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Mathematics for teaching in a professional learning network: Complexity inquiry

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Mathematics for teaching in a professional learning network: Complexity inquiry

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Abstract: *Numerous mathematics teachers nowadays have turned to online professional learning networks (PLNs) to carry on their professional learning. Very few related studies have revealed the implications of mathematics teachers' participation in PLNs for their professional development. This research is intended to address that gap by investigating participants' actions in a blog and disclosing the emergence therefrom based on the model of Mathematics-For-Teaching* (M4T). *I analyzed the data from a blog and its comments concerning the discussion about the Handshake Problem from two textbooks under the interpretation circles developed from Ellis' (1998) hermeneutics circles. The results exhibited a divergent conversation pattern and the emerged four-layer* M4T. *Participating in online PLNs involved individual and collective actions. Individual actions initiatively referred to posting comments. The comments related to the suggestions, expectations, visions, criticisms, experiences, and comparisons that induced the collective conversation. The collective conversation gave rise to the emergence of four layers of* M4T*: presentations of textbooks, students' thinking skills, classroom conversation, and geometry learning. Their emergence indicated not only the professional knowledge but also the knowing of the Handshake Problem for the participants. The collective conversation, interaction, or communication enabled visualization of the roles of the individual actions played within and deepened understanding of the presentations of the Handshake Problem. The results pointed to implications for teachers' participation in PLNs for their professional learning.*

Keywords: networks; M4T; emergence; interaction; professional learning

In the recent years, online professional learning communities have become prominent for teachers' professional development (Trust, 2012) and championed as an "anytime, anywhere" option for flexible, professional pursuits (Stanford-Bowers, 2008). A large number of teachers have extended their learning by participating in online PLNs.

From there, they could seek and provide help and support for other people by presenting their knowledge and capability, and communicating with individuals about issues, information, or feedback (Trust, 2012). Also, from there, they could access important resources that they could not afford or even avail in the local communities (Dede, Breit, Ketelhut, McCloskey, & Whitehouse, 2005).

However, even though the number of online learning communities is increasing, few related studies have been conducted to investigate mathematics teachers' professional learning through online PLNs. This research is intended to address that gap by investigating mathematics teachers' actions in a blog based on the model of M4T (Davis & Renert, 2014).

Mathematics-for-Teaching

Teachers' disciplinary knowledge of mathematics has experienced decades of intensive research but is not yet well-formulated (Davis, 2015; Davis & Renert, 2013/2014). From complexity theory, Davis and Renert (2013) argued that teachers' disciplinary knowledge of mathematics could not be a defined and well-connected set as basis rather than "sophisticated and largely enactive mix of familiarity with various realizations of mathematical concepts and awareness of the complex processes through which mathematics is produced" (p. 247). Accordingly, they developed a related notion of "profound understanding of emergent mathematics" (p. 247), highlighting "emergent" as different from "fundamental": vast not limited, intricate not straightforward, and evolving not static. Also, they marked the distinct character of teachers' disciplinary knowledge of mathematics with the term M4T (Davis & Renert, 2013; Davis & Simmt, 2006) and defined it as:

> A way of being with mathematics knowledge that enables a teacher to structure learning situations, interpret student actions mindfully, and respond flexibly, in ways that enable learners to extent understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice. (Davis & Renert, 2014, p. 4).

In addition, teachers' disciplinary knowledge of mathematics was tentatively identified by other researchers who were triggered and encouraged by Shulman's work. Ball and her colleagues (i.e. Ball & Bass, 2009; Ball, Thames, & Phelps, 2008), for example, proposed domains of knowledge of mathematics for teaching (KMT) in their work (see Figure 1). Davis and Simmt (2006) generated a different image (see Figure 2) in collaboration with practicing teachers to highlight the intertwining nature of KMT.

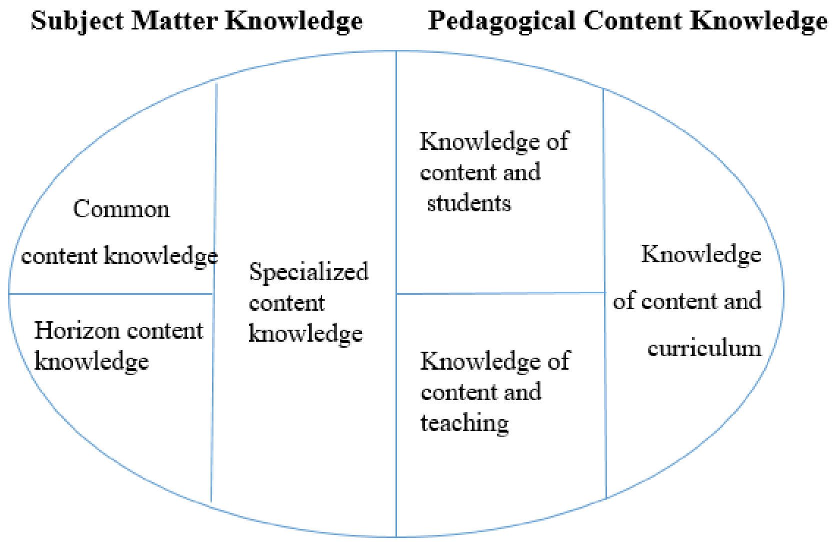


Figure 1. Domains of KMT (adapted from Ball, Thames, & Phelps, 2008, p. 403).

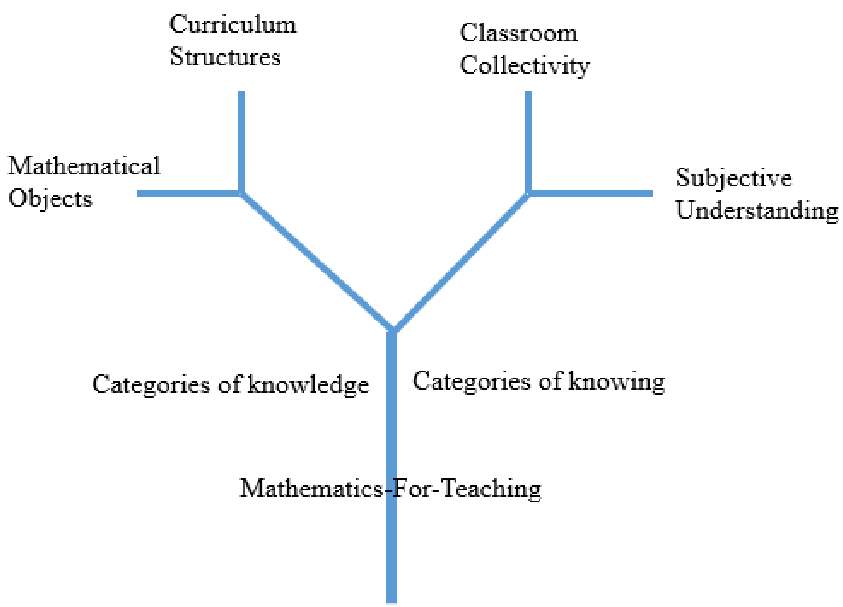


Figure 2. Perceived relationships among some aspects of teachers' M_4T (adapted from Davis & Simmt, 2006, p. 298).

Davis and Renert (2014) further pointed out the analogous and distinct aspects between the categories of KMT and of M_4T. The analogy between these two models is that categories of knowledge of M_4T are parallel to subject matter knowledge of KMT, while categories of knowing of M_4T are parallel to pedagogical content knowledge of KMT. Their distinction is that M_4T signals the differentiation of knowledge from knowing, while KMT does not. Mathematical knowledge at the collective level is described as relatively stable because it evolves "at a pace and a scale" (p. 90), while mathematical knowing at the personal level is characterized as "volatile and unstable" (p. 90) because it can self-transform in a rapid way. Compatible with complexity science, the categories of M_4T were creatively portrayed by Davis and Renert (2014) as nested layers on the M_4T tree (see Figure 3). They argued that the nested layers were better than the neighboring domains in the model of KMT (see Figure 1).

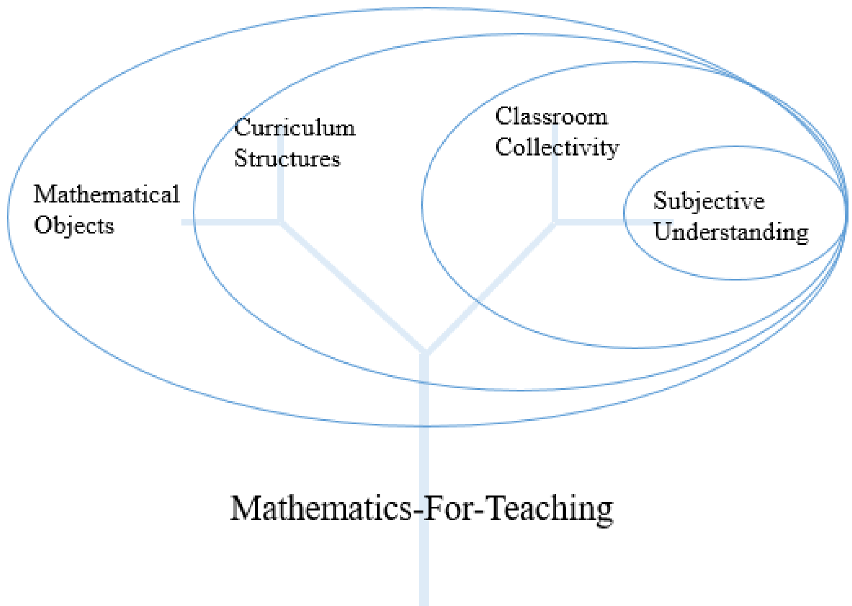


Figure 3. Interpreting knowing and knowledge as nested phenomena (adapted from Davis & Renert, 2014, p. 92).

Methodology

The Data: The Initial Blog Post and its Comments

At the beginning of the blog, its blogger introduced the Handshake Problem as a classic math problem: "If everybody here has to shake everyone else's hand, how many handshakes do we need?" She invited other participants to share how they use it. As it is a well-known problem, the sharing of how different teachers used it offered various interesting perspectives.

The blogger posted two presentations in the initial blog (337 words and 4-page textbook photocopy) of the Handshake Problem from two textbooks (T1 and T2). The textbooks described them as: (a) finding out a function rule about the number of handshakes at a party under the guidance of eight-step scaffolding from acting out the problem to modeling the problem using polygons and diagonals, and (b) looking for patterns from handshakes at a party and the diagonals of any regular polygon to find out the similarity between the patterns. The blogger also invited other participants to reflect upon the question of what takes on geometry these presentations would bring about. She addressed her own observations about the presentations regarding the aspects of modeling process supports, the connection between the handshake and the geometrical problem (such as diagonals), and the appearance orders of the problem in the textbooks (i.e., the problem appears in the first section of T1 and in the subsection 2.4 of T2). She also believed that those two presentations of the problem implied two assumptions: T1 implied that informal experiences are necessary for students to shape their reasoning capabilities and T2 implied that an explicit modeling process is essential for students to learn how to reason on their own.

Followed by the initial blog post were 11 comments (1,320 words). Of them, one (Comment 9) was removed by its author and the 10 remainders (130 words on average) were my subject matter.

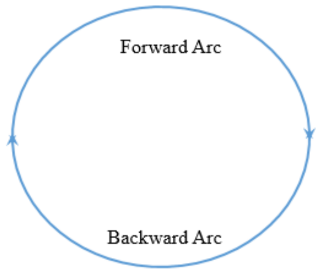
The M_4T Model

The nested M_4T (see Figure 3) (Davis & Renert, 2014; Davis & Simmt, 2006) was used as a framework to examine the emergent professional knowledge of participants from the online interactions and discussions threads in the PLN. What emerged from the interactions and discussions, however, was unpredictable, as it was dynamic. The interactions could also be deemed as a process of knowledge generation or an evolution process. Thus, the nested M_4T model was set to be able to illustrate what emerged from, and how the knowledge was produced through, the interactions.

The Interpretation Circles

The analysis of the blog and its comments (BCs) went through a process of interpretation circles derived from Ellis' (1998) hermeneutics circle (see Table 1).

Table 1
The Hermeneutics Circle (adapted from Ellis, 1998, p. 27)

The visualized circle	Descriptions
	**Forward Arc**: Entails making sense of a research participant, situation, or a set of data by drawing on one's forestructure, which is the current product of one's autobiography (belief, value, interests, interpretive framework) and one's relationship with the question or problem. (p. 27) **Backward Arc**: Entails endeavoring to see what went unseen in the initial interpretation resulting from projection. The data are re-examined for contradictions, gaps, omissions, or confirmations of the initial interpretation. Alternate interpretive frameworks are purpose-fully searched for and 'tried on'. (p. 27)

Before conducting the analysis, I read through and summarized the BCs to gain a preliminary sense of what they dealt with. The generalized BCs became the primary data used for the analysis during which I could go back to the original data from time to time, if necessary, to double check the relevant details. The process of going back and forth was presented by a double arrow in Figure 4. At the beginning of holding the data in hand, I considered the question of how the comments together with the blog expanded the conversation as a whole. Bearing this question in mind drove me to make a map of the BCs, and from the map arose a divergent pattern of conversation implying that participants ruminated from multiple perspectives about the issues or viewpoints derived from the blog. My further consideration was about what could emerge from such conversation. To elaborate on the emergence, I re-examined the summarized data and framed it by using the model of M_4T. The analysis showed the occurrence of a nested M_4T, which rendered me into further inquiry about what roles the BCs played in the emergence. This question motivated me to trace back to the summarized data once again to utilize relationship analysis to understand how the emergence happened. The result suggested that there existed an interaction between the BCs and the emergence. Going through the whole analysis allowed me to understand what the individual and collective actions of participants really were and what emerged from those actions in this case. To date, the inquiry process is ongoing, which is represented by the dotted line in Figure 4. The following sections communicate the results of the full analysis.

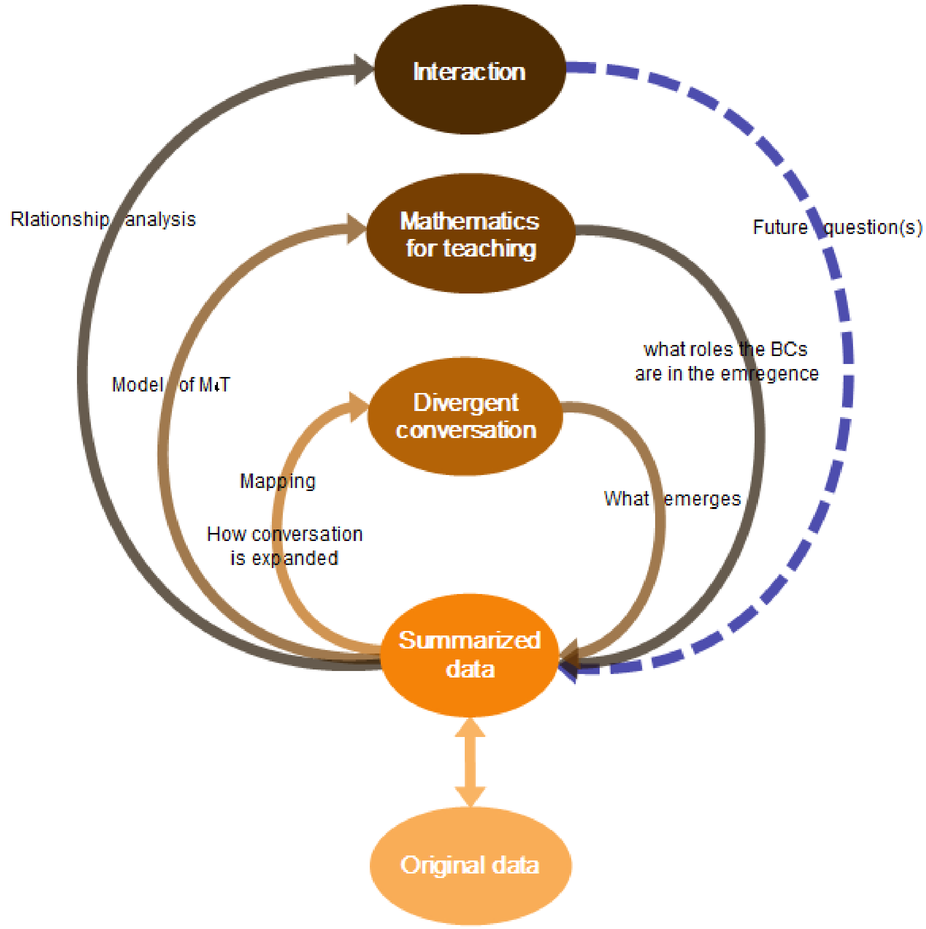


Figure 4. The analysis process.

Numbering Comments

Each comment was assigned a number representing its occurring sequence among the comments. If a comment came after the initial blog post, its number was considered an integer. For example, a comment numbered "3" was indicative of the third comment in the sequence of comments accompanying the initial blog post. If occurring after other comments instead of the initial blog post, the comment was designated as a decimal, which expressed its responding number and the order among the comments. For example, a comment was designated "3.2" if it responded to Comment 3 and it was the second among all the responses to Comment 3.

Results

Divergent Conversation

Eight out of ten comments responded directly to the initial blog post presenting a divergent conversation pattern (see Figure 5). The following was mainly to understand the 10 comments so as to see how they expanded the conversation about the presentations of the Handshake Problem.

Comment 1 proposed certain expectations on the support of students' thinking from the two presentations, such as taking away the procedural support from T1 and removing the explicit connection between the handshake and the geometrical problems from T2. Likewise, Comment 2 demanded an alternative way of calling students to build the connection between the handshake and diagonals problems after their reviewing the presentation from T2.

Comment 3 examined the two presentations from a higher viewpoint--helping students gain skills in such areas as problem-solving or mathematical modeling. A potential take could look to other fields such as literacy, because the skill of problem solving has similarity to that of reading or writing.

Comment 4 put forward two visions about the two presentations. One suggested leaving out the connection between the handshake and diagonals problems, which could provide opportunities for students to make their own connection. It would be more proper to show the connection in the teacher's reference book. The other proposed to separately position the two problems in the different chapters to leave the space for students to build the connection. Under these two visions, it was true that the student textbook provided too much scaffolding. It would be appropriate that the textbook was skinny while teachers' edition was thick. Fully favouring Comment 4's suggestions, Comment 5 advised that the support work be provided for the teacher rather than the student, so that the teacher would have various ideas for guiding the student to explore the problem in an organic way. Its commenter also shared his own experiences of working with his colleagues in their own geometry text. They tried to let the conversation among the students flow naturally and smoothly and to pay more attention to the support issue in their teaching practice.

Comment 6 posted a diagonals problem different from the one in T1 and T2. The problem was presented in a geometrical way without taking the handshake problem or other problems as prompt activities. The comment also asked the reason for the discussion in question.

Comment 7 highlighted one fundamental idea about the support issues (i.e. scaffolding two key Standards for Mathematical Practice: Understanding and Modeling problems). The comment further argued that, even if imperfect, the two key standards could still offer general scaffolds for students' problem solving and become thinking tools which the students could resort to when struggling with the problems. Less scaffolding could be conducive to promote students' thinking when they have troubles with their background knowledge and comprehensive capabilities. However, once failing to solve a problem under less scaffolding, the students with low mathematics achievement could reinforce such a belief that they were fundamentally bad at math as what the commenter had witnessed quite often in his own teaching.

Comment 8 further compared the purposes of the two presentations in T1 and T2. It revealed that T1 had established certain expectations of students' thinking. Particularly, the presentation (as Questions 1, 2 and 3) in T1 demonstrated a thinking flow from a specific case to a generalized situation. Following the presentation or the questions, students could be directed to experience the inductive reasoning which had been assumed by T1. However, the presentation in T1 indicated that, being seemingly significant, a gap existed between Questions 2 and 3, while the presentation in T2 did not make its purpose clear. Responding to Comment 8, Comment 8.1 attempted to dispel the confusion of the gap noted in Comment 8. It elaborated that in practice many students could not make the connection between counting handshakes and diagonals, but as a prompt, Question 3 could explicitly guide students to make that connection.

Comment 10 posted various ways of solving the Handshake Problem such as geometrically drawing the diagonals in a polygon, algorithmically listing all handshakes, algebraically counting the number of subsets of size 2 in a set, and recursively asking the number of handshakes when an additional person was added. However, it was true that few questions could cope with such a broad scope of solutions and connect solutions with important mathematical concepts, ideas, or thinking.

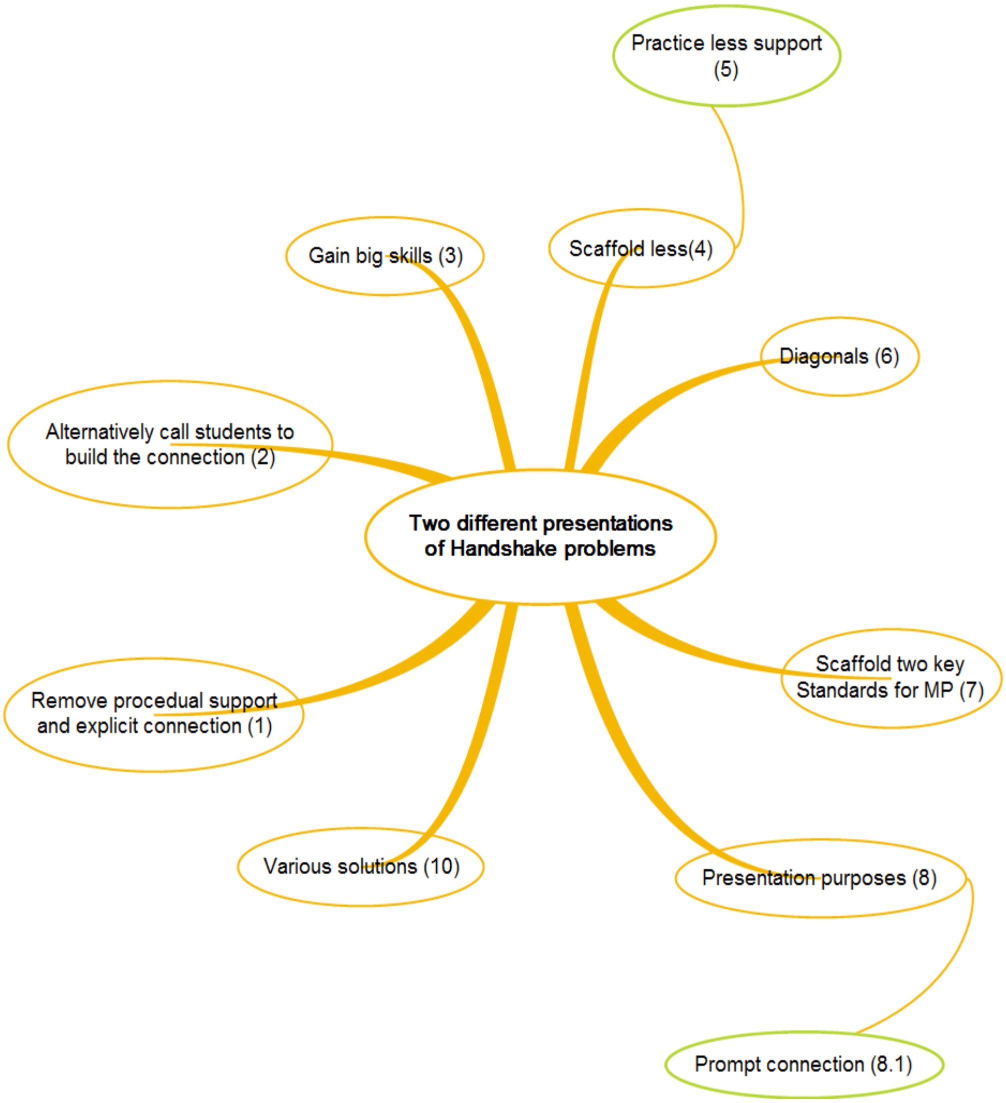


Figure 5. Divergent conversation pattern of comments.

Mathematics-For-Teaching

Looking back to the blog and its comments, their related conversation touched upon students' learning, teaching practice, geometry, as well as textbook presentation. According to the model of M4T, what emerged from the conversation could be illustrated as the nested four layers of M4T (see Figure 6): presentations of textbooks, students' thinking skills, classroom conversation, and geometry learning.

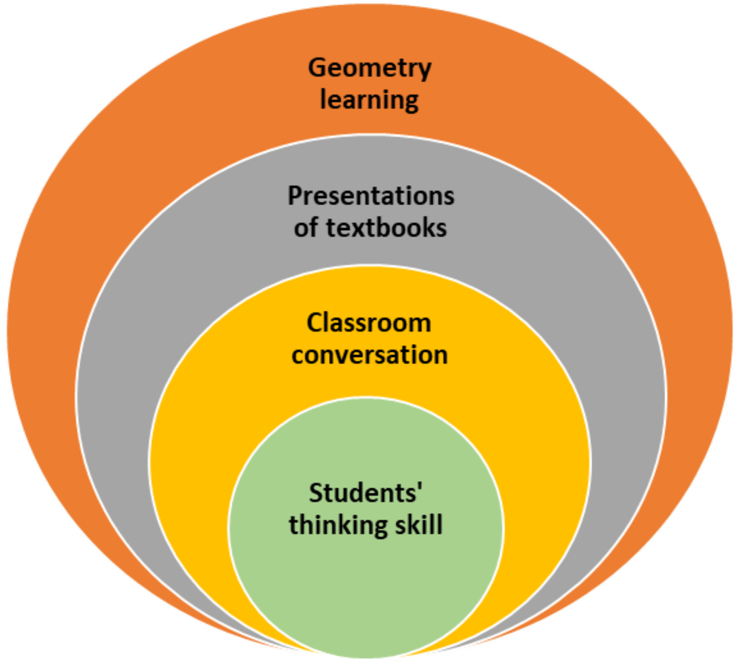


Figure 6. The nested M_4T of the blog and its comments.

Presentations of textbooks. The conversation from the initial blog was initiated by the various presentations of the Handshake Problems in T1 and T2. Six comments (1, 2, 4, 5, 8, and 10) focused on presentations of textbooks, which is taken as one layer of M_4T. For example, Comment 1 suggested two kinds of actions on the two presentations--removing the procedural support and taking away the explicit prompt of building the connection between the handshake and diagonals problems; Comment 2 called for an alternative way of prompting students to build the connection; Comment 4 proposed that there be less scaffolding from the presentations; Comment 5 agreed with Comment 4 and presented an ongoing work on the presentation design of geometry text in the school context; Comment 8 reviewed the purposes of the two presentations and noted confusion about the gap between problems, which was dispelled by Comment 8.1; and Comment 10 demonstrated various approaches to solve the Handshake Problem and criticized that in textbooks few questions could accommodate the broad scope of approaches.

Students' thinking skills. In addition to its major concern about the problem presentations in textbooks, the conversation also referred to students' thinking skills under the presentations of textbooks. For example, Comment 3 considered how to help students obtain big skills through the different presentations, and Comment 7 presented tools to support students' thinking when they struggle with problems.

Classroom conversation. Classroom conversation arose when the author of Comment 5 shared his practical experiences of providing less support for students' thinking skills. He and his colleagues designed their own geometry text and allowed the classroom conversation to flow in a natural way (without any control) to achieve the target of their presentation design.

Geometry learning. Geometry learning was implicated in the conversation, which went beyond the presentations of textbooks touching upon the purpose of geometry learning. Comment 6 posted a geometrical problem about diagonals from a textbook. The diagonals problem was very similar to the ones in T1 and T2 but without the Handshake Problem or other problems as prompt activities. Consequently, the geometrical problem was thought of as a direct way of learning geometry, which called me to reflect upon why textbooks needed the Handshake Problem. In fact, the initial blog already invited participants to think about the benefits of the presentations for geometrical learning. However, the ensuing conversation paid less attention to the topic of their benefits than to the differences of both the two presentations and the scaffolding issue. Comment 10 reviewed that few questions could carry a wide range of solutions and pointed to the critical concepts, ideas, or thinking in math, particularly in geometry. Thus, the questions involved in the presentations should have a clear purpose, just as Comment 8 mentioned that the three questions in T1 would lead students to experience inductive reasoning from a specific case to a generalized situation.

The four layers of M_4T did not emerge in order but enfolded in nature. For example, the most comments were put on presentations of textbooks which had been regarded as the topic of the blog. Being involved in presentations of textbooks, participants thought about how the different presentations of textbooks develop students' thinking skills. With such concern, one participant shared his experience of building classroom conversation under a new presentation of a textbook to support the development of students' thinking skills. It was clear that presentations of textbooks enfolded classroom conversation while classroom conversation enfolded students' thinking skills. Eventually, the participants thought about the purpose of geometry learning by going beyond the textbook, classroom, and students' thinking skills. Thus, geometry learning enfolded presentations of textbooks, classroom conversation, and students' thinking skills.

In addition, the four layers could be seen as an epitome of the model of M_4T (see Figure 3 and 6). Students' thinking skills was analogous to subjective understanding;

classroom conversation to classroom collectivity; presentations of textbook to curriculum structures; geometry learning to mathematical objects.

Discussion

Going back to the blog and its comments and looking through the divergent conversation and the emergent M_4T, there appears to be an interaction between individual and collective actions (i.e., posting and responding to comments) and the emergence. The individual actions—including the suggestions, expectations, visions, criticisms, experiences, and comparisons—not only deepen our understanding of the presentations of the Handshake Problem but construct the collective conversation as well. The collective conversation produces the emergence of M_4T. Through the emergence, we can realize the roles of the individual comments or actions played within the collective conversation.

A large portion of the individual comments or actions introduce various suggestions and visions about the two presentations from T1 and T2. For example, Comment 1 brings forward suggestions about lowering the support for students' thinking from the presentations such as dislodging the explicit prompt of calling students to build the connection between handshake and diagonals problems. The suggestions coincide with the notions from Comments 4 and 5--less scaffolding for students' thinking. It is therefore necessary to call for an alternative way of prompting students to construct the connection as Comment 2 advocates. One possible way is presenting questions with multiple solutions or approaches connecting with the key ideas or concepts of geometry as Comment 10 suggests. In the meantime, a clear presentation of purpose is also critical because it would guide students to experience the geometrical thinking that Comment 8 is concerned about. The above suggestions, notions, advocations, and concerns from the comments are integrated into the concept of presentations of textbooks being a layer of the nested M_4T (see Figure 6).

Some other comments try to touch upon the geometry in relation to the Handshake Problem. For instance, Comment 6 provides a geometrical presentation, and invites participants to reflect upon the role of the Handshake Problem in geometry but no response follows up. However, from another perspective, Comment 10 indicates that it is essential to present questions with multiple solutions pointing to the key concepts, ideas or thinking in math; nevertheless, it is still a challenging way of presenting questions. It is also necessary to make a clear presentation of purpose just as Comment 8 illustrates that the questions from T1 could direct students to experience the assumed thinking process. As it is, the initial blog explicitly asks participants to consider the benefits of different presentations for geometrical learning, but no comment expressly centers on this topic. The related individual comments or actions seem to just scratch the surface of the topic from different perspectives without going into depth. However, they actually open doors for us from different orientations to attend to and to explore the topic for the purpose of providing "new interpretive possibilities" (Davis & Simmt, 2006, p. 299). They also converge on the concept of geometry learning as a layer of M_4T (see Figure 6).

A small number of comments involve students' thinking skills under the presentations of the Handshake Problem in textbooks. For instance, helping students do better at solving hard problems or modeling should not be ignored when they are submerged in the detailed and multifarious presentations as Comment 3 believes. Meanwhile, providing thinking tools such as two key Standards for Mathematical Practice from Comment 7 should be taken as a potential way of supporting students' thinking when they are exposed to the different presentations. Thus, such comments as Comment 3 and 7 shape the layer of students' thinking skills in M_4T (see Figure 6).

Classroom conversation is referred to only in the shared teaching experience of Comment 5. Since this is the only reference, the concept of classroom conversation does not possess a rich interpretation, but it extends the scope of the conversation and presents a layer of M_4T (see Figure 6).

Taking the BCs as a collective conversation gives shape to the body of M_4T. Looking through the emerged body of knowledge, we could regard the BCs as parts dwelling in the body which supplies a larger picture, background, or context for the living of the BCs. For instance, Comment 8 is concerned about the presentation purpose, revealing that the presentation from T1 has set up the goals of students' thinking. Its viewpoints integrate with the ideas created from other comments (e.g. Suggestions on the support for students' thinking from Comment 1, 2, and 4, and visions/experiences from Comment 5 and 10) and their integration shapes up to be a conceptual blend on the presentations of textbooks. Thus, Comment 8 becomes an element of the conceptual blend. Meanwhile, under the presentations of textbooks, students' thinking skills involved in the presentation is regarded as another conversational focus, as Comment 8 expresses. The involved conversation presents the ways of supporting students' thinking (e.g., Comment 7) and the skills targeted by the problem (e.g., Comment 3). A potential way of implementing these ideas is to make classroom conversation naturally flow without any control (e.g., Comment 5). Therefore, the classroom environment is considered to be vital in the conversation. Looking back to the three integrated concepts, I come to know that the concept of classroom conversation "incorporates" the concept of students' thinking skills and "is incorporated into" (Davis & Simmt, 2006, p. 301) the concept of presentations of textbooks.

Looking forward, I have realized that those three concepts aim to facilitate students reaching an understanding of polygons (geometry). The conversation around the geometry also exposes the presentation challenges (e.g. Comment 10) and the potential supporting strategies (e.g. Comment 8). Thus, the integration of the aim, the challenges, and the strategies converge into the concept of geometry learning, incorporating the concepts of students' thinking skills, classroom conversation, and presentations of textbooks.

Our understanding of the above could allow us to diagram a blended concept as a layer "enfolded in and unfolding from" (Davis & Simmt, 2006, p. 296) a broader one in the emerged M4T (see Figure 6). The nested M4T accommodates the blog and the individual comments and offers them a larger picture, background, or context about the Handshake Problem.

Evidently, the individual comments or actions construct the layers of M4T when regarded as "conceptual blends" (Davis & Simmt, 2006, p. 301) based on their attached concepts, ideas, or notions; they produce the body of M4T when taken as a collective conversation; and they endow M4T with specific connotations by the multiple viewpoints that they carry. In turn, the emerged M4T accommodates the individual comments or actions as layers and brings them into a larger picture, background, or context about the topic in question.

References

Davis, B. (2015). The mathematics that secondary teachers (need to) know. *Revista Espanola de Pedagogia, 73*(261), 321-342.

Davis, B., & Renert, M. (2013). Profound understanding of emergent mathematics: Broadening the construct of teachers' disciplinary knowledge. *Educational Studies in Mathematics, 82*, 245-265.

Davis, B. & Renert, M. (2014). *The math teachers know: Profound understanding of emergent mathematics*. New York, NY: Routledge.

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*, 293-319.

Dede, C., Breit, L., Ketelhut, D. J., McCloskey, E., & Whitehouse, P. (2005). *An overview of current findings from empirical research on online teacher professional development*. Cambridge, MA: Harvard Graduate School of Education.

Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures*. Paper prepared based on keynote address at the 43rd Jahrestagung für Didaktik der Mathematik held in Oldenburg, Germany, March 1-4, 2009.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education, 59*, 389–407.

Ellis, J. (1998b). Interpretive inquiry as a formal research process. In J. Ellis (Ed.), *Teaching from understanding: Teachers as interpretative inquirer* (pp. 15-32). New York, NY: Garland Publishing.

Mitchell, M. (2009). *Complexity: A guided tour*. New York, NY: Oxford University Press.

Stanford-Bowers, D. E. (2008). Persistence in online classes: A study of perceptions among community college stakeholders. *Journal of Online Learning and Teaching, 4*(1).

Trust, T. (2012). Professional learning networks designed for teacher learning. *Journal of Digital Learning in Teacher Education, 28*(4), 133-138.