Statistical Analysis of Atrial Fibrillation Electrograms

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Abstract

Atrial fibrillation (AF) is the single most prevalent sustained cardiac rhythm disorder, arising when the normal electrochemical action potential propagating through the atria is interrupted by randomly firing foci. Current therapies rely on the analysis of electrocardiograms taken inside the atria to determine the amount of atrial activation at any given site on the endocardium. Atrial activation is measured by the appearance of peaks in an endocardial signal, deflections occurring close together correspond to sites of greater activation and may be closer to the foci in which the disturbance originates. It is the purpose of this study to use signal processing techniques to determine the occurrence times of the peaks in a digitized electrocardiogram (ECG) signal and to generate from this meaningful statistics about the atrial activation of the site where the ECG was taken. Currently, mean cycle length (CL) of a signal is the most widely used statistic for atrial activation. Frequency domain methods and spectrum analysis give basis to claims that AF is not completely chaotic and that its mechanism can be explained by the substrate through which the signals propagate. Frequency domain analysis is used liberally in this paper to support the development of an algorithm for deflection detection. Little is known presently about the mechanism of AF and algorithms such as the one proposed in this paper will provide more quantitative information about the disease process.
In Memory of K.D.C. Haley
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All pictures in this document are the property of the author unless otherwise stated. Graphs were drawn using Matlab 2008.

2 Introduction

2.1 Objective

The purpose of this paper is to help researchers quantify atrial activation from a given endocardial electrogram. We plan to develop a realistic and robust method for detecting deflections in the endocardial signals.

2.2 Organization

This paper is organized into seven sections. Section 3 contains preliminary information on the physiology of the heart and the normal cardiac action potential. It explains how a normal 12-lead ECG is taken and how it can be analyzed. Also in this section is an introduction to AF, the way an endocardial signal is measured and the technology and methods used.
Section 4 gives mathematical background to the signal processing methods which will be used in the analysis. Analysis in the frequency domain is given, followed by a review of digital filters and windowing. Section 5 gives the development of a paradigm for deflection detection based on digital filters introduced in sections two and three. Section 6 is a collection of examples of the algorithm designed in section four. We give analysis and discussion of its performance on various signals. These examples will bring up the following question about the quantification of the ECG features: is current software displaying meaningful statistics about the data? Hence, we suggest some meaningful statistics in section 7. Section 8 concludes with further work.

Sections 5 and onward represent the author’s original work. Some of the preliminary material given in sections 3 and 4 is not used in the analysis but is presented for completeness. Many of the sources which contribute in the preliminary sections continue to inspire the work in sections 5 to 8. This paper contributes to the field in that it gives a novel application of surface ECG signals to endocardial electrograms. In the next section we shall see what we mean by that.

3 Preliminaries

3.1 Physiology of the Heart

The heart is the main functional organ of the circulatory system. It is responsible for the continued circulation of blood to the tissues of the body which will provide cells with oxygen and nutrients. It is situated just left of the sternum within the chest cavity, and is the size of a large fist.

The heart is comprised of four primary chambers: two atria, and two ventricles. The atria sit atop the ventricles, and they are divided into left and right atrium/ventricle pairs by a fleshy septum. Over the course of a heartbeat, blood enters first the right atrium from the veins that enter it, whereupon it is pumped to the right ventricle. It then exits the heart through the pulmonary artery to the lungs where it acquires oxygen. It re-enters the heart via the pulmonary vein to the left atrium, and is sent to the left ventricle. Thereafter, blood leaves the left ventricle through the aorta to be sent to body tissues where it is needed. Figure 1 gives the author’s representation of the human heart. The atria and ventricles are labelled right and left from the patient’s point of view, i.e. left and right are reversed when
The wall of the heart, the myocardium, is composed mainly of striated cardiac muscle cells which contract to force blood through the chambers of the heart. It is also composed of specialized cells which conduct minute electrochemical charge to trigger the contraction of the muscles. This conduction system allows an impulse to propagate rapidly through the tissues of the heart, producing the cardiac cycle. In this way, the tissues of the myocardium go through two phases to produce a heart beat. The first delivers an electrochemical impulse and the second is the contraction of the muscle as a result of that impulse [38].

Along with this visible propagation of muscle contraction, there is always
an impulse through the conduction system which precedes it. The cardiac action potential, as it is called, consists of a depolarization and a repolarization phase, in which the cells experience a rapid change in membrane potential eventually returning to a baseline potential. This corresponds to the physical counterpart of contraction and relaxation of the muscles in the heart [10].

The signal originates in the sinoatrial (SA) node, a small patch of pacemaker cells which are able to spontaneously fire an electrical impulse. These cells are situated in the upper right atrium (RA). The wavefront propagates through the two atria, causing depolarization. Once it reaches the atrioventricular (AV) node, it is held and delayed momentarily before it is allowed to spread to the ventricles. Once released from the AV node, it collects in the septum at the bundle of His before branching into a network of specialized Purkinje fiber cells.

The automaticity of the pacemaking system is due to the spontaneous activation of the SA node. The SA node responds to information given by the autonomic nervous system, and its rate is determined by this information. The SA node has the fastest activation rate of the entire conduction system and it thus sets the limiting rate on the rest of the activation sequence. It is not uncommon, however, for there to arise another mass of cells, called an ectopic focus, to send off disruptive signals, sometimes overpowering or anticipating the wavefront that has originated at the SA node [38].

3.2 The twelve lead ECG

An ECG is a way of visualizing and monitoring the electrical activity in the heart. It consists of a series of waveforms of data collected by electrodes at particular sites on the body. The potential difference in the heart can be thought of as the collective difference in potential of the many current conducting cells in the myocardium added together to form a vector having magnitude and direction. An ECG measures this vector over time, from various sites inside or outside the body. The current standard noninvasive method to obtain an ECG is via the 12-lead configuration.

A 12-lead ECG is taken by placing nine electrodes on the surface of the skin which record the electrochemical potential differences in the heart over time. First, electrodes are placed on both wrists and the left foot of the patient. They are set up to form an equilateral triangle referred to as Einthoven’s triangle [10].

The signals from the first three electrodes provide six time series graphs
on the ECG map, I, II, III, aVR, aVF, and aVL. The leads I, II, and III are given by the combination of signals:

\[ I = V_{LA} - V_{RA} \]  
\[ II = V_{LL} - V_{RA} \]  
\[ III = V_{LL} - V_{LA}. \]

The signals \( V_{LA}, V_{RA} \) and \( V_{LL} \) are measurements taken from the left arm, right arm and left leg electrodes, respectively. Leads labelled aVR, aVL, and aVF are the signals from the right wrist, left wrist, and left foot from which we subtract the average potential of the remaining two electrodes, i.e.

\[ aVR = V_{RA} - (V_{LA} + V_{LL})/2 \]  
\[ aVL = V_{LA} - (V_{RA} + V_{LL})/2 \]  
\[ aVF = V_{LL} - (V_{RA} + V_{LA})/2. \]

Figure 2 serves to explain the generation of the six leads I, II, III, aVR, aVL, and aVF from the three electrodes \( V_{LA}, V_{RA}, V_{LL} \) placed in Einthoven’s triangle.

![Figure 2: Electrode placement for the 12-lead ECG.](image)

Figure 2 (a) shows the generation of leads I, II, and III. Schematically, we see lead I on the straight line between the figure’s two wrists, representing the subtraction from left to right. It is interesting to note that lead III
can be generated from II and I together. Figure 2(b) shows a schematic representation of the subtraction of the average of the other two leads from the first. For example, we see that aVR is the difference between the right wrist electrode reading and the average of the left leg and left wrist. These are called bipolar because they are calculated with reference to the other three leads on the triangle. The next six leads, the unipolar leads, are so called because they are calculated with reference to an independent reference point, \( V_{WCT} \).

![Figure 3: Unipolar Leads](image)

The electrodes, \( V_1, V_2, V_3, V_4, V_5, \) and \( V_6 \) are placed on the left side of the chest, monitoring the left anterior and posterior sides of the heart as seen in Figure 3. These unipolar leads have their potential differences measured with respect to this reference point:

\[
V_{WCT} = \frac{(V_{LA} + V_{LL} + V_{RA})}{3}.
\]  

(7)

The idealized ECG waveforms

The salient features of the ECG are best described with a diagram. From a glance at the schematic in Figure 4, we see the idealized ECG wave, the peaks of which are named P, Q, R, S, and T. The positive amplitude P wave at the beginning of the signal represents the depolarization of the atria just before contraction, activated by the SA node. The electrochemical signal propagates through the heart until it reaches the atrioventricular (AV) node. This, in turn, triggers the fast depolarization of the ventricles in the QRS complex. The potential voltage difference occurring during the QRS complex
ranges between -20mV and 90mV over a 80-90ms duration [38]. Post-QRS complex comes the T wave, which represents the repolarization of the ventricles and the return of the membrane potential to baseline.

It is important to keep in mind that the potential at any given time must be considered a vector. The lead signals I, II, and III give only the change in potential as viewed from one side of the body, mediated mathematically by subtraction with an adjacent lead, so a thorough analysis takes into account all views of the potential vector. In particular, a negative signal can be interpreted as a net negative charge moving away from the lead, and a positive signal as a negative charge moving toward the lead.

Some properties of a patient’s health can be immediately discerned from the ECG. For example, heart rate is usually measured via the distance between R-waves. Since the R-waves are of the greatest amplitude, these are not often obscured. Also, one can identify irregular rhythm and rate, irregular PQRST-wave structure, and arrhythmias.

3.2.1 The Endocardial Electrogram

It may be necessary to internally monitor the electrical impulses in the chambers of the heart. The endocardial electrogram differs greatly from that of the 12-lead (surface ECG) configuration. In such a procedure, electrodes are placed directly on the surface of the heart via small catheters. These
catheters have segments alternating in polarity along the length of the device. When a catheter touches the myocardium, the potential difference is calculated as the difference in potential between segments. Since the catheter dipoles are quite close together, this allows for the calculation of an (almost) instantaneous potential difference at the site of contact. In contrast with the signals produced by the 12-lead noninvasive procedure, the endocardial electrogram looks very different. The potentials given by leads I, II, and III, for example, give an idea of the global activity of the heart, that is, the changes in the cardiac action potential vector over time over the entire surface of the heart. An endocardial electrode gives a much more localized view of the electrical activity at any particular site. Depending on the catheter placement, in the atria or in the ventricles, the site monitored by the internal electrode can be relatively active or quiet.

It is endocardial electrograms that we will be considering for the rest of this paper.

3.3 Introduction to Atrial Fibrillation

Atrial fibrillation (AF) is the most prevalent sustained cardiac rhythm disorder, and 80% of people diagnosed are over the age of 65. One to two million Americans suffer from chronic AF, and it is increasing in prevalence [10, 3]. Furthermore, AF is the most common cardiogenic cause of stroke. The hallmark of AF is in the absence of the P wave in a surface ECG. This represents a rapid, erratic depolarization and repolarization of the atria in a seemingly random configuration.

The current hypothesis, thus far supported experimentally, is that the cause of the random signal is that one or more localized centres in the heart produces signals which interfere with the normal depolarization/repolarization cycle. Like the cells in the AV node, this tissue is able to spontaneously begin giving off impulses. The center may be situated inside the atria or outside, often originating in the veins entering the atria.

The tissue may give off signals in any number of shapes, depending on the properties of the origin of the disturbance, and/or conducting ability of the surrounding tissue. In particular, these disturbances may manifest themselves like ripples on a pond, propagating uniformly outwards from the site, or they may also activate as rotors or spirals [17]. An endocardial electrogram can be used to find these areas and to identify their properties. The next section elaborates on the mechanism for AF. The mechanism for
AF justifies the use of catheter ablation therapy, and vice versa, and its understanding will also be enlightening for subsequent mathematical study.

### 3.3.1 The Multiple Wavelet Hypothesis as a Mechanism for AF

Worth mentioning at this point is the proposed mechanism for AF. The literature offers a rich backlog of mathematical and physical theory to the study of random signals, harmonic analysis, waveform propagation, and application of fluid dynamical models to help with the development of an electrophysiologic model of wave propagation in the atrial tissue. The diversity of mechanisms include perpetuation of AF via

- rapid firing of ectopic foci, especially in the pulmonary veins
- multiple re-entrant circuits circulating within the atria, and
- wandering rotors [7, 19].

Current favoured theories for perpetuation of random signals are rooted in the research of Moe, Rheinboldt, and Abildskov in the early 1960’s. Moe et al. proposed what has become the multiple wavelet hypothesis. In it, they proposed that fibrillation is maintained by the fractionation of waveforms passing through inhomogeneous tissue, i.e. cells of varying excitability, refractory periods, and conduction capacity. The model showed that the heterogeneity of the refractory parameters in a patch of cells was essential to the continuation of random signals. In effect, the heterogeneity of the tissue was a key to the mechanism of AF.

An impulse initiated in an excitable muscle does not propagate uniformly outward from the site of activation. Because the cells in atrial tissue have different recovery (refractory) times, the wavefront spread may have a jagged configuration. For multiple impulses,

Since conduction velocity is low in relatively refractory tissue, a second response initiated at the site of origin of the primary response will be irregularly propagated [...] the advancing wave front of the second response must tend to conform itself to the retreating edge of the preceding response, and must also become serrated in contour. If this process is repeated a second or a third time, temporal dispersion of the processes of excitation and recovery must become accordingly greater [23].
The research of Moe et al. was backed by a computer simulation type model for the excitation of the cells in the heart [24]. What was interesting to them, and to the scientific community, was that the maintenance and perpetuation of AF appeared independent of its initiation [23].

M.A. Allessie et al. mapped the electrical wavefront in the canine heart, providing the first living demonstration of the multiple propagating wavelets which were proposed by the computer model. Further investigation by Allessie provided two major underlying mechanisms of AF, namely multiple wavelet reentry, and leading circle reentry of electrical impulses [13].

A reverberator-based model was proposed, in which the mechanism of AF was in the creation of tiny rotor-shaped wavefronts. These rotors would be created by straight waves propagating though a heterogeneous medium. The waves would get delayed by blockages of cells which would set off a set of waves with different direction and morphology. In particular, the wavefront would peel off from the blockage to form a spiral. Decay and reproduction of these rotors would cause them to appear and disappear almost at random, forming a plausible mechanism for the basic features of AF [20].

The multiple wavelet hypothesis has perpetuated the idea of chaos in the mechanism of AF. What was described above, of course, is the tip of a giant iceberg leading into the study of differential equations, stability of dynamical systems, and great attractors, see [26]. Some researchers, however, argue that AF retains a great many deterministic components.

Statistical methods applied to endocardial electrograms have given evidence that AF is not completely chaotic. It is not inconsistent with observed phenomena that repetitive activations of atrial tissue display preferred routes of propagation [17] and have a high level of spatiotemporal organization. The study of dominant frequency analysis tends to support this statement. The consensus among researchers today is that AF is not completely chaotic. Though the original hypothesis of Moe et al. supposes a completely random model of wave propagation, this part of the theory is no longer widely accepted. It also rejected the circus movement theory, in which electrical activity propagates through the heart in a complete circuit. This theory has since come back into vogue [15]. We look forward to a unified scientific consensus on this matter in the future.
3.3.2 Catheter Ablation

Pharmacologic methods for the regulation of AF have been remarkably unsuccessful. The long term success rate is approximately 50% [40]. A successful treatment procedure, based upon the mechanism of AF, is catheter ablation. The ablation procedure is an invasive operation intended first to find the site of the abnormal signals, and second, to burn a small amount of tissue at the periphery of the area to isolate and interrupt any outgoing signals. The precursor to ablation therapy involved surgical procedures which often removed large amounts of cardiac tissue.

Catheter ablation involves an apparatus of internal catheters which monitor the electrical activity of the heart in real time. These catheters come in several different types. They are thin tubes used for monitoring sites on the endocardial surface. There may be straight, basket, lasso-shaped, or they may be irrigated or non-irrigated tip ablation catheters. They have alternating dipoles spaced a few millimetres apart. The tip of a basic catheter is small, between 2 and 4 mm, and it can give an accurate picture of local activation time of a site on the surface endocardial tissue. A lasso\(^{TM}\) catheter (Biosense Webster) is a catheter bent into a lasso shape, has segmented dipoles, and is useful for locating centres of electrical activity, particularly within the pulmonary veins [22]. The ablation catheter is a much larger catheter, having a 5-8mm tip [42]. It is not generally used for data acquisition but is able to administer a radiofrequency signal equivalent to about 45 degrees Celsius, creating a lesion where it touches any tissue [45]. When irrigated, however, the ablation catheter tip ranges from only 3.5-4mm. The irrigation cools the catheter tip and allows for better penetration of energy into the myocardium.

In preparation for the procedure, catheters are inserted through veins leading to the right atrium. A trans-septal puncture is used to feed catheters from the right to the left atrium. During the first half of the procedure, the internal catheters are used to collect data about the internal geometry of the heart, and a EnSite NavX software system\(^1\) interpolates between the points collected to give a 3-dimensional surface. The reliability of this geometry is crucial for the next step of the process. Figure 5 gives an idea of the size of the geometric data set and the way it is visualized.

Once the geometry has been established, a cardiologist examines a time series evolving on a screen, representing the measurements of both the 12-lead

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\(^1\)EnSite NavX software, Saint Jude Medical - the software system to be used for the collection of data in this study.
If atrial fibrillation is present during the procedure, it will be characterized by its behaviour in the internal catheter measurements. Spiky deflections in the signal appearing above the noise level correspond to times of atrial activation. In healthy atria, these signals manifest themselves regularly, with identifiable onset and end. Nademanee et al. [25] identified fractionated signals, ones with complex fractionated electrograms (CFEs) as targets for ablation. These sites can be identified by the following characteristics of their electrograms.

1. Atrial electrograms that have fractionated electrograms composed of two deflections or more, and/or perturbation of the baseline with continuous deflection of a prolonged activation complex.

2. Atrial electrograms with a very short cycle length, less than 120ms [40, 25].
Often, re-entrant sites or rotors originate in the left pulmonary vein [38]. An ablation catheter may be used to apply a radiofrequency signal of between 20 and 40W to create a region of altered tissue, a lesion, with a radius of approximately 5mm. This lesion, provided sufficient power and duration of contact with the ablation catheter, will be unable to conduct electrical signal. Since the source of the ectopic focus often lies in one of the pulmonary veins, ablation is often performed circumferentially around the base of the veins to isolate them. This practice, called circumferential pulmonary vein isolation, has become widespread for the treatment of AF. Ablation within the vein can cause it to stenose, or close up slightly, which in turn can cause minor complications. Circumferential pulmonary vein isolation has been successful for isolation of the signal, and was one of the earliest significantly successful methods for AF therapy [19].

If the fractionated signals cease in the rest of the heart after ablation, an attempt is made to re-trigger the signal. If this does not work, the operation is considered a success. This situation is ideal. Though termination of the arrhythmia is the best case scenario, if this is not possible, the regularization of the atrial signal and lengthening of the mean time between deflections is also beneficial to the patient [42, 38].

It may be the case, however, that tissue scarred during the operation will serve to interrupt fractionated signals only until the damaged tissue heals. Unfortunately, this is only one of many complications that may arise post-procedure. In addition, as Moe’s hypothesis suggests, the arrhythmia may perpetuate itself even if the source of the activation has been isolated. However, ablation treatment for AF patients has been remarkably successful.

The difficulties associated with catheter ablation are numerous. First of all, the software described above generates a non-moving geometry, a shell that may or may not give an accurate description of the contact points of the catheters on the surface of the tissue at all points in the cardiac and respiratory cycles. Due to this movement, it is possible for a catheter to shift position relative to its reference frame leaving a confidence interval of up to 5-10 millimeters [11]. The process of isolation of the signal becomes more difficult with this consideration. Since a lesion due to contact with the ablation catheter is approximately 5-8mm, undetected gaps may remain between lesions, and insufficient isolation of the focus may occur.
3.3.3 The EnSite NavX Software

EnSite NavX customized software (St. Jude Medical, Austin, Tex) is a software system designed for the development of a real-time anatomical model of the chambers of the heart during catheter-based invasive procedures. It consists of a set of three pairs of grounding electrode patches, a reference patch, ten ECG electrodes, a data module and a display. Mapping is achieved through the location of catheters within the thorax by the use of externally applied currents. That is, catheter electrode position can be determined via voltage drops and impedance within the field [14].

The three electrode patch pairs are placed orthogonally so as to create, to a good approximation, an \((x, y, z)\) potential coordinate system. This particular placement of leads is analogous to the Frank Lead system [30]. These patches are used to send independent, alternating, low-power currents through the chest at slightly different frequencies. The difference in frequency -30Hz is used to mathematically separate the signals to measure the amplitude of each of the frequency components. The \((x, y, z)\) coordinates of each electrode are calculated by dividing the signal amplitudes (V) by the electrical field strength (V/cm). In this way, the position of a catheter can be determined with reference to a base patch, in terms of \((x, y, z)\) coordinates.

NavX allows up to 64 endocardial electrodes to be visualized at a time [14]. The development of a model of the cardiac anatomy of the patient can be done quickly with the use of the NavX software. Catheters probing an area of the endocardium are able to record their coordinates at a rate of 96 points/sec. A geometry is created using several thousand points. It is important that points be removed during the collection process, otherwise coordinates lying outside the endocardial wall may be collected. Since the EnSite system’s internal algorithm defines the surface by the most distant point in any given angle from the designated geometric center, the catheter may easily protrude out from the wall when collecting data. Without point erasure, the atrial geometry is generally oversized [45]. An interpolation algorithm uses the points collected to create a smooth surface, a triangulation consisting of tiny 3D patches, onto which activation levels can be projected.

The operator of the system is able to designate fixed points which cannot be removed by the algorithm that creates the surface, which begets a geometry more suited to a previously collected CT scan. This is facilitated by a Digital Image Fusion (DIF) package which allows for the simultaneous display of CT or MR segmented cardiac scans to be displayed alongside the
evolving geometry. Markers are placed on the shell as anatomical landmarks are identified, such as the sites of entry to pulmonary veins and the location of the left atrial appendage. Also, lesions where ablation has occurred can be tagged and identified [42].

Advantages of the NavX software are numerous in comparison with previous technologies. The fast point acquisition rate -96pt/s gives great resolution to an atrial chamber, and gives fast acquisition of a geometry consisting of several thousand points. In addition, up to 12 catheters and 64 electrodes can be visualized from the moment they are put into use. This greatly reduces the time a patient and operators must stay exposed to harmful X-rays during a procedure [45, 9].

3.3.4 Understanding Fractionation

During an episode of atrial fibrillation, an endocardial electrode will measure frequent and randomized depolarization and repolarization of the cells in the endocardium. For the determination of the amount of atrial activation, a measurement of the fractionation of the signal is key. Fractionation is a measure of how active the tissue is over any given time. It is quantified by current software in terms of mean cycle length. If the average amount of time between peaks is at or below 120ms, it is potentially near the site of the disturbance, and is a target for ablation therapy [19].

Many different deflection morphologies may be detected in the electrogram, with fluctuations grouped into segments or more or less spread over the length of the observation window. The more fractionated sites, the ones with more deflections or shorter mean CL, are likely to be closer to the rotors of depolarization and repolarization that cause the disturbance [25]. Closely placed internal electrodes can show fractionation over the atria. Fractionation is a difficult concept to be presented with at first, so examples will be given.

It is also worth mentioning that normal atrial ECGs contain deflections, but these are much more regular, and they occur with longer cycle length than those in Figure 6.

Figure 6 gives two examples of fractionated endocardial ECGs. The dotted lines are annotations suggesting where peaks may be. They correspond to sites of atrial activation. However, when atrial activation is high - that is, intervals between peaks come in under 120ms - as we can see from both signals in the diagram, we declare the signal fractionated.
Fractionation comes in all shapes and sizes. A signal may be continuously fractionated or it may have pockets of fractionation along its length. It may be difficult, or relatively easy to detect fractionation.

### 3.3.5 The CFE Mean Map

For each endocardial electrode within the heart, a small area can be monitored over a 4-6 second segment. Using this information, a fractionation quantity for that 3D coordinate can be calculated, and this data can be displayed on a Complex Fractionated Electrogram (CFE) map. The CFE data is visualized on the 3D map by colouring the surface vertices to correspond to the mean cycle lengths calculated by a threshold detection algorithm.

Deflection detection is done by the EnSite software package by specifying two specific settings for width and refractory. Width refers to the permissible length of the deflection peak from onset to end, while the refractory settings determine the permissible time between the end of one deflection to the beginning of the next. If the width and refractory settings are too large or too small, it is possible to over- or under-detect deflections, leading to a less reliable CFE map. The weakness of the current CFE map-making process lies in the unreliability of significant deflection detection.

Dr. Atul Verma of the Southlake regional health care centre offers his
advice in the collection of data for a CFE map in an online video for the EnSite NavX software system [42]. Once the catheters have been inserted into the patient and the software initialized, the threshold noise level can be estimated by placing a catheter in the middle of the atrial chamber; there, one can gauge the amount of ambient noise being picked up. Then, from a short signal collected from the endocardial wall, the initial mean cycle length at that site can be estimated. Over the course of the procedure, this cycle length is monitored to determine if it has changed. The lengthening and regularization of the time between deflections means a lower atrial activation rate and a healthier long term outcome for the patient. Studies by Nademane et al. define a CFE site as one in which the mean CL registers 120 ms or less [25]. Catheter ablation is recommended for CFE sites in order of shortest to longest mean cycle length.

During the mapping procedure, one must determine the settings for the CFE detection algorithm. First of all, it is important that since deflections are detected on an amplitude threshold based system, the threshold be set above the level of the ambient noise. The threshold voltage for the (P-P sensitivity) detection of deflections should not be too high or else the low-voltage deflections will not be detected, but on the other hand, if the threshold is too low, noise may be confused for the actual signal. This is why the ambient noise is determined at the onset of the procedure. This threshold level can range from 0.02-0.1mV depending on the noise present. The width criteria on the algorithm should be approximately 20ms, again, too narrow or too wide a width setting may pick up spurious waves or ignore true peaks. The refractory setting is designed to avoid double counting of a single electrogram. There ought to be a blanking period after a detected peak, for which no new peak can be detected. Dr. Verma suggests a 40-45ms setting for refractory. He also suggests that the segment length should be at least 5 seconds, ideally 6 seconds to ensure a consistent and reproducible snapshot of the signal, based on studies performed validating signal acquisition length.

When using NavX for the development of geometry, more parameters must be set so as to ensure a reliable 3D surface: interpolation, interior projection and exterior projection for collection of points. The mapping is done with several thousand points, colour coded by the mean cycle length. To highlight the regions of most interest, the regions with mean cycle length less than 120ms (CFE sites) are colour coded in terms of highest priority. These regions can then be targeted for ablation therapy.
3 PRELIMINARIES

The CFE map is a now an additional tool for cardiologists performing ablation. In [42], Dr. Verma gives a whole case study for a patient with paroxysmal AF, from the determination of NavX system settings to the actual ablation procedure itself. If the reader is interested in getting a better idea of the procedure, Dr. Verma’s three videos are readily available online.

3.3.6 An Example CFE Map

A sample CFAE map is given in Figure 7. The mean cycle length statistic is plotted as a colour value on the geometry. Cycle length on a CFE map can register from 30-450ms between deflections, but it is the regions with cycle length under 120ms which are targets for ablation. Those regions are clearly visible from the map. The geometry is in millimetres.

Figure 7: Sample CFE map - colour coded by mean cycle length between atrial deflections

It is clear that the CFE map is useful for clinicians who need a quick analysis of cycle lengths throughout the heart. The map is a clean, informative, easy-to-read description of the atrial activity throughout a procedure. The Achilles heel of the CFE map, of course, is in the algorithms working to produce meaningful statistics about the atrial activation. In a lab with a
lot of noise it is easy to set noise settings too low and to over-detect signals, or worse, set the settings too high and under-detect signals. Noise is merely one of the many challenges algorithm designers face when trying to detect signals which may often be obscured.

3.3.7 Objectives

Today’s NavX technology automatically identifies the fractionated signals characteristic of AF and places these statistics on a CFE map, though it is mainly up to the cardiologist to analyze the data quickly as it passes in real time. An experienced cardiologist can identify and interpret these signals with little error. It is important, though, that automated techniques be developed and improved as this process may take several hours to perform by visual inspection alone. The objectives of this paper are to first outline heuristics for atrial activation measurement, second, to reliably detect deflections on an electrogram trace, and third, to propose a robust method for fractionation quantification. The topics in the next section are in preparation for this analysis.

4 Signal Processing in ECG Analysis

4.1 Motivation for the use of Digital Signal Processing

In the past, digital signal processing has been used by [38] in ECG analysis to

- Condition the signal with respect to noise
- Extract basic salient ECG features
- Compress the signal for storage or transmission.

These problems lend themselves well to frequency domain techniques. The ECG waveform suggests a frequency domain approach primarily due to obvious periodicities in the data. Dominant frequency analysis [28] has been applied with enthusiasm in the past, and the multiple wavelet hypothesis for the mechanism of atrial fibrillation [23] suggests a rich diversity of spectral components in the data.
4 SIGNAL PROCESSING IN ECG ANALYSIS

Not such a great deal of literature exists on the endocardial electrogram as it does on the surface ECG, but methods used on surface ECGs can be adapted to better suit the specific qualities of the endocardial electrogram. In particular, a whole branch of electrocardiographic analysis is devoted to detecting the QRS complex. As seen before, the QRS complex has a distinct morphology, but may be severely distorted due to noise and artefacts. This is analogous to the problem of deflection detection in the atria, however it is perhaps more delicate to broach the latter problem as the mechanism of AF is not entirely known and the signal is less predictable.

It is important during the preprocessing stage to filter the noise from the signal as well as possible. The design of a filter must be done carefully in order to minimize distortion in the output. For example, it is easy to introduce ringing by the use of a simple linear (FIR or IIR) filter, which may cause the appearance of spurious waves.

To test filters, real data databases are available full of ECGs annotated by specialists, in particular [21]. It is good to test filters on annotated data before they are tried on ECGs with unknown properties.

4.2 A Statistical Arsenal

4.2.1 The Frequency Domain

For the benefit of readers with little engineering or mathematical background, we will refer to some more substantial reading throughout this section. Admittedly, some previous knowledge of frequency domain analysis will be helpful.

4.2.2 The Fourier Transform

The Fourier transform is the operation that makes analysis in the frequency domain possible. It relies on the fact that a periodic continuous signal can be approximated by a weighted sum of sinusoids of different frequencies. For a continuous signal \( x(t) \), we can consider its continuous Fourier transform, a function of frequency \( \omega \), defined by

\[
X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt
\]

\[
= \int_{-\infty}^{\infty} x(t)[\cos(\omega t) - j \sin(\omega t)]dt
\]

(8)
$X(\omega)$ is a complex number, having real and imaginary parts, and can be thought of as a vector in the complex plane having magnitude (length) and phase (angle). The magnitude characteristic $|X(\omega_0)|^2$ shows how much a given frequency $\omega_0$ contributes to the signal $x$. By convention, we capitalize $x$ when we wish to refer to its Fourier transform. For our purposes, we will refer to the discrete Fourier transform more often, which is the Fourier transform of a discretization of a continuous process $x(t)$. It is defined as follows.

Given a sequence $\{x(n)\}_{n=0}^{N-1}$ of $N$ discrete samples of a continuous process $x(t)$, the discrete Fourier transform (DFT) is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}, \ k = 0, 1, \ldots, N - 1. \ (10)$$

The term $X(k)$ is a complex function of frequency, and it gives the complex voltages (amplitude and phase) as a function of frequency that are present in the original signal. We can view (10) as a correlation between the signal $x(n)$ and the cosine (real part) and sine (imaginary part) functions at frequency $\omega = 2\pi k/N$. It can be shown that the output $X(k)$ of the DFT are the complex Fourier series coefficients for $x(n)$ [31]. It is in this way that the DFT contains all of the frequency domain information necessary to reconstruct the signal - i.e. the Fourier series representation of the signal.

As the DFT of a signal requires considerable computation time, $O(N^2)$ operations, programmers use the fast Fourier transform, or FFT. When $N$ is highly composite, then redundancies in the terms can be used to group terms together and reduce the computational complexity to $O(N\log(N))$ operations. Furthermore, the FFT computation gives a more accurate result, since a computer contains a finite register, fewer multiplications and additions means less rounding error [31].

A graph of $|X(k)|^2$ - the periodogram estimate of the spectrum - allows for a detailed examination of the frequency content of $x(n)$. A single sinusoid at frequency $f$ Hz, for example, will have a periodogram with a distinct peak at $f$ showing that most of the energy in the signal is concentrated at $f$. Signals which are sums of sinusoids of different frequencies will have peaks in their spectra corresponding to those frequencies which make them up.

Discretization of a continuous process involves invoking the Shannon sampling theorem which states that the sampling rate of $x(t)$ must be at least twice as large as the largest frequency component contained in the signal.
This critical frequency is called the Nyquist frequency, and it is equal to $1/2\Delta t$ Hz or $2/\Delta t$ rad/s, where $\Delta t$ is the time interval between samples (in seconds). If the sampling theorem is not adhered to, a phenomenon known as aliasing may be observed in which undersampling gives a distorted reconstruction of the continuous signal.

In the context we will be using it, it is estimated that the desired frequency content of an (endocardial) episode of AF rarely exceeds 250Hz. By the sampling theorem, the sampling frequency necessary to avoid distortion is 500Hz [33]. Since the sampling frequency of the EnSite NavX software is 1200Hz, we can expect to have captured the most important features of the signal, along with some high frequency noise. The sampling frequency of 1200Hz allows us to discriminate frequencies below 600Hz which gives enormous sensitivity for our analysis.

In any case, frequency domain analysis will provide a basis for the removal of noise. In the next section we will give a brief overview of linear filters.

4.3 Digital Filtering

The objective for the use of a digital filter is to separate the components of the signal, to remove the unwanted characteristics, and to obtain a signal of greater purity. There are two types of linear filter, the nonrecursive - FIR filter, or the recursive - infinite impulse response (IIR) filter. Both types have advantages and disadvantages for noise removal that can be seen in both the amplitude and phase characteristics of the DFT. Both types of filters have characteristics seen most easily in the frequency domain, as they isolate certain frequencies present in the signal and attenuate them.

Four types of filters are used most often: lowpass, highpass, bandpass, and bandstop. Their power gain$^2$ characteristics are illustrated in Figure 8.

In Figure 8: The lowpass, highpass, bandpass, and bandstop filters. a) Lowpass filters enhance the components of the spectrum with the lowest frequency, b) Highpass filters enhance high frequency components of the spectrum. c) Bandstop filters attenuate all frequency components lying outside a certain band. d) Bandpass filters attenuate frequency components within a prescribed band.

It is possible to create digital filters of both the FIR and IIR type with any of the above characteristics.

$^2$square of amplitude gain, measured on a log-scale (in dB)
The design of a filter requires the appropriate design of impulse response, \( h \), which is necessary to transfer the signal \( x \) into the output \( y \). Consider the following causal linear system,

\[
\sum_{m=0}^{M-1} a(m) y(t - m) = \sum_{n=0}^{N-1} b(n) x(t - n),
\]

where \( t \) represents time (now discrete), and in which the coefficients \( \{a(i)\}_{i=0}^{M-1} \) and \( \{b(j)\}_{j=0}^{N-1} \) are real, and we call \( x(n) \) the independent variable, or input, and \( y(m) \) the dependent variable, or output. Noting that both sides of the equation have been given as a convolution\(^3\), and taking the DFT, gives

\[
A(z)Y(z) = B(z)X(z),
\]

where the capital letters denote the DFTs of each of the component sequences.

---

\(^3\)It is known that the Fourier transform of the convolution product of two sequences corresponds to the product of the transform of each sequence separately, and that linear systems can be written in the form of a convolution product.
quences. Applying cross multiplication, we obtain

\[
\frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)} = H(z).
\]  

(13)

The function \(H(z)\) is called the transfer function because it contains all of the frequency domain information required to send the input \(x\) to the output \(y\). That is,

\[
H(z) = \frac{b(0) + b(1)z^{-1} + \ldots + b(N-1)z^{N-1}}{1 + a(1)z^{-1} + \ldots + a(M-1)z^{M-1}}
\]  

(14)

where we allow that \(a_0 = 1\). \(H(z)\) is the ratio of the DFTs of the vectors \(a = a_1, 2, \ldots, a_{M-1}\) and \(b = b_0, b_1, \ldots, b_{N-1}\).

Of interest in filter design are the amplitude and phase response of the transfer function \(H\), defined as follows:

**Amplitude Response:**

\[|H(z)| = \left| \frac{B(z)}{A(z)} \right|.
\]

**Phase Shift:**

\[\angle(H(z)) = \angle(B(z)) - \angle(A(z)),\]

where the angle function is shorthand for the angle of the argument with respect to the horizontal. The transfer function can be interpreted in terms of its poles and its zeros. That is, since the complex function \(H(z)\) is a ratio of two other complex functions \(B\) and \(A\), each of these component functions will have roots. The function \(B\) will have roots corresponding to zeros in \(H\), and \(A\) will have roots corresponding to poles in \(H\). These can be plotted on the complex plane to give a description of \(H\)'s properties. For an example of a pole/zero and amplitude gain plot see Figure 14.

When designing a digital filter, zeros and poles are placed strategically to modify \(H\)'s characteristics. Since the amplitude response is the ratio of the magnitudes of \(B\) and \(A\), when \(z\) approaches a root of \(A\) along the unit circle, the amplitude response will tend to infinity, and when \(z\) approaches a root of \(B\), the amplitude response will tend to zero.

Hence, considering the amplitude gain plots given in Figure 8, it is possible to design a filter by deciding on the amplitude gain characteristics needed, and placing the appropriate poles and zeros on the complex plane to design functions \(A\) and \(B\) that will define the transfer function.

Figure 8 was generated from four different types of FIR filters each with \(N = 101\) weights. They illustrate the characteristics of the lowpass, highpass, bandpass, and bandstop filters. For a lowpass filter, we must specify
which cutoff frequencies we require, the passband and the stopband, which determine the interval in which the filter gain must rapidly drop to zero.\textsuperscript{4} In the figure, we have chosen a passband of 100Hz, and a stopband of 200Hz. The bandpass and bandstop filters have two values each of bandpass and bandstop, and these are given by 100, 200, 300, and 400Hz. As we see in Figure 8, the bandpass filter changes power from practically zero to one between the frequencies 100 and 200 Hz, and then decreases from one to zero between the frequencies 300 to 400Hz.

It is also important to characterize the phase response. When $z$ approaches a root of $B$, the phase shift decreases, but when it approaches a root of $A$, the phase shift increases. Depending on the characteristics of $A$ and $B$, the phase may shift linearly or nonlinearly. It will become clear that this is a problem in the design of an appropriate IIR filter.

To use the filter, all we need do is apply an inverse discrete Fourier transform to $H$ to obtain $h$. Figure 9 shows the impulse response functions, $h$, of each of the filter types given in Figure 8, for comparison. The convolution of $h$ and $x$ gives the output $y$. Depending on the characteristics of $H$, certain frequencies present in the original signal can be discarded. One also notices

\textsuperscript{4}Close to zero, that is, approx. $10^{-14}$. 

Figure 9: Filter coefficients for lowpass, highpass, bandpass, and bandstop filters from Figure 8.
from Figure 9, the ringing effects of linear filter in the filter coefficients.

4.3.1 FIR Filters

FIR filters are simpler than IIR filters in that the linear system that they describe is nonrecursive, i.e. the output $y$ is not dependent on its own past values. If we suspect this is the case, we design a filter for $x$ in which the coefficients in the vector $a$ are all zero except the first, $a_0$, and the model equation 11, becomes

$$y(k) = \sum_{n=0}^{N-1} b(n)x(k-n), \quad k \in \{1, 2, \ldots, M\}$$  \hspace{1cm} (15)

Where, by convention, we assume that with a negative argument, elements of the sequence $x(n)$ are zero. Though the coefficients $b_n$ are unknown, the use of a transfer function corresponding to a causal system such as the one above, imply that the transfer function must have inverse Fourier transform of the form $[b(0) b(1) \ldots b(N-1)]$.

For example, the transfer function of the idealized (square) lowpass FIR filter with cutoff frequency $\nu$, and sampling interval $T$, is

$$H(\omega) = \begin{cases} 
1, & 0 \leq \omega \leq 2\pi\nu/T, \\
0, & \text{otherwise.} 
\end{cases} \tag{16}$$

The inverse Fourier transform of which is given by

$$h(kT) = \begin{cases} 
2\nu, & k = 0, \\
\frac{\sin(2\pi\nu k)}{\pi k}, & 1 \leq k \leq \infty. 
\end{cases} \tag{17}$$

The function $h$ is continuous, extending infinitely in both directions. In order to make the filter realizable, we discretize the filter, shift it, and truncate it to $N$ samples. Without loss of generality, we will assume $N$ is odd. Then the weights of the filter are given by

$$h(k) = \begin{cases} 
2\nu, & k = pN, p \in \mathbb{Z}, \\
\frac{\sin(2\pi\nu(k-N/2+1))}{\pi(k-N/2+1)}, & \text{else.}
\end{cases} \tag{18}$$

For $N$ large, the filter weights will approach the idealized lowpass filter power gain values desired.
4.3.2 Windowing

For a number of reasons, merely truncating the transfer function of the linear filter is not acceptable. In particular

- The choice of \( L = (N - 1)/2 \), the length of the filter, must be large enough to give an acceptably rectangular gain function. As \( L \) is increased, more and more computations are necessary to give the output.

- Ripples pile up by the boundary frequencies specified, that is, at the passband and stopband.

Many alternatives to truncation are available so that these negative effects are minimized. Among them is downsampling: if the sampling frequency can be decreased without introducing aliasing, then it is possible to create a more rectangular filter with fewer weights. However, it is usually disadvantageous to reduce the resolution of the signal, and in this case we use a windowing function. A windowing function will have the effect of truncating the infinite weight vector given by the transfer function \( h \) to a vector of length \( 2L + 1 \) for some chosen length \( L \).

The simplest window of all, the rectangular window, is equivalent to that of truncating the weight vector. The window is given by

\[
    w(n) = \begin{cases} 
        1, & n \in \{-L, -(L - 1), \ldots, 0, \ldots, L - 1, L\} \\
        0, & \text{otherwise.} 
    \end{cases} \tag{19}
\]

A nonrectangular window can give a better effect by smoothing weights closer to the end of the window. This can minimize the ripple which appears in the passband and stopband of the filter gain function. Some effective windows are

- The Hamming window,

\[
    w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N - 1}\right), \quad \tag{20}
\]

- not to be confused with the Hanning window,

\[
    w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N - 1}\right). \quad \tag{21}
\]
• The Blackman window,

\[ w(n) = 0.42 - 0.5 \cos\left(\frac{2\pi n}{N - 1}\right) + 0.08 \cos\left(\frac{4\pi n}{N - 1}\right). \]  \hspace{1cm} (22)

• The Kaiser window,\(^5\)

\[ w(n) = \frac{I_0(\beta \sqrt{1 - \left(\frac{2n}{N - 1} - 1\right)^2})}{I_0(\beta)}. \]  \hspace{1cm} (23)

where the expression \(I_0\) refers to the zero-order modified Bessel function of the first kind, given by

\[ I_0(x) = 1 + \sum_{n=1}^{\infty} \left(\frac{x/2}{n!}\right)^n. \]  \hspace{1cm} (24)

We will introduce these windows in the next section in specific examples. Often we can avoid the use of windows altogether and use other means; this is often preferable as the behaviour of a window may produce unpredictable results.

### 4.3.3 IIR Filters

IIR filters improve on the FIR filter design in that they introduce poles into the transfer function \(H\) and allow for sharper changes in the filter amplitude gain function. As stated before, a stable linear filter shall have poles lying inside the unit circle. This stability criterion shall pose problems in the design of a proper IIR filter. This introduces distortion in the phase aspect of the filtered data, and thus an IIR filter is only applicable for use when small amounts of phase distortion are allowable. This is not the case for the analysis of electrocardiograms, as the phase distortion will modify the length between peaks, and will thus alter mean cycle length and activation rate statistics.

It is, however, possible to reverse the nonlinear phase characteristic of the IIR filter by running the filter forwards and backwards. This technique finally makes the IIR filter practical. If a digital filter is not required to operate in real time, it is possible to store the data and run the filter forwards

\(^5\)The Kaiser window was discovered by Steve Rice.
and backwards offline. This is shown in [39]. The forward-backward operation for obtaining a linear phase characteristic from a transfer function $H$ with nonlinear phase characteristic is equivalent to using the absolute value squared of the transfer function $H$ to filter the input.

IIR filters are more versatile than the FIR filters in that they have more parameters. The addition of poles to the transfer function allows for greater control over the shape of the amplitude and phase gain plots. A filter designer can create virtually any type of filter he or she desires by strategically placing poles in the transfer function.

Two of the most commonly used IIR filters are the Butterworth and Chebyshev type I and II filters. It is unfortunate that the IIR filters are not useful in real time for the purposes of our detector.

### 4.3.4 Advantages and Disadvantages of FIR and IIR filters

FIR Filters have a great deal of advantages over IIR filters, the most important of which is the linear phase property. If the input signal $x(n)$ is shifted $m$ steps in the time domain, its Fourier transform shifts in phase proportionally with $m$. That is, each of the components of $x(n)$ is shifted in phase to produce a delay of $m$ time steps. Linear phase is not possible in a stable IIR filter operating in real time.

Since FIR filters have poles only at the origin and zeros anywhere, it is more difficult to create a filter with steep enough edges near the boundaries needed. That is, since zeros are always placed so far from poles, only a gradual decrease in amplitude gain is possible as $e^{j\omega}$ in the transfer function moves along the boundary of the unit circle toward a zero. If a steep transition is absolutely necessary, an IIR filter is more practical.

FIR filters, having only zeros, are not unstable. Stability is a crucial characteristic to keep in mind.\(^6\)

---

\(^6\)A system is unstable if its response to a transient input increases without bound. It is known that the poles of the transfer function of a linear system must be within the unit circle in order to maintain stability [39].
4.3.5 The 60Hz Powerline Interference: A Paradigm for Preliminary Noise Removal

Electromagnetic fields caused by a powerline may enter the electrogram signal traces in the form of a sinusoidal 60Hz\(^7\) disturbance, possibly accompanied by some harmonics. Analysis of the properties of the electrogram is made much more difficult by the appearance of this noise: spurious waves may be registered, while more low-amplitude signals may, in error, be ignored. As an example of FIR bandstop filtering, we challenge the problem of powerline noise removal. Though this technique and more sophisticated techniques have been developed in [38], this particular paradigm gives a novel introduction to the preprocessing stage of cardiac electrograms.\(^8\)

To design a filter, we first consider the placement of zeros on the complex plane. We will use a pair of complex-conjugated zeros placed at the interfering frequency \(\omega_0\),

\[ z_{1,2} = e^{\pm j \omega_0}. \]

Then we design a transfer function, \(H\) that will be zero at \(z_1\) and \(z_2\), namely

\[
H(z) = (1 - z_1 z^{-1})(1 - z_2 z^{-1})
      = 1 - z_1 z^{-1} - z_2 z^{-1} + z_1 z_2 z^{-2}
      = 1 - z^{-1}(z_1 + z_2) + z^{-2}
      = 1 - 2 \cos(\omega_0) z^{-1} + z^{-2},
\]  

where we have used the complex cosine identity to simplify the equation. Here we have a second order, notchband filter that, in theory, should eliminate frequencies not at \(\omega_0\). However, it is the case that the FIR filter design makes it difficult to sufficiently isolate \(\omega_0\) and frequencies nearby will likely be eliminated. To make up for this, we can switch our design to that of an IIR filter, and add some poles to the transfer function. We will add poles at complex conjugated points

\[
p_{1,2} = re^{\pm j \omega_0}, \ 0 < r < 1.
\]

\(^7\)The powerline disturbance may be at 50Hz, depending on where the signal was recorded.

\(^8\)This powerline interference removal is done automatically by the EnSite software suite, so it will not be necessary to perform this preliminary filtering on data obtained from this software.
The new transfer function we obtain is

\[ H(z) = \frac{(1 - z_1 z^{-1})(1 - z_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})} = \frac{1 - 2 \cos(\omega_0) z^{-1} + z^{-2}}{1 - 2r \cos(\omega_0) z^{-1} + r^2 z^{-2}}. \] (26)

Clearly, as \( r \) approaches 1, the notch becomes steeper and the filter is more selective, however, as \( re^{j\omega} \) approaches the boundary of the unit circle, the filter’s transient response increases, and the filter becomes unstable for \( r \geq 1 \).

We note again that it is important when using an IIR filter, to perform forward/backward filtering in order to preserve the linear phase property. Figures 10 and 11 show the magnitude response and pole/zero plot of the filter we have just designed. Note that the magnitude response function dips at 60Hz, the interfering frequency, and the poles and zeros have been placed as complex conjugate pairs, with \( r \) sufficiently close to 1. X’s on the complex plane refer to poles, while O’s refer to zeros.

![Magnitude Response (dB)](image)

Figure 10: The notchband filter for the removal of powerline interference: magnitude response.

Now we evaluate the performance of this filter. As the transient response increases (\( r \to 1 \)), a ringing artefact can be introduced into the filter output,
which in some cases, may significantly alter the morphology of the waveform. This may manifest itself as small spurious waves appearing at the boundaries of the waveform, and may mimic late potentials. These waveforms are largely unwanted, and provoke the use of more sophisticated filtering, that is, the use of a nonlinear filter.

4.3.6 Nonlinear Filtering with respect to the removal of Powerline interference

We can expect the powerline interference to be a sinusoid present in the signal with frequency 60Hz. With this in mind, we choose a model equation in which the signal of interest, or the output of the filter, \( y(n) \), is the observed signal \( x(n) \) with a sinusoidal component \( v(n) \) subtracted, that is,

\[
y(n) = x(n) - v(n).
\]  

Let \( v(n) \) be modelled by the equation

\[
v(n) = \omega_0 \sin(\omega_0 n),
\]
where the amplitude $\omega_0$ is unknown and may change over time. We will use an adaptive approach at each time step to update the estimate $\hat{v}(n)$ by changing $\omega_0$.

We choose a transfer function with complex conjugated poles situated at $\omega_0$,

$$H(z) = \frac{V(z)}{U(z)} = \frac{1}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}. \tag{29}$$

Hence, $v(n)$ may be generated by the difference equation,

$$v(n) = u(n) + 2\cos(\omega_0)v(n - 1) - v(n - 2) \tag{30}$$

with initial conditions $v(-1) = v(-2) = 0$, and in which

$$u(n) = \delta(n) \tag{31}$$

$$= \begin{cases} 
1, & n = 0 \\
0, & n = 2, 3, \ldots, N - 1, 
\end{cases} \tag{32}$$

is the unit impulse function.

Now consider the error function associated with how well the estimate $v(n)$ approximates the true powerline interference.

$$e(n) = x(n) - v(n) \tag{33}$$

Computing the first difference of $e(n)$ gives an error function independent of the DC level present in the signal

$$e'(n) = e(n) - e(n - 1) \tag{34}$$

$$= x(n) - x(n - 1) - (v(n) - v(n - 1)). \tag{35}$$

Depending on whether the error function is positive or negative, we may choose to add or subtract, respectively, a small increment $\alpha$ from the estimate $\hat{v}(n)$. Hence we can update the estimate for the powerline interference via the equation,

$$\hat{v}(n) = v(n) + \alpha(e'(n)). \tag{36}$$
The choice of $\alpha$ is a critical parameter. If $\alpha$ is chosen too small, then the adaptive amplitude method may not converge quickly enough to the desired amplitude. If $\alpha$ is too large, then the output may reflect sudden large jumps present in $v(n)$.

The desired output is given by (27) with $\hat{v}(n)$ substituted for $v(n)$. At each time unit, the error is calculated and $\hat{v}(n)$ is updated.

Via this nonlinear method, there is no ringing such as that which is caused by the linear filter; however, an improper choice of the parameter $\alpha$ may leave much of the sinusoid still present in the result.

### 4.3.7 Noise reduction of the endocardial ECG

Noise reduction thus far has been specific to the cause of the noise. Noise may be a result of

- powerline interference
- baseline wander, or
- muscle noise.

The first of these problems we have tackled in the preliminary analysis of the last section. Baseline wander and muscle noise manifest themselves as very low frequency components in the spectrum. In the case of baseline wander, many complicated techniques exist for its removal. In case of muscle noise disturbance, the noise may be less regular, and may be minimized by signal averaging. Muscle noise components are unlikely to be present in the data we are presented with because of limited movement of the patient during a procedure and our assumption of adequate electrode contact. Baseline wander, however, will be present, and we will treat it separately in our scheme.

What is important to do with the signal now is to isolate the components we are interested in for the purposes of wave delineation and feature extraction. The filters we have covered up to this point all introduce some kind of distortion in the signal; the choice of an appropriate filter depends primarily upon what kind of distortion is tolerable in the situation at hand.

### 5 A Framework for Deflection Detection

In this section we will proceed as in the detection of QRS complexes given in [38] on the surface ECG, only with adaptations to the low-amplitude signals
Figure 12: Deflection detector Schematic

5 A FRAMEWORK FOR DEFLECTION DETECTION

and variable morphologies of the endocardial traces. We will take a variable time/frequency domain-based approach and will evaluate our detection scheme in terms of accuracy and practicality.

The paradigm we will follow is best described with a diagram. In Figure 12, we see the structure of our detector. It consists of two preliminary processing steps. The first of which, the preprocessor, is designed to accentuate the signals of interest and to attenuate the smaller deflections without eliminating them. It contains a nonlinear transformation, which occupies itself with the accentuation of high peaks, and a linear filter with bandpass characteristics which is designed to remove high and very low frequency noise.

The next stage of the process is in rectification of the signal, that is, turning each of the deflections into a single positive peak. This involves finding the \textit{envelope} of the signal, which we shall see shortly involves a special kind of filter. Also involved in the rectification is the smoothing of the envelope which we accomplish with the use of a lowpass filter.

The most important step is the decision rule which may be a simple memoryless thresholding procedure, or a more complex adaptive threshold with memory. The thresholding scheme we make use of requires two variable parameters, width and refractory, that the EnSite system also uses. We will see in the next few sections how all of these components take shape.

5.1 Part 1: The Preprocessor

The \textbf{Nonlinear transformation} Returning to the paradigm with which we began, the first part of the preprocessor is a nonlinear transformation. This process must be computationally simple, memoryless, and its purpose must be to enhance the higher peaks present in the signal $x(n)$ while attenuating
lower ones. It is for this reason that we may choose a nonlinear transformation of the form

\[ y(n) = sgn(x(n))(e^{|x(n)|} - 1) \]  

(37)

where the \( sgn \) function evaluates to -1 if its argument is negative, and 1 if its argument is positive, defined as

\[ sgn(x(n)) = \begin{cases} \frac{x(n)}{|x(n)|}, & x(n) \neq 0 \\ 0, & \text{if } x(n) = 0. \end{cases} \]  

(38)

Along the same vein, we may also choose the following nonlinear transformation

\[ y(n) = sinh(x(n)) = \frac{1}{2}(e^{ix(n)} - e^{-ix(n)}). \]  

(39)

Both of these equations serve the same purpose, but in the examples we prefer (37) because it allows for greater peak enhancement. In Figure 13 we can see the distortion of signal amplitude characteristics.

![Comparison of sinh and exponential nonlinear transformations](image)

**Figure 13**: Nonlinear transformation comparisons

**The linear filter** A linear filter at this stage serves the purpose of eliminating high frequency noise. For this, we require a filter with lowpass characteristics. If, in addition, we would like the filter to be able to run in real time with linear phase shift, it is necessary that this be an FIR filter. In this section, we introduce the FIR filter with such characteristics. We will make use of this filter in step 2 as well.
First in the construction of such a filter, we choose the passband and stopband characteristics we would like to see in the result. We are confident in setting the bandstop characteristic at $f_{\text{stop}} = 200\text{Hz}$. In addition, since we want a relatively simple filter design, that is, with low order (number of zeros in the transfer function) we have chosen $f_{\text{pass}} = 100\text{Hz}$.

Now is the crucial positioning of zeros on the unit circle to give our frequency domain transfer function $H(z)$. For the purpose of attenuating high frequencies we choose $K$ complex conjugated values of the form

$$z_{k,k+K} = e^{\pm j\omega_k}, k = 1, \ldots, K$$

where the set $\Omega = \{\omega_0, \omega_1, \ldots, \omega_K\}$ contains $\omega$’s between $2\pi \frac{200}{1200} = \frac{\pi}{3}$ and $\pi$ radians.

In addition, we choose the following two additional poles on the real line

$$z_{2K+1} = r, \quad z_{2K+2} = \frac{1}{r}$$

where $0 < r < 1$ is real-valued. The order of the filter is clearly $2(K+1)$, and we can map its poles and zeros as in the upper plot in Figure 14 for a filter of order 10. We obtain the following frequency domain transfer function

$$H(z) = \prod_{k=0}^{2K+2} (z^{-1} - z_k). \quad (40)$$

This is truncated by way of a rectangular window in the time domain. In the filter response shown (which we will be using throughout the filtering process), we have poles on the unit circle situated at the minima of the magnitude response function, see Figure 14. It is this filter that will serve to remove the high frequency noise in our examples.

### 5.2 Part 2: Rectification

The purpose of righting the signal is to simplify it for detection by changing each waveform into a single positive peak. To do this, we may use a simple memoryless operation, such as a squarer. Since a squaring operation will introduce additional spurious peaks and valleys in the signal, the squarer will also need to be accompanied by a lowpass filter, yielding output

$$u(n) = \sum_{k=n-L+1}^{N-1} h(n - k)z^2(n), \quad (41)$$
Figure 14: The order 10 lowpass FIR filter pole/zero plot and magnitude gain function
where \( h \) denotes the impulse response of a lowpass linear filter with length \( L \). This operation is not terribly computationally costly, as the filtering of a signal amounts to the convolution of its impulse response function \( h \) with the input vector. If the impulse response, \( h \), is of length \( T \), the computation is of order \( O(TN) \) operations. It may also be implemented in real time. If a time delay of 1 sample (1/1200 th of a second) is permissible, then a superior technique, the computation of the envelope, \( u(n) \) of the signal \( z(n) \) is preferable.

Consider a signal in which a peak \( s(n) \) can be described by the following model equation

\[
s(n) = u(n) \cos(\omega_m n + \phi) \tag{42}
\]

where \( u(n) \) denotes a lowpass signal -the envelope of the signal \( s(n) \) - modified in amplitude by a deterministic cosine curve with parameters (angular) modulation frequency \( \omega_m \) and phase \( \phi \). The many different choices of parameters \( \omega_m \) and \( \phi \) can give rise to a host of different peak morphologies. In order to obtain the envelope of \( s(n) \) without the knowledge of \( \omega_m \) and \( \phi \), we first rewrite (42) as

\[
s(n) = \frac{u(n)}{2} \left( e^{(\omega_m n + \phi)} + e^{-(\omega_m n + \phi)} \right). \tag{43}
\]

Then taking the Fourier transform of both sides gives

\[
S(e^{j\omega}) = \frac{1}{2}(U(e^{j(\omega-\omega_m-\phi)}) + U(e^{j(\omega+\omega_m+\phi)})). \tag{44}
\]

We will use the convention that the capitalized signals, \( S(e^{j\omega}) \), \( U(e^{j(\omega)}) \), denote the Fourier transforms of \( s(n) \) and \( u(n) \). To isolate \( U(e^{j\omega}) \), we must cancel out negative frequencies present in the spectrum, and shift the half spectrum to center about \( \omega = 0 \). Consider first the Hilbert transformer, a linear time-invariant filter which has unit magnitude frequency response and phase response equal to \( -\pi/2 \), given by

\[
H(e^{j\omega}) = \begin{cases} -j, & 0 \leq \omega < \pi; \\
0, & -\pi \leq \omega < 0. \end{cases} \tag{45}
\]

The output of the Hilbert transformer is a 90 phase shifted version of \( s(n) \). Hence we apply the Hilbert transformer to (44) in the following way
to obtain the one sided spectrum, denoted in [38] as \( S_A(e^{j\omega}) \).

\[
S_A(e^{j\omega}) = S(e^{j\omega}) + jH(e^{j\omega})S(e^{j\omega}) \quad (46)
\]

\[
S_A(e^{j\omega}) = S(e^{j\omega}) + \tilde{S}(e^{j\omega}) \quad (47)
\]

Where we let \( \tilde{S}(e^{j\omega}) \) be defined as \( jH(e^{j\omega})S(e^{j\omega}) \). The result of this can be found to be

\[
S_A(e^{j\omega}) = \begin{cases} 
2S(e^{j\omega}), & 0 \leq \omega < \pi; \\
0, & -\pi \leq \omega < 0
\end{cases} \quad (48)
\]

\[
S_A(e^{j\omega}) = \begin{cases} 
U(e^{j\omega-\omega_m}), & 0 \leq \omega < \pi; \\
0, & -\pi \leq \omega < 0
\end{cases} \quad (49)
\]

Performing an inverse Fourier transform here, we get

\[ s_A(n) = u(n)e^{j\omega_m n}. \]

When we take the absolute value of \( s_A(n) \) in the equation above, we obtain

\[
u(n) = |s_A(n)| \quad (50)
\]

\[
u(n) = |s(n) + i\tilde{s}(n)| \quad (51)
\]

\[
u(n) = \sqrt{s^2(n) + \tilde{s}^2(n)}. \quad (52)
\]

In which we have used (43) in the last equality to remove the dependence on \( \omega \) and \( \phi \). Hence, we have isolated the desired envelope function \( u(n) \). This method, however, poses problems in implementation in discrete time. Since the impulse response of the Hilbert transformer is infinite, given by

\[
h(n) = \begin{cases} 
\frac{2}{\pi} \frac{\sin(\pi n/2)}{n}, & n \neq 0; \\
0, & n = 0,
\end{cases} \quad (53)
\]

it is impossible to use this filter without truncation or windowing. Instead of doing either of those things, we prefer to use the simple 'city block' approximation to (43), appropriate in this case only because it is computationally simpler. Note, it is the mathematical equivalent of adding the lengths of two sides of a right angle triangle instead of calculating the length of its hypotenuse from the knowledge of the length of both sides. That being said, we can expect the approximation to be quite a bit larger than the output of a Hilbert-transformed version.

\[
\sqrt{s^2(n) + \tilde{s}^2(n)} \approx |s(n)| + |\tilde{s}(n)|. \quad (54)
\]
Which, after truncating the Hilbert transformer of equation (53) to length 3, yields the approximate envelope

$$\hat{u}(n) = s(n) + \frac{2}{\pi}|s(n + 1) - s(n - 1)|.$$  

(55)

The truncation of this filter introduces some spurious activity in the signal. This can be removed from $u(n)$ with the same lowpass filter used in the preprocessing stage. We call the output of the rectification stage $v(n)$.

5.3 Part 3: The Decision Rule

Once all the filtering has been completed, we would like to define a rule which decides where peaks should be detected. This can be a simple thresholding operation, registering peaks which fall above a certain preset noise value, or it can be an operation with memory. The threshold detection system we propose here is an algorithm designed to catch peaks with

- Amplitude exceeding a certain base value $\eta$ believed to be on or just below the ambient noise level.
- Width greater than $\alpha$ observations, that is, presence over the threshold value $\eta$ for more than $\alpha$ consecutive values.
- Peaks that are outside the refractory period, or blanking period $\beta$, of a neighboring detected peak of higher amplitude.

We shall see the decision rule in action in the example section. What makes this algorithm design more sophisticated than a simple threshold is that the width and refractory parameters $\alpha$ and $\beta$ require that the waveform have a particular duration and morphology in order to be detected. The threshold can be visualized as a sort of step function that steps up to the value at the top of a detected peak, lingers for $\beta$ observations, and then returns to a baseline threshold value $\eta$ until a new detection occurs. We will see in the following sections that these parameters are to be set by the evaluation of their performance in practice. One would expect that $\beta$ should be fixed because the refractory period of a cell is fixed. We will, however, regard $\beta$ as a variable in this model, since the refractory period of each individual cell in the heart can vary quite immensely from cell to cell, and we expect that $\beta$ should act as a sort of global refractory period of the atrial tissue.
In addition, the threshold detector is designed to check after a detection whether a peak with higher amplitude occurs during its refractory period. In this case, the peak with the higher amplitude is chosen.

6 Examples

The detection scheme is best illustrated with a few examples. We treat them with the tools developed in the last few sections, and we give some examples with different width and refractory settings.

6.1 Example 1 - A typical (non fractionated) signal

Consider the uppermost signal in Figure 15. It registers as consistently fractionated, as the EnSite system software has distinguished the site where it was taken as a target for ablation, with CFE mean cycle length as 49.55ms. In the lower two graphs we see the output of the preprocessor, \( z(n) \), and the rectifier, \( v(n) \). We see that the peaks in the signal \( z(n) \) are much enhanced by way of the nonlinear transformation, and furthermore, small ringing artefacts have been smoothed by the linear filter. The envelope \( u(n) \) retains all of these peaks, only in a new, positively oriented morphology. The envelope too has been smoothed by the same lowpass filter we used in the preprocessing stage to give \( v(n) \) shown in the bottom graph.

The next stage, the detector itself, requires some tinkering. As mentioned in the last section, we need to choose three parameters: \( \eta \), the base cutoff amplitude, \( \alpha \), the width setting, and \( \beta \), the refractory period. The first parameter, \( \eta \), can be small as we expect little noise at this stage, and noise appearing above the threshold can be additionally tested for adequate width and refractory. We have chosen \( \eta = 0.02 \). In this way, we can also allow the parameters \( \alpha \) and \( \beta \) to be small. In Figure 16 we see three different plots of the signal \( v(n) \) overlaid with the threshold signal \( f \) plotted in magenta. The blue circles correspond to the detections made.

The mean cycle length calculated by the algorithm is also given at the top of each plot. As we can see, for higher width and refractory values, fewer detections are made, corresponding to a higher mean cycle length. The top plot gives the thresholding and detection procedure for width and refractory values \( \alpha = 20\text{ms} \) and \( \beta = 40\text{ms} \). These are suggested default settings for the EnSite software detection system. As we can see, the mean cycle length
6 EXAMPLES

Figure 15: Signal refinement of a non fractionated process

registered by this algorithm is much higher than that given by EnSite, and we get a much more conservative estimate. Decreasing the parameters $\alpha$ and $\beta$ in the next two plots lets in more detections and brings down the mean cycle length statistic.

6.2 Example 2 - A continuously fractionated signal

For contrast with example 1 we will give a signal that is continuously fractionated, one for which Ensite has calculated a mean cycle length of 37.64ms. This signal is given in Figure 17. Consider in the top graph the original signal. It is easily seen to correspond to a rapid rate of atrial activation. In the second graph, the preprocessed signal shows a reduction in the amount of noise present in the baseline. In the final graph we see the prepared signal for threshold detection in which the peaks have much more definition.

After cleaning the signal, we prepare the thresholding process. In Figure 18, we see the results. In the top graph the parameters $\alpha$, $\beta$, and $\eta$ have been set so that width and refractory are 20 and 40 ms, respectively, and the threshold is 0.02. We obtain a mean CL of 124.19ms. In the bottom graph, when the parameters have been changed so that width and refractory are 10
6.3 Example 3 - An alternating signal

In an episode of AF, often a signal may alternate between periods of quiet and of fractionated activation. This is illustrated by the signal trace in Figure 19. The entire four second segment is shown to give a more global look at the activation. In the bottom two signals, we proceed to clean the data as in the algorithm described. EnSite has calculated a mean cycle length of 110.76ms for this signal - just barely fractionated.

Now, setting the detector’s width and refractory settings back to 20ms and 40ms, respectively, and the baseline threshold at $\eta = 0.02$, we obtain
detected which look like that in the upper part of Figure 20. The second plot has $\eta = 0.04$ with width and refractory held constant. It appears we can lessen the detection of spurious events by increasing $\eta$ in this case.

In the first and second cases respectively, we get a mean cycle length of 124.76ms and 184.28ms. Neither of which statistic would register the signal as fractionated - though in some parts it is clear that it is, where in other patches the signal is relatively quiet.

What is most interesting about this is that though this signal is clearly fractionated in places, the whole signal does not come in with mean CL below 120ms. This is the underlying weakness of the mean cycle length statistic - that it does not differentiate between continuously non-fractionated signals versus signals which alternate between periods of quiet and fractionation. It appears there is more to the signal than its mean cycle length. A statistic that may characterize the difference between continuously fractionated signals and those in this example is that of percent fractionation - for what percentage of the time is the signal fractionated over the entire segment? This question will come up later.
6.4 Example 4 - Signal with high peaks

This example is an anomaly in that the signal itself behaves oddly. The top graph in Figure 21 shows a bizarre signal morphology: extreme peaks with voltage higher than we have seen in the previous examples, with barely any activity in between except for a few accompanying harmonics visible behind each of the wavefronts. The EnSite registered CFE mean is 282.95ms. The objective with this example is to make sure that peaks are not going over-detected.

Figure 21 shows the filtering and priming of the signal for threshold detection. If we set width and refractory parameters at the prescribed 20ms and 40ms levels, with baseline threshold of 0.02, we obtain the threshold given in the upper graph of Figure 22.

The second plot in Figure 22 helps to correct for the over-detection we see in the upper plot. By resetting the baseline threshold from 0.02 to 0.05, we eliminate the detection of some small (perhaps irrelevant) peaks. It is also worth mentioning that this signal, in any case, would not be a target for catheter ablation because its cycle length is well above 120ms.
6.5 Example 5 - A case of Baseline Wander

An interesting trend in the results of the detection scheme is that it tends to under-detect peaks in signals in which baseline wander is present. For example, consider the signals plotted in Figure 23. The CFE mean of each of the signals, from top to bottom of the figure, as presented by the EnSite software system, are 243.64ms, 246.67ms, 333.86ms, and 348.70ms.

The striking thing about these cases is that it is difficult to tell if fractionation would be present were the baseline wander removed. We run our algorithm on each of the signals with the appropriate subtraction of the median value from the signal. Figure 24 shows the result of our detection scheme on each of the signals presented in Figure 23, in the same order. In these we have set the parameters as follows $\alpha = 20\text{ms}$, $\beta = 40\text{ms}$, and $\eta = 0.03$.

It is unclear whether a threshold detector would do better on these baseline wander example signals with the preprocessing steps used by our algorithm. We still can see remnants of the baseline wandering through the preprocessed signals, and so arises the question: How does one go about properly removing the baseline wander in real time? As mentioned before, the algorithm we use is one operating offline, so we subtract the signal me-
dian before any filtering or preprocessing takes place to collect the signal about zero. However, this will rarely be the case while operating in real time. In this case, we may choose to subtract a running median from the data. That is, before preprocessing we may subtract from the signal, a causal median-smoothed version of the signal itself with a high degree of memory.

Baseline wander is an extremely interesting anomaly. Techniques for its removal can be quite intricate. This is a subject for the next section.

6.6 Analysis

In this section, we have looked at the basic results of the detector. We have discussed its behaviour with various types of data from continuously fractionated to mixed form signals, to high peaked signals to baseline wander. Of most interest to us in these examples was the proper setting of the parameters $\alpha$, $\beta$, and $\eta$. We would like to develop better techniques to deal with the more erratic or interesting data.

It also remains to be tested whether or not this detection scheme is of use in a clinical setting. For this, it is necessary that this routine be run on signals annotated by professional readers. Its ability to come significantly close to the readings given by human eyes will determine its relative success.
7 Meaningful Statistics

7.1 Baseline Wander

We saw in the last example of Section 6 that any threshold detection system will work surprisingly badly on electrocardiograms with baseline wander. In this section, we provide the simplest possible approach for its removal. Recall that in our algorithm, as we were operating offline, we were able to take the median value of the entire ECG segment and subtract it from the series. This is not appropriate for a detector operating in real time; so we propose a simple, realizable median smoother. A median smoother, depending on the length $L$ we choose, will approximate the baseline. Subtracting this smoothed series from the data will collect the series about zero, giving a signal with wander absent. Let the wandering ECG of interest be denoted by the series \( \{x(n)\}_{n=0}^{N-1} \), and the median smoothed series be \( \{y(n)\}_{n=0}^{N-1} \), defined by

\[
y(k) = \begin{cases} 
\text{median}\{x(k - L), x(k - L + 1), \ldots, x(k)\} & \text{for } N - 1 \geq k > L \\
\text{median} \{x(0), x(1), \ldots, x(L)\} & \text{for } k = 1, 2, \ldots, L 
\end{cases} 
\]  

(56)

Then the series of interest, denoted \( x'(n) \), will be given by
50

Figure 22: The choice of a proper threshold value

\[ x'(n) = x(n) - y(n), \]

for \( n = 0, 1, \ldots, N - 1. \)

To illustrate this method’s efficacy, let us consider the examples from the last section; refer to Figure 23. In Figure 25 we see the median smoother’s result with parameter \( L \) set at 100ms (120 observations.)

In the figure we see the interesting result that the 100ms baseline wander smoother works quite well for the first and third of the ECGs, but it performs quite badly on the second and fourth. One option would be to lengthen \( L \) until its approximation were ‘sufficiently fuzzy’ to give a good picture of the baseline. This, however, lowers the smoother’s realizability. The more observations the smoother has to take into account, the greater the delay before the smoother output can be given. That is, since the first \( L \) of the sequence \( y(n) \) are given by the median of the first \( L \) observations, we must wait \( L/1.2 \)ms before the first \( L \) of \( y(n) \) can be given. This delay may be unacceptable in some circumstances.

In Figure 26 we see an example where \( L \) is set to give the median of 500ms of the signal. This is an example where the length is clearly too long and the median statistic lags behind the baseline of the original signal. As we can see, there is a tradeoff between length and lag when choosing \( L \).

An additional downfall to the median smoother approach is that it may remove more than just baseline, at small lengths, picking up true cardiac signals. When this is subtracted from the original sequence, true peaks may be removed. This is a genuine concern for us. For these reasons, we do not
recommend the smoother with memory. It seems that the most reliable approach we have seen is the one we set out with - subtraction of the median of the entire sequence from the original signal. Indeed, this median value will not be available until the whole segment has been read, but this approach may not be entirely impractical. The creation of a CFE map can be a long process, and some delay following the recording of a segment may be acceptable.

The subtraction of the sample median is also the simplest of the methods we have seen, and this in itself is an advantage. There exist more interesting and complicated methods for wander removal, for example, linear time-invariant filtering, cubic spline fitting, and polynomial curve fitting to the data (introduced in [38].) But these methods are difficult to implement in real time. Though the it is minority of signal traces that actually have baseline wander, it poses a hazard even so. It is irksome that threshold detection should be so erratic with cases of baseline wander, and it is the opinion of the author that under no circumstances should wander be ignored; even if the only step to prevent it is the subtraction of the median value.
7.2 The Mean Cycle Length statistic

It is questionable whether the mean cycle length statistic contains enough information about the fractionation of the signal. It has been suggested by health professionals that more information is needed. We saw in Example 3 of section 5 that it is possible that a signal will alternate between periods of fractionation and periods of quiet. It is for this reason that it may be more useful to characterize the amount of time (in percent of the observation time length) over which the signal is fractionated. It is not difficult to calculate this.

7.2.1 Percent Fractionation

To justify calculating the amount of time for which a signal is fractionated, we must take into account the mechanism of AF. A signal may appear fractionated for a short time, quiet, and then fractionated again because the wavefront of a passing spiral rotor is affecting it. A measure of percent fractionation might give some indication as to the proximity of foci or rotors from...
Figure 25: Baseline wander smoothed results for $L = 120$ observations

the observation site. This information is clearly valuable, and is not given by a CFE mean cycle length statistic. A percent fractionation statistic would also be easily visualized on a colour-coded 3D map. Clinicians hope that this information will help give depth to the limited amount of information that a mean cycle length statistic offers.

Calculation of a percent fractionation statistic may be done after the analysis done in the previous section. Given a signal $x(n)$, and a set of occurrence times $\Omega = \{\omega_0, \omega_1, \ldots\}$ of atrial deflections, we would like to design an algorithm which will find the time between each deflection and the one after it, and for each of the cycle lengths measuring below 120ms, will register fractionation for the amount of time for which it occurs. Then the algorithm will divide the amount of time for which the signal is fractionated by the total length of the signal ($4s_t$) and give this value as a percentage for that site.$^9$

$^9$An alternative definition of a CFE site requires that the mean cycle length of the
Figure 26: Smoothed signal with $L$ set to the equivalent of 500ms.

We do this by first calculating the number of observations between peaks detected. This gives a sequence of cycle lengths that we can plot as in the bottom plot of Figure 27 (this figure and its CFE mean were also given in example 1 of section 5.) The dotted line in the figure represents the 120ms cutoff.

We added the fractionated cycle lengths altogether to get a fractionation value of 58%. This may be surprising at first. We might expect a higher fractionation value, but this statistic does not mean that 58% of the cycle lengths are below 120ms, instead this is the percentage aggregate of the time between deflections when they occur less than 120ms apart. This perspective makes the percent fractionation of 58% make more common sense.

It is clear that we will have to relax this definition for the purposes of this algorithm. It is interesting that among papers read, the definition of a CFE site varies. Reference [25] requires that a 10s recording be taken, while [40] gives the same definition in which the 10s requirement is removed. We will proceed as in the definition given in [40].
8 Conclusions

8.1 Summary

Given data from studies done at the Kingston General Hospital, we were able to develop and test an algorithm for the detection of atrial deflections in the endocardial ECG. This algorithm was carefully designed to be implementable in real time using only causal, realizable digital filters and linear/nonlinear transformations. Using the sequence of occurrence times generated by our algorithm, we were able to calculate a mean cycle length statistic to characterize the atrial activation at various sites throughout the heart. This mean cycle length statistic can be used to colour-code a CFE map, and is a useful visualization for clinical practitioners.

8.2 Future Work

It is clear from our analysis that much more is left to be done with this data, in particular, we have expressed doubts with respect to the mean cycle length statistic. Does it contain enough information about the signal, or is it merely something we use since it can be easily presented on a colour coded map? We also need tools to qualify the spatio-temporal information present in the data. How correlated are neighboring points? What can mathematically be
said about the spatial organization of the atrial deflections over time? Could this information link to a mechanism of AF?

Answering these questions is a lofty goal for analysts. More realistically, there is much potential for the detection scheme presented in this paper. These next steps include determining meaningful statistics from the detections generated by our algorithm. There is an abundance of information related to QRS complex detection and monitoring of R-R interval lengths on surface ECG processing, that is, the analysis of heart rate variability (HRV.) HRV techniques have been applied with variable success, but they may yet be applicable to the study of endocardial signals. It is of considerable interest whether or not these techniques can be adapted and whether or not they will yield meaningful conclusions.

8.2.1 Spatial and Temporal Organization

The data given by the EnSite system is not used to its fullest extent. Each trace is analyzed independently of its neighboring traces, and this wastes significant spatio-temporal information present in the geometry of the heart. If the spatial information could be incorporated into the analysis it could lead to a better peak detection algorithm, more consistent/reproducible results, and it may indeed help to give insight into the mechanism of AF. If it were possible to visualize rotors propagating across the inside surface of the heart, it would be difficult to reject the mother rotor hypothesis for the mechanism of AF. Needless to say, this information is contained in data we already have. All we need is an arsenal of statistical methods to describe it.

8.3 The application of Heart Rate Variability techniques to endocardial signals

It remains to be seen whether analysis of the lengths between RR intervals in the surface ECG can be applied to the endocardial signals to produce meaningful results about the dispersion of the cycle lengths. Heart rate variability (HRV) techniques have been applied with success to surface ECGs taken over a long time frame. It requires an overview of spectral analysis which, surprisingly, has not yet been used in this paper. The simplest possible measure of cycle length dispersion is the standard deviation from the CFE mean; it is questionable whether more complicated techniques for determining dispersion could give a more meaningful result. The algorithm presented computes
I am interested to see whether or not HRV techniques could be used, even offline, to give quantitative evidence for the degeneration or improvement of AF. It is unlikely, however, that much meaningful information can be gleaned from a segment as short as four seconds. This will be interesting to explore.

8.4 Conclusion

Endocardial electrograms offer vast potential for time and frequency domain analysis techniques, digital signal processing, and nonlinear methods. As we have seen, the detection scheme developed in this paper can form merely a basis for the characterization of these signals and an understanding of the disease process.

We developed a paradigm for detecting atrial activations in real time. It involved the use of linear FIR filters and nonlinear transformations which primed the signal for threshold detection of peaks. Analysis of the properties of the detector revealed its strengths and weaknesses. Improving and building upon the measurements of the detector will be the next steps to the development of an automated analysis paradigm.

Clinically, little is known about AF and we have just begun to scratch the surface in developing techniques for its analysis. Doctors look to technology to help understand the signals registered in a more objective and quantitative way. The advantage of these systems is in its ability to interpret complex information quickly and to display it meaningfully and visually. We look forward to more sophisticated algorithms in the future to better explain the diagnostics and quantification of AF. This information will allow us to better understand AF and will help form a basis for new therapeutic techniques.
References


REFERENCES


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