ENERGY DISSIPATION FROM INTERNAL SOLITARY WAVES SHOALING ON FLAT AND MILDLY SLOPING BEDS

by

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Abstract

Boundary layer separation-driven near-bed instability beneath internal solitary waves (ISWs) of depression may be a significant source of wave-energy dissipation and drive localized mixing and resuspension in coastal regions. In chapter 2, the details of estimates of energy flux by internal waves to quantify the dissipation of ISWs as they propagate over a flat bottom. Wave flume experiments were carried out to measure the dissipation of turbulent kinetic energy in the boundary layer beneath shoaling ISWs. Wave dissipative length scale of $O(100)$ wavelengths for unstable waves; which is in agreement with limited field observations. Stable waves propagated significantly further ($>1000$ wavelengths). Internal solitary wave energetics were used to model wave dissipation lengthscales in terms of the wavelength and momentum thickness Reynolds number $L^* = 100\lambda + 2.5 \times 10^{10} \lambda Re_{ISW}^{-3.7}$. In chapter 3, laboratory experiments were performed to measure dissipation within the turbulent bottom boundary layer beneath an ISW. Velocity vector maps were measured using Particle Image Velocimetry. Different methods of estimating dissipation from spatial gradients of the vector maps were compared, based on measured components of the dissipation tensor. Estimates based on assumptions of isotropy are typically larger than those based on methods using available velocity gradients with least number of assumptions (e.g., direct method). All the methods reproduce the same order-of-magnitude estimates for the dissipation rate ($\varepsilon \approx 10^{-7} - 10^{-6}$ W kg$^{-1}$) but the differences varied from 5% to 80% than the corresponding direct estimates. Chapter 4 describes an experimental investigation on the dissipative phase of ISWs as they propagate over uniformly sloping topography ($S = 0.04$) in a 20 m flume. Velocity profiles were recorded with three acoustic Doppler velocimeters. As each wave propagated up the slope, the wave of depression steepened and developed into an elevation-like wave before finally dissipating through forming a discrete vortex of dense fluid (bolus) which propagated for some distance up the slope. The dissipation length scale was parameterized in terms of the internal Iribarren number ($\xi = S/\sqrt{a/\lambda}$) as $L^* = (302.7 \times \xi - 10.4) \times a$ and requires only a knowledge of incident ISW amplitude $a$ and wavelength $\lambda$. 
Co-Authorship

The work included in the thesis is utterly my own, while carried out under the supervision of Dr. Leon Boegman. Significant contributions have been made by Leon Boegman in interpreting the results and commenting on the original work of the author for journal preparation.

Chapter 2 of this thesis has been submitted to *Physical Review Fluids* as: S. Zahedi, P. Aghsae, and L. Boegman, Energy flux and turbulent dissipation by internal solitary waves. The original lab data were provided by Dr. Aghsae, but not analyzed and published.

Chapter 3 of this thesis will be submitted to *Nonlinear Processes in Geophysics* as: S. Zahedi, P. Aghsae, and L. Boegman, Laboratory measurement of turbulent kinetic energy dissipation in shoaling internal solitary waves. The original lab data were provided by Dr. Aghsae, but not analyzed and published.

Chapter 4 of this thesis will be submitted to *Atmosphere-Ocean* as: S. Zahedi, A. Ghassemi, and L. Boegman, Energetics of internal solitary wave shoaling and bolus formation on a uniform slope. A. Ghassemi assisted with collection of the lab data.
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### List of Abbreviations

**Acronyms**

<table>
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<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADV</td>
<td>Acoustic Doppler Velocimeter</td>
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<tr>
<td>ADCP</td>
<td>Acoustic Doppler Current Profiler</td>
</tr>
<tr>
<td>BBL</td>
<td>Bottom Boundary Layer</td>
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<tr>
<td>ISW</td>
<td>Internal Solitary Wave</td>
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<tr>
<td>MODIS</td>
<td>Moderate Resolution Imaging Spectroradiometer</td>
</tr>
<tr>
<td>NLIW</td>
<td>Nonlinear Internal Wave</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle Image Velocimetry</td>
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<tr>
<td>RANS</td>
<td>Reynolds-averaged Navier-Stokes</td>
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<tr>
<td>SAR</td>
<td>Synthetic Aperture Radar</td>
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<tr>
<td>SMF</td>
<td>Structure function method</td>
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<tr>
<td>TKE</td>
<td>Turbulent Kinematic Energy</td>
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<tr>
<td>2D</td>
<td>Two-Dimensional</td>
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<tr>
<td>3D</td>
<td>Three-Dimensional</td>
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### Roman symbols

- $a$: Amplitude
- $c$: Phase velocity
- $c_0$: Linear long-wave speed
- $g'$: Reduced gravity
- $H$: Total depth of the fluid
- $h$: Interface thickness
- $h_1$: Thickness of the lower layer
- $h_2$: Thickness of the upper layer
- $\kappa$: Von Kármán constant
- $z$: Height above the bed
- $L_w$: Half of wavelength
- $S$: Boundary slopes
- $S_w$: Wave slope
- $L$: Dissipation Length scale
- $E$: Total wave’s energy
- $u_*$: Friction velocity
- $u_{max}$: Maximum wave velocity
- $Fr$: Froude number
- $Ri$: Richardson number
Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Nonlinearity coefficients</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Dispersion coefficients</td>
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<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
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<td>$\kappa$</td>
<td>Horizontal length scale</td>
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<td>Nonlinearity parameter</td>
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<td>$\nu$</td>
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<td>$\varepsilon$</td>
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<td>$\xi_{in}$</td>
<td>Internal Iribarren number</td>
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<tr>
<td>$\rho_1$</td>
<td>Upper density</td>
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<tr>
<td>$\rho_2$</td>
<td>Lower density</td>
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Chapter 1

Introduction

1.1 Introduction and pertinent literature

1.1.1 Introduction to internal waves

Internal waves propagate along density gradients within a multi-layer stratified fluid, owe their existence to the stratified density structure of the ocean, with a sharp density change that can be found within the pycnocline that acts as a waveguide. Internal solitary waves (ISWs) of depression are naturally characteristic of coastal environments and most likely to dissipate tidal energy, transport nutrients and sediments and are believed to resuspend sediments where they shoal (Bogucki, Dickey et al. 1997; Lamb 1997; Aghsae and Boegman 2015; Boegman and Stastna 2019). These waves impact forces on offshore oil and gas rigs and undersea pipelines (Osborne and Burch 1980). Any disturbance to the pycnocline can generate internal waves; however, they are most commonly generated by tidal flow over bottom topography (Ostrovsky and Stepanyants 1989; Dewan, Picard et al. 1998; Apel 2003; Nash and Moum 2005).

Internal wave properties are recognizably different in nature from surface waves because of the density difference between the water masses above and below the pycnocline. Internal waves propagate horizontally like surface waves, but commonly do so at slower speeds and have higher amplitudes and much lower frequencies (Helfrich and Melville 2006; Bourgault, Blokhina et al. 2007; Farmer, Li et al. 2009). Their amplitude is limited by a balance between dispersion and nonlinear steepening, depending on the background stratification (Figure 1.1). Generally these waves can form either as waves of depression, when the pycnocline is near the surface (e.g., \( h_1 < h_2 \)), or elevation when the pycnocline is located closer to the bottom, in shallow water, and also in winter stratification (e.g., \( h_2 < h_1 \)) (Osborne and Burch 1980; Helfrich 1992; Grimshaw, Pelinovsky et al. 2004; Vlasenko and Stashchuk 2007; Shroyer, Moum et al. 2010).
Figure 1.1 Schematic of an internal solitary wave of depression in a two-layer stratified fluid. Dashed lines are lines of constant water particle speed. A small surface solitary wave accompanies the ISW. Edited from Osborne A. R. and BurchT. L. (1980) Internal solitons in the Andaman Sea. Science 208 (4443): 451-460.

1.1 Literature review

1.1.1 Generation and formation of internal waves

The earliest recognition of a surface solitary wave was reported on the formation of a single, unchanging hump or mound propagating off the bow of a barge in the shallow water of a Scottish canal (Russell 1844). In the 1950-60s during in-situ measurements of internal waves, it appeared that they coexisted with surface slicks (Figure 1.2). Presently, we can view the surface signatures (Figure 1.3) on most oceanic coastlines worldwide using aircraft and satellite imaging sensors (Apel, Gasparovic et al. 1988; Jackson 2007).
Figure 1.2 Satellite image of the Banda Sea acquired on 24 February 2004. Four groups of internal waves are visible in the central sea radiating from a generation point in the Ombai Strait (Edited from Jackson, 2007)
Figure 1.3 Internal solitary wave observations in MODIS from Aug. 2002-May. 2004. Black dots show well-known occurrence sites, new areas of activity are shown in red and areas of geographically expanded activity are shown in blue (Edited from Jackson, 2007).

ISWs are below the grid-scale of most coastal ocean models (Dorostkar, Boegman et al. 2017). Moreover, to reproduce their shoaling mechanism requires models at high Reynolds numbers that also resolve the no slip bottom boundary condition (Aghsaee, Boegman et al. 2010). Therefore, it is computationally prohibitive to simulate ISW shoaling in three-dimensions with numerical simulations, because this would require a fine grid with a small time step. As a result, there have been many laboratory studies investigating ISW shoaling (Michallet and Ivey 1999). Laboratory experiments on ISW dynamics, however, do not accurately scale to the ocean due to the low Reynolds numbers, which are significantly smaller than oceanic values. Reynolds numbers are limited by the flume depth and viscosity. The only alternative to increase Reynolds numbers by increasing the wave speed through the addition of salt to the lower fluid layer to a practical limit value of $\sim 1100$ kg m$^{-3}$ (Aghsaee, Boegman et al. 2010; Aghsaee and Boegman 2015; Boegman and Stastna 2019).
1.1.2 Overview of ISWs propagation and shoaling

The propagation of internal solitary waves in a two-layer system over slope-shelf topography typically results in solitary waves of depression. However, waves of elevation are formed in the nearshore regions where the upper layer can be thinner than the lower layer. As the solitary wave shoals upslope, its rear face steepens (Klymak and Moum 2003; Scotti and Pineda 2004; Boegman, Ivey et al. 2005; Bourgault, Blokhina et al. 2007). In the vicinity of the shelf break, the rear face can become unstable (interfacial shear) and mixing occurs (Kao, Pan et al. 1985; Aghsae, Boegman et al. 2012). The waves on the shelf are dissipated mainly because of strong bottom interactions as the water shoals. There is some evidence of shear instability and boundary shear in shallow water (e.g., of order 30 to 35 m in the New York Bight) (Apel 2003; Orr and Mignerey 2003). It is also possible to observe significant increases in the velocity to their mean value when internal waves interact with ocean boundaries (Bluteau, Jones et al. 2011).

Examination of the propagation of first-mode solitary waves over slope-shelf topography considered the situation where the relative layer depths change as the wave moves from a deep region over a uniform slope onto the shelf. It was shown that the incident wave could be unstable in the neighborhood of the shelf break and instabilities result in localized vertical mixing. Numerical investigations (Bourgault, Morsilli et al. 2014) and laboratory experiments (Helfrich 1992; Michallet and Ivey 1999; Boegman and Ivey 2009) indicate that these waves can release a large part of their energy in the shoaling process. Experiments show that an internal solitary wave of depression on a uniform slope will break and produce multiple boluses and imply that the number of boluses generated increases as wavelength is decreased (Barad and Fringer 2010; Arthur and Fringer 2016).

1.1.3 Instability process and breaking mechanism of ISWs

Internal solitary waves degenerate through four distinct instability processes (e.g, shear instabilities; convective instabilities; global instabilities; wave breaking) that induce mixing and transport of buoyant
particles and sediments on continental shelves (Stastna and Lamb 2002; Zhang, Fringer et al. 2011; Lamb 2014; Boegman and Stastna 2019). Shear instabilities occur if the ISW-induced current shear is sufficient to generate density overturns (Kelvin-Helmholtz billows) through the pycnocline, which typically results in the dissipation of the wave energy. The shear is considered critical for the formation of billows if Richardson number is below critical value of 0.25 (Thorpe 1978; Lamb and Farmer 2011). Convective instabilities exist if the ISW induced current velocity exceeds the phase velocity of the wave which generates a trapped recirculating wave core (Helfrich and White 2010; Lamb 2014). In addition, waves with trapped cores can form near the surface if the stratification extends to the surface or background currents have strong near-surface shear (Lamb 2003; Carter, Gregg et al. 2005). The large amplitude ISWs associated with strong currents make them subject to global instabilities in the bottom boundary layer (Bogucki, Dickey et al. 1997). The instability is global because it is driven by properties of the ‘global’ flow; it depends on pressure gradients that vary along the length of the wave. Lab observations (Carr, Fructus et al. 2008) and numerical simulations (Diamessis and Redekopp 2006) as well as field measurement in the ocean (Moum, Farmer et al. 2007; Davis and Monismith 2011) suggest boundary layer global instability happens in the vicinity of separated flow in the adverse pressure gradient region directly underneath the rear shoulder of the wave and is characterized by a sudden burst of velocities leading to vortex shedding.

The dynamics of breaking ISWs have been studied extensively in both the laboratory (Imberger and Ivey 1991; Boegman, Ivey et al. 2005) and numerical simulations (Holt, Koseff et al. 1992; Aghsae, Boegman et al. 2010; Arthur, Koseff et al. 2017). ISW breaker type can be classified (e.g., Collapsing, Plunging, Surging and Fission) (Figure 1.4) according to wave slope (amplitude/wavelength) and bottom slope, s (Aghsae, Boegman et al. 2010). Field observations show gradual bottom slopes (s ~ 0.01) are likely to generate boluses through the fission of ISWs as they shoal. However, lab experiments have been performed on bottom slopes of 0.07 < s < 0.4 where fission will not occur. To relate experimental results with oceanic observations, experiments need to be conducted where boluses, generated through fission,
shoal on gradual slopes (s=0.04). Over mild slopes, an incident wave of depression, as it passes through the turning point \((h_1 = h_2)\), will degenerate through fission forming a packet of waves of elevation and there is little reflection (Orr and Mignerey 2003; Moum, Klymak et al. 2007; Shroyer, Moum et al. 2011). The fission mechanism associated with emerging ISWs is recognizably different in nature from that associated with turbulent bores, where wave inertia pushes a mixed breaking region shoreward (Helfrich 1992; Aghsaee, Boegman et al. 2010) and the bore-like waves of elevation move upslope until they ultimately dissipated through three-dimensional instability (Wallace and Wilkinson 1988; Helfrich and Melville 2006; Venayagamoorthy and Fringer 2007).

**Figure 1.4** Regime diagram showing ISW breaker types according to wave slope, \(S_w\) and bottom slope, \(S\) (Edited from Aghsaee et al. 2010). Colored circles show data from field observations. Horizontal blue and red lines show bottom slopes employed in laboratory experiments.

### 1.1.4 Dissipation and energy transport of ISWs

Field studies have focused on deeper shelf waters (e.g., 50+ m depths), whereas the primary fate of ISWs occurs in shallow regions (e.g., ~20 m, i.e., the nearshore) (Alford, Mickett et al. 2012; Walter,
Stastna et al. 2016). Observation from the Sulu Sea show roughly a two-day dissipation timescale for internal wave packets in deep water; similar to observations obtained from Gibraltar and open ocean observations made northeast of New Britain (Apel, Gasparovic et al. 1988; Apel 2003). Observations of nonlinear internal waves propagating through deep water show that they are only weakly turbulent, however, turbulence plays an important role in wave energy losses over long distances (e.g., 100 s of wavelengths) and in shallow water propagating over the continental shelf (Klymak and Moum 2003; Chang, Lien et al. 2006). Measurements of the internal solitary waves passing shoreward over the Oregon continental shelf show progression from a simple form to a more complex form as the waves become unstable, breaks and forms turbulent flow (Klymak and Moum 2003). The decay time scale of the ISWs was roughly 12 hours and the dissipation length scale estimated to be approximately 35 km (O (100) wavelengths) resulting in large energy loss to turbulent mixing (Klymak and Moum 2003; Moum, Farmer et al. 2007; Shroyer, Moum et al. 2010).

Dissipation length scales can be estimated by the ratio of wave energy to the rate of energy dissipation. Previous studies (Michallet and Ivey 1999) estimated wave energy by assuming that kinetic energy, $KE$ is equal with available potential energy, $APE$ (Helfrich 1992; Michallet and Ivey 1999; Bourgault, Blokhina et al. 2007); however, a solitary wave solution always has $KE > APE$. Simulations and estimates in the field found that $KE/APE$ was large as 2, confirming that the $APE$ flux always exceeded the $KE$ flux (Turkington, Eydeland et al. 1991; Klymak, Pinkel et al. 2006; Moum, Klymak et al. 2007; Lamb and Nguyen 2009).

Recent studies have estimated turbulent dissipation rates in field observations (Klymak and Moum 2007; Shroyer, Moum et al. 2010; Woodson, Barth et al. 2011; Walter, Stastna et al. 2016). ISW carries notable energy (e.g., 3 GJ/m) with a significant energy flux (e.g., 8.5 kW/m) (Chang, Lien et al. 2006). As the wave propagates (Orr and Mignerey 2003; Shroyer, Moum et al. 2009) and shoals shoreward (Carter, Gregg et al. 2005), estimated dissipation levels increased dramatically and the wave rapidly lost energy (e.g., 14 kW/m). These events are consistent within the range estimated from measurements of turbulent
dissipation (e.g., 10 W/m) which give a mean dissipation of $10^{-7} - 10^{-6}$ in an ISW packet (Klymak and Moum 2003; Lien, Tang et al. 2005). When waves break by fission, they form boluses (waves with an unsteady trapped core propagating along the bottom, often detached from the pycnocline) shoaling with elevated dissipation levels (e.g., $10^{-5} - 10^{-4}$ W/kg). Boluses are potentially an important cross-shelf transport mechanism in the coastal ocean (Stastna and Lamb 2002; Orr and Mignerey 2003; Bourgault, Blokhina et al. 2007; Helfrich and White 2010; Richards, Bourgault et al. 2013). Internal solitary waves of elevation with elevated turbulence levels (TKE dissipation rates of $10^{-6}$ W/kg) are commonly observed during winter downwelling (Klymak and Moum 2003; Scotti and Pineda 2004; Carter, Gregg et al. 2005).

Several measurement techniques have been developed to estimate the turbulent kinetic energy dissipation (TKE) rate including spectrum fitting, structure functions, and direct calculation using velocity gradients from (Bertucchioli, Roth et al. 1999; Doron, Bertucchioli et al. 2001; Variano and Cowen 2008; Hult, Troy et al. 2011). Additionally, these approaches can also map the flow structure and vorticity distribution associated with wave motion and breaking, leading to turbulence (Bertucchioli, Roth et al. 1999; Doron, Bertucchioli et al. 2001).

Particle Image Velocimetry (PIV) is a quantitative flow field mapping technique that can facilitate both the extraction of measurement data and the visualization of flow structures. The advantage of PIV is the ability to non-intrusively acquire instantaneous two-dimensional velocity distributions with a resolution that captures the peak dissipation scales (Dong, Chu et al. 1992; Lin, Vorobieff et al. 1995; Bertucchioli, Roth et al. 1999; Doron, Bertucchioli et al. 2001). Single camera 2D-PIV is limited to the measurement of the two in-plane velocity components while Stereo-PIV has the ability to investigate the three-dimensionality of instability beneath ISWs within a much larger spatial domain, continuously recording images at a sufficiently high frame rate to observe the dynamical evolution of the flow pattern (Prasad and Adrian 1993; Wieneke 2015). Acoustic Doppler Velocimeters (ADVs) can also be applied to non-intrusively measure profiles of flow velocity in both laboratory and field applications.
1.1.5 Ecological impacts of ISWs

Several studies verify the major role of ISWs for boundary mixing by associating observations of increased bottom boundary turbulence to ISW breaking (Thorpe 1978; Michallet and Ivey 1999; Klymak and Moum 2003; Lorke 2007). Moreover, field observations demonstrate that shoaling ISWs are able to resuspend sediment (Bogucki, Dickey et al. 1997; Hosegood and van Haren 2004; Moum, Klymak et al. 2007) and to transport resuspended sediment along sloping boundaries (Klymak and Moum 2003; Scotti and Pineda 2004). As a result of these observations the sediment resuspension mechanisms associated with ISWs were investigated numerically and in the laboratory (Stastna and Lamb 2008; Boegman and Ivey 2009; Aghsaee and Boegman 2015).

As ISWs shoal along a sloping bottom (Figure 1.5) they evolve into waves of elevation as they pass through the turning point (Grimshaw, Pelinovsky et al. 1999; Klymak and Moum 2003; Scotti and Pineda 2004; Shroyer, Moum et al. 2009; Nam, Kim et al. 2011). In time the rear face of each wave would steepen, and overturn creating a turbulent vortex that resuspends sediment leading to both upslope bolus transport and drainage of mixed fluid back down the slope (Helfrich 1992; Hosegood and van Haren 2004; Lamb 2014). Periodic shoaling forms bottom and intermediate nepheloid layers (Bourgault, Morsilli et al. 2014). Particle transport from ISW instability impacts biogeochemical cycling in the ocean (Scotti and Pineda 2004; Wang, Dai et al. 2007), with boluses suspected to be one of the main mechanisms of sediment resuspension and transport in coastal oceans (Cacchione and Southard 1974; Helfrich 1992; Hosegood and van Haren 2004; Diamessis and Redekopp 2006; Bourgault, Blokhina et al. 2007; Venayagamoorthy and Fringer 2007; Boegman and Ivey 2009; Aghsaee, Boegman et al. 2012).
**Figure 1.5** Schematic of a shoaling mode 1 solitary wave steepening and forming a train of much shorter, strongly non-linear ISWs. These waves are a common feature on the shelf (Jackson et al. 2012), usually occurring in packets of several waves (Edited from Lamb, 2014). $D, \varepsilon, E = APE + KE$ are total dissipation, rate of TKE dissipation and total energy of ISW in that order.

### 1.2 Research objectives

ISWs are often observed from offshore moorings, but their transformation from ISWs to boluses, between moorings, and the dissipation length scales remain unknown. These are likely a function of if the bottom boundary layer is stable or unstable. The laboratory-based objectives of the proposed research are:

1. to measure ISW wave energy and energy dissipation to allow for parameterization of the ISW dissipation length scale.
2. to apply PIV to investigate the three-dimensional structure of ISW instability and the degree of isotropy in ISW dissipation of turbulent kinetic energy from ISWs.
3. to characterize the transformation of ISWs to boluses, over sloping bottom topography, and estimate the bolus dissipation length scales.
1.2.1 Layout and significance of proposed research

Chapter 2- ISWs are commonly observed at an offshore mooring and their ultimate fate remains unknown. The central issue of this paper is to characterize the dissipation length scales for ISWs and compare these to the limited field observations. Target journal: Physical Review Fluids

Chapter 3- In environmental flows, it is difficult to measure dissipation over such large domains while directly resolving turbulent fluctuations at the Kolmogorov scale. Therefore, velocity gradients are often only measured in one or two coordinate directions. PIV laboratory experiments are undertaken to investigate dissipation estimates with different assumptions of isotropy. The results will be informative to field oceanographers, who often only measure velocity gradients in one direction only with an ADCP. Target journal: Nonlinear Processes in Geophysics

Chapter 4- Continental shelves are an important contributor to the dissipation of turbulent kinetic energy in the ocean. The ability to measure, and potentially parameterize, turbulent dissipation by these features would be extremely helpful in the understanding of the transition from wave to bolus and how energy is lost in this process. Propagation of boluses is typically observed in the ocean at moorings that are ~100 km apart and, therefore, oceanographers cannot track a bolus from one mooring to the next. Boluses are also thought to carry sediments and nutrients from deep water onto the continental shelf. Knowing how far boluses propagate and loose energy will help to better understand this process. Target journal: Atmosphere-Ocean
References


Chapter 2

Energy flux and turbulent dissipation by internal solitary waves

2.1 Introduction

Internal waves occur along the density gradient within the pycnocline of lakes and oceans. Observations of internal solitary waves (ISWs) abound in oceanography studies (Apel, Gasparovic et al. 1988; Holt, Koseff et al. 1992; Helfrich and Melville 2006; Jackson, Da Silva et al. 2012). In the coastal ocean, ISWs are frequently generated from tide-topography interaction and in lakes from nonlinear steepening of basin-scale internal seiches (Apel, Holbrook et al. 1985; Horn, Imberger et al. 2002; Lamb and Farmer 2011); where an initial disturbance of the pycnocline gradually steepens until it decomposes into a train of ISWs (Apel, Holbrook et al. 1985; Colosi, Beardsley et al. 2001).

The propagation of internal solitary waves in a two-layer system, with an upper layer that is shallower than the lower layer, results in solitary waves of depression. Although, in nearshore regions, the upper layer can be thinner than the lower layer and waves of elevation can exist. As the solitary waves shoal upslope, the rear face steepens leading to wave breaking that drives localized mixing and energy dissipation (Kao, Pan et al. 1985; Helfrich 1992). ISWs propagating through deep water are weakly turbulent (Klymak and Moum 2003; Chang, Lien et al. 2006); however, turbulence plays an important role in wave energy loss as they shoal into shallow water (Moum and Smyth 2006). Measurements of ISWs propagating shoreward over the Oregon continental shelf, show progression from a simple to a more complex form as the waves become unstable, break and form turbulence (Klymak and Moum 2003). Waves on the continental shelf are dissipated largely as a result of strong bottom interaction during shoaling, with some evidence of
shear and gravitational instability in shallow water (e.g., 30 to 35 m depth) (Helfrich and Melville 1986; Vlasenko and Hutter 2002; Moum, Farmer et al. 2003; Boegman, Ivey et al. 2005; Aghsae, Boegman et al. 2010). Significant increases in local velocity, compared to the mean value, occur where ISWs shoal and break (Michallet and Ivey 1999; Apel 2003; Lamb 2014).

The large amplitudes and strong currents associated with ISWs make them subject to a near-bed (global-type) instability, where a jet-flow separates from the bottom boundary in the adverse pressure gradient region beneath the rear shoulder of the ISW. This causes enhanced turbulence and sediment resuspension (Sandstrom, Elliot et al. 1989; Bogucki, Dickey et al. 1997). ISWs carry significant energy shoreward (Chang, Lien et al. 2006; Huang, Chen et al. 2016) and recent studies have estimated the rate of the dissipation of turbulent kinetic energy from field observations $\varepsilon \sim 10^{-7} – 10^{-6}$ W kg$^{-1}$ (Moum, Farmer et al. 2003; Lien, Tang et al. 2005). As ISWs propagate (Orr and Mignerey 2003; Shroyer, Moum et al. 2009) and shoal, the wave energy is ultimately dissipated (Carter, Gregg et al. 2005; Rayson, Ivey et al. 2011).

ISWs are generated and propagate on nearly every coastline worldwide (Jackson 2007), but they are below the grid-scale of most coastal ocean models (Dorostkar, Boegman et al. 2017) and difficult to track between moorings in the ocean (personal communication, N. Jones, U. West Aust.). Most field observations studies have focused on deeper shelf waters (e.g., >50 m depths); whereas the primary fate of ISWs occurs in shallow regions (e.g., ~20 m) (Alford, Mickett et al. 2012; Walter, Stastna et al. 2016), leading to uncertainty in estimating the ISW dissipation length- and timescales. Observations from the Sulu Sea show a ~ 2 days decay timescale for ISWs from deep water, similar to observations from Gibraltar and the open ocean northeast of New Britain (Apel 2003; Woodson, Barth et al. 2011). On the New Jersey continental shelf, the decay timescale was roughly 12 hours, giving a dissipation length scale of ~35 km ($O(100)$ wavelengths) (Shroyer, Moum et al. 2010). The objective of the present study is to perform laboratory experiments to
parameterize the ISW dissipation length scale in terms of readily measurable wave and water column properties.

2.2 Methods

2.2.1 Experimental setup

Experiments were carried out in the 6 m × 0.75 m × 0.65 m glass-walled Internal Wave Flume at the Queen’s University Coastal Engineering Research Laboratory (Figure 2.1). The flume has a bed of finished concrete (~1 mm roughness), which is smoother than the ocean floor (~1 cm roughness) but rougher than typical Perspex or glass walled flumes (~ 0.01 mm roughness), which are typically used for internal wave shoaling experiments (e.g., Helfrich 1992; Michallet and Ivey 1999; Boegman et al. 2005); future experiments may consider the effects of bottom roughness. The raw data were collected as part of an earlier study (Aghsaee and Boegman 2015). To generate ISWs of depression a two-layer stratification of depth $h_1$ and density $\rho_1$ overlaying depth $h_2$ and density $\rho_2$ (total depth $H = h_1 + h_2$) was introduced with a thinner upper layer of fresh water ($h_1 < h_2$). Salt (Cargill CMF) added to the lower layer to reach the desired density ($\rho_2$) in a stirring tank, with the density measured using a high-precision densimeter (Anton Paar DMA-6000). The flume was filled with lower layer fluid to the desired height ($h_2$) and then freshwater was carefully added to the thickness $h_1$ for the upper layer of fluid, using the floating sponge method to minimize mixing between two layers (pycnocline ~2 cm). A fluorescent tracer was added to the upper layer fluid to allow for flow visualization. Waves were generated using the gate-step method, whereby fluid is pumped behind a gate to create a step, and the gate is suddenly removed to forms an ISW of depression (Kao, Pan et al. 1985; Michallet and Ivey 1999; Carr, Fructus et al. 2008; Boegman and Ivey 2009; Aghsaee and Boegman 2015).
Figure 2.1 Schematic of the experimental setup, showing both the initial condition behind the gate and the resultant wave. ISWs of depression were generated within the two-layer fluid by suddenly lifting the gate (Edited from Aghsae and Boegman).

To measure the three-dimensional (3D) velocity components of the resulting ISWs, a profiling acoustic Doppler velocimeter (ADV; Nortek Vectrino II; 1 mm s\(^{-1}\) and ±5% measurement error) was located 2 meters downstream the wave-generating gate and recorded at \((1/\Delta t) = 25\) Hz, over a 3 cm profile with \(\Delta z = 1\) mm resolution. To increase the acoustic backscatter signal-to-noise ratio, the fluid was locally seeded with talcum powder. A high-definition camera (Canon VIXIA HV30) recorded the experiments that allowed measurement of the wave propagation speed, horizontal wavelength, and amplitude.

Each generated wave began to travel from left to right, passing through the camera field-of-view, and then the ADV, until it reflected from the end wall and then continued travelling back and forth until it lost its energy via viscous damping (Michallet and Ivey; Aghsae and Boegman 2015). To reduce sidewall friction and interfacial shear, the tank width was > 3.3 times the fluid layer depths (Troy and Koseff 2006).
### 2.2.2 Wave calculations

Properties of ISWs in a two-layer system can be theoretically estimated using the Korteweg-de Vries (KdV) equation (Osborne and Burch 1980; Apel 2002), if the background stratification and amplitude of the ISW are known (Osborne and Burch 1980; Apel 2002):

\[
\frac{\partial \eta}{\partial t} + c_o \left( \frac{\partial \eta}{\partial x} + \alpha \eta \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} \right) = 0
\]  

(2.1)

where \( \eta \) is the pycnocline displacement. The KdV equation models weakly nonlinear, weakly dispersive waves with nonlinearity and dispersion coefficients:

\[
\alpha = \frac{3c_o}{2} \frac{h_1 - h_2}{h_1 h_2} \quad \beta = \frac{c_o^2}{6} h_1 h_2
\]  

(2.2)

Here, \( c_o = \sqrt{g' h_2 / (h_1 + h_2)} \) is the linear phase speed and \( g' = g (\rho_2 - \rho_1) / \rho_2 \) is the reduced gravity due to stratification (Djordjevic and Redekopp 1978).

The evolution of ISWs, as described by the KdV equation, are often compared to laboratory experiments. The characteristic horizontal wavelength \( L_w \) maybe estimated either from the fluid volume behind the gate, in the event there are no trailing waves, or by integrating pycnocline displacement and dividing by the wave amplitude.

\[
L_w = \frac{1}{a} \int_{-\infty}^{+\infty} \eta(x) \, dx
\]  

(2.3)

where the wave amplitude \( a = \eta_{\text{min}}(x) \). Lab observations show that the actual wavelength is \( \lambda \approx 2L_w \) (Michallet and Ivey 1999; Boegman, Ivey et al. 2005; Carr, Fructus et al. 2008; Aghsaee and Boegman 2015).

Recent attempts to classify ISW stability have applied a wave Reynolds number \( Re_W \), pressure gradient parameter \( P_{ISW} \) and momentum thickness Reynolds number \( Re_{ISW} \), based on the characteristics of the wave-induced bottom boundary layer at the wave trough. In general, oceanographers use \( Re_W \) which is composed of wave properties that are readily measured from a typical mooring (wave amplitude and linear phase speed) (Aghsaee, Boegman et al. 2012):
\[ Re_W = \frac{ac_o}{\nu} \quad P_{ISW} = \frac{(|U_2|+c)|U_2|}{L_wg} \quad Re_{ISW} = \frac{|U_2|\sqrt{L_w/(\nu(|U_2|+c))}} \] (2.4)

Here, \( \nu \) is the kinematic viscosity and \( U_2 \) is an absolute value of the maximum horizontal velocity at the wave trough (obtained from the ADV data). This parameter may also be theoretically estimated from the bulk wave properties, if direct observations are not available (Ostrovsky and Stepanyants 2005; Aghsae, Boegman et al. 2012):

\[ U_2 = \frac{-ca}{h_2 + a} \] (2.5)

The nonlinear phase speed \( c \) and wave amplitude \( a \) were calculated from the video images.

**Table 2.1** Experimental parameters where \( \rho_2 \) is the lower layer fluid density, \( h_1 \) is the upper layer thickness, \( h_2 \) is the lower layer thickness, \( a \) is the wave amplitude, \( L_w \) is the half wavelength, and \( c \) is the wave phase speed. \( P_{ISW} \) and \( Re_{ISW} \) are the pressure gradient parameter and the wave Reynolds number, respectively (Aghsae, Boegman et al. 2012). \( D \) and \( L \) are the dissipation rate and the dissipation length scale. The upper layer fluid density was \( \rho_1 = 998 \text{ kg m}^{-3} \) for all experiments.

| Exp. | \( \rho_2 \) (kg m\(^{-3}\)) | \( h_1 \) (m) | \( h_2 \) (m) | \( a \) (m) | \( L_w \) (m) | \( c \) (m s\(^{-1}\)) | \( Re_{ISW} \) | \( P_{ISW} \) | \( E \) (J m\(^{-1}\)) | \( D \times 10^{-3} \) (W m\(^{-1}\)) | \( L \times 10^3 \) (m) | Unstable boundary layer |
|------|-----------------|------|------|-----|-------|--------|--------|--------|--------|----------------|----------------|----------------|------------------|
| 1    | 1090            | 0.085| 0.268| 0.083| 0.89  | 0.287  | 132    | 0.051  | 2.2    | 0.94            | 0.67           | No              |
| 2    | 1090            | 0.09 | 0.275| 0.086| 1.09  | 0.291  | 193    | 0.066  | 2.8    | 3.3             | 0.24           | Yes             |
| 3    | 1090            | 0.1  | 0.29 | 0.095| 1.4   | 0.3    | 235    | 0.059  | 3.8    | 4.9             | 0.23           | Yes             |
| 4    | 1090            | 0.103| 0.28 | 0.066| 1.08  | 0.298  | 146    | 0.047  | 2.1    | 0.91            | 0.67           | No              |
| 5    | 1090            | 0.108| 0.29 | 0.071| 1.15  | 0.293  | 166    | 0.05   | 2.5    | 1.2             | 0.56           | No              |
| 6    | 1090            | 0.118| 0.29 | 0.089| 1.51  | 0.296  | 239    | 0.053  | 3.9    | 3.4             | 0.34           | Yes             |
| 7    | 1090            | 0.125| 0.30 | 0.092| 1.51  | 0.302  | 225    | 0.051  | 4.0    | 2.5             | 0.48           | Yes             |
| 8    | 1096            | 0.06 | 0.30 | 0.073| 0.66  | 0.276  | 93     | 0.049  | 1.5    | 0.25            | 1.64           | No              |
| 9    | 1096            | 0.067| 0.30 | 0.082| 0.71  | 0.276  | 131    | 0.069  | 1.8    | 0.96            | 0.52           | No              |
| 10   | 1096            | 0.076| 0.295| 0.086| 1.01  | 0.293  | 145    | 0.046  | 2.6    | 1.3             | 0.58           | No              |
| 11   | 1096            | 0.087| 0.303| 0.116| 1.34  | 0.305  | 253    | 0.069  | 4.6    | 6.4             | 0.22           | Yes             |
| 12   | 1096            | 0.1  | 0.315| 0.12 | 1.37  | 0.304  | 265    | 0.072  | 4.7    | 6.7             | 0.21           | Yes             |
| 13   | 1096            | 0.11 | 0.327| 0.113| 1.38  | 0.299  | 255    | 0.067  | 4.5    | 5.3             | 0.29           | Yes             |
| 14   | 1096            | 0.12 | 0.338| 0.117| 1.54  | 0.305  | 260    | 0.056  | 5.2    | 6.3             | 0.25           | Yes             |
| 15   | 1096            | 0.13 | 0.338| 0.106| 1.55  | 0.308  | 258    | 0.058  | 4.5    | 5.1             | 0.27           | Yes             |
2.2.3 Energy calculations

Wave energy has historically been estimated by assuming an equipartition between kinetic energy ($KE$) and the readily measured available potential energy ($APE$) (Helfrich 1992; Michallet and Ivey 1999; Bourgault, Blokhina et al. 2007).

\[
APE = c g \Delta \rho \int_{t_0}^{t_1} \eta^2(t) \, dt \tag{2.6}
\]

Where $\Delta \rho = \rho_2 - \rho_1$ and $\eta(t)$ is the displacement of the pycnocline with time during the passage of the ISW. However, a solitary wave solution always has $KE > APE$ (Turkington, Eydeland et al. 1991); Lamb and Nguyen (Lamb and Nguyen 2009), found $KE/APE \sim 1.3$ confirming that the $KE$ flux exceeded the $APE$ flux. Estimates in the field show the ratio of $KE/APE$ to be large as 1.4 - 1.5 (Klymak, Pinkel et al. 2006; Moum and Smyth 2006). In the present study, we compute the $KE$ using a Dubreil-Jacotin-Long (DJL) solver (pers. comm. M. Stastna) that matches the $KE$ solution to the initial $APE$ of each experimental wave (Figure 2.2b). The observed dissipation lengthscale $L$ was experimentally computed from the linear decrease in wave energy against travelled distance (see below), following Michallet and Ivey ((Michallet and Ivey 1999); their Figure 7).

2.2.4 Dissipation calculations

Reynolds decomposition is challenging due to the unsteady nature of the flow and obtaining turbulence statistics through time, space, or ensemble averaging remains difficult (Boegman and Ivey 2009). The mean velocity field ($U, V, W$) was obtained by low-pass filtering the instantaneous velocity data ($u, v, v$) with a spectral cutoff between peaks associated with the ISW and instabilities (Aghsaee and Boegman 2015). The fluctuating components were then obtained through Reynolds decomposition (e.g., $u = U + u'$), where $(u', v', w')$ are the fluctuating components of the streamwise, spanwise and vertical velocity components (Figure 2.1).
In the bottom boundary layer, the velocity gradient from the bed, to the outer layer, leads to production of turbulent kinetic energy and dissipation $\varepsilon$ (Pope 2001). Assuming isotropy, $\varepsilon$ may be estimated from the three measured velocity gradients as (Piccirillo and Van Atta 1997):

$$\varepsilon = v \left[ 5 \left( \frac{\partial w'}{\partial z} \right)^2 + 5/2 \left( \frac{\partial u'}{\partial z} \right)^2 + 5/2 \left( \frac{\partial v'}{\partial z} \right)^2 \right]$$  \hspace{1cm} (2.7)

Where, $z$ is the vertical coordinate direction and $\left( \ldots \right)$ denotes time averaging.

The near-bed mean (filtered) velocity ($U$) profile follows the well-known logarithmic law when the flow is turbulent (Grant, Williams et al. 1984):

$$U_{\text{log}}(z) = \frac{1}{k} \ln \frac{z}{z_0}$$  \hspace{1cm} (2.8)

Here, $u_*$ is the friction velocity, $z_0$ is roughness length, $z$ is the height above the bed and $k = 0.41$ is the Von Kármán constant. To solve Eq. (2.8), we calculated $u_*$ using observed velocity data from the ADV. The vertical gradient of $u_*$ above the bed ($du_*/dz$) was then computed, locating the heights where the logarithmic profile is valid (constant $u_*$). A least-square fit of Eq. (2.8) provided time series of $u_*$ and $z_0$ (Lueck and Mudge 1997):

$$u_* = k z \frac{du}{dz}$$  \hspace{1cm} (2.9)

which were used to compute profiles of $\varepsilon$ within 5 mm of the bed, where the ADV suffered from a poor signal-to-noise ratio (Figure 2.2a) (Perlin, Moum et al. 2005).

$$\varepsilon = \frac{u_*^3}{kz}$$  \hspace{1cm} (2.10)

The dissipation values computed from Eqs. (2.7) and (2.10) were both qualitatively and quantitatively similar at $z= 5$ mm, where the data were smoothed using cubic splines. The measured dissipation ($\varepsilon$, W kg$^{-1}$) was converted to total observed dissipation ($D$, W m$^{-1}$) by integration over the waveform boundary layer $D = \Delta z \Delta t c \rho \int_{0}^{BBL} \int_{0}^{\lambda} \varepsilon dx dz$. 

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2.3 Results

2.3.1 Wave observations

An example of a stable ISW (Figure 2.3) shows little difference between the mean (filtered) velocity field (Figure 2.3b) and the instantaneous measurements (Figure 2.3a). Near-bed $\varepsilon$ (Figure 2.3d) peaks beneath the wave trough ($\sim 10^{-4} \text{ W kg}^{-1}$) and is small trailing the wave ($\sim 10^{-10} \text{ W kg}^{-1}$). These data show this experiment to have a stable ISW with little velocity fluctuation. Similarly, there was no sign of instability in the velocity fields of experiment 8 and no spanwise velocity fluctuations within the ISW-wake for experiments 1, 4, 5, 9, and 10 where $Re_{ISW} \leq 166$ (Table 2.1).
Figure 2.3 Example of a stable wave (Experiment 9). (a) Instantaneous streamwise velocity $u$, (b) mean horizontal velocity $U$, (c) mean vertical velocity $W$, and (d) dissipation rate profile ($\log_{10}$ scale). Velocity data is not given within 5 mm of the bed, due to the poor signal-to-noise ratio in this region. Here, dissipation was estimated using the log-law (Equation 2.10).

An example of an unstable ISW (Figure 2.4) shows the lower layer fluid to move from right to left beneath the leading half of the wave, with a favorable pressure gradient. After the wave trough an adverse pressure gradient is accompanied by three-dimensional velocity fluctuations (Aghsae and Boegman 2015) within a jet (Carr and Davies 2006) beneath the rear half of the wave (Figure 2.4a-c). As the flow decelerates in the adverse pressure gradient region, the boundary layer thickens by pushing fluid upward away from the bed flow due to flow separation, which indicates global instability within the boundary layer (e.g. Carr and Davies (Carr and Davies 2006); Aghsae et al. (Aghsae, Boegman et al. 2012)). The sudden burst and vortex shedding within the boundary layer
were accompanied by elevated $\varepsilon$ (Figure 2.4d). Transverse spanwise vortices form following streamwise vortices, confirming the three-dimensional nature of the instability.

The formation of secondary transverse instability greatly enhances dissipative losses (Fringer, Gerritsen et al. 2006; Aghsae, Boegman et al. 2012). The dissipation during the passage of the wave was approximately $10^{-7} - 10^{-5}$ W kg$^{-1}$, with the highest dissipation ($\sim 10^{-5}$ W kg$^{-1}$) for unstable waves within the boundary layer instability. The mean value of dissipation was $10^{-6}$ W kg$^{-1}$ the same order of magnitude compared with the field measurements (Moum, Farmer et al. 2003; Lien, Tang et al. 2005).

**Figure 2.4** Example of an unstable wave (Experiment 13). (a) Instantaneous streamwise velocity $u$, (b) instantaneous spanwise velocity $v$, (c) instantaneous vertical velocity $w$, and (d) $\varepsilon$ rate profile (log$_{10}$ scale).
2.3.2 Comparison of the present observations to published laboratory and numerical data

ISWs of depression become unstable when the boundary layer separates in the adverse pressure gradient region beneath the rear shoulder of the wave. This can lead to biogeochemical fluxes, such as sediment resuspension (Bogucki, Dickey et al. 1997; Boegman and Stastna 2019). Therefore, it is desirable to predict resuspension from readily measurable wave parameters (Diamessis and Redekopp). The laboratory data in Figure 2.5a, show the critical amplitude $a/H$ required for global instability to decrease as a function of increasing $Re_W$; as $a \rightarrow H$ conservation of volume will drive higher velocities beneath the wave trough and stronger pressure gradients. This is qualitatively consistent with two-dimensional DNS (Diamessis and Redekopp (Diamessis and Redekopp 2006); Aghsaee et al. (Aghsaee, Boegman et al. 2012)) and other laboratory data (Carr et al. (Carr, Fructus et al. 2008)). The critical amplitudes observed here are approximately ($a/H \sim 0.22-0.29$) and are similar to the previous experimental investigation (Carr et al. (Carr, Fructus et al. 2008)), but half that predicted by two-dimensional DNS (see discussion in Diamessis and Redekopp (Diamessis and Redekopp 2006) and Aghsaee et al. (Aghsaee, Boegman et al. 2012)). However, the $Re_W$ associated with instability for the laboratory data sets are not in quantitative agreement (Figure 2.5c).

Aghsaee et al. (Aghsaee, Boegman et al. 2012) proposed that the potential for instability and vortex shedding can be categorized according to $Re_{ISW}$ and $P_{ISW}$ at the point of separation ($P_{ISW} = 50 \times Re_{ISW}^{-1.3}$). The proposed 2D DNS-based criterion was compared to published laboratory experiments and field observations (Figure 2.5b, Table 2.2), showing that instability could occur for smaller values of $Re_{ISW}$ in the laboratory, compared to the 2D DNS and field observations. However, unlike the $a/H$ vs. $Re_{ISW}$ parameterization (Figure 2.5a), the laboratory data from the present study and Carr et al. (Carr, Fructus et al. 2008) show agreement in $Re_{ISW}$ vs. $P_{ISW}$ space, showing a transition from stable to unstable waves near $Re_{ISW} \sim 200$ (Figure 2.5d). The reasons for the discrepancies between the laboratory, 2D DNS and field data remain unclear.
(e.g., Figure 2.5b); however, the increased bandwidth between the large-scale energy containing and small-scale dissipative eddies at higher Reynolds number may play a role, as this has been shown to alter the ISW breaking mechanisms (Aghsae, Boegman et al. 2010). The lack of secondary spanwise eddies in the case of 2D DNS (Fringer, Gerritsen et al. 2006) has also been shown to alter breaking dynamics (Aghsae, Boegman et al. 2012).

**Figure 2.5** (a) Stable and unstable waves (present study) in $a/H$ vs. $Re_{ISW}$, showing laboratory observations and numerical simulations. The solid line is the power law best fit from the work of Diamessis and Redekopp (Diamessis and Redekopp 2006). (b) Stable and unstable waves in $Re_{ISW}$ vs. $P_{ISW}$ showing laboratory observations, numerical simulations, and field observations (Table 2.2). (c) Detail of laboratory data in panel a. (d) Detail of laboratory data in panel b.
Table 2.2 Parameters calculated from published field observations used to test the criterion shown in Figure 2.5b (Aghsaee, Boegman et al. 2012).

<table>
<thead>
<tr>
<th>Wave</th>
<th>$L_w$ (m)</th>
<th>$c$ (m/s)</th>
<th>$Re_{ISW}$</th>
<th>$P_{ISW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Vlasenko <em>et al.</em> (Vlasenko, Brandt et al. 2000), North of the Strait of Messina</td>
<td>300</td>
<td>1.00</td>
<td>3162</td>
<td>0.042</td>
</tr>
<tr>
<td>2: Vlasenko <em>et al.</em> (Vlasenko, Brandt et al. 2000), South of the Strait of Messina</td>
<td>200</td>
<td>1.20</td>
<td>1240</td>
<td>0.019</td>
</tr>
<tr>
<td>3: Bourgault &amp; Kelley (Bourgault and Kelley 2003), St. Lawrence River</td>
<td>124</td>
<td>0.70</td>
<td>2845</td>
<td>0.040</td>
</tr>
<tr>
<td>4: Moum <em>et al.</em> (Moum, Farmer et al. 2003), Oregon Shelf</td>
<td>150</td>
<td>0.60</td>
<td>2147</td>
<td>0.040</td>
</tr>
<tr>
<td>5: Orr &amp; Mignerey (Orr and Mignerey 2003), South China Sea</td>
<td>170</td>
<td>0.99</td>
<td>2901</td>
<td>0.047</td>
</tr>
<tr>
<td>6: Quaresma <em>et al.</em> (Quaresma, Vitorino et al. 2007), Portuguese shelf</td>
<td>110</td>
<td>0.34</td>
<td>3414</td>
<td>0.140</td>
</tr>
<tr>
<td>7: Shroyer <em>et al.</em> (Shroyer, Moum et al. 2009), New Jersey Coast</td>
<td>200</td>
<td>0.73</td>
<td>3571</td>
<td>0.018</td>
</tr>
</tbody>
</table>

2.3.3 Energetics

The $KE$ and $APE$ range from ~0.7–2.8 (J m$^{-1}$) of along wave-axis distance. In general, $KE \neq APE$ for the ISWs, with $KE$ approximately 15% greater than $APE$ (Figure 2.2b). The ratio $KE/APE \approx 1.15$ is smaller than observations of 1.4 and 2 (e.g., South China Sea and Oregon Continental Shelf) (Klymak, Pinkel et al. 2006; Moum, Klymak et al. 2007). Numerically, Lamb and Nguyen, (Barad and Fringer) found $KE/APE \leq 5$.

2.3.4 Dissipation

ISWs are commonly observed at offshore moorings that are spaced 10s of km apart, making it difficult to determine their ultimate fate as they shoal. We may estimate the dissipative energy loss as (e.g. Moum *et al.* (Moum, Farmer et al. 2007)):

$$D^* = \frac{cE}{L}$$  \hspace{1cm} (2.11)
over the water column depth and across the wave, where the decay rate \( \mu \approx c/L, E = KE + APE \) is the total wave energy and \( L \) is the dissipation lengthscale.

A linear regression (Figure 2.6a) of measured dissipation \( D \) [W m\(^{-1}\), from (2.7) and (2.10)] vs. \( E \) gives a decay rate of 0.0012 s\(^{-1}\), allowing parametrization of the dissipative loss, \( D^\ast = \mu \times E = 0.0012 \, \text{s}^{-1} \times E \). Figure 2.6b shows the decrease in \( E (E/E_i) \) with nondimensional distance travelled (Michellet and Ivey 1999) for most stable and unstable waves, compared to observations of an ISW propagating across the Oregon continental shelf (32 km onshore over a period of 12 h; Moum et al. (Moum, Farmer et al. 2007). The Oregon ISW loses energy at the same rate as the unstable laboratory wave; a surprising observation, given the Reynolds number discrepancies shown in Figure 2.5a. As expected, the unstable ISW lost energy more rapidly than the stable wave, which traveled further.

**Figure 2.6** (a) Observed dissipation \( D \) compared to total wave energy \( E \). (b) Decrease of wave energy, \( E/E_i \) against the nondimensional distance traveled (after Michellet and Ivey 1999) for stable (Experiment 1) and unstable (Experiment 16) ISWs. Also shown are field data from Moum et al (2007). \( E \) is the total energy scaled by the initial total energy \( E_i \) at location \( x \).

Following Aghsae et al. (Aghsae, Boegman et al. 2012), we plot \( D \) vs. \( Re_{ISW} \) and \( P_{ISW} \) (Figure 2.7). In general, \( D \leq 0.002 \, \text{W m}^{-1} \) for stable waves and \( \sim 0.002-0.007 \, \text{W m}^{-1} \) for unstable waves (Figure 2.7a). There is a positive correlation between \( D \) and \( Re_{ISW} \), showing enhanced
dissipation with increased Reynolds number – as expected. Dissipation was also regressed against wave amplitude (Diamessis and Redekopp 2006) and wavelength (Shroyer, Moum et al. 2010). The non-dimensional amplitude and wavelength show a positive correlations with dissipation (Figure 2.7c,d); larger waves have greater dissipation at higher $Re_{ISW}$ and the flow become more unstable in the bottom boundry layer.

Figure 2.7 Parameterization of measured dissipation. (a) $D$ vs. $Re_{ISW}$; (b) $D$ vs. $P_{ISW}$, (c) $D$ vs $\alpha/H$ and (d) $D$ vs $\lambda/H$.  

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2.4 Discussion

2.4.1 Instabilities in the bottom boundary layer

The separation of the stable and unstable data in $P_{ISW}$ vs $Re_{ISW}$ was in good agreement with those of Carr et al. (Carr, Fructus et al. 2008). However, 3D flow characteristics allow near-bed instability to form at lower pressure gradients and Reynolds numbers, compared to the 2D numerical simulations. Qualitative visualizations of the bottom region suggest instability occurs for experiments when $Re_{ISW} > \sim165$, but physical scale limitations in our laboratory do not allow generation of ISWs with $Re_{ISW} > \sim320$ (Figure 2.3, Figure 2.4 and Figure 2.5). The stable waves, with no instability in their boundary layer, represent the open ocean where waves travelled unhindered without losing significant energy before reaching the continental shelf. Waves with stable boundary layers eventually become unstable as they shoal on a sloping bottom (Aghsaee, Boegman et al. 2010).

2.4.2 Parameterization of dissipation and dissipation lengthscale

We parameterize the integrated dissipation and dissipation lengthscale in terms of wave and water column variables. The waves, whether stable or unstable, covered a wide range of stratification variables, with apparent correlations between the dissipation length scale being $L \sim \lambda$ (Shroyer et al. (Shroyer, Moum et al. 2010) proposed $L = \sim100\lambda$) and $L \sim \frac{cE}{D^*} \sim \frac{c}{\mu} \sim \frac{c}{0.0012}$ (from (2.11). Figure 2.7 suggests the parameterizations should be a function of $Re_{ISW}$, not $P_{ISW}$. There is a negative correlation between $L$ and $Re_{ISW}$ (Figure 2.8a), with an asymptote to $L \sim 100\lambda$ for unstable waves, in agreement with Shroyer et al (Shroyer, Moum et al. 2010), giving a parameterized dissipation lengthscale $L^* = 100\lambda + 2.5 \times 10^{10} \lambda Re_{ISW}^{-3.7}$. Increasing the Reynolds number causes more dissipation and reduces ISW travel. This suggests unstable waves will propagate ~100 wavelengths as they dissipate their energy from enhanced turbulence in the bottom boundary layer (Figure 2.4), but stable waves will propagate further, as commonly observed in the
open ocean. The variables $a$, $H$ and $c/\mu$ also show a negative correlation with $Re_{ISW}$; however, there is more scatter in the data (Figure 2.8). No direct relationship between $P_{ISW}$ and $D$ or $L$ was found (not shown).

Dissipation was parameterized from Eq. (2.11) as $c$ replacing the observed $L$ with the modelled value $L^*$ from Figure 2.8a. $D^*$ shows good agreement with $D$ (Figure 2.9b) over both stable and unstable waves. Using $L^* = 100\lambda$ overestimates dissipation for stable waves (Figure 2.9a).

**Figure 2.8** Normalized dissipation length scale vs $Re_{ISW}$ for stable and unstable waves: (a) observed dissipation length $L$ scaled with wavelength, $\lambda$. The horizontal line represents $L/\lambda=100$ (Shroyer, Moum et al. 2010), and the fit is $L^*/\lambda=100+2.5e10 \times Re_{ISW}^{-3.7}$; (b) dissipation length scaled
with wave amplitude, \( a \); (c) dissipation length scaled with total fluid depth, \( H \). The horizontal line represents 1000\( H \); (d) dissipation length scaled with \( c/\mu \) (from Eq. (2.11)).

**Figure 2.9** Parametrization of total dissipation using (a) \( D' = cE/100\lambda \) and (b) \( D' \approx cE/\lambda \times (100 + 2.5 \times 10^{10} \times Re_{ISW}^{-3.7}) \). Here \( c, E \) and \( \lambda \) are wave speed, total energy (\( E = KE + APE \)) and wavelength, respectively, \( Re_{ISW} \) is the momentum thickness Reynolds number \( Re_{ISW} \), based on the characteristics of the wave-induced bottom boundary layer at the wave trough.

### 2.5 Conclusions

Internal solitary wave energetics were used to model wave dissipation lengthscales in terms of the wavelength and momentum thickness Reynolds number \( L' = 100\lambda + 2.5 \times 10^{10} \lambda Re_{ISW}^{-3.7} \). The rate of dissipation of turbulent kinetic energy, in the bottom boundary layer, was larger for unstable ISWs (\( Re_{ISW} > \sim200 \)), relative to stable waves. Consequently, dissipation lengthscales for stable waves were \( O(100\lambda) \) wavelengths, in agreement with published observations; whereas stable waves propagated significantly further (> 1000\( \lambda \)). Future work should be directed at capturing missing dynamics such as 3D flow characteristics using volumetric PIV or DNS, which allow measurement of instantaneous velocity within a much larger spatial domain.
References


Chapter 3

Laboratory measurement of turbulent kinetic energy dissipation in shoaling internal solitary waves

3.1 Introduction

Internal solitary waves (ISWs) occur on nearly every coastline worldwide (Jackson 2007). They can travel 100s of kilometers from their generation sites, eventually shoaling on bottom topography (Helfrich and Melville 2006; Lamb 2014). ISWs are below the feasible grid-scale of most coastal ocean models (Dorostkar, Boegman et al. 2017) and difficult to resolve with field observations (Boegman and Stastna 2019). At low Reynolds number, ISW shoaling can be investigated through laboratory-scale experiments and Direct Numerical Simulations (DNS). However, resolving the no-slip bottom boundary condition with three-dimensional DNS remains a computational challenge (Aghsaee, Boegman et al. 2010). As a result, there have been many laboratory studies investigating ISW shoaling (Helfrich 1992; Michallet and Ivey 1999; Boegman, Ivey et al. 2005).

Large amplitude ISWs with strong currents, generate instability in the bottom boundary layer, beneath the waves (Bogucki, Dickey et al. 1997) and also through the pycnocline (Sandstrom, Elliot et al. 1989). Laboratory observations (Carr, Fructus et al. 2008; Aghsaee and Boegman 2015), numerical simulations (Diamessis and Redekopp 2006; Stastna and Lamb 2008) and ocean observations (Johnson, Weidemann et al. 2001; Nash and Moum 2005) show boundary layer instability to occur in the vicinity of the separated region directly beneath the rear shoulder of the wave, characterized by a sudden velocity burst leading to vortex shedding. As the waves shoal (Orr
and Mignerey 2003; Shroyer, Moum et al. 2009), instability causes wave energy to be lost to dissipation of turbulent kinetic energy (TKE; e.g., $\varepsilon \sim 10^{-7} - 10^{-6}$ W kg$^{-1}$; Lien, 2005) and turbulent mixing (e.g., $\sim 10^{-2}$ m$^2$ s$^{-1}$; Moum et al. 2003).

Acoustic Doppler current profilers (ADCPs) and profiling acoustic Doppler velocimeters (ADVs) provide a readily deployable means to evaluate $\varepsilon$ from velocity gradients, including the structure function (Wiles, Rippeth et al. 2006) and vertical component of the dissipation tensor (Piccirillo and Van Atta 1997; Saggio and Imberger 2001). However, the impact of anisotropy on the latter method remains unknown. Particle image velocimetry (PIV) provides another methodology to measure $\varepsilon$, from spatial derivatives of the fluctuating velocity components (Bertuccioli, Roth et al. 1999; Saarenrinne and Piirto 2000), over small domains ($\sim 1$ m) in the oceanic boundary layer (Doron, Bertuccioli et al. 2001), which are much smaller than the $\sim 1$ km ISW length-scale. Therefore, in high Reynolds number environmental flows, it remains impossible to simultaneously resolve the large-scale forcing dynamics and turbulent fluctuations at the Kolmogorov scale; however, these may be simultaneously measured in the lab (Boegman and Ivey 2009; Aghsaee and Boegman 2015).

Several techniques have been developed to utilize PIV measurements to estimate $\varepsilon$ (Variano and Cowen 2008) including: spectral fitting, structure function, and direct calculation using spatial velocity gradients in the dissipation tensor (Melville, Veron et al. 2002; Hult, Troy et al. 2011). These methods can also resolve the large-scale flow structure, albeit at low Reynolds number, and vorticity fields associated with wave shoaling and breaking, leading to turbulence production (Tanaka and Eaton 2007; Boegman and Ivey 2009). However, estimates of $\varepsilon$ using planar PIV measurements require an assumption of isotropy (Doron, Bertuccioli et al. 2001), because all the spatial gradients in the dissipation tensor cannot be simultaneously measured at the Kolmogorov scale. This is evident for breaking surface waves, where the experiments by Melville et al. (2002) found that TKE in the streamwise component was $\sim 50\%$ greater than in the vertical component,
but differences between $\varepsilon$ estimates, from the dissipation tensor and strain-rate estimates, were
within the scatter of the data (Veron and Melville 1999; Melville, Veron et al. 2002). Anisotropy
also occurs in field-scale geophysical boundary layers (Jabbari, Boegman et al. 2015; Jabbari,
Rouhi et al. 2016); however, $\varepsilon$ estimated from various methods (inertial fitting, structure function,
Batchelor fitting) converge to within $\sim 20\%$ of each other (Jabbari, Boegman et al. 2020).

In the present study, we determine $\varepsilon$ beneath a shoaling ISW using planar PIV in the lab. Results
when applying different assumptions of anisotropy are compared, with differences in measured $\varepsilon$
related to the characteristics of the flow field. These results should help guide computation and
interpretation of $\varepsilon$ from acoustic field observations.

3.2 Methods

3.2.1 Experimental methods:

The experiments were conducted in the glass-walled rectangular Internal Wave Flume (6 m
long $\times$ 0.75 m wide $\times$ 0.60 m deep) at the Queen’s University Coastal Engineering Research
Laboratory. The raw data were collected as part of an earlier study (Aghsaee and Boegman 2015).
A two-layer stratification was introduced by filling the lower layer fluid to the desired thickness
($h_2 = 0.295$ m) and adding salt (Cargill CMF), while stirring with a submersible pump, to reach
the desired lower layer density ($\rho_2 = 1050$ kg m$^{-3}$ as measured with an Anton-Paar densimeter
(DMA-6000). A thinner upper layer of fresh water was carefully added to a thickness of $h_1 = 0.1$ m using a floating sponge ($h_1 < h_2$ to generate ISWs of depression, total depth $H= h_1 + h_2$).
This method minimized mixing between two layers and generated a pycnocline of $\sim 2$ cm. ISWs
were then generated using the gate-release method (Kao, Pan et al. 1985; Michallet and Ivey 1999;
Carr, Fructus et al. 2008; Aghsaee and Boegman 2015). The amplitude and half wavelength of the
resulting waves were estimated to be 0.098 m and 1.03 m from image processing, respectively.
The two-dimensional planar velocity field (streamwise $x$ and vertical $z$) as well as the out-of-plane (spanwise $y$) component of the flow was measured using a LaVision Stereo Particle Image Velocimetry (PIV) system. Individual frames were captured at 14 Hz over a 40×40 cm field of view using two angled CCD cameras at 90° (Imager Pro X 4M, 14-bit image depth, 2112×2072-pixel resolution) equipped with 50 mm Nikon lenses and bandpass 532 nm filters. A laser light sheet was generated by pulsing an Nd:YAG Dual Cavity laser (Big Sky PIV 200; 10 ns exposure and 10 $\mu$s between pulses) onto a cylindrical lens arrangement. The light sheet illuminated locally seeded Eliokem pliolite particles, with an average diameter of 100 $\mu$m diameter and density of 1050 kg m$^{-3}$ (Figure 3.1). The PIV algorithm employed an adaptive multi-pass procedure where an interrogation window with 75% overlap performed two passes at 32 x 32 pixels. Vectors were smoothed with the LaVision software (DaVis 8.0.1) using a sliding-average nonlinear filter. The resultant velocity estimate was thus calculated at a resolution of $\sim$1.51 mm, which roughly matches the Kolmogorov length scale $\sim$1.50 mm using $\varepsilon$$\sim$2 $\times$ 10$^{-7}$ W kg$^{-1}$ (see below).

We compare the Stereo PIV results to those from a similar experiment in Aghsaee and Boegman (2015; Exp. 5), who measured 3D velocity profiles under ISWs using a profiling Acoustic
Doppler Velocimeter (ADV; Nortek Vectrino II; 1 mm s\(^{-1}\) resolution and ±5% measurement error) located 2.5 meters downstream from the wave generating gate. The ADV recorded at 25 Hz, over a 3.3 cm profile with 1 mm resolution, enabling \(\varepsilon\) to be computed, although their study focussed on sediment resuspension not dissipation. The experiments were chosen to match wave Reynolds number \(Re_{ISW} = |U_2| \sqrt{L_w/(\nu(|U_2| + c))} \approx 165\) (Aghsae et al. 2012; Table 2.1). Here \(U_2 = \frac{-c a}{h_2+a}\) is the maximum horizontal velocity at the wave trough. The ISW nonlinear phase speed \(c\) and wave amplitude \(a\) were calculated from video images. The characteristic horizontal wavelength \(L_w\), maybe estimated either from the fluid volume behind the gate, in the event there are no trailing waves, or by integrating the pycnocline displacement and dividing by the wave amplitude (Michallet and Ivey 1999); the actual wavelength is \(\lambda \approx 2L_w\) (Djordjevic and Redekopp 1978; Ostrovsky and Stepanyants 1989; Aghsae, Boegman et al. 2012; Aghsae and Boegman 2015).

Table 3.1 Experimental Parameters where \(\rho_2\) is the lower layer fluid density, \(h_1\) is the upper layer thickness, \(h_2\) is the lower layer thickness, \(a\) is the wave amplitude, \(L_w\) is the half wave length, \(c\) is the nonlinear wave phase speed, \(Re_{ISW}\) is the wave Reynolds number. The upper layer fluid density \(\rho_1\) was 998 (kg m\(^{-3}\)) for all experiments.

<table>
<thead>
<tr>
<th>Exp</th>
<th>(\rho_2) (kg m(^{-3}))</th>
<th>(h_1) (m)</th>
<th>(h_2) (m)</th>
<th>(a) (m)</th>
<th>(L_w) (m)</th>
<th>(c) (m s(^{-1}))</th>
<th>(Re_{ISW})</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIV</td>
<td>1050</td>
<td>0.1</td>
<td>0.295</td>
<td>0.098</td>
<td>1.03</td>
<td>0.2</td>
<td>175</td>
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<tr>
<td>ADV</td>
<td>1090</td>
<td>0.108</td>
<td>0.29</td>
<td>0.071</td>
<td>1.15</td>
<td>0.29</td>
<td>165</td>
</tr>
</tbody>
</table>

3.2.2 Analytical methods:

3.2.2.1 Reynolds decomposition:

Production of turbulent eddies in the bottom boundary layer was observed and \(\varepsilon\) computed from individual PIV vector maps. Different methods of estimating \(\varepsilon\), with different assumptions of isotropy are compared. A Reynolds decomposition was first applied to the instantaneous velocity.
to decompose into the mean and fluctuating components (Cantwell and Coles 1983; Boegman and Ivey 2009; Hult, Troy et al. 2011):

\[ u_i(x, z, t) = U_i(x, z) + u_i'(x, z, t) \]  

(3.1)

where the terms \( U_i \) and \( u_i' \) are, respectively, the mean component and the random turbulent component in direction \( i \). The decomposition was accomplished by high-pass filtering the energetic small-scale fluctuations, which occur at the Kolmogorov timescale \( T_\eta = (\nu/\varepsilon)^{1/2} \sim (10^{-6}/2 \times 10^{-7})^{1/2} \sim 2 \text{ s.} \)

### 3.2.2.2 Dissipation from PIV data:

Single-camera 2D-PIV is limited to the measurement of the two in-plane velocity components. Stereo-PIV adds a second camera to enable measurement of the in-plane transverse velocity, by using the two viewing angles. Stereo PIV experiments, therefore, provide the ability to investigate the spanwise variability of near-bed instability beneath ISWs (Prasad and Adrian 1993; Wienke 2015) and to measure the gradients of the spanwise velocity \( \frac{\partial v}{\partial x}, \frac{\partial v}{\partial z} \).

#### 3.2.2.2.1 Approach one: Direct method

For direct calculation, the total dissipation of TKE per unit mass can be written as (Hinze 1972):

\[ \varepsilon = \nu \left( \frac{\partial u_i'}{\partial x_i} \left( \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_i'}{\partial x_j} \right) \right) \]  

(3.2)

where \( \nu \) is the kinematic viscosity, double indices indicate summation over all three coordinate directions and the overbar denotes a time or space average.

In the three-dimensional case without any assumptions, each term on the right-hand-side of Eq. (3.2) is listed in Figure 3.2.
\[
\begin{array}{c|c|c|c}
 j = 1 & i = 1 & i = 2 & i = 3 \\
\hline
 j = 2 & 2 \left( \frac{\partial u'}{\partial x} \right)^2 & \left( \frac{\partial v'}{\partial x} \right)^2 + \frac{\partial u' \partial v'}{\partial y \partial x} & \left( \frac{\partial w'}{\partial x} \right)^2 + \frac{\partial u' \partial w'}{\partial z \partial x} \\
\hline
 j = 3 & \left( \frac{\partial u'}{\partial y} \right)^2 + \frac{\partial u' \partial w'}{\partial z \partial y} & 2 \left( \frac{\partial v'}{\partial y} \right)^2 + \frac{\partial v' \partial w'}{\partial z \partial y} & 2 \left( \frac{\partial w'}{\partial z} \right)^2 \\
\end{array}
\]

Directly calculated from \( u', w' \)

From continuity

Assuming similar shear magnitude

**Figure 3.2** Calculation of each term in the definition of dissipation tensor Eq. (3.2). The \( \overline{\ldots} \) is time averaged fluctuation velocity of \( u,v,w \) gradients in each \( x,y,z \) direction.

Using planar PIV, we can measure five of the components \( \left( \frac{\partial u'}{\partial x} \right)^2, \left( \frac{\partial w'}{\partial x} \right)^2, \left( \frac{\partial u'}{\partial z} \right)^2, \left( \frac{\partial u' \partial w'}{\partial z \partial x} \right) \) in Eq. (3.2) directly. This allows the terms related to the streamwise and vertical directions only (blue terms) to be determined. The term related to \( \frac{\partial v'}{\partial y} \) (green term) can be obtained from continuity:

\[
\left( \frac{\partial v'}{\partial y} \right)^2 = \left( - \frac{\partial u'}{\partial x} - \frac{\partial w'}{\partial z} \right)^2
\]

(3.3)

\[
\left( - \frac{\partial u'}{\partial x} - \frac{\partial w'}{\partial z} \right)^2 = \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 + 2 \frac{\partial u' \partial w'}{\partial x \partial z}
\]

(3.4)

For the terms related to the lateral direction \( (i,j = 2, \text{red terms}) \), the assumption is that all lateral fluctuations are of similar average magnitude. Stereo PIV allows measurement some of these terms directly so ultimately, can check if \( \left( \frac{\partial v'}{\partial x} \right)^2 = \left( \frac{\partial v'}{\partial z} \right)^2 \) (Doron, Bertuccioli et al. 2001).

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\[ \left( \frac{\partial u'}{\partial y} \right)^2 = \left( \frac{\partial w'}{\partial y} \right)^2 = \left( \frac{\partial v'}{\partial x} \right)^2 = \left( \frac{\partial v'}{\partial z} \right)^2 \]

\[ = \frac{1}{2} \left[ \left( \frac{\partial u'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial x} \right)^2 \right] \]

\[ \left( \frac{\partial u'}{\partial y} \right)^2 \frac{\partial v'}{\partial x} - \left( \frac{\partial v'}{\partial y} \right)^2 \frac{\partial u'}{\partial x} = \left( \frac{\partial u'}{\partial y} \right)^2 \frac{\partial w'}{\partial z} - \left( \frac{\partial w'}{\partial y} \right)^2 \frac{\partial u'}{\partial z} \]

\[ = \frac{1}{2} \left( \frac{\partial u'}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial w'}{\partial x} \right)^2 \]

\[ = \frac{1}{2} \left( \frac{\partial u'}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial w'}{\partial x} \right)^2 \]

\[ = \frac{1}{2} \left( \frac{\partial u'}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial w'}{\partial x} \right)^2 \]

(3.5)

In this case, the assumption that the cross-stream gradients are of the same average magnitude as the measured in-plane spatial gradients are supported by similar magnitudes of the measured gradients, \( \frac{\partial u'}{\partial z} \) and \( \frac{\partial w'}{\partial x} \) (see below).

All the directly measured gradients are used in the calculations. Therefore, the assumptions leading to Eq. (3.6) are weaker than those involved in isotropy-based methods described in the following approach (Fincham, Maxworthy et al. 1996; Doron, Bertuccioli et al. 2001). Introducing these assumptions into Eq. (3.2), direct 2D estimate of the dissipation rate of TKE is obtained:

\[ \varepsilon_D = 3\nu \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 + \left( \frac{\partial v'}{\partial z} \right)^2 + \left( \frac{\partial w'}{\partial x} \right)^2 + 2 \left( \frac{\partial u'}{\partial z} \frac{\partial v'}{\partial x} \right) + 2 \left( \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial z} \right) \right] \]

(3.6)

In the stereo-PIV set up we also measured \( \frac{\partial u'}{\partial x} \) and \( \frac{\partial v'}{\partial z} \) and by applying the same assumptions as applied in the direct method, a closer estimate to Eq. (3.2) is obtained:

\[ \varepsilon_S = 3\nu \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial z} \right)^2 + \frac{2}{3} \left( \frac{\partial u'}{\partial z} \right)^2 + \frac{2}{3} \left( \frac{\partial w'}{\partial x} \right)^2 + \frac{1}{3} \left( \frac{\partial v'}{\partial x} \right)^2 + \frac{1}{3} \left( \frac{\partial v'}{\partial z} \right)^2 + \frac{2}{3} \left( \frac{\partial u'}{\partial z} \frac{\partial v'}{\partial x} \right) + \frac{2}{3} \left( \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial z} \right) \right] \]

(3.7)

3.2.2.2.2 Approach two: isotropic assumption

Another way to estimate \( \varepsilon \) is based on a stricter assumption of isotropic turbulence or homogeneous isotropic turbulence. For isotropic turbulence, the terms in Eqs. (3.3) and (3.4) add up to 8/15 of the total dissipation. Additional terms are estimated, assuming if that all lateral
fluctuations have similar average magnitudes. There is no preferred spatial direction, and the relation is simplified to the two-dimensional form which gives:

$$\varepsilon = 6 \nu \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial y} \right)^2 + \frac{\partial^2 u'}{\partial y \partial x} \right]$$

(3.8)

From continuity, and then following Taylor (1935) \((\frac{\partial u'}{\partial x})^2 = \left( \frac{\partial v'}{\partial y} \right)^2 = \left( \frac{\partial w'}{\partial z} \right)^2 \) and \((\frac{\partial u'}{\partial y})^2 = \left( \frac{\partial w'}{\partial z} \right)^2 = \frac{\partial^2 u'}{\partial y \partial x} ; \)

\[
\begin{align*}
\left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 &= -2 \left( \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial y} + \frac{\partial v'}{\partial y} \frac{\partial w'}{\partial z} + \frac{\partial w'}{\partial z} \frac{\partial u'}{\partial x} \right) \\
\end{align*}
\]

(3.9)

Applying isotropy, and integrating the dissipation over a closed volume

\[
\left( \frac{\partial u'}{\partial x} \right)^2 = -2 \left( \frac{\partial u'}{\partial x} \frac{\partial v'}{\partial y} \right) = -2 \left( \frac{\partial u'}{\partial y} \frac{\partial w'}{\partial z} \right) \\
\]

(3.10)

Eventually pure straining can be related to simple shear by rotating the coordinate axis and applying isotropy to the resulting partial differential terms, for this reason:

\[
\left( \frac{\partial u'}{\partial x} \right)^2 = \frac{1}{2} \left( \frac{\partial v'}{\partial x} \right)^2 \\
\]

(3.11)

Substituting (10) and (11) into (7) gives the total dissipation in terms of \(\left( \frac{\partial u'}{\partial x} \right)^2, \left( \frac{\partial w'}{\partial z} \right)^2\) and \(\frac{\partial^2 u'}{\partial y \partial x}\)

$$\varepsilon = 15 \nu \left( \frac{\partial u'}{\partial x} \right)^2 = 7.5 \nu \left( \frac{\partial u'}{\partial x} \right)^2$$

(3.12)

where i, j = 1, 2, 3 and i≠j. This leads to four ways of calculating \(\varepsilon\) based on the 2D turbulent data as:

$$\varepsilon_{iso,1} = 15 \nu \left( \frac{\partial u'}{\partial x} \right)^2$$

(3.13)

$$\varepsilon_{iso,2} = 7.5 \nu \left( \frac{\partial u'}{\partial z} \right)^2$$

(3.14)
\[ \epsilon_{iso,3} = 7.5 \nu \left( \frac{\partial w'}{\partial x} \right)^2 \]  \hspace{1cm} (3.15) \\
\[ \epsilon_{iso,4} = 15 \nu \left( \frac{\partial w'}{\partial z} \right)^2 \]  \hspace{1cm} (3.16)

In the stereo-PIV set up we also measured \( \left( \frac{\partial v'}{\partial x} \right)^2 \) and \( \left( \frac{\partial v'}{\partial z} \right)^2 \) and by applying the same assumptions as in the isotropy 2D method, to Eq. (3.11):

\[ \epsilon_{iso,5} = 7.5 \nu \left( \frac{\partial v'}{\partial x} \right)^2 \]  \hspace{1cm} (3.17) \\
\[ \epsilon_{iso,6} = 15 \nu \left( \frac{\partial v'}{\partial z} \right)^2 \]  \hspace{1cm} (3.18)

3.2.2.3 Dissipation from ADV data:

The PIV results were compared to dissipation computed from the profiling Vectrino II ADV using vertical velocity gradients (Piccirillo and Van Atta 1997; Saggio and Imberger 2001). Here only \( \left( \frac{\partial w'}{\partial z} \right)^2 \), \( \left( \frac{\partial v'}{\partial z} \right)^2 \) and \( \left( \frac{\partial u'}{\partial z} \right)^2 \) are measured then assuming isotropy, dissipation may be estimated based on the velocity gradients in the vertical direction only:

\[ \epsilon = \nu \left[ 5 \left( \frac{\partial w'}{\partial z} \right)^2 + 5/2 \left( \frac{\partial v'}{\partial z} \right)^2 + 5/2 \left( \frac{\partial u'}{\partial z} \right)^2 \right] \]  \hspace{1cm} (3.19)

3.2.2.4 Dissipation from log-law data:

Dissipation was also computed by assuming a logarithmic velocity profile, according to the law-of-the-wall:

\[ \epsilon \sim \frac{u_*^3}{kz} \]  \hspace{1cm} (3.20)

where \( u_* \) is the friction velocity and \( \kappa = 0.41 \) is the Von Kármán constant, and \( z \) is the height above the bed.
The log-law uses a least-squares fit to a measured mean ADV velocity profile to estimate the friction velocity (Grant, Williams et al. 1984; Huntley and Hazen 1988). Data with poor signal-to-noise ratios, near the bed were neglected (bottom ~ 5 mm; Aghsaee and Boegman 2015).

3.3 Results

3.3.1 Flow field:

Each generated wave travels from left to right passing through the field of view before reflecting from the end wall. The waves travel back and forth until they lose their energy via viscous damping (Michallet and Ivey 1999; Aghsaee and Boegman 2015). Only the part of the data corresponding to the bottom boundary layer is shown, from the instant when the lower-layer fluid starts to pass through the wave trough, to the point when the separation region becomes fully formed and large-scale eddies are evident near the bed.

To investigate the near-bed instability and vortex shedding, within ISWs boundary layer, Figure 3.3 shows the instantaneous velocity field in the x-direction (streamwise velocity), y-direction (transversal velocity) and z-direction (vertical velocity). The near-bed instability and separation bubble region are shown as the solitary wave of depression propagates from left to right accompanied with vortex production. Strong velocities are under the wave trough, near the bottom boundary layer, in the negative x-direction and strong vertical velocities are after the wave, in the z-direction, where flow separation and instability occur. The instantaneous velocity components show \( u > w \) and \( v \). The horizontal velocity field shows a ~2 cm near bed streamwise jet and shear-layer (Carr and Davies 2006), which becomes unstable to periodic vertical ejections 2 to 10 cm in height; roughly is 1/3 of the total water column. The ejections have three-dimensional structure, with 1–2 cm components in x-direction across their height (Figure 3.3). These are consistent with ADV timeseries observations (Aghsaee and Boegman 2015).
Figure 3.3 Instantaneous velocity vector fields $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ nondimensionalized by $c_o$. The nondimensional time showing the location of the spatial images along the ISW waveform is shown in the inset panels as a vertical line at $(t \times c_o/\alpha) = 20$ and 25. Here $c_o$ is the phase speed and $\alpha$ is the wave amplitude. The colour bars show $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ and vectors are $\mathbf{u}$ and $\mathbf{w}$.

3.3.2 Dissipation of TKE

The dissipation of TKE ($\varepsilon$) was calculated using the various forms of equation (Figure 3.4). Instantaneous fields are shown at the times of interest corresponding to Figure 3.3. Despite the numerous assumptions involved in some of the estimation methods, all the methods reproduce the same order-of-magnitude estimates for the dissipation rate ($\varepsilon \sim 10^{-7} - 10^{-6}$ W kg$^{-1}$) above the jet; $z > \sim 0.05$ m from the bottom boundary (Figure 3.4). Weak dissipation of energy is noticed in instantaneous flows using the other assumption of isotropic turbulence. Using the direct method (Figure 3.4m,n; Eq. (3.6)), $\varepsilon$ is elevated ($\sim 10^{-7}$ W kg$^{-1}$) within the instabilities and vortex shedding occurring after the jet. The TKE dissipation estimate $\varepsilon_S$ obtained from stereo-PIV (Figure 3.4o,p; Eq. (3.7)) are slightly larger (> 15%) than the direct 2D method $\varepsilon_D$ (Figure 3.4m,n; Eq. (3.6)), suggesting that the cross-stream gradients are slightly larger than the measured in-plane
spatial gradients. The dissipation rate obtained from isotropy assumptions by Eq. (3.13) are typically higher than the direct method Eq. (3.6) except Eq. (3.15). The dissipation calculated from Eq. (3.14) gives the highest values with elevated $\varepsilon \sim 10^{-4}$ W kg$^{-1}$ in the jet-driven shear layer near the bed (Figure 3.4c,d). This strong vertical shear of the streamwise $u$-velocity in the $z$-direction corresponds to flow separation within boundary layer.

The accuracy of the isotropy-related assumptions can be assessed by comparing the magnitudes of available velocity gradients ($\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial z}$, $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial z}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial z}$) where matches in high dissipation accompany a large change in the velocity or shear between two layers. Streamwise $u$-velocity and vertical $w$-velocity gradients are not in same average magnitude as the measured transversal $v$-velocity gradients. We observed that above the jet, the instantaneous velocity gradients are considerably smaller, therefore dissipation within the boundary layer contributes to most TKE dissipation over the flat bottom. The maximum velocity gradient $\frac{\partial u}{\partial z}$ was over 0.02 m s$^{-1}$ observed near the bed at $z \leq 0.01$ m (Figure 3.5). The high dissipation from Eq. (3.14) in Figure 3.4(c),(d) is shown to directly result from vertical shear (Figure 3.5c,d). The vertical bursts in $w'$ (Figure 3.4c,d) also lead to enhanced shear (Figure 3.5e,f) that drives dissipation (Figure 3.4,f) through Eq. (3.15). Different patterns (or wavelengths) are observed in the dissipation maps and these correspond to the direction of the velocity gradients taken in the dissipation tensor. Elongated streaks in the $x$ (Figure 4c,d,g,h) and $z$ (Figure 4a,b,e,f) directions result from derivatives in the $z$ (Equation 3.14 and 3.16) and $x$ (Equations 3.13 and 3.15) directions, respectively.
Figure 3.4 Instantaneous dissipation rate of TKE $\varepsilon$ (W kg$^{-1}$) for isotropic turbulence $\varepsilon_{\text{iso},1-6}$ estimated with Eq. (3.13), 2D-direct method $\varepsilon_D$ with Eq. (3.6) and from Stereo-PIV method $\varepsilon_S$ from Eq. (3.7) at the two chosen time associated with images shown in the inset panels $(t \times c_o/\alpha) = 20$ and 25. Note that the color bars are log10 of $\varepsilon$ and $c_o$ is the phase speed and $\alpha$ is the wave amplitude.
Figure 3.5 Available instantaneous streamwise and vertical-velocity gradient $\frac{\partial u'}{\partial x}, \frac{\partial u'}{\partial z}$ (from 2D-PIV) and also transversal-velocity gradients $\frac{\partial v'}{\partial x}, \frac{\partial v'}{\partial z}$ (Stereo-PIV) at the two chosen time instants of $(t \times c_o/\alpha) = 20$ and 25 associated with images shown in the inset panels. Here $c_o$ is the phase speed and $\alpha$ is the wave’s amplitude.
3.4 Discussion

3.4.1 Estimations of temporal evolution of TKE dissipation

The dissipation estimations from stereo-PIV, ADV and log-law were compared for an unstable wave (Exp. 5; Aghsae and Boegman, 2015) within the boundary layer. The temporal evolution of the spatial-average magnitudes, of these derivatives, and the TKE dissipation rate estimated for different methods are compared (Figure 3.6). There are multiple peaks in time series of spatial-average dissipation associated with peaks in measured spatial-average shear magnitudes obtained at same times \( t \times c_o/a=15 \), \( t \times c_o/a=18 \) (3.18); \( t \times c_o/a=20 \) Eq. (3.20). The sudden increase in shear (Figure 3.6b) confirms that dissipation obtained from isotropy are overestimated compared to direct methods Eq. (3.6) and (3.7) with discrepancy as high as 80%. The data suggests a typical value of \( \varepsilon \) calculated from Eq. (3.15) and or Eq. (3.19) from Vectrino II ADV were much smaller than other estimates because \( \frac{\partial w'}{\partial x}, \frac{\partial w'}{\partial z} \) are small compared to other derivatives (Figure 3.6a). Eq. (3.14) and (3.18) give much higher dissipation because the shear values are higher.

![Figure 3.6](image)

**Figure 3.6** (a) Time series of spatial-average dissipation obtained from direct calculation Eqs. (3.6) and (3.7) or from isotropic-turbulent assumption Eqs. (3.13), Vectrino II ADV Eq. (3.19) and law of the wall Eq. (3.20). (b) Time series of available measured spatial-average shear magnitudes.
3.4.2 Comparison with dissipation observations in the coastal ocean

When we compared dissipation estimated from isotropy assumption, that is small to cause significant dissipation and instabilities (Lin, Vorobieff et al. 1995). Our measurements of the turbulent dissipation rate are consistent with earlier measurements in field observations using law of the wall method within the range estimated from measurements of the turbulence dissipation in an ISW packet, estimating a mean dissipation of $10^{-7} - 10^{-6}$ W kg$^{-1}$ (Moum, Farmer et al. 2007; Shroyer, Moum et al. 2010; Woodson, Barth et al. 2011). However, based on our results the dissipation rate from ADV data, Eq. (3.19) underestimated dissipation by 40%, and the log-law data (Eq. (3.20)) overestimated by nearly 24%. The discrepancies, compared to the corresponding direct method, are consistent with the ADV data measuring fewer of the secondary dissipation losses as result of transverse and vertical fluctuations. However, log-law assumes a steady turbulent flow, which is different from the unsteady ISW, with bursts of shear and associated bursts of dissipation.

The direct methods give more reasonable estimates of dissipation rates as they assume that turbulence is locally axisymmetric (Figure 3.5, Figure 3.6). The best $\varepsilon$ estimates are provided by the direct methods (Eq. (3.6), (3.7)), however Eq. (3.7) give a closer estimate than Eq. (3.6) to three-dimensional since it involves the least assumption of isotropy (e.g. measured $\frac{\partial u'}{\partial x} \frac{\partial u'}{\partial z}$). These observations agree with field observation taken at the bottom of the coastal ocean (Doron, Bertucioeli et al. 2001) and as well as laboratory observation (Fincham, Maxworthy et al. 1996).

Multiple peaks in $\varepsilon$ occur as a result of vorticity that grows from the instability. Three-dimensional motions will develop only after overturns are created because the initial instabilities are two-dimensional (Fringer, Gerritsen et al. 2006). The spanwise longitudinal rolls account for at least $\sim 20\%$ of the dissipation loss (Figure 3.3e,f). Dissipation obtained from stereo-PIV (Eq. (3.6), (3.7)) and from the ADV (Eq. (3.20)) shows significant loss occurring after the wave passes and the jet
and vortex shedding form. We could not see this increase with other isotropy methods and or for law of the wall method (Eq. (3.19)) because of a lack of secondary spanwise instabilities (Fringer, Gerritsen et al. 2006; Aghsae, Boegman et al. 2012). The formation of secondary transverse instability observed after jet enhances dissipative losses that also is in agreement with numerical simulations of ISW of depression propagating over a flat boundary (Diamessis and Redekopp 2006; Fringer, Gerritsen et al. 2006; Aghsae, Boegman et al. 2012) and recent experimental study of boundary layer unstable waves (Carr, Fructus et al. 2008; Aghsae and Boegman 2015).

3.5 Conclusion

I have experimentally investigated how dissipation changes, in the boundary layer beneath an ISW, depending on which components of the dissipation tensor are included and the associate’s assumptions of isotropy. Dissipation estimates obtained using various methods (Eq. (3.6),7 and Eq. 13-20), part of which rely on assumptions of isotropy, provide same order-of-magnitude values. The rate of dissipation when depending on integration of one-dimensional spectrum of velocity fluctuations (homogeneous isotropic turbulence assumption) using streamwise and vertical velocities are over and underestimated (~5-80%) as well as ADV (Eq. (3.19); 40%) data and log-law data (Eq. (3.20); roughly 24%).

The results will be also informative to field oceanographers, who often only measure velocity in one direction with an ADCP and not 2D or 3D. However, using of vertical velocity \( \frac{\partial w'}{\partial z} \) (Eq. (3.16); 5%) and transversal \( \frac{\partial v'}{\partial x} \) (Eq. (3.17); 7%) is better than streamwise velocity (u). The assumption of horizontal homogeneity of the turbulence (Eq. (3.13), (3.14)) introduces 30-50% errors in the dissipation. Comparison with estimates based on measured terms of the dissipation tensor provided by the 2-D (Eq. (3.6)) and Stereo-PIV data (Eq. (3.7)) yields differences of less
than 15%. Future work, should compare these 2D dissipation estimates to those obtained from the full 3D dissipations tensor from DNS or volumetric PIV.
References


Chapter 4

Energetics of internal solitary wave shoaling and bolus formation on a uniform slope

4.1 Introduction

Internal solitary waves (ISWs) are generated and propagate on nearly every coastline (Jackson, Da Silva et al. 2012; Li, Jackson et al. 2013), dissipating their energy as they shoal. Propagation of ISWs between ocean moorings that are ~100 km apart is difficult and ISWs are below the grid-scale of most coastal ocean models (e.g., 22-454m) (Aghsae, Boegman et al. 2010; Dorostkar, Boegman et al. 2017; Boegman and Stastna 2019); therefore, it is difficult to resolve them in three-dimensions with observations or numerical simulations. Field observations (e.g., New Jersey shelf, Oregon continental shelf, South China Sea and Gulf of Saint Lawrence) show gradual bottom slopes which are predicted to generate boluses through the fission of ISWs as they shoal, which is the dominant type of breaking in the nature. Lab experiments (Helfrich 1992; Michallet and Ivey 1999; Boegman, Ivey et al. 2005; Boegman and Ivey 2009; Sutherland 2013) have been performed on steeper bottom slopes 0.07 < S < 0.4 and fission will not occur (Figure 4.1). Therefore, the physical limitations from flume length have resulted in ISWs shoaling experiments that do not physically reproduce oceanic conditions.

The dynamics of breaking ISWs have been studied extensively in both the laboratory (Imberger and Ivey 1991; Hult, Troy et al. 2011) and numerical simulations (Holt, Koseff et al. 1992; Aghsae, Boegman et al. 2010). High-resolution 2D simulations of breaking of ISWs of depression shoaling upon closed slopes in a two-layer density field indicated four different breaking processes (e.g. collapsing, plunging, surging and fission). The ISW breaker type can be classified according to the bottom slope, S and wave slope ($S_w = a/\lambda$) where $a$ and $\lambda$ are the ISW amplitude and
wavelength, respectively (Figure 4.1). Collapsing, plunging and surging breakers occurred for moderate and steep wave slopes. However, over mild slopes ($S < -0.5S_w + 0.1$), an incident wave of depression will degenerate through fission along the slope (Wallace and Wilkinson 1988; Venayagamoorthy and Fringer 2007). As an ISW shoals over uniform slope, the water depth changes slowly resulting in slow steeping of the rear face of the wave, which is insufficient to cause overturning. Therefore, nonlinearity gradually shifts to zero before the turning point, then changes sign to form a wave of elevation and increases rapidly thereafter. These dynamics, in the region of positive polarity, move the wave of elevation onshore and this process of wave generation through fission continues periodically. The ratio of $S$ to $S_w$, or the internal Iribarren number, should predict fission by regulating the rate of nonlinear steepening.

Numerical investigations (Bourgault, Blokhina et al. 2007; Lamb 2014) and laboratory experiments (Helfrich 1992; Michallet and Ivey 1999; Boegman, Ivey et al. 2005) indicate that ISWs can release significant energy in the shoaling process. Experiments show that an ISW of depression, on a uniform slope, will break and produce multiple boluses and the number of boluses generated increases as the incident ISW wavelength, $\lambda$ is decreased (Helfrich 1992; Arthur and Fringer 2016). On the shelf, a bolus with a trapped core of fluid will propagate up the slope until it is ultimately dissipated through three-dimensional instability with elevated dissipation levels (e.g., $10^{-5} – 10^{-4}$ W kg$^{-1}$) (Grimshaw, Pelinovsky et al. 1999; Klymak and Moun 2003; Hosegood and van Haren 2004; Scoti and Pineda 2004; Moun, Klymak et al. 2007; Venayagamoorthy and Fringer 2007; Nam, Kim et al. 2011). The core continually drains fluid at the back of the wave, causing circulation, biogeochemical cycling, sediment transport and resuspension in the coastal ocean, which is known to be crucial to the coastal ecosystem (Helfrich 1992; Scoti and Pineda 2004; Wang, Dai et al. 2007). The interaction of the incident wave with the return flow, from the preceding wave, may cause wave breaking. Therefore, in the vicinity of the shelf break, the rear face can become unstable due to interfacial shear and localized vertical mixing results, with
dissipation occurring because of strong bottom interactions (Kao, Pan et al. 1985; Wallace and Wilkinson 1988; Helfrich and Melville 2006).

Figure 4.1 Regime diagram showing ISW breaker types according to wave slope, $S_w$ and bottom slope, $S$ (Edited from Aghsaee et al. 2010). Colored circles show data from field observations. Horizontal blue and red lines show bottom slopes employed in laboratory experiments.

The objective of this study is to overcome the steep slope restrictions in previous laboratory studies by using a longer flume that allows for a mild slope where fission will occur. In order to correlate experimental results with oceanic observations, experiments were conducted where boluses were generated through fission on gradual slopes (e.g., $S=0.04$).

The central issue is to characterize the dissipation properties of ISWs over the continental shelf. Knowing how far boluses propagate and loose energy will help to better understand this process. The ability to measure, and potentially parameterize, turbulent dissipation by these features would be extremely informative to coastal ecosystem management and helpful in the understanding of the
transition from wave to bolus and how energy is lost in this process (Horn, Imberger et al. 2002; Nash, Alford et al. 2005; Helfrich and Melville 2006; Boegman and Stastna 2019).

4.2 Methods

4.2.1 Experimental methods

Shoaling boluses were generated through fission on a slope \( (S = 0.04) \) installed in the 20 m × 0.75 m × 0.65 m glass-walled Internal Wave Flume at the Queen’s University Coastal Engineering Research Laboratory. Periodic ISWs were forced with a flapping airfoil type wavemaker, located at the end of the tank, producing internal waves, which propagated towards the slope as shown in Figure 4.2 (Thorpe 1978; Nakayama and Imberger 2010). The paddle was driven by an adjusting shaft attached to a rotating disk with variable-speed drive unit and an electric motor that could produce waves with periods \( T \sim 10\text{-}30\text{s} \) with different amplitudes, \( a \sim 0.025\text{-}0.085\text{m} \). The tank was filled with a two-layer stratification consisting of a fresh water upper layer above the denser layer premixed with salt (Cargill CMF) in a stirring tank to reach to the desired density of \( \rho_2 \) and desired depth of \( h_2 \). For visualization purposes, the lower layer fluid was dyed with food colouring then, fresh water was pumped into the flume and slowly added through a floating sponge to minimize mixing to the desired height of \( h_1 \). The thickness of the pycnocline between the layers was sharp typically 10-20 mm. The distance between the internal-wave generator and the toe of the slope was 10 m.

Eight wave cases are considered here with different wave amplitudes and stratifications. Experimental variables considered in this study are given in Table 1. Wave amplitude and wave period were measured with a 4-channel wave gauge system (HRWG-0090) and twin-wire wave probe (HRIA-1013) located 3 m away from the toe of the slope. Velocity profiles were recorded with Nortek Vectrino II acoustic Doppler velocimeters (ADV), mounted on pointer gauges, and sampling with rate of 50 Hz at three different locations upslope: where the interface intersects the
slope and 90 cm above and below the intersection point (Figure 4.2). We utilized, three cameras to capture ISWs shoaling on the slope. Two cameras (Canon D650) capturing the shoaling waves in a vertical-streamwise ($z, x$) planar field of view and one camera (Canon VIXIA HV30) looking downward recording the spanwise-streamwise ($y, x$) top view of the shoaling bolus. Steady-state periodic wave conditions allowed velocity profiling, of incident boluses, over 2 cm vertical bins with 1 cm overlap through the entire water column. Five waves at each height were recorded by the ADVs and the velocity profile from the middle 3 waves was analyzed, to minimize transient start-up and run-down effects. Therefore, it was possible to measure the entire velocity profile as the ISWs underwent fission and formed boluses at three critical locations after the turning point.

**Figure 4.2** Schematic of the experimental setup. Periodic ISWs of depression are generated within the two-layer fluid by flapping the wave paddle.

### 4.2.2 Analytical methods

Properties of ISWs in a two-layer system can be theoretically estimated using the Korteweg-de Vries (KdV) equation, if the background stratification and amplitude of the ISW are known (Osborne and Burch 1980; Apel 2002):

$$\frac{\partial \eta}{\partial t} + c_0 \left( \frac{\partial \eta}{\partial x} + \alpha \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} \right) = 0$$

(4.1)
where \( \eta \) is the pycnocline displacement. The KdV equation models weakly nonlinear, weakly dispersive waves with nonlinearity and dispersion coefficients:

\[
\alpha = \frac{3c_0}{2} \frac{h_1 - h_2}{h_1 h_2}, \quad \beta = \frac{c_0}{6} h_1 h_2
\]

Here, \( c_0 = \sqrt{g' h_1 h_2 / (h_1 + h_2)} \) is the linear phase speed and \( g' = g \rho_2 - \rho_1 / \rho_2 \) is the reduced gravity due to stratification (Djordjevic and Redekopp 1978).

The evolution of ISWs, as described by the KdV equation, are often compared to laboratory experiments. Relevant dimensionless parameters depend on incoming wave classified in terms of the wave Reynolds number (Troy and Koseff 2006) and Iribarren number (Boegman, Ivey et al. 2005) respectively as:

\[
Re_w = a^2 \omega_0 / \nu, \quad \xi = S / \sqrt{a / \lambda}
\]

Here \( \omega_0 \) is the wave frequency and the nonlinear phase speed \( c \), incident wave amplitude \( a \) and wavelength \( \lambda \) were calculated from the wave probe before the toe of slope. Local wavelength computed at each ADV from recorded velocity profile at each measurement point (Table 4.1; Figure 4.2).
Table 4.1 Experimental parameters for generated waves where $\rho_2$ is the lower layer fluid density, $h_1$ is the upper layer thickness, $h_2$ is the lower layer thickness, $a$ is the wave amplitude, $\lambda$ is the wave length, $c$ is wave phase speed and $E$ is the total wave’s energy. $Re_w$ is the wave Reynolds number [Troy and Koseff, 2005], $\xi$ is the Iribarren number [Boegman et al. 2005]. $D$ and $L$ are the dissipation rate and the dissipation length-scale (wave runup from the turning point upslope), respectively. The upper layer fluid density $\rho_1$ was 998 (kg m$^{-3}$) for all experiments. $H_b$ and $L_b$ are the bolus height and length observed in each ADVs from the left to right in field of view.

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4.2.3 Energy calculations

Wave energy has historically been estimated by assuming an equipartition between kinetic energy ($KE$) and the readily measured available potential energy, $APE$ (Helfrich 1992; Michallet and Ivey 1999; Bourgault, Blokhina et al. 2007).

\[
APE = c g \Delta \rho \int_{t_0}^{t_1} \eta^2(t) \, dt \tag{4.4}
\]

Where $\Delta \rho = \rho_2 - \rho_1$ and $\eta(t)$ is the displacement of the pycnocline with time during the passage of the ISW.

In the present study, we compute the $KE$ using a Dubreil-Jacotin-Long (DJL) solver (pers. comm. M. Stastna) that matches the $KE$ solution to the initial $APE$ of each experimental wave. The dissipation lengthscale $L$, the distance that waves travel after turning point until the bolus was fully degenerated, was experimentally observed from the field of view then compared with an energetics-based estimate of travelled distance from the linear decrease in wave energy (e.g. $L_E = \frac{cE}{D}$, here $c$ is the wave speed, $D$ is the integral dissipation (defined below) and $E = APE + KE$ is the total wave energy; Moum et al. 2007a) and laboratory measurements from Helfrich (1992) on a uniform slope.

4.2.4 Dissipation calculations

Reynolds decomposition is challenging due to the unsteady nature of the flow and obtaining turbulence statistics through time, space, or ensemble averaging remain difficult (Boegman and Ivey 2009). The mean velocity field ($U, V, W$) was obtained by low-pass filtering the instantaneous velocity data ($u, v, w$) with a spectral cutoff between peaks associated with the ISW and instabilities (Aghsae et al. 2015). The fluctuating components were then obtained through Reynolds decomposition (e.g., $u = U + u'$), where $(u', v', w')$ are the fluctuating components of the streamwise, spanwise and vertical velocity components (Figure 4.2).
The velocity gradient inside the trapped core and to the outer layer, leads to production of turbulent kinetic energy and dissipation $\varepsilon$ (Pope 2001). Assuming isotropy, $\varepsilon$ may be estimated from the three measured velocity gradients as (Piccirillo and Van Atta 1997):

$$
\varepsilon = \nu \left[ 5 \left( \frac{\partial u'}{\partial z} \right)^2 + \frac{5}{2} \left( \frac{\partial v'}{\partial z} \right)^2 + \frac{5}{2} \left( \frac{\partial w'}{\partial z} \right)^2 \right]
$$

(4.5)

Where, $z$ is the vertical coordinate direction and $()$ denotes time averaging. The dissipation values ($\varepsilon$, W kg$^{-1}$) computed from (5) was converted to total observed dissipation ($D$, W m$^{-1}$) by integration over the incident bolus at each ADV as:

$$
D = \Delta z \Delta t \rho_2 \int_{0}^{\lambda} \varepsilon \, dx \, dz
$$

(4.6)

4.3 Results

4.3.1 Flow field

The flow-field resulting from the fission, breaking and run-up of the solitary waves of depression is discussed (Figure 4.3) where the observed waves are travelling from left to right up the slope. The ISWs continue to steepen as they shoal, before fission occurs at the turning point ($h_1 = h_2$) and the waves of depression disperse into one or more waves of elevation, then continue steepening and form boluses (Figure 4.3b,c). Figure 4.3d shows the wave 5 seconds later where overturning is clearly visible, which resulted in the formation of the bolus propagating upslope. Shear instability and mixing developed in the region of high shear near the bolus crest (Figure 4.4a). A thin layer of backflow connected the boluses (Figure 4.3e-h). Our observations from the camera looking downward on the flume suggests that the front of the bolus has two-dimensional instability; however, spanwise velocity fluctuations indicate that three-dimensional instability is likely degenerating the shoaling wave (Figure 4.4b). The instantaneous velocity fields, with
dissipation profile (Figure 4.5), clearly show a vortex and a return flow is generated by the bolus with the lower-layer mass transported upslope, passing the field of view due to wave propagation.

Figure 4.3 Snapshot of laboratory experiment showing shoaling bolus taken from experiment 7 in the long flume with $\rho_2 = 1020 \text{ kg m}^{-3}$, $a = 0.083 \text{ m}$, $h_1 = 0.15 \text{ m}$, $h_2 = 0.25 \text{ m}$. The red circle shows the locations of the ADVs.
Figure 4.4 (a) Image of laboratory experiment 6 (with $\rho_2 = 1040 \text{ kg m}^{-3}$, $a = 0.058 \text{ m}$, $h_1 = 0.15 \text{ m}, h_2 = 0.25 \text{ m}$) showing shoaling bolus associated with shear instability (b) Top view of shoaling bolus from the camera looking downward on the flume.

Figure 4.5 (a) Horizontal velocity $u$, (b) transversal velocity $v$, (c) vertical velocity $w$, (d) magnitude of the dissipation rate, $\varepsilon$ associated with experiment 8 where $z$ is the height above the bed.
4.3.2 Dissipation of turbulent kinetic energy

The dissipation rate, $\varepsilon$ (W kg$^{-1}$) was computed at the three different ADVs at 90 cm intervals. The dissipation was elevated due to shear instability and vortices through the pycnocline and particularly along the trailing edge of bolus (Figure 4.5). Figure 4.6 shows total value of dissipation rate as a function of Reynolds number, $Re_w$ and local Iribarren number, $\xi$ for all cases at each ADV (Eq. (4.5)). The rate of dissipation at the first and second ADVs was $\sim 10^{-4}$ W kg$^{-1}$, which was an order of magnitude larger than last ADV, with most wave energy dissipated after the turning point as shear instability was initiated when the bolus was fully formed and then wave energy decreased ($\varepsilon$ reduced to $<10^{-6}$ W kg$^{-1}$) through increased turbulent mixing and vortex formation. During the strong turbulence conditions after the turning point, $\varepsilon$ is order of magnitude larger (Figure 4.6a).

The dependence of dissipation with $Re_w$ and local $\xi$ is presented in Figure 4.6b. Results suggest that, the bolus size grows (Exp 8; $H_b$: 0.041 – 0.089) with an increase $Re_w$ and decreases with increasing $\xi$. It is interesting to note that for $Re_w \leq 792$ the bolus is produced after first the ADV before the interaction point, but for $Re_w \geq 980$, the bolus was fully formed before the first ADV. The results obtained indicate that boluses tend to be larger and travel for longer distances as the amplitude is increased (Table 4.1).
Figure 4.6 From left to right, rate of dissipation, $\varepsilon$ at last ADV, interaction point and 1st ADV (Figure 4.2) scaled with (a) Reynolds number, $Re_w$ (b) local Iribarren number at each ADV, $\xi$ for each case with different ISW amplitude $\alpha$.

The relationship between integral dissipation rate $D$ and non-dimensional wave properties for different experiments is shown Figure 4.7. It is apparent that energy loss was higher for waves with smaller Iribarren number (larger amplitude waves). In general dissipation increases with $Re_w$ and decreases with $\xi$; therefore, at higher Reynolds numbers the flow is more likely to be turbulent which leads to stronger near-bottom vorticity and bottom shear stress, confirming that larger waves lose energy more rapidly than smaller waves (Figure 4.7a,b).

$D$ was proportional to the incident wave energy leading to energy flux estimates of 0.01-0.1 W m$^{-1}$. The stability and dissipation length scales of ISWs have been modelled according to nondimensional wavelength (Shroyer, Moum et al. 2010) and amplitude (Diamessis and Redekopp 2006). Figure 4.7c,d compares the change in dissipation over $a/H$ and $\lambda/H$. The total dissipation increases directly with $a/H$, which implies that larger waves lose energy more rapidly than the smaller waves. There is a positive, non-linear association between $D$ and $a/H$, the relationship is strong as there is little scatter in the data points (Figure 4.7c). However, no apparent relationship was found between $D$ and $\lambda/H$ (Figure 4.7d).
The dissipation rate $D$ ($W \text{ m}^{-1}$) relationship with (a) Reynold’s number, $Re_w$ (b) internal Iribarren number $\xi$ for incident wave. $D$ also, compared to non-dimensional wave properties of (c) amplitude $a/H$ and (d) wavelength $\lambda/H$. (Here $H=h_1+h_2$ and $h_1$ is the upper layer thickness and $h_2$ is the lower layer thickness).

4.3.3 Dissipation length scale

The energetics-based dissipation length scale, $L_E = cE/D$, (Moum, Farmer et al. 2007) from each ADV along the slope, may be compared with the observed value $L$. Although the results do not show good relative agreement, the measured dissipation lengthscales are all $L\sim 3-6$ m, but the computed values $L_E \sim 8-10$ m are considerably larger (Figure 4.8a). This suggests we have
underestimated the dissipation, or other processes are acting to degenerate the bolus. For example, the continual loss of fluid beneath the bolus (Figure 4.3) may drain the bolus of its volume, before the energy is dissipated and because boluses are travelling upslope some KE transfer to APE through diapycnal mixing. Therefore, $L_E = cE/D$ is a poor estimator of dissipation lengthscale over uniform slope however, it works well for ISWs shoaling on flat bottom (See chapter 2).

Figure 4.8b shows a comparison between the lab observation and experiments performed by Helfrich et al. 1992 for the run-up distance from interaction point (where the pycnocline intersects the slope) and its relationship with the amplitude of the incident wave. The data sets are qualitatively consistent indicating a positive relationship between dissipation length scale and the waves amplitude. The run-up is a linear function of incident amplitude, $a$ and the best fit to the data is $X_o = 19 \times a$.

**Figure 4.8** (a) Maximum run-up past the undisturbed interface-slope intersection, $X_o$ vs $a$, the amplitude for the lab observation and results from Helfrich, 1992 (b) Observed dissipation length scale ($L$) vs estimates from, $L_E = cE/D$ as a function of total observed dissipation.
4.4 Discussion

4.4.1 Evolution and run-up of the bolus

We have observed the energy decay and dissipation length scale of boluses generated from ISW fission over gradually sloping topography. Our analysis is consistent with breaking regimes classified in numerical simulations (Aghsaee, Boegman et al. 2010) over mild slopes ($S < -0.5S_w + 0.1$), where an incident wave of depression passes through the turning point and degenerates through fission, forming a packet of waves of elevation that evolved into boluses (Figure 4.1). As the boluses shoaled, overturning took place in both the leading face and tail associated with shear instability (Figure 4.3 and Figure 4.4). A near-bed return flow of lower layer fluid continually drained the boluses of mass, diapycnal mixing was observed and the bolus travelled upslope, likely leading to poor estimates of dissipation lengthscales from energy calculations. Secondary lateral instabilities appeared to be the ultimate stage of bolus degeneration (Arthur and Fringer 2016).

The experiments were intended to provide insight into the behaviour of a train of internal solitary waves as they travel shoreward onto a continental shelf. The fission event produced high velocities close to the bed, consistent with field observations showing sediment entrained into the vortex and transported considerable distances upslope (Helfrich and White 2010; Richards, Bourgault et al. 2013; Boegman and Stastna 2019). Elevated dissipation was also observed, primarily along the margins of the bolus, similar to field observations in the coastal ocean (e.g.$10^{-5} - 10^{-4}$ W kg$^{-1}$; (Klymak and Moum 2003; Lien, Tang et al. 2005; Moum, Farmer et al. 2007)).

The boluses in the present study evolved with an aspect ratio (bolus height/bolus length; $H_b/l_b \sim 0.3$) that was also noted in prior lab experiments and did not change with run-up (Figure 4.9a); the dynamics regulating the constant aspect ratio remain unclear (Wallace and Wilkinson 1988; Helfrich 1992). However, when normalized by the initial height $H_o$, the bolus height decreases linearly with position (Figure 4.9b). The data was consistent with prior experiments (Wallace and
Wilkinson 1988; Helfrich 1992) and the maximum run-up is a linear function of incident amplitude $a$ with a best fit to the data $X_o = 19 \times a$ (Figure 4.8b).

4.4.2 Decay criterion

Due to the inability to predict the observed dissipation lengthscale using energetics $L_E = cE / D$ (Figure 4.8a); we plot the observed dissipation lengthscale normalized by the incident ISW amplitude versus the internal Iribarren number (Eq. (4.3)), which has been shown to characterize internal wave breaking on steeper slopes (Boegman, Ivey et al. 2005; Sutherland 2013), but has not been applied to bolus shoaling (Figure 4.10a). This parameterization for the dissipation lengthscale $L^* = (302.7 \times S / \sqrt{a / \lambda} - 10.4) \times a$ requires only a knowledge of incident ISW amplitude and wavelength to inform oceanographer how far these waves travel and possibly deposit sediment and nutrients trapped in a bolus core. The data are clustered tightly along the observed dissipation lengthscale ($R^2 = 0.9742$) and equivalent to a dissipative length scale $\sim 300 \times a$. There is a negative correlation between $L$ and $Re_w$, showing enhanced dissipation (smaller $L/a$) with increased Reynolds number; however, there is more scatter in the data (Figure 4.10b).
Our parameterization for $L$ (Figure 4.10) and also the aspect ratio $H_b/L_b = 0.3 \pm 0.1$ (Figure 4.9) are in good agreement with field observation in St. Lawrence Estuary (Bourgault, Blokhina et al. 2007; Richards, Bourgault et al. 2013) and the laboratory measurements (Helfrich 1992).

**Figure 4.10** (a) Normalized observed dissipation length scale, $L$ vs (a) Iribarren number $\xi$, (b) wave Reynolds number $R_{e\omega}$. Parametrization of dissipation length scale on continental slope as a function of incident wave amplitude $a$, wavelength $\lambda$ and slope $S$ as: $L^* = (302.7 \times S/\sqrt{a/\lambda} - 10.4) \times a$ compared with observations in the St. Lawrence Estuary.

### 4.5 Conclusions

The rate of dissipation of turbulent kinetic energy was largest for boluses from large amplitude ISWs and the instantaneous rate of dissipation decreased by an order of magnitude as the boluses propagated up the slope. Unlike for ISWs propagating on flat bottoms, the bolus dissipation lengthscale could not be predicted using energetics arguments (wave energy/dissipation rate), due to the complex wave slope interaction during shoaling. In agreement with field observations, the bolus dissipation lengthscale was roughly $\leq O(300)a$, was parameterized as a function of the internal Iribarren number $L^* = (302.7 \times \xi - 10.4) \times a$ and requires only a knowledge of incident ISW amplitude, $a$ and wavelength, $\lambda$ to inform an oceanographer how far these waves travel and
possibly deposit sediment and nutrients trapped in a bolus core. Future work should further validate the parameterization against field data and verify if sediment deposits where the bolus degenerates.


Chapter 5

Conclusion and future work

In this chapter I will more discuss the implication of the most important outcomes of this thesis, to outline a path for future numerical and experimental work in coastal ocean management. This thesis has investigated decay models for ISWs over flat and sloping boundaries in lakes and oceans by performing scaled experiments.

In chapter 2 I have discussed the details of estimates of energy flux by internal waves to quantify the dissipation of ISWs as they propagate over a flat bottom. The typical situation in the ocean is where waves propagate unhindered until they shoal upon the shore. I aim to predict the dissipation length scale, calculated the rate of TKE dissipation, potential energy, kinetic energy, and total energy for stable waves vs unstable waves in the ISW boundary. The transverse velocity fluctuations within the jet occur in the separated region beneath the rear half of the wave, cause the notable velocity gradient between zero velocity at the bed and stronger velocity in the outer layer, which can be directly linked to production of turbulent kinetic energy because of the rise in shear velocity.

Energy flux has been parameterized as $cE/100\lambda$, where $c$ is the phase speed, $E$ is the total wave energy and $\lambda$ is the wavelength. I parameterized the data according to the Shroyer et al. (2010) field observations over New Jersey’s continental shelf. ISW energetics were used to model wave dissipation lengthscales in terms of the wavelength $\lambda$ and momentum thickness Reynolds number $Re_{ISW}$ resulting in a dissipative lengthscale parameterization $L^* = 100\lambda + 2.5 \times 10^{10}\lambda Re_{ISW}^{-3.7}$. The rate of dissipation of turbulent kinetic energy, in the bottom boundary layer, was larger for unstable ISWs ($Re_{ISW} > \sim 200$), relative to stable waves. Consequently, dissipation lengthscales for stable waves were $O(100\lambda)$ wavelengths, in agreement with published observations; whereas
stable waves propagated significantly further (> 1000 $\lambda$). Future work should be directed at capturing missing dynamics such as 3D flow characteristics that allow global instability using Stereo PIV or DNS which allow measurement of instantaneous velocity within a much larger spatial domain to investigate the three-dimensionality of global instability beneath ISWs.

In chapter 3, I first give a brief review of different methods used to estimate the turbulent dissipation rate. Particle image velocimetry (PIV) provides an instantaneous two-dimensional and three-dimensional velocity distribution over the entire sample area, which are sequence of measurements that presents a time series of the spatial distribution, not just a time series of velocity at a single point. The aim of the present research was to examine turbulence characteristics and dissipation estimates in the coastal ocean beneath bottom boundary layer from PIV data. This chapter offers a new possibility for turbulence kinetic energy dissipation rate measurements, which include spatial derivatives of the velocity fluctuation component.

The results from the direct calculations (Eq. (3.6), (3.7)) and the six approaches based on isotropic turbulence (Eq. (3.13)) and also Vectrino II ADV and law of the wall estimation of dissipation (Eq. (3.19), (3.20)) are compared as their spatial averages vary in time. All approaches give $\varepsilon$ within the same order of magnitude but isotropic estimation based on the derivatives of $u$, $v$ (Eq. (3.14), (3.18)) is $\sim 50, 80\%$ higher than the direct assumption (Eq. (3.6)). The direct method is the most accurate assumption, which is in agreement with dissipation from the Vectrino II ADV for unstable waves and log-law; however, stereo PIV (Eq. (3.7)) give a closer estimate than 2D-PIV (Eq. (3.6)) to three-dimensional estimate (Eq. (3.2)) since it involves the least assumption of isotropy. Based on our results the dissipation rate from ADV data, Eq. (3.19) underestimated dissipation by 40\%, and the log-law method Eq. (3.20) overestimated by nearly 24\%. The assumption of vertical velocity ($\frac{\partial w'}{\partial z}$, Eq. (3.16)) and transversal velocity ($\frac{\partial v'}{\partial x}$, Eq. (3.17)) homogeneity of the turbulence introduces 5-7\% errors in the dissipation compared to 2D direct
method (Eq. (3.6)). The assumption of horizontal homogeneity of the turbulence (Eq. (3.13), (3.14)) give 30-50% errors in the dissipation.

The results will be informative to field oceanographers, who often only measure velocity in one direction with an ADCP and not 2D or 3D, which gives ~5-80% error. Future work on the development of direct method with less isotropy assumptions using tomographic PIV is recommended.

Chapter 4 focusses on shoaling ISWs and the ability to measure and parameterize turbulent dissipation over continental shelves. This study should be helpful in the understanding of the transition from wave to bolus and how energy is lost in this process. Propagation of boluses is typically observed in the ocean at moorings that are ~100 km apart and, therefore, a bolus could not be tracked from one mooring to the next. Boluses are also thought to carry sediments and nutrients from deep water onto the continental shelf. Knowing how far boluses propagate and loose energy will help to better understand this process. The field observations show gradual bottom slopes ($S \sim 0.01$), which are predicted to generate boluses through the fission of ISWs as they shoal. Published lab experiments have been performed on bottom slopes $0.07 < S < 0.4$ and fission will not occur.

To relate experimental results with oceanic observations, experiments conducted where boluses generated through fission on gradual slopes ($S=0.04$). Energetics of bolus formation of internal waves were investigated experimentally and verified with field observation and previous lab experiments. The energy dissipation model from Chapter 2 was extended to predict the run up on continental shelf. The rate of dissipation of turbulent kinetic energy was largest for boluses form large amplitude ISWs and the instantaneous rate decreased by an order of magnitude as the boluses propagated up the slope. Unlike for ISWs propagating on flat bottoms, the bolus dissipation lengthscale could not be predicted using energetics arguments (wave energy/dissipation rate), due to the complex wave slope interaction during shoaling. The observed bolus dissipation lengthscale
was \( \leq O(300) \) and parameterized as a function of the internal Iribarren number (wavelength, amplitude and sloping boundary) to predict the dissipation lengthscale as \( L' = (302.7 \times S/\sqrt{a/\lambda} - 10.4) \times a \). Our model is directly applicable to ISWs propagating over continental shelf slope, which requires only a knowledge of incident ISW amplitude and wavelength to inform oceanographer how far these waves travel and possibly deposit sediment and nutrients trapped in a bolus core. Future work on the investigation of sediment transport and resuspension is recommended to verify if the sediment deposit where the bolus degenerates, based on our model, and also capturing missing dynamics using PIV, which allows measurement of instantaneous velocity within a much large spatial domain to investigate the three-dimensionality of a shoaling bolus.
Appendix A

Log fit code

clc
clear all;
close all;

% load the velocitmetry data
load('test.mat');

u=double(Data.Profiles_VelX);
t=double(Data.Profiles_TimeStamp);
range=double(Data.Profiles_Range);
rho2=1096;

j=size(u);
z=range(j(2))-range;

for i=1:j;
    velocity_depth=z(1:27);
    ur=u(i,1:27);
    % For u
    modelFun = @(p,z) (p(1)./0.41).*log(z./p(2)); % define the log equation
    startingVals = [0 3e^-5]; % initial value for u* and Z0
    Z0_initial=3e^-5;
    [coefEsts(i,:),resnorm(i,:)]=lsqcurvefit(modelFun,startingVals,[velocity_depth,Z0_initial],[ur,0]); % apply the log fit "velocity_depth" are the depths of observed velocities and ur are the observed velocity profile
    u_star_ur(i,:)=real(coefEsts(i,1)); % shear velocity
    Z0_ur=real(coefEsts(i,2)); % bottom roughness
    U_logfit(i,:)=u_star_ur(i,:)/0.41.*log(z./Z0_ur); % this line uses the estimated u* and Z0 to generate a theoretical velocity profile
    delta_z(i,:)=(gradient(z));
deltau_z(i,:)=gradient(U_logfit(i,:));
Uz(i,:)=deltau_z(i,:)./delta_z(i,:);
tau(i,:)=(rho2*1.26*10^-6.*Uz(i,:));
ta(i,:)=u_star_ur(i,:).^2.*rho2;
end
Appendix B

TKE dissipation code

% Cal_epsilon.m -- calculate dissipation

function [eps, Eps,Eps_iso1,Eps_iso2,Eps_iso3,Eps_iso4,uy2,vx2] = Cal_epsilon(u,v,x,y)
% input: turbulent fluctuations: uf, vf; (m/s)
% coordinate (m)
% output: 2D: eps(nj,ni)
% spatial mean: Eps(1,1)
% the value based on isotropic assumption: Eps_iso1,
% Eps_iso2,Eps_iso3,Eps_iso4
% fluid: water;

eps = zeros(size(u));

% convert units:
scale = 1/100;   % cm-->m
u = u*scale;
v = v*scale;
x = x*scale;
y = y*scale;

dx = x(1,2) - x(1,1);
dy = y(2,1) - y(1,1);

nu = 1.30e-6;   % (m/s^2)

% Derivatives:
[ux,uy] = gradient(u,dx,dy);
[vx,vy] = gradient(v,dx,dy);

% Assume isotropic turbulent:

Eps_iso1 = 15 * nu * squeeze(mean(mean(ux.^2)));
Eps_iso2 = 15 * nu * squeeze(mean(mean(vy.^2)));
Eps_iso3 = 7.5 * nu * squeeze(mean(mean(uy.^2)));
Eps_iso4 = 7.5 * nu * squeeze(mean(mean(vx.^2)));

eps(:,:,1) = 3*nu * (...
    ux.^2 + vy.^2 + uy.^2 + vx.^2 + 2*uy.*vx + 2/3*ux.*vy... );

Eps = squeeze(mean(mean(eps)));

uy2 = squeeze(mean(mean(uy.^2)));
vx2 = squeeze(mean(mean(vx.^2)));

end
Appendix C
Bolus evolution

Figure C.1 Snapshot of laboratory experiment showing shoaling bolus taken from experiment 1 in the long flume with $\rho_2 = 1020 \text{ kg m}^{-3}$, $\alpha = 0.025 \text{ m}$, $h_1 = 0.1 \text{ m}$, $h_2 = 0.3 \text{ m}$. The red circle shows the locations of the ADVs. The blue arrow shows the tongue of lower layer fluid attaching the first bolus to the pycnocline. The orange arrow shows the second ISW of elevation and the green arrow shows the second bolus. The third ADV is outside the field of view.
Figure C.2 Snapshot of laboratory experiment showing shoaling bolus taken from experiment 2 in the long flume with \( \rho_2 = 1040 \text{ kg m}^{-3} \), \( \alpha = 0.029 \text{ m} \), \( h_1 = 0.1 \text{ m} \), \( h_2 = 0.3 \text{ m} \). The red circle shows the locations of the ADVs. The blue arrows in b, d, g and k show the incident ISW of depression, the evolved ISW of elevation after the turning point, the bolus, and the point where the bolus has completely dissipated, respectively.
Figure C.3 Snapshot of laboratory experiment showing shoaling bolus taken from experiment 6 in the long flume with $\rho_2 = 1040 \text{ kg m}^{-3}$, $a = 0.058 \text{ m}$, $h_1 = 0.15 \text{ m}$, $h_2 = 0.25 \text{ m}$. The red circle shows the locations of the ADVs. The blue arrow shows the location where the rear face of the ISW of elevation reached maximum steepness.
Figure C.4 Time series of instantaneous velocity profiles (a&b) Horizontal velocity $u$, (c&d) vertical velocity $w$, (e&f) transversal velocity $v$, associated with experiment 1 captured by the first and second ADVs, respectively.