ARTICULAR ASPHERICITY OF THE ARTHRITIC HIP

by

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Abstract

The predominant model of the human hip is a mechanical ball-and-socket joint. This description has two key implications: that the motion of the hip is purely rotational, and that the rigid articulating geometry of the hip is a sphere-on-sphere contact. Since the widespread adoption of this model, in the late 1960s, there has however been a persistent thread of literature suggesting that the articulating geometry of the hip is aspherical. The recent widespread availability of three-dimensional medical imaging now makes it possible to empirically assess the applicability of the predominant model.

For this research dissertation, two arthritic groups were examined: patients either had primary early-life osteoarthritis of the hip, or hip dysplasia with secondary osteoarthritis. Computed tomography scans, taken as part of routine preoperative preparation, served as the source data for this work. The scans were manually segmented to produce 3D models of the bones of the hip, which were further refined to isolate the bony articular surfaces. These surfaces were fit to general ellipsoids and to spheres, the latter being the ball-and-socket model.

The arthritic hips examined had comparable fitting accuracy for both ellipsoids and spheres; however, sixteen of nineteen hips formed geometrically incompatible ball-and-socket joints. The dysplastic hips examined had a notable difference in fitting accuracy, with ellipsoids being a statistically significantly better fit to the hip
geometry. The ellipsoid shapes in all cases were aspherical, and in each population formed a statistically significantly aspherical group. There were no trends relating the ellipsoid shapes of bones of an individual joint, nor were there practical differences between the ellipsoid shapes between the two populations.

Despite patient groups not being controlled for age, sex, or race, and accounting for typical manual segmentation errors, these results suggest that the hip is aspherically shaped. Thus, the geometric foundation of the ball-and-socket motion may be unsupported, and the conventional kinematic description of the hip may be called into question.
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Chapter 1

Introduction

The hip is a key anatomical structure that is distinctive of our species – it provides our bipedal gait, the foundation of human motion [1]. Examined mechanically, the hip has two key features: it is a major load-bearing joint during both activities of daily living and sporting activities; and, it is highly mobile, allowing for bipedal gait as well as sporting or gymnastic movements with a large range of motion. The mechanical structure and function of the joint thus have competing sets of requirements, because the hip must transmit significant forces through the joint surfaces while maintaining a large-exursion motion. The durability and wear characteristics of the joint are an important mechanical aspect given the allowed motion and substantial force transmission. Arthritic wearing of the hip can affect mobility during activities of daily living, such as walking, exiting a car, sitting and standing.

Currently, the hip is predominantly described and modeled as a mechanical ball-and-socket joint across a wide range of academic fields, including biomechanics [2], kinesiology [3, 4], gait [5], and anatomy [6, 7]. The mechanical ball-and-socket joint has two important implications: it implies a rotational motion about any axis, with no translatory component; and it implies a sphere-on-sphere articulating joint geometry.
between the acetabulum (the hip “socket” in the pelvis) and the femoral head (the hip “ball”, a feature of the femur or thigh bone). Geometrically, a sphere has rotational symmetry about any axis; as such, a joint with an articulation formed by two rigid, conforming spheres would allow relative rotation without any translation \(^1\).

Although the exact origin of this biomechanical description of the hip is unclear, modern work heavily cites Hammond and Charnley \(^2\) in the late 60s [8], whose model superseded previous aspherical descriptions of the hip. Hammond and Charnley meticulously examined ten cadaver specimens using a variety of methods including direct measurement, radiographic measurement, examination of sectioned joints, measurement of contact between cartilaginous joint surfaces, and measurement of translation during motion. They concluded the shapes of the interacting joint surfaces were spherical, and that consequently the motion of the hip was that of pure rotation about a single point.

Hammond and Charnley’s conclusion identified a key link between the geometry of a joint and the resulting kinematics. The description of the hip as a rigid mechanical ball-and-socket joint intrinsically implied both the spherical morphology of the articulating surfaces and the resulting, purely rotational, motion.

The success of the ball-and-socket model since its introduction by Hammond and Charnley should not be unexpected. Nominally, the geometry of the bony hip joint is the contact of one sphere on another, a description that is especially useful when describing the hip in contrast with other articulating joints in the human skeleton.

\(^{1}\) A “ball-and-socket” type joint with an aspherical shape with rotational symmetry would lead to a uniaxial relative rotation about the axis of rotational symmetry. For example, a rotational ellipsoidal joint is kinematically equivalent to a hinge. Congruent “ball-and-socket” type joints with no rotational symmetry of the interacting shapes lead to an immobile joint.

\(^{2}\) Sir John Charnley, a prominent orthopaedic surgeon, went on to introduce the total hip replacement, which provides a mechanical ball-and-socket in place of the natural hip anatomy; he was knighted for his contributions to orthopaedics.
The bony anatomy of the joint does indeed incorporate a rounded protrusion that articulates in a deep depression to produce a relative rotation about any axis. When observing passive and active movements of the hip, no translation is obvious to a casual observer. This is also true when manipulating intact, partially dissected, or disarticulated cadaver joints. The ball-and-socket model is further supported by the success of the total hip prosthetics and the hip resurfacing prosthetics, both of which replace the articulating hip anatomy with a manufactured mechanical ball-and-socket joint. The mechanical ball-and-socket model is also advantageous because of the simplicity of the mechanical model – and its manufacture – deriving from symmetry of the spherical shape.

However, the anatomical foundation of the ball-and-socket biomechanical model is not entirely solid. Anatomical descriptions of isolated joint surfaces notably use less definitive terms, such as “approximately hemispherical” and “spheroidal” to describe the acetabulum and femoral head, respectively. Quantitative aspherical descriptions of the hip have been reported sparsely but consistently in the literature ever since the spherical description was first presented. A variety of aspherical joint geometries have been proposed by authors, encompassing a range of specimens and methods. 2D measurements of cadaver hips have used plain radiographs and physical casts of cadavers; 3D methods, including physical measurements of specimens as well as 3D imaging, have led to descriptions of the hip as ellipsoidal, conchoidal, or simply incongruent or irregular with a varying joint space. These previous methods are described in detail below in Section 2.3.

The majority of these proposed aspherical models have spherical shapes as a special case of more complex geometrical models. If the *de facto* spherical geometry of
the hip is truly an incomplete or insufficient description of the true geometry of the hip, it follows that the spherical kinematics of the joint are called into question as a direct result. This is because the interacting bony geometry does not provide enough constraint to exclude a translatory component in the relative motion of the hip.

A change in the geometric and kinematic model of the hip could have implications in each of the range of fields in which it is currently applied. Illustrative examples can be hypothesized in a number of these fields. In biomechanics, the contact geometry of the hip directly affects the calculated stress distributions in the joint. In gait and kinesiology, the ball-and-socket motion of the hip is often used to identify a kinematic femoral-head center to define anatomical coordinate frames, so a change in the model could have kinematic and kinetic consequences in the calculated joint angles and forces transmitted. To an anatomist, the physical and mechanical understanding of the function of the hip would alter; some authors suggest incongruity plays a role in lubrication and nutrition of the joint surfaces, for example. To a physical therapist, athletic therapist, or doctor, it may be that translational motion could be a natural consequence of the joint structure, to be allowed or even encouraged as part of healthy joint function. To doctors and scientists treating and researching osteoarthritis, the understanding of wear mechanisms inside the joint may change if the understanding of the motions causing that wear were to change.

As a more detailed example, total hip replacements have a very high failure rate (73% in patients younger than 45 years of age after 16 years [9]). A Charnley-style total hip replacement has a stemmed femoral component with a metal sphere, articulating in a spherical polyethylene or metal socket that replaces the native acetabulum. Some authors attribute early failures to the material quality of the polyethylene insert
used in the prosthesis [10]. However, if it was found that the hip had some transla-
tory component to its motion that was being over-constrained by the ball-and-socket
prosthesis, it is possible that these prosthetic joints may have been wearing differ-
ently than predicted using the idealized isocentric rotary motion for which they were
designed. This purely hypothetical example is but one illustration of how a distinct
biomechanical model of the hip could have an impact beyond core biomechanics.

It is clear that a changed biomechanical model of the hip involving aspherical
kinematics may ultimately have a significant impact in a number of fields. As Ham-
mond and Charnley identified, the kinematic behavior is closely tied to, and follows
from, the geometrical description of the hip.

Although much of the literature assessing hip geometry used direct measurement
of cadaver joints or 2D radiographs, the more recent development of 3D imaging
modalities such as computed tomography (CT), 3D fluoroscopy, and magnetic res-
onance imaging (MRI) have allowed the 3D geometry of a patient’s anatomy to be
visualized and measured non-invasively. These imaging modalities work by captur-
ing a series of cross-sectional images of the anatomy. As in a digital image, each
cross sectional “slice” of the 3D medical image is divided into pixels. Incorporat-
ing the spacing between slices, this 2D rectangular area can be extended to a 3D
rectangular prism, called a volumetric pixel, or “voxel.” These images can then be
manually masked to define a set of voxels containing some anatomy of interest in a
process called segmentation. The resulting voxel sets can be converted into digital
3D models, which can then be analyzed numerically for shape.

Three-dimensional medical imaging, which has obvious advantages in studies on
living patients, is not without problems when segmentation-based measurements are
made. Of special concern is an arthritic joint, where the joint space may be unclear and often is undetectable, making the definition of the margin between the bones of such a joint difficult and potentially inaccurate. In any imaging study that requires human judgment there may be concerns about intra-observer and inter-observer repeatability, which may affect the final 3D models. These limitations are often accepted because of the medical images may be easier to acquire and process than cadaveric specimens are, and non-invasive clinical measurements also may be more easily extended prospectively to clinical use on patients.

As it happened, observations inspired by 3D medical images were the genesis of this thesis. Orthopaedic surgeons at Kingston General Hospital noted qualitative asphericity of both hip surfaces of their arthritic patients, from CT scans and on direct examination during surgical procedures. These observations, and the connection between shape and joint kinematics, led to qualitative investigation of the shape of the bony articular surfaces of the hip. When the author joined the research, he was posed with the problem of quantifying these qualitative observations and chose to use least-squares fitting of ellipsoids to bony articular surfaces that were derived from CT scans.

To do this, two arthritic patient groups were examined: patients with early life osteoarthritis (specifically, those who underwent hip resurfacing) and patients with hip dysplasia with secondary osteoarthritis (specifically, those who underwent a periacetabular osteotomy). These patient groups were significantly younger than the broad population of patients who underwent hip-replacement surgery; further description of the medical conditions and surgical procedures are given below in Section 2.1.1.
The segmentations of the preoperative scans of the subjects were further processed to isolate the bony articular surfaces of the hip joint: the femoral head and the acetabulum. These bony articular surfaces were then fit to general ellipsoids using an adjusted least-squares method. These ellipsoids were assessed to determine their degree of asphericity. A general ellipsoid can be thought of as a sphere scaled independently in three orthogonal directions, with the relative scaling in these three orthogonal directions leading to a measure of asphericity of the general ellipsoid shape. These scalings were analyzed statistically for similarity between femoral head and acetabulum, between arthritic and dysplastic patients, and to Hammond and Charnley’s spherical description of the shape of the hip.

In every patient examined, both bony hip surfaces were significantly aspherical, both practically and statistically. There were no clear trends relating the shapes of matching femoral head-acetabulum pairs, suggesting an incongruous joint geometry. Further, there was no clear distinction between arthritic and dysplastic hip shapes for either the femoral head or acetabulum. Geometrically, the results suggest that the shape of the bony articular surfaces of the hip are both aspherical and unique among the patients studied.

These limited findings on arthritic hips brings into question the geometric foundation of the mechanical ball-and-socket model of the hip. At the time of writing it was unclear whether the motion of the arthritic hip is a pure rotational motion, for the bony anatomy does not appear to provide such a kinematic constraint. Further work that might arise from this thesis could include quantitative investigation of hip

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3Fitting a general ellipsoid to a partial set of points is not a trivial exercise; the details of the adjusted least-squares algorithm used in this thesis [11] is described in Section 3.3.2.
kinematics, in both arthritic and non-arthritic cases, to determine whether translational movement occurs and under what circumstances the human hip is confined to a truly spherical motion.

1.1 Objective

The focus of this thesis was to investigate the sphericity of the bony articular hip anatomy in two arthritic patient populations based on routine preoperative CT scans, by fitting the bony articular surfaces of the hip to portions of general ellipsoids. The potential contributions are twofold: establishing an aspherical geometrical model of the bony articular anatomy of the arthritic hip, and demonstrating a method by which such asphericity can be quantified on a living patient as part of routine pre-operative orthopaedic care.

This dissertation is organized as by chapters:

Chapter 2 gives background information on the anatomy of the hip, outlines previous quantitative work on spherical and aspherical descriptions of the hip joint, and briefly overviews fitting general ellipsoids to a partial set of points.

Chapter 3 details the methods used in the study, including patient selection details, CT segmentation methods, the shape fitting algorithms, and the metrics used to quantify asphericity of the bony articular surfaces of the hip.

Chapter 4 contains the results of the study methods presented in Chapter 3.

Chapter 5 discusses the results of the analysis, summarizes the findings, and presents potential future areas of research.
Appendix A provides a glossary of medical and anatomical terms that may be unfamiliar to a mechanical-engineering audience.
Chapter 2

Background and Literature Review

The ball-and-socket model of the hip has two distinct but closely related aspects: sphere-on-sphere contact geometry, and purely rotational (or isocentric) motion. If the contact geometry were found to be aspherical, the question of ball-and-socket kinematics would follow directly – and certainly, in the absence of additional constraints exterior to the joint, a more complex motion could not be ruled out.

The scope of this thesis, however, is restricted to the sphericity of arthritic hip joints. Although briefly discussed below, to provide a conceptual context for shape analysis, the kinematic consequences of aspherical contact geometry will not be examined in detail here.

Here, asphericity of the hip joint was investigated by fitting general ellipsoids to the 3D bony articular surfaces of the hip that were derived from segmented CT scans. In order to provide context for discussion of the methods and results, some background information is needed. Section 2.1 provides an overview of the bony anatomy of the hip, with a brief summary of the soft tissues comprising and surrounding the joint. This section also discusses the hip pathologies relevant to the patient populations included in this study. Section 2.2 recapitulates the ball-and-socket biomechanical
model of the hip and discusses some qualitative evidence in conflict with the model. Section 2.3 reviews the literature on quantitative aspherical descriptions of the hip joint, especially discussing Hammond and Charnley’s key paper that helped to establish the spherical model of the hip as the currently accepted joint morphology.

2.1 Morphology and Biomechanics of the Human Hip Joint

The hip joint is the articulation between the femur (or “thigh bone”) and the acetabulum (the “socket” of the pelvis, or “hip bone”). The articulating surfaces of the bones are covered by a layer of cartilage.

The hip has two main roles in the biomechanics of human motion: it provides load transfer between the leg and the upper body and allows relative motion between the pelvis and the thigh. A more thorough discussion of the most commonly accepted mechanical model of the hip is presented in Section 2.2, with a brief description of the primary motions of the hip given in Appendix B.2.

The hip joint consists of two bones: the hip bone (or innominate bone) and the femur. The hip bones meet at the pubis in a fibrous joint, and posteriorly connect to the sacrum (or “tail-bone”); these three bones form the pelvis. The acetabulum is a rounded \(^1\) depression in the hip bone, which opens in an inferior-lateral direction. The articular portion of the acetabulum roughly has a horseshoe-shaped or crescent-shaped articular area, surrounding a yet deeper depression called the acetabular notch. A picture of a model pelvis is shown in Figure 2.1.

The femur (or “thigh bone”) runs the length of the thigh, from the hip to the knee. A picture of a left proximal femur model is shown in Figure 2.2. The shaft of

\(^1\)“Rounded” in this document means without sharp edges or corners, without implying a circular or spherical shape.
the femur, which runs the length of the thigh between the hip and the knee, starts from the bottom of the image. Protruding medially and superiorly (and highlighted in green) is the femoral neck, which provides support for the femoral head. The femoral head, highlighted in red, has a cartilage layer covering it, and articulates in the hip joint.

Both the femoral head and the acetabulum are covered with a layer of cartilage (nonuniform and 2 mm at the thickest on the femoral head [12]). These provide smooth interacting surfaces that articulate in the joint. The joint space contains a lubricating fluid, called synovial fluid.

A ligamentous structure called the acetabular labrum runs along the rim and across the notch of the acetabulum. The entire joint (femoral head, acetabulum and labrum) is contained in a ligamentous pouch called the capsule.

There are a variety of muscles that cross the hip, inserting on both the pelvis
Figure 2.2: An anterior view of a plastic model of a left proximal femur, with some prominent features highlighted. The femoral head (which is fit to ellipsoids and spheres in this study) is highlighted in red; the femoral neck is highlighted in green.

and the femur and serving to create primarily rotational motion of the joint. Finally, surrounding the muscles are layers of fat and skin. These structures, which are critical to the operation of the joint, are beyond the scope of this thesis and will not be discussed further.

2.1.1 Common Hip Pathologies

The hip, like all anatomical structures, may be deformed from the normal shape or may malfunction. These differences may be attributable to genetics, early development, age-related degeneration, or other factors. The human subjects whose
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

anatomy was studied for this work all suffered from some form of hip disease, so a brief accounting of the major pathologies may provide useful context. The main relevant pathologies are osteoarthritis and dysplasia, with osteophytes and impingement being major symptoms that may or may not be classified as diseases.

Osteoarthritis (OA) of the hip is a disease that affects the articular cartilage in the joint. Its causes are not well understood. The cartilage in the hip may be torn or cut subsequent to injury, or may more gradually wear down to the underlying bone. Degradation of the articular cartilage layer leads to pain and reduced mobility [13]. Because adult articular cartilage has only a limited capacity to self-repair, osteoarthritis is a degenerative disease [13]. Severe cases may be treated by replacing the articulating surfaces of the joint with a prosthesis, as either a hip resurfacing or hip replacement. Because the expected lifespan of a prosthetic implant is approximately 10-15 years, hip resurfacings tend to be chosen for younger or more active patients [14]; a hip resurfacing can be followed by a conventional hip replacement [15], which is a simpler procedure than a revision surgery on an existing hip replacement. Most of the patients whose anatomy was studied for this work had some degree of hip osteoarthritis.

Developmental dysplasia of the hip (DDH) is a condition in which the orientation of the acetabulum provides insufficient acetabular coverage of the femoral head. This reduced coverage means that biomechanical loads are distributed over a smaller area, increasing stress, potentially leading to premature osteoarthritis [16]. DDH can be treated by major surgical intervention, such as surgically excising and reorienting the acetabulum to increase coverage [17]. This does not treat the existing arthritis but can, instead, redistribute biomechanical loads relative to the acetabulum, potentially
changing the primary load-bearing area and slowing arthritic development by lowering contact stresses [17]. An important sub-group of the patients whose anatomy was studied for this work had DDH.

Osteophytes are bony growths that occur at the margins of arthritic joints of diseased bones [13]. Osteophyte formation is correlated with bone and cartilage loss from within the joint [13]. Because they form on the outside of cortical bone, as opposed to being incorporated into regular bone, they tend to be weakly attached. In the context of this thesis, osteophytes were a concern during the segmentation of bone from soft tissue and were considered to not form part of the shape of the joint in question (see Section 3.2).

Femoroacetabular Impingement (FAI) is a class of conditions in which the femur makes contact with the acetabulum outside of the articular areas of the joint. A pincer impingement describes over-coverage of the femoral head by the acetabulum, leading to opposing impingement sites at high ranges of motion [18]. A cam impingement is when asphericity of the femoral head-neck junction causes the femoral head to lever out of the acetabulum near the end of the ranges of motion [18]. These impingements are shown schematically in Figure 2.3. It is speculated that increased stresses due to FAI may contribute to osteoarthritis.

Osteophytes on the femoral neck or acetabulum can alter the geometry sufficiently to introduce FAI in diseased joints.

Many of the patients whose anatomy was studied for this work had radiological evidence that was consistent with FAI; the medical records of these patients were not systematically investigated because FAI was not the primary object of study.

The most important effect (with respect to arthritis) of both DDH and FAI is that
they can increase the stress in the articular cartilage of the hip. This increased stress is believed to contribute to increased wear of the cartilage surface, and thus OA. Damage to the articular cartilage, whatever the cause (injury, normal age-related wear, accelerated pathological wear), leads to increased friction in the joint and resulting in pain for the patient.
CHAPTER 2. BACKGROUND AND LITERATURE REVIEW

2.2 The Ball-and-Socket Model of the Hip Joint

The hip is predominantly described as a mechanical ball-and-socket joint in the literature. The model rose to prominence after being suggested by Sir John Charnley shortly after he introduced the total hip replacement prosthesis [8]. This assumption is predominant in anatomical [6, 7], biomechanical [2], kinesiology [3, 4] and gait [5] texts as well as academic literature for these fields.

The ball-and-socket model has two important implications. Kinematically, a ball-and-socket joint allows relative rotation about any axis, but does not allow any relative translation. Geometrically, assuming that the motion of the joint is driven by the rigid geometry of interacting bony joint surfaces, this implies that the joint geometry is a highly congruent sphere-on-sphere contact. In such a joint, the motion is that of pure sliding; no rolling behavior is observed.

Despite the widespread assumption of the ball-and-socket model of the hip, the majority of the literature gives aspherical qualitative descriptions of the femoral head and acetabulum when describing the anatomy of the bones of the pelvis. For example, in the prominent Gray’s Anatomy textbook, the femoral head is described as spheroidal [6], with the acetabulum described as cup-shaped and approximately hemispherical [6].

2.3 Previous quantitative descriptions of the hip

2.3.1 Walmsley 1927

Several authors have quantitatively described the shape of the femoral head and acetabulum in humans. Prior to Charnley’s predominant ball-and-socket model, the
accepted morphological and kinematic description of the hip was that presented by Walmsley in a lecture published in 1927 [19]. Walmsley described human diarthroses (mobile joints, as opposed to fixed joints like the sutures of the skull) as having two simultaneous functions, load transmission and allowing motion— which must both be accommodated by the structure of the joint. He described the passive motion of the hip joint as being driven by two articular mechanisms: the joint capsule and the shape of the subchondral bone in the femoral head and acetabulum. The capsule is described as a highly complex, inelastic, non-contractile ligamentous structure. As the normal hip extends, the capsule tightens across the anterior surface of the femoral head and femoral neck, pulling the head medially into the acetabulum where the articular surfaces fit in an area with high congruence and “lock” into place as the extension reaches its limit. In other joint orientations, the acetabular fat pad and synovial fluid are pulled into the joint space to fill the volume between the femoral head and acetabulum; in the orientation of 70° flexion, 5° abduction, and 10° external rotation, Walmsley found that the capsule laxity allowed up to 2 cm of translation of the femoral head within the acetabulum.

Walmsley described the capsule as working in conjunction with the aspherical geometry of the subchondral femoral head and acetabulum. The normal articular surfaces are described as “definite and non-[reciprocally] shaped”, so that they are congruent and fit tightly in the load-bearing position of full extension and are otherwise incongruent. Walmsley cited earlier work by German authors Aeby (1863) and Schmidt (1874) who described the hip as a prolate rotational ellipsoid with the greater axis directed horizontally. He noted further that these deviations from spherical varied between subjects but were consistent within a single subject.
2.3.2 Hammond and Charnley 1967

Walmsley’s work formed the accepted wisdom until it was superseded in 1967 by Hammond and Charnley’s study describing a spherical hip [8]. Using ten cadaveric specimens (7 Formalin-fixed, 3 fresh frozen; one 19 years old, 9 over 60 years old) and a variety of tests, Hammond and Charnley’s work claimed to refute each of Walmsley’s conclusions. Hammond and Charnley used seven different methods of examination to look for congruence and shape of the hip, including replicating some methods described by Walmsley intended to demonstrate joint incongruence.

First, they described a test done by applying Engineer’s Blue on the femoral head to examine areas of contact with the acetabulum on specimens with the capsule, ligament and labrum removed. They found continuous contact around the edge of the acetabulum, but no contact with the deeper central acetabulum. They interpreted this result as the interference of a larger femoral head into a smaller acetabulum due to post-mortem swelling of articular cartilage, that is, they admitted that their cadaveric samples did not exhibit shapes consistent with normal motion.

The second method described was (as suggested by Walmsley) examination of plaster and acrylic casts of the articular anatomy. They did not discuss any results from this test, again citing potential articular cartilage swelling and preferring to work with cadaveric specimens.

Next, they reported having several observers articulate the cadaveric joints to determine whether an orientation where the joint fit together with greater stability or “locked” existed. No such results were reported by the observers. The dissection state in which this test took place was not described.

Fourth, Hammond and Charnley sectioned one preserved and two fresh joints on a
plane perpendicular to the medial-lateral axis to physically examine the congruence. In these three specimens they found no spaces between the femoral head and acetabulum. In a fourth specimen there was a gap in flexion which disappeared as the joint was brought into extension; the authors attributed this gap entirely to osteoarthritic thinning of the acetabular cartilage.

The femoral head was also tested for medial-lateral movement during a flexion/extension motion using a frame that held the acetabulum immobile and allowed uniaxial motion of the femoral head, with translation permitted along this axis (and measured with a micrometer). This test was performed on 4 cadavers (1 preserved) in various dissection states. Translation along this externally defined axis had an average range of 0.022 in (0.6 mm), with a maximum range of 0.053 in (1.3 mm); the maximum range corresponded to the only specimen tested with an intact joint.

Next, they directly measured femoral heads using a vernier caliper and a lens to detect contact between the calipers and the articular cartilage. Comparing oblique (superior-lateral to inferior-medial) diameters and horizontal (anterior to posterior) diameters, they found an average difference of 0.007 in (0.2 mm) across a 1.9 in average femoral head. Maximum deviation was 0.033 in (0.8 mm) which represented a 2% difference in diameter.

The final method used was to project radiographs of the femoral head onto a projection screen with a perfect circle drawn on it, to assess sphericity. Hammond and Charnley described the examination as being focused “in the arc or quadrant concerned in transmitting load during walking”. They examined the radiographs of the femoral head in three principal planes, and recorded deviations from sphericity.
compared to the “oblique” and “horizontal” diameters identified in the direct measurement. Measurements were taken at the eight principal compass points (that is, at 45° intervals). This method found maximum deviations of 1.6% in the bony shape and 1.2% in the cartilage shape over 9 subjects.

Based on this data, Hammond and Charnley concluded that the hip is “spherical with minor and inconsistent deviations from perfection”, in which “very perfect contact of all parts of the cartilaginous surfaces is [...] expected in any position of the hip joint”. They challenged the rotational ellipsoid model on the grounds that in an ellipsoid “the shape would be perfect and the axis of symmetry would be constant in direction and would need to be defined in relation to the hip joint”.

To the best of the author’s knowledge, this paper by Hammond and Charnley is the keystone in the perfect spherical geometric model of the hip and the resulting ball-and-socket biomechanical model. It is, however, limited in both scope and potential bias. Hammond and Charnley did attempt to describe the healthy hip, but selectively excluded the results of 3 specimens (of the original ten) they deemed to have arthritic changes (for example, they were excluded from the physical sections but not from measurements of sphericity using radiographs).

Hammond and Charnley did find highly spherical results using sections, direct measurements and radiographs; however, each of these methods is limited to examining cross-sections in particular planes (for which a rigorous definition was not described) and, in the case of direct measurements and radiographs, was very sparsely sampled. They found little medial-lateral movement using their frame, perhaps suggesting spherical motion, but this frame restricted rotation and translation to occur about and along a manually selected medial-lateral axis. Even in this case, a range
of 1.3 mm was observed in one specimen. Although these results do suggest sphericity, they do not preclude the possibility of an aspherical “normal” hip. Throughout their description of procedure and discussion, Hammond and Charnley held aspherical descriptions of the hip to an unprecedented standard of geometric consistency – in particular, they described a near-perfect rotational ellipsoid with an anatomically constant axis of symmetry as the only other quantitative model (though an egg-shaped femoral head was mentioned); however, other simple morphological measurements of the hip (neck-shaft angle, head-neck offset, version, etc) show a great deal of variation between subjects in other studies. This foundational paper did not discuss their findings of consistent spherical shape in the context of highly variable related structures.

Although broad in methods, this study suffered from a very limited sample size (limited further still by the selective exclusion) and lacked conclusive evidence to exclude the possibility of an aspherical hip. In particular, most of the geometrical models that arise later in the literature (for example, rotational ellipsoids and conchoïds) have special cases in which the geometry is spherical – as such, near-spherical hips could very easily form part of a spectrum of aspherical hip geometries.

### 2.3.3 Rebuttals to Hammond and Charnley

Bullough et al. published a counter-article in *Nature* shortly thereafter, in 1968 [20]. They cited previous aspherical descriptions of the hip (including Walmsley). Using a three-legged gauge, they measured four diameters across both the femoral head and acetabulum for 53 cadaver hips obtained at necropsy. They quantified asphericity as the largest difference between diameters divided by the average diameter. Although
their data had substantial scatter, they noted asphericity in each subject (ranging to about 0.3 for the bulk of the data, with outliers up to about 0.45) had a general trend of increasing sphericity with age. They concluded that the asphericity means the joint cannot be geometrically congruent in all orientations of the joint, although congruency may improve in certain orientations. They additionally gave a counter example where a young male hip was dyed to examine contact areas and clearly did not cover the entire cartilaginous surface, directly at odds with the conclusions of Hammond and Charnley. Bullough et al. postulated that the earlier results may have been affected by their subject selection, with nine of ten subjects over the age of sixty.

Blowers et al. examined the sphericity of the femoral head using 70 cadaver specimens and a Rank-Taylor-Hobson “Talyrond” instrument [21]. This instrument used a long stylus that glided tangentially along the outside of the femoral head to measure sphericity. Cadaver specimens were extracted and placed in a 4% formaldehyde solution until measurement. Repeated measurements taken a week later showed no change. The experimental design meant that measurements were taken in a series of parallel planes approximately perpendicular to the femoral neck. The contour was fit to a circle and an ellipse using least squares methods.

The non-circularity of the ellipsoid shapes was quantified as “ovality”. Ovality was calculated by measuring difference between the major ellipsoid axis and spherical radius, then taking the sum of the two measurements. Ovality was found to have a mean of 0.009 in (0.22 mm), ranging from 0.026 in to 0.001 in and with a wide spread in the inclination of the axes. They found no correlation between ovality and age. The authors concluded that their specimens had a wide range of elliptical cross-section,
but that their entire range detected ovalities were much less than those suggested by Walmsley.

Blowers et al. also discussed trying to use a three-legged probe, similar to the methods of Bullough et al, but found that their measurements were not adequately repeatable.

It is important to note that both the studies of Bullough et al. and Blowers et al. were limited in the cross-sections they examined; Bullough et al. examined four cross-sectional diameters while Blowers et al. measured a series of parallel contours. The results of Blowers et al., regarding the highly variable inclination of the ellipsoidal cross-section, may suggest a 3D variability in the inclination of aspherical deviations – and as such, the limited sampling of this small set of cross-sections allowed for aspherities directed obliquely to the measurement planes to be underestimated or to go entirely undetected.

2.3.4 Studies of Joint Incongruity

Greenwald and O’Connor also noted incongruity of the hip joint [22] as part of a study of load transmission through the hip. Building on the partial contact of the cartilage observed by Bullough et al., Greenwald and O’Connor examined cadaveric hip joints loaded with a hydraulic ram in a fluid bath, intended to emulate the conditions in the body. The femurs were attached to the hydraulic ram, with the the acetabulum mounted on a plate attached with a ball-and-socket joint (for orientation flexibility) and with position adjustments made with a screw thread. While the joints were in contact, a dye solution was run through the bath to stain any cartilaginous areas of non-contact; non-contact area estimates were made by covering surfaces with
gauze and counting squares. A neutral position was defined with the joint capsule intact. Cross-referencing marks were also made on both bones, which were then fully disarticulated for the experiments.

Fifty-one hips were tested, with joint positions in neutral and in five walking positions (20% increments in the gait cycle) using biomechanical loads from a cited study. Hips that showed evidence of cartilage degradation (e.g., osteoarthritis) were excluded from their study.

The results suggested a thorough contact of the acetabular cartilage under walking loads. The femur had thorough contact as well, but contact did not occur in a keyhole shape that encircled the fovea by a small margin and proceeded inferiorly and around the edge of the femoral head. As the applied loads were reduced, areas of non-contact appeared in the superior surface of the acetabulum and femoral head. Older specimens tended to have more complete contact of the cartilage, even as loads decreased. The authors speculated that joint incongruity allows for the joint to separate at low loads to allow for lubrication.

Afoke et al. measured incongruity in the hip with a casting study in 1980 [23]. Mounting 22 cadaveric specimens (37 to 90 years, disarticulated and cleaned of all soft tissue) in a hydraulic servo-controlled testing machine, each joint was brought into contact with surgical cement in the joint space under estimated biomechanical loads. Afoke et al noticed a wide variation in joint space sizes and patterns, with spaces ranging up to 0.4 mm. Spaces occurred in the superior aspect of the joint but also anterior and posterior under different loadings and positions. Within a single specimen, joint space varied widely with the joint position and load. Afoke et al. found no correlation between joint space and age.
Most recently, Lequesne et al. conducted a thorough study of plain radiographs of 223 patients (446 hips, 127 female, 96 male, mean age 51.3 years) who did not present with lumbar or hip pain. Anterior-posterior X-ray images were used to measure joint space at the lateral edge of the joint, the most superior point, and where the acetabular fossa began. Measurements were made using a ruler. The authors found highly variable joint space at all three sites; the space tended to increase from the medial to the lateral measurement positions, and was lower in females than males. No correlation with age was found. Further, Lequesne et al. also found that 20% of their subjects had a “dysmorphic” acetabulum – either an angular or a flattened acetabular roof [24], with a qualitatively very aspherical shape.

Greenwald and O’Connor’s work, as well as that of Lequesne, suffered from limitations in the data collection (e.g., sampling only a very small number of joint orientations and locations within the joint); however, the most significant finding with respect to this study is that each of these studies found some incongruency of the joint surfaces, which in some cases trended anatomically. This finding may be limited in significance by the focus of these studies, which were directed at examining load transmission and cartilage contact in addition to geometry. Regardless, the finding of joint incongruency is geometrically at odds with the mechanical ball-and-socket model and with the spherical geometry described by Hammond and Charnley.

2.3.5 Existing Classification Systems for Aspherical Hips

Some sphericity classification systems were developed to describe grossly aspherical hips in the case of Legg-Calvé-Perthes disease. This disease occurs when the femoral
head loses blood flow during development, so patients can suffer from severely de-
formed hip joints. Mose [25] gave reference data collected by examining plain ra-
diographs using a transparent template on which a series of concentric circles were
printed; a variation of 1 mm in the radius was allowed before the hip was classified as
aspherical. Mose noted, however, that even within this threshold, flattening of parts
of the femoral head did occur.

Stulberg et al. [26] described five classes of hip shapes affected with Legg-Calvé-
Perthes disease, with one of the primary determinants being the femoral head shape.
Shape was classed as “spherical”, “ovoid” or “flat” using Mose’s template method.
This study showed a differentiation in outcomes (particularly related to osteoarthritis)
based on its classification, which suggests a relationship between joint shape and
biomechanics.

2.3.6 3D Descriptions of the Shape of the Hip

More recently, mathematical models have been proposed to describe the 3D shape of
the femoral head and acetabulum.

Menschik described the shape of the femoral head as a rotational conchoid in
1997 [27]. A conchoid is a polar shape given by the formula $r = a + b \cos(\theta)$, from
which made a 3D shape can be generated by rotating about a vertical axis; an example
conchoid is shown in Figure 2.4.

Menschik used 10 cadaver specimens from 5 cadavers (33±14 years of age) based
on recordings of cadaver femurs made with a coordinate measuring machine. With
cartilage intact, he made a silicone mold of the disarticulated femoral heads and
acetabulums; these molds were next used to make plaster casts of the bones with
Figure 2.4: An example planar conchoid with parameters $a=5$, $b=4$. It is has been rotated so that the axis of symmetry is vertical.

cartilage. Cartilage was then removed from the cadaveric specimens chemically to expose the subchondral bone for direct measurement. Surfaces were measured in air using a CNC coordinate measuring machine with 0.0001 mm resolution and 0.001 mm uncertainty; Menschik sampled between 80-120 points per surface, and each point measurement was repeated 3 times.

Menschik found an approximate rotational axis for each specimen, that is, an approximate circular cross-section perpendicular to this axis (with radii $0.15\pm0.06$ mm). In each 3D case, conchoids had less fitting error than spheres; for the femoral head, both the conchoid and sphere better fit the bony surface than the cartilage surface; the opposite was found for the acetabulum. Menschik noted that for his fits, there was often a distinct gap in the zones of the joint that did not bear much or any load.

Xi et al. performed shape fitting of point cloud data from 3D laser scans of 38 cadaver acetabulums (12 male, 26 female) of Chinese race [28]. These data were fit to spheres, rotational conchoids, and rotational ellipsoids. Conchoids and spheres were found to fit comparably well, but ellipsoids fit the point cloud data statistically significantly better by approximately 0.1 mm. Although this magnitude may appear
minor, Xi et al. noticed there was a consistent area of poor fit at the apex of the acetabulum with both conchoids and spheres; this area had much higher fit quality with the rotational ellipsoid. They concluded the rotational ellipsoid was the most appropriate shape for the acetabulums they studied. This example, with a consistent localized area of poor fit not strongly reflected in the overallfitting errors, highlights a limitation of using overall RMS error (or similar statistics) to assess goodness of fit. Explicit examination of these fits can be advantageous to screen for such trends.

Xi et al. also studied the surface curvature across the cartilage surface of the acetabulum. The authors created sub-meshes for each point, incorporated each connected point, and fit a general quadric surface to each sub-mesh. Each surface was described as a peak, pit, ridge, valley, plane, minimal, saddle ridge, or saddle valley based on the local Gaussian curvature and mean curvature. This analysis found no ridges, meaning that the entire acetabulum could be treated as one quadric surface. They also found that, using surface curvature, the most appropriate shape was a rotational ellipsoid with major axis directed horizontally (between the hip centers).

Gu et al. conducted a very similar study using 25 cadaver acetabulums (45 ± 7 years; 12 male, 13 female) [29]. The cartilage surface of the specimens were sampled using a 3D laserscanner and local surface curvature (Gaussian and mean) were calculated, as well as shape fitting. As did Xi, they found a uniform single “pit” structure, with no ridges (aside from some rare points they deemed spurious) that was best modeled as a rotational ellipsoid. Fitting error differences between spherical (0.498 mm) and rotational ellipsoid (0.446 mm) fitting were statistically significantly different. The same high-error zone in the acetabular roof was present in the spherical fit but absent in the ellipsoids, more or less replicating the results of Xi.
Most recently, Pienkowski et al. used MRI imaging on ten pediatric patients (all male, age 8 ± 1 years) [30] with unilateral Legg-Calvé-Perthes disease. Cartilage surfaces of the femoral head and acetabulum were segmented and fit to spheres. They quantified the quality of the joints using shape deformity (spherical fit error), joint fit (relative scaling of femoral and acetabular spheres), and joint incongruity (the center distance between femoral and acetabular spheres). Despite the highly irregular morphology of these severely pathological joints, they found mean error of 1±0.3mm for the femoral head and 1±0.2mm for the acetabulum; the asymptomatic contralaterals fit with 0.5 ± 0.1mm and 0.7 ± 0.1mm, respectively. Joint incongruity was 3.0 ± 1.3mm for pathological joints, and 1.2 ± 0.5mm for asymptomatic joints. Despite these figures being superficially reasonable, figures included by the authors (for example, Figure 2.5) seem to suggest that spherical geometry is inappropriate from a biomechanical perspective. If the fitting error was a result of different morphology, as opposed to random variation or measurement noise, this could draw the applicability of their geometrical model into question.

The work for this thesis continued the development of quantitative aspherical models of the hip by modeling both the femoral head and acetabular bony articular surfaces with general ellipsoids, based on CT segmentation. The methods will be described below in Chapter 3.

2.4 Ellipsoid Fitting

General ellipsoid fitting is a non-trivial mathematical challenge. Because ellipsoids are in the family of quadrics, all with the same form, general least-squares methods
do not always restrict the fit to an ellipsoid.

A general quadric surface has the form:

\[
a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz
+2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0
\]  

(2.1)

which can be is optimized by minimizing the sum of squares of the distance of each
datum to a quadric surface [28, 29]. In matrix form, with

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}, \quad X = \begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}, \quad U = \begin{bmatrix}
  a_{14} \\
  a_{24} \\
  a_{34}
\end{bmatrix}, \quad A^* = \begin{bmatrix}
  A \\
  U \\
  U^T \\
  a_{44}
\end{bmatrix}
\]

(where \( A \) and \( A^* \) are symmetric and real), Equation 2.1 can be written in matrix form as

\[
X^T A X + 2U^T X + a_{44} = 0
\]  

(2.2)

The type of quadric shape is determined by the rank of \( A \), the rank of \( A^* \), the eigenvalues of \( A \), and the determinant of \( A^* \). For a real general ellipsoid, \( A \) and \( A^* \) are both full rank, \( A \) has all eigenvalues of the same sign, and \( A^* \) has a negative determinant. A sphere or rotational ellipsoid occurs when there are equal eigenvalues: if all eigenvalues are equal, the quadric is a sphere; if two are equal, the ellipsoid has an axis of rotation.

Ordinary least-squares fitting of a quadric shape, however, does not restrict the fit to a general ellipsoid. Any other quadric shape (for example, a paraboloid, hyperboloid, etc.) could just as well be fit. Ordinary least-squares methods are also a statistically inconsistent estimator [11], meaning that adding more data from an ideal shape does not ensure that the fitting error is thereby reduced. Geometric approaches can also be highly sensitive to initial conditions.

The work for this thesis used the Adjusted Least-Squares (ALS) algorithm of
Markovsky et al [11], which optimized data to an implicit form of the ellipsoid equation. This method improved on previously published algebraic approaches, which were found to be ineffective in some special cases, and on orthogonal fitting approaches, which were found to be sensitive to initial condition and have several local minima [11]. Markovsky showed that this estimator was consistent for normally distributed measurement error, independent of initial conditions, and computationally cheaper than an ordinary geometric method. This algorithm is examined in more detail in Section 3.3.2.

2.5 Summary

The previous sections summarized the anatomy relevant to the hip and the spherical geometry and rotational motion of the mechanical ball-and-socket model of the joint. The relevant literature was summarized, including the early description of Walmsley, the foundational paper by Hammond and Charnley describing a spherical hip joint, and a number of articles since suggesting aspherical and incongruous joint geometry. The methods and analyses used to perform general ellipsoid fitting of the hip are outlined in the following chapter.
Chapter 3

Methods

This chapter describes the methods used to conduct the work of this thesis. Spherical fitting and general ellipsoid fitting of the bony articular surfaces of the hip (the femoral head and acetabulum) were performed for two different pathologies in order to evaluate the validity of the prevalent ball-and-socket model of the literature.

Volumetric medical images of arthritic and dysplastic patients served as the source data for this study; scan parameters and patient selection details for the studied populations are given in Section 3.1. 3D models of the femoral head and acetabulum were created from the scans via a two-stage manual segmentation, using a procedure outlined in Section 3.2. The vertices of these 3D models were extracted and fit to both spheres and general ellipsoids, as described in Section 3.3. The validity of the ball-and-socket geometry was assessed by comparing spherical and ellipsoidal fit quality, as well as examining the asphericity of the ellipsoid fits (Section 3.4).

3.1 Data Collection

Computed tomography (CT) scans were taken in routine preparation for orthopaedic procedures on the hip at Kingston General Hospital in Kingston, Ontario, Canada.
These scans were segmented manually to produce 3D triangulated surface models of the bony hip anatomy; these models were the source data for this study. Segmentations were performed either in preparation for computer assisted procedures, or post-hoc for inclusion in this study. Two pathologies were included in this study, early life osteoarthritis (treated with a hip resurfacing procedure) and hip dysplasia (treated with a periacetabular osteotomy, or PAO).

### 3.1.1 Patient Selection

Patient demographics were determined solely by the selection criteria for the surgical procedure in question (and were thus otherwise uncontrolled). Hip resurfacings were indicated for younger patients (who were expected to outlive the anticipated duty span of a conventional total hip replacement) with early-life osteoarthritis. PAOs were indicated for severe dysplasia of the hip, with secondary osteoarthritis as a result of the dysplastic deformity.

Nineteen early arthritic and seventeen dysplastic patients were included in this study. The patient demographics are listed in Table 3.1. Note that these demographics were calculated across the population of hips as opposed to patients; thus, patients with bilateral disease were counted twice.

<table>
<thead>
<tr>
<th>Pathology</th>
<th>Sex</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early osteoarthritis (Resurfacing)</td>
<td>15 M, 4 F</td>
<td>52 ± 6</td>
</tr>
<tr>
<td>Dysplasia (PAO)</td>
<td>2 M, 17 F</td>
<td>35 ± 10</td>
</tr>
</tbody>
</table>
3.1.2 CT Protocol

The scans were acquired at Kingston General Hospital under a protocol approved by the Research Ethics Board. CT datasets were acquired with slice thickness of 2.50 mm (reconstructed to 1.25 mm) at an axial spacing of 1.25 mm with an approximate in-plane spatial resolution of 0.6 mm per pixel. Peak voltage through the X-ray generator was 120 kV. The scanning area contained the entire pelvis (and thus also the proximal femur). Certain datasets did not include the pelvis proximal of the anterior superior iliac spine (ASIS).

3.2 CT Segmentation

The CT data were segmented manually to produce the 3D triangulated models that served as the source data for this study. The models were created using commercial software (MIMICS by Materialise, Leuven, Belgium).

3.2.1 Reconstructing the Bony Anatomy of the Femur and Hip Bone

Segmentation of the femur and hip bone was performed by creating masks, which identified CT voxels that contained the desired anatomy. Each mask was then used by the software to generate a triangulated surface mesh of the bone. The following subsections outline the typical steps for creating a bone model, from importing a CT scan to final mesh. The particular workflow varied from technician to technician. It was not uncommon to perform several iterations of a step to refine a model.
 Thresholding and Cropping

The first operations on the CT data were thresholding and cropping operations, which were used to produce a rough starting mask. The thresholding operation created a mask including all voxels in the dataset that fall within a certain range of Hounsfield Units (HU). The Hounsfield unit describes the linear X-ray attenuation coefficient measured in a single voxel by the CT scanner. It is scaled such that distilled water is assigned 0 HU and air is assigned -1023 HU. In a pelvic CT, the typical range was from -1023 HU to 1850 HU.

The software had a thresholding preset for bone, which included all voxels greater than 225 HU. This captured dense cortical bone readily, but was less reliable for cancellous bone or low-density cortical bone. An example CT slice, overlaid with a thresholding mask, is shown in Figure 3.1. At this stage, although the majority of the bone surface was included, gaps frequently remained in the surface. Also, the interior of the bone was typically only sporadically included in the mask.

The cropping operation simply restricted a mask to a manually defined rectangular prism within the CT volume, excluding all voxels outside the prism. Figure 3.2 shows the threshold mask of Figure 3.1 cropped to isolate the left femur. Notice that the mask still included portions of the pelvis and other undesired voxels after the cropping operation.

Defining the Edges of a Bone

Each slice of the incomplete mask that was produced by the thresholding and cropping was manually edited. Voxels that were clearly not part of the femur were erased from the mask, then noncontiguous surface curves were completed and smoothed manually,
Figure 3.1: Axial slice of a CT mask resulting from a thresholding operation to isolate bone.

Figure 3.2: Axial slice of a CT mask that had been thresholded and cropped.
using the HU values of the voxels and anatomical background knowledge as guides.

Figure 3.3: Inspection prior to manual definition of the edge of the femur. The pixels belonging to the pelvis, in yellow, remained from the coarse thresholding and cropping operations and needed to be erased. Pixels highlighted in green were not part of any bone and also needed to be erased.

Figure 3.3 shows the thresholded and cropped mask; areas of the mask that required editing are highlighted. The voxels in the blue regions were not part of the femur and required removal from the mask. Pixels highlighted in yellow were not part of any bone and also needed to be excluded. Further, the bony articular surface of the femur (as well as some non-articular regions) were rough and required manual smoothing. Figure 3.4 shows the results of these manual edits, as well as some manual filling of the bone interior.

The manual smoothing process included some pixels of lower HU than the “bone” range defined by the software. The decision to include these pixels derived from the clinical experience of the investigators, because their experience with the anatomy
indicated that even with osteophytic or arthritic hips, a jagged bone margin was extremely unlikely in OA patients; rather, it was considered more likely that the HU of these voxels were low because of poor bone quality or subtleties of the scanning process.

**Osteophytes and Ossifications**

Osteophytes, a type of bony growth that can occur on the surface of bone, presented a challenge while segmenting. They often produced low to moderate HU and thus presented ambiguous cases to segmenters, who were forced to choose between inclusion or exclusion of the feature. An example of an osteophyte can be seen in Figure 3.5.

Segmentations produced for computer assisted surgeries would err on the side of inclusion or exclusion, depending on the procedure. For segmentations intended for use in computer aided surgery, the decision to include or exclude an osteophyte
CHAPTER 3. METHODS

Figure 3.5: Osteophyte in an axial CT slice. The osteophyte, highlighted in blue, was included by the threshold operation, but may or may not have been rigid and/or attached to the underlying cortical bone.

from a segmentation was driven by the registration method. For a physical surface registration using a patient-specific template (e.g., Kunz et al. [31]), erring on the side of inclusion produced an area of non-contact with the template should the osteophyte not have been rigid or fixed. This preserved the physical registration should the inclusion have been in error. Conversely, for a navigated procedure that relied on a probe-based registration, erring on the side of exclusion was preferable. Were the osteophyte to be present but not included in the segmentation, it could be manually removed by the surgeon to expose underlying cortical bone, or avoided altogether in the selection of surface points for registration. Segmentations created specifically for this study included or excluded osteophytes based on the best judgement of the segmenter.

For the purposes of this study, segmentation accuracy was required on the bony
articular surfaces only. Osteophytes were considered non-articular, and thus any osteophytes that were present were removed in the next stage of data processing (see Section 3.2.2).

Filling

After the edges of the bone were defined in each slice, the mask had its interior filled. This was done to simplify the final 3D model, so that the internal bony structure of the femur was not examined in this study. This was accomplished in one of several ways, varying according to technician preference. In some cases, manual filling was performed. Mimics also provided the ability to generate polylines that traced the outer margin of a mask and could be filled using a fill tool. The polylines tool was used with caution in areas of high curvature, because the tool preferred to create gradual curves and thus its use risked erroneously including or excluding voxels in some cases. These cases required manual editing to correct the edge after a polyline fill.

3D Model Generation

3D models were created using algorithms built into Mimics to generate triangulated meshes from the voxel masks. The particular parameters of the model creation will again vary between technicians. The 3D surface model of the same femur shown in the previous figures of this chapter can be seen in Figure 3.6.

The cross sections of the 3D models (which were of higher resolution than the CT scans) were superimposed on the CT slices to verify accuracy and check for interference between the bones of a joint. This allowed technicians to use varying
methods of segmentation and model creation to their preference with a common means of verification. A sample of this view is seen in Figure 3.7.

The 3D models were exported from MIMICS as triangulated meshes in the STL (stereolithography) file format.

Verification

Because the specific segmentation workflow may have varied between technicians, it was necessary to verify the resulting models as opposed to the procedures. Generally speaking, a “ground truth” could not be established because the segmented CT scans were from living patients whose anatomy could not be examined directly. The primary method of verification was to examine the 3D contours superimposed over the CT images (e.g., shown in Figure 3.7) through all slices in all three anatomical views to verify accuracy in defining bone margins and check for interference between the
bones of a joint. This was of particular concern in arthritic hip joints, where the joint spacing was limited or nonexistent and thus extremely difficult to detect on the CT. Additionally, contours of the bony articular surfaces of the bones were expected to be smooth – any spikes or jagged features were deemed unlikely to reflect patient anatomy. Finally, the 3D models themselves were visually inspected to ensure they were reasonable in appearance. Again, any sharp corners, spikes, jagged features, or “stair-stepping” artefacts were indicators of a poor segmentation or model generation. A speckled or mottled model surface texture was acceptable but not desired. This procedure allowed technicians to use different methods of segmentation and model creation to their preference, yet provided a common means of verification.

Efforts were undertaken by other members of the research group to quantitatively
evaluate the accuracy of this segmentation procedure using cadaveric specimens. Results on 6 cadaver hips, using laser scanned bones after disarticulation, suggested that the segmentation accuracy using this procedure was of sub-millimeter accuracy [32].

3.2.2 Isolating the Bony Articular Surfaces

Because the best-fit ellipsoids calculated in this study were to be fit to the bony articular surfaces only, the articular surfaces of the bone segmentations were next isolated. This processing was done using commercial software (MAGICS by Materialise, Leuven, Belgium).

After importing the STL file of the bone, the “hollow” tool of the software was used\(^1\) to create an internal wall just below the surface. A rough cut was then made using the polyline cut tool, either across the femoral neck or around the acetabulum. Figure 3.8 shows a femur imported into Mimics, which then was hollowed and had the portions distal to the femoral neck removed to produce the model in Figure 3.9. Rough edges and indentations around the perimeter of the initial cuts, mainly produced from processing osteophytes, required exclusion using further cuts. These edges were iteratively refined until all osteophytic portions and any portions with outward inflection (forming a saddle as opposed to an ellipsoid) were removed.

Next, the fovea or acetabular notch were removed, if present in the segmentation. Figure 3.10 shows the fovea isolated but not removed.

Finally, any segmentation artefacts were also manually cut out, producing the finished 3D shell. Figure 3.11 and Figure 3.12 show the final isolated bony articular surface superimposed over the original femur. The 3D model of the isolated bony

---

\(^1\)For this study, a wall thickness of 0.01 mm was chosen, with detail size 0.1 mm. A smaller wall thickness would have been preferable, but concerns about the file size and computation time precluded this extra accuracy.
Figure 3.8: Triangulated mesh from a CT segmentation was imported into Magics for articular-surface isolation. The “ridged” or “tiered” appearance at the top of the figure is an example of a mild stair-stepping artefact; more severe cases would require resegmentation or manual exclusion of the artefact.

Articular surface was also saved as a triangulated mesh in STL format.

3.3 Shape Fitting

All further data analysis was conducted using MATLAB (Mathworks, Natick, MA, USA). STL file importation was accomplished using a function publicly available on the MATLAB Central [33]. Figure 3.13 shows the triangulated surface mesh imported
The points of the triangulated meshes of the isolated bony articular surfaces (femoral head and acetabulum) were used to fit both spheres and general ellipsoids using custom MATLAB scripts.

### 3.3.1 Spherical Fit

Spherical fits were calculated with a custom MATLAB implementation of a least-squares estimation. Starting with the implicit formulation of a point on a sphere of radius $r$ centered at $[a, b, c]$: 

$$
(a - c)^2 + (b - e)^2 + (c - f)^2 = r^2
$$

Figure 3.9: Femoral head after a preliminary cut to remove the distal portions of the bone. Note the thin shell produced by the hollowing operation and the irregularity around the cut portion due to osteophytes.
Figure 3.10: Femoral head with the fovea isolated, but not removed, from the femoral head shell.

\[
r^2 = (x - a)^2 + (y - b)^2 + (z - c)^2
\]

\[
0 = x^2 - 2xa + a^2 + y^2 - 2yb + b^2 + z^2 - 2zc + c^2 - r^2
\]

\[
2xa + 2yb + 2zc - d = x^2 + y^2 + z^2
\]

(3.1)

with \(d = a^2 + b^2 + c^2 - r^2\). For a single point on the sphere, Equation 3.1 can be rewritten in a matrix form:

\[
\begin{bmatrix}
2x & 2y & 2z & -1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
= x^2 + y^2 + z^2
\]
Figure 3.11: Isolated articular portions of the femoral head (mottled cyan and red) superimposed with the original femur (blue). Note that the fovea and osteophytes on the inferior portion of the head were excluded.

Extending this matrix form to include $n$ points,

$$
\begin{bmatrix}
2x_1 & 2y_1 & 2z_1 & -1 \\
2x_2 & 2y_2 & 2z_2 & -1 \\
\vdots & \vdots & \vdots & \vdots \\
2x_n & 2y_n & 2z_n & -1 \\
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
\end{bmatrix}
= 
\begin{bmatrix}
x_1^2 + y_1^2 + z_1^2 \\
x_2^2 + y_2^2 + z_2^2 \\
\vdots \\
x_n^2 + y_n^2 + z_n^2 \\
\end{bmatrix}.
$$
Figure 3.12: Isolated articular portions of the femoral head (mottled cyan and red) superimposed with the original femur (blue).

MATLAB’s `mldivide` (or \\() function provided a least squares estimate of \[
\begin{bmatrix}
a & b & c & d
\end{bmatrix}^T,
\]
which was used to extract the parameters of the fit.

Fit error for a point was calculated as the difference between the fit radius and the distance from the fit center to the point. Error was expressed as the root-mean-squared (RMS) error.
Figure 3.13: Femur (blue) and isolated femoral head (cyan) imported into MATLAB as triangulated meshes. The triangulated structure, including the vertices used to fit spheres and ellipsoids, can be easily seen.

### 3.3.2 Ellipsoidal Fit

The technical difficulty of fitting an ellipsoid to a partial set of points was markedly higher than fitting a sphere. Geometric approaches, such as orthogonal distance regression, were highly sensitive to initial conditions and the deviation of the data from a pure ellipsoidal shape. Conversely, if ordinary least squares algebraic methods
had been used for optimization, the governing equations could have been ambiguous (fitting a general quadric surface such as a paraboloid, hyperboloid, or cone), and statistically, the estimator could have been inconsistent.

To overcome these challenges, an Adjusted Least-Squares (ALS) algorithm developed by Markovsky et al. [11] was used to calculate ellipsoidal fits\textsuperscript{2}. An example of such an ellipsoid fit is shown in Figure 3.14. Markovsky’s ALS algorithm relied on an implicit formulation of an ellipsoid.

A point $\vec{x}$ was constrained to lie on an ellipsoid centered at a point $\vec{c}$ if and only if it satisfied

$$(\vec{x} - \vec{c})^T A (\vec{x} - \vec{c}) = 1$$ (3.2)

where, for an ellipsoid, $A$ was symmetric and positive-definite.

Given a set $\{\vec{x}_i\}$ of points, the ALS algorithm found the matrix $A$ and vector $\vec{c}$ that minimized the objective function

$$\arg\min_{A, \vec{c}} \sum_{i=0}^{n} (\vec{x} - \vec{c})^T A (\vec{x} - \vec{c}) .$$ (3.3)

This implicit formulation has a straightforward relationship to the usual parametric formulation of an ellipsoid. The parametric form was found from a point $\vec{p}$ on the unit sphere centered at the origin, for which

$$\vec{p}^T \vec{p} = 1 .$$

\textsuperscript{2}Markovsky et al. kindly contributed a MATLAB implementation of their algorithm to MATLAB Central, a repository of publicly available code. This public implementation was used to find $A, \vec{c}$ and the RMS error for the ellipsoid fits.
Figure 3.14: An example ellipsoid fit for a dysplastic acetabulum, found by fitting the vertices of the triangulated mesh to an ellipsoid using Markovsky’s ALS algorithm. The major axis of the ellipsoid is shown in blue.
A corresponding point $\bar{x}$ on an ellipsoid was found by performing three operations on the unit sphere centered at the origin. First, the sphere was scaled to a general ellipsoid by premultiplying by a scaling matrix $S$ that was diagonal and positive-definite. The ellipsoid was then rotated about the origin to a general orientation by premultiplying by a rotation (special orthogonal group, or $SO(3)$ group) matrix $R$. Finally, the ellipsoid was translated to be centered off-origin by adding a center vector $\bar{c}$. Thus, a point $\bar{x}$ on a general ellipsoid was related to a point $\bar{p}$ on the unit sphere as

$$\bar{x} = R S \bar{p} + \bar{c}$$

$$\equiv \bar{p} = S^{-1} R^T (\bar{x} - \bar{c})$$

$$\implies \bar{p}^T = (\bar{x} - \bar{c}) R S^{-T}$$

Note that combinations of $S$ and $R$ were not unique. For example, an ellipsoid with both minor axes of unit length and major axis aligned with the $X$ coordinate axis could alternately be created by rotating an ellipsoid with major axis along the
Y coordinate axis about the Z coordinate axis by ±π/2:

\[
RS = I_{3\times3} = \begin{bmatrix}
a & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\[
= R_Z(\pi/2) = \begin{bmatrix}
1 & 0 & 0 \\
0 & a & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

\[
= R_Z(-\pi/2) = \begin{bmatrix}
1 & 0 & 0 \\
0 & a & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

With this in mind, the parametric form of Equation 3.4 was found from the implicit form of Equation 3.2 by using the Schur decomposition of \( A \):

\[A = RDR^T\]

where \( R \) was orthogonal and, because \( A \) was symmetric and positive-definite, \( D \) was a diagonal matrix with all positive values. This could readily be found by formulating the matrix \( S \) as \( S = \sqrt{D^{-1}} \), or in component terms, \( s_{ii} = 1/\sqrt{d_{ii}} \). Because \( S \) was diagonal and therefore symmetric,

\[D = S^{-T}S^{-1}\]

In some cases, the Schur decomposition of \( A \) gave an orthogonal matrix \( R \) where \( |R| = -1 \); this corresponded to a reflection about a primary plane and then a rotation.
Because of the primary-plane symmetry of the origin-centered ellipsoids (produced as the unit sphere scaled along primary axes in Equation 3.4), the reflection had no effect on the ellipsoid of best fit. \( R \) could thus be converted to a rotation matrix by post-multiplying by an arbitrary primary plane reflection.

Thus,

\[
(\vec{x} - \vec{c})^T A (\vec{x} - \vec{c}) = (\vec{x} - \vec{c})^T R DR^T (\vec{x} - \vec{c})
\]

\[
= (\vec{x} - \vec{c})^T RS^{-T} S^{-1} R^T (\vec{x} - \vec{c})
\]

\[
= \vec{p}^T \vec{p}
\]

\[
= 1
\]

Thus, from the estimated structural matrix \( A \) given by Equation 3.3, the rotation \( R \) and scaling \( S \) of the unit sphere formed the ellipsoid follow directly from the Schur decomposition of \( A \).

### 3.4 Characterization of Ellipsoidal Asphericity

Asphericity of the ellipsoidal fits was quantified and compared by computing the major and minor eccentricity of the ellipsoids. If the elements of the diagonal scaling matrix \( S \) were ordered such that \( s_{11} \geq s_{22} \geq s_{33} \), then the major eccentricity \( E_U \) and minor eccentricity \( E_L \) could be defined as

\[
E_U = s_{11}/s_{33} \quad (3.5)
\]

\[
E_L = s_{22}/s_{33} \quad (3.6)
\]
3.4.1 Comparison with Spherical Shape

The eccentricities calculated for each ellipsoidal fit were grouped and compared to spherical fits. For a sphere, $E_U = E_L = 1$ because $s_{11} = s_{22} = s_{33}$. Fits were compared to an idealized spherical shape by using a one-sample $t$-test with a confidence level of $\alpha = 95\%$ for both major and minor eccentricity. Additionally, RMS error was compared for both fits to compare the ability of the two shapes to fit the data accurately.

3.4.2 Testing for Trends in Asphericity

Eccentricity data were also tested for correlations relating major and minor eccentricity, and relating eccentricities of matching femur-acetabular pairs. A relationship between major and minor eccentricity would indicate that the anatomy in question naturally varies within a subgroup of ellipsoids, as opposed to a purely general selection. A relationship between femoral and acetabular eccentricities (for major and/or minor eccentricities) would indicate a relationship between the shape of the femoral head and the acetabulum with which it articulates.

Eccentricity data were also compared by testing for differences of means with two-sample $t$ tests with confidence level of $\alpha = 95\%$.

3.5 Summary

The previous sections detailed the methods of this thesis: the data acquisition, initial segmentation, bony articular surface isolation, shape fitting, and ellipsoid characterization. The results of these analyses are presented in Chapter 4 and discussed in Chapter 5.
Chapter 4

Results

This chapter presents the numerical results of this study on the morphology of the hip joint. First, fitting error of spherical and ellipsoid fitting are presented in Section 4.1. Next, details of the spherical fitting parameters are given in Section 4.2 and the parameters of ellipsoid fitting are given in Section 4.3. Interpretations of these results, and the validity of the ball-and-socket model of the hip joint, will be deferred to Chapter 5.

4.1 Fitting Error

Spherical fitting was performed using an ordinary least-squares method. RMS error was quantified by comparing the distance from each vertex to the fit center with the fit radius. General ellipsoid fitting was performed using Markovsky’s ALS algorithm [11], with the RMS error taken to be one of the outputs of the function provided by Markovsky.

Table 4.1 summarizes the error in fitting both shapes. A two-sample paired \( t \)-test was used to compare spherical and ellipsoidal error. Spherical fitting error was \( 0.68 \pm 0.13 \, mm \) for arthritic acetabulums, \( 0.69 \pm 0.18 \, mm \) for arthritic femoral heads,
Table 4.1: Shape fitting error for spherical and general ellipsoid fitting of bony articular hip surfaces. All values are in mm.

<table>
<thead>
<tr>
<th></th>
<th>Fitting Error, Arthritic (mm)</th>
<th>Fitting Error, Dysplastic (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Acetabulum</td>
<td>Femoral Head</td>
</tr>
<tr>
<td>Sphere</td>
<td>0.68 ± 0.13</td>
<td>0.69 ± 0.18</td>
</tr>
<tr>
<td>Ellipsoid</td>
<td>0.51 ± 0.11</td>
<td>0.49 ± 0.10</td>
</tr>
<tr>
<td>t-test</td>
<td>p &lt; 0.01*</td>
<td>p &lt; 0.01*</td>
</tr>
</tbody>
</table>

0.71 ± 0.25 mm for dysplastic acetabulums and 0.72 ± 0.23 mm for dysplastic femoral heads. Ellipsoid fitting error was 0.51 ± 0.11 mm for arthritic acetabulums, 0.49 ± 0.10 mm for arthritic femoral heads, 0.43 ± 0.08 mm for dysplastic acetabulums and 0.42 ± 0.09 mm for dysplastic femoral heads.

For the bony articular hip surfaces of both arthritic and dysplastic patients, the spherical fit was clearly worse: it had a p value of less than 0.01, which was statistically significant.

### 4.2 Spherical Fitting

Spherical fitting was completed in order to evaluate the geometric model the hip as a ball-and-socket shape. The spherical ratio was defined as the ratio of the radii of the spheres of best fit for a single hip joint. The center distance was defined as the 3D distance between the centers of the spheres of best fit.

In order for the two spheres of best fit to form a plausible (or “compatible”) ball-and-socket joint, the femoral head sphere had to fit within the acetabular sphere. To meet this criterion, the radius of the acetabular sphere needed to be greater than the
Table 4.2: Least-squares spherical fitting results for arthritic and dysplastic hip joints. The spherical ratio was found to be statistically significantly different between arthritic and dysplastic populations.

<table>
<thead>
<tr>
<th></th>
<th>Arthritic</th>
<th>Dysplastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical ratio (Acetabulum / Femoral Head)*</td>
<td>1.04 ± 0.05</td>
<td>1.12 ± 0.06</td>
</tr>
<tr>
<td>Center distance (mm)</td>
<td>3.34 ± 1.62</td>
<td>2.36 ± 1.42</td>
</tr>
<tr>
<td>Incompatible ball-and-socket joints (of n=19)</td>
<td>15</td>
<td>6</td>
</tr>
</tbody>
</table>

sum of the center distance and the femoral-head sphere radius.

These results are summarized in Table 4.2. The spherical ratio, the ratio of acetabular radius to femoral head radius, was 1.04 ± 0.05 for arthritic hips and 1.12 ± 0.06 for dysplastic hips. The center distance was 3.34 ± 1.62 mm for arthritic hips and 2.36 ± 1.42 mm for dysplastic hips. Fifteen of nineteen arthritic hips formed incompatible ball-and-socket joints, while six of nineteen dysplastic joints formed incompatible ball-and-socket joints.

The spherical ratio was found to be statistically significantly different ($p < 0.0001$) between arthritic and dysplastic populations using a two-sample t-test ($\alpha = 0.05$). The center distances in the spherical model of the joints were not found to be statistically significantly different or similar ($p = 0.4181$).

### 4.3 General Ellipsoid Fitting

General ellipsoid fitting was completed as a less constrained geometric model of the acetabulum and femoral head.

Each ellipsoidal shape was characterized by analyzing ellipsoid eccentricity. Major eccentricity was defined as the ratio of the major axis to the minor axis; minor
eccentricity was defined as the ratio of the semi-major axis to the minor axis; thus all eccentricities are greater than (or equal to) 1. Histograms of these eccentricities are shown in Figure 4.1 and Figure 4.2.

Figure 4.1: Histogram of major eccentricities for arthritic (top) and dysplastic (bottom) acetabulums and femoral heads. Note that for dysplastics, one acetabulum (of eccentricity 2.57) was omitted so that the remaining values could be presented on the same scale as arthritics.

Ellipsoid eccentricity was used as a measure of asphericity by comparing to a hypothetical value of 1 using a one-sample t-test at $\alpha = 0.05$ (as for a sphere, all primary axes are of equal length). For arthritic acetabulums, major and minor eccentricities were $1.20 \pm 0.08$ and $1.06 \pm 0.03$, respectively; arthritic femoral heads had major and minor eccentricities of $1.11 \pm 0.06$ and $1.03 \pm 0.03$. For dysplastic acetabulums, major and minor eccentricities were $1.35 \pm 0.36$ and $1.09 \pm 0.07$; dysplastic femoral heads had major and minor eccentricities of $1.13 \pm 0.06$ and $1.08 \pm 0.04$, respectively.
CHAPTER 4. RESULTS

Figure 4.2: Histogram of minor eccentricities for arthritic (top) and dysplastic (bottom) acetabulums and femoral heads.

Table 4.3: Adjusted least-squares general ellipsoid fitting results, showing the mean and standard deviation for major and minor eccentricity in arthritic and dysplastic hip joints.

<table>
<thead>
<tr>
<th></th>
<th>Arthritic Acetabulum</th>
<th>Arthritic Femoral Head</th>
<th>Dysplastic Acetabulum</th>
<th>Dysplastic Femoral Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major Ecc.</td>
<td>1.20 ± 0.08</td>
<td>1.11 ± 0.06</td>
<td>1.35 ± 0.36</td>
<td>1.13 ± 0.06</td>
</tr>
<tr>
<td>Minor Ecc.</td>
<td>1.06 ± 0.03</td>
<td>1.03 ± 0.03</td>
<td>1.09 ± 0.07</td>
<td>1.08 ± 0.04</td>
</tr>
</tbody>
</table>

Ellipsoid eccentricity results are summarized in Table 4.3. Note that a minor eccentricity of 1 (as is nearly the case with arthritic femoral heads) indicates a prolate ellipsoid, with the two shortest axes of equal length.
Table 4.4: Relationships between ellipsoid major eccentricities are tested using two-sample t-tests at $\alpha = 0.05$.

<table>
<thead>
<tr>
<th></th>
<th>Arthritic</th>
<th>Dysplastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arthritic</td>
<td>-</td>
<td>$p &lt; 0.01$</td>
</tr>
<tr>
<td>Fem. Head</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dysplastic</td>
<td>Acetab.</td>
<td>-</td>
</tr>
<tr>
<td>Fem. Head</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 4.3.1 Trends in Asphericity

Ellipsoid major eccentricities were also compared with two-sample t-tests to look for any relationships intra-anatomically and intra-pathologically. A matrix of this cross-referencing is seen in Table 4.4. None of the populations (arthritic or dysplastic, acetabulums or femoral heads) had shapes related to another population in this test pool.

Scatter plots were also used to look for links within populations in a paired sense. Figure 4.3 and Figure 4.4 show scatter plots of major vs minor eccentricity for arthritic and dysplastic hips, respectively. A trend in these scatter plots would suggest a relationship between the two eccentricities, and thus would imply the anatomy is not a general ellipsoid but rather some subset. No such trend appears to exist for either anatomical feature in either pathology.

Figure 4.5 and Figure 4.6 compare acetabular and femoral eccentricities for, respectively, arthritic and dysplastic hips. A trend in these plots would suggest a consistent relationship in the geometry of matching femoral head - acetabulum pairs.
Figure 4.3: Scatter plot of major and minor eccentricities for arthritic acetabulums and femoral heads. A trend in this plot would suggest the anatomy falls into some subset of the general ellipsoid shape.

Figure 4.4: Scatter plot of major and minor eccentricities for dysplastic acetabulums and femoral heads. Again, a trend in this plot would suggest the anatomy falls into some subset of the general ellipsoid shape.
Figure 4.5: Scatter plot of matching arthritic acetabular and femoral head eccentricities for major and minor eccentricity. A trend in this plot would suggest a predictive relationship between the bones of a particular hip joint.

Figure 4.6: Scatter plot of matching dysplastic acetabular and femoral head eccentricities for major and minor eccentricity. Again, a trend in this plot would suggest a predictive relationship between the bones of a particular hip joint.
There are no apparent trends for either arthritic or dysplastic populations.
Chapter 5

Discussion and Conclusions

This chapter presents an interpretation of the results from Chapter 4, plus a discussion of the strengths and weaknesses of the methods of this study. It also summarizes the findings and outline potential future work in the vein of this study.

The primary objective of this study was to examine whether the ball-and-socket model of the hip is appropriate, by assessing the sphericity of the bony articular surfaces; this will be discussed in detail in Section 5.1. Confounding and limiting factors affecting accuracy and the ability to detect trends in the ellipsoid parameters are discussed in Section 5.2. The results and discussion are summarized in Section 5.3. Ideas for expanding the study for future work are discussed in Section 5.4.

5.1 Physical Significance of Results

Physical interpretations of the numerical results can be characterized by the numerical error of the fits, implications of a spherical model of the joint, and implications of an ellipsoidal model of the joint.
5.1.1 Fitting Error

The lower fitting errors for ellipsoids were statistically significantly lower for both the arthritic and dysplastic populations; in contrast to the significance, the absolute magnitude of the variation was very low (approximately 0.2 mm for arthritics and 0.3 mm for dysplastics; on the order of 0.3 – 0.5 pixels).

For the early-arthritic population, this difference in fit quality does not seem to be of practical significance. That is, there does not seem to be a strong numerical argument that aspherical ellipsoid fits are better than simple spherical fits.

For the dysplastic population, the ellipsoids provided a statistically and practically superior fit. Thus, overall, ellipsoids provided at least as good – and, for dysplastics, distinctly better – model when compared to spheres as surfaces of fit.

5.1.2 Spherical Fitting

Spherical fitting was done to validate the prevalent ball-and-socket model. A mechanical ball-and-socket joint has a congruent sphere-on-sphere contact, yielding pure rotation about an isocenter (the common center of both spheres). This leads to two geometric constraints on the resulting spherical fits: the acetabular sphere must at least as large as the femoral sphere, and the centers of the spheres must be aligned. If a less strict model is applied, allowing for some relative translation in the joint, a different geometric constraint arises: to avoid interference between the two spheres, the radius of the acetabular sphere must be greater than the sum of the femoral head spherical radius and the center-center distance between the two spheres of best fit. Table 4.2, above, gives the numerical results of these analyses.

In conjunction with comparing the fitting error between ellipsoidal and spherical
models, these results bring the conventional ball-and-socket model into question. For arthritic populations, ellipsoidal and spherical geometries fit the bony articular surfaces comparably well; however, the spherical fitting only resulted in a compatible ball-and-socket joint in four of nineteen cases. Although the dysplastic hips had a total of thirteen potentially compatible ball-and-socket joints, their ellipsoid fit quality was clearly statistically significantly better, limiting the applicability of conclusions derived from the spherical fit results. Taken together, it appears questionable that a mechanical ball-and-socket joint is an adequate geometrical (and thus kinematic) model of the anatomy for either the arthritic or dysplastic populations included in this study.

5.1.3 Ellipsoid Fitting

Every ellipsoid fit was highly aspherical; for each pathology, acetabulums were overall more aspherical than femoral heads, and dysplastics were overall more aspherical than arthritics.

Major and minor ellipsoid eccentricities were compared using scatter plots in Figure 4.3 and Figure 4.4; neither plot suggested any relationship between major and minor eccentricity for the acetabulum or femoral head. However, when examining the eccentricity of femoral heads – particularly those of arthritics – minor eccentricity was found to be centered and distributed very close to $E_L = 1$ for a large number of cases. This finding, summarized in Table 4.3, suggests that the arthritic femoral head may be modeled as a portion of a rotational ellipsoid, rather than as a more general ellipsoid.

Trends were also sought in comparing major eccentricity both intra-pathologically
and inter-pathologically, using two-sample t-tests with $\alpha = 0.05$. These results are presented in Table 4.4. In most cases, the populations were statistically significantly different; this was not the case for intra-pathological comparisons of acetabular and femoral head shape. However, with $p = 0.10$ and $p = 0.19$ respectively, it is possible that anatomically meaningful correlations may appear if larger populations are sampled.

Femoral and acetabular eccentricities were also compared in a paired sense, that is, examination of the major and minor eccentricities from a matching femur-acetabulum pair was performed. A trend in this plot would suggest a relationship between matching femur-acetabulum pairs; phrased another way, it would imply an ability to predict the geometry of one shape knowing that of the paired shape.

No such relationship was apparent in the populations included in this study. This lack of a consistent geometrical relationship is consistent with the notion that the aspherical model of the hip has variable, non-congruent interacting geometry. As concluded in the discussion of the spherical fitting, the evidence given by the ellipsoid fitting also draws into question the implied geometry and kinematics of the mechanical ball-and-socket model; because the geometry did not provide the requisite constraint, it follows that the kinematics of these subject may have been more complex than the idealized isocentric rotation.

It is difficult to define a compatible ellipsoidal joint that analogous to the definition of a compatible spherical joint. This is because, from fundamental geometry, an ellipsoidal joint cannot be conforming; if it were to be conforming then it would either be immobile or it would function as a hinge, in the case of a rotational ellipsoid. Furthermore, because of the non-conforming geometry, it was not easy to define an
area of coverage of the femoral ellipsoid by the acetabular ellipsoid; this alternative way of establishing whether an ellipsoidal joint is compatible or not seems fraught with difficulties.

5.2 Confounding Factors

The analyses presented in this study did not account for a number of confounding factors that may have adversely affected the results. This is of particular note because of the lack of apparent trends; it may be that the ellipsoid fitting parameters were affected by factors not considered in the analysis.

5.2.1 Patient Selection

There were a number of variables in patient selection that were not controlled in this study, for practical reasons. Each subject was a patient at Kingston General Hospital who had either a hip resurfacing or a periacetabular osteotomy, with CT scans routinely acquired as part of the preoperative care. These patients were included in this study based on the surgical procedure, which gave an implied diagnosis. However, there is known to be gross skeletal variation due to the race, age, or sex [34, 1]. Forensic anthropologists use these skeletal variations to identify human remains – for example, there are well known anatomical variances in the skull that depend on race, and there are pronounced differences in the overall pelvic shape which can be used to determine sex [35]. Further, there is a correlation between height, weight and sex, building on the confounding effect of this variable. Specific to the shape of the hip, the literature is unclear on these trends, with some supporting hip sphericity trending with age [20] or sex [24] but others not supporting these conclusions [23, 24, 21]. It
is reasonable to conclude that gross skeletal variations across different demographics may also be found the anatomy of the hip joint and thus confound results, especially in defining the shape of the bony articular surfaces using ellipsoids.

Other factors that may have affected the geometry of the joint may be the activity level and medical history of each patient. It is well understood that, especially early in life, bone will remodel in response to applied loads. This is seen at both the microstructural level where Wolff’s Law describes cancellous bone as aligning its structure to best support the applied loads, and at a macroscopic scale in practices such as orthodontics, where torques and forces applied to the teeth cause the sockets in the underlying bones of the jaw to remodel [36]. The hip joints of each patient may have been affected by the level and nature of physical activity in the patients’ lifestyle. It is also possible that the medical history of each patient may have had an effect. Any factors that might cause interference in bone growth or development may have affected the final shape of the joint.

5.2.2 Segmentation Accuracy

Because segmentation is a manual process, there is inherently some intra-observer or inter-observer variability in segmentations. Although previous work in the laboratory at Queen’s University [32] suggested that this error was sub-millimeter (i.e. on the order of individual voxels), this magnitude of error may have been up to approximately 5% of the spherical radius of the femoral head. Also, segmentations were done by a variety of technicians; some of these personnel were less experienced and may have been not only less accurate, but also differently biased, producing unknown variation in the results.
During segmentation, especially in the case of arthritic joints, the margin between femoral head and acetabulum can be extremely difficult to identify; an example of such a CT slice is in Figure 5.1. In cases such as this, the segmenters used their anatomical background knowledge to defined an assumed border in each slice; because the study of segmentation accuracy (such as in [32]) was beyond the scope of this thesis, the consequences for accuracy are unknown.

Figure 5.1: Example CT slice of an arthritic hip demonstrating the difficulty in defining the margin between the femoral head and acetabulum. Especially without context (for example, in the isolated view in the yellow box), the margin is nearly impossible to distinguish; a best guess must be made based on the segmenter’s experience and knowledge of anatomy.

5.2.3 Articular Surface Isolation

The 3D models produced by segmentation were further processed to isolate the bony articular surface, using the procedure described in Section 3.2.2. As described, the
choice of margin and excluding features was done manually, based on the observer’s best judgement. Thus, it could not be verified that this manually selected area was truly articular within the joint. This step was performed by only one observer, so there was no inter-observer variability; however, the intra-observer repeatability was not quantified, and the sensitivity of the ellipsoid fitting algorithm to variations in the chosen bony articular surface was not studied. It is possible that these factors may have introduced artificial variation in eccentricity that does not reflect the anatomy.

5.3 Summary of Findings

Whether a spherical or a general ellipsoid was fit to either an arthritic and dysplastic hip, the same conclusion could be reached: these populations had aspherical hips that were not adequately described by the mechanical ball-and-socket model.

Although significantly better in a statistical sense, the fitting error of a general ellipsoid was not notably better in a physical sense; however, upon examining the spherical fitting, in a majority of cases joint surfaces did not form a valid (non-interfering) mechanical ball-and-socket joint. It should also be noted that the ellipsoid fitting results were highly aspherical. Assuming the rigid shape of the subchondral bone drives the joint kinematics, these results taken together suggest that the morphology and kinematics of the hip are more complex than the pure rotational motion of the prevalent ball-and-socket model. These kinematics may be further altered by the cartilage and synovial fluid interposed in the joint, as well as contributions of soft tissues external to the joint.

Investigation into the relationships within joints and between pathologies yielded only one clear trend: in these populations, arthritic femoral heads were approximately
prolate ellipsoids, i.e., they had two shorter axes of approximately equal length. The arthritic acetabulums and dysplastic acetabulums, and the dysplastic femoral heads, showed no trend in shape; this suggests that each joint was better modeled as a general ellipsoid, rather than being part of a pathology-defined subset. There was also no correlation between the shapes of matching acetabulum and femoral head pairs; this suggests that the interacting articular geometry varied from hip to hip in both pathologies.

Overall, it is difficult to generalize any of the results of the ellipsoid fitting. It can, however, be concluded that both the arthritic and the dysplastic hip joint appears to be geometrically more complex than the ball-and-socket model, and that there was high variability between patients, both within the pathology and between pathologies. This suggests that each hip may be both geometrically and kinematically unique.

5.4 Future Work

These results, when combined with the rather limited extant literature, provide impetus to further explore the biomechanics of the hip joint with the goal of establishing a more accurate biomechanical model\(^1\). The work presented here could be extended in a number of ways, including: expanding on the general ellipsoid fitting, incorporating the fitting of other shapes, including imaging from other modalities as source data, and expanding to passive kinematics, active kinematics and kinetics.

\(^1\)Alternatively, it might be possible to demonstrate that non-articular constraints provide sufficient limitations to the motion that the ball-and-socket model is approximately correct.
5.4.1 Expanding on General Ellipsoid Fitting

Because the results of this study were limited to relatively small populations ($n = 19$) and two specific hip pathologies, increasing the subject pool may help to clarify results. As discussed in Section 5.2.1, there were a number of potential confounding factors that may have affected the shape fitting results; prominent among these factors was that trends were observed in the correlations of matching joint surfaces but these trends were not statistically significant. It is entirely possible that larger sample sizes may allow for better detection of these factors.

There is a clear opportunity to compare the results found in this study with shapes of “normal” or “healthy” hips, as well as with other hip pathologies. A so-called “normal” hip population may turn out to be difficult to establish. For example, a young patient may be predisposed to some sort of hip pathology in later life, with joint changes simply not having had time to take effect.

A combination of increased sample size, both for adequate control of confounding factors and statistical significance, and an expansion of the examined pathology has the potential to lead to a shape-based classification of hip disease. This classification might be useful within a given pathology, between pathologies, or over time for an individual subject.

There has also not yet been a formal evaluation of segmentation sensitivity, both in terms of the initial segmentation and the bony articular surface isolation. As such, it is unclear whether this type of shape fitting requires careful hand segmentation as was done for this study, or whether some form of automated segmentation would produce an adequate result. Because segmentation was by far the most time-consuming
part of this study, partially or fully automating the segmentation would be highly advantageous. It is possible that an intra-observer and inter-observer sensitivity analysis would contribute to better quantifying the precision of the methods used in this study.

5.4.2 Fitting to Other Geometries

This study fit the bony articular surfaces to spheres and general ellipsoids. Although general ellipsoids are advantageous in that they encompass spheres, plus prolate and oblate rotational ellipsoids, there are an infinite number of other shapes that could be fit to the femoral head and acetabulum. Drawing from previous work, rotational ellipsoids could be fit explicitly, as was done in the work of Xi et al. [28] and Gu et al. [29]; joint space in the CT images could be quantified to follow the work of Greenwald and O’Connor [22], Afoke et al. [23] and Lequesne et al. [24]; or the anatomy could be fit to rotational conchoids as done by Menschik [27]. Shape fitting that has been described by previous work would not only allow for a verification of the present results, but the literature would then serve as a reference for comparison in developing classifications of the human hip.

Introducing new geometry for shape fitting of the hip may more accurately represent the geometry of bony articulating surfaces. Rotational shapes, including rotational polar shapes, may be of use if rotational symmetry is found; Fourier techniques, principal component techniques, or other statistical shape modeling may allow for more general shape fitting. Further, surface tangents or normals could be examined instead of surface points explicitly.
5.4.3 Use of Other Imaging Modalities

This study exclusively used CT scans taken preoperatively for surgical procedures performed at Kingston General Hospital. These scans were medically necessary, so the patients received no additional radiation exposure as part of their inclusion in this study. However, 3D images of the hip could be captured using non-ionizing imaging modalities, especially MRI and potentially ultrasound. It is also possible that CT scan settings might be adjusted to minimize the radiation dose of the scan to limit each patient’s exposure. A study involving multiple imaging modalities could to quantify the similarities and differences in the resulting 3D images under different modalities, and different settings within a modality, with such flexibility in imaging potentially easing the collection of data in the future.

5.4.4 Kinematics and Kinetics

The shape-fitting results of this study can be a precursor to an investigation of hip kinematics, and more generally, hip biomechanics. Because these results provide reason to question the accepted model, a direct investigation of the joint kinematics would be a logical next step. While a rigid aspherical interacting geometry of the hip does not provide sufficient kinematic constraint to produce a purely rotational motion, it is possible that this constraint is provided by other anatomy (e.g. soft tissue constraints). Directly quantifying how much, if any, translation occurs in normal hip motion would thus be a natural progression in clarifying the biomechanical model of the hip.

Hip-joint kinematics can be recorded, passively, for either cadaver specimens or orthopaedic patients during surgery. Direct measurement of the bone motion is more
accurate than detection based on skin markers, with direct measurement probably being preferable for initial work because joint translations may turn out to be relatively modest in magnitude. The relative position of the bones can be established during a range of motion, specifically to determine whether there is femoral head translation, and more generally to search for kinematic patterns in the motion of the joint. In particular, it would be of interest to determine when – if at all – the motion of the hip can be approximated as a pure rotation, and what ranges or motion produce high translation. Translation data may be then related to soft tissue contributions, such as muscle and ligament actions. The effect of a compressive force, emulating active motions, could also be of interest. A growing understanding the link between soft and rigid anatomical structures and resulting motion of the joint could prove valuable for planning orthopaedic interventions to improve the quality of life of a patient.

Because the hip is most commonly modeled as a ball-and-socket joint, further study of the consequences of hip translations in gait analysis would be of value. It is possible that joint forces and moments are sensitive to hip translation; if so, either direct measurement of hip translations or a well-established connection between the joint orientation and predicted translation would be useful in accounting for non-rotational contributions to motion. The hip may prove difficult to track using skin markers, so new techniques or protocols may need to be developed to account for skin-motion artifacts in tracking the hip during gait.

Joint motion can also be investigated in an active kinematic sense with cadaver patients. By physically actuating the muscles or tendons that cross the joint, more biologically accurate motions can be produced. These kinematics can similarly be examined for translation and kinematic patterns.
Knowledge of the kinematics and kinetics may prove valuable in surgical planning and development of prostheses. As the understanding of the joint evolves, the model of “ideal kinematics” may evolve alongside it; it is possible that a prosthetic implant that allows certain translations may be desirable, or that kinematic goals could lead to certain placements of prosthetic implants, for example.

5.5 Conclusions

Since the introduction of the spherical model of the hip by Hammond and Charnley in 1967 [8], the hip has been prevalently modeled as a ball-and-socket joint that has a conforming sphere-on-sphere joint geometry. There has, however, been a small but persistent thread of authors observing asphericity and nonconformity of the joint, starting as early as 1968 with Bullough et al. [20]. Authors have quantified these observations by examining the sphericity of cross-sections of the hip anatomy [21, 26, 25], by examining incongruity within the joint [23, 24], and by shape fitting articular surfaces to rotational conchoids [27] and rotational ellipsoids [28, 29]. Although these analyses were not explicitly replicated in this study, the aspherical nature described by the general ellipsoid fitting results is consistent with each of these articles in describing a non-congruent, aspherical hip. This study compared the shapes of two pathologies, which was not considered in the cited literature. This study also examined both the femoral head and the acetabulum, giving more extensive results than did studies that only examined the femoral head [27]. This study analyzed CT images of living patients, which was advantageous both in that it was a 3D analysis (improving on the published 2D studies [20, 21, 24, 26, 25]) and in that it was not restricted to cadaveric specimens. Finally, the general ellipsoid geometry used in this study is
the least constrained geometrical model of the cited literature, in particular by not requiring an axis of symmetry (as must any rotational shape fitting [27, 28, 29]).

Because each of the 38 hips included in this study was best modeled by a non-congruent and aspherical articular geometry, this study suggests that each hip was a uniquely shaped and kinematically aspherical joint. This result gives motivation to look at a wider range of patient pathologies to determine if this result is found outside of early arthritic and dysplastic patients. It also gives motivation to directly measure the kinematics of the hip, with the expectation that there may be small translations during its motion.
References


Appendix A

Glossary

For more detail about anatomical structure, a brief overview of the bony anatomy of the hip bone and proximal femur is provided in Appendix 2.1.

**Acetabular notch** A depression in the inferior aspect of the acetabulum, where a fat pad sits; the labrum traverses this gap as an unsupported ligament.

**Acetabulum** A rounded depression in the hip in which the femoral head articulates. The acetabulum forms the ‘socket’ half of the hip joint.

**Anterior** Towards the front of the body. For example, the sternum is anterior of the spine. Antonym: Posterior.

**Arthritis** Literally, inflammation of a joint. See Section 2.1.1.

**Articular surface** General term for the part of the joint which slides, rolls, or otherwise interacts with another component of the joint. In the case of the hip, the articular surfaces are the femoral head and acetabulum.

**Anterior superior iliac spine (ASIS)** Two bony landmarks on the anterior surface of the iliac crest. These landmarks can be physically felt on most patients,
slightly below the navel and near the edge of the body.

**Bilateral** On both sides - as opposed to just left or right.

**Cancellous (or ‘spongy’) bone** A bone structure that forms the interior of the femoral head (as well as the interior of other bones). The structure resembles that of a sponge, with thin, short bony ‘struts’ that align with the primary loads experienced by the bone.

**Cortical bone** The hard, rigid bone that forms the ‘shell’ of long bones. It is of uniform, solid structure.

**Computed Tomography (CT) Scan** A form of 3D imaging that uses X-rays to produce a set of 2D cross-sectional images.

**Distal** Located further to the central body mass. For example, the ankle is distal of the knee. Antonym: Proximal.

**Dysplasia** Developmental Dysplasia of the Hip (DDH) is a condition where the acetabulum provides insufficient coverage of the femoral head. See Section 2.1.1.

**Ellipsoid** A geometric shape created by scaling a sphere independently in three orthogonal directions.

**Femoral Head** A rounded structure that sits on the femoral neck and articulates as the distal component of the hip joint.

**Femoral Neck** A bony structure that extends medially from the femur to support the femoral head.
**Fovea** A depression located medially in the femoral head. This provides a non-articular attachment point for a ligament that runs to the acetabulum.

**Hounsfield Unit** Unit of measuring X-ray attenuation in a CT image. This provides the primary information used to perform a segmentation (as well as in reading CT scans).

**Inferior** Towards the bottom of the body. For example, the chin is inferior of the nose. Antonym: Superior.

**Major eccentricity** For an ellipsoid, the ratio of the major (longest) axis of the ellipsoid to its minor (shortest) axis.

**Medial** Towards the centerline of the body. For example, the eyes are medial of the ears. Antonym: Lateral.

**Minor eccentricity** For an ellipsoid, the ratio of the semi-major (second-longest) axis of the ellipsoid to its minor (shortest) axis.

**Lateral** Further from the centerline of the body. For example, the ears are lateral of the eyes. Antonym: Medial.

**Ossification** A process by which a non-bony structure changes into bony tissue. For the hip, this may be seen in the ligamentous structure (the labrum or the capsule).

**Osteoarthritis** Inflammation of bony tissues in a joint. See Section 2.1.1.

**Pathology** Term for the type of disease or illness.
**Posterior** Towards the back of the body. For example, the spine is posterior of the sternum. Antonym: Anterior.

**Proximal** Located closer to the central body mass. For example, the knee is proximal of the ankle. Antonym: Distal.

**Revision** A surgical procedure to correct or repair a previous procedure. For example, if a prosthetic breaks or loosens, it may be replaced with a similar component or refixated; these would both be revisions.

**Segmentation** Procedure for creating a 3D model from a set of 3D images. See Section 3.2.

**Subchondral bone** Bone located beneath a cartilage layer. Both the femoral head and acetabulum have a cartilage layer covering the articular surfaces, supported by comparatively rigid subchondral bone.

**Superior** Towards the top of the body. For example, the nose is superior of the chin. Antonym: Inferior.
Appendix B

Supplementary Information

These appendices provide some supplementary information intended to situate a reader with purely mechanical engineering background into the context of hip biomechanics. Appendix B.1 describes the types of bony tissue that comprise the femur and pelvis. Appendix B.2 describes the primary motion of the hip. Appendix A gives a glossary of terms which may be unfamiliar to the mechanical engineering field.

B.1 Types of bone tissue

Generally speaking, there are two types of bone found in the bones of the hip joint: cortical and cancellous bone. Cortical bone is a dense, solid bony material. Cancellous (also called “trabecular” or “spongy”) bone occurs as a sponge-like or truss-like structure, with the bone forming in narrow struts called trabeculae. According to Wolff’s law, the trabeculae are organized such that they best support the primary loading of the bone. Generally speaking, cortical bone forms the outer shell of a bone. A bone may be solid, hollow, or have cancellous bone in its interior.

A cross-sectional view of the proximal femur is given in Figure B.1. Both cortical and cancellous bone can be seen.
The bony anatomy of the femur and pelvis is described in Section 2.1.

**B.2 Primary motions of the hip**

The most prominent motion of the hip during walking (and most activities of daily living) is the flexion-extension motion, which occurs about a medial-lateral axis. Flexion orients the thigh forward in a lateral plane (for example, when one initiates a step); extension orients the thigh backwards in the lateral plane (for example, right before a leg breaks contact with the ground during walking). Photographs of model bones of the hip in these positions are shown in Figure B.2.

Another primary motion of the hip is the adduction-abduction motion, which
Figure B.2: Flexion (left pane) and extension (right pane) motions of the hip, relative to a neutral position (center pane). The hip is viewed from the left side.

Figure B.3: Adduction (left pane) and abduction (right pane) motions of the hip, relative to a neutral position (center pane). The hip is viewed from the front (i.e., along an anterior-posterior axis.)

occurs about an anterior-posterior axis. Adduction brings the knees closer together in a frontal plane; abduction spreads them apart (for example, as when adopting a wide stance). Photographs of model bones of the hip in these orientations are shown in Figure B.3.

The final primary motion of the hip is the internal rotation-external rotation motion, which occurs about a superior-inferior axis. If the knee and ankle do not rotate, internal rotation would bring the toes closer together; external rotation moves
Figure B.4: Internal rotation (left pane) and external rotation (right pane) motions of the hip, relative to a neutral position (center pane). The hip is viewed from the front (i.e., along an anterior-posterior axis.)

them apart (into a “duck-footed” stance). Photographs of model bones of the hip in these orientations are shown in Figure B.4.
Appendix C

Raw Data

The tables below (Tables C.1, C.2, C.3, C.4 present a subset of the data available on the joints included in this study. Spherical parameters include spherical radius $r$ and spherical fitting error $e$, both in $mm$. Ellipsoid fitting parameters are the three orthogonal scaling parameters $s_{11}$, $s_{22}$, and $s_{33}$; ellipsoid fitting error $e$; major eccentricity $E_U$ and minor eccentricity $E_L$. Major and minor eccentricity are given as:

\[
E_U = \frac{s_{11}}{s_{33}} \\
E_L = \frac{s_{22}}{s_{33}}
\]

Sphere and ellipsoid fitting errors are plotted in histograms in Figure C.1.
Table C.1: Raw Data for Osteoarthritic Acetabulums

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Table C.2: Raw Data for Osteoarthritic Femoral Heads

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## APPENDIX C. RAW DATA

Table C.3: Raw Data for Dysplastic Acetabulums

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Table C.4: Raw Data for Dysplastic Femoral Heads

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<th>Spherical Fitting</th>
<th>Ellipsoid Fitting</th>
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<td>$e$ (mm)</td>
<td>$s_{11}$ (mm)</td>
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Figure C.1: Histograms comparing the fitting error of ellipsoids and spheres to the bony articular hip surfaces.