Frictional Markets, Asset Liquidity and Business Cycles

by

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Abstract

This thesis consists of three essays that use dynamic stochastic general equilibrium (DSGE) models to study the interactions between the goods sector and the financial sector of the economy in the presence of search frictions in the asset market.

In Chapter 2, I construct a real business cycle model with search frictions in the asset market to endogenize asset liquidity. In the model, asset liquidity depends on the probability of matching a seller with an appropriate buyer. In this way, an exogenous shock in the real sector can cause asset liquidity to fluctuate by changing the tightness of the asset market. In this chapter, I demonstrate the existence of steady states, and examine the properties of the equilibria.

In Chapter 3, I used the model discussed in Chapter 2, together with productivity and liquidity shocks estimated using a structural vector autoregression (SVAR) on U.S. data to investigate how asset liquidity and asset prices fluctuate in response to productivity and liquidity shocks, and how, in turn, these fluctuations magnify the impact of productivity shocks on economic activity. In addition, I compare the business cycle properties generated by the baseline model to models without search frictions in the asset market in order to assess the importance of search frictions. I find that productivity and liquidity shocks are equally important in explaining business cycles. The model also shows that productivity shocks generate pro-cyclical
movements of hours worked and asset prices, which is not the case for liquidity shock.

In Chapter 4, I compare two models with search frictions in the asset market that differ in terms of their assumptions regarding the supply and the demand sides of the market. Two factors discussed in the paper are adjustment costs and government bonds. After calibrating the models, I find that the model with both adjustment costs and government bonds generates pro-cyclical movements in asset prices. In addition, the inclusion of adjustment costs significantly enlarge the volatility of asset prices because the adjustment costs prevent the asset price from immediately regressing to the steady state.
Dedication

To my family for their endless love, support and patience throughout my life, especially during my studies abroad.
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All errors in this dissertation are my own.
Contents

Abstract i

Dedication iii

Acknowledgments iv

Contents v

List of Tables vi

List of Figures vii

Chapter 1: Introduction 1

1.1 General Introduction ........................................... 1
1.2 Literature Review ............................................. 4

Chapter 2: Search Frictions and Endogenous Asset Liquidity 13

2.1 Introduction .................................................. 13
2.2 The Model Environment ....................................... 17
   2.2.1 The household’s problem ............................... 22
   2.2.2 The firm’s problem ...................................... 29
2.3 Stationary symmetric search equilibrium ....................... 29
Chapter 5: Concluding Remarks

Bibliography

Appendix A: Proof of Propositions from Chapter 2

Appendix B: The Model without Search Frictions

Appendix C: Chapter 3: Figures for a Robustness Check

Appendix D: Proof of Propositions from Chapter 4
List of Tables

3.1 Unit Root Tests. ................................................. 46
3.2 Unconditional Correlations ..................................... 47
3.3 model without asset liquidity ................................. 48
3.4 A Model with Asset Liquidity ................................. 50
3.5 Parameters ....................................................... 55
3.6 Steady State Values ............................................ 56
3.7 Correlations and Volatilities ................................. 66
3.8 Decompositions of GDP and Asset Liquidity ............... 68
4.1 Volatilities ....................................................... 95
4.2 Equity Price Volatilities ....................................... 95
D.1 Parameters from the Model with Government Bonds ...... 135
D.2 Parameters from the model with both Frictions ........... 136
List of Figures

2.1 Steady State Equilibrium ................................. 32

3.1 HP Detrended: Percentage Deviation of GDP, Employment, Productivity, Asset Prices and Spread from Trend .................. 45

3.2 Historical Decomposition ................................. 51

3.3 Impulse Responses to Productivity and Liquidity Shocks .............. 57

3.4 Impulse Responses to a One Standard Deviation Productivity Shock . 60

3.5 Impulse Responses of Labor Input .......................... 62

3.6 Impulse Responses: a Productivity Shock and a Liquidity Shock .... 64

3.7 A Comparison among Models with and without Search Frictions: Impulse Responses to One Standard Deviation Productivity Shocks and Accompanied Liquidity Shocks ........................................ 65

3.8 Simulated GDP Decomposition .............................. 67

3.9 Entry Rate of the Asset Market .............................. 71

3.10 Responses of Output, Asset Prices and Labor Input ................ 73

4.1 Impulse Responses to Productivity and Liquidity Shocks in the Model with Government Bonds ........................................ 93

4.2 Impulse Responses of Equity Price ............................ 97
4.3 Impulse Responses to Productivity and liquidity Shocks in the Model with Both Adjustment Costs and Government Bonds .......................... 97

A.1 Steady state value of K ................................................................. 116

C.1 Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\gamma$ 124

C.2 Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\phi_0$ 125

C.3 Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\pi$ 126

C.4 Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\rho$ 127

C.5 Impose Responses to One Standard Deviation of Productivity Shocks with different $\gamma$ ............................................................... 128

C.6 Impose Responses to One Standard Deviation of Productivity Shocks with different $\phi_0$ ............................................................... 129

C.7 Impose Responses to One Standard Deviation of Productivity Shocks with different $\pi$ ............................................................... 130

C.8 Impose Responses to One Standard Deviation of Productivity Shocks with different $\rho$ ............................................................... 131
Chapter 1

Introduction

1.1 General Introduction

Motivated by the recent financial crisis, there has been a popular literature that emphasizes the role of financial shocks in driving economic activity. Indeed, several researchers have argued that shocks to asset liquidity can be viewed as the main source of business fluctuations. A sudden drop in asset market liquidity will limit the amount of funds that firms can raise. Thus investment drops and output falls, which constitutes an economic downturn. Recently, however, Shi (2015) points out that there is a fundamental problem with this perspective: in these models, negative liquidity shocks cause asset prices to rise, whereas, in the data, asset prices are quite clearly pro-cyclical in the past three decades. This is because in these models a negative liquidity shock reduces equity supply without changing equity demand. The contradiction suggests that business cycles cannot be explained solely by liquidity shocks.

In retrospect many previous analysis in the financial market frictions and business cycles literature, such as these of Kiyotaki and Moore (2008) (henceforth, KM),
1.1. GENERAL INTRODUCTION

Christiano, Motto and Rostagno (2008), and Ajello (2016), account quite well for the importance of financial sector shocks in business cycles. However, they do not consider the intertemporal correlation between productivity and financial shocks. Jermann and Quadrini (2012) impose an intertemporal correlation between asset liquidity and productivity. However, they do not explain where the intertemporal correlation is coming from. This is the major contribution of this dissertation. I develop several dynamic stochastic general equilibrium models with search frictions in the asset market to study the asset price puzzle presented by Shi (2015).

This dissertation is organized as follows. In Chapter 2, I develop a DSGE model to the asset market. The key feature of the model is the existence of search frictions in the asset market, which implies that asset liquidity is endogenously determined. I use the model to study how labor input, asset liquidity and asset prices fluctuate in response to productivity and liquidity shocks, and how, in turn, these fluctuations magnify the impact of productivity shocks on economic activity. A household’s investment is restricted by its holdings of liquid assets in the model. When a negative productivity shock hits the economy, entry into the asset market declines. This effect discourages investment because of the low liquidity of assets, thereby magnifying the decrease in total output.

In Chapter 3, I estimate a structural vector autoregression (SVAR) for productivity, asset liquidity, labor input and GDP per capita, using U.S. quarterly data covering the period from 1980q1 to 2016q4. The key feature of my empirical specification is that productivity generates a contemporaneous effect on asset liquidity. When the economy experiences a negative productivity shock, the benefit from trading assets
declines, which, in turn, reduces asset market participation, thereby lowering asset liquidity and discouraging investment. Thus, in addition to being a source of fluctuations themselves, frictions in the asset market appear to magnify the impact of productivity shocks. After estimating this SVAR model, I identify the following features in U.S. data: (1) Productivity has a positive effect on asset liquidity; (2) Productivity and labor input are positively related; (3) Productivity explains about 6% of the variance of asset liquidity; (4) Productivity and asset liquidity account for more than 50% of GDP volatility, which is consistent with previous work. However, productivity shocks are evenly more important than liquidity shocks in explaining business cycles, which contradicts previous findings.

After developing and calibrating a stochastic version of the model discussed in Chapter 2 to match key features in U.S. data, I find that productivity shocks are also very important in explaining business cycles, which is consistent with my empirical observations. In addition, I show that productivity shocks can generate pro-cyclical movements in labor input, asset prices and liquidity, which is not the case for liquidity shocks. The model also generates significant positive correlations between productivity and output, between productivity and labor input and between productivity and asset liquidity, in agreement with my empirical observations. Although some of the quantitative results of the model can match the data very well, the price effect generated by productivity shocks is not strong enough to solve the puzzle of counterfactual movements in asset prices, especially during the financial crisis. However, the model with search frictions significantly reduces counter-cyclical movements of asset prices. Therefore, although the model does not solve the puzzle of counter-cyclical movements in asset prices in the current financial crisis, it points in the right
In Chapter 4, I compare the abilities of two extensions of my basic DSGE model with search frictions in the asset market to account for the co-movements in asset prices. The models differ in terms of the factors affecting the supply of and demand for assets. In the first model government bonds are introduced as investment substitutes for risky assets. When asset liquidity decreases, buyers adjust their investment portfolios by holding more government bonds and reducing asset demand. In the second model, costs are imposed on investment adjustment. The adjustment costs mitigate the reduction of asset supply during economic downturns, while government bonds magnify the reduction of asset demand during an economic downturn.

I calibrate each model to match key features of U.S. quarterly data and find the following results: (1) The model with government bonds generates pro-cyclical movements of asset prices. However the model overstates both the volatilities of output and labor. Moreover, the calibrated model generates too little volatility in asset prices. (2) The model with both adjustment costs and government bonds generates pro-cyclical movements of asset prices and significantly enlarges the volatility of asset prices. This is a significant improvement over the existing literature, although, the calibrated model still understates the volatility of asset prices.

1.2 Literature Review

The thesis first fits into the large literature on business cycles. The early literature studying real business cycle (RBC) models, such as Kydland and Prescott (1982), Barro and King (1984), and Prescott (1986), accounts well for some real sector fluctuations. For instance, Kydland and Prescott (1982), first introduce a growth model to
study business cycles. The co-movements and the serial correlation properties of output for the model are quantitatively consistent with several corresponding moments in U.S. data. Time-separable preference models used by Barro and King (1984) are good for doing macro level analysis. However, this type of separation restricts intertemporal substitution between leisure and consumption to relative price-income effects. The model also performs poorly in accounting for the relative responses of current decisions to prospective shocks. Prescott (1986) introduces Solow residuals in the growth model to study the large fluctuations in output and employment over short time periods during peacetime.

There are two well-known challenges to the basic RBC models. The first is explaining the behavior of the equity premium. The equity premium puzzle is first presented by Mehra and Prescott (1985). They find that basic RBC models cannot explain the large difference between the average return to risky equities and risk-free bonds. In the historical data, the average return on equity was about 7%, while the average yield on debt was less than 1%. The model generates a yield difference that is much lower than in the data. This contradiction is called the “equity premium puzzle”. There are many papers studying this topic. For instance, Constantinides (1990), and Constantinides, Donaldson and Mehra (2002). However, none of them have fully resolved the equity premium puzzle.

The second challenge is understanding what causes business cycles. My dissertation belongs to this literature. Prescott (1986) computes total factor productivity (TFP) and uses it as a measure of technology shocks. He argues that technology shocks are the main driving force of business cycles. However, TFP is not a purely exogenous technology shock, but has some endogenous components. Thus, the true
technology shocks is significantly magnified in Prescott (1986). Burnside and Eichenbaum (1996), King and Rebelo (1999) argue that although the technology shocks are smaller than TFP, technology shocks are still very important in explaining business cycles because some mechanisms such as capacity utilization and markup variation significantly amplify the effects of technology shocks.

Later, Gali (1999) redefines technology shocks as only those shocks that have permanent effects on output and productivity. He finds that hours worked fall when technology increases. However, hours worked is typically pro-cyclical in the data. Thus, he argues that technology shocks cannot be considered as the main driving force of business cycles. Kim and Loungani (1992) and Finn (2000) study the effects of energy price shocks. Christiano and Eichenbaum (1992) and Braun (1994) study the effects of fiscal shocks such as tax rate and government spending shocks. These shocks improve the performance of RBC models. However, these shocks are not large enough to be a major source of business fluctuations.

During the late 1990s and early 2000s, many authors study monetary shocks in DSGE models. Sticky prices and collateral constraints were the most important frictions introduced into the RBC model. Kiyotaki and Moore (1997), Altig, Christiano, Eichenbaum, and Linde (2011) and Gali, Lopez-Salido, and Valles (2004) find that, while technology shocks are still the most important source of business cycle fluctuations, monetary frictions significantly improve the performance of technology shocks in business cycles.

During the most recent financial crisis, real output and employment dropped significantly whereas TFP changes appear to have been quite small. Motivated by this phenomenon, there has been a popular literature emphasizing the role of financial
shocks in driving economic activity. Indeed, several of these researchers have argued that shocks to asset liquidity can be viewed as the main source of business fluctuations.\(^1\) A negative shock to asset liquidity reduces asset prices and stability in asset markets, which leads to declines in output and investment. KM formulates this idea using a monetary model with liquidity constraints in which a firm’s investment capability is restricted by the holding of liquid assets. A negative liquidity shock reduces capital investment, and thus reduces labor input and aggregate production by decreasing the marginal productivity of labor.

The liquidity shock hypothesis has become popular in macroeconomic models. Several authors have argued that there are good reasons to expect that financial shocks to asset liquidity are much more important than productivity shocks in accounting for fluctuations. Christiano, Motto and Rostagno (2008) build a standard monetary DSGE model with financial markets, and introduce shocks to capital producers and entrepreneurs originating in the financial sector. After calibrating the model to European and U.S. data, they find that financial shocks account for a significant portion of business cycle fluctuations. Jermann and Quadrini (2012) suggest that imposing an intertemporal correlation between asset liquidity and productivity, sizable adjustment costs to investment can result in pro-cyclical movements of equity value with liquidity shocks by adjusting the amount of equity in the market. However, the adjustment cost still cannot resolve the issue of counter-cyclical movements of the equity price. Iacoviello (2015) claims that business cycles are financial. He finds that Investor’s defaults of financial obligations are the main driver of business cycles. However, his paper does not explain what causes investors’ defaults. Ajello (2016)

\(^1\)Asset liquidity is defined as the probability that an entrepreneur can resell his equity holding before he misses the investment opportunity
applies a “Financing Gap” to measure financial shocks and finds that financial sector shocks account for a significant amount of output and investment volatility.

However, Shi (2015) shows that there is a fundamental problem in the liquidity shock model — it implies that asset prices are counter-cyclical. In the models, negative liquidity shocks cause asset prices to rise. In contrast, asset prices are quite clearly pro-cyclical in U.S. data. This is because a negative liquidity shock reduces equity supply without changing equity demand. This suggests that business cycles cannot be explained solely by liquidity shocks.

Motivated by this contradiction, in the second chapter of the dissertation, I develop a variant of the Shi (2015) model by allowing search frictions in the asset market. Search frictions generate a linkage between real sectors and financial sectors. When the economy experiences a negative productivity shock, asset market participation declines by reducing the marginal benefit of trade in the asset market, thereby discouraging investment. Thus, in addition to being a source of fluctuations themselves, frictions in the asset market magnify the impact of productivity shocks. I show that this indirect effect of productivity shocks on business cycles acting through the asset market is significant in the model environment where asset liquidity is endogenously determined. This is precisely why models with exogenous liquidity constraints typically underestimate the quantitative importance of productivity shocks in business cycles.

This dissertation also fits into a broad literature on search frictions. One strand of monetary research, pioneered by Kiyotaki and Wright (1991 and 1993), adopts the view that trading frictions and the mechanics of trade are important for understanding the essential function of money as a medium of exchange. Duffie, Garleanu
and Pedersen (2005 and 2007) extend the trading frictions that arise through search and bargaining into financial markets. Duffie, Garleanu and Pedersen (2005) study how trading frictions affect intermediation and asset prices in the over-the-counter market. They show that search-based inefficiencies affect prices through equilibrium allocations and through the effect of search on agents’ bargaining positions. Asset prices are higher if investors can find each other more easily in the frictional markets. The intuition is that improving an investor’s outside options based on their ability to trade with other investors forces market-makers to give better prices. Duffie, Garleanu and Pedersen (2007) extend the model to characterize the role of search frictions on asset prices in the over-the-counter markets in manipulations with risk aversion and risk limits. They compute steady-state prices both with risk-neutral and risk-averse agents and find that illiquidity discounts are higher if risk aversion is higher.

Weill (2003), and Vayanos and Wang (2007) consider cross sectional asset pricing in extensions with multiple assets. In Vayanos and Wang (2007), there are only two assets. Buyers can decide which one to search. They show the existence of a “cliente” equilibrium where investors who want to resell the asset shortly search for the same asset. This asset market has more buyers and sellers, lower search times, and higher trading price relative to its identical-payoff counterpart. Weill (2003) relaxes the assumption of only two assets by allowing investors to search simultaneously for several assets, and finds that cross-sectional variation in asset returns is explained by the distribution of ownership, which is consistent with the result from Vayanos and Wang (2007) in the two-asset model. Vayanos and Weill (2008) explore the interplay between liquidity and specialness by introducing short-selling activity. In their model,
the more agents short an asset, the greater the asset’s seller pool becomes. The asset’s buyer pool also increases because of the short-sellers who also need to buy the asset back. A larger buyer and seller pool in the searching market implies shorter search times, higher liquidity and lower yields. Thus, their model explain differences in liquidity and yields between otherwise identical assets.

The search-based approach is appealing for studying asset markets because it can rationalize standard measures of liquidity, such as bid-ask spreads and trading delays, and can be used to study the influence of market conditions on these measures. Duffie, Garleanu and Pedersen (2005) and followers develop tractable frameworks to capture these measures, however, they impose a stark restriction on asset holdings: agents can only hold either 0 or 1 unit of the asset. Lagos and Rocheteau (2007 and 2009) develop a search based model of liquidity in the asset markets with no restrictions on investor’s asset holdings. This extension makes it possible to discuss several dimensions of frictional markets, such as trading costs, trade volume and delays. Lagos and Rocheteau (2007) study the role of financial intermediaries and find that reducing the market power of intermediaries can lead to lower trading costs and higher trade volume. Lagos and Rocheteau (2009) study the role of investor behavior in frictional markets. They find that a reduction in trading frictions leads to an increase in the dispersion of asset holdings and trade volume. Lagos (2010 and 2011) uses versions of the Lagos and Wright (2005) search-based model of trade to study the effect of liquidity and monetary policy on asset prices. He shows that assets are valued for their liquidity in an environment that all agents participate in asset markets. In contrast in my analysis, limited participation in asset markets play an important role in the impact of productivity shocks on asset liquidity. Afonso
(2011) also relaxes the assumption of unrestricted asset holdings and extends Lagos and Rocheteau (2007) by allowing investors to make entry decisions. This extension allows him to study the extensive margin, such as the number of matches, as well as the intensive margin in a match.

There are multiple approaches to explain why the matching process is not instantaneous in the asset market. In over-the-counter markets, buyers and sellers might be heterogenous in terms of their portfolio needs. Moreover it takes time for an investor to locate someone on the other side of the market. Another approach is to introduce informational asymmetries with regard to the fundamental values of the assets. Buyers and sellers might be asymmetrically informed about the characteristics of the asset that is traded. These informational constraints prevent all orders from being matched instantly. This idea is first adopted by Lester, Postlewaite and Wright (2012) in the financial market literature. They introduce private information regarding the quality of assets into search models and show that there are circumstances under which a buyer will refuse to trade with a seller who holds an asset of unknown quality. Rocheteau (2008) studies the effects of private information on the sizes of the trades. He finds that a riskier asset tends to be less liquid.

These trading frictions are also useful for understanding the fundamental trading process of other financial derivatives. He and Milbradt (2014) study the interaction between default and liquidity for corporate bonds that are traded in an over-the-counter secondary market with search frictions. The endogenous liquidity through search frictions allows them to study the positive feedback loop between liquidity and default. They find the illiquidity of the secondary corporate bond market feeds back to the fundamentals of corporate bonds by edging the firm closer to bankruptcy.
Search frictions are also relevant in the federal funds markets. Afonso and Lagos (2015) develop a model with search frictions in the federal funds market that explicitly accounts for two distinctive features of the over-the-counter market: search for counterparties and bilateral negotiations.

Although these articles differ from one to another in terms of the methodologies and assumptions they adopt, they share the common view that search frictions are important in studying individuals’ trading behavior in the asset market. In accordance with this related literature, in chapter 2, I introduce search frictions in the asset market as a bridge to connect the goods and the financial sectors. In chapter 3, I extend the model by introducing stochastic shocks to study business cycles. I find that this connection between the goods and the financial sectors amplifies the effect of productivity shocks in the goods sector on business cycles. In chapter 4, I further extend the model by introducing government bonds and investment adjustment costs. I find that search frictions combined with government bonds and investment adjustment costs can go a long way to solve the asset price puzzle proposed by Shi (2015).
Chapter 2

Search Frictions and Endogenous Asset Liquidity

2.1 Introduction

In this chapter, I construct a RBC model with search frictions in the asset market to endogenize asset liquidity. In the model, asset liquidity depends on the matching probability of a seller with an appropriate buyer in the asset market. In this way, an exogenous shock in the goods sector can cause asset liquidity to fluctuate by changing the tightness of the asset market. In this chapter, I show that steady states exist in the model, provide the conditions under which buyers only partially participate in the asset market, and examine the properties of the equilibria.

Recently, many economists\footnote{Such as Iacoviello (2015), Jermann and Quadrini (2012) and Christiano, Motto and Rostagno (2014).} have argued that shocks to financial markets account for a large proportion of fluctuations in output and asset prices. In particular, a negative shock to asset liquidity reduces asset prices in asset markets, which leads to declines in output and investment. KM formulates this idea using a monetary model with liquidity constraints in which a firm’s investment capability is restricted by the holding of liquid assets. A negative liquidity shock reduces capital investment,
and thus reduces labor input and aggregate production by decreasing the marginal productivity of labor.

The liquidity shock hypothesis has become popular in macroeconomic models. Several authors have argued that there are good reasons to expect that financial shocks to asset liquidity are much more important than productivity shocks in accounting for fluctuations (See, for example, Christiano, Motto and Rostagno (2008), Jermann and Quadrini (2012), Del Negro, Eggertsson, Ferrero and Kiyotaki (2017), Ajello (2016), etc.) However, Shi (2015) shows that there is a fundamental problem with this argument — these models imply that asset prices are counter-cyclical. He demonstrates that a negative financial shock to asset liquidity always generates an increase in the asset price. These counterfactual movements of asset prices is generated because a negative liquidity shock reduces asset supply, and thus drives up the asset price.

In the light of the above literature, I construct a RBC model in which asset liquidity is determined by search frictions in the asset market. The main deviation of my model from the literature is that fluctuations of asset liquidity are not exogenous, but are endogenously determined. The framework is based on KM and Shi (2015). A representative household consists of a large number of members who trade in the frictional asset market. A cost is incurred when a buyer enters the asset market. Buyers make tradeoffs between the expected trading surplus and the entry cost when choosing whether to participate in the asset market. The greater the number of buyers entering the market, the higher the probability that sellers will sell their assets in each period. At the same time, however, more entry costs have to be paid by buyers.

\[2\] See Mankiw and Zeldes (1991), and Hallassos and Bertaut (1995) for empirical evidence of limited participation of households in the asset markets.
Therefore, a shock that changes the trading surplus in the asset market will affect market tightness (i.e., buyer to seller ratios) and asset liquidity.

In this paper, I demonstrate the existence of steady states of the model and analyze their properties. In equilibrium, buyers’ optimal decisions to enter the asset market will ensure that the expected trading surplus of participating in the asset market just covers fixed entry costs. If the fixed entry cost is small enough, specifically lower than the expected trading surplus in the asset market when all buyers participate, there is an unique equilibrium in which all buyers enter the asset market. As the entry cost increases, buyers’ trading surpluses decline. When the entry cost is large enough, some buyers will not participate in the asset market.

Productivity affects the terms of trade in the asset market as follows. First, a higher productivity implies that the marginal productivity of capital is higher. Thus investments are more valuable, and buyers are willing to pay higher prices to trade assets. This is the direct effect of productivity shocks. Second, when the productivity goes up, new investments can generate more benefits in the following period. Sellers optimally offer better deals (i.e. a lower asset price) in the asset market in order to sell more assets and make more new investments. These better deals attract more buyers to participate in the asset market. Thus, asset liquidity increases. This is the indirect effect of productivity shocks. Most dynamic stochastic general equilibrium (DSGE) models with exogenous financial shocks fail to account for the effects of productivity shocks on the terms of trade in the asset market, and thus, significantly underestimate the importance of productivity shocks in business cycles.

The basic structure of the model is closely related to that of KM (2008) and Shi (2015), who study short-term dynamics, driven by productivity and liquidity shocks,
in a model in which entrepreneur's investment capability is restricted by holdings of liquid assets. They do not, however, consider the impact of productivity on asset liquidity. Their models have significant success in accounting for the fluctuations in aggregate production and labor input with joint shocks to both productivity and asset liquidity. However, their model can not explain pro-cyclical movements of asset prices.

This chapter is related to the literature on limited participation in the asset market. Mankiw and Zeldes (1991), and Haliassos and Bertaut (1995) present evidence of limited participation by households in the asset markets. Vissing-Jorgensen (2002) and Attanasio and Vissing-Jorgensen (2003) provide empirical evidence that limited asset market participation is important for estimating the elasticity of intertemporal substitution in U.S. data. Attanasio, Banks and Tanner (2002) find support for this fact in U.K. data. My approach differs from these papers in that I develop a process of entry decision to endogenize the participation rate of households in the asset market.

This paper is also related to a large literature on the study of markets with search frictions. Kiyotaki and Wright (1989), Hosios (1990), Shi (1995, 1999), Lagos and Wright (2005), present a search environment with random match and price bargaining. However, none of these papers generate endogenous asset liquidity. Lagos (2010) develops an asset pricing model with search frictions. He shows assets are valued for their liquidity in an environment in which all agents participate in asset markets. Lagos (2011) proposes extensions of the asset pricing model to an environment with money to study monetary policy. In contrast to these analyses, I show that limited asset markets participation plays a key role in driving the impact of productivity shocks on asset liquidity.
The rest of the chapter is organized as following: the environment of the model is described in section 2.2. Equilibrium is defined in section 2.3. The analytical characteristics of equilibria are then discussed in section 2.4. My concluding remarks are offered in section 2.5.

2.2 The Model Environment

Time is discrete and infinite, and is indexed by $t$. There is a continuum of households in the economy with measure one. Each household consists of a unit measure of members, divided into two groups according to an independent draw from a binomial distribution each period. With probability $\pi \in (0, 1)$ a member is an entrepreneur, and with probability $1 - \pi \in (0, 1)$ the member is a worker. Members’ realizations are independent through time. Entrepreneurs have the ability to make new investments but cannot provide labor, while workers can provide labor but do not have investment opportunities. All members, who belong to the same household, share consumption and disutility from labor supply.

At the beginning of each period, households distribute their asset endowments to their members before they realize their types. Hence, an asset endowment is assigned to each member with instructions on its choices. A member’s decisions follow the instructions which he/she receives from his/her household. Entrepreneurs sell equities and make investment into new projects. Workers provide labor, buy consumption goods, and also buy equities if they get matched with “brokers”. Workers’ disutility of labor is incurred at the end of the period. In each period, brokers collect equities from sellers and sell them to buyers in the frictional equity market. To simplify the exposition, I assume households act as brokers. As a result, any unsold equity accrues
to households at the end of the period.

At date $t$, a household’s preferences are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) - h((1 - \pi)l^w_t) - d_t(1 - \pi)c^f_t\}$$

(2.1)

where $c$ denotes the total consumption of a household, which is a summation of the consumption of workers, $(1 - \pi)c^w$, and the consumption of entrepreneurs, $\pi c^e$. Here, the superscripts $w$ and $e$ denote respectively workers and entrepreneurs. Entrepreneurs must divide their endowments between consumption and capital investment. In cases where capital investment is more valuable than the consumption, the consumption of entrepreneurs is zero, and total consumption, $c$, equals the consumption of workers, $(1 - \pi)c^w$. The value of $l^w_t$ denotes labor supply per worker. The functions $u(c)$ and $h(l)$ are twice continuously differentiable.\(^3\) The last term in (2.1), $d_t(1 - \pi)c^f$, is the total participation cost of the asset market, where $d_t$ is the fraction of workers that participate in the asset market, and $c^f$ is the fixed participation cost per worker in terms of utility. This cost includes all expenses incurred in undertaking trade in the asset market, such as the costs of learning about the assets, effort costs, commissions, etc. Most of the expenses depend on time and effort spent, rather than money spent. Therefore, the entry cost is considered as a utility cost. It is assumed to be fixed for simplicity. The participation cost is incurred whenever a worker enters the asset market. Paying the cost does not mean that the worker gets to trade for sure. A worker’s trading probability depends on the matching protocol in the asset market. The protocol will be discussed in more detail later. Households discount the

\(^3\)The utility functions have the following properties: $u'(c) > 0$, $u''(c) < 0$, $\forall c$; $h'(l) > 0$, $h''(l) > 0$, $\forall l$; $u'(0) = \infty$, $u'(\infty) = 0$, $h(0) = 0$, $h(\infty) = \infty$, $h'(0) = 0$, $h'(\infty) = \infty$. 
future at rate $\beta \in (0, 1)$.

The economy is also populated by a large number of firms who behave competitively. A firm operates a technology that uses labor and capital to produce consumption goods. Capital depreciates at rate $(1 - \sigma)$ after production, where $\sigma \in (0, 1)$. A firm’s production function in period $t$ is given by

$$y_t = A_t F(k^d_t, l^d_t), \quad (2.2)$$

where $y_t$ denotes consumption goods produced in period $t$, $A_t$ is the productivity level in period $t$, which follows a Markov process and $k^d_t$ and $l^d_t$ are firm’s demands for capital and labor in period $t$.

In each period, a labor market, a capital market, a consumption goods market and an equity market operate. Firms participate in the labor market, capital market and the consumption goods market. The labor, capital and consumption goods markets are Walrasian. Taking the wage rate $w$ and capital rental rate $r$ as given, each firm chooses labor and capital input, each worker chooses labor and capital supply and each entrepreneur supplies capital only. In each period, firms sell consumption goods to workers and entrepreneurs. Consumption goods are the numeraire and they are non-storable across periods.

Search is random in the equity market. Sellers and buyers interact in the market with anonymous bilateral matching. The matching probability is determined by the matching function, $M(B_t, S_t)$. The values of $B_t$ and $S_t$ respectively represent the measures of buyers and sellers in the market in period $t$. The function $M(B_t, S_t)$ returns the total number of matches. Let $\phi^s_t$ and $\phi^b_t$ denote respectively the seller’s probability of selling and the buyer’s probability of buying in the equity market. It
follows that

\[
\begin{align*}
\phi^s_t &= \frac{M(B_t, S_t)}{S_t} = \phi^s(m_t) \\
\phi^b_t &= \frac{M(B_t, S_t)}{B_t} = \frac{\phi^s(m_t)}{m_t},
\end{align*}
\]

where \(m_t = B_t/S_t\) is the tightness of the equity market. I assume sellers make a take-it-or-leave-it offer \((x_t, q_t)\) to matched buyers. The value of \(x_t\) is the amount of consumption goods which the seller requires for one unit of equity and \(q_t\) is the amount of equity which the seller offers the participating worker. For buyers in a desirable match, they take the offer \((X_t, Q_t)\) as given and decide whether to accept it.

Entrepreneurs are granted a special ability which allows them to make new investments. One unit of investment costs one unit of the consumption good, and it takes one period for investment to generate installed capital. Entrepreneurs need to raise funds to make new investments. Two possible ways are available to them: They lend and sell their asset endowment or they can issue new equity in the asset market. Entrepreneurs’ ability to raise funds is subject to two restrictions. The first constraint is a liquidity constraint, which states that only a fraction \(\theta \in (0, 1)\) of total new investments can be financed by issuing new equity.\footnote{This assumption is also made in KM and Shi (2015).} An entrepreneur’s stock off equity holdings evolves according to

\[
s^e_{t+1} = i^e_t + \sigma s_t - q_t \geq (1 - \theta)i^e_t,
\]

where \(s_t\) and \(s^e_{t+1}\) represent respectively entrepreneur’s asset holding before and after
trading with a broker in period $t$. Because the household distributes assets evenly to all members, each of them holds $s_t$. The first equality in (2.5) states that the entrepreneur’s equity holding at the end of period $t$, $s_{t+1}^e$, equals the sum of new investment, $i_t^e$, and old capital after depreciation, $\sigma s_t$, minus equities sold in the market, where $\sigma$ is the capital survival rate.

The inequality in (2.5) states that an entrepreneur has to hold at least a fraction $(1 - \theta)$ of new investments. To make the liquidity constraint effective, I suppose that funds cannot be reallocated between household members until all markets close. Otherwise, workers would shift more assets to matched entrepreneurs until (2.5) does not bind. The liquidity constraint can be rewritten as

$$q_t - \sigma s_t \leq \theta i_t^e,$$  \hspace{1cm} (2.6)

where the left-hand side is the amount of new equity issued. This has to be lower than the upper limit on investment that can be financed by issuing equity. This restriction prevents entrepreneurs from issuing more equity than they can buy back in the following period. The fraction $(1 - \theta)$ can also be interpreted as the down payment requirement for a new investment.

The entrepreneur’s ability to raise funds is also restricted by the liquidity of equities. The equity market is frictional. If an entrepreneur cannot raise enough funds, he/she will lose the investment opportunity for the current period. To avoid this, entrepreneurs sell their asset endowment and new equity to brokers at a discounted price, which reflects the risk of not being able to sell them (both existing assets and new issuing equities) immediately. The discounted price depends on brokers’ matching probabilities in the equity market. A broker offers a higher price to entrepreneurs
only if he/she has a higher probability to resell the equities. After buying equities from entrepreneurs, brokers represent entrepreneurs to trade equities with workers. Workers who intend to trade in the equity market need to pay a fixed participation cost $c^f$.

I take $\theta$ as a constant. But asset liquidity $\phi^s$ is endogenous. A change in the value of $\phi^s$ is considered as “a change in asset liquidity”. Asset liquidity fluctuates with changes in market tightness. A shock to the real sector, which affects households’ willingness to trade in the equity market, changes asset liquidity and affects the stability of the equity market. This indirect effect of a shock to the real sector is absent in models where asset liquidity is exogenous.

2.2.1 The household’s problem

The household’s state variables are its equity holdings $s_t$ and the aggregate variables $(K, A)$, where $A$ is the realization of current productivity and $K$ is the aggregate capital stock. In each period, a household chooses investment $i_t$, consumption $c_t$, labor supply $l_t$, and trading decisions $(e_t, x_t, q_t)$ to maximize its value $v_t(s_t; A_t, K_t)$. The measure of workers is $(1 - \pi)$, so the total labor supply is $l_t = (1 - \pi)l^w_t$. After suppressing the time subscript $t$ and using $+1$ to denote variables in $t+1$, I formulate the household’s Bellman equation as

$$v(s; K, A) = \max_{(q, e, l, d, s_{+1})} u(c) - h(l) - d(1 - \pi)c^f + \beta Ev(s_{+1}; K_{+1}, A_{+1})$$ (2.7)

A model without search frictions is studied in Appendix B
subject to

\begin{align*}
rs + \omega l^w - \frac{\pi \phi^s}{1 - \pi} QX &= c^w \quad (2.8) \\
rs + x^d q^d &= i^e + c^e \quad (2.9) \\
q - \sigma s &\leq \theta i^e \quad (2.10) \\
\frac{\Omega}{U'(c)} &\geq X \quad (2.11) \\
c^w &\geq 0 \quad s_{i+1} \geq 0 \quad q \geq 0 \quad l \geq 0 \quad X \geq 0 \quad (2.12) \\
d &\in [0, 1]. \quad (2.13)
\end{align*}

Equation (2.8) is a worker’s total resource constraint. Only the matched workers purchase equities, Q, at a price, X. According to (2.4), a worker’s probability of getting a match is \( \pi \phi^s / (1 - \pi) \). The funds include capital rental income, \( r_t s_t \), and wage income, \( \omega l^w \).

Equation (2.9) is an entrepreneur’s resource constraint. Entrepreneurs raise funds from renting capital and selling equities. The funds are used to finance new investment and consumption. The term \( q^d \) represents the amount of equity a entrepreneur sells to a broker, and \( x^d \) is the discounted price the broker pays to the worker. For simplicity, I assume a broker can only purchase assets from one entrepreneur and a household can freely hire as many brokers as he/she want. Thus the measure of brokers is the same as the measure of entrepreneurs, and the trading quantity, \( q \), satisfies \( q = q^d \).

With free entry of brokers, the following condition must hold:

\[ x^d = \phi^s X. \quad (2.14) \]
2.2. THE MODEL ENVIRONMENT

The discounted price may be much lower than the equity price, $X$, because the unsold equities are assumed to have no future value for brokers. In this paper, the asset price refers to the value of $X$. To simplify the exposition, I refer to both entrepreneurs and brokers as “sellers”. A seller trades $q$ units of equities at a unit price $X$ with a buyer with a probability $\phi^s$.

The price $X$ that a seller posts to the matched worker must give a non-negative surplus to the worker. A worker’s benefit from the trade equals the expected marginal value of capital in the next period, discounted to the current period, $\Omega$. The cost associated with the trade is the forgone utility of consumption in the current period. Hence, the worker’s surplus is given by

$$\Omega - XU'(c).$$  \hfill (2.15)

For a trade to take place, the asset price $X$, needs to satisfy

$$XU'(c) \leq \Omega.$$  \hfill (2.16)

Rearranging it yields (2.11).

A household’s capital at the beginning of period $t + 1$ equals the summation of old capital after depreciation, new investments and the net trade in the equity market:

$$s_{t+1} = s_t + \pi e + \pi \phi^s Q - \pi \phi^s q.$$  \hfill (2.17)

---

6 The discounted price will be larger if unsold equities have values to brokers in the future. Since the discounted price $x^d$ is not a main force of this paper, to simplify the computation, I assume that the unsold equities have no value for brokers.
Using $s_{+1}$ in (2.17) to replace $t^e$ in (2.9) yields

$$\pi rs + \pi x^d q = s_{+1} - \sigma s + \pi \phi^s q - \pi \phi^s Q + \pi c^e. \quad (2.18)$$

Combining the liquidity constraint, (2.10), and the budget constraint of the entrepreneur yields

$$(r + \frac{\sigma}{\theta})s + (x^d - \frac{1}{\theta})q \geq c^e. \quad (2.19)$$

Equations (2.9) and (2.10) can be replaced by (2.18) and (2.19) in the household’s problem. Let the multipliers of (2.18) and (2.19) be $\lambda e$ and $\lambda m \beta E v_{s_{+1}} \pi \phi^s$, the optimal choices $(s_{+1}, c^e, l^w, q, d)$ must then satisfy the following first-order conditions:

$$\beta E v_{s_{+1}} = \lambda e \quad (2.20)$$

$$\pi u'(c) - \lambda e \pi - \lambda m \beta E v_{s_{+1}} \pi \phi^s \leq 0, \text{ and } c^e \geq 0 \quad (2.21)$$

$$w u'(c) = h'((1 - \pi)l^w) \quad (2.22)$$

$$\lambda e \pi \phi^s (X - 1) + \lambda m \beta E v_{s_{+1}} \pi \phi^s (\phi^s X - \frac{1}{\theta}) \leq 0, \text{ and } q \geq 0 \quad (2.23)$$

$$\begin{cases} 
(\lambda e \pi \frac{\partial \phi^s}{\partial d} + \lambda m \beta E v_{s_{+1}} \pi \phi^s \frac{\partial \phi^s}{\partial d}) Xq - \pi u'(c) Q X \frac{\partial \phi^s}{\partial d} - (1 - \pi) c^f = 0, & 0 < d < 1 \\
(\lambda e \pi \frac{\partial \phi^s}{\partial d} + \lambda m \beta E v_{s_{+1}} \pi \phi^s \frac{\partial \phi^s}{\partial d}) Xq - \pi u'(c) Q X \frac{\partial \phi^s}{\partial d} - (1 - \pi) c^f > 0, & d = 1 \\
(\lambda e \pi \frac{\partial \phi^s}{\partial d} + \lambda m \beta E v_{s_{+1}} \pi \phi^s \frac{\partial \phi^s}{\partial d}) Xq - \pi u'(c) Q X \frac{\partial \phi^s}{\partial d} - (1 - \pi) c^f < 0, & d = 0
\end{cases} \quad (2.24)$$

and the envelope condition:

$$v_s = u'(c)(1 - \pi)r + \lambda e (\pi r + \sigma) + \lambda m \beta E v_{s_{+1}} \pi \phi^s (r + \frac{\sigma}{\theta}). \quad (2.25)$$

Condition (2.20) shows that a household’s expected marginal discounted benefit
from holding capital to the next period has to be equal to the benefit from using it in the current period. Conditions (2.21) and (2.22) are standard conditions for optimal consumption and labour supply. The two inequalities in (2.23) hold with complementary slackness. It captures the fact that an equity trade $q$ is positive only if the expected benefit from the trade is non-negative. Condition (2.24) is the condition for optimal entry rate. The first term on the right hand side captures the benefits from relaxing the entrepreneur’s budget and liquidity constraints. The following two terms capture the worker’s trading cost and the fixed equity market entry cost. Using (2.20) to replace the shadow price $\lambda^e$ in (2.23) yields

$$\lambda^m \beta E v_{s+1} (1 - \theta \phi^s X) \geq \beta E v_{s+1} \theta (X - 1).$$ (2.26)

The cost of an investment is one and a fraction $\theta$ of new investments can be financed by issuing new equities. The equity is traded at the discounted price, $\phi^s X$. The left hand side of (2.26) equals the down payment on investment multiplied by the marginal discounted value of equity holdings, and hence represents the cost of investment in terms of utility. The equity trading surplus is $(X - 1)$. Since only a fraction $\theta$ of new investment will be financed through equity, the trading benefit of one unit of investment in terms of utility can be expressed as $E v_{s+1} \beta (X - 1)$. Entrepreneurs are willing to trade equities only if the benefit is enough to compensate the cost. For the liquidity constraint to bind, the shadow price $\lambda^m$ must be positive. Clearly from (2.26), a positive $\lambda^m$ requires that the asset price $X$ be greater than 1 and less than $1/(\theta \phi^s)$. In this paper, I will focus on the case where the liquidity constraint binds.

**Lemma 1.** If the liquidity constraint is binding, the entrepreneur’s consumption,

---

*I will derive the conditions under which the liquidity constraint binds later.*
$c^e = 0$ and the leverage ratio satisfies

$$\frac{q_t}{s_t} = \frac{r + \sigma/\theta}{1/\theta - x^d}. \quad (2.27)$$

When the liquidity constraint is binding, the equity price is greater than one. Since sellers have full market power, the price $X$ actually reflects the household’s marginal rate of intertemporal substitution between future consumption and current consumption. If $X$ is greater than 1, the marginal benefit from future consumption is larger than the marginal benefit from current consumption. Therefore, entrepreneurs will allocate no resources to current consumption, and, instead, will hold assets for future consumptions. The detailed proof is provided in appendix A.

This ratio, $q_t/s_t$, can also be interpreted as the leverage rate of the equity market. It depends on the down payment, $\theta$, and the fundamental value of the asset, $(x^d, r)$. As the price or the expected payoff of the asset increases, entrepreneurs can issue more equities based on the asset. There is no leverage restriction if the discount price reaches $\frac{1}{\theta}$ or higher. A higher price level returns more profits to sellers. If the price is high enough, the trading surplus is enough to pay the down payment of a new investment. In this case, sellers can make as many investments and trade equities as they want.

Condition (2.24) reflects a worker’s benefit and cost in the equity market. In a trade, a worker has to give up some current consumption to trade equity holdings in the following period. The trading benefit to a worker in terms of utility is $Xq^e\lambda^e$. So the first term in (2.24) represents the worker’s marginal benefit of trading. The second term of (2.24) is the expected cost of the consumption which needs to be given
up if the trade happens. $(1 - \pi)c^f$ is the fixed participation cost. The entry rate is an interior solution only if the benefit and the cost are equal in the interval where $e \in (0, 1)$. Using (2.20) to replace $\lambda^e$ in (2.24) yields

$$
\pi X \frac{\partial s^*}{\partial d} (\beta Ev_{s+1}q + \lambda^m \beta Ev_{s+1} \phi^* q - u'(c)Q) \begin{cases} 
> (1 - \pi)c^f, & \text{and } d = 1. \\
= (1 - \pi)c^f, & \text{and } 0 < d < 1. \\
< (1 - \pi)c^f, & \text{and } d = 0.
\end{cases} 
$$

(2.28)

The right hand side of equation (2.28) captures the household's net benefit in the equity market. The entry rate is positive only if the net benefit in the equity market is no less than the fixed entry cost. Combining the optimality condition (2.20), the optimal asset price condition (2.11), and the envelope condition (2.25), yields the asset-pricing equation:

$$
X = \beta E \left\{ \frac{u'(c+1)}{u'(c)} [X_{s+1} \sigma + ((1 - \pi)r_{s+1} + X_{s+1} \pi r_{s+1}) + \lambda^m X_{s+1} \pi \phi^*_s (r_{s+1} + \sigma \theta)] \right\}. 
$$

(2.29)

The first two parts on the right-hand side are the expected value of equities after depreciation and the expected rental income. The last term can be rewritten as

$$
\lambda^m_{s+1} X_{s+1} \pi \phi^*_s (r_{s+1} + \sigma \theta) = \lambda^m_{s+1} X_{s+1} \pi \phi^*_s \left( \frac{q_{s+1}}{s_{s+1}} \theta - \phi^* X_{s+1} \right) = X_{s+1} \pi \phi^*_s \left( \frac{q_{s+1}}{s_{s+1}} (X_{s+1} - 1) \right) 
$$

(2.30)

The first equality is from the equation for the leverage ratio. If the liquidity constraint does not bind, $\lambda^m$ is 0. Otherwise, the equation for the leverage ratio holds with equality, and $c^e$ is zero. This is why the first equation in (2.30) must hold.
2.3. STATIONARY SYMMETRIC SEARCH EQUILIBRIUM

The second equality is from (2.26), when the liquidity constraint is binding. The trading surplus is \((X_{+1} - 1)\). Thus, the last term in (2.29) represents the expected trading surplus when one extra unit of equity is held. If the liquidity constraint is not binding, the last term of (2.29) is zero. Multiplying the sum of these three components by the household’s discount factor, \(\beta u'(c_{+1})/u'(c)\), I obtain the discounted value of future dividends, which has to be equal to the equity price \(X\).

2.2.2 The firm’s problem

In each period, firms choose capital and labor supply to minimize costs, subject to the production function. Output prices adjust so that profits are zero. It follows that:

\[
\begin{align*}
  r_t &= A_t F_1'(k^d_t, l^d_t) \\
  w_t &= A_t F_2'(k^d_t, l^d_t).
\end{align*}
\]

2.3 Stationary symmetric search equilibrium

I focus on a stationary equilibrium where households are symmetric and make identical decisions. The time index is dropped and +1 is used to denote variables in \(t + 1\).

**Definition.** A symmetric search equilibrium consists of price functions \((x, X, r, \omega)\), a household’s policy functions \((i, q, Q, l, c^w, c^e, e, s_{+1})\), the demand factors of final goods producers, \((k^d, l^d)\), and the law of motion of aggregate capital stock, such that the following requirements are satisfied:

1. Given price functions and the aggregate state \((K, A)\), a household’s value and policy functions are solved from the household’s optimization problem.
2. Optimal conditions of firms, \( r = AF_1'(k^d, l^d) \), and \( \omega = AF_2'(k^d, l^d) \) hold.

3. All markets clear:

\[
\begin{align*}
goods: & \quad (1 - \pi)c^w + \pi c^e + \pi i^e = AF(k^d, l^d) \quad (2.33) \\
labor: & \quad l^d = (1 - \pi)l^w = l \quad (2.34) \\
capital: & \quad k^d = K = s \quad (2.35) \\
equity: & \quad s_{+1} = \sigma s + \pi i^e. \quad (2.36)
\end{align*}
\]

4. Aggregate capital \( K_{+1} \) satisfies the law of motion of capital:

\[
K_{+1} = \sigma K + \pi i^e. \quad (2.37)
\]

5. In the frictional asset market, the trading price satisfies:

\[
X = \frac{\beta Ev_{+1}}{u'(c)}.
\]

The trading offer posted by an seller has to be consistent with the offer accepted by the matched worker.

\[
x = X, \quad \text{and} \quad q = Q. \quad (2.38)
\]

6. Symmetry condition: All households are identical. They make symmetric decisions in each period.
2.4 Analytical Results

To understand the workings of the model it is useful to obtain some analytical results using specific functional forms. In particular, I consider the case with linear utility over household consumption,\(^8\) given by:

\[
u(c) = u_0 c,
\]

where \(u_0\) is a parameter. In addition, I assume that the disutility of labor and the production function have the following simple forms:

\[
h(l) = l^{\eta} \quad \text{and} \quad F(K, L) = K^\alpha L^{1-\alpha}.
\]

\(\eta > 1\) and \(0 < \alpha < 1\).

**Lemma 2.** In a steady state with a binding liquidity constraint, the equity price, \(X^*\), and the interest rate, \(r^*\), are positively related.

The superscript \(*\) indicates the value in steady state. Suppose the liquidity constraint is binding, asset prices can be rewritten as

\[
X^* = F(r^*) = \frac{\beta}{1-\beta} r^* - \frac{\beta}{1-\beta} (1 - \sigma), \quad (2.39)
\]

The right-hand side is the life-time capital return which is the difference between capital rental income and the depreciation cost. As the steady state interest rate

---

\(^8\)In order to get some analytical results, linear utility function is applied to simplify the model structure. In the next chapter, I relax this assumption and apply a more general utility function.

\(^9\)If the liquidity constraint does not bind, the multiplier \(\lambda^{m*}\) is zero. The equation (2.26) implies the equilibrium equity price \(X^*\) equals its fundamental value 1.
increases, the expected return to capital increases. Hence, buyers are willing to pay a higher price for equity, and vice versa. The proof of lemma 2 is discussed in Appendix A.

**Proposition 1.** There exists an equilibrium with \( \lambda^m > 0 \) and \( d = 1 \), if

\[
(1 - \phi^*\theta)(1 - \sigma) > \frac{\pi}{\beta} \quad (2.40)
\]

and

\[
c^f \leq \frac{\pi}{1 - \pi} X(K^* : A)q u_0 [X(K^0 : A) - 1
+ \frac{\theta X(K^* : A) - 1}{1 - \theta \phi^*X(K^* : A)} X(K^* : A)\phi^*] \phi_0 \left( \frac{1 - \pi}{\pi} \right)^\gamma \gamma, \quad (2.41)
\]

where \( K^* \) is the unique equilibrium capital at \( d = 1 \) and the value of \( q \) and \( \phi^* \) are defined in Appendix A.2. The equilibrium capital \( K^* \) is strictly increase in productivity.

Capital depreciation, \( 1 - \sigma \), is constant. Capital investment, which equals \( D(K) \),\(^{10}\)

\(^{10}\)It is defined in Appendix A.2
is strictly decreasing in $K$ at $d = 1$. Figure 2.1 depicts the steady state capital, $K^*$, as the intersection of the “break-even” level of investment and actual capital investment. This solution represents a corner equilibrium in the sense that all workers participate in the equity market because the participation cost is low. Workers always make a positive expected profit from entering the equity market. Condition (2.40) requires that $\theta$ and $\sigma$ cannot be too large. As the value of $\theta$ increases, the down payment required for an investment decreases. This process will stimulate new investments and trades in the equity market. Therefore, the trading benefit of an extra worker entering the market decreases relative to the fixed entry cost. If $\theta$ is large enough, the equilibrium entry rate $d^*$ will deviate from 1. Similarly, a higher value of $\sigma$ increases the entrepreneur’s asset holding from the previous period, which in turn increases his investment capacity and equity trade. The upper limit on $c^f$ is also necessary to obtain this result. If $c^f$ is too high, the expected benefit of entering the equity market is insufficient to cover the large participation cost. Some workers will leave the equity market to avoid the entry cost. Therefore, the entry rate, $d = 1$, is no longer a steady state value.

**Proposition 2.** If condition (2.40) holds and $c^f$ satisfies

$$c^f > \frac{\pi}{1 - \pi} X(K^* : A) g u_0 [X(K^* : A) - 1]$$

$$+ \frac{\theta (X(K^* : A) - 1)}{1 - \theta \phi^0 X(K^*: A)} X(K^1 : A) \phi^0 (\frac{1 - \pi}{\pi}) \gamma,$$

(2.42)

there is at least one equilibrium in which liquidity constraints are binding and the participation rate is strictly greater than zero and less than one.
Proposition 2 characterizes an equilibrium in which the entry rate decision \( e \) is less than 1. Condition (2.40) is required to ensure that liquidity constraints are binding. The solution pair \((K^2, d^2)\) is smaller than the solution from the case where the entry rate is equal to 1. Intuitively, if the entry rate is lower, the matching probability of a seller is also lower, and hence so are trading prices which brokers offer to entrepreneurs. Therefore, the total funds available for new investments are reduced. In a steady state, new investments have to be sufficient to offset depreciation. The depreciation rate is constant, which means capital is lower whenever the entry rate is smaller. A more-detailed proof of the Proposition 2 is provided in appendix A.3.

2.4.1 Equilibrium response to an productivity shock

Suppose there is a permanent increase in productivity. Higher productivity increases the value of asset investment, pushes more buyers to trade in the asset market, and incurs more entry costs. Although the entry costs restrict buyers’ incentive to enter the market, these costs do not restrict the amount of assets traded in a match.

Case 1: the equilibrium entry rate \( d^* = 1 \).

Suppose that the economy is initially in the stationary equilibrium with \( d^* = 1 \). There is permanent increase in productivity from \( A \) to \( A^1 \). Since the entry cost \( c_f \) is small enough and the equilibrium entry rate \( d = 1 \), the new equilibrium matching probability \( \phi^{e*}(d) \) remains at the highest level \( \phi^{e*}(d = 1) \).

As productivity increases, capital investment becomes more valuable, so the new equilibrium capital is higher. A productivity shock has two effects on the return
2.4. ANALYTICAL RESULTS

on capital $r$ and the asset price $X$. One is the wealth effect. As the productivity increases, capital productivity also increases, which in turn, raises the return on capital $r^*$ and the asset price $X^*$. A increase in capital reduces the marginal product of capital, and hence, reduces the return on capital $r^*$ and the asset price $X^*$. This is the substitution effect. If the substitution effect dominates the wealth effect, both the return on capital $r^*$ and the asset price $X^*$ decrease with productivity. The capital investment function $D(k)$ implies the the equilibrium capital $K^*$ decreases, which is contradicted to the result from Proposition 1. Thus, the wealth effect dominates the substitution effect. Both the return on capital $r^*$ and the asset price $X^*$ increases in the new equilibrium.

Since $r^*$ and $X^*$ increase, the new leverage ratio increase as well. The capital stock $K^*$ increases and equation (2.27) implies that asset trade $q^*$ increases. Since total output increases, investment $I^*$, labor input $L^*$ and the wage rate $\omega^*$ all increase. The movement of the consumption $C^*$ is ambiguous.

Case 2: the equilibrium entry rate $0 < d^* < 1$.

In this instance households can also adjust the entry rate $d$ when the economy experiences an exogenous shock. This adjustment makes it possible for multiple equilibria to exist. A productivity shock would shift the economy from one equilibrium to another. Shifting among different equilibria is out of the scope of this paper. In this section, I focus on parameter values such that there is a unique equilibrium where the equilibrium entry rate is lower than 1, but higher than 0.\textsuperscript{11} The unique solution of $d^*$ must satisfy condition (2.28) when $0 < d < 1$.

Suppose as before that the economy is initially in the stationary equilibrium with

\textsuperscript{11}In appendix A.3, I provide conditions under which unique solution exists.
2.4. ANALYTICAL RESULTS

0 < \( d^* < 1 \). A permanent positive shock hits the economy which increases productivity from \( A \) to \( A^1 \). Without any change, the fixed entry cost \( c^f \) is smaller than the potential trading benefit in the asset market. Therefore, more workers enter the asset market and the entry rate \( d^* \) increases.

As productivity increases, capital investment becomes more valuable, so the new equilibrium capital \( K^* \) is higher. There are still two effects of productivity shocks on the return on capital \( r^* \) and the asset price \( X^* \): the wealth effect and the substitution effect. If the substitution effect dominates (or just equals) the wealth effect, both \( r^* \) and \( X^* \) decrease (or remain unchanged) in the new equilibrium. This implies new capital investment \( D(K) \) at the old equilibrium level is lower than capital depreciation which contradicts the movement of \( K^* \). Thus, the only possible equilibrium is that the wealth effect dominates the substitution effect, both \( r^* \) and \( X^* \) increase in the new equilibrium.

Since both productivity \( A^* \) and capital \( K^* \) increase, the wage rate \( \omega^* \), labor input \( L^* \) and investment \( I^* \) all increase. The liquidity constraint implies that the amount of asset trade in a match, \( q^* \), equals the sum of existing capital, \( \sigma k^* \), and the fixed amount of new investment , \( \theta i^* \). Since the new equilibrium value of \( k^* \) and \( i^* \) both increase, the amount of trade \( q^* \) also increases. Consumption equals total output minus new investment:

\[
C = AF(K, L) - \pi i^*.
\] (2.43)

When productivity increases, A household’s wealth \( AF(K, L) \) increases, which in turn, increases consumption. This is the wealth effect of a productivity shock on consumption. A household chooses between consumption goods and new investment. As
2.5. CONCLUDING REMARKS

productivity increases, the marginal profit of investment increases. Thus, the household distributes more funds on new investment $\pi i^e$ and reduces consumption. This is the substitution effect. These two effects drive consumption in opposite directions. Thus, the movement of consumption is ambiguous.

Proposition 3. Assume conditions (2.40) and (2.41) hold.

Case 1: If the equilibrium entry rate $d^*$ equals 1, then, when productivity increases permanently, the new equilibrium interest rate $r$ and the asset price $X$, capital $K$, the labor input $L$, the wage rate $\omega$, the amount of trade $q$ and investment $i$ are higher in the steady state equilibrium. The movement of consumption is ambiguous.

Case 2: If the sum of the marginal effect of the entry rate on the marginal change of seller’s matching probability and the marginal effect on the tightness of the liquidity constraint dominates the marginal effect of the entry rate on the internal margin of trade, the equilibrium entry rate $d^*$ is unique and $0 < d^* < 1$. A permanent increase in productivity results in increases in capital $K$, the labor input $L$, the wage rate $\omega$, the amount of trade $q$ and investment $i$. The movement of consumption is ambiguous.\(^{12}\)

2.5 Concluding Remarks

In this paper, I have constructed and analyzed a tractable general equilibrium model with search frictions in the asset market where asset liquidity is endogenously determined. Matching is random, and take-it-or-leave-it offers are provided by sellers. My model has two key features: (1) search frictions in the equity market cause the asset price to depend on market tightness; (2) only part of the population participates in the asset market. The asset market participation rate depends on the expected

\(^{12}\)A detailed proof is provided in Appendix A.3
trading benefit in the market.

I show that stationary equilibria exist. I proved that a unique steady state with full participation in the asset market exists if the participation cost in the asset market is low. As the participation cost increases, the benefit received by buyers' from trading declines and some buyers decide not to enter the asset market. I also show that there is at least one stationary equilibrium in which buyers partially participate in the asset market if the participation cost is large.

Search frictions in the asset market serve as a channel through which changes in the real sector affect financial activity. In particular, a one-time increase in the productivity can have persistent effects on asset liquidity in the short run. The short-run effects arise from the frictions in the asset market. When productivity decreases, a buyer's participation rate in the asset market declines because of the lower return to investment. This lower participation rate reduces the probability that sellers match with suitable buyers, and thus decreases asset liquidity and future capital investment. These responses further reduce total output. Thus, this model has the potential to magnify the importance of productivity shocks in business cycles.

By extending the model to allow for stochastic shocks, it can also be used to investigate business cycles. In addition to being a source of shocks themselves, search frictions in the asset market actually amplify the effect of productivity shocks in business cycles. Since most DSGE models fail to account for the link between the real sector and the financial sector, the model with search frictions in the asset market might be a good candidate to resolve the gap between RBC models and models with financial shocks in explaining the importance of productivity shocks in business cycles.
Chapter 3

Search, Asset Liquidity and Business Cycles

3.1 Introduction

In this chapter, I develop a stochastic version of the baseline model developed in Chapter 2. I use this model, together with productivity and equity liquidity shocks estimated from U.S. data, to investigate how asset liquidity and asset prices fluctuate. Moreover, I quantify the extent to which the frictions which generate endogenous fluctuations in asset liquidity also magnify the impact of productivity shocks on economic activity. Finally, I compare the business cycle properties generated by the baseline model to models without search frictions in the asset market in order to gauge the importance of search frictions.

In the baseline model, a household’s investment is restricted by its holdings of liquid assets. When a negative productivity shock hits the economy, entry into the asset market declines. This effect discourages investment because of the reduced liquidity of assets, thereby amplifying the decrease in total output. After calibrating the model to match key features in U.S. data, I find that productivity and liquidity shocks are equally important in explaining business cycles, which is contrary to the findings of
several recent studies.\(^1\) In addition, I show that productivity shocks can generate pro-cyclical movements of labor, asset prices and liquidity, which is not the case for liquidity shocks. The model also generates significant positive correlations between productivity and output, between productivity and labor and between productivity and asset liquidity, which are quantitatively consistent with empirical observations.

In recent years, there has been considerable interest in understanding the interaction between financial markets and the macroeconomy. In particular, after the 2008 financial crisis, many economists have argued that shocks to financial markets might account for a large proportion of fluctuations in output and asset prices. They believed the contribution of productivity shocks to business cycle fluctuations is very small because they assume that the variations in asset liquidity are driven purely by financial shocks. These include Christiano, Motto and Rostagno (2008), Jermann and Quadrini (2012), Del Negro, Eggertsson, Ferrero and Kiyotaki (2017), Ajello (2016), etc. However, if the changes in liquidity can be caused by shocks other than financial shocks, are financial shocks still as important in explaining business cycles? More specifically, if the other shocks are productivity shocks, can they dominate the role of financial shocks in business cycles?

To answer these questions, I estimate a structural vector auto-regressive model, in which productivity is allowed to have contemporary effects on asset liquidity,\(^2\) using U.S. quarterly data from 1980q1 to 2016q4. I find the following observations of U.S. business cycles: First, hours worked, asset prices and asset liquidity (as measured by the inverse of the bid-ask spread) are pro-cyclical. Second, productivity and hours

---

\(^1\)such as Jermann and Quadrini (2012), Iacoviello (2015), Ajello (2016), etc.

\(^2\)The assumption that productivity has effects on asset liquidity is introduced in the model from chapter one. In order to testing the theoretical model developed in the previous chapter, I impose the same assumption on the empirical study.
worked are positively correlated. Moreover, productivity, asset liquidity and output are also positively related. Third, the combination of productivity and liquidity shocks accounts for more than 50% of GDP volatility. Finally, productivity shocks are still very important in explaining the variances in GDP.

Some of these observations have been documented previously. For example, basic RBC models, such as those of Barro and King (1984), Prescott (1986) and King and Rebelo (1999), account well for co-movements among productivity, hours worked and output. However, these models are silent about the asset market. The models of Jermann (1994), Campbell (1999), and Bernanke and Gertler (2000) do a good job in accounting for asset prices, but they did not explore the importance of shocks that originate from financial markets in driving business cycles.

The literature on liquidity shocks shows that financial shocks\(^3\) to asset liquidity are important in business cycles. My analysis confirms that the sum of productivity and liquidity shocks accounts for a half of the variations in GDP. However, instead of showing that liquidity shocks are the main driving force, I find that productivity shocks are as important as liquidity shocks in explaining business cycles. In addition, my model also documents that hours worked is pro-cyclical in response to productivity shocks, which is not the case in KM under reasonable parameter choices.\(^4\) However, my model still cannot solve the puzzle of counter-cyclical movements of asset prices. Although negative productivity shocks reduce asset prices by reducing asset demand, they are unable to compensate for the positive effect generated by asset liquidity.

---

\(^3\)Such as unexpected regulation changes that affect the entrepreneurs' ability to borrow, non-transparent information makes works to believe that the entrepreneurs' default rates on loans change, etc. All these shocks can be considered as financial shocks to asset liquidity.

\(^4\)In my model, endogenous asset liquidity mitigates the reduction of the substitution effect on labor which caused by the liquidity constraint. The substitution effect still dominates the wealth effect, and thus, leads to pro-cyclical movements of labor. The detail discussion is provided in section 3.4.2.
3.1. INTRODUCTION
during the most recent financial crisis.\(^5\)

The analysis and basic structure of my model are closely related to those of KM and Shi (2015), who also study short-term dynamics driven by productivity and liquidity shocks in a model in which entrepreneur’s investment capability is restricted by holdings of liquid assets. They do not, however, consider the impact of productivity on asset liquidity. Their models have significant success in accounting for the fluctuations of aggregate production and labor with joint shocks to both productivity and asset liquidity. However, they underestimate the impact of productivity shocks on aggregate variables because they fail to include the indirect effect of productivity shocks acting through asset liquidity.

Ajello (2016) applies the “Financing Gap” to measure financial shocks and finds that financial sector shocks account for a significant amount of output and investment volatility.\(^6\) Gertler and Kiyotaki (2015) develop a model that features both balance-sheet and financial accelerator effects to study banking instability. Again they do not consider the impact of productivity on asset liquidity. Jermann and Quadrini (2012) suggest that imposing an intertemporal correlation between asset liquidity and productivity, plus a sizable adjustment cost to investment can generate pro-cyclical movements of the equity value with liquidity shocks by adjusting the amount of equity in the market. However, the adjustment cost still cannot resolve the issue of countercyclical movements of the equity price. In addition, they do not provide a microfoundation for this intertemporal correlation between asset liquidity and productivity,

---

\(^5\)The positive effect is coming from the effect of liquidity shocks on equity supply. In the KM types of model, a liquidity shock reduces equity supply without changing the equity demand. This effect is called the positive effect generated by asset liquidity.

\(^6\)Financing Gap is defined as the different between internal funds generated by business operation and investment costs. If the gap is negative, corporations are lack of liquidity asset to make investments. Financial gap is one way to measure asset liquidity. There is no unanimous way to measure asset liquidity. I used the bid-ask spread to capture asset liquidity in this dissertation.
which is the main contribution of this paper.

The model structure is also similar to Cui and Radde (2014). They incorporate intertemporal portfolio choice to a dynamic model with endogenous asset liquidity. Their models have significant success in accounting for the flight to liquidity during recessions. In contrast, the motivation for this paper is to study the respective contributions of productivity shocks and liquidity shocks to business cycle fluctuations. Thus, I build a search model to provide a micro-foundation of the linkage between the real sector and the financial sector.

The rest of the paper is organized as follows: The key features in U.S. data are documented in section 3.2. Calibration and simulation are discussed in section 3.3. In section 3.4, I discuss the importance of search frictions. My concluding remarks are offered in section 3.5.

### 3.2 Data and Facts

In this section, I estimate a structural vector autoregression (SVAR) for productivity (a), asset liquidity (s), hours worked (h) and GDP per capita (y), using U.S. quarterly data covering the period from 1980q1 to 2016q4. Hours worked is defined as the total hours worked by non-farm employees from Bureau of Labor Statistics. Asset liquidity is defined as the ratio of the bid-ask spread to the average asset price.\(^7\) The bid-ask spread is a reflection of the supply and demand for a asset. Specifically, the bid-ask spread decreases in the trading volume in all markets. Chakravarty and Sarkar (1999) find that the bid-ask spread is highly related to liquidity. If the asset is more illiquid,

\(^7\)There is no unanimity in the literature on how to empirically measure asset liquidity. The bid-ask spread is one of the most popular measurements of illiquidity in the finance literature. It has been used by many researchers, such as Amihud and Mendelson (1986), Chakravarti and Sarkar(1999) and Gopalan, Kadan and Pevzner (2010).
sellers pay a higher premium to attract more buyers and increase the probability of selling assets immediately. Therefore, the spread increases. The spread cost greatly depends on the time taken to complete a transaction. Therefore, this cost is an appropriate measurement of liquidity cost. With a one percent decrease in asset liquidity, I assume that the trading premium offered by the seller will also increase by approximately 1%. Therefore, I assume that the spread and asset liquidity are perfectly negatively correlated with equal variances. The bid-ask spreads are observed from COMPUSTAT and are available at an individual firm level.\textsuperscript{8} Hence, I take an average of the spreads from all firms in each quarter and use the average as the bid-ask spread in this paper.

Capital is defined as the sum of undepreciated capital from the previous period, net fixed investment, changes in private inventory and nominal holding gains or losses.\textsuperscript{9} I back out total factor productivity (TFP) directly from the production function and define it as productivity in this paper.

Figure 3.1 illustrates the dynamic paths of productivity, output, hours worked, asset prices and the spread. To present the relationships clearly, I have multiplied deviations of GDP, hours worked and productivity by 10. Output and hours worked clearly move up and down together with nearly the same amplitudes. Asset prices and the spread negatively co-move, especially after 2001, which implies that asset prices and liquidity should be positively correlated. Productivity is pro-cyclical and seems to have lead business cycles in the last decade.

\textsuperscript{8}The data set includes more than 20 thousands firms in each quarter. The small-cap firms is not included.

\textsuperscript{9}This is the definition used in NIPA tables.
3.2. DATA AND FACTS

3.2.1 Short-Run VAR and Impulse Responses

In this section, I build an SVAR model for the mechanism described in the baseline model in the Chapter two. The structure is based on Sims (1986) and Bernanke (1986). This empirical model includes four variables: GDP per capita, $y$, hours worked, $n$, the bid-ask spread, $s$, and productivity, $a$. The causal relations are based on the assumptions in Chapter 2. Productivity shocks are ordered first.\footnote{I put productivity first to keep the assumption consistent with the assumption in the theoretical model.} Productivity has a contemporaneous effect on asset market tightness and, hence, asset liquidity and the bid-ask spread. However, there is no potential contemporaneous impact...
of liquidity on the measure of productivity. Productivity and asset liquidity, then, combine to affect hours worked and output.

Augmented Dickey-Fuller (ADF) unit root tests are applied to the levels, the first differenced and HP-detrended data. The results are reported in Table 3.1. The test fails to reject the null of a unit root in the levels, but it does reject the null when applied to the first differenced and HP-detrended data.\textsuperscript{11}

<table>
<thead>
<tr>
<th></th>
<th>Level Results</th>
<th>First Diff results</th>
<th>HP results</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2.65</td>
<td>dy -4.58</td>
<td>hp_y -4.97</td>
</tr>
<tr>
<td>h</td>
<td>-1.98</td>
<td>dh -4.37</td>
<td>hp_h -4.93</td>
</tr>
<tr>
<td>a</td>
<td>-3.73</td>
<td>da -4.16</td>
<td>hp_ah -4.48</td>
</tr>
<tr>
<td>s</td>
<td>-2.69</td>
<td>ds -6.23</td>
<td>hp_s -4.50</td>
</tr>
</tbody>
</table>

The estimated correlations are reported in table 3.2. GDP is positively correlated with productivity and hours worked, while it is negatively correlated with the asset spread and, hence, positively correlated with asset liquidity. As in Prescott (1989) and Christiano, Eichenbaum and Vigfusson (2003, 2004), labor and productivity are positively correlated. The magnitude of the correlation between the spread and productivity is approximately 11% in the first-differenced case. If I use the HP-filter to detrend the data, the negative correlation increases significantly to 35%.

The model can be written in matrix format as

\[ AY = BX + E, \]

where \( A \) and \( Y \) are the \( K \times K \) matrix, including all contemporary coefficients and

\textsuperscript{11}Note: The test is based on an ADF test with four lags. The 1% significance critical value is \(-3.503\), and the 5% significance critical value is \(-2.889\)
3.2. DATA AND FACTS

Table 3.2: Unconditional Correlations

<table>
<thead>
<tr>
<th></th>
<th>da</th>
<th>ds</th>
<th>dh</th>
<th>dy</th>
<th>hp_a</th>
<th>hp_s</th>
<th>hp_h</th>
<th>hp_y</th>
</tr>
</thead>
<tbody>
<tr>
<td>da</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ds</td>
<td>-0.11</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dh</td>
<td>0.08</td>
<td>0.05</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dy</td>
<td>0.83</td>
<td>-0.05</td>
<td>0.60</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hp_a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hp_s</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.35</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hp_h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.06</td>
<td>-0.04</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>hp_y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.57</td>
<td>-0.15</td>
<td>0.85</td>
<td>1.00</td>
</tr>
</tbody>
</table>

contemporary variables. $K$ is the number of variables. $B$ is a $K \times (Lk + 1)$ matrix containing coefficients of all past variables. $X$ is a $(Lk+1)$ vector including a constant and all past variables. $E$ is a matrix of unconditional errors. The distribution of $E$ is discussed later.

The basic model without asset liquidity

In the basic model, asset liquidity is removed.

$$hp_a = \beta_1 lag(G) + e_x$$

$$hp_h + a_{21} hp_a = \beta_2 lag(G) + e_n$$

$$hp_y + a_{21} hp_a + a_{32} hp_h = \beta_3 lag(G) + e_y.$$  

The function $lag(G)$ represents the lags of variable $G$.\(^{12}\) The matrix $A$ is a $3 \times 3$ matrix including all contemporary coefficients. The variables are HP-detrended. The estimated coefficients are listed in (3.1).

\(^{12}\)The optimal number of lag is 4 in this model. I tested higher level of lags. I found that the effects of lags after 4 periods are very small and can be ignored.
The value of $a_{21}$ is negative but not significant, which implies that the contemporary effect of productivity on hours worked is weakly positive. This result is consistent with previous findings in standard RBC models. The values of $a_{31}$ and $a_{32}$ are negative and significant, implying that productivity and hours worked have a positive contemporary effect on output.

The decompositions of the variance of GDP are shown in Table 3.3. Productivity explains approximately 33% of the GDP variances, and labor demand shocks\textsuperscript{13} explain more than 55% of the total variances. The last 12% is explained by undefined shocks. Without asset liquidity, labor demand shocks are the major driving force of business cycles. In the next section, I will discuss how liquidity shocks change the pattern of the variance decompositions.

Table 3.3: model without asset liquidity

<table>
<thead>
<tr>
<th>Response</th>
<th>GDP</th>
<th>Impulse</th>
<th>Prod</th>
<th>Labor</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decomposition</td>
<td>32.8%</td>
<td>55.5%</td>
<td>11.7%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The model with asset liquidity

In this section, I use the bid-ask spread to model asset liquidity and include it in the SVAR model. As in the theory discussed in Chapter 2, I assume that productivity generates a contemporaneous effect on the bid-ask spread, but not the reverse. In addition, productivity and asset liquidity affect hours worked and total output contemporaneously. The time interval \((1981q2 - 2016q4)\) is used. I assume that the shocks are orthogonal. The estimated matrix \(A\) is listed in equation 3.2

\[
A_{80} = \begin{bmatrix}
1 \\
6.33 & 1 \\
(2.70) \\
-0.01 & 0.005 & 1 \\
(0.08) & (0.002) \\
-0.99 & -0.00 & -0.64 & 1 \\
(0.01) & (-0.00) & (0.01)
\end{bmatrix}.
\]  

(3.2)

The values in parentheses represent the standard deviations. The positive value of \(a_{21} = 6.33\) implies that productivity has a negative effect on the spread and, hence, a positive effect on asset liquidity. The coefficient \(a_{31} = -0.01\). Thus, the productivity and hours worked are still positively related. The coefficients \(a_{41}\) and \(a_{43}\) are strictly negative, which means both productivity and demand shock increase total output immediately. The coefficients \(a_{32}\) and \(a_{42}\) are approximately 0. Therefore, asset liquidity does not have a large contemporaneous effect on hours worked or total output.

The results of the variance decomposition are shown in Table 3.4. Because asset
liquidity and the spread are perfectly negatively correlated, the decomposition results of asset liquidity should be identical with the results of the spread. To make the comparison with the model results more clear, I report the decomposition of the spread as the decomposition of asset liquidity. Columns 2 – 5 are the results of the

<table>
<thead>
<tr>
<th>Response</th>
<th>Asset Liquidity</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse</td>
<td>Prod Liq Emp GDP</td>
<td>Prod Liq Emp GDP</td>
</tr>
<tr>
<td>Decomposition</td>
<td>5.8% 82.0% 7.0% 5.2%</td>
<td>29.1% 25.2% 42.0% 3.7%</td>
</tr>
</tbody>
</table>

variance decomposition of asset liquidity. Columns 6 – 9 record the results of the variance decomposition of the total output. The majority of the variance of asset liquidity is explained by itself. Productivity explains 5.8% of the variance. The variance of GDP is explained primarily by three components: productivity (29.1%), asset liquidity (25.2%), and hours worked (42.0%). The sum of productivity and asset liquidity accounts for more than 50% of the GDP volatility. Productivity shocks are at least as important as liquidity shocks in explaining business cycles, which is contrary to recent findings. In these recent studies, the indirect effect of productivity shocks acting though the financial market is omitted, which causes a severe underestimation of the impact of productivity shocks.

Figure 3.2 provides a time series representation of the evolution of quarterly output growth from 1981q1 to 2016q4. The solid black line represents the observed data series. The blue dashed line represents the output fluctuations which the empirical model identifies as being driven by productivity shocks. The red dashed-dotted line represents the fluctuations driven by liquidity shocks. Liquidity shocks are indicated as the only financial shocks in the paper. The combination of productivity and liquidity shocks explains approximately 50% of the output variations.
3.3 Calibration and Simulation

In this section, I calibrate the model built in Chapter 2 and study the nature of the fluctuations in asset prices, liquidity, investments and output with productivity shocks. The following functional forms are used:

\[ u(c_t) = \frac{c_t^{1-\rho} - 1}{1 - \rho} \]
\[ h(l_t) = h_0 l_t^\eta, \quad \eta > 1 \]
\[ F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha} \]
\[ M(B_t, S_t) = \phi_0 B_t^\gamma S_t^{1-\gamma} \left( \frac{1}{1 + e^{\mu t}} \right) \]
\[ \phi_t^s = \phi_0 \left( \frac{b_t(1 - \pi)}{\pi} \right)^\gamma. \]

The matching function takes a Cobb-Douglas form. A productivity shock changes buyers’ willingness to trade and the market tightness and thus indirectly affects asset
liquidity. There are also exogenous shocks that change asset liquidity directly, such as shocks to broker’s balance sheets, shocks to broker’s future expectations and shocks to fixed investment costs.\textsuperscript{14, 15} The value of $\mu$ is used to capture the effect of these exogenous shocks on asset liquidity. A shock to $\mu$ is named a liquidity shock rather than a matching efficiency shock in order to making a consistent definition with the data, although, a shock to $\mu$ and a shock to the matching efficiency $\phi_0$ are equivalent.\textsuperscript{16} A decrease in $\mu$ reduces the matching probability and thus reduces asset liquidity. Assume that quarterly TFP and liquidity processes are given by

$$
\log A_{t+1} = (1 - \delta_A) \log A^* + \delta_A \log A_t + \varepsilon_{A,t+1}
$$

$$
\mu_{t+1} = (1 - \delta_\mu) \mu^* + \delta_\mu \mu_t + \varepsilon_{\mu,t+1},
$$

where $\delta_A$ and $\delta_\mu$ measure the persistence of TFP and liquidity shocks, respectively. The value of $A^*$ is the steady state TFP, which is normalized to 1. The steady state value of $\mu^*$ is normalized to 0. The values of $\varepsilon_A$ and $\varepsilon_\mu$ represent the error terms of the two shocks. The variance-covariance matrix of the error terms is given by

$$
cov = \begin{bmatrix}
\text{var}(\varepsilon_A) & \text{cov}(\varepsilon_A, \varepsilon_\mu) \\
\text{cov}(\varepsilon_\mu, \varepsilon_A) & \text{var}(\varepsilon_\mu)
\end{bmatrix}.
$$

The model is calibrated to a quarterly frequency. I partition the model parameters

\textsuperscript{14}Refer to KM, Del Negro, Eggertsson, Ferrero and Kiyotaki (2017) for further discussions about the shocks on liquidity.

\textsuperscript{15}These shocks are different from productivity shocks. Productivity shocks are realized at the beginning of each period. When households make entry decisions in each period, productivity is observable. In contrast, an exogenous shock to asset liquidity is realized after workers enter asset markets.

\textsuperscript{16}Liquidity shocks include all non-productivity shocks that can generate fluctuations of asset liquidity.
into two groups: a general group and a special group. The general group includes 
\((\beta, \alpha, \rho, \eta, \sigma, u_0, h_0, \delta_A, \var(\varepsilon_A))\), which are standard parameters in the business cycle 
literature. The special group includes \((\pi, \phi_0, c_f, \gamma, \delta_\mu, \var(\varepsilon_\mu), \cov(\varepsilon_A, \varepsilon_\mu))\).

The values of the discount factor, \(\beta\); the capital share, \(\alpha\); and the relative risk 
aversion, \(\rho\), are set equal to 0.992, 0.36 and 2, respectively. The curvature of disutility 
of labor, \(\eta\), equals to 1.5, is determined from the target where the labor elasticity is 
2. The annual investment to capital ratio, \(4(1 - \sigma)\), is set equal to 0.066. This ratio 
is used to solve the survival rate \(\sigma\). Given that the annual capital to output ratio is 
3, I can solve for the steady state capital rental rate, \(r\); the wage rate, \(w\); and the 
ratio of capital to labor, \(\frac{K}{L}\). The target for total hours worked is 0.2, which is used 
to determine the steady state values of capital and output. The value of \(h_0\) can be 
solved from

\[
h_0 = \frac{c^\omega \eta}{\eta^n - 1}.
\]

I obtain the steady state value of the broker’s matching rate, \(\phi^*\), from the capital’s 
law of motion.\(^ {17}\) Given the value of \(\phi^*\), I can get a relationship between the power 
of the matching function \(\gamma\) and the constant \(\phi_0\) from the matching function. The 
estimated range of the fraction of firms that adjust their capital annually is between 
0.2 and 0.4.\(^ {18}\) In this paper, I set the value equal to 0.24 as in Shi (2015). Hence, 
the quarterly value, \(\pi\), equals 0.06. The participation rate in the asset market is 
approximately 50%. The entry cost of the equity market, \(c^f\), is determined by the 
free entry condition.

\(^ {17}\) I actually use two targets, \(k/y\) and hours of work, to determine two parameters, \(h_0\) and \(\phi_0\), 
simultaneously.

\(^ {18}\) These values are from Doms and Dunne (1999) and Cooper, Haltiwanger and Power (1999).
The variance and the persistence of TFP shock are observed from the data. However, the matching function and the distribution of liquidity shocks cannot be observed from the data. In the absence of direct observations, I jointly calibrate the power of the matching function, $\gamma$, the constant, $\phi_0$, the variance and the persistence of liquidity shocks, $(\text{var}(\varepsilon_\mu), \delta_\mu)$, and the covariance between TFP and liquidity shocks, $\text{cov}(\varepsilon_A, \varepsilon_\mu)$ to match five targets: the broker’s matching rate, $\phi^*$; the relative volatility of asset liquidity, $\sigma_{\phi^*}/\sigma_y$; the correlation between TFP and asset liquidity, $\text{corr}(A, \phi^*)$; the correlation between TFP and hours worked, $\text{corr}(A, l)$; and the correlation between TFP and output, $\text{corr}(A, Y)$.

All parameters are listed in Table 3.5. The value of $\delta_A$ is strictly greater than the value of $\delta_\mu$, which means that productivity is more persistent. The covariance between the two shocks is positive, which implies that a negative productivity shock will more likely accompany a negative liquidity shock.

The steady state values implied by this calibration are listed in Table 3.6. Asset liquidity is strictly less than 1, which means that brokers need time to liquidate their equity holdings. In this paper, the probability that a broker matches a desired buyer and executes a trade is 14.3%, which is similar to the result of Del Negro, Eggertsson, Ferrero and Kiyotaki (2017). The asset price, $X$, is strictly greater than 1, which implies that the expected benefit of holding the asset is greater than the cost. Therefore, buyers have an incentive to trade and hold the asset.

\footnote{In Del Negro, Eggertsson, Ferrero and Kiyotaki (2017), the liquidity of assets equals 15%, which is solved from the target where the relative quantity of liquid and illiquid assets is approximately 14%. In Shi (2015), the liquidity is solved from the target where the annual return on liquid assets is approximately 2%, which gives $\phi^* = 27\%$.}
### Table 3.5: Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value(general)</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$: discount factor</td>
<td>0.992</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\rho$: relative risk aversion</td>
<td>2</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$h_0$: constant in labor disutility</td>
<td>9.16</td>
<td>target labor supply L=0.2</td>
</tr>
<tr>
<td>$\eta$: curvature in labor disutility</td>
<td>1.5</td>
<td>labor supply elasticity=2</td>
</tr>
<tr>
<td>$\alpha$: capital share</td>
<td>0.36</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\sigma$: survival rate of capital</td>
<td>0.984</td>
<td>annual investment/capital=0.066</td>
</tr>
<tr>
<td>$A^*$: steady-state TFP</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>$\delta_a$: persistence of TFP</td>
<td>0.96</td>
<td>persistence of TFP</td>
</tr>
<tr>
<td>$\theta$: fraction of new equity</td>
<td>0.143</td>
<td>equals the price discount rate</td>
</tr>
<tr>
<td>$\text{var} (\varepsilon_A)$: variance of TFP</td>
<td>$0.367 \times 10^{-4}$</td>
<td>observable from data</td>
</tr>
<tr>
<td>$\pi$: fraction of entrepreneurs</td>
<td>0.06</td>
<td>fraction of investing firms=0.24</td>
</tr>
<tr>
<td>$c_f$: entry cost of workers</td>
<td>0.25</td>
<td>worker’s entry rate e=0.5</td>
</tr>
<tr>
<td>$\gamma$: power of the matching function</td>
<td>0.25</td>
<td>relative volatility $\sigma_{\phi^*} / \sigma_Y = 21$</td>
</tr>
<tr>
<td>$\phi_0$: constant in the matching fun.</td>
<td>0.17</td>
<td>ratio of capital to annual output=3</td>
</tr>
<tr>
<td>$\delta_\mu$: persistence of the liq.</td>
<td>0.5</td>
<td>$\text{corr} (A, \phi) = 0.35$</td>
</tr>
<tr>
<td>$\text{var} (\varepsilon_\mu)$: variance of the liq.</td>
<td>0.1</td>
<td>$\text{corr} (A, L) = 0.06$</td>
</tr>
<tr>
<td>$\text{cov} (\varepsilon_\mu, \varepsilon_A)$: covariance of (TFP liq.)</td>
<td>$0.8 \times 10^{-4}$</td>
<td>$\text{corr} (A, Y) = 0.57$</td>
</tr>
</tbody>
</table>
3.3. CALIBRATION AND SIMULATION

Table 3.6: Steady State Values

<table>
<thead>
<tr>
<th>Variables</th>
<th>Steady State Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y ): total output</td>
<td>0.8</td>
</tr>
<tr>
<td>( K ): total capital supply</td>
<td>9.71</td>
</tr>
<tr>
<td>( L ): total labor supply</td>
<td>0.2</td>
</tr>
<tr>
<td>( C ): total consumption</td>
<td>0.65</td>
</tr>
<tr>
<td>( I ): total investment</td>
<td>0.16</td>
</tr>
<tr>
<td>( c_w ): consumption per worker</td>
<td>0.69</td>
</tr>
<tr>
<td>( i ): investment per entrep.</td>
<td>2.67</td>
</tr>
<tr>
<td>( l_w ): labor supply per worker</td>
<td>0.21</td>
</tr>
<tr>
<td>( e ): worker’s entry rate</td>
<td>0.50</td>
</tr>
<tr>
<td>( X ): asset price</td>
<td>1.67</td>
</tr>
<tr>
<td>( q ): asset trade per match</td>
<td>9.93</td>
</tr>
<tr>
<td>( \omega ): wage rate</td>
<td>2.59</td>
</tr>
<tr>
<td>( r ): capital rental rate</td>
<td>0.03</td>
</tr>
<tr>
<td>( \phi^* ): asset liquidity</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The implied impulse response functions following joint shocks (i.e. simultaneously shocks to both productivity and the liquidity) are depicted in Figure 3.3.\(^{20}\) The magnitude of the productivity shock is one standard deviation, and liquidity shocks are imposed according to the covariance between the two shocks. In this economy, a negative shock to productivity reduces worker’s labor income. Fewer workers enter the equity market due to a relatively expensive entry cost. The tightness of the equity market decreases, reducing both asset liquidity and prices. Entrepreneurs use initial asset holdings and funds obtained from brokers to finance new investments. The lower the asset price, the tighter the entrepreneur’s resource constraint. Thus, investments made by entrepreneurs decrease, and consequently, total output and consumption

\(^{20}\)At the given parameter set, the Blanchard-Kahn condition holds. The number of eigenvalues is greater than one and equals the number of forward-looking variables. Thus, the steady state is unique. I also simulate the model 1000 times. The probability of the entry decision \( e \), which hits the upper and lower bounds, is 0. Thus, the occasionally binding scenario rarely happens in the model and can be ignored.
3.4 Importance of Search Frictions

The importance of frictions depends on whether the frictions can help significantly improve the calibration results. Figure 3.2 shows that the combination of productivity and liquidity shocks explains approximately 50% of the output variations. Although a number of other researchers have found this result, they have also noted that liquidity decrease. At the same time, a negative matching efficiency shock hits the economy, which further reduces asset liquidity and, hence, further reduces total investment and output.

Figure 3.3: Impulse Responses to Productivity and Liquidity Shocks
3.4. IMPORTANCE OF SEARCH FRICTIONS

shocks are the main driving force of business cycles, which is contradicted by the result in Figure 3.2, where the red line does not show more features similar to the black line than the blue line.\textsuperscript{21} In contrast, I find that productivity shocks are as important as liquidity shocks in business cycles. In this section, I explore the reasons why search frictions in the asset market are important for explaining business cycles. I find that market tightness in the asset market is the key element in linking productivity to the financial market.

3.4.1 Asset Liquidity

Asset prices are pro-cyclical. Shi (2015, pp. 27-28) argues that “liquidity shocks cannot be the main driving force of business cycles because a negative liquidity shock leads asset price to rise rather than fall, and hence suggests that a negative liquidity shock must be accompanied by other concurrent shocks, such as productivity shocks, which reduce the demand for asset sufficiently.” In my model, the asset market tightness links productivity to asset liquidity. A negative productivity shock reduces capital productivity and the measure of buyers in the asset market and, hence, lowers asset liquidity. One can question whether productivity shocks are still strong enough to result in pro-cyclical movements of the asset price when they generate persistent fluctuations of asset liquidity.

An unexpected decrease in productivity reduces investments and total output by reducing the marginal productivity of capital. These reductions decrease a household’s consumption and drive up the marginal utility of consumption. Meanwhile, lower levels of investment and asset liquidity make the liquidity constraint (Eqn.

\textsuperscript{21}Two lines can not fully capture the movements of GDP because productivity and liquidity only account for 50% GDP variances. The other 50% that is generated by undefined shocks is not described in the Figure 3.2.
(2.19)) tighter in the subsequent period and, hence, increase the expected marginal
benefit of asset holdings, \( \Omega \). According to (2.11), the asset price falls if the increase in
the marginal utility of consumption dominates the increase in the expected marginal
benefit of asset holding.

To illustrate this possibility, I compute the response of the equilibrium output to
negative productivity shocks only. The six panels in Figure 3.4 depict the impulse
response function for productivity, asset liquidity, consumption, investment, the ex-
pected future utility of asset holdings and the asset price. As explained above, the
negative shock to productivity decreases asset liquidity, consumption and investment.
A decrease in asset liquidity tightens the liquidity constraint, which drives up the
expected utility of asset holdings. The last panel shows that the asset price falls,
which means the negative productivity shock does generate a significant decrease in
the demand of the asset, even though it also generates a concurrent decrease in as-
set liquidity. Thus, productivity shocks play a visible role in explaining pro-cyclical
movements of the asset price. Figure 3.3 further confirms this result by showing that
the asset price still falls with joint shocks to both productivity and asset liquidity.

3.4.2 Employment

Another notable feature of business cycles is the positive co-movement between GDP
and hours worked. Any shock that fails to generate this feature could not be con-
sidered a primary driving force of business cycles. In this section, I discuss whether
a productivity shock can generate this feature in my model. This issue needs to be
discussed because the liquidity constraint reduces the substitution effect of produc-
tivity shocks on hours worked. If the reduction is strong enough, then productivity
shocks push hours worked to move in the opposite direction. Hours worked, $l$, satisfies (2.22). Using the marginal product of labor to replace the wage rate $w$ and taking a log form yields

$$(\eta - 1 + \alpha) \log(l) = \log(A) + \alpha \log(K) - \rho \log(c) + \log(1 - \alpha) - \log h_0. \quad (3.6)$$

The term, $\log(1 - \alpha) - \log h_0$, is constant. The value of $\eta$ is greater than 1. Thus, hours worked decreases in $c$ and increases in $K$. A negative productivity shock reduces total income and consumption and, hence, increases the marginal utility of consumption, leading to an increase in labor supply. This reflects the wealth effect on hours worked. However, the productivity shock reduces the marginal productivity of capital. Thus households prefer to reallocate more funds to consumption. This reallocation leads
to a decrease in labor productivity and wages, and thus, the hours worked falls. This process is referred to as the substitution effect.

The substitution effect on hours worked depends on the change in the expected marginal benefit of capital. According to (2.25), the expected marginal benefit can be expressed as

$$v_{s+1} = u'(c_{+1})(1 - \pi)r_{+1} + \lambda_{+1}^c(\pi r_{+1} + \sigma) + \lambda_{+1}^m \beta Ev_{s+2} \pi \phi_{+1}^s(r_{+1} + \frac{\sigma}{\theta}).$$  \hspace{1cm} (3.7)

The first two terms are the marginal benefits of rental and selling incomes, respectively, which are exactly the same as those in the model without the liquidity constraint. The last term represents the benefit from relaxing the liquidity constraint (called the tightness premium), existing only in the model with liquidity constraints. As discussed above, a lower level of productivity tightens the liquidity constraint and, hence, increases the tightness premium. This increase raises the expected marginal benefit of capital and, hence, reduces the substitution effect on hours worked. If the reduction is large enough, then the wealth effect dominates the substitution effect. Therefore, the hours worked rises.

Search frictions are important in moderating the unexpected reduction in the substitution effect. A negative productivity shock reduces the measure of buyers in the equity market because of the relatively high entry cost, which reduces entrepreneurs’ investment capacity and capital. This effect is reflected as a reduction of the seller’s matching probability, $\phi_{+1}^s$. According to (3.7), a lower value of $\phi_{+1}^s$ reduces the tightness premium and thus mitigates the unexpected reduction in the substitution effect.
3.4. IMPORTANCE OF SEARCH FRICTIONS

Figure 3.5: Impulse Responses of Labor Input

Figure 3.5 illustrates a comparison of the effects of a one standard deviation decline in productivity on hours worked in different models. The solid blue line displays the dynamic path of hours worked from the search model. The green dashed line represents the impulse response of hours worked from a version of KM,\(^{22}\) and the brown dashed-dotted line represents the results from a standard RBC model. Clearly, the

\[ h^l((1-\pi)l^w) = \frac{\partial F(K, L)}{\partial L} u'(c). \]  

(3.8)

Thus, hours worked depends on three functions: the disutility of labor supply, the production function, and the utility function. The strict convexity of the disutility function implies that the left hand side of (3.8) strictly increases in labor supply, \( l^w \). The production function is a general Cobb-Douglas function, and the parameter values are standard. Thus, the only feasible way to alter the moving direction of hours worked is by changing the format of the utility function of consumption, particularly the relative risk aversion. For instance, if \( \rho = 1 \), hours worked is pro-cyclical. In my paper, the relative risk aversion equals 2. The assumption that households are risk averse is widely used in the related literature.
liquidity constraint and search frictions in the asset market have a substantial effect on hours worked, which can be traced to the effect of a decrease in productivity on the substitution effect of hours worked. In this version of KM, the liquidity constraint reduces the substitution effect of productivity on hours worked significantly. Thus, the wealth effect dominates the substitution effect and increases hours worked. In my model, search frictions in the asset market discourage buyers’ entry, which reduces market tightness and, hence, reduces the tightness premium and mitigates the reduction of the substitution effect. Consequently, hours worked decrease.

Figure 3.6 compares impulse responses to productivity shocks and to exogenous liquidity shocks in the search model. The three panels on the left report the impulse responses of hours worked and the asset price to productivity shocks, and the three panels on the right report the impulse responses to liquidity shocks. It is clear that an liquidity shock generates a pro-cyclical movement of hours worked, but it pushes asset prices counter-cyclically, which is counterfactual. This result implies that direct shocks to asset liquidity cannot be an independent source of business cycles. In contrast to liquidity shocks, productivity shocks with search frictions can generate positive co-movement between asset liquidity and asset prices as well as pro-cyclical movements of hours worked. Therefore, productivity shocks alone can be considered a main source of business cycles.

3.4.3 Cyclical Properties

Figure 3.7 offers a comparison of the dynamic paths between the models with and without search frictions. Two exogenous shocks, namely, productivity and liquidity shocks, are introduced in Section 3.3. The blue solid line represents the impulse
3.4. IMPORTANCE OF SEARCH FRICIONS

Figure 3.6: Impulse Responses: a Productivity Shock and a Liquidity Shock

responses from the model with search frictions. The resaleability of assets is endogeneously determined by market tightness and restricts the liquidity of both the new and existing assets. The green dashed line represents the dynamic results from KM. The resaleability is fixed and restricts the liquidity of the existing asset only. To enable an appropriate comparison, I compute another non-search model in which the resaleability constraint is imposed on both old and new assets. The dynamic results are represented by brown dashed-dotted lines in Figure 3.7. As the figure shows, the impulse responses for output are similar, even though different models are applied. This consistency suggests that search frictions do not significantly affect the joint impact of productivity and liquidity shocks on GDP. This result is also consistent with my empirical result that although the different assumptions have been used, the aggregate effect of productivity and asset liquidity shocks always accounts for approximately 50% of the variations in GDP. However, this result does not mean that
search frictions cannot help clarify the components of business cycles. In this section, I document the importance of search frictions in matching certain cyclical properties.

Figure 3.7: A Comparison among Models with and without Search Frictions: Impulse Responses to One Standard Deviation Productivity Shocks and Accompanied Liquidity Shocks

My estimated model with search frictions accounts well for the behaviour of asset liquidity, total output and hours worked. Table 3.7 reports the correlations and relative volatilities among some key business cycle variables for different models. In the table, the first column reports the results from my empirical analysis, the second column reports the results from the search model, and the third column lists the results from the model without search frictions in the asset market. The results from KM are reported in the fourth column.

Search frictions visibly increase the correlations of productivity with some of the

23This model forces on the equity market activities. It cannot capture the dynamic activities in the other market (i.e. labour, investment, goods markets) very well. Therefore, I did not report all moments here. In the next chapter, more features are included in the baseline model introduced in chapter 2. More moments will be presented and discussed in that chapter.
3.4. IMPORTANCE OF SEARCH FRICTIONS

key business cycle variables, such as correlations between productivity and asset liquidity, $corr(A, \phi^s)$ and between productivity and hours worked, $corr(A, L)$. The search model is also able to generate positive correlations between productivity and output and between hours worked and output, $corr(A, L)$ and $corr(L, Y)$ that are nearly identical to those observed in the data. The search model also replicates relatively well both asset liquidity volatility and productivity volatility, although the volatility of labour input is somewhat overstated. Although The standard RBC models can generate these productivity, output and labour volatilities very well. They are silence about the asset market. Thus, search frictions seem to be necessary for an accurate assessment of the importance of productivity shocks in business cycles.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>US data</th>
<th>Search Model</th>
<th>Without Search</th>
<th>Without Search (KM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$corr(A, \phi^s)$</td>
<td>0.35</td>
<td>0.22(0.13)</td>
<td>0.21(0.15)</td>
<td>0.15(0.15)</td>
</tr>
<tr>
<td>$corr(A, L)$</td>
<td>0.06</td>
<td>0.11(0.12)</td>
<td>-0.31(0.15)</td>
<td>0.04(0.15)</td>
</tr>
<tr>
<td>$corr(A, Y)$</td>
<td>0.57</td>
<td>0.46(0.13)</td>
<td>0.10(0.13)</td>
<td>0.47(0.13)</td>
</tr>
<tr>
<td>$corr(L, Y)$</td>
<td>0.84</td>
<td>0.93(0.01)</td>
<td>0.90(0.03)</td>
<td>0.89(0.03)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatilities</th>
<th>$\sigma_{\phi^s}/\sigma_A$</th>
<th>$\sigma_A/\sigma_Y$</th>
<th>$\sigma_L/\sigma_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\phi^s}/\sigma_A$</td>
<td>21</td>
<td>21(0.04)</td>
<td>24(3.75)</td>
</tr>
<tr>
<td>$\sigma_A/\sigma_Y$</td>
<td>0.61</td>
<td>0.59(0.01)</td>
<td>0.42(0.06)</td>
</tr>
<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>1.1</td>
<td>1.42(0.00)</td>
<td>1.65(0.09)</td>
</tr>
</tbody>
</table>

Numbers in parentheses are standard deviations over 1000 simulations

I simulate the model 1000 times. In each simulation, I randomly select 600 draws from the distributions of shocks. The artificial time series are detrended using an HP-filter with $\lambda = 1600$.

Having shown that the estimated search model fits key moments of the data quite well, I use the model to address a related question of how productivity shocks have contributed to the variations in GDP in the U.S. Figures 3.2 and 3.8 provide a visual representation. I simulate each model (with and without search frictions) for 1000 periods. In each period, two shocks are randomly drawn from the distributions
discussed in Section 5.2. Then, I compute the variations in GDP and illustrate them in Figure 3.8. The first panel shows the result from the model with search frictions, and the decomposition results from the non-search models are displayed in panels 2 and 3. The blue areas are the variations in GDP explained by productivity shocks, and the red areas are the variations explained by the efficiency shocks. As Figure 3.8 shows, the contribution of productivity shocks to variations in GDP is notable in the search model which is qualitatively similar to the result of the empirical model illustrated in Figure 3.2. In contrast, productivity shocks seem irrelevant to the variations in GDP in the non-search models.

![Figure 3.8: Simulated GDP Decomposition](image)

Table 3.8 presents the numerical results of the variance decomposition. The first panel reports the accumulated decomposition of the variance of GDP, and the second panel reports the accumulated decomposition of the variance of asset liquidity. The
first row shows the results from the data.\textsuperscript{24} The second row reports the results of the search model, and the third and the forth rows report the results of the models without search frictions. In the search model, productivity shocks account for approximately 69% of the variations in GDP, which is higher than what is observed in the data. However, these results are much better than the results from the model without search frictions. Moreover, in contrast to the models without search frictions, the search model is able to account for the contribution of productivity shocks on asset liquidity, although this contribution is somewhat overstated.

Table 3.8: Decompositions of GDP and Asset Liquidity

<table>
<thead>
<tr>
<th>Shocks</th>
<th>GDP</th>
<th>Asset Liquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Productivity</td>
<td>Matching eff.</td>
</tr>
<tr>
<td>Data</td>
<td>0.54</td>
<td>0.46</td>
</tr>
<tr>
<td>Search Model</td>
<td>0.69</td>
<td>0.31</td>
</tr>
<tr>
<td>Without Search</td>
<td>0.18</td>
<td>0.82</td>
</tr>
<tr>
<td>Without Search (KM)</td>
<td>0.21</td>
<td>0.79</td>
</tr>
</tbody>
</table>

### 3.4.4 Why are the Indirect Effects of Productivity Shocks on GDP so Important?

As the results in Table 3.8 document, the contribution of productivity shocks to the volatility of GDP significantly increases to 69%. In addition, only 7% of the movement in liquidity is driven by the endogenous response to productivity shocks. But are

\textsuperscript{24}Because only two shocks are studied in each model, to enable a proper comparison, I redefine the variables as the components explained by productivity and liquidity shocks only when I compute the results in the first row.

\textsuperscript{25}I assume that liquidity variations, which are not explained by productivity shocks, are generated by the liquidity shocks. This approach is consistent with my theorem result. Therefore, the variations in GDP, which are led by the standard error of asset liquidity in the empirical model, are considered to be the contribution of the matching efficiency to GDP fluctuations.
3.4. IMPORTANCE OF SEARCH FRICTIONS

the indirect effects of productivity shocks through asset liquidity still important in boosting their effects on GDP? I discuss this question further below.

The production function implies that the effect of a shock on GDP depends on its contributions to the fluctuations of capital investment and hours worked. In the model without search frictions, an unexpected decrease in productivity reduces hours worked by reducing the marginal productivity of labor. However, asset liquidity is fixed. Thus, capital investment changes very little. The restriction is imposed that hours worked cannot undergo a large decrease that leads to insufficient funds to clear capital market. However, an unexpected decrease in asset liquidity reduces capital investment by reducing entrepreneur’s investment capacity. In response to the large decline in capital investment, the hours worked also decreases significantly. Therefore, liquidity shocks dominate productivity shocks in explaining business cycles. In contrast to the model without search frictions, an unexpected productivity shock reduces the buyer’s willingness to trade by reducing the marginal productivity of capital in the search model, thereby reducing the entry rate. These reductions explain approximately 7% of the variance in asset liquidity and only approximately 7% of the variances in capital investment and hours worked. By inference, productivity shocks may explain an additional 7% of the variations in GDP. Why are the effects of productivity shocks on GDP so large, as shown in Table 3.8? In fact, exogenous liquidity shocks to matching efficiency have lesser of an effect on capital and hours worked than productivity shocks. An unexpected decrease in matching efficiency reduces entrepreneurs’ investment capability without reducing the marginal productivity of capital. Thus, more buyers will enter the asset market to increase capital investment. Figure 3.9 compares the entry rates from a model with a productivity shock and a
3.4. IMPORTANCE OF SEARCH FRICTIONS

model with a liquidity shock. The blue solid line represents the entry rates driven by a productivity shock, and the green dashed line represents those driven by a matching efficiency shock. The liquidity shock raises the entry rate. This is because the liquidity shock reduces the supply of equity without reducing the expected benefit from trades in the equity market. Workers are harder to get matched in the equity market. The expected trading benefit increases, which leads to more workers entering the equity market. This positive effect on the entry rate reduces the impact of the liquidity shocks on capital and hours worked. In contrast, an unexpected negative productivity shock reduces the marginal productivity of capital and hours worked, which, in turn, decreases asset liquidity by reducing the measure of buyers in the asset market. Therefore, although both shocks reduce asset liquidity, they act through different channels. A productivity shock reduces buyers’ entry rate and, hence, further decreases capital investment and hours worked. Thus, the indirect effects of productivity shocks acting through asset liquidity are still important in explaining business cycles, even though productivity shocks can only explain approximate 7% of the variation in asset liquidity.

In summary, it is clear that the search model with productivity shocks can generate pro-cyclical movements of hours worked and asset prices. Moreover, imposing search frictions in the asset market is necessary for precisely estimating the importance of productivity shocks in business cycles. In contrast to the model with search frictions, the non-search model understates the correlations of productivity with some key business cycle variables and the volatility of productivity and, hence, severely underestimates the contribution of productivity shocks to business cycles.
3.4. IMPORTANCE OF SEARCH FRICTIONS

3.4.5 Simulating the Financial Crisis: the Impact on Output, Asset Prices and Hours Worked

Having shown that the estimated search model explains well the behaviors of total output, asset liquidity and asset prices under general conditions, another open question is whether this model can capture certain specific facts in the recent financial crisis. Particularly pertinent is the question of whether the search model can still generate pro-cyclical movements of asset prices and hours worked given huge declines in asset liquidity and total productivity. The top two panels of Figure 3.10 depict productivity and liquidity shocks from U.S. quarterly data in the period 2008q1 – 2011q3 with an assumption that productivity shocks can generate some fluctuations of asset liquidity endogenously. These are the shocks used in the search model. Productivity shocks have no effect on asset liquidity in the model without search frictions. The two panels in the middle of Figure 3.10 depict the same shocks, but in this case, exogenous
shocks to asset liquidity are the only shocks that can change asset liquidity. The bottom three panels show the responses of output, asset prices and hours worked to productivity and the matching efficiency shocks. It is clear that both total output and hours worked decrease with negative joint shocks to productivity and the matching efficiency. However, the asset price is counter-cyclical in all cases. That is, while the search model generates more realistic fluctuations in general, it still has a hard time accounting for the observed movements during the Great Recession.

Comparing the dynamic paths of asset prices from the models with and without search frictions, we can see that the search model always generates smaller counter-cyclical movements of asset prices than the non-search model. In the search model, a negative productivity shock reduces asset demand by decreasing the marginal product of capital, and thus reducing the asset price. At the same time, a lower marginal product of capital also reduces buyers’ incentives to trade in the asset market, and thus decreases their participation rate in the market. This lower participation rate reduces asset demand and, hence, further reduces the asset price. In contrast, the decrease on asset demand generated by a productivity shock acting through the buyer’s participation rate does not exist in the model without search frictions. Therefore, although my model does not solve the puzzle of counter-cyclical movements of asset prices in the current financial crisis, I believe it points in the right direction toward solving the puzzle.

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26 The first two rows are not identical because search frictions create a link between equity liquidity and productivity shocks. With search friction, productivity changes, equity liquidity also changes. While without search friction, all fluctuations of equity liquidity is from exogenous liquidity shocks.

27 The Taylor approximation has been used to generate the dynamic paths. This approximation is accurate only if the shocks are in the neighborhood of the steady state. Clearly, both shocks are far from the steady state during the crisis. However, to my knowledge, for dynamic estimations with sequential shocks, this is most likely the best approach.
3.4. IMPORTANCE OF SEARCH FRICTIONS

3.4.6 Robustness

Four alternative parameter sets are evaluated to test the robustness of my results. Figures C.1 - C.8 in appendix C provide visual representations. In Figures C.1 - C.4, two shocks, the productivity and matching efficiency shocks, are imposed simultaneously in the model. The imposed responses generated only by productivity shocks are illustrated in Figures C.5-C.8. In each case, I find the following: 1). Productivity shocks generate large effects on asset liquidity; 2). Asset liquidity always positively comoves with the asset price; 3). The asset price and hours worked decrease when productivity falls. These results are qualitatively consistent with the results from the basic model. In this sense, my results are robust.

In the first case, I vary the power $\gamma$ in the matching function. In the second case, I vary the constant $\phi_0$ in the matching function. In the third case, I vary the fraction of entrepreneur $\pi$. In the last case, I test the relative risk aversion $\rho$. The widely used values of $\rho$ are in the range $[1, 2]$. Thus, I choose $\rho = 1$ and $\rho = 1.5$ as two alternative cases.
3.5 Conclusion

I estimate a SVAR model to characterize the interactions between productivity, asset liquidity and the aggregate economy. I then specify and calibrate the baseline model from section 3.3. I find that the model does well in quantitatively matching some key properties of the U.S. data. First, productivity and liquidity shocks are equally important in explaining business cycles. Second, productivity shocks generate pro-cyclical movements of hours worked and also generate a positive co-movement between asset prices and liquidity, which is not the case for liquidity shocks. However, the price effect generated by productivity shocks is still not strong enough to solve the puzzle of counterfactual movements of asset prices. Third, the model generates significant positive correlations between productivity and output, between productivity and hours worked and between productivity and asset liquidity, which are in line with my empirical findings.

My model could be used to study a number of other issues associated with business cycles. In particular, one could allow for money in the model and assess the effects of various monetary policies, such as policies that can increase asset liquidity by allowing the government to buy illiquid assets from private firms. These policy interventions increase the amount of liquid assets circulating in the economy, stimulate firms’ investments, and boost asset prices. The higher the asset prices, the lower the incentives that private buyers will have to trade assets. Therefore, the effects of monetary policies may not be so optimistic because they are mitigated by a reduction in the number of private buyers.
Chapter 4

Asset Prices, Adjustment Costs and Government Bonds

4.1 Introduction

In the baseline model studied in Chapters 2 and 3, search frictions were introduced in the asset market. Asset liquidity is endogenously determined by the asset market tightness. When a negative productivity shock hits the economy, buyers’ asset market participation rates decline, which reduce sellers’ matching probabilities in the asset market. Thus, both asset liquidity and prices decline. This model is successful in generating pro-cyclical movements of asset prices with exogenous productivity shocks only.\footnote{If there are only liquidity shocks, asset prices are counter-cyclical, which is contradicted with the empirical observation.} However, asset prices are still pro-cyclical when exogenous productivity and liquidity shocks coexist, especially, during the financial crisis.

There are two possible ways to improve the baseline model’s performance. The first way is to introduce government bonds — a substitute for risky assets as proposed by Cui and Radde (2014). Government bonds are risk free. When equity liquidity
4.1. INTRODUCTION

decreases, buyers will shift their attention from risky assets to risk-free government bonds, often referred to as a flight-to-liquidity. This action further reduces the demand for risky assets. If the reduction of asset demand dominates the decline of asset supply, the model generates pro-cyclical movements of asset price. The other approach is to introduce frictions on the asset supply. For example the convex adjustment costs in the gross rate of change in investment (Gertler and Kiyotaki, 2015). When asset liquidity decreases, investment adjustment costs dampen the resulting reduction in asset supply, and thus reduce the increase in the asset price caused by supply shortage, ceteris paribus.

In this section, I compare two models with search frictions in the asset market. In the first model, government bonds are introduced as an investment substitute for risky assets. In the second model, costs are imposed on asset investment adjustments. The underlying framework is related to the baseline model introduced in Chapter 2 and that of Cui and Radde (2014). The key feature of the baseline framework is the inclusion of search frictions in the asset market, which makes asset liquidity endogenous. Cui and Radde (2014) introduced risk-free government bonds in a search model. This chapter further extends the model by introducing investment adjustment costs.

The main purpose of this chapter is to show how the performance of the model can be improved. After calibrating the models, I find the following results: First, the model with only government bonds generates pro-cyclical movements of asset prices. However, this version of the model overstates the volatilities of both output and hours worked. Moreover, the calibrated model generates too little volatility in asset prices.
Second, the model with both adjustment costs and government bonds generates procyclical movements in asset prices. In addition, the adjustment costs significantly enlarge the volatility of asset prices because the adjustment costs prevent asset prices from the immediately adjusting to the steady state level, and increase the persistence of asset prices.

The two features introduced here have previously been used in other contexts. In the context of risk-free assets, KM and Shi (2015) study short-term dynamics driven by productivity and liquidity shocks in a model in which entrepreneur’s investment capability is restricted by holdings of liquid assets. Although, the models include risk-free assets as a substitute for illiquid assets, adjustment of investment portfolio has no effect on asset liquidity because the models assume that asset liquidity is exogenous. Cui and Radde (2014) propose extensions of the models to an environment with search frictions in the asset market. Search frictions make asset liquidity endogenous. When a negative productivity shock hits the economy, asset returns decline. This decreases the asset market participation rate, and thus reduces asset liquidity and price. Risk-free assets are investment substitutes for illiquid assets. During the economic downturn, investors adjust their investment portfolio by increasing holdings of risk-free assets and decreasing investment on illiquid assets, which in turn, further reduces asset liquidity and prices. However, the model has not been used to study business cycle co-movements, such as those amongst output, hours worked and asset prices.

The adjustment costs have been used in many business cycle models, such as Cooper and Haltiwanger (2006), Eisfeldt and Rampini (2006), and Gertler and Kiyotaki (2015), etc. Their models have significant success in describing the nature of
4.1. INTRODUCTION

the capital reallocation. However, capital liquidity is exogenously specified in their models.

This chapter also belongs to the recent strand of literature in introducing financial frictions and shocks to explain business cycles. Nezafat and Slavik (2015) shown that financial shocks are important in explaining not only business cycle fluctuations but also the high volatility of asset prices observed in the data. However, their model fails to generate pro-cyclical movements of asset prices. Ajello (2016) estimated that a financial intermediation technology shocks in a model with liquidity constraints and wage rigidities accounts for 25 percent of GDP and 30 percent of investment volatility. However, he did not consider the interactions between productivity and financial shocks, which are the key elements that set my work apart. Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) worked with a liquidity shock modeled as in KM. Again, they did not consider the impact of productivity on asset liquidity. In addition, Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) focus was on the period of the recent financial turmoil. I studied the past 35 years of U.S. data using SVAR.

The rest of the chapter organized as follows. Section 4.2 presents the basic model assumptions and structures. Sections 4.4 and 4.3 discuss characteristics of the models with government bonds, and with both adjustment costs and government bonds. Equilibrium is defined in section 4.5. Calibration and simulation are discussed in section 4.6. In section 4.7, I discuss and compare dynamic results across models with the different assumptions. Section 4.8 concludes by summarizing the results.

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2The model with adjustment cost only still cannot generate the pro-cyclical movement of asset price. And the model results are similar to the baseline model. So I did not discuss it separately.
4.2 Model Environment

The model structure is similar to the baseline models developed in Chapters 2 and 3. Time is discrete and continues forever. A household consists of two types of members: entrepreneurs and workers. With probability $\pi \in (0, 1)$ a member is an entrepreneur, and with probability $(1 - \pi)$ the member is a worker. Entrepreneurs sell equities and invest in new projects. Workers provide labor, buy consumption goods, and also buy equities if they participate and get matched in the equity market. Any unsold equity accrues to households at the end of the period. All members, who belong to the same household, share consumption and disutility from labor supply.

At date $t$, a household’s preferences are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) - h((1 - \pi)l_t^w) - d_t(1 - \pi)c_f\}$$

(4.1)

where $c_t$ and $l_t$ denote, respectively, the total consumption and labor supply of a household. The functions $u(c)$ and $h(l)$ are same as these functions in chapter 2. The last term, $e_t(1 - \pi)c_f$, is the total participation cost to the asset market, where $(1 - \pi)$ is the proportion of members who are workers and $d_t$ is the fraction of workers that participate in the asset market, and $c_f$ is the fixed participation cost per worker in terms of utility.

A large number of firms operated in a competitive market. Labor and capital are used by firms to produce consumption goods. A firm’s production function in period $t$ is given by

$$y_t = A_t F(k_t^d, l_t^d),$$

(4.2)

where $y_t$ denotes consumption goods produced in period $t$, $A_t$ is the productivity
level in period $t$, which follows a Markov process and $k_d^t$ and $l_d^t$ are firm’s demands for capital and labor in period $t$.

Search frictions exist in the equity market only. Search is random. The interactions between sellers and buyers in the market involve anonymous bilateral matching. The matching function, $M(B_t, S_t)$, is exactly same as the one discussed in Chapter 2. Let $\phi^s(m_t)$ and $\phi^b(m_t)$ denote, respectively, the seller’s and the buyer’s matching probabilities in the equity market, where $m_t = B_t / S_t$ is the tightness in the equity market. Sellers make a take-it-or-leave-it offer $(x_t, q_t)$ to matched buyers. The value of $x_t$ is the amount of consumption goods which the seller requires for one unit of equity and $q_t$ is the amount of equity which the seller offers the trading partner. Buyers take the offer $(X_t, Q_t)$ as given and decide whether to accept it.

Entrepreneurs make new investments using consumption goods. One unit of investment costs one unit of consumption goods. Entrepreneurs raise funds to make new investments. Entrepreneurs’ ability to raise funds is subject to two restrictions which are exactly same as those discussed in Chapter 2. The first constraint is the collateral constraint, which states that only a fraction $\theta \in (0, 1)$ of total new investments can be financed by issuing new equity. The second constraint is the liquidity constraint, which states that only the sellers that are matched in the equity market can trade equities. Entrepreneurs can sell their asset endowment and new equity to brokers at a discounted price. The discounted price depends on matching probabilities in the equity market. A broker offers a higher price to entrepreneurs only if he/she has a higher probability to resell the equities.

An entrepreneur’s equity holding in period $t + 1$, $s_{t+1}^e$, is expressed as

$$s_{t+1}^e = i_t + \sigma s_t - q_t + (1 - \phi^s)q_t \geq (1 - \theta)i_t + (1 - \phi^s)q_t,$$  \hspace{1cm} (4.3)
which equals the sum of new investment, \( i_t \), and old capital after depreciation, \( \sigma s_t \), minus equities traded in the market, and plus unsold equities.

The inequality in (4.3) is the liquidity constraint, which states that an entrepreneur has to hold at least a fraction \( (1 - \theta) \) of new investments as collateral, plus \( (1 - \phi^*) q_t \) of unsold equities. To make the liquidity constraint effective, I suppose that funds cannot be reallocated between household members within a period until all markets close. Otherwise, workers would shift more assets to matched entrepreneurs until (4.3) does not bind.

Members realizing their type as entrepreneurs make new investments. There are the convex adjustment costs in the gross rate of change in investment. The total resources of entrepreneurs are divided amongst new investment, \( i_t \), consumption, \( c^e_t \), investment adjustment costs, government bonds, \( b^e_t \) and lump-sum taxes, \( \tau \). Government bonds are risk free and can be a substitute for the risky equity as a store of value. Households can adjust their investment portfolio to reduce risk during an economic downturn. Same as equity, bonds are evenly distributed to all members belonging to the same household. Let \( b_t \) denote the bond holdings of a representative member at the beginning of the period \( t \). Government bonds are traded in a Walrasian market. The real bond price in period \( t \) is \( p_t \). In the following period, the bond issuer pays one unit of the consumption good to the bond buyers to buy back each unit of bonds. The budget constraint for entrepreneurs is

\[
ri_t s_t + x^d_t q_t + b_t = i^e_t + c^e_t + p_t b^e_{t+1} + f\left(\frac{i^e_t}{i^e_{t-1}}\right)i^e_t + \tau, \quad (4.4)
\]

where \( f\left(\frac{i^e_t}{i^e_{t-1}}\right)i^e_t \) reflects the adjustment costs on new investment, with \( f(1) = f'(1) = 0 \) and \( f''(1) > 0 \). The value \( \tau \) is the lump-sum tax charged by the government. The
4.2. MODEL ENVIRONMENT

The functional form of the adjustment costs captures the idea that adjustment is likely to be more costly when the change in investment is large. Entrepreneurs raise funds from renting capital \( r_t s_t \) and selling equities \( x_t^d q_t \), where \( x_t^d \) is the discounted trading price that brokers offer to entrepreneurs in the equity market.

Workers provide labor \( l_t^w \), capital \( s_t \) and bond income \( b_t \) in exchange for consumption \( c_t^w \) and new bonds \( b_t^{w+1} \). If a worker is matched in the equity market, he/she pays \( X_t \) for \( Q_t \) units of equity. It follows that the budget constraint of a worker satisfies

\[
r_t s_t + w_t l_t^w - \frac{\pi \phi_t}{1 - \pi} Q_t X_t + b_t^w = c_t^w + p_t b_t^{w+1} - \tau,
\]

where \( w_t \) is wage income in period \( t \). The equity price \( X \) that a seller posts to the matched worker must offer a non-negative surplus to the worker, which implies

\[
X_t \leq \frac{\Omega_t}{u'(c_t)},
\]

where \( \Omega_t \) is the expected marginal value of capital in the next period, discounted to the current period. The denominator of (4.6) is the forgone utility of consumption in the trade.

A household’s bond holdings at the end of period \( t \) are given by

\[
b_{t+1} = \pi b_{t+1}^c + (1 - \pi) b_{t+1}^w.
\]

The government runs a balanced budget in each period. It issues new debt \( B_t \), and collects lump-sum taxes, \( \tau \), to finance public expenditures, \( G_t \), and payments on
matured debt, $B_{t-1}$:

$$G_t + B_{t-1} = \tau + p_t B_t. \quad (4.8)$$

Let $v(s_t; k_t, A_t)$ be the value function for households. After suppressing the time subscript $t$ and using $+1$ to denote variables in $t+1$, the value function in period $t$ satisfies the Bellman equation

$$v(s; K, A) = \max u(c) - h(l) - d(1 - \pi)c^f + \beta Ev(s_{+1}; K_{+1}, A_{+1}). \quad (4.9)$$

A household’s capital at the beginning of period $t+1$ equals the summation of entrepreneurs’ and workers’ capital at the end of period $t$.

$$s_{+1} = \pi s_{t+1}^e + (1-\pi)s_{t+1}^w = \pi(\sigma s + \phi q) + (1-\pi)(\sigma s + \frac{\pi \phi}{1 - \pi} Q) = \sigma s + \pi \phi q + \pi \phi Q. \quad (4.10)$$

### 4.3 The Model with Government Bonds Only

In the section, government bonds are kept in the model, but, to emphasize their effect and make comparison more clear, I remove the adjustment costs. The budget constraint for entrepreneurs and workers, respectively, are simplified to

$$rs + x^d q + b = c^e + pb_{+1}^e + \nu + \tau \quad (4.11)$$

$$rs + w l^w - \frac{\pi \phi}{1 - \pi} Q X + b = c^w + pb_{+1}^w + \tau. \quad (4.12)$$

Using the resource constraint (4.3) to replace $q$ and the discounted price function
(2.14) to replace \( x^d \) in (4.11) yields

\[
(r + \phi^s X \sigma)s + b - \tau \geq c^e + (1 - \phi^s X \theta)i^e + pb_{t+1}^e. \tag{4.13}
\]

Multiplying (4.11) by \( \pi \), (4.12) by \( 1 - \pi \) and adding them up yields

\[
rs + b + (1 - \pi)\omega l^w - pb_{t+1} - \pi i - \tau + (\pi x^d q - \pi \phi^s QX) = c, \tag{4.14}
\]

where \( b \) and \( c \) are defined as

\[
b = \pi b^e + (1 - \pi)b^w \tag{4.15}
\]

\[
c = \pi c^e + (1 - \pi)c^w. \tag{4.16}
\]

Matched workers hold \( \sigma s \) units of assets from the last period and receive \( Q \) units of new equities from trade, while unmatched workers and workers not entering the asset market only hold \( \sigma s \) units of assets from the last period. Let \( s^w_{t+1} \) represent workers’ asset holdings at the end of the period \( t \). Then \( s^w_{t+1} \) is given by:

\[
s^w_{t+1} = \frac{\pi \phi^s}{1 - \pi} (Q + \sigma s) + (1 - \frac{\pi \phi^s}{1 - \pi}) \sigma s \tag{4.17}
\]

Using (4.3) and (4.17) to replace \( q \) and \( Q \) in (4.14) yield

\[
(r + X \sigma)s - X s' + (b - pb_{t+1}) + (1 - \pi)\omega l + (X - 1)\pi i - \tau = c. \tag{4.18}
\]

Let the Lagrange multipliers of (4.13) and (4.18) be \( \lambda^{mgb,\pi u'}(c) \) and \( \lambda^{e9b} \), respectively, the optimal choices must satisfy the following first-order conditions...
4.3. THE MODEL WITH GOVERNMENT BONDS ONLY

\[ \beta e v'_{s+1} \leq \lambda^{egb} X, \text{ and } s_{+1} \geq 0 \]  
\[ \pi u'(c) - \lambda^{mgb} \pi u'(c) \leq \lambda X, \text{ and } c^e \geq 0 \]  
\[ (1 - \pi)u'(c) \leq \chi(1 - \pi), \text{ and } c^w \geq 0 \]  
\[ \omega \lambda^{egb} = h'((1 - \pi)l^w) \]  
\[ \lambda^{egb}(X - 1) - \lambda^{mgb} u'(c)(1 - \phi^s X \theta) \leq 0 \]  
\[ \lambda^{mgb} u'(c)X(\sigma s + \theta i)\frac{\partial \phi^s}{\partial c} - (1 - \pi)c^f \begin{cases} > 0 & e = 1 \\ = 0 & 0 < e < 1 \\ < 0 & e = 0 \end{cases} \]  
\[ \beta E v'_{b+1} \leq \lambda^{egb} p + \lambda^{mgb} u'(c)p, \text{ and } b_{+1}^e \geq 0 \]  
\[ \beta E v'_{b+1} \leq \lambda^{egb} p, \text{ and } b_{+1}^w \geq 0, \]  

and the envelope conditions

\[ v_s = \lambda^{egb}(r + X\sigma) + \lambda^{mgb} \pi u'(c)(r + \phi^s X\sigma) \]  
\[ v_b = \lambda^{egb} + \lambda^{mgb} \pi u'(c). \]

**Lemma 3.** As long as the asset market stays active, the liquidity constraint (4.3) binds and entrepreneur’s spending on consumption \(c^e\) and bonds \(b_{+1}^e\) are always zero in the model with government bonds only.

Members belonging to the same family share consumption goods, equities and bonds at the end of each period. Workers participate in the equity market only if asset holdings can benefit their households more than consumptions and bonds can
do. Entrepreneurs have investment opportunities and can get new assets at a lower cost than workers. Therefore, entrepreneurs would never buy consumption or bonds, but only make new investments. See Appendix D.1 for a detailed proof.

The optimal asset price conditions in a trade and the envelope conditions on asset holdings together give rise to the asset pricing equations below

\[ X = \beta E\left\{ \frac{\nu'(c+1)}{u'(c)}[r_{t+1} + X_{t+1}\sigma + \lambda^m_{t+1}\pi(r_{t+1} + \phi_{t+1}X_{t+1}\sigma)]\right\} \quad (4.29) \]

\[ p = \beta E\left\{ \frac{\nu'(c+1)}{u'(c)}[1 + \lambda^m_{t+1}\pi]\right\} \quad (4.30) \]

### 4.4 The Model with Both Adjustment Costs and Government Bonds

In this version of the model, both adjustment costs and government bonds are retained. Investment adjustment costs mitigate declines in asset supply during economic downturns, while government bonds give investors a new channel to adjust their investment portfolio in order to lower risk. These two features working together help the model to generate pro-cyclical movements of asset prices, and magnify the fluctuations in asset prices.

Using the resource constraint (4.3) to replace \( q \) in (4.4) yields

\[ (r + \phi^sX\sigma)s + b - \tau \geq c^e + [1 - \phi^sX\theta + f\left(\frac{\epsilon^e}{\epsilon^{-1}}\right)]\epsilon^e + pb^e + 1 - \pi[1 + f\left(\frac{\epsilon^e}{\epsilon^{-1}}\right)]\epsilon^e - \tau + (\pi x^d q - \pi \phi^s Q X) = c, \quad (4.31) \]

Multiplying (4.4) by \( \pi \), (4.12) by \( 1 - \pi \) and adding them up yields

\[ rs + b + (1 - \pi)\omega l^w - pb_{t+1} - \pi[1 + f\left(\frac{\epsilon^e}{\epsilon^{-1}}\right)]\epsilon^e - \tau + (\pi x^d q - \pi \phi^s Q X) = c, \quad (4.32) \]

where \( b \) and \( c \) are defined in (4.15) and (4.16), respectively.
4.4. THE MODEL WITH BOTH ADJUSTMENT COSTS AND GOVERNMENT BONDS

Using (4.3) and (4.17) to replace \( q \) and \( Q \) in (4.32) yields

\[
(r + X\sigma)s - Xs' + (b - pb_{+1}) + (1 - \pi)\omega l + [X - 1 - f\left(\frac{i}{i_{-1}}\right)]\pi i - \tau = c. \quad (4.33)
\]

Letting the lagrange multipliers of (4.31) and (4.33) be \( \lambda_{mbh}\pi u'(c) \) and \( \lambda_{ebh} \), respectively, the optimal choices must satisfy the following first order conditions

\[
\beta Ev_{s+1}' \leq \lambda_{ebh}X, \text{ and } s' \geq 0 \quad (4.34)
\]

\[
\pi u'(c) - \lambda_{mbh}\pi u'(c) \leq \lambda_{ebh}, \text{ and } e^e \geq 0 \quad (4.35)
\]

\[
(1 - \pi)u'(c) \leq \lambda_{ebh}(1 - \pi), \text{ and } e^w \geq 0 \quad (4.36)
\]

\[
\omega \lambda_{ebh} = h'((1 - \pi)t^w) \quad (4.37)
\]

\[
\lambda_{ebh}[X - 1 - f\left(\frac{i}{i_{-1}}\right) - f'(\frac{i}{i_{-1}})i] - \lambda_{mbh}u'(c)[1 - \phi^s X\theta]
\]

\[
+f\left(\frac{i}{i_{-1}}\right) + f'(\frac{i}{i_{-1}})i \leq 0 \quad (4.38)
\]

\[
\lambda_{mbh}\pi u'(c)X(\sigma s + \theta i)\frac{\partial\phi^s}{\partial e} - (1 - \pi)c^f = 0, \quad 0 < e < 1
\]

\[
< 0, \quad e = 0 \quad (4.39)
\]

\[
\pi\beta Ev_{b+1}' \leq \lambda_{ebh}p + \lambda_{mbh}\pi u'(c)p, \text{ and } b_{b+1} \geq 0 \quad (4.40)
\]

\[
(1 - \pi)\beta Ev_{b+1}' \leq \lambda_{ebh}(1 - \pi)p, \text{ and } b_{w+1} \geq 0. \quad (4.41)
\]

Investment adjustment costs have no effect on envelope conditions, so conditions (4.27) and (4.28) still hold. Asset prices \( X \) and \( p \) satisfy (4.29) and (4.30).

**Lemma 4.** As long as the asset market stays active, the liquidity constraint (4.3)
binds and entrepreneur’s spending on consumption $c^e$ and bonds $b_{+1}^e$ are always zero in the model with both adjustment costs and government bonds.

The proof is essentially the same as the proof of Lemma 3. The equity market provides opportunities for households to redistribute funds among their members with costs. Households decide to pay the entry costs and send workers to participate in the equity market only if benefits from investments are higher than benefits from consumption and holding government bonds. Thus, entrepreneurs, as the only type of household member that has investment opportunities, will spend all funds on new investments. See Appendix D.2 for detailed proof.

### 4.5 Definition of Symmetric Search Equilibrium

Throughout this paper, I focus on a symmetric search equilibrium where households make identical decisions.

**Definition.** A symmetric search equilibrium\(^3\) consists of price functions $(x, X, r, \omega, p)$, a household’s policy functions $(i, q, Q, l, c^w, c^e, e, s_{+1}, b_{+1}^c, b_{+1}^w)$, the demand factors of consumption goods producers, $(k^d, l^d)$, and the law of motion of the aggregate capital stock, such that the following requirements are satisfied:

1. Given price functions and the aggregate state $(K, A)$, a household’s value and policy functions are solved from the household’s optimization problem.

2. Optimal conditions of firms, $r = AF_1'(k^d, l^d)$, and $\omega = AF_2'(k^d, l^d)$ hold.

\(^3\)The equilibrium is defined based on the model with both adjustment costs and government bonds. In the model with government bonds, the only thing that needs to be removed is the adjustment costs $[\ln(\frac{1}{1-i})]^2 \pi i$ in the goods market clearing condition (4.42). The rest of the definition is identical.
3. Markets clear:

- **goods:** \( c + \pi t^e + \left[ \ln \left( \frac{t^e}{\varepsilon} \right) \right]^2 \pi t^e + G = AF(k^d, l^d), \) (4.42)
- **labor:** \( l^d = (1 - \pi) l^w = l, \) (4.43)
- **capital:** \( k^d = K = s, \) (4.44)
- **equity:** \( s_{+1} = \sigma s + \pi i^e, \) (4.45)
- **bonds:** \( B_{+1} = (1 - \pi) b_{w+1} + \pi b_{e+1}. \) (4.46)

4. Government’s budget:

\[ G + (1 - p) B = \tau. \] (4.47)

5. Aggregate capital \( K_{+1} \) satisfies the law of motion:

\[ K_{+1} = \sigma K + \pi i. \] (4.48)

6. In the frictional equity market, the trading price satisfies:

\[ X = \frac{\beta E v_{s+1}}{u'(c)}. \]

The trading offer posted by an seller has to be consistent with the offer accepted by the matched worker.

\[ x = X, \quad \text{and} \quad q = Q. \] (4.49)

7. Symmetry condition: All households are identical. They make symmetric decisions in each period.
4.6 Calibration

I calibrate the model and study the nature of the fluctuations in asset prices, liquidity, investments and output resulting from productivity and liquidity shocks. The following functional forms are used:

\[
\begin{align*}
    u(c_t) &= \frac{(c_t)^{1-\rho} - 1}{1 - \rho} \\
    h(l_t) &= h_0 l_t^\eta, \quad \eta > 1 \\
    F(K_t, L_t) &= K_t^\alpha L_t^{1-\alpha} \\
    M(B_t, S_t) &= \phi_0 B_t^\gamma S_t^{1-\gamma} (1 - \frac{1}{1 + e^{\mu}}) \\
    \phi^*_t &= \phi_0 \left( \frac{b_t (1 - \pi)}{\pi} \right)^{(1-\gamma)}. \\
    f \left( \frac{\tilde{i}_t^e}{\tilde{i}_{t-1}^e} \right) &= \left[ \ln \left( \frac{\tilde{i}_t^e}{\tilde{i}_{t-1}^e} \right) \right]^2
\end{align*}
\]

The matching function is Cobb-Douglas. The function of the adjustment costs satisfies \( f(1) = f'(1) = 0 \) and \( f''(1) > 0 \). I assume that quarterly TFP and liquidity processes\(^4\) are same as those defined in Chapter 3.

The model is calibrated to a quarterly frequency. The values of the discount factor, \( \beta \), the relative risk aversion, \( \rho \) and the capital share, \( \alpha \) are exogenously chosen and are equal to 0.092, 2 and 0.36, respectively. The elasticity of labor supply equals 2, which implies that the curvature of the disutility of labor is \( \eta = 1.5 \). The survival rate, \( \sigma \), is set so that the annual investment to capital ratio, \( 4(1 - \sigma) \), is 0.076. Given that the annual capital to output ratio is 3.26, I can solve for the steady state capital rental rate, \( r \), the wage rate, \( w \), and the capital-labor ratio, \( \frac{K}{L} \). The total hours

\(^4\)Liquidity shocks include all non-productivity shocks that can generate fluctuations of asset liquidity.
work in the steady state are 0.25. As set in Shi (2015), government spending $g$ is 18% of the steady state level of output. The value of $h_0$ is solved from the FOC

$$h_0 = \frac{\rho \omega}{\eta \ln -1}.$$

A broker’s matching rate, $\phi^s$, is determined from the capital stock’s law of motion. The estimated range of the fraction of firms that adjust their capital in a quarter is between 0.2 and 0.4. In this paper, I set the value equal to 0.24 as in Shi (2015). Hence, the quarterly value, $\pi$, equals 0.06. The participation rate in the asset market is approximately 50%.

The variance and the persistence of TFP process are $0.37 \times 10^{-4}$ and 0.96, respectively. The hardest part is to estimate the matching function and the distribution of liquidity shocks, because there are no direct observations. I apply the same method to jointly calibrate the power of the matching function, $\gamma$, the constant, $\phi_0$, the variance and the persistence of liquidity shocks, $(\text{var}(\varepsilon_\mu), \delta_\mu)$, and the covariance between TFP and liquidity shocks, $\text{cov}(\varepsilon_A, \varepsilon_\mu)$ to match five targets: the broker’s matching rate, $\phi^s$; the relative volatility of asset liquidity, $\sigma_{\phi^s}/\sigma_y$; the correlation between TFP and asset liquidity, $\text{corr}(A, \phi^s)$; the correlation between TFP and hours worked, $\text{corr}(A, l)$; and the correlation between TFP and output, $\text{corr}(A, Y)$.

Calibrated parameters are listed in Tables D.1 and D.2. The parameters from the model with only government bonds are similar to those of the model with both adjustment costs and government bonds because the adjustment costs are 0 in the

$^5$Few targets are adjusted in order to introduce government bonds to models. The annual investment to capital ratio increases to 0.076, and the total hours worked adjusted to 0.25. Those targets are identical to values in Shi (2015). The annual capital to output rate is about 3. The value of 3.26 is used in this paper.

$^6$These values are from Doms and Dunne (1998) and Cooper, Haltiwanger and Power (1999).
steady state. The distributions of liquidity shocks are calibrated based on dynamic results. Therefore, estimated parameters associated with the distributions of liquidity shocks are different.

The calibrated entry cost \( c_f \) from the model with both adjustment costs and government bonds is higher than the result from the model with government bonds. Thus, the fluctuation of worker’s entry rate to shocks in the model with both adjustment costs and government bonds is lower than in the other model. The buyer’s power in the matching function \( \gamma \) are about twice as large in the model with both adjustment costs and government bonds as in the other model. Thus, the impacts of worker’s entry costs on the matching probability are much smaller in the model with both adjustment costs and government bonds. Since most of these calibrated parameters are similar to those of the baseline model, So I do not spend much of time to discuss those two tables.

4.7 Dynamic Results

4.7.1 Model with Government Bonds

Asset price fluctuations

Asset prices are pro-cyclical in U.S. data. In the baseline model introduced in chapter 2, endogenous asset liquidity is introduced. When a negative productivity shock hits the economy, capital productivity and the buyers’ participation in the asset market declines. Hence, both asset liquidity and price decrease. However, productivity shocks are not strong enough to lead pro-cyclical movements of the asset price when productivity shocks and exogenous liquidity shocks coexist, especially during the financial crisis.
Introducing government bonds is a potential option to solve the asset price puzzle. Government bonds are substitutes for risky assets. When asset liquidity decreases, buyers can adjust their portfolios by holding more risk free government bonds, which is often called “flight-to-liquidity”. This action reduces the demand for risky assets. Figure 4.1 illustrates the impulse responses of total output and the asset price following joint shocks, namely simultaneously shocks to productivity and liquidity shocks. As explained above, government bonds provide an investment opportunity to households. Households choose to hold bonds in order to avoid risk during an economic downturn. This demand shift from risky asset to risk free bonds further reduces the asset demand, which drives down the asset price. The last panel shows that the asset price falls, which implies that the introduction of government bonds does significantly reduce the demand for the risky assets. Thus government bonds plays a visible role in explaining pro-cyclical movements of asset prices. This result is consistent with Cui and Raddle (2014).

Figure 4.1: Impulse Responses to Productivity and Liquidity Shocks in the Model with Government Bonds
4.7. DYNAMIC RESULTS

Relative volatilities

Table 4.1 reports the relative volatilities among some key business cycle variables. The first column reports the results from my empirical analysis in chapter 3. The second and the third columns list the results from the baseline model and the model with government bonds, respectively. In the baseline model, the relative volatilities of capital are 0.1, which are much lower than the data results. The baseline model also underestimates the volatility of productivity relative to labour, and overestimates the volatility of labor relative to output.

After introducing government bonds, the calibrated model generates volatilities of productivity relative to output, capital relative to output and productivity relative to labor which are all much lower than what is observed in the data. Some of the relative volatilities are even lower than the counterparts generated by the baseline model. The government bonds model also result in an increase in the volatility of labor relative to that of output that is about 1.26 times that observed in the data. Therefore, even though the model with government bonds can generate pro-cyclical movements of equity prices, the model does not capture the relative volatilities of productivity and capital.

Another notable feature of business cycles is the large fluctuation in the equity price. The last panel of Figure 4.1 depicts the implied impulse response of the equity price for joint shocks. In response to a 0.6% productivity shock and simultaneous 1.5% liquidity shock, the equity price exhibits a pro-cyclical decline. However, the maximum deviation of the equity price fluctuation is only about 0.02%, which is much

\[7\] In this chapter, I did not report volatilities of all variables, because this current model only focuses on frictions on the equity market. The one of the future works of this chapter will be including one friction markets and studying more variables.
4.7. DYNAMIC RESULTS

Table 4.1: Volatilities

<table>
<thead>
<tr>
<th>Volatilities$^{ab}$</th>
<th>US data</th>
<th>Basic Model</th>
<th>With GB</th>
<th>With Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_A/\sigma_Y$</td>
<td>0.6</td>
<td>0.4(0.01)</td>
<td>0.3(0.03)</td>
<td>0.3(0.11)</td>
</tr>
<tr>
<td>$\sigma_L/\sigma_Y$</td>
<td>1.1</td>
<td>1.4(0.01)</td>
<td>1.4(0.02)</td>
<td>1.4(0.17)</td>
</tr>
<tr>
<td>$\sigma_K/\sigma_Y$</td>
<td>0.6</td>
<td>0.1(0.05)</td>
<td>0.1(0.01)</td>
<td>0.2(0.03)</td>
</tr>
<tr>
<td>$\sigma_A/\sigma_L$</td>
<td>0.6</td>
<td>0.3(0.07)</td>
<td>0.2(0.02)</td>
<td>0.2(0.07)</td>
</tr>
<tr>
<td>$\sigma_K/\sigma_L$</td>
<td>0.5</td>
<td>0.1(0.17)</td>
<td>0.1(0.01)</td>
<td>0.2(0.02)</td>
</tr>
</tbody>
</table>

$^a$Numbers in parentheses are standard deviations over 1000 simulations

$^b$I simulate the model 1000 times. In each simulation, I randomly select 600 draws from the distributions of shocks. The artificial time series are detrended using an HP-filter with $\lambda = 1600$.

less than the productivity shock. Table 4.2 contains the equity price volatilities. In the table, the first column reports the values from my empirical analysis. The second column reports the results for the government bonds model. The equity price, that is observed in the data, is 12 times more volatile than output and labor, and more than 20 times more volatile than capital. In contrast, the volatility generated by the model with government bonds is much smaller.

Table 4.2: Equity Price Volatilities

<table>
<thead>
<tr>
<th>Volatilities</th>
<th>US data</th>
<th>With GB</th>
<th>With BOTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_X/\sigma_Y$</td>
<td>12.5</td>
<td>0.02(0.03)</td>
<td>0.10(0.10)</td>
</tr>
<tr>
<td>$\sigma_X/\sigma_L$</td>
<td>11.3</td>
<td>0.01(0.03)</td>
<td>0.07(0.10)</td>
</tr>
<tr>
<td>$\sigma_X/\sigma_K$</td>
<td>21.1</td>
<td>0.13(0.02)</td>
<td>0.40(0.04)</td>
</tr>
</tbody>
</table>

In summary, a riskless asset in the form of government bonds seem to be helpful in generating pro-cyclical movements in equity prices. However, the model with government bonds overstates somewhat both the relative volatilities of output and labor. Moreover, the model generates too little volatility in the equity price.
4.7. Model with both Adjustment Costs and Government Bonds

Asset price fluctuations

In this section, both adjustment costs and government bonds are included in the model. The adjustment costs mitigate the reduction in asset supply but the effect is not strong enough to make asset prices pro-cyclical. Government bonds, on the other hand, magnify the reduction in asset demand, making asset prices pro-cyclical, but the resulting volatility of asset prices is much lower than what is observed in the data. Combining the adjustment costs with government bonds is therefore more likely to generate pro-cyclical movements of asset prices with relatively large fluctuations.

Figure 4.2 illustrates a comparison of the effects of joint productivity and liquidity shocks on equity prices from different models. The solid blue line displays the dynamic path of the equity price from the model with both adjustment costs and government bonds. The green dashed line represents the impulse response of the equity price from the model with government bonds only. Both lines clearly move up and down together, but the dynamic path from the model with both frictions exhibits triple the amplitude of the dynamic path from the model with government bonds only.

Relative volatilities

The implied impulse response functions following joint shocks are depicted in Figure 4.3. The two panels in the top row illustrate the two exogenous shocks: productivity and liquidity shocks. The two panels in the middle row show the impulse responses of total output and labor input, while the two panels in the last row depict dynamic responses of asset liquidity and prices.

In Shi (2015), a negative liquidity shock reduces equity supply without changing
4.7. DYNAMIC RESULTS

Figure 4.2: Impulse Responses of Equity Price

Figure 4.3: Impulse Responses to Productivity and liquidity Shocks in the Model with Both Adjustment Costs and Government Bonds

equity demand, which in turn leads to an counterfactual increase in the equity price. This model incorporates three features to resolve the equity price puzzle. A negative productivity shock combined with search frictions in the equity market makes the entry cost of the equity market relatively more expensive. Fewer buyers enter
the equity market, which lowers the equity market tightness, and thus reduces both equity liquidity and price. At the same time, government bonds as a substitute for equities provide a safer investment opportunity. During an economic downturn, more workers choose to hold government bonds instead of the risky equity. The equity demand decreases, which further reduces the equity liquidity and price. Moreover, the adjustment costs make changes in investment more sluggish. Entrepreneurs use initial equity holdings and funds obtained from brokers to finance new investments. In order to minimize the adjustment costs, entrepreneurs restrict the adjustments of new equity issuing, which lowers the reduction of equity supply generated by a negative liquidity shock. This lowers the increase of the equity price following a negative liquidity shock. In addition, the adjustment costs also prevent the equity price from immediately reverting to the steady state, and thereby increase its persistence in the short-run.

The relative volatilities from the model with both adjustment costs and government bonds are reported in the last column of Table 4.1. The calibrated model results are similar to the results from the model with government bonds only. The relative volatilities of productivity to output, capital to output, productivity to labor and capital to labor are much lower than those are observed in the data. Although these relative volatilities are still much lower than in the data, the model with both government bonds and adjustment costs is able to increase the volatility of capital relative to output and to hours worked. The model also overestimates the volatility of labor relative to output.

The calibrated model results related to asset prices with both adjustment costs and government bonds are listed in the last column of Table 4.2. Clearly, incorporating
4.8 Conclusion

I have analyzed two different model structures to study the movement of asset prices. In the first model, I introduced government bonds in the model with search frictions in the asset market. Government bonds are risk-free substitutes for risky assets. When asset liquidity decreases, buyers adjust their investment portfolios by holding more government bonds. These actions further reduce the asset demand during an economic downturn, and thus improve the model performs on the asset price dynamics. In the second model, I incorporated both government bonds and adjustment costs, the adjustment costs inhibit the reduction of asset supply. The adjustment costs make changes in investment sluggish. Investors decrease the reduction of asset supply during an economic downturn in order to minimize these costs, and thus reduce the increase in the asset price due to the decline of asset supply. At the same time, government bonds increase the reduction in asset demand. Thus, the model with the adjustment costs improves the calibrated volatility of the equity price. However, the model still understates its volatility. As illustrated in figure 4.2, the amplitude of the dynamic fluctuations in equity prices from the model with both frictions is three times as large as the fluctuation from the model with government bonds only. However, the calibrated relative volatilities of equity price from the model with both features are still much lower than in the data.

In summary, it is clear that the version of the model with government bonds and adjustment costs can generate pro-cyclical movements of equity prices. Moreover, the adjustment costs significantly increase the volatility of equity prices. However, the model calibration still understates the volatility of equity prices.
both adjustment costs and government bonds can generate pro-cyclical movements of asset prices.

I calibrated these models and found the following results: (1). The model with government bonds generates pro-cyclical movements in the asset price. However the model overstates both the volatilities of output and labor. Moreover, the calibrated model, like almost all models in the literature, generates too little volatility in asset prices. (2). The model with both government bonds and adjustment costs generates pro-cyclical movements in asset prices. Moreover, the adjustment costs significantly enlarge the volatility of asset prices. While a substantial improvement over the existing literature,\(^8\) the calibrated model still understates the volatility of asset prices.

These models could be used to study a number of other issues associated with business cycles. In particular, one could allow search frictions in the labor market to reduce volatilities of output and labor. When a negative productivity shock hits the economy, the wage rate decreases. Workers provide more labor in the market in order to smooth consumptions and investments, which in turn reduces the firm-worker ratio and the labor market tightness. Firms find it is easier to hire workers and keep output production smoothly. Thus, search frictions in the labor market may help to lower volatilities of output and labor calibrated from the model with both adjustment costs and government bonds.

Alternatively, one could use money to substitute government bonds and assess the effect of various monetary policies. Thus, the model can also be used for policy study by central banks and governments.

\(^8\)Such as KM (2008), Jermann and Quadrini (2012) and Shi (2015).
Chapter 5

Concluding Remarks

In this dissertation, I develop several DSGE models with search frictions in the asset market to allow for interaction between the real sector and the financial sector. Most of the previous literature on business cycles and financial shocks does not explicitly consider simultaneous shocks originated from the real sector and the financial sector. In particular, it fails to distinguish fluctuations in financial markets caused by real shocks from those caused by exogenous financial shocks. As a result, this literature overstates the contribution of financial shocks to business cycles. Thus, an extension of the existing business cycle models by including interactions between real and financial sectors provides us with a better framework to study business cycles along many dimensions.

In this dissertation, search frictions are introduced into a RBC model. By allowing for these frictions in asset market, productivity shocks affect financial activity, which in turn, magnifies total output fluctuations. Quantitatively speaking, it would be interesting to investigate further to what extent productivity shocks can contribute to business cycle fluctuations in a model with endogenous asset liquidity.

In Chapter 2, I discussed a theoretical framework of a model with search frictions
in the asset market. The model displays two key features: (1) search frictions in the asset market cause the asset price to depend on the market tightness; (2) only part of the population participates in the asset market. The asset market participation rate depends on the expected trading benefit in the market. I show that steady states exist, provide the conditions under which buyers only partially participate in the asset market, and examine the properties of the equilibria.

In Chapter 3, I used the baseline model discussed in Chapter 2, together with productivity and liquidity shocks estimated using SVAR to investigate how asset liquidity and prices fluctuate in response to productivity shocks and liquidity shocks, and how, in turn, these fluctuations magnify the impact of productivity shocks on economic activity. After calibrating the model to match key features in U.S. data, I found that productivity and liquidity shocks are equally important in explaining business cycles, which is contrary to recent findings from the financial shocks literature.

The baseline model is successful in capturing some key features of business cycles. However, asset prices are still counter-cyclical when productivity and liquidity shocks coexist, especially, during the financial crisis. In Chapter 4, I incorporate two more features into the baseline model to study movements of asset prices: investment adjustment costs and government bonds. Given the adjustment costs, investors decrease asset supply during an economic downturn in order to avoid large levels of the adjustment costs, and thus reduce the increase of the asset price due to the decline of asset supply. Government bonds are substitutes for risky assets. When asset liquidity decreases, buyers adjust their investment portfolios by holding more government bonds. These actions further reduce the asset demand during an economic downturn,
and thus improve the model performance on the asset price dynamics. After calibrating these models, I find that the model with both government bonds and adjustment costs successfully generates pro-cyclical movements of asset prices. Moreover, the calibrated model also substantially improves the match of asset price volatility in the model with that in the data.

Future work will involve two directions. The first direction will be the continued study the dynamics of asset prices. While pro-cyclical movements of asset prices can be addressed in a search model with risk-free assets and the adjustment costs, the volatility of asset prices is still much lower than in the data. Understanding and modeling the asset price paths are very important because the activities and functioning of the asset market are important to capture key features of business cycles. In particular, the dynamics of capital investment, labor supply and consumption are very sensitive to the asset price movements.

The second direction of my future research concerns monetary and fiscal policies. My focus is to explore the effects of monetary and fiscal policies on asset markets if there are search frictions. As my current work exemplifies, allowing for search frictions in asset markets can have significant effects on the way we understand business cycles. Therefore, I expect that the interactions between monetary and fiscal policies and search frictions in the asset market will generate interesting implications.
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Appendix A

Proof of Propositions from Chapter 2

A.1 Proof of Lemmas 1 and 2

Lemma 1: According to (2.21), if \( c^e \) is 0, the following condition holds

\[
\pi u'(c) < \lambda^e \pi + \lambda^m \beta E v_{a+1} \pi \phi^*.
\]  

(A.1)

The last term of (A.1) is nonnegative. So if \( \pi u'(c) \) is less than \( \lambda^e \pi \), condition (A.1) always holds. From (2.11) and (2.20), I obtain

\[
\pi u'(c) = \pi \frac{\lambda^e}{X},
\]

The condition that \( \pi u'(c) \) is less than \( \lambda^e \pi \) requires \( X > 1 \). The asset price \( X \) is greater than 1 only if the liquidity constraint is binding. Therefore, if the liquidity constraint is binding, condition (A.1) holds and \( c^e \) is zero. Equation (2.27) is obtained from (2.19). QED

Lemma 2: Replacing \( i \) in capital market clearing condition with the entrepreneurs’
budget constraint yields

\[ k^* = \sigma k^* + \pi r^* k^* + \pi \phi^* X^* q^*. \]  (A.2)

The superscript \(*\) indicates the value at the steady state. The liquidity constraint is binding. Thus, the equity trade, \(q\), can be rewritten from (2.27) as

\[ q^* = \frac{(r^* + \frac{\sigma}{\theta})k^*}{\frac{1}{\theta} - \phi^* X^*} > 0. \]  (A.3)

Using (A.3) to replace \(q\) in (A.2) yields

\[ (1 - \sigma)(\frac{1}{\theta} - \phi^* X^* ) = \pi r^*(\frac{1}{\theta} - \phi^* X^*) + \pi \phi^* X^* (r^* + \frac{\sigma}{\theta}). \]  (A.4)

Equation (2.26) holds with equality because \(q^*\) is positive. Using (2.26) to replace \(\lambda^m\) in the asset price in equation (2.29) evaluated at the steady state gives

\[ \frac{1}{\theta} - \phi^* X^* )X^* = \beta \{(\frac{1}{\theta} - \phi^* X^* )r^* + (\frac{1}{\theta} - \phi^* X^*)X^* \sigma \]
\[ + (X^* - 1)\{\pi r^*(\frac{1}{\theta} - \phi^* X^*) + \pi \phi^* X^* (r^* + \frac{\sigma}{\theta})\}\}. \]  (A.5)

Combining (A.4) and (A.5) yields the asset price function at the steady state as

\[ X^* = F(r^*) = \frac{\beta}{1 - \beta} r^* - \frac{\beta}{1 - \beta} (1 - \sigma). \]  (A.6)

The discount factor \(\beta\) is positive and less than 1. Therefore, the asset price \(X\) and the return on capital \(r\) are positively correlated. \textbf{QED}
A.2  Proof of proposition 1

There are three steps to prove proposition 1. In step 1, I show there is an equilibrium capital when the liquidity constraint is binding and \( e = 1 \). Next, I show that the equilibrium asset price is within the domain \((1, \frac{1}{\pi})\). So the liquidity constraint binds.

In the last step, I show that the condition \( \pi Xqu'(c)(X + \lambda^m X \phi^s - 1) \frac{d\phi^s}{dc}_{|e \in [0,1]} \geq (1 - \pi)c^l \) holds. The left hand side of the condition is the marginal benefit of entering the market, and the associated cost is listed on the right hand side. If the marginal benefit is no less than the marginal cost for all \( e \) in the domain \([0, 1]\), all workers will enter the equity market.

Step 1: Suppose the liquidity constraint is binding and the entry rate is fixed at 1. From (2.22), I obtain the labor supply as

\[
L = \left(\frac{u_0 A (1 - \alpha)}{\eta}\right)^{\frac{1}{\eta - 1 + \alpha}} K^{\frac{\alpha}{\eta - 1 + \alpha}}. \tag{A.7}
\]

Define BB as:

\[
BB = \left(\frac{u_0 A (1 - \alpha)}{\eta}\right)^{\frac{1}{\eta - 1 + \alpha}}.
\]

The return on capital can be rewritten as

\[
r = r(K : A) = A\alpha \left(\frac{L}{K}\right)^{1-\alpha} = A\alpha BB^{1-\alpha} K^{\frac{1-\alpha (1-\eta)}{\eta - 1 + \alpha}}. \tag{A.8}
\]

According to Lemma 2, the equity price \( X \) equals

\[
X = X(K : A) = \frac{\beta}{1 - \beta} A\alpha BB^{1-\alpha} K^{\frac{1-\alpha (1-\eta)}{\eta - 1 + \alpha}} - \frac{\beta}{1 - \beta} (1 - \sigma), \tag{A.9}
\]
where $\eta$ is strictly greater than 1. Given any productivity level, $r$ and $X$ are decreasing in $K$. In a steady state, capital is identical across the period. Suppose that $K^*$ is the steady state capital. Canceling $K^*$ on both sides of the capital law of motion (2.17) and replacing $q$ by (2.27) yields

$$(1 - \sigma) = \pi r(K^* : A) + \pi \phi^s X(K^* : A) \frac{r(K^* : A) + \frac{\pi}{1/\theta - \phi^s X(K^* : A)}}{1/\theta - \phi^s X(K^* : A)}.$$  \hspace{1cm} (A.10)$$

RHS(LHS) is denoted as the right (left) hand side of equation (A.10). LHS is a constant. $r(K : A)$ and $x(K : A)$ decrease in $K$, and $\bar{\phi}^s$ is a constant with $e = 1$. This, RHS is strictly decreasing in $K$. As $K \to 0 \Rightarrow RHS \to \infty$ and $K \to \infty \Rightarrow RHS \to 0$, $K^*$ is the unique solution that satisfies (A.10). Note, all the results in this step are based on the assumptions that the liquidity constraint is binding and the entry rate is 1.

Step 2: Suppose $K = \bar{K}$ when $X(K : A) = \frac{1}{\theta \phi^s}$. It implies that $RHS(\bar{K}) = \infty$, and hence, $K < K^*$. Suppose $K = \bar{K}$ when $X(K : A) = 1$. From (A.6), I obtain

$$r(\bar{K} : A) = \frac{1}{\beta} - \sigma.$$  

The difference between $LHS(\bar{K})$ and $RHS(\bar{K})$ is

$$\left(\frac{1}{\theta} - \bar{\phi}^s\right)(1 - \sigma) - \frac{\pi}{\beta \theta} + \sigma \pi \frac{1}{\theta} (1 - \bar{\phi}^s),$$

where $\bar{\phi}^s$ represents seller’s matching rate at $K = \bar{K}$. Part 2 is positive. Part 1 is also positive if condition (2.40) holds. Thus, $\bar{K}$ is strictly greater than $K^*$. The value of $\pi$ is the fraction of entrepreneurs and should be a very small number. Thus, the
A.2. PROOF OF PROPOSITION 1

The steady state capital, \( K^* \), is in the domain \((\bar{K}, \bar{K})\). The price function \( X(K : A) \) decreases in \( K \), which implies that \( X(K^* : A) \) is greater than 1 and less than \( \frac{1}{\theta \phi_s} \).

Thus, the liquidity constraint is binding at \( K^* \).

Step 3: The marginal benefit of entering the market at \( e = 1 \) is

\[
\frac{\pi}{1 - \pi} X(K : A)q(K : A)u_0[X(K : A) - 1 + \frac{\theta(X(K: A) - 1)}{1 - \theta \phi_s X(K : A)} X(K : A) \bar{\phi}^s] \frac{\partial \phi_s}{\partial e}
\]

(A.11)

The marginal benefit depends on the values of \( X(K : A), K \) and \( q \). According to the conditions (A.9) and (A.10), the values of \( X(K : A) \) and \( K \) decrease in \( e \). The equilibrium \( q \) is always greater than the value of \( q_{\bar{e}} \). \( q \) is defined as

\[
q = \frac{r(\bar{K}) + \frac{\sigma}{\theta} K}{1 - \bar{e}}
\]

which is solved from (A.3) when \( r, K \) and \( \phi^s X \) are at the lowest levels. Using \( X(K^1 : A), K^1 \) and \( q \) to replace the values of \( X(K : A), K \) and \( q \) with the lowest values, I obtain the critical value of the marginal cost of entering the market \( c^{f1} \) as

\[
c^{f1} = \frac{\pi}{1 - \pi} X(K^* : A)q u_0[X(K^* : A) - 1 + \frac{\theta(X(K^* : A) - 1)}{1 - \theta \phi_s X(K^* : A)} X(K^* : A) \bar{\phi}^s] \frac{\partial \phi_s}{\partial e} \bigg|_{e=1}.
\]

(A.12)

The value of \( c^{f1} \) is strictly lower than the smallest marginal benefit of entering the market for all \( e \) in the domain \([0, 1]\). Therefore, if the marginal cost, \( c^f \), is no more than \( c^{f1} \), the optimal entry rate decision is 1. Because \( X(K^* : A) - 1 \) and \( q \) are positive, \( c^{f1} \) is strictly greater than 0. Thus there always exists a positive \( c^f \) that is

\footnote{This condition is derived from the first-order condition (2.24).}
no more than $e^{f_1}$. Consumption, investment and trade decisions can be solved from first order conditions and market clearing conditions.

Suppose equilibrium capital $K^*$ increases as productivity $A$ decreases. Equations (A.8) and (A.9) imply that the new equilibrium return on capital $r^*$ and the new asset price $X^*$ decrease as well. The RHS of equation (A.10) decreases. As shown in Figure A.1 a negative productivity shock shifts $D(K)$ to the left, which leads to a decrease in the steady state capital.\footnote{The function $D(K)$ represents RHS(K).} This result is contradicted the assumption that equilibrium capital $K^*$ increases. Therefore, equilibrium capital $K^*$ decreases as productivity $A$ decreases. **QED**

---

**Figure A.1: Steady state value of $K$**
A.3 Proof of Proposition 2

Two steps are needed to prove the existence of an unique equilibrium. In the first step, I calculate equilibrium under a assumption that the liquidity constraint always binds. I also solve a feasible range of \( c^f \) with which there is an equilibrium. In the second step, I show that the liquidity constraint is binding at this equilibrium. It requires to show that the equilibrium price \( X \) is greater than 1 and lower than \( \frac{1}{\theta} \).

Step 1: Suppose the liquidity constraint binds, according to the capital market clearing condition (A.10), capital can be rewritten as

\[
K = f(d),
\]

where \( f(d) \) is a continuous function in the domain \( d \in [0, 1] \). According to (A.16), (A.8) and (A.9), the quantity of trade, the return on capital and asset prices can also be modeled as a continuous function of \( d \) in its domain. The marginal benefit of entering the market is

\[
\frac{1}{1 - \pi} \frac{\pi X(d) q(d) u_0[X(d)]}{1 - \pi} + \frac{\theta (X(d) - 1)}{1 - \theta \phi^s X(d)} X(d) \phi^s(d) - 1 | \phi_0 \gamma \left( \frac{1 - \pi}{\pi} \right) \gamma (d) \gamma - 1. \tag{A.13}
\]

The marginal benefit is a product of continuous functions, and hence, it is also a continuous function of \( d \) in the domain \( d \in [0, 1] \). When \( d \) equals zero, the marginal benefit equals infinity. Suppose \( c^{f2} \) satisfies

\[
c^{f2} = \frac{1}{1 - \pi} \pi X(K^*) q(K^*) u_0[X(K^*)] \]
\[
+ \frac{\theta (X(K^*) - 1)}{1 - \theta \phi^s X(K^*)} X(K^*) \phi^s - 1 | \phi_0 \gamma \left( \frac{1 - \pi}{\pi} \right) \gamma | d = 1 > 0. \tag{A.14}
\]
A.3. PROOF OF PROPOSITION 2

The right hand side is the marginal benefit when \( d \) equals one. Because \( X \) is strictly greater than 1, \( c^{f2} \) is positive. If the entry cost, \( c^f \), is larger than \( c^{f2} \), according to the fixed point theorem, there is at least one \( d \) at where the marginal benefit of entering the market equals the marginal cost.

Step 2: Suppose \( d^2 \) is one of the equilibrium entry rates from step 1. The values of \( K^2 \) and \( X^2 \) represent the associated capital holding and the equity price, respectively. In this step, I show that the liquidity constraint binds at \( (d^2, K^2) \), which requires the equilibrium price \( X^2 \) in the domain \((1, \frac{1}{\theta})\). The procedures are the same as in the proof of proposition 1. Given \( d = d^2 \), \( \phi^*(d) \) is a constant, and RHS is a strictly decreasing function of \( K \). At \( K \), the price is \( \frac{1}{\theta} \), and RHS is equal to infinity. At \( \bar{K} \), the price is 1, and LHS is greater than RHS if the condition

\[
(1 - \theta \phi^*)(1 - \sigma) > \frac{\pi}{\beta} \tag{A.15}
\]

is satisfied. In this case, the broker’s matching rate, \( \phi^* \), is not a constant but depends on the entry rate \( d \). Hence, condition (A.15) changes as the value of \( d \) changes. When \( d \) equals one, the right hand side of equation (A.15) reaches its lowest level. Using \( \bar{\phi}^* \) to replace \( \phi^* \) in (A.15), I obtain (2.40). If condition (2.40) holds, LHS is greater than RHS for all \( d \) in the domain \([0, 1]\). Therefore, there is at least one \( K \) between \( K \) and \( \bar{K} \) where RHS is equal to LHS. The equilibrium asset price is a decreasing function of \( K \). Therefore, \( X^2 \) is in a domain \((1, \frac{1}{\theta})\). \( K^2 \) is one of the \( K \) that makes RHS equal to LHS. Substituting \( K^2 \) and \( d^2 \) into (A.13), I obtain a unique positive \( c^{f2} \) where \((K^2, d^2, x^2)\) are equilibrium values of capital, entry rate and the asset price. QED
A.4  Proof of proposition 3

Case 1: According to (A.8) and (A.9), capital and productivity have opposite effects on the capital return $r$ and the asset price $x$. Suppose the impacts of capital on the return on capital and the asset price dominates the impact of productivity when productivity increases, both the return on capital $r^*$ and the asset price $X^*$ decrease as productivity $A$ increases. The curve $D(K)$ shifts to the left, which leads to a decrease in the steady state capital. This result is a contradiction to the proposition 1 result. Thus, the productivity effect dominates the capital effect. Both the return on capital $r^*$ and the asset price $X^*$ increases in the new equilibrium.

Both productivity and capital increase, equation (A.7) implies labour supply $L^*$ also increases in equilibrium. Rewriting equation (LR), I get

\[
q = \frac{(r(A) + \frac{\sigma}{\theta}K(A))}{\frac{1}{\theta} - \phi X(A)}. \tag{A.16}
\]

The return on capital and the asset price all increase, equation (A.16) implies asset trade $q^*$ also increases in the new equilibrium. Investment $i^*$ and the wage rate $\omega^*$ satisfy

\[
i = r(A)K(A) + \phi X(A)q(A) \tag{A.17}
\]
\[
\omega = A(1 - \alpha)(BB)^{-\alpha}K^{\frac{\alpha(q-1)}{\eta+\alpha}}. \tag{A.18}
\]

Since $K$, $r$, $X$ and $q$ are all increase, Investment $i^*$ and the wage rate $\omega^*$ also increase. Entrepreneur’s consumption is zero. So total consumption equals $(1 - \pi)e^\omega$. From
workers budget constraint, we get

\[ c^w = r(A)K(A) + \omega l^w(A) - \frac{\pi \phi^s}{1 - \pi} q(A)X(A). \]  \tag{A.19}

Total funds \( r(A)K(A) + \omega l^w(A) \) increase with a higher productive. Total new asset trade \( q(A)X(A) \) also increases. So the movement of the consumption is ambiguous.

**Case 2.** A internal solution of the entry rate implies

\[ \frac{\pi}{1 - \pi} X(e^*)q(e^*)u_0[X(e^*)] + \frac{\theta(X(e^*) - 1)}{1 - \theta \phi^s(e^*)X(e^*)} X(e^*)\phi^s(e^*) - 1|\phi_0 \gamma \left(1 - \frac{\pi}{\pi}\right)^\gamma (e^*)^{\gamma - 1} = c^f, \]  \tag{A.20}

where \( e^* \) is the equilibrium entry rate, which satisfies \( 0 < e^* < 1 \). Rewriting (A.20), I get

\[ \frac{\gamma}{1 - \pi}(\gamma - 1)e^{-1}X(e)q(e^*)U_0(X(e) - 1) - \frac{\theta X(e)}{1 - \phi^s(e)X(e)} X(e) = c^f, \]  \tag{A.21}

When \( e = 0 \) the left hand side of (A.21) is infinity, which is strictly greater than \( c^f \).

Equation (A.12) implies that the left hand side of (A.21) is strictly less than \( c^f \) when \( e = 1 \). If the left hand side of (A.21) strictly decreases in \( e \), the equilibrium is unique. The left hand side of (A.21) consists of three parts. The part 1 is the marginal probability of trade, which is strictly decreasing in \( e \). The part 3 is the shadow price of the liquidity constraint. As the entry rate increases, the liquidity constraint becomes less tightly, and thus, the part 3 is also decreasing in \( e \). The part 2 represents the internal margin of trade. If the impacts of the entry rate on the parts 1 and 3 dominates the impact on part 2, the left hand side of (A.21) is strictly decreasing in \( e \). The fixed point theorem implies that there is an unique equilibrium.

Supposing that the equilibrium capital \( K^* \) does not increase when productivity \( A \)
increases. Equations (A.8) and (A.9) imply that both of the return on capital $r^*$ and the asset price $x^*$ increases. The right hand side of equation (A.10) increases for any value of $K$. The curve D(K) in Figure A.1 shifts out, which leads to a increase in the new equilibrium of capital $K^*$. This result is a contradiction. Thus, the equilibrium capital $K^*$ increases as productivity $A$ increases. Equation (A.16) implies asset trade $q^*$ also decreases.

Increases of $q^*$ and $X^*$ imply that parts 2 and 3 of (A.21) increase. In order to retrieve equilibrium in (A.21), the entry rate $e^*$ must increase. Both productivity and capital increase, equation (A.7) implies labour supply $L^*$ increases in equilibrium as well. Since $K^*$, $r^*$, $X^*$ and $q^*$ are all increase, equations (A.17) and (A.18) imply both investment $i^*$ and the wage rate $\omega^*$ increase. Total funds $r(A)K(A) + \omega l^w(A)$ increase with productivity. Total new asset trade $q(A)X(A)$ also increases. So the movement of the new equilibrium consumption is ambiguous.

QED
Appendix B

The Model without Search Frictions

A representative household’s Bellman equation is:

\[ v(s : K, A) = \max_{(l, i, s_{+1})} u(c) - h(l) + \beta Ev(s_{+1}; K_{+1}, A_{+1}) \quad (B.1) \]

subject to

\[ rs + x(i^e + \sigma s - s^e_{+1}) \geq i^e + c^e \quad (B.2) \]
\[ rs + \omega l^w + x(\sigma s - s^w_{+1}) \geq c^w \quad (B.3) \]
\[ s^e_{+1} \geq (1 - \theta)i^e + (1 - b)\sigma s \quad (B.4) \]
\[ c^w \geq 0 \quad s_{+1} \geq 0 \quad q \geq 0 \quad l \geq 0 \quad x \geq 0 \quad i \geq 0 \quad (B.5) \]
\[ c = (1 - \pi)c^w, \quad l = (1 - \pi)l^w, \quad s_{+1} = \pi s^e_{+1} + (1 - \pi)s^w_{+1}, \quad (B.6) \]

where \( s^e_{+1} \) and \( s^w_{+1} \) are the entrepreneurs’ and workers’ asset holdings at the end of the period. The value of \( b \) represents the resaleability of the existing equity. Adding
(B.2) × π and (B.3) × (1 − π) and replacing $s_{+1}^e$ in (B.2) by (B.4) gives

$$rs + \omega l + (x - 1)\pi i^e + x(\sigma s - s_{+1}) \geq c \tag{B.7}$$

$$ (r + b\sigma x)s - (1 - \theta x)i^e \geq c^e. \tag{B.8}$$

Supposing the multipliers of (B.7) and (B.8) are $\lambda^{ee}$ and $\lambda^{mm}\pi u'(c)$, the FOCs for $(c, s_{+1}, l, i^e)$ are

$$u'(c) - \lambda^{mm}u'(c) \leq \lambda^{ee}, \quad c^e \geq 0 \tag{B.9}$$

$$u'(c) \leq \lambda^{ee}, \quad c^w \geq 0 \tag{B.10}$$

$$\beta E v_{s+1} \leq \lambda^{ee}x, \quad \text{and } s_{+1} \geq 0 \tag{B.11}$$

$$\omega u'((1 - \pi)c^w) \leq h'((1 - \pi)l^w), \quad \text{and } l \geq 0 \tag{B.12}$$

$$(x - 1) \leq (1 - \theta x)\lambda^{mm}, \quad \text{and } i \geq 0. \tag{B.13}$$

The envelope condition is

$$v_s = u'(c)(r + x\sigma) + \lambda^{mm}\pi u'(c)(r + b\sigma x). \tag{B.14}$$

Combining (B.11) and (B.14), I obtain the asset price as

$$x = \beta E\left\{ \frac{u'(c+1)}{u'(c)}[r_{+1} + \sigma x_{+1} + \lambda^{mm}_{+1}\pi(r_{+1} + b\sigma x_{+1})] \right\}. \tag{B.15}$$

Supposing the liquidity constraint is binding, the asset price $x$ is greater than 1 and less than $1/\theta$. Equations (B.9) and (B.10) imply that the entrepreneurs’ consumption $c^e$ is 0.
Appendix C

Chapter 3: Figures for a Robustness Check

Figure C.1: Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\gamma$
Figure C.2: Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\phi_0$. 
Figure C.3: Impulse Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\pi$
Figure C.4: Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different $\rho$
Figure C.5: Impose Responses to One Standard Deviation of Productivity Shocks with different $\gamma$
Figure C.6: Impose Responses to One Standard Deviation of Productivity Shocks with different $\phi_0$
Figure C.7: Impose Responses to One Standard Deviation of Productivity Shocks with different $\pi$
Figure C.8: Impose Responses to One Standard Deviation of Productivity Shocks with different $\rho$
Appendix D

Proof of Propositions from Chapter 4

D.1 Proof of Lemmas 3

When the asset market is active, buyers’ entry decision \( e \) is greater than 0. The entry cost \( c^f \) is positive. Equation (4.24) implies that \( \lambda^{mb} \) is positive. Therefore, the liquidity constraint binds.

The next period of capital \( s_{+1} \) is positive, the FOC (4.19) implies \( X = \frac{\beta E v'_{s+1}}{\lambda^{gb}} \). The Nash bargaining solution in the equity market implies \( X = \frac{\beta E v'_{s+1}}{u'(c)} \). Then we have

\[
\lambda^{gb} = u'(c). \tag{D.1}
\]

Using (D.1) to replace \( \lambda^e \) in the first equation of (4.20) gets

\[
u'(c) \leq u'(c) + \lambda^{mb} u'(c), \quad \text{and} \quad c^e \geq 0. \tag{D.2}
\]

As shown above, \( \lambda^{mb} > 0 \). Two inequalities in (D.2) hold with complementary slackness. Thus, equation (D.2) implies that entrepreneurs’ consumptions, \( c^e \), are always zero.
Suppose the bond holding, $b_{e+1}$, is positive. The FOC (4.25) implies

$$\beta E v_{b+1}' = \lambda^{eg} p + \lambda^{mgb} u'(c)p.$$  \hspace{1cm} (D.3)

Using (D.3) to replace $\lambda^{eg} p$ in the first equation of (4.26) gets

$$\beta E v_{b+1}' \leq \beta E v_{b+1}' - \lambda^{mgb} u'(c)p,$$  \hspace{1cm} (D.4)

which is in contradiction to the conditions that the resource constraint binds, and $\lambda^{mgb} > 0$. Therefore, $b_e$ is always zero. QED

**D.2 Proof of Lemmas 4**

When asset market is active, buyers’ entry decision $e$ is greater than 0. The entry cost $c^f$ is positive. Equation (4.39) implies that $\lambda^{mhh}$ is positive. Therefore, the resource constraint binds.

The next period of capital $s_{+1}$ is positive, the FOC (4.34) implies $X = \frac{\beta E v_{+1}'}{u'(c)}$. The Nash bargaining solution in the equity market implies $X = \frac{\beta E v_{+1}'}{u'(c)}$. Then we have

$$\lambda^{eh} = u'(c).$$  \hspace{1cm} (D.5)

Using (D.5) to replace $\lambda^{eh}$ in the first equation of (4.35) gets

$$u'(c) \leq u'(c) + \lambda^{mhh} u'(c), \text{ and } e^c \geq 0.$$  \hspace{1cm} (D.6)

As shown above, $\lambda^{mhh} > 0$. Two inequalities in (D.6) hold with complementary slackness. Thus, equation (D.6) implies that entrepreneurs’ consumptions, $e^c$, are
always zero.
Suppose the bond holding, $b_{e+1}$ is positive. The FOC (4.40) implies

$$\beta E v'_{h_{b+1}} = \lambda_{ebh} p + \lambda_{mbh} u'(c)p.$$  \hspace{1cm} (D.7)

Using (D.7) to replace $\lambda_{ebh} p$ in the first equation of (4.41) gets

$$\beta E v'_{b+1} \leq \beta E v'_{b+1} - \lambda_{mbh} u'(c)p,$$  \hspace{1cm} (D.8)

which is in contradiction to the conditions that the resource constraint binds, and $\lambda_{mbh} > 0$. Therefore, $b_e$ is always zero. QED

D.3 Calibrated parameters
Table D.1: Parameters from the Model with Government Bonds

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model with GBs</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$: discount factor</td>
<td>0.992</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\rho$: relative risk aversion</td>
<td>2</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$h_0$: constant in labor disutility</td>
<td>9.8</td>
<td>target labor supply $L=0.2$</td>
</tr>
<tr>
<td>$\eta$: curvature in labor disutility</td>
<td>1.5</td>
<td>labor supply elasticity=2</td>
</tr>
<tr>
<td>$\alpha$: capital share</td>
<td>0.36</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\sigma$: survival rate of capital</td>
<td>0.981</td>
<td>annual investment/capital=0.076</td>
</tr>
<tr>
<td>$A^*$: steady-state TFP</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>$\delta_a$: persistence of TFP</td>
<td>0.96</td>
<td>persistence of TFP</td>
</tr>
<tr>
<td>$\theta$: fraction of new equity</td>
<td>0.217</td>
<td>equals the price discount rate</td>
</tr>
<tr>
<td>$\text{var}(\varepsilon_A)$: variance of TFP</td>
<td>$0.367 \times 10^{-4}$</td>
<td>observable from data</td>
</tr>
<tr>
<td>$\pi$: fraction of entrepreneurs</td>
<td>0.06</td>
<td>fraction of investing firms=0.24</td>
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<td>$c_f$: entry cost of workers</td>
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<td>annual return to the liq. bond $=0.02$</td>
</tr>
<tr>
<td>$\gamma$: power of the matching function</td>
<td>0.28</td>
<td>relative volatility $\sigma_{\phi^*}/\sigma_y = 21$</td>
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<td>0.12</td>
<td>$\text{corr}(A, \phi) = 0.35$</td>
</tr>
<tr>
<td>$\text{var}(\varepsilon_\mu)$: variance of the liq.</td>
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<td>$\text{corr}(A, L) = 0.07$</td>
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<tr>
<td>$\text{cov}(\varepsilon_\mu, \varepsilon_A)$: covariance of (TFP liq.)</td>
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<td>$\text{corr}(K, \phi) = 0.41$</td>
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