LUMPED ELEMENT TRANSDIRECTIONAL AND CODIRECTIONAL COUPLER

by

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A thesis submitted to the Department of Electrical and Computer Engineering
In conformity with the requirements for
the degree of Master of Applied Science

Queen’s University
Kingston, Ontario, Canada
(May, 2018)

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Abstract

Directional couplers are an essential component in several microwave applications. Lumped element directional couplers gained a lot of attention due to their compact size. This thesis represents a lumped element model for transdirectional and codirectional couplers. Unlike the classical inductive coupling used for modelling, capacitors are used to represent the coupling mechanism between the two coupled lines. Transdirectional couplers are recommended for many applications such as power dividers and power combiners because of the direct current (DC) isolation between the input and output ports. Generic design equations are demonstrated at any frequency with arbitrary coupling coefficients for the lumped element transdirectional coupler. In addition, design equations for lumped element codirectional couplers at any frequency with 3 dB coupling coefficient are presented. Lumped element transdirectional coupler and codirectional coupler are designed and fabricated at 1 GHz to verify the design methodology. Both are fabricated using FR-4 substrate. There is good agreement between the simulated and measured results for both designs.
Acknowledgements

I would like to thank my two supervisors, Prof. Yahia Antar and Prof. Alois Freundorfer, for all their support and guidance throughout my degree. Their patient advice and expertise helped me throughout my research process. I would also like to thank my family and friends for their unwavering support and for always believing in me.
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<tr>
<td>TEM</td>
<td>Transverse Electromagnetic</td>
</tr>
<tr>
<td>MICs</td>
<td>Microwave Integrated Circuits</td>
</tr>
<tr>
<td>MMICs</td>
<td>Monolithic Microwave Integrated Circuit</td>
</tr>
<tr>
<td>ADS</td>
<td>Advanced Design System</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>VNA</td>
<td>Vector Network Analyzer</td>
</tr>
<tr>
<td>VIP</td>
<td>Vertically Installed Planar</td>
</tr>
<tr>
<td>TL</td>
<td>Transmission Lines</td>
</tr>
<tr>
<td>S-parameter</td>
<td>Scattering Parameters</td>
</tr>
<tr>
<td>PCB</td>
<td>Printed Circuit Board</td>
</tr>
<tr>
<td>CPW</td>
<td>Coplanar Waveguide</td>
</tr>
<tr>
<td>SM</td>
<td>Surface Mount</td>
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Chapter 1

Introduction

1.1 Directional Coupler Overview

Directional couplers are an essential component in many microwave applications such as balanced amplifiers [1], phase shifter modulators, beam forming networks for antennas [2], microwave mixers [3], and filters [4]. Directional couplers are four-port passive devices used for power combination or power division [5]. They have the property that an incident wave in port 1 couples the power into port 2 and 3 but keeps port 4 isolated as shown in Figure 1.1 [5]. Similarly, when an incident wave is on port 4, power couples to port 2 and 3 and keeps port 1 isolated because of its symmetry [3]. Directional couplers have an isolated port and are terminated with a matched load [5]. This leads to a three-port network as shown in Figure 1.2. Figure 1.2(a) shows a coupler being used as a power divider, where the input power is divided into two output ports. In Figure 1.2(b), the coupler is being used as a combiner, where two input signals are combined into one output. Directional couplers can be designed at arbitrary coupling coefficients and different output port locations based on the input port; this will be discussed in detail in this chapter.

![Directional Coupler Schematic](image)

Figure 1.1: Directional coupler schematic [5].
1.2 Applications of the Directional Coupler

Directional couplers can be used for several applications including power dividers, power combiners, feeding networks for antennas, and power amplifiers. Figure 1.3 shows an example for power amplifiers [1] using branch line couplers as power dividers and power combiners. As shown in Figure 1.3, the input power is split to feed the power amplifiers. The output power is then combined to produce a single output with high power.
Figure 1.3: Branch-line coupler in balanced power amplifiers [1].

Figure 1.4: The schematic of a 4×4 Butler matrix [2].
Another common application for the directional couplers is beam forming networks for an array of antennas such as the Butler matrix [2]. A 4×4 Butler matrix is designed by using directional couplers and phase shifters as shown in Figure 1.4. It takes four inputs and provides four outputs with the same amplitude but with different phases. The hybrid directional couplers generate half of the input power at each port as well as phase difference between the output ports. In Figure 1.5, the Lange coupler is used for power division and power combination for the power amplifier. Four large capacitors are used to block the DC bias through the Lange couplers to the input and output ports.

![Figure 1.5: The circuit design of integrated balanced amplifier with two Lange couplers [1].](image)

1.3 Types of Directional Couplers

There are three types of directional couplers: contradirectional (conventional), codirectional, and transdirectional. The type of directional coupler is dictated by the relative location of the isolated port to the input port [6]. In the three cases, the input port is port 1. The first type is the contradirectional (conventional) coupler, as shown in Figure 1.6(a). In this type, the input port is port 1; the output ports are ports 2 and 4; and port 3 is the isolated port. The second type is the codirectional coupler which is the most commonly used type. As shown in Figure 1.6(b), in codirectional couplers, port 1 is the input port, port 2 and port 3 are the output ports, and port 4 is the isolated port. The third type is the transdirectional coupler in which port 1 is
the input port, port 3 and port 4 are the output ports, and port 2 is the isolated port, as shown in Figure 1.6(c). The phase difference between the output ports can be $90^\circ$ or $180^\circ$ according to the directional coupler type; this will be discussed later. In this thesis, both the transdirectional and codirectional types are implemented using the lumped element model. The reason for not implementing the contradirectional coupler will be discussed later.

![Diagram of directional coupler types](image)

**Figure 1.6: Schematic of directional coupler types based on the relative location to the input port.** (a) Contradirectional. (b) Codirectional. (c) Transdirectional.

### 1.4 Directional Couplers Review

Directional couplers have been implemented using the transmission line (distributed), lumped element, or hybrid (combination of transmission line and lumped element) models. In this section, previous works of the three types of directional couplers are discussed. In addition, the latest implementation of directional couplers using the lumped element is also discussed. In this thesis, we are presenting lumped element circuits for two directional couplers: transdirectional and codirectional. The lumped element circuit of the transdirectional coupler has not yet been implemented; the only implementation was a hybrid circuit using
transmission lines and lumped elements. The contradirectional (conventional) coupler’s implementation requires both inductive and capacitive coupling between the two lines. Since we do not want to use inductors for coupling, which has been done in [7], we will not implement the contradirectional coupler. Here in this thesis, we only want to use capacitive coupling and this leads to the codirectional and transdirectional couplers. The following sections discuss the state of art for the three types of couplers and demonstrate the thesis contribution along with each type.

1.4.1 Contradirectional (Conventional) Coupler

The contradirectional (conventional) coupler is the most common type of two coupled transmission line coupler [5]. Figure 1.1 shows the output ports configuration of this directional coupler. The contradirectional coupler can be implemented using the transmission line and the lumped element models. The transmission line model of this coupler is shown in Figure 1.7. There is inductive and capacitive coupling between the two transmission lines of the conventional coupler.

![Contradirectional Coupler Schematic](image)

*Figure 1.7: The schematic of contradirectional coupler [5].*

The implementation of this contradirectional coupler in lumped element circuit is done by representing the inductive coupling using inductors and the capacitive coupling using capacitors. The authors in [7] used the lumped element model for each transmission line and represented the inductive coupling between the two coupled lines by an inductor as shown in Figure 1.8. A prototype of the contradirectional coupler that works
at 1 GHz with 3dB coupling coefficient using FR4 with 1.57 mm thickness has been fabricated with the measured S-parameters shown in Figure 1.9.

**Figure 1.8: Conversion from the distributed model of the contradirectional coupler to the lumped element model [7].**

**Figure 1.9: S-parameters of contradirectional coupler [7].**

The implementation of the inductive coupling using an inductor in the lumped element does not represent the transmission line version of the contradirectional coupler. This inductor allows the DC to go through port 3 and port 4 which does not happen in this version of the coupler. More detail about the
conversion between the transmission lines to the lumped element model of the contradirectional coupler will be illustrated in Chapter 2.

1.4.2 Codirectional Coupler

The codirectional coupler is a type of codirectional coupler that has the output ports from the sides as shown in Figure 1.6(b). The branch line coupler has been implemented using the transmission line and the lumped element models.

In [5], the author presented a transmission line model for the branch line model as shown in Figure 1.10. The presented design has the output ports 2 and 3 from the side with a 90° phase difference between them. Figure 1.11 shows the S-parameters of the branch line coupler at the desired frequency $f_0$ [5].

![Figure 1.10: The geometry of a branch line coupler [5].](image-url)
This design could be converted to a lumped element model by representing the impedance of each transmission line branch with inductance and capacitance as shown in Figure 1.12 [8]. The authors in [8] converted the transmission line model of the branch line into a lumped element model which is smaller in size and more compact than the transmission line model. Figure 1.13 shows the S-parameters of the lumped element model of the branch line coupler [8].

![Diagram](image)

**Figure 1.12:** The lumped elements model of the branch line coupler [8].
1.4.3 Transdirectional Coupler

This section illustrates designs for transdirectional couplers. All the implementations of the transdirectional coupler used either the transmission line using Vertically Installed Planar (VIP) or a hybrid between the transmission line and lumped element models.

In [9], the authors published a transdirectional coupler using periodical capacitors. The schematic of the transdirectional coupler is shown in Figure 1.14. $Z_{oe}$ and $\theta_e$ are the characteristic impedance and the electrical impedance of the even mode, respectively. $Z_{oo}$ and $\theta_o$ are the characteristic impedance and the electrical impedance of the odd mode, respectively.
The output ports are port 3 and port 4 as shown in Figure 1.14. The configuration of these capacitors, shown in Figure 1.15, causes complete transfer of power from the upper transmission line to the lower one.

![Transdirectional coupler with three shunt capacitors](image)

Figure 1.15: Transdirectional coupler with three shunt capacitors [9].

The published design was fabricated using FR4 substrate as shown in Figure 1.15. The presented transdirectional coupler works at 3.6 GHz with 3 dB coupling [9]. The authors in [9] reduced the size of the transdirectional coupler by adding three open subs that act as capacitors to the ground as shown in Figure 1.16. The S-parameters of the published design are shown in Figure 1.17.
In [10], the authors presented a transdirectional coupler without the capacitors previously mentioned in [9]. This is illustrated in Figure 1.18. The authors claimed to use vertically installed planar (VIP) circuit structure to design the transdirectional coupler. For the transdirectional coupler, there is impedance and phase velocity coupling that require control for the values of these four parameters: $\theta_e$, $\theta_o$, $Z_{0e}$, and $Z_{0o}$. The reason the authors in [10] used the VIP is that it can be controlled almost independently for $\theta_e$, $\theta_o$, $Z_{0e}$, and $Z_{0o}$. The structure of VIP is shown in Figure 1.19 [10]. The fabricated model and the S-parameters of the
proposed design in [10] are shown in Figure 1.20 and Figure 1.21, respectively. Clearly from Figure 1.19 and Figure 1.20, the VIP is not a practical design from a fabrication point of view.

**Figure 1.18: Schematic of a transdirectional coupler in [10].**

1- Horizontal dielectric
2- Feeder to the circuit
3- Vertical installed dielectric substrate
4- Broadside coupled line on the vertical substrate
5- Blind aperture
6- Ground plate.

**Figure 1.19: The structure of VIP in [10].**

**Figure 1.20: The fabricated VIP transdirectional coupler [10].**
In [11], the authors presented a coupled line transdirectional coupler which is constructed using two coupled transmission lines and four capacitors as shown in Figure 1.22. The authors in [11] divided the parallel coupled line into four parts with different capacitor values as shown in Figure 1.23. The main purpose of that was to control the effective parameters $\theta_e$, $\theta_o$, $Z_{oe}$, and $Z_{0o}$ [11]. The presented design works at 1 GHz with 3 dB coupling coefficient using FR-4 substrate with a relative permittivity $\varepsilon_r=3.5$ as shown in Figure 1.24. The results of the fabricated design are shown in Figure 1.25.

![Figure 1.21: The measured S-parameters of VIP transdirectional coupler [10].](image)

![Figure 1.22: The schematic of the proposed design in [11].](image)
A published study [12] on the transdirectional coupler with adjustable coupling coefficients. The presented design consisted of capacitors, varactors, and inductors as shown in Figure 1.26. The authors
claimed that by connecting the inductors with switches using the varactors, the coupling coefficient can be controlled. They designed the transdirectional coupler with two coupled transmission lines and three capacitors between the two coupled lines, as shown in Figure 1.26. Figure 1.27 shows the fabricated transdirectional model published in [12]. The measured and the simulated S-parameters are shown in Figure 1.28 [12].

![Figure 1.26: The layout of the proposed transdirectional coupler in [12].](image1)

![Figure 1.27: The fabricated transdirectional model in [12].](image2)
Figure 1.28: The measured and simulated S-parameters of transdirectional model in [12].

A pure lumped element model of the transdirectional coupler has not been implemented yet. This thesis presents a pure lumped element model for the transdirectional coupler and associated theory in Chapter 3.

1.5 Thesis Motivation

Transdirectional couplers have been previously implemented using hybrid between transmission line model and lumped element. The transdirectional coupler using lumped element model has not been implemented yet. The lumped element coupler is compact and suitable for systems that have size constraints. A derivation for purely lumped element transdirectional coupler conditions and design equations that operate at any frequency with arbitrary coupling coefficients will be given. A design example for a transdirectional coupler that works at 1 GHz with 3 dB coupling coefficient is presented. The transdirectional coupler
provides isolation between the input ports and the output ports. The simulation is done using ADS and a fabricated model is presented as well.

A codirectional lumped element coupler is also presented. The codirectional coupler provides isolation and DC blocking between the two output ports. Both the codirectional and transdirectional couplers have the same lumped element model. Capacitors are used to present the coupling mechanism between the two transmission lines for both designs. The design conditions for both designs are derived. Like the transdirectional coupler, design equations are presented for the codirectional coupler at any frequency with 3dB coupling coefficient. Simulation and fabrication of both the transdirectional and codirectional couplers are done to verify the design methodology.

1.6 Thesis Contribution

This thesis presents lumped element circuit for the transdirectional and codirectional couplers. Capacitors are used to represent the coupling between the two transmission lines. These capacitors provide isolation and DC blocking between output ports. The design conditions for transdirectional and codirectional couplers are derived. These design conditions for each coupler and the lumped element circuits are used to derive design equations for each coupler. Design equations for the transdirectional coupler that work at any frequency with arbitrary coupling coefficients are demonstrated. The design equations for the codirectional coupler that work at any frequency with a 3 dB coupling coefficient are presented. The PCB layout of transdirectional and the codirectional couplers is the same which makes it convenient to switch between the two designs. The only change is in the component values that can be calculated according to type of coupler.

1.7 Thesis Structure

Chapter 2 discusses the fundamentals of the directional coupler. The coupled line directional coupler and the branch line directional coupler are explained in detail because their basic theory is used in the derivations for the presented transdirectional and codirectional couplers. The lumped element models of the types of directional couplers are also presented.
In Chapter 3, the analyses and formulation for the transdirectional coupler are demonstrated. In addition, the required design conditions for the transdirectional coupler are derived. The design equations for the lumped element model of the transdirectional coupler at any frequency with an arbitrary coupling coefficient are demonstrated. The design equations are proven through simulated and fabricated models for the proposed transdirectional coupler. The simulated and measured results are discussed.

Chapter 4 presents the analyses and the formulation as well as the required design conditions for the codirectional coupler. The design equations for the lumped element model of the codirectional coupler at any frequency with 3 dB coupling coefficient are derived. Simulated and measured results of the codirectional coupler are given.

Finally, Chapter 5 includes the discussion/conclusions and future work. There is also a discussion about the extent to which the proposed designs affect conventional applications in terms of reducing sizes and enhancing performance.
Chapter 2

Directional Couplers Fundamentals

The properties of directional couplers are discussed and several forms of directional couplers are demonstrated and categorized based on the three types (contradirectional, codirectional, and transdirectional) of couplers. General equations for the four-port directional couplers are derived. These equations will be used in Chapter 3 and Chapter 4 to derive the design conditions for the transdirectional and codirectional couplers.

A lumped element model of the coupled line coupler is presented using capacitive coupling to represent the coupling mechanism between two transmission lines. This lumped element model will be used for the presented transdirectional coupler in Chapter 3 and codirectional coupler in Chapter 4.

2.1 Basic Properties of Directional Couplers

The scattering matrix of the four-port directional coupler is derived. The ideal directional coupler is a reciprocal lossless four-port network matched at all ports giving the following scattering matrix [5]:

\[ [S] = \begin{bmatrix}
0 & S_{12} & S_{13} & S_{14} \\
S_{12} & 0 & S_{23} & S_{24} \\
S_{13} & S_{23} & 0 & S_{34} \\
S_{14} & S_{24} & S_{34} & 0
\end{bmatrix} \quad (2.1) \]

By multiplying the conjugate of row 1 by row 2, and multiplying the conjugate of row 4 by row 3 [5]:

\[ S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \quad (2.2) \]

\[ S_{14}^* S_{13} + S_{24}^* S_{23} = 0 \quad (2.3) \]

Eq. (2.2) is multiplied by \( S_{24}^* \), and Eq. (2.3) by \( S_{13}^* \), and then subtracting them we get [5]:

\[ S_{14}^* (|S_{13}|^2 - |S_{24}|^2) = 0 \quad (2.4) \]

By multiplying the conjugate of row 1 by row 3, then multiplying the conjugate of row 4 by row 2 the result is [5]:

20
\[ S_{12}^*S_{23} + S_{14}^*S_{34} = 0 \]  \hspace{1cm} (2.5)

\[ S_{14}^*S_{12} + S_{34}^*S_{23} = 0 \]  \hspace{1cm} (2.6)

By multiplying Eq. (2.5) by \( S_{12} \), and Eq. (2.6) by \( S_{34} \), and then subtracting them we get [5]:

\[ S_{23} (|S_{12}|^2 - |S_{34}|^2) = 0 \]  \hspace{1cm} (2.7)

Eq. (2.4) and Eq. (2.7) are satisfied if \( S_{14} = S_{23} = 0 \), which results in a directional coupler.

By taking the self-product of the rows in Eq. (2.1), we obtain the following [5]:

\[ |S_{12}|^2 - |S_{13}|^2 = 1 \]  \hspace{1cm} (2.8)

\[ |S_{12}|^2 - |S_{24}|^2 = 1 \]  \hspace{1cm} (2.9)

\[ |S_{13}|^2 - |S_{34}|^2 = 1 \]  \hspace{1cm} (2.10)

\[ |S_{24}|^2 - |S_{34}|^2 = 1 \]  \hspace{1cm} (2.11)

These result in [1]:

\[ |S_{13}| = |S_{24}| \]

\[ |S_{12}| = |S_{34}| \]

More simplification is done by giving the phase references on three of the four-port such that

\[ S_{12} = S_{34} = \alpha, S_{13} = \beta e^{j\theta}, \text{ and } S_{24} = \beta e^{j\phi} \]

Where \( \alpha \) and \( \beta \) are real, and \( \theta \) and \( \phi \) are phase constants.

Taking the dot product of row 2 and row 3 results in [5]:

\[ S_{12}^*S_{13} + S_{24}^*S_{34} = 0 \]  \hspace{1cm} (2.12)

This yields [1]:

\[ \theta + \phi = \pi \pm 2\pi \]  \hspace{1cm} (2.13)
There are two options for the phase difference between the output ports [5]:

1. A Symmetric coupler when $\theta = \phi = \pi/2$. The scattering matrix is the following:

$$
[S] = \begin{bmatrix}
0 & \alpha & j\beta & 0 \\
\alpha & 0 & 0 & j\beta \\
j\beta & 0 & 0 & \alpha \\
0 & j\beta & \alpha & 0
\end{bmatrix}
$$  \hspace{1cm} (2.14)

2. An Antisymmetric coupler $\theta = 0, \phi = \pi$. The scattering matrix is the following:

$$
[S] = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0
\end{bmatrix}
$$  \hspace{1cm} (2.15)

The four main characteristics for the directional coupler are:

1. **Coupling factor**: the ratio between the input power and the coupled power.

2. **Directivity**: the ratio between the output power at the coupled port and the power delivered to the isolated port.

3. **Isolation**: the ratio between the input power and the power delivered to the isolated port.

4. **Insertion loss**: the ratio between the input power to the power delivered to the through port.

Further discussion about the positions of the coupled and isolated ports will be presented later.

### 2.2 Forms of Directional Couplers

This section discusses several forms of the directional coupler. We classify each one according to the type of directional coupler. In addition, the S-parameters at each port for four-port directional couplers are derived. These equations will be used in the next two chapters.
2.2.1 The Quadrature (90°) Hybrid (Codirectional)

The quadrature (90°) hybrid is also called branch line hybrid, which is codirectional according to its relative location to the input port as shown in Figure 1.6(b). Quadrature (90°) hybrid is a 3dB coupler, which means that half of the power goes to the through port and the other half goes to the coupled port, with 90° between the two ports. The geometry for the branch line coupler is shown in Figure 1.10. The power enters at port 1 and is then divided into the output ports (port 2 and port 3) while no power goes to port 4 (the isolated port) [5].

The scattering matrix of the Quadrature (90°) hybrid by substituting $\alpha = \beta = \frac{1}{\sqrt{2}}$ and $\theta = \phi = \pi /2$ in Eq. (2.14) is [5]:

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$  (2.16)

A branch line coupler has a symmetric structure as shown in Figure 1.10 [5]. Branch line coupler can be analyzed by even and odd modes [5].

**Even-Odd Mode Analyses for four-port networks**

Even and odd excitation modes can be used to decompose a symmetrical four-port network as shown in Figure 2.1 [5]. A symmetrical four-port network can be decomposed into two-port networks as shown in Figure 2.2 [5]. When applying the same amplitude $\frac{1}{2}$ and in phase due to the symmetry, the voltage will be equal at all ports. Furthermore, the open circuit will be between the upper ports (port 1 and port 2) and the lower ports (port 3 and port 4); this is known as the even mode as shown in Figure 2.2(a). The odd excitation has the same amplitude $\frac{1}{2}$ but with negative phase as shown in Figure 2.2(b) [5].
By merging the amplitudes at each port, the total reflection coefficient, \( S_{ij} \) (where \( i, j = 1, 2, 3, \ldots n \)), at each port can be expressed by [5]:

\[
S_{11} = \frac{1}{2} \Gamma_e + \frac{1}{2} \Gamma_o = \frac{1}{2} (S_{11}^e + S_{11}^o)
\] (2.17)

\[
S_{21} = \frac{1}{2} \Gamma_e + \frac{1}{2} \Gamma_o = \frac{1}{2} (S_{21}^e + S_{21}^o)
\] (2.18)

\[
S_{31} = \frac{1}{2} \Gamma_e - \frac{1}{2} \Gamma_o = \frac{1}{2} (S_{31}^e - S_{31}^o)
\] (2.19)

\[
S_{41} = \frac{1}{2} \Gamma_e - \frac{1}{2} \Gamma_o = \frac{1}{2} (S_{41}^e - S_{41}^o)
\] (2.20)
$S^e_{21}$ and $S^e_{31}$ ($T_e$) are the transmission coefficients in the even mode, $S^o_{11}$ and $S^o_{41}$ ($T_o$) are the transmission coefficients in the odd mode, $S^e_{11}$ and $S^o_{41}$ ($T_e$) are the reflection coefficients in the even mode, and $S^o_{11}$ and $S^o_{41}$ ($T_o$) are the reflection coefficients in the odd mode. Eqs. (2.17), (2.18), (2.19) and (2.20) are general equations that show the reflection coefficient at each port of the four-port network. These equations will be used in Chapter 3 to derive the conditions for the transdirectional coupler and will be used in Chapter 4 to derive the codirectional coupler.

**Even-Odd Mode Analyses for Branch Line Coupler (Codirectional coupler)**

The even and odd analyses are applied to the branch line coupler to get an expression for the S-parameters at each port. The resultant S-parameters will be used in Chapter 4 to derive design equations for the presented codirectional coupler. First, the reflection and transmission coefficients for the even mode are calculated by ABCD matrices of each cascaded component [5]. Assuming that the branch line coupler works at a center frequency of $f_0$, the length of each branch can be calculated from [5]:

$$l = \frac{\lambda_0}{4} = \frac{c}{4f_0} = \frac{\pi c}{2\omega_0}$$

(2.21)

Where $\lambda_0$ is the wavelength at the center frequency, $\omega_0$ is the frequency in radian, and $c$ is the speed of light. According to [5], the ABCD matrix of a transmission line with characteristic impedance $Z_c$ and length $l$ can be expressed as follows [5]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \beta l & jZ_c \sin \beta l \\ jY_c \sin \beta l & \cos \beta l \end{bmatrix}$$

(2.22)

Where $\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ and $l = \frac{\lambda}{4}$. Therefore, the phase constant is $\beta l = \frac{\pi}{2}$. By substituting the impedance values from Figure 2.1, the ABCD matrix becomes [5]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \frac{\pi}{2} & j\frac{1}{\sqrt{2}} \sin \frac{\pi}{2} \\ j\sqrt{2} \sin \frac{\pi}{2} & \cos \frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & j/\sqrt{2} \\ j/\sqrt{2} & 0 \end{bmatrix}$$

(2.23)

The characteristic impedance of the open circuit stub is [5]:

$$Z_{oc} = -jZ_c \cot \beta l$$

(2.24)

Where $l = \frac{\lambda_0}{8}$ and $\beta = \frac{2\pi}{\lambda_0}$. Therefore, $\beta l = \frac{\pi}{4}$. 

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The ABCD matrix of the shunt open circuit stub is [5]:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
\tan \frac{\pi}{4} & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
j & 1
\end{bmatrix}
\]  

(2.25)

For the even excitation mode, the ABCD matrix is the following:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_e = \begin{bmatrix}
1 & 0 \\
j \tan \frac{\pi}{4} & 1
\end{bmatrix} \begin{bmatrix}
\cos \frac{\pi}{2} & j \frac{1}{\sqrt{2}} \sin \frac{\pi}{2} \\
\frac{1}{\sqrt{2}} \sin \frac{\pi}{2} & \cos \frac{\pi}{2}
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
j \tan \frac{\pi}{4} & 1
\end{bmatrix}
\]  

(2.26)

For the even mode, the reflection and transmission coefficients are the following [5]:

\[
T_e = \frac{2}{A + B + C + D}
\]

(2.27)

\[
\Gamma_e = \frac{A + B - C - D}{A + B + C + D}
\]

(2.28)

For the shunt, the short circuit stub is [5]:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
-j \cot \frac{\pi}{4} & 1
\end{bmatrix}
\]  

(2.29)

Similarly, for the odd excitation mode, the ABCD matrix is the following:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_o = \begin{bmatrix}
1 & 0 \\
-j \cot \frac{\pi}{4} & 1
\end{bmatrix} \begin{bmatrix}
\cos \frac{\pi}{2} & j \frac{1}{\sqrt{2}} \sin \frac{\pi}{2} \\
\frac{1}{\sqrt{2}} \sin \frac{\pi}{2} & \cos \frac{\pi}{2}
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
-j \cot \frac{\pi}{4} & 1
\end{bmatrix}
\]  

(2.30)

For the odd mode, the reflection and transmission coefficients are the following [5]:

\[
T_o = \frac{2}{A + B + C + D}
\]

(2.31)

\[
\Gamma_o = \frac{A + B - C - D}{A + B + C + D}
\]

(2.32)

By substituting the admittance of the shunt open circuit stub with \(\frac{\lambda}{8}\) in Eqs. (2.27) and (2.28), they become [5]:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_e = \begin{bmatrix}
1 & 0 \\
j \frac{1}{\sqrt{2}} & 1
\end{bmatrix} \begin{bmatrix}
0 & j \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & 1
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
1 & -1
\end{bmatrix}
\]

(2.33)

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}_o = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & j \\
j & 1
\end{bmatrix}
\]

(2.34)
These two equations imply that Eqs. (2.27), (2.28), (2.31) and (2.32) are [5]:

\[ T_e = S_{21}^e = S_{31}^e = \frac{2}{A+B+C+D} = \frac{2}{(-1+j+j-1)/\sqrt{2}} = -\frac{1}{\sqrt{2}}(1+j) \]  
(2.35)

\[ I_e = S_{11}^e = S_{41}^e = \frac{A+B-C-D}{A+B+C+D} = \frac{(-1+j-j+1)/\sqrt{2}}{(-1+j+j-1)/\sqrt{2}} = 0 \]  
(2.36)

\[ T_o = S_{21}^o = S_{31}^o = \frac{2}{A+B+C+D} = \frac{2}{(-1+j+j-1)/\sqrt{2}} = \frac{1}{\sqrt{2}}(1-j) \]  
(2.37)

\[ I_o = S_{11}^o = S_{41}^o = \frac{A+B-C-D}{A+B+C+D} = 0 \]  
(2.38)

Given \( A_1 = 1 \), substituting Eqs. (2.27), (2.28), (2.31) and (2.32) into Eqs. (2.35), (2.36), (2.37) and (2.38) yields [5]:

\[ S_{11} = \frac{B_1}{A_1} = B_1 = \frac{1}{2} (S_{11}^e + S_{11}^o) = 0 \]  
(2.39)

\[ S_{21} = \frac{B_2}{A_1} = B_2 = \frac{1}{2} (S_{21}^e + S_{21}^o) = -\frac{j}{\sqrt{2}} \]  
(2.40)

\[ S_{31} = \frac{B_3}{A_1} = B_3 = \frac{1}{2} (S_{31}^e - S_{31}^o) = -\frac{1}{\sqrt{2}} \]  
(2.41)

\[ S_{41} = \frac{B_4}{A_1} = B_4 = \frac{1}{2} (S_{41}^e - S_{41}^o) = 0 \]  
(2.42)

The derivations from Eqs. (2.35) to (2.42) will be used to get the design equations for the presented codirectional coupler in Chapter 4.

### 2.2.2 Lumped Element Circuits of Codirectional Coupler

The transmission line model of the branch line hybrid coupler was previously mentioned and its schematic is shown in Figure 1.10. This transmission line model could be converted to lumped element model. The basic structure of the branch line coupler consists of four quarter wave length transmission lines with different impedances as shown in Figure 2.3. Each of these \( \frac{\lambda}{4} \) transmission lines can be converted to the \( \pi \) equivalent circuit as shown in Figure 2.4. This gives perfect matching between the four ports. The transmission line length is calculated based on the centered frequency of the application [13].
The values of the lumped element can be calculated by equating the ABCD matrices for the two structures [5]. The ABCD matrix of a lossless transmission line section $Z_c$ and electrical length $\theta$ is the following [5]:

\[
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} = \begin{bmatrix}
\cos \theta & jZ_c \sin \theta \\
j \frac{\sin \theta}{Z_c} & \cos \theta \\
\end{bmatrix}
\]  

(2.43)

The ABCD matrix of $\pi$ equivalent lumped element is the following [5]:

\[
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
j \omega L & 1 \\
\end{bmatrix} \begin{bmatrix}
1 & j \omega L \\
1 & 1 \\
\end{bmatrix}
\]

(2.44)

which can be simplified to [5]:

\[
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{j \omega C} & 0 \\
0 & 1 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 - \omega^2 LC & j\omega L \\
2j\omega C - j\omega^3 LC^2 & 1 - \omega^2 LC
\end{bmatrix}
\]  

(2.45)

Where \( \omega \) is the frequency in radian.

By equating Eq. (2.43) with Eq. (2.45) we get [5]:

\[
L = \frac{Z_c\sin\theta}{\omega}
\]  

(2.46)

\[
C = \frac{1}{\omega Z_c} \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}
\]  

(2.47)

Where \( \theta = 90^\circ \) because the transmission line is quarter-wavelength [5]. The lumped element values can be calculated from the following equations [5]:

\[
L = \frac{Z_c}{\omega}
\]  

(2.48)

\[
C = \frac{1}{\omega Z_c}
\]  

(2.49)

By replacing each circuit with the equivalent circuits, we get the equivalent lumped element model of branch line hybrid coupler as shown in Figure 2.5.
Figure 2.5 shows the equivalent lumped element circuit of the branch line coupler where [5]:

\[ Z_{01} = Z_0 = 50 \, \Omega \] (2.50)

\[ Z_{02} = \frac{Z_0}{\sqrt{2}} = 35 \, \Omega \] (2.51)

The following equations were used to obtain the component values of the lumped element model:

\[ L_{01} = \frac{Z_{01}}{\omega_0} \] (2.52)

\[ L_{02} = \frac{Z_{02}}{\omega_0} \] (2.53)

\[ C_{01} = \frac{1}{Z_{01}\omega_0} \] (2.54)

\[ C_{02} = \frac{1}{Z_{02}\omega_0} \] (2.55)

Substituting Eqs. (2.50) and (2.51) into Eqs. (2.52), (2.53), (2.54), and (2.55) we get [5]:

Figure 2.5: The lumped element model of the codirectional coupler [15].
\[ L_{01} = \frac{z_0}{\sqrt{2}\omega_0} \]  
\[ L_{02} = \frac{z_0}{\omega_0} \]  
\[ C_{01} = \frac{\sqrt{2}}{z_0\omega_0} \]  
\[ C_{02} = \frac{1}{z_0\omega_0} \]

The design shown in Figure 2.5 could be simplified to be as shown in Figure 1.12, to reduce the number of lumped element and the ease of fabrication.

The transdirectional coupler has not been implemented so far in lumped elements. All the designs that have been published are a hybrid of the lumped element and the transmission line models. In the next chapter, we will show the lumped element model of the transdirectional coupler. In addition, the design equations will be derived.

### 2.2.3 Coupled Line Directional Coupler

Coupled line directional coupler is a type of directional coupler which is formed by closing two unshielded transmission lines. Due to the electromagnetic interaction, the coupler can be coupled from one line to the other. Several examples of coupled transmission lines are shown in Figure 2.6.
The most common coupled line directional coupler is the conventional (contradirectional) coupler that is shown in Figure 1.7. In the next section, the even and odd analyses will be applied in a coupled line directional coupler to get the S-parameters at each port. These S-parameters will be used to derive the presented transdirectional and codirectional coupler in Chapter 3 and Chapter 4, respectively.

**Analysis of two coupled transmission lines**

The even and odd analyses are applied to a general, symmetric four-port directional coupler to get the scattering at each port. For the four-port network shown in Figure 2.7, each port is terminated with $Z_o$ and port 1 uses a voltage generator with $2V_o$ as shown in Figure 2.8.
In this analysis, the operation of each port is not named. The focus here is just studying and analyzing a four-port network coupled line directional coupler. The function of each port is mentioned in the next chapter with more detail about the type of coupler in terms of the relative location to the input port. By superposition, the excitation at port 1 in Figure 2.8 can be a sum of the excitation of even and odd modes as shown in Figure 2.9.

Figure 2.8: Analysis of two coupled lines [5].
Figure 2.9: Even and odd modes excitations [5].
Due to the symmetry for the even excitation mode [5]:

\[ V_1^e = V_4^e \]
\[ V_2^e = V_3^e \]
\[ I_1^e = I_4^e \]
\[ I_2^e = I_3^e \]

For the odd mode we apply the same magnitude with negative phase of voltage so we can see that:

\[ V_1^o = -V_4^o \]
\[ V_2^o = -V_3^o \]
\[ I_1^o = -I_4^o \]
\[ I_2^o = -I_3^o \]

The input impedance at port 1 from Figure 2.9 can be expressed as:

\[ Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o} \quad (2.60) \]

As shown in Figure 2.9, the characteristic impedance for the even and odd modes are \( Z_{oe} \) and \( Z_{oo} \), respectively. Letting \( Z_{in}^e \) be the input impedance for the even mode and \( Z_{in}^o \) be the input impedance of the odd mode, both are expressed using Eqs. (2.61) and (2.62):

\[ Z_{in}^e = Z_{oe} \frac{Z_{oe} + jZ_{oe} \tan(\theta)}{Z_{oe} + jZ_o \tan(\theta)} \quad (2.61) \]
\[ Z_{in}^o = Z_{oo} \frac{Z_{oo} + jZ_{oo} \tan(\theta)}{Z_{oo} + jZ_o \tan(\theta)} \quad (2.62) \]

Since for each mode we have a transmission line of characteristic impedance \( Z_{oe} \) and \( Z_{oo} \) terminated with load impedance \( Z_o \), the voltage division can be expressed by:

\[ V_1^e = V_o \frac{Z_{in}^e}{Z_{in}^e + Z_o} \quad (2.63) \]
\[ V_1^o = V_o \frac{Z_{in}^o}{Z_{in}^o + Z_o} \quad (2.64) \]
\[ I_1^e = \frac{V_o}{Z_{in}^e + Z_o} \quad (2.65) \]
\[ I^0 = \frac{V_o}{Z_{in}^0 + Z_o} \quad (2.66) \]

Substituting Eqs. (2.63), (2.64), (2.65), and Eqs. (2.66) into (2.60) yields:

\[ Z_{in} = Z_o + \frac{2(Z_{oe}^0 - Z_o Z_{in}^0)}{Z_{in}^0 + Z_o^0 + 2Z_o} \quad (2.67) \]

To get an expression for \( Z_{in} \) for each mode in terms of \( Z_{oe} \) and \( Z_{oo} \) we let:

\[ Z_o = \sqrt{Z_{oe} Z_{oo}} \quad (2.68) \]

\[ Z_{oe} Z_{oo} = Z_{in}^e Z_{in}^o = Z_o^2 \quad (2.69) \]

Eq. (2.67) can then be reduced to:

\[ Z_{in} = Z_o \quad (2.70) \]

As long as we satisfy Eq. (2.68), we get Eq. (2.70) at each port which means that port 1 and all other ports are matched.

Due to the matching at port 1, we have the following:

\[ V_o = V_1 \quad (2.71) \]

By taking the voltage division at port 1 [5]:

\[ V_1 = V_1^e - V_1^o = V_o \left[ \frac{Z_{in}^e}{Z_{in}^e + Z_o} - \frac{Z_{in}^o}{Z_{in}^o + Z_o} \right] \quad (2.72) \]

\[ \frac{Z_{in}^e}{Z_{in}^e + Z_o} = \frac{Z_o + j Z_{oe} \tan(\theta_e)}{2Z_o + j (Z_{oe} + Z_{oo}) \tan(\theta_e)} \quad (2.73) \]

\[ \frac{Z_{in}^o}{Z_{in}^o + Z_o} = \frac{Z_o + j Z_{oo} \tan(\theta_o)}{2Z_o + j (Z_{oe} + Z_{oo}) \tan(\theta_o)} \quad (2.74) \]

\[ V_1 = V_o \frac{j (Z_{oe} - Z_{oo}) \tan(\theta)}{2Z_o + j (Z_{oe} + Z_{oo}) \tan(\theta)} \quad (2.75) \]

The definition of the coupling coefficient is therefore expressed as [5]:

\[ C = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \quad (2.76) \]

By dividing both sides of Eq. (2.75) by \( V_o \), we get the reflection coefficient in terms of even and odd characteristic impedance. We need to have an expression for the reflection coefficient in terms of the characteristic impedance in the even mode and another expression for the reflection coefficient in terms of
the characteristic impedance in the odd mode. To make it clear, if the reflection coefficient of the even mode is on the left-hand side then all parameters on the right-hand side should be for the even mode. This can be done by substituting Eq. (2.69) in Eq. (2.75) [5]. Let \( S_{i1}^e \) and \( S_{ij}^o \) be the even and odd mode S-parameters of the coupler, respectively, where \( i,j=1,2,3,4 \).

\[
S_{11}^e = S_{41}^e = \Gamma_e = \frac{j\left(\frac{Z_{ae}}{Z_0} - \frac{Z_o}{Z_{ae}}\right)\sin(\theta_e)}{2\cos(\theta_e) + j\left(\frac{Z_{ae}}{Z_0} + \frac{Z_o}{Z_{ae}}\right)\sin(\theta_e)} \tag{2.77}
\]

\[
S_{11}^o = S_{41}^o = \Gamma_o = \frac{j\left(\frac{Z_{ao}}{Z_0} - \frac{Z_o}{Z_{ao}}\right)\sin(\theta_o)}{2\cos(\theta_o) + j\left(\frac{Z_{ao}}{Z_0} + \frac{Z_o}{Z_{ao}}\right)\sin(\theta_o)} \tag{2.78}
\]

Similarly, for the other ports:

\[
S_{21}^e = S_{31}^e = \Gamma_e = \frac{2}{2\cos(\theta_e) + j\left(\frac{Z_{ae}}{Z_0} + \frac{Z_o}{Z_{ae}}\right)\sin(\theta_e)} \tag{2.79}
\]

\[
S_{21}^o = S_{31}^o = \Gamma_o = \frac{2}{2\cos(\theta_o) + j\left(\frac{Z_{ao}}{Z_0} + \frac{Z_o}{Z_{ao}}\right)\sin(\theta_o)} \tag{2.80}
\]

These general equations Eqs. (2.77), (2.78), (2.79) and (2.80) will be used in the next two chapters to begin the main derivation for the presented transdirectional and codirectional directional couplers.

### 2.2.4 Lumped Element Model of Contradirectional Coupler.

The contradirectional coupler is the most mentioned coupler in literature. The schematic and the lumped element model of the contradirectional coupler was shown in Figure 1.8, where the inductor was used to represent the mutual inductance between the two coupled lines. The transmission line circuit of the contradirectional coupler was implemented in [5]. As we mentioned previously in the literature review, there are two types of coupling between the two coupled lines: inductive and capacitive. The only way to fabricate a lumped element model of the contradirectional coupler is to use an inductor. This inductor, shown in Figure 1.8, represented the mutual inductance between the two lines. The authors in [7] derived formulas for calculating the lumped element model of the contradirectional coupler. The authors in [7] used the even and odd modes to derive generic equations for their lumped element model of the contradirectional coupler. Since the directional coupler is vertically and horizontally symmetrical, the lumped element model and the
distributed model can be analyzed using even and odd modes. Figure 2.10 shows how the authors applied the even and odd modes on the distributed and lumped models of the contradirectional coupler.

![Figure 2.10: Applying the even and odd modes on the proposed contradirectional coupler in [7].](image)

It can be seen in Figure 2.10 that the authors in [7] cut the distributed and the lumped element into four planes. The first plane applied the even mode vertically and horizontally, the second plane applied the odd mode horizontally and vertically, the third one applied the odd mode vertically and the even mode horizontally, and the fourth mode applied the even mode vertically and the odd mode horizontally. After that, they started to calculate the input impedance for each mode for the distributed and lumped elements models and equated them. Finally, they got generic equations for the lumped element components in terms of the electrical length, even and odd impedances, and frequency. The four equations are the following [7]:

\[
C_e = \frac{\tan (\frac{\theta}{2})}{\omega_0 Z_{oe}} \quad (2.81)
\]

\[
L_e = \frac{Z_{oe} \sin (\theta)}{2\omega_0} \quad (2.82)
\]
\[ C_\alpha = \frac{\omega_0 L_e + M \tan \left( \frac{\theta}{2} \right) - \omega_0 \tan \left( \frac{\theta}{2} \right)}{-2 L_e \tan \left( \frac{\theta}{2} \right) \omega_0 Z_\alpha} \]  

(2.83)

\[ L_\alpha = \frac{-\omega_0 L_e \tan \left( \frac{\theta}{2} \right) \tan \left( \frac{\theta}{2} \right) + M + 2 \omega_0 Z_\alpha L_e C_\alpha - Z_\alpha}{-2 C_\alpha \omega_0 Z_\alpha + \omega_0 \tan \left( \frac{\theta}{2} \right) Z_\alpha - \omega_0 \omega_0 C_e} \]  

(2.84)

Where:

\[ M = Z_\alpha \omega_0^2 L_e C_e \]  

(2.85)

The presented design in [7] used an inductor to represent the inductive coupling between the two lines. This inductor allows the DC to go through port 3 and port 4 which does not happen in the transmission line model of the conventional coupler that is shown in Figure 1.8. The difficulty with this design is that the DC can go through all ports, which is not the case for the original coupled line.

### 2.2.5 The Lumped Element Model of the Coupled Line Directional Coupler

The transmission line is represented by two wires as shown in Figure 2.11 [14].

![Figure 2.11: Equivalent transmission line circuit](image)

In Figure 2.11, the characteristic impedance and the phase velocity of the transmission line are \( Z_0 \) and \( V_{ph} \), respectively, and \( d \) is the length of the line. This transmission line can be equivalent to the series
inductance, \( L \), and shunt capacitance per unit length, \( C \). \( l \) and \( C \) are the actual inductance and shunt capacitance in Henry and Farad, respectively. The equivalent circuit of the transmission line is used to get the lumped element model of the coupled line directional coupler that is shown in Figure 2.7. This model will be implemented for both transdirectional and codirectional couplers. Before we represent the lumped element of the coupled line directional coupler, we need to model the capacitive coupling between the two coupled lines.

As mentioned previously, by closing two unshielded transmission lines, the power can be coupled from one transmission line to another because of the electromagnetic field interactions. The equivalent circuit of these coupled lines of Figure 2.6 is shown in Figure 2.12. As shown in Figure 2.12, \( C_{12} \) represents the capacitance between the two lines, and \( C_{11} \) and \( C_{22} \) represent the capacitance between each line and the ground. We can assume that the two lines have the same size and location relative to the ground conductor.

![Figure 2.12: Coupled transmission line and its equivalent capacitance circuit](image)

An even and odd analysis was conducted on the transmission line model. In the even mode excitation, the currents in both lines have the same magnitude and direction as shown in Figure 2.13 (a). There are no current flows between the two lines. This yields \( C_e = C_{11} = C_{22} \), where \( C_{12} \) is open circuit, and \( C_e \) represents the capacitance in the even mode. The characteristic impedance in the even mode is given by [5]:

\[
Z_{0e} = \frac{\sqrt{C_e}}{\sqrt{C_e}}
\]

(2.86)

\[
Z_{0e} = \frac{1}{\nu_{ph} C_e}
\]

(2.87)
In the odd mode, the currents have the same magnitude but in opposite directions as shown in Figure 2.13(b). Therefore, there is a voltage null between the two lines in the middle of $C_{12}$. This yields to an effective capacitance in the odd mode between the two lines which is represented by:

$$C_o = C_{11} + 2C_{12} \quad (2.88)$$

The characteristic impedance in the odd mode is [5]:

$$Z_{0o} = \frac{C_o}{\sqrt{C_o}} \quad (2.89)$$

$$Z_{0o} = \frac{1}{\nu_{ph} C_o} \quad (2.90)$$

These equations will be used in Chapter 3 and Chapter 4 to get the component values of the lumped element for the presented designs.

*Figure 2.13: Even and odd excitations for coupled line [5]. (a) Even mode excitation. (b) Odd mode excitation*
It is evident that the capacitive coupling is used to represent the coupling mechanism (electromagnetic interactions) between the two lines. The presented lumped element model of the coupled line directional coupler is shown in Figure 2.14. The lumped element model will be the same for the transdirectional and codirectional couplers. We will illustrate the reason the contradirectional (conventional) coupler can be implemented using this lumped element model in later chapters.

In Figure 2.14, $L_e$ is the series inductance of the transmission line, $C_e$ is the capacitance between each transmission line and the ground in the even mode, and $C_{12}$ represents the capacitance between the two transmission lines. Note that Figure 2.14 behaves from a DC point of view exactly like the original coupled transmission line. As will be shown later, this structure will produce the codirectional and transdirectional couplers.
2.3 Summary

In this chapter, we derived the S-parameters for four-port networks at each port. We then demonstrated the S-parameters for the branch line coupler. Next, we illustrated general S-parameters for the coupled line directional coupler. The derived S-parameter conditions will be used to derive conditions for the transdirectional coupler in Chapter 3 and the codirectional coupler in Chapter 4. Finally, the general lumped element model based on the capacitive coupling between the two coupled lines is presented and used for both the codirectional and transdirectional coupler circuits. Design conditions and design equations for the codirectional and transdirectional couplers will be derived.
Chapter 3

Transdirectional Coupler

3.1 Motivation

A design of the lumped element transdirectional coupler with required design conditions for arbitrary coupling coefficients is presented. Capacitors are used to represent the coupling mechanism between the coupled lines. A 3 dB transdirectional coupler is designed and fabricated using FR-4 substrate at 1 GHz. A comparison between the simulated and measured results will be given.

3.2 Conditions of Transdirectional Coupler

Transdirectional coupler is a type of directional coupler that has the output from port 3 and port 4 as shown in Figure 3.1. Since directional couplers are symmetrical four-port network, we can recall the following S-parameters from Eqs. (2.17), (2.18), (2.19) and (2.20) [5]:

\[
S_{11} = \frac{1}{2} (S_{11}^e + S_{11}^o) = 0
\]

(3.1)

\[
S_{21} = \frac{1}{2} (S_{21}^e + S_{21}^o) = 0
\]

(3.2)

\[
S_{31} = \frac{1}{2} (S_{31}^e - S_{31}^o)
\]

(3.3)

\[
S_{41} = \frac{1}{2} (S_{41}^e - S_{41}^o)
\]

(3.4)

![Figure 3.1: Transmission line model of transdirectional coupler.](image-url)
For the transdirectional coupler geometry, port 1 is the input port and port 2 is the isolated port. This means that there is no power reflected at port 1 and no power delivered to port 2. Therefore, Eqs. (3.1) and (3.2) become [5]:

\[
S_{11}^0 = -S_{11}^0 \quad (3.5)
\]

\[
S_{21}^0 = -S_{21}^0 \quad (3.6)
\]

By substituting Eqs. (2.77) and (2.78) into Eq. (3.5):

\[
\frac{j\left(\frac{Z_{oe}}{Z_0} - \frac{Z_o}{Z_{oe}}\right)\sin(\theta_e)}{2\cos(\theta_e) + j\left(\frac{Z_{oe}}{Z_0} + \frac{Z_o}{Z_{oe}}\right)\sin(\theta_e)} = -\frac{j\left(\frac{Z_{oo}}{Z_0} - \frac{Z_o}{Z_{oo}}\right)\sin(\theta_o)}{2\cos(\theta_o) + j\left(\frac{Z_{oo}}{Z_0} + \frac{Z_o}{Z_{oo}}\right)\sin(\theta_o)} \quad (3.7)
\]

By substituting Eqs. (2.79) and (2.80) into Eq. (3.6):

\[
\frac{2}{2\cos(\theta_e) + j\left(\frac{Z_{oe}}{Z_0} + \frac{Z_o}{Z_{oe}}\right)\sin(\theta_e)} = \frac{2}{2\cos(\theta_o) + j\left(\frac{Z_{oo}}{Z_0} + \frac{Z_o}{Z_{oo}}\right)\sin(\theta_o)} \quad (3.8)
\]

To satisfy Eqs. (3.7) and (3.8), the transdirectional coupler should have the conditions:

\[
Z_{oe}Z_{oo} = Z_o^2 \quad (3.9)
\]

\[
\theta_e - \theta_o = (2n + 1)\pi \quad (3.10)
\]

Eq. (3.9) and Eq. (3.10) are the design conditions for the transdirectional coupler. These design conditions will be taken into consideration when deriving the design equations for lumped element design.

### 3.3 Lumped Element Model of Transdirectional Coupler

The presented lumped element in Chapter 2 is used to implement the transdirectional coupler, as shown in Figure 3.2. Each line of the transmission line model in Figure 3.1 is represented by two inductors \(L_e\) and three capacitors \(C_e\) that represent the interaction between the line and the ground as shown in Figure 3.2. The capacitive coupling is represented by the capacitors \(C_{12}\).
3.4 Design Equations for the Lumped Element Transdirectional Coupler

The design equations for a lumped element transdirectional coupler should satisfy the derived conditions in Eqs. (3.9) and (3.10). These derived design equations work at any frequency with arbitrary coupling coefficients.

The design procedure begins by relating the characteristic impedance in the even mode \( Z_{oe} \) and the characteristic impedance in the odd mode \( Z_{oo} \) with the desired coupling coefficient \( C \). This can be obtained by substituting Eqs. (3.9) into (2.76) [5]:

\[
Z_{oe} = Z_0 \sqrt{\frac{1+C}{1-C}} \quad [\Omega] \quad (3.11)
\]

\[
Z_{oo} = Z_0 \sqrt{\frac{1-C}{1+C}} \quad [\Omega] \quad (3.12)
\]
Since we know the desired coupling coefficient between the output ports and the desired characteristic impedance for the design $Z_0$, we can obtain $Z_{oo}$ and $Z_{oe}$. These values of $Z_{oo}$ and $Z_{oe}$ satisfy condition Eq. (3.9). The following equations are used to calculate the values of $\mathcal{L}_e, \mathcal{C}_e, \mathcal{C}_{12}$ [5]:

$$\mathcal{C}_e = \frac{1}{u_{ph} Z_{oe}} \quad [F/m] \quad (3.13)$$

$$u_{ph} = \frac{c}{\sqrt{\varepsilon_r}} \quad [m/s] \quad (3.14)$$

$$Z_{oe} = \frac{Z_{oe}}{\sqrt{\varepsilon_e}} \quad [\Omega] \quad (3.15)$$

$$\mathcal{L}_e = Z_{oe} \times \mathcal{C}_e \quad [H/m] \quad (3.16)$$

$$\mathcal{C}_o = \frac{1}{u_{ph} Z_{oa}} \quad [F/m] \quad (3.17)$$

$$\mathcal{C}_o = \mathcal{C}_e + 2\mathcal{C}_{12} \quad [F/m] \quad (3.18)$$

Where $u_{ph}$ is the phase velocity, $\mathcal{L}_e$ is the inductance per unit length in the even mode, $\mathcal{C}_e$ represents the capacitance per unit length between the line and the ground in the even mode, $\mathcal{C}_{12}$ represents the coupling capacitance per unit length, and $\mathcal{C}_o$ is the odd capacitance (effective capacitance) per unit length between the two lines in the odd mode. We have relative permittivity of $\varepsilon_r = 1$ since we are dealing with the lumped element. Therefore, Eq. (3.14) reduces to $u_{ph} = c$ where $c$ is the speed of light ($3 \times 10^8 \ m/s$).

![Figure 3.3: Transdirectional coupler even and odd mode circuits.](image-url)
The second step is to get the actual values of the capacitors and the inductors that are shown in Figure 3.2. In Figure 3.3, $\theta_e$ and $\theta_o$ are the electrical lengths in the even and odd modes, respectively, and $d_e$ and $d_o$ are the transmission line length in the even and odd modes, respectively. Eq. (3.10) defines a condition for the electrical length in the even and odd mode. This condition should be applied to design the lumped element transdirectional coupler. By assuming any values of $\theta_e$ and $\theta_o$ that satisfy condition Eq. (3.10), we obtain the transmission line length in the even mode, $d_e$, and odd mode, $d_o$:

$$\theta_e = \beta d_e \quad (3.19)$$

$$d_e = \frac{\theta_e}{\beta} \quad (3.20)$$

Where $\beta$ is the propagation constant and can be calculated by:

$$\beta = \frac{2\pi}{\lambda} \quad (3.21)$$

$$\lambda = \frac{c}{f} \quad (3.22)$$

Where the speed of light $c = 3 \times 10^8 \text{ m/s}$ and $f$ is the desired frequency.

$$\theta_o = \beta d_o \quad (3.23)$$

$$d_o = \frac{\theta_o}{\beta} \quad (3.24)$$

We can then obtain:

$$C_e = C_e \ast d_e \quad \text{[F]} \quad (3.25)$$

$$l_e = L_e \ast d_e \quad \text{[H]} \quad (3.26)$$

$$C_{12} = C_{12} \ast d_o \quad \text{[F]} \quad (3.27)$$

All the parameters for the transdirectional coupler can now be calculated according to the derived design conditions in Eq. (3.9) and Eq. (3.10).

The following is the design procedure at frequency $f$ with coupling coefficient $C$:
1. Calculate the characteristic impedances in the even and odd modes:

\[ Z_{0e} = Z_0 \sqrt{\frac{1+c}{1-c}}, \quad Z_{0o} = Z_0 \sqrt{\frac{1-c}{1+c}} \]

2. Calculate the capacitance and the inductance per unit length in the even mode:

\[ C_e = \frac{1}{v_{ph} Z_{0e}}, \quad L_e = Z_{0e}^2 \times C_e \]

3. Calculate the capacitance per unit length in the odd mode: \( C_o = \frac{1}{v_{ph} Z_{0o}} \)

4. Calculate the coupling capacitance per unit length: \( C_{12} = \frac{C_o - C_e}{2} \)

5. Assume values of \( \theta_e \) and \( \theta_o \) that satisfy Eq. (3.10)

6. Calculate the transmission line length in the even and odd modes: \( d_e = \frac{\theta_e}{\beta}, \quad d_o = \frac{\theta_o}{\beta} \)

where \( \beta = \frac{2\pi}{\lambda}, \quad \lambda = \frac{c}{f} \)

7. Calculate the actual value for \( C_e, l_e, C_{12} \): \( C_e = C_e \times d_e, \quad l_e = L_e \times d_e, \quad C_{12} = C_{12} \times d_o \)

These steps are summarized in the flowchart in Figure 3.4.
Figure 3.4: Design procedure for the transdirectional coupler at frequency \( f \) with coupling coefficient \( C \).
3.5 The Presented Design

A design of a lumped element transdirectional coupler that works at 1GHz with a 3dB coupling coefficient is presented. The design equations in the last section were used to design this lumped element transdirectional coupler. The simulation was done using the ADS software. The fabrication was done using FR-4 substrate with 1.6 mm thickness. Simulated and measured results are given.

The following equation is used to obtain the coupling coefficient $C$:

$$-3dB = 20\log (C)$$

$$C = 10^{\frac{-3}{20}}$$

$$C = 0.7$$

We then obtain $Z_{oe}$ and $Z_{oo}$ using Eqs. (3.11) and (3.12), where $Z_0 = 50\Omega$:

$$Z_{oe} = Z_0 \sqrt{\frac{1 + C}{1 - C}} = 120 \Omega$$

$$Z_{oo} = Z_0 \sqrt{\frac{1 - C}{1 + C}} = 20 \Omega$$

We then calculate $C_e$, $L_e$, $C_o$ and $C_{12}$ using Eqs. (3.13), (3.16), (3.17), and (3.18):

$$C_e = \frac{1}{Z_{oe} V_{ph}} = 27 * 10^{-12} F/m$$

$$L_e = Z_{oe}^2 * C_e = 393 * 10^{-9} H/m$$

$$C_o = \frac{1}{Z_{oo} C_o} = 161 * 10^{-12} F/m$$

$$C_{12} = \frac{C_o - C_e}{2} = 44.6 * 10^{-12} F/m$$

To get the actual component values and satisfy Eq. (3.10), we can assume any values for $\theta_e$ and $\theta_o$. The electrical lengths in even and odd modes are assumed to be the following:

$$\theta_e = 25^\circ$$

$$\theta_o = 205^\circ$$

Eqs. (3.20) and (3.24) are then used to get the following values at $f = 1GHz$:
$d_e = 0.02 \text{ m}$

$d_0 = 0.16 \text{ m}$

The actual values can then be calculated from Eqs. (3.25), (3.26), and (3.27):

$C_e = 0.5 \times 10^{-12} \text{ F}$

$L_e = 7.8 \times 10^{-9} \text{ H}$

$C_{12} = 9.26 \times 10^{-12} \text{ F}$

### 3.5.1 RF Model of the Presented Design

The RF (ideal) model, shown in Figure 3.5, was implemented by substituting the previously calculated component values in the ADS schematic of the presented design. Figure 3.5 shows that each port is terminated with 50 $\Omega$ to provide matching for all ports. Figure 3.6 shows the S-parameters of the RF model. $S_{11}$ is matched and $S_{21}$ is isolated at 1 GHz with -42 dB. $S_{31}$ and $S_{41}$ are -3 dB at 1 GHz, which is the desired performance.

![Figure 3.5: The presented RF model in ADS for the lumped element transdirectional coupler.](image-url)
A PCB layout is designed to be used as a surface for the 0603 Surface Mount (SM). Grounded CPW technology was used to feed the mounted lumped elements on the PCB board as shown in Figure 3.7. The substrate is FR-4 with a thickness of 1.6 mm and $\varepsilon_r=4.3$. The physical design dimensions are $23\times28\times1.6\,mm^3$. The large number of VIAs in the layout were used to define good ground for top CPW line. The design was ADS Momentum EM simulated to include the 2nd order coupling effects (between the interconnection transmission lines and the SM parasitics) on the PCB board. The values of the SM lumped components must be optimized to reach the desired results. The EM simulation file was imported as an S-parameters file to the ADS circuit simulation and into the 30-port network. The Murata library components were used to run the simulation with the real capacitors and the inductors. This was done to include the self-
resonance of the lumped element. The co-simulation using Murata library is shown in Figure 3.8. The results are influenced by the parasitic effects of lumped capacitors and inductors and therefore, optimization of the component values is needed to get the desired performance. Table 3.1 lists the Murata component values after optimization and the ideal lumped element values at 1 GHz calculated using the derived design equations. The value of $C_e$ in Murata library is reduced to compensate for the increased capacitance coupling through the gaps between the transmission line sections and the ground. In addition, $L_e$ in Murata library was reduced to compensate for adding inductance in the PCB connectors.

Figure 3.9(a) shows that $S_{11}$ and $S_{21}$ at 1 GHz are -48 dB and -30 dB, respectively, while $S_{31}$ and $S_{41}$ at 1 GHz are -3.2 dB and -3.5 dB, respectively. Table 3.2 compares the measured and simulated S-parameters. The theoretical phase difference between the output ports should be 90°. In this designed coupler, the phase difference between the output ports is around 89°. This is acceptable after including the coupling and parasitic effects as shown in Figure 3.10(a).

![Figure 3.7: ADS Momentum EM layout of the transdirectional coupler.](image)
Table 3.1: Transdirectional coupler ideal and optimized lumped components values at 1 GHz.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Ideal Value</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_e$</td>
<td>0.5 pF</td>
<td>0.3 pF</td>
</tr>
<tr>
<td>$L_e$</td>
<td>7.8 nH</td>
<td>7.1 nH</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>9.26 pF</td>
<td>8.1 pF</td>
</tr>
</tbody>
</table>

Table 3.2: The simulated and measured S-parameters of the transdirectional coupler at 1 GHz.

<table>
<thead>
<tr>
<th>S-parameters</th>
<th>Simulated values for S-parameters in dB.</th>
<th>Measured S-parameters in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>-48</td>
<td>-22.7</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>-30</td>
<td>-24.8</td>
</tr>
<tr>
<td>$S_{31}$</td>
<td>-3.2</td>
<td>-2.9</td>
</tr>
<tr>
<td>$S_{41}$</td>
<td>-3.5</td>
<td>-3.42</td>
</tr>
</tbody>
</table>

Figure 3.8: Co-simulation and the EM model using Murata library.
Figure 3.9: The simulated and the measured S-parameters. (a) Simulated S-parameters of co-simulation using Murata library. (b) The measured S-parameters.

Figure 3.10: The simulated and the measured Phase response. (a) Simulated S-parameters of phase response using Murata library. (b) The measured phase response.
3.6 Vector Network Analyzer (VNA) Calibration

Calibration is a setting up process for the Vector Network Analyzer (VNA) to correct it from its own error and consider the effects of the cables. Calibration standards are sets of connectors that are necessary for the calibration. Calibration standards are done by measuring precisely known devices. In addition, there is a software calibration kit installed into the VNA. The calibration is done by connecting several open, short, and thru standards to the cables and then pressing the appropriate button according to the type of the standard.

3.7 Fabrication and Measurements

The circuit is fabricated using FR-4 substrate with 1.6 mm thickness. The circuit is designed to accommodate the 0603 surface mount components (1.6mm×0.8mm) to fit the pick and replace soldering machine capabilities in the lab. The measurements of the design were done using Agilent E5071C 2-port VNA. The measured S-parameters of the fabricated design is shown in Figure 3.9(b). There is good agreement between the simulated and the measured design. Figure 3.9 shows that $S_{11}$ and $S_{21}$ at 1 GHz are -22.7 and -24.8 dB, respectively. In addition, $S_{31}$ and $S_{41}$ at 1 GHz are -2.9 dB and -3.42 dB, respectively. Insertion loss is the delivered power out of the through port relative to the input port. Therefore, the insertion loss \( \frac{1}{S_{21}} = \frac{1}{\text{Gain}} \). Table 3.2 shows a comparison between the simulated and the measured S-parameters. The measured phase difference between port 3 and port 4 is 86.2˚ as shown in Figure 3.10(b). The fabricated transdirectional coupler is shown in Figure 3.11. The physical dimensions of the circuit are 28×23×1.6 mm³. The measured results have some differences than the simulated results. In the simulated results, the effects of the SMA connectors are not included due to the simulation time and since they do not affect the performance by much. On the other hand, the VNA cables and the measurement environment affect the performance of the measured results. For these reasons, the simulated and the measured results are not identical. However, these slight differences do not affect the overall performance of the coupler.
3.8 Summary

In this section, we used the S-parameters of the four-port network and the S-parameters of the coupled line directional coupler to derive design conditions for the transdirectional coupler. Generic design equations that work at any frequency with arbitrary coupling coefficients were demonstrated. Furthermore, this section presented a simulated and fabricated model to prove the design equations. There is good agreement between the simulated and the measured S-parameters as well as the phase response. In the next chapter, the design conditions for the codirectional coupler will be derived using the same methodology as the transdirectional coupler. The implementation of the transdirectional and codirectional coupler is done using the same lumped element circuit and thus the same PCB board can be used. Design equations for the codirectional coupler at any frequency with 3 dB coupling coefficient will be outlined. Finally, the simulated and measured results will be demonstrated to verify the design equations.
Chapter 4

Codirectional Coupler

4.1 Motivation

A design of a lumped element codirectional coupler with 3dB coupling is presented. Capacitors are used to produce the coupling mechanism between the coupled lines. The codirectional coupler has a similar RF function as the branch line coupler. Unlike the branch line coupler, the presented codirectional coupler provides DC isolation between port 2 and port 3. The S-parameters for the branch line and coupled line coupler are used to derive the design conditions for the codirectional coupler. Design equations are presented that work at any frequency with 3 dB coupling coefficient. An example of a codirectional coupler design simulation and fabrication at 1 GHz with 3 dB coupling coefficient is presented. The simulations were done using the ADS software. Simulated and measured results are discussed.

4.2 The Conditions of Codirectional Coupler

For the codirectional coupler, port 1 is the input port (matched) and port 4 is isolated. Therefore, there is no power reflected at port 1 and no power delivered to port 4 as shown in Figure 4.1.

The following conditions can be obtained from Eqs. (2.17) and (2.20):

\[ S_{11} = \frac{1}{2} (S_{11}^e + S_{11}^o) = 0 \]  

(4.1)
\[ S_{41} = \frac{1}{2} (S_{41}^e - S_{41}^o) = 0 \]  

(4.2)

Eqs. (4.1) and (4.2) are set to 0 since there is no power at port 1 or port 4. To satisfy Eqs. (4.1) and (4.2), since \( S_{11}^e = S_{11}^e \) and \( S_{11}^o = S_{11}^o \) from Eqs. (2.77) and (2.78), the following condition must be true:

\[ S_{11}^e = S_{11}^o = 0 \]  

(4.3)

Substituting Eqs. (2.77) and (2.78) into Eq. (4.3) yields the following condition:

\[
\frac{j\left(\frac{Z_{oe}}{Z_a}\right)\sin(\theta_e)}{2\cos(\theta_e)+j\left(\frac{Z_{oe}}{Z_a}\right)\sin(\theta_e)} = \frac{j\left(\frac{Z_{oo}}{Z_a}\right)\sin(\theta_o)}{2\cos(\theta_o)+j\left(\frac{Z_{oo}}{Z_a}\right)\sin(\theta_o)} = 0
\]

(4.4)

To satisfy Eq. (4.4), we need:

\[ Z_{oe} = Z_{oo} = Z_{oe} \]  

(4.5)

This is the impedance condition for the codirectional coupled line coupler where \( Z_a \) is the characteristic impedance of the design, and \( Z_{oe} \) and \( Z_{oo} \) are the characteristic impedances in the even and odd modes, respectively.

The codirectional coupler has the same functions as the branch line coupler: port 1 is an input port (matched), port 4 is isolated, and port 2 and port 3 are the output ports with 3 dB and 90° phase difference. Therefore, the S-parameters for the branch line coupler can be used to obtain the second design conditions at the output ports.

From Eqs. (2.40) and (2.41), the S-parameters at port 2 and port 3 for the branch line coupler are the following [5]:

\[ S_{21} = \frac{1}{2} S_{21}^e + \frac{1}{2} S_{21}^o = -\frac{j}{\sqrt{2}} \]  

(4.6)

\[ S_{31} = \frac{1}{2} S_{31}^e - \frac{1}{2} S_{31}^o = -\frac{1}{\sqrt{2}} \]  

(4.7)

Using Eqs. (2.35) and (2.37), the transmission coefficients in the even and odd mode for the branch line coupler are the following:
\[ S_{21}^e = S_{31}^e = \frac{-1}{\sqrt{2}} (1 + j) \]  
\[ (4.8) \]

\[ S_{21}^o = S_{31}^o = \frac{1}{\sqrt{2}} (1 - j) \]  
\[ (4.9) \]

Using Eqs. (2.79) and (2.80), the transmission coefficient in the even and odd mode for the presented codirectional coupler:

\[ S_{21}^e = S_{31}^e = \frac{2}{2 \cos(\theta_e) + j \left( \frac{Z_{oe}}{Z_0} + \frac{Z_o}{Z_{oe}} \right) \sin(\theta_e)} \]  
\[ (4.10) \]

\[ S_{21}^o = S_{31}^o = \frac{2}{2 \cos(\theta_o) + j \left( \frac{Z_{oe}}{Z_0} + \frac{Z_o}{Z_{oe}} \right) \sin(\theta_o)} \]  
\[ (4.11) \]

By equating Eq. (4.8) with Eq. (4.10) and Eq. (4.9) with Eq. (4.11), we get:

\[ S_{21}^e = \frac{2}{2 \cos(\theta_e) + j \left( \frac{Z_{oe}}{Z_0} + \frac{Z_o}{Z_{oe}} \right) \sin(\theta_e)} = \frac{-1}{\sqrt{2}} (1 + j) \]  
\[ (4.12) \]

\[ S_{21}^o = \frac{2}{2 \cos(\theta_o) + j \left( \frac{Z_{oe}}{Z_0} + \frac{Z_o}{Z_{oe}} \right) \sin(\theta_o)} = \frac{1}{\sqrt{2}} (1 - j) \]  
\[ (4.13) \]

Substituting Eq. (4.5) into Eqs. (4.12) and (4.13) yields:

\[ S_{21}^e = S_{31}^e = \frac{2}{2 \cos(\theta_e) + 2j \sin(\theta_e)} = \frac{-1}{\sqrt{2}} (1 + j) \]  
\[ (4.14) \]

\[ S_{21}^o = S_{31}^o = \frac{2}{2 \cos(\theta_o) + 2j \sin(\theta_o)} = \frac{1}{\sqrt{2}} (1 - j) \]  
\[ (4.15) \]

By solving for \( \theta_e \) and \( \theta_o \), we end up with:

\[ \theta_e = \frac{5\pi}{4} \]  
\[ (4.16) \]

\[ \theta_o = \frac{7\pi}{4} \]  
\[ (4.17) \]

Eqs. (4.5), (4.16) and (4.17) represent the conditions of the codirectional coupler. These conditions will be applied for implementation in the following sections.

### 4.3 The Design Equations for Lumped Element Codirectional Coupler.

Since the transmission line coupler is the same for both the transdirectional and codirectional couplers, the lumped element of the codirectional coupler is the same as that of the transdirectional coupler. Figure 4.2 shows the lumped element circuit of the codirectional coupler.
In the presented design, we have \( C_o = C_e + 2C_{12} \) such that \( C_e \neq C_o \). We redefine Eqs. (2.87) and (2.90) as follows:

\[
C_e = \frac{1}{\nu_{ph}^e z_{oe}} \quad [F/m] \tag{4.18}
\]

\[
C_o = \frac{1}{\nu_{ph}^o z_{oo}} \quad [F/m] \tag{4.19}
\]

where \( \nu_{ph}^e \) is the phase velocity in the even mode, \( \nu_{ph}^o \) is the phase velocity in the odd mode, \( C_e \) is the capacitance for the even mode, and \( C_o \) is the effective capacitance in the odd mode.

Since \( \theta_e \) and \( \theta_o \) are known from Eqs. (4.16) and (4.17), \( \nu_{ph}^e \) and \( \nu_{ph}^o \) can be obtained:

\[
\theta_e = \frac{\omega}{\nu_{ph}^e} d \tag{4.20}
\]

\[
\theta_o = \frac{\omega}{\nu_{ph}^o} d \tag{4.21}
\]

where \( d \) is the transmission line length and \( \omega \) is the frequency in radian and can be obtained from \( \omega = 2\pi f \), where \( f \) is the desired frequency.

By solving Eqs. (4.20) and (4.21) we get:
\[ v_{ph}^o = \frac{5}{7} v_{ph}^e \quad (4.22) \]

Assuming that \( v_{ph}^e = c = 3 \times 10^8 \text{ m/s} \) we can obtain \( v_{ph}^o \).

We then can calculate \( C_e \) and \( C_o \) using Eqs. (4.18) and (4.19), respectively. With the calculated value of \( C_e \), we can calculate the inductance for the even mode per unit length, \( L_e \), from the following equation:

\[ Z_{0e} = \frac{\sqrt{\frac{L_e}{C_e}}}{2} \quad (4.23) \]

We then obtain the coupling capacitance per unit length, \( C_{12} \), as follows:

\[ C_o = C_e + 2C_{12} \quad (4.24) \]

Using Eqs. (4.20) or (4.21) and knowing \( v_{ph}^e \) and \( v_{ph}^o \), we can calculate \( d \) from the following:

\[ d = \frac{v_{ph}^e \theta_e}{\omega} = \frac{v_{ph}^o \theta_o}{\omega} \quad (4.25) \]

To get the actual values of the capacitors and the inductors in Farad and Henry, we use the following equations [1]:

\[ C_e = C_e \times d \quad [\text{F}] \quad (4.26) \]
\[ l_e = L_e \times d \quad [\text{H}] \quad (4.27) \]
\[ C_{12} = C_{12} \times d \quad [\text{F}] \quad (4.28) \]

The following summarizes the design procedure at frequency \( f \) with 3 dB coupling coefficient:

1. Calculate \( v_{ph}^o \): \( v_{ph}^o = \frac{5}{7} v_{ph}^e \), where \( v_{ph}^e = 3 \times 10^8 \text{ m/s} \)
2. Assume that: \( Z_{0e} = Z_{0o} = Z_0 \)
3. Calculate the capacitance and the inductance per unit length in the even mode:
   \[ C_e = \frac{1}{v_{ph}^e Z_{0e}}, L_e = Z_{0e}^2 \times C_e \]
4. Calculate the capacitance per unit length in the odd mode: \( C_o = \frac{1}{v_{ph}^o Z_{0o}} \)
5. Calculate the coupling capacitance per unit length: \( C_{12} = \frac{C_o - C_e}{2} \)
6. Obtain the transmission line length \( d \): \( d = \frac{v_{ph}^e \theta_e}{\omega} \), where \( \omega = 2\pi f \) and \( \theta_e = \frac{5\pi}{4} \).
7. Calculate the actual value for $C_e$, $l_e$, $C_{12}$: $C_e = C_e \times d$, $l_e = L_e \times d$, and $C_{12} = C_{12} \times d$

These steps are summarized in the flowchart in Figure 4.3.
Figure 4.3: Design procedure for codirectional coupler at frequency $f$ with 3dB coupling coefficient.
4.4 The Proposed Design

An example for a codirectional coupled line coupler at 1 GHz with 3 dB coupling coefficient is demonstrated. All the component values shown in Figure 4.2 will be calculated using the previous design equations.

We start with the impedance condition for the codirectional coupler:

\[ Z_{0e} = Z_{oo} = Z_0 = 50\Omega \]

By applying the phase conditions Eqs. (4.16) and (4.17) and solving Eqs. (4.20) and (4.21), we obtain:

\[ \psi_{ph} = \frac{5}{\sqrt{2}} \psi_{ph} \]

Assuming that \( \psi_{ph} = c = 3 \times 10^8 \text{m/s} \), we get:

\[ \psi_{ph} = 2.1 \times 10^8 \quad [\text{m/s}] \]

We then obtain \( C_e \) and \( L_e \) from Eqs. (4.18) and (4.23):

\[ C_e = 6.6 \times 10^{-11} \text{F/m} \quad [\text{F/m}] \]
\[ L_e = 1.6 \times 10^{-7} \text{H/m} \quad [\text{H/m}] \]

We then calculate \( C_o \) and \( C_{12} \) using Eqs. (4.19) and (4.24):

\[ C_o = 9.5 \times 10^{-11} \text{F/m} \quad [\text{F/m}] \]
\[ C_{12} = 1.45 \times 10^{-11} \text{F/m} \quad [\text{F/m}] \]

All the calculated values are per unit length. To obtain the actual component values for inductance in Henry and capacitance in Farad, we use the transmission line length, \( d \), as shown in Figure 4.4.

By solving Eq. (4.25) we obtain \( d \). We then calculate \( C_e, l_e, C_{12} \) from Eqs. (4.26), (4.27) and (4.28).

\[ C_e = C_e d = 2.56 \times 10^{-12} \quad [\text{F}] \]
\[ l_e = L_e d = 5.95 \times 10^{-9} \quad [\text{H}] \]
\[ C_{12} = C_{12} d = 7.428 \times 10^{-12} \quad [\text{F}] \]
The lumped element model of the codirectional coupler with the previously calculated values is simulated using ADS as shown in Figure 4.5(a). Both the transdirectional and codirectional couplers have the same layout shown in Figure 4.5(b). The EM model was simulated using ADS Momentum. The EM simulation file was imported as an S-parameters file to the ADS circuit simulation and inputted into the 30-port network. The Murata library components were used to run the simulation with the real capacitors and the inductors that would be mounted on the surface of the PCB. The co-simulation using Murata library is shown in Figure 4.5(c). Table 4.1 lists the ideal and the optimized lumped element values. The value of $C_e$ in Murata library is reduced to compensate for the increased capacitance coupling through the gaps between the transmission line sections and the ground. In addition, $L_e$ in Murata library was reduced to compensate for adding inductance in the PCB connectors.

Figure 4.4: Codirectional even (a) and odd (b) mode circuits.

4.5 ADS Simulation of the Presented Codirectional Coupler

The lumped element model of the codirectional coupler with the previously calculated values is simulated using ADS as shown in Figure 4.5(a). Both the transdirectional and codirectional couplers have the same layout shown in Figure 4.5(b). The EM model was simulated using ADS Momentum. The EM simulation file was imported as an S-parameters file to the ADS circuit simulation and inputted into the 30-port network. The Murata library components were used to run the simulation with the real capacitors and the inductors that would be mounted on the surface of the PCB. The co-simulation using Murata library is shown in Figure 4.5(c). Table 4.1 lists the ideal and the optimized lumped element values. The value of $C_e$ in Murata library is reduced to compensate for the increased capacitance coupling through the gaps between the transmission line sections and the ground. In addition, $L_e$ in Murata library was reduced to compensate for adding inductance in the PCB connectors.
Figure 4.5: The ADS Simulation of the presented codirectional coupler. (a) The lumped element model. (b) PCB layout (c) The co-simulation using Murata library components.
4.6 Simulation Results.

In this section, all ADS simulation results will be shown starting with the RF model, then the co-simulation between the EM model and the lumped element model. Figure 4.6 shows the S-parameters of the RF model of the presented lumped element codirectional coupler. It is evident that the input port (port 1) is perfectly matched and the isolated port (port 4) is isolated. Both $S_{11}$ and $S_{41}$ are -31 dB at 1 GHz. Both $S_{21}$ and $S_{31}$ are -3 dB at 1 GHz. Figure 4.7(a) shows the S-parameter results of the co-simulation using Murata library, where $S_{11}$ is -26.5 dB, $S_{21}$ is -3.8 dB, $S_{31}$ is -3.4 dB, and $S_{41}$ is -35 dB. Insertion loss is the delivered power out of the through port relative to the input port. Insertion loss is +3.8 dB. Therefore, the insertion loss $= \frac{1}{S_{21}}$. The S-parameter results changed slightly after including the coupling and parasitic effects between the interconnection transmission lines and the SM lumped components. Figure 4.8(a) shows the simulated phase response of the codirectional coupler. The theoretical phase difference between port 2 and port 3 is 90° while the simulated phase difference is 87.9°. The phase error is therefore 2.1°.
Figure 4.7: The simulated and the measured S-parameters. (a) Simulated S-parameters of co-simulation using Murata library. (b) The measured S-parameters.

Figure 4.8: The simulated and the measured Phase response. (a) Simulated S-parameters of phase response using Murata library. (b) The measured phase response.
4.7 Fabrication and Measurements

The circuit is fabricated using FR-4 substrate with 1.6 mm thickness. The physical dimensions of the circuit are 28×23×1.6 mm³. The measurements of the design were done using Agilent E5071C 2- port VNA. The PCB circuit for the presented lumped element codirectional coupler is the same as that of the transdirectional coupler. The only difference lies in the components values. The measured S-parameters of the codirectional coupler are shown in Figure 4.7(b). Table 4.2 shows a comparison between the simulated and measured S-parameters at 1 GHz. The measured phase response is shown in Figure 4.8(b). There is good agreement between the simulated and measured results for the codirectional coupler. The measured phase difference between port 2 and port 3 is 86.1°, yielding a phase error of 3.9°. The fabricated codirectional coupler is shown in Figure 4.9. The measured results have some differences than the simulated results. In the simulated results, the effects of the SMA connectors are not included due to the simulation time and since they do not affect the performance by much. On the other hand, the VNA cables and the measurement environment affect the performance of the measured results. For these reasons, the simulated and the measured results are not identical. However, these slight differences do not affect the overall performance of the coupler.

### Table 4.1: Codirectional coupler ideal and optimized lumped components values at 1 GHz.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Ideal Value</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_e$</td>
<td>2.56 pF</td>
<td>1.7 pF</td>
</tr>
<tr>
<td>$L_e$</td>
<td>5.95 nH</td>
<td>4.5 nH</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>7.428 pF</td>
<td>7 pF</td>
</tr>
</tbody>
</table>
Table 4.2: The simulated and measured S-parameters of the codirectional coupler at 1 GHz.

<table>
<thead>
<tr>
<th>S-parameters</th>
<th>Simulated [dB]</th>
<th>Measured [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>-26.5</td>
<td>-27.8</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>-3.8</td>
<td>-3.2</td>
</tr>
<tr>
<td>$S_{31}$</td>
<td>-3.4</td>
<td>-3.6</td>
</tr>
<tr>
<td>$S_{41}$</td>
<td>-35</td>
<td>-27.8</td>
</tr>
</tbody>
</table>

Figure 4.9: The fabricated codirectional coupler.

4.8 Summary

In this chapter, we derived the design conditions for the codirectional coupler. Furthermore, design equations that work at any frequency with a 3dB coupling coefficient were derived. The general S-parameters for four-port networks were used. Simulated and fabricated results have been presented for design verification. The upcoming chapter will summarize the thesis contribution and future work.
Chapter 5

Conclusion and Future Work

This thesis presents a lumped element transdirectional coupler. Using the S-parameters of the four-port network and the coupled line coupler, the required design conditions of a transdirectional coupler are derived. These design conditions are satisfied by setting the characteristic impedance and electrical length in both the even and odd mode analyses. Through these design conditions, the design equations for the presented lumped element transdirectional coupler are derived. The presented lumped element model allows control over the characteristic impedance and electrical length in the even and odd modes. The presented design conditions are generic and work at any frequency with an arbitrary coupling coefficient. Previous implementations of the transdirectional coupler used a hybrid between the transmission line and lumped element [9] or a VIP [10]. Both of these implementations are larger in size and more complex compared to the presented lumped element transdirectional coupler.

The coupling between the two transmission lines is done using capacitors in the lumped element circuit of the transdirectional coupler. This capacitive coupling provides isolation and DC blocking between the input and output ports unlike inductive coupling which allows the DC to go through the output ports.

A simulated design for the presented transdirectional coupler that works at 1 GHz with 3 dB coupling coefficient was created to verify the design methodology. All lumped element component values were calculated using the presented design equations. Furthermore, the PCB layout was electromagnetically simulated to include the coupling and parasitic effects between the lumped surface mount components and the transmission lines sections. The lumped element component values were optimized after including the coupling and parasitic effects to achieve good performance at the desired frequency. The measurements were done using Agilent E5071C 2- port VNA. Comparison of the fabricated and simulated models showed good agreement at the measured S-parameters and phase response.
The second design in this thesis is the codirectional coupler. The codirectional coupler was implemented previously using a branch line coupler. The lumped element circuit of the codirectional and branch line couplers are different but they both have a 3 dB coupling coefficient and 90° phase difference between the output ports. The S-parameters at each port of the four-port network and the S-parameters of the branch line coupler were used to derive design conditions for the codirectional coupler. Design equations that satisfy the design conditions for the codirectional coupler are demonstrated. These design equations work at any frequency with 3 dB coupling coefficient.

The presented codirectional coupler was implemented using the same lumped element model as the transdirectional coupler where the capacitive coupling was used to represent the coupling mechanism. Previous implementations of the lumped element branch line (codirectional) coupler converted each transmission line of the branch line coupler to its equivalent lumped element model [8]. The presented lumped element model provides isolation between port 2 and port 3 (the output ports) due to the capacitive coupling. Furthermore, the presented codirectional coupler was implemented using the same PCB layout of the transdirectional coupler. The difference between them was the component values which are based on the individual design methodologies. Like the transdirectional coupler, the PCB layout was electromagnetically simulated to include the coupling and parasitic effects between the lumped surface mount components and the transmission lines sections. Comparison of the fabricated and simulated models showed good agreement at the measured S-parameters and phase response.

Both presented designs for the transdirectional and codirectional couplers were done using the same lumped element circuit and PCB layout. This simplifies the switching between the two coupler designs which is done by only changing the lumped surface mount components.

In this thesis, only transdirectional and the codirectional couplers are implemented. The reason for not implementing the contradirectional (conventional) coupler is that it requires both capacitive and inductive coupling. The lumped element model requires both inductors and capacitors to implement the capacitive and inductive coupling. The inductors allow the DC to go through port 3 and port 4, which does not happen in the
transmission line model of the conventional coupler. This thesis presented lumped element circuit for the
transdirectional and codirectional coupler. The capacitors are used to represent the coupling between the two
transmission lines. These capacitors provide isolation and DC blocking between output ports. The design
conditions as well as the design methodologies for both the transdirectional and codirectional coupler are
demonstrated.

The recommended next steps include deriving generic design equations that work for the presented
codirectional coupler at any coupling coefficient, rather than just 3 dB. This will help generalize the design
and allow it to be implemented in other applications such as power amplifiers and feeding network for
antenna arrays that will reduce the circuit size and complexity. Furthermore, it is recommended that the PCB
layout used in this thesis be investigated for implementation on other forms of directional couplers. The
compact size and reduced complexity will allow for a wider array of application.
References


