

THEORY, RESEARCH, AND PRACTICE: DEVELOPING A MODEL FOR
TEACHING MATHEMATICS

by

Dianne LaFortune

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ABSTRACT

Dewey provides a rich context in which to develop an understanding of education as growth. By developing an understanding of education as growth, educational research can be incorporated into that context so that a more comprehensive model of education may be considered. Education as growth suggests that education is about progress rather than end states. The knowledge and skills of inquiry must be understood as part of an interconnected whole that includes the physical, social, and intellectual growth of the individual and the community. The role of inquiry in the development of concepts and habits that foster the intellectual and cultural growth of the individual and community are discussed. The work of researchers on math learning disabilities is presented and examined in light of Dewey's concepts of growth and inquiry so that the educational needs of students with math learning disabilities might be included in a model of education. The quality of the educational experiences of students with math learning disabilities has significant implications for the growth and development of all students, parents, teachers, researchers, and the community.

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CHAPTER 1: INTRODUCTION

The education system has evolved as the demands for education have changed over time. Some of the factors that have influenced the direction and focus of education include women joining the work force, the increased requirement for training and education of workers, and the human rights movement developing. However, there have always been some students who were excluded from or who were determined not capable of succeeding in the education system. The failure or limited ability of the education system to educate some students creates an ambiguous situation which becomes a dilemma for the student, the parent, the teacher, and the researcher and the larger community. Separate educational systems, one for general education students and another for special education students, reflect our limited understanding of what constitutes learning and growth. Recognizing and accepting that the needs of students with exceptionalities are a result of human variance suggests that everyone grows and develops. That said, no one is outside education. Through education, students acquire the knowledge and skills to grow (i.e. learn) and the knowledge and skills that allow them to participate in their community. Through education, parents, teachers, researchers, and the community continue to grow.

Educational policies and teaching practices that promote inclusion, such as differentiation, attempt to broaden the borders of models of education to meet the needs of students with exceptionalities. Even though every province has a policy of inclusion, the needs of students with exceptionalities remain outside the current models of education and outside general education. Special education departments are separate from

curriculum departments. Curriculum documents do not include the needs of students with exceptionalities, even when exceptionalities have a significant population being educated in general education. Throughout their education, students with exceptionalities are subjected to a wide range of teachers with an equally wide range of understanding of exceptionalities and effective teaching practices. Our understanding of what constitutes growth (i.e. learning) is limited to methods of teaching and comparisons of individuals' ability to reach goals successfully within specific timeframes or by specific methods, rather than one based on growth or progress made by the individual. The dialectic of general vs. special education becomes irrelevant once growth becomes the goal of education.

As an elementary and special education teacher, I have worked with many students with a variety of exceptionalities. However, those students who are diagnosed with a learning disability hold a unique position in education. In being diagnosed, they are assessed to have average to above average intelligence and yet typically perform 2 years below grade level. These students typically have demonstrated (through their work and behaviour) an understanding of school as being not only a frustrating experience, but a place where they might not belong. Students with math learning disabilities (MD) have an especially difficult time being understood, as "not being good at math" is generally accepted. The student who first made me aware of this situation in education, could not count beyond 7 or 8. In Grade 3, this student had virtually no understanding of number sense—magnitude (greater than, less than) or quantity, but read and wrote at grade level. A Grade 6 student did not know her basic facts, despite nightly practice, but could solve some of the most complicated problems if she was permitted to use a calculator. Other

students, could count and knew their basic facts, but did not know how to use this knowledge to solve “word” problems.

The purpose of this paper is to begin to develop a unified model of education based on the work of Dewey. Education as growth is based on Dewey’s concepts of continuity and the interconnectedness of the physical, social, and intellectual aspects of the individual and the community. The model is centred on the understanding that the education of all students is analogous and requires the understanding of exceptionality derived from research in order to meet the needs of students with exceptionality better, as well as continue to build a model of education that focuses on growth of all students. For the purposes of this paper, the discussion will be limited to the exceptionality of math learning disabilities (MD). The discussion of many of the issues related to special education research and exceptionality can be addressed through the discussion of MD. Dewey’s theoretical constructs of growth, inquiry, conceptual development, and habit formation are presented in the second chapter. A review of the current literature on MD is presented in the third chapter. The discussion presented in the following three chapters applies the work of Dewey to the research on number sense, number combinations, and problem solving. Strategy development and the role of working memory are addressed in each of these chapters. In the final chapter, the implications of Dewey’s work on developing a model of education based on growth that incorporates research findings for students with MD are discussed.

CHAPTER 2: DEWEY'S CONSTRUCTS

Dewey provides a framework that allows us to understand better the complexity of the relationships and the recurring patterns found in life, including education. He relates and integrates the pattern of physical growth found in biology to the intellectual and cultural growth of the individual to the growth or progress of community. In these patterns, common threads that are woven through the layers of human life are identified. These threads comprise the interconnected and integrated relationships of human life.

It is Dewey's recognition of the interconnectedness and integration of the various aspects of life that makes his work particularly appealing. Acknowledging that individuals are affected, to some degree, by all aspects of human life calls into question attempts to disentangle that which is acceptable or tolerated from that which is less than acceptable or not to be tolerated. The artificial dialectics of special education vs. general education, individual rights vs. societal rights, abled vs. disabled, scientific knowledge vs. experiential knowledge are dubious. To establish such dichotomies, suggests that we can disregard or ignore some aspect of humanity, of ourselves, and expect the impact will be insignificant, minimal, or worse yet, advantageous to some. The impact of limiting or isolating individuals from fully participating in the human experience limits the potential growth of both the individuals and the community. For Dewey, social and intellectual growth is based in the human experience of inquiry. The relationship between inquiry and growth—biological, cultural, intellectual—establishes the foundation upon which education and democracy are built.

Education promotes intellectual and cultural growth. Accordingly, education should be considered as developing knowledge or making progress rather than thought of as an individual being educated as an end state. The change or progress in the individuals' understanding and in the environment, as a result of inquiry, becomes the basis for subsequent inquiry (Dewey, 1927). Cultural growth occurs through the shared endeavour of inquiry, subsequent interactions between individuals or within the changed environment. The rate of growth may indicate more or less effective methods of inquiry, but it is dependent on both the conditions of inquiry and the quality of the experience.

The conditions of inquiry are established by factors external to the individual, as well as those that are internal. External or cultural factors include the environment and other individuals involved in inquiry. Internal or individual factors include conceptual understandings, habits brought to bear, and the method of inquiry selected. As interconnected and integrated parts of the community, external and internal factors assemble to delineate the situation, contextualize the method of inquiry and advance the conditions of inquiry for the future.

The quality of the experience is impacted by the conditions of inquiry, but largely determined by the cultural aspects of the community, including democracy. The shared values of a community are reflected in the external conditions of inquiry. The shared responsibility, participation, and decision-making capacity of individuals in the community speaks directly to democracy. According to Dewey, democracy improves the quality of the experience, such that growth and inquiry are fostered.

Growth

Growth reflects the experiential quality of continuity. There is a connection between the past and the future. Human growth - physical, intellectual, moral - occurs and continues as the result of changing conditions that are acted upon. "Continuity ... means continual readaptation of the environment to the needs of living organisms" (Dewey, 1916, p. 2). Patterns of growth, the ability to develop, are observable in the biological and cultural aspects of human life (Dewey, 1916). For Dewey, these aspects of human life serve as models for education. As such, education as schooling is the formal expression of biological and cultural experiences.

In this chapter, the work of Dewey is presented to illustrate and examine the natural human shared experience of learning as a form of growth. The forms of growth considered include biological, intellectual, and cultural. The intellectual and cultural aspects of growth that are integral to education are the focus of this work. Using Dewey's model of inquiry, the process of growth (e.g. education) is examined from conceptual development through habit formation to the more formalized stages of inquiry as an academic endeavour. Education as growth has significant implications for both the individual and the community.

Democracy

According to Dewey (1927), the ideal of democracy must consider:

the standpoint of the individual, it consists in having a responsible share according to capacity in forming and directing the activities of the groups to which one belongs and in participating according to need in the values which the groups sustain. From the standpoint of the groups, it demands liberation of the potentialities of members of a group in harmony with the interests and goods which are common. (p. 147)

The association of individuals, as participants of groups that interact and connect with other groups, speaks to the shared values of both individuals and groups (Dewey, 1927). According to Dewey, democracy is “primarily a mode of associated living, of conjoint communicated experience” (1938a, p. 87). Democracy requires that we consciously consider the outcome of our actions vis-a-vis the actions of others. Democracy is defined as community (Dewey, 1927).

The association with others, as individuals or through participation in interacting groups, is physical, communal, and moral (Dewey, 1927). Meeting the needs and fostering the development of all members of the community, including children and those less able, speaks to the strength of the community. Individual and communal changes are cumulative and occur as a result of reflection, discovery, invention, and experimentation (Dewey, 1927). The shared intelligence and education (e.g. knowledge) of the members of the community are reflected in the values, interests, means, and ends of the community. Knowledge “depends upon the tradition, upon the tools and methods socially transmitted, developed and sanctioned” (Dewey, 1927, p. 158). Interdependence and shared communication ensure that the outcomes sought contribute positively to the creation and continuity of the community. It is through communicated shared interests that a community of individuals develops common desires to act towards certain ends (Dewey, 1927). Those ends also speak to the shared values of the members of the community. Habits (see later in this chapter) are transmitted from one generation to the next, ensuring the continuity of the community.

Dewey argues that a truly democratic community requires “a kind of knowledge and insight which does not yet exist” (Dewey, 1927, p. 166). He states that freedom of

social inquiry and freedom of expression are two conditions or prerequisites of a democratic community. Genuine democracy assumes that communication and inquiry are integral to life in the community (Dewey, 1927).

Democracy requires that intellectual opportunities are equally and easily accessible to all (Dewey, 1938a). Equal access to knowledge suggests that the “material would have such an enormous and widespread human bearing that it ... would be an irresistible invitation to ... direct popular appeal” (Dewey, 1927, p. 183). Education facilitates adaptation (discussed later in this chapter) and fosters personal initiative (Dewey, 1938a). “Since each one of us is a member of social groups and since the latter have no existence apart from the selves who compose them, there can be no effective social interest unless there is at the same time an intelligent regard for our own well-being and development” (Dewey, 1932, p. 300). Through education, capacity for intelligent judgment and action is developed and it is this capacity that provides for democratic participation (Dewey, 1938a). Society develops and grows as a result of greater participation.

Biological Growth as Inquiry

Reproduction provides for the continuity of life or growth of the community through successive generations. The physical growth and maturity of any living organism illustrates continuity at the level of each particular organism. According to Dewey, biological functions and structures are necessary conditions of inquiry. The natural pattern of inquiry is recognizable in the organic life of an organism, including human physical growth.

At the most basic level, the integration of the environment with the living organism is naturally reciprocal. The interaction between the organism and the environment is not forced from without. The environment provides the means for living. The organism physically and biologically interacts with the environment. Simply put, biological growth occurs when less energy is spent than is restored. The intra-organic interaction of various organs and systems with the environment, both directly and indirectly, further illustrate the complexity of the biological-environmental relationship.

The biological-environmental relationship provides evidence of a general pattern of inquiry (Dewey, 1933). The pattern of observable behaviours—including, interaction, responses to stimuli, adaptation of the organism, change to the environment—all parallel and contribute to an understanding of inquiry as naturally occurring. Dewey (1938b), accordingly states that, inquiry ...

grows out of an earlier state of settled adjustment, which, because of disturbance, is indeterminate problematic (corresponding to the first phase of [organic] tensional activity), and then passes into inquiry proper, (corresponding to the searching and exploring activities of an organism); when the search is successful, belief or assertion is the counterpart, upon this level, of reintegration upon the organic level. (p.40)

The change in the environment and in the organic structures that impact further behaviour constitutes a habit. Habits compose organic learning and control over habit formation comprises reflective learning or inquiry. Stability, and individual and communal evolution are possible interim results of inquiry (Hickman, 1998). The products of inquiry are intermediate and instrumental. The pattern is sequential. “The aim of inquiry is therefore to reconstruct both found materials and available tools in ways that render them more richly meaningful” (Hickman, 1998, p. 168). There is no final and

comprehensive solution, as potential solutions to problems may become the basis of future problems.

Dewey and Memory

Dewey does not directly discuss the role of memory in education. However, the significant role that experiences play in his work, suggests that memory is central to growth. Growth, for Dewey, is based on the successive and related meanings of experiences. Experiences that are meaningful to individuals—aid in their functioning in the community—are built upon in future experiences. This suggests that individuals recognize or recall, and use information or patterns, consciously in the case of problem solving or unconsciously in the case of habitual responses. Memory is, for Dewey, found in meaningful experiences. Furthermore, Dewey's discussions about reflection and judgement support the idea that actions (including habits), intellectual or otherwise, are predicated, in part, on the knowledge based in past experiences.

In *The Later Works*, Dewey briefly describes the structure of memory. “Memory may provide a refrigerator in which to store a stock of meanings for future use, but judgement selects and adopts the one to be used ...” (Dewey, 1933, p. 215). Clearly, for Dewey, it is the use of judgement that is of paramount importance. The role of judgment in the selection of information from memory relates previous experiences to the current situation or emergency. Memories serve continuity of experience and, through shared memories that are the result of shared experience, continuity of the community is established. Access to memories appears to be immediate for Dewey, as it is judgment that requires the time to select appropriate or useful information (presumably from memory).

It is argued here that Dewey distinguishes between memories and memory. Memories constitute the information that is developed through meaningful experiences, and this includes information that is integral to concepts, habits, and inquiry. The form the information takes in terms of knowledge held or of ways of acting is critical in understanding *memories* and their role in education (see sections on working memory in chapters 4, 5, and 6). While Dewey recognizes the physical structure of memory as a container that holds memories, he does not address these structures and their role in education, any more than he recognizes the role of the hand in learning. However, it is this recognition, paired with the forms of information that adds credibility to using the work of Dewey to begin to consider the implication of research on the education of students with MD or more specifically, how to include students with MD in general education authentically. It is the process of acquiring and using memories that results in growth that is central to Dewey, rather than storage and retrieval of memories.

Cultural Growth

Both the physiological and social continuity of life are renewed through experience (Dewey, 1933). Adults not only meet the physical needs of children, but initiate them into the social group or community. Parents respond to the helplessness and dependency of their child by providing affection and a nurturing environment that fosters development (Dewey, 1938a). The first key trait of immaturity, dependency, is satisfied as a result of the social interaction between parents and their children. Immaturity creates the impetus to grow for both child and parent. Independence of the child, a failure to recognize immaturity or an unwillingness or inability to meet the needs of a child may

decrease social reciprocity and thus, capacity for growth. Parents and children develop interdependence that ideally fosters growth.

The second key trait of immaturity is plasticity. Plasticity "... means power to modify actions on the basis of the results of prior experiences, the power to develop dispositions" (Dewey, 1916, p. 44). It makes the acquisitions of habits possible. As society becomes increasingly complex, the period of childhood or dependency on parents lengthens and plasticity increasingly develops. Dewey (1916) argues that this indicates a push towards social growth:

In directing the activities of the young, society determines its own future in determining that of the young. Since the young at a given time will at some later date compose the society of that period, the latter's nature will largely turn upon the direction children's activities were given at an earlier period. This cumulative movement of action toward a later result is what is meant by growth. (p. 41)

The cultural environment is the transformation from organic to intellectual behaviour.

Dewey argues that culture is both a condition and a product of language. Language is key to transformation and is itself, as a cultural institution, in a state of transformation.

Cultural Growth as Inquiry

Children come to understand and share the aims and the habits of their social group through the process of transmission by means of communication. "The communication which insures participation in a common understanding is one which secures similar emotional and intellectual dispositions – like ways of responding to expectations and requirements" (Dewey, 1916, p. 5). Through communication, people become aware of, share an interest in, and work toward a common end to form and sustain the social continuity of their community. Social life is communication and

genuine communication is educative (Dewey, 1916). Each individual grows through their experiences and, in turn, shapes others and their environment.

Problems which induce inquiry grow out of the relations of fellow beings to one another, and the organs for dealing with these relations are not only the eye and ear, but the meanings which have developed in the course of living, together with the ways of forming and transmitting culture with all its constituents of tools, arts, institutions, traditions and customary beliefs. (Dewey, 1938b, p.48)

Language is the means of transmitting information (e.g. facts, ideas, experiences) within and between all other cultural institutions. It permeates all cultural activity. And yet language maintains its own distinctive structure. The form of language allows for abstraction. Language must be considered in its widest sense as a mode of communication. “Language did not originate association, but when it supervened, as a natural emergence from previous forms of animal activity, it reacted to transform prior forms and modes of associated behaviour in such a way as to give experience a new dimension” (Dewey, 1938b, p.62).

The commonality of language, social or intellectual, allows for communication that reflects already shared meanings. Misunderstanding occurs when there are no shared meanings. Language, insofar as it results in shared consequences, is operational. The occasion or conditions under which language is used contributes to the meanings shared. Meaning is gained in the shared experience through the use of language.

Signs and Symbols

Dewey distinguishes natural signs from artificial signs. The representative capacity of signs is limited because they exist in an actual spatial-temporal context. Inference creates the meaning or significance of a sign based on the evidence it provides.

Artificial signs are symbols which are synonyms for words. A symbol does not necessarily provide evidence of existence, it has intellectual property. “[W]ords mean what they mean in connection with conjoint activities that effect a common, or mutually participated in, consequence” (Dewey, 1938b, p. 59). It is through the cultural context that shared meanings are developed (e.g. names of objects; acts – physical, emotional, moral). Symbol-meanings may be related to one another. Inference is possible when symbol-meanings may be connected to or involved with existence.

The capacity of symbols for manipulation is more practically valuable to inquiry than the representative capacity of signs. The combination of recollections of experience with expectations that lead to intellectual exercise is possible only through the existence of symbols (Dewey, 1938b). Words or symbols make possible reasoning or ordered discourse. That is to say, that ideas and hypotheses are only possible through them. The implication of reasoning or ordered discourse suggests an existential operation if it satisfies the intellectual conditions of the propositions (Dewey, 1938b).

The use of meaning-symbols for institution of purposes or ends-in-view, for deliberation, as a rehearsal through such symbols of the activities by which the ends may be brought into being, is at least a rudimentary form of reasoning in connection with solution of problems. The habit of reasoning once instituted is capable of indefinite development on its own account. (Dewey, 1938b, p. 63)

The substantial capacity and use of symbols in social and academic endeavors transpire in experiences in the environment.

Environment

“The environment ... is whatever conditions interact with personal needs, desires, purposes, and capacities to create the experience which is had” (Dewey, 1938a, p. 44).

The teaching and learning of socially accepted attitudes and values that foster the

continuity of social life requires experiences in an environment. “[T]he environment consists of those conditions that promote or hinder, stimulate or inhibit, the characteristic activities of a living being” (Dewey, 1916, p. 11). By establishing the conditions which stimulate learning, adults create opportunities for children to become active participants in an experience. Through participation, the child will become aware of the ends and the means to meet them successfully. In the social environment, participation in a joint-activity means that all parties involved in the experience share or come to share similar values, interests, and ends.

The unconscious influence of the environment, as described by Dewey, has a significant impact on the development of the character and mind. Both are attitudes that develop and are shaped by social interaction (Dewey, 1916). “Mind is ... intentional purposeful activity controlled by perception of facts and their relationship to one another” (Dewey, 1916, p. 103). The habits of language, manners, good taste, and aesthetic appreciation that have been formed through relationships with others are examples of environmental condition that influence or shape experience. Individuals and communities are not always cognizant of the impact of access to resources or tools or the impact of modes of life that are the conditions that influence the quality of experiences.

The development and use of skills and tools is said to be intelligent when it enables humans do the three following things: to control their environment, to allow for greater meaning to be derived, and to make richer experiences possible. The internal and objective conditions of a transaction between an individual and the environment are realized in the application of skills and tools (Hickman, 1990). “The purpose of the tool is to reorganize the experience in some way that will overcome its disparity, its

incompatibility, or its inconsistency” (Hickman, 1990, p. 21). Tools, physical or psychological, are the assets brought to bear on a situation through inquiry (Hickman, 1990). The identification of a tool, its type, level, or function becomes apparent in experiences that result in learning (Hickman, 1990). The worth of a tool is found in the work it will do or the change that it will generate (Hickman, 1990). Habitual uses of tools that promote laziness, rote practice or are simply mechanical in nature are functional, lacking in intelligence, and lessen the potential value of the experience.

Experience and Growth

“The more definitively and sincerely it is held that education is a development within, by, and for experience, the more important it is that there shall be clear conceptions of what experience is” (Dewey, 1938a, p. 28). The significance of an experience, either agreeable or disagreeable, lies in its influence on the environment and subsequent experiences. An experience may be engaging but require no reflection (e.g. quenching a thirst) or be a problematic situation requiring reflection (Hickman, 1990). For Dewey, reflective experiences or situations can be either educative or mis-educative. The former results in growth, whereas, the latter arrests or distorts growth. The experiential continuum is realized in the interconnectedness of successive experiences.

Two qualities of experience recognized by Dewey are continuity and interaction. “... [C]ontinuity of experience means that every experience both takes up something from those which have gone before and modifies in some way the quality of those which come after” (Dewey, 1938a, p. 35). Through or as a result of an experience, the objective conditions of that or future experiences are actively changed. Interaction “assigns equal rights to both factors in experience – objective and internal conditions” (Dewey, 1938b,

p. 42). Together, internal and objective conditions constitute a situation, and, as such, the concepts of situation and interaction are inseparable. A mis-educative or non-educative experience is the result of internal or objective conditions failing to adapt. Continuity and interactions are integrated aspects of experience. Together, they establish the educative and qualitative value of the experience. Dewey asserts that democratic social arrangements promote a better quality of experience (Dewey, 1938a). The value of the experience to the learner impacts the development of the desire to continue with the task at hand and to go on learning. “An experience is always what it is because of a transaction taking place between an individual and what, at the time, constitutes his environment” (Dewey, 1938a, p. 43).

Developed through experience, tools are an extension and expression of the human experience (Hickman, 1990). As an expression of experience, tools provide a mode of language that frames the *operationalization* of the problem, the consequences of that *operationalization*, and the resulting possible solutions (Hickman, 1990). Thus the selection and use of a tool becomes an integral part of a situation, rather than standing outside it (Hickman, 1990). The appropriateness of the selection and use of a tool is determined through inquiry until such time as inquiry ceases (Hickman, 1990). The subsequent development, selection, and use of tools speak to the values and potential growth of the experiences of individuals and the community (Hickman, 1990).

As learners are engaged in a shared experience they form a community of learners. Attitudes towards learning are developed through and reflected in the principle of social control. “[T]he primary source of social control resides in the very nature of the work done as a social enterprise in which all individuals have an opportunity to

contribute and [in] which all feel responsible” (Dewey, 1938a, p. 56). Social control is actually located in the mind, intellect, and shared meanings (Dewey, 1916). Experiences that are educative effectively combine the principles of continuity and interactions such that growth of individuals and the community are reflected in the principle of social control. The opportunity to participate and share in inquiry is democracy. Thus, educative experiences are social and democratic.

Conceptions

The development of concepts is based in experience (Dewey, 1938b). According to Dewey, the origin of concepts is the result of a sorting of the shared and unlike qualities of a number of different particular things (Dewey, 1938b, p. 240). The search for the shared characteristics of concepts is inquiry (Dewey, 1938b). Experiences are carried over to subsequent experiences, such that expectations and anticipation arise so that the concept becomes increasingly refined (Dewey, 1938b). Through inquiry, the identity and stability of a concept is formed (Dewey, 1938b). “Synthesis is the operation that gives extension and generality to an idea, as analysis makes the meaning distinct” (Dewey, 1933, p. 242). Conceptions, stored in memory, are general because of their use and application in novel situations that further knowledge (Dewey, 1933). “Dewey located generality in the activity of productive inquiry, in operations performed with a view to determining relationships, that is, in *ways* (including the expression of habits and the use of instruments) of undertaking particular inquiries” (Hickman, 1990, p. 129).

As “*standards of reference*,” concepts, stable and standardized meanings are shared between individuals (Dewey, 1938b).

[F]or practical purposes it is often enough to know what *kind* of thing it is; knowing that fact, we can bring into play the habits of thought and behaviour that belong to every member of the entire class. The concept calls into play whatever is appropriate to a large number of cases previously known, thus freeing thought from the preoccupation with finding out what *this* is. (Dewey, 1933, p. 236)

All information or knowledge of the concept can be related to all other information or knowledge of the concept at both the level of the individual and that of the community (Dewey, 1933). Individuals with different cultural, social, and generational experiences are able to communicate as a result of shared meanings of concepts. It is the shared understanding of a concept or the shared accepted standardized meaning that permits individuals to communicate and to act similarly to resolve situations (Dewey, 1933). And in turn, that knowledge becomes a part of the greater system of knowledge of individuals and the community.

Habits

Whereas, conceptions allude to the *knowing*, habits entail the *doing* (Hickman, 1990). “[H]abit means an ability to use natural conditions as means to ends” (Dewey, 1916, p. 46). Both the individual and the environment are modified when there is active control of means to achieve an end. Habits are actively formed and practiced as we encounter stimuli in the environment.

[W]hat individuals are inclined to select as important from the busy environment in which they find themselves, and the values or goals that they are capable of projecting and toward which they are willing to work, are specific activities of production that are conditioned by the larger pattern of production called an occupation. Perceptions, goals, and ideals will thus vary among hunters, shepherds, military types, traders, those engaged in crafts, and those involved in manufacture. What seems not to vary, however, is that there are occupations that condition habits, which in turn condition specific activities. (Hickman, 1990, pp. 85-6)

Habit formation requires both efficiency and effective control of the environment. An environment that provides stimuli which promote the use of intelligence in the process of developing habits fosters growth. According to Dewey, adaptation refers to both “... adaptation *of* the environment to our own activities [and] of our activities *to* the environment” (Dewey, 1916, p. 47).

“The basic characteristic of habit is that every experience enacted and undergone modifies the one who acts and undergoes, while this modification affects, whether we like it or not, the quality of subsequent experiences” (Dewey, 1938a, p. 35). Habit formation requires plasticity, making possible the “formation of intellectual and emotional disposition as well as an increase in ease, economy, and efficiency of action” (Dewey, 1916, p. 48). Habits require levels of thought or reasoning, observation and reflection in accordance with the demands of the task. According to Dewey, judgement is required in an emergency to select and use data or ideas stored in memory (Dewey, 1933). The varied and elastic use of a habit speaks to the intellectual and internal component of habits, as well as to continued growth. External components of habits are the intentional and considered action or interaction with the environment.

[I]ncreased human freedom lies in the continuing reconstruction and reassessment of the results of previous decisions in order to maintain appropriate adjustment with respect to the overlapping environmental demands. This task is rendered difficult wherever specific tools [skills, institutions, and apparatus] are habitually coupled with particular tasks ... because such situations facilitate acquiescence to solutions that are not adequately adjustive. There is a tendency in such situations to muddle through with existing tools, rather than face the task of developing new ones. (Hickman, 1990, p. 164)

“Habituation is ... our adjustment to an environment which at the time we are not concerned with modifying, and which supplies leverage to our active habits” (Dewey, 1916, p. 47). An aversion to change or a lack of new stimuli required to foster the

development of habits limits growth. “Routine habits, and habits that possess us [bad habits] instead of our possessing them, are habits which put an end to plasticity” (Dewey, 1916, p. 49). The efficient repetitive and routine practice of habits, unconsidered responses, limits growth. “[G]rowth depends upon the presence of difficulty to be overcome by the exercise of intelligence” (Dewey, 1938a, p. 79).

Inquiry

When rules or habits are insufficient to respond satisfactorily to a situation, reason is required. The situation then, requiring inquiry, becomes problematic. A problem that does not arise out of a situation is busy work. “A problem represents the partial transformation by inquiry of problematic situation into a determinate situation” (Dewey, 1938b, pp. 111-112). The problem, as defined, determines in large part the conceptual development of parts of the problem, selection of data to be used, and the criteria by which hypotheses are selected.

In developing a pattern of inquiry considerations of the kinds of inquiry that have and have not worked is made. Dewey states that there are some methods of inquiry that are better than others. Previous experience dictates which methods are best suited to achieve certain ends (Dewey, 1938b). These methods then become the basis for further endeavours. “Inquiry is the controlled or directed transformation of an indeterminate situation into one that is so determinate in its constituent distinctions and relations as to convert the elements of the original situation into a unified whole” (Dewey, 1938b, p. 108).

The indeterminate situation is one in which there is an imbalance in the interaction between organism and environment. An indeterminate situation is a doubtful or troubling situation. Active and operational endeavours reduce the indeterminate situation to a unified situation to be acted upon temporally (Dewey, 1938b). As a practical and productive skill, inquiry requires the use of tools, whether abstract or concrete (Hickman, 1990, p. 47). Inquiry at this stage requires that existential consequences, possible impact of environmental conditions and potential responses be considered in light of the unified situation.

The formation of a genuine problem requires that its constituents be identified. Constituents are the operational conditions that arise out of ordinary human transactions or interactions with each other and the environment; they are the observable facts of the case that must be taken into account in resolving the problem (Dewey, 1938b). The existence of constituents makes them observable in real terms or symbolically through language. Other constituents may come to bear on the situation and are also considered. Suggestions of possible solutions occur as impulsive, not necessarily logical, responses to the problem (Dewey, 1938b). Through reasoning, both the constituents observed and the means to resolving the problem are logically developed. Then a suggestion becomes an idea. "Ideas are anticipated consequences (forecasts) of what will happen when certain operations are executed under and with respect to observed conditions" (Dewey, 1938b, p. 113).

Suggestions and ideas exist symbolically, and it is this existence which makes them observable and mutable. It is the use of symbols in rational discourse that ties a particular problem to previous experience. Without symbols, there can be no idea

(Dewey, 1938b). “[T]he idea or meaning when developed in discourse directs the activities which, when executed, provide needed evidential material” (Dewey, 1938b, p. 115). For Dewey, the problem, as it is perceived, and the suggestions or ideas as they are conceived are determinates of the situation and the whole is valued only insofar as the combined capacity allows us to arrive at a unified situation.

“Ideas are operational in that they instigate and direct further operations of observation; they are proposals and plans for acting upon existing conditions to bring new facts to light and to organize all the selected facts into a coherent whole” (Dewey, 1938b, p. 116). Facts are operational in the sense that they are selected, described and, as such, provide meaning for the problem (in and of themselves and as they relate to one another), its resolution and the validity of the resolution. Being functional, facts “serve as evidence and their evidential quality is judged on the basis of their capacity to form an ordered whole in response to operations prescribed by the ideas they occasion and support” (Dewey, 1938b, p. 117).

Ideas and facts interact through their operational forces in experiment. Both are tested and may be dismissed, reordered or modified. Suggestions, ideas, and possible solutions are expressed in symbolic form. Facts are formulated as propositions (see Chapter 6, Problem Solving and the Environment), in symbolic form, to facilitate inquiry. The operations involved in an inquiry result in an objectively unified existential situation. According to Dewey, facts and ideas in symbolic form are operational, as they are the conditions of inquiry required to carry out experiment, converting the elements of the original situation into a new unified whole.

Discussion

It is through the work of Dewey that the thread can be woven from biological growth through cultural growth to more formal of expressions of growth, such as education. Growth is the result of an individual's experience. The experience is partially described by the use of language and symbols, availability and use of tools, interaction with other individuals and with the environment. The experience or situation, as it may come to be, is further delineated by the conceptual understandings, habits, and skills of inquiry brought to bear by the individual. The interconnectedness of the various forms of growth is reflected in the concepts of interaction and continuity.

The experiences of individuals become the basis for future experiences. Individual growth, development or change is characterized in changes in conceptual understandings, habits, and skills of inquiry. Social growth, development or change is observable in the changing systems of language, symbols, tools, and the environment. The results of previous experiences become the basis for future experiences. It can be difficult to distinguish individual from cultural growth when both are integral to growth. The experiences of individuals woven together contribute to and comprise cultural growth, which in turn impacts environmental factors in which future experiences occur.

Individual growth is dependent on participation in experiences. Access and adeptness to use language, symbols, tools, as well as interacting with others in the environment determine the ability to participate, as well as the quality of the experience for individuals. Individuals' formal education determines in large part their adeptness and access to resources, including their own conceptual understandings, habit formations, and skills of inquiry. Formal education is an extension of and reflects culture. The more

democratic an experience, whether formal or informal, the greater the potential is for growth of individuals and society.

CHAPTER 3: REVIEW OF RESEARCH ON MATH DISABILITIES

The purpose of this chapter is to develop an understanding of the research findings on math learning disabilities (MD) in primarily elementary school aged children, and the possible implications for instruction or remediation. The research presented in this chapter does not address students with physical disabilities, profound intellectual disabilities or emotional behavioural disorders. After considering the many possible definitions of MD, math disabilities are defined, for the purposes of this paper, as a significant weakness in mathematics relative to performance in other academic domains or in cognitive domains, and in comparison to normally achieving peers, and may co-exist with a similar weakness in reading. The definition of MD and its diagnosis, the implications of deficits in working memory, and math performance of students with MD are discussed. Deficits in working memory and its component parts are identified and discussed as possible explanations for MD and for the identification of MD subtypes. For the purpose of this discussion, math performance is limited to number sense, number combinations, problem-solving, and strategy development.

Research in the field of MD continues to develop and provide insight into MD. It provides less than a comprehensive understanding of the learning experience of students with MD, the underlying causes, or effective teaching methods. Defining and identifying MD is problematic. Various cut-off points for identifying MD, as well as numerous research methodologies, make comparisons between studies difficult. The impact of other disabilities on math performance is not always fully developed or considered in the literature. The operationalization of key terms or concepts (i.e. number sense, number

combinations, problem-solving, problem) is often inconsistent from study to study. Furthermore, the interconnectedness of number sense, number operations, and problem solving is not well developed in the literature. For research purposes, difficulties in learning math are often isolated from learning in the general education classroom, as well as from the community of learners that comprise the classroom.

Math Learning Disabilities

There appears to be consensus in the literature that approximately 6% of students may be identified as having a math learning disability (MD) (Geary, 1993; Ginsburg, 1997; Kosc, 1974, Mazzocco, 2001; Shalev, 2001). Given the amount of time devoted to math instruction in schools it is difficult to understand why failure in math is not considered as disabling as failure to read (Kosc, 1974). In fact, Ansari and Karmiloff-Smith state that “dyscalculia is more prevalent than dyslexia” (2002, p. 511). Elementary school teachers (classroom and special education teachers), administrators, and parents appear to have limited knowledge of MD and of the co-morbidity with reading disabilities (RD). It appears that special education resources tend to be directed at students with reading difficulties, behavior disorders or more generalized learning disabilities (LD) (Ginsburg, 1997).

Neurological and Neuro-psychological Perspectives

Stated simply, the neurological approach to developmental dyscalculia (DC) strives to localize the areas of the brain involved in performing math. In 1974, Kosc (p. 165) defined DC as: “... a structural disorder of mathematical abilities, which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct

anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions.”

Recognition of and research on DC has a relatively long albeit sparse history. Rourke and Conway (1997) provided a brief overview of the neurological perspective and included the work of Gall in the 1700s, Broca in the 1800s, and Berger and Gerstmann in the 1920s and 1930s. More recent research on brain development indicates that brain development moves from back to front, that the brain (temporal, parietal, and frontal lobes) continues to develop well into early adulthood and, that through the process of “pruning,” specific areas of the brain develop while other areas of the brain do not (Giedd, et al., 1999). Identifying behaviours (e.g. types of mathematical errors) with respective corresponding areas of the brain has and will continue to further the research on MD (Geary, 1993). Additionally, the identification of the involvement of specific areas of the brain for component processes of math may aid in the identification of MD subtypes and may provide a possible explanation for the observable disassociation of particular math skills and abilities apparent in students with MD (Rourke & Conway, 1997).

Drawing conclusions from studies by Rourke and colleagues from the 1970s until 1993, Rourke and Conway (1997, p. 40) concluded that: “Arithmetic LD can result from at least two very broad classes of neuropsychological impairment, one based on verbal deficiencies (probably reflecting relatively dysfunctional left-hemisphere systems) and one based on nonverbal deficiencies (which appear to reflect the phenotypical outcome of early impairment of, or lack of access to, systems within the right hemisphere).” They argued that it may be the case that some children displaying arithmetic difficulties have

difficulties retrieving number facts from semantic memory or have difficulties with computation. It appeared that both of these difficulties were attributable to verbal factors and may constitute two or more subtypes of MD. In contrast, "... children whose difficulties with arithmetic are attributable to primarily non-verbal deficits may represent both a visual-spatial subtype and a nonverbal concept formation/adaptive reasoning subtype" (Rourke & Conway, 1997, p. 38).

Corroborating the work of Geary on the identification and classification of MD subtypes, Mazzocco (2001) considered three and found support for two subtypes in her research on students with Turner Syndrome, Fragile X Syndrome or Neurofibromatosis (Type 1). The impact of the different neurological profiles on math performance enabled Mazzocco (2001) to identify subtypes according to the different areas of the brain affected by the various diagnoses. The identification of subtypes was based on both student performance profiles and neuro-psychological profiles. Semantic memory math disability may coexist with RD, and was characterized by poor retrieval of math facts in terms of quality of answers and response times. Procedural math disability was characterized by use of immature problem-solving strategies, poor implementation of strategies and poor understanding of associated concepts. Visual-spatial math disability (characterized by difficulty with sign confusion, omission or rotation of numbers, difficulty aligning numbers and general difficulty with place value) may be different both cognitively and neurologically from the other two subtypes and could not be supported by the findings. Both Geary (2004) and Mazzocco (2001) argued that genetics does contribute to MD.

Having cited the work of researchers, in twin studies, on the co-morbidity of MD and RD, Shalev (2001) posited that genetic factors contributed to MD or developmental dyscalculia (DC). In fact, in her study of students identified with DC, Shalev found a significantly higher prevalence (10 times more likely) of DC amongst family members than the expected 6% found in the general population. Not surprisingly, she also found an association between reading and math difficulties. However, environmental factors, such as the quality of instruction at school and availability of resources at home, as well as a limited number of fathers participating in the study may have confounded results.

Definitions and Identification of Math Learning Disabilities

Understanding of the cognitive features required for development of age appropriate math skills remains somewhat limited. The co-morbidity of math disabilities with other disabilities, particularly RD and generalized learning disabilities, makes defining and identifying MD all the more difficult (Geary, 1993; Kosc, 1974; Mazzocco, 2001). There may be core deficits that impact both reading and mathematic skills and development. “Children with primary deficits in areas other than mathematics may demonstrate secondary deficits that are indistinguishable from primary math difficulties when evaluated for MD with standardized tests” (Mazzocco, 2005, p. 320).

Using rating scales to identify students with learning disabilities (LD) who also demonstrate weaknesses in math, Bryant, Bryant and Hammill (2000) concluded that classroom teachers of students aged 8 –18 were able to identify the math behaviours indicative of MD that are often observed by researchers in controlled settings. A group of characteristic math behaviours of students with LD and with weaknesses in math was found to be statistically significantly different from students with LD without difficulties

in math (Bryant et al., 2000). The top ranked behaviours of students with math weaknesses included difficulties with solving word problems, difficulties with multi-step problems, difficulties with the math language, and failure to check or verify their answers (Bryant et al., 2000). Co-morbidity may have been implicated in the observable behaviours of the students with LD and math weaknesses. Furthermore, students who are not identified with LD may present MD differently.

Math achievement assessments can be based on either criteria or on discrepancy from the norm. Using various IQ-based discrepancy methods of identification with different cut-off points, Silver, Pennett, Black, Fair and Balise (1999) demonstrated that regardless of the method of identification or the definition of learning disability used for identifying students aged 9-13, there were very few practical differences in the number of children who would be identified. These findings, while not specifically related to the definition or identification of MD, highlight the complexities of defining and identifying LD that appear to extend to MD. Geary (2005) suggested that a cut-off at the 35th percentile and below would reflect our current understanding and ability to measure or identify students with MD. Various cut-off points, especially higher than the 35th percentile, may be identifying students with low achievement scores for reasons other than MD (Geary, 2005). In fact, Silver et al. found that the identification of students with both MD and RD was more stable than that of students identified with only MD (1999).

Geary (2005) cautioned that standardized achievement tests sample and average across a broad range of skills and as a result tend to overestimate competencies in some areas and underestimate them in others. Research in the field of MD and math skill development provides “evidence of continuous development and of normally distributed

performance scores” (Mazzocco, 2005, p. 320). However, resistance to instructional interventions vis-à-vis retrieval deficit continues to appear to be a useful indicator of arithmetical forms of MD (Geary, 2004). The lack of homogeneity of learning and performance profiles of students with MD suggests that subtypes may be identifiable. Many students thought to have MD or to be at risk of failing math, later show marked improvement and can no longer be identified as having MD (Kauffman, Handl & Thony, 2003). However, Jordan et al. (2003) posited, after having assessed Grade 2 and 3 students longitudinally, those Grade 2 children with persistent deficits in fact mastery show weaknesses in nonverbal reasoning. “Children who have lower than expected achievement scores across successive academic years often have some form of memory or cognitive deficit, and a diagnosis of MD is often warranted” (Geary, 2004, p. 4).

Working Memory

Numerous studies have examined the role of working memory (WM) in students performing poorly in math. Deficits in WM may be directly related to deficits in the underlying cognitive systems of the central executive (Geary, 2004). In either the language system or the visuo-spatial system, deficits may be found in information representation or information manipulation (i.e. WM).

Short-term memory refers to limited capacity for temporal storage of information, whereas working memory combines both storage and processing of information (Baddeley, 1998, 2001; Sluis, Leij & Jong, 2005).

Working memory refers to a temporary storage of information while other cognitive tasks are being performed. ... [W]orking memory requires both the simultaneous processing of incoming information and the retrieval of other information. ... [W]orking memory may be somewhat domain specific, and ... the nature of the material to be remembered may determine the efficiency of memory. (Siegel & Ryan, 1989, p. 973)

Working memory is comprised of the central executive (CE) that interacts with two storage systems - the phonological loop (PL) and the visuospatial sketchpad (VSSP) (Baddley, 1998, 2001; Geary, 2004). The CE is a limited capacity system that regulates the processes comprising WM. According to Baddley, it is “a controlling attentional system [that] supervises and coordinates a number of subsidiary slave systems” (1998, p. 52). Based on research, Baddley (2001) more recently described four roles of the CE—focus attention, divide attention, switch attention, and “form an interface between the subsystems and LTM” (p. 857). The episodic buffer is “... capable of combining information from the LTM with that from the slave systems” (Baddeley, 2001, p. 859). The role of the CE in the sharing of information between the two slave systems has yet to be established (Baddeley, 2001).

Ashcraft, McLean and Hitch (1999) describe the role of the central executive as that of initiating and directing processing, comprehension, and retrieval from long-term memory (LTM). The CE coordinates performance on two or more tasks, selecting and switching strategies for retrieval of information, selectively filtering different inputs and locating and manipulating information in long term memory in complex arithmetic problems (McLean & Hitch, 1999). Measures of CE functioning include digit span backward where participants are required to transform or manipulate stimuli before they are recalled or used to complete a task (Sluis et al., 2005).

The PL is used for the storage of verbal information (Baddeley, 1998, 2001). Research indicates that it plays an important role in the comprehension of language, the acquisition of vocabulary, and learning to read (Baddeley, 1998, 2001). “The phonological loop is assumed to comprise two components, a phonological store that is

capable of holding speech-based information, and an articulatory control process based on inner speech” (Baddeley, 1998, p. 52). Baddeley (1998, 2001) posits that factors that influence memory span include word length, unattended speech, articulatory suppression, and acoustic similarity. PL capacity is measured by span tasks where participants are asked to repeat strings of material, including digits (Sluis et al., 2005). During the arithmetic process, the PL is thought to play a role in the memorization of numbers (Sluis et al., 2005). Comparing the performance of poorly performing math students (cut off of 25% on Graded Arithmetic-Mathematics Test) to age matched students on the digit span task, McLean and Hitch (1999) concluded that the phonological loop is involved in counting and holding information during calculations.

The visuospatial sketch-pad (VSSP) is used for the storage and manipulation of visuo-spatial information (Baddeley, 1998, 2001). Reporting the results of various studies, Baddeley (1998) concluded that brain imaging research supports the hypothesis that there may be separate visual and spatial components of imagery. It may comprise two areas responsible for the storage of either static visual information (i.e. colour) or dynamic visual information (i.e. direction of movement) (Sluis et al., 2005). “[I]t can be fed either directly through perception ... or indirectly ... through the generation of a visual image” (Baddeley, 1998, p. 79). Problems with the spatial representation of multi-digit (columnar positioning) appear to involve the VSSP (McLean & Hitch, 1999).

“The multicomponent approach to working memory aims to understand the way in which information is temporarily stored and maintained in the performance of complex cognitive processing” (Baddeley, 2001, p. 859). Working memory and its component parts appear to have specialized roles in math. Deficits in WM present a far greater

problem for students than simply requiring additional instruction. Poor math performance due to deficits in WM would dictate a different approach to instruction and remediation than poor performance due to developmental delay or poor instruction.

Number Sense

Gersten, Jordan and Flojo (2005) indicate that there is little consensus with regards to the definition of number sense. “[N]umber sense reputedly constitutes an awareness, intuition, recognition, knowledge, skill, ability, desire, feel, expectation, process, conceptual structure, or mental number line” (Berch, 2005, p. 333). Genetically inherited or the result of experience, number sense may be critical to the field of MD. The former position creates an understanding of number sense that is biologically based. The latter creates an understanding of the role of number sense in the conceptual development of mathematics and as having a role in higher order thinking. The relationship between visuo-spatial reasoning and number sense development is unclear (Mazzocco, 2005). Number sense, as a representational system, interacts with cognitive systems (Berch, 2005).

Citing the work of Anasari and Karmiloff-Smith and others, Chiappe (2005) explains how young infants demonstrate an understanding of quantity and that that understanding changes and develops. As early as 4 months, infants are sensitive to changes in quantities of 3 or fewer and at 11 months, can compare and discriminate large differences in sets of stimuli (Chiappe, 2005). These precursor representations provide a foundation for the development of mature representations of number. “Delays or failures in the acquisition of mature representations [of number] may underlie the range of deficits shown by children with MD” (Chiappe, 2005, p. 314). In fact, Anasari and

Karmiloff-Smith (2002) posited that two systems are involved in the representation of numbers – an analog system and an exact system. The analog system approximates quantity and is independent of language and culture. The exact system is accurate and depends on language to represent numbers.

After reviewing a number of studies, Gersten et al. (2005) concluded that there are three promising measures that may predict MD: quantity discrimination or magnitude comparison, identifying the missing number in a sequence, and number identification. However, as Dowker (2005) points out, there is little known about the order of the development of concepts and skills in young children and about later development of those same concepts and skills (Mazzocco, 2005). Developmental delay in procedural strategies suggests students experiencing difficulties may catch-up or be accommodated. Developmentally different students may experience difficulties with procedural strategies that do not necessarily reflect weak conceptual understanding, poor skill development or specific cognitive deficits (Geary, 1993; Gersten & Jordan, 2005; Jordan, Hanich, & Kaplan, 2003). Rather, retrieval deficits may be impacting their development (Geary, 1993; Jordan et al., 2003).

Research findings suggest that a pre-K-2 math program, *Number Worlds*, is effective in promoting children's conceptual understanding of number and number sense (Griffin, 2004). Designed to teach number sense to children from low income communities and identified as at-risk, it provides teachers with a substantial set of tools that enables them to develop a better understanding of children's mathematical development and to address gaps or deficits in that development effectively (Griffin, 2004). Based on theories of learning and cognitive development, it builds upon children's

current knowledge and provides hands-on problem solving opportunities, fostering the development of mathematical language by requiring children to respond to several questions throughout the problem solving activities (Griffin, 2004). Furthermore, it exposes children to the many ways in which *number* is represented in society and teaches computational fluency (Griffin, 2004). “In contexts such as these children receive ample opportunity to use the formal symbol system in increasingly efficient ways to make sense of quantitative problems they encounter in the course of their own activity [inside and outside of school]” (Griffin, 2004, p. 177). The opportunity to learn through participation in *Number Worlds* suggests that experiences can be created that teach, or at the very least promote the development of number sense.

Reviewing research on students with low SES, Gersten et al. (2005) argue that the development of number sense may be related to the informal math instruction or experiences at home before students attend school. Consequently, Gersten et al. (2005) suggest that number sense, like phonemic awareness, should be teachable. There are several preschool and primary intervention programs, such as *Number Worlds*, that appear to be promising (Dowker, 2005). However, most programs are still undergoing development and do not specifically target children with MD. Comparison of research on the effectiveness of such programs is complicated by the lack of consensus regarding the definition of number sense. The long-term effectiveness of any remedial program or intervention has yet to be established.

Math Performance: Counting

In the 1970s and 1980s researchers studied the underlying skills that comprise the ability to count. Gelman and Gallistel identified five principles of counting that preschool

children adhere to: one-to-one correspondence, stable order, order irrelevance, abstraction, and cardinality (Geary, 1993). The first three principles are subsumed under word-object correspondence for Briars and Siegler who argued that children count first by rote and then develop the conceptual understanding (Geary, 1993). Interestingly, Dowker (2005) found in her work with 40 four year old children that counting proficiency was unrelated to one-to-one correspondence to establish equivalency between sets. Furthermore, she found that one third of the children used an internal counting system to solve simple addition and subtraction questions, another third were used the “counting all strategy” to solve problems, and the last third were unable to cope with the tasks. Citing the work of Okamoto and Case, Gersten et al. (2005) state that “in young children, counting ability and number sense develop relatively independently” (p. 300). Regardless of how students develop the conceptual understanding of counting, many students with MD (with or without RD) continue to have counting difficulties into Grade 2 when compared to normally achieving peers or those with RD only (Geary, 2004).

Geary, Hamson and Hoard (2000), studied Grade 1 students with learning disabilities and found that students with consistently low math scores understood most basic counting principles. Of course, students with more general learning disabilities (MD and RD) were more delayed in number production and number comprehension skills than students with MD only, RD only or normally achieving peers (Geary et al., 2000). Citing the work of Briars and Siegler (1984), Geary, Hoard and Hamson (1999) reported that students with both RD and MD understood counting as a rote skill and those with only MD showed age-appropriate counting knowledge. This suggests that students with MD only may have procedural subtype MD and may be developmentally delayed, whereas

those with RD and MD may have semantic memory subtype MD and may be developmentally different. Students with MD only might be expected to respond to remedial teaching.

When Grade 2 students used counting to solve number-fact questions, students with MD only were successful (Jordan & Hanich, 2000). In contrast, students with combined MD and RD were reported as often responding incorrectly. Geary et al. (1999) found a similar pattern in Grade 1 students asked to identify if a puppet had correctly counted. This suggests that students with MD only may have stronger counting skills than students with co-morbid RD and MD.

However, students with MD, with and without RD, were less able to detect counting errors than students with only RD or no learning disabilities (Geary et al., 1999; Geary et al., 2000). Findings reported by Geary (2000) were not statistically significant and he concluded that ability to detect counting errors may be dependent on the length of the count. Students with MD may have specific deficits in the central executive component of WM, whereas students with both MD and RD may have deficits in the phonological loop component of working memory (Geary et al., 1999). Counting as a strategy to solve problems, regardless of the specific learning deficits or ability, is used by students with math learning difficulties.

Math Performance: Number Combinations

Often cited by others, the work of Geary (1993, 2004) and Geary et al. (1999, 2000) stated that a persistent deficit in basic math fact retrieval is a strong indicator of MD. Jordan and Hanich (2000) reported that students in Grade 2, regardless of math ability, did not use automatic retrieval more than 25% of the time. When children answer

quickly, they are confident that they know the answer (Jordan & Hanich, 2000). Students with poor fact mastery at the end of Grade 3 showed little growth in these skills during the second and third grades and continued to use counting as a back-up strategy (Jordan et al., 2003).

Students with MD who are good readers are more likely to have deficits with regards to math facts than their normally achieving peers (Geary et al., 1999, 2000). Grade 2 students with MD only have been found to perform better on un-timed basic math fact assessments than students with both RD and MD (Jordan & Hanich, 2000). Students with both reading difficulties and math disabilities displayed weaknesses in mastery of basic math facts and problem solving (Jordan et al., 2003). The pattern that emerges is unclear, but for some students deficits in both math and reading skills appear to be related. Those with deficits in both RD and MD appear to have more severe difficulties in math learning.

Jordan and Hanich (2000) also found that Grade 2 students with both reading and math disabilities had difficulty with regrouping. Error analysis indicated that the errors made by students with co-morbid disabilities reflected poor reasoning or judgment. In contrast, errors made by students with no disability or one disability (either reading or math) reflected minor miscalculations or employment of incorrect operation. So not surprisingly, Bryant et al. (2000) found LD students, aged 8-18, identified as weak in math had significant difficulty with multi-step problems, including making computational errors.

Experience in schools confirms the concerns teachers have about students having good working knowledge (automatic retrieval) of basic facts. Failure to master basic facts

persists and appears to be independent of language and reading abilities, but not independent of their disabilities (Jordan, Hanich & Kaplan, 2002). Research has suggested that students' failure to develop long-term memory for basic facts is a result of quick decay of information in working memory, such that relevant associations cannot be made (McLean & Hitch, 1999). Geary (1993) suggested that number facts are encoded into memory through the language system, but this was not supported by Jordan et al. (2003), who suggested that deficits in facts may be attributed to weaknesses in non-verbal reasoning. As stated earlier, it appears that knowledge of basic facts may facilitate higher order mathematical thinking (Fuchs, Fuchs, Hamlet, Powell, Capizzi, Seethaler, 2005) and may create space in working memory to allow for other information to be represented or manipulated. However, children with poor knowledge of basic facts have demonstrated relatively strong problem-solving skills when using problem-solution rules (e.g. schematics) to solve word problems (Fuchs, Fuchs & Prentice, 2004; Jordan & Hanich, 2000).

Strategies

Both Tournaki (2003) and Ostad (1997) reported that research conducted by Siegler and others indicates that students' use of strategies evolves and can be used as a predictor of early success in mathematics. Students typically move away from counting strategies to retrieval strategies to develop more sophisticated strategies (Ostad & Sorensen, 2007). The use of the minimum addend strategy by students in Kindergarten, Grade 1 and 2 is a predictor of later success in math (Tournaki, 2003). Most students deduce and use the minimum addend counting strategy on their own.

In a study of primary students, Geary et al. (2000) found that students with combined MD and RD or MD only were more likely to depend on finger counting as a strategy than their peers with no LD or those with only RD and this pattern continued into Grade 2. Students with MD and RD or MD only also made more counting errors and fact retrieval errors than their peers. Interestingly, errors in retrieval and counting diminished for students with MD only and MD and RD and they did begin to use the minimum addend strategy in Grade 2 (Geary et al., 2000).

Retrieval errors continued to be made by students with MD and RD or only MD, when compared to their peers with only RD who also demonstrated a low frequency of retrieval (Geary et al., 1999; Geary et al., 2000). Students with MD are more likely to develop and employ more skilled counting procedures or strategies in Grade 2 than their peers with both RD and MD (Geary et al., 1999; Geary et al., 2000). Based on the work of Jordan and Montani (1997), Geary et al. (1999) suggested that speed of arithmetic problem solving be used to distinguish MD students from students with both RD and MD. However, retrieval errors remain relatively high for students with MD, regardless of their reading ability, when compared to their normally achieving peers (Geary et al., 1999; Torbeyns et al, 2004). Geary et al., (1999) suggested that the students with RD have difficulties accessing information from long term semantic memory and that students with MD show different types of working memory deficits, such as executive control of attentional resources.

Students with MD are delayed in the development of simple addition and subtraction strategies when compared to their peers (Torbeyns, Verschaffel & Ghesquiere, 2004). Rote skill practice is less often effective than it appears (Gersten et

al., 2005; Woodward & Brown, 2006). Citing Geary, Gersten et al. (2005) stated that rote skill practice places significant demands on associative memory, an area of weakness in many students with MD. Unlike their peers, students with a learning disability do not develop strategies independently or spontaneously and continue to use fingers to count into later grades (Jordan et al., 2003; Ostad, 1997; Tournaki, 2003). Research supports the instruction of shortcut strategies (i.e. near doubles) that result in students employing minimal cognitive effort and increasing response time to answer questions involving number combinations correctly (Gersten et al., 2005). “[S]trategy characteristics of children with MD simply reflect their acquired mathematical ability level, and this suggests that their strategy development is delayed” (Torbeyns et al., 2004).

Ostad and Sorensen (2007) concluded that children with MD are developmentally different from typical math achievers. One hundred and thirty-four students, Grade 1 to 7, half with MD, were observed solving computational problems to assess their use of private speech vis-à-vis strategy use. Students with MD used backup strategy (i.e. counting) almost exclusively, whereas children without MD increasingly used retrieval strategies as they progressed through the grades. Strategy development of students with MD appeared to stop at the inaudible private speech back-up strategy. Students without MD increasingly used the silent private speech retrieval strategy. Ostad and Sorensen (2007) suggest that the difficulty with fact retrieval for students with MD may be attributable to difficulties with the phonetic representations in long term memory (Geary, 1993).

Working Memory and Strategies

In a study of elementary school students with and without MD, Keeler & Swanson (2001) did appear to find that there was a relationship between strategy choices and working memory and math performance. The mean age of the students was 10.8 years with a SD of 2.8 years. Participants were asked to complete a number of tasks, including selecting strategies from a menu of strategies. Procedural knowledge of strategies was not assessed, nor was it clear that students would have selected strategies without a menu or employed strategies on their own.

However, they did find that “the stability of strategy choices was related to WM performance” (Keeler & Swanson, 2001, p. 424). In addition, strategy choice appears to be related to mathematics performance. And lastly, selection of higher order strategies (i.e. rehearsal or visual-spatial strategies) was related to WM span scores. That is to say, students who demonstrated a preference for verbal type strategies had higher verbal WM scores than students who demonstrated a similar preference for visual-spatial strategies. Similarly, these students who preferred visual-spatial strategies had higher visual-spatial WM scores.

Keeler & Swanson (2001) also found that both verbal and visual-spatial WM contributed independently to math performance. They suggested that the issue for students with MD is one of storage and not processing, as these students draw resources from both verbal and visual-spatial WM. However, they were not able isolate the role of the central executive processor vis-à-vis the impact of finite resources stored in verbal and visual-spatial WM and the requirement for retrieval of information from long-term memory. They concluded that predictors of future success in math include both verbal

and visual-spatial WM, as well as knowledge of strategies. These authors proposed that one method of improving math achievement may be to develop students' knowledge of strategies.

Remedial Instruction of Strategies

In 1974 Kosc (p. 167) argued that students with MD "... could often reach much higher levels of skills in mathematical operations than they actually do, if their instruction could be appropriately organized and, above all, their compensational mechanisms could be used to the fullest." Teachers should try to facilitate the development of basic math skills, including the use of addition and subtraction strategies as early as possible (Torbeyns et al., 2004). Using computer assisted instruction, Fuchs et al. (2006) were able to improve addition number combination skills in Grade 1 students identified as at risk (25th percentile in arithmetic) at the beginning of the school year. Failure to improve subtraction number combination skills and to transfer to story problems was attributed to less time spent on these tasks. Fuchs et al. (2006) suggested that follow-up activities directed by adults or peers may improve results.

Ostad (1997) proposed that students with MD need to be taught strategies and how and where to apply them. He attributed the limited number of strategies used by students with MD to insufficient knowledge of strategies per se and describes this pattern of strategy use as strategic rigidity, as opposed to strategic flexibility. In contrast, Male (1996) argues that students with moderate learning difficulties are not necessarily unaware of strategies, but rather that the appropriate employment of strategies is influenced by both cognitive and motivational factors. The knowledge and selection of strategies of 20 twelve year old students with moderate learning disabilities were

compared to those of 20 twelve year old students with no learning disabilities. When compared to students with no learning disabilities, students with a moderate learning disability were more likely to employ strategies similar to their typical peers when confronted with an authentic task, such as recalling 7 items on a shopping list (Male, 1996).

Tournaki (2003) compared the outcomes of Grade 2 students who were taught one-digit addition facts either by drill and practice (direct instruction) or by the minimum addend strategy (cognitive behaviour modification), and a third no-intervention control group. Half of the 84 students who participated in the study, attended Grade 2 general education classes and the remaining 42 student attended self-contained special education classes and were identified as having LD. Students with LD improved regardless of method of instruction; however, improved performance (accuracy and response time) was greater for those students taught the strategy. Students who received strategy instruction, with and without LD, were more successful at transferring their skill. Students in the control group, having received neither direct instruction nor strategy instruction, appeared to make no improvement. The findings of this study support the “proposition made by a number of researchers about the need for direct and explicit instruction of strategies” (Tournaki, 2003).

Montague (1997) also reported on the results of cognitive strategy instruction. Six secondary students with MD were taught strategies using a combination of direct instruction and cognitive-behavioral procedures (e.g. modeling, rehearsal, feedback, etc.). Refining the teaching method and adding a meta-cognitive strategy component appeared to be more effective for 72 junior high school students with LD than the initial

intervention. However, students did require “periodic booster sessions” to promote maintenance. Effective use of number combination strategies would be expected to facilitate the development of higher level math skills for both computations and problem solving.

Math Performance: Problem Solving

For the purposes of this paper, problem solving refers to word problems rather than arithmetical or computational problems. Word problems may be loosely structured, require the student to analyze, and interpret, select information and may involve any number of steps. “Problem solving skills are primarily an improvement in children’s ability to represent the relationships among quantities described in the problem situation” (Garcia, Jimenez & Hess, 2006).

As problems become more difficult, students’ performance deteriorates regardless of ability (Fuchs & Fuchs, 2002). However, at the Grade 2 level, increasing demands of story problems impacted the performance of students with MD only (Jordan & Hanich, 2000). At the end of third grade, students with deficits in fact mastery performed as well on math story problems as students who demonstrated no such deficits (Jordan et al., 2003). In Fuchs and Fuchs (2002) study of Grade 4 students, students with MD performed better on arithmetic story problems and only slightly better on complex and authentic (real world) story problems than students with co-morbid RD and MD. However, Parmar, Cawley and Frazita (1996) found that from Grade 3 to 8, students with learning disabilities or behavior disorders had low success rates on even simple and direct problems. Garcia et al. (2006) found that students aged 7 to 9 with MD performed one or two levels of problem solving difficulty below that of their typically achieving peers.

The differences in performance for students with MD only or with co-morbid MD and RD on complex story problems and authentic problems were negligible regardless of abilities (Fuchs & Fuchs, 2002). The significant finding of the Fuchs & Fuchs (2002) study was that while students with MD, regardless of RD, performed comparably on operations on the complex and authentic problems, students with MD only outperformed the other group on the problem-solving dimension of the tasks (Fuchs & Fuchs, 2002). Indirect problems, where the language does not cue a particular operation, proved to be more difficult for Grade 3 to 8 students with learning disabilities and behavior disorders than problems with extraneous information (Parmar et al., 1996). Beginning in Grade 3, students with math weaknesses, learning disabilities and behavior disorders are largely unable to solve multi-step problems (Bryant et al., 2000; Parmar et al., 1996).

In contrast, the language in complex and authentic problems, as well as the poor performance on operations, may have been the cause of difficulties for Grade 4 students with both reading and math disabilities (Fuchs & Fuchs, 2002). Given that questions were read to Grade 4 students, comprehension and/or limited ability to refer back to re-read the questions (as the MD only students were most capable of) may have had important implications for students with both RD and MD (Fuchs & Fuchs, 2002). Elbaum (2007) found that the effect of reading the questions on a multiple choice 60 item math test for 643 students in Grades 6 to 10 was greater for students without LD. Jordan and Hanich (2000) found that Grade 2 students with RD only did not show any weakness on story problems and that students with MD only performed significantly worse than normally achieving peers on complex math problems. As suggested by Hanich and Jordan (2000), studies need to differentiate between decoding and comprehension. Underlying deficits in

working memory for students with both RD and MD may be different from those of students with either MD or RD only. However, “mathematics difficulties (or the underlying deficits associated with mathematics difficulties) may contribute more to mathematical problem-solving learning problems than do reading comprehension difficulties (or the underlying deficits associated with reading comprehension difficulties)” (Fuchs, Fuchs & Prentice, 2004, p. 298).

It must be noted that before Grade 3, students normally would not be exposed to meaningful word problems. Ideally, students beyond Grade 3 have received both sufficient instruction and experience in word problem solving, such that researchers can exclude lack of either as impacting outcomes. The quality of simpler problems at earlier grade levels may be insufficient to evaluate mathematical thinking or transference of basic skills. Nevertheless, Garcia et al. (2006) concluded in their study of un-timed problem solving skills of 143 students aged 7 to 9, with and without MD, that when word problems “are easier for typically achieving children [they] are moderately difficult for students with arithmetic learning disabilities, and those that are moderately difficult for the more skilled group mostly become high-difficulty items for students with low achievement” (p. 277).

Problem-Solving and Schematics

Supporting the work of Hegarty and Kozhevnikov, Garderen (2006) found that “the use of schematic images was positively related to success in mathematical problem solving, whereas the use of pictorial images was negatively related to success in mathematical problem solving” (p. 497). Schematic images comprise graphic organizers (i.e. flow chart) that organize information in such a way that it relates directly to

mathematical representation of information. “Schemata are the basis for understanding the appropriate mechanism for the problem solver to capture both the patterns of relations and their linkages to operations” (Garcia et al., 2006). Based on her study of 66 sixth-grade students and the research of others, Garderen (2006) posited that there is a positive correlation between visual spatial measures and mathematical problem solving measures. Researchers, including Garderen (2006), have found that students with LD use fewer representational strategies (i.e. visualization, pictorial imagery, paraphrasing) than their typically achieving peers. Furthermore, Geary (2004) suggests that a deficit in the ability to generate and use a mental number line (number sense) is related to deficits in visual-spatial skills.

Instruction in the application of schematics in problem solving for students with learning problems in Grades 6 through 8 improved significantly on measures in acquisition, maintenance and generalization of math skills when compared to those students who received general strategy instruction (Xin, Jitendra and Deatline-Buchman, 2005). Students were taught the structure of different problem types and provided direct instruction on the relationship between the schematic map and the math sentence. This visual strategy facilitated students participating in higher order thinking, as well as developed conceptual understanding (Xin et al., 2005). The stronger results of the study by Xin et al. (2005) are reported to support previous research and are reportedly stronger as a result of additional instructional time, linking schematic map and math sentence, and small group pull-out design of the program.

Discussion

The definition of MD and its identification seem to be rather unimportant issues for teachers to consider. However, specific difficulties with learning math may be attributable to specific learning profiles of students with MD only when compared to students with co-morbid disabilities. Understanding the underlying causes of these difficulties is of paramount importance to classroom and special education teachers when they are planning for instruction.

Findings from researchers appear to support the thinking that deficits in WM are integrally involved in MD. In addition, there may be a genetic or congenital component of MD. Implications of etiology for instruction and remediation are significant and appear to be supported by the work of researchers. While it is important to provide students with opportunities to solve problems we must insure that students have the prerequisite conceptual and procedural knowledge.

When students arrive at school, there needs to be a strong math program that fosters the development of number sense, counting skills and the conceptual understanding of addition and subtraction. The development of these skills should consider visual-spatial and semantic components of learning. Research suggests concrete models and written formats should be developed side by side throughout elementary school. Strategy instruction should be an integral part of math programs.

The need for students to know basic facts, as part of a larger set of problem solving skills, cannot be understated. Exercises to strengthen recall should be explored, along with strategies that facilitate recall. Spelling, as the backbone of writing, is comparable to the requirement to know basic facts. While spelling does not facilitate

ideas, it does free the author to concentrate on the larger picture. Basic facts and strategies must be taught explicitly and relationships should be pointed out and discussed. Math is about solving problems, but one needs to have an understanding of language and symbols, and of relationships between them to be able to solve problems.

A building block model for math programs reflects research in the field of MD. However, research is limited in providing insight into interventions or remediation that benefits learners with MD. Difficulties with identifying MD and its subtypes, implication of co-morbid disabilities, differing research methodologies, lack of consistency in operationalization of terms and other difficulties adversely affect the synthesis of the research of MD. In the following chapters, discussions focus on the interconnectedness of number sense, number combinations and problem solving as concepts, and on habits and inquiry. A theoretical construct, based on the work of Dewey, which incorporates instruction of students with MD into general math education, requires that research findings be used to benefit the education of teachers and all learners in the classroom. The theoretical principles developed in the following chapters may also be used to inform further research.

CHAPTER 4: APPLYING DEWEY'S FRAMEWORK TO NUMBER SENSE

Conceptions and Number Sense

The education of children is first based on the early experiences in the home. These experiences foster the development of concepts, habits and inquiry. Infants' opportunities to grow (i.e. learning) originate in first attempts of adults to meet their physical needs. Through relationships with adults, infants develop early conceptual understanding related to interacting with others to have their needs met. Infants and young children acquire concepts related to language, math (e.g. number sense), and other subject areas, but more importantly the concepts related to inquiry (i.e. learning) through their relationships with others. The social environment of the home and later the school provide children the experiences to develop the concepts of the community which includes both home and school. Education as growth demands that education be seen in the broadest light. Early concepts become the foundation on which future growth is based. Through shared cultural experiences at home and at elementary school, children further develop the concepts required to continue to grow—including those concepts related to the formation of habits (see Chapter 5) and to inquiry (see Chapter 6).

According to Dewey, conceptual development occurs through meaningful experiences. Through the shared experiences of inquiry (see Chapter 6), individuals come to have an awareness and understanding of the common characteristics of *something* that results in the development of concepts. Concepts become increasingly refined and stable with each subsequent experience. The universality of concepts is found in the knowledge that can be applied to a plethora of situations, as well as to the shared knowledge amongst

members of a community. Regardless of cultural and generational differences, or differences in experiences and environments, individuals come to share similar conceptual understandings. For example, the concept of sharing, a universally understood concept amongst members of different communities, is understood culturally as taking turns and mathematically as division.

Concepts, and their development, are the foundation on which much is built. The shift from physical maturity to intellectual and social maturity signals increased ability to interact with others and with the environment, as well as the development of concepts related to intellectual development and culture. Early childhood growth, from infancy to school age, begins in the home. The need for social interaction requires that infants begin to acquire an understanding of their culture. In striving to have their needs met, infants and children come to know their culture. Through their developing awareness of their culture, infants and children develop concepts that are both specific (e.g. number sense) and general (e.g. relate to inquiry). The development of concepts occurs through the on-going physical and intellectual growth of children that is dependent on their interaction within the cultural context of their home to have their needs met.

Number Sense and Biological Growth

According to Dewey (1938b), organic *education* is the basis of biological growth. Physical structures, including biological and neurological structures, are the tools used by organisms to grow or to adapt to their environments. Organic growth is non-reflective growth. In addition, physical structures also provide the tools, as well as the foundation (model), for intellectual growth (Dewey, 1938b). The interaction of intra-organic

structures within individual organisms and with the environment, suggests that physiological structures are actively involved in education, both organic and reflective. Hands, ears and eyes are the tools with which individuals interact with their environments, not only to meet their physical needs, but also their social and intellectual needs. For humans, reflective growth—adjustment to the environment and transformation of individuals and the environment—is the result of interaction of the individual with the environment. The physical experiences of individuals coincide with cultural and intellectual experiences that may result in meaningful experiences stored as memories in memory to be drawn upon in the future.

During the period of gestation, the biological and physical needs, and development of the fetus are paramount. After birth, the needs of the infant begin to shift. For example, infants respond to their own physical need for food. They cry out and parents respond to meet their needs. In doing so, parents also hold infants and talk to them. The exchange is not simply about physical needs or a transaction—food for silence—but rather is a complex exchange that directly meets the physical needs of infants so that they may physically grow, and indirectly meets their emotional, social, and cultural needs. As infants develop and become more independent, a change in the roles of both parents and children occurs. Interactions between parents and children shift from primarily meeting physical needs (e.g. feeding, changing, physically moving the infant so that they are more comfortable) to meeting emotional, social, and cultural needs (e.g. talking about experiences with friends). The exchange changes as the abilities of children develop; children begin to use tools (e.g. hands, spoon) rather than depending on the parents (e.g. to put food in their mouths to eat). The maturing relationship is founded on

the earlier relationship, which was heavily tied to meeting the physical needs of infants. Children are slowly transformed by the continuity of their interactions with their environment, including the relationship between parents and children. It should be noted that parents are also transformed by this relationship and this is discussed later.

Research in the area of number sense involves the issue of the *teachability* of number sense. Do genetics determine the development of conceptual understanding? What role does genetics play in the acquisition or development of number sense? Anasari and Karmiloff-Smith's (2002) research of very young infants' math abilities suggest that genetics may be involved and may become the basis for future math skill and knowledge development. As stated in Chapter 3, Geary, Mazzocco and Shalev all posit that genetics may play a part in the development of math knowledge and skills. However, the physiological structures of young infants may be simply responding to stimuli, biologically speaking, and the ability to reflect on that stimulus may simply be inferred rather than truly ascertainable. Nevertheless, poor understanding of concepts appears to be related to deficits in the right hemisphere of the brain, although the evidence is tentative at this time (Mazzocco, 2001; Rourke & Conway, 1993). The basis for future education may be attributable, in part, to the development of physical structures of the brain. However, interaction with the environment may contribute to the development of the structures of the brain, especially later in life through the pruning process.

Programs such as *Number Worlds* (Griffin, 2004) provide evidence that suggests that active and reflective situational experiences may compensate for the lack of earlier experiences. A delay in developing number sense, as the result of limited experiences or perhaps due to physiological immaturity (or deficits), suggests that number sense can be

taught and may not be as developmentally time sensitive as suggested by some researchers. A developmental difference, regardless of the quality of experiences or physiological maturity, suggests that number sense may be acquired differently for some or that the ability to develop number sense may be limited in some cases. The limitation may be attributed to researchers' imperfect understanding of the concept of number sense (see Chapter 3) and the experiences required to develop it, or to deficient physiological structures, as yet not fully understood. Experiences that promote the development of number sense at a young age may also facilitate the development of physiological structures or tools, for example through the pruning process, required to develop number sense.

Working Memory and Number Sense

The role of WM in the development of the concept of number sense is not explicitly discussed in the research. The lack of consensus surrounding the definition of number sense impacts the discussion about the sorts of knowledge or skills that comprise it. As such, the relationship between WM and the development and subsequent application of number sense does not appear to be well understood. However, there does appear to be some consensus that number sense is the foundation on which other math knowledge and skills are built.

Gersten et al. (2005) suggest that early indicators of MD include deficits in the ability to discriminate quantities (magnitude) on the one hand, and identifying numbers in isolation or identifying numbers missing in a sequence on the other. These may in fact constitute two related, but yet distinct, forms of knowledge. The VSSP (visuo-spatial sketchpad), static and dynamic visual storage of information, may be implicated in the

ability to discriminate quantities. And in contrast, the PL—the use of language to store and manipulate information in the phonological and articulatory stores—may be implicated in the identification of numbers. Furthermore, the analog-exact system of representing numbers described by Anasari and Karmiloff-Smith (2002) suggests that knowledge of numbers is dual based. Knowledge of numbers is based on an understanding of magnitude that is independent of language and on an understanding that is dependent on language (i.e. name of the number). Chiappe (2005) appears to support this position in arguing that the understanding of magnitude becomes the basis for the mature representation of numbers.

Magnitude is an understanding that is based on approximation, whereas, specific counts require specification (language). Indicators of MD may be related to difficulties with either or both informational representation and informational manipulation. It would seem that difficulties with information representation not requiring language, visuo-spatial information (VSSP), may impact the development of knowledge and skills related to numeracy through language (PL), such that the development of math knowledge and skills would be at a superficial level or at the rote level (i.e. counting, basic facts). In contrast, a strong conceptual understanding that is limited to visuo-spatial information would impact the ability to count, compute and, presumably, to discuss more advanced levels of math at an abstract level. The ability to manipulate both visual-spatial and language-based information is clearly required to solve problems, but perhaps more importantly, it may be the development of the relationship between these forms of information that is crucial to advancement of math knowledge and skills.

Dewey: Conceptions and Working Memory

The continuum of growth, for humans, extends well beyond that of the physical and biological. However, the physical and biological are the starting point for the development of the emotional, intellectual, and cultural characteristics of the individual. At conception, the biological characteristics develop and come to influence, if not determine to a certain degree, the future growth of the individual. At birth, the physical development of the infant outweighs the concern about all other areas of growth. Parents are primarily concerned with the physical growth of their baby (e.g. gains in weight) and continue to be concerned with physical growth well into childhood (e.g. height). However, a shift does occur over time. Parents begin to be concerned with the emotional and social interactions of their infant, smiling and cooing. As the infant becomes a toddler and, later, a child, the attentiveness to the development of the child comes to include much more than the physical, extending to the emotional and social characteristics of the child. Parents come to focus on the development of the intellect of the child, especially as they prepare for and enter school.

Before school, this focus is predominately on the ability to communicate (i.e. use language) and meet the physical demands of the environment (e.g. walk, run, etc.), including the use of tools (e.g. crayons, spoon, etc.). As children develop physically, they are better able to interact with the environment, such that the intellectual development of children is dependent on, to a certain extent, their ability to interact physically with their environment or to develop tools to facilitate their interaction with the environment. The growth of the child, to this point and for some time, is dependent on their physical development (including physical co-ordination), such that they are able to use their eyes,

hands, ears, legs and so on to enable them to interact in their environment and it is this interaction that facilitates the development of concepts.

It is through interacting with their environment to resolve situations that concepts are developed. An infant cries out, initially, because of a physical need (e.g. food), but, eventually, will cry out to have their emotional and social needs met. Concepts begin to develop as a result of similar successive interactions with the environment. The concept as a standard of reference and the generality of concepts are aspects of concepts that develop separately and yet are indiscernible. For example, the concept of sharing may initially be developed by the infant as the infant and parent being together. The concept of sharing, as a standard of reference, is the time spent together. The generality of the concept is found in the extension of the concept of sharing from sharing time together when hungry to sharing time together to meet emotional and social needs. The development of this concept is the result of the infant interacting with their environment, experimenting to have their needs met as their needs expand as a result of previous experiences. It is just this sort of inquiry—crying as a form of communication that results in needs being met— that is the basis of the development of concepts that become increasingly refined (sharing to mathematical computation of division) and that fosters future growth, a continuum of development from organic to intellectual, from physical to reflective.

The information that comprises a concept is the result of meaningful experiences. For Dewey (1933), that information is represented in the concept as a standard of reference (knowing). The information is manipulated in the generality of the concept (doing). Information representation is at the core of Dewey's standard of reference of

concept. Similarly, information manipulation is integral to Dewey's generality of concept. Concepts, drawn from memory, are applied in novel situations such that the concepts are refined to include ever more sophisticated levels of information in terms of information representation and information manipulation. Conceptual understanding reflects both the experiences had and the intellectual facilities of the individual, at that point in time. As concepts are shared by members of the community, concepts also reflect that knowledge of the community. Insofar as concepts are specific to certain intellectual and cultural areas, the level of sophistication of a concept held by an individual or community speaks to the level of understanding of that individual or community. Conceptual understanding—as a standard of reference and in the generality of the application of the concept—is the basis of understanding from which the individual and community may grow.

According to Dewey, co-ordination of the dual aspects of a concept (standard of reference and generality of concept) rests in the demands of the situation rather than in the central executive (CE). The concept is a tool to be used to facilitate interaction with the environment. Through interactions with the environment, some experiences come to be the sorts of situations that place demands on the individual to use their conceptual understanding (tools) that result in the growth of the individual and perhaps the community. In recognizing the demands of a given situation and analyzing the information, the individual selects and uses concepts to work to resolve the situation. The situation for Dewey is comparable to the CE.

Growth from physical development to conceptual understanding is initially gradual and yet becomes observable as an infant becomes a child (e.g. all four legged

animals are no longer called dogs). Conceptual understanding facilitates the speed with which information is analyzed to solve specific situations and synthesized to solve this and future situations. Concepts, such as knowledge of sharing, allow children to understand increasingly sophisticated problems as being similar to past problems, and to develop and apply their growing conceptual understanding to even more sophisticated future problems. It is this understanding that enables students to approach a problem and sort or classify it so as to be able to relate it to past problems. It gives them somewhere to start—a rule or maxim or principle to begin with. In cases when students become bogged down in trying to figure out what kind of problem is in front of them, their ability to interact with their environment to meet the demands of a situation is impacted. They become bogged down in trying to find out what the problem is and how to proceed. Memories of previous experiences and concepts arising out of them, as well as hands, eyes, and ears need to be co-ordinated in the environment to meet the demands of a situation. Similarly, the episodic buffer “... allows multiple sources of information to be considered simultaneously, creating a model of the environment that may be manipulated to solve problems and plan future behaviour” (Baddeley, 2001, p. 858). The principle of continuity suggests that the shift from organic interaction to reflective interaction (development of conceptual understanding) effects maturity in the form of intellectual adaptation.

Number Sense and Cultural Growth

Parents caring for infants and young children meet the physical, intellectual, emotional and social needs of their children. The lines between needs are blurred. An

example of the process of transformation from the organic environment to the cultural environment is readily apparent in the act of feeding an infant, as well as the increasing ability of children to feed themselves and to participate in discussions around the dinner table as they mature both physically and culturally. Feeding an infant or child is not only an act of meeting their physical needs so that physical growth results, but it is also a cultural experience. Through this repeated experience, children are initiated into the social group and develop dispositions. The dependency typified in the adult-child relationship is based on interaction which impacts the quality of experiences. The experience, including affection shown through touch, discussion and so on, fosters not only the cultural growth of the child, but the cultural growth of the adult and the community. The parent, responding to the immaturity of their child, draws on their previous experiences to meet the needs of their child. The adult continues to grow as a result of building on previous experiences, including reflecting on how their parents may have responded to similar situations. Through repeated experiences and interaction with adults, the cumulative experiences of children determine the growth of the community as the child grows from infant to adult.

The gradual shift from organic to intellectual behaviour that is characteristic of the cultural environment corresponds with both the development of number sense from infancy to early childhood as the person grows, as well as from informal math instruction to formal math instruction as the environment is altered. Informal math instruction occurs prior to attending school and typically happens in the home or at preschool (Gersten et al, 2005). In the home, real life experiences and situations typically provide the cultural and physical material unwittingly used for education in number sense. Building on the

example of feeding an infant, through language parents impart number sense. Offering a child *more* food, showing them an empty bowl and saying *all gone*, or asking that they have *three more bites* of food is using the language of math and developing an awareness of quantity and magnitude. The language used transforms the experience from one of simply meeting the needs for nutrition to a cultural experience based on the interaction of the parent with the child that provides opportunity to develop an understanding of number sense. The language used speaks to both the cultural context in which it occurs, as well as the *culture of math*, if you will. The language of math used by the child changes as the child's understanding develops from number sense to problem solving. Furthermore, as a field of study and research, the language of math continues to develop as math itself changes. And in turn this impacts the culture of society (e.g. the term google).

Regardless of the form of math instruction, the relationship between adults and children contributes substantially to the process of acquiring and developing number sense. For children at-risk, participating in formal pre-school math instruction programs, such as *Number Worlds* (Griffin, 2004), appears to promote conceptual understanding—number sense—and to develop dispositions towards math and education as students continue to grow, building on their experiences in the program (Dowker, 2005). In such formal programs, the interaction of the adult with the children determines the quality of the experience. Researchers provide teachers with formal tools to understand the continuum of math development and methods of addressing gaps or deficits in knowledge or skill. Formally, adults construct the environment, provide a model of language used and act as a model to children when resolving situations. Both the child and the teacher grow as a result of the interdependency, as teachers develop a greater understanding of

their students and students develop number sense. Researchers responsible for developing *Number Worlds* (Griffin, 2004) and other similar programs also grow as a result of their experiences, leading them to future research, and the development or improvement of programs. The possibility of improving the quality of programs delivered by teachers is heightened by ongoing research. In turn, the growth of teachers as educators is enhanced, as is the quality of the educational programs that promote the intellectual growth of children is improved. As an extension of the cultural environment of the home, the cultural environment of education—the interdependence of child, teacher and researcher in the school environment—is both more formal and complicated.

Number Sense and Cultural Growth as Inquiry

Culture is transmitted through and transformed by communication. The communication used in formal math programs and in real life situations reflects culture insofar as both *programs* use language (communication) to assess situations, determine the requirements, and respond accordingly. That is, the *programs* meet the generally accepted method of resolving the problem—children, wanting to interact with others, come to respond in like ways when compared to adults and their peers. Children’s interest in working towards resolving situations forms and sustains continuity of the community.

According to Dewey (1933), concepts are derived at as a result of inquiry. Conceptual development is the result of inquiry occurring in a cultural context. The degree to which young children are dependent on adults suggests that the socio-cultural aspects of growth may be of comparable, perhaps even greater, importance to growth as is inquiry. The habit (see Chapter 5) of inquiry is developed through experiences that initially require significant support from parents and other adults. As children develop the

habit of inquiry, they become more independent vis-a-vis inquiry. As children come to develop the concepts and habits related to inquiry, the required support of adults to develop concepts (related to inquiry, number sense, etc.) diminishes as the role of their peers, who share similar conceptual understandings, increases. Inquiry that results in conceptual development at an early age is different, but forms the basis of inquiry that results in resolving a situation as an adult. In Chapter 6, inquiry, as it relates to number sense, number combinations and problem solving, will be discussed in more detail.

Through communication, parents share the aims and habits of their family and their community with their infants and young children. Emotional and intellectual dispositions are developed through shared experiences of children and their parents. The shift from the social and human centre to the objective and intellectual is required for the development of reason that leads to inquiry (Dewey, 1938a). However, it is in the social and cultural context of the home, a microcosm of the community, that children are first introduced to and begin to develop patterns of inquiry. Through social interaction and transmission of culture, parents and children share in their response to situations, demonstrate an interest in resolving situations, share in the work to be done, and celebrate the results of inquiry. For example, a young child wants another cookie, one more cookie. Rolling their fingers in and out, alternating making a fist with an outstretched hand, they demonstrate (communicate) to their parent they want *more*. The hand and the shared meaning of the gesture are tools used by the child and parent. In the past, the parent has demonstrated to the child that this is an acceptable way to ask for *more* and this time the child has been rewarded with both a cookie and a hug for not crying, but rather using a symbol. This situation facilitated the development of number sense, reinforcing the

concept of *more*. In this simple situation, the child used language and interacted with their parent to resolve that situation. Through their relationship and use of tools, the social and communicative aspects of education are realized—culture is transmitted (Dewey, 1938b). Through culture, children are educated and their community (i.e. home, school) grows through increased interaction and participation of its members.

Programs such as *Number Worlds* (Griffin, 2004), are designed to meet the needs of children identified as at-risk. Without such programs, the continuity of the community and the growth of the individual child may be at-risk. The tried and true methods of solving problems using math knowledge and skills are explicitly communicated to the children. Information is communicated through the presentation of the problem to be solved, in the manipulatives or tools provided, and in the discussion or questions that take place during the process of solving the problem. Children and teachers formally communicate their experience and understanding or share meaning in the process of solving problems with each other, possibly resulting in the growth of the individuals involved. Communication and activities (problems) are program driven. The shift from the socially centred home to the objective and intellectual focus of the school is apparent. The experiences of children in such formal programs are different than the experiences of children in informal programs of the home. The long term impact of such programs has yet to be studied, although the short term results have been promising.

Number Sense, Signs, and Symbols

For Dewey, both natural signs and artificial signs are used to communicate. The usefulness of natural signs is limited insofar as they exist in spatial-temporal context. They provide evidence from which one can infer. Artificial signs or symbols, on the other

hand, are infinitely more useful. The intellectual property of symbols is derived from their meaning which is constructed and shared through the cultural context. Inference is possible when meanings are involved with existence. The use of symbols allows for recollection of experiences, deliberations, and rehearsal of ideas, abstract reasoning, and consideration of hypotheses. Symbols are invaluable to inquiry.

Number sense begins to be developed using natural signs. Infants' ability to compare small counts of objects and the development of one-to-one correspondence is dependent on natural signs. At an early age, number sense clearly requires that individuals begin to acquire the abstract language of math (i.e. names of numbers). It is through early experiences in the home that most children begin to use the language of math. Rote counting, like singing the alphabet song, indicates that children have clearly been exposed to, but do not yet use, the language to resolve situations. Meaning is limited to a shared social experience, rather than applied to resolve a situation. As number sense develops, children are less dependent on signs and able to use symbols more effectively. For example, the concepts of *greater than* and *less than* may initially be developed through comparisons of different magnitudes of natural signs, with no count required. Using their eyes, children observe that the taller pile of unifix cubes has more than the little pile of unifix cubes. As one-to-one correspondence is developed, counting the number of objects in each pile allows for comparison of counts. The Arabic symbols of counts are introduced and paired with the piles of unifix cubes. Eventually, unifix cubes are not required when *greater than* and *less than* are used. Developed in a social context, the shared meanings of *greater than* and *less than* are the consequences of the shared activities. Clearly the meanings of *greater than* and *less than*, are related not only to one

another as a result of comparisons, but also to the Arabic symbols of counts. These meanings can be used in other situations and developed further.

Formal math programs, such as *Number Worlds* (Griffin, 2004) and those offered at school, explicitly teach the abstract language of math. *Number Worlds* (Griffin, 2004) provides children with opportunities to learn the language of numbers and various representations of numbers found in society, the language of computation by addressing computational fluency and using formal math symbols. In the case of math skills and knowledge, real experiences are the basis for symbols or artificial signs, which then can be used to solve future real situations abstractly or to solve abstract situations. The development of concepts, initially tied to signs, continues as the symbolic representation of information develops and becomes ever more refined and sophisticated.

Number Sense and the Environment

The needs, desires, purposes, and capacities of individuals interact with their environment to create experiences (Dewey, 1938a). The cumulative effect of experiences in the environment results in the continuity of social life. Those conditions that promote or hinder growth constitute the environment (Dewey, 1916). Some of the environmental conditions that influence or shape experience include the customs, traditions, and values that have been formed over time and through interaction with others. The significant impact of the environment on the development of the character and mind may not always be obvious (Dewey, 1916). The quality of experiences is influenced by environmental conditions that include the mode of life of a community, and access to resources and tools (Dewey, 1916). The development, use or function, and worth of a tool stems from the ability of individuals to control their environment (Hickman, 1990). Through

participating and interacting in the environment, individuals come to share in and comprise that community. They share in experiences that result in the development of common interests, values, and ends.

The environment that initially promotes and stimulates number sense development is typically the home. At home the child participates socially with their parents and the influence of the environment on development of number sense is not necessarily obvious to those living in that environment. Parents interact with their children, showing pleasure, when the child acts or demonstrates appropriate knowledge. For example, a child demonstrates the socially accepted concept of sharing by breaking a cookie in half and giving one half to their sibling. The parent reacts positively and increases the likelihood that the child will repeat this behaviour. The child, having demonstrated the cultural concept of sharing, has also demonstrated a mathematical concept (division of the cookie into two equal parts). The quality of the experiences is impacted by many factors in the home environment that extends beyond the relationship between members of the home. These include the education of the parent, the cultural values and traditions, and the access, selection, and use of tools. Today, educational television programs, educational DVDs, and home computers are considered tools that facilitate educating young children, and may contribute to their acquiring number sense. In participating in the community that is the home, infants come to be integral members of the community and their development is influenced, as well as influences, that community. When at a young age, the community comes to include the larger community outside the home via television or computers, the impact of the environment on the

development of the child or their education is not yet understood. The impact on the continuity of the community is also not well understood.

Participating in programs, such as *Number Worlds* (Griffin, 2004) places children in environments created to foster learning, specifically, number sense. The activities designed and the manipulatives provided are tools selected to promote and stimulate education. The facilitation of educational opportunities in such programs is meant to compensate for impoverished home environments. Manufactured programs delivered to children, in a somewhat artificial environment, are furthering the development of number sense in children thought to be at-risk (Dowker, 2005). Formal programs target specific areas of learning, such as number sense, as well as children at-risk. The artificial environment mimics the informal experience by providing an adult (quasi-parent) and builds upon the informal environment by providing an enriched environment with access to tools and other resources.

The impact of providing a temporary environment with decidedly different cultural (educational) values and returning the child to their home environment does not address the issues related to the valuation of their home culture and its long term impact on the child's education. Continuity of the community may seem problematic in cases where children may have to choose between the values of communities, home, and school. In cases where parents support the efforts of the school and share similar values, the child and community appear more likely to benefit and in all probability both will grow. As a tool for promoting number sense, it appears that at the level of the individual, *Number Worlds* (Griffin, 2004) is successful. As a tool for promoting the continuity of the community, the continued participation of individuals in the larger community, once

they reach adulthood, establishes the success of the program and has yet to be determined.

Number Sense, Experience, and Growth

Growth is the result of educative experiences (Dewey, 1938a, p. 28). It is furthered by successive and interconnected experiences. Continuity and interaction are integrated aspects of experience. Integral to experience, tools are extensions and expressions of experiences (Hickman, 1990). A better quality of experience is a democratic social experience that is first established by continuity and interaction (Dewey, 1938a). It is the opportunity to develop the concepts and habits (tools) that is democracy for children. It is the promise of participating in inquiry that speaks to democracy for all members of the community. By taking responsibility and contributing to the demands of the situation, individuals are sharing the work of inquiry. Social control is found in the work of inquiry (Dewey, 1938a). The responsibility to create the opportunities for children to develop the concepts and habits that are integral to inquiry, as well as the democratic dispositions that results in a community that grows democratically, rests with adults.

It is through various interconnected experiences – at home, in school and in the community—that children develop number sense. Typically, infants begin to develop number sense through their experiences at home. Mis-education occurs in impoverished environments and may be addressed through programs such as *Number Worlds* (Griffin, 2004). It and other similar programs facilitate distinct experiences which foster the development of number sense. Regardless of the environment in which concept development begins, children are expected to advance their conceptual understanding

through subsequent experiences at school and throughout their lives. The continuum of experiences – continuity of experiences—is realized in the *developmental continuum of math skills and knowledge* that appears to begin with number sense.

An experience is what it is, based on interaction and the situation (Dewey, 1938a). The parent that applauds their child sharing a cookie speaks to the values (cultural and mathematical) found in the solution to the problem of what to do when there is one cookie and two children. An infant or child continues to develop number sense because there is value in the present experience – the shared interaction has value or meaning for the child and it relates to the meanings already held by the child. A child may not develop number sense as a result of wanting interactions with others or the environment, or limited opportunities to resolve situations that are meaningful to them as they do not relate to the meanings held by the child. The failure to grow through these sorts of experiences is observable in the lack of change in the environment or in limited growth of the individual child. Experiences that encompass positive interactions with others or the environment may impact the development of positive dispositions towards developing knowledge and skills related to specifically to math or to inquiry in general. Additionally, experiences that result in growth convey to children that they have a role or place in the work of inquiry in their community.

The situation, in part, furnishes the foundation for interaction. Assuming interest in a particular situation, the participation of individuals to resolve the situation is determined in part by the interaction with those present and the education of the individual up to that point in time. For example, children are asked to find out how much money is required if it costs \$1.00 for each child to attend a show. Developmentally

delayed students with MD may need more time or more experiences to develop counting proficiency. Developmentally different students with MD may need more time or experiences developing the concept of one-to-one correspondence before they can count accurately. In either case, the response of others to the participation of students with MD in solving a problem speaks to the quality of the interaction for all students. The conceptual understanding and skill set brought to the problem by the students with MD speaks to their education to that point in time. Furthermore, parents and teachers are responsible to ensure that the previous experiences of all the children working to solve that particular problem have prepared them to participate in resolving that problem with others. The problem goes beyond math to include cultural concepts and values. Those cultural concepts and values include the earlier opportunities to learn the concepts that relate to inquiry in general, which includes developing shared meanings and tools that facilitate participation, as well as the development of democracy.

As the situation for researchers continues (meeting the needs of children with MD), the apparent success of programs, such as *Number Worlds* (Griffin, 2004), suggests that communication between researchers and teachers results in more meaningful interactions for children struggling to develop number sense. Putting aside the specific educational needs of children with MD, failure to meet their needs means limiting their ability to resolve situations in their own lives and to participate in their community. By not developing the shared meanings that comprise number sense, children with MD experience difficulties in both solving math problems and in sharing in the experiences that promote growth – developing concepts related to inquiry in general. The development and delivery of programs, such as *Number Worlds* (Griffin, 2004), speaks to

the continuity of experience from young pre-school children to researchers. The continued growth of both researchers and educators is reflected in the expansion of math experiences to include students with MD so that they (children, teachers, researchers) continue to develop conceptual understandings of number sense, inquiry, and democracy by participating in the program. Expanding math programs to include practices, such as those found in *Number Worlds* (Griffin, 2004), reflects the depth and the breadth of continuity of experiences that includes the growth of adults (parents, teachers, researchers) and children, including those with MD, so that all children come to develop shared meanings, better enabling them to participate in the classroom. Participating in math problem solving activities provides experiences that educate and prepare children to participate in their community as adults. The quality of the experience of participating in problem solving activities and programs, such as *Number Worlds* (Griffin, 2004) exemplifies a stage in the development of democracy.

Discussion

The development of the concept of number sense is the beginning or the foundation on which much is built. The shift in growth from organic to reflective is readily apparent in the development of number sense. As the infant matures physically, so does their intellectual and social ability mature so that they are increasingly able to interact with others and their environment. Education, at this point, is the continuation of the physical maturity of the infant into childhood. The shift in the focus of growth from meeting the physical needs to meeting the socio-cultural needs is reflected in the tools developed by the child, the experiences had, the development and use of language, the

interaction with the environment and the development of dispositions towards education. The shift in growth occurs in the cultural context of the home and perhaps, pre-school. The need for social interaction arises out of the physical maturity of the infant and it is this need that requires that the infant begin to develop an understanding of the culture in which they live. At home and at pre-school, children work to solve situations to meet their various needs – physical (hunger), emotional, social, intellectual and cultural—in their environment. In working to meet their needs, children come to know their culture—how do I get what I need, what is valued by others, how to treat others, how to treat adults, how to adapt to the environment and so on. These early lessons foster the growth of the child, physically (i.e. crying to get food), intellectually (i.e. the development of number sense and concepts related to inquiry) and culturally as they work to become part of their community. Through their own growth and development, children are inquiring about their culture.

CHAPTER 5: APPLYING DEWEY'S FRAMEWORK TO NUMBER

COMBINATIONS

Habits and Number Combinations

In an early elementary classroom, more is occurring than the study of any particular subject. The experiences in the classroom have far reaching effects beyond the subject matter and form the base of the community as children become the leaders of the community. Education as growth demands that education be seen in the broadest light. Young children develop conceptual understandings and habits, not only about math, but about forming concepts and habits and they do so in the social environment of school. They are initiated into the culture by developing the basic concepts and habits of their larger community, including both home and school. Through shared cultural experiences at elementary school, children further develop the concepts and habits required to continue to grow—including those concepts and habits related to inquiry (see Chapter 6).

Number combination skills appear to be based on conceptual understandings of quantity, counting, and computation. For example, the conceptual understanding of what constitutes addition is realized in the application of the skilled habit of counting. With experience, the habit of counting is replaced with the skilled application of knowledge of number combinations, by developing strategies or by using retrieval from memory. In developing the habit, number combinations are actively formed and practiced. The habit of number combinations is developed and applied to new situations as they arise in real life situations and in math class. The often repeated experiences involved in developing knowledge of number combinations build upon one another until the child has established

the habit, allowing the child to perform basic operations automatically to solve future similar problems. It is expected that the formation of this habit will facilitate increasingly sophisticated computations and higher order thinking skills involved in problem solving.

Automaticity with regards to fact retrieval of number combinations allows the child to become efficient in solving basic algorithms. Students become more effective in controlling their environment by efficiently and accurately applying basic math facts with few errors. Accordingly, knowing number combinations and applying them automatically and accurately requires that students are able to apply judgment to a situation, selecting and using data or ideas stored in memory. Moreover, the skilled application of number combinations in successive situations reinforces the child's ability to solve algorithmic problems, thereby promoting further interaction with the environment by developing dispositions towards math, building knowledge and skills and impacting the environment of the child.

Number Combinations and Biological Growth

The involvement of biological structures and functions in the development of number combinations and their retrieval is supported by neuro-psychological researchers' focus on the relationship between structures of the brain and deficits in WM. The development of the conceptual understanding of number combinations is based on the appropriate development of number sense in the both hemispheres of the brain (Mazzocco, 2001; Rourke & Conway, 1993). As stated in the previous chapter, poor number sense appears to be more strongly related to deficits in the right hemisphere of the brain (Mazzocco, 2001; Rourke & Conway, 1993). Poor fact retrieval appears to be

more strongly related to deficits in the left hemisphere of the brain (Mazzocco, 2001; Rourke & Conway, 1993). Difficulties with number combinations may be based on deficits in number sense and counting, in which memory plays a role (Geary et al., 1999).

However, some students with no significant difficulties with number sense and counting appear to struggle with number combinations (Jordan et al., 2003) and this may be attributed to a deficit in spatial rather than visual components of the VSSP (Baddeley, 1998), both of which are located in the right hemisphere of the brain. Deficits in basic math fact retrieval are used as an indicator of deficits in the processing of WM (Geary, 2004). The observable patterns of performance of students experiencing difficulties with math may indicate that deficits in memory may be attributable to deficits in the structures of the brain. Understanding the relationship between organic structures and intellectual endeavours is improving with advancements in brain imaging technology.

The environments in which children develop math skills, including number combinations, can and do appear to have an impact, as established in the study involving *Number Worlds* (Griffin, 2004). Homes that do not provide the support required to master basic math facts may impact the development of those habits. Some impoverished home environments may also be the result of genetic factors rather than strictly socio-economic factors (Shalev, 2001). Children may not only be impacted by the limited ability of parents to provide an environment conducive to developing math knowledge and skills, but also by the genes that they inherit. The impact of genetic factors on the development of number combinations continues to be a significant area of research.

Working Memory and Number Combinations

The role of WM in the retrieval of math facts has been an area of study for some time and has continued to be a persistent indicator of MD (Geary, 2004). Intellectual growth, in this case in the area of development of math knowledge and skills, is dependent on WM, as supported by research and discussed in Chapter 3. The role of WM and its components may be implicated differently in number combinations than in number sense. All the same, difficulties with retrieval of number combinations are generally understood to be related to deficits in WM.

Deficits may occur in either information representation or information manipulation in WM in either the PL or the VSSP. Verbal information is stored in the PL in the phonological store, whereas static visual information is stored in the VSSP. Manipulation of information occurs through the articulatory control process in the PL and through dynamic spatial imagery in the VSSP. The accuracy of fact retrieval may speak to issues related to information representation in either the PL or VSSP. Geary (1993) suggested that deficits in encoding number facts into the PL was implicated in poor math performance. Error analysis of the work of students, with no disability or with either RD or MD, indicated errors were either attributable to minor miscalculations or use of incorrect operation (Jordan & Hanich, 2000). This suggests that issues related to language, such as the reading of the question or the encoding of facts, may account for these errors. The efficiency of fact retrieval may speak to information manipulation in either the PL or VSSP. Jordan et al. (2003) suggested that deficits in non-verbal reasoning accounted for difficulties with math fact retrieval. Students with MD performed better on untimed math fact assessments than students with both MD and RD

(Jordan & Hanich, 2000), suggesting that these students were better able to manipulate information given more time.

Deficits in math fact retrieval may also be the result of deficits in the CE, which regulates processing. The difficulty with making relevant associations in WM may be attributed to processing deficits in the CE, as suggested by Fuchs et al. (2006). The role of the CE in strategy development and selection is complicated by the role of LTM, attentional issues (Geary et al., 1999), and the rate of development of math abilities of students with MD (Torbeys et al., 2004).

The representation of information in both the PL and VSSP suggests that they both have a role to play in the development of math knowledge, from number sense to number combinations. Similarly, the manipulation of information in both the VSSP and PL suggests that they both have a role to play in the development of math skills, from number sense to number combinations. For example, the concept of quantity may be developed in the VSSP as information is visually represented as an image of an unknown small count. The development of the knowledge of counting may be developed in the PL, as information is represented as a specific count using language. Information manipulation in the VSSP may take the form of spatial comparison of unknown different counts and, in the PL, may take the form of one-to-one correspondence, initially using audible speech or self-talk to count. What remains unclear is the extent to which the knowledge and skills associated with the development of number combinations are related to and dependent on the knowledge and skills implicated in number sense. Additionally, to what degree do the VSSP and PL, and the representation and manipulation of information, interact to solve algorithms or problems? Furthermore, the

extent to which deficits in information representation or information manipulation in either the PL or VSSP can be overcome is unclear. The ability to overcome such deficits may reside in corresponding strengths in the PL or VSSP, in the co-ordinating aspects of the CE, or as a result of quality of or the interest in solving an algorithm or problem.

Dewey: Habits and Working Memory

Growth occurs through successive meaningful experiences, the continuity of experiences. Growth, as a multi-layered continuum that includes concepts and habits, depicts development as the continued adaptation of the physical, emotional, and intellectual characteristics of an individual in response to the demands of their environment. Adaptation requires that the memories from meaningful previous experiences be used to address the demands of the environment in any given situation. For example, drawing on previous experiences and having acquired some conceptual understanding of quantity, the shift to the development of habits of counting and automatic retrieval of number combinations occurs gradually through successive experiences. The development of math skills suggests that the line between conceptual understanding and habit formation is not strictly set. As there is a shift between organic and reflective learning, there is a similar corresponding shift in from conceptual understanding to habit formation and practice. Habits must initially be comprised, in part, of the aspects of concepts. Dewey's internal and external components of habits reflect a continuum of development from concept to habit, from knowing to doing.

Dewey's knowing and doing of concepts and habits coincide with researchers' models of working memory. Information representation is the essence of Dewey's facets of knowing, in the standard of reference of concept and in the internal components of

habit. Similarly, information manipulation is fundamental to Dewey's facet of doing, in the generality of concept and in the external components of habit. Required and related concepts and habits are drawn from memory and applied in novel situations, such that the development of concepts and habits are realized in increasingly sophisticated representations of information and highly developed skilled information manipulation. In representing information, concepts develop and become increasingly refined, from standards of reference to intellectual and internal components of habits. In manipulating information, concepts continue to develop and become increasingly discernable from generality of the concept to the external components of habits.

However, for Dewey, co-ordination rests in the demands of the situation rather than in the CE. The demands of the task or the situation require that judgement is used to apply habits. Judgement requires that knowledge from previous experiences stored in memory be drawn on and applied in a given situation. Habits, resulting from previous experiences, are tools insofar as they are ways of knowing and acting to be used in the pursuit of increasing knowledge universally and specifically. Furthermore, the demands of the situation determine the selection and co-ordination of tools, including concepts and habits, which produce a meaningful experience that fosters growth. Co-ordination occurs when an individual recognizes the demands of a given situation and uses judgement to select and use tools, including those in memory or from the environment, to work to resolve the situation.

Growth from conceptual understanding to habit formation is gradual and yet, observable. Habit formation facilitates the speed and accuracy with which information, knowledge and skills are applied to solve specific algorithms and future situations.

Habits, such as knowledge of basic facts, allow students to solve increasingly sophisticated algorithms and future problems. Without the habit of accessing and applying basic fact knowledge, students become bogged down in solving the algorithm at a pre-habit level or perhaps even at a conceptual level. Memories, concepts, habits, hands, eyes, and ears need to be co-ordinated in the environment to facilitate counting or to solve a basic fact with speed and accuracy to meet the demands of a situation. In cases when students become bogged down, interaction has failed at some level—the ability of the student to interact with their environment to meet the demands of that situation was limited. The principle of continuity suggests that the shift from conceptual understanding to habit formation effects maturity in the form of adaptation. Intellectual growth occurs when habits are selected and applied successfully to situations.

Number Combinations and Cultural Growth

In order to grow, children are dependent on adults to recognize and meet their needs. The physical, intellectual, emotional, and social needs of young children are primarily met by parents and, later, by teachers. In creating an environment to nurture children, adults act to maintain the continuity of their community. Adults transmit the culture and cultural values of their community through their selection of materials, the dispositions they model, and the situations they create or select for children. The quality of experiences is influenced and may even be determined by the interdependency typified in the adult-child relationship. Often working with or supervised by adults, young children work to find solutions to situations, which may or may not be of their own making. Plasticity means that children, through repeated opportunities to practice, are

able to modify their actions so that they increasingly approximate the desired outcome. These repeated experiences foster the development of habits. In fostering the development of habits, adults help groom children to function in their community.

The context in which children live and grow is determined by the adults (directly and indirectly) in their lives and in the environment in which they live. It is the adults, both individually and as a community, who are accountable for the context of education. The starting point, if you will, for educating children is based on both the previous experiences of the individual adult and the community, as well as on the environment. Individual adults build on their own life experiences, which influence their future interactions with others and with the environment. Continuing to grow, adults use inquiry to resolve situations, including finding effective methods of educating children. As the community has developed and grown over time, the age at which a child begins to be initiated is different than it was at other times in history. The community grows as a result of interaction between its members, including children, and the environment. Furthermore, the cumulative experiences of individual children continue to influence the growth of their community as the child grows from young child to adult.

The movement from informal math instruction to formal math instruction reflects the shift from number sense to number combinations, from concepts to habits. Unlike number sense, it is the school that plays the primary role in providing the cultural and physical material used for instruction and practice for number combinations. Teachers bring their personal and professional experiences with them to the classroom, as well as the expectations of the school community and the community beyond. Similarly, parents bring their previous experiences to their role as parents and to formal education. The

previous experiences of teachers and parents impact the number combination experiences they create for and with children. Through the elementary school years, teachers and parents help children to develop automatic and accurate retrieval of number combinations. Through number combination activities, parents and teachers model and use the language of math (oral and written), model dispositions, select and provide materials, and demonstrate the importance of math fact knowledge by the amount of time dedicated to it. Culture is transmitted through education in general and in this case, math education specifically.

Through repeated experiences, a situation, that requires conceptual understanding of number combinations and requires sustained effort to resolve in Grade 1, becomes a habit in later grades. Building on the example of feeding an infant from the previous chapter, the child developing the habit of number combinations can be compared to a child learning to prepare food when hungry. The child is able to go to the refrigerator, select what is needed, and make a sandwich to satisfy their hunger. Initially, it may take support from an adult, but with each additional experience the child requires less support and becomes more proficient at making a sandwich. In the case of math, the child is hungry to learn and to share in the experience with the adult. Initially the adult may need to provide materials, such as holding up fingers. However, the child eventually will use their own fingers to count and will gradually become more proficient at solving algorithms.

Experiences that foster the development of habits, such as automatic retrieval of math facts, are based on those experiences that facilitate conceptual understanding of number sense. When students quickly respond to number combination questions, they are

not only confident that they know the answer but also have established the habit of retrieving basic facts (Jordan & Hanich, 2000). The child has developed a disposition towards education, and specifically for math that fosters confidence in their capabilities. The child has moved along the continuum of math from conceptual understanding to habit formation with regards to number combinations. In school and at home, adults reaffirm the values of math, education, hard work (the work of inquiry), and ends (automatic and accurate answers in the case of number combinations) that are held by the community. Inquiry, used in the development of habits, will be discussed in the next chapter.

Students who have weak number sense may simply be developmentally delayed in developing the habit of automatic retrieval of number combinations as some students require more time to *find* an answer. Lack of experiences or lack of quality experiences to foster the development of number sense and subsequently, mastery of number combinations, suggests a developmental delay—additional experiences or more experiences are needed. Then again, children who continue to use counting as a back-up strategy or worse yet, guessing, may have established a less effective habit. In effect, habituation may have stunted the development of efficient and effective habits. Again, the quality of the experiences is implicated insofar as students are not engaged sufficiently or have not established the disposition necessary to develop efficient and effective habits. Conversely, deficits of the structures of the brain may be involved in the limited ability of students to develop habits related to fact retrieval. Regardless of the reason for the lack of or limited development of habits, educators need to develop an understanding of the significance of the role of habit formation for more advanced

problem solving. As habits later become the tools used to solve situations, it is of paramount importance that the habit of math fact retrieval be developed in one form or another. Furthermore, at school and at home, adults reaffirm the values of math, education, hard work (the work of inquiry) and ends (automatic and accurate answers in the case of number combinations) that are held by the community. In all cases, it would be expected that dispositions towards math, at the very least, would be negatively formed.

As an elementary school subject, math provides opportunities for children to develop concepts and habits related specifically to math and generally to inquiry. Education is a social endeavour; a child's community is expanded so that they and their community grow through their participation and contributions. The community is expanded from home to home and school. Parents' involvement in the formal education of their children speaks to the contribution each *institution* makes to the growth of children and of the community. As the cultural environment uses language to transform organic behaviour to intellectual behaviour, education similarly uses language to *transform* children to adults. Education is cultural growth, for the individual child and the community, as adults participate in education and children leave school to participate in the larger community.

The language of math is specific to its field and is abstract (See Number Combinations, Signs and Symbols, this chapter). The facility with which a student is able to express themselves mathematically, in this case the speed and accuracy of fact retrieval, speaks to the integration of that knowledge and skill. The internal component of the habit of number combination retrieval suggests that the language of math has become internalized allowing for self-talk (audible or inaudible) and reflection, as well as for the

varied and elastic use of the habit. Habits allow for students to communicate in shared activities with their peers and adults. In the case of activities that depend on fact retrieval, speed and accuracy speak to the issue of communication required to participate in the community. For example, children who are slower to *retrieve* facts are left behind as others move on to the next step or to the next problem. Additionally, the ability to communicate is often taken as a sign of maturity or intelligence. Students not able to quickly or accurately respond to number combination questions may withdraw, isolating themselves, or be excluded by members of the group or by the demands of the task. Again, it would be expected that dispositions towards math, at the very least, would be negatively formed.

Number Combinations and Cultural Growth as Inquiry

Through communication, children come to understand and share the aims and habits of their social group. Adults transmit the aims and habits of the community at home and at school. By providing a nurturing environment, adults encourage children to become aware of, share an interest in, and work toward a common end. In doing so, the social continuity of the community is formed in the young, re-formed (reaffirmed) in the adults, and sustained at the level of the community. The education provided through communication of aims and habits occurs in the social and cultural environment, and it too is shaped. The transmission of facts, ideas, experiences and so on, within and between cultural institutions, is only possible through communication. Language as a tool, permits transformation through the development of shared meanings and is itself developed as tool through its use as a mode of communication.

Through education, students grow to learn how to participate generally as a member of a community and specifically as a mathematician. Not unlike other subjects, math problems (algorithms) are presented or occur in a social environment. The relationships between the student and teacher, between students, between home and the school, between the school and the larger community, and hopefully between their parents and teacher are all present in the classroom. The language of math developed and used in the classroom develops out of the history of math and of the relationships in the classroom. As such, mastery of basic math facts has long been held as a valued habit in elementary level math, often appearing as a curriculum expectation and used as an indicator of success at elementary school grade levels. The tools used in education “... have developed in the course of living together with the ways of forming and transmitting culture” (Dewey, 1938b, p. 48). The use of language (tool) to reason and respond to questions in math, in both oral and in written formats, transmits a facet of culture, causing it to develop and grow in both the individual and the community. The *occupation of math* dictates the required habits to be transmitted and formed to be successful in math and in the larger context of the community.

Culture is transmitted through communication, which extends beyond the use of words and symbols. The value of math in modern society is reflected in the time spent in school in the subject area, as well as the resources available in the classroom. Not surprisingly, those who are successful in math are certainly regarded as more capable and perhaps more intelligent in the area of math. In the case of math fact retrieval, speed and accuracy are valued. Correct answers that are quickly applied correlate with success. Furthermore, knowledge of basic facts appears to facilitate higher order thinking skills or

free up WM space required for problem solving (Fuchs et al., 2005; Geary, 2004). The issues related to the value of quick and accurate fact retrieval as a habit speak to a student's efficient and effective control of their environment through the use of materials and language. Students develop the ability to select and apply tools to a specific math situation, having refined their ability to do so, as well as being able to do so in situations in general. However, it also speaks to the student having acquired the cultural knowledge and skills (aims and habits) of their community. Solving algorithms in the classroom, and expectedly later in real life, happens in a social context and is culturally bound. In developing the habit of number combinations, math becomes a tool so that the child also develops the habits related to math, social interaction, interaction with the environment, and the customs and values of the community, social continuity.

When developing habits, basic algorithms may be considered a math problem or, in the language of Dewey, constitute a situation. The habit of applying knowledge of facts, the internal component of a habit and the action required to solve algorithms, the external component of a habit, require that students use judgement in directing their reasoning efforts to observe and reflect on the demands of the task. The accuracy of recalling math facts speaks to the internal component of the habit, whereas the speed of recall speaks to the external component of the habit. The internal and external components of the habit are epitomized as information representation and information manipulation in both the PL and the VSSP. The abstract language used in representing numbers and operations to recall and respond to questions involving basic facts is by and large developed and used by each individual in shared activities in the classroom. The abstract methods of manipulating mathematical information, such as operations or

strategies, are similarly developed and used by each individual in shared activities in the classroom. The abstraction permitted by the language and manipulations of math creates experiences that have a dimension different from other subject areas in elementary education and, yet, share in that students are becoming aware of the aims and habits of their community. These abstract tools continue to be developed through the shared meanings and culture of the classroom and facilitate future activities that foster growth, as well as foster communication between students, between teachers and students, and, in the report card, between the institution of the school, the student and their parents.

Students with MD, co-morbid MD/RD or RD appear to develop number combination skills differently than their normally achieving peers. Students with MD only appear to experience greater difficulties with efficiency rather than effectiveness when compared to the performance of students with co-morbid MD/RD on untimed tests (Jordan & Hanich, 2000). Students with both MD and RD may have difficulties with conceptual understanding that impact the development of tools of habit. That is to say, limited number sense impacts the development of the knowledge of basic facts. Furthermore, error analysis indicated that the types of errors made by students with MD or RD more closely reflect the errors of students who are normally achieving, suggesting that the difficulty lies in efficiency or effectiveness of habit, whereas types of errors made by students with both MD and RD suggests that difficulties lie within conceptual understanding (Jordan & Hanich, 2000).

Dispositions towards math can only be assumed to be formed differently in students with no difficulties in math than in students with MD only, RD only or in students with both MD and RD. Continued dependency on counting as a back-up strategy

suggests that plasticity, related to the quality of the experiences or the development of tools (e.g. language, concept, habit), plays a role in the shift from conceptual understanding to habit formation. Students with MD or RD only appear to be different in terms of their ability to use judgement in applying habits from students with both MD and RD. Students with co-morbid MD/RD appear to have not benefited from experiences that foster habit formation, whereas students with either MD or RD appear to have not developed the tools associated with habit formation. Students with both disabilities appear to have limited understanding that requires further conceptual development, while students with either disability appear to need more time to develop and apply habits. Creating experiences that provide additional time in fostering habit formation would provide students with either disability opportunities to participate in classroom activities. Students with either disability may initially produce positive dispositions towards math but become frustrated with the tools they develop in their experiences in class. Educators may facilitate further growth vis-a-vis habit formation by providing students with either disability further opportunities to develop tools, such as strategies, or providing them with alternative tools, such as assistive technology. By not developing the habit of number combinations similar to others in the social group that makes up a class (a model or microcosm of the larger community), *math* places limits on the development of other habits related to math, social interaction, interaction with the environment, and the customs and values of the community, social continuity.

Number Combinations, Signs, and Symbols

As stated in earlier chapters, Dewey distinguishes natural signs from artificial signs, but recognizes that both are used to communicate. The fact that natural signs exist

in spatial-temporal context limits or impacts the ability to communicate. They provide evidence from which one can infer. The usefulness of artificial signs or symbols is infinitely far greater. The cultural context provides the means by which symbols are developed and used. It is the shared meaning, developed out of the social interaction with others and with the environment, which holds the intellectual property of symbols. Inference is possible when meanings are related to things in existence. Symbols may be used to reason or think about things that have the possibility of existing or are themselves abstract (e.g. gravity). In using symbols, individuals are able to recollect experiences, deliberate and rehearse ideas, abstractly reason, and consider hypotheses, individually or in sharing with others. Symbols are invaluable to inquiry.

Dewey's distinctions between signs and symbols have particular significance for the development of habits of number combinations. Counting is initially dependent on things to be counted. Signs depict those things that are counted. Symbols, the number stated or presented in print (Arabic symbol), depict a count, not necessarily associated with any particular item that is countable or measurable. Similarly, mathematical operations involved in solving number combinations (addition, subtraction, multiplication and division) shift from the concrete to the symbolic, from putting together groups of physical items to be added to recalling $4+3=7$. Inference is possible because the abstraction is based on real and tangible experiences. Inference is also possible because, mathematically speaking, items can be related to one another at a level of abstraction that is not possible in real terms. The symbolic language of math is uniquely abstract and creates an intellectual property that is also unique, a tool specific to the occupation of math. The language of math is also used in other fields, such as science, but may not

necessarily share the same exact meaning. The language of math is also used and has meaning in the larger cultural context. Many math terms or symbols are used in economics, banking, technology, baking and so on. Math, and its language, has both an academic context and a cultural context.

The capacity to use mathematical symbols is readily apparent in the speed and accuracy of retrieval of basic facts. The use of symbols allows for a level of manipulation that is practical and allows for the testing of hypotheses or ideas. At the level of basic facts, symbols allow a student to recall that $10 \times 10 = 100$ rather than making 10 groups of 10 to solve. The habit of number combinations is possible because of the abstract tools – math symbols – that are used. The application of the habit of automatic fact retrieval has real value in resolving situations in the classroom and for real life situations.

If the difficulty with math for students with MD lies within their capability to develop and use the language of math, the situation for educators, parents and the community is quite significant. If the difficulty lies in the shift from the concrete to the abstract or from the use of signs to the use of symbols, the growth of the individual with MD may be slower or more limited than for their peers without MD. In either case, the ability for students with MD to participate in the social context is impacted. They are limited in their ability to share in and later recollect experiences, deliberate and rehearse ideas, abstractly reason, and consider hypotheses, individually or in sharing with others in both the classroom and in the community. The growth of the community is also impacted by the limitations of individuals that comprise that community. As a cultural institution, schools sustain the continuity of the community and as such, are responsible for the growth of the community, as well as of individual students.

Number Combinations and the Environment

The environment interacts with personal needs, desires, purposes and capacities to create experiences (Dewey, 1938a, p. 44). Continuity of social life is shaped by the experiences had in the environment. The environment is comprised of those conditions that promote or hinder growth (Dewey, 1916, p.11). The customs, traditions, and values that have been formed through interaction with others are some of the environmental conditions that influence or shape experience. The impact of the environment on the development of the character and of mind is significant, but may not always be obvious (Dewey, 1916). The mode of life of a community, and access to resources and tools are also conditions that influence the quality of experiences (Dewey, 1916). The necessity for the development, the usefulness, the function, and the worth of a tool becomes apparent in the ability of humans to control their environment (Hickman, 1990). By actively participating in the social environment, individuals come to share in the functions and the resolution of situations in that community. In doing so, they engage in experiences that result in sharing similar interests, values, and ends.

Two specific environments may impact the development of number combinations. While the home appears to play a greater role in conceptual development of number sense, the school appears to play a greater role in the development of the habits of automatic and accurate retrieval of number combinations. A third, larger environment, also impacts the development of number combinations. The larger environment of the community includes the relationship between the home and the school, and reaches far beyond. All of the individuals that comprise these environments have their own habits based on their own experiences, and these habits include the habit of inquiry. These

environments may collaborate or interfere, to a lesser or greater degree, with one another in creating the conditions necessary for the development of habits. Parents rehearsing basic number facts with their children at home reflect or support the school environment. If the home environment is not supportive of education in general or de-values math specifically, the child may experience school quite differently than others. In the case of students with MD, they may find that not possessing the prerequisite skills, and perhaps dispositions, or not responding to interventions may also result in school experiences different from others. The continuity of the community, as well as the growth of children, is dependent on the collaboration of the home and school to educate according to the needs of the children.

In the development of the habit of automatic and accurate fact retrieval, students are encouraged to use concrete materials and the language of math. A shift from the use of concrete materials alone, to the use of concrete materials and the language of math combined, to the use of the language of math indicates a similar shift from concept to habit. Language is a powerful tool used in the social environment of the classroom by both the teacher and the students. Students with MD appear to use inaudible private speech, whereas students without MD use silent private speech to solve problems (Ostad & Sorensen, 2007). The habits related to the development of the habit of automatic and accurate math fact retrieval include self-talk, using fingers, counting, and so on. The development of habits is clearly dependent on the use of tools to do so. These tools include concepts, other habits, language, concrete materials (manipulatives), eyes, ears, fingers, and memory (see Chapter 3).

Gersten et al. (2005) reported that rote drill practice was not effective in developing the ability to recall basic facts. The reasons for difficulties in acquiring basic facts may include the following. First, memory as a tool may not be developed or may be faulty as a structure of the brain. Second, children may not find meaning in the situations that comprise questions related to basic facts or algorithms. Third, the child may need more time and experiences to develop the tool or habit of basic facts. Fourth, the language used to develop basic fact knowledge may present difficulties for the child, and these difficulties may be related to the shift in development from concept to habit. Keeler and Swanson (2001) found that students who had high verbal WM scores demonstrated a preference for verbal strategies, and, similarly, those who had higher visual-spatial WM scores preferred visual-spatial strategies. Building basic fact knowledge and strategies may require increased pairing of the representation of information in visual-spatial and language (oral and written) formats.

The development and use of tools that facilitate the habit of automatic fact retrieval include strategies. As strategies typically develop independently in most students, the environment clearly plays some role in promoting and stimulating the development of the tool. The delayed development of strategies (Torbeyns et al., 2004), coupled with the research on the effectiveness of instruction of shortcut strategies (Gersten et al., 2005), suggests that the classroom environment may not promote and stimulate strategy development in students with MD. However, the explicit and direct instruction of strategies seems to be somewhat effective in strategy development. The unconscious influence of the environment may be clearly seen here. The child may need more time or experiences, such that the demands of the environment are incongruous

with the needs of the child with MD in terms of time allotted to master skills or to develop tools (habits).

The use of speech or any form of communication in directing the activities of students is of paramount importance. It integrates the specific subject matter information (math) to be transmitted with social interaction in a cultural environment. The tools used to educate in the classroom environment reflect the culture of that community. The situations selected and the social interaction with others and the created environment all speak to the culture of the community. The use of teaching practices that recognize and include the needs of students with MD speak to democratic values of the community.

Number Combinations, Experience, and Growth

Educative experiences result in growth (Dewey, 1938a, p. 28). Successive and interconnected experiences further growth. The integrated aspects of experience are continuity and interaction. According to Dewey (1938a), the quality of experience is established by continuity and interaction, but a better quality of experience is a democratic social experience. Tools are extensions and expressions of experiences, as well as integral to them (Hickman, 1990). In sharing the work of inquiry, individuals take responsibility and contribute according to the demands of the situation. Social control rests within the mind and work of inquiry (Dewey, 1938a). It is the opportunity to develop the concepts and habits required for inquiry that is democracy for children, and it is the prospect of effectively participating in inquiry that speaks to democracy for all members (children and adults) of the community. Adults are responsible to create the opportunities for children to grow and develop the concepts and habits that are integral to

inquiry, as well as democratic dispositions, such that the community grows democratically.

For many students, the development of strategies appears to happen automatically. Numerical patterns or relationships are perhaps more obvious to these students. Number sense does appear to perhaps play some role in the development of strategies, as strategies require the decomposition of numbers and manipulation. While strategy development appears to facilitate the ability to respond more quickly and accurately to number combination questions, the space used in WM to do so would appear to be greater than simple recall. Both the habits of automatic recall and strategy use are dependent on the concept of number sense. This may speak to the development of a tool (habit of strategy use) to facilitate the future development of the habit of automatic recall of basic facts. Habits can develop out of habits, as the demands of the situations require, with repeated experiences to promote the *maturation* of habits. Using a calculator is a habit when it is intelligently selected and used. Researchers must consider the use of alternative habits, such as calculators, and the impact on inquiry (problem solving).

As early as 1974, researchers were calling for the development of more effective teaching practices for strategies. In Chapter 3, the findings of researchers suggest that the remedial instruction of strategies is effective in promoting the accurate and efficient recall of basic facts. Tournaki (2003) found that the control group, comprised of students with and without LD that did not receive additional instruction, demonstrated no improvement when compared to the students with MD who received either direct instruction or cognitive behaviour modification. Not surprisingly, instructional approaches such as cognitive behaviour modification proved to be more effective than

direct instruction in the acquisition of strategies (Tournaki, 2003). Both the use of computers (Fuchs et al., 2006) and the use of authentic tasks (Male, 1996) also appear to be effective. These studies suggest that there are a variety of effective methods of instruction that promote the development of habits. That is to say, experiences are required for growth to occur; however, the quality of the experiences may either enhance or limit growth.

Students participating in the many studies on LD and MD, in particular, have received some of the best instruction available. The experiences epitomize continuity in that researchers clearly establish the needs of participants, identifying where to begin and the deficits to be addressed. The environment includes the tools developed to address the needs of the participants, as well as adults with positive dispositions towards their own inquiry into MD and those able to participate. The children in the study may share similar experiences at school and home, with regards to math skill development. The interaction of the children in these studies with one another, the adults and the environment would appear to promote the growth of math skills and perhaps dispositions towards education. The transference and maintenance of skills appears to be questionable in light of Montague's (1997) finding that students need periodic booster sessions after returning to the classroom. The instruction in the classroom may not be building on the experiences of children that promote the development of math skills and knowledge, as well as growth.

The instructional needs of children to develop math strategies are documented in the research literature on MD. Given that some students require math strategy instruction, failure to provide it not only impacts those students vis-a-vis math, but also the community of learners in the classroom, students and teacher. If math strategy instruction

were integrated into all classroom math instruction, all students would be given the opportunity to develop, select, and use the tools that are strategies. Continuity of experience would be established in each particular year, but also throughout years of education. These tools—strategies—are integral to solving number combinations. They are part of the environment in which inquiry occurs. The ability of students with MD to contribute to situations and become responsible for situations is dependent on the experiences that have led to a given situation. The situation determines the work to be done and if one is not able to contribute, one is not able to share in the experience – academically, socially or otherwise. It is the responsibility of the teachers to provide children with experiences that develop habits integral to the specific modes of inquiry (i.e. math, history, mechanics) and the universal habits of inquiry, as well as the dispositions created in the classroom. Through education, children develop dispositions towards education, but perhaps more importantly towards others.

Discussion

The multi-layered interconnectedness of the model of growth and education put forward by Dewey provides a rich context in which to place educational research of children. Growth allows for the consideration of much that impacts early education. As education is the continuation of the initiation (from home) and the transmission of culture through experiences at school, culture is really the only subject at school in the earliest grades. It encompasses all others. The tools selected, the experiences created, the development and use of language, the environment, and the dispositions of teachers all reflect culture. The shift in growth from concepts to habits occurs in a cultural context.

The need for social interaction with others and the environment requires that children develop an understanding of the culture in which they live. In elementary school, as children work to solve situations in various subject areas, they are working to solve the situations of understanding what is important, how do I do this, what is valued, how to treat others, how to treat adults, how to adapt to the environment, and so on. Early elementary math class fosters the growth of both concepts and habits, specific to math and general to inquiry, but that speaks directly to the cultural growth of children as they work to become members of the community. In the classroom, through the development of math knowledge and skills, children are inquiring about their culture and the adults should at the very least be reflecting on it.

CHAPTER 6: APPLYING DEWEY'S FRAMEWORK TO PROBLEM SOLVING

Inquiry and Problem Solving

To this point, the discussion has been primarily about the shift from organic growth to reflective growth. It is the shift and the difference between *learning to read*, *reading to learn* that illustrates the point of this chapter. The maturity of physiological structures, the development of concepts, and the establishment of habits advance the capabilities of inquiry for children. For Dewey, the development of concepts and habits is the result of inquiry. Children develop the concepts and habits required to resolve situations in the development of those concepts and habits through the process of inquiry. Inquiry results in the growth of concepts and habits, and, consequently, the growth of the individual and of the community.

For the present purposes, it is essential to take into account the development of the knowledge and skills related to problem solving (inquiry) through the development of number sense (concepts) and number combinations (habits). What constitutes a problem (situation) at various points along the *developmental continuum of math skills and knowledge* relies upon the extent of previous meaningful experiences of children. The recognition of a problem as a situation requiring inquiry is grounded in a specific experience that relates to previous experiences. Based on relatively few experiences, infants come to develop an awareness of magnitude (see Chapter 4). The concept of quantity begins to take form. Further along the *developmental continuum of math skills and knowledge*, children begin to count, having developed rote skills. The habit of counting begins to take form. In both cases, the child involved has a repertoire of

experiences to draw on. Presented with a troubling situation, the infant or young child initially required the intervention of an adult to resolve the situation. Later, the support of an adult facilitated the outcome, until such time as the child could work to resolve the situation independently. How the infant or very young child developed the capacity to solve the situation independently speaks to their development of the knowledge and skills related to inquiry, as well as the interaction with others and with the environment. Furthermore, it speaks directly to the cultural initiation of the young into the community through the development of the knowledge and skills of inquiry.

In order for inquiry to occur, there must first be a situation to be resolved. From infancy through childhood, children come to identify a situation as such. As Dewey describes, “[t]hinking begins [with] ... a situation that is ambiguous, that presents a dilemma, that proposes alternatives” (1938b, p. 122). In uncertainty, a search for additional facts or the relationship between facts requires reflection (Dewey, 1938b). The search for a solution guides inquiry (Dewey, 1938b). A situation, as described by Dewey, differs significantly from the problems described in Chapter 3. Namely, the difference lies in the *assignment* of problems are that are described in terms of requirement of the student to analyze, interpret, select information, and that may involve any number of different steps. A problem only becomes a situation when the student or child finds it meaningful, otherwise it may simply occupy the child’s time. Not surprisingly, both Dewey and researchers are concerned with the development of problem solving skills. Dewey’s stages of inquiry, presented in Chapter 2, provide the context in which the research on math problem solving skills for students with MD is considered.

Inquiry and Biological Growth

As growth is the result of intra-organic structures' interaction with the environment to resolve a situation, it provides the basis for and a model of reflective growth and inquiry (Dewey, 1938a). Genetics may be implicated in the development of number sense (Anasari & Karmiloff-Smith, 2002). As discussed in Chapters 4 and 5, the human brain is implicated in the development of number sense and number combinations. The right hemisphere of the brain appears to play a significant role in the development of number sense (Mazzocco, 2001; Rourke & Conway, 1993), whereas, both hemispheres contribute to the conceptual development of number combinations (Mazzocco, 2001; Rourke & Conway, 1993). However, poor fact retrieval may be more strongly related to deficits in the left hemisphere of the brain (Mazzocco, 2001; Rourke & Conway, 1993). Associated with the right hemisphere of the brain, deficits in visual-spatial abilities have been attributed to students who struggle with number combinations (Jordan et al., 2003), as well as those who struggle with problem solving (Garderen, 2006; Geary, 2004). Neuro-psychological researchers' focus on the relationship between structures of the brain and deficits in WM supports the involvement of biological structures and functions in the development of number combinations and their retrieval. Furthermore, Mazzocco's (2001) work clearly supports a relationship between brain function and a procedural math disability which is indicated by immature problem solving strategy.

Inquiry, as it is presented in this chapter as math problem solving, is based on the conceptual understanding of number sense and the habit of efficient and accurate retrieval of math facts. Inquiry begins with the concepts and habits developed, and results with their further development. Research into the development of the human brain indicates

that brain development moves from back to front, with the frontal lobes (the seat of executive functions of reasoning, planning and impulse control) not fully maturing until early adulthood (Giedd et al., 1999). The biological basis for development of problem solving knowledge and skills is based on the growth of the human brain. Clearly, there is a biological basis for inquiry that results in the development of the growth of concepts and habits. In the next section, the role of the CE (central executive) of WM in problem solving is discussed.

Working Memory and Problem Solving

The section on working memory in Chapter 3, albeit brief, provides sufficient information on the central executive (CE) for the present purpose. The functions of the CE presented in Chapter 3 included supervising, co-ordinating, initiating and directing processing, comprehending, filtering, locating, manipulating, and selecting and switching strategies for retrieval of information. As these skills are required to solve problems, it would seem that problem solving requires that the CE functions well, as well as drawing on both the PL and the VSSP. However, children with MD appear to have deficits in the PL, VSSP or the CE. The function of and the impact of possible deficits in either the PL or the VSSP appear to be better understood than either the function or impact of deficits in the CE.

A review of the literature on problem solving skills of students with MD, clearly indicates that research has focused on the implications of number combination skills on problem solving. Jordan et al. (2003) found that fact mastery had no impact on problem solving performance of Grade 3 students. Jordan and Hanich (2000), Parmer et al. (1996), and Garcia et al. (2006) have all found that students with MD find it increasingly difficult

to meet the demands of problem solving. Fuchs et al. (2004) concluded that math difficulties may contribute more to problem solving difficulties than reading difficulties. However, difficulties with problem solving are not attributed to the CE or deficits within the CE. Rather, it appears that deficits in either the PL or VSSP explain difficulties with problem solving.

Nevertheless, problem solving requires that students analyze, interpret, select information and perform any possible number of steps to solve. The idea that the *universal* steps used to solve math problems can be isolated and taught is appealing. The use of schematics suggests that there are observable and set procedures that can be taught to students with MD so that they become better able to solve problems. The possibility of reassigning the skills required to solve problems from the CE to the external demands of the problem is debatable. A longitudinal study would be required to see if students taught to use schematics are able to generalize the format to increasingly demanding problems as they mature. Regardless, as problems may be classified as complex, authentic (real world), simple, direct, multi-step or situational, it becomes clear that problems must be designed for *learning* schematics rather than *learning* problem solving. At any rate, these problems are usually computational problems, focusing on only one area of math.

As problems become more difficult, performance of all children deteriorates (Fuchs & Fuchs, 2002). The limited experience of children in primary and elementary grades clearly impacts their performance with regards to problem solving. The performance of students with MD appears to be impacted by both limited experiences and deficits in WM. Deficits in WM appear to have an even greater impact on the

performance of students with co-morbid MD/RD. The role of the CE in problem solving needs to be better understood.

Dewey: Inquiry and Working Memory

The continuity of experiences that result in growth reflects the development of the knowledge and skills of inquiry of individuals. The facility with which individuals engage in inquiry is a direct result of their previous experiences in resolving situations that resulted in the formation of concepts and habits. The development of specific concepts and habits comes about as the result of resolving a specific situation or as a result of resolving a series of successive situations. These concepts and habits may then be used in a general way in future experiences or to resolve future situations. But the development of these same concepts and habits also speaks to the development of the knowledge and skills of inquiry. “The aim of inquiry is ... to reconstruct both found materials and available tools in ways that render them more richly meaningful” (Hickman, 1998, p. 168). These enhanced tools (concepts and habits) may then better enable the individual to participate in future situations, interacting with others and with the environment. The possibility of future growth is heightened through experiences that require inquiry, both specifically and generally, and for the individual and the community (discussed later in this chapter).

As a simple example, a young child needs to verify that they have enough money to buy a present. They have conceptual understanding of counting, money and so on. They have observed and participated in shopping experiences so that they know what to do when they go to a store (established habits). However, the child has never been responsible for shopping and ensuring there is enough money to make a purchase. In this

specific situation, the child is required to interact with others and the environment in a new way. In taking on this challenge, the child uses knowledge and skills from previous experiences to try to resolve this specific situation. They compare the price ticket to the money they have in their pocket, and decide to proceed to the checkout. The child observes others in line, confirming the order of events from previous experiences held in memory and the appropriate responses from their memories of how their parents responded during earlier shopping trips. After the item is put through the cash, the total amount requested from the child is more than the ticket price. Using more cash than the child had planned, the child adjusts to the demands and the item is purchased. The child could have chosen to ask a parent to shop or found some other solution to the specific situation. However, their previous experiences (memories) prepared them for this experience. The solution to this specific situation resulted in the growth of the child. Generally, the child grew as they expanded their ability to resolve different kinds of situations (commerce) and through increased participation in their community.

As discussed in Chapter 2, Dewey does not directly address the role of memory or importance of memories to education. However, meaningful experiences (memories) are a resource used by individuals to facilitate their participation in the community and that fosters growth. Memories are tools that are used in future experiences. Accordingly, one function of memories is to further the development of knowledge and skills that make adjustment and adaptation to the environment possible. Knowledge that forms concepts and habits are the standard of reference of concepts and the internal components of habits. Skills that form concepts and habits are the generality of concepts and the external components of habits. A meaningful experience that results in a memory or contributes to

an individual's pool of memories, may be drawn upon to resolve any situation that later may result in growth, including the development of concepts and habits.

Memories, as concepts and habits, undoubtedly play a role in adaptation. Drawing on memories from similar previous experiences, individuals may know how to proceed in the given circumstance (transactions, interactions). Having no such similar experiences to draw on, a situation may develop into a determinate situation (Dewey, 1938b). The individual uses judgment to select information from memory that they consider will aid in the resolution of the situation. The transformation of an indeterminate situation into a determinate situation suggests that the information held in memory has been judged by the individual as inadequate in some way. Memories may be organized in such a way or so limited as to moderate an individual's ability to continue without considerable effort. Through the initial stage of inquiry, the conceptual development of the parts of the determinate situation takes place, resulting in the selection of data and tools to be used. The tools selected from the environment and from memory, including concepts and habits, are aligned to the pattern of inquiry that has been developed by the individual (cultural pattern of inquiry is discussed below) to that point in time. As a tool, memory stores the knowledge of knowing and doing of concepts and habits, which are also tools.

For Dewey, the situation itself suggests the resources and actions required to be taken. The use of judgment refines the selecting of specific information and other tools to be used in the resolution of a particular determinate situation. The co-ordinating, filtering, manipulating, and selecting and switching strategies for retrieval of that information in memory is a function of memory as a tool. Experiences provide the exercises that foster the development of memory as a tool.

Problem Solving and Cultural Growth

In meeting the needs of infants and children, adults create a nurturing environment based on their own experiences and their interaction with children, transmit culture and cultural values, and, in doing so, maintain the continuity of their community. Through interaction in both informal and formal educational environments, children grow as a result of their experiences in the community. Adults (parents and teachers) grow as they reflect on their roles as *educators* of children. The relationship between children and adults has the potential to stimulate the development of growth in a number of possible directions beyond the immediate situation. The direction of growth speaks to the culture and inherent cultural values of the community. Responsibility for the children's growth shifts from adult to child as the child comes to develop the ability to reason (independence) through their education. As their awareness and understanding of the culture and cultural values of their community develops, children demonstrate that awareness and understanding through their ever more independent participation in the community. The ability of a child or any individual to participate in the community speaks to their understanding of the culture of their community, as well as to the culture of the community. The culture of the community is reflected in its capacity for participation of its members. It is that capacity that demonstrates democracy.

Through their development of concepts and habits, children practice and acquire the knowledge and skills required to participate in their community. Children acquire the basic concepts and habits that are required to resolve routine situations earlier in life. Based on the acquisition of these basic concepts and habits, children begin to develop the

concepts and habits that are specific to a variety of occupations (e.g. math, language arts, history). Through the development of basic and more specific concepts and habits, children also acquire and develop tools, as well as the concepts and habits that comprise the culture of their community. Ideally, children should be educated so that they can both function in their community with relatively little effort (using basic and specific concepts and habits) and grow in their community through opportunities to participate when relatively considerable effort is required (inquiry).

Opportunities to develop and practice concepts and habits require plasticity to grow. The continued development of basic concepts and habits to more mature concepts and habits is dependent on plasticity. Neurological research suggests that plasticity continues into early adulthood (Giedd et al., 1999) suggesting that growth, at least biologically speaking, continues well beyond early childhood. Apart from biological development, individual growth occurs as a result of opportunities presented in the cultural and social environment. In developing concepts and habits, children acquire more than those specific concepts and habits. They acquire the ways of knowing and doing that are valued in their community. The repeated experiences that result in the development of concepts and habits also foster the development of concepts and habits of inquiry. In developing the knowledge and skills of inquiry, the capacity to grow is heightened.

The *continuum of development of math knowledge and skills* from number sense to number combinations to problem solving imparts a pattern of growth that is similarly reflected in the shift in the roles of adults and children in the growth of concepts to habits to inquiry in children. Furthermore, this pattern is similarly portrayed in the shift in the environment in which a child participates, from the home to the school to the larger

community. The development of the conceptual understanding of number sense occurs in the informal educational environment of the home. The development of the habits related to number combinations occurs primarily at school, a formal environment. In either environment, the child interacts with both the environment and adults. In these contexts, children are acquiring the basic skills required for inquiry. Adults construct, demonstrate, guide, and aid the child in co-ordinating their activities of inquiry so that concepts (number sense) and habits (number combinations) are formed. At a cultural level, it could be said that the adults play the role of the CE in infancy and early childhood as the child develops basic concepts and habits. As the child develops concepts and habits through repeated activities, the opportunities to develop the knowledge and skills of inquiry are simultaneously occurring. Through these repeated activities and opportunities, the child increasingly assumes responsibility for co-ordinating their own activities.

The parents and teachers, having co-ordinated efforts to develop basic math skills, have also modelled and supported the development of the knowledge and skills of inquiry. Initially, formal math problem solving is often nothing more than computational problems. For example: Johnny had 2 apples and Jenny had 3 apples. How many apples did they have altogether? Teachers and parents would provide support to the child in their illustration of the problem pictorially, in Arabic format, and in language through the reading of the question and in the written response. In developing number combinations, these problems also speak to the cultural knowledge and skills held by the community. The problem speaks to the culture of the community at a level that is appropriate to the child. The use of apples rather than kiwi, as well as the names used, is somewhat telling of the community. The fact that a boy and a girl both play roles in this problem is telling

of the community. And lastly, that we would want to know *how many they have altogether* suggests that *sharing* is something to be valued in this community. At more sophisticated computational levels, the written format (e.g. French format of long division is the opposite of the English format of long division) or demonstration of computational steps (e.g. regrouping) may be culturally specific. Furthermore, these early problems serve as models of the work that is the occupation of math, initiating children to the *culture of math* and the methods of inquiry, both specific to math problem solving and to inquiry in general.

Inquiry requires the co-ordination of concepts (knowing) and habits (doing). It is the child (person with an indeterminate situation) who must take responsibility to co-ordinate their efforts to resolve situations. They must draw on their previous experiences in solving problems in math and in the methods of inquiry in general. They must consider the knowledge and methods they have at their disposal, as well as their observations of teachers and parents working to resolve situations. The resources and tools selected by children to use for inquiry speak to their knowledge and skill development with regards to math, inquiry, and culture. The availability of resources and tools for inquiry speaks to the level of knowledge and skill development in the fields (occupations) of math, inquiry, and culture of the community. Typically, as a child's knowledge and skills of inquiry develop, the environment in which that child acts to resolve situations expands from home to school and increasingly to the larger community. The opportunities to continue to develop and grow the knowledge and skills of a particular occupation (math) and inquiry in general reside partially in exposure to environments that foster that growth.

The opportunities provided in the school (e.g. access to tools), and, later in the community, are significantly different than those in the home.

Building on the example from previous chapters, a child goes to the refrigerator to select what is needed and to make a sandwich to satisfy their hunger rather than asking their parent to help them make a sandwich. The shift in responsibility is readily apparent. The child is able to make a sandwich, as an adult might, with no support. That same child, with the cultural knowledge of a sandwich is also able to order a sandwich in a restaurant (community) and have their expectations met. In the case of math, the adult may initially need to identify the problem, its parts, the materials required to resolve it, and what constitutes an acceptable solution. However, the child eventually will assume responsibility to identify the problem, select materials, and work to solve the problem. Through repeated experiences, the child will gradually develop a pattern of inquiry so that they may solve situations independently. It is through their relationships with others (including the observations of others) that the child comes to develop the concepts and habits that allow them to share in experiences in their community, including working to resolve situations—inquiry.

Inquiry requires that the child make several judgements. As the child moves along *the continuum of math knowledge and skills development* from conceptual understanding (number sense) to habit formation (number combinations), the knowledge and skills of inquiry are developed. The compilation of judgements used in particular situations of inquiry come together to comprise the concepts and habits that form a pattern of inquiry. Judgement is required in identifying a situation as indeterminate, the selection of abstract and concrete tools to be used, the possible impact of various courses

of action, the identification of the constituents of the indeterminate situations, suggestions of possible solutions (ideas), the *operationalization* of the situation vis-a-vis its symbolic formation, and so on. At every turn in inquiry, judgements are required and made. The facility with which judgments are made speaks to the concepts and habits associated with the development of the pattern of inquiry that are specific to math and general to inquiry. As the general pattern of inquiry is applied to all indeterminate situations and judgement is required for inquiry, judgements made are specific to the occupation (e.g. math) of the specific indeterminate situation and in keeping with the culture of the community. If you will, the continuity of an occupation and the culture of a community require that judgements made are in keeping with the historical or developmental continuum of knowledge and skills (concepts and habits) of the particular occupation or culture.

The purpose of identifying a doubtful or troubling situation as an indeterminate situation is to re-establish balance between the individual and the environment (Dewey, 1938b). However, this requires that the individual judge the situation as being doubtful or troubling. Some infants or children may not find a particular situation or type of situation to be troubling or doubtful. Formulating (*operationalizing*) the situation so that its constituents can be identified or so that the situation is workable requires that judgements be made. Difficulty judging the parts of the situation, the correct tools or resources, or the correct methods (steps) to be used in inquiry may cause children to take inappropriate actions. Judging the impact of actions or assessing the worthiness of various ideas may be challenging for some children, especially as they are in the process of developing concepts or habits. Recognizing that the indeterminate situation has been resolved also

requires judgement that must be based on the individual first judging that there was a situation requiring inquiry.

The opportunity to participate in inquiry is initially determined by the ability of individuals to judge a situation as indeterminate. The facility with which an individual participates in any particular act of inquiry is the result of the concepts and habits acquired through education that relate to both the type of inquiry (occupation) and inquiry in general. The capacity of any individual to participate in inquiry is consistent with the social and cultural concepts and habits of inquiry of the community. Integration of the individual into the community, as they mature from infant to adult, resembles the shift and growth of the knowledge and skills developed through inquiry from concept (number sense) to habit (number combinations) to inquiry (problem solving).

Problem Solving and Cultural Growth as Inquiry

The aims and habits of the social group come to be understood and shared through communication. As the child matures, the aims and habits of the community come to be transmitted in the child's ever expanding environment. Children, in the process of assuming the aims and habits of the community, continue to develop greater awareness of, share an interest in, and work toward a common end through shared experiences with both adults and other children. The social continuity of the community is initially based on the shared relationship with adults and children. The commitment to the social continuity of the community is formed in the young through their relationships with others as they work together in various environments to resolve situations. The social and cultural environment is shaped through the education of aims and habits as children are transformed and take greater responsibility for resolving situations. Transformation

occurs through communication, which makes the transmission of facts, ideas, experiences, and so on within (internal dialogue) and between children possible.

Language as a form of communication and as a tool transmits information, making the transformation of individuals, institutions, and the community possible and as a cultural institution is transformed as a product of shared experiences.

Growth is the result of inquiry. Without inquiry, infants and young children would not develop the concepts and habits required to begin to function within their community. Afterwards, through participation in the community, increasingly sophisticated concepts and habits are developed so that as children mature they gradually become further integrated into their community. As their environment is expanded from home to school to the community, their opportunities to participate are expanded and they become even further integrated into the community. Eventually, children come to assume responsibility for their community as they develop the knowledge and skills to resolve situations of their community. The social and cultural context in which inquiry occurs informs the concepts and habits that comprise both inquiry and culture.

For example, the social and cultural contexts of a community shape the very rich concept of *sharing*. Individuals are repeatedly exposed to this concept. Biologically speaking, sharing begins as the parental contribution to the genetic make-up of the child and develops as the parents share the responsibility of raising a child. The line between meeting physical needs and being responsible for social, emotional, and cultural needs shifts and blurs as the infant becomes a child. The level of responsibility of the adult shifts as the child comes to share in the responsibility for their own growth. When hungry, the infant depends on the parent to provide food and, in doing so, share in the

experience. As the infant matures, they interact with the parent to communicate their needs and begin to feed themselves. In sharing the experience, the parent and child are in the process of developing the concepts and habits that comprise both inquiry and culture in their relationship and their environment. Initially, the parent uses inquiry to come to understand what is meant by the infant's cry for food at a certain time of the day.

Similarly, as the infant is uncomfortable, they come to act in such a way as to have their needs met. After several similar experiences, the parent and infant come to establish and share in the habits that comprise meal time. They share in the history of experiences that comprise the shared meanings that are the concepts and habits of their community—their home. These concepts and habits are shared in a general sense with the larger community as the infant and parent comprise part of it.

At a very young age, children have an understanding of sharing that is based on their relationships with others. Sharing is a concept rooted in strong social and cultural contexts. The experiences that shape the concept of sharing, as well as the habit of sharing, require that individuals use inquiry to resolve situations, such as sharing a cookie with a friend. At a basic level, the concept of sharing is expressed mathematically as dividing. Again, at a basic level, the habit of sharing is expressed mathematically as number combinations (basic facts). The mathematical concept and habit of division is based on the cultural concept of sharing, which is well understood long before the child learns to divide. It may be argued that at a basic level the mathematical concept and habit of division (sharing) provide the symbolic organization of the constituents for significantly more complex situations, such as the collection of taxes to provide public education and universal health care. It is the cultural value of sharing that is expressed

mathematically. The concept originates at the cultural level and it is at this level that it is first understood. The habit of sharing is one that is encountered every day by individuals through their social exchanges with one another. The need to resolve ever more complex situations requires the advancement of the concept and habit of sharing to ever more sophisticated mathematical levels that are expressed by symbols.

Problem Solving, Signs, and Symbols

Dewey's distinction of the communicative value of natural signs from artificial signs is particularly valuable in the development of concepts and habits of inquiry in general. Inference, required for deriving of meaning or significance from a natural sign, is also required to develop artificial signs (words or symbols). The intellectual property of a symbol is first based on and associated with the spatial, natural signs that exist in the spatial-temporal context. The relationship between signs and symbols and the evolution of symbols is evidenced, at the level of the community, through the historical advancement of occupations and, at the level of the individual, through the growth of the child. The shared cultural meaning of symbols develops through common experiences of individuals of the community (Dewey, 1938b); the meaning of symbols must similarly develop for the child through successive experiences with others (including adults) in their environment. It is through the cultural context that shared meanings are developed for each individual that shares in that cultural context. The shared meaning not only makes communication possible but also speaks to the participation of individuals in that cultural context. Having shared in that context, the participants are able to use the symbols developed for future endeavours, which may include inquiry.

Symbols, as stated earlier, allow for a level of abstraction that is not possible with signs. The linking of symbols and their meaning with other symbols furthers the advancement of abstraction through inference. The capacity to manipulate symbols to resolve situations creates opportunities to consider ideas that might not otherwise be possible. Symbols allow for the recollections of experience, reasoning or ordered discourse and the suggestion of existential operations to resolve situations. Symbols reveal and advance the possibility of growth of individuals through shared experiences.

For the present purpose, a situation is recognized by an infant as requiring inquiry – there is an indeterminate situation. The infant still needs/wants more food after being fed and through the infant’s actions there appears to be recognition of the constituents (food, parent, need for communication) of the problem. The infant opens their mouth and receives nothing. The infant attempts to reach for the food and receives nothing. The infant bangs a toy on their table and still receives nothing. The infant begins to cry, getting the attention of their parent. The parent now has a situation that they need to resolve. In the end, the parent asks the child “Do you want more?” This word or symbol *more* is paired with the natural sign of another or additional spoon of food. At an introductory level, the concept of magnitude is paired with a sign (spoon of food) that is paired with a symbol (more). This is possible because of the relationship, shared experience, and cultural context shared between the parent and the infant. The parent clearly has an understanding of the concept, the sign, and the symbol. It is passed down, if you will, from one generation to the next. The concept and symbol exist in the cultural context in a small environment (home) and later become the basis for the development of

more sophisticated concepts and symbols that are developed and used in the larger community.

As the environment of the child is expanded from home to school, the education of the child is *enhanced* by the combined history of and the incremental effect of both informal and formal exercises (shared experiences). Typically, the school uses prepared problems (i.e. provincially approved math programs like *Math Makes Sense*) that target and promote the development of math problem solving skills. As stated in Chapter 3, “[p]roblem solving skills are primarily an improvement in children’s ability to represent the relationships among quantities described in the problem situation” (Garcia, Jimenez & Hess, 2006). The consensus that math problems provided by teachers are indeterminate situations to be resolved is assumed by the school system, teachers in school, and by most parents. Math problems may appear to be accepted by students as genuine problems, but it may be the shared cultural experiences or dispositions that are being sought and developed, especially in the early grades. The need to participate may be the influencing factor, rather than the need to resolve a situation.

The pairing of Arabic symbols with specific counts is based on the development of number sense (concepts and habits of quantities and counting). For example, a child is asked to pair a count with an Arabic number symbol. After repeated exercises, the child readily counts four objects (signs) and is able to write the symbol 4 to respond to the question. Eventually, the child will be able to picture 4 objects in their head to represent the meaning of 4. It is this mental picture that is the shift in development from the possible inference of a sign to the intellectual development of meaning of a symbol. This meaning was developed through their repeated experiences shared with adults (teacher,

parent, etc.) and peers (siblings or classmates). In due course, the child will know what 4 and other counts are without having to develop a mental picture. In solving future situations, the child will be able to draw on the intellectual property of the symbol, not requiring the signs to be present. In developing and using basic symbols, the child is furthering their use of meanings to a level of abstraction that becomes the concepts and habits used in resolving increasingly demanding and complex situations.

As problems become increasingly complex, students' performance deteriorates regardless of ability (Fuchs & Fuchs, 2002). The impression that students with LD perform significantly below that of their normally achieving peers on problem solving has and continues to be supported in the research literature (Garcia et al., 2006; Parmar, Cawley & Frazita, 1996). The ability to solve math problems requires the application of many different types of knowledge and skills (concepts and habits), including those related to the abstract use of language (symbols) for problem solving (inquiry). What remains unclear is whether the difficulty lay in the identification of the constituents (see Chapter 2) of the problem, the expression of the constituents as abstract symbols, the reasoning of ideas of how one might respond to the situation or in the selection of the methods (means) that are best suited to achieve the desired ends.

Grade 2 students with MD may have demonstrated both or either their limited development of symbols or their limited problem solving skills when the increasing demands of story problems impacted their performance (Jordan & Hanich, 2000). However, students' ability to recall number combinations (habits) did not appear to impact the performance of third grade students on math story problems (Jordan et al., 2003). This suggests that students with and without MD were able to demonstrate the use

of symbols to solve problems (inquiry). However, both the work of Elbaum (2007) on the effect of reading math questions and the work of Jordan and Hanich (2000) on the ability of Grade 2 students with RD only to solve story problems, suggests that difficulties with problem solving may not be directly related to symbols (language). Nevertheless, as Grade 4 students with MD performed better on arithmetic story problems than students with co-morbid RD and MD (Fuchs & Fuchs, 2002), it appears that students experiencing greater difficulties with the development of symbols (RD) also have greater difficulties demonstrating concepts and habits related to inquiry (problem-solving dimension of the task).

As indirect problems proved to be more difficult for Grade 3 to 8 students with LD and behaviour disorders than problems with extraneous information (Parmar et al., 1996), it appears that as symbols (language) become more intellectually abstract (less obviously tied to natural signs), students with LD and behaviour disorders have difficulties demonstrating problem-solving knowledge and skills (inquiry). However, the difficulty experienced by students may also be related to their ability to develop the possible suggestions or ideas of how one might respond to the situation or in the selection of the methods (means) that are best suited to achieve the desired ends. Then again, as the performance of Grade 4 students with MD was only slightly better on complex and authentic (real world) problems than students with co-morbid RD and MD (Fuchs & Fuchs, 2002); and given that Grade 3 students with math weaknesses, learning disabilities and behavior disorders are largely unable to solve multi-step problems (Parmar et al., 1996; Bryant et al., 2000); and that students with MD only performed significantly worse than normally achieving peers on complex math problems (Hanich & Jordan, 2000), it

appears that knowledge and skills of inquiry (concepts and habits of inquiry) of students experiencing difficulties with math need further development to resolve more demanding problems (situations). Whether the school defines a problem (situation) as an indeterminate one or a determinate one, students with MD appear to need to build on earlier experiences in such a way as to develop problem solving skills (concepts and habits related to inquiry) to be able to participate in the cultural context that comprises math activities—problem solving.

Problem Solving and the Environment

Experiences, the interaction of the environment with the personal needs, desires, purposes, and capacities, shape the continuity of social life (Dewey, 1938a, p. 44). Environmental conditions that promote or hinder growth include the customs, traditions, and values that have been formed through interaction with others, as well as access to resources and tools (Dewey, 1916). The stimulus for the development and use of tools is the capacity of individuals to manage their environment (Hickman, 1990). The worthiness of a tool is proven in the work it will do, the value of the work it will do, and in the ensuing growth of the individual or community (Hickman, 1990). As the social facet of shared experiences of resolving situations cannot be separated from the act of inquiry, the value to individuals of participating in the resolution of situations extends beyond the resolution to include the development and continuation of shared interests, values, and ends of the community.

Through successive experiences, individuals develop an awareness or knowledge of the customs, traditions, and values of their community that includes the accessibility of resources and tools for their use. Experiences differ from situations insofar as the

interaction of the individual with the environment does not meet the personal needs, desires, purposes, and capacities, and the customs, traditions and values of the community may be inadequate to respond to the situation. The perception of a situation requiring inquiry informs the identification of relevant facts and these facts become the basis for the development of proposed ways of responding—propositions. For the present purpose, propositions are discussed at a rudimentary level when compared to the more comprehensive account offered by Dewey. Propositions may be either generic or universal (Dewey, 1932). “Whereas generic propositions are said to do with “kinds,” universal propositions are said to do with “categories”” (Hickman, 1990, p. 133). Simply put, generic propositions are concerned with identifying or “knowing” the characteristics or traits of kinds (Dewey, 1932). Universal propositions are concerned with ways of acting or “doing” (Dewey, 1932). “Universal propositions function as definitions; generic propositions relate different kinds” (Hickman, 1990, p. 133). In addition, Dewey makes a distinction between two types of universal propositions—laws of science and mathematical propositions (Dewey, 1932). Propositions become tools of inquiry that are developed through experience and transmitted through interaction with others (parents, teachers).

Insofar as the perception of a situation requiring inquiry takes into consideration both the customs, traditions and values of the community, and the factual situation, the propositions considered by an individual are an extension of the experiences of the individual and their community. For example, a student is asked to solve a math problem: *It is Susan’s birthday and at her birthday party, one of her friends gives her 7 flowers and another friend gives her 7 more. How many flowers does Susan have?* The problem

expresses the cultural value placed on friendship and birthdays, as well as the shared traditions and values of the community (i.e. the context of the problem is genuine and not a problem in and of itself). The student solving the math problem needs to consider the facts of the problem (constituents)—there are 7 flowers and there are another 7 flowers, not that there is 1 friend and another friend or that it is Susan’s birthday, possibly her 7th birthday. Based on previous experiences, the student may know that *how many* expresses the type or kind of problem to be solved (generic proposition). Additionally, the student may recognize that this problem requires the grouping together of similar items so that they may be counted (universal proposition). The student may not need to draw a picture (natural sign) to pair with the symbolic Arabic expression of the facts (constituents) of $7+7$. Knowing from previous experiences, the student may know that the correct response is 14 and that $7+7$ is always 14 (mathematical proposition). The propositions held and exercised by the student disclose their knowledge of their culture (what constitutes a problem) and their knowledge of inquiry (how to solve a problem). The competency with which an individual is able to generate propositions speaks to their previous experiences that include the development of concepts and habits related to the specifics of the situation and to inquiry in general.

The difficulties with problem solving for student with MD may be related to conceptual understanding (number sense), habits (number combinations) or inquiry (problem solving skills). The use of strategies (a tool), specifically schematics, in problem solving appears to be an area not fully developed in students with MD (Garderen, 2006). From Chapter 3, “[s]chemata are the basis for understanding the appropriate mechanism for the problem solver to capture both the patterns of relations

and their linkages to operations” (Garcia et al., 2006). Instruction in the application of schematics in problem solving for students with learning problems in Grades 6 through 8 appears to be effective in developing problem solving skills and higher order thinking skills (Xin et al., 2005). It appears that the work of Xin et al. (2005) supports previous research that suggests that additional instructional time, linking schematic map and math sentence, and small group pull-out design are effective remediation methods for developing math problem solving skills.

The academic development of math problem solving skills requires that students arrive at school with sufficient experiences to have developed an understanding of informal mathematics that permits them to function in the classroom. Participation in programs such as *Number Worlds* (Griffin, 2004), not only develop number sense, but also create a community of learners that may be a predecessor to the larger community outside the school. Nevertheless, the social and cultural interaction experienced through participation in programs such as *Number Worlds* (Griffin, 2004) appears to be effective in developing number sense. As number sense develops through the process of inquiry, it stands to reason that students are not only developing number sense, but also the concepts and habits that relate to inquiry and social interaction with their peers and adults.

Similarly, the habits (including number combinations) that facilitate and comprise inquiry and social interaction are developed in the social and cultural context of the classroom. The impact of the social and cultural context (environment) on the development of concepts and habits related to math explicitly and to inquiry implicitly does not appear to be well understood. The explicit instruction of strategies to facilitate habit formation for both number combinations and problem solving (schematics) appears

to be somewhat effective. The development of these tools (strategies) impacts the ability of students with MD to participate in their classroom at both an academic and social level. Through participating in inquiry, students are also participating in the social and cultural environment. The environmental conditions that promote the development of strategies in all students impact the growth of concepts and habits of math, of inquiry and of social interaction at the level of the individual and of the community.

It is through recognizing the environmental conditions as possible constituents of the genuine problem of educating children that adults (parents, teachers and researchers) develop possible solutions (ideas). Not surprisingly, the research supports the idea that the *developmental continuum of math skills and knowledge* begins before formal education. The shift and the interconnection of environments in developing math skills and knowledge extends to the whole community, as well as to developing knowledge and skills of inquiry and participating in the community. Using the research on the math performance of students with MD to develop propositions, the shared experience of teachers and researchers to educate students with MD moves beyond the issues related to math and inquiry to include the continuation and development of shared interests, values, and participation of all members of the community—including, students with MD.

Problem Solving, Experience, and Growth

Growth – social, cultural, and academic—is the result of educative experiences (Dewey, 1938a, p. 28). During the elementary school years of education, it is the succession and interconnections of each experience that furthers growth. That is to say, specialization in one area of education has not yet necessarily begun to occur. Experiences are not only successive and interconnected in terms of the specific paths of

education (e.g. math), but in terms of education as an integrated whole directed at the whole person. Growth refers to education in the social and cultural aspects of life in the community, as well as in specific subject areas of education. It is through education in the specific subject areas, at least in early education, that continuity and interaction are realized. Continuity and interaction, as the values, traditions and practices of the community, establish the democratic social quality of experience (Dewey, 1938a).

Once recognizing a situation as one requiring inquiry, the individuals' perception of a situation determines the course of action to be taken, including the development, selection, and use of tools to meet the demands of the situation (Hickman, 1990). The knowledge and skills acquired through previous experiences, as well as the demands of the current situation, direct the work of inquiry (Dewey, 1938a). In order to resolve a situation into a unified whole, facts and ideas are *operationalized* (in symbolic form) so that the solutions to completed (perhaps conclusive) experiments may be expressed in symbolic form. Solutions then become the basis for future experiences.

As a child grows through experiences, they develop the concepts and habits as they relate directly to specific types or kinds of situations. They also develop the concepts and habits that relate to social interaction, cultural traditions and values, and to inquiry. For example, a child at school is asked to calculate: *how many chairs are needed in the classroom?* The child begins to count all the students in the room. The child answers that there are 15 students present and, so, 15 chairs are needed. The child has understood the math question and answered with an appropriate answer. The child has used all the concepts and habits that relate to math (used counting), social interaction (answer was polite) and culture (both boys and girls get chairs). However, the other children in the

Grade 3 class snicker at the response as there is clearly an issue with the answer to the problem. The child was presented with a situation. However, other children had the knowledge that there are 21 students in the class and 1 adult (teacher). Others in the class readily accounted for the children who were absent or out of the room, accounted for the teacher and would have solved the problem (if in fact, it posed a problem) differently. All of the children in this class have learned a great deal more than their class requiring 22 chairs. The contributions of Grade 3 children who are still counting to solve problems, and counting incorrectly, are not necessarily valuable. The development of concepts and habits as tools for future use extend beyond the math problems and extend to issues related to interaction with others, and to continuity of the current values and traditions of the community.

The concepts, habits, and ideas (propositions) used to resolve situations are all tools that are developed through successive experiences. It is of considerable importance that children are provided the opportunities to develop and use concepts and habits that relate to math. It is of paramount importance that the ideas of all children are valued, are allowed to be expressed, and are given the time to be tested. Through these experiments all children will grow. Through these experiments the community will grow.

Through interaction with students, teachers and researchers readily identify the students who are struggling to achieve when compared to their peers. However, the situations for the students and the adults are quite different. How is it that some children acquire math skills and others do not? The development of problem solving (inquiry) skills is based on previous experiences—using concepts, habits, propositions to carry out experiments. Individuals must recognize the situation as one requiring inquiry, perceive

the situation appropriately, and be willing to take risks to solve the situation. Also, previous experiences must be closely related to the current situation so that the knowledge and skills applied move the individual closer to the resolution.

Discussion

With maturity, the brain of a child develops as a result of physical growth and as a result of the experiences had. The concepts and habits developed relate to specific academic areas and inquiry in general, but also to the social and cultural aspects of the community. In educating children about math, adults are also educating them about their community. The values and traditions of the community are transmitted through interaction with others in the environment. The continuity of the values and traditions of the community is ensured as those values and traditions are passed down from generation to generation. The transmission and development of concepts and habits of inquiry are possible through communication, which includes signs, symbols and tools. The inherent social and cultural value of concepts, habits, inquiry, signs, symbols and tools extends beyond their utility as they are integral to the situation. Similarly, the utility of facts and ideas extends beyond their relationship to the solution of the particular situation to the links between experiences of the individual and to the connection between the individual and their community. Not unlike children, teachers and researchers use concepts, habits, facts, and ideas in their inquiry. All are involved in recognizing a situation requiring inquiry (indeterminate situation), identifying its constituents (facts), developing ideas (propositions), experimenting until a unified whole is realized. The difference may lie in the perception of the problem. Is it about experience and growth through the development

of math concepts and habits? And how closely related it is to previous experiences, and as successful people in education, how critically are we willing to look at education?

CHAPTER 7: THEORY, RESEARCH, AND PRACTICE

What is called for here is not a revolution in education, but rather an evolution that is consistent with the shifts in development as a result of growth in our knowledge of education, especially as it pertains to students with MD. “[T]hinking begins [with] ... a situation that is ambiguous, that presents a dilemma, that proposes alternatives” (Dewey, 1938b, p. 122). That students with average to above average intelligence are not able to succeed is an ambiguous situation that creates a dilemma for the student, the teacher, and the researcher. Alternative solutions or interventions are being proposed by researchers. Dewey’s model of education as growth creates an opportunity to see the situation as a whole and, in doing so, highlights areas of possible focus for future growth.

The concept of continuity allows us to consider the interconnectedness of the physical, intellectual, and social development of individuals. The interconnectedness of the home, school, and community allows us to relate the development of the individual to the development of the community. The development of the individual relates to the development of the community through the interconnection of the child, the parent, the teacher, and the researcher. It is this interconnection that represents education as a whole. For example, when the home is not seen as a microcosm or extension of the school and community, but rather as a haven from them or far removed from them, the education of the child and their community is impacted. Consideration of the whole of education—its interconnectedness—serves to develop a better understanding of what constitutes education.

The issue of educating students with MD is complicated by the interconnectedness of many factors and those inherent relationships. Through inquiry, children develop conceptual understanding. Children, through routine practice, develop the habits related to math and inquiry. All the while, the knowledge and skills that relate to inquiry are being developed. Programs designed to develop number sense, such as *Number Worlds* (Griffin, 2004), point to the possibility that the home environment is less able to provide the opportunities to develop math knowledge and skills. This may be the result of poverty, genetics, lack of education on the part of parents and so on. However, the problem may not be the development of number sense, but rather the development of the knowledge and skills of inquiry at that level. The research on the development of habits, such as number combinations, suggests that there is only one way of knowing or doing. Are there alternative habits that can be developed that allow the continued development of knowledge and skills of math? Of inquiry? Inquiry integrates and applies the knowledge and skills that comprise the concepts and habits required to resolve situations. The repeated integration of concepts and habits to resolve situations creates more opportunities for children to develop those concepts and habits, as well as the concepts and habits of inquiry.

The interventions designed to promote the development of math skills have added to our understanding of MD. The long term effect of these interventions needs to be determined. The impact of programs such as *Number Worlds* (Griffin, 2004), using assistive technology or using schematics or authentic problems to develop problem solving skills, needs to be evaluated vis-a-vis the ability of the adult to function and participate in the community. The use of interventions in the general education classroom

by students with MD fosters the growth of all the students and their community. The goals of researchers include the development of math skills, but also the increased ability of students with MD to participate in the classroom and in their community. As the classroom is an extension of the community, education fosters the growth of democracy through increased participation.

If education is about growth, it cannot be about end states. Then, education is about progress. Dewey's model of education as growth is education as progress. The continued progress made in the research in the field of MD supports and requires that students with MD are increasingly included in the discussion of general education and educational theory. As our understanding (concepts) of what constitutes education develops and as our ability to meet the needs (habits) of students develops, the situation of educating students with MD becomes less ambiguous.

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