OPTIMIZATION OF BATTERIES FOR PLUG-IN HYBRID ELECTRIC VEHICLES

by

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Abstract

This thesis presents a method to quickly determine the optimal battery for an electric vehicle given a set of vehicle characteristics and desired performance metrics. The model is based on four independent design variables: cell count, cell capacity, state-of-charge window, and battery chemistry. Performance is measured in seven categories: cost, all-electric range, maximum speed, acceleration, battery lifetime, lifetime greenhouse gas emissions, and charging time. The performance of each battery is weighted according to a user-defined objective function to determine its overall fitness.

The model is informed by a series of battery tests performed on scaled-down battery samples. Seven battery chemistries were tested for capacity at different discharge rates, maximum output power at different charge levels, and performance in a real-world automotive duty cycle. The results of these tests enable a prediction of the performance of the battery in an automobile. Testing was performed at both room temperature and low temperature to investigate the effects of battery temperature on operation.

The testing highlighted differences in behavior between lithium, nickel, and lead based batteries. Battery performance decreased with temperature across all samples with the largest effect on nickel-based chemistries. Output power also decreased with lead acid batteries being the least affected by temperature. Lithium-ion batteries were found to be highly efficient (>95%) under a vehicular duty cycle; nickel and lead batteries have greater losses.

Low temperatures hindered battery performance and resulted in accelerated failure in several samples. Lead acid, lead tin, and lithium nickel alloy batteries were unable to complete the low temperature testing regime without losing significant capacity and power capability. This is a concern for their applicability in electric vehicles intended for cold climates which have to maintain battery temperature during long periods of inactivity.

Three sample optimizations were performed: a compact car, a truck, and a sports car. The compact car benefits from increased battery capacity despite the associated higher cost. The truck returned the smallest possible battery of each chemistry, indicating that electrification is not advisable. The sports car optimization resulted in the largest possible battery, indicating large performance from increased electrification. These results mirror the current state of the electric vehicle market.
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Chapter 1

Introduction

Electric cars can increase their well-to-wheels efficiency compared to traditional automobiles by replacing some or all of the energy sourced from liquid fuels with grid-sourced electricity. Electric models will become increasingly widespread in the near future as a result of California’s Zero-Emissions Mandate and the US Corporate Average Fuel Economy standard. In 2012, 1892 plug-in electric cars were sold in Canada [1]. This number is projected to increase to 107,146 by 2020 [2].

Plug-in electric vehicles (PEVs) can be divided into two classes, full electric vehicles (EVs), which rely entirely on electric power; and plug-in hybrid electric vehicles (PHEVs), which combine an electric and combustion power source. PHEVs can be used with two drivetrains, as on the Toyota Prius; or with a combustion generator supporting electric propulsion, as on the Chevrolet Volt.

The design of electric vehicles requires the selection of an appropriate battery to meet the desired performance of the vehicle. The high cost of the battery means that the overall viability of the vehicle is closely tied to the effectiveness of its battery. Matching the battery to the specific needs of a vehicle ensures that the vehicle can meet its intended goals without wasting resources and increasing cost.

The aim of this thesis is to present a method for quickly selecting a battery to meet the needs of a PHEV in the design stage. The principle output of this project is an optimization program to determine the ideal battery for a PHEV. This program is informed by testing performed on scaled battery samples. Testing was deemed necessary due to limited and unreliable data from battery manufacturers.

The purpose of the experiments performed is to characterize the performance of batteries in ways relevant to automotive applications. This includes standard tests outlined by the US Advanced Battery Consortium (USABC) for capacity and power output as well as the response of batteries to a representative duty cycle for a hybrid car. Batteries were tested at room and low temperatures to determine what, if any, performance decrease will occur in low temperature operation. High temperature operation was not evaluated in this study; it should still be noted that thermal management of batteries is an important aspect of PHEV battery design. Information gathered from this testing is used to inform the optimization tool; it is not intended to be an exhaustive evaluation of battery performance.
The optimization program considers a range of candidate batteries, evaluates their performance based on theoretical and empirical analysis, and calculates their overall fitness for the application. This process can be performed quickly and frequently to determine the effect of changes in other aspects of the vehicle on the electric powertrain. It also enables new battery technologies to be evaluated in a PHEV-specific context, potentially reducing the latency between innovation and adoption.
2.1 Battery Chemistries

2.1.1 Lithium Ion Batteries
Lithium ion batteries are the most common battery type found in advanced electronics. Their popularity is a result of their high energy density and low maintenance requirements. Lithium ion batteries are used in the current models of mainstream electric vehicles.

The electrolyte in lithium ion batteries can be either a liquid solvent (commonly referred to as lithium-ion batteries) or a polymer (commonly called lithium-polymer batteries). Polymer electrolytes allow the cell to be prismatic rather than cylindrical, improving its packing factor. Lithium polymer batteries also require less exterior packaging resulting in a lower overall weight.

Lithium ion batteries encompass many different anode and cathode materials with the unifying characteristic of transferring lithium ions between the electrodes. Most commercial lithium-ion batteries use a graphite cathode. Different cathode and anode materials for lithium ion batteries are described in the subsections below.

2.1.1.1 Lithium Cobalt Oxide
Lithium cobalt oxide (LiCo) batteries use cobalt oxide (CoO$_2$) as the anode. The half-cell reactions are shown below [3]:

\[
\text{LiCoO}_2 + 0.5\text{Li}^+ + 0.5\text{e}^- \leftrightarrow \text{Li}_{0.5}\text{CoO}_2 \quad \Delta V = -0.7\text{V} \quad (2.1)
\]

\[
\text{LiC}_6 \leftrightarrow \text{C}_6 + \text{Li}^+ + \text{e}^- \quad \Delta V = 2.8\text{V} \quad (2.2)
\]

Lithium cobalt oxide batteries have a theoretical specific energy of 448Wh/kg, commercially available batteries have specific energies around 200Wh/kg [4]. LiCo is the oldest and most commonly used type of lithium ion battery having been introduced by Sony in 1991 [5]. Current manufacturers of LiCoO$_2$ batteries include Sony and Panasonic [6].

LiCo batteries are particularly prone to thermal runaway from overcharging and overheating [7]. This has resulted in multiple consumer recalls in laptops [8] and cell phones [9]. The grounding of the Boeing 787 was also a result of failure in LiCo batteries [10].

2.1.1.2 Lithium Iron Phosphate
Lithium iron phosphate (LFP) batteries use iron phosphate (FePO$_4$) as the cathode. The half-cell reactions during discharge are shown below [11] [3].
\[ \text{FePO}_4 + \text{Li}^+ + e^- \leftrightarrow \text{LiFePO}_4 \quad \Delta V \approx -0.4 \text{V} \quad (2.3) \]

\[ \text{LiC}_6 \leftrightarrow \text{C}_6 + \text{Li}^+ + e^- \quad \Delta V = 2.8 \text{V} \quad (2.4) \]

LFP batteries have a theoretical specific energy of 544Wh/kg, commercially available batteries have capacities from 120-160Wh/kg [4]. The stability of the cathode allows for higher currents to be used yielding a higher specific power and lower charging time than other li-ion types [7].

LFP batteries are stable when overcharged due to the stability of phosphate in the cathode; unlike other materials it will not release oxygen when overheated or overcharged. This stability also reduces side reactions during discharge and at low states-of-charge, increasing cycle life. A123 Systems quotes 7000 cycles at 100% depth-of-discharge before the capacity drops to 80% of the initial [12].

Manufacturers of LFP batteries include A123 Systems and BYD. The BYD e6 and Chevrolet Spark use lithium iron phosphate batteries [13] [14].

2.1.1.3 Lithium Manganese Spinel

Lithium manganese spinel (LiMn) batteries use a cathode of Mn₂O₄. The half-cell reactions are shown below:

\[ \text{LiMn}_2\text{O}_4 \leftrightarrow \text{Mn}_2\text{O}_4 + \text{Li}^+ + e^- \quad \Delta V \approx -0.7 \text{V} \quad (2.5) \]

\[ \text{C}_6 + \text{Li}^+ + e^- \leftrightarrow \text{LiC}_6 \quad \Delta V = 2.8 \text{V} \quad (2.6) \]

LiMn cells have a theoretical specific energy of 1001Wh/kg, commercially available cells have a specific energy of 120Wh/kg [4].

LiMn batteries are more resistant to thermal runaway than LiCo cells [7]. This is a result of its cubic close packed structure which allows the cathode to retain its structure when discharged in contrast to the layered structure used in lithium cobalt oxide cathodes [15]. Despite this, lithium manganese spinel batteries have limited cycle lives as a result of manganese dissolution to the electrolyte [16].

Lithium manganese spinel batteries are used in the Chevrolet Volt and Hyundai Sonata Hybrid [17].

2.1.1.4 Lithium Nickel-Cobalt-Manganese

Lithium nickel-cobalt-manganese (Li-NCM) batteries use an alloy of oxidized metals as an anode to provide better characteristics than any single metal oxide. Different metal alloys can also be used including nickel-cobalt-aluminum [18]. The particular redox reactions, and thus the specific capacity and voltage, depend on the alloy used. Battery manufacturers can adjust the ratios of these metals to achieve different combinations of energy capacity, cycle life, material cost, and manufacturability.

Lithium NCM batteries are planned to replace lithium manganese spinel in the Chevrolet Volt [19]. Lithium nickel alloy batteries are also used in the Tesla Model S [20].
2.1.1.5 Lithium Titanate
Lithium titanate cells can use any of the cathode materials described in the previous sections with an anode of titanate (such as Li\textsubscript{1/3}Ti\textsubscript{5/3}O\textsubscript{4} or Li\textsubscript{4}Ti\textsubscript{5}O\textsubscript{12}). The anode reaction of a lithium titanate cell is given below [21].

\[ Li_4Ti_5O_{12} + xe^- + Li^+ \leftrightarrow Li_{x+4}Ti_5O_{12} \quad \Delta V = \sim 1.5V \quad (2.7) \]

Lithium titanate has improved performance at low temperatures because the performance drop in carbon anode based cells is caused by slow diffusion of Li through graphite [22]. Lithium titanate structures have been created which do not expand with lithium insertion, reducing strain on the anode and extending cycle life [23]; and which bond lithium pseudocapacitively, enabling higher charge and discharge rates [24]. This allows for much faster charge and discharge rates.

Lithium titanate batteries are used in the Honda Fit EV and Mitsubishi i-MiEV [25] [26].

2.1.2 Nickel Cadmium
Nickel cadmium (NiCd) batteries are formerly common rechargeable batteries, although their market share has decreased due to the increased capacity, lower cost, and lower toxicity of nickel metal hydride batteries.

NiCd batteries use hydroxyl ions as the mobile ions between nickel oxide-hydroxide and cadmium plates. The half-cell reactions are shown below [4]:

\[ 2NiO(OH) + 2H_2O + 2e^- \leftrightarrow 2Ni(OH)_2 + 2OH^- \quad \Delta V = 0.45V \quad (2.8) \]
\[ Cd + 2OH^- \leftrightarrow Cd(OH)_2 + 2e^- \quad \Delta V = 0.81V \quad (2.9) \]

Nickel cadmium batteries have a theoretical specific energy of 244Wh/kg, available batteries have capacities of 40Wh/kg [4].

Nickel cadmium batteries are banned in the European Union with exceptions for emergency systems, medical equipment, and power tools [27]. As a result they cannot be used in vehicles intended for sale in Europe.

2.1.3 Nickel Metal Hydride
Nickel metal hydride (NiMH) batteries use the same nickel oxyhydroxide (NiOOH) anode as found in nickel cadmium cells. The cadmium cathode is replaced by a hydrogen absorbing alloy. The half-cell reactions during discharge are shown below:

\[ NiO(OH) + H_2O + e^- \leftrightarrow Ni(OH)_2 + OH^- \quad \Delta V = 0.45V \quad (2.10) \]
\[ MH + OH^- \leftrightarrow H_2O + M + e^- \quad \Delta V = 0.83V \quad (2.11) \]

NiMH batteries have a theoretical specific energy of 240Wh/kg, available batteries have specific energies of 100Wh/kg [4].
Current commercial NiMH batteries use LaNi₅ or AB₂ alloys (with A and B representing two of Zr, Ti, or V) as hydrogen absorbers. Mg based alloys have been investigated as they offer greater hydrogen storage potential and lower cost than rare earth based alloys. Additional studies have looked at AB₃ and A₂B₇ with different rare earths as A and transition metals as B. These alternate materials have shown improved capacity over traditional metal hydrides [28]. Nickel metal hydride batteries were first used for automobiles in the Honda EV Plus in 1997 [29]. They have more recently been used in hybrid cars with low battery capacities such as the Lexus CT200h [30] and Toyota Prius C [31].

2.1.4 Lead Acid

Lead acid batteries are the oldest and most widely used secondary cells. The half-cell reactions of a lead acid battery during discharge are shown below [32].

\[
Pb + SO₄^{2-} \rightarrow PbSO₄ + 2e^- \quad \Delta V = 0.36V \tag{2.12}
\]

\[
PbO_2 + SO₄^{2-} + 4H^+ + 2e^- \rightarrow PbSO₄ + 2H_2O \quad \Delta V = 1.69V \tag{2.13}
\]

Lead acid batteries have a theoretical specific energy of 252Wh/kg, commercially available batteries have specific energy near 35Wh/kg [4].

Recent developments in lead acid batteries include replacing the traditional lead grid electrode with a highly porous lead-coated carbon plate. This reduces the mass of non-active components of the battery, thereby increasing energy density to 50Wh/kg [33]. Similar studies have investigated lead-plated carbon honeycomb structures, finding an energy density of 97Wh/kg (considering only the electrode mass) [34].

The General Motors EV1 used lead acid batteries initially but later switched to NiMH. Today, lead acid batteries are commonly used to power starter motors in automobiles as well as providing backup power in stationary applications.

2.1.5 Comparison Table

Table 1 shows a brief comparison of all battery types considered above:
Table 1: Comparison of battery types for PHEVs

<table>
<thead>
<tr>
<th>Battery Type</th>
<th>Benefits</th>
<th>Drawbacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium Cobalt Oxide</td>
<td>High specific energy</td>
<td>High risk of thermal runaway</td>
</tr>
<tr>
<td></td>
<td>Established manufacturing</td>
<td></td>
</tr>
<tr>
<td>Lithium Iron Phosphate</td>
<td>Long cycle life</td>
<td>Lower nominal voltage</td>
</tr>
<tr>
<td></td>
<td>High charge/discharge currents</td>
<td>Lower specific energy</td>
</tr>
<tr>
<td></td>
<td>Stable at high and low SOCs</td>
<td>Requires cylindrical cells</td>
</tr>
<tr>
<td>Lithium Manganese Spinel</td>
<td>High specific energy</td>
<td>Lower cycle life</td>
</tr>
<tr>
<td>Lithium Nickel Alloy</td>
<td>High specific energy</td>
<td>High cost</td>
</tr>
<tr>
<td></td>
<td>Longer cycle life than Li-Ni</td>
<td></td>
</tr>
<tr>
<td>Lithium Titanate</td>
<td>High charge/discharge currents</td>
<td>Lower nominal voltage</td>
</tr>
<tr>
<td></td>
<td>Long cycle life</td>
<td>Low commercial availability</td>
</tr>
<tr>
<td>Nickel Metal Hydride</td>
<td>Lower cost than Li-ion</td>
<td>High self-heating</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Requires cylindrical cells</td>
</tr>
<tr>
<td>Nickel Cadmium</td>
<td>Low cost</td>
<td>Cadmium is banned in the EU</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Requires cylindrical cells</td>
</tr>
<tr>
<td>Lead Acid</td>
<td>Well established recycling infrastructure</td>
<td>Risk of acid spill during accidents</td>
</tr>
<tr>
<td></td>
<td>Lowest cost</td>
<td>Lowest specific energy</td>
</tr>
</tbody>
</table>

2.1.6 Future Battery Chemistries

2.1.6.1 Lithium Air
Lithium air batteries function by reacting lithium ions with atmospheric oxygen. This requires an electrolyte which will transport lithium ions between a lithium metal anode to a porous O$_2$ cathode. It also requires a membrane which is permeable to atmospheric O$_2$ but blocks other gases, particularly water vapour. The net discharge reaction of a lithium air battery is shown below [35].

$$2Li^+ + 2O_2^- \leftrightarrow Li_2O_2 + O_2$$ (2.14)

Discounting the mass of the oxygen (which is atmospheric and not contained within the battery) lithium-air batteries have a theoretical energy density of 11680Wh/kg – comparable to that of gasoline (13000Wh/kg).
There are still several issues which must be overcome before lithium air batteries can become commercialized. The $O_2^-$ used in the cathode reaction is highly basic and will degrade many electrolytes [36].

2.1.6.2 Silicon-based Lithium Ion
By using silicon in place of carbon the charge density of the cathode can be increased to 4200mAh/g compared to 372mAh/g with graphite. This is because lithium forms a LiC$_6$ structure with carbon and a Li$_{22}$Si$_5$ structure with silicon, allowing a higher mass fraction of lithium ions [37]. Several silicon structures have been examined, including silicon nanowires [38] [39], nanotubes [40], thin films [41], and amorphous silicon [42].

Current silicon anodes have short cycle lives as a result of high volumetric expansion (>300%) [43]. At macroscopic scales this results in mechanical fracturing of the anode, decreasing the amount of usable material. For nano-scale anodes, a solid-electrolyte interphase layer forms around the anode, slowing lithium transport and reducing battery capacity. Additionally, the expansion of the silicon structure can cause it to separate from the current collector [39] [40].

2.1.6.3 Flow Batteries
Vanadium redox batteries use two electrodes containing vanadium in different oxidization states in a sulphuric acid solution. The discharge half-cell reactions are shown below [44]:

\[
\begin{align*}
\text{VO}_2^+ + 2H^+ + e^- & \leftrightarrow VO^{2+} + H_2O \\
V^{2+} & \leftrightarrow V^{3+} + e^- 
\end{align*}
\]

(2.15)  (2.16)

Current vanadium redox batteries have very low energy densities (20-35Wh/kg) [45]. The use of liquid electrolytes in the reaction allows flow batteries to be recharged quickly by simply replacing the discharged electrolyte. The spent electrolyte can then be recharged electricity when not in use. Vanadium redox batteries are not currently used in any portable application; however, there are pilot projects of grid-level energy storage using the technology [46].

2.2 Previous Studies
Previous studies have been performed examining the performance of different batteries in PHEVs and attempting to determine suitable batteries.

One of the earliest studies tested four battery types (NiMH, NiCd, lead acid, and zinc-bromide) during constant current discharges, constant power discharges, peak power tests, and a limited simulated driving cycle. This study found niche applications in PHEVs for each battery type. Considering the advances in battery technology since the publication of this study its results should not be considered representative of today’s batteries [47].
A more recent study examined the requirements for batteries in PHEVs considering different curb weights, drag area, and driving cycles. A Powertrain System Analysis Toolkit (PSAT) model was created to simulate battery operation and validated using hardware-in-the-loop testing. The specific ranges estimated in this study were from 6.4km/kWh for a midsize car to 4.2km/kWh for a midsize SUV [48].

Another study examines the effect of real world driving cycles on PHEV performance. The study considered battery packs of 4, 8, 12, and 16 kWh. It found diminishing returns with respect to fuel savings as battery size increases. No battery characteristics are specified beyond capacity. The results from this study suggest that the benefit of increased battery capacity for a PHEV varies significantly depending on the daily distance driven [49].

While these prior studies were informative, they do not address the key goal of this thesis. It was determined that actual test data on current battery technology was required to provide reliable data for the optimization tool.

### 2.3 Vehicle Driving Cycles

Batteries in electric vehicles are subjected to unique loading scenarios relative to other high-performance electronics. These are characterized by frequent, rapid changes between charge and discharge. Accurately assessing the battery’s performance under these conditions is vital to predicting the overall effectiveness of a PHEV.

The duty cycle used in this experiment was developed by monitoring 40 hybrid vehicle drivers in Winnipeg over one month. The duty cycle selected for battery testing is a 20km cycle lasting 29:47; it is explained further in section 3.3.3. The duty cycle used for all-electric range and cycle life estimation is a full weekday cycle consisting of 6 trips totaling 37km. It is explained further in section 6.2.5 [50]. Considering only the Greater Toronto Area, the median trip length of all drivers is 5.3km with a mean of 3.9 trips per vehicle each day – an average of 20.5km travelled per vehicle [51]. The driving cycle under consideration thus corresponds to above-average usage for an urban driver.

### 2.4 Battery Aging

Battery aging occurs during charge and discharge. However, most tests and studies only account for changes in discharge patterns and assume optimal charging conditions. The method used in this optimization is based on that presented in [52] and outlined below.

In this model, each battery has a charge life, \( \Gamma \), equal to the sum of the effective amp-hours discharged during its lifetime. The rated charge life, \( \Gamma_R \), is given by the equation [52]:
The rated charge life of the battery, in Ah

The rated cycle life of the battery, in cycles

The rated discharge capacity of the battery, in Ah

The rated discharge current of the battery, in C

This rated charge life must then be adjusted for changing depths-of-discharge and discharge currents. It is assumed that the operating conditions (charging pattern, temperature, and rest times) are the same as in the reference tests. The relationship between depth-of-discharge and cycle life is defined by the equation [52]:

\[ L = u_2 \left( \frac{D_R}{D_A} \right)^{u_0} e^{u_1(1 - \frac{D_A}{D_R})} \]  

where:

- \( L \) is the cycle life of the battery, in cycles
- \( u_2, u_0, u_1 \) are coefficients
- \( D_R \) is the rated discharge capacity of the battery, in Ah
- \( D_A \) is the actual discharge capacity of the battery, in Ah

The coefficients \( u_0, u_1, \) and \( u_2 \) are found by fitting the curve to cycle life vs. depth of discharge which can be either provided by the manufacturer or produced from cyclic testing.

The relationship between discharge rate and cycle life can be similarly interpreted using the equation [52]:

\[ d_{cycle} = \left( \frac{C_R}{C_A} \right)^{v_0} e^{v_1 \left( \frac{C_A}{C_A} - 1 \right) d_{actual}} \]  

where:

- \( d_{cycle} \) is the effective discharge over the course of the cycle, in Ah
- \( C_R \) is the rated discharge rate of the battery, in C
- \( C_A \) is the actual discharge rate of the battery, in C
- \( v_0, v_1 \) are coefficients

Where \( C_A \) is the actual capacity at the given discharge rate. The coefficients \( v_0 \) and \( v_1 \) can be found by fitting a curve to cycle life vs. discharge rate from data provided from the manufacturer or
determined in testing. This data is not currently available for the batteries tested so a simplified version of this equation, where \( v_0 = 1 \) and \( v_1 = 0 \), is used for further analysis. Combining equations 2.18 and 2.19 yields the effective charge life decrease for a given discharge period. This can be combined with equation 1 to give the cycle life of the battery, shown below:

\[
L = \frac{L_R D_R C_R}{\sum \left( \frac{D_A}{D_R} \right)^{u_0} e^{u_1 \left( \frac{D_A}{D_R} \right)} \left( \frac{C_R}{C_A} \right)^{v_0} e^{v_1 \left( \frac{C_R}{C_A} - 1 \right)}}
\]  

(2.20)

\( L \) The cycle life of the battery, in cycles
\( L_R \) The rated cycle life of the battery, in cycles
\( D_R \) The rated discharge capacity of the battery, in Ah
\( C_R \) The rated discharge current of the battery, in C
\( C_A \) The actual discharge rate of the battery, in C
\( D_A \) The actual discharge capacity of the battery, in Ah
\( v_0, v_1 \) Coefficients
\( u_2, u_1, u_0 \) Coefficients

Equation 2.20 allows the cycle life of the battery to be predicted for a series of discharge events; in this case that is the daily driving cycle described in section 6.2.5.

2.5 Life-Cycle Emissions

Emissions from battery manufacture and electricity production are often cited as reasons why PHEVs are not a “green” choice compared to conventional automobiles. Recent studies have found that battery electric and hybrid electric vehicles have lower cradle-to-grave emissions than internal combustion vehicles [53] [54]. Given that reducing greenhouse gas emissions is the primary driving force behind PHEV development, determining the carbon footprint of a vehicle is a good idea. Traditionally, life cycle assessments examine five stages of the products life: materials creation, manufacture, transportation, use, and end-of-life [55]. Greenhouse gas emissions from the first three phases, referred to collectively as cradle-to-gate emissions, are given in Table 2:
Table 2: Global warming potentials from the manufacture of batteries

<table>
<thead>
<tr>
<th>Chemistry</th>
<th>Global Warming Potential (GWP(<em>{100})) (kg/Wh(</em>{\text{nominal}}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium iron phosphate</td>
<td>0.25 [56]</td>
</tr>
<tr>
<td>Lithium nickel-cobalt-magnesium</td>
<td>0.20 [56]</td>
</tr>
<tr>
<td>Nickel metal hydride</td>
<td>0.35 [56]</td>
</tr>
<tr>
<td>Nickel cadmium</td>
<td>0.16 [57]</td>
</tr>
<tr>
<td>Lead acid</td>
<td>0.35 [57]</td>
</tr>
</tbody>
</table>

The emissions for each of the lead acid batteries are assumed to be equal. Cradle-to-gate emissions for the combustion engine, electric motor, and other components are assumed to be negligible. Emissions from combustion are a result of the battery not being able to provide the power or energy required to meet the desired driving cycle. In this case the energy is derived from gasoline with an emissions intensity of 67.1g/MJ [58]. The combustion engine is assumed to be 24.6% efficient [59], resulting in an effective emissions intensity of 272.9g/MJ.

Emissions from electricity generation are dependent on the jurisdiction in which the vehicle will be used. For further calculations it is assumed that the car will be charged in Ontario with an emissions intensity of 27.8g/MJ [60].
Chapter 3

Experimental Setup

3.1 Testing Purpose and Scope
The goal of the battery testing is to determine the performance of different batteries in a plug-in hybrid electric vehicle. This requires both standard tests (capacity and maximum power) as well as duty cycle efficiency. These testing results, combined with the optimization program described in section 5, allow the future behavior of a vehicle to be estimated without the need for expensive prototypes.

The testing results presented in the section are a combination of these previously performed (room temperature evaluations of LFP, NiMH, NiCd, PbSn, and VRLA for capacity, power output, and duty cycle efficiency) [61] and new results (all results for Li-NCM and spiral LA and all low temperature results).

3.2 Apparatus

3.2.1 Battery Testing Unit
The battery testing unit (BTU) used is an Arbin Instruments BT200. This BTU provides two channels with voltage control from 2V to 15V and current control from -100A to 100A. The unit is computer controlled with schedule files defining voltage, current, or power at different stages of the test.

Voltage measurements are made at the terminals of each battery. Current measurements are made at the BTU. The battery leads are 6ft cables of 2AWG stranded copper wire. The maximum power loss along the leads is 19W under full load. The wire loss calculations can be found in Appendix A.1.

Low temperature tests were performed with the battery in a consumer chest freezer. The temperature inside the freezer varies from -16°C to -22°C. Batteries were placed in the same location inside the freezer for each test. For test scheduling purposes “low temperature” defined as when the mean thermocouple temperature reached -17.5°C. Figure 1 shows the ambient and battery temperatures during a test of the PbSn battery pack.
Figure 1: Ambient and battery temperatures prior to low temperature testing of the PbSn battery pack

High temperature testing was not performed because the requisite equipment was not available. Ideally, testing would be performed at temperatures near the upper range of the battery’s operating range. This testing would provide valuable information for determining the safety margins required to ensure the safety of the battery pack in operation. Additional temperature points would also allow relationships between temperature and performance to be determined more precisely.

Figure 2 shows the battery testing unit:
3.2.2 Battery Systems

The batteries tested are commercially available samples from a variety of manufacturers. Two lithium-based batteries were tested (LiFePO4 and Li-NCM), two nickel-based (NiMH and NiCd) and three lead-based (absorbed glass mat, lead-tin, and spiral cell). The specifications of each testing unit are shown below.

Table 3: Battery systems used in testing

<table>
<thead>
<tr>
<th>Chemistry</th>
<th>Manufacturer</th>
<th>Cell Arrangement</th>
<th>Voltage</th>
<th>Capacity</th>
<th>Max C-rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>A123 Systems</td>
<td>4S 4P</td>
<td>13.2V</td>
<td>9.2Ah</td>
<td>30C*</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>Ener1 Korea</td>
<td>3S</td>
<td>11.1V</td>
<td>16Ah</td>
<td>5C</td>
</tr>
<tr>
<td>NiMH</td>
<td>SAFT</td>
<td>10S</td>
<td>12V</td>
<td>15Ah</td>
<td>3.3C</td>
</tr>
<tr>
<td>NiCd</td>
<td>SAFT</td>
<td>10S 2P</td>
<td>12V</td>
<td>14Ah</td>
<td>3C</td>
</tr>
<tr>
<td>VRLA AGM</td>
<td>Unknown - Chinese</td>
<td>6S</td>
<td>12V</td>
<td>12Ah</td>
<td>3C</td>
</tr>
<tr>
<td>Pb-Sn</td>
<td>Enersys</td>
<td>6S</td>
<td>12V</td>
<td>16Ah</td>
<td>10C*</td>
</tr>
<tr>
<td>Spiral LA</td>
<td>Unknown - Spanish</td>
<td>6S</td>
<td>12V</td>
<td>50Ah</td>
<td>2C*</td>
</tr>
</tbody>
</table>

* - Maximum discharge current limited by BTU
Temperature was measured using one to four K-type thermocouples attached to the battery pack. The thermocouples were placed in the centre of the cell pack when possible. For the sealed lead acid batteries the thermocouples were placed on the outer casing.

Figure 3 shows four of the battery samples used in testing:

![Battery samples](image)

**Figure 3:** Photo of battery samples used for testing. Clockwise from top left: spiral cell lead acid, lithium iron phosphate, lithium nickel alloy, nickel metal hydride

### 3.3 Methodology

Tests were performed to or above the minimum number of repetitions defined in USABC Manual Appendix A: Generic Test Plan Outline for USABC Battery Testing [62]. The battery testing unit allows for the battery to be loaded with a defined current, voltage, or power. Capacity was tested using a constant current while maximum power and duty cycle testing was performed with defined powers.

The battery testing unit records the voltage, current, and temperature during the test. It also provides calculated values for charge and discharge capacities (Ah) and energies (Wh) and instantaneous power (W).

#### 3.3.1 Battery Charging

Batteries were charged at room temperature according to the manufacturer’s recommendations. Lithium and lead batteries were charged by applying a constant current until the maximum voltage was reached followed by a constant voltage until the charge current dropped to near zero. This method is not appropriate for nickel-based batteries because the cell voltage does not increase notably when fully charged. Instead, the increase in temperature is used to determine when the charge is complete. The charging patterns used are outlined in Table 4 on the next page.
Table 4: charging patterns by battery chemistry

<table>
<thead>
<tr>
<th>Chemistry</th>
<th>Charging Pattern</th>
<th>Charging Time (from 100% DOD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>Current at 12A to 14.4V, Voltage at 14.4V to &lt;0.05A</td>
<td>90 minutes</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>Current at 5.3A to 12.3V, Voltage at 12.3V to &lt;0.05A</td>
<td>4 hours</td>
</tr>
<tr>
<td>NiMH</td>
<td>Current at 4A to ΔV &lt; -0.02V/min or ΔT &gt; 0.5°C/min</td>
<td>5 hours</td>
</tr>
<tr>
<td>NiCd</td>
<td>Current at 1.5A to ΔV &lt; -0.02V/min or ΔT &gt; 0.5°C/min</td>
<td>4 hours</td>
</tr>
<tr>
<td>VRLA AGM</td>
<td>Current at 4A to 14.7V, Voltage at 14.7V to &lt;0.05A</td>
<td>8 hours</td>
</tr>
<tr>
<td>Pb-Sn</td>
<td>Current at 8A to 14.4V, Voltage at 14.4V to &lt;0.05A</td>
<td>6 hours</td>
</tr>
<tr>
<td>Spiral LA</td>
<td>Current at 30A to 14.4V, Voltage at 14.4V to &lt;0.05A</td>
<td>8 hours</td>
</tr>
</tbody>
</table>

3.3.2 Capacity Testing
Battery capacity was tested by cycling each battery pack at several discharge rates. This allows the relationship between discharge rate and capacity, known as the rate capacity effect, to be determined [63]. Batteries were discharged at 0.5C, 1C, 2C (1C is equal to the current required to discharge the nominal voltage in 1 hour), and the maximum recommended current at room temperature. Batteries were also discharged at 2C at low temperature. The procedure for testing capacity is outlined below:

1. Fully charge the battery following the manufacturer’s instructions. Allow the battery to reach ambient temperature before proceeding.
2. Discharge the battery at the desired current until the voltage drops below the manufacturer’s recommended cutoff voltage.
3. Allow the battery to rest and reach room temperature before recharging according to the manufacturer’s specifications.

Capacity tests were performed five times at room temperature for each discharge rate and three times at low temperature.

3.3.3 Maximum Power Testing
Maximum power is considered to be the amount of power the battery can deliver over a sustained period of time. It should not be confused with pulse power which is the power output of the battery.
over a short duration (<1s). It was tested according to the following procedure, adapted from the USABC Manual Appendix H: Procedure to Measure Actual Peak Power [64].

1. Fully charge the battery following the manufacturer’s instructions. Allow the battery to rest for one hour before proceeding.
2. Discharge the battery at a rate of C/3 to the desired state-of-charge. Allow the battery to reach ambient temperature. Record the open-current voltage at this time.
3. Ramp the discharge current to a value which decreases its voltage to 2/3 of the open-current voltage. This step should occur quickly (<5 seconds). Record this current.
4. Recharge the battery at room temperature. Allow the battery to rest before proceeding.
5. Discharge the battery to the desired state-of-charge. Allow the battery to reach ambient temperature.
6. Discharge the battery for 30 seconds at the current measured in step 3. The average power over this time is the maximum power output of the battery.

Each battery was tested at the top, middle, and bottom of its state-of-charge window. Some batteries could not discharge for 30 seconds at the bottom of their SOC window under cold conditions; in this scenario the SOC window floor was increased to allow the test to complete. The SOC windows for each maximum power test are shown below:

<table>
<thead>
<tr>
<th></th>
<th>Room Temperature (20 – 25°C)</th>
<th>Low Temperature (-16 – -18°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max SOC</td>
<td>Mid SOC</td>
</tr>
<tr>
<td>LiFePO4</td>
<td>80%</td>
<td>50%</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>80%</td>
<td>50%</td>
</tr>
<tr>
<td>NiMH</td>
<td>80%</td>
<td>55%</td>
</tr>
<tr>
<td>NiCd</td>
<td>80%</td>
<td>55%</td>
</tr>
<tr>
<td>VRLA AGM</td>
<td>80%</td>
<td>60%</td>
</tr>
<tr>
<td>Pb Sn</td>
<td>80%</td>
<td>60%</td>
</tr>
<tr>
<td>Spiral LA</td>
<td>80%</td>
<td>60%</td>
</tr>
</tbody>
</table>

Maximum power was tested three times at room temperature and once at low temperature for each SOC.

### 3.3.4 Duty Cycle Testing
Duty cycle testing was performed to determine the cumbolic efficiency of different batteries under PHEV usage patterns. The duty cycle used was the PHEV20 duty cycle described in section 3.5. The
duty cycle was scaled for each test so that the maximum power required corresponded to the maximum power available from the battery at that temperature and state-of-charge.

The procedure for duty cycle testing is outlined below. The testing was repeated multiple times for each battery and state-of-charge.

1. Fully charge the battery according to the manufacturer’s instructions. Allow the battery to rest for one hour before proceeding.
2. Discharge the battery at a rate of C/3 to the desired state-of-charge (80%). Allow the battery to reach ambient temperature before proceeding.
3. Run the power simulation.

Three capacities are measured during a duty cycle test: discharge (the gross charge used during the duty cycle), charge (the gross charge regained during the duty cycle), and recharge (the charge required to bring the battery to the original SOC after the duty cycle). The duty cycle efficiency is then calculated according to the equation:

\[
\text{efficiency} = \frac{C_{\text{discharge}} - C_{\text{charge}}}{C_{\text{recharge}}}
\]  

\(C_{\text{discharge}}\) The discharge capacity during the duty cycle
\(C_{\text{charge}}\) The charge capacity during the duty cycle
\(C_{\text{recharge}}\) The charge capacity to return to the original state-of-charge following the duty cycle

Duty cycle tests were performed three times at room temperature and once at low temperature for each battery.
Chapter 4

Testing Results

4.1 Physical Properties

Table 6 gives the manufacturer’s stated mass of each battery sample and its actual measured mass:

<table>
<thead>
<tr>
<th></th>
<th>Manufacturer (kg)</th>
<th>Measured (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>1.15</td>
<td>3.293</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>1.35</td>
<td>1.389</td>
</tr>
<tr>
<td>NiMH</td>
<td>3.18</td>
<td>5.252</td>
</tr>
<tr>
<td>NiCd</td>
<td>5.04</td>
<td>9.007</td>
</tr>
<tr>
<td>Lead Acid</td>
<td>4.82</td>
<td>8.632</td>
</tr>
<tr>
<td>Lead Tin</td>
<td>6.1</td>
<td>12.798</td>
</tr>
<tr>
<td>Spiral Lead Acid</td>
<td>22.7</td>
<td>23.488</td>
</tr>
</tbody>
</table>

The discrepancies are partly a result of the additional materials present in the samples that are not accounted for by the manufacturer (e.g., current collectors, padding, quick-release terminals).

4.2 Capacity Results

Figure 4 on the next page shows the battery capacity at various discharge rates at room temperature obtained using the method detailed in section 3.3.2 [61].
Each battery tested experienced a decrease in capacity as the discharge current. This effect is most pronounced in the lead acid samples are minimal in the LFP battery. This rate capacity effect is discussed further in section 4.1.

Figure 5 shows the battery capacity at room and low temperatures at a 2C discharge rate.

Capacity decreased with temperature across all battery types with the most significant decrease being in the nickel-based batteries.
Following these tests the VRLA AGM battery was unable to match its previous performance for either power output or capacity. Consequently, it was not tested for low temperature maximum power or duty cycle efficiency.

4.3 Maximum Power Results

Figure 6 shows the maximum output power of each battery sample at room temperature [61]:

![Bar chart showing maximum power at room temperature](image)

**Figure 6: Maximum output power at room temperature**

The results from this test indicate that for some battery samples maximum power is available at the middle of the SOC window rather than at max SOC as expected. This could be a result of an increased battery temperature as a result of a longer pre-discharge increasing the available power. It may also be a statistical anomaly caused by the low number of tests performed.

For this test the LiFePO4, Lead tin, and spiral cell lead acid batteries were limited by the maximum current of the BTU. As a result, the decrease in power available is not as significant as if more power could be drawn.

Figure 7 shows the maximum power output of each battery sample at low temperature:
The maximum power dropped significantly for all battery types tested. The most significant decreases (>80%) occurred in the lithium batteries. These were also notable for being unable to successfully complete the procedure from section 2.3.3 because the voltage would rapidly drop below the cutoff level. To prevent this, the 30s discharge current was reduced in 10% increments until the test could be completed.

The spiral cell lead acid battery remained constrained by the maximum current of the BTU.

### 4.4 Duty Cycle Results

Figure 8 shows the gross discharge and charge capacities of each battery during a room temperature duty cycle. Also shown in the recharge capacity (the charge required to return to 100% SOC).

Efficiency is measured by dividing the net discharge over the cycle (discharge-charge) by the post-cycle recharge. A discussion of duty cycle efficiencies is presented in section 4.2.
For the case of the NiCd battery packs (which must be charged separately) the sum of the recharge capacities was used. There is no reason to believe that this changes the results of the testing.

Figure 9 shows the discharge, charge, and recharge capacities from a low temperature duty cycle.

The capacities are decreased as a consequence of the scaling of the duty cycle to match each battery’s reduced power capability. The lead acid and Li-NCM battery samples were unable to complete a low temperature duty cycle and have been omitted from this analysis.
Chapter 5

Discussion

5.1 Rate Capacity Effect

Using the results presented in Figure 2 in section 4.2, the magnitude of the rate capacity effect (the decrease in available capacity at high discharge rates) at room temperature can be found for each battery. These values are shown in Figure 10 below:

![Figure 10: Decrease in capacity per 1C discharge rate](image)

Each battery tested showed a significant decrease in capacity as the discharge rate increased with the exception of the lithium iron phosphate sample. This is consistent with the manufacturers’ specifications where available [61].

The rate capacity effect calculated in this section is used in determining battery life during the optimization, presented in section 6.3.4.

5.2 Duty Cycle Efficiency

The efficiency of the battery over the testing duty cycle is a key factor in its viability in electric vehicles. It can be calculated by dividing the net discharge by the recharge required after running the cycle. Figure 11 shows the duty cycle efficiency of each battery sample at high and low temperatures:
Figure 11: Duty cycle efficiency at high and low temperatures

Figure 11 demonstrates a significant decrease in efficiency at low temperatures. This effect is greater in the nickel-based and lead-tin samples. Conversely, the spiral cell lead acid did not change efficiencies. This could be a unique characteristic of this battery type or a result of the BTU limiting its output power.

The standard lead acid and Li-NCM samples are not included in this analysis because a low temperature duty cycle could not be completed.

5.3 Internal Resistance Changes with Temperature
Each test scenario shows a decrease in capacity, voltage under load, and maximum power available at low temperatures. The discharge curves for each battery discharging at 2C at high and low temperatures are shown below:
The plots in Figure 12 indicate that there is a very large decrease in voltage at low temperatures in nickel-based batteries. The shape of the discharge curve remains similar for all chemistries with the exceptions of LFP, which has a voltage increase, and nickel cadmium, which has a very short discharge time (<30s). The voltage increase in the LFP graph is due to the battery’s self-heating raising its temperature over the course of the test.

The internal resistance can be calculated using the recorded voltage and current and each point over a duty cycle. The change in voltage per change in current is the internal resistance of the battery (V=IR). For this test the internal resistance is averaged over the full discharge window. Table 7 shows the high and low temperature internal resistances for each battery chemistry.
Table 7: Internal resistances at high and low temperatures

<table>
<thead>
<tr>
<th>Chemistry</th>
<th>Warm Resistance (Ω)</th>
<th>R-Squared</th>
<th>Cold Resistance (Ω)</th>
<th>R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>0.0123</td>
<td>0.9914</td>
<td>0.0974</td>
<td>0.943</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>0.0065</td>
<td>0.2413</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>NiCd</td>
<td>0.0361</td>
<td>0.9817</td>
<td>0.1168</td>
<td>0.680</td>
</tr>
<tr>
<td>NiMH</td>
<td>0.0362</td>
<td>0.9927</td>
<td>0.1000</td>
<td>0.930</td>
</tr>
<tr>
<td>VRLA AGM</td>
<td>0.0259</td>
<td>0.5526</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Pb Sn</td>
<td>0.0134</td>
<td>0.9310</td>
<td>0.0436</td>
<td>0.562</td>
</tr>
<tr>
<td>Spiral LA</td>
<td>0.0076</td>
<td>0.7779</td>
<td>0.0351</td>
<td>0.445</td>
</tr>
</tbody>
</table>

The $R^2$ value for the Li-NCM battery indicates that there is a non-linear internal resistance relationship. Figure 13 shows that this relationship is linear, however, there is a significant amount of noise.

![Figure 13: Voltage-current plot for the Li-NCM battery sample during a room temperature duty cycle](image)

$V = 0.0065I + 10.757$
The internal resistance of the spiral cell battery is linear at room temperature with a moderate amount of noise. At low temperatures the internal resistance is constant during discharge; however, it changes significantly during charge. This is shown in Figure 14:

![Figure 14: Voltage-current plot for the spiral cell lead acid battery during a low temperature duty cycle](image)

The same phenomenon occurs with the lead-tin battery and the nickel-cadmium battery, shown below:

![Figure 15: Voltage-current plot for lead-tin battery during a low temperature duty cycle](image)
In both scenarios the charging voltage is limited to 16V by the battery testing unit. This phenomenon was investigated by running each battery through a series of ramped current patterns and monitoring the voltage response. The results of this testing are given below.

Figure 17 highlights the non-linearity between the change in current and the corresponding change in voltage. The internal resistance increases as the discharge current approaches zero and increases sharply when the load switches to charging. This relationship is shown below.
Figure 18: Calculated internal resistances in a spiral cell lead acid battery during a current ramp test (see also Figure 16)

The negative data points could be a result of hysteresis, in which case the voltage continued to drop as the discharge current lessened, or from a decrease in the state-of-charge of the battery counteracting the change in internal resistance. The battery response depends not only on its current state but also on its past states.

5.4 Cycle Life Decrease with Temperature

The initial room-temperature tests were successfully completed using a single testing unit. However, three of the seven batteries tested (lithium nickel alloy, lead tin, and lead acid) were unable to complete the low temperature testing regime despite being within their stated cycle life. These results indicate that aging is accelerated during low temperature discharges.

This effect limited the amount of testing that could be performed at low temperatures. In particular, alternate discharge rates were not examined and therefore no low-temperature rate capacity effect term is available. Two different Li-NCM samples failed suddenly during low-temperature maximum power tests, highlighting the need for proper thermal management.

This effect is not modeled in the optimization program because predicting battery temperature during operation and the resulting effect on cycle life is beyond the scope of this project.
Chapter 6

Optimization Tool

The testing results from the previous sections form the basis of an optimization program to determine the optimal battery for a given plug-in hybrid electric vehicle. This section describes the function of that program.

The goal of the optimization is to find the maximum value of the objective function (described in section 6.4). The design space is defined by user-specified limits on battery cost, weight, volume, and voltage.

The optimization tool is coded in MATLAB. All sample optimizations were performed on a 4-core personal computer with 8GB of memory. Execution times ranged from 200 to 2,800 seconds. The full MATLAB code is given in Appendix C.

Figure 19 shows the flow of information in the program.

![Figure 19: Block diagram of the optimization tool](image)

The optimization tool uses a full combinatorial analysis (brute force) in lieu of a gradient-based or evolutionary method. This is possible due to the low computation intensity of evaluating potential batteries and the high proportion of invalid points which can be quickly neglected. Gradient-based algorithms cannot be used for this evaluation due to the non-differentiable nature of the design space.
as a result of the need for integer cell counts. Adopting such an algorithm would require a workaround to allow decimal cell counts during operation, potentially harming accuracy for a minimal increase in execution time.

6.1 Background Information

The first step of the program is to load the relevant information required for the optimization. This can be divided into four categories: battery properties, vehicle information, constraints, and design goals.

Battery information consists of fifteen characteristics for each battery chemistry, listed below:

1. Specific energy (Wh/kg)
2. Cell voltage (V)
3. Maximum discharge current (C-rate)
4. Maximum charge current (C-rate)
5. Cradle-to-gate GHG emissions (gCO₂,eq/Wh)
6. Cost per cell ($/cell)
7. Cost per Ah ($/Ah)
8. Coulombic charge efficiency (%)  
9. Rated charge life (Ah)
10. Aging term u₀
11. Aging term u₁
12. Rated depth of discharge (%)  
13. Rated discharge current (C-rate)
14. Rate capacity effect (%/C)
15. Calendar degradation rate (%/year)

Vehicle information consists of five characteristics of the vehicle itself, listed below:

1. Vehicle mass (kg)
2. Drag coefficient
3. Frontal drag area (m²)
4. Tire coefficient of friction
5. Tire coefficient of rolling resistance

Constraints are used to limit the size of the design space. These are divided into hard constraints, which must be met for the battery to be considered; and soft constraints, for which performance is limited to a specified value. For example, a battery with a mass greater than the constraint will not be evaluated while a battery that provided more power than the constraint is considered to only provide the constrained amount. The constraints are listed in Table 8 below:
Table 8: Constraints used in optimization

<table>
<thead>
<tr>
<th>Hard constraints</th>
<th>Soft constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>All-electric range</td>
</tr>
<tr>
<td>Cost</td>
<td>Acceleration</td>
</tr>
<tr>
<td>Mass</td>
<td>Maximum all-electric speed</td>
</tr>
<tr>
<td>SOC window</td>
<td>Output power</td>
</tr>
</tbody>
</table>

Design goals define the desired performance of the vehicle. There are seven categories, each given a weight, sensitivity, and reference value to match the objective function described in section 6.4. The performance criteria are listed below:

1. All-electric range (km)
2. Cost ($) 
3. Acceleration (s)
4. Maximum all-electric speed (km/h)
5. Battery life (years)
6. Greenhouse gas emissions (kgCO$_2$,eq)
7. Charging time

Of these criteria, acceleration is unique because it can have several different intervals defined. For example, the time to acceleration from 0-40km/h, 0-100km/h, and 80-120km/h could all be calculated and used in the objective function.

6.2 Design Space

The design space is the set of all possible combinations of the four independent design variables: battery chemistry, cell count, cell capacity, and SOC window. For each battery chemistry it is divided into $\leq$100 cell counts, 100 cell capacities, and 5 SOC windows.

Cell count is limited by the defined voltage constraints. The minimum and maximum cell counts are the number of cells which match the minimum and maximum voltages. The remaining cell counts are evenly distributed between these two values. Because cell counts must be integers it is possible that there will be fewer than 100 values meeting these constraints. In this case the design space is reduced in size.

Cell capacity is limited by both cost and mass. The minimum cell capacity is the greater of that required to meet the minimum cost or minimum mass with the maximum cell count. Minimum costs and masses, while not realistic engineering constraints, are used to limit the size of the design space.
and achieve greater resolution in the relevant area. The maximum cell capacity is the lesser of that which exceeds the maximum cost or mass with the minimum cell count.

SOC windows are divided evenly between the maximum and minimum SOC windows.

This method of generating the design space results in many areas which will not meet the design constraints. To reduce computation time each battery is assessed against the constraints in each category. If a battery is found to exceed the constraints its evaluation is stopped and the evaluation of larger battery of that category is avoided. For example, if a battery with 120 cells of 15Ah is found to exceed the maximum mass, each battery with 120 cells and more than 15Ah is not considered.

6.3 Evaluation Methods

6.3.1 Battery Cost

Battery cost is a function of the number of cells required, the capacity of each cell, and the number of battery replacements required over the car’s lifetime. The equation is given below:

\[
Cost = \frac{L_{\text{car}}}{L_{\text{battery}}} \times n_{\text{cells}} \times (\text{cost}_{\text{cell}} + \text{cost}_{\text{Ah}} \times \text{capacity})
\]

(6.1)

\[L_{\text{car}}\] The expected life of the car, in years
\[L_{\text{battery}}\] The expected life of the battery, in years
\[n_{\text{cells}}\] The number of cells in the battery
\[\text{cost}_{\text{cell}}\] The cost per cell, neglecting capacity
\[\text{cost}_{\text{Ah}}\] The cost of 1Ah of capacity in a cell
\[\text{capacity}\] The nominal capacity of the battery

Using this equation gives greater accuracy than assuming a set cost per kWh because it can account for the additional battery management equipment required to balance the charge across every cell. A battery using fewer, higher capacity batteries will be less expensive than a higher celled battery of equivalent capacity. The values used for cost per cell and cost per Ah are given below:

<table>
<thead>
<tr>
<th>Batteries</th>
<th>Cost per cell ($)</th>
<th>Cost per Ah ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lithium Iron Phosphate</td>
<td>0.76</td>
<td>2.95</td>
</tr>
<tr>
<td>Lithium NCM [66], [67]</td>
<td>1.00</td>
<td>3.00</td>
</tr>
<tr>
<td>Nickel Cadmium [68]</td>
<td>0.94</td>
<td>1.27</td>
</tr>
<tr>
<td>Nickel Metal Hydride [69]</td>
<td>0.50</td>
<td>0.98</td>
</tr>
</tbody>
</table>
The method for determining battery life is given in section 6.2.4.

### 6.3.2 All-Electric Range

All-electric range is a function of the battery efficiency, battery capacity, and vehicle mass. As the battery capacity, and by extension the battery mass, increases the power required increases, leading to higher losses and a less efficient driving cycle. This efficiency is determined using a timestep method over a sample duty cycle. The load on the battery for each timestep is determined using the equation:

\[
P = \begin{cases} 
  P_{\text{charge}} & \text{if } P_{\text{demand}} > P_{\text{charge}} \\
  -P_{\text{AR}} - P_{\text{RR}} - ma & \text{if } P_{\text{charge}} > P_{\text{demand}} > P_{\text{discharge}} \\
  P_{\text{discharge}} & \text{if } P_{\text{demand}} < P_{\text{discharge}} 
\end{cases}
\]  

(6.2)

Where:

- \( P \) Load across the battery
- \( P_{\text{charge}} \) Maximum charging power of the battery
- \( P_{\text{AR}} \) Power required to overcome air resistance
- \( P_{\text{RR}} \) Power required to overcome rolling resistance
- \( P_{\text{discharge}} \) Maximum output power of the battery
- \( P_{\text{demand}} \) Power required to follow the duty cycle on electric power

The energy efficiency of the duty cycle in can be found by dividing the total energy expenditure by the distance travelled. Multiplying this energy efficiency by the duty cycle efficiency of the battery yields the range per kWh of usable capacity. The relationship between mass and specific range is shown below:
This relationship is linearized over the range of battery masses in the design space for each optimization. It can be adjusted by adjusting the coefficients of drag and rolling resistance in vehicle characteristics. The all-electric range can then be calculated using the equation:

$$ AER = n_{cells} \times c_{cell} \times V_{cell} \times SOC_{window} \times \text{efficiency}_{cycle} $$  \hspace{1cm} (6.3)

- **AER**: All-electric range, in km
- **$n_{cells}$**: Number of cells in the battery
- **$c_{cell}$**: Capacity of the battery, in Ah
- **$V_{cell}$**: Single cell voltage
- **$SOC_{window}$**: Percentage of the battery capacity available for use
- **$\text{efficiency}_{cycle}$**: Specific range, accounting for battery duty cycle efficiency and automobile mass

This method can be verified by comparing it against current vehicles. Table 10 shows the US EPA rated electricity consumption of different EVs and PHEVs, the vehicle’s mass, and the model predicted cycle efficiency based on Figure 18 [73].

---

**Figure 20: Specific all-electric range for different vehicle masses**

---
Table 10: Predicted and actual specific ranges of electric vehicles

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Mass (kg)</th>
<th>EPA Range (km/kWh)</th>
<th>Model Range (km/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013 Chevrolet Volt</td>
<td>1715</td>
<td>4.57</td>
<td>5.52</td>
</tr>
<tr>
<td>2013 Nissan Leaf</td>
<td>1500</td>
<td>5.52</td>
<td>6.25</td>
</tr>
<tr>
<td>2013 Tesla Model S</td>
<td>2110</td>
<td>4.44</td>
<td>4.53</td>
</tr>
<tr>
<td>2013 Toyota Prius PHEV</td>
<td>1420</td>
<td>5.52</td>
<td>6.57</td>
</tr>
</tbody>
</table>

The ranges predicted by the model are higher than those given by the EPA. This can be attributed to differences in the driving cycle used for evaluation. The EPA tests include high speed driving (>90km/h) which is not found in the model duty cycle. The EPA also tests with the air conditioner and heater in use, these factors are not considered in this optimization [74].

Payload mass is not considered in this analysis. In actual operation the range will be lower; assuming a 100kg payload the modeled ranged decreases by 0.18km/kWh for the Tesla Model S to 0.35km/kWh for the Toyota Prius PHEV.

6.3.3 Maximum Speed and Acceleration

Maximum speed and acceleration are limited by the maximum discharge rate of the battery pack.

This rate is found using the equation:

$$P_{max} = I_{max} * V$$  \hspace{1cm} (6.4)

Where:

- \( P_{max} \) Maximum output power
- \( I_{max} \) Maximum output current
- \( V \) Battery voltage

This can be compared to the losses due to drag on the car from rolling and air resistance, the equations for which are shown below:

$$P_{air \ resistance} = \frac{1}{2} * \rho * V^3 * C_d * A_f$$  \hspace{1cm} (6.5)

$$P_{rolling \ resistance} = M * C_{rr} * V$$  \hspace{1cm} (6.6)
\[ p_{\text{max}} = \frac{1}{2} \rho V^3 c_d A_f + MC_{rr} V \]  

\[ \rho \]  
Air density in kg/m\(^3\)

\[ V \]  
Vehicle speed in m/s

\[ c_d \]  
Coefficient of drag

\[ A_f \]  
Frontal drag area in m\(^2\)

\[ C_{rr} \]  
Coefficient of rolling resistance

\[ M \]  
Total vehicle mass (empty weight + battery)

The maximum current for each battery type is taken from the testing results. The maximum power at the bottom of the SOC window is used to ensure even performance across all SOCs when the car is in use. The maximum speed is then found using the equation:

Acceleration capability is determined at 5km/h intervals from zero to the maximum speed. In addition to battery output it is limited by the maximum traction of the car’s drive tires. The maximum acceleration without wheelspin can be found with the equation:

\[ a_{\text{max}} = C_{f,tire} \times 4.9 \]  

\[ C_{f,tire} \]  
Coefficient of friction of the tires

\[ a_{\text{max}} \]  
Maximum acceleration in m/s\(^2\)

The acceleration at each speed step is found using the equation:
\[ a = \begin{cases} \frac{a_{\text{max}}}{(P_{\text{max}} - P_{\text{AR}} - P_{\text{RR}})/(m \cdot V)} & a > a_{\text{max}} \\ a < a_{\text{max}} & \end{cases} \] (6.9)

- \( a \) Vehicle acceleration in m/s\(^2\)
- \( a_{\text{max}} \) Maximum vehicle acceleration in m/s\(^2\)
- \( P_{\text{max}} \) Maximum output power
- \( P_{\text{AR}} \) Power required to overcome air resistance
- \( P_{\text{RR}} \) Power required to overcome rolling resistance
- \( m \) Total vehicle mass
- \( V \) Vehicle speed in m/s

This acceleration model assumes the weight of the car is evenly distributed across all four wheels and that the car is two wheel drive. It also neglects any input from the combustion motor making it applicable to parallel hybrids only.

### 6.3.4 Battery Lifetime

Battery life is estimated using a variant of the method outlined in section 3.3 adapted for hybrid vehicles. The algorithm for determining cycle life is outlined below:

1. Determine the current draw (in amperes) across the battery for every time step of the duty cycle. Limit high discharge and charge currents to the specified maximum.
2. Determine the nominal change in charge for each time step.
3. Determine the actual change in charge for each time step, adjusting for capacity reduction at high currents.
4. Determine the effective C-rate over the cycle by dividing the net effective discharge by the time spent discharging.
5. Calculate the effective charge life impact using the method outlined section 2.3.

This method assumes that the vehicle will be fully charged at the end of each daily cycle. It accounts for the increased damage caused by high currents by simulating capacity suppression at high currents; higher currents cause greater effective discharges and thus higher effective c-rates and depths-of-discharge.

The driving cycle used is based on data collected from weekday drivers in Winnipeg. It consists of 6 trips totaling 37km. It is shown with rest gaps removed in Figure 21:
The driving cycle results in 50.5 minutes of discharge daily, assuming sufficient battery capacity. The daily gross discharge ($D_A$) (adjusted for capacity reduction) is divided by this to determine the daily effective discharge rate ($C_A$). The daily gross discharge and effective discharge rate are used to determine the charge life decrease using the equation:

$$d_{cycle} = \left( \frac{D_A}{D_R} \right)^{u_0} e^{u_1 \left( \frac{D_A}{D_R} - 1 \right)} \left( \frac{C_R}{C_A} \right)^{v_0} e^{v_1 \left( \frac{C_R}{C_A} - 1 \right)}$$  \hspace{1cm} (6.10)

- $d_{cycle}$: The charge life decrease for one duty cycle
- $D_R$: The rated discharge capacity of the battery, in Ah
- $C_R$: The rated discharge current of the battery, in C
- $C_A$: The actual discharge rate of the battery, in C
- $D_A$: The actual discharge capacity of the battery, in Ah
- $v_0, v_1$: Coefficients
- $u_2, u_1, u_0$: Coefficients

A calendar degradation term is added to the cycle degradation term described above. This assumes a linear reduction in cycle life as a percentage of nominal capacity per year. It is assumed that the battery is never stored for a long period of time at a low state-of-charge or outside of normal operating temperatures, both of which would accelerate degradation.
The effective discharge due to calendar degradation

\[ d_{calendar} = \frac{calendar \ rate}{365} \times \Gamma_R \]  

\( d_{calendar} \)  The effective discharge due to calendar degradation
\( calendar \ rate \)  The rate of degradation independent of use, in %/day
\( \Gamma_R \)  The charge life of the battery, in Ah

The sum of the two degradation terms (\( d_{cycle} \) and \( d_{calendar} \)) is the effective charge life decrease for one day of use. The effective lifespan can then be calculated using the equation:

\[ L = \frac{L_R D_R C_R}{d_{cycle} + d_{calendar}} \]  

\( L \)  Battery lifetime, in days
\( L_R \)  The rated cycle life of the battery, in cycles
\( D_R \)  The rated discharge capacity of the battery, in Ah
\( C_R \)  The rated discharge current of the battery, in C
\( d_{cycle} \)  The charge life decrease for one duty cycle
\( d_{calendar} \)  The effective discharge due to calendar degradation

\( L \) is the expected cycle life of the battery; dividing it by 365 yields the battery lifetime in years. This does not account for the difference in driving patterns between weekdays, weekends, and holidays.

### 6.3.5 Lifetime CO₂ Emissions

Lifetime CO₂ emissions (EM) are a combination of the cradle-to-gate emissions discussed in section 2.4, direct emissions from the combustion engines, and indirect emissions from grid-sourced electricity.

The cradle-to-gate emissions are based on the size of the battery and the emissions intensity of the particular battery type. In the case of the battery needing replacement before the end of the car’s intended lifespan additional cradle-to-gate emissions will be incurred. The total emissions for over the car’s lifetime are given by the equation:
The total cradle-to-gate emissions of the battery, \( EM_{ctg} \) can be calculated using the equation:

\[
EM_{ctg} = \frac{L_{car}}{L_{battery}} \times em_{chemistry} \times C_{battery} \tag{6.13}
\]

- \( EM_{ctg} \): The total cradle-to-gate emissions of the battery
- \( L_{car} \): The expected lifetime of the car
- \( L_{battery} \): The expected lifetime of the battery
- \( em_{chemistry} \): The specific emissions from production for the particular battery chemistry, in kg/Wh
- \( C_{battery} \): The capacity of the battery, in Wh

Emissions from grid-sourced electricity are based on the specific emissions intensity of the electrical grid being considered. For purposes of this optimization, Ontario’s electricity generation profile was used with an emissions intensity of 100gCO\(_2\)eq/kWh. The daily electricity consumed is based on an average day of driving following the cycle given in section 6.2.4 and the cycle energy efficiency found in section 6.2.1. It can be calculated using the equation:

\[
EM_{electricity} = \begin{cases} 
em_{grid} \times AER \times efficiency_{cycle} & \text{AER} < 37 \\
em_{grid} \times 37 \times efficiency_{cycle} & \text{AER} > 37 
\end{cases} \tag{6.14}
\]

- \( EM_{electricity} \): Total emissions from electricity production for one duty cycle
- \( efficiency_{cycle} \): Specific energy consumption of the duty cycle-battery combination, in kWh/km
- \( em_{grid} \): Emissions intensity of the grid, in kg/kWh
- \( AER \): All-electric range, in km

Direct combustion emissions are generated when the car’s speed exceeds the maximum all-electric speed of the car or if the daily driving distance exceeds the car’s all-electric range. In these scenarios the energy is sourced entirely from gasoline at an emissions intensity of 272.9gCO\(_2\)eq/MJ as described in section 2.4. The combustion emissions can then be calculated with the equation:

\[
\]
\[ EM_{\text{combustion}} = 272.9 \]  

\[ \sum \left( \text{efficiency}_{\text{cycle}} \times (37 - \text{AER}) + \sum_{P>P_{\text{discharge}}} (P_{\text{AR}} + P_{\text{RR}} + ma) \times t \right) \]  

- \( EM_{\text{combustion}} \): Total emissions from combustion for one duty cycle
- \( \text{efficiency}_{\text{cycle}} \): Specific energy consumption of the duty cycle, in MJ/km
- \( \text{em}_{\text{grid}} \): Emissions intensity of the grid, in kg/kWh
- \( \text{AER} \): All-electric range, in km
- \( P_{\text{AR}} \): Power required to overcome air resistance
- \( P_{\text{RR}} \): Power required to overcome rolling resistance

The total life cycle emissions are then found using the equation:

\[ EM = EM_{\text{ct,g}} + EM_{\text{electricity}} + EM_{\text{combustion}} \]  

### 6.3.6 Charging Time

Charging time is calculated as the time it takes for a battery at the bottom of its SOC window to be fully charged. It is calculated with the equation:

\[ CT = C_{\text{charge}} \times SOC_{\text{window}} \]  

- \( CT \): Charging time
- \( C_{\text{charge}} \): Maximum charging rate, in C
- \( SOC_{\text{window}} \): Percentage of battery capacity available for use

This equation neglects the constant-voltage charging phase used on lithium and lead based batteries. From the testing performed this charging phase is responsible for approximately 5% of the total charge. As a result, this equation is accurate for SOC windows of 95% and under. As all currently available PHEVs use an SOC window of less than 90% this is appropriate for the likely design space. If the constant voltage charge is required it will increase the charging time from that predicted.
6.4 Objective Function

The goal of the optimization is to maximize the value of the objective function while remaining within the defined constraints. The objective function is adapted from prospect theory, a model of how humans evaluate decisions [75]. This function summarizes all the performance characteristics into a single value – referred to as fitness. The function can be represented as:

\[
Fitness = \sum W \cdot \tanh \left( \frac{S \cdot (P_a - P_r)}{P_r} \right) + 1
\]  

(6.18)

**Fitness** Measure of the battery’s effectiveness in meeting the design goals

- **W** Category weight
- **S** Category sensitivity
- **P_a** Actual performance
- **P_r** Reference performance

The variable term in this equation is the fractional difference between the actual performance and the reference, or desired, performance.

The weighting term allows the importance of different goals to be adjusted independently. Figure 22 shows the shape of the optimization function at different weights:

![Figure 22: Fitness function variation with weight](image-url)
Negative weights are used for performance categories in which higher values indicate poor performance (e.g., cost). Higher absolute values of weights are given to categories which are more important to the design of the car.

The sensitivity term dictates how much of an effect incremental improvements will have. The graph below shows the shape of the optimization function at different sensitivities:

![Graph showing fitness function variation with sensitivity]

**Figure 23: Fitness function variation with sensitivity**

Higher sensitivities result in decreasing marginal gains in fitness as performance increases. This represents categories in which small gains are significant but larger gains do not return the same improvement. For example, increasing the maximum all-electric speed of a car from 50km/h to 60km/h greatly expands its capabilities; increasing the all-electric speed from 120km/h to 130km/h does not have the same effect.

The fitness is calculated for every potential battery. If the fitness is greater than that of every battery of that chemistry yet evaluated the battery properties are saved. When the design space has been fully evaluated the batteries with the highest fitness are written to an output file.

### 6.5 Sample Results

Three sample optimizations were performed using the tool: a small urban vehicle based on the Toyota Prius, a lightweight sports car based on the Lotus Elise, and a utility truck based on the Ford F150. The vehicle characteristics and design goals are adjusted for each scenario as described in the following subsections.
### 6.5.1 Compact car

This scenario used a small economy car as its baseline. As the majority of PHEVs on the market fall into this category it allows the results to be verified against commercially available vehicles. Table 11 provides the vehicle characteristics and constraints used in this optimization.

#### Table 11: Vehicle characteristics and constraints used for compact car optimization

<table>
<thead>
<tr>
<th>Vehicle Characteristics</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Mass (excl. battery)</td>
<td>1100 kg</td>
<td></td>
</tr>
<tr>
<td>Drag Coefficient</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Frontal Area</td>
<td>2.2 m²</td>
<td></td>
</tr>
<tr>
<td>Tire Coefficient of Friction</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Tire Coefficient of Rolling Res.</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Battery Constraints</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>100 V</td>
<td>350 V</td>
</tr>
<tr>
<td>Volume</td>
<td>0 L</td>
<td>250 L</td>
</tr>
<tr>
<td>Mass</td>
<td>100 kg</td>
<td>500 kg</td>
</tr>
<tr>
<td>Cost</td>
<td>1000 $</td>
<td>12000 $</td>
</tr>
<tr>
<td>SOC window</td>
<td>10 %</td>
<td>80 %</td>
</tr>
<tr>
<td>Output power</td>
<td>0 kW</td>
<td>50 kW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance Constraints</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>20 km</td>
<td>100 km</td>
</tr>
<tr>
<td>Acceleration 1</td>
<td>3 s</td>
<td>12 s</td>
</tr>
<tr>
<td>Acceleration 2</td>
<td>3 s</td>
<td>12 s</td>
</tr>
<tr>
<td>Max all electric speed</td>
<td>90 km/h</td>
<td>175 km/h</td>
</tr>
</tbody>
</table>

Table 12 shows the sensitivities, weights, and reference values used in the optimization. Battery life is given the highest weight; all-electric range and cost are also weighted highly. This is done to reflect
common concerns with electric vehicles. Other performance categories are weighted less, the least important being 80-120km/h acceleration and charging time.

Table 12: Optimization parameters used for compact car optimization

<table>
<thead>
<tr>
<th>Consideration</th>
<th>Weight</th>
<th>Sensitivity</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-electric range (km)</td>
<td>0.5</td>
<td>0.5</td>
<td>15</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>-0.5</td>
<td>0.4</td>
<td>3000</td>
</tr>
<tr>
<td>0-100 acceleration (s)</td>
<td>-0.15</td>
<td>0.6</td>
<td>8</td>
</tr>
<tr>
<td>80-120 acceleration (s)</td>
<td>-0.1</td>
<td>0.4</td>
<td>10</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>0.2</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>Battery life (years)</td>
<td>0.6</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>Lifetime emissions (kg CO2)</td>
<td>-0.05</td>
<td>0.2</td>
<td>8000</td>
</tr>
<tr>
<td>Charging time (min)</td>
<td>-0.1</td>
<td>0.4</td>
<td>210</td>
</tr>
</tbody>
</table>

Table 13 shows the ideal cell count, cell size, and SOC window for each battery chemistry:

Table 13: Optimal batteries and fitnesses from the compact car optimization

<table>
<thead>
<tr>
<th>Chemistry</th>
<th>Cell Count</th>
<th>Cell Capacity (Ah)</th>
<th>SOC Window</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>23</td>
<td>110.7</td>
<td>80%</td>
<td>0.90</td>
</tr>
<tr>
<td>NiMH</td>
<td>48</td>
<td>124.9</td>
<td>80%</td>
<td>0.81</td>
</tr>
<tr>
<td>NiCd</td>
<td>88</td>
<td>12.56</td>
<td>80%</td>
<td>0.75</td>
</tr>
<tr>
<td>Lead Acid</td>
<td>32</td>
<td>18.93</td>
<td>80%</td>
<td>0.74</td>
</tr>
<tr>
<td>Spiral Lead Acid</td>
<td>50</td>
<td>11.35</td>
<td>80%</td>
<td>0.74</td>
</tr>
<tr>
<td>Lead Tin</td>
<td>75</td>
<td>15.94</td>
<td>80%</td>
<td>0.75</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>38</td>
<td>59.93</td>
<td>80%</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 13 indicates that lithium iron phosphate is the most desirable chemistry for this application. Of the seven cases presented three (NiCd, lead acid, spiral lead acid) returned batteries with
capacities near 1kWh while two (LiFePO4 and Li-NCM) resulted in batteries with capacities near 8kWh. This suggests that the design space is multimodal with one peak representing low cost-low performance batteries and another for high cost-high performance batteries. The performance of four of the batteries is given in Table 14 below. The spiral lead acid figures are similar to those of the batteries not represented.

| Table 14: Predicted performance of an optimal electrified compact car |
|---------------------------------|---------|---------|---------|---------|
| All-electric range (km)         | LiFePO4 | NiMH    | Spiral LA | Li-NCM |
| Cost ($)                        | $7527.52 | $5922.35 | $1238.55 | $6870.02 |
| Maximum Speed (km/h)            | 175      | 115     | 60       | 160     |
| 0-100km/h (s)                   | 12.7     | 41.3    | N/A      | 16.6    |
| 80-120 km/h (s)                 | 12.4     | N/A     | N/A      | 18.0    |
| Battery life (years)            | 7.36     | 5.26    | 3.57     | 4.81    |
| Lifetime emissions (tCO2eq)     | 408.6    | 1078    | 2880     | 483.7   |
| Charging time (min)             | 50.4     | 624     | 11.4     | 50.5    |

The decrease in lifetime emissions as electric range increases is evident in these results. Table 15 shows the breakdown of emissions by source for each of the batteries considered:

| Table 15: Greenhouse gas emissions by source for optimal electrified compact cars (tCO2eq) |
|---------------------------------|---------|---------|---------|
|                                  | LiFePO4 | NiMH    | Spiral LA | Li-NCM |
| Production                       | 4.2     | 6       | 0.8      | 5.0    |
| Electricity                      | 404.4   | 312.7   | 60.7     | 478.6  |
| Combustion                       | 0       | 759.7   | 2818     | 0      |
| TOTAL                            | 408.6   | 1078.4  | 2879.5   | 483.6  |

The values in Table 15 correspond to the total emissions over the ten year life of the vehicle assuming 37 kilometres of daily driving (135,000km overall) with battery replacements as necessary.
Combustion emissions are the most significant contribution where present while production emissions are negligible.

6.5.2 Pickup Truck

This scenario considers a large, heavy vehicle similar to a Ford F150. There are currently no electric vehicles available in this market. However, commercial trucks are strong candidates for future electrification because of their predictable usage, ample recharging time, and high torque requirements. Table 16 shows the vehicle properties in this optimization:

**Table 16: Vehicle characteristics and constraints used for truck optimization**

<table>
<thead>
<tr>
<th>Vehicle Characteristics</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Cost</td>
<td>45000</td>
<td>$</td>
</tr>
<tr>
<td>Vehicle Mass (excl. battery)</td>
<td>2125</td>
<td>kg</td>
</tr>
<tr>
<td>Drag Coefficient</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>Frontal Area</td>
<td>3</td>
<td>m²</td>
</tr>
<tr>
<td>Tire Coefficient of Friction</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Tire Coefficient of Rolling Res.</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Battery Constraints</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Voltage</td>
<td>150</td>
<td>500</td>
</tr>
<tr>
<td>Volume</td>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>Mass</td>
<td>150</td>
<td>750</td>
</tr>
<tr>
<td>Cost</td>
<td>2000</td>
<td>15000</td>
</tr>
<tr>
<td>SOC Window</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>Output Power</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>Performance Constraints</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Range</td>
<td>25</td>
<td>200</td>
</tr>
<tr>
<td>Acceleration 1</td>
<td>5</td>
<td>110</td>
</tr>
<tr>
<td>Acceleration 2</td>
<td>5</td>
<td>110</td>
</tr>
<tr>
<td>Max AE Speed</td>
<td>35</td>
<td>175</td>
</tr>
</tbody>
</table>

Table 17 shows the optimization parameters. The weights and sensitivities are largely unchanged from the previous optimization (range is weighted slightly less). The reference values are set towards a larger battery. The target range has a greater proportional increase than cost indicating a shift in preference towards less expensive options.
Table 17: Optimization parameters used for truck optimization

<table>
<thead>
<tr>
<th>Consideration</th>
<th>Weight</th>
<th>Sensitivity</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-electric range (km)</td>
<td>0.45</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>-0.5</td>
<td>0.4</td>
<td>7500</td>
</tr>
<tr>
<td>0-100 acceleration (s)</td>
<td>-0.15</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td>80-120 acceleration (s)</td>
<td>-0.1</td>
<td>0.4</td>
<td>8</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>0.2</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>Battery life (years)</td>
<td>0.6</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>Lifetime emissions (t CO₂eq)</td>
<td>-0.05</td>
<td>0.2</td>
<td>700</td>
</tr>
<tr>
<td>Charging time (min)</td>
<td>-0.1</td>
<td>0.4</td>
<td>210</td>
</tr>
</tbody>
</table>

Table 18 shows the optimal batteries:

Table 18: Optimal batteries and fitness from the truck optimization

<table>
<thead>
<tr>
<th></th>
<th>Cell Count</th>
<th>Cell Capacity (Ah)</th>
<th>SOC Window</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>38</td>
<td>48.54</td>
<td>80%</td>
<td>0.67</td>
</tr>
<tr>
<td>NiMH</td>
<td>53</td>
<td>117.73</td>
<td>80%</td>
<td>0.61</td>
</tr>
<tr>
<td>NiCd</td>
<td>90</td>
<td>75.81</td>
<td>80%</td>
<td>0.54</td>
</tr>
<tr>
<td>Lead Acid</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>Spiral LA</td>
<td>65</td>
<td>62.88</td>
<td>80%</td>
<td>0.52</td>
</tr>
<tr>
<td>Lead Tin</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>43</td>
<td>39.00</td>
<td>80%</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Table 19 shows the performance of each battery:

Table 19: Predicted performance of optimal electrified truck

<table>
<thead>
<tr>
<th></th>
<th>LiFePO4</th>
<th>NiMH</th>
<th>NiCd</th>
<th>Spiral LA</th>
<th>Li-NCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-electric range (km)</td>
<td>25.0</td>
<td>25.0</td>
<td>25.1</td>
<td>25.0</td>
<td>25.1</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>$5470</td>
<td>$6165</td>
<td>$8710</td>
<td>$9041</td>
<td>$5074</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>95.0</td>
<td>75.0</td>
<td>50.0</td>
<td>70.0</td>
<td>90.0</td>
</tr>
<tr>
<td>Battery life (years)</td>
<td>6.1</td>
<td>4.8</td>
<td>5.2</td>
<td>4.3</td>
<td>3.7</td>
</tr>
<tr>
<td>Lifetime emissions (tCO$_2$eq)</td>
<td>755.2</td>
<td>757.2</td>
<td>730.4</td>
<td>615.4</td>
<td>764.8</td>
</tr>
<tr>
<td>Charging time (minutes)</td>
<td>36.5</td>
<td>594.8</td>
<td>435.4</td>
<td>62.9</td>
<td>37.2</td>
</tr>
</tbody>
</table>

Each optimal battery delivers 25km of all-electric range (the bottom of the constraint window) indicating that a smaller, less expensive battery is preferable. This result also suggests that trucks are poor candidates for electrification.

None of the batteries presented are capable of reaching 100km/h, as a result the acceleration times are not applicable.

Counter-intuitively, the lithium-ion batteries are less expensive than those of the traditionally less expensive chemistries. This is a result of the higher performance of lithium-ion batteries requiring less capacity to meet the minimum range because of their improved duty cycle efficiency (see section 4.2).

This optimization uses a duty cycle developed from daily drivers rather than commercial users. This may result in some inaccuracy in the performance prediction, particularly in terms of range and battery life.

### 6.5.3 Sports Car

This case uses the Lotus Elise, a lightweight sports car, as its vehicle model. The Elise was also used as the basis of the Tesla Roadster. Table 20 shows the vehicle properties for the optimization.
Table 20: Vehicle characteristics and constraints used for sports car optimization

<table>
<thead>
<tr>
<th>Vehicle Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Mass (excl. battery)</td>
<td>910 kg</td>
</tr>
<tr>
<td>Drag Coefficient</td>
<td>0.29</td>
</tr>
<tr>
<td>Frontal Area</td>
<td>2 m²</td>
</tr>
<tr>
<td>Tire Coefficient of Friction</td>
<td>0.9</td>
</tr>
<tr>
<td>Tire Coefficient of Rolling Res.</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Battery Constraints</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>200 V</td>
<td>400 V</td>
</tr>
<tr>
<td>Volume</td>
<td>0 L</td>
<td>200 L</td>
</tr>
<tr>
<td>Mass</td>
<td>20 kg</td>
<td>400 kg</td>
</tr>
<tr>
<td>Cost</td>
<td>4000 $</td>
<td>20000 $</td>
</tr>
<tr>
<td>SOC Window</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>Output Power</td>
<td>0 kW</td>
<td>150 kW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance Constraints</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>60</td>
<td>200</td>
</tr>
<tr>
<td>Acceleration 1</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Acceleration 2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Max AE Speed</td>
<td>60</td>
<td>175</td>
</tr>
</tbody>
</table>

This scenario uses higher values for the tire coefficients of friction and rolling resistance to represent high performance tires.

Table 21 shows the fitness function parameters for this optimization. Acceleration is given greater importance than previous optimizations reflecting market desires.
Table 21: Optimization parameters used for sports car optimization

<table>
<thead>
<tr>
<th>Consideration</th>
<th>Weight</th>
<th>Sensitivity</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-electric range (km)</td>
<td>0.4</td>
<td>0.6</td>
<td>100</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>-0.4</td>
<td>0.25</td>
<td>6000</td>
</tr>
<tr>
<td>0-100 acceleration (s)</td>
<td>-0.3</td>
<td>0.6</td>
<td>6</td>
</tr>
<tr>
<td>80-120 acceleration (s)</td>
<td>-0.2</td>
<td>0.4</td>
<td>5</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>0.2</td>
<td>0.6</td>
<td>120</td>
</tr>
<tr>
<td>Battery life (years)</td>
<td>0.5</td>
<td>0.7</td>
<td>10</td>
</tr>
<tr>
<td>Lifetime emissions (kg CO2)</td>
<td>-0.05</td>
<td>0.2</td>
<td>8000</td>
</tr>
<tr>
<td>Charging time (min)</td>
<td>-0.1</td>
<td>0.4</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 22 shows the optimization results:

Table 22: Optimal batteries and fitnesses from the sports car optimization

<table>
<thead>
<tr>
<th></th>
<th>Cell Count</th>
<th>Cell Capacity (Ah)</th>
<th>SOC Window</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>49</td>
<td>136.46</td>
<td>0.8</td>
<td>0.753</td>
</tr>
<tr>
<td>NiMH</td>
<td>128</td>
<td>145.70</td>
<td>0.8</td>
<td>0.645</td>
</tr>
<tr>
<td>NiCd</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Lead Acid</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Spiral LA</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Lead Tin</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>40</td>
<td>166.21</td>
<td>0.8</td>
<td>0.763</td>
</tr>
</tbody>
</table>

The results of this optimization indicate that lead and nickel based batteries do not have the energy or power density to meet the constraints outlined here. Table 23 below shows the performance of the optimal batteries:
Both batteries in this scenario provide similar performance. Both are at the upper limit of the constraint window for cost indicating that larger batteries may perform better. This is supported by the actual battery found in the Tesla Roadster (a 53kWh lithium-ion battery).

### Table 23: Predicted performance of optimal electrified sports car

<table>
<thead>
<tr>
<th></th>
<th>LiFePO4</th>
<th>NiMH</th>
<th>Li-NCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-electric range (km)</td>
<td>94.4</td>
<td>78.8</td>
<td>108.6</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>$19,762.76</td>
<td>$18,397.43</td>
<td>$19,984.60</td>
</tr>
<tr>
<td>0-100km/h (s)</td>
<td>6.1</td>
<td>9.2</td>
<td>6.1</td>
</tr>
<tr>
<td>80-120 km/h (s)</td>
<td>3.4</td>
<td>8.7</td>
<td>3.6</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>175.0</td>
<td>175</td>
<td>175.0</td>
</tr>
<tr>
<td>Battery life (years)</td>
<td>9.2</td>
<td>8.1</td>
<td>8.2</td>
</tr>
<tr>
<td>Lifetime emissions (kgCO2eq)</td>
<td>12005.7</td>
<td>16638.6</td>
<td>11493.4</td>
</tr>
<tr>
<td>Charging time (minutes)</td>
<td>132.4</td>
<td>728.5</td>
<td>147.6</td>
</tr>
</tbody>
</table>
Chapter 7

Conclusion

This project sought to evaluate current commercially available batteries for suitability for use in PHEVs and to develop a method of determining the optimum battery for a given vehicle. Both of these goals have been achieved; eight battery samples have been evaluated and the resulting information used to inform an optimization program. The results of this optimization closely match the current trend in the design of PHEVs, suggesting that the program is accurate.

The effects of low temperature operation are greater than was initially expected. With few exceptions, batteries had greatly reduced capacities, power outputs, and cycle lives and an increased internal resistance when operating at low temperatures. The low performance and accelerated aging suggest that many manufacturers overstate the operating window of their batteries. This presents an issue for PHEVs in operation in colder climates where maintaining battery operating temperature is more important and will require more energy.

The optimization tool is capable of determining the optimal battery for a vehicle and its expected performance on a timescale of minutes. This allows designers to quickly integrate new technology and information into their decision making and to easily perform sensitivity analyses regarding design goals and battery properties. Ultimately, however, the results of the optimization are based on the user’s opinion of the importance of different performance criteria.

The evaluations presented account for only the battery portion of the electric powertrain and ignore the battery management systems and other power electronics necessary in a PHEV. While modern battery management systems have a negligible mass and volume in comparison to the battery itself, their cost is significant [76], particularly for lithium-ion batteries. The cost scales more steeply with voltage than capacity, favoring high capacity, low voltage batteries. However, this setup leads to higher currents and thus higher losses in the system. This balance is not addressed by the current optimization program.

Similarly, the optimization ignores thermal management of the battery packs. Adding a thermal management system will increase the cost, weight, and volume of the battery pack. This is particularly important for lithium battery chemistries for which thermal runaway is a significant concern.
Chapter 8

Recommendations

The use of small battery samples neglects the effects of thermal management, cell balancing, and mass addition found in full-scale battery packs. Furthermore, the variable sizes and capabilities of the battery samples may have led to a degree of inaccuracy in the results. Small battery samples were justified for use in this study for the purpose of developing the optimization protocol. However, for more accuracy in application, testing should ideally be performed using equivalent battery packs scaled from the predicted size required for a vehicle.

The cycle life estimates used are based on purely mathematical models and have not been experimentally verified. Given sufficient resources, cyclic testing should be performed to verify the accuracy of the model and determine the proper aging coefficients. This would allow for more accurate and confident estimates of battery life.

The cycle life estimates presented here indicate that automotive batteries will likely reach their end-of-life before the car itself. This reveals a need to determine an end-of-life scenario before vehicles are brought to market. An effective used battery management program would reduce consumer anxiety when considering electric vehicles and potentially lower the environmental footprint of the automobile.

The optimization currently does not allow for complex control algorithms controlling the use of the internal combustion engine to supplement electric power under normal use. In practice, the combustion and electric motors could be used in series to reduce the strain on the battery while operating the engine at peak efficiency. This would result in increased cycle life, faster acceleration, and greater top speeds than currently predicted.

Low temperature testing reveals greatly accelerated failure in several battery samples. This indicates that temperature plays an important role in battery aging, a factor which is not considered in battery life estimates. While proper thermal management can mitigate this effect, drivers in cold climates are likely to demand use of their vehicle before working temperature can be reached. Understanding how temperatures (particularly low temperatures) affect battery aging is necessary to accurately predict battery life in these conditions.

Many of the tests were not repeated due to practical limits on the time and resources available. When performed (such as on 2C low temperature discharges) the results were found to be highly similar. Additional testing repetition, ideally performed on multiple samples of each chemistry, would be
required to ensure the accuracy of this protocol. However, the testing performed was sufficient to inform the optimization procedure.

The optimization tool presented assumes the automobile is driven 37km per day. In reality, driving distances vary significantly between drivers and for the same driver on different days. Therefore, even a well-optimized car will only be optimal for a small portion of drivers. This could be improved by automakers providing several options for the battery. Of currently available electric vehicles only the Tesla Model S offers this capability. Offering multiple batteries for each vehicle would greatly increase their appeal for a minimal investment in design capital.
Works Cited


[37] T. Hatchard and J. Dahn, "In situ XRD and electrochemical study of the reaction of lithium with


[50] E. Tara, S. Shahidinejad, S. Filizadeh and E. Bibeau, "Battery Storage Sizing in a Retrofitted Plug-


63 D. Panihrahi, C. Chiasserini, S. Dey, R. Rao, A. Raghnathan and K. Lahiri, "Battery life


Appendix A
Sample Calculations

A.1 Wire Loss Calculations
The power loss in the leads was calculated using American Wire Gauge resistance calculations and electrical relationships. The wire resistance is calculated below.

\[ R_{1000} = 10^{\frac{AWG-10}{10}} \]
\[ R_{1000}' = 10^{\frac{2-10}{10}} \]
\[ R_{total} = \frac{0.15848\Omega}{1000} \times 12' = 0.0019\Omega \]

Using \( P = I^2R \) the wire losses under full load are calculated below.

\[ P_{loss} = I^2R_{total} \]
\[ P_{loss} = 100^2 \times 0.0019 \]
\[ P_{loss} = 19W \]

A.2 Cycle Life Calculation
The cycle life of a battery was evaluated according to the method in section 6.3.4. The numbers presented below are for a 100V, 27.5Ah battery with a rated discharge current \((C_R)\) of 1/3C, aging terms \(u_0\) and \(u_1\) of 2 and -1.6, rate capacity effect of 4%/C, and calendar degradation rate of 3%/year.

The charge life, \( \Gamma_R \), was found according to the manufacturer’s information:

\[ \Gamma_R = L_R D_R C_R \]
\[ \Gamma_R = 1250 \times 27.5 \times \frac{1}{3} = 34,375Ah \]

The next step is to evaluate the discharge at each second of the duty cycle. For a timestep with a desired power draw of 2.98kW:

\[ C_A = \frac{P}{V \times D_R} \]
\[ I = \frac{2980}{100 \times 27.5} = 29.8A \]
\[ C_{nominal} = \frac{29.85}{27.5} = 1.08 \]
\[ D_{effective} = 27.5 \times (1 - 0.04 \times 1.08) = 26.3Ah \]
\[ C_A = \frac{1.08}{26.3} = 1.13 \]
Repeating the process for the entire duty cycle gives a total discharge of 18.7Ah with 0.843 hours of discharge. This allows the daily charge life decrease from use to be found as follows:

\[ C_A = \frac{18.7}{0.843} = 0.843 \]

\[ D_A = (\frac{C_R}{C_A})^{\frac{C_R}{C_A} - 1} \cdot D_{nominal} \]

\[ D_A = \left(\frac{1}{3 \cdot 0.843}\right) e^{\frac{1}{3 \cdot 0.843 - 1}} \cdot 18.7 = 10.98Ah \]

Adding this term to the calendar decrease term gives the total daily charge life decrease:

\[ D_{total} = D_A + D_{calendar} \]

\[ D_{total} = 10.98 + \frac{3}{365} \cdot L_R \cdot D_R \cdot 5 \]

\[ D_{total} = 14.12Ah \]

The total cycle life is then found by dividing the charge life by the daily decrease

\[ L = \frac{\Gamma_R}{D_{total}} \]

\[ L = \frac{34375}{14.12} = 2434.5 \text{ days} \]

The battery will last 2435 days or 6.67 years at this level of usage, assuming it is driven daily. At this point it will hold 80% of its original capacity.

### A.3 Fitness Calculation

The fitness of each valid battery is calculated using the method described in section 5.4. Sample values for the fitness function and performance levels are given below:

<table>
<thead>
<tr>
<th>Consideration</th>
<th>Weight</th>
<th>Sensitivity</th>
<th>Reference</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-electric range (km)</td>
<td>0.5</td>
<td>0.5</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>-0.5</td>
<td>0.4</td>
<td>3000</td>
<td>3500</td>
</tr>
<tr>
<td>0-100 acceleration (s)</td>
<td>-0.15</td>
<td>0.6</td>
<td>8</td>
<td>9.3</td>
</tr>
<tr>
<td>80-120 acceleration (s)</td>
<td>-0.1</td>
<td>0.4</td>
<td>10</td>
<td>6.4</td>
</tr>
<tr>
<td>Maximum speed (km/h)</td>
<td>0.2</td>
<td>0.6</td>
<td>100</td>
<td>140</td>
</tr>
</tbody>
</table>
The all-electric range fitness term is found using the equation:

\[ \text{Fitness} = W \cdot \tanh \left( \frac{S \cdot (P_a - P_r)}{P_r} \right) \]

\[ \text{Fitness} = 0.5 \cdot \tanh \left( 0.5 \cdot \frac{18 - 15}{15} \right) = 0.45 \]
Appendix B
Results Summary

B.1 Low Temperature Discharge Capacity

<table>
<thead>
<tr>
<th>Battery</th>
<th>T0</th>
<th>T0</th>
<th>D (Ah)</th>
<th>T0</th>
<th>T0</th>
<th>D (Ah)</th>
<th>Lead Tin</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4 (9.4Ah nominal)</td>
<td>-19.2</td>
<td>14</td>
<td>7.85</td>
<td>-19.2</td>
<td>14</td>
<td>7.85</td>
<td>-17.4, 14.6, 6.113</td>
</tr>
<tr>
<td>NiMH (15Ah nominal)</td>
<td>-17</td>
<td>14.6</td>
<td>8.089</td>
<td>-17</td>
<td>14.6</td>
<td>8.089</td>
<td>-19, 14.7, 6.551</td>
</tr>
<tr>
<td>NiCd (14Ah nominal)</td>
<td>-19.2</td>
<td>14.3</td>
<td>7.759</td>
<td>-19.2</td>
<td>14.3</td>
<td>7.759</td>
<td>-16, 13.6, 6.509</td>
</tr>
<tr>
<td>Li-NCM (17Ah nominal)</td>
<td>-15.5</td>
<td>13.8</td>
<td>7.883</td>
<td>-15.5</td>
<td>13.8</td>
<td>7.883</td>
<td>-17.1, 13.3, 6.445</td>
</tr>
<tr>
<td>NiCd (14Ah nominal)</td>
<td>-18.2</td>
<td>15.9</td>
<td>7.812</td>
<td>-18.2</td>
<td>15.9</td>
<td>7.812</td>
<td>-17.2, 15.6, 2.262</td>
</tr>
</tbody>
</table>

B.2 Low Temperature Power Output

<table>
<thead>
<tr>
<th>Battery</th>
<th>Charge Level</th>
<th>Current (A)</th>
<th>30s Power (W)</th>
<th>Charge Level</th>
<th>Current (A)</th>
<th>30s Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiFePO4</td>
<td>80% SOC</td>
<td>1265</td>
<td></td>
<td>80% SOC</td>
<td>43</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>50% SOC</td>
<td>1229</td>
<td></td>
<td>50% SOC</td>
<td>36.4</td>
<td>333</td>
</tr>
<tr>
<td></td>
<td>20% SOC</td>
<td>1150</td>
<td></td>
<td>20% SOC</td>
<td>31.7</td>
<td>270</td>
</tr>
<tr>
<td>Li-NCM</td>
<td>80% SOC</td>
<td>116</td>
<td>1070</td>
<td>80% SOC</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50% SOC</td>
<td>116</td>
<td>880</td>
<td>50% SOC</td>
<td>78.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20% SOC</td>
<td>78</td>
<td>593</td>
<td>20% SOC</td>
<td>15</td>
<td>95</td>
</tr>
<tr>
<td>NiMH</td>
<td>80% SOC</td>
<td>824</td>
<td></td>
<td>80% SOC</td>
<td>33.5</td>
<td>267</td>
</tr>
<tr>
<td></td>
<td>50% SOC</td>
<td>851</td>
<td></td>
<td>50% SOC</td>
<td>34.8</td>
<td>295</td>
</tr>
<tr>
<td></td>
<td>20% SOC</td>
<td>641</td>
<td></td>
<td>20% SOC</td>
<td>30</td>
<td>253</td>
</tr>
<tr>
<td>NiCd</td>
<td>80% SOC</td>
<td>921</td>
<td></td>
<td>80% SOC</td>
<td>16.7</td>
<td>281</td>
</tr>
<tr>
<td></td>
<td>50% SOC</td>
<td>784</td>
<td></td>
<td>50% SOC</td>
<td>13.7</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>20% SOC</td>
<td>550</td>
<td></td>
<td>20% SOC</td>
<td>13.4</td>
<td>115</td>
</tr>
<tr>
<td>Lead Tin</td>
<td>80% SOC</td>
<td>1319</td>
<td></td>
<td>80% SOC</td>
<td>116</td>
<td>929</td>
</tr>
<tr>
<td></td>
<td>50% SOC</td>
<td>1205</td>
<td></td>
<td>50% SOC</td>
<td>100</td>
<td>867</td>
</tr>
<tr>
<td></td>
<td>20% SOC</td>
<td>1115</td>
<td></td>
<td>20% SOC</td>
<td>62.5</td>
<td></td>
</tr>
<tr>
<td>Lead Acid</td>
<td>80% SOC</td>
<td>1245</td>
<td></td>
<td>80% SOC</td>
<td>0.4</td>
<td>40.6</td>
</tr>
<tr>
<td></td>
<td>50% SOC</td>
<td>1252</td>
<td></td>
<td>50% SOC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spiral Lead Acid</td>
<td>20% SOC</td>
<td>476</td>
<td>20% SOC</td>
<td>80% SOC</td>
<td>50% SOC</td>
<td>20% SOC</td>
</tr>
<tr>
<td>------------------</td>
<td>---------</td>
<td>-----</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>80% SOC</td>
<td>116</td>
<td>1186</td>
<td>80% SOC</td>
<td>116</td>
<td>116</td>
<td>1077</td>
</tr>
<tr>
<td>50% SOC</td>
<td>116</td>
<td>1155</td>
<td>50% SOC</td>
<td>116</td>
<td>116</td>
<td>1073</td>
</tr>
<tr>
<td>20% SOC</td>
<td>116</td>
<td>1106</td>
<td>20% SOC</td>
<td>116</td>
<td></td>
<td>960</td>
</tr>
</tbody>
</table>
Appendix C
MATLAB Code

function [designspace] = spawn(batteryinfo,constraints)

% ROWS
% (1) Voltage
% (2) Volume
% (3) Mass
% (4) Cost
% COLUMNS
% (1) Minimum
% (2) Maximum

designspace = zeros(7,4);

for h = 1:7
    designspace(h,1) = ceil(constraints(1,1)/batteryinfo(h,3)); % min cell count
    designspace(h,3) = (constraints(1,2) - constraints(1,1))/(batteryinfo(h,3)*100); % cell multiplier
    designspace(h,2) = constraints(3,1)/(designspace(h,1)+designspace(h,3)*100)*batteryinfo(h,1)); % min cell capacity
    designspace(h,4) = ((constraints(3,2)/(designspace(h,1)*batteryinfo(h,3)/batteryinfo(h,1)) - designspace(h,2))/100; % cell capacity multiplier
    maxcost = (designspace(h,1) + designspace(h,3)*100)*batteryinfo(h,11) + (designspace(h,2) + designspace(h,4)*100)*batteryinfo(h,12); % max cost
    if maxcost > constraints(4,2)
        designspace(h,2) = (constraints(4,1) - (designspace(h,1)+designspace(h,3)*100*batteryinfo(h,11))/batteryinfo(h,12));
        designspace(h,2) = 0;
    end
    designspace(h,4) = (constraints(4,2) - (designspace(h,2)*batteryinfo(h,11)))/(100*batteryinfo(h,12));
end
    designspace(h,5) = constraints(5,1)/100; % minimum SOC window
    designspace(h,6) = (constraints(5,2)-constraints(5,1))/500; % SOC window multiplier
end

function [AER] = FindAER(cellcount,cellcap,batteryinfo,maxefficiency,efficiencyslope,batmash,SOC)
    efficiency = (maxefficiency + efficiencyslope*battmass)*batteryinfo(h,8);
    AER = cellcap * cellcount * batteryinfo(h,2) * efficiency * SOC /1000;
End
function [zero_hundred,eighty_onetwenty,maxspeed] = 
FindAccel(cellcount,cellcap,batteryinfo,carinfo,battmass,h,constraints) 
speed = 
[0;5;10;15;20;25;30;35;40;45;50;55;60;65;70;75;80;85;90;95;100;105;110; 
115;120;125;130;135;140;145;150;155;160;165;170;175]; 
totalmass = carinfo(2)+battmass; 
maxtraction = 4.9*carinfo(5); %Assume 2WD 
maxpower = batteryinfo(h,2)*batteryinfo(h,3)*cellcount*cellcap; 

goal

zero_hundred = 100; 
eighty_onetwenty = 100; 
if maxpower > constraints(6,2)*1000 
    maxpower = constraints(6,2)*1000; 
end 
for i = 1:36 
    Prr(i) = totalmass*9.8*speed(i)*carinfo(6)/3.6; 
    Par(i) = 0.5*1.2*(speed(i)/3.6)^3*carinfo(4)*carinfo(3); 
    Pnet(i) = maxpower - Prr(i) - Par(i); 
    accel(i) = Pnet(i)/(speed(i)/3.6*totalmass); 
    if accel(i) > maxtraction 
        accel(i) = maxtraction; 
    end 
    if i>=2 
        time(i) = ((speed(i)-speed(i-1))/3.6)/accel(i); 
    end 
    if Pnet(i) <= 0 
        maxspeed = speed(i); 
        break 
    end 
    maxspeed = speed(i); 
end 
if maxspeed > 100 
    zero_hundred = sum(time(1:20)); 
end 
if maxspeed > 120 
    eighty_onetwenty = sum(time(16:24)); 
end 
end 

end 

function [life] = 
cyclelife(powerprofile,ratedDOD,ratedC,u0,u1,u2,dischargeC,chargeC,voltage,cellcap,ratecapacityeffect,SOCwindow,calendarrate) 
capacity = voltage*cellcap; 
maxdischarge = -dischargeC*capacity/voltage; 
maxcharge = chargeC*capacity/voltage; 
houroffdischarge = 0.843; 
desiredcurrentdraw = powerprofile .* 1000 / voltage; %In Amperes 
while min(desiredcurrentdraw) < maxdischarge %Limits discharges to the maximum discharge current
\[ I = \min(\text{desiredcurrentdraw}); \quad \text{\%Find the location of the maximum discharge draw} \]
\[ \text{desiredcurrentdraw}(I) = \max(\text{discharge}) \]
end
while \ max(\text{desiredcurrentdraw}) > \maxcharge \ \%Limits charges to the maximum charge current
\[ I = \max(\text{desiredcurrentdraw}); \quad \text{\%Find the location of the maximum charge current} \]
\[ \text{desiredcurrentdraw}(I) = \maxcharge \]
end
nominalcapacitychange = desiredcurrentdraw ./ 3600; \ %convert to capacity changes at 1second intervals

nominaldischargerate = desiredcurrentdraw ./ cellcap;
while \ max(\text{nominaldischargerate}) > 0
\[ I = \max(\text{nominaldischargerate}); \quad \text{\%Find the location of the maximum charge current} \]
\[ \text{nominaldischargerate}(I) = 0; \]
end

effectivecapacity = cellcap - cellcap * \text{nominaldischargerate} * \text{ratecapacityeffect};
effectivecrate = desiredcurrentdraw ./ effectivecapacity;
depthofdischarge = -\sum(\text{effectivecrate}) / 3600;
if \ depthofdischarge > \text{SOCwindow}
\[ \text{depthofdischarge} = \text{SOCwindow}; \]
end

totalcrate = depthofdischarge / hoursofdischarge;
totaleffectivecapacity = cellcap + cellcap * \text{totalcrate} * \text{ratecapacityeffect};
cycledegradation = \((\text{depthofdischarge}/\text{ratedDOD})^u0)\exp(ul*(\text{depthofdischarge}/\text{ratedDOD}-1))\{\text{ratedC}/-\text{totalcrate}\}*\sum(\text{nominalcapacitychange});
calendardegradation = \text{calendarrate}/365*u2*cellcap*5;
life = u2*cellcap*\text{ratedDOD}/(\text{cycledegradation+calendardegradation})/365; \ %returns cycle life in years, assumes driving every day
end

tic
batteryinfo = xlsread('battery values.xlsx', 'Values', 'B3:P9');  %ROWS
 \%(1) Lithium Iron Phosphate (LiFePO4)
 \%(2) Nickel Metal Hydride (NiMH)
 \%(3) Nickel Cadmium (NiCD)
 \%(4) Lead Tin (PbSn)
 \%(5) Spiral Cell LA
 \%(6) Traditional lead acid

73
constraints = xlsread('Input.xlsx', 'Vehicle Parameters', 'B16:C21');

%ROWS
% (1) Voltage
% (2) Volume
% (3) Mass
% (4) Cost
% (5) SOC Window
% (6) Output Power
%COLUMNS
% (1) Minimum
% (2) Maximum

performancelimits = xlsread('Input.xlsx', 'Vehicle Parameters', 'B24:C26');

%ROWS
% (1) AE Range
% (2) Acceleration 1
% (3) Acceleration 2
% (4) Maximum AE Speed
%COLUMNS
% (1) Minimum
% (2) Maximum

carinfo = xlsread('Input.xlsx', 'Vehicle Parameters', 'B2:B7');
% (1) vehicle cost
% (2) vehicle mass
% (3) drag coefficient
% (4) frontal area
% (5) Tire Cf
% (6) Tire Crr

desiredperformance = xlsread('Input.xlsx', 'Design Parameters', 'B2:D9');
%ROWS
% (1) weighting
% (2) sensitivity
% (3) reference value
% COLUMNS
% (1) desired all-electric range (km)
% (2) desired cost ($) 
% (3) desired 0-100km/h time (all-electric)
% (4) desired 80-120km/h time (all-electric)
% (5) desired maximum speed
% (6) desired vehicle lifetime (years)
% (7) lifetime GHG emissions (kg CO2-eq)
% (8) charging time (minutes)

designspace = spawn(batteryinfo, constraints); %Creates the multipliers
for the design space

powerprofile = xlsread('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', 'Data + Analysis', 'Y4:Y3954'); %Reads the duty cycle

power profile

%Find effect of mass on the efficiency of this particular car
xlswrite('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', 'Vehicle Info', 'B2'); %Writes the drag coefficient to the

Excel file
xlswrite('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', 'Vehicle Info', 'B3'); %Write the frontal area to the Excel

file
for i = 1:6
    xlswrite('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', (constraints(3,2)-constraints(3,1))/5*i, 'Vehicle Info', 'H2'); %Writes

the 0,20,40,60,80,100 percentile battery mass to the Excel file to
spawn data points for the efficiency/mass function

efficiencies(i) = xlsread('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', 'Vehicle Info', 'G18');
energyflux(i) = xlsread('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', 'Vehicle Info', 'H6');
energy(i) = xlsread('Power Analysis Weekday Driving Cycle -
Trimmed.xlsx', 'Vehicle Info', 'E18');
end
maxefficiency = efficiencies(1); %Finds the lowest-mass duty cycle

efficiency (in km/kWh)
efficiencyslope = (efficiencies(5)-efficiencies(1))/(constraints(3,2)-

constraints(3,1)); %Finds the slope of the efficiency vs. mass

relationship
minenergyflux = energyflux(i); %Finds the average power going through

the battery (kWh/km)
energyfluxslope = (energyflux(5)-energyflux(1))/(constraints(3,2)-

constraints(3,1));
minnetenergy = energy(1); %Finds the net energy used over the driving

cycle (kWh)
netenergyslope = (energy(5)-energy(1))/(constraints(3,2)-

constraints(3,1));

%Fitness function loop
% (1) Lithium Iron Phosphate (LiFePO4)
% (2) Nickel Metal Hydride (NiMH)
peakfitness = zeros(1,7);
toc
parfor h = 1:7 %Cell chemistries
    for i = 0:75 %Cell counts
        cellcount = floor(designspace(h,1)+designspace(h,3)*i);
        voltage = cellcount*batteryinfo(h,2);
        for j = 0:75 %Cell capacities
            cellcap = designspace(h,2)+designspace(h,4)*j;
            for k = 4:5 %SOC windows
                %Define SOC window
                SOC = designspace(h,5)+designspace(h,6)*k;

                %Find battery mass
                battmass = cellcount * cellcap * batteryinfo(h,2) / batteryinfo(h,1);

                %Find all-electric range
                AER = FindAER(cellcount,cellcap,batteryinfo, maxefficiency, efficiencyslope, battmass,h,SOC);
                if (AER < performancelimits(1,1)) || (AER > performancelimits(1,2))
                    continue
                end

                %Find fitness based on AER
                AERfitness = desiredperformance(1,1) * tanh(desiredperformance(1,2)*AER/desiredperformance(1,3));

                %Find acceleration times
                [accel1,accel2,maxspeed] = FindAccel(cellcount,cellcap,batteryinfo,carinfo,battmass,h,constraints);
                if (accel1 < performancelimits(2,1)) || (accel1 > performancelimits(2,2)) || (accel1 < performancelimits(3,1)) || (accel1 > performancelimits(3,2))
                    continue
                end

                %Find fitness based on acceleration times and maximum speed
                accell1fitness = desiredperformance(3,1) * tanh(desiredperformance(3,2)*accel1/desiredperformance(3,3));
                accel2fitness = desiredperformance(4,1) * tanh(desiredperformance(4,2)*accel2/desiredperformance(4,3));
                speedfitness = desiredperformance(5,1) * tanh(desiredperformance(5,2)*speed/desiredperformance(5,3));
%Find cycle life
life = cyclelife(powerprofile,batteryinfo(h,12),batteryinfo(h,13),batteryinfo(h,10),batteryinfo(h,11),batteryinfo(h,9),batteryinfo(h,3),batteryinfo(h,5),voltage,cellcap,batteryinfo(h,14),SOC,batteryinfo(h,15));

%Find fitness based on cycle life
cyclelifefitness = desiredperformance(6,1) * tanh(desiredperformance(7,2)*life/desiredperformance(6,3));

%Find battery cost
cost = cellcount*(batteryinfo(h,6) + cellcap * batteryinfo(h,7))*(desiredperformance(6,3)/life);
if cost > constraints(4,2)
    continue
end

%Find fitness based on cost
costfitness = desiredperformance(2,1) * tanh(desiredperformance(2,2)*cost/desiredperformance(2,3));

%Find lifecycle emissions
productionemissions = batteryinfo(h,4) * cellcount * cellcap * batteryinfo(h,2) * ceil(desiredperformance(6,3)/life);
combustionemissions = 0.2729 * ((maxefficiency+efficiencyslope*battmass)*(37-AER))*(desiredperformance(5,3)*365);
if combustionemissions < 0
    combustionemissions = 0;
end
electricityemissions = 0.168 * (1+maxefficiency+efficiencyslope*battmass) /((-minnetenergy-netenergyslope)*battmass) * min(AER,37) * desiredperformance(5,3)*365;
%kg CO2/kWh * total electrical use (motive + losses) / kWh/km * km/lifetime.
emissions = productionemissions+electricityemissions+combustionemissions;

%Find fitness based on emissions
emissionsfitness = desiredperformance(7,1) * tanh(desiredperformance(7,2)*emissions/desiredperformance(7,3));

%Find charging time
chargingtime = cellcount*cellcap*SOC/batteryinfo(h,5);

%Find fitness based on charging time
chargingfitness = desiredperformance(8,1) * tanh(desiredperformance(8,2)*chargingtime/desiredperformance(8,3));

%Sum the objective function terms
fitness = AERfitness + costfitness + accel1fitness + accel2fitness + speedfitness + cyclelifefitness + emissionsfitness + chargingfitness + 8;
%Evaluate objective function
if fitness > peakfitness(h)
    peakfitness(h) = fitness;
    peakcellcount(h) = cellcount;
    peakcellcap(h) = cellcap;
    peakSOC(h) = SOC;
    peakAER(h) = AER;
    peakcost(h) = cost;
    peakaccel1(h) = accel1;
    peakaccel2(h) = accel2;
    peakspeed(h) = maxspeed;
    peaklife(h) = life;
    peakemissions(h) = emissions;
    peakct(h) = chargingtime;
end
end
end
end

output = [peakcellcount; peakcellcap; peakSOC; peakfitness; peakAER;
           peakcost; peakaccel1; peakaccel2; peakspeed; peaklife; peakemissions;
           peakct];
output = transpose(output)
xlswrite('Output.xlsx',output,'Raw Output','B3:L9');
toc