Two-digit Number Comparison:

Digit Similarity Effect on Reaction Time in Two-digit Number Comparison

by

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ABSTRACT

Magnitudes of numbers influence numerical inequality judgments of people. Do symbols representing numbers also affect numerical inequality judgments? To answer the question, I manipulated digit similarity in two-digit number comparison tasks. During the experiment, the participants took part in two comparison tasks – the judging-larger task and the judging-smaller task. Given pairs of two-digit numbers, the participants were required to make numerical inequality judgments (judging larger or judging smaller). To investigate the effect of digit similarity, two kinds of number pairs were used. Two-digit number pairs consisting of same-digits numbers (e.g., 21 – 12) and two-digit number pairs consisting of different-digits numbers (e.g., 21 – 30) were presented at random. The participants needed more time to compare the same-digits number pairs than the different-digits pairs. The result was independent of the findings in number comparison studies such as the numerical-distance effect (Moyer & Landauer, 1967) and the unit-decade compatibility effect (Nuerk, Weger, & Willmes, 2001). The present study poses challenge to the current theories of two-digit number comparison.

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CHAPTER ONE: INTRODUCTION

How do people represent numbers in their mind? How do they make judgments on quantities that numbers represent? How do they manipulate multi-digit numbers so easily? Although people's competence of dealing with numerical concepts and mathematical operations seems to be one of the major components of their cognitive abilities in general, we do not yet have a clear picture of the mechanisms and processes underlying the numerical-mathematical abilities. Are the abilities innate or acquired by experience? What part of the brain is responsible for numerical processing? How can we teach numerical concepts to children effectively? What is an effective intervention for people with mathematical difficulty? These are some of the questions that numerical/mathematical cognition research needs to answer. Thus, the scope and implication of numerical-mathematical cognition include several areas such as mathematics, biology and education.

The present study focuses on one mathematical ability – i.e., comparing numerical quantities. Judging numerical inequality between two or more quantities is deeply ingrained in our every day life. Indeed, we are required to make numerous decisions about which quantity is greater or lesser. Although such judgments do not seem to demand strenuous cognitive effort, we have little understanding of how we make the necessary comparisons.

CHAPTER TWO: LITERATURE REVIEW

An early, but still influential breakthrough in numerical inequality judgment was made by Moyer and Landauer (1967). They reported that response times (RTs) to judge the relative sizes of two single-digit numbers were affected by the numerical difference between the numbers to be compared – i.e., the smaller the difference between the two numbers, the more time people needed to make a correct inequality judgment – the numerical-distance effect. However, given the same numerical difference, comparison of smaller number pairs required less time than larger pairs – the size effect. Moyer and Landauer reported that RT in comparing single-digit numbers approximated a logarithmic function, the same function that had been observed in discriminating physical quantities such as the length of lines and the pitch of tones. They suggested that the numbers were "converted to analogue magnitudes" and the comparison between the magnitudes was made "in much the same way" as between physical stimuli (Moyer & Landauer, 1967, p. 1520). Their interpretation suggests that magnitudes of numbers are represented in a logarithmically compressed fashion, hence yield the numerical-distance effect and the size effect in numerical inequality judgments.

The distance and size effects led researchers to study numerical inequality judgments with multi-digit numbers. A viable theory of numerical judgment is required to address multi-digit comparisons equally well, but the explanation for multi-digit number comparison is not simple, in part, because of the complexity of multi-digit numbers. From simple calculations to sophisticated mathematical reasoning, our quantitative skill and competence depend heavily on the systemic operations of symbols, namely Arabic numerals (Dantzig, 1954). As a matter of fact, multi-digit numbers written in this system to the power of 10 are rich in information, and we use this system extremely often. Our intensive practice in manipulating the information in Arabic numerals may well affect our cognitive processes in multi-digit number comparison.

So far, studies of multi-digit number comparison have focused mainly on two-digit numbers because it is a first step to multi-digit number comparison, but researchers disagree how people represent them. There are three distinctive views that try to account for two-digit number comparison – namely the holistic, decomposition, and hybrid view (Korvorst & Damian, 2008; Verguts & De Moor, 2005). Even though they were not first to make such classification (and the literature has not agreed on the names), the essence of the classification was not fundamentally different than earlier taxonomies (see Hinrichs, Yurko, & Hu, 1981).

The holistic view of two-digit number comparison is an extension of the claim made by Moyer and Landauer (1967). The proponents of this view contend that the magnitudes of two-digit numbers are represented in analogue form, and the magnitudes are compared to each other as integrated quantities. The main support for this position is that RT patterns in comparison of two-digit numbers are approximately the same as comparison of single-digit numbers – the logarithmic numerical distance effect (Brysbaert, 1995; Dehaene, Dupoux, & Mehler, 1990; Hinrichs et al., 1981).

The decomposition view, by contrast, maintains that people decompose two-digit numbers into their constituent digits and have separate magnitude representations for the constituent digits. The proponents of this view argue that the decade digits plays a far greater role than unit digits in two-digit number comparison. This means that the overall magnitudes of the numbers are not required in comparison process (Poltrock & Schwartz, 1984; Verguts & De Moor 2005; Verguts & Fias, 2004; Verguts, Fias, & Stevens, 2005). For instance, Verguts and De Moor (2005) reported that an overall distance effect was observed in same-decade number pairs (e.g., 64 – 68), but not in different-decade number pairs (e.g., 64 – 72). They argued that the apparent overall distance effect in previous two-digit comparison studies resulted from the confounding of the numerical difference between decade digits – i.e., difference between decade digits is related to the overall numerical difference.

Finally, the hybrid view, as its name suggests, contends that both holistic processing and decomposition processing occur at the same time in two-digit number comparison. (e.g., Nuerk & Willmes, 2005). The hybrid view started with a study which reported, in addition to an overall distance effect, the independent influences of decade digits and unit digits in two-digit number comparison, namely the unit-decade compatibility effect (Nuerk, Weger, & Willmes, 2001). Nuerk et. al. (2001) called a number pair *compatible* if the decade magnitude comparison concurred the unit magnitude comparison (e.g., 42 – 57, since $4 < 5$ and $2 < 7$). If the two magnitude comparisons did not agree (e.g., 47 – 62, since $4 < 6$ but $7 > 2$), the number pair was called *incompatible*. They found that people took longer in judging inequality with unit-decade incompatible pairs than with unit-decade compatible pairs. Nuerk and Willmes (2005) argue that three distinctive comparisons take place in two-digit number comparison – comparison between the overall magnitudes, comparison between the decade magnitudes, and comparison between the unit magnitudes. The three comparisons occur in parallel, but have different impacts on response activation and, therefore, on the overall response latency – the overall magnitude comparison has a larger impact than the unit magnitude comparison.

CHAPTER THREE: EXPERIMENT

Studies of two-digit number comparison have emphasized the quantity information (magnitude) of the two-digit numbers. As a result, differences in RT have been interpreted solely in terms of the magnitude difference. We may question the wisdom of this emphasis because a multi-digit number conveys information other than a quantity representation. For example, 25 is an arithmetic representation that is more complex than a single-quantity representation. It can be considered as a linguistic representation of the quantity twenty-five, too. Needless to say, it is an object with physical attributes when it is visually presented.

The present study hypothesized that physical attributes of multi-digit numbers, not only magnitude of the numbers, play a role in judging numerical inequality. In this vein, the present study addressed one of the physical attributes of two-digit numbers, namely digit similarity.

One of the most salient physical attributes of multi-digit numbers is their constituent digits. By manipulating their constituent digits, the physical attribute of multi-digit numbers (digit similarity) can be controlled. Specifically, I investigated the effect of digit similarity on RT in two-digit number comparison. By digit similarity, I mean that some two-digit numbers are symbolically more similar than other two-digit numbers.

For example, 31 is symbolically more similar to 13 than to 49 because it shares the same symbols (digits) with 13, but not with 49. If symbolic similarity is taken into account during number comparison, people should need more time to make correct inequality judgments with pairs of symbolically similar numbers than with pairs of symbolically different numbers. This prediction is in accordance with the findings in research of discriminating physical quantities, that is, people need more time and make more errors when they compare physically similar stimuli. To exclude the distance effect as a possible explanation of the result, the numerical differences in similar number pairs and in different number pairs are held constant.

Participants

Thirty-two students from the undergraduate subject pool of the Department of Psychology at Queen's University participated in the present study. All of the participants reported normal or corrected-to-normal vision. They took part in the experiment for credit in the introductory psychology course.

Apparatus

The experiment was administered by a personal computer equipped with a 19-inch cathode ray tube monitor. A response box, which had two horizontally arranged response buttons, was connected to the parallel port of the computer. The response and response times (RTs) were recorded by DirectRT (Empirisoft Corporation, New York).

Materials

Of all two-digit numbers, 24 are appropriate for the purpose of this study. Hereafter, the 24 numbers will be referred to as 'standard numbers.' Each standard number was composed of two different digits. The important property of the standard number is its numerical differences from its corresponding same-digits number and different-digits number. How I used the numerical difference will be explained in the next paragraph. For example, *21* is a standard number.

To manipulate digit similarity, a same-digits number and a different-digits number were defined for each standard number. The same-digits number was composed of the same digits as the standard number but in reversed order. For example, the standard number 21 had the same-digits number 12; notice that the numerical difference between the two is nine. The different-digits number was composed of two digits different from the digits of the standard number. The critical attribute of the different-digits number was that it had the same numerical distance from the standard number as the same-digits number derived from the standard number. In case of the standard number 21, the different-digits number is 30. The numerical difference between the two is nine, which is the same as that between 21 and 12. Table 1 shows the numbers used to make the number pairs.

To make comparison stimuli, each standard number was paired with its same-digits number, and its different-digits number – e.g., 21 – 12 (same-digits pair) and 21 – 30 (different-digits pair), resulting in 48 number pairs. Another 48 pairs were constructed by reversing the order of the numbers in the pair – e.g., 12 – 21 and 30 – 21. The entire set of the 96 number pairs was presented twice, resulting in 192 trials in each comparison task. The order in which the number pairs were presented was randomized within the 96 number pair set.

Table 1. Digit similarity manipulation.

Different-digits Numbers	Standard Numbers	Same-digits Numbers
49	31	13
68	41	14
20	47	74
12	48	84
87	51	15
79	52	25
31	58	85
23	59	95
98	62	26
50	68	86
42	69	96
61	79	97
30	21	12
14	23	32
17	35	53
60	42	24
52	43	34
28	46	64
63	54	45
90	63	36
74	65	56
58	67	76
93	75	57
69	78	87

The number pairs used in this experiment have a noticeable attribute with respect to the decomposition view, which seems to be gaining more support recently, than other number comparison views. If we calculate the numerical differences between the two numbers in pairs only with the decade digits, the same-digits pairs have greater differences than or the same differences as the different-digits pairs (see Table 1). For example, the first stimuli set 49 – 31 (different-digits pair) vs. 31 – 13 (same-digits pair) can be regarded as 4X – 3X vs. 3X – 1X. If the decade digits are the sole or dominant factor in comparison, as the decomposition view suggests, people will respond faster to same-digits pairs (e.g., 3X – 1X) than to different-digits pairs (e.g., 4X – 3X).

Design

I manipulated two variables, namely digit similarity and comparison task. As I noted earlier (in the Materials section), digit similarity referred to whether the numbers to be compared consisted of the same digits or consisted of different digits. On the same-digits trials, the participants were shown number pairs including a standard number and its same-digits number. By contrast, on the different-digits trials, the participants were shown pairs consisting of a standard number and its different-digits number.

The variable, Comparison Task, refers to whether the task was to judge which of two numbers was larger or smaller. In the judging-larger task, the participants were required to decide which of the two numbers was larger. In the judging-smaller task, by contrast, the participants were asked to decide which of the two numbers was smaller. The participant completed both tasks, and the order of task was randomized for each participant.

Reaction times of the participants were measured, and the effects of the two variables on response times were analyzed, entailing a 2 (digit similarity) × 2 (comparison task) factorial, within-subject design.

Procedure

The participants were seated in front of a computer monitor. A response box was placed on the desk between the monitor and the participants, and there were two horizontally arranged buttons on the box. The participants were instructed to press the left button with their left-hand index finger if the target (larger or smaller) number appeared on the left side of the screen or to press the right button with their right-hand index finger if the target number appeared on the right side of the screen. The design of the experiment ensured that the correct responses were distributed equally on the two buttons. The standard numbers were the correct responses in a half of the trials of the experiment. The participants were asked to respond as accurately and as quickly as possible; they were told that both their accuracy and RT would be recorded.

After they had received the task instruction, the participants had a brief practice session for their first task. Practice involved 20 pairs of two-digit numbers. At the beginning, the phrase "Press the button to get started..." appeared in blue on the black screen. On each trial, a blue cross was shown at the centre of the screen for one second, and, then, a pair of horizontally arranged two-digit numbers replaced the cross. The numbers were written in Times New Roman font and subtend the visual angle of about 1° in a comfortable viewing posture on the chair.

The number pair was presented on the screen until the participants made a response. The practice was identical to the experimental task except that it consisted of fewer trials, and the number pairs were made of two-digit numbers that were not used in the experimental trials. After the practice, all the participants expressed confidence in their understanding of the task. Then, the participants proceeded to the first experimental task. After completing the first task, the participants had a two-minute break and, then, proceeded to the practice for the second task. The details of the second practice and experimental task were the same as the first task except that the participants responded to the opposite target number.

CHAPTER FOUR: RESULTS

Analysis of Results

The overall error rate was 4.77%, a value that indicates that participants performed the tasks with high accuracy. Accuracy in comparing the same-digits number pairs was high at 95.54% (*SD* = 3.58). Accuracy with the different-digits pairs was also high at 94.92% (*SD* = 3.75). An analysis of variance (ANOVA) confirmed that null difference, $F\,(1, 31) = 3.23$, $p > .05$. Accuracy in both the judging-larger task and the judging-smaller task was high at 95.23% (*SD* = 3.44) and 95.23% (*SD* = 3.91), respectively. ANOVA confirmed that the accuracy in the two tasks was not statistically different, $F\,(1, 31) < 1$. The preliminary analysis shows that neither digit similarity nor comparison task altered the participants' accuracy. Because of its high level, accuracy was not analyzed further.

In reaction time analysis, error trials were excluded. The median RT of each number pair was used to calculate the mean RT of the participant in each comparison condition, hence *M* in the analysis indicates the mean of participants' median RTs. The participants responded faster in the different-digits comparison (*M* = 595.66 ms, *SD* = 66.01) than in the same-digits comparison (*M* = 633.60 ms, *SD* = 70.19), $F\,(1, 31) = 113.85$, $MS_e = 404.52$, $p < .001$. The participants also needed less time in the judging-larger task (*M* = 595.84 ms, *SD* = 67.67) than in the judging-smaller task (*M* = 633.43 ms, *SD* = 68.69), $F\,(1, 31) = 29.88$, $MS_e = 1513.57$, $p < .001$. As Figure 1 shows, the advantage in the different-digits trials was independent of the type of comparison task, – i.e., the two variables did not interact, $F(1, 31) = 1.63$, $p > .05$ (See Appendix A for the details of analysis).

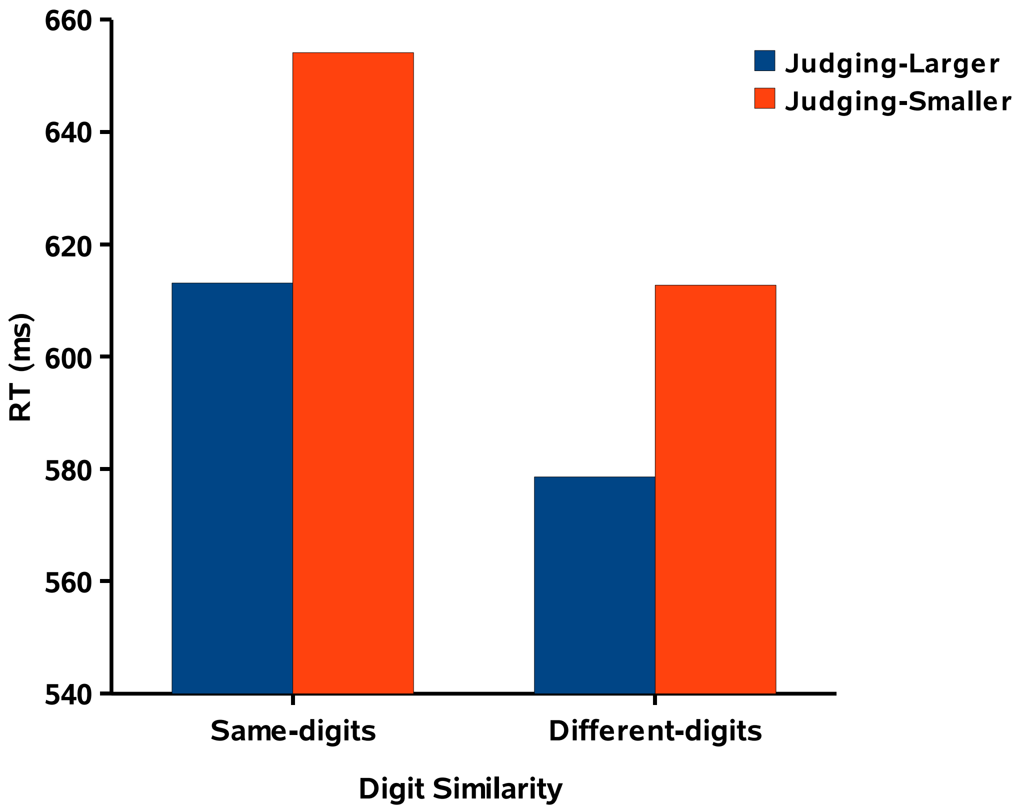

Figure 1. Mean RTs in digit similarity and comparison task manipulation

The experiment demonstrated that people had more difficulty making correct inequality judgments when they were dealing with symbolically similar two-digit numbers than with symbolically dissimilar two-digit numbers. I will call this finding "the digit-similarity effect" hereafter. Because digit similarity has not been a topic of great interest in the two-digit number comparison paradigm, it may be beneficial to investigate the data of the experiment in the context of other findings of previous number-comparison studies.

The data were analyzed in order to see whether the experiment replicated prominent findings of other two-digit number-comparison studies. For the re-analysis, the stimulus set was divided into four groups based on the numerical differences (i.e., 9, 18, 27, and 36) between the two numbers in a pair. The means of median RTs were calculated for the four groups of numerical distance, two groups of digit similarity, and two groups of judging task. Mean RT decreased as the numerical difference increased (see Figure 2). As numerical difference was added to analysis – i.e., 2 (digit similarity) × 2 (comparison task) × 4 (numerical difference), the results of the previous analysis were confirmed – the disadvantage of the same-digits pairs (M = 629.53 ms, SD = 84.07) over the different-digits pairs (M = 595.03 ms, SD = 75.12), F (1, 31) = 37.75, MS_e = 4034.95, p < .001, and the advantage of the judging-larger task (M = 596.96 ms, SD = 76.70) over the judging-smaller task (M = 627.59 ms, SD = 83.39), F (1, 31) = 27.25, MS_e = 4406.25, p < . 001.

With respect to numerical distance, the group difference in RT was found significant, F (3, 93) = 61.18, MS_e = 1666.36, p < .001. The participants responded faster to the number pairs with a larger numerical difference than to the number pairs with a smaller numerical difference – an overall distance effect. This inverse linear relation between RT and numerical difference was confirmed by a trend analysis, F (1, 31) = 157.66, MS_e = 1915.61, p < .001 (see Figure 2). The graph shows that the interaction between digit similarity and numerical distance, $F(1, 31) = 23.10$, $MS_e = 1843.77$, $p < .001$, indicating that RTs in the different-digits number pairs decreased faster than RTs in the same-digits number pairs. (See Appendix B for the details of analysis).

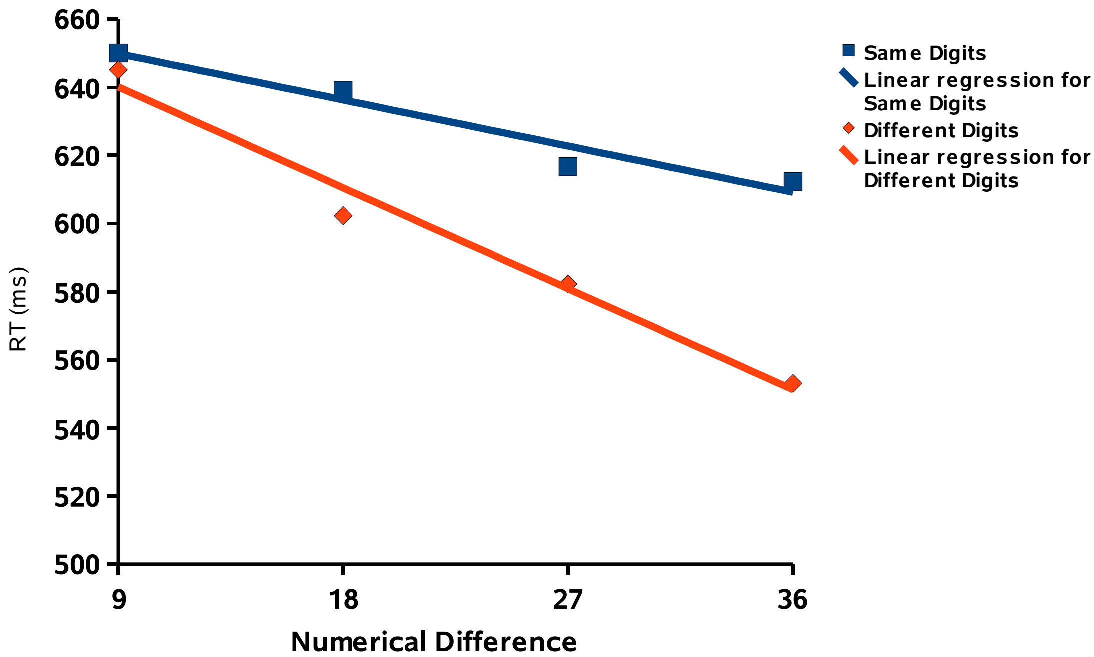

Figure 2. Mean RTs in four numerical-difference groups

Extending analysis of the Results

Nuerk and his colleagues argue that the longer RT in the same-digits number comparison can be explained by the unit-decade compatibility effect. Indeed, it is possible that the digit-similarity effect resulted from the unit-decade compatibility. In the present study, the same-digits pairs were always unit-decade incompatible because one number was in reversed order of the other number in the pair – e.g., 31

– 13 and 78 – 87. In contrast, a half of the different-digits trials consisted of unit-decade compatible pairs – e.g., 31 – 49, whereas the other half of the different-digits trials consisted of unit-decade incompatible pairs – e.g., 78 – 69 (see Table 1, p. 9). The disparity in composition may have affected the participants' performance; the extra incompatible pairs may have contributed to the longer RT in the same-digits number comparison. Subsequent analysis revealed that this was not the case.

More specifically, to check whether the disadvantage of RT in the same-digits comparison was independent of the unit-decade compatibility, I conducted two separate analyses based on the unit-decade compatibility of the different-digits pairs (Recall that all the same-digits pairs were unit-decade incompatible). One was for the number pairs that were unit-decade incompatible in the different-digits trials – the unconfounded trials (the pairs in the bottom half of Table 1). The other was for the number pairs that were unit-decade compatible in the different-digits trials – the confounded trials (the pairs in the top half of Table 1). Table 2 shows that two analyses produced remarkably parallel results, regardless of confounding in unit-decade computability (See Appendix C and D for the details of analyses).

As Figure 3 shows, I found the same pattern as before: digit similarity hampered the participants' performance in RT, while the judging-larger task was easier than the judging-smaller task in both analyses. The null interaction results suggest that the digit-similarity effect is independent of unit-decade compatibility.

Table 2. Results of Trials based on Unit-decade Compatibility

Confounded Trials

	Digit Similarity		Judging Task	
	Same-Digits	Different-Digits	Judging-Larger	Judging-Smaller
M	618.21	575.23	580.05	613.38
SD	69.48	63.45	69.45	66.36
F	71.68 (< .001)		26.35 (< .001)	

Unconfounded Trials

	Digit Similarity		Judging Task	
	Same-Digits	Different-Digits	Judging-Larger	Judging-Smaller
M	651.41	620.53	617.42	654.52
SD	72.19	71.66	71.45	70.91
F	60.86 (< .001)		22.65 (< .001)	

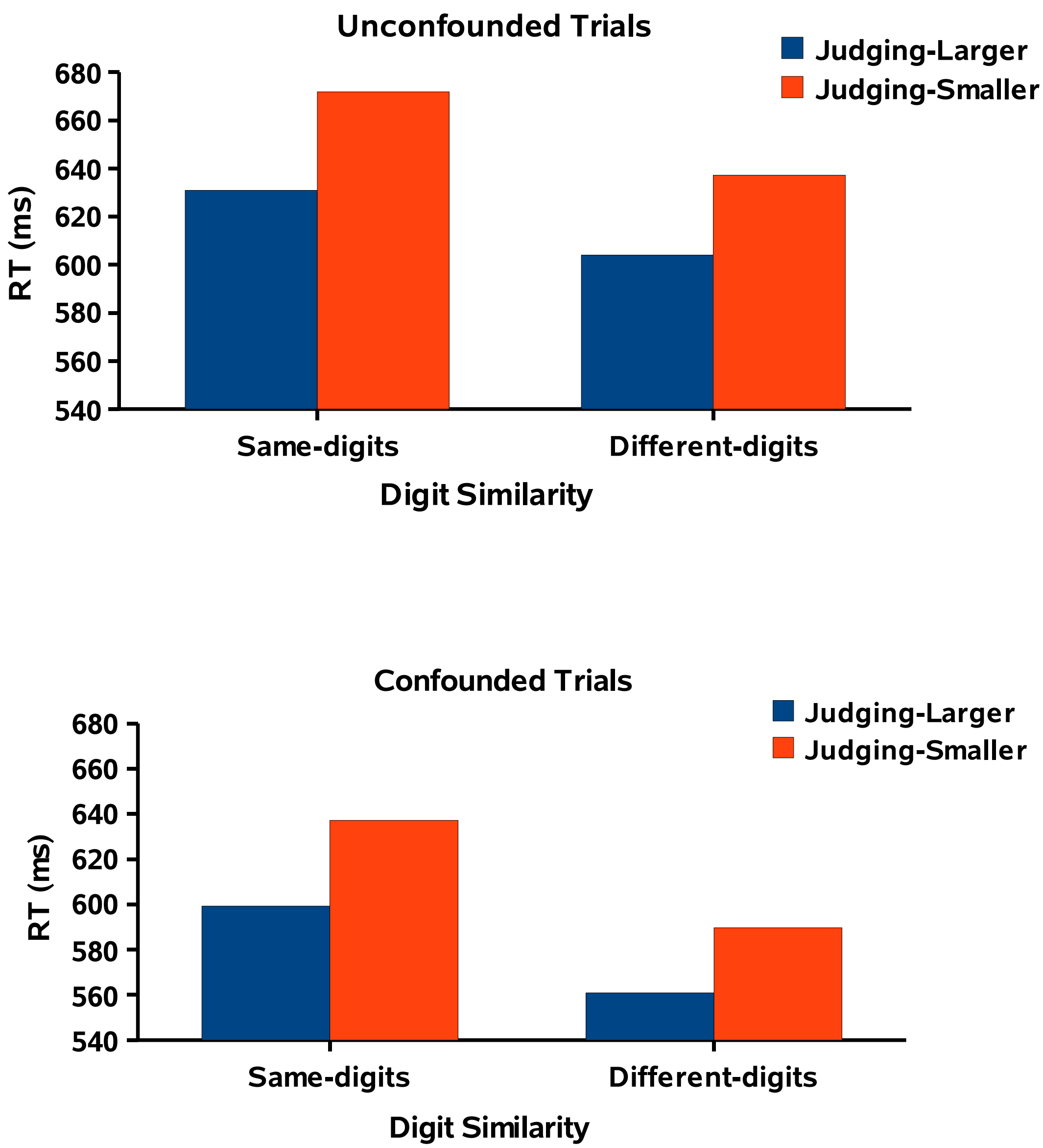

Figure 3. Mean RTs in unconfounded trials and confounded trials with unit-decade compatibility

CHATER FIVE: CONCLUSIONS

The digit-similarity effect poses a challenge to the current theories of two-digit number comparison. Because the holistic view holds that the magnitude of a two-digit number is represented in an analogue form without references to its decade and unit digits, the constituent digits of two-digit numbers should have little, if any, influence on number comparison task. Hence, the proponents of a strictly holistic view have no way of explaining the digit-similarity effect within their theoretical framework.

The present study demonstrated, however, that the participants took longer in comparing symbolically similar number pairs than in comparing symbolically dissimilar number pairs. As the constituent digits of the numbers were the source of similarity in this study, it is clear that constituent digits play a role in two-digit number comparison. In addition, the present data show that the influence of constituent digits on two-digit number comparison is independent of the magnitude information they convey.

The results of the present study do not support the decomposition view, either. According to the decomposition view, decade digits should have a dominant effect in two-digit number comparison, while unit digits should have a minimal effect. In a strict decomposition view, unit digits are irrelevant in comparison process if people can make inequality judgment only with decade digits. Nevertheless, the data of the present experiment indicate that the composition of digits matters in two-digit number comparison. A noticeable attribute of the number pairs used in this experiment was explained in the Materials section – the same-digits pairs have greater differences than or the same differences as the different-digits pairs if we consider only the difference between decade digits (see Table 1). A strict decomposition view should predict that people respond faster in judging inequality of the same-digits pairs because they argue that the decade digits are the dominant factor in comparison process. The results of the present experiment contradict this prediction of the decomposition view.

The hybrid view by Nuerk et. al. assumes three distinctive processes in two-digit number comparison – i.e., people compare the overall magnitudes, the decade digits, and the unit digits. The assumption is intuitive, but their notion of separate and systemic comparisons is not consistent with the results of the present study because the same digit in a different place affects comparison processing. This suggests that their assumption is not comprehensive enough, if not wrong, to account for the effect of digit similarity in two-digit number comparison. To explain the findings of the present study within their theoretical framework, the proponents of the hybrid view may need to incorporate another process into their assumption – comparison between decade digits and unit digits, which will make their theory more complicated than it is now.

Since the study of Moyer and Landauer (1967), the patterns of response time in number comparison studies have been understood in terms of the magnitude information of numbers. This one-dimensional interpretation may have led researchers to the assumption of analogue form in magnitude representation and number comparison. The most prominent example may be "the mental number line" – an analogy pervasively used in number-comparison literature (Dehaene, 2003; Gallistel & Gelman, 1992). However, the results of the present study suggest that it might be useful to look for other factors influencing number comparison. Furthermore, it may be insightful if we interpret the findings of number comparison studies in terms of other well-established constructs in cognitive psychology rather than positing idiosyncratic explanations for them in haste.

If we assume that number comparison process works in a way similar to other cognitive processes, we can present very different explanations for number comparison from the current theories. For instance, the data of the present study may be better explained by a long-term memory model by Collins and Loftus (1975) rather than the current views on two-digit number comparison.

The spreading-activation theory by Collins and Loftus maintains that the currently attended item primes other items in long-term memory, and the strength of association between items vary. While the association between items facilitates recognition, it may hinder discerning the difference between them. In terms of numerical representation, it is reasonable to assume that the association between the number 1 and 2 is stronger than the association between the number 2 and 5. The association between two numbers can be arithmetical, spatial and linguistic in nature as well as quantitative. There could be associations between numbers that have not yet been addressed by mathematical cognition research. Compared to the weaker association between the numbers with a larger difference (e.g., 1 – 5), the stronger association between the numbers with a smaller difference (e.g., 1 – 2) can be a source of the disadvantage in RT to discriminate the relative sizes of two numbers. Thus, the strength of the association between numbers is correlated with the magnitude difference between them. In this sense, the strength of the association between numbers can be an alternative explanation for the distance effect. The strength of the association between items was the fundamental concept in developing the latent semantic association (LSA) model by Landauer and Dumais (1997).

Another issue concerns our familiarity with numbers. One of the findings of long-term memory studies is that practice has reliable and large effects on memory retrieval – i.e., practice improves people's recognition performance in terms of RT. In addition, the pattern of the improvement is particularly interesting as recognition time is a power function (Newell & Rosenbloom, 1981; Pirolli & Anderson, 1985) or more precisely an exponential function of the amount of practice (Heathcote, Brown, & Mewhort, 2000). The practice effect and RT patterns may be directly associated with another major finding in number comparison paradigm – the size effect. Numbers are not used equally often; the smaller the number, the more frequently it appears in our language (Dehaene & Mehler, 1992). That means that we practice smaller numbers more often and have greater familiarity with them as a result. That probably leads to faster processing of smaller numbers. In that sense, the difference in the extent of practice rather than comparisons between logarithmically represented analogue forms may account for the advantage of RT in comparing pairs with smaller numbers (e.g., 1 – 2) over pairs with larger numbers (e.g., 2 – 3).

Applying the spreading-activation model (Collins and Loftus, 1975) makes it easier to explain why the same-digits pairs required longer time to be compared in the present study. We may assume that the numbers in a different-digits pair activate four distinct items (digits) whereas the numbers in a same-digits pair activate the same two digits in the long-term memory. The greater overlap between same-digits numbers requires more effort to control comparison process, which leads to longer RT. How the decade digits and unit digits are represented is a separate issue beyond the scope of this paper.

Another potential source of misunderstanding in the number comparison paradigm is semantic confusion of some numerical concepts. We need to make distinctions between numerical concepts which are closely related to each other but do not necessarily refer to the same entities. Numerosity refers to the numeric property of a set of objects in the world independent of the observer. The numerical notations are entities that convey numerical information via particular symbolic codes usually in systemic forms. Numerical representations refer to the entities internal to the observer which correspond to numerosity or numerical notations (Fayol & Seron, 2005). For example, one question we may ask is whether the internal representation of numerosity is the same as that of its corresponding numerical notation. The semantic or linguistic closeness (or confusion) does not guarantee that they are the same entities. It is not plausible to hold that the representation of large numerosity is the same as its counterpart of numerical notation (e.g., 5876). What about small numbers? Even though it has not been addressed specifically, many researchers posit that numerical notations are limited to single-digit numbers. Several studies utilized a quantity-comparison task with materials other than numbers. They reported the same RT patterns as number-comparison studies. For instance, RT in comparing dot-arrays was a logarithmic function of the numerical difference between dot-arrays (Buckley & Gillman, 1974). The same pattern appeared in comparing the sizes of named objects (Kosslyn, Murphy, Bemesderfer, & Feinstein, 1977). The ubiquitous distance effect has led to the idea of analogue form in number representation and comparison.

We need to be careful in interpreting the results of such studies because there are alternative ways of making numerical inequality judgments with materials rather than numerosity comparison. For example, when people compare dot-patterns, numerosity is closely related to the size or density of the stimuli. People do not have to rely on numerical representations to make necessary comparisons. In this vein, comparing materials may have little or nothing to do with number comparison process per se. They may actually compare physical stimuli just as they discriminate the length of lines – i.e., there is no surprise in finding the logarithmic distance effect. The same patterns of RT in comparing materials and comparing single-digit numbers may result from different underlying mechanisms; the former results from discriminating physical stimuli and the latter results from the difference in practice as mentioned in this section above. This insight has a significant implication on the current models of two-digit number comparison. If we cannot assume the same representation for numerosity and its corresponding number notation, then the holistic view of number comparison stands on very shaky ground. The results of the present study reveal another different attribute between the representation of numerosity and the representation of numerical notation, namely symbolic similarity.

We know that surface similarity has a significant role in perceptual processes such as discriminating physical stimuli. It is also true that surface similarity affects cognitive processes such as recognition and recall. The digit-similarity effect found in the present study may well be an instance of the wide-ranged application of similarity in human perception and cognition. Unless we consider numbers as pure magnitude representations, we should expect a great deal of overlap between number representation and other cognitive processes. In that sense, we may be able to explain numerical comparisons in the context of general cognitive phenomena. Furthermore, the number comparison paradigm can be an effective tool to study topics in cognition other than magnitude representation. What information numbers represent other than magnitude could be an interesting topic of future studies in number representation. Like other processes of human cognition, number comparison must be a multi-faceted phenomenon.

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Appendix A

ANOVA of RT (Similarity by Task)

# of Subjects (per group) = 32

# of factors = 2

Within-Subject = 2

Factor	Levels	Name
1	2	1 = Digit Similarity (Same-digits / Different-digits)
2	2	2 = Judging Task (Judging-larger / Judging-smaller)

Source	*SS*	*df*	*MS*	*F*	*p*
Ss	472544.0000	31	15243.3545		
1 = Digit Similarity (Same-digits / Different-digits)					
	46056.1250	1	46056.1250	113.8526	< .001
	12540.2500	31	404.5242		
2 = Judging Task (Judging-larger / Judging-smaller)					
	45225.2812	1	45225.2812	29.8799	< .001
	46920.5938	31	1513.5675		
1 × 2 (Digit Similarity × Judging Task)					
	381.5703	1	381.5703	1.6269	~ .21
	7270.8047	31	234.5421		

Means (Standard Deviations below each mean)

~~~~~~

1 = Digit Similarity (Same-digits / Different-digits)

633.60	595.66
70.19	66.01

2 = Judging Task (Judging-larger / Judging-smaller)

595.84	633.43
67.67	68.69

1 × 2 (Digit Similarity × Judging Task)

613.08	578.59
68.88	62.83
654.12	612.73
66.31	65.62
~~~~~~

Appendix B

ANOVA of RT (Distance by Similarity by TASK)

# of Subjects (per group) = 32

# of factors = 3

Within-Subject = 3

Factor	Levels	Name
1	4	1 = Distance (9, 18, 27, 36)
2	2	2 = Digit Similarity (Same-digits / Different-digits)
3	2	3 = Judging Task (Judging-larger / Judging-smaller)

Weights Table :

Factor 1

Level 1 :	-3.00	-1.00	1.00	3.00	(Linear)
Level 2 :	-1.00	1.00	1.00	-1.00	(Quadratic)
Level 3 :	-1.00	3.00	-3.00	1.00	(Cubic)

Factor 2

Level 1 : -1.00 1.00

Factor 3

Level 1 : -1.00 1.00

Source	*SS*	*df*	*MS*	*F*	*p*
Ss	1834176.0000	31	59166.9648		
1 = Distance (9, 18, 27, 36)					
	306210.3750	3	102070.1250	61.1797	< .001
	155157.9375	93	1668.3649		
Comparisons					
Linear					
	302010.0938	1	302010.0938	157.6573	< .001
	59383.9688	31	1915.6118		
Quadratic					
	4154.7310	1	4154.7310	1.9196	~ .18
	67094.7500	31	2164.3467		
Cubic					
	45.5556	1	45.5556	0.0492	~ .83
	28679.2168	31	925.1360		
2 = Digit Similarity (Same-digits / Different-digits)					
	152300.2500	1	152300.2500	37.7453	< .001
	125083.3125	31	4034.9456		
3 = Judging Task (Judging-larger / Judging-smaller)					
	120065.3125	1	120065.3125	27.2489	< .001
	136593.6094	31	4406.2456		

1 × 2 (Distance × Digit Similarity)

47883.8164	3	15961.2725	10.8028	< .001
137408.4062	93	1477.5098		

Comparisons

~~~~~~~~~~~

Distance (Linear) × Digit Similarity

42596.0195	1	42596.0195	23.1027	< .001
57156.9023	31	1843.7710		

Distance (Quadratic) × Digit Similarity

725.5669	1	725.5669	0.4157	~ .52
54111.3555	31	1745.5276		

Distance (Cubic) × Digit Similarity

4562.2290	1	4562.2290	5.4104	~ .03
26140.1562	31	843.2308		

1 × 3 (Distance × Judging Task)

1137.3569	3	379.1190	0.2392	~ .87
147419.0000	93	1585.1505		

Comparisons

~~~~~~~~~~~

Distance (Linear) × Judging Task

993.1368	1	993.1368	0.4442	~ .51
69314.4453	31	2235.9497		

Distance (Quadratic) × Judging Task

87.3677	1	87.3677	0.0487	~ .83
55670.0547	31	1795.8081		

Distance (Cubic) × Judging Task

56.8524	1	56.8524	0.0786	~ .78
22434.5078	31	723.6938		

2 × 3 (Digit Similarity × Judging Task)

335.5669	1	335.5669	0.1941	~ .66
53593.4180	31	1728.8199		

1 × 2 × 3 (Distance × Digit Similarity × Judging Task)

2512.8687	3	837.6229	0.4492	~ .72
173398.6719	93	1864.5018		

Comparisons

~~~~~~~~~~~~

Distance (Linear) × Digit Similarity × Judging Task

842.3798	1	842.3798	0.3586	~ .55
72817.9297	31	2348.9653		

Distance (Quadratic) × Digit Similarity × Judging Task

1050.8247	1	1050.8247	0.4333	~ 0.52
75172.5312	31	2424.9204		
~~~~~~~~~~~~

Distance (Cubic) × Digit Similarity × Judging Task				
619.6642	1	619.6642	0.7560	~ .39
25408.2070	31	819.6196		

Means (Standard Deviations below each mean)

~~~~~~

1 = Distance (9, 18, 27, 36)

647.58	620.69	598.17	582.68
91.24	72.55	69.46	76.77

2 = Digit Similarity (Same-digits / Different-digits)

629.53	595.03
84.07	75.12

3 = Judging Task (Judging-larger/Judging-Smaller)

596.96	627.59
76.70	83.39

1 × 2 (Distance × Digit Similarity)

650.06	639.05	616.68	612.30
107.15	73.18	71.24	75.53
645.09	602.33	579.66	553.05
72.73	67.61	62.88	66.27
~~~~~~

1 × 3 (Distance × Judging Task)

634.70	605.14	582.27	565.76
73.80	71.63	67.34	77.31
660.46	636.24	614.07	599.59
104.88	70.64	68.40	72.94

2 × 3 (Digit Similarity × Judging Task)

613.40	580.53
74.38	75.73
645.65	609.54
90.18	71.91

1 × 2 × 3 (Distance × Digit Similarity × Judging Task)

640.02	620.36	599.44	593.80
72.94	71.28	69.44	77.87
629.38	589.92	565.09	537.72
75.43	69.77	61.52	66.80
660.11	657.75	633.92	630.81
133.42	71.28	69.85	69.44
660.81	614.73	594.22	568.38
67.46	64.06	61.76	63.08

Appendix C

ANOVA of Confounded Trials

# of Subjects (per group) = 32

# of factors = 2

Within-Subject = 2

Factor	Levels	Name
1	2	1 = Digit Similarity (Same-digits / Different-digits)
2	2	2 = Judging Task (Judging-larger / Judging-smaller)

Source	*SS*	*df*	*MS*	*F*	*p*
Ss	442336.0000	31	14268.9023		
1 = Digit Similarity (Same-digits / Different-digits)					
	59125.0078	1	59125.0078	71.6846	< .001
	25568.6172	31	824.7941		
2 = Judging Task (Judging-larger / Judging-smaller)					
	35544.4453	1	35544.4453	26.3508	< .001
	41815.6797	31	1348.8929		
1 × 2 (Digit Similarity × Task)					
	648.0000	1	648.0000	1.7023	~ .20
	11800.6250	31	380.6653		

Means (Standard Deviations below each mean)

~~~~~~

1 = Digit Similarity (Same-digits / Different-digits)

618.21	575.23
69.48	63.45

2 = Judging Task (Judging-larger / Judging-smaller)

580.05	613.38
69.45	66.36

1 × 2 (Digit Similarity × Judging Task)

599.30	560.81
69.03	65.37
637.12	589.64
65.61	58.99
~~~~~~

Appendix D

ANOVA of Unconfounded Trials

# of Subjects (per group) = 32

# of factors = 2

Within-Subject = 2

Factor	Levels	Name
1	2	1 = Digit Similarity (Same-digits / Different-digits)
2	2	2 = Judging Task (Judging-larger / Judging-smaller)SD

Source	*SS*	*df*	*MS*	*F*	*p*
Ss	512888.0000	31	16544.7734		
1 = Digit Similarity (Same-digits / Different-digits)					
	30504.5000	1	30504.5000	60.8603	< .001
	15537.8750	31	501.2218		
2 = Judging Task (Judging-larger / Judging-smaller)					
	44030.2812	1	44030.2812	22.6568	< .001
	60244.0938	31	1943.3579		
1 × 2 (Digit Similarity × Judging Task)					
	472.7812	1	472.7812	0.7823	~ .38
	18733.5938	31	604.3094		

Means (Standard Deviations below each mean)

~~~~~~

1 = Digit Similarity (Same-digits / Different-digits)

651.41	620.53
72.19	71.66

2 = Judging Task (Judging-larger / Judging-smaller)

617.42	654.52
71.45	70.91

1 × 2 (Digit Similarity × Judging Task)

630.94	603.91
71.03	70.37
671.88	637.16
68.42	70.10
~~~~~~