ON THE DETECTABILITY OF DENSITY DISTRIBUTIONS OF ASTEROIDS AND SAGD RESERVOIRS USING GRAVIMETRY AND MUON TOMOGRAPHY

by

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Abstract

In this thesis, the procedures of modelling density distributions for different targets are investigated. In the process of evaluating density distributions of asteroids, gravity forward modelling shows that homogeneous and heterogeneous distributions are distinguishable. Also, this study demonstrates that muon tomography forward and inversion modelling of SAGD reservoirs could be used for monitoring fluid migration and bitumen depletion in the subsurface.

Two different asteroids and spherical models with homogeneous and heterogeneous density distributions are investigated and the resulting surface gravitational accelerations are calculated. The asteroid forward modelling assists in the analysis of surface gravimetry sensitivity, by studying the heterogeneity of asteroids and surface boulder detectability. Results indicate that asteroids with heterogeneous and homogeneous density distributions are distinguishable. However, a gravimeter with sub-mGal accuracy is needed to sense asteroid surface gravity. Results show that a boulder of 8 m$^3$ causes gravitational accelerations on the order of µGal and is not detectable using gravimetry with sub-mGal accuracy.

A real SAGD density model is used at two time steps and forward modelled using muon tomography. This results in maps of opacity and muon count in a 2D plane view of the reservoir. The forward modelled 2D image arrays, used to construct the 2D reservoir plane view at surface, estimate bitumen depletion around wellbores during the raising phase and the spreading phase. The depletion patterns in the reservoir are studied by testing different arrays of muon sensors in the in-line and cross-line directions. Results show that depletion trends can be resolved with high resolution by using 120 sensors and also when using 60 sensors with a reduced resolution. In the inversion modeling approach, 3D density change models are estimated using 2D opacity image arrays as observations. The depletion trends around wellbores are determined individually until the reservoir reaches the spreading phase when depletion areas become interconnected.
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List of Abbreviations

1D one-dimensional
2D two-dimensional
3D three-dimensional
4D four-dimensional
nGal nano-Gal
µGal micro-Gal
mGal milli-Gal
m/s² metre per Second Squared
m metres
km kilometres
m² metre square
m³ cubic metre
cm³ cubic centimetre
m.w.e metre water equivalent
cP centipoise
SAGD steam assisted gravity drainage
SI international system
ESA European space agency
NASA national aeronautics and space administration
JAXA Japan aerospace exploration agency
GRASP gravimetric asteroid surface probe
HERA  human exploration research analog
VEGA  vector gravimeter for asteroid
ARRM  asteroid redirect robotic mission
IGMAS+ interactive geophysical modelling assistant system
Chapter 1

Introduction

This thesis addresses two specific problems, both related to the determination of density distributions, namely, i) Asteroid density distribution from surface gravimetry, and ii) Steam Assisted Gravity Drainage (SAGD) reservoir density monitoring from Muon tomography. The goal of this thesis is to demonstrate the potential of the two innovative methods applied to two targets which have not been addressed before. The technical challenges involved in applying the two methods to the two targets in the field still have not been overcome. Therefore, this thesis consists of theoretical and numerical approaches attempting to show the potential and the feasibility of the two methods.

1.1 Problem

1.1.1 Problem I: Asteroid density distribution

A number of recent missions by national space agencies (NASA, JAXA, and ESA) have been conducted to study and explore asteroids; however, the density distributions of these asteroids are still ambiguous. The density distribution of asteroids provides important information concerning the formation of the solar system, potential asteroid mining and redirect strategies for asteroid impact hazards. This research addresses all three aspects by modelling the density distribution of asteroids through simulated surface gravity surveys. Determining the
density distribution is a non-trivial problem which cannot be uniquely solved and retains inherent non-uniqueness (Scheeres, Khushalani and Werner, 2000). This research focuses on the integrated fields of potential field geophysics, space geodesy and Earth systems monitoring. The density distribution of small bodies of asteroids have not been studied in detail due to the lack of asteroid visits.

To study the characteristics and properties of asteroids, parameters such as mass, size, shape as well as gravity field must be determined. One of the fundamental parameters is their density distribution, which is considered to be a source of valuable information to determine the composition and structure of asteroids. Gravity field modelling as used in geophysics is the process of determining internal density distributions of the Earth or parts of the Earth, or other celestial objects consistent with the observed gravity field (Forsberg, 1984). Applying those modelling techniques to asteroids, specifically forward modeling for sensitivity analysis, is conducted in the first part of this study.

1.1.2 Problem II: SAGD Reservoir Monitoring

A second application area concerns the monitoring of density changes in hydrocarbon reservoirs due to bitumen (or heavy oil) recovery and fluid migration, usually referred to as spatio-temporal reservoir monitoring, which can be conducted using different geophysical techniques. Constraints and limitations are presented by different geophysical monitoring techniques based on measurement process, signals, device sensitivity, noise level and reservoir properties. For example, gas monitoring experiments conducted in the North Sea and Paris Basin using seismic surveys were successful, but it became clear that the interaction of the reservoir
rock, fluid and gas components could enhance or degrade the time-lapse seismic signal depending on specific reservoir conditions (Lumley, 2001). The cost of such surveys can be significant; can cost a few million dollars especially in offshore fields, and as a result, seismic surveys are mostly conducted only every other year. In 2001, there were about 75 active projects worldwide, and more than 100 cumulative projects in the 1990s. The total annual expenditures on 4-D seismic projects were on the order of $50–100 million US (Lumley, 2001). Alternative reservoir monitoring methods using gravity gradiometry have been proposed by Reitz (2015) and Elliott and Braun (2017). New geophysical techniques such as muon tomography can also provide important information of the sub-surface density distribution. Herein, we study the utilization of muon tomography to detect density changes of Steam Assisted Gravity Drainage (SAGD) reservoirs.

Muons are elementary particles that are produced when cosmic rays collide with molecules in the upper atmosphere. Cosmic rays originating from either within the galaxy or outside the galaxy reaching the Earth’s atmosphere are composed of 90% protons, 9% helium nuclei and 1% heavier nuclei. The rate of cosmic particles hitting the atmosphere per square metre per second is about 1000 particles, which will generate a constrained number of elementary particles but enough to explore density distributions of shallow reservoirs (Rodriguez, 2017). Generated particles such as muons can penetrate 10-100s of metres into rocks and other matter before experiencing any significant effects of attenuation. Muon tomography has recently been used to detect density anomalies in mines (Davis, 2012), but has the potential to be used in other application areas as well.
1.2 Outline

In this thesis, the density distribution of asteroids and the density changes in SAGD reservoirs are studied using gravimetry and muon tomography, respectively. After a general introduction of the problem in Chapter 1, a literature review on asteroids, gravimetry and muon tomography follows in Chapter 2. In Chapter 3, the asteroid’s voxels density values are forward modeled to estimate surface gravity and the required gravimeter is estimated. Chapter 4 discusses forward modeling of density data sets for a real SAGD reservoir at two time steps and are constructed to estimate muon counts and opacity in the sub-surface from certain depth and angles. Synthetic muon data are then used in inverse modeling and 3D models of density changes due to fluid depletion and steam injection in SAGD reservoirs are presented in Chapter 5.

1.3 Scope

The study aims to demonstrate how density distribution is resolved and mapped by utilizing muon tomography and gravimetry observations. The developed methodology focuses on the resolvability of gravitational acceleration to distinguish homogeneous or heterogeneous asteroid density distributions. To achieve that, we model random asteroid density distributions that are dependent on mean density, density variance and correlation length. Models were developed for asteroids 25143 Itokawa and 2008EV5.

In order to spatially and temporally monitor different types of reservoirs, the density distribution of SAGD reservoirs is resolved using muon tomography. The research is focused on resolving the density distribution of one specific operational SAGD reservoir in the McMurray formation in Alberta after depletion times or 1.25 and 5 years, respectively.
1.4 Motivation

Density is one of the most fundamental physical parameters which governs many processes in science and engineering. While gravimetry is able to estimate density distributions of a variety of Earth systems, its spatio-temporal change has only been addressed from dedicated gravity missions (GRACE, GRACE-FO) and localized point measurements (e.g. observatories). Estimating the density distribution of asteroids is of significance in both asteroid and Earth sciences. Asteroid mass and density knowledge is limited due to the limited capabilities of existing radar, photogrammetric and spectrographic observation methods (Zuber, 1998). The limited number of space missions to asteroids has also made it difficult to enhance our knowledge of asteroids and solar system formation (Cheng, 1997). Density distribution is a fundamental physical parameter in asteroid science for various reasons including estimation of mining potential and the implementation of re-direct strategies to mitigate asteroid impacts. The second chapter discusses some of the basic data accumulated on asteroid bulk density, asteroid shape, density distribution and gravity forward modeling. Using these data, we will estimate the density distribution of asteroids, outline the implications of different density variance and correlation length between voxels forming asteroid models. The internal structure of asteroids can be assessed by studying various density distribution models, e.g. radially symmetric versus homogeneous or random density distributions potentially shed light on the mechanisms of asteroid formation. Interpretation of surface gravity data would help to characterize the distribution of areas with high and low gravity signals for future landing sites and boulder retrieval missions (Cheng, 1997b). Asteroid boulder retrieval missions will help to better study asteroids and their density distribution and start a new era of potential mineral exploration and redirect strategies.
Furthermore, density distribution is of importance in petroleum engineering and geophysics to monitor fluid migration (here hydrocarbon migration) in the sub-surface. The detection of density distribution in SAGD reservoirs could utilize naturally occurring particles, herein muons, produced in the upper atmosphere. This study will estimate the density distribution in a SAGD field by modelling the intensity of those elementary particles using a forward model with an array of sub-surface muon sensors. Sub-surface fluid monitoring techniques can help to reduce oil field operational costs, increase production efficiency and optimize field operations (ex. Hydraulic fracturing, CO2 injection, matrix acidizing) (Emberley, 2005). 4-D seismic surveys can monitor the progress of costly injected fluid fronts (water, gas, steam, CO2, etc.) that can save hundreds of millions of dollars by optimizing injection programs. 4-D seismic can map reservoir compartmentalization and the fluid-flow properties of faults (sealing versus leaking), which can be very useful for optimal design of production facilities and well paths in complex reservoir flow systems (Lumley, 2001). However, conducting 4-D seismic can be very costly.

The properties of muons make it a potential monitoring technique that can be used to reveal density change even in specific directions, as muons only sense density along their trajectories through the medium as opposed to potential fields such as gravity. Muons are charged particles that are similar to electrons and positrons. They are about 200 times heavier than electrons and travel nearly at the speed of light. They have long life times (approximately 2.2 µ sec), and muon flux at sea level is approximately 10000/m²·min, which means that many muons penetrate the Earth’s surface and travel deeper. Muons lose more energy in denser material (Figure 1). Muon tomography was developed in the 1970 (Pesente, 2009) and represents a promising monitoring technique to economically increase production efficiency and environmentally mitigate out-of-zone hazards (Figure 2).
Figure 1 Muons travel into the sub-surface losing energy as they travel in dense matter. Denser regions (red) reduce the muon flux from a particular direction for each detector (blue).

Figure 2 Steam Assisted Gravity Drainage (SAGD) reservoir. The steam injected from injector wells to reduce viscosity of bitumen and fluid (Bitumen and steam) is produced from producing wells.
1.5 Objectives

The main objective of this thesis is to identify the density distribution of asteroids from gravity forward modelling and of SAGD reservoirs to monitor fluid migration in the sub-surface using muon tomography. The bulk of the thesis is concerned with revealing density distribution of different targets by applying different geophysical techniques.

Studying the density distribution of asteroids and their randomness starts with simulating target asteroids and then creating 3-D random density models with known and changeable parameters (e.g. variance, correlation length). The density distributions of selected asteroids and target boulders are studied from simulated surface gravity observations. The internal density distribution of an asteroid can be studied from its gravity field and shape (Scheeres, Khushalani and Werner, 2000). The objective of this study is also to distinguish asteroids with homogeneous and heterogeneous density distributions. Using this approach, it should be possible to gain additional insight into the internal structure of asteroids, or any irregularly shaped body for which we have a gravity field and a shape (Scheeres, Khushalani and Werner, 2000). A homogeneous asteroid was likely generated at the same time, while a heterogeneous density body indicates potential accretion of material over time. Hence, the density structure may reveal insights into the formation of asteroids, an important part of solar system formation.

The goal of seismic reservoir monitoring is to image fluid flow in a reservoir during production. This is possible because, as fluid saturation and pressure in the reservoir change, the seismic impedance properties change accordingly (White, 1975). Alternatively, detection of fluid flow in the sub-surface could be achieved using muon tomography or gravity gradiometry (Elliott and Braun, 2017). The focus in this SAGD reservoir study will be on applying muon tomography
forward and inverse modeling to monitor fluid flow in the sub-surface. Results are compared with Elliott and Braun (2017) forward models of gravity gradiometry of the same site.

Muon tomography has applications in mineral exploration in existing mines and in undeveloped sites as well as potential applications in block caving monitoring and coal resource estimation (Davis, 2012). Applying muon tomography to study fluid flow in SAGD reservoirs could lead to a new, more economically viable geophysical monitoring technique in the petroleum industry. The current limitation of applying muon tomography to oil and gas reservoirs is the size of muon detectors which are significantly larger than standard well diameters. However, industry is working on developing smaller sensors which eventually would be able to fit inside a well.

Figure 3 Workflow of modeling steps starting from input data into different modeling procedures to finally obtain models that describe the density distribution.
The workflow of the thesis starting from input data into inverse and forward modeling algorithms to derive 2-D and 3-D models that can be used to describe the density distributions of different targets is illustrated in Figure 3. To summarize, the emphasis of this thesis will be on the utilization of modelled density distributions from surface gravity field modelling and local opacity modeling derived from muon flux simulations. The first part will review principles for the utilization of known and unknown density models, then the practical computation of such effects - especially asteroids with topographic effects - will be outlined. Finally, the influence of steam injection and bitumen production on density change will be studied through investigations of empirical functions and measured muon flux for various test areas in a real SAGD field using forward and inverse modeling. The opacity calculated from muon data of a reference geological model is compared with the opacity of a corresponding model. Actual applications of known asteroid shapes and given density models to calculate gravity signals on the asteroid surface are presented as practical evidence of the working methodology.
Chapter 2

Literature review

2.1 Gravimetry

Gravity is an important pillar in geophysics that is used to measure forces exerted by one mass on another. Of all the forces existing in nature, gravity is clearly the most universal one. Gravity influences and is influenced by everything that carries energy and has a mass (Hofmann-Wellenhof and Moritz, 2006). Four of the most fundamental forces in the universe are the strong forces, the weak forces, electromagnetic forces and gravity, which is considered to be the weakest but the most dominant force for shaping large structures such as galaxies and stars. Newton’s law of universal gravitation states that the forces between two masses is attractive, acting along the line joining them and that it is inversely proportional to the square of the distance separating them; i.e. Equation 1 (see Figure 4).

![Newton's law of universal gravitation](image)

Figure 4 Newton's law of universal gravitation. Attractive forces act along the line joining the two masses and are inversely proportional the distance between them.
\[ \vec{F} = G \frac{Mm}{r^2} \hat{r} \]  \hspace{1cm} (1)

\( \vec{F} \) (N) is the force acting between two masses \( M \) and \( m \) (kg), separated by distance \( r \) in metres (m). \( \hat{r} \) is the unit vector which represents the direction of the force. The gravitational constant \( G \) is equal to \( 6.673 \times 10^{-11} \) N m\(^2\)/kg\(^2\).

Newton’s second law is represented in the form of gravitational acceleration that describes the causes of motion and can be used to connect mass with weight:

\[ \vec{F} = m \vec{\ddot{a}} \]  \hspace{1cm} (2)

\( F \) is the force acting on a mass in (N), \( m \) is the mass of object in (kg), and \( \vec{\ddot{a}} \) is the gravitational acceleration of a mass object in m/s\(^2\).

Gravitational acceleration on Earth at the equator is calculated given the following:

Earth’s radius (r) 6371 km, Earth’s mass (M) 5.98 x 10\(^{24}\) kg and is equal to 9.78 m/s\(^2\) (or 978000 mGal).

To calculate the net gravitational forces of a unit of mass on a rotating body such as the Earth, centrifugal acceleration is incorporated with gravitational acceleration. The resultant gravitational force and centrifugal force of Earth are the only forces imposed on a body, resting on the Earth’s surface (Hofmann-Wellenhof and Moritz, 2006). The centrifugal force on a unit of mass is given by angular velocity and distance from the spin axis, equation 3 (see Figure 5).
\[ f = \omega^2 \sqrt{x^2 + y^2} = \frac{v^2}{\sqrt{x^2 + y^2}} \] (3)

Where "\( f \)" is the centrifugal acceleration (m/s\(^2\)), "\( \omega \)" is the angular velocity of a rotating body (rad/s). "\( \sqrt{x^2 + y^2} \)" is the orthogonal distance on the spin axis in metres and "\( v \)" is body’s linear speed in (m/s). Centrifugal acceleration may be re-written in terms of linear speed \( v \) (m/s).

![Figure 5 Centrifugal force (blue) of an object at a distance \( \sqrt{x^2 + y^2} \) from the spin axis.](image)

The sum of gravitational acceleration and centrifugal acceleration is called gravity, equation 4.

\[ g_{\text{total}} = \ddot{a} + f \] (4)

\( g_{\text{total}} \) is gravity, \( \ddot{a} \) is the gravitational acceleration of an object mass while \( f \) is the centrifugal acceleration of an object in motion.
2.1.1 Gravity Gradients

It is important to clarify the differences between mass, density and weight. Mass is a property of a physical body and a measure of a body’s resistance to acceleration while density is a physical property defined as mass per unit volume. Weight is the force experienced by a mass in the presence of a gravitational field. Gravity surveys are usually undertaken to measure variations in the density distribution of the sub-surface. Gravity anomalies can be modeled as vector fields with three directional components (x, y, z), referred to as gravity vector components, and an associated magnitude. Gravity components on the x and y axes measure the change in density in the lateral directions and are equivalent to the horizontal derivative of gravity while the vertical gravity component measures the change in density in the vertical direction. Gravity components are measured by modern-day gravimeters, which have sensitivity levels between 0.05 to 5 µGal.

The gravity gradient tensor is used to define 9 gravitational component derivatives (the derivatives of a gravity vector component in a certain direction) as expressed in equation 5. The gravity gradient tensor is symmetric about its trace, and the sum of the diagonal components (g_{xx}, g_{yy}, g_{zz}) is identically equal to zero in source-free regions, according to Laplace’s equation (Sandwell, 2002). The gravity gradient components g_{xz}, g_{yz} and g_{zz} are computed by taking the vertical derivative of g_{x} and g_{y} and g_{z} respectively. This causes further sharpening of anomalies and enhances the high frequencies of g_{x}, g_{y} and g_{z} (see equation 5).
Gravity Gradient Tensor (T) = \begin{bmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{yx} & g_{yy} & g_{yz} \\ g_{zx} & g_{zy} & g_{zz} \end{bmatrix}

The horizontal derivative of potential field data is a technique used to aid interpretation. The derivatives along the x-axis and y-axis of gravity components are often used to define the anomalous body boundaries. The horizontal derivative produces maximum ridges between different density bodies (Kusumah and Wibowo, 2010). Horizontal derivative filtering was applied on gravity data acquired from a geothermal field located in West Jaffa, Indonesia, to locate density anomalies and analyze results for well targeting (Kusumah and Wibowo, 2010). Gravity gradients and their derivatives enhance and aid with the interpretation of data since it sharpens gravity signals of the anomalies with different densities. However, the noise is also amplified by the derivative and can sometimes lead to larger noise levels overshadowing the signals of interest.

The horizontal vertical gravity gradient (HGz) is sensitive to changes in density along the lateral and vertical directions. It can improve detection and delineation of shallow anomalies (Kusumah and Wibowo, 2010). Equation 6 shows how the horizontal vertical gravity gradient is defined.

\[
HG_z = \sqrt{G_{zx}^2 + G_{zy}^2}
\]

Where \(HG_z\) is the horizontal vertical gravity calculated from the derivatives of vertical gravity in the x-direction and the y-direction.

The gravity signals presented in this study illustrate the gravitational acceleration that indicate the intensity of the gravitational field due to the object’s mass combined with the centrifugal
accelerations due to rotation. The individual gravitational components are computed and the resultant gravitational acceleration is calculated. The centrifugal effects on asteroids are explained in more detail in the following chapter, however, those effects will cancel out when the differences between homogeneous and heterogeneous density models are analyzed.

2.2 Background on Asteroids

Most asteroids in our solar system are located in the asteroid belt between the orbits of Mars and Jupiter at a heliocentric distance of between 2.1 and 2.3 AU (astronomical units). An interesting group of dark and distant asteroids in the outer asteroid belt is located on the leading and trailing LAGRANGIAN points of Jupiter’s orbit (60° ahead of and behind Jupiter) and are called Trojan asteroids (Morbidelli, 2005). An overview of the distribution of asteroids observed in the solar system in and around the asteroid belt is illustrated in Figure 6. In addition, the term near-Earth object (NEO) is defined for asteroids with a perihelion within 1.3 AU from Earth. As of 2013, around 10000 NEOs have been identified and classified, with most NEOs having radii ranging from 4 to 15 km (Lissauer and de Pater, 2013).
Lissauer and de Pater (2001) proposed that binary asteroid systems exist, as well as systems with multiple asteroids, both types have been identified by optical telescopes and spacecrafts. The binary bodies have different amplitudes of light curves and orbital periods. The reflected light curves recorded in the case of the first binary system (NEO AW1) observed in 1994, have different amplitudes. The largest fraction of binary systems is located at the NEO’s category where 15% of the NEO’s are part of a binary system (Lissauer and de Pater, 2013). Figure 7 shows two examples of binary systems.

A satellite sent to orbit an asteroid could provide additional data on the density and mass distribution of the main asteroid. The artificial or natural satellite(s) orbiting the primary
asteroid will give information on the gravitational forces between the two bodies, from which the celestial body’s mass and density can be determined, if its shape and size are known.

It is crucial to consider the possibility of encountering an asteroid in a binary system in any future asteroid mission. Binary asteroid system indicates that both asteroids have enough mass and therefore gravitational force to attract and orbit each other.

Figure 7 (a) Binary C-type asteroid 90 Antiope as imaged with the adaptive optics system on the very large telescope (VLT) on two different dates in 2004; 7 days difference. The two components are equal in size; each with radius of 43 km, separated by 171 km. (b) Closer view of the binary system. (Descamps, 2007)

2.2.1 Asteroid Taxonomic Types

The various asteroid compositional types are found to be systematically distributed in the solar system based on heliocentric distance. Asteroid diameter and composition can have a large influence on the rotational period of an asteroid and its angular velocity. Hurris and Burns (1979) have concluded after analyzing 182 asteroids for their composition and diameter that asteroids with smaller diameters tend to rotate faster than larger ones. But C type asteroids typically rotate 80% slower than S type asteroids (Dermott and Murray, 1982). Seven distinct compositional
types of asteroids and their properties are presented in Table 1, are found from 1.8 to 5.2 AU (Dermott and Murray, 1982).

Table 1 Seven distinct types of asteroids. For each asteroid type, an explanation is provided.

<table>
<thead>
<tr>
<th>Type</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Carbonaceous asteroids; similar in surface composition to CI and CM meteorites. Dominant on outer belt (&gt;2.7 AU). Comprises 75% of known asteroids and known for its low albedo.</td>
</tr>
<tr>
<td>D</td>
<td>Extreme outer belt and Trojans. Red featureless spectrum with very low albedo, possibly due to organic matter</td>
</tr>
<tr>
<td>P</td>
<td>Outer and extreme outer belt. Spectrum is flat to slightly reddish, similar to M types, but lower albedo.</td>
</tr>
<tr>
<td>S</td>
<td>Stony asteroids. With spectral type same as siliceous mineralogical composition asteroids. A major class in the inner-central belt.</td>
</tr>
<tr>
<td>M</td>
<td>Stony-iron asteroids or iron asteroids; featureless flat to reddish spectrum.</td>
</tr>
<tr>
<td>W</td>
<td>Visible light spectra similar to those of M types but have an absorption band near 3 µm (indicative of hydration).</td>
</tr>
<tr>
<td>V</td>
<td>Similar to basaltic achondrites. About 7% of the main belt asteroids are V asteroids type. Example: 4Vesta</td>
</tr>
</tbody>
</table>

1 Albedo is the measure of the diffuse reflection of solar radiation out of the total solar radiation received by an astronomical body.


2.2.2 Asteroid Density Distribution and Composition

Studies of the physical properties of asteroid systems are of fundamental importance when understanding the formation and early evolution of the solar system (Cloutis, Binzel and Gaffey, 2014). A body’s density can yield invaluable information on the asteroid’s internal density structure, composition and origin. In addition, information of processes regarding solar system formation and early planetesimals may still be present within asteroids. Such evidence of the primitive composition of the solar system cannot be seen on planets due to surface thermal erosion, tectonic movement and volcanism (Carry, 2012). Theoretical and physical studies have always emphasized the idea that asteroids, which are much smaller than planets and most natural satellites, represent the remnants of a population of planetesimals as they undergo relatively little geochemical alteration since their formation (Gardie and Tedesco, 1982). Thus, it is more likely to observe evidence of the formation of early planetary and solar systems by examining the density of asteroids since their inner composition is neither exposed to high temperature or pressure nor segregation of elements due to their small diameter, unlike planets.

The analysis of surface properties of small bodies (e.g. asteroids) such as spectral and light curve reflections can provide information of the body’s surficial density, but, as with any technique, there is uncertainty in the data, and the bulk composition of the body may not be truly reflected. One factor contributing to the introduction of uncertainty into the data is that surface properties are subject to changes by solar winds, radiation, cosmic rays and other space weather events.

Accurate derivation of asteroid density is effectively controlled by how accurate mass and volume can be measured. In most asteroid cases, the estimated density can vary significantly, from 0.5 to nearly 6 g/cm³ (Krasinsky, 2002).
The mass of an object can be derived from the gravitational force that it exerts on other objects. Methods which are used to estimate asteroid mass include: orbit deflection during a close encounter with another celestial body, planetary ephemeris (which takes into consideration gravitational influences with numerical models), spacecraft tracking using radio signals emitted from a spacecraft, and monitoring the orbit of a satellite which can be determined from optical and radar images (Carry, 2012). Each method presents data uncertainties, but can still provide significant information on asteroid mass. More details on the uncertainties associated with the estimation of asteroid mass can be found in Carry (2012).

Asteroid volume is directly proportional to asteroid diameter and to how precise the diameter can be calculated. Another important parameter related to density and mass distribution is the porosity, which can reach significant values of 20% to 50%; objects with high porosity values (exceeding the latter value) are often considered loose-rubble.

How precise an asteroid’s density is determined directly depends on the techniques used to measure its volume and mass. Also, knowledge of the asteroid’s porosity can significantly improve density measurements and our understanding of the asteroid’s density distribution. In this study, the density distribution of asteroids is simulated to be normally randomly distributed and the mean density of each model is selected based on pre-made assumptions. Forward and inversion models of different asteroids and density distributions are investigated.
2.3 Forward and Inverse Modelling

2.3.1 Forward Modelling Theory and Concept

Forward modeling is a technique to simulate, using algorithms, a sensor’s response (e.g. gravity gradiometer, magnetometer etc.). Forward modelling is a useful geophysical tool that simulates observations based on a presumed model of material properties. In addition, forward modelling can develop a physical meaning of the simulated model properties. Gravity, magnetics, resistivity, and induced polarization are methods used in applied geophysics to probe the Earth's subsurface. Generally, forward modelling is used to infer the structure and composition of the sub-surface and compare with measurements taken on the Earth’s surface, from boreholes or from an airborne platform (Butler and Sinha, 2012).

After identifying the forward problem, forward modelling calculates the model anomaly to design the experiment. Hence, signals are simulated from the model and processed through theoretical equations. In many case studies, forward modelling has been used to investigate and interpret geophysical properties. For example, density models of a SAGD reservoir in the McMurray formation in northern Alberta, were forward modelled and results investigated for bitumen depletion trends around wellbores (Elliott and Braun, 2017). Also, 3D resistivity forward modelling was conducted to investigate for leaks of saline water from a pond in Laughlin, Nevada (Zhang, Mackie and Madden, 1995). Many other forward modelling examples of different geophysical properties for different purposes can be found in other scientific papers (Torres-verd and Habashy, 1994). Forward modelling software is developed based on the physical property of interest and environment. In this study, asteroid gravity forward modelling
is conducted using Interactive Geophysical Modelling Assistant Software, IGMAS+ (Schmidt and Gözte, 2014).

2.3.1.1 IGMAS+ Software - Forward Modeling

IGMAS+ is an interactive modelling software that simulates the expected signal response of pre-defined models, herein density models. It provides a tool for 3D modelling of potential field data, including physical parameters such as density or susceptibility. The software allows the user to build simulated models, and to assign and edit physical parameters and properties. It also calculates the gravitational or magnetic response, interactively modifies, and visualizes models in 2D or 3D view, updating the calculated fields in real-time.

In IGMAS+, gravity and gravity gradients are synonymous with the gravitational acceleration and its gradients, as it does not include the effects of centrifugal acceleration. In this study, gravity is thus identical with gravitational acceleration unless otherwise noted. Centrifugal accelerations for asteroids are calculated outside of IGMAS+. Gravity signals for the entire volume are calculated by transferring the volume integral into a sum of line integrals denoted as sections in IGMAS+ which are user defined. For a homogeneous sphere, equations 7, 8, and 9 are used in IGMAS+ to calculate the gravitational components and gradients for each voxel. IGMAS+ results can be visualized in 2D cross section view, a 2D surface planar view, and a 3D view. Model voxels and interfaces can be imported to IGMAS+ via file import or modified through a graphical user-interface manually (Schmidt and Barrio-Alvers, 2014). Each model
presented in this research is divided into voxel cubes and each has one uniform specific mass; the center of mass of each voxel is the center of the voxel cube.

\[ g_z = \frac{4}{3} \pi G \rho R^3 \frac{-z}{\sqrt{(r^2 + z^2)^3}} \]  \hspace{1cm} (7)

\[ g_{zx} = \frac{4}{3} \pi G \rho R^3 \frac{3xz}{\sqrt{(r^2 + z^2)^5}} \]  \hspace{1cm} (8)

\[ HGz = \sqrt{g_{zx}^2 + g_{zy}^2} \]  \hspace{1cm} (9)

R is the radius from station to center of mass (for sphere), \( R = \sqrt{(x^2 + y^2)} \)

\( G \) is gravitational constant 6.67\( \times \)10\(^{-11}\) \( \text{kg}^{-1} \text{m}^3 \text{s}^{-2} \)

\( \rho \) is the model density

\( x, y, z \) are distance vector components between center of mass and station coordinates

\( g_z \) is the vertical gravity component

\( g_{zx} \) is the derivative of the vertical gravity with respect to \( x \)

\( g_{zy} \) is the derivative of the vertical gravity with respect to \( y \)

\( HGz \) is horizontal vertical gravity gradient
2.3.2 Inverse Modelling Theory and Concept

Inversion is the opposite procedure of forward modelling; it estimates possible models using available observations/data. The goal of inversion modelling is to generate a reasonable model that simulates a property through the inverse algorithm. The input data to the inversion are field survey details, measurements, constraints and any other information that can minimize misfits and estimate an inverse model. Thus, the inversion process starts with an initial estimated model, and then proceeds with a forward calculation on the initial model to predict what measurements would be recorded if a survey were carried out over that initial model. The resulting data set is called the predicted data. The predicted data are then compared to the observed “actual” data from the field. Then disagreements with the actual model are calculated, and the process is iterated until the misfit limit is satisfied or the misfit does not improve anymore. The best model is the one that is close to the true model and would result in the same signals as the measured responds. It is important to note that no unique solution exits. In inversions, the observed data that are used to construct a model anomaly, can result in more than one solution, see Figure 8. Additional information, often referred to as constraints, can help to eliminate some non-unique solutions and lower the number of admissible models.
Figure 8 Forward Modelling, multiple solutions of anomaly models result in the same measured responds.

This issue is referred to as the problem of non-uniqueness. All problems with more unknowns than data will have an infinite number of solutions. Therefore, there will always be errors associated with every data point in the predicted inverse model. Errors will increase as more unknowns are presented in the inversion problem. Some physical measurements do not contain adequate information. For example, potential field data (including magnetic and gravity surveys) do not contain information about the distance between cause and effect.

The following steps outline the steps of the inversion process:

**Input:** data collected from field surveys, estimation of errors and uncertainty, algorithm used for forward modeling within the inversion algorithm. All integrated with estimations of errors and constraints/assumption about the model.
**Discretization:** discretization of the body of interest into voxels. Each voxel has the same dimensions and its own physical property value.

**Decision Criteria:** Comparison of data predicted and survey measurements. With the integrated information added to the model, the decision is made for fitting a best model based on inversion algorithms and tolerance. Comprehensive understanding of assumptions that are included in the models and errors associated with the input data are crucial in generating meaningful results.

**Inversion:** A set of equations and mathematical tools that are used to find the predicted value of each model cell, with the goal of being consistent with the field measurements.

**Evaluation:** Evaluation of inversion results. Producing the optimal model which is consistent with the measured data and pre-defined information. This also includes information regarding the environmental properties of the survey and area of interest (on Earth or in space).

**Iteration:** Iterations will not produce a perfect model, but rather test a range of different possible/admissible models.

**Interpretation:** The models produced should be simple and easy to understand. The outcome model can be interpreted by different perspectives; geological, geotechnical, geophysical etc. based on the analysis needed.

To constrain the infinite number of solutions and to overcome the issue of non-uniqueness, data of observed measurements and assumptions made in models are integrated to select the best model information that can help to compute a better inversion are summarized as follow:

**Geological Information:** For instance, knowing the host rock can be very important. For example, considering the density and the structural characteristics of the host rock and faults in
the area of interest provide a better understanding of the targeted body. Basic structural information may be known from boreholes, surface mapping and other surveys.

**Geophysical Information:** Magnetic susceptibility and density contrast are geophysical properties, which are used to infer the nature of the sub-surface materials. Density contrast can be positive or negative depending on the relative density of the host rock and the targeted body.

**Logical Information:** The inverted model should simply explain the observed data. Simple models that may have a pattern are more preferable than complicated ones.

### 2.4 Muon Tomography

Thousands of sub-atomic particles are generated in the atmosphere and travel towards the Earth, penetrating our bodies and the Earth continuously. One such type of particles is the muon; they are second-generation leptons and are produced in the upper atmosphere when cosmic rays interact with molecules in the upper atmosphere (Figure 9). The muon flux at sea level is about one muon per cm per minute (Schultz, 2007). Cosmic rays are a form of high-energy radiation coming from space, outside the solar system, that penetrate the Earth’s atmosphere. Pions, which result from the interaction of cosmic rays with the atmosphere, are short-lived particles that decay into longer-lived muons (Schultz, 2007). Muons are heavier particles than electrons and do not get absorbed by materials as quickly. Instead, they lose some of their energy slowly as they travel through matter and the denser matter is, the more energy is lost. Thus, muons are known to penetrate deep into the sub-surface in excess of hundreds of metres. Muons penetrate deeper into materials than X-rays or other forms of radiation (Borozdin, 2003). Because of this, muons are excellent for probing anomalies hidden below the subsurface.
For decades, the properties of muons have been exploited for imaging purposes and particle identification. Many muon radiographies use Coulomb Scattering of Muons to infer density information in the sub-surface and to produce 3D images of density contrast. Primarily, muons interact with matter through Coulomb force, with no nuclear interaction and less radiation than electrons (Schultz, 2007). Coulomb force states that opposite charges attract each other and that same charges repel. As negatively charged muons pass through a volume, they interact with the negatively charged electrons in the material and are deflected. The angle of deflection can be analyzed before and after passing through a volume of interest and can be used to gather information about the material. Coulomb scattering of muons can also provide information about particles with different atomic numbers (Z) based on the scattering angle (Perry, 2014). Algorithms have been developed, using the information acquired from the scattering angle, to
search for high atomic number materials such as uranium (Perry, 2014). However, some obstacles using this methodology have also been realized. For instance, low energy muons penetrating thin anomalies will result in large scattering angles, as will muons with high-energy penetrating thick anomalies. To overcome such problems, only the variance in scattering angle is considered. (Schultz, 2007 and Perry, 2014).

In addition, the attenuation of muons is used to detect anomalies that have high-density contrast with the surroundings. Large contrast values indicate good differential attenuation of muons and the possibility to optimize muon tomography (Bevelacqua, 2016). Muons are used to probe dense anomalies without the use of artificial radiation and provide tomographic radio images of penetrated layers before attenuation.

Muon geo-tomography depends on the principle that muon attenuation through matter is directly related to its intensive properties, specifically its density $\rho$ and more generally, its nuclear structure. Considering this principle, the measured muon flux is related to the material’s intensive property (density) and the trajectory taken by the muon from the surface to a muon detector. The measured flux at depth, beneath the subsurface, is proportional to the integral of mass-length traversed by the muons, (i.e. equation 10). (Schouten and Furseth, 2016)

$$\text{flux} \propto \int_{\text{path}} \rho \, dl$$

(10)

2.4.1 Applications of Muon Technology

Muon technology has been used for environmental, safety and security purposes since it was discovered in the 1950s (Clarkson, 2014). Different sectors have been utilizing muons to help
keep cities and environment safer and more secure (Fishbein, 2003). Many potential applications of developed muon detection devices are considered convenient and easy to install. Such applications use muon techniques for monitoring purposes. For example, such a technique is installed at a port in the Bahamas and has been working effectively to scan cargo and detect contraband trying to enter the port (Rhodes, 2015). Using this technology has improved the reliability, safe security inspection time and enhanced the functionality of the port.

In addition, cosmic ray muons are used to monitor and control nuclear waste and fuel containers (Fishbein, 2003). After nuclear fuel is used in a nuclear reactor, it must be stored in special containers and at a safe location. Monitoring those sealed containers is necessary to avoid any potential environmental hazards. Muons can be utilized to inspect and monitor the content of those containers without opening them and risking harm.

U.S. Department of Homeland Security funded research in 2007 to utilize muon rays for homeland security. To prevent any unsafe radioactive material from crossing borders, muon detectors were developed. Muons can detect highly radioactive material such as plutonium and some isotopes of uranium. Problems arise when such materials are contained in lead or iron, but to overcome this the scattering angle can be measured and will help to distinguish materials (Borozdin, 2003). Muons are highly sensitive to the changes of fluid density in the sub-surface. Injection of steam into the subsurface to enhance oil reservoir productivity will cause drops in the bitumen density and will enhance fluid mobility. Muon tomography is utilized to investigate density changes in a SAGD reservoir and the potential of monitoring fluid migration in the sub-surface.
2.5 Steam Assisted Gravity Drainage (SAGD) Reservoirs

Due to the growing demand for oil and the depletion of many conventional oil resources and fossil fuel resources, intensive research has been focused on recovering bitumen through oil sands. Oil sands in Canada are one of the largest and most challenging oil resources to recover because of the environmental and economic challenges, which are unlike those experienced by the conventional oil resources in the Middle East (Rashedi, 2018). However, the depletion of conventional oil reserves has resulted in an energy demand/supply unbalance. Therefore, attention has switched to alternative sources, of which heavy crude oil and natural-bitumen are perhaps the most readily available to meet short- and long-term needs (Shah, 2010).

Methods to enhance heavy oil recovery from oil sands, mostly located in Alberta, Canada, are developed using techniques such as cyclic steam stimulation (CCS) and steam assisted gravity drainage (SAGD). The gravity drainage method was invented by Butler in the late 1970’s (Hosseini, 2017). The SAGD process has been implemented in the oil industry since the mid-1980s and it is known to improve the recovery rate (Rashed, 2018).

The number of naturally flowing oil reservoirs compared to the number of heavy oil and oil sand resources worldwide is small and does not exceed 15% worldwide. Statistically, recovery rates from cold heavy oil (oil with same temperature as the reservoir) range from 5% to 15%; this number dramatically changes to 25 to 60% when in-situ steam injection is applied. Geological heterogeneities and fluid properties such as the high viscosity and low pressure of fluid are the two main reasons for the low oil mobility compared to conventional oil. Therefore, the SAGD process is used to increase oil motility in the reservoir for better recovery processes with lower energy and emission intensities (Gates and Larter, 2014).
Approximately 20% of oil sand reservoirs are considered shallow reservoirs at a depth of only 40-75m. Such reservoirs are seen to be suitable to extract and mine; however, mining those shallow reservoirs can cause high environmental, biological and landscape damage. Preferentially, underground in-situ recovery methods such as SAGD processes are used to enhance oil recovery, as is the case for deeper oil sand reservoirs that can only be extracted by drilling vertical and horizontal injection and producing wells (Rashedi, 2018). SAGD is a continuous process that requires a high quantity and quality of hot steam with a minimum pressure down the reservoir to push the fluids to the wellbore (Rashedi, 2018). Gates and Chakrabarty (2005) observed that the quality of injected steam should be as high as possible to avoid any condensate formation in the fluid injected into the reservoir, which will negatively affect the quality of heat transferred to the reservoir, hence constraining fluid mobility to the producing well. During the SAGD process, steam releases its remaining latent heat through condensation at the edge of the steam chamber, heating the oil sands (oil, sand, and water), see Figure 9. The viscosity of the heated oil drops and under the influence of gravity, it drains through the steam chamber, or down its edges, to the production well at the base of the chamber (Gates and Larter, 2014).
Figure 10 Steam is injected from injector wells and heat is transferred to the reservoir (oil, water and sands). Condensation of fluid takes place at the edge of the steam chamber and accumulates around the producing well to be produced to the surface.

Currently, the most effective method to mobilize and produce bitumen from heavy oil reservoirs (e.g. Athabasca oil sands reservoirs) is SAGD. Large amounts of energy, water and pumps are required to produce steam and inject it down the wellbore. However, an adverse effect of the SAGD process is the significant volume of emitted greenhouse gases, and can be seen as environmentally damaging (Gates and Larter, 2014).

Worldwide, oil reserves are estimated to be over 6 trillion barrels, mostly stored in the form of heavy oil and oil sands (Gates and Larter, 2014). Bitumen with a quality degree of less than 10 API is a highly viscous fluid with a viscosity ranging from tens of thousands to over 10 million cP under reservoir conditions. The API (American Petroleum Institute) gravity unit is used to measure how heavy or light the petroleum liquid is compared to water while cP (centipoise) unit is used to describe the fluid viscosity and its resistance to deform when stress is applied.
Heating up the reservoir and increasing its temperature by several tens of degrees lowers the viscosity of the oil by several orders of magnitude (Gates and Larter, 2014). For a successful in situ bitumen recovery process in oil sands, two requirements must be met. First, it is necessary to raise the oil mobility, which is often done by lowering its viscosity by raising its temperature, until it can be moved by natural forces such as gravity. Secondly, it is necessary to move the mobilized oil to a production wellbore so it can be produced to the surface. (Gates and Larter, 2014). The recovered fluid is tested for its cumulate steam to oil ratio (cSOR), and it is determined whether it is economically feasible to extract. Recent studies show that the differential pricing of oil and natural gas vs. steam generated is still profitable even in extreme cases (Gates and Larter, 2014).

Some factors that influence the productivity and efficiency of the SAGD process are summarized below.

The rate of reduction in oil viscosity with increased temperature, i.e. the extent of viscosity reduction compared to heat added. In addition, oil viscosity is affected by the natural oil composition. Oil with low viscosity at the top of the oil column, and a more viscous oil at the bottom can affect the development of the steam chamber. Thermal conductivity of the host rock and fluid influence the heat transformation rate to the edge of the thermal chamber. Steam is also absorbed by the cap rock and thief zones or faults will significantly affect heat transformation to the bitumen. Geological layers (cap rock) with low permeability such as shale or siltstone will prevent steam rise and limit oil mobility. The oil relative permeability in the hydrocarbon reservoir at the edge of the steam chamber will affect the mobility of oil. Maintaining a safe zone between the injector and producer wells is needed to prevent live steam production. Density
contrast between the produced well (condensate steam and bitumen) and reservoir fluids is a key factor of SAGD process efficiency (Gates and Larter, 2014).

Parameters inherent to the reservoir can also constrain the SAGD process. SAGD performance is significantly affected by the geometric reservoir parameters, such as host rock properties. For example, horizontal and vertical rock permeability, reservoir heterogeneity, and oil reservoir thickness are all crucial parameters in the SAGD process. In addition, operational conditions such as the reservoir depth, distance between wells, well length and steam injection rate may influence the SAGD process efficiency (Barillas, Dutra and Mata, 2006). Nevertheless, the enhanced bitumen recovery, economic and environmental benefits are the main concerns in the evolution of the SAGD process, which demands a controlled steam distribution in the reservoir resulting in uniform steam chamber growth (Rashedi, 2018). Such factors will raise the importance of a continuous monitoring technique of fluid migration and steam growth in the SAGD reservoir.
Chapter 3

Resolvability of Asteroid Density Distribution from Surface Gravimetry – Forward Modelling

3.1 Introduction

Asteroid science contributes to a deeper understanding of the universe. Information crucial to advance this understanding includes data on asteroids composition, geometry, orbits and physical properties. Small celestial bodies are of interest to the space community for a number of reasons. The depletion of resources on Earth is leading some prospectors to consider asteroids for future mining potential. Studying asteroids can also provide evidence of how the solar system was formed, and maintain a record of its history. Small bodies have a small diameter that would limit the amount of radiogenic nuclides within the body’s interior. This would in turn limit the amount of internal heat generated, which would inhibit compositional alterations in the interior of the asteroid. However, small changes in composition may still result from exogenous factors such as collisions, external heating and bombardment by high-energy particles (Carry, 2012).

Considering the abundance of asteroids and small bodies in the solar system, asteroids are considered to present potential collision hazards and as such have been the subject of initiatives aimed to identify, characterize and redirect them (Pearl and Hitt, 2016). This study aims to inform missions to characterize asteroids by modeling potential density distributions, which is a fundamental property, containing significant information of asteroid composition and potentially formation. Missions to visit asteroids can further this understanding by collecting samples and
conducting on-site surveys to enhance asteroid knowledge. However, to date there have been only a limited number of asteroid rendezvous due to the high cost, technological requirements and time commitment required for preparation. Some current asteroid missions include the JAXA Hayabusa-1 mission, launched in 2013 to Itokawa, Osiris-Rex to Bennu that is due to complete its task in August 2018, and JAXA Hayabusa-2 to Ryugu asteroid. Hayabusa-2 dropped three lander vehicles to the surface between the end of September and early October 2018 and will be finishing its mission by 2020 with the return of asteroid samples. At the time of this thesis, only two other asteroid missions are being considered, the HERA mission by ESA (European Space Agency) and the GRASP mission (Canada, the initial mission concept study proposal was declined in Summer 2018). The Gravimetric Asteroid Surface Probe (GRASP) is a proposed micro satellite mission, developed by a group of Canadian/US space scientists and companies dedicated to the development of asteroid surface observations. It includes an asteroid lander and a hopping rover equipped with a gravimeter and magnetometers as the main science sensors. The forward modelling study aims at assisting to identify the requirements for a gravimeter’s sensitivity as well as to design a surface survey.

This research evaluates the feasibility of measuring the density distribution of asteroids, and the detectability of boulders by simulating surface gravity surveys. Models utilizing real asteroid data are constructed and gravitational forces are mapped to investigate density distribution and distinguish between asteroids that exhibit high-density heterogeneity and homogeneous asteroids. Gravity forward models are simulating asteroids with different designed density models and boulders of differing mass, to model gravity signals on the asteroid’s surface. In the forward models, the gravity stations imitate rover locations on the surface for which measurements would become available during the mission. These simulated gravity stations are
created on the asteroid’s surface with a constant local elevation of 2m above the asteroid, which approximates the length of the rover legs.

On Earth, terrestrial gravimeters can measure relative gravity with high accuracy during surveys. However, most field deployable gravimeters have a drift-compensated repeatability within ~ 5-10 μGal over the course of a day. Their size, weight and temperature sensitivity makes them unsuitable for small body’s gravimetry applications in space (Carroll, 2015). The simulated asteroids models tested here are considered to be in the category of small asteroids with a radius of 250~500 metres.

The surface gravitational acceleration modelled for the simulated asteroid models are on the order of mGal in magnitude and to survey such small bodies, a highly precise and accurate gravimeter sensitive to sub-mGal is required. The VEctor Gravimeter for Asteroids (VEGA) developed for space geophysical exploration is capable of measuring gravity signals at the μGal level in space (Carroll, Spencer and Zee, 2015).

To date, not much is known about the density distributions of asteroids or their gravity fields, as this requires sufficient orbital or surface data. In theory, after sufficient time, asteroids tend to reach harmonic density distribution as surficial regolith migrates and reshapes its body for stabilization. Kanamaru and Sasaki (2001) indicate that meteorites can be a seismic source on asteroids that will result in a tendency of loosing regolith and small gravel to move to areas of lower gravity potential. The surface topography of an asteroid becomes close to an equipotential surface over sufficient time (Kanamaru and Sasaki, 2001). This implies that density variations within the asteroid tend to be small compared to the mean density of the asteroid. This is taken into consideration while building the density models for the simulated asteroids.
To measure the mass of a boulder on an asteroid for potential scientific analysis after pick-up (as proposed in the NASA ARRM Mission, which was cancelled in 2017), studies on gravity and density distributions of asteroids are necessary to mitigate any undesired mission failure caused by the boulder’s lack of integrity or excessive weight. Based on the assumed density, the simulations can yield information on the sensitivity required by the gravimeter, density and minimum detectable mass of a boulder for retrievable missions, and can be used to gain a better understanding of the material composition of the asteroid. Finally, these simulations can determine the optimal locations for gravity surveys and reduce the station number.

3.2 Methodology

This research project focused on two real asteroid models, 2008 EV5 and 25143 Itokawa as well as a perfect sphere model. Models of the centrifugal acceleration and density structures for these asteroids were created (using Matlab). The software used for calculating the surface gravity of the asteroid models was IGMAS+. The asteroid models are simulated to realistic density values statistically, and the density values are either normally randomly distributed or uniform throughout. Asteroid surface models are obtained from NASA’s 3D Asteroid Catalogue (Durech and Sidorin, 2008) as faces and vertices that describe the surface geometry of the simulated asteroids. A 3D polyhedron is created by interpolating faces and vertices to create a 3D triangulated surface model. It is important to note that for this study, the accuracy of the surface models obtained has no impact in the accuracy of the forward models, as they are constant for all models. It is therefore not important if the asteroid surface model is precise or not. The 3D triangulated surface model is then converted to a set of voxels that form the asteroid body. The
discretized voxels have equal dimensions (10m × 10m × 10m) and each voxel can have different density values (heterogeneous models) or a uniform density value (homogeneous density models), see Figure 11.

Figure 11 Model components for an Itokawa model with correlation length of 20m.

In the heterogeneous asteroid models, two parameters were varied: the variance of density and the correlation length. Density variance is the density variation within the normally distributed heterogeneous density model; the correlation length is the scale within which the density values of voxels are correlated. In the homogenous density models, a uniform density value equal to the mean density of the heterogeneous density models is used. The specifications of the simulated asteroids and sphere models are summarized in Table 2.
Table 2 Asteroid Models and Sphere Model Specifications.

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Itokawa</th>
<th>2008 EV5</th>
<th>Sphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-min (m)</td>
<td>-255</td>
<td>-225</td>
<td>-205</td>
</tr>
<tr>
<td>$x$-max (m)</td>
<td>305</td>
<td>225</td>
<td>205</td>
</tr>
<tr>
<td>$y$-min (m)</td>
<td>-155</td>
<td>-225</td>
<td>-205</td>
</tr>
<tr>
<td>$y$-max (m)</td>
<td>145</td>
<td>215</td>
<td>205</td>
</tr>
<tr>
<td>$z$-min (m)</td>
<td>-115</td>
<td>-205</td>
<td>-205</td>
</tr>
<tr>
<td>$z$-max (m)</td>
<td>125</td>
<td>195</td>
<td>205</td>
</tr>
<tr>
<td>Mean Density (g/cm$^3$)</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>#of voxels</td>
<td>17736</td>
<td>34868</td>
<td>38544</td>
</tr>
<tr>
<td>Heterogeneous Models</td>
<td>40</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Density Input-Variance (g/cm$^3$)</td>
<td>0.04, 0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Correlation length (m)</td>
<td>10, 20, 50, 100</td>
<td>10, 20, 50, 100</td>
<td>10, 20, 50</td>
</tr>
</tbody>
</table>

Forward models of a boulder of 8m$^3$ with different uniform density values are constructed to analyze boulder gravity effects. The boulder gravity signals are simulated using gravity stations at different distances away from the boulder’s center of mass. As the distance increases, the gravity signals drop exponentially.

Given a mesh composed of a finite number of voxels of equal volume (1000m$^3$), and with a known density distribution, the gravitational components are measured over irregularly shaped asteroid bodies. For each gravity station the static gravitational forces from the center of each voxel are calculated and the sum of the contributions of all voxels is integrated. First, a sphere with radius of 205 metres discretized to voxels of 10m × 10m × 10m with a constant density of 3 g/cm$^3$ is used to validate the functionality of the IGMAS+ software, as well as to estimate errors.
when calculating gravity components with a coarse voxel volume which approximates a sphere with cubes. This is achieved through comparison with an analytical solution for the gravity effect of a uniform density sphere.

IGMAS+ was originally designed to model potential fields for limited size geological targets in the subsurface with approximately flat surfaces. This is taken into consideration while adopting the software to model entire asteroids, as the stations will be around the entire body, and not only on one side. IGMAS+ allows the user to design arbitrary voxel volumes as well as stations surrounding it or even inside the volume. IGMAS+ imports the 3D objects as voxel files and voxels of equal dimensions are each assigned material properties, here density.

![Figure 12](image.png)

Figure 12 The sphere on the left has a radius of 205m and a uniform density of 3g/cm³. Each voxel has equal density and volume. The figure on the right shows the sphere body voxels and gravity stations at the surface (red points).
In the 3D model, the imported voxel files have x, y and z coordinates representing the center of each cube voxel (Figure 12). Simulating a gravimeter landing on an asteroid, the gravimeter sensor itself is placed at an elevation of 2m above ground level. A set of points representing the measurement stations are distributed around the surface of the asteroid are defined and imported to IGMAS+ as a .csv file (Figure 12). The gravity components and total gravity at each station are calculated. For the spherical homogeneous model, gravity signals are between 17-18 mGal (see Figures 13 & 14).

![Sphere with 3 g/cm³ Uniform Density - Total Gravity](image)

**Figure 13** Histogram of total gravity signals on the surface of 205m radius sphere with 3 g/cm³ uniform density.
Figure 14 Total surface gravity of a sphere with a radius of 205 m and uniform density of 3 g/cm³.

The results were also compared to analytical solutions by calculating the gravity using mass from volume and density (see equations 11, 12 & 13).

\[ g = \frac{G m}{r^2} \]  

(11)

g is the gravitational acceleration in m/s².

G is the gravitational constant and it is equal to 6.67×10⁻¹¹ m³ kg⁻¹ s⁻².

m is the mass of the spherical asteroid in kg.

r is the distance of the station from the sphere center in metres.

\[ V_{sphere} = \frac{4}{3}\pi r^3 \]  

(12)

\( V_{sphere} \) is the volume of the spherical asteroid in cubic metres.

r is the radius of the spherical asteroid in metres.
\[ m = V_{sphere} \rho \]  

(13)

\( m \) is the mass of the spherical asteroid in kg.

\( V_{sphere} \) is the volume of the spherical asteroid in cubic metres.

\( \rho \) is the density of the spherical asteroid in kg/m\(^3\).

The total number of voxels in the sphere model is 38544, each of them with a volume of 1000 m\(^3\) for a total volume of 38544000 m\(^3\). The corresponding radius of a perfect sphere would be 209.550833 metres. From the analytical solution, equation 11, the median gravity at each station is equal to 1.73\times10^{-4} \text{ m/s}^2 (or 17.3 mGal) since it is a sphere with uniform density and constant distance from any station to the center of mass. The volume of the perfect sphere of 205 m radius is 36086951.21 m\(^3\) and its total gravity is 1.69\times10^{-4} \text{ m/s}^2 or 16.9 mGal from 207m away from sphere center of mass; 205m of spheroid and 2m for station elevation. The differences between the analytical and the numerical solution is 0.4 mGal, which is caused by the non-spherical shape of the voxelized sphere. The voxel size of 10m\(^3\) does not allow for a smooth spherical surface which leads to a varying radius of the numerical sphere. For further analysis, this difference will cancel out as we only compare numerical models which are based on the same voxel model. The reason that the stations are distributed irregularly is due to the fact that some stations are located inside a voxel as the voxels can be up to 5 m above the radius of the sphere. Those stations were removed from the analysis.

Figure 14 illustrates the total surface gravity signals at each station in the z-y plane view. The majority of gravity signals measured from these stations show fluctuations in the total gravity of
around 17.5 mGal. In Figure 12, the histogram shows that most stations (45000+ stations out of ≃ 54000 stations) measure between 17-18 mGal with low variance around the mean, which is consistent with the analytically calculated gravity value. However, some stations show higher gravity signals than expected. The stations deviating from the calculated value are influenced by a variety of factors: the volume of discretized spherical model with voxels is not identical with the volume of the perfect sphere, some artifacts related to using the center of mass instead of the full extent of the voxel volume, singularity at the poles at which a function can take an infinite value, and the dimension and orientation of the voxels which can create lower or higher gravity signals especially at the surface in proximity to the stations. By comparing the simulated results in Figure 14 and the calculated values, it is clear that the accuracy of the software is acceptable on the order of sub-mGal. When comparing different density distributions for the same asteroid with the same discretization, most of the numerical errors should cancel out and the uncertainty in the calculated surface gravity differences can be neglected. That also includes the gravitational effects due to solar radiation pressure and tidal induced gravity perturbations.

3.2.1 Asteroid surface gravity stations

Stations are placed on the asteroid’s surface by creating a normal vector to the local surface of the point using an algorithm that takes into account the point location and its surrounding points. This process is repeated for each point until the the asteroid surface is covered, resulting in a high number of stations, here 0.1 station/m². Some stations are located inside the voxels and removed. Creating a normal vector can yield stations inside the voxel volume due to the
discretized surface using cubes. The stations inside the asteroid are identified and then removed. (see Figure 15)

![Figure 15: The left figure shows a spherical body with a radius of 205 m and heterogeneous density distribution; mean density is 3 g/cm³. Gravity stations are represented by red dots on the sphere’s surface; the figure on the right shows the stations covering](image)

3.2.2 Boulder Mass Calculations

This study also addresses gravitational calculations of a boulder on the asteroid surface with different density values. The gravitational signals of the boulder are estimated using Newton’s law of gravitation from the boulder’s center of mass to the gravimeter in order to assess the detectability of the boulder mass from surface gravity stations. The boulder has dimensions of 2m x 2m x 2m and its density is varied between 0.4 and 8 g/cm³. The gravity signals sensed from the 8m³ boulder are small at approximately 1 μGal compared to variations between different density models. Gravity signals from the boulder are likely not
detectable on a realistic asteroid with a heterogeneous density distribution as gravity effects due to local density variations can exceed the boulder signal. However, a larger boulder with a high average density could potentially be detected on an asteroid with homogeneous density distribution, or one with low-density variations. Information on shape, location, size and most importantly, gravity signals, about the boulder can be obtained from landing and placing gravity stations around the targeted boulder. However, when calculating the asteroid gravity and the boulder gravity, which can be located anywhere on the asteroid surface, centrifugal acceleration can be a confounding variable as it can be as high as 1 mGal at the equator or very small (close to zero) on the rotational axis. The gravity signals caused by a boulder mass are calculated using absolute distance between the gravimeter and the boulder’s center of mass (see equation 11).

3.2.3 Centrifugal effect

Every celestial body in space is subject to rotation. The angular velocity of the rotation has a direct effect on the body’s gravity field. The effect of centrifugal acceleration and its amplitude varies across the celestial body based on the orthogonal distance from the rotational axis. Based on data downloaded from the 3D Asteroid Catalogue (Durech and Sidorin, 2008), the rotational axis is defined to be aligned with the z-axis. Asteroids can have multiple rotational axes, however, the asteroids included in this study are considered to have one rotational axis only without any tumbling. Thus, the distance of each station from the spin axis is calculated based on the x and y coordinates of each station.

\[ f = \omega^2 \sqrt{x^2 + y^2} \]  

(14)

\( f \) is the centrifugal acceleration in m/s\(^2\).
\( \omega \) is the angular velocity in rad/s\(^{-1}\).

\( \sqrt{x^2 + y^2} \) is the orthogonal distance from the spin axis in metres.

\[ g_{total} = g_{acc} + f \]  \hspace{1cm} (15)

\( g_{total} \) is the total gravitational acceleration of rotating object in m/s\(^2\).
\( g_{acc} \) is the gravitational acceleration of body’s mass in m/s\(^2\).
\( f \) is the centrifugal acceleration in m/s\(^2\).

The SI unit of gravitational acceleration is m/s\(^2\). However, the gravity values in this study are presented in mGal units, which is more appropriate to the problem.

Based on data from (Ryan and Ryan, 2009), 984 asteroids are listed with their angular velocity and maximum diameter and the centrifugal effect can be calculated using equation 1. The maximum centrifugal acceleration is calculated for Bennu, 2008 EV5 and Itokawa, and the results show centrifugal acceleration up to 4.5 and 6 mGal for Bennu and 2008 EV5, respectively, and about 0.6 mGal for Itokawa (see Table 2). Itokawa is known to complete a full revolution every 12.17 hours (Greenberg, 2009).
Table 3 Centrifugal Effect at the equator of different asteroids and spin rate (rev/hr).

<table>
<thead>
<tr>
<th>Asteroid</th>
<th>Asteroid Radius</th>
<th>Spin Rate</th>
<th>Centrifugal Acc.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m)</td>
<td>(rev/hr)</td>
<td>(mGal)</td>
</tr>
<tr>
<td>Itokawa</td>
<td>267.5</td>
<td>0.0824</td>
<td>0.5</td>
</tr>
<tr>
<td>Bennu</td>
<td>272</td>
<td>0.2327</td>
<td>4.5</td>
</tr>
<tr>
<td>2008 EV5</td>
<td>210</td>
<td>0.2685</td>
<td>6</td>
</tr>
</tbody>
</table>

The strength of the centrifugal effect acting on each station located on Itokawa’s surface changes based on the distance from the spin axis from 0 to 0.65 mGal for Itokawa (see Figure 16). The total surface gravitational acceleration of Itokawa with a uniform density distribution of 2 g/cm³ varies from 5-9 mGal (see Figure 17).

Figure 16 Centrifugal acceleration at the surface of Itokawa. The spin axis is along z-axis.
3.3 Results

Following the methodology used in the gravity forward modelling of the spherical body, some of the model results are outlined here. Two asteroids are modelled for their static gravitational acceleration, Itokawa and 2008 EV5, and also a spherical body with a radius of 205m. The models’ discretized voxels are modelled for their surface gravitational acceleration. First, homogeneous density models of 2 g/cm\(^3\) and 3 g/cm\(^3\) are forward modelled for Itokawa and 2008 EV5, respectively. In addition, heterogeneous density models following a Gaussian autocorrelation function are generated by varying the input density variance between 0.04 g/cm\(^3\) and 0.08 g/cm\(^3\), and the correlation length from 10, 20, 50 and 100 m. The correlation length is the proximity within which the densities of voxels are correlated, according to a Gaussian distribution (Figure 11).
3.3.1 Asteroid 25143 Itokawa

The Itokawa model is voxelized to 17736 voxels. Itokawa’s homogeneous model has a uniform density of 2 g/cm\(^3\) while the heterogeneous density models have mean densities of 2 g/cm\(^3\) and input density variance of 0.04 or 0.08 g/cm\(^3\), and all heterogeneous density models follow a Gaussian distribution. Forty heterogeneous density models for the Itokawa asteroid were constructed; the first 20 models have a density variance of 0.04 g/cm\(^3\) and the other 20 models have a variance of 0.08 g/cm\(^3\). Within the 20 models, five different correlation lengths were used, 10m, 20m, 50m and 100m, so that, for each correlation length five-density models are constructed. Running the same algorithm to create four different density models with the same input will result in different output as values are randomly generated using different seed numbers. The forward models calculate Itokawa’s surface gravitational acceleration, which varies between 6 to 9 mGal. Surface gravity is strongly influenced by density variance, distribution and topography, but the values produced here are consistent with the results published previously by Greenberg (2009) and Mukai, Nakamura and Sakai (2006), who considered homogeneous density models only.
Figure 18 (a) Itokawa density model [2] with a mean density of 2 g/cm$^3$, calculated variance of 0.155 g/cm$^3$ and correlation length of 10m. (b) Gravity forward model of Itokawa model [2]. Total gravity measured at the surface of Itokawa. (c) Difference between the surface gravitational acceleration of model [2] and the surface gravitational acceleration of Itokawa with a uniform density of 2g/cm$^3$.

For example, model [2] of Itokawa has density values between 0.3 and 3.3 g/cm$^3$ and a mean density of 2 g/cm$^3$. The forward models are generated using the following steps; creating a 3D density model, calculate gravitational acceleration at the surface stations, and subtract gravitational acceleration of the homogeneous model from the heterogeneous density model (see Figures 18a, b and c). The gravity values at the surface are then interpolated to create a surface grid as shown in Figure 18. The correlation length of the density model is 10m and its calculated density variance is 0.155 g/cm$^3$. The difference between input variance and calculated variance are due to the code used to generate the density models and randomness of density values. The limited number of voxels in some scenarios are due to the large correlation length compared to the dimensions of the asteroid. The gravity forward model shows that Itokawa’s surface total gravity varies between 6 to 9 mGal and with high gravity signals in the middle part of the
asteroid, and lower gravity signals towards the ends along the x-axis. The difference in gravity between homogeneous and heterogeneous models are between -0.5 and 0.5 mGal (Figure 18c). The trend of the gravity difference on the surface follows the density distribution of the heterogeneous model as seen at the surface. Large differences in gravity corresponds to large density anomalies. Although only the surface of the density model is shown, the correlations length used defines that the density in the subsurface is correlated, which projects the surface density distribution into the subsurface. The second set of 20 heterogeneous models of Itokawa have an increased input variance of 0.08 g/cm$^3$. Figure 19 shows model [7] of this group and the results. In model [7], the correlation length increased to 20m and the calculated variance is 0.082 g/cm$^3$ which is very close to the input variance of 0.08 g/cm$^3$. The outlined results in Figure 19c show the same total gravity difference trend as in its corresponding heterogeneous density model. The rest of the models and input data can be found in appendix (A).
3.3.2 Asteroid 2008 EV5

This asteroid is more symmetrically shaped than the Itokawa asteroid, which makes it a useful model for comparison. The same procedure that was applied in the modelling of Itokawa is applied to 2008 EV5. The number of voxels is high compared to Itokawa, due to the larger volume of 2008 EV5, resulting in 34868 voxels.

20 models of 2008 EV5 with heterogeneous density distributions are created with different correlation length, 10, 20, 50 and 100 m. The input variance of these models is 0.08 g/cm³ with a mean density of 3 g/cm³. The gravity forward models of 2008 EV5 with a uniform density of 3 g/cm³ and the heterogeneous density models exhibit gravity magnitudes between 14 and 17 mGal. Figure 20 shows the gravitational acceleration of the uniform density model.

![Figure 20 Surface gravity model of the 2008 EV5 asteroid with a uniform density model of 3 g/cm³.](image)

Model [14] of 2008 EV5 has a correlation length of 50m and a mean density of 3 g/cm³. The calculated density variance is 0.075 g/cm³ that is close to the input variance of 0.08 g/cm³.

Figure 21a shows model [14] of 2008 EV5, which has density values ranging from 2.45 to 3.67 g/cm³. After forward modeling, the gravitational acceleration of the uniform gravity model is
subtracted from the heterogeneous gravity model. The difference in magnitudes between both models ranges between -1 and 1 mGal and clearly shows the spatial distribution of the heterogeneous density distribution across the asteroid, especially when large correlation length is assigned (see Figures 21a and 20b).

Figure 21 (a) 2008 EV5 Density model [14] with a mean density of 3 g/cm³, variance of 0.075 g/cm³ and correlation length of 0.05 km. (b) Gravitational Acceleration of model [14] of 2008 EV5. Total gravity varies between 14 and 17 mGal. (c) Difference in Itokawa surface gravity of model [14] and surface gravity of uniform density model of 3 g/cm³.

The results for the rest of the 2008 EV5 models are presented in Appendix A. All model results reveal the spatial distribution of density distribution from the simulated surface gravitational acceleration.
3.3.3 Boulder Mass Sensitivity

The gravity signals of a boulder on an asteroid surface with 8m$^3$ volume and constant density of 2.5 g/cm$^3$ are calculated for increasing distances between the boulder and station. The gravity signals drop exponentially as the gravimeter moves away from the boulder’s center of mass. In Table 4, the gravity signals of the boulder are listed. The boulder mass effect is small and varies around 1 µGal and increases with increasing proximity to the boulder to a few µGal (Figure 22). Calculating the gravitational signals of the same boulder volume but with different uniform density values yields the results shown in Figure 22. At a distance of 10m away from the boulder’s centre of mass results in amplitudes between 1 to 10 µGal for different density values, which indicates that in principle, the boulder mass can be detected by a space gravimeter as sensitive as VEGA. However, other sources of gravity signals could overprint this small signal.
Table 4 Gravity signal of a 8 m³ boulder cube with a density of 2.5 g/m³. Distance is the absolute distance between the boulder's centre of mass and the gravimeter station.

<table>
<thead>
<tr>
<th>Absolute Distance (m)</th>
<th>Gravity (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1334</td>
</tr>
<tr>
<td>10</td>
<td>0.001334</td>
</tr>
<tr>
<td>50</td>
<td>0.00005336</td>
</tr>
<tr>
<td>100</td>
<td>0.00001334</td>
</tr>
<tr>
<td>200</td>
<td>0.00003335</td>
</tr>
<tr>
<td>500</td>
<td>5.336E-07</td>
</tr>
<tr>
<td>800</td>
<td>2.08438E-07</td>
</tr>
<tr>
<td>1000</td>
<td>1.334E-07</td>
</tr>
<tr>
<td>1500</td>
<td>5.92889E-08</td>
</tr>
<tr>
<td>2000</td>
<td>3.335E-08</td>
</tr>
</tbody>
</table>
Figure 22 8m$^3$ Boulder gravity signal vs. distance along the x-axis, from boulder centre of mass and the gravimeter. 8m$^3$ Boulder with varying densities.

3.4 Discussion

Itokawa was visited by the Japanese Space Agency (JAXA) Hayabusa-1 mission (Saito, 2006). During the mission, the spacecraft stayed in proximity to the asteroid; about 7 km away, for three months. Data collected by the mission includes the asteroid’s mass ($3.58 \times 10^{10}$ kg ±5%) and its volume ($1.84 \times 10^{-2}$ km$^3$ ± 7%), hence the bulk density for the asteroid is $1.95 \pm 0.14$ g/cm$^3$ (Saito, 2006). Those physical properties were determined using the light detection and ranging instrument (LiDAR). Considering the information obtained from this asteroid mission, the mean density of the simulated Itokawa’s density models was estimated to be 2 g/cm$^3$ and the gravitational acceleration results are consistent with previously published results by Mukai,
Nakamura and Sakai (2006). However, Itokawa is a highly irregularly shaped asteroid; formed by the collision of two masses (Demura, 2006), and the need to investigate more asteroids with less surface variation arise. The shape of the 2008 EV5 asteroid is very close to a spheroid. This is due to the sufficient time (millions of years), minor collisions, and the tendency of celestial objects to reshape themselves to a more stable configuration (Tanga, 2009). The assumed mean density of the 2008 EV5 density models is 3 g/cm$^3$ since it is considered to be C-type asteroids (Alí-Lagoa, 2013). The average gravity acceleration simulated on the surface is compatible with given density and density variances. The difference of the gravitational acceleration of the homogeneous and heterogeneous models of 2008 EV5 asteroid is approximately within the range of 2 mGal. In comparison, Itokawa shows results within the range of 1 mGal. For the spheroid model, 15 heterogeneous density models are created with different variance and correlation lengths. The gravitational acceleration calculated varies between 16 and 18 mGal. The difference in total gravity between the heterogeneous and uniform density models ranges from -1 mGal to 1 mGal (2 mGal range). Results show that there is higher variation in surface gravitational acceleration as the asteroid’s size and/or mean density increases. The Itokawa model shows the least variation in difference between homogeneous and heterogeneous models, however, the difference is sufficiently large to distinguish between heterogeneous or homogeneous density distributions.

The resolution of these models can be increased by reducing the voxel dimensions. Creating voxels of 1-meter dimension will improve the results and accuracy and will reduce the surface discretization errors, however, the number of voxels defining the asteroid body would increase by a factor of $10 \times 10 \times 10$ and exceed the IGMAS+ limits of 30 million voxels (Schmidt and
The asteroids studied here have gravitational acceleration on the order of mGal; therefore, the accuracy required of a gravimeter to study the asteroid’s surface gravity would be on the order of sub-mGal. A space gravimetry developed by GEDEX Inc., the VEGA space gravimeter, has a predicted sensitivity on the order of µGal in space conditions (1 mGal on Earth) and therefore would be capable of mapping asteroid gravity studied herein.

Besides the asteroid mass, other forces can influence the gravity field and include solar radiation pressure and gravity from other celestial bodies, such as the sun and planets. The gravitational acceleration from the Sun and planets are sufficiently well known and can therefore be estimated by using the distances between asteroid and sun and other planets (Kobzar, 2012). The effect of solar radiation pressure can be about 20 times higher than that of the gravitational attraction by the asteroid (Greenberg, 2009). In this paper, solar radiation pressure and the imposed gravitational attraction from other celestial bodies are not corrected for, as the difference of homogeneous and heterogeneous models is at the focus herein and those effects would cancel out. The gravitational signal of a boulder on the asteroid’s surface was determined to be on the order of µGal and is not likely to be detectable, especially on an asteroid that has a highly heterogeneous density distribution. Finally, asteroids with homogeneous and heterogeneous 3D density distributions are distinguishable and differences are on the order of mGal. The density distribution of an asteroid can reveal significant information on the composition and the creation of the asteroid. Further studies can enhance the work presented here such as inverse modeling and a closer look into the gravitational gradient tensor, which in principle can be determined from a vector gravimeter such as VEGA. As a next step, the minimum number of gravity stations required to distinguish homogeneous and heterogeneous density distribution can be investigated as the number of stations are limited to about 100 for the proposed GRASP lander mission.
To sum up, modeling density distributions of asteroids can reveal significant information on asteroid composition and density structure. Studies of gravity forward modeling of those asteroids reveals that:

- Homogenous and heterogeneous density distributions are distinguishable (order of mGal) and would shed light on the composition/creation of asteroid.
- Surface boulder signals are at the order of μGal and are likely not detectable, especially on asteroids with heterogeneous density distributions.
- Asteroid gravity requires a gravimetry with sub-mGal accuracy.
- Asteroid gravity could be mapped sufficiently well with the VEGA space gravimeter (GEDEX Inc.); VEGA sensitivity is at the order of μGal level.
Chapter 4

Resolvability of SAGD Density Distribution Using Muon Tomography

4.1 Introduction

For the purpose of enhancing oil reservoir productivity index, different techniques have been developed to enhance the efficiency of heavy oil extraction such as SAGD and hydraulic fracturing (Hunt, 1988). Heavy oil reservoirs are produced using SAGD techniques, where steam is injected from injector wells to reduce the oil’s viscosity and displace bitumen to an underlying parallel (or producing) well. Detecting changes in density due to the extraction of bitumen is significantly important to improve production efficiency as large changes may indicate that the sub-surface volume is depleted of bitumen and further injection of steam would have no further economic benefit. The objectives of this research are to develop a procedure to monitor heavy oil production and to mitigate any environmental hazards from out-of-zone flow. The SAGD process requires a high amount of energy and a large volume of water, both of which can be decreased through more efficient resource allocation by identifying regions with low and high permeability and avoiding steam propagation in the geological layers with low permeability (Reitz, Krahenbuhl and Li, 2015). Traditional techniques used to model the density distribution of a reservoir after a period of depletion include microseismic monitoring, 4D & 3D seismic, Gravity Gradiometry, Borehole Resistivity, Reservoir Temperature and Pressure surveys (Maxwell, 2010; Asa, 2013; Elliott and Braun, 2017). Seismic monitoring techniques provide valuable information on bitumen migration, however, these techniques are considered costly and
are therefore only conducted once every one to three years. Gravity surveys measure a continuous field and require highly sensitive gravimeters. Gravity surveys are also susceptible to uncertainty, a small anomaly close to the surface generate the same gravitational signals as a big anomaly buried deeper beneath the subsurface (Elliott and Braun, 2017). Borehole surveys only provide information on the surrounding area of the wellbore.

Tomographic methods are usually used to build a model or construct an image of buried objects by multiple projections of signals from different directions (Schultz, 2007). Herein, tomography using scattered cosmic ray muons is discussed. Each muon carries information along the path of travel through matter and when measuring the paths and effects of multiple muons, the density properties of those objects can be estimated. Muon tomography utilizes the continuously occurring natural event of muons being generated in the upper atmosphere to measure changes in the density distribution of a SAGD reservoir over time (Figure 23). Muon tomography provides information on the material properties through which the muon travelled, from the surface to the muon sensor. In SAGD reservoirs, density change due to steam injection and bitumen production can be monitored by measuring the intensity of muons coming from different directions penetrating the reservoir rocks or depleted areas. Muons lose energy as they travel in dense material, therefore by measuring the number of muons arriving from different directions it is possible to distinguish areas with high and low densities, which may reveal areas with different depletion rates. It is important to note that this study only counts muons arriving at a detector and its trajectory, but not their individual energy levels. In the following, muon counts and muon intensity are used synonymously. Muon particles at their typical energy level at sea level are considered to have energy to penetrate into few hundred metres thick rock layers (Taiuti, 2011). The archaeological investigation conducted in the Egyptian Chephren pyramid
used muon tomography to find unknown chambers (Marteau, 2012). In addition, muon tomography techniques have been used to image density contrast to explore for compact uranium deposits (Bryman and Jansen, 2015).

![Diagram of cosmic-ray muons in the atmosphere](https://example.com/diagram.png)

**Figure 23** The production of cosmic-ray muons in the atmosphere: $\pi^+(+/-)$, $\pi^0$ = pions; $N$ = nucleons; $\gamma$ = gamma radiation; $e^+, e^-$ = electrons; $\mu$ = muons. From Malmqvist et al. (1979).

Muon sensors placed in the subsurface will register each muon, as well as its direction, penetrating its emulsion surface. The intensity signature of muons in the subsurface is directly proportional to the integrated density of the overburden along the muon’s path (see equation 16). This integral can be used to derive the opacity of the geological layers along that path in metre
water equivalent (m.w.e). The opacity of the geological material revealed from the collected muon data can be converted to density by inverting the integral equation.

$$I(\theta, \phi) = \frac{O}{\int_{ray} \rho \, dl}$$

(16)

Where $\theta$ is incidence angle in the Easting direction (x-axis), $\phi$ is the incidence angle in the Northing direction (y-axis) and both represent the direction of the ray path. "$I_{vertical}$" is the vertical intensity of muons reaching the sensor in (cm$^2$sr$^{-1}$s$^{-1}$) and “sr” is the SI unit of the solid angle which quantify planar angle while "$O$" is opacity measured in m.w.e. $\rho$ is the average density of the material along the ray path "$l$".

The methodology is applied to SAGD reservoirs and, advantages and constraints are investigated. Herein, realistic SAGD density models are used to investigate how the number of sensors, the observation time for sensors in the sub-surface, and the spacing between sensors in the cross-line and in-line directions affects the resolvability. Muon imaging constitutes one of the most promising tools to obtain information on the density distribution inside geological objects if sensors can be placed underneath the targets (Taiuti, 2011; Marteau, 2012)

4.2 Methodology

The muon flux is uniform on the surface and as muons penetrate the surface of the Earth, they lose energy at a rate that is proportional to the density of material that is traversed. Muon
sensors are placed under a region of interest, hence, they are upward-looking sensors opposite to most geophysical techniques which look down from the Earth surface or perform tomography across boreholes. Current muon detectors/sensors can map the number of muons coming from all directions above them, within a field of view and create a two-dimensional radiograph image. Running 2D detector images through an inversion algorithm produces a 3D map of density anomalies for the target volume. In this chapter, the focus is on the forward modelling of muon trajectories through a pre-defined density model. Output of those models includes muon count, opacity and density anomalies in metre water equivalent.

The distance traversed by muons is typically longer than radiographs produced through other radiography techniques. However, a much longer exposure time must be used by the muon sensors because of the limited number of muons which reach the sensors at depth. Quantitatively, the first step is to use a model of the surface level spectrum of muons; an average number of muons which reach the surface, that is derived from decades of muon measurements, complied from the 1940s to the present (Gorringe and Hertzog, 2015). The intensity of muons at certain depth changes based on the rock type (density) traversed. The intensity of muons compared to depth in an environment, such as sea water, lake water and standered rocks are shown in Figure 23 (Bugaev, 1998). Also, more information on muon flux collected from conducted experiments in different types of rocks can be found in Bugaev (1998).
Figure 24 Vertical muon intensity versus depth (km water equivalent) measured using data acquired from experiments conducted by MACRO in standard rock, DUMAND, NESTOR, ANTARES in sea water, BAIKAL in lake water, AMANDA in ice (from Bugaev, 1998)

The spectrum is propagated through a geological model, and the intensity of muons is predicted as a function of depth (Schouten and Furseth, 2016). The energy of a muon is usually not measured, because doing so with any precision for high-energy muons requires strong magnetic fields (larger than 1 Tesla, about 16000 times larger than Earth’s magnetic field). Instead, muon intensity is measured in each pixel of the sensor’s image array as the rate of muons per unit area per unit solid angle (equation 17).
\[ I = \frac{N}{A \cdot \alpha \Omega \Delta t} \]  

(17)

Where \( I \) is muon intensity defined as the rate of muons per unit area per solid angle, \( N \) is the rate of muons, \( A \) is sensor’s active surface area, \( \alpha \) is the geometric acceptance and efficiency of the sensor, \( \Omega \) is the solid angle (pixel’s size), \( \Delta t \) is the exposure time over which muons are counted.

The intensity of muons decreases with increasing zenith angle away from the vertical. That is because the length of the ray path grows with a rate of \( 1/\cos \theta \) which results in longer ray paths and consequently less muons. This is taken into consideration in the forward modelling algorithm.

4.2.1 Muon Radiographs and Solid Angle Sections

The observed intensity of muons is used to interpret, interrogate and improve a geological model. The field of view of a muon sensor is determined, then divided into subsections by solid angles from the detector to the surface. This looks similar to an inverted pyramid in rectilinear coordinate (30 × 30 or 900 pixels), or when using polar coordinates, a cone (see Figure 25).
The sensor’s surface is divided into solid angles which results in the field of view from a particular point (Figure 26). The rate of muon detection by the sensors is sensitive to the average density of the overburden rocks of a solid angle. Each solid angle in the detector is represented here as a pixel in the spectrum image (also known as image array) (Figure 27). The raw data of a muon sensor consists of the rate of muons $N$ recorded in each pixel of the image array over the exposure time.
Figure 26 The registration of cosmic ray muons with muon detectors in the horizontal well of a SAGD reservoir. Detectors register only muons reaching sensors from a specified range of directions. Both detectors have different solid angle sizes (different spacing between sensor registration plates).

Figure 27 The sensor view in rectilinear coordinates in easting and northing directions. The opacity of different regions is projected to the surface. The sensor has a rectangular surface of 30cm × 30cm and 900 pixels.
The direction of the muons is defined by two angles, either $\theta, \phi$ (zenith and azimuth) in polar coordinates, or $\tan \theta_x, \tan \theta_y$ in rectilinear coordinates (see equations 18 and 19).

\[
\tan \theta_x = \frac{\Delta x}{\Delta z}
\]

\[
\tan \theta_y = \frac{\Delta y}{\Delta z}
\]

Where $z$ is the vertical direction, and $x$ and $y$ are the easting and northing directions respectively. $\tan \theta_x = \tan \theta_y = 1$ is 45 degrees off axis in both the easting and northing directions.

The difference between muon tomography and other geophysical techniques is that muon tomography measures discrete particles instead of continuous fields. Over a given period, although the rates of muons are predictable, muons are discrete particles that arrive at random intervals. Therefore, over a given exposure time, the number of recorded muons follows a Poisson distribution and as the detectors are exposed for longer period, the underlying geological structure emerges and the uncertainty is reduced. The relative statistical uncertainties decrease at a rate of $1/\sqrt{\Delta t}$ as the exposure time increases and the radiograph image becomes less fuzzy.

A complete simulation of a muon detector array and the subsequent data analysis is used to study the arrays capability to monitor the density distribution of SAGD reservoirs throughout production time. The localization of an anomalous density distribution in the overlaying rocks is possible using muon tomography technique and can be done with good accuracy as a change of
one percent in mean rock density corresponds to a change of about three percent in the counting rate (Malmqvist et al., 1979).

### 4.2.2 Forward Model of Muon Tomography

The SAGD reservoir of interest is located north of Edmonton, Alberta, and the bitumen is extracted from the McMurray Formation. The oil sand basin is located 60km northeast of Fort McMurray in Athabasca and SAGD operations were implemented to mobilize the bitumen. The density data were collected from delineation wells, 3D seismic, core analysis, log analysis, geological and reservoir modeling, production forecasting and regional comparison. The reservoir model is subjected to steaming and bitumen depletion and the resulting density models are used for our muon tomography forward modelling. Real reservoir density models are forward modelled at three time steps: before production takes place, after 1.25 years of production and after 5 years of production and opacities are simulated using modelled muon rates. The provided density model of each time scenario is imported to a python script developed by CRM Inc. which calculates the opacity in m.w.e for each pixel according to equation 20 (Figure 27).

\[
O = < \rho > \cdot L + \sum_{i=1}^{m} \Delta \rho_i \cdot L_i
\]  \hspace{1cm} (20)

Where \( O \) is the opacity of the overburden rocks, calculated in m.w.e. \(< \rho >\) is the mean rock density and \( \Delta \rho \) is the difference in rock density from the mean, for each geological structure in the model (m number of geological structures). \( L \) is the ray path.
The calculated opacity is then used to calculate a corresponding intensity for each pixel using the parameters of a given muon sensor (see equation 16). Finally, the expected raw muon counts can be calculated using equation 17. For the analysis of field data, the information collected by muon sensors, which are the observed count of muons detected for each pixel over the exposure time, is used to calculate muon intensity from which the opacity is determined. Uncertainty in the data can be calculated by taking the square root of the muon count ($\sqrt{N_i}$). From observed and expected muons, the intensity ($\Delta I_i$) and opacity ($\Delta O_i$) uncertainties can be determined (see Figure 28).

Figure 28 Flowchart of muon opacity, intensity, counts and uncertainties calculation steps.

Realistic density data derived from data collected by seismic surveys and known geological parameters are forward modeled. In the results section, models of muon sensors in the SAGD field showing muon counts and the opacity of each sensor are combined together to create an overall image of density change in the SAGD field.
The model results include all density changes represented as opacity in m.w.e and as a z value, e.g. the difference in muon counts corrected by the muons standard deviation in the reference model across the field (see equation 21). Although both opacity and z value images show the same bitumen depletion trend, using the z values illustrates a better depletion trend because of applying a sliding window algorithm to the counts image (a smoothing kernel). The kernel window is $3 \times 3$ with a weight of $[[1, 1, 1], [1, 1, 1], [1, 1, 1]]$.

$$z = \frac{(N_{\text{reservoir}} - N_{\text{reference}})}{\sigma(N_{\text{reference}})}$$ (21)

Where $z$ is the difference in muon counts between depleted model and reference model, corrected by muon count standard deviation. $N_{\text{reservoir}}$ is the muon count of the depleted model and $N_{\text{reference}}$ is the muon count at time zero before production. $\sigma$ is the standard deviation of $z$ for the reference model.

The measured opacity in synthetic data approaches the “true” opacity as the exposure time of the muon sensor is extended. Given an infinite time, the measured opacity and the true opacity would be equal. Herein, the illustrated Figures show the true opacity of the density model since synthetic data are used. In the real case scenario, muon sensors are recommended to be set with an adequate exposure time, up to a few months, to collect enough data and to reduce uncertainty. Since simulated opacity in the presented models is true opacity, it is independent of time and infinite time is assumed.
4.3 Results

The generated results represent the change in density within the SAGD field due to bitumen depletion. The reference opacity model is subtracted from the depleted opacity model of 1.25 years of production or 5 years of production. For example, the opacities revealed due to depletion after 1.25 and 5 years are negative, as the produced bitumen will result in lower opacity, as a result of less density due to production, around the wells. Areas of zero change in opacity representing no production and therefore no density change in non-depleted areas. The absolute density distribution ranges between 1.00 g/cm³ and 3.67 g/cm³ and the average absolute density is 2.16 g/cm³. The iso-surface of the density changes after 1.25 years and 5 years are plotted to visualize the depletion trend in the field of 600m×1000m. After 1.25 years the density changes indicate depletion trends around the 6 well pairs unlike density changes after 5 years, where depletion is expanded and depletions trends around the 6 well pairs are connected, see Figures 29 & 30.

![Figure 29 Iso-surfaces of density changes in the SAGD reservoir after 1.25 years of production.](image-url)
Figure 30 Iso-surfaces of density changes in the SAGD reservoir after 5 years of production.

The range of density change in the absolute model is approximately 2.67 g/cm³ and when the forwarded reference opacity model is subtracted from the depleted opacity model, the static opacity areas cancel out. The resultant opacity is in metre water equivalent (m.w.e) and only shows density changes due to steam injection and bitumen depletion.

The SAGD field has six well pairs and for each density model, the muon sensors are placed in the producing wells below the injection wells. Hence, the locations of the sensors in the cross-line is constrained by the locations of the 6 well pairs. In the in-line direction, the sensors are constrained to be in-between the coordinates of the heel and toe of each horizontal well. The coordinates of the wells’ heels on the cross-line are: (800m, 1500m), (880m, 1500m), (960m, 1500m), (1040m, 1500m), (1120m, 1500m) and (1220m, 1500m) and the wells’ toes are located at 2550m in-line. The depth is fixed to 230m, which assumes that the producing wells are at 230m depth. The sensors are left for a period of time to collect muon data. Each density model consists of 20 models, and the coordinates of each sensor in every model can be derived from table 5. The spacing between sensors in the in-line direction vary and different in-line spacing are investigated. An example model can be seen in Figure 31.
Table 5 List of all sensor distributions in all models in a SAGD field per density model.

<table>
<thead>
<tr>
<th>In-line Sensors</th>
<th>In-line Spacing (m)</th>
<th>All wells</th>
<th>Well_1 &amp; 3 &amp; 5</th>
<th>Well_2 &amp; 5</th>
<th>Well_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2500:-50:1550</td>
<td>120 sensors</td>
<td>60 sensors</td>
<td>40 sensors</td>
<td>20 sensors</td>
</tr>
<tr>
<td>10</td>
<td>2450:-100:1550</td>
<td>60 sensors</td>
<td>30 sensors</td>
<td>20 sensors</td>
<td>10 sensors</td>
</tr>
<tr>
<td>7</td>
<td>2450:-150:1550</td>
<td>42 sensors</td>
<td>21 sensors</td>
<td>14 sensors</td>
<td>7 sensors</td>
</tr>
<tr>
<td>5</td>
<td>2550:-250:1550</td>
<td>30 sensors</td>
<td>15 sensors</td>
<td>10 sensors</td>
<td>5 sensors</td>
</tr>
<tr>
<td>3</td>
<td>2550:-500:1550</td>
<td>18 sensors</td>
<td>9 sensors</td>
<td>6 sensors</td>
<td>3 sensors</td>
</tr>
<tr>
<td>1</td>
<td>2000</td>
<td>6 sensors</td>
<td>3 sensors</td>
<td>2 sensors</td>
<td>1 sensors</td>
</tr>
</tbody>
</table>

Figure 31 Example Model: Field view with 21 sensors spread in the first, third and fifth wells. Depth of 230m and 150m in-line spacing.
4.3.1 Observation Time of Sensors in the Sub-surface.

In order to investigate whether individual well pairs can be delineated when all six well pairs are included in the model, 120 muon sensors are placed in all 6 producing wells with a 50m in-line spacing (see Table 4). The signal responses in plane view for the six well pairs generally show broad depletion trends around the well pair location after 1.25 years (Figure 32). After 1.25 years of production, the calculated muon opacity difference and the difference in counts between depleted and reference models can differentiate density change for individual well pairs. After 1.25 years of production, opacity (Figure 32a) and muon count (Figure 32b) illustrate higher depletion at the heel, the southwest part of the well pairs and varying degrees of density change in the middle of the reservoir. The opacity difference variation ranges between -0.25 and -13.25 m.w.e indicating loss of density and the difference in muon count “z value” corrected by muons standard deviation, varies between -1.5 and 5 muons. The opacity loss of 0.25 m.w.e in the entire reservoir that it is reasonable to the true density change model.

After 5 years of production, the opacity (Figure 33a) and the z values (Figure 33b) indicate that the reservoir is evenly depleted, with some variability in the south east corner of the well pairs. Neither opacity nor the z value can differentiate density changes for individual well pairs as the density changes are spread out at the top of the reservoir, which is known as the spreading phase. Opacity and z value are illustrative of the variability in density change along the well path, and the increase in depletion from 1.25 years (Figure 32) to 5 years of production (Figure 33).
Figure 32 Density change due to bitumen depletion and steam injection, 50m in-line spacing of sensors. (a) Opacity of the reservoir after 1.25 years of production, (b) z value (counts) of all wells after 1.25 years of production.

Figure 33 Density change due to bitumen depletion and steam injection after 5 years of production. (a) Opacity, (b) z value.
The 2D image arrays indicate that after 1.25 years of production (Figure 32), the highest density changes occur near the heel of the wells, and after 5 years of production (Figure 33), are concentrated in the northwestern area of the well pairs. These trends are apparent in the plan view plots of opacity and z values. Areas with low z values are directly linked to regions with the smallest density changes in the reservoir while high absolute opacity values are linked to regions of high-density changes.

4.3.2 In-Line Spacing

A 50m in-line spacing separates the muon sensors discussed earlier. Increasing in-line spacing to 100m will reduce the number of sensors in the field from 120 sensors to 60 sensors. The results in this section are obtained by the same analysis as before but with less sensors, the true opacity and z value still demonstrate the same trends in bitumen depletion with good accuracy after 1.25 and 5 years of production, illustrated in Figure 34 and Figure 35 respectively.

In Figure 33b the depletion trend of the individual wells can be seen more clearly than in Figure 34b. Nevertheless, the provided resolution with 100m spacing is good, sufficient to demonstrate the depletion trends and delineate individual well pairs and can be compared with the results shown in 50m spacing. The results from models with sensors of 50m spacing provide better resolution of depletion trends than the obtained results of 100m sensors spacing. However, image arrays projected to the surface will result in pixels overlapping which may bias results. Herein, results of image arrays are projected to a certain depth (smaller than sensors depth) to prevent any pixels overlapping.
Figure 34 Muon sensors image arrays of the SAGD field with 100m sensor spacing in-line after 1.25 years of production. (a) Opacity, (b) z value.

Figure 35 Muon sensors image arrays of the SAGD field with 50m spacing in-line after 5 years of production. (a) Opacity, (b) z value.
4.3.3 Cross-line Spacing

Muon sensors are placed in the producing wells, which are parallel to and beneath the injector wells. The spacing between the wells controls spacing between sensors in the cross-line direction. To perform a full field investigation, muon sensors are placed in all 6 producing wells as discussed above. Increasing the distance between sensors in the cross-line direction will reduce the number of wells used to investigate the SAGD field. In Figure 36, seven muon sensors are placed in each of the first, third and fifth well and are evenly distributed with a spacing of 150m in the in-line direction (see Table 4). The opacity and z value show only parts of the density change in the reservoir. The image arrays did not capture density changes around the second, fourth and sixth well. The field of view of the remaining sensors does not capture muons coming through the three well pairs without sensors. However, after 5 years of production, the resultant image arrays generated from these 21 sensors can still image the general trend of depletion across the field (Figure 37).
Figure 36 Muon sensor image arrays of the SAGD field with sensors placed in the 1st, 3rd and 5th wells with an in-line spacing of 150m after 1.25 years of production. (a) Opacity, (b) $z$ value.

Figure 37 Muon sensors image arrays of the SAGD field with sensors placed in the 1st, 3rd and 5th wells with an in-line spacing of 150m after 5 years of production. (c) Opacity, (d) $z$ value.
4.4 Discussion

During a SAGD process, the steam is injected into the reservoir to mobilize bitumen. The density changes after 1.25 years range between 0.0002 and -0.2859 g/cm$^3$ and after 5 years, the density changes range increases from 0.0001 to -0.2978 g/cm$^3$. However, production after 5 years expanded and more areas were depleted in the reservoir. After 1.25 years of production, the highest density changes are at the heel of the well pairs and directly above the injection well path, opacity values in these areas reach -10 m.w.e. After five years of production, the density changes extend between the well paths, and are mostly north and south-west of the reservoir. Opacity in some of these areas increased to -13 m.w.e and the depleted area has expanded. These trends occur because the SAGD steam chamber undergoes two growth phases. In the first phase, the steam chamber rises until it encounters bitumen or a cap rock barrier and this can be seen clearly in the results found after 1.25 years of production. During the second phase the steam spreads laterally and develops into connected depletion areas around well pairs, individual wells cannot be identified after 5 years. However, some areas in the SAGD field show almost no depletion, particularly in the south-eastern heels of the reservoir and therefore steam injection could have been stopped there to reduce energy use. Potentially, the wellbore may not hit the reservoir in that particular location so no steam reached the bitumen volume. The permeability of the host rock can also be a significant factor in preventing steam from spreading. The results obtained by muon tomography analysis are consistent with analysis conducted by Elliot and Braun (2016, 2017) which used gravity and gravity gradients. Muons sense density anomalies along their path only, which provides a more localized sensitivity. Gravity gradiometry fails to provide an accurate model of density distribution because it is sensitive to all surrounding masses. Density measurements based on muon tomography can complement ordinary gravity.
measurements, or they may be applied under circumstances where the gravity method loses resolvability.

Models with 120 muon sensors and an in-line spacing of 50m provide high-resolution models of all individual wells and of the field as a whole. However, increasing the spacing to 100m has also provided opacity and z values with good resolution. The resolvability of the SAGD field drops significantly when increasing the in-line and cross-line spacing between sensors to 150m; this can clearly be seen in the models of 1.25 years of production, Figure 36. Nevertheless, the number of sensors in-line and cross-line can be adjusted based on the survey requirements and the area of interest.

Muon sensors can be manufactured in different shapes and with different geometries. However, it can affect the available emulsion surface area, and hence image resolution. Large emulsion surface areas mean that more muons can be sensed. Smaller emulsion surface areas can be made feasible by increasing the exposure time of the sensor to achieve the same quality (Bryman and Jansen, 2015). Increasing exposure time and/or receiving more muons would reduce errors in muon registry, uncertainty in muon count, and noise coming from overlying aquifers or surface human activities. The longer the sensors are placed in the sub-surface will result in higher resolution image arrays (Malmqvist et al., 1979). There is a trade-off between image array resolution quality and muon sensors exposure time. Another parameter that influences data resolution is the sensor depth. The muon count is inversely proportional to the sensor depth. In Canada, SAGD reservoirs are typically found at depths between 50-200m (Government of Canada, 2013). Muon sensors can be used at a depth as sufficient muons will
penetrate into such depth (Roos, 2004). The sensor depth for all generated models is set to 230m where the producing wells are located in the realistic SAGD reservoir used herein.

Muon tomography measures discrete particles and to probe an anomaly a muon sensor has to be placed beneath it. The area covered by a sensor is governed by the solid angle size, which will also influence the number of muons captured. The widest angle of sensors used in the presented models is 45°, which can be considered the optimal image size to maintain a high resolution while reducing the impact of blind areas between sensors. The size of blind zones depends on the size of the solid angle (pixel) of the muon sensor, where narrow solid angles result in a larger blind zone between sensors (Figure 26). However, wide solid angles will result in lower resolution images due to the longer distance that muons have to penetrate, particularly for the pixels along the edge of the image.

Sometimes a target could be imaged from two different sensors and it is not clear whether they are two separate depletion regions or whether they represent the same area of depletion. In this case, another sensor should be installed under the area of interest to map muon intensity from different directions. Muon sensors should be stable and robust in harsh working conditions and can either be capable of continuously transferring data remotely using Ethernet Wi-Fi or store them safely till they are pulled out. The number, weight, geometry, power consumption, robustness and cost of the muon sensors can be customized and optimized for the specific requirements of a survey (Bryman and Jansen, 2015). However, there are currently no muon sensors available which could be deployed in a SAGD well.
To sum up, muon tomography can be used in monitoring and distinguishing individual wells and depletion zones developed around wells. Using muon tomography would enable regular monitoring of fluid migration in the reservoir, prevent unnecessary energy consumption, aid to detect out-of-zone fluid migration and enhance bitumen recovery. Ultimately, the goal is to develop high resolution, precise models of density distribution with a short exposure time for increased temporal resolution. All these requirements are, however, impossible to attain at the same time (Malmqvist et al., 1979). There is a trade-off between these traits and it further depends on the circumstances in the field and the particular constraints of individual surveys. Optimal survey designs are case-specific. Taking into consideration influencing factors can help to improve survey design and results. A unique property of a muon sensor is its spatial resolution, which means that a sensor from one position can make measurements in different directions. This characteristic is complementary to gravity observations and a joint inversion would clearly benefit from the lateral resolution provided by muon tomography as well as the depth sensitivity of gravimetry.
5.1 Introduction

The goal of this chapter is to perform inversion modelling of synthetic muon tomography observations to predict density changes around wellbores in a SAGD reservoir. The key task of inversion problems is to infer knowledge about the structure of some system from measurements (Nakamura and Potthast, 2015). The inversion algorithms change based on field properties, determined parameters and modeling environment. Usually, inversion algorithms and data assimilation are based on theoretical equations, which model the underlying physical, chemical or biological processes, and it reconstructs sources or structural information. Evaluation of inversions depends on the analysis of initial state, in addition to structural information and underlying parameter functions. Data assimilation algorithms calculate initial states on which forecasts can be based (Nakamura and Potthast, 2015; Liversidge, Cundy and Bishop, 1980).

Two types of inversions are used in this chapter, an algorithm developed by CRM Inc. (Schouten and Furseth, 2016) called CRM Inversion and the Totomz inversion developed by the Geological Survey of Canada (GSC). Both inversions use different mathematical algorithms and approaches, which also results in different modelling outputs. Although, both inversion models result in realistic estimates of density changes in the SAGD reservoir, there are clear differences between them in terms of distribution within the reservoir, especially the volume of constant density iso-surfaces. The initial model state is based on the average field opacity and/or high-opacity areas.
Determination of initial states to construct initial models has been of crucial importance for the application from the very beginning, since forecasts are possible only when reliable estimates for the initial conditions are available (Nakamura and Potthast, 2015).

Simulation of density changes due to SAGD processes has been in the focus of the oil industry in order to monitor sub-surface fluid migration and mitigate out-of-zone flow (Economides and Martin (2008); Reitz, 2015; Elliott and Braun, 2017). The SAGD simulation is based on forward modeled data and scientific reasoning, which leads to the qualitative and quantitative description of these processes (Nakamura and Potthast, 2015). Inversion of muon tomography will use synthetic muon data and estimate a 3D density model using an initial model that is close to a geologically reasonable opacity model. The inversion problem is applied to models of a SAGD reservoir at specific time intervals, in order to monitor bitumen depletion and steam replacement via estimated density changes. Using inversion, density changes are estimated using muon sensors 2D image arrays of calculated opacity resulting from the muon tomography forward modelling described in the previous chapter. From the image arrays, an initial opacity model (initial state) is estimated to start the inversion. The initial model goes into the inversion algorithm and several iterations to finally output 3D reservoir models of density changes. The output density changes model is verified within the inversion by recalculating opacity and muon intensity from the output density change model and based on tolerance range (difference between the forward modeled output and the data), the inversion process is repeated.

In the inversion models, the true density distribution is known, but this is not available in most cases. These density models have a resolution of 2m and cover the area from 700m to 1322m Easting direction and 1500m to 2948m Northing. The vertical coverage starts at 162m depth to
226m. The true density changes models after 1.25 years and 5 years of depletion are shown in Figure 38 and 39, respectively. Herein, inversion modelling uses 21 forward modeled image arrays of muon counts as they would be registered by 21 muon sensors spread in the first, third and fifth producer wells. Uncertainties in the output density models will always be present since probability is part of the inversion process. Probability theories are used to do some field estimation of physical properties and data assimilation. The methodology steps and results of both inversion types are presented in the following sections.

![Image](image.png)

**Figure 38** The true density changes model of the SAGD reservoir with 2m resolution. The data show density changes after 1.25 years of production. The model is clipped at depth of 190m along the vertical direction (z-axis) to visualize the interior density changes.
Figure 39 The true density change data model of the SAGD reservoir with 2m resolution. The model shows density changes after 5 years of production. The model is clipped at depth of 190m along the vertical direction (z-axis) to visualize the interior density changes.

5.2 Methodology

To invert muon data, one needs to start with an initial model that is reasonably close to the real model in order for the inversion algorithm to converge. This is because the inversion is typically highly underdetermined. Therefore, an overall average opacity of the rock in the image volume needs to be set in the initial model provided to the inversion. Alternatively, it is equivalent and
easier to subtract the mean density and invert for density changes as shown in this chapter. For other computational reasons related to the fact that the space is discretized into voxels, using density change is more convenient and will directly provide the same information about reservoir fluid depletion. To run an inversion, the opacity image arrays for the reservoir model are taken from the forward models and used as the inversion input. A corresponding intensity map is calculated from the count map, including statistical fluctuation effects caused by vertical and non-vertical muons. The opacity map is then calculated from the intensity map while taking into consideration muon count statistical uncertainties. The inverted volume in the SAGD field has to be specified as it plays an important role for the computational time of the inversion. After specifying an inversion volume to cover the SAGD field, inversion algorithms of Totomz and CRM are used to invert opacity data and estimate density models.

As explained in the previous chapter, muon counts follow a Poisson distribution and uncertainty of the muon counts is calculated by taking the square root of muon counts in each pixel of the sensor image. The corresponding uncertainty of muon intensity can be calculated using equation 17 and since muon intensity and opacity have a one to one correspondence, opacity uncertainty is estimated using intensity uncertainty (Figure 24). Then, it is added to the observed opacity and difference from expected opacity is taken to finally obtain the opacity changes caused by steam injection and fluid migration (Equation 22).

\[
\delta O_i = (O_i \pm \Delta O_i)^{(observed)} - O_i^{(expected)}
\] (22)

\(\delta O_i\) is the opacity changes caused by density changes. \(O_i^{(observed)}\) is the opacity observed by muons sensors.
\( \Delta O_i \) is the opacity uncertainty obtained from muon intensity uncertainty that is calculated from muon counts uncertainty.

\( O_i^{(expected)} \) is the model’s expected opacity.

The data of opacity changes can be inverted to solve directly for density changes which is the difference between the assumed density in the forward models and real world density (Equation 23).

\[
\delta \rho = \rho^{(observed)} - \rho^{expected}
\]  

(23)

The 3D volume of the SAGD field is discretized into voxels of equal sizes, then the opacity change of a pixel can be estimated from muon ray paths and overlying voxels, as expressed in equation 24:

\[
\delta O_i = \sum_{j \in \text{ray}_i} \delta \rho_j \cdot L_j
\]  

(24)

Where the sum is over all voxels intersected by the ray, and the distance the ray passes across each voxel. Those linear equations can be inverted and 3D models of density change can be estimated. The CRM inversion code minimizes the difference to minimize the model misfit based on equation 23. The inversion algorithm converges and stops iterations when \( x \) is close to zero.
\[ x^2 = \sum_i \left( \frac{\delta O_i - \sum_{j \in \text{ray}_i} \delta \rho_j \cdot L_j}{\Delta O_i} \right)^2 \] (23)

5.3 Results

The data collected from 21 muon sensors located in the 1st, 3rd and 5th wells mainly show density changes around those wells. Information about density changes around the 2nd, 4th and 6th wells are not sensed and that is because density changes around those wells are located in the blind zones where sensors in 1st, 3rd and 5th wells cannot collect information due to solid angle size. The resulting models show the density change after two time steps; 1.25 years of production and 5 years of production.

5.3.1 After 1.25 years of Production

To investigate density change models from muon inversions, the true density changes of the SAGD reservoir whose thickness is 64m is compared with inversion results and visualized by Geoscience Analyst Software. The true density changes over the SAGD field after 1.25 years are shown previously in Figure 38 and the density changes range between 0.0002 and -0.2859 g/cm³.

By using the two inversion algorithms, the estimated density changes over the SAGD field can be cross-validated. Differences in the estimated density change models are based on the different methodologies used by each inversion algorithm and due to the undetermined nature of the inversion process. The CRM Inversion predicted density change over the SAGD field as presented in Figure 40, the shown voxels of density change are from 0 to 0.024 g/cm³. To
investigate higher density change voxels, voxels with a density change less than 0.01 are removed and it is clear from Figure 41 that the highest density changes are surrounding the well pairs.

By using the two inversion algorithms, the estimated density changes over the SAGD field can be seen from the following Figures. Differences in the estimated density change models are based on the different methodologies used by each inversion algorithm and due to the undetermined nature of the inversion process. CRM Inversion has predicted density changes over the SAGD field and are presented in Figure 40, the shown voxels of density change are from 0 to 0.024 g/cm$^3$. To investigate higher density changes voxels, voxels with density changes less than 0.01 are removed and it is clear from Figure 41 that the highest density changes are surrounding the well pairs.

![Figure 40 Density change over the SAGD field from the CRM inversion of 21 sensors in 3 wells. Density change changes from 0 to 0.024 g/cm$^3$. The interesting areas are thus covered by voxels with small density change.](image-url)
Figure 41 Same as in Figure 39, but only density changes between -0.01 g/cm$^3$ and -0.024 g/cm$^3$ are shown.

The Totomz Inversion has also predicted density changes in the SAGD field. Although density change increases to 0.05 g/cm$^3$ compared to results in the CRM inversion, most density changes voxels are located around 0.02 g/cm$^3$. In Figure 42, voxels of density changes starting from 0 to 0.05452 g/cm$^3$ and when removing voxels with density changes less than 0.01 g/cm$^3$, all voxels of density changes higher than 0.01 g/cm$^3$ will show depletion trends around the well pairs (Figure 43). All voxels which have higher density changes than 0.02482 g/cm$^3$ (the maximum density change in the CRM inversion) are removed in (Figure 44) and no significant changes between density changes trends in Figures 43 and 44 can be seen. The reason is that voxels with relatively high-density changes are few and located very close to wellbore locations (see Figure 45).
Figure 42 Totomz Inversion. Density changes in the SAGD field between 0 and -0.05452 g/cm³.

Figure 43 Totomz Inversion. Voxels with density changes from -0.01 to -0.05452 g/cm³.
Figure 44 Totomz Inversion. Voxels with density changes from -0.01 to -0.0248 g/cm³.

Figure 45 Histogram of density change in SAGD voxels. Histogram shows the different distributions of density change in the reservoir by CRM (yellow) and Totomz inversions (teal) after 1.25 years of production and the true density change (Blue).
In Figure 45, the density change results from both inversions and the real density change data are plotted in a histogram to analyze the differences in density change assigned by the inversion algorithms. Results from both inversions imply that density changes predicted for depleted bitumen are relatively equal but differently distributed through the reservoir. The mean predicted density changes from the CRM inversion are $-0.0059$ g/cm$^3$ and $-0.0043$ g/cm$^3$, respectively. The reason is that both methods have different approaches to the inversion problem and have artifacts caused by the lack of depth resolution since muons are independent of depth and only provide the average density change along their path. However, when comparing the mean real density change of $-0.0094$ g/cm$^3$ with the predicted ones, it is roughly double the magnitude predicted by both inversions. That is expected since muon detectors are only placed in the 1$^{st}$, 2$^{nd}$ and 3$^{rd}$ wells (representing half the wells) and depletions around the other wells are not sensed as they are located in the blind zones. Overall, the inversions provide satisfying results that describes steam injection and fluid migration after 1.25 years of production around the 1$^{st}$, 2$^{nd}$ and 3$^{rd}$ wells.

5.3.2 After 5 Years of Production

The true density change data after 5 years of production is shown above in Figure 39. Density changes in the SAGD field range between 0.0001 and $-0.2978$ g/cm$^3$. The mean value of density changes in the whole reservoir has increased from $-0.0094$ g/cm$^3$ to $-0.0455$ g/cm$^3$. The inversion results indicate that the density change has expanded through the reservoir and even increased in some places (Figure 46&49).
The density changes over the SAGD field after 5 years of production show an overall increasing trend. As it can be seen from Figure 46, the depletion around wells has expanded and depletions areas between well pairs are connected. Using the CRM inversion, only high depletion rates above 0.03 g/cm$^3$ are shown in Figure 47. The depletion trends show only density changes around the three muon sensors hosting wells. Removing voxels of density changes above 0.04 g/cm$^3$ will only remove inner voxels of high-density changes and they are located near the wellbores (Figure 48). No significant difference can be seen between Figure 47 and 48.

Figure 46 CRM Inversion, density changes over the SAGD field after 5 years of production. Density changes range from 0 to 0.046 g/cm$^3$. 
Figure 47 CRM Inversion, density changes over the SAGD field after 5 years of production. Density changes range from 0.03 to 0.046 g/cm³.

Figure 48 CRM Inversion, density changes over the SAGD field after 5 years of production. Density changes range from 0.03 to 0.04 g/cm³.
Totomz inversion is applied and density changes after 5 years of production are used. The resulting 3D model of density changes consists of voxels with a higher range of density change values than the CRM inversion, ranging from 0 to 0.0542 g/cm$^3$ (Figure 49). All voxels of density changes less than 0.03 g/cm$^3$ are removed and only voxels with higher density changes are kept in Figure 50. The output model is different than the model generated using CRM inversion and that is normal, since they are two different inversion algorithms. However, depletion trends can still be identified around the wellbores using Totomz inversion. Removing voxels with higher density changes (over 0.04 g/cm$^3$) will only remove few voxels surrounding the wellbores as mentioned previously; Figure 51.

Figure 49 Totomz Inversion. Density changes range from 0 to 0.0542 g/cm$^3$ after 5 years of production time.
Figure 50 Totomz Inversion. Density changes range from 0.03 to 0.0542 g/cm$^3$ after 5 years of production time.

Figure 51 Totomz Inversion. Density changes range from 0.03 to 0.04 g/cm$^3$ after 5 years of production time.
Figure 52 Histogram of density changes in SAGD voxels. Histogram shows the mass changes in the reservoir estimated by CRM (yellow) and Totomz (tea) inversions as well as the true density change (blue) after 5 years of production.

In the histogram shown in Figure 52, the predicted density changes by Totomz and CRM inversions are shown with mean density changes of $-0.0171 \text{ g/cm}^3$ and $-0.0178 \text{ g/cm}^3$, respectively. The overall density changes predicted by each inversion in the reservoir can be calculated and represent the area under the curve. Both inversions predict relatively similar total density change in the reservoir, however, different distributions can be noticed from the results. The true mean density change is calculated at -0.0455, and it is almost 3 times larger than the predicted ones which is caused by two factors, i) muon detectors are only placed in half of the six wells, and ii) the blindzones between the wells (see Figure 33) are not detected. The inversion models are all unconstrained, which leads to density changes in areas where density changes do not occur naturally, for instance above the bitumen layer. Constraining the inversion algorithm to
stop assigning density changes to voxels located above the reservoir would improve inversion results and assign density change to the depletion areas. However, constraining the inversion is beyond the scope of this thesis as the goal was to validate that muon tomography is able to resolve depletion areas in a SAGD reservoir. This goal was met by both inversion algorithms.

5.4 Discussion

After 1.25 years of production, density changes around wellbores are well predicted using CRM and Totomz inversions. Obtained 3D density changes models identify clearly the depleted areas around the 1st, 3rd and 5th wellbores individually. However, depletions around 2nd, 4th and 6th wellbores were not captured, additional sensors are required to be placed in those wells in order to collect information of depletion trends around them. Although, the thickness of the SAGD reservoir is 64m starting from 162m cap rock depth, inversions show depletions at depths shallower than the cap rock depth. The reason is that the inverted muon tomographic 2D image arrays are not sensitive to depth instead they give the average density change at a specific direction. Constraints can be added to the inversion algorithms to limit the predicted density changes geometry in the models. In more specific environments, inverse problems are adjusted to fit particular conditions and it is of crucial importance to know the application and the properties of the reservoir, its settings and environment (Nakamura and Potthast, 2015).

Nevertheless, the inversion models show the first phase in the SAGD process called rising phase. When steam expands and reaches the cap rock, it starts to develop in the lateral directions and the spreading phase starts. Inversion results after 5 years of production show major depletion trends in the reservoir. The depleted areas are now interconnected and no wells can individually
be identified, especially in Totomz inversion results. In the CRM inversion, the algorithm predicted density changes around wellbores to increase and limit density changes between wells. However, Totomz inversion results seem to agree more with previously found results in chapter 4 and in Braun and Elliott (2017). In terms of depletion areas, results from the Totomz inversion show no depletion trend in the south east area of the reservoir but more depletion is predicted in the south west and north locations of the reservoir. Also, voxels with higher density changes than 0.03 g/cm³ are predicted after 5 years of production with higher than the highest density change recorded after 1.25 years of production, 0.02482 g/cm³.

Increasing exposure time of sensors in the sub-surface will improve quality of data collected, hence better forward and inverse modeling results. However, within the inversion the number of sensors and inversion volume play important roles in inversion time. Increasing inversion sensors number will increase inversion time linearly and increasing inversion volume will cubically increase inversion time. In this chapter, the inversion modelling is only applied to one field scenario of 21 sensors located in three wells. The analysis could be conducted with more muon sensors, but optimization for inversion time is required.

To conclude, monitoring density changes caused by fluid migration and steam injection in heavy oil reservoirs during SAGD process is crucial to optimize production rates, enhance the productivity index (by decrease fluid viscosity), mitigate any out-of-zone flow, and protect the environment. Muon tomography could be used to monitor fluid migration in SAGD reservoirs by measuring density changes above muon sensors from different directions. Some other geophysical monitoring techniques measure continuous fields such as gravity, seismic and magnetic surveys, and that causes a lack of localization, which could translate into
misinterpretations especially in reservoirs that exhibit high structural heterogeneity. “The internal structure of the Earth's crust is commonly studied by seismological, electromagnetically or gravitational geophysical observations. However, these measurements are rather indirect and have substantial intrinsic uncertainties” Hiroyuki K.M. Tanaka (Tanaka, 2007). Therefore, conventional electromagnetic or seismic techniques may not directly determine reservoir structure. Muon tomography could be used as a complementary technique to other geophysical monitoring techniques. Joint inversion of gravity gradiometry and muon tomography could certainly build more constrained inversion models that would be closer to the true density model (Jourde, Gibert and Marteau, 2014). Synergies can be harnessed by combining muon and gravity monitoring techniques. Example of such a joint inversion done by CRM Inc. inside a mine for mineral exploration show improvement in the ability to localize density anomalies (Schouten and Furseth, 2016).
Chapter 6

Conclusions

This research assessed the feasibility for estimating density distribution using gravimetry and muon tomography. The density distributions of two applications were discussed, asteroid density distribution from surface gravimetry, and fluid migration in a SAGD reservoir. Forward and inversion modelling of different asteroid and density distributions, and muon sensor image arrays were outlined.

In chapter 3, the density distribution of 25143 Itokawa, 2008 EV5 and a spheroid shaped asteroid were studied. Three-dimensional density models of asteroids with known variance and correlation length were created, with the aim to assess if an asteroid with homogeneous density can be distinguished from heterogeneous asteroids. Results showed that heterogeneity of asteroids can be revealed, and homogeneous asteroids are distinguishable using a gravimeter with sub-mGal accuracy. The asteroid’s gravity signals are on the order of mGal; Itokawa shows gravity variability of 7-9 mGal while the 2008 EV5 model ranges between 14-17 mGal. From gravity forward modeling, we concluded that an asteroid’s associated gravity signal could be mapped with a VEGA space gravimeter that has a sensitivity of one µGal. However, the results indicate that surface boulders cannot be detected, especially if an asteroid exhibits a heterogeneous density distribution near the surface, which is likely the case even for asteroids, which exhibit a homogeneous density interior.
In chapter 4, the density changes in a SAGD reservoir caused by fluid migration and bitumen depletion was evaluated using muon tomography. Forward models of SAGD density data were conducted by measuring muon intensity coming from different directions. Muon sensors were spaced in-line and cross-line inside the producer wells, and synthetic opacity and muon counts were derived. Density models of two time steps, after 1.25 years of production and after 5 years of production were investigated. After 1.25 years of production, muon opacity and muon counts normalized by standard deviation can resolve the individual well pair depletion and results show the opacity trend is around 1 m.w.e. around wellbores and goes up to 13 m.w.e. in some areas near wellbores. When depletion lies in the blind region where sensors can no longer receive muons, information of such depletion is not acquired. After 5 years of production, depletion regions become interconnected, and individual well pairs can no longer be identified. However, the opacity image of the SAGD field shows a realistic and larger depleted area with higher depletion rates and z values than the 1.25 year model. Forward modeling of muon tomography can be used to analyze the sensitivity of muon tomography towards depleted areas and help to enhance SAGD operations and bitumen production.

In chapter 5, the synthetic opacity images and muon intensity from the forward models were used for inversion modelling. Two inversion algorithms were used, CRM and Totomz. Opacity is calculated from muon counts, which are sensitive to the density of the overburden. The inversions were run for two separate time steps, after 1.25 years of production and after 5 years of production. The inversions are performed on one muon array model with 21 muon sensors placed in the first, third and fifth well. CRM inversion after 1.25 years of production estimates the density change around the sensor wells very well, but depletion around the second, fourth
and sixth wells were not observed. The reason is that no sensors are in those wells, hence no information could be collected. In the Totomz inversion, depletion around the three wells are also identified. Although both inversion algorithms yield density changes between 0.01 and 0.025 g/cm³, Totomz density models show a larger depletion volume. This can be explained by the fact that there were different inversion algorithms. After 5 years of production, the general trend of density changes in the SAGD field can be seen, but no individual well pair depletion is identified. Depleted areas around the wellbores expand while producing bitumen and this will result in connecting depleted areas and individual well pair depleted areas can no longer be identified. The density changes increase as more bitumen is produced, so after 5 years of production the density changes reach up to 0.045 g/cm³. Inverse modeling of muon tomography can provide information about the unknown density distribution and this emphasizes that this method is a feasible monitoring technique. Muon tomography has demonstrated that the lateral density changes can be effectively monitored, but the depth of the depletion volumes is less well constrained. This is an intrinsic limitation in muon tomography, which potentially could be overcome by incorporating surface or borehole gravimetry for improved depth sensitivity.
References


Tanaka, H. K. M. (2007) ‘High Resolution Imaging in the Inhomogeneous Crust with Cosmic-ray Muon Radiography: The density Structure Below the Volcanic Crater Floor of Mt. Asama,


Appendix A

Asteroids Gravity Forward Models

Table 6 Gravity Forward Models of Itokawa. 1 homogeneous and 20 heterogeneous density models of 2 g/cm³ mean density and input variance of 0.04 g/cm³.

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<th>Density Model (g/cm³)</th>
<th>Surface Grav. Acc. (mGal)</th>
<th>Difference in Grav. Acc. (mGal)</th>
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Table 7 Gravity Forward Models of Itokawa. 1 homogeneous and 20 heterogeneous density models of 2 g/cm³ mean density and input variance of 0.08 g/cm³.

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Table 8 Gravity Forward Models of 2008 EV5. 1 homogeneous and 20 heterogeneous density models of 3 g/cm³ mean density and input variance of 0.08 g/cm³.

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<th>Corr. variance (g/cm³)</th>
<th>Density Model (g/cm³)</th>
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<th>Difference in Grav. Acc. (mGal)</th>
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Table 9 Gravity Forward Models of spheroid body. 1 homogeneous and 15 heterogeneous density models of 3 g/cm³ mean density and input variance of 0.08 g/cm³.

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<th>Surface Grav. Acc. (mGal)</th>
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