SEARCH AND INFORMATION FRICTIONS IN DECENTRALIZED MARKETS

by

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Abstract

This thesis studies the importance and implications of information asymmetry in decentralized markets with search frictions. The first chapter provides an introduction and literature review. In the next chapter, I propose a model of the housing market using a search framework in which sellers are unable to commit to asking prices announced ex ante. Relaxing the commitment assumption prevents sellers from using price posting as a signalling device to direct buyers’ search. Adverse selection and inefficient entry on the demand side then contribute to housing market illiquidity. Real estate agents that can facilitate the search process can segment the market and alleviate information frictions. In Chapter 3, I further study the importance and implications of the commitment assumptions embedded in directed search models. I eliminate commitment to take-it-or-leave-it trading mechanisms in a model of the labour market with worker heterogeneity and a matching process that allows for multiple firms to match with a single worker. When workers and firms cannot commit to ex ante offers, to an allocation rule, or to an ex post bargaining strategy, the equilibrium is necessarily inefficient. This is true for a broad class of protocols for wage determination, of which bilateral bargaining and Bertrand competition are special cases. Finally, Chapter 4 presents a theory of land market activity for settings where there is uncertainty and private information about the security of land tenure.
sellers match with buyers in a competitive search environment, and an illiquid land market emerges as a screening mechanism. The implications of the theory are tested using household level data from Indonesia. As predicted, formally titled land is more liquid than untitled land in the sense that ownership rights are more readily transferable.
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Chapter 1

Introduction

1.1 Motivation

Search frictions are relevant in a large set of markets: housing and land markets, labour markets, and markets for physical goods and financial securities. It takes time and resources for buyers and sellers, firms and workers, or borrowers and lenders to match with an appropriate trading partner and determine the terms of a transaction. Search-theoretic frameworks have been developed to study trading frictions in various markets and these models have met with some quantitative success in helping to understand how these markets function. Often times, however, the search and matching process in these decentralized economies leads to situations with unobservable heterogeneity and problems of hidden information. In labour markets, for instance, the pre-match announcements (job listings from employers, application cover letters from workers, etc.) contain inadequate and presumably misleading information. In real estate markets, there are good reasons to suspect that some idiosyncratic characteristics
of the seller or the property are unobservable to a potential buyer. These possibilities introduce new modelling complications related to the implications of asymmetric information on search strategies and the determination of terms of trade. This thesis consists of three essays that study efficiency and liquidity in markets with both search and information frictions.

The next chapter develops a model of the housing market using a search framework with asymmetric information in which sellers are unable to commit to asking prices announced ex ante. Relaxing the commitment assumption prevents sellers from using price posting as a signalling device to direct buyers’ search. Consequently, housing is illiquid due to adverse selection and inefficient entry or insufficient search effort. Since real estate brokerage is common, I study whether introducing agents as service providers that can help alleviate the burden of the search process can segment the market and mitigate the information frictions. By modelling the listing contract between a seller and her agent I find that, in some circumstances, real estate agents can offer incentive compatible contracts to segment the market by seller type. This resolves the information problem and increases liquidity in the housing market.

Chapter 3 further studies the efficiency implications of relaxing commitment in a directed search theoretic model, this time in the context of the labour market. I propose a framework without commitment to pre-match announcements or post-match negotiation strategies and assignment rules. When firms match with workers, the wage is determined via a multilateral bargaining process wherein bargaining strategies do not necessitate or permit commitment to an offer as a means of influencing the assignment rule (i.e, the determination of which firm ultimately recruits the worker). I show that multilateral matching and bargaining can hinder market separation when
the terms of trade are sufficiently sensitive to the number of firms in a match.

In Chapter 4, I construct a theory of land market activity in settings where land tenure is insecure. I argue that uncertainty and private information about the security of property rights over land can help explain the widely varied volumes of transactions across developing country land markets. Land sellers match with buyers in a competitive search environment, and an illiquid land market emerges as a screening mechanism. The implications of the theory are tested using difference-in-difference and probit estimation of household level data from Indonesia. As predicted, ownership rights of formally titled land are more readily transferable. This is consistent with the theory given that possession of a legal land certificate improves ownership security, and access to a land registry reduces the asymmetry of information.

1.2 Literature Review

This thesis is related to the directed search literature. The seminal papers focus on environments with full information, and with commitment to posted prices [65, 60, 66]. Other directed search models use different pricing protocols, but maintain the assumption of full information. For example, Julien et al. have a series of papers [45, 46, 47] that study the implications on wage dispersion of coordination frictions when workers’ wages are determined by an auction mechanism. More recently, several authors have extended directed search frameworks to settings with incomplete information. Guerrieri et al. [35] present a search environment with adverse selection and show that screening can at least partly alleviate the symptoms of private information in a competitive search environment when the uninformed party can commit to a take-it-or-leave-it trading mechanism. Delacroix and Shi [24] study a model with adverse
selection where sellers can post non-negotiable prices as a means of directing search, and also as a signal of the quality of their asset. This thesis contributes to this new branch of the literature by developing directed search frameworks for housing, land, and labour markets in settings with asymmetric information.

The directed search literature is founded on the notion that market participants can commit to take-it-or-leave-it trading mechanisms. In many decentralized markets these commitment assumptions are not trivially satisfied. Some recent papers explore the implications of relaxing the commitment assumption in directed search models. Camera and Selcuk [14] partly relax commitment by allowing the terms of trade to be “renegotiated” ex post, although they remain agnostic about the process through which renegotiation transpires. Kim [48] shows that non-binding messages can generate a partially separating equilibrium in a decentralized asset market when there is private information about the quality of the asset. Menzio [58] completely relaxes the commitment assumption in a model of the labour market in a search environment that restricts the matching process to bilateral meetings. He shows that equilibrium market (partial) separation is still possible when firms are heterogeneous. The next two chapters of this dissertation are closely related to this strand of the literature. In Chapter 2, the absence of commitment reduces housing liquidity when the method of price determination is modelled to reflect features that are commonly observed in North American residential real estate markets. In Chapter 3, I show that multilateral matching and price determination can hinder market separation when the terms of trade are sufficiently sensitive to the number of economic agents in the match.

Chapter 2 is related to the recent literature that applies search theory to model the housing market [76, 7, 50, 3, 25, 37]. My approach differs from these papers in
that I develop a process of price determination that reflects the following stylized facts: sometimes the terms of sale are determined through bilateral bargaining, other times the house is sold in an auction with multiple bidders. Moving away from Nash bargaining and non-negotiable price posting towards a setting that more closely resembles the pricing mechanism observed in North American real estate markets has important implications for housing liquidity and market efficiency. The model is perhaps closest to Albrecht et al. [4]. They also depart from benchmark search models and allow for multilateral matches with terms of trade determined through auctions. Their framework imposes commitment to sell when a buyer offers the list price. I demonstrate the importance of this type of assumption for achieving a fully separating equilibrium and constrained efficiency. In Canada and the U.S., there is no such commitment mechanism, at least in the form of a legal obligation associated with the list price that compels a seller to accept an offer. Nevertheless, there could be other market institutions, such as agency or intermediation, that have emerged to help cope with the inefficiencies.

The chapter on land markets in developing country contexts, Chapter 4, is related to a large literature on the importance of a well-defined and secure system of property rights over land. The literature has focused on several benefits of tenure security and well-functioning land markets: (i) the appropriate incentives for landowners to engage in long-term productivity enhancing investments [10, 12, 43, 21, 29]; (ii) the ability to use land as collateral, thus improving landowners’ access to credit [27, 69]; and (iii) the allocation of land to more productive cultivators [73, 20]. In contrast, this chapter focuses specifically on the role of land transferability in the efficient allocation of workers between agricultural and off-farm activities. In less developed
countries, infrequent land transfers are often accomplished through inheritance and reallocation by village leaders. As non-agricultural sectors start to emerge and population densities increase, so does the need for land sales markets or rental transactions. A few other papers acknowledge this important dimension of efficient land markets [77, 78, 51]. For example, Yang [77] uses a static household model to argue that the prohibition of farmland sales adversely affects the incentives for rural-urban migration, and Kung [51] estimates a significant relationship between the emergence of off-farm labour markets and land rental market activity in rural China. My search theoretic approach fills a gap in the literature by applying a framework that allows one to understand the mechanism by which tenure insecurity leads endogenously to illiquidity in the land market.

1.3 Organization of Thesis

The next chapter studies illiquid housing markets and the information role of real estate agents. Chapter 3 focuses on the issue of commitment in directed search models in the context of the labour market. Chapter 4 establishes the theoretical and empirical links between tenure security and land market activity. Chapter 5 concludes. All proofs are relegated to Appendices A, B, and C.
Chapter 2

Information, Commitment, and Separation in Illiquid Housing Markets

2.1 Introduction

In this chapter, I develop a search-theoretic model of the housing market that (i) employs a method of price determination that accounts for the strategic interaction between buyers and sellers; (ii) incorporates the documented heterogeneity in seller motivation and asymmetry of information; and (iii) provides insight about the role of real estate agents and intuition for the seemingly puzzling structure of listing contracts. I first show that satisfying the first two requirements leads to an equilibrium with adverse selection and inefficient entry of buyers. I then focus on the potential role of real estate agents in overcoming information frictions and improving market efficiency.
Extensive empirical work has established several stylized facts about housing market prices and selling times.\textsuperscript{1} The correlation between prices and liquidity and the observed price dispersion in housing markets point to search theory as an appropriate modelling technique. While existing search models of the housing market can account for a wide range of the empirical trends, I argue that off-the-shelf search frameworks are not consistent with casual observations of the real estate market. For instance, some of these models do not allow for multiple offers by competing bidders, while others ignore the possibility of renegotiating offers announced ex ante when there are ex post incentives to do so. I show that accounting for these phenomena in the pricing protocol of a search and matching model has implications for liquidity and efficiency, and introduces the informational role of agency in illiquid markets.

There is good reason to suspect that sellers of identical houses differ in terms of their reservation price. Glower et al. \cite{33} conduct a survey of home sellers and find substantial heterogeneity in terms of motivation to sell: some sellers have a strong desire to sell quickly, while other sellers are much more patient. A seller’s degree of patience can be a reflection of a job opportunity elsewhere or the seller’s arrangement to purchase her next home (i.e., the seller might have already bought a new home, and wants to sell the first home quickly in order to avoid double mortgage payments). Accordingly, I introduce heterogeneity on the seller side of the market to reflect differences in reservation values. Importantly, the seller’s willingness to sell is unobservable to the buyer. Market participants would benefit if this information could be credibly conveyed, for example, by means of list prices. In Canada and the U.S., there is no legal obligation associated with a list price that compels a seller to

\textsuperscript{1}See for example, Glower et al. \cite{33}, Merlo and Ortalo-Magné \cite{59}, Krainer \cite{50}, and Leung et al. \cite{53}.
accept an offer. The inability to commit to a list price prevents sellers from using price posting as a signalling device. Instead, patient sellers mimic impatient sellers in order to drive up the final sale price by increasing the probability of a bidding war. Consequently, illiquidity in the housing market is rendered more severe because of adverse selection and inefficient entry on the demand side.

I extend the model to include real estate agents as service providers that can alleviate the burden associated with the process of searching for a home. Agents help buyers find suitable properties and provide expert advice and marketing services to sellers. In North American housing markets, sellers typically pay the real estate commission fees, while much of agents’ efforts and services are aimed at facilitating home buying. By modelling the listing contract between a seller and her agent, I find that in some circumstances, real estate agents can offer incentive compatible contracts to segment the market by seller type. This alleviates the information problem and increases liquidity in the housing market. Even if real estate agents face the same information frictions as other market participants and possess no technological advantage in the matching process, incentive compatible listing contracts are implementable as long as housing is sufficiently illiquid; i.e., a house is not readily saleable due to search and information frictions.

In the theory, incentive compatibility does not rely on exogenously imposed assumptions on preferences or technologies to satisfy a Spence-Mirrlees sorting condition, since sellers need not benefit directly from real estate services and the cost is independent of a seller’s type. Instead, the housing market is characterized by a directed search environment in which real estate agents play the role of market makers as in Mortensen and Wright [63]. Designing a new real estate listing agreement
creates a new submarket in the search framework that can potentially attract sellers and buyers. Sellers respond differently to changes in the arrival rate of buyers, which in turn is related to the endogenous composition of sellers. Anxious sellers might be willing to over-invest in real estate services if it allows them to distinguish themselves from relaxed sellers and attract more potential buyers. Market separation is therefore the result of a sorting condition that arises endogenously because of the beliefs and equilibrium search strategies of buyers. These theoretical predictions are consistent with the recent empirical evidence of endogenous sorting and service differentiation between full-commission full-service realtors, and low-cost limited-service agents [9, 55, 38]: sellers represented by full-commission agents tend to exhibit characteristics consistent with high motivation to sell, and consequently experience shorter selling times and a higher probability of sale.

The next section presents the model of the housing market with heterogeneity in seller motivation but without real estate agents. A comparison of the market equilibrium with the constrained efficient allocation leads to a discussion of how information frictions give rise to housing illiquidity. Real estate agents are introduced in Section 2.3. Section 2.4 concludes.

2.2 The Model

There is a fixed measure $S$ of sellers, and a measure $B$ of buyers determined by free entry. Buyers pay a cost $\kappa_0$ to enter the market for housing and visit a home listed for sale. Buyers are homogeneous, and assign value $v$ to home ownership. Heterogeneity on the seller side reflects differences in willingness to sell. Consistent with the evidence documented by Glower et al. [33], some sellers are desperate to sell quickly, while other
sellers are more relaxed. In a dynamic setting, preferences over price and liquidity would reflect in the discount rate. In a static setting, heterogeneity in reservation values is sufficient for capturing this phenomenon. A fraction $\sigma_0$ of sellers are anxious or impatient sellers with a low reservation value, $c_A$. The remaining $1 - \sigma_0$ of sellers are relaxed/patient, with a high reservation value, $c_R \in (c_A, v)$. Differences in sellers’ willingness to sell is an important source of asymmetric information in the housing market, since reservation values are unobservable to buyers.

If a buyer meets a seller and a transaction takes place at price $p$, the payoff to the buyer is $v - p$, and the payoff to the seller is $p - c$, where $c \in \{c_A, c_R\}$ refers to the reservation value of the seller. Buyers are unable to coordinate their search activities, which generates both an unsold stock of housing and bidding wars in equilibrium. The matching process of buyers and sellers is governed by the urn-ball matching function. Let $\theta = B/S$ denote the ratio of buyers to sellers, or market tightness. The probability that a seller is matched with exactly $k$ buyers follows a Poisson distribution,

$$e^{-\theta} \cdot \frac{\theta^k}{k!}, \quad k = 0, 1, 2, ...$$

I depart from the price determination mechanisms typically used in off-the-shelf search models. Nash bargaining is inappropriate for modelling the interaction between

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2For instance, a seller moving to another city to start a new job is likely willing to sell at a low price if it means a shorter time on the market. On the other hand, a seller hoping to move to a different neighbourhood in the same town is more inclined to hold out for a higher sale price. The fact that most sellers are also buyers in the housing market is likely another source of heterogeneity in seller motivation. Some sellers might have already submitted offers to purchase another home. Illiquidity in the housing market means that they may either find themselves servicing two mortgages, or have the purchase fall through if it was a conditional-on-sale offer.

3These matching probabilities are calculated for a large market with $B, S \to \infty$ and $B/S = \theta$. Search frictions therefore arise because of a lack of coordination among buyers (see Burdett et al. [13]).
buyers and sellers in housing markets with multilateral matches (i.e., when several buyers visit the same house), especially in settings with private information. Price posting by sellers requires commitment, even though ex post there are incentives for sellers to allow buyers to bid the price up above the posted price. Instead, I propose a different mechanism to reflect these important dimensions of house price determination. In a bilateral match, the buyer negotiates directly with the seller, but if other buyers are interested in the same house, they bid competitively for the purchase.

2.2.1 Buyers’ Bidding Strategies

Consider a housing market characterized by the buyer-seller ratio \( \theta \), and the fraction of highly motivated sellers \( \sigma \). If a buyer is the only one to visit a particular house (a bilateral match), he is free to make an offer without worrying about competing bidders. In such cases, the buyer is a monopsonist, and makes a take-it-or-leave-it offer of either \( c_A \) or \( c_R \), whichever yields the highest expected payoff. If \( \sigma(v - c_A) > v - c_R \), there is a selection problem, and a monopsonist offers \( c_A \), knowing that if the seller is of type \( R \), the offer is rejected and there is no transaction. Otherwise, the monopsonist makes a safer offer of \( c_R \), and trade will occur regardless of the seller’s type. When more than one buyer arrives (a multilateral match), they compete for the house in a private value sealed bid auction.\(^4\) A potential buyer can observe the number of

\(^4\)The theoretical results in this paper are robust to perturbations of the process of price determination. For example, it is straightforward to show that the expected payoff functions are unaltered when buyers are permitted to submit bids with escalator clauses, or when sellers run simultaneous multiple round auctions. Incorporating a more sophisticated bilateral bargaining game instead of a take-it-or-leave-it offer, such as the one studied by Grossman and Perry [34] and used by Menzio [58], does not change the theoretical implications of the model.
competing bidders,\footnote{This assumption is consistent with a survey of recent home buyers, conducted by Genesove and Han [32]. A seller has a vested interest in disclosing this information, since the presence of other buyers bids up the price of her house. Sellers/real estate agents have strategic ways to credibly convey this information to competing bidders. For example, a home listing can specify a date and time when offers will be accepted and reviewed. This leads to a scenario with competing bidders in the same location at the same time, where buyers can condition their bidding strategy on the number of other buyers interested in the same house. Alternatively, sellers can inform potential buyers after the initial offer submission that there are \( k \) competing offers and provide opportunity to resubmit. Intermediation by real estate agents adhering to a code of ethics would prevent sellers from being untruthful about the existence of competing offers. Instead, permitting buyers to submit bids with escalator clauses would circumvent the issue of truthful disclosure regarding the participation of other bidders (see footnote 4).} so that buyers compete à la Bertrand and bid their valuation, \( v \). The seller randomly selects among the buyers, so that each bidder has an equal probability of purchasing the home.

### 2.2.2 Expected Payoffs and Free Entry

The expected payoff to the buyer is

\[
U(\sigma, \theta) = e^{-\theta} \max \{\sigma(v - c_A), v - c_R\} = \begin{cases} 
  e^{-\theta} \sigma(v - c_A) & \text{if } \sigma > \frac{v-c_R}{v-c_A} \\
  e^{-\theta} (v - c_R) & \text{if } \sigma \leq \frac{v-c_R}{v-c_A}
\end{cases}
\] (2.1)

This is just the payoff in the monopsony case, which occurs with probability \( e^{-\theta} \). The expected payoff in a multilateral match with \( k = 1, 2, \ldots \) other buyers is zero since the equilibrium bid is \( v \). Two cases arise because the cut-off for offering \( c_A \) in a bilateral match depends on the fraction of anxious sellers. The adverse selection problem must be severe before the buyer risks offering \( c_A \). In such cases, no transaction will occur if the seller happens to be the relaxed type, since the offer is below her reservation value, \( c_R \). The expected payoff function (2.1) and the free entry of buyers, \( U(\sigma, \theta) = \kappa_0 \), determine the equilibrium buyer-seller ratio, \( \theta \).
The expected payoff to a relaxed seller is

\[ V_R(\sigma, \theta) = e^{-\theta} \sum_{k=2}^{\infty} \frac{\theta^k}{k!} (v - c_R) = \left[ 1 - (1 + \theta)e^{-\theta} \right] (v - c_R) \]  

(2.2)

The final expression recognizes the McLaurin series of the exponential function. The simplicity of this expression arises because the payoff to a type \( R \) seller in a bilateral match is zero regardless of whether or not a transaction takes place. A motivated seller, on the other hand, has the following expected payoff:

\[ V_A(\sigma, \theta) = \left[ 1 - (1 + \theta)e^{-\theta} \right] (v - c_A) + \begin{cases} 0 & \text{if } \sigma > \frac{v-c_R}{v-c_A} \\ \theta e^{-\theta} (c_R - c_A) & \text{if } \sigma \leq \frac{v-c_R}{v-c_A} \end{cases} \]  

(2.3)

The last term reflects the positive surplus for a type \( A \) seller in a bilateral match whenever the buyer offers \( c_R > c_A \). Anxious sellers only benefit from the bilateral \( c_R - c_A \) bonus if \( \sigma \leq (v - c_R)/(v - c_A) \).

### 2.2.3 Full Information Benchmark

If sellers’ reservation values were observable, buyers could condition their search strategy and bilateral offers on the seller’s willingness to sell. The expected payoffs to sellers in a housing market with observable \( c_A \) and \( c_R \), according to (2.2) and (2.3), are

\[ V_A(1, \theta_A) = \left[ 1 - (1 + \theta_A)e^{-\theta_A} \right] (v - c_A) \]  

(2.4)

and

\[ V_R(0, \theta_R) = \left[ 1 - (1 + \theta_R)e^{-\theta_R} \right] (v - c_R) \]  

(2.5)
with \( \{\theta_A, \theta_R\} \) determined by the free entry conditions according to (2.1):

\[
U(1, \theta_A) = e^{-\theta_A}(v - c_A) = \kappa_0 \tag{2.6}
\]
\[
U(0, \theta_R) = e^{-\theta_R}(v - c_R) = \kappa_0 \tag{2.7}
\]

This full information separating equilibrium is constrained efficient. The pricing mechanism is efficient in the sense that a house is always transferred to the highest bidder, and no buyer-seller match leaves positive surplus on the table. Efficiency of the separating equilibrium further requires that \( \theta_R \) and \( \theta_A \) maximize social surplus.

To show that buyer entry is optimal, denote by \( \Pi_A \) the social surplus from putting a house on the market when the seller has reservation value \( c_A \). As long as one or more potential buyers show up, the surplus is \( v - c_A \).

\[
\Pi_A(\theta) = e^{-\theta} \sum_{k=1}^{\infty} \frac{\theta^k}{k!} (v - c_A) = (1 - e^{-\theta})(v - c_A) \tag{2.8}
\]

Define \( \Pi_R \) in the analogous manner for houses available for purchase from relaxed sellers. Constrained efficiency means the social planner is also subject to the same coordination frictions faced by market participants. Taking the measures of sellers as given, the social planner has only to choose the measures of buyers visiting sellers of each type to maximize total social surplus less entry costs. Equivalently, the social planner can choose \( \theta_A \) and \( \theta_R \) to maximize the average social surplus per house.

\[
\max_{\theta_A, \theta_R} \sigma_0 [\Pi_A(\theta_A) - \kappa_0 \theta_A] + (1 - \sigma_0) [\Pi_R(\theta_R) - \kappa_0 \theta_R] \tag{2.9}
\]
After substituting for $\Pi_A$ using the definition in equation (2.8) and likewise for $\Pi_R$, the first order conditions for the planner’s problem are

$$e^{-\theta_A}(v - c_A) = \kappa_0$$  \hspace{1cm} (2.10)

$$e^{-\theta_R}(v - c_R) = \kappa_0$$  \hspace{1cm} (2.11)

These are the same equations as the free entry conditions for buyers in the full information benchmark housing market, equations (2.6) and (2.7). When sellers’ reservation values are observable, the equilibrium free entry conditions imply that the arrival rates of buyers are efficient. The intuition for this result is as follows: Buyers are the ones paying the search cost, $\kappa_0$. With take-it-or-leave-it offers in bilateral matches, buyers are also the ones reaping the benefits of search. Finally, since house prices are bid higher in multilateral matches, buyers also bear the cost of congestion. Since buyers face undistorted incentives in searching for a house, their entry decisions are consistent with the solution to the constrained planner’s problem.

2.2.4 Equilibrium and Efficiency Under Asymmetric Information

In contrast to the full information equilibrium, the equilibrium of this model with unobservable reservation values is a random search equilibrium with both types of sellers attracting buyers in a single market. Equilibrium payoffs are given by (2.1), (2.2), and (2.3) with $\theta$ determined by a single free entry condition and the share of anxious sellers in the market equal to the aggregate fraction of motivated sellers, $\sigma_0$. The information problem generates illiquidity in the housing market due to adverse selection.
and inefficient entry. Figure 2.1 illustrates the liquidity of housing (as measured by the average probability of a transaction) in the housing market equilibrium relative to the full information benchmark in terms of the composition of sellers. When $\sigma_0$ is high ($\sigma_0 > (v - c_R)/(v - c_A)$), the adverse selection problem is severe in the sense that buyers make take-it-or-leave-it offers in bilateral matches that get rejected whenever the seller is less motivated to sell. Failure to trade in a match even when the surplus is positive reduces the number of transactions in the real estate market relative to the efficient allocation. Even when $\sigma_0$ is low ($\sigma_0 \leq (v - c_R)/(v - c_A)$), the private information about the seller’s motivation makes housing less liquid. When buyers offer $c_R > c_A$ in a bilateral match and their share of the surplus in a transaction with an impatient seller is reduced, fewer buyers find it worthwhile to participate in the housing market. This is an implication of the free entry condition.

![Figure 2.1: Housing liquidity in equilibrium relative to the full information benchmark.](image)

The full information equilibrium and solution to the social planner’s problem
establish that it is efficient for sellers with different reservation values to be distinguishable. With $c_A$ and $c_R$ unobservable, there could be efficiency gains associated with a mechanism that allows sellers to reveal their type. If sellers can differentiate themselves, buyers can direct their search. More buyers will visit the impatient sellers, knowing that a lower offer will be accepted in a bilateral match. Past studies have proposed the list price as a means of signalling private information [4, 24]. Menzio [58] shows that non-contractual messages in job listings can sometimes credibly convey information when wages are determined through bilateral bargaining. In my framework, the list price is not a credible signalling device: Type $R$ sellers will list their house at a low price, mimicking the type $A$ sellers in order to attract more buyers. This increases the probability that a bidding war will drive the selling price upward. Unlike in Menzio’s model of partially directed search, the process of price determination is not rigid enough to discourage such mimicking. In the event of a bilateral match, a type $R$ seller’s payoff is zero regardless of whether the buyer offers $c_R$ (leaving the seller with none of the surplus) or $c_A$ (in which case the seller simply rejects the offer). This result is stated formally in Proposition 2.2.1. All proofs are relegated to Appendix A.

**Proposition 2.2.1.** Suppose sellers can costlessly communicate with buyers through negotiable list prices. A correlation between the list price and the seller’s reservation value is unsustainable, and the equilibrium reduces to random search with uninformative list prices.

With the inability to commit to list prices, market separation violates incentive compatibility. The housing market equilibrium is inefficient, and housing units are
illiquid relative to the full information benchmark. Even with asymmetric information, however, the separating allocation is implementable by the social planner as long as the planner can commit not to alter the trading mechanism ex post. That is, the planner can design a mechanism to achieve market separation, increase social surplus, and circumvent both the welfare loss of unconsummated matches generated by the adverse selection problem and the inefficient entry resulting from information asymmetry. Implementing the separating allocation is accomplished, for example, using auctions with publicly observable and binding reserve bids. The planner therefore imposes a commitment to ex ante announcements which is absent in the market equilibrium. Submitting appropriate reserve bids is incentive compatible for sellers, and the endogenous arrival rates of buyers to sellers of either type are then efficient. These results are summarized in Proposition 2.2.2.

**Proposition 2.2.2.** Consider the following price-posting game: a seller sets a list price, and the planner sells the home by sealed bid auction using the posted price as an unsealed reserve bid. Then, sellers’ optimal list prices are \( \{p_A, p_R\} = \{c_A, c_R\} \), and buyers’ search and bidding strategies are identical to those in the full information benchmark. The constrained efficient allocation is therefore implementable even when reservation values are unobservable.

This is similar to the efficient equilibrium in Albrecht et al. [4], which imposes partial commitment to posted prices as part of the environment. In their housing market model, sellers are forced to sell whenever a buyer offers her asking price, even in the decentralized equilibrium. They suggest that the commitment to sell when a *bona fide* offer arrives could be part of the contract with a real estate agent, although real estate agents are not explicitly part of their model. In the next section,
I investigate whether agency can fulfil the role of a signalling mechanism in the housing market. I derive conditions that permit real estate agents to offer distinct incentive compatible listing agreements to segment the market, allow buyers to direct their search, and help overcome the problem of asymmetric information. It turns out that in some cases, the type of real estate contract that is often observed in housing markets is conducive to market separation.

2.3 Real Estate Agents

I add real estate agents to the model as a way of endogenizing $\kappa_0$: the buyer’s cost of searching for a house. Intuitively, real estate agents (REAs) have access to more detailed information about the characteristics of houses and the idiosyncratic preferences of prospective buyers. Acquiring and using this knowledge can reduce the informational burden of searching for a home. Detailed listings, databases of relevant real estate information, and advertisements are created to help guide buyers throughout the search process. In addition, REAs work with a sellers to showcase the features of a unit by decluttering, painting, repairing, renovating, decorating, and staging the home. Let $a \in [0, \infty)$ denote the level of services supplied by a REA, and let the search cost be a decreasing function of $a$, $\kappa : [0, \infty) \rightarrow [0, \kappa_0]$, with $\kappa(0) = \kappa_0$ and $\lim_{a \to \infty} \kappa(a) = 0$. Of course, providing services to decrease $\kappa$ is costly for the real estate agent. Let $\phi : [0, \infty) \rightarrow [0, \infty)$ be the cost function associated with supplying service level $a$. The cost function satisfies the following properties: $\phi(0) = 0$, $\phi'(a) > 0$ for all $a \in [0, \infty)$, and $\lim_{a \to \infty} \phi(a) = \infty$.

A REA offers a contract $(a, z) \in C$ to be accepted by a seller: $a$ is the extent of the REA’s marketing efforts, which can also be expressed in terms of $\kappa$ (the cost borne by
a buyer that searches among the houses listed with agents providing service level \(a\); \(z\) is the REA’s commission, expressed as an upfront non-refundable fee; and \(C = [0, \infty)^2\) is the set of all possible contracts. The flat fee assumption is made for tractability, and is sufficient for deriving results that are robust to changes in the structure of the REA’s commission. A fixed rate commission structure would better reflect the listing contracts commonly observed in residential real estate markets. Most REAs in large U.S. cities charge a commission rate between 5 and 7 percent of the sale price [42, 28].

I return to fixed rate contracts in Section 2.3.4 and show that features common in real world listing contracts are important for incentive compatibility. Assume that the market for REAs is frictionless and perfectly competitive. While this assumption may seem implausible given the allegations in the report by the Federal Trade Commission and U.S. Department of Justice [28], there is evidence that barriers to entry in the real estate brokerage industry are minute [8].

I study the equilibria of the following two stage game: in the first stage, REAs enter the housing market by posting contracts; in the second stage, sellers sort themselves by selecting a contract/REA, and buyers enter submarkets which are identifiable by the supply of real estate services, \(a\). When buyers match with sellers, they implement competitive bidding strategies to purchase the house. Equilibria are constructed by solving backward. An equilibrium of the second stage subgame takes as given the set of real estate contracts. This pins down the arrival rate of buyers and the expected number of sellers of each type attracted to a particular contract. In the first stage, REAs correctly anticipate the search behaviour of buyers and sellers in

---

6Most, but not all real estate brokers adopt fixed-rate fee structures. There appears to be an emergence of flat-fee, limited-service brokers in real estate markets [38, 55]. Moreover, even REAs with an ostensible fixed-rate commission structure will demand that most of it be paid as an upfront non-refundable fee, effectively transforming the contract into a flat-fee contract.
the second stage subgame. Taking as given the contracts posted by other agents, a REA enters the market and posts contract \((a, z)\) if it is profitable to do so.

Adding REAs to the model in this manner introduces several more layers of analytical complexity. A useful intermediate step is to imagine that the services provided by REAs are completely valueless but observable by other market participants. A straightforward way to impose such an environment is to set \(\kappa(a) = \kappa_0\) for all \(a\). Increasing \(a\) has no direct benefit to a potential buyer or the seller, but with \(a\) observable it becomes feasible for sellers to spend resources on REAs as a means of signalling their type. I proceed by investigating when even ineffective REAs play a role in the housing market. The intuition developed from the analytical results derived in this simpler environment carry through to the version of the model with \(\kappa'(a) < 0\).

**2.3.1 Real Estate Agents in an Environment with \(\kappa(a) = \kappa_0\)**

In this environment, the level of real estate services, \(a\), has no economic interpretation except that it can act as an observable market signal and affect beliefs about the buyer-seller ratio, \(\theta\), and the composition of sellers, \(\sigma\). The real estate market can be characterized by a directed search framework. REAs post contracts, effectively creating submarkets that can be distinguished by the observable real estate services, \(a\). Buyers and sellers then direct their search to the different submarkets. What follows is a formal definition of the second stage equilibrium of the housing market model, taking as given a set of real estate contracts, \(\mathcal{C}_P\). Definition 2.3.1 already takes into account the optimal bidding strategies of buyers and the optimal accept/reject decisions of sellers and focuses instead on equilibrium search behaviour. Next, a definition of an equilibrium at the first stage determines the optimal set of contracts, \(\mathcal{C}_P\).
Definition. Given a set of real estate contracts $C_P$, a second stage equilibrium of the housing market is a distribution of buyers $\Gamma$ on $C$ with support $C_P$, buyer-seller ratios $\{\theta_a\}$, and compositions of sellers $\{\sigma_a\}$ across submarkets satisfying the following:

1. Buyers’ optimal entry: $U(\sigma_a, \theta_a) = \kappa_0$ for all $(a, z) \in C_P$.

2. Sellers’ optimal search:

   (i) If $\sigma_a > 0$ for some $(a, z) \in C_P$, then $V_A(z, \sigma_a, \theta_a) = \max_{(a', z') \in C_P} V_A(z', \sigma_a', \theta_a')$.

   (ii) If $\sigma_a < 1$ for some $(a, z) \in C_P$, then $V_R(z, \sigma_a, \theta_a) = \max_{(a', z') \in C_P} V_R(z', \sigma_a', \theta_a')$.

3. Market clearing:

   $$\int_{C_P} \frac{\sigma_a}{\theta_a} d\Gamma(a, z) = \sigma_0 S \quad \text{and} \quad \int_{C_P} \frac{1 - \sigma_a}{\theta_a} d\Gamma(a, z) = (1 - \sigma_0) S$$

The first two parts of Definition 2.3.1 specify optimal search behaviour on the part of buyers and sellers. For instance, 2(i) requires that anxious sellers do not enter a submarket unless it enables them to achieve their highest possible payoff. Part (ii) is the analogous requirement for type $R$ sellers. The final part of Definition 2.3.1 ensures that every seller enters a submarket.\(^7\)

What is missing from Definition 2.3.1 is the equilibrium behaviour of REAs. In any equilibrium, perfect competition and free entry in the market for REAs ensure that commission fees will be bid down to earn zero profit. Let $C_0$ denote the set of zero profit contracts:

$$C_0 = \{(a, z) \mid a \geq 0, \ z = \phi(a)\}$$

\(^7\)Definition 2.3.1 ignores the possibility that a REA posts a contract that attracts neither buyers nor sellers. An implicit assumption is that sellers find it worthwhile to list their house for sale in at least one of the submarkets.
The zero profit fee schedule result is stated formally in the following Lemma.

**Lemma 1.** With perfect competition and free entry in the market for REAs, every real estate contract posted in equilibrium must earn zero profit, \( C_P \subset C_0 \).

While Lemma 1 restricts the set of contracts that REAs can post in equilibrium, further restrictions are needed to characterize the set of zero profit equilibrium contracts. REAs play a market-making role, creating submarkets by constructing new listing agreements. An equilibrium set of contracts must be such that no other contract can be introduced to earn positive profit. This restriction requires specifying the beliefs about submarket tightness, \( \theta \), the composition of sellers, \( \sigma \), and the commission fee, \( z \), for real estate contracts that are not offered in equilibrium. An equilibrium at stage one is such that no REA can offer an out-of-equilibrium listing contract and earn a positive profit given the equilibrium behaviour of buyers and sellers in the stage two subgame. An equivalent characterization of equilibrium at stage one rules out a candidate set of contracts \( C_P \) if there exists a zero profit deviation that can improve the expected payoffs to sellers participating in the new submarket.\(^8\)

**Definition.** A stage one equilibrium in the housing market with REAs is a set of real estate contracts \( C_P \) with \((0,0) \in C_P\), a distribution of buyers \( \Gamma \) on \( C \) with support \( C_P \), a function \( \theta : [0, \infty) \to [0, \infty] \), and a function \( \sigma : [0, \infty) \to [0, 1] \) satisfying the following:

\(^8\)Equivalence follows from the following argument: a listing contract that attracts some sellers and makes them strictly better off can be restructured to divide the extra surplus between the seller and the agent. Inversely, if a profitable deviation is possible, the real estate agent could instead pass some of the surplus on to his clients. This equivalent characterization is applied here in order to avoid introducing extra notation for beliefs regarding submarkets with real estate contracts that earn strictly positive profit. The assumption is maintained that upon observing \( a \), a prospective buyer deduces that the commission charged to the seller is \( \phi(a) \).
1. REAs offer zero profit contracts: $\mathcal{C}_P \subset \mathcal{C}_0$; and $\{\Gamma, \theta, \sigma\}$ satisfy Definition 2.3.1 given the set of contracts $\mathcal{C}_P$.

2. Let $\{\nabla_A, \nabla_R\}$ denote a pair of seller values associated with an equilibrium:

$$
\nabla_A = \max_{(a, z) \in \mathcal{C}_P} V_A(z, \sigma(a), \theta(a)) \quad \text{and} \quad \nabla_R = \max_{(a, z) \in \mathcal{C}_P} V_R(z, \sigma(a), \theta(a))
$$

(2.13)

For any $(a', z') \in \mathcal{C}_0 \setminus \mathcal{C}_P$,

$$
V_A(z', \sigma(a'), \theta(a')) \leq \nabla_A \quad \text{and} \quad V_R(z', \sigma(a'), \theta(a')) \leq \nabla_R
$$

(2.14)

where $\{\Gamma, \sigma, \theta\}$ satisfy Definition 2.3.1 given $\mathcal{C}_P \cup (a', q')$.\footnote{For completeness, part 2 of Definition 2.3.1 should also require the following: If $V_A(z', \sigma(a'), \theta(a')) < 0$ and $\sigma(a') > 0$, then $\theta(a') = \infty$. If $V_R(z', \sigma(a'), \theta(a')) < 0$ and $\sigma(a') < 1$, then $\theta(a') = \infty$. This allows REAs to consider contracts that would not attract any sellers in stage two.}

First note that $(0, 0) \in \mathcal{C}_P$, which means that sellers always have the option not to hire a REA: the for-sale-by-owner option. Part 1 of the definition then states that the entry and search behaviour of buyers and sellers is a second stage equilibrium given the posted set of zero profit listing agreements. Part 2 states that no out-of-equilibrium contract can benefit sellers. This requires beliefs about $\theta$ and $\sigma$ for out-of-equilibrium submarkets to be consistent with the search behaviour of buyers and sellers in the subgame that includes the additional deviation under consideration. The resulting buyer-seller ratio, $\theta(a')$, has to be consistent with the free entry condition for buyers. Similarly, the resulting composition of sellers, $\sigma(a')$, must reflect the equilibrium search strategies of sellers following the posting of contract $(a', z') \in \mathcal{C}_0$. 

25
There is a local single-crossing property that can arise endogenously which introduces the possibility of signalling. Paying for ineffective real estate services is not a traditional sorting variable, as it directly affects both types of sellers in the identical manner. In other words, the REA technology does not satisfy a Spence-Mirrlees single crossing property for exogenous reasons. Instead, the endogenous composition of sellers and buyer-seller ratio can initiate sorting. To see this, consider a pooling equilibrium without real estate agents, and imagine a real estate agent deciding to enter the housing market and offer a listing agreement \((a, z)\) with zero profit commission \(z = \phi(a) > 0\). The payoff functions for sellers in the new submarket would be

\[
\begin{align*}
V_R(\phi(a), \sigma(a), \theta(a)) &= \left[1 - (1 + \theta(a))e^{-\theta(a)}\right](v - c_R) - \phi(a) \\
V_A(\phi(a), \sigma(a), \theta(a)) &= \left[1 - (1 + \theta(a))e^{-\theta(a)}\right](v - c_A) - \phi(a)
\end{align*}
\]

where I have assumed \(\sigma(a) > (v - c_R)/(v - c_A)\). Differentiating the payoff functions with respect to \(a\) yields

\[
\begin{align*}
\frac{dV_R}{da} &= \theta e^{-\theta}(v - c_R) \frac{d\theta}{da} - \frac{d\phi}{da} \\
\frac{dV_A}{da} &= \theta e^{-\theta}(v - c_A) \frac{d\theta}{da} - \frac{d\phi}{da}
\end{align*}
\]

Consider the following conceptual adjustment process after the new contract is introduced. Relaxed sellers have no signalling incentive and are initially uninterested in the new listing agreement. The real estate agent therefore expects anxious sellers to be the first to accept the listing agreement in an attempt to signal their type and attract a high number of buyers. With \(\sigma(a) = 1\) and a high buyer-seller ratio, relaxed sellers might thereafter find it worthwhile to mimic the anxious types by entering the hotter submarket and signing the new listing agreement. Type \(R\) sellers continue
to flow into the new submarket, and $\sigma(a)$ adjusts until relaxed sellers are indifferent between the two markets. The type $R$ indifference condition is

$$V_R(\phi(0), \sigma(0), \theta(0)) = [1 - (1 + \theta(0))e^{-\theta(0)}](v - c_R)$$

$$= [1 - (1 + \theta(a))e^{-\theta(a)}](v - c_R) - \phi(a) = V_R(\phi(a), \sigma(a), \theta(a))$$

(2.18)

Differentiation yields

$$\theta e^{-\theta}(v - c_R) \frac{d\theta}{da} - \frac{d\phi}{da} = 0$$

(2.19)

which can be substituted into (2.17) to obtain

$$\frac{dV_R}{da} = 0 \quad \text{and} \quad \frac{dV_A}{da} = \theta e^{-\theta}(c_R - c_A) \frac{d\theta}{da} = \left(\frac{c_R - c_A}{v - c_R}\right) \frac{d\phi}{da} > 0$$

(2.20)

Therefore, the endogenously determined composition of sellers and arrival rate of buyers generate a single crossing property: the expected payoff to a type $A$ seller is increasing in $a$, while type $R$ sellers remain indifferent between the two submarkets.

The piecewise nature of the payoff function for type $A$ sellers in (2.3) introduces another complication. If $\sigma_0 \leq (v-c_R)/(v-c_A)$, type $A$ sellers get a positive payoff even in a bilateral match because buyers are making cautious take-it-or-leave-it offers to ensure the purchase of a home regardless of the seller’s motivation. Thus, even though $V_A$ is locally increasing in $a$, they might still prefer the original pooling submarket because of the bilateral bonus. Even if $\sigma_0 > (v-c_R)/(v-c_A)$, a fully separating equilibrium might not be feasible. There are two offsetting effects. First, type $A$ sellers are attracted to a submarket with real estate fees because $\sigma$ is increasing in $a$ and therefore so is $\theta$. A higher buyer-seller ratio improves the likelihood of a multilateral
match and a payoff of \( v - c_A \). On the other hand, the bilateral bonus of \( c_R - c_A \) in a type \( R \) submarket is appealing to an anxious seller. A fully separating equilibrium is only achievable if the first effect dominates. This occurs whenever the buyer-seller ratios are sufficiently low (i.e., if housing is sufficiently illiquid) that the benefit from an increase in market tightness, \( \theta \), is large. When \( \theta \) is too high, the benefit of further increasing market tightness inadequately offsets the appeal of the bilateral bonus in the type \( R \) market. The parameter most directly (but inversely) related to market tightness is \( \kappa_0 \), the entry cost for buyers. When \( \kappa_0 \) is high, buyers are scarce and the potential benefit from signalling a high motivation to sell is sizeable. I proceed by characterizing the housing market equilibrium in terms of the parameters \( \kappa_0 \) and \( \sigma_0 \).

**Lemma 2.** Type \( R \) sellers select the for-sale-by-owner contract \((a_R, z_R) = (0, 0)\).

**Lemma 3.** A pair of fully separating submarkets with contracts \((a_R, z_R) = (0, 0)\) and \((a_A, z_A)\) is incentive feasible if and only if

\[
\kappa_0 \geq (v - c_A) \exp \left( \frac{c_A - c_R}{v - c_R} \right) \equiv \kappa
\]

where \((a_A, z_A)\) is the zero profit real estate contract that binds the type \( R \) incentive compatibility constraint for full separation, \( V_R(0, 0, \theta_R) = V_R(z_A, 1, \theta_A) \).

Lemmas 2 and 3 specify the necessary and sufficient conditions for which the for-sale-by-owner option \((a_R, z_R) = (0, 0)\) and the listing contract \((a_A, z_A)\) induce search behaviour by buyers and sellers that is consistent with a fully separating equilibrium. The final criterion for a stage one equilibrium is to determine the parameter restrictions under which no other real estate contract can generate better expected payoffs to sellers deviating to the new submarket. The deviation of interest is a full
pooling contract. The following Lemma characterizes the conditions necessary and sufficient for sellers to prefer a full pooling submarket over the pair of fully separating submarkets.

**Lemma 4.** Assume the parameters of the model satisfy (2.21) so that the pair of fully separating contracts is incentive feasible. A full pooling contract \((a_P, z_P) \to (0, 0)\) can increase the expected payoffs for both types of sellers (strict for at least one type) if and only if \(\kappa_0 \in [\kappa, \kappa(\sigma_0)]\) and \(\sigma_0 \in [\kappa^{-1}(\kappa), 1]\), where

\[
\kappa(\sigma_0) \equiv \exp \left( \frac{(v-c_A)[1+\log(v-c_A)]-\sigma_0(v-c_A)[1+\log(v-c_A)]-\sigma_0(c_R-c_A)[1+\log(v-c_A)]}{(1-\sigma_0)(v-c_A)-\sigma_0(c_R-c_A)} \right) \tag{2.22}
\]

When \(\kappa_0 \geq \kappa\), the single crossing property precludes a pooling equilibrium. If the conditions of Lemma 4 are satisfied, a pooling contract can nonetheless be welfare improving. This leads to the typical equilibrium non-existence problem as in Rothschild and Stiglitz [71]. Lemmas 2, 3, and 4 combine to form the necessary and sufficient conditions for a fully separating equilibrium in the housing market with REAs, which are stated in the following Proposition.

**Proposition 2.3.1.** The pair of incentive feasible contracts, \((a_R, z_R) = (0, 0)\) and \((a_A, z_A)\), constitute a fully separating equilibrium if and only if

\[
\kappa_0 \geq \begin{cases}
\kappa & \text{if } \sigma_0 < \kappa^{-1}(\kappa) \\
\kappa(\sigma_0) & \text{if } \sigma_0 \geq \kappa^{-1}(\kappa)
\end{cases} \tag{2.23}
\]

Proposition 2.3.1 is consistent with the intuition developed earlier. The ratio of buyers to sellers in the housing market must be low in order for anxious sellers to engage in costly signalling by accepting real estate agreements with positive commission.
fees. When the entry cost $\kappa_0$ is low, the buyer-seller ratios are sufficiently high that the benefit of signalling is not enough to provide anxious sellers with the incentive to give up the bilateral bonus. Proposition 2.3.1 also points to a relationship between the aggregate composition of sellers, $\sigma_0$, and the existence of a fully separating equilibrium. When most sellers are anxious to sell, the full pooling submarket closely resembles the separating type $A$ submarket: market tightness is high, and buyers make low offers of $c_A$ in the event of a bilateral match. Therefore, as the population of sellers becomes relatively homogeneous, it becomes harder to justify paying agency fees to achieve full market segmentation.

Proposition 2.3.1 is reminiscent of the endogenous market segmentation result in Fang [26]. In Fang’s paper, social culture is a seemingly irrelevant activity that can be used as an endogenous signalling device to partially overcome an information problem in the labour market. Here, if the parameters are conducive to separation, the hiring of irrelevant but costly real estate agents is used to signal type. Buyers form different beliefs about the composition of sellers in each separate submarket. Given these beliefs, anxious and relaxed sellers face different incentives to join a particular submarket. The advantage of listing a house with a costly REA is a higher arrival rate of buyers, which results in a higher probability of trade. Because sellers differ in their reservation values, $(a_A, z_A)$ can be carefully chosen by REAs so that relaxed sellers are just indifferent between the two submarkets, while anxious sellers strictly prefer the one with REAs. Embedding Fang’s result in a directed search framework with profit maximizing market makers thus rules out Pareto inferior signalling equilibria.

With parameters that violate (2.23), an incentive feasible contract $(a_A, z_A)$ can no longer be constructed. When $\sigma_0 > \bar{\pi}^{-1}(\kappa)$ and $\kappa_0 \in (\kappa, \bar{\pi}(\sigma_0))$, the entire group
of anxious sellers prefer to enter the submarket with $a = 0$, along with the relaxed sellers. It is of interest to know what happens in the housing market when condition (2.21) is violated (i.e., when $\kappa_0 < \kappa$). For example, under what parameter restrictions is there a full pooling equilibrium? Proposition 2.3.2 fills in the details, and Figure 2.2 provides a graphical representation.

**Proposition 2.3.2.** Suppose $\kappa_0 < \kappa$. Then,

1. if $\sigma_0 > \sigma$, the model has no equilibrium; and

2. if $\sigma_0 \leq \sigma$, there exists a full pooling equilibrium.

![Graphical characterization of the housing market equilibrium with REAs.](image)

Figure 2.2: Graphical characterization of the housing market equilibrium with REAs.

If $\sigma_0 > \sigma$ and $\kappa_0 < \kappa$, a pooling contract does not constitute an equilibrium because a deviating REA can offer a listing agreement with a positive commission to attract only the anxious sellers. Once the anxious sellers exit the pooling submarket, buyers alter their bidding strategy and offer $c_R$ instead of $c_A$ in a bilateral match.
This change in buyers’ behaviour affects the expected payoffs such that anxious sellers’ search behaviour is no longer optimal. This is the intuition behind the equilibrium non-existence problem in part 1 of Proposition 2.3.2. When $\sigma_0 \leq \sigma$, the share of anxious sellers is low enough that buyers cautiously offer $c_R$ in a bilateral match even in a pooling submarket in order to guarantee a successful home purchase. If $\kappa_0 < \kappa$, there is no deviation that will attract only the motivated sellers.

2.3.2 Real Estate Agents in an Environment with $\kappa'(a) < 0$

The intuition developed in the previous section is still relevant when the economic importance of real estate services, $a$, is derived from the monotonic relationship with $\kappa$, the buyer’s search cost. For notational convenience, the signalling role of real estate services $a$ and the direct economic benefit of decreasing $\kappa$ via $a$ can be collapsed by imagining that $\kappa$ itself is observable. One can therefore consider REA contracts of the form $(\kappa, z)$. Let $\psi(\kappa)$ denote the implicit cost function REAs face when supplying the level of service required to reduce the search cost from $\kappa_0$ to $\kappa$.

When $\kappa'(a) < 0$, the effect of REA services on $\kappa$ affects market tightness $\theta$ directly via the free entry condition, which then enters the sellers’ payoff functions. Unlike in the environment with $\kappa'(a) = 0$, the REA technology directly imposes a local sorting condition.\(^{10}\) Sellers hiring real estate agents are effectively paying for higher matching probabilities by subsidizing the entry of buyers. Anxious sellers, by their very nature, benefit relatively more from the marginal reduction in $\kappa$. Consequently, anxious sellers should tolerate higher commission fees in equilibrium for two reasons: they have a higher willingness to pay REAs to increase $\theta$ (i) directly by lowering buyers’

\(^{10}\)The sorting condition is local, and REA contracts do not automatically generate equilibrium market separation for the same reasons pooling and separation are both possibilities in Section 2.3.1.
search costs, and (ii) indirectly by affecting buyers’ beliefs about the composition of sellers. The fact that $\kappa$ is now a traditional sorting variable therefore reinforces the signalling role of REAs. This environment is an appropriate fit for the North American housing markets if the marketing efforts of a realtor do not directly warrant compensation between 5 and 7 percent of the price. Hiring a full service agent can still be worthwhile for motivated sellers because listing the house on the Multiple Listing Service (MLS) signals a high willingness to sell and generates additional visits from potential buyers.

This provides a theoretical foundation for the empirical results of Hendel et al. [38]. They compare housing market transactions on two different marketing platforms: the MLS and the newly established low cost FSBO Madison. They find that after controlling for observable house characteristics, the precommission sale prices are similar between the two platforms, but that homes listed with a traditional real estate broker have shorter times on the market and are more likely to ultimately result in a transaction. They also find evidence of endogenous sorting and report that impatient sellers are more likely to list with the high commission, high service option. These findings are consistent with the main theme of this paper. A higher buyer-seller ratio for houses listed on the MLS and the higher level of services provided by full-commission REAs lead to a higher probability of a sale.

Levitt and Syverson [55] similarly compare limited-service and full-service REAs. Time on the market is longer for houses sold with the assistance of less costly realtors, but sale prices are not significantly different. Bernheim and Meer [9] study Stanford Housing listings and find that sellers realize similar prices but sell less quickly when they select not to hire an agent. These empirical observations and the predictions
of the theory point to REAs and the MLS as primarily fulfilling a liquidity role in the housing market, rather than directly affecting the expected sale price of a home. In contrast, Johnson et al. [44] compare house prices when the seller’s REA decides not to advertise the listing with the MLS. They calculate the average sale price of a house to be more than 6 percent higher for homes that are marketed without the MLS, after controlling for the documented characteristics of homes. Unfortunately, they do not present any results related to transaction probabilities or time on the market. Nevertheless, their main finding is consistent with the idea that separate bundles of real estate services are offered to attract different types of sellers.

### 2.3.3 Constrained Efficiency with Real Estate Agents

With endogenous real estate services, the constrained planner chooses market tightness, $\theta$, and entry cost $\kappa$ to maximize the (per seller) social surplus:

$$\max_{\theta, \kappa} [1 - e^{-\theta}] (v - c) - \theta \kappa - \psi(\kappa)$$  \hspace{1cm} (2.24)

The first order conditions with respect to $\theta$ and $\kappa$ are

$$e^{-\theta} (v - c) = \kappa \hspace{1cm} (2.25)$$

$$\theta = \psi'(\kappa) \hspace{1cm} (2.26)$$

The first condition pins down the optimal buyer-seller ratio by equating the cost of entering the market, $\kappa$, with the marginal social surplus of having an additional buyer searching for a house. The additional condition stemming from the optimal choice of $\kappa$ equates the marginal benefit of real estate services for all buyers with the marginal
cost to REAs, $\psi'(\kappa)$. The efficient allocation with heterogeneous sellers is simply the separating allocation described in Section 2.2.4 with the additional restriction that $\kappa$ satisfy equation (2.26).

In a fully separating equilibrium with REAs, the type $R$ submarket achieves the efficient level of real estate services and efficient buyer entry. The type $A$ submarket, on the other hand, involves excess spending on REAs, which is the signalling cost required to induce efficient buyer entry. Full separation in an equilibrium with REAs is welfare improving, but does not achieve the solution to the constrained planner’s problem because of the inefficiencies required to make the type $A$ real estate contract incentive compatible. These welfare results contrast those of Albrecht et al. [4]. In their paper, partial commitment to the list price yields an efficient separating equilibrium without the efficiency loss from costly signalling. They conjecture that the contract between a seller and her agent leads to market segmentation. I have shown that while separation is possible under certain parameters, the first best allocation remains unattainable.

### 2.3.4 Fixed Rate Commissions

The analysis thus far deals with flat fee commissions charged by REAs. In practice, however, a fixed rate commission structure is more common: real estate contracts in North America typically specify a commission of 5 to 7 percent of the sale price [42, 28]. From a principal-agent perspective,\(^{11}\) a real estate fee that increases with

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\(^{11}\)Many theoretical models of real estate agents focus on the principal-agent relationship between the seller (the principal) and the realtor (the agent) [81, 5, 31]. Yavaş [79] and Yavaş and Yang [80] analyze the search effort of the real estate agent, while Arnold [6] considers the incentives for conveying truthful information about the conditions of the real estate market to aid with setting an appropriate list price. The attention of empiricists has also been aimed at the principal-agent problem in the market for real estate services. Levitt and Syverson [54] and Rutherford et al. [72]
the sale price is more likely to induce effort on the part of the real estate agent, whereas upfront non-refundable fees are least effective at motivating the agent. While I abstract from principal-agent matters in this paper, it is important to check the robustness of the results when listing contracts are modelled to reflect the type of contract commonly observed between a seller and her agent.

Restricting the analysis to fixed rate contracts introduces two additional effects that further hinder full market separation. First, when the commission is specified as a fraction of the sale price, a buyer has to increase his take-it-or-leave-it offer in a bilateral match until the seller deems it acceptable after real estate fees are deducted. More specifically, when the commission is \( z \) percent and the seller is willing to accept \( c \), the offer must be at least \( c/(1 - z) \). This reduces the payoff to a buyer in a bilateral match and implies that buyer entry is affected by the commission rate. Higher fees result in fewer buyers, which offsets the desired effect of attracting more buyers with REAs. Second, fixed rate contracts affect the incentive for relaxed sellers to mimic because payment to REAs is contingent on a transaction. To see why this is important, compute the expected real estate fee to be paid by an anxious seller in a type \( A \) submarket with commission rate \( z_A \):

\[
\theta_A e^{-\theta_A} \frac{z_A c_A}{1 - z_A} + \left[1 - (1 + \theta_A) e^{-\theta_A}\right] z_A v
\]  

(2.27)

The first term is the commission paid on the take-it-or-leave-it offer of \( c_A/(1 - z_A) \) in a bilateral match, and the second term is the commission paid when two or more buyers arrive. When a relaxed seller accepts the \((\kappa_A, z_A)\) contract and lists her home, find evidence to support the hypothesis that sellers’ and their agents’ incentives are misaligned by comparing the selling prices and duration on the market in transactions when the real estate agent is a third party and when the agent is also the owner of the home.
in the type $A$ submarket, the expected commission fee is only

$$\left[1 - (1 + \theta_A)e^{-\theta_A}\right] z_Av$$

(2.28)

An offer is rejected by a mimicker in a bilateral match since $c_R > c_A$, and the REA only collects the commission when two or more buyers visit a relaxed seller’s house in the anxious sellers’ submarket. Since both types of sellers receive zero payoff in a bilateral match, this does not affect the incentive compatibility constraint directly. Instead, the zero profit conditions in the market for REAs imply that mimicking sellers can essentially free ride on the commissions paid by anxious sellers. This makes the type $A$ submarket relatively more appealing compared to the type $R$ submarket, where relaxed sellers bear the full burden of real estate costs. The two effects just described work against incentive compatibility and full market separation. However, the analysis is not fundamentally altered when fixed rate contracts are imposed: it merely implies that a smaller parameter space generates a fully separating equilibrium.

Listing agreements typically specify a list price. What if the REA’s commission can be made contingent on procuring a “ready, willing, and able” buyer (i.e., contingent on receiving an offer at or above the list price)? This form of contract is often observed in North American real estate markets.\footnote{For example, a listing agreement with the Toronto Real Estate Board stipulates that “the Seller agrees to pay the Listing Brokerage a commission of ........% of the sale price of the Property or ........ for any valid offer to purchase the Property from any source whatsoever obtained during the Listing Period and on the terms and conditions set out in this Agreement.”} Even if the seller rejects an offer equal to or above the list price, it is considered that the REA has provided the agreed upon services and the seller must still pay the commission. The “ready, willing, and able” clause (hereinafter, the RWA clause) is useful for generating a separating...
equilibrium. This structure of real estate contract dissuades patient sellers from mimicking impatient ones and entering the market with the higher buyer-seller ratio. The contract introduces a cost to rejecting a take-it-or-leave-it offer in a bilateral match.

**Proposition 2.3.3.** Adding the list price and a RWA clause to the real estate contract tightens the incentive compatibility constraint for relaxed sellers. In other words, it becomes more costly for relaxed sellers to mimic, and hence less costly for anxious sellers to signal their type.

Consider a listing agreement designed for type A sellers with the list price $p_A = c_A/(1 - z_A)$. The RWA clause has no effect on type A sellers’ payoff since they are already willing to accept an offer of $c_A/(1 - z_A)$. Type R sellers, on the other hand, now pay a cost in a bilateral match if they choose to list their house at $p_A$. The extra cost to mimickers makes it easier for REAs to offer incentive compatible contracts that separate sellers by type. If list prices are determined strategically, it is possible that an anxious seller’s expected payoff can be further enhanced. While a list price in $[c_A, c_A/(1 - z_A))$ does adversely affect even anxious sellers, it might sting less than the direct cost of the agency fee. In other words, there is the possibility that simultaneously lowering $z_A$ and $p_A$ improves the expected payoff to type A sellers without attracting type R sellers. Thus, RWA clauses effectively mitigate both of the unfavourable incentive effects associated with fixed rate commissions.
2.4 Concluding Remarks

In this paper I present a model of the market for housing using a search framework that captures the realistic and strategic interaction between buyers and sellers in determining transaction prices. The model reflects differences in sellers’ willingness to make a sale. Private information about a seller’s motivation leads to an inefficient equilibrium with illiquid real estate. Some buyer-seller matches fail to result in a transaction despite the positive gains from trade. Reduced entry of buyers can further impact the volume of trade in the housing market. By introducing real estate agents into the model, there is a potential for housing market segmentation to alleviate the information problem and increase housing market efficiency. When the adverse selection problem is too severe, or the equilibrium buyer-seller ratio is too high, market segmentation can break down and situations arise wherein patient sellers mimic impatient sellers in order to drive up the final sale price by increasing the probability of a bidding war.

The model can qualitatively account for many of the observed realtor facts in residential real estate markets. For instance, 88 percent of home sellers choose to enlist the services of a real estate agent according to a 2010 survey conducted by the National Association of Realtors. This percentage has remained high in recent years, despite evidence suggesting that the value of the services provided by real estate agents, measured in terms of transaction prices and time on the market, is not enough to justify a high commission rate between five and seven percent. With seller heterogeneity and incomplete information, the theory sheds light on the demand for real estate services. Realtors not only provide valuable marketing/matching services, but can also structure their contracts in a way that offers a potential solution to the
adverse selection problem. A motivated seller can select a particular listing agreement as a means of signalling a high willingness to sell, thus attracting more buyers. In fact, I show that the demand for agency can, in some circumstances, be maintained even when the services offered by agents provide no direct benefit to buyers or sellers.

The listing agreement between a seller and her real estate agent typically specifies the commission as a fixed percentage of the sale price, and outlines the broker’s right to earn a commission when a suitable buyer is found. In the real estate agent literature, the listing contract is often studied from the perspective of a principal-agent relationship. Many have noted the apparent inefficiency in applying a fee structure that fails to completely align the incentives of the seller and the agent. A better commission structure in a simple (linear) model of incentive compensation would require a lower intercept and a steeper slope. Nevertheless, real estate contracts with a constant percentage commission have persisted for decades. In this paper, I analyze the listing contract from a different perspective and offer an alternative explanation for the structure of real estate contracts. I highlight the “ready, willing, and able” clause as a mechanism to induce sellers to truthfully reveal their willingness to sell. Interestingly, the clause would be less effective if the fee structure was altered according to the solution to an agency problem.
Chapter 3

Commitment to Offers and Trading Mechanisms in Directed Search Models

3.1 Introduction

There are typically assumptions embedded in a directed search environment that allow market participants to commit to offers or take-it-or-leave-it trading mechanisms. Often times, this takes the form of non-negotiable wage or price posting, where firms/workers or buyers/sellers can commit to the terms of trade before matching with a suitable trading partner. They agree not to renegotiate after a match occurs even when it is in their interest to do so. Directed search models are also notable for their tendency to yield constrained efficient equilibria. Efficiency arises both in terms of market entry (or search intensity) and market separation. The latter is important when there is unobservable heterogeneity, since endogenous market separation
resolves the issue of asymmetric information. This paper studies the efficiency implications of relaxing commitment in a directed search framework. I propose a model without commitment to pre-match announcements or post-match negotiation strategies and assignment rules. I show that the usual constrained efficiency results cannot be sustained in the absence of commitment.

A model of the labour market is developed that allows for the possibility of directed search: workers can make pre-match announcements in the form of cheap talk messages (the content of an application cover letter, for example), and firms can condition their search strategies on these signals. Search frictions take the form of coordination frictions à la Burdett et al. [13], which leads to an exponential matching process with multilateral matches. Once firms match with workers, the wage is determined via a multilateral bargaining process wherein bargaining strategies do not necessitate or permit commitment to an offer as a means of influencing the assignment rule (i.e., the determination of which firm ultimately recruits the worker).

Consider the following examples of negotiation strategies that are precluded by the inability to commit. The first and most obvious tactic to rule out is the pre-match proposal of non-negotiable terms of trade. Workers and firms cannot commit to pre-match wage demands or offers, since it might be in one party’s interest to renegotiate after a match. A worker could advertise a low wage demand in order to attract firms and increase the matching probability. Once a worker is matched, however, there is a hold-up problem and an incentive to renege on the promise to supply labour at the low wage. Certain post-match pricing mechanisms are also incompatible with the absence of commitment. Suppose several firms match with and aim to recruit a worker, and the worker must select and begin negotiations with one of the firms.
At this stage, a firm might consider promising a higher wage in order to influence the worker’s choice of firm. The inability to commit removes the credibility of such promises. Suppose further that a worker can switch and initiate negotiations with different firms. A firm might then consider offering a higher wage in exchange for the worker’s guarantee not to continue negotiations with the other firms. Once again, the absence of commitment invalidates such agreements.

I develop a multilateral bargaining structure that can capture the competition among firms within a match without allowing firms or workers to derail the pricing protocol or assignment rule with take-it-or-leave-it offers. Avoiding commitment in the determination of prices or wages requires departing from standard auction mechanisms. An auction is a complex set of rules about the assignment rule, bidders’ strategy space, bidders’ information sets, and the auctioneer’s disclosure of information. When the auctioneer is also engaged in the transaction, it might be unrealistic to assume commitment to the auction mechanism since there might be ways to exploit ex post opportunities or use private information to one’s advantage.\(^1\)

The next section presents the model of the labour market without worker heterogeneity and derives a condition for the efficient level of recruitment. Section 3.3 introduces worker heterogeneity and establishes conditions on the model parameters that are consistent with an incentive compatible fully separating equilibrium. It is this section that establishes the impossibility of efficiency without commitment. Section 3.5 concludes.

\(^1\)For example, Kim and Kircher [49] consider a labour market without commitment to firms’ pre-match announcements, but with commitment by firms to running a first-price auction and by workers to their take-it-or-leave-it wage demands. Since firms are also auctioneers, there is good reason to suspect that a firm might deviate from the structure of the auction mechanism by, for example, revealing the number of workers competing for the job to increase the degree of competition among workers, or revealing the bids ex post and allowing for subsequent rounds of bidding to further enhance the terms of the transaction.
3.2 The Model: Identical Workers

The model of the labour market is one of recruitment: firms must spend resources in order to search for a currently unemployed worker to fill a job vacancy. Specifically, an attempt to recruit a worker costs the firm $\kappa$. A firm produces output $y$ when matched with a worker, and 0 otherwise. Firms are anonymous and cannot coordinate their recruiting efforts. Several firms might therefore match with a single worker even though the worker can accept at most one job. These coordination frictions justify an exponential matching process to capture the decentralized nature of the labour market (see Burdett et al. [13]). Let $\theta$ denote the ratio of recruiting (unmatched) firms to unemployed workers (i.e., labour market tightness). A worker is contacted by $k$ firms with probability

$$e^{-\theta \frac{\theta^k}{k!}}, \quad k = 0, 1, 2, \ldots$$

(3.1)

In this section, workers are ex ante identical. Their opportunity cost of working, or reservation wage, is denoted $c$. The measure of unemployed workers is fixed, and the level of recruitment is determined by the free entry of firms.

In a bilateral match, the wage is determined by means of an alternating offer bargaining game. Following Binmore et al. [11], the worker and firm bargain strategically when there is an exogenous risk of negotiation breakdown between offers. In the first round, the firm offers $w_1$ which the worker can either accept or reject. If $w_1$ is accepted, the worker’s payoff is $w_1 - c$, while the firm retains $y - w_1$ of the output. If $w_1$ is rejected, negotiations breakdown with probability $1 - \exp(-(1 - \beta) \Delta)$ and both parties receive a payoff of zero. If bargaining continues, the worker proposes a
wage $w_2$ to be accepted or rejected by the firm. If rejected, negotiations can again be exogenously terminated, this time with probability $1 - \exp(-\beta \Delta)$. The bargaining game continues indefinitely until either negotiations are exogenously aborted or a mutually agreeable offer is proposed and accepted. Figure 3.1 displays the timing of the alternating offer bargaining game. As in Binmore et al. [11], I focus on the limiting equilibrium outcome by letting the length of the bargaining rounds, $(1 - \beta)\Delta$ and $\beta \Delta$, decrease to zero for a constant $\beta \in [0, 1]$.

**Proposition 3.2.1.** There exists a unique subgame perfect equilibrium. In the limit as $\Delta \to 0$, the equilibrium outcome is

$$w = (1 - \beta)c + \beta y$$  \hspace{1cm} (3.2)

The worker’s payoff is

$$U_1 = w - c = \beta(y - c)$$  \hspace{1cm} (3.3)

and the firm’s payoff is

$$V_1 = y - w = (1 - \beta)(y - c)$$  \hspace{1cm} (3.4)
When matches are not bilateral, rather than arbitrarily choosing one firm and ignoring the others, having two or more firms interested in filling a vacancy can be used to the worker’s advantage. While there is a wide literature on multilateral negotiations, the bilateral bargaining game discussed above can be extended to a setting with multilateral matches by allowing workers to negotiate bilaterally in a sequential manner. The worker’s negotiated outcome with one firm can be used as the outside option in a new round of negotiations with a second firm. To capture a variety of wage determination protocols, another exogenous source of negotiation breakdown risk is introduced when a worker switches from negotiating with one firm to another. Specifically, each successive round of bargaining breaks down with probability \( \eta \). Breakdown is firm specific, so the offer from the previous firm does not become void before a new round of bargaining begins. For example, when a worker is matched with two firms, the worker first negotiates wage \( w^{(1)} \) with one of the firms. With probability \( 1 - \eta \), the worker is able to bargain anew with the other firm, eliciting a wage offer denoted \( w^{(2)} \). At this stage, the worker can return to the first firm for renegotiation, this time with \( w^{(2)} \) as his outside option. This process continues until bargaining with one of the firms is exogenously terminated, leaving the worker with the last negotiated offer. Figure 3.2 is a graphical representation of the sequential bargaining game when a worker matches with exactly two firms. For a worker matched with three firms, the multilateral negotiation can break down twice before the worker is left with the offer currently in hand (see Figure 3.3).
The equilibrium wage outcomes are

\[
\begin{align*}
    w^{(1)} &= c + \beta(y - c) \\
    w^{(2)} &= w^{(1)} + \beta(y - w^{(1)}) = c + [1 - (1 - \beta)^2] (y - c) \\
    w^{(3)} &= w^{(2)} + \beta(y - w^{(2)}) = c + [1 - (1 - \beta)^3] (y - c) \\
    \vdots
    w^{(i)} &= c + [1 - (1 - \beta)^i] (y - c)
\end{align*}
\] (3.5)
The expected payoff to a worker in a match with two recruiting firms is therefore

\[
U_2 = \eta(w^{(1)} - c) + \eta(1 - \eta)(w^{(2)} - c) + \eta(1 - \eta)^2(w^{(3)} - c) + \cdots
\]

\[
= \eta [1 - (1 - \beta)] (y - c) + \eta(1 - \eta) [1 - (1 - \beta)^2] (y - c)
\]

\[+ \eta(1 - \eta)^2 [1 - (1 - \beta)^3] (y - c) + \cdots
\]

\[
= \left[ 1 - (1 - \beta) \left( \frac{\eta}{\eta + \beta(1 - \eta)} \right) \right] (y - c)
\]

(3.6)

In a match with three firms, the worker’s multilateral bargaining strategy continues even after the first breakdown, and only ends after two of the three firms drop out of the bargaining process. The expected payoff is therefore

\[
U_3 = \binom{1}{0} \eta^2 (w^{(1)} - c) + \binom{2}{1} \eta^2 (1 - \eta)(w^{(2)} - c) + \binom{3}{2} \eta^2 (1 - \eta)^2(w^{(3)} - c) + \cdots
\]

\[
= \binom{1}{0} \eta^2 [1 - (1 - \beta)] (y - c) + \binom{2}{1} \eta^2 (1 - \eta) [1 - (1 - \beta)^2] (y - c)
\]

\[+ \binom{3}{2} \eta^2(1 - \eta)^2 [1 - (1 - \beta)^3] (y - c) + \cdots
\]

\[
= \left[ 1 - (1 - \beta) \left( \frac{\eta}{\eta + \beta(1 - \eta)} \right)^2 \right] (y - c)
\]

(3.7)

This continues for all possible multilateral matches. The unemployed worker’s ex ante expected payoff, which takes into account the probability of matching with \( k \) firms using the matching probabilities in (3.1), is

\[
U(\theta) = \theta e^{-\theta} U_1 + e^{-\theta} \frac{\theta^2}{2!} U_2 + e^{-\theta} \frac{\theta^3}{3!} U_3 + \cdots
\]

\[
= \left[ 1 - e^{-\theta} \right] (y - c) - e^{-\theta} \left( \frac{\eta}{\eta + \beta(1 - \eta)} \right)^{-1} \left[ \frac{\eta \theta}{e^{\eta + \beta(1 - \eta)} - 1} \right] (1 - \beta)(y - c)
\]

(3.8)
From here it is straightforward to derive the expected payoff associated with recruiting. The firm’s payoff is simply the share of the surplus that is not captured by the worker or by competing firms. In a multilateral match with \( k \) firms, each recruiter has a \( \frac{1}{k} \) chance of hiring the worker.\(^2\) The firm’s expected payoff in a match with \( k - 1 \) other firms is therefore

\[
\begin{align*}
V_1 &= (1 - \beta)(y - c) \\
V_2 &= \left( \frac{\eta}{\eta + \beta(1 - \eta)} \right) \frac{(1 - \beta)(y - c)}{2} \\
V_3 &= \left( \frac{\eta}{\eta + \beta(1 - \eta)} \right)^2 \frac{(1 - \beta)(y - c)}{3} \\
&\vdots \\
V_k &= \left( \frac{\eta}{\eta + \beta(1 - \eta)} \right)^{k-1} \frac{(1 - \beta)(y - c)}{k}
\end{align*}
\] (3.9)

Again using the matching probabilities in (3.1), the ex ante expected payoff for a recruiting firm is

\[
V(\theta) = e^{-\theta}V_1 + \theta e^{-\theta}V_2 + e^{-\theta} \frac{\theta^2}{2!} V_3 + \cdots
= e^{-\theta} \left( \frac{\eta\theta}{\eta + \beta(1 - \eta)} \right)^{-1} \left[ e^{\frac{\eta\theta}{\eta + \beta(1 - \eta)}} - 1 \right] (1 - \beta)(y - c)
\] (3.10)

Modelling multilateral wage determination in this manner allows one to characterize a variety of different settings by changing the parameter values for the risks of negotiation breakdown, \( \beta \) and \( \eta \). Propositions 3.2.2 and 3.2.3 establish that the present set-up nests two well-known static search and matching models of the labour market.

\(^2\)This follows from the fact that the firm involved in the first round of bargaining is chosen at random. The ultimately successful recruiter is then determined by the breakdown of negotiations with the other firms, which is determined by the realization of idiosyncratic random variables.
Proposition 3.2.2. When \( \eta = 1 \), so that there is only one round of bargaining even in a multilateral match, the model reduces to a static version of the Pissarides [67, 68] random search model.

When \( \eta = 1 \), the worker’s and firm’s expected payoffs are

\[
U(\theta) = \beta [1 - e^{-\theta}] (y - c) \quad (3.11)
\]
\[
V(\theta) = (1 - \beta) \left[ \frac{1 - e^{-\theta}}{\theta} \right] (y - c) \quad (3.12)
\]

The worker’s payoff is \( \beta(y - c) \) whenever matched with at least one firm, which occurs with probability \( 1 - e^{-\theta} \) with the urn-ball matching function, and 0 otherwise. Similarly, the firm gets \( (1 - \beta)(y - c) \) with probability \( 1/\theta \) conditional on a match, and zero otherwise. This is the same as the Nash bargaining outcome in the Pissarides [67, 68] search model with \( \beta \) as the worker’s bargaining strength parameter.

Proposition 3.2.3. When \( \eta \to 0 \), so that competition among recruiters in a multilateral match is the most severe, the worker’s expected payoff is

\[
\lim_{\eta \to 0} U(\theta) = [1 - e^{-\theta}] (y - c) - \theta e^{-\theta} (1 - \beta)(y - c) \quad (3.13)
\]

and the firm’s expected payoff is

\[
\lim_{\eta \to 0} V(\theta) = e^{-\theta} (1 - \beta)(y - c) \quad (3.14)
\]

In addition, if \( \beta = 0 \), the model resembles the search model developed by Julien et al. [47].

The first term in (3.13) is the entire surplus, \( y - c \), in the event of a multilateral
match (a worker matched with two or more firms) which occurs with probability $1 - e^{-\theta} - \theta e^{-\theta}$. The second term is the outcome from a single round of bargaining in the event of a bilateral match, $\beta(y - c)$, which happens with probability $\theta e^{-\theta}$. In (3.14), the firm’s payoff comes only from the possibility of a bilateral match, since multi-round bargaining reduces the firm’s surplus to zero when $\eta \to 0$. If $\beta = 0$, the firm captures all of the surplus if no other firm matches with the same worker, and zero otherwise.

### 3.2.1 Efficiency

A constrained social planner$^3$ chooses the level of recruitment to maximize the total social surplus from matching firms with workers, less the cost of recruiting. Equivalently, the social planner chooses labour market tightness, $\theta$, to maximize the per worker social surplus:

$$\max_{\theta} \left[ 1 - e^{-\theta} \right] (y - c) - \theta \kappa$$

(3.15)

The first order condition is

$$e^{-\theta}(y - c) = \kappa$$

(3.16)

This condition states that the societal benefit of an additional vacancy must equal the cost of recruiting. The total benefit of generating another vacancy is simply the total surplus of a match times the probability that an otherwise unmatched worker will now be recruited by the additional firm.

In general, the decentralized equilibrium will not satisfy (3.16). However, there are parameter values for $(\beta, \eta)$ that do lead to an efficient equilibrium. These can

$^3$The social planner is constrained in the sense that the planner is also subject to the search/coordination frictions that are present in the labour market environment.
be found by substituting for \( \kappa \) in (3.16) using the equilibrium free entry condition, \( V(\theta) = \kappa \). Doing so and rearranging yields

\[
\beta = 1 - \frac{\eta \theta}{\eta + \beta(1-\eta)} e^{\eta \theta / (\eta + \beta(1-\eta))} - 1
\]  

(3.17)

Figure 3.4 maps out the parameter combinations in \((\eta, \beta)\)-space that satisfy the efficiency condition (3.17) for a particular numerical example. Notice that when \( \eta = 1 \), the equilibrium is constrained efficient if and only if

\[
\beta = 1 - \frac{\theta e^{-\theta}}{1 - e^{-\theta}}
\]  

(3.18)

That is, in the absence of multilateral bargaining, the efficiency requirement reduces to the well-known Mortensen–Hosios condition\(^4\) with the urn-ball matching function. When \( \eta \to 0 \) and competition among recruiters is most severe, the equilibrium is constrained efficient if and only if \( \beta \to 0 \). This highlights an important difference between the competitive search model of Moen [60] and the directed search model of Julien et al. [47]. In the former, efficiency arises because agents can commit to offers ex ante and search is directed. In Julien et al. [47], the equilibrium is efficient even without commitment because the pricing protocol happens to satisfy the efficiency condition. In the present environment with \( \eta \to 0 \) and \( \beta \to 0 \), which is equivalent to a first price auction with full information or a Bertrand competition determining wages, the private cost of posting a vacancy is aligned with the social benefit. Therefore, no pre-match announcements or commitments are required to obtain efficiency.

\(^4\)See Mortensen [62] and Hosios [41].
when workers are identical. With heterogeneous workers, directed rather than random/undirected search is also necessary for efficiency. This is addressed in the next section.

![Figure 3.4: The parameter combinations that generate efficient equilibria.](image)

### 3.3 Worker Heterogeneity

There are good reasons to believe that workers differ in their opportunity cost of working. To capture this form of worker heterogeneity, suppose workers differ in terms of their reservation wages: some workers have a low reservation wage, $c_L$, and the remaining workers have a high reservation wage, $c_H > c_L$. In the presence of heterogeneity, a constrained efficient allocation is not possible in a random search environment. The efficiency condition cannot hold for both types with only one matching function. I therefore allow for the possibility of directed search. Workers can make cheap talk announcements (the content of an application cover letter, for
example), and recruiters can use these messages to shape their recruitment strategies. If workers could commit to an ex ante wage demand, firms could condition their search efforts on the announced wage, and the directed search equilibrium would be constrained efficient, as in Montgomery [61] or Moen [60]. However, the pricing protocol studied here does not allow market participants to commit in this manner.

An interesting question related to market efficiency is the following: in an environment without commitment, are there \((\beta, \eta)\) parameter values that render workers’ ex ante messages informative?

In order to answer this question, I consider a candidate separating equilibrium and check for profitable deviations.\(^5\) To do so, the bargaining game described in Section 3.2 must first be extended to a setting with incomplete information. Such a bargaining game was studied by Gul and Sonnenschein [36], who find a continuum of sequential equilibria in which the worker’s strategy is stationary and monotonic. Following Grossman and Perry [34], one can select the unique sequential equilibrium that satisfies a monotonicity restriction on the firm’s out-of-equilibrium beliefs. This mirrors the methodology employed by Menzio [58], but extended to allow for multilateral matching with sequential rounds of bargaining. These refinements have the following implications: a deviating worker – a worker that makes an announcement to mimic a type other than his true type – negotiates as if he is the worker he pretended to be at the communication stage. Only if the worker’s final offer is such that his payoff is negative will the mimicking worker reveal his true type by rejecting the offer in favour of unemployment.

\[
\text{If } c_H \leq w^{(1)} = (1 - \beta)c_L + \beta y \iff \beta \geq (c_H - c_L)/(y - c_L), \text{ negotiations will }
\]

\(^5\)If pre-match announcements are informative, then the equilibrium will be fully separating. Verifying the incentive compatibility of a candidate separating equilibrium allows me to check for informative messages and directed search without explicitly modelling the message posting stage.
always result in agreement (i.e., no offers are ever rejected). In a fully separating equilibrium, the free entry conditions are \( V(\theta_L) = \kappa \) and \( V(\theta_H) = \kappa \), or

\[
e^{-\theta_L}(1 - \beta)(y - c_L) \left[ \frac{\eta \theta_L}{\eta + \beta(1 - \eta)} - 1 \right] = \frac{\eta \theta_L}{\eta + \beta(1 - \eta)} \kappa \tag{3.19}
\]

\[
e^{-\theta_H}(1 - \beta)(y - c_H) \left[ \frac{\eta \theta_H}{\eta + \beta(1 - \eta)} - 1 \right] = \frac{\eta \theta_H}{\eta + \beta(1 - \eta)} \kappa \tag{3.20}
\]

The expected payoffs for workers can then be written

\[
U_L(\theta_L) = \left[ 1 - e^{-\theta_L} \right] (y - c_L) - \theta_L \kappa \tag{3.21}
\]

\[
U_L(\theta_H) = \left[ 1 - e^{-\theta_H} \right] (y - c_L) - \theta_H \kappa \tag{3.22}
\]

\[
U_H(\theta_H) = \left[ 1 - e^{-\theta_H} \right] (y - c_H) - \theta_H \kappa \tag{3.23}
\]

\[
U_H(\theta_L) = \left[ 1 - e^{-\theta_L} \right] (y - c_H) - \theta_L \kappa \tag{3.24}
\]

The incentive compatibility constraint for type L workers, \( U_L(\theta_L) \geq U_L(\theta_H) \), is

\[
[1 - e^{-\theta_L}](y - c_L) - \theta_L \kappa \geq [1 - e^{-\theta_H}](y - c_L) - \theta_H \kappa \\
\Rightarrow \quad [e^{-\theta_H} - e^{-\theta_L}](y - c_L) \geq (\theta_L - \theta_H) \kappa \tag{3.25}
\]

Similarly, the incentive compatibility constraint for type H workers is \( U_H(\theta_H) \geq U_H(\theta_L) \), or

\[
[e^{-\theta_H} - e^{-\theta_L}](y - c_H) \leq (\theta_L - \theta_H) \kappa \tag{3.26}
\]

Substituting for \( \kappa \) in (3.25) using (3.19) yields

\[
\frac{\eta \theta_L}{\eta + \beta(1 - \eta)} \left[ e^{-\theta_H} - e^{-\theta_L} \right] \geq e^{-\theta_L}(1 - \beta)(\theta_L - \theta_H) \left[ e^{\eta \theta_L / (\eta + \beta(1 - \eta))} - 1 \right] \tag{3.27}
\]
or, after rearranging,

\[ \beta \geq 1 - \left( \frac{e^{\theta_L - \theta_H} - 1}{\theta_L - \theta_H} \right) \left( \frac{\eta \theta_L}{\eta + \beta(1 - \eta)} \right) \left[ e^{\eta \theta_L/\eta + \beta(1 - \eta)} - 1 \right]^{-1} \]  

This inequality implicitly defines a lower bound for the bilateral bargaining parameter \( \beta \). Let \( \underline{\beta} \) denote this lower bound. The same procedure using (3.26) and (3.20) leads to the following inequality:

\[ \beta \leq 1 - \left( \frac{1 - e^{\theta_H - \theta_L}}{\theta_L - \theta_H} \right) \left( \frac{\eta \theta_H}{\eta + \beta(1 - \eta)} \right) \left[ e^{\eta \theta_H/\eta + \beta(1 - \eta)} - 1 \right]^{-1} \]  

This determines the upper bound, \( \overline{\beta} \). Therefore, as long as \( \beta \in [\underline{\beta}, \overline{\beta}] \) and \( \beta \geq (c_H - c_L)/(y - c_L) \), there exists a fully separating equilibrium. Figure 3.5 displays these upper and lower bounds for \( \beta \) (the dashed lines) for a particular numerical example.

Figure 3.5: The upper and lower bounds for \( \beta \) that achieve separating equilibria.
There are two dimensions of efficiency to be considered: market separation so that buyer arrival probabilities adjust for workers of different types, and efficient firm entry or recruitment as discussed in Section 3.2.1. Figure 3.5 also traces out the efficiency condition for the hypothetical worker with reservation wage \((c_L + c_H)/2\), which lies between the \(\beta\)-thresholds for most of its domain. The two dimensions of efficiency are related: the same parameters that align the costs and benefits of vacancy creation in the case with identical workers also tend to generate market separation when worker heterogeneity is introduced. The next proposition establishes that as worker heterogeneity tends to zero, the conditions for market separation reduce to the modified Mortensen–Hosios condition.

**Proposition 3.3.1.** Let \(c_L = c - \varepsilon/2\) and \(c_H = c + \varepsilon/2\), \(\varepsilon > 0\). Given any \(\eta \in [0,1]\), let \(\beta^*\) denote the bargaining parameter value for \(\beta\) that satisfies (3.17) when the reservation wage is \(c\). If \(\beta \geq \varepsilon/(y - c_L)\), then

\[
\lim_{\varepsilon \to 0} \beta = \lim_{\varepsilon \to 0} \beta^* = \beta^*
\]  

(3.30)

An interesting part of Figure 3.5, however, is the region with \(\beta\) and \(\eta\) close to zero. As \(\eta \to 0\), the threshold \(\overline{\beta}\) no longer lies above \(\beta\) and \(\beta^*\). This has to do with the mimicking worker’s option to reject offers that are deemed unsuitable. Recall that in deriving conditions (3.28) and (3.29), it was assumed that \(c_H \leq (1 - \beta)c_L + \beta y \Leftrightarrow \beta \geq (c_H - c_L)/(y - c_L)\). This ensured that even a type \(H\) worker pretending to be a type \(L\) would find the negotiated outcome worth accepting over unemployment. As \(c_H - c_L \to 0\), this condition is satisfied trivially. With non-negligible worker heterogeneity, there can arise situations wherein a type \(H\) worker mimicking a type \(L\) worker ends up with only wage offers below his reservation wage.
Let $\rho^{(i)}$ denote the probability that a worker’s negotiations end with a wage offer of $w^{(i)}$. Using the matching probabilities (3.1) and the exogenous probability of breakdown, $\eta$, these probabilities can be written as follows:

$$\rho^{(1)} = \theta e^{-\theta} + \binom{0}{0} \eta e^{-\theta} \frac{\theta^2}{2!} + \binom{1}{0} \eta^2 e^{-\theta} \frac{\theta^3}{3!} + \cdots$$

(3.31)

$$\rho^{(2)} = \binom{1}{1} \eta (1 - \eta) e^{-\theta} \frac{\theta^2}{2!} + \binom{2}{1} \eta^2 (1 - \eta) e^{-\theta} \frac{\theta^3}{3!} + \cdots$$

(3.32)

$$\rho^{(3)} = \binom{2}{2} \eta (1 - \eta) e^{-\theta} \frac{\theta^2}{2!} + \binom{3}{2} \eta^2 (1 - \eta) e^{-\theta} \frac{\theta^3}{3!} + \cdots$$

(3.33)

\[\vdots\]

The expected payoff for a deviating type $H$ worker is therefore

$$U_H(\theta_L) = \left[1 - e^{-\theta_L}\right] (y - c_H) - \theta_L \kappa$$

$$+ \begin{cases} 
0 & \text{if } c_H \leq w^{(1)} \\
\rho^{(1)} (c_H - w^{(1)}) & \text{if } c_H \in (w^{(1)}, w^{(2)}) \\
\rho^{(1)} (c_H - w^{(1)}) + \rho^{(2)} (c_H - w^{(2)}) & \text{if } c_H \in (w^{(2)}, w^{(3)}) \\
\vdots & 
\end{cases}$$

(3.34)

Since workers are not compelled to accept an offer that is unsuitable, the expected payoff (3.34) should be used in place of (3.24) in deriving $\beta$. Doing so gives rise to the kink in the upper bound for $\beta$ in Figure 3.5. Therefore, the lack of commitment to pre-match announcements or bargaining strategies in combination with multilateral matching/bargaining can unravel market separation. Consequently, an equilibrium that is both constrained efficient and fully separating is unsustainable.

**Proposition 3.3.2.** With $c_H > c_L$, there do not exist parameter values for $\beta$ and $\eta$
that generate a fully separating and constrained efficient equilibrium.

When $\eta > 0$, the efficiency condition for $\beta$ requires different values for type $L$ and type $H$ workers. The implication is that the modified Mortensen–Hosios condition cannot hold for both type $L$ and type $H$ workers. When $\eta \to 0$, efficiency requires $\beta = 0$ for any type of worker. As shown in Figure 3.5, however, a separating equilibrium with $\eta \to 0$ and $\beta \to 0$ is not incentive compatible. A formal proof is relegated to Appendix B.

### 3.4 Relationship to the Literature

Some recent papers explore the issue of commitment in directed search models. Menzio [58] relaxes the commitment assumption in a model of the labour market in a search environment that restricts the matching process to bilateral meetings. He shows that non-contractual announcements can support market separation with one-sided heterogeneity as long as the market is neither too tight nor too slack. This finding is analogous to the upper and lower bounds on $\beta$, with the restriction that $\eta = 1$. Allowing for multilateral matching and competition among recruiting firms ($\eta < 1$) is an important element of this paper. In many decentralized labour markets, the incidence of multilateral meetings is high given the coordination problem that gives rise to search and matching frictions. I extend the bargaining game with incomplete information to allow for multilateral matches, and show that as competition among recruiters in the bargaining process becomes more severe (i.e., as $\eta \to 0$), the incentive compatibility of market separation becomes infeasible. In other words, the interaction of multilateral matching with the inability to commit to announcements, offers or bargaining strategies prevents endogenous market separation.
Kim and Kircher [49] ask a different but related question: does there exist a pricing mechanism that, along with exponential matching probabilities, can give rise to a constrained efficient directed search equilibrium in the absence of commitment to pre-match announcements? They find that a first price auction when the number of bidders is unobservable delivers a constrained efficient and fully revealing equilibrium. While they do relax commitment to pre-match messages, they maintain commitment to the rules of an auction. In particular, workers submit take-it-or-leave-it wage demands post-match, and firms commit to a single round of offers, to the allocation rule (the worker that submits the lowest wage demand is hired), and also to not revealing the true number of bidders. In the bargaining process described in this paper, submitting more appealing offers to influence the allocation rule is never an optimal strategy. Whenever possible, a worker continues to negotiate more favourable terms of employment with the next firm in a multilateral match since there is no cost to doing so. Furthermore, the fact that offers are submitted without knowing the number of agents involved in the match is crucial for the existence of a fully revealing equilibrium in the model studied by Kim and Kircher [49]. When the auctioneer is also involved in the economic transaction, there is an incentive to convey this information to the bidders in a credible way if at all possible. Commitment to the information revelation strategy of the auctioneer is not an issue in this paper given the mechanical and sequential nature of the multilateral bargaining procedure.
3.5 Concluding Remarks

Typically, directed search models require commitment to pre-match announcements, a post-match assignment rule and/or post-match negotiation strategies. In many illiquid and decentralized markets, there is no legislated commitment to pre-match offers, and no credible commitment mechanism to support a complicated pricing protocol. Furthermore, multilateral matches are commonly observed in decentralized markets (labour, housing, art, vintage wine, etc.), and market participants tend to engage in successive rounds of bilateral negotiations rather than implementing a complicated multilateral trading mechanism. This paper proposes a directed search model with features consistent with these observations, and shows that the usual constrained efficiency results cannot be sustained.
Chapter 4

Tenure Insecurity, Adverse Selection, and Liquidity in Rural Land Markets

4.1 Introduction

In this chapter I construct a theory of land market activity in settings where land tenure is insecure. I argue that differences in the security of property rights over land can help explain the widely varied volumes of trade across developing country land markets. The framework establishes that asymmetric information about land tenure insecurity can reduce the volume of transactions in a land lease or sales market. As a useful application of the model, I analyze the effect of the performance of the land market on workers’ migration decisions. Thinness in the land market can prevent the efficient allocation of workers between the farm and off-farm sectors.

It is a widely held view that systems of property rights have important effects
on the functioning of agricultural land markets. In particular, policies that improve tenure security are often argued to improve land transferability and hence strengthen a landowner’s capacity to capitalize on the value of his land should he decide to migrate or accept off-farm employment [69, 74]. This is of economic importance because the process of economic development typically involves a shift in labour from agricultural sectors to more modern manufacturing or service sectors. Workers’ incentives for making the transition partly depend on the functioning of the rural land market.

The main theoretical contribution of the paper is establishing the link between tenure security and land market activity. It is the hidden information about the security of land ownership that renders land illiquid. The transferability of a particular plot is determined endogenously by the number of land market participants. A low buyer-seller ratio implies a low probability of selling or leasing out land, which acts as a screening mechanism that allows the demand side of the market to determine the quality of the property rights. The endogenous mechanism is incentive compatible because owners of relatively secure land are willing to accept a lower probability of trade if payment is more favourable in the event of a land transfer.

This paper focuses specifically on the role of land transferability in the efficient allocation of workers between agricultural and off-farm activities. In less developed countries, infrequent land transfers are often accomplished through inheritance and reallocation by village leaders. As non-agricultural sectors start to emerge and population densities increase, so does the need for land sales markets or rental transactions. Accordingly, I model the land market in an environment where a fraction of landowners receive an opportunity to work more productively in a modern sector. With no further need for land as a productive input, an emigrating landowner could benefit
from the ability to lease or sell his plot.

The theory presented here is related to the household models of land rental markets with transaction costs [10, 73, 15, 22]. In Deininger and Jin [20] for example, demand for land transferability is driven by off-farm employment opportunities and differences in agricultural ability. By imposing exogenous transaction costs in the land market, illiquidity is introduced in a reduced-form fashion. An increase in transaction costs results in a larger set of self-cultivating agents not participating in the market. Deininger and Jin [20] point to the costly acquisition of information and the risk of expropriation by village leaders as key determinants of transaction costs. Exactly how tenure insecurity translates into transaction costs that hamper land market participation is not modeled. That is, transaction cost models assume rather than explain the crucial determinants of land market inactivity. Recently, there has been a push in the field of development economics towards the understanding of underlying mechanisms [17, 18]. This paper fills a gap in the literature by applying a framework that allows one to understand the mechanism by which tenure insecurity leads endogenously to illiquidity in the land market.\footnote{In a dynamic setting, a measure of land activity is the number of transactions within a fixed interval of time. In a static context like the environment studied here, the analogous notion of land market activity or liquidity is the probability of trade: i.e., the likelihood of selling a particular plot.}

To evaluate the predictions of the model, I present empirical evidence using household level data from Indonesia. As the model predicts, owners of rural land parcels are more active in the supply side of the land market if their land is registered. Approximately 17 percent of unregistered landowners supplied land to the market, while over 25 percent of certificate holders supplied some or all of their farmland to the market. This is consistent with the theory given that possession of a legal land certificate improves ownership security, and access to a land registry reduces the asymmetry of
information. The data also suggest a link between the operation of the land market and non-farm business activity and labour force participation. Rural households with land certificates are more likely to have members earning wages in a non-agricultural sector compared to households with unregistered land. More rigorous analyses with difference-in-difference estimation and probit models support these relationships. Additional implications of the theory are empirically validated by constructing a proxy variable for land tenure security and studying the differences between markets for untitled land across Indonesian provinces.

The plan for the paper is as follows. Section 4.2 describes the details of the model, Section 4.3 characterizes the equilibrium, and Section 4.4 presents the main theoretical results including efficiency implications. Section 4.5 presents an empirical analysis of land markets in Indonesia to verify the testable implications of the theory. Section 4.6 concludes. The proofs are presented in Appendix C. Appendix D outlines the derivation of the proxy variable for land tenure insecurity.

4.2 The Model

There exist two sectors of production: the traditional agricultural sector (the rural economy) and the emerging off-farm economy with modern sector employment (the urban economy).\(^2\) Rural workers match with urban firms before production takes place. Let \(q\) be the exogenous probability that a rural worker receives an urban job offer. The demand for land market transactions is driven by the potential gains from transferring land from an emigrant to a rural worker.

\(^2\)The sectors may not be separated geographically. The urban sector could therefore refer to the non-agricultural activity in the rural area.
There are two periods. The land market is active in the first period, and production takes place in the second. In the final period, landless rural workers earn the wage rate $w^R$, and urban workers receive $w^U > w^R$. A rural landowner earns labour income as well as land rent, $\pi$. If a landowner fails to sell (or a lessee fails to transfer his lease), he can maintain possession of the land even if he accepts employment in the off-farm sector. In this case, the discounted continuation value of land ownership is $\alpha \pi$, $\alpha \in (0, 1)$. The continuation value is intended to mimic a dynamic setting, wherein an unsuccessful seller could try again to transact the following period.\(^3\) Migration occurs at the beginning of the second period. A worker with an urban job offer decides whether or not to accept the job and migrate to the urban sector. The land market operates when potential emigrants try to transfer their land, and landless workers are willing to purchase/lease farmland. The model is constructed so as to be appropriate for both land lease and sales markets, since both buyers and lessees are exposed to the risk of losing agricultural output. To keep the terminology clear, I continue with the description of the model in terms of sales markets, even though land lease agreements are common for transferring farmland in developing countries.

There is a fixed number of indivisible farm plots and an initial distribution of landownership among the rural population. To reflect tenure insecurity, suppose the owner of a plot of land faces a probability $\lambda \in (0, 1)$ of losing ownership of the land.\(^4\) Therefore, a rural landowner’s total expected income is $w^R + (1 - \lambda)\pi$ in

---

\(^3\)In a dynamic context, $\pi$ can be thought of as the discounted present value of future land rents. Owners of unsold land lose rent in period two, but can still profit from the value of their land from the end of period 2 onward, $\alpha \pi$. While it is useful to interpret the model in a way that mimics a dynamic version, it will become clear that a fully dynamic model would be significantly complicated by the evolving distribution of land ownership and the possibility of learning under asymmetric information.

\(^4\)The notion of tenure insecurity is often thought of as a random probability of losing ownership rights to a particular plot of land. This modelling approach is consistent with the evidence of forced
period two, and an emigrant landowner earns $w^U + \alpha(1 - \lambda)\pi$ in expectation. Land is heterogeneous in terms of ownership security. There are only two types: less secure land (type $L$ with $\lambda_L \in (0, 1)$), and land with high tenure security (type $H$ with $\lambda_H \in [0, \lambda_L]$). The rural population, with measure normalized to one, is made up of landless workers and landowners. For simplicity, households are restricted to own and operate at most one plot of land. A share $n_L \in (0, 1)$ of the rural population own type $L$ land, and a share $n_H \in (0, 1)$ own type $H$ land, with $n_L + n_H < 1$. A fraction $1 - n_L - n_H$ of the rural population consists of ex ante homogeneous landless workers.

The quality of the land title is the landowner’s private information. Hence there is the potential for a situation of adverse selection in the land market [2]. I propose a framework with multiple submarkets for agricultural land market participants as an institution to help overcome the adverse selection problem. Submarkets arise endogenously, and each one is characterized by the price at which trade occurs in that particular subdivision of the market. Buyers and sellers observe the set of submarkets and decide in which to participate. Buyers and sellers meet bilaterally, and land transactions can occur at the price specific to that submarket. To highlight the mechanism by which adverse selection and an insecure system of property rights stifle land market transactions, I assume there are no matching frictions. The short side of the market matches with probability one. If, for example, there are more

appropriations by local government leaders in rural China [56] and the perceived threat of land reallocations in Ethiopia [21]. Alternatively, $\lambda$ can be interpreted as the share of land rent spent on costly land disputes [19, 40]. This might be appropriate for understanding the litigation costs in Cameroon for example, where title disputes make up a large number of cases brought before village-level and provincial courts [30].

Institutional differences across communities can bring about heterogeneous land titles, a phenomenon which has been documented in terms of land transfer rights in rural China [57]. On the other hand, Deininger et al. [23] observe a variety of legal and informal documents that convey different levels of tenure security in Nicaragua. As is discussed in greater detail in Section 4.5, overlapping property rights regimes have brought about heterogeneity in tenure security in Indonesia.
sellers than buyers in a particular submarket, buyers randomly select a seller to trade with.

Potential sellers are the landowners with urban job offers: $S_L \equiv qn_L$ are type $L$ (owners of insecure land), and $S_H \equiv qn_H$ are type $H$ (owners of secure land). Sellers can choose not to enter any submarket when the expected payoff is negative. The pool of potential buyers includes all landless workers without outside labour market opportunities, $(1 - q)(1 - n_L - n_H)$. Not all potential buyers choose to enter the land market: buyers enter until the expected benefit of doing so is equal to the entry cost, $c$.

Assumption 1. $q < 1 - n_L - n_H$. The measure of non-migrant landless workers is enough to ensure that the free entry conditions for buyers hold in equilibrium.

Note that the migration decision in period two may depend on land market outcomes in the previous period. The accept/reject decisions of each type of agent can be characterized as follows:

1. A landless worker with no opportunity to purchase land will migrate if

   \[ w^U - w^R \geq 0. \]  \hspace{1cm} (4.1)

2. A landless worker with an opportunity to purchase type $i$ rural land at price $p$, and a type $i$ landowner with the option of a land sale at price $p$ will migrate if

   \[ w^U - w^R \geq (1 - \lambda_i)\pi - p. \]  \hspace{1cm} (4.2)
3. A type $i$ landowner with no opportunity to sell land will migrate if

$$w^U - w^R \geq (1 - \alpha)(1 - \lambda_i)\pi.$$  \hspace{1cm} (4.3)

Migration occurs among the landless whenever there exists an urban-rural wage gap. A precondition for land market activity is for the price of land, $p$, to satisfy inequality (4.2). Otherwise, landowners would never accept urban employment and there would be no need for a land market. Finally, if condition (4.3) holds for $i \in \{L, H\}$, all urban job offers are accepted.

A buyer’s payoff in a land market transaction involving type $i$ land for price $p$ is $(1 - \lambda_i)\pi - p$. A type $i$ seller’s gain from a land market transaction is

$$p - \max\{\alpha(1 - \lambda_i)\pi, (1 - \lambda_i)\pi - (w^U - w^R)\}$$  \hspace{1cm} (4.4)

where the last term represents the opportunity cost of the transaction to the seller, and the maximization operator ensures that expected payoffs are consistent with optimal migration decisions. That is, the migration decisions of potential sellers are consistent with conditions (4.2) and (4.3). To simplify the notation, let

$$\xi_i = \max\{\alpha(1 - \lambda_i)\pi, (1 - \lambda_i)\pi - (w^U - w^R)\}, \text{ for } i \in \{L, H\}$$  \hspace{1cm} (4.5)

represent the type-dependent opportunity costs of land transactions. $\xi_H > \xi_L$ is an important property for deriving the results that follow. Choosing not to enter the land market (denoted by submarket $p = \emptyset$), yields a payoff of zero.

**Assumption 2.** $c < \min\{(1 - \alpha)(1 - \lambda_L)\pi, w^U - w^R\}$. The cost of entering the land...
market, c, is small enough that expected gains from trade are always positive.6

4.3 The Land Market

**Definition.** Given a set of submarkets, \( \mathbb{P} \), a land market equilibrium is a measure \( \{b(p)\}_{p \in \mathbb{P}} \), and measures \( \{s_L(p)\}_{p \in \mathbb{P} \cup \{\emptyset\}} \), and \( \{s_H(p)\}_{p \in \mathbb{P} \cup \{\emptyset\}} \) such that

1. **Buyers’ participation is consistent with free entry:** for any \( p \in \mathbb{P} \),

\[
\min \left\{ 1, \frac{s_L(p) + s_H(p)}{b(p)} \right\} \left[ \frac{s_L(p)(1 - \lambda_L) + s_H(p)(1 - \lambda_H)}{s_L(p) + s_H(p)} \pi - p \right] - c \leq 0
\]

(with equality if \( b(p) > 0 \))

2. **Sellers enter submarkets optimally:** for \( i = L,H \),

(i) if \( s_i(p) = 0 \) for all \( p \in \mathbb{P} \), then \( s_i(\emptyset) = S_i \) and

\[
\max_{p \in \mathbb{P}} \left\{ \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} (p - \xi_i) \right\} = 0
\]

(ii) if \( s_i(p) > 0 \) for some \( p \in \mathbb{P} \), then

\[
\min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} (p - \xi_i) \geq \max_{p \in \mathbb{P}} \left\{ \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} (p - \xi_i) \right\}
\]

6The introduction of entry costs is necessary on the demand side of the market to pin down the number of buyers in each submarket using the free entry condition. If entry costs are too high, expected gains from trade can become negative and the land market shuts down. Assumption 2 is a sufficient condition to ensure that a land market exists.

On the supply side of the market, entry costs are unnecessary because there is a fixed measure of potential sellers that enter the market whenever the expected gains from trade are positive. The results are unaffected by the introduction of entry costs on the supply side of the market, except for Proposition 4.4.3 (see footnote 8, p.78).
3. Aggregation: for any \( i \in \{L, H\} \),

\[
\sum_{p \in P \cup \{\emptyset\}} s_i(p) = S_i
\]

The last part of the definition says that every potential seller either chooses a submarket or decides not to enter the land market. Part 2 requires that sellers choose optimally between submarkets. The first part of the definition says that free entry drives a buyer’s expected payoff down to the cost of entering the market. Notice from part 2 of the definition that an equilibrium allocation of buyers and sellers determines a pair of type-dependent utilities for sellers. Denote these market utilities \( \{\bar{U}_L, \bar{U}_H\} \).

As previously noted, migration decisions in period two may hinge on land market outcomes in period one. Accordingly, the opportunity cost of a land transaction reflects the anticipated migration decision. Depending on the parameter values, the environment can be classified according to the pattern of equilibrium migration decisions. Suppose for example that all job offers are accepted, even without land market transactions. Call this a case 1 environment. A case 2 environment is one in which only type \( H \) landowners who fail to sell land reject urban job offers. Finally, a case 3 environment manifests when both types of landowners reject job offers when unable to sell their land. Equilibrium in the land market can be classified in this manner according to the urban-rural wage gap:

Case 1 if \( w^U - w^R \geq (1 - \alpha)(1 - \lambda_H)\pi \)

Case 2 if \( w^U - w^R \in \left[ (1 - \alpha)(1 - \lambda_L)\pi, (1 - \alpha)(1 - \lambda_H)\pi \right) \)

Case 3 if \( w^U - w^R < (1 - \alpha)(1 - \lambda_L)\pi \)
The equilibrium definition does not impose restrictions on prices (i.e., submarkets) that do not appear in the land market. Consequently, many sets of prices and allocations of agents across submarkets satisfy the equilibrium conditions. For instance, an equilibrium with only one submarket with price \( p \in (\xi_L, \min\{\xi_H, (1 - \lambda_L)\pi - c\}) \) is an equilibrium with only a market for type \( L \) land: type \( L \) sellers choose to enter since \( p > \xi_L \); type \( H \) sellers choose not to enter whenever \( p < \xi_H \); and buyers choose to enter because \( p \leq (1 - \lambda_L)\pi - c \). Type \( L \) land is readily sold, since \( p < (1 - \lambda_L)\pi - c \) implies \( b(p) > S_L \), while type \( H \) land is completely illiquid. Moreover, if \( p < (1 - \lambda_L)\pi - c \), it seems reasonable that the price would be bid upward, since \( p' \in (p, (1 - \lambda_L)\pi - c) \) would still attract type \( L \) sellers and the deviation would yield a strictly positive payoff. To restrict the set of equilibria, consider the following refinement.

**Equilibrium Refinement 1.** The set of submarkets (prices \( \mathbb{P} \), and allocations \( \{b(p)\}_{p \in \mathbb{P}}, \{s_L(p)\}_{p \in \mathbb{P} \cup \{\emptyset\}}, \) and \( \{s_H(p)\}_{p \in \mathbb{P} \cup \{\emptyset\}} \)) satisfying equilibrium conditions 1, 2, and 3 represents a competitive search land market equilibrium if, given the associated seller utilities \( \{\bar{U}_L, \bar{U}_H\} \), there is no deviating offer \( p' \in \mathbb{R}_+ \setminus \mathbb{P} \) that yields a strictly positive expected payoff to the subset of buyers bidding \( p' \).

The equilibrium refinement turns the land market equilibrium into a competitive search equilibrium like that of Moen [60], but extended to a setting with asymmetric information. The framework is similar to that developed in Guerrieri et al. [35], but without the standard single-crossing condition. In other words, there is no sorting variable allowing buyers to attract type \( H \) sellers without also attracting type \( L \) sellers. Instead, there is a trade-off between the probability of trade and the price of land that endogenously sorts sellers into submarkets. As in other models of competitive search, there is the implicit restriction on out-of-equilibrium beliefs that agents...
correctly anticipate the ratio of buyers to sellers in all possible submarkets, not just those that appear in equilibrium. There is also an assumption that buyers can fully commit to their offers, so that there is no renegotiation ex post.

**Proposition 4.3.1.** Under Assumptions 1 and 2, there exists a unique competitive search land market equilibrium with

1. land prices: \( p_L = (1 - \lambda_L)\pi - c \) and \( p_H = (1 - \lambda_H)\pi - c \)

2. seller allocations: \( \{s_L(p_L), s_L(p_H)\} = \{S_L, 0\} \) and \( \{s_H(p_L), s_H(p_H)\} = \{0, S_H\} \)

3. buyer allocation: \( b(p_L) = S_L \) and

\[
b(p_H) = S_H \times \begin{cases} 
(1 - \alpha)(1 - \lambda_L)\pi - c & \text{if } w^U - w^R \geq (1 - \alpha)(1 - \lambda_L)\pi \\
(1 - \lambda_H)\pi - \alpha(1 - \lambda_L)\pi - c & \text{if } w^U - w^R < (1 - \alpha)(1 - \lambda_L)\pi \\
\frac{w^U - w^R}{w^U - w^R - c} & \text{if } w^U - w^R < (1 - \alpha)(1 - \lambda_L)\pi \\
w^U - w^R + (\lambda_L - \lambda_H)\pi - c & \text{if } w^U - w^R > (1 - \alpha)(1 - \lambda_L)\pi 
\end{cases}
\]

4. type-dependent seller utilities:

\[
\bar{U}_L = \begin{cases} 
(1 - \alpha)(1 - \lambda_L)\pi - c & \text{if } w^U - w^R \geq (1 - \alpha)(1 - \lambda_L)\pi \\
w^U - w^R - c & \text{if } w^U - w^R < (1 - \alpha)(1 - \lambda_L)\pi 
\end{cases}
\]

and

\[
\bar{U}_H = \begin{cases} 
\frac{b(p_H)}{S_H}[(1 - \alpha)(1 - \lambda_L)\pi - c] & \text{if } w^U - w^R \geq (1 - \alpha)(1 - \lambda_H)\pi \\
\frac{b(p_H)}{S_H}[w^U - w^R - c] & \text{if } w^U - w^R < (1 - \alpha)(1 - \lambda_H)\pi 
\end{cases}
\]

In a full information benchmark economy, the land market is fully liquid: \( b(p_i) = S_i \) for \( i \in \{L, H\} \). In an equilibrium with asymmetric information, however, \( b(p_H) < S_H \),
implying that there are too few land market transactions in the type $H$ submarket. Some type $H$ landowners with valuable urban job offers fail to sell their land.

In the presence of adverse selection due to private information regarding tenure insecurity, type $H$ land plots are made illiquid as a screening mechanism. The illiquidity arises because too few buyers enter the market for type $H$ land. The mechanism functions appropriately because type $L$ landowners prefer a liquid market in order to avoid being stuck with an insecure land title when production takes place. Type $L$ sellers will therefore enter submarket $p_L$ in order to sell more readily, albeit at a lower price. On the other hand, type $H$ owners are more likely to maintain ownership of their land if they fail to sell. Sellers of secure land are therefore more willing to tolerate a lower probability of trade if it means a chance at receiving a fair price.

4.4 The Effects of the Property Rights System

In this section I analyze how changes to the system of property rights affect the functioning of the land market and the allocation of workers across sectors. Let $\lambda_L = \lambda + \varepsilon$ and $\lambda_H = \lambda - \varepsilon$. Since all type $L$ sellers trade in equilibrium and only a fraction $\theta_H \equiv b(p_H)/S_H \in (0,1)$ of type $H$ sellers trade, the total number of land market transactions is $S_L + \theta_H S_H$. The following proposition summarizes the linkages between the system of land rights (governed by parameters $\lambda$ and $\varepsilon$) and the functioning of the land market in terms of the number of transactions.

**Proposition 4.4.1.** Equilibrium comparative statics with respect to the system of ownership rights over land imply the following:
(i) A deterioration of land tenure security undermines the transferability of land:

\[
\frac{\partial \theta_H}{\partial \lambda} \leq 0, \quad \text{(strict for high values of } \lambda) \]

(ii) An increase in the heterogeneity of land titles curtails land market activity:

\[
\frac{\partial \theta_H}{\partial \varepsilon} < 0
\]

The first result describes the effect of a change in (unweighted) average land tenure security on the rural land market. As average land tenure insecurity increases, it becomes more likely that a case 1 or case 2 land market emerges; i.e., it becomes less likely that land market outcomes influence subsequent migration decisions. Once this is the case, type \( H \) land plots become less liquid, and land market activity declines. This is an important but intuitive result. It says that tenure security makes it easier for households to conduct land market transactions. The relationship between land market activity and the security of landowner rights supplements the existing literature on the importance of property rights in land markets. While other researchers focus on the impact of tenure insecurity on incentives for investment and the ability to use land as collateral [10, 69, 12, 21], I emphasize its effect on land transferability. Establishing the link between tenure insecurity and the equilibrium volume of transactions in a land market is one of the main theoretical contributions of the paper. Note that without private information, the security of ownership would not affect the number of land sales.\(^7\)

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\(^7\)Under perfect information, changes in the \( \lambda \) parameters would affect equilibrium prices, but markets would clear for any \( \{\lambda_L, \lambda_H\} \) distribution.
The second part of Proposition 4.4.1 describes how a land market is adversely affected by the variability in land title quality. Greater dispersion in land tenure security (the $\lambda$s) makes it more difficult for landowners with more secure property rights to trade. Substantial differences in tenure security between plots renders the information asymmetry problem more severe. Land market transactions are rare in such environments because the ratio of buyers to sellers adjusts as an effective sorting device.

An accessible system of clear property titles allows land market participants to counteract the effect of tenure uncertainty. It is conceptually possible for a comprehensive land titling initiative to eliminate the asymmetric information problem altogether. A complete and widespread system of property titles in conjunction with a mechanism for land dispute resolution could avoid the manifestation of illiquidity as a screening device if potential buyers can access records of conclusive land title at a low cost. Even when land registration is less comprehensive, land reforms are likely to have several intermediate effects. For example, land legislative reforms should help reduce the incidence and cost of land disputes, as well as reduce the probability of losing ownership due to forced land takings. Proposition 4.4.1(i) therefore supports land titling initiatives and other land reforms if the aim is to increase the transferability of land via enhanced security of landownership. In addition, land titling efforts should reduce the differences in land tenure security across plots, which is more symptomatic of informal institutions with complex systems of customary land rights. A formal land title is more likely to pin down the property rights for all owners of registered plots, especially if land registration involves a cadastral map. Proposition 4.4.1(ii) would then imply reinforced land transferability as a result of the reduction in
uncertainty surrounding tenure security. These arguments are supported empirically in Section 4.5 with estimates of land market participation for owners of registered and unregistered land.

4.4.1 Efficiency

Inefficiencies in the land market distort workers’ migration decisions and lead to an inefficient allocation of agents across sectors. The flow of workers from farm to off-farm employment stalls when sellers face a low probability of a successful land transaction. From a labour allocation point of view, all urban job offers should be accepted whenever $w^U - w^R > c$. This efficient allocation of agents across sectors only occurs in a case 1 equilibrium. The case 1 outcome is efficient in this sense only because the urban wage is so high that the land market becomes irrelevant to the migration decision. In case 2 and case 3 environments, only a fraction $b(p_H)/S_H < 1$ of job offers received by type $H$ landowners are accepted in equilibrium. Migration flows are therefore sub-optimal whenever $w^U - w^R < (1 - \alpha)(1 - \lambda_H)\pi$.

Illiquidity in the land market enables buyers to distinguish between the types of land plots. It allows plots of all types (at least some plots of each type) to be traded in equilibrium even in an environment with tenure insecurity and adverse selection. The endogenous screening mechanism therefore partly corrects for market failure, moving the equilibrium closer to the full information benchmark: an equilibrium where all beneficial land transactions take place. In certain situations, however, it’s possible that the inefficiencies that arise from screening outweigh the benefits of treating both types distinctly. As Guerrieri et al. [35] show, a pooling allocation can Pareto dominate the separating competitive search equilibrium. This occurs when there are not
too many insecure plots. In other words, the land market equilibrium might not be constrained Pareto efficient. Proposition 4.4.2 summarizes the efficiency results.

**Proposition 4.4.2.** The equilibrium labour flow from agricultural to off-farm sectors is inefficiently low whenever the wage gap is not too large; that is, if

\[ w^U - w^R < (1 - \alpha)(1 - \lambda_H)\pi. \]

The competitive land market equilibrium is constrained Pareto optimal if the proportion of type L plots of land is sufficiently large:

\[
\frac{S_L}{S_L + S_H} > \begin{cases} 
\frac{(1 - \alpha)(1 - \lambda_H)\pi - c}{(1 - \lambda_H)\pi - \alpha(1 - \lambda_L)\pi - c} & \text{if case 1} \\
\frac{w^U - w^R - c}{(1 - \lambda_H)\pi - \alpha(1 - \lambda_L)\pi - c} & \text{if case 2} \\
\frac{w^U - w^R - c}{w^U - w^R + (\lambda_L - \lambda_H)\pi - c} & \text{if case 3}
\end{cases}
\]

The next proposition describes how a land market might not function at all if land tenure becomes too insecure, or the information asymmetry becomes too extreme. Intuitively, the land market shuts down when the risk of land appropriation drives the gains from trade to zero. Interestingly, even when there are gains from transferring the relatively more secure land, all land transactions can be rendered infeasible because the screening mechanism unravels when the gains from trading the least secure land fall to zero.

**Proposition 4.4.3.** If land tenure becomes too insecure, such that \( \lambda \) rises above the

---

8When both buyers and sellers pay entry costs to participate in the land market, the market for secure land can shut down before the entire market collapses. This is because the cost of illiquidity can dominate the gains from trade in the type \( H \) market, while type \( L \) land continues to be readily transferable.
threshold

\[ \bar{\lambda} \equiv \frac{(1 - \varepsilon)(1 - \alpha)\pi - c}{(1 - \alpha)\pi}, \]

the land market shuts down completely. Moreover, a high degree of heterogeneity in land rights, \( \varepsilon \), restricts the range of \( \lambda \) for which a land market can operate.

Proposition 4.4.3 offers an explanation as to why land markets have failed to emerge in some developing countries. Low tenure security and asymmetric information might also explain why administrative land reallocations are sometimes used to transfer land when land sales markets are absent and the incidence of rental activity is low.

This result is related to other studies of asset markets in settings with private information. Guerrieri et al. [35] show that an entire asset market can shut down when the gains from trading the bad asset fall below zero. Lester et al. [52] construct an environment with information frictions in which some assets become non-transferable when the seller cannot recognize their quality. Chiu and Koeppel [16] model a financial asset market using search theory in a setting with adverse selection and show that the market “freezes” when the average quality of the asset falls below a certain threshold.

4.5 Evidence from Indonesian Land Markets

I test whether registered ownership is associated with a higher incidence of land market participation using an interesting micro dataset from Indonesia. The Indonesian data is particularly appropriate because of the way the government is implementing its land registry program. Indonesian farmland is being titled in a sporadic manner so that land with varying degrees of tenure security are spread across the country,
rather than having land certificates concentrated in specific areas. This provides a useful environment for examining the link between tenure security and land market participation.

4.5.1 Background

Over the past few decades, population growth, declining land fertility, and the conversion of agricultural land for non-farm utilization have made it more difficult for farming households in Indonesia to cultivate enough land to achieve a sufficient standard of living. Well-functioning land markets could allow households to seize off-farm labour market opportunities and transfer land to those who remain. As I have argued in this paper, land tenure insecurity and an ill-defined system of property rights over land reduce the scope of market transactions.

The system of land ownership rights in rural Indonesia exhibits many of the salient features of the theoretical model described above, including the following:

Land Tenure Insecurity: Sources of tenure insecurity in Indonesia include under-compensated land confiscation by governments and frequent land disputes. Land disputes among rural residents arise because of ill-defined boundary definitions, a complex system of land rights, and overlapping land deeds.

Heterogeneous Land Titles: Indonesia’s complicated system of land rights is the result of colonial governments instituting statutory law when land titles under traditional (adat) law already existed. The Basic Agrarian Law (Undang-undang Pokok Agraria/UUPA) of 1960 was established to unify both the traditional and the statutory land laws. The UUPA recognizes a variety of land rights, the most secure of which is Hak Milik, or the right of perpetual ownership and use.
Asymmetric Information: Article 19 of the UUPA states the government’s intention to register all land in the Republic of Indonesia. The purpose of the land registry is not only to protect the rights of landowners and users, but also to make available land right information for potential buyers, banks, real estate firms, and other interested individuals or organizations. As of 2006, however, only 36 percent of land had been registered, or about 34 million land certificates issued [70]. Until all land is registered, land title under customary law and land transactions of unregistered land are still recognized. Informal transfers are acknowledged in the court system but rely on private conveyances and possibly the testimony of a witness of the land transaction. “Private conveyancing is inefficient and potentially dangerous since it can be subject to fraud as there is no easy proof that the vendor is the true owner” [75].

An overview of the empirical section of the paper is as follows. First I describe the Indonesian data. Then I outline the empirical methods used to study the relationship between the possession of a land certificate and a farmer’s participation in the land market on the supply side. Comparing the land market participation of registered landowners and owners without legal documentation reveals to what extent tenure insecurity and asymmetric information influence the volume of land market transactions. To investigate further, I analyze the land market participation of unregistered owners using a proxy variable for tenure insecurity. By exploiting regional differences in tenure security in rural Indonesia, I find evidence to support Proposition 4.4.1; specifically, I find that regional land markets tend to be more active when tenure security is high (on average) and when there is minimal heterogeneity in tenure security across plots.
4.5.2 Data

The data used in the analysis come from the 2000 and 2007 rounds of the Indonesian Family Life Survey (IFLS). The IFLS is a longitudinal household panel dataset that began in 1993, but has expanded after four rounds to cover over 15,000 households in 13 of the 27 Indonesian provinces. The surveys cover standard household and individual level characteristics, farm assets, labour earnings, and land use data. The IFLS collects additional community level information about land use and off-farm sectors. Only the most recent survey directly asked landowners about land certificates. Fortunately, respondents were also asked when the land title/document was obtained, which allows one to determine which households held land certificates in previous IFLS waves. I use the two most recent cross sections of the IFLS because of the expanded sample size relative to the first two waves. Since there are missing values in the date of certification variable in the most recent survey, most of the econometric analysis focuses on the 2007 cross section (IFLS4), with one lagged land market variable from the 2000 wave (IFLS3).

I limit the sample to IFLS households that own farmland. The IFLS sample is large enough that even after this restriction, 3,607 IFLS4 households and 2,746 IFLS3 households are left for the analysis. The sales market for land in rural Indonesia is almost non-existent: less than one percent of the sample sold land in the last year. Instead, households conduct land transactions in the lease market. It could be that rental or profit-sharing arrangements are preferred when potential buyers are borrowing constrained. Table 4.1 provides basic descriptive statistics for the relevant subset of the IFLS4 sample. The data reveal a moderate level of activity on the supply side of the land lease market: 25.1 percent of households with land certificates lease

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out land, while only 16.5 percent of households without formal title documentation rent out or profit-share land. Conditional on renting out land, the total area of land rented out does not appear to be constrained for unregistered landowners relative to registered owners. The data suggest that land market frictions affect the extensive but not the intensive margin.\(^9\)

Moreover, households with at least one member operating an off-farm enterprise are more likely to possess land certificates: 41.1 percent of households with registered farmland are involved in non-agriculture business, while only 33.7 percent of unregistered households are operating businesses in the off-farm economy. Furthermore, households with land certificates tend to supply more labour to the non-agricultural economy (0.55 members relative to 0.41 members for households with unregistered land). Descriptive statistics therefore illustrate the linkages between land certification, land market participation, and off-farm economic activity. One interpretation of the summary evidence is that the owners of registered land experience greater tenure security and possess credible documentation to convey their secure ownership. The market for certified land is consequently more active than the market for untitled land, allowing farmers with land certificates to lease out land and accept off-farm employment more readily.

### 4.5.3 Empirical Analysis

To investigate the effect of land certificates on supply side land market participation, a difference-in-difference methodology is applied to the IFLS data. In addition to land

\(^9\)This is inconsistent with land models with transaction costs proportional to the area rented, as in [20], where land market frictions affect both the participation decision and the amount of land to rent in or out.
Table 4.1: Summary Statistics

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>landowners with certificates</th>
<th>landowners without certificates</th>
<th>t test</th>
<th>H_A: diff &gt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>std. error</td>
<td>obs.</td>
<td>mean</td>
</tr>
<tr>
<td>supply side land market participation rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rental market</td>
<td>0.131</td>
<td>0.0102</td>
<td>1094</td>
<td>0.0961</td>
</tr>
<tr>
<td>lease market</td>
<td>0.121</td>
<td>0.00985</td>
<td>1094</td>
<td>0.0673</td>
</tr>
<tr>
<td>both markets</td>
<td>0.251</td>
<td>0.0131</td>
<td>1100</td>
<td>0.165</td>
</tr>
<tr>
<td>size and value of household farmland endowments</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>size (in hectares)</td>
<td>2.184</td>
<td>0.780</td>
<td>1084</td>
<td>1.697</td>
</tr>
<tr>
<td>value (in Rupiah) per hectare</td>
<td>4.92e+08</td>
<td>5.39e+07</td>
<td>1077</td>
<td>2.87e+08</td>
</tr>
<tr>
<td>size (in hectares) of land rented out</td>
<td>0.683</td>
<td>0.0905</td>
<td>143</td>
<td>2.888</td>
</tr>
<tr>
<td>non-agricultural economic activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>participation in off-farm business</td>
<td>0.411</td>
<td>0.0148</td>
<td>1100</td>
<td>0.337</td>
</tr>
<tr>
<td>members earning off-farm wages</td>
<td>0.552</td>
<td>0.0232</td>
<td>1100</td>
<td>0.405</td>
</tr>
</tbody>
</table>

Notes: The sample is restricted to Indonesian households that owned farmland in 2007. Single asterisk denotes statistical significance at the 90% level of confidence, double 95%, triple 99%.
Source: 2007 Indonesian Family Life Survey (IFLS4), calculations by author.
registration, one would expect a household’s demographic characteristics to affect the decision to participate in the land market. The age, gender, and educational attainment of the household head are included as independent variables. To account for market imperfections for non-land factors of production, the number of adult males and females in the household are included, as well as the market value of livestock and farm equipment. If land market transactions are driven by off-farm labour market opportunities as in the theoretical model, supplying land in the lease market should also be related to the participation of household members in non-agricultural sectors and the value of non-farm assets. Off-farm economic variables are therefore included as well. Market transactions might correct for an unequal distribution of land by transferring land from households with large landholdings to households lacking sufficient acreage. Accordingly, the area of owned land per household member is included among the explanatory variables. Finally, provincial dummy variables are included to account for differences in population densities and the possibility that off-farm economic opportunities differ across regions. The probit model to be estimated is therefore

\[
Pr(y_i = 1) = \Phi(\beta_0 + \beta_1 c_i + \beta_2 d_2007 + \beta_3 X_i + \beta_4 Y_i + \beta_5 Z_i + \beta_6 P_i),
\]

where \(i\) indexes households, \(y_i\) is an indicator variable for supply side land market participation, the \(\beta\)s are coefficients or vectors of coefficients, \(c_i\) indicates possession of a land certificate, \(d_2007\) is an indicator for the 2007 survey, \(X_i\) is a vector of household demographic characteristics, \(Y_i\) is a vector of farm-related variables, \(Z_i\) is a vector of off-farm economic variables, \(P_i\) is a vector of provincial dummies, and \(\Phi\) is the standard normal distribution function.
The variable of interest is \( c_i \), which indicates the ownership of registered land. More specifically, \( c_i \) is a dummy variable for the possession of a land certificate securing the *Hak Milik* title. This type of land right is intended to increase tenure security by providing perpetual ownership, user rights, and transferability. In the context of the theory, land certification and the presence of an accessible land registry reduce the asymmetric information problem and eliminate the need for illiquidity as a screening mechanism. Therefore, the market for registered land should be more active than the market for unregistered land, since unregistered land is subject to the inefficiencies associated with tenure insecurity and private information.

Rental and profit-sharing arrangements can be long lasting, and past participation in the land lease market is likely related to the current participation decision. To account for this, I estimate probit models for the binary decision to lease out land in 2007 (\( y_i \) in IFLS4, or \( y_{2007i} \)) including the land variable from 2000 (\( y_i \) in IFLS3, or \( y_{2000i} \)) on the right-hand side. The probability of supplying land to the lease market in 2007 is represented by the probit equation

\[
Pr(y_{2007i} = 1) = \Phi(\beta_0 + \beta_1 c_i + \beta_2 y_{2000i} + \beta_3 X_i + \beta_4 Y_i + \beta_5 Z_i + \beta_6 P_i), \quad (4.7)
\]

where \( i \) indexes households; \( X_i, Y_i, Z_i \), and \( P_i \) are defined as before; and \( \Phi \) is the standard normal distribution function.
4.5.4 Empirical Findings

The Impact of Land Certification

The land market participation difference-in-difference estimation is presented in Table 4.2. Since equation (4.6) is a nonlinear model, the interaction effect can be computed according to Ai and Norton [1] so as to avoid misinterpreting the difference in the land market outcomes of registered versus unregistered landowners and the statistical significance of the result. Applying their method of estimating the magnitude and standard errors of the interaction effects for the first specification yields a mean interaction effect of 0.0732 with a mean z-statistic of 2.46 (significant at the 95 percent confidence level). For the remaining specifications, the estimated mean interaction term is similar in magnitude (between 0.0610 and 0.0675), and the mean z-statistics establish significance at the 95 percent confidence level. Moreover, for all four specifications, the interaction effect is positive for the entire sample; that is, for all combinations of independent variables that exist in the sample.

The implication of the above is that a household having acquired a legal land title between the IFLS3 and IFLS4 surveys has a higher propensity to lease out land. This empirical trend is robust to the inclusion of farm variables (columns (2) and (4)) and non-farm economic variables (columns (3) and (4)). Proponents of land certification programs argue that registered land offers greater tenure security and reduces the asymmetry of information in land market transactions. The findings are therefore consistent with the idea that tenure insecurity and asymmetric information reduce liquidity in land markets. There is no attempt to control for the possibility that landowners acquire certificates in anticipation of supply side land market participation. This would be important if the objective was to evaluate the impact of
exogenous land titling initiatives, as in Holden et al. [39]. Regardless of the direction of causation, the fact that land certificates are associated with more active land markets supports the notion that tenure insecurity and asymmetric information affect how easily land transactions can be carried out.

As expected, farm asset ownership is negatively related to land market participation on the supply side, although household labour endowments are not significant predictors of land market participation, except for the number of adult males in column (4). Off-farm business activity, off-farm employment, and non-farm asset ownership are positively related to leasing out land. These results are in line with the presupposition that the demand for land market transactions is stimulated by off-farm economic opportunities, as in the theoretical framework. The size of owned farmland is important in the participation decision, as households with greater per capita land endowments are more likely to rent out land.

Table 4.3 presents the comparable estimates from probit regressions for the most recent cross section of the IFLS, using a binary variable indicating land market participation on the supply side in 2007. Some of the land rental and profit-sharing arrangements in the IFLS4 could simply be the continuation of prior arrangements. To account for this, the land market variable from 2000 (i.e., supply side land market participation in the IFLS3) is included as an explanatory variable. Past participation is a significant predictor of current participation in all four specifications. Focusing again on the variable of interest, the probit results reinforce the findings from the difference-in-difference analysis. Possession of a land certificate is associated with an increased propensity to supply land by 3.3 to 4.3 percentage points.

The difference-in-difference and probit results have established a clear link between
land certificates and land market activity. Tenure insecurity, unclear boundary definitions, and the complexity of the customary system of property rights are preventing unregistered land from being readily transferred in the rental and sales markets. Acquiring a formal legal title improves the liquidity of farmland. Further analysis of unregistered land is required to establish whether the theoretical model presented in Section 4.2 is a reasonable representation of land markets with uncertainty and private information about tenure security.

**Tenure Insecurity in the Market for Unregistered Land**

The degree of tenure security associated with a particular plot is unobservable. It is precisely this hidden information that renders land illiquid in order for potential buyers to distinguish the ownership rights associated with a vendor’s plot. Although private information makes it difficult for econometric analysis, a proxy variable can be backed out of the IFLS data which, according to the theory, is correlated with the $\lambda$ parameters. Appendix D provides the details for the construction of the proxy variable, $\lambda$. Essentially, land tenure is assumed to be insecure if annual farm output is high relative to the value of the plot. This is based on the premise that owners of insecure land discount future farm profit at a higher rate, which lowers their self-assessed valuation of their land. Taking the ratio of land rent to land value is one way of backing out the degree of ownership insecurity.

Proposition 4.4.1 can be tested empirically using $\lambda$ as a proxy variable for unobservable tenure insecurity. Proposition 4.4.1(i) describes the negative relationship between average tenure insecurity and land market activity, while Proposition 4.4.1(ii) characterizes a negative relationship between the dispersion of the $\lambda$s and
Table 4.2: Difference-in-Difference Results for Supply Side Land Market Participation

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>land certificate × 2007 indicator</td>
<td>0.297**</td>
<td>0.300**</td>
<td>0.269**</td>
<td>0.279*</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
<td>(0.137)</td>
<td>(0.136)</td>
<td>(0.142)</td>
</tr>
<tr>
<td>land certificate</td>
<td>-0.0991</td>
<td>-0.119</td>
<td>-0.101</td>
<td>-0.118</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.122)</td>
<td>(0.123)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>2007 indicator</td>
<td>0.0754</td>
<td>0.0328</td>
<td>0.0874</td>
<td>0.0401</td>
</tr>
<tr>
<td></td>
<td>(0.0569)</td>
<td>(0.0598)</td>
<td>(0.0591)</td>
<td>(0.0619)</td>
</tr>
<tr>
<td>per capita farmland (log)</td>
<td>0.0293**</td>
<td>0.0496***</td>
<td>0.0404***</td>
<td>0.0571***</td>
</tr>
<tr>
<td></td>
<td>(0.0145)</td>
<td>(0.0156)</td>
<td>(0.0151)</td>
<td>(0.0163)</td>
</tr>
<tr>
<td>head’s age</td>
<td>0.0237**</td>
<td>0.0274**</td>
<td>0.0162</td>
<td>0.0243*</td>
</tr>
<tr>
<td></td>
<td>(0.0118)</td>
<td>(0.0129)</td>
<td>(0.0121)</td>
<td>(0.0132)</td>
</tr>
<tr>
<td>head’s age squared</td>
<td>-0.000165</td>
<td>-0.000218*</td>
<td>-9.10e-05</td>
<td>-0.000185</td>
</tr>
<tr>
<td></td>
<td>(0.000121)</td>
<td>(0.000131)</td>
<td>(0.000123)</td>
<td>(0.000133)</td>
</tr>
<tr>
<td>head’s gender (female = 1)</td>
<td>0.387***</td>
<td>0.199**</td>
<td>0.452***</td>
<td>0.238***</td>
</tr>
<tr>
<td></td>
<td>(0.0756)</td>
<td>(0.0852)</td>
<td>(0.0771)</td>
<td>(0.0867)</td>
</tr>
<tr>
<td>head’s years of schooling</td>
<td>0.0596***</td>
<td>0.0439***</td>
<td>0.0492***</td>
<td>0.0367***</td>
</tr>
<tr>
<td></td>
<td>(0.00552)</td>
<td>(0.00591)</td>
<td>(0.00602)</td>
<td>(0.00640)</td>
</tr>
<tr>
<td>household males 15-60 years</td>
<td>-0.0557</td>
<td>-0.0783**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0357)</td>
<td>(0.0370)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>household females 15-60 years</td>
<td>0.0628*</td>
<td></td>
<td>0.0378</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0370)</td>
<td></td>
<td>(0.0383)</td>
<td></td>
</tr>
<tr>
<td>indicator for ownership of livestock or farm assets</td>
<td>-1.211***</td>
<td>-1.172***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0823)</td>
<td></td>
<td>(0.0851)</td>
<td></td>
</tr>
<tr>
<td>value of farm assets (log)</td>
<td>-0.0220*</td>
<td></td>
<td>-0.0167</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0127)</td>
<td></td>
<td>(0.0131)</td>
<td></td>
</tr>
<tr>
<td>non-farm business indicator</td>
<td>0.312***</td>
<td>0.250***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0505)</td>
<td>(0.0535)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>value of non-farm assets (log)</td>
<td>0.0540***</td>
<td>0.0456***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0146)</td>
<td>(0.0157)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>share of household members with off-farm employment</td>
<td>0.365***</td>
<td>0.287***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0830)</td>
<td>(0.0874)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>provincial indicators</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>observations</td>
<td>4,219</td>
<td>4,066</td>
<td>4,053</td>
<td>3,911</td>
</tr>
</tbody>
</table>

Notes: Parameter estimates from the probit estimation of equation (4.6). Standard errors in parentheses. Single asterisk denotes statistical significance at the 90% level of confidence, double 95%, triple 99%.

Source: Indonesian Family Life Survey (IFLS), calculations by author.
Table 4.3: Probit Results for Supply Side Land Market Participation in 2007

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>land certificate</td>
<td>0.169**</td>
<td>0.152**</td>
<td>0.145**</td>
<td>0.136*</td>
</tr>
<tr>
<td></td>
<td>(0.0720)</td>
<td>(0.0758)</td>
<td>(0.0740)</td>
<td>(0.0776)</td>
</tr>
<tr>
<td>supply side land market participation in 2000</td>
<td>0.645***</td>
<td>0.569***</td>
<td>0.640***</td>
<td>0.578***</td>
</tr>
<tr>
<td></td>
<td>(0.0831)</td>
<td>(0.0886)</td>
<td>(0.0848)</td>
<td>(0.0901)</td>
</tr>
<tr>
<td>per capita farmland (log)</td>
<td>0.0414*</td>
<td>0.0653**</td>
<td>0.0427*</td>
<td>0.0603**</td>
</tr>
<tr>
<td></td>
<td>(0.0229)</td>
<td>(0.0255)</td>
<td>(0.0237)</td>
<td>(0.0261)</td>
</tr>
<tr>
<td>head’s age</td>
<td>0.00686**</td>
<td>0.00527*</td>
<td>0.00615**</td>
<td>0.00506*</td>
</tr>
<tr>
<td></td>
<td>(0.00269)</td>
<td>(0.00286)</td>
<td>(0.00277)</td>
<td>(0.00293)</td>
</tr>
<tr>
<td>head’s gender (female = 1)</td>
<td>0.409***</td>
<td>0.260**</td>
<td>0.467***</td>
<td>0.293**</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.116)</td>
<td>(0.105)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>head’s years of schooling</td>
<td>0.0594***</td>
<td>0.0386***</td>
<td>0.0492***</td>
<td>0.0318***</td>
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<tr>
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<td>(0.00776)</td>
<td>(0.00842)</td>
<td>(0.00837)</td>
<td>(0.00900)</td>
</tr>
<tr>
<td>household males 15-60 years</td>
<td>-0.00525</td>
<td>-0.0347</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0484)</td>
<td>(0.0502)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>household females 15-60 years</td>
<td>0.0651</td>
<td>0.0443</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0522)</td>
<td>(0.0533)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>indicator for ownership of livestock or farm assets</td>
<td>-1.311***</td>
<td>-1.259***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>value of farm assets (log)</td>
<td>-0.0302*</td>
<td>-0.0218</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0182)</td>
<td>(0.0186)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>non-farm business indicator</td>
<td></td>
<td></td>
<td>0.295***</td>
<td>0.222***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.0704)</td>
<td>(0.0746)</td>
</tr>
<tr>
<td>value of non-farm assets (log)</td>
<td>0.0584***</td>
<td>0.0462**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0214)</td>
<td>(0.0233)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>share of household members with off-farm employment</td>
<td>0.345***</td>
<td>0.255**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>provincial indicators</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>observations</td>
<td>2,218</td>
<td>2,146</td>
<td>2,160</td>
<td>2,094</td>
</tr>
</tbody>
</table>

Notes: Parameter estimates from the probit estimation of equation (4.7). Standard errors in parentheses. Single asterisk denotes statistical significance at the 90% level of confidence, double 95%, triple 99%.

Source: Indonesian Family Life Survey (IFLS), calculations by author.
the thickness of the land market. These implications of the theory can be verified in the IFLS data by exploiting differences in the distributions of \( \lambda \) across regions. Figure 4.1 displays the disparities between two Indonesian regions for unregistered land. Land in Java is relatively secure, while the distribution of \( \lambda \) in Sumatra suggests that land is more insecure.

Table 4.4 contains the first and second moments of the distributions of \( \lambda \), as well as the share of landowners participating on the supply side of the market for five geographical regions. It is easy to detect a pattern of high land market participation in regions where tenure is on average more secure. Land markets are most active in Java, Nusa Tenggara, and Sulawesi, where land is secure (low average \( \lambda \)) and the information problem is limited (small standard deviation of \( \lambda \)). The two regions with the least active land markets among unregistered landowners, Kalimantan and Sumatra, have the most severe average tenure insecurity as well as the distributions with the most dispersion.

To investigate further, variables can be generated by computing the mean and standard deviation of \( \lambda \) in the household’s province of residence. The distributions are highly skewed, and the first two moments are highly sensitive to outliers. To circumvent this issue, a transformation of \( \lambda \) is computed in order to derive a proxy variable for land tenure security \( LT_{proxy} \equiv - \log(\lambda) \). The distribution of \( LT_{proxy} \) has nicer properties than the distribution of \( \lambda \). The variables \( LT_{proxy\text{MEAN}} \), \( LT_{proxy\text{SD}} \), and \( LT_{proxy\text{CV}} \) are generated by computing the mean, standard deviation, and coefficient of variation of \( LT_{proxy} \) in the household’s province of residence. Including average tenure security, and the dispersion of tenure
security across plots within each province in the probit models of land market partic-
ipation allows Proposition 4.4.1 to be tested formally. While the coefficients have the
appropriate signs (not shown), the significance levels are low (significant only at the
10 percent level) when both variables are included. This is likely because provinces
with low tenure security also tend to suffer from high degrees of heterogeneity in
land rights. Instead of including both the mean and the standard deviation, only the
coefficient of variation, \(LTSproxyCV\), is included in the probit models. In a given
province, heterogeneity of ownership rights across plots (high \(LTSproxySD\)) implies
a high \(LTSproxyCV\), and should limit land market participation according to Propo-
sition 4.4.1(ii). Low average tenure security (low \(LTSproxyMEAN\)) also implies a
high \(LTSproxyCV\), and should curb participation in the land rental and lease markets
according to Proposition 4.4.1(i).

The probability of supplying unregistered land to the lease market in 2007 is
represented by the probit equation

\[
Pr(y_{2007i} = 1) = \Phi(\beta_0 + \beta_1 LTSproxyCV_j + \beta_2 y_{2000i} + \beta_3 X_i + \beta_4 Y_i + \beta_5 Z_i),
\]  

(4.8)

where \(i\) indexes households; \(j\) indexes provinces; \(y_{2007i}, y_{2000i}, X_i, Y_i,\) and \(Z_i\) are de-

defined as before; and \(\Phi\) is the standard normal distribution function. Table 4.5 presents
the results for the sample of landowners without legal documentation of ownership.
The standard errors are adjusted to allow for clustering based on the 13 provinces
included in the IFLS sample. Households living in provinces with high \(LTSproxyCV\)
are much less likely to be conducting land market transactions. For example, a de-
crease in the coefficient of variation from the highest to the lowest provincial statistic
is associated with an increase in the probability of supply side land market participation of roughly 8 or 9 percentage points. This is highly significant even with farm and non-farm related variables included on the right-hand side. Since $LTSproxyCV$ is proportional to the standard deviation of the proxy variable and negatively related to the provincial average degree of tenure security, this result supports both parts of Proposition 4.4.1.

Figure 4.1: The kernel density estimation of $\lambda$ by region for unregistered land
Table 4.4: Land Market Activity and *lambda* by Region

<table>
<thead>
<tr>
<th>REGION</th>
<th>share of landowners supplying land (%)</th>
<th>lambda</th>
<th>mean</th>
<th>std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sumatra</td>
<td>14.267</td>
<td>0.238</td>
<td>0.234</td>
<td></td>
</tr>
<tr>
<td>Java</td>
<td>18.182</td>
<td>0.162</td>
<td>0.181</td>
<td></td>
</tr>
<tr>
<td>Nusa Tenggara</td>
<td>15.194</td>
<td>0.142</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>Kalimantan</td>
<td>12.935</td>
<td>0.410</td>
<td>0.296</td>
<td></td>
</tr>
<tr>
<td>Sulawesi</td>
<td>22.283</td>
<td>0.204</td>
<td>0.191</td>
<td></td>
</tr>
<tr>
<td>All Regions</td>
<td>16.394</td>
<td>0.206</td>
<td>0.204</td>
<td></td>
</tr>
</tbody>
</table>

Source: 2007 Indonesian Family Life Survey (IFLS4), calculations by author.

Table 4.5: Probit Results for Supplying Unregistered Land in 2007

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTSproxyCV</td>
<td>-1.150***</td>
<td>-1.103***</td>
<td>-1.029**</td>
<td>-1.025***</td>
</tr>
<tr>
<td></td>
<td>(0.434)</td>
<td>(0.404)</td>
<td>(0.418)</td>
<td>(0.367)</td>
</tr>
<tr>
<td>per capita farmland (log)</td>
<td>0.0333</td>
<td>0.0446**</td>
<td>0.0434*</td>
<td>0.0477*</td>
</tr>
<tr>
<td></td>
<td>(0.0209)</td>
<td>(0.0194)</td>
<td>(0.0260)</td>
<td>(0.0252)</td>
</tr>
<tr>
<td>supply side land market participation in 2000</td>
<td>0.704***</td>
<td>0.611***</td>
<td>0.697***</td>
<td>0.624***</td>
</tr>
<tr>
<td></td>
<td>(0.0851)</td>
<td>(0.116)</td>
<td>(0.0931)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>household variables</td>
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<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>farm variables</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>non-farm variables</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>provincial indicators</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>observations</td>
<td>1,533</td>
<td>1,479</td>
<td>1,495</td>
<td>1,446</td>
</tr>
</tbody>
</table>

Notes: Parameter estimates from the probit estimation of equation (4.8). Standard errors in parentheses. Single asterisk denotes statistical significance at the 90% level of confidence, double 95%, triple 99%. Household variables include head’s age, head’s gender, and head’s years of schooling. Farm variables include household males and females 15-60 years, indicator for ownership of livestock or farm assets, and value of farm assets (log). Non-farm variables include non-farm business indicator, value of non-farm assets (log), and share of household members with off-farm employment.

Source: Indonesian Family Life Survey (IFLS), calculations by author.
4.6 Concluding Remarks

This paper proposes a theory to explain the varied levels of activity across land markets in developing countries in terms of the security of property rights over land. Despite the discussion of land market thinness in the literature, tenure insecurity in most land market models either affects the price of land [27] or the presence of transaction costs [10, 20]. In contrast, I construct a model that allows one to explicitly characterize important aspects of the land market such as the volume of trade and the likelihood of selling a particular plot. The results show how the number of land transactions in an economy with an emerging off-farm labour market can be hindered by tenure insecurity: a characteristic of many property rights regimes over land. Moreover, when ownership security becomes more varied across plots, the adverse selection problem worsens and land market activity declines.

Enhancing tenure security is often cited as a motivation for land titling programs in developing countries. Land titles allow for the verification of ownership, which reduces the incidence of land disputes as well as land market transaction costs in general. The availability of records reduces asymmetric information about ownership and the quality of the land title. In smallholder farming communities in Indonesia, land registration has endowed land users with more secure rights and triggered a higher volume of land transfers. The result that landowners without legal documentation cannot transact land as readily as certified owners is evidence that tenure insecurity and private information about property rights lead to thin land markets. Empirically, a novel contribution of this study is the derivation of a proxy variable for land tenure insecurity, which has traditionally been difficult to measure. This widened the scope of the study to include analyses of supply side land participation among unregistered landowners.
Chapter 5

Summary and Conclusions

5.1 Summary

A number of authors have proposed search-theoretic frameworks to study trading frictions in labour and real estate markets. While existing search models can account for a wide range of the empirical trends, I argue that off-the-shelf search frameworks are often inconsistent with actual market interactions and sometimes ignore important features of the environment. For instance, some models do not allow for multiple offers by competing bidders, while others ignore the possibility of renegotiating offers announced ex ante when there are ex post incentives to do so. Instead, most analyses have adopted standard random or competitive search frameworks in which prices are determined via bilateral bargaining or non-negotiable price-posting. In other contexts, full information is assumed when certain features of the trading environment would be more appropriately modelled as private information.

This thesis develops search theoretic models of decentralized markets (labour,
housing and land) with features consistent with real world observations. In Chapters 2 and 3, I emphasize the importance of commitment for market efficiency and liquidity when there is private information. Without it, the volume of transactions in a market is reduced because of adverse selection and distorted entry incentives. In some markets, institutions have emerged to cope with these issues. For example, real estate brokerage has the potential to segment the housing market, alleviate the information problem and increase housing market efficiency. Chapter 4, in contrast, focuses on the case where the informed seller’s private information directly affects the uninformed buyer’s utility. A competitive search equilibrium in the land market allows market participants to illicit private information about insecure property rights and reduce the severity of the adverse selection problem. Together, the three essays contained in this thesis demonstrate the significance and far-reaching implications of asymmetric information in a directed search framework. This thesis contributes to the literature by developing frameworks that allow one to understand the mechanisms by which the combination of search and information frictions leads endogenously to illiquidity.

5.2 Future Work

While the details of labour, land or housing markets are typically very different from other markets, some key features are similar. For example, many financial securities are traded in a decentralized manner in over-the-counter markets. Similarly, debt contracts are often negotiated bilaterally when borrowers and lenders are brought together through a process of search and matching. In the labour market, firms and workers must spend time and money implementing a search strategy, and there is
often imprecise information about productivity, match quality, and compensation.

Future work will address the determinants and dynamics of the endogenous liquidity of other markets. While equilibrium price paths and the functioning of markets over time is important for macroeconomic analysis, considerable attention will be paid to the matching process and pricing protocols so as to capture key features of actual markets. This is important because price movements can be very sensitive to these microeconomic details. A second feature that my projected research agenda will continue to address is the nature of the matching equilibrium if there is hidden information about asset quality, search effort, and the trading partner’s willingness to carry out a transaction. As my current work exemplifies, search and information frictions can interact in interesting ways.
Bibliography


[37] Allen Head, Huw Lloyd-Ellis, and Amy Sun. Search, liquidity and the dynamics of house prices and construction. Working Papers 1276, Queen’s University, Economics Department, 2011.


Appendix A

Omitted Proofs from Chapter 2

Proof of Proposition 2.2.1. Suppose (for the sake of contradiction) that list prices are informative. The housing market can then be characterized by two submarkets: One submarket for sellers with list prices in $P_1$, and another submarket for sellers with list prices in $P_2$, $P_1 \cap P_2 = \emptyset$.

Consider a type $R$ seller in a submarket with $\sigma \in [0, 1]$, and $\theta$ determined by the free entry condition, $U(\sigma, \theta) = \kappa_0$, with

$$U(\sigma, \theta) = \begin{cases} e^{-\theta} \sigma(v - c_A) & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\ e^{-\theta} (v - c_R) & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} \end{cases}$$  \hspace{1cm} (A.1)

The seller's payoff would be

$$V_R(\sigma, \theta) = [1 - (1 + \theta)e^{-\theta}](v - c_R)$$ \hspace{1cm} (A.2)
Differentiating yields
\[ \frac{dV_R}{d\sigma} = \theta e^{-\theta}(v - c_R) \frac{d\theta}{d\sigma} \] (A.3)

The derivative \( d\theta/d\sigma \) can be obtained by differentiating the free entry condition and rearranging.
\[
\frac{d\theta}{d\sigma} = \begin{cases} 
1/\sigma & \text{if } \sigma > \frac{v - c_R}{v - c_A} \\
0 & \text{if } \sigma \leq \frac{v - c_R}{v - c_A} 
\end{cases} \] (A.4)

Therefore,
\[
\frac{dV_R}{d\sigma} > 0 \text{ if } \sigma > \frac{v - c_R}{v - c_A}, \quad \text{and} \quad \frac{dV_R}{d\sigma} = 0 \text{ if } \sigma \leq \frac{v - c_R}{v - c_A} \] (A.5)

Relaxed sellers prefer the submarket with the highest buyer-seller ratio. According to the free entry conditions, the submarket with the highest share of anxious sellers will have the highest buyer-seller ratio. This rules out a fully separating equilibrium since \( V_R(0, \theta_R) > V_R(1, \theta_A) \) requires \( \theta_R > \theta_A \), while the free entry conditions imply \( \theta_A > \theta_R \). It also rules out partial pooling equilibria with informative list prices, since two distinct submarkets can only be an equilibrium if \( \sigma_1, \sigma_2 < (v - c_R)/(v - c_A) \). Otherwise, the relaxed sellers have an incentive switch to the hotter submarket. Even in such cases that \( \sigma_1, \sigma_2 < (v - c_R)/(v - c_A) \), the free entry conditions imply \( \theta_1 = \theta_2 \), and buyer entry and bidding strategies are such that list prices are meaningless: this equilibrium is indistinguishable from random search.

Proof of Proposition 2.2.2. Assuming full separation, the expected payoff to a seller with reservation value \( c \) setting a reserve bid of \( p \in [0, v] \) is
\[
V(p, \theta) = \theta e^{-\theta}(p - c) + [1 - (1 + \theta)e^{-\theta}](v - c) \] (A.6)
with bidders arriving according to the free entry condition

\[ U(p, \theta) = e^{-\theta}(v - p) = \kappa_0 \]  \hspace{1cm} (A.7)

The optimal reserve bid is uniquely defined by the first order condition:

\[ \frac{dV}{dp} = \theta e^{-\theta} + \left[ e^{-\theta}(p - c) + \theta e^{-\theta}(v - p) \right] \frac{d\theta}{dp} = 0 \]  \hspace{1cm} (A.8)

The expression for \( d\theta/dp \) can be obtained by differentiating the free entry condition. After substituting this into the first order condition it becomes

\[ \frac{dV}{dp} = -e^{-\theta}(p - c) \frac{v - p}{v - p} = 0 \]  \hspace{1cm} (A.9)

The optimal reserve bid is therefore equal to the seller’s true reservation value, \( p = c \).

Since the arrival rate of buyers is a function of the reserve bid and not the seller’s type, there is no reason for any seller to deviate from their optimal reserve bid. In other words, relaxed sellers submit the reserve bid \( p_R = c_R \) in order to optimally trade-off transaction probability and expected price. Anxious sellers are more concerned about the probability of trade and so prefer to set a lower reserve bid of \( p_A = c_A \).

A separating allocation is thus implementable by the social planner. Moreover, the free entry conditions ((A.7) with \( \{p_A, p_R\} = \{c_A, c_R\} \)) are the same as equations (2.6) and (2.7), which implies that the arrival rates of buyers are efficient. The constrained efficient allocation described in Section 2.2.3 is thus implementable by a social planner even when reservation values are unobservable.

\[ \square \]

Proof of Lemma 1. Suppose that some REA offering \((a, z)\) earns positive profit. Free
entry of REAs implies that a new REA can offer \((a, z - \varepsilon)\) with \(\varepsilon > 0\). With perfect competition in the market for REAs, every seller in submarket \(a\) will then choose the new contract over the original one. Moreover, \(a\) is unchanged so buyers’ beliefs about seller types and submarket tightness remain the same. Finally, since \(\varepsilon\) can be arbitrarily small, it can be chosen so that the new real estate agent earns positive profit. REA entry remains profitable until \(z = \phi(a)\).

**Proof of Lemma 2.** Suppose (for the sake of contradiction) that \(a_R > 0\). By Lemma 1, \(z_R = \phi(a_R)\). Consider a new contract with \(a'_R < a_R\). If \(z'_R < z_R\), it attracts (at least) all the sellers that were attracted to the original contract. Moreover, the free entry condition for buyers imply that \(\theta'_R \geq \theta_R\) (with equality if the same set of sellers accept the new contract). The REA can set \(z'_R\) close enough to \(z_R\) that it earns a positive profit: a contradiction by Lemma 1.

**Proof of Lemma 3.** The relevant expected payoffs, assuming a separating equilibrium, are

\[
V_A(0, 0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_A) + \theta_R e^{-\theta_R} (c_R - c_A)
\]

(A.10)

\[
V_R(0, 0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}] (v - c_R)
\]

(A.11)

\[
V_A(z_A, 1, \theta_A) = [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_A) - z_A
\]

(A.12)

\[
V_R(z_A, 1, \theta_A) = [1 - (1 + \theta_A)e^{-\theta_A}] (v - c_R) - z_A
\]

(A.13)
The incentive compatibility constraint for relaxed sellers pins down the optimal commission, \( z_A \):

\[
V_R(0, 0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}](v - c_R)
\]

\[
= [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A = V_R(z_A, 1, \theta_A)
\]

Substituting the payoff functions from above, the incentive compatibility constraint can be rewritten as

\[
V_A(0, 0, \theta_R) - [1 - e^{-\theta_R}](c_R - c_A) = V_A(z_A, 1, \theta_A) - [1 - (1 + \theta_A)e^{-\theta_A}](c_R - c_A)
\] (A.15)

The fully separating market arrangement is incentive compatible for anxious sellers if

\[
V_A(z_A, 1, \theta_A) \geq V_A(0, 0, \theta_R)
\]

Using the condition derived above, a separating equilibrium requires

\[
[e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}](c_R - c_A) > 0 \iff e^{\theta_A - \theta_R} \geq 1 + \theta_A
\] (A.16)

The free entry conditions can be used to to solve for \( \theta_A \) and \( \theta_R \) explicitly.

\[
U(0, \theta_R) = e^{-\theta_R}(v - c_R) = \kappa_0 \iff \theta_R = \log \left( \frac{v - c_R}{\kappa_0} \right)
\] (A.17)

\[
U(1, \theta_A) = e^{-\theta_A}(v - c_A) = \kappa_0 \iff \theta_A = \log \left( \frac{v - c_A}{\kappa_0} \right)
\] (A.18)

The inequality above reduces to

\[
\kappa_0 \geq \exp \left( -\frac{c_R - c_A}{v - c_R} \right) (v - c_A)
\] (A.19)
As long as (2.21) is satisfied, the pair of submarkets is incentive compatible for both types.

Proof of Lemma 4. First consider the case where \( \sigma_0 \leq (v - c_R)/(v - c_A) \equiv \sigma \). Since buyer entry and bidding strategies are identical in both the type R submarket and the pooling submarket, relaxed sellers are indifferent between the two: \( V_R(0, 0, \theta_R) = V_R(0, \sigma_0, \theta_P) \). Moreover, a full pooling contract cannot strictly benefit anxious sellers because \( V_A(z_A, 1, \theta_A) \geq V_A(0, 0, \theta_R) = V_A(0, \sigma_0, \theta_P) \) by Lemma 3. Since neither seller type can achieve a strictly better expected payoff, (2.21) is a sufficient condition to rule out the possibility that sellers can benefit from pooling when \( \sigma_0 \leq \sigma \).

Next consider the case where \( \sigma_0 > (v - c_R)/(v - c_A) = \sigma \). The relevant expected payoffs, assuming a separating equilibrium, are (A.10), (A.11), (A.12), (A.13), and

\[
V_A(0, \sigma_0, \theta_P) = [1 - (1 + \theta_P)e^{-\theta_P}](v - c_A) \\
V_R(0, \sigma_0, \theta_P) = [1 - (1 + \theta_P)e^{-\theta_P}](v - c_R)
\]  

(A.20)  

(A.21)

With \( \sigma_0 > \sigma \), relaxed sellers prefer the pooling submarket because of the higher buyer-seller ratio: \( V_R(0, \sigma_0, \theta_P) > V_R(0, 0, \theta_R) \). Whether anxious sellers prefer the pooling submarket depends on \( \kappa_0 \) and \( \sigma_0 \).

The incentive compatibility constraint for relaxed sellers pins down the optimal commission rate for full separation, \( z_A \):

\[
V_R(0, 0, \theta_R) = [1 - (1 + \theta_R)e^{-\theta_R}](v - c_R) \\
= [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A = V_R(z_A, 1, \theta_A)
\]  

(A.22)
Substituting the pooling payoff (A.21) from above, the constraint becomes

\[
V_R(0, \sigma_0, \theta_P) + [(1 + \theta_P)e^{-\theta_P} - (1 + \theta_R)e^{-\theta_R}](v - c_R) \\
= [1 - (1 + \theta_A)e^{-\theta_A}](v - c_R) - z_A
\]

(A.23)

Substituting the type A payoff functions (A.12) and (A.20) from above, the constraint can be rewritten and rearranged to obtain

\[
V_A(0, \sigma_0, \theta_P) - V_A(z_A, 1, \theta_A) = [(1 + \theta_R)e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}](v - c_R) \\
+ [(1 + \theta_A)e^{-\theta_A} - (1 + \theta_P)e^{-\theta_P}](v - c_A)
\]

(A.24)

The preference for pooling among anxious sellers, \(V_A(0, \sigma_0, \theta_P) \geq V_A(z_A, 1, \theta_A)\), therefore requires

\[
\frac{v - c_R}{v - c_A} \geq \frac{(1 + \theta_P)e^{-\theta_P} - (1 + \theta_A)e^{-\theta_A}}{(1 + \theta_R)e^{-\theta_R} - (1 + \theta_A)e^{-\theta_A}}
\]

(A.25)

Using the free entry conditions, we can substitute for the buyer-seller ratios to obtain

\[
\frac{v - c_R}{v - c_A} \geq \left[1 + \log \left(\frac{\sigma_0(v - c_A)}{\kappa_0}\right)\right] \frac{\kappa_0}{\sigma_0(v - c_A)} - \left[1 + \log \left(\frac{v - c_A}{\kappa_0}\right)\right] \frac{\kappa_0}{v - c_A}
\]

(A.26)

Rearranging to isolate \(\kappa_0\) yields

\[
[(1 - \sigma_0)(v - c_A) - \sigma_0(c_R - c_A)] \log \kappa_0 \\
\geq [(1 - \sigma_0)(v - c_A) - \sigma_0(c_R - c_A)] - \sigma_0(c_R - c_A) \log(v - c_A) \\
+ (v - c_A)\left[\log (\sigma_0(v - c_A)) - \sigma_0 \log (v - c_R)\right]
\]

(A.27)
Dividing both sides by the multiplier 
\[ (1 - \sigma)(v - c_A) - \sigma(c_R - c_A) \]

affects the inequality depending on its sign: If \( \sigma_0 < \bar{\sigma} \), the condition is \( \kappa_0 \geq \overline{\kappa} \), where \( \bar{\sigma} \) and \( \overline{\kappa} \) are defined as
\[
\bar{\sigma} \equiv \left[ 1 + \frac{c_R - c_A}{v - c_A} \right]^{-1} \quad (A.28)
\]
and
\[
\overline{\kappa} \equiv \exp \left( 1 + \frac{(v - c_A)[\log(\sigma_0(v - c_A)) - \sigma_0 \log(v - c_B)] - \sigma_0 (c_R - c_A) \log(v - c_A)}{(1 - \sigma_0)(v - c_A) - \sigma_0 (c_R - c_A)} \right) \quad (A.29)
\]

Otherwise, if \( \sigma_0 > \overline{\sigma} \), the inequality is reversed, \( \kappa_0 \leq \overline{\kappa} \). I proceed by showing that in the first case, the inequality can never be satisfied because it would imply an entry cost that prohibits buyer entry.

**Claim 1:** If \( \sigma_0 \in (\overline{\sigma}, \bar{\sigma}) \), \( \kappa_0 \geq \overline{\kappa} \) implies \( \kappa_0 > \sigma_0(v - c_A) \).

**Proof of Claim 1.** The condition that \( \overline{\kappa} > \sigma_0(v - c_A) \) can be written
\[
1 + \frac{(v - c_A)[\log(\sigma_0(v - c_A)) - \sigma_0 \log(v - c_B)] - \sigma_0 (c_R - c_A) \log(v - c_A)}{(1 - \sigma_0)(v - c_A) - \sigma_0 (c_R - c_A)} > \log(\sigma_0(v - c_A)) \quad (A.30)
\]
Condition (A.30) can be rearranged so that \( \sigma_0 \) appears only on one side of the inequality:
\[
v - c_A > \sigma_0(v - c_A) \left[ 1 + \log \left( \frac{v - c_R}{\sigma_0(v - c_A)} \right) \right] + \sigma_0(c_R - c_A) \left[ 1 + \log \left( \frac{v - c_A}{\sigma_0(v - c_A)} \right) \right] \quad (A.31)
\]
Claim 1 requires that the right hand side of (A.31) remains strictly less than the left hand side when evaluated at any \( \sigma_0 \in (\overline{\sigma}, \bar{\sigma}) \). The right hand side is concave in \( \sigma_0 \):
\[
\frac{\partial^2 \text{RHS}}{\partial \sigma_0^2} = - [ (v - c_A) + (c_R - c_A) ] \frac{1}{\sigma_0} < 0 \quad (A.32)
\]
Moreover, it is straightforward to show that the right hand side attains a maximum at some $\sigma' \in (\sigma, \bar{\sigma})$. At $\sigma'$,

$$\left. \frac{\partial \text{RHS}}{\partial \sigma_0} \right|_{\sigma_0 = \sigma'} = (v - c_A) \log \left( \frac{v - c_R}{v - c_A} \right) - [(v - c_A) + (c_R - c_A)] \log (\sigma') = 0 \quad (A.33)$$

Evaluating condition (A.31) at $\sigma'$ yields

$$(1 - \sigma')(v - c_A) - \sigma'(c_R - c_A) > 0 \quad (A.34)$$

which is true by the fact that $\sigma' < \bar{\sigma}$. This proves that $\kappa_0 \geq \bar{\kappa}$ can never hold when $\sigma \in (\sigma, \bar{\sigma})$ without the entry cost prohibiting buyers from participating in the full pooling submarket altogether.

Claim 1 proves that a pooling contract cannot improve the expected payoff to anxious sellers when $\sigma_0 \in (\sigma, \bar{\sigma})$. When $\sigma \in (\sigma, 1]$ on the other hand, the condition becomes $\kappa_0 \leq \bar{\kappa}$, which can be satisfied depending on the parameters of the model.

To gain further insight, I first prove the following Claim about the properties of the threshold $\bar{\kappa}$.

Claim 2: When $\sigma_0 \in (\sigma, 1]$, the threshold $\bar{\kappa}(\sigma_0)$ exhibits the following properties: (1) $\partial \bar{\kappa}/\partial \sigma_0 > 0$, (2) $\lim_{\sigma_0 \to \sigma^+} \bar{\kappa}(\sigma_0) = 0$, and (3) $\bar{\kappa}(1) > \kappa$.

Proof of Claim 2. To prove part 1, simply differentiate $\bar{\kappa}$ with respect to $\sigma_0$:

$$\frac{\partial \bar{\kappa}}{\partial \sigma_0} = \bar{\kappa}(v - c_A) \left[ (v - c_A) - \sigma_0(v - c_A) \left[ 1 + \log \left( \frac{v - c_R}{\sigma_0 (v - c_A)} \right) \right] - \sigma_0(c_R - c_A) \left[ 1 + \log \left( \frac{v - c_A}{\sigma_0 (v - c_A)} \right) \right] \right] \left[ (1 - \sigma_0)(v - c_A) - \sigma_0(c_R - c_A) \right]^{-2} \quad (A.35)$$
which is positive if and only if

\[ v - c_A > \sigma_0(v - c_A) \left[ 1 + \log \left( \frac{v - c_R}{\sigma_0(v - c_A)} \right) \right] + \sigma_0(c_R - c_A) \left[ 1 + \log \left( \frac{v - c_A}{\sigma_0(v - c_A)} \right) \right] \quad (A.36) \]

This is the same condition as (A.31). The concavity in \( \sigma_0 \) of the right-hand side and the proof of Claim 1 therefore establish that (A.36) is satisfied for all \( \sigma_0 \in [\bar{\sigma}, 1] \), and therefore \( \bar{\kappa} \) is increasing in \( \sigma_0 \).

To prove part 2, recall the expression for \( \bar{\kappa}(\sigma_0) \):

\[ \exp \left( \frac{(v - c_A)[1 + \log(\sigma_0(v - c_A))] - \sigma_0(v - c_A)[1 + \log(v - c_R)] - \sigma_0(c_R - c_A)[1 + \log(v - c_A)]}{(1 - \sigma_0)(v - c_A) - \sigma_0(c_R - c_A)} \right) \quad (A.37) \]

For \( \sigma_0 > \bar{\sigma} \), the denominator is negative, but as \( \sigma_0 \to \bar{\sigma}^+ \), the denominator approaches zero. Property 2 then requires that the numerator remain positive as \( \sigma_0 \to \bar{\sigma}^+ \). The numerator at \( \bar{\sigma} \) is

\[ (v - c_A) \log (\bar{\sigma}(v - c_A)) - \bar{\sigma}(v - c_A) \log(v - c_R) - \bar{\sigma}(c_R - c_A) \log(v - c_A) \quad (A.38) \]

Dividing by \( (v - c_A) \) and using the definition of \( \bar{\sigma} \), the inequality becomes

\[ \log (\bar{\sigma}(v - c_A)) - \bar{\sigma} \log(v - c_R) - (1 - \bar{\sigma}) \log(v - c_A) > 0 \quad (A.39) \]

Applying Jensen’s inequality yields

\[ \bar{\sigma} \log(v - c_R) + (1 - \bar{\sigma}) \log(v - c_A) < \log (v - c_A - \bar{\sigma}(c_R - c_A)) \]

\[ = \log (\bar{\sigma}(v - c_A)) \quad (A.40) \]

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where the equality follows from the definition of $\bar{\sigma}$.

The proof of part 3 requires an expression for $\bar{\kappa}(1)$:

$$\pi(1) = \exp \left( 1 + \log(v - c_A) - \frac{v - c_A}{c_R - c_A} \log \left( \frac{v - c_A}{v - c_R} \right) \right)$$  \hspace{1cm} (A.41)

Part 3 of Claim 2 therefore states that

$$\exp \left( 1 + \log(v - c_A) - \frac{v - c_A}{c_R - c_A} \log \left( \frac{v - c_A}{v - c_R} \right) \right) > \exp \left( -\frac{c_R - c_A}{v - c_R} \right) (v - c_A)$$  \hspace{1cm} (A.42)

Taking the logarithms of both sides and simplifying yields

$$\frac{c_R - c_A}{v - c_R} > \log \left( 1 + \frac{c_R - c_A}{v - c_R} \right)$$  \hspace{1cm} (A.43)

which must hold because $(c_R - c_A)/(v - c_R) > 0$.

Claim 2 states that as $\sigma$ increases over the interval $(\bar{\sigma}, 1]$, $\pi$ increases from 0 to a value above $\bar{\kappa}$. Therefore, there exists a unique $\sigma^* \in (\bar{\sigma}, 1)$ such that $\bar{\kappa}(\sigma^*) = \bar{\kappa}$. For any $\sigma_0 \in (\bar{\sigma}, \sigma^*)$, the incentive feasible pair of submarkets dominate a full pooling submarket; and for any $\sigma_0 \in [\sigma^*, 1]$, a full pooling submarket dominates the pair of fully separating contracts if and only if $\kappa_0 \in [\bar{\kappa}, \bar{\kappa}]$.

Claim 3: The $\sigma^* \in (\bar{\sigma}, 1)$ satisfying $\bar{\kappa}(\sigma^*) = \bar{\kappa}$ is given by the following expression:\footnote{\textit{W}_{-1} is the $-1$ branch of the Lambert $W$ function.}

$$\sigma^* = \exp \left( -\frac{v - c_A}{v - c_R} \right) \exp \left( \text{\textit{W}_{-1} \left( \exp \left( -\frac{v - c_A}{v - c_R} \right) \left[ \log \left( \frac{v - c_A}{v - c_R} \right) - \frac{(v - c_A) + (c_R - c_A)}{v - c_R} \right] \right) \right)$$  \hspace{1cm} (A.44)
Proof of Claim 3. The proof of Claim 3 consists of setting $\kappa(\sigma^*) = \kappa$ and solving for $\sigma^*$.

Claims 1, 2, and 3 complete the proof of Lemma 4.

Proof of Proposition 2.3.1. There are no possible deviations that attract only type $R$ sellers, since they refrain from paying real estate fees in the type $R$ submarket. Moreover, there are no out-of-equilibrium contracts that can attract only type $A$ sellers, since their commission rate $z_A$ is as low as possible without attracting some relaxed sellers. Existence of a fully separating equilibrium can therefore only be compromised by a full pooling or partial pooling submarket. By Definition 2.3.1, this pooling or partial pooling submarket must offer greater expected payoffs to both types of sellers, and a strictly greater payoff to at least one type. Type $R$ sellers benefit from pooling, but type $A$ sellers do not benefit as long as

$$V_A(z_A, 1, \theta_A) \geq V_A(0, \sigma, \theta), \quad \text{for all } \sigma \in [0, \sigma_0]$$

(A.45)

Potential submarkets with $\sigma > \sigma_0$ can be ignored because the endogenous sorting condition guarantees that the expected payoff to anxious sellers is increasing in the level of real estate services, $a$, even when relaxed sellers are indifferent, which makes $(a_A, z_A)$ the most preferred incentive compatible contract by anxious sellers.

Given the properties of $V_A(z, \sigma(a), \theta(a))$, conditions (A.45) reduce to

$$V_A(z_A, 1, \theta_A) \geq \max\{V_A(0, 0, \theta_R), V_A(0, \sigma_0, \theta_P)\}$$

(A.46)

The proof therefore follows from Lemma 4.
Proof of Proposition 2.3.2. Proof of part 1: $\kappa_0 < \kappa$ rules out a fully separating equilibrium. Therefore, a candidate equilibrium must involve a pooling submarket. Moreover, $\sigma_0 > \sigma$ and $\kappa_0 < \kappa$ require that there be a pooling submarket with $\sigma_P > \sigma$. The endogenous sorting condition ensures that the expected payoff to anxious sellers is increasing in $a$ when relaxed sellers are indifferent. Consequently, when $V_R(\phi(a), 1, \theta(a)) = V_R(0, \sigma_P, \theta_P)$, it must be that $V_A(\phi(a), 1, \theta(a)) > V_A(0, \sigma_P, \theta_P)$. In other words, an incentive compatible zero profit contract with $a > 0$ can be designed to attract the type $A$ sellers without attracting the type $R$ sellers. This rules out a equilibria with pooling when $\sigma_0 > \sigma$. This proves non-existence of equilibria, since an equilibrium must involve either separation or pooling and both are incompatible with $\kappa_0 < \kappa$ and $\sigma_0 > \sigma$.

Proof of part 2: Take as given a full pooling equilibrium $(a_P, z_P) = (0, 0)$ with $\sigma_0$ and $\theta_0$, and consider a deviation: a submarket $(a, z)$, with $a > 0$, $\sigma \equiv \sigma(a) > \sigma_0$, and $\theta \equiv \theta(a) > 0$. If $\sigma \leq (v - c_R)/(v - c_A)$, the new submarket does not attract any sellers. If $\sigma > (v - c_R)/(v - c_A)$, the relevant payoff functions are

\begin{align}
V_A(0, \sigma_0, \theta_0) &= [1 - (1 + \theta_0)e^{-\theta_0}](v - c_A) + \theta_0 e^{-\theta_0}(c_R - c_A) \quad \text{(A.47)} \\
V_R(0, \sigma_0, \theta_0) &= [1 - (1 + \theta_0)e^{-\theta_0}](v - c_R) \quad \text{(A.48)} \\
V_A(z, \sigma, \theta) &= [1 - (1 + \theta)e^{-\theta}](v - c_A) - z \quad \text{(A.49)} \\
V_R(z, \sigma, \theta) &= [1 - (1 + \theta)e^{-\theta}](v - c_R) - z \quad \text{(A.50)}
\end{align}

Type $R$ sellers enter the new submarket as long as $V_R(z, \sigma, \theta) \geq V_R(0, \sigma_0, \theta_0)$. Similarly, the flow of type $A$ sellers into the new submarket continues as long as $V_A(z, \sigma, \theta) \geq V_A(0, \sigma_0, \theta_0)$. Using the same approach as in the proof of Lemma 3, we can obtain the
parameter restrictions under which a deviation violates incentive feasibility. Starting with the binding incentive compatibility constraint for relaxed sellers and substituting for the anxious sellers’ expected payoffs yields

\[
V_A(z, \sigma, \theta) > V_A(0, \sigma_0, \theta_0) \iff e^{\theta - \theta_0} > 1 + \theta \tag{A.51}
\]

The free entry conditions \(e^{-\theta_0}(v - c_R) = \kappa_0\) and \(e^{-\theta} \sigma(v - c_A) = \kappa_0\) can be used to substitute for \(\theta_0\) and \(\theta\) in the condition above. This yields

\[
\kappa_0 > \sigma(v - c_A) \exp \left( \frac{v - c_R - \sigma(v - c_A)}{v - c_R} \right) \tag{A.52}
\]

The deviation \((a, z)\) therefore attracts both types of sellers and violates the equilibrium conditions of Definition 2.3.1 if

\[
\kappa_0 \in \left( \sigma(v - c_A) \exp \left( \frac{v - c_R - \sigma(v - c_A)}{v - c_R} \right), (v - c_A) \exp \left( -\frac{c_R - c_A}{v - c_R} \right) \right) \tag{A.53}
\]

A contradiction obtains if this interval is empty; that is, if

\[
\sigma(v - c_A) \exp \left( \frac{v - c_R - \sigma(v - c_A)}{v - c_R} \right) \geq (v - c_A) \exp \left( -\frac{c_R - c_A}{v - c_R} \right) \iff \frac{v - c_A}{v - c_R} - \log \left( \frac{v - c_A}{v - c_R} \right) \geq \frac{\sigma(v - c_A)}{v - c_R} - \log \left( \frac{\sigma(v - c_A)}{v - c_R} \right) \tag{A.54}
\]

Given that \((v - c_A)/(v - c_R) > 1\) and \(\sigma(v - c_A)/(v - c_R) > 1\), the condition above leads to a contradiction because \(\sigma \leq 1\). Intuitively, this result means that if a fully separating submarket is not incentive feasible, then neither is any other possible deviation when \(\sigma_0 < (v - c_R)/(v - c_A)\). Thus, the full pooling submarket constitutes
an equilibrium.

Proof of Proposition 2.3.3. I prove Proposition 2.3.3 for the version of the model with $\kappa'(a) = 0$. The analysis is similar when $\kappa'(a) < 0$, except with the payoff functions from Section 2.2.

Consider a listing agreement designed for type A sellers with the list price $p_A = c_A/(1 - z_A)$. The RWA clause has no effect on buyers’ bidding strategies in a type A submarket, and does not impact anxious sellers’ expected payoff because they are willing to accept an offer of $c_A/(1 - z_A)$ regardless of the clause. Type R sellers, on the other hand, now pay a cost in a bilateral match if they choose to mimic type A sellers by entering the type A submarket. The cost is $\min \{ z_A c_A/(1 - z_A), c_R - c_A \}$, where the minimization operator allows mimicking sellers to choose between rejecting the offer but paying the REA’s commission, $z_A c_A/(1 - z_A)$, or going through with the transaction at the low price in a bilateral match, resulting in a negative payoff. Since commissions must be paid upon receiving an offer at or above $p_A$ regardless of whether a transaction takes place, a mimicking seller would optimally choose to refuse the offer and pay her REA. This is true as long as $c_A/(1 - z_A) < c_R$, which must be the case for type A sellers to find signalling worthwhile.\footnote{When $\kappa'(a) < 0$ and both types of sellers choose to hire REAs such that $\kappa_A, \kappa_R > 0$, it is not always the case that a mimicking seller will opt not to transact in a bilateral match. Nevertheless, the RWA clause tightens the incentive compatibility constraint because the cost to mimicking is positive: $c_R - c_A > 0$. Then, type A contracts need not be so distorted to achieve type R incentive compatibility.}

Recall the type R incentive compatibility constraint in the absence of a RWA
clause:

\[
V_R(0, 0, \theta_R) = \left[ 1 - (1 + \theta_R)e^{-\theta_R} \right] (v - c_R) \\
= \left[ 1 - (1 + \theta_A)e^{-\theta_A} \right] [(1 - z_A)v - c_R] = V_R(z_A, 1, \theta_A)
\]  \hspace{1cm} (A.55)

With the RWA clause, this same condition becomes

\[
V_R(0, 0, \theta_R) = \left[ 1 - (1 + \theta_R)e^{-\theta_R} \right] (v - c_R) \\
> \left[ 1 - (1 + \theta_A)e^{-\theta_A} \right] [(1 - z_A)v - c_R] - \theta_A e^{-\theta_A} \frac{z_A c_A}{1 - z_A} = V_R(z_A, 1, \theta_A)
\]  \hspace{1cm} (A.56)

The extra term on the right hand side implies that the inequality is no longer binding.
A lower commission rate than \( z_A \) will therefore induce market separation, which directly benefits anxious sellers in a fully separating equilibrium. \( \Box \)
Appendix B

Omitted Proofs from Chapter 3

Proof of Proposition 3.2.3. Taking the limit of $U(\theta)$ as $\eta \to 0$ yields

$$
\lim_{\eta \to 0} U(\theta) = \left[ 1 - e^{-\theta} \right] (y - c) - e^{-\theta}(1 - \beta)(y - c) \lim_{\eta \to 0} \left[ \frac{e^{\frac{\eta^\theta}{\eta + \beta(1-\eta)}} - 1}{\eta} \right] \tag{B.1}
$$

Using l’Hôpital’s rule,

$$
\lim_{\eta \to 0} \left[ \frac{e^{\frac{\eta^\theta}{\eta + \beta(1-\eta)}} - 1}{\eta} \right] = \lim_{\eta \to 0} \frac{\eta^\theta}{\eta + \beta(1-\eta)} = \theta \tag{B.2}
$$

which yields

$$
\lim_{\eta \to 0} U(\theta) = \left[ 1 - e^{-\theta} \right] (y - c) - \theta e^{-\theta}(1 - \beta)(y - c) \tag{B.3}
$$

Then, if $\beta = 0$, the worker’s payoff is 0 with probability $e^{-\theta} + \theta e^{-\theta}$ and $y - c$ probability with $1 - e^{-\theta} - \theta e^{-\theta}$, as in the model proposed by Julien et al. [47]. Similarly, from
the firm’s perspective, the limit of $V(\theta)$ as $\eta \to 0$ is

$$\lim_{\eta \to 0} V(\theta) = e^{-\theta} (1 - \beta) (y - c) \lim_{\eta \to 0} \left[ e^{\eta \beta (1 - \eta)} - 1 \right] (B.4)$$

$$= e^{-\theta} (1 - \beta) (y - c) \quad (B.5)$$

If $\beta = 0$, the firm captures all of the surplus if no other firm matches with the same worker, and zero otherwise.

Proof of Proposition 3.3.1. The result can be obtained by taking the limit of (3.28) and (3.29) as $\varepsilon \to 0$. For instance, applying l’Hôpital’s rule to (3.28) yields the following, where $\beta(0)$ denotes $\lim_{\varepsilon \to 0} \beta(\varepsilon)$:

$$\beta(0) = 1 - \left( \frac{\eta \theta}{\eta + \beta(0)(1 - \eta)} \right) \left[ e^{\eta \beta(0)(1 - \eta)} - 1 \right]^{-1} \lim_{\varepsilon \to 0} \left( e^{\theta L - \theta H} \left[ \frac{d\theta_L}{d\varepsilon} - \frac{d\theta_H}{d\varepsilon} \right] \right) (B.6)$$

Dividing out the common factor $\left[ \frac{d\theta_L}{d\varepsilon} - \frac{d\theta_H}{d\varepsilon} \right]$ and using the result that $\lim_{\varepsilon \to 0} e^{\theta L - \theta H} = 1$ reproduces the efficiency condition (3.17), implying the result that $\lim_{\varepsilon \to 0} \beta(\varepsilon) = \beta^\ast$.

The same exercise can be applied to (3.29) to obtain $\lim_{\varepsilon \to 0} \overline{\beta}(\varepsilon) = \beta^\ast$.

Proof of Proposition 3.3.2. When $\eta > 0$, the efficiency condition for $\beta$ requires different values for type $L$ and type $H$ workers. The implication is that the modified Mortensen–Hosios condition cannot hold for both type $L$ and type $H$ workers. When $\eta \to 0$, efficiency requires $\beta = 0$ for any type of worker. It remains to be shown that a separating equilibrium with $\eta \to 0$ and $\beta \to 0$ is not incentive compatible.

When $\eta \to 0$, the probabilities that a worker ends up with $w^{(i)}$ as the highest offer reduce to $\rho^{(i)} = \theta e^{-\theta}$ and $\rho^{(i)} = 0$ for all $i > 1$. When $c_H > w^{(1)} \iff \beta < \frac{(c_H - c_L)}{(y - c_L)}$, the expected payoff when a type $H$ worker mimics a type $L$
worker is therefore

\[
U_H(\theta_L) = \left[1 - e^{-\theta_L}\right] (y - c_H) - \theta_L \kappa + \theta_L e^{-\theta_L} (c_H - w^{(1)})
\]

\[
= \left[1 - (1 + \theta_L) e^{-\theta_L}\right] (y - c_H)
\]

The incentive compatibility constraint, \(U_H(\theta_H) \geq U_H(\theta_L)\), in this case is

\[
\left[1 - e^{-\theta_H}\right] (y - c_H) - \theta_H e^{-\theta_H} (1 - \beta) (y - c_H) \geq \left[1 - (1 + \theta_L) e^{-\theta_L}\right] (y - c_H) \quad (B.7)
\]

which cannot be satisfied with any \(\beta \in [0, (c_H - c_L)/(y - c_L)]\) since \(\theta_L > \theta_H\) is implied by the free entry conditions. Hence, an equilibrium with \((\beta, \eta) \to (0, 0)\) violates incentive compatibility. \(\square\)
Appendix C

Omitted Proofs from Chapter 4

Proof of Proposition 4.3.1

It is straightforward to check that these prices and allocations satisfy the equilibrium conditions 1, 2, and 3. Claim 1 establishes that the proposed prices, allocations, and utilities satisfy the equilibrium refinement. Lemma 5 shows that there cannot exist a pooling submarket. Finally, Claims 2, 3, and 4 establish uniqueness.

Claim 1. The prices, allocations, and utilities in Proposition 4.3.1 satisfy the equilibrium refinement.

Proof. Consider all possible deviations, \( p' \in \mathbb{R}_+ \setminus \mathbb{P} \). If \( p' > (1 - \lambda_H)\pi - c = p_H \), the payoff to the buyer in a land transaction is negative. If \( p' < (1 - \lambda_L)\pi - c = p_L \), no seller will enter the new submarket since submarket \( p_L \) is strictly preferred. Finally, consider \( p' \in ((1 - \lambda_L)\pi - c, (1 - \lambda_H)\pi - c) \). Let \( \theta \) denote the buyer-seller ratio in the
new submarket $p'$. Type $L$ sellers are attracted to the new submarket if

$$
\min\{\theta, 1\}(p' - \xi_L) \geq (1 - \lambda_L)\pi - \xi_L - c = \bar{U}_L
$$

$$
\Rightarrow \quad \theta \geq \frac{(1 - \lambda_L)\pi - \xi_L - c}{p' - \xi_L} \equiv \theta_L
$$

Similarly, type $H$ sellers are attracted to the new submarket if

$$
\min\{\theta, 1\}(p' - \xi_H) \geq \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \frac{[(1 - \lambda_H)\pi - \xi_H - c]}{[(1 - \lambda_L)\pi - \xi_L - c][p' - \xi_H]} \equiv \theta_H
$$

One can rank the cut-offs $\theta_L$ and $\theta_H$ defined above for any $p' \in ((1 - \lambda_L)\pi - c, (1 - \lambda_H)\pi - c)$.

$$
\theta_L = \frac{(1 - \lambda_L)\pi - \xi_L - c}{p' - \xi_L} \geq \frac{[(1 - \lambda_L)\pi - \xi_L - c][1 - \lambda_H]\pi - \xi_H - c}{[(1 - \lambda_H)\pi - \xi_L - c][p' - \xi_H]} = \theta_H
$$

$$
\frac{p' - \xi_H}{p' - \xi_L} \leq \frac{(1 - \lambda_H)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c}
$$

Let $p' = \mu[(1 - \lambda_L)\pi - c] + (1 - \mu)[(1 - \lambda_H)\pi - c] \Rightarrow p' = (1 - \lambda_H)\pi - c - \mu(\lambda_L - \lambda_H)\pi$, $\mu \in (0, 1)$. The inequality from above becomes

$$
\frac{(1 - \lambda_H)\pi - \xi_H - c - \mu(\lambda_L - \lambda_H)\pi}{(1 - \lambda_H)\pi - \xi_L - c - \mu(\lambda_L - \lambda_H)\pi} < \frac{(1 - \lambda_H)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c}
$$

and the inequality is strict for any $\mu \in (0, 1)$. This establishes $\theta_L < \theta_H$ for any new submarket $p' \in ((1 - \lambda_L)\pi - c, (1 - \lambda_H)\pi - c)$. Since type $H$ sellers correctly anticipate that type $L$ sellers will enter submarket $p'$ until the buyer-seller ratio is $\theta_L$, the type $H$ sellers choose not to enter. Since the deviation $p' > (1 - \lambda_L)\pi - c$ attracts only
sellers of insecure land, the deviation is not profitable. □

**Lemma 5.** There are no submarkets that attract both type $L$ and type $H$ sellers in any competitive land market equilibrium.

**Proof.** Suppose (for the sake of contradiction) that there exists a submarket with price $p$ that attracts $s_L(p)$ type $L$ sellers and $s_H(p)$ type $H$ sellers. The type-dependent utilities are

$$
\bar{U}_L = \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} (p - \xi_L)
$$

$$
\bar{U}_H = \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} (p - \xi_H)
$$

and buyers in submarket $p$ earn a payoff of zero in expectation because of the free entry condition, which requires $p \leq \left[ 1 - \frac{s_L(p)}{s_L(p) + s_H(p)} \lambda_L - \frac{s_H(p)}{s_L(p) + s_H(p)} \lambda_H \right] \pi - c$. Consider a deviation to $p' \in (p, (1 - \lambda_H)\pi - c)$. Let $\theta$ again denote the buyer-seller ratio in the new submarket. A type $L$ seller prefers the new submarket if

$$
\min\{\theta, 1\} (p' - \xi_L) \geq \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} (p - \xi_L)
$$

$$
\Rightarrow \quad \min\{\theta, 1\} \geq \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} \left( p' - \xi_L \right)
$$

Similarly, a type $H$ seller prefers the new submarket if

$$
\min\{\theta, 1\} \geq \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} \left( \frac{p - \xi_L}{p' - \xi_L} \right)
$$

Since

$$
\frac{p - \xi_H}{p' - \xi_H} < \frac{p - \xi_L}{p' - \xi_L}
$$

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type $H$ sellers enter the new submarket until the buyer-seller ratio is

$$\theta = \min \left\{ \frac{b(p)}{s_L(p) + s_H(p)}, 1 \right\} \left( \frac{p - \xi_H}{p' - \xi_H} \right) \in (0, 1)$$

which is low enough that the new submarket makes type $L$ sellers worse off. Consequently, the deviation attracts only type $H$ sellers, which yields a positive payoff to the buyer since $p' < (1 - \lambda_H)\pi + c$ and $\theta < 1$.

Lemma 6. In any competitive land market equilibrium, there is a submarket $p_L \in \mathbb{P}$ with $s_L(p_L) > 0$.

Proof. Suppose (for the sake of contradiction) that there is no such submarket, and therefore $s_L(p) = 0$ for all $p \in \mathbb{P}$ and $\bar{U}_L = 0$. A subset of buyers can offer $p_L = \xi_L + \delta$ with $\delta > 0$ close to zero. All potential type $L$ sellers will choose to enter the market since $p_L - \xi_L > 0$. The expected payoff to buyers in the new submarket is at least $(1 - \lambda_L)\pi - \xi_L - \delta - c$ (even higher if $H$ types enter the submarket as well). With $\delta$ small enough, this payoff becomes arbitrarily close to $(1 - \lambda_L)\pi - \xi_L - c$, which is strictly positive by Assumption 2.

Claim 2. In any competitive land market equilibrium, $\bar{U}_L = (1 - \lambda_L)\pi - \xi_L - c$, and there is a submarket with price $p_L = (1 - \lambda_L)\pi - c$.

Proof. By Lemma 6, there is some $p_L \in \mathbb{P}$ with $s_L(p_L) > 0$ such that

$$\bar{U}_L = \min \left\{ \frac{b(p_L)}{s_L(p_L)}, 1 \right\} (p_L - \xi_L) \geq 0$$

Suppose (for the sake of contradiction) that $\bar{U}_L \neq (1 - \lambda_L)\pi - \xi_L - c$. First, suppose
\[ \bar{U}_L > (1 - \lambda_L)\pi - \xi_L - c. \] Since

\[
\min \left\{ \frac{b(p_L)}{s_L(p_L)}, 1 \right\} (p_L - \xi_L) \leq (p_L - \xi_L)
\]

this can only occur if \( p_L > (1 - \lambda_L)\pi - c \). Any transaction involving type \( L \) land in a submarket with price \( p_L > (1 - \lambda_L)\pi - c \) will yield a negative payoff to the buyer. This is a contradiction since, according to Lemma 5, there can only be one type of seller in an equilibrium submarket.

Suppose then that \( \bar{U}_L < (1 - \lambda_L)\pi - \xi_L - c \). Since it’s already been established that \( p_L \leq (1 - \lambda_L)\pi - c \), then either \( b(p_L)/s_L(p_L) < 1 \) or \( p_L < (1 - \lambda_L)\pi - c \). If \( b(p_L)/s_L(p_L) < 1 \), then the buyers’ free entry condition implies \( p_L = (1 - \lambda_L)\pi - c \).

A buyer can post a deviating offer \( p = p_L - \delta \), with \( \delta > 0 \) small enough that some type \( L \) sellers prefer the new offer:

\[
p_L - \delta - \xi_L > \frac{b(p_L)}{s_L(p_L)} (p_L - \xi_L)
\]

\[ \Rightarrow \delta < \left[ 1 - \frac{b(p_L)}{s_L(p_L)} \right] (p_L - \xi_L) \]

Since the deviating offer attracts sellers and involves a smaller payment for land, the deviation is profitable.

If \( p_L < (1 - \lambda_L)\pi - c \), consider the deviating offer \( p \in (p_L, (1 - \lambda_L)\pi - c) \). Since

\[
p - \xi_L > p_L - \xi_L \geq \bar{U}_L
\]
some type $L$ sellers prefer the new submarket. Moreover, since $p < (1 - \lambda_L)\pi - c$, the deviation is profitable. This completes the proof that $\bar{U}_L = (1 - \lambda_L)\pi - \xi_L - c$. Moreover, the possibility that $p_L \neq (1 - \lambda_L)\pi - c$ has been ruled out. \hfill \Box

**Lemma 7.** In any competitive land market equilibrium, there is a submarket $p_H \in \mathbb{P}$ with $s_H(p_H) > 0$.

**Proof.** Suppose (for the sake of contradiction) that there is no such submarket, and therefore $s_H(p) = 0$ for all $p \in \mathbb{P}$ and $\bar{U}_H = 0$. A subset of buyers can offer any $p_H = (1 - \lambda_H)\pi - c - \delta$, with $\delta > 0$ close to zero. Type $L$ sellers will prefer the new submarket if

$$
\min\{\theta, 1\}(p_H - \xi_L) \geq (1 - \lambda_L)\pi - \xi_L - c = \bar{U}_L
\Rightarrow \theta \geq \frac{(1 - \lambda_H)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_L - c - \delta} \equiv \theta_L
$$

For $\delta$ close to zero, $\theta_L < 1$. Type $H$ sellers will enter the new submarket, since

$$
\theta_L(p_H - \xi_H) = \frac{(1 - \lambda_H)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_L - c - \delta} [(1 - \lambda_H)\pi - \xi_H - c - \delta] > 0
$$

where the last inequality follows from Assumption 2 and $\delta$ small. Since type $L$ sellers correctly anticipate that type $H$ sellers will enter submarket $p_H$ until the buyer-seller ratio is below $\theta_L$, the type $L$ sellers choose not to enter. Since the deviation attracts only sellers of secure land and $p_H < (1 - \lambda_H)\pi - c$, the deviation is profitable. \hfill \Box

**Claim 3.** In any competitive land market equilibrium,

$$
\bar{U}_H = \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) [(1 - \lambda_H)\pi - \xi_H - c]
$$
and there is a submarket with $p_H = (1 - \lambda_H)\pi - c$.

Proof. By Lemma 7, there is some $p_H \in \mathbb{P}$ with $s_H(p_H) > 0$ such that

$$
\bar{U}_H = \min \left\{ \frac{b(p_H)}{s_H(p_H)}, 1 \right\} (p_H - \xi_H) \geq 0
$$

Suppose (for the sake of contradiction) that $\bar{U}_H \neq \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) [(1 - \lambda_H)\pi - \xi_H - c]$. First, suppose $\bar{U}_H > \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) [(1 - \lambda_H)\pi - \xi_H - c]$. Then,

$$
\bar{U}_H = \min \left\{ \frac{b(p_H)}{s_H(p_H)}, 1 \right\} (p_H - \xi_H) > \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) [(1 - \lambda_H)\pi - \xi_H - c]
$$

$$
\Rightarrow \min \left\{ \frac{b(p_H)}{s_H(p_H)}, 1 \right\} > \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) \left( \frac{(1 - \lambda_H)\pi - \xi_H - c}{p_H - \xi_H} \right) \quad (C.1)
$$

From Claim 2, $\bar{U}_L = (1 - \lambda_L)\pi - \xi_L - c$. Condition 2(ii) of the equilibrium definition requires

$$
\bar{U}_L = (1 - \lambda_L)\pi - \xi_L - c \geq \min \left\{ \frac{b(p_H)}{s_H(p_H)}, 1 \right\} (p_H - \xi_L)
$$

$$
\Rightarrow \min \left\{ \frac{b(p_H)}{s_H(p_H)}, 1 \right\} \leq \frac{(1 - \lambda_L)\pi - \xi_L - c}{p_H - \xi_L} \quad (C.2)
$$

Combining (C.1) and (C.2) gives

$$
\left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) \left( \frac{(1 - \lambda_H)\pi - \xi_H - c}{p_H - \xi_H} \right) < \frac{(1 - \lambda_L)\pi - \xi_L - c}{p_H - \xi_L}
$$

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This reduces to

\[ p_H > (1 - \lambda_H)\pi - c \]

Any transaction at \( p_H > (1 - \lambda_H)\pi - c \) yields a negative payoff to the buyer, which contradicts the equilibrium free entry condition for buyers.

Suppose then that \( \bar{U}_H < \left( \frac{(1-\lambda_H)\pi - \xi_L - c}{(1-\lambda_H)\pi - \xi_H - c} \right) \left( (1 - \lambda_H)\pi - \xi_H - c \right) \), so either \( p_H < (1 - \lambda_H)\pi - c \) or \( b(p_H)/s_H(p_H) < \frac{(1-\lambda_H)\pi - \xi_L - c}{(1-\lambda_H)\pi - \xi_L - c} \). If \( p_H < (1 - \lambda_H)\pi - c \), the buyers’ free entry condition implies \( b(p_H)/s_H(p_H) > 1 \). The type \( L \) seller’s equilibrium condition 2(ii) requires

\[ p_H - \xi_L \leq p_L - \xi_L = \bar{U}_L \quad \Rightarrow \quad p_H \leq p_L \]

This is a contradiction since it means either a single submarket for both types, or if the inequality is strict, that type \( H \) sellers would prefer the type \( L \) submarket.

If \( b(p_H)/s_H(p_H) < \frac{(1-\lambda_H)\pi - \xi_L - c}{(1-\lambda_H)\pi - \xi_L - c} \), the buyers’ free entry condition implies \( p_H = (1 - \lambda_H)\pi - c \). Moreover, condition 2(ii) requires that type \( H \) sellers prefer submarket \( p_H \) over submarket \( p_L \):

\[
\bar{U}_H = \frac{b(p_H)}{s_H(p_H)} (p_H - \xi_H) \geq p_L - \xi_H \\
\frac{b(p_H)}{s_H(p_H)} [(1 - \lambda_H)\pi - \xi_H - c] \geq (1 - \lambda_H)\pi - \xi_H - c \\
\Rightarrow \quad \frac{b(p_H)}{s_H(p_H)} \geq \frac{(1 - \lambda_L)\pi - \xi_H - c}{(1 - \lambda_H)\pi - \xi_H - c} \tag{C.3}
\]

Consider the deviating offer \( p(\mu) = (1 - \mu)p_L + \mu p_H, \mu \in (0, 1) \). The new offer can
be written

\[ p(\mu) = (1 - \mu)(1 - \lambda_L)\pi + \mu(1 - \lambda_H)\pi - c = (1 - \lambda_L)\pi - c + \mu(\lambda_L - \lambda_H)\pi \]

Type \( H \) sellers prefer the new offer as long as

\[ \min\{\theta, 1\}(p(\mu) - \xi_L) \geq \frac{b(p_H)}{s_H(p_H)}(p_H - \xi_H) \]

\[ \min\{\theta, 1\}[(1 - \lambda_L)\pi - \xi_H - c + \mu(\lambda_L - \lambda_H)\pi] \geq \frac{b(p_H)}{s_H(p_H)}[(1 - \lambda_H)\pi - \xi_H - c] \]

\[ \Rightarrow \quad \min\{\theta, 1\} \geq \frac{b(p_H)}{s_H(p_H)} \left( \frac{(1 - \lambda_H)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c + \mu(\lambda_L - \lambda_H)\pi} \right) \equiv \theta_H \quad \text{(C.4)} \]

Similarly, type \( L \) sellers prefer the new submarket if

\[ \min\{\theta, 1\}(p(\mu) - \xi_L) \geq p_L - \xi_L = \bar{U}_L \]

\[ \min\{\theta, 1\}[(1 - \lambda_L)\pi - \xi_L - c + \mu(\lambda_L - \lambda_H)\pi] \geq (1 - \lambda_L)\pi - \xi_L - c \]

\[ \Rightarrow \quad \min\{\theta, 1\} \geq \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_L - c + \mu(\lambda_L - \lambda_H)\pi} \equiv \theta_L \quad \text{(C.5)} \]

The new offer \( p(\mu) \) yields a positive expected payoff to the deviating buyer if \( \theta_H < \theta_L \) and \( \theta_H \leq 1 \): only type \( H \) sellers enter the new market if \( \theta_H < \theta_L \); the buyer makes a purchase with probability one if \( \theta_H \leq 1 \); and since \( p(\mu) < p_H = (1 - \lambda_H)\pi - c \), there is a strictly positive payoff to the buyer in a transaction. From the definition of \( \theta_L \) in (C.5), one can see that \( \theta_L < 1 \). The necessary condition for a contradiction
is therefore $\theta_H < \theta_L$. As before, the cut-offs can be ranked for any $p(\mu), \mu \in (0, 1)$.

$$\theta_H \geq \theta_L$$

$$\frac{b(p_H)}{s_H(p_H)} \left( \frac{(1 - \lambda_H)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c + \mu(\lambda_L - \lambda_H)\pi} \right) \geq \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_L - c + \mu(\lambda_L - \lambda_H)\pi}$$

By assumption and from (C.3),

$$\frac{b(p_H)}{s_H(p_H)} \in \left[ \frac{(1 - \lambda_L)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c}, \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_H - c} \right]$$

Denote the particular buyer-seller ratio in submarket $p_H$ by the convex combination

$$\theta(\gamma) = (1 - \gamma) \left( \frac{(1 - \lambda_L)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c} \right) + \gamma \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_H - c} \right)$$

with $\gamma \in [0, 1)$. Substituting this into the inequality above yields

$$\frac{(1 - \gamma) \left( \frac{(1 - \lambda_L)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c} \right) + \gamma \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_H - c} \right)}{(1 - \lambda_L)\pi - \xi_L - c + \mu(\lambda_L - \lambda_H)\pi) \geq}$$

The left-hand side of (C.6) is a monotonic function of $\gamma$,

$$L : [0, 1] \rightarrow \left[ \frac{(1 - \lambda_L)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c}, \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_L)\pi - \xi_H - c} \right]$$
Similarly, the right-hand side of (C.6) is a monotonic function of $\mu$,

$$R : [0, 1] \to \left[ \frac{(1 - \lambda_L)\pi - \xi_H - c}{(1 - \lambda_L)\pi - \xi_L - c}, \frac{(1 - \lambda_H)\pi - \xi_H - c}{(1 - \lambda_H)\pi - \xi_L - c} \right]$$

By the intermediate value theorem, for any left-hand side value evaluated at $\gamma \in [0, 1]$, there is a $\mu \in [0, 1]$ such that $L(\gamma) = R(\mu)$. Since $R(\cdot)$ is strictly increasing, for any $\gamma \in [0, 1)$, there is a $\mu \in (0, 1)$ such that $L(\gamma) < R(\mu)$. This means that a profitable deviating offer is always possible. This completes the proof that

$$\bar{U}_H = \left( \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} \right) \left[ (1 - \lambda_H)\pi - \xi_H - c \right]$$

Moreover, the possibility that $p_H \neq (1 - \lambda_H)\pi - c$ has been ruled out. \hfill \Box

**Claim 4.** In any competitive land market equilibrium, $s_L(p_L) = S_L$, $b(p_L) = S_L$, $s_H(p_H) = S_H$, and

$$b(p_H) = \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c} S_H$$

*Proof.* Claim 2 and the definition of $\bar{U}_L$,

$$\bar{U}_L = (1 - \lambda_L)\pi - \xi_L - c = \min \left\{ \frac{b(p_L)}{s_L(p_L)}, 1 \right\} (p_L - \xi_L)$$

imply $b(p_L) \geq s_L(p_L)$. However, $b(p_L) > s_L(p_L)$ violates the free entry condition for buyers, so it must be that $b(p_L) = s_L(p_L)$. Since $\bar{U}_L > 0$, equilibrium condition 2(ii) requires that all type $L$ sellers enter the land market, $s_L(p_L) = S_L$. 

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Claim 3 and the definition of $\bar{U}_H$ imply

$$b(p_H) = \frac{(1 - \lambda_L)\pi - \xi_L - c}{(1 - \lambda_H)\pi - \xi_L - c}s_H(p_H)$$

With $\bar{U}_H > 0$, all type $H$ sellers enter the land market, $s_H(p_H) = S_H$. □

By Claims 2, 3, and 4, the unique competitive land market equilibrium is characterized by the proposed prices, allocations of buyers and sellers across submarkets, and type-dependent payoffs. This completes the proof of Proposition 4.3.1.

**Comparative Statics and Efficiency Results**

It is convenient to introduce some additional notation. Let $\lambda_L = \lambda + \varepsilon$ and $\lambda_H = \lambda - \varepsilon$. Then the system of property rights over land can be summarized by the two parameters: unweighted average tenure insecurity, $\lambda$, and a measure of the dispersion with respect to the quality of land titles, $\varepsilon$. Let $\theta_L$ and $\theta_H$ denote the share of type $i$ sellers trading in equilibrium, $i \in \{L, H\}$. Then $\theta_L(\lambda, \varepsilon) = 1$, and

$$\theta_H(\lambda, \varepsilon) = \left(\frac{(1 - \lambda - \varepsilon)\pi - \xi_L(\lambda, \varepsilon) - c}{(1 - \lambda + \varepsilon) - \xi_L(\lambda, \varepsilon) - c}\right)$$

where

$$\xi_L(\lambda, \varepsilon) = \max\{\alpha(1 - \lambda - \varepsilon)\pi, (1 - \lambda - \varepsilon)\pi - (w^U - w^R)\}$$

In order to derive useful comparative statics results, it is useful to first state the following:

**Lemma 8.** Let $\lambda_L = \lambda + \varepsilon$ and $\lambda_H = \lambda - \varepsilon$. The fraction of type $H$ sellers trading
in equilibrium, \( \theta_H \), is continuous in \( \lambda \) and \( \varepsilon \).

**Proof of Lemma 8.** Let \((\lambda^*, \varepsilon^*)\) be any pair \((\lambda, \varepsilon)\) satisfying \( w^U - w^R = (1 - \alpha)(1 - \lambda^* - \varepsilon^*) \pi \). It is straightforward to check that

\[
\lim_{(\lambda, \varepsilon) \to (\lambda^*, \varepsilon^*)^-} \theta_H(\lambda, \varepsilon) = \lim_{(\lambda, \varepsilon) \to (\lambda^*, \varepsilon^*)^+} \theta_H(\lambda, \varepsilon) = \theta_H(\lambda^*, \varepsilon^*)
\]

Since \( \theta_H(\lambda, \varepsilon) \) is also continuous off the boundary, \( \theta_H \) is a continuous function. \( \square \)

**Proof of Proposition 4.4.1(i).** Let \( \lambda_L = \lambda + \varepsilon \) and \( \lambda_H = \lambda - \varepsilon \), and analyze the impact of a change in \( \lambda \).\(^1\) Proposition 4.4.1(i) holds if the number of type \( H \) land transactions, \( \theta_H S_H \), declines with \( \lambda \). Differentiating \( \theta_H \) with respect to \( \lambda \) yields

\[
\frac{\partial \theta_H}{\partial \lambda} = \begin{cases}
\frac{-2(1 - \alpha)\pi^2 \varepsilon}{[(1 - \lambda + \varepsilon)\pi - \alpha(1 - \lambda - \varepsilon)\pi - \varepsilon]^2} < 0 & \text{if } w^U - w^R \geq (1 - \alpha)(1 - \lambda - \varepsilon)\pi \\
0 & \text{if } w^U - w^R < (1 - \alpha)(1 - \lambda - \varepsilon)\pi
\end{cases}
\]

Therefore, if \( w^U - w^R < (1 - \alpha)(1 - \lambda - \varepsilon) \), an increase in \( \lambda \) does not affect activity in the land market. The cut-off \( (1 - \alpha)(1 - \lambda - \varepsilon)\pi \) decreasing in \( \lambda \) implies that as \( \lambda \) increases, at some point \( w^U - w^R = (1 - \alpha)(1 - \lambda - \varepsilon)\pi \) and the buyer-seller ratio in the type \( H \) submarket declines, reducing the total number of land transactions. \( \square \)

**Proof of Proposition 4.4.1(ii).** Let \( \lambda_L = \lambda + \varepsilon \) and \( \lambda_H = \lambda - \varepsilon \), with \( \varepsilon > 0 \), and analyze the effect of \( \varepsilon \), since an increase in \( \varepsilon \) represents greater heterogeneity in the quality of land market titles. Proposition 4.4.1(iii) holds if the number of type \( H \)

---

\(^1\)This is a way of increasing the \( \lambda \)s in manner that preserves the degree of heterogeneity in the quality of land titles. Otherwise it would be impossible to separate the effect of the land tenure insecurity problem and the the effect of heterogeneity.
land transactions, \( \theta_H S_H \), declines with \( \varepsilon \). Differentiating \( \theta_H \) with respect to \( \varepsilon \) yields

\[
\frac{\partial \theta_H}{\partial \varepsilon} = \begin{cases} 
-2\pi \left[ (1 - \alpha)(1 - \lambda)\pi - c \right] \\ [(1 - \lambda + \varepsilon)\pi - \alpha(1 - \lambda - \varepsilon)\pi - c]^2 < 0 & \text{if } w^U - w^R \geq (1 - \alpha)(1 - \lambda - \varepsilon)\pi \\
-2\pi \left[ w^U - w^R - c \right] \\ [w^U - w^R + 2\varepsilon\pi - c]^2 < 0 & \text{if } w^U - w^R < (1 - \alpha)(1 - \lambda - \varepsilon)\pi
\end{cases}
\]

This establishes the result that the number of type \( H \) land transactions declines with \( \varepsilon \).

\( \square \)

**Proof of Proposition 4.4.2.** To determine whether a Pareto improving market intervention is possible given the asymmetric information in the land market, consider an allocation that treats all landowner types identically, regardless of the level tenure security associated with their plot. The number of buyers equals the number of sellers, \( b(p) = S_L + S_H \), and the price of land, \( p \), must satisfy a free-entry condition:

\[
p = \left( 1 - \frac{S_L}{S_L + S_H} \lambda_L - \frac{S_H}{S_L + S_H} \lambda_H \right) \pi - c
\]

The expected payoff for a seller of type \( i \in \{L, H\} \) is therefore

\[
U^i_P = \left( 1 - \frac{S_L}{S_L + S_H} \lambda_L - \frac{S_H}{S_L + S_H} \lambda_H \right) \pi - \xi_i - c
\]

Type \( L \) sellers prefer the pooling allocation over the competitive land market equilibrium if \( U^L_P > \bar{U}_L \), or

\[
\left( 1 - \frac{S_L}{S_L + S_H} \lambda_L - \frac{S_H}{S_L + S_H} \lambda_H \right) \pi - \xi_L - c > (1 - \lambda_L)\pi - \xi_L - c
\]
The inequality is satisfied because $\lambda_L > \lambda_H$. Type $L$ sellers prefer the pooling allocation because the price is higher than the market price for type $L$ land. In addition, there is no trade-off in terms of market liquidity for type $L$ sellers, since type $L$ land is perfectly liquid even in the market equilibrium.

Type $H$ sellers may or may not prefer the pooling allocation. Their preference for the pooling regime requires a low proportion of type $L$ land plots:

$$U^P_H = \left(1 - \frac{S_L}{S_L + S_H} \lambda_L - \frac{S_H}{S_L + S_H} \lambda_H \right) \pi - \xi_H - c > \left(\frac{(1 - \lambda_L) \pi - \xi_L - c}{(1 - \lambda_H) \pi - \xi_L - c}\right) \left[(1 - \lambda_H) \pi - \xi_H - c\right] = \bar{U}_H$$

$$\Rightarrow \frac{S_L}{S_L + S_H} < \frac{(1 - \lambda_H) \pi - \xi_H - c}{(1 - \lambda_H) \pi - \xi_L - c}$$

Type $H$ sellers prefer the price of land in the type $H$ equilibrium submarket over the price of land in under the pooling regime. However, type $H$ land is perfectly liquid in the pooling regime, but not in equilibrium. The greater efficiency in trade outweighs the cost of subsidizing type $L$ sellers as long as there is not too much insecure land in the economy.

Proof of Proposition 4.4.3. Land market transactions take place as long as there are gains from trade, taking into account illiquidity and buyers’ cost of entering the land market. With $w^U - w^R$ fixed, an economy starting from a case 3 environment will eventually switch to a case 2 environment as $\lambda$ increases, and then to a case 1. This is because rural residents develop greater incentives to accept urban job offers as the land tenure problem worsens. Proposition 4.4.3 then asserts that a case 1 land market will break down as $\lambda$ increases further.

The surplus to a seller is $p - \alpha (1 - \lambda_i) \pi$, and the surplus to the buyer is $(1 - \lambda_i) \pi - p$,
$i \in \{L, H\}$. The total surplus of a type $i$ land market transaction, including the entry cost, is therefore $(1-\alpha)(1-\lambda_i)\pi - c$. A land market exists only if $(1-\alpha)(1-\lambda_L)\pi \geq c$. Suppose that $(1-\alpha)(1-\lambda_L)\pi < c$. If $(1-\alpha)(1-\lambda_H)\pi < c$, there are no gains from trading either type of land plot. Instead, if $(1-\alpha)(1-\lambda_H)\pi \geq c > (1-\alpha)(1-\lambda_L)\pi$, the land market shuts down entirely even though there remain gains from trading type $H$ land because buyers can no longer screen for the type $L$ land. Type $L$ sellers will be attracted to the type $H$ market, and potential buyers, anticipating this outcome, choose not to enter. With $\lambda_L = \lambda + \varepsilon$ and $\lambda_H = \lambda - \varepsilon$, the land market shuts down whenever

$$(1-\alpha)(1-\lambda - \varepsilon)\pi < c \quad \Rightarrow \quad \lambda > \frac{(1-\varepsilon)(1-\alpha)\pi - c}{(1-\alpha)\pi}$$
Appendix D

Deriving the Proxy Variable for Tenure Insecurity Used in Chapter 4

The degree of tenure security associated with a particular plot is unobservable. It is precisely this hidden information that renders land illiquid in order for potential buyers to distinguish the ownership rights associated with a vendor’s plot. Although private information makes it difficult for econometric analysis, a proxy variable can be backed out of the IFLS data which, according to the theory, is correlated with tenure insecurity (the $\lambda_i$s). Recall from the model that the price of a plot of land before transaction costs is

$$p_i = (1 - \lambda_i)\pi$$  \hspace{1cm} (D.1)
The analogous pricing equation in an infinite horizon framework is

\[ p_{i,t} = \pi + \beta(1 - \lambda_i)E_t[p_{i,t+1}] \]  \hspace{1cm} (D.2)

where \( t \) indexes time, and \( \beta \) is the discount factor. Equation (D.2) can be rearranged to express the price of land as the present discounted value of the stream of per period agricultural output.

\[ p_i = \sum_{j=0}^{\infty} \beta^j (1 - \lambda_i)^j \pi = \frac{\pi}{1 - \beta(1 - \lambda_i)} \]  \hspace{1cm} (D.3)

The value of insecure land (i.e., land with a high \( \lambda \)) is lower because future farm output is discounted according to the risk of expropriation. Figure D.1 plots the densities of the log of self-reported value per hectare of household land separately for registered and unregistered land to see if this might be true in the data. On average, the perceived value of unregistered land is less than the value of land when ownership is secured by land certificates.\(^1\)

The \( \pi \) in equations (D.2) and (D.3) is broadly interpreted as land rent. If we assume that \( \pi_{i,t} \) for household \( i \) at time \( t \) is a function of farm assets \( k_{i,t} \), labour endowments \( n_{i,t} \), and landholdings \( l_{i,t} \), we can express land rent as \( \pi(k_{i,t}, n_{i,t}, l_{i,t}) \). Moreover, if factor inputs and land rent are relatively stable over time, then

\[ p_i = \frac{\pi(k_i, n_i, l_i)}{1 - \beta(1 - \lambda_i)} \]  \hspace{1cm} (D.4)

\(^1\)This is not the only interpretation. For example, if land differs in terms of soil quality, it may be that owners of poor quality land do not find it worthwhile to obtain a certificate. However, the property rights explanation becomes more plausible when I compare the ratio of the land value to annual farm output and find that the relationship still holds.
and a proxy variable for $\lambda$ can be defined as

$$\lambda_i \equiv \frac{\pi(k_i, n_i, l_i)}{p_i}$$  \hfill (D.5)

if the discount factor $\beta$ is close to one.

In the IFLS, farming households were asked to estimate the market value of their land, and to report their annual farm output. To recover a variable representing land rent, the contributions of farm assets and labour must first be subtracted from farm output. To do so, I first estimate the parameters of an agricultural production function:

$$a_i = \exp(A)k_i^{\rho_k}n_i^{\rho_n}l_i^{\rho_l}$$  \hfill (D.6)

$a_i$ is farm output for household $i$; $l_i$, $n_i$, and $k_i$ are land, labour, and capital used by household $i$; and $\exp(A)$ is total factor productivity. Taking the logarithms of both sides of (D.6) gives an equation that can be estimated by ordinary least squares:

$$\log a_i = A + \rho_k \log k_i + \rho_n \log n_i + \rho_l \log l_i + \epsilon_i$$  \hfill (D.7)

Estimates of the wage rate and rental rate on capital are obtained using marginal products:

$$w = \hat{\rho}_n \bar{a}/\bar{n} \quad \text{and} \quad r_k = \hat{\rho}_k \bar{a}/\bar{k}$$  \hfill (D.8)

where $\bar{a}$, $\bar{n}$, and $\bar{k}$ are the average levels of farm output, employment, agricultural capital stock. Land’s contribution to farm output can be written

$$\pi(k_i, n_i, l_i) = a_i - wn_i - r_k k_i$$  \hfill (D.9)
which can be calculated from the data using farm output, farm labour endowment, and farm assets. Plugging this into the numerator of (D.5) and the reported land value into the denominator yields a proxy variable for $\lambda_i$. Notice that $\pi$ is computed using $a_i$ and not the fitted values from the production function estimation since the unexplained part of agricultural output likely reflects undocumented land characteristics such as soil quality and topography. Then, according to equation (D.5), any unobserved characteristics that affect land productivity (the numerator) will also be reflected in the value of the land (the denominator). In the construction of $\lambda$, these effects will cancel out, so that the proxy variable is less sensitive to land characteristics such as soil quality and topography. Any factor that influences the perceived value of a plot of land without affecting annual farm output will however generate misleading $\lambda$ values. I therefore remove the effect of proximity to the provincial capital center on reported land values before constructing the proxy variable.\(^2\)

According to the theory, $\lambda$ is an estimate of $\lambda_i$. As such, one would expect $\lambda$ to be distributed between 0 and 1. Moreover, since even a small probability of losing ownership in a given year reflects a substantial amount of insecurity, most of the mass should be located close to zero. Figure D.2 displays the kernel density of the derived proxy variable, $\lambda$. The distribution is as expected.

\(^2\)The log of the reported value per hectare of land, log $v_i$, is regressed on a constant and the distance from the provincial capital center, $d_i$:

$$\log v_i = \gamma_0 + \gamma_1 d_i + \epsilon_i$$

The estimated coefficient on the distance variable, $\hat{\gamma}_1$ is negative and significant at the 1 percent level. Then, the adjusted total land value used to construct the proxy variable for tenure insecurity is adjusted based on the proximity to the provincial capital center:

$$\lambda_i = \frac{y_i - w_i - r_k k_i}{\exp(\log v_i - \hat{\gamma}_1(d_i - \bar{d})) l_i}$$

where $\bar{d}$ is the average distance between a farm plot and the capital center.
Figure D.1: The kernel density estimation of the per hectare value of household land (log)

Figure D.2: The kernel density estimation of the proxy variable for tenure insecurity, $\lambda$