Relay Selection in Two-Hop Wireless Communications

by

MinChul Ju

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Abstract

Relay communication has been shown to be effective to extend service coverage and mitigate channel impairments. This thesis focuses on relay selection (RS) of both unidirectional and bidirectional relay networks employing the amplify-and-forward (AF) and decode-and-forward (DF) protocols.

This thesis presents four works on RS in two-hop relay networks. In the first work, we study opportunistic relaying (OR) and selection cooperation (SC) in DF-based unidirectional multi-antennas relay networks. We first propose two joint relay-and-antenna selection schemes which combine OR and SC, respectively, with transmit antenna selection. For each joint selection scheme, a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at this selected relay are jointly determined. Then we derive the outage probabilities, and show that the two schemes achieve the same outage performance.

In the second work, we study RS with the physical-layer network coding (PNC) in DF-based bidirectional relay networks. By modifying the well-known SC and OR, we first propose two RS schemes for the PNC network: SC-PNC and OR-PNC. Then we derive the outage probability and diversity order of the SC-PNC. Finally, we show that the OR-PNC achieves the same outage performance as the SC-PNC.

In the third work, we study RS with the analog network coding (ANC) and time division broadcast (TDBC), in AF-based bidirectional relay networks. We first consider RS schemes for the ANC and TDBC protocols based on a max-min criterion. Then we derive outage probabilities for the ANC and TDBC protocols.

In the fourth work, we study joint relay-and-source selection in an AF-based bidi-
rectional relay network. Since RS and opportunistic source selection (OSS) could individually improve performance of relay networks, we propose a joint RS-OSS protocol. In this network, a best source is selected to transmit data to the other source with the help of a selected best relay. Then, we derive the outage probability and average bit-error rate.

The considered RS schemes and obtained outage probability expressions will help the design of two-hop wireless communications in determining the system parameters such as relay location and the transmission power at each terminal.
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6.3 Outage probabilities against $d_{S_1,R}$ of the joint RS-OSS, RS with ANC, and RS with TDBC. $L = 1, 4$. $R = 1.5$ bps/Hz. $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ for $l = 1, \cdots, L$. $\mathcal{E}_s = \mathcal{E}_r/2 = 10$ dB for the joint RS-OSS, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_r = 10$ dB for the RS with ANC and TDBC. $N = 0$ in $\hat{F}_{W_l}^{(N)}(w)$ for $F_{W_l}(w)$ in (5.19).
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>AF</td>
<td>Amplify-and-Forward</td>
</tr>
<tr>
<td>ANC</td>
<td>Analog Network Coding</td>
</tr>
<tr>
<td>BC</td>
<td>Broadcast Channel</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CRC</td>
<td>Cyclic Redundancy Check</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DF</td>
<td>Decode-and-Forward</td>
</tr>
<tr>
<td>MAC</td>
<td>Multiple Access Channel</td>
</tr>
<tr>
<td>MGF</td>
<td>Moment Generating Function</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combining</td>
</tr>
<tr>
<td>OR</td>
<td>Opportunistic Relaying</td>
</tr>
<tr>
<td>OSS</td>
<td>Opportunistic Source Selection</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PNC</td>
<td>Physical-layer Network Coding</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>RS</td>
<td>Relay Selection</td>
</tr>
<tr>
<td>SC</td>
<td>Selection Cooperation</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>Acronym</td>
<td>Full Form</td>
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<tr>
<td>---------</td>
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</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>STC</td>
<td>Space-Time Coding</td>
</tr>
<tr>
<td>TAS</td>
<td>Transmit Antenna Selection</td>
</tr>
<tr>
<td>TDBC</td>
<td>Time Division BroadCast</td>
</tr>
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</table>
Chapter 1

Introduction

1.1 Relay Networks

Wireless communications face many challenges such as power consumption, service coverage, and channel impairments due to multi-path, fading, and shadowing [1]. In order to overcome these difficulties, relay communication has been proposed [2]. The fundamental idea of relay communication is that several terminals participate in communications by relaying a signal from a source to a destination and/or by forming a distributed multi-antenna system [3, 4]. Relay communication has been shown to be an effective means of saving power, attaining broader coverage range, and mitigating channel impairments resulting from fading [3, 4].

Depending on the number of concurrent information flows, there are two different network configurations: unidirectional relay network and bidirectional relay network. In the literature, considerable research attention has been put on both network configurations. In unidirectional relay networks, information is transmitted in one direction from the source to the destination; and in bidirectional relay networks, information is concurrently transmitted in two different directions such that two end-source terminals can exchange information. In what follows, the concepts of unidirectional and bidirectional relay networks will be discussed.
1.1.1 Unidirectional Relay Networks

The meaning of *unidirectional* suggests that information flow in the network can only go in one direction, i.e., from a source to a destination, possibly with the help of intermediate relay(s) \[5\]. A typical unidirectional relay network consists of a source, a destination, and a single relay, which is depicted in Fig. 1.1. In the first time slot, the source terminal broadcasts an information-bearing symbol, which is then forwarded to the intended destination with the help of relay(s) in the second time slot.

There are two well-known relaying strategies: *decode-and-forward* (DF)-based relaying and *amplify-and-forward* (AF)-based relaying \[5\]. In the DF-based relaying, each relay terminal detects the incoming signal and retransmits the detected symbol, whereas in the AF-based relaying, each relay terminal simply amplifies the incoming signal and retransmits the amplified signal. Note that for both relying strategies, independent replicas of the source signal are received by the destination, and thus, spatial diversity can be achieved.

There has been a lot of work done on DF-based relaying because it can be easily combined with channel codes and incorporated into network protocols \[6\]. Many of the works assumed that relay terminals could determine whether the detected symbols were correct or not and did not retransmit detected symbols with errors. Thus, the destination received only the correctly detected symbols from the relay terminals \[7\]–\[13\]. However, this scheme requires hardware for cyclic-redundancy-check (CRC)
codes and additional resource to decode and re-encode CRC codes at the relay terminals. For these reasons, relay networks with CRC codes might not be good choices for some applications such as real-time services and sensor networks, which are equipped only with a small and simple hardware \[16\]. Therefore, many other researchers have considered relay networks without CRC codes, which however induce error propagation effects at the relays \[3\] \[14\] \[15\]. The optimum detection at the destination with DF-based relaying is generally complex: the maximum-likelihood (ML) receiver has an exponential detection complexity. Therefore, most of the works focused on sub-optimum receivers: \(\lambda\) maximum-ratio-combiner (\(\lambda\)-MRC) \[3\], cooperative MRC (C-MRC) \[14\], and link-adaptive regeneration (LAR) \[15\]. Recently, the ML performance was analyzed for \(M\)-pulse amplitude modulation (PAM) and \(M\)-quadrature amplitude modulation (QAM) constellations \[17\]. It is well-known that the DF-based relaying achieves the full diversity order two when there exists a single relay \[6\].

AF-based relaying has been also studied extensively in the literature \[6\], \[18\]–\[21\]. Although the destination receives the noisy version of relayed signal, the AF-based relaying can also achieve the full diversity order two because the destination receives two independent signals from the direct-path and the relay-path \[6\]. The performance of AF-based relay networks has been widely discussed in the literature. Hasna and Alouini analyzed the end-to-end performance of two-hop wireless communication systems \[18\]. Hua et al. discussed diversity gain using Hurwitz-Radon matrices \[19\]. Anghel and Kaveh derived the exact average symbol-error rate (SER) \[20\]. Ribeiro et al. derived SERs for general multi-hop relay networks \[21\].

On the other hand, relay networks with multi-antenna terminals have recently received considerable attention \[22\]–\[27\]. In fact, a multi-antenna relay network is a generalized form of a single-antenna relay network. Wang et al. analyzed capacity of a multi-antenna relay network \[22\]. Fan et al. discussed different signaling and routing methods for two-hop multi-antenna relay networks in terms of the capacity \[23\]. Yuksel et al. presented diversity-multiplexing tradeoff for a multi-antenna relay
network [24]. Lee et al. derived the outage probability of the AF protocol in a multi-antenna relay network adopting transmit antenna selection (TAS) [27], which has widely been used in the classical multi-input-multi-output (MIMO) systems because it exhibited excellent performance with full diversity while it was simple to implement and it required only very low feedback signaling overhead.

1.1.2 Bidirectional Relay Networks

In many applications of relay networks, two different sources may need to exchange information with the help of relay(s) [28]. For those applications, several bandwidth efficient bidirectional protocols have been proposed: the physical-layer network coding (PNC) [29–37], the analog network coding (ANC) [38–42], and the time division broadcast (TDBC) [43, 44]. A typical bidirectional relay network consists of two end-sources and a single relay, which is depicted in Fig. 1.2. The PNC and ANC protocols require two time slots to exchange information between the two end-sources. In the first time slot, the two end-sources transmit their signals; in the second time slot, the relay retransmits the combined version of two incoming signals. Under a half-duplex constraint, however, the PNC and ANC cannot exploit the direct-path between the two end-sources even if such direct-path physically exists. On the other hand, the TDBC protocol can utilize the direct-path even under the half-duplex constraint. However, the TDBC needs three time slots for the exchange of information. In the first time slot, the first source transmits its signal; In the second time slot, the second source transmits its signal; in the third time slot, the relay retransmits the combined version of two incoming signals. Therefore, the PNC and ANC can have better bandwidth efficiency, while the TDBC can have better reliability due to the utilization of the direct-path. For instance, in a bidirectional relay network with a single relay where the direct-path between two end-sources exists, the diversity order of the PNC and ANC protocol is still one due to the half-duplex constraint, whereas that of the TDBC protocol is two.
Although the PNC, ANC, and TDBC have shown to be effective protocols for bidirectional communication, they do not intelligently exploit time-varying channel fluctuations, which can be potentially utilized to further improve the performance. In order to exploit the channel fluctuations, recently, Yi et al. proposed the opportunistic source selection (OSS) protocol with a single relay for an AF-based bidirectional network \[45\]. In this protocol, two-way communication between two end-sources was supported with the help of the single relay in an opportunistic manner depending on channel conditions. That is, a best source out of two end-sources is selected to transmit data to the other source with the help of the single relay. Let $\gamma_1$ denote the instantaneous signal-to-noise ratio (SNR) of the signal transmitted from $S_1$ and received by $S_2$ via $R$; $\gamma_2$ denote the instantaneous SNR of the signal transmitted from $S_2$ and received by $S_1$ via $R$. At the beginning of each two-time-slot, the two SNRs $\gamma_1$ and $\gamma_2$ are calculated at both sources. Then, depending on the SNRs, only one source transmits the signal to the other source with the help of the single relay. Specifically,
If $\gamma_1 \geq \gamma_2$

If $\gamma_1 < \gamma_2$

Figure 1.3: OSS protocol in bidirectional relay network consisting of two end-source terminals $S_1$ and $S_2$ and a relay $R$.

if $\gamma_1 \geq \gamma_2$, only $S_1$ transmits the signal to $S_2$ with the help of $R$. If $\gamma_1 < \gamma_2$, only $S_2$ transmits the signal to $S_1$ with the help of $R$. The authors derived lower bounds of the outage probability and average bit-error rate (BER), and they numerically showed that the OSS protocol provided higher reliability compared with the ANC and TDBC protocols in terms of the outage probability and average BER. Although the OSS protocol supports one-way communication either from $S_1$ to $S_2$ or from $S_2$ to $S_1$ at a given time instance, it can support two-way communication between $S_1$ and $S_2$ over time-varying channels in a long term. Very recently, Liu et al. considered the OSS protocol with a single relay for a DF-based bidirectional network, and they derived its outage probability [40]. The system model for the OSS protocol is depicted in Fig. 1.3.

1.1.3 Relay Selection

*Opportunistic transmission* has been considered as an attractive technique in time-
varying channels [47, 48]. Opportunistic transmission exploits time-varying channel fluctuations by selecting a single best user associated with the best channel gain at a given instant. Recently, many researchers have applied opportunistic transmission to relay networks, particularly for relay selection (RS) [49–64].

When there exist multiple relays in a network, there are many different strategies to exploit multiple relays: distributed space-time coding (STC) [65–72], distributed beamforming [37, 73, 74], and RS. Among those strategies, RS has been widely studied in the literature, because it exhibits excellent performance with full diversity while it is simple to implement and it requires only low feedback signaling overhead [49–62]. On the other hand, distributed STC and distributed beamforming require ideal frequency/time synchronization across the relays; however, this is not the case for RS. In the following, we summarize previous works on RS in DF-based unidirectional networks, RS in AF-based unidirectional networks, and RS in bidirectional networks.

1.1.3.1 Relay Selection in DF-based Unidirectional Networks

For DF-based unidirectional single-antenna relay networks, there have been two major RS schemes: opportunistic relaying (OR) and selection cooperation (SC).

**Opportunistic Relaying**: OR has been studied extensively for a unidirectional relay network consisting of a source, a destination, and multiple relays [49–62]. In this scheme, a single best relay was selected such that the minimum of two instantaneous SNRs of the first and second hops for each relay-path was maximized. That is, the selection was made based on “max-min” criterion, which was also referred to as “bottleneck” criterion [49] and “best-worst” criterion [52]. Since Bletsas et al. proposed OR [49], this max-min criterion has been applied to numerous systems [50, 59, 62]. Michalopoulos et al. derived the outage probability and average BER of the OR when there was a direct path [59]. Recently, some works studied RS based on the max-min criterion in the presence of interference [61, 62].
Selection Cooperation: As another relay selection scheme for a unidirectional relay network consisting of a source, a destination, and multiple relays, SC had been developed independently and it has also been studied extensively \cite{50,57}. In this scheme, at the first time slot, the source transmits a symbol to all the relays, and a set of relays that correctly decode the symbol is determined. At the second time slot, among the relays in the determined set, only a single best relay associated with the maximum SNR in the second hop retransmits the decoded symbol to the destination. Beres et al. proposed the SC and showed that SC outperformed distributed space-time codes in terms of outage probability \cite{57}. Since then, many works have been devoted to the SC \cite{50,58,59}. Bletsas et al. obtained interesting results that SC and OR had the same outage performance, and that the two schemes were outage-optimal under the aggregate power constraint at the relays \cite{50}. Michalopoulos et al. derived the outage probability and average BER of the SC when there was a direct path \cite{59}.

1.1.3.2 Relay Selection in AF-based Unidirectional Networks

Many works have been done on RS in AF-based unidirectional single-antenna relay networks. Bletsas et al. proposed opportunistic AF-based relaying \cite{49} and showed that it was outage-optimal among single-RS schemes \cite{50}. Zhao et al. showed that the AF-based relaying with RS provided lower outage probability and higher throughput than the AF-based relaying without RS except in the low SNR regime \cite{51}. Jing et al. derived the diversity order of many single-RS schemes and proposed several SNR-suboptimal multi-RS schemes \cite{52}. Recently, for AF-based multi-antenna relay networks, Lee et al. derived the outage probability for a system adopting TAS \cite{27}, which can be easily combined with RS.

1.1.3.3 Relay Selection in Bidirectional Networks

As in the unidirectional relay networks, when there exist multiple relays, RS can be adopted in the bidirectional relay networks. However, very few works have investigated RS in bidirectional networks \cite{63,64}, because the concept of the bidirectional
networks was proposed only recently. For DF-based bidirectional relay networks, Oechtering et al. considered RS for a system adopting superposition-encoding at relays, and they proposed a criterion for RS to maximize the weighted rate sum for any bidirectional rate pair and they showed that RS achieved the same multiuser diversity order as distributed beamforming \cite{63}. On the other hand, for AF-based bidirectional relay networks, Hwang et al. proposed a criterion for RS to maximize the instantaneous sum-rate of two opposite traffic flows in the ANC protocol, and they derived the performance bounds in terms of the average sum-rate, average SER, and outage probability \cite{64}.

1.1.4 Performance Measures

To evaluate the performance of RS in two-hop wireless communications, in this subsection, we introduce some general system performance measures that are widely used in any digital communication systems.

1.1.4.1 Outage Probability

Outage probability is an information-theoretic performance measure of communication systems. By Shannon’s channel coding theorem, vanishing probability of error can be achieved by using powerful channel coding as long as the data rate of the system is lower than channel capacity. On the other hand, for a system transmitting with a higher rate than channel capacity, error-free communication is impossible, and the system is called in outage. Therefore, outage probability is defined as the probability that a target transmission rate $R$ exceeds channel capacity $C$, i.e.,

\[ P_{\text{out}} = \Pr(C < R). \quad (1.1) \]

As the channel capacity is usually monotonically increasing in instantaneous SNR $\gamma$, the outage probability of (1.1) may be rewritten as

\[ P_{\text{out}} = \int_{0}^{\gamma_{th}} f_{\gamma}(x) dx, \quad (1.2) \]
where $\gamma_{th}$ is a predetermined threshold value related with the target rate $R$. As \ref{1.2} demonstrates, the outage probability is cumulative distribution function (CDF) of $\gamma$, evaluated at $\gamma = \gamma_{th}$. Furthermore, since the moment generating function (MGF) is just the Laplace transform of the PDF with argument reversed in sign, the outage probability can be found through the MGF of $\gamma$. We see that the CDF or MGF of $\gamma$ plays an important role in analyzing the performance of communication systems.

1.1.4.2 Error Probability

Error probability is a fundamental performance measure of any digital communication systems \cite{86}. The SER and BER are two widely used error probability measures. Consider a communication system with instantaneous SNR $\gamma$ at the receiver, the instantaneous BER or SER for many signalling schemes can be bounded or exactly expressed via an expression of Gaussian Q-function \cite{86}:

$$P_0 = a Q\left(\sqrt{b\gamma}\right),$$

(1.3)

where $a$ and $b$ are modulation-dependent constants, $\gamma$ is a function of the channel coefficient. As an example, the instantaneous SNR of a dual-hop cooperative diversity network with CSI-assisted relay is given by $\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}$, where $\gamma_1$ and $\gamma_2$ are the instantaneous SNRs of the first and second hops, respectively.

As the instantaneous BER/SER varies with random variable $h$, it would be more desirable to find the average BER/SER, which is the expectation of the instantaneous BER/SER, as shown as follows:

$$E[P_0] = a \int_0^{\infty} Q(\sqrt{bx}) f_{\gamma}(x) dx,$$

(1.4)

where $f_{\gamma}(x)$ is the PDF of $\gamma$. As the average error probability of \ref{1.4} makes use of the PDF $f_{\gamma}(x)$ of $\gamma$, the approach of \ref{1.4} was referred to as the PDF-based approach \cite{86}. Through this PDF-based approach, the average BER/SER, if not able to be solved in closed-form, can be represented with a single-integral with infinite limits.
An alternative way to derive the average BER/SER is to take advantage of the MGF of $\gamma$. By representing the Gaussian Q-function with the Craig’s formula, the average BER/SER can be represented with a single integral with finite limits and an integrand composed of the MGF of $\gamma$, and this approach was referred to as the MGF-based approach. The MGF-based approach is described as follows:

$$E[P_0] = a[Q(\sqrt{b\gamma})]$$

$$= E\left[\frac{a}{\pi} \int_0^{\pi/2} \exp\left(-\frac{b\gamma}{2\sin^2 \theta}\right) d\theta\right]$$

$$= \frac{a}{\pi} \int_0^{\pi/2} M_{\gamma}\left(-\frac{b}{2\sin^2 \theta}\right) d\theta,$$

where $M_{\gamma}(s)$ is the MGF of $\gamma$. As (1.5) shows, using the MGF-based approach, the average BER/SER, if not able to be solved in closed-form, can be represented in a single-integral with finite limits.

### 1.2 Motivation of Thesis

In the literature, extensive works have been done on RS in AF/DF-based unidirectional/bidirectional relay networks. Nevertheless, many important issues have not been addressed yet. In the following, we will present those issues and explain the motivation of our work.

For unidirectional single-antenna relay networks, there have been many works on the RS. For unidirectional multi-antenna relay networks, however, very limited results have been reported on the RS. Specifically, for the AF-based unidirectional multi-antenna relay networks, Lee et al. recently studied a multi-hop system adopting TAS. With a simple modification of [27], one can consider RS in the AF-based multi-antenna network adopting TAS. To the best of our knowledge, however, there has been no prior work done on RS in DF-based unidirectional multi-antenna relay networks. This has motivated our work to propose joint relay-and-antenna selection.

---

1. For a multi-antenna relay network adopting TAS, the required selection is a combination of a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at this selected relay. Thus, the selection is actually the “joint relay-and-antenna selection.”
in DF-based unidirectional multi-antenna relay networks, which will be discussed in Chapter 2.

For the DF-based bidirectional relay networks, Oechtering et al. recently considered RS for superposition-encoding based on a criterion to maximize the weighted rate sum for any bidirectional rate pair [63]. To the best of our knowledge, however, RS has never been studied for the PNC with XOR-encoding, nor has the outage performance been analyzed. This has motivated our work to propose RS in the PNC protocol, where each relay adopts the XOR-encoding to combine two received symbols from the two end-sources, which will be discussed in Chapter 3.

For the AF-based bidirectional relay networks, Hwang et al. recently considered RS based on a criterion to maximize the instantaneous sum-rate of two opposite traffic flows in the ANC protocol [64]. This sum-rate maximization criterion is useful to maximize the whole system capacity. On the other hand, in a multiuser system, the whole system is in outage if any user is in outage [47, 75, eq. (25)]. In order to minimize the outage of the multiuser system, therefore, the minimum mutual information among all the users must be maximized. In the ANC and TDBC protocols, there are two opposite traffic flows from two different users in the same channel; therefore, the two protocols can be considered as multiuser systems, specifically two-user systems. To minimize the outage of the ANC and TDBC protocols, therefore, the minimum mutual information of two opposite traffic flows must be maximized. This problem can be formulated as a max-min problem. Consequently, when RS is combined with ANC or TDBC, it is very useful to adopt the max-min criterion. To the best of our knowledge, however, there has been no work in which the max-min criterion was applied to RS in the ANC and TDBC protocols. This has motivated us to propose RS in the ANC and TDBC protocols based on the max-min criterion, which will be discussed in Chapter 4.

For the AF-based bidirectional relay networks, Yi et al. recently proposed the OSS protocol with a single relay [45]. In the OSS protocol, a best source out of two
end-sources is selected to transmit data to the other source with the help of the single relay. In [45], the authors numerically showed that OSS provided higher reliability compared with the ANC and TDBC protocols in terms of the outage probability and average BER. Since OSS and RS could individually improve the performance of relay networks, optimum combining of OSS and RS is an interesting and important issue to further improve the performance considerably. This has motivated our work to propose joint relay-and-source selection in a bidirectional relay network, which will be discussed in Chapter 5.

Finally, we present Table 1.1 to summarize the previous works and our works for RS in both unidirectional and bidirectional relay networks.

Table 1.1: Summary of previous works and our works for RS in both unidirectional and bidirectional relay networks.

<table>
<thead>
<tr>
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<th>Unidirectional single-antenna relay network</th>
<th>Unidirectional multi-antenna relay network</th>
<th>Bidirectional single-antenna relay network</th>
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<td>[27]</td>
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1.3 Thesis Contribution and Overview

The primary contributions of this thesis are briefly summarized as follows.

- For the DF-based unidirectional multi-antenna relay networks, we study joint relay and antenna selection. Specifically, we first propose two joint relay-and-antenna selection schemes which combine OR and SC, respectively, with TAS: joint OR-TAS and joint SC-TAS. For each joint selection scheme, a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at this selected relay are jointly determined in an optimum sense. Then we derive the outage probability of joint OR-TAS, and we prove that the outage probability of joint SC-TAS is identical to that of joint OR-TAS.
For the DF-based bidirectional relay networks, we study RS. Specifically, we propose two RS schemes for the PNC network: SC-PNC and OR-PNC. We derive the exact outage probability in closed-form and diversity order of the SC-PNC. Finally, we show that the OR-PNC achieves the same outage performance as the SC-PNC.

For the AF-based bidirectional relay networks, we study RS. Specifically, we first consider RS schemes in the ANC and TDBC protocols based on a max-min criterion to minimize the outage probabilities. Then, for the RS in the ANC protocol, we derive a closed-form expression of the outage probability; for the RS in the TDBC protocol, we derive a one-integral form of the outage probability and its tight lower bound in closed-form.

For the AF-based bidirectional relay networks, we study joint relay and source selection. Specifically, a best source is selected to transmit data to the other source with the help of a selected best relay in an opportunistic manner depending on channel conditions. Then, for the joint relay and source selection, we derive the outage probability and average BER for $M$-quadrature amplitude modulation (QAM).

The remainder of this thesis is organized as follows. In Chapter 2, we study joint relay and antenna selection in DF-based unidirectional multi-antenna relay networks. In Chapter 3, we study RS with the PNC, the major DF-based protocol in bidirectional relay networks. In Chapter 4, we study RS with the ANC and TDBC, two major AF-based protocols in bidirectional relay networks. In Chapter 5, we study joint relay and source selection in the AF-based bidirectional relay networks. Finally, some conclusions of this thesis are drawn and future works are discussed in Chapter 6.
Chapter 2

Joint Relay-and-Antenna Selection

For the decode-and-forward (DF) protocol in relay networks, opportunistic relaying (OR) and selection cooperation (SC) are two major relay selection schemes, which have been studied only for single antenna terminals. In this chapter, we study OR and SC in a multi-antenna relay network where each terminal has multiple antennas. To fully exploit multiple antennas without incurring high feedback overhead, we adopt transmit antenna selection (TAS). Specifically, we first propose two joint relay-and-antenna selection schemes which combine OR and SC, respectively, with TAS: joint OR-TAS and joint SC-TAS. For each joint selection scheme, a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at this selected relay are jointly determined in an optimum sense. In this network, at the first time slot, the selected antenna at the source transmits a symbol to the selected relay; at the second time slot, the selected antenna at the selected relay retransmits the detected symbol to the destination. We derive the outage probability of joint OR-TAS. Also, we obtain the outage probability of joint SC-TAS by proving that the outage probability of joint SC-TAS is identical to that of joint OR-TAS.

2.1 Introduction

Relay communication has been shown to be an effective means to extend service coverage and increase system capacity [2]. One of the most well-known relaying strategies
is the decode-and-forward (DF) relaying where each relay detects the incoming signals and retransmits the detected symbols. There has been a lot done on DF relaying because it can be easily implemented in current digital systems, and it can be combined with channel codes and adaptive modulation. A key issue in relay communication is relay selection [49–62] because it can improve system performance with low feedback signaling overhead. There have been two major relay selection schemes for the DF protocol: opportunistic relaying (OR) [49, 50] and selection cooperation (SC) [50, 57]. Bletsas et al. proposed OR based on the “max-min” criterion [49]. Beres et al. showed that SC outperformed distributed space-time codes in terms of outage probability [57]. In [50], Bletsas et al. obtained an interesting result that OR and SC had the same outage performance. However, all the works on OR and SC have been limited to relay networks with single-antenna terminals.

On the other hand, relay networks with multi-antenna terminals have recently received considerable attention [22–27]. Wang et al. analyzed capacity of a multi-antenna relay network [22]. Yuksel et al. presented diversity-multiplexing tradeoff for a multi-antenna relay network [24]. Lee et al. derived the outage probability of the AF protocol in a multi-antenna relay network adopting transmit antenna selection (TAS) [27]. For multi-antenna relay networks, however, there has been no prior work done on relay selection in the form of OR and SC. This has motivated our work. In this chapter, we extend OR and SC to a multi-antenna relay network where the source, the destination, and the multiple relays have multiple antennas.

Among many different strategies to exploit multiple antennas, TAS has widely been used in the classical multi-input multi-output (MIMO) systems because it exhibits excellent performance with full diversity while it is simple to implement and it requires only very low feedback signaling overhead. In this chapter, therefore, we adopt TAS and combine it with OR and SC in a multi-antenna relay network. Specifically, we first propose two joint relay-and-antenna selection schemes, namely, joint OR-TAS and joint SC-TAS. For each joint selection scheme, a single best transmit
antenna at the source, a single best relay, and a single best transmit antenna at this
selected relay are jointly determined in an optimum sense. We derive the outage
probability of joint OR-TAS. Also, we obtain the outage probability of joint SC-TAS
by proving that the outage probability of joint SC-TAS is identical to that of joint
OR-TAS.

The remainder of this chapter is organized as follows. In Section 2.2, we describe
the system model for a multi-antenna relay network. In Section 2.3, we propose
two joint relay-and-antenna selection schemes: joint OR-TAS and joint SC-TAS. In
Section 2.4, we derive the outage probabilities of joint OR-TAS and joint SC-TAS. In
Section 2.5, we present simulation results. Finally, conclusions are drawn in Section
2.6.

2.2 System Model

Consider a multi-antenna two-hop relay network consisting of one source, one desti-
ation, and \( L \) relays, where each terminal operates in a half-duplex mode and the relays
adopt the DF protocol. We use \( S \), \( D \), and \( R_l \) to denote the source, the destination,
and the \( l \)-th relay for \( l = 1, \cdots, L \). In the network, \( S \) is equipped with \( M_S \) antennas;
\( D \) with \( M_D \) antennas; and \( R_l \) with \( M_{R_l} \) antennas. A direct link from \( S \) to \( D \) is not
considered assuming the distance between \( S \) and \( D \) is very long or there are some ob-
stacles between the two terminals.\(^1\) The transmission power from \( S \) is denoted by \( E_S \);
and the transmission power from \( R_l \) is denoted by \( E_{R_l} \). The \( M_{R_l} \times M_S \) channel matrix
from \( S \) to \( R_l \) is denoted by \( H_{1,l} \), whose element is \( h_{1,l}(m_S, m_{R_l}) \) for \( m_S = 1, \cdots, M_S \)
and \( m_{R_l} = 1, \cdots, M_{R_l} \); and the \( M_D \times M_{R_l} \) channel matrix from \( R_l \) to \( D \) is denoted by
\( H_{2,l} \), whose element is \( h_{2,l}(m_{R_l}, m_D) \) for \( m_{R_l} = 1, \cdots, M_{R_l} \) and \( m_D = 1, \cdots, M_D \). We
assume that all the channel coefficients are fixed during two time slots; each element
\( h_{1,l}(m_S, m_{R_l}) \) of \( H_{1,l} \) is independent and modeled by \( h_{1,l}(m_S, m_{R_l}) \sim \mathcal{CN}(0, \Omega_{1,l}) \),

\(^1\) In many scenarios, there might be no practically meaningful performance gain even if we add
a weak direct link due to path attenuation and shadow fading. Furthermore, a weak direct link is
very difficult to be exploited because synchronization is extremely difficult in the low SNR regime.
where \( h \sim \mathcal{CN}(0, \Omega) \) indicates that \( h \) is a circularly symmetric complex-valued Gaussian random variable with zero mean and variance \( \Omega \); each element \( h_{2,l}(m_{R_l}, m_D) \) of \( H_{2,l} \) is independent and modeled by \( h_{2,l}(m_{R_l}, m_D) \sim \mathcal{CN}(0, \Omega_{2,l}) \); and \( H_{1,l} \) and \( H_{2,l} \) are mutually independent. Also, we assume that \( D \) knows all the channel matrixes \( \{H_{1,l}, H_{2,l} : l = 1, \cdots, L \} \), and \( R_l \) knows the channel matrix \( H_{1,l} \). The noise associated with each channel is modeled as a mutually independent additive white Gaussian noise (AWGN) with zero mean and unit variance.

In the classical MIMO systems, there have been many schemes to exploit multiple antennas at the transmitter. Among them, TAS requires only very low feedback signaling overhead while exhibiting excellent performance with the maximum diversity order \([82, 83]\). Also, it is well-known that a maximal-ratio combiner (MRC) at the receiver maximizes the diversity order and received signal-to-noise ratio (SNR). Accordingly, in this chapter, we adopt TAS at the transmitter and an MRC at the receiver.\(^2\) Note that in our system, there exist two transmitter/receiver pairs; hence, both TAS and an MRC are adopted not only at the first hop but also at the second hop. In the first hop, if we suppose that the \( m_S \)-th antenna at \( S \) transmits a symbol and \( R_l \) receives the symbol, then the output instantaneous SNR \( \gamma_{1,l}(m_S) \) of the MRC at \( R_l \) is given by

\[
\gamma_{1,l}(m_S) = \sum_{m_{R_l}=1}^{M_{R_l}} \gamma_{1,l}(m_S, m_{R_l}), \quad \text{where} \quad \gamma_{1,l}(m_S, m_{R_l}) = \mathcal{E}_S|h_{1,l}(m_S, m_{R_l})|^2
\]

denotes the instantaneous SNR of the link from the \( m_S \)-th antenna at \( S \) to the \( m_{R_l} \)-th antenna at \( R_l \). In the second hop, if we suppose that the \( m_{R_l} \)-th antenna at \( R_l \) retransmits the detected symbol and \( D \) receives the symbol, then the output instantaneous SNR \( \gamma_{2,l}(m_{R_l}) \) of the MRC at \( D \) is given by

\[
\gamma_{2,l}(m_{R_l}) = \sum_{m_D=1}^{M_D} \gamma_{2,l}(m_{R_l}, m_D), \quad \text{where} \quad \gamma_{2,l}(m_{R_l}, m_D) = \mathcal{E}_R|h_{2,l}(m_{R_l}, m_D)|^2
\]

denotes the instantaneous SNR of the link from the \( m_{R_l} \)-th antenna at \( R_l \) to the \( m_{D} \)-th antenna at \( D \).

\(^2\)Due to the higher signaling overhead, the complete channel state information (CSI) might be hard for some scenarios such as in fast fading channels and/or with a number of relays. On the other hand, in many other scenarios such as in slow fading channels and with small number of relays, it becomes easier and reasonable. Since channel estimation and signaling need to be done every channel coherence time, not every transmission, the burden of channel information signaling is not much if the channel coherence time is sufficiently longer than a symbol duration.\(^3\) This setting has been made in numerous previous works on the classical MIMO systems \([84, 85]\).
In the classical relay selection for a single-antenna relay network \( (M_S = M_D = M_{R_l} = 1 \text{ for } l = 1, \ldots, L) \) considered in the literature, only a single best relay is selected to help communication from S to D \([49]-[52]\). On the other hand, for a multi-antenna relay network considered in this chapter, a single best transmit antenna at S, a single best relay, and a single best transmit antenna at this selected relay must be jointly determined.

### 2.3 Two Joint Relay-and-Antenna Selection Schemes

In this section, we propose two joint relay-and-antenna selection schemes which combine OR and SC, respectively, with TAS, namely, joint OR-TAS and joint SC-TAS.

#### 2.3.1 Joint OR-TAS for Multi-Antenna Relay Networks

OR has been studied extensively for a single-antenna relay network \([49, 50]\). In this case, since each terminal had a single antenna, it was sufficient to select a single best relay without selecting transmit antennas at S and at the selected relay. Depending on channel conditions, the index of a single best relay was determined by the max-min criterion. On the other hand, for a multi-antenna relay network, the required selection is a combination of a single best transmit antenna at S, a single best relay, and a single best transmit antenna at this selected relay. Adopting the max-min criterion, we propose a joint relay-and-antenna selection scheme, which will be referred to as “joint OR-TAS” in this chapter. In this scheme, we jointly determine the index \( \hat{m}_S \) of the selected transmit antenna at S, the index \( \hat{l} \) of the selected relay, and the index \( \hat{m}_{R_l} \) of the selected transmit antenna at \( R_l \) as follows:

\[
(\hat{m}_S, \hat{l}, \hat{m}_{R_l}) = \arg \max_{m_S=1, \ldots, M_S} \min_{l=1, \ldots, L} [\gamma_{1,l}(m_S), \gamma_{2,l}(m_{R_l})].
\]

The system model for the joint OR-TAS is depicted in Fig. 2.1. At the first time slot, the \( \hat{m}_S \)-th antenna at S transmits a symbol to the selected relay \( R_{\hat{l}} \), and \( R_{\hat{l}} \) detects the
symbol using its MRC. At the second time slot, the $\hat{m}_{R_i}$-th antenna at $R_i$ retransmits the detected symbol to $D$, and $D$ makes the final detection using its MRC.

### 2.3.2 Joint SC-TAS for Multi-Antenna Relay Networks

As another relay selection strategy for a single-antenna relay network, SC has also been studied extensively [50, 57]. In this strategy, at the first time slot, $S$ transmits a symbol to all the relays $\{R_i\}_{i=1}^L$, and a set of relays that correctly detect the symbol is determined. At the second time slot, among the relays in the set, only a single best relay associated with the maximum SNR in the second hop retransmits the detected symbol to $D$. In a multi-antenna relay network, however, the set of relays that correctly detect the transmitted symbol generally varies depending on which antenna
is used at S to transmit the symbol. The maximum SNR in the second hop also varies depending on which antenna is used at Rₙ to retransmit the detected symbol. To extend the SC to a multi-antenna relay network, therefore, relay-and-antenna selection must be done jointly.

We first define \( \Gamma_{1,l} \overset{\text{def}}{=} \max_{m_S=1,\ldots,M_S} \gamma_{1,l}(m_S) \) as the maximum SNR value among all the output instantaneous SNRs \( \{\gamma_{1,l}(m_S)\}_{m_S=1}^{M_S} \) of the MRC at Rₙ. We also define \( \Gamma_{2,l} \overset{\text{def}}{=} \max_{m_{R_l}=1,\ldots,M_{R_l}} \gamma_{2,l}(m_{R_l}) \) as the maximum SNR value among all the output instantaneous SNRs \( \{\gamma_{2,l}(m_{R_l})\}_{m_{R_l}=1}^{M_{R_l}} \) of the MRC at D. Note that \( \Gamma_{1,l} \) can be obtained by transmitting a symbol through a single best transmit antenna at S and combining the received signals by the MRC at Rₙ, and \( \Gamma_{2,l} \) can be obtained by transmitting a symbol through a single best transmit antenna at Rₙ and combining the received signals by the MRC at D. Therefore, for a multi-antenna relay network adopting TAS at the transmitter and an MRC at the receiver, \( \Gamma_{1,l} \) is the equivalent SNR of the link from S to Rₙ, and \( \Gamma_{2,l} \) is the equivalent SNR of the link from Rₙ to D. We now sort the equivalent SNRs \( \{\Gamma_{2,l}\}_{l=1}^{L} \) of the links from \( \{R_l\}_{l=1}^{L} \) to D in a descending order as follows: \( \Gamma_{2,(1)} \geq \Gamma_{2,(2)} \geq \cdots \geq \Gamma_{2,(L)} \), where \( \Gamma_{2,(l)} \) denotes the \( l \)-th largest equivalent SNR in the second hop. Furthermore, we let \( R_{(l)} \) denote a relay terminal associated with \( \Gamma_{2,(l)} \). Finally, we let \( \Gamma_{1,(l)} \) denote the equivalent SNR of the link from S to \( R_{(l)} \).

Note that \( \Gamma_{1,(l)} \) is not the \( l \)-th largest equivalent SNR of the first hop; but it is the equivalent SNR of the first hop, which is associated with the \( l \)-th largest equivalent SNR, \( \Gamma_{2,(l)} \), of the second hop.

Based on the sorted equivalent SNRs \( \{\Gamma_{2,(l)}\}_{l=1}^{L} \) and the equivalent SNRs \( \{\Gamma_{1,(l)}\}_{l=1}^{L} \), we jointly determine a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at the selected relay. To this end, we first consider the \( (1) \)-st relay-path from S via \( R_{(1)} \) to D. Since \( \Gamma_{2,(1)} \) is the largest equivalent SNR in the second hop, we select \( R_{(1)} \) as the best relay if the link from S to \( R_{(1)} \) is not in outage, i.e. \( \Gamma_{1,(1)} \geq \bar{R} \), where \( \bar{R} = 2^{2^R} - 1 \) and \( R \) denotes the end-to-end spectral efficiency in bps/Hz. Then we consider the \( (k) \)-th relay-path from S via \( R_{(k)} \) to D for
$k = 2, \ldots, L$. Since $\Gamma_{2,(k)}$ is the $k$-th largest equivalent SNR in the second hop, we select $R_{(k)}$ as the best relay if all the links from $S$ to $\{R_{(l)}\}_{l=1}^{k-1}$ are in outage and the link from $S$ to $R_{(k)}$ is not in outage, i.e. $\{\Gamma_{1,(l)} < \bar{R}\}_{l=1}^{k-1}$ and $\Gamma_{1,(k)} \geq \bar{R}$. Summarizing the steps, the index $\bar{l}$ of the selected relay is determined as follows:

$$
\bar{l} = \begin{cases} 
(1), & \text{if } \Gamma_{1,(k)} \geq \bar{R}, \\
(k), & \text{if } \{\Gamma_{1,(l)} < \bar{R}\}_{l=1}^{k-1} \text{ and } \Gamma_{1,(k)} \geq \bar{R} \text{ for } k = 2, \ldots, L, \\
\emptyset, & \text{otherwise},
\end{cases}
$$

(2.2)

where $\emptyset$ means that no relay is selected. Finally, the index $\bar{m}_S$ of the selected transmit antenna at $S$, and the index $\bar{m}_{R_{l}}$ of the selected transmit antenna at $R_{l}$ are determined as follows:

$$
\bar{m}_S = \arg \max_{m_S=1,\ldots,M_S} \gamma_{1,\bar{l}}(m_S),
$$

(2.3)

$$
\bar{m}_{R_{l}} = \arg \max_{m_{R_{l}}=1,\ldots,M_{R_{l}}} \gamma_{2,\bar{l}}(m_{R_{l}}).
$$

(2.4)

The system model for the joint SC-TAS is depicted in Fig. 2.1 At the first time slot, the $\bar{m}_S$-th antenna at $S$ transmits a symbol to the selected relay $R_{l}$, and $R_{l}$ detects the symbol using its MRC. At the second time slot, the $\bar{m}_{R_{l}}$-th antenna at $R_{l}$ retransmits the detected symbol to $D$, and $D$ makes the final detection using its MRC. Note that joint OR-TAS and joint SC-TAS have exactly the same signaling overhead and almost the same computational complexity.

### 2.4 Outage Probabilities of Joint OR-TAS and Joint SC-TAS

In this section, we first derive the outage probability and diversity order of the joint OR-TAS. Then we obtain the outage probability and diversity order of the joint SC-TAS.

---

4 When $M_S = M_D = M_{R_l} = 1$ for $l = 1, \ldots, L$, the relay selection rule in (2.2) reduces to the SC for a single-antenna relay network [57].

5 Although (2.2)–(2.4) may appear to be individual selections, they are a truly joint selection. In (2.2), the index $\bar{l}$ was determined based on the equivalent SNRs $\{\Gamma_{1,(l)}, \Gamma_{2,(l)}\}_{l=1}^{L}$. Also, $\Gamma_{1,(l)}$ was obtained by selecting a single best transmit antenna at $S$; $\Gamma_{2,(l)}$ was obtained by selecting a single best transmit antenna at $R_l$. Thus, the selection rule in (2.2) implicitly included the selection of the single best transmit antennas at $S$ and $R_l$. 

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22
TAS by proving that the outage probability of the joint SC-TAS is identical to that of the joint OR-TAS.

### 2.4.1 Outage Probability of Joint OR-TAS

In this subsection, we derive the outage probability and diversity order of the joint OR-TAS of (2.1). Since $\Gamma_{1,l}$ and $\Gamma_{2,l}$ are the equivalent SNRs of the links from S to $R_l$ and from $R_l$ to D, respectively, the outage probability $P_{\text{out}}^{\text{OR-TAS}}(R)$ of the joint OR-TAS is given by

$$P_{\text{out}}^{\text{OR-TAS}}(R) = \Pr \left[ \max_{l=1,\ldots,L} \min[l, \Gamma_{1,l}, \Gamma_{2,l}] < \bar{R} \right]$$

$$= \prod_{l=1}^{L} \Pr \left[ \min[l, \Gamma_{1,l}, \Gamma_{2,l}] < \bar{R} \right],$$

where $\bar{R} = 2^R - 1$. We solve the probability in (2.6) and present the outage probability in the following theorem.

**Theorem 2.1:** The exact closed-form outage probability $P_{\text{out}}^{\text{OR-TAS}}(R)$ of the joint OR-TAS is given by

$$P_{\text{out}}^{\text{OR-TAS}}(R) = \prod_{l=1}^{L} \left[ F_{\gamma_{1,l}}(\bar{R}) + F_{\gamma_{2,l}}(\bar{R}) - F_{\gamma_{1,l}}(\bar{R})F_{\gamma_{2,l}}(\bar{R}) \right].$$

In the above equation, $F_{\gamma_{1,l}}(x) = \left[ 1 - \exp(-x/\bar{\gamma}_{1,l}) \sum_{s=0}^{M_{S}-1} \frac{1}{\pi} \left( \frac{x}{\bar{\gamma}_{1,l}} \right)^{s} \right]^{M_{S}}$ and $F_{\gamma_{2,l}}(x) = \left[ 1 - \exp(-x/\bar{\gamma}_{2,l}) \sum_{r=0}^{M_{R}-1} \frac{1}{\pi} \left( \frac{x}{\bar{\gamma}_{2,l}} \right)^{r} \right]^{M_{R_l}}$, where $\bar{\gamma}_{1,l} = \varepsilon_{S} \Omega_{1,l}$ and $\bar{\gamma}_{2,l} = \varepsilon_{R_{l}} \Omega_{2,l}$ denote the average SNRs of the link from any antenna of S to any antenna of $R_l$ and the link from any antenna of $R_l$ to any antenna of D, respectively. Also, $F_X(\cdot)$ denotes a cumulative distribution function (CDF) for a random variable $X$.

**Proof:** Using [86 eq. (9.441)], one can obtain the CDF $F_{\gamma_{1,l}(m_{S})}(x)$ of $\gamma_{1,l}(m_{S})$. Then the CDF $F_{\Gamma_{1,l}}(x)$ of $\Gamma_{1,l}$ can be obtained by multiplying all the CDFs $\{F_{\gamma_{1,l}(m_{S})}(x)\}_{m_{S}=1}^{M_{S}}$. Similarly, one can obtain the CDF $F_{\Gamma_{2,l}}(x)$ of $\Gamma_{2,l}$. Finally, using [93 eq. (6.81)], one can obtain $P_{\text{out}}^{\text{OR-TAS}}(R)$ in (2.7). □

Note that $P_{\text{out}}^{\text{OR-TAS}}(R)$ in (2.7) reduces to [50 eq. (20)] when $M_{S} = M_{D} = M_{R_{l}} = 1$ for $l = 1, \ldots, L$. Therefore, our outage probability analysis in this subsection can
be considered as a generalization of that of [50]. In the following lemma, we present the diversity order of the joint OR-TAS.

**Lemma 2.1:** The joint OR-TAS achieves full diversity order, and its diversity order $D_{\text{OR-TAS}}$ is given by

$$D_{\text{OR-TAS}} = \min[M_S, M_D] \sum_{l=1}^{L} M_{R_l}. \quad (2.8)$$

**Proof:** See Appendix 2-A. □

Note that when $M_S = M_D = M_{R_l} = 1$ for $l = 1, \cdots, L$, the diversity order $D_{\text{OR-TAS}}$ in (2.8) reduces to $L$, and it is identical to [52] Theorem 2] which considered only a single-antenna relay network. Therefore, our diversity order analysis in this subsection can be considered as a generalization of that of [52].

### 2.4.2 Outage Probability of Joint SC-TAS

In this subsection, we derive the outage probability and diversity order of the joint SC-TAS. We first present the outage probability expression of the joint SC-TAS in the following lemma.

**Lemma 2.2:** The outage probability of the joint SC-TAS is given by

$$P_{\text{out}}^{\text{SC-TAS}}(R) = \Psi_1(R; L) + \Psi_2(R; L), \quad (2.9)$$

where

$$\Psi_1(R; L) = \sum_{k=1}^{L} \left( \prod_{l=1}^{k-1} \Pr[\Gamma_{1,(l)} < \bar{R}] \right) \Pr[\Gamma_{1,(k)} \geq \bar{R}] \Pr[\Gamma_{2,(k)} < \bar{R}], \quad (2.10)$$

$$\Psi_2(R; K) = \prod_{l=1}^{K} \Pr[\Gamma_{1,(l)} < \bar{R}]. \quad (2.11)$$

**Proof:** The outcomes of the joint SC-TAS in Section 2.3.2 can be classified into two cases: Case 2.1: one of the relays is selected; and Case 2.2: no relay is selected. For Case 2.1, if the link from the selected relay $R_i$ to $D$ is in outage, i.e. $\Gamma_{2,j} < \bar{R}$, then a system is in outage. Using (2.2), therefore, one can obtain (2.10). For Case 2.2, since all the links from $S$ to $\{R_i\}_{i=1}^{L}$ are in outage, it is obvious that a system is in
outage. Therefore, one can obtain (2.11). Then adding (2.10) and (2.11) yields (2.9).

\[\Omega_{1}(R; l) = \Omega_{1}(R; l) + \Omega_{2}(R; l)Pr[\Gamma_{1,\ell} \geq \bar{R}]Pr[\Gamma_{2,\ell} < \bar{R}], \quad (2.12)\]
\[\Omega_{2}(R; l) = \Omega_{2}(R; l)Pr[\Gamma_{1,\ell} < \bar{R}], \quad (2.13)\]

where the initial conditions of \(\Omega_{1}(R; 0)\) and \(\Omega_{2}(R; 0)\) are given by \(\Omega_{1}(R; 0) = 0\) and \(\Omega_{2}(R; 0) = 1\).

Directly solving (2.9) with order statistics [94], it is possible to obtain a closed-form outage probability expression of the joint SC-TAS. However, the final expression obtained this way is extremely lengthy and complicated. In this chapter, therefore, we take another approach which is useful and more insightful. Specifically, by proving that the outage probabilities of the joint SC-TAS and joint OR-TAS are identical, we obtain the outage probability of the joint SC-TAS. In the following theorem, we show the identicalness.

**Theorem 2.2:** For a multi-antenna relay network, the outage probability of the joint SC-TAS is also given by (2.7), which is the outage probability of the joint OR-TAS.

**Proof:** See Appendix 2-B.

It follows from Theorem 2.2 that the outage probability \(P_{\text{out}}^{\text{SC-TAS}}(R)\) in (2.9) does not need to be directly solved, which leads to a lengthy and complicated form. Instead, one can use the simple and closed-form outage probability expression of (2.7) even for joint SC-TAS. Also, the joint SC-TAS achieves full diversity order and its diversity order \(D_{\text{SC-TAS}}\) is identical to that of the joint OR-TAS in (2.8).

**Remark 2.1:** In [50], for a single-antenna relay network, the authors showed that two outage probabilities of SC and OR were the same by comparing two final outage
probability expressions. In our work, for a multi-antenna relay network, we show that two outage probabilities of the joint SC-TAS and joint OR-TAS are still the same, but without actually deriving the final two expressions. Thus, in a sense, Theorem 2.2 is a generalization of [50] to a multi-antenna relay network, although our approach is different from theirs. This is a very interesting result and it gives an insight into joint relay-and-antenna selection, joint OR-TAS and joint SC-TAS, in multi-antenna relay networks.

2.5 Simulation Results

We compare the outage probabilities obtained by our analysis with those obtained by Monte Carlo simulations using Matlab. First, we investigate the effect of SNR values. We set \( L = 1, 2, 3, 4 \), \( R = 2 \) bps/Hz, \( M_S = M_D = M_{R_l} = 2 \), \( \mathcal{E}_S = \mathcal{E}_{R_l} = \mathcal{E} \), and \( \Omega_{1,l} = \Omega_{2,l} = 1 \) for \( l = 1, \cdots, L \). Thus, \( \bar{\gamma}_{1,l} = \bar{\gamma}_{2,l} = \mathcal{E} \). Fig. 2.2 shows the outage probabilities against \( 10 \log_{10} \mathcal{E} \) of the joint OR-TAS and joint SC-TAS in a multi-antenna relay network. As the number \( L \) of relays increases, one can see that the performance improves, because the diversity order increases.

Secondly, we investigate the effect of relay locations. Let \( d_{S,R} \) denote the distance between \( S \) and \( R_l \), and \( d_{D,R} \), the distance between \( D \) and \( R_l \), both of which are normalized by the distance between \( S \) and \( D \). Therefore, we have \( d_{S,R} + d_{D,R} = 1 \). Furthermore, we set the path loss exponent as four to model radio propagation in urban areas [79]. As a result, we set \( \Omega_{1,l} = d_{S,R}^{-4} \) and \( \Omega_{2,l} = (1 - d_{S,R})^{-4} \) for \( l = 1, \cdots, L \). Also, we set \( L = 1, 2, 3, 4 \), \( R = 2 \) bps/Hz, \( M_S = 3 \), \( M_D = 2 \), \( M_{R_l} = 1 \), and \( \mathcal{E}_S = \mathcal{E}_{R_l} = 5 \) dB for \( l = 1, \cdots, L \). Thus, \( \bar{\gamma}_{1,l} = \mathcal{E} d_{S,R}^{-4} \) and \( \bar{\gamma}_{2,l} = \mathcal{E}_{R_l} (1 - d_{S,R})^{-4} \). Fig. 2.3 shows the outage probabilities against \( d_{S,R} \) of the joint OR-TAS and joint SC-TAS in a multi-antenna relay network. Irrespective of SNR in Fig. 2.2 and relay location in Fig. 2.3 we can see (2.7) exactly matches with simulation results.
Figure 2.2: Outage probabilities against $10 \log_{10} E$ of the joint OR-TAS and joint SC-TAS in a multi-antenna relay network. $L = 1, 2, 3, 4$. $R = 2$ bps/Hz. $\bar{\gamma}_{1,l} = \bar{\gamma}_{2,l} = \mathcal{E}$ and $M_S = M_D = M_{R_l} = 2$ for $l = 1, \ldots, L$. 

Analysis using eq. (2.7)
Simulation for joint OR−TAS
Simulation for joint SC−TAS
Figure 2.3: Outage probabilities against $d_{S,R}$ of the joint OR-TAS and joint SC-TAS in a multi-antenna relay network. $L = 1, 2, 3, 4$. $R = 2$ bps/Hz. $\hat{\gamma}_{1,l} = \mathcal{E}d_{S,R}^{-4}$ and $\hat{\gamma}_{2,l} = \mathcal{E}(1 - d_{S,R})^{-4}$ with $\mathcal{E} = 5$ dB, and $M_S = 3, M_D = 2, M_{R_l} = 1$ for $l = 1, \cdots, L$. 

\[ \text{Outage probability} \]

\[ \text{Outage probability} \]

\[ \begin{align*}
\text{Outage probability} &= \text{Analysis using eq. (2.7)} \\
&\text{Simulation for joint OR-TAS} \\
&\text{Simulation for joint SC-TAS} \\
\end{align*} \]

\[ \begin{align*}
\text{Outage probability} &= \text{Outage probability} \\
&\text{Outage probability} \\
\end{align*} \]
2.6 Conclusions

In this chapter, we have extended two well-known relay selection schemes, OR and SC, to a multi-antenna relay network. Specifically, combining OR and SC, respectively, with TAS, we have first proposed two joint relay-and-antenna selection schemes, namely, joint OR-TAS and joint SC-TAS. For each joint selection scheme, a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at this selected relay were jointly determined in an optimum sense. We have derived the outage probability of joint OR-TAS. Also, we have obtained the outage probability of joint SC-TAS by proving that the outage probability of joint SC-TAS was identical to that of joint OR-TAS.
Chapter 3

Relay Selection in PNC Protocol

In this chapter, we study relay selection for the physical-layer network coding (PNC) in a bidirectional relay network consisting of two different end-sources and multiple relays. By modifying the well-known selection cooperation (SC) and opportunistic relaying (OR), we propose two relay selection schemes for the PNC network, and the proposed schemes are referred to as SC-PNC and OR-PNC. For the SC-PNC, in the multiple access channel (MAC) phase, a set of relays that correctly decode two received symbols from the two end-sources is determined; in the broadcast channel (BC) phase, among the relays in the determined set, a single best relay is selected such that the minimum mutual information of the two links from each relay to the two end-sources is maximized. For the OR-PNC, a single best relay is selected such that the minimum mutual information of both the MAC phase and the BC phase is maximized. We derive the exact outage probability in closed-form and diversity order of the SC-PNC. Finally, we show that the OR-PNC achieves the same outage performance as the SC-PNC.

3.1 Introduction

Relay communication is an effective method to attain broader coverage range and to mitigate channel impairments [3, 5]. In many applications of relay networks, two different end-sources may need to exchange information with the help of relays. For
those applications, several bandwidth efficient bidirectional relaying protocols were proposed [28]. In particular, the well-known decode-and-forward (DF)-based protocol has been applied to bidirectional relay communication, and it has been referred to as the physical-layer network coding (PNC) [29]–[37]. The PNC protocol requires two time slots to exchange information between two end-sources. At the first time slot, the two end-sources transmit simultaneously their symbols to a relay over a multiple access channel (MAC); at the second time slot, the relay forwards a PNC-encoded version of the two received symbols to the two end-sources over a broadcast channel (BC).

Two different PNC encoding rules at relays have been discussed in the literature: XOR-encoding and superposition-encoding. In almost all the works on the PNC, it has been assumed that the XOR-encoding is used at the relays [29]–[37]. In the XOR-encoding, a relay decodes two received symbols in the MAC phase, and broadcasts the XORed version of the two decoded symbols in the BC phase. Zhang et al. presented maximum likelihood (ML) decoder for the XOR-encoding scheme, and analyzed the instantaneous symbol error probability [29], [30]. Kim et al. analyzed the capacity region for the XOR-encoding scheme [43]. On the other hand, only in very limited works, the superposition-encoding was assumed at the relays [37], [63]. In the superposition-encoding, a relay decodes two received symbols in the MAC phase, and broadcasts a linear combination of two decoded symbols in the BC phase. In this paper, we will focus only on the widely-adopted XOR-encoding at relays.

Relay selection in relay networks has also received considerable attention in the literature [49]–[62] because it can enhance system performance with simple hardware. There have been two well-known relay selection schemes for DF-based unidirectional networks: selection cooperation (SC) [50], [57], and opportunistic relaying (OR) [49], [50]. Beres et al. showed that the SC outperformed distributed space-time codes

\footnote{Recently, Yi et al. obtained optimum distributed beamforming (BF) vectors for both XOR-encoding and superposition-encoding strategies in the BC phase, and numerically demonstrated that, in the distributed BF, the XOR-encoding outperformed the superposition-encoding [37].}
in terms of outage probability \cite{57}. Bletsas et al. proposed the OR based on the “max-min” criterion \cite{49}. In \cite{50}, Bletsas et al. obtained an interesting result that the SC had the same outage performance as the OR. All these works have focused on unidirectional networks \cite{49}–\cite{62}. To the best of our knowledge, however, relay selection has never been studied for the PNC with XOR-encoding, nor has the outage performance been analyzed. This has motivated our work.

In this paper, we consider relay selection with the PNC in a bidirectional relay network consisting of two end-sources and multiple relays, where each relay adopts the XOR-encoding to combine two received symbols from the two end-sources. By modifying the well-known SC, we first propose a relay selection scheme for the PNC network. The proposed scheme is working as follows: in the MAC phase, a set of relays that correctly decode two received symbols from the two end-sources is determined; in the BC phase, among the relays in the determined set, a single best relay is selected such that the minimum mutual information of the two links from each relay to the two end-sources is maximized. Then we derive the exact outage probability and diversity order. Furthermore, by modifying the well-known OR, we propose another relay selection scheme for the PNC network. The proposed scheme based on the OR is working as follows: a single best relay is selected such that the minimum mutual information of both the MAC phase and the BC phase is maximized. We show that the two proposed schemes have the same outage performance.

The remainder of this paper is organized as follows. In Section II, we describe the system model. In Section III, we first propose a relay selection scheme by modifying the original SC, and then derive its exact outage probability and diversity order. In Section IV, we first propose another relay selection scheme by modifying the original OR, and then show the two proposed relay selection schemes have the same outage performance. In Section V, we present simulation results. Finally, conclusions are drawn in Section VI.

\textit{Notation:} We use \( A := B \) to denote that \( A \), by definition, equals \( B \), and we use...
A =: B to denote that B, by definition, equals A. Also, for a random variable \( X \), \( f_X(\cdot) \) denotes its probability density function (PDF), and \( F_X(\cdot) \) denotes its cumulative distribution function (CDF). For two conditions \( Y \) and \( Z \), \( Y \Leftrightarrow Z \) denotes \( Y \) is equivalent to \( Z \). Finally, \( x \sim \mathcal{CN}(m, \Omega) \) indicates that \( x \) is a circularly symmetric complex-valued Gaussian random variable with mean \( m \) and variance \( \Omega \).

### 3.2 System Description and Achievable Rates

In this section, we first describe the system model, and then present achievable rates of the PNC.

#### 3.2.1 System Model

Consider a bidirectional relay network consisting of two different end-sources and \( L \) relays, where each terminal has a single antenna and operates in a half-duplex mode, and the relays adopt the DF protocol and XOR-encoding rule to combine two received symbols from the two end-sources in the MAC phase. We use \( S_1, S_2, \) and \( R_l \) to denote the first source, the second source, and the \( l \)-th relay for \( l = 1, \cdots, L \), respectively. The transmission power at \( S_i \) is denoted by \( E_i \) for \( i = 1, 2 \); and the transmission power at \( R_l \) is denoted by \( E_R \). The complex channel coefficient between \( S_1 \) and \( R_l \) is denoted by \( h_{1,l} \); and the complex channel coefficient between \( S_2 \) and \( R_l \) is denoted by \( h_{2,l} \). Under the half-duplex constraint, since the direct-path between the two end-sources can not be exploited even if such direct-path physically exists, we do not consider the complex channel coefficient between \( S_1 \) and \( S_2 \). We assume that all the channels are reciprocal and they are modeled as follows: \( h_{1,l} \sim \mathcal{CN}(0, \Omega_{1,l}) \) and \( h_{2,l} \sim \mathcal{CN}(0, \Omega_{2,l}) \).

Also, all the channel coefficients are assumed to be fixed over two time slots. The noise associated with each channel is modeled as a mutually independent additive white Gaussian noise (AWGN) with zero mean and unit variance. Finally, we assume that both end-sources know all the channel coefficients\(^2\) \( \{h_{1,l}, h_{2,l} : l = 1, \cdots, L\} \), and

\(^2\) This is an essential assumption for bidirectional communications, and thus, it has been widely-adopted in numerous previous works on the PNC [44 63 90].
that each relay $R_l$ knows its own channel coefficients $h_{1,l}$ and $h_{2,l}$.

The PNC with multiple relays operates in two consecutive phases: a MAC phase and a BC phase. In the MAC phase, $S_1$ and $S_2$ broadcast simultaneously their symbols to all the relays $\{R_l\}_{l=1}^L$ at the first time slot. In the BC phase, only a selected relay broadcasts the XORed version of the two received symbols to $S_1$ and $S_2$ at the second time slot. The system model for the PNC with multiple relays is depicted in Fig. 3.1.

### 3.2.2 Achievable Rates of PNC

In this subsection, focusing on a single relay $R_l$ who is assumed to participate in the BC phase, we present achievable rates of the PNC. For the PNC, the achievable rate region must be determined by considering both the MAC and BC phases. In the MAC phase, $S_1$ and $S_2$ transmit simultaneously their symbols to all the relays $\{R_l\}_{l=1}^L$ at the first time slot. The two channels from $S_1$ and $S_2$ to the $l$-th relay, $R_l$, constitute a multiple access channel, and this will be referred to as the “$l$-th MAC” in this paper. It is well-known that the achievable rate region for the $l$-th MAC is given by [90, eqs. (8)–(10)]

\[
R_1 \leq \frac{1}{2} \log_2 (1 + \mathcal{E}_1 X_l) =: \mathcal{T}_{1,l}^{\text{MAC}},
\]

\[
R_2 \leq \frac{1}{2} \log_2 (1 + \mathcal{E}_2 Y_l) =: \mathcal{T}_{2,l}^{\text{MAC}},
\]

\[
R_1 + R_2 \leq \frac{1}{2} \log_2 (1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l) =: \mathcal{T}_{\text{sum},l}^{\text{MAC}},
\]

Figure 3.1: System model for the PNC with multiple relays.
where $R_1$ and $R_2$ denote the transmission rates of $S_1$ and $S_2$, respectively, with $X_l = |h_{1,l}|^2$ and $Y_l = |h_{2,l}|^2$.

In the BC phase, $R_l$ first decodes two received symbols from $S_1$ and $S_2$, and then forwards the XORed version of the two received symbols to $S_1$ and $S_2$. The two channels from $R_l$ to $S_1$ and $S_2$ constitute a broadcast channel, and this will be referred to as the “$l$-th BC” in this paper. The achievable rate region for the $l$-th BC is given by [90, eqs. (11) and (12)]

\[
R_2 \leq \frac{1}{2} \log_2 (1 + \mathcal{E}_{R}X_l) =: \mathcal{I}_{BC}^{1,l},
\]

\[
R_1 \leq \frac{1}{2} \log_2 (1 + \mathcal{E}_{R}Y_l) =: \mathcal{I}_{BC}^{2,l}.
\]

In the above equations (3.4)–(3.5), we use the pre-log factor $1/2$ because communication between $S_1$ and $S_2$ is done during two time slots.

The end-to-end achievable rates should be determined by considering both the MAC and BC phases. The combination of the $l$-th MAC and the $l$-th BC constitute a bidirectional end-to-end channel between the two end-sources through $R_l$, and this will be referred to as the “$l$-th bidirectional channel” in this paper. Combining (3.1)–(3.5), therefore, the achievable rate region for the $l$-th bidirectional channel is given by [90, eqs. (13)–(15)]

\[
R_1 \leq \min\{\mathcal{I}_{MAC}^{1,l}, \mathcal{I}_{BC}^{2,l}\},
\]

\[
R_2 \leq \min\{\mathcal{I}_{MAC}^{2,l}, \mathcal{I}_{BC}^{1,l}\},
\]

\[
R_1 + R_2 \leq \mathcal{I}_{sum,l}^{MAC}.
\]

Note that this achievable rate region is equivalent to many other works such as [43, Theorem 2], [44, eqs. (6)–(8)], and [63, eq. (4)].

### 3.3 SC-PNC

In this section, by modifying the well-known SC, we first propose a relay selection scheme for the PNC network. Then we derive the exact outage probability and
diversity order of the proposed scheme.

### 3.3.1 Proposed SC-PNC

SC has been studied extensively for a *unidirectional* relay network consisting of a source, a destination, and multiple relays \[57\]–\[59\]. In this scheme, at the first time slot, the source transmits a symbol to all the relays, and a set of relays that correctly decode the symbol is determined. At the second time slot, among the relays in the determined set, only a *single best* relay associated with the maximum signal-to-noise ratio (SNR) in the second hop retransmits the decoded symbol to the destination.

In a *bidirectional* relay network, however, the original SC designed for the unidirectional relay network is not directly applicable, because there are two traffic flows between the two end-sources. In this paper, therefore, the SC is modified for use in a bidirectional relay network as follows. Firstly, in the MAC phase, a set of relays that correctly decode *two* received symbols from *both* end-sources must be determined. To this end, we define \( D \) as a set of indices of relays that correctly decode *two* received symbols from both end-sources in the MAC phase, and it is given by

\[
D = \left\{ l \in \{1, \cdots , L\} : I_{1,l}^{\text{MAC}} \geq R_1, I_{2,l}^{\text{MAC}} \geq R_2, I_{\text{sum},l}^{\text{MAC}} \geq R_1 + R_2 \right\}. \quad (3.9)
\]

Since the two end-sources are equivalent terminals, it is fair to set the target rate of each end-source as \( R/2 \), where \( R \) denotes a target rate in bps/Hz for the whole network. Therefore, we set \( R_1 = R_2 = R/2 \) in (3.9).

Secondly, in the BC phase, a *single best* relay among the relays in the set \( D \) must be selected such that the minimum of *two* mutual information of the *two* links from each relay to the two end-sources is maximized. That is, the index \( l^{\text{SC-PNC}} \) of the selected relay must be determined such that \( \min[I_{1,l}^{\text{BC}}, I_{2,l}^{\text{BC}}] \) is maximized over the set \( D \) as follows:

\[
l^{\text{SC-PNC}} = \arg\max_{l \in D} \min[I_{1,l}^{\text{BC}}, I_{2,l}^{\text{BC}}]. \quad (3.10)
\]

\[^3\] If the set \( D \) has no element, i.e. \( D = \emptyset \), then no relay is selected.
In this paper, this selection scheme will be referred to as the “SC-PNC.”

Bidirectional communication reduces to unidirectional communication by allowing only $S_1$ to transmit its symbol at the first time slot and a selected relay to retransmit the decoded symbol to $S_2$ at the second time slot. In this case, the set $\mathcal{D}$ of (3.9) is simplified to $\mathcal{D} = \{l \in \{1, \ldots, L\} : I_{1,l}^{\text{MAC}} \geq R_1\}$ and the index $l^{\text{SC-PNC}}$ of (3.10) is simplified to $l^{\text{SC-PNC}} = \arg\max_{l \in \mathcal{D}} I_{2,l}^{\text{BC}}$. Then this becomes identical to the unidirectional SC [50, eq. (11)]. In this sense, therefore, the relay selection of (3.10) can be considered as a generalization of the relay selection in unidirectional relay networks [50, eq. (11)].

### 3.3.2 Outage Probability and Diversity Order of SC-PNC

In this subsection, we derive the outage probability of the SC. To this end, we first determine the outage probability of the SC-PNC focusing only on a relay $R_l$. It is evident that the $l$-th bidirectional channel is in outage if the $l$-th MAC is in outage and/or the $l$-th BC is in outage. In the MAC phase, if the index $l$ is not an element of the set $\mathcal{D}$, i.e. $l \notin \mathcal{D}$, then the $l$-th MAC is in outage. In the BC phase, if the minimum of two mutual information, $I_{1,l}^{\text{BC}}$ in (3.4) and $I_{2,l}^{\text{BC}}$ in (3.5), is smaller than a target rate $R/2$, i.e. $\min[I_{1,l}^{\text{BC}}, I_{2,l}^{\text{BC}}] < R/2$, then the $l$-th BC is in outage. Combining the two possibilities, therefore, the outage probability $P^{\text{SC-PNC}}_{\text{out,}l}(R)$ of the SC-PNC for the $l$-th bidirectional channel is given by

$$P^{\text{SC-PNC}}_{\text{out,}l}(R) = \Pr \left[ l \notin \mathcal{D} \text{ or } \min[I_{1,l}^{\text{BC}}, I_{2,l}^{\text{BC}}] < \frac{R}{2} \right].$$  

(3.11)

---

4 In this case, since there is only one traffic flow in a system, the transmission rate $R_1$ of $S_1$ must be $R$, not $R/2$. 

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Then the outage probability $P_{\text{out}}^{\text{SC-PNC}}(R)$ of the SC-PNC can be obtained by taking the minimum of $\{P_{\text{out}}^{\text{SC-PNC}}(R) : l = 1, \ldots, L\}$ as follows:

$$P_{\text{out}}^{\text{SC-PNC}}(R) = \min_{l=1, \ldots, L} P_{\text{out}}^{\text{SC-PNC}}(R)$$ (3.12)

$$= \Pr \left[ \max_{l \in \mathcal{D}} \min \left[ I_{1,l}^{\text{BC}}, I_{2,l}^{\text{BC}} \right] < \frac{R}{2} \right]$$ (3.13)

$$= \Pr \left[ \max_{l \in \mathcal{D}} \left( E_R \min[X_l, Y_l] \right) < T_1 \right],$$ (3.14)

where $T_1 = 2^R - 1$. In the step from (3.12) to (3.13), we use $P_{\text{out}}^{\text{SC-PNC}}(R) = 1$ if $l \notin \mathcal{D}$. Note that the selection rule in (3.10), which maximizes the minimum mutual information of $I_{1,l}^{\text{BC}}$ and $I_{2,l}^{\text{BC}}$ in the set $\mathcal{D}$, also minimizes the outage probability in (3.13).

In the following theorem, the outage probability of the SC-PNC is derived.

**Theorem 3.1:** The exact closed-form outage probability $P_{\text{out}}^{\text{SC-PNC}}(R)$ of the SC-PNC is given by

$$P_{\text{out}}^{\text{SC-PNC}}(R) = \prod_{l=1}^{L} \Phi_l(R),$$ (3.15)

where

$$\Phi_l(R) = \begin{cases} 
\Xi_1(T_1, T_1, E_2, E_1), & \text{Case 1,} \\
\Xi_1(T_1, T_1, E_2, E_1), & \text{Case 2,} \\
\Xi_2(T_1, T_1, E_1, E_2, E_1), & \text{Case 3,} \\
\Xi_2(T_1, T_1, E_1, E_2, E_1), & \text{Case 4,} \\
\Xi_2(T_1, T_1, E_1, E_2, E_1), & \text{Case 5,} \\
\Xi_2(T_1, T_1, E_1, E_2, E_1), & \text{Case 6,} \\
\Xi_2(T_1, T_1, E_1, E_2, E_1), & \text{Case 7.} 
\end{cases}$$ (3.16)

In the above equation, Case 1: $E_R \geq \max[E_1, E_2]$; Case 2: $E_2 \geq E_R \geq E_1$ and $E_2 \leq E_R(T_1+1)$; Case 3: $E_2 \geq E_R \geq E_1$ and $E_2 > E_R(T_1+1)$; Case 4: $E_1 \geq E_R \geq E_2$ and $E_1 \leq E_R(T_1+1)$; Case 5: $E_1 \geq E_R \geq E_2$ and $E_1 > E_R(T_1+1)$; Case 6: $E_R \leq \min[E_1, E_2]$ and $E_1 + E_2 \leq E_R T_2/T_1$; Case 7: $E_R \leq \min[E_1, E_2]$ and $E_1 + E_2 > E_R T_2/T_1$, where
\[ T_2 = 2^{2R} - 1. \] Also,

\[
\Xi_1(\xi_1, \xi_2, \xi_3, \xi_4) = \begin{cases} 
\frac{1}{\xi_1 \Omega_1, l - \xi_2 \Omega_2, l} \left( \frac{T_1}{\xi_1 \Omega_1, l} + \frac{T_2 - \xi_1}{\xi_2 \Omega_2, l} \right) - \Xi_2 \Omega_2, l \Xi_2 \left( \frac{T_1}{\xi_1 \Omega_1, l} + \frac{T_2 - \xi_2}{\xi_2 \Omega_2, l} \right) , & \xi_1 \Omega_1, l \neq \xi_2 \Omega_2, l, \\
\Xi_2 \left( \frac{T_2}{\xi_1 \Omega_1, l} \right) - \left( \frac{T_2 - \xi_1}{\xi_1 \Omega_1, l} \right) \exp \left( - \frac{T_2}{\xi_1 \Omega_1, l} \right) , & \xi_1 \Omega_1, l = \xi_2 \Omega_2, l, 
\end{cases}
\]

(3.17)

\[ \Xi_2(\xi_1) = 1 - \exp(-\xi_1). \] (3.18)

**Proof:** See Appendix 3-A.

It is important to note that Theorem 3.1 can be considered as a generalization of previous works. Specifically, allowing only \( S_1 \) to transmit its symbol at the first time slot and a selected relay to retransmit the decoded symbol to \( S_2 \) at the second time slot, and taking steps similar to Appendix 3-A, \( P_{\text{SC-PNC}}^{\text{out}}(R) \) of (3.15) reduces to

\[ P_{\text{out}}^{\text{SC-PNC}}(R) = \prod_{l=1}^{L} \Xi_2 \left( \frac{T_2}{\xi_1 \Omega_1, l} + \frac{T_2}{\xi_2 \Omega_2, l} \right). \]

Then this becomes identical to the outage probability for the unidirectional SC \([50], \text{eq. (15)}\]. In this sense, therefore, our outage probability analysis in this subsection can be considered as a generalization of that of \([50] \). Furthermore, note that \( P_{\text{out}}^{\text{SC-PNC}}(R) \) of (3.15) reduces to \([46], \text{eq. (18)}\] when \( L = 1 \) and \( \mathcal{E}_1 = \mathcal{E}_2 \). Therefore, our outage probability analysis in this subsection can be considered as a generalization of that of \([46] \). In the following lemma, we present the diversity order of the SC-PNC.

**Lemma 3.1:** The SC-PNC with \( L \) relays achieves the full diversity order \( L \).

**Proof:** For simplicity, we assume \( \mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_R = \mathcal{E} \). Note that this assumption does not change the diversity order of the system \([91], [92] \). When \( \mathcal{E} \) goes to infinity, we have

\[ \lim_{\mathcal{E} \to \infty} P_{\text{out}}^{\text{SC-PNC}}(R) = \frac{C_1}{\mathcal{E}^L} + O \left( \frac{1}{\mathcal{E}^2} \right), \] (3.19)

where \( C_1 \) is a constant. That is, \( P_{\text{out}}^{\text{SC-PNC}}(R) \) decays with \( \mathcal{E} \) as \( 1/\mathcal{E}^L \). Thus, the diversity order of the SC-PNC with \( L \) relays is \( L \). \( \square \)

\( ^5 \) In \([46], \) the authors derived the outage probability of the PNC protocol when there existed a single relay in a network and the two end-sources had the same transmission power.
3.4 OR-PNC

In this section, by modifying the well-known OR, we first propose another relay selection scheme for the PNC network. Then we show this scheme achieves the same outage performance as the SC-PNC. This implies that the proposed scheme also achieves the full diversity order.

3.4.1 Proposed OR-PNC

As another relay selection scheme for a unidirectional relay network consisting of a source, a destination, and multiple relays, OR has also been studied extensively [50]–[62]. In this scheme, a single best relay was selected such that the minimum of two instantaneous SNRs of the first and second hops for each relay-path was maximized.

Similarly to the SC, however, the original OR designed for the unidirectional relay network is not directly applicable to a bidirectional relay network. In the following, therefore, the OR is modified for use in a bidirectional relay network.

We first focus on a single relay $R_l$. The $l$-th bidirectional channel involves three mutual information $\{I_{\text{MAC}}^{1,l}, I_{\text{MAC}}^{2,l}, I_{\text{MAC}}^{\text{sum},l}\}$ of (3.1)–(3.3) in the MAC phase and two mutual information $\{I_{\text{BC}}^{1,l}, I_{\text{BC}}^{2,l}\}$ of (3.4) and (3.5) in the BC phase. Note that the target rate associated with any of $\{I_{\text{MAC}}^{1,l}, I_{\text{MAC}}^{2,l}, I_{\text{BC}}^{1,l}, I_{\text{BC}}^{2,l}, I_{\text{MAC}}^{\text{sum},l}/2\}$ is $R/2$, whereas the target rate associated with $I_{\text{MAC}}^{\text{sum},l}$ is $R$. Considering the different target rates, a single best relay is selected such that $\min[I_{\text{MAC}}^{1,l}, I_{\text{MAC}}^{2,l}, I_{\text{BC}}^{1,l}, I_{\text{BC}}^{2,l}, I_{\text{MAC}}^{\text{sum},l}/2]$ is maximized over all the relays. That is, the index $l^{\text{OR-PNC}}$ of the selected relay is determined as follows:

$$l^{\text{OR-PNC}} = \arg \max_{l=1,\ldots,L} \min[I_{\text{MAC}}^{1,l}, I_{\text{MAC}}^{2,l}, I_{\text{BC}}^{1,l}, I_{\text{BC}}^{2,l}, I_{\text{MAC}}^{\text{sum},l}/2]. \quad (3.20)$$

---

6 Originally, the SC and OR were developed independently for unidirectional relay networks. Since Beres et al. proposed the SC [57], many works have been devoted to the SC [58, 59]. As another relay selection method, Bletsas et al. proposed the OR based on the “max-min” criterion [49]. Since then, this max-min criterion has been applied to numerous systems [50, 59, 62]. Recently, Bletsas et al. showed that, for the unidirectional networks only, the SC and OR had the same outage performance [50]. However, the SC or OR has never been studied for the PNC with XOR-encoding, which is the motivation of our work.

7 That is, the selection was made based on “max-min” criterion, which was also referred to as “bottleneck” criterion [49] and “best-worst” criterion [52].
In this paper, this selection scheme will be referred to as the “OR-PNC.”

As in the SC-PNC, if we assume only $S_1$ transmits its symbol at the first time slot and a selected relay retransmits the decoded symbol to $S_2$ at the second time slot, then the index $l^{\text{OR-PNC}}$ of (3.20) is simplified to $l^{\text{OR-PNC}} = \arg \max_{l=1, \ldots, L} \min[I_{1,l}^{\text{MAC}}, I_{1,l}^{\text{BC}}]$, which is identical to the unidirectional OR [50, eq. (18)]. In this sense, therefore, the relay selection of (3.20) can be considered as a generalization of the relay selection in unidirectional relay networks [50, eq. (18)].

### 3.4.2 Outage Probability of OR-PNC

In this subsection, we derive the outage probability of the OR-PNC. We first determine the outage probability of the OR-PNC focusing only on a relay $R_l$. From (3.6), one knows that the $l$-th bidirectional channel is in outage if any of the three inequalities of (3.6) is not satisfied. Therefore, the outage probability $P^{\text{OR-PNC}}_{\text{out}, l}(R)$ of the OR-PNC for the $l$-th bidirectional channel is given by

$$P^{\text{OR-PNC}}_{\text{out}, l}(R) = \Pr \left[ \min[I_{1,l}^{\text{MAC}}, I_{1,l}^{\text{BC}}, I_{2,l}^{\text{MAC}}, I_{2,l}^{\text{BC}}, I_{\text{MAC sum}, l}/2] < R/2 \right].$$

(3.21)

Then the outage probability $P^{\text{OR-PNC}}_{\text{out}, l}(R)$ of the OR-PNC is obtained by taking the minimum of \{ $P^{\text{OR-PNC}}_{\text{out}, l}(R) : l = 1, \ldots, L$ \} as follows:

$$P^{\text{OR-PNC}}_{\text{out}}(R) = \min_{l=1, \ldots, L} P^{\text{OR-PNC}}_{\text{out}, l}(R)$$

$$= \Pr \left[ \max_{l=1, \ldots, L} \min[I_{1,l}^{\text{MAC}}, I_{1,l}^{\text{MAC}}, I_{1,l}^{\text{BC}}, I_{1,l}^{\text{BC}}, I_{\text{MAC sum}, l}/2] < R/2 \right]$$

(3.22)

$$= \Pr \left[ \max_{l=1, \ldots, L} \min \left( 1 + \varepsilon_1 X_l, 1 + \varepsilon_2 Y_l, 1 + \varepsilon_R X_l, 1 + \varepsilon_R Y_l, \sqrt{1 + \varepsilon_1 X_l + \varepsilon_2 Y_l} \right) < 2R \right].$$

(3.23)

Note that the selection rule of (3.20), which maximizes the minimum of \{ $I_{1,l}^{\text{MAC}}, I_{2,l}^{\text{MAC}}, I_{1,l}^{\text{BC}}, I_{2,l}^{\text{BC}}, I_{\text{MAC sum}, l}/2$ \}, also minimizes the outage probability of (3.22). We define $W_l$ as
the minimum of \( \{ 1 + \mathcal{E}_1 X_l, 1 + \mathcal{E}_2 Y_l, 1 + \mathcal{E}_R X_l, 1 + \mathcal{E}_R Y_l, \sqrt{1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l} \} \) as follows:

\[
W_l := \min \left[ 1 + \mathcal{E}_1 X_l, 1 + \mathcal{E}_2 Y_l, 1 + \mathcal{E}_R X_l, 1 + \mathcal{E}_R Y_l, \sqrt{1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l} \right].
\] (3.24)

Then \( P_{\text{out}}^{\text{OR-PNC}}(R) \) of (3.23) can be rewritten as

\[
P_{\text{out}}^{\text{OR-PNC}}(R) = \Pr \left[ \max_{l=1,\ldots,L} W_l < 2^R \right]
= \prod_{l=1}^{L} F_{W_l}(2^R).
\] (3.25)

In the following theorem, by deriving \( P_{\text{out}}^{\text{OR-PNC}}(R) \), we show the OR-PNC is equivalent to the SC-PNC in terms of outage performance.

**Theorem 3.2:** The outage probability \( P_{\text{out}}^{\text{OR-PNC}}(R) \) of the OR-PNC is also given by (3.15), which is the outage probability \( P_{\text{out}}^{\text{SC-PNC}}(R) \) of the SC-PNC.

**Proof:** See Appendix 3-B. \(\square\)

From Theorem 3.2, one knows that the OR-PNC also achieves the full diversity order. In fact, one may intuitively expect the result of Theorem 3.2 as follows. Due to \( \min \left[ I_{1,l}^{\text{MAC}}, I_{2,l}^{\text{MAC}}, I_{\text{sum},l}^{\text{MAC}}/2 \right] \) in the OR-PNC of (3.20), the relay selection is essentially done only over the set \( \mathcal{D} \), not all the relays in the network, with probability one. Specifically, if \( \min \left[ I_{1,l}^{\text{MAC}}, I_{2,l}^{\text{MAC}}, I_{\text{sum},l}^{\text{MAC}}/2 \right] < R/2 \), then the \( l \)-th MAC is in outage, which is equivalent to \( l \notin \mathcal{D} \). As a result, the OR-PNC actually searches a single best relay over only the set \( \mathcal{D} \), and thus, the outage probability of the OR-PNC is identical to that of the SC-PNC.

**Remark 3.1:** In [50], for a unidirectional relay network, the authors showed that two outage probabilities of the original SC and OR were the same. In our work, for a bidirectional relay network, we show that two outage probabilities of the SC-PNC and OR-PNC are still the same. In this sense, Theorem 3.2 is a generalization of [50] to a bidirectional relay network. This is a very interesting result and it gives an insight into relay selection in bidirectional relay networks.
3.5 Simulation Results

In this section, we check the accuracy of the obtained outage probability of (3.15) by comparing Monte Carlo simulations. Fig. 3.2 shows the outage probabilities against $10 \log_{10} E$ of the SC-PNC and OR-PNC, where we set $L = 1, 2, 3, 4$, $R = 1$ bps/Hz, $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_R = \mathcal{E}$ and $\Omega_{1,l} = \Omega_{2,l} = 1$ for $l = 1, \cdots, L$. Fig. 3.3 shows the outage probabilities against $10 \log_{10} E$ of the SC-PNC and OR-PNC, where we set $L = 1, 2, 3, 4$, $R = 1.5$ bps/Hz, $\mathcal{E}_1 = \mathcal{E}_2/2 = \mathcal{E}_R/1.5 = \mathcal{E}$, and $\Omega_{1,l} = 1.5$ and $\Omega_{2,l} = 1$ for $l = 1, \cdots, L$. Irrespective of the SNR in Figs. 3.2 and 3.3, one can see that (3.15) exactly matches with the simulation results.

We now investigate the effect of relay location. Let $d_{S_1,R}$ denote the distance between $S_1$ and $R$, and $d_{S_2,R}$, the distance between $S_2$ and $R$, both of which are normalized by the distance between $S_1$ and $S_2$. Therefore, we have $d_{S_1,R} + d_{S_2,R} = 1$. Furthermore, we set the path loss exponent as four to model radio propagation in urban areas [79]. As a result, we set $\Omega_{1,l} = d_{S_1,R}^{-4}$ and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ where $l = 1, \cdots, L$. Fig. 3.4 shows the outage probabilities against $d_{S_1,R}$ of the SC-PNC and OR-PNC, where we set $L = 1, 2, 3, 4$, $R = 2$ bps/Hz, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_R = 10$ dB. Fig. 3.5 shows the outage probabilities against $d_{S_1,R}$ of the SC-PNC and OR-PNC, where we set $L = 1, 2, 3, 4$, $R = 3$ bps/Hz, and $\mathcal{E}_1/1.5 = \mathcal{E}_2/2 = \mathcal{E}_R = \mathcal{E}$ with $\mathcal{E} = 13$ dB. Irrespective of relay location in Figs. 3.4 and 3.5, one can see that (3.15) exactly matches with the simulation results.

3.6 Conclusions

In this paper, we have studied relay selection for the PNC bidirectional relay network consisting of two end-sources and multiple relays, where each relay adopted XOR-encoding to combine two received symbols from the two end-sources. Specifically, we have proposed two relay selection schemes for the PNC: SC-PNC and OR-PNC. For the SC-PNC, in the MAC phase, we have determined a set of relays that correctly
Figure 3.2: Outage probabilities against $10 \log_{10} \mathcal{E}$ of the SC-PNC and OR-PNC. $L = 1, 2, 3, 4$. $R = 1 \text{ bps/Hz}$. $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_R = \mathcal{E}$. $\Omega_{1, l} = \Omega_{2, l} = 1$ where $l = 1, \cdots, L$. 
Figure 3.3: Outage probabilities against $10 \log_{10} E$ of the SC-PNC and OR-PNC. $L = 1, 2, 3, 4$. $R = 1.5$ bps/Hz. $E_1 = E_2/2 = E/R/1.5 = E$. $\Omega_{1,l} = 1.5$ and $\Omega_{2,l} = 1$ where $l = 1, \cdots, L$.
Outage probabilities against $d_{S_1,R}$ of the SC-PNC and OR-PNC. $L = 1, 2, 3, 4$. $R = 2$ bps/Hz. $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_R = 10$ dB. $\Omega_{1,l} = d_{S_1,R}^{-4}$ and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ where $l = 1, \ldots, L$. 

Figure 3.4: Outage probabilities against $d_{S_1,R}$ of the SC-PNC and OR-PNC. $L = 1, 2, 3, 4$. $R = 2$ bps/Hz. $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_R = 10$ dB. $\Omega_{1,l} = d_{S_1,R}^{-4}$ and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ where $l = 1, \ldots, L$. 

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Figure 3.5: Outage probabilities against $d_{S_1,R}$ of the SC-PNC and OR-PNC. $L = 1, 2, 3, 4$. $R = 3$ bps/Hz. $E_1/1.5 = E_2/2 = E_R = E$ with $E = 13$ dB. $\Omega_{1,l} = d_{S_1,R}^{-4}$ and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ where $l = 1, \cdots, L$. 

Analysis using eq. (3.15)

Simulation for SC−PNC

Simulation for OR−PNC
decoded two received symbols from both end-sources; in the BC phase, among the relays in the determined set, we have selected a single best relay such that the minimum mutual information of the two links from each relay to the two end-sources was maximized. For the OR-PNC, we have selected a single best relay such that the minimum mutual information of both the MAC phase, including the sum capacity constraint, and the BC phase was maximized. For these two relay selection schemes, we have derived the exact outage probabilities, which were identical. Furthermore, we have shown that the SC-PNC and OR-PNC achieved the full diversity order.
Chapter 4

Relay Selection in ANC and TDBC Protocols

In this chapter, we study relay selection (RS) with the analog network coding (ANC) and time division broadcast (TDBC), which are two major amplify-and-forward (AF)-based protocols in bidirectional relay networks. We consider a bidirectional network consisting of two different end-sources and multiple relays, where each terminal has a single antenna and operates in a half-duplex mode. In this network, a single best relay is selected depending on channel conditions to help bidirectional communication between the two end-sources. Specifically, we first consider RS schemes for the ANC and TDBC protocols based on a max-min criterion to minimize the outage probabilities. Then, for the RS in the ANC protocol, we derive a closed-form expression of the outage probability; for the RS in the TDBC protocol, we derive a one-integral form of the outage probability and its lower bound in closed-form. Numerical results confirm that the closed-form expression of the ANC protocol and the one-integral form of the TDBC protocol are very accurate, and that the closed-form lower bound of the TDBC protocol is also accurate.

4.1 Introduction

Relay communication is an effective method to attain broader coverage range and to mitigate channel impairments by relaying signals in a network [3, 5]. One of the most
well-known relaying strategies is the amplify-and-forward (AF) relaying where each relay terminal samples, amplifies, and retransmits the incoming signals. In many applications of relay networks, two different end-sources may need to exchange information with the help of relays. Since this bidirectional communication can increase bandwidth efficiency, it has recently received considerable attention [28]. Many researchers have studied two major AF-based protocols in bidirectional relay networks: the analog network coding (ANC) protocol [38–42] and the time division broadcast (TDBC) protocol [43]. The ANC protocol requires two time slots to exchange information between the two end-sources. Under a half-duplex constraint, however, the ANC cannot exploit the direct-path between the two end-sources even if such direct-path physically exists. On the other hand, the TDBC protocol can utilize the direct-path even under a half-duplex constraint. However, the TDBC needs three time slots for the exchange of information. Therefore, the ANC can increase bandwidth efficiency, while the TDBC can improve reliability due to the utilization of the direct-path. For instance, in a bidirectional relay network with a single relay where the direct-path between two end-sources exists, the diversity order of the ANC protocol is still one due to a half-duplex constraint, whereas that of the TDBC protocol is two.

Relay selection (RS) in relay networks has also received considerable attention in the literature [49–52] because it can enhance system performance with simple hardware. Bletsas et al. proposed opportunistic relaying [49] and showed that opportunistic AF relaying was outage-optimal among single-RS schemes [50]. Zhao et al. showed that the AF relaying with RS provided lower outage probability and higher throughput than the AF relaying without RS except when signal-to-noise ratio (SNR) was very low [51]. Jing et al. derived the diversity order of many single-RS schemes and proposed several SNR-suboptimal multi-RS schemes [52]. However, these works have focused on RS in unidirectional networks.

Very recently, Hwang et al. considered RS in bidirectional networks [64]. In [64],
the authors proposed a criterion for RS to maximize the instantaneous sum-rate of two opposite traffic flows in the ANC protocol, and they derived the performance bounds in terms of the average sum-rate, average symbol-error rate, and outage probability. This sum-rate maximization criterion is useful to maximize the whole system capacity. On the other hand, in a multiuser system, the whole system is in outage if any user is in outage [47, 75 eq. (25)]. In order to minimize the outage of the multiuser system, therefore, the minimum mutual information among all the users must be maximized. In the ANC and TDBC protocols, there are two opposite traffic flows from two different users in the same channel; therefore, the two protocols can be considered as multiuser systems, specifically two-user systems. To minimize the outage of the ANC and TDBC protocols, therefore, the minimum mutual information of two opposite traffic flows must be maximized. This problem can be formulated as a max-min problem. Consequently, when RS is combined with ANC or TDBC, it is very useful to adopt the max-min criterion. To the best of our knowledge, however, there has been no work in which the max-min criterion was applied to RS in the ANC and TDBC protocols and the outage performance was analyzed based on this criterion. This has motivated our work. In this chapter, we consider RS for the ANC and TDBC protocols in a bidirectional relay network consisting of two different end-sources and multiple relays.

The contributions of this chapter can be summarized as follows.

- We propose RS schemes for the ANC and TDBC protocols to minimize the outage probabilities. Specifically, we adopt the max-min criterion to maximize the minimum mutual information of two opposite traffic flows.
- For the RS in the ANC protocol, we derive closed-form outage probability.
- For the RS in the TDBC protocol, we derive outage probability in a one-integral form and its lower bound in closed-form.

In particular, our work is different from [64]. In [64], a single best relay was
selected such that the sum of two mutual information of two opposite traffic flows was maximized, whereas in our work, a single best relay is selected such that the minimum mutual information of two opposite traffic flows is maximized. Note that the sum-rate maximization criterion is useful when one focuses on the total traffic flows of bidirectional communication, whereas the max-min criterion is useful when one focuses on the individual traffic flow of bidirectional communication. As a result, the two criterions complement each other, not exclude each other.

The remainder of this chapter is organized as follows. In Section 4.2, we describe the system model. In Section 4.3, we consider RS schemes based on the max-min criterion for the ANC and TDBC protocols. In Section 4.4, we derive the outage probability of the RS in the ANC protocol. In Section 4.5, we derive the outage probability of the RS in the TDBC protocol. In Section 4.6, we present simulation results. Finally, conclusions are drawn in Section 4.7.

**Notation:** We use $A := B$ to denote that $A$, by definition, equals $B$, and we use $A =: B$ to denote that $B$, by definition, equals $A$. Also, for a random variable $X$, $f_X(\cdot)$ denotes its probability distribution function (PDF), and $F_X(\cdot)$, its cumulative distribution function (CDF). Finally, $x \sim \mathcal{CN}(m, \Omega)$ indicates that $x$ is a circularly symmetric complex-valued Gaussian random variable with mean $m$ and variance $\Omega$.

### 4.2 System Model

Consider a bidirectional relay network consisting of two different end-sources and $L$ relays, where each terminal has a single antenna and operates in a half-duplex mode, and the relays adopt the AF protocol. We use $S_1$, $S_2$, and $R_l$ to denote the first source, the second source, and the $l$-th relay for $l = 1, \cdots, L$, respectively. We let $x_1$ and $x_2$ denote the information-bearing symbols transmitted from $S_1$ and $S_2$, respectively, and both symbols have unit average power. The complex channel coefficient between $S_1$ and $S_2$ is denoted by $h_0$; the complex channel coefficient between $S_1$ and $R_l$ is denoted by $h_{1,l}$; and the complex channel coefficient between $S_2$ and $R_l$ is denoted by
we assume that all the channels are reciprocal and they are modeled as follows: 
\[ h_0 \sim \mathcal{CN}(0, \Omega_0), \ h_{1,t} \sim \mathcal{CN}(0, \Omega_{1,t}), \text{ and } h_{2,t} \sim \mathcal{CN}(0, \Omega_{2,t}). \] 
Also, all the channel coefficients are assumed to be fixed over two time slots for the ANC protocol and three time slots for the TDBC protocol.

In the ANC protocol, there are two opposite traffic flows: one is from \( S_1 \) via \( R_t \) to \( S_2 \), and the other is \( S_2 \) via \( R_t \) to \( S_1 \). For the first traffic flow, \( R_t \) and \( S_2 \) are the receivers. Thus, it is reasonable to assume that \( R_t \) knows \( h_{1,t} \), and \( S_2 \) knows \( \{h_{1,t}, h_{2,l} : l = 1, \cdots, L\} \) as in the traditional unidirectional relay network \([20]–[21]\). Similarly, considering the second traffic flow, it is reasonable to assume that \( R_t \) knows \( h_{2,t} \), and \( S_1 \) knows \( \{h_{1,t}, h_{2,l} : l = 1, \cdots, L\} \). In total, we assume that both end-sources know all the channel coefficients \( \{h_{1,t}, h_{2,l} : l = 1, \cdots, L\} \), and that each relay \( R_t \) knows the channel coefficients \( h_{1,t} \) and \( h_{2,t} \). In a similar way, for the TDBC protocol, we assume that both end-sources know all the channel coefficients \( \{h_0, h_{1,t}, h_{2,l} : l = 1, \cdots, L\} \), and that each relay \( R_t \) knows the channel coefficients \( h_{1,t} \) and \( h_{2,t} \). Compared with the ANC, the TDBC needs additional channel information on \( h_0 \) at both end-sources because the TDBC utilizes the direct-path between the two end-sources, while the ANC cannot utilize the direct-path due to the half-duplex constraint even if such direct-path physically exists. The channel information assumption adopted in this chapter has been made in almost all previous works on AF-based bidirectional protocols \([38]–[42]\).

1. Note that \( R_t \) is the receiver associated only with the path from \( S_1 \) to \( R_t \); thus, \( R_t \) knows \( h_{1,t} \). On the other hand, \( S_2 \) is the receiver associated with all the paths from \( S_1 \) via \( \{R_t\}_{t=1}^L \) to \( S_2 \); thus, \( S_2 \) knows \( \{h_{1,t}, h_{2,l} : l = 1, \cdots, L\} \).
2. In some works on AF-based bidirectional protocols, even a stronger channel assumption has been made. In \([76]–[77]\), it is assumed that each relay \( R_t \) knows all the channel coefficients even for other relays. That is, each \( R_t \) knows \( \{h_{1,t}, h_{2,l} : l = 1, \cdots, L\} \). In this case, the relays may use beamforming techniques, but under the assumption of ideal frequency/time synchronization across all the relays, which does not need to be assumed in the RS.
3. Due to the higher signaling overhead, the perfect instantaneous channel knowledge of all the links might be hard for a system with a large number of relays. In practice, however, the number of relays should be restricted to a small number due to some reasons: the nature of indoor home/office applications may need only a small number of relays; timing, frequency, and phase synchronizations become very difficult for a large number of relays; and the complexity of protocol becomes very high for a large number of relays. That is, assuming a small number of relays is practical and reasonable. As a result, the burden on total signaling overhead may not be too much.

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Figure 4.1: System models for the ANC and TDBC protocols with multiple relays. In the ANC protocol, the direct-path between $S_1$ and $S_2$ is not plotted, because it cannot be utilized due to the half-duplex constraint even if such direct-path physically exists.
4.2.1 ANC Protocol with Multiple Relays

The ANC protocol has widely been studied [38]–[42], [64]. The system model for the ANC protocol with multiple relays is depicted in Fig. 4.1(a). In this protocol, at the first time slot, $S_1$ and $S_2$ transmit simultaneously their symbols, $x_1$ and $x_2$, over the multiple access channel, and the signal $r_l^{\text{ANC}}$ received by the $l$-th relay $R_l$ is given by

$$r_l^{\text{ANC}} = \sqrt{E_s h_{1,l} x_1} + \sqrt{E_s h_{2,l} x_2} + n_l^{\text{ANC}},$$

where $E_s$ is the transmission power at $S_1$ and $S_2$, and $n_l^{\text{ANC}}$ is the additive white Gaussian noise (AWGN) at $R_l$ with $n_l^{\text{ANC}} \sim \mathcal{CN}(0, 1)$. At the second time slot, $R_l$ amplifies $r_l^{\text{ANC}}$ with an amplifying coefficient $\rho_l$ and then broadcasts it to $S_1$ and $S_2$ over the broadcast channel, and the signal $y_{i,l}^{\text{ANC}}$ received by $S_i$ is given by

$$y_{i,l}^{\text{ANC}} = \sqrt{E_r \rho_l h_{i,l} r_l^{\text{ANC}}} + v_{i,l}^{\text{ANC}}$$

$$= \sqrt{E_s E_r \rho_l h_{i,l} h_{1,l} x_1} + \sqrt{E_s E_r \rho_l h_{i,l} h_{2,l} x_2} + \sqrt{E_r \rho_l h_{i,l} n_l^{\text{ANC}}} + v_{i,l}^{\text{ANC}},$$

where $E_r$ is the transmission power at $R_l$, and $v_{i,l}^{\text{ANC}}$ is the AWGN at $S_i$ with $v_{i,l}^{\text{ANC}} \sim \mathcal{CN}(0, 1)$ for $i = 1, 2$. In order to ensure that the transmission power at $R_l$ is $E_r$, the amplifying coefficient $\rho_l$ should be determined as follows:

$$\rho_l = \frac{1}{\sqrt{E_s (|h_{1,l}|^2 + |h_{2,l}|^2) + 1}}.$$  \hspace{1cm} (4.3)

With the above $\rho_l$, however, the analysis becomes very difficult. Adopting a widely-used approximation [20]–[21], therefore, we approximate $\rho_l$ of (4.3) as follows:

$$\rho_l \approx \frac{1}{\sqrt{E_s (|h_{1,l}|^2 + |h_{2,l}|^2)}}.$$  \hspace{1cm} (4.4)

This approximation was originally proposed for the high SNR regime; but it was also numerically demonstrated that the approximation was still very accurate even if the SNR was as low as 0 dB [20]. Through numerical results, in Section 4.6, we will also demonstrate that the approximate $\rho_l$ in (4.4) yields very accurate results over the entire SNR range.
Since $S_i$ knows its own symbol $x_i$ and all the channel coefficients, it can remove the self-interference, i.e. $\sqrt{E_s}E_r \rho_i h_{i,l}^2 x_1$ for $S_1$ and $\sqrt{E_s}E_r \rho_i h_{2,l}^2 x_2$ for $S_2$, from $y_{i,l}^{\text{ANC}}$ in (4.2). Therefore, $S_1$ and $S_2$ can obtain new interference-free signals $\tilde{y}_{1,l}^{\text{ANC}}$ and $\tilde{y}_{2,l}^{\text{ANC}}$, respectively, as follows:

$$\tilde{y}_{1,l}^{\text{ANC}} = \sqrt{E_s}E_r \rho_i h_{1,l} h_{2,l} x_2 + \sqrt{E_s}E_r \rho_i h_{1,l} x_1$$
$$\tilde{y}_{2,l}^{\text{ANC}} = \sqrt{E_s}E_r \rho_i h_{1,l} h_{2,l} x_1 + \sqrt{E_s}E_r \rho_i h_{2,l} x_1$$

Consequently, using the approximate $\rho_i$ in (4.4), the instantaneous SNR $\gamma_{1,l}^{\text{ANC}}$ of the signal $\tilde{y}_{1,l}^{\text{ANC}}$ in (4.5) is given by

$$\gamma_{1,l}^{\text{ANC}} = \frac{\mathcal{E}_s}{\mathcal{E}_s X_{1,l} + (\mathcal{E}_s + \mathcal{E}_r) X_{2,l}}$$

where $X_{1,l} = |h_{1,l}|^2$ and $X_{2,l} = |h_{2,l}|^2$ for $l = 1, \ldots, L$. Similarly, the instantaneous SNR $\gamma_{2,l}^{\text{ANC}}$ of the signal $\tilde{y}_{1,l}^{\text{ANC}}$ in (4.6) is given by

$$\gamma_{2,l}^{\text{ANC}} = \frac{\mathcal{E}_s}{(\mathcal{E}_s + \mathcal{E}_r) X_{1,l} + \mathcal{E}_s X_{2,l}}$$

In Section 4.3.1, based on the instantaneous SNRs $\gamma_{1,l}^{\text{ANC}}$ in (4.7) and $\gamma_{2,l}^{\text{ANC}}$ in (4.8), we will study RS for the ANC protocol.

### 4.2.2 TDBC Protocol with Multiple Relays

The TDBC protocol has also widely been studied \[43, 44\]. The system model for the TDBC protocol with multiple relays is depicted in Fig. 4.1(b). In this protocol, there are two different signal components: direct-path signals and relay-path signals. We first consider the direct-path signals. The signal $y_{1,0}^{\text{TDBC}}$ transmitted from $S_2$ and received by $S_1$ at the second time slot and the signal $y_{2,0}^{\text{TDBC}}$ transmitted from $S_1$ and received by $S_2$ at the first time slot are given by, respectively,

$$y_{1,0}^{\text{TDBC}} = \sqrt{\mathcal{E}_s} h_0 x_2 + v_{1,0}^{\text{TDBC}}; \quad y_{2,0}^{\text{TDBC}} = \sqrt{\mathcal{E}_s} h_0 x_1 + v_{2,0}^{\text{TDBC}},$$

where $v_{i,0}^{\text{TDBC}}$ is the AWGN at $S_i$ with $v_{i,0}^{\text{TDBC}} \sim \mathcal{CN}(0, 1)$ for $i = 1, 2$. Consequently, the instantaneous SNRs of the two direct-path signals, $y_{1,0}^{\text{TDBC}}$ and $y_{2,0}^{\text{TDBC}}$ in (4.9), are identical and they are denoted by $\gamma_0^{\text{TDBC}}$, where $\gamma_0^{\text{TDBC}} = \mathcal{E}_s |h_0|^2$. 

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We then consider the relay-path signals. We let $r^{TDBC}_{i,l}$ denote the signal transmitted from $S_i$ and received by $R_l$ at the first time slot, and let $r^{TDBC}_{2,l}$ denote the signal transmitted from $S_2$ and received by $R_l$ at the second time slot. Then $r^{TDBC}_{i,l}$ is given by

$$r^{TDBC}_{i,l} = \sqrt{E_s} x_i + n^{TDBC}_{i,l},$$

where $n^{TDBC}_{i,l}$ is the AWGN at $R_l$ with $n^{TDBC}_{i,l} \sim CN(0,1)$ for $i = 1, 2$. At the third time slot, $R_l$ first combines the two received signals $r^{TDBC}_{1,l}$ and $r^{TDBC}_{2,l}$, and then it broadcasts the combined signal. Specifically, the two received signals $r^{TDBC}_{1,l}$ and $r^{TDBC}_{2,l}$ are combined as follows:

$$u^{TDBC}_l = \zeta^{TDBC}_{1,l} r^{TDBC}_{1,l} + \zeta^{TDBC}_{2,l} r^{TDBC}_{2,l}. \quad (4.11)$$

Since $E[|r^{TDBC}_{i,l}|^2] = E_s X_{i,l} + 1$, we can make $u^{TDBC}_l$ in (4.11) be unit power by selecting $\zeta_{i,l}$ as follows:

$$\zeta_{i,l} = \sqrt{\frac{\alpha_{i,l}}{E_s X_{i,l} + 1}}, \quad (4.12)$$

for $i = 1, 2$. In the above equation, the value $\alpha_{i,l}$ is positive but smaller than one, i.e. $0 < \alpha_{i,l} < 1$, and the values $\alpha_{1,l}$ and $\alpha_{2,l}$ determine how $R_l$ combines the two signals $r^{TDBC}_{1,l}$ and $r^{TDBC}_{2,l}$, where $\alpha_{1,l} + \alpha_{2,l} = 1$. Note that $u^{TDBC}_l$ in (4.11) always has unit power irrespective of the values $\alpha_{1,l}$ and $\alpha_{2,l}$. As we approximated $\rho_l$ in (4.4), we also approximate $\zeta_{i,l}$ of (4.12) in the same way [20–21]:

$$\zeta_{i,l} \approx \sqrt{\frac{\alpha_{i,l}}{E_s X_{i,l}}}. \quad (4.13)$$

Through numerical results, in Section 4.6, we will also demonstrate that the approximate $\zeta_{i,l}$ in (4.13) yields very accurate results over the entire SNR range.

After combining the two received signals, $R_l$ broadcasts the combined signal $u^{TDBC}_l$ in (4.11) to $S_1$ and $S_2$, and the signal $y^{TDBC}_{i,l}$ received by $S_i$ is given by

$$y^{TDBC}_{i,l} = \sqrt{E_s} h_{i,l} u^{TDBC}_l + v^{TDBC}_{i,l}, \quad (4.14)$$

where $v^{TDBC}_{i,l}$ is the AWGN at $S_i$ with $v^{TDBC}_{i,l} \sim CN(0,1)$ for $i = 1, 2$. Similar to the ANC protocol, since $S_i$ knows its own symbol $x_i$ and all the channel coefficients, it can
remove the self-interference. Consequently, $S_1$ and $S_2$ can obtain new interference-free signals $\tilde{y}_1^{\text{TDBC}}$ and $\tilde{y}_2^{\text{TDBC}}$, respectively, as follows:

$$\tilde{y}_1^{\text{TDBC}} = \sqrt{E_s E_r} \zeta_1, l h_1,l x_2 + \sqrt{E_r} \zeta_1, l h_2,l m_1,l + v_1^{\text{TDBC}}, \quad (4.15)$$

$$\tilde{y}_2^{\text{TDBC}} = \sqrt{E_s E_r} \zeta_1, l h_1,l h_2,l x_1 + \sqrt{E_r} \zeta_1, l h_2,l m_2,l + \sqrt{E_r} \zeta_2, l h_2,l m_2,l + v_2^{\text{TDBC}}. \quad (4.16)$$

Then, using the approximate $\zeta_i, l$ in (4.13), the instantaneous SNRs $\gamma_1^{\text{TDBC}}$ of the signal $\tilde{y}_1^{\text{TDBC}}$ in (4.16) and $\gamma_2^{\text{TDBC}}$ of the signal $\tilde{y}_2^{\text{TDBC}}$ in (4.15) are given by, respectively,

$$\gamma_1^{\text{TDBC}} = \frac{\alpha_1,l E_s E_r X_1,l X_2,l}{(E_s + \alpha_2,l E_r) X_1,l + \alpha_1,l E_r X_2,l}; \quad \gamma_2^{\text{TDBC}} = \frac{\alpha_2,l E_s E_r X_1,l X_2,l}{\alpha_2,l E_r X_1,l + (E_s + \alpha_1,l E_r) X_2,l}. \quad (4.17)$$

In Section 4.3.2, based on the instantaneous relay-path SNRs $\gamma_1^{\text{TDBC}}$ and $\gamma_2^{\text{TDBC}}$ in (4.17), we will study RS for the TDBC protocol.

### 4.3 RS in ANC and TDBC Protocols

In this section, two RS schemes for the ANC and TDBC protocols are proposed based on a max-min criterion to minimize the outage probabilities.

#### 4.3.1 RS in ANC Protocol

In this subsection, we propose an RS scheme for the ANC protocol. We first formulate mutual information of two opposite traffic flows for the $l$-th relay-path of the ANC protocol: from $S_1$ via $R_l$ to $S_2$ and from $S_2$ via $R_l$ to $S_1$. For the traffic flow from $S_1$ via $R_l$ to $S_2$, using (4.7), the mutual information $I_{1,l}^{\text{ANC}}$ is given by:

$$I_{1,l}^{\text{ANC}} = \frac{1}{2} \log_2 (1 + \gamma_{1,l}^{\text{ANC}}), \quad (4.18)$$

where we use the pre-log factor $1/2$ because communication from $S_1$ to $S_2$ is done during two time slots. For the traffic flow from $S_2$ via $R_l$ to $S_1$, using (4.8), the mutual information is limited by the sum-rate constraint as well as individual rate constraints at the MAC phase because the relay detects/decodes the incoming signals from two end-sources.
information $I_{2,l}^{\text{ANC}}$ is given by

$$I_{2,l}^{\text{ANC}} = \frac{1}{2} \log_2(1 + \gamma_{2,l}^{\text{ANC}}).$$

(4.19)

In a multiuser system, the system is in outage if any user is in outage [73, eq. (25)], [47]. Since there are two opposite traffic flows from two different users in the same channel, the ANC protocol can be considered as a two-user system, which is a special case of a multiuser system. Therefore, the $l$-th relay-path of the ANC protocol is in outage if either $I_{1,l}^{\text{ANC}}$ in (4.18) or $I_{2,l}^{\text{ANC}}$ in (4.19) is smaller than a target rate. Since the two end-sources are equivalent terminals, it is fair to set the target rate of each source as $R/2$, where $R$ denotes a target rate in bps/Hz for the whole network. Then the outage probability $P_{\text{out},l}^{\text{ANC}}(R)$ of the $l$-th relay-path of the ANC protocol is given by

$$P_{\text{out},l}^{\text{ANC}}(R) = \Pr[I_{1,l}^{\text{ANC}} < R/2 \text{ or } I_{2,l}^{\text{ANC}} < R/2]$$

$$= \Pr[\min[I_{1,l}^{\text{ANC}}, I_{2,l}^{\text{ANC}}] < R/2].$$

(4.20)

Based on this outage probability for each relay-path, we now consider RS in the ANC protocol with multiple relays. The objective of RS is to choose a single relay which minimizes the outage probability of the system. To this end, a relay must be selected such that $\min[I_{1,l}^{\text{ANC}}, I_{2,l}^{\text{ANC}}]$ is maximized over all the relays. That is, the index $\hat{l}$ of the selected relay must be determined as follows:

$$\hat{l} = \arg \max_{l=1,\ldots,L} \min[I_{1,l}^{\text{ANC}}, I_{2,l}^{\text{ANC}}].$$

(4.21)

The RS in the ANC protocol is implemented in a system as follows. At the beginning of every two-time-slot, the mutual information $\{I_{i,l}^{\text{ANC}} : i = 1, 2, l = 1, \ldots, L\}$ is calculated at both sources. Then one of the sources broadcasts selection information.

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5 This RS criterion can be considered as a generalization of [12, eq. (26)], which considered RS in AF-based unidirectional relay networks.

6 In practice, RS is done every channel coherence time. Also, we can calculate the total number of pilot signals, and it is $2 + 2L$. Consequently, the signaling overhead of this selection is “$2 + 2L$” per channel coherence time.

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to all the relays, and only the selected relay $R_i$ helps bidirectional communication between $S_1$ and $S_2$ over two time slots. Specifically, at the first time slot, $S_1$ and $S_2$ simultaneously broadcast their signals. At the second time slot, only the selected single relay $R_i$ among all the relays multiplies the incoming signal, which is the sum of two signals transmitted from $S_1$ and $S_2$, with an amplifying coefficient $\rho_i$. Then $R_i$ broadcasts it to $S_1$ and $S_2$.

Using (4.20) with (4.21), the achieved system outage probability $P_{\text{out}}^{\text{ANC}}(R)$ of the RS in the ANC protocol is given by

$$
P_{\text{out}}^{\text{ANC}}(R) = \Pr \left[ \max_{i=1,\ldots,L} \min\{I_{\text{ANC}}^{\text{ANC}}, I_{2,l}^{\text{ANC}}\} < \frac{R}{2} \right].
$$

(4.22)

Note that the selection rule in (4.21), which maximizes the minimum mutual information between $I_{1,l}^{\text{ANC}}$ and $I_{2,l}^{\text{ANC}}$, also minimizes the outage probability in (4.22). In Section 4.4, using (4.22), we will derive the outage probability of the RS in the ANC protocol.

### 4.3.2 RS in TDBC Protocol

In this subsection, we propose an RS scheme for the TDBC protocol. Similar to the ANC protocol, there are two opposite traffic flows between $S_1$ and $S_2$. For the traffic flow from $S_1$ to $S_2$, two channels are involved: the relay-path channel from $S_1$ via $R_i$ to $S_2$ and the direct-path channel from $S_1$ directly to $S_2$. Then the mutual information $I_{1,l}^{\text{TDBC}}$ of the two channels is given by

$$
I_{1,l}^{\text{TDBC}} = \frac{1}{3} \log_2 (1 + \gamma_0^{\text{TDBC}} + \gamma_{1,l}^{\text{TDBC}}),
$$

(4.23)

where we use the pre-log factor $1/3$ because communication from $S_1$ to $S_2$ is done during three time slots. In the same way, for the traffic flow from $S_2$ to $S_1$, the mutual information $I_{2,l}^{\text{TDBC}}$ is given by

$$
I_{2,l}^{\text{TDBC}} = \frac{1}{3} \log_2 (1 + \gamma_0^{\text{TDBC}} + \gamma_{2,l}^{\text{TDBC}}).
$$

(4.24)
Similar to (4.20), the outage probability $P_{\text{out, } l}^{\text{TDBC}}(R)$ of the $l$-th relay-path with the direct-path of the TDBC protocol is given by

$$P_{\text{out, } l}^{\text{TDBC}}(R) = \Pr \left[ \min[I_{1,l}^{\text{TDBC}}, I_{2,l}^{\text{TDBC}}] < \frac{R}{2} \right]. \quad (4.25)$$

Taking a step similar to (4.21), we select a single relay to maximize the value $\min[I_{1,l}^{\text{TDBC}}, I_{2,l}^{\text{TDBC}}]$ over all the relays. That is, the index $\tilde{l}$ of the selected relay must be determined as follows:

$$\tilde{l} = \arg \max_{l=1, \ldots, L} \min[I_{1,l}^{\text{TDBC}}, I_{2,l}^{\text{TDBC}}]. \quad (4.26)$$

The RS in the TDBC protocol is implemented in a system as follows. At the beginning of every three-time-slot, the mutual information $\{I_{i,l}^{\text{TDBC}} : i = 1, 2, l = 1, \ldots, L\}$ is calculated at both sources. Then one of the sources broadcasts selection information to all the relays, and only the selected relay $R_{\tilde{l}}$ helps bidirectional communication between $S_1$ and $S_2$ over three time slots. Specifically, at the first time slot, $S_1$ broadcasts its signal; and at the second time slot, $S_2$ broadcasts its signal. At the third time slot, only the selected single relay $R_{\tilde{l}}$ among all the relays combines the two received signals and broadcasts the combined signal to $S_1$ and $S_2$.

Using (4.25) with (4.26), the achieved system outage probability $P_{\text{out}}^{\text{TDBC}}(R)$ of the RS in the TDBC protocol is given by

$$P_{\text{out}}^{\text{TDBC}}(R) = \Pr \left[ \max_{l=1, \ldots, L} \min[I_{1,l}^{\text{TDBC}}, I_{2,l}^{\text{TDBC}}] < \frac{R}{2} \right]. \quad (4.27)$$

Note that the selection rule in (4.26), which maximizes the minimum mutual information between $I_{1,l}^{\text{TDBC}}$ and $I_{2,l}^{\text{TDBC}}$, also minimizes the outage probability in (4.27).

In Section 4.5, using (4.27), we derive the outage probability of the RS in the TDBC protocol.
4.4 Outage Probability for RS in ANC Protocol

In this section, we derive the outage probability of the RS in the ANC protocol. Using (4.18) and (4.19), we first rewrite the outage probability \( P_{\text{ANC}}^{\text{out}}(R) \) in (4.22) as follows:

\[
P_{\text{ANC}}^{\text{out}}(R) = \Pr \left[ \max_{l=1,\ldots,L} \log_2(1 + \gamma_{1,l}^{\text{ANC}}), \log_2(1 + \gamma_{2,l}^{\text{ANC}}) < R \right]
\]

(4.28)

\[
= \Pr \left[ \max_{l=1,\ldots,L} \log_2 \left( 1 + \min[\gamma_{1,l}^{\text{ANC}}, \gamma_{2,l}^{\text{ANC}}] \right) < R \right]
\]

(4.29)

\[
= \Pr \left[ \log_2 \left( 1 + \max_{l=1,\ldots,L} \min[\gamma_{1,l}^{\text{ANC}}, \gamma_{2,l}^{\text{ANC}}] \right) < R \right].
\]

(4.30)

In order to solve the probability in (4.30), it is essential to know the CDF of \( \min[\gamma_{1,l}^{\text{ANC}}, \gamma_{2,l}^{\text{ANC}}] \). Let us define \( W_l \) as the minimum value between \( \gamma_{1,l}^{\text{ANC}} \) and \( \gamma_{2,l}^{\text{ANC}} \) as follows:

\[
W_l := \min[\gamma_{1,l}^{\text{ANC}}, \gamma_{2,l}^{\text{ANC}}].
\]

(4.31)

In the following theorem, we derive the CDF of \( W_l \).

**Theorem 4.1:** The CDF \( F_{W_l}(w) \) of \( W_l \) is given by

\[
F_{W_l}(w) = \Psi_1(w; E_s, E_s, E_r, \Omega_{1,l}, \Omega_{2,l}) + \Psi_1(w; E_s, E_s, E_r, \Omega_{2,l}, \Omega_{1,l}).
\]

(4.32)

In the above equation,

\[
\Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5)
\]

\[
= \frac{\xi_5}{\xi_4 + \xi_5} + \frac{\xi_4}{\xi_4 + \xi_5} \exp \left( -w(\xi_4 + \xi_5)(\xi_1 + \xi_2 + \xi_3) \right)
\]

\[
- \exp \left( -w \frac{\xi_2 + \xi_3}{\xi_1 \xi_3} + \frac{1}{\xi_3} \right) \left[ 2w \sqrt{\frac{\xi_2 + \xi_3}{\xi_1 \xi_3}} \frac{\xi_2 + \xi_3}{\xi_1 \xi_3} K_1 \left( 2w \sqrt{\frac{\xi_2 + \xi_3}{\xi_1 \xi_3}} \right) \right.
\]

\[
\left. - \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} \left( \frac{w(\xi_2 + \xi_3)}{\xi_1 \xi_3} \right)^{i+1} E_{i+2} \left( \frac{w}{\xi_1 \xi_3} \right) \right]
\]

(4.33)

where \( \epsilon = \xi_1(\xi_1 + \xi_3 - \xi_2)/(\xi_2(\xi_2 + \xi_3 - \xi_1)) \), \( K_\nu(\cdot) \) is the modified Bessel function of the second kind with order \( \nu \), and \( E_i(\cdot) \) is the exponential integral function.

**Proof:** See Appendix 4-A. \( \square \)

Note that \( \Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5) \) in (4.33) involves only standard functions: the exponential function, the modified Bessel function, and the exponential integral function. Also, one can calculate recursively the exponential integral function \( E_{i+1}(\mu) \) for
\(i = 1, 2, \cdots\), as follows \cite{78}:

\[
E_{i+1}(\mu) = \frac{1}{i} \left( \exp(-\mu) - \mu E_i(\mu) \right), \quad \text{for } \mu > 0.
\]

Thus, only with \(E_1(\mu)\), one can evaluate \(E_{i+1}(\mu)\) for \(i = 1, 2, \cdots\). Since \(\Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5)\) in (4.33) is given in infinite series, we test the convergence of the infinite series expression in the following lemma.

**Lemma 4.1:** The infinite series expression in (4.33) is convergent.

**Proof:** See Appendix 4-B. \(\square\)

It follows from Lemma 1 that the CDF \(F_{W_l}(w)\) of \(W_l\) in (4.32) is convergent. Using (4.30) with the CDF \(F_{W_l}(w)\) in (4.32), we derive outage probability in the following theorem.

**Theorem 4.2:** The outage probability \(P_{\text{ANC, out}}(R)\) of the RS in the AF protocol is given by

\[
P_{\text{ANC, out}}(R) = \prod_{l=1}^{L} F_{W_l}(\bar{R}), \quad (4.34)
\]

where \(\bar{R} = 2^R - 1\).

**Proof:** Let \(U_1 = \max_{l=1,\cdots,L} W_l\). Since the CDF \(F_{U_1}(w)\) is given by \(F_{U_1}(w) = \prod_{l=1}^{L} F_{W_l}(w)\), one can obtain (4.34). \(\square\)

### 4.5 Outage Probability for RS in TDBC Protocol

In this section, taking a step similar to (4.30), we first rewrite the outage probability expression \(P_{\text{out}}^{\text{TDBC}}(R)\) in (4.27) for the RS in the TDBC protocol as follows:

\[
P_{\text{out}}^{\text{TDBC}}(R) = \Pr \left[ \log_2 \left( 1 + \gamma_0^{\text{TDBC}} + \max_{l=1,\cdots,L} \min[\gamma_{1,l}^{\text{TDBC}}, \gamma_{2,l}^{\text{TDBC}}] \right) < 1.5 R \right]. \quad (4.35)
\]

Using (4.35), we derive the outage probability in a one-integral form for the RS in the TDBC protocol and its lower bound in closed-form.
4.5.1 Outage Probability in One-Integral Form

Similar to $\mathcal{W}_l$ in (4.31), we define $\mathcal{V}_l$ as the minimum value between $\gamma_{1,l}^{\text{TDBC}}$ and $\gamma_{2,l}^{\text{TDBC}}$ as follows:

$$\mathcal{V}_l := \min[\gamma_{1,l}^{\text{TDBC}}, \gamma_{2,l}^{\text{TDBC}}].$$  \hfill (4.36)

In the following theorem, we derive the CDF of $\mathcal{V}_l$.

**Theorem 4.3:** The CDF $F_{\mathcal{V}_l}(w)$ of $\mathcal{V}_l$ is given by

$$F_{\mathcal{V}_l}(w) = \begin{cases} 
\Psi_1(w; \alpha_{2,l} E_r, \alpha_{1,l} E_r, \mathcal{E}_s, \Omega_{1,l}, \Omega_{2,l}) + \Psi_1(w; \alpha_{1,l} E_r, \alpha_{2,l} E_r, \mathcal{E}_s, \Omega_{2,l}, \Omega_{1,l}), & \text{Case 4.1,} \\
\Psi_2(w; \mathcal{E}_s + \alpha_{2,l} E_r, \Omega_{1,l}, \alpha_{1,l} E_r, \Omega_{2,l}, 1 + \alpha_{2,l} E_r / \mathcal{E}_s), & \text{Case 4.2,} \\
\Psi_2(w; \alpha_{2,l} E_r, \Omega_{1,l}, (\mathcal{E}_s + \alpha_{1,l} E_r) \Omega_{2,l}, 1 + \alpha_{1,l} E_r / \mathcal{E}_s), & \text{Case 4.3,}
\end{cases}$$  \hfill (4.37)

where Case 4.1: $0 \leq \alpha_{1,l} \leq 1/2$ and $\mathcal{E}_s \geq (1 - 2\alpha_{1,l}) E_r$ or $1/2 < \alpha_{1,l} \leq 1$ and $\mathcal{E}_s \geq (2\alpha_{1,l} - 1) E_r$; Case 4.2: $0 \leq \alpha_{1,l} \leq 1/2$ and $\mathcal{E}_s < (1 - 2\alpha_{1,l}) E_r$; and Case 4.3: $1/2 < \alpha_{1,l} \leq 1$ and $\mathcal{E}_s < (2\alpha_{1,l} - 1) E_r$. In the above equation,

$$\Psi_2(w; \xi_1, \xi_2, \xi_3) = 1 - \frac{2w \xi_3}{\sqrt{\xi_1 \xi_2}} \exp \left( -w \xi_3 \left( \frac{1}{\xi_1} + \frac{1}{\xi_2} \right) \right) K_1 \left( \frac{2w \xi_3}{\sqrt{\xi_1 \xi_2}} \right).$$  \hfill (4.38)

**Proof:** See Appendix 4-C.

Using (4.35) with the CDF $F_{\mathcal{V}_l}(w)$ in (4.37), we derive outage probability in the following theorem.

**Theorem 4.4:** The outage probability $P_{\text{out}}^{\text{TDBC}}(R)$ of the RS in the TDBC protocol is given by

$$P_{\text{out}}^{\text{TDBC}}(R) = \int_0^R \left( \prod_{l=1}^L F_{\mathcal{V}_l}(\tilde{R} - \eta) \right) \frac{\exp \left( -\eta / (\mathcal{E}_s \Omega_0) \right) \mathcal{E}_s \Omega_0}{\mathcal{E}_s \Omega_0} d\eta,$$  \hfill (4.39)

where $\tilde{R} = 2^{1.5R} - 1$.

**Proof:** Let $U_2 = \max_{l=1,\ldots,L} \mathcal{V}_l$. Then $P_{\text{out}}^{\text{TDBC}}(R)$ in (4.35) can be given by

$$P_{\text{out}}^{\text{TDBC}}(R) = \Pr \left[ \log_2 \left( 1 + \gamma_0^{\text{TDBC}} + U_2 \right) < 1.5R \right]$$

$$= \int_0^R \Pr[U_2 < \tilde{R} - \eta] f_{\gamma_0^{\text{TDBC}}}(\eta) d\eta.$$  \hfill (4.40)
Since the CDF $F_{\gamma_2}(w)$ is given by $F_{\gamma_2}(w) = \prod_{i=1}^{L} F_{\gamma_i}(w)$ and $\gamma_{\text{TDBC}}^{\text{TD}}$ is an exponentially distributed random variable with mean $E_s \Omega_0$, one can obtain (4.39). □

Although the obtained outage probability $P_{\text{out}}^{\text{TDBC}}(R)$ in (4.39) is very accurate, the final expression is given in a one-integral form, which requires numerical integration. Therefore, it should be also useful to derive a closed-form bound of the outage probability. In the next subsection, we derive a lower bound of the outage probability in closed-form, which does not involve any numerical integration.

### 4.5.2 Lower Bound of Outage Probability in Closed-Form

In the performance analysis of relay networks, the following inequality has widely been used [49, 20]: $\theta_1 \theta_2 / (\theta_1 + \theta_2) < \min[\theta_1, \theta_2]$, where $\theta_1 > 0$ and $\theta_2 > 0$. Using this inequality, the minimum value between $\gamma_{1,l}^{\text{TDBC}}$ and $\gamma_{2,l}^{\text{TDBC}}$, $\min[\gamma_{1,l}^{\text{TDBC}}, \gamma_{2,l}^{\text{TDBC}}]$, is upper-bounded by

$$\min[\gamma_{1,l}^{\text{TDBC}}, \gamma_{2,l}^{\text{TDBC}}] < \min \left[ \min \left[ \frac{E_s X_{1,l}}{E_s + \alpha_2,l E_r X_{2,l}}, \frac{\alpha_1,l E_s E_r X_{1,l}}{E_s + \alpha_1,l E_r} \right], \min \left[ \frac{\alpha_2,l E_s E_r X_{1,l}}{E_s + \alpha_1,l E_r}, \frac{E_s X_{1,l}}{E_s + \alpha_2,l E_r X_{2,l}} \right] \right]$$

$$= \min \left[ \frac{E_s X_{1,l}}{E_s + \alpha_2,l E_r X_{2,l}}, \frac{\alpha_2,l E_s E_r X_{1,l}}{E_s + \alpha_1,l E_r} \right] =: \mathcal{V}_l^U. \tag{4.41}$$

In the following lemma, we derive the CDF $F_{\mathcal{V}_l^U}(w)$ of $\mathcal{V}_l^U$.

**Lemma 4.3:** The CDF $F_{\mathcal{V}_l^U}(w)$ of $\mathcal{V}_l^U$ is given by

$$F_{\mathcal{V}_l^U}(w) = 1 - \exp \left( -w \beta(l) \right), \tag{4.42}$$

where

$$\beta(l) = \begin{cases} \frac{\alpha_2,l(E_s + \alpha_2,l E_r)\Omega_{1,l} + \alpha_1,l(E_s + \alpha_1,l E_r)\Omega_{2,l}}{\alpha_1,l \alpha_2,l E_s E_r \Omega_{1,l} + \alpha_1,l \alpha_2,l E_s \Omega_{2,l}}, & \text{Case 4.1,} \\ \frac{\alpha_1,l \alpha_2,l E_s E_r \Omega_{1,l} + \alpha_1,l \alpha_2,l E_s \Omega_{2,l}}{\alpha_1,l E_s \Omega_{1,l} + \alpha_2,l \Omega_{1,l} + \alpha_1,l \Omega_{2,l}}, & \text{Case 4.2,} \\ \frac{\alpha_1,l \alpha_2,l E_s \Omega_{1,l} + \alpha_1,l \Omega_{2,l}}{\alpha_2,l \alpha_1,l E_s \Omega_{1,l} + \alpha_1,l \Omega_{2,l}}, & \text{Case 4.3.} \end{cases} \tag{4.43}$$

**Proof:** See Appendix 4-D. □

Note that the CDF $F_{\mathcal{V}_l^U}(w)$ in (4.42) is expressed by the exponential function. Taking a step similar to (4.39) and using the CDF $F_{\mathcal{V}_l^U}(w)$ in (4.42), we derive a closed-form lower bound of outage probability in the following theorem.
Theorem 4.5: The closed-form lower bound \( P_{\text{out, LB}}^{\text{TDBC}}(R) \) of the outage probability of the RS in the TDBC protocol is given by

\[
P_{\text{out, LB}}^{\text{TDBC}}(R) = 1 - \exp \left( - \frac{\hat{R}}{\mathcal{E}_s \Omega_0} \right) + \sum_{l=1}^{L} (-1)^l \sum_{m_1=1}^{L} \cdots \sum_{m_l=1}^{L} \frac{\exp \left( - \hat{R} \sum_{i=1}^{l} \beta(m_i) \right) - \exp \left( - \hat{R} / (\mathcal{E}_s \Omega_0) \right)}{1 - \mathcal{E}_s \Omega_0 \sum_{i=1}^{l} \beta(m_i)}.
\]

(4.44)

Proof: See Appendix 4-E.

Note that since the lower bound \( P_{\text{out, LB}}^{\text{TDBC}}(R) \) of (4.44) is given in closed-form, it does not require any numerical integration.

4.6 Simulation Results

In this section, we first discuss the convergence speed of the series expression in (4.33), which was analytically shown to be convergent in Section IV. Then, for the proposed RS in the ANC and TDBC protocols, we check the accuracy of the obtained outage probabilities by comparing our analysis with Monte Carlo simulations. Finally, we compare the RS in the ANC protocol with the RS in the TDBC protocol.

Firstly, we check the convergence speed of the series expression in (4.33). We let \( \hat{\Psi}^{(N)}_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5) \) denote a truncated version of \( \Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5) \) with truncation window size \( N \), i.e., \( \Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5) \) of (4.33) with \( \sum_{i=0}^{\infty} \) replaced by \( \sum_{i=0}^{N} \). Then we define \( \Xi(N) \) as the truncation error normalized by \( \Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5) \) as follows:

\[
\Xi(N) := \left| \frac{\Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5) - \hat{\Psi}^{(N)}_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5)}{\Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5)} \right| = \left| \exp \left( -w \left( \frac{\xi_2 + \xi_4}{\xi_1 \xi_2 \xi_3} + \frac{1}{\xi_3 \xi_5} \right) \right) \right| \sum_{i=N+1}^{\infty} \frac{(-1)^i}{i!} \frac{\left( \frac{w (\xi_2 + \xi_4)}{\xi_1 \xi_3 \xi_5} \right)^{i+1}}{\Psi_1(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5)} \cdot (4.45)
\]

Fig. 4.2 shows the normalized truncation error \( \Xi(N) \) against truncation window size \( N \). We set \( w = 2, \xi_1 = \xi_2 = \xi_3 = \mathcal{E}, \) and \( \xi_4 = \xi_5 = 1 \). For various \( \mathcal{E} \)
values, $\mathcal{E} = 10, 20, 30$ dB, we can see the normalized truncation error is very small even with a small value $N$. For instance, the normalized truncation error is smaller than $10^{-7}$ when $\mathcal{E} = 20$ dB and $N = 1$. Therefore, even with a small value $N$, substituting $\hat{\Psi}^{(N)}_i(w; \xi_1, \xi_2, \xi_3, \xi_4, \xi_5)$ into (4.32) and (4.37) gives very accurate closed-form approximations $\hat{F}^{(N)}_{W_i}(w)$ and $\hat{F}^{(N)}_{V_i}(w)$ to actual $F_{W_i}(w)$ and $F_{V_i}(w)$, respectively. Using $\hat{F}^{(N)}_{W_i}(w)$ and $\hat{F}^{(N)}_{V_i}(w)$, in the following, we calculate outage probabilities of (4.34) and (4.39).

Secondly, we check the accuracy of the obtained outage probability of (4.34). Fig. 4.3 shows the outage probability against $10 \log_{10} \mathcal{E}$ of the RS in the ANC protocol. We set $L = 1, 2, 3, 4$, $R = 1$ bps/Hz, $\mathcal{E}_s = \mathcal{E}_r = \mathcal{E}$, and $\Omega_{1,l} = \Omega_{2,l} = 1$ where $l = 1, \cdots, L$. Also, we set $N = 1$ in $\hat{F}^{(N)}_{W_i}(w)$ for $F_{W_i}(w)$ of (4.34). From Fig. 4.3, we can see that (4.34) is very accurate even with $N = 1$. In Fig. 4.4 we investigate the effect of relay
location. Let $d_{S_1,R}$ denote the distance between $S_1$ and $R_l$, and $d_{S_2,R}$, the distance between $S_2$ and $R_l$, both of which are normalized by the distance between $S_1$ and $S_2$. Therefore, we have $d_{S_1,R} + d_{S_2,R} = 1$. Furthermore, we set the path loss exponent as four to model radio propagation in urban areas. As a result, we set $\Omega_{1,l} = d_{S_1,R}^{-4}$ and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ where $l = 1, \cdots, L$. Fig. 4.4 shows the outage probability against $d_{S_1,R}$ of the RS in the ANC protocol. We set $L = 1, 2, 3, 4$, $R = 1.5$ bps/Hz, and $\mathcal{E}_s = \mathcal{E}_r = 10$ dB. Also, we set $N = 1$ in $\hat{F}^{(N)}_{W_l}(w)$ for $F_{W_l}(w)$ of (4.34). Irrespective of relay location in Fig. 4.4, we can see that (4.34) is very accurate even with $N = 1$.

Thirdly, we check the accuracy of the obtained outage probabilities of (4.39) and (4.44). Fig. 4.5 shows the outage probability against $10 \log_{10} \mathcal{E}$ of the RS in the TDBC
Figure 4.4: Outage probability against $d_{S_1,R}$ of the RS in the ANC protocol. $L = 1, 2, 3, 4$. $R = 1.5$ bps/Hz. $E_s = E_r = 10$ dB. $\Omega_{1,l} = d_{S_1,R}^{-4}$ and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ where $l = 1, \ldots, L$. $N = 1$ in $F_{W_l}(w)$ for $F_{W_l}(w)$ of (4.34).
Figure 4.5: Outage probability against $10 \log_{10} E$ of the RS in the TDBC protocol. $L = 1, 2, 3, 4$. $R = 1$ bps/Hz. $E_s = E_r = E$. $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$ and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ where $l = 1, \cdots, L$. $N = 2$ in $\hat{F}_{V_i}^{(N)}(w)$ for $F_{V_i}(w)$ of (4.39).

The TDBC protocol. We set $L = 1, 2, 3, 4$, $R = 1$ bps/Hz, $E_s = E_r = E$, $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ where $l = 1, \cdots, L$. Also, we set $N = 2$ in $\hat{F}_{V_i}^{(N)}(w)$ for $F_{V_i}(w)$ of (4.39). From Fig. 4.5 we can see that (4.39) is very accurate even with $N = 2$, and that (4.44) is also accurate. As in Fig. 4.4 we investigate the effect of relay location. Fig. 4.6 shows the outage probability against $d_{S_1,R}$ of the RS in the TDBC protocol. We set $L = 1, 2, 3, 4$, $R = 2$ bps/Hz, $E_s = E_r = 15$ dB, $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ where $l = 1, \cdots, L$. Also, we set $N = 2$ in $\hat{F}_{V_i}^{(N)}(w)$ for $F_{V_i}(w)$ of (4.39). Irrespective of relay location in Fig. 4.6 we can see that (4.39) is very accurate even with $N = 2$, and (4.44) is also accurate.

Finally, we compare the RS with the ANC protocol and the RS with the TDBC protocol. Fig. 4.7 shows the outage probabilities against $10 \log_{10} E$ of the RS with
Figure 4.6: Outage probability against $d_{S_1, R}$ of the RS in the TDBC protocol. $L = 1, 2, 3, 4$. $R = 2$ bps/Hz. $E_s = E_r = 15$ dB. $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ where $l = 1, \cdots, L$. $N = 2$ in $\hat{V}_l^{(N)}(w)$ for $F_{V_l}(w)$ of (4.39).
the ANC protocol and the RS with the TDBC protocol. We set $L = 4$, $R = 1, 2, 3, 4$ bps/Hz, $\mathcal{E}_s = \mathcal{E}_r = \mathcal{E}$, $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ for the RS in the TDBC protocol where $l = 1, \cdots, L$. Also, we set $N = 2$ in both $\hat{F}_{W_l}(w)$ for $F_{W_l}(w)$ of (4.34) and $\hat{F}_{V_l}(w)$ for $F_{V_l}(w)$ of (4.39). For the RS with the ANC protocol and the RS with the TDBC protocol, it is well-known that the diversity orders are $L$ and $L + 1$, respectively, and the multiplexing gains are 1 and $2/3$, respectively. Thus, in the high SNR regime, the RS with the TDBC protocol always outperforms the RS with the ANC protocol irrespective of the target rate $R$. In the low and medium SNR regimes, however, one scheme outperforms the other scheme depending on $R$ and SNR value. As in Figs. 4.4 and 4.6, we investigate the effect of relay location. Fig. 4.8 shows the outage probabilities against $d_{S_1,R}$ of the RS with the ANC protocol and the RS with the TDBC protocol. We set $L = 2$, $R = 1, 2, 3$ bps/Hz, $\mathcal{E}_s = \mathcal{E}_r = 15$ dB, $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ for the RS in the TDBC protocol where $l = 1, \cdots, L$. Also, we set $N = 2$ in both $\hat{F}_{W_l}(w)$ for $F_{W_l}(w)$ of (4.34) and $\hat{F}_{V_l}(w)$ for $F_{V_l}(w)$ of (4.39). Irrespective of relay location in Fig. 4.8, as the target rate $R$ increases, we can see the RS with the ANC protocol outperforms the RS with the TDBC protocol.

### 4.7 Conclusions

In this chapter, we have studied RS for the ANC and TDBC protocols in a bidirectional relay network consisting of two end-sources and multiple relays. A single best relay was selected depending on channel conditions to help bidirectional communication between the two end-sources. Specifically, we have selected a single best relay based on a max-min criterion to minimize the outage probabilities of the ANC and TDBC protocols. Then, for the RS in the ANC protocol, we have derived a closed-form expression of the outage probability. Also, for the RS in the TDBC protocol, we have derived a one-integral form of the outage probability and its lower bound in closed-form. For future work, we will investigate RS with optimal power allocation.
Figure 4.7: Outage probabilities against $10\log_{10} E$ of the RS with the ANC protocol and the RS with the TDBC protocol. $L = 4$. $R = 1, 2, 3, 4$ bps/Hz. $E_s = E_r = E$. $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ for the RS with the TDBC protocol where $l = 1, \cdots, L$. $N = 2$ in both $\hat{F}^{(N)}_{V_l}(w)$ for $F_{V_l}(w)$ of (4.34) and $\hat{F}^{(N)}_{V_l}(w)$ for $F_{V_l}(w)$ of (4.39).
Figure 4.8: Outage probabilities against $d_{S_1,R}$ of the RS with the ANC protocol and the RS with the TDBC protocol. $L = 2$. $R = 1, 2, 3$ bps/Hz. $\mathcal{E}_s = \mathcal{E}_r = 15$ dB. $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$, and $\alpha_{1,l} = \alpha_{2,l} = 1/2$ for the RS with the TDBC protocol where $l = 1, \cdots, L$. $N = 2$ in both $\hat{F}_W^\mathcal{W}_l(w)$ for $F_W(w)$ of (4.34) and $\hat{F}_V^\mathcal{W}_l(w)$ for $F_V(w)$ of (4.39).
when each terminal has a single antenna, and RS with precoding or beamforming when each terminal has multiple antennas.
Chapter 5

Joint Relay-and-Source Selection

Relay selection (RS) has widely been studied in the literature, and opportunistic source selection (OSS) protocol with a single relay was recently proposed [15]. Since RS and OSS could individually improve the performance of relay networks, optimum combining of RS and OSS is an interesting topic. In this chapter, we combine RS and OSS optimally in the sense that the mutual information is maximized, and we propose a joint RS-OSS protocol in an amplify-and-forward (AF)-based bidirectional relay network, which consists of two different end-sources and multiple relays. In this network, a best source is selected to transmit data to the other source with the help of a selected best relay in an opportunistic manner depending on channel conditions. Then, in order to show the performance of the joint RS-OSS, we derive the outage probability and average bit-error rate (BER) for $M$-quadrature amplitude modulation (QAM). Numerical results confirm that the derived outage probability and average BER expressions are very accurate. Also, we find that the proposed joint RS-OSS considerably outperforms both RS and OSS in terms of the outage probability and the average BER, and that the performance is highly dependent on relay location. The obtained outage probability and average BER will help the design of reliable bidirectional relay networks in determining the system parameters such as relay location and the transmission power at source and relay.
5.1 Introduction

Relay communication has been shown to be an effective means to attain broader coverage range and to mitigate channel impairments [3, 5]. For many applications in relay networks, two different end-sources may need to exchange information with the help of relays. This bidirectional communication has recently received considerable attention [28]. Many researchers have studied several different protocols in bidirectional relay networks: the ANC [38–42], the PNC [29–37], while the TDBC [43, 44]. Although the ANC and PNC protocols increase the bandwidth efficiency, their diversity order is only one because they do not utilize the direct-path between two end-sources due to a half-duplex constraint even if such direct-path physically exists. On the other hand, the TDBC protocol achieves full diversity order two.

Although the ANC, PNC, and TDBC have shown to be effective protocols for bidirectional communication, they do not exploit time-varying channel fluctuations, which can be potentially utilized to further improve the performance. In order to exploit the channel fluctuations, recently, Yi et al. proposed the opportunistic source selection (OSS) protocol with a single relay for an amplify-and-forward (AF)-based bidirectional network [45]. In this protocol, two-way communication between two end-sources was supported with the help of the single relay in an opportunistic manner depending on channel conditions. That is, a best source out of two end-sources is selected to transmit data to the other source with the help of the single relay. The authors derived lower bounds of the outage probability and average bit-error rate (BER), and they numerically showed that the OSS protocol provided higher reliability compared with the ANC and TDBC protocols in terms of the outage probability and average BER. Very recently, Liu et al. considered the OSS protocol with a single relay for a decode-and-forward (DF)-based bidirectional network, and they derived the outage probability [46].

On the other hand, relay selection (RS) has also widely been considered as an
attractive technique in unidirectional networks [49]–[62]. Bletsas et al. showed that opportunistic DF relaying was outage-optimal, and that opportunistic AF relaying was also outage-optimal among single-RS schemes [50]. Beres et al. showed that RS outperformed distributed space-time codes in terms of the outage probability [57]. Very recently, Oechtering et al. considered RS in a DF-based bidirectional network to maximize the achievable rate, and they showed that RS realized the same multiuser diversity order as distributed beamforming [63].

Since OSS and RS could individually improve the performance of relay networks, optimum combining of RS and OSS is an interesting and important issue to further improve the performance considerably. This has motivated our work. In this chapter, to maximize the mutual information, we first propose joint RS-OSS in a bidirectional relay network consisting of two different end-sources and multiple relays. It is assumed that the relays adopt the AF protocol and the two end-sources have the same transmission power. In this network, a best source is selected to transmit data to the other source with the help of a selected best relay in an opportunistic manner depending on channel conditions. Then we derive the outage probability in a one-integral form and its lower bound in closed-form. Finally, for $M$-quadrature amplitude modulation (QAM), we obtain the average bit-error rate (BER) in a one-integral form and its lower bound in closed-form.

The remainder of this chapter is organized as follows. In Section 5.2, we describe the system model and review the previous RS and OSS. Then we propose joint RS-OSS. In Section 5.3, we analyze cumulative distribution functions (CDFs) and probability density function (PDF) of a certain random variable, which is essential for derivation of outage probability and average BER. In Section 5.4, we derive the outage probability of joint RS-OSS for a network with and without a direct-path. In Section 5.5, we derive the average BER of joint RS-OSS for a network with and without a direct-path adopting $M$-QAM. In Section 5.6, we present extensive simulation results. Finally, conclusions are drawn in Section 5.7.
\textit{Notation:} We use \( A := B \) to denote that \( A \), by definition, equals \( B \), and we use \( A =: B \) to denote that \( B \), by definition, equals \( A \). Also, for a random variable \( X \), \( f_X(\cdot) \) denotes its PDF, and \( F_X(\cdot) \), its CDF. Finally, \( x \sim \mathcal{CN}(m, \Omega) \) indicates that \( x \) is a circularly symmetric complex-valued Gaussian random variable with mean \( m \) and variance \( \Omega \).

\section{System Model, RS, OSS, and joint RS-OSS}
\subsection{System Model}

Consider a bidirectional relay network consisting of two different end-sources and \( L \) relays, where each terminal has a single antenna and operates in a half-duplex mode. We use \( S_1, S_2, \) and \( R_l \) to denote the first source, the second source, and the \( l \)-th relay for \( l = 1, \cdots, L \), respectively. We assume that the relays adopt the AF protocol. The transmission power either at \( S_1 \) or at \( S_2 \) is identical and it is denoted by \( E_s \); and the transmission power at \( R_l \) is denoted by \( E_r \). The complex channel coefficient between \( S_1 \) and \( S_2 \) is denoted by \( h_0 \); the complex channel coefficient between \( S_1 \) and \( R_l \) is denoted by \( h_{1,l} \); and the complex channel coefficient between \( S_2 \) and \( R_l \) is denoted by \( h_{2,l} \). We assume that all the channels are reciprocal\(^1\) and that each channel coefficient is fixed over channel coherence time\(^2\) with \( h_0 \sim \mathcal{CN}(0, \Omega_0) \), \( h_{1,l} \sim \mathcal{CN}(0, \Omega_{1,l}) \), and \( h_{2,l} \sim \mathcal{CN}(0, \Omega_{2,l}) \). Also, for coherent detection at the two end-sources, we assume that both end-sources know all the channel coefficients \( \{h_0, h_{1,l}, h_{2,l} : l = 1, \cdots, L\} \) and each relay \( R_l \) knows the channel coefficients \( h_{1,l} \) and \( h_{2,l} \); these are assumptions which have been made in almost all previous works\(^3\) on the AF-based bidirectional protocols.

\(^1\) Under the assumption that the channels are not reciprocal, our system model is similar to the case of having two separate source-destination pairs: one is from \( S_1 \) via \( R_l \) to \( S_2 \), and the other is \( S_2 \) via \( R_l \) to \( S_1 \).

\(^2\) Each channel might have different channel coherence time, and each channel coherence time should be greater than at least two time slots.

\(^3\) In some works on AF-based bidirectional protocols, even a stronger channel assumption has been made. In [76, 77], it is assumed that each relay \( R_l \) knows all the channel coefficients even for other relays. That is, each \( R_l \) knows \( \{h_{1,l}, h_{2,l} : l = 1, \cdots, L\} \). In this case, the relays may use beamforming techniques, but under the assumption of ideal frequency/time synchronization across all the relays, which does not need to be assumed in the RS.
such as the ANC and TDBC protocols [38–44]. Finally, the noise associated with every channel is modeled as a mutually independent additive white Gaussian noise (AWGN) with zero mean and unit variance.

There are two different signal components: a direct-path signal component and a relay-path signal component. For the direct-path signal, the instantaneous signal-to-noise ratio (SNR) either from $S_1$ to $S_2$ or from $S_2$ to $S_1$ is identical and it is denoted by $\gamma_0$, where $\gamma_0 = \mathcal{E}_s|h_0|^2$. For the relay-path signal, the instantaneous SNRs are different depending on which source transmits the signal. If we assume $S_1$ is the transmitter and $S_2$ is the receiver, the instantaneous SNR $\gamma_{1,l}$ of the signal transmitted from $S_1$ and received by $S_2$ via $R_l$ is given by [20]

$$\gamma_{1,l} = \frac{\mathcal{E}_s\mathcal{E}_r X_l Y_l}{\mathcal{E}_s X_l + \mathcal{E}_r Y_l + 1} \approx \frac{\mathcal{E}_s\mathcal{E}_r X_l Y_l}{\mathcal{E}_s X_l + \mathcal{E}_r Y_l}, \quad (5.1)$$

where $X_l = |h_{1,l}|^2$ and $Y_l = |h_{2,l}|^2$ for $l = 1, \cdots, L$. In the step from (5.1) to (5.2), we adopt a widely-used approximation [20–21]. Although the authors of [20] originally proposed this approximation for the high SNR regime, they numerically demonstrated that it was still very accurate even if the SNR was as low as 0 dB. In Section 5.6, we will also numerically demonstrate that the approximated SNR of (5.2) yields very accurate results over all the SNR range. On the other hand, if we assume $S_2$ is the transmitter and $S_1$ is the receiver, the instantaneous SNR $\gamma_{2,l}$ of the signal transmitted from $S_2$ and received by $S_1$ via $R_l$ is given by [20]

$$\gamma_{2,l} = \frac{\mathcal{E}_s\mathcal{E}_r X_l Y_l}{\mathcal{E}_s Y_l + \mathcal{E}_r X_l + 1} \approx \frac{\mathcal{E}_s\mathcal{E}_r X_l Y_l}{\mathcal{E}_s Y_l + \mathcal{E}_r X_l}, \quad (5.3)$$

Due to the higher signaling overhead, knowing all the channel coefficients might be hard for some scenarios such as in fast fading channels and/or with a number of relays. On the other hand, in many other scenarios such as in slow fading channels and with small number of relays, it becomes easier and reasonable. Since channel estimation and signaling need to be done every channel coherence time, not every transmission, the burden of channel information signaling may not be much if the channel coherence time is sufficiently longer than a symbol duration.

The approximation can be obtained by selecting the amplifying coefficient at $R_l$ as $\rho = \sqrt{1/(\mathcal{E}_s X_l)}$ [20].
In this chapter, based on the instantaneous SNRs $\gamma_{1,l}$ in (5.2) and $\gamma_{2,l}$ in (5.4) for the relay-path signal, we will propose joint RS-OSS. Before this, in the next two subsections, we briefly review the previous works on RS and OSS: RS in unidirectional networks and the OSS protocol in bidirectional networks.

5.2.2 RS in Unidirectional Networks

The RS has been widely studied in the unidirectional network [49]–[62]. Without loss of generality, we suppose $S_1$ is the transmitter and $S_2$ is the receiver. At the beginning of each minimum coherence time, the instantaneous SNRs $\{\gamma_{1,l}\}_{l=1}^{L}$ are calculated at $S_2$. In order to maximize the mutual information, the best relay $R_{l_{RS}}$ associated with the maximum $\gamma_{1,l_{RS}}$ is selected [50], eq. (26), where the index $l_{RS}$ of the selected relay is determined by $l_{RS} = \arg \max_{l=1,\ldots,L} \gamma_{1,l}$. The selected relay $R_{l_{RS}}$ helps communication from $S_1$ to $S_2$ over minimum coherence time. At the first time slot, $S_1$ transmits the signal to $S_2$ and $R_{l_{RS}}$; at the second time slot, $R_{l_{RS}}$ multiplies the incoming signal with an amplifying coefficient $\rho$ and retransmits it to $S_2$. A system with RS gives a higher throughput and lower outage probability than a system without RS [51]. In this chapter, RS in the unidirectional network will be referred to as “conventional RS.”

5.2.3 OSS in Bidirectional Networks

The OSS protocol has been recently proposed for a single-relay bidirectional communication system in order to exploit the channel fluctuations [45]. When there is a single relay ($L = 1$) as assumed in [45], there are only two relay-path SNRs $\gamma_{1,1}$ from $S_1$ via $R_1$ to $S_2$ in (5.2), and $\gamma_{2,1}$ from $S_2$ via $R_1$ to $S_1$ in (5.4). At the beginning of each minimum coherence time, the two SNRs $\gamma_{1,1}$ and $\gamma_{2,1}$ are calculated at both sources. Then, depending on the SNRs, only one source transmits the signal to the other source with the help of the single relay. Specifically, if $\gamma_{1,1} \geq \gamma_{2,1}$, only $S_1$ trans-

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6 Minimum coherence time means the minimum of coherence time values of all the channels.
mits the signal to $S_2$ with the help of $R_1$. If $\gamma_{1,1} < \gamma_{2,1}$, only $S_2$ transmits the signal to $S_1$ with the help of $R_1$. Although the OSS protocol supports one-way communication either from $S_1$ to $S_2$ or from $S_2$ to $S_1$ at a time instance, it can support two-way communication between $S_1$ and $S_2$ over time-varying channels in a long term.

5.2.4 Proposed Joint RS-OSS in Bidirectional Networks

In this subsection, we now propose a novel joint RS-OSS scheme in bidirectional networks. In the conventional RS, a single best relay is selected; in the OSS with a single relay, a single best source is selected. In joint RS-OSS, therefore, both a best relay and a best source must be jointly selected. That is, the joint RS-OSS is a joint relay-source selection problem. In this chapter, we make the joint selection optimally in the sense that the mutual information is maximized.

There are two possible traffic flows: one is from $S_1$ to $S_2$ with the help of $R_l$, and the other is from $S_2$ to $S_1$ with the help of $R_l$. For the first traffic flow, two channels are involved: the relay-path channel from $S_1$ via $R_l$ to $S_2$ and the direct-path channel from $S_1$ directly to $S_2$. Similarly, for the second traffic flow, two channels are involved: the relay-path channel from $S_2$ via $R_l$ to $S_1$ and the direct-path channel from $S_2$ directly to $S_1$. For the $i$-th traffic flow, the mutual information $I_{i,l}$ is given by \[ I_{i,l} = \frac{1}{2} \log_2(1 + \gamma_0 + \gamma_{i,l}), \] \[ (5.5) \]

where $i = 1, 2$ and $l = 1, \ldots, L$. In the above equation, we use the pre-log factor $1/2$ because communication from $S_1$ to $S_2$ is done within two time slots. The joint

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7 Because the OSS allows only a best end-source to transmit, it can achieve the so-called *multiuser diversity*.

8 Traditional bidirectional protocols such as the PNC, ANC, and TDBC protocols support two-way communication concurrently, while the OSS protocol supports two-way communication opportunistically based on instantaneous channel conditions. This is the main difference between the traditional bidirectional protocols and the OSS protocol, and the difference makes the OSS protocol more reliable than the traditional bidirectional protocols. However, the bandwidth efficiency of the OSS protocol is inferior to the traditional bidirectional protocols. More detailed comparisons between the OSS and the traditional bidirectional protocols are well addressed in [45].
RS-OSS selects the particular combination of a source and a relay in order to attain the maximum mutual information. That is, the index $i_S$ of the selected source and the index $l_R$ of the selected relay are jointly determined as follows:

$$(i_S, l_R) = \arg \max_{i=1,2, \ldots, L} \mathcal{I}_{i,l}.$$  \hfill (5.6)

It is easy to see that this problem reduces to the following SNR maximization problem:

$$(i_S, l_R) = \arg \max_{i=1,2} \gamma_{i,l}. \hfill (5.7)$$

The joint RS-OSS is implemented in a system as follows. At the beginning of each minimum coherence time, the SNRs $\{\gamma_{i,l} : i = 1, 2 \text{ and } l = 1, \ldots, L\}$ are calculated at both sources. Then, depending on the SNRs, only the selected source $S_{i_S}$ transmits the signal to the other source with the help of the selected relay $R_{l_R}$. \footnote{The joint RS-OSS changes the source in different amount of times based on channel conditions. As explained in \cite{45}, however, we can control selection time and also achieve fairness between two sources if we implement the proportional fairness algorithm proposed in \cite{48}.} Specifically, if $\gamma_{1,l_R} \geq \gamma_{2,l_R}$, only $S_1$ transmits the signal to $S_2$ with the help of $R_{l_R}$; if $\gamma_{1,l_R} < \gamma_{2,l_R}$, only $S_2$ transmits the signal to $S_1$ with the help of $R_{l_R}$. The joint RS-OSS is depicted in Fig. 5.3

5.3 Analysis of CDF and PDF of $\max[\gamma_{1,l}, \gamma_{2,l}]$

In this section, we first derive the CDF and PDF of $\max[\gamma_{1,l}, \gamma_{2,l}]$, which will be used to derive one-integral expressions of the outage probability and average BER. Then we obtain the CDF of an upper bound of $\max[\gamma_{1,l}, \gamma_{2,l}]$, which will be used to derive closed-form lower bounds of the outage probability and average BER.

5.3.1 CDF and PDF of $\max[\gamma_{1,l}, \gamma_{2,l}]$

In this subsection, we derive the CDF and PDF of $\max[\gamma_{1,l}, \gamma_{2,l}]$. Let us define $\mathcal{W}_l$ as the maximum value between $\gamma_{1,l}$ and $\gamma_{2,l}$ as follows:

$$\mathcal{W}_l := \max[\gamma_{1,l}, \gamma_{2,l}].$$  \hfill (5.8)
If $\gamma_{1,t} \geq \gamma_{2,t}$

Figure 5.1: System model for the joint RS-OSS.
Then we derive the CDF and PDF of $\mathcal{W}_i$ in the following theorem.

**Theorem 5.1:** The CDF $F_{\mathcal{W}_i}(w)$ and PDF $f_{\mathcal{W}_i}(w)$ of $\mathcal{W}_i$ are given respectively by

\[
F_{\mathcal{W}_i}(w) = 1 + \exp(-w\alpha_1) - \Psi_1(w; \Omega_{1,i}, \Omega_{2,i}) - \Psi_1(w; \Omega_{2,i}, \Omega_{1,i}), \tag{5.9}
\]

\[
f_{\mathcal{W}_i}(w) = -\alpha_1 \exp(-w\alpha_1) + \Psi_2(w; \Omega_{1,i}, \Omega_{2,i}) + \Psi_2(w; \Omega_{2,i}, \Omega_{1,i}), \tag{5.10}
\]

where $\alpha_1 = (1/\Omega_{1,i} + 1/\Omega_{2,i})(1/\mathcal{E}_s + 1/\mathcal{E}_r)$. In the above equations,

\[
\Psi_1(w; \xi_1, \xi_2) = \exp(-w\alpha_2) \left[ \frac{2w}{\sqrt{\xi_1 \xi_2 \mathcal{E}_s \mathcal{E}_r}} K_1 \left( \frac{2w}{\sqrt{\xi_1 \xi_2 \mathcal{E}_s \mathcal{E}_r}} \right) - \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} (w\alpha_3)^{i+1} E_{i+2}(w\alpha_4) \right], \tag{5.11}
\]

\[
\Psi_2(w; \xi_1, \xi_2) = \exp(-w\alpha_2) \left[ \frac{2w\alpha_2}{\sqrt{\xi_1 \xi_2 \mathcal{E}_s \mathcal{E}_r}} K_1 \left( \frac{2w}{\sqrt{\xi_1 \xi_2 \mathcal{E}_s \mathcal{E}_r}} \right) + \frac{4w}{\xi_1 \xi_2 \mathcal{E}_s \mathcal{E}_r} K_0 \left( \frac{2w}{\sqrt{\xi_1 \xi_2 \mathcal{E}_s \mathcal{E}_r}} \right) \right. \\
- \left. \sum_{i=0}^{\infty} \frac{(-1)^i(w\alpha_3)^i}{i!} \left[ (w\alpha_2 - (i + 1))\alpha_3 E_{i+2}(w\alpha_4) + w\alpha_3\alpha_4 E_{i+1}(w\alpha_4) \right] \right], \tag{5.12}
\]

where $\alpha_2 = 1/(\xi_1 \mathcal{E}_{\text{max}}) + 1/(\xi_2 \mathcal{E}_{\text{min}})$, $\alpha_3 = 1/(\xi_2 \mathcal{E}_{\text{max}})$, and $\alpha_4 = 1/(\xi_1 \mathcal{E}_{\text{min}})$ with $\mathcal{E}_{\text{max}} = \max[\mathcal{E}_s, \mathcal{E}_r]$ and $\mathcal{E}_{\text{min}} = \min[\mathcal{E}_s, \mathcal{E}_r]$. Also, $K_\nu(\cdot)$ is the modified Bessel function of the second kind with order $\nu$, and $E_\nu(\cdot)$ is the exponential integral function.

**Proof:** See Appendix 5-A. \(\square\)

Note that the CDF in (5.9) and the PDF in (5.10) involve only standard functions: the exponential function, the modified Bessel function, and the exponential integral function. Also, the exponential integral function $E_{i+1}(x)$, $i = 1, 2, \cdots$, can be recursively calculated [78]: $E_{i+1}(x) = \frac{1}{x} \left( \exp(-x) - x E_i(x) \right)$ for $x > 0$. Therefore, one needs to compute only $E_1(x)$ for the evaluation of $E_{i+1}(x)$ with $i = 1, 2, \cdots$.

Since the CDF in (5.9) and the PDF in (5.10) are given in infinite series, we test the convergence of the infinite series expressions in the following lemma.

**Lemma 5.1:** The infinite series expressions in (5.11) and (5.12) are convergent.

**Proof:** We first show that the series expansion in (5.11) is convergent. Let

\[
\tilde{M}_{1,i} = \frac{(w\alpha_3)^{i+1}}{i!} E_{i+2}(w\alpha_4) \exp(-w\alpha_2).
\]

Then the series expansion in (5.11) is given
by \(\sum_{i=0}^{\infty} (-1)^i \hat{M}_{1,i}\). Since the exponential integral function \(E_i(\cdot)\) is monotonically decreasing in \(i\), it can be shown that
\[
\lim_{i \to \infty} \frac{\hat{M}_{1,i+1}}{\hat{M}_{1,i}} = \lim_{i \to \infty} \frac{w_{\alpha_3} E_{i+3}(w_{\alpha_4})}{E_{i+2}(w_{\alpha_4})} < \lim_{i \to \infty} \frac{w_{\alpha_3}}{i+1} = 0.
\]
By the ratio test \(80\), thus, it can be shown that the series expansion in (5.11) is convergent. Secondly, taking steps similar to those used above, we can show that the series expansion in (5.12) is also convergent.

It follows from Lemma 5.1 that the CDF in (5.9) and the PDF in (5.10) are convergent.

As a special case, when the selected source and the selected relay have the same transmission power \((\mathcal{E}_s = \mathcal{E}_r)\), the series expressions of the CDF in (5.9) and the PDF in (5.10) reduce to closed-form expressions (without infinite series summation) as in the following corollary.

**Corollary 5.1 (Equal power allocation):** When \(\mathcal{E}_s = \mathcal{E}_r\), the CDF \(\hat{F}_{W_l}(w)\) in (5.9) and the PDF \(\hat{f}_{W_l}(w)\) in (5.10) can be given in closed-form as follows:
\[
\hat{F}_{W_l}(w) = 1 - \frac{2w}{\mathcal{E}_s \mathcal{E}_r \Omega_{1,l} \Omega_{2,l}} \exp \left(-w \left(\frac{1}{\mathcal{E}_s \Omega_{1,l}} + \frac{1}{\mathcal{E}_r \Omega_{2,l}}\right)\right) K_1 \left(\frac{2w}{\sqrt{\mathcal{E}_s \mathcal{E}_r \Omega_{1,l} \Omega_{2,l}}}\right),
\]
\[
\hat{f}_{W_l}(w) = \frac{2w}{\mathcal{E}_s \mathcal{E}_r \Omega_{1,l} \Omega_{2,l}} \exp \left(-w \left(\frac{1}{\mathcal{E}_s \Omega_{1,l}} + \frac{1}{\mathcal{E}_r \Omega_{2,l}}\right)\right) \times \left[2 K_0 \left(\frac{2w}{\sqrt{\mathcal{E}_s \mathcal{E}_r \Omega_{1,l} \Omega_{2,l}}}\right) + \frac{\mathcal{E}_s \Omega_{1,l} + \mathcal{E}_r \Omega_{2,l}}{\sqrt{\mathcal{E}_s \mathcal{E}_r \Omega_{1,l} \Omega_{2,l}}} K_1 \left(\frac{2w}{\sqrt{\mathcal{E}_s \mathcal{E}_r \Omega_{1,l} \Omega_{2,l}}}\right)\right].
\]

*Proof:* When \(\mathcal{E}_s = \mathcal{E}_r\), it is evident that \(\gamma_{1,l} = \gamma_{2,l}\). Therefore, one knows that \(W_l = \gamma_{1,l}\). Taking the same steps in Appendix 5-A, one can derive the CDF in (5.13) and the PDF in (5.14).

When there is no source selection in the joint RS-OSS, which is equivalent to the conventional RS, the CDF in (5.9) and the PDF in (5.10) reduce to the well-known results [20, eqs. (8) and (12)] as in the following corollary.

---

10 For a series \(\sum_{i=0}^{\infty} g_i\), we define \(L := \lim_{i \to \infty} |g_{i+1}/g_i|\). If \(L < 1\), then the series converges absolutely; if \(L > 1\), then the series diverges; and if \(L = 1\), then the series is inclusive [80].
**Corollary 5.2 (Conventional RS only):** For the conventional RS, the CDF \( F_{W_l}(w) \) in (5.9) reduces to \( \tilde{F}_{W_l}(w) \) in (5.13), which is equivalent to [20] eq. (8); and the PDF \( f_{W_l}(w) \) in (5.10) reduces to \( \tilde{f}_{W_l}(w) \) in (5.14), which is equivalent to [20] eq. (12).

**Proof:** It is evident that \( W_l = \gamma_{1,l} \). Therefore, one can obtain (5.13) and (5.14). \( \square \)

The above corollary shows that the analysis in this subsection can be considered as a generalization of [20].

Using the CDF \( F_{W_l}(w) \) in (5.9) and the PDF \( f_{W_l}(w) \) in (5.10), we will derive the outage probability in Section IV and the average BER in Sections 5.5. Although they are very accurate, the final expressions will be given in a one-integral form, which requires numerical integration. Therefore, it should be also useful to derive closed-form *bounds* of the outage probability and average BER. To this end, in the next subsection, with a modification of [45], we present a CDF of an upper bound of \( \max[\gamma_{1,l}, \gamma_{2,l}] \), which enables us to derive closed-form bounds of the outage probability and average BER.

### 5.3.2 CDF and PDF of Upper Bound of \( \max[\gamma_{1,l}, \gamma_{2,l}] \)

In the performance analysis of relay networks, the following inequality has widely been used [20, 45, 49]: \( ab/(a+b) < \min[a, b] \). Using this inequality, \( \max[\gamma_{1,l}, \gamma_{2,l}] \) is upper-bounded by

\[
\max[\gamma_{1,l}, \gamma_{2,l}] < \max \left[ \min[\mathcal{E}_sX_l, \mathcal{E}_rY_l], \min[\mathcal{E}_sY_l, \mathcal{E}_rX_l] \right] =: W^U_l. \tag{5.15}
\]

For the OSS with a single relay, based on \( W^U_l \) in (5.15), a lower bound of the outage probability was derived in [45], which was equivalent to the CDF of an upper bound of the sum of the relay-path SNR and the direct-path SNR. For the joint RS-OSS, however, it is essential to know the CDF of the relay-path SNR only (without the direct-path SNR). With a simple modification of [45, Theorem 1], therefore, we present the CDF of an upper bound of the relay-path SNR in the following lemma.
Lemma 5.2 ([45 Theorem 1]): The CDF $F_{\mathcal{W}_1^U}(w)$ of $\mathcal{W}_1^U$ is given by

$$F_{\mathcal{W}_1^U}(w) = 1 - \sum_{i=1}^{3} (-1)^{\delta[i-1]} \exp(-w\beta_i(l)),$$

(5.16)

where $\beta_1(l) = 1/(\mathcal{E}_{\text{min}}\Omega_{1,l}) + 1/(\mathcal{E}_{\text{min}}\Omega_{2,l})$, $\beta_2(l) = 1/(\mathcal{E}_{s}\Omega_{1,l}) + 1/(\mathcal{E}_{r}\Omega_{2,l})$, and $\beta_3(l) = 1/(\mathcal{E}_{r}\Omega_{1,l}) + 1/(\mathcal{E}_{s}\Omega_{2,l})$. Also, $\delta[\cdot]$ is the Kronecker delta function: $\delta[0] = 1$ and $\delta[i] = 0$ for $i \neq 0$.

Proof: With a simple modification of the proof of [45 Theorem 1], one can easily obtain (5.16). □

Note that the CDF $F_{\mathcal{W}_1^U}(w)$ in (5.16) is expressed in a linear combination of only exponential functions. Using the CDF in (5.16), we will derive closed-form bounds of the outage probability and average BER in Sections IV and V.

As a special case, when there is no source selection in the joint RS-OSS, it is evident that $\mathcal{W}_1^U = \min[\mathcal{E}_{s}X_l, \mathcal{E}_{r}Y_l]$. Since $\min[\mathcal{E}_{s}X_l, \mathcal{E}_{r}Y_l]$ is an exponentially distributed random variable with mean $1/\beta_2(l)$, the CDF $F_{\mathcal{W}_1^U}(w)$ in (5.16) reduces to

$$\bar{F}_{\mathcal{W}_1^U}(w) = 1 - \exp(-w\beta_2(l)).$$

(5.17)

5.4 Outage Probability of Joint RS-OSS

One of the most important performance measures for coded systems is the outage probability. In this section, we first present outage probability expression of the joint RS-OSS. From (5.6) with (5.5), it can be shown that the outage probability $P_{\text{out}}^{\text{RS-OSS}}(R)$ of the joint RS-OSS is given by

$$P_{\text{out}}^{\text{RS-OSS}}(R) = \Pr\left[\frac{1}{2}\log_2\left(1 + \gamma_0 + \max_{l=1,\cdots,L} \left[\max[\gamma_{1,l}, \gamma_{2,l}]\right]\right) < R\right].$$

(5.18)

Note that the selection rule maximizing the mutual information in (5.6) also minimizes the outage probability in (5.18) because the maximum SNR value among $\{\gamma_{i,l} : i = 1, 2 \text{ and } l = 1, \cdots, L\}$ leads to minimization of the outage probability. Using (5.18), we first derive the outage probability in a one-integral form and then its lower bound.
in closed-form. Finally, for a special case of a network where a direct-path between the two end-sources is not available, we present the outage probability in closed-form.

### 5.4.1 Outage Probability in One-Integral Form

Using (5.18) with the CDF $F_{W_l}(w)$, we derive the outage probability in the following theorem.

**Theorem 5.2 (Joint RS-OSS):** The outage probability $P_{out}^{RS-OSS}(R)$ of the joint RS-OSS is given by

$$P_{out}^{RS-OSS}(R) = \int_0^{\bar{R}} \left( \prod_{l=1}^{L} F_{W_l}(\bar{R} - z) \right) \frac{\exp \left( -\frac{z}{(\mathcal{E}_s\Omega_0)} \right)}{\mathcal{E}_s\Omega_0} dz, \quad (5.19)$$

where $\bar{R} = 2^{2R} - 1$.

**Proof:** Let $U = \max_{l=1,\ldots,L} W_l$. Then $P_{out}^{RS-OSS}(R)$ in (5.18) can be given by

$$P_{out}^{RS-OSS}(R) = \int_0^{\bar{R}} \Pr[U < \bar{R} - z] f_{\gamma_0}(z) dz. \quad (5.20)$$

Since the CDF $F_U(u)$ is given by $F_U(u) = \prod_{l=1}^{L} F_{W_l}(u)$ and $\gamma_0$ is an exponentially distributed random variable with mean $\mathcal{E}_s\Omega_0$, one can obtain (5.19). □

In the following corollary, we show that our analysis in this subsection can be considered as a generalization of that of the conventional RS.

**Corollary 5.3 (Conventional RS only):** For the conventional RS, replacing $F_{W_l}(\cdot)$ in (5.19) with $F_{W_l}(\cdot)$ in (5.13) gives the outage probability $P_{out}^{RS}(R)$. □

In the next subsection, we derive a lower bound of the outage probability in closed-form, which does not involve any numerical integration.

### 5.4.2 Lower Bound of Outage Probability in Closed-Form

Taking a step similar to (5.19) and using the CDF $F_{W_l}(w)$ in (5.16), we derive a closed-form lower bound of the outage probability in the following theorem.
Theorem 5.3 (Joint RS-OSS): The closed-form lower bound $P_{\text{out,LB}}^{\text{RS-OSS}}(R)$ of the outage probability of the joint RS-OSS is given by

$$
P_{\text{out,LB}}^{\text{RS-OSS}}(R) = \sum_{(l,i,j,n_1,n_2,n_3)} \exp \left( - d(l, i, j, n_1, n_2, n_3) \hat{R} \right) - \exp \left( - R / \left( E_s \Omega_0 \right) \right) / \left( 1 - E_s \Omega_0 d(l, i, j, n_1, n_2, n_3) \right).$$  \hspace{1cm} (5.21)

In the above equation,

$$
\sum_{(l,i,j,n_1,n_2,n_3)} = \sum_{t=0}^{L} \sum_{i=0}^{l} \sum_{j=0}^{i} (-1)^i \sum_{n_1(1)=1}^{L} \cdots \sum_{n_1(l)=1}^{L} \sum_{n_2(i)=1}^{L} \cdots \sum_{n_2(i)=1}^{L} \sum_{n_3(j)=1}^{L} \cdots \sum_{n_3(j)=1}^{L} , \hspace{1cm} (5.22)
$$

$$
d(l, i, j, n_1, n_2, n_3) = \sum_{t=1}^{l} \beta_1(n_1(t)) + \sum_{s=1}^{i} \left( \beta_2(n_1(n_2(s))) - \beta_1(n_1(n_2(s))) \right) + \sum_{r=1}^{j} \left( \beta_3(n_1(n_2(n_3(r)))) - \beta_2(n_1(n_2(n_3(r))) \right), \hspace{1cm} (5.23)
$$

where $n_1 = [n_1(1), \cdots, n_1(l)]$, $n_2 = [n_2(1), \cdots, n_2(i)]$, and $n_3 = [n_3(1), \cdots, n_3(j)]$.

Proof: One can obtain the lower bound $P_{\text{out,LB}}^{\text{RS-OSS}}(R)$ from (5.19) with $F_{W_l}^l(\bar{R} - z)$ replaced by $F_{W_l}^l(\bar{R} - z)$ in (5.16). Using the product identity $\prod_{l=1}^{L} (1 - x(l)) = \sum_{t=0}^{L} (-1)^t \sum_{n(1)=1, \cdots, n(l)=1}^{L} \prod_{t=1}^{L} x(n(t))$, the expression $\prod_{l=1}^{L} F_{W_l}^l(w)$ can be rewritten as

$$
\prod_{l=1}^{L} F_{W_l}^l(w) = \sum_{(l,i,j,n_1,n_2,n_3)} \exp(-d(l, i, j, n_1, n_2, n_3)w).$$

Then, substituting the above equation into $P_{\text{out,LB}}^{\text{RS-OSS}}(R)$ and taking integration, one can obtain the final closed-form result in (5.21). \hfill \Box

Note that since the lower bound $P_{\text{out,LB}}^{\text{RS-OSS}}(R)$ in (5.21) is given in closed-form, it does not require any numerical integration. With a simple modification of (5.21), we obtain the following result for the conventional RS.

Corollary 5.4 (Conventional RS only): For the conventional RS, the lower bound
\(P_{\text{out,LB}}^{\text{RS}}(R)\) of the outage probability can be given by

\[
P_{\text{out,LB}}^{\text{RS}}(R) = \sum_{l=0}^{L} (-1)^l \sum_{n(1)=1}^{L} \cdots \sum_{n(l)=1}^{L} \frac{\exp \left( -\bar{R} \sum_{t=1}^{l} \beta_2(n(t)) \right) - \exp(-\bar{R}/(\mathcal{E}_s \Omega_0))}{1 - \mathcal{E}_s \Omega_0 \sum_{t=1}^{l} \beta_2(n(t))}.
\]

(5.24)

**Proof:** Using (5.17) and taking steps similar to the proof of Theorem 3, one can obtain (5.24).

Until now, we have considered a network with a direct-path. If the distance between the two sources is too long or there are some obstacles between the two sources, a direct-path may not be available. In such cases, only a relay-path is available. In the next subsection, we consider such a network without any direct-path.

### 5.4.3 Special Case: Network without Direct-Path

Since a network without a direct-path can be considered as a special case of a network with a direct-path, with a simple modification of (5.19), we can derive a closed-form outage probability.

**Corollary 5.5 (Joint RS-OSS):** When there is no direct-path, \(P_{\text{out}}^{\text{RS-OSS}}(R)\) in (5.19) can be simplified into

\[
P_{\text{out}}^{\text{RS-OSS}}(R) = \prod_{l=1}^{L} F_{W_l}(\bar{R}).
\]

(5.25)

**Proof:** Since there is no direct-path, the outage probability of the joint RS-OSS can be obtained by simply multiplying the outage probabilities for all the relay-paths.

\(\Box\)

Note that the above outage probability in (5.25) is given in closed-form. Therefore, we do not present a closed-form lower bound of the outage probability, although it can be easily obtained by simply modifying (5.21).
5.5 Average BER of Joint RS-OSS

The average BER is a very important system performance measure for uncoded systems. In this section, for a joint RS-OSS network with a direct-path, we first derive the average BER for $M$-QAM in a one-integral form and its lower-bound in closed-form. Then, assuming there is no direct-path, we present some BER results.

5.5.1 Average BER in One-Integral Form

In this subsection, we derive the average BER. Let $V = U + \gamma_0$, where $U = \max_{l=1,\ldots,L} W_l$.

We first derive the moment generating function (MGF) of $V$ in the following lemma.

**Lemma 5.3 (Joint RS-OSS):** The MGF $M_{RS-OSS}^V(s)$ of $V$ can be given by

$$M_{RS-OSS}^V(s) = \frac{1}{1 - sE_s \Omega_0} \int_0^\infty \left( \sum_{l_1=1}^L f_{W_{l_1}}(u) \prod_{l_2=1, l_2 \neq l_1}^L F_{W_{l_2}}(u) \right) \exp(su)du. \tag{5.26}$$

**Proof:** Since the CDF $F_U(u)$ of $U$ is given by $F_U(u) = \prod_{l=1}^L F_{W_l}(u)$, it can be shown that the PDF $f_U(u)$ is given by $f_U(u) = \sum_{l_1=1}^L f_{W_{l_1}}(u) \prod_{l_2=1, l_2 \neq l_1}^L F_{W_{l_2}}(u)$. Then, using $[86$, eq. (2.4)], one can obtain $M_{\gamma_0}(s) = 1/(1 - sE_s \Omega_0)$ and $M_U(s) = \int_0^\infty f_U(u) \exp(su)du$. Finally, since $\gamma_0$ and $U$ are independent, $M_{RS-OSS}^V(s)$ can be obtained by multiplying $M_{\gamma_0}(s)$ with $M_U(s)$, and it is given in (5.26). □

With a simple modification of (5.26), we obtain the following result for the conventional RS.

**Corollary 5.6 (Conventional RS only):** For the conventional RS, replacing $F_{W_l} (\cdot)$ in (5.26) with $\bar{F}_{W_l} (\cdot)$ in (5.13) and replacing $f_{W_l} (\cdot)$ in (5.26) with $\bar{f}_{W_l} (\cdot)$ in (5.14) give the MGF $M_{RS}^V(s)$ of $V$. □

Note that it can be shown that the MGF $M_{RS}^V(s)$ reduces to [20, eq. (7)] when there is no direct-path and $L = 1$. Therefore, the analysis in this subsection can be considered as a generalization of [20]. Using the MGF in (5.26), we now derive the average BER for $M$-QAM in the following theorem.

---

$^{11}$For an uncoded joint RS-OSS system, the source-relay selection criterion should be the maximization of SNR instead of the maximization of mutual information, although these two are mathematically the same in our system as can be seen in (5.6) and (5.7).
**Theorem 5.4 (Joint RS-OSS):** For M-QAM, the average BER $P_{b}^{RS-OSS}$ of the joint RS-OSS is given by

$$
P_{b}^{RS-OSS} = \frac{4}{\sqrt{M \log_2 M}} \sum_{m=1}^{(\log_2 M)/2} \sum_{k=0}^{(1-2^{-m})\sqrt{M}-1} \frac{B_{M,m,k}}{\pi} \int_{0}^{\pi/2} \mathcal{M}_{V}^{RS-OSS} \left( - \frac{A_{M,k}}{2 \sin^2 \theta} \right) d\theta, \tag{5.27}
$$

where $A_{M,k} = 3(2k+1)^2/(M-1)$, $B_{M,m,k} = (-1)^{\lfloor k \cdot 2^{-m-1}/\sqrt{M} \rfloor} (2^{m-1} - \lfloor k \cdot 2^{-m-1}/\sqrt{M} + 1/2 \rfloor)$, and $\lfloor \varrho \rfloor$ denotes the largest integer less than or equal to $\varrho$.

**Proof:** Using Craig’s formula for $Q$-function [86], eq. (4.2)], the BER expression for M-QAM [87], and $\mathcal{M}_{V}^{RS-OSS}(s)$ in (5.26), one can obtain the BER result in (5.27).

Since $P_{b}^{RS-OSS}$ of (5.27) is given in the double-integral form, we approximate $P_{b}^{RS-OSS}$ in the following. To this end, we adopt the following widely-used very accurate approximation for $Q$-function [88]:

$$
Q(x) \approx \frac{1}{12} \exp(-x^2/2) + \frac{1}{4} \exp(-2x^2/3), \quad x \geq 0. \tag{5.28}
$$

Using the above approximation in (5.28), we approximate $P_{b}^{RS-OSS}$ of (5.27) as in the following lemma.

**Lemma 5.4 (Joint RS-OSS):** For M-QAM, the average BER approximation $\hat{P}_{b}^{RS-OSS}$ of the joint RS-OSS is given by

$$
\hat{P}_{b}^{RS-OSS} = \frac{4}{\sqrt{M \log_2 M}} \sum_{m=1}^{(\log_2 M)/2} \sum_{k=0}^{(1-2^{-m})\sqrt{M}-1} B_{M,m,k}
\times \left[ \frac{1}{12} \mathcal{M}_{V}^{RS-OSS} \left( - \frac{A_{M,k}}{2} \right) + \frac{1}{4} \mathcal{M}_{V}^{RS-OSS} \left( - \frac{2A_{M,k}}{3} \right) \right]. \tag{5.29}
$$

**Proof:** Using (5.27) with (5.28) and taking MFG-approach [86], eq. (5.6)], one can obtain (5.29).

The average BER approximation in (5.29) is very accurate, which will be numerically demonstrated in Section 5.6. With a simple modification of (5.29), we obtain the following result for the conventional RS.

---

12 One integral is in the final BER expression of (5.27) and the other is in $\mathcal{M}_{V}^{RS-OSS}(s)$ of (5.26).
Corollary 5.7 (Conventional RS only): For the conventional RS, replacing $M_{V}^{RS-OSS}(\cdot)$ in (5.29) with $M_{V}^{RS}(\cdot)$ in Corollary 5.6 gives the average BER approximation $\hat{P}_{b}^{RS}$. □

Although the average BER approximation in (5.29) is very accurate, it still requires one numerical integration because of the single integral in $M_{V}^{RS-OSS}(\cdot)$. In the next subsection, we present a lower bound of the average BER in closed-form, which can be computed without any numerical integration.

5.5.2 Lower Bound of Average BER in Closed-Form

Let $V^{U} = U^{U} + \gamma_{0}$, where $U^{U} = \max_{l=1,\cdots,L} W_{l}^{U}$. We derive the MGF of $V^{U}$ in the following lemma.

Lemma 5.5 (Joint RS-OSS): The MGF $M_{V^{U}}^{RS-OSS}(s)$ of $V^{U}$ is given by

\[
M_{V^{U}}^{RS-OSS}(s) = \sum_{(l,i,j,n_{1},n_{2},n_{3})} \frac{1}{1 - E_{s}^{\Omega_{0}}d(l,i,j,n_{1},n_{2},n_{3})} \times \left( \frac{1}{1 - sE_{s}^{\Omega_{0}}} - \frac{1}{1 - s/d(l,i,j,n_{1},n_{2},n_{3})} \right). \tag{5.30}
\]

Proof: Using (5.23), the CDF $F_{V^{U}}(v)$ can be given by $F_{V^{U}}(v) = P_{out,LB}^{RS-OSS}((\log_{2}v + 1)/2)$. Then, taking a derivative of $F_{V^{U}}(v)$ with respect to $v$ and taking Laplace transform, one can obtain (5.30). □

With a simple modification of (5.30), we obtain the following result for the conventional RS.

Corollary 5.8 (Conventional RS only): For the conventional RS, the MGF $M_{V^{U}}^{RS}(s)$ in (5.30) is simplified into

\[
M_{V^{U}}^{RS}(s) = \sum_{l=0}^{L} (-1)^{l} \sum_{n(1)=1}^{L} \cdots \sum_{n(l)=1}^{L} \frac{1}{E_{s}^{\Omega_{0}} \sum_{t=1}^{l} \beta_{2}(n(t))} \times \left( \frac{1}{1 - sE_{s}^{\Omega_{0}}} - \frac{1}{1 - s/d(n(1) < \cdots < n(l))} \right). \tag{5.31}
\]

Proof: Taking the same steps in the proof of Lemma 5.5 with $P_{out,LB}^{RS}(\cdot)$ in (5.24), one can obtain (5.31). □

\footnote{When there is no direct-path and $L = 1$, (5.31) reduces to [20 eq. (17)].}
Using the MGF in \([5.30]\), we derive a closed-form lower bound of the average BER in the following theorem.

**Theorem 5.5 (Joint RS-OSS):** For \(M\)-QAM, the closed-form lower bound \(P_{b,LB}^{RS-OSS}\) of the average BER of the joint RS-OSS is given by

\[
P_{b,LB}^{RS-OSS} = \frac{4}{\sqrt{M \log_2 M}} \sum_{m=1}^{(\log_2 M)/2} \frac{1}{(1-2^{-m})\sqrt{M-1}} \sum_{k=0}^{(l,i,j,n_1,n_2,n_3)} \frac{B_{M,m,k}}{2(1 - \mathcal{E}_s \Omega_0 d(l, i, j, n_1, n_2, n_3))} \times \left( \sqrt{\frac{A_{M,k}}{2d(l, i, j, n_1, n_2, n_3)}} + A_{M,k} \right) \left( 1 + A_{M,k} \right) = \frac{B_{M,m,k}}{2(1 - \mathcal{E}_s \Omega_0 \sum_{t=1}^{L} \beta_2(n(t)))} \left( \sqrt{\frac{A_{M,k}}{2\sum_{t=1}^{L} \beta_2(n(t))}} + A_{M,k} \right) \left( 1 + A_{M,k} \right).
\]

**Proof:** The MGF \(\mathcal{M}_{RS-OSS}^V(s)\) in \([5.30]\) is expressed in a linear combination of a form \(c_1/(1 - c_2 s)\), where \(c_1 = \pm 1/(1 - \mathcal{E}_s \Omega_0 d(l, i, j, n_1, n_2, n_3))\) and either \(c_2 = \mathcal{E}_s \Omega_0\) or \(c_2 = 1/d(l, i, j, n_1, n_2, n_3)\). Therefore, replacing \(\mathcal{M}_{RS-OSS}^V(\cdot)\) in \([5.27]\) with \(\mathcal{M}_{RS-OSS}^V(\cdot)\) in \([5.30]\) and taking MGF-approach \([5.6]\), one can obtain the BER result in \([5.32]\). \(\square\)

With a simple modification of \([5.32]\), we obtain the following result for the conventional RS.

**Corollary 5.9 (Conventional RS only):** For the conventional RS, the lower bound \(P_{b,LB}^{RS}\) of the average BER is given by

\[
P_{b,LB}^{RS} = \frac{4}{\sqrt{M \log_2 M}} \sum_{m=1}^{(\log_2 M)/2} \frac{1}{(1-2^{-m})\sqrt{M-1}} \sum_{k=0}^{(l,i,j,n_1,n_2,n_3)} \frac{B_{M,m,k}}{2(1 - \mathcal{E}_s \Omega_0 \sum_{t=1}^{L} \beta_2(n(t)))} \left( \sqrt{\frac{A_{M,k}}{2\sum_{t=1}^{L} \beta_2(n(t))}} + A_{M,k} \right) \left( 1 + A_{M,k} \right) = \frac{B_{M,m,k}}{2(1 - \mathcal{E}_s \Omega_0 \sum_{t=1}^{L} \beta_2(n(t)))} \left( \sqrt{\frac{A_{M,k}}{2\sum_{t=1}^{L} \beta_2(n(t))}} + A_{M,k} \right) \left( 1 + A_{M,k} \right).
\]

**Proof:** Taking a step similar to \([5.32]\) with \(\mathcal{M}_{RS}^V(\cdot)\) in \([5.31]\), one can obtain \([5.33]\). \(\square\)

Until now, we have considered a network with a direct-path. In the next subsection, we consider a network without a direct-path.
5.5.3 Special Case: Network without Direct-Path

We first present the average BER in a one-integral form. In [89], Chen and Tellambura derived a new expression for average error probability as follows: 

\[ E_Z[Q(\sqrt{C}Z)] = \sqrt{\frac{C}{8\pi}} \int_0^\infty \exp(-Cz/2) F_Z(z)dz, \]

where \( F_Z(z) \) is the CDF of \( Z \). Using this equation, we derive the average BER in the following lemma.

Lemma 5.6 (Joint RS-OSS): When there is no direct-path, for \( M \)-QAM, the average BER \( P_{RS-OSS}^b \) of the joint RS-OSS is given by

\[
P_{RS-OSS}^b = \frac{4}{\sqrt{M \log_2 M}} \sum_{m=1}^{(\log_2 M)/2} \sum_{k=0}^{(1-2^{-m})\sqrt{2M}-1} B_{M,m,k} \frac{A_{M,k}}{8\pi} \times \int_0^\infty \frac{\exp(-A_{M,k}z^2/2)}{\sqrt{z}} \left( \prod_{l=1}^L F_{W_l}(z) \right) dz. \tag{5.34}
\]

Proof: Using the BER expression for \( M \)-QAM [87] and the CDF \( F_U(u) = \prod_{l=1}^L F_{W_l}(u) \), one can obtain the BER result in (5.34).

With a simple modification of (5.32), we also present a closed-form lower bound of the average BER in the following corollary.

Corollary 5.10 (Joint RS-OSS): When there is no direct-path, for \( M \)-QAM, \( P_{RS-OSS}^b \) in (5.32) can be simplified into

\[
P_{b, LB}^{RS-OSS} = \frac{4}{\sqrt{M \log_2 M}} \sum_{m=1}^{(\log_2 M)/2} \sum_{k=0}^{(1-2^{-m})\sqrt{2M}-1} \left( \sum_{l,i,j,n_1,n_2,n_3} B_{M,m,k} \right) \frac{A_{M,k}}{2 \left( 2d(l, i, j, n_1, n_2, n_3) + A_{M,k} \right)}. \tag{5.35}
\]

Proof: Taking steps similar to those used from (5.30) to (5.32), one can obtain (5.35). \( \square \)

\( ^{14} \) Taking steps similar to those used from (5.26) to (5.29) including the Q-function approximation in (5.28), it is possible to obtain an average BER in a one-integral form. In this subsection, however, we present a one-integral form of average BER without adopting the Q-function approximation.
5.6 Simulation Results

We first check the convergence speed of the series expressions in (5.11) and (5.12). We let \( \hat{\Psi}_1^{(N)}(w; \xi_1, \xi_2) \) and \( \hat{\Psi}_2^{(N)}(w; \xi_1, \xi_2) \) denote the sum of the first \( N + 1 \) terms, \( \sum_{i=0}^{N} \Psi_1(w; \xi_1, \xi_2) \) of (5.11) and \( \Psi_2(w; \xi_1, \xi_2) \) of (5.12). Fig. 5.2 shows the normalized truncation error \( |(\Psi_j(w; \xi_1, \xi_2) - \hat{\Psi}_j^{(N)}(w; \xi_1, \xi_2)) / \Psi_j(w; \xi_1, \xi_2)| \) against truncation window size \( N \) for \( j = 1, 2 \). We set \( w = \xi_1 = \xi_2 = 1 \) and \( \mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E} \). For various \( \mathcal{E} \) values, \( \mathcal{E} = 0, 10, 20 \) dB, we can see the two normalized truncation errors are very small even with a small value \( N \). Therefore, even with a small value \( N \), substituting \( \hat{\Psi}_1^{(N)}(w; \xi_1, \xi_2) \) into (5.9) gives a very accurate approximation \( \hat{F}_W(w) \) in closed-form for \( F_W(w) \); and substituting \( \hat{\Psi}_2^{(N)}(w; \xi_1, \xi_2) \) into (5.10) yields a very accurate approximation \( \hat{f}_W(w) \) in closed-form for \( f_W(w) \). Using \( \hat{F}_W(w) \) and \( \hat{f}_W(w) \), in the following, we calculate outage probabilities in (5.19) and (5.25), and average BERs in (5.27), (5.29), and (5.34).

Secondly, we compare the proposed joint RS-OSS with the conventional RS

Fig. 5.3 shows the outage probabilities against \( 10 \log_{10} \mathcal{E} \) of the joint RS-OSS and the conventional RS when there is a direct-path. We set \( L = 1, 2, 4, R = 1.5 \) bps/Hz, \( \mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E} \), and \( \Omega_0 = \Omega_{1, l} = \Omega_{2, l} = 1 \) for \( l = 1, \cdots, L \). Also, we set \( N = 0 \) in \( \hat{F}_W(w) \) for \( F_W(w) \) in (5.19). Even with \( N = 0 \), we can see that (5.19) is very accurate. Fig. 5.4 shows the average BERs against \( 10 \log_{10} \mathcal{E} \) of the joint RS-OSS and the conventional RS when there is a direct-path for QPSK. We set \( L = 1, 2, 4, \) \( \mathcal{E}_s/3 = \mathcal{E}_r = \mathcal{E} \), and \( \Omega_0 = \Omega_{1, l} = \Omega_{2, l} = 1 \) for \( l = 1, \cdots, L \). Also, we set \( N = 0 \) in \( \hat{F}_W(w) \) and \( \hat{f}_W(w) \) for \( F_W(w) \) and \( f_W(w) \) of \( \mathcal{M}_V^{RS-OSS}(s) \) in (5.26), which in turn is substituted into (5.27) and (5.29). Even with \( N = 0 \), we can see that (5.27) and (5.29) are very accurate. Irrespective of SNR value in Figs. 5.3 and 5.4, we can see the proposed joint RS-OSS considerably outperforms the conventional RS in terms of the outage probability and average BER.

\footnote{Note that our proposed scheme can be considered as unidirectional communications with two separate source-destination pairs. In that sense, our scheme is a generalization of conventional RS}
Figure 5.2: Normalized truncation error $|\left(\Psi_j(w; \xi_1, \xi_2) - \hat{\Psi}_j^{(N)}(w; \xi_1, \xi_2)\right)/\Psi_j(w; \xi_1, \xi_2)|$ against truncation window size $N$ for $j = 1, 2$. $w = \xi_1 = \xi_2 = 1$ and $E_s/2 = E_r = E$. 
Figure 5.3: Outage probabilities against $10 \log_{10} E$ of the joint RS-OSS and the conventional RS when there is a direct-path. $L = 1, 2, 4$. $R = 1.5$ bps/Hz. $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$ for $l = 1, \cdots, L$. $E_s/2 = \mathcal{E}_r = \mathcal{E}$. $N = 0$ in $\hat{F}_{w_l}^{(N)}(w)$ for $F_{w_l}(w)$ in (5.19). Conventional RS means RS in unidirectional networks.
Figure 5.4: Average BERs against $10 \log_{10} E$ of the joint RS-OSS and the conventional RS when there is a direct-path. QPSK is used. $L = 1, 2, 4$. $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$ for $l = 1, \cdots, L$. $\mathcal{E}_s/3 = \mathcal{E}_r = \mathcal{E}$. $N = 0$ in $\hat{F}_{\mathcal{W}_l}^{(N)}(w)$ and $\hat{f}_{\mathcal{W}_l}^{(N)}(w)$ for $F_{\mathcal{W}_l}(w)$ and $f_{\mathcal{W}_l}(w)$ of $\mathcal{M}_V^{RS-OSS}(s)$ in (5.26), which in turn is substituted into (5.27) and (5.29). Conventional RS means RS in unidirectional networks.
Then we investigate the effect of relay location. Let $d_{S_1, R}$ denote the distance between $S_1$ and $R_l$, and $d_{S_2, R}$, the distance between $S_2$ and $R_l$, both of which are normalized by the distance between $S_1$ and $S_2$. Therefore, we have $d_{S_1, R} + d_{S_2, R} = 1$.

Furthermore, we set the path loss exponent as four to model radio propagation in urban areas [79]. As a result, we set $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1, R}^{-4}$, and $\Omega_{2,l} = (1 - d_{S_1, R})^{-4}$ for $l = 1, \cdots, L$. $\mathcal{E}_s = \mathcal{E}_r / 2 = 15$ dB. $N = 0$ in $\hat{F}_W^{(N)}(w)$ for $F_W(w)$ in (5.25). Conventional RS means RS in unidirectional networks.

Fig. 5.5 shows the outage probabilities against $d_{S_1, R}$ of the joint RS-OSS and the conventional RS when there is no direct-path. $L = 1, 2, 3$, $R = 1$ bps/Hz. $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1, R}^{-4}$, and $\Omega_{2,l} = (1 - d_{S_1, R})^{-4}$ for $l = 1, \cdots, L$. $\mathcal{E}_s = \mathcal{E}_r / 2 = 15$ dB, and $N = 0$ in $\hat{F}_W^{(N)}(w)$.

Fig. 5.6 shows the outage probabilities against $10 \log_{10} \mathcal{E}$ of the joint RS-OSS and in unidirectional networks.
Figure 5.6: Outage probabilities against $10 \log_{10} E$ of the joint RS-OSS and the conventional RS when there is a direct-path. $L = 1, 2, 4$. $R = 1.5$ bps/Hz. $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^4$, and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ for $l = 1, \cdots, L$ with $d_{S_1,R} = 0.8$. $\mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E}$. $N = 2$ in $\hat{F}_{W_l}^{(N)}(w)$ for $F_{W_l}(w)$ in (5.19). Conventional RS means RS in unidirectional networks.

The conventional RS when $d_{S_1,R} = 0.8$ and there is a direct-path. We set $L = 1, 2, 4$, $R = 1.5$ bps/Hz, $\mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E}$, and $N = 2$ in $\hat{F}_{W_l}^{(N)}(w)$. Fig. 5.7 shows the average BERs against $10 \log_{10} E$ of the joint RS-OSS and the conventional RS when $d_{S_1,R} = 0.8$ and there is a direct-path for QPSK. We set $L = 1, 2, 4$, $\mathcal{E}_s/3 = \mathcal{E}_r = \mathcal{E}$, and $N = 0$ in $\hat{F}_{W_l}^{(N)}(w)$ and $\hat{f}_{W_l}^{(N)}(w)$. Compared with Figs. 5.3 and 5.4, we can see in Figs. 5.6 and 5.7 that the difference between the joint RS-OSS and conventional RS becomes much larger. Irrespective of relay location in Fig. 5.5 and SNR value in Figs. 5.6 and 5.7, we can see the proposed joint RS-OSS considerably outperforms the conventional RS in terms of the outage probability and average BER.
Figure 5.7: Average BERs against $10 \log_{10} \mathcal{E}$ of the joint RS-OSS and the conventional RS when there is a direct-path. QPSK is used. $L = 1, 2, 4$. $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ for $l = 1, \cdots, L$ with $d_{S_1,R} = 0.8$. $\mathcal{E}_s/3 = \mathcal{E}_r = \mathcal{E}$. $N = 2$ in $\tilde{F}^{(N)}_{W_l}(w)$ and $\tilde{j}^{(N)}_{W_l}(w)$ for $F_{W_l}(w)$ and $f_{W_l}(w)$ of $\mathcal{M}^{\text{RS-OSS}}_V(s)$ in (5.26), which in turn is substituted into (5.27) and (5.29). Conventional RS means RS in unidirectional networks.
5.7 Conclusions

In this chapter, we have proposed the joint RS-OSS protocol in an AF-based bidirectional relay network consisting of two end-sources and multiple relays. In this protocol, RS and OSS were combined optimally in the sense that the mutual information was maximized. Specifically, a best source was selected to transmit data to the other source with the help of a selected best relay in an opportunistic manner depending on channel conditions. Then, we have derived the outage probability of the joint RS-OSS in a one-integral form and its lower bound in closed-form. Also, for M-QAM, we have derived the average BER of the joint RS-OSS in a one-integral form and its lower bound in closed-form. We have found that the proposed joint RS-OSS considerably outperformed both RS and OSS in terms of the outage probability and the average BER. Also, we have found that the performance was highly dependent on relay location.
Chapter 6
General Discussion

In this chapter, we will discuss all RS schemes proposed for AF-based bidirectional protocols. Specifically, we compare the proposed RS-OSS with the RS with ANC and TDBC. Fig. 6.1 shows the outage probabilities against $10 \log_{10} \mathcal{E}$ of the joint RS-OSS, the RS with ANC, and the RS with TDBC. We set $L = 1, 4$, $R = 1$ bps/Hz, and $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$ for $l = 1, \ldots, L$. To satisfy the same transmission power consumption, we set $\mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E}$ for the joint RS-OSS, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_r = \mathcal{E}$ for the RS with ANC and TDBC. Also, we set $N = 0$ in $\hat{F}_{\mathcal{W}_l}(w)$ for $F_{\mathcal{W}_l}(w)$ in (5.19). Fig. 6.2 shows the average BERs against $10 \log_{10} \mathcal{E}$ of the joint RS-OSS, the RS with ANC, and the RS with TDBC. We set $L = 1, 4$, $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$ for $l = 1, \ldots, L$, $\mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E}$ for the joint RS-OSS, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_r = \mathcal{E}$ for the RS with ANC and TDBC. To make the spectral efficiency the same, we assume the joint RS-OSS adopts 16-QAM, the RS with ANC adopts QPSK, and the RS with TDBC adopts 8-QAM. Also, we set $N = 0$ in $\hat{F}_{\mathcal{W}_l}(w)$ and $\hat{f}_{\mathcal{W}_l}(w)$ for $F_{\mathcal{W}_l}(w)$ and $f_{\mathcal{W}_l}(w)$ of $\mathcal{M}_{\mathcal{V}}^{\text{RS-OSS}}(s)$ in (5.20), which in turn is substituted into (5.27).

Also, we investigate the effect of relay location. Fig. 6.3 shows the outage probabilities against $d_{S_1,R}$ of the joint RS-OSS, the RS with ANC, and the RS with TDBC. We set $L = 1, 4$, $R = 1.5$ bps/Hz, $\Omega_0 = 1$, $\Omega_{1,l} = d_{S_1,R}^{-4}$, and $\Omega_{2,l} = (1 - d_{S_1,R})^{-4}$ for $l = 1, \ldots, L$, $\mathcal{E}_s = \mathcal{E}_r/2 = 10$ dB for the joint RS-OSS, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_r = 10$ dB for the RS with ANC and TDBC. Also, we set $N = 0$ in $\hat{F}_{\mathcal{W}_l}(w)$ for $F_{\mathcal{W}_l}(w)$ in (5.19). Irrespective of the SNR value in Figs. 6.1 and 6.2 and relay locations in Fig. 6.3, we
Figure 6.1: Outage probabilities against $10 \log_{10} \mathcal{E}$ of the joint RS-OSS, the RS with ANC, and the RS with TDBC. $L = 1, 4$. $R = 1 \text{ bps/Hz}$. $\Omega_0 = \Omega_{1,l} = \Omega_{2,l} = 1$ for $l = 1, \cdots, L$. $\mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E}$ for the joint RS-OSS, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_r = \mathcal{E}$ for the RS with ANC and TDBC. $N = 0$ in $\hat{F}_{W_l}(w)$ for $F_{W_l}(w)$ in (5.19).
Figure 6.2: Average BERs against 10 log₁₀ $E$ of the joint RS-OSS, the RS with ANC, and the RS with TDBC. $L = 1, 4$. $\Omega_0 = \Omega_{1,1} = \Omega_{2,1} = 1$ for $l = 1, \cdots, L$. $\mathcal{E}_s/2 = \mathcal{E}_r = \mathcal{E}$ for the joint RS-OSS, and $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_r = \mathcal{E}$ for the RS with ANC and TDBC. 16-QAM for the joint RS-OSS, QPSK for the RS with ANC, and 8-QAM for the RS with TDBC. $N = 0$ in $F_{\mathcal{W}_0}^{(N)}(w)$ and $f_{\mathcal{W}_0}^{(N)}(w)$ for $F_{\mathcal{W}_1}(w)$ and $f_{\mathcal{W}_1}(w)$ of $\mathcal{M}^{\text{RS-OSS}}_{\mathcal{V}}(s)$ in (5.26), which in turn is substituted into (5.27).
can see the proposed joint RS-OSS considerably outperforms both the RS with ANC and the RS with TDBC in terms of the outage probability and average BER.

In general, when there exist $L$ relays in a network, the RS with ANC can not utilize the direct channel between the two sources. Thus, the protocol can achieve diversity order $L$. On the other hand, since the RS with TDBC can utilize the direct channel, it can achieve full diversity order $L+1$. Also, the joint RS-OSS exploits time-varying channel fluctuations, which can be utilized to further improve the reliability. On the other hand, for the information exchange between two end-sources, the RS with ANC requires two time slots, the RS with TDBC requires three time slots,
Table 6.1: Comparison of the joint RS-OSS, RS with ANC, and RS with TDBC in terms of reliability and bandwidth efficiency, where the notation $A \succ B$ means $A$ outperforms $B$.

<table>
<thead>
<tr>
<th>Reliability</th>
<th>joint RS-OSS $\succ$ RS with TDBC $\succ$ RS with ANC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth efficiency</td>
<td>RS with ANC $\succ$ RS with TDBC $\succ$ joint RS-OSS</td>
</tr>
</tbody>
</table>

and the joint RS-OSS requires four time slots. Therefore, the joint RS-OSS achieves the highest reliability and lowest bandwidth efficiency; the RS with TDBC achieves medium reliability and bandwidth efficiency; and the RS with ANC achieves lowest reliability and highest bandwidth efficiency, which is given in Table 6.1.
Chapter 7

Conclusions and Future Work

7.1 Conclusions

This thesis carried out a comprehensive study on RS in bidirectional single-antenna relay networks as well as RS in unidirectional multi-antenna relay networks.

Firstly, we have extended two well-known RS schemes, OR and SC, to a DF-based unidirectional multi-antenna relay network. Specifically, combining OR and SC, respectively, with TAS, we have first proposed two joint relay-and-antenna selection schemes, namely, joint OR-TAS and joint SC-TAS. For each joint selection scheme, a single best transmit antenna at the source, a single best relay, and a single best transmit antenna at this selected relay were jointly determined in an optimum sense. We have derived the outage probability of joint OR-TAS. Also, we have obtained the outage probability of joint SC-TAS by proving that the outage probability of joint SC-TAS was identical to that of joint OR-TAS.

Secondly, we have studied RS with the PNC protocol in a DF-based bidirectional relay network consisting of two end-sources and multiple relays, where each relay adopted XOR-encoding to combine two received symbols from the two end-sources. Specifically, we have proposed two RS schemes for the PNC: SC-PNC and OR-PNC. For the SC-PNC, in the MAC phase, we have determined a set of relays that correctly decoded two received symbols from both end-sources; in the BC phase, among the relays in the determined set, we have selected a single best relay such that the
minimum mutual information of the two links from each relay to the two end-sources was maximized. For the OR-PNC, we have selected a single best relay such that the minimum mutual information of both the MAC phase, including the sum capacity constraint, and the BC phase was maximized. For these two RS schemes, we have derived the exact outage probabilities, which were identical. Furthermore, we have analytically shown that the SC-PNC and OR-PNC achieved the full diversity order.

Thirdly, we have studied RS with the ANC and TDBC protocols in an AF-based bidirectional relay network consisting of two end-sources and multiple relays. A single best relay was selected depending on channel conditions to help bidirectional communication between the two end-sources. Specifically, we have selected a single best relay based on a max-min criterion to minimize the outage probabilities of the ANC and TDBC protocols. Then, for the RS in the ANC protocol, we have derived a closed-form expression of the outage probability. Also, for the RS in the TDBC protocol, we have derived a one-integral form of the outage probability and its lower bound in closed-form.

Fourthly, we have proposed the joint RS-OSS protocol in an AF-based bidirectional relay network consisting of two end-sources and multiple relays. In this protocol, RS and OSS were combined optimally in the sense that the mutual information was maximized. Specifically, a best source was selected to transmit data to the other source with the help of a selected best relay in an opportunistic manner depending on channel conditions. Then, we have derived the outage probability of the joint RS-OSS in a one-integral form and its lower bound in closed-form. Also, for $M$-QAM, we have derived the average BER of the joint RS-OSS in a one-integral form and its lower bound in closed-form. We have found that the proposed joint RS-OSS considerably outperformed both RS and OSS in terms of the outage probability and the average BER.

The proposed RS schemes could help the design of reliable bidirectional single-antenna relay networks as well as unidirectional multi-antenna relay networks. Specif-
ically, the obtained outage probabilities and average BER will help the design of reliable relay networks in determining the system parameters such as relay location and the transmission power at source and relay.

7.2 Future Work

The work presented in this thesis can be extended further in many directions.

In our works, we considered only two-hop wireless networks. This two-hop network can be considered as a special case of multi-hop networks. Therefore, one can investigate RS for both unidirectional and bidirectional multi-hop networks.

Also, in the work of Chapter 2, for joint relay-and-antenna selection in the DF-based unidirectional multi-antenna relay network, we optimized system performance based on the outage probability. However, to the best of our knowledge, there has been no work on joint relay-and-antenna selection based on the BER or SER. Therefore, this can be considered as a possible future work. We conjecture that joint relay-and-antenna selection based on the BER or SER is a very challenging job. Furthermore, one can also investigate RS with precoding and beamforming when each terminal has multiple antennas and transmitters have the full CSI.

In the work of Chapters 3 and 4, for RS in the PNC, ANC, and TDBC protocols, we assumed that two end-sources had the same target rate. It will be very interesting to investigate the case when two end-sources have different target rates. Also, one can also investigate RS with optimal power allocation when each terminal has a single antenna, and RS with precoding and beamforming when each terminal has the full CSI with multiple antennas.

In the work of Chapter 5, for joint RS and OSS protocol, we assumed that two end-sources had the same transmission power and same target rate. What if the two end-sources have different transmission power and/or different target rates? Then it must be interesting to investigate joint RS and OSS under the different transmission power and different target rates. Furthermore, it is also interesting to investigate RS
with precoding and beamforming when each terminal has the full CSI with multiple antennas. All of these considerations are interesting and therefore can be considered as future work.
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Appendixes

Appendix 2-A
Proof of Lemma 2.1

First, we show that the diversity order of the joint OR-TAS is given in the form of (2.8). Since all the channels are independent, without loss of generality, we focus on the $l$-th relay-path from S via $R_l$ to D. We let $P_{\text{out},k}^{\text{OR-TAS}}(R)$ denote the outage probability of the $l$-th relay-path, and it is given by $P_{\text{out},l}^{\text{OR-TAS}}(R) = F_{\Gamma_1,l}(\bar{R}) + \ldots$
To simplify the derivation of the diversity order of the l-th relay-path, we present upper and lower bounds of $P_{\text{out,}l}^{\text{OR-TAS}}(R)$. We first consider an upper bound. Since $F_{\Gamma_{1,l}}(\bar{R})$ and $F_{\Gamma_{2,l}}(\bar{R})$ are non-negative, it is evident that an upper bound $P_{\text{out,}l}^{\text{OR-TAS}}(R)$ can be given by $P_{\text{out,}l}^{\text{OR-TAS}}(R) \leq P_{\text{out,UB,}l}^{\text{OR-TAS}}(R) \overset{\text{def}}{=} F_{\Gamma_{1,l}}(\bar{R}) + F_{\Gamma_{2,l}}(\bar{R})$. Applying the asymptotic results for incomplete Gamma functions in (2.7), we can show that $F_{\Gamma_{1,l}}(\bar{R})$ and $F_{\Gamma_{2,l}}(\bar{R})$ decay as fast as $1/(M_SM_R k)$ and $1/(M_DM_R l)$, respectively, which follows the upper bound $P_{\text{out,UB,}l}^{\text{OR-TAS}}(R)$ decays as fast as $1/(\min[M_S, M_D]M_R l)$. Then we consider a lower bound. Using two inequalities, $P_{\text{out,}l}^{\text{OR-TAS}}(R) = F_{\Gamma_{1,l}}(\bar{R}) + F_{\Gamma_{2,l}}(\bar{R})(1 - F_{\Gamma_{1,l}}(\bar{R})) \geq F_{\Gamma_{1,l}}(\bar{R})$ and $P_{\text{out,}l}^{\text{OR-TAS}}(R) = F_{\Gamma_{2,l}}(\bar{R}) + F_{\Gamma_{1,l}}(\bar{R})(1 - F_{\Gamma_{2,l}}(\bar{R})) \geq F_{\Gamma_{2,l}}(\bar{R})$, we can obtain a lower bound $P_{\text{out,LB,}l}^{\text{OR-TAS}}(R)$ as follows: $P_{\text{out,}l}^{\text{OR-TAS}}(R) \geq P_{\text{out,LB,}l}^{\text{OR-TAS}}(R) \overset{\text{def}}{=} \max[F_{\Gamma_{1,l}}(\bar{R}), F_{\Gamma_{2,l}}(\bar{R})]$. Taking similar steps in the derivation of the upper bound, we can prove the lower bound $P_{\text{out,LB,}l}^{\text{OR-TAS}}(R)$ decays as fast as $1/(\min[M_S, M_D]M_R l)$. From the upper bound and the lower bound, we know that the diversity order of the l-th relay-path is $\min[M_S, M_D]M_R l$. Finally, since all the channels are independent, we can obtain (2.8).

We now prove that (2.8) represents full diversity order. To this end, we derive the diversity order of a genie-aided system that has side information about a symbol. Suppose that the genie informs about either a transmitted symbol at S to the best relay $R_i$ if $M_S \geq M_D$ or a detected symbol at the best relay $R_i$ to D if $M_S < M_D$. That is, if $M_S \geq M_D$, then the transmission from S to $R_i$ is error-free; if $M_S < M_D$, then the transmission from $R_i$ to D is error-free. Obviously, this system serves as a performance upper bound of the joint OR-TAS, and its diversity order is given by $\min[M_S, M_D] \sum_{l=1}^L M_{R_l}$, which is identical to (2.8). Therefore, the joint OR-TAS achieves full diversity order.
Appendix 2-B
Proof of Theorem 2.2

We let \( \varepsilon[A] \) denote an event \( A \). Then \( \varepsilon[\min[\Gamma_{1,l}, \Gamma_{2,l}] < \bar{R}] \) in (2.6) can be rewritten as

\[
\varepsilon[\min[\Gamma_{1,l}, \Gamma_{2,l}] < \bar{R}] = \varepsilon_{\mathcal{U}} - \varepsilon[\Gamma_{1,l} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,l} \geq \bar{R}]
\]

\[
= \varepsilon[\Gamma_{1,l} < \bar{R}] \cup (\varepsilon[\Gamma_{1,l} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,l} < \bar{R}]),
\]  

(2-B.1)

(2-B.2)

where \( \varepsilon_{\mathcal{U}} \) is the entire event, and \( \cup \) and \( \cap \) denote the union and intersection operators, respectively. We let \( \varepsilon_{\Psi_i}(l) \) denote an event for the probability \( \Psi_i(R; l) \) with \( i = 1, 2 \), i.e. \( \Pr[\varepsilon_{\Psi_i}(l)] = \Psi_i(R; l) \). Then we can rewrite (2.12) and (2.13) in terms of events as follows:

\[
\varepsilon_{\Psi_1}(l) = \varepsilon_{\Psi_1}(l - 1) \cup (\varepsilon_{\Psi_2}(l - 1) \cap \varepsilon[\Gamma_{1,(l)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(l)} < \bar{R}]),
\]

(2-B.3)

\[
\varepsilon_{\Psi_2}(l) = \varepsilon_{\Psi_2}(l - 1) \cap \varepsilon[\Gamma_{1,(l)} < \bar{R}],
\]

(2-B.4)

where \( \varepsilon_{\Psi_1}(0) = \emptyset, \varepsilon_{\Psi_2}(0) = \varepsilon_{\mathcal{U}}, \) and \( \emptyset \) denotes an empty event. In the following, using (2-B.1)–(2-B.4), we prove the outage event \( \varepsilon_{\text{out-OR-TAS}} \) of the joint OR-TAS is identical to that \( \varepsilon_{\text{out-SC-TAS}} \) of the joint SC-TAS.

We first consider a network with a single relay \( (L = 1) \). In this case, using (2-B.2)–(2-B.4), it is easy to show that \( \varepsilon_{\text{out-OR-TAS}} \) is identical to \( \varepsilon_{\text{out-SC-TAS}} \). We then consider a network with two relays \( (L = 2) \). In this case, using (2-B.1) and (2-B.2), the outage event \( \varepsilon_{\text{out-OR-TAS}} \) of the joint OR-TAS is given by

\[
\varepsilon_{\text{out-OR-TAS}} = \varepsilon[\min[\Gamma_{1,1}, \Gamma_{2,1}] < \bar{R}] \cap \varepsilon[\min[\Gamma_{1,2}, \Gamma_{2,2}] < \bar{R}]
\]

\[
= (\varepsilon[\Gamma_{1,1} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,1} < \bar{R}] \cap (\varepsilon_{\mathcal{U}} - \varepsilon[\Gamma_{1,2} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,2} \geq \bar{R}]))
\]

\[
\cup (\varepsilon[\Gamma_{1,1} < \bar{R}] \cap (\varepsilon[\Gamma_{1,2} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,1} < \bar{R}]) \cup (\varepsilon[\Gamma_{1,1} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,2} < \bar{R}])).
\]

(2-B.5)

Similarly, the outage event \( \varepsilon_{\text{out-OR-TAS}} \) can be given in another form as follows:

\[
\varepsilon_{\text{out-OR-TAS}} = (\varepsilon[\Gamma_{1,2} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,2} < \bar{R}] \cap (\varepsilon_{\mathcal{U}} - \varepsilon[\Gamma_{1,1} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,1} \geq \bar{R}]))
\]

\[
\cup (\varepsilon[\Gamma_{1,2} < \bar{R}] \cap (\varepsilon[\Gamma_{1,1} < \bar{R}] \cup (\varepsilon[\Gamma_{1,1} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,1} < \bar{R}]))).
\]

(2-B.6)
Note that (2-B.5) and (2-B.6) are equivalent but have different forms. Using (2-B.5) if \( \Gamma_{2,1} \geq \Gamma_{2,2} \) and (2-B.6) if \( \Gamma_{2,1} < \Gamma_{2,2} \), \( \varepsilon_{\text{out-TAS}}^{\text{OR}} \) can be rewritten as:

\[
\varepsilon_{\text{out-TAS}}^{\text{OR}} = (\varepsilon_{\Psi_1}(1) \cap (\varepsilon_{\mathcal{U}} - \varepsilon[\Gamma_{1,(2)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(2)} \geq \bar{R}])
\]

\[
\cup (\varepsilon_{\Psi_2}(1) \cap (\varepsilon[\Gamma_{1,(2)} < \bar{R}] \cup (\varepsilon[\Gamma_{1,(2)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(2)} < \bar{R}])))
\]

\[
= \varepsilon_{\Psi_1}(2) \cup \varepsilon_{\Psi_2}(2) - (\varepsilon_{\Psi_1}(1) \cap \varepsilon[\Gamma_{1,(2)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(2)} \geq \bar{R}])
\]

\[
= \varepsilon_{\Psi_1}(2) \cup \varepsilon_{\Psi_2}(2) = \varepsilon_{\text{out-TAS}}^{\text{SC}}.
\]

(2-B.9)

In the step from (2-B.7) to (2-B.8), we use (2-B.3) and (2-B.4). Also, in the step from (2-B.8) to (2-B.9), since \( \varepsilon[\Gamma_{2,(1)} < \bar{R}] \varepsilon[\Gamma_{2,(2)} \geq \bar{R}] = \emptyset \) which contradicts the assumption \( \Gamma_{2,(1)} \geq \Gamma_{2,(2)} \), the term \( \varepsilon_{\Psi_1}(1) \cap \varepsilon[\Gamma_{1,(2)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(2)} \geq \bar{R}] = \varepsilon[\Gamma_{1,(1)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(1)} < \bar{R}] \cap \varepsilon[\Gamma_{1,(2)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(2)} \geq \bar{R}] = \emptyset \). Finally, we consider a network with more than two relays \( (L \geq 3) \). Taking steps similar to those used from (2-B.5) to (2-B.9), we can show that

\[
\varepsilon_{\text{out-TAS}}^{\text{OR}} = (\varepsilon_{\Psi_1}(L - 1) \cap (\varepsilon_{\mathcal{U}} - \varepsilon[\Gamma_{1,(L)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(L)} \geq \bar{R}])
\]

\[
\cup (\varepsilon_{\Psi_2}(L - 1) \cap (\varepsilon[\Gamma_{1,(L)} < \bar{R}] \cup (\varepsilon[\Gamma_{1,(L)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(L)} < \bar{R}])))
\]

\[
= \varepsilon_{\Psi_1}(L) \cup \varepsilon_{\Psi_2}(L) - (\varepsilon_{\Psi_1}(L - 1) \cap \varepsilon[\Gamma_{1,(L)} \geq \bar{R}] \cap \varepsilon[\Gamma_{2,(L)} \geq \bar{R}])
\]

\[
= \varepsilon_{\Psi_1}(L) \cup \varepsilon_{\Psi_2}(L)
\]

\[
= \varepsilon_{\text{out-TAS}}^{\text{SC}}.
\]

(2-B.10)

Therefore, it is evident that \( P_{\text{out-TAS}}^{\text{SC}}(R) \) in (2.9) is identical to \( P_{\text{out-TAS}}^{\text{OR}}(R) \) in (2.6).

---

1 In this chapter, we use (2-B.5) if \( \Gamma_{2,1} \geq \Gamma_{2,2} \) and (2-B.6) if \( \Gamma_{2,1} < \Gamma_{2,2} \). It is also possible to prove that (2-B.5) reduces to (2-B.9) even if \( \Gamma_{2,1} < \Gamma_{2,2} \), and that (2-B.6) reduces to (2-B.9) even if \( \Gamma_{2,1} \geq \Gamma_{2,2} \). However, this proof requires long derivation. Also, our approach is more intuitive, and it can be easily extended to a network with more than two relays \( (L \geq 3) \).
Appendix 3-A
Proof of Theorem 3.1

Since the outage probability of (3.14) involves the set $D$, it is very difficult to directly solve the probability. Therefore, we take another approach, which involves two dimensional region $(X_l, Y_l)$. Using (3.1) and (3.3), we first define $R_{MAC}$ as the two-dimensional region $(X_l, Y_l)$ satisfying \{$E_1 X_l \geq T_1, E_2 Y_l \geq T_1, E_1 X_l + E_2 Y_l \geq T_2$\} as follows:

$$R_{MAC} := \{(X_l, Y_l) : E_1 X_l \geq T_1, E_2 Y_l \geq T_1, E_1 X_l + E_2 Y_l \geq T_2\}. \quad (3-A.1)$$

Also, using (3.4), we define $R_{BC}$ as the two-dimensional regions $(X_l, Y_l)$ satisfying \{$E_R X_l \geq T_1, E_R Y_l \geq T_1$\} as follows:

$$R_{BC} := \{(X_l, Y_l) : E_R X_l \geq T_1, E_R Y_l \geq T_1\}. \quad (3-A.2)$$

Note that $R_{MAC}$ and $R_{BC}$ denote the non-outage regions for the MAC and BC phases, respectively.

We now rewrite the outage probability $P_{out}^{SC-PNC}(R)$ of (3.14) in terms of $R_{MAC}$ and $R_{BC}$. To this end, we first let $\epsilon_{out}^{SC-PNC}$ denote the outage event of the SC-PNC; let $\epsilon[D]$ denote the event of $D$, where $\epsilon[A]$ means an event $A$; and let $\epsilon_{out[D]}^{SC-PNC}$ denote the outage event of the SC-PNC given the set $D$. Taking a step similar to [50, eq. (7)], the outage event $\epsilon_{out}^{SC-PNC}$ of the SC-PNC can be expressed in terms of $\epsilon_{out[D]}^{SC-PNC}$ and $\epsilon[D]$ as follows:

$$\epsilon_{out}^{SC-PNC} = \bigcup_{|D|=0}^L \left( \epsilon_{out[D]}^{SC-PNC} \cap \epsilon[D] \right), \quad (3-A.3)$$

where $\bigcup$ denotes the union operator, and $\cap$ denotes the intersection operator. In the above equation,

$$\epsilon_{out[D]}^{SC-PNC} = \bigcap_{i \in D} \epsilon[(X_i, Y_i) \in R_{BC}^C], \quad (3-A.4)$$

$$\epsilon[D] = \left( \bigcap_{i \in D} \epsilon[(X_i, Y_i) \in R_{MAC}] \right) \bigcap \left( \bigcap_{j \notin D} \epsilon[(X_j, Y_j) \in R_{MAC}] \right). \quad (3-A.5)$$
Substituting (3-A.4) and (3-A.5) into (3-A.3) yields

\[ \epsilon_{SC-PNC}^\text{out} = \bigcup_{|D|=0}^L \left[ \left( \bigcap_{i \in D} \epsilon[(X_i, Y_i) \in \mathcal{R}_{MAC} \cap \mathcal{R}_{BC}^C] \right) \cap \left( \bigcap_{j \notin D} \epsilon[(X_j, Y_j) \in \mathcal{R}_{MAC}^c] \right) \right]. \]  

(3-A.6)

Note that the two events \( \epsilon[(X_l, Y_l) \in \mathcal{R}_{MAC} \cap \mathcal{R}_{BC}^C] \) and \( \epsilon[(X_l, Y_l) \in \mathcal{R}_{MAC}^c] \) in (3-A.6) are disjoint, because \( \epsilon[\mathcal{R}_{MAC} \cap \mathcal{R}_{BC}^C \cap \mathcal{R}_{MAC}^c] = \emptyset \), where \( \emptyset \) denotes an empty event.

Then it can be shown that the outage event \( \epsilon_{SC-PNC}^\text{out} \) in (3-A.6) is rewritten as

\[ \epsilon_{SC-PNC}^\text{out} = \bigcup_{|D|=0}^L \left[ \bigcap_{l \in D} \epsilon[(X_l, Y_l) \in (\mathcal{R}_{MAC} \cap \mathcal{R}_{BC}^C) \cup \mathcal{R}_{MAC}^c] \right] \]

\[ = \bigcap_{l=1}^L \epsilon[(X_l, Y_l) \in (\mathcal{R}_{MAC} \cap \mathcal{R}_{BC}^C) \cup \mathcal{R}_{MAC}^c] \]

\[ = \bigcap_{l=1}^L \epsilon[(X_l, Y_l) \in \mathcal{R}_{MAC}^c \cup \mathcal{R}_{BC}^c]. \]  

(3-A.7)

From (3-A.7), one can obtain \( P_{out}^{SC-PNC}(R) \) as follows:

\[ P_{out}^{SC-PNC}(R) = \prod_{l=1}^L \Phi_l(R). \]  

(3-A.8)

In the above equation,

\[ \Phi_l(R) = \Pr[(X_l, Y_l) \in \mathcal{R}_{MAC}^c \cup \mathcal{R}_{BC}^c], \]  

(3-A.9)

where \( \mathcal{R}^C \) denotes the complementary region of \( \mathcal{R} \).

Since the random variables \( \{X_l, Y_l : l = 1, \ldots, L\} \) are independent one another, we focus on \( \Phi_l(R) \). Depending on \( (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_R, R) \), the region \( \mathcal{R}_{MAC}^c \cup \mathcal{R}_{BC}^c \) in (3-A.9) has different forms. We first consider \( \mathcal{E}_2 \geq \mathcal{E}_1 \), and there are five possible cases:

\[ \mathcal{E}_R \geq \mathcal{E}_2 \geq \mathcal{E}_1; \mathcal{E}_2 \geq \mathcal{E}_R \geq \mathcal{E}_1 \text{ and } \mathcal{E}_2 \leq \mathcal{E}_R(T_1 + 1); \mathcal{E}_2 \geq \mathcal{E}_R \geq \mathcal{E}_1 \text{ and } \mathcal{E}_2 > \mathcal{E}_R(T_1 + 1); \]

\[ \mathcal{E}_2 \geq \mathcal{E}_1 \geq \mathcal{E}_R \text{ and } \mathcal{E}_1 + \mathcal{E}_2 \leq \mathcal{E}_R T_2/T_1; \mathcal{E}_2 \geq \mathcal{E}_1 \geq \mathcal{E}_R \text{ and } \mathcal{E}_1 + \mathcal{E}_2 > \mathcal{E}_R T_2/T_1. \]

One can
calculate the probability $\Phi_1(R)$ as follows:

\[
\Phi_1(R) = \begin{cases} 
\Xi_1 \left( T_1, T_1, \xi_2, \xi_1 \right), & \xi_R \geq \xi_2 \geq \xi_1, \\
\Xi_2 \left( \xi_{2R} T_1, T_1, \xi_R, \xi_1 \right), & \xi_2 \geq \xi_R \geq \xi_1 \text{ and } \xi_2 \leq \xi_R(T_1 + 1), \\
\Xi_3 \left( \xi_{1R}, \xi_{1R} T_1, \xi_{2R}, \xi_R \right), & \xi_2 \geq \xi_R \geq \xi_1 \text{ and } \xi_2 > \xi_R(T_1 + 1), \\
\Xi_4 \left( \xi_{2R} T_2, \xi_{2R} T_1, \xi_R, \xi_R \right), & \xi_2 \geq \xi_R \geq \xi_1 \text{ and } \xi_1 + \xi_2 \leq \xi_R T_2 / T_1, \\
\Xi_5 \left( \xi_{2R} T_1, \xi_{2R} T_2, \xi_R \right), & \xi_2 \geq \xi_1 \geq \xi_R \text{ and } \xi_1 + \xi_2 > \xi_R T_2 / T_1.
\end{cases}
\] (3-A.10)

Taking integration, one can solve the probability $\Phi_1(R)$ of (3-A.10) as follows:

\[
\Phi_1(R) = \begin{cases} 
\Xi_1 \left( T_1, T_1, \xi_2, \xi_1 \right), & \xi_R \geq \xi_2 \geq \xi_1, \\
\Xi_2 \left( \xi_{2R} T_1, T_1, \xi_R, \xi_1 \right), & \xi_2 \geq \xi_R \geq \xi_1 \text{ and } \xi_2 \leq \xi_R(T_1 + 1), \\
\Xi_3 \left( \xi_{1R}, \xi_{1R} T_1, \xi_{2R}, \xi_R \right), & \xi_2 \geq \xi_R \geq \xi_1 \text{ and } \xi_2 > \xi_R(T_1 + 1), \\
\Xi_4 \left( \xi_{2R} T_2, \xi_{2R} T_1, \xi_R, \xi_R \right), & \xi_2 \geq \xi_R \geq \xi_1 \text{ and } \xi_1 + \xi_2 \leq \xi_R T_2 / T_1, \\
\Xi_5 \left( \xi_{2R} T_1, \xi_{2R} T_2, \xi_R \right), & \xi_2 \geq \xi_1 \geq \xi_R \text{ and } \xi_1 + \xi_2 > \xi_R T_2 / T_1.
\end{cases}
\] (3-A.11)

Taking steps similar to those used from (3-A.10) to (3-A.11), one can calculate $\Phi_1(R)$ for $\xi_2 < \xi_1$. Combining $\xi_2 \geq \xi_1$ and $\xi_2 < \xi_1$, one can obtain $\Phi_1(R)$ of (3.10).

Finally, substituting $\Phi_1(R)$ into (3-A.8), one can obtain the exact outage probability $P_{\text{out}}^{\text{SC-PNC}}(R)$ of (3.13) in closed-form.

**Appendix 3-B**

**Proof of Theorem 3.2**

Since it is very difficult to directly derive $F_{W_i}(2R)$ of (3.25), we divide $W_i$ of (3.24) into two parts as follows:

\[
W_i = \min \left[ W_i, \sqrt{1 + \xi_1 X_i + \xi_2 Y_i} \right],
\] (3-B.1)
where $\mathcal{V}_l = 1 + \min \left[ \mathcal{E}_1 X_l, \mathcal{E}_2 Y_l, \mathcal{E}_R X_l, \mathcal{E}_R Y_l \right]$. Depending on $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_R)$, $\mathcal{V}_l$ can be given by

$$
\mathcal{V}_l = \begin{cases} 
1 + \min[\mathcal{E}_1 X_l, \mathcal{E}_2 Y_l], & \mathcal{E}_R \geq \max[\mathcal{E}_1, \mathcal{E}_2], \\
1 + \min[\mathcal{E}_1 X_l, \mathcal{E}_R Y_l], & \mathcal{E}_2 \geq \mathcal{E}_R \geq \mathcal{E}_1, \\
1 + \min[\mathcal{E}_R X_l, \mathcal{E}_2 Y_l], & \mathcal{E}_1 \geq \mathcal{E}_R \geq \mathcal{E}_2, \\
1 + \mathcal{E}_R \min[X_l, Y_l], & \mathcal{E}_R \leq \min[\mathcal{E}_1, \mathcal{E}_2].
\end{cases}
$$

(3-B.2)

For each case, we derive the CDF $F_{\mathcal{W}_l}(2^R)$ of $\mathcal{W}_l$ in the following.

1. $\mathcal{E}_R \geq \max[\mathcal{E}_1, \mathcal{E}_2]$

We first consider $\mathcal{E}_R \geq \mathcal{E}_2 \geq \mathcal{E}_1$. In this case, $\mathcal{V}_l$ of (3-B.2) can be simplified into

$$
\mathcal{V}_l = \begin{cases} 
1 + \mathcal{E}_1 X_l, & \mathcal{E}_1 X_l \leq \mathcal{E}_2 Y_l, \\
1 + \mathcal{E}_2 Y_l, & \mathcal{E}_1 X_l > \mathcal{E}_2 Y_l.
\end{cases}
$$

(3-B.3)

Substituting (3-B.3) into (3-B.1) yields

$$
\mathcal{W}_l = \begin{cases} 
1 + \mathcal{E}_1 X_l, & (X_l, Y_l) \in \mathcal{R}_{A,1}, \\
\sqrt{1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l}, & (X_l, Y_l) \in \mathcal{R}_{A,2} \cup \mathcal{R}_{A,3}, \\
1 + \mathcal{E}_2 Y_l, & (X_l, Y_l) \in \mathcal{R}_{A,4},
\end{cases}
$$

(3-B.4)

where $\mathcal{R}_{A,1} = \{(X_l, Y_l) : \mathcal{E}_1 X_l \leq \mathcal{E}_2 Y_l, 1 + \mathcal{E}_1 X_l \leq \sqrt{1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l} \} = \{(X_l, Y_l) : X_l \geq 0, Y_l \geq \mathcal{E}_1 X_l(1 + \mathcal{E}_1 X_l)/\mathcal{E}_2 \};$ $\mathcal{R}_{A,2} = \{(X_l, Y_l) : \mathcal{E}_1 X_l \leq \mathcal{E}_2 Y_l, 1 + \mathcal{E}_1 X_l > \sqrt{1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l} \} = \{(X_l, Y_l) : (X_l, Y_l) \in \mathcal{R}_{A,2} \cup \mathcal{R}_{A,3}, \\
1 + \mathcal{E}_2 Y_l, & (X_l, Y_l) \in \mathcal{R}_{A,4}.
\end{cases}
$$

\begin{align}
F_{\mathcal{W}_l}(2^R) \ &= \mathbb{P}[\mathcal{W}_l < 2^R] \\
&= \mathbb{P}[1 + \mathcal{E}_1 X_l < 2^R, (X_l, Y_l) \in \mathcal{R}_{A,1}] + \mathbb{P}[1 + \mathcal{E}_2 Y_l < 2^R, (X_l, Y_l) \in \mathcal{R}_{A,4}] \\
&\quad+ \mathbb{P}[\sqrt{1 + \mathcal{E}_1 X_l + \mathcal{E}_2 Y_l} < 2^R, (X_l, Y_l) \in \mathcal{R}_{A,2} \cup \mathcal{R}_{A,3}] \\
&= \mathbb{P}[\mathcal{E}_1 X_l < T_1, (X_l, Y_l) \in \mathcal{R}_{A,1}] + \mathbb{P}[\mathcal{E}_2 Y_l < T_1, (X_l, Y_l) \in \mathcal{R}_{A,4}] \\
&\quad+ \mathbb{P}[\mathcal{E}_1 X_l + \mathcal{E}_2 Y_l < T_2, (X_l, Y_l) \in \mathcal{R}_{A,2} \cup \mathcal{R}_{A,3}].
\end{align}

(3-B.5)
One can calculate the CDF \( F_{W_l}(2^R) \) of (3-B.5) as follows:

\[
F_{W_l}(2^R) = \int_{x=0}^{\frac{T_1}{2}} \int_{y=-\frac{E_1 x (1+\varepsilon_1 x)}{\varepsilon_1 T_2}}^{\infty} f_{X_l}(x)f_{Y_l}(y)dydx + \int_{y=0}^{\frac{T_1}{2}} \int_{x=-\frac{E_1 x (1+\varepsilon_1 x)}{\varepsilon_1 T_2}}^{\infty} f_{X_l}(x)f_{Y_l}(y)dydx
+ \int_{x=0}^{\frac{T_2}{2}} \int_{y=-\frac{E_2 x (1+\varepsilon_2 x)}{\varepsilon_2 T_2}}^{\infty} f_{X_l}(x)f_{Y_l}(y)dydx
+ \int_{y=0}^{\frac{T_2}{2}} \int_{x=-\frac{E_2 x (1+\varepsilon_2 x)}{\varepsilon_2 T_2}}^{\infty} f_{X_l}(x)f_{Y_l}(y)dydx
= \Xi_1(T_1, T_1', \varepsilon_2, \varepsilon_1),
\]  

(3-B.6)

where we use \( \sqrt{T_1 + T_2} = T_1 + 1 \). For \( \varepsilon_2 \geq \varepsilon_1 \geq \varepsilon_2 \), taking steps similar to those used from (3-B.3) to (3-B.6), one can obtain \( F_{W_l}(2^R) = \Xi_1(T_1, T_1', \varepsilon_2, \varepsilon_1) \). For Case 1, therefore, one knows that \( F_{W_l}(2^R) \) is identical to \( \Phi_l(R) \) of (3.16).

2. \( \varepsilon_2 \geq \varepsilon_R \geq \varepsilon_1 \) and \( \varepsilon_1 \geq \varepsilon_R \geq \varepsilon_2 \)

We first consider \( \varepsilon_2 \geq \varepsilon_R \geq \varepsilon_1 \). Taking steps similar to those used from (3-B.3) to (3-B.4), one can rewrite \( W_l \) of (3-B.1) as follows:

\[
W_l = \left\{ \begin{array}{ll}
1 + \varepsilon_1 X_l, & (X_l, Y_l) \in \mathcal{R}_{B.1}, \\
\sqrt{1 + \varepsilon_1 X_l + \varepsilon_2 Y_l}, & (X_l, Y_l) \in \mathcal{R}_{B.2} \cup \mathcal{R}_{B.3}, \\
1 + \varepsilon_R Y_l, & (X_l, Y_l) \in \mathcal{R}_{B.4}.
\end{array} \right.
\]  

(3-B.7)

where \( \mathcal{R}_{B.1} = \{(X_l, Y_l) : 0 \leq X_l < (\varepsilon_2 - \varepsilon_R)/(\varepsilon_1 \varepsilon_R), Y_l \geq \varepsilon_1 X_l/\varepsilon_R\} \cup \{(X_l, Y_l) : X_l \geq (\varepsilon_2 - \varepsilon_R)/(\varepsilon_1 \varepsilon_R), Y_l \geq \varepsilon_1 X_l(1 + \varepsilon_1 X_l)/\varepsilon_2\} \cup \{(X_l, Y_l) : X_l \geq (\varepsilon_2 - \varepsilon_R)/(\varepsilon_1 \varepsilon_R), Y_l \geq \varepsilon_1 X_l(1 + \varepsilon_1 X_l)/\varepsilon_2\} =: \mathcal{R}_{B.1,1} \cup \mathcal{R}_{B.1,2}; \mathcal{R}_{B.2} = \{(X_l, Y_l) : X_l \geq (\varepsilon_2 - \varepsilon_R)/(\varepsilon_1 \varepsilon_R), \varepsilon_1 X_l/\varepsilon_R \leq Y_l < \varepsilon_1 X_l(1 + \varepsilon_1 X_l)/\varepsilon_2\}; \mathcal{R}_{B.3} = \{(X_l, Y_l) : \varepsilon_2 Y_l/\varepsilon_1 < X_l < Y_l(2 \varepsilon_R - \varepsilon_2 + \varepsilon_2^2 Y_l)/\varepsilon_1, Y_l \geq (\varepsilon_2 - \varepsilon_R)/\varepsilon_2^2\} \cup \{(X_l, Y_l) : X_l \geq Y_l(2 \varepsilon_R - \varepsilon_2 + \varepsilon_2^2 Y_l)/\varepsilon_1, Y_l \geq \varepsilon_2 - \varepsilon_R/\varepsilon_2^2\} =: \mathcal{R}_{B.4,1} \cup \mathcal{R}_{B.4,2}. \) It can be shown that

\[
\frac{\varepsilon_2 - \varepsilon_R}{\varepsilon_1 \varepsilon_R} \leq \frac{T_1}{\varepsilon_1} \leq \frac{\varepsilon_2 - \varepsilon_R}{\varepsilon_2^2} \leq \frac{T_1}{\varepsilon_R} \leq \frac{\varepsilon_2 - \varepsilon_R}{\varepsilon_R} + \frac{\varepsilon_2 (\varepsilon_2 - \varepsilon_R)}{\varepsilon_R^2} \leq T_2 \leq \frac{\varepsilon_2}{\varepsilon_R} \leq T_1 + 1.
\]  

(3-B.8)
Using (3-B.8), the CDF $F_{W_i}(2^R)$ of $W_i$ is given by

$$F_{W_i}(2^R) = \begin{cases} 
\Pr[\mathcal{E}_iX_i < T_1, (X_i, Y_i) \in \mathcal{R}_{B,1}] + \Pr[\mathcal{E}_iY_i < T_1, (X_i, Y_i) \in \mathcal{R}_{B,2}] \\
+ \Pr[\mathcal{E}_iX_i + \mathcal{E}_iY_i < T_2, (X_i, Y_i) \in \mathcal{R}_{B,3} \cup \mathcal{R}_{B,4}], & \text{Case 2}, \\
\Pr[\mathcal{E}_iX_i < T_1, (X_i, Y_i) \in \mathcal{R}_{B,1}] + \Pr[\mathcal{E}_iY_i < T_1, (X_i, Y_i) \in \mathcal{R}_{B,4}], & \text{Case 3}. 
\end{cases}$$

(3-B.9)

One can calculate the CDF $F_{W_i}(2^R)$ of (3-B.9) as follows:

$$F_{W_i}(2^R) = \begin{cases} 
\int_{x=0}^{T_1} \int_{y=\frac{x_1}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx + \int_{x=\frac{T_1}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} \int_{y=0}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx \\
+ \left[ \int_{x=\frac{T_1}{\xi_i}}^{\frac{T_2}{\xi_i}} \int_{y=\frac{x_1}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx \right] + \int_{x=\frac{T_1}{\xi_i}}^{\frac{1+\sqrt{T_1T_2}}{\xi_i}} \int_{y=\frac{x_1}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx \\
+ \int_{x=\frac{T_1}{\xi_i}}^{\frac{T_2}{\xi_i}} \int_{y=\frac{x_1}{\xi_i}}^{\frac{1+\sqrt{T_1T_2}}{\xi_i}} \int_{x=\frac{T_2}{\xi_i}}^{T_1} \int_{y=\frac{y}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx, & \text{Case 2}, \\
\int_{x=0}^{T_1} \int_{y=\frac{x_1}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx + \int_{x=\frac{T_1}{\xi_i}}^{\frac{x_2-\xi_i}{T_1}} \int_{y=0}^{\frac{x_2-\xi_i}{T_1}} f_X(x) f_Y(y) dy dx, & \text{Case 3}. 
\end{cases}$$

(3-B.10)

Taking integration, one can obtain the closed-form expression of $F_{W_i}(2^R)$ in (3-B.10) as follows:

$$F_{W_i}(2^R) = \begin{cases} 
\Xi_1 \left( \frac{\xi_i T_1}{\xi_1}, T_1, \mathcal{E}_i, \mathcal{E}_1 \right), & \text{Case 2}, \\
\Xi_2 \left( \frac{T_1}{\xi_1}, \frac{T_1}{\xi_{1i}}, \frac{T_1}{\xi_{1i+1}} \right), & \text{Case 3}. 
\end{cases}$$

(3-B.11)

Taking steps similar to those used from (3-B.7) to (3-B.11), one can obtain $F_{W_i}(2^R) = \Xi_1(T_1, \xi_i T_1/\xi_R, \mathcal{E}_2, \mathcal{E}_R)$ for Case 4, and $F_{W_i}(2^R) = \Xi_2(T_1/(\xi_R \xi_{1i} + T_1/(\xi_{2i+1} \mathcal{E}_R)))$ for Case 5. For Case $i$ with $i = 2, 3, 4, 5$, therefore, one knows that $F_{W_i}(2^R)$ is identical to $\Phi_i(R)$ of (3.10).
3. $\mathcal{E}_R \leq \min[\mathcal{E}_1, \mathcal{E}_2]$

We first consider $\mathcal{E}_2 \geq \mathcal{E}_1 \geq \mathcal{E}_R$. Taking steps similar to those used from (3-B.3) to (3-B.4), one can rewrite $\mathcal{W}_i$ of (3-B.1) as follows:

$$
\mathcal{W}_i = \begin{cases} 
1 + \mathcal{E}_R X_i, & (X_i, Y_i) \in \mathcal{R}_{C,1}, \\
\sqrt{1 + \mathcal{E}_4 X_i + \mathcal{E}_2 Y_i}, & (X_i, Y_i) \in \mathcal{R}_{C,2} \cup \mathcal{R}_{C,3}, \\
1 + \mathcal{E}_R Y_i, & (X_i, Y_i) \in \mathcal{R}_{C,4},
\end{cases}
$$

(3-B.12)

where $\mathcal{R}_{C,1} = \{(X_i, Y_i) : 0 \leq X_i < (\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R)/\mathcal{E}_R^2, Y_i \geq X_i\} \cup \{(X_i, Y_i) : X_i \geq (\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R)/\mathcal{E}_R^2, Y_i \geq X_i(2\mathcal{E}_R - \mathcal{E}_1 + \mathcal{E}_R^2 X_i)/\mathcal{E}_2\} =: \mathcal{R}_{C,1} \cup \mathcal{R}_{C,1,2}$; $\mathcal{R}_{C,2} = \{(X_i, Y_i) : X_i \geq (\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R)/\mathcal{E}_R^2, Y_i \leq Y_i(2\mathcal{E}_R - \mathcal{E}_1 + \mathcal{E}_R^2 X_i)/\mathcal{E}_2\}$; $\mathcal{R}_{C,3} = \{(X_i, Y_i) : Y_i < X_i < Y_i(2\mathcal{E}_R - \mathcal{E}_2 + \mathcal{E}_R^2 Y_i)/\mathcal{E}_1, Y_i \geq (\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R)/\mathcal{E}_R^2\}$; and $\mathcal{R}_{C,4} = \{(X_i, Y_i) : X_i > Y_i, 0 \leq Y_i < (\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R)/\mathcal{E}_R^2\} \cup \{(X_i, Y_i) : X_i \geq Y_i(2\mathcal{E}_R - \mathcal{E}_2 + \mathcal{E}_R^2 Y_i)/\mathcal{E}_1, Y_i \geq (\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R)/\mathcal{E}_R^2\}$.

It can be shown that

$$\frac{\mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R}{\mathcal{E}_R^2} \leq \frac{T_1}{\mathcal{E}_R} \leftrightarrow \frac{(\mathcal{E}_1 + \mathcal{E}_2) \mathcal{E}_1 + \mathcal{E}_2 - 2\mathcal{E}_R}{\mathcal{E}_R^2} \leq \frac{T_2}{\mathcal{E}_R} \leftrightarrow \frac{\mathcal{E}_1 + \mathcal{E}_2}{\mathcal{E}_R} \leq \frac{T_2}{T_1}. \quad (3-B.13)
$$

Using (3-B.13), the CDF $F_{\mathcal{W}_i}(2^R)$ of $\mathcal{W}_i$ is given by

$$
F_{\mathcal{W}_i}(2^R) = \begin{cases} 
\Pr[\mathcal{E}_R X_i < T_1, (X_i, Y_i) \in \mathcal{R}_{C,1}] + \Pr[\mathcal{E}_R Y_i < T_1, (X_i, Y_i) \in \mathcal{R}_{C,4}], \\
+ \Pr[\mathcal{E}_1 X_i + \mathcal{E}_2 Y_i < T_2, (X_i, Y_i) \in \mathcal{R}_{C,2} \cup \mathcal{R}_{C,3}], \\
\frac{\mathcal{E}_1 + \mathcal{E}_2}{\mathcal{E}_R} \geq \frac{\mathcal{E}_1}{\mathcal{E}_R} \geq \mathcal{E}_R \quad \text{and} \quad \frac{\mathcal{E}_1 + \mathcal{E}_2}{\mathcal{E}_R} \leq \frac{T_2}{T_1},
\end{cases}
$$

(3-B.14)
One can calculate the CDF $F_{W_l}(2R)$ as follows:

$$
F_{W_l}(2R) = \left\{ \begin{array}{l}
\Xi_1 \left( \frac{\xi_1 + \xi_2 - 2\xi_{R}}{\xi_{R}}, T_1, T_2, \xi_{R}, \xi_{R} \right), \quad \xi_2 \geq \xi_1 \geq \xi_{R} \text{ and } \frac{\xi_1 + \xi_2}{\xi_{R}} \leq \frac{T_2}{T_1}, \\
\Xi_2 \left( \frac{T_1}{\xi_{R} T_1 \Omega_{1,t}}, \frac{T_2}{\xi_{R} T_2 \Omega_{2,t}} \right), \quad \xi_2 \geq \xi_1 \geq \xi_{R} \text{ and } \frac{\xi_1 + \xi_2}{\xi_{R}} > \frac{T_2}{T_1}.
\end{array} \right.
$$

(3-B.16)

Taking integration, one can obtain the closed-form expression of $F_{W_l}(2R)$ in (3-B.15) as follows:

$$
F_{W_l}(2R) = \left\{ \begin{array}{l}
\Xi_1 \left( \frac{\xi_1 + \xi_2 - 2\xi_{R}}{\xi_{R}}, T_1, T_1, \xi_{R}, \xi_{R} \right), \quad \xi_2 \geq \xi_1 \geq \xi_{R} \text{ and } \frac{\xi_1 + \xi_2}{\xi_{R}} \leq \frac{T_2}{T_1}, \\
\Xi_2 \left( \frac{T_1}{\xi_{R} T_1 \Omega_{1,t}}, \frac{T_2}{\xi_{R} T_2 \Omega_{2,t}} \right), \quad \xi_2 \geq \xi_1 \geq \xi_{R} \text{ and } \frac{\xi_1 + \xi_2}{\xi_{R}} > \frac{T_2}{T_1}.
\end{array} \right.
$$

(3-B.16)

For $\xi_1 \geq \xi_2 \geq \xi_{R}$, taking steps similar to those used from (3-B.12) to (3-B.16), one can obtain $F_{W_l}(2R) = \Xi_1(\xi_2 T_1/\xi_{R}, \xi_1 T_1/\xi_{R}, \xi_1/\xi_{R}, \xi_{R}, \xi_{R})$ for $\xi_1 \geq \xi_2 \geq \xi_{R}$ and $\xi_1 + \xi_2 \leq \xi_{R} T_2/T_1$; $F_{W_l}(2R) = \Xi_2(T_1/(\xi_{R} T_1 \Omega_{1,t}) + T_1/(\xi_{R} T_2 \Omega_{2,t}))$ for $\xi_1 \geq \xi_2 \geq \xi_{R}$ and $\xi_1 + \xi_2 > \xi_{R} T_2/T_1$. For Case 6 and Case 7, therefore, one knows that $F_{W_l}(2R)$ is identical to $\Phi_l(R)$ of (3.16).

Finally, since $F_{W_l}(2R)$ is identical to $\Phi_l(R)$ of (3.16) for Case $i$ with $i = 1, \cdots, 7$, it is obvious that $P_{\text{out-PNC}}^{\text{OR-PNC}}(R) = P_{\text{out-PNC}}^{\text{SC-PNC}}(R)$. 

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Appendix 4-A
Proof of Theorem 4.1

From (4.7) and (4.8), the difference $\gamma_{1,l}^{\text{ANC}} - \gamma_{2,l}^{\text{ANC}}$ is given by

$$\gamma_{1,l}^{\text{ANC}} - \gamma_{2,l}^{\text{ANC}} = \frac{E_s E_r X_{1,l} X_{2,l} (X_{1,l} - X_{2,l})}{(E_s X_{1,l} + (E_s + E_r) X_{2,l})((E_s + E_r) X_{1,l} + E_s X_{2,l})}. \quad (4-A.1)$$

Depending on the values $(X_{1,l}, X_{2,l})$, $W_l$ is given by

$$W_l = \begin{cases} \gamma_{1,l}^{\text{ANC}}, & X_{1,l} < X_{2,l}, \\ \gamma_{2,l}^{\text{ANC}}, & X_{1,l} \geq X_{2,l}. \end{cases} \quad (4-A.2)$$

Using (4-A.1), the CDF $F_{W_l}(w)$ of $W_l$ is given by

$$F_{W_l}(w) = \Pr[\gamma_{1,l}^{\text{ANC}} < w, X_{1,l} < X_{2,l}] + \Pr[\gamma_{2,l}^{\text{ANC}} < w, X_{1,l} \geq X_{2,l}]$$

$$= \Pr\left[\frac{E_s E_r X_{1,l} X_{2,l}}{E_s X_{1,l} + (E_s + E_r) X_{2,l}} < w, X_{1,l} < X_{2,l}\right]$$
$$+ \Pr\left[\frac{E_s E_r X_{1,l} X_{2,l}}{(E_s + E_r) X_{1,l} + E_s X_{2,l}} < w, X_{1,l} \geq X_{2,l}\right]$$
$$=: P_1 + P_2. \quad (4-A.3)$$

We first solve $P_1$ as follows:

$$P_1 = \Pr[(E_s E_r X_{2,l} - E_s w) X_{1,l} < (E_s + E_r) X_{2,l} w, X_{1,l} < X_{2,l}]. \quad (4-A.4)$$

Since the first condition, $(E_s E_r X_{2,l} - E_s w) X_{1,l} < (E_s + E_r) X_{2,l} w$, of the probability in the right-hand side of (4-A.4) is always satisfied for $X_{2,l} \leq w/E_r$, the probability $P_1$ in (4-A.4) can be divided into two parts:

$$P_1 = \Pr[X_{1,l} < X_{2,l}, X_{2,l} \leq \frac{w}{E_r}] + \Pr[X_{1,l} < \min\left\{\frac{(E_s + E_r) X_{2,l} w}{E_s E_r X_{2,l} - E_s w}, X_{2,l} > \frac{w}{E_r}\right\}, X_{2,l} > \frac{w}{E_r}]. \quad (4-A.5)$$

It can be shown that

$$\min\left\{\frac{(E_s + E_r) X_{2,l} w}{E_s E_r X_{2,l} - E_s w}, X_{2,l}\right\} = \begin{cases} X_{2,l}, & \frac{w}{E_r} < X_{2,l} \leq \frac{w}{E_r} \left(1 + \frac{E_s + E_r}{E_s}\right), \\ \frac{(E_s + E_r) X_{2,l} w}{E_s E_r X_{2,l} - E_s w}, & X_{2,l} > \frac{w}{E_r} \left(1 + \frac{E_s + E_r}{E_s}\right). \end{cases} \quad (4-A.6)$$
Substituting (4-A.6) into (4-A.5) yield

\[
P_1 = \Pr \left[ X_{1,t} < X_{2,t}, X_{2,t} \leq \frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right) \right] \\
+ \Pr \left[ X_{1,t} < \frac{(E_s + E_r)X_{2,t}w}{E_sE_rX_{2,t} - E_sw}, X_{2,t} > \frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right) \right].
\]

(4-A.7)

Then \( P_1 \) in (4-A.7) can be calculated by

\[
P_1 = \int_{t=0}^{\infty} \int_{q=0}^{\infty} f_{X_{1,t}}(q) f_{X_{2,t}}(t) dq dt \\
+ \int_{t=\frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right)}^{\infty} \int_{q=0}^{\infty} f_{X_{1,t}}(q) f_{X_{2,t}}(t) dq dt \\
= \int_{0}^{\infty} f_{X_{2,t}}(t) dt - \int_{0}^{\infty} \frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right) F_{X_{1,t}}(t) f_{X_{2,t}}(t) dt \\
- \int_{0}^{\infty} \frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right) \bar{F}_{X_{1,t}} \left( \frac{(E_s + E_r)tw}{E_sE_r t - E_s w} \right) f_{X_{2,t}}(t) dt \\
= : P_{11} - P_{12} - P_{13},
\]

(4-A.8)

where \( F_{X_{1,t}}(t) = 1 - F_{X_{1,t}}(t) \). Since \( X_{1,t} \) and \( X_{2,t} \) are exponentially distributed random variables with mean \( \Omega_{1,t} \) and \( \Omega_{2,t} \), respectively, the difference \( P_{11} - P_{12} \) in (4-A.8) can be solved into

\[
P_{11} - P_{12} = \frac{\Omega_{2,t}}{\Omega_{1,t} + \Omega_{2,t}} + \frac{\Omega_{1,t}}{\Omega_{1,t} + \Omega_{2,t}} \exp \left( -\frac{w(\Omega_{1,t} + \Omega_{2,t})(2E_s + E_r)}{E_sE_r \Omega_{1,t} \Omega_{2,t}} \right).
\]

(4-A.9)

Also, \( P_{13} \) in (4-A.8) can be rewritten as

\[
P_{13} = \int_{\frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right)}^{\infty} \exp \left( -\frac{(E_s + E_r)tw}{(E_sE_r t - E_s w) \Omega_{1,t}} \right) \exp \left( -\frac{t}{\Omega_{2,t}} \right) dt \\
= \exp \left( -w(E_s + E_r)/(E_sE_r \Omega_{1,t}) \right) \int_{\frac{w}{E_r} \left( 1 + \frac{(E_s + E_r)}{E_s} \right)}^{\infty} \exp \left( -\frac{(E_s + E_r)tw^2}{E_sE_r^2 \Omega_{1,t} \Omega_{2,t} z} - \frac{z}{\Omega_{2,t}} \right) dz,
\]

(4-A.10)

where \( A = \exp \left( -w((E_s + E_r)/(E_sE_r \Omega_{1,t} + 1/(E_r \Omega_{2,t}))) \right) \). Since we have no closed-form solution to (4-A.10), we rewrite (4-A.10) as follows:

\[
P_{13} = \frac{A}{\Omega_{2,t}} \int_{0}^{\infty} \exp \left( -\frac{(E_s + E_r)w^2}{E_sE_r^2 \Omega_{1,t} z} - \frac{z}{\Omega_{2,t}} \right) dz \\
- \frac{A}{\Omega_{2,t}} \int_{0}^{\infty} \frac{w(E_s + E_r)}{E_sE_r} \exp \left( -\frac{(E_s + E_r)w^2}{E_sE_r^2 \Omega_{1,t} z} - \frac{z}{\Omega_{2,t}} \right) dz.
\]

(4-A.11)
Using [78, eq. (3.324.1)], the first part of the right-hand side of (4-A.11) can be solved into $2wA\sqrt{\frac{\xi_s+\xi_r}{\xi_s^2\Omega_{1,1}^2\Omega_{2,l}}} \times K_1\left(2w\sqrt{\frac{\xi_s+\xi_r}{\xi_s^2\Omega_{1,1}^2\Omega_{2,l}}}\right)$. For the second part of the right-hand side of (4-A.11), since we have no closed-form solution, we tackle this problem by adopting a series expansion of exponential function, $\exp(x) = \sum_{i=0}^{\infty} \frac{x^i}{i!}$. Substituting the series expansion of $\exp(-z/\Omega_{2,l})$ into the second part of the right-hand side of (4-A.11) yields

$$\frac{A}{\Omega_{2,l}} \int_{0}^{w(\xi_s+\xi_r)} \exp\left(-\frac{w^2}{\xi_s^2\Omega_{1,1}^2\Omega_{2,l}} - \frac{z}{\Omega_{2,l}}\right)dz = A \int_{0}^{1} \sum_{i=0}^{\infty} \frac{(-1)^i a_1^{i+1} z^i}{i!} \exp\left(-\frac{a_2}{z}\right)dz$$

(4-A.12)

$$= A \sum_{i=0}^{\infty} \frac{(-1)^i a_1^{i+1}}{i!} \int_{0}^{1} z^i \exp\left(-\frac{a_2}{z}\right)dz$$

(4-A.13)

$$= A \sum_{i=0}^{\infty} \frac{(-1)^i a_1^{i+1}}{i!} \int_{1}^{\infty} \frac{1}{z^{i+2}} \exp(-a_2\bar{z})d\bar{z}$$

$$= A \sum_{i=0}^{\infty} \frac{(-1)^i a_1^{i+1}}{i!} E_{i+2}(a_2),$$

(4-A.14)

where $a_1 = w(\xi_s+\xi_r)/(\xi_s\xi_r\Omega_{2,l})$ and $a_2 = w/(\xi_s\Omega_{1,1})$. From (4-A.12) to (4-A.13), we switch the order of integral and infinite series, which is not generally possible. In order to show the validness of the step from (4-A.12) to (4-A.13), we will show that the infinite series of the integrand of the right-hand side of (4-A.12) is uniformly convergent in the following.\footnote{A sequence $\{g_n(x)\}$ of functions converges uniformly to a limiting function $g(x)$ if the speed of convergence of $g_n(x)$ to $g(x)$ does not depend on $x$. Also, if $g_n$ is a uniformly convergent sequence on $[a, b]$ with limit $g$, then Riemann integrability of all $g_n$ implies Riemann integrability of $g$. That is, $\int_{a}^{b} g(x)dx = \int_{a}^{b} \lim_{n \to \infty} g_n(x)dx = \lim_{n \to \infty} \int_{a}^{b} g_n(x)dx$.}

Let $b_i = \frac{a_1^{i+1}}{i!} \exp(-a_2/\bar{z})$. Then the infinite series of the integrand of the right-hand side of (4-A.12) is given by $\sum_{i=0}^{\infty} (-1)^i b_i$. For $0 < \bar{z} \leq 1$, we can choose an upper bound $M_i$ to satisfy

$$b_i \leq \frac{a_1^{i+1}}{i!} \exp(-a_2/\bar{z}) \leq \frac{a_1^{i+1}}{i!} \exp(-a_2) =: M_i.$$  

(4-A.15)

Note that $M_i$ is independent of $\bar{z}$. Then it can be shown that

$$\lim_{i \to \infty} \frac{M_{i+1}}{M_i} = \lim_{i \to \infty} \frac{a_1}{i + 1} = 0.$$  

(4-A.16)
By the ratio test \[80\] it is evident that the series \(\sum M_i\) is convergent. Also, by the Weierstrass M-test \[81\] it can be shown that \(\sum_{i=0}^{\infty}(-1)^i b_i\) is uniformly convergent. Therefore, the step from (4-A.12) to (4-A.13) can be justified. Finally, substituting (4-A.14) into (4-A.11) yields \(P_{13}\), which in turn is substituted into (4-A.8) with \(P_{11} - P_{12}\) of (4-A.9). Taking steps similar to those used from (4-A.4) to (4-A.14), one can obtain \(P_2\). Then, using (4-A.3) with \(P_1\) and \(P_2\), one can obtain the final expression of the CDF \(F_{W_1}(w)\), which is given in (4.32).

**Appendix 4-B**

**Proof of Lemma 4.1**

We show that the series expansion in (4.33) is convergent. Let \(\bar{M}_{1,i} = \exp(-wc_1)\frac{(wc_2)^{i+1}}{i!} \times E_{i+2}(wc_3)\), where \(c_1 = (\xi_2 + \xi_3)/(\xi_1 \xi_3 \xi_5) + 1/(\xi_3 \xi_5), c_2 = \epsilon(\xi_2 + \xi_3)/(\xi_1 \xi_3 \xi_5),\) and \(c_3 = 1/(\epsilon \xi_3 \xi_4)\). Then the series expansion in (4.33) is given by \(\sum_{i=0}^{\infty}(-1)^i \bar{M}_{1,i}\). Since exponential integral function \(E_i(\cdot)\) is monotonically decreasing in \(i\), taking a step similar to (4-A.16), it can be shown that

\[
\lim_{i \to \infty} \frac{\bar{M}_{1,i+1}}{\bar{M}_{1,i}} = \lim_{i \to \infty} \frac{wc_2}{i+1} \frac{E_{i+3}(wc_3)}{E_{i+2}(wc_3)} < \lim_{i \to \infty} \frac{wc_2}{i+1} = 0. \tag{4-B.1}
\]

By the ratio test \[80\], therefore, it can be shown that the series expansion in (4.33) is convergent.

\[^3\text{For a series } \sum_{i=0}^{\infty} g_i, \text{ we define } \mathcal{L} := \lim_{i \to \infty} |g_{i+1}/g_i|. \text{ If } \mathcal{L} < 1, \text{ then the series converges absolutely; if } \mathcal{L} > 1, \text{ then the series diverges; and if } \mathcal{L} = 1, \text{ then the series is inclusive } 80.\]

\[^4\text{If } \sum_{i=0}^{\infty} M_i \text{ is convergent, then the original series is uniformly convergent } 81.\]
Appendix 4-C
Proof of Theorem 4.3

From (4.17), the difference $\gamma_{1,l}^{\text{TDBC}} - \gamma_{2,l}^{\text{TDBC}}$ is given by

$$\gamma_{1,l}^{\text{TDBC}} - \gamma_{2,l}^{\text{TDBC}} = \mathcal{E}_s \mathcal{E}_r X_{1,l} X_{2,l} \left( \alpha_{1,l}(\mathcal{E}_s + (2\alpha_{1,l} - 1)\mathcal{E}_r)X_{2,l} - (1 - \alpha_{1,l})(\mathcal{E}_s - (2\alpha_{1,l} - 1)\mathcal{E}_r)X_{1,l} \right) / \left( (\mathcal{E}_s + (1 - \alpha_{1,l})\mathcal{E}_r)X_{1,l} + \alpha_{1,l}\mathcal{E}_r X_{2,l} \right) \left( (1 - \alpha_{1,l})\mathcal{E}_r X_{1,l} + (\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r)X_{2,l} \right).$$

(4-C.1)

Depending on the values $(\mathcal{E}_s, \mathcal{E}_r, \alpha_{1,l}, X_{1,l}, X_{2,l})$, $\mathcal{V}_l$ in (4.36) is given by

$$\mathcal{V}_l = \begin{cases} \min[\gamma_{1,l}^{\text{TDBC}}, \gamma_{2,l}^{\text{TDBC}}], & \text{Case } 4.1, \\ \gamma_{1,l}^{\text{TDBC}}, & \text{Case } 4.2, \\ \gamma_{2,l}^{\text{TDBC}}, & \text{Case } 4.3. \end{cases}$$

(4-C.2)

For Case 4.1 of (4-C.2), the CDF $F_{\mathcal{V}_l}(w)$ of $\mathcal{V}_l$ is given by

$$F_{\mathcal{V}_l}(w) = \Pr[\gamma_{2,l}^{\text{TDBC}} < w, \mathcal{E}_{eq} X_{1,l} < X_{2,l}] + \Pr[\gamma_{1,l}^{\text{TDBC}} < w, \mathcal{E}_{eq} X_{1,l} \geq X_{2,l}],$$

(4-C.3)

where $\mathcal{E}_{eq} = (1 - \alpha_{1,l})(\mathcal{E}_s - (2\alpha_{1,l} - 1)\mathcal{E}_r) / (\alpha_{1,l}(\mathcal{E}_s + (2\alpha_{1,l} - 1)\mathcal{E}_r))$. Then, taking steps similar to those used from (4-A.3) to (4-A.16), one can obtain the first case of (4.37).

For Case 4.2 of (4-C.2), taking steps similar to the proof of [23, Lemma 1], one can obtain the CDF $F_{\mathcal{V}_l}(w)$ of $\mathcal{V}_l$ as follows:

$$F_{\mathcal{V}_l}(w) = \Pr \left[ \frac{\alpha_{1,l}\mathcal{E}_r X_{1,l} X_{2,l}}{(\mathcal{E}_s + \alpha_{2,l}\mathcal{E}_r)X_{1,l} + \alpha_{1,l}\mathcal{E}_r X_{2,l}} < w \right] = \Psi_2 \left( w; (\mathcal{E}_s + \alpha_{2,l}\mathcal{E}_r)\Omega_{1,l}, \alpha_{1,l}\mathcal{E}_r\Omega_{2,l}, 1 + \alpha_{2,l}\mathcal{E}_r / \mathcal{E}_s \right),$$

(4-C.4)

where $\Psi_2(w; \cdot, \cdot, \cdot, \cdot, \cdot)$ is given in (4.38). Finally, for Case 4.3 of (4-C.2), taking a step similar to (4-C.4), one can solve the CDF $F_{\mathcal{V}_l}(w)$ of $\mathcal{V}_l$, which is given by $F_{\mathcal{V}_l}(w) = \Psi_2 \left( w; \alpha_{2,l}\mathcal{E}_r, \Omega_{1,l}, (\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r)\Omega_{2,l}, 1 + \alpha_{1,l}\mathcal{E}_r / \mathcal{E}_s \right)$.  

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Appendix 4-D
Proof of Lemma 4.3

Depending on the values \((\mathcal{E}_s, \mathcal{E}_r, \alpha_{1,l})\), \(V^U_l\) in (4.41) is given by

\[
V^U_l = \begin{cases}
\min \left[ \frac{(1-\alpha_{1,l})\mathcal{E}_s\mathcal{E}_r}{\mathcal{E}_s+\alpha_{1,l}\mathcal{E}_r} X_{1,l}, \frac{\alpha_{1,l}\mathcal{E}_s\mathcal{E}_r}{\mathcal{E}_s+(1-\alpha_{1,l})\mathcal{E}_r} X_{2,l} \right], & \text{Case 4.1,} \\
\min \left[ \mathcal{E}_s X_{1,l}, \frac{\alpha_{1,l}\mathcal{E}_s\mathcal{E}_r}{\mathcal{E}_s+(1-\alpha_{1,l})\mathcal{E}_r} X_{2,l} \right], & \text{Case 4.2,} \\
\min \left[ \frac{(1-\alpha_{1,l})\mathcal{E}_s\mathcal{E}_r}{\mathcal{E}_s+\alpha_{1,l}\mathcal{E}_r} X_{1,l}, \mathcal{E}_s X_{2,l} \right], & \text{Case 4.3.}
\end{cases}
\]  

(4-D.1)

For Case 4.1 of (4-D.1), the CDF \(F_{V^U_l}(w)\) of \(V^U_l\) is given by

\[
F_{V^U_l}(w) = \Pr \left[ \frac{(1-\alpha_{1,l})\mathcal{E}_s\mathcal{E}_r}{\mathcal{E}_s+\alpha_{1,l}\mathcal{E}_r} X_{1,l} < w, X_{1,l} < \frac{\alpha_{1,l}(\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r)}{(1-\alpha_{1,l})(\mathcal{E}_s + (1-\alpha_{1,l})\mathcal{E}_r)} X_{2,l} \right] \\
+ \Pr \left[ \frac{\alpha_{1,l}\mathcal{E}_s\mathcal{E}_r}{\mathcal{E}_s+(1-\alpha_{1,l})\mathcal{E}_r} X_{2,l} < w, X_{1,l} \geq \frac{\alpha_{1,l}(\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r)}{(1-\alpha_{1,l})(\mathcal{E}_s + (1-\alpha_{1,l})\mathcal{E}_r)} X_{2,l} \right] \\
= \int_{q=0}^{\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r} \int_{t=\frac{(1-\alpha_{1,l})(\mathcal{E}_s + (1-\alpha_{1,l})\mathcal{E}_r)}{\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r}}^{\infty} f_{X_{1,l}}(q) f_{X_{2,l}}(t)dt dq \\
+ \int_{t=0}^{\infty} \int_{q=\frac{(1-\alpha_{1,l})(\mathcal{E}_s + (1-\alpha_{1,l})\mathcal{E}_r)}{\mathcal{E}_s + \alpha_{1,l}\mathcal{E}_r}}^{\infty} f_{X_{1,l}}(q) f_{X_{2,l}}(t) dq dt. 
\]  

(4-D.2)

(4-D.3)

Taking integration, one can obtain (4.42) with the first case of (4.43). Taking steps similar to those used from (4-D.2) to (4-D.3), one can obtain (4.42) with the second and third cases of (4.43).

Appendix 4-E
Proof of Theorem 4.5

Taking a step similar to (4.39), one can obtain the lower bound \(P_{out, \text{LB}}^{TD/BC}(R)\) as follows:

\[
P_{out, \text{LB}}^{TD/BC}(R) = \int_0^R \left( \prod_{l=1}^L F_{V^U_l}(\tilde{R} - \eta) \right) \frac{\exp \left(-\eta/(\mathcal{E}_s\Omega_0)\right)}{\mathcal{E}_s\Omega_0} d\eta. 
\]  

(4-E.1)

Using the product identity \(\prod_{l=1}^L (1-d(l)) = 1 + \sum_{l=1}^L (-1)^l \sum_{m_1=1, \ldots, m_l=1}^{L} \prod_{i=1}^l d(m_i)\), one can rewrite the expression \(\prod_{l=1}^L F_{V^U_l}(w)\) in (4-E.1) as follows:

\[
\prod_{l=1}^L F_{V^U_l}(w) = 1 + \sum_{l=1}^L (-1)^l \sum_{m_1=1}^{\sum_{l=1}^L m_i} \cdots \sum_{m_l=1}^{\sum_{l=1}^L m_i} \exp \left(-w \sum_{i=1}^l \beta(m_i)\right). 
\]  

(4-E.2)
Then, substituting \((4-E.2)\) into \((4-E.1)\) and taking integration, one can obtain the final closed-form result in \((4.44)\).
Appendix 5-A

Proof of Theorem 5.1

We first derive the CDF $F_{\mathcal{W}_i}(w)$ of $\mathcal{W}_i$. From (5.2) and (5.4), the difference $\gamma_{1,t} - \gamma_{2,t}$ is given by

$$\gamma_{1,t} - \gamma_{2,t} = \frac{\mathcal{E}_s \mathcal{E}_r X_i Y_i}{(\mathcal{E}_s X_i + \mathcal{E}_r Y_i)(\mathcal{E}_s Y_i + \mathcal{E}_r X_i)}((\mathcal{E}_r - \mathcal{E}_s)(X_i - Y_i)). \quad (5-A.1)$$

Depending on the values $(\mathcal{E}_s, \mathcal{E}_r, X_i, Y_i)$, therefore, $\mathcal{W}_i = \gamma_{1,t}$ for $(\mathcal{E}_r > \mathcal{E}_s$ and $X_i > Y_i)$ or $(\mathcal{E}_r < \mathcal{E}_s$ and $X_i < Y_i)$; $\mathcal{W}_i = \gamma_{2,t}$, otherwise. Then, using (5.2) and (5.4), $\mathcal{W}_i$ can be rewritten as

$$\mathcal{W}_i = \frac{\mathcal{E}_s \mathcal{E}_r X_i Y_i}{\mathcal{E}_{\max} \min[X_i, Y_i] + \mathcal{E}_{\min} \max[X_i, Y_i]]. \quad (5-A.2)$$

From (5-A.2), the CDF $F_{\mathcal{W}_i}(w)$ of $\mathcal{W}_i$ is given by

$$F_{\mathcal{W}_i}(w) = \Pr \left[ \frac{\mathcal{E}_s \mathcal{E}_r X_i Y_i}{\mathcal{E}_{\max} X_i + \mathcal{E}_{\min} Y_i} < w, X_i < Y_i \right] + \Pr \left[ \frac{\mathcal{E}_s \mathcal{E}_r X_i Y_i}{\mathcal{E}_{\max} Y_i + \mathcal{E}_{\min} X_i} < w, X_i \geq Y_i \right]$$

$$=: P_1 + P_2. \quad (5-A.3)$$

We first solve $P_1$ as follows:

$$P_1 = \Pr \left[ (\mathcal{E}_s \mathcal{E}_r Y_i - \mathcal{E}_{\max} w) X_i < \mathcal{E}_{\min} Y_i w, X_i < Y_i \right]. \quad (5-A.4)$$

Since the first condition, $(\mathcal{E}_s \mathcal{E}_r Y_i - \mathcal{E}_{\max} w) X_i < \mathcal{E}_{\min} Y_i w$, of the probability in the right-hand side of (5-A.4) is always satisfied for $Y_i \leq \mathcal{E}_{\max} w/(\mathcal{E}_s \mathcal{E}_r) = w/\mathcal{E}_{\min}$, $P_1$ in (5-A.4) can be divided into two parts:

$$P_1 = \Pr \left[ X_i < Y_i, Y_i \leq \frac{w}{\mathcal{E}_{\min}} \right] + \Pr \left[ X_i < \min \left[ \frac{\mathcal{E}_{\min} Y_i w}{\mathcal{E}_s \mathcal{E}_r Y_i - \mathcal{E}_{\max} w}, Y_i \right], Y_i \geq \frac{w}{\mathcal{E}_{\min}} \right]. \quad (5-A.5)$$

It can be shown that

$$\min \left[ \frac{\mathcal{E}_{\min} Y_i w}{\mathcal{E}_s \mathcal{E}_r Y_i - \mathcal{E}_{\max} w}, Y_i \right] = \begin{cases} Y_i, & \frac{w}{\mathcal{E}_{\min}} < Y_i \leq w \left( \frac{1}{\mathcal{E}_s} + \frac{1}{\mathcal{E}_r} \right), \\ \frac{\mathcal{E}_{\min} Y_i w}{\mathcal{E}_s \mathcal{E}_r Y_i - \mathcal{E}_{\max} w}, & Y_i > w \left( \frac{1}{\mathcal{E}_s} + \frac{1}{\mathcal{E}_r} \right). \end{cases} \quad (5-A.6)$$

Substituting (5-A.6) into (5-A.5) yields

$$P_1 = \Pr \left[ X_i < Y_i, Y_i \leq w \left( \frac{1}{\mathcal{E}_s} + \frac{1}{\mathcal{E}_r} \right) \right] + \Pr \left[ X_i < \frac{\mathcal{E}_{\min} Y_i w}{\mathcal{E}_s \mathcal{E}_r Y_i - \mathcal{E}_{\max} w}, Y_i > w \left( \frac{1}{\mathcal{E}_s} + \frac{1}{\mathcal{E}_r} \right) \right]. \quad (5-A.7)$$
Then $P_1$ in (5-A.7) can be calculated by

$$P_1 = \int_{y=0}^{\infty} f_{X_1}(x) f_{Y_1}(y) dy + \int_{y=w\left(\frac{1}{\bar{E}_x} + \frac{1}{\bar{E}_r}\right)}^{\infty} \int_{x=0}^{\infty} f_{X_1}(x) f_{Y_1}(y) dx dy$$

$$= \int_{0}^{\infty} f_{Y_1}(y) dy - \int_{0}^{w\left(\frac{1}{\bar{E}_x} + \frac{1}{\bar{E}_r}\right)} \bar{F}_{X_1}(y) f_{Y_1}(y) dy$$

$$- \int_{w\left(\frac{1}{\bar{E}_x} + \frac{1}{\bar{E}_r}\right)}^{\infty} \bar{F}_{X_1}(y) \left(\frac{\mathcal{E}_{\min} y w}{\mathcal{E}_s \mathcal{E}_r y - \mathcal{E}_{\max} w}\right) f_{Y_1}(y) dy$$

$$=: P_{11} - P_{12} - P_{13}, \quad (5-A.8)$$

where $F_{X_1}(x) = 1 - F_{X_1}(x)$. Since $X_i$ and $Y_i$ are exponentially distributed random variables with mean $\Omega_{1,i}$ and $\Omega_{2,i}$, respectively, the difference $P_{11} - P_{12}$ in (5-A.8) can be solved into

$$P_{11} - P_{12} = \frac{\Omega_{2,i}}{\Omega_{1,i} + \Omega_{2,i}} + \frac{\Omega_{1,i}}{\Omega_{1,i} + \Omega_{2,i}} \exp\left(-w\left(\frac{1}{\Omega_{1,i}} + \frac{1}{\Omega_{2,i}}\right)\left(\frac{1}{\bar{E}_s} + \frac{1}{\bar{E}_r}\right)\right). \quad (5-A.9)$$

Also, $P_{13}$ in (5-A.8) can be rewritten as

$$P_{13} = \int_{0}^{\infty} \exp\left(-\frac{\mathcal{E}_{\min} y w}{\mathcal{E}_{1,k} (\mathcal{E}_s \mathcal{E}_r y - \mathcal{E}_{\max} w)}\right) \frac{\exp(-y/\Omega_{2,i})}{\Omega_{2,i}} dy$$

$$= A \int_{0}^{\infty} \exp\left(-\frac{w^2}{\mathcal{E}_s \mathcal{E}_r \Omega_{1,i} \tilde{z}} - \frac{z}{\Omega_{2,i}}\right) dz, \quad (5-A.10)$$

where $A = \exp\left(-w(1/(\mathcal{E}_{\max} \Omega_{1,i}) + 1/(\mathcal{E}_{\min} \Omega_{2,i}))\right)$. Since we have no closed-form solution to (5-A.10), we rewrite (5-A.10) as follows:

$$P_{13} = \frac{A}{\Omega_{2,i}} \int_{0}^{\infty} \exp\left(-\frac{w^2}{\mathcal{E}_s \mathcal{E}_r \Omega_{1,i} \tilde{z}} - \frac{z}{\Omega_{2,i}}\right) dz - \frac{A}{\Omega_{2,i}} \int_{0}^{\infty} \exp\left(-\frac{w^2}{\mathcal{E}_s \mathcal{E}_r \Omega_{1,i} \tilde{z}} - \frac{z}{\Omega_{2,i}}\right) dz. \quad (5-A.11)$$

Using [78] eq. (3.324.1), the first part of the right-hand side of (5-A.11) can be solved into $\sqrt{\mathcal{E}_s \mathcal{E}_r \Omega_{1,i} \Omega_{2,i}} K_1(\sqrt{\frac{2w}{\mathcal{E}_s \mathcal{E}_r \Omega_{1,i} \Omega_{2,i}}})$. For the second part of the right-hand side of (5-A.11), since we have no closed-form solution, we tackle this problem by adopting a series expansion of exponential function, $\exp(x) = \sum_{i=0}^{\infty} \frac{x^i}{i!}$. Substituting the series expansion of $\exp(-z/\Omega_{2,i})$ into the second part of the right-hand side of (5-A.11)
yields
\[
\frac{A}{\Omega_{2,l}} \int_0^{\nu_{\text{max}}} \exp \left( - \frac{w^2}{E_s E_r \Omega_{1,l} z} - \frac{z}{\Omega_{2,l}} \right) dz = A \int_0^1 \sum_{i=0}^{\infty} (-1)^i a_1^{i+1} \frac{z^i}{i!} \exp \left( - \frac{a_2}{z} \right) d\bar{z}
\]
\[
= A \sum_{i=0}^{\infty} (-1)^i a_1^{i+1} \frac{1}{i!} E_{i+2}(a_2),
\]
(5-A.12)

where \(a_1 = w/(E_{\text{max}} \Omega_{2,l})\) and \(a_2 = w/(E_{\text{min}} \Omega_{1,l})\). Finally, substituting (5-A.12) into (5-A.11) yields \(P_{13}\), which in turn is substituted into \(P_1\) of (5-A.8) with \(P_{11} - P_{12}\) of (5-A.9). Taking steps similar to those used from (5-A.4) to (5-A.12), one can obtain \(P_2\). Then, using (5-A.3), one can obtain the final CDF \(F_{W_k}(w)\) in (5.9).

Secondly, by taking a derivative with respect to \(w\), one can obtain the PDF \(f_{W_k}(w)\) of \(W_k\) in (5.10).