

PERFORMANCE ANALYSIS OF DECODE-AND-FORWARD  
PROTOCOLS IN UNIDIRECTIONAL AND BIDIRECTIONAL  
COOPERATIVE DIVERSITY NETWORKS

by

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# Abstract

Cooperative communications have the ability to induce spatial diversity, increase channel capacity, and attain broader cell coverage with single-antenna terminals. This thesis focuses on the performance study of both unidirectional and bidirectional cooperative diversity networks employing the decode-and-forward (DF) protocol.

For the unidirectional cooperative diversity network, we study the average bit-error rate (BER) performance of a DF protocol with maximum-likelihood (ML) detection. Closed-form approximate average BER expressions involving only elementary functions are presented for a cooperative diversity network with one or two relays. The proposed BER expressions are valid for both coherent and non-coherent binary signalling. With Monte-Carlo simulations, it is verified that the proposed BER expressions are extremely accurate for the whole signal-to-noise ratio (SNR) range.

For the bidirectional cooperative diversity network, we study and compare the performance of three very typical bidirectional communication protocols based on the decode-and-forward relaying: time division broadcast (TDBC), physical-layer network coding (PNC), and opportunistic source selection (OSS). Specifically, we derive an exact outage probability in a one-integral form for the TDBC protocol, and exact closed-form outage probabilities for the PNC and OSS protocols. For the TDBC protocol, we also derive extremely tight upper and lower bounds on the outage

probability in closed-form. Moreover, asymptotic outage probability performance of each protocol is studied. Finally, we study the diversity-multiplexing tradeoff (DMT) performance of each protocol both in the finite and infinite SNR regimes.

The performance analysis presented in this thesis can be used as a useful tool to guide practical system designs for both unidirectional and bidirectional cooperative diversity networks.

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# Chapter 1

## Introduction

### 1.1 Cooperative Communications

Wireless communications suffer from great challenges due to detrimental fading effects of wireless channels [1]. In order to combat fading effects and boost system performance of wireless communications, some techniques known as diversity techniques have been proposed and widely adopted in practice. Among these diversity techniques, the *spatial diversity* is of particular interest because it can be readily combined with the well-known multiple-input-multiple-output (MIMO) signaling processing techniques. MIMO communication through space-time coding (STC) has been widely accepted as an effective means to increase Quality of Service (QoS) of point-to-point communications in terms of combating fading effects, increasing error performance, and boosting throughput by using multiple collocated antennas at the transmitter and/or the receiver [2], [3]. However, due to size, cost, and hard complexity limitations, MIMO techniques with multiple collocated antennas might not be feasible in many applications such as ad hoc or sensor networks [4].

A new way of realizing spatial diversity has recently been proposed and named *cooperative diversity* [5]–[8]. The main idea of cooperative diversity is that multiple terminals equipped with single antenna cooperate to form a virtual MIMO array realizing spatial diversity in a distributed manner. Due to the imitation of classic MIMO, cooperative communication is also known as distributed space-time coding (DSTC). Cooperative diversity avoids the cost, size, and hardware complexity limitations and achieves spatial diversity in a distributed fashion, and therefore, it has been widely accepted as one of the most promising techniques in future wireless communication networks.

In a cooperative diversity network, each user can be either a source terminal transmitting its own signals to others, or a relay terminal assisting in transmissions initiated by other terminals. The relaying signal processing is very important as it helps to provide additional diversity to improve the performance of source transmission. It is desirable to develop simple but effective relay protocols due to the hardware complexity constraints on cooperative communications. To this end, two cooperative protocols, the amplify-and-forward (AF) protocol and the decode-and-forward (DF) protocol, have been proposed in [7]. In the AF protocol, the relay first amplifies the incoming signal and then forwards it to the destination. At the destination, multiple independent replicas of the transmitted signal are received, and therefore, spatial diversity is achieved. In the DF protocol, the relay decodes the incoming signal and transmits an estimate of the incoming signal to the destination. Once again, independent replicas of the source signal are received at the destination and therefore spatial diversity can be achieved.

In principle, signal transmission in a cooperative diversity network can occur in

any direction between any two terminals. However, the medium-access control protocol to coordinate transmissions in possibly arbitrary directions becomes very complex and it would be prohibitively complex to study the performance. For simplicity and analytical tractability, considerable research attention has been put on *unidirectional cooperative diversity networks* where information is transmitted in one direction from the source to the destination and *bidirectional cooperative diversity networks* where information is transmitted in two different directions such that two end-source terminals can exchange information. In what follows, the concepts of unidirectional cooperative diversity networks and bidirectional cooperative diversity networks will be introduced.

### 1.1.1 Unidirectional Cooperative Diversity Networks

The underlying meaning of *unidirectional* suggests that information flows in the network can only travel in one direction, that is, from the source to the destination, possibly through the help of some intermediate relays. A typical unidirectional cooperative diversity network with a source, a destination, and multiple relays is shown in Fig. 1.1, where the single-ended arrows capture the direction in which the information flows travel in the network. In a unidirectional cooperative diversity network, the source terminal initiates an information flow by broadcasting an information-bearing symbol, which is then forwarded to the intended destination by adjacent relays through either AF or DF protocols.

For the AF protocol, each relay uses an amplifying coefficient to scale the incoming signal and the scaled signal is then forwarded to the destination. For CSI-assisted relay, i.e., relay with knowledge of the source-relay channel  $h_{sr}$ , the following amplifying

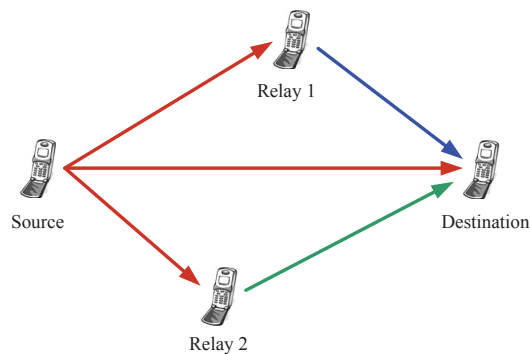


Figure 1.1: Unidirectional cooperative diversity network where source transmits information to destination, with the help of adjacent relays.

coefficient  $\alpha$  is often used [9]–[11]

$$\alpha = \sqrt{\frac{E_r}{E_s|h_{sr}|^2 + N_0}}, \quad (1.1)$$

in order to maintain a fixed instantaneous transmission power at the relay, where  $E_s$  and  $E_r$  represents the transmission power at the source and the relay, respectively, and  $N_0$  is the additive noise power. Although the above choice of  $\alpha$  in (1.1) ensures that the instantaneous transmission power at the relay is always fixed to  $E_r$ , irrespective of the source-relay channel  $h_{sr}$ , it requires the relay to estimate the channel coefficient  $h_{sr}$ , which adds more overhead to the relay. As an alternative, one may also consider a blind-relay case, where the relay has no need to estimate the source-relay channel, and hence, the relay has no knowledge of  $h_{sr}$ . For such blind-relay case, in order to ensure that each relay satisfies a long-term power constraint, the following coefficient is often used [12], [13]

$$\alpha = \sqrt{\frac{E_r}{E_s E[|h_{sr}|^2] + N_0}}, \quad (1.2)$$

where  $E[\cdot]$  is the expectation operator. For the blind-relay case, the instantaneous

transmission power at the relay might fluctuate with  $h_{sr}$ ; however, the average transmission power at the relay in a long-term is still fixed to  $E_r$ . Moreover, the blind-relay scheme also prevents the need of channel estimation.

In an uncoded system adopting DF protocol, each relay first decodes the incoming signal and then re-modulates and forwards the decoded signal to the destination. As the relay always forwards the signal to the destination whether the signal is detected correctly or not, an erroneous decoded signal might be forwarded by the relay, which might deteriorate the performance at the destination. To cope with this problem, DF cooperation incorporated with cyclic-redundancy-check (CRC) codes has been studied in many works [4], [14]. With CRC codes at the relay, only a correctly decoded signal is allowed to be forwarded by the relay; this way the error propagation due to relay decoding error can be completely avoided. Note that while the use of CRC codes prevents error propagation caused by the relay decoding error, it also increases the hardware complexity of the relay and requires additional time to decode and re-encode CRC codes at the relay.

### 1.1.2 Bidirectional Cooperative Diversity Networks

In a bidirectional cooperative diversity network, two end-source terminals intend to exchange information with each other, through the help of intermediate relays working in either AF or DF mode, as shown in Fig. 1.2, where the arrows represent the possible directions of information flows in the network.

Recently, bidirectional communication has received considerable attention because of its improved bandwidth efficiency compared with unidirectional communication [15]. Many spectral efficient bidirectional protocols combined with the concept of

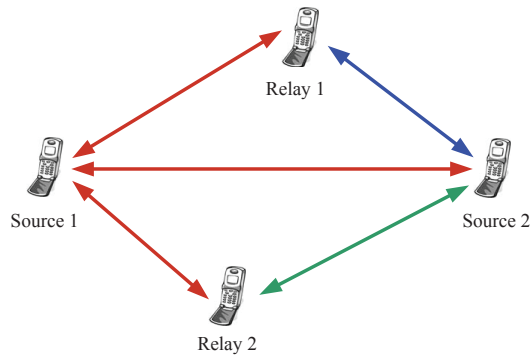


Figure 1.2: Bidirectional cooperative diversity network where two end-source terminals A and B exchange information, with the help of a relay terminal R.

network coding have been proposed. In particular, Zhang *et al.* proposed the physical-layer network coding (PNC) [16], Katti *et al.* studied the analog network coding (ANC) [20], and Kim *et al.* studied the time division broadcast (TDBC) protocol [18]. In general, these bidirectional protocols have better spectral efficiency than unidirectional protocols.

In addition, bidirectional communication has been considered in an opportunistic manner such that the well-known *multiuser diversity* can be exploited to increase the reliability. In [21], Zhihang and Kim proposed an *opportunistic source selection* (OSS) protocol in which only one end-source user with the best channel condition is allowed to transmit in each time slot. Owing to the opportunistic manner of determining the active transmitting source, the OSS protocol has a high reliability, which was demonstrated numerically in [21].

### 1.1.3 Performance Measures

To evaluate the performance of cooperative diversity communications, in this section, we introduce some general system performance measures that are widely used in any digital communication systems.

#### Error Probability

Error probability is a fundamental performance measure of any digital communication systems [1], [23]. The symbol-error rate (SER) and the bit-error rate (BER) are two widely used error probability measures. Consider a communication system with instantaneous signal-to-noise ratio (SNR)  $\gamma$  at the receiver, the instantaneous BER or SER for many signalling schemes can be bounded or exactly expressed via an expression of Gaussian Q-function [2], [23]:

$$P_0 = aQ(\sqrt{b\gamma}), \quad (1.3)$$

where  $a$  and  $b$  are modulation-dependent constants,  $\gamma$  is a function of the channel coefficient. As an example, the instantaneous SNR of a dual-hop cooperative diversity network with CSI-assisted relay is given by  $\gamma = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}$ , where  $\gamma_1$  and  $\gamma_2$  are the instantaneous SNRs of the first and second hops, respectively.

As the instantaneous BER/SER varies with random variable  $h$ , it would be more desirable to find the average BER/SER, which is the expectation of the instantaneous BER/SER, as shown as follows:

$$E[P_0] = a \int_0^\infty Q(\sqrt{bx}) f_\gamma(x) dx, \quad (1.4)$$

where  $f_\gamma(x)$  is the PDF of  $\gamma$ . As the average error probability of (1.4) makes use of the PDF  $f_\gamma(x)$  of  $\gamma$ , the approach of (1.4) was referred to as the PDF-based approach



[23]. Through this PDF-based approach, the average BER/SER, if not able to be solved in closed-form, can be represented with a single-integral with infinite limits.

An alternative way to derive the average BER/SER is to take advantage of the moment generating function (MGF) of  $\gamma$ . By representing the Gaussian Q-function with the Craig's formula, the average BER/SER can be represented with a single integral with finite limits and an integrand composed of the MGF of  $\gamma$ , and this approach was referred to as the MGF-based approach [23]. The MGF-based approach is described as follows:

$$\begin{aligned} E[P_0] &= a[Q(\sqrt{b\gamma})] \\ &= E \left[ \frac{a}{\pi} \int_0^{\pi/2} \exp \left( -\frac{b\gamma}{2 \sin^2 \theta} \right) d\theta \right] \\ &= \frac{a}{\pi} \int_0^{\pi/2} M_\gamma \left( -\frac{b}{2 \sin^2 \theta} \right) d\theta, \end{aligned} \tag{1.5}$$

where  $M_\gamma(s)$  is the MGF of  $\gamma$ . As (1.5) shows, using the MGF-based approach, the average BER/SER, if not able to be solved in closed-form, can be represented in a single-integral with finite limits.

### Outage Probability

Outage probability is an information-theoretic performance measure of communication systems. By Shannon's channel coding theorem [24], vanishing probability of error can be achieved by using powerful channel coding as long as the data rate of the system is lower than channel capacity. On the other hand, for a system transmitting with a higher rate than channel capacity, error-free communication is impossible, and the system is called in *outage*. Therefore, *outage probability* is defined as the

probability that a target transmission rate  $R$  exceeds channel capacity  $C$ , i.e.,

$$P_{\text{outage}} = \Pr(C < R). \quad (1.6)$$

As the channel capacity is usually monotonically increasing in  $\gamma$ , the outage probability of (1.6) may be rewritten as

$$P_{\text{outage}} = \int_0^{\gamma_{\text{th}}} f_{\gamma}(x) dx, \quad (1.7)$$

where  $\gamma_{\text{th}}$  is a predetermined threshold value related with the target rate  $R$ . As (1.7) demonstrates, the outage probability is cumulative distribution function (CDF) of  $\gamma$ , evaluated at  $\gamma = \gamma_{\text{th}}$ . Furthermore, since the MGF is just the Laplace transform of the PDF with argument reversed in sign, the outage probability can be found through the MGF of  $\gamma$ . Once gain, we see that the CDF or MGF of  $\gamma$  plays an important role in analyzing the performance of communication systems.

### **Diversity-Multiplexing Tradeoff**

Diversity gain and multiplexing gain are two important measures of system performance in two different aspects [3]. Diversity gain measures system reliability in terms of error probability or outage probability, whereas multiplexing gain measures system performance in terms of spectral efficiency. Many practical communication systems have aimed to achieve either maximal diversity gain or maximal multiplexing gain, depending on the applications. For example, the well-known vertical Bell Labs space-time (VBLAST) architecture was developed for MIMO systems to achieve the highest multiplexing gain. Also, through orthogonal space-time block coding (OSTBC), MIMO communication is able to achieve maximal (full) diversity gain [2], [3].

However, system designs based purely on the diversity criterion or the multiplexing criterion might not be sufficient. This has been demonstrated by Zheng and Tse

by discovering a fundamental tradeoff between diversity gain and multiplexing gain, which was referred to as the *diversity-multiplexing tradeoff* [25]. This fundamental tradeoff suggests that increasing diversity gain decreases multiplexing gain, and increasing multiplexing gain also decreases diversity gain. Also, it was demonstrated that maximal diversity gain comes at the price of sacrificing multiplexing gain, and maximal spatial multiplexing gain comes at the price of sacrificing diversity gain [25]. Therefore, it is more reasonable to compare different communication systems in terms of the diversity-multiplexing tradeoff rather than diversity gain or multiplexing gain only. Since the work of [25], numerous works have started to study the diversity-multiplexing tradeoff of different communication systems, especially the MIMO and cooperative communication systems.

## 1.2 Motivation and Thesis Overview

Theoretical performance analysis is a useful tool to guide communication system designs in practice. By optimizing the derived analytical performance expressions, such as minimizing error probability, minimizing outage probability, maximizing spectral efficiency, and so on, some optimum system parameters may be determined, which can be then used to guide practical communication system designs. For this reason, numerous works have been devoted to the performance analysis of digital communication systems. In particular, performance analysis of communication protocols in cooperative diversity networks has attracted a lot of interest.

For unidirectional cooperative diversity networks, many works have analyzed the performance of AF protocol, including outage probability analysis and average BER analysis (see [12], [13], and [26]), and average SER analysis (see [9]–[11]). However,

little progress has been made on the average BER analysis of DF protocol in uncoded unidirectional cooperative diversity networks. This has motivated us to analyze the average BER of DF protocol in uncoded unidirectional cooperative diversity networks.

Bidirectional cooperative protocols are relatively newer than unidirectional cooperative protocols, and therefore, much less research attention has been put on the performance analysis of bidirectional cooperative protocols. To the best of our knowledge, for the DF-based bidirectional protocols such as the TDBC, PNC, and OSS protocols, there have been no analytical results on the outage probability or diversity-multiplexing tradeoff performance analyses in the literature, this has motivated our work on the performance study of the bidirectional cooperative protocols, with emphasis on outage probability and diversity-multiplexing tradeoff.

The remainder of this thesis is organized as follows. In Chapter 2, we study the average BER of the maximum likelihood (ML) detection in uncoded unidirectional networks employing the DF protocol. Closed-form average BER expressions are derived and shown to be valid for general dissimilar networks adopting both coherent and noncoherent binary signalings. In Chapter 3, we carry out performance analysis of bidirectional DF-based protocols including TDBC, PNC, and OSS. Closed-form outage probability (either exact or bounded) expressions and the diversity-multiplexing tradeoff performance in both the finite and infinite SNR regimes are reported. Finally, some conclusions of this thesis are drawn in Chapter 4.

### 1.3 Thesis Contribution

This thesis aims to analyze the performance of unidirectional and bidirectional cooperative protocols based on DF relaying. The primary contributions of the thesis are

summarized as follows:

- For unidirectional cooperative diversity networks, we analyze the average BER performance of the ML detection with uncoded DF protocol. Specifically, we derive closed-form approximate average BER expressions for both the single-relay and two-relay cooperative diversity networks with general dissimilar channel settings. The derived BER expressions are valid for coherent and noncoherent binary signalings. Furthermore, numerical results demonstrate that our BER expressions are extremely accurate in the whole SNR range.
- For bidirectional cooperative diversity networks, we study the outage probability and diversity-multiplexing tradeoff performance of three bidirectional protocols based on DF relaying: TDBC, PNC, and OSS. Specifically, we derive an exact outage probability in a one-integral form for the TDBC protocol, and exact closed-form outage probability expressions for the PNC and OSS protocols. For the TDBC protocol, we also derive extremely tight upper and lower bounds on the outage probability in closed-form. Moreover, we derive asymptotic outage probability expression in closed-form and study the diversity gain for each protocol. To provide more insight, we also study the diversity-multiplexing tradeoff of each bidirectional protocol in both the finite and infinite SNR regimes. With the diversity-multiplexing tradeoff analysis, we demonstrate that depending on the spectral efficiency, any one of the three protocols might outperform other protocols in terms of diversity-multiplexing tradeoff.

## Chapter 2

# Unidirectional Cooperative Diversity Networks

In this chapter, the average bit-error rate (BER) performance is analyzed for uncoded *decode-and-forward* (DF) unidirectional cooperative diversity networks. We consider two typical networks: a single-relay cooperative network with the direct source-destination link and a two-relay cooperative network with the direct source-destination link, under *dissimilar* network settings, i.e., the fading channels of different relay branches may have different variances. We first derive a closed-form approximate average BER expression of binary signalling including noncoherent binary frequency shift keying (BFSK), coherent BFSK, and coherent binary phase shift keying (BPSK), for the single-relay network. We then generalize our analysis to the two-relay network, and a closed-form approximate average BER expression for binary signalling is derived. We also show that our BER expressions can be considered as generalizations of previously reported results in the literature. Throughout our analysis, only one approximation, so-called the piecewise-linear approximation, is made.

Simulation results are in excellent agreement with the theoretical analysis, which validates our proposed BER expressions.

## 2.1 Introduction

Diversity has been acknowledged as one of the most effective techniques to combat fading effects in wireless communications [1]. Recently, a new kind of diversity, cooperative diversity [5]–[8], has been proposed as a means of attaining broader cell coverage, mitigating channel impairments, and increasing the channel capacity without using multiple antennas at each terminal. One of the most well-known cooperative protocols is the decode-and-forward (DF) protocol, in which each relay receives and decodes the signal transmitted by the source and then forwards the decoded signal to the destination. There has been a lot of works carried out on the DF protocol incorporated with channel codes, for which the name *coded cooperation* is given [14], [27], [28]. Some bit-error rate (BER) and frame-error rate bounds have been derived in [14], [27], [28]. On the other hand, many works have focused on *uncoded DF cooperation* where no channel codes are used [29]–[33].

Recently, many researchers have started to analyze the performance of ML detection for uncoded DF cooperation in unidirectional cooperative diversity networks [30]–[33]. However, the *exact* closed-form average BER analysis is extremely difficult even for *single*-relay cooperative networks with *binary* modulations [29], [30].<sup>1</sup> Therefore, most of the works have been devoted to the *approximate* average BER analysis. Nonetheless, still very limited results have been reported in the literature. In [30],

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<sup>1</sup>In [30], the authors commented “Analysis of DF diversity transmission is challenging since it must treat the nonlinear behavior of (7), which significantly complicates the effort of obtaining a closed-form solution for BER.”.

Chen and Laneman have proposed an accurate approximation, which was referred to as the piecewise-linear (PL) approximation, and they obtained an accurate approximate closed-form average BER expression for single-relay systems with noncoherent binary frequency shift keying (BFSK). For single-relay systems with coherent BPSK, two approximate average BER expressions have been derived in [32] and [33] based on the PL approximation;<sup>2</sup> however, the BER expressions are expressed in *double-integral* forms, which are not truly closed-form in a strict sense and are quite difficult to use in practice. For single-relay systems with coherent BFSK, no average BER expression has been reported, and it has been acknowledged that it is hard to analyze the average BER performance of the ML detection in closed-form for coherent BFSK even for *single-relay* cooperative networks [30].<sup>3</sup> For *two-relay* cooperative networks, only one BER expression has been reported in the literature [31, eq. (4.14)]. However, this expression is only applicable for noncoherent BFSK and for *symmetric* networks where the channel variances of the first hop for different branches are equal. However, the symmetric settings is not practical as the author acknowledged [31].

To the best of our knowledge, no closed-form average BER expressions of coherent binary signalling for single-relay uncoded DF cooperative networks have been found, and no closed-form BER expressions of binary signalling for two-relay uncoded DF *dissimilar* cooperative networks have been found either. This motivated our work.

In this chapter, we analyze the average BER performance of the ML detection for uncoded DF unidirectional cooperative networks with dissimilar settings. Specifically, we consider two typical cooperative diversity networks: the single-relay cooperative

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<sup>2</sup>In [32] and [33], the ML receivers are termed optimum and sub-optimum, respectively, depending on whether the instantaneous source-relay channel coefficients are available at the destination.

<sup>3</sup>In [30], the authors commented “Note that, at present, we have no simple, closed-form expression for the BER of coherent DF ...”.



diversity network [30]–[33] and the two-relay cooperative diversity network [31], [34], [35]. First, we derive the probability density functions (PDFs) and cumulative distribution functions (CDFs) of the sufficient statistics for the ML decision-making at the destination. Then we apply the accurate PL approximation and derive closed-form approximate average BER expressions with the help of the obtained PDFs and CDFs. Our BER expressions are shown to be valid for the general dissimilar uncoded DF networks adopting both coherent and noncoherent binary signalling. We also show that our BER expressions can be considered as generalizations of the previously reported results in the literature. The numerical results demonstrate that our BER expressions are extremely accurate.

The remainder of this chapter is organized as follows. In Section 2.2, we describe the system model and review the ML detection for uncoded DF networks. In Section 2.3, we derive a closed-form approximate average BER expression for single-relay uncoded DF networks. In Section 2.4, a closed-form approximate average BER expression for two-relay uncoded DF networks is proposed. Section 2.5 presents some numerical results and Section 2.6 concludes this chapter.

*Notation:* For a real-valued random variable  $X$ ,  $X \sim \mathcal{N}(\mu_x, \Omega_x)$  indicates that  $X$  is a real-valued Gaussian random variable with mean  $\mu_x$  and variance  $\Omega_x$ . For a complex-valued random variable  $Z$ ,  $Z \sim \mathcal{CN}(\mu_z, \Omega_z)$  indicates that  $Z$  is a circularly symmetric complex-valued Gaussian random variable with mean  $\mu_z$  and variance  $\Omega_z$ .

## 2.2 System Model

Consider a unidirectional cooperative diversity network consisting of a source,  $K$  relays, and a destination as shown in Fig. 2.1. The source and the destination are

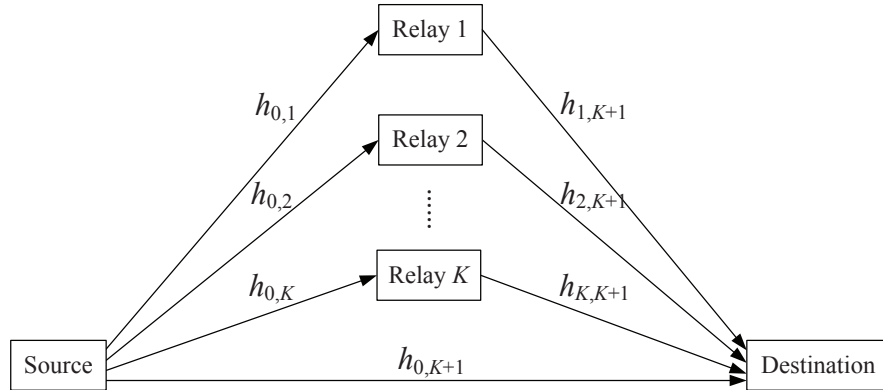


Figure 2.1: Unidirectional cooperative diversity network with  $K$  relays and the direct source-destination link.

denoted by terminal 0 and terminal  $K + 1$ , respectively, and relays are denoted by terminal  $i$ ,  $i = 1, 2, \dots, K$ . In this chapter, we consider the *uncoded cooperation* with DF protocol, i.e., no channel codes are used. Each terminal in the network is equipped with a single antenna working in the half-duplex mode. We consider an *orthogonal* transmission scheme in which only one terminal is allowed to transmit at each time slot [7], [29], [30]. Therefore, the data transmission consists of two phases. In the first phase, the source broadcasts the signal, and each relay attempts to decode the signal. In the second phase, different relay terminals transmit their own remodulated signals in different time slots.

The channel coefficients  $h_{i,j}$  for the link between terminal  $i$  and terminal  $j$  are modeled as complex Gaussian random variables with  $h_{i,j} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$ ,  $i, j \in \{0, 1, 2, \dots, K + 1\}$ . In this chapter, we consider three different modulation schemes: *coherent* BPSK, BFSK, and *noncoherent* BFSK. For coherent schemes, we assume that the destination has knowledge about the instantaneous source-destination channel coefficient  $h_{0,K+1}$  and all the instantaneous relay-destination channel coefficients  $h_{i,K+1}$ ,

$i = 1, 2, \dots, K$ ; but only the statistical knowledge (i.e., channel distribution) about the source-relay channels  $h_{0,i}$ ,  $i = 1, 2, \dots, K$ .<sup>4</sup> For the noncoherent scheme, no instantaneous CSI is available at the destination; only the statistical distributions of all channels are known at the destination. Furthermore, the channel coefficients  $h_{i,K+1}$ ,  $i = 0, 1, \dots, K$ , are assumed to be mutually independent. The additive noise associated with  $h_{i,j}$  is denoted by  $n_{i,j}$  for BPSK and denoted by  $n_{i,j,k}$ ,  $k \in \{0, 1\}$ , for BFSK, where the third subscript  $k$  is the index of the two frequency subbands for BFSK signalling. We model the noise terms as mutually independent additive white Gaussian noise (AWGN) with zero mean and variance  $N_0$ . Consequently, the instantaneous signal-to-noise ratio (SNR)  $\gamma_{i,j}$  of the channel from terminal  $i$  to terminal  $j$  is given by  $\gamma_{i,j} = E_i|h_{i,j}|^2/N_0$  and the average SNR is given by  $\bar{\gamma}_{i,j} = E_i\sigma_{i,j}^2/N_0$ , where  $E_i$  is the average transmission power at terminal  $i$ .

For BFSK signalling, the signals received by terminal  $i$ ,  $i = 1, 2, \dots, K + 1$ , which were transmitted from the source, through the first frequency subband  $y_{0,i,0}$  and through the second frequency subband  $y_{0,i,1}$  are given by

$$\begin{aligned}
 y_{0,i,0} &= (1 - x_0)\sqrt{E_0}h_{0,i} + n_{0,i,0}, \\
 y_{0,i,1} &= x_0\sqrt{E_0}h_{0,i} + n_{0,i,1},
 \end{aligned} \tag{2.1}$$

where  $x_0 = 0$  if the first frequency subband is used and  $x_0 = 1$  if the second frequency subband is used.

For BPSK signalling, the signal  $y_{0,i}$  received by terminal  $i$ ,  $i = 1, 2, \dots, K + 1$ , which was transmitted from the source, is given by

$$y_{0,i} = (1 - 2x_0)\sqrt{E_0}h_{0,i} + n_{0,i}, \tag{2.2}$$

---

<sup>4</sup>The same channel state information (CSI) assumption was made in [30], [31] and [33] for coherent detection.

where  $x_0 \in \{0, 1\}$ .

In this chapter, we will refer to  $x_0$  as the transmitted signal at the source for BFSK and BPSK modulations, because  $x_0$  can represent the two possibilities of the binary transmission. For DF, we suppose that, at relay  $i$ , the signal  $x_0$  is decoded into  $x_i$ ,  $x_i \in \{0, 1\}$ . Similarly, we will refer to  $x_i$  as the transmitted signal at relay  $i$ . Then for BFSK signalling, the signals received by the destination, which were transmitted from relay  $i$ , through the two frequency subbands, are given by

$$\begin{aligned} y_{i,K+1,0} &= (1 - x_i)\sqrt{E_i}h_{i,K+1} + n_{i,K+1,0}, \\ y_{i,K+1,1} &= x_i\sqrt{E_i}h_{i,K+1} + n_{i,K+1,1}. \end{aligned} \quad (2.3)$$

For BPSK signalling, the signal  $y_{i,K+1}$  received by the destination, which was transmitted from relay  $i$ , is given by

$$y_{i,K+1} = (1 - 2x_i)\sqrt{E_i}h_{i,K+1} + n_{i,K+1}. \quad (2.4)$$

The log-likelihood ratio (LLR) of the ML detection for the uncoded DF cooperation with binary modulations has been shown to be given by [29], [30]:

$$\text{LLR} = t_0 + \sum_{i=1}^K \psi(t_i), \quad (2.5)$$

where

$$\psi(t_i) = \ln \frac{(1 - \epsilon_i)e^{t_i} + \epsilon_i}{\epsilon_i e^{t_i} + (1 - \epsilon_i)}. \quad (2.6)$$

In this equation,  $\epsilon_i$  represents the *average*<sup>5</sup> BER at relay  $i$ , and it is given by

$$\epsilon_i = \begin{cases} \frac{1}{2} \left( 1 - \sqrt{\frac{\bar{\gamma}_{0,i}/2}{1 + \bar{\gamma}_{0,i}/2}} \right), & \text{for coherent BFSK,} \\ \frac{1}{2} \left( 1 - \sqrt{\frac{\bar{\gamma}_{0,i}}{1 + \bar{\gamma}_{0,i}}} \right), & \text{for coherent BPSK,} \\ \frac{1}{2 + \bar{\gamma}_{0,i}}, & \text{for noncoherent BFSK,} \end{cases} \quad (2.7)$$

---

<sup>5</sup>We assume that the destination only has the statistical CSI rather than the instantaneous CSI of the source-relay channels. Therefore, the destination only knows the average BER at the relays.

for  $i = 1, 2, \dots, K$ . The sufficient statistics  $t_i$ ,  $i = 0, 1, \dots, K + 1$ , are given by

$$t_i = \begin{cases} \frac{2\sqrt{E_i}\Re\{h_{i,K+1}^*(y_{i,K+1,0}-y_{i,K+1,1})\}}{N_0}, & \text{for coherent BFSK,} \\ \frac{4\sqrt{E_i}\Re\{h_{i,K+1}^*y_{i,K+1}\}}{N_0}, & \text{for coherent BPSK,} \\ \frac{\tilde{\gamma}_{i,K+1}}{(1+\tilde{\gamma}_{i,K+1})N_0}(|y_{i,K+1,0}|^2 - |y_{i,K+1,1}|^2), & \text{for noncoherent BFSK.} \end{cases} \quad (2.8)$$

It has been acknowledged that the exact closed-form BER analysis is very challenging even for single-relay systems with binary signalling due to the nonlinear behavior (2.6) of the ML detection [30]. Therefore, to facilitate the BER analysis, the PL approximation has been proposed [30]:

$$\psi(t_i) \approx \psi_{PL}(t_i) = \begin{cases} T_i, & \text{for } t_i \geq T_i, \\ t_i, & \text{for } -T_i < t_i < T_i, \\ -T_i, & \text{for } t_i \leq -T_i, \end{cases} \quad (2.9)$$

where  $T_i = \ln \frac{1-\epsilon_i}{\epsilon_i}$ , and  $T_i > 0$  since  $\epsilon_i < 1/2$ . It has been demonstrated that the above PL approximation is very accurate [30]. For a symmetric network with identical channel variance of the first hop for each branch,  $T_i$ ,  $i = 1, 2, \dots, K$ , are identical. This simplified case was considered in [31] for a two-relay network. In this chapter, we consider a general dissimilar network with different  $T_i$  for different branches.

In this chapter, the PL approximation (2.9) which has been extensively used in many papers, e.g., [30]–[33], is also applied, and it is the only approximation we adopt in our analysis.

## 2.3 Closed-Form Approximate Average BER for Single-Relay Networks

In this section, we derive a closed-form approximate average BER expression for the single-relay DF cooperative diversity network. Note that  $t_i$ ,  $i = 0, 1, \dots, K + 1$ , represent the sufficient statistics for ML detection. Hence, in order to assess the ML detection performance, it is needed to find the distribution of  $t_i$ . To this end, we first develop the following lemma.

**Lemma 2.1.** *Consider a random variable  $Y = |Z|X + |Z|^2$ , where  $X \sim \mathcal{N}(0, \sigma_x^2)$  and  $Z \sim \mathcal{CN}(0, \sigma_z^2)$  are independent random variables. Then the PDF  $f(a, b, c, y)$  of random variable  $Y$  is given by*

$$f(a, b, c, y) = \begin{cases} a \exp(cy), & y \leq 0, \\ a \exp(by), & y > 0, \end{cases} \quad (2.10)$$

and the CDF  $F(a, b, c, y)$  of random variable  $Y$  is given by

$$F(a, b, c, y) = \begin{cases} \frac{a}{c} \exp(cy), & y \leq 0, \\ 1 + \frac{a}{b} \exp(by), & y > 0, \end{cases} \quad (2.11)$$

where the parameters  $a$ ,  $b$ , and  $c$  are given by

$$a = \frac{1}{\sigma_z \sqrt{\sigma_z^2 + 2\sigma_x^2}}, \quad (2.12)$$

$$b = \frac{1}{\sigma_x^2} \left( 1 - \frac{\sqrt{\sigma_z^2 + 2\sigma_x^2}}{\sigma_z} \right), \quad (2.13)$$

$$c = \frac{1}{\sigma_x^2} \left( 1 + \frac{\sqrt{\sigma_z^2 + 2\sigma_x^2}}{\sigma_z} \right). \quad (2.14)$$

*Proof.* From the standard probabilistic analysis, PDF of  $Y$  can be derived by taking expectation over the conditional PDF of  $Y$  given  $Z$ , and CDF of  $Y$  can be derived by integrating the PDF of  $Y$ .  $\square$

**Corollary 2.1.** *Suppose  $Y = |Z|X - |Z|^2$ , where  $X \sim \mathcal{N}(0, \sigma_x^2)$  and  $Z \sim \mathcal{CN}(0, \sigma_z^2)$  are independent random variables. Then the PDF and CDF of random variable  $Y$  are given by  $f(a, b, c, -y)$  and  $1 - F(a, b, c, -y)$ , respectively.*

*Proof.* Rewrite  $Y = |Z|X - |Z|^2 = -(|Z|^2 - |Z|X)$ , and denote  $X' = -X$ , then  $Y = -(|Z|^2 + |Z|X')$ . Since  $X'$  has the same distribution as  $X$ , by applying Lemma 2.1, the PDF of the term  $|Z|^2 + |Z|X'$  is given by  $f(a, b, c, y)$ . Therefore, it can be easily shown that the PDF of  $Y$  is  $f(a, b, c, -y)$ , and CDF is  $1 - F(a, b, c, -y)$ .  $\square$

Using Lemma 2.1 and Corollary 2.1, in the following theorem, we now derive the distribution of  $t_i$ ,  $i = 0, 1, \dots, K$ , given the transmitted signal  $x_i$ .

**Theorem 2.1.** *The PDFs of  $t_i$  conditioned on  $x_i = 0$  and  $x_i = 1$  are given by  $f(a_i, b_i, c_i, t_i)$  and  $f(a_i, b_i, c_i, -t_i)$ , respectively. The CDFs of  $t_i$  conditioned on  $x_i = 0$  and  $x_i = 1$  are given by  $F(a_i, b_i, c_i, t_i)$  and  $1 - F(a_i, b_i, c_i, -t_i)$ , respectively. For coherent binary modulations, the parameters  $a_i$ ,  $b_i$ , and  $c_i$  are given by (2.12), (2.13), and (2.14), respectively, with  $\sigma_z$  and  $\sigma_x$  replaced by  $\sigma_{z_i} = 2\sqrt{\gamma_{i,K+1}}$  and  $\sigma_{x_i} = \sqrt{2}$  for coherent BPSK; by  $\sigma_{z_i} = \sqrt{2\gamma_{i,K+1}}$  and  $\sigma_{x_i} = \sqrt{2}$  for coherent BFSK. For noncoherent BFSK,  $a_i$ ,  $b_i$ , and  $c_i$  are given by  $a_i = \frac{\lambda_i \lambda'_i}{\lambda_i + \lambda'_i}$ ,  $b_i = -\lambda_i$ ,  $c_i = \lambda'_i$ , where  $\lambda_i = 1/\gamma_{i,K+1}$  and  $\lambda'_i = 1 + \lambda_i$ .*

*Proof.* For coherent BFSK and BPSK, we can rewrite the expression of  $t_i$  in the same form as random variable  $Y$  in Lemma 2.1. Therefore, the PDFs and CDFs

for coherent BFSK and BPSK can be obtained by applying Lemma 2.1. For non-coherent BFSK,  $t_i$  is the subtraction of two exponential random variables. In this case, the distribution of  $t_i$  given  $x_i$  has been derived by Chen and Laneman in [30]. By comparing eq. (17) of [30] with our expression of (2.11), we observed that our obtained CDF expression of (2.11) reduce to eq. (17) of [30] when replacing  $a$ ,  $b$ , and  $c$  with  $a_i = \frac{\lambda_i \lambda'_i}{\lambda_i + \lambda'_i}$ ,  $b_i = -\lambda_i$ ,  $c_i = \lambda'_i$ , respectively, where  $\lambda_i = 1/\bar{\gamma}_{i,K+1}$  and  $\lambda'_i = 1 + \lambda_i$ .  $\square$

It can be seen from (2.5) that the ML detection for cooperative diversity networks with  $K$  relays involves  $K + 1$  sufficient statistics  $t_0, t_1, \dots, t_K$ . In order to derive the average BER for single-relay cooperative networks, we develop the following lemma which deals with the probability involving two different statistics  $t_i$  and  $t_j$ .

**Lemma 2.2.** *Define  $Pr(t_i + t_j < S_1, -S_2 < t_j < S_2 | x_i = 0, x_j) = g_{x_j}(a_i, b_i, c_i, a_j, b_j, c_j, S_1, S_2)$ , where  $x_j \in \{0, 1\}$ ,  $S_1$  is any arbitrary real number, and  $S_2 > 0$ . Also,  $t_i$  and  $t_j$  are two different statistics taken from (2.8) for a common modulation scheme with  $i, j \in \{0, 1, \dots, K\}$  and  $i \neq j$ . Define  $\hat{b}_j = (1 - x_j)b_j - x_j c_j$  and  $\hat{c}_j = (1 - x_j)c_j - x_j b_j$ .*



Then  $g_{x_j}(\cdot)$  is given as follows: If  $b_i \neq \hat{b}_j$  and  $c_i \neq \hat{c}_j$ ,

$$\begin{aligned}
 & g_{x_j}(a_i, b_i, c_i, a_j, b_j, c_j, S_1, S_2) \\
 &= \begin{cases}
 \begin{aligned}
 & F(a_j, \hat{b}_j, \hat{c}_j, S_2) - F(a_j, \hat{b}_j, \hat{c}_j, -S_2) + \frac{a_i a_j \exp(b_i S_1)}{b_i(b_i - \hat{c}_j)} \left\{ \exp[(b_i - \hat{c}_j)S_2] - 1 \right\} \\
 & + \frac{a_i a_j \exp(b_i S_1)}{b_i(b_i - \hat{b}_j)} \left\{ 1 - \exp[(\hat{b}_j - b_i)S_2] \right\}, \quad \text{for } S_1 \geq S_2 > 0,
 \end{aligned} \\
 \\
 \begin{aligned}
 & F(a_j, \hat{b}_j, \hat{c}_j, S_1) - F(a_j, \hat{b}_j, \hat{c}_j, -S_2) + \frac{a_i a_j \exp(b_i S_1)}{b_i(b_i - \hat{c}_j)} \left\{ \exp[(b_i - \hat{c}_j)S_2] - 1 \right\} \\
 & + \frac{a_i a_j \exp(c_i S_1)}{c_i(c_i - \hat{b}_j)} \left\{ \exp[(\hat{b}_j - c_i)S_1] - \exp[(\hat{b}_j - c_i)S_2] \right\} \\
 & + \frac{a_i a_j \exp(b_i S_1)}{b_i(b_i - \hat{b}_j)} \left\{ 1 - \exp[(\hat{b}_j - b_i)S_1] \right\}, \quad \text{for } 0 \leq S_1 < S_2,
 \end{aligned} \\
 \\
 \begin{aligned}
 & F(a_j, \hat{b}_j, \hat{c}_j, S_1) - F(a_j, \hat{b}_j, \hat{c}_j, -S_2) + \frac{a_i a_j \exp(c_i S_1)}{c_i(c_i - \hat{b}_j)} \left\{ 1 - \exp[(\hat{b}_j - c_i)S_2] \right\} \\
 & + \frac{a_i a_j \exp(b_i S_1)}{b_i(b_i - \hat{c}_j)} \left\{ \exp[(b_i - \hat{c}_j)S_2] - \exp[(\hat{c}_j - b_i)S_1] \right\} \\
 & + \frac{a_i a_j \exp(c_i S_1)}{c_i(c_i - \hat{c}_j)} \left\{ \exp[(\hat{c}_j - c_i)S_1] - 1 \right\}, \quad \text{for } -S_2 \leq S_1 < 0,
 \end{aligned} \\
 \\
 \begin{aligned}
 & \frac{a_i a_j \exp(c_i S_1)}{c_i(c_i - \hat{b}_j)} \left\{ 1 - \exp[(\hat{b}_j - c_i)S_2] \right\} \\
 & + \frac{a_i a_j \exp(c_i S_1)}{c_i(c_i - \hat{c}_j)} \left\{ \exp[(c_i - \hat{c}_j)S_2] - 1 \right\}, \quad \text{for } S_1 < -S_2 < 0.
 \end{aligned}
 \end{cases}
 \end{aligned} \tag{2.15}$$

If  $b_i = \hat{b}_j$ , the last terms in the first and second cases of (2.15) are replaced by  $\frac{a_i a_j S_2 \exp(b_i S_1)}{b_i}$  and  $\frac{a_i a_j S_1 \exp(b_i S_1)}{b_i}$ , respectively. If  $c_i = \hat{c}_j$ , the last terms in the third and fourth cases of (2.15) are replaced by  $-\frac{a_i a_j S_1 \exp(c_i S_1)}{c_i}$  and  $\frac{a_i a_j S_2 \exp(c_i S_1)}{c_i}$ , respectively.

*Proof.* By Theorem 2.1 and performing some mathematical manipulations, it is not hard to prove this lemma.  $\square$

From the expressions of  $a_i$ ,  $b_i$ , and  $c_i$  given in Theorem 2.1, it can be easily verified that  $a_i > 0$ ,  $b_i < 0$ , and  $c_i > 0$ . By some simple manipulations, it can be

easily verified that  $a_i \neq c_i$ ,  $b_i + c_i \neq 0$ ,  $b_i + b_j \neq 0$ ,  $b_i - c_j \neq 0$ , and  $c_i - b_j \neq 0$  for any  $i, j \in \{0, 1, \dots, K\}$ ; but it is possible that  $b_i = b_j$ ,  $c_i = c_j$ , or  $b_i + c_j = 0$  for  $i, j \in \{0, 1, \dots, K\}$  and  $i \neq j$ . Therefore, the possibilities  $b_i = \hat{b}_j$  and  $c_i = \hat{c}_j$  need to be considered, and the possibilities  $b_i - \hat{c}_j = 0$  and  $c_i - \hat{b}_j = 0$  need not be considered in Lemma 2.2.

With Theorem 2.1 and Lemma 2.2, in the following theorem, we now derive an average BER expression of single-relay systems.

**Theorem 2.2.** *For binary modulations, a closed-form approximate average BER  $P_{B,1}$  of the single-relay ( $K = 1$ ) cooperative diversity network is*

$$\begin{aligned}
 P_{B,1} = & (1 - \epsilon_1) \{ g_0(a_0, b_0, c_0, a_1, b_1, c_1, 0, T_1) + F(a_0, b_0, c_0, -T_1) [1 - F(a_1, b_1, c_1, T_1)] \\
 & + F(a_0, b_0, c_0, T_1) F(a_1, b_1, c_1, -T_1) \} \\
 & + \epsilon_1 \{ g_1(a_0, b_0, c_0, a_1, b_1, c_1, 0, T_1) + F(a_0, b_0, c_0, -T_1) F(a_1, b_1, c_1, -T_1) \\
 & + F(a_0, b_0, c_0, T_1) [1 - F(a_1, b_1, c_1, T_1)] \},
 \end{aligned} \tag{2.16}$$

where  $\epsilon_1$ ,  $T_1$ ,  $a_i$ ,  $b_i$ , and  $c_i$ ,  $i = 0, 1$ , are modulation-dependent parameters as described before.

*Proof.* Applying the total probability theorem, Theorem 2.1, and Lemma 2.2 yields this theorem.  $\square$

Note that (2.16) is truly a *closed-form* approximate average BER expression, which does not involve any numerical computations. In the expression of (2.16), by choosing  $a_i$ ,  $b_i$ , and  $c_i$  as described in Theorem 2.1,  $\epsilon_1$  as given by (2.7), and  $T_1 = \ln \frac{1-\epsilon_1}{\epsilon_1}$ , we obtain the closed-form approximate average BERs of single-relay cooperative diversity networks for different binary modulation schemes including noncoherent BFSK,

coherent BFSK, and coherent BPSK. In [30, eq. (14)], a closed-form approximate average BER expression was reported for the single-relay network with noncoherent BFSK. By simple manipulations, it can be shown that our BER expression of (2.16) reduces to eq. (14) of [30]. In this sense, therefore, our BER expression of (2.16) can be considered as a generalization of the expression of [30]. For coherent BPSK with a single relay, two *approximate* average BER expressions have been derived based on the PL approximation [32], [33]. However, these expressions involve numerical integrations, and thus, they are not truly closed-form. On the other hand, with the same PL approximation, our BER expression provides a truly closed-form approximate average BER expression for coherent BPSK. For coherent BFSK with a single relay, no average BER expression has been reported in the literature (see footnote 3). Therefore, our expression of (2.16) is the closed-form approximate average BER expression reported in the literature for the first time for coherent BFSK. Numerical results in Section 2.5 will demonstrate that (2.16) provides extremely accurate error probabilities for the ML detection in single-relay cooperative diversity networks adopting both coherent and noncoherent binary signalings.

In the next section, we will extend our analysis to the two-relay cooperative diversity networks.

## 2.4 Closed-Form Approximate Average BER for Two-Relay Networks

In this section, we consider the two-relay *dissimilar* cooperative diversity network in which the channel variances of different relay branches are different in general.

We derive a closed-form approximate average BER expression for the two-relay DF cooperative network again based on the PL approximation. Recall that the BER performance analysis of the single-relay case involved the computation of the probability for two statistics  $t_0$  and  $t_1$ . For the two-relay case, one more statistic  $t_2$  is involved, and therefore, the BER performance analysis becomes even harder. In this analysis, Theorem 2.1 and Lemma 2.2 of Section 2.3 are still needed. In the following, we first derive two lemmas which deal with three sufficient statistics.

**Lemma 2.3.** *Let  $t_i, t_j$ , and  $t_k$  be three different statistics taken from (2.8) for a common modulation scheme with  $i, j, k \in \{0, 1, \dots, K\}$  and  $i \neq j \neq k$ ,  $x_j, x_k \in \{0, 1\}$ . Define  $Pr(t_i + t_j + t_k < 0, -T_j < t_j < T_j, -T_k < t_k < T_k | x_i = 0, x_j, x_k) = W_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k)$  assuming  $0 \leq T_j \leq T_k$ . Define  $\hat{b}_j = (1 - x_j)b_j - x_j c_j$ ,  $\hat{c}_j = (1 - x_j)c_j - x_j b_j$ ,  $\hat{b}_k = (1 - x_k)b_k - x_k c_k$ , and  $\hat{c}_k = (1 - x_k)c_k - x_k b_k$ . Then  $W_{x_j, x_k}(\cdot)$  is given as follows:*

$$\begin{aligned}
 & W_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k) \\
 &= \frac{(-1)^{x_k} a_i (b_i - c_i)}{b_i c_i} \left\{ g_m(a_k, b_k, c_k, a_j, b_j, c_j, 0, T_j) - F(a_k, b_k, c_k, (-1)^{x_k+1} T_k) \right. \\
 & \quad \left. \times \left[ F(a_j, b_j, c_j, T_j) - F(a_j, b_j, c_j, -T_j) \right] \right\} + \eta_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k) \\
 & \quad + \varphi_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k).
 \end{aligned} \tag{2.17}$$

In this equation,  $m = x_j \oplus x_k$ , where  $\oplus$  represents the modulo 2 addition. Function

$\eta_{x_j, x_k}(\cdot)$  is defined as follows: If  $b_i \neq \hat{b}_j$  and  $b_i \neq \hat{b}_k$ ,

$$\begin{aligned}
 & \eta_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k) \\
 &= \frac{a_i a_j a_k}{b_i(\hat{c}_j - b_i)(\hat{c}_k - b_i)} \left\{ 1 - \exp[(b_i - \hat{c}_j)T_j] \right\} \left\{ 1 - \exp[(b_i - \hat{c}_k)T_k] \right\} \\
 &+ \frac{a_i a_j a_k}{b_i(\hat{c}_k - b_i)} \left\{ \frac{\exp[(\hat{b}_j - \hat{c}_k)T_j] - 1}{\hat{b}_j - \hat{c}_k} - \frac{\exp[(b_i - \hat{c}_k)T_k] \left\{ \exp[(\hat{b}_j - b_i)T_j] - 1 \right\}}{\hat{b}_j - b_i} \right\} \\
 &+ \frac{a_i a_j a_k}{b_i(\hat{b}_k - b_i)} \left\{ \frac{1 - \exp[(b_i - \hat{c}_j)T_j]}{b_i - \hat{c}_j} - \frac{1 - \exp[(\hat{b}_k - \hat{c}_j)T_j]}{\hat{b}_k - \hat{c}_j} \right\}.
 \end{aligned} \tag{2.18}$$

If  $b_i = \hat{b}_j$ , the second term of (2.18) is replaced by  $\frac{a_i a_j a_k}{b_i(\hat{c}_k - b_i)} \left[ \frac{\exp(b_i - \hat{c}_k)T_j - 1}{b_i - \hat{c}_k} - T_j \exp((b_i - \hat{c}_k)T_k) \right]$ . If  $b_i = \hat{b}_k$ , the last term of (2.18) is replaced by  $\frac{a_i a_j a_k}{b_i(b_i - \hat{c}_j)^2} \left\{ 1 + [(b_i - \hat{c}_j)T_j - 1] \exp((b_i - \hat{c}_j)T_j) \right\}$ . Also, function  $\varphi_{x_j, x_k}(\cdot)$  is defined as follows: If  $c_i \neq \hat{c}_j$ ,  $c_i \neq \hat{c}_k$ ,

$$\begin{aligned}
 & \varphi_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k) \\
 &= \frac{a_i a_j a_k}{c_i(\hat{b}_j - c_i)(\hat{b}_k - c_i)} \left\{ \exp[(\hat{b}_j - c_i)T_j] - 1 \right\} \left\{ \exp[(\hat{b}_k - c_i)T_k] - 1 \right\} \\
 &+ \frac{a_i a_j a_k}{c_i(\hat{b}_k - c_i)} \left\{ \frac{\exp[(\hat{b}_k - c_i)T_k] \left\{ 1 - \exp[(c_i - \hat{c}_j)T_j] \right\}}{\hat{c}_j - c_i} - \frac{1 - \exp[(\hat{b}_k - \hat{c}_j)T_j]}{\hat{c}_j - \hat{b}_k} \right\} \\
 &+ \frac{a_i a_j a_k}{c_i(\hat{c}_k - c_i)} \left\{ \frac{\exp[(\hat{b}_j - c_i)T_j] - 1}{\hat{b}_j - c_i} - \frac{\exp[(\hat{b}_j - \hat{c}_k)T_j] - 1}{\hat{b}_j - \hat{c}_k} \right\}.
 \end{aligned} \tag{2.19}$$

If  $c_i = \hat{c}_j$ , the second term of (2.19) is replaced by  $\frac{a_i a_j a_k}{c_i(\hat{b}_k - c_i)} \left[ T_j \exp((\hat{b}_k - c_i)T_k) + \frac{1 - \exp((\hat{b}_k - c_i)T_j)}{\hat{b}_k - c_i} \right]$ . If  $c_i = \hat{c}_k$ , the last term of (2.19) is replaced by  $\frac{a_i a_j a_k}{c_i(\hat{b}_j - c_i)^2} \left\{ 1 + [(\hat{b}_j - c_i)T_j - 1] \exp((\hat{b}_j - c_i)T_j) \right\}$ .

*Proof.* We can prove this lemma with the help of Theorem 2.1 and Lemma 2.2; but the proof is very long. Thus, we will just present the main ideas for the proof. Since

three random variables  $t_i, t_j$ , and  $t_k$  are involved in the definition of  $W_{x_j, x_k}(\cdot)$ , a triple-integral needs to be solved. Because of the asymmetry of the PDF of  $t_i$  given  $x_i$ , it is needed to divide the integral region for each integral such that a fixed PDF function is determined in each integrand. After dividing the integral regions according to this requirement, the expression of  $W_{x_j, x_k}(\cdot)$  can be derived through some simple manipulations.  $\square$

For the same reason as in Lemma 2.2, only the possibilities  $b_i = \hat{b}_j$  and  $b_i = \hat{b}_k$  are considered for (2.18), and  $c_i = \hat{c}_j$  and  $c_i = \hat{c}_k$  are considered for (2.19) in Lemma 2.3. Using Lemma 2.3, we can prove the following lemma.

**Lemma 2.4.** *Let  $t_i, t_j$ , and  $t_k$  be three different statistics taken from (2.8) for a common modulation scheme with  $i, j, k \in \{0, 1, \dots, K\}$  and  $i \neq j \neq k$ ,  $x_j, x_k \in \{0, 1\}$ . Define  $Pr(t_i + \psi_{PL}(t_j) + \psi_{PL}(t_k) < 0 | x_i = 0, x_j, x_k) = U_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k)$  assuming  $0 \leq T_j \leq T_k$ . Then*

$$\begin{aligned}
 & U_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k) \\
 = & A_{1, (x_j, x_k)} F[a_i, b_i, c_i, -(T_j + T_k)] + A_{2, (x_j, x_k)} F(a_i, b_i, c_i, T_k - T_j) \\
 & + A_{3, (x_j, x_k)} F(a_i, b_i, c_i, T_j - T_k) + A_{4, (x_j, x_k)} F(a_i, b_i, c_i, T_j + T_k) \\
 & + B_{1, (x_j, x_k)} g_{x_j}(a_i, b_i, c_i, a_j, b_j, c_j, -T_k, T_j) + B_{2, (x_j, x_k)} g_{x_j}(a_i, b_i, c_i, a_j, b_j, c_j, T_k, T_j) \\
 & + B_{3, (x_j, x_k)} g_{x_k}(a_i, b_i, c_i, a_k, b_k, c_k, -T_j, T_k) + B_{4, (x_j, x_k)} g_{x_k}(a_i, b_i, c_i, a_k, b_k, c_k, T_j, T_k) \\
 & + W_{x_j, x_k}(a_i, b_i, c_i, a_j, b_j, c_j, a_k, b_k, c_k, T_j, T_k),
 \end{aligned} \tag{2.20}$$

and the coefficients  $A_{i, (x_j, x_k)}$  and  $B_{i, (x_j, x_k)}$ ,  $i = 1, 2, 3, 4$ , are presented in Table 2.1.

*Proof.* Applying the total probability theorem according to the possibilities of  $t_j$  and  $t_k$ ,  $U_{x_j, x_k}(\cdot)$  can be written as a combination of nine terms. We observed that the nine

Table 2.1: Coefficients of  $A_{i,(x_j,x_k)}$  and  $B_{i,(x_j,x_k)}$ .

$x_j = 0$ $x_k = 0$	$A_{1,(0,0)} = [1 - F(a_j, b_j, c_j, T_j)] [1 - F(a_k, b_k, c_k, T_k)]$
	$B_{1,(0,0)} = [1 - F(a_k, b_k, c_k, T_k)]$
	$A_{2,(0,0)} = [1 - F(a_j, b_j, c_j, T_j)] F(a_k, b_k, c_k, -T_k)$
	$B_{2,(0,0)} = F(a_k, b_k, c_k, -T_k)$
$x_j = 0$ $x_k = 0$	$A_{3,(0,0)} = F(a_j, b_j, c_j, -T_j) [1 - F(a_k, b_k, c_k, T_k)]$
	$B_{3,(0,0)} = [1 - F(a_j, b_j, c_j, T_j)]$
$x_j = 0$ $x_k = 0$	$A_{4,(0,0)} = F(a_j, b_j, c_j, -T_j) F(a_k, b_k, c_k, -T_k)$
	$B_{4,(0,0)} = F(a_j, b_j, c_j, -T_j)$
$x_j = 0$ $x_k = 1$	$A_{1,(0,1)} = A_{2,(0,0)}, B_{1,(0,1)} = B_{2,(0,0)}$
	$A_{2,(0,1)} = A_{1,(0,0)}, B_{2,(0,1)} = B_{1,(0,0)}$
	$A_{3,(0,1)} = A_{4,(0,0)}, B_{3,(0,1)} = B_{3,(0,0)}$
	$A_{4,(0,1)} = A_{3,(0,0)}, B_{4,(0,1)} = B_{4,(0,0)}$
$x_j = 1$ $x_k = 0$	$A_{1,(1,0)} = A_{3,(0,0)}, B_{1,(1,0)} = B_{1,(0,0)}$
	$A_{2,(1,0)} = A_{4,(0,0)}, B_{2,(1,0)} = B_{2,(0,0)}$
	$A_{3,(1,0)} = A_{1,(0,0)}, B_{3,(1,0)} = B_{4,(0,0)}$
	$A_{4,(1,0)} = A_{2,(0,0)}, B_{4,(1,0)} = B_{3,(0,0)}$
$x_j = 1$ $x_k = 1$	$A_{1,(1,1)} = A_{4,(0,0)}, B_{1,(1,1)} = B_{2,(0,0)}$
	$A_{2,(1,1)} = A_{3,(0,0)}, B_{2,(1,1)} = B_{1,(0,0)}$
	$A_{3,(1,1)} = A_{2,(0,0)}, B_{3,(1,1)} = B_{4,(0,0)}$
	$A_{4,(1,1)} = A_{1,(0,0)}, B_{4,(1,1)} = B_{3,(0,0)}$

terms can be classified as three types: the first four terms including the probability of a single random variable, the following four terms including the joint probability of two random variables, and the last including the joint probability of three random variables. Therefore, the first four terms can be determined by the individual CDFs (2.11) of  $t_i$ ,  $t_j$ , and  $t_k$ , given  $x_i$ ,  $x_j$ , and  $x_k$ , respectively. The following four terms can be determined through Lemma 2.2. The last term can be determined through Lemma 2.3. Finally, the expression of (2.20) can be derived.  $\square$

Using Lemma 2.4, we now derive an average BER expression for the two-relay cooperative networks in the following theorem.

**Theorem 2.3.** *For binary modulations, a closed-form approximate average BER  $P_{B,2}$  of the two-relay ( $K = 2$ ) cooperative diversity network is*

$$\begin{aligned}
 P_{B,2} = & \sum_{x_{n_1} \in \{0,1\}} \sum_{x_{n_2} \in \{0,1\}} \{[(1 - x_{n_1})(1 - \epsilon_{n_1}) + x_{n_1}\epsilon_{n_1}][(1 - x_{n_2})(1 - \epsilon_{n_2}) + x_{n_2}\epsilon_{n_2}] \\
 & \times U_{x_{n_1}, x_{n_2}}(a_0, b_0, c_0, a_{n_1}, b_{n_1}, c_{n_1}, a_{n_2}, b_{n_2}, c_{n_2}, T_{n_1}, T_{n_2})\}.
 \end{aligned} \tag{2.21}$$

where  $n_1$  and  $n_2$  denote the two relays with  $n_1 = 1$  and  $n_2 = 2$  if  $0 \leq T_1 \leq T_2$ , and  $n_1 = 2$  and  $n_2 = 1$  if  $0 < T_2 < T_1$ ;  $x_{n_1}$  and  $x_{n_2}$  are the possible transmitted signals at relay  $n_1$  and relay  $n_2$ , respectively; The modulation-dependent parameters  $\epsilon_i$ ,  $T_i$ ,  $i = 1, 2$ ,  $a_j$ ,  $b_j$ , and  $c_j$ ,  $j = 0, 1, 2$ , are described as before.

*Proof.* For binary modulations with equal priors (the signals 0 and 1 are transmitted by the source with equal probability), the average BER equals the probability of error when either 0 or 1 is transmitted [1]. Without loss of generality,  $P_{B,2}$  can be written



as follows:

$$\begin{aligned}
 P_{B,2} &= \Pr(t_0 + \psi_{PL}(t_1) + \psi_{PL}(t_2) < 0 | x_0 = 0) \\
 &= \Pr(t_0 + \psi_{PL}(t_{n_1}) + \psi_{PL}(t_{n_2}) < 0 | x_0 = 0),
 \end{aligned} \tag{2.22}$$

where  $n_1$  and  $n_2$  are described as in Theorem 2.3. We have rewritten the expression of (2.22) in the second equality to give  $0 \leq T_{n_1} \leq T_{n_2}$  so that Lemma 2.4 can be applied. Then applying the total probability theorem based on possibilities of  $x_1$  and  $x_2$  yields the following expression:

$$\begin{aligned}
 P_{B,2} &= (1 - \epsilon_{n_1})(1 - \epsilon_{n_2})\Pr(t_0 + \psi_{PL}(t_{n_1}) + \psi_{PL}(t_{n_2}) < 0 | x_0 = 0, x_{n_1} = 0, x_{n_2} = 0) \\
 &\quad + (1 - \epsilon_{n_1})\epsilon_{n_2}\Pr(t_0 + \psi_{PL}(t_{n_1}) + \psi_{PL}(t_{n_2}) < 0 | x_0 = 0, x_{n_1} = 0, x_{n_2} = 1) \\
 &\quad + \epsilon_{n_1}(1 - \epsilon_{n_2})\Pr(t_0 + \psi_{PL}(t_{n_1}) + \psi_{PL}(t_{n_2}) < 0 | x_0 = 0, x_{n_1} = 1, x_{n_2} = 0) \\
 &\quad + \epsilon_{n_1}\epsilon_{n_2}\Pr(t_0 + \psi_{PL}(t_{n_1}) + \psi_{PL}(t_{n_2}) < 0 | x_0 = 0, x_{n_1} = 1, x_{n_2} = 1).
 \end{aligned} \tag{2.23}$$

This gives the expression of (2.21).  $\square$

Note that (2.21) is truly a closed-form approximate average BER expression for dissimilar two-relay cooperative networks. Again, by choosing  $a_j, b_j$ , and  $c_j$  as described in Theorem 2.1,  $\epsilon_i$  as given by (2.7), and  $T_i = \ln \frac{1-\epsilon_i}{\epsilon_i}$ , we obtain the average BERs for two-relay cooperative diversity networks with different binary modulation schemes including coherent BPSK, coherent BFSK, and noncoherent BFSK. For two-relay systems, the only BER expression reported in the literature was the closed-form approximate average BER expression in [31, eq. (4.14)]; however, it is valid only for symmetric networks with noncoherent BFSK. On the other hand, our expression of (2.21) is the closed-form approximate average BER expression for the general dissimilar two-relay networks with noncoherent BFSK. In this sense, therefore, our

BER expression of (2.21) can be considered as a generalization of the BER expression of [31]. For coherent BFSK and coherent BPSK, no average BER expressions have been reported for two-relay systems. Therefore, our expression of (2.21) is the closed-form approximate average BER expression reported in the literature for the first time for coherent BFSK and coherent BPSK. Numerical results in Section 2.5 will demonstrate that (2.21) provides extremely accurate error probabilities for the ML detection in two-relay dissimilar cooperative diversity networks adopting both coherent and noncoherent binary modulations.

## 2.5 Numerical Results

We compare the proposed approximate BER expressions with the exact BER obtained by Monte-Carlo simulations of the exact ML detection of (2.5) along with (2.6). We use the alphabetic indices  $\{s, r, d\}$  to characterize the single-relay network, and the numeric indices  $\{0, 1, 2, 3\}$  as in Section 2.2 to characterize the two-relay network. We consider an equal power allocation with total transmission power of the whole network normalized to 1. Specifically,  $E_s = E_r = 1/2$  for single-relay systems and  $E_0 = E_1 = E_2 = 1/3$  for two-relay systems. For ease of exposition, we assume that the source, relays, and the destination are located in a straight line.<sup>6</sup> The distance  $d_{i,j}$  from node  $i$  to node  $j$  is normalized by the distance between the source node and the destination node, and hence,  $d_{s,r} + d_{r,d} = 1$  for  $K = 1$ , and  $d_{0,i} + d_{i,3} = 1$ ,  $i = 1, 2$ , for  $K = 2$ . The channel variances are modeled as  $\sigma_{i,j}^2 = d_{i,j}^{-4}$ . We plot the average BER with respect to the ratio of total power over the noise variance, which is  $1/N_0$ .

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<sup>6</sup>This assumption has also been made in [29]–[31] because it is very convenient to model the fading parameters for both dissimilar and symmetric cooperative diversity networks by simply choosing the positions of the relays in a straight line.

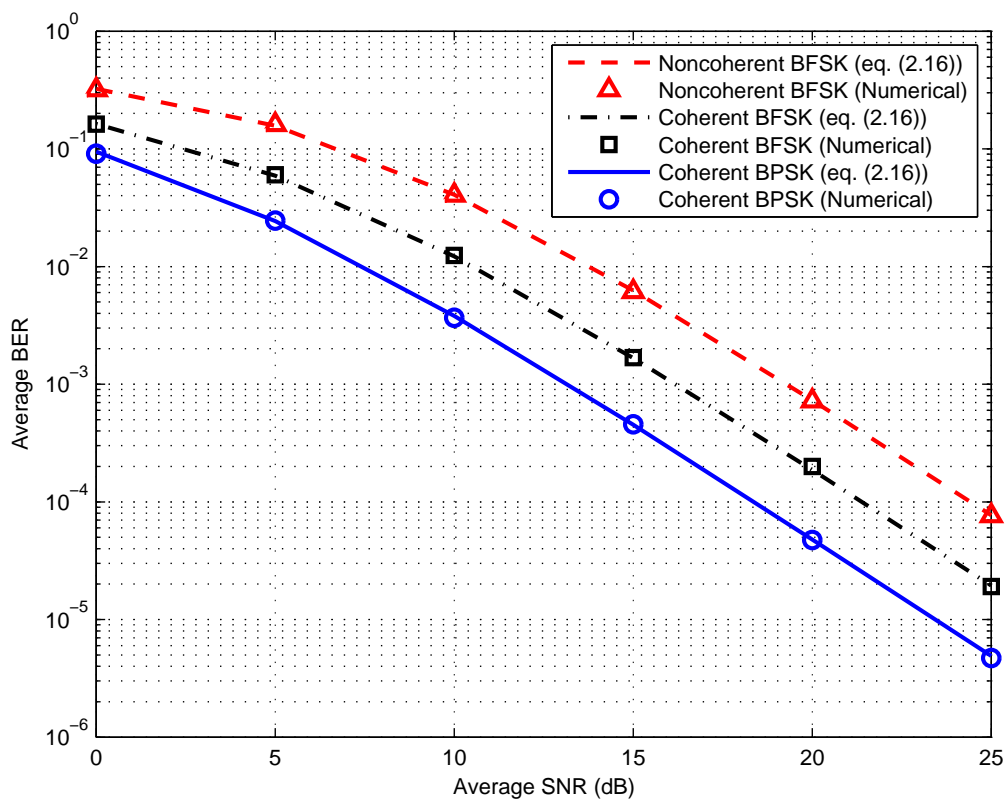


Figure 2.2: Average BER of the single-relay cooperative network with  $d_{s,r} = 0.1$ .

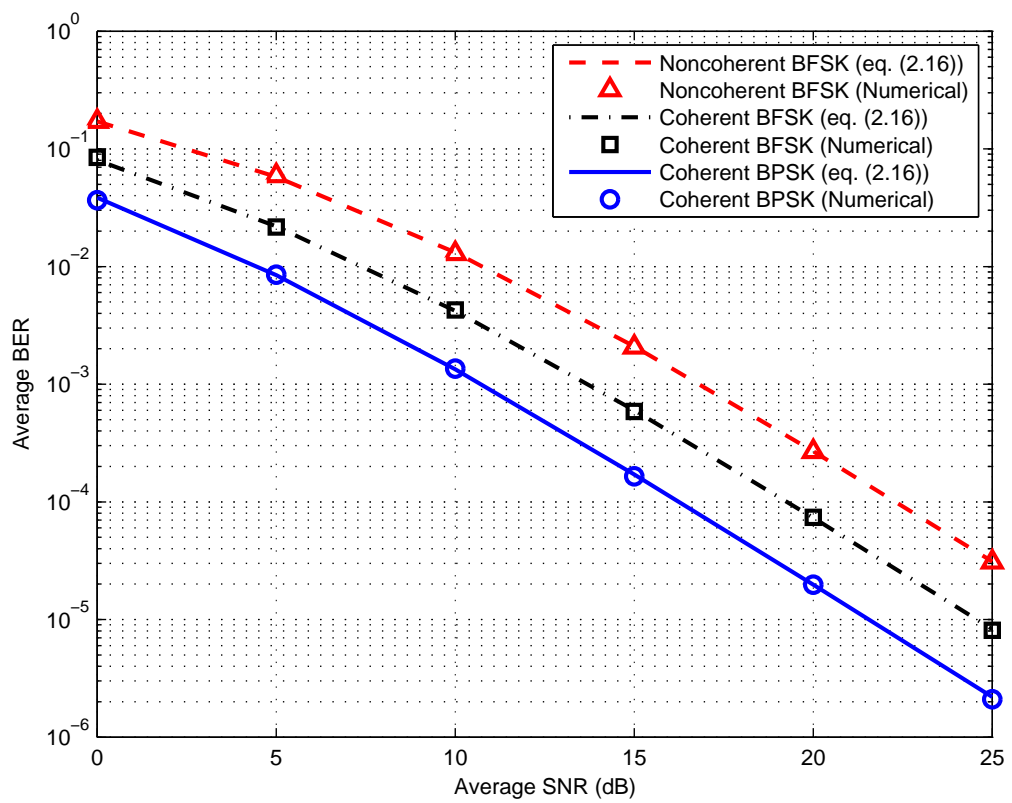


Figure 2.3: Average BER of the single-relay cooperative network with  $d_{s,r} = 0.5$ .

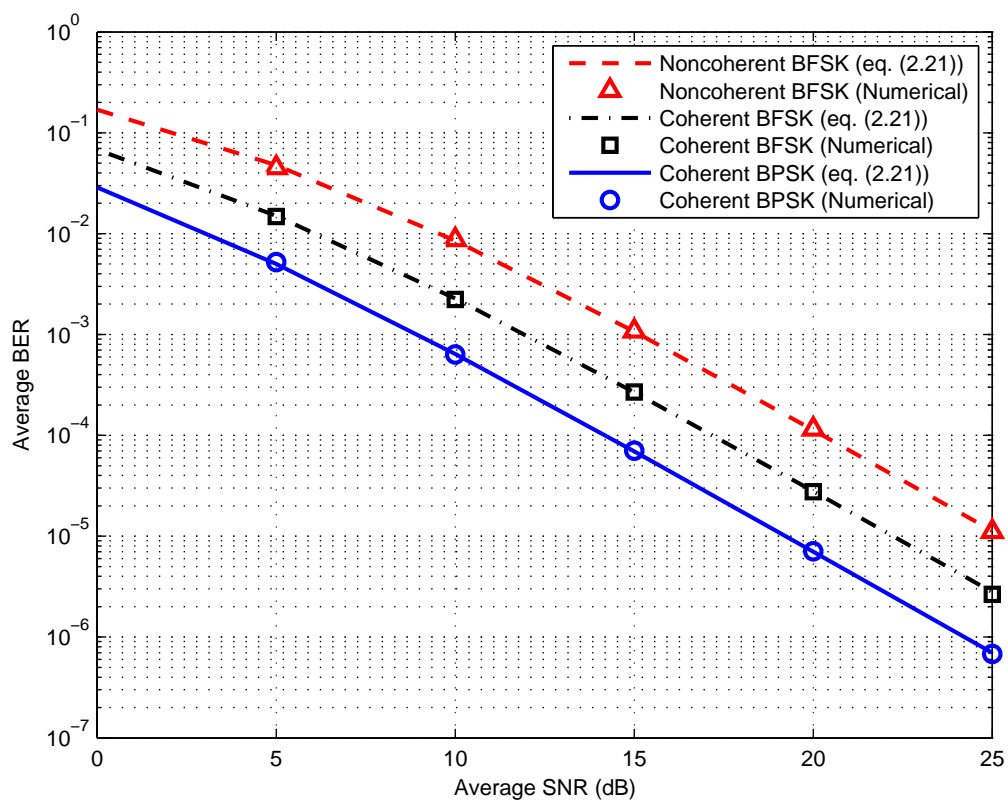


Figure 2.4: Average BER of the two-relay symmetric network with  $d_{0,1} = 0.5$  and  $d_{0,2} = 0.5$ .

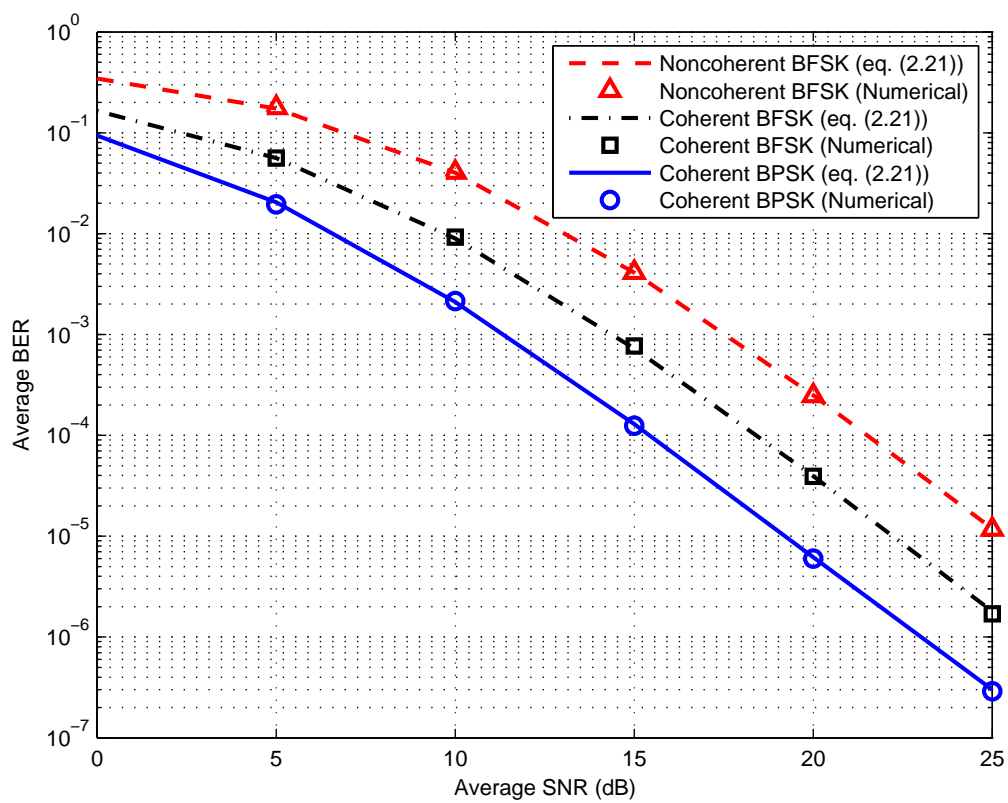


Figure 2.5: Average BER of the two-relay dissimilar network with  $d_{0,1} = 0.1$  and  $d_{0,2} = 0.9$ .

Firstly, cooperative diversity networks with a single relay are considered. Figs. 2.2 and 2.3 show the results for cooperative networks with  $d_{s,r} = 0.1$  and  $d_{s,r} = 0.5$ , respectively. Secondly, we consider cooperative diversity networks with two relays. Figs. 2.4 and 2.5 show the simulated average BER for two cases: 1) the symmetric network with  $d_{0,1} = 0.5$ ,  $d_{0,2} = 0.5$ ; 2) the dissimilar network with  $d_{0,1} = 0.1$ ,  $d_{0,2} = 0.9$ . It can be seen that the proposed BER expressions overlap the simulation results. Moreover, the BER curves for both the single-relay and two-relay networks demonstrate 3 dB shifts to the left, from noncoherent BFSK to coherent BFSK and from coherent BFSK to coherent BPSK, which can be expected from coherent and noncoherent communication theories.

## 2.6 Conclusion

In this chapter, we have analyzed the average BER of the ML detection for uncoded DF unidirectional cooperative diversity networks. Specifically, two typical cooperative diversity networks were considered: the single-relay cooperative diversity network with the direct source-destination link and the two-relay cooperative diversity network with the direct source-destination link. First, we derived the PDFs and CDFs of the sufficient statistics for the ML detection at the destination. Then we applied the accurate PL approximation and derived closed-form approximate average BER expressions with the help of the obtained PDFs and CDFs. Our BER expressions were shown to be valid for the general dissimilar DF networks adopting both coherent and noncoherent binary signalings. We also showed that our BER expressions can be considered as generalizations of the previously reported results in the literature. Throughout our analysis, only one approximation, i.e., the accurate PL approximation

was made. Simulation results match excellently with the theoretical analysis, which validates our proposed BER expressions.



## Chapter 3

# Bidirectional Cooperative Diversity Networks

In this chapter, we study and compare the performance of three very typical bidirectional communication protocols based on the decode-and-forward relaying: time-division broadcast (TDBC), physical-layer network coding (PNC), and opportunistic source selection (OSS). We first derive an exact outage probability in a one-integral form for the TDBC protocol, and exact closed-form outage probabilities for the PNC and OSS protocols. For the TDBC protocol, we also derive extremely tight upper and lower bounds on the outage probability in closed-form. Moreover, we derive asymptotic outage probability expressions in closed-form for all protocols, with which we prove that the TDBC and OSS protocols achieve full diversity order two and the PNC protocol achieves diversity order one. Finally, we study the diversity-multiplexing tradeoff (DMT) performance of each protocol both in the finite and infinite signal-to-noise ratio (SNR) regimes. With the DMT analysis, we demonstrate that the PNC protocol is able to achieve the highest multiplexing gain, whereas the TDBC and OSS

protocols are able to achieve the highest diversity gain.

### 3.1 Introduction

Cooperative communication has been extensively studied due to its ability to induce spatial diversity, increase channel capacity, and attain broader cell coverage even with single-antenna terminals [5]–[8]. In the literature, the *unidirectional* cooperative communications have received a lot of attention. In a unidirectional cooperative network, the source transmits information-bearing signals to the destination with the help of one or multiple relays, which may adopt either an amplify-and-forward (AF) or a decode-and-forward (DF) relaying [7], [8].

Recently, cooperative communication has also been studied for a *bidirectional* cooperative network, where two different end-sources  $S_1$  and  $S_2$  intend to exchange information with the help of a relay R. One can directly apply the unidirectional protocols to the bidirectional network. That is,  $S_1$  transmits information to R and  $S_2$  in the first time slot; in the second time slot, R forwards (either with AF or DF) the incoming signal to  $S_2$ ; in the third time slot,  $S_2$  transmits information to R and  $S_1$ ; and in the fourth time slot, R forwards the signal to  $S_1$ . However, this naive application of the unidirectional protocols requires four time slots for one information exchange, which indeed leads to a low spectral efficiency [15]. In this chapter, the naive scheme will be referred to as the *traditional* approach.

It has been shown that the spectral efficiency can be improved by exploiting the shared broadcast channel (BC) nature of the wireless medium through the network coding concept [16], [36], [37]. A very good example is the *time-division broadcast* (TDBC) protocol, which combines the transmissions of the third and fourth time

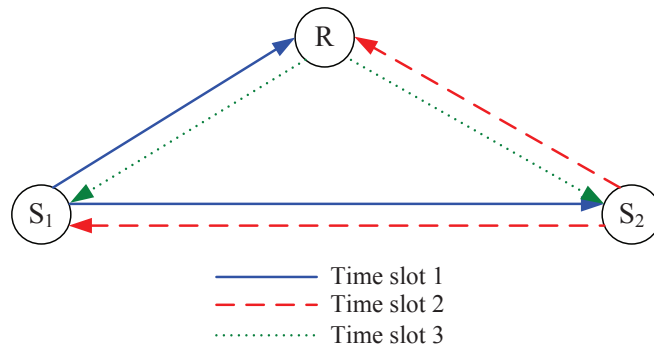


Figure 3.1: The TDBC protocol in the bidirectional networks; either AF or DF may be used at the relay.

slots in the traditional approach into one single transmission in a broadcast manner [16]–[18].<sup>1</sup> Specifically,  $S_1$  and  $S_2$  transmit their own signals in the first and second time slots, respectively. In the third time slot,  $R$  amplify-and-forwards the sum of the two received signals to  $S_1$  and  $S_2$  through the BC as depicted in Fig. 3.1. Another possibility is that  $R$  decodes the received signals first and then forwards an XORed version of the two information symbols to  $S_1$  and  $S_2$  in the third time slot. Through the network coding techniques, it is possible for each end-source to detect the signal from its counterpart. This way the TDBC protocol can accomplish one information exchange in three time slots, meaning that the spectral efficiency is increased by 33% over the traditional approach [16].

It has recently been shown that the spectral efficiency can be further increased by allowing the two end-sources to transmit simultaneously over a medium access channel (MAC), and hence, only two time slots are needed for one information exchange. This

<sup>1</sup>Note that the TDBC protocol was also called *straightforward network coding* in [16]. In [17], TDBC-I was used to specify a TDBC protocol without using the direct link between the two end-sources, and TDBC-II was used to specify a TDBC protocol which utilizes the direct link. In this chapter, we consider the TDBC protocol which utilizes the direct link.

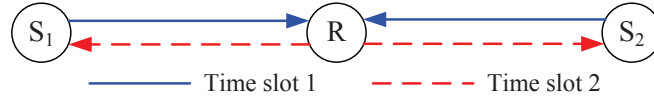


Figure 3.2: The PNC (when DF is used at the relay) and ANC (when AF is used at the relay) protocols in the bidirectional networks.

protocol is widely known as *physical-layer network coding* (PNC) (see [16]–[19]<sup>2</sup>) if the DF protocol is employed at the relay or *analog network coding* (ANC) (see [20]–[22]) if the AF protocol is employed at the relay, as depicted in Fig. 3.2. It is easy to see that the PNC and ANC protocols achieve the highest spectral efficiency since two-traffic flows are concurrently supported at each time slot. Due to the half-duplex constraint, however, the direct link between S<sub>1</sub> and S<sub>2</sub> can not be utilized even if it physically exists, and therefore, the PNC and ANC protocols can only achieve *diversity order one*. On the other hand, the direct link can be utilized in the TDBC protocol since the two end-sources transmit in two different time slots. Therefore, the TDBC protocol can achieve *diversity order two*.<sup>3</sup>

When multiple users exist, it has been demonstrated that the time-varying channel fluctuations can be exploited to achieve the well-known *multiuser diversity* by allocating all resources to the best user whose instantaneous channel quality is near the peak [38], [39]. In the TDBC, PNC, and ANC protocols, however, the time-varying channel fluctuations are not exploited at all. To overcome this problem, an *opportunistic source selection* (OSS) protocol for a bidirectional cooperative network with a single AF relay has been proposed [21], as depicted in Fig. 3.3. In the OSS

<sup>2</sup>Note that the PNC protocol was also called *multiple access broadcast* (MABC) in [17] and [18].

<sup>3</sup>The fact that the PNC protocol achieves diversity order one and the TDBC protocol achieves diversity order two was first stated by inferring from whether or not the direct link can be used, and numerically verified by Kim *et al.* in [17]. However, there is no analytical proof for this.

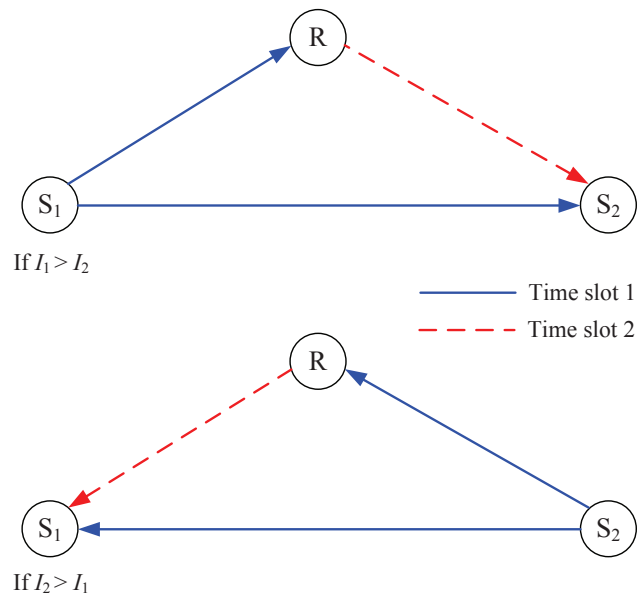


Figure 3.3: The OSS protocol in the bidirectional networks.  $I_1$  is the mutual information between the transmitted signal from  $S_1$  and the received signal at  $S_2$  when  $S_1$  is selected as the transmitting source;  $I_2$  is the mutual information between the transmitted signal from  $S_2$  and the received signal at  $S_1$  when  $S_2$  is selected as the transmitting source. In [21], only AF-based OSS protocol was considered. In this chapter, however, the DF-based OSS protocol is considered.

protocol,  $S_1$  and  $S_2$  transmit their own information in an *opportunistic* manner such that the network throughput is maximized. That is to say, at each time slot, only a single end-source with the best channel condition is allowed to transmit. Therefore, the multiuser diversity is achieved in the OSS protocol. Numerical results of [21] also demonstrated that the AF-based OSS protocol can achieve higher reliability in terms of both the average BER and the outage probability performance than other AF-based bidirectional protocols such as AF-based TDBC and ANC at some low data rates. The outage probability and the diversity-multiplexing tradeoff (DMT) performance of the AF-based bidirectional protocols such as the AF-based TDBC, ANC, and the AF-based OSS have been well studied in the literature [21], [22]. For the DF-based bidirectional protocols including the DF-based TDBC, PNC and the DF-based OSS, however, there have been no analytical results on the outage probabilities and DMT performance in the literature, to the best of our knowledge. This has motivated our work.

In this chapter, we analyze and compare the performance of the DF-based bidirectional protocols including the DF-based TDBC, PNC, and the DF-based OSS protocols. We first derive an exact outage probability in a one-integral form for the TDBC protocol, and exact closed-form outage probabilities for the PNC and OSS protocols. Asymptotic outage probability expression of each protocol is also derived, with which we analytically prove that the TDBC and OSS protocols achieve full diversity order two and the PNC protocol achieves diversity order one. To facilitate the outage probability analysis for the TDBC protocol, we also derive extremely tight closed-form upper and lower bounds on the outage probability. Moreover, we study the DMT performance of each bidirectional protocol both in the finite and infinite

signal-to-noise ratio (SNR) regimes. With the DMT analysis, we demonstrate that the PNC protocol is able to achieve the highest multiplexing gain, whereas the TDBC and OSS protocols are able to achieve the highest diversity gain. Finally, numerical results perfectly match with the derived analytical expressions, which confirm the validity of the analytical results.

The remainder of this chapter is organized as follows. We describe the system model for a bidirectional cooperative network with a single DF relay in Section 3.2. In Section 3.3, we study the outage probability and asymptotic performance for each protocol. In Section 3.4, we study the finite-SNR DMT and infinite-SNR DMT for each protocol. Section 3.5 presents some numerical results and Section 3.6 concludes this chapter.

*Notation:* We use  $A := B$  to denote that  $A$ , by definition, equals  $B$  and we use  $A =: B$  to denote that  $B$ , by definition, equals  $A$ . For two functions  $g(x)$  and  $h(x)$ ,  $g(x) = O(h(x))$  means  $\lim_{x \rightarrow \infty} g(x)/h(x) = c$ , where  $c$  is a non-zero constant. For random variable  $X$ ,  $f_X(x)$  denotes its probability density function (PDF). Also,  $X \sim \mathcal{CN}(\mu, \Omega)$  indicates that  $X$  is a circularly symmetric complex Gaussian random variable with mean  $\mu$  and variance  $\Omega$ . Finally,  $\log(\cdot)$  denotes the base-2 logarithm, and  $\ln(\cdot)$  the natural logarithm.

## 3.2 System Model

Consider a bidirectional cooperative network consisting of two different end-sources and a single relay, where each terminal is equipped with a single antenna and operates in a half-duplex mode. We use  $S_1$ ,  $S_2$ , and  $R$  to denote the two end-sources and the relay, respectively. It is assumed that relay  $R$  adopts the DF protocol and all channels

are reciprocal and quasi-static over at least three time slots. In the remainder of this chapter, we will refer to the DF-based TDBC and the DF-based OSS protocols simply as the TDBC and OSS protocols, respectively, whenever there is no ambiguity. Let  $h_1$  denote the channel coefficient between  $S_1$  and R;  $h_2$  the channel coefficient between  $S_2$  and R;  $h_0$  the channel coefficient for the direct link, if any, between  $S_1$  and  $S_2$ . The channel coefficients are mutually independent with  $h_i \sim \mathcal{CN}(0, \Omega_i)$  for  $i = 0, 1, 2$ . The complex noise associated with each channel is modeled as a mutually independent additive white Gaussian noise (AWGN) with zero mean and variance  $N_0$ . Let  $E$  denote the total transmission power of the whole bidirectional network. We use  $E_r$  to denote the average transmission power at R. For the TDBC and PNC protocols, it is assumed that the average transmission power for  $S_1$  and  $S_2$  is identical and the total transmission power of  $S_1$  and  $S_2$  is denoted by  $E_s$ . For the OSS protocol, we use  $E_s$  to denote the average transmission power at the selected end-source. Therefore,  $E_s + E_r = E$  holds for all three protocols. When we compare the performance of the three bidirectional protocols, we will constrain that the total transmission power  $E$  is the same for each protocol. For convenience, we define the average SNR  $\rho$  as the ratio of the total transmission power  $E$  to the noise power  $N_0$ , i.e.,  $\rho := \frac{E}{N_0}$ .

In the bidirectional protocols such as the TDBC, PNC, and OSS, the two end-sources intend to exchange information. There are two traffic flows in the bidirectional cooperative networks. In the coherent detection, the receiver must know the channel information. Therefore, with coherent detection, it is reasonable to adopt the full channel state information (CSI) assumption for the bidirectional cooperative networks. Specifically, we assume that relay R knows both  $h_1$  and  $h_2$ , and both end-sources know  $h_1$  and  $h_2$  in the PNC protocol, and  $h_0$ ,  $h_1$ , and  $h_2$  in the TDBC and



OSS protocols. Note that this full CSI assumption has been widely adopted in all previous publications on the bidirectional cooperative networks [15], [21], [22].

### 3.3 Outage Probability Analysis

In this section, we study the outage probability performance of three bidirectional protocols: TDBC, PNC and OSS. We also study the asymptotic performance in the high-SNR regime.

#### 3.3.1 TDBC Protocol

In the TDBC protocol, one information exchange between two end-sources is accomplished in three time slots, as depicted in Fig. 3.1. Let  $R_1$  and  $R_2$  denote the data rates of  $S_1$  and  $S_2$ , respectively. We denote the target data rate of the whole network as  $R$ . It is fair to set the target data rate for each end-source as  $R/2$ . Then an achievable rate region of the TDBC protocol is the closure of the convex hull of the set of points  $(R_1, R_2)$  satisfying the following inequalities [17, eqs. (8) and (9)]:

$$R_1 < I_1^{\text{TDBC}}, \quad (3.1)$$

$$R_2 < I_2^{\text{TDBC}}, \quad (3.2)$$

where  $I_1^{\text{TDBC}}$  and  $I_2^{\text{TDBC}}$  are defined as

$$I_1^{\text{TDBC}} = \begin{cases} \frac{1}{3} \log \left( 1 + \frac{\beta \rho |h_0|^2}{2} \right), & \text{if } |h_1|^2 < \frac{2T_1}{\beta}, \\ \frac{1}{3} \log \left( 1 + \frac{\beta \rho |h_0|^2}{2} \right) + \frac{1}{3} \log [1 + (1 - \beta) \rho |h_2|^2], & \text{if } |h_1|^2 > \frac{2T_1}{\beta}, \end{cases} \quad (3.3)$$

$$I_2^{\text{TDBC}} = \begin{cases} \frac{1}{3} \log \left( 1 + \frac{\beta \rho |h_0|^2}{2} \right), & \text{if } |h_2|^2 < \frac{2T_1}{\beta}, \\ \frac{1}{3} \log \left( 1 + \frac{\beta \rho |h_0|^2}{2} \right) + \frac{1}{3} \log [1 + (1 - \beta) \rho |h_1|^2], & \text{if } |h_2|^2 > \frac{2T_1}{\beta}. \end{cases} \quad (3.4)$$

In the above equations, the pre-log factor  $1/3$  is because each traffic flow takes three time slots in the TDBC protocol [17]. Also,  $\beta$  is defined as the ratio of the total transmission power of  $S_1$  and  $S_2$  to the total transmission power of the whole network, i.e.,  $\beta := E_s/E$ , and  $T_1$  is a threshold related with the target rate and it is given by  $T_1 := \frac{2^{1.5R}-1}{\rho}$ .

The system is in outage when the rate pair  $(R_1, R_2)$  falls out of the capacity region. Therefore, the outage probability of the TDBC protocol is given by

$$P_{\text{out}}^{\text{TDBC}}(R, \rho) := \Pr \left( I_1^{\text{TDBC}} < \frac{R}{2} \text{ or } I_2^{\text{TDBC}} < \frac{R}{2} \right). \quad (3.5)$$

In the following theorem, we derive an exact expression of the outage probability for the TDBC protocol.

**Theorem 3.1.** *For the TDBC protocol, an exact outage probability expression is given by*

$$P_{\text{out}}^{\text{TDBC}}(R, \rho) = \begin{cases} 1 + \exp \left[ -\frac{2(\lambda_0 + \lambda_1 + \lambda_2)T_1}{\beta} \right] - \exp \left( -\frac{2\lambda_0 T_1}{\beta} \right) - \exp \left[ -\frac{2(\lambda_1 + \lambda_2)T_1}{\beta} \right], & \text{if } 0 < \beta \leq 2/3, \\ 1 + \exp \left[ -\frac{2(\lambda_0 + \lambda_1 + \lambda_2)T_1}{\beta} \right] - \exp \left( -\frac{2\lambda_0 T_1}{\beta} \right) - \exp \left[ -\frac{(\lambda_1 + \lambda_2)T_1}{1-\beta} \right] \\ - \Psi(a, b, c, \theta), & \text{if } 2/3 < \beta < 1. \end{cases} \quad (3.6)$$

The function  $\Psi(a, b, c, \theta)$  is defined as

$$\Psi(a, b, c, \theta) := c \int_{\theta}^1 \exp \left[ -\left( ax + \frac{b}{x} \right) \right] dx, \quad (3.7)$$

with parameters given as follows:  $a = \frac{(\lambda_1 + \lambda_2)(1 + L_1)}{(1 - \beta)\rho}$ ,  $b = \frac{2\lambda_0}{\beta\rho}$ ,  $c = a \exp \left[ \frac{\lambda_1 + \lambda_2}{(1 - \beta)\rho} + \frac{2\lambda_0}{\beta\rho} \right]$ , and  $\theta = \frac{\beta + 2(1 - \beta)L_1}{\beta + \beta L_1}$ , where  $L_1 = 2^{1.5R} - 1$  and  $\lambda_i = 1/\Omega_i$ , for  $i = 0, 1, 2$ .

*Proof.* See Appendix 3-A. □

For  $0 < \beta \leq 2/3$ , we have an exact truly closed-form expression for the outage probability of the TDBC protocol; for  $2/3 < \beta < 1$ , we have an exact outage probability expression in a one-integral form. Note that the integration lower limit  $\theta$  of (3.7) is a constant irrespective of  $\rho$ , and it can be shown that  $0 < \theta < 1$  by using the fact  $2/3 < \beta < 1$ . Solving  $\Psi(a, b, c, \theta)$  in closed-form is extremely hard, to the best of our knowledge; however, this integration can be easily computed with numerical approaches since the integration region is between 0 and 1.<sup>4</sup> Furthermore, to facilitate the outage evaluation of the TDBC protocol for the case  $2/3 < \beta < 1$ , we derive tight upper and lower bounds in closed-form for this case in what follows.

**Corollary 3.1.** *For the case  $2/3 < \beta < 1$ , a closed-form upper bound on the outage probability of the TDBC protocol is given by*

$$P_{UB}^{TDBC} = \begin{cases} 1 + \exp\left[-\frac{2(\lambda_0 + \lambda_1 + \lambda_2)T_1}{\beta_U}\right] - \exp\left(-\frac{2\lambda_0 T_1}{\beta_U}\right) - \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1 - \beta_U}\right] \\ - \frac{(3\beta_U - 2)(\lambda_1 + \lambda_2)T_1}{\beta_U(1 - \beta_U)} \exp\left[-\frac{2\lambda_0 T_1}{\beta_U + 2(1 - \beta_U)L_1}\right], & \text{if } \beta = \beta_U, 2/3 < \beta_U < 1, \\ 1 + \exp\left[-\frac{2(\lambda_0 + \lambda_1 + \lambda_2)T_1}{\beta}\right] - \exp\left(-\frac{2\lambda_0 T_1}{\beta}\right) - \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1 - \beta}\right] \\ + \frac{(\lambda_1 + \lambda_2)[\beta + 2(1 - \beta)L_1]}{2(1 - \beta)\lambda_0 - (\lambda_1 + \lambda_2)[\beta + 2(1 - \beta)L_1]} \left\{ \exp\left[\frac{2(2 - 3\beta)\lambda_0 T_1}{\beta[\alpha + 2(1 - \beta)L_1]} - \frac{2(\lambda_1 + \lambda_2)T_1}{\beta}\right] \right. \\ \left. - \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1 - \beta}\right] \right\}, & \text{otherwise,} \end{cases} \quad (3.8)$$

---

<sup>4</sup>In standard softwares such as Matlab, such integration can be easily computed by the built-in function *quad*.

where  $\beta_U := \frac{2\lambda_0 - (\lambda_1 + \lambda_2)2L_1}{2\lambda_0 - (\lambda_1 + \lambda_2)(2L_1 - 1)}$ . Also, a closed-form lower bound on the outage probability of the TDDBC protocol is given by

$$P_{LB}^{TDDBC} = \begin{cases} 1 + \exp\left[-\frac{2(\lambda_0 + \lambda_1 + \lambda_2)T_1}{\beta_L}\right] - \exp\left(-\frac{2\lambda_0 T_1}{\beta_L}\right) - \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1 - \beta_L}\right] \\ - \frac{(3\alpha_L - 2)(\lambda_1 + \lambda_2)T_1}{\beta_L(1 - \beta_L)} \exp\left[-\frac{2\lambda_0 T_1}{\beta_L(1 + L_1)}\right], & \text{if } \beta = \beta_L, 2/3 < \beta_L < 1, \\ 1 + \exp\left[-\frac{2(\lambda_0 + \lambda_1 + \lambda_2)T_1}{\beta}\right] - \exp\left(-\frac{2\lambda_0 T_1}{\beta}\right) - \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1 - \beta}\right] \\ + \frac{\beta(\lambda_1 + \lambda_2)(1 + L_1)}{2(1 - \beta)\lambda_0 - \beta(\lambda_1 + \lambda_2)(1 + L_1)} \left\{ \exp\left[\frac{2(2 - 3\beta)\lambda_0 T_1}{\beta^2(1 + L_1)} - \frac{2(\lambda_1 + \lambda_2)T_1}{\beta}\right] \right. \\ \left. - \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1 - \beta}\right] \right\}, & \text{otherwise,} \end{cases} \quad (3.9)$$

where  $\beta_L := \frac{2\lambda_0}{2\lambda_0 + (\lambda_1 + \lambda_2)(1 + L_1)}$

*Proof.* Firstly, we use the expression of (3.A-8) to derive lower and upper bounds for  $\Psi(a, b, c, \theta)$ . A lower bound on  $\Psi(a, b, c, \theta)$  can be derived by replacing  $x_2$  in the denominator of the exponent of  $e^{-\frac{2\lambda_0}{\beta} \frac{T_1 - (1 - \beta)x_2}{1 + (1 - \beta)\rho x_2}}$  by the integration lower limit  $2T_1/\beta$ . Similarly, an upper bound on  $\Psi(a, b, c, \theta)$  can be derived by replacing  $x_2$  in the denominator of the exponent of  $e^{-\frac{2\lambda_0}{\beta} \frac{T_1 - (1 - \beta)x_2}{1 + (1 - \beta)\rho x_2}}$  by the integration upper limit  $T_1/(1 - \beta)$ . Secondly, the upper bound  $P_{UB}^{TDDBC}$  can be obtained by substituting the lower bound on  $\Psi(a, b, c, \theta)$  into (3.6) for the case  $2/3 < \beta < 1$ . Also, the lower bound  $P_{LB}^{TDDBC}$  can be obtained by substituting the upper bound on  $\Psi(a, b, c, \theta)$  into (3.6) for the case  $2/3 < \beta < 1$ .  $\square$

To provide more insight into the TDDBC protocol, we analyze the asymptotic high-SNR performance in what follows.

**Corollary 3.2.** *The TDBC protocol achieves full diversity order two, and the asymptotic high-SNR outage probability is given by*

$$P_{out}^{TDBC}(R, \rho) = \begin{cases} \frac{4\lambda_0(\lambda_1+\lambda_2)L_1^2}{\beta^2} \frac{1}{\rho^2} + O\left(\frac{1}{\rho^3}\right), & \text{if } 0 < \beta \leq 2/3, \\ \left[ \left( \frac{2(\lambda_0+\lambda_1+\lambda_2)^2 - 2\lambda_0^2}{\beta^2} - \frac{(\lambda_1+\lambda_2)^2}{2(1-\beta)^2} \right) L_1^2 - \frac{2\lambda_0(\lambda_1+\lambda_2)(1+L_1)(1-\theta+\ln \theta)}{\beta(1-\beta)} \right. \\ \left. + \frac{(\lambda_1+\lambda_2)^2(1+L_1)(1-\theta)[\theta(1+L_1)-(1-L_1)]}{2(1-\beta)^2} \right] \frac{1}{\rho^2} + O\left(\frac{1}{\rho^3}\right), & \text{if } 2/3 < \beta < 1. \end{cases} \quad (3.10)$$

*Proof.* By very typical asymptotic analysis as in [7], it is not hard to derive (3.10) from (3.6). Furthermore, it can be seen from (3.10) that the TDBC protocol achieves diversity order two.  $\square$

### 3.3.2 PNC Protocol

The PNC protocol consists of two phases, namely the MAC phase and the BC phase, as depicted in Fig. 3.2. In each phase, two traffic flows are simultaneously supported. We use  $R_1$  and  $R_2$  to denote the transmission rates of  $S_1$  and  $S_2$ , respectively. An achievable rate region of the PNC protocol is the closure of the convex hull of the set of points  $(R_1, R_2)$  satisfying the following inequalities [17], [18]:

$$R_1 < I_1^{\text{PNC}}, \quad (3.11)$$

$$R_2 < I_2^{\text{PNC}}, \quad (3.12)$$

$$R_1 + R_2 < I_{\text{sum}}^{\text{PNC}}, \quad (3.13)$$

where

$$I_1^{\text{PNC}} = \min \left\{ \frac{1}{2} \log \left( 1 + \frac{\beta \rho |h_1|^2}{2} \right), \frac{1}{2} \log [1 + (1 - \beta) \rho |h_2|^2] \right\}, \quad (3.14)$$

$$I_2^{\text{PNC}} = \min \left\{ \frac{1}{2} \log \left( 1 + \frac{\beta \rho |h_2|^2}{2} \right), \frac{1}{2} \log [1 + (1 - \beta) \rho |h_1|^2] \right\}, \quad (3.15)$$

$$I_{\text{sum}}^{\text{PNC}} = \frac{1}{2} \log \left[ 1 + \frac{\beta \rho (|h_1|^2 + |h_2|^2)}{2} \right]. \quad (3.16)$$

In eqs. (3.14)–(3.16), the pre-log factor  $1/2$  is because each traffic flow takes two time slots in the PNC protocol [17], [18]. Again, we set the target data rate for each end-source as  $R/2$  assuming the target data rate for the whole network is  $R$ . The system is in outage when the rate pair  $(R_1, R_2)$  falls out of the capacity region. Therefore, the outage probability of the PNC protocol is given by

$$P_{\text{out}}^{\text{PNC}}(R, \rho) := \Pr \left( I_1^{\text{PNC}} < \frac{R}{2} \text{ or } I_2^{\text{PNC}} < \frac{R}{2} \text{ or } I_{\text{sum}}^{\text{PNC}} < R \right). \quad (3.17)$$

In the following theorem, we derive an exact closed-form expression of the outage probability for the PNC protocol.

**Theorem 3.2.** *For the PNC protocol, an exact closed-form outage probability expression is given by*

$$P_{\text{out}}^{\text{PNC}}(R, \rho) = \begin{cases} 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp \left[ \frac{2(\lambda_2 - \lambda_1)T_2 - 2\lambda_2 T_3}{\beta} \right] + \frac{\lambda_2}{\lambda_1 - \lambda_2} \exp \left[ \frac{2(\lambda_1 - \lambda_2)T_2 - 2\lambda_1 T_3}{\beta} \right], & \text{for Case A,} \\ 1 - \exp \left( -\frac{2\lambda_2 T_3}{\beta} \right) - \frac{2\lambda_2 (T_3 - 2T_2)}{\beta} \exp \left( -\frac{2\lambda_1 T_3}{\beta} \right), & \text{for Case B,} \\ 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp \left[ \frac{(\lambda_2 - \lambda_1)T_2}{1 - \beta} - \frac{2\lambda_2 T_3}{\beta} \right] + \frac{\lambda_2}{\lambda_1 - \lambda_2} \exp \left[ \frac{(\lambda_1 - \lambda_2)T_2}{1 - \beta} - \frac{2\lambda_1 T_3}{\beta} \right], & \text{for Case C,} \\ 1 - \exp \left( -\frac{2\lambda_2 T_3}{\beta} \right) - 2\lambda_2 \left( \frac{T_3}{\beta} - \frac{T_2}{1 - \beta} \right) \exp \left( -\frac{2\lambda_1 T_3}{\beta} \right), & \text{for Case D,} \\ 1 - \exp \left[ -\frac{(\lambda_1 + \lambda_2)T_2}{1 - \beta} \right], & \text{for Case E,} \end{cases} \quad (3.18)$$

where  $T_2$  and  $T_3$  are defined as:  $T_2 := \frac{2^R-1}{\rho}$  and  $T_3 := \frac{2^{2R}-1}{\rho}$ . Also, Case A represents  $0 < \beta \leq 2/3$  and  $\lambda_1 \neq \lambda_2$ ; Case B:  $0 < \beta \leq 2/3$  and  $\lambda_1 = \lambda_2$ ; Case C:  $2/3 < \beta \leq \beta_0$  and  $\lambda_1 \neq \lambda_2$ ; Case D:  $2/3 < \beta \leq \beta_0$  and  $\lambda_1 = \lambda_2$ ; Case E:  $\beta_0 < \beta < 1$ , where  $\beta_0 = \frac{T_3}{T_3+T_2}$ .<sup>5</sup>

*Proof.* See Appendix 3-B. □

Note that the outage probability of (3.18) for the PNC protocol only involves standard elementary functions such as the exponential function which can be easily computed. Therefore, it is an exact and truly closed-form expression. To provide more insight into the PNC protocol, we also study the asymptotic high-SNR performance in what follows.

**Corollary 3.3.** *The PNC protocol achieves diversity order one, and the asymptotic high-SNR outage probability is given by*

$$P_{out}^{PNC}(R, \rho) = \begin{cases} \frac{2(\lambda_1+\lambda_2)L_2}{\beta} \frac{1}{\rho} + O\left(\frac{1}{\rho^2}\right), & \text{if } 0 < \beta \leq 2/3, \\ \frac{(\lambda_1+\lambda_2)L_2}{1-\beta} \frac{1}{\rho} + O\left(\frac{1}{\rho^2}\right), & \text{if } 2/3 < \beta < 1, \end{cases} \quad (3.19)$$

where  $L_2 = 2^R - 1$ .

*Proof.* Using similar asymptotic analysis as in Corollary 3.2 yields (3.19). Furthermore, it can be seen from (3.19) that the PNC protocol achieves diversity order one. □

### 3.3.3 OSS Protocol

The fundamental idea of the OSS protocol is that one source out of two end-sources is selected based on the instantaneous channel conditions such that the *multiuser*

<sup>5</sup>By definitions of  $T_3$  and  $T_2$ , it is easy to show that  $T_3 > 2T_2$ . Using the fact  $T_3 > 2T_2$ , it can be easily shown that  $2/3 < \beta_0 < 1$ .

*diversity* of the time-varying channels is exploited. The traffic flow in the OSS protocol is actually *unidirectional* in each time instant because only one of the two end-sources is selected and the selected source transmits information in one directional only.<sup>6</sup> Let us first consider the case when  $S_1$  is selected as the transmitting source. Then the signal transmitted from  $S_1$  is received at  $S_2$  through two paths: the path from  $S_1$  directly to  $S_2$  and the relay path from  $S_1$  to  $S_2$  via  $R$ . Note that this is equivalent to the traditional three-node unidirectional DF cooperative network [41, eq. (19)]. Therefore, the mutual information  $I_1^{\text{OSS}}$  between the transmitted signal from  $S_1$  and the received signal at  $S_2$  when  $S_1$  is selected as the transmitting source is given by

$$I_1^{\text{OSS}} = \begin{cases} \frac{1}{2} \log(1 + \beta\rho|h_0|^2), & \text{if } |h_1|^2 < \frac{T_3}{\beta}, \\ \frac{1}{2} \log[1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_2|^2], & \text{if } |h_1|^2 > \frac{T_3}{\beta}, \end{cases} \quad (3.20)$$

where the pre-log factor  $1/2$  is because each traffic flow takes two time slots, and  $\beta = E_s/E$  denotes the ratio of the average transmission power at the selected source to the total transmission power of the whole network. Secondly, let us consider the case when  $S_2$  is selected as the transmitting source. Then, in the same way, the mutual information  $I_2^{\text{OSS}}$  between the transmitted signal from  $S_2$  and the received signal at  $S_1$  when  $S_2$  is selected as the transmitting source is given by

$$I_2^{\text{OSS}} = \begin{cases} \frac{1}{2} \log(1 + \beta\rho|h_0|^2), & \text{if } |h_2|^2 < \frac{T_3}{\beta}, \\ \frac{1}{2} \log[1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_1|^2], & \text{if } |h_2|^2 > \frac{T_3}{\beta}. \end{cases} \quad (3.21)$$

In the OSS protocol, the two end-sources are opportunistically selected as the transmitting source such that the mutual information of the whole network is maximized. That is,  $S_1$  is selected as the transmitting source if  $I_1^{\text{OSS}} > I_2^{\text{OSS}}$ , and the

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<sup>6</sup>In the long term, however, the traffic flow in the OSS protocol is bidirectional over time-varying channels because the two end-sources are opportunistically selected as the transmitting source.



mutual information  $I^{\text{OSS}}$  of the OSS protocol equals  $I_1^{\text{OSS}}$ , i.e.,  $I^{\text{OSS}} = I_1^{\text{OSS}}$ . On the other hand, if  $I_2^{\text{OSS}} > I_1^{\text{OSS}}$ ,  $S_2$  is selected as the transmitting source and  $I^{\text{OSS}} = I_2^{\text{OSS}}$ . If  $I_1^{\text{OSS}} = I_2^{\text{OSS}}$ ,  $S_1$  and  $S_2$  are selected as the transmitting source with equal probability.

In the following lemma, a unified expression of the mutual information  $I^{\text{OSS}}$  combining (3.20) and (3.21) is derived.

**Lemma 3.1.** *For the OSS protocol, the mutual information  $I^{\text{OSS}}$  between the transmitted signal from the selected source and the received signal at the other end-source is given by*

$$I^{\text{OSS}} = \begin{cases} \frac{1}{2} \log (1 + \beta \rho |h_0|^2), & \text{if } \max \{|h_1|^2, |h_2|^2\} < \frac{T_3}{\beta}, \\ \frac{1}{2} \log [1 + \beta \rho |h_0|^2 + (1 - \beta) \rho |h_1|^2], & \text{if } \frac{T_3}{\beta} < |h_2|^2 < |h_1|^2 \text{ or } |h_1|^2 < \frac{T_3}{\beta} < |h_2|^2, \\ \frac{1}{2} \log [1 + \beta \rho |h_0|^2 + (1 - \beta) \rho |h_2|^2], & \text{if } \frac{T_3}{\beta} < |h_1|^2 < |h_2|^2 \text{ or } |h_2|^2 < \frac{T_3}{\beta} < |h_1|^2. \end{cases} \quad (3.22)$$

*Proof.* See Appendix 3-C. □

Using the mutual information in Lemma 3.1, we now study the outage probability performance of the OSS protocol. The OSS protocol is in outage when the target data rate  $R$  exceeds the maximum achievable rate  $I^{\text{OSS}}$ , that is,

$$P_{\text{out}}^{\text{OSS}}(R, \rho) := \Pr (I^{\text{OSS}} < R). \quad (3.23)$$

In the following theorem, we derive an exact closed-form outage probability expression for the OSS protocol.

**Theorem 3.3.** *For the OSS protocol, a closed-form exact outage probability expression is given by*

$$P_{out}^{OSS}(R, \rho) = (1 - e^{-\lambda_0 T_3/\beta}) (1 - e^{-\lambda_1 T_3/\beta}) (1 - e^{-\lambda_2 T_3/\beta}) + G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3) + G(\beta, \lambda_0, \lambda_2, \lambda_1, T_3), \quad (3.24)$$

where  $\lambda_i = 1/\Omega_i$ ,  $i = 0, 1, 2$ . The function  $G(x, y_0, y_1, y_2, z)$ ,  $0 < x < 1$ ,  $z > 0$ ,  $y_i > 0$ ,  $i = 0, 1, 2$ , is defined as follows:

$$G(x, y_0, y_1, y_2, z) := \left\{ \begin{array}{l} \exp\left(-\frac{y_2 z}{x}\right) + \frac{xy_1}{(1-x)y_0 - xy_1} \exp\left[-\frac{(y_0+y_2)z}{x}\right] - \frac{(1-x)y_0}{(1-x)y_0 - xy_1} \exp\left[-\left(\frac{y_1}{1-x} + \frac{y_2}{x}\right)z\right], \\ \text{for Case 1,} \\ \\ \exp\left(-\frac{y_2 z}{x}\right) + \frac{xy_1}{(1-x)y_0 - xy_1} \exp\left[-\frac{(y_0+y_2)z}{x}\right] - \frac{(1-x)y_0}{(1-x)y_0 - xy_1} \exp\left[-\left(\frac{y_1}{1-x} + \frac{y_2}{x}\right)z\right] \\ - \frac{y_1}{y_1+y_2} \exp\left[-\frac{(y_1+y_2)z}{x}\right] - \frac{xy_1}{(1-x)y_0 - x(y_1+y_2)} \exp\left[\left(\frac{1-2x}{x}y_0 - y_1 - y_2\right)\frac{z}{x}\right] \\ + \frac{(1-x)y_0 y_1}{(y_1+y_2)[(1-x)y_0 - x(y_1+y_2)]} \exp\left[-\frac{(y_1+y_2)z}{1-x}\right], \\ \text{for Case 2,} \\ \\ \exp\left(-\frac{y_2 z}{x}\right) - \frac{y_1 z}{1-x} \exp\left[-\frac{(y_0+y_2)z}{x}\right] - \exp\left[-\left(\frac{y_1}{1-x} + \frac{y_2}{x}\right)z\right], \\ \text{for Case 3,} \\ \\ \exp\left(-\frac{y_2 z}{x}\right) - \frac{y_1 z}{1-x} \exp\left[-\frac{(y_0+y_2)z}{x}\right] - \exp\left[-\left(\frac{y_1}{1-x} + \frac{y_2}{x}\right)z\right] \\ - \frac{y_1}{y_1+y_2} \exp\left[-\frac{(y_1+y_2)z}{x}\right] - \frac{xy_1}{(1-x)y_0 - x(y_1+y_2)} \exp\left[\left(\frac{1-2x}{x}y_0 - y_1 - y_2\right)\frac{z}{x}\right] \\ + \frac{(1-x)y_0 y_1}{(y_1+y_2)[(1-x)y_0 - x(y_1+y_2)]} \exp\left[-\frac{(y_1+y_2)z}{1-x}\right], \\ \text{for Case 4,} \\ \\ \exp\left(-\frac{y_2 z}{x}\right) - \frac{(1-x)y_0}{(1-x)y_0 - xy_1} \exp\left[-\left(\frac{y_1}{1-x} + \frac{y_2}{x}\right)z\right] + \frac{xy_1}{(1-x)y_0 - xy_1} \exp\left[-\frac{(y_0+y_2)z}{x}\right] \\ + \frac{y_1}{y_1+y_2} \left\{ \exp\left[-\frac{(y_1+y_2)z}{1-x}\right] - \exp\left[-\frac{(y_1+y_2)z}{x}\right] \right\} + \frac{(2x-1)y_1 z}{x(1-x)} \exp\left(-\frac{y_0 z}{x}\right), \\ \text{for Case 5,} \end{array} \right. \quad (3.25)$$

where Case 1 represents  $0 < x \leq 1/2$  and  $x \neq \frac{y_0}{y_0+y_1}$ ; Case 2:  $1/2 < x < 1$ ,  $x \neq \frac{y_0}{y_0+y_1}$ ,

and  $x \neq \frac{y_0}{y_0+y_1+y_2}$ ; Case 3:  $x = \frac{y_0}{y_0+y_1}$  and  $y_0 \leq y_1$ ; Case 4:  $x = \frac{y_0}{y_0+y_1}$  and  $y_0 > y_1$ ; Case 5:  $x = \frac{y_0}{y_0+y_1+y_2}$  and  $y_0 > y_1 + y_2$ .<sup>7</sup>

*Proof.* See Appendix 3-D. □

Although (3.24) along with (3.25) provides an exact closed-form expression for the outage probability of the OSS protocol, it is rather long in form. To provide more insight into the OSS protocol, we study the asymptotic high-SNR performance of the OSS protocol in what follows.

**Corollary 3.4.** *The OSS protocol achieves full diversity order two, and the asymptotic outage probability is given by*

$$P_{out}^{OSS}(R, \rho) = [g(\beta, \lambda_0, \lambda_1, \lambda_2) + g(\beta, \lambda_0, \lambda_2, \lambda_1)] \frac{L_3^2}{\rho^2} + O\left(\frac{1}{\rho^3}\right), \quad (3.26)$$

where  $L_3 = 2^{2R} - 1$ , and  $g(x, y_0, y_1, y_2)$  with  $0 < x < 1$ ,  $y_i > 0$ , and  $i = 0, 1, 2$  is defined as follows:  $g(x, y_0, y_1, y_2) := \frac{y_0 y_1}{2x(1-x)}$  for Case 1;  $\frac{(3x-1)y_0 y_1}{2x^3}$  for Case 2;  $\frac{2y_0 y_1(1-x) - x y_1^2}{2x(1-x)^2}$  for Case 3;  $\frac{(2x^3 - 6x^2 + 5x - 1)y_0 y_1 - x^3 y_1^2}{2x^3(1-x)^2}$  for Case 4; and  $\frac{(3x^2 - 5x + 2)y_0 y_1 + (2x - 1)y_1 y_2 + (2x - 1)y_1^2}{2x^2(1-x)^2}$  for Case 5.

*Proof.* Using similar asymptotic analysis as in Corollary 3.2 yields (3.26). Furthermore, it can be seen from (3.26) that the OSS protocol achieves full diversity order two. □

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<sup>7</sup>The case when  $x = \frac{y_0}{y_0+y_1+y_2}$  and  $y_0 \leq y_1 + y_2$  is included in Case 1.

### 3.4 Diversity-Multiplexing Tradeoff

In order to compare the reliability as well as the spectral efficiency of the three bidirectional protocols, we consider the diversity gain and the multiplexing gain simultaneously. The *diversity-multiplexing tradeoff* which is a fundamental tradeoff for any communication systems provides a whole view on the diversity gain and multiplexing gain [25]. Recently, the DMT performance of multiple-antenna systems has also been studied in the finite SNR regime [40]. In this section, we study both the finite-SNR DMT and the infinite-SNR DMT performance for the three bidirectional protocols.

We first study the finite-SNR DMT. The finite-SNR DMT for a single-antenna system is defined as [40]

$$d(r, \rho) := -\frac{\partial \ln P_{\text{out}}(r \log(1 + \rho), \rho)}{\partial \ln \rho}, \quad (3.27)$$

where  $r$  is the multiplexing gain, and  $d(r, \rho)$  is the corresponding diversity gain at a finite SNR  $\rho$ . With the derived outage probability expressions, it is straightforward to derive the finite-SNR DMT for the bidirectional protocols by using (3.27). Specifically, for the TDBC with  $0 < \beta \leq 2/3$ , PNC, and OSS protocols, exact closed-form expressions of the finite-SNR DMT can be easily derived by substituting their respective outage probability expressions of (3.6), (3.18), and (3.24) into (3.27). For the TDBC protocol with  $2/3 < \beta < 1$ , however, the finite-SNR DMT expression is more complex than other cases because taking derivative of  $\Psi(a, b, c, \theta)$  yields another integral whose integral region is between  $\theta$  and 1, as in  $\Psi(a, b, c, \theta)$ . Unfortunately, it is extremely hard to solve the resulting integral in closed-form. However, the integral can be easily computed with similar numerical approaches used for  $\Psi(a, b, c, \theta)$ . Also,

closed-form lower and upper bounds on the finite-SNR DMT for  $2/3 < \beta < 1$  can be easily derived by using the extremely tight bounds  $P_{\text{UB}}^{\text{TDBC}}$  of (3.8) and  $P_{\text{LB}}^{\text{TDBC}}$  of (3.9), respectively. In this chapter, we do not present all these derived finite-SNR DMT expressions due to a length limit. Instead, as an example, we present the exact finite-SNR DMT expression of the TDBC protocol for  $0 < \beta \leq 2/3$  in the following theorem.

**Theorem 3.4.** *The finite-SNR DMT  $d^{\text{TDBC}}(r, \rho)$  of the TDBC protocol for  $0 < \beta \leq 2/3$  is given as follows:*

$$d^{\text{TDBC}}(r, \rho) = [(3r\rho - 2)\xi + 3r/\beta] \times \frac{(\lambda_0 + \lambda_1 + \lambda_2)e^{-2(\lambda_0 + \lambda_1 + \lambda_2)\xi} - \lambda_0 e^{-2\lambda_0\xi} - (\lambda_1 + \lambda_2)e^{-2(\lambda_1 + \lambda_2)\xi}}{1 + e^{-2(\lambda_0 + \lambda_1 + \lambda_2)\xi} - e^{-2\lambda_0\xi} - e^{-2(\lambda_1 + \lambda_2)\xi}}, \quad (3.28)$$

where  $\xi = \frac{(1+\rho)^{1.5r} - 1}{\rho\beta}$ .

*Proof.* Substituting (3.6) for the case  $0 < \beta \leq 2/3$  into (3.27) and taking derivative yields (3.28).  $\square$

We now consider the infinite-SNR DMT. By definition, the infinite-SNR DMT can be obtained from the finite-SNR DMT by letting  $\rho$  go to infinity [22], [25], [40]. However, as the finite-SNR DMT expressions are very long, it becomes involved to take the limit. As an alternative approach, we use the asymptotic high-SNR outage probability expressions to derive the infinite-SNR DMT, as given in the following theorem.

**Theorem 3.5.** *The infinite-SNR DMT  $D^{\text{TDBC}}(r)$  of the TDBC protocol,  $D^{\text{PNC}}(r)$  of*

the PNC protocol, and  $D^{OSS}(r)$  of the OSS protocol are given as follows:

$$D^{TDBC}(r) = 2 - 3r, \quad (3.29)$$

$$D^{PNC}(r) = 1 - r, \quad (3.30)$$

$$D^{OSS}(r) = 2 - 4r. \quad (3.31)$$

*Proof.* We first derive the infinite-SNR DMT for the TDBC protocol. We rewrite the asymptotic outage probability of the TDBC protocol irrespective of the values of  $\beta$  as follows:

$$P_{\text{out}}^{\text{TDBC}}(R, \rho) = \frac{(aL_1^2 + bL_1 + c)}{\rho^2} + O\left(\frac{1}{\rho^3}\right), \quad (3.32)$$

where  $a$  is a non-zero constant,  $b$  and  $c$  are constants which can possibly take zero values. By the definition of the infinite-SNR DMT, we substitute  $R = r \log \rho$  into (3.32) and let  $\rho$  go to infinity. Then  $P_{\text{out}}^{\text{TDBC}}(R, \rho)$  is accurately approximated as  $P_{\text{out}}^{\text{TDBC}}(R, \rho) \approx a\rho^{3r-2}$  by ignoring higher order terms. Therefore,  $D^{\text{TDBC}}(r) = 2 - 3r$ . Also, the infinite-SNR DMT for the PNC and OSS protocols can be derived in the same manner as the TDBC protocol by using the asymptotic outage probability expressions we have derived earlier.  $\square$

## 3.5 Numerical Results

This section gives extensive numerical results on the outage probability performance of the TDBC, PNC, and OSS protocols. To model a general three-node cooperative network, we assume all three terminals are located in a straight line.<sup>8</sup> We fix the distance between  $S_1$  and  $S_2$  as one and let  $d$  denote the distance between  $S_1$  and R.

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<sup>8</sup>This assumption has also been widely adopted in many publications including [29], [30], [21], and so on.

Furthermore, the channel variances are modeled as  $\Omega_0 = 1$ ,  $\Omega_1 = d^{-4}$ , and  $\Omega_2 = (1 - d)^{-4}$  [29], [30], [21], [43]. We note that the derived exact outage probability expressions of (3.6), (3.18), and (3.24) and the upper bound and lower bound expressions of (3.8) and (3.9) are invariant under interchanging of  $\lambda_1$  and  $\lambda_2$ . Therefore, it is sufficient and no loss of generality to consider the case  $0 < d \leq 0.5$  only.

Firstly, we fix the data rate as  $R = 1$  bps/Hz and compare the reliability of the TDBC, PNC, and OSS protocols. Each protocol is compared under the condition that the total average transmission power of the whole network is  $E$ . As there are several factors such as the average SNR  $\rho$ , the relay position  $d$ , and the power ratio  $\beta$  that can affect the outage probabilities, we consider each of the factors separately by fixing the other two factors. Specifically, we consider the following three cases:

1. We vary the average SNR  $\rho$  while fixing the relay position  $d$  and setting  $E_s = 2E/3$  and  $E_r = E/3$  for each protocol. Note that the total transmission power of the OSS protocol is  $E$ , which is the same as in the TDBC and OSS protocols, because only one end-source is allowed to transmit whenever one end-source out of two is selected as the transmitting source. Figs. 3.4 and 3.5 show the outage probabilities against the average SNR  $\rho$  for  $d = 0.3$  and  $d = 0.5$  respectively. It is easy to see that the simulated outage probabilities perfectly match with the exact outage probabilities obtained by the derived expressions. Also, asymptotic outage probabilities approach the exact outage probabilities in the high-SNR regime. Therefore, it is easy to see that the OSS and TDBC protocols achieve full diversity order two and the PNC protocol achieves diversity order one. Moreover, the outage probability curve for the OSS protocol lies strictly below the outage probability curves for the PNC and TDBC protocols, which indicates

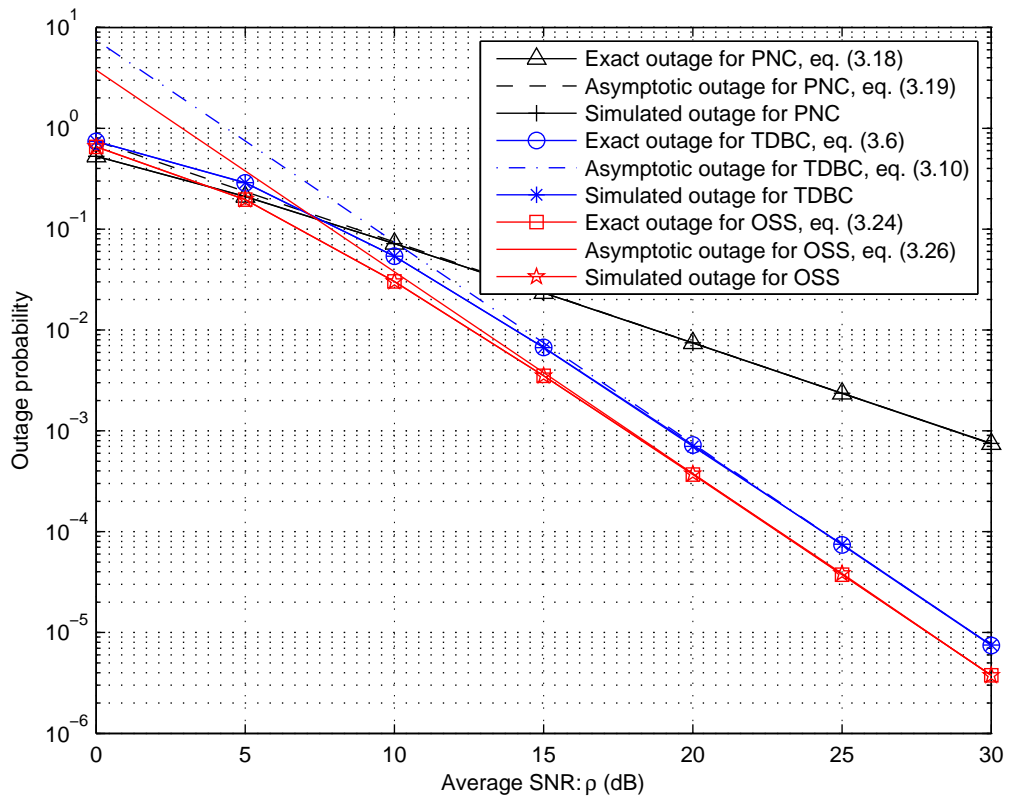


Figure 3.4: Outage probabilities against average SNR  $\rho$  for the TDBC, PNC, and OSS protocols with  $d = 0.3$ ,  $R = 1$  bps/Hz,  $E_s = 2E/3$ , and  $E_r = E/3$ .



that the OSS protocol is more reliable.

2. We vary the relay position  $d$  while fixing the average SNR  $\rho$  and setting  $E_s = 2E/3$  and  $E_r = E/3$  for each protocol. Fig. 3.6 shows the outage probability against the relay position  $d$  when  $\rho = 20$  dB. We see once again that numerical results perfectly match with analytical results. Also, the outage probability of the OSS protocol is lower than those of the TDBC and PNC protocols, which again indicates that the OSS protocol is more reliable.
3. We vary the power ratio  $\beta$  while fixing the relay position  $d$  and the average SNR  $\rho$ . For each protocol, the total average transmission power of the whole network is  $E$ . We compare the outage probability performance by varying  $\beta$  for each protocol. Figs. 3.7 and 3.8 show the outage probabilities with  $\rho = 20$  dB against various  $\beta$  for  $d = 0.3$  and  $d = 0.5$ , respectively. Once again, we see that numerical results perfectly match with analytical results and the OSS protocol is more reliable.

Moreover, we investigate the tightness of the derived upper bound (3.8) and lower bound (3.9) on the outage probability of the TDBC protocol for  $2/3 < \beta < 1$ . As Fig. 3.9 shows, the derived upper and lower bounds are extremely tight to the exact outage probability, irrespective of the average SNR  $\rho$  and the relay position  $d$ .

We have demonstrated that the OSS protocol is more reliable than other protocols for low data rate, e.g.,  $R = 1$  bps/Hz. In the following, we investigate the reliability for higher data rates. In Fig. 3.10, we consider three data rates:  $R = 1$  bps/Hz,  $R = 3$  bps/Hz, and  $R = 6$  bps/Hz. We fix the relay position as  $d = 0.3$ . We set  $E_s = 2E/3$  and  $E_r = E/3$  for all three protocols as before. We observe that the

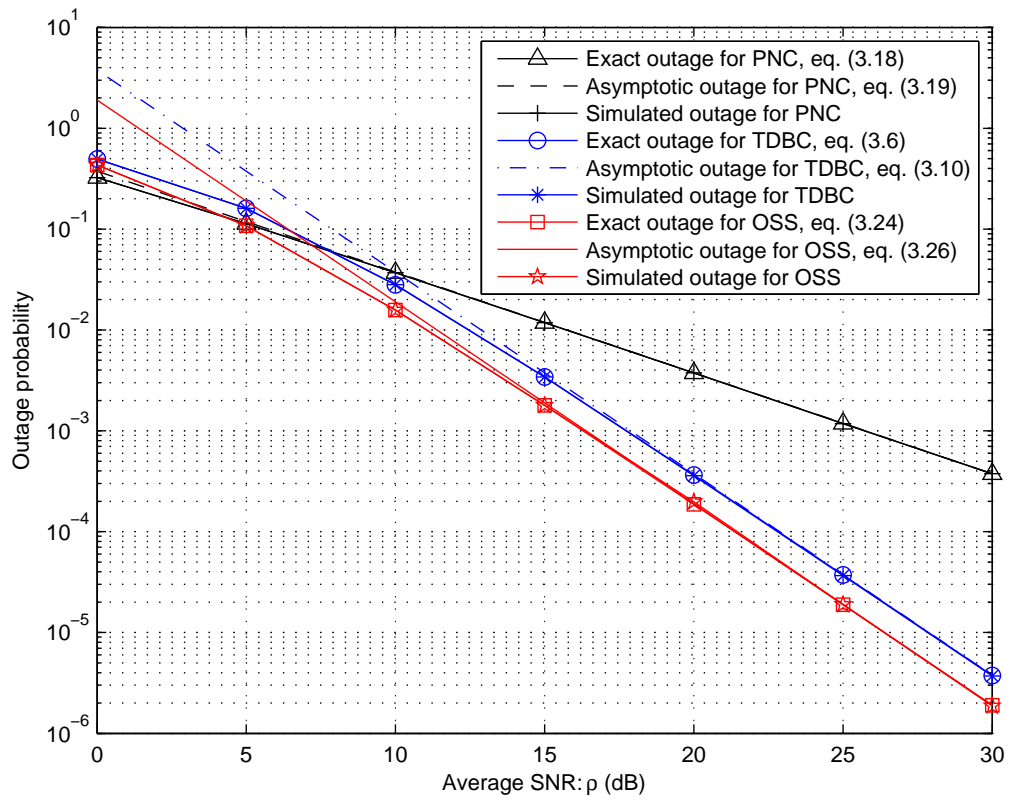


Figure 3.5: Outage probabilities against average SNR  $\rho$  for the TDBC, PNC, and OSS protocols with  $d = 0.5$ ,  $R = 1$  bps/Hz,  $E_s = 2E/3$ , and  $E_r = E/3$ .

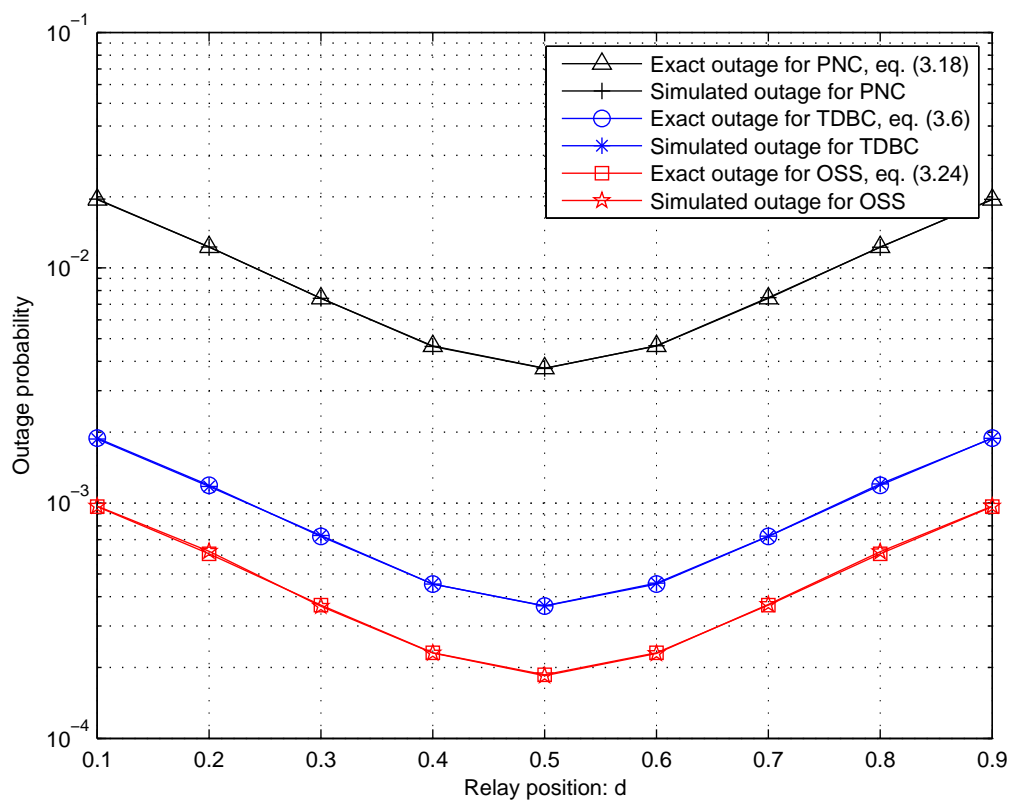


Figure 3.6: Outage probabilities of the TDBC, PNC, and OSS protocols for various relay positions  $d$  while  $\rho = 20$  dB,  $R = 1$  bps/Hz,  $E_s = 2E/3$ , and  $E_r = E/3$ .

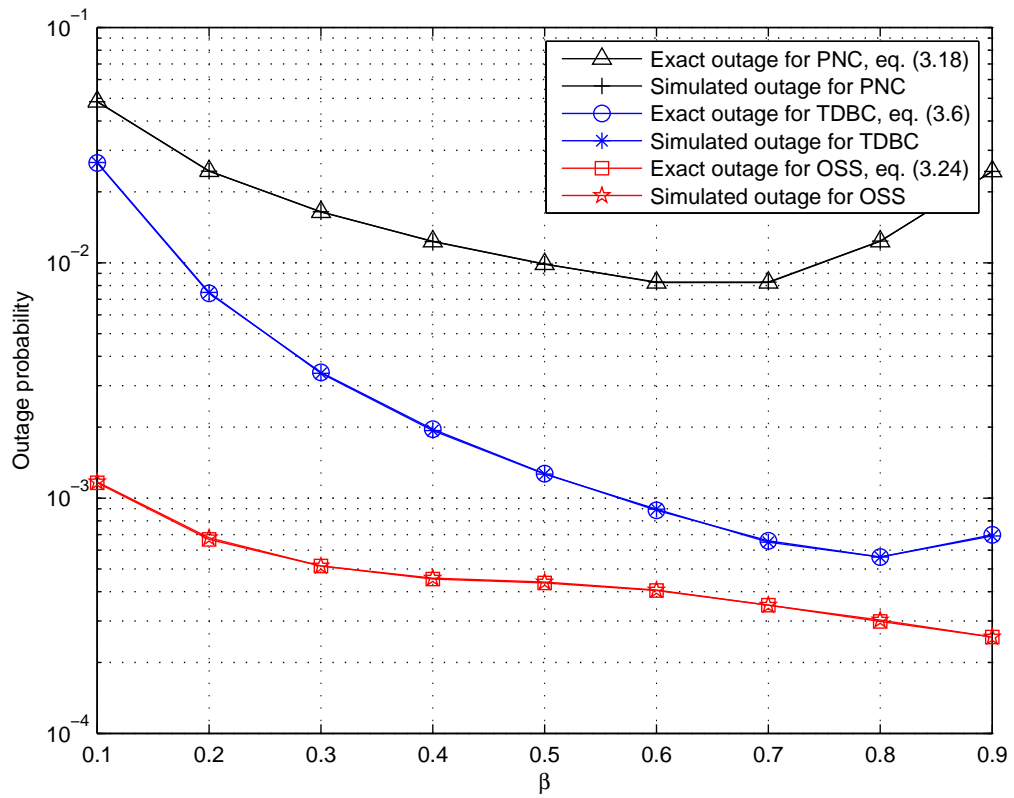


Figure 3.7: Outage probabilities of the TDBC, PNC, and OSS protocols for various  $\beta$  while  $\rho = 20$  dB,  $d = 0.3$ , and  $R = 1$  bps/Hz.

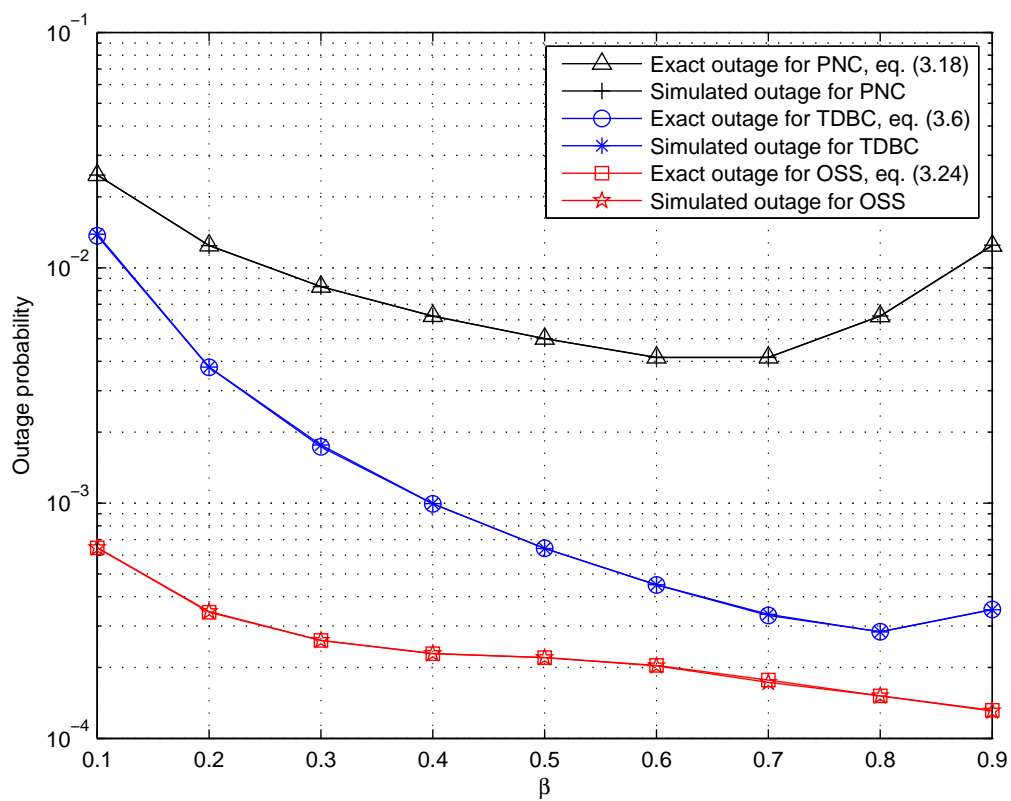


Figure 3.8: Outage probabilities of the TDBC, PNC, and OSS protocols for various  $\beta$  while  $\rho = 20$  dB,  $d = 0.5$ , and  $R = 1$  bps/Hz.

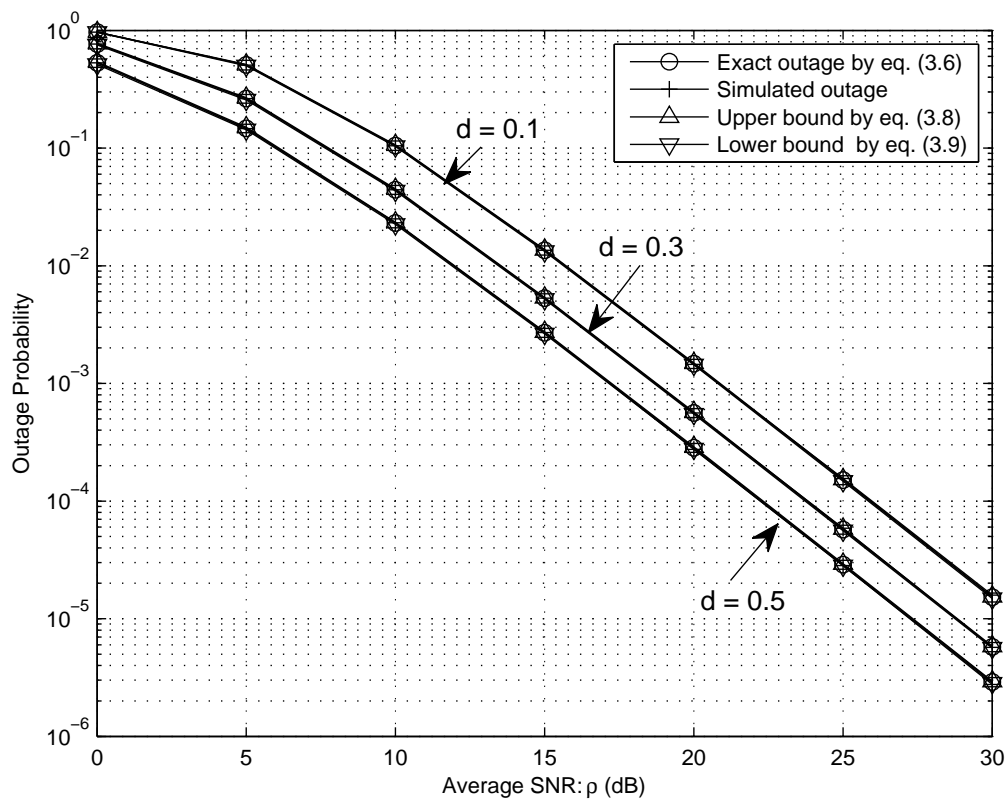


Figure 3.9: Outage probabilities and bounds for the TDBC protocol with  $\beta = 0.8$  and  $R = 1$  bps/Hz.

OSS protocol has the highest reliability in the whole SNR range when data rate is very low, e.g.,  $R = 1$  bps/Hz, as demonstrated by Fig. 3.10. This is because only the OSS protocol can exploit the multiuser diversity, as discussed in Section 3.1. However, as the data rate increases, both TDBC and PNC can outperform the OSS. As an example, we consider the case when outage probability equals  $10^{-2}$ . If  $R = 1$  bps/Hz, the OSS protocol requires the lowest SNR to achieve the same performance as other protocols; If  $R = 3$  bps/Hz, the TDBC becomes the best protocol which achieves the same performance as others at the lowest SNR; If  $R = 6$  bps/Hz, the best protocol is the PNC protocol. A detailed comparison is presented in Table. 3.1. We also observe that the TDBC and OSS protocols outperform the PNC protocol at the asymptotic high-SNR regime, irrespective of the data rate. All these observations can be explained by the the concept of the diversity-multiplexing tradeoff, which is discussed in the following.

Table 3.1: Comparison of the bidirectional protocols when the outage probability is equal to  $10^{-2}$ , where the notation  $A \succ B$  means  $A$  outperforms  $B$  in terms of outage performance.

Low Rate, e.g., $R = 1$ bps/Hz	OSS $\succ$ TDBC $\succ$ PNC
Medium Rate, e.g., $R = 3$ bps/Hz	TDBC $\succ$ OSS $\succ$ PNC
High Rate, e.g., $R = 6$ bps/Hz	PNC $\succ$ TDBC $\succ$ OSS

We plot the DMT curves in both the finite and infinite SNR regimes in Fig. 3.11. We see that the finite-SNR DMT curves converge to the infinite-SNR DMT curves, as expected by definitions. Moreover, the OSS and TDBC protocols achieve higher maximal diversity gain than the PNC protocol. On the other hand, the PNC protocol achieves higher maximal multiplexing gain than the TDBC and OSS protocols. For

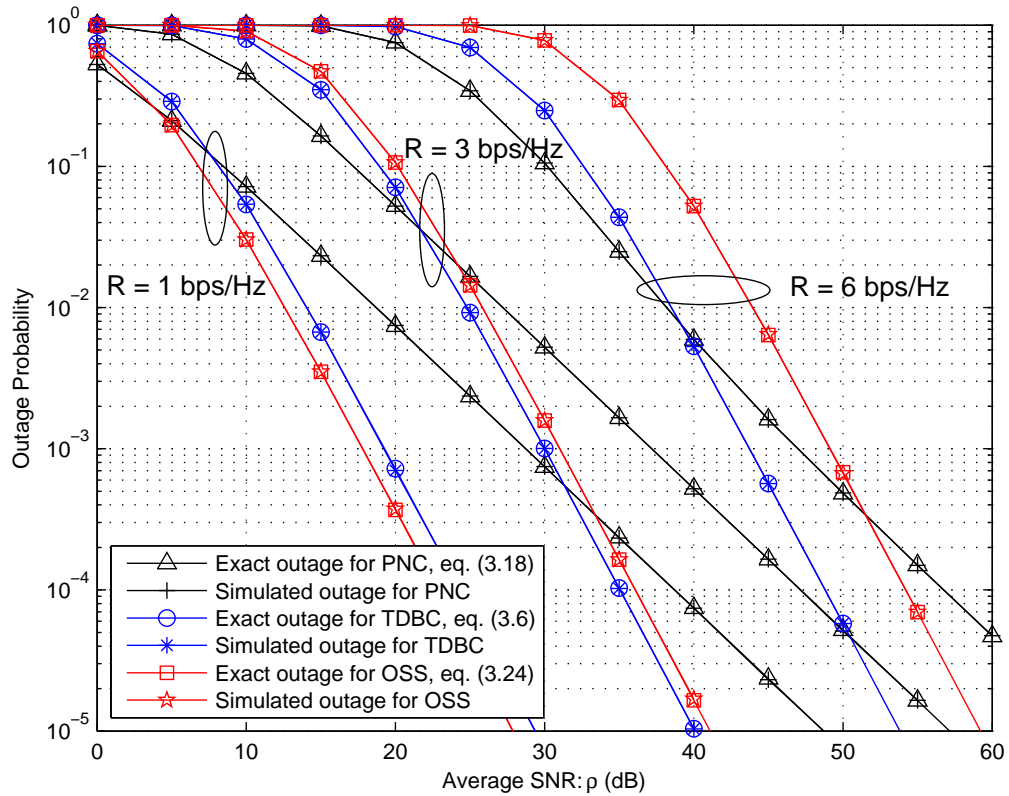


Figure 3.10: Outage probabilities of the TDBC, PNC, and OSS protocols for various rates with  $d = 0.3$ ,  $E_s = 2E/3$ , and  $E_r = E/3$ .

low data rate, the effect of the diversity gain dominates the effect of the multiplexing gain, and hence, TDBC and OSS outperform the PNC protocol. For high data rate and finite SNR, the effect of the multiplexing gain dominates the effect of the diversity gain, and therefore, PNC outperforms other protocols. However, for the asymptotic high-SNR regime, the effect of the diversity gain dominates the effect of the multiplexing gain, irrespective of the data rate. Due to this reason, the TDBC and OSS protocols outperform the PNC protocol in the asymptotic high-SNR regime.



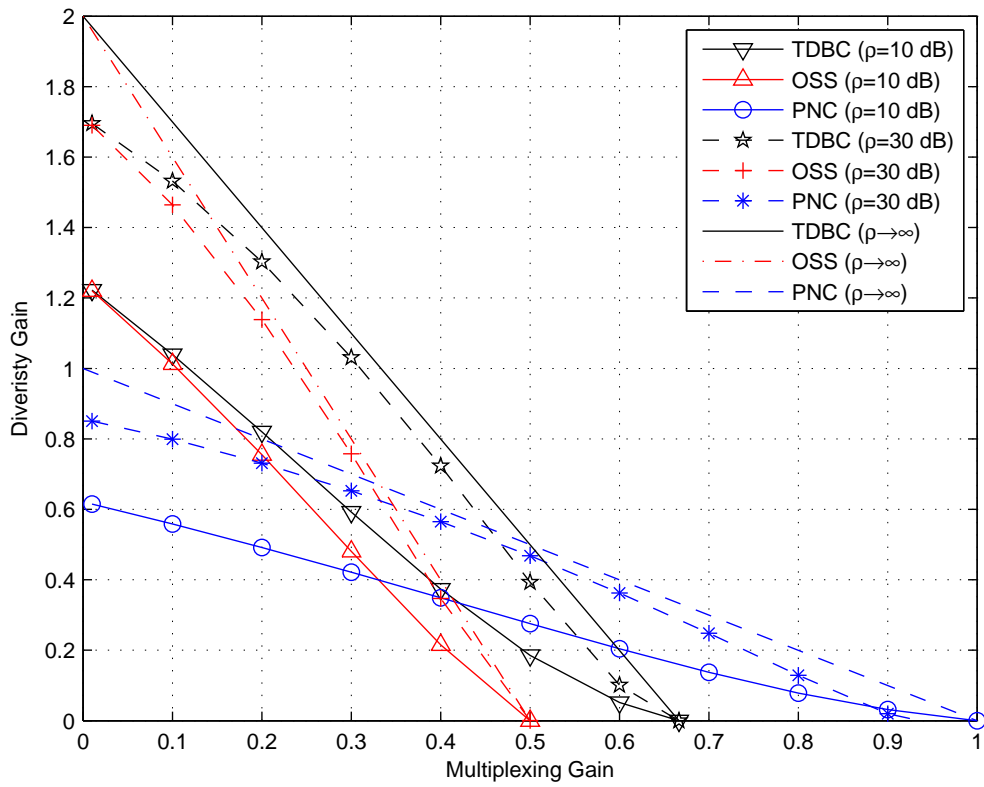


Figure 3.11: Finite-SNR and infinite-SNR DMT curves of the TDBC, PNC, and OSS protocols for  $d = 0.1$ ,  $E_s = 2E/3$ , and  $E_r = E/3$ .

*Remark:* For two protocols A and B, even if A always outperforms B in the sense of DMT, it does not necessarily guarantee that A always has better performance than B in terms of outage probability. For instance, even if we suppose A always has a higher multiplexing gain than B with the same diversity gain, A can be inferior to B in outage probability for low data rates. In this chapter, we demonstrated that OSS has a lower multiplexing gain than TDBC while they have the same diversity gain; however, for low data rates, OSS can achieve better outage probability performance than TDBC, as demonstrated in Figs. 3.4–3.8, and 3.10, although TDBC is eventually superior to OSS in outage probability for high data rates.<sup>9</sup>

### 3.6 Conclusion

In this chapter, we have analyzed and compared the performance of the DF-based bidirectional protocols including the TDBC, PNC, and OSS protocols for a bidirectional cooperative network with a single relay. We derived an exact outage probability in a one-integral form for the TDBC protocol, and exact closed-form outage probabilities for the PNC and OSS protocols. We derived asymptotic outage probability expression for each protocol and demonstrated analytically that the TDBC and OSS protocols achieve full diversity order two and the PNC protocol achieves diversity order one. To facilitate the outage analysis of the TDBC protocol, we also derived extremely tight closed-form upper and lower bounds on the outage probability of the TDBC protocol. Moreover, we studied the finite-SNR and infinite-SNR DMT

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<sup>9</sup>A similar observation was made in [42], where the non-orthogonal AF (NAF) protocol and the distributed Alamouti coding protocol were compared. It was demonstrated that even though the NAF protocol always has a higher multiplexing gain than the distributed Alamouti coding with the same diversity gain, the NAF protocol has worse outage probability performance at low data rates, although the NAF is eventually better than the distributed Alamouti coding in high data rates.

performance and demonstrated that the PNC protocol is able to achieve the highest multiplexing gain, whereas the TDBC and OSS protocols are able to achieve the highest diversity gain. Therefore, for applications with high data rate, the PNC protocol is the best choice. On the other hand, the OSS and TDBC protocols can be better choices than the PNC protocol when data rate is low. Moreover, we demonstrated that the OSS protocol is the best choice for applications with very low data rate (e.g.,  $R = 1$  bps/Hz) because it can exploit the multiuser diversity.

### 3.7 Appendix 3-A: Proof of Theorem 3.1

Let  $X_i = |h_i|^2$ ,  $i = 0, 1, 2$ . Then  $X_i$ 's are independent exponential random variables with parameters  $\lambda_i$ ,  $i = 0, 1, 2$ . Using Total Probability Theorem, the outage probability  $P_{\text{out}}^{\text{TDBC}}$  of (3.5) can be written as

$$\begin{aligned}
 P_{\text{out}}^{\text{TDBC}} &= 1 - \Pr \left( X_0 > \frac{2T_1}{\beta}, X_1 < \frac{2T_1}{\beta}, X_2 < \frac{2T_1}{\beta} \right) \\
 &\quad - \Pr \left( X_0 > \frac{2T_1}{\beta}, X_1 < \frac{2T_1}{\beta}, X_2 > \frac{2T_1}{\beta} \right) \\
 &\quad - \Pr \left( X_0 > \frac{2T_1}{\beta}, X_1 > \frac{2T_1}{\beta}, X_2 < \frac{2T_1}{\beta} \right) \\
 &\quad - \Pr \left( \frac{\beta}{2}X_0 + (1 - \beta)X_2 + \frac{\beta(1 - \beta)\rho}{2}X_0X_2 > T_1, X_1 > X_2 > \frac{2T_1}{\beta} \right) \\
 &\quad - \Pr \left( \frac{\beta}{2}X_0 + (1 - \beta)X_1 + \frac{\beta(1 - \beta)\rho}{2}X_0X_1 > T_1, X_2 > X_1 > \frac{2T_1}{\beta} \right) \\
 &=: 1 - A_1 - A_2 - A_3 - A_4 - A_5,
 \end{aligned} \tag{3.A-1}$$

where  $A_1$ ,  $A_2$ , and  $A_3$  can be easily derived by using the fact that  $X_i$ 's are independent exponential random variables,

$$A_1 = \exp\left(-\frac{2\lambda_0 T_1}{\beta}\right) \left[1 - \exp\left(-\frac{2\lambda_1 T_1}{\beta}\right)\right] \left[1 - \exp\left(-\frac{2\lambda_2 T_1}{\beta}\right)\right], \quad (3.A-2)$$

$$A_2 = \exp\left(-\frac{2\lambda_0 T_1}{\beta}\right) \left[1 - \exp\left(-\frac{2\lambda_1 T_1}{\beta}\right)\right] \exp\left(-\frac{2\lambda_2 T_1}{\beta}\right), \quad (3.A-3)$$

$$A_3 = \exp\left(-\frac{2\lambda_0 T_1}{\beta}\right) \exp\left(-\frac{2\lambda_1 T_1}{\beta}\right) \left[1 - \exp\left(-\frac{2\lambda_2 T_1}{\beta}\right)\right]. \quad (3.A-4)$$

Also,  $A_4$  and  $A_5$  are identical in form except that  $X_1$  and  $X_2$  are interchanged. Therefore,  $A_5$  can be obtained by swapping  $\lambda_1$  and  $\lambda_2$  in the expression of  $A_4$ . In the following, we first derive  $A_4$ . If  $0 < \beta \leq 2/3$ , with some manipulations,  $A_4$  can be simplified as

$$A_4 = \Pr\left(X_1 > X_2 > \frac{2T_1}{\beta}\right) = \frac{\lambda_2}{\lambda_1 + \lambda_2} \exp\left[-\frac{2(\lambda_1 + \lambda_2)T_1}{\beta}\right]. \quad (3.A-5)$$

If  $2/3 < \beta < 1$ ,  $A_4$  can be obtained as follows:

$$\begin{aligned} A_4 &= \int_{x_2=\frac{2T_1}{\beta}}^{\infty} \Pr\left(X_1 > x_2, X_0 > \frac{T_1 - (1-\beta)x_2}{\frac{\beta}{2} + \frac{\beta(1-\beta)\rho}{2}x_2}\right) f_{X_2}(x_2) dx_2 \\ &= \frac{\lambda_2}{\lambda_1 + \lambda_2} \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1-\beta}\right] + \int_{x_2=\frac{2T_1}{\beta}}^{\frac{T_1}{1-\beta}} \lambda_2 e^{-(\lambda_1 + \lambda_2)x_2} e^{-\frac{2\lambda_0}{\beta} \frac{T_1 - (1-\beta)x_2}{1 + (1-\beta)\rho x_2}} dx_2. \end{aligned} \quad (3.A-6)$$

Therefore,

$$A_4 + A_5 = \begin{cases} \exp\left[-\frac{2(\lambda_1 + \lambda_2)T_1}{\beta}\right], & 0 < \beta \leq 2/3, \\ \exp\left[-\frac{(\lambda_1 + \lambda_2)T_1}{1-\beta}\right] + \Psi, & 2/3 < \beta < 1, \end{cases} \quad (3.A-7)$$

where

$$\Psi := (\lambda_1 + \lambda_2) \int_{x_2=\frac{2T_1}{\beta}}^{\frac{T_1}{1-\beta}} e^{-(\lambda_1 + \lambda_2)x_2} e^{-\frac{2\lambda_0}{\beta} \frac{T_1 - (1-\beta)x_2}{1 + (1-\beta)\rho x_2}} dx_2. \quad (3.A-8)$$

By some manipulations such as change of variable and some normalization techniques,  $\Psi$  can be written in a more compact form as follows:

$$\Psi = c \int_{\theta}^1 \exp \left[ -\left(ax + \frac{b}{x}\right) \right] dx =: \Psi(a, b, c, \theta), \quad (3.A-9)$$

where  $a$ ,  $b$ ,  $c$ , and  $\theta$  are given in Theorem 3.1. Finally, substituting (3.A-2)–(3.A-4) and (3.A-7) into (3.A-1) yields (3.6).

### 3.8 Appendix 3-B: Proof of Theorem 3.2

We rewrite the outage probability  $P_{\text{out}}^{\text{PNC}}$  of (3.17) for the PNC protocol as follows:

$$P_{\text{out}}^{\text{PNC}} = \Pr \left( \min(I_1^{\text{PNC}}, I_2^{\text{PNC}}) < \frac{R}{2} \text{ or } I_{\text{sum}}^{\text{PNC}} < R \right). \quad (3.B-1)$$

As in Appendix 3-A, let  $X_i = |h_i|^2$ ,  $i = 1, 2$ . Then  $X_1$  and  $X_2$  are independent exponential random variables with parameters  $\lambda_1$  and  $\lambda_2$ , respectively. From (3.14) and (3.15), it can be shown that

$$\begin{aligned} \min(I_1^{\text{PNC}}, I_2^{\text{PNC}}) &= \frac{1}{2} \log \{1 + \min[\beta\rho/2, (1-\beta)\rho] \min[X_1, X_2]\} \\ &= \begin{cases} \frac{1}{2} \log \{1 + \frac{\beta\rho}{2} \min[X_1, X_2]\}, & 0 < \beta \leq 2/3, \\ \frac{1}{2} \log \{1 + (1-\beta)\rho \min[X_1, X_2]\}, & 2/3 < \beta < 1. \end{cases} \end{aligned} \quad (3.B-2)$$

In the following, therefore, we divide the discussion into two cases:  $0 < \beta \leq 2/3$  and  $2/3 < \beta < 1$ .

*A. Case 1:  $0 < \beta \leq 2/3$*

From (3.B-1) and (3.B-2), the outage probability  $P_{\text{out}}^{\text{PNC}}$  for the case  $0 < \beta \leq 2/3$

can be written as

$$\begin{aligned}
 P_{\text{out}}^{\text{PNC}} &= \Pr \left( \min[X_1, X_2] < \frac{2T_2}{\beta} \text{ or } X_1 + X_2 < \frac{2T_3}{\beta} \right) \\
 &= 1 - \Pr \left( X_1 > \frac{2T_2}{\beta}, X_2 > \frac{2T_2}{\beta}, X_1 + X_2 > \frac{2T_3}{\beta} \right) \\
 &= 1 - \Pr \left( X_1 > \frac{2T_2}{\beta}, X_2 > \frac{2T_2}{\beta} \right) + \Pr \left( X_1 > \frac{2T_2}{\beta}, X_2 > \frac{2T_2}{\beta}, X_1 + X_2 < \frac{2T_3}{\beta} \right) \\
 &= 1 - \exp \left[ -\frac{2(\lambda_1 + \lambda_2)T_2}{\beta} \right] + \Pr \left( X_1 > \frac{2T_2}{\beta}, X_2 > \frac{2T_2}{\beta}, X_1 + X_2 < \frac{2T_3}{\beta} \right).
 \end{aligned} \tag{3.B-3}$$

Using the fact  $T_3 > 2T_2$ , the last term of (3.B-3) can be derived as follows:

$$\begin{aligned}
 &\Pr \left( X_1 > \frac{2T_2}{\beta}, X_2 > \frac{2T_2}{\beta}, X_1 + X_2 < \frac{2T_3}{\beta} \right) \\
 &= \int_{x_2=\frac{2T_2}{\beta}}^{\frac{2(T_3-T_2)}{\beta}} \int_{x_1=\frac{2T_2}{\beta}}^{\frac{2T_3}{\beta}-x_2} \lambda_1 e^{-\lambda_1 x_1} \lambda_2 e^{-\lambda_2 x_2} dx_1 dx_2, \\
 &= \begin{cases} \exp \left[ -\frac{2(\lambda_1 + \lambda_2)T_2}{\beta} \right] + \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp \left[ \frac{2(\lambda_1 - \lambda_1)T_2 - 2\lambda_2 T_3}{\beta} \right] \\ \quad + \frac{\lambda_2}{\lambda_1 - \lambda_2} \exp \left[ \frac{2(\lambda_1 - \lambda_2)T_2 - 2\lambda_1 T_3}{\beta} \right], & \lambda_1 \neq \lambda_2, \\ \exp \left[ -\frac{2(\lambda_1 + \lambda_2)T_2}{\beta} \right] - \exp \left( -\frac{2\lambda_2 T_3}{\beta} \right) - \frac{2\lambda_2(T_3 - 2T_2)}{\beta} \exp \left( -\frac{2\lambda_1 T_3}{\beta} \right), & \lambda_1 = \lambda_2. \end{cases}
 \end{aligned} \tag{3.B-4}$$

Therefore, the the outage probability  $P_{\text{out}}^{\text{PNC}}$  for  $0 < \beta \leq 2/3$  is given as follows:

$$P_{\text{out}}^{\text{PNC}} = \begin{cases} 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp \left[ \frac{2(\lambda_2 - \lambda_1)T_2 - 2\lambda_2 T_3}{\beta} \right] + \frac{\lambda_2}{\lambda_1 - \lambda_2} \exp \left[ \frac{2(\lambda_1 - \lambda_2)T_2 - 2\lambda_1 T_3}{\beta} \right], & \lambda_1 \neq \lambda_2, \\ 1 - \exp \left( -\frac{2\lambda_2 T_3}{\beta} \right) - \frac{2\lambda_2(T_3 - 2T_2)}{\beta} \exp \left( -\frac{2\lambda_1 T_3}{\beta} \right), & \lambda_1 = \lambda_2. \end{cases} \tag{3.B-5}$$

*B. Case 2:  $2/3 < \beta < 1$*

From (3.B-1) and (3.B-2), the outage probability  $P_{\text{out}}^{\text{PNC}}$  for the case  $2/3 < \beta < 1$  can be written as

$$P_{\text{out}}^{\text{PNC}} = 1 - \Pr \left( X_1 > \frac{T_2}{1-\beta}, X_2 > \frac{T_2}{1-\beta}, X_1 + X_2 > \frac{2T_3}{\beta} \right). \quad (3.B-6)$$

If  $\frac{2T_2}{1-\beta} > \frac{2T_3}{\beta}$ , i.e.,  $\frac{T_3}{T_3+T_2} < \beta < 1$ , (3.B-6) is simplified as

$$P_{\text{out}}^{\text{PNC}} = 1 - \Pr \left( X_1 > \frac{T_2}{1-\beta}, X_2 > \frac{T_2}{1-\beta} \right) = 1 - \exp \left[ -\frac{(\lambda_1 + \lambda_2)T_2}{1-\beta} \right]. \quad (3.B-7)$$

If  $\frac{2T_2}{1-\beta} < \frac{2T_3}{\beta}$ , i.e.,  $2/3 < \beta < \frac{T_3}{T_3+T_2}$ , the outage probability  $P_{\text{out}}^{\text{PNC}}$  can be derived in the same manner as in the case  $0 < \beta \leq 2/3$ , and the derived expression for this case is given as follows:

$$P_{\text{out}}^{\text{PNC}} = \begin{cases} 1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} \exp \left[ \frac{(\lambda_2 - \lambda_1)T_2}{1-\beta} - \frac{2\lambda_2 T_3}{\beta} \right] + \frac{\lambda_2}{\lambda_1 - \lambda_2} \exp \left[ \frac{(\lambda_1 - \lambda_2)T_2}{1-\beta} - \frac{2\lambda_1 T_3}{\beta} \right], & \lambda_1 \neq \lambda_2, \\ 1 - \exp \left( -\frac{2\lambda_2 T_3}{\beta} \right) - 2\lambda_2 \left( \frac{T_3}{\beta} - \frac{T_2}{1-\beta} \right) \exp \left( -\frac{2\lambda_1 T_3}{\beta} \right), & \lambda_1 = \lambda_2. \end{cases} \quad (3.B-8)$$

Finally, a combination of (3.B-5), (3.B-7), and (3.B-8) yields the closed-form expression  $P_{\text{out}}^{\text{PNC}}$  of (3.18).

### 3.9 Appendix 3-C: Proof of Lemma 3.1

From the source selection criterion, the mutual information  $I^{\text{OSS}}$  of the whole network in the OSS protocol is given by

$$I^{\text{OSS}} := \max \{ I_1^{\text{OSS}}, I_2^{\text{OSS}} \}. \quad (3.C-1)$$

Based on the rule of (3.C-1), the instantaneous channel conditions are compared at  $S_1$  and  $S_2$  as follows:

1. If  $\frac{T_3}{\beta} < |h_1|^2 < |h_2|^2$ ,  $I_1^{\text{OSS}} = \frac{1}{2} \log [1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_2|^2]$  and  $I_2^{\text{OSS}} = \frac{1}{2} \log [1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_1|^2]$ . Since  $|h_1|^2 < |h_2|^2$ ,  $I_1^{\text{OSS}} > I_2^{\text{OSS}}$ . Therefore,  $S_1$  is selected as the transmitting source. The mutual information  $I^{\text{OSS}}$  in this case is  $I^{\text{OSS}} = I_1^{\text{OSS}}$ .
2. If  $\frac{T_3}{\beta} < |h_2|^2 < |h_1|^2$ ,  $I_1^{\text{OSS}} = \frac{1}{2} \log [1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_2|^2]$  and  $I_2^{\text{OSS}} = \frac{1}{2} \log [1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_1|^2]$ . Since  $|h_1|^2 > |h_2|^2$ ,  $I_1^{\text{OSS}} < I_2^{\text{OSS}}$ . Therefore,  $S_2$  is selected as the transmitting source. The mutual information  $I^{\text{OSS}}$  in this case is  $I^{\text{OSS}} = I_2^{\text{OSS}}$ .
3. If  $|h_2|^2 < \frac{T_3}{\beta} < |h_1|^2$ ,  $I_1^{\text{OSS}} = \frac{1}{2} \log [1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_2|^2]$  and  $I_2^{\text{OSS}} = \frac{1}{2} \log (1 + \beta\rho|h_0|^2)$ . It is obviously that  $I_1^{\text{OSS}} > I_2^{\text{OSS}}$ . Therefore,  $S_1$  is selected as the transmitting source. The mutual information  $I^{\text{OSS}}$  in this case is  $I^{\text{OSS}} = I_1^{\text{OSS}}$ .
4. If  $|h_1|^2 < \frac{T_3}{\beta} < |h_2|^2$ ,  $I_1^{\text{OSS}} = \frac{1}{2} \log (1 + \beta\rho|h_0|^2)$  and  $I_2^{\text{OSS}} = \frac{1}{2} \log [1 + \beta\rho|h_0|^2 + (1 - \beta)\rho|h_1|^2]$ . It is easy to see that  $I_1^{\text{OSS}} < I_2^{\text{OSS}}$ . Therefore,  $S_2$  is selected as the transmitting source. The mutual information  $I^{\text{OSS}}$  in this case is  $I^{\text{OSS}} = I_2^{\text{OSS}}$ .
5. If  $|h_1|^2 < \frac{T_3}{\beta}$  and  $|h_2|^2 < \frac{T_3}{\beta}$ ,  $I_1^{\text{OSS}} = I_2^{\text{OSS}} = \frac{1}{2} \log (1 + \beta\rho|h_0|^2)$ . Therefore,  $S_1$  and  $S_2$  are selected as the transmitting source with equal probability and the mutual information  $I^{\text{OSS}}$  is  $I^{\text{OSS}} = I_1^{\text{OSS}} = I_2^{\text{OSS}}$ .

In all, the mutual information  $I^{\text{OSS}}$  is given by (3.22).



### 3.10 Appendix 3-D: Proof of Theorem 3.3

As in Appendix 3-A, we let  $X_i = |h_i|^2$ ,  $i = 0, 1, 2$ . Then  $X_i$ 's are independent exponential random variables with parameters  $\lambda_i$ ,  $i = 0, 1, 2$ . Using Total Probability Theorem, the outage probability  $P_{\text{out}}^{\text{OSS}}$  of (3.23) can be written as follows:

$$\begin{aligned}
 P_{\text{out}}^{\text{OSS}} &= \Pr \left( \left( X_0 < \frac{T_3}{\beta} \right) \cap \left( \max(X_1, X_2) < \frac{T_3}{\beta} \right) \right) \\
 &\quad + \Pr \left( \left( X_0 + \frac{1-\beta}{\beta} X_1 < \frac{T_3}{\beta} \right) \cap \left( \left( \frac{T_3}{\beta} < X_2 < X_1 \right) \cup \left( X_1 < \frac{T_3}{\beta} < X_2 \right) \right) \right) \\
 &\quad + \Pr \left( \left( X_0 + \frac{1-\beta}{\beta} X_2 < \frac{T_3}{\beta} \right) \cap \left( \left( \frac{T_3}{\beta} < X_1 < X_2 \right) \cup \left( X_2 < \frac{T_3}{\beta} < X_1 \right) \right) \right).
 \end{aligned} \tag{3.D-1}$$

Since  $X_i$ 's are independent exponential random variables, the first probability of (3.D-1) can be derived as follows:

$$\begin{aligned}
 &\Pr \left( \left( X_0 < \frac{T_3}{\beta} \right) \cap \left( \max(X_1, X_2) < \frac{T_3}{\beta} \right) \right) \\
 &= \left[ 1 - \exp \left( -\frac{\lambda_0 T_3}{\beta} \right) \right] \left[ 1 - \exp \left( -\frac{\lambda_1 T_3}{\beta} \right) \right] \left[ 1 - \exp \left( -\frac{\lambda_2 T_3}{\beta} \right) \right].
 \end{aligned} \tag{3.D-2}$$

We note that the second and third probabilities of (3.D-1) are identical in form except that  $X_1$  and  $X_2$  are interchanged. Therefore, their final expressions are identical in form except that  $\lambda_1$  and  $\lambda_2$  are interchanged. We denote the second probability as  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3)$ . Then the third probability is denoted as  $G(\beta, \lambda_0, \lambda_2, \lambda_1, T_3)$ . For convenience, we define sets  $A$ ,  $B$ , and  $C$  as:  $A := \left( X_0 + \frac{1-\beta}{\beta} X_1 < \frac{T_3}{\beta} \right)$ ,  $B := \left( \frac{T_3}{\beta} < X_2 < X_1 \right)$ ,  $C := \left( X_1 < \frac{T_3}{\beta} < X_2 \right)$ . Then we have  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3) = \Pr(A \cap (B \cup C))$ , where  $\cap$  and  $\cup$  denote the intersection and union operators, respectively. It can be further shown that  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3) = \Pr(A \cap B) + \Pr(A \cap C)$  since  $B \cap C = \emptyset$ , where  $\emptyset$  denotes the empty set. In the following, we now derive  $\Pr(A \cap B)$  and  $\Pr(A \cap C)$  separately.

A.  $Pr(A \cap B)$

If  $\frac{1-\beta}{\beta} \geq 1$ , i.e.,  $0 < \beta \leq 1/2$ , it can be shown that  $A \cap B = \emptyset$ , and therefore,  $Pr(A \cap B) = 0$ . In the flowing, we now focus on the case with  $1/2 < \beta < 1$ . We proceed as follows:

$$\begin{aligned} Pr(A \cap B) &= \int_0^\infty Pr(A \cap B | X_1 = x_1) f_{X_1}(x_1) dx_1 \\ &= \int_0^\infty Pr(A | X_1 = x_1) Pr(B | X_1 = x_1) f_{X_1}(x_1) dx_1. \end{aligned} \quad (3.D-3)$$

By some mathematical manipulations, we get

$$\begin{aligned} Pr(A \cap B) &= e^{-\lambda_2 T_3 / \beta} [e^{-\lambda_1 T_3 / \beta} - e^{-\lambda_1 T_3 / (1-\beta)}] - \frac{\lambda_1}{\lambda_1 + \lambda_2} [e^{-(\lambda_1 + \lambda_2) T_3 / \beta} - e^{-(\lambda_1 + \lambda_2) T_3 / (1-\beta)}] \\ &\quad - \lambda_1 e^{-(\lambda_1 + \lambda_2) T_3 / \beta} \int_{T_3 / \beta}^{T_3 / (1-\beta)} \exp \left[ \left( \frac{1-\beta}{\beta} \lambda_0 - \lambda_1 \right) x_1 \right] dx_1 \\ &\quad + \lambda_1 e^{-\lambda_0 T_3 / \beta} \int_{T_3 / \beta}^{T_3 / (1-\beta)} \exp \left[ \left( \frac{1-\beta}{\beta} \lambda_0 - \lambda_1 - \lambda_2 \right) x_1 \right] dx_1, \end{aligned} \quad (3.D-4)$$

where the integrals of (3.D-4) can be easily solved. The exponents of the exponential functions in the above integrals can be possibly equal to zero because  $\beta$  can take any values satisfying  $1/2 < \beta < 1$ . Specifically, if  $\beta = \frac{\lambda_0}{\lambda_0 + \lambda_1}$  and  $\lambda_0 > \lambda_1$ , the exponent in the first integral equals zero; If  $\beta = \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2}$  and  $\lambda_0 > \lambda_1 + \lambda_2$ , the exponent in the second integral equals zero.<sup>10</sup> Therefore, the solution to the integrals of (3.D-4) can be possibly different depending on the values of  $\beta$ . Because the final expression of  $Pr(A \cap B)$  is very long, we do not present the final expression of  $Pr(A \cap B)$ .

<sup>10</sup>Note that  $\lambda_0 > \lambda_1$  ensures that  $\frac{\lambda_0}{\lambda_0 + \lambda_1}$  is between 1/2 and 1, and  $\lambda_0 > \lambda_1 + \lambda_2$  ensures that  $\frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2}$  is between 1/2 and 1.

B.  $Pr(A \cap C)$ 

In order to derive  $Pr(A \cap C)$ , we proceed as follows:

$$Pr(A \cap C) = \int_0^\infty Pr(A|X_1 = x_1) Pr(C|X_1 = x_1) f_{X_1}(x_1) dx_1. \quad (3.D-5)$$

Comparing (3.D-5) and (3.D-3), we see that the derivation of  $Pr(A \cap C)$  is similar to the derivation of  $Pr(A \cap B)$ . Specifically, we also need to discuss two cases separately:  $0 < \beta \leq 1/2$  and  $1/2 < \beta < 1$ . For each case, there also exists a special subcase where the exponent of a certain term in the integral equals zero. Therefore, a piecewise solution to  $Pr(A \cap C)$  can be derived for different  $\beta$  values. Due to a space limitation, we do not present the final expression of  $Pr(A \cap C)$  because it is very long.

Therefore, using  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3) = Pr(A \cap B) + Pr(A \cap C)$ , we can obtain  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3)$ . Also, the last term of (3.D-1) can be derived by swapping  $\lambda_1$  and  $\lambda_2$  in the expression of  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3)$ . We have presented a general expression  $G(x, y_0, y_1, y_2, z)$  of (3.25) instead of presenting specific expressions  $G(\beta, \lambda_0, \lambda_1, \lambda_2, T_3)$  and  $G(\beta, \lambda_0, \lambda_2, \lambda_1, T_3)$ . Finally, with (3.D-1), we derive the closed-form expression of  $P_{\text{out}}^{\text{OSS}}$  for the OSS protocol, which is given by (3.24).

# Chapter 4

## Conclusions and Future Work

### 4.1 Conclusions

Performance analysis is a useful theoretical tool in designing practical communication systems as it can guide system designs by choosing optimum parameters. For this reason, this thesis carried out a comprehensive study on the performance analysis of various DF protocols in the unidirectional and bidirectional cooperative diversity networks.

Firstly, we studied the average BER performance of ML detection for the DF protocol in uncoded unidirectional cooperative diversity networks. Two typical cooperative diversity networks were considered: the single-relay cooperative diversity network with the direct source-destination link and the two-relay cooperative diversity network with the direct source-destination link. First, we derived the PDFs and CDFs of the sufficient statistics for the ML detection at the destination. Then we applied the accurate PL approximation and derived closed-form approximate average

BER expressions with the help of the obtained PDFs and CDFs. Our BER expressions were shown to be valid for the general dissimilar DF networks adopting both coherent and noncoherent binary signalings. We also showed that our BER expressions can be considered as generalizations of the previously reported results in the literature. Throughout our analysis, only one approximation, i.e., the accurate PL approximation was made. Simulation results match excellently with the theoretical analysis, which validates our proposed BER expressions.

Secondly, we analyzed and compared the performance of the DF-based bidirectional protocols including the TDBC, PNC, and OSS protocols for a bidirectional cooperative network with a single relay. We derived an exact outage probability in a one-integral form for the TDBC protocol, and exact closed-form outage probabilities for the PNC and OSS protocols. We also derived asymptotic outage probability expression for each protocol and demonstrated analytically that the TDBC and OSS protocols achieve full diversity order two and the PNC protocol achieves diversity order one. To facilitate performance evaluation of the TDBC protocol, we also derived extremely tight closed-form upper and lower bounds on the outage probability of the TDBC protocol. Moreover, we studied the finite-SNR and infinite-SNR DMT performance and demonstrated that the PNC protocol is able to achieve the highest multiplexing gain, whereas the TDBC and OSS protocols are able to achieve the highest diversity gain.

Finally, our work can be used to guide communication system designs. For example, with the derived BER expressions for the unidirectional cooperative diversity network, we may find optimum power allocation scheme by minimizing the average BER. This power optimization can be done at least through numerical search if it

is not analytically tractable. Also, with the theoretical analysis of the bidirectional protocols, we can design a hybrid bidirectional protocol that adaptively choose the three bidirectional protocols depending on the applications. For applications with high data rate, the PNC protocol was shown to be the best choice. On the other hand, the OSS and TDBC protocols can be better choices than the PNC protocol when data rate is low. Moreover, we demonstrated that the OSS protocol is the best choice for applications with very low data rate (e.g.,  $R = 1$  bps/Hz) because it can exploit the multiuser diversity.

## 4.2 Future Work

There are still enough rooms for performing future work on the performance analysis in cooperative diversity networks. The work presented in this thesis can be developed further in many directions.

The work of Chapter 2, the BER analysis of DF protocol in unidirectional cooperative diversity networks, may be developed further in the following ways. In the analysis presented in Chapter 2, we assumed that the destination has instantaneous knowledge of the source-destination and relay-destination channels, but only the statistical knowledge (i.e., channel variance) of the source-relay channel. What if the destination also has instantaneous channel information about the source-relay channel? Then it might be interesting to analyze the average BER under the new CSI assumption. Also, as mentioned in Section 4.1, it is useful and interesting to determine the optimum power allocation scheme by minimizing the average BER, which can be done at least through numerical search. Moreover, we studied the BER performance only for single-relay and two-relay networks. It would be more useful to

generalize the BER analysis to a cooperative diversity network with arbitrary number of relays.

The work of Chapter 3, performance analysis of bidirectional protocols, may be also developed further. The outage probability and DMT analyses are already completely done by our work. However, to the best of our knowledge, there has been no work on the BER or SER analysis of the DF-based bidirectional protocols. Therefore, this can be considered as a possible future work. We conjecture that the BER analysis of the DF-based bidirectional protocols is a very challenging job. Also, it is possible to consider a bidirectional cooperative diversity network consisting of multiple relays. With multiple relays, how to coordinate transmissions of different relay terminals to maintain good spectral efficiency or diversity gain performance? To answer this question, we conjecture that a single-relay selection combined with network coding may provide good performance. Another possibility to deal with multiple relays may be adopting beam-forming at relays. All of these initial considerations are interesting and therefore can be considered as future work.

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