Slemon

A dissertation on the Nature and Educational value of induction
THE
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OF
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A DISSERTATION

ON THE

NATURE AND EDUCATIONAL VALUE

— OF —

INDUCTION.

SUBMITTED TO THE FACULTY OF EDUCATION, OF QUEEN'S UNIVERSITY,
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DOCTOR OF PEDAGOGY.

— BY —

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CHAPTER I.

THE DISTINCTION BETWEEN INDUCTION AND DEDUCTION.

There are two quite distinct processes by which the mind moves from one acquisition to another in the pursuit of truth, the Inductive and the Deductive process. These processes though distinct, so frequently alternate in any course of thought that it is possible to describe only parts of any prolonged effort in reasoning, as purely inductive or deductive, though the whole may be designated by either of these terms. By saying that any effort in reasoning is inductive or deductive, we mean that in the main, the movement of thought is either inductive, that is passing from particular facts, brought to light in experience, to general truths, or from general truths to particular cases included under the general truths. In any inductive treatment of a subject, there will be found deductive links, and in any deductive treatment the results of induction are constantly used.

The inductive mode of reasoning is best and most typically shown in the study of the physical sciences, while the deductive mode is that almost exclusively used and most characteristically set forth in mathematics.
In any inductive science, we are constantly basing our steps on particular facts of experience, and though we may use inductions as facts for higher induction, we can never leave experience out of sight. In a deductive science, on the other hand, having once established the premises, the reasoning could go on equally well, were we rational beings quite removed from the world of matter and experience.

Since the time of Bacon, much has been written concerning Induction. Possibly no one has devoted more thought to the subject than J. S. Mill. In his System of Logic, he says, “When from the observation of a number of individual instances, we ascend to a general proposition, or when combining a number of general propositions, we conclude from them another still more general, the process which is substantially the same in both cases is commonly called Induction.”—(J. S. Mill—System of Logic, page 107). In reference to the deductive process, he says, “The operation is not a process of inference, but a process of interpretation.”—(Mill—System of Logic, page 127). That is, it is an investigation to find out if the general truth basic to the deductive process is rightly interpreted as covering the case that is cited.

But no one so emphasized the importance of the inductive process and its possibilities as did Francis Bacon. In it, there lay for him, the hope of man’s complete mastery over nature. He says, “There are only two ways for the investigation and discovery of truth. One flies from the senses and particulars to the most general axioms and from these principles and their infallible truths, determines and discovers intermediate axioms. And this is the way now in use. The other constructs axioms from the senses and particulars by ascending continually and gradually, so as to reach the most general axioms last of all. This is the true way, but it is as yet untried.” He saw that deduction as known and applied in nature, was based on axioms got by “flying” from particulars as contrasted with those got by the continual and gradual ascent from particulars; that the true way of induction began not with hastily formed axioms, but with the facts of experience, and ended with axioms laboriously established. (Novum Ὅ. Aphorism 19).

But Bacon’s ideal plan of induction involved impossibilities and though of almost inestimable suggestiveness and value to subsequent scientific effort, it will never do what in his pansophic conception, he thought it would do, for mankind. In Bacon’s plan, the search for truth must proceed by a perfectly exhaustive process from the lowest particulars, through a consideration of all the facts to general principles. His induction was to be the key by which
man might unlock the secret chambers of nature and use the secrets thus discovered to make nature absolutely his servant.

But nature has not yielded up her secrets in response to Bacon's method, for it is not a fact, as he supposed, that an enormous collection of facts, followed by careful comparison and classification, will discover the hidden truths of nature. He overlooked the importance of the work of the mind itself. The whole work of science does not consist in observation. The body of knowledge, which constitutes science, is not something external to the mind, and independent of its activities, and Bacon's idea that his inductive method "leaves not much to acumen and strength of wit, but nearly levels all wits and intellects," shows how entirely this misconception has seized him. (Novum O. I. 61.)

The great value of Bacon's method, lies in the stress he laid upon the importance of verifying theory and facts; in his warnings against fanciful speculations; against the subjection of the mind to mere authority and against over hasty generalization. "Our intellects want not wings, but rather weights of lead to moderate their course."—(Novum O.) In his ardour to overthrow false anticipations of nature, he undervalued entirely the value of those which call forth penetrating investigations, and often result in scientific discovery. Bacon seems to have overlooked the fact that induction, as well as deduction is largely a matter of interpretation. It is true, we start with observed facts of experience, but these facts must be seen in their context in nature, and the significance of the facts is limited by this context; hence adequate knowledge is necessary as well as that analytic capacity which will determine whether or not the facts are germane and warrant the generalization. For science, something more is needed than a mind accustomed to accurate observation and comparison. The mind must be stored with general conceptions previously acquired, of the sorts which bear affinity to the subject of the particular inquiry. The advance of knowledge in any sphere is dependent on advance in other spheres, on wide and accurate knowledge of nature. It was just such lack of adequate knowledge of facts seen in many relations that prevented the marvellously methodical and constructive intellect of Bacon, from discovering more of the secret 'forms' or laws of nature. Knowledge in Bacon's day, was not adequate for his ambitious scientific aims. If, however, his knowledge of related facts was inadequate to bring his general method into fruitage, his broad schematic conception of method was indeed marvellous for his age. In this, he anticipated to a great extent, the best thought of modern scientists. His general method is briefly this:—A, phenomenon, 'P' is under investigation.
In any inductive science, we are constantly basing our steps on particular facts of experience, and though we may use inductions as facts for higher induction, we can never leave experience out of sight. In a deductive science, on the other hand, having once established the premises, the reasoning could go on equally well, were we rational beings quite removed from the world of matter and experience.

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(1) Construct a table by collecting as many instances as possible where P. occurs.

(2) Construct a second table by collecting as many instances as possible, where P. does not occur, but where other phenomena found in (1) do occur.

(3) Construct a table in which P. is found in different degrees of intensity as other phenomena are found in varying intensity.

The explanation of P. is got by a process of elimination. Those several qualities which are not found where P. is found, are rejected; also those qualities which are found when P. is absent or are found most prominent where P. is less prominent, or less prominent when P. is more prominent. After these eliminations some hypothesis may be found and this hypothesis tested by observation and experiment. A striking similarity is readily seen between (1), (2) and (3), above, and the modern methods of Agreement, of Difference, and of Concomitant Variations. There is, however, a lack of definiteness in Bacon's method. There is, too, with him, very little use made of deduction after the inductive process, and no attempt made to determine quantitative relations between cause and effect, and in consequence, none of the possibility of predictions by calculations, so wonderful in modern science.

Like Bacon, J. S. Mill emphasizes the importance of induction. With him, it is the only means of arriving at new truths, since he regards deduction as merely a means of systematizing and arranging what we already know. Deduction is not inference, it is interpretation.

His meaning may be got from his analysis of a formal syllogism as:

(a) All men are mortal.
(b) Gladstone is a man.
(c) Therefore Gladstone is mortal.

How is the general truth (a), obtained? From observation and inference. From observed instances a conclusion is reached which is held to hold true in all similar instances. The same observation concerning the different men, A, B., C., &c., that each is mortal, may through the contrivance of language, be recorded in one short sentence, All men are mortal. When concluding that Gladstone is mortal from knowledge of the death of A., B. and C., we may pass through the generalization—'All men are mortal,' as an intermediate step, but in this latter half of the process, that is in the descent from "All men," to "Gladstone," there is no inference. The inference
is finished when we have gone from the particular cases to the general statement; the rest is merely interpreting and deciphering our own notes as given in the generalization. The inference is found in the "Anodic" process from the particulars up to our generalization, which is used but to obviate the necessity of going over the particulars from which the generalization was made. The mortality of A., B., C., is the only evidence we have for the mortality of Gladstone. Mill says of this—"I cannot see why it should be impossible to journey from one place to another unless we march up a hill and then march down again." "It may be the safest road and there may be a resting place at the top of the hill, affording a commanding view of the surrounding country, but for the mere purpose of arriving at our journey's end, our taking that road is perfectly optional; it is a question of time, trouble and danger." (Mill System of Logic, page 123). "Not only may we reason from particular to particular without passing through the generals, but we perpetually do so reason."—(Page 123). What then is the use of (1) The general form? The generalization enables us to hold together in one view, the particulars through which we have worked our way up to it. And (2) the syllogism? It is a convenient form in which to set forth our reasoning, and since it is necessary that the generalization, the record of facts, which are sometimes forgotten, should be correctly read, the rules of the syllogism are a set of precautions to insure our doing so. Moreover, though inference be from particular to particular with or without reference to a generalization, that is, in the form:—

\[
\begin{align*}
(1) \text{ All men are mortal (A., B., C.)} \\
(2) \text{ Gladstone is a man.} \\
(3) \text{ Therefore Gladstone is mortal.}
\end{align*}
\]

(OR)

\[
\begin{align*}
\text{A., B., C. &c. are mortal.} \\
\text{Therefore Gladstone is mortal,}
\end{align*}
\]

there is a greater probability of care being exercised in the inference if the result is stated in a general form, than if the inference is made without it. The syllogistic form then tends to insure the conclusiveness or detect the inconclusiveness of the evidence on which inference is based.

Induction starts with particular facts and by analysing these facts discovers universal relations which they embody and thereby constructs a system which explains them. Deduction brings particulars under some general principle which is assumed or admitted to be true. All men of "All men are mortal," includes Gladstone and therefore, Gladstone is mortal.
Induction is an insight into a system from a close analysis and investigation of its parts. Deduction is an insight into the significance of a supplied system in reference to the parts rightly included in this system. In Induction, we bring to light, a system within which the particulars which we are considering, fall. While in Deduction, given the general nature of a system, we bring to light the particular facts properly included in the system. Both processes end in the same result,—an insight into a bond of relations necessary among particulars, that is, in an extension of scientific knowledge. Both processes tend to a rationalization of the otherwise isolated and comparatively meaningless facts of the world. We begin with particulars unsatisfactory to the intellect. We climb up continually by an alternation of analysis and synthesis to wider views of the laws which are operating in them. As we go up, we carry the particulars along and these become at each upward step, more law-full, more satisfactory to the intellect. When we have reached a point from which a whole set of facts is seen as clearly interpreted through law and therefore intelligible, we can come down bringing our highest insight with us, illuminating thereby further particulars and labelling and classifying them. The light by which we descend, is found in the particulars through which we laboured up, and in which it was found. The light came not from some supersensible sphere, but was always in the facts of experience. The only difference is, that in the inductive, upward process there was laborious discovery, while in the deductive descent, there is comparatively easy application.

Each of these processes implies the other, and is dependent upon it. In induction, how starting with particular facts could a universal generalization be reached? What but some such underlying principle as the "Uniformity of Nature," or "The universe is a national system," or "Thought is objective and has in itself its own standard of truth," could bridge over the "Inductive hazard" and make the generalization logically justifiable?

In deduction, as for example in Geometry, are not the axiomatic truths, the starting point of the process, themselves derived from previous inductions from particulars found in every one's experience and from which induction is inevitable? Besides, the great difficulty in deductive reasoning is to form the syllogism, a process implying many inductions. If we have clearly before us a fact—this S. is M.; and have also a universal truth already in our possession as all M. is P., the middle term expressed by M. at once suggests and is the ground of the conclusion, S. is P. The ease with which the syllogism is here formed, is due to the readiness and clearness with which the minor premise is found. If we always have such power
of insight as to see S. in M. as a significant mark amid all the other properties of M., the inclusion of this fact under the major premise, M. is P., is immediate, and the conclusion comes at once. A fact A. is simply placed under its formula B., where it properly belongs, through its characteristic mark.

Generally, however, the matter is not at all so simple as this. The fact A. if brought under the formula D., say, would complete a syllogism and answer the question before us; but there seems no connection between them. There is, however, a formula B., established by a previous induction under which A. falls; but A., from other marks in it might also be regarded as falling under B₁, B₂, &c. Again, B. falls under C., by some of its marks and by others under C₁, and C₂. Once more, C. falls under D., which brings A. under D., and we have our syllogism and therefore the answer to our question. The difficulty then consists in getting the minor premise necessary to complete the required syllogism. Thus such deductive inference is possible only for those whose memories hold many intermediate formula by which A. may be mediated with D., and whose power of imagination can, out of all possibilities, select the proper ones for this mediation. It is because beginners in the study of Geometry have so few of these servicable intermediate formula and so little of servicable imagination that they find such difficulty with deductions, that is, in finding the relation between that which is to be proved and any axiomatic truths or previously established inductions.

The ability to make inferences is essentially relative to the amount of servicable, that is, appropriate knowledge already possessed. It is just because of this that one man reasons better than another. The man possessing wide, related knowledge can form syllogisms and therefore make inferences which are impossible for another not so informed.

Induction and deduction are complementary. Particulars are observed, a universal law is assumed and it is enunciated as an hypothesis. But deductions from this basis lead to conclusions unsupported by facts. There follows a re-examination of particulars and a correction of the inductive inference. It is seldom possible in any complicated inquiry, to go much beyond the initial steps without calling in the instrument of deduction and the temporary aid of hypothesis.
CHAPTER II.
RELATED SUBSIDIARY PROCESSES—ANALYSIS AND SYNTHESIS.

Since the great body of study consists in a mastery of what nature furnishes the student, and since nature supplies only concrete individuals, it would seem natural to begin study with individuals, that is, with concrete wholes as presented in experience.

Sir William Hamilton says in his Lectures on Metaphysics, "The first procedure of the mind in the elaboration of its knowledge is always analytic. It descends from the whole to its parts, from the vague to the definite." Generalizations are comprehensible only in the light of the data from which they are developed. Generalizations and disassociated parts are artificial products, so that the proper point of departure in teaching the first truths to immature minds is neither the deductive method, involved in the application of generalizations to particulars, nor the synthesis of disassociated parts to form a significant unit, but rather the concrete wholes of experience.

In the beginnings of knowledge, thought does not travel outside the particular object, to show its connection with others. Both induction and deduction, therefore, must be preceded by subsidiary mental activity—the study of single things, or rather of things seen as single. When the object of study is to understand any particular whole, the order of presentation should be, first Analysis and then Synthesis. This Analytic-synthetic activity of mind—based on its fundamental power of noting differences and likenesses—brings into view new features and their relation, and thus changes a vague whole merely apprehended, into a more definite whole more or less fully comprehended. And a further defining and clarifying of the first view goes on with every subsequent new analysis and synthesis.

When the object of study is a number of individuals from which a classification or generalization, to be afterwards applied to new cases, is to be made, the order of procedure is, first induction and then deduction. This inductive-deductive activity of mind, based too, on the same fundamental faculty of noting likenesses and differences, brings into intelligible and meaningful relation the individuals of experience which otherwise would remain in meaninglessness isolation.

Thus the natural order for the study of things may be given as follows:—

1. When dealing with an individual thing,—

(a) Analysis, to bring into relief the involved contents or elements of the thing.
(b) Synthesis, to hold these discovered elements together in one view, thus constituting a fuller knowledge of the thing, and changing it from a vague, to a definite whole.

II. When dealing with a number of wholes comprehended as in I, to understand their relation,—

(a) Inductive inference based on comparison founded on likenesses and differences, leading to a generalization.

(b) The deductive application of this generalization to particulars legitimately included under it.

Thus we begin with individuals and return to individuals,—the four steps leading us from particulars, vague and merely apprehended, to particulars illuminated and rationalized by law.

There is grave danger among educators of neglecting either I., or II., both of which are important. So much is made of the present day "training in sense observation," that the importance of II., is overlooked, and the learner is left in possession of only isolated facts, the world of the savage. It is not seen that these facts, however clearly known, do not form scientific knowledge. There is an oversight of the fact, that while the learner must begin with analysis, he must end in system; that he must pass out of the concrete and into the realm of the abstract to return again into the concrete with the light of the abstract, before the concrete is understood.

Other educators neglect I., and beginning with abstractions and generalizations, deal with these only, and keep the learner forever divorced from the concrete. These fail to see that the abstract is but a key to the true understanding of the concrete and not properly an end in itself.

Through either of these defects there is danger of an arrested development which would defeat the true end of education, to make the learner at home in the world by understanding it through a unification of the manifold into a rationalized system.

Since the immature mind sees things first as unrelated wholes, if growth in knowledge of the thing takes place, there must first be analysis,—a differentiation of the comparatively meaningless homogeneity which makes up the thing, into parts which must again be held together to form the whole. How does this compound operation take place?

First, by very small steps, that is, only very slowly do the parts emerge and the mind be conscious of them as constituting the whole. There may be, and generally are, many periods of rest.
between these steps. The growth of knowledge in this respect in a child’s mind may be compared to his representations of some individual object, say, a man. Here we have successive and progressive efforts exhibiting the fact that there are resting stages, but in all a growth and gradual development. At each stage the picture symbolizes the child’s idea for the time.

I. Two circles, one for the head and one for the body, with two lines for the legs. For the child, this at first is all there is to draw. After rests of days or weeks he goes on to—

II. Where there is the addition of arms.

III. Where the features of the face begin to emerge and the neck is seen.

IV. Where more features are seen.

V. Where the hands and feet are added.

VI. Where less striking features of the face are brought out, in the eye-brows and eyelashes.

VII. The last of the series where the articulation of the limbs is shown.
The length of the resting periods and the exact order are not determinable. The one outstanding fact is, growth in perception from fundamental to accessory features.

Second, when an object is being analysed there must be a selection of some of its elements, a discrimination in favour of these and a neglect, for the time at least, of the others. What determines this? The contents of the mind of the learner. Mary's doll had moveable eyes; Emma's had not. At a doll store Emma's mother bought her a doll with moveable eyes which was immediately wrapped up. On the way home Emma could tell hardly another feature of her doll. What made this discrimination? Evidently the idea of Mary's doll in Emma's mind. A man on the street passes a dentist, a tailor and a barber. The dentist sees his teeth, the tailor the cut of his coat, and the barber the cut of his hair. Why? "To him that hath shall be given."

Not only does the mental content of the observer determine what is seen in an object, but the meaning and significance of this feature is determined not by what is now before the eye, but by what is not present to the sense at all, some related idea or group of ideas used as a standard or gauge. The feature observed has its meaning assigned to it largely because it is one of a class or is supposed to be one of a class of which the other members are already in the mind's possession and organized. The word 'largely' is used above, because the selection of any particular feature and the significance it carries is sometimes determined by its complete unlikeness to anything in the mind. This feature unrelated to anything now in the mind will hereafter not be unrelated, but related to the object in which it is found and the idea of the object will be modified thereby. If a teacher could rightly read the whole contents of the mind of his pupil, he could predict both the feature of any object which the child would select, and the significance he would attach to it.

In the case referred to above, the dentist sees the teeth of the man he passes and not the cut of his coat, because he is a dentist and not a tailor. He does more; there is significance assigned to what he sees. He infers (say) the teeth are artificial:—

\[
\begin{align*}
\text{A row of teeth with a certain regularity, &c., is artificial.} \\
\text{This row has such marks.} \\
\text{Therefore, these teeth are artificial.}
\end{align*}
\]

He probably has no such syllogism explicitly in his mind, but this or something like this must be the logical ground of the significance given in "This man has artificial teeth." This inference might be incorrect. A trial might show the teeth to be natural and this would result in the revision of the major premise above,
and a new meaning being put into the word "certain." This is analagous to the corrections made in hasty inductions by subsequent tests, through deductive applications, already noted. If it be remembered, too, what in this dissertation has been said of the option of using the major premise and throwing reasoning into the syllogistic form or of omitting it and reasoning from particulars to any particular bearing the same marks, it will appear that the above analytic-synthetic process may be thought of as a process of induction or deduction, in which the one element, part or feature observed forms with other pertinent possessions of the mind a set of elements from which inference is made. This granted, it will also appear that the bond which thus binds together the analytic-synthetic and induction and deduction processes is the one fundamental and irreducible fact of mind—the noting of likenesses and differences. This fact is basic to all thought. The laws of thought—identity, contradiction and distributed middle, respectively, simply state that if mind acts it must do so by noting likenesses and differences, and that if objects are not alike they are different, that is, there is no undistributed middle ground and they cannot be both like and unlike at the same time. If this irreducible element of mind be fully seized, it will simplify much of the theory usually given regarding the logical basis of thinking and will relate and unify the most primary teaching with the most advanced.

The relation of analysis, synthesis, induction and deduction may be indicated by the following diagram when read downwards:—

I. When dealing with individual wholes.

II. When dealing with groups of individuals.
II.

Induction

Deduction

Comparison.
The diagram brings into relief the following points:

1. That the natural starting point is the unanalysed concrete individual wholes—the A., B., C., D., of the diagram.

2. That the first mental operation is a distinguishing of parts within the wholes—A₂, B₂, C₂, D₂, that is analysis based on the fundamental faculty of noting differences.

3. That the elements thus disassociated by analysis, must be mentally reconstructed into wholes—A₃, B₃, C₃, D₃—that is, there is synthesis based on the fundamental faculty of noting likenesses. In this case the likeness consists in that of being parts of the same whole.

4. That by this analytic-synthetic act (for though analysis and synthesis may be thought of separately, they are but two aspects of the one act of mind), the change from A., to A₃ B., to B₃; &c., shows that knowledge has been made more specific and concrete.

5. That the analysis-synthesis of I., must take place before the work of Induction, the basis of which is comparison, can take place.

6. That generalizations co-ordinate and explain an indefinite number of facts. The generalization of the diagram explains, at least in one respect, the concrete wholes represented not only by A₃, B₃, C₃, D₃, but also by A₄, B₄, C₄, D₄, and an indefinite number of others.

7. That the generalization, the result of a true induction and not a mere colligation must transcend the limits of actual investigation, that of A₃, C₃, B₃, D₃, and infer, under the assumption of the uniformity of nature, that what is true of A₃, B₃, C₃, D₃, will be true of A₄, B₄, C₄, D₄, and of all others under similar conditions.

From the whole it is seen how study increases both the complexity and simplicity of the world; exhaustive analysis and synthesis giving infinite richness and complexity, and wide generalization giving comprehensive simplicity.
CHAPTER III.

GENERAL METHODS OF INDUCTION.

Before treating in detail the application of induction (involving deduction), analysis and synthesis to particular school subjects, it will be necessary to give with some fulness, the nature of the different methods employed in Inductive Research.

These may be classified as follows:

General Methods:

I. The method of Simple Enumeration.
II. The method of Analogy.
III. The method of Scientific Analysis.

Special Methods under III.:

1. The method of Agreement.
2. The method of Difference.
3. The method of Concomitant Variations.
4. The method of Residues.

Each of these divisions and sub-divisions will, for the sake of simplicity, be discussed under the heads—Definition, Illustration, and Use.

I. The first method is that against which Bacon said so much.

Definition.—It consists in ascribing the character of general truths to all propositions which are true in every instance that we happen to know. We note that two things, A. and B., are found together in various experiences. It is not necessary that we record our observation by giving any particular number, but from this evidence we conclude A. and B. will always be found together in similar experiences.

Illustration.—A nurse has nursed six or eight cases of Diphtheria, and has observed some form of paralysis to follow. She mentally runs over the cases and infers that the next case will have subsequent paralysis. If all the cases possible are noted, and the result stated, the Induction is called perfect Induction. This seems to me to be a misnomer as induction implies the projection of thought beyond the actual cases examined.

A rather interesting instance in which the simple enumeration suggests the law, the ground of which is afterwards independently found, is given in what is called ‘Mathematical Induction.’ This, would seem to the writer, to be more properly called “Perfect Induction.”
Thus by trial, \(a^2 - b^2\), \(a^4 - b^4\), \(a^6 - b^6\), are found to be exactly divisible by \(a + b\). Here by simple enumeration, it is suggested that \(a^{2n} - b^{2n}\) is exactly divisible by \(a + b\).

If the division of \(a^{2n} - b^{2n}\) by \(a + b\), be set down as follows:

\[
\begin{array}{c|cc}
& a^2n + b^{2n} - ab^{2n-1} & \\
\hline
a + b & a^{2n} - b^{2n} & a^{2n-1} - b^{2n-1} \\
\hline
\text{Rem} = - a^{2n-1}b + ab^{2n-1} & - ab(a^{2n-2} - b^{2n-2})
\end{array}
\]

it will appear that if the remainder \(a^{2n-2} - b^{2n-2}\), or \(a^{2n} - b^{2n}\) is divisible by \(a + b\) then \(a^{2n+2} - b^{2n+2}\) is also divisible by \((a + b)\).

So that, if \(a^2 - b^2\) be divisible by \(a + b\), then \(a^2 + 2 - b^2 + 2\), or \(a^4 - b^4\) will be so divisible and \(a^6 - b^6\), \(a^8 - b^8\) \(\ldots\) \(a^{2n} - b^{2n}\)

Here the possibility for such a ground of inference lies in the nature of number which as found in the integers 2, 4, 6, 8 \(\ldots\) \(2^n\), the exponents, forms an indefinite series homogeneous throughout.

Use.—It is purely an observational method and is suggestive of fruitful lines of investigation. There is in it no ground for inference, nothing of a universal character, to bridge over or act as a medium of identity from one case or relation to another.

In the case of paralysis following diphtheria, it suggests an investigation into the relation between the toxine of diphtheria and paralysis.

One negative case upsets the induction. The actual counting of cases for and cases against the first induction, lays the ground for a calculation of possibilities, assuming that the conditions under which observations were made and all others are similar.

It is a method used with effect in popular addresses, \textit{ex. gratia}, discussions re Reciprocity and Anti-Reciprocity, and its easy conclusions should be carefully scrutinized.

It is a method often used by children and if exceptions not covered by the inference, are given by the teacher through his wider acquaintance with facts, these will teach pupils to exercise greater care in drawing conclusions.
II.—Induction by Analogy:—

Definition.

Let A. and B. be two sets of phenomena having:

1. Resemblances, a, b, c, d and a, b, c, d.
2. Differences, f., g., h., i. and m., n., o., p.
3. A region of unascertained facts, 3.3.
4. And X., an observed part of A.

Then X. is inferred by Analogy, for B.

That is, that since A. resembles B., what is true of A., is true of B.

Illustration.—An individual as an active agent lives for,

\{ (1) Preservation.
\} (2) Advancement or amelioration.

Society, too, has these two objects:—

\{ (1) Protection or Preservation.
\} (2) Advancement.

Brain power in the individual corresponds to general intelligence in Society.

And since advancement for the individual comes through an increase in knowledge, it is inferred that advancement in society depends on knowledge.

Illustration 2.—Every mind if it acts at all, acts by noting likenesses and differences, sequences or co-existences. This activity of thought underlies all knowledge, perception as well as inference. Its cause is some force.

Again in the outer world, matter and motion are manifestations of force.

What is the character of this force? Spencer says, that in both cases it must be conceived of, in analogy with our feelings of exertion when we voluntarily act.
Criticism.—It is to be noted concerning this loose sort of inductive process:

(1) That, like the strictest induction, it is based on the supposition that when A. resembles B. in one or more properties, it does so in a certain other property.

(2) But that the ground of inference in analogy is mere resemblance, while in the stricter inference, it is invariable relation.

(3) That in the diagram if 1.1 are very large, compared with the whole content of A. and B., the probability of X being in B. is increased; while as 2.2 grows larger it is decreased. Also if 3.3, are large compared with the whole content of A. and B., the inference from 1.1 and 2.2 taken separately or in conjunction, is less reliable.

Before any inference is made there should, therefore, be close observation of, not only 1.1., but of 2.2., and even of the extent of 3.3., the unascertained regions.

The chief Use of this principle is found in suggesting explanations. For example, reproduction in plants is understood as explained by considering certain organs of the plant as analogous to those of reproduction in animals.

III.—The method of Analysis.

The methods of Enumeration and Analogy are methods of observation. This method combines observation and experimentation as the basis of a more thorough induction.

1. Method of Agreement.

Definition.—When, by comparing different instances which agree in manifesting both the phenomena under consideration and a constant circumstance, this constant element is regarded as the cause of the phenomenon, the inference is made by the Method of Agreement.

In A., B., C. (instances), P. is found and also among the varying circumstances of A., B. and C., a constant circumstance (m), is found. Then by this method (m) is said to be the cause of P.
It will appear that here as in the method of enumeration, there is a collecting of instances, but that here there is a greater need of analysis and discrimination of the essential from the non-essential in reference to the phenomenon. It is, however, the most purely observational of all the methods of scientific analysis.

Illustration.—Let us suppose that we wish to ascertain the cause of dew-deposit.

Similar cases which agree in a like deposit are found in:

(1) That which appears on the outside of a pitcher of cold water when brought into a warm room.

(2) That which collects on the stone of cellar walls when the cellar is warm.

(3) The deposit on wall paper on “wash days” in winter.

(4) That which beclouds our glasses when we breathe on them.

In each and all of these cases, however else they may differ, two things are found in agreement:

\begin{align*}
(a) & \text{ A deposition of moisture.} \\
(b) & \text{ A difference in temperature between the atmosphere and the surface moisture.}
\end{align*}

These two things are also found in any case of dew-deposit. The difference in temperature may easily be shown by using the thermometer on the grass, say, and in the air a few feet above the grass.

By the method of Agreement, then, we should in consequence, conclude that dew is caused by a difference in temperature between the object bedewed and the surrounding air. This example serves to show not only the character of this method, but also its limitation. For it is easily shown that dew deposit depends on:

(a) The nature of the substance bedewed. If polished brass or iron be exposed beside glass, the glass is bedewed but not the metals.

(b) The power of conducting heat,—the poor conductors having the greatest dew deposit. The dew deposit on a lead bar is greater than that on a silver bar.

(c) The nature of the surface bedewed,—rough or smooth. The rough surface, i.e., the better radiator of heat brings the greatest deposit of dew.

(d) The texture of the substance,—close or loose. The loose texture, that is, the poorer conductors, favouring dew deposit.
The facts given in (a), (b), (c), (d) are disclosed by the aid of the methods of Difference and Concomitant Variation. Thus by these methods in conjunction with the initial method of Agreement, the more significant and general conclusion is reached, that whenever bodies lose heat from their surfaces more readily than it can be supplied from within, that is, when they are better radiators than conductors of heat, dew will be deposited.

Use.—The use and value of this method are seen in the above example—to collate instances and by a first analysis lead towards the cause of any phenomenon under consideration and thus to suggest more exhaustive methods of research.

This method alone does not prove. The causal relation between (m) and P. (of the diagram) becomes more probable as:

(a) The number of coincidences increase.
(b) The character of these instances in other respects greatly vary, as both of these conditions favour the elimination of the non-causal element.


Definition.

Let A. and B. be two instances which differ only in that (H) is present in A., but wanting in B., B. having been produced by eliminating (H) from A. Then if when (H) disappears from A. to form B., P. also disappears, (H) is called the cause of P., we are said to infer by the method of Difference.

If, starting with B., when (H) is introduced P. appears, we have the same method with the same inference.

In this method but two instances are compared, and the emphasis of attention is on their difference. In the method of Agreement many instances are compared and the chief attention is directed towards their agreement.
This is a method too, where experimentation is pre-eminent, while in the method of Agreement observation is the chief basis for inference.

The method of Agreement can only substantiate its inference by numerous and varied examples and then never beyond a probability. The method of Difference tests at once its supposedly correct inference by experimentation.

Illustrations.—Suppose we have a luminous coal gas flame. We suspect that the luminosity is due to incandescent particles of carbon in the flame, the presence of which may be shown by the deposit on any cold object thrust into the flame. If now we introduce oxygen into the flame and with the removal of the incandescent carbon particles, thereby, the luminosity disappears, we then infer that our suspicion was correct and that luminosity depend on incandescent particles floating in the flame.

Or reversing the process, and starting with a non-luminous flame as that of hydrogen in oxygen, suppose we dust carbon dust into the flame, the introduction of this element is accompanied by luminosity and we, by this method of Difference, infer that the luminosity is due to the incandescent particles unburnt and floating in the flame.

The method of comparison of two instances which differ in only one circumstance is common in our every day inferences.

A tooth aches;—some medicine is applied; the pain is gone. The inference is that the medicine is the cause, mediate or immediate, of the change in feeling.

Illustration 2.—Take a pigeon and watch its movements, and register what it does under certain circumstances.

Next take the same pigeon and remove its hemispheres, as they are ordinarily cut out for a lecture-room demonstration.

Now register its conduct under the same circumstances as were used before the removal of its hemispheres. The bird, if expressly excited thereto, will perform all the movements natural to it. In the ability to perform acts there is no defect, no change. But the removal of the hemispheres is accompanied by a removal of spontaneity and initiative in action. By the method of Difference it is inferred that the function of the hemispheres is to determine objects and ends of action.

Use and Criticism.—It has been shown to be of common use in inferring causes. Its chief use for scientific purposes is to bring to a more rigid test the conclusions reached at by the methods of Analogy
and Agreement, or by keen insight. For instance, by a number of agreements of \((m)\) and \(P\), \((m)\) is regarded as the cause of \(P\). The method of Difference immediately sets to work to confirm or negate this by showing that when \((m)\) disappears, \(P\) also disappears. This implies in this method, artificial experiment, which is a characteristic of the method and as this elimination or introduction of the element which is the supposed cause of the phenomenon under consideration, is not always possible, this method is not always applicable.

Care must be exercised in applying this method to ascertain:

1. That the cases do differ only in the one particular we select as the cause.
2. That this particular be the cause and not the cause of the cause.
3. That in eliminating or introducing an element, we do not introduce or eliminate inadvertently some other element essential to the phenomenon.

Sometimes the methods of Agreement and Difference are combined,—that is, a number of instances are found in which \(P\), a phenomenon, is always connected with \((m)\), a common element, the supposed cause; also a number of instances which agree in nothing save the absence of \((m)\), and the absence of \(P\). Then \((m)\) is said to be the cause of \(P\). There is, however, nothing new in this combined method and nothing more need be said of it.


Definition.—When causal relation is discovered on the evidence that any quantitative variation in the supposed cause is accompanied by a corresponding variation in the phenomena under consideration, the method of inference is that of Concomitant Variation.

If \(A\) be one phenomenon and \(B\) another phenomenon in causal relation of this character, the simplest relation may be symbolically expressed thus:

\[ A = m B \]

If, however, \(B\) be not the whole cause of \(A\), for example if \(A\) be due to some constant cause along with the variable cause \(B\), the relation would be expressed \[ A = m B + C \], where \(C\) is the constant.

Illustration.—For example the pressure, of any definite surface immersed in any liquid, water (say) varies as the depth of this definite surface below the surface of the water.
Here, \( A. = m \cdot B. \), where \( A. \) is pressure and \( B. \) the depth. If the atmospheric pressure be regarded, then we have \( A. = m B. + P. \), where \( P. = \) atmospheric pressure.

**Illustration.**—Probably the best example is that of Boyle's law, where the volume of a gas under pressure varies as the pressure, symbolized thus, \( V. = \frac{m}{P.} \). Taken in connection with the variation of volume due to variation of temperature, the total result may be expressed, \( V. = m \left( \frac{T}{P.} \right) \).

Other illustrations are common.—As the variation in inclination of a magnetic needle as the pole is approached or the variation in intensity of light on a given surface as that surface changes its distance from the source of light.

There are, of course, innumerable cases of concomitancy where no constant value for \( m. \) can be found, that is, where the law cannot be symbolically expressed. In such cases the induction may go no further than to say that with an increase in \( B. \) there will be an increase in \( A. \), or the reverse.

**Use.**

```
<table>
<thead>
<tr>
<th>A.</th>
<th>B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>G. (H.) (SUPPOSED CAUSE OF P.)</td>
<td>G. (H. + dH.)</td>
</tr>
<tr>
<td>(P.)</td>
<td>(P. + dP.)</td>
</tr>
</tbody>
</table>
```

This method is often of use when the method of Difference is inapplicable; as when the element (H) of the circumstance A. (supposed to be the cause of P.), cannot be eliminated, but can only be changed to (H. + d H.) with corresponding change of P. to (P. + d P.), when \( d \text{ H.} \) and \( d \text{ P.} \) are increments of \( \text{H.} \) and \( \text{P.} \) respectively.

Thus in the above circumstance, A., if \( H. \) be the pressure and \( P. \) the volume of a gas, \( H. \) the pressure, can never be eliminated for gas must be under some pressure, and therefore the method of difference cannot be applied. But \( H. \) may be made to vary, that is, may take on an increment and become \( (H. + d \text{ H.}) \) and in consequence \( P. \) become \( (P. + d \text{ P.}) \)
From a study of the relation of the rate of change of $H$ to the rate of change of $P$, laws more or less definite have been found. The induction of course, lies in the fact that from a limited number of concomitant values of the variables, a universal relation has been discovered.

Illustration.—Suppose a gold bob to hang suspended by a flexible string. If it be drawn to one side and released, it will vibrate for a few seconds and come to rest. What reasoning could lead to the conclusion that if friction were removed, the motion would never cease, since there never was a case where friction was removed entirely?

(1) Let the support be as flexible as possible and let the bob be suspended successively in the following liquids whose densities are given:

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sulphuric Acid</td>
<td>1.840</td>
</tr>
<tr>
<td>Molasses</td>
<td>1.426</td>
</tr>
<tr>
<td>Water</td>
<td>1.000</td>
</tr>
<tr>
<td>Oil of Turpentine</td>
<td>0.870</td>
</tr>
</tbody>
</table>

It will be found that as the densities decrease, there is an increase in the time which it takes the bob to come to rest. Let the experiment be continued in the following gases:

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chlorine</td>
<td>2.45</td>
</tr>
<tr>
<td>Sulphur dioxide</td>
<td>2.21</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>1.52</td>
</tr>
<tr>
<td>Air</td>
<td>1.00</td>
</tr>
<tr>
<td>Marsh gas</td>
<td>0.55</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.069</td>
</tr>
</tbody>
</table>

and air (in an air pump) of various densities down to .00055. It will again be found that the length of time increases with the decrease in densities, from a few seconds to more than thirty hours.

(2) If the same medium be used and strings of different flexibilities be used, the time increases with the flexibility of the string. From both of these, it is inferred that were both the density of the medium and the resistance of the string to be reduced to the limit zero, the time before coming to rest would reach the limit infinity, that is, that the original motion would continue without change.

III.—4. The Method of Residues.

Definition.—Suppose we have an instance in which are active operating causes $a$, $b$, $c$, of which $a$ and $b$ are known, but $c$ is not recognized, and suppose we have in consequence known and
measured effects \(a, b, c\), of which \(a, b\), are known by previous 
inductions to be effects of \(a, b\); then by the method of residues, \(c\) 
must be the effect of \(c\).

Here then we have an effect \(c\) of known magnitude for which we 
must seek a cause. Now it is generally the case that \(c\) is very small 
and unexpected. If, however, it is noted and properly estimated, 
valuable inferences often follow.

*Illustration 2.*—In 1845 it was found by astronomers that 
Uranus failed to move precisely in the path which the computers 
predicted for it. It was misguided by some unknown force, so that 
the difference between its observed place and its prescribed place 
was nearly as much as the ‘intolerable’ quantity of two minutes 
of an arc. Here (the reference is to the definition above) \(a, b\) were 
known to the astronomer and the consequent effects \(a, b\) were 
known, but the unexpected though minute discrepancy must be 
accounted for in some other way. With this discrepancy and 
known physical laws as data, the direction of the unknown cause \(c\) 
was determined, and by means of the telescope the cause was found 
to be the planet Neptune.

*Use.*—Its use is largely suggestive. The known causal elements 
do not account for all the effects, and additional causes must be 
sought. The result is a more careful analysis by the methods 
already given.

With the modern delicate means for detecting and measuring 
very small changes in quantity or motion, this method is used more 
and more to extend scientific knowledge by pointing out its inade-
quacy in certain cases to account for all the facts observed, and 
thereby leading to a successful search for the true explanation.
CHAPTER IV.
APPLICATION TO PARTICULAR SCHOOL SUBJECTS.

The foregoing methods of induction and analysis, find their application separately and in various combinations in the best presentation of school subjects, and in the following pages an attempt is made to illustrate such applications as the writer thinks may be useful to the practical teacher. A general theory of induction which finds in its treatment no application to particular instances, is often of very little practical value to the teacher, for whom the whole difficulty frequently lies in his inability to connect the concrete work of the class room with the theory.

No attempt, however, is made at an exhaustive treatment of the different subjects. There is rather, an effort to give a few illustrations in detail, believing that these when fully grasped along with the general meaning of induction, will result, in the hands of the thoughtful teacher, in the wise extension of the method to other subjects. It must not be forgotten too, that the inductive is not always the best method to be used. It is the instrument for gaining general principles. When these are in possession, knowledge may be extended by their deductive application to new realms of fact and the application and use of these general principles will always form a considerable part of the teacher's work. This latter is, however, a comparatively easy task and the more difficult work of disentangling from the medley of particulars, useful and comprehensive generalizations, has therefore been emphasized.

From time immemorial, Geometry has been regarded as one of the subjects best suited to set forth the deductive method of reasoning. In this subject, definitions were first given, that there might be clear notions of the terms used; the axioms, the so-called self-evident truths were then learned and these formed the basis upon which all the superstructure of derived truth was raised. Great care was taken to shut out any aid from measurements and only such demands or postulates as were absolutely necessary to make the figure, were allowed. These postulates granted the use of compasses and straight edge only. Not only were the evidences of the senses unheeded that the conclusions of reason alone might be relied on, but the figures were often designedly so drawn as to make such evidence, if regarded at all, contradictory of the findings of reason. Now we are far from condemning such treatment of the subject, for it has great worth as an expository method and for this purpose will never be superseded. But it is not the beginner's method; it is not the method of discovery. It is a method that convinces; one in which the learner gives assent. In every point
and at every step the learner must concur in the conclusion reached, but just why the steps chosen and not others were taken, is not known till the end is reached. There is in consequence an uncomfortable sense of mystery growing out of the very perfection of the process,—a desire to know the steps by which such a proposition was first suggested—a feeling that the problem “Hath had elsewhere its setting, and cometh from afar.”

The inductive method of treatment has invaded even this stronghold of deduction and to-day we find the experimental or inductive study of Geometry used both as an end in itself and a preparation for its subsequent formal and demonstrative treatment, and there is pretty general accord that this practical treatment should be continued during the period of the elementary course.

The advantages of the method in this subject are not hard to see. For all our general knowledge we are indebted to induction or deduction, or to processes involved in these. And Geometry treated as here suggested, will enable a student from a number of observations to reach general results and again from these generalizations to draw conclusions and must therefore have distinct educational value. Through accurate measurements repeated in a number of cases, the learner as a matter of observation, feels himself justified in making some general proposition. In this process nothing is hidden from the student. After the aim is set before him, he cooperates in every act and the initiative may lie with him. His intellectual interest and curiosity are excited in reaching conclusions almost as if he were a discoverer, and if subsequently when suitable cases come before him, he finds he can utilize his findings, he feels keenly the value of his induction. He has, in fact, much of the sense of ownership and the intellectual emotions of the successful investigator.

This mode of studying the subject implies exact measurements, without which no useful conclusions could be drawn. Inaccurate measurements inevitably give unsatisfactory and even misleading results, and always lead to failure. And to measure accurately, is to see clearly and know the quantitative aspect thoroughly. The eye and the hand are thus trained, and judgments are made to depend upon the results. This is of great importance, this learning through self exercise of hand and eye by first hand contact with reality, the exact character of the facts from which judgments are to be formed. Then too, if the exercise has been done carefully, the practical work will leave firmly fixed in the learner's mind, the leading facts of elementary geometry, which in themselves, aside from the logical exercise involved, will be useful knowledge. If too,
a more extensive course be subsequently taken, this knowledge will form an excellent introduction to the more adequate and completely exhaustive proofs found in deductive geometry.

Too much can hardly be said of the advantage of having clear conceptions of the general enunciations of the deductive method, based on particular cases, and this the experimental method supplies. Not unfrequently, another advantage comes from what might be called the negative result of the inductive method in this subject. Students gradually feel the inadequacy of the so-called inductive proofs and are therefore often led to inquire for and appreciate the perfectly adequate and general proofs deductively established.

The value of the inductive process may be best brought out by dealing at close quarters with topics in which it finds suitable application. Topics in Geometry and Grammar have been chosen for close treatment mainly for two reasons:—(1) The examples from which generalizations are made are easily obtained and of such a definite character that there can be no difference of opinion as to their exact nature. (2) Heretofore these subjects have usually been treated deductively, and if the inductive method can be shown of marked value here, it may reasonably be inferred to be of quite general value and wide application. It might be added as a further reason that these subjects lend themselves well to this method, since our knowledge in them is held in the form of abstractions,—rules and general propositions.

In the following example from Geometry the teacher's purpose is to get the class to find and set forth a set of conditions, sufficient to warrant the conclusion, that these conditions being found, two triangles may be said to be equal in every respect. Only one case of the congruency of triangles is taken, and the conditions are, that the two sides and contained angle of one triangle be equal to the two sides and the contained angle of another triangle.
(a) PREPARATION.

(1) What is an equilateral triangle?

(2) If the angles of two triangles are equal, are the triangles necessarily equal in every respect?

Ans.—No. Reason.—Because the angles of all equilateral triangles are equal and all equilateral triangles are not equal.

This focusses the mind on one aspect of the question—the conditions of equality of triangles, and prepares the way for the aim.

(b) AIM.

Let us try to find out a set of conditions sufficient to warrant our saying that therefore the two triangles are equal in every respect.

The statement of the aim gives definite direction to the pupils' research and makes their answers to the teacher's questions more relevant, while at the same time, the mere statement of it defines for the teacher his task.

(c) PRESENTATION.

(1) Draw as in fig. I. the lines 2.1 inches, 2.1 inches.

(2) Make the angles 56°, 56° at one end of these lines.

(3) Cut off other arms 3.1 inches, 3.1 inches.

Look at these figures. If these parts were given, would the triangles formed by joining B. C. and E. F. be equal in every respect?

(4) Find by measurement B C. and E F.

Result, B C. = 2.6 in.; E F. = 2.6 in.

(5) Find by measurement angles B., E., C. and F.

Result, B. = 82°, E. = 82°; C. = 42°, F. = 42°.

(6) Cut out triangle A B C., or trace it on paper and apply it, or its trace, to triangle D E F.

Result, they fit and therefore have same area.

Let us look at the figures again:—

(7) What parts had we given equal? Ans.—Two sides equal to two sides and one angle equal to one angle. Or A B. = 2.1 = D E.; A C. = 3.1 = D F.; and angle A. = 56° = angle D.

What position does the angle occupy with respect to the given sides? Ans.—It is between these.
(8) What parts did we find by measurement to be equal?
B C. = E F. = 2.6 by measurement.
Angle B. = angle E. = 82° by measurement.
Angle C. = angle F. = 42° by measurement.
Area triangle A B C. = area triangle D E F. by measurement.

What position do the sides, proved equal, hold with respect to the angles given equal? Ans.—Opposite.

What position do the angles shown to be equal hold with respect to the sides given equal?
Answer—Equal angles are found opposite equal sides.

Under each pupil's drawings should be expressly stated:—
(1) What conditions he had to begin with.
(2) What he found by measuring and experiments.

There should be thorough analysis of the cases treated that all the facts may be brought out. There should be just such thoroughness here in laying bare the facts as finds its counterpart in the careful experimentation of science where the induction method is typically shown. At this stage the basis is laid for subsequent comparison and generalization and any haste or lack of thoroughness here will prevent sound conclusions.

The cases in figures II., III., IV., which should be made to vary as much as possible, should be separately and fully gone over as in case I., no attempt at this stage being made to do anything more than to define each case and find out facts relative to this case alone.

(d) ASSOCIATION (or) COMPARISON, AND ABSTRACTION.

1. What in each and all of I, II., III., IV. cases do we find as to the length of the third sides?
   In each and every case they are equal.
2. What in regard to the remaining angles?
3. What in regard to the areas?
4. In all of these cases what were the conditions with which we began? We had two sides of one triangle = to two sides of another triangle, and the contained angles were equal.
5. In all of these, what have we discovered?
   (a) That the third sides are equal.
   (b) That the remaining angles of one triangle are equal to the remaining angles of the other triangle.
   (c) That the areas of the compared triangles are equal.
There should be a going over of all the cases to establish likeness in points and to note differences, the emphasis, however, being put upon the likenesses.

This step if well done, will almost force the generalization of the next step.

(e) SYSTEM (or) GENERALIZATION.

(1) From the consideration of what we had given in these cases, and what you have found, what statement of relation between these could you make?

Answer.—In these cases when two sides and the contained angle of one triangle were equal to the two sides and contained angle of another triangle, the two triangles are equal in every respect.

(2) Do you feel warranted in making any broader statement? If not, more cases in which the conditions are supplied by the students, should be measured.

(3) What statement can you now make?

If two triangles, or if any two triangles have the two sides and the contained angle of the one equal to the two sides and the contained angle of the other, these triangles shall be equal in every respect.

This last statement should be written by the pupils in red ink, to distinguish it from the findings of the individual cases. Often if the four cases make up the four quarters of a page, this conclusion can be written across the bottom.

This should be treated as the pupils’ own discovery.

(f) APPLICATION.

If the conclusion above is tentatively held and certain new cases are used to verify it, this step may be regarded as a part of the induction, or as a deductive step within an inductive process not yet completed. If, however, the conclusion is accepted as general, before which of course there would be no real induction, then this step is deductive. Thus the first and last steps of this process are deductive. Assuming this step to be deductive, such questions as the following may be used:—

(1) Two circles have equal radii, the angle between two radii of the first circle is 6° and the distance between the outer ends of the radii is 4 m.m. What is the distance between the outer ends of two radii of the second circle when the contained angle is 6°?

Answer.—4 m.m.
How do you know?

*Answer.*—The general conclusion of (e) is cited.

This and other uses or applications of the general principle learned, will give the students an appreciation of what they have done. As many varied examples as convenient should be cited to which this principle might be applied.

It was stated above, that Grammar had been taught deductively. We have only to recall such grammars as Murray's to verify this. On the first page we meet this definition—"*English Grammar is the art of speaking and writing the English Language with propriety.*" *English Grammar is divided into four parts, Orthography, Etymology, Syntax and Prosody.*" Definitions of these were then given. Subsequently rules were given and the whole work of the student of grammar was to learn these definitions and rules, and find their application. The learner was not an investigator to discover rules and relations. His attitude was receptive. And though this method will always form a part of the general method in the treatment of grammar, the greater stress in now laid on the discovery of relations and the expression of these as they are exhibited in sentences. The result of the new treatment of the subject is that Grammar, once so distasteful, is now among the best liked subjects, provided the study is not begun at too early an age.

The fact that from certain data, numerous examples in the form of sentences, the different definitions and rules of grammar may be drawn and formulated to be used again in application to other examples, makes the teaching of Grammar almost a perfect example of the inductive-deductive method of instruction.

The following illustrative lesson is given to show this and to bring out the educational value of this method. The lesson chosen is the 'preposition,' its Use and Name.

**Suitable Examples.**

1. The man *on skates* is a racer.
2. The book *against the table* is mine.
3. The marble *in the cup* is blue.
4. The man ran *through the corn.*
5. The horse stood *beside the fence.*
6. The dog crawled *under the gate.*
7. The cow ran *towards him.*
8. The hunter shot *beyond them.*
9. The wind blew *over me.*
10. He wandered *earth around.*
(a) PREPARATION.
(1) Give an example of a phrase.
(2) What is a noun? A pronoun? A phrase?
(3) Select the phrases from the sentences one to ten (1-10).
(4) What words do these phrases modify?

Here use is made of the principle of apperception. Old and related knowledge is adjusted to the new material to be presented. Apperceptive systems are brought into the focus of consciousness.

Although the general trend of thought in this method is inductive, since particular facts here presented are brought under general truths already known—ex. gratia,—“What is a phrase?” “Name the phrases in sentences 1-10,” this part of the whole is deductive.

Ziller recognized the advantage of making this step quite explicit as that concerned with old knowledge, and divided Herbart's first step, “Clearness” into this step of “Preparation,” and another step called “Presentation,” the latter treating of the new material to be apperceived. Rein added a sub-step called the “Aim,” which he placed before the step called “Preparation,” but which seems more naturally to follow this step. And if the statement of the aim cannot be briefly and succinctly made, it had better be omitted. Its value is shown both in the greater relevancy of the pupils' answers to the teacher's questions, and the greater zest with which the pupils carry on their research with the teacher. The aim of this lesson may be stated as follows:—

(b) AIM.

Let us find out the uses of the words beginning the phrases mentioned and a suitable name for them.

(c) PRESENTATION.

(1) In what subdivision of the subject or predicate is the word on found? Answer.—In the phrase.

This question must be asked in reference to 'against,' 'in,' 'through,' 'beside,' 'under,' 'towards,' 'around,' of the several sentences.

(2) Where in the phrase is each to be found?

Each word is the first word of a phrase, except in No. 10, which is not in the natural order.

(3) In sentence 1, what word shows the relation between the word 'skates,' which is in the phrase, and 'man,' a word not in the phrase? Answer.—The word 'on.'
In sentence 2, between what words does 'against' show a relation? Between 'table' and 'book.'

(The actual relation may be illustrated here by putting a book against the table).

Similar questions should be put regarding each of the sentences. This will bring out in 10, the peculiar position of 'around.'

(4) In sentence 1, what part of speech is the word in the phrase between which and some other word, 'on' shows a relation?

Answer.—A noun.

(5) Look at sentences 2, 3, 4, 5, 6, 10, and tell what part of speech the corresponding word is. Answer.—A noun.

(6) In sentences 7, 8, 9, what is the part of speech of the word in the phrase, between which and some other word, 'towards,' 'beyond,' 'over,' show relations? Answer.—A pronoun.

This second step is of very great importance if the inductive process is subsequently to be of much use in giving trustworthy results. The essential preparation for subsequent induction is the clearness with which these relevant particulars are seen. This was no doubt the fact that made Herbart give this step, for which Ziller substituted steps one and two, the name 'Clearness.' On this step the teacher must spend most time, both in selecting examples before the class discussion and in framing those questions which will lead the class to make the proper analysis of the examples selected.

These examples:—

(a) Should contain varying instances otherwise they would not be representative.

(b) Should be so arranged in order that those which most typically exhibit the main idea to be grasped should come first, for first consideration.

(c) Should be arranged in groups, or in columns or in some way to help in the next step called by Herbart, Association and by Ziller, Comparison and Abstraction.

There will in this step be a tendency to make comparisons and of course this is almost necessary to the thorough understanding of any examples, but this should not be the main purpose here.

(d) COMPARISON AND ABSTRACTION OR ASSOCIATION.

(1) From each and all of the examples, what have we observed as to the position of the words under consideration? Each and all hold the position of the first word in a phrase, except in example 10, where the order is not the natural order.
(2) What relational use of these words may be observed in all of these examples?

These words show a relation between some word in the phrase and some other word not in it.

(3) What part of speech may this word in the phrase be? It may be a noun or a pronoun.

(Note.—It may be any substantive).

(4) Where are these words placed with reference to the noun or pronoun referred to in (3)?

Usually before the noun or pronoun.

This position gives them their name—from 'pre—before,' and 'pono—'I place,' and they are called prepositions.

The wording of the answers will be determined to some extent by the form of the questions put. The essential thing here is that there be no oversight of any of the particulars common to all the examples; and that these observations be made by the learners, and be given in their own words.

Of course as yet there is no induction, but merely what Dr. Whewell calls a 'colligation.'

(e) GENERALIZATION—DEFINITION.

What then is a preposition?

A preposition is a word which stands usually first in a phrase, and shows a relation between the noun or pronoun of that phrase and some other word in the sentence.

This is of course, just what we are forced to say when we have a common name for the subject of our definition. It is, however, an induction as the conclusion is a general one concerning not only these cases, from which it arose, but all other parallel cases.

(f) APPLICATION.

Exercises such as:—In the following examples, select the prepositions and say why they are held to be prepositions.

Here under 'application' we have particulars classified under general conceptions and the step is deductive. It may be thrown into syllogistic form thus:—

\[
\begin{align*}
(1) & \text{ Major premise—Definition of a preposition.} \\
(2) & \text{ Minor premise—'At' (say) is such a word.} \\
(3) & \text{ Conclusion—(or therefore) 'at' is a preposition.}
\end{align*}
\]
In giving in detail the five formal steps of inductive lessons, we have restricted ourselves to Geometry and Grammar. And these steps may not be so fully exemplified in every subject but the inductive trend here somewhat fully set forth is applicable in almost every subject of school study except those in which the acquisition of skill is the aim, such as the special arts—Writing, Music, Drawing, etc.

In whatever subjects we have rules or general principles, the inductive method should be used to establish or verify these by individual examples, that they may be grounded in the understanding rather than merely held in the memory.

For instance, in Arithmetic those principles found in the complex operation of multiplying by numbers expressed by two or more digits, should be inductively established by preliminary exercises.

Reference is made to the following:—

(a) That to multiply by the parts or addends of a number, and then add the partial products, is to multiply by the number itself.

(b) That to multiply by a number gives the same result as to multiply by the factors of that number in succession:—

\[(a)\] Thus:—
\[8 \times 9 = 72.\] This is known by the tables.

Now \[9 = (4 + 3 + 2)\ldots\ldots (other addends might be used.)\]

And
\[
\begin{align*}
8 \times 4 &= 32 \\
8 \times 3 &= 24 \\
8 \times 2 &= 16
\end{align*}
\]

\[8 \times (4 + 3 + 2) = 72\]

From this and similar examples the general fact expressed in \((a)\) is established.

\[(b)\] Thus:—
\[8 \times 9 = 72\]

And \[(8 \times 3) \times 3\] = 24 \times 3 = 72, i.e. \(8 \times\) factors of 9 in succession.

From this and other similar examples \((b)\) is inductively established.

Now having \((a)\) and \((b)\) such questions as 432 \times 23 may be explained.
For (1) To multiply by 23 is to multiply by 3 and then by 20 and add results.

And (2) To multiply by 20 is to multiply by 2 and then by 10, which is to multiply by 2 and move the result one place to the left.

The multiplication then becomes deductive, based on the general principles (a) and (b), which have been established by induction.

\[
\begin{align*}
432 \\
23 \\
\hline
1296 &= 432 \times 3 \\
8640 &= 432 \times 20 = 432 \times 10 \times 2 \\
9936 &= 432 \times (20 + 3) = 432 \times 23
\end{align*}
\]

Unless such inductions are made, Arithmetic becomes, what it too often is, nothing more than the application of rules, that is, becomes wholly deductive. The application of rules is essential, for only by using these as a point of departure can progress be made, but they should always stand for the result of steps which could at any call be made explicit.

In Geography the primary object of teaching is to train the "outlook faculty" to image forms and forces beyond the horizon; to see the forms of land and water and the climatic forces acting on these, and more especially both these forms and forces in relation to man. Geography puts under tribute the facts and findings of all the natural sciences and is in reality little more than the correlation of these facts to illuminate and explain man's actual or possible relation to his environment.

As a concrete science it begins with particulars and follows the course—observation, comparison, inference and verification; that is, the inductive deductive method is used.

First, the facts of the child's homeland are observed, the forms of land and water and the animal and plant life connected therewith, together with the climatic forces acting thereon. Such an acquaintance is made with these that their definition or representation is satisfactorily made:

(1) As to outline—by maps and drawings.

(2) As to surface—levels, slopes, hills, &c., by modelling in wet sand, &c.
(3) As to climatic relation of earth, air and ocean, by verbal accounts to show how the sun lifts the water from the ocean, how the air carries it and drops it on the land, and how the land returns it to the ocean, and the profits man derives from these.

These facts, thus expressed, must first be impressed by out-of-door, first-hand contact with nature in both her static and dynamic aspects. This must be so not only in the interest of truth and accuracy in its bearing on the child's immediate surroundings that he may actually know these, but rather because these observations are those in terms of which all conditions beyond, are to be interpreted and understood. Nothing so insures vividness and accuracy of observation as the child's own doings with these forms of land and water and climatic forces—sailing round an island; waiving a flag on a headland or cape; walking round a pond or small lake, or catching a fish therein; sailing up or down a river, branching off into a tributary; sliding down a hill or standing on its summit feeling its breezes; watching the sailing clouds; catching rain-drops or hail, &c., &c. It is not enough that children have in a casual way observed; there must be close observation in which the interests are active.

Man's relation, that is the children's relation of these things must be emphasized; how they are used by man to contribute to his life.

In this early observation stage which is characteristically one for acquiring data, only the simplest and easiest generalizations, provisional half-truths, are possible. But in answer to the child's "why?" in a limited way, there may be comparison, inference and verification. The facts observed, too, should be selected by the teacher who ought to have an eye on the significance of the facts selected for wider knowledge.

Having secured adequate knowledge of the child's home world to serve as a constant standard of judgment for the world beyond his horizon, before the next step, comparison, is taken, there must be a great increase in knowledge of what lies beyond his immediate neighborhood. This knowledge may now be got by various means.

(1) Vivid descriptions of things by the teacher, in which his words are not remembered descriptions, but the clothing of his own efforts at imaging these things.

(2) An interpretation of maps, now possible because of the pupil's acquaintance with maps already made by his own hands and representing what he actually saw and therefore, when seen, suggesting images of realities.
(3) Pictures and specimens.
(4) Word images from books.

Comparison and inference based on data thus obtained, but always supplemented and interpreted by first hand knowledge, are now possible with resulting generalizations. When these generalizations have been reached, their application completes the inductive-deductive method.

There may, of course, be certain modifications to be made in these generalizations as knowledge grows. But in course of time such information as can be conveyed to the eye by a good map will be sufficient to lead a student to infer at once generalizations and to deduce therefrom very important facts.

Below we submit an illustration of inductive inference. The particulars used may be obtained from two maps:

(1) A relief map. (2) A good wall map on which rainfall is shown by colour shading, the direction of winds by arrow heads, and the fact of their blowing from warmer to cooler areas or vice versa, by latitude.

I.—British Columbia.—Features noted.

\[
\begin{align*}
(1) & \quad \text{The coast range of mts. almost unbroken, about 10,000 ft. high.} \\
(2) & \quad \text{The wind blowing off the Pacific at right angles to the mountains.} \\
(3) & \quad \text{The wind blows from warmer to colder areas.} \\
(4) & \quad \text{Rainfall on (a) Windward side of mts. is 68 in.} \\
& \quad \text{(b) Leeward side of mts. is 10 to 14 in.}
\end{align*}
\]

II.—India, South West Coast.—Features noted.

\[
\begin{align*}
(1) & \quad \text{Western Ghats almost unbroken (3,000-8,000) ft. high.} \\
(2) & \quad \text{The wind blowing at right angles to the mountains.} \\
(3) & \quad \text{The wind blows from warmer to cooler areas.} \\
(4) & \quad \text{Rainfall on (a) Windward side, 100-250 in.} \\
& \quad \text{(b) Leeward side, 35 in.}
\end{align*}
\]

III.—India, South East Coast.—Features noted.

\[
\begin{align*}
(1) & \quad \text{Eastern Ghats, broken by open spaces—height, 1,500 ft.} \\
(2) & \quad \text{Wind from ocean blowing nearly at right angles to mts.} \\
(3) & \quad \text{The wind blows from warmer to cooler areas.} \\
(4) & \quad \text{Rainfall on (a) Windward side is 40 in.} \\
& \quad \text{(b) Leeward side is 18 in.}
\end{align*}
\]
IV. — United States.

1. Eastern side—Appalachian—2,500 ft. high.
2. Wind blowing parallel to the ranges of mts.
3. Warmer to colder areas.
4. Rainfall on (a) one side, 30 in.
   (b) other side, 30 in.

V. — Africa—Morocco.—Features noted.

1. The Atlas Mountains, unbroken—9,000 ft. high.
2. Wind from ocean at right angles or nearly so to mts.
3. The wind blowing from cooler to warmer areas.
4. Rainfall on (b) Windward side—none of any moment.
   (b) Leeward side—none of any moment.

VI. — South America—Peru.—Facts:

1. Andes Range, unbroken, height 12,000 ft.
2. The wind blowing from east, over Brazil.
3. The wind blows from slightly warmer regions.
4. Rainfall on (a) Windward side is 50 in.
   (b) Leeward side is almost nil.

By comparing (1) and (4) of I., II., III., it might be concluded by simple Enumeration that "When mountains run parallel to the coast:

(a) There is a greater rain fall on the sea-ward side caused by the sea."

A negative case like that of VI., where the greater rain fall is on the land-ward side, would by the method of Difference correct this and if the direction of the winds were noted, the corrected inference would run:

(b) When mountains parallel the coast there is a greater rainfall on the windward side, caused by the interception of vapour-laden winds.

By the method of Concomitant Variation this would be confirmed for:

VI., I. and II. show the rainfall differs most when the wind blows at right angles and the mountains are highest, and therefore when there is greatest interception.

III. shows the difference less when the interception is less, due to broken range and lower mountains.
IV. shows difference nil when wind is parallel to mountains, and therefore when there is no interception.

Again inference (b) would be overthrown by a case like V. (Atlas Mountains in Morocco), where the rainfall is nil on the windward side. This would lead to a new analysis of the facts and if it was noted whence the winds came, the conclusion might now be:

(c) When mountains parallel the coast there is a greater rainfall on the windward side, caused by the interception of vapour-laden winds when these winds come from a warmer to a colder region.

Further elimination of the condition "When mountains parallel the coast," might be made, but enough has been given to illustrate the inductive method.

A more easily established generalization is that the temperature of a place depends on (1) Its latitude i.e. if A. has a higher latitude than B., all other conditions being equal, A. will be colder than B.

(2) Its altitude.
(3) Its humidity.
(4) Its proximity to the sea.
(5) The prevailing winds.

Each of these conditions may be shown to be a causal factor by the method of Concomitant Variations, by keeping all other factors constant while the one under consideration is made to vary.

Illustrations of the use of Deductive inference in geography:

\[
\begin{align*}
\text{Data} & \quad \begin{align*}
(1) & \text{A good map.} \\
(2) & \text{Some knowledge of the soil of the place.} \\
(3) & \text{Some knowledge of the minerals.}
\end{align*}
\end{align*}
\]

The inferences are obtained by the application of general principles to particulars.

1. The map gives the position and the physical features.
2. The position and physical features give the climate.
3. The climate, soil and physical features give the vegetation.
4. The climate and vegetation give the animal life.
5. The climate, vegetation, animal life, soil (minerals), position and physical features give man's occupations.

Special Application.—To infer the nature of life in north eastern England, say, Yorkshire.

Position.—In north temperate zone, therefore the climate is such that the land is habitable for the white man. It is in the line of the Gulf Stream, and therefore not only is it a habitable land, but
it has a pleasant climate, somewhat humid, but for that very reason without the extremes of temperature. The "North Easters," however, modify the eastern part of England and coming from higher latitudes, cold and dry, make winters here harsher and in consequence make winter grazing for cattle, horses and even sheep, less profitable than in the milder and more humid west.

Physical Features.—The mountain ranges running north and south intercept the heat and moisture, so that though the whole county being on a small island, is green, yet no great rainfall comes here. The rivers, however, carry the water from the higher western part, through to the North Sea, so that the land is well watered.

The Soil, is varied, but in the main is naturally adapted for grazing—on the higher parts for sheep and the lower for cattle. The limestone character of the rocks and the short slopes, favour falls along the rivers, and therefore give power. The vegetation is neither scanty nor profuse, and the animals would be such as sheep, horses and cattle.

The Minerals, Coal and Iron, taken in connection with the position of the whole island, so favourable for commercial interchange, would give rise to manufacturers of steel and iron; while the great quantities of wool and the power supplied by the rapid streams and native coal, would give rise to woollen manufactures. The proximity to the Northern Sea almost inland, would suggest fisheries.

We should therefore expect here a dense population, occupied mainly in manufactures of iron, steel and woollen goods, and in a less degree in agriculture and in fisheries.

When that which is to be studied is not a general principle, as in the above examples where induction is the mode of inference, but an individual unit of a more or less complex character, the mental movement involved follows a certain course which may be described as analytic-synthetic, which begins with simple apprehension and always returns upon itself with more or less complete comprehension of the character of the unit with which it began. And as much that forms the subject of study in elementary education is of individual things which it is the object of the learner to understand, and which for the time seem to have very little relation to other things, a clear conception of the process involved is very desirable and to this end the following illustrative example is given. It will be remembered that the bond of connection between this mental procedure and that involved in induction, was previously
pointed out as consisting in the fundamental faculty of the mind—the power of noting likenesses and differences and this basis will prominently appear in the illustration.

Let the aim be to know "Six," the addition facts of "Six" (say):

In I.—A. is a solid cylindrical block. B. consists of a lower cylindrical block as shown 5 in. high, on which stands a 1 in. block. C., D., are as indicated. The lower blocks are dark and the upper light in colour.

If the pupils have begun their work in Arithmetic by measuring with units of different lengths, the solid block A., marked in inches may be placed before the class and the aim be stated something as follows:—"What different lengths go to make up this six-inch block?"

Place all the other columns as shown in I., so that to understand any of them is to understand A.

Here we have the first requisite—a simple apprehension of the whole as such. The name by which to refer to A., may be given, or it may be found out by the pupils by measuring with a ruler.
The next step, analysis, is possible because parts may be discriminated. In B. we have 5 dark and 1 light coloured inch lengths. The colour helps the discrimination. The size would make discrimination easy here if there were no difference in colour; and if size and colour were the same, position would make discrimination possible. The children might be made to make the analysis by being asked to take down the columns. The result would be shown as in II. 1, 5; 2, 4; 3, 3.

If the children be now asked to build again the columns, the mechanical reconstruction will appear as in III. The corresponding mental synthesis would find expression in words like:

\[
\begin{align*}
\{ & 5 \text{ inches and } 1 \text{ inch make } 6 \text{ inches.} \\
\{ & 4 \text{ inches and } 2 \text{ inches make } 6 \text{ inches.} \\
\{ & 3 \text{ inches and } 3 \text{ inches make } 6 \text{ inches.} \\
\{ & 6 \text{ inches is made up of } 5 \text{ inches and } 1 \text{ inch.} \\
\{ & \&c., \&c. \\
\end{align*}
\]

Now the basis on which depends the possibility of grouping or synthesizing is the likeness expressed by the word "inch" or "inches." If the objects were such as:—Book, bell, pencil, crayon, pen and pointer, the likeness would consist in mere "oneness," or that which remains after abstracting all concrete qualities, that is mere 'being'. The difficulty in getting a concept of number lies in this abstraction.

If line III. represents not only the mechanical but the mental synthesis, there is a return of the mental movement from a vague apprehension of A., to such a comprehension of it as finds abstract expression thus, \(6 = 5 + 1 = 4 + 2 = 3 + 3\).

The analogy between the procedure shown in the preceding example and that involved in an effort to know a poem is very striking.

The whole is first read, preferably aloud. This gives an apprehension as above, more or less vague. Some of the outstanding features are recognized:—the general movement of the verse, the music, more or less of suggestiveness awakening the imagination or all of these fused into a feeling, a sort of love of the life which the poem reflects and radiates.

Within this whole, differences are discernible. The sources of the effects dimly felt at the first view, gradually emerge and with their emergence the effects become enhanced. Discrimination is aided by the stanza arrangement and by the various subtle arts of the poet, to the most prominent of which attention might be directed. No simple waiting without mental activity, will bring into relief
the content of the poem. There must be analysis, to discover, at least for most students, the fuller suggestiveness possible. No mere standing by the pool can bring its fullest healing influence—one must move in its waters.

Here too, as in the preceding illustration, the chief difficulty lies in discovering the common factor among the elements which analysis has brought to light. And here the difficulty is greater, since in poetry this common element may consist in no definable thought, but rather in a kinship of emotions. The stanzas or other discriminated parts may have nothing more in common than the suggestiveness sufficient to influence the same kind of feelings from slightly different angles. But whatever this likeness among the different parts of the poem may be, it is essential that it should be either seen or felt, that the fitness of each for a place in the whole may be realized. Not until after the return of thought to the poem as a whole through synthesis following analysis, can the poem be adequately comprehended; and often this circle of activity must be repeated again and again before the full effect is felt.

But we may be told that Literature cannot be taught. That it is destroyed by analysis, that it is an art the quality of which is unexplainable and inexpressible in any language except its own. The fact remains nevertheless that it is and has been taught. When, however, we reflect on what a rare combination of qualities in the teacher the successful teaching of Literature demands, we are not so surprised that it is regarded by some as incapable of being taught. The lack of a strong sympathetic feeling for the young, on the one hand, and of a deep aesthetic insight into the value of literature through its ideals on the other, at once spell failure. While even the possession of these qualities does not of itself guarantee success. To create ideals that shall cast their imaginative spell over young hearts, beget therein a longing admiration for the noble and the good and shape conduct, demands of the teacher, not only sympathy and appreciation, but the power to communicate these, demands the highest art, the power to teach. And how shall this active feeling attitude towards the ideals of literature be secured? It must be developed, often quite slowly through the teacher's art. It will not arise of itself; it cannot be forced. The subtle influence of colour and music and movement must be tactfully and lovingly brought to bear upon the learner and there left in faith, to produce its effects. In one sense the teacher does very little and in another, he does much. The spiritual character of all teaching and especially of the teaching of literature, must never be undervalued, but at the same time the work of the teacher, including what he says and does in his class as well as what he is, must not be overlooked. We may
talk of the relation between the teacher and his class as analogous to electric or magnetic induction, and speak of his boys as becoming generous, magnanimous and noble through influence from the teacher's highly charged spiritual condition, and this is all well enough, to emphasize the great value of character, but if, for the purposes of teaching, it is to mean more than Utopian dreaming, there must be some sort of rational articulation of thought between the teacher and the class. The teacher may stand before his class, highly charged with feelings from some poem, but is this though a necessary condition, of itself, any guarantee of a similar feeling in his class? Surely not, unless he in some way effectively turn about into different lights that which in the poem gives him so much pleasure. Even after this, he may fail, but without it or something equivalent, he cannot succeed. A discovery of harmony among the parts of the poem, of an insight into the relevant significance of its various elements and of the fair unity of the whole, are surely needed, and this setting forth of the content of the author's work, to be seen at different angles and in different lights, by whatever means produced, is all we mean here by analysis, and that by which the final synoptic view is produced, is all that is here meant by synthesis.

Besides in well-knit ratiocinative prose, the central thought or principle to be established is often derived by a sort of quasi-induction.

The generalization grows partly out of elements named by the author, and partly out of kindred elements from the background of experience. It is not the sum of these elements; it is not the whole of any of them; but rather that which is common to them all in sentiment or in thought. It is broader than all the elements giving rise to it, and in this latter lies its inductive character. There is no doubt that as far as the author is concerned, this general conception was in his mind before the elements selected to give expression to it, but that which was first for the author is last for the reader and learner. Some authors actually set down this conception at the beginning, and follow it by giving the incidents and elements which substantiate it. Others as in the moral appended to some poems, lead up to its expression. In most writings, however, this obtrusion of the author's views is not made and the mind makes or feels its own inferences.

It is not, however, claimed that any formal inductive step can be generally applied to the study of literature. Even in the investigation of scientific subjects, the home realm of induction, no one set mode of procedure is possible, for the different methods of Agreement, Difference, Concomitant Variation, &c., either singly or in
conjunction are used according to the special nature of the problem. It is, however, held that the attitude of mind which grows out of the inductive habit, is often helpful in understanding, at least the intellectual element in literature. The search for likenesses and differences, the instituting of comparisons, and the systematic disposition of the consequent findings, the search for significant design-like relations in nature does, it is claimed, aid in the discovery of the thought and sentiment, artfully, that is designedly, embodied in literature.
CHAPTER V.

THE VALUE OF INDUCTIVE STUDIES FOR SOCIAL ADJUSTMENT.

What has been written thus far has been an attempt to show the nature of Induction and related subsidiary processes, and their application to the work of the school-room. The illustrative examples used have been introduced to give clearness to the meaning implied in the general statements as well as to increase the value of the discussion for practical pedagogies. Incidentally and for purposes of contrast, deduction has been treated and thus the character and limitations of induction have been better defined. In a word, it may be said that the inductive method as we have defined it and illustrated its use, seems to be what normally occurs, or ought to occur, when the teacher properly guides the pupil in the pursuit of general knowledge.

It is however pretty generally believed that the aim of education should be essentially practical; not in any narrow "make-a-living" sense, but in a large sense and therefore in valuing the inductive method of the schools, its relation to subsequent life must be considered. One of the best tests the educator can apply to his work is the ultimate effect of his teaching method on conduct or action,—during work and leisure, in the place of business and in the home, or under a sense of social or civic responsibility. For such action, it is not difficult to see the usefulness of the inductive treatment of particulars in deriving therefrom laws and principles, and in their application to new situations. If for a moment, we informally tabulate what is involved in the establishing of any general truth in the school room, and the mental operations called into play in man's every day life among the animate and inanimate forces of nature, we shall find that the one tabulation will serve for both cases:—

(a) Observing facts.
(b) Recording these,—either by writing them or by holding them in memory.
(c) Classifying.—This belongs here, that there.
(d) Suspending judgment till all the facts are considered. The opposite in life, marks the hasty judgment.
(e) Experimenting,—making tests of the evidence on hand.
(f) Keeping an openness of mind. Hardly any question is finally closed.
(g) Applying results to new cases. The wise profiting by experience.

In the case of the lawyer, this involves the sifting of a mass of evidence to find that which is relevant to the theory he wishes to
establish; in the case of the scientist, the elimination by careful observation and experimentation of all those cases in which the essential common element is not found; in the physician, a careful noting of all symptoms, a proper consequent classification and subsequent suitable prescription; in the ordinary man, the habit of close inspection that mere likeness or analogy may not mislead; and in all, a careful and critical attitude towards facts before accepting them as sufficient to warrant certain conclusions.

There is no doubt that in all this there are to be found deductive links, and there is no wish to minimize the importance of this mode of inference. It, too, is essential. The fact is that these two types of thinking continually alternate like the to-and-fro motion of the weaver's shuttle, together weaving the web of thought. But that men may deal wisely with the problems that every day presents, it is much more important that they know how to use the particular varying experiences they have as a means of arriving at general truth, than that they should be able to argue correctly from given premises to correct conclusions. The occasion for the exercise of the former sort of ability arises much more frequently than that for the latter. And even were the occasion for these equal in number, it would be found that men go wrong oftener by arriving at general truths too hastily and from insufficient data, than by illogical reasoning from given generalizations.

It is just this necessary critical attitude towards facts and their significance, that proper inductive school training gives, and therefore from this point of view it is one of the best preparations for adjustment to future environment.

In addition to the proper attitude of mind, which the school through induction produces for subsequent profitable use in the affairs of life, there is another element—the emotional effect of successful induction, which is quite as effective in life as in the school. And nothing is more manifest than the joy a class or individual feels in firmly grasping a general truth. It affords such a comportable sense of power, such a pleasant sense of easement in finding in this truth a string, as it were, by which to hold all the facts together in one aspect. To the teacher, too, comes the joy of having led the class to discover largely through their own self-activity this valued truth. There is in consequence, renewed interest and endeavour in the school room. On reflection it will at once appear that this must grow into inductive habit and bear fruit in after years, in lives of happy, independent and successful endeavour in the world.
CHAPTER VI.

SUMMARY—INFERRENCES—CONCLUSION.

A brief recapitulation of the main positions taken in this dissertation, will serve both to give a synoptic view of the whole and to furnish an opportunity of dealing with a few inferences which may be of some pedagogical value and which could not well have been treated in the several sections.

1. (a) In Chapter I., the nature of induction is given, which is, in brief, a process of inference in which the mind passes from one particular fact to others, and by analysis and comparison comes to a generalization by which it is able to hold these facts and all others of like character in one universal aspect. The starting point is particular facts, the process is mental transition through analysis and comparison, and the end is a universal relation discovered in these facts. In contrast, deduction is shown to be a process in which the mind interprets particular facts in the light of a universal relation which properly covers these facts. The starting point here, is a universal truth, the process is synthesis through comparison, and the end is the inclusion of a new fact under the universal truth. From one point of view, all inference may be regarded as inductive since the universals used in the so-called deduction are the results of inductions from experience, and are only registered as a convenience for further induction. From another point of view, all inference is deductive, since the grounds by which the mind in its generalizations goes beyond the particular facts actually observed, is a universal truth or assumption, such as the uniformity of nature.

But whatever be the ultimate ground on which inference rests, the mind in reasoning from one mental acquisition to another, constantly employs both of these movements and both result in giving system and intelligibility to the otherwise isolated and comparatively meaningless facts of experience.

(b) It will at once be seen that Induction is a much more difficult process than Deduction,—that reaching a generalization through analysis, comparison and abstraction, amid a multiplicity of particulars, is a course often devious and laborious, while the application of this generalization when once found, is a comparatively direct and easy one. It must also appear that these generalizations are of very great value because of their applicability over such a wide field of facts. They not only hold together our best mental possessions in a form convenient for ready inference, which is a necessity for thinking, but they also give the power for enlarging those possessions.
This forces an important question upon the teacher's attention—how shall these so important generalizations become the possession of the pupils? Shall the difficult inductive process be passed over and the result of induction, in the form of general truths, be given to them ready made. This would appear at first sight to be a course of true economy. But is it possible to become possessed of the true import and meaning of a generalization without an acquaintance with the particular notions out of which it grew? The failure to ask and answer this question resulted until a hundred years ago, in following this supposedly short and economic method of teaching to the almost utter exclusion of any other.

The method of the schoolmen of Medieval times, and even of modern times, amply exemplify this mode of procedure in teaching. It was a method in harmony with the almost universal and implicit reliance upon judgments enunciated by authority, characteristic of the times. The growth of modern science has, however, demanded direct evidence, rather than appeal to authority, as the basis of knowledge, and educational practice, in consequence, is slowly changing from a purely deductive to a more or less inductive procedure.

In contrast with the schoolmen, Herbert Spencer holds that as the race gained its knowledge inductively through experience, the individual must gain his in the same way,—by experience, experiment or observation, and that he can gain it in no other way.

It seems to the writer that the positions of the Schoolmen on the one hand, and of Spencer on the other, are extreme. The student should surely not be given what he can reasonably find out; he should not be deprived of the joy and development of independent effort; his mind should not be regarded as a merely passive receptacle for knowledge. On the other hand, the student should not discover or even rediscover by his own personal effort, what the race has learned during the centuries. Even in science where, if anywhere, it might be thought most feasible, it is utterly impossible; and had a student years enough, and were it possible, would it be desirable? When here, as in other departments of thought, the experience of the student makes generalizations intelligible, what is gained by refusing to accept them? Why refuse to be "An heir of all the ages?" Has not progress been brought about in the main, by accepting the discoveries made by exceptional investigators whose gifts to the race are these discoveries and an account of the steps which led to them?

For the practical schoolmaster, the truth lies in the mean between telling and developing. Many teachers who have heard
that "Telling is not teaching," make most lamentable efforts by attempting to carry "development" too far and waste time by "Dipping buckets into empty wells, and growing old in drawing nothing up." There are times when telling is the only and, therefore the truest teaching. Everything depends upon whether the mind of the learner is or is not in need of facts from without, and the skill of the teacher shows itself in knowing at every point whether his work at that point is one of diffusion, or of development of knowledge.

2. (a) Generalizations we have seen involve comparison of particulars and would in consequence imply more or less accurate knowledge of these particulars, to make comparison possible. In the second Chapter, an attempt is made to show how knowledge of particulars is obtained, and how the process of obtaining it is subsidiary to induction and with it, forms the whole cycle of thought processes by which the world is scientifically known.

That with which mind is first engaged is single objects, or objects regarded as single. Generalizations and disassociated parts are artificial products, and exist only in mind and follow previous perceptions of objects as individual wholes.

The mind in obtaining knowledge of wholes, follows a certain order in its movements based upon certain fundamental powers. This order is analysis followed by synthesis, and the foundational principle upon which these are based is found in the power of the mind to note likenesses and differences.

The analysis necessary in dealing with any object to understand it, involves a differentiation of a part or parts from other parts within the whole and this differentiation depends upon the irreducible power of the mind to discriminate or note differences. The synthesis of these discriminated parts necessary to reconstruct the object and change it from an object merely apprehended to an object more or less fully comprehended, depends on the mind's power to identify or note likenesses.

These activities of the mind, though distinguishable in thought, are in reality, not separable and are just two aspects of one analytic-synthetic process and constitute a perpetual paradox in that, the distinguishable elements of a thing may be said to be held apart and at the same time held together.

The knowledge of an object grows as this analysis-synthesis becomes more and more exhaustive. This growth is usually slow and covers a length of time in which there are many periods of rest. The advance to fuller knowledge from any resting stage depends
upon the acquisitions already made, which determine very largely what is further observed and how it is interpreted. From what is presented to the mind at any particular stage, together with representations from the mind's previous possessions, the ground for a syllogism is supplied and inference follows. Further experience tests the accuracy of this inference and confirms or corrects it, and thus the knowledge of an object grows.

The characteristic and proper marks of an object discovered by the analytic-synthetic process form the basis for the comparison of different objects and thus make possible subsequent induction and deduction. Induction and deduction, too, depend on the power to discriminate and identify.

An attempt is also made to show by means of a diagram and its analysis, both the subsidiary relation which analysis and synthesis bear to induction and deduction, and the common fundamental principle on which these related processes rest.

It is thus shown how these interrelated processes explain and connect the infinite richness and complexity of truth and its comprehensive simplicity.

(b) Elementary education has to do mainly with individual things or at least with things seen only in limited relations. Here the chief work of the educator should be to secure for the child, through analytic and synthetic processes, an acquaintance with the characteristic marks and properties of objects. These objects need not be material things, but may consist in any of the individual experiences of the child.

And since this analytic-synthetic activity is based on the pupil's power to note differences and likenesses, it follows that knowledge can only arise when these powers are actually called into exercise and that therefore nothing can take the place of the self-activity of the child. Much attention, therefore, in the early stage of education, must be given to direct perception, and no second hand verbal substitutes should be in any case allowed. This is the time and place for concrete experience.

If too, as was shown, the acquired content of the mind makes our perceptions largely inferences, the importance of what is learned in determining character is evident; and the necessity for a judicious choice of material becomes a matter of great moment to the teacher.

Further, since mental content largely determines observation even when the things observed are directly before the senses it will be seen that induction which grows out of a comparison of these observations and consequently deduction and the higher modes of
thought are all influenced by the earlier processes. This forces on the teacher the constant need "To look before and after," to see, as it were, the bud in the seed, the flower in the bud and the fruit in the flower. And the consciousness of this need should result in a greater emphasis on proper correlation and sequence of studies and on the mastering of those educational principles upon which such sequence and correlation rationally depend.

3. (a) Of the different methods employed in Inductive research:

I.—The Method of Enumeration.
II.—The method of Analogy.
III.—The method of Scientific Analysis,

the first two may be regarded as loose, uncertain and popular methods while the third, as its name indicates, is an exact, well established one, and for that reason, in the interest of truth, far too uncommon.

In simple Enumeration, we have that tendency of mind long ago referred to by Bacon in his Idola Tribus, of supposing greater order and regularity in nature than there actually is, a tendency to generalize experiences if these point in the one direction.

In Analogy, we have another of Bacon's Idola, in an excessive tendency of the mind to note resemblances among things. From resemblances in certain particulars, two things are said to be alike in other particulars, and therefore a certain proposition which is true of the one is probably true of the other.

Both of the methods are of value and may result in conclusions which are valid. They are sometimes sufficient for the ordinary guidance of conduct. But they do not constitute proof and may result in any degree of fallacy. They mark the limit of the purely observational methods of unreflecting and uncultivated minds.

On the other hand, in the method of Scientific Analysis, we have the instruments used by investigators who are not satisfied by surface resemblances and their indications, but who "interrogate nature," and find out her secrets.

This Scientific Analysis involves two fundamental principles. "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number; one is by comparing together different instances in which the phenomenon occurs. The other is by comparing together instances in which the phenomenon does occur with instances in other respects similar, in which it does not occur. These two
methods may be respectively denominated the method of Agreement and the method of Difference." The methods of Concomitant Variations and Residues, too, are covered by these principles.

These latter are of very great importance in modern science, because by them the quantitative relations between phenomena can often be established and the result may then be made the ground of deductive inference of most useful and far reaching consequences.

(b) The inferences for pedagogics would seem to be:—

(i) That the natural tendency to generalize from few particulars so common among children, should be made the starting point from which the true nature of inference might be taught.

(ii) To show the inadequacy of a generalization too hastily made to explain the facts legitimately falling under it. This would not be a difficult matter for a teacher with his wider knowledge. It is what must be constantly done in the growth of conceptions, or in the developing of definitions from examples.

(iii) To amend and modify the generalization through a closer scrutiny of marks, or an extension of examples, and to show the satisfactory, because sufficient, character of the corrected generalization.

This will, if properly done, make students more careful in observing and comparing, and tend towards accuracy in thought and expression. By pointing out false analogies, the teacher may make students anxious to search for these resemblances which are due to an identity in essential elements, that is, to lead them to search for those marks of a thing which mainly constitute it, what it is.

The great care involved in observations and experiments necessary for the establishment of a law must beget in teachers and students alike, the useful habit of closely scrutinizing the grounds for any inference. It may also serve that great purpose of creating an ideal of perfection in reasoning, which shall afterwards function in efforts to realize that ideal, not only in reasoning, but in other matters, a result of paramount importance in education.

4. (a) The application of general principles to particular instances is here given somewhat in detail to aid the teacher in connecting the concrete work of the school room with theory.

Those subjects which lend themselves most readily to inductive treatment are chosen, but wherever general truths from facts are to be established, this method is applicable.
It will be observed that in every instance the complementary deductive method is also used and in the case of Geography, its application is made with some fulness of treatment.

There are however always, but especially in primary work, facts which must be studied in their individual rather than in their related, aspects, and here the mental operations, involved are mainly those of analysis and synthesis.

An outline of the application to Arithmetic and Literature illustrative of this analytic-synthetic treatment is given. In all such lessons the facts or feelings presented, should make direct appeal to the learner with a minimum of indirect aid; but it is held that except in the simplest cases, some aid is necessary to secure the best results.

(b) Since in this Chapter the discussion deals with the application of principles, no further inferences need be given.

5. (a) The inductive-deductive processes of the school room by securing in any study, a satisfactory grasp of details which enable the student to cope successfully with related facts has a real value for subsequent out-of school life. The ideal procedure of the school room is carried over into practical affairs in man's every day life, among the animate and inanimate forces of nature. This factor in the preparation of the individual for the immediate future helps to secure that adjustment to man's physical and social environment, which it is the business of a proper system of education to provide. If, in addition, those subjects for treatment by this method be chosen which deal with the facts which subsequent experiences must face directly or indirectly, education will in a yet fuller sense, be the "preparation of the individual for successful participation in the economic, political and social activities of his fellows." (Munroe, Hist. of Education, page 369).

In general conclusion, it is held, that if the nature and function of Induction be understood, its province be well defined, and the necessity of its being constantly supplemented by Deduction and subsidiary operations, be not overlooked, we shall find in it an instrument of great educational value both as—

(1) A school agency for the acquisition of knowledge and

(2) A fitting preparation for subsequent practical life.