A Two-Dimensional Horizontal Wave Propagation and Mud Mass Transport Model on Muddy Coastal Regions

by

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To

Malihe & Anahid
Abstract:

It is well known that surface water waves interact with fluid mud on the sea bed. Wave-mud interaction results in high wave energy dissipation and mud mass transport. This kind of wave energy dissipation, which generally is much more significant than wave dissipation due to bottom friction, should be considered in the simulation of wave evolution and transformation in muddy coastal environments.

In this research, a two-dimensional horizontal wave propagation and morphodynamic model for muddy coasts was developed. The model can be applied on a general three dimensional bathymetry of a soft muddy coast to calculate wave damping, fluid mud transport and resulting bathymetry change under wave action. In addition to the effect of wave-mud interaction on wave propagation, the dissipation due to wave-mud interaction was also implemented in SWAN (a third generation numerical model for Simulating WAves Nearshore) using a multilayered wave-mud interaction model. These two models combined, can be used for generation and propagation of waves in muddy coastal areas. The nonlinear constitutive equations of the visco-elastic-plastic model are adopted for the rheological behavior of fluid mud in this research.

The results of the numerical model are compared against a series of wave-basin experiments, wave-flume experiments and field observations. Comparisons between the simulated results with the both field and laboratory data reveal the capability of the proposed model to predict the wave transformation and mud mass transport.
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# Table of contents:

1. Introduction ...................................................................................................................... 1  
   1.1. General introduction ................................................................................................. 1  
   1.2. Definition of fluid mud ............................................................................................. 2  
   1.3. Mud profile processes in coastal area ....................................................................... 3  
   1.4. Objectives and scope of the study ............................................................................. 7  
   1.5. Outline of dissertation ............................................................................................... 8  
   References ............................................................................................................................ 9  

2. Numerical Modeling of Wave Transformation on Muddy Coasts ................................. 12  
   2.1. Abstract ................................................................................................................... 12  
   2.2. Introduction ............................................................................................................. 13  
   2.3. Rheological model of mud ...................................................................................... 14  
   2.4. Wave transformation model .................................................................................... 16  
      2.4.1. Governing equations ......................................................................................... 16  
      2.4.2. Mud dissipation ............................................................................................... 17  
      2.4.3. Breaking dissipation ....................................................................................... 19  
      2.4.4. Discretization ................................................................................................. 20  
      2.4.5. Boundary conditions ....................................................................................... 22  
   2.5. Procedure of calculation ......................................................................................... 25  
   2.6. Laboratory investigations ........................................................................................ 25  
   2.7. Model performance ................................................................................................. 28  
      2.7.1. Wave transformation on a fixed bed ................................................................. 28  
      2.7.2. Comparison with the laboratory experiments ................................................... 32  
      2.7.3. Comparison with field data of Kumamoto Port ................................................ 35  
   2.8. Summary and Conclusion ....................................................................................... 37  
   References ............................................................................................................................ 37  

3. A Two-Dimensional Horizontal Wave Propagation and Mud Mass Transport Model . 40  
   3.1. Abstract ................................................................................................................... 40  
   3.2. Introduction ............................................................................................................. 41  
   3.3. Wave propagation model ........................................................................................ 43  
      3.3.1. Dissipation of wave energy due to wave-mud interaction ................................ 44  
      3.3.2. Dissipation of wave energy due to wave breaking ........................................... 45  
      3.3.3. Discretization of mild slope equations .............................................................. 46  
      3.3.4. Boundary conditions for mild slope equations ................................................ 49  
   3.4. Rheological model of mud ...................................................................................... 51  
   3.5. Wave-mud interaction model .................................................................................. 54  
      3.5.1. Wave-driven mud mass transport velocity ....................................................... 62  
   3.6. Gravity-driven mud mass transport ......................................................................... 64  
   3.7. Mud profile change ................................................................................................. 68  
   3.8. Procedure of calculation ......................................................................................... 70  
   3.9. Model performance ................................................................................................. 72  

References ............................................................................................................................ 73
List of figures:

Fig. 1.1. Typical vertical transport processes characterizing the variation of the density of mud-wave mixture .............................................................. 4
Fig.1.2. Schematic response of a mud bed to wave forcing and feed back to related response of mud bed to water waves.. ................................................... 5
Fig. 1.3. Stages in mud profile responses to wave episode........................................ 6
Fig. 2.1. Definition of multi-layered model. ................................................................ 19
Fig. 2.2. Staggered mesh scheme. ............................................................................ 21
Fig. 2.3. Offshore open boundary. ........................................................................... 23
Fig. 2.4. Flow chart of the simulation procedure. ....................................................... 26
Fig. 2.5. Experimental setup and modeling domain. ................................................... 27
Fig. 2.6. Computational region for wave refraction example. ...................................... 29
Fig. 2.7. Distribution of wave height and direction of refracted waves. (a) wave height contours (units: mm), (b) wave direction ........................................ 30
Fig. 2.8. Cross-shore distribution of wave height and wave angle at the middle section...... 31
Fig. 2.9. Wave height transformation on laboratory wave basin (mm). .......................... 33
Fig. 2.9. (Continued) ................................................................................................. 34
Fig. 2.10. Comparison between simulated and measured wave heights at the measuring points. ........................................................................... 35
Fig. 2.11. Bathymetry and the wave measurement stations (m). .................................. 36
Fig. 2.12. Simulated wave height at Kumamoto Port (m). ............................................ 37
Fig. 2.1. Staggered mesh scheme ............................................................................... 48
Fig. 3.2. Boundary conditions for the wave propagation model: a) Boundary with arbitrary reflectivity, b) Offshore open boundary................................................................. 51
Fig. 3.3. a) Definition sketch of multi-layered model for wave–mud interaction, b) Definition sketch of multi-layered model on a sloping bed. ......................................................... 55
Fig. 3.4. The momentum balance in the x-direction for an element with unit width in the y direction. .................................................................................. 66
Fig. 3.6. Profile of the mass transport velocity in the mud layer normalized by $u_1$, calculated velocity at the surface of mud. ...................................................... 73
Fig. 3.6. Continue ......................................................................................................... 74
Fig. 3.8. Comparison of simulated profile of water-mud interface with the measurement and simulated profile of Jiang (1993). ......................................................... 76
Fig. 3.9. Experimental setup and numerical modeling domain in the wave basin at the hydraulic laboratory of Yokohama National University. ........................................ 79
Fig. 3.10. Simulated wave height contours for case 4 (unit, mm). ............................... 80
Fig. 3.11. Comparison between simulated and measured wave heights at points 1 & 2. Error! Bookmark not defined.
Fig. 3.12. Comparison between simulated and measured profile of mud movement at point 81
Fig. 3.13. Time variation of deposition height in trench #1 ........................................ 82
Fig. 3.14. Bathymetric depth below datum in simulation domain and the location of the wave measurement stations (unit, m). ...................................................... 83
Fig. 3.15. Simulated wave height contours in meter at Kumamoto Port. ......................... 83
Fig. 3.16. Contours of simulated wave height and layout of the trenches in local modeling domain (unit, m)................................................................................................................................. 84
Fig. 3.17. Simulated mud thickness in trench #1, without submerge dike and trench #2, with submerged dike. ................................................................................................................................. 85
Fig. 4.1. Definition of multi-layered model .................................................................................................................. 94
Fig. 4.2. Louisiana coast and shelf, Coordinates are in Kilometers and contours in meter ...................................................................................................................... 102
Fig. 4.3. Ten selected wind measurement stations .................................................................................................. 102
Fig. 4.4. Spectral evolution, a) Measurement, b) Simulated wave height no fluid mud, c) Simulated wave height with fluid mud ........................................................................................................... 103
Fig. 5.1. Site of the Kumamoto Port, A and B are wave measurement stations (depth from datum) ............................................................................................................................. 109
Fig. 5.2. Definition of multi-layered model .................................................................................................................. 111
Fig. 5.3. Measured time series of wave height, tide and suspended sediment concentration in Trench # 1 ........................................................................................................................................ 116
Fig. 5.4. Vertical distribution of suspended sediment concentration at Station A and B ........................................................................................................................................................................ 117
Fig. 5.5. Wind time series measured at Kumamoto Airport .......................................................................................... 117
Fig. 5.6. Simulation domain and bathymetry ................................................................................................................ 118
Fig. 5.7. Wave height comparison (m) ......................................................................................................................... 119
List of tables:

Table 1-1- Mobile mud density and corresponding water content ratios ................................. 3
Table 2.1. The chemical composition of kaolinite ................................................................. 27
Table 2.2 Incident wave and fluid mud characteristics ........................................................... 28
Table 2.3 Wave heights and wave periods at measuring points ............................................. 28
Table 3.1 Incident wave and fluid mud characteristics ........................................................... 77
Table 3.2 Wave heights and wave periods at measuring points ............................................. 78
Chapter 1:

1. Introduction

1.1. General introduction

Many water pollutants, such as trace metals, petroleum and pesticides, are absorbed by the surface of cohesive mud deposited on the bed of water bodies; consequently, transport of soft marine mud is of great concern as it can carry the pollutants. These types of pollutants can be carried by the fluid mud and passed through water intakes in coastal areas. Moreover, the effects of pollutants on ecosystem health in estuaries attract the researchers interest to shallow water where the effects of surface water waves are significant on sediment transport processes. In addition, mud transport in harbors and coastal zones is often undesirable as the resulting bathymetric changes reduce navigability and make costly dredging operations necessary to keep the required navigable depth. Therefore, a numerical model is an important tool for the managers of harbors and coastal zones to study the undesired accumulation and movement of mud in these areas. Many examples of mud shorelines are found adjacent to large rivers, including San Francisco Bay (Liang and Williams, 1993), Western Louisiana coast (Morgan et al., 1953; Kemp, 1986), West coast of Peninsular Malaysia (Malaysian EPU, 1986; Hor, 1991), Northeast coast of China (Yu et al., 1987), Kumamoto Port in Japan (Tsuruya et al., 1990) and the Amazon River continental shelf (Cacchione et al., 1994).

When a surface water wave travels over a soft muddy bed, an interfacial wave between the water layer and the mud layer is generated resulting in significant dissipation of wave energy.
The inter-surface wave also results in mass transport in the mud layer. Mud mass transport under the influences of waves and gravity, and suspended mud transport in the water layer are the main mechanisms of mud transport in estuaries and coastal regions. Since sediment concentration in the mud layer is often in the order of a few hundred kg/m$^3$, compared to typical sediment concentrations in the water column which are in the order of a few tenths of kg/m$^3$ (Shibayama et al., 1990; Kessel et al., 1996), it is expected that the movement of the mud layer is the predominant transport mechanism under stormy conditions. However, long-term effects of suspended mud transport should not be overlooked, given the associated environmental impacts.

1.2. Definition of fluid mud

Cohesive sediments are composed of very small grains, generally in the range of silt ($<70 \mu m$) to clay ($<4 \mu m$), in which the attractive force, between sediment grains is strong. Mud, which is mostly composed of particles smaller than about $20 \mu m$ mixed with organic material, is much more cohesive than the one composed of coarser particles (Mehta and Lee, 1994). The inter-particle attraction force is a function of the grain mineralogy and water chemistry, particularly salinity. Thus, coarse silt that behaves like noncohesive fine sand in fresh water may become cohesive in an ocean environment. Consequently, small particles that have been carried by suspension from rivers to the sea can flocculate and be deposited in form of cohesive mud at the bottom of the ocean in coastal areas. Mobile mud is a sediment and water mixture with a density between 1030 and 1300 kg/m$^3$ (Table 1-1). Mobile mud can also be described as a fluid-like mud, in which the particles are largely fluid supported (Smith and Kirby 1984).
Table 1-1- Mobile mud density and corresponding water content ratios

<table>
<thead>
<tr>
<th>Researcher(s)</th>
<th>$\rho_u$ (kg/m$^3$)</th>
<th>$\rho_l$ (kg/m$^3$)</th>
<th>$W_l$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inglis and Allen (1957)</td>
<td>1030</td>
<td>1300</td>
<td>170</td>
</tr>
<tr>
<td>Krone (1962)</td>
<td>1010</td>
<td>1110</td>
<td>530</td>
</tr>
<tr>
<td>Wells (1983)</td>
<td>1030</td>
<td>1300</td>
<td>170</td>
</tr>
<tr>
<td>Kendrick and Derbyshire (1985)</td>
<td>1120</td>
<td>1250</td>
<td>210</td>
</tr>
<tr>
<td>Nichols (1985)</td>
<td>1003</td>
<td>1200</td>
<td>270</td>
</tr>
</tbody>
</table>

The density of the water mud mixture changes through depth and with the time of consolidation. The fluid mud layer can be distinguished by a sudden change of the density (Fig. 1.1). More often, several layers of fluid mud, each having different characteristics, can be found in coastal areas. The typical vertical transport process for cohesive sediment has been shown in Fig. 1.1.

1.3. Mud profile processes in coastal area

In coastal areas, waves are the main cause of fluid mud generation; when the waves stop the bed recovers again. Based on Maa and Mehta (1989), the response of a mud bed to wave forcing is schematized in Fig. 1.2, together with brief descriptions of the numbered pathways. The box with the dashed line represents the mud-water system, the inverted triangle represents forcing, rectangles are components of the mud-water system, ellipses are process transfer functions and circles represent responses. The pathways are as follows: 1,2) water waves determine the flow field; 3,4) flow field and bed properties together govern the character and the dynamics of the interface; 5,6) wave loading, consolidation, fluidization and thixotropy change mud properties with time; 7,8) flow field and interfacial character
determine the interfacial shear stress; 9,10) interfacial shear stress and interfacial properties determine the rate of particle entrainment, or interfacial erosion; 11,12) shear (and normal) stresses together with mud properties determine mud motion; 13,14) mud properties largely determine the rate of surface wave damping or attenuation. The response of the mud profile under wave action is more complex than the response of sandy profiles. Fig. 1.3 demonstrates the process that might be seen in a coastal area (Lee and Mehta 1997). Assuming that the bed is already consolidated after a period of calm sea, when a storm starts, the bottom shear stress due to wave action will increase and cause erosion. Consequently it will generate high turbidity, resulting in an increase of sediment concentration in the water
column and generation of a fluid mud layer. The fluid mud layer can also generate due to collapse of structure of loosely consolidated mud bed under wave action.

Fig.1.2. Schematic response of a mud bed to wave forcing and feed back to related response of mud bed to water waves. (after Maa and Mehta 1989).
Fig. 1.3. Stages in mud profile responses to wave episode (Lee and Mehta 1997)

The wave generated fluid mud is transported under the continued wave action and gravity (Fig. 1.3a). At this time, the surface of fluid mud can be eroded which is referred to the fluid mud entrainment. The effect of the wave orbital motion within the mobile mud results in wave-induced mass transport, i.e. Stokes drift and Eulerian velocity, in the direction of the wave propagation. In calm conditions it is expected that the mobile mud moves downward on the slope, however, if large waves exist, the mobile mud can move onshore due to effect of Stokes drift and Eulerian velocity. The change in bathymetry on the muddy coast is largely due to the mass transport of the fluid mud, rather than suspension. The wave energy dissipation due to fluid mud will also significantly decrease the wave height in this episode.
Usually the mud motion stops soon after large wave action stops, and the mud hardens by self-weight consolidation (Fig. 1.3b).

1.4. Objectives and scope of the study

Wave attenuation and mud mass transport on cohesive beds have long been topics of interest (e.g., Ross and Mehta, 1990; Shibayama and An, 1993). Kessel and Kranenburg (1998) examined mud mass transport on a sloping bed. Gravity driven flow and pressure gradients were included in their one-dimensional simulation. A constant wave height was used in their treatment of mud transport ignoring the effects of shoaling and wave attenuation. They could not offer a quantitative comparison between the flow model and measured values. A general formula for an equilibrium mud profile shape was offered by Lee and Mehta (1997). They also tried to relate the rate of mud transport to wave energy dissipation in order to calculate temporal profile changes. Soltanpour et al. (2003) proposed a cross-shore numerical model for wave propagation and mud mass transport considering shoaling and wave dissipation due to wave breaking and fluid mud. However, the 2D horizontal elements such as refraction and diffraction were absent in their model.

Although several two-dimensional horizontal wave generation and propagation models exist, they cannot usually be applied on soft muddy beds due the high rate of energy dissipation resulting from wave-mud interaction. Moreover, very few morphodynamic models include the mud mass transport under wave action.
The objective of this study is to develop an engineering and scientific numerical model for simulation of wave generation, propagation and resulting morphodynamic changes in coastal areas covered with mud. The main strength of the numerical model is the capability of this model in consideration of wave-mud interaction. A two-dimensional horizontal wave propagation and morphodynamic model is presented here. The wave propagation model includes the various effects of shoaling, refraction, diffraction, reflection, wave breaking and wave damping over a soft mud bed. The wave-induced and the gravity-driven mud mass transport is introduced to the mud morphodynamic model to calculate the resulting bathymetric change.

In addition to the effect of wave-mud interaction on wave propagation, the dissipation due to wave-mud interaction is also implemented in SWAN (a third generation numerical model for Simulating WAves Nearshore) using a multilayered wave-mud interaction model. The two above mentioned models combined can be used for generation and propagation of wave in muddy coastal areas. The nonlinear constitutive equations of the visco-elastic-plastic model are adopted for the rheological behavior of fluid mud in this research.

1.5. Outline of dissertation

This dissertation is in manuscript format and it includes four papers from Chapter 2 to Chapter 5. In chapter 2 the propagation of surface water waves on fluid mud is the main focus. The paper described the details of the development of a numerical model for wave propagation and interaction with fluid mud in coastal areas. In this paper, the results of the
model were compared with the experimental and field observations to prove capability of the model in prediction of wave characteristics in muddy coastal environment.

In chapter 3, along with the review of the wave propagation model, the main objective of the paper is the mud mass transport due to wave action and resulting bathymetric changes. The implementation of wave-induced and gravity-driven mud mass transport in a model is described in this paper. The consequent bed evolution is the outcome of the numerical model. Extensive comparisons have been done with the physical observation to demonstrate the efficiency of the numerical model in prediction of mud movement in coastal area.

In chapters 4 and 5, the implementation of the spectral dissipation of wave energy due to wave-mud interaction in SWAN (a third generation numerical model for Simulating WAves Nearshore) is explained. SWAN is a state of the art wave generation model, which can be used for prediction of wave generated by waves in coastal area. The proposed model was used to simulate several real field cases and the comparisons demonstrated a favorable agreement between prediction measurements.

General discussion and conclusions from the thesis follows the main chapters. The main contributions of this research and recommendation for future studies can be found in this chapter.

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Chapter 2

2. Numerical Modeling of Wave Transformation on Muddy Coasts


2.1. Abstract

The present study offers a two-dimensional horizontal wave transformation model that can be applied to a general three dimensional bathymetry of soft muddy coasts. The wave transformation model is based on time-dependent mild slope equations including the high energy dissipation of bottom mud layers as well as the combined effects of wave refraction, diffraction and breaking. The constitutive equations of visco-elastic-plastic model are adopted for rheological behavior of fluid mud. The results of the numerical model are compared with a series of wave-basin experiments. The performance of the application of the numerical model to a real field condition, i.e. Kumamoto Port of Japan, is also examined. Comparison between the simulated results and measured values in both laboratory and field conditions reveal the capability of the proposed wave transformation model to consider the wave attenuation on fluid mud layers.
Keywords: Fluid mud; wave height attenuation; wave transformation; visco-elastic-plastic model.

2.2. Introduction

Many examples of muddy coasts, usually adjacent to large rivers, are found including Ariake Sea of Japan (Yamada and Kobayashi, 2004), San Francisco Bay (Liang and Williams, 1993), Western Louisiana coast (Morgan et al., 1953), west coast of Peninsular Malaysia (Hor, 1991), northeast coast of China (Smith and Kirby, 1984) and northwest of the Persian Gulf (Soltanpour, 2004).

When surface water waves travel over a soft mud bed, an inter-surface wave between the water layer and the mud layer is generated resulting to high energy dissipation of waves. The inter-surface wave also results to mass transport in mud layer. These two major aspects of wave-mud interaction have long been drawing attention (eg., Maa and Mehta, 1990; Shibayama and An, 1993). However, most of the past studies have been conducted on horizontal beds under the laboratory conditions. Neglecting the refraction, there have also been few efforts to include the cross-shore bathymetry on wave transformation and mud mass transport (e.g., Soltanpour et al., 2003). In order to approach the real muddy field problems, where 2-D horizontal effects such as the changes of cross-shore profiles along the shoreline cannot be ignored, the influence of bottom configuration on wave height transformation in a general three-dimensional bathymetry should also be examined.
Although various two-dimensional horizontal wave models have been proposed during past decades, they are not capable of simulating wave attenuation on muddy sea beds. A hydrodynamic model is presented here considering the various effects of shoaling, refraction, wave breaking and energy absorption of soft mud bed on wave height transformation. Simulating the high energy dissipation of fluid mud layers, the proposed wave model can be applied on real field conditions.

2.3. Rheological model of mud

An suitable rheological model of mud should be adopted in order to investigate wave-mud interaction. A large variety of different constitutive equations have been employed for the prediction of the response of fluid mud layers. Here, the visco-elastic-plastic model has been adopted to develop a predictive behavior model for wave-mud interaction (Shibayama and An, 1993). This rheological model can be considered as a viscoplastic model where the elastic part has been replaced by a viscoelastic state. Considering that the elastic part of the Bingham model, in the upper part of mobile mud, is usually neglected in wave-mud interaction studies, the visco-elastic-plastic model seems to be a better choice that takes into account the behavior of mobile mud at both low and high shear stresses. The constitutive equations are expressed as:

\[ \sigma_{ij} = 2 \mu \varepsilon_{ij} \]  

\[ \mu_e = \mu + \frac{iG}{\sigma} \left( \frac{1}{2} \sigma_{ij} \sigma_{ij} \leq \tau^2 \right) \]  

\[ (2.1) \]

\[ (2.2a) \]
\[ \mu_e = \mu_2 + \frac{\tau_y}{\sqrt{4 |\Pi_e|}} \quad \left( \frac{1}{2} \sigma_{ij} \sigma_{ij} > \tau^2_y \right) \]  

(2.2b)

where \( i \) and \( j \) take the values 1 and 2 which correspond to \( x \) and \( z \) axis, respectively, \( \sigma \) is the angular frequency of wave, \( \mu_e \) is the apparent viscosity, \( \sigma_{ij} \) is the deviator part of stress tensor, \( \dot{\epsilon}_{ij} \) is the deviator part of strain rate tensor, \( G \) is the elastic modulus, \( \mu_1 \) is the viscosity of mud in the viscoelastic state, \( \mu_2 \) is the viscosity of mud in the viscoplastic state and \( \tau_y \) is the yield stress. \( |\Pi_e| \) is the objective of the deformation-rate tensor and expressed as

\[ |\Pi_e| = \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2 + \frac{1}{4} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 \]  

(2.3)

where \( u \) and \( w \) are velocity components in \( x \) and \( z \) direction, respectively.

The rheological viscoelastic parameters, i.e. shear modulus and viscosity, are calculated from the results of the laboratory experiments of Shibayama and An (1993) on commercial kaolinite

\[ \mu_1 = 10^{(3.353 - 9.56 \times 10^{-3} W)} \times T \]  

(2.4)

\[ \log G = 3.761 - 1.05 \times 10^{-2} W \]

\[ + (0.147 - 3.38 \times 10^{-3} W) \log(T - 0.522 - 1.23 \times 10^{-3} W) \]  

(2.5)
where $T$ is wave period (s), $\mu_i$ is the viscosity (in Pa.s), $G$ is the elasticity modulus (in Pa) and $W$ is the water content of fluid mud (%).

The viscoplastic parameters of kaolinite, i.e. $\tau_y$ and $\mu_2$, are evaluated from the laboratory experiments of Tsuruya et al. (1987)

\begin{align}
\tau_y &= 1.494 \times 10^6 W^{-2.452} \\
\mu_2 &= 8.465 \times 10^3 W^{-1.344}
\end{align}

where $\tau_y$ and $\mu_2$ are in Pa and Pa.s, respectively.

2.4. Wave transformation model

2.4.1. Governing equations

The governing equations are time dependent mild slope equations which were introduced by Nishimura et al. (1983)

\begin{align}
\frac{\partial Q_x}{\partial t} + c^2 \frac{\partial}{\partial x}(x) + f_D Q_x &= 0 \\
\frac{\partial Q_y}{\partial t} + c^2 \frac{\partial}{\partial y}(y) + f_D Q_y &= 0
\end{align}
\[
\frac{\partial \xi}{\partial t} + \frac{1}{n} \left\{ \frac{\partial (nQ_x)}{\partial x} + \frac{\partial (nQ_y)}{\partial y} \right\} = 0 \tag{2.10}
\]

where \(Q_x\) and \(Q_y\) are flow rates, \(\xi\) is water surface elevation and \(c\) is wave celerity, derived from wave-mud interaction sub-model. \(f_D\) is the total dissipation coefficient which includes energy loss caused by wave-mud interaction \((f_{DM})\) before breaking line and both energy dissipation of wave breaking and mud in the surf zone \((f_D = f_{DB} + f_{DM})\).

### 2.4.2. Mud dissipation

Water waves propagating over a mud bed are attenuated mainly due to the energy dissipation in mud layers. The attenuation of wave height on a horizontal bed is usually approximated by an exponential function

\[
H(x) = H_0 e^{-k_i(x-x_0)} \tag{2.11}
\]

where \(H_0\) is the incident wave height at \(x=x_0\), and \(k_i\) is the wave attenuation coefficient.

Eq. (2.11) has been supported by both theoretical and laboratory investigations (e.g., Keulegan, 1950; Gade, 1958; Iwasaki and Sato, 1972). The variety of applied rheological models shows that the reliability of the exponential decay approximation is not related to the assumed rheological behavior.

Applying the exponential wave height decay, Eq. (2.11), over a gentle bottom slope (Oveisy et al., 2006)
\[ \varepsilon_{Dm} = \nabla.(c_g E) = -2C_g k_i E \]  

(2.12)

where \( \varepsilon_D \) is the energy dissipation rate, \( E = \rho g H^2 / 8 \) is the wave energy per unit surface area based on linear wave theory, \( \rho \) is the water density and \( C_g \) is the group velocity. Considering

\[ \nabla.(c_g E) = -f_{DM} nE \]  

(2.13)

results in

\[ f_{DM} = 2k_i \cdot c \]  

(2.14)

Eq. (2.14) relates the fluid mud dissipation coefficient to the wave attenuation rate. \( k_i \) is a function of wave characteristics, mud characteristics and water depth that can be calculated from wave-mud interaction models. Here, the local values of \( k_i \) at grid points are calculated by a multi-layered wave mud interaction model which is described in the next section.

2.4.2.1 Wave-mud interaction model

Following Tsuruya et al. (1987), the fluid system above the stationary mud, i.e. the fluid mud layer and water layer, is divided into \( N \) sub-layers, in which the water layer is represented by \( N = 1 \) (Fig. 2.1). This wave-mud interaction model is the extended two-layered model of Dalrymple and Liu (1978). Different mud characteristics can be used in horizontal sub-layers to represent the vertical changes of fluid mud properties. The governing equations, i.e. linearized Navier-Stokes equations and the continuity equation are solved considering the boundary conditions at rigid bottom, water surface and the interfaces. 5N boundary
conditions are required for a fluid model of N layers. The details of wave-mud interaction model can be found in the original work of Tsuruya et al. (1987). The unknown complex wave number, \( k \), in which its imaginary part, \( k_i \), shows the wave attenuation rate, is calculated by this sub-model.

![Fig. 2.1. Definition of multi-layered model (after Tsuruya et al., 1987).](image)

### 2.4.3. Breaking dissipation

The breaking dissipation coefficient is approximately calculated by (Watanabe and Maruyama, 1986)

\[
f_{DB} = \alpha_D \tan \beta \sqrt{\frac{g}{h} \left( \frac{\dot{Q}}{Q_r} - 1 \right)} \tag{2.15}
\]

\[
\dot{Q} = \sqrt{\dot{Q}_x^2 + \dot{Q}_y^2}, \quad Q_r = \gamma' \sqrt{gh^3} \tag{2.16}
\]
where $\alpha_p$ and $\gamma'$ are set to 2.5 and 0.25, respectively and $\tan \beta$ is an average bottom slope. The breaking point is determined using the breaker index, i.e. the ratio of the wave height to water depth as a predefined value.

2.4.4. Discretization

Tanimoto and Kobune (1975) are followed for discretization of Eqs. (2.8) to (2.10). It is assumed that the $x$-axis is directed from offshore towards shore, with the $y$-axis perpendicular to it, i.e. along the trend of the shoreline. The study area is divided into grid cells of equal spacing of $\Delta s$ in both the $x$ and $y$ directions and the time increment is denoted by $\Delta t$. As shown in Fig. 2.2, a staggered mesh scheme is used employing the following notations (Horikawa, 1988)

$$Q_x^m = Q_x \left\{ i \Delta s, (j + \frac{1}{2}) \Delta s, m \Delta t \right\} \quad (2.17)$$

$$Q_y^m = Q_y \left\{ (i + \frac{1}{2}) \Delta s, j \Delta s, m \Delta t \right\} \quad (2.18)$$

$$\xi^m = \xi \left\{ (i + \frac{1}{2}) \Delta s, (j + \frac{1}{2}) \Delta s, (m + \frac{1}{2}) \Delta t \right\} \quad (2.19)$$

The discretization of Eq. (2.8) to Eq. (2.10) yields

$$Q_x^{m+1} = Q_x^m - \frac{\Delta t}{\Delta s} (c_{x,i,j})^2 (\xi^{m+1}_{i,j} - \xi^{m+1}_{i-1,j}) + \Delta t (Q_x^m f_D) \quad (2.20)$$
\[ Q_{yi,j}^{m+1} = Q_{yi,j}^m - \frac{\Delta t}{\Delta s} (c_{yi,j})^2 (\xi_{i,j}^m - \xi_{i,j-1}^m) + \Delta t (Q_{yi,j}^m f_D) \] (2.21)

\[ \xi_{i,j}^{m+1} = \xi_{i,j}^m - \frac{\Delta t}{\Delta s} n_{i,j} (n_{x_{i+1,j}} Q_{x_{i+1,j}}^{m+1} - n_{x_{i,j}} Q_{x_{i,j}}^{m+1} + n_{y_{i,j+1}} Q_{y_{i,j+1}}^{m+1} - n_{y_{i,j}} Q_{y_{i,j}}^{m+1}) \] (2.22)

Since the dissipation terms of Eq. (2.20) and Eq. (2.21) are nonlinear with respect to the flow rate, an iterative method is necessary to reach the final steady state solution for the whole area. The finite difference method can be employed using the wave height values of the preceding wave calculation in dissipation terms.

Fig. 2.2. Staggered mesh scheme.
In order to include the breaking dissipation, the term $\hat{Q}$ in the equations of motion in the $x$-direction and $y$-direction will be computed by Eqs. (23) and (24), respectively.

\[
\hat{Q}_{i,j} = \left[ (\hat{Q}_{xi,j})^2 + \left\{ (\hat{Q}_{yi,j} + \hat{Q}_{yi,j+1} + \hat{Q}_{yi-1,j+1} + \hat{Q}_{yi-1,j}) / 4 \right\}^2 \right]^{1/2} \tag{2.23}
\]

\[
\hat{Q}_{i,j} = \left[ \left\{ (\hat{Q}_{xi,j} + \hat{Q}_{xi+1,j} + \hat{Q}_{xi+1,j-1} + \hat{Q}_{xi,j-1}) / 4 \right\}^2 + (\hat{Q}_{yi,j})^2 \right]^{1/2} \tag{2.24}
\]

The grid length is typically set to be 1/20 to 1/40 of the wavelength around a representative breaking point (Section 2.4.6) in order to obtain an accurate solution. $\Delta t$ must satisfy the following stability condition

\[
\Delta t \leq \frac{\Delta s}{\sqrt{2c_{\text{max}}}} \tag{2.25}
\]

where $c_{\text{max}}$ is the maximum value of the phase velocity in the study area.

### 2.4.5. Boundary conditions

- **Nonreflective Virtual Boundaries**

Side boundaries of the computation domain are assumed as virtual open boundaries in the numerical simulation. Since these virtual boundaries should not affect the solution in the study area, no reflection, i.e. $k_R = 0$, is used for the lateral boundaries and the waves can pass freely through them. The open boundaries parallel to the $x$-axis and parallel to the $y$-axis are represented by Eqs. (26) and (27), respectively.
where

\[ \tau = \Delta s \cdot \cos \alpha_n / c \]  

(2.28)

\[ Q_x'(x_0, y_0) = Q_x^{i-r}(x_0 - \Delta s, y_0) \]  

(2.26)

\[ Q_y'(x_0, y_0) = Q_y^{i-r}(x_0, y_0 - \Delta s) \]  

(2.27)

- Offshore Incident Boundary

Incident wave conditions are necessary as the input parameters on the offshore boundary. On the other hand, the outgoing waves should be freely transmitted through the boundary. Applying the procedure adopted for a nonreflective virtual boundary, the flow rate on an offshore open boundary is set as

Fig. 2.3. Offshore open boundary (after Horikawa, 1988).
\[ Q^I(x_0, y_0) = a_i \cos \alpha_i \sin(kx_0 \cos \alpha_1 + ky_0 \sin \alpha_1 - \sigma) + Q^I_{sr}(x_0 + \Delta s, y_0) \] (2.29)

where

\[ Q^I_{sr}(x_0 + \Delta s, y_0) = Q^I(x_0 + \Delta s, y_0) - a_i \cos \alpha_1 \sin[k(x_0 + \Delta s) \cos \alpha_1 + ky_0 \sin \alpha_1 - \sigma] \] (2.30)

\( a_i \) is the amplitude of incident wave and \( \alpha_i \) represents its direction (Fig. 2.3).

### 2.4.6. Mesh size sensitive analysis

In order to find the proper mesh size for discretization a sensitive analysis is done in this section. A constant slope of 0.5% is considered for 1000 m long cross-shore seabed in these tests. The wave propagation model is run for 4 different mesh sizes and the incident wave height at the start of the slope is 1.0 m at the depth of 5 m. The wave heights at 1.9 m depth are compared with the wave height calculated from exact solution for shoaling of linear wave. For the mesh size equal to 1/5, 1/10, 1/20, 1/40 of the wave length at breaking point the results were 1.457 m, 1.26 m, 1.199 m, and 1.194 m respectively. 1.193 m is the exact solution for the wave height in this point. Therefore the relative error for the mesh size are equal to 1/5, 1/10, 1/20, 1/40 are 22.1%, 5.6%, 0.5%, 0.08% respectively.

The sensitive analysis suggests the mesh size between 1/20 to 1/40 of the wave length at the breaking point can be used with acceptable accuracy.
2.5. Procedure of calculation

Fig. 2.4 shows the flow chart of the simulation procedure. The input data consists of the necessary information including offshore wave height and wave period, bottom topography and fluid mud characteristics.

Assuming a fixed bed where no fluid mud exists, wave transformation model is first applied on the study area. No wave energy dissipation is included in this step. The values of wave attenuation rates and wave phase velocities at all grid points are then calculated by the wave-mud interaction model using the local values of water depth, wave characteristics and fluid mud properties of the nodes. The corresponding wave energy dissipation rate is used to calculate the wave height at the next grid point. Inside the surf zone, the energy dissipation of fluid mud layer is combined with the one due to wave breaking.

2.6. Laboratory investigations

The 2D horizontal laboratory experiments were carried out in a wave basin, measuring 9.3 m in length by 9.0 m wide at the hydraulic laboratory, Yokohama National University. Fig. 2.5 shows the sketch of experimental setup.

Commercial kaolinite was used as fluid mud bed since it has similar rheological behavior with natural mud (Otsubo and Muraoka, 1988), and also due to its reproducible properties as
well as easy handling. The distribution of particle sizes of the kaolinite passed through the points $D_{86.3} = 3 \, \mu m$ and $D_{99.98} = 10 \, \mu m$ where $D^*$ is the diameter at which $\%$ of the mud weight is finer. Table 2.1 lists the chemical composition of the used kaolinite. The test mud sample was prepared by careful mixing of kaolinite with tap water and it was put in a section in the wave basin that measured 1.0 m wide by 1.0 m length and 12 cm in height (Fig. 2.5).

Monochromatic waves of different combinations of wave heights, wave periods and incident wave directions were generated by the wave paddles at the edge of the basin. Table 2.2 shows the test conditions of the four conducted cases. The waves on the mud section at Point 1 and Point 2 which were located at 30 cm and 70 cm from the edge of the mud box,
respectively, were recorded by using electric capacitance-type wave gauges. Table 2.3 presents wave heights and wave periods at the measured points.

### Table 2.1. The chemical composition of kaolinite.

<table>
<thead>
<tr>
<th>Composition</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO₂</td>
<td>60.15</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>30.44</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>0.07</td>
</tr>
</tbody>
</table>

![Fig. 2.5. Experimental setup and modeling domain.](image)
Table 2.2 Incident wave and fluid mud characteristics

<table>
<thead>
<tr>
<th>Case</th>
<th>Incident Wave Height (cm)</th>
<th>Incident Wave Period (s)</th>
<th>Mud Water Content (%)</th>
<th>Water Depth at Point 1 (cm)</th>
<th>Wave Angle (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.61</td>
<td>1.00</td>
<td>160</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>3.80</td>
<td>1.00</td>
<td>130</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3.78</td>
<td>1.00</td>
<td>130</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4.47</td>
<td>1.00</td>
<td>130</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.3 Wave heights and wave periods at measuring points

<table>
<thead>
<tr>
<th>Case</th>
<th>Wave Height at Point 1 (cm)</th>
<th>Wave Period at Point 1 (s)</th>
<th>Wave Height at Point 2 (cm)</th>
<th>Wave Period at Point 1 (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.88</td>
<td>1.01</td>
<td>1.23</td>
<td>0.94</td>
</tr>
<tr>
<td>2</td>
<td>2.59</td>
<td>1.01</td>
<td>1.53</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>2.66</td>
<td>0.99</td>
<td>1.78</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>3.50</td>
<td>1.06</td>
<td>2.00</td>
<td>1.06</td>
</tr>
</tbody>
</table>

2.7. Model performance

The applicability of the proposed wave transformation model is evaluated in this section. The suitability of the model on a constant rigid slope, where the combinations of shoaling, refraction and wave breaking can be theoretically calculated, is first examined. The performance of the numerical model for the simulation of wave transformation on a fluid mud layer in laboratory conditions is then examined. The numerical model is also applied to a real field condition.

2.7.1. Wave transformation on a fixed bed

As an example, a condition is taken in which regular waves with a height of 6.6 cm, period of 1.0 s, and incident angle of 10° are incident to a beach with a constant slope of 1/30. The
water depth at the beginning of computational area is set at 0.187 cm. Fig. 2.6 shows the assumed dimensions for the wave transformation example.

Applying the numerical model, Fig. 2.7 shows the distribution of the wave height and wave direction. The comparisons of the simulated wave heights and wave angles with the ones calculated by Snell’s law are presented in Fig. 2.8. The agreement indicates the capability of the employed numerical model to simulate the wave transformation on fixed beds.

Fig. 2.6. Computational region for wave refraction example.
Fig. 2.7. Distribution of wave height and direction of refracted waves. (a) wave height contours (units: mm), (b) wave direction.
Fig. 2.8. Cross-shore distribution of wave height and wave angle at the middle section.
2.7.2. Comparison with the laboratory experiments

Fig. 2.9 shows the wave height contours for four laboratory cases of wave basin experiments assuming the breaker index of 0.68. The major processes that affect a wave as it propagates from deep into shallow water, i.e. shoaling, refraction and wave breaking can be distinguished in the figure. A remarkable reduction of wave height is seen over the mud section. The combined dissipation effects of wave breaking and mud bed may also be observed comparing the changes of wave height contours over mud section layer and outside it.

Fig. 2.10 shows the comparison between the simulated wave heights and the laboratory results of all the conducted cases at two measuring points. The agreement is acceptable in spite of the observed scatter. The differences between the simulated results and measured values may be mainly attributed to the experimental limitations such as the small size of mud box and the effects of the side walls.
Fig. 2.9. Wave height transformation on laboratory wave basin (mm).
Fig. 2.9. (Continued)
2.7.3. Comparison with field data of Kumamoto Port

The model is also applied for the entrance of Kumamoto Port in Japan. Fig. 2.11 shows the bathymetry of the modeling domain. The wave attenuation factor between station A and Station B is reported to be 0.56 for an incident significant wave of 1.25 m high and period of 4 s corresponding to 10 cm fluid mud layer with a water content ratio of 200% (Tsuruya et al., 1990).
Taking the significant wave height as a proper monochromatic representative wave for the irregular wave transformation on fluid mud layer (Soltanpour et al., 2007), Fig. 2.12 represents the simulated wave height contours. The yield stress and plastic viscosity of local fluid mud were adopted from the laboratory experiments of Tsuruya et al. (1987). The calculated wave height attenuation factor, i.e. the ratio of the simulated wave height at station A to the corresponding value of station B, is 0.58 which is in a good agreement with the reported value of 0.56.

Fig. 2.11. Bathymetry and the wave measurement stations (m).
2.8. Summary and Conclusion

Wave attenuation on a general fluid mud bathymetry was successfully simulated as a part of dissipation term in mild slope equations using a multi-layer wave-fluid mud interaction model for the estimation of wave attenuation rate. The wave transformation numerical model includes combined wave refraction, diffraction, reflection, breaking as well as wave attenuation due to wave mud interaction. Model performance was tested with the performed laboratory experiments showing a reasonable agreement. The application of the numerical model on a field data is promising. However, more laboratory and field data is necessary for better evaluation of the accuracy of the numerical model.

References

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Soltanpour M., 2004. A survey on fishery ports in Iran. 2nd Yokohama Symposium on Coastal Processes in Developing Countries, 72-76.

Chapter 3

3. A Two-Dimensional Horizontal Wave Propagation and Mud Mass Transport Model


The present study offers a two-dimensional horizontal wave propagation and morphodynamic model for muddy coasts. The model can be applied on a general three-dimensional bathymetry of a soft muddy coast to calculate wave damping, fluid mud transport and resulting bathymetry change under wave action. The wave propagation model is based on time-dependent mild slope equations including wave energy dissipation due to wave-mud interaction of bottom mud layers as well as the combined effects of the wave refraction, diffraction and breaking. The constitutive equations of the visco-elastic-plastic model are adopted for the rheological behavior of fluid mud. The mass transport velocity within the fluid mud layer is calculated combining the Stokes’ drift, the mean Eulerian velocity and the gravity-driven mud flow. The results of the numerical model are compared against a series of conducted wave-basin experiments, wave-flume experiments and field observations. Comparisons between the computed
results with the both field and laboratory data reveal the capability of the proposed model to predict the wave transformation and mud mass transport.

3.1. Introduction

Coastal areas covered with mud can be observed in many real field conditions such as Kumamoto Port in Japan (Tsuruya et al., 1990) and on the Amazon continental shelf (Cacchione et al., 1994). Mud transport in harbors and coastal zones is often undesirable as the resulting bathymetric changes reduce navigability and make costly dredging operations necessary to keep the required navigable depth. Thus, a numerical model is an important tool for the managers of harbors and coastal zones to prevent the undesired accumulation of mud and also to predict long term serviceability of the planned projects.

When a surface water wave travels over a soft muddy bed, an inter-surface wave between the water layer and the mud layer is generated resulting in significant dissipation of wave energy. The inter-surface wave also results in mass transport in the mud layer. Suspended mud transport in the water layer and mud mass transport under the influences of waves and gravity, are the main mechanisms of mud transport in estuaries and coastal regions. Since sediment concentration in the mud layer is often in the order of a few hundred kg/m$^3$ compared to typical sediment concentrations in the water column which are in the order of a few tenths of kg/m$^3$ (Shibayama et al., 1990; Kessel et al., 1996), it is expected that the movement of the mud layer is the predominant transport mechanism under stormy conditions. However, long–term effects of suspended mud transport should not be overlooked given the associated environmental impacts.
Wave attenuation and mud mass transport on cohesive beds have long been topics of interest (e.g., Ross and Mehta, 1990; Shibayama and An, 1993). Kessel and Kranenburg (1998) examined mud mass transport on a sloping bed. Gravity driven flow and pressure gradients were included in their one-dimensional simulation. A constant wave height was used in their treatment of mud transport ignoring the effects of shoaling and wave attenuation. They could not offer a quantitative comparison between the flow model and measured values. A general formula for an equilibrium mud profile shape was offered by Lee and Mehta (1997). They also tried to relate the rate of mud transport to wave energy dissipation in order to calculate temporal profile change. Soltanpour et al. (2003) proposed a cross-shore numerical model for wave propagation and mud mass transport considering shoaling and wave dissipation due to wave breaking and fluid mud. However, the 2D horizontal elements such as refraction and diffraction were absent in their model.

Although several two-dimensional horizontal wave propagation models exist, they cannot usually be applied on soft muddy beds due the high rate of energy dissipation resulting from wave-mud interaction. Moreover, very few morphodynamic models include the mud mass transport under wave action. A two-dimensional horizontal wave propagation and morphodynamic model is presented here. The wave propagation model includes the various effects of shoaling, refraction, diffraction, reflection, wave breaking and wave damping over a soft mud bed. The wave-induced and the gravity-driven mud mass transport are introduced to the mud morphodynamic model to calculate the resulting bathymetric change. The proposed model consists of five sub-models namely, a two-
dimensional horizontal wave propagation model, a rheological model for wave-mud interaction, a two-dimensional vertical wave-mud interaction model, a gravity-driven mud mass transport model and a morphodynamic model. The wave propagation model is used for the prediction of the wave height in the simulation domain. The apparent viscosity of the mud under wave action is calculated by the rheological model of the mud. The multi-layered wave-mud interaction model is then used to calculate the wave attenuation rate due to the wave-mud interaction. The wave-induced mud mass transport velocities in the mud sub-layers are also calculated in the wave-mud interaction model. The wave attenuation rate in each computational node is utilized in the wave propagation model to simulate wave propagation on fluid mud bed. The wave-driven and the gravity-driven mud mass transport velocities are finally introduced to the mass conservation equation to calculate the new bathymetry in morphodynamic model.

3.2. Wave propagation model

The governing equations for monochromatic wave propagation are the time dependent mild slope equations (Nishimura et al., 1983; Horikawa, 1988):

\[
\frac{\partial Q_x}{\partial t} + c^2 \frac{\partial}{\partial x} (\xi) + f_D Q_x = 0, \tag{3.1}
\]

\[
\frac{\partial Q_y}{\partial t} + c^2 \frac{\partial}{\partial y} (\xi) + f_D Q_y = 0, \tag{3.2}
\]

\[
\frac{\partial \xi}{\partial t} + \frac{1}{n} \left\{ \frac{\partial (nQ_x)}{\partial x} + \frac{\partial (nQ_y)}{\partial y} \right\} = 0, \tag{3.3}
\]

43
where \( Q_s \) and \( Q_y \) are flow rates, \( \xi \) is the water surface elevation, \( c \) is the wave celerity and \( n \) is the wave group velocity over wave celerity. The total wave energy dissipation coefficient \( f_D \) includes energy loss caused by wave-mud interaction \( (f_{DM}) \) before the breaking line and both energy loss due to wave breaking and wave-mud interaction within the surf zone \( (f_D = f_{DB} + f_{DM}) \). The governing equations are discretized and a staggered mesh scheme is employed. Because Eqs. (3.1) to (3.3) cannot be discretized into an explicit form due to the nonlinear dissipation term with respect to the flow rate, an iterative scheme is used to reach the steady state solution. Computation is made by finite difference method with discretization as presented in section 2.3

3.2.1. Dissipation of wave energy due to wave-mud interaction

Water waves propagating over a mud bed are attenuated mainly due to the energy dissipation in fluid mud layers. The attenuation of wave height on a horizontal bed is usually approximated by an exponential function

\[
H(x) = H_0 e^{-k_i(x-x_0)},
\]

where \( H_0 \) is the incident wave height at \( x=x_0 \), and \( k_i \) is the wave attenuation rate. The wave attenuation rate is a function of wave characteristics, mud characteristics and water depth; local values of \( k_i \) at grid points are calculated by the multi-layered wave-mud interaction model which is described in section 4.
Eq. (3.4) has been supported by both theoretical and laboratory investigations (e.g., Keulegan, 1950; Gade, 1958; Iwasaki and Sato, 1972). Applying the exponential wave height decay over a gentle bottom slope, the wave energy dissipation rate is calculated as (Oveisy et al., 2006)

\[ \varepsilon_{Dm} = \nabla (c_g E) = -2C_g k_i E, \]  

where \( C_g \) is the group velocity, \( E = \rho_w g H^2 / 8 \) is the wave energy per unit surface area based on linear wave theory, \( \rho_w \) is the water density. Considering

\[ \nabla (c_g E) = -f_{DM} n E, \]  

the wave energy dissipation coefficient due to fluid mud can be related to the wave attenuation rate \( (k_i) \) as follow:

\[ f_{DM} = 2k_i c. \]  

3.2.2. Dissipation of wave energy due to wave breaking

The wave energy dissipation coefficient due to wave breaking is calculated by (Watanabe and Maruyama, 1986)

\[ f_{DB} = \alpha_g \tan \beta \sqrt{\frac{g}{h} \left( \frac{Q}{Q_r} - 1 \right)}, \]
\[ \dot{Q} = Q^2 + Q^3, \quad Q = \gamma' \sqrt{gh}, \quad (3.9) \]

where \( \alpha_n \) and \( \gamma' \) are set to 2.5 and 0.25 respectively and \( \tan \beta \) represents average bottom slope at the breaking line. The breaking line is determined using the depth-limited breaker index (ratio of the wave height to water depth) as a user-defined value.

### 3.2.3. Discretization of mild slope equations

Methods of Tanimoto and Kobune (1975) are followed for the discretization of Eqs. (3.1) to (3.3). It is assumed that the \( x \)-axis is directed onshore, with the \( y \)-axis perpendicular to \( x \)-axis, representing the shoreline trend. The study area is divided into grid cells, each having a spacing of \( \Delta s \) in the both \( x \) and \( y \) directions; the time step is denoted by \( \Delta t \). As shown in Fig. 3.1, a staggered mesh scheme is used employing the following notations (Horikawa, 1988)

\[ Q_{xi,j}^m = Q_x \left\{ i\Delta s, (j + \frac{1}{2})\Delta s, m\Delta t \right\} \quad (3.10) \]

\[ Q_{yi,j}^m = Q_y \left\{ (i + \frac{1}{2})\Delta s, j\Delta s, m\Delta t \right\} \quad (3.11) \]

\[ \xi_{xi,j}^m = \xi \left\{ (i + \frac{1}{2})\Delta s, (j + \frac{1}{2})\Delta s, (m + \frac{1}{2})\Delta t \right\} \quad (3.12) \]

The discretization of Eq. (3.1) to (3.3) yields
\[
Q_{x,i,j}^{m+1} = Q_{x,i,j}^{m} - \frac{\Delta t}{\Delta s} \left( c_{x,i,j}^{m} \right)^2 \left( \xi_{i,j}^{m} - \xi_{i-1,j}^{m} \right) + \Delta t(Q_{x,i,j}^{m}) f_D ,
\]
(3.13)

\[
Q_{y,i,j}^{m+1} = Q_{y,i,j}^{m} - \frac{\Delta t}{\Delta s} \left( c_{y,i,j}^{m} \right)^2 \left( \xi_{i,j}^{m} - \xi_{i,j-1}^{m} \right) + \Delta t(Q_{y,i,j}^{m}) f_D ,
\]
(3.14)

\[
\xi_{i,j}^{m+1} = \xi_{i,j}^{m} - \frac{\Delta t}{\Delta s} \left( \frac{1}{n_{i,j}} \left( n_{x,i,j}^{m+1} Q_{x,i,j}^{m+1} - n_{x,i,j}^{m} Q_{x,i,j}^{m} + n_{y,i,j+1}^{m+1} Q_{y,i,j+1}^{m+1} - n_{y,i,j}^{m+1} Q_{y,i,j}^{m+1} \right) \right) ,
\]
(3.15)

where \(i\) and \(j\) are the grid numbers in the \(x\) and \(y\) directions respectively and \(m\) is the time step index. \(c_{x,i,j}, n_{x,i,j}\) and \(c_{y,i,j}, n_{y,i,j}\) are the values of \(c\) and \(n\) at the points where \(Q_{x,i,j}, Q_{y,i,j}\) are respectively defined, and \(n_{i,j}\) is the value of \(n\) at the location of \(\xi_{i,j}\).

Since the dissipation terms of Eq. (3.13) and Eq. (3.14) are nonlinear with respect to the flow rate, an iterative method is necessary to reach the final steady state solution for the computational domain. Flow rate values of the preceding wave calculation are used in the definition of the dissipation terms in the finite difference approach adopted for this model. In order to include dissipation rates due to wave breaking and wave-mud interaction, the term \(\hat{Q}\) in the equations of motion in the \(x\)-direction and the \(y\)-direction is computed by Eqs. (3.16) and (3.17), respectively

\[
\hat{Q}_{x,i,j} = [(\hat{Q}_{x,i,j})^2 + \{(\hat{Q}_{y,i,j} + \hat{Q}_{y,i,j+1} + \hat{Q}_{y,i-1,j+1} + \hat{Q}_{y,i-1,j}) / 4\}^2]^{1/2}
\]
(3.16)

\[
\hat{Q}_{y,i,j} = [(\hat{Q}_{x,i,j} + \hat{Q}_{x,i+1,j} + \hat{Q}_{x,i+1,j-1} + \hat{Q}_{x,i,j-1}) / 4]^2 + (\hat{Q}_{y,i,j})^2]^{1/2}
\]
(3.17)
The grid length is typically set to be 1/20 to 1/40 of the wavelength associated with the breaking wave in order to obtain an accurate solution. The time step must satisfy the following stability condition

$$
\Delta t \leq \frac{\Delta s}{\sqrt{2c_{\text{max}}}}
$$

(3.18)

where $c_{\text{max}}$ is the maximum value of the phase velocity in the study area.
3.2.4. Boundary conditions for mild slope equations

The boundary conditions for the wave propagation model are as follows:

- Reflective boundary

The following equations are used for reflective boundaries parallel to the $y$-axis and the $x$-axis respectively

\[ Q'_y(x_n, y_o) = A Q_{y,-}^{y,-} (x_o - \Delta s, y_o), \]  
\[ Q'_x(x_n, y_o) = A Q_{x,-}^{x,-} (x_o, y_o - \Delta s), \]

where

\[ A = \frac{1 - k_R}{1 + k_R^2 - 2k_R \cos(2k\Delta s \cos \alpha_n)} \sqrt{2}, \]
\[ \tan \sigma \tau = \frac{1 + k_R}{1 - k_R} \tan(k \Delta s \cos \alpha_n) \]
\[ \tau = \Delta s \cos \alpha_n / c, \]

where $k_R$ is the reflection coefficient, and $\alpha_n$ is the direction angle of the wave component, as shown in Fig. 3.2a.

- Nonreflective virtual boundaries
Lateral boundaries of the computation domain are assumed to be nonreflective open boundaries as they are assumed to have no effect on the solution in the simulation domain. Considering \( k_x = 0 \), the open boundaries parallel to the \( y \)-axis and the \( x \)-axis are represented by Eqs. (3.24) and (3.25), respectively

\[
Q'_i(x_0, y_0) = Q'_{i-\tau}(x_0 - \Delta s, y_0) \tag{3.24}
\]

\[
Q'_y(x_0, y_0) = Q'_{y-\tau}(x_0, y_0 - \Delta s) \tag{3.25}
\]

- Offshore Incident Boundary

The incident wave is used at the offshore boundary to drive the simulation, while the outgoing waves (i.e. reflected waves) should be freely transmitted through the boundary. Applying the procedure adopted for a nonreflective virtual boundary, the flow rate on the offshore open boundary is set as

\[
Q'_i(x_0, y_0) = a_i c \cos \alpha_i \sin(kx_0 \cos \alpha_i + ky_0 \sin \alpha_i - \sigma) + Q'_{i-\tau}(x_0 + \Delta s, y_0), \tag{3.26}
\]

where

\[
Q'_{i-\tau}(x_0 + \Delta s, y_0) = Q'_i(x_0 + \Delta s, y_0) - a_i c \cos \alpha_i \sin[k(x_0 + \Delta s) \cos \alpha_i + ky_0 \sin \alpha_i - \sigma], \tag{3.27}
\]

\( a_i \) is the amplitude of the incident wave and \( \alpha_i \) represents its direction (Fig. 3.2b).
3.3. Rheological model of mud

A proper rheological model of mud should be adopted in order to investigate wave-mud interaction. Mud in general can range from being a highly rigid and weakly viscous material to one that can be approximated as a purely viscous fluid, depending on the properties of the constituent sediment, the ambient fluid and the wave characteristics. Due to the complexity of mud behavior, different constitutive equations have been assumed for the response of muddy beds. Gade (1958) proposed a viscous model for the fluid mud, where overlying water considered inviscid, with the assumption of the shallow water. Ignoring the wave energy dissipation due to mud layer, Mallard and Dalrymple (1977) and Dawson (1978) used a linear elastic model to study the phase speed of the surface waves propagating over a deformable bed. Following the viscous model of Gade (1958), Dalrymple and Liu (1978) assumed a two layer viscous fluid system. Their model was valid for any depth upper layer and both deep and shallow lower fluid layers. Yamamoto and Takahashi (1985) proposed a poro-elastic model that was able to consider...
wave dissipation through the application of a complex shear modulus. However, the assumption of porous media is not suitable for low permeable mud beds. Mehta (1989) proposed a viscoelastic model to study the response of mud to water waves.

Some observations show that mud begins to deform at a distinct stress point. This behavior is best represented using yield stress of the mud and it cannot be explained by viscous or viscoelastic models. Tsuruya et al. (1986), Otsubo and Muraoka (1988) found that the mud also exhibits plastic behavior and that a viscoplastic model is appropriate to describe the behavior of mud under these conditions.

In order to overcome the limitations of the viscoelastic model, Shibayama et al. (1989) included the effect of yield stress in the numerical viscoelastic model. They introduced the visco-elastic-plastic terminology, combining viscoelastic and viscoplastic models. The state of mud was identified by comparing shear stress and yield stress. When the shear stress is less than the yield stress, the mud is in a viscoelastic state and it is in a viscoplastic state when the shear stress is greater than the yield stress. Thus the visco-elastic-plastic model can be considered as the viscoplastic model where the elastic part has been replaced by the viscoelastic state. The visco-elastic-plastic model seems to be a better choice as the behavior of mobile mud at both low and high shear stresses is taken into account. The constitutive equations are expressed as

$$\sigma_{ij} = 2\mu_e \dot{e}_{ij}$$  \hspace{1cm} (3.28)
\[ \mu_e = \mu_i + \frac{iG}{\sigma} \quad (\frac{1}{2} \sigma_{ij} \sigma_{ij} \leq \tau^2_y) \quad (3.29a) \]

\[ \mu_e = \mu_2 + \frac{\tau_y}{\sqrt{4|\Pi_e|}} \quad (\frac{1}{2} \sigma_{ij} \sigma_{ij} > \tau^2_y) \quad (3.29b) \]

where \( i \) and \( j \) take the values 1 and 2 which correspond to \( x \) and \( z \) axis, respectively, \( \sigma \) is the angular frequency of wave, \( \mu_e \) is the apparent viscosity, \( \sigma_{ij} \) is the deviator part of stress tensor, \( \dot{e}_{ij} \) is the deviator part of strain rate tensor, \( G \) is the elastic modulus, \( \mu_i \) is the viscosity of mud in the viscoelastic state, \( \mu_2 \) is the viscosity of mud in the viscoplastic state and \( \tau_y \) is the yield stress. \( |\Pi_e| \) is the objective of the deformation-rate tensor and expressed as

\[ |\Pi_e| = \frac{1}{2} (\frac{\partial u}{\partial x})^2 + \frac{1}{2} (\frac{\partial w}{\partial z})^2 + \frac{1}{4} (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x})^2 \quad (3.30) \]

where \( u \) and \( w \) are velocity components in \( x \) and \( z \) direction, respectively.

The rheological viscoelastic parameters, i.e. shear modulus and viscosity, are calculated from the results of the laboratory experiments of Shibayama and An (1993) on commercial kaolinite

\[ \mu_1 = 10^{(3.353-9.56\times10^{-3}W)} \times T \quad (3.31) \]

\[ \log G = 3.761 - 1.05 \times 10^{-2} W \]
\[ + (0.147 - 3.38 \times 10^{-3} W) \log(T - 0.522 - 1.23 \times 10^{-3} W) \]  

(3.32)

where \( T \) is wave period (s), \( \mu_1 \) is the viscosity (in Pa.s), \( G \) is the elasticity modulus (in Pa) and \( W \) is the water content of fluid mud (%).

The viscoplastic parameters of kaolinite, i.e. \( \tau_y \) and \( \mu_2 \), are evaluated from the laboratory experiments of Tsuruya et al. (1987)

\[
\tau_y = 1.494 \times 10^6 W^{-2.452}
\]

(3.33)

\[
\mu_2 = 8.465 \times 10^3 W^{-1.344}
\]

(3.34)

where \( \tau_y \) and \( \mu_2 \) are in Pa and Pa.s, respectively.

3.4. Wave-mud interaction model

Following the approach of Tsuruya et al. (1987), Fig. 3.3a shows the N-layered fluid system. The multi-layered bed model is based on the two-layered model of Dalrymple and Liu (1978), which is applicable when the wave amplitude is small enough that the equations of motion can be linearized. As it was also remarked by Ma and Mehta (1990), the linear theory would work well under conditions wherein large bed deformations are not involved, as in a mild wave environment. Different mud characteristics can be used in the sub-layers to represent the vertical distribution of fluid mud properties. Linearized Navier-Stokes equations and the continuity equation are the governing equations for an incompressible laminar fluid under wave action (MacPherson, 1980):
Fig. 3.3. a) Definition sketch of multi-layered model for wave–mud interaction (after Tsuruya et al., 1987), b) Definition sketch of multi-layered model on a sloping bed.

\begin{align}
\frac{\partial \vec{u}_j}{\partial t} &= -\frac{1}{\rho_j} \frac{\partial \vec{p}_j}{\partial x} + \nu_{e,j} \left( \frac{\partial^2 \vec{u}_j}{\partial x^2} + \frac{\partial^2 \vec{u}_j}{\partial z^2} \right), \\
\frac{\partial \vec{w}_j}{\partial t} &= -\frac{1}{\rho_j} \frac{\partial \vec{p}_j}{\partial z} + \nu_{e,j} \left( \frac{\partial^2 \vec{w}_j}{\partial x^2} + \frac{\partial^2 \vec{w}_j}{\partial z^2} \right),
\end{align}

(3.35) (3.36)
\[
\frac{\partial \tilde{u}_j}{\partial x} + \frac{\partial \tilde{w}_j}{\partial z} = 0,
\] (3.37)

where \(x\) and \(z\) are the horizontal and vertical coordinates respectively, the subscript \(j\) indicates the layer index, and the parameters \(t\), \(\rho\), \(v_e\), \(\tilde{u}\), \(\tilde{w}\) and \(\tilde{p}\) represent the time, density, apparent kinematic viscosity of mud, horizontal and vertical component of orbital velocity.

The separable, periodic solutions for \(\tilde{u}_j\), \(\tilde{w}_j\) and \(\tilde{p}_j\) are assumed as

\[
\tilde{u}_j(x, z; t) = u_j(z) \exp[i(kx - \omega t)]
\] (3.38)

\[
\tilde{w}_j(x, z; t) = w_j(z) \exp[i(kx - \omega t)]
\] (3.39)

\[
\tilde{p}_j(x, z; t) = p_j(z) \exp[i(kx - \omega t)]
\] (3.40)

where \(\omega\) is the wave angular frequency and \(k\) is the unknown complex wave number,

\[
k = k_r + ik_i,
\] (3.41)

The real part of the wave number, \(k_r\), provides the wave length \((L = 2\pi/k_r)\) and its imaginary part, \(k_i\), represents the wave attenuation rate. Displacements of water surface and interfaces, \(\eta_j\), are represented by

\[
\eta_j = a_j \exp[i(kx - \omega t)],
\] (3.42)
where $a_j$ is a complex unknown value to define the amplitude of the displacement of the $j^{th}$ layer. The water surface is expressed as $\eta_1$ and $\eta_j$, $j=2,\ldots,n$ are unknown complex values specifying mud sublayers displacements based on their amplitudes and phases. Substituting the real and imaginary parts of the wave number into Eq. (3.42), the expression of water surface and interfacial displacements can be obtained as

$$\eta_j = a_j \exp(-kx) \exp[i(kx - \omega t)], \quad (3.43)$$

Substituting Eqs. (3.38) and (3.39) into the continuity Eq. (3.37) results in

$$u_j = \frac{iw_j'}{k}, \quad (3.44)$$

where the prime represents the differential with respect to $z$. Introduction of Eq. (3.44) into Eq. (3.35) yields an expression for $p_j$,

$$p_j = \frac{\rho_j \nu_{e,j}}{k^2} (w_j''' - w_j' \lambda_j^2), \quad (3.45)$$

where

$$\lambda_j^2 = k^2 - i \sigma \nu_{e,j}^{-1} \quad (3.46)$$

Substituting $p_j$ into the vertical momentum equation, Eq. (3.36), yields the fourth order differential equation for $w_j$. 

57
\[ w_j ^ {''''} - (k^2 + \lambda_j^2) w_j ^ {''} + k^2 \lambda_j^2 w_j = 0. \] (3.47)

The solutions can be obtained as

\[ w_j(z) = A_j \sinh k \left( \sum_{n=1}^{j} d_n + z \right) + B_j \cosh k \left( \sum_{n=1}^{j} d_n + z \right) + 
\]

\[ C_j \exp[\lambda_j \left( \sum_{n=1}^{j-1} d_n + z \right)] + D_j \exp[-\lambda_j \left( \sum_{n=1}^{j} d_n + z \right)] \] (3.48)

where \( d_n \) is the thickness of \( n^{th} \) layer. The complex constants \( A_j, B_j, C_j, D_j \), and the unknown variables \( k, a_j \) are determined from the boundary conditions. 5N boundary conditions are required for a viscous fluid model of \( N \) layers. The unknown constants and variables are determined from the boundary conditions at the water surface, the interfaces and the rigid bottom as follows:

- At the water surface (\( z = \eta_1 \))

The kinematic boundary condition, requiring the surface particles to follow the surface, and the imposition of zero normal and tangential stresses can be written as

\[ \frac{\partial \eta_1}{\partial t} = \dot{\omega}_1 \] (3.49)

\[ \hat{p}_1 - 2 \rho_1 \nu_{e_1} \frac{\partial \dot{\omega}_1}{\partial z} = 0 \] (3.50)
\[ \rho_i \nu_{e,i} \left( \frac{\partial \hat{u}_i}{\partial z} + \frac{\partial \hat{w}_i}{\partial x} \right) = 0 \]  

(3.51)

or after Taylor’s expansion:

\[ A_i \sinh kd_i + B_i \cosh kd_i + C_i + D_i \exp(-\lambda_i d_i) = -i \omega a \]

(3.52)

\[ M_1 (A_i \cosh kd_i + B_i \sinh kd_i) - 2 \rho_i \nu_{e,i} \lambda_i [C_i - D_i \exp(-\lambda_i d_i)] = \rho_i g a_i \]

(3.53)

\[ 2A_i k^2 \sinh kd_i + 2B_i k^2 \cosh kd_i + (\lambda_i^2 + k^2) [C_i + D_i \exp(-\lambda_i d_i)] = 0, \]

(3.54)

where

\[ M_1 = \frac{i \rho_i \omega}{k} - 2 \rho_i \nu_{e,i} k, \]

(3.55)

\[ \lambda_i^2 = k^2 - i \omega \nu_{e,i}^{-1}, \]

(3.56)

and \( g \) is the gravitational acceleration.

- At the interfaces \( z_j = -\sum_{n=1}^{j-1} d_n \) where \( j = 1, \ldots, N - 1 \)

The kinematic conditions which, due to the assumed linearity of the problem, are applied at \( z_j = -\sum_{n=1}^{j} d_n \) can be written as
\[
\frac{\partial \eta_{j+1}}{\partial t} = \hat{w}_j, \quad (3.57)
\]

or

\[
B_j + D_j + C_j \exp(-\lambda_j d_j) = -i \omega a_{j+1}, \quad (3.58)
\]

The continuity of horizontal and vertical velocities are

\[
\hat{u}_j = \hat{u}_{j+1}, \quad (3.59)
\]

\[
\hat{w}_j = \hat{w}_{j+1}, \quad (3.60)
\]

or

\[
A_{j+1} \sinh k d_{j+1} + B_{j+1} \cosh k d_{j+1} + C_{j+1} + D_{j+1} \exp(-\lambda_{j+1} d_{j+1}) = B_j + D_j + C_j \exp(-\lambda_j d_j) \quad (3.61)
\]

\[
kA_{j+1} \cosh k d_{j+1} + kB_{j+1} \sinh k d_{j+1} + C_{j+1} \lambda_{j+1} + D_{j+1} \lambda_{j+1} \exp(-\lambda_{j+1} d_{j+1}) = kA_j + \lambda_j C_j \exp(-\lambda_j d_j) - \lambda_j D_j, \quad (3.62)
\]

The normal and tangential stresses are also continuous across the interfaces. Considering the Taylor series expansion about \( z_j = -\sum_{n=1}^{J} d_n \), they can be written as
\[ \dot{p}_j - 2\rho_j v_{e,j} \frac{\partial \hat{w}_j}{\partial z} - \rho_j g \eta_{j+1} = \dot{p}_{j+1} - 2\rho_{j+1} v_{e,j+1} \frac{\partial \hat{w}_{j+1}}{\partial z} - \rho_{j+1} g \eta_{j+1}, \quad (3.63) \]

\[ \rho_j v_{e,j} \left( \frac{\partial \hat{u}_j}{\partial z} + \frac{\partial \hat{w}_j}{\partial x} \right) = \rho_{j+1} v_{e,j+1} \left( \frac{\partial \hat{u}_{j+1}}{\partial z} + \frac{\partial \hat{w}_{j+1}}{\partial x} \right), \quad (3.64) \]

or

\[ M_j A_j - 2\rho_j v_{e,j} \lambda_j [C_j \exp(-\lambda_j d_j) - D_j] = \]

\[ M_{j+1} (A_{j+1} \cosh k d_{j+1} + B_{j+1} \sinh k d_{j+1}) \]

\[ - 2\rho_{j+1} v_{e,j+1} \lambda_{j+1} [C_{j+1} - D_{j+1} \exp(-\lambda_{j+1} d_{j+1})] - (\rho_{j+1} - \rho_j) g a_{j+1} \quad (3.65) \]

\[ \rho_j v_{e,j} [2k^2 B_j + (\lambda_j^2 + k^2)(C_j \exp(-\lambda_j d_j) + D_j)] \]

\[ = \rho_{j+1} v_{e,j+1} \{2k^2 (A_{j+1} \sinh k d_{j+1} + B_{j+1} \cosh k d_{j+1}) \]

\[ + (\lambda_{j+1}^2 + k^2)[C_{j+1} + D_{j+1} \exp(-\lambda_{j+1} d_{j+1})]\}, \quad (3.66) \]

where

\[ M_j = \frac{i\rho_j \omega}{k} - 2\rho_j v_{e,j} k, \quad (3.67) \]
At the rigid bottom \( z_j = -\sum_{n=1}^{N} d_n \)

The velocities in both the horizontal and vertical directions should be zero at the fixed bottom, i.e.

\[
\hat{u}_N = 0 \quad (3.68)
\]

\[
\hat{w}_N = 0 \quad (3.69)
\]

or

\[
kA_N - \lambda_N D_N + \lambda_N C_N \exp(-\lambda_N d_N) = 0 , \quad (3.70)
\]

\[
B_N + D_N + C_N \exp(-\lambda_N d_N) = 0 , \quad (3.71)
\]

### 3.4.1. Wave-driven mud mass transport velocity

The wave-driven mud mass transport velocity consists of two components, the Stokes’ drift \( \bar{u}_s \) and the mean Eulerian velocity \( \bar{u}_e \) which are calculated as a part of the wave-mud interaction model. The former component is caused by the orbital motion of the particle and the latter one is due to the viscosity of the fluid. The Stokes’ drift is expressed by

\[
\bar{u}_s = \frac{\partial \hat{u}}{\partial x} \int^t \hat{u} dt + \frac{\partial \hat{w}}{\partial z} \int^t \hat{w} dt , \quad (3.72)
\]
where the overbar denotes the time average over a wave period. Using the real parts of Eqs. (3.38) and (3.39), the Stokes’ drift is calculated by

\[ \bar{u}_s = \frac{1}{2\omega} \exp(-2k,x) \{ \cos \theta_k |k| u_j |^2 - \sin(\beta_j - \alpha'_j) |u'_j| |w_j| \}, \quad (3.73) \]

where \( \beta_j, \alpha_j, \beta_j, \theta_k \) are the arguments of \( w_j, u'_j, w'_j, k \) respectively. The mean Eulerian velocity at the \( j^{th} \) layer, \( \bar{u}_{E,j} \) can be calculated from the time averaged momentum equation (Mei, 1983; Sakakimata and Bijker, 1988)

\[ \frac{\partial}{\partial z} \left( \mu_{e,j} \frac{\partial \bar{u}_{E,j}}{\partial z} \right) = \frac{\partial \rho_j \hat{u}_j}{\partial x} + \frac{\partial \rho_j \hat{w}_j}{\partial z}, \quad (3.74) \]

where \( \rho_j \) is the density of the \( j^{th} \) layer of mud and \( \rho_w \) is the density of water. Eq. (3.74) is integrated with the following boundary conditions:

- No velocity at the rigid bottom:

\[ \bar{u}_{E,N} = 0. \quad (3.75) \]

- Continuity of velocities and shear stresses at the interfaces:

\[ \bar{u}_{E,j+1} = \bar{u}_{E,j} \quad (3.76) \]

\[ \mu_{e,j+1} \frac{\partial \bar{u}_{E,j+1}}{\partial z} = \mu_{e,j} \frac{\partial \bar{u}_{E,j}}{\partial z} \quad (3.77) \]
Outside the boundary layer:

\[ \frac{\partial u_E}{\partial z} \to 0 \]  

(3.78)

3.5. Gravity-driven mud mass transport

In addition to mud mass transport resulting from Stokes’ drift and the mean Eulerian velocity, mud flows down on a sloping bed or with small inclination of the mud surface. Fig. 3.3b shows the proposed multi-layered model where the \( x \) coordinate is parallel to the bed and \( \theta \) represents the inclination of the bed. The flow of the fluid mud layers is assumed to be laminar and uniform, due to the high mud viscosity and gradually varied flow along the \( x \)-axis (Kessel and Kranenburg, 1996). It is also assumed that the pressure field is hydrostatic and the velocity can be approximated from the local depth and slope, assuming a uniform flow approach (Coussot, 1994). Hence, the only shear stress considered is the one acting in planes parallel to the bed in the \( x \) direction. The momentum balance in the \( x \)-direction for an element with unit width in the \( y \) direction \( (\Delta x \times \Delta z) \) located in the \( j^{th} \) sub-layer is shown in Fig. 3.4. Considering the hydrostatic pressure and assuming a constant density in each sub-layer, the normal stress can be calculated by

\[
\sigma_{g,j} = (\rho_j - \rho_w)(z_j - z)g \cos \theta + \sum_{n=j+1}^{N}(\rho_n - \rho_w)d_ng \cos \theta
\]  

(3.79)
where $\sigma_{g,j}$ is normal stress due to gravity at $j^{th}$ sub-layer, $z_j$ is the height of the $j^{th}$ sub-layer and $d_n$ is the thickness of the $n^{th}$ sub-layer, i.e.

$$d_n = z_n - z_{n-1}$$

(3.80)

Neglecting the interfacial shear stress at the fluid mud-water interface, the vertical change of shear stress is expressed by

$$\frac{\partial \tau_{g,j}}{\partial z} = \frac{\partial}{\partial z} (\mu_{v,j} \frac{\partial u_{g,j}}{\partial z})$$

(3.81)

where $\tau_{g,j}$ and $u_{g,j}$ are the shear stress and velocity due to gravity at $j^{th}$ sub-layer in the $x$ direction. Consequently, the equation of momentum in the $x$ direction for the $j^{th}$ sub-layer of the fluid mud is

$$\frac{\partial (\rho u_x^2)}{\partial x} - \frac{\partial}{\partial z} (\mu_{v,j} \frac{\partial u_{g,j}}{\partial z}) + (\rho_j - \rho_w)g \sin \theta +$$

$$g \cos \theta \left\{ \sum_{n=j+1}^{N} [ (\rho_n - \rho_v) \left( \frac{\partial z_n}{\partial x} - \frac{\partial z_{n-1}}{\partial x} \right) + (\rho_j - \rho_w) \frac{\partial z_j}{\partial x} ] \right\} = 0$$

(3.82)

or after omitting the inertia forces

$$- \frac{\partial}{\partial z} (\mu_{v,j} \frac{\partial u_{g,j}}{\partial z}) = -(\rho_j - \rho_w)g \sin \theta -$$
\[ g \cos \theta \left[ \sum_{n=j+1}^{N} \left[ (\rho_u - \rho_w) \left( \frac{\partial z_{n+1}}{\partial x} - \frac{\partial z_{n-1}}{\partial x} \right) \right] + (\rho_j - \rho_w) \frac{\partial z_j}{\partial x} \right] \]  

(3.83)

where \( \theta \) is the slope of the solid bed. The governing equation can be easily integrated for each sub-layer. \( 2N \) unknown constants in \( N \) equations of velocity are determined from the boundary conditions at the fluid mud surface, the interfaces of fluid mud sub-layers and the rigid bottom. The following boundary conditions are applied:

\[ \tau_j + \frac{\Delta z}{2} \frac{\partial \tau_j}{\partial z} \]

\[ \sigma_{sj} = \frac{\Delta x}{2} \frac{\partial \sigma_{sj}}{\partial x} \]

\[ (\rho_j - \rho_w) g \sin \theta \]

\[ \rho_j u_j^2 - \frac{\Delta x}{2} \frac{\partial (\rho_j u_j^2)}{\partial x} \]

\[ \rho_j u_j^2 + \frac{\Delta x}{2} \frac{\partial (\rho_j u_j^2)}{\partial x} \]

\[ \tau_j = \frac{\Delta z}{2} \frac{\partial \tau_j}{\partial z} \]

Fig. 3.4. The momentum balance in the \( x \)-direction for an element with unit width in the \( y \) direction.

- At the mud surface \( (z_N = \sum_{n=1}^{N} d_n) \)

The imposition of zero tangential stresses are written as
\[
\mu_{e,N} \frac{\partial u_{g,N}}{\partial z} = 0 \quad (3.84)
\]

- At the interfaces \( z_j = \sum_{n=1}^{j} d_n \) where \( j = 1, \ldots, N-1 \)

The velocity profile should be continuous within the mud layer, i.e.

\[
u_{g,j} = u_{g,j+1}, \quad (3.85)
\]

and the continuity of tangential stresses is maintained by

\[
\tau_{g,j} = \tau_{g,j+1}, \quad (3.86)
\]

\[
\mu_{e,j} \frac{\partial u_{g,j}}{\partial z} = \mu_{e,j+1} \frac{\partial u_{g,j+1}}{\partial z}. \quad (3.87)
\]

- At the rigid bottom \( z = 0 \)

There is no movement at the fixed bottom, i.e.

\[
u_{g,1} = 0. \quad (3.88)
\]

The gravity-driven flow is added to the Stokes’ drift \( \bar{u}_s \) and the mean Eulerian velocity \( \bar{u}_E \) to determine the total mass transport velocity. Integrating the total mass transport
velocity from the fixed bed to the surface of the mud layer will result in the local volumetric sediment transport rate.

3.6. Mud profile change

Due to assumption of linearity for the problem, the wave-driven and gravity-driven velocities are calculated separately and algebraic summation of them is used for the total velocities. In calculation of the wave related velocities, a constant depth is assumed between two adjacent computational nodes. On the other hand, the gravity-driven velocities are obtained assuming the uniform and steady mud flow. This assumption can be justified considering the high viscosity of the fluid mud and very small inclination of the interfacial surfaces and the bed (Kessel and Kranenburg, 1996). The same apparent viscosity is used in both of the calculations.

Although the computation of mud mass transport under the combined effects of wave and gravity is affected by nonlinearity, the assumption of linearity which allows the separate treatment of mud mass transport due to wave and gravity has resulted to an acceptable estimation of mud transport here. It is possible to solve the general form of the equations of motion but it requires a numerical solution which results in much more computational effort. While the numerical solution might not be computational efficient, the offered explicit solution in this paper is much faster in computation procedure and a relatively large computational domain can be easily modeled.
The conservation equation of sediment mass is employed in order to calculate the change of the bathymetry

\[-\frac{\partial h}{\partial t} = \frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y},\]  \hspace{1cm} (3.89)

where \( h \) is the water depth and \( q_{sx} \) and \( q_{sy} \) are the rate of volumetric sediment transport per unit length in the \( x \) and \( y \) directions in units of volume/area/time. \( q_{sx} \) and \( q_{sy} \) can be calculated using the sum of Stokes’ drift, the mean Eulerian velocity in the direction of wave propagation and the gravity-driven flow. Eq. (3.89) is solved numerically using a finite difference method. The discretization of Eq. (3.89) is applicable on the same grid used for calculation of the wave propagation

\[h_{i,j}^{m+1} = h_{i,j}^{m} - \frac{\Delta t}{\Delta s} [(q_{sx,i+1/2,j}^{m} = q_{sx,i-1/2,j}^{m}) + (q_{sy,i,j+1/2}^{m} = q_{sy,i,j-1/2}^{m})],\] \hspace{1cm} (3.90)

where \( i, j \) are the grid numbers in the \( x \) and \( y \) directions respectively and \( m \) is the time step index. If the mud bottom is extended out of the modeling domain, it is assumed that the sediment can pass through the seaward boundary condition, allowing constant mud depth at this boundary. Thus, it is important to carefully define the length in the offshore zone. The shoreward boundary assumed coincident with the inshore end of surf zone, is specified by a user-defined water depth near the shoreline. Since the rates of mud mass transport are typically very large, it is necessary to choose small time intervals to ensure stable results.
3.7. Procedure of calculation

Fig. 3.5 shows the flowchart of the simulation procedure. The input data includes offshore wave height and wave period, bottom bathymetry and fluid mud characteristics. Assuming a fixed bed where no fluid mud exists, the wave propagation model is first applied over the simulation domain. No wave energy dissipation is included in this step. The complex wave number at all grid nodes are then calculated by the wave-mud interaction model using the local values of water depth, wave characteristics and fluid mud properties at the nodes. As discussed in section 2, the real part of the wave number, \( k_r \), provides the wave length \( L = 2\pi/k_r \) and its imaginary part, \( k_i \), represents the wave attenuation rate. Moving toward shoreline, the node specific wave energy dissipation rates are used to calculate the wave height at successive grid points. Inside the surf zone, the wave energy dissipation due to the fluid mud layer is combined with the energy dissipation caused by wave breaking. This iteration is continued until convergence, after which resulting wave heights in the solution domain are fed back into the wave-mud interaction model to calculate mud mass transport at each node. Introducing total mud mass transport due to the wave and gravity into the mass conservation equation, the bathymetry change is calculated. Following an update of the bathymetry and increment in time, the calculation returns to the wave propagation model. This morphodynamic loop continues for the period of the wave event.
Fig. 3.5. Flowchart of the simulation procedure.
3.8. Model performance

Computed results of wave propagation and mud mass transport are compared against the wave flume data of Sakakiyama and Bijker (1988), and Jiang (1993). The model results are also compared with the wave basin experiments conducted at the hydraulic laboratory, Yokohama National University, and the field measurements at Kumamoto Port in Japan.

3.8.1. Comparison with the wave flume laboratory experiment of Sakakiyama and Bijker (1988)

These laboratory experiments were performed in a wave flume 24.5 m long, 0.5 wide and 0.57 m deep, with a 12.0 m long flat mud bed section at the Laboratory of Fluid Mechanics, Delft University of Technology. Water depth was kept constant at 30 cm in all experiments while the mud depth was maintained between 9.0 and 9.5 cm. The mud was a mixture of 46.7% SiO$_2$, 38.0% Al$_2$O$_3$, 0.8% Fe$_2$O$_3$, and 1.7% K$_2$O. In these experiments the mud movement velocities were measured in four cases. Although the present model is two dimensional, it can also be applied for the simulation of these one dimensional cross-shore cases. The comparisons of measured and calculated profiles of mud mass transport velocities are depicted in Fig. 3.6. The figure also shows the calculated results of Sakakiyama and Bijker (1988). The mud characteristics were derived from the Sakakiyama and Bijker (1988) laboratory tests. In their calculation, the viscosity of the mud was assumed to be constant; the same assumption was considered here because viscoelastic and viscoplastic characteristics of the mud were not available.
The mud mass velocities calculated by the current model are generally less than values calculated by Sakakiyama and Bijker (1988) and they are closer to measured values. The difference is expected to be due to the fact that they didn’t consider gravity driven flow in their calculation. Fig. 3.7 shows the wave height measured in three experimental cases in comparison with the values predicted by the present numerical model. The accurate prediction of wave height is crucial, since the mud mass transport velocity is calculated based on the local predicted wave height at each node.

Fig. 3.6. Profile of the mass transport velocity in the mud layer normalized by $u_1$, calculated velocity at the surface of mud. $d_m$, $\rho_{mud}$, $H_0$, $T_w$, $X$ are mud thickness, mud density, incident wave height, wave period, and the location of the measured profile from the beginning of the mud section respectively. a) $\rho_{mud}=1380$ kg/m$^3$, $d_m=9.0$ cm, $H_0=3.8$ cm, $T_w=1.02$ s, $X=3.5$ m, $u_1=0.0456$ cm/s; b) $\rho_{mud}=1300$ kg/m$^3$, $d_m=9.3$ cm, $H_0=2.7$ cm, $T_w=1.01$ s, $X=5.0$ m, $u_1=0.0424$ cm/s; c) $\rho_{mud}=1230$ kg/m$^3$, $d_m=9.3$ cm, $H_0=3.2$ cm, $T_w=1.02$ s, $X=5.0$ m, $u_1=0.0868$ cm/s; d) $\rho_{mud}=1140$ kg/m$^3$, $d_m=9.2$ cm, $H_0=3.8$ cm, $T_w=1.02$ s, $X=6.0$ m, $u_1=0.1079$ cm/s.
Fig. 3.6. Continue

Fig. 3.7. Simulated and measured wave height comparison along the flume for three experimental cases of Sakakiyama and Bijker (1988), $\rho_{\text{mud}}=1240 \text{ kg/m}^3$, Mud depth=9 cm, Wave period= 1 s, Incident Wave Heights: 3.4, 2.0, 1.0 cm.
3.8.2. **Comparison with the wave flume laboratory experiment of Jiang (1993)**

The performance of the model for prediction of the cross shore profile change under the wave action was also examined using wave flume data of Jiang (1993). The laboratory experiment was carried out in the Coastal Engineering Laboratory of the University of Florida. The flume was 20 m long, 0.46 m wide and 0.45 m deep. An 8 m long recessed mud bottom with end slopes of the 1:12 was placed in the middle section of the model. The fluid mud was a mixture of 50% kaolinite, 50% attapulgite and water with density of 1.2 g/cm³. The wave height, wave period, water depth and mud depth were 3.0 cm, 1.0 s, 16.0 cm and 17.3 cm respectively. Starting with an initially horizontal mud bed, the wave maker was run for 10 hours. The water-mud interfacial slope reached an equilibrium condition after about 3 hours.

The numerical model was used to simulate the water-mud interfacial profile. Because the viscosity of the mud in both viscoelastic and viscoplastic states was not available the viscosity of mud was chosen manually in order to reproduce the same wave energy attenuation rate as measurement. Fig. 3.8 shows the comparison between the measured values and computed results. This figure also includes the water-mud interfacial profile calculated using the cross shore model of Jiang (1993). The laboratory measurements show a smooth initial slope for the interfacial surface which may be due to the effect of the front face of the mud box which prevents the generation of wave orbits in the mud layer in this area, thereby resulting in considerable reduction of mud movement. In other
words, the complicated trajectories of mud particles in the affected area near the front transition to the mud bed do not follow the assumed orbits of the wave-mud interaction model. The same phenomenon was observed in the experiments of Shen (1993) where the affected area was estimated to be about 1/3 of the wavelength. Therefore, no effect of wave-mud interaction was assumed at the leading transition and it was increased linearly to full interaction of wave-mud at a distance of 1/3 of the wave length from the transition.

![Graph showing comparison of water-mud interface changes](image)

**Fig. 3.8.** Comparison of simulated profile of water-mud interface with the measurement and simulated profile of Jiang (1993).

### 3.8.3. Comparison with the wave basin laboratory experiments

Since the proposed model is a two-dimensional horizontal model, laboratory experiments in a wave basin or field measurements can better demonstrate the capabilities of the model. A series of laboratory experiments were carried out in a wave basin, measuring
9.3 m by 9.0 m at the hydraulic laboratory, Yokohama National University. Fig. 3.9 shows the sketch of the experimental setup.

Commercial kaolinite was used for the fluid mud bed because it exhibits rheological behavior similar to natural mud (Otsubo and Muraoka, 1988). The particle size distribution of the kaolinite was characterized by $D_{86.3} = 3 \mu\text{m}$ and $D_{99.98} = 10 \mu\text{m}$. The mud was a mixture of 60.15% SiO$_2$, 30.44% Al$_2$O$_3$ and 0.07% Fe$_2$O$_3$.

The mud sample was prepared by carefully mixing kaolinite with tap water and the mud mixture was placed in a section in the wave basin measuring 1.0 m wide by 1.0 m long and 12 cm in height (Fig. 3.9). Monochromatic waves of various wave heights and incident wave directions were generated by the wave paddles at the boundary of the basin. Table 3.1 presents the test conditions of the four conducted cases. The waves on the mud section at Point 1 and Point 2, which are located at 30 cm and 70 cm from the edge of the mud box respectively, were recorded using electronic capacitance-type wave gauges. Table 2 presents wave heights and wave periods at the measured points.

Table 3.1 Incident wave and fluid mud characteristics

<table>
<thead>
<tr>
<th>Case</th>
<th>Incident Wave Height (cm)</th>
<th>Incident Wave Period (s)</th>
<th>Mud Water Content (%)</th>
<th>Water Depth at Point 1 (cm)</th>
<th>Wave Angle (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>6.61</td>
<td>1.00</td>
<td>160</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Case 2</td>
<td>3.80</td>
<td>1.00</td>
<td>130</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Case 3</td>
<td>3.78</td>
<td>1.00</td>
<td>130</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Case 4</td>
<td>4.47</td>
<td>1.00</td>
<td>130</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3.2 Wave heights and wave periods at measuring points

<table>
<thead>
<tr>
<th>Case</th>
<th>Wave Height at Point 1 (cm)</th>
<th>Wave Period at Point 1 (s)</th>
<th>Wave Height at Point 2 (cm)</th>
<th>Wave Period at Point 2 (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1.88</td>
<td>1.01</td>
<td>1.23</td>
<td>0.94</td>
</tr>
<tr>
<td>Case 2</td>
<td>2.59</td>
<td>1.01</td>
<td>1.53</td>
<td>0.70</td>
</tr>
<tr>
<td>Case 3</td>
<td>2.66</td>
<td>0.99</td>
<td>1.78</td>
<td>0.96</td>
</tr>
<tr>
<td>Case 4</td>
<td>3.50</td>
<td>1.06</td>
<td>2.00</td>
<td>1.06</td>
</tr>
</tbody>
</table>

The mud motion was investigated at point 1 using colored mud, with the same water content as the experimental mud, as a tracer (Sakakiyama and Bijker, 1988). The approach was originally used during experiments by Umita et al. (1986), in which the mud movements due to unidirectional flow were measured. The colored mud was prepared in a small box and was inserted into the mud layer at a right angle by a piston cylinder system. Then the box was removed leaving a sheet of colored mud in the mud layer without disturbance. After 60 seconds of wave generation the wave paddles were stopped and a transparent plastic box was used to remove a sample from the bed. The colored mud was observed through the transparent plate and the vertical profile of mud displacement was measured.

All four cases of the wave basin experiments were computed to be compared with measured values. In the numerical treatment, plastic parameters of kaolinite, i.e. yield stress and viscosity in the viscoplastic state, are calculated from Eqs. 3.43 and 3.44. Fig. 3.10 shows the wave height contours for case 4 of the laboratory experiments. The major processes that affect a wave train as it propagates from deep water into shallow water, i.e. shoaling, refraction and wave breaking, can be distinguished in the figure. Waves break just before the mud box between 50 mm contours. The combined dissipation effects due to wave breaking and mud bed influence may be observed by comparing the changes of
wave height contours over the mud section and outside of it. Fig. 3.11 shows the comparison between the calculated wave heights and the laboratory data for all the conducted cases at two measuring points. Although mud box lies in the breaker zone and both dissipations due to wave-mud interaction and wave breaking are combined over the mud box, the model is favorably successful in prediction of wave height in these experiments.

![Diagram](image)

Fig. 3.9. Experimental setup and numerical modeling domain in the wave basin at the hydraulic laboratory of Yokohama National University.

The measured mud movements at point 1 were also calculated. Fig. 3.12 presents a comparison of vertical profiles of the mud movement distributions at point 1 for the four experimental cases. A few discrepancies are observed and they are expected to be largely due to the effect of the relatively shallow water depth and occurrence of wave breaking in the laboratory experiments.
Fig. 3.10. Simulated wave height contours for case 4 (unit, mm).

Fig. 3.11. Comparison between simulated and measured wave heights at points 1 & 2 (after Soltanpour et al., 2008).
3.8.4. **Comparison with field observation at Kumamoto Port**

The numerical model has also been applied for the entrance area of Kumamoto Port in Japan during a storm event from 2:00 am to 6:00 am on August 31st, 1987. The reported value of wave attenuation factor between station A and Station B was 0.56 for an incident significant wave height and period of about 1.25 m and 4 s, respectively. This is considered to be consistent with the generation of a 10 cm fluid mud layer in the area (Tsuruya et al., 1990). In order to measure sediment deposition, two 70 m by 50 m trenches with a depth of 2 m were dug close to the harbor entrance. About 63 cm sediment deposition was measured during this event in one of the trenches (Fig. 3.13). In
the other trench, which was confined by a 1.0 m high submerged dike, there was almost no deposition. The sudden significant sediment deposition in the first trench is considered to be mainly due to mud mass transport induced by wave. The submerged dike in the other trench is shown to inhibit the mud movement significantly.

![Graph showing time variation of deposition height](image)

Fig. 3.13. Time variation of deposition height in trench #1 (after Tsuruya et al., 1990).

Fig. 3.14 shows the bathymetry of the computational domain. The simulation was carried out assuming the significant wave as a proper monochromatic representative wave for the irregular wave propagation on the fluid mud layer (Soltanpour et al., 2007). Fig. 3.15 represents the computed wave height contours. The yield stress and plastic viscosity of the fluid mud were adopted from the laboratory experiments of Tsuruya et al. (1990) conducted on the natural mud of the Kumamoto Port. The calculated wave height attenuation factor is 0.58 which is in good agreement with the reported value of 0.56.
Fig. 3.14. Bathymetric depth below datum in simulation domain and the location of the wave measurement stations (unit, m).

Fig. 3.15. Simulated wave height contours in meter at Kumamoto Port. (after Soltanpour et al., 2008).
Fig. 3.16. Contours of simulated wave height and layout of the trenches in local modeling domain (unit, m).

Fig. 3.16 shows the 400 m by 300 m local modeling domain for the calculation of the deposition in two trenches. The incident wave boundary condition of 0.84 m can be extracted from the wave propagation model, i.e. Fig. 3.15, for this inner domain. The 1.0 m high submerged dike in front of one of the trenches was also considered in the modeling domain. The wave height contours around the trenches and the resulting change of the bathymetry are depicted in Fig. 3.16 and 3.17, respectively. The height of sediment deposition in the unprotected trench is obvious, while there is almost no deposition shown in the protected one. Since the mud mass transport is directly related to wave height distribution and bottom slope, the complex pattern of wave height distribution and
bathymetry are reflected in the computed pattern of mud mass transport. The calculated deposition height in the middle of the unprotected trench, the point at which the deposition was measured at Fig. 3.13, is 37 cm. The difference between the calculated deposition of 37 cm and the measured deposition of 63 cm is expected to be largely due to deposition of suspended mud which is not included in this simulation. Based on the suspended sediment transport simulation which was completed by Tsuruya et al. (1990), the deposition due to suspended material during 27 hours of simulation was reported to be 39.8 cm. This would easily account for additional deposition in the trench.

Fig. 3.17. Simulated mud thickness in trench #1, without submerge dike and trench #2, with submerged dike.
3.9. Discussion

Reviewing various applications of the proposed numerical model, different level of accuracy in prediction of wave height and mud mass transport are observed. In general, prediction of wave height is more accurate than prediction of the mud mass transport, because the numerical model uses the predicted wave height to calculate mud mass transport. Moreover, mud mass transport is more sensitive to rheological behavior of mud, which is difficult to predict accurately.

Experimental conditions in different applications could explain some discrepancy between model predictions and physical observations. As a result of the laboratory conditions in the wave basin experiments, comparison of Fig. 3.6 and Fig. 3.12 shows that the predicted mud movement in the flume experiments of Sakakiyama and Bijker (1988) using the current model is more accurate than that of the wave basin experiment. Fig. 3.8 also reveals that the profile of water-mud interface for flume experiment of Jiang (1993) is predicted with an acceptable accuracy. The beds at wave flume experiments were flat and there were no wave breaking. On the contrary, the laboratory conditions for flume experiments were selected such that the effect of shoaling and wave breaking could be observed, in order to test the ability of the numerical model to predict wave height growth and wave energy dissipation due to both wave breaking and wave-mud interaction on sloping bed. However, a relatively small mud box (i.e. 1m by 1m), shallow depths and the occurrence of significant wave breaking in the experiments generate conditions that are not completely consistent with the theoretical assumptions of the model. For example, a relatively large incident wave height of 6.61 cm in case (1) resulted in considerable wave breaking. This could impose extra stress on
the fluid mud layer which is not accounted for in the numerical simulation. Moreover, considering small water depths (i.e. 7 cm) above the mud box, wave setup could induce significant change in water depth and consequently reduction of the mud mass transport in case (4). The change of water depth due to wave setup has not been considered in this simulation.

Although mud mass transport values are more sensitive than wave height attenuation rate to the rheological parameters, the high waves passing over the relatively low water depths in wave basin experiments are partially responsible for the relatively poor agreement between the predicted and measured mud movements. It is worth mentioning that the calculated wave induced mud mass transport velocities which are the sum of the Stokes’ drift $\overline{\nu}_s$ and the mean Eulerian velocity $\overline{u}_s$ for the conditions of the wave basin experiment are in the order of (cm/s), because of the high wave and low water depth on top of the mud box. These velocities are at least an order of magnitude greater than calculated velocities in the flume experiments; although the effect of gravity driven flow prevents large absolute displacement of the mud in the wave basin experiments. This difference in the magnitude of wave induced mud mass transport velocities could partly explain large absolute difference between measured and calculated mud displacements in the wave basin experiments compared with the flume experiments.

Application of the numerical model in Kumamoto Port (section 8.4) shows a good agreement for wave height simulations. However, the absence of suspended sediment transport in the current model results to the underestimation of the amount of deposition in the trench.
The following improvement could be considered to enhance the skills and applicability of the numerical model:

- The linear assumption reduces the accuracy of the model when large waves propagate over shallow waters. Nonlinear terms both in wave propagation model and wave-mud interaction model can be considered to overcome this inaccuracy.

- The phenomenon of wave breaking on muddy beds is still somewhat unknown. Better understanding and accurate simulation of this phenomenon which is highly simplified here, can improve model performance when the wave breaking is involved.

- A hydrodynamic model could be coupled with this model in order to accurately simulate other nearshore physical processes (i.e. wave set up, current shear stress, ...).

- The present model only considers mass transport in fluid mud layer. One major improvement is to include the suspended sediment transport. This is particularly important for long term prediction of sediment transport and bathymetry change.

3.10. Conclusion

Using a multi-layered wave-mud interaction model for the estimation of wave attenuation rate, the wave damping by fluid mud on a general bathymetry was successfully computed by imposing a dissipation term in the mild slope equations. The wave propagation model includes the combined effects of wave refraction, diffraction, reflection and breaking as well as wave attenuation due to wave-mud interaction. Moreover, considering wave-induced and gravity-driven velocities in the mud layers, the fluid mud mass transport and resulting bathymetry evolution were also modeled. The mud mass transport is particularly important in storm events, when a significant sedimentation occurs in a short period of time. The
comparisons of the calculated results with the performed laboratory data show a favorable agreement. The application of the numerical model on a real field condition was also presented. The comparisons between the calculated and measured wave heights usually offer a better agreement than comparisons between the calculated and measured mud mass transport. This may partially be explained by the high sensitivity of the calculated mud mass transport velocities to the input rheological parameters. Extensive applications of the model on laboratory and field cases suggest that although the current two-dimensional horizontal model is a step forward, some improvements are necessary to increase its accuracy and applicability. Coupling the present model with a hydrodynamic and suspended sediment transport model, it could be applied for long term sediment transport in real field conditions. Moreover, nonlinearity could be addressed by using a nonlinear wave propagation model as well as nonlinear terms in the wave-mud interaction model.

References


Chapter 4

4. Wave evolution on fluid mud bottom

This chapter is based on the following paper: Hall, K. & Oveisy, A., 2007. Wave evolution on fluid mud bottom. Coastal Sediment Conference 07. New Orleans, USA 1660-1668

4.1. Abstract:

It is well known that surface water waves interact with the fluid mud on the sea bed. Wave-mud interaction results in high wave energy dissipation and mud mass transport. This kind of wave energy dissipation, which generally is much more significant than wave dissipation due to bottom friction, should be considered in simulation of wave evolution and transformation in muddy coastal environments. This paper presents a numerical model of wave evolution and dissipation on fluid mud. For this purpose, the spectral decay of waves due to mud is implemented in SWAN (a third generation numerical model for Simulating WAves Nearshore) using a multilayered wave-mud interaction model. In this numerical model, it is assumed that mud layer has a visco-elastic-plastic behavior.

4.2. Introduction

Wave-mud interaction has been long the subject of the scientific research and some aspects of this interaction such as wave attenuation and mud mass transport have been
well known. In coastal areas covered with soft mud bed, traveling waves generate inter-surface waves between the water layer and the fluid mud layer resulting in high wave attenuation due to viscous damping of fluid mud layer. Wave energy dissipation due to wave-mud interaction should be considered in the simulation of wave evolution and transformation in coastal muddy environments (Gade 1957; Tubman and Suhayda 1976; Sheremet et al. 2003; Oveisy and Soltanpour 2005 and 2006). In this paper the spectral decay of waves due to mud is calculated in a multilayered wave-mud interaction model which considers the mud layer as a visco-elastic-plastic material. The calculated wave attenuation coefficient is used in SWAN in order to simulate wave evolution and dissipation on fluid mud.

4.3. Numerical model

The simulation of wave evolution on fluid mud is carried out by introducing a two dimensional vertical wave-mud interaction model two SWAN. The multi-layered vertical model is used to calculate the wave attenuation rate on mud. The value of wave attenuation rate is then used in SWAN in a similar fashion as the other dissipation factors to simulate wave dissipation on fluid mud bed.

4.4. Wave-mud interaction model

Following Tsuruya et al. (1987), the fluid system is divided into $N$ layers in which the water layer is represented by $N = 1$. Fig. 4 shows the schematic for the multi-layered model with the water layer on top.

93
The linearized Navier-Stokes equations, neglecting the convective accelerations and the continuity equation for an incompressible fluid can be expressed as

\[
\frac{\partial \bar{u}_j}{\partial t} = -\frac{1}{\rho_j} \frac{\partial \bar{p}_j}{\partial x} + \nu_{e,j} \left( \frac{\partial^2 \bar{u}_j}{\partial x^2} + \frac{\partial^2 \bar{u}_j}{\partial z^2} \right) \tag{4.1}
\]

\[
\frac{\partial \bar{w}_j}{\partial t} = -\frac{1}{\rho_j} \frac{\partial \bar{p}_j}{\partial z} + \nu_{e,j} \left( \frac{\partial^2 \bar{w}_j}{\partial x^2} + \frac{\partial^2 \bar{w}_j}{\partial z^2} \right) \tag{4.2}
\]

\[
\frac{\partial \bar{u}_j}{\partial x} + \frac{\partial \bar{w}_j}{\partial z} = 0 \tag{4.3}
\]

where \(x\) and \(z\) are the horizontal and vertical coordinates, the subscripts \(j\) indicate the layers, and the parameters \(t, \rho, \nu_e\) and \(p\) represent the time, density, kinematic viscosity of mud and dynamic pressure, respectively. The separable, periodic solutions for \(\bar{u}_j, \bar{w}_j\) and \(\bar{p}_j\) are assumed as
\begin{align}
\hat{u}_j(x, z; t) &= u_j(z) \exp[i(kx - \sigma t)] \quad (4.4) \\
\hat{w}_j(x, z; t) &= w_j(z) \exp[i(kx - \sigma t)] \quad (4.5) \\
\hat{p}_j(x, z; t) &= p_j(z) \exp[i(kx - \sigma t)] \quad (4.6)
\end{align}

where \( k \) is the unknown complex wave number, namely

\[ k = k_r + ik_i \quad (4.7) \]

Displacements of water surface and interfaces, \( \eta \), are represented by:

\[ \eta_j = a_j \exp[i(kx - \sigma t)] \quad (4.8) \]

where \( a_j \) is the amplitude of the displacement of the \( j \)th layer and the water surface is expressed as \( \eta \). Substituting the real and imaginary parts of wave number into Eq. (4.8), the expression of water surface and interfacial displacements can be obtained as

\[ \eta_j = a_j \exp(-k_r x) \exp[i(k_r x - \sigma t)] \quad (4.9) \]

Therefore, the real part of the wave number, \( k_r \), gives the wave length \( L = \frac{2\pi}{kr} \) and its imaginary part, \( k_i \), presents the wave attenuation rate, assuming the exponential wave height decay. Substituting Eqs. (4.4) and (4.5) into the continuity Eq. (4.3) results in

\[ u_j = \frac{i w_j'}{k} \quad (4.10) \]
where the prime represents the differential with respect to $z$. Introduction of Eq. (4.10) into Eq. (4.1) yields an expression for $p_j$

$$p_j = \frac{\rho_j v_{c,j}}{k^2}(w_j''' - w_j' \lambda_j^2)$$

(4.11)

where

$$\lambda_j^2 = k^2 - i\sigma v_{c,j}^{-1}$$

(4.12)

Substituting $p_j$ into the vertical momentum, Eq. (4.2), yields the fourth order differential equation for $w_j$

$$w_j''' - (k^2 + \lambda_j^2)w_j'' + k^2 \lambda_j^2 w_j = 0$$

(4.13)

The solutions can be obtained as

$$w_j(z) = A_j \sinh k(\sum_{n=1}^{i} d_n + z) + B_j \cosh k(\sum_{n=1}^{i} d_n + z) +$$

$$C_j \exp[\lambda_j(\sum_{n=1}^{i} d_n + z)] + D_j \exp[-\lambda_j(\sum_{n=1}^{i} d_n + z)]$$

(4.14)

where $d_n$ is the thickness of $n_{th}$ layer. The complex constants $A_j$, $B_j$, $C_j$, $D_j$, and the unknown variables $k$ and $a_j$ are determined from the boundary conditions.
boundary conditions are required for a viscous fluid model of $N$ layers. The unknown constants and variables are determined from the boundary conditions at the water surface, the interfaces and the rigid bottom. The details can be found in Tsuruya et al. (1987).

4.5. Rheological model of mud

Proper rheological model of mud should be adopted in order to investigate wave-mud interaction. Mud in general can range from being a highly rigid and weakly viscous material to one that can be approximated as a purely viscous fluid, depending on the properties of the constituent sediment and the ambient fluid. Considering the complexity of rheological behavior, the visco-elastic-plastic model has been adopted in the present study to develop a predictive behavior model for wave-mud interaction (Shibayama and An 1993). The constitutive equations are expressed as:

\[
\sigma_{ij} = 2\mu_c\dot{e}_{ij} \tag{4.15}
\]

\[
\mu_c = \mu_1 + \frac{iG}{\omega} \text{ for } \left(\frac{1}{2}\sigma_{ij}\sigma_{ij} \leq \tau_y^2\right) \tag{4.16}
\]

\[
\mu_c = \mu_2 + \frac{\tau_y}{\sqrt{4|\Pi_e|}} \text{ for } \left(\frac{1}{2}\sigma_{ij}\sigma_{ij} > \tau_y^2\right) \tag{4.17}
\]

where $\sigma_{ij}$ is the deviator part of stress tensor, $\dot{e}_{ij}$ is the deviator part of strain rate tensor, $G$ is the shear modulus of elasticity, $\mu_1$ is the viscosity of mud in the viscoelastic state, $\mu_2$ is the viscosity of mud in the viscoplastic state, $\tau_y$ is yield stress, $\omega$ is the angular frequency of wave and $4|\Pi_e|$ is expressed as
\[ 4|\Pi_r| = 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 \]  \hspace{1cm} (4.18)

where \( u \) and \( w \) are velocity components in \( x \) and \( z \) direction, respectively. The rheological viscoelastic parameters of mud, i.e. shear modulus and viscosity, are calculated from the empirical equations offered by Shibayama and An (1993):

\[ \mu_1 = 10^{(3.353 - 0.56 \times 10^{-3} W)} \times T \]  \hspace{1cm} (4.19)

\[ \log G = 3.761 - 1.05 \times 10^2 W + (0.147 - 3.38 \times 10^3 W) \times \log(T - 0.522 - 1.23 \times 10^{-3} W) \]  \hspace{1cm} (4.20)

where \( T \) is the wave period and \( W \) is the water content ratio of fluid mud (%). The viscoplastic parameters, \( \mu_2 \) and \( \tau_y \) were adopted from the laboratory experiments of Tsuruya et al. (1987)

\[ \tau_y = 1.494 \times 10^{14} W^{-5.796} \]  \hspace{1cm} (4.21)

\[ \mu_2 = 8.465 \times 10^4 W^{-1.436} \]  \hspace{1cm} (4.22)

In the viscoplastic state of the visco-elastic-plastic model, the objective \( 4|\Pi_r| \) of the representative viscosity can be approximated by a viscoelastic field using an iterative method. The real parts of Eqs. (4.4) and (4.5) are written as

\[ \hat{u}_j(z) = \left| u_j \right| \exp(-k_x x) \cos(k_x x - \sigma t + \alpha_j) \]  \hspace{1cm} (4.23)
\[ \hat{w}_j(z) = |w_j| \exp(-k_i x) \cos(k_i x - \alpha + \beta) \]  

(4.24)

where \( \alpha_j \) and \( \beta_j \) are the arguments of \( u_j \) and \( w_j \), respectively. Taking the partial derivatives of Eqs. (4.23) and (4.24) with respect to \( x \) and \( z \)

\[ \frac{\partial \hat{u}_j}{\partial x} = -|u_j| \exp(-k_i x) \sin(k_i x - \alpha + \alpha_j + \theta_k) \]  

(4.25)

\[ \frac{\partial \hat{u}_j}{\partial z} = -|u'_j| \exp(-k_i x) \cos(k_i x - \alpha + \alpha'_j) \]  

(4.26)

\[ \frac{\partial \hat{w}_j}{\partial x} = -|w_j| \exp(-k_i x) \sin(k_i x - \alpha + \beta + \theta_k) \]  

(4.27)

\[ \frac{\partial \hat{w}_j}{\partial z} = -|w'_j| \exp(-k_i x) \cos(k_i x - \alpha + \beta'_j) \]  

(4.28)

where \( \alpha'_j \) and \( \beta'_j \) and \( \theta_k \) are the arguments of \( u'_j \), \( w'_j \) and \( k \), respectively. Substituting Eqs. (4.25) to (4.28) into Eq. (4.3) leads to

\[ 4|\Pi| = \frac{1}{2} \{ |k|^2 (2|u_j|^2 + |w_j|^2) + (|u_j|^4 + 2|w_j|^4) \} \]

\[ -2|u_j||w_j||k|\sin(\beta + \theta_k - \alpha'_j) \exp(-2k_i x) \]  

(4.29)
4.6. Implementation in SWAN

The evolution of wave at SWAN is governed by wave action balance equation (Booij et al. 1999) expressed as,

\[
\frac{dN}{dt} = \frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(c_x N) + \frac{\partial}{\partial y}(c_y N) + \frac{\partial}{\partial \sigma}(c_\sigma N) + \frac{\partial}{\partial \theta}(c_\theta N) = \frac{S}{\sigma} \tag{4.30}
\]

\(N = E/\sigma\) is the action density (\(E\) is energy density), \(c_x\) and \(c_y\) are propagation velocities in \(x\) and \(y\) direction respectively, \(c_\sigma\) is propagation velocity in \(\sigma\) (relative frequency) space and \(c_\theta\) is propagation velocity in \(\theta\) (propagation direction) space. \(S\) includes all sources and sinks terms. A spectral sink term is needed to describe the wave-mud interaction (Zhang and Zhao 1999). Considering wave spectrum in space of \(\sigma\), \(\theta\) and exponential decay of wave height due to wave mud interaction,

\[
E(\sigma, \theta) = \frac{1}{\Delta \sigma \cdot \Delta \theta} \cdot \frac{1}{2} a_i^2 \tag{4.31}
\]

\[
a_{ix} = a_{i0} e^{-k_i(x-x_0)} \tag{4.32}
\]

where \(a_{i0}\) is the amplitude of \(i\)-th wave component at point \(x=x_0\), \(a_{ix}\) is the amplitude of \(i\)-th wave component at distant \(x\) from point \(x_0\), and \(\sigma_i, \theta_i\) are corresponding frequency and direction, the following equation can be derived for sink term in the right hand side of the action balance equation due to wave mud interaction.

\[
S_{\text{Mud}} = 2k_i cN \tag{4.33}
\]
where $c$ is the propagation velocity and $k_i$ is attenuation coefficient calculated in wave mud interaction model. The above mentioned dissipation term is implemented in SWAN to simulate wave-mud interaction.

4.7. Model performance and discussion

Since wave attenuation over muddy banks along the Louisiana coast has been well-known (Gade 1957; Tubman and Suhayda 1976) and significant measurement data are available, the model was used to simulate wave evolution along the Louisiana coast during a storm event which occurred from Jan 29 to Feb 01, 2001. In Fig. 4.2, stations CSI3 and CSI5 are parts of WAVCIS (Wave-Current Information System, wavcis.cis.lsu.edu), operated by Coastal Studies Institute of Louisiana State University. CSI3 is located at a muddy part of the coast while CSI5 is over a sandy bottom (Sheremet et al. 2003).

For this simulation, wind data are extracted and interpolated on a computational grid from ten metrological stations (Fig. 4.3) of the NDBC (National Data Buoy Center), and spectral wave measurements from Buoy 42001 were used as the offshore wave boundary condition.
Fig. 4.2. Louisiana coast and shelf, Coordinates are in Kilometers and contours in meter (Terra-1 MODIS satellite image; Earth Scan Lab, LSU; Adopted from Sheremet et al., 2005)

Fig. 4.3. Ten selected wind measurement stations

The simulation is conducted considering two scenarios. First, assuming no fluid mud layer and, second, using 100 (cm) of high sediment concentration layer above the bed covering 8 (Km) around the CSI3. It was assumed that the viscosity of the water-sediment mixture is two orders of magnitude greater than viscosity of water (Sheremet et al. 2005).
Fig. 4.4 shows spectral evolution during above mentioned storm at CSI3 and CSI5 for the two scenarios compared with the measured data. Measurements obviously shows less spectral energy density at CSI3 than CSI5 (Fig. 4.4-a). Fig. 4.4-b shows spectral evolution for the first scenario. With no mud dissipation there is still less intensive spectral density in CSI3 compared to CSI5. This is more likely due to the location of CSI3 which is surrounded by very shallow water (Fig. 4.2) resulting in significant refraction and bottom dissipation.

![Fig. 4.4. Spectral evolution, a) Measurement (Adopted from Sheremet and Stone 2003), b) Simulated wave height no fluid mud, c) Simulated wave height with fluid mud](image)

The simulation results for second scenario show significant wave attenuation. Higher attenuation is recognized for swells (waves with period greater than 5 seconds) since they
have more interaction with bottom. The attenuation of short waves in the measurements occurs due to surface-interface wave interaction and three-wave interaction (Sheremet et al. 2005).

4.8. Conclusion

The wave attenuation on fluid mud was introduced as a part of the dissipation terms in SWAN considering a multi-layer visco-elastic-plastic model. The model was used for simulation of a storm event from Jun 29 to Feb 01, 2001 in coast of Louisiana. The simulation result shows considerable wave height attenuation at CSI3 due to shoals around this point and wave-mud interaction.

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Chapter 5

5. Wave generation and interaction with fluid mud in Ariake Bay

(Under review in supplement to Journal of Continental Shelf Research for International Conference on Cohesive Sediment 2007)

5.1. Abstract

During a storm event, the wave measurements in Ariake Bay, Japan demonstrated that wave energy dissipation is significant due to fluid mud existence near Kumamoto Port in Ariake Bay (Fig. 5.1). The wave generation and its interaction with fluid mud within Ariake Bay were simulated here. The spectral decay of wave energy due to mud was implemented in SWAN (a third-generation numerical model for Simulating Waves Nearshore). A multilayered wave-mud interaction model was employed to calculate the wave height decay. A Visco-elastic-plastic model was used to represent the rheological behavior of the fluid mud layer. The preliminary application of the model shows its capability to predict wave height considering a soft muddy bed.

5.2. Introduction

In muddy coastal areas, propagating waves generate an inter-surface wave between the water layer and the fluid mud layer resulting in considerable wave attenuation due to the high viscosity of fluid mud. Wave-mud interaction has long been the subject of scientific
research and some aspects of this interaction such as wave attenuation and mud mass transport are well known (e.g., Gade, 1958; Dalrymple and Liu, 1978; Sakakiyama and Bijker, 1989; Maa and Mehta, 1990; Sheremet et al., 2003; Hall and Oveisy, 2007). In the storm events wave energy dissipation due to wave-mud interaction is generally more significant than wave dissipation due to the bottom friction. It is important to consider this type of wave energy dissipation in the modeling of wave propagation. Winterwerp et al. 2007 offered an implementation of surface wave damping on fluid mud in SWAN. However, they used the simple two-layer approach of Gade (1958). In this paper, the spectral decay of the wave energy due to fluid mud is calculated in a 2D vertical multilayered wave-mud interaction model assuming the visco-elastic-plastic rheological behavior of the fluid mud. The model is coupled with SWAN, introducing the wave attenuation as a sink term in the wave action balance equation.

5.3. Wave-mud interaction model

Following Tsuruya et al. (1987) the fluid system above the solid bed, i.e. the fluidized mud and water layer, is divided into \( N \) sub-layers in which the water layer is represented by \( N = 1 \) (Fig. 5.2). The linearized Navier-Stokes equations, neglecting the convective accelerations and the continuity equation for an incompressible fluid can be expressed as,

\[
\frac{\partial u_j}{\partial t} = -\frac{1}{\rho_j} \frac{\partial p_j}{\partial x} + \nu_{e,j} \left( \frac{\partial^2 u_j}{\partial x^2} + \frac{\partial^2 u_j}{\partial z^2} \right) \tag{5.1}
\]

\[
\frac{\partial w_j}{\partial t} = -\frac{1}{\rho_j} \frac{\partial p_j}{\partial z} + \nu_{e,j} \left( \frac{\partial^2 w_j}{\partial x^2} + \frac{\partial^2 w_j}{\partial z^2} \right) \tag{5.2}
\]
where $x$ and $z$ are the horizontal and vertical coordinates, the subscripts $j$ indicate the layers, and the parameters $t$, $\rho$, $v_c$ and $p$ represent the time, density, kinematic viscosity of mud and dynamic pressure, respectively. The separable, periodic solutions for $\hat{u}_j$, $\hat{w}_j$ and $\hat{p}_j$ are assumed as

\[
\hat{u}_j(x, z; t) = u_j(z) \exp[i(kx - \sigma t)] \tag{5.4}
\]

\[
\hat{w}_j(x, z; t) = w_j(z) \exp[i(kx - \sigma t)] \tag{5.5}
\]

\[
\hat{p}_j(x, z; t) = p_j(z) \exp[i(kx - \sigma t)] \tag{5.6}
\]

where $k$ is the unknown complex wave number, namely

\[
k = k_r + i k_i \tag{5.7}
\]

Displacements of water surface and interfaces, $\eta_j$, are represented by:

\[
\eta_j = a_j \exp[i(kx - \sigma t)] \tag{5.8}
\]

where $a_j$ is the amplitude of the displacement of the $j$th layer and the water surface is expressed as $\eta_1$. Substituting the real and imaginary parts of wave number into Eq. (5.8), the expression of water surface and interfacial displacements can be obtained as
\[ \eta_j = a_j \exp(-k_i x) \exp[i(k_r x - \sigma x)] \] (5.9)

Therefore, the real part of the wave number, \( k_r \), gives the wave length \( L = 2\pi/k_r \) and its imaginary part, \( k_i \), presents the wave attenuation rate, assuming the exponential wave height decay. \( 5N \) boundary conditions exist for a fluid model of \( N \) sub-layers as presented by Tsuruya et al. (1987). These boundary conditions at the water surface, the interfaces and the rigid bottom are used to determine the unknown constants and variables. The wave attenuation rate, \( k_i \), is calculated by this model.

Fig. 5.1. Site of the Kumamoto Port, A and B are wave measurement stations (depth from datum)
5.4. Rheological model of mud

A proper rheological model of mud should be adopted in the wave-mud interaction model. Mud in general can range from being a highly rigid and weakly viscous material to one that can be approximated as a purely viscous fluid, depending on the properties of the constituent sediment and the ambient fluid. Considering the complexity of rheological behavior, the visco-elastic-plastic model has been adopted in the present study to develop a predictive behavior model for wave-mud interaction (Shibayama and An, 1993). The constitutive equations are expressed as:

\[
\sigma_{ij} = 2\mu_e \dot{e}_{ij} \tag{5.10}
\]

\[
\mu_e = \mu_1 + \frac{iG}{\omega} \quad \text{for} \quad \left(\frac{1}{2}\sigma_{ij}\sigma_{ij} \leq \tau^2_y\right) \tag{5.11-a}
\]

\[
\mu_e = \mu_2 + \frac{\tau_y}{\sqrt{4|\Pi_e|}} \quad \text{for} \quad \left(\frac{1}{2}\sigma_{ij}\sigma_{ij} > \tau^2_y\right) \tag{5.11-b}
\]

where \(\sigma_{ij}\) is the deviator part of stress tensor, \(\dot{e}_{ij}\) is the deviator part of strain rate tensor, \(G\) is the shear modulus of elasticity, \(\mu_1\) is the viscosity of mud in the viscoelastic state, \(\mu_2\) is the viscosity of mud in the viscoplastic state, \(\tau_y\) is yield stress, \(\omega\) is the angular frequency of wave and \(4|\Pi_e|\) is expressed as

\[
4|\Pi_e| = 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 \tag{5.12}
\]
where \( u \) and \( w \) are velocity components in \( x \) and \( z \) direction, respectively. The rheological viscoelastic parameters of mud, i.e. shear modulus and viscosity, are calculated from the empirical equations offered by Shibayama and An (1993)

\[
\mu_1 = 10^{(3.353-9.56\times10^{-3}W)} \times T
\]
\[
\log G = 3.761 - 1.05\times10^2W + (0.147 - 3.38\times10^3W) \times \log(T - 0.522 - 1.23\times10^{-3}W)
\]

where \( T \) is the wave period and \( W \) is the water content ratio of fluid mud (%). The viscoplastic parameters, \( \mu_2 \) and \( \tau_y \) were adopted from the laboratory experiments of Tsuruya et al. (1987)

\[
\tau_y = 1.494 \times 10^{14} W^{-5.796}
\]
\[
\mu_2 = 8.465 \times 10^4 W^{-1.436}
\]

Fig. 5.2. Definition of multi-layered model
In the viscoplastic state of the visco-elastic-plastic model, the objective $4 | \Pi |$ of the representative viscosity can be approximated by a viscoelastic field using an iterative method. The real parts of Eqs. (5.4) and (5.5) are written as

$$\hat{u}_j(z) = |u_j| \exp(-k_i x) \cos(k_i x - \sigma t + \alpha_j)$$

(5.17)

$$\hat{w}_j(z) = |w_j| \exp(-k_i x) \cos(k_i x - \sigma t + \beta_j)$$

(5.18)

where $\alpha_j$ and $\beta_j$ are the arguments of $u_j$ and $w_j$, respectively. Taking the partial derivatives of Eqs. (5.23) and (5.24) with respect to $x$ and $z$

$$\frac{\partial \hat{u}_j}{\partial x} = -k |u_j| \exp(-k_i x) \sin(k_i x - \sigma t + \alpha_j + \theta_k)$$

(5.19)

$$\frac{\partial \hat{u}_j}{\partial z} = -|u'_j| \exp(-k_i x) \cos(k_i x - \sigma t + \alpha'_j)$$

(5.20)

$$\frac{\partial \hat{w}_j}{\partial x} = -k |w_j| \exp(-k_i x) \sin(k_i x - \sigma t + \beta_j + \theta_k)$$

(5.21)

$$\frac{\partial \hat{w}_j}{\partial z} = -|w'_j| \exp(-k_i x) \cos(k_i x - \sigma t + \beta'_j)$$

(5.22)

where $\alpha'_j$ and $\beta'_j$ and $\theta_k$ are the arguments of $u'_j$, $w'_j$ and $k$, respectively. Substituting Eqs. (5.25) to (5.28) into Eq. (5.3) leads to
\[ 4|\Pi| = \frac{1}{2} \left[ |k^2 (2|u_j|^2 + |w_j|^2) + (|u_j|^2 + 2|w_j|^2) \right] \]
\[ -2 |u_j||w_j||k|\sin(\beta_j + \theta_k - \alpha_j')\exp(-2k_x x) \]  

(5.24)

### 5.5. Implementation of wave damping in SWAN

SWAN is a state of the art wave propagation model which is developed by Delft University of Technology (Booij et al. 1999). As a third-generation numerical model for Simulating Waves Nearshore, SWAN includes the processes of wind generation, whitecapping, quadruplet wave-wave interactions, and bottom dissipation. The triad wave-wave interactions and depth-induced wave breaking are also considered in SWAN. SWAN solves the balance equation of wave action density in a two-dimensional grid using an implicit scheme. The governing equation in SWAN is expressed as,

\[
\frac{dN}{dt} = \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} (c_x N) + \frac{\partial}{\partial y} (c_y N) + \frac{\partial}{\partial \sigma} (c_\sigma N) + \frac{\partial}{\partial \theta} (c_\theta N) = \frac{S}{\sigma} \]  

(5.25)

Where \( N = E/\sigma \) is the action density (\( E \) is energy density), \( c_x \) and \( c_y \) are propagation velocities in the \( x \) and \( y \) directions respectively, \( c_\sigma \) is propagation velocity in \( \sigma \) (relative frequency) space and \( c_\theta \) is propagation velocity in \( \theta \) (propagation direction) space. \( S \) includes all sources and sinks terms,

\[ S = S_{in} + S_{ds,w} + S_{ds,b} + S_{ds,br} + S_{mud} \]  

(5.26)
$S_{in}$ is a source term for wind energy input and $S_{ds,w}$, $S_{ds,b}$, $S_{ds,br}$ are dissipation terms due to whitecapping, bottom friction and depth-induced breaking respectively. $S_{mud}$ is added here for dissipation of the wave energy due to wave-mud interaction. A spectral sink term is needed to describe the wave-mud interaction (Zhang and Zhao 1999). Considering the wave spectrum in the space of $\sigma$, $\theta$ and the exponential decay of wave height due to wave mud interaction,

$$E(\sigma_i, \theta_i) = \frac{1}{\Delta\sigma \cdot \Delta\theta} \cdot \frac{1}{2} a_{ix}^2$$  \hspace{1cm} (5.27)

$$a_{ix} = a_{i0} e^{-k_i(x-x_0)}$$  \hspace{1cm} (5.28)

where $a_{i0}$ is the amplitude of the $i$-th wave component at point $x=x_0$, $a_{ix}$ is the amplitude of the $i$-th wave component at distant $x$ from point $x_0$, and $\sigma_i, \theta_i$ are corresponding frequency and direction, the following equation can be derived for the sink term on the right hand side of the action balance equation due to wave mud interaction.

$$\left( \frac{\partial N}{\partial x} \right)_{mud} = \frac{1}{\sigma} \frac{\partial E}{\partial x} = -2k_i c N$$  \hspace{1cm} (5.28)

$$S_{mud} = -2k_i c N$$  \hspace{1cm} (5.29)

In these relationships $c$ is the propagation velocity and $k_i$ is the wave attenuation rate which is calculated in the wave mud interaction model (Sec. 5.2). Eq. (5.29) is implemented in SWAN to simulate wave-mud interaction.
5.6. Kumamoto Port

Kumamoto Port, at a site near Kumamoto City along the East coast of Arika Bay in Japan was under construction in 1988 (Fig. 5.1). A large amount of fine sediment was transported to this area mainly by two large rivers originating from Aso Mountain, which is an active volcano. Due to high sediment discharge and formation of a wide tidal flat in this area, the Fourth District Port Construction Bureau of Japan carried out various field observations to investigate sedimentation around the construction site to establish sufficient countermeasures against siltation. During the measurement period from December 1986 to March 1988 several parameters such as: tidal level, tidal current, wind, wave and vertical distribution of suspended sediment were measured. An example of measured time series has been shown in Fig. 5.3. Three trenches 70m long, 50m wide and 2m depth were also dug to observe sediment deposition.

There were two wave measurement stations, A and B, near the port. These stations were operating during a storm event on August 31\textsuperscript{st} 1987, from 12:00 am to 6:00 am (Tsuruya et al., 1990). A wave height reduction of about 56% was reported between the measurement stations during the storm event. Referring to Fig. 5.3, the increase in suspended sediment concentration in Trench\# 1 (next to station A) is obvious with the increase of wave height, particularly during low tide when the interaction of wave with bed is greater than that of high tide condition. The wave dissipation was simultaneous with formation of high suspended sediment concentration (greater than 10,000 mg/lit) near the bed at both wave measurement stations (Fig. 5.4). This sudden change in the vertical distribution of suspended sediment concentration can be interpreted as the formation of a fluid mud near the bed. With the
existence of fluid mud during the storm and consequently wave-mud interaction, the significant dissipation of wave between the two stations is expected.

Fig. 5.3. Measured time series of wave height, tide and suspended sediment concentration in Trench # 1 (After Tsuruya et al., 1990).
Fig. 5.4. Vertical distribution of suspended sediment concentration at Station A and B (After Tsuruya et al., 1990).

Fig. 5.5. Wind time series measured at Kumamoto Airport.
5.7. Model performance and discussion

The model is applied at Ariake Bay in Japan to predict the wave generation and attenuation. In order to simulate the storm event on August 31\textsuperscript{st} 1987, the model was forced by wind time series measured during that period. The wind data, obtained from the nearby station at Kumamoto Airport, is shown in Fig. 5.5. Typhoon No.8712 set upon the area in that period. Since Ariake Bay has the highest tidal range on the Japanese coast, it is necessary to consider changes in the water elevation for the simulation. The tidal fluctuation, which was measured during the storm event, was used for this simulation. The simulation domain is shown in Fig. 5.6.
The mud properties are adopted from the laboratory experiments of Tsuruya et al. (1987) for the local mud. These relations were discussed in section 3. The mud thickness and the water content ratio were reported to be 10 cm and 200% respectively (Tsuruya et al., 1990). The model was run for the period of measurement; the predicted significant wave height was extracted at stations A and B. Comparing the simulated wave heights with measured results at two stations, a significant dissipation due to fluid mud can be recognized (Fig. 5.7). Numerical simulation of the storm shows a wave height reduction of about 60% during the storm event between these two stations (compared with 56% measured).

![Wave height comparison](image)

**Fig. 5.7.** Wave height comparison (m)
5.8. Conclusion

The wave attenuation on a fluid mud was introduced through the introduction of a dissipation term in SWAN. In order to determine the dissipation term, the wave attenuation rate was calculated interactively in the multilayered wave-mud interaction model. In this model the effective viscosity of mud was defined based on the visco-elastic-plastic rheological model. The properties of mud were determined using laboratory tests on the local sediment. The model was used for simulation of a storm event on August 31st 1987 within Ariake Bay in Japan. The simulation result shows considerable wave height attenuation due to wave-mud interaction and agrees well with measured values.

References


General Discussions:

6. General discussions and Conclusion:

6.1. Summary and conclusions

In this study, different nearshore processes present in muddy coastal areas were investigated through numerical simulation. Wave-mud interaction was used to assist in the prediction of wave generation, transformation and mud mass transport. The capability of the numerical model was verified using several sets of both laboratory and field physical observations. The main contributions of the current study can be summarized as follows:

1. Following the general structure of the two-dimensional horizontal wave models based on time dependent mild slope equations, a new hydrodynamic model was proposed which considers the energy dissipation of muddy beds in the wave energy equation. Formulation of wave decay was also extended to the surf zone by combining both dissipation effects, i.e. due to the mud and wave breaking, in the surf zone.
2. The two-dimensional horizontal mud mass transport velocity resulting from wave action on a fluid mud bottom was calculated based on the wave transformation models. These velocities are the sum of the Stokes drift and Eulerian velocity in the direction of wave propagation.
3. The gravity-driven flow of mud layers on sloping bed was investigated. A multi-layered model was proposed and the local downward velocity was calculated from the numerical simulation.
4. The mud mass transport due to waves and gravity was used to calculate sediment transport under stormy conditions. The resulting bathymetric change under a storm event on muddy coastal environment is the outcome of this computation.
5. The wave-mud interaction was implemented in an existing wave generation numerical model, SWAN (a third generation numerical model for Simulating WAVes Nearshore), to calculate wave generation on a fluid mud bottom.

6. Several physical observations of wave transformation on fluid mud and resulting mud mass transport were gathered. The numerical simulation results were compared against these data to show the capability of the proposed numerical model.

6.2. Recommendations for further studies

Although some of the basic difficulties in approaching the modeling of muddy shore morphodynamics have been addressed in the present study, it is a very early stage of the practical application of numerical models on the cohesive sediment transport in coastal area and there are many problems remaining for future studies. Some of the recommendations for further research are presented here.

1. The linear assumption reduces the accuracy of the model when large waves propagate into shallow water. Nonlinear terms in both the wave propagation model and wave-mud interaction model can be considered to overcome this inaccuracy.

2. The phenomenon of wave breaking on muddy beds is still somewhat unknown. Better understanding and accurate simulation of this phenomenon which is highly simplified here, can improve model performance when wave breaking is involved.

3. A hydrodynamic model could be coupled with this model in order to accurately simulate other nearshore physical processes (i.e. wave set up, current shear stress, etc.).

4. The present model only considers mass transport in a fluid mud layer. One major improvement is to include the suspended sediment transport in the model. This is particularly important for long term prediction of sediment transport and bathymetric change.