Heat Transfer Modelling and Thermal Imaging Experiments in Laser Transmission Welding of Thermoplastics

by

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“The laser was invented in 1960 and was soon dubbed a solution looking for a problem. So new was the tool that our thinking had not caught up with the possibilities. Today the story is distinctly different…” William M. Steen [1]
Abstract

This thesis presents a comprehensive study on the thermal modelling aspects of laser transmission welding of thermoplastics (LTW), a technology for joining of plastic parts. In the LTW technique, a laser beam passes through the laser-transmitting part and is absorbed within a thin layer in the laser-absorbing part. The heat generated at the interface of the two parts melts a thin layer of the plastic and, with applying appropriate clamping pressure, joining occurs. Transient thermal models for the LTW process were developed and solved by the finite element method (FEM). Input to the models included temperature-dependent thermo-physical properties that were adopted from well-known sources, material suppliers, or obtained by conducting experiments. In addition, experimental and theoretical studies were conducted to estimate the optical properties of the materials such as the absorption coefficient of the laser-absorbing part and light scattering by the laser-transmitting part. Lap-joint geometry was modelled for semi-crystalline (polyamide - PA6) and amorphous (polycarbonate - PC) materials.

The thermal models addressed the heating and cooling stages in a laser welding process with a stationary and moving laser beam. An automated ANSYS® script and MATLAB® codes made it possible to input a three-dimensional (3D), time-varying volumetric heat-generation term to model the absorption of a moving diode-laser beam. The result was a 3D time-transient, model of the laser transmission welding process implemented in the ANSYS® FEM environment.

In the thermal imaging experiments, a stationary or moving laser beam was located in the proximity of the side surface of the two parts being joined in a lap-joint configuration. The side surface was then observed by the thermal imaging camera. For
the case of the stationary beam, the laser was activated for 10 s while operating at a low power setting. For the case of the moving beam, the beam was translated parallel to the surface observed by the camera. The temperature distribution of a lap-joint geometry exposed to a stationary and moving diode-laser beam, obtained from 3D thermal modelling was then compared with the thermal imaging observations. The predicted temperature distribution on the surface of the laser-absorbing part observed by the thermal camera agreed within 3°C with that of the experimental results. Predicted temperatures on the laser-transmitting part surface were generally higher by 15°C to 20°C. This was attributed to absorption coefficient being set too high in the model for this part. Thermal imaging of the soot-coated laser-transmitting part surface indicated that significantly more scattering and less absorption takes place in this part than originally assumed. For the moving laser beam, good model match with the experiments (peak temperatures predicted within 1°C) was obtained for some of the process conditions modelled for PA6 parts. In addition, a novel methodology was developed to extract the scattered laser beam power distribution from the thermal imaging observations of the moving laser beam.
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Nomenclature

Variables

\( a \) fit parameter

\( a_F \) mean free path of the soot particles (mm)

\( a_p \) pinhole area (mm\(^2\))

\( b \) fit parameter

\( c_0 \) speed of light in vacuum (m/s)

\( d \) laser beam diameter (mm)

\( d_p \) optical penetration depth (mm)

\( d_{par} \) soot particle diameter (mm)

\( d_x \) finite difference in the x direction (mm)

\( d_y \) finite difference in the y direction (mm)

\( d_z \) finite difference in the z direction (mm)

\( f_v \) volume fraction

\( g \) gravitational constant (m/s\(^2\))

\( h \) convective heat transfer coefficient (W/m\(^2\).°C), Planck constant (J.s)

\( h_r \) radiative heat transfer coefficient (W/m\(^2\).°C)

\( i_{\lambda} \) directional spectral radiation intensity (W/m\(^2\))

\( k \) thermal conductivity (W/m.°C), Boltzmann constant (J/K)

\( k_c \) volumetric absorption coefficient

\( k_{\lambda} \) spectral imaginary part of the refractive index

\( l \) characteristic dimension of sample (mm)

\( n_{\lambda} \) spectral real part of the refractive index
\( q \)  heat generation (W/m\(^3\))
\( q_0 \)  initial heat generation (W/m\(^3\))
\( t \)  time (s)
\( v \)  volume (mm\(^3\))
\( w \)  width of the laser scanned lines (mm), weld line width (mm), mass fraction
\( w_0 \)  beam radius measure at the beam waist (mm)
\( x \)  distance from the left corner of the sample (mm), axis along the laser beam narrow length, direction of the moving laser beam in lap-joint configuration
\( x_i \)  coordinate
\( X_{\text{beam}} \)  beam dimension along the x-axis (mm)
\( y \)  distance from the top surface of the sample (mm), axis along the depth of the sample
\( y_a \)  Depth to which the heat generation is defined within the laser-absorbing part (mm)
\( y_i \)  coordinate
\( y_T \)  laser-transmitting part thickness (mm)
\( z \)  distance from the rear surface of the sample (mm), axis along the laser beam wide length, direction of the moving laser beam in T-like joint configuration
\( z' \)  distance from the front surface of the sample along the z-axis (mm)
\( z_i \)  coordinate
\( Z_{\text{beam}} \)  beam dimension along the z-axis (mm)
\( A \)  area (mm\(^2\))
\( A_1 \)  parameter
\( A_i \) fitted parameter

C specific heat (J/kg \( \cdot ^\circ \)C)

\( E_{\lambda b} \) spectral emissive power (W/m\(^2\))

I beam power intensity (W/m\(^2\))

I\(_0\) maximum beam power intensity (W/m\(^2\)), beam power intensity before hitting the surface (W/m\(^2\))

K total extinction coefficient (mm\(^{-1}\)), absorption coefficient (mm\(^{-1}\))

K\(_a\) extinction coefficient due to absorption (mm\(^{-1}\))

K\(_s\) extinction coefficient due to scattering (mm\(^{-1}\))

K\(_T\) extinction coefficient of the laser-transmitting part (mm\(^{-1}\))

K\(_\lambda\) spectral extinction coefficient (mm\(^{-1}\))

L beam path length (mm)

N number of soot particles

P laser beam power (W), Pressure (MPa)

P\(_o\) maximum laser beam power (W)

P\(^*\) laser beam power reaching the laser-absorbing part surface (W)

R\(_1\) effective radius of a soot particle (mm)

T temperature (\( ^\circ \)C), absolute temperature of the body (K)

T\(_B\) fluid bulk temperature (\( ^\circ \)C)

T\(_g\) glass transition temperature (\( ^\circ \)C)

T\(_m\) melting temperature (\( ^\circ \)C)

T\(_s\) element surface temperature (\( ^\circ \)C)

T\(_\infty\) initial temperature (\( ^\circ \)C), surroundings temperature (\( ^\circ \)C)
\( T_{\text{amb}} \)  ambient temperature \(^\circ\text{C}\)

\( V \)            laser beam scanning speed (mm/s)

**Greek Symbols**

\( \alpha \)吸光率，热扩散率 \((m^2/s)\)

\( \beta \) 体积热膨胀系数 \((K^{-1})\)

\( \varepsilon \) 发射率

\( \lambda \) 波长 (nm)

\( \nu \) 速度粘度 \((m^2/s)\)

\( \theta \) 光束发散角半角（mrad）

\( \rho \) 密度 \((kg/m^3)\)，反射率

\( \sigma \) 斯特凡-玻尔兹曼常数 \((W/m^2\cdot K^4)\)，标准偏差

\( \tau \) 透射性

\( \zeta \) 转矩能量密度分布 \((J/mm^2)\)

\( \Delta t \) 时间步（s）

\( \Delta x \) 在x方向的步长（mm）

\( \Delta y \) 在y方向的步长（mm）

\( \Delta z \) 在z方向的步长（mm）

\( \Delta T \) 温度差 \((K)\)

\( \Phi \) 功率通量 \((W/mm^2)\)

\( \Lambda \) 线能量 \((J/mm)\)

\( \Lambda_\circ \) 临界线能量 \((J/mm)\)

\( \Psi \) 归一化功率通量分布 \((W/mm^2)\)

\( \Psi^*(0) \) 比例因子
\[ \Sigma \quad \text{summation} \]

*Subscripts*

- \( a \) absorption
- \( \text{amb} \) ambient
- \( g \) glass transition point
- \( m \) melting point
- \( \text{par} \) soot particle
- plastic plastic
- \( s \) scatter
- \( \lambda \) spectral
- \( \infty \) surroundings
### Glossary

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>ABS</td>
<td>Acrylonitrile Butadiene Styrene</td>
</tr>
<tr>
<td>ANOVA</td>
<td>Analysis of Variance</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
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<tr>
<td>Bi</td>
<td>Biot Number (hL/k)</td>
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<tr>
<td>BPP</td>
<td>Beam Parameter Product</td>
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<tr>
<td>CCD</td>
<td>Charged Coupled Device</td>
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<tr>
<td>CW</td>
<td>Continuous Wave</td>
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<tr>
<td>DBS</td>
<td>Diffusion Broadening Spectroscopy</td>
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<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
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<tr>
<td>DSC</td>
<td>Differential Scanning Calorimetry</td>
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<tr>
<td>EES</td>
<td>Engineering Equation Solver</td>
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<tr>
<td>FDM</td>
<td>Finite Difference Method</td>
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<tr>
<td>FEM</td>
<td>Finite Element Method</td>
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<tr>
<td>Fo</td>
<td>Fourier Number (kt/ρCL²)</td>
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<tr>
<td>Gr_L</td>
<td>Grashof Number</td>
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<tr>
<td>HDPE</td>
<td>High Density Polyethylene</td>
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<tr>
<td>HG</td>
<td>Heat Generation</td>
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<tr>
<td>JCG</td>
<td>Jacobi Conjugate Gradient</td>
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<tr>
<td>Laser</td>
<td>Light Amplification by Stimulated Emission of Radiation</td>
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<tr>
<td>LAZ</td>
<td>Laser Affected Zone</td>
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<tr>
<td>LDPE</td>
<td>Low Density Polyethylene</td>
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<td>LTW</td>
<td>Laser Transmission Welding</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
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<tr>
<td>NIR</td>
<td>Near Infra-Red</td>
</tr>
<tr>
<td>Nu&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Nusselt Number</td>
</tr>
<tr>
<td>PA</td>
<td>Polyamide</td>
</tr>
<tr>
<td>PB</td>
<td>Polybutylene</td>
</tr>
<tr>
<td>PBT</td>
<td>Polybutylene</td>
</tr>
<tr>
<td>PC</td>
<td>Polycarbonate</td>
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<tr>
<td>PE</td>
<td>Polyethylene</td>
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<tr>
<td>PEEK</td>
<td>Polyetheretherketone</td>
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<tr>
<td>PETG</td>
<td>Polyethylene Terephthalate</td>
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<tr>
<td>PLGA</td>
<td>Polylactic-co-Glycolic Acid</td>
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<tr>
<td>PMMA</td>
<td>Polymethylmethacrylate</td>
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<tr>
<td>POM</td>
<td>Polyoxyethylene, Polyacetal</td>
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<tr>
<td>PP</td>
<td>Polypropylene</td>
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<tr>
<td>PRK</td>
<td>Photorefractive Keratectomy</td>
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<tr>
<td>PS</td>
<td>Polystyrene</td>
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<td>PSU</td>
<td>Polysulfone</td>
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<tr>
<td>PVC</td>
<td>Polyvinyl Chloride</td>
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<tr>
<td>PVP</td>
<td>Polyvinyl Pyrrolidone, Povidone, Polyvidone</td>
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<tr>
<td>RAM</td>
<td>Random Access Memory</td>
</tr>
<tr>
<td>Ra&lt;sub&gt;L&lt;/sub&gt;</td>
<td>Rayleigh Number (ρv/μ)</td>
</tr>
<tr>
<td>TEM</td>
<td>Transmission Electron Microscopy</td>
</tr>
<tr>
<td>TGA</td>
<td>Thermogravimetry Analysis</td>
</tr>
<tr>
<td>VST</td>
<td>Vicat Softening Temperature</td>
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</table>
Chapter 1  Introduction

This introductory chapter presents background and motivation for this research (Section 1.1), briefly reviews material properties of plastics (Section 1.2), and then discusses welding of thermoplastics with the focus on laser welding (Sections 1.3 and 1.4). Objectives and scope of this thesis (Section 1.5) are followed by a thesis map (Section 1.6), which concludes this Chapter.

1.1  Background and Motivation

Plastics are extensively employed in manufacture of automotive components, consumer products, electronics, and medical devices [2,3]. The use of plastics continues to increase because of their good strength-to-weight ratio, low cost, and ease of recycling.

The ability to join thermoplastic components expands significantly the possibilities for a designer. Since making the same complex component in one piece is not always feasible, they are often made of smaller moulded parts that are then joined together. Various joining techniques have been developed, including adhesive joining, mechanical fastening, and fusion bonding or welding. Each method offers certain advantages and disadvantages [4].

This study will concentrate on a fusion bonding technology known as laser transmission welding (LTW). In the LTW technique, a laser beam passes through the laser-transmitting part and is absorbed within a thin layer in the laser-absorbing part. The heat generated at the interface of the two parts melts a thin layer at the interface and, with an appropriate applied clamping pressure, joining occurs [5].
LTW has been developed to address some of the shortcomings of the alternative approaches [6, 7, 8]. For example, during friction welding, vibration can damage installed electronic devices or plastic pieces. Furthermore, the part geometry, size, and orientation of the parts are limited since one part needs to move relative to the other during friction welding.

In the LTW technique, delivery of the thermal energy by the laser beam is influenced by process parameters, and optical and material properties. The presence of reinforcements and other additives can complicate the process by making these properties anisotropic. If the parts to be joined do not receive enough heat, weld gaps can occur. If the parts are exposed to excessive heat, polymer degradation may happen. None of these conditions is desirable; therefore, to form a strong bond and good quality weld, it is important that the weld interface be exposed to sufficient heat to melt the polymer. Thus, developing a heat transfer model to help understand LTW process, predict the laser-material interaction during this process, and optimize the process is very beneficial. Nevertheless, few accurate models of this process exist. This thesis will address the thermal aspects of the LTW process through numerical modelling and thermal-imaging-based experimental work.

1.2 Plastic Materials

Plastic materials are divided into two major groups: thermosets and thermoplastics [9]. Thermosets are the non-weldable plastics. They are composed of large chains of repeated chemical molecules, which undergo an irreversible reaction to form a closed networked structure with strong covalent bonds. This structure results in a rigid material
upon cooling, but cannot be softened again by heating. If excess heat is applied, thermostet materials will degrade.

Thermoplastics, on the other hand, can be welded. They consist of repeated chemical units called mers. The mers, when linked together, form long chains, i.e., polymers in a reaction known as polymerization. In a thermoplastic, there are no chemical links (cross-links) between the long-chain molecules; hence, molecular chains can diffuse and flow when exposed to heat. Thermoplastics can be further subdivided into amorphous and crystalline categories, which will be introduced in the following sections.

1.2.1 Crystalline Thermoplastics

Crystalline thermoplastics have long molecular chains that are arranged in an organized manner, thus forming crystallites (Figure 1) [10]. In semi-crystalline thermoplastics (e.g., PA), crystallization never reaches completion so that these plastics remain partly amorphous. For flow to occur in these plastics, the applied temperature must be above the crystalline melt temperature ($T_m$), and the temperature at the joint area should be higher than $T_m$ for diffusion of the molecules and consequently welding to happen.

![Figure 1- Internal structure of a semi-crystalline material [10].](image)
In this study, polyamide (also known as Nylon) was used. It is a synthetic semi-crystalline thermoplastic material that is well known for its strength, elasticity, resistance to abrasion and chemicals. Wallace Carothers at DuPont [11] invented PA in 1935. PA fibers are now used in fabrics and ropes, and solid PA is used as an engineering material [12]. One can employ extrusion and injection moulding to process engineering grades of PA. Frequently, reinforcements such as glass fibres are added to the PA to make it stronger with higher working temperature.

There are several grades of PA used in engineering applications (e.g., PA6, 11, and 12). PA6 is the most common commercial grade of PA. One can see a common link of -NHCO- in the amide family. For PA6, this common link is part of a monomer called caprolactam. When this monomer is polymerized, PA6 (~[CO(CH2)5NH]n~) is created (Figure 2) [13]. Despite this common link, various grades of PA possess individual properties.

The presence of the amide linkage causes PA to absorb water. Therefore, one should make sure that PA is dry during each experiment. In this study, care was taken to ensure that the PA samples were dry by preserving them in the vacuum-sealed aluminum-lined bags.
1.2.2 Amorphous Thermoplastics

Amorphous thermoplastics (e.g., PC) lack order in the arrangement of the long chain molecules (Figure 3). They have a narrow temperature band above which the plastics have viscous or rubbery properties, and below which they are in a hard and brittle glassy state. This temperature band is called the glass transition temperature ($T_g$). Amorphous plastics need to be above $T_g$ for molecular chains to flow and thus for welding to take place.

![Figure 3- Internal structure of an amorphous material [10].](image)

In this study, polycarbonate (PC) was used. PC is an amorphous thermoplastic material that is well known for its transparency, ease of molding, and resistance to high temperature and impact. Hermann Schnell from Bayer invented PC in 1953. PC is used in making safety glasses, compact discs, bulletproof glass, and automotive headlamps. PC has low scratch-resistance; therefore, a hard coating is applied to PC lenses.

In PC, carbonate groups (-O-(C=O)-O-) are linked together. PC is made by condensation polymerization from bisphenol A and phosgene ($COCL_2$). A common link of bisphenol A can be seen in the PC structure (Figure 4) [14].
1.3 Welding of Plastics

Laser transmission welding is one of about 20 thermoplastic joining techniques, which can be classified as fusion bonding. Fusion bonding techniques can be further subdivided into friction, conduction, and electromagnetic according to their heating mechanism. Type, size, and desired production rate as well as in-service requirements and manufacturing budget define a proper welding method [15, 16]. In all welding-based joining technologies, parts are placed and gripped in specially designed fixtures. Then, either the surfaces are brought into contact first and then heated or heated first and then brought into contact. Local melting of surfaces at the joined areas leads to molecular diffusion and formation of a joint. The joint interface then is cooled, and the welded part is removed from the work-holding fixture. This section briefly addresses available fusion bonding techniques; the next section concentrates on the laser transmission welding.

The method of energy delivery to the joint area defines the welding technique. For example, in friction welding methods (e.g., vibration welding, spin welding, ultrasonic welding, and friction stir welding), the heat is generated by the relative movement of components. In conduction welding methods (e.g., hot plate welding), heat is conducted from a preheated platen (usually metal). The platen itself may be heated by an external
heat source such as infrared radiation, laser, or other heat sources. Electromagnetic welding methods [17] (e.g., induction welding, electromagnetic welding, radio frequency welding, and laser welding) rely on the interaction of the electromagnetic radiation with a material to raise its temperature. Vibration, ultrasonic, and hot plate welding are commonly used plastic welding methods and they are briefly described herein.

Linear friction or vibration welding [18] brings components to be joined into contact under pressure and moves them relative to each other at a suitable frequency and amplitude to achieve a linear reciprocating motion. Frictional heat sufficient to melt the plastics is generated in the joint area. After the vibration is stopped, the parts are aligned and the molten polymer is allowed to solidify, creating the weld. This method can be applied to most polymers, amorphous and semi-crystalline. The technique is said to be particularly useful for welding of crystalline thermoplastics (e.g., polyethylene, PE, and polypropylene, PP), which are not easily welded by ultrasonic welding.

Ultrasonic welding [19] uses high frequency sound energy to melt plastics at the joint. An electrical signal of about 10 to 70 kHz drives a transducer that converts the electrical energy into mechanical energy. The transmitted sonic energy moves across the joint area and a melt zone is created. Special design features called energy directors are usually required to focus the energy path to the joint area. Heat is generated through a combination of friction and hysteresis by viscous dissipation. Ultrasonic welding joins amorphous thermoplastics more readily than semi-crystalline ones.

Hot plate welding [20] involves pressing components against a metal plate that has been heated to a temperature above glass transition or melt temperatures, removing the plate, and pushing the molten surfaces of the two components together to form a weld. The main welding parameters include the temperature of the hot plate, weld pressure, and
weld time. Weld time will vary with the volume of polymer to be fused and the thermal conductivity of the substrate, and normally falls in the time range of 10 s to 30 s or even longer. The mass of polymer that must be melted, and the corresponding cooling rates, will govern cycle times [21].

1.4 Laser Welding of Thermoplastics

LTW involves clamping laser-transmitting and laser-absorbing thermoplastic parts together [22]. A laser beam then passes through the laser-transmitting part. A very thin layer of the laser-absorbing part absorbs the laser beam near the weld interface. Heat generated at the surface conducts the heat both into the laser-transmitting and laser-absorbing parts. This energy melts a thin layer of plastic in both parts. Molecular diffusion occurs and a solid joint forms as the melt layer solidifies (Figure 5).

Figure 5- LTW process for: a) T-like joint configuration, b) lap-joint configuration.

Semi-crystalline plastics that appear opaque as well as coloured parts or black parts coloured with special non-absorbing pigments can be transmitting to the laser light [23, 24]. In LTW, the danger of overheating the laser-transmitting surface of the joint increases as the proportion of the transmitted energy decreases, this limits the thickness of the transmitting plastic part.
For LTW, it is critically important to achieve sufficient and consistent heating of the thermoplastics in the region of the joint during the pre-melt, melt and fusion phases to produce consistent thickness for the weld [25]. By adding carbon black or laser-absorbing dye to the natural plastic, a normally laser-transmitting material becomes laser absorbing [26]. If the laser-absorbing part contains a relatively small quantity of the laser-absorbing material, the radiated energy will be absorbed deep into the material, the surface temperature will rise more slowly, and the thermal gradient between the surface and the bulk of the part will be less steep. Therefore, the melt layer is mostly located within the absorbing part. On the other hand, if the carbon black content of the laser-absorbing material is high, the laser light is absorbed within a thin layer; therefore, the energy is conducted more equally into both parts. Consequently, the thickness of the melt layer will be similar in the laser-transmitting and absorbing parts.

Among the three common joint types (Figure 6), the LTW technique applies primarily to the lap and T-like joint configurations [27] as their geometry allows for a relatively short laser beam travel path through the laser-transmitting part. For semi-crystalline polymers in particular, scattering of the laser beam as it passes through the laser-transmitting part can have a significant impact on the process.

Figure 6- Common joint types: a) butt joint, b) T-like joint, c) lap joint.
In most applications, the joining of the two plastic components requires creation of a weld line consisting of several linear or curved segments [28]. One can create the weld line by one of the following methods: 1) contour welding, in which the laser beam moves at a relatively slow speed along a pre-programmed path, with only a fraction of the total weld line length molten at any time (Figure 7a); 2) mask welding, in which the part surface is masked to ensure that the laser beam reaches only the exposed joining area (Figure 7b); 3) simultaneous welding, in which the laser energy is shaped to produce the desired contour and the entire weld line is simultaneously exposed (Figure 7c); and 4) quasi-simultaneous welding, in which the laser beam is scanned at high speed to induce simultaneous melting along the weld line (Figure 7d) [29]. The quasi-simultaneous technique has been used for manufacture of automotive sensors [30], while contour [31] and simultaneous mask welding [32] have been used to manufacture medical microfluidic devices. This thesis concentrates on the contour welding technique.

Figure 7- Different methods for delivery of the laser beam to the joint interface: a) contour welding, b) mask welding, c) simultaneous welding, d) quasi-simultaneous welding [29].

Meltdown in fusion bonding refers to the relative movement of the two joined components that takes place under clamping pressure when the joint interface softens or
becomes molten. It is commonly monitored in real time, for example, in vibration welding; the meltdown distance is used to assess the progress and quality of the weld. In LTW, the meltdown is generally expected to occur in quasi-simultaneous and simultaneous techniques only. For contour welding, meltdown may occur only if the weld line length is very short (such as in a test specimen) and is generally not expected to take place in real-world components.

LTW has a number of advantages over other polymer joining methods. LTW does not require contact with a hot surface or any other tools and it can deliver precisely controlled energy to the surfaces of the components to be welded. In addition, it is flexible with regard to the welding geometry, since a computer can control the movement of the laser beam on the part. LTW has the capability of high processing rates, low total heat input into the weld area, good control of the melt-zone temperature and depth as well as weld dimensions and position. It also has low tooling costs, allows rapid change-over to different product designs, allows accurate control of part position through pre-assembly, makes possible welding of 3D joint lines, and keeps the outer surfaces of the components un-melted [33, 34, 35].

Nevertheless, the LTW technology is not widely used. One of the main concerns, which contributes to this fact is the cost associated with the LTW technology. The economic aspects of the LTW process were investigated by Klein et al. [36]. They found that there still exists some prejudice against using this technology in large-scale industrial applications, with the main perceived drawback being that this technology has higher capital costs compared to other joining methods. They compared laser welding and bonding in a particular application and found the LTW technique unit cost was 70% less than that of the adhesives [36].
1.5 Objectives and Scope

The overall objective of this thesis was to develop a 3D numerical thermal model for the LTW process that can predict the spatial and temporal weld characteristics in contour (or scanning) welding of thermoplastics. The specific objectives of this thesis were as follows:

- To develop 3D transient thermal models of a lap-joint geometry being joined by the scanning LTW process for the purpose of predicting the time varying temperature field in the specimen,
- To predict weld dimensions from the temperature field,
- To validate the above models by employing thermal imaging studies and weld dimension data.

The modelling was carried out using the commercial finite element code ANSYS®\(^1\). The experiments were conducted in full scale using a ThermoVision A40 infrared camera for measuring the spatial and temporal temperature distributions\(^2\) in the plastic samples while being laser welded with a Rofin-Sinar DLX16 diode laser.

1.6 Thesis Map

Chapter 2 gives an in-depth survey of the pertinent literature, including information about lasers, experimental and modelling work relating to LTW, optical properties of the parts, and radiative properties of carbon black particles. Chapter 3 presents the background theory required for understanding of the modelling performed as part of the

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\(^1\) The technical support for ANSYS® was provided by ROI Engineering Company. Assistance of Mr. P. Budgell and Dr. T. Firmin in particular is appreciated.

\(^2\) The technical support for thermal cameras was provided by FLIR Company. Mr. R. Milner assistance with the cameras is appreciated.
thesis. The experimental approach used for the thermal imaging is outlined in Chapter 4. Chapter 5 describes the experiments conducted and theoretical developments made for describing the laser-material interaction; this information was required to define accurate input for the thermal model. Chapter 6 outlines the computational approach used in the development of the thermal finite element models and the techniques used to assure the model validity. Chapters 7 and 8 present the experimental and computational results, respectively. Finally, Chapter 9 provides conclusions and recommendations for the future work.

Appendices A to I provide information about thermal modelling of a T-like joint performed during the initial stages of this research, about experiments conducted to obtain plastic material properties as well as additional information on the effect of coatings in thermal imaging, thermal modelling of soot particles, and experimental equipment specifications.
Chapter 2  Literature Review

The modelling of laser transmission welding (LTW) must consider several technical subject areas including laser characteristics, carbon black particle radiative properties, plastic thermal and radiative properties, conduction, convection, and radiative heat transfer, and thermodynamics. The information in this chapter has been organized into the following sections to present a brief summary of the literature available in each of these areas: laser transmission welding of plastics applications (Section 2.1), characteristics of lasers used for the LTW process (Section 2.2), scattering, transmission, and reflection in the laser-transmitting part (Section 2.3), absorption in the laser-absorbing part (Section 2.4), radiative properties of soot particles (Section 2.5), experimental work on the LTW process (Section 2.6), previous studies to model the LTW process (Section 2.7), and the contribution of this research (Section 2.8).

2.1 Laser Transmission Welding Applications

LTW has found numerous applications in the automotive and biomedical device manufacturing. There is also a continuous growth in the use of LTW in the manufacture of micro-parts. This section presents an overview of the LTW applications.

Laser welding of plastics was first attempted in the 70’s with CO₂ lasers [37, 38]. The LTW technique was patented in 1982 by Nakamata [39]. The first product that was industrially mass-produced using the LTW technology was a keyless entry device manufactured by a German company, Marquardt (Figure 8a) [40]. Another application of LTW was for fabrication of keyless entry cards (Figure 8b) [27].
Although the LTW technique was first introduced in 1998 at a trade show in Germany, it was well established within three years due to quickly increasing demand in electronic and medical applications [41]. Russek et al. [27] reported that 75 production plants in Germany were established and committed to the LTW technology during 1998 to 2003. They presented a number of applications for the LTW process in automotive industry: connectors, rear lights, liquid containers, bumpers, dashboards, liquid tanks (Figure 9a), floodlights (Figure 9b), camshaft sensors (Figure 9c), and pump or turbine housings (Figure 9d); in the electronics industry: sensors and switches; in the medical industry: dialysis components; in the building trade: large dowels and polymeric windows; and in the household goods industries: shavers and plastic dishes. Klein et al. [42] talked about the micro-pump by ThinXXS, Mainz, which uses the laser welding to join housing components.
Figure 9- Examples of automotive products assembled using the LTW technique: a) a liquid tank of glass fibre reinforced PA, b) a floodlight of polymethylmethacrylate (PMMA), Acrylonitrile Butadiene Styrene (ABS), c) a camshaft sensor of glass fibre reinforced PA, d) a pump or turbine housing of polyacetal (POM) [27].

Weber [43] introduced the LTW technique to integrate sensitive electronics into motor mounts. Bayer and BMW [23] presented automotive intake manifolds for the M3 model assembled using the LTW technique (Figure 10a). Other applications of the LTW include filter cases of motor vehicles (Figure 10b), electronic engine oil sensors by Audi [44], integrating electronics in composite foot pedals by Ford [45], automotive taillight assemblies, and air bag release sensors manufactured by Bosch [46, 23] (Figure 11).

The use of the LTW continues to expand to other applications such as electro-pneumatic valves [47] and heat exchangers [48].
Figure 10- a) Plastic air intake manifold for BMW M3: The top part (a) is moulded from coloured PA6 and base part (b) is filled with carbon black particles [23], b) Automotive filter cases made from PA [27].

Figure 11- a) An air bag sensor manufactured by Bosch, b) The cross section through an air bag sensor [46].

In the biomedical field, syringes have been assembled using the LTW technique by AltaMAR Laser and Control in conjunction with Coherent Inc. (Figure 12a) [49]. This technique made it possible for the parts to have clean lines and addressed sanitation and disposability concerns. Smolka et al. [50] encouraged the use of the LTW technology in the laboratories, particularly for the micro-joining applications. They believed that features of this technology, such as small thermal loads on the laser-welded parts, controllable and precise laser-beam-energy delivery, and the good equipment life expectancy are advantageous. Micro-fluidic devices used for DNA analysis and clinical diagnostics were joined using the LTW technique [51] as well. In addition, medical packaging fabrication (e.g., blood bags) has employed this technology [52].
Lu et al. [53] investigated welding of micro-fluidic devices using the LTW technology. They mentioned that one of the main challenges in the joining of these devices was to connect a cover to a channel to produce a tube-like geometry. They needed to apply the LTW technique very carefully since the flash escaping from the joint could potentially block the channel. They suggested using a water-soluble material that prevented flash from flowing inside the channel. They applied diode laser to polyvinyl pyrrolidone (PVP) and polylactic-co-glycolic acid (PLGA) samples. They concluded that their method was successful since the micro-channels passed the leak tests. However, they noticed that on several occasions the micro-channels were deformed due to the weld flash overheat or extreme weld force.

Hustedt et al. [54] investigated the possibility of using LTW technique for micro-laser welding of dental prostheses made of PC. The micro-welds were created with and without masks using a diode laser. They studied the influence of process parameters on the weld dimension and quality. They found that LTW technique could be successfully employed for this application; however, they recommended not using the masks in order to eliminate creating and then adjusting the masks to the shape of the cavities.
Bielomatik Inc. from Germany and Leister from Switzerland, and Branson from the USA are among the few companies that manufacture laser systems suitable for the LTW applications. For instance, Bielomatik patented a double laser system to expand the possible workspace area of quasi-simultaneous LTW applications (Figure 13a), Leister created Novolas C system to join thin polymer layers for medical applications using mask welding technique (Figure 13b), and Branson manufactures equipment for simultaneous welding (Figure 13c) [55].

![Figure 13- a) Double lasers for quasi-simultaneous welding of larger parts, b) Mask welding for micro-structures [41], c) IRAM200 system for simultaneous welding from Branson [55].](image)

### 2.2 Characteristics of Lasers Used for the LTW Process

Laser (Light Amplification by Stimulated Emission of Radiation) was first introduced in the 1960s. A laser generates a coherent optical beam with a constant wavelength and time-dependant phase and amplitude. A laser system consists of a lasing gain medium (gas, liquid, or solid), means to excite the gain medium, optical resonator, and heat sink. The laser wavelength can vary from less than 0.2 μm to more than 10 μm, and pulse duration can be from less than 1 nano-second to continuous waves. The
average power can be up to several kilowatts, and the peak power goes up to Megawatts. The laser beam can be delivered to the work-piece by fibre optics or using mirrors [56]. Since a laser emits a directional radiation at a particular wavelength, it can be focused specifically on a small area. In LTW, a laser beam is used to melt the plastic in the joint interface. This laser beam usually belongs to the infrared range of the electromagnetic spectrum [16, 57]. In addition, it needs to function in a continuous mode (as opposed to a pulsed mode). The wavelength of the beam defines the application of a given laser since energy absorption in the plastics is a function of the laser wavelength. Figure 14 shows spectral transmission of some polymer materials [58].

The main laser types of interest for plastic processing are CO₂ [59, 60], Nd:YAG, and diode [61]. Short (UV wavelength) lasers such as excimers are not well suited for plastic welding. Non-laser radiation sources such as infrared lamp (tungsten filament) are commonly used for plastic welding but have disadvantages of longer cycle times, less energy efficiency, and less control over energy input into the weld [23, 62]. Table 1 shows laser types, their characteristics and applicability to the LTW process.

![Spectral transmission for some polymer materials](image)

Figure 14- a) Spectral transmission for some polymer materials [58].
### Table 1 - Commercial lasers and their characteristics [23, 59-66]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Types of Lasers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CO\textsubscript{2}</td>
</tr>
<tr>
<td>Wavelength ((\mu)m)</td>
<td>10.6</td>
</tr>
<tr>
<td>Laser active medium</td>
<td>gas</td>
</tr>
<tr>
<td>Max. power, continuous wave (CW) (kW)</td>
<td>60</td>
</tr>
<tr>
<td>Interaction with plastics</td>
<td>complete absorption at surface in &lt; 0.5 mm</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>5 - 10</td>
</tr>
<tr>
<td>Space needed</td>
<td>very large</td>
</tr>
<tr>
<td>Laser transmission welding suitability</td>
<td>yes</td>
</tr>
<tr>
<td>Capital cost ($1000 CAD)</td>
<td>High power</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>160</td>
</tr>
<tr>
<td>Running cost (S/CAD)</td>
<td>(S/W)</td>
</tr>
<tr>
<td></td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>(S/hr)</td>
</tr>
<tr>
<td></td>
<td>0.5 - 10</td>
</tr>
</tbody>
</table>

CO\textsubscript{2} laser beam (10.6 \(\mu\)m wavelength) is absorbed by most natural polymers within a very short depth. In addition, CO\textsubscript{2} lasers can leave visible marks on the external surface of the natural plastics (i.e., laser-transmitting part). Therefore, this laser is mainly used in thin-film applications and packaging industry [63, 64]. Nd:YAG laser (1.064 \(\mu\)m wavelength) has lower absorption within the plastic parts and therefore has deeper
penetration depth within the natural plastics. Therefore, applications requiring welding of the thicker plastic parts and films can be addressed.

Improvements in diode lasers (0.75 μm to 1.8 μm wavelength) have been the primary reason for the expanding popularity of the LTW process. These lasers are relatively easier to integrate and handle in industrial applications compared to other types of lasers [65, 66] because they are compact, efficient, and reliable. Diode lasers can be guided by glass fibre cables and therefore can be used in industrial robots. Diode lasers used in the LTW processes usually function in the wavelengths of 0.8 μm, 0.94 μm, and 0.98 μm [65]. A small fraction of the diode-laser energy is absorbed by natural polymers. However, they could be employed in the LTW process where an additive is added to one of the joining partners to make it laser absorbing, or an opaque surface is placed between the natural joining parts. Diode laser wavelengths being around 1 μm (similar to the Nd:YAG lasers) means their radiation has lower absorption within the plastic parts than CO₂ lasers. This makes it possible to employ the diode lasers for both thin films and thick-part applications.

The first diode laser was built by Hall in 1962 [28]. His laser had a wavelength of 0.84 μm and operated in the pulsed mode. The first diode laser that was capable of operating continuously at room temperature was developed in 1970 [28]. Diode lasers achieve their high power by combining moderate-power radiation of a large number of individual laser diode emitters (1 W to 2 W) (Figure 15a). Complex micro-optical elements converge a high number of laser beams from emitters to a focal spot of approximately 1 mm [65]. These emitters can be combined into a semiconductor element to generate more power. The size of this semiconductor element can be 10 × 1 × 0.1
mm$^3$. This element is called a laser bar (Figure 15b). The efficiency of a laser bar, which is the ratio of the optical power output to the total electric power input to the system, can be up to 50%.

Since only a fraction (approximately 30% to 50%) of the electrical energy put into the system is emitted as light, a significant amount of heat is generated during laser operation. Safe dissipation of this heat without damaging the laser is one of the main challenges in construction of diode lasers. In a diode laser, the bar with laser emitters is soldered onto a heat sink with copper micro-channels, which carry cooling water. The size of the channels is approximately $0.3 \times 0.3$ mm$^2$. A maximum power of up to 50 W/bar can be generated without the bar is being damaged by overheating. The laser bars are stacked on top of each other and the strips of the laser beam are collimated by
micro-lenses (Figure 15c). The strips are then collected by a spherical lens and the final beam is generated (Figure 15d and Figure 15e). The beam parameter product, an indicator of the beam quality ($BPP = w_o \times \theta$, where $w_o$ is the beam radius measured at the beam waist and $\theta$ is the beam divergence half-angle), equal to about 60 mm mrad can be achieved for a 160 W diode laser usually used for plastic welding.

Researchers have attempted to expand the boundaries of laser welding applications by introducing new laser types or improving the capabilities of the existing ones by initiating novel techniques. For instance, fibre-coupled lasers have introduced new possibilities [67, 68, 69, 70]. Combined with special lenses, these lasers can be used for micro-welding applications. Researchers have been able to generate micro-weld lines (approximately 25 µm to 50 µm) for PC and polystyrene (PS) welding parts [67]. High-power diode laser light has been shaped using diffractive optics for micro-joining [68, 69]. The use of high power femto-second lasers (0.775 µm) with the application to the micro-electro-mechanical systems, in order to create circular micro-channels in dry PS samples filled with 0.6% carbon black, was reported [70]. It was stated that long exposures to the laser beam caused longer and deeper patterns on the polymer plaque.

### 2.3 Scattering, Transmission, and Reflection in the Laser-Transmitting Part

In an LTW process, laser-beam-energy distribution reaching the joint interface is affected significantly by the optical properties of the laser-transmitting part. Some of the energy is lost due to the material absorption and reflection (Figure 16). Researchers have reported the relation between the optical characteristics of natural polymers and their
welding performance as a function of material thickness, operation wavelength, and additives for LTW processes [23, 58, 71]. Polymers such as PMMA, PA6, PE, and PP have been mainly studied due to their broad applications [71, 58].

Until recently, the LTW has been applied mainly to joining of relatively small-scale components (e.g., sensor enclosures, medical devices) in which the laser-transmitting material would be relatively thin (under 2 mm). As LTW is being extended to larger components (e.g., under-the-hood automotive components), commonly fabricated from reinforced semi-crystalline polymers, larger thicknesses of the laser-transmitting material are needed, leading to scattering having a significant effect on the energy distribution within the beam reaching the weld interface.

![Figure 16- Transmission through plastic samples: a) amorphous materials, b) semi-crystalline materials [58].](image)

Glasslike polymers typically reflect from the surface only about 7% of the incident beam intensity; however, many thermoplastics, including PA6, are not glasslike due to additives or inherent crystallinity. Therefore, surface reflection still happens but is minor compared to scattering within the polymer matrix [72]. The difficulty here is in separating the scatter and the absorption. It is known that fillers can increase internal scatter of the laser light and this increases the effective beam diameter. A portion of transmitted light may scatter away from the incident beam direction. This causes the energy density at the weld interface to be reduced [72].
The level and type of additives have a strong influence on the optical properties of polymers. For example, if short glass fibre reinforcements and mineral fillers are used, scattering will increase. The influence of fillers and colour additives on the light transmission has been investigated (Figure 17) [58, 73, 74]. Pigmented (green, yellow, red, white, and black) and natural PA6 were used to study the effects of colorants on optical properties [73]. Optimized methods for pigment loading of natural and glass-fibre reinforced grades of PA have been suggested [73]. It is believed that organic additives have better transmission than the inorganic ones due to their smaller particle size resulting in reduced light scattering [73]. PA6 thermoplastics blended with about 1% (wt.) of white or green pigments transmit 60% to 65% less near infrared light than the uncoloured polymer for a 3.2-mm-thick sample. Similar concentration of red colorants and pigments do little to near-infrared transmission [75].

![Figure 17](image_url)

Figure 17- Transmission of different grades of natural PA materials vs. glass-fibre content, material thickness is 3.2 mm: a) 1.064 µm, b) 0.83 µm [58].

Impact modifiers and fire retardants, depending on type, can reduce light transmission even more than fillers and reinforcements. The small inhomogeneities
introduced by impact modifiers scatter light more effectively than glass fibres, resulting in a transmission that is 55% lower than that of natural PA6 for a 3.2-mm-thick sample.

The laser-transmitting part thickness has a significant effect on the fraction of the transmitted laser-beam energy for semi-crystalline polymers [71, 76, 77]. For example, when the thickness of uncoloured or red PA6 was increased from 0.8 mm to 6.25 mm, its near-infrared transmission dropped from 85% to 42%, while for yellow, green, and white PA6, it decreased from 50% to 2% [75]. Some researchers suggested that the transmission of 50% glass-fibre-filled PAmxD6 was linearly decreasing when plotting the natural logarithm of transmission versus the part thickness (i.e. constant extinction for Bouguert-Lamber law) [76]. They reported a transmission of 75% for a 0.5-mm-thick sample. For a 3-mm-thick sample, a transmission of 11% was reported. Lee et al. [77] conducted similar experiments to obtain the transmission for a 30% glass fibre filled PA6. They reported that the transmission of the plastic part linearly decreased with the increase in its thickness (Figure 18). Rhew et al. [78, 79] stated that reflectance or transmittance of some plastic materials (e.g., PC and HDPE) is thickness-independent if the incident diode-laser beam hits the surface of the polymer plaque perpendicularly.

When laser light passes through the laser-transmitting part, another factor to consider is the effect of the laser wavelength. Vegte et al. [80] presented data for spectral light transmission of several engineering plastics, which included 2-mm-thick specimens of PA6 and PC (Figure 19a). Addition of 30% glass fibre to PA6 was shown to decrease transmission by about 22% at 1 µm (Figure 19b). The PC was the most transmitting engineering material: 89% and constant over the range of wavelengths examined. For PA6, transmission increased nearly linearly from 54% at 0.83 µm to 63% at 1.046 µm. Other researchers reported transmission of 78% and 69% for 2 mm and 3.2-mm-thick
natural PA samples, respectively, which remained unchanged in the spectral range of 0.7 µm to 1.1 µm [23]. Kagan et al. [58] also stated that the transmission for PA6 and PC did not change considerably in the spectral range of 0.83 µm to 1.046 µm.

![Graph](image)

Figure 18- Transmission vs. nominal thickness for PA6 samples [77].

![Graph](image)

Figure 19- a) Spectral transmission of 2-mm-thick un-reinforced engineering plastics, b) The effect of glass fibre-filler on the spectral transmission of 2-mm-thick PA6 plaques [80].

Scattering significantly affects the laser energy distribution within the semi-crystalline materials (e.g., PA6) due to the interaction of the laser light and spherulites (Figure 20). Scatter radiates the energy by reflection, refraction, and diffraction within
the medium. While investigating the use of LTW for micro-welding, Haberstroh et al. [81, 82] noted the importance of minimizing the scattering effect of the laser-transmitting part in this application. They believed that the injection moulding parameters were very important, especially for the semi-crystalline materials such as polybutylene (PBT), since the laser-material interaction is dependant on the internal structure of the injection moulded polymer samples. They chose a number of injection mould speed and temperature combinations to produce plastic plaques. The plaques were then welded and it was observed that each of the plaques showed different optical properties. Plaques with fine spherulites showed higher transmittance than those with the coarse spherulites.

![Atomic force micro-graph of the spherulites in un-reinforced PA6](image)

Figure 20- Atomic force micro-graph of the spherulites in un-reinforced PA6 [80].

Researchers have presented several approaches to study of laser light scattering by materials; these include the use of an x-y scanning pinhole [83] or a small light sensor [84], as well as the use of a sensor array such as a charged coupled device (CCD) camera [80].

Becker et al. [83] described measurement of the power flux distribution at the weld interface after passing through a 5-mm-thick sample of un-reinforced PP. They placed a 1-mm-diameter pinhole directly below the laser-transmitting part and a power meter below the pinhole. They moved the pinhole radially across the beam and obtained a relatively Gaussian radial power flux profile. The incoming laser beam diameter of 4 mm
was diverged to 7 mm after passing through the laser-transmitting part. The large pinhole diameter (with respect to the beam diameter) raises the question as to the accuracy of the measurements. The authors observed that the total power measured by the power meter was 30% to 50% larger than the one obtained by integrating the observed beam profile. They believed this discrepancy was due to the fast divergence of the light upon exiting the laser-transmitting material.

Ilie et al. [84] studied laser scattering in a laser-transmitting amorphous medium containing artificially added particles. They exposed a PMMA plaque filled with particle-sized silica to a diode laser beam. The size of their particles varied between 0.007 µm and 7 µm. They adopted a computer code developed by CORIA laboratory (University of Rouen, France) and modified it to fit their experimental conditions to predict scattering of materials. The code combined the Mie theory and Monte Carlo method with the assumption that the scattering medium is incoherent and non-absorbing. Using their code, considering a Gaussian beam profile for the laser beam, and globalizing several correction factors implemented in their model, they were able to determine the laser beam profile within and after passing through their scattering plaque. In their measurements, the 3-mm-thick PMMA samples filled with smaller particles (i.e., 0.007 µm and 0.04 µm), when exposed to a diode laser (0.808 µm wavelength), did not scatter the laser beam: the transmitted profile was identical to the incident one. With larger particle diameters (i.e., 5.7 µm and 7 µm), some scattering was observed. Incident beam diameter of 2.22 mm increased to 2.84 mm.

Vegte et al. [80] measured the scattering by engineering plastics (e.g., PA6, PC) for 2-mm-thick polymer plaques. They observed the laser beam spread for the semi-crystalline part was considerably more for the glass-fibre-filled PA6 than the natural PA6.
2.4 Absorption in the Laser-Absorbing Part

The rate of energy absorption by the laser-absorbing part has a strong influence on the weld quality. As discussed earlier, the laser beam absorption is achieved by adding carbon black or laser-absorbing dyes to the polymer. A number of studies have investigated the influence of carbon black mass fraction and size on the weld quality and strength for a LTW process. In addition, some have tried to measure the absorption coefficient as a parameter that defines what percentage of the laser beam energy is absorbed within a thin layer of the laser-absorbing polymer.

The research group led by Dr. Haberstroh at the Institute of Plastics Processing, Aachen, Germany, has reported on the effect of carbon particle size in several publications. Klein et al. [42] presented the results of their study in 1999 related to the micro-joining technology using the LTW technique. They investigated the effect of the carbon black on the weld quality of PMMA and ABS samples exposed to a diode laser beam during a mask welding process. They concluded that the parts with smaller carbon black particles showed a lower absorption, higher penetration depth, and higher weld strength. Schulz et al. [85] in 2000 also described the influence of the size of the carbon black particles on the LTW of PMMA and ABS. They reported that the weld strength for the parts with smaller particles (20 nm) was 10% higher than that for the parts with larger ones (60 nm). Haberstroh and Luetzeler [86] in a 2001 publication reported on the effect of carbon black particle size on the laser weld quality of PP and POM. They added carbon black with an average particle diameter of either 20 nm or 60 nm to the plastics during the extrusion process. They proposed that smaller carbon black particles had a higher heat loss to the surrounding plastic and consequently resulted in a better weld
quality (i.e., decreasing the particle size from 60 nm to 20 nm increased the weld strength up to 25%).

Absorption rate can be characterized by the absorption coefficient, with higher values corresponding to greater rate of absorption. Penetration depth is inversely related to the absorption coefficient, so that shallower depths correspond to higher absorption. Section 3.2 discusses these concepts in more detail.

A number of researchers have reported on the effect that the carbon black concentration has on the penetration depth.

Wehner et al. [87] described application of diode laser in the manufacturing of micro-fluidic devices. They welded a transparent PC plate cover to the PC base plate with different carbon black contents (0.5% - 1.5%, wt.). The base plate contained micro-channels created using laser ablation. They recommended the use of higher carbon black content (1% - 1.5%, wt.) for the parts due to the shallower weld seam.

Haberstroh and Luetzeler [86] found that by increasing the carbon black content from 0.2% to 0.5% (wt.) the heat affected zone thickness decreased while the melt temperature and weld strength increased (i.e., increasing the carbon content from 0.1% to 0.5% (wt.) increased the weld strength up to 23%). In a 2006 publication, Haberstroh et al. [88] showed the influence of carbon black content on the formation of the weld seam when joining thermoplastics in micro-technology applications. They applied diode laser beam using masked welding technique to overlapped PC samples and changed the carbon black content from 0.5% to 1.5% (wt.). They noticed that, with increase in the carbon black content, the penetration depth decreased (from 29 µm to 9 µm).
Russek et al. [29, 89] measured optical penetration depth of carbon-black-filled PA6 and PC. They showed that, for 0.2% (wt.) carbon black content, the optical penetration depths were 61 µm (PA6) and 71 µm (PC) at 0.94 µm wavelength [89].

Watt et al. [90] measured the penetration depth of the laser beam passing through the laser-absorbing medium for the LTW process. To achieve this goal, they cut very thin layers of the laser-absorbing material. This task was not easy due to the existence of the carbon black filler. They employed 3.7 mm PP plaques and cut thin layers of plaques (0.05-mm to 0.5-mm-thick wedges). They placed the wedges between a layer of the PP and a piece of Santoprene rubber, and passed the diode laser beam through the centre. They peeled off the Santoprene piece after welding had occurred. Based on the trace of the heat on the wedge, and using a scanning acoustic microscope, they estimated the laser-beam-penetration depth.

Very few papers reported actual values of the absorption coefficients. Chen et al. [91] obtained absorption coefficient values by measuring light transmission for PC at several low carbon black concentrations (0.1% and lower) (Figure 21). To achieve this, they measured the defocused laser-beam power after it passed through a small hole and the PC sample enclosed by glass plates located below the hole. Range of sample thicknesses (from 0.25 mm to 3 mm) was obtained by microtome cutting. An absorption coefficient just below 8 mm\(^{-1}\) was found for 0.1% (wt.) carbon black and linear dependence of the absorption coefficient on the carbon black level was observed for the three CB levels tested.
Figure 21- a) Natural log of transmission as a function of thickness for natural PC, b) Absorption coefficient of PC vs. carbon black level [91].

Ilie et al. [92] used thermal imaging method to estimate the absorption coefficient of the laser-absorbing part. They fitted a two-dimensional (2D) FEM model to their experimental data obtained from thermal imaging observations and assumed that the absorption coefficient of the laser-absorbing part was the only unknown parameter. They calculated the absorption coefficient of $19.5 \text{ mm}^{-1}$ for the PC/ABS alloy. Carbon black content of the 2-mm-thick sample was not reported.

2.5 Radiative Properties of Soot Particles

Limited experimental data related to soot optical properties and size or distribution is available in the LTW field. On the other hand, extensive research has been conducted in the combustion studies due to the importance of soot in this field. The author of this thesis believes that the carbon black particles embedded in the laser-transmitting polymer are similar to the soot particles suspended in a transmitting gas medium, and can be modelled likewise. Thus, soot is used instead of carbon black in this section.
Knowing the heat transfer from soot particles in combustion studies is of great importance. The emission from the soot particles is not desirable in jet-combustion chambers or rocket-engine bases. The estimate of this heat transfer is complicated since it is hard to know the amount of soot particles generated, particle size, shape, or optical properties. Extensive research in this area has concentrated on investigating spectral optical constants of soot particles and presenting thermal models to study soot-laser interaction. Researchers have attempted to make a connection between the absorption coefficient and particle size, volume fraction, and composition while making some simplifying assumptions regarding the surface roughness and soot particle shape [93].

Techniques such as transmission electron microscopy (TEM) [94] and diffusion broadening spectroscopy (DBS) [95] have been developed to study soot shape, size, number of particles, and primary particle diameter. Other methods, are based on laser heating and cooling of particles [96, 97, 98, 99, 100].

Figure 22 shows a typical TEM photograph of soot particles obtained in the combustion studies [101]. Using the TEM photographs, one can distinguish the primary spherical soot particles, which have similar diameters and form soot aggregates. Agglomerates are the product of soot aggregate clustering.

Figure 22- Typical TEM photograph of soot in combustion studies [101].
Wang et al. [102] studied the morphology of the soot particles embedded in the polymers for LTW processes. They used PA6 and PC samples filled with 0.2% (wt.) carbon black. They cut very thin layers of the polymer plaques (70 nm) and observed them using TEM method. Figure 23a and Figure 24a show TEM images obtained for PA6 and PC, respectively. Using image-processing software, the figures have been processed (Figure 23b and Figure 24b) and analyzed. Figure 25 and Figure 26 show the distribution of the aggregate and particle diameter for PA6 and PC, respectively. The average aggregate size of 80 nm and 35 nm was observed for PA6 and PC. The average particle diameters of 40 nm (PA6) and 20 nm (PC) were observed.

![Figure 23- a) TEM photograph of CB/PA6, b) Analyzed image of the TEM photograph [102].](image)

![Figure 24- a) TEM photograph of CB/PC, b) Analyzed image of the TEM photograph [102].](image)
Figure 25- a) CB aggregates diameter distribution obtained from TEM in PA6, b) Average CB particle diameter distribution obtained from TEM in PA6 [102].

Figure 26- a) CB aggregates diameter distribution obtained from TEM in PC, b) Average CB particle diameter distribution obtained from TEM in PC [102].

Note, however, that Zhu et al. [99] pointed out that TEM analysis mainly focuses on the smaller agglomerates due to its high magnification, which excludes imaging of large agglomerates. Therefore, they recommended optical microscope analysis in conjunction with TEM analysis for soot shape and size studies.

The information obtained for the soot particle size, shape, or distribution has resulted in several approaches to obtain spectral optical constants of soot [94, 101, 103, 104, 105, 106, 107, 108]. The main difference between these methods is
the assumptions made regarding the shape of the primary soot particles and the interaction among them. For example, the Rayleigh approach assumes that the ratio of the soot particle diameter to the wavelength is smaller than some fixed value as will be discussed later [94, 103]. Some methods assume that the primary particles are spherical with identical diameters [101, 105, 106]. In addition, they are small enough to be included in the Rayleigh regime; they have the same refractive index, and just touch one another. Some methods include the Rayleigh criterion except for the fact that the positions of the primary spherical particles and their interaction are included [107, 108].

Several studies introduced numerical models for soot particles when exposed to heat sources [109, 110, 111, 112] in which either the soot absorption coefficient was a known parameter and the temperature distribution for the soot particles were obtained, or the temperature distribution for the soot particles were obtained using imaging techniques and fed into the thermal models to estimate the soot absorption coefficient. Navier-Stokes and radiative heat transfer equations as well as Mie theory had a key role in developing these models. These studies suggested that heat absorption by the soot particles was volume related. In addition, they believed that surface area determined the heat loss from the particles; therefore, smaller particles had better heat transfer [109]. Furthermore, large non-spherical particles were not heated uniformly [113].

2.6 Experimental Work on the LTW Process

The experimental work in the LTW field has focused on the process parameters and mechanical performance, moisture content and weld quality, process monitoring, thermal model validation, micro-scale welding, Clearweld® technique, joint design, and weld
strength. This section introduces a brief review for each of these topics based on their relevance to this research.

2.6.1 Process Parameters and Mechanical Performance

A number of studies have focused on the influence of the process parameters (e.g., weld pressure, laser beam power, and scanning speed) on the weld quality and mechanical performance of polymers.

Kagan and Pinho [114] investigated the mechanical performance (i.e., tensile strength) of welded PA as a function of process parameters at room temperature. The main purpose of this study was to promote the use of PA in LTW applications. They applied a high power diode laser (100 W, 0.8 μm) to a T-like joint geometry, and compared the efficiency of the LTW process and vibration welding for different commercial grades of PA.

Potente et al. [115] studied the effect of process parameters on the weld quality for the quasi-simultaneous welding of PEEK T-like joint specimens exposed to an Nd:YAG laser. They believed that contour welding offered very limited gap bridging when compared to the quasi-simultaneous welding. They reported that linearly increasing the laser beam intensity linearly decreased the welding time. However, the weld strength decreased. They reported an optimum joining displacement at which the weld strength showed its maximum value.

Abed et al. [116] used a diode laser with PP specimens to observe the weld seam micro-structure. They investigated how the micro-structure is affected by the variation of laser speed and power as well as carbon black concentration in the laser-absorbing part. Raising the carbon black content from 0.05% to 5% (wt.), caused increase of melt layer
ratio between laser-transmitting and absorbing parts from 0.2 to nearly 0.9. Above 1%, there was not much change in this ratio. Ratios approaching 1 mean that heat is nearly equally distributed between the two components and indicate low penetration depth. Using transmitted polarized light and TEM micro-graphs, they were able to identify three distinct micro-structure zones within the weld cross-section, starting from the innermost zone, which experienced the greatest heating, to the outermost zone. The inner zone was observed to have relatively large spherulites (50 μm) compared to the bulk material and the authors expressed concern that this may affect negatively the weld's mechanical properties.

Al-Wohoush et al. [117] investigated the weld cross section micro-structure for lap-joint welds of PA6, PA6 30%GF, and PC plaques made with a range of process parameters. The same laser system employed in this research (based on the DLX16 diode laser) was used to make these joints. Figure 27 shows a cross section of a PA6 lap joint exposed to a 70 W diode laser beam moving at 25 mm/s, with a Laser Affected Zone (LAZ), a term defined by the paper’s authors, is clearly visible. It extends mostly into the (darker) laser-absorbing part and has elliptical boundaries, which taper off towards the weld line edges. The paper also reports that the areas of the LAZ for all materials were a linear function of the energy input into the weld (also known as the line energy) (Figure 28).
Bierogel et al. [118] introduced extensometry in order to evaluate weld quality and dimensions for the LTW process. They obtained the deformation and strength of different grades of welded PA plaques by tensile tests. They mentioned that weld quality was usually quantified by the welding coefficient, defined as the ratio of the weld tensile strength to the tensile strength of the bulk material. However, this coefficient does not provide enough information on the elastic behaviour of the weld or the influence of the weld surrounding area. Therefore, they recommended the use of heterogeneity parameter,
which was defined as the ratio of the local strain differences (i.e., maximum local strain minus minimum local strain) to the integral strain. They showed that the heterogeneity increased with increase in the integral strain. They also found that the welded PA plaques with higher glass fibre contents had lower maximum strain.

For a lap-joint geometry, when using relatively thin and flexible specimens, applying tensile force to measure the shear weld strength leads to an undesirable peel stress component. On the other hand, if a standard T-like joint is used, there is a large stress concentration at the corner of the "T" where the joint ends. This tends to reduce the apparent weld strength, as the tensile stress is no longer evenly distributed across the weld interface. To address the stress concentration issue, Kagan [23] introduced the idea of using a modified T-like joint in which a rib is added to separate the faying surface from the corners where the stress concentration exists.

Prabhakaran [119] studied the effect of process parameters (e.g., weld pressure, laser beam scanning speed, power, and area) on micro-structure and weld strength for unreinforced and 30% glass-fibre-reinforced PA6. Modified T-like joint specimens were joined using a diode laser and then tensile tested. Due to short length of the specimens, meltdown occurred with the contour welding technique used. Meltdown was found to increase linearly with the line energy (ratio of laser power to scan speed, which equals to the amount of energy delivered to the weld line per unit length). For reinforced specimens, weld strength reached a maximum when the line energy was raised to about 10 J/mm and then started to fall off. The best strength results approached the weld strength of the natural nylon.

Atanasov et al. [120] investigated laser welding of the cylindrical PC and PP samples. They found that PC and PP showed different behaviour. The PP reached the
melt temperature faster than PC. In addition, a smaller amount of the laser energy had to be gradually applied to the PC samples to avoid discoloration of the material and promote greater penetration depth.

Wehner et al. [87] recommended using lower laser powers and line energies to obtain cleaner weld seams for manufacturing of micro-fluidic devices. In addition, they found that with increasing the scanning speed, the weld seam width decreased and thus resulted in a better weld quality.

2.6.2 Moisture Content and Weld Quality

Kocheny et al. [121] investigated the influence of the moisture on the LTW process of PA6. They reported that although the absorption of moisture by PA6 could have changed its thermo-physical and mechanical properties, the existence of the moisture did not prevent good quality welds from occurring. They used natural and 33% glass-fibre-filled injection moulded PA6 samples for their experiments. They conducted tests with the samples that were sealed before welding, were kept in a chamber with the fixed relative humidity of 62%, and were submerged in the water to result in samples with 100% relative humidity. They welded these samples and performed cross sectional studies. They did not observe any significant change in the laser energy transmission due to the water content of the plastic parts. Tensile tests showed identical weld strength for PA6 samples; however, a maximum 30% decrease of the weld strength was observed for the 30% glass-fibre-filled PA6 samples when the humidity level increased to 100%.

In another publication by the same research group as in [121], Kagan et al. [122] further discussed the influence of the moisture on the PA welded parts and compared laser-welded specimen strength to those welded by vibration and ultrasonic welding.
techniques. They mentioned the existing concern related to the power-train-component performance given its exposure to moisture. They believed that moisture was not an obstacle for LTW applications; in addition, it did not have a negative influence on the mechanical performance of the welded PA samples. They reported that, under optimized laser welding conditions, the tensile strength of the laser-welded damp PA6 samples was equal to that of vibration-welded parts and superior to that of ultrasonic-welded parts. However, the shear strength of the laser-welded samples depended on the moisture content.

2.6.3 Process Monitoring

Some studies have reported on monitoring of the welding process. Haferkamp et al. [123] introduced the factors that had a fundamental influence on the laser weldability of natural and reinforced semi-crystalline and amorphous plastics. They conducted contour welding experiments using the diode and Nd:YAG lasers for both lap joint and T-like joint geometries. They applied tensile shear tests for the lap joints and tensile tests for the T joints to obtain the joint strength. They applied a thermographic method to plastics (PA and polybutylene - PB) to assess their transmissivity by measuring the temperature difference of the sample surfaces exposed to a diode laser beam. They integrated their findings into a FEM model, without reporting any details.

Von Busse et al. [124] used thermography to optimize the LTW process. They used the thermal response of the plastic exposed to the laser beam to predict the weldability of the plastic parts. They found that there is a direct association between the material composition, process parameters, and material heating. They adopted an analytical equation to predict the heat rate during the welding process. However, they concluded
that the combination of the laser scanning speed and power (i.e., line energy) is not sufficient to determine the weldability or weld quality and a correction factor was needed as well. In addition, other factors (e.g., carbon black content) needed to be considered. They found that with increasing the level of the carbon black, the correction factor value needed to be increased as well. Furthermore, they concluded that higher welding time (i.e., slower laser beam scanning speed) and lower power will ensure a good quality weld for a contour LTW process.

Herzog et al. [125] presented a monitoring method for LTW of thermoplastics. They employed a CCD camera along with a pyrometer to capture an image of the weld seam during the welding. They mentioned that, given the fact that the weld seam is usually covered by a laser-transmitting plastic, it is not possible to see through the weld; therefore, a special laser was used to light up the weld seam. To achieve this, they selected a laser-transmitting part that was partly transmitting to this special lighting laser. They believed that plastics’ surface finish and moisture absorption could change the quality of the weld and cause weld defects. Their focus was to study and monitor the effect of the parameters that could cause weld defects. They exposed a lap-joint geometry made of PA66 (30% glass fibre filled) to a diode laser beam. The camera was located off-axis from the laser beam. The weld area was lit by another diode laser beam. They made some grooves on the surface of the welding area, used sand paper to roughen the surface, and submerged the parts in water to increase the moisture content. They were able to measure the image contrast change or temperature drop of the weld seam due to the defects.

Fargas et al. [126] reported that due to the limited knowledge in the field of the LTW process monitoring, choosing appropriate process parameters was challenging for
quasi-simultaneous applications. Therefore, they suggested an online process monitoring of the melt pool during welding. To achieve this, they exposed PA6 samples with different fillers to a diode laser beam during a quasi-simultaneous process. They embedded tracer particles in the joint weld area. A high-speed thermal camera along with an image converter was used to record the particle motion lit up by passing an x-ray beam from a micro-focus tube. They concluded higher line energies resulted in more dynamic tracer particles and consequently higher weld strength values.

Jansson et al. [127] used an infrared camera to monitor the temperature and weld seam during a quasi-simultaneous process. To achieve this, they measured these parameters for the laser-absorbing PP and PC parts (without the top part) when exposed to a diode-laser beam. They concluded that when the laser beam scanned the welding path several times, higher weld strengths were observed. They observed that with increasing the number of scans, the width and thickness of the welds increased.

2.6.4 Thermal Model Validation

Few studies have been able to validate the LTW thermal models directly. They have mainly used indirect methods such as comparing the predicted and actual molten zone depths. Direct measurements by thermocouples inserted into the plastic are unreliable due to the laser energy being directly absorbed by the wire and due to errors introduced by high thermal conductivity of the wires (conduction errors) [83, 128]. Non-contact surface temperature measurement by infrared thermal imaging offers one possible approach to thermal model validation for LTW. While use of thermal imaging in LTW has been reported, its application has focused on material optical property assessment and not on model validation [123, 89, 172, 129]. Haberstroh et al. [129] employed thermal
imaging to measure the optical properties of different grades of PA exposed to a pulsed diode laser. They found that the natural glass-fibre-filled PA6 absorbed more energy than the un-reinforced one and consequently its temperature rose significantly. A similar response was observed for the polymers with high degree of crystallinity. They related this temperature rise to the laser beam scattering by PA6.

2.6.5 Clearweld® Technique

Clearweld® process was invented in 1998 by Jones [130]. In this method, the components do not have to incorporate a black part. The process allows two similar natural or coloured parts to join by using a nearly invisible absorbing layer in the form of a thin film or a spreadable liquid. Therefore, the weld forms without changing the appearance of the polymer. Parts can be of the same material, and optically transmitting joints are possible. The disadvantage is that a consumable material must be used and it must be applied to the joint interface in a separate production step (similar to an adhesive), thus negating some of the economic advantages of the LTW.

A number of studies have focused on the Clearweld® technology. Kagan et al. [131] investigated the weld strength of the Clearweld® technology for PA parts. They conducted experiments on the T-like joint samples for which the weld areas were hand painted with liquid dyes. They considered a fixed power and scanning speed for their diode laser beam; however, they changed the laser beam size from 2.5 to 4.5 mm. They observed that the weld efficiency of the clear-welded laser parts was very similar to that obtained from other welding methods. Hartley et al. [132] investigated Clearweld® technique with the focus on process parameters and spectral characteristics of PC, PA,
Hilton et al. [133] used Clearweld® technology to join the lid of a container to the main body using a diode laser. The lids of the components were made of natural PC, PP, and ABS, while the main bodies of the components were made of carbon-black-filled PC, natural PP, and natural ABS. Therefore, they applied the regular LTW process to join PC components and Clearweld® technology to join the PP and ABS components. In the case of the Clearweld® welding, changes in the orientation of the polymer molecules were observed. They concluded that these changes were due to the residual stresses caused by the material heating cycles during the welding process.

Woosman et al. [134] used the Clearweld® technique and measured the weld strengths for several thermoplastics (e.g., PC, PMMA, LDPE, PVC, PSU, and PETG) that were butt-joint-welded. In continuation to their study, Woosman et al. [135] investigated the influence of embedding laser-absorbing additives into different polymers for the Clearweld® applications. They were able to conduct limited number of experiments since certain polymers were compatible with certain additives (e.g., attempts to weld LDPE with G20 additive to the natural polymer did not succeed). They suggested more tests be conducted to study the compatibility of the plastic parts and additives.

Burrell et al. [136] studied the weldability of PC parts in the LTW process. Instead of using a coating with an absorber, they combined natural polymer with the organic-based near infrared absorber normally used in the Clearweld® process. The benefit was that this absorber is relatively transparent in the visible wavelength range and thus the laser-absorbing part does not have the black colouration normally occurring with the carbon black filler. They conducted a series of tests to predict optimum welding
conditions based on the absorber concentration level. They concluded that the absorber concentration and laser beam intensity had the most influence (almost linear) on the weld strength.

2.6.6 Joint Design

Haberstroh and Luetzeler [86, 137] investigated the LTW technique for a contour welding process by developing an experimental method to form a 3D weld contour in polymer joining. They studied and optimized the process parameters on PA and POM. Using contour welding method in their experiment, they were able to melt the weld interface; however, presence of gaps or bad spots due to warpage remained a challenge.

Kouvo et al. [138] investigated the potential for the LTW technology in 3D-joint applications. They believed size and shape of the part as well as the production cycle times are important factors, which determine the type of the LTW process to be employed. They concluded the quasi-simultaneous technique was able to tolerate the weld gaps during a welding process better than contour welding for their 3D joints. In addition, this process was relatively cheaper especially when using 2D scanners. However, they reported that this method was not as flexible as contour welding and more preparation time between the weld setups was needed. The biggest expense in implementing quasi-simultaneous welding for 3D joints was the requirement to use the 3D scanner and mirror mechanism.

Xu et al. [139] used the contour LTW technique to make a hollow vessel from unreinforced PA6. They believed that the joints made using this technique were stronger than vibration welded seams based on pressure tests. They studied the effect of the joint design (i.e., joint angle, location, thickness, height, and clamping pressure) on weld
strength, and defined a good weld as the one where the entire weld area was molten and then resolidified.

Kirkland et al. [140] introduced several joint designs for the LTW process. They specifically examined the collapse and non-collapse types of joints. They stated that certain collapse laser welding techniques produced flash similar to the ones of the hot plate and vibration welding. However, in some cases, since the part movement was restricted by the surrounding material for a “contained welding,” no joint collapse occurred. They believed that the collapsed welding was ideal for welding with the gaps (approximately 0.2 mm); however, in this case, the influence of the weld pressure became significant. On the other hand, for the contained welding, a maximum weld gap of 0.1 mm was bridged; in addition, the weld pressure did not play a key role.

### 2.7 Previous LTW Modelling Studies

A number of studies attempted to model the physical processes of heating, melting, and resolidification during a LTW welding process. These studies concentrated on predicting the temperature distribution and heat flow. The majority of these studies have introduced simplifying assumptions that reduced the accuracy of their predictions. Some of the simplifications made were that: 1) the laser beam was absorbed completely at the interface of the parts, 2) the convection and radiation heat transfer at the boundaries were neglected, and 3) material properties throughout the process were temperature-independent, 4) laser beam profile was uniform and scattering was ignored. In addition, mainly 1D and 2D models were investigated.

The purpose of this study is to eliminate some of these simplifying assumptions and present a more accurate thermal model that is able to predict the temporal and spatial
temperature profiles and weld characteristics during a LTW process. Such model may reduce the time and expense involved in the trial and error screening studies experiments to obtain the process parameters. Furthermore, an accurate thermal model will assist the optimization of the LTW process. This section introduces the literature pertaining to analytical models followed by numerical models, presented in order of increasing complexity: one-dimensional (1D), two-dimensional (2D), and then three-dimensional (3D) computational domains.

2.7.1 Analytical Models

Analytical models allow quick modelling of the welding process at the expense of accuracy in many cases; they require the least computational resources. An analytical thermal solution for a welding process was first proposed by Rosenthal [141] in 1941. He introduced a moving heat source applied to a highly conductive semi-infinite plate (metal). He ignored the dependence of the thermal properties to the temperature. Temperature-dependant properties cause nonlinearity that cannot be evaluated using analytical methods [142]. Kennish et al. [143] introduced an analytical heat conduction model for a Clearweld® process. They investigated the effect of process parameters on the weld characteristics. They employed an amorphous material (PETG) and assumed a semi-infinite plate with constant thermo-physical properties exposed to a symmetrical diode laser beam. They concluded that a good quality weld happens only when the temperature of the weld area exceeds the melt temperature. In addition, their measured temperature profile was very similar to a regular LTW process in which a laser-transmitting plastic is joined to a laser-absorbing one. They hand-painted ink onto the
joining area to achieve absorption at the joint interface and found that with increasing the ink concentration, the laser beam absorption increased.

Potente et al. [144] analyzed the heating phase in the LTW of PA6 in 1999. For the case of the absorbing part, they assumed that the absorption coefficient was sufficiently high that all of the heat was generated at the interface and was conducted equally into the both parts, resulting in a symmetrical temperature distribution. Given the above approximation, they applied an analytical heat transfer model of single-sided impulse welding to the LTW process. For a laser-absorbing material with a low absorptivity, they defined a correction factor and supported their results by comparing the calculated and measured melt-layer thicknesses. They assumed that the material properties did not change with temperature and ignored the effect of heat convection during the welding process.

2.7.2 One-Dimensional Models (1D)

A numerical model is capable of accounting for the effects of a moving heat source and temperature-dependant thermo-physical properties. Modelling of the above effects causes nonlinearity that cannot be evaluated using analytical methods [145, 146, 147, 148].

One-dimensional (1D) numerical models are the simplest implementation of the numerical approach to model physical phenomena. They have low computational requirements and allow for obtaining quick results.

Grewell et al. [149] presented a 1D thermal model to investigate the LTW technique for thermoplastics. They adopted the power flux for the weld to occur using Stokes approach and chose PC, ABS, Acrylic, and PS samples for their studies. They
found that the welds made by LTW technique had very low residual stresses and uniform heating was achievable. Using their 1D model, they suggested the minimum power requirements for particular welding time and working distance as well as diode laser type. In addition, they recommended the optimum laser-transmitting part thickness for welding PC as a function of laser beam power and weld time. In continuation to their study, Grewell et al. [31, 150, 151] modelled heat flow for a moving heat source (laser) in a welding process. The model was applied to plastics being micro-welded by transmission method using fibre-coupled laser diodes. The samples were assumed semi-infinite with temperature-independent thermo-physical properties. Their moving coordinate system and Gaussian heat source had the same velocity, which resulted in a quasi steady-state heat conduction problem. The samples were assumed semi-infinite with temperature-independent properties.

Prabhakaran et al. [119] investigated a modified T-like joint made of PA6 exposed to a diode laser beam. A 1D finite difference model (FDM) was developed to predict the temperature distribution. The absorption of the laser-absorbing part was obtained by using it as a parameter to fit the 1D model to the experimentally observed heat affected zone thickness.

Kurosaki et al. [152] numerically modelled lap joint welding of thermoplastic parts (with the focus on low-density polyethylene - LDPE) exposed to a CO₂ laser. They introduced a simplified 1D transient heat transfer equation for the welding process; however, they did not specify the boundary conditions for their problem. They presented a numerical model to predict the melt depth for different processing conditions for amorphous materials (PC, PMMA, and PS samples) [153]. They concluded that thermal diffusion and absorption properties of the plastic parts had great influence on their melt
depths. They found that the change of the material did not change the temperature profile as much since the materials had similar thermo-physical properties. They concluded that with increasing the thickness of the laser-transmitting material, the peak temperature decreased and in order to obtain higher temperature peak, the laser beam intensity had to be increased. In addition, they studied the influence of absorption coefficient and found that with increase of the absorption coefficient the peak temperature increased.

2.7.3 Two-Dimensional Models (2D)

Two-dimensional (2D) models provide the possibility to study the LTW process within the plane transverse or parallel to the laser-beam moving direction. Several studies modelled a moving laser heat source by a 2D model in the plane of the moving beam. By using a coordinate system moving together with a heat source, a quasi-steady state model is obtained. Sato et al. [154] studied the effect of transmissivity and absorptivity of coloured plastics on the temperature distribution within the welded parts in 2002. They assumed a semi-infinite work-piece, stationary laser beam, and moving work-piece. They applied the FDM to model PS plates in a lap-joint configuration. They assumed that the diode laser beam had a circular cross section. They modelled a pair of transmitting and absorbing PS plaques held between transmitting PMMA plaques. Thermo-physical properties were assumed constant.

Becker et al. [83] modelled the heating phase of LTW process along the moving beam direction using FEM and compared the results with collected data from experiments with PP. They tried to measure the temperature distribution by a pyrometer; however, they had difficulty obtaining consistent emissivity at the surface. Their next attempt was to measure the temperature by finding the melt layer thickness in the joint zone using a
transmitted-light microscope. They studied the effects of beam scanning speed and laser power on the melt layer thickness. It was observed that with lower level of energy, the melt layer thickness decreased. However, they could only use this method for semi-crystalline thermoplastics, where the boundary of molten and solid plastics could be identified. Their experimental melt profile had a concave shape while their model led them to a convex melt profile. The main drawback of such 2D models is that they cannot predict the temperature distribution in the joint transverse to the beam travel direction.

Potente et al. [155] modelled the heating and cooling phase in a LTW process along the moving beam direction using the FEM based on a commercial code ABAQUS®. They considered PA6 as a case study and presented the temperature and flow profiles. They assumed an “effective absorption coefficient” for the laser-absorbing part and zero absorption for the laser-transmitting part. Therefore, they obtained a symmetrical temperature profile along the depth of the material. They proposed the use of higher order elements to better address the coupling of the thermal and mechanical models; however, they did not further pursue or present the mechanical modelling.

Huang et al. [156] proposed a heat conduction simulation along the moving beam direction and solved it using the FDM technique. Their lap-joint geometry made of natural PP (30% glass fibre filled) and black Santoprene was exposed to a diode laser beam. They assumed that 90% of the incident energy was absorbed in a very thin layer of the laser-absorbing part, thermo-physical properties were temperature-independent, and the phase change was ignored. In addition, the laser beam was assumed to have a Gaussian profile. They obtained the temperature distribution at the same line energies but different process parameters. They observed that the peak temperature increased from 261°C to 436°C at the same line energy (3.5 J/mm) but doubled power level. Their
model suggested peak temperatures that were significantly above the polymer degradation temperature.

Shaban et al. [157] presented a thermo-mechanical simulation of a LTW process along the moving laser beam direction, employing ABAQUS® to solve their FEM problem. They exposed a butt-joint geometry made of PC to a diode laser beam. They reported that the physical properties of the amorphous polymer changed with temperature and strain rate and this change was very sharp at the glass transition temperature ($T_g$). They assumed a Gaussian profile for the laser beam. They concluded that the maximum temperature happened 0.15 mm from the interface inside the laser-absorbing part. They found the horizontal stress at the upper surface and centerline, and concluded that at the centerline, the stress increased with increasing time up to the point where the $T_g$ was achieved and then dropped sharply to zero. The stress remained zero until the temperature was above the $T_g$. When the temperature went below the $T_g$, the stress again increased and then relaxed with time.

Fargas et al. [158] analyzed the weld seam quality experimentally and theoretically. They developed a 2D FEM simulation for a quasi-simultaneous LTW process to study the mechanical and morphological changes of the polymer during a welding process. They defined an effective absorption coefficient for their simulation, which was obtained by fitting the experimental results into the simulations. The details of the FEM simulations were not provided.

Ilie et al. [92] presented a 2D FEM model for the LTW process of an amorphous material (PMMA filled with laser-absorbing particles and PC/ABS carbon filled alloy). They assumed their axial model was symmetrical and used COMSOL software to solve
their problem. The main purpose of this model was to obtain the absorption coefficient as discussed earlier in Section 2.4.

Coelho et al. [159] developed a 2D simulation of the laser welding process for thin films of HDPE and LDPE in a lap-joint configuration exposed to high speed CO$_2$ laser beams. They studied the influence of welding process parameters and beam shape on the weld quality. They solved the heat equation using the Green function. They found that the energy delivered to the weld area by the elliptical laser beam cross section was higher than that of the circular cross section beam (for the same area), and the elliptical cross section resulted in better weld quality and higher processing rates. Laser scanning speed did not have a considerable effect if the beam intensity was kept constant. They did not provide any information on the temperature profile at any stage of the welding process.

Potente et al. [160] presented a simplified combined analytical-FEM model for a quasi-simultaneous welding of PMMA, PC, and PA6 parallel to the laser-beam moving direction to obtain the temperature and flow profiles. They assumed that the diode laser beam had a circular cross section with a symmetrical Gaussian profile. The peak intensity measurements to obtain Gaussian beam profile were 21% lower than the calculated power based on the formula they suggested. Therefore, they used a correction factor to address this discrepancy. They found that the peak temperature happened within the laser-absorbing part, 0.2 mm from the joint interface. In addition, the laser-absorbing part showed first signs of melting.

Mayboudi et al. [161] modelled the 2D heat transfer for a LTW process of a T-like joint made of PA6 in which some of the simplifying assumptions in the earlier models presented in the literature were eliminated. Thermo-physical properties were
temperature-dependent, and phase change was accounted for in the model. More information on this study are presented in Appendix A.2.

2.7.4 Three-Dimensional Models (3D)

Three-dimensional (3D) models impose minimum restrictions on the modelling of physical phenomena, but at the expense of significant increase in the computing resource requirements. However, with the rapid growth of the computing power, progressively more complex numerical models have become possible. 3D quasi-steady models for laser welding of materials – most notably metals – have been previously addressed [162,163]. Some of theses models have been simplified to 1D models based on the symmetry criteria [162]. Since the laser welding of metals is a very different process compared to the plastic welding due to the existence of a keyhole\(^1\) effect in metal welding, these studies could not be applied to the LTW process directly.

Nevertheless, some researchers believe that a keyhole can be produced with the LTW technology as well. However, plastics will degrade if they are exposed to this condition [164]. On the other hand, the plastics and metal welding have a similarity in the sense that both can employ Bouguer-Lambert law. In metal welding, the keyhole is considered as a black body, which absorbs energy [163]. Therefore, the laser beam intensity changes as it passes through the keyhole according to the Bouguer-Lambert law. The absorption coefficient of the keyhole though is relatively small (0.8 mm\(^{-1}\)) [163]. Researchers have presented bibliographies of finite element analysis and simulation of welding processes for all materials (mainly metals); these studies have been incorporated into the modelling in this research [165, 166, 167, 168].

\(^1\) A keyhole in welding is the vapour cavity formed when a high-power laser is used to scan along the joint. The cavity is filled with liquid metal behind the moving beam.
The 3D thermal modelling for the LTW of thermoplastics was first presented by Mayboudi et al. [169, 170, 171]. The lap-joint studies will be presented as part of this thesis. Speka et al. [172] introduced a simplified 3D model for the LTW of amorphous materials (PMM and PC/ABS carbon black filled alloy). The geometry was simplified by assuming a symmetry plane, and a circular cross section for the laser beam instead of an elliptical cross section was assumed. They employed COMSOL to solve their problem, and assumed a very small absorption coefficient (0.38 mm\(^{-1}\)) for the laser-absorbing part. Detailed information on the temperature profile was not provided. They used thermography to predict temperature-dependent physical and optical properties. They observed some discrepancies between the experimental and theoretical results that could have been due to the orientation of their thermal camera with respect to the sample surface.

An error approximation technique was suggested to carry out adaptive meshing in thermal modelling problems [173, 174]. Although the results obtained for the temperature distribution showed continuity, the heat flux field was discontinuous in some cases.

Grewell and Benatar [175, 176] introduced multiphysical coupled models for molecular diffusion, temperature, and squeeze flow for laser transmission micro-welding of plastics. In these models, a Gaussian distributed heat source for a semi-infinite plaque was assumed. The model was able to predict the weld size and quality at the joint area. The model was used to obtain the lower limit of the weld size as well. It was noted that the steady-state models were not able to describe the process. In addition, the weld strength as a function of transient temperature at the weld area was investigated [176]. They applied pulsed laser beam to PC and PS plaques in a lap-joint configuration. By
choosing a very thin layer of polymer, they assumed that the weld area was isothermal. They mainly focused on the rough surfaces of the parts to be joined and the melt flow, which filled the rough areas. To model the surface roughness, they assumed that a number of identical small cylinders were placed between the two surfaces. They made some simplifying assumptions for their flow model such as constant material properties, a Newtonian fully developed and symmetrical fluid, and solved the cylindrical Navier-Stokes equations for the flow between the joining plaques. They found that the strength and heating time to the $\frac{1}{4}$ power were linearly related.

2.8 Contribution of this Work

During a welding process, high temperature is produced in the weld area. This temperature falls rapidly with increasing distance from the weld interface. To form a strong bond, it is important that the weld interface be exposed to sufficient heat to melt a thin layer of polymer in order to form close contact between the two surfaces allowing the intermolecular diffusion to occur. On the other hand, excessive heat could degrade the polymer. Delivery of the thermal energy by the laser beam is affected by process parameters, such as laser beam power, scanning speed, beam spot size, and material properties, such as absorptivity, presence of reinforcements and other additives.

To understand and apply the LTW technique to any industrial applications, research engineers must be recruited and trained to operate lasers and carry on experiments in conjunction with the material suppliers. However, these pilot studies are expensive and time consuming. The knowledge generated and advanced in this research can be applied to the LTW industrial applications in which process and cost optimization is of great importance.
In theoretical aspects, developing and solving a full 3D model to address the laser-material interaction for a LTW process has always been considered a challenging task; especially, for the LTW of the semi-crystalline polymers whose scattering properties introduce uncertainty regarding the beam intensity profile at the joint interface.

In this research, 1D, 2D and 3D thermal models of a modified T-like joint geometry (Figure 29) and 3D thermal models for a lap-joint geometry (Figure 30) were developed to study laser-material interaction. The mathematical models for the LTW process were presented and a commercial FEM code (ANSYS®) was employed. The results of the finite element thermal model for the lap joint were compared to the experimental data obtained from thermal imaging and weld dimensions studies.

Figure 29- T-like joint geometry to study the LTW of thermoplastics.
Figure 30- Lap-joint geometry to study the LTW of thermoplastics.

The thermal model presented in this research addressed the major simplifying assumptions presented in the previous studies. Nevertheless, the potential for improvement exists, especially, to integrate thermal, structural, and flow in a multiphysics model.

The main contributions of this thesis are:

- This was the first time a full 3D transient thermal model was developed and validated for the stationary and scanning laser beam for the LTW process of semi-crystalline and amorphous plastics,

- This was the first time a full laser beam profile was measured and incorporated into a 3D thermal model for plastics laser welding applications,

- This was the first time that the scattering of the laser beam after passing through a scattering laser-transmitting medium was modelled and incorporated into a 3D thermal model,
- This was the first time that the absorption coefficient of the laser-absorbing part for the LTW applications was modelled, with the help of soot studies presented in the combustion field,

- Detailed theoretical data were generated for the welding process (e.g., weld pool size and temporal and spatial temperature distribution).
Chapter 3 Theory

This chapter presents the theory of conductive (Section 3.1) and radiative (Section 3.2) heat transfer, laser-material interaction and Bouguer-Lambert law (Section 3.3), Rayleigh approximation theory (Section 3.4), which describes interaction of laser light and soot particles, and finite element method (Section 3.5).

3.1 Heat Conduction and Convection Equation

The purpose of this section is to introduce some of the fundamental concepts, equations, and the parameters that influence the heat conduction required for thermal modelling of the laser transmission welding. Heat conduction is one of the heat transfer modes. Heat transfer by conduction occurs due to the molecular motion within a medium from a high-temperature region to a low temperature one. The main purpose of studying the heat conduction in a medium is to determine the spatial and temporal temperature profiles as well as the heat flow for that medium. Fourier’s law states that the heat flux (q) within an object is proportional to the temperature gradient in the direction of the heat transfer. For a 1D plane wall, this law can be written as:

\[ q_x = -k_x \frac{\partial T}{\partial x} \]  

(1)

where \( k_x \) is thermal conductivity along the x direction, and T is temperature distribution.

The general heat conduction equation defines the temperature distribution within a body based on the energy conservation law, which balances the rate of the internally generated heat within the body, body’s capacity to store this heat, and the rate of thermal
conduction to the boundaries (based on Fourier’s law). A general heat conduction equation in three-dimensional Cartesian space can be expressed as follows [177]:

\[
\frac{\partial}{\partial x}(k_x(T) \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k_y(T) \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z}(k_z(T) \frac{\partial T}{\partial z}) + q(x, y, z, t) = \rho(T)C(T)\frac{\partial T}{\partial t}
\]  

(2)

where \(k_x(T), k_y(T),\) and \(k_z(T)\) are anisotropic thermal conductivities, \(\rho(T)\) is temperature-dependent density, \(C(T)\) is temperature-dependent specific heat, \(t\) is time, and \(q(x, y, z, t)\) is internal heat generation rate per unit volume. A simplified version for Equation 2 can be obtained for the case of an isotropic material where thermal conductivity is independent of the direction, \(k_x(T) = k_y(T) = k_z(T) = k(T)\).

The heat generation in this research is due to a laser beam interaction with the plastic material as the beam passes through it. The heat generation is defined in terms of the laser power flux change caused by the absorption of the laser beam energy by the plastic and is a function of laser power, material absorption properties, laser-beam cross-section dimensions, laser beam scanning speed, and laser-beam power flux distribution. This heat generation term varies with \(x, y, z,\) and time.

The surfaces exposed to air transfer the energy by the convective heat transfer mode. This convective heat flux \(q_{\text{conv}}\) is expressed by Newton's law of cooling [177]:

\[
q_{\text{conv}} = A \cdot h \cdot (T_s - T_{\infty})
\]

(3)

where \(h\) is convection heat transfer coefficient, \(T_s\) is surface temperature, \(T_{\infty}\) is fluid temperature, and \(A\) is the area of the surface. Depending on the flow nature, convection can be categorized as a forced or free convection. If the flow is caused by external means (e.g., a fan), a forced convection takes place. In a free convection, the flow is caused by buoyancy forces caused by temperature difference in the fluid.
Convective heat transfer coefficient depends on the geometry and fluid motion. One can obtain this coefficient using free convection correlations for the flows near vertical or horizontal plates as a function of Nusselt and Grashof numbers [177]. Nusselt number \( (hL/k) \) is the dimensionless temperature gradient at the surface, where \( L \) is the characteristic length. Grashof number \( (g\beta(T_\infty - T_s)L^3/\nu^2) \) gives the ratio of the buoyancy force to the viscous force acting on the fluid, where \( g \) is gravitational constant, \( \beta \) is volumetric thermal expansion coefficient, and \( \nu \) is kinematic viscosity.

### 3.2 Radiation Heat Transfer

The thermal imager pertinent to this study detects the radiation emitted from the object due to the laser-material interaction. This section provides the background theory related to the radiation heat transfer. Radiation is a heat transfer mode that occurs due to the electromagnetic waves. Unlike the heat conduction, which requires a medium, thermal radiation can pass through vacuum. The mid-range of the electromagnetic spectrum that covers the spectral range from 0.1 µm to 100 µm, including the infrared range (i.e., 0.7 µm to 100 µm), generates thermal radiation. The magnitude of the energy radiated and the spectral distribution are important factors that are characteristics of the emitting body’s surface. Electromagnetic radiation is governed by the Planck function, which describes the spectral distribution of a blackbody:

\[
E_{\lambda b}(\lambda, T) = \frac{2 \pi h c_0^2}{\lambda^5[\exp(h c_0/\lambda k T) - 1]}
\]

where \( h \) \((6.625 \times 10^{-34} \text{ J·s})\) and \( k \) \((1.3805 \times 10^{-23} \text{ J/K})\) are the Planck and Boltzmann constants, respectively. \( c_0 \) \((2.998 \times 10^8 \text{ m/s})\) is the speed of light in vacuum, \( T \) is the
absolute temperature of the body, and $E_{sb}(\lambda, T)$ is the spectral emissive power (W/m$^2$·μm).

All objects emit energy at temperatures above their surroundings. The radiation flux emitted from the surface of objects ($q_{rad}$) is related to the thermal energy within their boundaries, which is described by the Stefan Boltzmann law:

$$q_{rad} = \sigma \varepsilon (T_s^4 - T_{\infty}^4)$$  \hspace{1cm} (5)

where $T_{\infty}$ is the absolute surroundings temperature, and $\sigma$ ($5.67 \times 10^{-8}$ W/m$^2$·K$^4$) is Stefan Boltzmann constant, and $\varepsilon$ is the emissivity. In a blackbody, 100% of the surface energy is emitted ($\varepsilon = 1.0$). For a real surface, the emitted energy is a fraction of that of the ideal surface ($\varepsilon < 1.0$). The ratio of these two energy levels is the emissivity, which is a radiative surface property and varies between zero and one. Equation 5 can be rewritten to match the form of the convective heat transfer equation (Equation 3) as follows:

$$q_{rad} = h_r (T_s - T_{\infty})$$  \hspace{1cm} (6)

where $h_r$ is the radiation heat transfer coefficient: $h_r = \sigma \varepsilon (T_s^2 + T_{\infty}^2)(T_s + T_{\infty})$.

Absorptivity ($\alpha$) is defined as the ratio of the radiated energy absorbed by a real surface to that of the black body’s and varies between zero and one. In a blackbody, 100% of the energy reaching the surface is absorbed ($\alpha = 1.0$). A gray surface has the same absorptivity and emissivity according to the Kirchhoff’s law ($\alpha = \varepsilon$). If $\alpha < 1$, then the surface is opaque and a fraction of the radiated energy is reflected (surface radiation). This is shown by reflectivity ($\rho$). For a transparent object, a fraction of the radiated energy is transmitted (volume radiation). The ratio of the transmitted energy to the total
incident energy is called transmissivity ($\tau$). The relation between these optical properties is as follows:

$$\tau + \alpha + \rho = 1 \quad (7)$$

### 3.3 Laser-Material Interaction and Bouguer-Lambert Law

The interaction of an electromagnetic wave passing through a medium and hitting the surface of another medium was first studied by Maxwell in 1864 [178]. He defined the relation between optical, radiative, and electrical properties of the wave passing through an ideal material where the surface is optically smooth and clean. Maxwell found the amplitude of electric intensity wave for a medium with a very large thermal conductivity was a function of spectral refractive index ($n_\lambda$) defined as the ratio of the speed of light in the vacuum to the speed of light in the medium. For a medium with a finite thermal conductivity, energy is absorbed gradually as it travels through the medium, which introduces a spectral extinction coefficient ($k_\lambda$) to Maxwell’s relations. The combination of the refractive index and extinction coefficient defines complex refractive index ($\tilde{n}_\lambda = n_\lambda - ik_\lambda$). $n_\lambda$ and $k_\lambda$, the real and imaginary parts of the spectral complex refractive index, are known as optical constants and are used for absorption coefficient calculations needed to obtain spectral intensity of a beam passing through a laser-transmitting medium. Spectral intensity can be described by the Bouguer-Lambert law [178]:

$$i'_\lambda(y) = i'_\lambda(0) \exp \left[ -\int_0^y k_\lambda(y') dy' \right] \quad (8)$$
where $i'_\lambda$ is the directional spectral radiation intensity, $y$ is the coordinate along path of radiation with $y = 0$ mm corresponding to the position of the incident surface, $\lambda$ is beam wavelength, and $K_\lambda$ is total spectral extinction coefficient and is a function of temperature ($T$), pressure ($P$), concentration of the absorbing species ($f_v$) and wavelength of the incident radiation ($\lambda$):

$$K_\lambda = K(\lambda, T, P, f_v)$$  \hspace{1cm} (9)

where $K_\lambda$ is the total extinction coefficient, which is a combination of the extinction due to absorption ($K_a$) and extinction due to scattering ($K_s$):

$$K_\lambda = K_a + K_s$$  \hspace{1cm} (10)

Beer’s law, which is a simplified version of the Bouguer-Lambert law, assumes that the total extinction coefficient ($K_\lambda$) is only dependent on the volume concentration of the absorbing species along the path in Equation 9 ($K_\lambda = K(f_v) = K$) [178]:

$$I(y) = I_0 e^{-K_y}$$  \hspace{1cm} (11)

where $I_0$ is the radiation intensity at the surface ($y = 0$).

If scatter is ignored, Equation 10 is simplified ($K_\lambda = K_a$). Optical penetration depth ($d_p$) is defined as the length within the material in the direction of the laser beam propagation where the initial laser beam energy is reduced to $1/e (= 0.368$). The extinction coefficient is inversely related to the optical penetration depth:

$$K = 1/d_p$$  \hspace{1cm} (12)
3.4 Rayleigh Approximation

Rayleigh theory can be used to describe the absorption of radiation energy by a particle (e.g., soot). A dimensionless quantity known as the size parameter describes the soot particle size in relation to the wavelength of the incident light. The size parameter is defined as the ratio $\frac{\pi d_{\text{par}}}{\lambda}$, where $d_{\text{par}}$ is the soot particle diameter. When the size parameter is less than 0.3, the scattering cross section, which is a function of $(\frac{\pi d_{\text{par}}}{\lambda})^4$, is negligible compared to the absorption cross section, which is a function of $(\frac{\pi d_{\text{par}}}{\lambda})$. Hence, the spectral extinction coefficient, which is a function of absorption and scattering cross sections, and is needed for absorption coefficient calculations, will only depend, in this case, on the absorption cross section. Therefore, the extinction coefficient can be obtained through the classical Rayleigh theory as follows [178]:

$$K_\lambda = \frac{36 \pi n_\lambda k_\lambda f_v}{(n_\lambda^2 - k_\lambda^2 + 2)^2 + 4n_\lambda^2 k_\lambda^2}$$

(13)

where $f_v$ is the soot volume fraction.

From the definition of the size parameter, for the 940 nm wavelength of a typical diode laser, the maximum particle size for which the Rayleigh theory is applicable is 90 nm. Equation 13 is applicable to the absorption coefficient of small particles suspended in a non-absorbing medium. The effect of the particle diameter has already been incorporated into this equation assuming the soot can be modelled as spherical particles of uniform diameter [178]. Table 2 shows the spectral optical constants for the propane soot, a type of soot commonly used in combustion studies. For 940 nm wavelength, the values of 1.61 and 0.54 can be obtained for $n$ and $k$, respectively, by linear interpolation from the data presented in Table 2.
### Table 2- Spectral optical constants of propane soot [178]

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>n</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>550</td>
<td>1.57</td>
<td>0.53</td>
</tr>
<tr>
<td>650</td>
<td>1.56</td>
<td>0.52</td>
</tr>
<tr>
<td>806.5</td>
<td>1.57</td>
<td>0.49</td>
</tr>
<tr>
<td>2500</td>
<td>2.04</td>
<td>1.15</td>
</tr>
<tr>
<td>3000</td>
<td>2.21</td>
<td>1.23</td>
</tr>
</tbody>
</table>

### 3.5 Finite Element Method (FEM)

The idea of representing a large domain in terms of sub-domains can be traced back to Babylonian mathematicians (1680 BCE), who estimated the value of π by using a large number of sub-domains (sides) of a polygon to approximate a circle. In modern era, Hrenikoff [179] first introduced the concept of the FEM in 1941. He introduced the framework method in which he defined the domain of a plane elastic medium in the form of sub-domains of bars and beams. Courant in 1943 used triangular elements to study the St. Venant torsion problem, and Clough used the term of finite element first in 1960 [179].

One needs to take the following steps in order to develop a thermal FEM model: 1) a geometrically complex domain of the problem is represented as the collection of geometrically simple sub-domains (i.e., finite elements). Each element is bounded by a set of nodes at which the solution variable is calculated; 2) the approximate equations satisfying the conservation of energy are derived for each element using a polynomial; 3) the continuity of the nodal value and its derivative is applied to the adjoining elements; and 4) algebraic relations among the unknown nodal values are obtained and solved simultaneously knowing that the governing equations over each element are satisfied.

To understand the FEM approach, assume the following linear differential equation [180]:
where \( L \) is a linear differential operator in the interval \([a, b]\) and \( \phi(x) \) is the solution (e.g., a temperature field). The goal is to find an approximate solution for Equation 14. The solution to this problem could be found by multiplying the equation by a weight function and integrating it with respect to \( x \) over the defined interval:

\[
\int_{a}^{b} (L\phi)v(x)dx = \int_{a}^{b} f(x)v(x)dx
\]

(15)

where \( v(x) \) is a weighting function, \( \phi_1(x), \phi_2(x), ... \) are linearly independent basis functions for the linear space \( x \).

There exist different techniques for solving this problem. One of them, which is employed in the ANSYS® software, is the Galerkin method. In the Galerkin approach, the weighting function is the same as the basis function. To solve the FEM problem using the Galerkin approach, a solution \( (\phi(x)) \) is approximated using a linear combination of the piecewise local basis functions. Afterwards, a weak equation that is not restricted in terms of inter-elemental continuity is obtained. Finally, the coordinates are transformed into isoparametric coordinates to simplify the formulation.

Assume the following 2D heat conduction equation with a heat source [180]:

\[
L\phi = \frac{\partial\phi}{\partial t} - \alpha \left( \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} \right) - S(x, y, t) = 0
\]

(16)

For this equation, the FEM formulation assumes an approximate steady-state solution, \( \bar{\phi}(x, y) \), for Equation 16:

\[
\bar{\phi}(x, y) = \sum_{j=1}^{J} N_j(x, y)\phi_j \quad j = 1, 2, ..., J
\]

(17)
Figure 31 shows an area that is subdivided into elements. The approximate solution for element e_{NE} is as follows [180]:

\[
\tilde{\phi}(x, y) = \phi_N N_j(x, y) + \phi_N N_N(x, y) + \phi_N N_{NE}(x, y) + \phi_E N_E(x, y) \quad x, y \in e_{NE} \tag{18}
\]

The same logic is applicable to all elements.

![Figure 31- Four square elements surrounding a typical interior point j [180].](image)

The basis function \( N_k(x, y) \) is made to be 1 at \((x_k, y_k)\) and zero at the three other vertices of the square element. Since \( \tilde{\phi}(x, y) \) is an approximate solution to Equation 16, its substitution into the Equation 16, generates a residual \( R \):

\[
L\tilde{\phi} = \frac{\partial^2 \tilde{\phi}}{\partial t^2} - \alpha \left( \frac{\partial^2 \tilde{\phi}}{\partial x^2} + \frac{\partial^2 \tilde{\phi}}{\partial y^2} \right) - S(x, y, t) = R(x, y, t) \tag{19}
\]

To minimize the residual for the steady-state solution, it must be orthogonal to all basis functions [180]:

\[
(R, N_j) = \int_{\Omega} R N_j(x, y) dx dy = 0 \tag{20}
\]

where \( \Omega \) is the domain.

One advantage of using the Galerkin approach is the lower requirement of inter-elemental continuity. In the general heat conduction equation (Equation 16), the function
\( \phi(x, y) \) needs to be twice differentiable; however, in Equation 20, the approximate function \( \tilde{\phi}(x, y) \) only needs to be continuous in itself and its first derivative. That is why it is called the weak form of Equation 16.

Another advantage of Galerkin approach is that the piecewise functions get their local support from the elements centred on the grid point (e.g., node \( j \) in Figure 31). Therefore, Equation 20 can be reduced to the sum of the four integrals.

\[
(R, N_j) = K_{NW} + K_{SW} + K_{SE} + K_{NE} = 0 \tag{21}
\]

where \( K_p \) is defined as follows [180]:

\[
K_p = \iint_{\Sigma_p} \left( \frac{\partial \tilde{\phi}}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial \tilde{\phi}}{\partial y} \frac{\partial N_j}{\partial y} - SN_j \right) \, dx \, dy \tag{22}
\]

where \( P = NW, SW, SE, NE \).

The last task is to solve Equation 22. To simplify this equation, the physical coordinates are transformed into isoparametric coordinates. To achieve this, the coordinate is transformed so that the basis function within the element can represent universally those in all the local elements.
Chapter 4 Experimental Approach

This section addresses the experimental approach taken in this research. To understand the experimental approach, one needs to understand the materials used in the experiments and their preparation method (Section 4.1), the laser and fixture apparatus (Section 4.2), thermal imaging (Section 4.3), and the approach taken to conduct thermographic measurement for the stationary (Section 4.4) and moving (Section 4.5) laser beam cases. Section 4.6 analyzes the error sources in thermal imaging observations.

4.1 Materials and Sample Preparation

Natural PA6 (Akulon, F223D) from DSM and PC (Makrolon, AL2647) from Bayer were employed for the laser-transmitting (natural) plastic parts. The laser-absorbing part was a pre-compound (natural PA6 and PC mixed with 0.2% (wt.) carbon black by the material supplier).

Natural PA6 is in the form of granules (Figure 32a). PA granules were heated for 36 hours prior to injection moulding in an oven at the Royal Military College of Canada (RMC) to ensure their dryness (Figure 32b). Granules were then injection moulded at RMC using a 55 ton Engel machine (Figure 33) in the form of $100 \times 100 \times 3$ mm$^3$ plastic plaques (Figure 34a)$^1$. The material supplier provided the injecting moulding data (Table 3). Moulding conditions were carefully chosen to ensure the uniformity of the material distribution inside the mould$^2$.

The plaques were then separated from the sprue (Figure 34b). The extra part of the runner was then removed using an electric band saw at RMC (Figure 35a), and the

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$^1$ Mr. J. Perreault, RMC technician’s help with the injection moulding of the samples is acknowledged.

$^2$ Mr. M. Chen’s, Queen’s Ph.D. student, assistance with making the samples is acknowledged.
unevenness of the edges of the sample was removed with a utility knife (Figure 35b). The plaques then were cut into smaller pieces \((83 \times 27 \times 3 \text{ mm}^3)\) using a milling machine located at the machine shop of the Department of Mechanical and Materials Engineering at Queen’s University\(^1\) and sanded to create smooth and even edges (Figure 36).

Figure 32- a) PA6 granules, b) Drying the PA6 granules in the oven.

Figure 33- Injection moulding machine (55 ton Engel located at RMC).

---

\(^1\) Mr. A. Bryson and other technicians in the Mech. Eng. machine shop are acknowledged for their assistance with cutting of the samples.
Figure 34- a) The injection moulded PA6 sample, b) Separating the sprue from the PA6 sample.

Table 3- Injection moulding conditions

<table>
<thead>
<tr>
<th>Injection Moulding Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection hold pressure, bar</td>
<td>60</td>
</tr>
<tr>
<td>Injection hold time, s</td>
<td>20</td>
</tr>
<tr>
<td>Cooling time, s</td>
<td>20</td>
</tr>
<tr>
<td>Screw feed speed profile, %</td>
<td>95,95,95,55,20</td>
</tr>
<tr>
<td>Back pressure, bar</td>
<td>5.0</td>
</tr>
<tr>
<td>Mould temperature, °C</td>
<td>70</td>
</tr>
<tr>
<td>Barrel temperature, °C</td>
<td>270</td>
</tr>
<tr>
<td>Nozzle temperature, °C</td>
<td>275</td>
</tr>
</tbody>
</table>

Figure 35- a) Cutting the edges of the PA6 samples using an electric band saw, b) Cleaning up the edges of the PA6 sample.
4.2 Laser System

The experiments presented in this thesis were carried out using a Rofin DLX16 160 W diode laser operating at 940 nm wavelength (Figure 37a). The laser produced a focal spot size of $0.7 \times 1.4 \text{ mm}^2$ at a working distance (WD) of 83 mm [181]. WD was defined as the distance from the laser lens to the material surface. Exposing a plastic surface to the laser beam well away from the focal plane revealed that the beam was produced by the output of 100 individual diode sources arranged in four groups of 25 (Figure 38). The diode laser was housed inside a safety enclosure (UW200 workstation, Figure 39). The laser head was mounted on a frame and was capable of moving vertically along the $y$ direction of a three-axis motion system (Figure 37b).

The sample was mounted on a clamping fixture that could be moved along the $x$ and $z$ directions using two linear motion stages (Figure 37b). The laser vertical movement and sample horizontal movements, as well as the exposure to the laser beam could be controlled manually or by means of a computer code programmed using the NView software (supplied by Aerotech). A built-in camera allowed observation of the laser beam spot on the surface of the sample (Figure 37a). A pneumatic clamp system
provided the appropriate pressure for keeping the samples in their place. See Appendix G for detailed laser system specifications.

Figure 37- Rofin-Sinar laser welding workstation interior: a) diode laser head DLX16, b) pneumatic clamping fixture and the three-axis motion system.

Figure 38- Individual diode lasers effect on the absorbing plastic plaque seen with the defocused laser beam.
4.3 Infrared Camera

Thermal imaging was used in this work as one method of validation of the thermal model. Thermal infrared (IR) imaging cameras are detector and precision optics platforms that produce a visual representation of the infrared energy emitted and reflected by all objects [182, 183].

All objects above absolute zero (-273°C) radiate infrared energy [184]. The radiation measured by the camera depends on the temperature, surface emissivity ($\varepsilon$), and surroundings conditions of the objects. Surroundings (i.e., the area around the object) partly reflected by the object are important. Volume emission and transmission are important because, if the material is not opaque, the camera will be able to see into the object. Surface emissivity affects the emission of thermal radiation by the object and the reflection of the surroundings ($\varepsilon = 1 - \rho$ if opaque). Radiation from the object is also influenced by the absorption in the atmosphere and any external optics that it passes through. To measure the surface temperature distribution, it is highly desirable that the
object be opaque. To minimize the effects of the surroundings and the internal volume, we want the surface emissivity to be near that of a black body (i.e., \( \varepsilon = 1.0 \)) [185]. Experiments were conducted to measure the emissivity of the plastic part. The emissivity of 0.95 was used for the plastic samples. For further details on the emissivity measurements, see Appendix B.2.1.

In order to observe and record the temperature distribution during the LTW process, the thermal imaging camera (ThermoVision A40) was mounted on a frame built inside the enclosure of the diode laser welding workstation (Figure 40). The fixture was secured to the enclosure floor and the camera’s position could be adjusted along the x, y, and z directions as required for alignment with the specimen. The camera had a spectral range of 7.5 \( \mu \text{m} \) to 13 \( \mu \text{m} \). ThermoVision A40 uses a \( 320 \times 240 \) uncooled micro-bolometer focal plane array. The image refresh rate is 60 Hz. In order to assure accuracy of the thermal imager, it needed to be calibrated. Appendix H provides camera specifications and describes the calibration procedure.
The thermal imager camera was connected to a portable computer using a FireWire (IEEE 1394) interface. The camera was controlled, images were captured and stored on a hard drive, and temperature readings were collected using ThermaCAM™ Researcher Pro 2.8 Software by FLIR SYSTEMS. This software provides several analysis tools (e.g. spot, area, and line), which can be employed for extraction of the temperature readings.

The thermal imager field of view for ThermoVision A40 thermal imaging camera is shown in Figure 41. A metal ruler (dimensions in cm) was held against the plastic sample to obtain the field of view dimensions. It is seen that the field of view window is a $25 \times 33 \text{ mm}^2$ rectangle. There are 320 pixels in the horizontal field of view and 240 pixels in the vertical field of view. Dividing the field of view dimensions by the number of pixels gives the spatial resolution of the image as 0.103 mm/pixel in the horizontal and 0.104 mm/pixel in the vertical direction.

---

1 The framing was supplied by Shelley Industrial Automation manufactured by Parker Automation. The models of framing components were downloaded from the manufacturer’s website and were used in Solid Edge assembly.
Temperature measurements were extracted from the video images by defining the lines and the spots where the data was to be collected. To position the lines and spots in the image accurately, the pixel coordinates of each spot as well as the start- and end-points of each line were used. The pixel coordinates were converted into the real world dimensions using the spatial calibration of the image. One can locate the start and end of each line or spot based on their pixel coordinates with a spatial resolution of 0.1 mm.

4.4 Thermal Imaging Experiments for the Stationary Laser Beam

Experiments were conducted with the stationary diode laser beam for the purpose of the thermal model validation. Stationary laser beam means that the sample was not moving with respect to the laser beam and the thermal imager. The heating process was prolonged (10 s) and the laser beam power was decreased (1 W) to accommodate the thermal camera data-acquisition speed limitations while avoiding material degradation.

The laser beam centre (x = 0 mm) was located at a distance “a” of 3 mm from the front edge of the plastic sample (Figure 42). This location of the laser beam made it possible to observe temperature rise on the front surface of the sample using the thermal camera.
Figure 42- The diagram of the stationary laser beam thermal imaging experiments: a) pictorial view, b) setup viewed from above and the side (all dimensions in mm, not to scale).
The thermal camera (equipped with a macro lens), had to be placed so that the lens was 20 mm from the sample front surface to achieve focus. The camera was directed perpendicular to the sample front surface (Figure 43 and Figure 44). The viewing axis was perpendicular to the laser beam axis.

Figure 43- The setup for the stationary laser beam thermal imaging experiments: a) the overall view inside the enclosure, b) the close-up view.

Figure 44- a) A PC sample mounted inside the fixture, b) The close-up of a PA6 sample.

An illustration of the image window in Figure 45 shows the location of 7 lines and 16 spots where the temperature readings were collected for these experiments. The
horizontal lines (LI01, LI02, LI03, and LI04) mark the location of the part boundaries within the thermal image.

![Diagram](image)

Figure 45- The lines and spots at which thermal imaging measurements were collected.

While carrying out thermal imaging experiments, it was important to assure that the true surface temperature of the object was captured as opposed to the temperature within the bulk of the material. This issue is particularly important in this case since the peak heat generation occurs close to the surface facing the camera (3 mm away) and high thermal gradients are expected. To address this issue, the recommended technique in thermography is to coat the surface facing the thermal camera with a material of known emissivity. Thus, several surface treatments were tested while conducting the thermal imaging observations of plastic samples exposed to a stationary laser beam (Table 4).
Table 4- Experimental matrix for the stationary laser beam experiments

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Test S1</th>
<th>Test S2</th>
<th>Test S3</th>
<th>Test S4</th>
<th>Test S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Power (W)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Laser beam exposure time (s)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>Distance from the edge (mm)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Material</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
</tr>
<tr>
<td>5</td>
<td>Surface treatment</td>
<td>Uncoated</td>
<td>Black tape</td>
<td>Soot</td>
<td>White tape</td>
<td>White correction ink</td>
</tr>
</tbody>
</table>

To ensure consistency of the results, all coatings were applied to the face of the same plaque. The plaque’s surface facing the camera was divided into five equal regions, each with one of the surface treatments applied: uncoated, black-tape coated, soot coated, white-tape coated, or white-correction-ink coated. The two long faces of each rectangular plastic plaque were used, and five different samples in total were tested to ensure repeatability of the experiments. Test S1 was the base condition where no coatings were used.

In Test S2, the plastic samples were covered with 0.2-mm-thick black electric tape ($\varepsilon = 0.95$) [186]. However, it was believed that the added thermal resistance at the joint between the tape and the plastic could have affected the accuracy of the readings. Thus, in Test S3, the polymer samples were coated with soot particles (HB lead from a pencil: 68% graphite, 26% clay, and 5% wax) on the surface facing the thermal camera ($\varepsilon = 0.95$) [178].

Temperature observations of the laser-transmitting part with black-tape and soot coatings indicated that the scattered laser light (as shown in Figure 46) might have been absorbed by the coatings, and their temperature increased as a result. To address this heating effect, in Tests S4 and S5, the surface was coated with materials having
emissivities close to 1.0, and which did not absorb significantly the laser beam energy (at the near-infrared wavelength). It is known that the white surfaces have similar emissivity to that of the black surfaces at the operational wavelengths of the thermal camera (i.e., $\varepsilon = 0.95$) [186].

![Diagram of the stationary laser beam thermal imaging experiments.](image)

**Figure 46-** The diagram of the stationary laser beam thermal imaging experiments.

In Test S4, the plastic parts were coated with a 0.2-mm-thick white electric tape, which had similar specifications to those of the black tape. Test S5 addressed the effect of the imperfect contact between the white tape and the surface of the plastic part. To achieve this, the front surface of the plastic parts was coated with a thin layer of white correction ink.

Another issue that could potentially affect accuracy of the thermal imaging observations was the existence of a small gap between the two plastics parts being joined due to surface imperfections. This gap might have allowed direct transmission of thermal radiation emitted by the centre of the heated zone at the joint interface (as indicated in Figure 46). To address this, the test parts were pre-joined by making several parallel weld lines across the area where the stationary beam was located for thermal imaging tests.
4.5 Thermal Imaging Experiments for the Moving Laser Beam

Experiments were conducted with the moving diode laser beam for the purpose of the thermal model validation. The setup for these experiments was the same as the one for the stationary laser beam (Figure 42 and Figure 43) except that the sample was moved with respect to the laser beam and the thermal camera horizontally and parallel to the part’s surface observed by the camera. An additional step was required to ensure that the distance of the laser beam centre from the sample edge was uniform as the sample moved under the beam. A dial gauge was used to validate that the sample edge remained at a fixed distance along z-axis. It was assumed, based on the experiments described earlier, that the temperatures indicated by the camera on this surface were the true surface temperatures.

Experiments were conducted for different power levels and laser beam scanning speeds and for two polymer materials (PA6 and PC). Table 5 shows the matrix for the moving laser beam thermal imaging experiments. A set of base process parameter values (P = 18 W, V = 30 mm/s) were selected (Test M1) based on the material weldability observed in the beam scattering measurement experiments (for PA6) reported in Section 5.2. Thermal imaging requirements played the primary role in determining the process parameters – achieving good weld quality was not the priority. Base power of 18 W and speed of 30 mm/s gave a line energy of 0.6 J/mm, which was observed to produce welds in the trial runs. The line energy was doubled to 1.2 J/mm by raising the power to 36 W while maintaining the same speed (Test M2). The influence of increasing the speed while keeping the line energy constant was tested (Test M3) by halving the power (18 W) and halving the speed (15 mm/s) compared to Test M2.
Table 5- Experimental matrix for the moving laser beam experiments

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Test M1</th>
<th>Test M2</th>
<th>Test M3</th>
<th>Test M4</th>
<th>Test M5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Power (W)</td>
<td>18</td>
<td>36</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Speed (mm/s)</td>
<td>30</td>
<td>30</td>
<td>15</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Distance from the edge (mm)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Material</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PC</td>
<td>PC</td>
</tr>
</tbody>
</table>

While primary focus was modelling for PA6, two additional sets were run for PC, in which case scattering was not taking place (Tests M4 and M5). Power and speed combination of 18 W and 30 mm/s were used. One set was for the laser beam located at 3 mm and another at 2 mm away from the sample edge. The smaller distance was tested since scattering was not taking place and thus the beam width was expected to be smaller than that of PA6 allowing the beam to be brought closer to the edge without significant beam scatter affecting the observations.

Each pair of laser-transmitting and laser-absorbing plaques was first scanned along one long side and then rotated 180 degrees and scanned along the opposite side. Five different sample pairs in total were tested for each test condition in order to assess the repeatability of the results.

### 4.6 Error Analysis for the Thermal Imaging Experiments

In this section, the errors associated with thermal imaging experiments are examined and quantified. When extracting temperature data from a thermal image, sample spots or lines are located in the image (as shown in Figure 45). The following can be identified as possible sources of error: 1) Uncertainty in the spatial location of these sampling points or lines in the image due to limitations of image spatial resolution and due to uncertainty in location of the sampling point or line, 2) Background noise of the
camera image sensor, and 3) Uncertainty about the emissivity of the specimen surface. For the stationary laser beam experiments, temperature readings for the uncoated PA6 samples were used. For the moving laser beam experiments, temperature readings for the base case with PA6 samples ($P = 18$ W, $V = 30$ mm/s) were used.

4.6.1 Spatial Position Uncertainty in the Thermal Image

As was described in Section 4.4, the $320 \times 240$ pixel thermal image was spatially calibrated by capturing images of a ruler scale (Figure 41). Assuming an error of $\pm 0.2$ mm for reading of the magnified ruler scale in the image, an average positioning error of $\pm 0.1$ mm may be expected near the centre of the image. In addition, because of the image spatial resolution of 0.1 mm, one can expect to have an error of 0.05 mm. A third source of spatial error is the location of the edges of the part or any other feature in the image. An error between 1 and 2 pixels (or 0.1-0.2 mm) can be expected.

The spatial error can be translated into temperature error by considering the highest expected temperature gradients in the image. Gradients up to $20^\circ$C/mm were observed in images of a moving laser beam (along the y-axis). Thus, 0.1 mm position error can result in 2-4$^\circ$C temperature error.

4.6.2 Background Noise in the Thermal Image

To assess the background temperature variability, a spot image analysis tool was placed in the thermal image video collected for an experiment with uncoated PA6 sample and a stationary laser beam. The spot was placed 5 mm from the left edge and 8 mm from the upper surface of the laser-transmitting part. Readings over 5 s near the start of the experiment were recorded and are plotted in Figure 47. The maximum range of
temperature variation was 1.36°C (from 23.49°C to 24.85°C), with an average temperature of 24.10°C and the standard deviation of 0.26°C based on 301 data points collected. A histogram of the temperature confirms that it is a normally distributed random variable.

Figure 47- The background temperature variability for the stationary laser beam thermal imaging experiments: a) transient temperature diagram, b) histogram.

Figure 48 shows the background temperature variability observed during the moving laser beam experiments. The spot sensor was located at 8 mm from the sample top surface. The temperature variation range was 1.31°C (23.91°C - 25.22°C), with average temperature of 24.54°C and the standard deviation of 0.23°C for 301 data points collected for the 5 s interval. The background variability of temperature appears to be similar in both cases and distributed normally (as can be seen in the histogram plot) with a maximum range of variation of about 1.3°C.
Figure 48- The background temperature variability for the moving laser beam thermal imaging experiments: a) transient temperature diagram, b) histogram.

In addition to the inherent background variability of the temperature, for the experiments where the laser beam and the sample were moving relative to one another, the thermal camera was fixed with respect to the laser while the part moved in the field of view. This resulted in a quasi-static image observed by the camera. It is then of interest to determine how stable this image is; for example, if any variations in the laser or the sample surface quality have caused variability in the thermal imaging data collected.

Figure 49 shows temperature profiles collected using a sampling line oriented along the depth of the sample (y-axis) and passing through the point where the peak temperature was observed. For this figure, temperature profiles were recorded approximately every 0.25 s over 1.5 s interval, giving seven profiles. It is seen that there is a good repeatability between the temperature profiles. Table 6 lists the peak temperatures for the temperature profiles in Figure 49. The average temperature obtained was 33.19°C with the standard deviation of 0.56°C. This standard deviation is only slightly higher than the inherent background variability with standard deviation of 0.23°C.
Figure 49- The temperature repeatability along the depth of the sample (y-axis) for the moving laser beam thermal imaging experiments.

Table 6- The peak temperature repeatability for the moving laser beam experiments

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Peak temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32.9</td>
</tr>
<tr>
<td>0.250</td>
<td>33.3</td>
</tr>
<tr>
<td>0.517</td>
<td>33.7</td>
</tr>
<tr>
<td>0.751</td>
<td>33.9</td>
</tr>
<tr>
<td>1.018</td>
<td>33.4</td>
</tr>
<tr>
<td>1.251</td>
<td>32.3</td>
</tr>
<tr>
<td>1.501</td>
<td>32.8</td>
</tr>
</tbody>
</table>

4.6.3 Uncertainty in the Emissivity Value

Thermal imager calculates the temperature based on its internal calibration and using the emissivity of the observed material surface. Thus, any error in the emissivity translates into inaccuracy of the temperature observations.

Figure 50 tracks temperature over 1.5 s interval at a spot where maximum temperature was observed in the moving laser beam experiment. Emissivity value of $\varepsilon$
= 0.95 was used in this thesis for the plastic surface (see Appendix B.2.1). For this setting, the difference between the maximum and minimum temperatures was 2.29°C, with the average of 33.07°C and the standard deviation of 0.539°C based on 91 data points. To assess the sensitivity of the temperature to the emissivity, the emissivity setting was varied in the thermal imaging software from 0.85 to 1.0 and corresponding temperature readings were collected and plotted in Figure 50 for each setting. Table 10 summarizes the data plotted in the figure for different emissivities. The average temperature decreases from 34.50°C to 32.46°C when the emissivity increases from 0.85 to 1.00. Thus, a range of temperature variation of about 2°C results when emissivity ranges from 0.85 to 1.0.

Figure 50- Temperature variability for the moving laser beam thermal imaging experiments.
Table 7- Temperature variability assessment for the moving laser beam experiments

<table>
<thead>
<tr>
<th>Emissivity</th>
<th>Minimum temperature (°C)</th>
<th>Maximum temperature (°C)</th>
<th>Average temperature (°C)</th>
<th>Standard deviation (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>33.46</td>
<td>35.99</td>
<td>34.50</td>
<td>0.59</td>
</tr>
<tr>
<td>0.90</td>
<td>32.76</td>
<td>35.16</td>
<td>33.75</td>
<td>0.57</td>
</tr>
<tr>
<td>0.95</td>
<td>32.13</td>
<td>34.42</td>
<td>33.07</td>
<td>0.54</td>
</tr>
<tr>
<td>1.00</td>
<td>31.55</td>
<td>33.75</td>
<td>32.46</td>
<td>0.52</td>
</tr>
</tbody>
</table>

4.6.4 Spatial Temperature Variation of the Image Background

Thermal images collected with ThermoVision A40 camera during the moving laser beam welding trials have displayed a common variation of the background temperature from left to right edge of the image: the middle third of the image was observed to be slightly cooler than the left and the right sides. This can be seen in a typical image taken during experiments with a moving laser beam joining PA6 samples (Figure 51). The image shows darker blue (cooler) region through the middle third of the image. The effect is quantified by the plots in Figure 52. Temperature profiles, with data averaged over 20 pixels (to remove random noise), are plotted for the horizontal line L1 located near the bottom of the image. One profile was collected from the PA6 test while another was collected from a test with PC specimens. Both profiles show similar gradual dip towards the middle, with the temperature varying over 2.3°C range for PC and 1.8°C for PA6. It is not known what causes this image artifact. It may be speculated that it is connected with the variation of the detector performance over its area or with the camera optics. Note that the camera manufacturer (FLIR) specifies camera accuracy as ±2°C or ±2% (Appendix H).
In summary, based on the assessment of the data variability presented in this section, an uncertainty of approximately $\pm 1.5^\circ C$ can be expected in the collected temperature values.

Figure 51- A “cool spot” artifact in the thermal image taken during the moving laser beam experiment with PA6.

Figure 52- Spatial temperature variation of the thermal image background for moving laser beam experiments with PA6 and PC.
Chapter 5 Laser-Material Interaction

This section presents the studies conducted in order to measure or calculate the inputs to the thermal model. These input parameters include the unscattered laser beam profile before hitting the laser-transmitting part (Section 5.1), the scattered laser beam profile at the interface of the two parts (Section 5.2), the absorption coefficient of the laser-absorbing part (Section 5.3), and the thermo-physical properties of the polymers from which the parts were moulded (Section 5.4).

5.1 Characterization of the Unscattered Laser Beam Profile

This section presents the experiments conducted to characterize the high power diode laser beam used in the LTW process employed in this research. These experiments made it possible to provide detailed 3D information on the laser-beam intensity distribution. The experimental method employed a pinhole moving under the laser beam while the power transmitted through the pinhole was measured. The results obtained from the pinhole technique were then compared with the ones obtained using an alternative beam profiling technique (the knife-edge method). To show the power-independence of the laser beam profile, beam profile measurements for two power levels were compared.

5.1.1 Beam Profiling

Many laser applications are sensitive to the laser beam profile since the laser-beam-energy distribution influences the performance of the laser beam [187]. In some cases, the desired laser beam profile is prefabricated by the manufacturer to meet certain application needs. In other cases, the beam profile is measured in order to better
understand and optimize the laser-material interaction [188]. Some medical applications for the laser beam such as Photorefractive Keratectomy (PRK) require regular monitoring of the laser beam to guarantee the operation reliability.

Beam power profiling is the determination of the spatial distribution of the power flux over the beam cross section. There are two alternatives for delivery of diode laser light to the work piece surface: 1) a fibre optic cable, or 2) light obtained directly from the laser-mounted optics. While the former method produces either an axially symmetric Gaussian or a top-hat beam power profile [189], the latter method can produce an elongated elliptical profile, which changes significantly once outside of the focal plane [190]. To control the dimensions of the weld line, a defocusing strategy is often used for laser welding, thus making the knowledge of the beam profile outside of the focal plane important. The beam power profile directly affects the heat distribution within the weld seam during LTW. Knowledge of this distribution is needed as the input for accurate thermal modelling as well as assuring weld quality from the heat transfer occurring during the LTW of thermoplastics.

Non-electronic tools (laser beam reflection observation, burning wood or paper, fluorescing plates, and acrylic mode burners) and electronic devices (mechanical scanning and camera-based beam analyzers) have been used for laser beam profiling [191]. The non-electronic tools, while quick and inexpensive, are highly limited in terms of consistency and intensity resolution, and depend on operator interpretation. On the other hand, CCD-camera-based techniques have severe limitations (micro-Watts per square cm) on the maximum beam intensity that can be tolerated without damage. For instance, at a 20 W setting, a 1 mm$^2$ beam produces a 2 kW/cm$^2$ average intensity. Although the beam can be sampled and attenuated, the high attenuation required
necessitates complex devices, which can be problematic for lasers with highly divergent beams (i.e., attenuating devices may be difficult or impossible to fit between the laser lens and the sensor while capturing the beam profile in the focal plane [191]).

Mechanical scanning techniques have been long used for beam profiling. These involve moving a small opening such as a pinhole [192], a thin slit or a knife-edge [193] under the beam while measuring the total beam power passing through the opening using an optical power meter. The advantages of these techniques are simplicity, ability to be carried out with relatively inexpensive equipment, and tolerance to high laser powers. Drawbacks are that they are time-consuming and have resolution limited by the number of data points collected.

Pinhole approach was employed for this research in order to measure the laser beam profile. Details of these measurements are presented herein1.

5.1.2 Pinhole Approach

A pinhole with an aperture diameter of 200 μm (Ealing Catalog #43-5727) was selected for this study. Selection of the pinhole size was a compromise between getting an acceptable level of power readings and acceptable spatial resolution. A gold-plated pinhole, which is highly reflective for the infrared laser beam, was selected to minimize pinhole heating. In addition, the pinhole was enclosed in an aluminum block heat sink (Figure 53). This pinhole also had a high damage threshold of 50 MW/cm². The pinhole was moved by the x and z linear stages with 1 μm resolution in 0.1 mm steps and in a raster scan pattern. The laser power passing through the hole was measured by a thermopile power meter (Coherent Air-Cooled Thermopile Sensor PM10, 10W max

1 Ms. I. Poposka’s, summer undergraduate student, assistance with the pinhole fixture design, is acknowledged.
power). A meter connected to the sensor displayed the power readings and produced an analogue 2V full-scale output, which was recorded by a data acquisition system. Figure 54 shows the experimental setup for the pinhole experiments. Since the thermopile sensor responds to wavelengths up to 11 μm, infrared radiation from the pinhole plate heated by the laser beam was a challenge for this experiment: the power readings could drift upwards during the exposure if the plate temperature increased while the measurements were collected.

Figure 53- a) Experimental setup for the pinhole method, b) The cross section of the experimental setup for the pinhole method.

Figure 54- Experimental setup for the pinhole method: a) top view of the pinhole, b) side view with the power meter visible, c) pinhole fixture inside the laser fixture, d) cutaway section view of the pinhole fixture assembly.
The meter output was collected over a grid consisting of 30 × 30 points (Figure 55). For each point, the pinhole was moved, the laser turned on, and, after a 5 s interval to assure no drifting in laser power or the meter reading, ten measurements were recorded at 100 ms intervals and averaged. For the 15 W laser power, a maximum of just over 1000 mW was recorded at the focal plane. Typically, a 1 mW to 2 mW standard deviation was obtained for each measurement and no drifting was observed.

![Figure 55- The diagram for the pinhole experiments (the grid points show the raster pattern of the pinhole movement; not to scale).](image)

A normalized power flux distribution (NPFD) is defined as power per unit area of the beam normalized so that its integral over the beam area is equal to unity. Thus, NPFD has units of one over the length squared. In order to obtain the NPFD, it was assumed that, for each point \((x_i, z_j)\), the measurement corresponds to the average power flux over the pinhole area given by:

\[
\phi_{ij} = \frac{P_{ij}}{a_p}
\]
where $P_{ij}$ is the power passing through the pinhole, and $a_p$ is the area of the pinhole. To validate the measurements, the power flux was summed for all measured points. The total beam power ($P$) was then obtained as follows:

$$P = \sum_{i} \sum_{j} \phi_{ij} \Delta x \Delta z$$

where $\Delta x$ and $\Delta z$ are the measurement increments along x and z-axes. If $\Psi_{ij}$ is the NPFD value at the location $(x_i, z_j)$, then it is expected that:

$$\sum_{i} \sum_{j} \Psi_{ij} \Delta x \Delta z = 1$$

Thus, from the Equations (23 to 25), NPFD can be found as:

$$\Psi_{ij} = \frac{P_{ij}}{\Delta x \Delta z \sum_{i} \sum_{j} P_{ij}}$$

5.1.3 Knife-Edge Technique

To double check the results obtained from pinhole experiments, they were compared with those of the knife-edge technique. The 2D NPFD obtained from the pinhole method was transformed to 1D normalized power distributions along x and z to make the comparison between the pinhole and knife-edge techniques possible:

$$\tilde{\Psi}_i = \Delta z \sum_{j} \Psi_{ij} \text{ and } \tilde{\Psi}_j = \Delta x \sum_{i} \Psi_{ij}$$

The integral of the power flux over the beam area is equal to the total power. The equations are given to express the same idea in terms of the measured discrete quantities $(P_{ij})$.

Note that the knife-edge technique is a mechanical method to measure laser-beam profile in which a sharp linear edge moves underneath a laser beam incrementally along

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1 The knife-edge experiments were conducted by Mr. M. Chen [194].
the x-axis in $\Delta x$ steps and then along the z-axis in $\Delta z$ steps (Figure 56). This movement happens from the point where the laser beam is completely obscured to the point where it is completely revealed while recording the power by means of a power meter. This way, the power readings increase from zero to the full power as the knife-edge is moved along each axis. Then, the differences of the consecutive beam power readings are obtained; these values are used for beam profiling measurements. The beam profile measurements will be presented in the next section.

Figure 56- The diagram for the knife-edge experiments.

5.1.4 Unscattered Laser Beam Profile

A total power of 15 W was used in these experiments unless otherwise indicated. To validate the pinhole-based measurements, the total power was extracted from the collected data using Equation 24 for each beam profile measurement conducted at seven working distances ranging from 82.5 mm to 88.5 mm. These total power estimates were then averaged to get $14.8 \pm 1.0$ W. This value was statistically equal to the incoming
beam power of 15 ± 0.2 W. Figure 57 to Figure 63 show the observed NPFD \((\Psi_{i, j})\) obtained from the pinhole method at working distances of 85.5 mm to 88.5 mm.

Examining the change in beam profile as a function of WD (working distance) in Figure 57 to Figure 60, one can observe above the focal plane (WD < ≈86 mm) the beam breaks up into a two-peak structure along X; this is clearly seen in Figure 60. Interestingly, the same double peak was not observed below the focal plane (>≈86 mm WD) (Figure 61 to Figure 63). This observation clearly shows the weld quality will be direction-dependent if defocused welding is attempted. It was concluded that, to achieve a wide contour weld, the beam should move along the x-axis.

Figure 57- The 3D NPFD contours at WD = 85.5 mm (focal plane).
Figure 58- The 3D NPFD contours at WD = 84.5 mm (1 mm above the focal plane).

Figure 59- The 3D NPFD contours at WD = 83.5 mm (2 mm above the focal plane).
Figure 60- The 3D NPFD contours at WD = 82.5 mm (3 mm above the focal plane).

Figure 61- The 3D NPFD contours at WD = 86.5 mm (1 mm below the focal plane).
Figure 62- The 3D NPFD contours at WD = 87.5 mm (2 mm below the focal plane).

Figure 63- The 3D NPFD contours at WD = 88.5 mm (3 mm below the focal plane).

Figure 64 to Figure 66 show the corresponding contour plots for the laser beam cross section. The contour plots aid in determining the beam length and width. Beam width is defined here based on a Gaussian distribution for which 95% of the energy is
contained within ± 2 standard deviation interval. At this point, the Gaussian distribution is at $\frac{1}{e^2} = 0.135$ of the peak value. Thus, the lowest contour bound corresponds to the 13.5% of the peak intensity value observed for that profile. Double peak contours for the location 3 mm above the focal plane are clearly seen in Figure 66.

The extreme x and z values of the lowest contour were then taken as the beam x and z dimensions. These dimensions are plotted against the WD in Figure 67. From Figure 67, one can determine the z dimension minimum is located at a working distance of 86.5 mm while that of x is at a working distance of 85.5 mm. The focal plane beam dimensions are thus determined as $0.72 \times 1.36 \text{ mm}^2$, which matches well with the manufacturer’s specifications ($0.7 \times 1.4 \text{ mm}^2$, Appendix G). Figure 67 indicates that the beam diverges faster along the x (narrow direction) axis.

Figure 64- NPFD contours at WD = 85.5 mm (focal plane).
Figure 65- NPFD contours at WD = 84.5 mm (1 mm above the focal plane).

Figure 66- NPFD contours at WD = 82.5 mm (3 mm above the focal plane).
5.1.5 Validation of the Experimental Results

As mentioned earlier in the experimental approach section, in order to compare the beam profiling results obtained from the pinhole and knife-edge methods, the 2D NPFD acquired from the pinhole method was transformed to 1D NPFD along the x and z directions. The 1D NPFD results at the focal point (WD = 85.5 mm) and 2 mm above it (WD = 83.5 mm) were then compared along the x and z dimensions (Figure 68 to Figure 71). It is seen that the results are in good agreement. The close agreement of the two independent test results validates both tests.
Figure 68- Comparison of the knife-edge [194] and pinhole NPFD plots along the x direction at WD = 85.5 mm (focal plane).

Figure 69- Comparison of the knife-edge [194] and pinhole NPFD plots along the z direction at WD = 85.5 mm (focal plane).
Figure 70- Comparison of the knife-edge [194] and pinhole NPFD plots along the x direction at WD = 83.5 mm (2 mm above the focal plane).

Figure 71- Comparison of the knife-edge [194] and pinhole NPFD plots along the z direction at WD = 83.5 mm (2 mm above the focal plane).

The beam profile was also measured at another power level (7 W) to confirm the power-independence of the laser beam profile. The results are shown in Figure 72 and Figure 73. A good agreement is seen between the beam profiles in the x and z directions for 7 W and 15 W power levels.
5.2 Characterization of the Scattered Laser Beam Profile

Light scattering by semi-crystalline thermoplastics affects the quantity and distribution of the heat input into the thermal model since the laser must pass through the laser-transmitting part before reaching the weld interface. While laser beam profiling techniques are available for measurement of the laser-beam power profile (as described in...
the previous section), such techniques are difficult to use in order to characterize the power profile of the beam after it has passed through the light-scattering semi-crystalline polymer (e.g., PA6).

The use of pinhole and CCD cameras has been reported in the literature as discussed in the previous section. However, each of these methods imposed their own challenges, which encouraged development of another method for laser beam scattering measurements. For example, to improve the measurement accuracy in the pinhole approach, one needs to decrease the pinhole diameter. This would reduce the amount of energy passing through the pinhole to the power meter, potentially below its measurement threshold. On the other hand, raising the laser power input to compensate is constrained by the melting of the laser-transmitting material due to excessive energy input: a scattering semi-crystalline polymer has some absorption of the laser beam energy.

Scattering of the beam in the laser-transmitting part will effectively diffuse the beam and thus broaden its distribution as it travels through the material. During the initial phase of this research, this effect was ignored and the process was modelled as absorption only. Methods for isotropic scatter were investigated at the later stages of this research and the laser beam scattering was addressed extensively whereby the profile of a high-power diode laser beam was measured before entering and after passing through a semi-crystalline plaque (i.e., PA6). This study proposed a technique that allowed the measurement of the scattering of the laser-beam energy after passing through a semi-crystalline laser-transmitting part. Details of this approach are presented herein.
5.2.1 Laser Beam Scattering Theory

The energy delivered per unit of weld line length is commonly referred to as the line energy. For contour welding, the primary focus of this research, the laser beam moves along the weld line. In this case, the line energy (Λ) is equal to the ratio of laser power (P) to the beam scanning speed (V) and is defined as follows:

\[ Λ = \frac{P}{V} \]  

(28)

While the line energy can be easily calculated, it does not completely describe the nature of the laser light reaching the joint interface. Before reaching the interface, the light must pass through the laser-transmitting material. Three kinds of interaction can occur when a light beam interacts with a material: the light can be scattered, absorbed, and transmitted [178]. Scatter is a combination of reflection, refraction, and diffraction. Surface quality, optical properties, and thickness of the laser-transmitting material will determine the degree to which each type of interaction will affect the laser beam.

For the purposes of the derivation presented here, a continuous form of the normalized power flux distribution (NPFD) was used instead of the discrete form defined earlier in Equation 25. The continuous form \( Ψ(x, z) \) defined over the beam area has the property that:

\[ \int_{\text{Beam Area}} \int Ψ(x, z) \, dx \, dz = 1 \]  

(29)

If the beam travels in the x direction, the derivation of the distribution will be obtained from the NPFD along the z direction (transverse to the weld line) (Figure 74).
Figure 74- Coordinates for the weld line on the laser-absorbing specimen surface.

Since the laser beam thermal effect on the material is due to the energy delivered by the beam as it moves along the x-axis, the quantity of interest is the NPFD perpendicular to the weld line direction (i.e., along the z-axis), $\Psi(z)$. This transverse NPFD function has units of one over the unit width of the beam and is such that:

$$\int_{\text{Beam Width}}^{\Psi(z)} dz = 1$$ (30)

Assuming that $\Psi(z)$ does not change significantly with the changes in the laser power (which is supported by the results of the laser beam profiling studies in Section 5.1), for a particular laser power setting of $P_k$, one can use $\Psi(z)$ to describe the power flux distribution along the z-axis by a product $P_k \Psi(z)$ (with units of power per unit width). Furthermore, the factor that will determine whether melting will take place at a particular point is the energy delivered per unit area of the weld or energy density ($J/m^2$). Therefore, $\xi(z)$, the transverse energy density distribution is defined as:

$$\xi(z) = \frac{P_k \Psi(z)}{V} = \Lambda_k \Psi(z)$$ (31)

In general, due to the absorption and reflection by the laser-transmitting part, the total power reaching the laser-absorbing part surface $P^*$ is less than the power $P$ reaching...
the transmitting upper surface. It can be assumed that the transmissivity, \( \tau \) (the ratio of the total beam power leaving the laser-transmitting part to the one entering the part), is constant and independent of the power.

Furthermore, the NPFD at the weld interface will be generally affected by the light scattering through the laser-transmitting part. To reflect that, \( \Psi^*(z) \) is defined as the transverse NPFD at the weld interface after passing through a laser-scattering-transmitting part while \( \Psi(z) \) is the corresponding transverse NPFD at the same surface in the absence of the laser-transmitting part or for the case when no scattering takes place. Then, from Equation 31, transverse energy density distribution reaching the absorbing part surface can be described by:

\[
\xi^*(z) = \frac{\tau P_k \Psi^*(z)}{V} = \tau \Lambda_k \Psi^*(z) \tag{32}
\]

Now, consider Figure 75 showing two transverse power flux distributions at the laser-absorbing part surface corresponding to the two different laser power levels, \( P_k \) and \( P_0 \), where \( P_k > P_0 \). Define the threshold power \( P_0 \) as the power for a particular scan speed \( V \) at which the polymer is just reaching the melting point at \( z = 0 \) mm (i.e., at the peak point of the distribution). When the power is increased from \( P_0 \) to \( P_k \), the weld line width is expected to increase to \( w_k \). In addition, the energy per unit area of the weld at point B should be equal to the one at the points A and C in the figure. For the case of a symmetrical transverse NPFD, it can thus be written that:

\[
\frac{\tau P_k \Psi^*(w_k / 2)}{V} = \frac{\tau P_0 \Psi^*(0)}{V} \tag{33}
\]

Rearranging the terms in Equation 33, for each power setting \( P_k > P_0 \), we can obtain a corresponding value of the NPFD transverse to the weld-line-scan direction:
\[ \Psi^*(w_k / 2) = \frac{P_0 \Psi^*(0)}{P_k} \]  \hspace{1cm} (34)

where \( \Psi^*(0) \) is a scale factor determined by the requirement that the integral of the distribution is equal to 1.0. \( \Psi^*(0) \) is calculated after numerical integration of the distribution data obtained from the measurements.

As can be seen from Equation 34, since \( \Psi^*(0) \) is a constant, for \( \Psi^*(w_k / 2) \) to approach zero, the ratio of \( P_0 / P_k \) must approach zero as well. However, this ratio is limited either by the maximum power available from the laser system or by the laser-transmitting or absorbing material’s ability to withstand the high power levels without excessive degradation. This means that the information about the extreme tails of the NPFD distribution cannot be obtained. The distribution tails were therefore approximated by linearly extrapolating the trends of the known data until \( \Psi^*(z) = 0 \) was reached.

![Conceptual diagram of the two transverse power flux distributions.](image)
These distribution tails needed to be included into the summation for the normalization of the distribution as shown below (using the trapezoidal rule for numerical integration) (Figure 76):

\[
\Psi^*(0) = \frac{2}{P_0 \left\{ \frac{w_1}{P_0} + \sum_{k=2}^{n} \left( \frac{w_k}{P_{k-1}} - \frac{w_{k-1}}{P_k} \right) + \frac{w_n}{P_n} \right\}}
\]

(35)

where \( w_i/2 \) is the z-value at which the extrapolated distribution tail crosses the z-axis \( (\Psi^*(z) = 0) \).

![Figure 76- Numerical integration for normalization of the NPFD data.](image)

5.2.2 Laser Beam Scattering Measurement Approach

The technique presented in this research allowed measurement of the NPFD transverse to the laser-beam scan direction produced by a DLX16 diode laser, after having passed through the light-scattering laser-transmitting plastic part.

Chen et al. [195] used laser scanned lines with increasing power to identify the threshold at which welding occurred or the material degraded. The possibility of using
this technique for light scattering measurements was hypothesized by our research group [196]. The author conducted a feasibility study, the experiments, as well as data analysis for the results presented herein.

To apply this technique, the laser-absorbing part must be exposed to a laser beam traversing across its surface. A sequence of scan lines of progressively increasing line energy (and constant scan speed) are made on the surface, starting with the lowest power, where no melting takes place, up to the highest power possible without significant degradation of the plastic. No contact should occur between the laser-transmitting and absorbing parts as the laser is passing over the surface to allow for easy examination of the weld line width (Figure 77).

![Figure 77- The diagram for the laser beam scattering measurements experiments.](image)

While keeping the speed constant, one must raise the laser power incrementally from the lowest level – when no visible mark is left on the laser-absorbing part surface – to the level at which the first thinnest line is created. The power setting at this point is the threshold power ($P_0$). The power is then increased in increments to obtain a sequence of increasing power settings $P_k$ ($k = 1…n$).

As the power is increased, wider lines should be visible on the laser-absorbing part surface. These lines are produced by melting of a thin layer at the polymer surface.
Image processing techniques can then be used to obtain a sequence of accurately measured line widths $w_k$ (k = 1…n), corresponding to the sequence of power settings $P_k$. Equation 34 can then be used to calculate the points of the transverse NPFD profile.

The following assumptions were made for this approach: 1) there exists a threshold amount of energy, which must be delivered to the surface of the laser-absorbing part in order for melting to happen; and 2) the weld line transverse NPFD profile has a single peak, with profile values decreasing monotonically away from the peak point.

Experiments were conducted to determine the transverse NPFD for laser light passing through the natural plastic parts. The specimens were held by a pneumatic clamp on the platform moved by the x and z stages with 1 µm resolution. The laser-transmitting parts were made from natural PA6 (unfilled and 30% glass-fibre filled). The laser-absorbing parts were made from the same natural un-reinforced PA6 filled with 0.2% (wt.) of carbon black.

To validate the proposed technique, the laser beam was scanned on the laser-absorbing plaque surface directly, without passing through the laser-transmitting part first. Using this approach, the transverse NPFD from the proposed line scan method could be compared with the corresponding known beam profile obtained by the pinhole approach for the unscattered laser beam, explained in the previous section.

For the validation experiments, in which the beam is not passing through a laser-transmitting part, the highest scanning speed of 150 mm/s was used. Parallel scan lines with progressively increasing laser power were made on the laser-absorbing plaque surface located approximately 3 mm below the focal plane (at 88.5 mm WD).

For measurements of transverse NPFD in the presence of a laser-scattering-transmitting part, a 3-mm-thick laser-transmitting part was placed on top of the laser-
absorbing part while separated by 0.3 mm metal shims (Figure 77). For the un-reinforced PA6, 50 mm/s and, for the 30% glass-fibre-filled PA6, 25 mm/s scanning speeds were used. Slower scanning speeds were required to compensate for the laser light absorption by the laser-transmitting part. Table 8 shows the experimental matrix used for the light scattering study.

### Table 8- Experimental matrix for the laser beam scattering measurements

<table>
<thead>
<tr>
<th>Scan along</th>
<th>Laser-transmitting part GF (wt %)</th>
<th>Power range (W)</th>
<th>Laser scanning speed (mm/s)</th>
<th>Line energy range (J/mm)</th>
<th>No of line sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>N/A</td>
<td>8 to 50</td>
<td>150</td>
<td>0.053 to 0.333</td>
<td>1</td>
</tr>
<tr>
<td>z</td>
<td>N/A</td>
<td>3 to 30</td>
<td>150</td>
<td>0.02 to 0.200</td>
<td>2</td>
</tr>
<tr>
<td>x</td>
<td>0</td>
<td>16 to 90</td>
<td>50</td>
<td>0.32 to 1.80</td>
<td>1</td>
</tr>
<tr>
<td>z</td>
<td>0</td>
<td>10 to 50</td>
<td>50</td>
<td>0.20 to 1.00</td>
<td>2</td>
</tr>
<tr>
<td>x</td>
<td>30</td>
<td>25 to 100</td>
<td>25</td>
<td>1.00 to 4.00</td>
<td>3</td>
</tr>
<tr>
<td>z</td>
<td>30</td>
<td>20 to 75</td>
<td>25</td>
<td>0.80 to 3.00</td>
<td>2</td>
</tr>
</tbody>
</table>

To obtain the widths of the scan lines (wk) made on the laser-absorbing part surface, an accurate measurement technique is required. A desktop scanner (HP ScanJet 3570c) captured the images of the scanned lines. The plaques were placed on the scanner bed and scanned in as 8-bit grayscale images at a resolution of 1200 dpi. The scanner was calibrated by scanning a ruler scale verifying that the scanned image scale matched the ruler’s dimensions. The dimensions matched within 2 pixels over 1200 pixels (corresponding to 1 inch on the ruler). The scanned resolution means that 47.2 pixels (1200/25.4) in the image correspond to 1 mm on the sample surface (or 0.021 mm per pixel). Figure 78 shows a sequence of scan lines produced directly on the laser-absorbing part, scanned along the x direction. The white stripes within the scanned lines are
hypothesized to be due to polymer degradation by the high-energy intensity around the centre of the beam cross section and are visible in the zoomed image (Figure 78b).

Figure 78- Laser scan lines (in the x direction) produced directly on the laser-absorbing part (V = 150 mm/s, P = 8 W to 50 W, as indicated): a) original image, b) zoomed in image.

While examining the results, it was found that in some cases, not only the width of the scan line, but also the location of the distribution peak could be extracted from the images. This information is not important for cases of symmetrical NPFD; however, as learned from the beam profiling studies conducted on the same laser system [194], the distribution is asymmetrical along the x direction. Thus, for scans along the z direction, it was noted that the white stripes seen in Figure 78b were located off centre (Figure 79). Their location was recorded - when possible - during the image analysis and used to estimate the location of the distribution peak.
5.2.3 Results for Laser-Beam-Scattering Measurements

The first group of the results focuses on the validation of the technique, and the second group shows the scattered beam profile obtained from the laser beam passing through a 3-mm-thick natural semi-crystalline PA6 plaques. Results for the 30% glass-fibre-filled PA6 plaques are given in Appendix B.2.5.

Figure 80 shows the results of the validation experiments for the case in which the laser beam made multiple scans across the laser-absorbing parts along the x direction. Therefore, the information about the power flux distribution along the z direction was measured. In the absence of scattering by the laser-transmitting part, the laser beam profile is expected to be identical to that of the unscattered laser beam. It can be seen that the results agree very well with the pinhole-based beam profiling data. Each data point in all plots is the result of averaging of three line width measurements. Standard deviations of these measurements were under 0.1 mm.

Figure 81 shows the results of the validation experiments when the laser scanned lines along the z direction directly on the surface of the laser-absorbing part. This provided information about the power flux distribution along the x direction. The results
from the proposed line scan technique are compared to the data collected using the pinhole method. Two sets of lines, each set scanned on a different plastic plaque, are shown. The agreement between the direct beam measurements by the pinhole method and the indirect measurements by the proposed line scan method is good. In addition, good sample-to-sample results consistency was seen.

Figure 80- A comparison between the z direction NPFD profiles from the proposed line scan method and the pinhole method (88.5 mm working distance, scanned along the z direction).

Figure 81- A comparison between the x direction NPFD profiles from the proposed line scan method and the pinhole method (88.5 mm working distance, scanned along the z direction).
Figure 82 and Figure 83 show the transverse NPFD results for line scans along the x and z directions, respectively, for the un-reinforced PA6 plaques. Both plots are compared to the unscattered incoming laser beam profiles obtained by the pinhole technique.

Figure 82- A comparison between the z-direction transverse NPFD profiles after scattering by the 3-mm-thick un-reinforced PA6 plaque and unscattered beam profile from the pinhole method (scanned along the x direction).

Figure 83- A comparison between the x-direction transverse NPFD profiles after scattering by the 3-mm-thick un-reinforced PA6 plaque and unscattered beam profile from the pinhole method (scanned along the z direction).
The scans along the x and z directions (Figure 82 and Figure 83) show the clear difference between the x and z-axis profiles as very different distribution shapes are obtained. Results show the wider z-axis beam profile and narrower x-axis beam profile. The scattering effect of the semi-crystalline polymer (PA6) is clearly shown by the spreading of the distribution tails.

### 5.3 Absorption by Soot Particles and Bouguer-Lambert Law

Light absorption by a laser-absorbing thermoplastic establishes the amount of heat absorbed by this material and it is thus an important parameter to the thermal model. Detailed data on the absorption properties of the plastic material were difficult to obtain. Originally, the limited data from previous studies were adopted for initial modelling. In these earlier studies, the absorptivity of the laser-absorbing plastic part (equal to 1.8 mm$^{-1}$) was obtained by fitting the experimental molten zone thickness into a 1D FDM model [119]. Later studies showed that the absorption coefficient of the laser-absorbing part could be up to 10 mm$^{-1}$ based on Rayleigh approximation theory and the soot-plastic block model (SPBM). The details of these methods are presented herein.

#### 5.3.1 Radiative Properties of Soot

Most thermoplastics are naturally transmitting to the 1 µm laser wavelength. Soot particles, with dimensions of approximately 0.1 µm, are used as additives in order to make the thermoplastics absorbing to the laser radiation. Soot is a mixture of elemental and organic carbon. It is mixed with plastics during extrusion with a weight fraction from 0.05% to approximately 2%, rendering the natural plastic laser absorbing. Radiative properties of soot depend on the size and the optical properties of the particles [197].
Optical constants depend on the chemical composition of soot as well as the radiation wavelength that hits the particle. The rate of heat absorption is described by the absorption coefficient of the laser-absorbing part. Knowing the rate of heat absorption is vital in thermal model development for predicting the molten zone dimensions, which in turn helps to predict the weld size and strength. Thermal and optical interaction of the soot particles and the plastic with the laser beam determines heating rate, the melting, and consequently the welding of plastics.

Studies show soot optical properties are independent of temperature even during a combustion process where soot is one of the main by-products [197]. Classical electromagnetic theory can be applied to predict radiative properties of particles. Existing optical models and measurements concentrate on the Rayleigh approximation to obtain optical properties of soot expressed by complex refractive index. Soot mass concentrations as well as optical constants are required for these calculations. Obtaining each of these parameters though could be challenging because of unknown soot shape and properties. There is little experimental information currently available regarding the relationship between the absorption coefficient and the volume fraction as well as the size and the shape of the soot particles. On the other hand, extensive literature is available on the subject of laser light absorption by soot particles suspended in a gas.

This study applies the classical Rayleigh theory used in combustion studies to estimate spectral soot absorption in order to predict the laser absorption coefficient for the equivalent concentration of soot particles in thermoplastics. In addition, a combined soot-plastic block model has been derived using energy conservation laws. This model describes the dependence of the absorption coefficient on the soot optical and physical properties as well as volume concentration when used as an additive for laser light
absorption in thermoplastics. This research assumes that soot aggregates can be modelled as spherical particles of uniform diameter and thus herein the term “soot particle” will be used.

5.3.2 Rayleigh-Approximation-Based Approach

Rayleigh approximation approach was used to predict the absorption coefficient of soot particles suspended in a transmitting medium based on their size. Soot additive is normally specified as a mass fraction during the mixture compounding. The following equation can be used to express the volume fraction used in Equation 13 as a function of the mass fraction:

$$f_v = [1 + \frac{\rho_{par}}{\rho_{plastic}} \left( \frac{1}{w_{par}} - 1 \right)]^{-1} \approx w_{par} \frac{\rho_{plastic}}{\rho_{par}}$$

(36)

where $\rho_{par}$ is the soot density, and $\rho_{plastic}$ is the plastic density.

Figure 84 shows absorption coefficient as a function of wavelength calculated using Equation 13 for $d_{par} \leq 90$ nm and for soot concentration of 0.2% (wt.). The $n_\lambda$ and $k_\lambda$ were adopted from the empirically obtained values reported in [178] and partially listed in Table 2.

As discussed in Section 3.4, from the definition of the size parameter ($\pi d_{par} / \lambda$), for the 940 nm wavelength of a diode laser, the maximum particle size for which the Rayleigh theory is applicable is 90 nm. The absorption coefficient is shown to be inversely dependent on the radiation wavelength.

Figure 85 shows the relation between the absorption coefficient at 940 nm wavelength predicted by the Rayleigh theory for PA6 and PC and the soot particle mass fraction (assuming propane-sourced soot, and densities of PA6, PC, and soot of
1060 kg/m³, 1190 kg/m³, and 2250 kg/m³, respectively [198]). The absorption coefficient is predicted to be a nearly linear function of the soot mass fraction. For example, for 0.2% (wt.) soot mass fraction, assuming that $n_\lambda = 1.61$ and $k_\lambda = 0.54$, the absorption coefficient of 4.8 mm⁻¹ and 5.4 mm⁻¹ are predicted for PA6 and PC at the 940 nm wavelength, respectively. This difference is due to the different densities of the two plastics.

Figure 84- Absorption vs. wavelength for 0.2% (wt.) soot, $d_{\text{par}} \leq 90$ nm.

Figure 85- Absorption coefficient vs. soot mass fraction at 94 nm, $d_{\text{par}} \leq 90$ nm.
5.3.3 Soot-Plastic Block Model (SPBM)

Work of Haberstroh et al. [86] suggests that soot particle size has considerable influence on absorption coefficient. Soot particles act as the primary light absorber inside the plastic matrix. This research modelled the absorption coefficient due to soot particles embedded in a plastic matrix for LTW applications. In this model, it is assumed that the soot particles are well separated and do not overlap to cause blockage of laser energy for other particles. The SPBM uses the concept of total absorption coefficient [199].

Assume an \( x \times z \times \Delta y \) block of plastic filled with spherical soot particles (Figure 86), with the \( \Delta y \) dimension being very small. Let the input power flux to this block be \( q_0 \) (W/m\(^2\)); \( q_0-q \) of this energy is absorbed by the block of the plastic and soot mixture [200].

\[
(q_0 - q) x \, z = \alpha q_0 x \, z = N \alpha_{\text{par}} q_0 A_{\text{par}} + \alpha_{\text{plastic}} q_0 A_{\text{plastic}} \tag{37}
\]

where \( \alpha \) is the effective absorptivity of the mixture, \( \alpha_{\text{par}} \) is the absorptivity of the individual soot particles, \( A_{\text{par}} \) is the projected area of individual soot particles, \( \alpha_{\text{plastic}} \) is
absorptivity of the laser-transmitting plastic, and $A_{\text{plastic}}$ is the projected area filled with laser-transmitting plastic, which is defined as follows:

$$A_{\text{plastic}} = xz - N \frac{\pi d_{\text{par}}^2}{4} \quad (38)$$

where $d_{\text{par}}$ is particle diameter and $N$ is number of soot particles in the mixture volume:

$$N = \frac{6v f_v}{\pi d_{\text{par}}^3} \quad (39)$$

where $v = x \times z \times \Delta y$.

Substituting Equations 38 and 39 into Equation 37 results in the following equation, assuming that all soot particles have the same absorptivity:

$$\alpha = \frac{3 \Delta y f_v}{2 d_{\text{par}}} (\alpha_{\text{par}} - \alpha_{\text{plastic}}) + \alpha_{\text{plastic}} \quad (40)$$

The transmissivity of the polymer matrix ($\tau$) could be obtained as follows based on Bouguer-Lambert law:

$$\tau = \frac{q}{q_0} = e^{-K_T \Delta y} \quad (41)$$

where $K_T$ is extinction coefficient for absorption of the matrix and is equal to the matrix absorption coefficient.

Designating the reflectivity of matrix surface as $\rho$, the absorptivity of polymer matrix is:

$$\alpha_{\text{plastic}} = 1 - \rho - \tau = 1 - \rho - e^{-K_T \Delta y} \quad (42)$$

It is known that the absorptivity of the soot particles is nearly 100% [29]. Herein, it is assumed that the soot particles act as black bodies (i.e., $\alpha_{\text{par}} = 1.0$). From Equations 40
to 42, the absorption coefficient of the mixture of soot and plastic can be determined as follows:

\[
K_{\tau} = K_{\tau} - \lim_{\Delta y \to 0} \frac{1}{\Delta y} \ln \left( 1 - \frac{3 \Delta y f_v}{2d_{par}} \right)
\]

(43)

By expanding the natural logarithm in Equation 43 using the Taylor series, this equation can be rewritten as follows:

\[
K = K_{\tau} + \lim_{\Delta y \to 0} \frac{1}{\Delta y} \left( \frac{3 \Delta y f_v}{2d_{par}} + \frac{1}{2} \left( \frac{3 \Delta y f_v}{2d_{par}} \right)^2 + \frac{1}{3} \left( \frac{3 \Delta y f_v}{2d_{par}} \right)^3 + \ldots \right) = K_{\tau} + \frac{3 f_v}{2d_{par}}
\]

(44)

In order to estimate the value of the \( K_{\tau} \), a transmission of 51% based on the laser-beam-power readings before and after hitting the plastic plaques, for the 3 mm of natural PA6 was assumed [119]. As the result, \( K_{\tau} = 0.2 \text{ mm}^{-1} \) is obtained from Equation 41 for the natural PA6.

Figure 87 shows the absorption coefficient predicted by the SPBM (Equation 44) plotted versus particle diameter for different soot-particle weight fractions. It is seen that the absorption coefficient rises quickly as the particle size decreases. This finding is supported by other studies [108, 178, 201]. Note that Rayleigh approximation theory is valid for the size parameter \( (\pi d_{par} / \lambda) \) under 0.3. For the diode laser wavelength of 940 nm, this means the soot particle diameters should be less than 90 nm for the Rayleigh theory to be valid. Therefore, for particles greater than 90 nm, the SPBM is more applicable.

Figure 88 shows a linear relationship between the absorption coefficient and the soot mass fraction predicted by the SPBM for different soot particle diameters.
Relationship between the mass and volume fractions in Equation 36 was used in the calculations. Rayleigh theory predicts a similar linear relationship.

![Figure 87](image1)

Figure 87- Absorption coefficient predicted by SPBM vs. soot particle diameter for mass fractions changing from 0.03% to 0.3% (wt.).

![Figure 88](image2)

Figure 88- Predicted absorption coefficient from SPBM vs. soot particle mass fraction for particle diameters from 50 nm to 200 nm.
Herein, a comparison is made between the theoretical results (i.e., Rayleigh approximation and soot-plastic block model) and the experimental data available in the literature. In order to predict the soot absorption coefficient using the SPBM approach, the soot particle diameter needs to be specified. Wang et al. [102] measured the soot aggregate size (for the PA6 used in this study) using TEM, and obtained an average value of 96 nm, which was therefore used for the SPBM model predictions. Figure 89 summarizes the theoretical predictions and adds experimental results for comparison purpose.

![Graph showing comparison of models with experimental data](image)

Figure 89- Comparison of models with experimental data from [29, 91, 89]

\(L_1 = 808 \text{ nm}, L_2 = 940 \text{ nm}\).

As reported in the literature section, several researchers conducted experiments in order to obtain absorption coefficient [91] or optical penetration depth [29, 89] for amorphous and semi-crystalline polymers. Note that the absorption coefficient is inversely related to the optical penetration depth (Equation 12). Chen et al. [91] measured the absorption coefficient of PC for three low soot concentration levels. These
measurements match well with the SPBM predictions. The optical penetration values measured by Russek et al. [89] correspond to the absorption coefficients of 16.4 mm\(^{-1}\) (PA6) and 14.1 mm\(^{-1}\) (PC). Russek et al. [89] reported optical penetration depths of 47 µm (PA6) and 55 µm (PC) at 0.8-µm wavelength for 0.3% (wt.) soot-filled plastics. These values correspond to the absorption coefficients of 21.3 mm\(^{-1}\) (PA6) and 18.2 mm\(^{-1}\) (PC). The SPBM prediction for the PA6 (22.3 mm\(^{-1}\)) compares well with these results. The model prediction for the PC (24.8 mm\(^{-1}\)) is 36% higher than the experimental value.

Figure 89 shows that the Rayleigh approach generally under-predicted the experimentally obtained absorption coefficients by 60%. This observation is supported by the results of the combustion studies, where the Rayleigh approach was found to underpredict the absorption coefficient of soot particles suspended in a transmitting medium [108, 197]. Note that the average particle size of 96 nm is above the upper limit (90 nm) of the Rayleigh theory applicability [178, 202, 203], which may be an additional reason why the Rayleigh predictions and experimental results do not match.

To understand the thermal interaction between the soot particles and the polymer matrix during the laser welding process, a micro-scale heat transfer model for a single soot particle was developed (Appendix D).

5.4 Material Properties

The main material adopted for this study was a grade of injection moulded PA called polycaprolactam (PA6) with the commercial name of Akulon (F223D) and supplied by DSM Engineering Plastics. An exploratory study of PC, supplied by Bayer Corporation under commercial name of Makrolon (AL2647), was conducted as well.
This section provides information on the materials’ thermo-physical and optical properties.

Material properties used in this research are subcategorized as thermal, physical, and optical. Thermo-physical properties at room temperature, which were provided by the material supplier, are listed for reference in Table 9 [204]. The laser-absorbing parts had the same thermo-physical properties as the natural parts (laser-transmitting parts) but contained 0.2% (wt.) of carbon black.

In this research, both temperature-dependent and temperature-independent thermal properties were assumed, as determined by the degree of temperature influence. Reliable temperature-dependent thermo-physical properties have been presented in the open literature only for a limited number of polymers due to high cost associated with conducting these experiments [205]. Therefore, properties such as heat capacity, and melt and degradation temperatures were determined by carrying out measurements on the specific materials used in this thesis.

<table>
<thead>
<tr>
<th>Property</th>
<th>Thermal conductivity (W/m·K)</th>
<th>Density (kg/m³)</th>
<th>Specific heat (J/kg·K)</th>
<th>Melting temperature (°C)</th>
<th>Thermal diffusivity (m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PA6 (0% GF)</td>
<td>0.25</td>
<td>1130</td>
<td>1600</td>
<td>200-220</td>
<td>7.07E-7</td>
</tr>
<tr>
<td>PC</td>
<td>0.2</td>
<td>1200</td>
<td>1172</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The specific and latent heats of the plastics increase with increasing temperature up to the melting temperature ($T_m$) for crystalline materials and up to the glass transition temperature ($T_g$) for amorphous materials. Knowing the temperature-dependent specific heat, one can derive the latent heat of fusion. Several researchers [119, 206, 207, 208, 209] have introduced experimental correlations for temperature-
dependent specific heat. For this study, differential scanning calorimetry (DSC) tests were conducted to determine temperature-dependant specific heat for PA6 and PC (Table 10). In addition, thermogravimetric analyses (TGA) were conducted to find the degradation temperatures.

Crystalline polymers have a higher thermal conductivity than amorphous polymers [206, 207]. Thermal conductivity shows a gradual increase up to $T_g$ for amorphous polymers followed by a gradual decrease [206, 207]. See Appendix B.1.3 for examples of the information presented in the literature. In this research, it was assumed that all materials were isotropic and, therefore, thermal conductivity was direction-independent. Thermal conductivity for PA6 was assumed temperature-independent and adopted from the material-supplier information given in Table 9. Temperature-dependent thermal conductivity for PC was adopted from the data presented in the open literature [210] (Figure 90).

<table>
<thead>
<tr>
<th>Property</th>
<th>Temperature range (°C)</th>
<th>Specific heat (J/kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td><strong>25°C &lt; T ≤ 194°C</strong></td>
<td><strong>Cp = -0.0509 T^2 + 18.217 T + 909.83</strong></td>
</tr>
<tr>
<td>PA6 (0% GF)</td>
<td><strong>194°C &lt; T ≤ 220°C</strong></td>
<td><strong>Cp = 8.3945 T^2 - 3301.9 T + 327236</strong></td>
</tr>
<tr>
<td></td>
<td><strong>220°C &lt; T ≤ 229°C</strong></td>
<td><strong>Cp = 95.356 T^2 - 43397 T + 5E+06</strong></td>
</tr>
<tr>
<td></td>
<td><strong>229°C &lt; T ≤ 380°C</strong></td>
<td><strong>Cp = 0.0276 T^2 - 14.702 T + 4244.6</strong></td>
</tr>
<tr>
<td></td>
<td><strong>25°C &lt; T ≤ 140°C</strong></td>
<td><strong>Cp = 3.53 T + 1149.1</strong></td>
</tr>
<tr>
<td>PC</td>
<td><strong>140°C &lt; T ≤ 148°C</strong></td>
<td><strong>Cp = 27.106 T - 2177.5</strong></td>
</tr>
<tr>
<td></td>
<td><strong>148°C &lt; T ≤ 400°C</strong></td>
<td><strong>Cp = 3.92 T + 1183.2</strong></td>
</tr>
</tbody>
</table>
Researchers have shown that polymer density is temperature-dependent [206, 207], and decreases with increasing temperature. However, PA6 density changes by only 17% in a 275°C temperature range [211]. Therefore, a constant value for the PA6 density was adopted in this study based on the information provided by the material supplier [119] (Table 9). Temperature-dependent density for PC was adopted from the data presented in the open literature [210] (Figure 90).

Optical properties included emissivity, reflectivity, and transmissivity of the plastic samples. Note that the optical properties of plastics are wavelength-dependent and are different for the laser light and for the thermal radiation observed by the camera. The laser-transmitting part has high transmissivity to laser and low transmissivity to the thermal radiation. Plastics emissivity measurements were obtained using thermal imaging method and miniature thermocouples (Appendix B.2.1). Surface reflectivity of plastics (for thermal radiation) was estimated using thermal imaging observations (B.2.4). For the PA6 laser-transmitting part, transmission of 51% was adopted from the laser-light transmission measurements reported in the literature [119]. For the PC laser-
transmitting part, transmission of 93% was used assuming a reflection of 7% measured and supported by the literature [72]. Table 11 shows the materials’ optical properties employed in this research.

Table 11- Optical properties of thermoplastics (3-mm-thick plaque)

<table>
<thead>
<tr>
<th>Property</th>
<th>Material</th>
<th>ε (%) (thermal radiation)</th>
<th>α (%) (laser light)</th>
<th>τ (%) (laser light)</th>
<th>ρ (%) (laser light)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PA6 (0% GF)</td>
<td>95</td>
<td>43</td>
<td>51</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td>95</td>
<td>0</td>
<td>93</td>
<td>7</td>
</tr>
</tbody>
</table>
Chapter 6  Computational Approach

The FEM approach was adopted in this research since it is the basis for a well-established commercial code (ANSYS®), which was available for this research. In addition, the use of the FEM technique employed in this code makes it possible for the thermal analysis to be coupled to the structural analysis in future studies.

The ultimate purpose of a FEM analysis is to model an actual engineering system. Figure 91 shows the flowchart of the FEM analysis employed in this research. ANSYS® FEM software was employed for all the stages of this modelling including pre-processing, solution, and post-processing. Developing an ANSYS® transient thermal model requires the following information: 1) geometry, 2) material properties, 3) type and size of the elements, 4) boundary and initial conditions, 5) time step, and 6) solution method.

This section presents the steps taken to develop the FEM thermal model. First the approach used to model the geometry (Section 6.1) and the element types used (Section 6.2) is addressed. Section 6.3 describes the challenges, which had to be overcome with regard to the software as the model was developed. Section 6.4 describes the geometry, grid size, and the other parameters used for the modelling of welding with stationary and moving laser beams. Modelling of the heat generation by the stationary and moving laser beam within the amorphous and semi-crystalline plastics is described in Section 6.5. Section 6.6 outlines the approach taken to select appropriate grid size and time step for the solution to assure its accuracy while minimizing the use of computer resources.
6.1 Defining the Geometry

In this stage, the geometric configuration of the model, including the nodes and elements, are defined. Geometry can be either created within the ANSYS® program or imported from a computer-aided design software (CAD). Direct generation\(^1\) and solid modelling are the two methods used to create the finite element model inside the ANSYS® program.

In solid modelling, one describes the geometric shape of the model, and then instructs ANSYS® to mesh the geometry with nodes and elements automatically. The size and shape of the elements can be controlled this way. In this research, the solid modelling approach was adopted. This technique offered the following advantages:

\(^1\) In direct generation, one manually defines the location of each node and the connectivity of each element. This method is very time consuming, especially when making complicated models.
1) ease of use for large or complex models, 2) producing relatively small number of data and managing each group of data individually, and 3) the possibility for the manipulation of the geometric components (e.g., nodes and elements), employing Boolean operations, and geometry modifications.

The solid modelling approach can be carried out using either the top-down or bottom-up techniques. In the top-down technique, one can assemble the model using lines, areas, and volumes, which are the higher order solid model entities. As these features are created, the program automatically creates all lower-level entities associated with them (e.g., keypoints). In the bottom-up technique, the keypoints are created first, and lines, areas, and volumes are generated afterwards. The top-down and bottom-up approaches could be combined if needed. In this research, the bottom-up approach was employed to create the geometry. In addition, the Boolean operators were used to omit or join the higher order entities.

6.2 Element Types

Once the solid model has been completed, element attributes (e.g., element type, real constants, material properties, and coordinate system) need to be set and grid controls (e.g., the element shape, mid-side node placement, and element size) established. Elements in ANSYS® have 30 classifications and are assigned to 7 major subgroups of structural, thermal, electric, magnetic, dynamics, fluid, and coupled categories. ANSYS® element library consists of almost 200 element types, among which 40 elements can perform a steady state and transient thermal analysis. The thermal subgroup of elements, which is the focus of this research, covers point (e.g., MASS71), line (e.g., LINK32), plane (e.g., PLANE55), solid (e.g., SOLID70), shell (e.g., SHELL157), and electric (e.g.,
PLANE67) elements. Each element functions differently, depending on its type (e.g., thermal point elements have one degree of freedom, temperature, at their only node).

This research mainly concentrated on thermal solid elements. However, thermal line elements were used in the 1D thermal model of a T-like joint, which are presented in Appendix A.1. 2D thermal solid elements were used to develop the 2D thermal model of a T-like joint, presented in Appendix A.2.

3D thermal solid elements are either quadrilateral or tetrahedral with the capability of transferring heat by conduction, radiation, and convection between the nodes. Among elements available in the thermal solid library for 3D elements, only SOLID70 is a lower order element with the capability of transferring heat by conduction mode and ability to accommodate internal heat generation, convection (or heat flux), and radiation. Therefore, this element was adopted for the 3D analysis in this research.

SOLID70 has a 3D transient and steady state thermal conduction capability [212]. The element is defined by eight nodes, with a single degree of freedom (temperature) at each node and the orthotropic material properties (Figure 92). Prism, tetrahedral, and pyramid shaped elements are other optional features for SOLID70. Should thermal stress analysis be required, SOLID70 could be switched to its structural equivalent element (e.g., SOILD45). This capability makes it possible for the thermal model to be updated to a thermal-stress analysis for future studies [213, 214].

SOLID70 is also capable of modelling nonlinear steady-state fluid flow through a porous medium. Either convection or heat flux and radiation may be input as surface loads to the element faces shown by the circled numbers in Figure 92. This element is capable of addressing mass transport and heat flow from a constant velocity field if needed; these capabilities were not used in this study.
When employing SOLID70, one assumes that the element does not have zero volume. Note that a zero volume occurs when the element is not numbered properly. In addition, the specific heat is evaluated at each integration point to allow for quick changes due to melting within a grid. Furthermore, a free surface of the element, which is neither adjacent to another element nor subject to a boundary constraint, is assumed adiabatic.

6.3 ANSYS® Software Limitations

ANSYS® with the University Research license was adopted for this study. Among the different ANSYS® products (e.g., Multiphysics, Mechanical, and Structural), only ANSYS® Multiphysics, Mechanical, Professional, and FLOTRAN support transient thermal analysis. The ANSYS® University Research license gives access to the complete capabilities of the Multiphysics license in addition to the nonlinear mechanical and thermal capabilities. While the majority of the development work on this project was carried out, there was a 512,000 limitation on the number of nodes in the ANSYS® University Research license, which limited the grid size and the dimensions of this model. Thus, great care needed to be taken to optimize the solutions to work within this
limitation. Towards the end of the modelling work, ANSYS® removed the limitation on the number of nodes from this license. The three-dimensional modelling results presented herein were obtained using the software without the license-imposed restrictions. However, the solutions still benefited from the memory-use optimization as the computer hardware still imposed limits on the model size.

6.4 3D Thermal Models of a Lap-Joint Configuration

6.4.1 The Joint Exposed to A Stationary Laser Beam

This section presents a 3D FEM thermal model of a lap-joint geometry exposed to a stationary diode laser beam that was developed and solved with ANSYS®10 (Figure 93). The material modelled in this study was PA6 with the commercial name of Akulon F223-D [215]. The laser-absorbing part was modelled for the case of 0.2% (wt.) carbon black (CB) concentration.

![Figure 93- The 3D thermal model geometry for a lap-joint configuration for the LTW of plastics (dimensions in mm, the thickness of each part is 3 mm).](image-url)
A laser beam produced by the Rofin-Sinar DLX16 diode laser with a power capacity of 160 W was modelled. The beam has an elliptical cross-section shape with the focal plane dimensions of $1.4 \text{ mm} \times 0.7 \text{ mm}$ [194]. The focal plane is assumed to be at the weld interface. The off focal-plane beam hits the laser-transmitting part 3 mm above the focal plane where the laser beam dimensions are slightly larger ($1.5 \text{ mm} \times 0.9 \text{ mm}$) [194]. The long beam dimension (1.4 mm) was aligned with the z-axis (Figure 93).

In the first stage of this research, the beam was assumed to have a rectangular cross-section shape, with dimensions of $1.4 \text{ mm} \times 0.7 \text{ mm}$ and a uniform power flux distribution [181]. In the second stage, the information obtained from the laser-beam scattering by the semi-crystalline polymer (PA6), investigated in Section 5.2, was used to model the laser beam to take into account the effect of scattering. In this second model, spreading of the beam as it travels through the laser-transmitting part was represented by linear interpolation between the unscattered beam profile at the upper surface and scattered beam profile at the lower surface of the laser-transmitting part. Section 6.5 gives details on how the heat generation by the laser beam in this stage of research was implemented.

The lap joint used in the experiments was modelled in this study (Figure 93). The dimensions of the rectangular plaques were $83 \times 27 \times 3 \text{ mm}^3$. The laser-transmitting part (part A) was welded to the laser-absorbing part (part B) while another laser-absorbing part (part C) acted as an insulator. The beam centre was located at a distance “a” away from the front edge; this distance matched that of the experimental set up and was equal to either 2 or 3 mm.
Due to the symmetry assumption in the geometry and laser beam profile, the geometry was cut in the laser beam symmetry plane, at \( x = 0 \), by a plane normal to the \( x \)-axis. Preliminary models with full-size geometry helped to identify where in the part the temperature gradients were sufficiently small to allow reduction of the model size to decrease the solution time. The reduced model dimensions were \( 10 \times 12 \times 8 \text{ mm}^3 \) (\( b \times c \times 8 \text{ mm}^3 \)) (the blue shaded area in Figure 93).

Convective heat transfer has a minor effect on plastic welding compared to conductive heat transfer [25]. For the stationary model, the top and front faces of the geometry were assumed to transfer heat by convection to the environment since they were exposed to the surroundings. The free convection correlations presented in the literature [177] were used to obtain an average value of \( h = 5 \text{ W/m}^2\text{K} \), which was applied to the model. Same value was reported by Ilie et al. [92] in LTW modelling application. For more information, see Appendix D.

The rest of the surfaces were assumed adiabatic. The left vertical side was the plane of symmetry and was adiabatic. A second laser-absorbing plastic part C, located under part B, helped to insulate the joint from the aluminum fixture surface underneath. This extra part was partially included in the thermal model: 2 mm depth of this part was modelled to account for the conduction heat transfer to this part.

The absorption coefficient for the laser-absorbing part was adopted from Section 5.3 where it was suggested the absorption coefficient could be predicted by modelling CB as soot particles suspended in a laser-transmitting medium, leading to \( K = 4.8 \text{ mm}^{-1} \). For the laser-transmitting part, transmission of 51% was adopted from the laser-light transmission measurements reported in the literature [119]. By assuming that
the energy not transmitted or reflected is absorbed by the laser-transmitting part and by using the Beer’s law, the absorption coefficient for the laser-transmitting part ($K_T = 0.2 \text{ mm}^{-1}$) was calculated from:

$$K_T = -\frac{\ln(\tau/(1-\rho))}{\tau}$$  \hspace{1cm} (45)

where $\tau$ is the transmission of the laser-transmitting part, and $\rho$ is the surface reflection of the parts.

ANSYS®10 was used in all stages of the thermal model including the pre and post processing phases. The CPU time to process and solve the problem was approximately 12 hours on a 3 GHz PC with 2 GB of RAM. The model was meshed using SOLID70 elements (Figure 92, Figure 94, and Figure 95).

The Jacobi Conjugate Gradient (JCG) solver was used for the runs. The JCG solver starts with the element matrix formulation. It assembles the full global matrix instead of triangularizing the global matrix. The solver then calculates the degree of freedom (DOF) solution by iterating to convergence (starting with an assumed initial temperature value for all DOF). The JCG solver is advantageous when solution speed is crucial in single-field problems (e.g., thermal models).
Figure 94- The 3D meshed model for the lap-joint configuration.

Figure 95- The close up of the 3D meshed model along the x-y plane (front view).

Vertical element size varied throughout the height of the model (y dimension) (Table 12). A refined element size of 0.008 mm was used in the region around the interface of parts A and B; element vertical size increased to a maximum of 0.4 mm away from the interface. Horizontal element size was equal for z and x dimensions (0.08 mm). Grid sensitivity analysis was performed by progressive reduction of element sizes both in the refined mesh area in part B and in the rest of the model. In the refined area, element
sizes from 0.00625 to 0.2 mm were tested; in the rest of the model, element sizes from 0.125 to 0.4 mm were tested.

<table>
<thead>
<tr>
<th>Y start point (mm)</th>
<th>Y end point (mm)</th>
<th>Grid size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0</td>
<td>0.2</td>
</tr>
<tr>
<td>2.0</td>
<td>2.5</td>
<td>0.1</td>
</tr>
<tr>
<td>2.5</td>
<td>4.5</td>
<td>0.008</td>
</tr>
<tr>
<td>4.5</td>
<td>5.0</td>
<td>0.1</td>
</tr>
<tr>
<td>5.0</td>
<td>6.0</td>
<td>0.2</td>
</tr>
<tr>
<td>6.0</td>
<td>8.0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

For the stationary laser beam experiments modelled in this study, the heating process was prolonged to 10 s and power was decreased to 1 W to accommodate the thermal camera data-acquisition speed limitations during the heating and cooling phases. Thus, the model simulated a 10 s interval of laser-material interaction (i.e., heating) with a stationary laser beam followed by a 10 s cooling interval. A range of time steps were explored starting from 0.5 to 2 s. A time step of 1 s was chosen to satisfy solution convergence and stability.

6.4.2 The Joint Exposed to A Moving Laser Beam

This section presents a 3D, transient, thermal FEM model of a scanning LTW process for a lap-joint geometry. The model incorporated an accurate representation of a laser beam, which accounted for its spatial intensity distribution as well as scattering by the semi-crystalline polymer. The laser heating was treated as a time- and space-varying internal heat generation source. Laser energy absorption was considered in both the laser-transmitting and laser-absorbing parts being joined.
The geometry modelled was similar to the stationary case (Figure 93), consisting of three stacked rectangular plaques (83 × 27 × 3 mm³). The laser-transmitting part (part A in Figure 93) was welded to the laser-absorbing part (part B). Part C was added to minimize the conduction between part B and the aluminum support plate. The centre of the laser beam was located 3 mm from the edge of the sample as it moved along the x direction. The length of the model (c = 20 mm along the x-axis) was selected based on the length sensitivity studies, which will be discussed in Section 6.6.2. Model dimension b (z direction) was set to 9 mm and model height (y direction) was set to 7 mm (Figure 93). Model dimension b as well as model height were decreased by 3 mm and 1 mm compared to the stationary case in order to decrease the solution time. Modelling results validated that choice since, at y = 7 mm and at z = 0 mm, the temperature gradient was zero and temperatures were equal to the room temperature.

As discussed earlier for the case of the stationary laser beam model, heat transfer by convection has negligible effect in polymer welding. Benatar [25] estimated its effect to be one thousand times less than that of conduction. Furthermore, since peak temperatures are reached at least 3 mm away from the air-plastic boundary, temperature difference at the air-plastic interface remains relatively low. In the interest of minimizing the complexity of the model and thus reducing the solution time, convective and radiative heat losses due to contact with air at the front and top surfaces were ignored in the moving beam model. The rest of the exposed surfaces were assumed adiabatic.

The heat generation was defined as the laser-energy flux change caused by the absorption of the laser beam energy by the plastic, and was a function of the laser beam
power, scanning speed, material absorption properties, and beam intensity distribution. Details on heat generation implementation are provided in Section 6.5.

ANSYS®10 was used for pre and post processing stages. The model was meshed using SOLID70 thermal elements (Figure 92). The geometry was finely meshed near the joint interface to account for the steep thermal gradient while mesh size was gradually increased with the distance from the interface (Table 13). Horizontal element size was equal for z and x dimensions (0.125 mm). Details of grid sensitivity studies are given in Section 6.6.

Table 13- Vertical grid sizes used for the moving-laser-beam model

<table>
<thead>
<tr>
<th>Y start point (mm)</th>
<th>Y end point (mm)</th>
<th>Grid size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0</td>
<td>0.2</td>
</tr>
<tr>
<td>2.0</td>
<td>2.5</td>
<td>0.1</td>
</tr>
<tr>
<td>2.5</td>
<td>4.5</td>
<td>0.0125</td>
</tr>
<tr>
<td>4.5</td>
<td>5.0</td>
<td>0.025</td>
</tr>
<tr>
<td>5.0</td>
<td>5.5</td>
<td>0.05</td>
</tr>
<tr>
<td>5.5</td>
<td>6.0</td>
<td>0.1</td>
</tr>
<tr>
<td>6.0</td>
<td>7.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 14 shows the matrix describing the runs carried out for the moving laser beam modelling. The beam started with its leading edge just outside the modelled volume, with beam centre at x = -1 mm, and moved along the positive x direction (Figure 93). The experimental process conditions listed in Table 5 were modelled. Three different values of the absorption coefficient (from 4.8 mm⁻¹ to 14.8 mm⁻¹) and two different values of transmission (51% and 69%) were explored for the base parameter settings of 18 W power and 30 mm/s scanning speed.
Table 14- Matrix for the moving laser beam modelling runs

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
<th>Run 5</th>
<th>Run 6</th>
<th>Run 7</th>
<th>Run 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Power (W)</td>
<td>18</td>
<td>36</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>Speed (mm/s)</td>
<td>30</td>
<td>30</td>
<td>15</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>Distance from the front edge (mm)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Absorption coefficient (1/mm)</td>
<td>4.8</td>
<td>4.8</td>
<td>4.8</td>
<td>9.6</td>
<td>14.8</td>
<td>4.8</td>
<td>5.4</td>
<td>5.4</td>
</tr>
<tr>
<td>5</td>
<td>Transmission (%)</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>69</td>
<td>51</td>
<td>51</td>
<td>51</td>
</tr>
<tr>
<td>6</td>
<td>Material</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PA6</td>
<td>PC</td>
<td>PC</td>
</tr>
</tbody>
</table>

As for the case of the stationary-laser beam model, the Jacobi Conjugate Gradient (JCG) solver was used for all the runs. A time step of 0.008 s was selected (Section 6.6 gives details of time-step selection process). The model contained 3,402,126 nodes. The CPU time to process and solve this problem for the base parameter settings (Run 1) was approximately 72 hours on a Dell Precision 390 Workstation with Intel Core 2 Duo 1.86 GHz processor and 4 GB of RAM running Windows XP Pro 64-bit operating system. ANSYS multi-processor option was employed to take advantage of the two processors making up the CPU. For the case where the multi-processor option was activated, with 2742712 nodes, CPU time was 138 hours but the run time was 84 hours.

### 6.5 Heat Generation

Accurate heat generation input data is important for obtaining realistic thermal model predictions. Heat is generated in the plastic parts as the laser beam passes through them and the beam’s energy is absorbed. The rate of heat generation at any point then
will be determined by the spatial distribution of the laser-beam power flux in a horizontal plane, by variation of this spatial distribution due to scattering by a semi-crystalline polymer, and by change in the local power flux due to the absorption. In addition, for the case of a moving laser beam, the three-dimensional heat generation distribution is translated in the direction of movement at the scanning speed. Knowing the spatial variation of the beam power flux, one can define the heat generation as the power flux change caused by the absorption of the laser beam energy.

The following section addresses how the laser beam distribution data (for unscattered and scattered beams) and the absorption characteristics of the laser-transmitting and laser-absorbing parts were used to generate the variation of the beam intensity through the depth of the two parts and how this variation was used to produce the heat generation expressions. Subsequent section describes how the above information was used to generate input files for the ANSYS software. Finally, the heat generation plots for the cases of stationary and moving laser beam thermal models are presented.

6.5.1 Derivation of the Heat Generation Expressions

The incident laser-beam cross-sectional shape in general could be circular, elliptical, or rectangular, depending on the type of laser and on the beam delivery optics used. In the early studies, without access to the beam intensity distribution data for the DLX16 laser (as it was not provided by the manufacturer), the beam profile was approximated by a uniform intensity distribution over a rectangular area. Subsequent beam profile studies carried out by the author (Section 5.1) provided the beam-intensity-distribution data, which was then incorporated into the later thermal modelling work.
Distribution of the power flux within the laser beam can be described by \( \Psi(x, z)(1/\text{mm}^2) \), a 2D normalized power flux distribution (NPFD), defined in Section 5.2.1 (Equation 29). When describing the beam distribution, the \( x^* \) and \( z^* \) coordinates correspond to the frame \( F_B \) with the origin at the beam centre and moving with the laser beam (Figure 96). Let \( F_P \) be a frame attached to the part. Then, for a beam moving at a constant velocity \( V \) along the \( x \)-axis and offset by a constant shift \( \Delta z \) along the \( z \)-axis, the points \( (x^*, z^*) \) in the \( F_B \) frame can be located with respect to the \( F_P \) frame by:

\[
x = x^* + Vt \quad \text{and} \quad z = z^* + \Delta z
\]  

(46)

Figure 96- Laser-beam frame defined with respect to the part frame.

If the beam moves along the \( x \) direction, the 1D NPFD along \( x \)-axis, \( \Psi(x^*) \), and transverse to \( x \), \( \Psi(z^*) \), can be defined such that:

\[
\Psi(x^*) = \int \Psi(x^*, z^*) \, dz^* \quad \text{and} \quad \Psi(z^*) = \int \Psi(x^*, z^*) \, dx^*
\]  

(47)

As the beam propagates through the laser-transmitting part, it is scattered and attenuated. Scattering changes the NPFD shape, so that for each \( y \) value, one can define: \( \Psi(x^*|_y) \) and \( \Psi(z^*|_y) \). A linear increase of the scattered beam width as a function of laser-transmitting part thickness was reported in the literature [123]. Thus, the 1D NPFD
profiles were calculated using a linear interpolation between the unscattered and scattered distributions for the range $0 < y < y_T$ (where $y = 0$ mm corresponds to the laser-transmitting part upper surface and $y_T$ is the laser-transmitting part thickness):

$$
\Psi(z^*)|_y = (1 - \frac{y}{y_T})[\Psi(z^*)|_{y=0} - \Psi(z^*)|_{y=y_T}] + \Psi(z^*)|_{y=y_T} - 3 \leq z^* \leq 3
$$

$$
\Psi(x^*)|_y = (1 - \frac{y}{y_T})[\Psi(x^*)|_{y=0} - \Psi(x^*)|_{y=y_T}] + \Psi(x^*)|_{y=y_T} - 1.5 \leq x^* \leq 1.5
$$

(48)

For $y > y_T$, the scattered distribution profile was assumed to remain unchanged throughout the laser-absorbing part depth. The effect of this approximation should be negligible given that, for the laser-absorbing part, when $K = 4.8$ mm$^{-1}$, over 99% of energy is absorbed within 1 mm of the interface.

While the unscattered 2D NPFD for the DLX16 Rofin-Sinar diode laser was measured and results were presented earlier, only 1D NPFD profiles are available for the scattered beam at this point. Exact extraction of the 2D NPFD from the 1D distributions is not a trivial exercise for the irregular (non-Gaussian) beam profile.

Given that a moving beam was being modelled, the power per unit length transverse to the beam movement direction, $P \Psi(z^*)$, was of primary importance. Variation of the beam intensity along the scanning direction, $\Psi(x^*)$, only affects how the heat generation varies over time at a point. For the scanning speeds and beam dimensions considered herein, the maximum total exposure time for each point was very short. It is believed that the 2D NPFD can be well approximated by assuming rectangular beam bounds, taking into account the low conductivity of the polymer. Given the above assumption, the heat generation per unit volume (in W/mm$^3$) due to the laser beam expressed in the part coordinate system $F_P$ was obtained by:

$$
q(x, y, z, t) = q(y) \Psi(x - Vt)|_y \Psi(z - \Delta z)|_y
$$

(49)
where \( q(y) \) (W/mm) is the rate of internal heat generation along the depth of the sample, which was derived by differentiating the Beer’s law with respect to \( y \).

\[
q(y) = \begin{cases} 
K_T P (1 - \rho) e^{-K_T y} & 0 \leq y \leq y_T \\
K \tau P (1 - \rho) e^{-K(\gamma - \gamma_T)} & y_T < y \leq y_T + \gamma_A
\end{cases}
\]

(50)

where \( K_T \) is the absorption of the laser-transmitting part, \( K \) is the absorption of the laser-absorbing part, and \( P \) is the laser beam power. Equation 50 was incorporated into the parabolic heat conduction equation (Equation 2) as the internal heat generation term.

### 6.5.2 Defining Heat Generation Input for ANSYS

Heat generation input in ANSYS® was defined by reading from a file of comma-separated “q” values. However, this array can represent only up to three space-time dimensions. To input the three spatial and one time dimension of the heat generation data \( q(x, y, z, t) \) required breaking up of this data into individual input files along one of the dimensions. Figure 97 represents how this was accomplished for the stationary-laser-beam model. Each input file (marked “Input_#” on the drawing) contains a sequence of arrays, with each array describing the heat generation over the z-y plane for a particular time. The array for \( t_0 \) specifies the starting condition of zero heat generation; the array for \( t_1 \) specifies the heat generation values when the laser is turned on; the array for \( t_2 \) is all zeros and thus sets heat generation to zero when the laser is turned off. Each input file corresponds to a “strip” covering a range along the x-axis. For example, \([x_1, x_2]\) specifies that the heat generation input in the array is applied to the range of the elements between the values of \( x_1 \) and \( x_2 \).
Figure 97- The heat generation input data structure for the stationary laser beam thermal model.

Figure 98 describes how the heat generation data were input for the moving laser-beam model. For this case, each input file contained a sequence of arrays, with each array describing the heat generation in the x-y plane. Each array in the sequence corresponded to the state of heat generation at time $t_k$ ($k = 0 \ldots n$). The values of $t_k$ were increased by equal time increment $\Delta t$: $t_{k+1} = t_k + \Delta t$. To represent the beam movement in the x direction, the non-zero heat generation values corresponding to the laser beam were shifted incrementally by $\Delta x$ within the x-y array in the positive x direction (as represented by the dark band in Figure 98). The shift value was determined by the scan speed: $\Delta x = V \Delta t$. Each input file was applied to the elements within a certain range along the z-axis.
Figure 98- The heat generation input data structure for the moving laser beam thermal model.

Figure 99 shows the flowchart of the MATLAB® program used to calculate the heat generation input. First, the input parameters were specified: the laser beam scanning speed and power, laser beam dimensions, the distance of the centre of the laser beam with respect to the sample edge, and the absorption coefficients of the materials. Next, the user was prompted to specify whether this was a stationary or a moving laser beam case, whether a top (laser-transmitting) part was present, and, if present, whether the top part was scattering (PA6) or not (PC). After these selections were made, plots were generated for user visual validation. The ASCII comma-separated-value files with heat generation input were then written, and ANSYS® script lines were automatically generated for reading in these input files.
For example, for a moving laser beam model (PA6, 18 W, 30 mm/s, 0.008 s time step, K = 4.8 mm\(^{-1}\), and \(\tau = 51\%\)), the space along the z-axis was subdivided into 30 strips (requiring 30 files to be generated). Each file consisted of 97 time steps with \(84 \times 52\) arrays of x-y plane heat generation input data. The files and the ANSYS\(^\text{®}\) command lines were automatically generated using the MATLAB\(^\text{®}\) script described above.

For example, assume that an input data file named \texttt{hg_input_1.csv} was created containing heat generation data for the first strip. Given below is a sample of the ANSYS\(^\text{®}\) script commands used to read the data from this file into ANSYS\(^\text{®}\) and to define the heat-generation body force:

1) Define input array dimensions of \(52 \times 84 \times 97\):

   *DIM, hg_input_1, table, 52,84,97, y,x,time

2) Read the data from the file \texttt{hg_input_1.csv} into the array \texttt{hg_input_1}:
3) Select elements within the z interval \([3, 3.2]\) mm and apply the heat-generation body force from the array \(hg\_input\_1\) to the selection:

\[
\text{nse1, s, loc, z, 0.003, 0.0032} \\
\text{esln, s, 1} \\
\text{bfe, all, hgen, , %hg\_input\_1%}
\]

6.5.3 Heat Generation for the Stationary Laser Beam

This section gives examples of the heat generation input used for modelling a stationary laser beam interacting with semi-crystalline and amorphous plastic materials. Three different cases are presented: 1) the top part is amorphous; 2) the top part is semi-crystalline; 3) no top part is present.

6.5.3.1 Amorphous Top Part

The results presented in this section are for the case of an amorphous material (i.e., PC). Adjusting the reflectance and transmittance for the material, one can adopt the method to any amorphous (non-scattering) material. Figure 100 shows the laser beam normalized power flux distribution (NPFD) along the z direction at the laser-transmitting part’s top surface \((\Psi(z^*)|_{y=0})\), while Figure 101 shows the NPFD along the x direction \((\Psi(x^*)|_{y=0})\). Both distributions were measured using the pinhole approach (Section 5.1). Due to the symmetry, only one-half of the heat generation profile along the x direction was used. The beam profile remains unchanged after the laser beam has passed through the laser-transmitting part since the amorphous materials do not scatter the laser beam. Thus, for this case, \(\Psi(z^*)|_{y=0} = \Psi(z^*)|_{y=y_T}\) and \(\Psi(x^*)|_{y=0} = \Psi(x^*)|_{y=y_T}\).
Figure 100- The laser beam NPFD profile along the z direction.

Figure 101- The laser beam NPFD profile along the x direction.

Figure 102 shows the heat generation profiles along the z direction for different x values. The profiles are given for y = 3 mm (i.e., at the interface between the two parts). At this depth, the highest heat generation values were reached. Heat generation values
decrease quickly as one moves away from the beam centre at \( x = 0 \) mm towards the edge of the beam at \( x = 1.25 \) mm.

Figure 102- The heat generation profiles along the \( z \) direction for different locations along \( x \) (PC, \( y = 3 \) mm, \( P = 18 \) W, \( \rho = 7\% \), \( \tau = 93\% \), \( K = 5.4 \text{ mm}^{-1} \)).

Figure 103 shows the heat generation profiles along the \( z \) direction at \( x = 0 \) mm for different \( y \) values (depths). The profiles show the laser beam centre located 3 mm from the edge of the sample (the edge is at \( z = 9 \) mm). Heat generation is only present for \( y > 3 \) mm since there is no absorption in the laser-transmitting part. The plots show steep gradients in the heat generation along \( z \), as the beam intensity drops off sharply from its peak value at \( z = 6 \) mm. The heat generation also decreases quickly as the laser beam penetrates into the laser-absorbing part’s depth: the intensity decreases to one quarter of the maximum value after passing through 0.3 mm and to only 2% after 0.8 mm (at \( y = 3.8 \) mm).

Figure 104 shows the development of the heat generation profile along \( y \) (sample’s depth) at \( x = 0 \) mm (beam’s centre). Peak values occur at \( y = 3 \) mm, where the laser
beam enters the laser-absorbing part. The values decrease quickly to near zero at
$y = 4 \text{ mm}$, having passed through only 1 mm of the laser-absorbing part.

Figure 103- The heat generation profiles along the $z$ direction for different locations along $y$ (PC,
$x = 0 \text{ mm}, P = 18 \text{ W}, \rho = 7\%, \tau = 93\%, K = 5.4 \text{ mm}^{-1}$).

Figure 104- The heat generation profiles along the $y$ direction for different locations along $z$ (PC,
$x = 0 \text{ mm}, P = 18 \text{ W}, \rho = 7\%, \tau = 93\%, K = 5.4 \text{ mm}^{-1}$).
Figure 105 shows heat generation at the centre of the laser beam for absorption coefficient ranging from 1.8 mm\(^{-1}\) to 14.8 mm\(^{-1}\). The plots start at the interface of the two parts and end 2 mm into the laser-absorbing part. The plots show the sensitivity of the heat generation profile to the absorption coefficient of the laser-absorbing part. The heat generation penetrates further into the laser-absorbing part with smaller absorption coefficients while higher peak values are produced near the interface with larger absorption coefficients.

![Graph showing heat generation profiles for different absorption coefficients.](image)

Figure 105: The heat generation profiles along the depth of the sample for different absorption coefficients (PC, x = 0 mm, z = 6 mm, P = 18 W, \(\rho = 7\%\), \(\tau = 93\%\)).

### 6.5.3.2 Semi-Crystalline Top Part

The results presented in this section are for a semi-crystalline material (i.e., PA6). One can adopt the approach and adjust it to any other semi-crystalline material, if its reflectance, transmittance, and scattering are known. Figure 106 shows the laser beam NPFDs along the z direction of the laser beam for different locations along the depth of the laser-transmitting part. These curves result from linear interpolation between the...
unscattered beam profile at $y = 0$ mm ($\Psi(z^*)|_{y=0}$) and the scattered one at $y = y_T$ ($\Psi(z^*)|_{y=y_T}$) and are obtained using Equation 48. One can see that as one gets farther from the upper surface of the laser-transmitting part ($y = 0$ mm), the NPFD widens and consequently its peak magnitude decreases, thus reflecting the scattering effect. Note that the area under each curve is equal to one.

![Graph showing NPFD laser beam profiles along the z direction for different depths within the laser-transmitting part (PA6).](image)

Figure 106- The NPFD laser beam profiles along the $z$ direction for different depths within the laser-transmitting part (PA6).

Figure 107 shows the NPFD along the $x$ direction of the laser beam for different locations along the depth of the laser-transmitting part. One-half of the heat generation profile along the $x$ direction was used to satisfy the symmetry conditions. The beam profile changes as the laser beam passes through the laser-transmitting part due to the scattering by the semi-crystalline material. The curves, calculated using Equation 48, result from the linear interpolation between the unscattered beam profile at $y = 0$ mm ($\Psi(x^*)|_{y=0}$) and the scattered one at $y = y_T$ ($\Psi(x^*)|_{y=y_T}$).
Figure 107- The NPFD laser beam profiles along the x direction for different depths within the laser-transmitting part (PA6).

Figure 108 shows the heat generation along the z direction for different distances from the laser beam centre (x values). The profiles are at the interface between the two parts (y = 3 mm), where the highest heat generation values were attained. Heat generation magnitudes show a rapid decrease as one moves away from the beam centre at x = 0 mm towards the edge of the beam at x = 3 mm.

Figure 109 shows the heat generation along the depth of the sample (y) at z = 6 mm for different x-axis positions. It is seen that some heat is absorbed inside the laser-transmitting part. The heat generation values are high near the laser beam centre (x = 0 mm). The sharp heat generation gradient at the interface of the two parts (y = 3 mm) is apparent. The values decrease quickly to near zero at y = 4 mm, having passed through only 1 mm of the laser-absorbing part.
Figure 108- The heat generation profiles along the z direction for different locations along x  
(PA6, y = 3 mm, P = 18 W, ρ = 6%, τ = 51%, K = 4.8 mm⁻¹).

Figure 109- The heat generation profiles along the y direction for different locations along x  
(PA6, z = 6 mm, P = 18 W, ρ = 6%, τ = 51%, K = 4.8 mm⁻¹).

6.5.3.3 Without the Top Part

Note that the developed ANSYS® subroutine is capable of modelling problems in  
which the top part is absent. Such a model is helpful for laser-beam threshold studies in
which the laser-absorbing part and laser beam interaction is the subject of interest as well as the validation of the scattered beam profile measurements. To address these cases, knowing the heat generation profile in the absence of the top part is required. The NPFD profiles along the laser beam width (z) and length (x) directions are the same as the unscattered laser beam profiles introduced in Section 6.5.3.1.

6.5.4 Heat Generation for the Moving Laser Beam

This section presents examples of the heat generation input used for modelling a moving laser beam scanning semi-crystalline and amorphous polymers. The NPFD profiles along the z direction are the same as the ones given for the stationary laser beam in the absence and presence of the top part for both amorphous and semi-crystalline materials. Since the symmetry condition did not apply in this case, the complete NPFD along the x direction was defined for the amorphous (Figure 110) and semi-crystalline (Figure 111) polymers.

Figure 112 shows the heat generation profiles along the x direction for the moving laser beam for the semi-crystalline material. The time step of 0.008 s was used in this case. This time step gives a 0.24 mm step size in the x direction for the laser beam moving at 30 mm/s. The plots show how the laser beam enters the modelled sample volume on the left side and then moves from left to right.
Figure 110- The NPFD laser beam profile along the x direction (PC).

Figure 111- The NPFD laser beam profile along the x direction for different depths within the laser-transmitting part (PA6).
Figure 112- The heat generation profiles along the x direction for different times (PA6, y = 3 mm, z = 6 mm, P = 18 W, ρ = 6%, τ = 51%, K = 4.8 mm\(^{-1}\)).

6.6 Grid Size and Time Step Selection

Grid density is very important in a FEM analysis: too coarse a grid can result in large errors; too fine a grid wastes the computer resources, increases the running time, makes the model difficult to manipulate, and, beyond certain point, leads to increase of the round-off error.

According to the ANSYS\textsuperscript{®} manual [212], there is no direct way of determining the appropriate grid size for a model. The manual suggests that initially an analysis with as low a number of elements as possible is carried out. Then, the number of elements should be doubled and the solutions compared. If the results are sufficiently close, the grid size is adequate; otherwise, it needs to be further refined. This procedure is time consuming; in addition, any time the material properties (e.g., absorption coefficient) or process parameters (e.g., laser power) change, the procedure needs to be repeated.
In this research, a method was developed for selecting the time step and grid size based on an approach that combines the energy balance relations with the Beer’s law. This section presents the methodology developed for selecting the time step and grid size. This methodology then was applied to study the relationship between the optimized grid-size and time-step selections and the material properties and process parameters.

### 6.6.1 Methodology for Selecting Grid Size and Time Step

The time step and grid size settings affect each other: choosing appropriate settings for both is not a trivial task. As discussed earlier, there is a sharp heat generation gradient at the interface of the two parts at \( y = 3 \text{ mm} \). Capturing the heat generation at the interface is challenging; if the time step and grid size are not chosen properly, this sharp gradient can cause instability of the solution or problems with the convergence. Care must be taken to limit the temperature rise of an interface element over a single time step. As the absorption coefficient increases, the element size needs to be decreased to compensate for the steeper heat generation gradient at the interface. This requires a specific grid size and time step for given process parameters, optical, and thermo-physical properties.

Herein, an analytical-numerical program was developed using Engineering Equation Solver (EES) software, which focused on the choice of the grid size and time step with respect to the absorption coefficient, material properties, and process parameters. EES is a specialized software developed by Klein (University of Wisconsin–Madison) [216] to help solve problems in the thermal sciences.

Figure 113 shows the energy conservation diagram for the laser beam hitting an element \( (v = x \times \Delta y \times z) \) located within the laser-absorbing part, adjacent to the interface.
between the two parts. For the purpose of this derivation, a uniform laser-beam power flux $I_0 = P/x_{beam} z_{beam}$ was assumed, where $x_{beam}$ and $z_{beam}$ are the narrow and wide laser-beam dimensions. If the flux of $I_0$ hits the surface of the laser-transmitting part, then $\tau (1-\rho)$ fraction of this power flux reaches the laser-absorbing part element. Thus, the rate of energy entering the element is:

$$q_y = \tau (1-\rho) I_0 x z$$  \hspace{1cm} (51)

Due to the absorption of the laser energy inside the element, not all of the energy is passed through. According to the Beer’s law (Equation 11), one can calculate the energy exiting the element (see Section 3.3):

$$q_{y+\Delta y} = \tau (1-\rho) I_0 e^{-\kappa \Delta y} x z$$  \hspace{1cm} (52)

The exact absorbed energy then is the difference between the energy entering and exiting the element:

$$q_i = q_y - q_{y+\Delta y} = \frac{\tau (1-\rho) I_0 (1-e^{-\kappa \Delta y}) v}{\Delta y}$$  \hspace{1cm} (53)

An alternative approach can be used to calculate the absorbed energy. The approach uses derivative of the Beer’s law (Equation 11) to evaluate the linear
approximation (finite difference) of the absorbed energy. This finite difference approximation is:

\[ q_2 = K \tau (1 - \rho) I_o e^{-K \Delta y} \nu \]  \hspace{1cm} (54)

The absorbed energy defined in these equations increases the internal energy of the element. Herein, lumped capacitance method is employed, which connects the rate of the heat loss at the surface of the element volume to the rate of the change of the internal energy [177]. Therefore, the temperature change obtained from the exact energy balance approach (\( \Delta T_1 \)) and the finite difference approximation (\( \Delta T_2 \)) can be defined as follows:

\[ \Delta T_1 = \frac{\tau (1 - \rho) I_o (1 - e^{-K \Delta y}) \Delta t}{\Delta y k / \alpha} \] \hspace{1cm} (55)

\[ \Delta T_2 = \frac{K \tau (1 - \rho) I_o e^{-K \Delta y} \Delta t}{k / \alpha} \] \hspace{1cm} (56)

where \( \alpha \) is the thermal diffusivity and \( k \) is the thermal conductivity of the polymer. Equations 55 and 56 show that material thermo-physical and optical properties, and process parameters are the determining factors in choosing an appropriate time step and grid size for a LTW thermal analysis:

\[ \Delta T = f(K, \tau, \rho, P, x_{beam}, z_{beam}, \Delta t, \Delta y) \] \hspace{1cm} (57)

To determine the time step from the grid size, the temperature change per time step must be limited. High temperature gradients result in a rapid temperature rise, which causes instability.

The temperature rise obtained based on exact calculation \( \Delta T_1 \) can be compared to the one based on the finite difference approximation \( \Delta T_2 \). The difference between the two quantities provides an indication of how fast the heat generation value changes along \( y \). The temperature rise estimate error can be described as \( E = (\Delta T_1 - \Delta T_2) / \Delta T_1 \). This
error must be kept to a relatively low level (below 10%) to capture accurately the heat generation. As a second requirement, the temperature rise ($\Delta T$) should be limited (approximately 100°C). If $\Delta T$ is large, the solution may be unstable. The two limits given above were proven through numerous case studies presented in the following sections. Note that the lumped capacitance assumption results in the order-of-magnitude estimates of $\Delta T$. The actual $\Delta T$ is smaller due to the thermal conductivity.

Figure 114 indicates a nearly linear dependence of the temperature rise ($\Delta T_i$) on the absorption coefficient for the range of values considered here. For the same absorption coefficient, the temperature rise decreases as the time step is decreased. Figure 115 shows that the temperature rise increases as a function of absorption coefficient. As the grid size decreases, the temperature rise increases for the same absorption coefficient. For the grid sizes smaller than $\Delta y = 0.0125$ mm, the temperature rise dependence on the absorption coefficient becomes nearly linear.

![Figure 114- Temperature rise ($\Delta T_i$) obtained from the exact approach vs. absorption coefficient (dy = 0.0125 mm, P = 18 W, $\tau = 51\%$, $\rho = 6\%$).](image-url)
Figure 115- Temperature rise ($\Delta T_1$) obtained from the exact approach vs. absorption coefficient
(dt = 0.008 s, P = 18 W, $\tau$ = 51%, $\rho$ = 6%).

Figure 116 shows the error (E) in the lump capacity estimate of the temperature rise
as a function of the absorption coefficient for different grid sizes. One can see that the
error increases with increasing absorption coefficient for the same grid size ($\Delta y$). This is
expected since the Beer’s law predicts steeper heat generation gradient with higher
absorption coefficients. This leads to greater error due to the linear approximation. As
the grid size ($\Delta y$), and thus the spatial resolution, decreases, this error decreases as well.
The results are presented for the time step of 0.008 s, where the temperature difference of
100°C for the absorption coefficient of 4.8 mm$^{-1}$ was obtained. For the same grid size,
changing the time step did not affect the error; however, with increasing the absorption
coefficient, the time step needs to decrease so that the temperature rise does become
excessive. Examples of the parametric tables obtained from this approach are presented
in Appendix E.
This study clearly indicates that as the absorption coefficient increases, the grid size must decrease to maintain the solution accuracy. Furthermore, as the grid size decreases, the time step needs to decrease to limit the temperature increase per time step.

6.6.2 Length Sensitivity Studies

The experimental set up modelled used three stacked samples, each with dimensions $83 \times 27 \times 3$ mm$^3$. To reduce the running time and memory requirements, the modelled geometry dimensions were reduced. To study the effect of the model length in the x direction on the results, several cases were run for the lengths of 10, 20, and 30 mm. The results present both temporal (Figure 117 to Figure 121) and spatial (Figure 122 and Figure 123) temperature distributions. Length sensitivity analysis showed that the length of the model could be reduced to 20 mm to save the computational time.

The sample was exposed to an 18 W diode laser beam, which scanned the sample at 30 mm/s. The time step was 0.008 s and grid size was 0.1 mm. Figure 117 shows the
transient temperature at the centre of the laser beam for the middle of the samples. For a 30-mm-long model, the temperature profile shows an increase up to 242°C at 0.42 s where the laser beam has just passed the middle of the sample (x = 15 mm) and then gradual decrease down to 110°C at 1.5 s where the laser beam has left the sample (x = 30 mm). In addition, the point reached the melting temperature (200°C) at t = 0.55 s and remains above this temperature for 0.18 s. The temperature profiles’ peaks for the three model lengths are very similar. Note that the plots have been shifted to facilitate the comparison.

Figure 117- Predicted transient temperature profiles for different lengths of the model at the centre of the laser beam (middle of the samples, y = 3 mm, z = 6 mm, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹, dt = 0.008 s, dy = 0.1 mm).

Figure 118 compares the peak temperatures for different model lengths presented in Figure 117. It shows that for the lengths of 20 mm and greater, the peak temperature remains nearly constant. The small temperature difference (under 0.1°C) does not justify the significantly increased run times associated with longer models.
Figure 119 shows the predicted transient temperature profiles at the front surface and at the left edge of the model, for different model lengths. The peak temperature of 35.5°C was predicted. It is seen that the low thermal conductivity of the plastic has caused a small amount of energy to be transferred to the model front surface, which is 3 mm away from the laser beam centre in this case. The profiles are very similar except for the L = 10 mm, which shows a 1.3°C (16%) difference in the peak temperature change. The temperature profiles obtained for L = 20 mm and L = 30 mm are very close.

At points away from the left edge (x > 0 mm), the temperature difference observed between different model lengths decreases. Figure 120 shows the transient temperature profiles at x = 5 mm for different model lengths; the profiles are very similar. Similar trend for transient temperature was observed as one moved inside the model (Figure 121). Figure 121 shows the peak temperatures in Figure 119 and Figure 120. The convergence of peak temperatures validates the choice of L = 20 mm.
Figure 119- Predicted transient temperature profiles for different lengths of the model at the front surface of the model (x = 0 mm, y = 3 mm, z = 9 mm, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹, dt = 0.008 s, dy = 0.1 mm).

Figure 120- Predicted transient temperature profiles for different lengths of the model at the front surface of the model (x = 5 mm, y = 3 mm, z = 9 mm, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹, dt = 0.008 s, dy = 0.1 mm).
Figure 121- Predicted peak temperature profile vs. length of the model at the front surface of the model for different locations along x (y = 3 mm, z = 9 mm, P = 18 W, V = 30 mm/s, $\tau = 51\%$, $K = 4.8 \text{ mm}^{-1}$, $dt = 0.008 \text{ s}$, $dy = 0.1 \text{ mm}$).

Figure 122 shows the temperature profiles along x for different model lengths. Note that the plots for shorter models have been shifted along x to superimpose the peak temperature points in order to facilitate the comparison. It is seen that the temperature profile at the leftmost coordinate for $L = 10 \text{ mm}$ is different from the ones of the $L = 20 \text{ mm}$ and $L = 30 \text{ mm}$. It could be concluded that for smaller lengths, the solution is affected by the boundary conditions. As the length increases, the temperature profiles converge and show very similar values for the $L = 20 \text{ mm}$ and $L = 30 \text{ mm}$ cases. The temperature profiles close to the peak are nearly identical in all three cases.

The temperature profiles along the z-axis are shown in Figure 123. The peak temperature ($242^\circ \text{C}$) is observed at the centre of the laser beam (at $z = 6 \text{ mm}$). The results obtained for the two longer models ($L = 20 \text{ mm}$ and $L = 30 \text{ mm}$) are very similar.
6.6.3 Time-Step-Sensitivity Studies

Reducing the time step decreases the temperature rise per time step (as seen in the results from the EES-based study in Figure 114). This can improve the stability of the
solution. Decreasing the time step, on the other hand, increases the solution time and can introduce round off errors to the solution. This section presents the results obtained for the time-step-sensitivity studies. Results for time steps of 0.004 s, 0.008 s, and 0.016 s are presented. Figure 124 and Figure 125 show the temporal results and Figure 126 to Figure 127 show the spatial results. The time step of 0.008 s was found to be the optimum. The model length used for these runs was 20 mm; this value was obtained from the length sensitivity studies explained in Section 6.6.2.

Figure 124 shows the transient temperature profiles for a point located at \( x = 10 \text{ mm}, \ y = 3.2 \text{ mm}, \) and \( z = 6 \text{ mm} \) for different time steps. The temperature profiles show an increase up to 242°C at 0.42 s, at which point the laser beam has just passed the mid-point of the model \( (x = 10 \text{ mm}) \), and then gradual decrease down to 150°C at 0.8 s, at which point the laser beam has just left the model \( (x = 20 \text{ mm}) \). It is seen that the temperature profiles are very similar. The peak temperatures for different time steps presented in Figure 124 are very similar and, based on this figure, it is not possible to distinguish between the time steps.

Figure 125 shows the transient temperature at the front surface \( (z = 9 \text{ mm}) \), at the left edge of the model \( (x = 0 \text{ mm}) \). The results obtained for the \( dt = 0.004 \text{ s} \) and \( dt = 0.008 \text{ s} \) are very similar while the peak temperature is about 0.8°C (9% of the total temperature rise) higher for \( dt = 0.016 \text{ s} \).

Figure 126 shows the temperature profiles along the length of the model for different time steps. The result obtained for the \( dt = 0.016 \text{ s} \) is slightly higher than the rest, with deviation increasing close to the model’s left edge \( (x = 0 \text{ mm}) \). Figure 127 shows the temperature profiles along the width of the model at the interface of the two
parts \( (y = 3 \text{ mm}) \). The maximum temperature of \( 172^\circ \text{C} \) is observed at the centre of the laser beam \( (z = 6 \text{ mm}) \) for the two smaller time steps; for \( dt = 0.016 \text{ s} \), the peak temperature increases to \( 177^\circ \text{C} \).

![Figure 124](image1.png)

Figure 124- Predicted transient temperature profiles for different time steps at the centre of the laser beam \( (x = 10 \text{ mm}, y = 3 \text{ mm}, z = 6 \text{ mm}, P = 18 \text{ W}, V = 30 \text{ mm/s}, \tau = 51\%, K = 4.8 \text{ mm}^{-1}, dy = 0.1 \text{ mm}, L = 20 \text{ mm}) \).

![Figure 125](image2.png)

Figure 125- Predicted transient temperature profiles for different time steps at the front surface of the model \( (x = 0 \text{ mm}, y = 3 \text{ mm}, z = 9 \text{ mm}, P = 18 \text{ W}, V = 30 \text{ mm/s}, \tau = 51\%, K = 4.8 \text{ mm}^{-1}, dy = 0.1 \text{ mm}, L = 20 \text{ mm}) \).
Figure 126- Predicted temperature profiles along the length of the model for different time steps at the front surface of the model ($y = 3$ mm, $z = 9$ mm, $P = 18$ W, $V = 30$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$, $dy = 0.1$ mm, $L = 20$ mm).

Figure 127- Predicted temperature profiles along the width of the model for different time steps at the centre of the laser beam ($x = 0$ mm, $y = 3$ mm, $P = 18$ W, $V = 30$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$, $dy = 0.1$ mm, $L = 20$ mm).

Note that ANSYS® is equipped with automatic time stepping feature that can adjust the future time step based on the previous and current time steps. Several cases were run for the fixed and automatic time stepping to see if this feature was useful. For the
automatic time stepping to take place, one needs to define the minimum and maximum time steps as well as the time step startup. The solution starts with the defined time step and automatically adjusts itself to the end of the program in which the transient solution becomes steady. In this case, the time step increases to its maximum value. This way, as long as the ultimate time step is between the minimum and maximum time step, the solution converges.

Running numerous cases showed that the runs with a fixed time step were less time consuming than the ones with the varying time step. However, the challenge is to find the appropriate time step. If the fixed time step is not chosen properly, the solution will not converge at one of the time sub-steps. This convergence problem wastes all the time that was spent for the solution to get to that time sub-step. Therefore, using the automatic feature helps to gain some insight to choose the appropriate time step for a fixed time step solution. Nevertheless, automatic time stepping caused instability in some cases, especially, at the beginning and end of the runs. For stability reasons, it was decided not to adopt automatic time stepping.

6.6.4 Grid-Size-Sensitivity Studies

Grid size needs to be sufficiently fine to accurately represent the temperature gradient near the interface (as shown in Section 6.6.1); at the same time unnecessarily small grid size wastes computer resources. This section presents the results of the grid-size-sensitivity study. Models with the grid sizes of 0.2, 0.1, 0.05, 0.025, 0.0125, and 0.00625 mm near the interface of the two parts were investigated (with constant time step of 0.008 s). The methodology presented in Section 6.6.1 indicates that these conditions
avoid excessive temperature rise (Figure 114 and Figure 115 show a rise in the range of 100°C).

The greatest predicted temperature variation among the grid sizes can be seen at the interface of the two parts and centre of the laser beam (Figure 128). As the grid size near the interface of the two parts decreases, the results converge at a peak temperature of 186°C and the plot shapes become more and more similar. Figure 129 shows the predicted overall peak temperatures as a function of the grid size. It is seen that the results obtained for the 0.0125 mm grid size are within 2.4°C (1.5% of the temperature rise) of those for the 0.00625 mm grid size. Thus, a grid size of 0.0125 mm was selected.

Figure 130 shows how the two parameters estimated in Section 6.6.1 to help assess the grid size and time step (temperature rise ($\Delta T_i$) and percent error ($E$)) vary as grid size and time step change in the two studies reported above. The arrows in the plot indicate directions of decreasing grid size and time step values. It can be seen that as the grid size is decreased in the study, the error ($E$) decreases to a sufficiently small level (below 10%) near the grid size value selected (0.0125 mm). For the time-step study, the highest value tested (0.016 s) corresponds to a relatively high temperature rise of $\Delta T_i = 175$°C. Given that the overall peak temperature rise predicted by the model is near 200°C, the above value is clearly too high. The two runs where converging results were observed both display reasonable temperature rises of 100°C or less.
Figure 128- Predicted transient temperature profiles for different grid sizes in the middle of the model and at the centre of the laser beam (x = 10 mm, y = 3 mm, z = 6 mm, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹, dt = 0.008 s, L = 20 mm).

Figure 129- Predicted peak temperature for different grid sizes in the middle of the model and at the centre of the laser beam (x = 10 mm, y = 3.2 mm, z = 6 mm, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹, dt = 0.008 s, L = 20 mm).
Figure 130- Temperature rise (ΔT₁) vs. error (E) for the runs carried out for grid and time step sensitivity studies as well as for higher absorption coefficient values.

The grid size and time step selection methodology presented in Section 6.6.1 was used to narrow the search range for suitable grid and time step settings when models with higher absorption coefficient values were run as part of the investigation into the effect of material optical properties (Section 8.2.1.2). Figure 130 plots the points corresponding to the runs made for two higher absorption coefficients of 9.6 mm⁻¹ and 14.8 mm⁻¹. The plot shows that the error (E) was under 5% for all the runs; the run conditions finally selected were in the circled area. Results of numerous runs showed that the proposed methodology for selecting the time step and grid size as a function of material thermo-physical and optical properties is a useful tool that saves considerable computational time.
Chapter 7  Experimental Results

This chapter presents the results of thermal imaging observations for plastic specimens being welded in a lap-joint configuration using stationary (Section 7.1) and moving (Section 7.2) laser beam. In Section 7.1, PA6 specimens only, while in Section 7.2 both PA6 and PC specimens were used.

7.1 Experimental Results for the Stationary Laser Beam

The experimental results presented in this section are based on the 10 s exposure of the natural PA6 samples to a 1 W stationary diode laser beam. In addition to the thermal imaging observations, the weld dimensions at the interface between the laser-transmitting and the laser-absorbing parts were measured. Figure 131 shows an image of the laser-absorbing part’s joining surface after the laser-joined parts were separated. The light-coloured oval-shaped spot corresponds to the weld produced by the stationary laser beam. The weld spot dimensions are 1.0 × 1.5 mm² and its centre is located 3 mm away from the front surface of the sample. The surface facing the camera is seen as an edge at the bottom of the image. The two dark horizontal lines were produced by the two line scans made to pre-join the samples prior to the thermal imaging observation.

For the thermal imaging results presented here, the sample surfaces that faced the camera either were coated with soot particles or were left uncoated. Other coatings were also explored (black tape, white tape, and correction ink), and the results are reported in Appendix C. The transient and spatial temperature distributions for the soot-coated and untreated samples are compared.
Figure 131- Weld spot produced by the stationary laser beam (P = 1 W, V = 0 mm/s).

Figure 132 shows typical thermal images of the soot-coated PA6 plaques. The beam centre was located 3 mm from the sample’s front surface (identified as Surface A in Figure 94). The thermal image capture was started when the laser was turned on and continued at least 20 s after it was turned off. Line L2 marks the top surface of part A (y = 0 mm) and line L3 marks the lower surface of part C (y ≈ 9 mm). The image above line L3 shows the second laser-absorbing plastic part (part C). L1 marks the location of the laser-beam centre line. Figure 132a displays the temperature image at the end of the laser exposure (at 10 s elapsed time), where the image peak temperature of 66°C is observed. Figure 132b shows the temperature distribution after 10 s of cooling (at 20 s elapsed time), where the image peak temperature drops to 55°C.

Figure 133 shows thermal images of the uncoated PA6 plaques. Here the beam centre is located at the same distance as the coated sample (3 mm from Surface A). Figure 133a displays the temperature image at the end of the heating phase (10 s), where the temperature shows its maximum value (73.6°C) on the viewed surface. Figure 133b shows the temperature distribution after 10 s of cooling (t = 20 s), where the joint temperature drops to 52°C, and heat conducts through the components.
At the end of the laser exposure, the soot-coated samples show significant heating throughout the height of Part A, around the beam centre line. On the other hand, the uncoated samples show the temperature rise concentrated at the interface of Parts A and B. It is believed that the soot, which coated the front surface of Part A, absorbed the scattered laser beam energy, causing the surface temperature to rise. Normally, in LTW, most of the heat generation is expected to take place near the interface of the laser-transmitting and the laser-absorbing parts (A and B); the uncoated sample images are in better agreement with this observation.
Note that, assuming the scattered laser light is reaching the surface facing the thermal camera, the camera should not be detecting this light as false temperature reading since the laser light is emitted near 1 µm wavelength while the camera is sensitive to the wavelengths in the 7.5 µm to 13 µm range as described in Section 4.3. The heated soot, on the other hand, would radiate broadband and thus would be highly visible to the thermal imager.

Figure 134 and Figure 135 compare the transient temperatures at several points located along the laser-beam centre line (line L1 in the thermal images). Figure 134 shows the temperatures within the laser-absorbing part and Figure 135, within the laser-transmitting part. In both figures, solid lines show the transient temperature of the soot-coated sample (with the letter “s” in the diagram legend) and the triangle points show the transient temperature of the uncoated sample.

In Figure 134, it can be seen that the transient temperatures within the laser-absorbing parts (B and C) have similar magnitudes and curve shapes for the soot-coated and uncoated samples. The temperature closest to the interface of A and B (y = 3.5 mm) shows the fastest temperature rise (about 4.7°C/s) for the both cases, and reaches the highest value (74°C for soot-coated and 70°C for uncoated case) at 10 s, at the end of the laser exposure.

Temperature sample points further away from the interface show more gradual temperature rise as the distance from the interface increases. The peak temperature is reached at progressively later times as the distance (y) increases. These trends match well the expected temperature variation for the case where the heat source is located near the interface of parts A and B.
Figure 134- Transient temperatures obtained from the thermal imaging experiments with uncoated and soot-coated samples for different locations along the y-axis of the laser-absorbing part (s: soot-coated, P = 1 W).

Figure 135- Transient temperatures obtained from the thermal imaging experiments with uncoated and soot-coated samples for different locations along the y-axis of the laser-transmitting part (s: soot-coated, P = 1 W).
The transient temperatures for points on the front surface of the laser-transmitting part \((0 < y < 3)\) are shown in Figure 135. In this case, the soot-coated and uncoated samples show significantly different curve shapes. For the soot-coated sample, the temperatures are seen to rise at a higher rate than for the uncoated sample. The soot-coated sample points are also seen to start cooling immediately upon turning off of the laser beam. On the other hand, the temperatures recorded for the uncoated sample continue increasing for some time after the laser beam is turned off and then begin to decrease. The further the point is from the part interface, the longer this delay is.

The transient temperatures recorded for the soot-coated sample show behaviour matching that of a heat source located at the surface where the temperatures are observed by the thermal camera. In addition, this heat source is associated with the laser since it starts and stops at the same time as the laser is turned on and off. This behaviour suggests that the laser beam energy scattered by the semi-crystalline plastic is reaching the soot-coated surface and is being absorbed. This generates the heat detected by the thermal imager.

Note that the temperature distributions near the upper surface of the laser-transmitting part \((y = 0 \text{ mm})\) are very similar for the soot-coated and uncoated samples. It is likely that this is due to less laser scattering taking place near the top surface of the laser-transmitting part. In addition, temperature profiles show similar values some time after the laser beam turns off \((t > 10 \text{ s})\), which provides additional confirmation that the laser beam side scattering phenomenon by this semi-crystalline plastic (PA6) is affecting the observations.

Figure 136 shows the effect of the soot coating more clearly. The plot shows temperature profiles along the y-axis (along the line L1) at 10 s (when maximum heating
is achieved) for all the coatings tested. Within the laser-transmitting part, the soot-coated sample, shows an average temperature that is about 25°C higher than that of the uncoated sample. Within the laser-absorbing part, the soot-coated and uncoated temperature profiles are very close (within 1°C for y > 3.5 mm).

![Figure 136](image)

Figure 136- A comparison between the temperature profiles along the y-axis of the plastic samples obtained from the thermal imaging at the laser beam centre (at L1, P = 1 W, t = 10 s).

Examining the results for other coatings shown in Figure 136, the black-tape coated sample showed similar temperature rise in the laser-transmitting part as the soot-coated one. This is as expected because the black tape absorbs the scattered laser light, similarly to the soot. The main difference is that the black-tape-coated sample does not have the sharp temperature peak near the interface seen in the soot-coated sample. It is possible that an air gap near the interface insulated the tape.

Coatings such as white tape or correction ink do not appear to absorb the laser light. Within the laser-transmitting part, temperatures close to that of the uncoated sample were
observed. On the other hand, these coatings appear to smooth out the sharp temperature rise at the interface of the two parts, which is likely due to the contact resistance at the boundary between the sample surface and the coatings. Away from the weld interface, these materials match the uncoated results reasonably well.

Based on the thermal imaging observations and application of the various surface coatings, it can be concluded that, for the light-scattering laser-transmitting part, absorbing coatings (soot and black tape) cannot be used for the purpose of validation of the finite element modelling results since they absorb the scattered light energy and this effect was not included into the model.

The two non-laser-light-absorbing coatings resulted in temperatures on the laser-transmitting part surface that are similar to those in the uncoated case. This supports the assumption that the thermal camera is observing the true surface temperature when the laser-transmitting part is uncoated. For the laser-absorbing part, the surface can be used without coating and the thermal imaging observations can be used for model validation. Thus, for the experiments with the moving laser beam, reported in Section 7.2, no coating was used.

As suggested above, the transient temperature profiles during the first 10 s shown for the soot-coated laser-transmitting part in Figure 135 can be assumed to be caused by heating of the soot by the scattered laser beam. If this is the case, the observed temperature-time relationship can be used to estimate the surface heating rate that caused this temperature increase. Assuming that this heating was caused by the scattered laser beam radiation, it can be used as an indication of how the laser beam is scattered over the surface facing the camera.
To estimate the heating rate, it is assumed that the material surface is being heated by a constant heat flux, that the surface is part of a semi-infinite solid, and that there is no convection heat loss at the surface. Given these assumptions, an analytical relationship can be derived describing the 1D transient-temperature distribution as a function of the distance from the sample surface [177]:

\[
T(z',t) = T_\infty + \frac{2q_0(\alpha t / \pi)^{1/2}}{k} \exp\left(-\frac{z'^2}{4\alpha t}\right) - \frac{q_0z'}{k} \operatorname{erf}(\frac{z'}{2\sqrt{\alpha t}})
\]

where \( z' \) is the distance from the front surface of the sample along the z-axis. For a location at the surface (\( z' = 0 \) mm), this relationship becomes:

\[
T(0, t) = T_\infty + \frac{2q_0(\alpha / \pi)^{1/2}}{k} t^{1/2} = T_\infty + A_1 t^{1/2}
\]

where \( A_1 \) is a parameter \( (\frac{2q_0(\alpha / \pi)^{1/2}}{k}) \) depending on the material thermo-physical properties and laser beam heat flux.

The above equation can be used to fit the initial 10 s of thermal imaging data, which corresponds to the time the laser beam was turned on. For example, the plots in Figure 135 for the soot-coated sample represent such data. To fit Equation 59 using least-squares technique [180], the parameter \( A_1 \) can be estimated by the following equation:

\[
\hat{A}_1 = \frac{\sum T_i t_i^{1/2}}{\sum t_i}
\]

Selected curves in Figure 135 for several points along the y-axis are shown in Figure 137 together with the fitted curves. Note that the data in the range \([0.9, 10]\) s was used for the fit in order to avoid the initial transient. It can be seen that the fit is very good given that the same relationship is fitted to all curves, with only parameter \( A_1 \) being
To estimate the heating rate, it is assumed that the material surface is being heated by a constant heat flux, that the surface is part of a semi-infinite solid, and that there is no convection heat loss at the surface. Given these assumptions, an analytical relationship can be derived describing the 1D transient-temperature distribution as a function of the distance from the sample surface [177]:

$$T(z', t) = T_\infty + \frac{2q_0 (\alpha / \pi)^{1/2}}{k} \exp\left(\frac{-z'^2}{4\alpha t}\right) - \frac{q_0 z'}{k} \text{erfc}\left(\frac{z'}{2\sqrt{\alpha t}}\right)$$  \hspace{1cm} (58)

where $z'$ is the distance from the front surface of the sample along the z-axis. For a location at the surface ($z' = 0$ mm), this relationship becomes:

$$T(0, t) = T_\infty + \frac{2q_0 (\alpha / \pi)^{1/2}}{k} t^{1/2} = T_\infty + A_1 t^{1/2}$$  \hspace{1cm} (59)

where $A_1$ is a parameter ($= \frac{2q_0 (\alpha / \pi)^{1/2}}{k}$) depending on the material thermo-physical properties and laser beam heat flux.

The above equation can be used to fit the initial 10 s of thermal imaging data, which corresponds to the time the laser beam was turned on. For example, the plots in Figure 135 for the soot-coated sample represent such data. To fit Equation 59 using least-squares technique [180], the parameter $A_1$ can be estimated by the following equation:

$$\hat{A}_1 = \frac{\sum T_i t_i^{1/2}}{\sum t_i}$$  \hspace{1cm} (60)

Selected curves in Figure 135 for several points along the y-axis are shown in Figure 137 together with the fitted curves. Note that the data in the range [0.9, 10] s was used for the fit in order to avoid the initial transient. It can be seen that the fit is very good given that the same relationship is fitted to all curves, with only parameter $A_1$ being
adjusted. This provides further evidence in support of the hypothesis that surface heating is taking place.

![Figure 137](image.png)

Figure 137- Transient temperatures for locations along the y-axis of the laser-transmitting part (at L1) and curve fits obtained by assuming surface heating (P = 1 W, t = 10 s).

Using Equation 59, the fitted parameter values $\hat{A}_1$ can be used to estimate the heat flux that would result in this surface heating rate. Using PA6 thermal properties given in Table 9, the heat flux was estimated by:

$$q_0 = \frac{\hat{A}_1 k}{2 (\alpha / \pi)^{1/2}}$$

(61)

The fitted parameter values $\hat{A}_1$ and the heat flux estimates for the curves shown in Figure 135 are given in Table 15 and plotted in Figure 138. The figure indicates that closer to the upper surface of the laser-transmitting part (y = 0), there is less heating, but within 0.5 mm of the surface, the heating rate rises quickly. The heating rate continues to increase more slowly as the distance from the upper surface increases. This supports the
idea that the beam entering the upper surface scatters more as it propagates towards the laser-absorbing part located at 3 mm. A slight drop in the heating rate near the interface could be attributed to decrease of scattered light energy, as it is partly absorbed at the interface with the laser-absorbing part.

Table 15- Estimates of parameter A₁ and surface heat generation

<table>
<thead>
<tr>
<th>y (mm)</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁ (°C/s(^{1/2}))</td>
<td>1.85</td>
<td>9.16</td>
<td>10.47</td>
<td>10.71</td>
<td>11.79</td>
<td>12.32</td>
<td>11.82</td>
</tr>
<tr>
<td>q₀ (W/m(^2))</td>
<td>1101</td>
<td>5458</td>
<td>6240</td>
<td>6379</td>
<td>7024</td>
<td>7341</td>
<td>7040</td>
</tr>
</tbody>
</table>

Figure 138- Surface heating rate estimate vs. depth of the sample (P = 1 W).

7.2 Experimental Results for the Moving Laser Beam

This section presents the thermal imaging experimental results for the moving laser beam for different materials, laser-scanning speeds, and power levels as per the experimental matrix presented in Table 5. Note that the surface of the plastic parts facing the thermal camera was not coated.
Also, note that the relative movement between the laser and the part is achieved by translation of the part under the laser beam (Figure 42 and Figure 43). The thermal camera is stationary with respect to the laser beam and is aimed so that the laser beam is offset by approximately one quarter of the image width from the right side of the image. The part is thus observed to move in the camera’s field of view from right to left. Once the part’s left edge moves out of view, the thermal image remains nearly unchanged (in a quasi-steady-state condition) until the right edge of the part passes through the image.

7.2.1 Thermal Imaging Experiments for the Semi-Crystalline Material

Figure 139 shows the weld lines obtained in the moving laser beam experiments with PA6 samples. The lines were observed on the laser-absorbing part’s joint surface after the laser-transmitting part was removed. The images were obtained by scanning on a flat-bed scanner at 1200 dpi resolution.

![Figure 139- Weld lines produced by the moving laser beam on PA6 samples.](image)

Table 16 shows the results from measurements of the weld lines shown in Figure 139. The weld width and line centre distance from the edge (identified as distance “a” in
Figure 42) were measured from the image at five points distributed evenly over the length of each line. A mean and standard deviation (SD) for the five measurements of each line are presented. The results indicate that the line width was not varying significantly through the length of the line and that the line was accurately positioned at 3 mm from the edge. Greater line width was obtained for the tests with higher line energy. For the case when a scan speed was decreased by half while keeping the same line energy (1.2 J/mm), a line width smaller by 0.33 mm was obtained when comparing Test M3 to average of Test M2 results. The difference is expected to be due to reduced temperature as the more heat is conducted from the weld with the slower scan speed.

<table>
<thead>
<tr>
<th>Item</th>
<th>P (W)</th>
<th>V (mm/s)</th>
<th>LE (J/mm)</th>
<th>Width_{mean} (mm)</th>
<th>Width_{SD} (mm)</th>
<th>a_{mean} (mm)</th>
<th>a_{SD} (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test M1</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>1.17</td>
<td>0.05</td>
<td>3.06</td>
<td>0.13</td>
</tr>
<tr>
<td>Test M2a</td>
<td>36</td>
<td>30</td>
<td>1.2</td>
<td>1.92</td>
<td>0.06</td>
<td>2.98</td>
<td>0.09</td>
</tr>
<tr>
<td>Test M2b</td>
<td>36</td>
<td>30</td>
<td>1.2</td>
<td>1.81</td>
<td>0.06</td>
<td>3.01</td>
<td>0.07</td>
</tr>
<tr>
<td>Test M3</td>
<td>18</td>
<td>15</td>
<td>1.2</td>
<td>1.54</td>
<td>0.06</td>
<td>2.95</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 140 shows the temperature contours for the PA6 samples observed with the thermal imaging camera for the laser beam power of 18 W and scanning speed of 30 mm/s. Line L1 indicates the location of the laser beam centre (which remained fixed with respect to the image throughout the test). In this figure, the temperatures have reached a quasi-steady-state condition. A bright line at the boundary between parts A and B is due to the heating by the laser at the part interface. The width of this line is seen to increase from right to left. This corresponds to conduction of the heat away from the weld interface. Note that, since the part is moving at 30 mm/s, it takes 1.1 s for a point to
move from the right edge to the left edge of the image. Thus, points at the left edge of the image have passed by the laser beam centre only 0.9 s earlier.

Figure 140- Thermal image for a moving laser beam (P = 18 W, V = 30 mm/s).

Figure 141 shows the temperature distributions along the vertical lines L4 to L7 shown in Figure 140. Temperature data in these plots have been smoothed out using a moving average with a period of two data points. The temperature reaches its peak inside the laser-absorbing part and close to the interface of the two parts. The temperature profile along line L4 shows the start of laser heating, with temperature being close to ambient at 25°C and the peak near the interface just starting to appear. Line L5 crosses the point where the overall surface A peak temperature of 34°C was reached. This line corresponds to the point where the heating due to the laser beam ends. Lines L6 and L7 show the decrease of peak temperature as cooling starts to the left of L5. Note that there is a slight decrease of average temperatures for points at y > 4 mm. This decrease becomes noticeable for L5 (about 0.5°C) and increases to about 1°C. Looking at the thermal image in Figure 140, it is clear that there is general cooling in the lower and
upper areas of the image close to the horizontal centre. It is believed that this is due to a thermal imaging artifact, which was discussed in Section 4.6.4.

![Temperature profile graph](image)

Figure 141- Temperature profiles along the depth of the plastic plaque obtained from the thermal imaging experiments (P = 18 W, V = 30 mm/s).

Figure 142 shows temperature contours obtained for the PA6 sample when the laser scanning speed was halved to 15 mm/s while maintaining the same power of 18 W. Line L1 in the figure shows the location of the laser-beam centre. The peak temperature increased by only a few degrees (from about 34°C for 30 mm/s to 38.5°C for 15 mm/s); location where peak temperature was observed is indicated in the figure.

Figure 143 shows temperature distribution along the horizontal line located 0.2 mm below the interface of parts A and B, where the overall peak temperature was observed. The laser centre location is indicated by the red arrow in the figure. The segment of each curve to the right of the peak corresponds to the temperature rise due to the laser beam exposure. Note that the laser heating effect is spread over a much greater length along the x direction (approximately 8 mm) than the width of the unscattered laser beam (under
1 mm). This indicates the extent of scattering taking place inside the laser-transmitting part. The part of the curve to the left of the peak corresponds to the material cooling after the end of the exposure to the laser beam heating.

![Thermal image for a moving laser beam](image)

**Figure 142-** Thermal image for a moving laser beam (P = 18 W, V = 15 mm/s).

![Temperature profile](image)

**Figure 143-** Temperature profile along the length of the plastic plaque obtained from the thermal imaging experiments (P = 18 W, V = 30 mm/s, y = 3.2 mm).
It is of interest to examine the segment of the profiles shown in Figure 143 where the laser is heating the part. It can be stated with certainty that the progressive rise in the temperature between the points at \( x = 33 \text{ mm} \) and \( x = 24 - 25 \text{ mm} \) is due to the heating by the laser beam. It can also be observed that the heating rate changes within the interval, starting near zero at the right, increasing to higher rate in the middle, and decreasing to zero at the left. This variation in the heating rate can be connected with the variation in the laser beam intensity profile. Assuming that the scattered laser beam is heating the laser-absorbing part as it moves under the laser, the rise in the temperature is the result of cumulative heating by the laser beam energy. It may thus be possible to extract the intensity distribution profile of this laser energy from the temperature vs. \( x \) location curves.

If the laser intensity profile is assumed Gaussian in nature, then the integral of this profile is a form of an “error function” (erf (x)). Figure 144 shows the result of fitting the following form of the error function to the curve segments described above.

\[
T(x^*) = a \, \text{erf}(b x^*) + c
\]  

(62)

where \( a, b, \) and \( c \) are fit parameters.

Note that, to aid in fitting, each curve segment was shifted along the \( x \)-axis so that the centre of the segment can correspond to zero. The zero location should correspond to the centre of the laser beam as it corresponds to the point where the highest heating rate is occurring. Table 17 shows the result of the fit for the three curves observed for PA6 with different process parameters (power and speed). Figure 144 and Table 17 indicate that the fit of the data to the error function is very good (with \( R^2 \) values approximately 0.99).
The fit parameter $b$ is related to the standard deviation ($\sigma$) in the Gaussian distribution

$$\sigma = \frac{-1}{b \sqrt{2}}$$

, with Gaussian distribution equation with mean equal to zero given by [217]:

$$n(x^*, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left[ \frac{x^*}{\sigma} \right]^2}$$

(63)

Figure 144- Temperature profile along the length of the plastic plaque obtained from the thermal imaging experiments (PA6, $P = 18$ W, $V = 30$ mm/s, $y = 3.2$ mm).

Table 17- The error function fitted parameters for the PA6 samples

<table>
<thead>
<tr>
<th>Item</th>
<th>P (W)</th>
<th>V (mm/s)</th>
<th>LE (J/mm)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test M1</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>3.689</td>
<td>-0.4278</td>
<td>30.40</td>
<td>1.65</td>
</tr>
<tr>
<td>Test M2</td>
<td>36</td>
<td>30</td>
<td>1.2</td>
<td>9.489</td>
<td>-0.3464</td>
<td>34.52</td>
<td>2.04</td>
</tr>
<tr>
<td>Test M3</td>
<td>18</td>
<td>15</td>
<td>1.2</td>
<td>5.200</td>
<td>-0.3887</td>
<td>32.01</td>
<td>1.82</td>
</tr>
</tbody>
</table>

Table 17 shows the standard deviation estimates obtained from the fits and used to produce the Gaussian distribution plots shown in Figure 145. The three curves show that they are similar to one another, with an average standard deviation of 1.84 mm. This
implies a beam width of $4 \times \sigma = 7.36$ mm [56]. This is notably wider than the dimension of the scattered laser beam used in the model. Figure 146 allows the comparison of the distributions. This result indicates that there is significantly more scattering taking place in the semi-crystalline laser-transmitting material (PA6) than was assumed in the model based on the available experimental measurements.

Figure 145- NPFD along the x-axis obtained from the thermal imaging experiments (PA6).

Figure 146- NPFD along the x-axis obtained from the thermal imaging experiments (PA6).
7.2.2 Thermal Imaging Experiments for the Amorphous Material

Experiments similar to those conducted for the semi-crystalline materials were also performed for the amorphous material (PC). PC samples in a lap-join configuration were exposed to a moving diode laser beam with a power of 18 W and a speed of 30 mm/s. The first set of experiments was conducted with the laser beam located 3 mm from the sample front face; for the second set of experiments, the laser beam was located 2 mm from the front face.

Figure 147 shows two welded PC samples, one with the weld line located 3 mm (Test M4) and the other with the weld line located 2 mm (Test M5) from the edge. The lines are visible on the joint interface without the laser-transmitting part being removed. Close up views of the weld line shown in the figure were taken from scanned images of the samples. Table 18 shows the results from measurements of the weld lines shown in the close-up images in Figure 147. The weld width and line centre distance from the edge (identified as distance “a” in Figure 42) were measured from the image at five points distributed evenly over the length of each line. A mean and standard deviation (SD) for the five measurements of each line are presented. An overall average line width of 1.47 mm with standard deviation of 0.044 mm was obtained for the 20 measurements taken from the four weld lines. The results indicate that the line width was not varying significantly through the length of the line and, based on an analysis of variance (ANOVA) test [217] of the data, there was no statistically significant difference between the line widths of the four lines ($\alpha = 0.05$). In addition, the results show that the lines were accurately positioned at 3 mm (Figure 147a) and 2 mm (Figure 147c) from the edge.
Table 18- Weld dimensions for the moving laser beam experiments on PC samples

<table>
<thead>
<tr>
<th>Item</th>
<th>P  (W)</th>
<th>V (mm/s)</th>
<th>LE (J/mm)</th>
<th>Width_{mean} (mm)</th>
<th>Width_{SD} (mm)</th>
<th>a_{mean} (mm)</th>
<th>a_{SD} (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test M4a</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>1.49</td>
<td>0.04</td>
<td>2.98</td>
<td>0.07</td>
</tr>
<tr>
<td>Test M4b</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>1.46</td>
<td>0.04</td>
<td>2.94</td>
<td>0.06</td>
</tr>
<tr>
<td>Test M5a</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>1.49</td>
<td>0.04</td>
<td>2.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Test M5b</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>1.44</td>
<td>0.05</td>
<td>1.94</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Figure 147- Welded PC samples (P = 18 W, V = 30 mm/s); a) sample photo of Test M4; b) scanned close-up image of Test M4 weld line; c) sample photo of Test M5; d) scanned close-up image of Test M5 weld line.

Figure 148a shows a thermal image of the laser beam scanning the PC sample when the beam centre was located 3 mm from the sample front face. A very low maximum temperature of 26.5°C was observed. Given the difference of less than two degrees compared to the ambient temperature, these observations would not be very useful for the purpose of thermal model validation.

For the next set of experiments, the laser beam centre was moved 1 mm closer to the front surface in order to raise the observed peak temperature. Figure 148b shows the thermal image for this experiment. The peak temperature of 46.7°C is observed at the front surface of the material, achieving about 20°C increase compared to the case of
3 mm distance from the sample front surface. Given the measured laser beam intensity
distribution along the z-axis in the focal plane (see Figure 69), locating the sample’s front
surface 2 mm from the beam centre places the surface within about 1.2 mm from the edge
of the beam.

Figure 149 shows the temperature profiles along the depth (y-axis) of the sample for
the cases where the centre of the laser beam is located at 3 mm and 2 mm from the
sample edge. Profiles along line L1 are shown. As expected, the maximum temperature
happens within the laser-absorbing part and very close to the interface of the parts.

Figure 148- Thermal image for the laser beam centre (P = 18 W, V = 30 mm/s): a) 3 mm from the
sample edge, b) 2 mm from the sample edge.

Figure 150 shows temperature distribution along the horizontal line located 0.2 mm
below the interface of the two PC parts being joined, where the overall peak temperature
was observed. The laser centre location is indicated by the red arrow in the figure. The
segment of the curve to the right of the peak value corresponds to heating by the laser
beam. The temperature rise in this case, however, is much faster than the one observed in
the PA6 sample. The difference can be attributed to the scattering of the laser beam.
Figure 149- Temperature profiles along the depth of the sample for different locations of the laser beam centre from the sample edge.

Figure 150- Temperature profile along the length of the plastic plaque obtained from the thermal imaging experiments (PC, P = 18 W, V = 30 mm/s, y = 3.2 mm).

A fit to the error function (integral of a Gaussian distribution) was also performed for the segment of the curve where the temperature is increasing from right to left. The points in this segment are plotted in Figure 151 together with the fitted curves using
Equation 63. Two curves are shown: each taken from a thermal imaging observation of a different PC sample carried out under the same experimental conditions (a = 2 mm, P = 18 W, V = 30 mm/s).

The fitted parameters are shown in Table 19. Again, a good fit was obtained with $R^2$ values of 0.97 and 0.99. Figure 152 shows the intensity distributions acquired based on the above fits. An average standard deviation of 0.14 mm was obtained, giving an estimate of the beam width ($4 \times 0.14 = 0.56$ mm). The modelled beam distribution is shown on the same figure for comparison. The model represents a beam of about 0.8 mm width. Note that the smaller width obtained indicates that only a narrow part of the elliptical beam cross section comes in contact with the laser-absorbing part at the front surface. Thus, the acquired width is reasonable given the known dimensions of the unscattered laser beam (1.4 mm $\times$ 0.8 mm).

Figure 151- Temperature profile along the length of the plastic plaque obtained from the thermal imaging experiments (PC, P = 18 W, V = 30 mm/s, y = 3.2 mm).
<table>
<thead>
<tr>
<th>Item</th>
<th>P (W)</th>
<th>V (mm/s)</th>
<th>LE (J/mm)</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replicate 1</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>8.572</td>
<td>-5.74</td>
<td>35.27</td>
<td>0.123</td>
</tr>
<tr>
<td>Replicate 2</td>
<td>18</td>
<td>30</td>
<td>0.6</td>
<td>7.955</td>
<td>-4.607</td>
<td>33.84</td>
<td>0.153</td>
</tr>
</tbody>
</table>

Figure 152- NPFD along the x-axis obtained from the thermal imaging experiments (PC, P = 18 W, V = 30 mm/s, y = 3.2 mm).
Chapter 8   Computational Results

This section presents results of the transient 3D thermal models of a lap-joint configuration for stationary (Section 8.1) and moving (Section 8.2) laser beam cases. The numerical results are compared with the experimental results from the weld line geometrical characterization and from the thermal imaging observations.

8.1 Sample Exposure to the Stationary Laser Beam

This section presents the results where exposure of a semi-crystalline material (PA6) to a stationary diode laser beam was modelled. The results are presented in two sections, each defined by how the laser beam was modelled: 1) unscattered (focused) laser beam and 2) scattered laser beam. In the first case, results of a simplified model ignoring the scattering are presented; in the second case, information about scattered beam profile is incorporated into the model so that beam profile varies from unscattered at the upper surface of the laser-transmitting part to scattered at the interface between the two parts being joined. The model results were compared first to the dimensions of the molten zone observed on the joint surface and second to the thermal imaging observations on the vertical surface facing the camera.

8.1.1 Model-1: Unscattered Stationary Laser Beam

For the results shown in this section, it was assumed that the laser beam energy was uniformly distributed within a rectangular area of the focused laser beam (1.4 × 0.7 mm²). This uniform beam distribution served as a first-order approximation of the unscattered laser beam distribution in the focal plane. The 3D lap-joint geometry in Figure 93 was exposed to a 1 W stationary diode laser beam for a 10 s duration. The 3D temperature
Contours at the end of the laser heating (at 10 s) are shown in Figure 153. Corresponding temperature contours on the symmetry plane of the geometry \((x = 0 \text{ mm})\) are shown in Figure 154. The highest temperature of 345°C was predicted at the symmetry plane and at 0.2 mm below the interface \((y = 3.2 \text{ mm})\). If 217°C melting point for PA6 is assumed, the red area in the contour plot shows the predicted molten area shape.

Figure 153- Temperature contours at the end of the heating phase (PA6, Model-1, \(t = 10 \text{ s}, \quad P = 1 \text{ W}, \quad V = 0 \text{ mm/s}, \quad \tau = 51\%, \quad K = 4.8 \text{ mm}^{-1}\)).

Figure 154- Temperature contours at the symmetry \((y-z)\) plane at the end of the heating phase (PA6, Model-1, \(x = 0 \text{ mm}, \quad t = 10 \text{ s}, \quad P = 1 \text{ W}, \quad V = 0 \text{ mm/s}, \quad \tau = 51\%, \quad K = 4.8 \text{ mm}^{-1}\)).
Thus, the model predicts that some melting should take place at the interface due to laser heating. This result was supported by the experimental observation of the weld interface, which indicated that a molten area was created when the joint was exposed to a stationary laser beam for the modelled geometry and process parameters (Figure 131).

Figure 155 shows the temperature distribution versus time at the centre of the laser beam \((z = 9 \text{ mm})\) for different locations along the \(y\) direction. The colour-coded diagram in the top right corner shows the locations of the points for which temperatures are plotted. The temperature rises up to 345°C at 10 s and then decreases quickly at the joint interface (where the heat generation peaks). Temperature at the upper surface \((y = 0 \text{ mm})\) peaks at 162°C while temperature at 3 mm below the joint interface remains below 38°C throughout the modelled time interval. From Figure 155, the melting at the joint interface starts after 1.8 s and continues up to 10.8 s, assuming a 217°C melting point.

Figure 155- Predicted temperature vs. time along the \(y\) direction of the geometry (PA6, Model-1, \(x = 0 \text{ mm}, z = 9 \text{ mm}, \) centre of the laser beam, \(P = 1 \text{ W}, V = 0 \text{ mm/s}, \tau = 51\%, K = 4.8 \text{ mm}^{-1}\)).

Figure 156 shows spatial temperature distributions along the \(x, y,\) and \(z\)-axes at 10 s. The distributions are shown for the lines passing through the laser beam centre and at the horizontal plane where the peak temperature of 345°C was observed \(([x, y, z] = [0, 0, 9])\).
3.2, 9 mm). The plot along the y-axis shows that the peak temperature is reached at y = 3.2 mm. This corresponds to 0.2 mm from the interface, inside the laser-absorbing part. The plot along the z-axis shows how the temperature varies between the surface facing the thermal camera (42°C at z = 12 mm) and the point where peak temperature was reached and the beam centre was located (345°C at z = 9 mm).

Using the data in these plots and by assuming a particular melting point, model-predicted molten-zone dimensions along each of the three axes can be calculated. Assuming 217°C, the weld dimensions are [x = 0.8 mm, y = 1 mm, z = 1.35 mm]. The molten spot at the joint interface visible in Figure 131 has the x × z dimensions of 1.0 × 1.5 mm², which are close to those predicted by this thermal model.

Figure 156- Predicted temperature distribution along the x, y, and z-axes (PA6, Model-1, centre of the laser beam, t = 10 s, P = 1 W, V = 0 mm/s, τ = 51%, K = 4.8 mm⁻¹).

For the prediction of the molten zone dimensions from the model, it was assumed that there was a visible change in the plastic sample caused by melting at a particular temperature and subsequent resolidification. As described in Section 5.4 and shown by
the DSC tests (Appendix B.1.1), melting of PA6 takes place over a range of temperatures (200°C - 220°C). Thus, the temperature at which a visible change in the polymer will occur during laser welding is not known precisely. Data presented in Figure 157 show the effect of varying the assumed melting temperature value on the predicted molten zone dimensions. The plots show the weld dimensions along the x, y, and z-axes as a function of the assumed melting temperature. In addition, the molten zone cross section area in the x-z plane is plotted. The area is calculated by assuming elliptical shape and multiplying the major and minor radii of the ellipse by $\pi$ ($\approx 3.1415$). As expected, as the assumed melting temperature increases, all the weld dimensions decrease. The dotted lines are the linear fits to the weld dimension data. For comparison, the molten zone area observed experimentally (Figure 131) is 1.18 mm$^2$. Thus, the model slightly under-predicts the experimentally observed value.

![Figure 157](image_url)

Figure 157- Predicted molten zone dimensions as a function of the melting temperature (PA6, Model-1, $P = 1$ W, $V = 0$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).
Figure 158 compares the temperatures predicted by the model with those obtained from the thermal imaging observations presented in Figure 133 and described in Section 7.1). The camera was directed at the surface referred to as “surface A” in Figure 94 and the sample surface was uncoated. Figure 158 shows temperatures as a function of time for several points along the y direction at the symmetry plane (x = 0 mm) and on the surface A (z = 12 mm).

![Graph showing predicted temperature vs. time from thermal imaging experiments at different locations along the y direction (PA6, Model-1, x = 0 mm, z = 12 mm, surface A, P = 1 W, V = 0 mm/s, τ = 51%, K = 4.8 mm⁻¹).](image)

Note that points along the y-axis where 0 < y < 3 mm are within the laser-transmitting part (part A), while points where y > 3 mm are within the laser-absorbing part (part B). It is seen that the model does not predict the nearly linear temperature rise to 70°C observed slightly below the interface of the parts A and B (where the peak
heating was observed in thermal imaging). The model predicts a more gradual rise up to a maximum of about 48°C. The thermal camera results show a sudden slope change when the laser is turned off at 10 s. One possible explanation is that the scattered laser beam light is reaching the viewed surface and heating the laser-absorbing part. The unscattered laser beam model does not account for this effect. There is also a possibility that the thermal camera is seeing into the gap between the two parts.

At y = 0 mm (at the upper surface of part A), thermal camera results show very little temperature rise (below 33°C); on the other hand, the model predicts a temperature rise up to 50°C and a similar curve trend for other points within part A. It is likely that less absorption is taking place in the laser-transmitting part than assumed by the model. The best agreement between the model and the thermal imaging observations is obtained for y = 6 mm. The two curves match each other within 1°C. At this point, the primary source of heat at the interface of parts A and B (Figure 95) is far enough that any effects due to beam scattering not being modelled are not significant.

Data shown in Figure 159 examines the sensitivity of the temperature on the surface A to the distance of the laser beam centre from this surface (distance “a”). The figure shows the temperature variation for three different “a” distances. The surface temperature is expected to increase as the laser is moved closer (shorter distance “a” in Figure 93). Figure 159 clearly shows how sensitive the surface A temperature is to the position of the laser. When the distance was increased from 2 mm to 4 mm, the model-predicted peak face temperature decreased from 73°C to 37°C.
Figure 159- Predicted temperature vs. time for different distances of the laser beam centre from the front surface (PA6, Model-1, x = 0 mm, y = 3.2 mm, z = 12 mm, surface A, P = 1 W, V = 0 mm/s, \( \tau = 51\% \), K = 4.8 mm\(^{-1}\)).

8.1.2 Model-2: Scattered Laser Beam

Scattering is an important factor in the LTW process of PA6. Scattering causes the beam to widen as it passes through the laser-transmitting part. Therefore, the beam hitting the interface of the laser-transmitting and absorbing parts (parts A and B in Figure 95) is spread over a wider area than the unscattered beam. To more accurately model this effect, a scattered laser beam distribution was introduced for the modelling studies presented in this section as discussed in Section 6.5. This scattered laser beam distribution was obtained in the experiments described in Section 5.2.

The thermal model presented in this section assumed the part was exposed to a diode laser beam whose power flux distribution changed from the unscattered shape at the upper surface of the laser-transmitting part to the scattered shape at the interface with the laser-absorbing part. Details of the modelling approach are given in Section 6.4.1.
The 3D temperature contours at the end of the laser heating (at 10 s) are shown in Figure 160. The contours indicate location of the beam close to the surface A (which is observed by the thermal imaging camera). Figure 161 shows the cross section through the weld zone centre (at x = 0) along the z-y (vertical) plane. Assuming a 217°C melting temperature for PA6, the red area shows the predicted molten zone shape. The cross section has more elongated shape than that of the unscattered laser beam presented in the previous section (Figure 154). The shape also more closely resembles the weld zone cross sections reported elsewhere for the scanning laser beam (Figure 27) [117].

Figure 160- Temperature contours at the end of the heating phase (PA6, Model-2, t = 10 s, $P = 1$ W, $V = 0$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).
Figure 161- Temperature contours at the symmetry (y-z) plane at the end of the heating phase (PA6, Model-2, x = 0 mm, t = 10 s, P = 1 W, V = 0 mm/s, τ = 51%, K = 4.8 mm\(^{-1}\)).

Figure 162 shows the temperature distribution at 10 s along the x and z directions in the plane at y = 3.2 mm where the peak temperature of 305\(^\circ\)C was reached (near the interface of parts A and B). Assuming melting point of 217\(^\circ\)C, the predicted molten zone dimension in the horizontal plane along the z direction is 1.23 mm and along the x direction, the half-width dimension is 0.29 mm (or 0.58 mm full width). Compared with the experimental molten zone dimensions shown in Figure 131 (1.0 \(\times\) 1.5 mm\(^2\)), the model predicts lower values along both dimensions. The values are also lower than those of the unscattered laser beam model presented in the previous section. It is likely that spreading the same power input over the larger area has resulted in a smaller molten zone.
Figure 162- Predicted temperature distribution along the x, y, and z-axes (PA6, Model-2, centre of the laser beam, t = 10 s, P = 1 W, V = 0 mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).

Data presented in Figure 163 show the effect of varying the assumed melting temperature value on the predicted molten zone dimensions. The plots show the weld dimensions along the x, y, and z-axes as a function of the assumed melting temperature. In addition, the molten zone cross section area in the x-z plane is plotted. The area is calculated by assuming elliptical shape for the molten zone and multiplying the major and minor radii of the ellipse by $\pi$ ($\approx 3.1415$). As expected, as the assumed melting temperature increases, all the weld dimensions decrease. The dotted lines are the linear fits to the weld dimension data. For comparison, the molten zone area observed experimentally (Figure 131) is 1.18 mm$^2$. Thus, the model under-predicts the experimentally observed value.
Figure 163- Predicted molten zone dimensions as a function of the melting temperature (PA6, Model-2, P = 1 W, V = 0 mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).

Figure 164 and Figure 165 compare the transient temperatures predicted by thermal modelling with those observed by thermal imaging for different locations along the $y$ direction on the surface A (Figure 94) for the laser-absorbing (Figure 164) and laser-transmitting (Figure 165) parts. The points are located along the symmetry plane ($x = 0$). The sample surface was uncoated for the imaging results used here.

For points on the laser-absorbing part surface (Figure 164), beyond the point where maximum temperature was reached ($y > 3.2$ mm), the agreement between the model and the experiment is very good. During the time the laser is on ($t < 10$ s), the agreement for all three curves shown is within 1°C. For $10 \, s < t < 20$ s, when the sample is cooling, deviation increases to a maximum of 2.2°C around $t = 18$ s for $y = 4$ mm and 4.5 mm. At $y = 6$ mm, the model and the experiment agree to within 1°C throughout. Similar good agreement at this location was obtained for the unscattered laser beam model (Model-1).
At the point on surface A where maximum temperature is reached ($y = 3.2 \text{ mm}$), the model predicts peak temperature ($67^\circ\text{C}$), which is close to that observed in thermal imaging ($70^\circ\text{C}$). This deviation is much less than the one observed for the unscattered laser beam model (Model-1). In addition, the curve shape is much closer to the imaging observation. This indicates that scattering of the laser light brings it close to surface A and thus creates a heat source close to that surface. This results in a sharp change in slope when the laser beam is turned off at 10 s, which is now observed in the model and confirmed by the experiment.

![Figure 164](image-url)  
*Figure 164* - A comparison between the transient temperature profiles obtained from the thermal imaging and modelling along the depth of the laser-absorbing part (PA6, Model-2, $z = 9 \text{ mm}$, $P = 1 \text{ W}$, $V = 0 \text{ mm/s}$, $\tau = 51\%$, $K = 4.8 \text{ mm}^{-1}$).

On the laser-transmitting part’s front surface (Figure 165) ($0 < y < 3 \text{ mm}$), the model and the experimental values deviate significantly at all points along $y$. For
example, at $y = 2\, \text{mm}$, the model predicts temperature rise up to a peak temperature of 56.8°C at 10 s, while the experimental observation shows rise to only 40.8°C at the same time. Experimental observation shows temperature continuing to rise after the laser is turned off at 10 s. This indicates that the source of heat contributing to this temperature rise is located away from this front surface. This observation supports the hypothesis that little laser light absorption takes place inside the laser-transmitting part and that most absorption occurs at the interface of parts A and B.

Figure 165- A comparison between the transient temperature profiles obtained from the thermal imaging and the modelling along the depth of the laser-transmitting part (PA6, Model-2, $z = 9\, \text{mm}$, $P = 1\, \text{W}$, $V = 0\, \text{mm/s}$, $\tau = 51\%$, $K = 4.8\, \text{mm}^{-1}$).

Figure 166 compares the predicted spatial temperature distribution and the thermal imaging observations along the $y$-axis. It can be seen that the temperatures within the laser-absorbing part are very close for the model and the experiment; the temperatures in
the laser-transmitting part are higher by 14°C to 17°C for points between 0 and 2.5 mm along y. The latter difference is likely due to too much absorption being incorporated into the model of the laser-transmitting material, which causes temperature rise in the laser-transmitting part not observed in the experiments.

![Figure 166](image)

Figure 166- A comparison between the predicted temperature profile along the y-axis and that of the thermal imaging observations (PA6, z = 9 mm, t = 10 s, P = 1 W, V = 0 mm/s, τ = 51%, K = 4.8 mm⁻¹).

Table 20 summarizes the results presented for the two models and compares them with the experimental results. It shows that the scattered-beam model (Model-2) predicts better the peak temperature at the surface observed by the thermal camera. The table shows that the overall peak temperature predicted by Model-2 is lower by 40°C than that predicted by Model-1 (305°C vs. 345°C). Distribution of the energy over a greater area due to incorporation of scattering into the model accounts for this. Similarly, the molten zone dimensions are decreased for Model-2 compared to Model-1 due to the broadening
of the laser beam distribution’s tails. This transfers energy from the beam centre to the tails, thus resulting in smaller molten zone.

Both models under-predicted the experimentally observed molten zone dimensions at the joint interface. It is possible that the under-prediction could be explained by low joining pressure between the two parts during the experiment. The parts were pre-joined by two line scans but they were clamped only from one side (to allow for the laser beam to reach the upper surface and to allow for thermal imaging observation from the opposite side). If there were increased contact resistance between the two parts due to low joining pressure, the laser-absorbing part’s upper surface might have reached higher temperature than predicted by the model.

Table 20- Summary of the computational results for the PA6 sample exposure to the stationary laser beam (assumed melting temperature of 200°C - 230°C, P = 1 W, V = 0 mm/s, τ = 51%, K = 4.8 mm⁻¹)

<table>
<thead>
<tr>
<th>Property</th>
<th>Model-1 Unscattered</th>
<th>Model-2 Scattered</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molten zone dimension (x-axis) mm</td>
<td>0.91 - 0.76</td>
<td>0.67 - 0.53</td>
<td>1.0</td>
</tr>
<tr>
<td>Molten zone dimension (y-axis) mm</td>
<td>1.14 - 0.87</td>
<td>0.87 - 0.57</td>
<td>N/A</td>
</tr>
<tr>
<td>Molten zone dimension (z-axis) mm</td>
<td>1.42 - 1.29</td>
<td>1.41 - 1.11</td>
<td>1.5</td>
</tr>
<tr>
<td>Molten zone area (xz-plane) mm²</td>
<td>1.02 - 0.77</td>
<td>0.75 - 0.47</td>
<td>1.18</td>
</tr>
<tr>
<td>Molten zone area (yz-plane) mm²</td>
<td>1.28 - 0.88</td>
<td>0.97 - 0.49</td>
<td>N/A</td>
</tr>
<tr>
<td>Molten zone volume (xyz-volume) mm³</td>
<td>0.77 - 0.44</td>
<td>0.43 - 0.18</td>
<td>N/A</td>
</tr>
<tr>
<td>Peak temperature (overall) (°C)</td>
<td>345</td>
<td>305</td>
<td>N/A</td>
</tr>
<tr>
<td>Peak temperature (surface A) (°C)</td>
<td>50</td>
<td>67</td>
<td>70</td>
</tr>
</tbody>
</table>

8.2 Sample Exposure to the Moving Laser Beam

This section presents the results of modelling the exposure of semi-crystalline (e.g., PA6) and amorphous (e.g., PC) plastic materials to a moving laser beam. Details of the
implementation of the beam profile in the moving-laser-beam model are given in Section 6.5.4.

8.2.1 Thermal Model for Semi-Crystalline Materials

This section presents the results of thermal model where a semi-crystalline plastic (PA6) was exposed to a moving diode laser beam for different processing parameters (e.g., laser scanning speed and power level). The laser beam profile was assumed to be unscattered as it entered and scattered as it exited the laser-transmitting part.

8.2.1.1 Low Line Energy Case (P = 18 W, V = 30 mm/s)

The 3D lap-joint geometry in Figure 93 was exposed to an 18 W diode laser beam scanning the sample at 30 mm/s. Figure 167 displays temperature contours on the horizontal plane located 0.2 mm below the joint interface (y = 3.2 mm) where the peak temperature along y (214.2°C) was predicted. The highest temperature range (red colour) corresponds to the range of 200°C to 250°C and thus approximates the extent of the heat-affected zone (HAZ). Note that in the welding tests carried out during thermal imaging observations, an easily broken joint was formed using the above process parameters.

Figure 168 shows the temperature contours along the vertical (x-y) plane passing through the beam centre, and indicates the maximum vertical extent of the heat-affected zone. Most of the melting takes place in the laser-absorbing part. The length of the molten volume is 1.6 mm, assuming a melting temperature of 200°C.
Figure 167- Temperature contours on the horizontal x-z plane (PA6, y = 3.2 mm, t = 0.3 s, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹).

Figure 168- Temperature contours on the vertical x-y plane (PA6, z = 10 mm, t = 0.3 s, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹).
Figure 169 shows temperature contours in the vertical (z-y) plane, transverse to the beam scan direction. The plot indicates that the thickness of the molten zone diminishes from centre to the outer edges.

Figure 169- Temperature contours on vertical y-z plane (PA6, \( x = 10 \) mm, \( t = 0.3 \) s, \( P = 18 \) W, \( V = 30 \) mm/s, \( \tau = 51\% \), \( K = 4.8 \text{ mm}^{-1} \)).

Figure 170 shows transient temperature for the location in y-z plane where the peak temperature is reached and for particular value of x: \([x, y, z] = [10, 3.2, 6] \text{ mm}\). The maximum temperature of 214.2°C is obtained at a location that is 0.2 mm below the interface.
Figure 170- Predicted temperature vs. time (PA6, x = 10 mm, y = 3.2 mm, z = 6 mm, P = 18 W, 
V = 30 mm/s, τ = 51%, K = 4.8 mm\(^{-1}\)).

Spatial temperature profiles along the x direction are shown in Figure 171 for different times. The line along which the temperature profile is shown passes through the point where peak temperature was observed: \([y, z] = [3.2, 6]\) mm. This point is located at 0.2 mm depth inside the laser-absorbing part and at the centre of the laser beam path. The plot shows that, as the laser beam moves along the x direction, the same peak temperature is predicted for all points along the x direction.
Figure 171- Predicted temperature distributions along the x direction for different times (PA6, \(y = 3.2\) mm, \(z = 9\) mm, \(P = 18\) W, \(V = 30\) mm/s, \(\tau = 51\%\), \(K = 4.8\) mm\(^{-1}\)).

### 8.2.1.2 Effect of Material’s Optical Properties (\(P = 18\) W, \(V = 30\) mm/s)

In this section, the effect of two important material properties on the model predictions is examined. The first property is the absorption coefficient of the laser-absorbing part (\(K\)) and the second property is the transmission of the laser-transmitting part (\(\tau\)).

Figure 172 to Figure 174 show the temperature contours and weld zones predicted after assuming different absorption coefficient values (\(K = 4.8\) mm\(^{-1}\), 9.6 mm\(^{-1}\), and 14.8 mm\(^{-1}\)) and with the transmission of the laser-transmitting part set to \(\tau = 51\%\). The length of the molten zone in the scanning direction (along the x-axis) increases from 1.6 mm to 7.3 mm with the absorption coefficient increase to 14.8 mm\(^{-1}\) (when 200°C melting temperature is assumed).
Figure 172- Temperature contours obtained from the thermal modelling (PA6, P = 18 W, V = 30 mm/s, τ = 51%, K = 4.8 mm⁻¹), a) x-y plane, z = 6 mm, b) y-z plane, x = 10 mm, c) x-z plane, y = 3.2 mm.

Figure 173- Temperature contours obtained from the thermal modelling (PA6, P = 18 W, V = 30 mm/s, τ = 51%, K = 9.6 mm⁻¹), a) x-y plane, z = 6 mm, b) y-z plane, x = 10 mm, c) x-z plane, y = 3.2 mm.

Figure 174- Temperature contours obtained from the thermal modelling (PA6, P = 18 W, V = 30 mm/s, τ = 51%, K = 14.8 mm⁻¹), a) x-y plane, z = 6 mm, b) y-z plane, x = 10 mm, c) x-z plane, y = 3.2 mm.

Figure 175 zooms in on the molten areas obtained for the three absorption coefficient values using the assumed melting temperature of 200ºC. As the absorption
coefficient is raised, the laser beam penetration depth decreases and most of the laser energy is absorbed in a progressively thinner layer of the laser-absorbing part. This results in higher temperature being reached at the interface. The effect is increase of the molten zone width since the non-uniform laser beam energy distribution leads to progressively wider area reaching the melting point.

The molten areas were measured for each of the absorption coefficients tried. In order to estimate the area, the central regions shown in red were assumed to have an elliptical shape. The molten area was calculated by measuring the horizontal and vertical dimensions of the region. These dimensions were then used as the major and minor diameters of an ellipse to calculate the molten area.

![Molten zone profiles for the absorption coefficients (PA6, x = 10 mm, P = 18 W, V = 30 mm/s, τ = 51%): a) 4.8 mm⁻¹, b) 9.6 mm⁻¹, c) 14.8 mm⁻¹.](image)

Figure 175- Molten zone profiles for the absorption coefficients (PA6, x = 10 mm, P = 18 W, V = 30 mm/s, τ = 51%): a) 4.8 mm⁻¹, b) 9.6 mm⁻¹, c) 14.8 mm⁻¹.

Figure 176 shows the molten zone depth (y), width (z), and area as a function of absorption coefficient for the assumed melting temperature of 200ºC. With increasing absorption coefficient, the weld zone width increases from 0.47 mm to 1.26 mm while the depth increases from 0.12 mm to 0.17 mm. The molten area increases from 0.13 mm² to 0.17 mm². The predicted weld line widths can be compared with the experimental weld line width measurements reported in Table 16 (Section 7.2.1). Average width of 1.17 ± 0.05 mm was found. The width predicted for the lowest absorption coefficient is only 40% of the experimental width. It is possible that the larger experimental weld line
width is due to the low joint pressure (weld pressure was estimated to be 0.7 MPa) when
the samples were clamped by their edge in order to allow thermal imaging observations.
Higher pressure should improve conduction from the laser-absorbing to transmitting part
and thus reduce the temperature rise and consequently the weld line width.

Figure 176- Predicted molten zone dimensions vs. the absorption coefficient (PA6, P = 18 W,
V = 30 mm/s).

The predicted molten zone shape can be compared with the weld cross section
micro-structure obtained by Al-Wohoush et al. [117] for the joints made on the same laser
system and similar materials (Figure 27). The image shown in Figure 27 is for a higher
line energy than that used in the model (2.8 J/mm vs. 0.6 J/mm) and thus results in a
wider and deeper weld. The aspect ratio and general shape of the experimental cross
section is closest to the prediction for the lowest absorption coefficient value tried
(4.8 mm⁻¹).
The predicted molten zone area can also be compared with the results reported by Al-Wohoush et al. [117] (Figure 28). Extrapolating the linear trend shown in the figure to 0.6 J/mm line energy for the modelled experimental conditions gives a value of 0.007 mm$^2$, which is lower than the 0.04 mm$^2$ predicted by the model for the smallest absorption coefficient value tried.

Transmission of 51% was used to generate the model results reported above based on the earlier tests carried out by this research group [119]. However, there is evidence that higher transmission for the same material and specimen thickness is possible. Kagan and his colleagues reported in several papers transmission of 69% for natural PA6 plaques of 3.2-mm thickness [23, 71, 73].

Figure 177 shows the temperature contours for transmission of 69% and absorption of 4.8 mm$^{-1}$. Figure 178 shows close-up views of the molten area for the two transmission values used. A depth of 0.18 mm and width of 0.93 mm of the molten zone were obtained, which result in 0.13 mm$^2$ molten zone area (for 200°C assumed melting temperature) (Figure 176).

Figure 177- Temperature contours predicted by thermal modelling (PA6, P = 18 W, V = 30 mm/s, $\tau = 69\%$, $K = 4.8$ mm$^{-1}$), a) x-y plane, $z = 6$ mm, b) y-z plane, $x = 10$ mm, c) x-z plane, $y = 3.2$ mm.
Figure 178- Molten zone profiles for two different transmission values (PA6, x = 10 mm, P = 18 W, V = 30 mm/s, K = 4.8 mm⁻¹): a) τ = 51%, b) τ = 69%.

Figure 179 shows the temperature profiles on the front surface (surface A) along the depth of the plastic sample obtained from the thermal imaging and modelling. The thermal modelling results are presented for different absorption coefficients assuming a transmission of 51% for the laser-transmitting part as well as a case where the transmission was assumed 69% for the absorption coefficient of 4.8 mm⁻¹. Note that the peak temperature increases from 31.6°C to 36.8°C as the absorption coefficient increases. For the same absorption coefficient (4.8 mm⁻¹), increasing transmission from 51% to 69% also leads to increase in peak temperature from 31.6°C to 33.7°C as more laser energy is reaching the joint interface. Figure 180 expands the y-axis scale of Figure 179 to zoom in on the joint interface and thus make the comparison easier. It can be clearly seen how the temperature peak spreads as the absorption coefficient decreases.
Figure 179- A comparison between the transient temperature profiles obtained from thermal imaging experiments and modelling (PA6, $P = 18$ W, $V = 30$ mm/s).

Looking at the results obtained from thermal imaging and thermal modelling, one can present the following interpretations:
Increasing the absorption coefficient (for a constant transmission of 51%) results in increasing peak temperature. For the absorption coefficient of 9.6 mm\(^{-1}\), the modelling and thermal imaging results show the best agreement (in terms of the peak temperature value). However, the shape of the curves differs. Higher coefficient produces sharper peak than that observed in the experiments.

Increasing the transmission to 69% and assuming that the absorption coefficient is 4.8 mm\(^{-1}\), also leads to a close match with the observed peak temperature. Therefore, one can suggest that the possibility of higher transmission for the laser-transmitting part is likely as reported in the literature [23].

On the other hand, if the side scatter of the laser beam near the weld interface causes false elevated peak temperature readings, the model with combination of 51% transmission and 4.8 mm\(^{-1}\) absorption coefficient matches the thermal imaging results better for points deeper within the laser-absorbing part (y > 3.6 mm, Figure 180).

Figure 181 uses the peak temperatures presented in Figure 180 and presents these temperatures as a function of the absorption coefficient. The temperature values show an increase from the ambient temperature of 25°C to 31.6°C for the lowest up to 36.8°C for the highest absorption coefficient value. A dashed horizontal line shows the peak temperature obtained from thermal imaging for comparison. For the absorption coefficient of zero, no increase from the ambient temperature would be expected. Thus, a point at [0, 25] was added to the plot. The dashed line joining this point to the model-predicted values is extrapolated to predict the peak temperature for lower absorption coefficient values. The relationship shows a smoothly increasing trend for the peak temperature as a function of the absorption coefficient.
8.2.1.3 High Line Energy and High Speed Case (P = 36 W, V = 30 mm/s)

The 3D lap-joint geometry in Figure 93 was exposed to a 36 W diode laser beam scanning the sample at 30 mm/s. This combination of power and speed gives line energy twice that for the results reported in the previous section (1.2 vs. 0.6 J/mm). Figure 182 shows the temperature contours obtained from the thermal modelling, with the peak temperature of 365.4°C predicted. The red regions in the diagram designate the heat-affected zone (where temperature over 200°C was predicted). The shape of the heat-affected zone for the z-y plane (weld line cross section) is nearly elliptical.

Predicted weld dimensions change depending on the assumed melting temperature. Figure 183 shows the temperature contours along the weld cross section obtained from the thermal modelling. Molten zones were obtained by assuming 200°C and 230°C melting temperatures and are indicated by the red area. Using similar plots for 200°C,
210°C, 217°C, and 230°C, and employing an image analysis software, one can obtain the relation between the weld dimension and the assumed melting temperature value (Figure 184). Linear fits matched these results well ($R^2 > 0.95$). The predicted weld width ranged from 1.04 mm to 1.34 mm. For comparison, the average experimental weld width reported in Table 16 for the same process conditions was $1.87 \pm 0.06$ mm. Figure 183 also shows by black dotted lines contours of the experimental weld cross section (PA6, $P = 70$ W, $V = 25$ mm/s) obtained from Figure 27 [117]. The experimental contour dimensions are matched in scale to the model output image. The contours are very similar in shape, but the experimental contour is larger in size, as expected, given the higher line energy of 2.8 J/mm (vs. 1.2 J/mm for the modelled conditions).

![Figure 182- Temperature contours predicted by the thermal modelling (PA6, $P = 36$ W, $V = 30$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$), a) x-y plane, $z = 6$ mm, b) y-z plane, $x = 10$ mm, c) x-z plane, $y = 3.2$ mm.](image)

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Figure 183- Molten zone profiles for different assumed melting temperature values (PA6, P = 36 W, V = 30 mm/s, τ = 51%, K = 4.8 mm$^{-1}$, black dotted lines show experimental weld cross sections: a) $T_m = 200^\circ C$, b) $T_m = 230^\circ C$.

The predicted molten zone area in Figure 184 can be compared with the results reported by Al-Wohoush et al. [117]. Extrapolating the linear trend shown in Figure 28 to 1.2 J/mm line energy for the modelled experimental conditions gives a value of 0.06 mm$^2$. The molten zone areas predicted by the model range from 0.082 mm$^2$ for the highest assumed melting temperature to 0.342 mm$^2$ for the lowest temperature.

Figure 184- Predicted molten zone dimensions vs. the melting temperature (PA6, P = 36 W, V = 30 mm/s, τ = 51%, K = 4.8 mm$^{-1}$).
Figure 185 compares the temperature profile along the depth of the sample (y-axis) obtained from thermal imaging experiments and thermal modelling. The peak temperature from the thermal imaging readings is about 7.3°C higher than that of the thermal modelling. Note that for 4.8 mm\(^{-1}\) absorption coefficient and lower line energy case, thermal imaging also showed higher peak temperature values versus predicted ones. Only by increasing the transmission or the absorption coefficient better match was obtained.

![Temperature profile comparison](image)

Figure 185- A comparison between the transient temperature profiles obtained from thermal imaging experiments and modelling (PA6, P = 36 W, V = 30 mm/s, \(\tau = 51\%\), K = 4.8 mm\(^{-1}\)).

### 8.2.1.4 High Line Energy and Low Speed Case (P = 18 W, V = 15 mm/s)

Figure 186 shows temperature contours obtained from the thermal modelling for P = 18 W and V = 15 mm/s. The red regions in the contour plots indicate where the temperature is over 200°C. A peak temperature of 325.2°C was obtained. Although the line energy (P/V) in this case is identical to the previous run, where the laser beam power
and scanning speed were twice as much (1.2 J/mm), the peak temperature is lower by 11% (324.2°C vs. 365.4°C).

Predicted weld zone dimensions change with the assumed melting temperature. Figure 187 shows the temperature contours for the plane transverse to the scan direction. Molten zone areas indicated by red colour were obtained by assuming 200°C and 230°C melting temperature. Employing an image analysis software for such plots at 200°C, 210°C, 217°C, and 230°C, one can obtain the relation between the weld dimension and assumed melting point (Figure 188). Linear fits matched the data well ($R^2 > 0.99$). The predicted weld width ranged from 0.93 mm to 1.21 mm. For comparison, the average experimental weld width reported in Table 16 for the same process conditions was $1.54 \pm 0.06$ mm.

The predicted molten zone areas in Figure 188 range from 0.079 mm$^2$ for the highest assumed melting temperature to 0.268 mm$^2$ for the lowest temperature. These values are higher than the experimental observation of 0.06 mm$^2$ reported by Al-Wohoush et al. [117] (Figure 28) when their results are extrapolated to 1.2 J/mm line energy.

Figure 186- Temperature contours predicted by the thermal modelling (PA6, $P = 18$ W, $V = 15$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$), a) x-y plane, $z = 6$ mm, b) y-z plane, $x = 10$ mm, c) x-z plane, $y = 3.2$ mm.
Figure 187- Molten zone profiles for different assumed melting temperature values (PA6, $P = 18$ W, $V = 15$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$) a) $T_m = 200^\circ$C, b) $T_m = 230^\circ$C.

Figure 188- Predicted weld dimensions as a function of the melting temperature obtained from the thermal modelling (PA6, $P = 18$ W, $V = 15$ mm/s, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).

Figure 189 shows the transient temperature profiles obtained from the thermal imaging experiments and modelling for the surface facing the camera (Surface A). The temperature distributions have similar profiles and the peak temperatures are very close. Note that for the same line energy and higher speed, the thermal imaging showed higher peak temperature than the model by 7.3°C.
Figure 189 - A comparison between the transient temperature profiles obtained from thermal imaging experiments and modelling (PA6, P = 18 W, V = 15 mm/s, τ = 51%, K = 4.8 mm⁻¹).

Figure 190 compares the transient temperature obtained from thermal modelling for two combinations of power and scanning speed that give the same line energy (1.2 J/mm). Figure 190 compares the transient temperature profiles at the laser-beam centre (z = 6 mm), where the maximum temperature is observed. The higher power and speed combination are predicted to produce a 41.2°C higher peak temperature than the lower power and speed combination. The part of the curve where the temperature rises corresponds to the point being heated as the beam passes over it. Start of cooling corresponds to the time when the beam has moved completely past this point.

As the beam moves further from the point, the temperature difference between the two power-speed combinations diminishes. It appears that at the lower scan speed, conduction begins to have greater influence as the time scale of the heating process increases. Similar speed-related effect was observed in the experimental work conducted by Chen et al. [195].
Figure 190- Predicted temperature vs. time obtained from thermal modelling in the middle of the sample (PA6, x = 10 mm, y = 3.2 mm, z = 6 mm, LE = 1.2 J/mm, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).

Note that the predicted weld width for the high line energy and low speed (1.21 mm) is less than that for the same high line energy and high speed (1.31 mm) (for the 200°C assumed melting temperature). In addition, the weld depth for the high line energy and low speed (0.28 mm) is less than that of the high line energy and high speed (0.33 mm).

Table 21 summarizes the results presented for the PA6 thermal models and compares them with the experimental results. The model generally under-predicted the width of the molten zone compared with the experimental weld line width measurements. However, model correctly reflected the relative change in the width as the process parameters were changed. The model predicted the smallest dimension for the low line energy case and showed that, for the same line energy, the lower scan speed resulted in narrower line width due to increased conduction heat loss. The peak temperatures predicted by the model also reflected the difference between the process parameter
settings, which matched the line width predictions. The smallest peak temperature was for the lowest line energy and, for the two cases with the same line energy, the one with the lower speed resulted in lower peak temperature as expected.

Table 21- Summary of the computational results for the PA6 sample exposure to the moving laser beam (assumed melting temperature of 200°C - 230°C, $\tau = 51\%$, $K = 4.8$ mm$^{-1}$).

<table>
<thead>
<tr>
<th>Property</th>
<th>Model $^*$</th>
<th>Experiment</th>
<th>Model</th>
<th>Experiment</th>
<th>Model</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molten zone dimension (y-axis) mm</td>
<td>0.12</td>
<td>N/A</td>
<td>0.33 - 0.24</td>
<td>N/A</td>
<td>0.28 - 0.24</td>
<td>N/A</td>
</tr>
<tr>
<td>Molten zone dimension (z-axis) mm</td>
<td>0.47</td>
<td>1.17 ± 0.05</td>
<td>1.34 - 1.04</td>
<td>1.92 ± 0.06</td>
<td>1.21 - 0.93</td>
<td>1.54 ± 0.06</td>
</tr>
<tr>
<td>Molten zone area (yz-plane) mm$^2$</td>
<td>0.04</td>
<td>0.007$^{**}$</td>
<td>0.34 - 0.19</td>
<td>0.06$^{**}$</td>
<td>0.27 - 0.17</td>
<td>0.06$^{**}$</td>
</tr>
<tr>
<td>Peak temperature (overall) ($^\circ$C)</td>
<td>214.2</td>
<td>N/A</td>
<td>365.4</td>
<td>N/A</td>
<td>324.2</td>
<td>N/A</td>
</tr>
<tr>
<td>Peak temperature (surface A) ($^\circ$C)</td>
<td>31.6</td>
<td>34.2</td>
<td>38.5</td>
<td>45.8</td>
<td>37.0</td>
<td>38.1</td>
</tr>
</tbody>
</table>

$^*$ For the low-line-energy case, results are reported only for 200°C assumed melting temperature since the peak temperature reached was only 214°C.

$^{**}$ Molten zone experimental areas were extrapolated from Al-Wohoush et al. results [117].

Looking at comparisons with the thermal imaging observations, the model correctly predicted higher temperature at the surface observed by the camera as the line energy was increased. For the two high line energy cases, the model predicted nearly the same peak temperature at the surface while the thermal imaging observations showed nearly 8°C higher temperature for the high-speed case.

Comparisons of the molten zone shape with the experimentally observed weld line cross section obtained by Al-Wohoush et al. [117] on the same laser welding equipment...
showed that the model predicted weld line profiles of closely matching shape. This indicated good accuracy in the laser beam profile modelling and good spatial resolution of the model.

8.2.2 Thermal Model for Amorphous Materials

The 3D lap-joint geometry shown in Figure 93 was used to model an 18 W diode laser beam scanning the amorphous material (PC) sample at 30 mm/s. The laser-transmitting part was assumed to pass the laser beam without scattering or absorption and with 7% transmission loss due to reflection.

Figure 191 shows the temperature contours for the three orthogonal planes passing through the point where the peak temperature was predicted. Red areas in figures in the left column indicate regions with the temperature above 200°C; red areas in figures in the right column indicate regions with the temperatures above 267°C.

Since PC is an amorphous polymer, it does not display a melting point like crystalline thermoplastics – an amorphous polymer above $T_g$ gradually softens instead. On the other hand, during the welding trials with PC, a weld line can be clearly observed through the transparent laser-transmitting part (see Figure 147). Since it is not certain at what minimum temperature joining takes place, a range of temperatures were examined. The lowest temperature considered was 143°C, which is the Vicat softening temperature (VST)\(^1\) rating (ASTM D1525 standard) for Makrolon AL2647 PC [218]. The highest temperature considered is 288°C. It is specified in the product literature for Makrolon as the “melt temperature” [218]. The plastic’s injection moulding is recommended at this

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\(^1\) The Vicat softening temperature is the temperature at which a $1 \text{mm}^2$ flat-ended needle will penetrate $1 \text{mm}$ into a material under a specified load and heating rate. The VST can be used to compare the heat-softening characteristics of some polymers.
temperature. Thus, it is expected that the material will be capable of flowing at this temperature and the joining should be able to take place.

Figure 191- Temperature contours (PC, P = 18 W, V = 30 mm/s, τ = 93%, K = 5.4 mm⁻¹): lower (200°C) joining temperature at: a) x-y plane, z = 6 mm, b) x-z plane, y = 3.2 mm, c) y-z plane, x = 10 mm; higher (267°C) joining temperature at: d) x-y plane, z = 6 mm, e) x-z plane, y = 3.2 mm, f) y-z plane, x = 10 mm.
Using the assumed joining temperatures of 143°C, 200°C, 267°C, and 288°C, one can determine the weld dimensions along the y and z directions (Figure 192). For instance, a weld width of 1.50 mm and cross section area of 0.70 mm² is obtained transverse to the laser beam scan direction if joining occurs at 143°C; the width and the area decrease to 0.98 mm and 0.22 mm², respectively, assuming the joining temperature is 288°C. The average experimental weld dimension along z, obtained by measurements of the weld line width, was 1.47 mm (reported in Section 7.2.2). This value falls in the upper range of the weld widths predicted by the model (0.98 mm to 1.50 mm). Al-Wohoush et al.’s [117] experimental results indicate a weld area of 0.91 mm². This value was obtained by interpolating between the reported experimental line energies of 0.5 J/mm and 0.64 J/mm and speed of 50 mm/s to fit the modelled conditions of 0.6 J/mm (at 30 mm/s). The experimental evidence suggests that the VST is the temperature at which joining occurs.

Figure 193 shows the temperature profiles for different times along the length of the sample and parallel to the scan direction (x-axis). The profiles are for a line passing through the central axis of the laser beam (z = 6 mm) and through a horizontal plane where peak temperature was achieved (y = 3.2 mm). The peak temperature of 485°C is predicted. Note that the TGA tests show that mass loss of PC starts at approximately 450°C (see Appendix B.1.2); however, no weld degradation was observed at these conditions in the experiments with PC. It is conjectured that, given the very short time of the exposure during welding compared to the TGA test conditions, the 485°C predicted peak temperature will not lead to any degradation.
Figure 192- Predicted molten zone dimension vs. assumed joining temperature (PC, P = 18 W, V = 30 mm/s, \(\tau = 93\%\), \(K = 5.4 \text{ mm}^{-1}\)).

Figure 193- Predicted temperature profiles along the length of the sample for the moving laser beam (PC, \(y = 3.2 \text{ mm}\), \(z = 6 \text{ mm}\), \(P = 18 \text{ W}\), \(V = 30 \text{ mm/s}\), \(\tau = 93\%\), \(K = 5.4 \text{ mm}^{-1}\)).
Figure 194 shows transient temperature for the centre of the laser beam at 0.2 mm below the interface of the two parts. Fast heating rate (nearly 9000°C/s) as the beam travels over a point is followed by cooling after the beam passes.

![Graph showing predicted transient temperature profile](image)

Figure 194- Predicted transient temperature profile at the centre of the laser beam (PC, $x = 10\text{ mm}$, $y = 3.2\text{ mm}$, $z = 6\text{ mm}$, $P = 18\text{ W}$, $V = 30\text{ mm/s}$, $\tau = 93\%$, $K = 5.4\text{ mm}^{-1}$)

To compare the model results with thermal imaging observations of surface A, Figure 195 shows the temperature profiles along the width of the sample for different locations of the laser beam centre at the interface of the two parts. For the case where laser beam centre is located 2 mm from the sample’s front surface observed by the camera (surface A), the peak temperature decays to the ambient temperature (25°C) at a point 0.8 mm from the surface A ($z = 8.2\text{ mm}$). This means that the temperature of the front surface of the sample never increases beyond the ambient temperature during the modelled time interval of 0.8 s. For comparison, the thermal imaging observations of the front surface (surface A) for the case of $a = 2\text{ mm}$ showed an increase of temperature to 46.5°C (Figure 149). The model predicts that the temperature of 46.5°C is reached at a
location 1.05 mm from the sample’s front surface. A possible reason for the difference is that the model of the laser beam distribution did not include the low-power tails present in a real beam. Note that the thermal imaging experiments for the case of $a = 3$ mm showed near-zero temperature rise at the front surface. Thus, it is possible that the laser beam distribution tails will extend up to 1 mm further than in the model.

Figure 195- Predicted temperature profiles along the depth of the sample for different locations of the laser beam centre from the sample edge (PC, $x = 10$ mm, $y = 3$ mm, $t = 0.3$ s, $P = 18$ W, $V = 30$ mm/s, $\tau = 93\%$, $K = 5.4$ mm$^{-1}$).

Table 22 summarizes the results presented for the PC thermal models and compares them with the experimental results.
Table 22- Summary of the computational results for the PC sample exposure to the moving laser beam (assumed joining temperature of 143°C - 288°C, P = 18 W, V = 30 mm/s, LE = 0.6 J/mm, \( \tau = 93\% \), K = 5.4 mm\(^{-1}\)).

<table>
<thead>
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<th>Property</th>
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<th></th>
<th>a = 2 mm</th>
<th></th>
</tr>
</thead>
<tbody>
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<td>Model</td>
<td>Experiment</td>
</tr>
<tr>
<td>Molten zone dimension (y-axis) mm</td>
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<td>0.59 - 0.28</td>
<td>N/A</td>
</tr>
<tr>
<td>Molten zone dimension (z-axis) mm</td>
<td>1.50 - 0.98</td>
<td>1.48 ± 0.04</td>
<td>1.50 - 0.98</td>
<td>1.46 ± 0.05</td>
</tr>
<tr>
<td>Molten zone area (yz-plane) mm(^2)</td>
<td>0.70 - 0.22</td>
<td>0.91*</td>
<td>0.70 - 0.22</td>
<td>0.91*</td>
</tr>
<tr>
<td>Peak temperature (overall) (°C)</td>
<td>485</td>
<td>N/A</td>
<td>485</td>
<td>N/A</td>
</tr>
<tr>
<td>Peak temperature (surface A) (°C)</td>
<td>25.1</td>
<td>26.7</td>
<td>25.1</td>
<td>46.7</td>
</tr>
</tbody>
</table>

* Molten zone experimental areas were taken from Al-Wohoush et al. [117].
Chapter 9  Conclusions and Recommendations

This chapter presents the concluding remarks (Section 9.1) and recommendations (Section 9.2) for the future work. The concluding remarks summarize the research conducted and highlight the contributions this research work has brought to the LTW field. The recommendations suggest the topics that can be developed further in the area of LTW modelling.

9.1 Conclusions

This research aimed to develop a 3D transient thermal model of a lap joint being welded using contour Laser Transmission Welding process. The time-varying temperature field predicted by the model could then be employed to predict the weld dimensions, the minimum heat input into the weld required for welding, or the conditions leading to weld degradation.

Work on this thesis proceeded along the following three directions: 1) characterization of the process and materials to identify accurate model inputs, 2) model development, and 3) thermal imaging for the purpose of direct validation of the model.

9.1.1 Model Input Identification

Accurate information about the modelled process and material is required to produce accurate model results. The primary process input into the model is the laser beam. However, no detailed information about the laser beam intensity distribution for the diode laser used in this research was available prior to commencement of this work, and thus a set of carefully designed and executed experiments were carried out to
characterize the laser beam intensity distribution over the beam cross section area and over a range of working distances. To achieve this, a pinhole approach was employed [194].

Two materials were employed in experiments and modelling: PA6, a semi-crystalline thermoplastic, and PC, an amorphous thermoplastic. Two types of properties were required: optical, for modelling of the heat generation by the laser beam, and thermo-physical, for thermal modelling.

Optical material properties included absorption and scattering by the laser-transmitting part and absorption by the laser-absorbing part. Scattering by the laser-transmitting part was characterized using a novel technique in which widths of lines scanned on the laser-absorbing part surface were varied by adjusting the laser power and then related to the scattered laser beam intensity profile.

To quantify the rate at which the energy was absorbed in the laser-absorbing part, a methodology, new to the LTW applications, was developed to estimate the soot absorption coefficient using the plastic thermo-physical properties and the knowledge available in the literature in the field of combustion studies [200]. The absorption coefficient data obtained were combined with the beam profiling studies to produce accurate heat generation as the beam entered the laser-transmitting and laser-absorbing parts.

The heat capacity was measured by carrying out DSC testing of the polymer samples. Other required (temperature-dependent) thermo-physical properties data were obtained through information available in the open literature, or provided by the material’s supplier.
9.1.2 Model Development

The model development commenced with creation of a simplified 1D thermal finite element model of a T-like joint – this geometry was used in the earlier work conducted in this research group. 2D [161] and 3D [169] finite element models of the same geometry were created subsequently, allowing comparison between these models as well as comparison with the earlier experimental work carried out in this research group. The results confirmed the ability of the finite element thermal model to match earlier observations [119]. However, it was concluded that the T-like joint geometry was not suitable for the planned thermal imaging observations and thus the development switched to the lap-joint geometry. This configuration allowed thermal imaging observations of the side surface adjacent to the weld line.

The lap-joint geometry models were set up to match the conditions under which the thermal imaging observations were made. Thus, as a first step, a model of a stationary laser beam slowly heating the joint area located near the edge of the two parts was developed to assess the accuracy of the model's input parameters [170, 171]. The model development concluded with the 3D thermal model of a laser beam moving parallel and in close proximity to the sample's edge [219].

One of the key challenges in the model development was accurate representation of the heat generation by the laser beam. Heat generation techniques for the stationary laser beam and for the moving laser beam cases were produced. Challenges overcame included defining the input of the heat generation for a stationary and moving laser beam, and dealing with ANSYS data structure limitations for inputting of the heat generation that varied in three spatial dimensions as well as in time (for the moving beam).
MATLAB programming and ANSYS parametric design language (PDL) programming were employed to produce the required input heat generation. These automated scripts helped to minimize the possibility for human error during the manual data entry.

The incoming laser beam profile, the scattered laser beam distribution, and the absorption properties, all obtained as part of the model input identification work, were used to describe the heat generation in the two parts being joined. This work resulted in the novel technique for representation of the beam broadening as it passed through the laser-scattering upper part.

The mesh generation was a significant challenge for this problem. Very high thermal gradients (up to 700°C/mm) develop near the joint interface as the laser beam enters the laser-absorbing part. These gradients required fine mesh spacing to produce accurate solutions. The need for the fine mesh size conflicted with the limitations of the computer resources in terms of time needed to run the solution and memory requirements. Thus, significant effort was expended on improving the efficiency of the solution by reducing the number of elements and increasing the time step as much as possible without compromising the accuracy.

A novel approach, based on the thermo-physical and optical properties of the polymer and the process parameters, was developed to identify appropriate range of time step and grid size. This method was implemented as an aid to the LTW modelling. Numerous computational runs verified the use of this methodology for different absorption coefficients and process parameters.

In addition to validation by thermal imaging, model predictions were compared with the weld dimension data such as weld line width, weld area and shape. For the
stationary laser beam and PA6 samples, the molten zone dimensions predicted by the model were generally lower by 5% to 20% than those observed after the two joined samples were broken apart. The predicted dimensions approached those of the experiments when lower melting temperatures were assumed (near 200°C).

For the moving laser beam and PA6 samples, predicted weld zone dimensions were generally lower than the measured ones. Assuming 200°C melting temperature, the predicted weld zone width varied from 40% of the experimental width for the low line energy case to 70% to 80% for the high-line-energy case. Possible reasons for lower predicted values may be greater heating in the experiments due to contact resistance not being included in the model or higher actual transmission value compared to the one used in the model. For the moving laser beam and PC samples, predicted weld zone widths matched experimental results closely when assumed softening temperature of 143°C was used.

Furthermore, a micro-scale finite volume model of the laser heating of the individual soot particles was developed to help with further understanding of the LTW process [220].

9.1.3 Model Validation by Thermal Imaging

Validation of the thermal models in the LTW field is a challenge due to quickly changing temperature fields, small weld dimensions, and the presence of laser radiation. Thermal imaging was selected as a means for model validation as it allowed simultaneous capture of the temperature field at the weld and the surrounding area at a rate allowing real-time monitoring of the laser welding process. However, the challenge with the thermal imaging of the laser welding is that the joining takes place at the interface not
visible to the camera. It was then hypothesized that, by locating the joint close to the vertical surface observable by the camera, a “side view” of the process could be captured and used to validate the model.

A number of steps were taken to assure validity and accuracy of the thermal imaging observations. The camera’s temperature readings were calibrated against a reference blackbody heat source over a range from 25°C to 400°C. The emissivity of the plastic surface was measured by calibration with thermocouples. The opacity of the plastic material to the long infrared wavelength detected by the camera was evaluated by placing a heat source behind the plastic plaque and observing it with the thermal camera. Precise control of the distance between the beam and the front surface observed by the camera as the part moves is very important to avoid drift of temperatures which would occur if the beam moved closer to or farther away from the front surface. Care was taken to align the sample face precisely with the motion direction. Line position measurements after the scans confirmed accuracy of the alignment to within 0.1 mm over approximately 80 mm of the sample travel distance.

The use of coatings on the surface of the plastic plaques facing the thermal camera was investigated to ensure the opacity of the surface [221] and then developed to measure the temperature rise at the soot-coated surface of the laser-transmitting part facing the thermal camera. An analytical model was fitted to the temperature rise on this surface to obtain the rate of surface heating attributed to the laser beam’s side-scattered energy.

Thermal imaging observations were carried out 1) for the case of a stationary laser beam with PA6 samples, and 2) for the case of a plaque moving with respect to the laser beam with PA6 and PC samples. For the stationary laser beam, the beam centre was
located 3 mm behind the surface facing the camera and laser power was set to 1 W for the duration of 10 s. The reduced power and thus prolonged heating times prior to sample degradation, facilitated collection of the thermal imaging data. Surface coatings were investigated for this case and the results with no coating were found to be the ones allowing most accurate readings of the surface temperature.

Comparison with the model results showed that the stationary-laser-beam model incorporating the scattering effect was able to predict accurately surface temperatures on the laser-absorbing part surface. Peak temperature was predicted within 3°C and temperatures matched within 2.5°C over the majority of locations examined and the time of observation. Predictions of temperature on the laser-transmitting part surface were generally higher by 15°C to 20°C than the thermal imaging results. The higher model predictions were attributed to the absorption coefficient being set too high in the model of the laser-transmitting part. The thermal imaging results, as well as other work by this research group, indicated that significantly more back- and side-scattering and less absorption were taking place in the laser-transmitting part than originally estimated.

For the case of samples moving with respect to the laser, the beam centre was located 3 mm behind the surface facing the camera for PA6 and either 2 mm or 3 mm behind the surface for PC. For PA6, two laser power settings (18 W and 36 W) and two speed settings (15 mm/s and 30 mm/s) were used to carry out trials with three different power-speed combinations: 1) low line energy of 0.6 J/mm with low power and high speed, 2) high line energy of 1.2 J/mm with high power and high speed, and 3) high line energy of 1.2 J/mm with low power and low speed.
For the low-line-energy case, several settings of the optical material properties were explored in the model and the results then compared to the ones of the thermal imaging. Temperature profile observed along a vertical line through the interface between the two parts was compared to the model predictions. A transmission value of 69% (higher than the base setting of 51%) and the laser-absorbing part’s absorption coefficient setting of 4.8 mm\(^{-1}\) were found to give the closest match of the peak temperature (34°C observed values was matched within 1°C) and the curve shape. For the case of high line energy and low speed, model predictions matched the thermal imaging observations well for peak temperature on the surface observed by the camera (within 1°C). For the case of high line energy and high speed, the predicted peak temperature was about 7°C lower.

For PC, a single combination of power and speed (18 W and 30 mm/s) was used, giving 0.6 J/mm line energy, since higher line energy setting caused excessive degradation of the material. When the laser beam was located 3 mm from the surface observed by the camera, very little temperature rise (< 2°C) was observed by the camera. Moving the beam by only 1 mm closer to the front surface, the heating effect of the laser became clearly visible near the interface of the two parts and the peak surface temperature of 47°C was observed. However, the model did not predict any temperature rise at the surface. These results indicate that the unscattered laser beam distribution for the amorphous polymer has a very sharp drop off at its boundary. Thus, there is very little margin for error between the modelled beam distribution and the actual one. Also, since there is little absorption in the laser-transmitting PC part, even the low power in the laser distribution tails (which were ignored in the beam model) can cause appreciable rise in the surface temperature.
It can be thus concluded that thermal imaging observations of the weld’s side surface are effective for the semi-crystalline (scattering) polymer such as PA6 but are not as useful for the amorphous polymer such as PC since they are very sensitive to the relative position of the beam and the front surface in the latter case.

Thermal imaging observations of the moving welded specimen’s side surface provided additional information, which was not anticipated at the outset of this research. The rapid temperature rise seen at the weld interface can be assumed to occur primarily due to direct heating by the laser, with conduction having a relatively insignificant effect. Based on this idea, a methodology was developed to extract the information about the laser beam intensity distribution parallel to the direction of the part movement and at the front surface of the part. Assumption of a Gaussian laser beam profile resulted in excellent data fit for PA6 and a very good fit for PC data. The results for PA6 indicated that the beam scattering is significantly greater than that incorporated into the model (actual width of 8 mm vs. 2 mm for the model). It is most likely that the tails of the distribution are much wider than originally assumed. Note that the line scanning technique used to obtain the scattered beam distribution does not provide information about the extreme tails of the distribution and thus an approximation must be made.

### 9.2 Recommendations

Accurate information about the optical properties of the thermoplastic is critical in order to obtain accurate model results. In particular, scattering by a semi-crystalline polymer as well as absorption by the laser-absorbing part are a challenge to determine precisely.
Two new techniques that can aid in characterizing the scattering phenomenon have been developed based on the thermal imaging work presented in this thesis: 1) a technique based on the observation of the soot-coated side surface of the laser-transmitting part while exposed to a stationary laser beam; and 2) a technique based on the observation of the joint interface heated by the laser beam as the sample moves past.

For the first case, as an extension of the method implemented in the thesis, one can obtain the temperature rise on the surface facing the thermal camera within the laser-transmitting part, and then use the analytical model in order to extract the heat generated for the entire visible surface. If one assumes some fraction of the laser energy is absorbed at the surface facing the thermal camera, then the laser flux can be calculated for the points on the surface based on the thermal imaging observations. This method could be useful for evaluation of the side-scattering of the laser beam as it passes through the laser-transmitting part. To see the scattering effect more clearly, the experiment can be carried out without the laser-absorbing part. Thermal modelling can then be compared to the experiment while the scattering model is refined.

For the second case, the information about the laser beam intensity profile obtained from the thermal images of a moving sample is particularly valuable for the laser-scattering material such as PA6. An experiment can be proposed where scans are made at progressively varying distances between the laser beam centre and the surface observed by the camera as the beam distribution profiles are extracted for each distance. This would provide the ability to observe the complete scattered beam profile (including the distribution tails) after the beam passes through the laser-transmitting part.

Several directions can be pursued in the quest to refine the modelling of the contour laser transmission welding process. The thermal model can be extended to include the
mechanical deformation effects as well as material flow. Inclusion of the contact resistance at the joint interface would further enhance accuracy of the model.

However, considering the long running times of the models solved for this thesis (up to 84 hours), additional modelling complexity would necessitate availability of significantly greater computing resources (e.g., those available through the High Performance Computing Virtual Laboratory (HPCVL)).

In the current work, lap joints (in addition to the T-like joint) were modelled since the purpose was to validate the model by thermal imaging. It would be of interest to develop models for more complex designs, such as 3D joints or joints where joining surface is curved or not parallel to the incident surface of the laser-transmitting part or where the joint path is not straight.
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Appendix A: Thermal Modelling of a T-like Joint

In this section, results of T-like joint thermal models with increasing complexity are presented: transient 1D with stationary laser beam input with insulated boundary conditions (Section A.1), transient 2D with stationary and moving laser beam input with convective and radiative boundary conditions (Section A.2), and transient 3D with stationary laser input with insulated, convective, and radiative boundary conditions (Section A.3). This work was carried out during the early stages of this research. Based on this modelling work, the ANSYS® FEM commercial software was adopted for preprocessing, solution, and post processing of the later work in LTW heat transfer modelling.

A.1 One-Dimensional Thermal Model of a T-like Joint

A 1D thermal model of a T-like joint geometry was developed for un-reinforced PA6 specimens using ANSYS®. The laser-absorbing part was assumed to have 0.2% (wt.) carbon black additive. The model was compared with a simplified 1D FDM thermal analysis carried out as part of the earlier work in this research group [119].

A.1.1 Thermal Line Elements

Thermal line elements have one degree of freedom (i.e., temperature) at each of their two nodes. They are uniaxial with the capability of transferring heat by conduction, radiation, and convection between the nodes. Among elements available in the thermal line elements library, only LINK32 was capable of transferring heat by conduction mode.
and accommodating the internal heat generation feature; therefore, it was adopted in this work.

The geometry, node locations, and the coordinate system for LINK32 are shown in Figure 196. LINK32 is defined by two nodes and the material properties. Specific heat and density are ignored for steady-state analyses; however, they are incorporated into a transient analysis along with the thermal conductivity. LINK32 assumes heat flows only in the longitudinal element direction (1D thermal model). Employing LINK32, one incorporates the following assumptions automatically: 1) the geometry is the x-y plane shown in Figure 196; 2) the element has finite length; and 3) a free end of the element, which is neither adjacent to another element nor subjected to a boundary condition, is adiabatic.

![Figure 196- LINK32 geometry](image)

**A.1.2 Modelling Approach**

The 1D T-like joint geometry in Figure 197 was exposed to a 60 W diode laser beam moving at 10 mm/s scanning speed. The geometry was meshed with 490 nodes, closely spaced at the interface. Physical and thermal properties of the material were defined, and loads (boundary conditions and heat generation) were applied (Figure 197 and Figure 198).
Maximum heat generation happened in the laser-absorbing part in the neighborhood of the contact surface of the two materials (Figure 198). Heat generation in the model decayed to zero for $y = 17.5$ mm as one gets farther from the interface into the laser-absorbing part (Figure 199). The model was then introduced as transient and solved taking into account the effect of the phase change.
A.1.3 Computational Results

The results of the FEM thermal model are shown in Figure 200. Note that 0.27 s heating time indicates the exposure of a point along the length of the sample to a 2.7 mm length of a defocused diode laser beam moving at 10 mm/s. The comparison of the results obtained from the FEM analysis and that of the FDM analysis shows that they are generally in good agreement. The slight difference between the results occurs where the peak temperature and weld depth predicted by the FEM analysis are 4°C and 0.08 mm greater than those of the FDM analysis. These minor differences could be associated with the following reasons: 1) Heat generation term was discretized using the central difference method to acquire accurate results, compared to the backward method reported [119]; 2) Using the backward difference to calculate the heat generation term in the laser-transmitting part leads to a zero value on the boundary; forward difference method is needed to obtain this term.
A.2 Two-Dimensional Thermal Model of a T-like Joint

This section addresses the 2D FEM thermal model solved using ANSYS®FEM software. The use of the 2D model requires the assumption that no significant heat flow existed normal to the geometry plane. A modified T-like joint geometry was modelled for un-reinforced PA6 specimens. The laser-absorbing part was filled with 0.2% (wt.) carbon black additive. This thermal model addressed the heating and cooling stages for the LTW process. The 2D model was capable of predicting the molten zone depth as well as transient temperature distribution transverse to the weld line (in the x direction).

A.2.1 2D Thermal Solid Elements

Thermal solid elements have one degree of freedom (temperature) at each node. Depending on the number of the nodes each element has, they could be of lower order (4 nodes) or higher order (8 nodes). 2D thermal solid elements are either triangular or...
rectangular with the capability of transferring heat by conduction, radiation, and convection between the nodes. Among elements available in the thermal solid library for 2D elements, only PLANE55 is a lower-order element capable of transferring heat by conduction mode and can accommodate internal heat generation, convection (or heat flux), and radiation. Therefore, this element was used for the 2D analysis in this research.

PLANE55 can be used as a plane element with a 2D thermal conduction capability. The element has four nodes with a single degree of freedom (i.e., temperature) at each node (Figure 201). The element is applicable to a 2D, steady state, or transient thermal analysis. Specific heat and density are ignored for steady-state solutions. In case of thermal stress analysis, this element could be replaced by an equivalent structural element (e.g., PLANE42). This capability makes it possible for the thermal model to be updated to a thermal-stress analysis for the future studies [213, 214]. Although it is recommended not to use the higher order elements in order to optimize the solution efficiency, PLANE77 can be employed as a higher order form of PLANE55 with mid-side nodes if needed. Convection or heat flux and radiation can be input as surface loads to the element faces shown by the circled numbers in Figure 201.

![Figure 201 - PLANE55 geometry](image_url)

**Figure 201 - PLANE55 geometry [212].**
Employing PLANE55, one assumes that the element does not have a zero area (e.g., zero area can occur when the element is not numbered properly). It is also assumed that the specific heat is evaluated at each integration point to allow for sudden changes (e.g., at the melting point) within a grid of elements.

A.2.2 Modelling Approach

The 2D T-like joint geometry in Figure 202 was exposed to a 60 W diode laser beam defocused to give dimensions of 2.7 by 3.8 mm and moving at 10 mm/s scanning speed along z-axis. The heating time (0.27 s) was obtained based on the speed and the length of the moving laser beam along z (2.7 mm). Figure 202 shows the geometry for this 2D transient thermal model. This T-like joint configuration is beneficial for assessing weld strength in tension without inducing significant stress concentration. It consists of a flat plate (flange) with a 3.8 mm wide rib that rises 1 mm above the plate surface. A second plate (web) is welded to the rib face. Unlike T-like joints without a rib, the weld zone can be subjected to near uniaxial stress in a tension test as any clamping-related bending moments are located off the plane [222].

Convective and radiative boundary conditions were applied to the geometry (Figure 202). The vertical sides (1, 3, 5, and 7) were assumed to transfer heat by convection and radiation, assuming the heat transfer coefficient for a vertical plate. The horizontal surfaces (4 and 8) were assumed to transfer heat by convection and radiation, assuming the heat transfer coefficient for the horizontal flat plate. The surface 2 was assumed adiabatic. The surface 6 was assumed at the ambient temperature, which was reasonable due to the length of the sample and low thermal conductivity of the laser-absorbing part. Convective heat transfer coefficient was assumed 5 W/m²K. This coefficient is typical of
natural convection values near vertical or horizontal plates [177]. Sensitivity studies showed that radiation and convection were negligible.

Figure 202- The 2D thermal model geometry for a T-like joint configuration for the LTW of plastics (dimensions in mm).

The heat generation distribution in the y direction was assumed to be the same as for the 1D transient finite element thermal model (Figure 198). The heat generation in the x direction was assumed uniform, which assumes the beam to have a rectangular shape. Maximum heat generation happens at the contact surface of the parts. Heat generation was defined by an input ASCII file in the form of a table and was a function of x, y, and time. Note that the beam profile changes along the y direction (depth of the sample) and it is applied uniformly to the length x of the sample. The laser beam remains turned on for a period of time which is related to the beam dimensions and scan speed and is then turned off.

Defining the appropriate heat generation term was challenging due to the high gradients near the interface. The heat generation undergoes a discontinuity at the interface between the two parts (as the absorption coefficient makes a step increase). As
a result, there is a very steep gradient of heat generation once inside the laser-absorbing part. This fast change poses a problem for accurate numerical modelling and thus requires careful consideration when defining the grid. The larger the grid size along the y direction, the more averaging takes place with the heat generation in the volume of the sample, which causes the heat generation term to be incorrectly applied at the weld interface. Care was taken to make sure that the chosen grid size accurately captured the heat generation term at the weld interface.

ANSYS®9 was used in all stages of the thermal model including pre and post processing phases. The CPU time to process and solve the problem was approximately 30 minutes on a 3 GHz PC with 2 GB of random access memory (RAM). The model was meshed using PLANE55 elements (Figure 201 and Figure 202). The heating time was assumed 0.27 s based on the laser beam speed of 10 mm/s and the laser beam length of 2.7 mm. The model assumed defocused beam dimensions of 3.8 mm by 2.7 mm [119]. A laser power of 60 W, resulting in the energy flux of 5.58 W/mm², was used, unless otherwise noted.

Figure 203- Close-up of the 2D meshed model of a T-like joint.
A.2.3 Computational Results

Figure 204 and Figure 205 illustrate temperature contours inside the geometry at the end of the laser heating (Figure 204) and during the cooling stage (Figure 205). The highest temperature of 297°C was calculated in the middle of the joining area; temperature decreases towards the outer edges of the interface (Figure 204). The temperature contours obtained from the total welding time of 1 s are shown in Figure 205. 1 s is approximately the time that the laser beam takes to traverse the 12-mm-long specimen used in the earlier experiments [119]. 1 s weld time corresponds to a heating time of 0.27 s and a cooling time of 0.73 s. Figure 205 thus shows the temperature distribution just before the meltdown collapse at the end of the weld cycle.

Figure 204- Temperature contours at the end of laser heating for the 2D model of a T-like joint (t = 0.27 s, P = 60 W, V = 10 mm/s, t = 0.27 s, K = 1.8 mm⁻¹).

Temperature as a function of time for different locations along the y-axis is shown in Figure 206. The small diagram on the bottom right corner of this figure is colour coded to assist finding the location of each point. The temperature increase rate is highest for the area located very close to the interface inside the laser-absorbing part. The temperature increases initially with the applied heat and then decreases after the laser
beam is turned off. The temperature increase in areas farther from the joining area is considerably smaller than that of the joining area.

Figure 205- Temperature contours during cooling stage for the 2D model of a T-like joint \( (t = 1 \text{ s}, P = 60 \text{ W}, V = 10 \text{ mm/s, } t = 1 \text{ s, } K = 1.8 \text{ mm}^{-1}) \).

Temperature variation as a function of location along the x direction for different times is shown in Figure 207. It is seen that temperature gradient is larger near the exposed edges during the heating process, while there is a gentler trend during the cooling
process. It is believed that these gradients, along with clamping pressure will have a significant effect on the meltdown (i.e., the amount of the molten plastic squeezed out of the weld area).

![Graph showing temperature distribution along the x direction for different times](image)

**Figure 207** - Temperature distribution at the interface along the x direction for the 2D model of a T-like joint ($y = 4$ mm, $P = 60$ W, $V = 10$ mm/s, $K = 1.8$ mm$^{-1}$).

The temperature distribution as a function of location along the y direction for different times is shown in Figure 208. Note that the model predicts a significant temperature rise (up to $175^\circ$C) at the surface of the laser-transmitting part. This means that if, for example, we were to raise the laser power to compensate for a thicker laser-transmitting part, there is a point where melting may occur on the upper surface where the laser enters the laser-transmitting part. The plot indicates the maximum temperature lies in the laser-absorbing part, 0.2 mm from the joining plane. Using this 2D thermal model, theoretical molten zone depth can be estimated from the temperature profiles by using the melt temperature of PA6 ($217^\circ$C). Figure 208 shows the molten zone depth at the centre of the sample at 0.27 s predicted by the model is 0.586 mm. In the previous experimental work, meltdowns in the range of 0.33 to 0.44 mm were observed when applying a weld
clamping pressure of 0.7 MPa [119]. This suggests that the assumed absorption coefficient for the laser-absorbing part (1.8 mm\(^{-1}\)) may not be sufficiently high.

The molten zone near the exposed edges of the weld interface and the clamping pressure determines the meltdown. For low clamping pressures, the meltdown can be predicted from the dimensions of the molten zone at the exposed edges of the interface (i.e., the corners). The molten zone profiles based on the adopted absorption coefficient (1.8 mm\(^{-1}\)) as a function of time is shown in Figure 209. The difference between the molten zone depths of the centre and corners should define the meltdown under conditions of low clamping pressure. The results obtained from the 2D modelling were compared to the cross sectional studies presented in the literature where the results were presented for an un-reinforced injection moulded grade of PA6 with the commercial name of Zytel®7301 [119]. The maximum molten depth calculated by this 2D model is 0.586 mm that is close to the measured value of 0.569 mm. However, the molten zone
depth in the corners is calculated to be zero, suggesting no meltdown. The absorption coefficient plays an important role in the temperature distribution and consequently molten zone depth. It was concluded that the absorption coefficient employed for the laser-absorbing part from [119] (1.8 mm\(^{-1}\)) was not sufficiently high.

A higher absorption coefficient of 4.8 mm\(^{-1}\) was adopted for the laser-absorbing part compared to that of 1.8 mm\(^{-1}\) adopted in the earlier work by Prabhakaran [119]. Applying this value to the thermal model resulted in the temperature profiles of Figure 210, which shows the molten zone depth in the laser-transmitting and absorbing plastic parts. The maximum molten depths of 0.69 mm and 0.44 mm are calculated for the centre and the corners of the weld interface. The maximum meltdown obtained in this case is assumed to equal the thickness of the molten area at the corners (0.44 mm) and this is in good agreement with the experimental observations (0.44 mm) when minimum pressure of 0.7 MPa was applied [119]. The difference between these two molten
values \((0.69 - 0.44 = 0.25 \text{ mm})\) is only half of the maximum value \((0.57 \text{ mm})\) reported earlier [119] for the experimental molten zone depth. This suggests that the assumed absorption coefficient could be on the low side. It should be noted that one would expect the 2D model to over-predict meltdown because the plane conduction and the support from the un-molten material away from the laser-heated zone are ignored. This is why a full 3D model is needed to predict meltdown for the moving laser beam more accurately.

Figure 210- Transient molten zone depth profile \((y = 4 \text{ mm}, P = 60 \text{ W}, V = 10 \text{ mm/s}, K = 4.8 \text{ mm}^{-1})\).

A.3 The 3D Thermal Model of a T-like Joint

The focus of this work was to develop a 3D thermal model for the LTW process that can predict the extent of the molten zone in space and over time as accurately as possible for a T-like joint configuration exposed to a moving diode laser beam. Such models are essential for in-depth understanding of the LTW process and consequently its optimization. This model was solved using the FEM technique by employing the ANSYS® commercial software. This model addressed the heating and cooling stages in a
LTW process with a moving laser beam for a T-like joint configuration. The 3D thermal model had the flexibility to adopt a variety of plastic materials, boundary conditions, optical properties, and geometry. Unlike the 2D model, the 3D model accounted for the heat conduction along the beam travel direction.

### A.3.1 Computational Approach

Figure 211 shows the assumed modified T-like joint geometry for the 3D transient thermal model. The model was exposed to a 60 W diode laser beam moving at 10 mm/s scanning speed. Due to the assumption of the symmetry in the geometry and laser beam profile, only half of the model was simulated to save the computing time.

![3D thermal model geometry for a T-like joint configuration for the LTW of plastics](image)

Figure 211- The 3D thermal model geometry for a T-like joint configuration for the LTW of plastics (dimensions in mm).

The top, bottom, and vertical faces of the geometry were assumed adiabatic. Sensitivity studies showed that free convection and radiation effects from exposed surfaces were negligible.
The heat generation distribution in the y direction was mapped into the x, y, and z space of the plastic parts. This profile is translated with \( z = z^* + Vt \), where \( V \) is the laser beam scanning speed and \( t \) is time. The heat generation profile along the y direction was assumed the same as the one in the 2D and 1D models (Figure 198). A rectangular defocused laser beam profile (2.7 mm \( \times \) 3.8 mm) with uniform power flux distribution was assumed in this model for the diode laser. The long beam dimension (3.8 mm) was aligned with the x-axis (Figure 211).

The heat generation in the x and z directions was assumed uniform. A moving laser beam with the scanning speed of 10 mm/s was modelled in this study. Maximum heat generation happened at the contact surface of the parts. Heat generation was defined by an input ASCII file in the form of a 4-D array and was a function of x, y, z, and time. A MATLAB® computer code was developed to create the heat generation input file as a function of the beam dimensions, laser beam scanning speed, laser power, and absorption coefficient of the laser-transmitting and absorbing parts.

ANSYS®10 was used in all stages of the thermal model including pre and post processing phases. The CPU time to process and solve the problem was approximately 15 hours on a 3 GHz PC with 2 GB of RAM. The model was meshed using SOLID70 element (Figure 92). An element size of 0.2 mm was used for the model (Figure 212). Grid sensitivity analysis confirmed this choice of element size. When the grid size was decreased to 0.1 mm and 0.05 mm, a change of less than 3°C was observed from the total range of 350°C.

The welding process was modelled for 2 s, starting from the time the leading edge of the laser beam entered the part until 0.48 s after the trailing edge of the beam left the
part. The beam leaves the specimen after 1.52 s, based on the laser beam speed of 10 mm/s, the laser beam length of 2.7 mm, and the specimen depth of 12.5 mm. 0.48 s was added to model the specimen cooling. A time step size of 0.004 s was used. It was found to satisfy the numerical stability criterion. The heat generation input file updated the beam location at 0.008 s intervals, corresponding to 0.08 mm increments in the z direction and thus modelling the 10 mm/s beam travel speed. A laser power of 60 W, resulting in the energy flux of 5.58 W/mm², was used.

Figure 212- 3D meshed model for the T-like joint configuration.

A.3.2 Computational Results

3D temperature contours, after the beam’s leading edge scanned 10 mm of the total of 12.5 mm travel, are shown in Figure 213. Corresponding temperature contours on the symmetry plane of the geometry (x = 0 mm) are shown in Figure 214. The highest temperature of 298°C was predicted within the symmetry plane.
Figure 213- Temperature contours after 10 mm of beam travel for the 3D model of a T-like joint (t = 1 s, P = 60 W, V = 10 mm/s, t = 1 s, K = 1.8 mm⁻¹).

Figure 214- Temperature contours on the symmetry plane after 10 mm of beam travel for the 3D model of a T-like joint (x = 0 mm, t = 1 s, P = 60 W, V = 10 mm/s, t = 1 s, K = 1.8 mm⁻¹, side view).

Temperature as a function of time for different locations along the y direction where welding starts (z = 0 mm) is shown in Figure 215. The small diagram on the bottom-right corner of this figure is colour coded to help with finding the locations. The temperature
increase rate is highest at the surface located at \( y = 4.2 \) mm. This corresponds to 0.2 mm from the interface, inside the laser-absorbing part. The temperature increases initially (up to \( t = 0.27 \) s) while the beam passes over the specimen edge and then decreases after the laser beam leaves. The temperature increase of the specimen in areas farther from the joining area is considerably smaller than that of the joining area. Also note that the peak temperature in this case (about 287°C) is lower than the one predicted by the 2D model (300°C). It is believed that the difference can be attributed to the heat conduction in the z direction, which cannot be accounted for by the 2D model.

Figure 215- Temperature vs. time along the y direction of the geometry for the 3D model of a T-like joint \((x = 0 \text{ mm}, z = 0 \text{ mm}, P = 60 \text{ W}, V = 10 \text{ mm/s}, K = 1.8 \text{ mm}^{-1})\).

Figure 216 shows the transient temperature for a location 0.2 mm below the joining surface for different locations along the x and z directions. The temperature at the symmetry plane \((x = 0 \text{ mm})\) reaches the melt temperature of the plastic in contrast with the temperature of the specimen edges at the furthest location from the symmetry plane \((x = 1.925 \text{ mm})\).
Temperature variation as a function of location along the x direction for different times is shown in Figure 217 for z = 0 mm, where the welding starts. It is seen that the temperature gradient is larger near the weld edges (x = 1.925 mm) after 0.27 s of heating while there is a more moderate gradient after 1.25 s of cooling. It is believed that these gradients, along with the clamping pressure will have a significant effect on the meltdown (i.e., the vertical collapse of the joint due to the molten plastic being squeezed out of the weld area).

Temperature distribution as a function of location along the y direction at the symmetry plane (x = 0 mm) is shown in Figure 218 and at x = 1.925 mm in Figure 219. Note that the model predicts a significant temperature rise (up to 179°C) at the upper surface of the laser-transmitting part. This means that if, for example, we were to raise the laser power to weld a thicker laser-transmitting part, degradation may occur on the upper surface of the laser-transmitting part. The plots indicate that the maximum temperature lies in the laser-absorbing part, close to the weld interface.
Figure 217- Temperature distribution at the interface along the x direction for the 3D model of a T-like joint (z = 0 mm, y = 4 mm, P = 60 W, V = 10 mm/s, K = 1.8 mm\(^{-1}\)).

Figure 218- Temperature distribution along the y direction for the 3D model of a T-like joint (x = 0 mm, z = 0 mm, P = 60 W, V = 10 mm/s, K = 1.8 mm\(^{-1}\)).
Using this 3D thermal model, the molten zone depth can be estimated from the temperature profiles. Figure 220 shows the temperature contours on the symmetry plane in the y direction range ±0.5 mm from the joint interface at $t = 1.52$ s. Inspection of this figure allows one to estimate molten zone thickness as a function of $z$. The molten zone is identified by the highest temperature range in the contour plot ($217^\circ$C - $350^\circ$C). Maximum molten zone thickness of 0.5 mm was predicted. If the temperature at which joining occurs is assumed $200^\circ$C, the yellow contour will show the boundary of the molten area.

Further examination of the model results also shows that the molten zone depth at the farthest edges from the symmetry plane ($x = 1.925$ mm) is zero. It is expected that the meltdown can only occur if the molten zone near the weld edges is non-zero. However, note that the heat deflection temperature for this grade of PA6 is $160^\circ$C (at 0.45 MPa). Figure 219 indicates a maximum temperature of $154^\circ$C, suggesting the possibility of
meltdown taking place given the 0.7 MPa clamping pressure used in the experiments [23].

Figure 220- Close-up of the temperature contours at the symmetry plane for the 3D model of a T-like joint (x = 0 mm, 3.5 mm < y < 4.5 mm, t = 1.52 s).

Figure 221 to Figure 223 present the temperature profiles along the depth (Figure 221 and Figure 222) and the length (Figure 223) of the sample for different absorption coefficients during heating and cooling. Using a higher absorption coefficient value of 4.8 mm\(^{-1}\) results in the peak temperature increase. The results presented are for the weld time of 1.52 s at which the laser beam has just finished scanning the joint. Therefore, one expects to obtain higher temperatures for z = 12.5 mm.

Figure 221- Temperature distribution along the y direction for different absorption coefficients for the 3D model of a T-like joint (x = 0 mm, t = 1.52 s, P = 60 W, V = 10 mm/s).
Note that as one gets farther from the laser beam centre (x = 0 mm), the peak temperature drops. In addition, the lower absorption coefficient for the locations farther from the laser beam centre results in slightly higher peak temperature, which is associated with the small thermal conductivity and more chance for the heat to penetrate through the sample depth.

Figure 222- Temperature distribution along the y direction for different absorption coefficients for the 3D model of a T-like joint (x = 1.925 mm, z = 0 mm, t = 1.52 s, P = 60 W, V = 10 mm/s).

Figure 223- Temperature distribution at the interface along the x direction for different absorption coefficients for the 3D model of a T-like joint (z = 0 mm, y = 4 mm, t = 1.52 s, P = 60 W, V = 10 mm/s).
In earlier experimental studies conducted by this research group for the T-like joint configuration and the process parameters introduced earlier in the computational results, a meltdown of 0.33 mm (using 0.7 MPa clamping pressure) was observed while a weld strength of 25 MPa was achieved in tensile tests [119]. Figure 224 shows a weld cross-section for the above experimental conditions obtained by viewing a microtomed slice under an optical microscope using polarized light. No polymer degradation is visible in the image (degradation is expected to occur at 425°C as determined by TGA analysis of our PA6 samples). This observation is supported by the model’s prediction of peak temperature of just under 300°C.

![Figure 224- Micro-graph of the weld interface transverse to the weld path (x-y plane) for a T-like joint [119].](image)

To facilitate comparison with the experimental results, extent of the molten zone predicted by the model can be visualized by plotting temperature contours transverse to the weld path at the time when the trailing edge of the beam just left the part (Figure 225). The highest temperature range corresponds to temperature above the melting point of PA6. Since the meltdown and molten material flow out of the interface have not been modelled, the predicted molten zone includes the material that would be ejected as flash from the weld interface upon meltdown. The predicted molten zone shape has more uniform thickness than the experimental one. This is most likely because a uniform beam intensity profile was assumed for the model whereas actual beam profile should be closer
to a Gaussian distribution shape (due to near-Gaussian shape of the incoming beam profile [194] and due to scattering by the semi-crystalline polymer [129]).

![Figure 225- Temperature contours in the x-y plane for the 3D model of a T-like joint (t = 1.52 s, z = 12.5 mm).](image)

Since the experimental and the model-predicted molten zones differ in shape, it would not be correct simply to measure their maximum thickness for the comparison purposes. Thus, to compare the two, molten zone cross-sectional area was calculated for both cases and then divided by the absorbing part thickness (3.85 mm). For the experimental case, the apparent molten zone area from the image in Figure 224 (1.21 mm²) was added to the measured meltdown distance (0.33 mm) multiplied by the part width (A_{exp} = 1.21 + 0.33 \times 3.85 = 2.48 \text{ mm}^2). For the model case, the molten zone area was obtained from Figure 225 (after multiplying by two), which is 1.92 mm². Dividing both numbers by the part width, the normalized thickness of 0.64 mm for the experimental case and 0.5 mm for the model was calculated. The slightly higher number for the experimental case could be due to some plastic deformation of the two parts upon the meltdown as well as the ejection of the molten material near the joint edges.
Appendix B: Plastic Properties

B.1 Thermo-Physical Properties

B.1.1 Differential Scanning Calorimetry (DSC)

DSC is a thermal analysis technique in which difference in energy input to a material is measured as a function of temperature as the temperature of the process is increased in a controlled fashion. A reference material is also used for which there is a general agreement regarding its reaction to the input energy. This technique reveals qualitative and quantitative information about temperature-dependent physical and chemical changes concerning endothermic and exothermic processes that occur during heating. Researchers usually use this technique to extract information on glass transition temperature, melting point, specific heat, and so on.

DSC tests were conducted based on ASTM 1269-05 standard [223]. Figure 226 shows the heat flow as a function of temperature obtained from the DSC tests for PA6 (Akulon F223D from DSM). At the beginning of the heating process, there is a large endothermic start-up hook, which is due to the heat capacity difference between the sample and the reference. Since the reference is usually lighter than the sample, the sample weight is hardly offset. For fast heating rates (e.g., 20°C/min), this effect is more pronounced. The heating rate of this test was set to 10°C/min. Since the results of the first 2-3 minutes are usually not reproducible, this data is ignored in the heat capacity measurements. The heat flow becomes relatively stable at some point, which means that
the sample is receiving heat with increasing temperature up to the start of melting (the intersection of the two tangential lines in Figure 226, 205°C).

Figure 226- Heat flow vs. temperature obtained from DSC test for PA6 (Akulon F223D).

Knowing the weight of the sample used for this test (5.602 mg) and incorporating the correction factor, the heat flow data presented in Figure 226 are modified to obtain the temperature-dependent specific heat (Figure 227). The area under the triangular section of this plot (70 J/g) is the latent heat of fusion. The correlations between the specific heat and temperature were obtained from Figure 227 for PA6 and used as input to the thermal model. They are presented in Table 10.

Some studies define specific heat for solid and liquid states of polymers and introduce correlations between these values. Prabhakaran et al. measured the specific heat for a particular grade of PA6 (Zytel®7301) shown in Figure 228 [119].
Figure 227- Specific heat vs. temperature obtained from DSC test for PA6 (Akulon F223D).

Figure 228- Specific heat vs. temperature for PA6 (Zytel 7301) [119].

Similar DSC tests with the heating rate of 10°C/min were conducted to measure the temperature-dependant specific heat for PC (Makrolon AL2647 from Bayer). Knowing the weight of the sample used for this test (6.948 mg) and incorporating the correction
factor, the plot in Figure 229, which is the relation between the heat flow and temperature, is modified to obtain the temperature-dependent specific heat shown in Figure 230. The correlations between the specific heat and temperature were obtained from Figure 230 for PC and used as input to the thermal model. They are listed in Table 10.

Figure 229- Heat flow vs. temperature obtained from DSC test for PC (Makrolon AL2647).

Figure 230- Specific heat vs. temperature obtained from DSC test for PC (Makrolon AL2647).
Bathe et al. [208] believed that latent heat had considerable effects on temperature distribution during phase changes and solid-phase transformations. They recommended an iteration method for enthalpy changes during the process to evaluate this term. It was suggested latent heat could be considered constant during a welding process. They believed the effects of phase transformations were only considered when the transformation happens at very low temperatures. Few reliable data are available for latent heat of polymers because of the difficulties of defining this parameter. Krevelen [206], Bicerano [207], and Baird [209] presented a temperature-dependant specific heat for solid and melt states of polymers as follows:

\[
_{s}C_{p}(T) = _{s}C_{p}(25^\circ C)(0.106 + 3 \times 10^{-3}T) \] \tag{64}
\]

\[
_{m}C_{p}(T) = _{m}C_{p}(25^\circ C)(0.64 + 1.2 \times 10^{-3}T) \] \tag{65}
\]

where \( T \) is temperature in Kelvin. \(_{s}C_{p}(25^\circ C)\) and \(_{m}C_{p}(25^\circ C)\) are experimental heat capacities of polymers for the solid and melt states of the material that were presented in a table for different polymers [206]. For PA6, these values were 1470 and 2140-2470 J/kg\(^{\circ}\)C, respectively.

They suggested Equation 64 could be adopted for semi-crystalline materials when the temperature is below the melt temperature. For temperatures above the melt temperature, Equation 65 can be used. For amorphous materials though, Equation 65 can be employed for temperatures beyond the glass transition temperature.
B.1.2 Thermogravimetry Analysis (TGA)

TGA is a thermal analysis technique in which change of the material mass is recorded as the material is heated with a fixed rate of temperature increase. The degradation temperature is obtained when the heated material starts losing its mass or the heat flow starts decreasing. TGA tests were carried out for the PA6 (Akulon F223D from DSM) and PC (Makrolon AL2647 from Bayer) samples. The heating rate of both tests was set to 10°C/min. TGA results suggest that the estimated degradation temperature for PA6 is 425°C (Figure 231). TGA test for PC suggests that the estimated degradation temperature for PC is 446°C (Figure 232). The author of this thesis is aware that the degradation temperature for the TGA tests is much smaller and the plastic part is exposed to heating for a longer time; in addition, degradation does not always happen with the weight loss.

![Heat flow vs. temperature obtained from TGA test.](image)

Figure 231- PA6 heat flow vs. temperature obtained from TGA test.
Figure 232- PC heat flow vs. temperature obtained from TGA test.

B.1.3 Thermal Conductivity

Temperature-dependant thermal conductivity for a semi-crystalline material (e.g., PA6) with 35% crystallinity is shown in Figure 233. There are other studies as well that suggest a constant thermal conductivity could be used for PA6 due to the small change of thermal conductivity with respect to temperature [119].

Figure 233- Thermal conductivity vs. temperature for PA6 [206, 207].
B.1.4 Density

Figure 234 shows the temperature-dependent density for some grades of PA.

Figure 234- Density vs. temperature for [211].

B.2 Optical Properties

This section describes the experiments conducted to measure the optical properties of the polymers used in this study. These properties are surface emissivity, transmission, and surface reflectivity.

B.2.1 Plastics Emissivity Measurements

For the thermal camera to determine surface temperature, surface emissivity of the plastic plaques needs to be specified in the ThermaCAM® Researcher software. Experiments were conducted to determine the emissivity of the plastic samples. For the thermal imaging experiments to be accurate, the plastics must be opaque to the radiation in the wavelengths important in this study. The radiation wavelength depends on the temperature as described by the Planck function [224]. In this study, plastic surface temperatures up to approximately 70°C for the lap-joint tests were of interest.
The plastic surface emissivity was estimated by heating the laser-transmitting and absorbing PA6 plaques to a known temperature monitored by thermocouples embedded in the samples and then viewing the surfaces with the thermal camera (ThermaCAM SC 1000). Miniature thermocouples (type K, 0.005” diameter, and 1 s response time) were inserted in PA6 plaques (laser-transmitting and absorbing parts) (Figure 235a). To ensure accurate positioning of thermocouples in the plastic, a stepper-motor was used to move the thermocouples1. The junction of each thermocouple was gripped between two plates of aluminum and copper. An electric current of 0.75 A (determined by trial-and-error experiments) was then passed through the thermocouple joint; the temperature of the joint increased consequently. The plates connected to a stepper-motor moved the thermocouple joint very slowly towards the plaque. When the joint touched the plaque, it melted the spot and entered the sample. After the plastic cooled down, the thermocouple location was secured (see Appendix I for further information).

The samples were then put and secured inside a desiccator (Figure 235b). The sample holder that was mounted on top of a one-quarter-inch-thick aluminum plate made it possible to rotate the sample to study the effect of the sample orientation on the test results. A heat source (Figure 236a) inserted in a cylindrical heat sink was attached to the aluminum plate to warm up the insulated desiccator (Figure 236b). The temperature of the heater was controlled by means of a temperature control unit.

The temperature shown by the thermocouple reader (Figure 237a) was then compared with that obtained from the thermal camera (Figure 237b). The surface emissivity setting in the camera was adjusted until the temperature indicated by the

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1 Mr. Gregory Park, ex-CAMM research associate, for his assistance with inserting the thermocouples in the plastic samples in the early experimental work, is acknowledged.
imager agreed with the known sample temperature. This method showed the emissivity of the plastic to be near 0.95 for object temperatures from 22°C to 250°C, matching the thermocouples’ temperatures within 0.5°C.

Figure 235- a) The PA6 sample and inserted thermocouples, b) The PA6 sample located inside a desiccator.

Figure 236- a) The heater, b) The insulated desiccator.

Figure 237- a) The thermocouple reader, b) The thermal imaging setup for emissivity measurements.
B.2.2 Assessment of Plastics Total Transmissivity

Tests were conducted using a black body source (Figure 264) to make sure the 3-mm-thick natural plastic samples (PA6 and PC) used in the experiments were opaque to the thermal radiation at the thermal imaging wavelength (3-5 µm and 7-13 µm). In the black body test, the 3-mm-thick plastic sample was placed in front of a blackbody source with the temperature changing from 25°C to 400°C and then viewed by both thermal cameras used in the experiments (ThermaCAM SC 1000 and ThermoVision A40); no temperature increase from the initial condition was recorded in any of these experiments.

B.2.3 Plastics Transmissivity Measurements for Near Infrared Wavelength

In this study, the transmission coefficient of the laser-transmitting part (un-reinforced sample) was adopted from earlier experimental studies using a 940 nm diode laser (160 W Rofin-Sinar) conducted by Prabhakaran [119]. The diagram of this test is shown in Figure 238. Optical transmissions of PA6 and PC were obtained based on the power readings of the laser beam energy before and after passing the laser beam through the plastic plaque. The plastic plaque was 3-mm-thick and a sensitive power meter was used to record the power readings. A transmission of 51% was obtained for PA6. This value is 20% lower than the corresponding value acquired by Kagan [73]. PC showed very good transmitting properties (approximately 95%).
B.2.4 Plastics Total Reflectivity Measurements

Surface reflectivity of plastics is an input parameter to the thermal model since it directly determines the amount of heat reaching the surface of the parts to be joined. Some studies reported that surface reflection where the beam enters the laser-transmitting semi-crystalline plastic is negligible for semi-crystalline polymers [72]. In this study, the reflectivity of the plastic surfaces at the visible range was measured using a thermal imaging technique. The sample was held motionless in front of the camera and the heat source was moved to the side. As the sample was rotated, the reflection of the soldering iron on the sample surface was captured by the thermal camera meaning the sample was to some extent reflective. The temperature of the reflected iron image was recorded as 92°C (Figure 239). Using the Planck’s radiation law, a reflectivity of 6% for the laser-transmitting PA6 was obtained.
In the case of PC, it was not possible to see through the sample when holding the iron behind the sample. Reflection of the iron was observed though when moving the heat source to the side and capturing a temperature of 160°C by the camera, which resulted in a 8% total reflection for PC, which is similar to the value reported for PC in the near-infrared range [72]. These values are in the range of the blackbody (i.e., the entire spectrum).

Similar experiments were conducted with ThermoVision A40 infrared camera, which showed no light transmission from laser-transmitting or absorbing parts (PA6 and PC) for temperatures up to 450°C. In addition, similar results for reflection were observed.

**B.2.5 Plastic Scattering Measurements (30% Glass Fibre Filled Natural PA6)**

Figure 240 and Figure 241 show the transverse NPFD for line scans along the x and z directions, respectively, when the laser beam passed through 30% glass-fibre-filled PA6 plaques. Both are compared to the unscattered incoming laser beam profiles obtained by the pinhole method. As expected, the scattering induced by the presence of the glass
fibres is more significant than that without glass fibres in the case of the natural PA6 (Figure 82 and Figure 83). Despite the large scattering effect, the results still show a difference between the scattered profiles along the x and z directions, with the z-direction profile being broader due to the wider incoming beam distribution along the z direction.

Figure 240- A comparison between the z direction transverse NPFD profiles after scattering by 3 mm 30% GF PA6 plaque and unscattered laser beam profile from the pinhole method) (scanned along the x direction).

Figure 241- A comparison between the x direction transverse NPFD profiles after scattering by 3 mm 30% GF PA6 plaque and unscattered laser beam profile from the pinhole method) (scanned along the z direction).
Appendix C: Effect of Coatings for the Thermal Imaging Experiments

This appendix provides further details about the investigation into effect of coating on the thermal imaging observations of the side surface of the PA6 samples being joined by a stationary laser beam. For all coatings, the laser beam centre was positioned 3 mm away from the surface facing the ThermoVision A40. The laser was set to 1 W power and turned on for 10 s and then turned off. Thermal images were recorded from the time the laser was turned on for 100 s.

C.1 Black-Taped Plastic Plaques

Plastic samples were coated with black electric tape (0.2-mm-thick) made of Polyvinyl Chloride (PVC) film coated with a non-corrosive rubber adhesive (LR31971). The PVC tape is capable of handling the temperature range of 18°C to 90°C with an adhesion of 20 N/100 mm.

Figure 242a displays the temperature image at the end of the heating phase (t = 10 s), where the temperature shows its maximum value (71.6°C) on the viewed surface. Figure 242b shows the temperature distribution after 10 s of cooling (t = 20 s), where the joint temperature drops to 53°C.
Figure 242-Thermal imaging window for the black-taped sample at: a) 10 s (laser on), b) 20 s (laser off).

The transient temperature for the black-taped plastic samples along the laser beam centre axis (along y) is shown in Figure 243 for the laser-absorbing part and in Figure 244 for the laser-transmitting part. Curves for the laser-transmitting part show similar shape to those for the soot-coated sample (Figure 135) during the period of laser heating (up to 10 s). The sharp change in slope at 10 s (when the laser is turned off) indicates that the tape covering the laser-transmitting part is being heated directly by the laser’s side-scattered radiation.

Figure 243-Transient temperature profiles obtained from the thermal imaging experiments for different locations along the depth of the black-taped laser-absorbing part (P = 1 W).
Figure 244- Transient temperature profiles obtained from the thermal imaging experiments for different locations along the depth of the black-taped laser-transmitting part (P = 1 W).

Figure 245 shows temperature profile along the y-axis through the peak temperature point observed on the surface. The curve shape is similar to that for the soot-coated specimen except for the absence of the sharp temperature peak for the black-taped samples. It is possible that, due to the slight indentation at the joining plane between the two plaques, the air gap insulates the tape from the sample surface.

Figure 245- Temperature profiles along the depth of the black-taped plastic plaque (P = 1 W, t = 10 s).
Figure 246 shows temperature profiles along the length of the sample (x-axis) for the surface of the laser-transmitting part (y = 0 mm), interface of the parts (y = 3.2 mm), and the bottom of the laser-absorbing part (y = 6.4 mm). These profiles show similar trends to those of the soot-coated samples.

![Figure 246- Temperature profiles along the length of the black-taped plaque for different locations along the depth of the ample (y = 3.2 mm, P = 1 W, t = 10 s).](image)

**C.2 White-Taped Plastic Plaques**

To address the heating due to scattered laser light absorption by the black tape, treating the surface of the plastic parts with a coating that does not absorb energy at the wavelength of the laser and has a known emissivity at the thermal camera’s operating wavelength was the next step. It is known that the white colour behaves similarly to the black colour in the infrared range (i.e., $\varepsilon = 1.0$). The plastic parts were coated with a white electric tape (0.2-mm-thick), which had similar specifications to those of the black tape coating explained earlier.

Figure 247a displays the temperature image at the end of the heating phase ($t = 10$ s), where the temperature shows its maximum value ($47.1^\circ$C) on the viewed
surface. Figure 247b shows the temperature distribution after 10 s of cooling (t = 20 s), where the joint temperature drops to 46°C.

Figure 247- Thermal imaging window for the white-taped sample at: a) 10 s (laser on), b) 20 s (laser off).

The transient temperature for the white-taped plastic plaque is shown in Figure 248 for points on the laser-absorbing part surface and in Figure 249 for points on the laser-transmitting part surface.

Figure 248- Transient temperature profiles obtained from the thermal imaging experiments for different locations along the depth of the white-taped laser-absorbing part (P = 1 W).
Figure 249- Transient temperature profiles obtained from the thermal imaging experiments for different locations along the depth of the white-taped laser-transmitting part (P = 1 W).

Figure 250 shows the temperature profile along the y axis at the centre of the laser beam. The temperature rise within the laser-transmitting part shows less absorption of the side-scattered laser beam energy by the white tape compared to the equivalent data for the black tape. Figure 251 shows the temperature profiles along the length of the sample (x-axis) for different points along y.

Figure 250- Temperature profile along the depth of the white-taped plastic plaque at the laser beam centre (P = 1 W, t = 10 s).
C.3 Correction-Ink-Coated Plastic Plaques

The next step was to study the effect of the imperfect contact of the white tape and the surface of the plastic plaques on the thermal imaging temperature readings. In addition, it was desirable to know if the white tape absorbed some side-scattered laser beam energy. To achieve this, the side surface of the plastic plaques facing the thermal camera was coated with a thin layer of white correction ink.

Figure 252a displays the temperature image at the end of the heating phase (10 s), where the temperature shows its maximum value (44°C) on the viewed surface. Figure 252b shows the temperature distribution after 10 s of cooling (20 s), where the joint temperature drops to 42°C.
Figure 252-Thermal imaging window for the correction-ink-coated sample at: a) 10 s (laser on), b) 20 s (laser off).

The transient temperatures for the plastic plaques are shown in Figure 253 (laser-absorbing part) and Figure 254 (laser-transmitting part). Figure 255 shows the temperature profile along the y axis through the centre of the laser beam. In this case, the temperature profile shows a gradual increase through the laser-transmitting part, which is similar to the one for the white-taped plaques.

Figure 253- Transient temperature profiles obtained from the thermal imaging experiments for different locations along the depth of the correction-ink-coated laser-absorbing part (P = 1 W).
Figure 254- Transient temperature profiles obtained from the thermal imaging experiments for different locations along the depth of the correction-ink-coated laser-transmitting part (P = 1 W).

Figure 256 shows the temperature profile along the x-axis for different locations along y. This diagram shows very similar trend to the temperature profiles of the white-taped plaques.

Figure 255- Temperature profiles along the depth of the correction-ink-coated plastic plaque at the laser beam centre (P = 1 W, t = 10 s).
Figure 256- Temperature profiles along the length of the correction-ink-coated plastic plaque at the laser beam centre for different locations along the depth of the sample (y = 3.2 mm, P = 1 W, t = 10 s).
Appendix D: Convective Heat Transfer Coefficient

Figure 257 was obtained based on the correlations for free convection heat transfer coefficient presented in the open literature [177]. As it can be seen, the convective heat transfer coefficients for 10-mm-long and 20-mm-long horizontal planes are very similar (≈ 4 W/m·K). For a 9-mm-long vertical plane, a convective heat transfer coefficient of 6 W/m·K was obtained. In this study, the average of the two values was adopted. Table 23 presents the correlations used to obtain the convective heat transfer coefficient. In the table, $N_u L$ is the Nusselt number ($\frac{hL}{k}$) and $Ra$ is the Rayleigh number ($\frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu}$).

![Convective heat transfer coefficient vs. temperature for horizontal and vertical plates](image)

Table 23- Convective heat transfer coefficients [177]

<table>
<thead>
<tr>
<th>Plate Orientation</th>
<th>Correlations</th>
<th>Rayleigh Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>$N_u L = 0.54Ra_L^{0.25}$</td>
<td>$10^4 \leq Ra_L \leq 10^9$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$N_u L = 0.68 + \frac{0.670Ra_L^{0.25}}{1 + (0.492/Pr)^{9/16}}^{4/9}$</td>
<td>$Ra_L \leq 10^9$</td>
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Appendix E: Parametric Tables for the EES Runs

Table 24 shows an example for the grid size and time step selection for the LTW process using the proposed method for different power levels and absorption coefficients. Table 24 shows that as the absorption coefficient increases, the values of $\Delta T_1$, $\Delta T_2$, and $E$ increase (Figure 114, Figure 115, and Figure 116). As the time step decreases, the value for $E$ does not change for the same absorption coefficient and grid size; however, the temperature difference decreases. As the grid size decreases, $E$ decreases almost linearly; however, the temperature difference increases. The increase in the temperature difference continues until the $\Delta T_2$ and $\Delta T_1$ values obtained from the two approaches become nearly equal. Modelling results showed that the combination of $E$ under 10% with $\Delta T_2$ and $\Delta T_1$ on the order of 100°C could be adopted as the criteria for a good run.

Table 24- Parametric table for the time step and grid size studies for different power levels and absorption coefficients.

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<th>$K$ (1/mm)</th>
<th>$\Delta t$ (s)</th>
<th>$\Delta y$ (mm)</th>
<th>$P$ (W)</th>
<th>$q_2$ (W)</th>
<th>$q_1$ (W)</th>
<th>$\Delta T_2$ (K)</th>
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Table 25- Parametric table for the time step and grid size studies $\Delta t \in [0.0005 \text{ s}, 0.1 \text{ s}]$, $\Delta x \in [0.00625 \text{ mm}, 0.1 \text{ mm}]$, $P = 18 \text{ W}$. 

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Table 26- Parametric table for the time step and grid size studies, $\Delta t \in [0.002 \text{s}, 0.016 \text{s}]$, $\Delta x \in [0.00625 \text{mm}, 0.1 \text{mm}]$, $P = 18 \text{W}$.

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Table 27- Parametric table for the time step and grid size studies, $\Delta t \in [0.002 \text{ s}, 0.016 \text{ s}]$, $\Delta x \in [0.00625 \text{ mm}, 0.1 \text{ mm}]$, $P = 36 \text{ W}$.

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Table 27- Parametric table for the time step and grid size studies, $\Delta t \in [0.002 \text{ s}, 0.016 \text{ s}]$, $\Delta x \in [0.00625 \text{ mm}, 0.1 \text{ mm}]$, $P = 36 \text{ W}$.
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<td>0.00001874</td>
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<td>0.00002865</td>
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<td>1329</td>
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<td>162.4</td>
<td>166.2</td>
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</table>
Appendix F: Thermal Modelling of Soot Particles

Natural thermoplastics are to various degrees transmitting at the wavelength of the lasers used for LTW (around 1 μm) [73]. To create a laser-absorbing component, a small quantity of an absorbing agent (0.05% to 2% by weight) is normally mixed with a natural thermoplastic during extrusion. Soot particles (also known as carbon black) are the most common absorbing agents used. Thermal and optical interaction of soot particles and plastic with laser beam determines heating, melting, and consequently welding of plastics. However, there exists no thermal model, to the best knowledge of the author, in which the soot-plastic interaction has been examined for the LTW process on the micro-scale.

On the other hand, numerical studies for laser-soot interaction in combustion studies have been extensively discussed. Knowing the heat transfer from soot particles in combustion studies is of great importance. The emission from the soot particles is not desirable in jet-combustion chambers or rocket-engine bases. The estimate of this heat transfer though is complicated since it is hard to know the amount of soot particles generated, particle size, shape, or optical properties of the generated soot. Extensive research in this area has concentrated on investigating spectral optical constants of soot particles and presenting thermal models to study soot-laser interaction. Researchers have attempted to make a connection between the absorption coefficient and particle size, volume fraction, composition, and thermal response while making some simplifying assumptions regarding the surface roughness and soot particle shape [93]. The soot particles are not expected to vaporize in a LTW process as the ones involved in combustion studies since polymer begins to degrade at much lower temperature.
The purpose of this section is to investigate the thermal response of soot particles to a laser heat source for LTW applications. Some of the thermal modelling techniques suggested in the literature for the soot combustion studies have been incorporated into this thermal model. Such a model will describe the relationship between soot optical and physical properties as well as LTW process parameters. Herein, a finite volume technique is suggested to model the thermal response of soot particles surrounded by a polymer (e.g., PA6). MATLAB®6.5 was employed to solve this problem. In addition, the results are compared to the ones obtained from FEM analysis solved with a commercial code (ANSYS®). For an additional validation, the micro-scale thermal model results are compared with the macro-scale thermal models of a LTW process with similar process parameters.

F.1 Modelling Approach

Assume that a soot particle is located on the vertices of a cube (Figure 258). The side of the cube “a” is equal to the effective mean free path of the soot particles. The mass fraction and volume fraction of soot particles are related as follows:

\[ f_v = \frac{V_{\text{soot}}}{V} \approx w_{\text{soot}} \frac{\rho_{\text{plastic}}}{\rho_{\text{soot}}} \]  

where \( V_{\text{soot}} \), \( w_{\text{soot}} \), and \( \rho_{\text{soot}} \) are soot particle volume, mass fraction, and density. \( \rho_{\text{plastic}} \) is the matrix density, and \( v \) is the total volume occupied by the soot particles and matrix. For example, for a 0.2\% (wt.) soot concentration in the PA6 polymer, the volume fraction of 0.094\% is obtained (see Table 9 for material properties).

Assuming that the plastic matrix is a transmitting medium with suspended soot particles, an effective mean free path (\( a_F \)) can be defined for the soot particles, which
equals the average distance between closest neighbouring particles. This parameter is related to the soot particle volume fraction as follows:

\[ a_F = \left[ \frac{4 \pi}{3 f_v} \right]^{1/3} R_1 \]  

(67)

where \( R_1 \) is the effective radius of a soot particle. For example, for the volume fraction of 0.094%, the mean free path is 16.5 \( R_1 \).

If the effective particle diameter ranged from 13 nm to 300 nm, the mean free path would change from 0.11 \( \mu \text{m} \) to 2.47 \( \mu \text{m} \). The soot particle and the surrounding plastic were assumed to form concentric spheres, with the soot particle located at the centre. In this study, it was assumed that the soot particles do not overlap.

![Figure 258- Soot particles on the vertices of a plastic block.](image)

**F.2 Thermal Model and Boundary Conditions**

The following parabolic heat conduction equation can be adopted for the concentric spheres of this problem:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( k(r) r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k(\phi) \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k(\theta) \sin \theta \frac{\partial T}{\partial \theta} \right) + q = \rho C_p \frac{\partial T}{\partial t} 
\]  

(68)
where T is temperature, r is radial distance, k is thermal conductivity, $C_p$ is specific heat, and $q$ is heat generation per unit volume.

Due to the symmetry in the $\theta$ and $\phi$ directions of the concentric spheres, Equation 68 can be simplified as follows:

$$
\frac{1}{r^2} \frac{\partial}{\partial r} \left( k(r) r^2 \frac{\partial T}{\partial r} \right) + \frac{\partial q}{\partial t} = \rho C_p \frac{\partial T}{\partial t}
$$

To account for melting, the multiplier $\delta$ is incorporated into the heat conduction equation (Equation 69) with the values of zero at the matrix melting point and one at other temperatures as follows:

$$
\frac{1}{r^2} \frac{\partial}{\partial r} \left( k(r) r^2 \frac{\partial T}{\partial r} \right) + \frac{\partial q}{\partial t} = \delta (\rho C_p \frac{\partial T}{\partial t})
$$

In this problem, it is assumed that the material is isotropic; this assumption simplifies Equation 70 as follows:

$$
\frac{2}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial q}{\partial t} = \delta \left( \frac{1}{\alpha} \frac{\partial T}{\partial t} \right)
$$

where $\alpha$ is the heat diffusivity ($= \frac{k}{\rho C_p}$).

The discretized form of Equation 71 can then be expressed as follows:

$$
T_{i}^{j} = \frac{T_{i-1}^{j} \left( \frac{2}{r_{i}} + 1 \right) + T_{i}^{j-1} \frac{\Delta r}{k} + \delta \left( \frac{1}{Fo} T_{i}^{j-1} \right) - \frac{\Delta t}{\alpha} \frac{\Delta r}{n}}{\left( \frac{2}{r_{i}} + 2 + \frac{\delta}{Fo} \right)}
$$

where $Fo$ is a dimensionless parameter (Fourier number) and is defined as $Fo = \frac{\alpha \Delta t}{\Delta r^2}$.

$T_{i}^{j}$ is discretized temperature at time index $j$ and at radial location index $i$ (1 ... n).
Heat is transferred by conduction from the soot particle to the interface of the soot particle-matrix and from there conducted away to the surrounding matrix. In addition, heat is radiated from the interface of the soot particle-matrix to the surrounding.

It was assumed that the temperature at the centre of the sphere is finite:

\[-k \frac{\partial T}{\partial r}\bigg|_{r=0} = 0 \quad \therefore \quad T_1^j = T_2^j \quad (73)\]

In addition, the outer boundary of the surrounding plastic was assumed adiabatic:

\[-k \frac{\partial T}{\partial r}\bigg|_{r=R_1 + a} = 0 \quad \therefore \quad T_n^j = T_{n+1}^j \quad (74)\]

The temperature and heat flux continuity conditions were applied to the interface of the soot particle and plastic matrix. The heat generated at the exterior surface of the soot particle due to the laser beam energy plus the conduction of the laser beam energy coming from the inside of the soot particle, transfer heat by conduction and radiation to the surrounding plastic, which result in energy storage inside the plastic matrix as follows:

\[-k \frac{\partial T}{\partial r}\bigg|_{\text{soot-exterior_surface}} + q_{\text{soot}} = \sigma \varepsilon_{\text{soot}} (T^i - T_{\text{amb}}) - k \frac{\partial T}{\partial r}\bigg|_{\text{plastic-interior-surface}} + \delta (\rho C_p \frac{\partial T}{\partial t})_{\text{soot}} \quad (75)\]

where \( \sigma \) is the Stephan-Boltzmann’s constant \( (= 5.67 \times 10^{-8} \text{ W·m}^{-2}·\text{K}^{-4}) \), \( \varepsilon_{\text{soot}} \) is soot particles emissivity, and \( T_{\text{amb}} \) is the ambient temperature. The effect of the radiated energy absorbed is almost instant, as this mode of energy transfer does not require a medium with a high thermal conductivity to facilitate the heat transfer process.

Equation 75 can be discretized as follows:
\[
T_m^j \left( \frac{k_{\text{soot}} + k_{\text{plastic}}}{\Delta r_1} + \frac{\delta (\rho_{\text{soot}} C_{\text{p,soot}} T_m^{j-1})}{\Delta t} \right) + \frac{\Delta \rho_{\text{soot}}}{\Delta t} \left( \frac{T_m^{j-1}}{\Delta r_2} \right)
\]

where \( m \) is the radial location index at the interface of the soot particle and plastic matrix.

The volumetric heat generation inside the soot particle (\( \dot{q}_{\text{soot}} \)) is a function of the ratio of the projected area to the volume of the soot particle and plastic matrix and is defined as follows:

\[
\dot{q}_{\text{soot}} = \frac{3 \varepsilon_{\text{plastic}} \tau_{\text{plastic}} \varepsilon_{\text{soot}} (1 - \rho_{\text{plastic}})^3 P}{4 A_{\text{beam}} R_1}
\]

where \( A_{\text{beam}} \) is the laser beam area (beam width \( \times \) beam length), \( P \) is the laser beam power, \( \tau_{\text{plastic}} \) is the transmission for the plastic matrix. \( \varepsilon_{\text{plastic}} \) is the emissivity for the plastic, and \( \rho_{\text{plastic}} \) is the reflectivity of the matrix.

The laser power \( P \) is the power incident upon the upper laser-transmitting-part surface. A fraction of the power (equal to \( P \tau_{\text{plastic}} \)) reaches the upper surface of the laser-absorbing part where the soot particle being modelled is assumed to be located. The power reaching this surface is reduced by absorption and scattering in the laser-transmitting part and by reflection from its upper and lower interfaces, giving the power reaching the laser-absorbing part as \( \tau_{\text{plastic}} (1 - \rho_{\text{plastic}})^2 P \). This power is assumed to be uniformly distributed over the area of the rectangular laser beam spot.
F.3 Material Properties

Soot particles and un-reinforced PA6 were chosen for this study. Soot particle effective diameter of 1 µm was assumed. While an average individual soot particle diameter of 40 nm was reported [102], the particles generally attach to each other and create agglomerates up to several micro-meters in size when embedded during an injection moulding process.

The heat capacity of the polymer was assumed temperature-dependant and adopted from the DSC test experiments conducted by the author. Emissivity of the soot particles was assumed to be 1.0 [178].

F.4 Solution

Introducing an appropriate time step and grid size was the main challenge imposed by this problem mainly because of the small size of the soot particles. Grid sensitivity studies were conducted for the thermal model.

Defining the appropriate heat generation term was challenging due to the high gradients of the heat generation at the interface of the two concentric spheres. To address this challenge, the grid size and time step needed to be defined very carefully. A grid size of 1 nm was eventually obtained that satisfied the grid sensitivity analysis. Stable and reliable results were obtained at a very fine time step of 10^{-4} s. The solution time was approximately 15 hours on a P4 machine with 2 GB of RAM.

F.5 Results

The process parameters used for the model results presented here are based on the LTW welding tests conducted by this research group with a T-like joint configuration
2D and 3D macro-scale transient thermal models of the same process were reported by the author in [161, 169]. The T-like joint configuration is shown in Figure 211. The laser beam passes through 4 mm of laser-transmitting part before reaching the joint interface. The laser beam was defocused for these experiments and its approximate dimensions were 2.7 mm along the scanning direction and 3.8 mm in the transverse direction. The laser was set to produce 60 W power and a scan speed was 10 mm/s. The laser-absorbing part was soot-filled with 0.2\% (wt.) concentration. In the experiments, these parameters produced good welds.

Given the beam dimension in the scan direction and the scan speed, an exposure time of 0.27 s results for each point on the material surface over which the beam passes. A transmission value $\tau_{\text{plastic}}$ of 51\% is used with a reflectivity $\rho_{\text{plastic}}$ of 6\% [73].

Transient temperature for the case of 0.27 s of heating by laser and 0.1 s of cooling is shown in Figure 259. A peak temperature at the boundary of the soot particle and the plastic (0.5 µm) of 333°C is predicted. When the laser passes by the soot particle at 0.27 s, temperature within the particle begins to decrease. At the same time, the temperature inside the plastic is continuing to increase.

Figure 260 shows the radial temperature distribution when heating stops at 0.27 s. The plot shows nearly constant temperature within the soot particle, which quickly drops within 1 µm of the soot-plastic interface to about 75°C. A finite element thermal model was developed and solved using ANSYS®9 for comparison between the FEM and FVM results. An example of the results of this analysis is shown in Figure 260. It is seen that there is a good agreement between the FEM and FVM results, which verifies the FVM
approach adopted in this study. To help visualize the extent of heating around the soot particle, a colour contour plot of temperature distribution is shown in Figure 261.

Figure 259- Predicted temperature vs. time for different radial locations in the micro-scale model of a soot particle heated by a laser (r = 0.5 µm, P = 60W).

It is instructive to examine the results of the 3D macro-scale model for the same process conditions [169]. Figure 216 shows temperature distribution as a function of time near the joint interface (at y = 4.2 mm), where peak temperature of 287°C is predicted. On the other hand, the peak temperature at the soot particle centre of 338°C is predicted by the micro-scale model. It is expected that the soot particle should reach higher local temperature than the average temperature predicted by the macro-scale model.
Figure 260- Radial temperature distribution predicted for a soot particle heated by a laser 
\( (P = 60 \text{ W}, t = 0.27 \text{ s}). \)

Figure 261- Contour plot of temperature distribution predicted for a soot particle heated by a laser 
\( (P = 60 \text{ W}, t = 0.27 \text{ s}). \)
Appendix G: Laser System Specifications

The setup used for the studies reported in this thesis contained a laser source and a motion system. For the laser source, a laser was required which could provide power up to about 150 W and which could operate in CW mode (since pulsed operation is not well suited for welding of plastics). A three-axis motion system was used to move the beam and the specimen with respect to one another at a controlled speed. The speed range was up to 250 mm/s. Table 28 presents the laser system specifications.

Table 28- Rofin-Sinar DLX16 diode laser specifications

<table>
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<tr>
<th>Category</th>
<th>Characteristic</th>
<th>Specification</th>
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</thead>
<tbody>
<tr>
<td><strong>Performance</strong></td>
<td>Wavelength</td>
<td>940 nm</td>
</tr>
<tr>
<td></td>
<td>Output power</td>
<td>160 Watts</td>
</tr>
<tr>
<td></td>
<td>Output power range</td>
<td>0.5 to 160 Watts</td>
</tr>
<tr>
<td></td>
<td>Minimum focus size</td>
<td>$1.4 \times 0.7$ [mm]</td>
</tr>
<tr>
<td></td>
<td>Work distance</td>
<td>84 mm</td>
</tr>
<tr>
<td><strong>Physical Dimensions</strong></td>
<td>Laser head (length x Width x height)</td>
<td>$6.5 \times 8.37 \times 24.5$&quot;</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>19” rack mount, 6.5” high</td>
</tr>
<tr>
<td></td>
<td>Chiller</td>
<td>19” rack mount, 15.38” high</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td>Laser Head</td>
<td>50 lbs</td>
</tr>
<tr>
<td></td>
<td>Control</td>
<td>35 lbs</td>
</tr>
<tr>
<td></td>
<td>Chiller</td>
<td>85 lbs</td>
</tr>
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</table>
Appendix H: Thermal Imaging Cameras

ThermaCAM SC 1000 (Figure 262) and ThermoVision A40 (Figure 263) infrared systems were used for the experiments. The cameras were capable of observing a temperature range of up to 500°C. Table 29 and Table 30 present the specifications of the two cameras.

Figure 262- ThermaCAM SC 1000 focal plane array (FPA) infrared system.

Figure 263- ThermoVision A40 focal plane array (FPA) infrared system.

Table 29- ThermaCAM SC 1000 technical specification data (FLIR)

<table>
<thead>
<tr>
<th>Imaging performance</th>
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</thead>
<tbody>
<tr>
<td>Field of view / min focus distance</td>
<td>17°×16° / 0.25 m</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>1.2 mrad</td>
</tr>
<tr>
<td>Thermal sensitivity</td>
<td>&lt;0.07°C at 30°C</td>
</tr>
<tr>
<td>Image frequency</td>
<td>50 / 60 Hz non-interlaced</td>
</tr>
<tr>
<td>Electronic zoom function</td>
<td>2:1 , 4:1</td>
</tr>
<tr>
<td><strong>Detector</strong></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td><strong>Type</strong></td>
<td>PtSi / CMOS 256×256 FPA with variable integration</td>
</tr>
<tr>
<td><strong>Spectral range</strong></td>
<td>3.4 to 5 µm</td>
</tr>
<tr>
<td><strong>Detector cooling</strong></td>
<td>Stirling cooled to 70 K, cool down time &lt;6 minutes</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Measurement</strong></th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature range</strong></td>
<td>-10°C to 450°C (-14°F to 842°F)</td>
</tr>
<tr>
<td><strong>Extended temperature range</strong></td>
<td>up to +1500°C (2732°F), standard up to +2000°C (3632°F), optional</td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td>±2°C, ±2%</td>
</tr>
<tr>
<td><strong>Atmospheric transmission correlation</strong></td>
<td>inputs for distance, atmospheric temperature, and relative humidity</td>
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</table>

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<td><strong>File</strong></td>
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<tr>
<td><strong>Operating temperature range</strong></td>
<td>-15°C to +50°C (5°F to 122°F)</td>
</tr>
<tr>
<td><strong>Storage temperature range</strong></td>
<td>-40°C to +70°C (-40°F to 158°F)</td>
</tr>
<tr>
<td><strong>Humidity</strong></td>
<td>operating and storage 10% to 95%, non-condensing</td>
</tr>
<tr>
<td><strong>Encapsulation</strong></td>
<td>IP 54 IEC 359 (metal casting)</td>
</tr>
<tr>
<td><strong>Shock</strong></td>
<td>Operational: 25G, IEC 68-2-29</td>
</tr>
<tr>
<td><strong>Vibration</strong></td>
<td>Operational: 2G, IEC 68-2-6</td>
</tr>
</tbody>
</table>

<table>
<thead>
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</tr>
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<td>1/4&quot;-20</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th><strong>Lenses (Optional)</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Field of view / min focus distance</strong></td>
<td>2°×2.1° / 15 m 4°×4.2° / 5 m 8°×8.5° / 0.7 m 32°×34° / 0.5 m 25 mm×24 mm / 67 mm (close-up w/16° lens) 6.3 mm×6.0 mm (microscope)</td>
</tr>
<tr>
<td><strong>Lens identification</strong></td>
<td>Filters 3.9µm, 5.0µm, 3.42µm</td>
</tr>
<tr>
<td><strong>Imaging performance</strong></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>Field of view / min focus distance</td>
<td>24°×18° / 0.3 m</td>
</tr>
<tr>
<td>Spatial resolution</td>
<td>1.3 mrad</td>
</tr>
<tr>
<td>Thermal sensitivity</td>
<td>0.08°C at 30°C</td>
</tr>
<tr>
<td>Image frequency</td>
<td>50 / 60 Hz</td>
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<tr>
<td>Focusing</td>
<td>Built in focus motor</td>
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</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>Spectral range</td>
<td>7.5 to 13 µm</td>
</tr>
<tr>
<td>Detector cooling</td>
<td>uncooled micro-bolometer</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Measurement</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature range</td>
<td>-40°C to +500°C (-40°F to 932°F)</td>
</tr>
<tr>
<td>Extended temperature range</td>
<td>up to +2000°C (3632°F), standard</td>
</tr>
<tr>
<td>Accuracy</td>
<td>±2°C or ±2%</td>
</tr>
<tr>
<td>Atmospheric transmission correlation</td>
<td>inputs for distance, atmospheric temperature, and relative humidity</td>
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<table>
<thead>
<tr>
<th><strong>Image Presentation</strong></th>
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<tbody>
<tr>
<td>FireWire Ethernet output</td>
<td>8/16-bit monochrome and 8-bit color</td>
</tr>
<tr>
<td>Video output</td>
<td>RS170 EIA/NTSC or CCIR/PAL composite video</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Environmental Specification</strong></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Operating temperature range</td>
<td>-15°C to +50°C (5°F to 122°F)</td>
</tr>
<tr>
<td>Storage temperature range</td>
<td>-40°C to +70°C (-40°F to 158°F)</td>
</tr>
<tr>
<td>Humidity</td>
<td>operating and storage 10% to 95%, non-condensing</td>
</tr>
<tr>
<td>Encapsulation</td>
<td>IP 40 (determined by connector type)</td>
</tr>
<tr>
<td>Shock</td>
<td>Operational: 25G, IEC 68-2-29</td>
</tr>
<tr>
<td>Vibration</td>
<td>Operational: 2G, IEC 68-2-6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Physical Characteristics</strong></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>1.4 kg (3.0 lbs.)</td>
</tr>
<tr>
<td>Size</td>
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</tr>
<tr>
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</tr>
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<td>Lenses (Optional)</td>
<td></td>
</tr>
<tr>
<td>------------------</td>
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</tbody>
</table>
| Field of view / min focus distance | 7°×5.3° / 4 m  
12°×9° / 2 m  
45°×34° / 0.1 m  
80°×60° / 0.1 m  
Close-up: 64/150 mm (FOV=64×48 mm at 150 mm);  
34/80 mm (FOV=34×25 mm at 80 mm) |
| Lens identification | Automatic lens recognition and measurement corrections |

**H.1 FLIR Systems Research Software**

FLIR Systems Research (FSR) was the software used, first, as the interface between the user and thermal cameras and, second, for the analysis of the collected thermal image sequences. After the camera is turned on (and the internal cooler lowers the temperature of the sensor to about 70 K for the case of ThermaCAM SC 1000), the user adjusts camera’s temperature range to that of the experiment and the camera is ready to be used. The FSR captures the temperature of the specified points (or spots), lines, and areas on the surface of the object. In addition, maximum and minimum temperatures for each data collection tool (e.g., point or line) can be obtained. Using the “Histogram” function, the user can estimate the proportion of the specified line or area that is occupied by a certain temperature interval.

**H.2 Thermal Imaging Calibration**

A thermal camera captures the energy radiated from the objects in the form of the object signals. The manufacturer then translates the object signal to the temperature values by defining an internal conversion equation when setting up the camera. To obtain thermal cameras’ calibration curve and to confirm the accuracy of the camera’s internal conversion equation, each camera (ThermaCAM SC 1000 and ThermoVision A40) was
directed toward an infrared calibrator (Figure 264), which provides a target with a precisely controlled temperature and with a known emissivity ($\varepsilon = 0.95$). As the target temperature was changed from 40°C to 400°C, the temperature readings of the camera were recorded. Figure 265 presents the error of the camera temperature reading (i.e., temperature of the heat source target indicated by the camera minus the set temperature) plotted versus the set temperature. Thermal camera calibration showed that the camera temperature readings were within 2°C of the heat source set temperature over the calibration range. Note that the manufacturer specifies the target temperature accuracy as $\pm 1.4^\circ$C.

![Infrared calibrator (Omega BB703).](image)

**Figure 264-** Infrared calibrator (Omega BB703).

![Camera temperature indication error vs. temperature for the ThermoVision A40.](image)

**Figure 265-** Camera temperature indication error vs. temperature for the ThermoVision A40.
Appendix I: Thermocouples

Thermocouples were used to measure the temperature of the plastic surface during the emissivity measurements reported in Appendix B.2.1. Thermocouples are divided into the following categories based on their calibration and metal combination: E, J, K, and T thermocouples. Selection of appropriate type depends on the desired temperature range and on the type of environment where thermocouple will be installed. The response time is the time required for a thermocouple sensor to reach 63.2% of a step change in temperature. The response time depends on the thermocouples wire diameter: the thinner the wire, the faster the response time [225]. On the other hand, the thinner the wire is, the lower the maximum service temperature is [225]. Hence, a compromise needs to be considered between the time response and temperature range when selecting a thermocouple.

Both thermocouples type K and J could be applied to the experiments; however, type K was selected for the following reasons: 1) The material from which the type K thermocouples are made is stronger than that of type J, 2) For a thin thermocouple wire required (0.005"), the maximum service temperature of type J thermocouples is too low (300°C) compared to 600°C for the type K, and 3) Type K thermocouples are more linear than type J.

Type K thermocouples (Omega CHAL-005-24") with the wire diameters of 0.005" were used. Type K Data acquisition cartridges (DAQ, Omega 5B37-K-02) were used to collect the data. 24" type K extension wires (Omega RECK4-K) were employed due to the short length of the thermocouples. Type K high temperature connectors (Omega, HMPW-K-MF) were used to connect the extension wires to thermocouples.