DEVELOPMENT OF A NEW METHODOLOGY FOR
MEASURING DEFORMATION IN TUNNELS AND SHAFTS WITH
TERRESTRIAL LASER SCANNING (LIDAR) USING ELLIPTICAL
FITTING ALGORITHMS

by

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Abstract

Three dimensional laser scanning, also known as Light Detection and Ranging (LiDAR) has quickly been expanding in its applications in the field of geological engineering due to its ability to rapidly acquire highly accurate three dimensional positional data. Recently it has been shown that LiDAR scanning can be easily integrated into an excavation sequence in an underground environment for the purpose of collecting rockmass and discontinuity information. As scans are often taken multiple times of the same environment, the next logical application of LiDAR scanning is for monitoring for change and deformation.

Traditionally, deformation and change in an underground environment is measured using a series of five or more permanent control points installed around the profile of an excavation. Using LiDAR for profile analysis provides many benefits as compared to traditional monitoring techniques. Due to the high density of the point cloud data, the change in profile is able to be fully characterized, and areas of anomalous movement can easily be separated from overall closure trends. Furthermore, monitoring with LiDAR does not require the permanent installation of control points, therefore monitoring can be completed more quickly after excavation, and scanning is non-invasive therefore no damage is done during the installation of temporary control points.

The main drawback of using LiDAR scanning for deformation monitoring is that the raw point accuracy is generally the same magnitude as the smallest level of deformations that need to be measured. To overcome this, statistical techniques for profile analysis must be developed. This thesis outlines the development one such method, called the Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (EFA) for tunnel and shaft convergence analysis. Testing of the EFA and LPA has proved the robustness of this technique in its ability to deal with accuracy and precision issues associated with LiDAR scanning.
Co-Authorship

The thesis “Development of a new methodology for measuring deformation in tunnels and shafts with terrestrial laser scanning (LiDAR) using elliptical fitting algorithms” is the product of the formal research of Dani Delaloye. However, the support of Mark Diederichs and Jean Hutchinson largely helped guide her ideas and writing. Complete references for submitted journal papers are included in Chapter 7.
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# Table of Contents

Abstract .................................................................................................................................................. ii
Co-Authorship ......................................................................................................................................... iii
Acknowledgements ................................................................................................................................. iv

Chapter 1 Introduction ............................................................................................................................. 1
  1.1 Project motivation and overview ................................................................................................. 1
  1.2 Thesis format ............................................................................................................................... 2
  1.3 Synopsis of findings ...................................................................................................................... 2
    1.3.1 Examination of LiDAR accuracy and precision ................................................................. 2
    1.3.2 Development of technique for tunnel and shaft profile analysis using LiDAR data ......... 3
    1.3.3 Testing the sensitivity of the newly developed LiDAR profile analysis technique ...... 3
    1.3.4 Workflow for tunnel and shaft deformation analysis using LiDAR data ...................... 4
  1.4 Thesis summary ............................................................................................................................ 4

Chapter 2 Background Information and Preliminary Experimentation ................................................... 5
  2.1 Preliminary Analysis of LiDAR Application to Change detection .............................................. 6
  2.2 Scoping analysis of shaft deformation ....................................................................................... 11

Chapter 3 Development of an Elliptical Fitting Algorithm for Tunnel Deformation Monitoring with Static Terrestrial LiDAR Scanning .......................................................................................... 17
  3.1 Abstract ....................................................................................................................................... 17
  3.2 Introduction ................................................................................................................................. 18
  3.3 Deformation monitoring in tunnels ........................................................................................... 19
    3.3.1 Traditional monitoring techniques .................................................................................... 21
    3.3.2 Georeferencing and absolute positioning ......................................................................... 24
    3.3.3 Permanent referencing in an active tunneling environment ........................................... 25
    3.3.4 Back analysis using deformation measurements ............................................................ 26
  3.4 Accuracy and noise in LiDAR scanning ....................................................................................... 26
    3.4.1 Non scan related noise ...................................................................................................... 27
  3.5 Terrestrial LiDAR scanning for deformation monitoring in tunnels and shafts .................. 28
    3.5.1 Applications of LiDAR scanning in Geological Engineering ......................................... 28
    3.5.2 Deformation monitoring with LiDAR ............................................................................... 29
3.5.3 Previous work in LiDAR scanning for monitoring tunnel deformation ........ 31
3.5.4 Benefits of LiDAR scanning for deformation monitoring.......................... 33
3.5.5 Limitations and errors in current analysis techniques............................. 34
3.6 Ellipse fitting for tunnel deformation analysis............................................ 36
  3.6.1 Methods of ellipse fitting ........................................................................ 38
    3.6.1.1 Algebraic ellipse fitting ................................................................. 38
    3.6.1.2 Geometric ellipse fitting ............................................................... 40
3.7 Tunnel profile analysis with LiDAR.............................................................. 43
  3.7.1 Elliptical Fit Analysis ............................................................................ 46
  3.7.2 LiDAR Profile Analysis ......................................................................... 47
  3.7.3 Data filtering ......................................................................................... 52
3.8 Workflow ..................................................................................................... 53
3.9 Application of EFA and LPA to real data.................................................... 56
3.10 Conclusions................................................................................................. 61

Chapter 4 Sensitivity Testing of the Newly Developed Elliptical Fitting Method for the Measurement of Convergence in Tunnels and Shafts .................................................. 65
  4.1 Abstract..................................................................................................... 65
  4.2 Introduction............................................................................................... 66
  4.3 Deformation monitoring in tunnels ............................................................ 66
    4.3.1 Traditional monitoring techniques ......................................................... 67
  4.4 Terrestrial laser scanning ......................................................................... 68
    4.4.1 Time of flight scanners ....................................................................... 69
    4.4.2 Phase shift scanners ........................................................................... 70
  4.5 Sources of error in LiDAR scanning .......................................................... 71
    4.5.1 Sources of range error ........................................................................ 73
      4.5.1.1 Noise created by the scanner ......................................................... 74
      4.5.1.2 Beam divergence and spot size effect ........................................... 76
      4.5.1.3 Surface reflectivity ....................................................................... 78
      4.5.1.4 Scan density and spatial resolution .............................................. 80
      4.5.1.5 Angle of incidence ...................................................................... 80
    4.5.2 Sources of angular error ...................................................................... 81
    4.5.3 Other sources of error ........................................................................ 82
      4.5.3.1 Spurious scan points .................................................................. 82
4.5.3.2 Ambiguity interval ............................................................................... 82
4.5.3.3 Georeferencing and alignment error ......................................................... 83
4.6 Sources of noise in tunneling ......................................................................... 83
4.7 Positioning in LiDAR scanning ......................................................................... 84
4.8 Tunnel profile analysis with LiDAR data using elliptical fitting ...................... 84
  4.8.1 Generating synthetic LiDAR tunnel profiles ................................................. 85
  4.8.2 Expected outcomes of EFA and LPA analysis ............................................ 88
    4.8.2.1 Radial uniform deformation ................................................................. 89
    4.8.2.2 Elliptical uniform deformation ............................................................. 89
    4.8.2.3 Non-uniform deformation ..................................................................... 92
4.9 Sensitivity testing of the EFA and LPA ............................................................. 92
  4.9.1 Results of sensitivity testing with synthetic LiDAR data ......................... 96
  4.9.2 Implications of occluded data ................................................................. 100
4.10 Conclusions .................................................................................................. 100

Chapter 5 A New Workflow for LiDAR Scanning for Change Detection in Tunnels and Caverns
................................................................................................................................. 102
  5.1 Abstract ....................................................................................................... 102
  5.2 Introduction .................................................................................................. 102
  5.3 LiDAR scanning for deformation measurement ............................................. 104
    5.3.1 Noise, accuracy and resolution of LiDAR scanning ................................ 105
    5.3.2 Other sources of noise underground ...................................................... 108
  5.4 Workflow for tunnel convergence measurement with LiDAR ..................... 109
    5.4.1 Data collection ........................................................................................ 110
      5.4.1.1 Scan resolution .................................................................................. 111
      5.4.1.2 Scan spacing and locations ............................................................... 112
    5.4.2 Initial data processing ............................................................................. 117
      5.4.2.1 Alignment of scans ........................................................................... 117
      5.4.2.2 Filtering of data ................................................................................ 117
    5.4.3 Cross section extraction .......................................................................... 119
  5.5 Convergence measurement ........................................................................... 119
  5.6 Conclusions .................................................................................................. 123

Chapter 6 General discussion .................................................................................... 124
  6.1 Discussion ..................................................................................................... 124

vii
6.2 Limitations ..................................................................................................................... 124
6.3 Future work ................................................................................................................... 125

Chapter 7 Summary and Conclusions .............................................................................. 127
7.1 Summary ....................................................................................................................... 127
7.2 Contributions ............................................................................................................... 128
  7.2.1 Refereed Journal Articles (Submitted) ................................................................. 128
  7.2.2 Refereed Conference Papers ................................................................................. 128
  7.2.3 Non-refereed Conference Presentations ............................................................... 129
  7.2.4 Courses instructed ............................................................................................... 129
7.3 References ..................................................................................................................... 129

Appendix A Matlab code for EFA, LPA, NP and synthetic data generation ...................... 137
Appendix B Complete graphical results of sensitivity analysis of EFA and LPA ................. 171
List of Figures

Figure 2-1: Set of drawers scanned for initial testing of LiDAR point cloud point to point comparison for change detection (point cloud intensity data shown). ........................................ 7

Figure 2-2: Comparison of LiDAR scans to determine the amount of change measured from shortest distance point to point comparison with: a) the same scan, b) two scans taken at the same location, c) two scans taken from different locations, and d) two scans taken from the same location with a drawer opened by 20 mm before the second scan was taken................................................................. 8

Figure 2-3: Comparison between different scans: a) the same scan compared to itself, showing there is no error in comparison; b) two separate scans taken from the same location showing a distribution in the change measured when they are compared; c) two scans taken from different locations showing a distribution in the change measured when they are compared; and d) values of the mean and standard deviation (in mm) of change measured between scans compared to themselves and to each other. ........................................ 9

Figure 2-4: Shortest distance point to point comparison of temporal scans taken within a mining environment where the rockmass was covered with steel screen. Areas where rocks have moved behind the screen are indicated with arrows. ........................................................................... 10

Figure 2-5: Example of shaft deformation model created in Phase 2 (RocScience 2011). Direction of displacement zone and boundary displacement progression (white arrows show direction of joint failure and deformation zone progression with increased stress ratio, black arrows show direction of boundary deformation). ............................................... 12

Figure 2-6: Average displacements for Queenston Shale analyses with no joints dependent upon the stress ratio, K. .................................................................................................................................. 13

Figure 2-7: Normalized minimum and maximum displacements of each rock type for different K values. ............................................................................................................................... 15

Figure 3-1: Traditional monitoring of tunnel section using control points. Barla’s technique defines a) the location of points for chainage array measurements, and b) an example of displacements of array measurements over time (after Barla 2008). In another plot of tunnel convergence data (after Schubert et al, 2002), absolute displacement vectors of five control points are shown in: c) cross section to show vertical movement, and d) longitudinal section to show lateral translation......................................................... 23

Figure 3-2: a) The tunnel profile shown has been translated but has experienced no convergence. b) The measured radial convergence between the tunnel profiles shown in a) based on
radial measurement from the centroid of the first shape. c) The measured relative
diametric convergence between the tunnel profiles shown in a).

Figure 3-3: Parameters that characterize an ellipse. The distance that is minimized by a geometric ellipse fit is the distance between the \(i^{th}\) data point \((x_i, y_i)\) and the corresponding point on the ellipse \((x, y)\) (after Ray & Srivastava, 2008).

Figure 3-4: Example of elliptical fitting for profile analysis: a) two ellipse fits \((E_1\) and \(E_2)\) with orientation of long axis displayed, b) distributed radial displacement profile (DRDP) shown in purple at five times exaggeration, and c) diametric tunnel displacement profile (DDP).

Figure 3-5: Example of profile analysis with LiDAR point data: a) two LiDAR profiles for comparison (CS\(_1\) in green and CS\(_2\) in red), which are elliptically centered, b) distributed radial displacement profile (DRDP) shown in purple at five times exaggeration, and c) the diametric tunnel displacement profile (DDP).

Figure 3-6: Areas of volume increase and decrease of profile overlain on top of the distributed radial displacement profile (DRDP) from the LiDAR Profile Analysis (LPA). A positive volume indicates an area of convergence; a negative value indicates an area of expansion or divergence. The green line indicates the first LiDAR cross section and the purple line shows the DRDP.

Figure 3-7: Deviation of LiDAR diametric displacement profile from elliptical displacement profile for characterization of random and systematic noise and anomalous movement of the tunnel profile. LiDAR diametric displacement profile is displayed as the smoothed curve using a fifty point moving average calculation, shown here in red.

Figure 3-8: a) Scan data collected on the wall of a tunnel covered with screen. b) Close up of the raw LiDAR point cloud data, and c) data after it is meshed, showing how the screen and rock surface are integrated as one surface.

Figure 3-9: Meshing errors are created when near surface anomalies are not filtered from the data set prior to meshing a) LiDAR data meshed without filtration, b) sketch showing how errors are created during meshing without filtering.

Figure 3-10: Steps for elliptical fitting and alignment of cross sections used for EFA and LPA: a) CS\(_1\), the extracted cross section from the first LiDAR scan (or set of scans), b) CS\(_2\), the extracted cross section from the second LiDAR scan (or set of scans), c) CS\(_1\) fit with an ellipse, \(E_1\), d) CS\(_2\) fit with an ellipse, \(E_2\), e) alignment of CS\(_1\), CS\(_2\), \(E_1\) and \(E_2\) using
centroids of ellipses, f) $E_1$ and $E_2$ aligned for EFA, and g) $CS_1$ and $CS_2$ aligned for LPA.

Figure 3-11: Workflow for deformation monitoring of tunnels with LiDAR data

Figure 3-12: LiDAR scan of a Martello tower, Fort Frederick, located in Kingston, Ontario

Figure 3-13: Cross sections taken at a vertical spacing of 0.5 m through LiDAR model of Fort Fredrick tower

Figure 3-14: Cross section of tower intersecting windows a) prior to filtering, and b) after using a 5% filter cutoff to remove anomalies

Figure 3-15: Example output from application of EFA and LPA analysis to real LiDAR data. a) Initial tower cross section shown in green with DRDP at 5 times exaggeration displayed in purple, b) Deviation of LiDAR diametric displacement profile from elliptical displacement profile for characterization of random and systematic noise and anomalous movement of the tunnel profile. LiDAR diametric displacement profile is displayed as the smoothed curve, shown here in red. The noise profile (NP) is shown in purple.

Figure 4-1: Time of flight LiDAR scanner. These scanners send out a single pulse and measure the time elapsed before the pulse is returned to the scanner to calculate distance

Figure 4-2: Phase shift LiDAR scanner, which sends out a continuous beam and has an ambiguity interval (maximum scan distance) equal to the longest modulation of the laser

Figure 4-3: The distance to an object is measured by the phase shift between the transmitted and received signals

Figure 4-4: Example of the sinusoidal amplitude modulated signals emitted at one time by an AMCW laser in a phase shift LiDAR scanner

Figure 4-5: Four sets of point cloud data showing the differences between high and low precision and high and low accuracy. When LiDAR scanning is complete, both high precision and high accuracy are desired

Figure 4-6: Two scans of the same limestone brick within a tunnel: one at “medium” resolution, and one at “super high” resolution with the Leica HDS 6000. Cross sections of the brick show the super high resolution data is noisier than the medium resolution data

Figure 4-7: Gaussian profile of laser beam intensity showing both Gaussian and FWHH measurements of laser footprint size
Figure 4-8: Section of point cloud data from the wall of a tunnel covered with steel screen and rockbolts. A curtain of points can be seen trailing away from the edge of the mesh towards the rock surface. .................................................................78

Figure 4-9: Scan of a two tone target fit with a best-fit plane. Deviations from the plane are shown in meters. .................................................................79

Figure 4-10: Sketch showing drop off in the accuracy of data acquired by LiDAR scanner as surface becomes more oblique to the scanner. Lighter arrows indicate reduced accuracy and reduced return. .................................................................81

Figure 4-11: Application of the EFA and LPA to LiDAR scan data of a partially occluded tunnel profile. a) sketch showing possible sources of occlusion in LiDAR profile, b) two occluded tunnel profiles (40% occlusion), CS1 shown in green, CS2 in red, c) diametric displacement profile (DDP) from elliptical fits to occluded data, d) DDP from LiDAR profile data, and e) deviation of LiDAR diametric displacement profile from elliptical fit displacement profile for analysis of random and systematic noise .................................86

Figure 4-12: Steps in the process of generating synthetic LiDAR data. a) generation of a circle with a radius and number of points both defined by the user, b) introduction of random noise with a set standard deviation, c) introduction of systematic noise with a set magnitude and period, d) introduction of skew with a set magnitude, e) rotation of the profile by a user defined amount, f) occlusion of the profile where the percentage, average length, and standard deviation of that length are all defined by the user. ..........87

Figure 4-13: The three primary types of deformation that can be expected in a tunnel depending on the rockmass and in situ stress conditions. .................................................................88

Figure 4-14: Outcomes of the EFA and LPA from the analysis of a tunnel profile undergoing radial uniform deformation. a) parameters of best fit ellipse to each tunnel profile, b) two LiDAR tunnel profiles fit with ellipses, c) DRDP at three times exaggeration from EFA, d) DRDP at three times exaggeration from LPA, e) DDP from EFA, f) DDP from LPA, g) NP. ..................................................................................................................90

Figure 4-15: Outcomes of the EFA and LPA from the analysis of a tunnel profile undergoing elliptical uniform deformation. a) parameters of best fit ellipse to each tunnel profile, b) two LiDAR tunnel profiles fit with ellipses, c) DRDP at three times exaggeration from EFA, d) DRDP at three times exaggeration from LPA, e) DDP from EFA f) DDP from LPA, g) NP. ..................................................................................................................91
Figure 4-16: Outcomes of the EFA and LPA from the analysis of a tunnel profile undergoing non-uniform deformation. a) parameters of best fit ellipse to each tunnel profile, b) two LiDAR tunnel profiles fit with ellipses, c) DRDP at three times exaggeration from EFA, d) DRDP at three times exaggeration from LPA, e) DDP from EFA, f) DDP from LPA, g) NP. ................................................................. 93

Figure 4-17: a) MACE from the LPA showing an increase in error with an increase in random noise and an increase in beta b) MACE from the EFA showing the same trend as the LPA, but with a much smaller magnitude of error. .................................................. 97

Figure 4-18: a) MACE from the LPA showing an increase in error with an increase in random noise and minimal impact of skew on the error b) MACE from the EFA showing the same trend as the LPA, but with a much smaller magnitude of error................................. 99

Figure 4-19: MACE from LPA and EFA for occluded data showing clearly that occlusion only begins to impact the measurement error at about 60% occlusion. The allowable occlusion is lowered by an increase in the random noise................................................. 100

Figure 5-1: a) mapping of geological structure in the tunnel face over multiple blast rounds b) measurement of joint spacing c) extraction of discontinuity orientations d) seepage mapping in lined and unlined area of the tunnel e) as built modeling of tunnel environment (a-e after Fekete, (2010)). ................................................................. 105

Figure 5-2: Accuracy of single point and modeled surface from LiDAR scan data as a function of the distance from the scanner (all measurements are one standard deviation). ............. 106

Figure 5-3: Spot spacing of collected data by Leica HDS 6000 at standard resolution settings.. 107

Figure 5-4: Measured diameter of the laser beam emitted by the Leica HDS 6000 as a function of distance from the scanner.................................................................................. 108

Figure 5-5: Complete workflow for the collection and processing of LiDAR data for tunnel deformation measurement.................................................................................. 109

Figure 5-6: Factors affect the collection of data during LiDAR scanning field work. .............. 110

Figure 5-7: General alignment of scans spaced at one diameter along the axis of the tunnel to ensure full coverage of the tunnel walls........................................................................... 112

Figure 5-8: Modification to standard scan set-up to account for expected occlusion in the tunnel face and roof where two scans are set up at every point on both sides of the tunnel centerline........................................................................................................... 113

Figure 5-9: Calculation of maximum tunnel radius that can be scanned by only setting up along centerline of tunnel using a Leica HDS 6000. ................................................................. 114
Figure 5-10: Two scans of a static rough surface can result in a measured deformation that does not actually exist due to the resolution of the scan not being high enough to accurately capture the surface. ................................................................................................................................. 115

Figure 5-11: Scans must be positioned in order to ensure that structure within the rockmass does not affect the collection of the full tunnel surface. ............................................................................................................. 115

Figure 5-12: Modification to standard scan set-up to account for expected occlusion in the tunnel walls where scans are set up at a distance of every radius along the tunnel centerline. 116

Figure 5-13: Where screen has been installed to support an excavation meshing of the LiDAR point cloud creates an erroneous surface combining the wire screen and rock surface into the mesh. Meshing of the bare rock surface produces a good result. 118

Figure 5-14: Results of the MICE from the EFA showing that an increase in the number of points used in the analysis decreases the error until the number of points exceed about 20000. The change in radius being measured has no impact on the error in the results of convergence measurement. ............................................................................................................. 121

Figure 5-15: Results of the MICE from the EFA showing that a higher number of points is required to reduce the error in measurement of convergence when the magnitude of random noise is higher. ................................................................................................................................. 122
List of Tables

Table 2-1: Rockmass properties used for parametric study of shaft deformation .......................... 13
Table 4-1: Parameters varied for sensitivity testing of the EFA and LPA analysis ......................... 94
Table 5-1: Scanning times and data sizes produced by the Leica HDS 6000 for standard
resolution settings ......................................................................................................................... 111
Table 5-2: Number of points in one LiDAR cross sectional profile based on different scan
resolutions of the Leica HDS6000 terrestrial LiDAR scanner .................................................... 120
Chapter 1

Introduction

1.1 Project motivation and overview

Light Detection and Ranging (LiDAR) is a laser scanning technology used for quickly capturing hundreds of thousands of points that have both positional (x,y,z) and intensity (i) data. Due to LiDAR’s fast acquisition of high accuracy high precision point cloud data, the number of applications of this technology is continually growing.

Previous research of LiDAR applications to geological and geotechnical engineering have proved the usefulness of LiDAR for measuring discontinuity information and for rockmass characterization in slopes and rock outcrops (Lato, 2010). Further research demonstrated that LiDAR scanning could be easily implemented into a tunneling excavation sequence. The research conducted by Fekete (2010) showed that LiDAR is robust and rugged enough to be well suited to data collection in an underground environment. Scan data collected underground has been used for discontinuity measurement and rockmass characterization, but no method for change detection has been demonstrated. The next logical application for LiDAR in an underground setting is for change detection. Hence this thesis focused on developing a method for using LiDAR to measure deformation in underground environments.

Deformation monitoring in underground environments is an essential part of any excavation project. The measurement of deformations allows for:

- Comparison of measured deformations with those modelled and accounted for in initial support design
• Refinement of support design and excavation rate and method
• Prediction of future deformations or large failures
• Back calculation of rockmass deformability and other rockmass parameters

The goal of this thesis was to develop and test a method for applying LiDAR in underground excavations, specifically near circular tunnels and shafts, and to develop a workflow for completion of deformation measurements. Special consideration was given to the accuracy of LiDAR scan data and the implications this has on the user’s ability to measure deformation and change.

1.2 Thesis format

This thesis has been prepared in manuscript form in accordance with the guidelines established by the School of Graduate Studies at Queen’s University, Kingston, Ontario, Canada. Chapter 2 provides project background and a summary of preliminary investigation completed prior to the main body of research for this thesis. Chapters 3-4 are manuscripts that have been submitted to international journals and Chapter 5 is a published conference proceedings that may be submitted to international journals at a later date. Chapter 6 provides a discussion of the limitations of the developed analysis techniques and workflow, and suggestions for future work. Chapter 7 includes a summary of the conclusions that have stemmed from this thesis and the contributions made as a result of research conducted.

1.3 Synopsis of findings

The major findings of this thesis are summarized in the following sections.

1.3.1 Examination of LiDAR accuracy and precision
Before any analysis into application of LiDAR scanning for deformation monitoring and change detection could be completed it was important to complete a thorough examination of the accuracy and precision associated with LiDAR scanning. Initial analyses were also completed to determine the magnitude of change that would be needed to measure with LiDAR scanning and how this relates to the raw LiDAR point cloud accuracy. From the initial analysis it was concluded that the magnitude of the level of change that would need to be measured was the same as the raw LiDAR point cloud accuracy.

1.3.2 Development of technique for tunnel and shaft profile analysis using LiDAR data

As the analysis of LiDAR accuracy found that levels of change within the single point raw accuracy of LiDAR point cloud data were necessary, it was determined that statistical analysis of data for change detection using LiDAR data was needed. To meet this requirement a new analysis technique using elliptical fitting to LiDAR profile data was created. The development of this analysis technique has been submitted to an international journal, Tunneling and Underground Space Technology (Delaloye, D., Walton, G., Hutchinson, J., Diederichs, M. (2012) Development of an Elliptical Fitting Algorithm for Tunnel Deformation Monitoring with Static Terrestrial LiDAR Scanning. *Tunneling and Underground Space Technology*).

1.3.3 Testing the sensitivity of the newly developed LiDAR profile analysis technique

With the development of any new analysis technique, it is important to test the robustness of the technique and ensure it can accommodate errors associated with real data. A sensitivity test of the newly developed profile analysis techniques was completed with results submitted to an international journal, Rock Mechanics and Rock Engineering (Delaloye, D., Hutchinson, J.,

**1.3.4 Workflow for tunnel and shaft deformation analysis using LiDAR data**

To ensure that the newly developed profile analysis technique is applied properly to LiDAR data, a workflow was developed. The workflow outlines all the steps necessary from initial data collection, through data processing and management, to final data analysis. The workflow has been presented in the conference proceedings of the 46th U.S. Rock Mechanics Geomechanics Symposium (Delaloye, D., Hutchinson, J., Diederichs, M. (2012) A New Workflow for LiDAR Scanning for Change Detection in Tunnels and Caverns. *46th U.S. Rock Mechanics Geomechanics Symposium* ARMA 2012. Chicago, United States).

**1.4 Thesis summary**

The research presented in this thesis was completed to develop an accurate and easily applicable method for determining change and deformation within tunnels and shafts using LiDAR scanning. The work was motivated by the successful application of LiDAR scanning in underground environments for rockmass characterization. The aim of the research is to provide a more comprehensive technique for deformation characterization of tunnel and shaft profiles and to provide the ability to discern anomalous movement from general convergence. To ensure the method of profile analysis developed in this thesis is easily and properly applied, a workflow for data collection, management and analysis for change detection was created as a capstone to the project.
Chapter 2

Background Information and Preliminary Experimentation

As new technologies emerge it is important to explore their application potential in different fields, and to test these applications and design appropriate workflows so the technologies are used properly. Light Detection and Ranging (LiDAR) has been around in one form or another for almost sixty years. Terrestrial based high resolution systems have become popular in the past ten years, and have a wide range of applications. As the technology has become more popular, the geological and geotechnical engineering community has begun to apply it in various projects, primarily due to its ability to rapidly acquire large high accuracy, high precision three dimensional point cloud data sets.

Previous work completed by the Geomechanics Research group within the Department of Geological Sciences and Geological Engineering at Queen’s University gave an introduction to the application of LiDAR in slopes (Lato, 2010) and then proceeded by taking those applications and demonstrating their applicability in an underground environment (Fekete, 2010). The major applications for use in tunnels demonstrated by Fekete (2010) are for:

- Quality control of bolt spacing and shotcrete thickness
- Seepage mapping
- Measurement of overbreak (the difference between the design and the excavated tunnel wall)
- Mapping of geological features and structures
• Joint spacing measurement
• Large scale joint roughness calculation
• Discontinuity orientation measurement and extraction for input into DEM modeling

One of the newest applications of LiDAR scanning is for change detection. Multiple scans of the same area are taken at different time periods and compared to determine areas and quantities of change. Clearly change detection is possible with LiDAR data (Lato et al. 2009, Fekete et al. 2010) but little work has been done to quantify the smallest level of change that can be measured within two data sets, and where errors in change measurements stem from. The goal of this thesis is to determine if movements and change within the same level of magnitude as the raw accuracy of the scanner can be measured, and if they can be measured to determine and appropriate workflow for change detection and deformation monitoring with LiDAR.

Prior to development of a technique for change detection, it was important to determine the raw accuracy of LiDAR scanning as it pertains to the measurement of change. Preliminary work was done in this thesis to assess the amount of deformation that can be measured accurately (Section 2.1) and the magnitude of change that would need to be detected in an underground shaft sinking program in Canadian Shield rocks (Section 2.2).

2.1 Preliminary Analysis of LiDAR Application to Change detection

To determine the accuracy of LiDAR for levels of change measurement, an initial experiment was completed prior to the main body of research for this thesis. The goal of the experiment was to determine what level of change was measurable within LiDAR point cloud
data, and the error in this measurement. In other words, is there a level of change measured within the data that does not actually exist, which is generated by instrument error?

The experiment was conducted by scanning a set of wooden drawers (Figure 2-1). The drawers remained static and unchanged throughout the initial part of the experiment. To test the repeatability of LiDAR scanning, multiple scans were completed from the same location, and then from slightly different locations. The results were compared to determine the level of change detected.

Figure 2-1: Set of drawers scanned for initial testing of LiDAR point cloud point to point comparison for change detection (point cloud intensity data shown).
change measured within the data (Figure 2-2). The final step of the experiment was to move one of the drawers out by a controlled amount and repeat the scan to see if the amount the drawer was moved correlated well to the amount that was measured by hand.

The scans of the drawer set were all compared using a shortest point to point distance calculation comparing the shortest Euclidean distance between a given point in point cloud A to a point in point cloud B. Clearly this does not provide a correct answer to the problem of change detection, because change was measured between scans when no actual change had occurred (Figure 2-2). This error stems from the point to point comparison. When using LiDAR scanning

![Figure 2-2: Comparison of LiDAR scans to determine the amount of change measured from shortest distance point to point comparison with: a) the same scan, b) two scans taken at the same location, c) two scans taken from different locations, and d) two scans taken from the same location with a drawer opened by 20 mm before the second scan was taken.](image)

8
it is impossible to ensure that points collected from different scans are located in exactly the same positions, even with exactly the same scan set-up. Hence, point to point comparisons yield some measure of deformation even if there is no actual change in the surface being scanned.

Estimates of the probability destiny functions have been plotted to show the distribution in the level of change measured between scans where no change should occur (Figure 2-3).

**Figure 2-3**: Comparison between different scans: a) the same scan compared to itself, showing there is no error in comparison; b) two separate scans taken from the same location showing a distribution in the change measured when they are compared; c) two scans taken from different locations showing a distribution in the change measured when they are compared; and d) values of the mean and standard deviation (in mm) of change measured between scans compared to themselves and to each other.
To avoid point to point comparison problems and errors, LiDAR data is generally meshed. The process of meshing creates a surface, and the comparison of two surfaces allows for the normal or shortest distance between the two surfaces to be compared. This provides a more accurate comparison of data sets than point to point comparison.

If large changes have occurred, point to point comparison can highlight areas of change. This methodology is not accurate for quantifying change on a millimeter to sub millimeter scale, but can be used for mapping of locations of change, especially when creation of a mesh is not practical. For example, scans were taken at a mine where the walls are covered in steel screen (Figure 2-4). Point to point comparison from two temporal scans highlight areas where rocks

![Image](image_url)

**Figure 2-4:** Shortest distance point to point comparison of temporal scans taken within a mining environment where the rockmass was covered with steel screen. Areas where rocks have moved behind the screen are indicated with arrows.
have moved behind the screen.

It is important to determine the level of change that is going to be measured within the data sets, and to determine an appropriate method for measuring this deformation. The focus of this thesis is primarily on the development and testing of the method for measuring convergence, change and deformation, but initial analysis to determine the level of change expected was conducted.

2.2 Scoping analysis of shaft deformation

To determine the expected level of change that a LiDAR scanning program for change detection may be required to measure, a two-dimensional analysis of shaft deformation was completed using a 10 m diameter shaft model created in Phase2 (RocScience 2011) (Figure 2-5). The model was built to simulate conditions expected when sinking a shaft in rocks characteristic of the Canadian Shield. A sensitivity analysis and parametric study of parameters were completed. The sensitivity analysis tested the impact of stress distribution, boundary conditions, boundary type, mesh type and discretization on the deformation of the shaft.

The parametric study tested three different rock types at four stress ratios (from K=1 to K=3) with and without joints to determine the amount of deformation expected in the shaft. A summary of the varied rockmass parameters can be found in Table 2-1.

The results of the parametric study show that an increased stress ratio produced higher deformations within the shaft. It has also been shown that stiffer rockmasses produce smaller deformations. In models without joints, the maximum displacement simulated in the Sudbury Granite is 5 mm (0.1% radial closure), in the Cobourg Limestone is 10 mm (0.2% radial closure) and in the Queenston Shale is 35 mm (0.7% radial closure). With joints, the maximum of
Figure 2-5: Example of shaft deformation model created in Phase 2 (RocScience 2011).

Direction of displacement zone and boundary displacement progression (white arrows show direction of joint failure and deformation zone progression with increased stress ratio, black arrows show direction of boundary deformation).

Displacement simulated in the Sudbury Granite is 7.0 cm (1.4% radial closure), in the Cobourg Limestone is 10.6 cm (2.1% radial closure), and in the Queenston Shale is 19.6 cm (3.9% radial closure) (Figure 2-6).
Table 2-1: Rockmass properties used for parametric study of shaft deformation

<table>
<thead>
<tr>
<th>Rockmass Properties</th>
<th>Sudbury Granite</th>
<th>Queenston Shale</th>
<th>Cobourg Limestone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Weight (MN/m³)</td>
<td>0.027</td>
<td>0.0269</td>
<td>0.0269</td>
</tr>
<tr>
<td>GSI</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>mi</td>
<td>25</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Young's Modulus (MPa) intact</td>
<td>70000</td>
<td>15290</td>
<td>37330</td>
</tr>
<tr>
<td>Young's Modulus (MPa) rockmass</td>
<td>61624</td>
<td>13460</td>
<td>32863</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.25</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Intact Compressive Strength (MPa)</td>
<td>250</td>
<td>48</td>
<td>110.5</td>
</tr>
<tr>
<td>mb Parameter peak (residual)</td>
<td>12.24 (7.6)</td>
<td>2.94 (1.82)</td>
<td>5.87 (3.65)</td>
</tr>
<tr>
<td>s Parameter peak (residual)</td>
<td>0.11 (0.05)</td>
<td>0.11 (0.05)</td>
<td>0.11 (0.05)</td>
</tr>
<tr>
<td>a Parameter peak (residual)</td>
<td>0.50 (0.5)</td>
<td>0.50 (0.5)</td>
<td>0.50 (0.5)</td>
</tr>
</tbody>
</table>

Figure 2-6: Average displacements for Queenston Shale analyses with no joints dependent upon the stress ratio, K.
The greatest displacements in the models were always found parallel to the orientation of maximum stress, while minimum displacements were parallel to the orientation of minimum stress. In models without joints the zone of greatest displacement is parallel to maximum stress, whereas in models with joints the zone of greatest deformation extends along the orientation of the joints (Figure 2-7).

Based on the results of the analysis in some cases of rock deformation it will be important to measure millimeter scale change over a range of meters, therefore point to point comparison clearly is not an option. Current surface meshing processing techniques also do not lend themselves to conveniently analyzing underground convergence measurements and provide no methodology for quantifying convergence or analyzing different types of deformation. Therefore, new methods need to be developed that allow for measurement of deformation that is useful both for convergence analysis and for back calculation of excavation performance.
Figure 2-7: Normalized minimum and maximum displacements of each rock type for different K values.
Chapter 3

Development of an Elliptical Fitting Algorithm for Tunnel Deformation Monitoring with Static Terrestrial LiDAR Scanning

3.1 Abstract

Terrestrial laser scanning, also known as Light Detection and Ranging (LiDAR) is an emerging technology that has many proven uses in the geotechnical and geomechanical engineering community including rockmass characterization, discontinuity measurement and landslide monitoring. One of the up-and-coming applications of LiDAR scanning is deformation monitoring and change detection.

Traditionally deformation in tunnels is measured using a series of five or more control points installed around the diameter of the tunnel and measured at regular time intervals. LiDAR provides the ability to get a more complete characterization of the tunnel surface, allowing for determination of the mechanism and magnitude of tunnel deformation, as the entire surface of the tunnel is being modeled rather than just a fixed set of points. The change in deformation pattern over time is also much more easily extracted from LiDAR profile analysis.

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1 This chapter appears as submitted to an international journal with the following citation:

This paper discusses LiDAR scanning for deformation mapping of a surface and for cross-sectional closure measurements within an active tunnel using elliptical fitting for profile analysis.

3.2 Introduction

Light Detection and Ranging (LiDAR) is a technology used for quickly acquiring three dimensional point cloud data. By emitting a laser and measuring either the time elapsed or the phase shift in the returned beam (depending on the scanner type) the scanner collects both three dimensional (3D) positional data \((x, y, z)\) and intensity return \((i)\). The newest scanners can collect up to one million points per second.

Multiple LiDAR scans taken at different times and distances from an advancing tunnel face can be compared to determine the deformation profile of a tunnel and how it is changing over time. This information can be useful in back analysis for analyzing stress orientations and determining the amount and location of support to be installed.

To use a LiDAR point cloud for deformation measurement, appropriate workflows and data analysis techniques must also be developed. Authors have begun to develop techniques for deformation monitoring in various geological environments such as tunnels (Van Gosliga et al. 2006), landslides (Jaboyedoff et al. 2009), volcanic environments (Nguyen et al. 2011) and rockfalls (Abellan et al. 2011). To use LiDAR scan data for tunnel profile analysis and back calculation, a new method of data analysis involving elliptical fitting has been proposed by the author. This new method allows for both the best fit ellipses to multiple scans as well as the raw LiDAR profiles to be analyzed to gain the maximum amount of information from the scan data.
3.3 Deformation monitoring in tunnels

During the excavation of a tunnel or other underground space, the in situ stresses are redistributed around the excavated area. The redistribution of the stress field results in a tendency for closure of the void created by the excavation. This closure or deformation is generally referred to as convergence when it occurs in a tunneling environment. The magnitude of deformation is related to the rock mass conditions, the magnitude, orientation and ratio of the in situ stresses, the excavation method and rate, and the type and location of installed support. Monitoring tunnel convergence is an integral part of modern design and construction techniques.

The objectives and requirements of deformation monitoring programs are different for different tunneling environments. In mountain tunneling environments, small deformations of the tunnel walls are generally acceptable (centimeters of tunnel closure over meter scale tunnels). The magnitude of the deformation is monitored to ensure that the temporary support installed within the tunnel is adequate. Deformations on the scale of centimetres over the tunnel diameter are allowable in mountain or other remote tunnel environments as the tunnels general have a large degree of overburden therefore closure of the tunnel has little to no impact at ground surface. Small tunnel deformations also do not generally influence the final functionality of the tunnel. Hence in mountainous and remote environments the accuracy of the deformation measurements can be on the order of a few millimetres.

In contrast, when tunneling in urban environments, even small deformations (on the scale of millimeters over meter excavations) are generally unacceptable. Any movement will affect surface infrastructure and the area surrounding the excavation. Therefore in urban environments, high accuracy, high precision measurements of deformation are necessary. It is of the utmost
importance to monitor deformations as close to the face as possible. The goal of monitoring
deformations near the face is to capture as much of the initial deformation (deformation occurring
immediately after excavation) of the profile as possible. Capturing the initial deformation allows
for rapid response and the installation of increased support and modification of excavation
methods if deformations are higher than expected. Traditional monitoring markers cannot be
installed until the excavation has been supported, therefore monitoring can only begin about 2-4
m away from the tunnel face. At this point 60-80% of the deformation has already occurred
(Kavvadas, 2005).

In many tunneling environments, it is important that deformation data is processed in real
time, or near real time. Having real time deformation monitoring data allows for rapid response
to any identified hazards or potential upcoming events indicated by deformations. Although real
time monitoring is extremely beneficial, it is associated with increased expenditure and
necessitates the implementation of more advanced technologies. These drawbacks generally limit
the use of real time monitoring to urban tunneling environments, where the consequences of
excessive deformation are higher than in mountainous or remote tunneling environments.

Monitoring deformations is beneficial when the New Austrian Tunneling Method
(NATM) for tunnel excavation is being used (Wittke et al., 2006). In this method, if deformation
measurements are not consistent with those determined through modelling during initial design,
more support or different excavation methods can be implemented. The magnitude of
deformations recorded can also help refine the final liner design. Furthermore, in squeezing
ground it is difficult at the design stage to predict the magnitude, location and orientation of
convergence, given typical uncertainties in material properties and stress regime. Squeezing
conditions may occur over very short distances (Barla et al., 2008); therefore deformation
monitoring is crucial to ensure support is properly installed to maintain the safety and stability of excavations. In addition, deformations can serve as an early warning for more severe failures. Results obtained from monitoring programs can also be used to back calculate deformability and other ground strength data.

3.3.1 Traditional monitoring techniques

There are many existing technologies for measuring tunnel wall convergence. These include, but are not limited to: tape extensometers, total station surveying (or other geodetic methods), and laser profilers. Historically deformation in tunnels has been measured using extensometers or INVAR tapes, which provide the relative displacement between two points (Kontogianni & Stiros, 2003). With the advent of theodolites and total stations, absolute distances can be measured and deformations can therefore be linked to external coordinate systems. Today convergence control points are generally established and measured soon after excavation and the installation of support. Measurements are then repeated daily until movements stabilize. In tunneling environments, measurements of deformation are generally made during construction, and are sometimes continued well after construction is completed if deformations have not stabilized and continue to pose problems during operation (Kontogianni & Stiros, 2005).

Tape extensometers are the most accurate of all the commonly used deformation monitoring technologies (Figure 3-1 a). They have an accuracy of +/- 0.2 mm over 10-15 m (Kavvadas, 2005). The main drawback with tape extensometers is that they only have the ability to measure movement along a specific line. They also must be permanently installed in the walls of the tunnel, which interrupts the construction process of the tunnel. Barla et al. (2008) has
demonstrated the technique and utility of measuring convergence by the measurement of 
chainages from five control points around a tunnel cross section.

Total station surveying makes use of optical reflectors installed around the tunnel profile 
(usually five to seven points) as well as a stable reference point outside the zone of convergence 
to measure deformations (Figure 3-1 c). The station must be moved progressively forward from 
the location of the stable reference points to the tunnel face, as it progresses. Total stations have 
an accuracy of about +/- 2.5 mm over 100 m (Kavvadas, 2005).

Photogrammetric devices require that a number of reference points be installed on the 
surface of the tunnel. Photogrammetric devices, where the camera’s position is located using a 
total station, have an accuracy of about +/- 5 mm (Kavvadas, 2005).

Some innovative methods for remotely measuring deformation in tunnels include the use 
of broken ray videometrics, where a series of relay stations are compared to a reference target 
(Qifeng et al., 2008), and the use of charge coupled device (CCD) cameras (digital cameras that 
use sensors that convert light to electrical charge instead of film) with targets (Nakai et al., 2005). 
The “tunnel convergence monitoring system” developed by Hashimoto et al. (2006) involves the 
use of connecting rods, displacement gauges and inclinometers installed at certain cross-sectional 
locations. The device places universal displacement gauges in an octagonal shape around the 
tunnel profile. The length of the rods and the angle between them is measured, with seven points 
to one reference. This method is very accurate, but requires permanent installation of 
measurement devices.
Figure 3-1: Traditional monitoring of tunnel section using control points. Barla’s technique defines a) the location of points for chainage array measurements, and b) an example of displacements of array measurements over time (after Barla 2008). In another plot of tunnel convergence data (after Schubert et al, 2002), absolute displacement vectors of five control points are shown in: c) cross section to show vertical movement, and d) longitudinal section to show lateral translation.
3.3.2 Georeferencing and absolute positioning

For the purposes of performance monitoring or back analysis of single or non-interacting tunnels or shafts, either relative or absolute measurements of tunnel convergence and deformation can be made. Before the advent of georeferencing, relative measurements were the only option for measuring tunnel convergence. Today, as total stations and global positioning systems (GPS) are very common, absolute positioning has become the state of practice for convergence measurements. The main advantage that absolute positioning provides in tunnel deformation monitoring is that it adds the ability to measure vertical and lateral shift in the tunnel, as well as allowing referencing to an external coordinate system (Kontogianni & Stiros, 2003).

Although it is often assumed that absolute positioning is necessary to characterize deformation, this is not the case. Data examined using relative convergence techniques provides the same quality of stability information and back analysis capability as using absolute positioning measurements. In a single bore tunnel, where diametric convergence is being measured, there is no direct benefit to implementing absolute positioning. Absolute positioning is necessary in twin bore tunnels, however, when the vertical and lateral shift in one tunnel can affect the stability of the other tunnel. Absolute positioning and displacement measurement may also be required where surface settlement or building interaction is being monitored.

When using relative positioning to characterize tunnel convergence it is important to consider the diametric convergence instead of the radial convergence, to ensure any errors due to the misalignment of the centroid of the tunnel are eliminated (Figure 3-2).
Figure 3-2: a) The tunnel profile shown has been translated but has experienced no convergence. b) The measured radial convergence between the tunnel profiles shown in a) based on radial measurement from the centroid of the first shape. c) The measured relative diametric convergence between the tunnel profiles shown in a).

3.3.3 Permanent referencing in an active tunneling environment

To implement any type of deformation monitoring near the face in a tunneling environment where a tunnel boring machine (TBM) is being used, absolute positioning is likely not possible. Data will need to be collected rapidly, and possibly with little warning or time for preparation, when a break in the TBM operations permit access for scanning the tunnel. The locations for measurement must also be flexible. Occlusion of areas of the tunnel wall will be inevitable as parts of the tunnel wall will be blocked by machinery, ventilation, conveyor systems or the TBM itself.

Furthermore, in an active tunnelling environment, it is not always practical or possible to maintain permanent survey markers. For total station measurements of convergence, prisms are usually installed at certain sections around the perimeter of the tunnel. If one of these prisms is
knocked out of place or comes loose, convergence measurements can no longer be made from that point. Also, if one of the prisms were installed in an area of anomalous movement, the data collected would be useful for characterizing that section of deformation, but would not provide information that could be used for back analysis.

3.3.4 Back analysis using deformation measurements

One of the main reasons deformation measurements are collected is for back analysis. Back analysis is useful for monitoring the stability of excavations, for calculation and interpretation of field stresses, and for calculation of rockmass parameters such as deformability. Many authors have explored different methods for using monitoring results for back analysis in tunnels and other underground excavations (Chan & Stone, 2005, Vardakos et al., 2006, Sakuri & Takeuchi, 1983, Russo et al., 2009, Oreste, 2004, Zhifa et al., 2004). The development of a complete methodology for back calculation of rockmass parameters and back analysis of stress orientations from LiDAR data is beyond the scope of this paper, but LiDAR data can be of added benefit to existing techniques.

3.4 Accuracy and noise in LiDAR scanning

Before LiDAR data is collected, the necessary accuracy and resolution of that data to be collected must be determined. The end use of the data is the main factor that contributes to the maximum allowable accuracy and minimum allowable resolution. In general, the level of error in a data set must be less than the resolution of the object or quantity being measured. To minimize error in LiDAR data it is important to be familiar with the sources of error associated with data acquisition. These can be broken into:
- the error in the range measurement
- the error in the vertical angle measurement
- the error in the horizontal angle measurement

The error in range measurement increases with distance from the scanner. The main components are the spot size of the laser, the reflectivity of the object being scanned, and the angle of incidence between the scanner and the object. Error in range measurement is minimized by choosing appropriate scan locations and settings.

Angular error, broken into the error in the vertical and horizontal angle measurement, is associated with the scanner’s measurement of one or more of its moving parts, and therefore can be minimized by using a properly calibrated scanner.

Other errors within LiDAR data sets can be introduced during field procedures for scanning and processing of the LiDAR point cloud data such as errors in positioning measurement in the field and poor alignment of scans during processing. Erroneous points, also referred to as spurious scan points (Jacobs, 2009), are additional sources of error in LiDAR scanning and are often associated with poor atmospheric conditions, interfering radiation, or highly reflective surfaces such as spectral reflectors.

### 3.4.1 Non scan related noise

There are other sources of “noise” that impact LiDAR scanning in an underground environment that are not associated with the scanning itself. “Noise” here is defined by the author to be anything that creates deviation from a perfectly circular tunnel design profile. These sources of noise include:

- Roughness of the rock surface
• Systematic undulation of the profile (caused by excavation type)
• Noise created by structure within the rockmass
• Anomalies on the tunnel surface (e.g. rockbolts, faceplates, screen)
• Occlusion of the profile by obstructions

3.5 Terrestrial LiDAR scanning for deformation monitoring in tunnels and shafts

As LiDAR scanning is becoming a more common surveying device, new applications for LiDAR scanning are arising. The most recent and promising application of LiDAR scanning is for deformation monitoring. Experiments have been conducted to determine if the error within LiDAR data is small enough to measure millimetric deformations (Abellan et al., 2009). In general, the level of error within survey data must be less than the minimum level of change of the physical quantity measured (Simeoni & Zanei, 2009). Due to the large number of high accuracy, high precision data points collected during LiDAR scanning it is possible to use statistical techniques to measure levels of change within the level of precision of the LiDAR scan data. What this means is that even if the LiDAR point cloud only has millimeter precision, millimeter scale change can be measured using statistical analysis of the data. A review of previously completed work demonstrating the use of LiDAR scanning for geological applications and for deformation monitoring is discussed in the following sections.

3.5.1 Applications of LiDAR scanning in Geological Engineering

Many authors have demonstrated the use of LiDAR scanning for geological engineering applications, the most common use being the extraction of rockmass discontinuity data (Fekete et al. 2010, Gigli & Casagli, 2011, Lato et al., 2009, Otto et al., 2011, Poropat, 2006, Slob, 2010,
Slob et al., 2007, Sturzenegger & Stead, 2009). LiDAR has also been used for large-scale roughness calculation using the roughness length measurement technique (Rahman et al, 2006, Tesfamariam, 2007) and recently, LiDAR scan data has been used for discrimination between lithological units in sedimentary rocks (Burton et al., 2010). The main applications of LiDAR scanning in tunnels and shafts to date has been for as-built modeling, rockmass characterization (Fekete et al., 2010), and for quality control and assurance, such as the calculation of shotcrete thickness and rockbolt spacing. Fekete et al. (2010, 2012) have also used LiDAR data to locate areas of seepage in the rockmass, map structures in the face of the tunnel, structurally map and document sections of overbreak, and to extract joint models for implementation into discrete element models.

Seo et al. (2008) used tunnel cross section analysis to measure overbreak and underbreak, and showed the economic viability of this measurement. The extracted cross-sections were analyzed in CAD and compared to design sections. Volume of overbreak was calculated using a range in the number of data points, but the method for cross sectional volume calculation was not demonstrated.

The use of LiDAR in a tunneling environment has also been shown by Decker & Dove (2008) in the Devil’s Slide tunnels in California. Here, LiDAR was used to compare excavation profiles against design profiles for quality control of shotcrete thickness, profile and smoothness, for geological documentation, and for rockmass classification.

3.5.2 Deformation monitoring with LiDAR

Deformation monitoring using terrestrial laser scanning is gaining attention due to the high point density and spatial resolution that can be acquired in a minimal amount of time, but the
use of laser scanning for deformation monitoring is still in its infancy. Authors have begun to develop techniques for using LiDAR scanning for deformation monitoring of various objects, such as a television tower (Schneider, 2006), an old cooling tower (Ioannidis et al., 2006), concrete and wooden beams (Gordon & Lichti, 2007) and in geological environments such as landslide areas (Hui et al., 2010, Jaboyedoff et al., 2009, Monseratt & Crosetto, 2008, Oppikofer et al., 2009, Slob & Hack, 2004), volcanic environments (Nguyen et al., 2011) and rockfalls zones (Abellan et al., 2011, Scaioni & Alba, 2010).

Schneider (2006) demonstrated the use of terrestrial laser scanning for deformation monitoring of a television tower by taking sections of the tower in the z plane, the plane defined by a normal parallel to the tower axis, and calculating the area of the sections to determine the amount of bending in the tower.

Monseratt & Crosetto (2008) estimated deformation of landslides using terrestrial laser scanning data processed using local surface matching techniques. The methodology involves co-registration of point clouds, using a least squares fitting algorithm, for areas that are assumed to be stable. After the surfaces from two different time periods are matched, the displacement vectors are extracted. An assumption is made that the slide is a rotational failure with a simple circular slip surface.

Slob & Hack (2004) demonstrated the use of time of flight scanning for calculating volume change in earth fill and soil heaps. They showed about 5 to 10 mm resolution in the measurements of deformation. Although the measurement of deformation of a landslide is a similar problem to scanning within tunnels, measuring the change in a soil heap is actually a 2.5D problem, not a truly 3D problem as the change in volume can be measured relative to the z plane, which here is the horizontal plane. In a tunnel, change must be measured relative to the centroid
of the tunnel therefore the change cannot be compared using a projection onto the z plane as in the case of a landslide.

Tsakiri et al. (2006) has suggested that deformations smaller than the magnitude of single-point precision of the scanner can be measured by deriving best-fitting surface models. To make use of best-fitting surfaces, pre-processing to eliminate noisy and erroneous data must be completed. The method proposed by Tsakiri et al. (2006) has been applied to planar surfaces successfully, but has not been extended to rough, undulating, or curved surfaces as would be found in underground excavations.

3.5.3 Previous work in LiDAR scanning for monitoring tunnel deformation

Completing full surface deformation characterization in tunnels is beneficial as it can allow for greater understanding into the mechanism through which rockmasses react to excavation, and allow for the prediction of potential stability issues (Lemy et al., 2006). As such, the next logical application of LiDAR in underground environments is for the measurement of deformation. Authors have begun to demonstrate methods for applying LiDAR scanning to tunnel deformation monitoring (Van Gosliga et al., 2006, Lemy et al., 2006, Lindenberg et al., 2005, Lindenberg et al., 2009, Nuttens et al., 2010).

Van Gosliga et al. (2006) are one of the first in the literature to demonstrate a methodology for measuring deformation in tunnels using terrestrial laser scanning. Van Gosliga et al. (2006) decimated the LiDAR scan to deal with only 1% of the data, reducing the point cloud data to a 15 cm by 15 cm grid. Scanning was completed in a concrete segment lined tunnel, where roughness of the surface was of minimal concern. Their methodology used raster partitioning over a cylinder to compare scans of different epochs using a computational program
written in Matlab. The scan data is partitioned into cells and the raster is laid out over the object. During the experiment scans were taken hours apart with objects added to the surface of the tunnel to simulate deformation. The scan data was fit to the cylinder using a least squares adjustment.

Van Gosliga et al. (2006) indicated that it remained to be determined if the amount of deformation measured was real or due to measurement noise, and did not test whether using a higher percentage of the original data set changed the results of the measurement of deformation in a meaningful way. Furthermore, they suggested that the results of deformation analysis using terrestrial laser scanning would be improved when the behavior of scan accuracy was better understood.

Lemy et al. (2006) proposed a method for monitoring deformation in LiDAR scans by matching scan points to total station survey markers. Similar to Van Gosliga et al. (2006), Lemy et al. (2006) characterized deformation by fitting a surface to the LiDAR data and interpolating the scan points over a localized grid.

A methodology for measuring deformation with LiDAR in rectangular tunnels was created by Lindenberg et al. (2009). A simple region grown from a seeded point, similar to region grows used for joint extraction (Slob et al., 2007), was used to define segments of data. A data point is chosen and the normal vector of the point is estimated based on fitting a plane to its nearest neighbors. If the points defining the plane result in a large root mean square error (RMSE) the point is discarded as a single point, and another data point is chosen to start the process. The region grow continues until the threshold for the RMSE is reached. The data was segmented by Lindenberg et al. (2009) to allow for selection of near planar regions of points that were then fit with a best fit plane, and the deviation of the points from the plane were calculated.
Nuttens et al. (2010) has also demonstrated a method for determining deformation within a concrete segment lined tunnel using LiDAR data. A triangular irregular network (TIN) was created from the LiDAR data and then a cross section of the tunnel was extracted from the TIN. The locations of sequential cross sections based on different scans of the tunnel were taken through reference targets installed on the tunnel walls to ensure that the sections were extracted from the same locations. The main limitation of this approach is that a TIN is not truly three dimensional, therefore is not a completely accurate representation of the tunnel profile. During the creation of a TIN, a surface is created by projecting from a single direction. No two points can exist with a third coordinate equal in this direction. For example, if a TIN were created by projecting onto a surface in the x,y direction, no two points could have the same z coordinate, hence a TIN is not truly 3D.

Nuttens et al. (2010) also used a 51 point moving average calculation to smooth the surface of the TIN, which had little impact on the accuracy of the analysis due to the smoothness of concrete segments, but would have a large impact on results for deformation in an unlined tunnel.

To date, no systematic methodology for using LiDAR in an active or unlined tunneling environment to measure deformation has been created.

3.5.4 Benefits of LiDAR scanning for deformation monitoring

As previously mentioned, the high point density and spatial resolution of LiDAR data allows for full surface characterization. There are additional benefits to using LiDAR scanning for deformation measurement including its ability to be implemented without interruption of the construction sequence, and the lack of necessity for permanent referencing. Furthermore, it is not
important to carefully select sections for subsequent deformation analysis, prior to monitoring, as any location within the tunnel can be extracted from the LiDAR scan for analysis, as long as scans are taken at spacing close enough to provide full coverage.

The ability of LiDAR scanning to be implemented into tunnel construction, without interruption of the excavation sequence, has been shown by Fekete et al. (2010). Although Fekete et al. (2010) were not using the scanned data to determine deformation, little change in the scanning protocol they developed is necessary to permit deformation detection from temporal scans.

As shown in this thesis, with LiDAR, it is possible to measure deformation and convergence without using a total station to locate the device and without installation of any permanent reference markers.

As LiDAR scanning offers less raw single point precision than traditional tunnel convergence monitoring techniques (Lindenberg et al., 2005), it is best suited for broader surface analysis rather than for measurement of specific points of movement, particularly because it is impossible to exactly locate and scan the same point in subsequent LiDAR scans. Hence, LiDAR must use either full surface analysis or statistical techniques for measurements of change.

3.5.5 Limitations and errors in current analysis techniques

When dealing with LiDAR data for quantitative analysis or, specifically, change detection, it is not sufficient to look at raw point cloud data. Post processing is necessary to generate usable data. Only in recent years have computing and software developments become robust enough to allow users to handle and process massive data sets. Buckley et al. (2008) provided a typical workflow used for collecting and processing LiDAR data for geological
applications. Although they defined a workflow specifically for outcrop mapping, many of the
same steps can be used for underground LiDAR mapping.

LiDAR scanning it is best suited for surface analysis, rather than specific point analysis. It should also be noted that it is generally not possible to exactly locate and scan the same
individual surface point in a LiDAR scan.

General purpose algorithms are often not sufficient for specific purposes such as change
detection. It is not appropriate, for example, to use the built-in PolyWorks® V 11.0.28
(InnovMetric, 2011) calculation of shortest distance as, in this case, shortest distance is normally
an underestimation of the total displacement or change. It is more appropriate to use the normal
distance from a specified reference surface (tunnel cylinder or arc), comparing the distance to the
old surface (original profile) with the distance to the new surface (deformed profile).

In standard LiDAR data processing using software like PolyWorks® V 11.0.28
(InnovMetric, 2011), the data is interpolated twice before any measurements are extracted. The
data is initially interpolated when loaded into the program to allow each interpolated point to be
assigned a normal direction based on its relation to its nearest neighbors. A second interpolation
occurs when the data is meshed. Every interpolation introduces a small amount of error into the
data, and means that the user is one step further away from working with the initial collected data
set.

Another drawback of current deformation monitoring techniques using LiDAR scanning
is the necessity for meshing the LiDAR data. Not only is this a second interpolation of the data,
which in itself adds a level of error to the data, but also meshing itself can create erroneous
surfaces. A clear example of this is shown when a scan of a tunnel shaft where the walls have
been covered in screen. It is impractical to try and pre-process the data to eliminate the screen
within a scan, as it is almost impossible to discern between the screen and the tunnel wall behind. Currently no filtering algorithms exist for removing one surface layer (screen) from another (rock surface) in LiDAR data sets. If the data is meshed without removing the screen, the screen and the bare rock are meshed as one surface. This is clearly an erroneous surface, and cannot be used for any type of deformation or profile analysis.

Another limitation of current software based deformation mapping techniques is that it is limited to color display of shortest distances. As previously mentioned, using shortest distance calculation between profiles leads to an underestimation in the amount of deformation of the tunnel. The color display is a useful visualization technique but it is more beneficial to actually extract quantitative deformation profile data. The Matlab code written as part of this thesis automatically produces deformation profile graphs.

The current programs do allow for a volume change calculation, but this is a unidirectional volume change calculation and must be taken in reference to a given plane. This methodology works well, for example, when shotcrete volume needs to be calculated for an area sprayed on the roof of the tunnel. It is not beneficial though for calculating the volume change of a tunnel that is squeezing in every direction.

### 3.6 Ellipse fitting for tunnel deformation analysis

Tunnel convergence measurements are usually analyzed by plotting each point or chainage measurement separately. Terzaghi (1942) introduced a method to instead fit an ellipse to the displaced points and determine the mean deformation ellipse. The mean deformation ellipse allows for different periods of deformation to be more easily characterized than by looking at single measurements of displacement. For example, one can clearly discern periods of vertical
compression from those of horizontal compression. Terzaghi’s (1942) ellipse fitting method allows for the identification of patterns within convergence measurements more easily than by looking at convergence measurements alone (Stiros & Kontogianni, 2009).

Historically, use of the Terzaghi’s (1942) mean deformation ellipse has been limited due to a limited number of profile measurements. With the advent of LiDAR and other fast point acquisition technologies, this method is becoming more feasible.

To use an ellipse for convergence measurement, it is generally assumed, for the purpose of back calculation, that plane strain conditions exist, which is valid as this assumption is common practice at distances greater than two diameters from the tunnel face. The use of the deformation ellipse also assumes an initially circular section that deforms gradually over time. One of the main benefits of using an ellipse to define deformations is that the change in the elliptical profile over time defines a two dimensional tensor that can be used to characterize deformation.

The long and short axes of the ellipse correspond to the principal deformation directions within the tunnel. To calculate the amount of normalized tunnel convergence, uniform deformation between measurements must be assumed. The orientation of the ellipse can easily be calculated, and is important to identify because a change in the orientation of the ellipse suggests a change in the deformation pattern. In other words, the directions of minimum and maximum deformation have changed over time.

Changes in deformation pattern are not easy to discern from convergence measurements when they are analyzed alone. The shape and the orientation of the ellipse, along with back analysis of these parameters allow for information on the mechanisms of deformation to be gained, which can be applied to future decision making for a given tunneling project.
The implementation of the mean deformation ellipse for assessing convergence measurement in a tunnel has been demonstrated by Stiros and Kontogianni (2009). Ellipses were fit to profile measurements made with tapes, and it was suggested by the authors that this technique could be used for laser scanning point cloud data.

3.6.1 Methods of ellipse fitting

When fitting an ellipse to a cross section of data there are different methods for creating the best fit ellipse. These methods can be separated into algebraic ellipse fits and geometric ellipse fits. Algebraic methods are simpler, but have curvature bias. Geometric solutions, using genetic algorithms are more complicated, but deal better with noisy data and have no curvature bias.

3.6.1.1 Algebraic ellipse fitting

An algebraic ellipse fit works on the basis of using a linear or non-linear least squares regression. The formula for an ellipse is:

\[ ax^2 + bxy + cy^2 + dx + ey + f = 0 \]

Where:

\[ b^2 - 4ac \leq 0 \]

To fit a set of data with an ellipse, the formula for an ellipse can be generalized to:

\[ ax_i^2 + bx_iy_i + cy_i^2 + dx_i + ey_i + f = r_i \]

Where the set of data to fit points to is:

\( (x_i, y_i, \ldots, x_n, y_n) \)

The set of equations can then be written in matrix form:
Or:

\[
\begin{bmatrix}
{x_1^2} & {x_1 y_1} & {y_1^2} & {x_1} & {y_1} & 1
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots
{x_i^2} & {x_i y_i} & {y_i^2} & {x_i} & {y_i} & 1
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots
{x_n^2} & {x_n y_n} & {y_n^2} & {x_n} & {y_n} & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f \\
\end{bmatrix}
= 
\begin{bmatrix}
r_1 \\
r_2 \\
r_3 \\
r_4 \\
r_5 \\
r_n \\
\end{bmatrix}
\]

Or:

\[Xu = r\]

Where:

\[u = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}\]

To find the best fit ellipse, r must be minimized therefore:

\[\|Xu\| = \min\]

Essentially, the algebraic solution attempts to find the set of \(u\) such that the system of equations is as close as possible to the equation of a perfect ellipse.

Fitzgibbon et al. (1996) presented an efficient algebraic fitting method that minimizes the algebraic distance using the constraint:

\[b^2 - 4ac = 1\]

This incorporates an ellipticity constraint into the normalization factor; therefore the system of equations is not solved for a general conic. Fitzgibbon et al.’s (1996) method is robust and easy to implement, and therefore it is the method that has currently been integrated into the Matlab code written for ellipse fitting to LiDAR data. Fitzgibbon et al. (1996) demonstrated that the non-iterative ellipse specific algorithm is better than other algebraic fitting algorithms for noisy and/or occluded data sets.
3.6.1.2 Geometric ellipse fitting

Genetic algorithms are useful for solving complicated multi-variable non-linear inverse problems. To complete a geometric ellipse fit, the goal is to minimize the Euclidean distance from any point to the corresponding point on an ellipse (Figure 3-3).

The parametric equation for an ellipse is:

\[
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} a \cos \varphi \\ b \sin \varphi \end{bmatrix}
\]

Where:

- \((x_0, y_0)\) is the centre of the ellipse
- \(a\) is the length of the longest axis
- \(\alpha\) is the angle of inclination of the axis of length \(a\) from the \(x\) axis
- \(b\) is the length of the axis that is perpendicular to \(a\)
- \(\varphi\) is the parameter that runs from 0 to \(2\pi\) anti-clockwise from the major axis (to point)
Figure 3-3: Parameters that characterize an ellipse. The distance that is minimized by a geometric ellipse fit is the distance between the $i^{th}$ data point $(x_i, y_i)$ and the corresponding point on the ellipse $(x,y)$ (after Ray & Srivastava, 2008).

A given ellipse is therefore characterized by the following parameters:

$$ u = \begin{bmatrix} x_o \\ y_o \\ a \\ b \\ \alpha \end{bmatrix} $$

If a set of data does not lie directly on the ellipse, $d_i(u)$ can be used to define the distance from the $i^{th}$ data point to the center of the ellipse:

$$ d_i(u) = \left\| \begin{pmatrix} x_i \\ x_o \\ y_i \\ y_o \end{pmatrix} \right\| $$
The distance from the center of the ellipse along the same line that connects the center to 
\(d_i(u)\) to a point on the ellipse is:

\[
c_i(u) = \left\| \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} a \cos \varphi_i \\ b \sin \varphi_i \end{bmatrix} \right\|
\]

\[= \sqrt{(a^2 \cos^2 \varphi_i + b^2 \sin^2 \varphi_i)}
\]

Thus we can define the distance from the \(i^{th}\) data point to the point on the ellipse as:

\[
F_i(u) = ||d_i(u) - c_i(u)||
\]

The goal of the ellipse fitting algorithm is to minimize \(F_i(u)\).

This involves solving the non-linear least squares problem. To minimize \(F_i(u)\), the model needs to be found such that:

\[
G(u) = \sum_{i=1}^{m} F_i^2(u) = \text{min}
\]

To solve this equation \(\varphi\) must be known. With \((x_i, y_i)\) the angle of \(\varphi_i\) from \(\alpha\) is calculated using the point on the ellipse \((x, y)\) and the centre of the ellipse \((x_o, y_o)\).

The slope of the line \((p)\) that connects \((x, y)\) and \((x_o, y_o)\) is:

\[
p = \frac{y - y_o}{x - x_o} = \frac{y_o - y_i}{x_o - x_i}
\]

As \(x_i, y_i\) are known and \(x_o, y_o\) are known it is easy to calculate \(p\).

To use the genetic algorithm for solving complicated multi-variable non-linear inverse problems, some initial conditions and estimations must be made. For ellipse fitting it is clear that \((x_o, y_o)\) must lie within the area defined by the point cloud. The length of the long axis also cannot exceed the diagonal of the most extreme \((x, y)\) coordinates. \(\alpha\) can be restricted to \((0, \frac{\pi}{2})\) as either the short axis or the long axis must lie in the first quadrant.
Ray & Srivastava (2008) have demonstrated that the geometric solution is more robust for ellipse fitting to noisy data, where only partial data sets are acquired. Although genetic algorithms are very powerful, and the geometric ellipse fitting algorithm has proven to produce better results than algebraic fitting algorithms, it is not considered useful at this point for processing LiDAR data. The genetic algorithm is computationally much more intensive and is preferable when only a few data points exist. Genetic algorithms are of a greater advantage when the number of points acquired is low, and the noise in the points is high. However this approach has not been considered to be the best option for fitting to LiDAR profile data due to the large number of data points available from a single scan. It is suggested however, that testing of genetic algorithm fits on LiDAR data be performed to test this hypothesis in the future.

3.7 Tunnel profile analysis with LiDAR

To analyze tunnel convergence it is appropriate to look at multiple cross sections throughout the tunnel. To complete cross section analysis of LiDAR data, a new methodology has been developed in Matlab. Prior to cross section extraction, the following steps must be taken:

- Collect LiDAR scan data in the field
- Upload scan data
- Trim scans to the area of interest
- Clean and filter data
- Align multiple scans of same area
- Create surface or mesh from point cloud data
- Extract cross sections
The data must be cleaned to remove all erroneous and spurious scan points. Objects within the scan that are not part of the tunnel profile, such as machinery or people must also be removed. If there is screen or other support installed on the surface of the tunnel walls this does not have to be removed during pre-processing. A filter has been built into the Matlab code, as part of this thesis, that will serve to remove screen or other physical objects on the tunnel surface that are not easily filtered out in pre-processing.

After the initial cleaning of the LiDAR data, the scan of the tunnel must be aligned such that the z axis is the up direction, and the tunnel axis is aligned parallel to either the x or y axis. Alignment in such a manner allows for cross sections to be accurately extracted along the plane perpendicular to the heading direction of the tunnel. This coordinate orientation is also necessary for the cross section to be compatible with the Matlab code.

When comparing multiple scans it is important to extract a cross section from the same location within the tunnel. It is therefore desirable, but not necessary, to use a best fit algorithm to align the two point clouds. This ensures that as close as possible to the same cross section is extracted from both scans. The Iterative Closest Point (ICP) algorithm for alignment of point clouds is a data driven registration method first proposed by Besl & McKay (1992). This is a widely used algorithm, and is the built in registration method for most software programs, including PolyWorks® V 11.0.28 (InnovMetric, 2011). The goal of the ICP is to create the best alignment between the data shape \((D)\) and the model shape \((M)\). In the case of LiDAR point cloud alignment, the first point cloud for alignment \((PC_1)\) will be used in place of the model shape, and the second point cloud \((PC_2)\) will be used in place of the data shape. The distance \((d)\)
between an individual point \((p_1)\) in the first point cloud and its closest point in \(PC_2\) is minimized by the algorithm:

\[
d(p_1, p_2) = \min_{p_2 \in P_2} \|p_2 - p_1\|
\]

Each point in \(PC_1\) and its closest point in \(PC_2\) form a correspondence. The ICP algorithm completes a least squares regression to minimize the sum of the mean square distance errors between the correspondences. A detailed description of the algorithm used to complete the transformation is beyond the scope of this paper but can be found in Besl & McKay (1992).

Alignment of scans from multiple different time periods allows for easier extraction of the same cross section for comparison. As no absolute positioning is incorporated in this method of LiDAR deformation analysis, the best way to align scans is by ICP algorithm. The algorithm used by the author is built in to PolyWorks® V 11.0.28 (InnovMetric, 2011). To complete an alignment, a large look window is initially used for rough alignment (i.e. 1 m). This window is progressively decreased until it reaches a level equal to the calculated noise level within the data. By progressively decreasing the look window to the level of data noise, non-deformed areas should automatically be the only areas used by the end of the alignment. It is still important for the user to check the data to ensure a logical alignment of the data has been made and to select the areas of cross section extraction with care.

There are two proposed methods of cross section extraction. Either a mesh can be created from the LiDAR scan data, and a cross section extracted from the new surface, or a narrow swath of point cloud data can be used. Both of these methods allow for elimination of errors associated with an attempt at direct point to point comparison, and reduction of errors associated with noise in the data.
Once cross sections are extracted, they can be input into the new method of profile analysis, the combined Elliptical Fit Analysis and LiDAR Profile Analysis.

### 3.7.1 Elliptical Fit Analysis

The first stage of the developed methodology for profile analysis is the Elliptical Fit Analysis (EFA). In the first step of the EFA, two cross sections of a tunnel are imported into Matlab. In general these would be cross sections of the same area of a tunnel extracted from two temporal LiDAR scans, but can be any profile point cloud data. The code assumes that the cross sections are two dimensional data. Hence, proper alignment of the tunnel model during cross section extraction is important.

Each tunnel profile is fit with an ellipse using a non-iterative ellipse specific algorithm developed by Fitzgibbon et al. (1996). The ellipses are then translated so they are centered about the origin. The parameters that define each ellipse (the length of both axes, the location of the centroid, and the rotation of the axes) are extracted for comparison. The change in the elliptical profile is also displayed as the distributed radial displacement profile (DRDP) (Figure 3-4 b). The DRDP is calculated as:

\[
DRDP = E_1 - (\Delta E_1 E_2) \ast \gamma
\]

Where:

- \( E_1 \) is the best fit ellipse to the first LiDAR cross section
- \( E_2 \) is the best fit ellipse to the second LiDAR cross section
- \( \gamma \) is the number of times the displacements are to be exaggerated
is the change between the first and second best fit ellipse, a positive number indicates the tunnel profile has converged between the first and second scan.

The DRDP allows for areas of convergence and expansion to be easily discerned within the profile. The DRDP is also a specific tensor that characterizes the deformation between the first and second LiDAR scan.

The EFA also produces a diametric displacement profile (DDP) (Figure 3-4 c). The DDP is calculated as:

\[ DDP = E_1 - (\Delta E_{1,2}) \]

The DDP allows for measurement of the maximum convergence and expansion of the tunnel profile.

As this analysis is completed using a best fit, and the fit itself is being analyzed, raw point cloud cross section data can be imported without any prior interpolation for surface creation.

**3.7.2 LiDAR Profile Analysis**

The second stage of the methodology developed is the LiDAR Profile Analysis (LPA). The steps of the LPA are very similar to the EFA. Firstly, two cross sections of a tunnel are imported and each profile is fit with an ellipse using a non-iterative ellipse specific algorithm (Figure 3-5). The ellipses are then used to translate the LiDAR profiles so they are centered about the origin.
Figure 3-4: Example of elliptical fitting for profile analysis: a) two ellipse fits ($E_1$ and $E_2$) with orientation of long axis displayed, b) distributed radial displacement profile (DRDP) shown in purple at five times exaggeration, and c) diametric tunnel displacement profile (DDP).
LPA uses the LiDAR point data instead of the elliptical fit to the data. The change in the LiDAR profile is used to calculate the DRDP and the DDP, exchanging $E_1$ and $E_2$ for $CS_1$ and $CS_2$, where:

- $CS_1$ is the first LiDAR cross section
- $CS_2$ is the second LiDAR cross section

Therefore the DRDP (Figure 3-5 b) becomes:

$$DRDP = CS_1 - (\Delta_{CS_1CS_2}) \gamma$$

And the DDP (Figure 3-5 c) is calculated as:

$$DDP = CS_1 - (\Delta_{CS_1CS_2})$$

For the LPA analysis, $CS_1$ and $CS_2$ are extracted from surface models of the LiDAR data, so $CS_1$ and $CS_2$ are not raw points. By definition the comparison of $CS_1$ and $CS_2$ is a surface to surface or mesh to mesh comparison.

Additionally, the LPA calculates the areas of volume increase and volume decrease from the LiDAR tunnel profiles (Figure 3-6). Instead of a unidirectional comparison, the volumes are calculated with respect to the center of the tunnel, therefore a more accurate volume is found than if a unidirectional shortest distance comparison were completed.

To characterize the noise of the LiDAR scan data, the LPA analysis also includes the generation of a noise profile (NP). The noise profile is the calculated deviation between the LiDAR profile and the elliptical fit to that LiDAR data (Figure 3-7):

$$NP_{CS_1} = DDP_{E_1} - DDP_{CS_1}$$
Figure 3-5: Example of profile analysis with LiDAR point data: a) two LiDAR profiles for comparison (CS$_1$ in green and CS$_2$ in red), which are elliptically centered, b) distributed radial displacement profile (DRDP) shown in purple at five times exaggeration, and c) the diametric tunnel displacement profile (DDP).
The NP allows for characterization of both the random uncorrelated noise and any systematic deviation from the elliptical profile. Areas of anomalous movement of the tunnel profile can also be easily identified using the NP.

To reduce the impact of noise within the LiDAR scan and/or to reduce the impact of roughness of the scanned surface so larger deformation trends can be considered, a moving average calculation of the DDP from the LPA is included in the NP analysis. The number of points to be included in the moving average should be chosen based on the point spacing of the

Figure 3-6: Areas of volume increase and decrease of profile overlain on top of the distributed radial displacement profile (DRDP) from the LiDAR Profile Analysis (LPA). A positive volume indicates an area of convergence; a negative value indicates an area of expansion or divergence. The green line indicates the first LiDAR cross section and the purple line shows the DRDP.
scan data and the roughness of the surface. If too large a number of points are chosen for the moving average calculation, real deviations in the surface that are important for convergence measurement may be eliminated.

**3.7.3 Data filtering**

The type of surface support installed on the walls of the tunnel analyzed by the EFA and LPA methods impacts the amount of filtering required prior to analysis. For example, during the course of this thesis, LiDAR scanning was completed in an area where the rockmass was covered

![Figure 3-7](image)

**Figure 3-7:** Deviation of LiDAR diametric displacement profile from elliptical displacement profile for characterization of random and systematic noise and anomalous movement of the tunnel profile. LiDAR diametric displacement profile is displayed as the smoothed curve using a fifty point moving average calculation, shown here in red.
in steel screen. In this area the screen is the minimum level of support, and hence a necessary safety precaution. Although screen presents additional challenges to using LiDAR scan data for change detection, the use of LiDAR scanning is still effective. However, meshing of the LiDAR data (seen in Figure 3-8), which includes the screen, introduces error into the final data set.

In order to remove the effect of the screen on the meshed data, the screen must be filtered prior to meshing (Figure 3-9). Initially it was thought that it might be possible to filter the screen out based on the intensity data collected by the LiDAR, but the results of the scanning show that the difference between the intensity return of the mesh and the rock is negligible. A filter has been included in the EFA and LPA that allows for near surface deviations from the rock profile (such as screen) to be removed based on the distance from the center of the ellipse to the surface. The user must choose the appropriate “look window” size and “filter cut-off” for filtration, based on point spacing and the size of deviation to be removed. The “look window” is the portion of the diameter of the tunnel to be filtered at a given time. The “filter cut-off” is the percentage of the tunnel profile data to keep. For large anomalies (i.e. rockbolts) a large look window is necessary. For near surface anomalies and rough profiles a smaller look window is best. Some trial and error may be required to properly filter the surface.

3.8 Workflow

To further explain the process that the Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (LPA) must follow, a workflow demonstrating the steps of the procedure has been created (Figure 3-10). As explained in sections 3.7.1 and 3.7.2, first two separate tunnel profiles are extracted from the LiDAR data sets (Figure 3-10 a & b). Each of the extracted LiDAR tunnel profiles,
Figure 3-8: a) Scan data collected on the wall of a tunnel covered with screen. b) Close up of the raw LiDAR point cloud data, and c) data after it is meshed, showing how the screen and rock surface are integrated as one surface.
Figure 3-9: Meshing errors are created when near surface anomalies are not filtered from the data set prior to meshing a) LiDAR data meshed without filtration, b) sketch showing how errors are created during meshing without filtering.
generally referred to as CS\textsubscript{1} and CS\textsubscript{2}, are best fit with an ellipse using a non-iterative algebraic elliptical fitting algorithm (Figure 3-10 c & d). The best fit ellipses are referred to generally as E\textsubscript{1} and E\textsubscript{2}, where E\textsubscript{1} is fit to CS\textsubscript{1} and E\textsubscript{2} is fit to CS\textsubscript{2}. The centroids of E\textsubscript{1} and E\textsubscript{2} are then used to align both the best fit ellipses and the LiDAR profiles of the two tunnel sections (Figure 3-10 e). The two aligned best fit ellipses are then extracted for use in the Elliptical Fit Analysis (EFA) (Figure 3-10 f), and the two aligned LiDAR profiles are extracted for use in the LiDAR Profile Analysis (LPA) (Figure 3-10 g).

A workflow has also been created that outlines all the steps in the analysis from LiDAR data collection to the analysis through the EFA and LPA is shown in Figure 3-11.

3.9 Application of EFA and LPA to real data

To demonstrate the application of the EFA and LPA analysis developed by the authors, the analyses have been tested on LiDAR scan data. A three-story 18.3m diameter Martello tower, Fort Frederick, located in Kingston, Ontario Canada was scanned using a Leica HDS 6000 system (Figure 3-12). Eight scans were taken from around the base of the tunnel at high resolution, and then stitched together in PolyWorks® V 11.0.28 (InnovMetric, 2011) using an ICP alignment. The tower was constructed in 1846, primarily of limestone bricks which are an average of 30 cm in height and range from 30 to 100 cm in width. The average roughness of the blocks is $\pm 4$ mm which was quantified based on the deviation from the surface of a best fit plane. The tower is roughly circular with flattened walls near the base, and four rounded rooms at each corner. There are also multiple windows located at varying heights in the central region of the tower, and a wooden bridge structure at the northwestern corner of the structure.
Figure 3-10: Steps for elliptical fitting and alignment of cross sections used for EFA and LPA: a) CS$_1$, the extracted cross section from the first LiDAR scan (or set of scans), b) CS$_2$, the extracted cross section from the second LiDAR scan (or set of scans), c) CS$_1$ fit with an ellipse, E$_1$, d) CS$_2$ fit with an ellipse, E$_2$, e) alignment of CS$_1$, CS$_2$, E$_1$ and E$_2$ using centroids of ellipses, f) E$_1$ and E$_2$ aligned for EFA, and g) CS$_1$ and CS$_2$ aligned for LPA.
Figure 3-11: Workflow for deformation monitoring of tunnels with LiDAR data
Figure 3-12: LiDAR scan of a Martello tower, Fort Frederick, located in Kingston, Ontario Canada, scanned using a Leica HDS 6000 system.

To test the EFA and LPA analysis, multiple cross sections, taken from the mesh in the central circular area of the tower were compared. Care was taken to avoid the lower area of the tower where the walls are flattened. Sections were taken every 0.5 m up the tower wall (Figure 3-13). The lowest section, taken at a height of 6 m above the ground surface, was used as the reference cross section for each comparison and will hereafter be referred to as CS$_1$. Each cross section was compared to CS$_1$ and the maximum, minimum and average amount of diametric change was measured. These measurements were compared to an “actual” amount of change, calculated using the slope of the external wall and the spacing of the cross sections.

The cross sections of the tower also allow for the testing of the filtering algorithm built into the profile analysis. Some of the cross sections of the tower were cut through the windows...
located in the midsection of the tower. These windows create anomalies in the section that must be filtered out prior to calculating the change in diameter (Figure 3-14 a). Using the filtering algorithm with an appropriate look window and filter cutoff, the anomalies created by the windows can be easily filtered out, without removing any other data from the tower.

Figure 3-13: Cross sections taken at a vertical spacing of 0.5 m through LiDAR model of Fort Fredrick tower.
Figure 3-14: Cross section of tower intersecting windows a) prior to filtering, and b) after using a 5% filter cutoff to remove anomalies.

profile (Figure 3-14 b). This method is the same as would be applied to remove rockbolts or other anomalies in a tunnel profile that may be present in an extracted profile, but do not represent an actual part of the tunnel profile.

By comparing sections taken at increasing elevations along the tower wall, the measurement of a converging tunnel is simulated. An example of the results from completing an EFA and LPA on the tower data and the development of a NP is shown in Figure 3-15.

3.10 Conclusions

Deformation monitoring is traditionally completed by measuring five or more permanently installed markers on the tunnel profile. Although this type of monitoring provides a high level of accuracy, it does not completely characterize the tunnel profile. Using LiDAR for
tunnel deformation monitoring provides the user with the ability to capture a much more complete picture of the location and pattern of deformations occurring.

A new method for tunnel profile analysis using elliptical fitting to LiDAR profiles has been developed to allow for analysis both of the general trends in deformation and for the analysis of anomalous movements. General trends of deformation can found using the Elliptical Fit Analysis (EFA) and can be used for back analysis of stresses and rockmass properties such as deformability. Using LiDAR Profile Analysis (LPA), areas shown to be moving anomalously can be analyzed separately from the general trends, and supported additionally to maintain stability.

In the future it will be important to test the sensitivity of the newly developed EFA and LPA to errors that occur within LiDAR data such as random noise, systematic noise, and occlusion of the tunnel profile.
Figure 3-15: Example output from application of EFA and LPA analysis to real LiDAR data. a) Initial tower cross section shown in green with DRDP at 5 times exaggeration displayed in purple, b) Deviation of LiDAR diametric displacement profile from elliptical displacement profile for characterization of random and systematic noise and anomalous movement of the tunnel profile. LiDAR diametric displacement profile is displayed as the smoothed curve, shown here in red. The noise profile (NP) is shown in purple.
Chapter 4

Sensitivity Testing of the Newly Developed Elliptical Fitting Method for the Measurement of Convergence in Tunnels and Shafts

4.1 Abstract

Light Detection and Ranging (LiDAR) is becoming more widely used in the geotechnical community as its number of applications increase. It has been shown to be useful in tunneling for applications such as rockmass characterization and discontinuity measurement. LiDAR data can also be used to measure deformation in tunnels, but before a comprehensive methodology can be developed, the accuracy issues associated with scanning must be fully understood. Once the accuracy issues associated with LiDAR are well understood, any analysis technique that uses LiDAR data must be tested to ensure the determined accuracy issues have minimal impact on the results of the analysis.

To prove the usefulness of the newly developed elliptical fitting method for the measurement of convergence in tunnels and shafts proposed by Delaloye et al. (2012), a comprehensive analysis of accuracy issues associated with LiDAR scanning was conducted and then a sensitivity test of the convergence measurement technique was completed. The results of

the analysis show that by using the statistical techniques built into the Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (LPA), levels of change within the level of random and systematic noise included in the data can be measured. Furthermore, the new analysis is robust enough to handle large amounts of occlusion within data sets.

4.2 Introduction

Deformation monitoring is an important part of any underground excavation project. Traditional monitoring generally involves measurement of either tapes or control points installed around the perimeter of the excavation. Light Detection and Ranging (LiDAR) is becoming more widely used in the geotechnical community, especially in underground environments, and there is growing interest in its application to change detection and deformation monitoring. As techniques for using LiDAR for deformation monitoring are developed, it is important they are tested to ensure they can accurately characterize and measure change within an environment. To do so, the accuracy issues associated with LiDAR scanning must be well understood and techniques must be tested for their sensitivity to these accuracy issues.

This paper provides an in depth review of accuracy issues associated with LiDAR scanning, and then tests the newly developed Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (LPA) (Delaloye et al., 2012) for their sensitivity to these accuracy issues.

4.3 Deformation monitoring in tunnels

Deformation in tunnels occurs as a result of the redistribution of stress around the excavation. The magnitude of deformation is related to the rockmass conditions, the stress ratio and orientation, the excavation method, the rate of excavation, the type of support, and the
locations and distance from the face at which support is installed. Monitoring deformation of tunnels is an important part of modern design and construction practices, and is the basis for the New Austrian Tunneling Method (NATM) (Wittke et al., 2006). When using the NATM method for tunnel excavation, if measurements of deformation are not consistent with those allowed for in the initial support design, more support or different excavation methods are implemented.

Other benefits of deformation monitoring are:

- deformation measurements can help refine the final liner design,
- deformations can serve as an early warning for more severe failures, and
- deformations can be used for back calculation of stress orientation and rockmass properties.

4.3.1 Traditional monitoring techniques

Current techniques for monitoring deformation in tunnels involve one or more of the following techniques (Kontogianni, 2003):

- Tape extensometers
- Geodetic surveys (ie. Total station)
- Photogrammetric devices

Tape extensometers are the most accurate of the surveying devices listed. They have an accuracy of +/- 0.2 mm over 10-15 m (Kavvadas, 2005). The main drawback with tape extensometers is that they only have the ability to measure along a specific line between anchor points which must be installed in the tunnel walls. The installation of permanent anchors often interrupts the construction process of the tunnel. In addition, the anchor points are installed at a time in the construction cycle when it is safe to access the excavation, which is usually after some
level of support has been installed. Therefore the monitoring commences at some distance back from the advancing tunnel face. At this point, a significant portion of the tunnel deformation has generally already occurred.

Total stations make use of optical reflectors installed around the tunnel profile (usually five to seven points), as well as a stable reference point outside the zone of deformation, to measure deformations. The station must be moved progressively forward from the area with the stable reference points towards the locations of the tunnel profile of interest. A total station has an accuracy of about +/- 2.5 mm over 100 m (Kavvadas, 2005). Total station reflectors are very susceptible to disturbance and destruction during construction processes.

Photogrammetric devices require that a number of reference points be installed on the surface of the tunnel. Photogrammetric devices, where the scanner is located using a total station, have an accuracy of at worst +/- 5 mm (Kavvadas, 2005) but with new developments the accuracy of these systems is increasing to the sub millimeter level.

4.4 Terrestrial laser scanning

LiDAR is a laser scanning technology used for quick acquisition of a dense point cloud of three dimensional position data. LiDAR scanners can collect millions of points that have both high accuracy, high precision positional data \((x,y,z)\) and uncalibrated intensity values \((i)\), in a matter of minutes. For example, the Leica HDS 6000 (the scanner used for this research) collects up to 500,000 points per second with a single point positional accuracy of 6mm at 1 m to 25 m, and 10 mm at 50 m (Leica Geosystems, 2007) (measurements are one standard deviation). Newer scanners can collect one million points per second, have increased accuracy, and include additional features such as the ability to collect RGB color data for every point.
There are two main types of terrestrial scanners: time of flight and phase shift, each with their own benefits and drawbacks.

4.4.1 Time of flight scanners

Time of flight scanners (Figure 4-1) measure the distance to an object by sending out a laser pulse and measuring the time elapsed before the pulse is returned. As the speed of light \((c)\) is a known constant, the distance to the object \((D_{obj})\) can be found:

\[
D_{obj} = \frac{ct}{2}
\]

The newest time of flight scanners have a range up to 4 km, hence time of flight scanners are often referred to as long range scanners. They are mainly used in geological engineering for scanning areas such as landslides and rockfall areas that are large and/or are not safe to access. Scan times range from about ten minutes up to an hour, depending on the speed of the scanner and the size of the area to be scanned.

![Figure 4-1: Time of flight LiDAR scanner. These scanners send out a single pulse and measure the time elapsed before the pulse is returned to the scanner to calculate distance.](image)
4.4.2 Phase shift scanners

Phase shift scanners (Figure 4-2) make use of amplitude modulated continuous wavelength (AMCW) lasers, and measure the phase shift between the emitted laser signal and the returned signal. More specifically, AMCW lasers operate by modulating the power of the emitted light with a sine wave of given frequency. The returned energy waveform is delayed by the travel time and appears phase shifted when compared to the emitted wave (Figure 4-3). The measured phase difference ($\theta$) is proportional to the time elapsed:

$$ t = \frac{\theta}{\omega} $$

Where:

$\omega$ is the angular frequency of the modulation

$$ D_{obj} = \frac{c\theta}{2\omega} $$

The accuracy of the measured distance is higher for higher modulation frequencies. Most phase shift scanners improve accuracy by using dual or triple frequency amplitude modulated signals (Figure 4-4).

The distance the scanner is able to measure is proportional to the longest phase modulation; hence phase scanners have a limited data acquisition range. The longest modulation determines the ambiguity interval, or the maximum distance the scanner can measure, usually around 100 m. For the Leica HDS6000 the maximum range is 79 m (Leica Geosystems, 2007).

Phase shift scanners acquire data much more quickly than time of flight scanners as measurements are taken continuously. As such, phase shift scanners are more appropriate for
underground work than TOF scanners due to their rapid acquisition rate and the lack of necessity for long range measurement underground.

Figure 4-2: Phase shift LiDAR scanner, which sends out a continuous beam and has an ambiguity interval (maximum scan distance) equal to the longest modulation of the laser.

4.5 Sources of error in LiDAR scanning

It is important to understand the difference between precision and accuracy when referring to LiDAR point cloud data. Accuracy refers to how close the mean of the measured points is located as compared to the position of the actual mean of the object in space. Precision on the other hand is a function of the resolution of a scan, and how noisy the data is; it is the variability of the measurements. Precision is often simply referred to as “noise”, where a less precise data set is referred to as a “noisy” data set. For LiDAR scan data to be useful it must be both accurate and precise (Figure 4-5). Collectively, the accuracy and precision of the scan data can be referred to as the error within the data set.
Figure 4-3: The distance to an object is measured by the phase shift between the transmitted and received signals.

Figure 4-4: Example of the sinusoidal amplitude modulated signals emitted at one time by an AMCW laser in a phase shift LiDAR scanner.
Figure 4-5: Four sets of point cloud data showing the differences between high and low precision and high and low accuracy. When LiDAR scanning is complete, both high precision and high accuracy are desired.

4.5.1 Sources of range error

A complete knowledge and understanding of where errors originate within LiDAR data is required to ensure that all necessary steps are taken to minimize these errors. The error of the point cloud data collected by a LiDAR instrument can be broken into three different components:

- the error in the range measurement,
- the error in the vertical angle measurement, and
- the error in the horizontal angle measurement.

The total error, \( E \), is equal to the square root of the sum of the squares of all the sources of error.

\[
E = \sqrt{(\Delta X^2 + \Delta Y^2 + \Delta Z^2)}
\]

Where:

- \( \Delta X \) is the horizontal angle error
- \( \Delta Y \) is the range error
- \( \Delta Z \) is the vertical angle error

There are also errors associated with field procedures for scanning and the processing of the LiDAR point cloud data.

The Leica HDS 600 has a single point positional accuracy of \( \pm 5 \) mm at a 1-25 m range, and \( \pm 9 \) mm up to 50 m. This can be broken into the accuracy in the range measurement (\( \leq 2 \) mm at 90\% albedo up to 25m; \( \leq 3 \) mm at 18\% albedo up to 25m; \( \leq 3 \) mm at 90\% albedo up to 50 m; \( \leq 5 \) mm at 18\% albedo up to 50 m) and the accuracy in angle measurement, where both horizontal and vertical are equal (125 \( \mu \)rads or 7.9 mgon) (Leica Geosystems, 2007).

4.5.1.1 Noise created by the scanner

Noise in LiDAR scanning refers to the degree of scattering of data around a best fit plane. Lower noise data will have better modeled precision but not necessarily better accuracy. Low noise data is very important for relative comparisons between scans, but accuracy itself is the most important component when absolute geometry is of greatest importance.

It is easy to determine the amount of noise within a scan when there are perfectly flat surfaces within the scan to use as reference planes. In this case a best fit plane is fit to the area of the flat
surface within the scan, and the deviation of the scan points from this surface are used to
determine the amount of noise within the scan. For rough surfaces, such as the wall of a tunnel,
the quantification of noise within a scan is more difficult. Noise can also be created in phase
based scanners by the speed of the scan. A higher rate of point capture causes the scan speed to
increase, and the data collected to be noisier (Figure 4-6).

Figure 4-6: Two scans of the same limestone brick within a tunnel: one at “medium”
resolution, and one at “super high” resolution with the Leica HDS 6000. Cross sections of
the brick show the super high resolution data is noisier than the medium resolution data.
4.5.1.2 Beam divergence and spot size effect

The spot size refers to the diameter of the laser as it strikes the object being surveyed. There are two different methods generally used by industry for measuring spot size. The first is the Gaussian method, where the laser beam is assumed to reduce in intensity in accordance to the Gaussian profile (Figure 4-7). In this measurement method the spot size is measured as the diameter of the beam where:

\[ l = \frac{1}{e^2} I_{\text{max}} \]

Where:
- \( e \) is the mathematical constant
- \( I_{\text{max}} \) is the maximum intensity of laser beam

The other method of spot size measurement is called full-width-half-height (FWHH) (Figure 4-7), where the spot size is equal to the diameter at half the maximum intensity. FWHH can be used for beams that do not have a Gaussian profile. It is important to note that a FWHH measure of spot size will always result in a smaller measure of the spot size than the Gaussian method. Both methods do not measure the entire beam diameter, but the spot size they define contains most of the energy of the beam.

The laser beam emitted by the LiDAR scanner has a specified diameter which diverges as it travels away from the scanner. For example, the Leica HDS 6000 has a beam with of 3mm at exit (based on Gaussian definition) and has a divergence of 0.22 mrad, hence a beam width of 8 mm at 25 m and 14 mm at 50m (Leica Geosystems, 2007). It is important to know the spot size of the beam at the location of the scan as the spot size has implications with respect to accuracy and precision. A larger spot size results in less definition and also reduces accuracy. Hence, the
Figure 4-7: Gaussian profile of laser beam intensity showing both Gaussian and FWHH measurements of laser footprint size.

Further an object is away from the scanner, the less accurate the measurement will be.

Furthermore, a larger spot size decreases precision as discrimination of features smaller than one third of the laser footprint is not possible.

An increase in spot size can also result in an increased number of erroneous points within a scan. When scanning with a phase shift scanner, if the beam width is distributed over two objects that are located at slightly different distances from the scanner, the difference in the phase shift between the two objects will be averaged and a point between the objects will be returned to the scanner. For example, a curtain of erroneous points that trail away from the edge of the physical screen towards the rock face can clearly be seen in Figure 4-8. These points are the result of the averaging of two phases returned to the scanner. The two partial beam returns are
the result of the laser’s expanded spot size being distributed over both the screen and the rock face.

Figure 4-8: Section of point cloud data from the wall of a tunnel covered with steel screen and rockbolts. A curtain of points can be seen trailing away from the edge of the mesh towards the rock surface.

4.5.1.3 Surface reflectivity

The amount of the laser beam reflected by a surface depends on the color, roughness, and other material properties such as electric permittivity, magnetic permeability and conductivity of
the material (Lichti, 2003). Of main importance to LiDAR scanning is the color of the surface. White surfaces have a high albedo and therefore yield a strong reflection of the laser light. In contrast, reflection from dark surfaces that have a low albedo is weak. White surfaces also produce less noise and less range error than dark surfaces (Jacobs, 2009). Figure 4-9 shows this effect clearly; the dark areas of the target have an error of ±3 mm whereas the white areas of the target have an error of ±1 mm. This affect is also shown in the previously quotes range accuracy of the Leica HDS 6000 which is ≤3 mm at 90% albedo (up to 50 m) and drops to ≤5 mm at 18% albedo (Leica Geosystems, 2007).

The effect of a colored surface on the reflectance of the beam depends on the spectral characteristics of the laser used by the scanner (Boehler et al., 2003). Spectral reflectors or other highly reflective objects tend to scatter the laser light and result in “black holes” within the scan.

Figure 4-9: Scan of a two tone target fit with a best-fit plane. Deviations from the plane are shown in meters. Inset shows sketch of light and dark areas of two tone target.
4.5.1.4 Scan density and spatial resolution

The scan density is a measure of how close the point spacing is in a given scan. For a phase based scanner the density is based on the frequency of the beam of the continuous laser source. Scan density can be increased either by sampling more frequently, or, where this is not an option, multiple scans can be done of the same area. In general, a higher scan density results in a higher resolution of the scan. The point spacing also increases as a function of distance from the scanner.

It is essential to know the spatial resolution of the scanner before acquiring scan data to determine the level of detail that can be resolved from the data. There are different methods for determining spatial resolution of a scanner. Lichti & Jamtsho (2006) proposed the effective instantaneous field of view (EIFOV), which more suppliers are moving towards for measuring resolution. The EIFOV is a function of the sampling density, the width of the laser beam (spot size) and the angle of incidence between the scanner and the surface being scanned. Traditionally an angular resolution and a range resolution are provided. A full explanation on how to measure resolution is beyond the scope of this paper, but the important issue to note is that resolution defines the smallest data sampling range-interval that can be extracted from data.

4.5.1.5 Angle of incidence

The angle at which the laser hits the object being scanned will affect the accuracy of the scan. At an angle of incidence perpendicular to the scanner, the accuracy of the data collected is greatest. As the angle increases and the scanned surface becomes more oblique to the scanner, the measurement of the object’s position becomes less accurate. Higher incidence angles and longer distances result in high noise levels (Soudarissanane et al., 2007, Lato, 2010). Data
collected at an incidence angle of more than 75° should be discarded, and it is suggested to limit scanning to a maximum incidence angle of 45° for the best results (Figure 4-10).

![Figure 4-10: Sketch showing drop off in the accuracy of data acquired by LiDAR scanner as surface becomes more oblique to the scanner. Lighter arrows indicate reduced accuracy and reduced return.](image)

4.5.2 Sources of angular error

The angular accuracy of LiDAR point cloud data is broken into the horizontal angle accuracy and the vertical angle accuracy. Both of these errors are a function of the measurement of one of its moving parts by the scanner: the rotating mirror for vertical accuracy and the mechanical rotation of the scanner head for horizontal accuracy. Methods for testing the angular error of a scanner have been developed by Boehler et al. (2003), recognizing that every scanner has a different angular error.
It is important to ensure that the scanner used for collection of LiDAR data is calibrated correctly as this is the only way to minimize angular error. If a scanner is not calibrated correctly, accuracy of the scan data will be reduced.

4.5.3 Other sources of error

4.5.3.1 Spurious scan points

Spurious scan points are points within a LiDAR point cloud that do not represent any real object within the scan. These points should not have been collected by the scanner and must be removed by the user or through a filter before the scan data can be used for deformation monitoring. There are many conditions that can result in spurious points which include, but are not limited to, atmospheric conditions, interfering radiation, reflective surfaces, edge effects, and the ambiguity interval of the scanner.

Atmospheric conditions such as dust or steam in the air can result in unwanted return points. Dust and vapor can scatter the laser, either increasing the noise upon return or returning spurious points. Interfering radiation such as the rays of the sun, which have a similar wavelength to that of the scanner, can be picked up as spurious points. Reflective surfaces such as mirrors or wet surfaces can reflect the laser beam. The beams will be returned from locations other than where they initially hit the object.

4.5.3.2 Ambiguity interval

Every phase based laser scanner has a stated ambiguity interval, beyond which distance the phase shift of the returned beam is ambiguous. If points are returned to the laser scanner from a distance farther than the ambiguity interval they will be positioned too close to the scanner. For
example, if the ambiguity interval of a scanner is 60 m and a point is returned from 62 m it will be stored as being 2 m away from the scanner.

4.5.3.3 Georeferencing and alignment error

Error associated with the georeferencing of a LiDAR scan is based on the error in the georeferencing measurements. The less error in the georeferencing measurements used to align the scan, the less error in the scan itself. Error in a scan can also be created if it is not aligned properly with other scans, or if it is aligned with a scan that is poorly georeferenced.

4.6 Sources of noise in tunneling

There are other sources of noise when scanning in a tunneling environment that are not associated with the LiDAR scanning itself. The first of these sources of noise is the roughness of the tunnel profile. This roughness can be broken into two different components: the roughness associated with the rock surface (generally a combination of the rock type and the excavation method and quality) and the roughness or systematic undulation caused by rock structures and other large (macro) scale features. Both of these sources of noise are unavoidable, and can vary drastically from one section of tunnel to another. It is important that any type of profile analysis for a tunnel be able to deal with these sources of noise. Random noise is generally considered to have a maximum centimeter scale magnitude, whereas systematic undulation of the profile can be up to meters in scale.

Another unavoidable source of noise within a tunneling environment is occlusion. When scanning in a tunneling environment, occlusion of part of the tunnel profile is generally unavoidable. Machinery, ventilation ducts, conveyor systems, tunneling equipment or workers in
the tunnel may obstruct the LiDAR scanner from collecting parts of the walls or roofs in the tunnel. These obstructions are most likely to be encountered in tunnels that are under construction, where convergence measurement is more important. For a convergence measurement technique to be effective, the analysis must be able to deal with partially occluded data sets.

4.7 Positioning in LiDAR scanning

When completing a LiDAR scanning program it is important to decide in advance what reference system will be used for alignment of scan data. Either absolute positioning or relative positioning can be used. Absolute positioning can be completed using GPS data of specific reference points within the scan, or by surveying in the laser scanner using a device such as a total station. Although it is often assumed that absolute positioning is necessary to characterize deformation, this is not the case. Using relative convergence provides the same level of stability information and back analysis ability as using absolute positioning measurement. The main limitation with not having absolute positioning data is that no lateral or vertical movement of the tunnel can be tracked, but these types of movement typically are only of concern in twin bore tunnels at depth.

4.8 Tunnel profile analysis with LiDAR data using elliptical fitting

Terzaghi (1942) introduced a method to fit an ellipse to the displaced points monitored in a tunnel. His calculation of the mean deformation ellipse replaced the traditional method of looking at each point individually. The mean deformation ellipse allows for different periods of deformation to be easily characterized. For example, one can discern periods of vertical and
horizontal compression much more clearly than by looking at each point by itself (Stiros & Kontogianni, 2009). To apply Terzaghi’s method of elliptical fitting, a new method for analyzing LiDAR data has been developed. The analysis comprises a combined Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (LPA) described in detail by Delaloye et al. (2012) (Figure 4-11).

4.8.1 Generating synthetic LiDAR tunnel profiles

To test the new methodology for profile analysis a synthetic LiDAR profile data generator was created. The use of synthetic data allows for the amount of deformation that is measured by the analysis to be compared to the amount of deformation that is mathematically included in the data. Once the new methodology, the combined EFA and LPA, is proved useful and its limitations defined using the synthetic data, it can be applied to real LiDAR scan data.

The synthetic data generator, created in Matlab as part of this thesis, allows for selection of the number of points (which are evenly spatially distributed) and for the radius of the tunnel profile to be chosen. Both random uncorrelated noise with a defined standard deviation and systematic noise (locally consistent deviation from the elliptical profile) can be added to the data (Figure 4-12 b, c).

Data can also be “elliptically skewed”: the short axis of the tunnel profile is contracted and the long axis of the profile is expanded radially by the amount of “skew” entered by the user (Figure 4-12 d). The new elliptical data set can then be rotated to simulate directions of minimum and maximum deformation that are not parallel to the x and y axis.

The generated tunnel profile data can also be occluded to simulate data that cannot be collected within a tunneling environment due to ventilation, machinery, workers, and other
Figure 4-11: Application of the EFA and LPA to LiDAR scan data of a partially occluded tunnel profile. a) sketch showing possible sources of occlusion in LiDAR profile, b) two occluded tunnel profiles (40% occlusion), CS$_1$ shown in green, CS$_2$ in red, c) diametric displacement profile (DDP) from elliptical fits to occluded data, d) DDP from LiDAR profile data, and e) deviation of LiDAR diametric displacement profile from elliptical fit displacement profile for analysis of random and systematic noise.
obstructions that may block some of the profile from being collected. The occlusion generator, created during the course of this thesis, allows for the percentage of the tunnel profile to be occluded to be chosen as well as the average circumferential length of an occluded section and the standard deviation of this length (Figure 4-12 f). By having user control of these parameters, the amount of occlusion of the tunnel profile is controlled but still randomized.

Figure 4-12: Steps in the process of generating synthetic LiDAR data. a) generation of a circle with a radius and number of points both defined by the user, b) introduction of random noise with a set standard deviation, c) introduction of systematic noise with a set magnitude and period, d) introduction of skew with a set magnitude, e) rotation of the profile by a user defined amount, f) occlusion of the profile where the percentage, average length, and standard deviation of that length are all defined by the user.
4.8.2 Expected outcomes of EFA and LPA analysis

The expected deformation in a tunnel can be broken into three primary types:

- radial uniform deformation
- elliptical uniform deformation
- non-uniform deformation

Based on the three primary types of deformation (Figure 4-13) the expected outcomes of the EFA and LPA are demonstrated. Quantitative amounts of deformation have been exaggerated to highlight the different trends that should be present in the analyses. The goal of this demonstration is to outline the useful aspects of the combined EFA and LPA. It is worth repeating that the EFA and LPA are intended to be a combined analysis technique, and neither analysis is as powerful on its own as when they are combined.

Figure 4-13: The three primary types of deformation that can be expected in a tunnel depending on the rockmass and in situ stress conditions.
4.8.2.1 Radial uniform deformation

Under hydrostatic stress conditions in homogeneous isotropic rock, radial uniform deformation of the rockmass (Figure 4-13 a) is expected. Radial uniform deformation is characterized by a near equal displacement of the tunnel wall at every point. The Distributed Radial Displacement Profile (DRDP) and Diametric Displacement Profile (DDP) from the EFA and the LPA, as well as the Noise Profile (NP) are shown in Figure 4-14 for a tunnel undergoing this type of deformation. If a tunnel is undergoing radial uniform deformation both the DDP from the EFA and the DDP from the LPA should be flat lines equal to the amount of diametric convergence, as deformation is consistent around the tunnel profile. There should be no deviation between the DDPs, therefore the NP should also be flat, which is consistent with the results shown in Figure 4-14 g.

4.8.2.2 Elliptical uniform deformation

Elliptical uniform deformation (Figure 4-13b) can be expected in a rockmass when either the stress conditions are non-homogeneous, or the rockmass is anisotropic. The output of the EFA and LPA analysis for a tunnel profile undergoing elliptical uniform deformation is shown in Figure 4-15. Both the DRDP from the EFA and LPA show the same pattern of deformation, an exaggerated “peanut” shape characteristic of an elliptical deformation tensor. The DDP of both analyses is a sinusoidal wave characteristic of elliptical deformation. The NP shows that the elliptical fit and the LiDAR profile provide similar DDPs, but there is a small amount of discrepancy. This discrepancy is associated with the fact that the LiDAR profile is not perfectly elliptical, not that anomalous movement is occurring.
Figure 4-14: Outcomes of the EFA and LPA from the analysis of a tunnel profile undergoing radial uniform deformation. a) parameters of best fit ellipse to each tunnel profile, b) two LiDAR tunnel profiles fit with ellipses, c) DRDP at three times exaggeration from EFA, d) DRDP at three times exaggeration from LPA, e) DDP from EFA, f) DDP from LPA, g) NP.
Figure 4-15: Outcomes of the EFA and LPA from the analysis of a tunnel profile undergoing elliptical uniform deformation. a) parameters of best fit ellipse to each tunnel profile, b) two LiDAR tunnel profiles fit with ellipses, c) DRDP at three times exaggeration from EFA, d) DRDP at three times exaggeration from LPA, e) DDP from EFA f) DDP from LPA, g) NP.
4.8.2.3 Non-uniform deformation

Non-uniform deformation (Figure 4-13c) occurs in anisotropic rockmasses exposed to either a homogeneous or a non-homogeneous stress field. When non-uniform deformation of a rockmass is occurring, the DRDP and the DDP from the EFA will not match those from the LPA as in previous deformation types. The EFA highlights the overall trends in deformation while the LPA shows all the areas of anomalous or non-uniform movement. The benefits of the NP are shown in the analysis of non-uniform deformation. The NP shows systematic deviation of the tunnel profile from an elliptical profile (Figure 4-16 g). Future analysis techniques should be designed to allow this type of systematic deviation to be filtered from the data.

4.9 Sensitivity testing of the EFA and LPA

A parametric study was conducted to determine the sensitivity of the EFA and the LPA to variability in the different parameters that affect the collected LiDAR profile data. The varied parameters included:

\[ \beta \] the amount of perturbation of the LiDAR profile from a perfectly elliptical/circular profile; the magnitude of systematic noise

\[ \gamma \] the periodicity of the perturbation of the profile; the number of systematic perturbations

**Random noise** the standard deviation of the random error in measurement of radius

**Radius** the radius of the tunnel profile

**Occlusion** the percentage of the tunnel profile not collected during the scan

**Skew** the amount of shortening in the short axis and lengthening of the long axis of the ellipse from a perfect circular profile
Figure 4-16: Outcomes of the EFA and LPA from the analysis of a tunnel profile undergoing non-uniform deformation. a) parameters of best fit ellipse to each tunnel profile, b) two LiDAR tunnel profiles fit with ellipses, c) DRDP at three times exaggeration from EFA, d) DRDP at three times exaggeration from LPA, e) DDP from EFA, f) DDP from LPA, g) NP.
Table 4-1: Parameters varied for sensitivity testing of the EFA and LPA analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of values tested</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0 – 0.6 m</td>
<td>0.001 m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0 – 30</td>
<td>2</td>
</tr>
<tr>
<td>Random noise</td>
<td>0 – 0.5 m</td>
<td>0.001 m</td>
</tr>
<tr>
<td>Radius</td>
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<td>0.0005 m</td>
</tr>
<tr>
<td>Occlusion</td>
<td>0 – 100%</td>
<td>2%</td>
</tr>
<tr>
<td>Skew</td>
<td>0 – 0.5 m</td>
<td>0.001 m</td>
</tr>
</tbody>
</table>

For tests with constant system noise, a $\beta$ equal to 0.05 was chosen and a $\gamma$ of 6. For constant change in tunnel size, a radius of 4.95 m was used. Seven tests for each combination of parameters were completed, and the mean and standard deviation of these seven tests were used for analysis.

For every test, the first set of data (reference profile) used in the comparison was a perfectly circular profile with a radius of 5 m. No random or systematic noise, occlusion, or skew was included in the reference profile. In each of the compared profiles one or more of the parameters show in Table 4-1 were varied to test the impact on the measurement of convergence of the tunnel profile. Parameters were tested individually and together to determine if any combination of parameters had a greater impact on error than each parameter tested alone.

The results of the parametric study were analyzed by looking at the “maximum change error” (MACE) the “minimum change error” (MICE) and the “average change error” (ACE), collectively known as the “change errors” or CEs.
The CE is defined as:

\[ CE = C_m - C_k \]

Where:

- \( C_m = \text{the measured change in tunnel diameter} \)
- \( C_k = \text{the known change in tunnel diameter} \)

By convention, if the tunnel being analyzed has converged between the first and second epoch, the measured change (\( C_m \)) is positive.

For the EFA:

\[ C_m = \Delta_{E_1E_2} \]

For the LPA:

\[ C_m = \Delta_{CS_1CS_2} \]

The MACE is the error in the measurement of the maximum change in diameter, which coincides with the greatest amount of convergence found in the profile. The MICE is the error in the measurement of the minimum change in diameter, or the error in measurement of the smallest convergence or largest divergence in the profile. The ACE is the error in the measurement of the average profile change.

In all cases the, the closer the MACE, MICE and ACE are to zero, the better the measured change correlates to the actual amount of change in the data, therefore the better the results of the analysis. The magnitude of the CEs allow for interpretation of which parameters have the greatest effect on the ability to accurately measure the amount of change in the LiDAR profile. In a perfectly circular profile analysis, the MACE, MICE and ACE should all be equal as there should be no difference in the maximum and minimum convergence of the profile.
It must be noted that average change error is only an appropriate metric for analysis when looking at circular deformation of the tunnel profile. When skew is introduced into the parametric study, only the MACE and MICE can be used for analysis. The standard deviation of each analysis was also determined.

4.9.1 Results of sensitivity testing with synthetic LiDAR data

Overall the results of the parametric study show that the EFA produces measurements of convergence that are well below the noise level of the data, and therefore the EFA is a beneficial methodology for the measurement of convergence. As expected, the LPA analysis produces a high amount of error in the measurement of minimum and maximum convergence, and is only useful if the measurement of average convergence is desired.

The results of the parametric study also show that each of the varied parameters is linearly independent in their effect on the amount of error they cause in the measurement of convergence.

For both the EFA and LPA, variation of the diameter had no effect on any of the CEs. The number of systematic deviations of the tunnel profile (gamma) and skew also had no effect on the CEs of the EFA and LPA. It must be noted that the results for a gamma value less than 4 must be discounted from the analysis, as this does not produce logical or possible tunnel profiles.

The variation of $\beta$ had a large impact on the MACE and MICE produced by the LPA, resulting in errors of up to 10 cm in the measurement of diametric convergence. An increased $\beta$ also had an impact on the MACE and MICE of the EFA, but the impact was much smaller. At a maximum systematic deviation of 6 cm the greatest CE was about 0.03 cm (Figure 4-17).
Figure 4-17: a) MACE from the LPA showing an increase in error with an increase in random noise and an increase in beta b) MACE from the EFA showing the same trend as the LPA, but with a much smaller magnitude of error.
The inclusion of random noise in the analysis again impacts both the EFA and LPA, but the LPA CEs are much greater than those of the EFA. When random noise is included with a standard deviation of 5 cm, the resulting error in the measurement of convergence by the LPA is about 23 cm, whereas for the EFA it is about 5 mm. As the measurement of convergence is an order of magnitude less than the noise included, this proves that the EFA produces accurate measurements of convergence even when the collected data is noisy.

When both random and systematic noise are included in the data, the impact on the CEs measured by the LPA and EFA is unchanged. The LPA produced a maximum error of 32 cm at the highest level of noise, where the EFA produced a maximum error of 4 mm.

From the parametric study it was also determined that a level of up to approximately 60% occlusion is acceptable for both the EFA and LPA analysis. When the distributed occlusion level is increased above 60%, the proficiency of both analysis techniques significantly degraded largely due to the inability to accurately fit an ellipse to the data, and then to position the ellipse or the raw data for analysis. The results also show that a slightly higher level of occlusion is allowable for data sets with lower noise levels.

As expected, for analysis of a perfect circle, the magnitude of the MACE and MICE are very similar. The accuracy of the analysis is further demonstrated by comparing the MACE and MICE that result from the EFA. When skew is included (Figure 4-18), the MACE and MICE are very similar, showing that the error in measurement is an absolute error, and not dependent on the direction or amount of deformation.
Figure 4-18: a) MACE from the LPA showing an increase in error with an increase in random noise and minimal impact of skew on the error b) MACE from the EFA showing the same trend as the LPA, but with a much smaller magnitude of error.
4.9.2 Implications of occluded data

For the parametric study, all test cross sections were compared back to a “perfect” tunnel section, which is a profile of points with no noise and no occlusion. The impact of increased noise and increased occlusion can be seen in Figure 4-19. It is clear that occlusion begins to have an impact on the accuracy of measurement at about 60% occlusion, and measurement accuracy decreases rapidly when occlusion reaches 80% of the tunnel profile. It can also be seen that an increase in noise results in occlusion having a higher impact, therefore for noisier data sets, the amount of allowable occlusion of the profile is lower.

Figure 4-19: MACE from LPA and EFA for occluded data showing clearly that occlusion only begins to impact the measurement error at about 60% occlusion. The allowable occlusion is lowered by an increase in the random noise.

4.10 Conclusions

Through a completed review of the accuracy issues associated with LiDAR scanning it is clear that there are levels of noise and errors within data sets that are unavoidable. For any
change detection analysis to be useful, it must be able to filter these errors and in some way deal with noise so that accurate levels of change can be measured. Previous analysis has shown that in tunnels and shafts, levels of change on the orders of millimeters must be measured. As the accuracy of single points in LiDAR data is of the same magnitude of this necessary level of change detection, statistical analysis techniques of profile data are needed. One such technique is the Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (LPA) developed by Delaloye et al. (2012). To ensure that this technique is robust enough to deal with random noise, systematic noise, and other errors unavoidable during LiDAR scanning, a sensitivity analysis was completed.

The results of the sensitivity analysis show that the combined EFA and LPA analysis can accurately measure levels of change within the level of raw accuracy of the LiDAR point cloud. The combined EFA and LPA also have the added benefit that they are able to deal with large amount of occlusion of the tunnel profile within the data sets. The analysis has shown the level of error in change detection does not become significant until the occlusion exceeds about 60% of the tunnel profile. Future analysis should be completed to test the EFA and LPA on real data sets, and compare the results of LiDAR monitoring results with traditional deformation analysis techniques.
Chapter 5

A New Workflow for LiDAR Scanning for Change Detection in Tunnels and Caverns\(^3\)

5.1 Abstract

Measuring change in underground environments is an important aspect of geological engineering. Recently, methods for using Light Detection and Ranging (LiDAR) to measure change and convergence in tunnels and other underground environments have been demonstrated. To properly apply these new methods, it is important that an appropriate workflow is followed. The workflow proposed in this paper includes recommendations for choosing scan resolution settings and scan locations based on the level of change to be measured. The workflow follows through to the extraction of cross-section data for convergence measurement and back calculation.

5.2 Introduction

The benefit of using Light Detection and Ranging (LiDAR) for geological engineering applications has been widely demonstrated and discussed in the literature over the past ten years.

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\(^3\) This chapter appears as submitted to a conference proceedings with the following citation:

For example, LiDAR can be used for rockmass characterization, discontinuity measurement, and change detection in underground spaces (Fekete, 2010; Lato, 2010). As the number of applications of LiDAR scanning is increasing and its use is becoming more commonplace, it is important to establish best practices for both data collection and data management. More recently it has been shown that LiDAR data can be used for monitoring tunnel deformations and for change detection (Delaloye et al., 2012). To ensure that the LiDAR data collected from within tunnels is of high enough quality for all of these applications, it is important that the appropriate amount of data is collected from the right locations at the correct resolution.

Previous workflows have been suggested for different geological engineering applications of LiDAR scanning. Buckley et al. (2008) created a practical workflow for scanning of large outcrops with long range time of flight laser scanners. The workflow proposed by Buckley et al. (2008) is very efficient for large scale feature extraction, but must be refined for application in underground environments, especially if scans are to be used for change detection. A workflow for discontinuity extraction from underground LiDAR scans for input into stability models was created by Fekete et al. (2010). Although this workflow is practical for collection of scans for rockmass characterization, it does not have all the necessary considerations for scanning for change detection and convergence measurement.

When completing multi-temporal LiDAR scans for change detection, more care has to be taken in the data acquisition than when only one scan set is collected. This paper will suggest a new workflow that can be used to ensure that the resolution, accuracy and coverage of LiDAR scans collected within a tunnel or other underground environment meet data requirements for all underground scanning applications, with special focus on scanning for change detection. The workflow focuses on the resolution settings and scan locations needed for optimal and complete
coverage of an underground area. Comments are also made on the accuracy issues associated with LiDAR scanning and how to minimize any manageable errors in the field during data collection.

Once LiDAR data sets are collected, it is important to use a proper workflow for data management and analysis. In addition to recommendations for data collection, this paper extends the workflow to include suggestions for managing multiple data sets and how they can be analyzed for the aforementioned underground applications. Implications of processing techniques on data accuracy and measurement resolution are also included in the workflow.

5.3 LiDAR scanning for deformation measurement

The applications of LiDAR scanning in underground excavations are continually growing. LiDAR has been demonstrated to be useful for discontinuity orientation measurement, rockmass characterization, and as built modeling Fekete et al. (2010). A summary of some of the main applications of LiDAR in tunneling environments to date is shown in Figure 5-1. One of the most recent LiDAR scanning applications is for the measurement of deformation in tunnels (Delaloye et al. 2012, van Gosliga et al., (2006), Lindenberg et al. (2009)). Before using LiDAR for tunnel deformation measurement, it is important to discuss the sources of error, as the level of accuracy and noise in the data has implications as to the level of convergence that can be measured in the tunnel or cavern.
Figure 5-1: a) mapping of geological structure in the tunnel face over multiple blast rounds b) measurement of joint spacing c) extraction of discontinuity orientations d) seepage mapping in lined and unlined area of the tunnel e) as built modeling of tunnel environment (a-e after Fekete, (2010)).

5.3.1 Noise, accuracy and resolution of LiDAR scanning

To define the scale of features that can be extracted from a LiDAR scan it is necessary to have a good understanding of the noise, accuracy and resolution of the scan data. The accuracy, resolution and noise of the scan data all contribute to the ability to measure convergence and change within the scan environment. The first contributor to the accuracy that must be considered is the accuracy of the position related to the LiDAR scan itself. An in depth discussion of the contributors to scanner accuracy is beyond the scope of this paper but can be found in Delaloye et al. (2011). The accuracy of the scanner used for this research, the Leica HDS 6000, decreases with distance from the scanner (as shown in Figure 5-2). The general
The implication of this finding is that the accuracy of the data is reduced when a larger space is scanned.

**Figure 5-2:** Accuracy of single point and modeled surface from LiDAR scan data as a function of the distance from the scanner (all measurements are one standard deviation, based on data provided by InnovMetric for Leica HDS 6000).

A major contributor to the resolution of a LiDAR scan is the spot spacing, in other words the density of the point cloud. The spot spacing varies based on distance from the scanner and the resolution settings of the scanner. The Leica HDS 6000 has five different resolution settings that can be chosen for data collection. The spot spacing as a function of distance for the different scan resolutions is shown in Figure 5-3. Due to the increase in spot spacing as a function of distance...
distance, larger areas may need to be scanned at a higher resolution to ensure spot spacing is sufficient for specific features and levels of convergence to be measured.

Figure 5-3: Spot spacing of collected data by Leica HDS 6000 at standard resolution settings (based on data provided by InnovMetric for Leica HDS 6000).

A final consideration in the accuracy and resolution of the scanner is the “spot size” or the diameter of the footprint of the laser when it reaches a surface. The diameter of the laser diverges as a function of distance from the scanner (as shown in Figure 5-4). It has been demonstrated by Soudarissanane et al. (2007) that features smaller than one third of the beam divergence cannot be extracted from the scan data. Again, with increased distance, the ability to measure smaller features is reduced.
Figure 5-4: Measured diameter of the laser beam emitted as a function of distance from the scanner (based on data provided by InnovMetric for Leica HDS 6000).

5.3.2 Other sources of noise underground

An underground environment presents other sources of noise, separate from the accuracy and resolution issues associated with the laser scanner, that impact the user’s ability to measure convergence and change in an underground tunnel or cavern environment. Sources of noise in underground excavations not associated with the LiDAR scanner itself include:

- Large scale roughness of the surface and structure within the rockmass
- Reflectivity of the rock surface
- Anomalies on the surface of the rock (i.e. screen, rockbolts, etc.)
- Anomalous movement of blocks of rock
• Occlusion created by obstructions within the tunnel

5.4 Workflow for tunnel convergence measurement with LiDAR

It is important to set up a proper scanning protocol before data collection for any LiDAR monitoring project. To assist in the selection of the appropriate scan resolution and scan spacing for optimum data collection, a study of the impact of spot spacing, accuracy and scan distance on the error in convergence measurement has been completed. The results of the study are a workflow for selecting the appropriate scanning protocol for a project (Figure 5-5). This paper

![Figure 5-5: Complete workflow for the collection and processing of LiDAR data for tunnel deformation measurement.](image)
will only focus in detail on the front end of the workflow (everything up to and including cross section extraction) (Figure 5-6).

![Diagram showing factors affecting data collection during LiDAR scanning field work.]

**Figure 5-6: Factors affect the collection of data during LiDAR scanning field work.**

### 5.4.1 Data collection

The major inputs to consider when selecting an appropriate scanning protocol are:

- The diameter of the tunnel (or size of the underground space)
- The roughness of the surface and location of structure within the rockmass
- The accuracy of the scanner being used
- The reflectivity of the scan surface
- The scale of deformation or change expected
Other factors that affect the resolution and scan spacing that should be considered are the data processing capability of the user and the time available for collection and processing of data.

5.4.1.1 Scan resolution

The effect of different scan resolutions on the point spacing of a scan as a function of distance has already been described in Section 2.1. If a larger area is being scanned, and the distance between the scanner and target is increased, a higher scan resolution must be used to maintain the same point density. The resolution settings of the scanner used for this research are shown in Table 5-1.

Table 5-1: Scanning times and data sizes produced by the Leica HDS 6000 for standard resolution settings

<table>
<thead>
<tr>
<th>Scan Resolution</th>
<th>Scan Time</th>
<th>Data Size (MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preview</td>
<td>25 sec</td>
<td>3</td>
</tr>
<tr>
<td>Medium</td>
<td>1 min 40 sec</td>
<td>50</td>
</tr>
<tr>
<td>High</td>
<td>3 min 22 sec</td>
<td>200</td>
</tr>
<tr>
<td>Super High</td>
<td>6 min 44 sec</td>
<td>800</td>
</tr>
<tr>
<td>Ultra High</td>
<td>26 min 40 sec</td>
<td>2400</td>
</tr>
</tbody>
</table>

With current computer processing power, it is only possible to import ¼ of the data collected from a Super High resolution LiDAR scan, and Ultra High scan data is completely impractical to process. Hence, it is generally only practical to scan between Medium to Super High resolution. The amount of time available to complete scanning may also have an impact on the selected scan resolution.
5.4.1.2 Scan spacing and locations

The optimum maximum angle of incidence between the scanner and the surface being scanned is 45°. To ensure optimum coverage of the tunnel walls, scans should be spaced at a maximum of every tunnel diameter along the axis of the tunnel (Figure 5-7). To ensure an area of overlap for alignment, scans should be trimmed to a length of one diameter centered about the scanner plus a minimum of 30% on either side (based on work by Bornaz et al. (2002) showing that 30% overlap is necessary for alignment). This equates to increasing the allowable angle of incidence between the scanner and surface to approximately 55°. If there are many joints or other structures present in the walls of the tunnel it may be necessary and appropriate to scan on either side of the tunnel centerline to reduce occlusion (Figure 5-8).

Figure 5-7: General alignment of scans spaced at one diameter along the axis of the tunnel to ensure full coverage of the tunnel walls.
Figure 5-8: Modification to standard scan set-up to account for expected occlusion in the tunnel face and roof where two scans are set up at every point on both sides of the tunnel centerline.

Although the Leica HDS 6000 scanner can collect data up to 79 m away, the recommended maximum scan distance for the area of interest is 17 m (Leica Geosystems, 2007); hence the scanner should not be used to scan tunnels larger than 20 m in diameter if scanning is to be completed only along the centerline of the tunnel (Figure 5-9). This should not present a problem as the largest TBM tunnel constructed to date is 16 m in diameter, and the largest tunnel proposed for the near future is 19 m in diameter (JSC, 2007).
Both the roughness of the rock surface and the surface condition due to rock structure and excavation quality affect the ability to use LiDAR scans for convergence measurement (Figure 5-10) when the position of the scan and the point of comparison is not exactly the same. The effects of small scale surface roughness on the error in convergence measurement can be minimized by ensuring that the scan resolution is set at a high enough level. This is achieved when the spot spacing is smaller than the small scale roughness, thereby returning a data point that is not affected by averaging of a rough surface, and the accuracy is at a high enough level to measure the anticipated levels of change in the rockmass. The surface condition due to rock structure and excavation quality primarily affects the amount of occlusion that can be expected from specific scan locations. If there is structure in the rock that is prevalent and will lead to occlusion, or if the excavation quality is poor, creating large deviations in the walls of the cavern or tunnel that will lead to occlusion, the scan spacing must be modified accordingly (Figure 5-9).
It may be necessary to space scans as close as one radius instead of one diameter to ensure 100% overlap and the greatest possible reduction in occlusion.

Figure 5-10: Two scans of a static rough surface can result in a measured deformation that does not actually exist due to the resolution of the scan not being high enough to accurately capture the surface.

Figure 5-11: Scans must be positioned in order to ensure that structure within the rockmass does not affect the collection of the full tunnel surface.
If scanning is completed at 100% overlap, and scans are spaced every tunnel radius (Figure 5-12), it is allowable to trim scans to a width of one diameter, as the extra scan length is no longer necessary for overlap and alignment.

Figure 5-12: Modification to standard scan set-up to account for expected occlusion in the tunnel walls where scans are set up at a distance of every radius along the tunnel centerline.

Scanning in caverns or other larger areas requires different scan spacing protocols. Scans must be spaced logically in the 3D environment. It is necessary to ensure coverage of all walls as well as the roof and potentially the floor of the area. If the cavern or underground space is significantly taller than 17 m, to ensure scan resolution is maintained at a high enough level for change measurement it is necessary to either scan from an elevated location within the cavern, or to increase the scan resolution to maintain a low enough point spacing on the surface. The same considerations for occlusion that are made in a tunneling environment must also be made when scanning in a cavern or other underground space.
5.4.2 Initial data processing

Once data is collected, it is important that the proper processing workflow be followed to ensure feature extraction and convergence measurement can be completed. The first steps in data processing are loading the scans, and trimming them to the area of interest. For tunneling it is suggested that scans be trimmed to one diameter plus 30% on either side of the scan to allow overlap for alignment.

5.4.2.1 Alignment of scans

An area of overlap between two scans must be used for alignment. The alignment can be done by matching targets captured in both scans, using objects within the scan as reference points for alignment or by using a computer driven algorithm alignment. The most common computer driven alignment is the Iterative Closest Point (ICP) algorithm. It was developed for the alignment of point clouds by Besl & McKay (1992) and is a data driven registration method. This is a widely used algorithm, and is the built in registration method for most software programs, including PolyWorks® V 11.0.28 (InnovMetric (2011). For alignment in true space, targets within the scan environment can be located using georeferencing.

5.4.2.2 Filtering of data

Initial data processing also includes the filtering of data. Filtering can be either user driven, or be completed using a filtering algorithm. User driven filtering can be done to remove large anomalies in the scan, such as pieces of equipment or machinery. It is not practical to use user driven filtration to remove smaller and near surface anomalies as it is too time consuming.
In the case where near surface anomalies are present, such as a screened rockmass, filtration algorithms must be used, such as developed by Delaloye et al. (2012).

Near surface anomalies must be filtered prior to meshing and cross section extraction. If these anomalies are not filtered out, they will be meshed with the rock surface as seen in Figure 5-13. The meshing of the anomalies and rock together creates an erroneous surface that cannot be used for analysis of deformation.

Figure 5-13: Where screen has been installed to support an excavation meshing of the LiDAR point cloud creates an erroneous surface combining the wire screen and rock surface into the mesh. Meshing of the bare rock surface produces a good result.
Once the scan data has been properly filtered, the point cloud can be turned into a surface model for the extraction of cross sections.

**5.4.3 Cross section extraction**

There are two different possible methods of cross section extraction. If a surface has been modeled from the point cloud, a cross section of that surface should be extracted for convergence measurement. Care should be taken to ensure cross sections taken from multi-temporal scans are extracted in the same (or as near as possible) location.

If the LiDAR data is not turned into a meshed surface it is still possible to use the raw point cloud for cross section extraction. In this case a narrow swath of data should be used for comparison. The use of a swath of data reduces the accuracy needed in the selection of the same cross section of data by allowing for averaging over a small area to be completed. Another benefit of using a swath of data is that the swath allows for some of the noise and roughness within the scan to be averaged out, as the points are just treated as one cross section throughout the rest of the analysis. It must be ensured that the section is taken perpendicular to the axis of the tunnel within +/− 1° to ensure that no skew in the profile is introduced.

**5.5 Convergence measurement**

To measure convergence within a tunnel, the Elliptical Fitting Analysis (EFA) and LiDAR Profile Analysis (LPA) was proposed by Delaloye et al., (2012). The analysis requires two tunnel profiles extracted from LiDAR data (or other point cloud data) as input. The robustness of the EFA and LPA analysis has previously been demonstrated, but the required resolution for extraction of specific measurements of convergence, and at different tunneling
scales has not been shown. A parametric study using synthetic LiDAR data was completed to test the necessary point spacing (based on the number of points in a given profile, Table 5-2) and allowable noise for the measurement of different levels of diametric change.

Table 5-2: Number of points in one LiDAR cross sectional profile based on different scan resolutions of the Leica HDS6000 terrestrial LiDAR scanner

<table>
<thead>
<tr>
<th>Scan Resolution</th>
<th>Number of Points in One Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 m diameter</td>
</tr>
<tr>
<td>Preview</td>
<td>3000</td>
</tr>
<tr>
<td>Medium</td>
<td>12000</td>
</tr>
<tr>
<td>High</td>
<td>25000</td>
</tr>
<tr>
<td>Super High</td>
<td>50000</td>
</tr>
</tbody>
</table>

The amount of error in the resulting measurement of convergence using both the EFA and LPA was characterized using the maximum change error (MACE), minimum change error (MICE) and average change error (ACE). Based on the results of the parametric study the protocols for choosing scan resolution settings have been suggested.

The results of the parametric study clearly demonstrate that the “Preview” resolution of the Leica HDS 6000 does not provide dense enough point spacing to minimize the error in convergence measurement. Although “Medium” resolution provides an improvement, at the expected accuracy level of the LiDAR scan data (standard deviation of 6 mm), High resolution provides the best balance of scan data resolution and measurement extraction accuracy in tunneling and other underground environments (Figure 5-14, Figure 5-15).
Figure 5-14: Results of the MICE from the EFA showing that an increase in the number of points used in the analysis decreases the error until the number of points exceed about 20000. The change in radius being measured has no impact on the error in the results of convergence measurement.
Figure 5-15: Results of the MICE from the EFA showing that a higher number of points is required to reduce the error in measurement of convergence when the magnitude of random noise is higher.

From the results of the analysis it can also be concluded that Super High resolution scan data is unnecessary to achieve practical levels of change measurement. This is beneficial as current computer software is not capable of handling the data size of Super High resolutions scans.
5.6 Conclusions

LiDAR is becoming an increasingly common tool for use in geological engineering due to its ability to rapidly acquire large, high accuracy, high precision positional data sets. To ensure the correct resolution and coverage of LiDAR scans are collected during a field program, it is important to develop and implement comprehensive workflows. The workflow created in this paper suggests techniques for selecting the appropriate locations and scan resolutions for data collected in tunnels and other underground environments to be later used for change detection and convergence measurement. In general scans should be spaced at a maximum of every diameter down the axis of the tunnel, and scan resolution should be set at “High” (for the Leica HDS 6000) or a similar setting such that the number of points on the profile is at least 20000.
Chapter 6

General discussion

6.1 Discussion

Two different profile analysis techniques have been developed in this thesis to take advantage of the different opportunities that each approach provides. The Elliptical Fit Analysis (EFA) analysis is to be used for extracting accurate information about the overall deformation of the tunnel, and can provide data that can be used for back analysis. The LiDAR Profile Analysis (LPA) is used for identification of local anomalous movements that deviate from the overall deformation profile trend.

6.2 Limitations

The current profile analysis code is limited to the analysis of circular and near circular tunnels and shafts. Mean deformation ellipse analysis is not considered to be practical at this point for other shape of excavations, as the author has developed the analysis specifically for diametric deformation and closure measurement. The code can be refined in the future to allow for application to other shapes of excavations such as horseshoe tunnels and square caverns.

The developed algorithm is limited in that it can only measure tunnel or shaft convergence, and cannot account for vertical or lateral shift. Although this is a limitation of the current analysis, it is rarely necessary to account for lateral or vertical movement for a single tunnel analysis; convergence is by far the most important component of tunnel displacement measurement. If the user does desire to measure vertical and lateral movement it is possible to
either survey in control points with a total station to five absolute positioning or survey in the location of the scanner during every scan. This does, however, reduce one of the benefits as suggested by the author that LiDAR scanning is beneficial due to the lack of necessity for absolute positioning.

A major concern with implementing LiDAR scanning for deformation measurement is the establishment of an appropriate stable reference system. Although it is often assumed that absolute positioning is necessary to characterize deformation this is not the case. Using relative convergence provides the same level of stability information and back analysis ability as using absolute positioning measurement.

### 6.3 Future work

One interesting topic that has not been examined in this project is the difference in the distribution of points collected by a terrestrial laser scanner in a tunnel and a shaft. If the laser scanner is set up in the center of a shaft the points will be evenly distributed about the profile of the shaft. In a tunnel, because of the way the scanner rotates, the number of points directly above the scanner is much greater than on the floor directly below the scanner. This effect reduces rapidly as you move away from the scanner, but will still result in a slightly unequal distribution of points about the profile of the tunnel when a section is taken.

The other issue that may arise when scanning in a shaft is the lack of constraint in two directions that is possible within a tunnel. In a tunneling environment the scanner has gravity as a reference to properly align the z direction in the scan, and the tunnel axis can be chosen as either the x or y axis (generally y). In a shaft however, the direction of gravity and the axis of the shaft are aligned, therefore there is no constraint in the x, y plane. Methods need to be developed that
will allow for the direction in the x, y plane to be constrained. This could be as simple as ensuring there is paint on the wall of the shaft located in the direction of the chosen z axis.

In future, it is suggested that Fourier analysis be incorporated into the analysis of the LiDAR profiles to assist in the removal of systematic noise from the data. Fourier analysis is commonly used in signal processing to remove background noise from data using the frequency spectrum. By removing background noise from LiDAR profiles, trends in deformation data will be easier to extract and use for back analysis.

The developed code is not limited to profile analysis of tunnels derived from LiDAR data. The code is robust enough to handle any point cloud data, and can be applied to any scale of measurement. As such, the code would be very valuable in its application to measuring deformation for boreholes collected with sonic logging. The code has the ability to show the difference between an ellipse and the actual profile, and therefore could be used to quantify borehole breakout compared to borehole deformation.

As the code has been developed to handle any general point cloud, it is also not limited to specifically using LiDAR data. In the future this code can be tested and applied to any point cloud. It may be useful to use the code to compare the results of LiDAR data collection to photogrammetry in underground environments.

To more easily deal with data sets with near surface anomalies, full surface filtering algorithms need to be developed. This thesis has presented a method for filtering sections of data, and the same logic of using a “look window” and “filter cutoff” could be applied to swaths of data, and be continued along the entire tunnel profile.
Chapter 7

Summary and Conclusions

7.1 Summary

The major advantages of using LiDAR for deformation monitoring that have been discussed in this thesis are:

1) Acquisition of large sets of high precision, high accuracy positional data is very fast: a scan can be set up and completed in less than seven minutes.

2) Geo-referencing of scans is not required for tunnel convergence measurement: relative convergence measurements are extremely useful for deformation monitoring and absolute positioning is not actually necessary.

3) Full surface characterization is possible due to the high point density acquired from single scan: rather than just monitoring single points, the full surface is characterized.

Top properly apply LiDAR scan data for the measurement of convergence in near circular tunnels and shafts, a method for profile analysis was created and coded in Matlab. The combined Elliptical Fit Analysis (EFA) and LiDAR Profile Analysis (EFA) developed in this thesis allows for complete profile analysis. In addition, the code allows for:

- noise to be quantified and areas of anomalous movement to be identified
- accurate calculation of volumes of convergence and divergence
- both elliptical fit analysis and raw LiDAR profile analysis

Although the EFA and LPA have only been demonstrated using LiDAR point cloud data, they can easily be used for other data sets such as photogrammetry point cloud data.
7.2 Contributions

The research conducted during the preparation of this thesis has been the source for many publications and presentations. Presented below is a full list of the contributions resulting from this research.

7.2.1 Refereed Journal Articles (Submitted)


7.2.2 Refereed Conference Papers


7.2.3 Non-refereed Conference Presentations


7.2.4 Courses instructed

Short Course Co-Instructor: Lidar Imaging for Mining - Expanding the state-of-practice with state-of-art tools. At the Centre for Excellence in Mining Innovation (CEMI). October 2011. Sudbury, Ontario, Canada.

7.3 References


Appendix A

Matlab code for EFA, LPA, NP and synthetic data generation

Elliptical Fit Analysis (EFA)

% user must input what type of data is being used as this affects the
% orientation
evalResponse = input('If you are using shaft data enter 0. If you are
using tunnel data aligned with the Y axis enter 1. If you are using
tunnel data aligned with the X axis enter 2.


data1 = dlmread('CS1.txt');
data2 = dlmread('CS2.txt');

if(evalResponse == 0);

    X1 = data1(:,1);
    Y1 = data1(:,2);
    X2 = data2(:,1);
    Y2 = data2(:,2);

elseif(evalResponse == 1);

    X1 = data1(:,1);
    Y1 = data1(:,3);
    X2 = data2(:,1);
    Y2 = data2(:,3);

elseif(evalResponse == 2);

    X1 = data1(:,2);
    Y1 = data1(:,3);
    X2 = data2(:,2);
    Y2 = data2(:,3);

else
    error('Please use either properly aligned tunnel or shaft
data.'

end

% elliptical fitting function is used to fit an ellipse to each data
set
% a = fitellip(X1,Y1);
\begin{verbatim}
mx1 = mean(X1);
my1 = mean(Y1);
sx1 = (max(X1) - min(X1)) / 2;
sy1 = (max(Y1) - min(Y1)) / 2;
x1 = (X1 - mx1) / sx1;
y1 = (Y1 - my1) / sy1;

% Build design matrix
D1 = [ x1.*x1  x1.*y1  y1.*y1  x1  y1  ones(size(x1)) ];

% Build scatter matrix
S1 = D1'*D1;

% Build 6x6 constraint matrix
C1(6,6) = 0; C1(1,3) = -2; C1(2,2) = 1; C1(3,1) = -2;

% Solve eigensystem
[gevec1, geval1] = eig(S1,C1);

% Find the negative eigenvalue
[NegR1, NegC1] = find(geval1 < 0 & ~isinf(geval1));

% Extract eigenvector corresponding to positive eigenvalue
A1 = gevec1(:,NegC1);

% unnormalize
a1 = [A1(1)*sy1*sy1, ...
      A1(2)*sx1*sy1, ...
      A1(3)*sx1*sx1, ...
      -2*A1(1)*sy1*sy1*mx1 - A1(2)*sx1*sy1*my1 + A1(4)*sx1*sy1*sy1, ...
      -A1(2)*sx1*sy1*mx1 - 2*A1(3)*sx1*sx1*my1 + A1(5)*sx1*sx1*sy1, ...
      A1(1)*sy1*sy1*mx1*mx1 + A1(2)*sx1*sy1*mx1*my1 + A1(3)*sx1*sx1*my1*my1 ...
      - A1(4)*sx1*sy1*sy1*mx1 - A1(5)*sx1*sx1*sy1*my1 ...
      + A1(6)*sx1*sy1*sy1*sy1 ...
    ]';

% a = fitellip(X2,Y2);
mx2 = mean(X2);
my2 = mean(Y2);
sx2 = (max(X2) - min(X2)) / 2;
sy2 = (max(Y2) - min(Y2)) / 2;
x2 = (X2 - mx2) / sx2;
y2 = (Y2 - my2) / sy2;

% Build design matrix
D2 = [ x2.*x2  x2.*y2  y2.*y2  x2  y2  ones(size(x2)) ];
\end{verbatim}
% Build scatter matrix
S2 = D2'*D2;

% Build 6x6 constraint matrix
C2(6,6) = 0; C2(1,3) = -2; C2(2,2) = 1; C2(3,1) = -2;

% Solve eigensystem
[gevec2, geval2] = eig(S2, C2);

% Find the negative eigenvalue
[NegR2, NegC2] = find(geval2 < 0 & ~isinf(geval2));

% Extract eigenvector corresponding to positive eigenvalue
A2 = gevec2(:,NegC2);

% unnormalize
a2 = [
    A2(1)*sy2*sy2,
    A2(2)*sx2*sy2,
    A2(3)*sx2*sx2,
    -2*A2(1)*sy2*sy2*mx2 - A2(2)*sx2*sy2*my2 + A2(4)*sx2*sy2*sy2,
    -A2(2)*sx2*sy2*mx2 - 2*A2(3)*sx2*sy2*my2 + A2(5)*sx2*sy2*sy2,
    A2(1)*sy2*sy2*mx2*mx2 + A2(2)*sx2*sy2*mx2*my2 +
    A2(3)*sx2*sy2*my2*my2
    - A2(4)*sx2*sy2*sy2*mx2 - A2(5)*sx2*sy2*sy2*my2 ...
    + A2(6)*sx2*sy2*sy2*sy2 ...
]';

% get ellipse orientation
thetal1 = atan2(a1(2),a1(1)-a1(3))/2;

% get scaled major/minor axes
ctl1 = cos(thetal1);
st1 = sin(thetal1);
ap1 = a1(1)*ctl1*ctl1 + a1(2)*ctl1*st1 + a1(3)*st1*st1;
cpl1 = a1(1)*st1*st1 - a1(2)*ctl1*st1 + a1(3)*ctl1*ctl1;

% get translations
T1 = [ [a1(1) a1(2)/2]' [a1(2)/2 a1(3)]' ];
t1 = inv(2*T1)*[a1(4) a1(5)]';
cxl1 = t1(1);
cyl1 = t1(2);

% get scale factor
vall1 = t1'*T1*t1;
scale1 = 1 / (vall1- a1(6));
% get major/minor axis radii
r11 = 1/sqrt(scale1*ap1);
r21 = 1/sqrt(scale1*cp1);
v1 = [r11 r21 cx1 cy1 theta1]
';

% get ellipse orientation
theta2 = atan2(a2(2),a2(1)-a2(3))/2;

% get scaled major/minor axes
t2 = inv(2*T2)*[a2(4) a2(5)]
';
t2 = t2(1);
ct2 = cos(theta2);
st2 = sin(theta2);
ap2 = a2(1)*ct2*ct2 + a2(2)*ct2*st2 + a2(3)*st2*st2;
cp2 = a2(1)*st2*st2 - a2(2)*ct2*st2 + a2(3)*ct2*ct2;

% get translations
t2 = t2'*T2*t2;
t2 = t2(1);

% get scale factor
val2 = t2'*T2*t2;
scale2 = 1 / (val2 - a2(6));

% get major/minor axis radii
r12 = 1/sqrt(scale2*ap2);
r22 = 1/sqrt(scale2*cp2);
v2 = [r12 r22 cx2 cy2 theta2]
';
dat = [v1 v2];

figure(1)
t =uitable;
cnames = {'Ellipse 1', 'Ellipse 2'};
rnames = {'Length of Minor Axis (m)', 'Length of Major Axis (m)', 'Centre of Ellipse x-coord', 'Centre of Ellipse y-coord', 'Theta'};
set(t, 'Data', dat, 'ColumnName', cnames, 'RowName', rnames,
'ColumnWidth', {78}, 'Position', [80 80 435 120]);
annotation(figure(1), 'textbox', [0.15 0.48 0.78 0.078],
'String', {'Properties of Ellipses Best Fit to LiDAR Data'},
'HorizontalAlignment', 'center', 'FontSize', 12, 'FitBoxToText', 'off',
'LineStyle', 'none', 'EdgeColor', 'none');
saveas(1, 'EO Figure 1.jpg');

figure(2)
plot(X1,Y1,.g', X2,Y2,.r');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Two Cross Sections of LiDAR Data for Comparison', 'FontSize', 14);
legend('Cross Section 1', 'Cross Section 2', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
saveas(2, 'EO Figure 2.jpg');

% create best fit ellipse data set for first best fit
N1 = 100; % this number of points can be changed
dx1 = 2*pi/N1;
elliptheta1 = v1(5);
Rad1 = [[cos(elliptheta1) sin(elliptheta1)]', [-sin(elliptheta1)
cos(elliptheta1)]]';
for i = 1:N1
    ang1 = i*dx1;
    x1 = v1(1)*cos(ang1);
    y1 = v1(2)*sin(ang1);
    d11 = Rad1*[x1 y1]';
ellipX1(i) = d11(1) + v1(3);
ellipY1(i) = d11(2) + v1(4);
end
figure(3)
plot(X1,Y1,'.g', ellipX1, ellipY1, 'b');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Cross Section 1 Fit With Ellipse', 'FontSize', 14);
legend('Cross Section 1', 'Ellipse Fit', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
saveas(3, 'EO Figure 3.jpg');

% create best fit ellipse data set for second best fit
N2 = 100;
dx2 = 2*pi/N2;
elliptheta2 = v2(5);
Rad2 = [[cos(elliptheta2) sin(elliptheta2)]', [-sin(elliptheta2)
cos(elliptheta2)]]';
for i = 1:N2
    ang2 = i*dx2;
    x2 = v2(1)*cos(ang2);
    y2 = v2(2)*sin(ang2);
    d12 = Rad2*[x2 y2]';
ellipX2(i) = d12(1) + v2(3);
ellipY2(i) = d12(2) + v2(4);
end
figure(4)
plot(X2,Y2,'r', ellipX2, ellipY2, 'b');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Cross Section 2 Fit With Ellipse', 'FontWeight', 'bold', 'FontSize', 14);
legend('Cross Section 2', 'Ellipse Fit', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
saveas(4, 'EO Figure 4.jpg');

% centre of ellipse is used to translate each data set so it is centred about the origin
newX1 = ellipX1 - v1(3);
newY1 = ellipY1 - v1(4);
ewX2 = ellipX2 - v2(3);
newY2 = ellipY2 - v2(4);

x1 = linspace(0, max(newX1), 100);
x2 = linspace(0, max(newY1), 100);

m1 = 0;
m2 = 0;

angle1 = abs(v1(5) + pi/2);
angle2 = abs(v2(5) + pi/2);

circtest1 = abs(v1(1)-v1(2));
circtest2 = abs(v2(1)-v2(2));

if (circtest1 < 0.00015);
m1 = 0;
y1 = m1*x1;
a = 1;
else if (angle1 < 0.0001);
m1 = 0;
y1 = m1*x1;
a = 2;
else if (abs(1 - (angle1/(pi/2))) < 0.0001);
y1 = x1;
x1 = zeros(size(x1));
a = 3;
else if (abs(1 - (angle1/pi)) < 0.00001);

m1 = 0;
y1 = m1*x1;
a = 4;
else
    m1 = tan(v1(5) + pi/2);
    y1 = m1*x1;
    a = 5;
end
end
end

if (circtest2 < 0.00015);
m2 = 0;
y2 = m2*x2;
b = 1;
else if (angle2 < 0.0001);
m2 = 0;
y2 = m2*x2;
b = 2;
else if (abs(1 - (angle2/(pi/2))) < 0.0001);
y2 = x2;
x2 = zeros(size(x2));
b = 3;
else if (abs(1 - (angle2/pi)) < 0.0001);
m2 = 0;
y2 = m2*x2;
b = 4;
else
    m2 = tan(v2(5) + pi/2);
y2 = m2*x2;
b = 5;
end
end
end

% calculate the volumes enclosed by the best fit ellipses
vol1 = abs(v1(1)*v1(2)*pi);
vol2 = abs(v2(1)*v2(2)*pi);

figure (5)
plot(newX1, newY1, '-g', x1, y1, '--g', newX2, newY2, '-r', x2, y2, '--r');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Comparison of two Ellipse Fits', 'FontWeight', 'bold', 'FontSize', 14);
legend('Ellipse 1', 'Long Axis Orientation of Ellipse 1', 'Ellipse 2', 'Long Axis Orientation of Ellipse 2', 'Location', 'NorthEastOutside');
xlim([min(ellipX1)-1 max(ellipX1)+1]);
ylim([min(ellipY1)-1 max(ellipY1)+1]);
set(gcf, 'Color', 'w');
text(max(ellipX1)+2, min(ellipY1), sprintf('Ellipse 1 = %0.3f m^2, Ellipse 2 = %0.3f m^2', vol1, vol2));
saveas(5, 'EO Figure 5.jpg');

[theta1, rho1] = cart2pol(newX1, newY1);
[theta2, rho2] = cart2pol(newX2, newY2);

theta1 = theta1';
theta2 = theta2';
rho1 = rho1';
rho2 = rho2';
E1 = [theta1, rho1];
E2 = [theta2, rho2];
E1 = sortrows(E1,1);
E2 = sortrows(E2,1);

theta1 = E1(:,1)';
theta2 = E2(:,1)';
rho1 = E1(:,2)';
rho2 = E2(:,2)';

% determine the change between the two LiDAR profiles
change = rho1 - rho2;

% chose an exaggeration to display the deformations at
stretch = input('By how many times would you like the deformations exaggerated?\n');
stretchdef = change*stretch;
def = rho1 - stretchdef;

[defX, defY] = pol2cart(theta1, def);

figure (6)
plot(newX1, newY1, 'g');
hold on;
plot(defX, defY, 'Color', [0.5 0 .5], 'LineWidth', 2);
axis equal;
title('Change in Ellipse from First Fit Profile', 'FontWeight', 'bold', 'FontSize', 14);
legend('Ellipse fit 1', 'Change in ellipse profile');
xlim([min(newX1) -.5 max(newX1)+.5]);
ylim([min(newY1) -.5 max(newY1)+.5]);
set(gcf, 'Color', 'w');
saveas(6, 'EO Figure 6.jpg');

% determine diametric change
acrossdiam = round((length(change)/2)) + 1;
deform = zeros(1, length(change)/2);
newtheta = zeros(1, length(change)/2);

for i = 1:(length(change)/2)
    deform(i) = change(i) + change(acrossdiam);
    acrossdiam = acrossdiam + 1;
    newtheta(i) = thetal(i);
end

figure (7)
plot(newtheta, deform, 'Color', [.5 0 .5], 'LineWidth', 2);
xlabel('Distance around tunnel (m) clockwise from sidewall', 'FontSize', 12);
ylabel('Total displacement (m) [positive is inwards movement]', 'FontSize', 12);
title('Diametric tunnel displacement profile', 'FontWeight', 'bold', 'FontSize', 14);
legend('Displacement profile', 'Location', 'NorthEastOutside');
set(gcf, 'Color', 'w');
line([-pi,0], [0, 0], 'Color', 'k');
xlim([min(newtheta),0]);
saveas(7, 'EO Figure 7.jpg');
LiDAR Profile Analysis (LPA)

% user must input what type of data is being used as this affects the % orientation
evalResponse = input('If you are using shaft data enter 0. If you are using tunnel data aligned with the Y axis enter 1. If you are using tunnel data aligned with the X axis enter 2.\n');
format long g;

data1 = dlmread('CS1.txt');
data2 = dlmread('CS2.txt');

if(evalResponse == 0)
    X1 = data1(:,1);
    Y1 = data1(:,2);
    X2 = data2(:,1);
    Y2 = data2(:,2);
elseif(evalResponse == 1);
    X1 = data1(:,1);
    Y1 = data1(:,3);
    X2 = data2(:,1);
    Y2 = data2(:,3);
elseif(evalResponse == 2);
    X1 = data1(:,2);
    Y1 = data1(:,3);
    X2 = data2(:,2);
    Y2 = data2(:,3);
else
    error('Please use either properly aligned tunnel or shaft data.\n');
end

% elliptical fitting function is used to fit an ellipse to each data set
a = fitellip(X1,Y1);
b = fitellip(X2,Y2);

% function is used to solve for parameters that define best fit ellipse
v1 = solveellipse(a);
v2 = solveellipse(b);

cx1 = v1(3);
cy1 = v1(4);
cx2 = v2(3);
cy2 = v2(4);

% centre of ellipse is used to translate each data set so it is centred
% about the origin
X1 = X1 - cx1;
Y1 = Y1 - cy1;
X2 = X2 - cx2;
Y2 = Y2 - cy2;

[theta1, rho1] = cart2pol(X1,Y1);
[theta2, rho2] = cart2pol(X2,Y2);

figure(1)
plot(X1,Y1, '.g', X2,Y2, '.r');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Two Cross Sections of LiDAR Data for Comparison', 'FontWeight', 'bold', 'FontSize', 14);
legend('Cross Section 1', 'Cross Section 2', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
saveas(gcf, 'ULS Figure 1.jpg');

% create best fit ellipse data set for first best fit
N1 = 100; % this number of points can be changed
dx1 = 2*pi/N1;
ellipthetal = v1(5);
Rad1 = [ [cos(ellipthetal) sin(ellipthetal)]', [-sin(ellipthetal) cos(ellipthetal)]' ];

for i = 1:N1

    ang1 = i*dx1;
    x1 = v1(1)*cos(ang1);
y1 = v1(2)*sin(ang1);

    d11 = Rad1*[x1 y1]';
ellipXi(i) = d11(1);
ellipYi(i) = d11(2); 
end
```matlab
figure(2)
plot(X1,Y1, '.g', ellipX1, ellipY1, 'b');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Cross Section 1 Fit With Ellipse', 'FontWeight', 'bold', 'FontSize', 14);
legend('Cross Section 1', 'Ellipse Fit', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
saveas(2, 'ULS Figure 2.jpg');

% create best fit ellipse data set for second best fit
N2 = 100;
dx2 = 2*pi/N2;
elliptheta2 = v2(5);
Rad2 = [[cos(elliptheta2) sin(elliptheta2)], [-sin(elliptheta2) cos(elliptheta2)]];
for i = 1:N2
    ang2 = i*dx2;
    x2 = v2(1)*cos(ang2);
    y2 = v2(2)*sin(ang2);
    d12 = Rad2*[x2 y2]';
    ellipX2(i) = d12(1);
    ellipY2(i) = d12(2);
end

figure(3)
plot(X2,Y2, '.r', ellipX2, ellipY2, 'b');
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title('Cross Section 2 Fit With Ellipse', 'FontWeight', 'bold', 'FontSize', 14);
legend('Cross Section 2', 'Ellipse Fit', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
saveas(3, 'ULS Figure 3.jpg');

figure(4)
plot(thetal,rhol, '.g');
xlabel('Theta', 'FontSize', 12);
ylabel('Radius (m)', 'FontSize', 12);
```

148
% a filter cutoff is used to filter anomalies from the interior of the data
% set, closest points are filtered
evalResponse = input('Enter the filter cutoff % (enter 100 to use all data)\n');

nwindows = 5; % the number of filter windows must be optimized based on the type of anomalies present

if(evalResponse == 100)
else
    counter1 = 1;
    counter2 = 2;
    windowwidth1 = round(length(theta1)/nwindows);
    windowwidth2 = round(length(theta2)/nwindows);
    for i = 1:nwindows
        if(counter1*windowwidth1 > length(theta1))
        else
            temp = sortrows([theta1((windowwidth1*(counter1-1)+1):counter1*windowwidth1),rho1((windowwidth1*(counter1-1)+1):counter1*windowwidth1)],2);
            temp1:round(((100-evaulResponse)*windowwidth1)/100),:) = 0;
            theta1((windowwidth1*(counter1-1)+1):counter1*windowwidth1) = temp(:,1);
            rho1((windowwidth1*(counter1-1)+1):counter1*windowwidth1) = temp(:,2);

    end
end
counter1 = counter1 + 1;
end
end

for i = 1:nwindows
    if (counter2*windowwidth2 > length(theta2))
    else
        temp = sortrows([theta2((windowwidth2*(counter2-1)+1):counter2*windowwidth2), rho2((windowwidth2*(counter2-1)+1):counter2*windowwidth2)],2);
        temp(1:round(((100-evalResponse)*windowwidth2)/100),:) = 0;
        theta2((windowwidth2*(counter2-1)+1):counter2*windowwidth2) = temp(:,1);
        rho2((windowwidth2*(counter2-1)+1):counter2*windowwidth2) = temp(:,2);
        counter2 = counter2 + 1;
    end
end
end

theta1 = nonzeros(theta1);
theta2 = nonzeros(theta2);
rhol = nonzeros(rhol);
rho2 = nonzeros(rho2);

[junk index] = unique(theta1,'first');
theta1 = theta1(sort(index));
rhol = rhol(sort(index));

[junk index] = unique(theta2,'first');
theta2 = theta2(sort(index));
rho2 = rho2(sort(index));

regtheta = linspace(min(theta1),max(theta1),length(theta1));
newrho2 = interp1(theta2,rho2,regtheta);
newrho1 = interp1(theta1, rho1, regtheta);

logical1 = isnan(newrho1);
logical2 = isnan(newrho2);

newrho1(logical1 == 1) = [];
newrho1(logical2 == 1) = [];
newrho2(logical1 == 1) = [];
newrho2(logical2 == 1) = [];
regtheta(logical1 == 1) = [];
regtheta(logical2 == 1) = [];

% find the difference between the data sets
diff = newrho1 - newrho2;

evalResponse = input('By how many times would you like the deformations exaggerated?\n');

stretchfactor = evalResponse;
stretchdiff = diff.*stretchfactor;
rhodef = newrho1 - stretchdiff;

[defx, defy] = pol2cart(regtheta, rhodef);

deforms = [regtheta; rhodef];
addstep = length(regtheta)/2;
lengthstretch = length(regtheta);

% need to evaluate diametric displacement
if mod(lengthstretch,2)==0
    rhodefdiam = zeros(1,length(stretchdiff)/2);
    for i = 1:length(rhodefdiam)
        rhodefdiam(i) = stretchdiff(i) + stretchdiff((length(stretchdiff)/2) + i);
    end
else
    rhodefdiam = zeros(1,(length(stretchdiff)-1)/2);
    for i = 1:length(rhodefdiam)
rhodefdiam(i) = stretchdiff(i) +
stretchdiff(((length(stretchdiff)-1)/2) + i);
end
end

if(mod(lengthstetch,2)==0)
    regthetadiam = regtheta(1:length(regtheta)/2);
else
    regthetadiam = regtheta(1:(length(regtheta)-1)/2);
end

% determine the average diametric displacement, the standard deviation,
% and the variance of displacement
disp = rhodefdiam/stretchfactor;
avg_disp = mean(disp);
std_dev = std(disp);
variance = var(disp);

figure (6)
plot(regthetadiam, disp, '.', 'Color', [.5 0 .5]);
xlabel('Distance around tunnel (m) clockwise from sidewall', 'FontSize', 12);
ylabel('Total displacement (m) [positive is inwards movement]', 'FontSize', 12);
title('Diametric tunnel displacement profile', 'FontSize', 14);
legend('Displacement profile', 'Location', 'NorthEastOutside');
text(0.1, max(disp)-(max(disp)/10), sprintf('Average displacement (diametric): %0.3f m', avg_disp));
text(0.1, max(disp)-(max(disp)/10)*2, sprintf('Standard deviation: %0.3f m', std_dev));
% text(0.1, max(disp)-(max(disp)/10)*3, sprintf('Variance: %0.3f m', variance));
set(gca, 'Color', 'w');
line([-pi,0], [0, 0], 'Color', 'k');
xlim([min(regthetadiam),0]);
saveas(6, 'ULS Figure 6.jpg');

figure(7)
plot(X1,Y1, '.g');
hold on;
plot(defx, defy, '.', 'Color', [.5 0 .5]);
axis equal;
xlabel('width (m)', 'FontSize', 12);
ylabel('height (m)', 'FontSize', 12);
title({'Cross Section 1 with Amount of Change'; 'Compared to Cross Seciton 2'}, 'FontWeight', 'bold', 'FontSize', 14);
legend('Cross Section 1', 'Change between CS1 and CS2', 'Location', 'SouthEast');
set(gcf, 'Color', 'w');
hold off;
saveas(7, 'ULS Figure 7.jpg');

dtheta = regtheta(2) - regtheta(1);
R = newrhol;

% determine the volumes of convergence and the locations that limit areas
% of convergence and divergence
area = zeros(1, 10000);
crossoverpoints = zeros(2, 10000);
changei = zeros(1, 10000);
areacount = 1;
if isempty(nonzeros(diff))
    error('There is no change in the data therefore volume changes cannot be calculated or displayed.
');
else
    for i = 1:(length(regtheta) - 1)
        area(areacount) = area(areacount) + dtheta*R(i)*diff(i);
        if(abs(sign(diff(i+1)) - sign(diff(i))) > 1 || sign(diff(i)) == 0)
            areacount = areacount + 1;
            crossoverpoints(:, areacount) = [regtheta(i); newrhol(i)];
            changei(areacount) = i;
        end
    end
end
area = nonzeros(area)';
logicalcrossover1 = (crossoverpoints(1,:) == 0);
logicalcrossover2 = (crossoverpoints(2,:) == 0);

logicalcrossover = logicalcrossover1.*logicalcrossover2;
crossoverpoints(:,logicalcrossover == 1) = [];
changei(logicalcrossover == 1) = [];

if (sign(diff(1)) == sign(diff(end)))
    if (length(area) ~= 1)
        area(1) = area(1) + area(end);
        area(end) = [];
    end
end

minarea = input('Minimum area change to consider (in m^2)?
');

logicalminarea = abs(area) < minarea;
area(logicalminarea == 1) = [];
crossoverpoints(:,logicalminarea == 1) = [];
changei(logicalminarea == 1) = [];

[crossx crossy] = pol2cart(crossoverpoints(1,:),crossoverpoints(2,:));
[newX1,newY1] = pol2cart(regtheta,newrho1);

figure(8)
plot(X1,Y1,'.g',X2,Y2,'.r');
axis equal;
xlabel('width (m)','FontSize', 12);
ylabel('height (m)','FontSize', 12);
title('Deformed Area Locations and Volumes Plotted on LiDAR Profiles','FontWeight','bold','FontSize', 14);
legend(['CS1', 'CS2','Location', 'SouthEast']);
set(gcf, 'Color', 'w');
saveas(8, 'ULS Figure 8.jpg');

for i = 1:length(crossx)
    text(crossx(i),crossy(i),sprintf(' %0.3f , %0.3f
',crossx(i),crossy(i)),'HorizontalAlignment','center','BackgroundColor',[0.8 0.8 0.8], 'EdgeColor', [0.8 0.8 0.8]);
if(i == 1)

    text(mean([newX1(round(mean([change1(i), (length(defx) - change1(end)])]))], defx(round(mean([change1(i), (length(defx) - change1(end)])**])), mean([newY1(round(mean([change1(i), (length(defx) - change1(end)])]))], defy(round(mean([change1(i), (length(defx) - change1(end)])**)))), sprintf('%0.3f m^2 ', area(i)), 'HorizontalAlignment', 'center', 'BackgroundColor', [1 1 1], 'EdgeColor', [0 0 0]);

else

    text(mean([newX1(round(mean([change1(i), change1(i-1)]))]), defx(round(mean([change1(i), change1(i-1)]))**)), mean([newY1(round(mean([change1(i), change1(i-1)]))]), defy(round(mean([change1(i), change1(i-1)]))**))), sprintf('%0.3f m^2', area(i)), 'HorizontalAlignment', 'center', 'BackgroundColor', [1 1 1], 'EdgeColor', [0 0 0]);

end

end

if(isempty(crossx))

    text(0,0,sprintf('%0.3f m^2', area), 'HorizontalAlignment', 'center', 'BackgroundColor', [0.8 0.8 0.8], 'EdgeColor', [0.8 0.8 0.8]);

end
Noise Profile (NP)

% Interpolation for noise and moving average calc
max_theta = max(regthetadiam);
num_interps = length(disp);
length_interp = linspace(-pi,0,num_interps);
new_deform = interp1(newtheta, deform, length_interp);
noise_calcplot = new_deform - disp;

% Moving average calculation for LiDAR profile data
pt_avgs = input('What number of points would you like to use for the
moving average?\n');
num_avgs = round(length(disp)/pt_avgs)-1;
first_seed = round(pt_avgs/2);
moving_average = zeros(1, num_avgs);
movavg_theta = zeros(1, num_avgs);
temp = zeros(1, pt_avgs);
pt_avg_count = 1;
pt_avgs2 = pt_avgs;
for i = 1:num_avgs
    start = pt_avg_count;
    finish = pt_avgs2;
    temp = disp(start:finish);
    moving_average(i) = sum(temp)/pt_avgs;
    movavg_theta(i) = regthetadiam(first_seed);
    pt_avg_count = pt_avg_count + pt_avgs;
    pt_avgs2 = pt_avgs2 + pt_avgs;
    first_seed = first_seed + pt_avgs;
end
plot(regthetadiam, noise_calcplot, 'Color', [0.5 0 0.5]);
hold on;
plot(newtheta, deform, 'Color', [0.8 0.2], 'LineWidth', 2);
plot(movavg_theta, moving_average, '-or', 'LineWidth', 1);
xlabel('Angle around tunnel', 'FontSize', 12);
ylabel('Total displacement (m) [positive is inwards movement]',
'SanitizeSize', 12);
title('Deviation from Best Fit Ellipse', 'FontWeight', 'bold',
'SanitizeSize', 14);
set(gcf, 'Color', 'w');
line([-pi,0], [0, 0], 'Color', 'k');
xlim([min(regthetadiam),0]);
format short;

% Determine the max, min, avg, and std deviation of diametric
convergence
% measured from the comparison of the elliptical best fit profiles as well
% as the volume change
max_diam_conv_ellip = max(deform);
min_diam_conv_ellip = min(deform);
avg_diam_conv_ellip = mean(deform);
std_diam_conv_ellip = std(deform);
vol_change_ellip = vol1 - vol2;

% determine the max, min, avg, and std deviation of diametric convergence
% measured from the comparison of the LiDAR profiles as well
% as the volume change
max_diam_conv_lidar = max(disp);
min_diam_conv_lidar = min(disp);
avg_diam_conv_lidar = mean(disp);
std_diam_conv_lidar = std(disp);
vol_change_lidar = sum(area);
empty = isempty(area);
if empty == 1
    vol_change_lidar = 0;
else
end

max_syst_deviat = max(noise_calcplot);
min_syst_deviat = min(noise_calcplot);

if abs(max_syst_deviat) >= abs(min_syst_deviat);
    max_syst_deviat = abs(max_syst_deviat);
else
    max_syst_deviat = abs(min_syst_deviat);
end

logic_noise = isnan(noise_calcplot);\nnoise_calcplot(logic_noise == 1) = [];
noise = std(noise_calcplot);

misfit = zeros(1, length(new_deform));
for i = 1:length(disp)
    misfit(i) = (disp(i) - new_deform(i))^2;
end

logic1 = isnan(misfit);
misfit(logic1 == 1) = [];

RMS = sqrt(sum(misfit)/length(misfit));
Excel = [ max_diam_conv_ellip max_diam_conv_lidar min_diam_conv_ellip min_diam_conv_lidar avg_diam_conv_ellip avg_diam_conv_lidar std_diam_conv_ellip std_diam_conv_lidar vol_change_ellip vol_change_lidar RMS max_syst_deviat]"
Synthetic Data Generator

clear
% user is asked for input values to create synthetic data set
A = input('How many data points would you like to use?\n');
radius = input('What is the radius of tunnel you would like\nto generate data for (in m)?\n');
std_noise = input('What is the standard deviation of the noise (in m)?\n');

syst_noise = input('Would you like to include systematic noise (enter 1 for yes)?\n');

if (syst_noise == 1)
    % beta is the magnitude of the maximum systematic deviation
    beta = input('What beta would you like to use?\n');
    % gamma is the number of systematic deviations
    gamma = input('What gamma would you like to use (must be >2)?\n');
end

squish = input('Would you like to skew or rotate data (enter 1 for yes)?\n');

if (squish == 1)
    % skew is the amount of extension of the long axis and shrinking of the
    % short axis
    skew = input('What level of elliptical skew would you like to\nadd to the data (m)?\n');
    rotate = input('What rotation would you like to give the data (in radians ccw)?\n');
end

occlude = input('Would you like to occlude data (enter 1 for yes)?\n');

if (occlude == 1)
    percent_occlude = input('What percent of your data would you like occluded?\n');
    avg_length_occlude = input('What would you like the average length of an occluded\nsection to be (in m)?\n');
    std_occlude = input('What would you like the standard deviation of an occluded\nsections length to be (in m)?\n');
end

CS = input('Is this CS1 data or CS2 data?\n');

% create a data set with the number of points input by the user
theta = linspace(0, 2*pi, A);
% convert this data set to x and y coordinates
x = radius*cos(theta);
y = radius*sin(theta);

% add random noise to the data based on the specified standard deviation
% input by the user
noise = random('Normal',0, std_noise, A, 1);

% systematic noise is created
if (syst_noise == 1)
    for n = 1:A
        x_noise(n) = (radius + noise(n) +
        beta*cos(gamma*theta(n)))*cos(theta(n));
    end
    for n = 1:A
        y_noise(n) = (radius + noise(n) +
        beta*cos(gamma*theta(n)))*sin(theta(n));
    end
else
    for n = 1:A
        x_noise(n) = (radius + noise(n))*cos(theta(n));
    end
    for n = 1:A
        y_noise(n) = (radius + noise(n))*sin(theta(n));
    end
end

% skew is added to the data
if (squish == 1)
    max_skew = radius + skew;
    min_skew = radius - skew;

    x_skew = x_noise;
    y_skew = y_noise;

    for n = 1:A
        x_skew(n) = (x_noise(n)/radius)*min_skew;
    end

    for n = 1:A
        y_skew(n) = (y_noise(n)/radius)*max_skew;
    end
end
[thetay_final, rho1] = cart2pol(x_skew, y_skew);

% data is rotated if a rotation is present
for n = 1:A
    thetay_final(n) = thetay_final(n) + rotate;
end

for n = 1:A
    if (thetay_final(n) <= 2*pi)
        else
            thetay_final(n) = thetay_final(n) - 2*pi;
    end
end

[x_rot, y_rot] = pol2cart(thetay_final, rho1);

else
    x_rot = x_noise;
    y_rot = y_noise;
end

% data is occluded if occlusion is entered
if (occlude == 1)
    avg_pts_occlude = avg_length_occlude*(A/(2*pi*radius));
    std_pts = std_occlude*(A/(2*pi*radius));
    num_occlude_seeds = (percent_occlude/100)*A/avg_pts_occlude;
    prob_seed = num_occlude_seeds/A;
    tol = 0.05;
    x_final = x_rot;
    y_final = y_rot;

    while ((length(x_final) > (A*(1-percent_occlude/100)+tol*A)) ||
           (length(x_final) < (A*(1-percent_occlude/100)-tol*A)))
        x_final = x_rot;
        y_final = y_rot;

        randnums = random('unif',0,100,A,1);
        logical = randnums <= prob_seed*100;
        start_pts = find(logical);
        occlude_lengths = zeros(1,nnz(start_pts));
        start = zeros(1,nnz(start_pts));

        count = 1;
        current_count = 100000000;
hit_first_seed = 0;
occlude_data = zeros(1,A);

for i = 1:A
    if(count <= length(start_pts))
        if(start_pts(count) == i)
            occlude_lengths(count) = round(normrnd(avg_pts_occlude,std_pts));
            start(count) = i;
            current_count = 1;
            count = count + 1;
            hit_first_seed = 1;
        end
    end
end

if(hit_first_seed == 1)
    if(current_count <= occlude_lengths(count-1))
        occlude_data(i) = 1;
    end
    current_count = current_count + 1;
end
end

x_final(occlude_data == 1) = []; y_final(occlude_data == 1) = [];
end
else
    x_final = x_rot;
    y_final = y_rot;
end
X = x_final';
Y = y_final';
CS_data = [X Y];

% data is written to a text file and saved
if (CS == 1)
    dlmwrite('CS1.txt', CS_data);
elseif (CS == 2)
    dlmwrite('CS2.txt', CS_data);
else
    error('Please either choose 1 or 2.
');
end

figure(1)
plot(x_final, y_final, '.b', 'MarkerSize', 15);
axis equal;
xlabel('width of tunnel (m)');
ylabel('height of tunnel (m)');
title('Synthetic tunnel data');
ylim([min(y_final) - 2, max(y_final) + 2]);
xlim([min(x_final) - 2, max(x_final) + 2]);
set(gcf, 'Color', 'w');
Synthetic Data Analysis

% example of creation and testing of multiple synthetic data sets for MACE, MICE, ACE analysis
radnom = 5;
circumf = pi*radnom*radnom;

Synthfunc(5500, radnom, 0, 0, 0, 0, 0, 0, 0, 0, 1);

rad2 = 4.9:0.0005:4.995;
% rad2 = radnom - 0.005;

std_noise = 0:0.001:0.05;
beta = 0;
gamma = 0;
k=1:7;

maxdisp = zeros(length(std_noise), length(rad2), length(k));
mindisp = zeros(length(std_noise), length(rad2), length(k));
avgdisp = zeros(length(std_noise), length(rad2), length(k));
maxdeform = zeros(length(std_noise), length(rad2), length(k));
mindenform = zeros(length(std_noise), length(rad2), length(k));
avgdeform = zeros(length(std_noise), length(rad2), length(k));

for (k=1:7)
    for (i=1:length(std_noise))
        for (j=1:length(rad2))
            Synthfunc(5500, rad2(j), std_noise(i), beta, gamma, 0, 0, 0, 0, 2)
            [regthetadiam disp newtheta deform] = lidarfunc();
            maxdisp(i,j,k) = maxdisp(i,j,k) + max(disp);
            mindisp(i,j,k) = mindisp(i,j,k) + min(disp);
            avgdisp(i,j,k) = avgdisp(i,j,k) + mean(disp);
            maxdeform(i,j,k) = maxdeform(i,j,k) + max(deform);
            mindeform(i,j,k) = mindeform(i,j,k) + min(deform);
            avgdeform(i,j,k) = avgdeform(i,j,k) + mean(deform);
        end
    end
end
maxdisp2d = zeros(length(std_noise),length(rad2));
mindisp2d = zeros(length(std_noise),length(rad2));
avgdisp2d = zeros(length(std_noise),length(rad2));
maxdeform2d = zeros(length(std_noise),length(rad2));
mindeform2d = zeros(length(std_noise),length(rad2));
avgdeform2d = zeros(length(std_noise),length(rad2));

stdmaxdisp2d = zeros(length(std_noise),length(rad2));
stdmindisp2d = zeros(length(std_noise),length(rad2));
stdavgdisp2d = zeros(length(std_noise),length(rad2));
stdmaxdeform2d = zeros(length(std_noise),length(rad2));
stdmindeform2d = zeros(length(std_noise),length(rad2));
stdavgdeform2d = zeros(length(std_noise),length(rad2));

for (i=1:length(std_noise))
    for (j=1:length(rad2))
        maxdisp2d(i,j) = mean(maxdisp(i,j,:));
        mindisp2d(i,j) = mean(mindisp(i,j,:));
        avgdisp2d(i,j) = mean(avgdisp(i,j,:));
        maxdeform2d(i,j) = mean(maxdeform(i,j,:));
        mindeform2d(i,j) = mean(mindeform(i,j,:));
        avgdeform2d(i,j) = mean(avgdeform(i,j,:));
        stdmaxdisp2d(i,j) = std(maxdisp(i,j,:));
        stdmindisp2d(i,j) = std(mindisp(i,j,:));
        stdavgdisp2d(i,j) = std(avgdisp(i,j,:));
        stdmaxdeform2d(i,j) = std(maxdeform(i,j,:));
        stdmindeform2d(i,j) = std(mindeform(i,j,:));
        stdavgdeform2d(i,j) = std(avgdeform(i,j,:));
    end
end

%need to use rad2sized if the length of rad2 is greater than 1
rad2sized = zeros(length(std_noise),length(rad2));

for (m = 1:length(std_noise))
    rad2sized(m,:) = rad2';
end

diffmaxdisp = maxdisp2d - (radnom - rad2sized)*2;
diffmindisp = mindisp2d - (radnom - rad2sized)*2;
diffavgdisp = avgdisp2d - (radnom - rad2sized)*2;
\text{diffmaxdeform} = \text{maxdeform2d} - (\text{radnom} - \text{rad2sized})*2;
\text{diffmindeform} = \text{mindeform2d} - (\text{radnom} - \text{rad2sized})*2;
\text{diffavgdeform} = \text{avgdeform2d} - (\text{radnom} - \text{rad2sized})*2;

\text{raddiff} = (\text{radnom}-\text{rad2})*2;

[X1 XZ] = \text{meshgrid}(\text{raddiff}, \text{std}\_\text{noise});

\% disp is from LiDAR data, deform is from Elliptical data
Synthetic Data Analysis Figures

% Example of figures used for analysis of synthetic data
figure (1) % Change error for the maximum diameter measurement from LPA
contour3(X1,X2,diffmaxdisp,300);
colormap jet;
title('MACE from LPA', 'FontWeight', 'bold', 'FontSize', 14);
axis([0 0.01 6000 11000 -0.04 0.04]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
zlabel('MACE (m)', 'FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylims = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylims, 'Position', [0.88 0.8 0.012 0.12]);

figure(2) % Change error for the maximum diameter measurement from EFA
contour3(X1,X2,diffmaxdeform,300);
colormap jet;
title('MACE from EFA', 'FontWeight', 'bold', 'FontSize', 14);
axis([4.9 5.0 0 0.06 -0.01 0.1]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
zlabel('MACE (m)', 'FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylims = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylims, 'Position', [0.88 0.8 0.012 0.12]);

figure(3) % Change error for the minimum diameter measurement from LPA
contour3(X1,X2,diffmindisp, 300);
colormap jet;
title('MICE for LPA', 'FontWeight', 'bold', 'FontSize', 14);
axis([4.9 5.0 0 0.06 -0.1 0.01]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
zlabel('MICE (m)', 'FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylims = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylims, 'Position', [0.88 0.8 0.012 0.12]);

figure(4) % Change error for the minimum diameter measurement from EFA
contour3(X1,X2,diffmindeform, 300);
colormap jet;
title('MICE from EFA', 'FontWeight', 'bold', 'FontSize', 14);
axis([4.9 5.0 0 0.06 -0.1 0.01]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
figure(5) % Change error for the average diameter measurement from LPA
contour3(X1,X2,diffavgdisp, 300);
colormap jet;
title('ACE from LPA','FontWeight','bold','FontSize', 14);
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)','FontSize', 12);
ylabel('Number of points','FontSize', 12);
zlabel('ACE (m)','FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylims = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylims, 'Position', [0.88 0.8 0.012 0.12]);

figure(6) % Change error for the average diameter measurement from EFA
contour3(X1,X2,diffavgdeform, 300);
colormap jet;
title('ACE from EFA','FontWeight','bold','FontSize', 14);
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)','FontSize', 12);
ylabel('Number of points','FontSize', 12);
zlabel('ACE (m)','FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylims = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylims, 'Position', [0.88 0.8 0.012 0.12]);

figure (7) % Standard deviation of MACE from LPA
surf(X1,X2,stdmaxdisp2d);
colormap jet;
title('Standard deviation of MACE from LPA','FontWeight','bold','FontSize', 14);
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)','FontSize', 12);
ylabel('Number of points','FontSize', 12);
zlabel('Standard Deviation (m)','FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylims = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylims, 'Position', [0.88 0.8 0.012 0.12]);

figure(8) % Standard deviation of MACE from EFA
surf(X1,X2,stdmaxdeform2d);
colormap jet;
title('Std Maximum change from Elliptical Fits', 'FontWeight', 'bold', 'FontSize', 14);
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
zlabel('Standard Deviation (m)', 'FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylim = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylim, 'Position', [0.88 0.8 0.012 0.12]);

figure(9)
surf(X1,X2, stdmindisp2d);
title('Std Minimum change from LiDAR Profiles', 'FontWeight', 'bold', 'FontSize', 14);
colormap jet;
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
zlabel('Standard Deviation (m)', 'FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylim = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylim, 'Position', [0.88 0.8 0.012 0.12]);

figure(10)
surf(X1,X2, stdmindeform2d);
title('Std Minimum change from Elliptical Fits', 'FontWeight', 'bold', 'FontSize', 14);
colormap jet;
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
zlabel('Standard Deviation (m)', 'FontSize', 12);
set(gcf, 'Color', 'w');
colorbar;
ylim = get(colorbar, 'YLim');
set(colorbar, 'YTick', ylim, 'Position', [0.88 0.8 0.012 0.12]);

figure(11)
surf(X1,X2, stdavgdisp2d);
title('Std Average change from LiDAR Profiles', 'FontWeight', 'bold', 'FontSize', 14);
colormap jet;
axis([4.9 5.0 0 0.06 -0.001 0.001]);
xlabel('Standard deviation of noise (m)', 'FontSize', 12);
ylabel('Number of points', 'FontSize', 12);
Appendix B

Complete graphical results of sensitivity analysis of EFA and LPA
Change in beta and diameter
Change in beta and diameter
Change in beta and diameter

ACE from LPA

ACE from EFA
Change in beta and gamma

MACE from LPA

MACE from EFA
Change in beta and gamma
Change in beta and gamma
Change in beta and random noise
Change in beta and random noise
Change in beta and random noise
Change in beta and random noise

Standard deviation of MACE from LPA

Standard deviation of MACE from EFA
Change in beta and random noise

**Standard deviation of MICE from LPA**

**Standard deviation of MICE from EFA**
Change in beta and random noise

Standard deviation of ACE from LPA

Standard deviation of ACE from EFA
Change in diameter and noise
Change in diameter and noise

MICE for LPA

MICE from EFA
Change in diameter and noise

ACE (m)

ACE from EFA

ACE from LPA

Random Noise (m)

Change in Diameter (m)

$10^{-3}$
Change in diameter and noise

Standard deviation of MACE from LPA

Standard deviation of MACE from EFA
Change in diameter and noise

Standard deviation of MICE from LPA

Standard deviation of ACE from LPA
Change in diameter and noise
Change in Diameter and Noise with System Noise Included
Change in Diameter and Noise with System Noise Included
Change in Diameter and Noise with System Noise Included

ACE from LPA

ACE from EFA
Change in Occlusion and Noise

MACE from LPA

Percent Occluded (m)  Random Noise (m)

MACE from EFA

Percent Occluded (m)  Random Noise (m)
Change in Occlusion and Noise

MICE for LPA

MICE from EFA
Change in Occlusion and Noise

ACE from LPA

ACE from EFA
Change in Occlusion and Noise

Standard deviation of MACE from LPA

Standard deviation of MACE from EFA
Change in Occlusion and Noise

Percent Occluded (m)

Standard Deviation (m)

Random Noise (m)

Standard deviation of MICE from EFA

Standard deviation of MICE from LPA
Change in Occlusion and Noise

Standard deviation of ACE from LPA

Standard deviation of ACE from EFA

Percent Occluded (m)

Random Noise (m)
Change in Noise and Skew
Change in Noise and Skew
Change in Noise and Skew
Change in Noise and Skew