GAS EJECTION FROM
SPIRAL GALAXY DISKS

by

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Abstract

We present the results of three proposed mechanisms for ejection of gas from a spiral arm into the halo. The mechanisms were modelled using magnetohydrodynamics (MHD) as a theoretical template. Each mechanism was run through simulations using a Fortran code: ZEUS-3D, an MHD equation solver. The first mechanism modelled the gas dynamics with a modified Hartmann flow which describes the fluid flow between two parallel plates. We initialized the problem based on observation of lagging halos; that is, that the rotational velocity falls to a zero at some height above the plane of the disk. When adopting a density profile which takes into account the various warm and cold H$_I$ and H$_{II}$ molecular clouds, the system evolves very strangely and does not reproduce the steady velocity gradient observed in edge-on galaxies. This density profile, adopted from Martos and Cox [1], was used in the remaining models. However, when treating a system with a uniform density profile, a stable simulation can result. Next we considered supernova (SN) blasts as a possible mechanism for gas ejection. While a single SN was shown to be insufficient to promote vertical gas structures from the disk, multiple SN explosions proved to be enough to promote gas ejection from the disk. In these simulations, gas ejected to a height of 0.5 kpc at a velocity of 130 km s$^{-1}$ from 500 supernovae, extending to an approximate maximum height of 1 kpc at a velocity of $6.7 \times 10^3$ km s$^{-1}$ from 1500 supernovae after 0.15 Myr, the approximate time of propagation of a supernova shock wave. Finally, we simulated gas flowing into the spiral arm at such a speed to promote a jump in the
disk gas, termed a hydraulic jump. The height of the jump was found to be slightly less than a kiloparsec with a flow velocity of $41 \text{ km s}^{-1}$ into the halo after 167 Myr.

The latter models proved to be effective mechanisms through which gas is ejected from the disk whereas the Hartmann flow (or toy model) mechanism remains unclear as the heliocentric velocity profile becomes unstable when run through a time-dependent simulation. Though the cause of this instability is unclear, pressure fluctuations in the system are suspected to play a role.
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Chapter 1

Introduction

The gas dynamics between galactic disks and halos has been observed and analysed in many studies. In particular, it is well known that there is an inflow/outflow of matter between the galactic disk and the halo as gas, dust, and heavy elements have been observed in the halos of spiral galaxies. The reasons behind this behaviour, however, remain unclear.

Numerous theoretical approaches have been made to predict matter flow between the disk and the halo. In order to properly model this phenomenon, one must first know the dynamical properties of the inflowing/outflowing matter. Further, since galaxies are magnetized gaseous disks, one must also observe the magnetic properties in the disk and halo.

Early observations of edge-on galaxies have revealed radio continuum emission extending to kiloparsec \((1 \text{ kpc} = 3.086 \times 10^{19} \text{ m})\) heights above galaxy disks [19], [20], [21], [22]. Figure 1.1 shows an H\(_{\alpha}\) (optical) image of NGC 5775 overlaid with the total radio intensity contour map. Other edge-on galaxies (e.g. NGC 891) show a similar extensive radio halo. The primary source of the halo radio emission is from synchrotron radiation (from relativistic electrons) which indicates the existence of magnetic fields perpendicular to the galactic plane by its polarization [23]. The
field strength, however, is inferred by the intensity of synchrotron emission whereas the magnetic field direction is inferred by how the galactic medium rotates polarized emission. The magnitude of this rotation is proportional to the rotation measure (RM) whose sign indicates whether the magnetic field component parallel to the line of sight is directed towards the observer (positive RM) or away from the observer (negative RM). Chapter 3 explains rotation measures in greater detail. Halos are now observed in H\textsubscript{\alpha} emission, in X-ray emission, in electronic transitions of CO (carbon monoxide), in the 21 cm H\textsubscript{I} line, and in infrared (IR) emission, all of which are normally observed in the interstellar medium (ISM) in the disk.

![Figure 1.1: Total intensity contour map at 8.35 GHz with polarization observed with the 100-m Effelsberg telescope, superimposed on an H\textsubscript{\alpha} image. The lines represent magnetic field vectors. Taken from Soida et al. [2].](image)

In the above list of emission, H\textsubscript{\alpha} refers to the photon emitted from the relaxation
of the electron in hydrogen from the n=3 to n=2 principle quantum level. This de-excitation produces a photon with a wavelength of 656 nm. This is observed in regions of ionized hydrogen (H\textsubscript{II} regions) and is suspected to be correlated with RM since these regions provide an ionized medium through which plane polarized radiation would be subjected to rotation. H\textsubscript{I}, on the other hand, refers to the much smaller energy photon that is emitted from the relaxation in energy within the hyperfine structure of the neutral hydrogen 1s ground state. Because of magnetic interactions between the central proton and the circulating electron, a slight excess of energy is introduced to a system where the spins of each species is parallel. When this relaxes to a lower energy state (where the spins are anti-parallel), a small energy photon is emitted with a wavelength of 21 cm. This emission is readily observed (with radio telescopes) given the high abundance of neutral hydrogen in the Universe. Figure 1.2 depicts the processes through which H\textsubscript{α} and H\textsubscript{I} emission originate.

![H\textsubscript{α} emission](image)

(a) H\textsubscript{α} emission

![H\textsubscript{I} photon emission](image)

(b) H\textsubscript{I} λ\textsubscript{21 cm} photon emission

**Figure 1.2:** The origin of H\textsubscript{α} and H\textsubscript{I} emission lines.

A study by Tripp et al. [24] observed the ultraviolet interstellar absorption spectra toward distant Galaxy stars (the capital ‘G’ refers to the Milky Way). They concluded that gas not only falls onto the more dense disk region from the halo, but it is also ejected from the disk as well at similar (but opposite) velocities. This is strange do
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...to the high gravitational potential at the disk. These ‘forbidden velocities’ have an extensive range, from +100 km s$^{-1}$ to −100 km s$^{-1}$.

Dynamics in the halo itself are of interest as the heliocentric velocity of the gas has a relatively constant and negative gradient above the disk, eventually falling to a systemic level. The systemic velocity is that of the center of the galaxy. This ‘lagging halo’ has been studied in many edge-on galaxies such as NGC 891 [3], NGC 5775 ([15], [25]), NGC 4302 [25], NGC 4157, NGC 3600, and NGC 2683 [26], but not yet explained. Figure 1.3 shows kinematic results of NGC 891 taken from Kamphuis et al. [3].

![NGC 891 close up by Hubble Space Telescope (HST)](image1)

![NGC 891 halo kinematics](image2)

**Figure 1.3:** The average rotational velocities indicating an increasing ‘lag’ at higher heights above the disk of NGC 891. *(Left)* Image credit: NASA/STScI/WikiSky. *(Right)* Plot taken from Kamphuis et al. [3].

In this report, we explore a simple toy model mechanism using Hartmann flow as a possible explanation for the lagging halo where Hartmann flow describes the fluid flow between two parallel plates. That is, we test whether effects such as viscous and resistive drag are sufficient to reproduce a decrease in heliocentric velocity with...
height.

The disk/halo vicinity is also home to magnetic loops perpendicular to the disk. These loops are predicted by the instability resulting from a vertical perturbation and the consequence of magnetic field lines being frozen into the disk (see Chapter 2). This is known as the Parker instability. Basically, as a magnetic flux tube starts to buckle, gas slides down the field lines towards the disk due to gravity. This creates a buoyancy force where the flux tube buckled, causing it to rise. As it rises, more gas slides down the field lines, creating an even stronger buoyancy force. This situation is illustrated in Figure 1.4.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{magnetic_loops.png}
\caption{Schematic picture showing the formation of magnetic loops. Taken from Machida et al. [4].}
\end{figure}

If you imagine the flux tube depicted above as having some direction, then as the buoyancy force pushes the top of the flux tube to greater heights, the bottom portions of the loop (which have opposing directions; one side is directed up, the other down) are brought closer together. As will be discussed in Chapter 3, this results in magnetic reconnection where the approaching field lines are pinched together and the topology of the field lines is changed. This is a mechanism through which magnetic energy is converted to kinetic energy. A familiar example of magnetic reconnection can be seen in solar flares (Figure 1.5).
Hanasz, Otmianowska-Mazur, and Lesch [27] predicted galactic magnetic loops via 3D resistive magnetohydrodynamic (MHD) modelling. They describe the evolution of a Parker unstable magnetic field under the influence of a Coriolis force and magnetic reconnection which leads to helically twisted magnetic flux tubes which are then agglomerated by reconnection forming a significant poloidal field component, consistent with a fast dynamo process proposed by Parker [28]. Other studies, however, suggest that the magnetic properties of disk and halo field symmetry is a result of strong galactic winds [29], [30]. In general, a mixture of quadrupole-like symmetry and dipole-like symmetry is observed in galactic halos and disks, respectively [31]. A depiction of a dipole magnetic field and anquadrupole electric field is shown in Figure 1.6.
Both the kinematic and magnetic field observations must be taken into account when forming a theory as to disk/halo matter behaviour. One study by Rivest [32] focused on the dust dynamics as a result of radiation pressure from starlight, the purpose of which was to predict the IR properties of galaxy halos since dust emits in the IR. It was found that the heights attained by grains of dust varied between 0.3 kpc and 3 kpc depending on the grain size and type. In general, larger grains (0.3 µm) tend to rise more quickly but then fall back to the disk over time due to their larger mass. Classical grains (0.1 µm) tend to rise and become stable at their new height which can be as high as 3 kpc. Smaller grains, however, tend to stay where they are.

The focus of this report, however, is on MHD models and their implications for gas dynamics in spiral arms. What follows are possible hydrodynamic mechanisms through which gas is ejected into the halo such as describing the gas dynamics by high
mass stars exploding as supernovae, tracking the density and magnetic field strength, and initializing gas flow into the spiral arm so as to promote a combination of a shock and a hydraulic jump. Hydraulic jumps occur when a fluid at high velocity discharges into a zone of lower velocity, resulting in a rather abrupt rise in the liquid surface. This is depicted in Figure 1.7. The latter model has been studied extensively. Martos and Cox [1] presented a 2D simulation as described above. Gomez and Cox [33] published a 3D MHD model of the gaseous structure of the Galaxy and noted density structures resembling a breaking wave pattern occurring with a period of about 60 Myr. Their model, however, was more global than what will be presented in this study and they do not consider the effects of self-gravity. Thus we aim to compare our results with the single-arm study performed by Martos and Cox [1]. These jumps/bores were also present in models of protoplanetary disks, extending to heights of approximately 1 AU (astronomical unit) [34].

![Figure 1.7](image)

**Figure 1.7:** Conditions which promote a hydraulic jump where $Q$ is the flow rate, $B$ is the channel width, $h_1$ and $h_2$ are the upstream and downstream depths, and $v_1$ and $v_2$ are the upstream and downstream velocities, respectively.

Following this introduction is a description of MHD theory and conditions in Chapter 2. A general outline of galactic magnetic fields, their structure and strength, and how they are detected is presented in Chapter 3. The report then focuses on the
mechanisms described above. We employ the fluid flow between two parallel plates (Hartmann flow) to explore the lagging halo puzzle. Setting up a Hartmann flow model (named the ‘toy model’) we analytically solve a simplified example in Chapter 4. The numeric models are simulated using an MHD equation solver: ZEUS-3D [35]. The mechanisms are initialized in Chapter 5 and the results are presented in Chapter 6. Finally, Chapter 7 summarizes the conclusions extracted from the simulations.
Chapter 2

Theory

Space physics is, to a large part, plasma physics. Most astrophysical objects can be described as a plasma: a quasi-neutral ensemble of ions and electrons in a co-moving fluid. Though technically there are two fluids, one of ions and another of electrons, plasmas are often treated as a single fluid. Plasmas resulting from ionization of neutral gases generally contain equal numbers of positive and negative charge carriers. In this situation, the oppositely charged fluids are strongly coupled, and tend to electrically neutralize one another on macroscopic length-scales. Such plasmas are termed quasi-neutral (‘quasi’ because the small deviations from exact neutrality have important dynamical consequences for certain types of plasma mode). As such, the equations describing fluid motion (hydrodynamics) are combined with Maxwell’s equations of electromagnetism to create the theory of magnetohydrodynamics (MHD).

Presented in this chapter are the properties and equations that describe a plasma as well as some physical consequences that arise from these equations. Characters in **bold** font denote vector quantities; that is, quantities with both a magnitude and direction.
2.1 MHD Equations

The equations governing the physics of magnetohydrodynamics will be presented in this section.

We begin with the fundamental definition of a plasma: an ensemble of $N$ particles describing $N$ equations plus Maxwell’s equations for electromagnetism,

$$m_i \frac{dv_i}{dt} = q_i [E(x_i, t) + v_i \times B(x_i, t)]$$  \hspace{1cm} (2.1)

where $v_i(t) = dx_i/dt$ is the velocity vector of the $i^{th}$ particle. The mass, charge, electric and magnetic fields are represented by $m_i$, $q_i$, $E$, and $B$, respectively. This is the equation of motion that describes the $i^{th}$ particle.

$$\nabla \cdot E = \frac{1}{\epsilon_0} \sum_{i=1}^{N} q_i \delta[x - x_i(t)]$$  \hspace{1cm} (2.2)

is Gauss’s law where $\nabla$ is a gradient operator equivalent to $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ in Cartesian coordinates, $\epsilon_0$ is the permittivity of free space, a universal constant, and $\delta$ is the Dirac delta function which is zero everywhere except when the term inside the square brackets equals zero. More formally,

$$\int_{-\infty}^{\infty} \delta(n) \, dn = 1.$$  \hspace{1cm} (2.3)

Gauss’s law describes the relationship between an electric field and the generating electric charges.

$$\nabla \times B = \mu_0 \sum_{i=1}^{N} q_i v_i(t) \delta[x - x_i(t)] + \frac{1}{c^2} \frac{\partial E}{\partial t}$$  \hspace{1cm} (2.4)

is Ampere’s law with Maxwell’s displacement current where $c$ is the speed of light and $\mu_0$ is the permeability of free space, another universal constant. Ampere’s law states that magnetic fields can be generated by an electric current and by a changing
electric field.

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.5) \]

is Faraday’s law of induction which describes how a time varying magnetic field creates an electric field.

\[ \nabla \cdot \mathbf{B} = 0 \quad (2.6) \]

is Gauss’s law for magnetism which implies that magnetic monopoles (charges) do not exist.

Obviously, with \( N \) being a very large number, tracing every particle is not feasible. One solution for reducing the complexity is to integrate over \( N - 1 \) particle variables \( x_i \) and \( v_i \) which leads to a distribution function for particles with zero correlation, \( f_s(x, v, t) \), for species \( s \) of \( n \) total. Basically, we have separated the system into two fluids: one described by the flow of electrons, and the other by the flow of ions, so \( n \) is equal to 2. Thus the Vlasov equation for a collisionless plasma is

\[ \left[ \frac{\partial}{\partial t} + v \cdot \nabla + \frac{q_s}{m_s} (E + v \times B) \cdot \frac{\partial}{\partial v} \right] f_s(x, v, t) = 0. \quad (2.7) \]

The densities and bulk velocities entering the current and charges are determined as the moments of the distribution function, \( f_s \),

\[ n_s = \int d^3v f_s(x, v, t) \quad (2.8) \]

and

\[ n_s v_s = \int d^3v vf_s(x, v, t). \quad (2.9) \]

Equation 2.7 may be used to derive fluid equations for the different particle components by multiplying the Vlasov equation successively by rising powers of the velocity \( v \) and integrating the resulting equation over the entire velocity space. The first
two moment equations are the continuity equation for the particle density and the momentum conservation equation,

\[ \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s v_s) = 0 \]  \hspace{1cm} (2.10)

and

\[ \frac{\partial (n_s v_s)}{\partial t} + \nabla \cdot (n_s v_s v_s) = n_s \frac{q_s}{m_s} (E + v_s \times B) - \frac{1}{m_s} \nabla p_s \]  \hspace{1cm} (2.11)

where the pressure, \( p_s \), is assumed to be isotropic.

Due to quasi-neutrality, the number densities of ions and electrons, \( n_p \) and \( n_e \), respectively, are about equal and thus the charge density, \( \rho_c = q(n_p - n_e) \), is approximately zero where the \( q \) outside of the brackets represents the elementary charge. Since the Universe is primarily composed of hydrogen (to approximately 90% by number), the number density of ions is usually presented as the number density of protons. The mass density, \( \rho \), is thus approximately equal to \( m_p n \), where \( m_p \) is the mass of a proton which is about 2000 times as massive as an electron (\( m_e \)). The current density is given by equation 2.4,

\[ j = q n (v_p - v_e) = \frac{1}{\mu_0} \nabla \times B. \]

Finally, the total pressure is given by the sum of the pressures contributed by the ions and electrons, \( p = p_p + p_e \).

We now move on to the second half of MHD theory - the fluid dynamics. MHD makes use of mainly two fluid equations: the continuity equation,

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]
and the equation of motion (the momentum equation),

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{j} \times \mathbf{B} - \nabla p + \mathbf{F}$$

which are derived from equations 2.10 and 2.11, respectively and where \( \mathbf{F} \) is any external force per unit volume.

The mass conservation equation can be easily obtained by integrating equation 2.10 over a volume \( V \),

$$\frac{dM}{dt} = \int_V \frac{\partial \rho}{\partial t} dV = -\int_V \nabla \cdot (\rho \mathbf{v}) dV = -\int_S (\rho \mathbf{v}) \cdot \mathbf{n} dS. \tag{2.12}$$

This states, not surprisingly, that a mass \( M \) inside a volume \( V \) changes if there is a net mass flux in or out through the surface \( S \).

The momentum conservation equation can be found similarly by integrating equation 2.11 over a volume \( V \),

$$\frac{d\mathbf{P}}{dt} = \int_V \frac{\partial (\rho \mathbf{v})}{\partial t} dV = -\int_S \mathbf{T} \cdot \mathbf{n} dS + \int_V \mathbf{F} dV, \tag{2.13}$$

where \( \mathbf{T} \) is a dyad which can be thought of as an extension of a vector and is described by [36],

$$\mathbf{T} = \rho \mathbf{v} \mathbf{v} + \left( p + \frac{B^2}{2\mu_0} \right) \mathbf{I} - \frac{\mathbf{BB}}{\mu_0}$$

where \( \mathbf{I} \) acts as an identity matrix. Thus the total momentum \( \mathbf{P} \) inside a volume \( V \) changes due to stresses on the boundary and external forces.

With the exception of the energy equation, our arsenal of equations is nearly complete. Before considering the energy equation, however, it is helpful to contrast the ideal and non-ideal forms of some familiar laws.
The first equation for analysis is Ohm’s law, which can be written as

\[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{R} \]  

(2.14)
in a moving medium. The vector \( \mathbf{R} \) represents different forms of Ohm’s law: in the ideal case, \( \mathbf{R} = 0 \). However in the resistive case, \( \mathbf{R} = \eta \mathbf{j} \) where \( \eta \) is the resistivity which can be thought of as representing ion/electron friction. It is usually treated as a constant for simplicity. Other forms include the electron pressure, the Hall term, inertial terms, etc. None of the latter cases will be considered or discussed in what follows as they are not relevant to astrophysical plasmas.

The induction equation (equation 2.5) can be added to Ohm’s law to give

\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \mathbf{R}) \]  

(2.15)

where \( \mathbf{R} \) is the same as before. Interestingly, the resistive form of the induction equation can be expanded, and noting that the divergence of \( \mathbf{B} \) (\( \nabla \cdot \mathbf{B} \)) is equal to zero,

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \Delta \mathbf{B}. \]  

(2.16)

Here we have used the definition of the current density (equation 2.1) in the resistive induction equation. The operator \( \Delta \) is the Laplacian and is equivalent to the divergence of the gradient.

Also in the dimensionless form of equation 2.16 is the magnetic Reynolds number, \( R_m \). This is achieved by setting \( \mathbf{B} = B_0 \hat{\mathbf{B}} \), \( \mathbf{v} = v_0 \hat{\mathbf{v}} \), and \( t = t_0 \hat{t} \). The result (using the fact that \( t_0 = L_0/v_0 \)) is

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\hat{\mathbf{v}} \times \hat{\mathbf{B}}) + \frac{1}{R_m} \Delta \hat{\mathbf{B}}. \]  

(2.17)
where
\[ R_m = \frac{\mu_0 L_0 v_0}{\eta}. \quad (2.18) \]

Usually \( R_m \gg 1 \) for astronomical purposes.

The last consideration is that of energy. The energy equation can be written in different forms depending on the thermodynamic variables used, however it is more commonly presented by an equation of state for an ideal gas and by internal energy
\[ e = \frac{p}{(\gamma - 1)p}, \quad (2.19) \]

where \( \gamma \) is the ratio of specific heats and \( p = k \rho \gamma \) where \( k \) is a constant. For resistive MHD, the energy equation is given as
\[ \frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = (\gamma - 1) \eta |\mathbf{j}|^2. \quad (2.20) \]

The right hand side of equation 2.20 represents Ohmic heating and is equal to zero in the ideal case. These equations, however, are not presented in conservative form. That is, a form in which the overall change to energy is zero. This can be attained, however, by multiplying the momentum equation (equation 2.13) by \( \mathbf{v} \) and combining it with the energy equation. The result is
\[ \frac{\partial}{\partial t} \left( \frac{1}{2} \rho v^2 + \rho e + \frac{B^2}{2\mu_0} \right) + \nabla \cdot \left[ \frac{\rho v^2}{2} \mathbf{v} + H \mathbf{v} + \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \right] = 0 \quad (2.21) \]

for both ideal and resistive MHD where the term \( H \) is the enthalpy and is a thermodynamic quantity equivalent to the total heat content of a system,
\[ H = \rho e + p. \quad (2.22) \]
Lastly, we note that magnetic torsion is propagated by Alfvén waves at a speed

\[ v_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}} \]  

(2.23)

where \( v_A \) is the Alfvén speed.

### 2.2 MHD Conditions

The description of magnetic fields in galaxies is based on MHD principles. The system, for the sake of MHD implementation, is treated as a charged plasma with differential rotation. In this section, two principles will be presented which are inherent in our understanding of the structure, function, and strength of both small and large-scale galactic magnetic fields: these are the principles of flux-freezing and magnetic reconnection.

#### 2.2.1 Flux-Freezing

Consider a magnetic flux, \( \psi \), through a contour, \( C \), which is co-moving with the plasma:

\[ \psi = \int_S \mathbf{B} \cdot d\mathbf{S}, \]

(2.24)

where \( S \) is a surface which spans \( C \). The rate of change of \( \psi \) with respect to time has two components; the first is the time variation of \( \mathbf{B} \) over the surface \( S \),

\[ \left( \frac{\partial \psi}{\partial t} \right)_1 = \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}. \]
Using Faraday’s law \( \frac{\partial \mathbf{B}}{\partial t} = - \nabla \times \mathbf{E} \), this expression becomes

\[
\left( \frac{\partial \psi}{\partial t} \right)_1 = - \int_S \nabla \times \mathbf{E} \cdot d\mathbf{S}.
\]

The second contribution to the changing flux is due to the motion of \( C \). Because this is a fluid, one must account for the motion of the surface itself. If \( d\mathbf{l} \) is an element of \( C \) then \( \mathbf{v} \times d\mathbf{l} \) is the area swept by \( d\mathbf{l} \) per unit time and the rate of change of flux is

\[
\left( \frac{\partial \psi}{\partial t} \right)_2 = \int_C \mathbf{B} \cdot \mathbf{v} \times d\mathbf{l} = \int_C \mathbf{B} \times \mathbf{v} \cdot d\mathbf{l}.
\]

Using Stoke’s theorem, the derivative becomes a surface integral,

\[
\left( \frac{\partial \psi}{\partial t} \right)_2 = \int_S \nabla \times (\mathbf{B} \times \mathbf{v}) \cdot d\mathbf{S}.
\]

Thus, the total derivative of the magnetic flux with respect to time is

\[
\frac{d\psi}{dt} = - \int_S \nabla \times (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{S} \tag{2.25}
\]

as long as \( \nabla \cdot \mathbf{B} = 0 \). The term in parentheses is simply Ohm’s law and for a perfect conductor, this term is equal to zero. Thus we arrive at an important conclusion,

\[
\frac{d\psi}{dt} = 0 \tag{2.26}
\]

in an ideal, non-resistive plasma. This means that the flux through any closed contour in a plasma, each element of which moves with the local plasma velocity, is a conserved quantity and the magnetic field lines are essentially frozen in to the plasma. This is the flux freezing condition.
2.2.2 Magnetic Reconnection

With the magnetic field locked into the plasma, there is no obvious mechanism for magnetic energy to be transformed into kinetic energy. In this section, we explore the conditions such that the frozen-in principle is broken. Resistive dissipation is more effective the more the electric current is localized to regions with a small spatial scale length. Thus, in reconnection a small-scale structure is generated in some region, such that the constraint of ideal dynamics is broken. The interesting aspect is that a local non-ideality can have a global effect. Under such circumstances highly conducting plasma structures are able to transform magnetic to kinetic energy in an efficient way and the magnetic topology can change. This is thought to be happening in solar flares (see Figure 1.5) and magnetospheric storms (the Earth’s magnetic field interaction with solar wind), and other plasma processes in the universe.

A formal derivation of this process is excluded in this report but can be derived by using MHD mass and momentum conservation laws as well as imposing incompressible flow, $\nabla \cdot \mathbf{v} = 0$. As two ideal fluids with opposing magnetic field directions flow into each other, a diffusion region forms at the surface of contact. This is needed because annihilation of the magnetic flux is prevented by the frozen-in condition and the fact that $R_m$ is large. Moreover, $\nabla \cdot \mathbf{B} = 0$ always, so in the diffusion region, particle motion and microscopic fields must be considered as the macroscopic picture breaks down. See Figure 2.1 for a two-dimensional depiction of this process. Within this diffusion region, the resistive term $j/\sigma$ in Ohm’s law is much larger due to an enhancement of the current density perpendicular to the plane and the frozen-in condition is broken. This results in the original magnetic field lines being broken and then reconnecting with the field lines on the opposite end of the diffusion region. This causes a massive acceleration of particles and an outflow perpendicular to the two inflow velocities in accordance with the change in magnetic field topology.
Figure 2.1: Two-dimensional representation of the magnetic reconnection process. Taken from Zweibel and Yamada 2009 [5].
Chapter 3

Galactic Magnetic Fields

The magnetic fields present within (and in between) galaxies vary in strength and structure depending on which component (disk or halo) one is observing. The origin and stability of these fields is currently under debate. Here we present the most common methods of field determination followed by a summary of Galactic as well as extragalactic field strength and structure. Finally, the galactic dynamo theory will be summarized as a proposal for the origin of galactic magnetic fields with $\mu$G strength.

3.1 Field Determination

In this section we describe how magnetic fields are determined. By taking advantage of three physical interactions between galactic plasmas and emitted radiation, we can determine the direction and strength of the magnetic field parallel (rotation measures) and perpendicular (polarized synchrotron radiation) to the line of sight.
3.1.1 Rotation Measure

When plane polarized radiation of wavelength $\lambda$ propagates through a plasma in which there is a parallel component of the magnetic field $B_\parallel$ to the direction of propagation, the radiation’s plane of polarization slowly rotates (see Figure 3.1). This is known as Faraday rotation. The angle of rotation is $\lambda$-dependent,

$$\psi = RM \lambda^2,$$

(3.1)

where

$$RM = \frac{q^3}{2\pi m_e^2 c^4} \int_0^D ds \ n_e B_\parallel$$

(3.2)

is the rotation measure. Here $q$ and $m_e$ represent the elementary charge and mass of an electron, respectively. The distance to the object is represented by $D$, $n_e$ is the number or column density of free electrons along the line of sight and $c$ is the speed of light. While any polarized radio-emitting body can be used to probe the magnetic field of the foreground gas, for most studies of Galactic magnetic field structure, the source of radiation are pulsars of a known distance behind the area of interest (otherwise no rotation would be measured). This radiation observed is usually in the radio region. Figure 3.2, for example, shows the halo of the Milky Way observed in H$_\alpha$ overlaid with RM data which clearly shows magnetic fields present in the disk as well as in the halo. RMs can be determined for external galaxies as well, again using any source that emits polarized radiation. A RM distribution map is shown for NGC 253 in Figure 3.3 [7].

A pulsar is a neutron star that emits beams of radiation that are observed at regular time intervals. The cause of this radiation is due to a misalignment of the star’s rotation axis and magnetic axis. The rotation of the neutron star causes the beam of radiation generated within the magnetic field to sweep in and out of our line of sight with a regular period. Thus, what is observed is a pulse of radiation when in
Figure 3.1: Faraday rotation - plane polarized light rotating through a medium.

reality the stream of light is continuous. The angle of rotation of the polarized pulsar beam, \( \psi \), can be determined from the ratio of the Stoke’s U and Q vectors, maps of which are standard outputs from observations of polarized emission,

\[
\psi = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right).
\]  

(3.3)

Plotting this angle against \( \lambda^2 \) will yield a slope equal to the \( RM \).

3.1.2 Dispersion Measure

The dispersion measure (\( DM \)) is also determined from electromagnetic signals from pulsars of known distance \( D \). This can be thought of as a broadening of an otherwise sharp pulse when a pulsar is observed over a finite bandwidth. The broadening is a result of the interaction between light and free electrons - the electrostatic interaction causes a delay in the propagation of the light. More energetic photons are not as affected by the presence of charged particles however radio waves are low frequency
and thus lower energy. Further, since protons are about 2000 times more massive than electrons, the amount of dispersion is dominated by free electrons. Another representation of the $DM$ is the number of free electrons between an observer and a pulsar per unit volume. By plotting the time of arrival for a pulse of frequency $\nu$ against $\nu^{-2}$, the slope of best-fitted curve can be used to determine $DM$ according to equation 3.4:

$$t(\nu) = \frac{D}{c} \left( 1 + \frac{\nu^2}{\epsilon_0 m_e D} \frac{DM}{\nu^2} \right),$$

(3.4)

where $\epsilon_0$ is the permittivity of free space. The dispersion measure can then be used to determine the column density, $n_e$, along the line of sight,

$$DM = \int_0^D ds \, n_e.$$  

(3.5)

A typical value for $n_e$ for the Milky Way is $\sim 3 \times 10^4$ m$^{-3}$ [10].
3.1.3 Field Parallel to Line of Sight

Equation 3.5 should look familiar. It is simply the integral presented in the equation for $RM$ assuming constant $B_\parallel$ along the line of sight. Thus in cgs units,

$$RM = 0.81 B_\parallel DM,$$

(3.6)

where the factor of 0.81 presents the magnetic field in units of $\mu G$. Simple rearrangement of equation 3.6 yields an expression for the component of the magnetic field parallel to the line of sight,

$$B_\parallel = 1.23 \frac{RM}{DM}.$$

(3.7)

So, by extracting information from background pulsars, one can calculate the magnetic field parallel to the line of sight by equation 3.7. Variations of equation 3.7 are used for statistical accuracy, such as studies done by Mao et al.[12] and Tayler et al.
Thus at low Galactic latitudes (i.e. looking along the plane of the Milky Way), rotation measures can be used to determine the magnetic field parallel to the plane of the disk, and at high Galactic latitudes can be used to determine, essentially, the magnetic field normal to the plane of the disk.

### 3.1.4 Field Parallel to the Plane of the Sky

A charged particle moving through a magnetic field will spiral around field lines and radiate due to the Lorentz force which accelerates it about the field line in centripetal acceleration (see Figure 3.4). Cosmic Ray (CR) electrons are very effective at generating synchrotron radiation due to their low mass and relativistic speeds. The number of particles $N(\beta)$ with Lorentz factors $\leq 10^6$ is given by Binney and Merrifield [10],

$$N(\beta) \propto \beta^{-2.5}.$$  \hspace{1cm} (3.8)

![Figure 3.4: Motion of a charged particle in a magnetic field.](image)

This corresponds to low frequencies/energies due to $\beta = E/m_0c^2$, where $E$ is the total energy and $m_0$ represents the rest mass of a particle. Low frequencies (i.e. radio
wavelengths) must be observed because the emission is strongest there, reflecting the fact that the cosmic ray energy spectrum increases at lower energies (see Figure 3.5 from Astrophysics: Decoding the Cosmos [8]). This generates synchrotron radiation

\[ j_\nu \sim \nu^{-0.75} \]

for a restrictive case in which the cosmic ray electron spectral index is \( \gamma = -2.5 \). This is what is observed for cosmic rays near the Earth but vary with energy as well as in other galaxies. Finding the field strength is no trivial task as \( j_\nu \) scales with the local number density of relativistic electrons. However, \( j_{\text{sync}} \propto \epsilon B^2 \), where \( \epsilon \) is the energy density of synchrotron-emitting electrons. \( \epsilon \) is estimated by assuming the energy density of all cosmic-ray particles is proportional

**Figure 3.5:** Cosmic ray spectrum for particles from \( 10^8 \) to \( 10^{21} \) eV. The ordinate, \( J(E) \), represents a specific intensity for particles and the abscissa represents the kinetic energy per particle. Taken from Astrophysics: Decoding the Cosmos [8].
to $B^2$. This “equipartition of energy” is widely accepted and used in computations of the magnetic field strength. Even if it does not hold in every case, it is clear from the above discussion that $B \propto (j_{\text{sync}})^{\frac{1}{4}}$, so significant errors in $j_{\text{sync}}$ lead to relatively small errors in $B$.

Just as Faraday rotation measures provide information on the magnetic field parallel to the line of sight, $B_\parallel$, synchrotron radiation can be used to determine the strength of the magnetic field perpendicular to the line of sight, $B_\perp$. Typical plots include $j_{\text{sync}}$ and radio map overlays like in Figure 3.6 which shows vectors of the magnetic field direction perpendicular to the line of sight taken from Beck et al. [9]. This method is used to measure the magnetic field strength near the Galactic centre which is essentially perpendicular to the disk and therefore perpendicular to the line of sight.

### 3.1.5 Davis-Greenstein Effect

When a dust grain’s spin axis is not parallel to $\mathbf{B}$, the direction of $\mathbf{B}$ relative to axes fixed in the grain changes as the grain rotates. Dissipative effects then torque the grain in such a way that the grain’s spin axis moves closer to $\mathbf{B}$. Since grains tend to spin about the axis that has the greatest moment of inertia, the net result is to preferentially align perpendicular to $\mathbf{B}$ (see Figure 3.7). As a wave passes a grain, the radiation scattered or absorbed by the grain tends to be polarized parallel to the grain’s long axis because the grain responds most readily to the component of $\mathbf{E}$ in the direction of its long axis. Conversely, transmitted radiation tends to be polarized parallel to $\mathbf{B}$. This enables us to map the direction of the interstellar magnetic field by measuring the apparent polarization of starlight due to foreground dust.

This effect is generally only used for distances no more than a few kpc from us as it relies on reflected starlight which is strongly attenuated by dust. Thus the radio means of quantifying field strengths is more universal.
3.2 The Milky Way

The Milky Way is thought to be a barred spiral galaxy as depicted in Figure 3.8. Being immersed in a large plasma makes it difficult as observers to see the large-scale magnetic field structure. After all, we are viewing the Milky Way from within the Milky Way. We can only piece together fragments of information on small scales at various parts of the Galaxy and attempt to interpret and explain the resulting picture based on the theoretical model presented in Section 2.2.1. Recall that a very conductive medium (where $\eta = 0$) is required for the magnetic flux to be frozen into the plasma. This is the case for galactic disks and thus resistivity is assumed to be
negligible for most astronomic purposes.

Unfortunately, and as will be presented in this chapter, it is not a simple matter of identifying one large scale structure. After all, the physical shape of the Galaxy can be broken into multiple components, each having their own unique characteristics: the disk, the bulge, and the halo. Do each of these components have their own magnetic field structure, or is there a common magnetic field shared between all components of the Milky Way? Perhaps the answer is somewhere in between.

### 3.2.1 Structure and Strength

From the previous section, it is interesting to note that $RM$ is proportional to $B_\parallel$ rather than $B^2$, so contributions from different points along a line of sight will cancel unless $B$ is ordered. Therefore, $B$ estimates derived from $RM$ will be lower than those from $j_{sync}$ unless the field is ordered. This is expected if $B$ wiggles to the degree that
the bottom panel of Figure 3.9 (which is the result of the Davis-Greenstein effect near the Sun) suggests. Moreover, $RMs$ are only sensitive to the ordered field. $j_{\text{sync}}$ from Galactic synchrotron emission provides probably the most reliable method to measure the total magnetic field, including the random component.

In this section, we explore the magnetic field structure and strength in various components of the Galaxy: the local neighborhood, the disk, the Galactic centre, and the halo. It should be noted that due to the difficulty of determining the field strength as well as the various statistical methods employed (data are taken from about 500 pulsars), the values found by different authors are not always in agreement.
Local

The solar neighbourhood is dominated by random magnetic field lines. This is not surprising because we as observers are submersed within the disk and while we predict that the magnetic field lines are trapped within the interstellar gas at large scales, there is no reason to assume that the field will be uniform throughout the entire body. It turns out that the random component is 2 - 3 times greater than the mean ordered component, which has a strength of about $6 \mu$G and is azimuthal and oriented at a longitude of $88 \pm 5^\circ$ [38].

**Figure 3.9:** Magnetic field lines derived from the alignment of dust grains (Davis-Greenstein Effect). **Bottom panel:** 100 - 200 pc scale. The field is quite random and unordered. **Top panel:** 1 - 2 kpc scale. The field lies within the plane of the disk due to the frozen-in condition. Taken from Binney and Merrifield [10].

The bottom panel of Figure 3.9 shows the field lines in the local region (within 200
pc of the Sun). This study takes advantage of the **Davis-Greenstein effect** (Section 3.1.5) in which spinning dust grains align their spin axes with \( \mathbf{B} \). Clearly, the field is very random at small scales and the structure has a lot of ‘wiggle’, the importance of which is quantified in terms of a picture in which the field is a *superposition* of an ordered large scale component and a random small scale component.

The local field is not only azimuthal (in the direction of Galactic rotation), but it seems to have a vertical component as well. A mean field of \( 0.31 \pm 0.03 \) \( \mu \text{G} \) pointing towards the south Galactic cap was determined by Taylor et al. [39]. As will be shown in the **Halo** section, this is consistent with the findings of Mao et al. [12].

**Disk**

The large scale Galactic magnetic field in the disk is contained within the disk and is azimuthal as the frozen-in condition would suggest given that the disk is conducting (with \( \eta \approx 0 \)) and has differential rotation. This condition is not necessarily met in the halo. The frozen-in condition is still met on small scales, however one needs to view the physical boundaries of the plasma within which the magnetic field is immersed to properly observe this effect. The top panel of Figure 3.9 shows the alignment of dust drains on kpc scales. However, this field is by no means uniform and in fact is rather complex.

From observations of over 500 pulsars at low Galactic latitude, a \( RM \) map has been produced which shows that the magnetic field reverses direction every so often. See Figure 3.10 for a \( RM \) map with arrows indicating the direction of the magnetic field. While the figure does not show any magnetic field near the Galactic centre, the reader should be reminded that rotation measures are only sensitive to magnetic fields parallel to the line of sight. Thus an absence of data simply means that there is no magnetic field component along the line of sight present near the Galactic centre (i.e. no radial component). To measure magnetic fields in that vicinity, another approach
is followed: measuring synchrotron emission. Recall that polarized synchrotron radiation is sensitive to $B_{\perp}$ and thus can be used to measure the vertical magnetic field near the GC at low Galactic latitudes as well as planar components perpendicular to the line of sight.

![Figure 3.10: RM map along spiral arms. The magnetic reversal seems to follow a sinusoidal curve. The direction of the magnetic field is indicated by the red arrows. The circles are positive RM values while x’s are negative. The strength of the RM value is represented by the relative sizes of the circles and x’s. Taken from Han 2008 [11]](image)

The mechanism through which the field reverses is not fully understood, however it seems to fit a sinusoidal curve as found by Rand and Kulkarni [40]. This model is by no means exact. In fact, it is now speculated that the magnetic field strength decreases with distance from the GC. So it is perhaps more appropriate to model the reversal as a damped sine curve.

The strength of the magnetic field varies, though it seems to be correlated with the gas density. The field is stronger in molecular clouds [41]. This study demonstrated that as the density increased from $10^9$ m$^{-3}$ to $10^{11}$ m$^{-3}$ in the Orion B cloud, the magnetic field also increased, from $30 \mu$G to $60 \mu$G. Also, $B$ is most highly ordered
between spiral arms and relatively disordered in spiral arms.

**Galactic Centre**

The Galactic centre is home to some of the brightest synchrotron sources in the Galaxy. The strongest magnetic fields are known to occur in filaments of synchrotron emission near the GC. These filaments are located about 30 pc from the GC and extend perpendicular to the plane for a few pc. From the brightness of synchrotron emission, $B \sim 1000 \, \mu G$.

Magnetic stresses scale as $B^2$, thus these fields would exert forces 10 000 times larger than those exerted by $B$ at a typical point in the ISM, where $B \leq 10 \, \mu G$. A clue to the strength of the magnetic field in the non-thermal radio filaments is provided by the near absence of deformation or bending along their lengths. Every filament that has been sufficiently well studied has been found to be associated with, and is probably interacting with, at least one molecular cloud. However, the magnetic filaments are not subject to large distortions at the interaction sites, in spite of the large velocity dispersion within Galactic centre molecular clouds, and in spite of the likelihood that, given the large intercloud velocity dispersion at the Galactic centre, most clouds have a typical velocity of at least a few tens of kilometers per second with respect to the ambient magnetic field. By equating the apparent turbulent pressures within clouds to the magnetic pressure, as a minimum condition on the strength of the magnetic field, Yusef-Zadeh and Morris [42] have determined that the magnetic field within the filaments has milligauss strength as indicated in the beginning of this section.

**Halo**

The Halo magnetic field is the most challenging to understand. The collected evidence does not seem to be sufficient to come to a consensus on its structure. Some of these
basic measurements are in disagreement and no present model can explain the nature of these results.

At low Galactic latitudes, the field is azimuthal, perhaps remnant of the disk field. If this is the case, then there should be no difference observed when pointing the line of sight slightly above or slightly below the plane. However, as some studies claim, there is a reversal in the field direction across the plane. This is depicted in Figure 3.11.

![Figure 3.11: Field reversal in Halo across plane. Left: the antisymmetric rotation measured sky, derived from RMs of extragalactic radio sources. Right: the magnetic field structure in the Galactic halo resulting from the RM map. Taken from Han 2008 [11].](image)

A study by Mao et al. [12] mapped RMs across the Galactic caps. If, indeed, the Halo’s magnetic field has dipole symmetry, then this should be most easily observed at the caps with Faraday rotation. Recall that RMs are sensitive to the magnetic field parallel to the line of sight, thus when observing the Galactic caps at high altitudes, one is essentially measuring the vertical magnetic field component. The results are inconsistent with a dipole symmetry model, but are consistent with other studies which advocate a quadrupole field symmetry ([37], [11]). They find essentially no horizontal component at the Galactic poles and no field at the North Galactic pole. A field of strength 0.46 $\mu$G is, however, present along the line of sight to the South pole. RM data overlaid with an H$\alpha$ emission map is shown in Figure 3.12. They show no correlation between RM and H$\alpha$ emission. That is, they show no correlation
between the column density of free electrons \( (n_e) \), which must be producing the RM, and the ionized hydrogen content (which are discrete regions of known high electron density) in the ISM. The latter is termed the Diffuse Ionized Gas (DIG) region. The largest circle corresponds to a RM value of +93 (+76) rad m\(^{-2}\) toward the North (South) Galactic pole. Sources with RMs consistent with zero at 1\( \sigma \) are denoted by asterisks. The resolution change between the top and bottom panels corresponds to the change of H\( \alpha \) data being used to create the composite map from the Wisconsin H-Alpha Mapper (WHAM) to the Southern H-Alpha Sky Survey Atlas (SHASSA), the latter has better spatial resolution but much poorer sensitivity. The conclusion they draw is that the magnetic field across the plane of the Galaxy seems to be directed towards the South pole and is inconsistent with dipole symmetry.

Dynamo studies have been performed to show whether the Halo has quadrupole symmetry, but these results do not agree with post-Galactic origin models for the magnetic field. See Section 3.4 for a summary of the proposed theories.
Figure 3.12: RM distribution overlaid on the Hα emission map toward the north Galactic pole in the top panel and towards the south Galactic pole in the bottom panel. Positive (negative) RMs are denoted by filled (open) circles with their diameters proportional to the magnitude of the RM. The blue cross in the top panel represents the centre of the Coma cluster. The dotted green curve in the bottom panel marks the projection of the LB wall. Taken from Mao et al. [12].

Summary

Table 3.1 provides a summary of the magnetic field strengths and orientations in various components of the Milky Way determined by various studies [41], [43], [39], [11], [12]. Surveys of the total synchrotron emission from the Milky Way yield total field strengths of 6 µG averaged over a radius of 1 kpc around the Sun. Near the
inner Galaxy, however, a field strength of about 100 $\mu$G is observed. Faraday RM and DM data of pulsars give an average magnetic field strength of the local ordered field of 6 $\mu$G, while the regular field strength in the inner Norma arm (see Figure 3.8) is $4.4 \pm 0.9$ $\mu$G [44]. A field reversal is required at 1 or 2 kpc from the Sun towards the Milky Way’s centre which is in agreement with the study of RMs from extragalactic sources near the Galactic plane by Brown et al. [45]. The structure of the regular field predicted by Han et al. [44] could not be confirmed by statistical tests [46]. Whether a field reversal across the Galactic plane occurs is under debate, however a study by Sun et al. [31] note a field reversal above and below the plane towards the inner Galaxy. While not much is known about the vertical magnetic field, the local vertical magnetic field is presented as 0.2 $\mu$G [11]. There is also a difference in field strength and order between the spiral arms and the interarm regions [47]. A much more ordered field is observed in the interarm regions whereas small-scale and turbulent field structures are enhanced in the spiral arms.

<table>
<thead>
<tr>
<th>Location</th>
<th>Orientation</th>
<th>Strength ($\mu$G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near Galactic centre</td>
<td>$\perp$</td>
<td>$\sim 1000$</td>
</tr>
<tr>
<td>Molecular HI clouds</td>
<td>azimuthal</td>
<td>$28 \pm 8$ at $v_{LSR} = 14.5$ km/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$63 \pm 8$ at $v_{LSR} = 6.1$ km/s</td>
</tr>
<tr>
<td>Local</td>
<td>azimuthal towards $l = 88 \pm 5^\circ$</td>
<td>mean 6</td>
</tr>
<tr>
<td></td>
<td>$\perp$</td>
<td>random 5 - 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.31 \pm 0.03$ towards South cap</td>
</tr>
<tr>
<td>Disk</td>
<td>azimuthal with field reversals</td>
<td>$2 - 6$ along arms</td>
</tr>
<tr>
<td></td>
<td>$\perp$</td>
<td>strongest in interarm region</td>
</tr>
<tr>
<td>Halo</td>
<td>azimuthal at low $</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>$\perp$ at high $</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sim 0.46$ towards South cap</td>
</tr>
<tr>
<td></td>
<td></td>
<td>no horizontal field</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of Galactic magnetic fields A comparison of the magnetic field orientations and strengths with various components of the Milky Way.
3.3 Extragalactic Fields

The magnetic field strength and structure discussed above are not unique to the Milky Way. Other galaxies show similar strengths and trends which have been compiled and reviewed by numerous authors ([48], [49]).

3.3.1 Magnetic Field Strength

Using the integrated radio continuum emission, Hummel [50] found $\langle B_{\text{tot}} \rangle = 8 \mu G$ for 65 Sbc galaxies, Fitt & Alexander [51] obtained $\langle B_{\text{tot}} \rangle = 10 \mu G$ for 146 late-type galaxies and Niklas [52] derived $\langle B_{\text{tot}} \rangle = 9 \mu G$ for 74 nearby galaxies. The mean total field for these data is $B_{\text{tot}} = 17 \mu G$ with a standard deviation of 14 $\mu G$. For the regular field the mean is $B = 5 \mu G$, with a standard deviation of 3 $\mu G$; thus the random component of the field is around 3 times the strength of the regular component. In spiral galaxies, the field strength is greater in the interarm regions than within the spiral arms which suggests that magnetic fields are disrupted by the gas and dust in the optical arms. Some galaxies have anomalously strong magnetic fields. M82, for example, has a magnetic field strength $\simeq 50 \mu G$ and has an extraordinarily high star formation rate.

3.3.2 Magnetic Field Structure

Face-on galaxies tend to show a spiral structure; a strong example being NGC 6946 (Figure 3.6 shows the magnetic spiral pattern overlaid on the optical image). A correlation between magnetic and optical spiral structures have been observed in many other spiral galaxies as well: M83, IC 324, and M81.

Radio observations of edge-on spiral galaxies suggest that the dominant component of the field at high galactic latitudes is perpendicular to the disk plane. Two galaxies, NGC 891 and NGC 4631, have been mapped by Hummel et al. [22] in lin-
early polarized emission and they found field strengths of 8 µG and 5 µG, respectively with scale heights ranging from 5 kpc to 10 kpc. Figure 3.13 shows the L (1 - 2 GHz) and C (4 - 8 GHz) band polarization maps of NGC 4631 taken with the Expanded Very Large Array (EVLA). These images show fields parallel to the plane within the optical disk which adopt a more vertical structure away from the plane. The global magnetic field structure, however, is very complex.

As alluded to in the discussion of the Milky Way, the symmetry of galactic magnetic fields is under debate. The general consensus is that the magnetic field through the disk and halo is of even symmetry which is consistent with a quadrupole-like structure. A recent study of the Milky Way by Sun et al. [31] reported a reversal in the halo field above and below the disk while the disk field maintained the same direction above and below the Galactic equator. Moss and Sokoloff [29] have investigated the effects of the relative disk and halo diffusivity value ($\eta_h/\eta_d = 5$) on the dynamo-driven magnetic field properties of galaxies (to be discussed in more detail in Section 3.4), the results of which suggest that a two-part dynamo (one in the disk and one in the halo) generates either a quadrupole-like or a dipole-like magnetic field both in the disk and the halo. This suggests that either the halo enslaves the disk, or vice-versa, depending on the relative magnitude of the dynamo numbers. However, a more recent study by Moss et al. [30] concludes that by considering galactic winds of speeds from 200 km s$^{-1}$ up to 600 km s$^{-1}$ at 10 kpc above the disk and allowing a larger contrast in the halo and disk diffusivities ($\eta_h/\eta_d = 25$) with a smoother transition between these regions, parity mixing such as observed by Sun et al. [31] can be obtained.
Figure 3.13: Top: NGC 4631 L band polarization map with B vectors over H\textalpha. Bottom: NGC 4631 C band polarization map superimposed on the Digitized Sky Survey (DSS2B) optical image. Image credit: Judith Irwin et al. image taken with the EVLA.
3.4 Galactic Magnetic Field Origin

The origin and preservation of galactic magnetic fields is thought to be the consequence of a dynamo mechanism, though details of the mechanism remain unclear and it does not seem to explain the existence of magnetic fields in elliptical galaxies and clusters [49]. A magnetic dynamo consists of electrically conducting matter moving in a magnetic field in such a way that the induced currents amplify and maintain the original field. The effect is most simply illustrated with reference to a system consisting entirely of solid (rather than fluid) conductors. This is the homopolar disk dynamo illustrated in Figure 3.14.

![Figure 3.14: The homopolar disk dynamo. Taken from [13].](image)

A solid copper disk rotates about its axis with angular velocity \( \omega \), and a current path between its rim and axle is provided by the wire twisted as shown in a loop around the axle. This system can be unstable to the growth of magnetic perturbations. Suppose that a current \( I(t) \) flows in the loop; this generates a magnetic flux \( \Phi \) across the disk, and, provided the conductivity of the disk is not too high, this flux is given by \( \Phi = M_0 I \) where \( M_0 \) is the mutual inductance between the loop and the rim.
of the disk. Rotation of the disk leads to an electromotive force \( \mathcal{E} = \omega \Phi / 2\pi \) which drives a current \( I \), and the equation for \( I(t) \) is then

\[
L \frac{dI}{dt} + RI = \mathcal{E} = M \omega I, \tag{3.9}
\]

where \( M = M_0 / 2\pi \) and \( L \) and \( R \) are the self-inductance and resistance of the complete current circuit. The device is evidently unstable to the growth of \( I \) from an infinitesimal level if

\[
\omega > R/M. \tag{3.10}
\]

In this circumstance the current grows exponentially, as does the retarding torque associated with the Lorentz force distribution in the disk. Ultimately the disk angular velocity slows down to the critical level \( \omega_0 = R/M \) at which point the driving torque just balances this retarding torque, and the current can remain steady. This type of example is certainly suggestive, but differs from the conducting fluid situation in that the current is constrained by the twisted geometry to follow a special path that is particularly conducive to dynamo action (the conversion of mechanical energy into magnetic energy).

From the discussion in the previous sections, a magnetic field frozen into a conducting, differentially rotating disk would annihilate itself on relatively short timescales. This is due to magnetic field lines being tightly wound near the galactic centre, where the rotational velocity is greater. Diffusion of these antiparallel fields leads to a relatively quick decay of the overall magnetic field. This is known as the winding problem and it illustrated in Figure 3.15.
To counter this, the field must be regenerated by a dynamo process. One of the more standard models for this amplification and regeneration of galactic magnetic fields includes an $\alpha\Omega$-dynamo, the essential features of which are as follows: small-scale velocity fluctuations in the interstellar medium (ISM) carry loops of toroidal magnetic field out of the plane of the disk. The mechanisms of these fluctuations is the major focus of this report which include hydromagnetic instabilities and supernova explosions. These loops are twisted into the poloidal plane by the Coriolis effect while the toroidal field is regenerated from the poloidal field by differential rotation. Upon magnetic reconnection, a magnetic field is created perpendicular to the prevailing field. This is known as the $\alpha$-effect (Figure 3.16) and is thought to be responsible for the regeneration of the azimuthal magnetic field in the disk.
This mechanism needs an already present magnetic field. Otherwise nothing could be amplified or regenerated. An early hypothesis was that the original, pregalactic magnetic field was a relic of the early Universe. A magnetic field that permeates a protogalactic medium will be compressed, and thus amplified, as the galaxy forms and then by differential rotation once the disk is developed. This was challenged by Parker [53] among others on the grounds that turbulent diffusion destroys a primordial field on a relatively short timescale. More recent studies suggest that the current magnetic field originated by amplification of $10^{-20}$ G seed fields. The seed field could have a pregalactic origin (as in the primordial field theory), or it could arise in the protogalaxy as a result of charge separation due to electrons interacting with the microwave background photons (known as the battery effect), or else it could originate in the first generation of stars and be expelled into the interstellar medium by their winds and/or supernova explosions. It is important to note, however, that observations do not support a dynamo alone; either several dynamo modes are superimposed and cannot be distinguished with the limited sensitivity and resolution of present-day telescopes, or no large-scale dynamo modes exist and most of the ordered fields traced by the polarization vectors are anisotropic due to shearing or compressing gas flows [54].
Chapter 4

Toy Model

Spectroscopic observations of Diffuse Ionized Gas (DIG) in the halos of galaxies reveal that the velocities at high galactic latitudes drop from the midplane value to reach the systemic velocity at $z \approx 9$ kpc for NGC 5775 [15] but varies by galaxy (this value is around 5 kpc for the Milky Way [17]). Kinematic studies of NGC 5775 [15] and NGC 891 [3] attempt to decipher the gradient at which the velocity drops and find that it is more or less constant. Gradients are also observed in H_I as in the study of the edge-on galaxies NGC 4157, NGC 3600, and NGC 2683 [26]. Though it varies by galaxy [26], the average gradient is around $-20$ km s$^{-1}$ kpc$^{-1}$. The bright emission lines of hydrogen, nitrogen, and oxygen are used to derive heliocentric velocities for the DIG as a function of $z$. This trend is explainable by assuming the DIG does not rotate at high galactic $z$. The study by Tullman et al. [15] indicates that the heliocentric velocity peaks at $z = -1.6$ kpc (where $z = 0$ is defined by the midplane) and steadily decreases above and below the plane as indicated by Figure 4.1. The peak at $z = -1.6$ kpc is explained by dust obscuration. That is, dust is obscuring the emission, creating the perception that the highest velocities are shifted by $-1.6$ kpc from the midplane. As they indicate in their article, an R-band (0.65 - 1.0 $\mu$m) image of NGC 5775 provides evidence for large amounts of dust extending 1.8 kpc
into the halo.

This chapter sets up and solves an analytic model for the lagging halo using Hartmann flow as the principal mechanism. We note that this is not a realistic model due to its simplicity and serves only to test the effects of magnetic drag due to viscous and resistive shearing in the halo.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{ngc5775_vhel.png}
\caption{Heliocentric velocities as a function of $z$ for NGC5775 taken from Tullman et al. 2000 [15]. The systemic velocity is given by the dashed horizontal line at 1681 km s$^{-1}$ [16].}
\end{figure}
CHAPTER 4. TOY MODEL

4.1 Setup

To study and isolate various possible effects, we consider Hartmann flow, which describes the flow of a fluid between two parallel plates. We begin with a disk that is a time-independent system of constant density, $\rho$, rotating at a constant rate, $\Omega$. The coordinate system is as follows: $x$ points in the direction of rotation, $y$ points radially towards the disk centre, and $z$ extends perpendicular to the disk in accordance with a right-hand system. The $x$-component of the velocity is a function of $z$ with $v_x(0) = V$ (the rotational velocity of the disk). The $x$-component of the magnetic field, $B_x$, is also a function of $z$. Both $B_z$ and $v_z$ are positive constants acting perpendicular to the plane and represent the vertical magnetic field and ejection velocity, respectively. This ejection velocity exists at the upper $Z$ boundary, of course, so technically this toy model follows a modified Hartmann flow. That is, gas is not necessarily restricted to between the plates. Finally, gravity acts on the system in the $z$ and $y$ directions as

$$\mathbf{g} = -g_z \hat{z} + g_y \hat{y}. \quad (4.1)$$

The equation of motion of the system in the inertial frame is given by

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi \rho} + \nu \nabla^2 \mathbf{v} + \mathbf{g}. \quad (4.2)$$

where $p$ represents pressure and $\nu$ is the viscosity which is a measure of a fluid’s resistance to flow.

The focus of this study is on the gradient of the rotational velocity (the $x$-component) with height to the systemic level; that is, the height at which the velocity is equal to the velocity of the nucleus of the galaxy. We ignore the $y$-component since we are only interested in the rotational velocity, not the radial velocity. Using an
inertial frame of reference and taking only the $x$-component, equation 4.2 reduces to

$$v_z \frac{dv_x}{dz} = \frac{B_z}{4\pi \rho} \frac{dB_x}{dz} + \nu \frac{d^2v_x}{dz^2}. \quad (4.3)$$

Similarly, the MHD conservation equation (equation 2.16) yields

$$B_z \frac{dv_x}{dz} - v_z \frac{dB_x}{dz} + \eta \frac{d^2B_x}{dz^2} = 0, \quad (4.4)$$

where $\eta$ is the resistivity, a measure of the potential electrical resistance of the conductive plasma.

All that is left is to combine equations 4.3 and 4.4 in order to isolate for the velocity gradient, $dv_x/dz$.

### 4.2 Velocity

#### 4.2.1 General Conditions

Some simplifications need to be applied to the system of equations in order to properly isolate the velocity gradient. Also, it is convenient to perform the analysis, both analytically and numerically, with unit-less quantities. Let

$$b_x = \frac{B_x}{B_z},$$

$$u_x = \frac{v_x}{V},$$

$$u_z = \frac{v_z}{V},$$

$$m_z^2 = \frac{4\pi \rho v_z^2}{B_z^2},$$
and

$$Z = \frac{z}{h},$$

where $h$ is the height above the plane at which the velocity decreases to the systemic velocity and the term $m_z^2$ is the Alfvénic Mach number. Substituting these into equations 4.3 and 4.4 yield the unit-less forms,

$$\frac{du_x}{dZ} = \frac{u_z}{m_z^2} \frac{db_x}{dZ} + \tilde{\nu} \frac{d^2 u_x}{u_z dZ^2},$$  \hspace{1cm} (4.5)

and

$$\frac{du_x}{dZ} = u_z \frac{db_x}{dZ} - \tilde{\eta} \frac{d^2 b_x}{dZ^2}.$$  \hspace{1cm} (4.6)

Here $\tilde{\nu} = \nu/Vh$ and $\tilde{\eta} = \eta/Vh$. The boundary conditions are such that $b_z(Z = 1) = 0, u_x(Z = 0) = 1$, and $u_x(Z = 1) = 0$. The magnetic field boundary condition arises from the assumption that the $z$-component of the magnetic field is dominant at the systemic level. That is, it is anchored in the intergalactic medium which is assumed to be at rest with respect to the systemic velocity of the galaxy. This is the mechanism being explored. The boundary conditions associated with the velocity stem from definitions already presented: the rotational velocity is $V$ at the plane of the disk and zero at the systemic level.

Integrating equations 4.5 and 4.6 produces two equivalent expressions for $u_x$ with integration constants $k_1$ and $k_2$, respectively,

$$u_x = \frac{u_z}{m_z^2} b_x + \frac{\tilde{\nu}}{u_z} \frac{du_x}{dZ} + k_1$$  \hspace{1cm} (4.7)

and

$$u_x = u_x b_x - \tilde{\eta} \frac{db_x}{dZ} + k_2.$$  \hspace{1cm} (4.8)
We then solve both equations 4.7 and 4.8 for $b_x$ and set them equal,

$$
\frac{u_x m_z^2}{u_z} - \frac{\tilde{\nu} m_z^2}{u_z^2} \frac{du_x}{d\mathcal{Z}} - \frac{m_x^2 k_1}{u_z} = \frac{u_x}{u_z} + \frac{\tilde{\eta} \, db_x}{u_z \, d\mathcal{Z}} - \frac{k_2}{u_z}.
$$

This expression is simplified and takes the form of an ordinary differential equation to the second order with $\mathcal{Z}$ after rearranging equation 4.5 for $db_x/d\mathcal{Z}$,

$$
\left( \frac{\tilde{\eta} \tilde{\nu} m_z^2}{u_z^2} \right) \frac{d^2 u_x}{d\mathcal{Z}^2} - \frac{m_z^2 (\tilde{\eta} + \tilde{\nu})}{u_z} \frac{du_x}{d\mathcal{Z}} + (m_z^2 - 1) u_x + m_z^2 \left( \frac{k_2}{m_z^2} - k_1 \right) = 0. \tag{4.9}
$$

Employing the boundary conditions to equations 4.7 and 4.8 allow us to solve for the integration constants. At height $h$ above the disk we have that $u_x(1) = 0$ and $b_x(1) = 0$. Using the $\mathcal{Z} = 1$ boundary conditions, equations 4.7 and 4.8 become

$$
0 = \tilde{\nu} \frac{du_x}{u_z \, d\mathcal{Z}} \bigg|_1 + k_1
$$

and

$$
0 = -\tilde{\eta} \frac{db_x}{d\mathcal{Z}} \bigg|_1 + k_2,
$$

respectively. At the plane of the disk we have that $u_x(0) = 1$ and $b_x(0)$ is an unknown constant. Using the $\mathcal{Z} = 0$ boundary conditions, equations 4.7 and 4.8 become

$$
1 = \frac{u_z}{m_z^2} b_x(0) + \frac{\tilde{\nu} \, du_x}{u_z \, d\mathcal{Z}} \bigg|_0 + k_1
$$

and

$$
1 = u_z b_x(0) - \frac{\tilde{\eta} \, db_x}{d\mathcal{Z}} \bigg|_0 + k_2,
$$

respectively. Rearranging the above expressions for $k_1$ and $k_2$ yield separate expres-
sions depending on the boundary condition and are given below,

\[
\begin{align*}
    k_1 &= - \tilde{\nu} \frac{du_x}{u_z \frac{dZ}{dz}} \bigg|_1 \\
    &= 1 - \frac{u_z}{m_z^2} b_x(0) - \tilde{\nu} \frac{du_x}{u_z \frac{dZ}{dz}} \bigg|_0
\end{align*}
\]

and

\[
\begin{align*}
    k_2 &= \tilde{\eta} \frac{db_x}{dZ} \bigg|_1 \\
    &= 1 + \tilde{\eta} \frac{db_x}{dZ} \bigg|_0 - u_z b_x(0).
\end{align*}
\]

Equation 4.9 is now represented in a more manageable form,

\[
\frac{d^2 u_x}{dZ^2} - u_z \left( \frac{\tilde{\eta} + \tilde{\nu}}{\tilde{\eta} \tilde{\nu}} \right) \frac{du_x}{dZ} + \frac{u_z^2}{\tilde{\eta} \tilde{\nu}} \left( \frac{m_z^2}{m_z^2 - 1} \right) u_x + \frac{u_z^2}{\tilde{\eta} \tilde{\nu}} \left( \frac{k_2}{m_z^2} - k_1 \right) = 0 \tag{4.10}
\]

where \( k_1 \) and \( k_2 \) are functions of the localized velocity and magnetic field gradients, respectively. From observations quoted in Sections 4.2.4 and 4.5, the velocity gradient is roughly constant and negative. Those measurements are used in this analysis to further simulate real physical conditions. Further, given the assumption that the magnetic field is anchored in the intergalactic medium, the gradient of \( b_x \) at the systemic level is assumed to be roughly zero.

### 4.2.2 Special Case with Zero Viscosity

It is tempting to say that the symmetry of equation 4.9 indicates that setting either viscosity or resistivity to zero will affect the velocity curve in the same way and yield a simple first order differential equation,

\[
\begin{align*}
    u_x' + \frac{u_z}{(\tilde{\eta} \text{ or } \tilde{\nu})} \left( \frac{1 - m_z^2}{m_z^2} \right) u_x - \left( \frac{u_z}{\tilde{\eta} \text{ or } \tilde{\nu}} \right) \left( \frac{k_2}{m_z^2} - k_1 \right) = 0 \tag{4.11}
\end{align*}
\]
which has the general solution

\[ u_x = \left( \frac{m_z^2}{1 - m_z^2} \right) \left( \frac{k_2}{m_z^2} - k_1 \right) + Ae^{- \frac{u_z}{\tilde{\eta} \text{ or } \tilde{\nu}}} \left( \frac{1 - m_z^2}{m_z^2} \right) Z. \] (4.12)

However, we must be careful with this statement as the constants \( k_1 \) and \( k_2 \) change depending on whether the system is non-viscous or non-resistive.

To determine the multiplying constant \( A \), we must employ a single boundary condition. We are free to choose either boundary to control and then let the system evolve to see how it differs from the normal case. We choose to keep the condition that the disk is rotating \((u_x(0) = 1)\) and let the system evolve. The other boundary condition, describing the fall of the velocity to systemic at \( Z = 1 \) is relaxed. For now, we will proceed with the derivation assuming zero viscosity \((\tilde{\nu} = 0)\), then present the function where there is zero resistivity \((\tilde{\eta} = 0)\).

Using the \( u_x(0) = 1 \) boundary condition,

\[ A = -1 - \left( \frac{m_z^2}{1 - m_z^2} \right) \left( \frac{k_2}{m_z^2} - k_1 \right), \]

and the constants \( k_1 \) and \( k_2 \) become

\[ k_1 = 1 - \frac{u_z}{m_z^2} b_x(0) \]

and

\[ k_2 = 1 + \frac{\tilde{\eta} db_x}{dZ} \bigg|_0 - u_z b_x(0). \]

Thus the final form of equation 4.12 for the case of zero viscosity is

\[ u_x = 1 + \frac{\tilde{\eta} db_x}{(1 - m_z^2) dZ} \left[ 1 - e^{- \frac{u_z}{\tilde{\eta}} \left( \frac{1 - m_z^2}{m_z^2} \right) Z} \right]. \] (4.13)
The gradient of the magnetic field ratio is assumed to be large and negative due to the frozen in condition (Section 2.2.1).

### 4.2.3 Special Case with Zero Resistivity

If we now consider the case where resistivity is removed, the constants $k_1$ and $k_2$ become

$$k_1 = 1 - \frac{\tilde{\nu}}{u_z} \frac{du_x}{dZ} \bigg|_0 - \frac{u_z}{m_z^2} b_x(0)$$

and

$$k_2 = 1 - u_z b_x(0).$$

Thus the final form of equation 4.12 for the case of zero resistivity is

$$u_x = 1 + \tilde{\nu} \left( \frac{m_z^2}{1 - m_z^2} \right) \frac{db_x}{dZ} \bigg|_0 \left[ 1 - e^{-\tilde{\nu} \left( \frac{1 - m_z^2}{m_z^2} \right) Z} \right]. \quad (4.14)$$

There is no reason to assume that $db_x/dZ|_0$ in equations 4.13 and 4.14 are equivalent. After all, the magnetic field should evolve differently under conditions of zero viscosity or zero resistivity. However, since the frozen-in condition is more strictly met with the case where resistivity is zero, it may be safe to assume that the absolute value of the gradient of $b_x$ will be larger.

### 4.2.4 Numerical Solutions

We note that equation 4.10 has the form of an inhomogeneous, damped harmonic oscillator:

$$u_x'' - a u_x' + b u_x + c = 0. \quad (4.15)$$
Using a realistic range of variables, we can estimate likely values for the parameters $a$, $b$, and $c$. We let $u_z$ be a constant ranging between 0.1 and 0.2, and assume that viscosity and resistivity are equal and are related to the turbulent scale velocity and the turbulent scale length ($v_t l_t / V h \approx 0.01 - 0.02$). The gas velocity range chosen corresponds to typical velocities the gas would have in a Parker instability (30 km/s) [32]. Viscosity and resistivity are not truly equal and this is a simplification. However, recall that the magnetic field is frozen into the disk due to the low value of $\eta$. Thus the current moves with the plasma as well and so any resistance to the flow (by viscosity) affects the current and vice versa (any resistance to the current will affect the plasma flow). Hence, the viscosity and resistivity of a conductive plasma complement each other and are about equal. For the sake of modelling, we consider cases in which the Mach number is greater than and less than unity.

Simply using these variables in equation 4.10, the likely parameters are found to be

\[ a \approx 20 - 40 \]
\[ b \approx \pm \left( \frac{100}{9} - \frac{400}{9} \right) . \]

Parameter $c$ is dependent on the derivatives of $u_x$ and $b_x$ evaluated at $Z = 1$,

\[ c = \frac{u_z^2}{\tilde{\eta} \tilde{\nu}} \left[ \frac{\tilde{\eta}}{m_z^2} \frac{dv_x}{dZ} \right]_1 + \frac{\tilde{\nu}}{u_z} \frac{du_x}{dZ} \bigg|_1 . \]  

(4.16)

This creates 8 distinct cases with $a$ and $b$. 
We note that Cases 5 - 8 produce an increasing velocity function which is forced to zero by the boundary conditions. These do not reproduce a smooth decline as is observed by Tullmann et al. [15] and Kamphuis et al. [3]. Of course, we have not yet optimized the parameter \(c\), which is defined mostly by the gradients of \(b_x\) and \(u_x\) at \(Z = 1\). Assuming that the magnetic field is essentially vertical at the height of systemic velocity (locked in the intergalactic medium), we can ignore the first term on the right hand side of equation 4.16. Further, using an average velocity gradient of \(-20\ \text{km s}^{-1} \ \text{kpc}^{-1}\) taken from the authors mentioned above, and assuming the gradient is constant with height, we conclude that \(du_x/dZ|_1 \approx -1\) and that \(c \approx -10\). The conditions which produce the smoothest declining curve in accordance with the observed velocity gradient is Case 2. In fact, all simulations with a Mach number greater than unity produce an increasing velocity profile (which is then forced to zero by the boundary condition). Figure 4.2 shows the velocity profile inspired by Case 5. Notice that despite the imposed negative gradient at the boundaries, the high mach number causes the velocity to increase with \(Z\) which is contrary to observation.

The parameters presented in Case 2 are carried over in all numeric experiments.

<table>
<thead>
<tr>
<th>Case</th>
<th>(a)</th>
<th>(b)</th>
<th>(m_z^2)</th>
<th>(\tilde{\eta}(=\tilde{\nu}))</th>
<th>(u_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>-(\frac{100}{9})</td>
<td>0.9</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>-(\frac{400}{9})</td>
<td>0.69</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>-(\frac{100}{9})</td>
<td>0.97</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>-(\frac{400}{9})</td>
<td>0.9</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>(\frac{100}{9})</td>
<td>1.13</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>(\frac{400}{9})</td>
<td>1.8</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>(\frac{100}{9})</td>
<td>1.03</td>
<td>0.01</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>(\frac{400}{9})</td>
<td>1.13</td>
<td>0.01</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 4.1: Cases under realistic conditions. Under likely conditions, the parameters \(a\) and \(b\) range from 20 to 40 and \(\pm\frac{100}{9}\) to \(\pm\frac{400}{9}\), respectively.
Figure 4.2: Case 5 velocity curve. With a Mach number greater than unity, the velocity increases with height. The curve is forced back to zero quite unnaturally as a result of the imposed boundary condition.

Figure 4.3 shows the evolution of $u_x$ with height under these conditions as well as when viscosity or resistivity is null. It is interesting to note that the system falls to systemic velocity much more quickly when either viscosity or resistivity is removed, falling at a normalized height of about 0.3. This makes sense as the restoring force becomes large in the case of zero resistivity so the behaviour of the velocity curve should be dominated by magnetic effects. With zero viscosity, diffusion effects would cause the velocity to drop more quickly.

This is in good agreement with the velocity profiles of NGC 5775 and the Milky Way given in Figures 4.1 and 4.4, though the toy model curve falls more quickly.
Figure 4.3: Optimized velocity curves. The scaled velocity smoothly decreases to 0 under likely physical parameters of viscosity, resistivity, and Mach number. The red curve represents the system under normal conditions and the green curve represents the system with zero viscosity, and the blue curve represents the system under zero resistivity.

4.3 Tan(θ) to Polarization Angle

4.3.1 General Conditions

We note that the tangent of the polarization angle, θ, is given by $B_x/B_z$ which we have defined as $b_x$. The magnetic field is derived in the same manner as before; equations 4.7 and 4.8 are set equal with $k_1$ and $k_2$ remaining the same as before. This yields a similar differential equation,

$$\frac{d^2 b_x}{dZ^2} - u_z \left( \frac{\tilde{\eta} + \tilde{\nu}}{\tilde{\eta} \tilde{\nu}} \right) \frac{db_x}{dZ} + \left( \frac{m_z^2 - 1}{m_z^2} \right) \frac{u_z^2}{\tilde{\eta} \tilde{\nu}} b_x + \frac{u_z}{\tilde{\eta} \tilde{\nu}} (k_2 - k_1) = 0. \quad (4.17)$$
Recall that $b_x$ represents the ratio of the planar and perpendicular components of the magnetic field. Again, it is tempting to say that, by equation 4.17, setting either viscosity or resistivity to zero will result in the same first order differential function,

$$b_x' + \frac{u_z}{\nu \text{ or } \eta} \left( \frac{1 - m_z^2}{m_z^2} \right) b_x - \frac{1}{\nu \text{ or } \eta} (k_2 - k_1) = 0.$$  \hspace{1cm} (4.18)

This is not the case, however, as the constants $k_1$ and $k_2$ change just as before.

### 4.3.2 Special Case with Zero Viscosity

We now set the viscosity to zero and evolve the system as a consequence of resistive effects. Following the same derivation as before, we arrive at

$$b_x' + \frac{u_z}{\eta} \left( \frac{1 - m_z^2}{m_z^2} \right) b_x - \frac{1}{\eta} (k_2 - k_1) = 0.$$  \hspace{1cm} (4.19)
where

\[ k_2 - k_1 = \tilde{\eta} \frac{db_x}{dZ} \bigg|_1. \]

Here we have employed the only boundary condition available for the magnetic field, that is that the field is anchored in the intergalactic medium as proposed by Tullman et al. [15] \((b_x(1) = 0)\).

Solving equation 4.19 yields a simple function,

\[
b_x = -\frac{\tilde{\eta}}{u_z} \left( \frac{m_z^2}{1 - m_z^2} \right) \frac{db_x}{dZ} \bigg|_1 \left[ \frac{u_z}{1 - e^{-\tilde{\eta} \left( \frac{1 - m_z^2}{m_z^2} \right) (1 - Z)}} \right].
\]

### 4.3.3 Special Case with Zero Resistivity

We now set the resistivity to zero and evolve the system as a consequence of viscous effects. Following the same derivation as before, we arrive at

\[
b_x' + \frac{u_z}{\nu} \left( \frac{1 - m_z^2}{m_z^2} \right) b_x - \frac{1}{\nu} (k_2 - k_1) = 0
\]

where

\[ k_2 - k_1 = -\tilde{\nu} \frac{db_x}{dZ} \bigg|_0 - \frac{u_z}{\tilde{\nu}} \left( \frac{1 - m_z^2}{m_z^2} \right). \]

Solving the differential equation yields

\[
b_x = -\frac{\tilde{\nu}}{u_z} \left( \frac{m_z^2}{1 - m_z^2} \right) e^{-\frac{u_z}{\tilde{\nu}} \left( \frac{1 - m_z^2}{m_z^2} \right) \frac{db_x}{dZ} \bigg|_0} \left[ \frac{u_z}{1 - e^{-\tilde{\nu} \left( \frac{1 - m_z^2}{m_z^2} \right) (1 - Z)}} \right].
\]
4.3.4 Numeric Solutions

Under the conditions of Case 2 presented in Table 4.1, equations 4.17, 4.20, and 4.22 are solved numerically and plotted in Figure 4.5. With the field anchored at

![Figure 4.5: Magnetic field ratio curves.](image)

The scaled magnetic field ratio smoothly decreases to 0 under likely physical parameters of viscosity, resistivity, and Mach number. The red curve represents the system under normal conditions, the green curve represents the system with zero viscosity, and the blue curve represents the system with zero resistivity.

the normalized height by the intergalactic medium, the different curves indicate a different behaviour of the magnetic field ratio. First, under normal conditions, the horizontal component of the magnetic field needs to be about 8.5 times as strong as the vertical component (as indicated by the horizontal intercept of the red curve). In both a non-viscous and non-resistive medium (green and blue curve, respectively), there seems to be a field reversal with \(x\)-component strengths of \(-10\) and \(-22\) at
the plane. Note that these reversals are a consequence of further simplifying the toy model by assuming either no resistivity or no viscosity in the system.

These curves make sense - without viscosity, the magnetic field is dragged more intensely by the disk (does more of the shear reduction); more so without resistivity as the field strength drops much more quickly with height. The field reversal can be explained by the fact that when \( b_x < 0, B_x < 0 \) (assuming \( B_z > 0 \) always) and the magnetic stress in the \( x \)-direction across the \( z \) surface is given by \( B_x B_z / 4\pi \). Thus when \( b_x < 0 \), the stress acts to slow the disk down.

### 4.4 Magnetic Field Lines

Equations 4.17, 4.20, and 4.22 were integrated with respect to \( Z \) to yield the magnetic stream lines under normal conditions, with zero viscosity, and zero resistivity, respectively. The case under normal conditions was solved numerically with Maple 13 while the latter two cases were solved analytically and are presented below.

For zero viscosity,

\[
x = - \frac{\tilde{\eta}}{u_z} \left( \frac{m_z^2}{1 - m_z^2} \right) \frac{db_x}{dZ} \bigg|_0
+ \left[ Z + \frac{\tilde{\eta}}{u_z} \left( \frac{m_z^2}{1 - m_z^2} \right) e \frac{u_z}{\tilde{\eta}} \left( \frac{1 - m_z^2}{m_z^2} \right) \left( 1 - Z \right) \right] + C_1 \tag{4.23}
\]

and for zero resistivity,

\[
x = - \frac{\tilde{\nu}}{u_z} \left( \frac{m_z^2}{1 - m_z^2} \right) e - \frac{u_z}{\tilde{\nu}} \left( \frac{1 - m_z^2}{m_z^2} \right) \frac{db_x}{dZ} \bigg|_0
+ \left[ Z + \frac{\tilde{\nu}}{u_z} \left( \frac{m_z^2}{1 - m_z^2} \right) e \tilde{\nu} \left( \frac{1 - m_z^2}{m_z^2} \right) \left( 1 - Z \right) \right] + C_2. \tag{4.24}
\]
The resultant curves are plotted in Figure 4.6. The appropriate integration constants were chosen as to allow all three curves to have the same value at the plane. This value is irrelevant but the result displays how the field lines evolve from the same point given differing environments. All three field lines are vertical at the normalized height, which was the only imposed boundary condition. However as a result of the field reversal, the magnetic field lines under the special conditions behave more as we would expect. With the disk rotating in the positive $x$-direction, one would expect the field lines at the plane to be dragged in the negative direction as is the case with zero viscosity and zero resistivity. Under normal conditions, the field lines actually repel the plasma flow, perhaps indicative of a strong Lorentz restoring force or a viscous effect which essentially releases the magnetic field from the plasma. While

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure46.png}
\caption{Magnetic field lines. The \textbf{red} curve represents the system under normal conditions, the \textbf{green} curve represents the system with zero viscosity, and the \textbf{blue} curve represents the system with zero resistivity.}
\end{figure}
the behaviour of the magnetic field lines is uncertain, the model warrants a more sophisticated analysis with an MHD program; running a time-dependent simulation with relaxed (and more realistic) conditions such as non-uniform density and self-gravity interactions. This is presented in Chapter 6.

4.5 Comparison to Observations

The analysis so far has been performed with unit-less quantities. In order to compare the velocity gradient with physical observations, we must assign values to the normalized height, \( h \), and the rotational velocity at the plane of the galaxy, \( V \). From the studies mentioned in this chapter, the diffuse ionized gas falls to systemic velocity at a range of heights depending on the galaxy though the average gradient is constant and around \(-20 \text{ km s}^{-1} \text{ kpc}^{-1}\). The average rotational velocity of the spiral galaxies NGC 4157, NGC 3600, and NGC 2683 is 157 km s\(^{-1}\). Thus the average velocity gradient used in this analysis is \(-15.7 \text{ km s}^{-1} \text{ kpc}^{-1}\).

A kinematic analysis of the three galaxies mentioned above was performed by H. Kennedy [26]. In her dissertation, high-latitude arcs are analyzed individually and their velocity gradients are presented. NGC 4157 displays 10 arc-like features which have an average velocity gradient of \(-16.6 \text{ km s}^{-1} \text{ kpc}^{-1}\). NGC 3600 displays 6 arc-like features which have an average velocity gradient of \(-10.5 \text{ km s}^{-1} \text{ kpc}^{-1}\). NGC 2683 displays 7 arc-like features which have an average velocity gradient of \(-16.4 \text{ km s}^{-1} \text{ kpc}^{-1}\). Thus the imposed normalized values produce an accurate estimation of the velocity gradient. This further produces a steadily declining velocity function using Hartmann flow as a mechanism under physically realistic conditions.
Chapter 5

Numerical Techniques

Evolving any physical system with time requires computers to perform many calculations very quickly. Especially when working with a system of complex equations such as those presented in Chapter 2, the use of computers to simulate MHD conditions is paramount and provides a quick yet deep insight into the evolution of the system. The conditions we wish to simulate are those which promote gas outflow from the Galactic disk. In particular, supernova explosion and hydraulic jump mechanisms will be simulated using an MHD program: ZEUS-3D. In this Chapter, a brief overview of ZEUS-3D’s inner workings will be provided as well as the initial setups for the three mechanisms mentioned above. The results of these simulations are saved for the next chapter.

5.1 ZEUS-3D

At its core, ZEUS-3D is a three-dimensional ideal (non-resistive, non-viscous, adiabatic) non-relativistic MHD fluid solver which solves the following coupled partial
differential equations as a function of time and space:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{5.1}
\]

\[
\frac{\partial \mathbf{s}}{\partial t} + \nabla \cdot (\mathbf{s} \mathbf{v}) = -\nabla p - \rho \nabla \Phi + \mathbf{J} \times \mathbf{B} \tag{5.2}
\]

\[
\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{v}) = -p \nabla \cdot \mathbf{v} \tag{5.3}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \tag{5.4}
\]

where \(\rho\) is the matter density, \(\mathbf{v}\) is the velocity flow field, \(\mathbf{s}\) is the momentum density vector field \(= \rho \mathbf{v}\), \(p\) is the thermal pressure, \(\Phi\) is the gravitational potential, \(\mathbf{J}\) is the current density, \(\mathbf{B}\) represents the magnetic induction, and \(e\) is the internal energy density. Also implied by the above equations is Gauss’s law for a magnetic field: \(\nabla \cdot \mathbf{B} = 0\). The code enforces this condition at all times to within machine precision.

There exist other versions of the ZEUS code, modified by various users to suit their scientific curiosities, performing simulations from comet-planet collisions to cosmology. The roots of these variants can be traced to the original 2-D code developed by Jim Stone [55], Michael Norman [56], and David Clarke [57]. The version used in the simulations that follow was developed and maintained by David Clarke, whose help and guidance was integral in understanding and running the code.

While ZEUS was developed specifically with astrophysical processes in mind, it can still be used for various fluid problems. However, the code assumes charge neutrality at all times and thus can not act as a plasma code. In its earlier version, the code was strictly Newtonian; no relativistic astrophysics could be simulated. Since then, the code has undergone a number of revisions over two decades (this study uses version 3.6), one of these included the installation of a two-fluid approximation for a relativistic fluid. In fact, it was not until one of the more recent versions (3.4) that one could specify any kinematic viscosity term and appropriately adjust equations 5.1
- 5.4 to include viscous effects (basically, adding a viscosity term $\nu \nabla^2 v$ to equation 2.11). The dynamics of the simulations relevant to this study, however, do not require any relativistic treatment. So while there is a wealth of possibilities in using the code, we have restricted the simulations to the conditions of the toy model, a supernova blast, and a hydraulic jump.

The user interface is mostly confined to three files: `myprob`, `zeus36.mac`, and `dzeus36.s`. The first file is where the user can create an initialization subroutine. That is, this is where one sets up the physical problem with the appropriate density, velocity, and magnetic profiles and atmospheres, creates the flow variables and parameters, and inputs the subroutine into the ZEUS code, where the problem is initiated.

The next file, `zeus36.mac`, is where the user specifies the physical processes to be included in the simulation (MHD, self-gravity, two-fluid, etc), the solving algorithms, and the problem to be initiated (usually whatever the user has named the subroutine contained in `myprob`) as well as the symmetry of the system (cartesian, cylindrical, spherical) and maximum error allowances.

The user will probably spend most of their time in the script file, `dzeus36.s`. It is here where the user creates the grid and specifies not only the boundary behaviours, but also the files to be output such as 1-D plots, 2-D slices, or 2-D integrated line of sight plots, 3-D images, etc. It is also here that the user specifies the computing time, the maximum number of iterations, the hydrodynamic controls (sets the parameters which control the hydrodynamics - most notably, whether to solve the internal (equation 2.19) or total energy equation (equation 2.21)), the equation of state, among many others. Care must be taken here, as an error in the controls or calculation of the time unit could lead to very unexpected (and wrong) results. All three initialization files are presented in Appendix A.

It is also necessary to ensure that the problem being undertaken can be solved completely by equations 5.1 - 5.4.
5.2 Setup and Initial Conditions

5.2.1 Toy Model

The setup for the toy model is explained in detail in Chapter 4. Here, the initial conditions, as used in the numeric simulations, are outlined. A 3-D Cartesian grid is set up by 100 x 20 x 100 zones corresponding to the $x$, $y$, and $Z$ directions, respectively. A zone is where a calculation of each variable (such as the velocity, the magnetic field, and whatever else the user specifies in the script file) is made at each time step. The disk with a height of 0.03 length units (where 1 length unit is defined as 10 kpc) is represented as a velocity flow in the $x$-direction. The $Z$-axis is perpendicular to the disk. This is essentially an open box which is fixed along a spiral arm as gas flows through in the positive $x$ direction. The use of a spiral arm is not significant for the simulation, however it a more realistic setting as it is where we observe small-scale vertical perturbations (from supernovae and fast inflowing gas) which are inherent in the model ($u_z$). The simulation acts as a stabilization run where the velocity and magnetic profiles are taken from equations 4.10 and 4.18. Further, we wish to test the stability based on the conditions of Case 2, which produced a smoothly declining velocity curve consistent with that of observations. The system is evolved under these conditions noting again that ZEUS is an ideal MHD solver and has no resistive component to it (although artificial viscosity can be simulated). This is the main motivation behind using the numerical profiles given in Chapter 4 - the ZEUS-3D code simply can not properly simulate resistive MHD (without microsurgery to the underlying physics).

The inner and outer $x$-boundary conditions produce an inflowing and outflowing gas flow, respectively as representative of a fixed box in space with the spiral arm flowing through it. The outer $Z$-boundary is reflective with $B$ continuous across the boundary whereas the inner $Z$-boundary is both reflective and conducting to better
simulate the conducting disk where the flux-freezing principle is obeyed. This is consistent with the Hartmann flow model with the conductive boundary representing the disk. The initial density profile is taken to be uniform. Given that one length unit is 10 kpc and the average rotation velocity \( V \) is of the order of 200 km s\(^{-1}\), one time unit corresponds to about 50 Myrs (50 million years).

The initial conditions are solved numerically and are presented in Figures 5.1 - 5.3 which show the density contours (with poloidal velocity vectors), the gas velocity in the \( x \)-direction \( (u_x) \) from Chapter 4), and the tangent of the polarization angle \( (b_x) \) from Chapter 4). The density is presented in units of hydrogen masses per cubic centimetre and is initially uniform. The velocity is presented as \( u_x \) which has units given by the rotation velocity at the midplane, \( V \). The evolution of this system is traced over 250 Myr (5 time units) which corresponds to about one revolution of the Sun about the Galactic centre. Figure 5.1 shows the fixed-box with the gas initialized according to Case 2. This is also the conditions under which the gas flows into the fixed box. So, despite the fact that ZEUS-3D is non-resistive, we force this condition at the inflow boundary (all gas flowing into the fixed box is resistive as described by Case 2). Note the localized \( u_z \) around \( x = 0.5 \). This is meant to represent a small-scale vertical perturbation along the spiral arm.

### 5.2.2 Supernova Blast

In a supernova, an energy of order \( 10^{51} \) erg (\( 10^{44} \) Joules) is released into the interstellar medium. An expanding spherical blast wave is formed as the explosion sweeps up the surrounding gas. Several good examples of these supernova remnants are observed in the Galaxy. The effect is similar to a bomb. We suppose that an energy \( E \) is released at \( t = 0, r = 0 \) and that the explosion is spherically symmetric. The external medium has density \( \rho_0 \) and pressure \( p_0 \). In the Sedov Taylor phase of the explosion, the pressure \( p \gg p_0 \). Then a strong shock is formed and the external pressure \( p_0 \) can
be neglected (formally set to zero). The shock is at \( r = R(t) \) and has a speed \( \dot{R} \), where the radius as a function of time is given by Sedov [58]:

\[
R(t) = \left( \frac{E}{a \rho} \right)^{1/5} t^{2/5} \tag{5.5}
\]

where \( E \) is the energy deposited at \( r = 0 \) and \( t = 0 \), \( \rho \) is an initial uniform density, \( a \) is a unitless constant (=0.49 for \( \gamma = 5/3 \), where \( \gamma \) is the ratio of specific heats), and \( t \) is the time since the explosion. The total energy of the explosion is given by

\[
E = \int_0^R \left( \frac{1}{2} \rho u^2 + \frac{p}{\gamma - 1} \right) 4\pi r^2 dr \tag{5.6}
\]

where \( u \) is the flow velocity.

A true point explosion is impossible to set up numerically. Thus, for example, one can deposit an energy \( E \) into a sphere of radius \( r = 0.1 \) resolved with 2.5 zones (and

\[\text{Figure 5.1: Toy model initial density contours with poloidal velocity vectors. The minimum velocity shown is } 0V \text{ and the maximum is } 0.964V.\]
thus a volume of about 65 cells) embedded in a box of $L = 10$ resolved with $250^3$ zones. By $t = 0.01$, $R(t) \approx 4.6$.

For the initial conditions, we deposit an energy $E \approx 10^{51}$ erg into a sphere of density $\rho_0 \approx 10^{-24}$ g cm$^{-3}$. The radius of the sphere is given by equation 5.5 and is equal to about 6 pc at $t = 1000$ yr with an expansion rate $\dot{R} = 3000$ km s$^{-1}$. This is the point at which we begin the simulation. In unit-less quantities, one density unit is equivalent to the mass of a hydrogen atom ($m_H = 1.67 \times 10^{-24}$ g) per cubic centimetre and thus the density within the spherically propagating shock front is equal to about $0.6$ m$_H$ cm$^{-3}$. The scale length chosen is 1 kpc. Using equation 5.6, the energy deposited in one supernova blast is $1.9 \times 10^{-7}$ kpc$^5$ m$_H$ cm$^{-3}$ (0.3 Myr)$^{-2}$. With $\dot{R} = 3000$ km s$^{-1}$ and 1 length unit defined as 1 kpc, one time unit corresponds to 0.3 Myr. The length resolution, however, is 0.01 kpc (1 kpc/100 zones). Finally, a magnetic field of strength $5 \mu$G acting in the azimuthal direction with a $2 \mu$G field acting in the vertical direction is added to the system given by the local magnetic
CHAPTER 5. NUMERICAL TECHNIQUES

Figure 5.3: Toy model initial contours of the tangent of the polarization angle.

Figure 5.4 depicts the density profile adopted from Martos and Cox [1] (see equation 5.12) with the blast sphere located at \((x, z) = (1, 0.1) \text{ kpc}\). All boundaries represent free outflow in this model.

Given that the shockwave of a supernova blast propagates for up to 100,000 years, the simulation was run for 0.5 time units. As will be seen in the next chapter, one supernova blast is not sufficient to promote any significant density perturbations or vertical structures. Thus multiple blasts can be simulated but one should not inject an unphysical amount of energy into a blast radius. The frequency of supernovae varies with the star formation rate (SFR) in a linear fashion as described by equation 5.7 [59],

$$\frac{\nu_{SN}}{\text{yr}^{-1}} \approx 0.041 \left[ \frac{\text{SFR}}{M_\odot \text{yr}^{-1}} \right]. \quad (5.7)$$

The SFR itself varies as much as a factor of 1000 between galaxies, as shown in Figure 5.5 taken from Kennicutt (2008) [18] which uses a 417 galaxy sample. Thus
the frequency of supernovae ranges from approximately 400 to 400,000 every 100,000 years within the entire galaxy (again, depending highly on the SFR of the galaxy). Since we are evolving the system over this timeframe, this puts an upper limit on the number of supernovae that one can deposit into the simulation. Given that we are ejecting energy into a bubble with a radius of 0.1 kpc, the volume of this bubble is approximately 0.4% of the total volume of a galaxy. Thus, the upper limit of SN within the conditions of the simulation reduces to 1675 for a galaxy with a high SFR. To justify confining this many supernovae into a single bubble, we note that the SNe must occur close in time compared to the time scale of the vertical structures, which is to the order of $10^7$ yr. The study by Kennicutt [18] refers to all stellar masses whereas the SNR applies only to massive stars. Correcting for this changes the result by only a factor of 2 [60]. Moreover, these observed vertical gas filaments/loops are not exclusive to star forming (and dying) regions. Perhaps the cause is through hydrodynamics independent of stellar evolution. This has been the subject of many

**Figure 5.4:** Supernova blast initial density contours with poloidal velocity vectors.
Figure 5.5: Comparison of integrated SFRs derived from integrated IRAS 25 μm plus Hα luminosities and reddening-corrected Hα luminosities, for a sample of star-forming galaxies. Taken from Kennicutt 2008 [18].

recent studies ([1], [61], [33], [34]) and we follow one of those studies in the next section.

5.2.3 Hydraulic Jump

The study presented in this section follows that of Martos and Cox [1]. In it, the authors describe a rotating spiral gaseous disk (around the solar neighbourhood - 8.5 kpc from the Galactic centre) with an interarm-to-arm gas flow. This inflow velocity, named $v_{\text{entry}}$, is greater than the gravity wave velocity, $v_{gw}$, which is a function of the perpendicular distance from the midplane, $z$,

$$v_{gw} = [zg(z)]^{1/2}. \quad (5.8)$$

The gravity wave velocity is similar to that of shallow water velocity for river bores on Earth with the difference being that the speeds of compressional sound and gravity
waves are likely to be comparable for a wide range of heights above the midplane for the Galactic disk. When $v_{\text{entry}} > v_{gw}$, a hydraulic jump is predicted by the mass and momentum conservation equations from Chapter 2:

\[(\sigma v)_1 = (\sigma v)_2 \quad (5.9)\]

and

\[\phi_2 - \phi_1 = (\sigma v)(v_1 - v_2), \quad (5.10)\]

respectively where $\sigma = \int_0^\infty \rho dz$, $\phi = \int_0^\infty pdz$, and $dp/dz = -\rho g$. The $\phi$ term comes from the conservation of momentum where $\nabla \cdot (\rho \mathbf{v} \mathbf{v} + p \mathbf{I}) = \oint (\rho \mathbf{v} \mathbf{v} + p \mathbf{I}) \cdot ds$, where $s$ is a surface. The subscripts 1 and 2 represent the two sides of the interface. With some substitutions and assuming that the density distributions on the two sides have the same functional shape (i.e., that they are isomorphic), they arrive at

\[\left(\frac{Y^\alpha - 1}{Y - 1}\right) = \frac{v_1^2}{\beta_1 g' h_1^2}, \quad (5.11)\]

where $Y = \sigma_2/\sigma_1$ is the enhancement of the surface density, $\beta$ is a shape factor, $h$ is a scale height, and $\alpha = (3\gamma - 1)/(\gamma + 1)$. They adopt $v_{gw} \approx [zg(z)]^{1/2}$ as an estimate of the speed of the gravity waves where $g(z)$ is taken from Bienayme et al. [62]. The density profile (used in the previous section) is $\rho(z) = 1.27 m_H n(z)$ where $m_H$ is the mass of a hydrogen atom and $n(z)$ is the number density given by

\[n(z) = 0.6 e^{-z^2/[2(70 \text{ pc})^2]} + 0.3 e^{-z^2/[2(135 \text{ pc})^2]} + 0.07 e^{-z^2/[2(135 \text{ pc})^2]} + 0.1 e^{-|z|/400 \text{ pc}} + 0.03 e^{-|z|/900 \text{ pc}} \quad (5.12)\]

in units of cm$^{-3}$. As mentioned earlier, the different terms correspond respectively to the contributions of $\text{H}_2$, cold $\text{H}_I$, warm $\text{H}_I$ in the clouds, warm intercloud $\text{H}_I$, and warm diffuse $\text{H}_II$. 

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**CHAPTER 5. NUMERICAL TECHNIQUES**
They calculate an entry velocity of approximately 16 km s$^{-1}$ based on the relative speed between the gas and arm of 86 km s$^{-1}$ at 8.5 kpc from the Galactic centre and a local pitch angle of 11° (taken from Heiles [63]).

The authors ran 2-D simulations over about 300 Myr varying the entry velocity, the direction of the magnetic field, and the grid boundary conditions. Here, we follow Case 28 from their study, with the magnetic field acting in the azimuthal direction (the only orientation to produce a stable hydrostatic equilibrium) and the inner/outer-z boundary being reflecting (and conducting)/open (promoting outflow). We, however, present a 3-D simulation of a Galactic bore under the conditions given by the above study. The authors also neglect self-gravity in their model, the results of which are presented in the next chapter. The description of self-gravity is analogous to that of electrostatic potential. The electric field $\mathbf{E}$ formed by a point charge $Q$ at a distance $r$ from the point source is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}. \quad (5.13)$$

Similarly, the gravitational acceleration due to a point mass $M$ at a distance $r$ is

$$\mathbf{g} = -G \frac{M}{r^2} \hat{r}. \quad (5.14)$$

Equation 5.14 is structured similarly to equation 5.13 with $-G$ the same as $1/4\pi\epsilon_0$ and $M$ the same as $Q$. With this, we can make inferences as to the divergence of the gravitational acceleration as Gauss’s Law for an electrostatic field is

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}, \quad (5.15)$$

where $\rho_e$ is the charge density. Another way to present equation 5.15 is to use an electrostatic potential $\phi_e$,

$$\nabla^2 \phi_e = -\frac{\rho_e}{\epsilon_0}. \quad (5.16)$$
The above equations allow us to infer the gravitational equivalents as

\[
\nabla \cdot \mathbf{g} = 4\pi G\rho
\]

(5.17)

and

\[
\nabla^2 \phi = 4\pi G\rho
\]

(5.18)

where \( \rho \) is the mass density distribution. Equation 5.18 is Poisson’s equation and describes how the gravitational potential, \( \phi \), is determined from the mass density distribution \( \rho \).

The initialized problem is presented in Figure 5.6 as a density plot. The disk is centred at \( z = 0 \) again with a fixed box model open at both ends of the \( x \)-axis, representing free gas flow through the box as the galaxy rotates.

**Figure 5.6:** Initial jump conditions - density contours with poloidal velocity vectors.
## 5.2.4 Summary

Table 5.1 summarizes the initial conditions for the three mechanisms described in this chapter.

<table>
<thead>
<tr>
<th></th>
<th>Toy Model</th>
<th>Supernova Blast</th>
<th>Hydraulic Jump</th>
</tr>
</thead>
<tbody>
<tr>
<td>inner/outer-x boundary</td>
<td>in/out</td>
<td>out/out</td>
<td>in/out</td>
</tr>
<tr>
<td>inner/outer-z boundary</td>
<td>conducting/continuous</td>
<td>out/out</td>
<td>conducting/out</td>
</tr>
<tr>
<td>zones ((x, y, z))</td>
<td>100, 20, 100</td>
<td>100, 20, 100</td>
<td>250, 100, 250</td>
</tr>
<tr>
<td>(x_{\text{min}}/x_{\text{max}})</td>
<td>0/1</td>
<td>-5/5 kpc</td>
<td>0/3 kpc</td>
</tr>
<tr>
<td>(z_{\text{min}}/z_{\text{max}})</td>
<td>0/1</td>
<td>-5/5 kpc</td>
<td>0/3 kpc</td>
</tr>
<tr>
<td>density (\text{m}_H \text{ cm}^{-3})</td>
<td>1.0 (uniform)</td>
<td>1.27\text{m}_H \times \text{eq. 5.12}</td>
<td>1.27\text{m}_H \times \text{eq. 5.12}</td>
</tr>
</tbody>
</table>

**Table 5.1:** Summary of initial conditions for gas-ejection mechanisms.
Chapter 6

Results and Discussion

The mechanisms described in the previous chapter are run through week-long simulations with ZEUS-3D. The results of which are what follows. To gain appreciation for the magnitude of these computations, Table 6.1 shows how long each simulation takes to complete.

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Simulation time (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toy Model (uniform density)</td>
<td>6</td>
</tr>
<tr>
<td>Toy Model (non-uniform density)</td>
<td>13</td>
</tr>
<tr>
<td>Supernova Blast</td>
<td>4</td>
</tr>
<tr>
<td>Hydraulic Jump</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of simulation times for the three mechanisms explored in this report.

6.1 Toy Model

6.1.1 Uniform Density

The system described in Section 5.2.1 is evolved over 5 time units (250 Myr). Significant mass loss occurs over the simulation period (over 40% mass loss, see Figure
6.1) resulting in a less dense (but non-uniform) profile. Mass loss is indicative that the inflow rate is less than the outflow rate through the fixed box. This is perhaps due to a pressure build up at the inflow boundary which causes the gas to speed up as it moves through the box. Figure 6.2 represents the density contours after 250 Myr. The gas flows in along $x = 0$ according to the initial conditions presented in the previous chapter.

The velocity profile shown in Figure 6.3 indicates that the initial gas velocity at the disk, $V$, increases as time evolves but decreases azimuthally. While the velocity decrease is still relatively stable, it does not decrease fast enough to reach 0 as predicted in the analytic model. Again, this increase in gas velocity could be a consequence of pressure building up at the inflow boundary.

While the velocity gradient is negative and more or less stable, it still predicts an unphysical value for the disk rotation velocity ($u_x(0) > 1$). We should be wary when comparing these gradients to observations due to this anomaly which the simulation predicts.
Figure 6.1: Toy model time evolution of the total mass.

Figure 6.2: Toy model density contours with poloidal velocity vectors at $t=5$. The minimum velocity shown is $0.183V$ and the maximum is $1.977V$. 
Figure 6.3: Toy model $u_x$ contours at t=5.

The magnetic field ratio ($B_x/B_z$) shown in Figure 6.4 stays relatively stable over time however, it does drop more quickly with height. At $t = 5$ units, there is a magnetic field reversal at about $Z = 0.8$. This is more likely an indication of a magnetic loop where $B_x$ changes direction ($B_z$ is a positive constant in the simulation). However, this could also be an indication of the simulation becoming unstable. If, indeed, there are magnetic loops extending to heights of $0.8h$, then that would show a correlation between the height at which the rotational velocity goes to 0 (which varies by galaxy) and the magnetic field properties in the halo.
Finally, it is interesting to look at the evolution of the total Alfvénic Mach number ($M_A = v/(B/\sqrt{4\pi\rho})$), shown in Figure 6.5. Recall that in the analytic model, a Mach number greater than unity produced an unstable velocity profile which increased with height. Therefore we determined that $m_z^2 < 1$, and indeed, the most stable model was produced with $m_z^2 = 0.69$. As the model evolves with time, however, the flow velocity becomes superalvénic. Upon comparing this to Figure 6.4, the stream of high Mach number contours traces the junction at which the magnetic field reverses direction, suggesting a large increase in $m_x^2$. The Mach number is about constant after 250 Myr at 1.173. Since the density of the system is decreasing, this is indicative of either an increasing flow velocity or a decrease in the magnetic field strength. The simulation seems to predict the former as demonstrated by the increase in flow velocity in Figure 6.3.
The kinetic energy, $E_K$, of the system stays relatively constant during the simulation. The magnetic energy, $E_{mag}$, is also dominated by the azimuthal gradient in $b_x$, decreasing for the first 50 Myr or so and steadily increasing for the rest of the simulation with an indication of reaching an equilibrium after 200 Myr ($t = 4$) shown by an increasingly horizontal energy curve. Figure 6.6 displays the energy components as well as the magnetic energy. The subscripts 1, 2, and 3 represent the x, y, and z components, respectively. The kinetic and magnetic energies are given by equations 6.1 and 6.2, respectively, the latter derived in Introduction of Electrodynamics [64]:

$$E_K = \frac{1}{2} \rho v^2$$  \hspace{1cm} (6.1)

and

$$E_{mag} = \frac{1}{2\mu_0} \int B^2 d\tau.$$  \hspace{1cm} (6.2)

The $v$ term in equation 6.1 represents the root-mean-squared (rms) speed and the
integral in equation 6.2 is over all space. Though the molecules in the gas exhibit a
distribution of speeds, the rms speed acts as an average and is calculated by equation
6.3,
\[ v_{rms} = \sqrt{\frac{3kT}{m}} \]  (6.3)
where \( m \) is the mass of one molecule of the gas, \( T \) is the temperature, and \( k \) is
Boltzmann’s constant \( (1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}) \). Note that equation 6.1 represents
the kinetic energy per unit volume which is analogous to the dynamic pressure, \( p \).

The energy steps could also be due to a pressure fluctuation as previously men-
tioned.
CHAPTER 6. RESULTS AND DISCUSSION

Figure 6.6: Top: Time evolution of the kinetic energy components. Bottom: Time evolution of the magnetic energy components.

From the simulated density and velocity profiles, the analytic toy model does not
CHAPTER 6. RESULTS AND DISCUSSION

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seem very plausible. The simulation predicts a large density increase at high $Z$ values which decreases along $x$ over time (see Appendix B for density contours at various time units). Further, an unphysical increase in $u_x$ is predicted by the simulation (doubling by $t = 5$ at low $Z$) despite the decrease in velocity with $Z$ maintaining semi-stability. However, we note that $u_x$ does not fall to systemic at high $Z$ except for at low $x$ values, where there is gas inflow that matches the analytic model anyway.

Recall that ZEUS-3D is non-resistive. Though we imposed resistive initial conditions which flowed into the fixed box at every time step, the system had evolved under no resistivity from one end of the fixed box to the other. Figures 6.7, 6.8, 6.9, and 6.10 show the system’s density, $u_x$, $b_x$, and $M_A$ profiles, respectively for the special case described in Sections 4.2.3 and 4.3.3 where $\eta = 0$. As before, there is mass loss however the $u_x$ and $b_x$ profiles reach an equilibrium after 150 Myr but the $u_x$ profile predicts an unphysical disk velocity ($u_x > 1$ at $Z = 0$). The Mach number is less than unity as predicted, however it is very small except at heights close to $Z = 1$.

Figure 6.7: Toy model special case with zero resistivity and initialized uniform density - density contours at $t = 5$. 
Figure 6.8: Toy model special case with zero resistivity and initialized uniform density - $u_x$ contours at $t = 5$.

Figure 6.9: Toy model special case with zero resistivity and initialized uniform density - $b_z$ contours at $t = 5$. 
Finally, we consider multiple perturbations averaged along the length of the arm. Thus, $u_z$ is no longer localized but extends over the $x$-axis from $x = 0.1$ to $x = 0.9$. This results in the disk maintaining its stability over the simulation which was not found in the previous cases. We also note that mass is not lost during this simulation. The velocity profile is shown in Figure 6.11.
6.1.2 Non-Uniform Density

We now initialize the density profile to match that given by Martos and Cox (see equation 5.12). This will provide a more realistic view of the effects of Hartmann flow in a galactic environment which is not uniform in density. The system is allowed to evolve over 250 Myr as before and the resulting density, velocity, $b_x$, and Mach number profiles are presented below.

Interestingly, the density profile (Figure 6.12) shows a density perturbation similar to a hydraulic jump at 2 kpc ($x = 0.2$) which extends to a height of 1 kpc. The gas outflow velocity at the jump is $0.42V$ which is equal to 84 km s$^{-1}$, more than double the initialized $u_z$. We note, however, that this may be a consequence of the modelling procedure; gas flows into the fixed box at a rate governed by the initial conditions. If the flow velocity decreases with $x$, then a faster flow rate from $x = 0$ (the inflow rate) would produce conditions for a hydraulic jump.

Figure 6.11: Toy model $u_x$ contours with $u_z$ averaged along the $x$-range.
Figure 6.12: Toy model initialized with non-uniform density profile - density contours after 250 Myr. The maximum velocity vector corresponds to $1.784 V$.

Figure 6.14 depicts the rotational velocity profile which seems to increase with height until about $Z = 0.5$, and then falls to zero at about $Z = 0.75$. Recall that this was the velocity profile predicted for Mach numbers greater than unity. The velocity profile at low values of $x$ is also peculiar as there is a very sudden increase in the rotational velocity at the scale height (top of the halo). This may be a consequence of the fact that the gas inflow at high $z$ is very small (close to zero) and that the boundary is closed at $Z = 1$. Though the disk velocity is rather stable (staying less than 1), the region of high velocity decreases in value and height over time (see Figure 6.13 for a $u_x$ contour plot at $t = 0.004$ (5 Myr)). It is suspected, therefore, that the region of high velocity eventually falls to values lower than unity and the velocity profile shows a steady velocity at low galactic latitudes followed by a quick decrease to systemic, perhaps overshooting, at high latitudes.
Figure 6.13: Toy model initialized with non-uniform density profile - $u_x$ contours after 5 Myr.

Figure 6.14: Toy model initialized with non-uniform density profile - $u_x$ contours after 250 Myr.
The Alfvenic Mach number profile is shown in Figure 6.15 and corresponds to the jump in the density profile where the Mach number is greater than unity. Everywhere else in the disk has a Mach number less than unity and the halo maintains a near null Mach number profile. This means that the flow velocity is much lower than the Alfven wave speed above the disk.

![Mach number contours after 250 Myr.](image)

**Figure 6.15:** Toy model initialized with non-uniform density profile - Mach number contours after 250 Myr.

Finally, $b_z$ ($\tan \theta = B_x/B_z$) behaves much as predicted by the analytic model, with the field decreasing with height to approximately zero at the scale height as depicted by Figure 6.16. This profile remains relatively stable throughout the simulation.
We now investigate the special case where $\eta = 0$ with a non-uniform density profile. Figure 6.17 shows a peculiar jump, similar to Figure 6.12 except in the opposite direction. The $u_x$ profile becomes unstable and, as shown in Figure 6.18, is very complex. The $b_x$ profile, however, remains stable throughout the simulation due to the fact that there is no resistivity in the system. As was the case with the uniform density profile, the total Mach number becomes very low throughout most of the system, however here it drops even more, to 0.24, and peaks at the disk (low $Z$) rather than near the systemic level.

**Figure 6.16:** Toy model with non-uniform density profile - $b_x$ contours after 250 Myr.
Figure 6.17: Toy model special case with zero resistivity and initialized density profile taken from Martos and Cox [1] - density contours at $t = 5$.

Figure 6.18: Toy model special case with zero resistivity and initialized density profile taken from Martos and Cox [1] - $u_x$ contours at $t = 5$. 
Figure 6.19: Toy model special case with zero resistivity and initialized density profile taken from Martos and Cox [1] - $b_x$ contours at $t = 5$.

Figure 6.20: Toy model special case with zero resistivity and initialized density profile taken from Martos and Cox [1] - $M_A$ contours at $t = 5$. 
With a non-uniform density initialization, the toy model is more plausible than if one initializes a uniform density profile. The velocity profile, however, evolves very strangely. Shortly after the simulation begins, the initialized velocity profile quickly destabilizes, adopting a very high flow velocity region at $Z = 1$. Recall that this was predicted for cases where $m_z^2 > 1$. After time, however, this region decreases in value and height. It is predicted that this trend continues until the velocity drops to or below unity and attains a decreasing pattern with height. A longer simulation would need to be performed in order to confirm this prediction, however the evidence at hand suggests that Hartmann flow can be used to test the effects of the field and viscosity on shear flow and that the observed negative velocity gradient is possible under these conditions. Evolving the system with non-resistive flow does not seem to match the shear flow predicted by the analytic model in Chapter 4.

This evolution is strange and is due, in part at least, to the pressure build up. However, this instability dissapears when we consider multiple vertical perturbations averaged over the $x$-range and used a uniform density profile. Almost no mass is lost over the time of the simulation which indicates an absence of the pressure problem described earlier.

### 6.2 Supernova Blast

A single supernova blast of energy $E = 10^{51}$ erg was deposited into a sphere with a radius of 2.4 pc and evolved over 0.15 Myr. The effect on the galactic system is negligible - there is no observed density perturbation and the maximum vertical gas flow velocity is to the order of $3.1 \times 10^{-6}$ km s$^{-1}$. Figure 6.21 depicts the density profile of the system after 0.15 Myr. There was no change in either the energy density or the magnetic field contour plots.
Figure 6.21: A single supernova blast density contour plot after 0.15 Myr. The maximum velocity vector corresponds to $3.1 \times 10^{-6}$ km s$^{-1}$

Clearly, one supernova blast is insufficient to create any vertical structures. The question remains: how many supernovae are necessary in order to promote gas outflow from the disk and is this a reasonable number to be confined to a spiral arm? Presented in Figure 6.22 is the density contour with the energy equivalent of 500 supernovae contained within a 100 pc radius bubble and evolved over 0.15 Myr. Recall that the upper limit of SN in this bubble was around 1600.
Figure 6.22: 500 supernova blasts - density contour plot after 0.15 Myr. The maximum velocity vector corresponds to $1.3 \times 10^2$ km s$^{-1}$.

Hundreds of supernovae are needed to promote gas outflow from the galactic disk. With 500, the gas flows into the halo at a relatively high speed, $130$ km s$^{-1}$, and extends to heights under a kiloparsec. Perhaps more interesting are the magnetic field contours shown in Figure 6.23. The figure depicts a magnetic loop created by the shock front of the blast which extends well into the halo up to $5$ kpc. The scale is presented in units of $\mu$G. Observations near the galactic centres of spiral galaxies indicate that these magnetic loops can extend far into the halo up to $8.5$ kpc [4].
As described in Chapter 6, we assigned an upper limit of $10^5$ supernova blasts within a galaxy. It is very unlikely that this many supernovae would erupt all along a single arm over such a small time frame, thus we adopt an upper limit given the size of the bubble containing the supernovae to 1500. The purpose of including this many SN in the simulation is to gauge how high the gas will flow into the halo. The resulting density contour plot after 0.15 Myr is shown in Figure 6.24.
Figure 6.24: 1500 supernova blasts - density contour plot after 0.15 Myr. The maximum velocity vector corresponds to $6.7 \times 10^3$ km s$^{-1}$.

A spherically propagating shock wave is predicted for this amount of energy which is powerful enough to blow gas from the disk up to about 1 kpc from the disk. Note that these plots are 2-D slices (through $y = 0$) and are not integrated through the line of sight. Thus Figure 6.22 only depicts the gas flow in the $x$-$z$ plane described by $y = 0$ and is not necessarily indicative of what is observed. However, since the density increase at high $z$ is substantial, the shock fronts at high $z$ would feasibly be observed.

As the gas is compressed due to the shock, the magnetic field lines frozen into the plasma are also compressed, thus an increase in magnetic field strength is expected. Indeed, Figure 6.25 shows an increase in the magnetic field strength at the shock fronts extending in the $z$ direction (the path of least resistance). The scale is again in units of $\mu$G.
Figure 6.25: 1500 supernova blasts - total magnetic field contour plot after 0.15 Myr.

Supernova blasts are a viable mechanism for gas outflow from the galactic disk, though hundreds are needed to achieve this. From the simulations presented above, it seems that a likely number of high mass stars exploding within the same region within 1000 years does not eject gas from the disk with a height much greater than a kiloparsec (see Figure 6.22). Obviously, a higher number of supernovae result in a higher flow velocity and the shock front carries the magnetic field lines with it, creating magnetic loops which extend into the halo to high galactic latitudes. Though with a high enough number of supernovae, the magnetic field is blown from the disk, leaving a rather large region of low field strength in the disk.
6.3 Hydraulic Jump

Gas outflow is not necessarily restricted to star forming regions. The study performed by Martos and Cox was replicated using ZEUS-3D. In this study, however, we include the effects of self gravity and perform the simulation in a 3-dimensional grid. Figure 6.26 represents the density contours in the $x-z$ plane with $y = 0$ after 167 Myr ($t = 5$). It compares the density contours of the study performed by Martos and Cox to the study performed using ZEUS-3D including self-gravity. The current study predicts a higher maximum vertical gas velocity than that of Martos and Cox - 41 km s$^{-1}$ compared to 29 km s$^{-1}$ reported by Martos and Cox. The jump in our simulation reaches a greater height and has a thinner distribution with $x$ (normal to the galactic arm and parallel to the plane of the disk).

The results are consistent when evolved in 2-D with self-gravity not considered, however it is clear that both self-gravity and modelling in 3-D have an impact on the jump height and shape. It also shows a higher gas velocity in the vertical direction which is consistent with our approximation of the vertical gas velocity, $u_z$, in the toy model. These effects are more likely due to the inclusion of self-gravity in the simulation which hinders the diffusion of mass density causing a sharper, more defined jump. As the mass is less diffuse, the jump strength is greater, causing a larger vertical velocity.

Observations have shown that the outflow velocity is as high as 100 km s$^{-1}$ [24]. All mechanisms presented in this report suggest an outflow velocity within this range.
Figure 6.26: Hydraulic jump density contour plot after 167 Myr. **Top:** A 2D reproduction of Case 28 from Martos and Cox. The maximum vertical velocity is 29 km s\(^{-1}\) [1]. **Bottom:** Study performed in 3D with self-gravity included. The maximum vertical velocity is 41 km s\(^{-1}\). 
Chapter 7

Conclusions

In this dissertation, we have explored three mechanisms that promote gas outflow from the disk. Under the constraints of MHD, each mechanism was modelled and tested using an MHD equation solver: the ZEUS-3D code. This chapter will summarize the essential results of the three models and comment on their practicality and usefulness.

The toy model was employed to represent the lagging halo by assuming a Hartmann flow mechanism. To begin, we treated the system as a time-independent, uniformly dense, viscous, and resistive fluid with a magnetic field acting azimuthally and perpendicular to the plates. We enforced boundary conditions to match the lagging halo observations; that is, the rotational velocity fell to zero at some height $h$. The system of equations constituting the equation of motion and the MHD induction equation was solved analytically using likely physical values for the viscosity and resistivity parameters resulting in a smoothly declining velocity function. A key note was made that the mach number, $m_z^2$, had to be less than unity. Otherwise, the velocity function actually increased with height and then was forced to zero by the boundary conditions. Upon solving the set of equations for the magnetic field lines, however, we encountered an unusual result: the field lines were being sheared in the direction opposite to rotation. Further analysis of the toy model was made, solving
the system of equations again but under the condition that viscosity/resistivity be set to zero. The resulting velocity curves dropped much more quickly and the field lines were sheared with the disk movement as expected.

The stability of the model was then tested with a time-dependent simulation where the disk was free to rotate through a box fixed in space. Significant mass loss was observed over the simulation due to a net velocity flux out of the box. Because of the velocity profile adopted from the analytical model, a build up of density was predicted at high \( z \). The velocity profile became destabilized over time, with the disk reaching an unphysical velocity. This is thought to be a consequence of pressure build up at the inflow boundary which accelerates the gas and causes unusual evolution. The system adopted a uniform mach number (which was greater than unity) except at high galactic latitudes where a magnetic field reversal was observed. Obviously, the toy model could not stand the test of time, perhaps because true galaxies do not have a uniform density profile. Therefore, we tested the toy model on a more physical and realistic density profile which took into account the warm and cold \( \text{H}_I \) and \( \text{H}_II \) regions above the disk. We ran this through the same simulation, the resulting density profile showed properties similar to a hydraulic jump. The velocity profile, however, immediately becomes unstable and adopts a profile very similar to that observed in the analytical model with \( m^2 z^2 \gg 1 \). Over time, however, the velocity settles as consistent with a decreasing mach number. It is expected that the profile eventually reaches an equilibrium with \( m^2 z^2 < 1 \) given its evolution pattern over time.

Given the fact that ZEUS-3D is a non-resistive MHD solver, we tested the special case where \( \eta = 0 \) for both a uniform and non-uniform density profile. The system reached an equilibrium with a uniform density profile, however the gas flow at the disk was unphysical. When a non-uniform density profile was adopted, the simulation became unstable, as it did before. The \( u_x \) profile’s evolution was very complex and a jump was observed, somewhat like before, only in the opposite direction. While
the $b_x$ profile remained stable, the total Mach number dropped to very low values, peaking at the plane of the disk with a value of 0.24. One should be careful, however, when using this model to compare to observations. While the velocity gradient is relatively stable for the uniform density case, it should be noted that the simulation predicts an unphysical increase in the disk gas velocity. Further, real galaxies do not have uniform density profiles. When adopting a more realistic density profile for a galaxy, the velocity gradient does not match those of kinematic studies of edge-on spiral galaxies such as NGC 5775 and NGC 891. Thus, the toy model is an implausible mechanism through which the rotational velocity of the gas in the halo is lagged due to a combination of viscous and resistive effects. Again, the reader should be reminded that the strange evolution of the toy model with ZEUS-3D could be a consequence of pressure fluctuations. One case which explored seemed to solve this pressure problem; when $u_z$ is averaged over the $x$ range, the velocity profile becomes stable and almost no mass is lost. This is reassuring as this is the closest time-dependent analog to the analytic model and the pressure no longer builds up to accelerate the disk.

The next mechanism that was tested was shock waves produced by supernova explosions, the theoretical basis of which included depositing some energy into a bubble of lower density in excess of the ambient internal energy. The lower density region was consistent with the shock wave carrying gas and dust with it as it propagates isotropically according to the Sedov Taylor model. However, most of the energy escaped perpendicular to the disk as the density decreased with $z$ (the path of least resistance). This is analogous to the traditional jet mechanism. Thus, enough energy deposited in the bubble could result in gas being ejected from the disk. A single supernova was insufficient to promote any significant density perturbations of the disk surface however hundreds of supernovae could. This was modelled by depositing the equivalent energy of hundreds of supernovae into the initialized bubble. Given that the simulation was run over 150 000 years (and the frequency of supernovae in this
time frame vary greatly depending on the galaxy), this is more or less acceptable in lieu of placing multiple individual supernovae along the spiral arm. Further, multiple supernovae promoted magnetic loops above the plane of the disk. The number of supernovae used in the simulation was well below the maximum predicted, though the range of supernova rates varies greatly between galaxies. This mechanism was deemed plausible on the basis that the number of supernova blasts (within a region of a spiral arm) required to promote gas outflow is not unphysical.

Finally, the hydraulic jump mechanism was tested and compared to previously published results. The comparison was with Martos and Cox [1]; we used the same parameters for the magnetic field strength and orientation, as well as for the relative velocity of the gas flowing into the spiral arm. The difference was that this simulation was run in 3D and considered self-gravity. This resulted in gas outflow, as expected, however the morphology and velocity of the jump was affected by the added dimensionality and the inclusion of self-gravity. As already concluded in published works, this is a plausible mechanism for gas ejection and is within the velocity range indicated by Tripp et al. [24].

This study strongly suggests that gas outflow from the disk is a consequence of hydrodynamic processes. All three tested mechanisms were built on the basis of MHD principles and while the toy model showed that the magnetic field and viscosity can affect the shear flow to produce a lagging rotational velocity in the halo, the latter two models proved to be viable mechanisms for gas ejection from the disk. Thus MHD seems to be a viable and useful tool for modelling galactic processes in the context of gas dynamics in the disk and halo.
Bibliography


Appendix A

Input Files

In Section 5.1, we outlined the procedure of running a simulation with ZEUS-3D. To summarize, the script file (dzeus36.s) reads the initialized problem file (myprob) and runs the simulation according to the physics controlled in the zeus36.mac file. Here we present the three subroutines which initialize the mechanisms (myprob for the toy model, supernova blast, and hydraulic jump).

A.1 Toy Model

```fortran
C=======================================================================
C            B E G I N S U B R O U T I N E //////////
C ////////// T O Y M O D E L \\
C=======================================================================
C
C LOCAL VARIABLES:

subroutine toymod2
C
C LOCAL VARIABLES:

C=======================================================================
C
```
*call comvar
  integer i , j , k
  real*8 da , db , e1a , e1b , e2a
  1 , e2b , v1a , v1b , v2a , v2b
  2 , v3a , b1a , b1b , b2a , v3b
  3 , b2b , b3a , b3b
*if def,MHD
  4 , q11 , q12 , q2 , q3
*endif MHD

c-----------------------------------------------------------------------
c
  c Input parameters:
c
  c da , db array and boundary values for density
c  e1a, e1b array and boundary values for first internal energy
c  e2a, e2b array and boundary values for second internal energy
c  v1a, v1b array and boundary values for 1-velocity
c  v2a, v2b array and boundary values for 2-velocity
c  v3a, v3b array and boundary values for 3-velocity
c  b1a, b1b array and boundary values for 1-magnetic field
c  b2a, b2b array and boundary values for 2-magnetic field
c  b3a, b3b array and boundary values for 3-magnetic field

c namelist / pgen /
  1 da , db , e1a , e1b , e2a
  2 , e2b , v1a , v1b , v2a , v2b
  3 , v3a , b1a , b1b , b2a , b2b
  4 , b3a , b3b

c
  c Set default values

da = 1.0d0
db = 1.0d0
e1a = 1.0d1
e1b = 1.0d1
e2a = 0.0d0
e2b = 0.0d0
v1a = 1.0d0
v1b = 1.0d0
v2a = 0.d0
v2b = 0.d0
v3a = 1.0d0
v3b = 1.0d0
b1a = 1.0d0
APPENDIX A. INPUT FILES

b1b = 1.0d0
b2a = 0.0d0
b2b = 0.0d0
b3a = 1.0d0
b3b = 1.0d0

c
Read namelist pgen.
c
read (ioin, pgen)
write (iolog, pgen)
c
do 30 k = ksmnm2, kemxp3
do 20 j = jsmnm2, jemxp3
do 10 i = ismm2, iemxp3
d (i, j, k) = 0.6d0 * exp(-x3a(k)**2 * 102.d1)
1 + 0.37d0* exp(-x3a(k)**2 * 27.4d1)
2 + 0.1d0 * exp(-abs(x3a(k)) * 2.5d1)
3 + 0.03d0* exp(-abs(x3a(k)) * 1.11d1)
v1(i, j, k) = 1.d0 - 1.33548387d0 * ( 1.d0 -
1 dexp(-4.49275d0*x3a(k)) )
v2(i, j, k) = v2a
if( x1a(i) .ge. 0.45d0 .and. x1a(i) .le. 0.55d0 ) then
  v3(i, j, k) = v3a
else
  v3(i, j, k) = 0.d0
endif
*if -def,ISO
  e1(i, j, k) = e1a
*endif -ISO
*if def, TWOFLUID
  e2(i, j, k) = e2a
*endif TWOFLUID
*if def, MHD
  b1(i, j, k) = 0.249d0 * ( 1.d0 - dexp(4.49275d0*
 1 (1.d0-x3a(k))) )
b2(i, j, k) = b2a * g2bi (i)
b3(i, j, k) = b3a
*endif MHD
10 continue
20 continue
30 continue
*if -def, ISYM
c
c Set inflow boundary arrays.
c
*if def, MHD
  q11 = ( v2b * b3b - v3b * b2b ) * dx1a(ism1)
  q12 = ( v2b * b3b - v3b * b2b ) * dx1a(ism2)
  q2 = ( v3b * b1b - v1b * b3b ) * g2a(is)
  q3 = ( v1b * b2b - v2b * b1b ) * g31a(is)
*endif MHD

  do 50 k=ksmm2,kemxp3
    do 40 j=jsmm2,jemxp3
      niib (j,k) = 10
      diib1 (j,k) = 0.6d0 * exp(-x3b(k)**2 * 102.d1)
      diib2 (j,k) = 0.6d0 * exp(-x3b(k)**2 * 102.d1)
    1 + 0.37d0* exp(-x3b(k)**2 * 27.4d1)
    2 + 0.1d0 * exp(-abs(x3b(k)) * 2.5d1)
    3 + 0.03d0* exp(-abs(x3b(k)) * 1.11d1)

    v1iib1 (j,k) = 1.d0 - 1.33548387d0 * ( 1.d0 -
    dexp(-4.49275d0*x3b(k)) )
    v1iib2 (j,k) = v1b
    v1iib3 (j,k) = v1b
    v2iib1 (j,k) = v2b
    v2iib2 (j,k) = v2b
    v3iib1 (j,k) = v3b
    v3iib2 (j,k) = v3b
  *if -def, ISO
    e1iib1 (j,k) = e1b
    e1iib2 (j,k) = e1b
  *endif -ISO
  *if def, TWOFLUID
    e2iib1 (j,k) = e2b
    e2iib2 (j,k) = e2b
  *endif TWOFLUID
  *if def, MHD
    emf1iib1(j,k) = q11
    emf1iib2(j,k) = q12
    emf2iib1(j,k) = q2 * dx2a(j)
    emf3iib1(j,k) = q3 * dx3a(k) * g32a(j)
  *endif MHD
40 continue
50 continue
*endif -ISYM

write (iotty, 2010)
write (iolog, 2010)
A.2 Supernova Blast

*if alias PROBLEM.eq.snova

subroutine snova

mml:zeus3d.blast <-------- initialises spherical supernova blast

LOCAL VARIABLES:

r initial radius of overpressured region
symm symmetry of overpressured sphere
1.0 => no symmetry; full sphere initialised on grid
2.0 => planar symmetry; half sphere initialised
4.0 => quadrantal symmetry; quarter sphere initialised
8.0 => octal symmetry; eighth sphere initialised
x10,x20,x30 coordinates of centre of overpressured region.
d0 density in ambient medium (default = 1.0)
p0 pressure in ambient medium (default = 0.6)
e10 int. energy density in ambient medium (default = 0.9)
v10 1-velocity in ambient medium (default = 0.0)
v20 2-velocity in ambient medium (default = 0.0)
APPENDIX A. INPUT FILES

v30  3-velocity in ambient medium (default = 0.0)

b10  1-magnetic field in ambient medium (default = 0.0)

b20  2-magnetic field in ambient medium (default = 0.0)

b30  3-magnetic field in ambient medium (default = 0.0)

d1   density in central region (default = 1.0)

drat ratio of density across blast front (default = 0.0)

p1   pressure in central region (default = 0.6)

prat ratio of pressure across blast front (default = 0.0)

e11  int. energy density in central region (default = 0.0)

nrg  energy deposited in central region (default = 0.0)

v11  1-velocity in central region (default = 0.0)

v21  2-velocity in central region (default = 0.0)

v31  3-velocity in central region (default = 0.0)

b11  1-magnetic field in central region (default = 0.0)

b21  2-magnetic field in central region (default = 0.0)

b31  3-magnetic field in central region (default = 0.0)

m,dr s,drc parameters for specifying a sphere whose surface is

sinusoidally perturbed (spherical coordinates only

For an unperturbed sphere, set all values to zero (default).

The central density may be specified with either drat or d1. If
drat .ne. 0, its setting overrides d1.

The central internal energy density may be specified in four ways:
p1, prat, e11, nrg, with non-zero values further in the list
overriding those before. If nrg is specified, it should be
thought of as the internal energy deposited in a complete sphere of
volume V = 4 pi r**3 / 3 (regardless of how "symm" is set) in
*excess* of the background (e10 * V).

-----------------------------------------------------------------------

*ca comvar

integer i,j,k,ip1,jp1
real*8 r,symm,x10,x20,x30
     ,d0,p0,e10,v10,v20
     ,v30,b10,b20,b30,d1
     ,drat,p1,prat,e11,nrg
     ,v11,v21,v31,b11,b21
     ,b31,drs,drc,rsq,rin
     ,rout,frac,cofrac,vol1,dvol
     ,nrg0,nrg1,nrg2,fact,one
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```fortran
integer iin (ijkx), iout (ijkx), jin (ijkx), jout (ijkx), kin (ijkx), kout (ijkx)

equivalence ( iin , wa1d ), ( iout , wb1d )
1 , ( jin , wc1d ), ( jout , wd1d )
2 , ( kin , we1d ), ( kout , wf1d )

data one / 1.0001d0 /

External statements

real*8 overlap
external overlap , round

i = ismn
j = jsmn
k = ksmn

r = 0.1d0
symm = 1.0d0
x10 = 1.0d0
x20 = 0.0d0
x30 = 1.d-1
d0 = 1.0d0
p0 = 0.d0
e10 = 1.0d0
v10 = 0.0d0
v20 = 0.0d0
v30 = 0.0d0
b10 = 0.0d0
b20 = 1.0d0
b30 = 0.0d0
d1 = 0.6d0
drat = 0.0d0
p1 = 0.d0
prat = 0.0d0
e11 = 0.0d0
nrg = 1.d+1
v11 = 0.0d0
v21 = 0.0d0
v31 = 0.0d0
b11 = 0.0d0
```
b21 = 1.0d0
b31 = 0.0d0
m = 0
drs = 0.0d0
drc = 0.0d0

c  namelist / pgen /
   1 r , symm , x10 , x20 , x30
   2 , d0 , p0 , e10 , v10 , v20
   3 , v30 , b10 , b20 , b30 , d1
   4 , drat , p1 , prat , e11 , nrg
   5 , v11 , v21 , v31 , b11 , b21
   6 , b31 , m , drs , drc
read (ioin, pgen)
if (iolog .gt. 0) write (iolog, pgen)

  c  Set up atmosphere.
  c
  if (e10 .ne. 0.0d0) p0 = e10 * gamm1(1)
  if (e10 .eq. 0.0d0) e10 = p0 / gamm1(1)
*if -def, KSYM
   do 30 k=ksmm2,kemxp3
*endif -KSYM
*if -def, JSYM
   *endif -JSYM
*if -def, ISYM
   *endif -ISYM
      d (i,j,k) = 0.6d0 * exp(-x3a(k)**2 * 102.d-1)
      1 + 0.37d0* exp(-x3a(k)**2 * 27.4d-1)
      2 + 0.1d0 * exp(-abs(x3a(k)) * 2.5d-1)
      3 + 0.03d0* exp(-abs(x3a(k)) * 1.11d-1)
   v1(i,j,k) = v10
   v2(i,j,k) = v20
   v3(i,j,k) = v30
*if -def, ISO
   e1(i,j,k) = e10
*endif -ISO
*if def, MHD
*endif MHD
10 continue
c Set up central region.
c *
if -def,ISYM
   do 40 i=ismn,iemxp1
*endif -ISYM
   ip1 = i + ione
   if ( abs(x1a(i)-x10) .lt. abs(x1a(ip1)-x10) ) then
      iin (i) = i
      iout(i) = ip1
   else
      iin (i) = ip1
      iout(i) = i
   endif
40 continue

c *
if -def,JSYM
   do 50 j=jsmn,jemxp1
*endif -JSYM
   jp1 = j + jone
   if ( abs(x2a(j)-x20) .lt. abs(x2a(jp1)-x20) ) then
      jin (j) = j
      jout(j) = jp1
   else
      jin (j) = jp1
      jout(j) = j
   endif
50 continue

c *
if -def,KSYM
   do 60 k=ksmn,kemxp1
*endif -KSYM
   kp1 = k + kone
   if ( abs(x3a(k)-x30) .lt. abs(x3a(kp1)-x30) ) then
      kin (k) = k
      kout(k) = kp1
   else
      kin (k) = kp1
      kout(k) = k
   endif
60 continue

c nrg = max ( 0.0d0, nrg )
c
  r = max ( onei*dx1a(is), onej*dx2a(js), onek*dx3a(ks), r )
  if (drat .ne. 0.0d0) d1 = d0 * drat
  if (nrg .gt. 0.0d0) e11 = nrg * 0.75d0 / ( pi * r**3 ) + e10
  if (prat .ne. 0.0d0) p1 = p0 * prat
  if (e11 .ne. 0.0d0) p1 = e11 * gamm1(1)
  if (e11 .eq. 0.0d0) e11 = p1 / gamm1(1)
*if -def,KSYM
  do 90 k=ksmn,kemxp1
*endif -KSYM
*if -def,JSYM
  do 80 j=jsmn,jemxp1
*endif -JSYM
*if -def,ISYM
  do 70 i=ismn,iemxp1
*endif -ISYM
*if def,XYZ
  rsq = r**2
  rin = ( x1a(iin(i)) - x10 )**2
    1 + ( x2a(jin(j)) - x20 )**2
    2 + ( x3a(kin(k)) - x30 )**2
  rout = ( x1a(iout(i)) - x10 )**2
    1 + ( x2a(jout(j)) - x20 )**2
    2 + ( x3a(kout(k)) - x30 )**2
*endif XYZ
*if def,ZRP
  rsq = r**2
  rin = ( x1a(iin(i)) - x10 )**2
    1 + ( x2a(jin(j)) - x20 )**2
  rout = ( x1a(iout(i)) - x10 )**2
    1 + ( x2a(jout(j)) - x20 )**2
*endif ZRP
*if def,RTP
  rsq = r**2 * ( 1.0d0 + drs * sin (m * x2a(j))
    1 + drc * cos (m * x2a(j)) )**2
  rin = ( x1a(iin(i)) - x10 )**2
  rout = ( x1a(iout(i)) - x10 )**2
*endif RTP
if ( (rin .lt. rsq) .and. (rout .le. rsq) ) then
  d (i,j,k) = d1
  v1(i,j,k) = v11
  v2(i,j,k) = v21
  v3(i,j,k) = v31
*if -def,ISO
  e1(i,j,k) = e11
*endif -ISO
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*if def, MHD
  b1(i,j,k) = b11
  b2(i,j,k) = b21
  b3(i,j,k) = b31
*endif MHD
endif
if ( (rin .lt. rsq) .and. (rout .gt. rsq) ) then
  *if def, XYZ
    frac = overlap ( 1, r, x10, x20, x30
                   , x1a(iin(i)), x2a(jin(j)), x3a(kin(k))
                   , x1a(iout(i)), x2a(jout(j)), x3a(kout(k)) )
  *else XYZ
    frac = ( rsq - rin ) / ( rout - rin )
  *endif XYZ
  cofrac = 1.0d0 - frac
  d(i,j,k) = d1 * frac + d0 * cofrac
*if -def, ISO
  e1(i,j,k) = e11 * frac + e10 * cofrac
*endif -ISO
endif
70 continue
80 continue
90 continue
*if -def, ISO
  c
  c If "nrg" was specified, adjust the energy density inside the
  c blast region so the total energy on the grid is as specified to
  c machine accuracy.
  c
  if (nrg .gt. 0.0d0) then
    vol1 = 0.0d0
    nrg1 = 0.0d0
  *if -def, KSYM
    do 120 k=ksmn,kemx
  *endif -KSYM
  *if -def, JSYM
    do 110 j=jsmn,jemx
  *endif -JSYM
  *if -def, ISYM
    do 100 i=ismn,iemx
  *endif -ISYM
  if (e1(i,j,k) .gt. one*e10) then
    dvol = dvl1a(i) * dvl2a(j) * dvl3a(k)
    vol1 = vol1 + dvol
    nrg1 = nrg1 + dvol * e1(i,j,k)
*endif -ISYM
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```fortran
endif
100   continue
110   continue
120   continue

   fact = ( nrg / symm + e10 * vol1 ) / nrg1
   vol1 = 0.0d0
   nrg1 = 0.0d0

*if -def,KSYM
   do 150 k=ksmn,kemx
*endif -KSYM

*if -def,JSYM
   do 140 j=jsmn,jemx
*endif -JSYM

*if -def,ISYM
   do 130 i=ismn,iemx
*endif -ISYM

   if (e1(i,j,k) .gt. one*e10) e1(i,j,k) = fact * e1(i,j,k)
   dvol = dvl1a(i) * dvl2a(j) * dvl3a(k)
   vol1 = vol1 + dvol
   nrg1 = nrg1 + dvol * e1(i,j,k)
130   continue
140   continue
150   continue

   nrg0 = e10 * vol1
   nrg2 = nrg1 - nrg0

*endif -ISO

c

c-----------------------------------------------------------------------
c----------------------- Write format statements -----------------------
c-----------------------------------------------------------------------

c2010 format('SNOVA : Total internal energy on grid = ',1pg21.15,/ 1   ,'SNOVA : (',1pg12.6,' in ambient, ',g12.6 2   ,', excess in central sphere).')
*endif

c    return
end

c=======================================================================
c\\\\\ END SUBROUTINE SNOVA \\\\\n
c======================================================================
```


A.3 Hydraulic Jump

```c
subroutine jump

LOCAL VARIABLES:

*call comvar
    integer i, j, k
    real da, db, e1a, e1b, e2a
1    , e2b, v1a, v1b, v2a, v2b
2    , v3a, b1a, b1b, b2a, v3b
3    , b2b, b3a, b3b
*if def,MHD
4    , q11, q12, q2, q3
*endif MHD

Input parameters:

da, db array and boundary values for density
e1a, e1b array and boundary values for first internal energy
e2a, e2b array and boundary values for second internal energy
v1a, v1b array and boundary values for 1-velocity
v2a, v2b array and boundary values for 2-velocity
v3a, v3b array and boundary values for 3-velocity
b1a, b1b array and boundary values for 1-magnetic field
b2a, b2b array and boundary values for 2-magnetic field
b3a, b3b array and boundary values for 3-magnetic field
	namelist / pgen /
1    da, db, e1a, e1b, e2a
2    , e2b, v1a, v1b, v2a, v2b
3    , v3a, b1a, b1b, b2a, v3b
4    , b2b, b3a, b3b
```
Set default values

da = 1.0
db = 0.1
e1a = 1.e1
e1b = 1.e1
e2a = 0.0
e2b = 0.0
v1a = 1.e1
v1b = 1.e1
v2a = 0.0
v2b = 0.0
v3a = 0.0
v3b = 0.0
b1a = 0.0
b1b = 0.0
b2a = 1.e0
b2b = 1.e0
b3a = 0.0
b3b = 0.0

Read namelist pgen.

read (ioin, pgen)
write (iolog, pgen)

do 30 k=ksmm2,kemxp3
do 20 j=jsmm2,jemxp3
do 10 i=ismm2,iemxp3
d(i,j,k) = 0.6e0 * exp(-x3a(k)**2 * 12.2e-1)
  1 + 0.37e0* exp(-x3a(k)**2 * 27.4e-1)
  2 + 0.1e0 * exp(-abs(x3a(k)) * 2.5e-1)
  3 + 0.03e0* exp(-abs(x3a(k)) * 1.11e-1)
v1(i,j,k) = sqrt(x3a(k) * 2.58e-3 *
  1 (1.e0 - 0.52e0 * exp(-1.e0 *
  2 abs(x3a(k)) / 0.325e0 ) -
  3 0.48e0 * exp(-1.e0 *
  4 abs(x3a(k)) / 0.9e0) ) )
v2(i,j,k) = v2a
v3(i,j,k) = v3a
*if -def,ISO
e1(i,j,k) = e1a
*endif -ISO
*if def,TWOFLUID

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$$e_2(i,j,k) = e_2 a$$

*endif TWOFLUID

*if def,MHD

$$b_1(i,j,k) = b_1 a$$
$$b_2(i,j,k) = b_2 a$$
$$b_3(i,j,k) = b_3 a * g_{31b}(i) * g_{32b}(j)$$

*endif MHD

10 continue
20 continue
30 continue

*if -def,ISYM

Set inflow boundary arrays.

*if def,MHD

$$q_{11} = (v_{2b} * b_3b - v_{3b} * b_2b) * dx_1a(ism1)$$
$$q_{12} = (v_{2b} * b_3b - v_{3b} * b_2b) * dx_1a(ism2)$$
$$q_2 = (v_{3b} * b_1b - v_{1b} * b_3b) * g_{2a}(is)$$
$$q_3 = (v_{1b} * b_2b - v_{2b} * b_1b) * g_{31a}(is)$$

*endif MHD

do 50 k=ksmmn2,kemxp3

do 40 j=jsmmn2,jemxp3

$$n_{ibb} (j,k) = 10$$

$$d_{iib1} (j,k) = 0.6 e_0 * \exp(-x_{3a}(k) ** 2 * 102. e^{-1})$$
$$+ 0.37 e_0 * \exp(-x_{3a}(k) ** 2 * 27.4 e^{-1})$$
$$+ 0.1 e_0 * \exp(-\text{abs}(x_{3a}(k)) * 2.5 e^{-1})$$
$$+ 0.03 e_0 * \exp(-\text{abs}(x_{3a}(k)) * 1.11 e^{-1})$$

$$d_{iib2} (j,k) = 0.6 e_0 * \exp(-x_{3b}(k) ** 2 * 102. e^{-1})$$
$$+ 0.37 e_0 * \exp(-x_{3b}(k) ** 2 * 27.4 e^{-1})$$
$$+ 0.1 e_0 * \exp(-\text{abs}(x_{3b}(k)) * 2.5 e^{-1})$$
$$+ 0.03 e_0 * \exp(-\text{abs}(x_{3b}(k)) * 1.11 e^{-1})$$

if( x_{3a}(k) .le. 0.3 e_0 ) then

$$v_{iib1} (j,k) = v_{1b}$$
$$v_{iib2} (j,k) = v_{1b}$$
$$v_{iib3} (j,k) = v_{1b}$$

else

$$v_{iib1} (j,k) = \sqrt{x_{3b}(k) * 2.58 e^{-3} *$$
$$1 (1.e0 - 0.52 e_0 * \exp(-1.0 e_0 *$$
$$2 \text{abs}(x_{3b}(k)) / 0.325 e_0 ) -$$
$$3 0.48 e_0 * \exp(-1.0 e_0 *$$
$$4 \text{abs}(x_{3b}(k)) / 0.9 e_0 ) )$$

$$v_{iib2} (j,k) = \sqrt{x_{3b}(k) * 2.58 e^{-3} *$$
$$1 (1.e0 - 0.52 e_0 * \exp(-1.0 e_0 *$$
$$2 \text{abs}(x_{3b}(k)) / 0.325 e_0 ) -$$
$$3 0.48 e_0 * \exp(-1.0 e_0 *$$
4           abs(x3b(k)) / 0.9e0 ) )
v1iib3 (j,k) = sqrt(x3b(k) * 2.58e-3 * 
1           (1.e0 - 0.52e0 * exp(-1.e0 * 
2           abs(x3b(k)) / 0.325e0 ) - 
3           0.48e0 * exp(-1.e0 * 
4           abs(x3b(k)) / 0.9e0 ) )
        endif
v2iib1 (j,k) = v2b
v2iib2 (j,k) = v2b
v3iib1 (j,k) = v2b
v3iib2 (j,k) = v2b
*if -def,ISO
        e1iib1 (j,k) = e1b
e1iib2 (j,k) = e1b
*endif -ISO
*if def,TWOFLUID
        e2iib1 (j,k) = e2b
e2iib2 (j,k) = e2b
*endif TWOFLUID
*if def,MHD
        emf1iib1(j,k) = q11
emf1iib2(j,k) = q12
        emf2iib1(j,k) = q2 * dx2a(j)
emf3iib1(j,k) = q3 * dx3a(k)
*endif MHD
40          continue
50          continue
*endif -ISYM
c
        write (iotty, 2010)
write (iolog, 2010)
2010 format('JUMP : Initialisation complete.')
c
        return
end
c
c=======================================================================
c
END SUBROUTINE JUMP

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Appendix B

Additional Plots

Here we present additional contour plots of the toy model at various time steps. This will give a better sense of the evolution of the toy model over time. Plots of density, $u_x$, and $b_x$ at $t=1..4$ will be presented here, with the $t=5$ plots presented in Chapter 6. We also present ZEUS-3D simulations of the special toy model case with no resistivity (see Sections 4.2.3 and 4.3.3) to see any significant differences in the evolution of the system. As noted in Section 5.1, the ZEUS-3D code is non-resistive. Though we imposed a resistive environment (from having the gas flow into the fixed box be resistive in accordance with Case 2), the system evolved from $x = 0$ to $x = 1$ under non-resistive MHD.
APPENDIX B. ADDITIONAL PLOTS

Figure B.1: Time evolution of the toy model with a uniform density profile - density contours.
Figure B.2: Time evolution of the toy model with a uniform density profile - $u_x$ contours.
Figure B.3: Time evolution of the toy model with a uniform density profile - $b_x$ contours.
Figure B.4: Time evolution of the toy model with a non-uniform density profile - $u_x$ contours.