
<td>Appropriate meaning – developing consistency Eg. “I would choose the Premium Plan because it is less expensive than the Gold Standard Plan. The Premium Plan has only a flat fee while the Gold Standard Plan has a flat fee and a rate.”</td>
<td>Consistent and Accurate use Eg. “I would choose the Premium Plan because it is less expensive than the Gold Standard Plan. The Premium Plan has only flat fee while the Gold Standard Plan has a flat fee and a rate. The Premium Plan will cost less once enough minutes have been used.”</td>
</tr>
</tbody>
</table>

**Implicit Logical Relations**

Types of Logical Relations
- a) Addition and Replacement
- b) Comparison and Contrast
- c) Exemplification and restatement
- d) Cause and Condition
- e) Time and Place

None

Inappropriate
Eg. “I would choose the Premium Plan because its cheaper.”

Appropriate
Eg. “I would choose the Gold Standard Plan if I wanted to spend less than 1000 minutes talking and I would choose the Premium Plan if I wanted to spend more than 1000 minutes talking.”

Consistent and Accurate
Eg. If I were going to purchase one of these two plans, I would choose the Premium Plus Plan. This is because you, as the customer, would have an additional 400 minutes for the same value as the Gold Standard Plan. The Gold Standard Plan is more expensive over time.

**Figure 5-3. Rubric for Student Written Work.** Adapted from “The linguistic challenges of mathematics teaching and learning: A research review.” by M.J. Schleppegrell, 2007, *Reading & Writing Quarterly*, 23(2), 139-159.
Appendix B: General Research Ethics Board (GREB) Clearance Letter

November 04, 2013

Ms. Kathleen Goslin
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Faculty of Education
Duncan McArthur Hall
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511 Union Street
Kingston, ON, K7M 5R7

GREB Ref #: GEDUC-701-13; Romeo # 6010931
Title: “GEDUC-701-13 Mathematical Discourse and Problem Solving”

Dear Ms. Goslin:

The General Research Ethics Board (GREB), by means of a delegated board review, has cleared your proposal entitled "GEDUC-701-13 Mathematical Discourse and Problem Solving" for ethical compliance with the Tri-Council Guidelines (TCPS) and Queen's ethics policies. In accordance with the Tri-Council Guidelines (article D.1.6) and Senate Terms of Reference (article G), your project has been cleared for one year. At the end of each year, the GREB will ask if your project has been completed and if not, what changes have occurred or will occur in the next year.

You are reminded of your obligation to advise the GREB, with a copy to your unit REB, of any adverse event(s) that occur during this one year period (access this form at https://eservices.queensu.ca/romeo_researcher/ and click Events - GREB Adverse Event Report). An adverse event includes, but is not limited to, a complaint, a change or unexpected event that alters the level of risk for the researcher or participants or situation that requires a substantial change in approach to a participant(s). You are also advised that all adverse events must be reported to the GREB within 48 hours.

You are also reminded that all changes that might affect human participants must be cleared by the GREB. For example you must report changes to the level of risk, applicant characteristics, and implementation of new procedures. To make an amendment, access the application at https://eservices.queensu.ca/romeo_researcher/ and click Events - GREB Amendment to Approved Study Form. These changes will automatically be sent to the Ethics Coordinator, Gail Irving, at the Office of Research Services or irvingg@queensu.ca for further review and clearance by the GREB or GREB Chair.

On behalf of the General Research Ethics Board, I wish you continued success in your research.

Yours sincerely,

Joan Stevenson, Ph.D.
Chair, General Research Ethics Board

c: Dr. Jamie Pyper, Faculty Supervisor
Dr. Don Klinger, Chair, Unit REB
Ms. Erin Wicklham, c/o Graduate Studies and Bureau of Research
Appendix C: Recruitment Letter

Dear Colleague,

I am working on my research for my M.Ed thesis. My study is called *Mathematical Discourse and Problem Solving*. This study is designed to examine how a teacher’s oral mathematical language use influences students’ discourse when talking through a problem solving situation, and then appears in students’ written solutions of word problems. The total time commitment for classroom observations, and a pre- and post-interview would be between 6 and 7 hours. Your time would also be spent completing a small demographic survey.

I have attached the Letter of Information and Consent forms to provide more details of my study. Please contact me if you are interested and we could meet and discuss the research more fully.

Sincerely,

Kathleen Goslin
Appendix D: Letters of Information and Consent Forms for Parents, Teachers, and Students

LETTER OF INFORMATION

Mathematical Discourse and Problem Solving

Dear Parent(s) or Guardian(s),

I am writing to seek permission to have your child participate in a formal research activity conducted as part of my graduate degree program at Queen's University. This study is called Mathematical Discourse and Problem Solving. This study will examine the effect of a teacher’s oral mathematical language use and its impacts on students’ written solutions when problem solving. This study has been granted clearance according to the recommended principles of Canadian ethics guidelines, and Queen's University policies.

The research will involve three consecutive days of classroom observations where the main focus is the teacher's and students' use of language within the classroom. I will be observing regular classroom activities in those three days of one course, which will have no interruption to class or instructional flow. The classroom dialogue will be audio-recorded during the observation period so that I am able to follow the teacher's and students' conversations accurately when discussing mathematical problems or concepts in class. As part of the normal classroom routine, your child will be asked to complete a word problem before and after the teacher's three days of lessons. If you agree for your child to participate in this study, their solutions to this problem will be collected as part of this study. If you give consent for your child to participate, your child may be invited to discuss his/her solutions in a twenty-minute interview at the end of the set of classroom observations. The students will be selected based upon the students' pre- and post-observational sample work and their level of participation in the group oral discourse. The interview will be audio-recorded. This interview would ask students to explain their process while problem solving and how their language use affected their solution to the problem. The interviews will take place at lunch or after school, based on your child's schedule and availability, in an office provided by the school. Your child will not be obligated to answer any questions at any point. There are no known risks to your child associated with this study.

Your child's name will be removed from all transcripts and written work collected as part of this study. Although teacher, school and students' names will be collected during the study, confidentiality will be protected to the fullest extent possible by assigning fictitious names to protect identities. Portions of the transcript (i.e. non-identifying quotes or paraphrased quotes) may be used as points of discussion within my thesis and the transcript in its entirety may be viewed by my supervisor, Dr. Jamie Pyper, and by your child's teacher. If you, or your child, does not wish to for your child to participate, they will remain in the class during the observation period as not to miss class. Recording devices will be removed from your child's work space. In the event that your child's voice is recorded, their classroom comments will not be transcribed. A video recording device will be placed in the room to ensure that only student participants' comments are recorded on the transcript. No video images will be used for anything other than to help identify students' participation in the classroom discourse. Your child's use of language while problem solving in group discussions in the classroom may be used within my thesis as a means of discussion of how students' problem solving abilities are influenced by their teachers' language use. Non-identifying quotes and aggregate results from this set of observations and interviews may be used in the publication of my thesis. If the data is used for secondary analysis it will contain no identifying information. In accordance with the Faculty of Education's policy, data will be retained for a minimum of five years. At the end of a term of five years the documentation will be shredded, audio and video tapes erased, and the computer files will be deleted. Video-tapes will be erased after transcripts have been created and reviewed by the classroom teacher and my supervisor.

In asking your child to participate in this exercise, I want to assure you that your child may withdraw from the study at any time without consequence (I will explain this to your child at the beginning of the set of classroom observations and interview). Should you, or your child, choose to withdraw you may request that all or part of your child’s data (their contribution to the recorded dialogues in class, their interview recording and transcript, and their documented work) be destroyed.

If at this point, or any point in the future, you have any questions about this research, you should feel free to contact me, Kathleen Goslin, at 9kmg@queensu.ca or my supervisor Dr. Jamie Pyper at 613-533-6000 x 77748 or pyperj@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613 533 6011 or chair.GRESB@queensu.ca.

If you allow your child to participate in this research, please sign the accompanying Consent Forms, returning one copy to me and retaining the second copy for your records.

Thank you,

Kathleen Goslin, kg
CONSENT FORM FOR PARENTS/GUARDIANS OF STUDENT PARTICIPANTS IN
MATHMATIC DISCOURSE AND PROBLEM SOLVING

Dear Parent(s) or Guardian(s),

If you are willing to allow your child to participate in the classroom observation and document collection, as described in the Letter of Information, please sign this form.

I have read the description of the classroom observations, interviews, and document collections for the research called Mathematical Discourse and Problem Solving and have retained a copy for my records. My questions have been answered. I understand that my child’s participation is voluntary, that he/she may withdraw at any time. I understand that my child is not obligated to answer any questions at any point. I understand that participants may request part or all of their data to be removed from the study at any time. I also understand that what transpires during the observations will be treated as confidential, and although my child’s first and last name may be called out in class, his/her identity will not be divulged and the information my child provides will be kept confidential to the extent possible. I understand that my child will be audio and video-recorded and what she/he says in class, as it pertains to mathematics and learning, will be transcribed. I understand that the video recording is going to be used to help identify voices on the audio recording, maintain flow of the discussion in class in the transcription, and that no images from the videos will ever be used for publication. I understand that my child will be taking part in a set of observations that will take place during her/his regular classroom time and that these observations may be used in the publication of the researcher’s graduate thesis. I understand that my child may be invited to take part in a separate interview to discuss their work from class that will take place during lunch or afterschool. Selection will be based upon the students’ pre- and post-observational sample work and their level of participation in the group oral discourse. I understand that the total time for my child to participate will be approximately four hours, most of which will occur during class. I understand that solutions to a word problem given at the start and at the end of the three days of observation will be collected, copied, and my child’s name removed. Further, I understand that at the end of a term of five years the documentation will be shredded, audio-tapes erased, and the computer files will be deleted and that videotapes will be erased after transcripts have been created and reviewed by the classroom teacher and my supervisor.

I am aware that if I have any questions about this research, I may direct those questions to Kathleen Goslin, at 9kdmg@queensu.ca or her supervisor, Dr. Jamie Pyper, at 613-533-6000 x 77748 or pyper@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613 533 6081 or chair.GREB@queensu.ca.

Please return one copy of your signed Consent form to Kathleen Goslin. Retain the second copy for your records.

YES [ X ] NO [ ] I consent to data from my child’s participation in the classroom activities being collected.

YES [ X ] NO [ ] I consent that my child may participate in an interview if he/she is selected.

Child’s Name: ____________________________

Signature of Parent/Guardian: __________________________________________

Date: ____________________________

YES [ X ] NO [ ] I would like to request a copy of the completed study. I have included my email below.

Email Address: __________________________________________

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LETTER OF INFORMATION

Mathematical Discourse and Problem Solving

Dear Colleague,

I am writing to request your consent to participate in a research study titled Mathematical Discourse and Problem Solving. This research is being conducted as part of my graduate degree program at Queen's University and focuses on my thesis project. This study will examine the effect of a teacher's oral mathematical language use and its impact on students' written solutions when problem solving. This study has been granted clearance according to the recommended principles of Canadian ethics guidelines, and Queen's University policies.

Your participation in this study will involve a short electronic survey, a set of pre- and post-observation interviews, and a set of classroom observations. The survey will be sent to you approximately one to two weeks prior to the scheduled observation period. A few days before the classroom observations I will conduct a brief pre-observation interview to explore your understanding of mathematical discourse and how you use it in class. If you agree to participate in the study, I will observe three consecutive days of one of your classes in a mathematics class of your choice that you are currently teaching. Each observation period will be one period long (or approximately 75 minutes in length) and will be audio and video-recorded. It will take place in your classroom during regular classroom hours. After the last day of observations a post-observation interview will be conducted. This interview will be focused on your reflection of the experience and views on mathematical discourse. Both interviews will be approximately 45 minutes and in a time and location of your choice. If you choose to participate you are under no obligation to answer any questions.

I will prepare a verbatim transcript of the conversations that occur in the interviews and classroom (as they relate to mathematics), concealing your identity and the identity of people that you might mention through the course of the observations by using pseudonyms, keeping your identity confidential to the extent possible. Portions of the transcript (i.e. quotes or paraphrased quotes that may be inadvertently identifying) may be used as points of discussion within my thesis and the transcript in its entirety may be viewed by my supervisor, Dr. Jamie Pyper, and by you, the classroom teacher. If some students do not wish to participate, they will remain in the class during the observation period as not to miss class. Recording devices will be removed from the students work space. In the event that non-participant students' voices are recorded, their classroom comments will not be transcribed. A video recording device will be placed in the room to ensure that only student participants' comments are recorded on the transcript. No video images will be used for anything other than to help identify students' participation in the classroom discourse and the order of discussion points. Your identity and any identifying locations will be removed before the transcript is reviewed by my supervisor and you will be asked to review the transcript. Your use of language, techniques in teaching, and strategies in the classroom may be used within my thesis as a means of discussion of how mathematical discourse is used within current classrooms and how it may affect students' problem solving abilities. The data from this interview (i.e. quotes or paraphrased quotes, teaching philosophy, and teaching experiences, etc.) may be used in the publication of my thesis. If the data is used for secondary analysis it will contain no identifying information. In accordance with the Faculty of Education's policy, data will be retained for a minimum of five years. At the end of a term of five years the documentation will be shredded, audio-tapes erased, and the computer files will be deleted. Video-tapes will be erased once the transcripts have been reviewed by my supervisor and you the classroom teacher.

In asking you to participate in this survey, set of observations, and set of interviews, I am assuming you that you may choose to stop participating at any point. There are known risks to you as the only teacher in this case study. Your identity will be protected to the extent possible but there are risks for you in that your philosophy of teaching, quotes from the interviews and classroom observation as they relate to mathematics education, and the events from the observed classes may be included in the publication of my thesis. Your participation is entirely voluntary. Also, you may withdraw from the research cycle at any time, without pressure or consequence of any kind. Should you choose to withdraw or you may request that all or part of your data be destroyed.

If at this point, or any point in the future, you have any questions about this research, you should feel free to contact me, Kathleen Goslin, at 2khling@queensu.ca or my supervisor Dr. Jamie Pyper at 613-533-6000 x 77748 or pyperj@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613 533 6081 or chair.GREP@queensu.ca

If you agree to participate in this research, please sign the accompanying Consent Forms, returning one copy to me and retaining the second copy for your records.

Thank you,

Kathleen Goslin, KG
CONSENT FORM FOR TEACHER PARTICIPANTS IN MATHEMATICAL DISCOURSE AND PROBLEM SOLVING

Dear Colleague,

If you are willing to participate in the survey, set of classroom observation, and set of interviews, as described in the Letter of Information, please sign this form.

I have read the description of the survey, set of classroom observation, and set of interviews for the research called Mathematical Discourse and Problem Solving and have retained a copy for my records. My questions have been answered. I understand that my participation is voluntary, and that I may withdraw at any time. I understand that participants may request part or all of their data to be removed from the study at any time. I also understand that what transpires during the interviews and observations will be treated as confidential, and although my name may be referred to in class, my identity will not be divulged and the information I provide will be kept confidential to the extent possible. I understand that there are known risks to me as I am the only teacher in this case study. My identity will be protected to the extent possible but there are risks for me in that my philosophy of teaching, quotes from the interviews and class as they relate to mathematics education, and events from the observed classes may be included in the publication of the researcher’s thesis. I understand that my voice will be audio and video recorded and what I say in class and during the interviews, as it pertains to mathematics and learning, will be transcribed. I understand that I will be taking part in a set of observations that will take place during their regular classroom time and that these observations may be used in the publication of the researcher’s graduate thesis. I understand that my participation in this study will take a total of approximately six and a half hours which includes class observation time. I understand that the researcher will be taking notes on my interactions with students as the classroom observations occur. Further, I understand that at the end of a term of five years the documentation will be shredded, audio-tapes erased, and the computer files will be deleted. Video-tapes will be erased once the transcripts have been reviewed by you, the classroom teacher, and my supervisor.

I am aware that if I have any questions about this research, I may direct these questions to Kathleen Goslin, at 613 529 8088, skdr@queensu.ca or her supervisor, Dr. Jamie Parker, at jparker@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613 533 6081 or chair.GREE@queensu.ca.

Please return one copy of your signed Consent Form to Kathleen Goslin. Retain the second copy for your records.

Name: ______________________________________
Signature: __________________________________
Date: ______________________________________

YES [ ] NO [ ] I would like to request a copy of the completed study. I have included my email below.

Email Address: ______________________________________
LETTER OF INFORMATION

Mathematical Discourse and Problem Solving

Dear Student,

I am writing to seek permission for your participation in a formal research activity conducted as part of my graduate degree program at Queen's University. This study is called Mathematical Discourse and Problem Solving. This study will examine the effect of a teacher's oral mathematical language use and its impact on students' written solutions when problem solving. This study has been granted clearance according to the recommended principles of Canadian ethics guidelines, and Queen's University policies.

The research will involve three consecutive days of classroom observations where the main focus is the teacher's and students' use of language within the classroom. I will be observing regular classroom activities in those three days of one course. There will be no interruption to class or instructional flow. The classroom dialogue will be audio and video-recorded during the observation period so that I am able to follow the teacher's and students' conversations accurately when discussing mathematical problems or concepts in class. As part of the normal classroom routine, you will be asked to complete a word problem before and after the three days of observed lessons. If you agree to participate in this study, your solutions to these problems will be collected as part of this study. If you give consent for your participation, you may be invited to discuss your solutions in a twenty minute interview at the end of the set of classroom observations. The students will be selected for the interview based upon the students' pre- and post-observational sample work and their level of participation in the group oral discourse. The interview will be audio-recorded.

This interview would ask students to explain their process while problem solving and how their language use affected their solution to the problem. The interviews will take place at lunch or after school, based on your schedule and availability. In an office provided by the school, you will not be obligated to answer any questions at any point. There are no known risks to you associated with this study.

Your name will be removed from all transcripts and written work collected as part of this study. Although teacher, school and students' names will be collected during the study, confidentiality will be protected to the fullest extent possible by assigning fictitious names to protect identities. Portions of the transcript (i.e. non-identifying quotes or paraphrased quotes) may be used as points of discussion within my thesis and the transcript in its entirety may be viewed by your classroom teacher and by my supervisor, Dr. Jamie Pyper. If you do not wish to participate, you will remain in the class during the observation period as not to miss class. No audio recording will be removed from your work space. In the event that your voice is recorded during classroom discussions, your classroom comments will not be transcribed. A video recording device will be placed in the room to ensure that only student participants' comments are recorded on the transcript. No video images will be used for anything other than to help identify students' participation in the classroom discourse. Your use of language while problem solving in group discussions in the classroom may be used within my thesis as a means of discussion of how students' problem solving abilities are influenced by their teachers' language use. Non-identifying quotes and overall results from this set of observations and interviews may be used in the publication of my thesis. If the data is used for secondary analysis it will contain no identifying information. In accordance with the Faculty of Education's policy, data will be retained for a minimum of five years. At the end of a term of five years the documentation will be shredded, audio and video-tapes erased, and the computer files will be deleted. Video-tapes will be erased after transcripts have been created and reviewed by the classroom teacher and my supervisor.

In asking you to participate in this exercise, I want to assure you that you may withdraw from the study at any time without consequence (I will explain this to you at the beginning of the set of classroom observations and interview). Should you choose to withdraw you may request that all or part of your data (their contribution to the recorded dialogues in class, their interview recording and transcript, and their documented work) be destroyed.

If at this point, or any point in the future, you have any questions about this research, you should feel free to contact me, Kathleen Goslin, at kgoslin@queensu.ca or my supervisor, Dr. Jamie Pyper at 613-533-7000, x77748 or pyperj@queensu.ca Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613 533 6081 or chair.GREE@queensu.ca

If you give your consent to participate in this research, please sign the accompanying Consent Forms, returning one copy to me and retaining the second copy for your records.

Thank you,

Kathleen Goslin, kg
CONSENT FORM FOR PARENTS/GUARDIANS OF STUDENT PARTICIPANTS IN
MATH DISCOURSE AND PROBLEM SOLVING

Dear Student,

If you are willing to participate in the classroom observation and document collection, as described in the Letter of Information, please sign this form.

I have read the description of the classroom observations, interviews, and document collections for the research called Mathematical Discourse and Problem Solving and have retained a copy for my records. My questions have been answered. I understand that my participation is voluntary, that she/he may withdraw at any time. I understand that I am not obligated to answer any questions at any point. I understand that participants may request part or all of their data to be removed from the study at any time. I also understand that whatever transpires during the observations will be treated as confidential, and although my first and last name may be called out in class, my identity will not be divulged and the information I provide will be kept confidential to the extent possible. I understand that I will be audio and video-recorded and what I say in class, as it pertains to mathematics and learning, will be transcribed. I understand that the video recording is going to be used to help identify voices on the audio recording, maintain flow of the discussion in class in the transcription, and that no images from the video will ever be used for publication. I understand that I will be taking part in a set of observations that will take place during my regular classroom time and that these observations may be used in the publication of the researcher’s graduate thesis. I understand that I may be invited to take part in a separate interview to discuss my work from class that will take place during lunch or after school. Selection for this interview will be based upon the students’ pre- and post-observational sample work and their level of participation in the group oral discourse. I understand that the total time for my participation will be approximately four hours, most of which will occur during class. I understand that solutions to a word problem given at the start and at the end of the three days of observation will be collected, copied, and my name removed. Further, I understand that at the end of a term of five years the documentation will be shredded, audio-tapes erased, and the computer files will be deleted and that video-tapes will be erased after transcripts have been created and reviewed by the classroom teacher and my supervisor.

I am aware that if I have any questions about this research, I may direct those questions to Kathleen Goslin, at 91dmg@queensu.ca or her supervisor, Dr. Jamie Pyper, at 613-533-6000 x 77748 or pyperj@queensu.ca. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at 613 533 6081 or chair.GREB@queensu.ca.

Please return one copy of your signed Consent Form to Kathleen Goslin. Retain the second copy for your records.

YES [ ] NO [ ] I consent to data from my participation in the classroom activities being collected.

YES [ ] NO [ ] I consent that I may participate in an interview if I am selected.

Student’s Name: ________________________________________________

Signature of Student: _____________________________________________

Date: __________________________________________________________

YES [ ] NO [ ] I would like to request a copy of the completed study. I have included my email below.

Email Address: __________________________________________________
Appendix E: Teacher Demographic Survey Questions

Teacher Participant Survey – Mathematical Discourse and Problem Solving

This survey is intended to collect demographic information about the teacher participants in research on Mathematical Discourse. The information in this survey will be collected, used, and discussed in publication. The participants’ names will be changed as a means to protect the participants’ identity. Names of the participants will not be published.

This survey is being sent to you electronically so that you may write your responses out in full in the space given beside each question. Please feel free to use more space than appears allotted for each response.

Name: ____________________  Date: ____________________

<table>
<thead>
<tr>
<th>Questions</th>
<th>Responses</th>
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<tbody>
<tr>
<td>1. Describe your experience taking secondary school mathematics courses as a student.</td>
<td></td>
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<tr>
<td>2. What were your motivations for becoming a mathematics teacher?</td>
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<td>3. Is teaching your second career?</td>
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<tr>
<td>4. What is your undergraduate degree?</td>
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<tr>
<td>5. What subject areas did you focus on most?</td>
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<tr>
<td>6. When did you complete your Education degree?</td>
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<td>What subject areas are you certified to teach?</td>
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<tr>
<td>What AQ courses (or equivalent) have you completed in the area of language or mathematics?</td>
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<tr>
<td>7. Is there anything else you can add about your mathematical background? (I.e. do you hold any further interests or courses in mathematics that cannot be described as a degree?)</td>
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<tr>
<td></td>
<td>Question</td>
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<td>--------------------------------------------------------------------------</td>
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<tr>
<td>8.</td>
<td>How many years have you been teaching?</td>
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<tr>
<td>9.</td>
<td>How many years have you been teaching mathematics?</td>
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<tr>
<td>10.</td>
<td>What pathway of courses have you been teaching (academic, applied, or workplace)?</td>
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<td></td>
<td>How long have you been teaching each pathway?</td>
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<td>11.</td>
<td>Do you employ literacy strategies within your teaching of mathematics?</td>
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<td></td>
<td>If so, what type of strategies?</td>
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<td>12.</td>
<td>Do you participate in professional development within your board of education, professional organizations, or independently?</td>
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<td></td>
<td>If so, what types of in-service training have you participated in related to mathematics or literacy?</td>
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</table>

Thank you for your time to fill out this survey. You will be contacted a few days before classroom observations to confirm an appointment for our pre-observational interview. When you are contacted an electronic set of Consent Forms, Letters of Information, and interview questions will be included.

Prior to this interview please remember to read over the Letter of Information regarding the pre- and post-observational interviews as well as the interview questions that will be sent to you a few days ahead of the interview.

Thank you,

Kathleen Goslin
Appendix F: Teacher Pre- and Post- Observational Interview Questions

**Mathematical Discourse and Problem Solving Study**
**Teacher Pre-Observation Interview**

<table>
<thead>
<tr>
<th>Line</th>
<th>Question/Response</th>
<th>Notes/Researchers' Thoughts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1.</td>
<td>During class, how do you use the terms and vocabulary that are associated with the unit of study that you are in?</td>
<td></td>
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<td>Q2.</td>
<td>What is the importance to using this academic vocabulary within mathematics?</td>
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<td>Q3.</td>
<td>How does your use of oral language compare to your students' use of oral language in the classroom?</td>
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<td><strong>Q4.</strong></td>
<td>How is problem solving incorporated into the lesson?</td>
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<td>Q5.</td>
<td>Do students often ask questions during class?</td>
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<td></td>
<td>If so, what is the nature of the questions they ask?</td>
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<td></td>
<td>Are they asking questions about vocabulary? About understanding the problem? About a mathematical concept? About a process of the solution?</td>
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<td>Q6.</td>
<td>How much of your course do you allocate for problem solving?</td>
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<td></td>
<td>What kind of percentage would you say are engaged in word problems or problem solving? You can break it down by pathway if you want.</td>
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<td>Q7.</td>
<td>How much time in class do you spend discussing a problem and possible strategies of solution?</td>
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<tr>
<td>Q8.</td>
<td>What are students spending time on during problem solving time?</td>
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<td></td>
<td>- are they spending time reading the question?</td>
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<td></td>
<td>- are they spending time planning how to solve the problem?</td>
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<td></td>
<td>- selecting tools? ...</td>
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<td>Q9.</td>
<td>Do you feel mathematics educators should be explicitly teaching mathematical discourse practices to students? Mathematical discourse practices are both the social and academic practices that allow one to communicate within the field of mathematics.</td>
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<tr>
<td></td>
<td>within the field of mathematics. These practices include oral, written, symbolic and gestural interactions between people in the field.</td>
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<tr>
<td>Q10.</td>
<td>Would you say you have a mathematical discourse community within your classroom?</td>
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<td></td>
<td>What are the elements of this discourse community an outsider to your classroom would see?</td>
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<tr>
<td>Q11.</td>
<td>Do you feel that there is a connection between how students respond orally to how they respond in written work within mathematics?</td>
<td></td>
</tr>
<tr>
<td>Q12.</td>
<td>That's the end of the interview. I just want to say that I really appreciate the depth of your answers and that you took the time to do this and I just wanted to ask if there is anything else you would like to add?</td>
<td></td>
</tr>
</tbody>
</table>

** can be skipped if question is answered through interpretation of previous questions.
## Mathematical Discourse and Problem Solving Study
### Teacher Post-Observation Interview

**Date of Interview:** ____________________  **Name of Participant:** ________________  **Notes/Researchers Thoughts:**

<table>
<thead>
<tr>
<th>Line</th>
<th>Question/Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1.</td>
<td>When planning the observed lessons what were your thoughts about the introduction of new vocabulary? How did this effect how you developed your problem solving lessons?</td>
</tr>
<tr>
<td>Q2.</td>
<td>Tell me about your sense of the mathematical rigor of your oral delivery during the lesson? What do you think is so?</td>
</tr>
<tr>
<td>Q3.</td>
<td>Do you always use the corresponding terminology and academic vocabulary during class/lessons? Can you tell me about a particular example and why this is important to you?</td>
</tr>
<tr>
<td>Q4.</td>
<td>What kind of responses do you remember your students giving back to you? What evidence can you remember from the classroom that tells you this?</td>
</tr>
<tr>
<td>Q5.</td>
<td>During the observations, what do you think was the impact of your language use on students’ oral participation habits in class?</td>
</tr>
<tr>
<td>Q7.</td>
<td>What were a few aspects to the group problem solving that students surprised and/or impressed you? Did they think of, or say anything new or different from previous years? Why did it surprise you?</td>
</tr>
<tr>
<td>Q8.</td>
<td>How do you think students’ oral participation in the group and partnered conversations during problem solving effect their individual written problem solving?</td>
</tr>
<tr>
<td>Q9.</td>
<td>Please provide me with an example of where there is a connection between how your students respond orally to how they respond in writing when problem solving. Can you expand on this?</td>
</tr>
</tbody>
</table>

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## Mathematical Discourse and Problem Solving Study
### Teacher Post-Observation Interview

<table>
<thead>
<tr>
<th>Line</th>
<th>Question/Response</th>
<th>Notes/Researchers Thoughts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q10.</td>
<td>What are a few aspects of your teaching you feel more validated in, or might change from your participation in this study? Do you feel this study has given you new insight into how students use language within your classroom? Could you please expand on that?</td>
<td></td>
</tr>
<tr>
<td>Q11.</td>
<td>That’s the end of the interview. I just want to say that I really appreciate the depth of your answers and that you took the time to do this and I just wanted to ask if there is anything else you would like to add?</td>
<td></td>
</tr>
</tbody>
</table>
## Appendix G: Student Interview Questions

### Mathematical Discourse and Problem Solving Study

#### Student Interview

<table>
<thead>
<tr>
<th>Line</th>
<th>Question/Response</th>
<th>Notes/Researchers Thoughts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1.</td>
<td>Would you please review your pre-observation problem and its solution?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Could you explain your solution to me?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Possible extensions:</td>
<td>What was the answer? How do you mean clarify? How might you clarify? And now, what would your point be in your solution? Would you write your solution or final answer out differently now? Do you think you would bring in more of the terminology that you learned in class now?</td>
</tr>
<tr>
<td>N/A</td>
<td>Would you please review your post-observation problem and its solution?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Could you explain your solution to me?</td>
<td></td>
</tr>
<tr>
<td>N/A</td>
<td>What similarities and differences do you notice about your two solutions?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Could you explain that point further please?</td>
<td></td>
</tr>
<tr>
<td>Q4.</td>
<td>What do you think is the importance to using academic vocabulary within mathematics?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>This is what I am understanding from your response... is that what you meant?</td>
<td></td>
</tr>
<tr>
<td>Q5.</td>
<td>How does understanding the language used in a problem impact your work (solution)?</td>
<td></td>
</tr>
<tr>
<td>Q6.</td>
<td>What are the key components/parts you feel are important when communicating your solution to your teacher?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Do you mean....?</td>
<td></td>
</tr>
<tr>
<td>Q7.</td>
<td>Do you feel it is important to understand the language used in class to understand how to solve word problems in class or on assignments and tests?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Could you please expand on that?</td>
<td></td>
</tr>
<tr>
<td>Q9.</td>
<td>Do you feel your teacher stresses your mathematical language use in class and/or in your written work?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Please explain that further.</td>
<td>When you aren't understanding something in class, what might you do? Do you ever ask for help with vocabulary? Do you feel that your homework is closely connected to the vocabulary that you learn in class? Does mathematics vocabulary every come up in other classes?</td>
</tr>
<tr>
<td>Q10.</td>
<td>That's the end of the interview. I just want to say that I really appreciate the depth of your answers and that you took the time to do this and I just wanted to ask if there is anything else you would like to add?</td>
<td></td>
</tr>
</tbody>
</table>
Appendix H: Sample Questions for Student Work

Pre-Observational Word Problem

1. A new cell phone company has just opened its door at the mall. They offer a plan called Premium that costs $35 per month inclusive. The Premium plan includes 500 minutes. Anything after the 500 minutes, a phone card is required. The cost equation for the Premium plan is \( C = 35 \).

2. The company offers another plan called Gold Standard that charges a flat fee of $15 per month plus $0.02 per minute. Gold Standard includes 100 minutes every month in their flat fee. The cost equation for Gold Standard plan is \( C = 0.02n + 15 \), where \( C \) represents cost and \( n \) represents the number of minutes used.

   Assume that the plans are only differing through their calling features. Text and data use are not applicable to this problem.

   a. Summarize each plan.

<table>
<thead>
<tr>
<th>Premium Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. What is the initial cost for each plan?

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>Initial Cost</td>
<td>Initial Cost</td>
</tr>
</tbody>
</table>

   c. How quickly does the cost increase in each plan?

<table>
<thead>
<tr>
<th>Premium Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of increase</td>
<td>Rate of increase</td>
</tr>
</tbody>
</table>


d. If there were no minutes being used, how much would the monthly bill be for each plan? What does this value represent mathematically?

<table>
<thead>
<tr>
<th>Premium Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at zero minutes</td>
<td>Cost at zero minutes</td>
</tr>
</tbody>
</table>


e. Determine how much it would cost if 500 minutes were used on each plan. Show your work, and steps. (BEDMAS)

<table>
<thead>
<tr>
<th>Premium Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at 500 minutes</td>
<td>Cost at 500 minutes</td>
</tr>
</tbody>
</table>


f. Determine how many minutes could be produced at a cost of $35 for the month for each plan. (Inverse order of operations – SAMDEB)

<table>
<thead>
<tr>
<th>Premium Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes at $35</td>
<td>Minutes at $35</td>
</tr>
</tbody>
</table>


g. If you were going to purchase one of these plans, which would you choose? Please explain your reasoning in full sentences.
Post-Observational Word Problem

1. Our new cell phone company has made a few adjustments to their plans. They now offer a plan called Premium Plus that charges a flat fee of $35 per month plus $0.01 per minute. The Premium plan includes 400 minutes. The cost equation for the Premium plan is \( C = 0.01n + 35 \), where \( C \) represents cost and \( n \) represents the number of minutes used.

The company offers another plan called Gold Standard that charges a flat fee of $15 per month plus $0.02 per minute. Gold Standard includes 100 minutes every month in their flat fee. The cost equation for Gold Standard plan is \( C = 0.02n + 15 \), where \( C \) represents cost and \( n \) represents the number of minutes used.

Assume that the plans are only differing through their calling features. Text and data use are not applicable to this problem.

a. Summarize each plan.

<table>
<thead>
<tr>
<th>Premium Plus Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. What is the initial cost for each plan?

<table>
<thead>
<tr>
<th>Premium Plus Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>Initial Cost</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Premium Plus Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Increase</td>
<td>Rate of increase</td>
</tr>
</tbody>
</table>
d. If there were no minutes being used, how much would the monthly bill be for each plan? What does this value represent mathematically?

<table>
<thead>
<tr>
<th>Premium Plus Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at zero minutes</td>
<td>Cost at zero minutes</td>
</tr>
</tbody>
</table>


e. Determine how much it would cost if 400 minutes were used on each plan. Show your work, and steps. (BEDMAS)

<table>
<thead>
<tr>
<th>Premium Plus Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost at 400 minutes</td>
<td>Cost at 400 minutes</td>
</tr>
</tbody>
</table>


f. Determine how many minutes could be produced at a cost of $55 for the month for each plan. (Inverse order of operations – SAMDEB)

<table>
<thead>
<tr>
<th>Premium Plus Plan</th>
<th>Gold Standard Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes at $55</td>
<td>Minutes at $55</td>
</tr>
</tbody>
</table>


g. If you were going to purchase one of these plans, which would you choose? Please explain your reasoning in full sentences.