GIGAHERTZ MODULATION OF A PHOTONIC CRYSTAL CAVITY

by

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Abstract

Photonic crystal (PtC) cavities are an increasingly important way to create all optical methods to control optical data. Not only must the data be controlled, but interfacing it with high frequency electrical signals is particularly interesting especially if this occurs in the 1.55\,\mu m telecom band. We present an experiment that uses Rayleigh surface acoustic waves (SAWs) to modulate the frequency of the guided mode of an L3-cavity PtC created on a silicon slab. This work has the potential to interface optical and electrical signals via a mechanical strain wave operating at gigahertz frequencies.

Defects are carefully designed into a triangular lattice PtC to realize a waveguide coupled optical cavity. The cavity can be experimentally accessed through grating couplers excited by polarized light at 10° incidence from normal. The optical components are fabricated on a silicon-on-insulator platform, with light confined to the silicon slab region. Through transmission experiments, the L3 cavity was found to have a narrow resonance characterized by a Lorentzian distribution. A quality factor of 165 centered at 6255\,\text{cm}^{-1}(1.599\mu m) was measured.

Aluminum interdigitated transducers (IDTs) were fabricated through a lithography liftoff process. Their ability to create SAWs requires a piezoelectric medium.
silicon does not have this property, growth of a thin ZnO film was required. The transducers were measured using a network analyzer and were found to produce Rayleigh SAWs at a frequency of 179MHz and a wavelength of 24µm. The acoustic energy traveled 70µm to the target optical device. The L3 cavity has dimensions of around 4µm a side - less than 1/2 a SAW wavelength.

Modulation of the L3 PtC resonant frequency was monitored through a repeat of the transmission experiment but with RF excitation of the IDTs at the SAW frequency. A broadening of the transmission spectrum was expected. Unfortunately no change in the fitting parameters could be measured. An HF etch was used to undercut the L3 PtC such that a silicon slab suspended in air could be realized. Simulations had been conducted showing an order of magnitude increase in the quality factor was possible. Broken wirebonds on the transducers created unintended etch channels rendering the SAW non-operational.
Acknowledgments

I would like to express my appreciation to our collaborators in Jeff Young’s research group at UBC. Especially Ellen who was always quick to help me with questions. Thank you to Paulo Santos’ research group at the Paul-Drude-Institute who helped us a great deal by providing the thin film growth. Without collaborators like these this project would not have been possible.

Many thanks to my research group at Queen’s: James, my supervisor, for guiding me along this path, and Golnaz and Ryan for working tirelessly in the trenches with me.

Lastly I’d like to thank my family for their love, support and encouragement. I wouldn’t have made it here without you. A special thanks to Kelly, my partner in crime - who makes every moment of my life amazing.
Statement of Originality

I hereby certify that all of the work described within this thesis is the original work of the author. Any published (or unpublished) ideas and/or techniques from the work of others are fully acknowledged in accordance with the standard referencing practices.

Aaron Karim Taylor Ali
April, 2013
List of Abbreviations

SAW - Surface Acoustic Wave
PtC - Photonic Crystal
PnC - Phononic Crystal
IDT - Interdigitated Transducer
EM - Electromagnetic
1D/2D/3D - 1, 2 or 3 Dimensions
FDTD - Finite Difference Time Domain
L3 - A partial line defect (3 point defects in a row)
AC(DC) - Alternating(Direct) Current
RF - Radio Frequency
SOI - Silicon on Insulator
DI - Distilled (Water)
IPA - Isopropyl Alcohol
ECR - Electron Cyclotron Resonance
TM(TE) - Transverse Magnetic(Electric)
FWHM - Full Width at Half Max
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Chapter 1

Introduction

Coincidentally, the field of surface acoustic motion in solids and that of photonic crystals find their origins in the same scientist: Lord Rayleigh. It is generally accepted that he matured the required theory necessary for the analysis of surface acoustic waves in “On Waves Propagated along the Plane Surface of an Elastic Solid” in 1885[31]. At the time, he foresaw applications in the study of earthquakes, where p- and s-waves (pressure/longitudinal and sheer/transverse) can couple into modes confined to the surface of the earth, now known as a Rayleigh surface acoustic wave (SAW). Two years later, Lord Rayleigh published another paper (partially) entitled “On the Propagation of Waves through a Medium endowed with Periodic Structure”[32]. In it, he attempts to explain the sharp colour spectrum of laminant layered plates, effectively one of the earliest 1D photonic crystals (PtC): a Bragg mirror.

Many years later, these fields would mature in relative isolation. Manufacturing constraints was the limiting factor gating the experimentation of PtCs. Photonic structures require features on the order of the wavelength of optical excitation. For
laser light created by fibre lasers of gallium arsenide based devices, this requires nanometer resolution. The advancement of semiconductor lithography techniques opened up this level of manipulation. Although generally restricted to two dimensions, many interesting experiments were devised. Berrier et al. fabricated a 2D PtC with a negative refractive index, resulting in the focusing of telecom wavelength light\cite{2}. Smith and coworkers pursued not just the properties of a pure crystal but introduced single defects in the lattice to localize modes\cite{35}. The study showed that complete photonic band gaps can be made in 2D and well chosen defects could push modes into that band, localizing them in the crystal. Further studies demonstrated the guiding of light through sharp bends using line defects (waveguiding)\cite{29}.

Combining these strategies, accompanied with design software, complete optical circuits can now be made on chip. Such an optical circuit was created by our collaborators in Jeff Young’s group at UBC\cite{34}. The central device in their experiment (and in this thesis) is an L3 cavity coupled to an input and output waveguide all created using carefully arranged defects in a single PtC (Figure 1.1). The device can be accessed by placing it in a larger circuit depicted in Figure 1.2. Grating couplers (also created using a PtC) couple light into the silicon slab. Adiabatic waveguides direct this light to the central L3 PtC. Light that satisfies the mode of the cavity exits the system through a mirrored circuit.

It was not until the turn of the century that the field of surface acoustics and photonic crystals in solids found a reason to cohabitate. Piezoelectric substrates proved an effective means to generate coherent SAWs on-chip electrically\cite{40}\cite{39}\cite{8}. With both a piezoelectric field and material deformation, SAWs provided interesting modulation schemes on semiconductor devices. In an experiment by McNeil et al.,
Figure 1.1: A layout representation of an L3 cavity photonic crystal (PtC) waveguide made by Jeff Young’s group at UBC. This device results in a highly narrow-band optical filter for light traveling from left to right (or right to left) through the structure.

Figure 1.2: The PtC filter of Figure 1.1 can be accessed experimentally through gratings that couple incident light in-plane. Adiabatic waveguides direct this energy into and out of the filter. Note this diagram is not to scale.
electrons bound in a 2D electron gas made using a GaAs/AlGaAs heterostructure were transported in the confining fields of a SAW\cite{25}. Studies have shown that not only can transport occur, the electron spin can be coherently transferred as well\cite{38}. Designed carefully, the SAW has the effect of confining the electron in an isolated moving potential well. Other recent studies have been conducted that employ PtCs that are simultaneously phononic crystals (PnCs). As these periodic structures have the ability to concentrate and direct oscillations, Eichenfield et al. demonstrated coupling between mechanical and optical modes\cite{9}.

With digital information being pushed to the optical domain, the modulation of optical structures by SAWs seemed a logical avenue to pursue. A detailed review of opto-acoustic modulation experiments using SAWs was published in 2005 by de Lima and Santos\cite{7}. Recently, many optical components have been investigated in this regard. Studies have shown SAW broadening of optical emission from quantum dots fabricated on substrates\cite{11}~\cite{26}. Using a piezoelectric substrate of LiNbO$_3$, Courjal et al. modulated an L3 PtC through elastic changes in the lattice parameter\cite{6}. Another study combined the previous two structures on a GaAs piezoelectric substrate; Fuhrmann and colleagues embedded a quantum dot within the L3 PtC, strengthening the effect of modulation on the dot’s emission\cite{10}.

This thesis centers around a collaboration with Jeff Young’s group at UBC. Their research group created and tested the L3 cavity PtC circuit shown in Figure 1.2. Fabricated on a silicon on insulator (SOI) substrate platform, the project targets application in the telecom optical domain. Our research group fabricated SAW transducers on this platform to attempt, as with the LiNbO$_3$ experiment, modulation of the crystal lattice parameter. Although the modulation scheme is not novel, realization in an
Figure 1.3: A layout representation of the electro-optical modulation scheme conducted in this thesis. SAWs are generated electrically through transducers. The acoustic energy modulates the lattice parameter of the L3 PtC from Figure 1.1.

industry supported platform (silicon) at telecom wavelengths (1550nm) justifies our endeavor. A diagram illustrating the modulation scheme being proposed is offered in Figure 1.3. The cavity portion of the filter has dimensions of around 4µm a side, less than 1/2 of the SAW wavelength, ensuring the entire cavity will compress (or stretch) concurrently.

Since the SAWs are generated electrically, this study aims to offer a strategy to modulate optical signals electrically. The frequency of modulation is dictated by fabrication and can reach into the gigahertz regime using micron feature sizes - well within the capabilities of modern lithography. This opens up the possibility of high speed information transfer from electrical to optical domains on-chip.
1.1 Thesis Layout

The layout of the remainder of this thesis follows a typical report style document. As the introduction explained, two distinct areas within physics are bridged in this research. Consequently, the reader will find nearly all sections are divided into two broad divisions: photonics and phononics. Chapter 2 builds a foundation in the theory underlying this project. The photonics are addressed mostly through eigen-mode solver simulations and band diagram interpretation. The phononic theory is approached by using numerical methods to solve approximations of the real system. Chapter 3 delves into the specific devices under test and how they were manufactured. It does this by explaining each distinct fabrication sequence (each composed of a number of processing steps) in the same order in which it was actually done to create the final device. The equipment and procedures used to characterize the devices are explained in Chapter 4. As with the previous chapters, this is done by explaining the photonic and phononic characterization separately, however, it concludes with a description of the modulation experiment which bridges the two. Finally the results are presented and discussed in Chapters 5 and 6.
Chapter 2

Theory

This study bridges two problems in physics: electro-magnetic (EM) and acoustic wave propagation. These two manifestations of dynamic energy respond to different material properties. EM fields are affected by the index of refraction \( n \), slowing in areas with \( n > 1 \). Acoustic fields have a similar relationship with material elastic stiffness \( c \), propagating faster with increased values.

The theory behind these two problems will be dealt with individually. It is interesting to note that despite the differences between them, the equations governing their motion both involve solving coupled partial differential equations in space and time. In fact, Auld shows that written in a particular way both EM and acoustic wave propagation show many similarities in form and results. It is not surprising to discover that similar computational methods (FDTD, eigenmode matrix solvers) are employed in the solution to both problems.

The next section uses Maxwell’s equations to understand how periodic \( n \) can be used to guide EM energy creating a PtC. Section 2.2 then introduces the acoustic
2.1 Theory of Photonic Crystals

Much of the theory explained in this section is developed using strategies detailed by Joannopoulos\[18\]. In the study of PtCs, the behaviour of EM fields in structured media is investigated. Much like the electronic crystals of semiconductor physics, a material with periodic properties can have interesting effects on a particle traveling through it. Since the particles of interest in this case are photons, the material property of interest is the index of refraction, \( n \) (at times it is instead convenient to consider the dielectric constant of the material, \( \varepsilon = n^2 \)). Perhaps the simplest periodic structure might be a 1D PtC shown in Figure 2.1.

It can be shown that when the lattice spacing (\( a \)) and photon wavelength (\( \lambda \)) meet the criteria specified in Equation\[2.1\] complete reflection of the incident light results\[12\].
\[ \lambda = 2n_e a \quad \text{(2.1)} \]

where \( n_e \) is the effective refractive index of the media\footnote{\( n_e \) is calculated by doing a weighted average of the refractive indices based on the energy density of the EM wave\cite{18}.} This complete reflection is equivalent to a photonic bandgap in the dispersion diagram of this material, not unlike an electronic bandgap in semiconductor physics. Extending this material periodicity to 2D, structures can be created that form a complete photonic bandgap - light cannot propagate in any direction within the plane of periodicity.

To understand these types of problems, as with most problems in EM theory, an appropriate starting point is Maxwell’s Equations in matter. It is convenient to employ a few built-in assumptions:

1. No bound current or charge density
2. Weak field strengths such that non-linear (\( \chi^2 \) and higher order) effects are ignored
3. Isotropic (electric displacement (\( \mathbf{D} \)) and electric (\( \mathbf{E} \)) fields are linearly related)
4. There is no material dispersion (no frequency dependence of \( \varepsilon \))
5. The material is non-absorbing making \( \varepsilon \) real and positive
6. Magnetic permeability (\( \mu \)) of the material is approximately unity such that the magnetizing (\( \mathbf{H} \)) and magnetic (\( \mathbf{B} \)) fields are equivalent

With these assumptions in place, Maxwell’s equations become Equations\footnote{Equations 2.2 to 2.5 in the text.} 2.2 to 2.5

\[ \nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0 \quad \text{(2.2)} \]

\[ \nabla \times \mathbf{E}(\mathbf{r}, t) + \mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} = 0 \quad \text{(2.3)} \]
\( \nabla \cdot [\varepsilon(r)E(r, t)] = 0 \) \hspace{1cm} (2.4)

\( \nabla \times H(r, t) - \varepsilon_0 \varepsilon(r) \frac{\partial E(r, t)}{\partial t} = 0 \) \hspace{1cm} (2.5)

\( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space respectively. These coupled partial differential equations can be further reduced by assuming the spacial and temporal solutions are separable. If both \( E \) and \( H \) are rewritten such that \( E(r, t) = E(r)e^{-i\omega t} \) and \( H(r, t) = H(r)e^{-i\omega t} \), making use of \( \varepsilon_0 = \frac{1}{\mu_0 c^2} \), Equations 2.3 and 2.5 combine to yield the master Equation 2.6. It is worth noting that this concise equation is quite similar to the familiar eigenvalue problem of quantum mechanics[18]. Rewritten as Equation 2.7 it becomes apparent the eigenvalue is simply \( \left( \frac{\omega}{c} \right)^2 \) and the eigenvector \( H \), with the hermitian operator \( \hat{\Theta} \).

\[ \nabla \times \left( \frac{1}{\varepsilon(r)} \nabla \times H(r) \right) = \left( \frac{\omega}{c} \right)^2 H(r) \] \hspace{1cm} (2.6)

\[ \hat{\Theta}H(r) = \left( \frac{\omega}{c} \right)^2 H(r) \] \hspace{1cm} (2.7)

### 2.1.1 Simulating The Master Equation

Although a few problems can be solved analytically through symmetry arguments, the vast majority are evaluated computationally[18]. The master Equation 2.6 as an eigenvalue problem is a convenient form as many computational methods already exist to solve it. Most software solutions for photonics often provide multiple simulation methods that can be used depending on what type of information is required. Some
software solutions include Lumerical FDTD/Mode, GMES and MIT’s MPB/MEEP. This section begins by solving for the simple cases of a uniform dielectric and a stacked dielectric analytically. The remainder of the section utilizes MIT’s MPB open source software[20] to solve the dispersion curve for the more complicated systems; a 2D square lattice and a 2D triangular lattice PtC. These two lattice types are used in the optical circuitry of this thesis and therefore serve as a solid foundation for understanding the results. It will be shown from simulation that photonic bands emerge, and in the triangular lattice case, photonic band gaps arise. The next Section 2.1.2 builds on this knowledge by including the effects of point and line defects on the triangular lattice, providing a strong transition to introduce the central device in this thesis; an L3 cavity inside a triangular lattice PtC.

The simplest case to be considered is that of a uniform dielectric. Intuition might lead one to expect the solution to Equation 2.6 is that of plane waves as shown in Equation 2.8, where \( H_0 \) is a constant and defines the polarization.

\[
H(r) = H_0 e^{(ik \cdot r)} \quad (2.8)
\]

Equation 2.2 imposes the transversality requirement, essentially stating the polarization must be perpendicular to the direction of propagation. Substituting Equation 2.8 into 2.6 and making use of the fact that \( \varepsilon \) is a constant yields the familiar condition that \( \frac{\omega}{c} = \frac{|k|}{\sqrt{\varepsilon}} \). Indeed the solution of uniform plane waves is a solution to the master equation.

A slightly more complicated case would be that of a silicon slab of finite thickness \( a \) and a refractive index \( n \) surrounded by air \((n_0)\) above and a substrate of SiO\(_2\) \((n_s)\) below. This is depicted in Figure 2.2.
In this case, intuition might suggest that the finite thickness of the slab leads to quantization of the allowed modes in this system. Clearly some modes may not meet these conditions and hence not be well confined within the slab. The plane wave solutions confined to the slab are of particular interest to this thesis, as it is within such a slab that a PtC will be created. The dielectric steps in the normal ($z$) direction are problematic, but if we consider only the in-plane wavevector $\rho$ (where we still have translational symmetry) we can attain some useful results. To simplify matters further, only transverse electric (TE) and transverse magnetic (TM) fields, perpendicular to both $\rho$ and $z$, traveling in the x-direction will be considered. To find the solution for modes in the slab an envelope function ($f$) is added to the proposed solution, Equation 2.9

\[
\begin{align*}
H_{TM}(r) &= H_0 e^{(i\rho x)} f(z) \hat{y} \\
E_{TE}(r) &= E_0 e^{(i\rho x)} f(z) \hat{y}
\end{align*}
\]  
\hspace{1cm} (2.9)

The TE case can be converted to $H_{TE}(r)$ using Equation 2.3. Substituting these
solutions into the master Equation 2.6 yields the same required condition for both TE and TM modes, Equation 2.10.

\[
\frac{\partial^2 f(z)}{\partial z^2} = \left( \rho^2 - \frac{\omega^2}{c^2 \varepsilon} \right) f(z)
\]

(2.10)

Boundary conditions are then applied by imposing continuity of the electric and magnetic fields at the interfaces. The resulting solutions are given in Equation 2.11.

\[
2v\sqrt{1 - b} = m\pi + \tan^{-1} \left( b \frac{\sqrt{n^2_s}}{1-b} \right) + \tan^{-1} \left( b \frac{\sqrt{n^2_n}}{1-b} \right) \quad \text{(for TE)}
\]

\[
2v\sqrt{1 - b} = m\pi + \tan^{-1} \left( b \frac{n^2_s}{n^2_s - n^2_s} \right) + \tan^{-1} \left( b \frac{n^2_s}{n^2_s - n^2_s} \right) + \tan^{-1} \left( b \frac{n^2_s}{n^2_s - n^2_s} \right) \quad \text{(for TM)}
\]

(2.11)

where

\[
v = \frac{1}{2} a \frac{w}{c} \sqrt{n^2 - n^2_s}
\]

\[
b = \frac{n^2_s}{n^2 - n^2_s}
\]

\[
\gamma = \frac{n^2_s}{n^2 - n^2_s}
\]

\[
n_e = \frac{\rho c}{w}
\]

\[
m = 0, 1, 2, \ldots
\]

Figure 2.3 shows a plot of the allowed modes in the system overlayed with light lines for air \( \omega = \frac{\rho}{c} \) and the substrate \( \omega = \frac{\rho}{cn_s} \). Wavevectors in the shaded regions have energies such that they can leak into plane wave modes in the SiO\(_2\) substrate (region 2) or in air (1), and so do not make for well guided modes of the slab. In this plot \( n = 3.4784, n_s = 1.444 \) and \( n_0 = 1 \), which are valid near wavelengths of 1550nm.
Figure 2.3: The allowed TE (red) and TM (blue) modes for the silicon slab structure of Figure 2.2. Light lines for air and the substrate are shaded implying a continuum of modes that are not well guided within the slab. The continuum of modes from the air (region 1) cannot couple into the slab (region 3) due to the continuum of leaky substrate modes (region 2) between the two regions. No modes exist below the light line for the slab material (region 4).
CHAPTER 2. THEORY

Figure 2.4: Bulk silicon ($\varepsilon = 12.1$) is perforated with columns of air ($\varepsilon = 1$) to form a square lattice. The structure is infinite in the normal direction.

The interesting result from this plot is that modes corresponding to plane waves in air cannot couple into modes of the slab (region 3) as they correspond to modes that will leak into the substrate. This motivated UBC’s design of the 2D square PtC to change the dispersion curve such that this coupling can occur[34]. This is the next system to be investigated and it is illustrated in Figure 2.4.

The medium is silicon with circular air columns on a square lattice. The filling fraction of the holes result in a ratio of hole radius to lattice parameter given by $r/a = 0.34$. This system emulates the grating couplers which couple incident light into and out of the slab. In this first simulation, the PtC is infinite in all directions. The $\Gamma$ - $M$ direction is simulated for both TE and TM modes; the first three bands are shown in Figure 2.5.

We can add in the effect of the slab by making some approximations in the simulation (see Appendix A) - however for the purpose of clarity (and at the expense of accuracy) simulations in this section are infinite in the $z$-direction. From the dispersion plot, it becomes apparent that plane wave modes in the air along the $\Gamma$ - $M$ direction, depending on their energy, can now couple to in-plane TE and TM modes.
Figure 2.5: 2D square lattice PtC simulations for both TE (red) and TM (blue) modes over the Γ - M direction. Only the first three bands are plotted. A similar structure forms the grating couplers used in experiment.
Figure 2.6: Bulk silicon ($\varepsilon = 12.1$) is perforated with columns of air ($\varepsilon = 1$) to form a triangular lattice. The structure is infinite in the normal direction.

of the slab. Light is incident anywhere from 5° to 15° from normal to create the required in-plane wavevector. For a total wavevector near $7500\text{cm}^{-1}$ both forward propagating TE and backward TM modes are possible.

Having coupled EM modes into the slab, waveguides direct this energy adiabatically into a mode that will couple well to the central triangular lattice PtC$^{34}$. A defect variant of this PtC is the target of acoustic modulation which is the motivation of this thesis. Before studying that case, simulating the pure crystal is a logical first step. Figure 2.6 illustrates the medium under consideration.

Again the medium is silicon with circular air columns. A filling fraction of $r/a = 0.28$ is used (consistent with the manufactured dimensions). The simulated band structure is given in Figure 2.7. Of particular interest is the complete TE photonic band gap that arises at the edge of the Brillouin zone between wavenumbers $4760-5960\text{cm}^{-1}$. Light cannot propagate along the $\Gamma$ - $K$ direction in such a structure. TE light with energy in the band gap that is incident on the PtC will be reflected - not very useful since the adiabatic waveguides are directing EM waves at this structure. In the next section, defects will be introduced so that discrete photon energies in the band gap can propagate through the PtC, thus making an effective waveguide.
Figure 2.7: Simulation of the 2D triangular lattice PtC of Figure 2.6 for both TE (red) and TM (blue) modes over in-plane wavevectors along Γ - K. Only the first four bands are plotted. A complete TE photonic bandgap is shaded in green between the 1st and 2nd bands. A defect variant of this PtC is used in experiment.
2.1.2 Point/Line Defects and L3 Cavities

Removing or changing the dielectric constant of the material at a single lattice site is known as a point defect\cite{18}. When this occurs in the 2D PtCs of the last section, the discrete translational symmetry that applied to the plane is no longer applicable. Fortunately some symmetry remains intact allowing the continued use of MPB simulations to understand the effects. For example, in the case of a single point defect 60° rotational symmetry is still present in the plane. A special kind of point defect has been introduced into the pure 2D triangular lattice of Figure 2.6 and is illustrated in Figure 2.8.

The three point defects in a row form what is known as an L3 cavity\cite{3}. The system still maintains mirror symmetry through the y=0 plane so we employ MPB to find a solution. A frequency that was previously in the complete photonic band gap is simulated, and the mode profile is displayed in Figure 2.9.

From the image, it is clear that this previously disallowed frequency is now strongly localized to the cavity. Intuitively, this can be understood as the unperturbed lattice acting as a perfectly reflecting mirrors for frequencies in the band gap\cite{18}. The cavity
allows a small area for the mode to exist but the surrounding crystal fixes it in place. Because the mode is not supported in the original PtC, it cannot leak energy into the surrounding crystal - the localized mode is highly resonant. A FDTD simulation using MEEP shows the quality factor for the center (most highly confined) mode to be $Q = 239$.

The difficulty in moving forward is how to excite this localized mode. Line defects will resolve this problem. Line defects are simply point defects arranged continuously in one direction. In this particular thesis, the line defect is created by removing an entire row of holes from the PtC. Using the reflecting walls line of reasoning, if the modes allowed in the vacated row exist in the photonic band gap, a waveguide structure results. The line defect is illustrated in Figure 2.10. Simulation results reveal a single mode is brought down from the 2nd band into the gap of the pure crystal at $5290\,\text{cm}^{-1}$. That mode profile is provided in Figure 2.11.

If the line defect is combined with an L3 cavity and matched in such a way
Figure 2.10: The PtC of Figure 2.6 with a line defect. The columns of air are now silicon.

Figure 2.11: Simulation of the line defect PtC of Figure 2.10 for TE modes for a fixed wavevector along the line defect axis. The z-component of the magnetic field intensity is shown for the mid-band mode at energy 5290 cm$^{-1}$. 
that the cavity frequency is also a guided mode, the resulting structure is a highly resonant filter\cite{34}. The 2D triangular lattice with integrated line defect and L3 cavity is depicted in Figure 2.12. A combined structure such as this can be treated using coupled mode theory. For the situation of a waveguide coupled to a cavity coupled to a waveguide, the transmission spectrum can be characterized by Equation 2.13\cite{18}.

Using the $Q$ and $\omega_0$ values found for the cavity through simulation, a plot of this equation (a Lorentzian) has been provided in Figure 2.13.

$$T(\omega) = \frac{1}{4Q^2} \left( \frac{\omega-\omega_0}{\omega_0} \right)^2 + \frac{1}{4Q^2}$$ (2.13)

This filter is to be the subject of modulation by the acoustic waves explained in the next section. A simulation to demonstrate the motivation of this modulation was conducted. The input parameters to this simulation are identical to the original filter, however the lattice parameter is modified by $\pm 0.01\%$. Such a change is within the capacity of a SAW on GaAs\cite{24,30}, which has a smaller piezoelectric constant than that of ZnO (used in this thesis). The resulting transmission is plotted along side the original in Figure 2.13. From the figure, it is clear that a noticeable change in the PtC filter frequency results from the electrically excited acoustic field. Acoustic frequencies on silicon can reach into the GHz regime\cite{5}, making possible a high-speed
Figure 2.13: The transmission spectrum for the PtC filter of Figure 2.6. The center frequency and quality factor are determined through simulation. The original design is the blue solid curve. The black dashed curves are a repeat of the procedure but with the lattice parameter changed by ±0.01%. A change in center wavenumber of ±0.5 cm\(^{-1}\) results.
**2.2 Elastic Motion in Solids and Surface Acoustic Waves**

While the last section dealt with the theory behind PtCs, this section explains the second half of the required theory: acoustic motion. The primary goal of this section is to outline the equations of elastic motion in a solid and apply those to identify our solution of interest - SAWs. There has been much work done in understanding elastic motion in solids. This section relies heavily on the theory of elastic motion as detailed by Auld[1] and Royer[33]. A common starting point is to define a variable representing the displacement of a point in the solid from its nominal position (its position in the absence of a deforming field).

Figure 2.14 illustrates the definition of \( u \), the particle displacement field.

At times, it is useful to think of this quantity \( u \) as being similar to particle position in Newton’s second law of motion \( F = ma \). In such a problem, it would be useful to define a force \( T \) acting to change the particle position. Equation 2.14 outlines this problem as it relates to elastic motion. \( T \) is the restoring force (stress field) always acting to bring the particle back to its nominal position, \( \rho \) is the density of medium.

\[
\nabla \cdot T = \rho \frac{\partial^2 u}{\partial t^2}
\]

(2.14)

As explained by Auld[1], the main problem with representing particle deformation by \( u \), is that rigid body motion (which is not deformation), results in non-zero values. This motivates Equation 2.15 which defines the strain field \( S \). This quantity
Figure 2.14: The dashed grid represents the material lattice with no deformation. The point at nominal position \( \mathbf{L} \) experiences a displacement to position \( \mathbf{l} \) at some later time \( t \). The deformation is characterized by the particle displacement field \( \mathbf{u} \) which represents the displacement of each point in space from its nominal position \( (\mathbf{l} - \mathbf{L}) \). Reproduced from [1].

is the symmetric component of the gradient of \( \mathbf{u} \) and is only non-zero under material deformation.

\[
\mathbf{S} = \nabla_s \mathbf{u} \tag{2.15}
\]

Equations 2.14 and 2.15 are two of a set of equations governing elastic motion in solids. A third equation is needed to complete the set and facilitate a solution. Equation 2.16 relates the stress field to the strain field by means of the elastic stiffness tensor \( \mathbf{c} \). A simple interpretation of this equation is Hooke’s Law\([1]\), where \( \mathbf{c} \) is the spring constant and \( \mathbf{S} \) represents the particle position.

\[
\mathbf{T} = \mathbf{c} : \mathbf{S} \tag{2.16}
\]
Written out explicitly with Christoffel symbols these become Equations \(2.17\), \(2.18\) and \(2.19\).

\[
\frac{\partial T_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{2.17}
\]

\[
S_{ij} = \begin{cases} 
\frac{\partial u_i}{\partial x_i} & \text{if } i = j \\
\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) & \text{otherwise}
\end{cases} \tag{2.18}
\]

\[
T_{ij} = c_{ijkl} S_{kl} \tag{2.19}
\]

With these three equations, we can solve a number of simple problems in elastic motion. The medium of acoustic propagation under consideration is a hexagonal lattice (that of ZnO) and is depicted in Figure \(2.15\). ZnO is actually a wurtzite crystal structure, with oxygen and zinc in distinct interlaced hexagonal lattices. Despite this the symmetries are such that the structure can be treated equivalently to a hexagonal lattice.

Due to crystal symmetries it has only five independent components to its elastic stiffness tensor, the form of which is provided in Equation \(2.20\).

\[
c = \begin{bmatrix} 
c_{11} & c_{12} & c_{13} \\
c_{12} & c_{11} & c_{13} \\
c_{13} & c_{13} & c_{33} \\
c_{44} & & \\
0 & c_{44} & \\
& & c_{66}
\end{bmatrix} \tag{2.20}
\]
Figure 2.15: The crystal lattice for ZnO is shown. The alternating zinc and oxygen atoms create distinct hexagonal lattices together forming a wurtzite crystal structure.

where \( c_{66} = \frac{c_{11} - c_{12}}{2} \). Non-transient solutions are of interest. Therefore a traveling wave solution, separable in space and time, is assumed and written explicitly in Equation 2.21.

\[
\mathbf{u}(r,t) = \mathbf{A}u_0 = \mathbf{A}e^{i(k \cdot r - \omega t)}
\]  

(2.21)

where \( \mathbf{A} \) is a constant vector representing polarization and \( \mathbf{k} \) is the wavevector. For simplicity only propagation along the x-axis (as depicted in Figure 2.15) is considered. Clearly, partial derivatives in space and time yield factors of \(-i\omega\) and \(ik_x = ik\) respectively. Substituting the trial solution into Equation 2.18 yields strain components listed in Equation 2.22.

\(\textsuperscript{2}\) The indices used exploit the symmetry of the system: \(1 = 11, 2 = 22, 3 = 33, 4 = 23 = 32, 5 = 13 = 31, 6 = 12 = 21\). Factors of 2 or 4 can also appear in stress and material property tensors. These conventions are used liberally and without notice (aside from the change of indices)[1].
$S_1 = ikA_1u_0$

$S_5 = ikA_3u_0$  \hspace{1cm} (2.22)

$S_6 = ikA_2u_0$

Employing Equation 2.19 gives the stress components listed in Equation 2.23:

$T_1 = ic_{11}kA_1u_0$

$T_2 = ic_{12}kA_1u_0$

$T_3 = ic_{13}kA_1u_0$  \hspace{1cm} (2.23)

$T_5 = ic_{44}kA_3u_0$

$T_6 = ic_{66}kA_2u_0$

Finally these results can be substituted into Equation 2.17 and evaluated to produce a system of three Equations 2.24:

$c_{11}k^2 = \rho w^2$

$c_{44}k^2 = \rho w^2$  \hspace{1cm} (2.24)

$c_{66}k^2 = \rho w^2$

Isolating for wave velocity, $v = w/k$ gives three distinct velocities corresponding to three possible bulk acoustic wave polarizations; two transverse in the y and z directions, and one longitudinal. These velocities are evaluated in Equation 2.25 using values from Table 2.1 and the resulting dispersion curves are plotted in 2.16:

$v_L = \sqrt{\frac{c_{11}}{\rho}} = 6.22\text{km/s}$

$v_{TZ} = \sqrt{\frac{c_{44}}{\rho}} = 2.99\text{km/s}$  \hspace{1cm} (2.25)

$v_{TY} = \sqrt{\frac{c_{66}}{\rho}} = 5.46\text{km/s}$
Table 2.1: ZnO material properties used in the calculation of acoustic wave values.

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>5.605 g/cm(^3)</td>
<td>[17]</td>
</tr>
<tr>
<td>(c_{11})</td>
<td>217 GPa</td>
<td>[13]</td>
</tr>
<tr>
<td>(c_{33})</td>
<td>225 GPa</td>
<td>[13]</td>
</tr>
<tr>
<td>(c_{44})</td>
<td>50 GPa</td>
<td>[13]</td>
</tr>
<tr>
<td>(c_{12})</td>
<td>117 GPa</td>
<td>[13]</td>
</tr>
<tr>
<td>(c_{13})</td>
<td>121 GPa</td>
<td>[13]</td>
</tr>
</tbody>
</table>

Another possible mode of significant interest are surface modes (SAWs). To attain this result, two requirements need to be met: a boundary condition defining a stress free surface and a complex wavevector component in the z-direction. The complex component of the wavevector will ensure the SAW solution exponentially decays into the bulk. The system under consideration is illustrated in Figure 2.17.

A trial solution of a Rayleigh SAW is given in Equation 2.26 and includes a complex z-component wavevector (\(\lambda\)). Substitution into Equations 2.18, 2.19 and 2.17 yields a system of two equations which can be written in matrix form as Equation 2.27. For a consistent solution the determinant of the matrix must vanish which yields a quadratic in \(\gamma^2 = (\lambda/k)^2\).

\[
\mathbf{u}(\mathbf{r}, t) = A\mathbf{u}_0 = (A_1, 0, A_3)e^{i(kx+\lambda z-wt)} \tag{2.26}
\]

\[
\begin{bmatrix}
  c_{11} + c_{55}\gamma^2 - \rho v^2 & (c_{13} + c_{55})\gamma \\
  (c_{13} + c_{55})\gamma & c_{55} + c_{33}\gamma^2 - \rho v^2
\end{bmatrix}
\begin{bmatrix}
  A_1 \\
  A_3
\end{bmatrix}
= [0] \tag{2.27}
\]

Once a velocity (\(v = \omega/k\)) is chosen, the equation can be solved for four possible \(\gamma\) solutions. Only solutions with positive imaginary components are kept as they allow for surface bound modes. At most, two solutions are possible; \(\gamma_1\) and \(\gamma_2\). A
Figure 2.16: Analytical solutions for acoustic modes in ZnO. Two transverse (blue) and one longitudinal (red) bulk modes are shown for acoustic fields traveling in the plane perpendicular to c-axis oriented ZnO. A surface mode (SAW) known as a Rayleigh wave is also shown (green) with depth profile plotted on the right.

Figure 2.17: A half-filled z-space boundary condition is enforced to discover a SAW as a solution to the elastic acoustic equations of motion. This condition imposes no stress along the z=0 boundary.
\[ u(r, t) = \sum_{n=1}^{2} (A_{1n}, 0, A_{3n}) e^{i(kx + \lambda_n z - wt)} \] (2.28)

Imposing the boundary conditions means all \( z \)-components of stress are zero \((T_3(z = 0) = T_4(z = 0) = T_5(z = 0) = 0)\). This yields two equations which are summarized in matrix form in Equation 2.29. The amplitude ratios \( \chi_n \) can be determined by imposing the equations of motion on the two superposition solutions individually, as in Equation 2.30. For a solution the determinant of the matrix must vanish.

\[
\begin{bmatrix}
  c_{13} \chi_1 + c_{33} \gamma_1 & c_{13} \chi_2 + c_{33} \gamma_2 \\
  c_{55} (\gamma_1 \chi_1 + 1) & c_{55} (\gamma_2 \chi_2 + 1)
\end{bmatrix}
\begin{bmatrix}
  A_{31} \\
  A_{32}
\end{bmatrix}
= [0] \tag{2.29}
\]

\[
\chi_n = \frac{A_{1n}}{A_{3n}} = -\frac{\gamma_n (c_{13} + c_{55})}{c_{55} \gamma_n^2 + c_{11} - \rho v^2} \tag{2.30}
\]

For the material constants of ZnO the Rayleigh SAW velocity was computed to be 2.79 km/s. The SAW dispersion curve and depth profile are provided in Figure 2.16.

Having discovered the SAW as a supported mode in the system, the question remains as to how they can be generated. Piezoelectricity is one method of exciting acoustic fields. Unfortunately, it complicates the situation to a degree as an electromagnetic field must propagate along with the acoustic field. This means solutions can only result by coupling Equations 2.14, 2.15 and 2.16 with equations governing the piezoelectric field. The next section introduces these equations and discusses the implications.
2.2.1 Creating SAWs using the Piezoelectric Effect

Piezoelectricity is an effect where the application of an electric field, due to a lack of inversion symmetry in the crystal lattice, causes the material to respond by deformation\textsuperscript{33}. Technically, this is called the inverse piezoelectric effect - however, throughout this document it will simply be referred to as the piezoelectric effect. The process can be reversed, applying a deformation to produce an electric field - an effect that is used in two-port network analysis which is discussed later in the thesis. In this experiment, the (inverse) piezoelectric effect is exploited to create SAWs on the surface of a microchip. If an electric field can be delivered periodically at a wavevector and frequency that is supported by the SAW mode of the piezoelectric material, such an acoustic mode should propagate through the system. This periodic electric field has been produced in numerous studies by patterning interdigitated transducers (IDTs) on the surface of the material \textsuperscript{11,37,36,6}. These IDTs are made of a metal and provide contact points for an alternating current (AC) signal to be applied. A typical IDT schematic is depicted in Figure 2.18.

A generated SAW traveling through a piezoelectric material abides by slightly alternate versions of the acoustic equations of motion given in \textsuperscript{2.17, 2.18 and 2.19}. An EM field is generated as the material deforms (and in fact itself acts to cause deformation) and must propagate along with the acoustic field. To account for this the previous Hooke’s law of Equation \textsuperscript{2.19} can be redefined to include the piezoelectric field to produce Equation \textsuperscript{2.31}

\[ T_{ij} = c_{ijkl} S_{kl} - e_{ijk} E_k \]  

(2.31)

where \( E \) represents the piezoelectric field and \( e \) is a material property known as the
CHAPTER 2. THEORY

Figure 2.18: A metal is patterned onto the surface of a piezoelectric material to create an IDT. The fingers of opposite pads alternate position at a specific period, corresponding to the wavelength of the excited field. If the frequency of the applied AC signal and the wavelength of the IDT are supported by the material along the chosen propagation direction a SAW is generated.

piezoelectric stress constant. A fourth equation (2.32) relates the piezoelectric field and the electric displacement field $D$ to the strain field.

$$D_i = \varepsilon_{ij}E_j + \epsilon_{ijk}S_{jk} \tag{2.32}$$

The dielectric constant of the material $\varepsilon$ appears again to relate $E$ to $D$. Because acoustic velocities are much smaller than EM velocities, the quasi-static approximation can be used which allows the electric field to be expressed as the gradient of the scalar piezoelectric potential; $E = -\nabla \Phi \tag{30}$. Substituting this into Equation 2.32 and making use of Equation 2.4, Equation 2.33 results.

$$\frac{\partial D_i}{\partial x_i} = -\varepsilon_{ij} \frac{\partial \Phi}{\partial x_j} + \epsilon_{ijk} \frac{\partial S_{jk}}{\partial x_i} \tag{2.33}$$

Combining two of the elastic Equations 2.17 and 2.31 common terms are identified between Equations 2.33 and 2.34.
\[
\frac{\rho \partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j} = c_{ijkl} \frac{\partial S_{kl}}{\partial x_j} - e_{ijk} \frac{\partial \Phi}{\partial x_j \partial x_k}
\] (2.34)

Isolating for \( \Phi \) in Equation 2.33 and substituting into Equation 2.34 yields Equation 2.35 (after some rearranging).

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = \left[ c_{ijkl} - e_{ijk} \left( \epsilon_{ikl} \right) \right] \frac{\partial S_{kl}}{\partial x_j}
\]

\[
\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} \left( 1 + K^2_{ijkl} \right) \frac{\partial S_{kl}}{\partial x_j}
\] (2.35)

It becomes clear that the addition of the piezoelectric effect amounts to an adjustment to the elastic stiffness constant by a material parameter \( K \), known as the electro-mechanical coupling constant [22]. Even the largest published values for \( K \) yield at most a 7% change in \( c \). As such the results found in the previous section remains valid for use in this thesis.

### 2.2.2 Lamb Waves and Plate Modes

A final consideration in this section is another flavour of surface mode called a Lamb wave. This type of wave is expected to be created when the Rayleigh SAW encounters the PtC area which (after undercutting of the SiO\(_2\)) is a slab geometry with finite thickness \( d \) as depicted in Figure 2.19. We will again consider propagation in the \( x \)-direction. This type of solution can be arrived at by imposing two stress free surface boundary conditions. Similar to the Rayleigh wave conditions, these are \( T_3(z = \pm d/2) = T_4(z = \pm d/2) = T_5(z = \pm d/2) = 0 \). Lamb described that by imposing such boundary conditions one arrives at a set of Equations 2.36[23]. Where \( \alpha = \frac{w^2}{v_x^2} - k^2 \) and \( \beta = \frac{w^2}{v_z^2} - k^2 \), functions of the bulk longitudinal (\( v_x \)) and transverse (\( v_z \)) wave velocities.
Figure 2.19: Lamb waves are expected to be created when SAWs encounter an undercut silicon slab (suspended in air). Both the lower and upper surface ($z = \pm d/2$) must satisfy stress free conditions ($z$-components of stress must be zero).

\[
\tan \left( \frac{\beta d}{2} \right) / \tan \left( \frac{\alpha d}{2} \right) = -\frac{4\alpha \beta k^2}{(k^2 - \beta^2)^2}
\]

\[
\tan \left( \frac{\beta d}{2} \right) / \tan \left( \frac{\alpha d}{2} \right) = -\frac{(k^2 - \beta^2)^2}{4\alpha \beta k^2}
\]

(2.36)

The top equation is arrived at by assuming a symmetric form (about the $z = 0$ plane) for the particle displacement field. The lower equation derives from an asymmetric assumption. Both equations can be solved computationally (see Appendix C), a plot of which has been provided in Figure 2.20. The values for silicon used in this computation are listed in Table 2.2. For the bulk wave velocities, the generic derivations from the previous section (Equation 2.25) allows them to be readily determined; $v_L = v_x = 8.43 \text{ km/s}$ and $v_T = v_z = 5.84 \text{ km/s}$. The axes chosen are typical for Lamb wave dispersion diagrams, allowing the dependence on slab thickness to be easily interpreted. Its not surprising that due to the finite thickness of the slab multiple modes arise for small values of $d$ (although not so small as to be below cut-off values for these modes). For the conditions in this thesis $d = 420 \text{ nm}$ and $\lambda = 24 \mu\text{m}$, which results in $\omega d/v_s = 0.081$. The region of interest in the dispersion diagram suggests that both a symmetric ($v_+ = 8.41 \text{ km/s}$) and asymmetric ($v_- = 1.01 \text{ km/s}$) mode can
Two classes of solutions exist, symmetric and asymmetric. As the slab thickness increases (or wavelength decreases) multiple modes arise. In the large $d$ limit the lowest modes converge to a Rayleigh SAW (dashed line marks the Rayleigh velocity for silicon). 

be generated. From the mismatch in values for Rayleigh SAW velocity and Lamb SAW velocities, it is apparent that some losses should be expected. These velocity values can be used in the final form of the Lamb wave solution given in Equation 2.37, where $\phi = 0$ for symmetric modes and $\pi/2$ for asymmetric modes. A plot of the $x$ component of the particle displacement field for two Lamb modes is given in Figure 2.21.
### Table 2.2: Silicon material properties used in the calculation of acoustic Lamb wave values.

<table>
<thead>
<tr>
<th>Property Name</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>2.329 g/cm$^3$</td>
<td>[16]</td>
</tr>
<tr>
<td>$c_{11}$</td>
<td>165.6 GPa</td>
<td>[16]</td>
</tr>
<tr>
<td>$c_{12}$</td>
<td>63.94 GPa</td>
<td>[16]</td>
</tr>
<tr>
<td>$c_{44}$</td>
<td>79.51 GPa</td>
<td>[16]</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    u_x &= qA[\cos(\beta z + \phi) - B_1 \cos(\alpha z + \phi)]e^{i(kx - wt)} \\
    u_z &= ikA[\cos(\beta z + \phi) - B_2 \sin(\alpha z + \phi)]e^{i(kx - wt)} \\
    B_1 &= \frac{2k^2 \cos(\beta d + \phi)}{k^2 - \beta^2 \cos(\alpha d + \phi)} \\
    B_2 &= \frac{2\alpha \beta \cos(\beta d + \phi)}{k^2 - \beta^2 \cos(\alpha d + \phi)}
\end{align*}
\]
Figure 2.21: Possible solutions to the slab problem of Figure 2.19 are Lamb waves. Both symmetric (top) and asymmetric (bottom) modes are possible. The $x$ component of the particle displacement field for the zeroth modes is shown.
Chapter 3

Device Fabrication

The central device in this thesis is an acousto-optical circuit on a microchip. Figure 3.1 shows an optical microscope image of the device, where the inset shows a magnified image of the photonic circuitry.

Clearly, the acoustic and optical components are substantially different in scale, with feature sizes similar in magnitude to their wavelength. The acoustic features are in the micron regime, whereas the optical are in the nanometer range. For this reason two distinct sequences of processing stages were used to fabricate the devices used in this thesis. This section is divided into four subsections, each detailing a recipe used to create the final device. The order of these subsections is the order in which fabrication was conducted. The first section describes how the optical circuitry used deep ultraviolet lithography to create the nanoscale features; devised by the Inter-University Microelectronics Center (IMEC) CMOS foundry in Leuven, Belgium. The second and third sections detail the necessary processing for the acoustic devices, including a thin film growth by Paulo Santos’ group at the Paul-Drude-Institute in Germany and transducer fabrication right here at Queen’s University. The final
Figure 3.1: An optical microscope image of the acoustic and optical devices used in this thesis. The substrate is a SOI wafer. The main image is taken at 5x magnification and shows the micron scale SAW transducers. The blue outlined inset is taken at 50x magnification and shows the nanometer scale optical components.
section details an optimization step to the optical components developed in-house based on previous work.

### 3.1 Photonic Devices with Deep-UV Lithography

The introduction section explained how the central experiment in this thesis was devised through a collaboration between institutions. The photonic devices were part of the project conducted by Jeff Young’s research group at UBC. The PtC design, fabrication and test details are not a part of this thesis. A brief overview of the fabrication is provided here for clarity. For additional details, refer to the Master’s thesis by Ellen Schlew of UBC[34].

The photonic components were fabricated through a CMOS foundry operated by IMEC. They provides a 6 inch wafer process to pattern the silicon on insulator (SOI) photonics wafers. The wafers have a bulk silicon substrate with a 2µm thick buried layer of silicon dioxide and a 220nm thick silicon top layer. The process involves deep UV lithography followed by a wet-etch to remove the top silicon layer in select areas to form the photonic structures.

Each chip is 10mm by 14mm allowing for multiple chips to be made on one wafer. Within each chip, the test structures were repeated in groups (shifts) of 6, itself repeated across 9 rows and 7 columns. In total 378 devices were provided on one chip. The layout of a chip is provided in Figure 3.2.

The chip was designed in such a way that the L3 PtC characteristics varied across the chip. As the rows increased the hole diameters increased by 10nm. As columns increased the lattice pitch increased by 5nm. The spacing between the pairs of holes that create the L3 defect within the line defect increases as the shift increases. A
CHAPTER 3. DEVICE FABRICATION

Figure 3.2: A layout of the optical components on the chip. A total of 9 rows x 7 columns x 6 shifts resulted in 378 unique devices. The L3 PtC hole diameter increases with row position and the lattice pitch with column position. The spacing between the pairs of holes in the line defect increased with shift position. Reproduced from [34].
device can therefore be uniquely indexed through a specific row, column and shift (eg. R1C2S3).

A final consideration in this section is a correction that had to be made to account for a manufacturing defect. The processing resulted in approximately 50nm larger hole diameters for the grating couplers than what was designed. To account for this UBC kept the grating couplers covered in photoresist. The difference in refractive index from what it would have been in air shifted the grating modes such that a compatible dispersion curve was retrieved. This was fortuitous as ZnO needed to be grown over the entire chip to accommodate the phononic circuitry. ZnO provides an equivalent refractive index adjustment.

3.2 ZnO Processing and Recovering PtC Performance

The acoustic components were largely fabricated at Queen’s University. Our research group at Queen’s specializes in SAW transducer fabrication and testing. The wafers used in this fabrication are often piezoelectric, being a required property for electro-acoustic transducers (IDTs). In this particular study, the wafers selected by the research group at UBC (for their optical properties) were SOI. Since silicon is not piezoelectric, a thin film of ZnO was grown onto the completed photonic devices of the previous section.

As was described in the theory section, SAWs propagate in the plane perpendicular to the ZnO c-axis. This necessitates a c-axis oriented crystalline structure. There are a variety of ways to grow orientation specific thin films. In the past, our research group
Figure 3.3: A RF magnetron deposition process used to create a thin film of c-axis oriented ZnO. The target material of high purity ZnO is evaporated by an RF power supply. The ZnO ions mix with the oxygen plasma created from the left by Electron Cyclotron Resonance (ECR). The ions travel in low pressure to the surface of the target. A magnetic field is maintained perpendicular to the target to ensure proper crystal orientation. Reproduced from [21].

has experimented with sol-gel processes alternated with annealing. Unfortunately the films resulting from these studies have been of poor quality and not piezoelectric. That motivated a more traditional process that had more control over the growth: RF/DC magnetron sputtering[21][8].

DC magnetron sputtering can be done by applying a DC current across a conducting target. For a ZnO film target materials are typically ZnO or Zn[39]. The evaporant is released into a controlled atmosphere at a fixed temperature and allowed to accumulate on the surface of the microchip. RF magnetron sputtering is required to limit charging if the target is non-conducting, as is the case with the ZnO target used in this process, but this can also be used with conducting targets[8][27]. RF Sputtering shows a preferential ZnO growth with a c-axis oriented normal to the surface[39][8]. The entire process is illustrated in Figure 3.3.
Unfortunately, ZnO sputtering was not readily available at Queen's, therefore the film growth had to be done off-site. Paul Santos’ research group at the Paul-Drude-Institute in Germany have experience in this process. We were fortunate that they agreed to assist us in growing a 1µm thick, c-axis oriented, ZnO film. Plain silicon wafers were also sent and received equivalent treatment so that they could be tested without the complications of the photonic devices.

A small discussion in the previous section (3.1) explained the necessity of a specific refractive index in the holes of the grating PtCs. The grating holes required a material having $n \approx 2$ while the holes in the L3 PtC required air ($n = 1$). The ZnO grown in this section was deposited uniformly across the surface of the chip. Coincidentally, it has a refractive index of $n = 2.008 \approx 2.008$. As such, it meets the needs of the grating holes, however it adversely affects the L3 PtC. To recover the performance of the original design, the ZnO covering the L3 PtC needed to be removed.

The PtC spans approximately 10µm in width. When the ZnO was etched, an entire shift group was opened simultaneously making the etch height actually six PtCs high (plus spacing) or 250µm. The large feature sizes permitted standard optical lithography techniques to be used to make an etch mask. This is the same processing technology used to pattern the IDTs described in the next section, and our in-house recipes have shown good pattern definition down to a 2µm feature size. Being an oxide, ZnO is easily etched by a variety of acids (buffered HF, HCl, FeCl$_3$)[40][41]. This processing step had the advantage of relaxed side-wall profiles so HCl was chosen as it is a fast, easily handled and accessible etchant. Tests were conducted to ensure our usual resist for IDT fabrication, Micro Resist Ma-N 405, would provide a suitable etch mask. The final recipe is outlined in Table 3.1 and schematically in Figure 3.4.
CHAPTER 3. DEVICE FABRICATION

<table>
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<tr>
<th>Step</th>
<th>Process</th>
<th>Procedure</th>
<th>Time</th>
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<tbody>
<tr>
<td>1</td>
<td>clean</td>
<td>rinse with acetone, IPA and DI water then dry</td>
<td></td>
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<tr>
<td>2</td>
<td>spin coat</td>
<td>apply Ma-N 405, spin at 3000rpm</td>
<td>30s</td>
</tr>
<tr>
<td>3</td>
<td>expose</td>
<td>expose with Xe/Hg lamp at 8.4mW/cm²</td>
<td>3s</td>
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<tr>
<td>4</td>
<td>develop</td>
<td>submerge in Ma-D 331S and stir</td>
<td>2m</td>
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<tr>
<td>5</td>
<td>rinse</td>
<td>rinse in DI water</td>
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<tr>
<td>6</td>
<td>dry</td>
<td>dry with N₂ gas</td>
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<tr>
<td>7</td>
<td>etch</td>
<td>submerge in 0.3% HCl at room temperature and stir</td>
<td>15s</td>
</tr>
<tr>
<td>8</td>
<td>clean</td>
<td>rinse in acetone, IPA and DI water</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>dry</td>
<td>dry with N₂ gas</td>
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Table 3.1: A recipe used to etch 10µm x 250µm rectangles through 1µm thick ZnO on an SOI wafer.

Figure 3.4: A schematic representation of the recipe detailed in Table 3.1. ZnO is processed using lithography for mask creation and a HCl wet etch for removal.
3.3 Acoustic Transducers using Lift-off Lithography

After growing the piezoelectric ZnO and removing it from the L3 PtC, the next step was to build the transducers. The IDT mask used in fabrication was a 3µm double figure width design (see Figure 3.5). This resulted in a SAW wavelength of \( \lambda = 8 \times 3\mu m = 24\mu m \). Its worth noting that this is larger than the extents of the L3 defect (a few microns across). A double finger pattern was chosen because previous studies have shown it to provide better acoustic power delivery[7]. The SAW position was chosen to minimize the distance to the L3 PtC (70µm) in order maximize the acoustic power available for modulation.

Aluminum is a common metal used in IDT fabrication on ZnO[40][39][8]. A lift-off process involving optical lithography and an evaporator were used in their construction. The initial steps involving mask creation (exposure and development) are identical to steps 1 to 6 in Table 3.1. After mask creation the sample was placed in an electron beam evaporator which deposited 40nm of aluminum evenly across the chip. Openings in the mask allow aluminum to adhere well to ZnO. The other areas “lift-off” when the sample is placed in an acetone ultrasound bath for a few minutes. An optical image of the final transducer is provided in Figure 3.5. A schematic representation of the process is illustrated in Figure 3.6.

3.4 Improving PtC Quality through Under-Cutting

Having completed fabrication of both the optical and acoustic components the remaining task was to undercut the SiO2 layer. Appendix A covers the MPB simulations
Figure 3.5: An optical image of a SAW IDT after fabrication taken with a 10x objective.
CHAPTER 3. DEVICE FABRICATION

Figure 3.6: A schematic representation of the process used to fabricate the SAW IDTs. A mask is created using lithography upon which aluminum is evaporated. The chip is submerged in an acetone ultrasound bath to finish the process.

and details the argument justifying this procedure. Essentially the index contrast is better when both layers surrounding the slab are air as opposed to the bottom being SiO$_2$. The overall effect is an increased quality factor for the L3 PtC resonance. Without the increase in quality factor, the effects of SAW modulation would likely not be detectable. Normally, the presence of the ZnO would be problematic as it previously covered the entire chip. Fortunately, the steps in the previous section remove the ZnO directly above the L3 PtC, partially exposing the section of buried SiO$_2$ of interest. The same photoresist mask used in the last section is used to protect all layers but the L3 PtC cavity.

Young’s group at UBC performed a similar fabrication step to also increase the quality factor of the L3 PtC\cite{34}. Those recipe steps were modified to accommodate the etchant and resist chemicals available at the Queen’s fabrication facility. A buffered oxide etchant (BOE) composed of 6 parts 40\% $\text{NH}_4\text{F}$ and 1 part 49\% HF was used. This was slightly different than the 10:1 solution used in the original recipe. Tests on similar samples suggested an etch time of 15 minutes was appropriate for
CHAPTER 3. DEVICE FABRICATION

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<td>rinse</td>
<td>rinse in DI water</td>
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<tr>
<td>6</td>
<td>dry</td>
<td>dry with N₂ gas</td>
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<tr>
<td>7</td>
<td>hard bake</td>
<td>bake at 100°</td>
<td>30m</td>
</tr>
<tr>
<td>8</td>
<td>expose</td>
<td>expose with no mask using Xe/Hg lamp at 8.4mW/cm²</td>
<td>15s</td>
</tr>
<tr>
<td>9</td>
<td>etch</td>
<td>submerge in 6:1 BOE (40% NH₄F:49% HF) at room temperature and stir</td>
<td>10m</td>
</tr>
<tr>
<td>10</td>
<td>clean</td>
<td>rinse in acetone, IPA and DI water</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>dry</td>
<td>dry with N₂ gas</td>
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Table 3.2: A recipe used to etch 10μm x 25μm rectangles through a partially buried 2μm thick SiO₂ layer.

the thickness of SiO₂ (2μm). These tests also served to verify that the Ma-N 405 photoresist could provide a suitable mask for the process. The final recipe is detailed in Table 3.2.

Based on supplier application notes, some additional steps were taken to improve the resist’s chemical stability in HF. The chip was hard baked at 100° for 30 minutes after the resist was developed. Following that was a flood exposure (no mask) for 15 seconds. The sample was then submerged in the BOE for 10 minutes. An optical image of the final device is provided in Figure 3.7. A schematic representation of the process is illustrated in Figure 3.8.
Figure 3.7: An optical image of the L3 PtC after undercutting of the buried SiO$_2$ layer using a BOE.
Figure 3.8: A schematic representation of the process used to undercut the L3 PtC. A mask is created using lithography with additional hard bake and flood exposure steps for hardening. The chip is submerged in HF, which etches the partially exposed SiO$_2$. 
Chapter 4

Experimentation

Similar to the fabrication section layout, experimental methods for testing the acoustic and optical components are explained in separate sections. The first section describes the characterization of SAW transducers by means of a network analyzer. A scanning Sagnac interferometer can also be used to measure the picometer scale vertical displacement caused by the SAW. The second section explains the various fibre and free-space optical transmission experimental setups and the reasons governing their design. The final section details the experiment that bridges the two; a hybrid of the free-space optical transmission setup is used to measure the SAW modulation effects.

4.1 SAW Performance and Deformation Imaging

The acoustic components fabricated in this thesis are IDTs, capable of transforming an electrical AC signal to an acoustic mode of the chip. The transducers used in this thesis are fabricated in pairs, with a mirror image of the transducer appearing
a couple millimeters along the SAW propagation direction. When a SAW is created at one transducer it travels along the surface of the chip until it activates the paired transducer. The receiving transducer works in reverse fashion, providing an electrical signal generated from the SAW. This arrangement allowed for a network analyzer to conduct two-port analysis.

In two-port analysis the network parameters $S_{11}$ (transducer 1 reflection), $S_{22}$ (transducer 2 reflection), $S_{21}$ and $S_{12}$ (transmission from 1 to 2 and 2 to 1) are measured as the frequency of the electrical signal is swept. The set of four network parameters are known as s-parameters and are standard characterization parameters in electrical networks. Figure 4.1 shows an s-parameter of IDTs similar to those used in this thesis, placed on a silicon wafer coated with 1 $\mu$m zinc oxide (no optical devices). At times discerning a SAW signal from the noise can be difficult. The network analyzer can also operate in time-of-flight mode, exciting at a fixed frequency while recording time correlated signals. This technique was used for the device sample due to the high levels of noise. Some of these plots have been provided in Appendix B.

A series of these s-parameter measurements were taken for transducers of different wavelength fabricated on a 2 $\mu$m zinc oxide thin film on a silicon substrate. The result of plotting wavevector versus Rayleigh SAW angular frequency is given in Figure 4.2. A linear regression gives the resulting speed of sound ($w = vk$); (3.43 ± 0.03)km/s. This is faster than the zinc oxide SAW velocity found in the theory section (2.79km/s), but slower than that of silicon (4.7km/s)[15]. This is expected as a faster velocity results from more of the acoustic energy being concentrated in the silicon layer[8] (recall the depth penetration is on the order of the wavelength).
Figure 4.1: The s-parameters are measured using a network analyzer operating in two-port mode on a pair of opposing IDTs. The solid blue line shows $S_{12}$ (scale on the left), while the dashed black line shows $S_{22}$ (scale on the right). Aside from selecting robust signals that correlate in both reflection and transmission spectra, Time-of-flight measurements are also used in identifying features. The Rayleigh SAW mode and Sewaza mode are annotated. The 1st harmonic of the Sewaza mode is just off scale.
Figure 4.2: The Rayleigh SAW fundamental frequency determined from s-parameter analysis plotted versus wavenumber. The IDTs were fabricated on a 2µm ZnO film deposited on a silicon substrate. A linear regression shows the speed of sound for this mode to be (3.43 ± 0.03)km/s. The residuals have been provided in the inset. A clear trend can be seen. This is attributed to the velocity dependence on the depth penetration of the SAW - which changes with wavelength.
Understanding how much deformation is imparted to the optical components is useful when analyzing the modulation data. Although the electrical power delivered to the IDTs can be measured, losses can occur through many channels before SAWs reach their destination. As no impedance matching was performed, losses are expected in the RF connections to the transducer. Transducer design can also affect conversion; acoustic power can escape through both the intended and opposite propagation directions out of the transducer. Horn shapes have been utilized in other studies to combat this effect[7]. Another source of loss occurs when the Rayleigh SAW impinges on the silicon slab (converting to a Lamb SAW - see Section 2.2.2).

In general, these losses are not a major concern as the electrical power can often be increased to compensate. However, it does mean an accurate calculation of deformation at the L3 PtC is difficult to infer. The scanning Sagnac interferometer offers a probe capable of measuring the magnitude of the vertical surface displacement using laser interference. The device was implemented using free-space optics by a previous master’s student of our research group, Reuble Mathew. For detailed information on its operation please refer to that thesis[24].

The principle behind the Sagnac interferometer is to sample the surface twice using coherent light traveling along different paths. The two paths are equal in total distance, but sample the surface at slightly different times. The interference between the two signals is measured and correlates to the difference in chip surface position at the two sample times (all else being equal). A schematic from Mathew’s thesis is reproduced in Figure 4.3 to illustrate the experiment. The interferometer splits a 532nm wavelength 10mW laser into two paths separated by a physical distance (2d) on approach to the sample. This distance is set to be equal to 1/2 the SAW
Figure 4.3: A block diagram of the Sagnac interferometer used to measure the vertical displacement of the chip surface caused by the Rayleigh SAW. This image was recreated from the thesis of a former master’s student in our group, Reuble Mathew[24].

The period ($\tau$) times the speed of light ($c$), Equation 4.1, so as to sample over maximum displacement. On return before measurement the signals travel the opposite path to remove the initial phase added leaving only the phase difference created by reflecting off of the sample.

$$2d = \frac{\tau \times c}{2} \quad (4.1)$$

The device can scan in the 2D plane of the chip to create a contour map of the vertical displacement caused by the SAW. The measurements have a spot size of approximately $(2.4\pm0.2)\mu$m and a position resolution of $0.3\mu$m. The interference has been shown to be highly sensitive - capable of measuring displacement with $(4\pm1)$ pm resolution[24]. During analysis these displacement measurements can be used to validate the magnitude of the optical modulation.
4.2 Optical Transmission Measurements

Optical transmission measurements were the method used to characterize the photonic components. A few variants on the implementation of this experiment were used. The strategy common to all involved focusing a 1550nm wavelength 5mW laser at $\sim 10^\circ$ incidence onto a grating coupler. The light emitted by the out-coupling grating was collected and focused onto an optical detector. Once a desirable configuration was attained the laser wavelength was swept. The range of wavelengths that represent supported modes of both the grating and the media within which it travels appeared as a resonance in the transmission plot.

The simulations of the PtC Section 2.1 show that the grating couplers support both a forward propagating TE mode and a backward coupling TM mode. When positioned in the regular fashion as depicted in Figure 4.4 the TE mode travels through the waveguide to the L3 PtC and out the matching grating. In this same position, a TM mode (also depicted) will travel backwards through the silicon slab to a grating of the adjacent column. As such, the experimental method involves first finding the transmitted TM mode and optimizing this signal to ensure proper beam focus and alignment. The TM mode coupling is advantageous as it is described only by the slab and grating coupler broad resonance ($\sim 125\text{cm}^{-1}$) as compared with that of the TE mode which is bound to the narrower L3 PtC resonance ($\sim 40\text{cm}^{-1}$). Once this is completed the TE mode is excited, and the signal of interest, L3 PtC transmission, can be more easily found.

A preliminary version of the transmission experiment was performed using the CMC Photonics Lab at Queen’s University. The facility is a fibre laser lab used mostly for electrical engineering experiments. We were fortunate this lab already had
CHAPTER 4. EXPERIMENTATION

Figure 4.4: During an optical transmission measurement either TE or TM modes can be excited. The TE mode forward couples through the L3 PtC as designed. A TM mode travels in the reverse direction and emits from a grating of a neighbouring column. The angle of incidence $\alpha$ can be varied slightly about 10°.

A transmission setup that was compatible with the overall strategy outlined above. Optical fibres were driven by a 1550nm wavelength laser (Agilent 81600B Tunable Laser). Two plastic arms were threaded with fibre cores (cladding removed), one used to shine laser light the other to collect. Each arm was equipped with three micron stage axis controls allowing motion over 3D. They could be configured to approach input and output fibres at 10° incidence to a sample mounted horizontally on a table. A microscope focused on the sample to allow visual feedback while placing the fibres. The collected light intensity was then measured with an Agilent 81634B Optical Power Sensor. A diagram of the setup is provided in Figure 4.5. Following the steps outlined above, a TM transmission signal was first found to properly position the input fibre (corresponding to slab transmission). Finding the transmission signal through the PtC can be problematic. The efficiency of the gratings were under 5% for both laser polarization[34], resulting in most of the light being reflected. The transmitted and reflected signals are only spatially separated for about a millimeter after leaving the sample. The fibre setup had a distinct advantage of collecting light
Figure 4.5: Using the fibre laser facilities at the CMC lab at Queen’s University, the optical transmission of the chip was measured. Fibre holders allowed input and output fibres to approach the sample at 10° incidence. The output fibre took light to a power meter. The driving laser allowed the wavelength to be swept once a suitable signal was found.

close enough to the sample such that signal light alone could be selected. Following identification of slab transmission a TE signal was applied, and the output fibre was moved to the corresponding grating. Once a transmission signal was established, the driving laser was swept about the resonance with step sizes of around 0.5nm. Ultimately, this particular setup suffered from a large excitation spot size. As the results section show, we were unable to excite a single grating coupler resulting in artificial broadening of the PtC transmission signal.

To probe individual devices, a customized setup was designed in our own facilities, which involved free-space optics. Figure 4.6 shows a block diagram of the experimental setup. Two lasers (Sacher 780nm and Ando 1520-1620nm Tunable) traveled coincident beam paths to the sample. The visible red laser was sampled just after reflection off the sample as the beam was diverging. In this manner, the excitation spot on the sample was configured. Three stage actuators controlled the spot position and size on the sample. The sample stage also rotated so an angle of incidence about 10° could be selected. Once complete, the visible laser was allowed to propagate through the
remaining experiment through to the detector. Proper alignment of the entire beam path could be done using this mode of operation.

Following the procedure developed for the optical measurements, a TM transmission signal was first identified. Unlike the fibre setup, the reflection and transmission signals were not spatially separated when collected by the prism mirror (M4 in diagram). The light from the chip was refocused to a second imaging plane where a 50µm pinhole (aperture in diagram) was used to block the reflection and permit transmission. The transmitted light could then be focused onto a photodiode (Newport InGaAs 818-BB-30). A lock-in amplifier (SR830) digitized the signal, which was referenced to an optical chopper placed in the optical path just before the polarizer. The frequency of the laser was swept and transmission versus wavelength plotted to reveal the resonance. As before, once TM transmission was identified and optimized, the laser polarization was changed to couple to the TE mode, and the L3 PtC transmission was searched for. Akin to moving the output fibre, the pinhole was moved to account for the change in the position of the output grating.

The task of moving the pinhole proved to be inefficient. To increase the usability of the setup, a camera was positioned after the pinhole (this followed a similar strategy used in UBC transmission experiments). The camera was a 1D 1024 InGaAs pixel array, attached to a spectrometer - Acton SP2500. The spectrometer also had a manual 1D slit aperture before the camera that replaced the functionality of the pinhole. With the slit fully opened, the transmission and reflection signals appeared as distinct, spatially separate excitation spots thus allowing multiple spatial positions to be probed simultaneously. The transmission signal could be isolated by reducing the slit size and tuning the beam axis. In addition, the signal could be optimized by
Figure 4.6: The bulk of optical transmission experiments were done using a free-space setup. The sample was mounted vertically. A zoom-in on the sample holder configuration is provided in the dash-dot inset. Four degrees of movement configured the sample stage. A rotation allowed the angle of incidence to be tuned about 10°, while the other three controlled the three standard axis. The driving laser allowed the wavelength to be swept once a suitable signal was found. This configuration of the experiment also included an optional RF generator to simultaneously excite SAWs while measuring transmission (dashed area).
adjusting the 3D micron actuators at the sample stage to properly excite the input grating. A typical reflection and transmission signal as seen by the camera is provided in Figure 4.7. The transmission and reflection signals were separated at the camera by -30 pixels in the case of TM and +90 in the case of TE. These bin separations equate to the small distance traveled by the TM slab mode back to the previous column and the larger traveled by the TE L3 PtC mode across a column. The actual spacial separation of the two signals from the reflection (at 0) is -250\(\mu\)m and +750\(\mu\)m. Using a 25\(\mu\)m pixel width the imaged plane experiences a 3x magnification.

Once the transmission signal is isolated, the spectrometer is placed into feed-through mode. In this state mirror 3 (M3 - inside the spectrometer) redirects the beam away from the camera and out the side of the spectrometer. The photodiode was positioned to catch the feed-through signal allowing measurement of the transmission intensity as the wavelength was swept.

4.3 SAW Modulation of Optical Transmission

The previous section offered a method of measuring optical transmission. With the Rayleigh SAW frequency identified from the s-parameter analysis (Section 4.1), all that remained was to excite the acoustic transducers and measure the modulation. As discussed in the photonics theory Section 2.1, broadening of the optical transmission spectrum was expected due to modulation of the PtC lattice parameter. A regular optical transmission frequency sweep was conducted as outlined in the previous section. Additionally, SAWs were excited with the RF generator (SG384l) at its full power of 9dBm. An RF amplifier (ZX60-V82) was used at times providing another 20dBm of gain resulting in a 30dBm RF signal exciting the SAW.
Figure 4.7: In the free-space optics experiment the transmission signals had to be selected from the (much larger) reflection signal. Because the TM and TE transmission signal were in different locations a 1D 1024 pixel array camera was used to perform the selection. The (saturated) reflection signal was separated by -30(+90) pixels for TM(TE) excitation corresponding to slab(PtC) transmission. The above image is actually a panorama using a series of shots while moving the focus point on the array.
Another possible measurement is to fix the wavelength and gather time-resolved optical transmission data to verify modulation. Since the signal should be periodic at the frequency of the SAW, a reference signal can be provided by the RF generator to trigger a high-speed oscilloscope. The modulation transmission experiment is shown in Figure 4.6 when the dashed areas are included. An annotated picture of the setup is given in Figure 4.8.
Chapter 5

Discussion and Results

Before fabrication of the acoustic components began (ZnO film growth and metal deposition) the optical components required characterization. These measurements were conducted at the CMC fibre optics lab at Queen’s University. The chip was delivered with a protective photoresist coating, which was removed before testing. Figure 5.1 is a plot of slab transmission from R8C4S1 to R8C5S1 at 10° incidence using TM polarized light. A Gaussian distribution (Equation 5.1) was fit to the signal yielding a center of $\mu = 6620\text{cm}^{-1}$, $\sigma = 50\text{cm}^{-1}$ and a FWHM (Equation 5.2) of 125cm$^{-1}$. The results are comparable to those attained by UBC with FWHM for uncoated samples ranging from 110 - 140cm$^{-1}$[34].

$$y(x) = \frac{1}{\sigma\sqrt(2\pi)} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$  \hspace{1cm} (5.1)

$$\text{FWHM} = 2\sqrt{2\ln 2}\sigma$$  \hspace{1cm} (5.2)

Due to the grating coupler holes being larger than designed, only a few of the
Figure 5.1: Using the fibre setup at CMC, slab transmission through R8C5S1 to R8C4S1 using TM polarized light is provided. The wide resonance of the grating couplers resulted in a Gaussian distribution width of $\sigma = 50$. 

\[
\mu = 6620, \quad \sigma = 50
\]
Figure 5.2: Using the fibre setup at CMC, PtC transmission through R8C5S1 using TE polarized light is provided. A narrower resonance than the slab was found; $\sigma = 40$, but this was wider than expected from UBC results. It is suspected that multiple PtC’s in the shift were excited resulting in artificial broadening and a Gaussian distribution.

circuits had compatible grating and PtC resonances. At the advice of UBC, efforts were concentrated around the R8C5 devices. PtC transmission was found for multiple devices in this area. That for R8C5S1 is plotted in Figure 5.2. The sample was excited at 10° incidence with TE polarized light.

$$Q \approx \frac{\mu}{FWHM}$$

(5.3)

As the PtC had not been undercut, a low quality factor was expected. Using
Equation 5.3 a quality factor of 70 was determined. This is lower than UBC results ranging from 150-800 for similar samples [34]. This wide resonance was found despite many attempts to optimize alignment. It was suspected that the beam spot size was too large. A larger spot size would mean multiple gratings couplers (and hence multiple PtCs) were being excited, artificially broadening the resonance. This also gave the spectrum a Gaussian profile instead of the expected Lorentzian shape. Distribution parameters yielded $\sigma = 40\text{cm}^{-1}$, FWHM of $94\text{cm}^{-1}$ centered at $6620\text{cm}^{-1}$.

With the initial optical characterization complete, the samples were sent to the Paul-Drude-Institute in Germany where the thin film ZnO was grown. Once returned, it was important to recharacterize the optical performance. Previous measurements had been conducted with grating coupler holes of air which were now instead filled with zinc oxide, offering a slightly higher index of refraction (as discussed in fabrication). The change in index caused a shift to lower wavenumbers which is reflected in Figure 5.3; a plot of slab transmission while covered in zinc oxide. This measurement was done using the free space optics arrangement at $10^\circ$ incidence with TM polarized light from R0C4S5 to R0C3S5. A Gaussian distribution was fit to the signal yielding a center of $6340\text{cm}^{-1}$, $\sigma = 45\text{cm}^{-1}$ and a FWHM of $106\text{cm}^{-1}$, within the range of similar devices.

Compatible PtCs were found by removing zinc oxide over likely candidates and looking for a transmission signal. Knowledge of the chip layout and resulting shifts in resonance between columns and rows [34] aided in narrowing down the R0C4 shift of devices as suitable matches. The S6 PtC circuit transmission is provided in Figure 5.4. A Lorentzian distribution (Equation 5.4) was fit to the results yielding parameters $\Gamma = \text{FWHM} = 38\text{cm}^{-1}$ and a center of $6255\text{cm}^{-1}$. The resonance improved significantly
Figure 5.3: Using the free space setup at Queen’s, slab transmission from R0C4S5 to R0C3S5 using TM polarized light is provided. A Gaussian distribution width of $\sigma = 45$ was found which is slightly narrower than the fibre result ($\sigma = 50$). The signal before accounting for system response is the dashed line.
Figure 5.4: Using the free space setup at Queen’s, PtC transmission through R0C4S5 using TE polarized light is provided. A Lorentzian distribution width of $\Gamma = 38$ was found. The signal before accounting for system response is the dashed line. A poorly configured transmission (R8C5S3) is offered in the inset. With better positioning of the input grating excitation the signal can be improved.
over the fibre measurements. A quality factor of 164 was found, consistent with UBC results.

\[ y(x) = \frac{1}{2\pi} \frac{\Gamma}{(x - \mu)^2 + (\frac{\Gamma}{2})^2} \]  

(5.4)

With the target PtC acquired, transducer fabrication was carried through. Transducers with a wavelength of 24μm were patterned using the fabrication facilities at Queen’s University. To identify the Rayleigh SAW mode frequency, s-parameter analysis was conducted. Time-of-flight measurements were also taken to help discern SAW signals - some of which have been provided in Appendix B. The results of analysis are provided in Figure 5.5. The frequency was found to be \((179 \pm 1)\)MHz yielding a speed of sound of \((4.29 \pm 0.3)\)km/s. Considering the ZnO thickness to wavelength ratio \((1/24)\) this is comparable to speeds found in other studies \((4.3\text{km/s})\) [15].
Figure 5.5: The network analyzer was used to find the SAW Rayleigh mode frequency (annotated). In the top plot, the solid blue line shows $S_{12}$ (scale on the left), while the dashed black line shows $S_{22}$ (scale on the right). The spectrum experienced a substantial amount of noise. Time-of-flight measurements were made at frequencies of interest. The bottom plot shows a transmission signal received at 220ns at the Rayleigh mode frequency of 179MHz. This value is used in both the Sagnac and optical modulation experiments that followed.
CHAPTER 5. DISCUSSION AND RESULTS

Having identified the SAW frequency, measurements to characterize the deformation imparted to the PtC were conducted using the Sagnac interferometer. Due to the significant difference in reflectivity of aluminum and zinc oxide, the transducers could be easily imaged. The intensity image of Figure 5.6 demonstrates the spacial resolution is sufficient to identify features on the order of the transducer fingers ($2\mu$m), providing confidence in resolving SAW features. Numerous attempts were made to acquire an interference measurement and quantify the magnitude of deformation in the PtC area. Unfortunately due to an inability to overcome noise issues this was not possible. It should be noted that deformation values well within the required amount ($\pm 0.01\%$) have been attained in other studies\cite{24,30}.
Figure 5.6: The Sagnac interferometer was used to image the mouth of the SAW IDT. A log-normalized raw intensity signal is shown. Since the aluminum transducer is significantly more reflective than the zinc oxide coating good image contrast results. The keen observer will notice a (unrelated) line feature in the buried silicon layer cutting across the IDT around the first finger.
With acoustic and optical components quantified in isolation, transmission modulation experiments could be conducted. Figure 5.7 shows optical transmission intensity with and without RF. A Lorentzian distribution was fit to the data yielding parameters $\Gamma = 50\text{cm}^{-1}$ and a center of $6276\text{cm}^{-1}$. The quality factor was calculated to be 146. The measurement with RF was performed over a smaller range and increased resolution but remains consistent with the original signal. No significant broadening could be identified. This was not entirely unexpected. The theory section shows a change in the center wavenumber of $\pm 0.5\text{cm}^{-1}$ could be possible. The actual amount of broadening appears to be too small to be visible given the quality factor. A narrower PtC resonance could be achieved by undercutting the sample. Two attempts were made to perform the HF etch but neither produced usable devices. Unfortunately after those attempts the sample was in a deteriorated state and could not be used. No further experimentation was possible. Figure 5.8 shows an optical image of the damaged IDT. Some ZnO damage was incurred as well.
Figure 5.7: Using the free space setup at Queen’s, PtC transmission through R0C4S6 using TE polarized light is provided. A Lorentzian distribution width of $\Gamma = 50$ was found. The spectrum without modulation is plotted in black (with system response is dotted). No discernible broadening results when compared with the modulated spectrum in dashed blue. The apparent increase of noise in the blue trace can be attributed to a change in wavelength step size from 0.5nm to 0.1nm.
Figure 5.8: The IDTs were wirebonded before the undercutting procedure to facilitate SAW measurements. They had to be broken before proceeding with the HF etch. The wirebond stubs resulted in a rough surface - resist was unable to provide a suitable barrier to the acid. HF dissolved the underlying ZnO, removing large chunks of aluminum. The device was rendered inoperable.
Chapter 6

Conclusion

This thesis investigated the union of two areas in condensed matter physics: acoustic motion in solids and its ability to modulate photonic structure through changes to the lattice constant. A triangular lattice PtC was made by our collaborators at UBC. Defects (linear and point) were designed into the PtC to realize a narrow optical filter \((Q \approx 5000)\). This structure could be accessed by shining light at input and output gratings couplers. The entire optical circuit was built on a SOI substrate. Fibre-based optical experiments were conducted to excite the gratings and measure transmission through the circuit. They could not resolve individual devices.

Free-space experiments were designed and carried out, providing the required resolution. TE polarized light incident at \(10^\circ\) normal to the surface, forward propagated through the structure to measure the narrow resonance of the L3 PtC. Analysis showed the resulting spectrum to be Lorentzian in shape - in accordance with coupled mode theory. For one particular device (R0C4S6) a center of \(6255\,\text{cm}^{-1}\) and FWHM of \(\Gamma = 38\,\text{cm}^{-1}\) was found. This corresponded to a optical filter with a quality factor of 165, which is consistent with similar devices. TM polarized light was also used...
in transmission measurements. It back-coupled traveling only through the silicon slab allowing characterization of the gratings in isolation. When exciting from device R0C4S5 to R0C3S5 a Gaussian shape was found to fit the spectrum. A center of 6340cm$^{-1}$ and FWHM of 106cm$^{-1}$ was found.

Acoustic transducers were fabricated on top of the optical devices. A thin film of ZnO had to be grown to satisfy the requirement of piezoelectric behaviour, a necessity for the operation of the aluminum IDTs. The transducers were found to produce a Rayleigh SAW with a wavelength of 24µm at 179MHz. There was evidence of a Sewaza SAW mode at 206MHz. These measurements were further verified with time domain analysis of the transmitted signal.

Modulation experiments were conducted in which the cavity transmission was measured as Rayleigh SAWs traveled through the device. The acoustic wave changes the lattice parameter of the entire structure at once, compressing and stretching its nominal value at the SAW frequency. Efforts were made to quantify the magnitude of these effects. Although outward displacement could not be measured, using intensity measurements the Sagnac interferometer spatial resolution was experimentally confirmed to be adequate. Calculations using deformation values from literature show that the quality factor of the filter was too small for a significant change in the transmission to be measured. An HF etch procedure was devised to undercut the L3 PtC, realizing the optimal structure of a silicon slab suspended in air. Simulations show that the quality factor of the resonance would increase by as much as fifty times. Such a narrow resonance would allow for SAW modulation effects to be measured. Unfortunately, the HF acid was found to critically damage the device due to etch channels made possible by the transducer wirebonds.
6.1 Future Work

Due to the damaged device, the strongest recommendations for future studies is a change in the order of fabrication. The HF undercutting procedure should be conducted before transducer fabrication. This was not attempted here as there was a concern that the undercut slab would be damaged during the ZnO deposition. The order was also chosen in the hopes that undercutting would not be required (some modulation could be measured even with a poor quality factor). Results have shown this not to be possible.

A second choice would be the SAW operational frequency. 179MHz was found to be a difficult frequency to measure using the network analyzer. Interference measurements using the Sagnac interferometer were also found to be challenging to attain - whether this is due to the SAW frequency is undetermined. A marginally higher frequency (> 200MHz) should be chosen in future studies to avoid this issue.
Bibliography


Appendix A

PtC Simulations with MPB

MIT’s MPB simulation software was used to model most of the behaviour of the optical structures in this thesis. MPB is an eigenmode solver that finds allowed energies at specific wavevectors. Band structures can be created by applying it at intervals along a direction, yielding information about the behaviour of the medium.

MPB relies on symmetry existing in order to classify modes. In the case of 2D simulations, continuous translational symmetry exists in the z-direction. This allows modes to be classified as either TE or TM based on the electric or magnetic fields being polarized in the plane perpendicular to $z$.

The actual photonic circuit is not infinite in $z$, but actually confined to a 220nm thick silicon slab. In practice the infinite simulations provide a suitable description of the experiment. They also ran substantially faster than their 3D counterparts. Adding in the effect of the slab had the result of raising all energies - which wasn’t used in characterizing the samples. To its detriment, the dispersion diagrams are also made more complicated, with quasi-bound modes existing above the light line. This technically also closed the band gap in the triangular PtC, but a functional
band gap still exists below the light line (see Figure A.2). The slab simulations were required, however, to appropriately model the quality factor of the cavity, having an order of magnitude effect on the result. Johnson et al. provide a good guide in understanding these slab simulations [19]. The code used to run the slab simulations has been provided at the end of this section. The 2D variants are really a subset of these with only a few modifications.

A FDTD simulation was also used to find the cavity quality factor. MEEP is a companion tool to MPB used to solve these types of problems. The familiar definition for quality factor is given in Equation 5.3. It can also be defined in terms of the cavity resonance ($\omega_0$) and lifetime ($\tau$), Equation A.1 [18].

\[ Q = \frac{\omega_0 \tau}{2} \]  \hspace{1cm} (A.1)

$\omega_0$ can be found using MPB. $\tau$ is determined by exciting a TE source at cavity center and measuring the field amplitude some time later. The field amplitude decays exponentially, proportional to $e^{-t/\tau}$. The code used to run that time domain MEEP simulation is among the excerpts below.
Figure A.1: Simulation of the square lattice PtC of Figure 2.4 for both TE (red) and TM (blue) modes over the Γ - M direction. The lattice is confined to a 220nm thick slab suspended in resist. Only the first four bands are plotted. The light line for resist is included.
Figure A.2: Simulation of the triangular lattice PtC of Figure 2.6. The lattice is confined to a 220nm thick slab. The first four bands of TE (red) and TM (blue) modes over in-plane wavevectors along the Γ - K are shown. In the region below the light line of air, a TE photonic bandgap is shaded in green between the 1st and 2nd bands with a (total) wavenumber range between 5998-7398 cm$^{-1}$. 
Figure A.3: Simulation of the L3 PtC of Figure 2.8 as a slab for TE modes over a
wavevector along the L3 defect axis. Energies previously in the bandgap
are now possible, three are displayed. The z-component of the magnetic
field is plotted for these modes with increasing energy from left to right
(6440, 6822 and 6841cm$^{-1}$). Field energy confinements within the area
of the cavity are 18.8%, 19.4% and 15.7%, which is much greater than
modes not in the band (< 2%).
Figure A.4: Simulation of the line defect PtC of Figure 2.10 as a slab for TE modes for a fixed wavevector along the line defect axis. The z-component of the magnetic field intensity is shown for the mid-band mode at energy 6611 cm$^{-1}$. 
MPB Simulation Code:

Grating Coupler: Square Lattice PtC Slab Dispersion

; grating coupler slab
; lattice
(define-param h (/ 220 795))
(define-param sh (* 8 (/ 220 795)))
(set! geometry-lattice (make
    lattice (size 1 1 sh)))

(set! default-material (make dielectric (index 2)))
(set! geometry (list
    (make block (center 0 0 0) (material (make dielectric (index 3.4784)) (size 1 1 h)))
    (make cylinder (center 0 0 0) (radius (/ (/ 535 795) 2)) (height h)
      (material (make dielectric (index 2))))))

; Gamma -> X
(set! k-points (list
    (vector3 0 0 0)
    (vector3 0.5 0 0))
)
(set! k-points (interpolate 32 k-points))

; run
(set! resolution 64)
(set! num-bands 4)
(run-zodd)
(run-seven)

Triangular Lattice PtC Slab Dispersion

; 2d triangular lattice slab ptc
; lattice
(define-param h (/ 220 400))
(define-param sh (* 8 (/ 220 400)))
(set! geometry-lattice (make
    lattice (size 1 1 sh)
    (basis1 (/ (sqrt 3) 2) 0.5)
    (basis2 (/ (sqrt 3) 2) -0.5)))

APPENDIX A. PTC SIMULATIONS WITH MPB

; geometry
(define-param rad (/ 110 400))
(set! geometry (list
    (make block (center 0 0 0) (material (make dielectric (index 3.4784))) (size 1 1 h))
    (make cylinder (center 0 0 0) (radius rad) (height infinity) (material air))
))
;
; run
(set! k-points (list
    (vector3 0 0 0) ; Gamma
    (vector3 1 0.5 0) ; K (x-dir equiv)
))
(set! k-points (interpolate 32 k-points))
;
(set! resolution 64)
(set! num-bands 4)
;
(run-zodd)
(run-zeven)
;
L3 PtC Field

; L3 PtC slab
; lattice
(define-param nx 8)
(define-param h (/ 220 400))
(define-param sh (* 8 (/ 220 400)))
(set! geometry-lattice (make
    lattice (size nx nx sh)
    (basis1 (/ (sqrt 3) 2) 0.5)
    (basis2 (/ (sqrt 3) 2) -0.5)
))
;
; geometry
(define-param rad (/ 110 400))
(define-param si 3.4784)
(set! geometry (list
    (make block (center 0 0 0) (material (make dielectric (index si))) (size 1 1 h))
    (make cylinder (center 0 0 0) (radius rad) (height infinity) (material air))
))
;
duplicate geometry over extended lattice
(set! geometry (geometric-objects-lattice-duplicates geometry 1 1 sh))
;
create L3 defects
(set! geometry (append geometry (list
    (make cylinder (center 0 0 0) (radius rad) (height h) (material (make dielectric (index si)))))
))
APPENDIX A. PTC SIMULATIONS WITH MPB

(make cylinder (center 1 0 0) (radius rad) (height h) (material (make dielectric (index si))))
(make cylinder (center -1 0 0) (radius rad) (height h) (material (make dielectric (index si))))

(set! k-points (list (vector3 (/ 0.5 nx) (/ nx) 0))
(set! num-bands 190)
(set! resolution 32 32 8)
(run-seven)

; extract field densities
(define (getfield i)
  (print "i:" i "\n")
  (output-hfield-z i)
  (get-dfield i)
  (compute-field-energy)
  (print "energy in cavity: ")
  (compute-energy-in-objects (make block (center 0 0 0) (size 3 1 h) (e2 -1 2 0)))
  "\n"
)
(getfield 80)
(getfield 87)
(getfield 88)

L3 PtC Q-Factor (MEEP)

; l3 ptc q-factor
; lattice
(define-param nx 10)
(define-param h (/ 220 400))
(define-param sh (* 8 (/ 220 400)))
(set! geometry-lattice (make
  lattice (size nx nx nx sh))
)

; geometry
(define-param rad (/ 110 400))
(define-param si 3.4784)
; slab
(set! geometry (list
  (make block (center 0 0 0) (material (make dielectric (index si))) (size nx nx h)))
)
; lattice
(set! geometry (append geometry
 (geometric-objects-duplicates (vector 3 0 (sqrt 3) 0) -2 2
 (geometric-object-duplicates (vector 3 1 0) -4 4
 (make cylinder (center 0 0 0) (radius rad) (height infinity) (material air))
 )
 )
 )
 (set! geometry (append geometry
 (geometric-objects-duplicates (vector 3 0 (sqrt 3) 0) 0 3
 (geometric-object-duplicates (vector 3 1 0) 0 9
 (make cylinder (center -4.5 (* -3 (/ (sqrt 3) 2)) 0) (radius rad)
 (height infinity) (material air))
 )
 )
 ))
 ; line defects (missing)
(set! geometry (append geometry
 (geometric-object-duplicates (vector 3 1 0) -1 1
 (make cylinder (center 0 0 0) (radius rad) (height h)
 (material (make dielectric (index si)))))
 )
 ))
 ; run
(define-param dpml 1) ; PML thickness
(set! pml-layers (list (make pml (thickness dpml))))
(set-param! resolution 64)
;
(define-param fcen 0.273) ; pulse center frequency
(define-param df 0.2) ; pulse width (in frequency)
(define-param nfreq 5000) ; number of frequencies at which to compute flux
;
(set! sources (list
 (make source
 (src (make gaussian-src (frequency fcen) (width df)))
 (component Hz) (center 0 0))
 ))
;
(run-sources+ 400
 (at-beginning output-epsilon)
 (after-sources (harminv Hz (vector 3 0) fcen df))
)
(run-until (/ 1 fcen) (at-every (/ 1 fcen 20) output-hfield-z))

Line Defect PtC Field

; line defect ptc slab
APPENDIX A. PTC SIMULATIONS WITH MPB

; lattice
(define-param nx 14)
(define-param h (/ 220 400))
(define-param sh (* 8 (/ 220 400)))
(set! geometry-lattice (make
  lattice (size 1 nx sh)
  (basis1 (/ (sqrt 3) 2) 0.5)
  (basis2 (/ (sqrt 3) 2) -0.5)
))

; geometry
(define-param rad (/ 110 400))
(define-param si 3.4784)
(set! geometry (list
  (make block (center 0 0 0) (material (make dielectric (index si)))) (size 1 1 h))
  (make cylinder (center 0 0 0) (radius rad) (height infinity) (material air))
))
(set! geometry (geometric-objects-lattice-duplicates geometry 1 1 sh))
; create a line defect along the basis1 direction
(set! geometry (append geometry (list
  (make cylinder (center 0 0 0) (radius rad) (height h) (material (make dielectric (index si))))
))

; run
(set! k-points (list
  (vector3 0.5 (/ nx) 0)
))

; run-seven

; extract field properties
(define (getfield i)
  (print "i:" i "\n"
  (output-hfield-z i)
)
(set! num-bands 35)
(set! resolution 256 256 128)


Appendix B

Time-Of-Flight Measurements

The network analyzer was capable of making time domain as well as frequency domain measurements, which were used in this thesis. This became particularly useful when making measurements on the device sample. The spectrum under consideration was substantially more noisy than previous measurements. In time domain analysis a frequency is excited by the instrument and the time of signal arrival is recorded. When measuring transmission the arrival time should equate to distance traveled divided by speed. The distance traveled is known to 1.2mm for the IDT pair. The 220ns travel time found for the Rayleigh mode at 179MHz (see Figure 5.5) does seem fast. It may be possible to attribute this to an increase in velocity from the ZnO removal over the PtCs. The time-of-flight measurements for the Sewaza mode and its harmonic (206MHz and 396MHz) are provided in Figures B.1 and B.2 respectively. The same measurement for a false SAW signal at 129MHz is provided in Figure B.3.
Figure B.1: Time-of-flight for transmission of the 206MHz Sewaza mode.

Figure B.2: Time-of-flight for transmission of the 396MHz Sewaza 1st harmonic mode.
Figure B.3: Time-of-flight for transmission at 129MHz. No signal could be discerned.
Appendix C

Computing Rayleigh and Lamb SAW Velocity

Computational methods were used to compute both the Rayleigh and Lamb SAW velocities. The problems essentially distilled down to finding the value of a variable that gave a zero result for a function. This is done by computing the function value \( y \) at various values of the variable \( x \) and looking for a zero crossing in the \( x-y \) plot. Octave was used to perform this task.

For the case of the Rayleigh SAW velocity of ZnO, the equation is the determinant of the matrix given in Equation 2.29. The determinant is calculated for a series of velocities \( v \). The other variables in the equation can be expressed in terms of \( v \) using Equations 2.27 and 2.30. The resulting plot is provided in Figure C.1. The octave code to perform this computation follows.
Figure C.1: A plot of the determinant of Equation 2.29 as a function of potential Rayleigh SAW velocity. The real value is the solid line and the imaginary dashed. The velocity that yields a value of zero for both components is the true velocity. While a solution may seem possible at 1.10km/s, the only true crossing is at 2.79km/s (marked with an x).
% constants
global p = 5.606; % g/cm^3
global c11 = 217; % GPa
global c33 = 225; % GPa
global c55 = 50 ; % GPa
global c13 = 121; % GPa

% calculating

% get gamma
function y = gam(v)
    global p c11 c33 c55 c13;
    % 0 = ax^2 + bx + c
    y = roots([c55 * c33 - (c33 + c55) * p * v^2 + c55^2 + c11 * c33 - (c13 + c55)^2 c11 * c55 - (c11 + c55) * p * v^2 + p^2 v^4]);
endfunction

% get chi (A1/A3)
function y = chi(v, g)
    global p c11 c55 c13;
    y = -g * (c13 + c55) / (c55 * g^2 + c11 - p * v^2);
endfunction

% get matrix determinant
function y = dt(g1, g2, x1, x2)
    global c33 c55 c13;
    y = det([c13 * x1 + c33 * g1 c13 * x2 + c33 * g2 ; c55 * (g1 * x1 + 1) c55 * (g2 * x2 + 1)]);
endfunction

% velocities in km/s
dv = 0.001;
v = 1.001:dv:2.97;

% find gamma (lambda/k)
% generally two g for each v
% 0 means root was not a physical solution
g = []; c = []; d = [];
for vi = v
    gi = [];
    ci = [];
    rs = sqrt(gam(vi));
    for r = transpose(rs)
        if (imag(r) > 0)
            gi = [gi; r];
            % get amplitude constants, chi=A1/A3
            ci = [ci; chi(vi, r)];
        else
            gi = [gi; 0];
            ci = [ci; 0];
        end
    end
end

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APPENDIX C. COMPUTING RAYLEIGH AND LAMB SAW VELOCITY

end if
end for

\[ g = [g, g_i]; \]
\[ c = [c, c_i]; \]

\% eval matrix determinant
\[ d_i = \det(g_i(1), g_i(2), c_i(1), c_i(2)); \]

\% look for zero crossings
\[ s = \text{size}(d)(2); \]
\[ \text{if}(s > 0) \]
\[ \text{if}(\text{imag}(d(s)) \cdot \text{imag}(d_i) < 0) \]
\[ \text{printf}("potential zero-crossing at index \%i: \text{va}=%f, \text{g1}=%f+i%f, \text{g2}=%f+i%f \ldots ", \]
\[ s, \text{vi} = \text{dv}, \text{real}(g(1, s)), \text{imag}(g(1, s)), \text{real}(g(2, s)), \text{imag}(g(2, s))) \]
\[ \text{printf}("\text{vb}=%f, \text{g1}=%f+i%f, \text{g2}=%f+i%f \ldots \), \text{vi}, \text{real}(g_i(1)), \text{imag}(g_i(1)), \text{real}(g_i(2)), \text{imag}(g_i(2))) \]
\[ \text{printf}("\text{chi1}=%f+i%f, \text{chi2}=%f+i%f\n", \text{real}(c_i(1)), \text{imag}(c_i(1)), \text{real}(c_i(2)), \text{imag}(c_i(2))) \]
\[ \text{end if} \]
\[ \text{end if} \]
\[ d = [d, d_i]; \]
end for

\% plot

\text{plot}(v, \text{real}(d))
\text{hold on}
\text{plot}(v, \text{imag}(d), ': ')
\text{ylim}([-100 100])
\text{h = figure}(1);
\text{FS} = \text{findall}(h,'-property','FontSize');
\text{set}(\text{FS}, 'FontSize', 20);
\text{FS} = \text{findall}(h,'-property','LineWidth');
\text{set}(\text{FS}, "linewidth", 4);
\text{set}(\text{gca}(), "linewidth", 2);
\text{title}(\text{'Rayleigh SAW Velocity Zero-Crossing'}, 'FontSize', 30, 'FontName', 'Bookman');
\text{ylabel}(\text{\text{'determinant'}}); 'FontSize', 20, 'FontName', 'Bookman');
\text{xlabel}(\text{'velocity (km/s)'}, 'FontSize', 20, 'FontName', 'Bookman');
\%
Computation of the Lamb SAW dispersion curve proceeded in a similar manner. Because many velocities had to be computed for varying values of $\omega d/v_s$, the process had to be repeated many times. Since there were both asymmetric and symmetric modes, two equations were solved simultaneously (Equation 2.36). A simplistic octave routine was written to determine appropriate zero crossings. This proved to be a difficult task as asymptotic behaviour could result in false zero crossings. Some adjustment of the program output was required to create the dispersion curve. It should be noted that the zeroth order modes required no adjustments (and were the only values used to determined the Lamb SAW velocity applicable to this thesis). One of the many zero crossing plots is given in Figure C.2. The full octave routine to generate the dispersion data follows.
Figure C.2: A plot of symmetric Equation 2.36 as a function of potential Lamb SAW velocity. The real value is the solid line and the imaginary dashed. The velocities that yield a value of zero for both components correspond to the true velocities of the various modes (marked with x). Many false positives appear due to asymptotic behaviour.
APPENDIX C. COMPUTING RAYLEIGH AND LAMB SAW VELOCITY

\( \text{\% (100) silicon constants} \)
\[
p = 2.329; \text{g/cm}^3
\]
\[
c_{11} = 165.6; \text{GPa}
\]
\[
c_{44} = 79.51; \text{GPa}
\]
\[
sigma = 0.34; \text{titanium & aluminum (example)}
\]
\[
\text{vs} = \left( \frac{c_{44}}{p} \right)^{1/2};
\]
\[
\text{vl} = \left( \frac{c_{11}}{p} \right)^{1/2};
\]
\[
\text{global vsl} = \left( \frac{\text{vs}}{\text{vl}} \right)^2;
\]

\% functions

\% seek zero

\text{function } y = \text{lambsym}(v, \text{wd})
\[
\text{global vsl;}
\]
\[
y = \frac{\tan\left( \sqrt{vsl - 1/v^2} \right) \cdot \text{wd}/2}{\tan\left( \sqrt{1 - 1/v^2} \right) \cdot \text{wd}/2} + \left( 1 - v^2 \right)^{1/2} \cdot \left( \frac{v^2}{vsl - 1/v^2} + \frac{\sqrt{vsl - 1/v^2}}{\sqrt{1 - 1/v^2}} \right)^2;
\]
\text{endfunction}

\text{function } y = \text{lambaasym}(v, \text{wd})
\[
\text{global vsl;}
\]
\[
y = \frac{\tan\left( \sqrt{vsl - 1/v^2} \right) \cdot \text{wd}/2}{\tan\left( \sqrt{1 - 1/v^2} \right) \cdot \text{wd}/2} + \left( v^2 \right) \cdot \frac{\sqrt{vsl - 1/v^2}}{\left( 1 - v^2/2 \right)^2} \cdot \frac{\sqrt{1 - 1/v^2}}{\left( 1 - v^2/2 \right)^2};
\]
\text{endfunction}

\% data correction

\text{function } s = \text{scorematch}(a, b)
\[
i = 0;
\]
\[
\text{score} = 0;
\]
\text{for } bi = b
\[
i = i + 1;
\]
\text{if } (a(i) > 0)
\[
\text{score} += \text{abs}(a(i) - bi);
\]
\text{endif}
\text{endfor}
\[
s = \text{score};
\]
\text{endfunction}

\text{function } v = \text{matchbands}(a, bs)
\[
sa = \text{size}(a)(2);
\]
\[
sb = \text{size}(bs)(2);
\]
\text{if } (sa < sb)
\[
\text{\% fix b so it has no zeros for comparisons}
\]
\[
i = \text{size}(bs)(1); \text{last row}
\]
\[
j = sb; \text{last col}
\]
\[
b = [];
\]
\[
nzflag = 0;
\]
\text{for } n = \text{sort}(1:j, 'descend')
\text{endfor}
\]
\text{endfunction}
if (bs(i,nj) > 0)
    nb = bs(i,nj);
else
    nb = 0;
    nimin = 1;
    if (i-5 > 1) % only go back so far to look for a non-zero (may have gone assymptotic)
        nimin = i-5;
    endif
    for ni = nimin:1:i-1
        if(bs(ni,nj) > 0)
            nb = bs(ni,nj);
        endif
    endfor
endif
b(nj) = nb;
endfor

% fix a so its only missing one 0, assuming only one mid-root is missed in a slice
while (size(a)(2) + 1 < sb)
    a = [a, 0];
 endwhile

% find the best 0 position
sa = size(a)(2);
s = 1000000;
for i = 0:1:sa
    ns = scorematch([a(1:i),0,a(i+1:sa)],b);
    if (ns < s)
        si = i;
        s = ns;
    endif
endfor

% si is the best position
tv = [a(1:si),0,a(si+1:sa)];
else
tv = a;
endif
v = tv;
endfunction

%%%%
%% calculating
%%

% angular frequency x slab thickness scaled by shear velocity
dwd = 0.1;
wd = sort(dwd:dwd:20,'descend');

% velocity scaled by shear velocity
vlim = 1/sqrt(vsl);
dv = 0.002;
vmax = 4.5;
APPENDIX C. COMPUTING RAYLEIGH AND LAMB SAW VELOCITY

\[ v = dv : dv : vmax; \]

\[
% \textit{find v (wd)}
\]
\[ sv = []; \]
\[ av = []; \]
\[ sr = []; \]
\[ ar = []; \]
\[ hard\_limit = -10; \]
\[ \text{for} \ wdi = wd \]
\[ sb = 0; \]
\[ ab = 0; \]
\[ svi = []; \]
\[ avi = []; \]
\[ \text{printf(}\%f:\n", wdi)\]
\[ \text{fflush(stdout);}\]
\[ \text{for} \ vi = v \]
\[ % \textit{compute and save function value} \]
\[ sr = [sr, lambasym(vi, wdi)]; \]
\[ ar = [ar, lambasym(vi, wdi)]; \]
\[ % \textit{if vi < 1 or > 1/sqrt(vsl) check for soft zero crossing on real component} \]
\[ \text{if} \ (vi < 1 | | vi > vlim) \]
\[ sp = \text{real}(sb) * \text{real}(sr(\text{size}(sr)(2))); \]
\[ ap = \text{real}(ab) * \text{real}(ar(\text{size}(ar)(2))); \]
\[ \text{if} \ (sp < 0 && ap > hard\_limit) \]
\[ \text{printf(\" sym real zero crossing at %f: %f \%f to %f \%f\n\",} \]
\[ \text{vi, real(sb), imag(sb), real(sr(\text{size}(sr)(2))), imag(sr(\text{size}(sr)(2)))} \]
\[ svi = [svi, vi]; \]
\[ \text{endif} \]
\[ \text{if} \ (ap < 0 && sp > hard\_limit) \]
\[ \text{printf(\" asym real zero crossing at %f: %f \%f to %f \%f\n\",} \]
\[ \text{vi, real(ab), imag(ab), real(ar(\text{size}(ar)(2))), imag(ar(\text{size}(ar)(2)))} \]
\[ avi = [avi, vi]; \]
\[ \text{endif} \]
\[ % \textit{otherwise check for hard/soft zero crossing on imaginary component} \]
\[ \text{else} \]
\[ sp = \text{imag}(sb) * \text{imag}(sr(\text{size}(sr)(2))); \]
\[ ap = \text{imag}(ab) * \text{imag}(ar(\text{size}(ar)(2))); \]
\[ \text{if} \ (sp < \text{hard\_limit}) \]
\[ \text{printf(\" sym hard imaginary zero crossing at %f: %f \%f to %f \%f\n\",} \]
\[ \text{vi, real(sb), imag(sb), real(sr(\text{size}(sr)(2))), imag(sr(\text{size}(sr)(2)))} \]
\[ \text{else if} \ (sp < 0) \]
\[ \text{printf(\" sym soft imaginary zero crossing at %f: %f \%f to %f \%f\n\",} \]
\[ \text{vi, real(sb), imag(sb), real(sr(\text{size}(sr)(2))), imag(sr(\text{size}(sr)(2)))} \]
\[ svi = [svi, vi]; \]
\[ \text{endif} \]
\[ \text{if} \ (ap < \text{hard\_limit}) \]
\[ \text{printf(\" asym hard imaginary zero crossing at %f: %f \%f to %f \%f\n\",} \]
\[ \text{vi, real(ab), imag(ab), real(ar(\text{size}(ar)(2))), imag(ar(\text{size}(ar)(2)))} \]
\[ \text{else if} \ (ap < 0) \]
\[ \text{printf(\" asym soft imaginary zero crossing at %f: %f \%f to %f \%f\n\",} \]
\[ v = dv : dv : vmax; \]

% find v (wd)
sv = [];
av = [];
sr = [];
ar = [];
hard_limit = -10;
for wdi = wd
    sb = 0;
    ab = 0;
    svi = [];
    avi = [];
    fprintf("%f:\n", wdi)
    fflush(stdout);
    for vi = v
        % compute and save function value
        sr = [sr, lambasym(vi, wdi)];
        ar = [ar, lambasym(vi, wdi)];
        % if vi < 1 or > 1/sqrt(vsl) check for soft zero crossing on real component
        if (vi < 1 || vi > vlim)
            sp = real(sb)*real(sr(size(sr)(2)));
            ap = real(ab)*real(ar(size(ar)(2)));
            if (sp < 0 && ap > hard_limit)
                fprintf(" sym real zero crossing at %f: %f \%f to %f \%f\n",
                    vi, real(sb), imag(sb), real(sr(size(sr)(2))), imag(sr(size(sr)(2)))
                svi = [svi, vi];
            endif
            if (ap < 0 && sp > hard_limit)
                fprintf(" asym real zero crossing at %f: %f \%f to %f \%f\n",
                    vi, real(ab), imag(ab), real(ar(size(ar)(2))), imag(ar(size(ar)(2)))
                avi = [avi, vi];
            endif
        % otherwise check for hard/soft zero crossing on imaginary component
        elseif
            sp = imag(sb)*imag(sr(size(sr)(2)));
            ap = imag(ab)*imag(ar(size(ar)(2)));
            if (sp < hard_limit)
                fprintf(" sym hard imaginary zero crossing at %f: %f \%f to %f \%f\n",
                    vi, real(sb), imag(sb), real(sr(size(sr)(2))), imag(sr(size(sr)(2)))
            else if (sp < 0)
                fprintf(" sym soft imaginary zero crossing at %f: %f \%f to %f \%f\n",
                    vi, real(sb), imag(sb), real(sr(size(sr)(2))), imag(sr(size(sr)(2)))
                svi = [svi, vi];
            endif
            if (ap < hard_limit)
                fprintf(" asym hard imaginary zero crossing at %f: %f \%f to %f \%f\n",
                    vi, real(ab), imag(ab), real(ar(size(ar)(2))), imag(ar(size(ar)(2)))
            else if (ap < 0)
                fprintf(" asym soft imaginary zero crossing at %f: %f \%f to %f \%f\n",
                    vi, real(ab), imag(ab), real(ar(size(ar)(2))), imag(ar(size(ar)(2)))
                endif
            endif
        endif
    endif
endfor
APPENDIX C. COMPUTING RAYLEIGH AND LAMB SAW VELOCITY

\[ \begin{align*}
    &v_1, \text{ real}(ab), \text{ imag}(ab), \text{ real}(\text{size}(ar)(2)), \text{ imag}(\text{size}(ar)(2)) \\
    &\text{ avi} = [\text{ avi}, v_1]; \\
    &\text{ endif} \\
    &\text{ endif} \\
    &\text{ % buffer last value} \\
    &\text{ if } (v_1 == 1) \\
    &\hspace{1em} sb = 0; \\
    &\hspace{1em} ab = 0; \\
    &\text{ else} \\
    &\hspace{1em} sb = \text{ size}(sr)(2); \\
    &\hspace{1em} ab = \text{ size}(ar)(2); \\
    &\text{ endif} \\
    &\text{ endfor} \\
    &\text{ % (smartly) zero pad for solutions that have gone asymptotic} \\
    &\text{ if } (\text{size}(sv)(1) > 0) \\
    &\hspace{1em} sv = [sv; \text{matchbands}(svi, sv)]; \\
    &\hspace{1em} av = [av; \text{matchbands}(avi, av)]; \\
    &\text{ else} \\
    &\hspace{1em} sv = [sv; svi]; \\
    &\hspace{1em} av = [av; avi]; \\
    &\text{ endif} \\
    &\text{ endfor} \\
    &\text{ % save generated data} \\
    &x = \text{transpose}(wd); \\
    &\text{ save vsym.dat sv;} \\
    &\text{ save vasymp.dat av;} \\
    &\text{ save x.dat x;} \\
    &\% \\
    &\% plot results \\
    &\% \\
    &\text{ function } \text{ plotv}(x, y, c) \\
    &\% make plotables with no 0s \\
    &\text{ for } yi = y \\
    &\hspace{1em} i = 0; \\
    &\hspace{1em} xp = []; \\
    &\hspace{1em} yp = []; \\
    &\text{ for } yi = \text{transpose}(yi) \\
    &\hspace{1em} i = i + 1; \\
    &\text{ if } (yi > 0) \\
    &\hspace{1em} xp = [xp, x(i)]; \\
    &\hspace{1em} yp = [yp, yi]; \\
    &\text{ endif} \\
    &\text{ endfor} \\
    &\% plot \\
    &\text{ plot}(xp, yp, c) \\
    &\text{ endfor} \\
    &\text{ endfunction} \\
\end{align*} \]
APPENDIX C. COMPUTING RAYLEIGH AND LAMB SAW VELOCITY

hold on
plotv(x, sv, 'b');
plotv(x, av, 'r');
hold off
%

Appendix D

Computing SOI Slab Dispersion

Computing of the SOI slab dispersion diagram proceeded in a similar fashion to other computation methods used for SAW velocity calculations. Equation 2.11 had to be solved. A simple rearrangement reduces the problem to finding the frequency ($\omega$) that yields a zero value for a function. One equation yields the TM values while the other TE. A simple octave routine was written to perform this task repeatedly for values of in-plane wavevector ($\rho$). To simplify the task the routine works with increasing values of $\rho$, using the previously found $\omega$ as a starting point (since values are always increasing). The code to produce the dispersion plot follows.
% constants
n0 = 1;
n = 3.4784;
ns = 1.444;
c = 299792458;

% calculating

% TE
function y = fte(w, p, m, n, ns, n0, c)
    y = (w/c * sqrt((n^2 - (p*c/w)^2)) - (m*pi) - \ 
        atan(sqrt(((p*c/w)^2 - ns^2)/(n^2 - (p*c/w)^2)) - \ 
        atan(sqrt(((p*c/w)^2 - n0^2)/(n^2 - (p*c/w)^2)));
endfunction

% TM
function y = ftm(w, p, m, n, ns, n0, c)
    y = (w/c * sqrt((n^2 - (p*c/w)^2)) - (m*pi) - \ 
        atan(n^2/ns^2 * sqrt(((p*c/w)^2 - ns^2)/(n^2 - (p*c/w)^2)) - \ 
        atan(n^2/n0^2 * sqrt(((p*c/w)^2 - n0^2)/(n^2 - (p*c/w)^2)));
endfunction

xmax = 2;
mmax = 4;
x = 0.01:0.01:xmax;
m = 0:1:mmax;
dw = 0.00001*c;

% TE
yte = [];
for mi = m
    printf("simulating TE band m=%i\n", mi)
    fflush(stdout);
    yi = [];
    i = 1;
    w = 0;
    maxflag = 0;
    for xi = x
        maxflag = 0;
        wmax = c*2*pi*xi;
        if (w < wmax/n)
            w = (wmax/n) + dw; %minimum bound
        endif
        a = -1;
        printf("searching with p=%f, w=%f ... " , xi, w)
        while (imag(a) != 0 || real(a) < 0)
            a = fte(w, xi*2*pi, mi, n, ns, n0, c);
            w = w + dw;
        if (w > wmax/ns) %maximum bound
            printf "hit max ... ")
maxflag = 1;
break
endif
endwhile
w = w - dw;
if (maxflag == 0)
    printf("value found: %f\n", w)
yi(i) = w;
else
    printf("\n")
yi(i) = -1;
endif
fflush(stdout);
i = i + 1;
endfor
yte = [ yte; yi ];
endfor

STM
ytm = [];
for mi = m
    printf("simulating TM band m=%i\n", mi)
    fflush(stdout);
yi = [];
i = 1;
w = 0;
maxflag = 0;
for xi = x
    maxflag = 0;
wmax = c*2*pi*xi;
    if (w < wmax/n)
        w = (wmax/n) + dw; %minimum bound
    endif
    a = -1;
    printf("searching with p=%f, w =%f . . . ", xi, w)
    while (imag(a) != 0 || real(a) < 0)
        a = ftm(w, xi*2*pi, mi, n, ns, n0, c);
        w = w + dw;
        if (w > wmax/ns) %maximum bound
            printf("hit max . . . ")
            maxflag = 1;
            break
        endif
    endwhile
endfor
w = w - dw;
if (maxflag == 0)
    printf("value found: %f\n", w)
yi(i) = w;
else
    printf("\n")
\[ yi(i) = -1; \]
\[ \text{endif} \]
\[ \text{fflush(stdout);} \]
\[ i = i + 1; \]
\[ \text{endfor} \]
\[ ytm = [ytm; yi]; \]
\[ \text{endfor} \]
\[
\% \text{ save generated data} \\
\text{save ytm.dat ytm} \\
\text{save yte.dat yte} \\
\text{save x.dat x} \\
\]
\[
\% \text{ plotting} \\
\% \text{ TE} \\
\text{for yi = transpose(yte)} \\
\text{i = 1;} \\
\text{for yii = transpose(yi)} \\
\text{if (yii < 0)} \\
\text{i = i + 1;} \\
\text{endif} \\
\text{endfor} \\
\text{plot(x(i:size(x)(2)),yi(i:size(x)(2))/(2*pi*c),'r')} \\
\text{hold on;} \\
\text{endfor} \\
\% \text{ TM} \\
\text{for yi = transpose(ytm)} \\
\text{i = 1;} \\
\text{for yii = transpose(yi)} \\
\text{if (yii < 0)} \\
\text{i = i + 1;} \\
\text{endif} \\
\text{endfor} \\
\text{plot(x(i:size(x)(2)),yi(i:size(x)(2))/(2*pi*c),'b')} \\
\text{hold on;} \\
\text{endfor} \\
\% \text{ ymax}=1.5 \\
\text{ylim([0 ymax])}; \\
\text{plot([0,xmax],[0,xmax],'--k',4);} \\
\text{plot([0,xmax],[0,xmax/ns],'--k',4);} \\
\text{plot([0,xmax],[0,xmax/n],'--k',4);} \\
\% \\
\text{h = figure(1);} \\
\text{FS = findall(h,'property','FontSize');} \\
\text{set(FS,'FontSize',20);} \\
\text{FS = findall(h,'property','LineWidth');} \\
\text{set(FS,'linewidth',4);}
set(gca(), "linewidth", 2);
title('Silicon Slab on Silicon Dioxide Substrate Dispersion – TE and TM Modes',
     'FontSize',30, 'FontName','Bookman');
set(gca(), 'position',[0.03,0.03,0.96,0.9]);
%