Mapping of Spatiotemporal Scalar Fields by Mobile Robots using Gaussian Process Regression

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Abstract—Spatiotemporal maps are data-driven estimates of time changing phenomena. For environmental science, rather than collect data from an array of static sensors, a mobile sensor platform could reduce setup time and cost, maintain flexibility to be deployed to any area of interest, and provide active feedback during observations. While promising, mapping is challenging with mobile sensors because vehicle constraints limit not only where, but also when observations can be made. By assuming spatial and temporal correlations in the data through kernel functions, this paper uses Gaussian process regression (GPR) to generate a maximum likelihood estimate of the phenomenon while also tracking the estimate uncertainty. Spatiotemporal mapping by GPR is simulated for a single fixed-path mobile robot observing a latent spatiotemporal scalar field. The learned spatiotemporal map captures the structure of the latent scalar field with the largest uncertainties in areas the robot never visited.

I. INTRODUCTION

Understanding naturally occurring environmental phenomena is vital in many areas of science and engineering, but data collection through an array of static sensors can be costly and time-consuming to complete. Mobile robots provide a new means to generate such data because they can provide responsive and targeted data collection through the deployment of a single mobile sensor in the region of interest. With advancement of mobile robotics in all domains (i.e., ground, water, and air), this solution can cater to many environmental observation tasks [1]. The key challenge in using data collected by a mobile sensor to model a phenomenon varying in space and time—referred to as a spatiotemporal phenomenon—is that: (1) observation positions are governed by the finite vehicle motion; and, (2) a single sensor can only provide measurements from a single time. A successful spatiotemporal mapping solution with mobile robots must accommodate both limitations of such data when attempting to estimate the unknown spatiotemporal field.

For an environment unchanging over the time of observation by a mobile robot, the phenomenon of interest can be sufficiently modelled as a static spatial map with no dependence in time. Bathymetry mapping using an uncrewed surface vessel (USV) is an example of such a static phenomenon. In this case, a survey by an array of static sensors would be identical to a survey by a mobile robot because the observations only vary by position. This is also possible when the timescale of the survey (i.e., length of time the robot requires to survey the entire area of interest) is significantly less than the rate of change of the environment. In contrast, this paper focuses on spatiotemporal fields that are changing at a rate similar to the motion of the vehicle, as exemplified in Fig. 1. As the vehicle moves through and observes a spatiotemporal field, the field itself is persistently changing. With sparse data, correlations in the data through all dimensions (i.e., both space and time) must be identified and leveraged.

Finding such data correlations lends itself well to learning techniques, such as Gaussian process regression (GPR). Use of a non-parametric GP model allows the mapping solution to remain generic and flexible, while the hyperparameters can be tuned to provide data-specific adjustments to find a suitable and practical model for the given application. GPR also maintains an estimate of the uncertainty of its prediction, which provides future opportunities to develop uncertainty-focused planning algorithms. This GPR spatiotemporal mapping technique can be used in any situation where a mobile robot is encountering a time and space varying phenomenon.

Fig. 1. A mobile robot moving through a spatiotemporal field (grey contours) is unable to directly make observations at multiple positions at the same time. Assuming that measurements are correlated to "nearby" positions and times, estimates are possible at states that could not be observed directly. In this illustration, predictions at $t_2$ across the entire field must be made assuming a correlation forward (red) and backward (purple) in time, as well as around the observed position (green).
robotics community, having the ability to model and predict disturbances, even if only on a limited time horizon, could be incredibly valuable for precision control.

II. RELATED WORK

Spatiotemporal fields can be found at a range of scales, from large-scale trends captured by remote sensing orbital platforms [2] to local effects that can only be monitored by in-situ observations. Much work has been done on the deployment of static sensors to understand a given local spatiotemporal environment. Sahu and Mardia [3] used data from static sensors to model air pollution and demonstrated an ability to make spatial and future predictions. Data in this work were collected simultaneously at 15 non-uniformly distributed sites, so the techniques they used are generally applicable to any static sensor network. Use of the Matérn basis function is identified as optimal only for spatial predictions, limiting their ability to perform temporal mapping. Erickson et al. [4] built a continuous model of indoor temperature and humidity using an array of sensors. Although static, sensor placement was optimized using a Halton sequence to provide a well-sampled spatial data set. Temporal mapping was not a focus of this work, but GPR was used to model three-dimensional space with a variety of common kernels.

Mobile sensors have been proposed as tools for data collection in environmental monitoring. Alam et al. [5] used uncontrolled aquatic sensors (so-called drifters) to persistently monitor water flow in a coastal environment. Without vehicle control, they required multiple drifters to capture sufficient data. These data were used as input to a standard ocean modeling system to generate a static, yet updating, vector field for water flow. In Fossum et al. [6], an underwater uncrewed aquatic robot was deployed to sample phytoplankton. Predetermined vehicle paths were used in the sampling procedure and volumetric estimates were created using GPR. No temporal component was considered, but a sequential update process for the prediction was proposed.

Most connected to the contents of this paper are works where partial developments of spatiotemporal mapping techniques were demonstrated with mobile robots. Lan and Schwager [7] used multiple mobile sensors in simulation to capture a spatiotemporal field using a finite number of spatial basis functions. The use of spatial basis functions makes an assumption about the geometric structure of the dynamic field, whereas GPR spatiotemporal mapping uses kernel combinations to capture any arbitrary shape. Path planning for optimum data collection is a major element of their work as well, which is outside the scope of our work. Whitman et al. [8] proposed GPs to capture the spatial distribution of a field with a squared exponential basis function that are then evolved over time using a weighted linear model. Multiple mobile robots were used in the data collection for a simulated weed growth example, where each robot can measure weed emergence across an entire row of the space. Ma et al. [9] used a sparse GPR formulation to learn a model in real-time, however it is only suitable for long-term environmental monitoring. Although they model spatiotemporal fields, the fields under investigation change sufficiently slowly that they can be considered pseudo-static during robot observations. Singh et al. [10] similarly use GPR and adaptive sampling with a mobile sensor to map a spatiotemporal field. They demonstrate GPR as a valid technique for spatiotemporal mapping in simulation and experiment, but use sensors not subject to nonholonomic constraints often encountered with mobile vehicles.

In much of the existing literature, squared exponential basis functions (also known as radial basis functions) are used to exclusively capture spatial variations in a scalar field [3], [6], [4], [8], [9]. Mapping of temporal variations is more commonly found in adjacent areas of robotics research such as control system error correction [11] and into global influenza tracking [12]. In Klenske et al. [11], they used a combination of squared exponential and periodic basis functions in their GPR model. Although limited to a single axis of mapping (time being the only input variable), the combination of kernels proved effective in mapping and extrapolating system behaviour. Senanayake et al. [12] created a complete spatiotemporal model using GPR with spatial squared exponential basis functions and temporal periodic kernels, leveraging previous knowledge about the seasonal variations of influenza. Data were collected from multiple static observation points, providing simultaneous observations across the entire spatial domain.

Research presented in this paper seeks to combine spatial mapping with mobile robots and temporal mapping found in robotics and other domains, in order to build continuous spatiotemporal maps using GPR. Such maps make minimal assumptions of the structure of the latent spatiotemporal field and are reliant on only a single mobile sensor platform.

III. MAPPING WITH MOBILE ROBOTS

A. Dynamic Field Observations

Consider a dynamic scalar field $\phi(q, t) : Q \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ where $Q \subset \mathbb{R}^s$ is the spatial domain of interest and time $t \in \mathbb{R}_{\geq 0}$. The objective of mapping is to find the best estimate $\hat{\phi}(q, t)$ by making a set of discrete observations taken from the $s$-dimensional space at multiple points in time.

Rather than collect data simultaneously from multiple sensors (i.e., at multiple locations), a mobile robot instead collects a sequence of samples of the spatiotemporal field at increasing times and at a variety of locations, as illustrated in Fig. 1. Each measurement made by the mobile sensor is assumed to follow the discrete model

$$\hat{y}_i = \phi(q_i, t_i) + \nu_r,$$

where $i \in \mathbb{Z}_{\geq 0}$ indicates the index of the discrete observations at positions $q_i$ and time $t_i$, and $\nu_r \sim \mathcal{N}(0, \sigma_r^2)$ is noise added by the sensor. Due to the robot’s inherent mobility, measurements can be made at any position $q_k$ and repeat measurements can occur at a later time (e.g., $q_i = q_p$, $p > i$). However, the time of measurement is always increasing (i.e., $t_{i+1} > t_i$) so a single mobile platform cannot make observations at multiple locations at the same point in time. Nor can it return to a previous time for a repeat
observation. This limitation of the data set must be overcome by the chosen spatiotemporal mapping technique.

B. GPR for Spatiotemporal Fields

With an ideal spatiotemporal map, predictions of the field can be made at positions never before visited or across time to estimate what happened in the past, what is happening now, and what might be happening next. The use of the map may vary by application. For environmental monitoring, which is the focus of this paper, mapping can be performed as a batch process after data collection is complete.

Using GPR for spatiotemporal mapping is a two-step learning and prediction process. The learning process makes use of a set of training data inputs \( X \) of \( m \in \mathbb{Z}_{>0} \) observations, \( X = \{x_i \mid i = 1, 2, ..., m\} \), where \( x \in \mathcal{X} \), \( \mathcal{X} \subset \mathbb{R}^n \) is a vector of input values, and observation target values \( y = \{\hat{y}_i \mid i = 1, 2, ..., m\} \), \( \hat{y} \in \mathcal{Y} \). We currently limit our analysis to spatiotemporal scalar fields, hence \( \mathcal{Y} \subset \mathbb{R} \).

Training data inputs are the position and time of observation, \( x_i = [q_i, t_i] \). In this paper we assume observations are independent of the robot orientation and that the traversable space is limited to two dimensions, so \( q_i = (q_1, q_2) \), the coordinates of the robot position, and \( n = 3 \) (i.e., two spatial dimensions and the single time dimension).

The predicted distribution \( f^* \) is found from noisy measurements through GPR as

\[
f^* | X, y, X^* \sim \mathcal{N} (f^*, \text{cov} (f^*))
\]

where

\[
f^* = \mathbb{E} [f^* | X, X^*] = K_* [K + \sigma^2 I]^{-1} y,
\]

\[
\text{cov} (f^*) = K_* - K_* [K + \sigma^2 I]^{-1} K_*,
\]

\( X^* \) is the testing set, and \( K = K(X, X) \), \( K_* = K(X^*, X) \), and \( K_* = K(X^*, X) \) are a shorthand for the covariance matrices evaluated by the kernel function; e.g., see [13] for a detailed introduction to GPR.

The testing set contains a list of position-time pairs \( X^* = \{x_{*, j} \mid j = 1, 2, ..., m_*\} \). For a spatiotemporal model, the entries of \( X^* \) are selected to provide complete coverage of the region and duration of interest. One such option is to select \( x_j = [q_j, t_j] \) as an evenly spaced grid of all combinations of positions and time, discretized by spatial resolution \( S \) and temporal resolution \( T \). Practical lower limits for the resolution can be found based on available runtime of the prediction. In a more targeted application, only the region of interest and time horizon desired need to be predicted, lending this technique to considerable computation efficiency in real-time applications.

C. Covariance Functions

GPR uses kernel functions to evaluate the covariance between two inputs, i.e., \( \kappa (x_a, x_b) : \mathcal{X} \times \mathcal{X} \to \mathbb{R} \). Selecting a kernel can be challenging without knowledge of the latent spatiotemporal field function, driving some researchers to investigate automated kernel selection; see Duvenaud et al. [14]. However, in this work we implement a combination of the common squared exponential kernel and the periodic kernel to capture long term and cyclic trends in the field. Kernel functions can be specifically selected to determine the correlation of inputs along any input direction while hyperparameters (discussed below) are tuned separately.

The squared exponential kernel (also known as the radial basis function) is a popular choice in spatial mapping and takes the form

\[
\kappa_{SE} (x_a, x_b) = \exp (\frac{-1}{2} (x_a - x_b)\T L_{SE} (x_a - x_b))
\]

where \( L_{SE} = \text{diag} (l_{SE}^2) \) are the length-scale hyperparameters. \( l_{SE} \in \mathbb{R}^3 \) is effectively a normalizing factor that weights the correlation between points relative to the Euclidean distance in any axis (space or time), so a larger \( l \) increases correlation to points farther away.

Periodic kernels allow for modeling of repeating functions as the field, such as a long term shift and oscillatory behaviour, a combination of kernels can be used with distinct hyperparameters to map the field [16]. If it is desirable to have values strongly correlated when nearby or periodic, a summation of kernels

\[
\kappa_{Per} (x_a, x_b) = \sigma^2 (\kappa_{SE} (x_a, x_b) + \kappa_{Per} (x_a, x_b))
\]

is used with signal variance \( \sigma^2 \). For correlation to reduce with distance regardless, but maintain periodicity, the product

\[
\kappa (x_a, x_b) = \sigma^2 \kappa_{SE} (x_a, x_b) \kappa_{Per} (x_a, x_b)
\]

is used. The relative effect of each kernel function in the resulting combination kernel varies with the length-scales chosen. As each axis of the input vector data is considered independently, either combination of kernels can capture periodic behaviour in one direction without affecting mapping in the other two.

D. Hyperparameters for Mobile Robot Mapping

Training the GP spatiotemporal map is achieved through the tuning of the hyperparameters. For the combined kernels (7) and (8) there are 10 hyperparameters, which we denote as \( \theta = [\sigma, l_{SE}, \lambda, l_{Per}] \). The optimum set of hyperparameters \( \theta^* \) is found by maximizing

\[
\theta^* = \arg \max_{\theta} \log p (y | X),
\]

where the log-likelihood function

\[
\log p (y | X) = -\frac{1}{2} y\T (K + \sigma_n^2 I)^{-1} y - \frac{1}{2} \log |K + \sigma_n^2 I| - m \log 2\pi.
\]
Hyperparameters are often left unconstrained in the optimization process, allowing the optimizing tool to find \( \theta^* \) that maximizes (9) without consideration for practical limitations. For a dataset collected by a mobile robot, we propose constraints based on vehicle path, vehicle speed, and sensor measurement frequency. This intends to reduce the likelihood of overfitting and improve our confidence in the generated spatiotemporal map.

IV. EXAMPLE SIMULATIONS

The proposed GPR-based spatiotemporal mapping technique was evaluated in simulated experiments with a single mobile robot sampling an unknown spatiotemporal scalar field while driving along a fixed path. The simulation takes place in a square spatial domain of area \( L^2 \) and runs in discrete time from \( t = 0, T, 2T, \ldots, (N-1)T \in \mathbb{R}^N \), where \( T > 0 \) is the sample time and \( N \in \mathbb{Z}_{>0} \) is the total number of time steps. In all trials, we use \( L = 10 \) m, \( T = 0.1 \) s, and a simulation time of \( t_N = 60 \) s.

A. Spatiotemporal Scalar Field

The spatiotemporal scalar field can be an arbitrary function dependent on space and time. One option, the approach chosen by Lan and Schwager [7], is to simulate the dynamic scalar field as the sum of time-varying Gaussian radial basis functions, which we follow here. Each of the \( n \in \mathbb{Z}_{>0} \) spatial functions takes the form

\[
g_i(q) = k_i \exp \left( -\frac{1}{2\sigma_i^2}(q - q_i)^2 \right),
\]

where \( q_i = (x_i, y_i) \) is the centre of the basis function, \( k_i \in \mathbb{R} \), and \( \sigma_i \) is the scaling parameter. The temporal component of the field is introduced with time-varying weights \( w_i(t) \)

\[
\phi(q, t) = g(q)w^T(t),
\]

where \( g(q) = [g_1(q), g_2(q), \ldots, g_n(q)] \) and \( w(t) = [w_1(t), w_2(t), \ldots, w_n(t)] \). The weights \( w_i(t) \) are chosen to vary according to the linear system

\[
\dot{w}(t) = Aw(t),
\]

where \( A \) determines the system’s behaviour. To create an illustrative dynamic scalar field for simulated experiments, \( n = 2 \) basis functions were used with randomly generated centre points \( q_i \) and scaling factors \( \sigma_i \). Oscillatory motion of the field is created by first choosing two complex-conjugate eigenvalues for \( A \) that lie on the imaginary axis, then oscillation frequency is varied by scaling the eigenvalues. Four oscillation frequencies

\[
f = \left( \frac{2\pi}{t_N}, \frac{4\pi}{t_N}, \frac{6\pi}{t_N}, \frac{10\pi}{t_N} \right)
\]

are considered and the performance of the GPR-based spatiotemporal mapping results are compared.

B. Mobile Sensor Platform

A simple kinematic unicycle model

\[
\dot{q} = G(q)u = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} u,
\]

was used. Inputs and initial position were selected to give the vehicle a circular trajectory, starting at \( q_0 = (L, \frac{L}{T}, 0) \) and moving counterclockwise with \( u_k = [2, \frac{3}{4}] \). It is unlikely that this would be an optimal path for mapping the spatiotemporal field, but it provides reasonable spatial coverage, a smooth path that a real vehicle could follow, and a consistent set of observation data for direct comparison of kernel affects on mapping output. Fig. 2 shows an overlay of the vehicle path on the dynamic scalar field, and illustrates how the robot position could affect observations.

Observations were collected at the same sample rate of the simulation following the discrete model (1) with \( \sigma_r = 0.05 \). At each observation, the robot position \( (x, y) \) and current time \( t \) are collected for training data \( x_t \) and the noisy measurement is stored in \( y_t \). Localization and timing knowledge are assumed perfect in this simulation. It is anticipated that noise on these inputs would degrade the GPR mapping performance, but that is untested here.

C. GPR Implementation

The GPY library [17] was used as a GPR process implementation. A multidimensional squared exponential (SE)
The shape of the spatiotemporal model is captured in Fig. 3 for Test 2 with a field oscillation period of 30 s. The squared exponential kernel provides a spatially smooth result. The added t-dimension periodic kernel gives similar performance at twice the processing time. If the RMSE results in Table III meet a specific need, selection of a kernel could fall to other external factors such as a desire for fewer hyperparameters, available computational power, or to specifically capture a periodic signal.

We anticipate that model uncertainty is the lowest along the path of the robot, which we see confirmed from Test 2 in Fig. 4. The structure of this plot was useful in identifying GPR models that successfully captured the shape of the spatiotemporal field; if the GPR training failed, this plot loses the radial pattern and uncertainties no longer track the vehicle path. This information could be leveraged as an input to a path planning algorithm, similar to [9], in order to reduce uncertainty in the spatiotemporal map in real time.

V. CONCLUSIONS

The ability to map a spatiotemporal field with a mobile sensor enables responsive environmental monitoring applications as well as full field surveys when individual sensors are cost prohibitive. In simulation, Gaussian process regression has been shown to be an effective tool for spatial and temporal correlation of observations collected by a mobile robot. A range of kernels and their combinations were identified and tested, with the squared exponential and additive periodic kernels providing the best relative performance. Additionally, GPR-based spatiotemporal mapping performance improved when hyperparameters were constrained to match the physical limits of our mobile robot survey and the result is a more accurate and robust model.

Upcoming collaborations with coastal engineers and marine life researchers provide an important and necessary demonstration for these techniques onboard an autonomous uncrewed surface vessel. This work is also the first step towards a number of robot control research objectives such as adaptive sampling with a real-time variant of this mapping technique and a closed-loop controller that can leverage spatiotemporal mapping of exogenous disturbances to improve controller performance. By sampling and mapping disturbances, predictions can be used to anticipate, reject, and even leverage disturbances for improved control performance and energy efficiency.
Fig. 3. A comparison of the spatiotemporal mapping performance in Test 3. Ground truth (top) and prediction (bottom) as made using GPR with a squared exponential kernel. The vehicle path is highlighted by a yellow dashed circle in each frame.

Fig. 4. Uncertainty distribution for a successfully generated spatiotemporal map (Test 2 at t = 30 s). Lowest uncertainties are found along the vehicle path (yellow dashed circle). This trend remains constant through time.

REFERENCES


