Essays on Public Good Contribution

by

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A thesis submitted to the Department of Economics
in conformity with the requirements for
the degree of Doctor of Philosophy

Queen's University
Kingston, Ontario, Canada
November, 2007

Abstract

This thesis explores some theoretical and empirical issues in the voluntary contributions to public good. Chapter I contains a brief motivation and introduction. In chapters II and III, we analyze two non-cooperative methods for either enhancing or mitigating externality-causing activities. Chapter II deals with positive externality in the public good contribution context, and chapter III with negative externality in the pollution abatement context. Chapter IV contains an empirical analysis of charitable donations by the elderly.

Chapter II models the so-called "corporate challenge gift" used in real world fund-raising, and adopts the concept to voluntary contributions to public goods more generally. We model the process as a sequential game in public good contributions. One of the agents sets a quantity-contingent matching scheme to leverage higher contributions from the other players. Under the assumption that the preferences of agents are public information and the assumption that the scheme setter can commit to the matching plan, we show that the scheme brings efficient levels of total contributions to the public good.

Chapter III applies some ideas from a joint work with Professor Robin Boadway and Professor Jean-François Tremblay on "Commitment and Matching Contributions to Public Goods" to the issue of reducing negative externality-causing activity. In particular, it adapts both the Guttman-Danziger-Schnytzer type of rate-matching mechanism and the quantity-contingent matching method for public good contributions to the international pollution abatement problem. In a simple two-country model, we find that both matching schemes induce the countries to internalize the negative externality imposed on the other country. However, perhaps due to the lack of enough policy instruments, they cannot equate the marginal costs of abatement across the countries, leaving room

for Pareto improvement. This further improvement can be achieved if the two countries also contribute to a conventional public good.

Chapter IV is an empirical exercise on some positive externality-generating activities by the elderly. It attempts to document the charitable giving of money and time by people aged 60 or above in the 2003 PSID data for the United States and analyze the influences of some economic and demographic factors on these activities. Income, wealth, the subjective rating of health status, and years in school are found to have statistically significant impacts. Income and wealth appear to have distinct influences. The tax price of money donation also has a statistically significant effect on money donations.

Acknowledgements

Perhaps the most relaxing part in drafting a thesis is to write the acknowledgements.

I sincerely thank Professor Robin Boadway and Professor Charles Beach for their excellent supervision, encouragement, and support. I would also like to thank all the teachers with whom I took courses, at Queen's and in China, for teaching and enlightening me.

Some people kindly offered their comments and help on this thesis. They include Professor Dan Usher, Professor John Hartwick, Professor Ruqu Wang, Mei Li, Joel Rodrigue, Junfeng Qiu, Xiaoting Wang, Jeremy Lise, and Jean-François Houde. I appreciate their help.

My classmates and the fellow students in and outside the department made the six years here interesting and rewarding in many other ways. Their kindness will be remembered.

My study would not have been possible without the financial supports from Queen's University, the Department of Economics, and the Ontario Graduate Scholarship (OGS). These are gratefully acknowledged.

I also thank all my family members for bearing with my absence for this many years and for enabling me to study here without worrying about anything at home.

Table of Contents

List of Tables

List of Figures

Chapter I

Introduction

The search for mechanisms to overcome the free rider problem is one of the most studied topics in public economics. Various cooperative and non-cooperative mechanisms have been proposed. In the context of voluntary contributions to public goods, these mechanisms can partially or completely change players' incentives to free ride and induce higher contributions. Batina and Ihori (2005) is an excellent recent review and extension of the huge literature.

Among the many mechanisms, one type that usually assumes full information about preferences is concerned with non-cooperatively improving upon the free riding situation in the absence of a "central authority" to coordinate and implement.

This thesis first explores two theoretical issues along the line of non-cooperative and self-implementing mechanisms, and then empirically documents the charitable donations by a particularly respectable group in our society—the elderly people. Chapter II models the "corporate challenge gift" in real world fundraising as a non-cooperative process for public good contributions, under the assumption that preferences are publicly known. It examines whether this quantity-contingent method can increase public good contributions relative to the Nash equilibrium quantity of voluntary contributions. It shows that such a mechanism leads to a Pareto efficient level of contributions, thereby fully overcoming the free rider problem. In a simple model with two countries and two or three stages, chapter III applies two non-cooperative methods for increasing public good contributions to the issue of international environmental pollution reduction. The first method is a two-stage one that implements the Lindahl allocation in public good contributions; the second one is the analogue of the quantity-contingent process in chapter II. Chapter IV estimates a reduced form two-equation econometric model to document the charitable contributions of money and time by the elderly and analyzes some of their determinants.

Now we discuss some related literature and provide a brief overview for each chapter.

A Quantity-contingent Method for Public Good Contribution

The so-called "corporate challenge gift" is a way of raising funds that is used in the real world. Sometimes, a large donor, who can be a wealthy individual, a company, or even a government agency, may want to use its gift to leverage overall contributions to some charitable organization. Before making its donation, the large donor can announce to the general public that it will donate a (significant) amount if the funds raised from the public exceed a certain level. We can show how the insights obtained from such a mechanism apply to voluntary contributions to public goods more generally, provided one of the participating agents is able to commit.

To our knowledge, only two papers by James Andreoni have studied situations that are similar to the corporate challenge gift. Andreoni (1998) shows that a small amount of seed money from the government can generate substantial additional private donations toward the provision of a public good, when there is a threshold level of contribution below which no benefit from the public good can be produced. The paper focuses on capital fundraising, where the threshold arises because of the non-convex production technology of the public good. In this context, zero provision can be one of the equilibria in a simultaneous contribution game, if there is no pledge of donation by some lead donors. The lead donors' pledge of seed money can help remove this non-provision equilibrium and thus bring success in fundraising. Andreoni (2006) studies another aspect of such lead donor pledges—the signaling role of the leadership giving. Here there is no technological threshold in producing the public good. However, the general public may be uncertain about the quality of the charity. Lead donors may try to convince other donors that the charity is of high quality, by pledging a large amount of seed money. In equilibrium, the leaders are the wealthy, since they have lower cost of providing the signal. Section 8 of the paper briefly mentioned the relation of the central theme of the paper and the challenge gifts. From the description in the paper, the challenge gifts there seem to include both the block quantity-matching type, such as the corporate challenge gift, and the rate-matching type, where the large donors match every dollar of the contributions from the general public by a certain number of dollars. Andreoni mentioned that, in the real world, the threat to withdraw the gift is usually not credible, and that the pledged challenge gift is similar to the binding gift modeled in his paper; however, he commented that to model a challenge gift requires a whole separate paper. For our purpose, the relevant point is that, in reality, challenge gift pledges are rarely withdrawn when the contributions from the general public do not meet the challenge threshold. As the paper mentioned, in those cases, extensions of time lines and assistance to the charities are often given by the lead donors, and sometimes the pledge is only partially withdrawn or reissued at a more realistic level in a future date. We will discuss more about this point in chapter II with respect to its relation to our assumption of the lead donor's commitment ability.

In our model of a corporate challenge gift, there is no technological threshold in public good production—the production technology is the usual linear one with continuous quantities as in a standard subscriptions model. There is also no asymmetric information about the quality of the public good. Our focus is on the use of a threshold by the donor who pledges the challenge gift to leverage contributions from the general public.

Focusing on leveraging or contingent matching in quantities, we build a simple model of a quantity-contingent method (QCM) for public good contribution. The model assumes public information about preferences. With two players, a QCM consists of three stages. In Stage 1, a player who sets the parameters of the QCM (the QCM setter) announces a contingent matching plan to the other players in the game. The plan specifies that, if Stage 2 contributions from the other players are no less than some threshold level, the QCM setter will contribute a certain (large) amount; however, if Stage 2 contributions do not meet the threshold, the QCM setter will only contribute a (small) amount, which is based on its *ex post* individual maximization given the Stage 2 contributions. In Stage 2, players other than the QCM setter actually contribute, simultaneously, to the public good. Then, in Stage 3, the QCM setter makes its contribution according to the contingent-matching plan announced before. Another important assumption in this model is that the QCM setter can commit to the QCM announced in Stage 1 and does not renege on it in Stage 3.

If there are more than two players, the game is modified. There are four stages now. The contingent matching plan announced in Stage 1 remains the same as above for the case where Stage 2 contributions by the other players are no less than the threshold. However, when Stage 2 contributions fall short of the threshold, the QCM setter now allows all the other players to simultaneously adjust their contributions in Stage 3—taking back or adding any amount as they wish, but now anticipating that, in Stage 4, the QCM setter will only contribute a (small) amount based on individual maximization, given the adjusted contributions from Stage 3. In Stage 2, players other than the QCM setter contribute. Stage 3 is necessary only when Stage 2 contributions do not meet the threshold level; otherwise, the game directly goes into Stage 4. In Stage 4, the QCM setter acts according to the contingent plan.

With this setup, we find that the QCM can bring an efficient level of public good contributions. In an equilibrium of the QCM game, the level of the public good is exactly an efficient one, and no player wants to contribute any more or less than his or her equilibrium quantity. This resembles the "point provision" result found by, for example, Bagnoli and Lipman (1989) and Andreoni (1998).

Then we consider five extensions. A QCM must offer the players no less than their reservation utilities in order to induce them to participate. The analysis above implicitly assumed that the fall-back situation that determines the reservations utilities is a Stackelberg game in public good contributions, as studied by Varian (1994). This need not always be the case. Thus, in the first extension, we analyze the effect of changing the fall-back situation into one where the QCM setter always commits to a zero contribution in the event that the QCM requirement is not satisfied. We find that in some cases, this can weaken the fall-back positions of some of the players other than the QCM setter and generate higher surplus for the latter. In the second extension, we ask whether, in addition to the commitment to the QCM, the commitment to a per unit subsidy to all other players could make the QCM setter even better off. We find that that is not possible. In the third extension, we consider, in a two-player model, what happens to the efficiency result in the main sections if there is now a second public good, to which no QCM is applied. The contributions to this second public good take place after those to the first one, for which a QCM is in place. If we assume that contributions to the second one are interior, the QCM is now of no use for the setter. This is due to the neutrality outcome in the contributions to the second public good. The neutrality theorem, as in Bergstrom, Blume, and Varian (1986) for example, implies that the incentives in the game are altered such that the total level of the first public good, rather than individual players' contributions, is the only relevant quantity. This is so because redistributions of the players' endowments do not change the players' consumptions of either the public good or the private good in a Nash equilibrium, as long as such redistributions do not alter the set of contributors. Thus, with our assumption of interior last stage contributions, only the sum of the players' endowments available for the last stage matters for determining the players' utilities; the total level of the first public good, rather than its composition, enters into this sum of endowments. This renders the QCM, which focuses on individual contributions in some sense, useless. The fourth and the fifth extensions relate to Boadway, Song and Tremblay (forthcoming). More specifically, in a two-player model, we compare in these two extensions the levels of the public good and the players' utilities under the rate-matching process studied by that paper and under the QCM in this chapter. Extension four deals with the case where both of the players in the rate-matching game can commit; extension five looks at a situation in which only one of the two players can commit in the rate-matching game.

Some Matching Methods for Pollution Abatement

Among the non-cooperative mechanisms for overcoming the free rider problem, there are some that implement the Lindahl allocation. As Danziger and Schnytzer (1991) commented, this type of mechanism in a sense transforms the public good in question into a kind of private good, and thus makes a player's decision on public good contribution similar to one on the purchasing of a private good.

Guttman (1978) proposed a two-stage process to implement the Lindahl allocation. In the first stage, each player simultaneously announces a matching rate at which the player will match all others' flat contributions in the second stage. In the next stage, given the matching rates, each player simultaneously contributes a flat amount to the public good, and the matching rates in the first stage are applied—each player's matching contribution beside the flat one is equal to the matching rate the player announced in the first stage times the sum of the second-stage flat contributions of all other players. Using a model with identical players whose utility functions are quasilinear-in-consumption and publicly known, Guttman showed that the efficient allocation emerges in a subgame perfect equilibrium. Danziger and Schnytzer (1991) changed the setup somewhat and generalized the Guttman mechanism to general preferences that are not necessarily identical. In their model, players subsidize, rather than match, all other players' subscriptions to the public good. Suppose that the production cost, or market price, of the public good in terms of the private good is p. In stage 1, players simultaneously announce per unit subsidy rates, s_i. In stage 2, given the subsidy rates, players simultaneously set their flat subscriptions, or "purchases", of the public good, q_i. The subsidy rates are then applied. A typical player i's total subsidy to player $j \neq i$ is $s_i q_j$, and player i's outlay on his or her own flat subscription/purchase of the public good is equal to $(p - \sum_{k \neq i} s_k) q_i$. Danziger and Schnytzer mentioned several technical advantages of this setup relative to that of Guttman's. Both these papers make the assumption that players can commit to the matching or subsidy rates announced in stage 1, when it comes to stage 2.

Chapter III is mainly an application of some ideas in the joint research, Boadway, Song and Tremblay (forthcoming). This paper examines the consequences for the Guttman-Danziger-Schnytzer mechanism of partially relaxing the commitment assumption and of introducing a second public good, to which there is no matching in place and to which the players can choose to contribute after contributions to the first one are resolved. Chapter III applies the Guttman-Danziger-Schnytzer type of matching and the QCM matching for increasing public good contributions, from the joint work and chapter II respectively, to the reduction of negative externality-causing activities. In particular, we consider international environmental pollution abatement.

We use a simple two-country model, where the countries may have different abatement cost functions. We show that the matching mechanisms can induce the countries to internalize the negative externality to the extent that a Samuelson condition holds. However, the matching mechanisms in general cannot equate the marginal abatement costs across countries, leaving room for Pareto improvement. This outcome is perhaps due to the lack of enough policy instruments in the particular model. However, if, after resolving the pollution abatement issue, the two countries need to contribute voluntarily to a public good, for which no matching scheme is in place, and if their contributions are both interior, full efficiency in the abatement issue can be achieved, although the public good contributions are still inefficient. This outcome is due to the neutrality result applying to the public good contributions and to the setup we adopt. That the public good contributions are interior implies that the neutrality theorem applies. The theorem guarantees that the sum of the two countries' endowments (rather than its composition) after the abatement issue is resolved is the relevant quantity. Under our setup, this makes the sum of the endowments of the two countries' the common objective function which both countries attempt to maximize when they choose the environmental policies. It is then not surprising that the private outcome coincides with the social optimum for pollution reduction.

Charitable Donations by the Elderly

A common way for many people to contribute to a public good is through charitable donations. The pervasiveness of this phenomenon in many countries is well documented. There may be many different motivations for people to make such giving, and some people may not at all view it as something like a contribution to a public good. Broadly speaking, it might be said that the giving of good things can take the form of wealth (external wealth, such as money and other belongings, and "internal wealth", such as physical and mental labour, etc.), wisdom (teaching of the right things or anything that can bring true benefit to others, advice that helps, and perhaps, supervising a student, etc.), and even fearlessness (removing others' discomfort, worry, or fear, etc. in some way). These forms may well overlap with each other. In economics, by "charitable giving", we often mean the giving of money and time (volunteering) to charitable organizations.

In chapter IV, we focus on the charitable giving of money and time by the elderly. The elderly are sometimes viewed as passive recipients of resources from the society. In the economics literature, it is perhaps true that the many formal and informal giving of resource, wisdom, and other good things by older people has not been very well documented and analyzed. In contrast, there seem to be quite a number of studies on the altruistic behaviour of the elderly at least in sociology and psychology.

Midlarsky and Kahana (1994) conducted several field experiments and surveys to study the various formal and informal helping behaviour by older people and the relation of helping to their well-being. The contributions to society by older Americans studied by Bass (1995) include working and other productive engagement, volunteering, care-giving, and support to children.

In economics, there are a large number of empirical studies on charitable giving behaviour, although the focus of many of them is perhaps not on older people. Clotfelter (2002) provides an excellent recent review of the empirical studies of money donation. Brown and Lankford (1992) and Andreoni et al (1996) contain reviews of empirical studies on both money donation and time donation.

In chapter IV, we try to document the charitable donations of money and time by the elderly, as well as to analyze some economic factors that affect these donations.

We use the year 2003 Panel Study of Income Dynamics (PSID) data for the United States, which contains the survey results from a philanthropy module. We use information on people who are aged 60 or above. Our econometric model has two equations, one for money giving and one for time giving. The dependent variable in each equation can potentially be censored, and thus the system is of the SUR Tobit type.

The factors that we analyze are in line with many previous studies. They include the tax price of money giving, which is one minus the donor's marginal income tax rate, the wage rate, which is intended to serve as a measure of the opportunity cost of volunteering for those working, income, wealth, and demographic factors like health, education, and age. The findings are broadly consistent with the existing literature. Income, wealth, subjective rating of health status, and years in school are found to have statistically significant impacts with the expected signs. The tax price also has a statistically significant effect on money donation. Besides, income and wealth appear to have distinct influences on money donation. On average, hours volunteered are highest among people between 70 and 80 years of age.

Chapter II

A Quantity Contingent Method for Increasing Contributions to Public Good

2.1 Introduction

A corporate challenge gift, a way of fund-raising, is "the process whereby individuals or companies seek to leverage their support for a philanthropic cause or nonprofit organization during a public fund-raising campaign by requiring that a specific amount be raised from the general public prior to making a sizable donation." (J. S. Ott, 2001) Real world examples can be found where individuals, companies, or government agencies use this method to raise funds for various purposes.

In this chapter, we try to model this mechanism and examine its properties. In particular, we examine whether it could increase the total amount contributed. We are able to show that it would result in efficient levels of contributions, under the assumption that preferences are public information. The next step we would like to attempt is to examine its properties when this assumption does not hold.

The mechanism can be thought of as a three-stage game. (In section 3, which deals with the general case, an extra stage will be introduced.) In Stage 1, the individual or company seeking to leverage its support announces the specific threshold amount that the public must contribute before the former provides its own contribution. In Stage 2, the public makes contributions to the public good simultaneously. In Stage 3, if Stage 2 contributions exceed the threshold, the individual or company contributes its promised amount; otherwise, some fall-back situation occurs. We will model fall-back situations in detail below. For ease of reference, we call this mechanism a quantity contingent method (QCM). As one might agree, a natural benchmark with which we could compare the mechanism would be a sequential contribution situation without a QCM. The latter was probably first studied by Varian (1994).

Through out the chapter, we focus on pure-strategy equilibria, since equilibria in mixed strategies in this particular game seem hard to interpret. The key part of the proof showing that efficiency obtains under the mechanism is readily found in at least two papers: Bagnoli and Lipman (1989) and Andreoni (1998). The first and some part of the second deal with voluntary contribution games involving "point provision" equilibria. In the first paper, if the sum of total simultaneous contributions is no less than a preannounced threshold, the public good is constructed; otherwise, proceeds are refunded. The second paper focuses on capital fund-raising campaigns, where the technology for producing a public good involves a threshold level of total contributions and is thus non-convex. Such non-convexity may arise because of fixed cost in production or increasing returns around the point at which total contributions approach the threshold. Thus, this chapter may be viewed as a simple application of these ideas and techniques to the sequential contribution setting with continuous outcomes.

The rest of the chapter is organized as follows. Section 2 shows how a QCM works and results in efficiency in a two-player public good contribution game. Section 3 shows the efficiency property of a QCM for the many-player case. Section 4 looks at some extensions. Section 5 concludes.

2.2 QCM in the Two-player Case[1]

In this section, we try to illustrate how a QCM would work in a two-player setup. Suppose the two players are two countries, Rome and Greece.[2] Rome's

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[1] This section draws on section 4.3 of Boadway, Song and Tremblay (forthcoming).

[2] In this chapter, when there are two players, we refer to them as Rome and Greece; when there are more than two players, we label them as players $1, \cdots, n$.

utility function is given by $u(G,x) = u(g+\gamma, w-g)$, and that of Greece's is $\upsilon(G,\chi) = \upsilon(g+\gamma, \omega-\gamma)$. $G = g+\gamma$ is the total contributions to public good; x and χ are the countries' respective consumption of a private good; w and ω are their respective initial endowments, which are used to finance the public good contributions and the private good consumptions. Throughout the chapter, we assume that the utility function are strictly concave and strictly increasing in both arguments; we also assume that both the public good and the private good are strictly normal. The strict normality assumption, combined with the other assumptions, just ensures that (the non-zero part of) the demand for the public good and the private good as functions of the "virtual income",[3] $w+\gamma$ for Rome and $\omega+g$ for Greece, have slopes strictly between 0 and 1. This implies that, as one player's contribution to the public good increases, the other player's demand for both goods increase. This point is not very important in this section, but it will be in some parts of the next section.

Suppose Rome sets the QCM and can commit to it. As mentioned above, under a QCM, Rome commits to a given contribution contingent on Greece's contribution being at least some threshold amount. If Greece does not meet the threshold, Rome no longer provides its committed amount, and the outcome reverts to some fallback situation. Rome takes into consideration the fallback situation and designs the QCM to induce Greece to participate. The fallback situation is thus important in determining the parameters of the QCM and therefore the payoffs attained by each player.

It is useful to characterize the QCM as a three-stage procedure. In Stage 1, Rome announces its QCM. This specifies that if Greece's contribution γ in Stage 2 is no less than some threshold level, $\tilde{\gamma}$, Rome will contribute an amount $\tilde{g}$ in Stage 3. Otherwise, Rome will no longer be obliged to contribute $\tilde{g}$, and, instead,

[3] See Bergstrom, Blume, Varian (1986).

it will contribute some fallback level of g (lower than $\tilde{g}$). In equilibrium, $\tilde{\gamma}$ and $\tilde{g}$ will in fact be realized, but the values set by Rome depend upon the fallback outcomes. In Stage 2, Greece contributes to the public good. In Stage 3, Rome makes its contribution to the public good according to the QCM announced in Stage 1.

In principle, the fallback can be different depending on the ability of the two countries to commit. For example, the fallback may be a natural Stackelberg game as studied by Varian (1994), where Greece can commit to a contribution, which Rome takes as given. Similarly, Rome may be able commit to a zero contribution, regardless of what Greece contribution is. Or, no one can commit and the fallback effectively becomes a simultaneous move game in contributions. The point is that, $\tilde{g}$ and $\tilde{\gamma}$ calculated by Rome and also g and γ that would emerge in the fallback situation depend on how we specify each country's commitment ability and the levels of discrete quantities to which they can commit. These differences affect the two countries' utilities in fallback situations, and thus affect the divisions of surplus between them when a QCM is actually implemented.

In this chapter, we mainly consider the case where the fallback is a natural Stackelberg game in contributions, with the QCM setter (Rome in this section) moving in the last Stage. In the first of the five extensions in section 4, we look at the outcome when Rome always commit to a zero contribution in the fallback.

The order of events is as follows. Rome announces $\tilde{\gamma}$ and $\tilde{g}$ first. Then, Greece chooses γ in Stage 2. In Stage 3, if $\gamma \geqslant \tilde{\gamma}$, Rome supplies $\tilde{g}$; otherwise, $g = \text{argmax}_g u(G, x)$, given γ. Whether Greece chooses $\gamma \geqslant \tilde{\gamma}$ depends upon the utility obtained in the fallback position. In this section, we assume that Greece has no bargaining power and its *reservation utility* in the game is just equal to the level that it can obtain in the fallback situation; in the next section, we allow the reservation utilities to be higher than those in the fallback, due perhaps to more bargaining power. Rome must ensure that the QCM gives Greece its reservation utility for the QCM to operate. We begin by specifying Greece's reservation utility. Rome will then use that to determine its choice of $\tilde{\gamma}$ and $\tilde{g}$.

2.2.1 **Greece's Reservation Utility**

In the fallback position, Greece contribute $\gamma < \tilde{\gamma}$ in Stage 2, and Rome fulfills its commitment and sets $g = \text{argmax}_g u(G, x)$ in Stage 3. This is the sequential contributions to public good game studied by Varian (1994). Because of Greece's ability to commit to a γ that is optimal for itself in the fallback, it is the Stackelberg leader in this game and can effectively free ride on Rome's subsequent contribution. In this subsection, we will only draw some relevant results from Varian's paper.

In Stage 3, Rome solves

$$\max_{\{g\}} \; u(g + \gamma, w - g), \tag{1}$$

which gives Rome's demand function, $g(\gamma)$. With our assumption above, $g(\gamma)$ is strictly decreasing in γ for γ between 0 and the level such that g is just crowded out, and $g(\gamma)$ remain at 0 for any γ greater than that crowding out level.

In Stage 2, Greece anticipates Rome's optimal response to any γ and selects as its optimal contribution

$$\gamma^* = \arg\max_{\{\gamma\}} \; \upsilon(g(\gamma) + \gamma, \omega - \gamma). \tag{2}$$

Depending on the specific forms of the utility functions and the endowments, Greece may or may not find it optimal to completely free ride on Rome's subsequently contribution, that is, γ^* may or may not be 0. However, Varian (1994) showed that the total contribution in this Stackelberg game is never larger than that arising in a simultaneous move game in contributions. That is, the free rider problem is exacerbated by the sequential nature of the game and Greece's ability to commit and free ride. Also because of its ability to commit and effectively free ride in this game, Greece is never worse off, and is sometimes better off, than in the simultaneous move game.

In any case, the Stackelberg game in the fallback determines $g^* \equiv g(\gamma^*)$, γ^*, and Greece's reservation utility:

$$\upsilon^{res} = \upsilon(g(\gamma^*) + \gamma^*, \omega - \gamma^*) \tag{3}$$

2.2.2 **Rome's Choice of a QCM**

Greece will only participate in the QCM if it obtains at least its reservation utility level. The problem for Rome in designing the QCM is therefore to choose the values of g and γ that maximize its own utility subject to the constraint that Greece obtains at least its reservation utility level (which will be binding). The Lagrangian for Rome's problem is:

$$\mathcal{L} = u(g + \gamma, w - g) + \lambda[\upsilon(g + \gamma, \omega - \gamma) - \upsilon^{res}]$$

The first-order conditions on g and γ are:

$$u_G - u_x + \lambda \upsilon_G = 0 \tag{4}$$

$$u_G + \lambda(\upsilon_G - \upsilon_\chi) = 0 \tag{5}$$

and the binding constraint (3). The solutions to this problem then give $\tilde{g}$ and $\tilde{\gamma}$ in the QCM. Thus, Rome commits to $g \geqslant \tilde{g}$ in Stage 3 if Greece contributes $\gamma \geqslant \tilde{\gamma}$ in Stage 2, but g^* if Greece contributes any less than $\tilde{\gamma}$.

It is straightforward to see that the QCM yields an equilibrium in which Greece contributes $\tilde{\gamma}$ and Rome contributes $\tilde{g}$.[4] First, neither Rome nor Greece

[4] This proof is analogous to the proof of Proposition 3 in Andreoni (1998).

would want to contribute more than $\tilde{g}$ or $\tilde{\gamma}$. To see this, note that by the first-order conditions with respect to $\tilde{g}$ and $\tilde{\gamma}$, (4) and (5) respectively, it must be the case that $u_G - u_x < 0$ and $\upsilon_G - \upsilon_\chi < 0$, since $\lambda > 0$. That is, total G is too high from each individual country's point of view. Second, neither country would contribute less than the amounts $\tilde{g}$ and $\tilde{\gamma}$ specified by the QCM. In the case of Rome, it is assumed that a commitment to the QCM is binding, so g cannot be reduced below $\tilde{g}$ (even though doing so would make Rome better off *ex post* since $u_G - u_x < 0$). If Greece sets γ lower than $\tilde{\gamma}$, anticipating Rome's response as specified by the announced plan, Rome acts according to the plan and each gets its reservation utility. Thus, Greece cannot be better off by lowering γ below $\tilde{\gamma}$, implying that $\tilde{g}$ and $\tilde{\gamma}$ can be sustained as an equilibrium.

Now we show that the QCM equilibrium is efficient. To see this, note that Rome's problem above is essentially a Pareto-optimizing one. Combining the first-order conditions with respect to $\tilde{g}$ and $\tilde{\gamma}$, we obtain the Samuelson condition:

$$\frac{u_G}{u_x} + \frac{\upsilon_G}{\upsilon_\chi} = 1 \tag{6}$$

In this efficient outcome, Greece obtains the reservation utility level υ^{res}, while Rome gets the remaining surplus from internalizing the free rider problem.

It follows immediately that whatever the reservation utility level that Greece must be given in order to willingly provide $\tilde{\gamma}$, the above analysis will apply. As we have seen, the reservation level of utility is determined by the fallback outcome, which in turn depends upon commitment ability in Stage 3. The implication is that, whatever we assume about commitment in Stage 3, the QCM will yield an efficient outcome. Different fallback outcomes will just divide this surplus from internalizing the free rider problem between Rome and Greece differently. We will consider another fallback situation in section 4. First, some comments on this QCM equilibrium should be made.

The QCM equilibrium resembles the point provision results found in Bagnoli and Lipman (1989), Admati and Perry (1991) and Andreoni (1998). In these papers, the technology of the public good is such that it is produced or purchased if and only if total contributions exceed a certain threshold. Then, in an important subset of equilibria found in Bagnoli and Lipman (1989) and in some of the cases analyzed by Admati and Perry (1991) and Andreoni (1998), the aggregate level of private provision is exactly at the threshold. Efficiency obtains in many of the cases analyzed by these papers. The experimental outcomes of Bagnoli and McKee (1991) and Cadsby and Maynes (1999) support the theoretical results in Bagnoli and Lipman (1989). A QCM does not involve a technological threshold in the production or purchase of the public good: continuous amounts can be supplied. However, the player who commits to a QCM effectively uses a threshold to leverage a given contribution by the other player. Baker, Walker and Williams (2006) use laboratory experiments to analyze the effects of conditional contributions in settings that resemble our matching and QCM cases. As with our theoretical results, they find that public good provision is higher when the conditional contribution takes the form of a fixed amount provided the contributions of others reach some threshold level.

We collect the points that have emerged in this section in the following statement: In a QCM with two players, under our assumptions on utility functions, the ability of the players to commit, and the reservation utilities, total contribution is efficient; the first mover gets its reservation utility, and the last mover gets all the surplus from internalizing the externality.

2.3 QCM in the Many-player Case

In this section, we assume that there are multiple players who contribute simultaneously in Stage 2, while there is still only one player in the first and the last stages. All players have general utility functions as before, and these are public information. In addition, we give earlier stage players some bargaining power in a reduced form way.

It turns out that, to take into account of the strategic interactions among earlier stage players, we need to modify the QCM for it to work. The modified QCM can now be characterized as a four-stage game. Suppose there are $i = 1, 2, \cdots, n$ players. Player n sets the QCM; the other players always move simultaneously. In Stage 1, player n announces a QCM, which specifies that, if Stage 2 total contribution by players 1 through $n-1$ is no less than some threshold $\tilde{G}^1$, nothing happens in Stage 3, and he can commit to contribute $\tilde{g}_n$ in Stage 4. On the other hand, if Stage 2 total contribution falls short of $\tilde{G}^1$, player n will no longer be obliged to contribute $\tilde{g}_n$ and will set his contribution to maximize his utility in Stage 4. In the latter case, Stage 2 players may adjust their contributions in Stage 3 if they like, anticipating now that player n will no longer be committed to contributing $\tilde{g}_n$ in Stage 4 and will only try to maximize his own utility.

Again, we first analyze the Stackelberg fall-back situation and then, with some additional assumption that guarantee its existence, examine the properties of a QCM equilibrium. The results are qualitatively similar to those found in section 2. The QCM can bring efficiency in the total contributions. It gives first movers (Stage 2 players 1 through $n-1$) their reservation utilities (which now may be higher than those obtained in the fall-back Stackelberg equilibrium, due to the modified assumption on bargaining powers), and leave the remaining surplus to the last mover (Stage 4 player n).

2.3.1 **The Fallback Position: Stackelberg Equilibrium**

In the fallback Stackelberg game, only Stages 3 and 4 of the four stage game above are relevant. In particular, players 1 to $n-1$ simultaneously make their contributions g_ℓ, $\ell = 1, 2, \ldots, n-1$, in Stage 3; player n then contributes g_n in Stage 4, given Stage 3 total contribution, $G^1 \equiv \sum_{\ell=1}^{n-1} g_\ell$. As in the two-player case typical player i, $i = 1, \cdots, n$, has the general utility function $U_i(G,\ x_i)$.[5] As in the previous section, the utility functions are strictly concave and strictly increasing in G and x_i, and both G and x_i are strictly normal. The endowment of player i is w_i. We focus on sub-game perfect equilibrium in pure strategies, and solve the game backwards.

Stage 4

Given G^1, player n solves

$$\begin{aligned} &\max_{\{g_n\}} U_n(G^1 + g_n,\ x_n) \\ &\text{s.t. } x_n + g_n \leqslant w_n \\ &\qquad g_n \geqslant 0. \end{aligned} \tag{7}$$

Following Bergstrom, Blume and Varian (1986), we can transform the problem into one where player n chooses the total level of $G \equiv G^1 + g_n$, in order to solve

$$\begin{aligned} &\max_{\{G\}} U_n(G,\ x_n) \\ &\text{s.t. } x_n + G \leqslant w_i + G^1 \\ &\qquad G^1 \leqslant G. \end{aligned} \tag{8}$$

As in Bergstrom, Blume, and Varian (1986), this standard consumer optimization problem gives the demand function for the public good

$$G(G^1; w_n) = f_n(w_n + G^1),$$

[5] In the rest of this section, whenever possible, we will use ℓ to indicate a typical player in the set of all earlier stage players (i.e., a typical player other than the QCM setter, player n) and i to indicate a typical player in the set of all players in the game (i.e., any player among players 1 through n.)

which is strictly increasing and $0 < f' < 1$, because of the strictly normality of both goods. Taking into account of the non-negativity constraint $G \geqslant G^1$, we can write the complete demand function for the public good as

$$G(G^1; w_n) = \max\{f_n(w_n + G^1), G^1\}. \tag{9}$$

Function $G(G^1; w_n)$ is equal to $f_n(w_n)$ at $G^1 = 0$, is strictly increasing in G^1 with slope between 0 and 1 up to a level $G^{1c} > 0$ such that $f_n(w_n + G^{1c}) = G^{1c}$ (G^{1c} is thus the level of G^1 that just crowds out g_n), and is increasing in G^1 with slope 1 for $G^1 > G^{1c}$. Figure 2.1 shows the shape of $G(G^1; w_n)$. In what follows, it is convenient to write this demand function simply as $G(G^1)$.

The total level of public good demanded, $G(G^1)$, corresponds to the level of individual contribution made by player n, $g_n(G^1)$, in the following way. $g_n = f_n(w_n) > 0$ at $G^1 = 0$; $g_n = f_n(w_n + G^1) - G^1$ is strictly decreasing in G^1 with slope between -1 and 0 up to G^{1c}, and $g_n = 0$ for all $G^1 > G^{1c}$. Curve LPG^1 in Figure 2.2 depicts the shape of such a demand function, with point P in the figure corresponding to G^{1c}. (Note that Figures 2.1 and 2.2 are not drawn according to the same scale. OG^{1c} and $Of_n(w_n)$ in Figure 2.1 should be equal to OP and $Of_n(w_n)$ in Figure 2.2, respectively.)

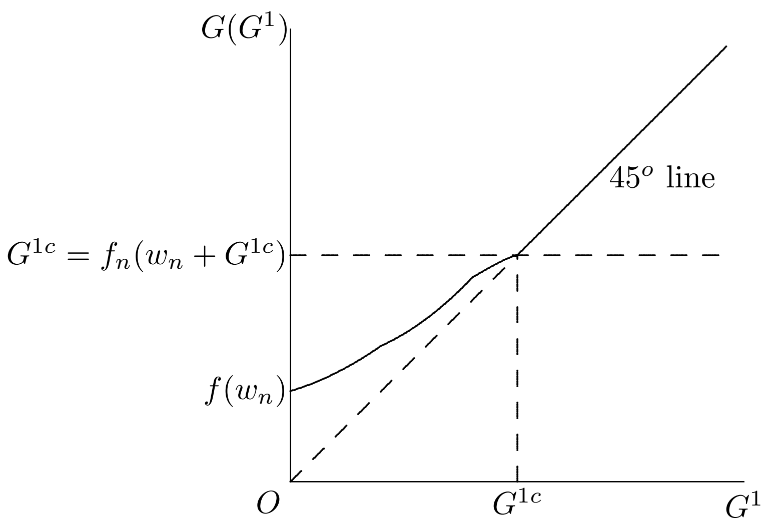

Figure 2.1

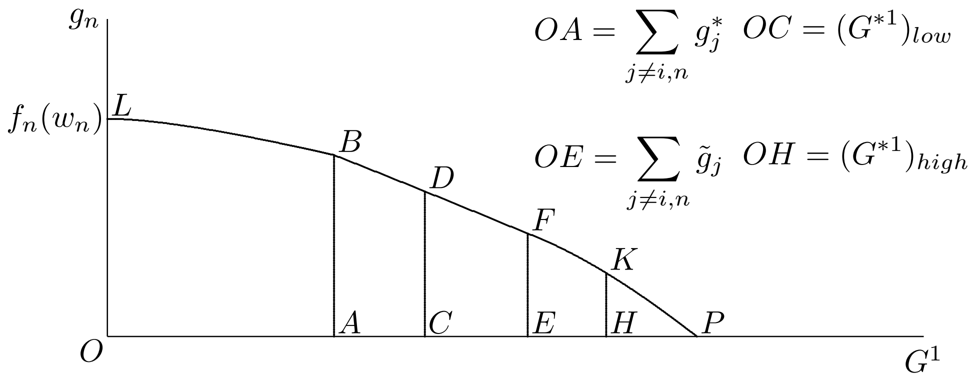

Figure 2.2

Stage 3

Given player n and all other Stage 3 players' strategies, each player ℓ, $\ell = 1, 2, \cdots, n-1$, chooses his optimal contribution. If an equilibrium in pure strategies exists for this stage, equilibrium in pure strategies exists for the whole game.

For a typical Stage 3 player ℓ, denote the total contributions by all other *Stage 3* players as $G^1_{-\ell}$, $\ell \in \{1, 2, \cdots, n-1\}$. Since Stage 3 contribution G^1 is equal to $g_\ell + G^1_{-\ell}$, where the function $G(G^1)$ is as introduced above, the total level of public good from the game can be written as $G(g_\ell + G^1_{-\ell})$.

In order to identify the Stage 3 pure strategy equilibrium, it turns out to be conceptually convenient to think of the entire four-stage game (essentially two-stage in the fallback in this section) as if there is only one stage—Stage 3, but with a modified "production function" for the public good. The idea is to realize that, for any Stage 3 player ℓ, whether the Stage 4 player is present merely affects the shape of the "production function" $G(G^1)$ faced by Stage 3 players. That is, from the point of view of the Stage 3 players, the "output" $G(G^1)$ is a linear function of the "input" G^1, $G(G^1) = G^1$, without the Stage 4 player, and it is a nonlinear function in G^1 with the Stage 4 player. One can easily see this from Figure 2.1.

Player ℓ's ($\ell = 1, \cdots, n-1$) problem is then

$$
\begin{aligned}
&\max_{\{g_\ell\}} U_\ell(G(g_\ell + G^1_{-\ell}), x_\ell) \\
&\text{s.t. } x_\ell + g_\ell \leqslant w_\ell \\
&\qquad g_\ell \geqslant 0,
\end{aligned}
\tag{10}
$$

which can be rewritten as

$$
\begin{aligned}
&\max_{\{G^1\}} U_\ell(G(G^1), x_\ell) \\
&\text{s.t. } x_\ell + G^1 \leqslant w_\ell + G^1_{-\ell} \\
&\qquad G^1 \geqslant G^1_{-\ell}.
\end{aligned}
\tag{11}
$$

Theorem 9.32 of Sundaram (1996, p. 247), for example, contains sufficient conditions for the existence of a Nash equilibrium in pure strategies in a finite game. There are two conditions: for each player, 1) the set of his strategies is a compact and convex set, and 2) his payoff function is quasi-concave on the set of his strategies for every fixed set of all other players' strategies.

For the game at hand, in terms of the first formulation of the maximization problem above, each player's strategy space, $[0, w_\ell]$, is a compact and convex set. However, it turns out that condition 2) may or may not be satisfied. Our assumption of the strict concavity of utility function $U_\ell(G, x_\ell)$ in $\{G, x_\ell\}$ and the properties of function $G(G^1)$ derived above together cannot guarantee that $U(G(g_\ell + G^1_{-\ell}), (w_\ell - g_\ell))$ is quasi-concave in g_ℓ. To see this, note that strict concavity of utility implies strict quasi-concavity. Thus, given any fixed level of all others' contribution in Stage 3, $G^1_{-\ell}$, two bundles of goods $\{G(g_{\ell 1} + G^1_{-\ell}), (w_\ell - g_{\ell 1})\}$ and $\{G(g_{\ell 2} + G^1_{-\ell}), (w_\ell - g_{\ell 2})\}$, and some $0 < \lambda < 1$, we have

$$
\begin{aligned}
&U_\ell(\lambda G(g_{\ell 1} + G^1_{-\ell}) + (1-\lambda) G(g_{\ell 2} + G^1_{-\ell}), \lambda(w_\ell - g_{\ell 1}) + (1-\lambda)(w_\ell - g_{\ell 2})) \\
&> min\{U_\ell(G(g_{\ell 1} + G^1_{-\ell}), (w_\ell - g_{\ell 1})), U_\ell(G(g_{\ell 2} + G^1_{-\ell}), (w_\ell - g_{\ell 2}))\}.
\end{aligned}
$$

What the existence condition 2) requires is that

$$U_\ell(G(\lambda g_{\ell 1} + (1-\lambda)g_{\ell 2} + G^1_{-\ell}), \lambda(w_\ell - g_{\ell 1}) + (1-\lambda)(w_\ell - g_{\ell 2}))$$
$$\geqslant min\{U_\ell(G(g_{\ell 1} + G^1_{-\ell}), (w_\ell - g_{\ell 1})), U_\ell(G(g_{\ell 2} + G^1_{-\ell}), (w_\ell - g_{\ell 2}))\}.$$

From the properties of the function $G(G^1)$, we know that

$$\lambda G(g_{\ell 1} + G^1_{-\ell}) + (1-\lambda)G(g_{\ell 2} + G^1_{-\ell}) \gtreqless G(\lambda g_{\ell 1} + (1-\lambda)g_{\ell 2} + G^1_{-\ell}),$$

depending on the exact shape of the function $G(G^1)$ and the value of λ. Thus, presumably, unless U_i is sufficiently concave in G and / or $G(G^1)$ is sufficiently close to linear (especially around the point G^{1c}) such that the difference between the left-hand sides of the first two inequalities above is not large, we may not have quasi-concavity of U_ℓ in g_ℓ and are left with only a mixed-strategy equilibrium.

Since a mixed-strategy equilibrium seems hard to interpret in such a game, to facilitate discussion, we assume that the conditions needed for the existence of a pure-strategy equilibrium hold in what follows. It may also be helpful to conduct simulations to see how hard or easy quasi-concavity obtains for commonly used utility function specifications.

In a pure-strategy equilibrium, as long as there is at least one player who will contribute some positive amount to the public good when no one else is contributing (i.e., $\partial U_i/\partial G - \partial U_i/\partial x_i > 0$ when evaluated at $G = 0$ for at least one i, $i = 1, 2, \cdots, n$), equilibrium total contributions will be positive. None or all of the Stage 3 players may be contributors; Stage 4 player will contribute if $G^1 < G^{1c}$. Any contributor will have $\partial U_i/\partial G - \partial U_i/\partial x_i = 0$ and any free rider will have $\partial U_i/\partial G - \partial U_i/\partial x_i < 0$. Since the sum of marginal rates of substitution between the public good and the private good, $\sum_{i=1}^{n} (\partial U_i/\partial G)/(\partial U_i/\partial x_i)$, would in general be greater than the marginal cost of providing the public good, which is 1 here, the level of the public good is less than efficient due to free riding.

In any case, any sub-game perfect equilibrium in pure-strategies in this Stackelberg game determines the reservation utilities for the QCM for all the players. These are taken into account by player n in setting the QCM.

There may also be multiple sub-game perfect equilibria in the fallback Stackelberg game. This makes it unclear which fallback position to use for the QCM. We shall assume that there will be some factor (for example, focal point) outside the game that can help all players decide which pure-strategy equilibrium is most likely to arise. Alternatively, if stage 3 contributions can be modeled as sequential contributions rather than simultaneous ones, the multiplicity of equilibria might disappear. In this case, the $n-1$ players in stage 3 contribute one by one. Those who contribute later observe the contributions of those who contribute earlier, and the earlier contributors anticipate the reactions of later contributors.

In this subsection, we showed that, under certain conditions, pure-strategy equilibrium can exist in the fallback Stackelberg game. We are now ready to look at the QCM.

2.3.2 **The QCM game: Point Provision Equilibrium**

As outlined above, a QCM consists of four stages. Player n announces the QCM in Stage 1. All other players simultaneously contribute in Stage 2. If the Stage 2 total contribution G^1 is less than the threshold level $\tilde{G}^1$, Stage 2 players can adjust their contributions in Stage 3, now anticipating that player n will no longer commit to any preannounced level of contribution. Then, in Stage 4, player n acts according to the QCM. As we will see below, Stage 3 ensures that the Stackelberg equilibrium is the proper fallback situation for the QCM.

We will show that a QCM leads to efficiency in public good contribution. The idea of the proof below is the same as that for the simpler two-player case in the previous section: on the one hand, since there is already too much public good from any individual player's point of view, no one wants to contribute more than the QCM amount; on the other hand, since everyone is pivotal, no one wants to deviate by contributing less and making everyone, including himself, worse off. The only new point we now need to make sure is that, in equilibrium, no Stage 2 player wants to free ride on the other Stage 2 players' contributions.

Unlike in the previous subsection, we analyze the stages of the QCM in their natural order.

Stage 1

Given the set of utilities that would emerge in the Stackelberg fallback situation, U_i^{Sta}, $i = 1, 2, \cdots, n$, player n computes the optimal threshold $\tilde{G}^1$ by maximizing his own utility subject to all others' participation

$$\max_{\substack{\{\tilde{g}_\ell, \ell=1,\cdots,\\ n-1;\tilde{g}_n\}}} U_n(\sum_{\ell=1}^{n-1} \tilde{g}_\ell + \tilde{g}_n, w_n - \tilde{g}_n) \tag{12}$$

$$\text{s.t. } U_\ell(\sum_{m=1}^{n-1} \tilde{g}_m + \tilde{g}_n, w_\ell - \tilde{g}_\ell) \geqslant k_\ell U_\ell^{\text{sta}} = k_\ell U_\ell(\sum_{m=1}^{n-1} g_m^* + g_n^*, w_\ell - g_\ell^*) \quad (\lambda_\ell)$$
$$(k_\ell \geqslant 1, \quad \ell = 1, \cdots, n-1),$$

where we use a tilde to denote contributions in the QCM and a star to denote those in the Stackelberg game equilibrium. λ_ℓ denotes the Lagrange multiplier associated with the ℓth constraint. Recall that in this section we allow Stage 2 players to have some bargaining power. To persuade them to participate in the QCM, player n may have to give some or all of the Stage 2 players utilities that are higher than those in the Stackelberg fallback. To represent this in a simple way, we introduce the positive constants, $k_\ell \geqslant 1$. Thus, $k_\ell U_\ell^{\text{sta}}$, which is equal to or greater than U_ℓ^{sta}, represents the reservation utility of player ℓ. Besides, we assume that player n's reservation utility is simply U_n^{Sta}, although it is equally well to assume that his reservation utility is $k_n U_n^{\text{sta}}$, where $k_n \geqslant 1$. When all k_ℓ's are equal to one, player n gets all the surplus from the QCM; when at least some of the k_ℓ's are greater than one, player n has to set the $\tilde{g}_\ell$ and $\tilde{g}_n$ such that those Stage 2 players get utilities higher than in the Stackelberg equilibrium. For the analysis to be meaningful, we need to assume that the k_i's are such that Pareto improvement is possible; this is to preclude any combination of bargaining powers where some player demands too much from the QCM such that it is not possible for some other player to be at least as well off in the QCM as in the Stackelberg equilibrium.

The first order conditions are

$$\frac{\partial U_n}{\partial G} + \lambda_\ell\left(\frac{\partial U_\ell}{\partial G} - \frac{\partial U_\ell}{\partial x_\ell}\right) + \sum_{\substack{m=1,\\ m\neq\ell}}^{n-1} \lambda_m \frac{\partial U_m}{\partial G} \leqslant 0, \quad \text{equality if } \tilde{g}_\ell > 0, \tag{13}$$

$$\left(\frac{\partial U_n}{\partial G} - \frac{\partial U_n}{\partial x_n}\right) + \sum_{\ell=1}^{n-1} \lambda_\ell \frac{\partial U_\ell}{\partial G} \leqslant 0, \quad \text{equality if } \tilde{g}_n > 0, \tag{14}$$

$$(\ell = 1, \cdots, n-1)$$

and the $n-1$ participation constraints.

This maximization problem is a Pareto optimizing problem, and the solution would correspond to an efficient level of G. Since the utility functions are strictly concave, the problem has a unique global maximum. That is, there is a unique set of $\tilde{g}_i,\ i = 1, \cdots, n$ that satisfies the Kuhn-Tucker conditions above. Player n then calculates the threshold $\tilde{G}^1$ as $\sum_{\ell=1}^{n-1} \tilde{g}_\ell$ and announces it to the Stage 2 players. The constraints will be binding at the optimum,[6] and the Stage 2 players get their reservation utilities.

[6] Suppose not. Suppose, for example, all players in $\{1, \cdots, n-1\}$, except player j, face binding participation constraints. Asking player j to increase his contribution slightly will not violate his participation constraint and will increase everyone else's utility and relax their participation constraints. Thus, a set of $\tilde{g}_\ell$ that gives some Stage 2 player a level of utility that is higher than his reservation utility would not be optimal.

Stage 2 and Stage 3

$\tilde{G}^1$ can potentially be met by a continuum of combinations of contributions by Stage 2 players. Will the Stage 2 players be willing and able to choose $\tilde{g}_i$ in equilibrium if player n only announces an aggregate threshold $\tilde{G}^1$? We reason below that this would be the case. The key is to note the following two points. First, the set of $\{\tilde{g}_\ell,\ \ell = 1, \cdots, n-1\}$ such that $\sum_{\ell=1}^{n-1} \tilde{g}_\ell = \tilde{G}^1$ is a Nash equilibrium in Stage 2 of the QCM game. Second, any $g_\ell \neq \tilde{g}_\ell,\ \ell = 1, \cdots, n-1$ will give some Stage 2 players utilities that are higher than their reservation utilities, but will leave some other Stage 2 players utilities that are lower than their reservation utilities; this will make the latter set of Stage 2 players unwilling to participate in the QCM and therefore bring everyone back to the Stackelberg fallback, where every Stage 2 player is weakly worse off than in the QCM.

Although the reasoning is straight forward, we present it in several steps below for clarity. Steps (1) to (3) characterize the properties of the solution to the maximization problem above; steps (4) and (5) show how such a solution can be sustained as Nash equilibrium in the QCM game. Steps (4) and (5) mainly come from the first-order conditions above and do not quite depend on steps (1) to (3); however, it helps to present (1) to (3) first.

(1) For U_ℓ^{QCM} to be no less than $k_\ell U_\ell^{\text{Sta}},\ \ell = 1, \cdots, n-1$ (and thus no less than U_ℓ^{Sta}) and for U_n^{QCM} to be no less than U_n^{Sta}, $\tilde{G}_{-i}$ must be higher than $G^*_{-i},\ i = 1, \cdots, n$.[7] That is, for every player in the game, the sum of all other players' contributions in the QCM must be strictly larger than that in the Stackelberg fallback.

[7] Recall that we use tilde for the QCM quantities and star for the Stackelberg fallback ones. Besides, when possible, we use i or j, $i, j \in \{1, \cdots, n\}$, to indicate typical players in the set of all players, and ℓ or m, $\ell, m \in \{1, \cdots, n-1\}$, to indicate typical players in the set of Stage 2 players.

We will show this point by way of contradiction. Before doing so, it is convenient to establish some intermediate results. Suppose the same n players play two different games in contributions to the public good. One is a simultaneous move game, and the other is the Stackelberg game in the fallback situation, which consists of Stage 3 and Stage 4, as we called them in the previous subsection. Suppose that, for some player ℓ who moves in Stage 3 in the Stackelberg game, the sum of all other players' contributions, $\sum_{i=1, i \neq \ell}^{n} g_i$, is the same in the two games. Then, we can show the following. 1) Player ℓ's contribution in the Stackelberg game is never large than that in the simultaneous move game. 2) If player ℓ is a non-contributor in the simultaneous move game (and, by 1), is also a non-contributor in the Stackelberg game), he is apparently equally well off in both games. 3) If player ℓ is a contributor in the simultaneous move game (so he may or may not be a contributor in a Stackelberg game), player ℓ is strictly better off in the Stackelberg game than in the simultaneous move game if the Stage 4 player, player n, makes a positive contribution in the Stackelberg game (and also in the simultaneous move game, by our setup), and player ℓ is equally well off in the Stackelberg game as in the simultaneous move game if player n makes no contribution in both games. These results are quite similar to a set of results for the two-player Stackelberg game of Varian (1994). The derivations below are essentially the same as those of Varian (1994).

We only need to analyze the case where player ℓ is a contributor in the simultaneous move game to show the above results. First, note that player ℓ's first-order condition for public good contribution in the simultaneous move game is

$$\frac{dU_\ell}{dg_\ell} = \frac{\partial U_\ell}{\partial G} - \frac{\partial U_\ell}{\partial x_\ell} = 0 \tag{15}$$

His first-order condition for public good contribution in the Stackelberg game is

$$\frac{dU_\ell}{dg_\ell} = \frac{\partial U_\ell}{\partial G}(1 + g_n'(G^1)) - \frac{\partial U_\ell}{\partial x_\ell} \tag{16}$$

Because g_i, $i \neq \ell$, are all the same across the two games and because $g_n'(G^1)$ is between -1 and 0 when $g_n > 0$ and is equal to 0 when $g_n = 0$, we know that the second first-order condition is less than or equal to 0 when evaluated at the simultaneous move game equilibrium contributions. Thus, if all others' contributions are the same across the two games, player ℓ's contribution in the Stackelberg game must be strictly less than that in the simultaneous move game if $g_n > 0$ in the Stackelberg game, and they are equal across the two games if $g_n = 0$ in the Stackelberg game. The statements 1), 2), and 3) follow directly.

Now we can complete step (1) by way of contradiction.

Suppose that the step (1) statement is not true. That is, suppose that, in the solution to player n's QCM maximization problem, $\tilde{G}_{-i} \leqslant G^*_{-i}$, for some $i \in \{1, \cdots, n\}$. Then the "virtual income" of player i, $w_i + G_{-i}$, which is the argument of the demand function for public good, $f_i(w_i + G_{-i})$, under the Bergstrom-Blume-Varian setup presented above, would not be larger than that in the Stackelberg fallback. We look at two cases of $\tilde{G}_{-i} \leqslant G^*_{-i}$ below.

Case 1: $\tilde{G}_{-i} < G^*_{-i}$ for some $i \in \{1, \cdots, n\}$.

Consider four situations. The first is the Stackelberg game in the fallback, where individual contributions are g_i^*. The second is a fictitious situation derived from the first one. Here, the contributions of all players other than i's are fixed at their Stackelberg levels as in the first situation, and only player i is allowed to re-maximize utility given the fixed Stackelberg contributions g^*_{-i}'s. The third situation is the QCM outcome, where individual contributions are $\tilde{g}_i$'s. The fourth situation is a fictitious situation derived from the third one. In this situation, the contributions of all players other than i's are fixed at their QCM levels as in the third situation, and only player i is allowed to re-maximize utility given the fixed QCM contributions $\tilde{g}^*_{-i}$'s.

Player i will be at least as well off in the second situation as in the first, and also at least as well off in the fourth situation as in third, because he is allowed to individually re-maximize holding other players' contributions fixed. The comparison between the first and the second situations follows from the intermediate result above. The comparison between the third and the fourth situations can be seen as follows. Note that $\partial U_i/\partial G - \partial U_i/\partial x_i < 0$ from the FOC's for the QCM, so the QCM allocation is not *ex post* utility maximizing from any individual player's point of view. Every player would like to reduce his contribution if he had a chance.

However, player i is strictly worse off in the fourth situation than in the second for the following reason. From Bergstrom, Blume, and Varian (1986), we know that in simultaneous move public good contribution games, player i's utility is strictly increasing in his virtual income. Besides, in the second situation for example, player i is maximizing utility given others' contributions, so player i's utility in this situations is the same as that in a simultaneous move contribution game where all other players' contributions sum to G^*_{-i}, since other players' contributions are held fixed; the same can be said for the fourth situation, where all other players' contributions sum to $\tilde{G}_{-i}$. Thus, since the virtual income in the second situation, $w_i + G^*_{-i}$, is greater than that in the fourth situation, $w_i + \tilde{G}_{-i}$, player i will be strictly better off in the second than in the fourth situation.

Therefore, player i is strictly better off in the first situation than in the third. That is, if $\tilde{G}_{-i} < G^*_{-i}$, we must have $U_i^{\text{QCM}} < U_i^{\text{Sta}}$, which violates player i's participation constraint.

Case 2: $\tilde{G}_{-i} = G^*_{-i}$ for some $i \in \{1, \cdots, n\}$.

From the reasoning above that utilizes a fictitious simultaneous move game, we know that the highest possible utility that the QCM allocation can give player i is exactly the same as the utility that player i can get in the Stackelberg fallback.[8] This is fine in the QCM game if $k_i = 1$, that is, if player i has no bargaining power or only wishes to get the same level of utility as in the Stackelberg fallback. However, total contributions $\tilde{G}$ must then be equal to G^*. If $\tilde{G} = G^*$, there would only be two possibilities: either some players are worse off in the QCM than in the Stackelberg fallback in order for some other players to be better off in the QCM than in the Stackelberg fallback; or, else, every player gets the same utility in the QCM as in the Stackelberg fallback. Thus, either some players' participation constraints are violated, or the QCM does not improve any player's utility upon the Stackelberg fallback. Therefore, this case is also not possible under the QCM.

Thus, for every player in the QCM game, the total contribution by all others is strictly higher than that in the Stackelberg fallback.

(2) $\tilde{G} \equiv \sum_{\ell=1}^{n-1} \tilde{g}_\ell + \tilde{g}_n > G^* \equiv \sum_{\ell=1}^{n-1} g_\ell^* + g_n^*$ must hold. That is, total contributions must be greater than that under the Stackelberg game.

Step (1) result implies that at least for some Stage 2 player ℓ, $\tilde{g}_\ell > g_\ell^*$, since otherwise it is impossible for the last stage player n to have $\tilde{G}_{-n} > G_{-n}^*$. Therefore, if the contrary of (2) is true, the participation constraint cannot be satisfied for this first-stage player ℓ, because player ℓ is contributing more in the QCM than in the Stackelberg game but is enjoying a lower level of public good in the QCM than in the Stackelberg.

(3) In principle, for a particular Stage 2 player ℓ, $\tilde{g}_\ell$ can be greater than, equal to, or less than g_ℓ^*, depending on the utility functions and on the constants k_ℓ's that represent bargaining powers.

[8] In fact, for the non-contributors in the QCM allocation, their QCM utility is the same as their Stackelberg utility; for the contributors, their QCM utility is strictly lower than their Stackelberg utility. One can see this from the QCM FOC $\partial U_i/\partial G - \partial U_i/\partial x_i < 0$, which holds true for both contributors and non-contributors.

Results in steps (1) through (3) should be fairly standard for this type of Pareto-optimizing problem. For ease of reference, we summarize them as follows: in the solution of player n's QCM maximization problem, which would lead to efficiency in total contributions if the allocation can be implemented in the QCM game, for every player, total contributions by all other players is higher in the QCM than that in the Stackelberg fallback; total contribution is higher in the QCM than that in the Stackelberg fallback; the contribution by an individual player may not necessarily be higher than that in the Stackelberg fallback.

With these results, we may proceed to show that $\tilde{G}^1$ would emerge in the equilibrium in Stage 2 contributions.

The proof below largely follows the proof of Proposition 3 of Andreoni (1998), who looked at a situation similar in nature to the one we are dealing with here. In Andreoni (1998), a capital investment type of public good can only be constructed if the total contributions reach a certain threshold, or else the contributions could be wasted, refunded, or used to construct some other public good that is less valuable to the players. His Proposition 3 deals with the case where this threshold exceeds the Nash equilibrium level of total contributions in a simultaneous contribution game by the same set of players but without the threshold technology for public good production. This situation is the same in nature as Stage 2 in our QCM. In fact, the structure of the entire proof contained in steps (1) through (5) here is essentially inspired by Andreoni's proof of his Proposition 3.

(4) With the QCM allocation $\{\tilde{g}_i, \ i = 1, \cdots, n\}$, no Stage 2 player wants to increase his contribution.

Starting from the optimum of the QCM maximization problem, an increase in any player ℓ's ($\ell \in \{1, \cdots, n\}$) contribution g_ℓ above $\tilde{g}_\ell$ by some small $dg_\ell > 0$

changes his utility by

$$\left.\frac{dU_\ell(G, x_\ell)}{dg_\ell}\right|_{\substack{g_i=\tilde{g}_i,\\ i=1,\cdots,n}} = \frac{\partial U_\ell}{\partial G} - \frac{\partial U_\ell}{\partial x_\ell} < 0, \quad \ell = 1, \cdots n-1. \tag{17}$$

This is strictly negative for the following reasons. 1) All the λ_ℓ's, $\ell = 1, \cdots, n-1$, in the first-order conditions are non-negative (since participation constraints are just binding or strictly binding, as we reasoned before). 2) At least some λ_ℓ's are strictly positive (corresponding to those strictly positive $\tilde{g}_\ell$'s and thus strictly binding constraints) 3) The $\partial U_\ell / \partial G$ in the first-order conditions are strictly positive. Therefore, no Stage 2 player wants to contribute any more than $\tilde{g}_\ell$. We will see in the subsection for Stage 4 below that this is also true for player n. Thus, the statement in this step applies to all players in the game.

Therefore, the nature of the allocation is as follows. Under the QCM allocation, every player's "virtual income", $w_i + \tilde{G}_{-i}$, is higher than that in the Stackelberg fallback, $w_i + G^*_{-i}$. Because both goods are strictly normal, each player i, $i = 1, \cdots, n$ must therefore prefer to consume more of both G and x_j than in the Stackelberg equilibrium if it is possible to do so, if one recalls the reasoning above using the Bergstrom-Blume-Varian (1986) setup of the consumer optimization problem. However, under the QCM allocation, each player i's consumption of the public good is higher that in the Stackelberg equilibrium, and each player i's consumption of the private good is such that, although it may be more than, less than, or equal to that in the Stackelberg equilibrium, every player has an incentive to "restore balance" in the first-order condition by decreasing the consumption of G and increasing the consumption of x_i through reducing his own contribution g_i.

(5) Every Stage 2 player is pivotal in the QCM, and no one is willing to provide any less than $\tilde{g}_\ell$. If any Stage 2 player deviates and provides less than $\tilde{g}_\ell$, $G^1 < \tilde{G}_1$ will occur. In this case, Stage 2 players have to adjust their contributions in

Stage 3, knowing that now player n will respond according to his self-interest only. Everyone would be back in the Stackelberg fallback resulting from the play in Stages 3 and 4, and every Stage 2 (Stage 3) player gets a utility lower than that under the QCM by $(k_i - 1)U^{\mathrm{Sta}}$. Thus, no Stage 2 player is willing to provide any less than $\tilde{g}_\ell$.

Adjustment of contributions by Stage 2 players in Stage 3, in the case where $G^1 < \tilde{G}^1$, is key for (5) to hold. Intuitively, the players most likely to contemplate deviation are those who have little bargaining power and can only get in the QCM allocation some levels of utilities that are close to their Stackelberg fallback utilities. Suppose no adjustment is allowed, that is, there is no Stage 3. Suppose one such Stage 2 player, ℓ, deviates downward (upward deviation is not optimal, as we showed in step (4) above), given the other Stage 2 players' contributions, $\tilde{g}_m$, $m \in \{1, \cdots, n-1\}$, $m \neq \ell$; player n then reacts in the Stage 4 by maximizing his own utility without worrying about the announced matching plan. We will show that, in this event, if the sum of all other Stage 2 players' contributions in the QCM is greater than that in the Stackelberg equilibrium ($\sum_{m \neq \ell} \tilde{g}_m > \sum_{m \neq \ell} g^*_m$), player ℓ's budget set in the QCM will be strictly larger than that in the Stackelberg game, and player ℓ can always find some deviation that makes himself better off than in the Stackelberg fallback; if the reverse holds ($\sum_{m \neq \ell} \tilde{g}_m < \sum_{m \neq \ell} g^*_m$, which is the case if ℓ is the only Stage 2 player who contribute more in the QCM than in the Stackelberg), player ℓ's budget set in the QCM will be strictly smaller than in the Stackelberg, and he cannot find a profitable deviation. As the first case is the most likely one, Stage 3 adjustment is important for the QCM allocation to be sustained as equilibrium.

Recall the Bergstrom-Blume-Varian setup of the individual player's maximization problem. In the Stackelberg game, player n solves the following problem when he contributes in the last stage, given the contributions G^1 in the previous stages

$$\max_{\{G\}} U_n(G,\ x_n)$$
$$\text{s.t. } x_n + G \leqslant w_i + G^1 \tag{18}$$
$$G \geqslant G^1.$$

The solution gives the demand function for the public good

$$G(G^1) = \max\{f_n(w_n + G^1), G^1\}. \tag{19}$$

As discussed in subsection 2.3.1, the properties of this function corresponds to the properties of player n's individual contribution, $g_n = \max\{f_n(w_n + G^1) - G^1, 0\}$: $g_n = f_n(w_n) > 0$ when $G^1 = 0$; g_n strictly decreases in G^1 with slope between -1 and 0 up to the level the crowding out level G^{1c}; $g_n = 0$ for all $G^1 \geqslant G^{1c}$.

This reaction function $g_n(w_n + G^1)$ was depicted as $LDPG^1$ in Figure 2.2. For ease of reference, we reproduce the same figure here. OL is equal to $f_n(w_n)$, the amount that player n will contribute if he is the only contributor in the game; OP is equal to G^{1c}, the amount of Stage 2 contribution that will just crowd out player n's contribution in Stage 4; OA is assumed to be the amount that all Stage 2 players other than player ℓ contribute in the Stackelberg game, while OE is the counterpart in the QCM game.

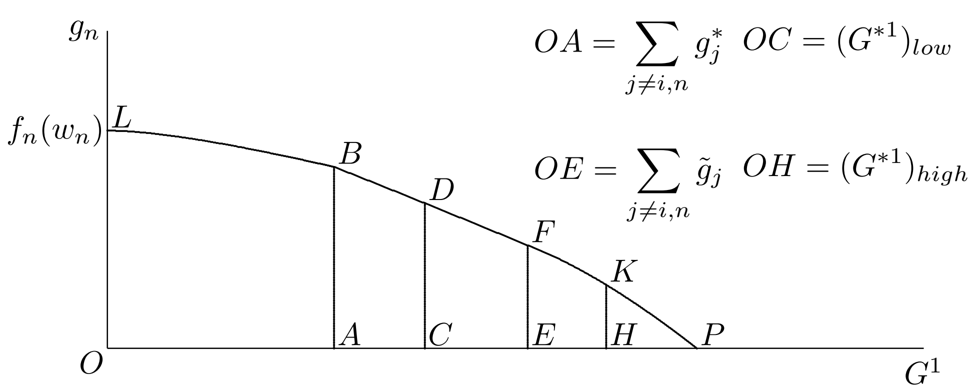

Figure 2.2

These properties of player n's reaction function define the shape of Stage 2 player ℓ's budget set, when all other Stage 2 players' contribution $\tilde{g}_m$, $m \neq \ell$ are given, and player ℓ contemplates deviation in anticipation of player n's response in Stage 4, in the absence of a Stage 3. Consider the consumption possibilities that player ℓ faces in the $(G, x_\ell)-$space, as depicted in Figure 2.3. 1) When the total contribution by the Stage 2 players other than ℓ is zero, player ℓ can enjoy the bundle $\{x_\ell = w_\ell,\ G = f_n(w_n)\}$ if he contributes nothing; this bundle is denoted as A in figure 2.3. As player ℓ increases his contribution g_ℓ, player n's contribution g_n will be crowded out gradually; suppose, for example, that player ℓ deviates to $g_\ell = EF$, then g_n will be FB and the point B on the boundary of the budget set is located. Once $g_\ell = G^{1c}$, player n's contribution is completely crowded out and the ratio of tradeoff between player ℓ's consumptions of the public good and the private good becomes 1. Thus, the boundary of the budget set in the first quadrant is given by $ABCD$, when $\sum_{m \neq \ell} g_m = 0$. 2) When the total contributions by the Stage 2 players other than ℓ is exactly G^{1c}, that is, $\sum_{m \neq \ell} g_m = G^{1c}$, player n's contribution will be completely crowded out, regardless of player ℓ's level of contribution, and player ℓ's budget line is given by AD in Figure 2.4. The ratio of tradeoff mentioned above is always 1. 3) When the total contribution of the Stage 2 players other than ℓ is between 0 and G^{1c}, player n contributes some positive amount between 0 and $f_n(w_n)$, depending on player ℓ's contribution, and player ℓ's budget line is given by AD in Figure 2.5.

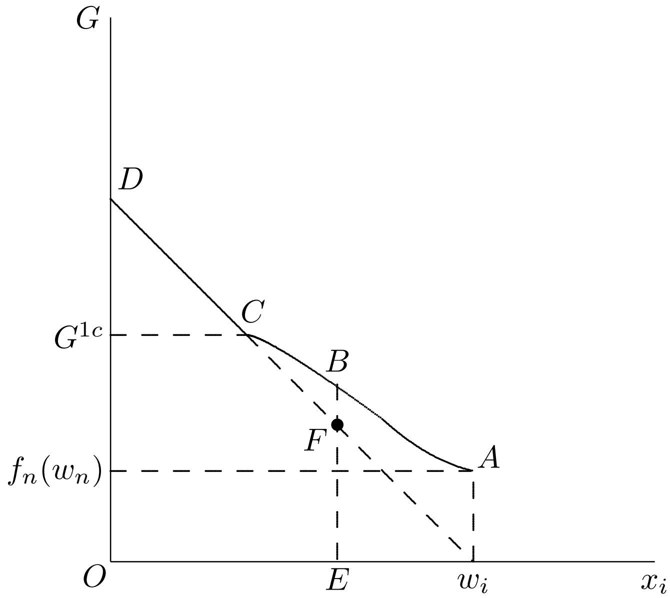

Figure 2.3

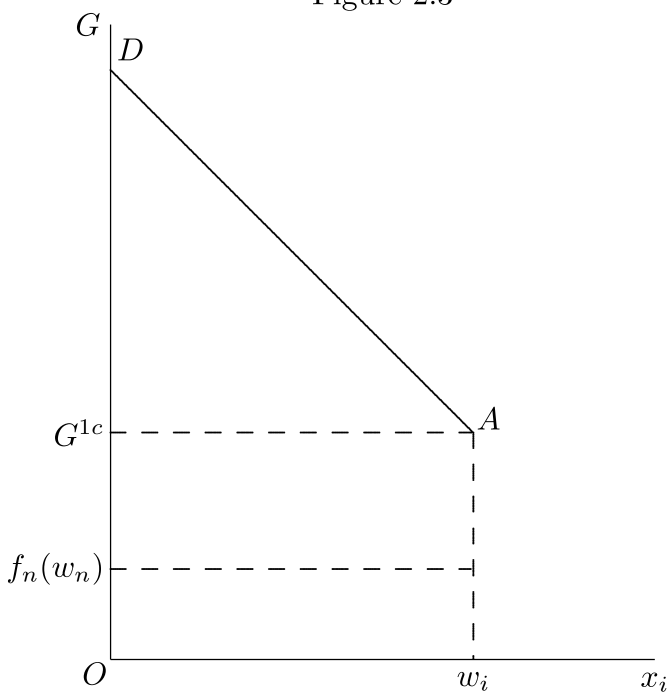

Figure 2.4

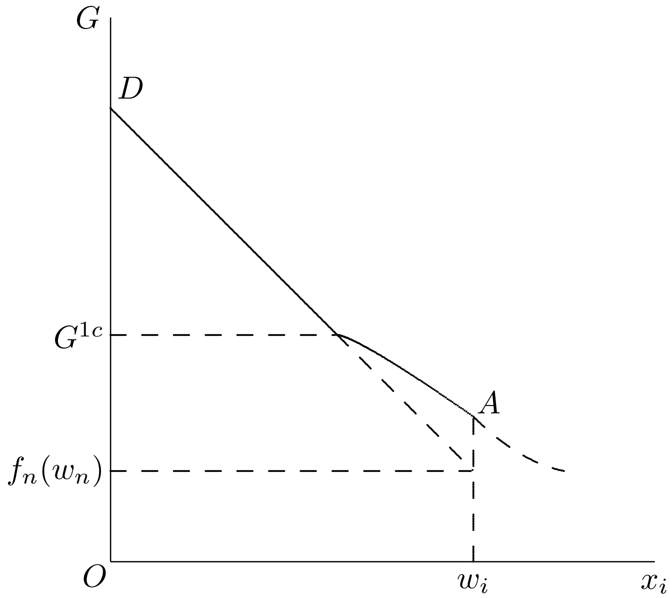

Figure 2.5

As we can see from these figures, as $\sum_{m\neq\ell} g_m$ increases (decreases), player ℓ's budget set becomes strictly larger (smaller). It is as if the boundary AD is pushed to the right as $\sum_{m\neq\ell} g_m$ increases from zero but does not exceed G^{1c}, and is pushed upward after $\sum_{m\neq\ell} g_m$ becomes greater than G^{1c}; vice versa. Thus, because the utility functions are strictly concave (corresponding to strictly convex preferences) and strictly increasing in both arguments, player ℓ will (will not) be able to find a profitable deviation, if $\sum_{m\neq\ell} \tilde{g}_m$ is greater (less) than $\sum_{m\neq\ell} g^*_m$. Since $\sum_{m\neq\ell} \tilde{g}_m > \sum_{m\neq\ell} g^*_m$ is more likely to happen for most players, it is important to have an adjustment Stage 3 for (5) to hold. This stage prevents any Stage 2 player to free ride, not on Stage 4 player n, but on the other Stage 2 players in a QCM.

From steps (4) and (5), we know that no Stage 2 player wants to deviate from $\tilde{g}_\ell$. Thus, $\{\tilde{g}_\ell,\ \ell = 1, \cdots, n-1\}$ in the solution to player n's maximization problem can be sustained as a Nash equilibrium in Stage 2, given that player n can commit to the QCM announced in Stage 1. Stage 3 will not be used in the actual play of the QCM game; it only serves as a "threat" to Stage 2 players so that the QCM can be implemented. Given the full information about preferences and the kind of perfect rationality these players have, when player n only announces the aggregate threshold $\tilde{G}^1$ to Stage 2 players, each Stage 2 player ℓ will be able to figure out and contribute exactly $\tilde{g}_\ell$, since this is the best he can do.

However, the fallback outcome is also an equilibrium of the game. If all $n-2$ stage 2players contribute their fallback amounts, it is optimal for the $n-1$st player to contribute his fallback amount. Consequently, player n will contribute his fallback amount and the overall outcome is the Stackelberg fallback.

Stage 4

We now turn to Stage 4. By reasoning similar to that in step (4) in the previous subsection, we know that player n does not want to contribute any more than $\tilde{g}_n$, since, in the optimum,

$$\frac{\partial U_n}{\partial G} - \frac{\partial U_n}{\partial x_n} < 0$$

by the QCM first-order conditions.

Because of his ability to commit, or because of the assumed large utility loss from reneging on the contingent matching plan, player n will not contribute any less than $\tilde{g}_n$ once $G^1 \geqslant \tilde{G}^1$. Besides, if Stage 2 players anticipate that player n may renege, they may not contribute $\tilde{g}_\ell$, $\ell = 1, \cdots, n-1$, too.

Thus, player n will contribute $\tilde{g}_n$, if Stage 2 contributions are no less than $\tilde{G}^1$, even though $\tilde{g}_n$ is not *ex post* utility-maximizing for player n.

Then, $\{\tilde{g}_i,\ i = 1, \cdots, n\}$ that solves the maximization problem can be sustained as a Nash equilibrium in the QCM game.

We collect some of the results in this section below:

In a QCM game with full information for all the players and the commitment by the player who sets the QCM and contributes in the last stage of the game, the contributions of all players in the game as desired by the QCM setter can be sustained as one subgame perfect Nash equilibrium; they lead to efficiency and maximize the QCM setter's utility, leaving the other players their reservation utilities. However, the fallback outcome can also be a subgame perfect equilibrium.

As we can see, the basic force at work is similar to that in the two-player game in section 2, and is thus again the same as that in the Bagnoli and Lipman (1989) paper. There are two main differences between the relevant game in that paper and the QCM game here. In the Bagnoli and Lipman paper, the fallback is determined by refund to the contributors in the case where the target level is not met; in the QCM, the fallback is determined by the Stackelberg equilibrium, which is made possible by allowing adjustment of contributions in Stage 3 by Stage 2 players in case the Stage 2 target level is not met. In addition, we need the commitment assumption to prevent the Stage 4 player from free riding on Stage 2 contributions and thus upset the mechanism. Both these differences are related to the fact that the game in Bagnoli and Lipman (1989) is a simultaneous move game and the QCM game here is a sequential one; the first difference is also related to the different technologies for producing the public good in the games, which was discussed earlier.

2.4 Some Extensions

In this section, we try to analyze five related issues.

First, given the QCM setter's ability to commit, we can equally well assume that he can commit to a zero contribution in the case where Stage 2 contributions do not meet the threshold level specified in the QCM. When the fallback was determined by the Stackelberg equilibrium in previous sections, the QCM setter's *ex post* utility-maximizing contribution may be greater than or equal to zero. When it would be greater than zero in the previous fallback, committing to zero contribution is not *ex post* utility maximizing for the QCM setter, *if* the fallback actually occurs. However, as we saw above, in the equilibrium outcome of a QCM, the fallback never occurs. We will show that, in some cases, committing to a zero contribution in the fallback can increase the QCM setter's utility in a QCM equilibrium, by weakening the Stage 2 players' fallback positions.

Second, we allow for the possibility of a subsidy from the QCM setter to the Stage 2 players, given that he is the only player who can credibly commit. We try to examine whether such a unilateral subsidy is to the advantage of the QCM setter. It turns out that such a subsidy cannot increase his utility above the level without such a subsidy.

The next three extensions directly draw upon Boadway, Song and Tremblay (forthcoming).

Third, sometimes voluntary contributions to multiple public goods are resolved sequentially. In a two-player setting, we introduce a second public good. The QCM is applied to the first public good only. As it turns out, if both players contribute to the second public good, the QCM will not be of any use, due to the neutrality outcome in the second public good.

In extensions 4 and 5, we use a two-player model and compare the levels of the public good and the players' utilities under the rate-matching process studied by the joint paper and under the QCM in this chapter. Extension four looks at a situation in which only one of the two players can commit in the rate-matching game; extension five deals with the case where both of the players in the rate-matching game can commit. Since the next chapter also analyzes a situation similar to that in extension five, we omit the details of derivations for this extension to save space.

In extensions 1 and 2, we still use the n-player model as in section 3. In the other three extensions, we use a two-player model, with the two players referred to as two countries, Rome and Greece.

2.4.1 **Changing the Fallback Position**

Suppose now that player n commits to the following matching plan: he will contribute $\tilde{g}_n$ if $G^1 \geqslant \tilde{G}^1$; but *zero* otherwise. In the case where $G^1 < \tilde{G}^1$, the adjustment of contributions by Stage 2 players is still allowed in Stage 3.

Now the fallback position is determined by the Nash equilibrium of the simultaneous move game played by the $n-1$ Stage 2 players, rather than by the Stackelberg equilibrium.

As can be seen from the analysis of Stage 2 players' budget sets in step (5) in the previous section, if player n contributes a positive amount in the Stackelberg equilibrium, the boundary of a typical Stage 2 player's budget set involves a non-linear portion as shown in Figure 2.5 (or 2.3); if player n now reduces his contribution to zero, with other things being held constant, this non-linear portion will shrink to a linear portion that extends from the negative 45^o line in, for example, Figure 2.5. Thus, if his optimal choice in the Stackelberg fallback was on the non-linear portion of the budget-set boundary, a typical Stage 2 player will be worse off in the new simultaneous move fallback than in the old Stackelberg one. Thus, as long as the following two conditions hold in the original Stackelberg fallback: 1) player n's contribution is positive (and consequently all Stage 2 players' budget-set boundaries in the fallback involve a non-linear portion), and 2) at least some Stage 2 players choose consumption bundles on the non-linear portion of their budget-set boundaries, player n can extract more surplus in a QCM game by committing to a zero contribution than by committing to a Stackelberg contribution.

2.4.2 **Allowing Player n to Subsidize**

Suppose player n can commit to giving the other players a subsidy at rate s for every unit of their contributions. Would this increase his utility further from the level in a QCM without such a subsidy?

We approach this question by examining whether, starting from a QCM allocation, increasing the subsidy rate slightly from zero could increase player n's utility in the QCM. Thus, we first setup a QCM maximization problem that has an expression for the subsidy rate s in the expressions, but bearing in mind that this s is equal to 0 now. After taking first-order conditions for the QCM maximization problem and working out some useful expressions, we apply the Envelope Theorem to the maximized utility function of player n to examine the effect of an incremental increase in s on player n's maximized utility.

With an expression for s in place, player n's problem is

$$\max_{\substack{\{\tilde{g}_\ell,\ell=1,\\ \cdots,n-1;\tilde{g}_n\}}} U_n(\sum_{m=1}^{n-1}\tilde{g}_m+\tilde{g}_n, w_n-\tilde{g}_n-s\sum_{m=1}^{n-1}\tilde{g}_m) \tag{20}$$

$$\text{s.t. } U_\ell(\sum_{m=1}^{n-1}\tilde{g}_m+\tilde{g}_n, w_\ell-(1-s)\tilde{g}_\ell) \geqslant k_\ell U_\ell^{\text{Sta}} = k_\ell U_\ell(\sum_{m=1}^{n-1} g_m^*+g_n^*, w_\ell-g_\ell^*), \quad (\lambda_\ell)$$
$$(k_\ell \geqslant 1, \quad \ell=1,\cdots,n-1).$$

The FOC's are

$$\frac{\partial U_n}{\partial G}-s\frac{\partial U_n}{\partial x_n}+\lambda_\ell(\frac{\partial U_\ell}{\partial G}-(1-s)\frac{\partial U_\ell}{\partial x_\ell})+\sum_{\substack{m=1,\\ m\neq\ell}}^{n-1}\lambda_m\frac{\partial U_m}{\partial G}\leqslant 0, \text{ equality if } \tilde{g}_\ell>0, \tag{21}$$

$$(\frac{\partial U_n}{\partial G}-\frac{\partial U_n}{\partial x_n})+\sum_{m=1}^{n-1}\lambda_m\frac{\partial U_m}{\partial G}\leqslant 0, \text{ equality if } \tilde{g}_n>0, \tag{22}$$

$$(\ell=1,\cdots,n-1).$$

By rearranging (21), the FOC's for $\tilde{g}_\ell,\ \ell = 1, \cdots, n-1$, we get

$$\lambda_\ell \geqslant \frac{\partial U_n/\partial x_n}{\partial U_\ell/\partial x_\ell}, \tag{23}$$

with equality if $\tilde{g}_\ell > 0$.

Denote the maximized utility functions as V_i, $i = 1, \cdots, n$, and denote the Lagrange function as $\mathcal{L}$. Also bear in mind that $\tilde{g}_i$, $i = 1, \cdots, n$, are the solutions to the maximization problem. Applying the Envelope Theorem to changes in the subsidy rate s, we have

$$\left.\frac{\partial V_n}{\partial s}\right|_{\tilde{g}_i, i=1,\cdots,n} = \frac{\partial \mathcal{L}}{\partial s} = -\frac{\partial U_n}{\partial x_n} \sum_{\ell=1}^{n-1} \tilde{g}_\ell + \sum_{\ell=1}^{n-1} \lambda_\ell \left(\frac{\partial U_\ell}{\partial x_\ell} \tilde{g}_\ell \right), \tag{24}$$

where $\mathcal{L}$ denotes the Lagrange function. Using the expressions above for the $\lambda_\ell,\ \ell = 1, \cdots, n-1$, we may note from the complementary slackness condition that

$$\lambda_\ell = \frac{\partial U_n/\partial x_n}{\partial U_\ell/\partial x_\ell}$$

for those strictly positive $\tilde{g}_i$'s, and that

$$\lambda_\ell \neq \frac{\partial U_n/\partial x_n}{\partial U_\ell/\partial x_\ell}$$

for those $\tilde{g}_i$'s that are zero. Thus,

$$\left.\frac{\partial V_n}{\partial s}\right|_{\tilde{g}_i, i=1,\cdots,n} = 0.$$

Thus, a unilateral subsidy by player n is redundant for him. This may be due to the fact that the QCM is already efficient and to the specifics of our setup.

2.4.3 **Adding a Second Public Good**

Suppose now that there are only two players, player 1 and player 2, and that there is a second public good, also financed by the voluntary contributions of the two players. For ease of reference, we think of the two players as two countries and call them Rome and Greece. The countries first simultaneously contribute to public good 1, to which a QCM, set by Rome, is applied, and then simultaneously contribute to public good 2, for which no QCM is in place. We would like to study whether the presence of the second public good affects the consequences of Rome being able to commit to a QCM for the first public good. It turns out that the addition of public good 2 in such a way has dramatic effects, if we assume an interior solution in the contributions to public good 2. Under this assumption, Rome's commitment to the QCM for public good 1 is rendered useless: it does not make Rome any better off than in a simple subgame perfect NE in contributions. This is the result of a neutrality outcome arising from contributions to the second public good, analogous to that in Bergstrom, Blume and Varian (1986).

In what follows, we use subscripts to distinguish between the two public goods. We use Roman and Greek letters to indicate the players. Rome's utility function is now $u(G_1, G_2, x)$ and Greece's is now $\upsilon(G_1, G_2, \chi)$. The sequence of events is now as follows. In Stage 1, Rome announces a QCM for public good 1, which is the same as that discussed before. In Stage 2, Greece contributes γ_1. In Stage 3, Rome acts according to the QCM and contributes g_1. Finally, in Stage 4, the players simultaneously contribute γ_2 and g_2 to public good 2, and consumption levels of the private good are determined as a residual. We begin with Stage 4.

Stage 4: Rome and Greece Choose g_2 and γ_2

Given g_1 and γ_1 from Stages 2 and 3, Rome and Greece choose g_2 and γ_2 as follows:

$$\max_{\{g_2\}} \; u(G_1, g_2 + \gamma_2, w - g_1 - g_2), \qquad \max_{\{\gamma_2\}} \; \upsilon(G_1, g_2 + \gamma_2, \omega - \gamma_1 - \gamma_2)$$

where $G_1 = g_1 + \gamma_1$. Rome and Greece have given amounts of net income available, $w - g_1$ and $\omega - \gamma_1$ respectively, to finance contributions g_2 and γ_2 and private consumption x and χ. Assume the NE is an interior one. It is then clear, following Bergstrom, Blume and Varian (1986), that the neutrality theorem applies to g_2 and γ_2, so a redistribution of net income has no effect on total contributions to G_2. The latter only depends upon aggregate income available in Stage 4, $w + \omega - g_1 - \gamma_1 = w + \omega - G_1$. Since $w + \omega$ is fixed, we can write the Stage 3 outcomes as $G_2(G_1)$, $x(G_1)$, and $\chi(G_1)$.

Stage 1: Rome Sets $\tilde{g}_1$ and $\tilde{\gamma}_1$ of the QCM

Anticipating $G_2(G_1)$, $x(G_1)$, and $\chi(G_1)$, where $G_1 = g_1 + \gamma_1$, Rome solves the following problem in setting a QCM:

$$\max_{\{\tilde{g}_1, \tilde{\gamma}_1\}} \mathcal{L} = u(G_1, G_2(G_1), x(G_1)) + \lambda[-v^{\text{res}} + v(G_1, G_2(G_1), \chi(G_1))]$$

We do not need to analyze the first-order conditions to characterize the solution. Due to the neutrality outcome in Stage 4, the utilities of both players depend on the level of G_1 only, as we have written them above. Suppose that there is a unique maximizer G_1 for Rome that solves

$$\max_{G_1} \ u(G_1, G_2(G_1), x(G_1))$$

and that there is also a unique maximizer for Greece that solves

$$\max_{G_1} \ v(G_1, G_2(G_1), \chi(G_1)).$$

Denote these maximizers as c_1 and ς_1. Suppose $c_1 < \varsigma_1$. If so, no matter what the threshold level $\tilde{\gamma}_1$ is, Greece would ignore Rome's QCM announcement and contribute ς_1 in Stage 2. Rome consequently contributes nothing, and $G_1 = \varsigma_1$. If $c_1 > \varsigma_1$, Rome might ask Greece to contribute some level of γ_1 and then make up the gap between γ_1 and c_1. This makes $G_1 = c_1$. However, this would give

Rome the same overall level of utility as if it directly contributes c_1 all by itself, because of the neutrality outcome in the last stage. Thus, a QCM would not make Rome any better off than in a setting without it. In fact, this is also true if the contributions to public good 1 is simultaneous and a rate-matching scheme committed to by one or both of the countries is in place, as studied by Boadway, Song and Tremblay (forthcoming).

The reason for these results is the neutrality outcome applying to G_2. It makes G_1, rather than its composition, the only quantity that matters.

2.4.4 Comparing with Rate-matching Where Only One Player Can Commit

The purpose of this and the next subsections is compare the outcomes under the QCM and those under the rate-matching methods analyzed in the joint work, Boadway et al (forthcoming). That paper considers the consequences of relaxing the full commitment assumption for the Guttman-Danziger-Schnytzer type of rate-matching process. In the two-country setup of the joint work, a rate-matching mechanism where both players can commit can be described as follows. There are two stages in the game. In Stage 1, Rome and Greece simultaneously announce matching rates, m and μ respectively, at which they will match the direct contributions of the other country. Then, in Stage 2, both simultaneously set their direct contributions, g and γ, respectively. Beside these direct contributions, Rome also needs to make a matching contribution of $m\gamma$, as it promised, and similarly Greece needs to make a matching contribution of μg. The full-commitment assumption means that both countries do not renege on their promised matching actions in Stage 2.

In this subsection, we assume that Rome, but not Greece, can commit to any matching rate promised in Stage 1. We will compare whether Rome and Greece are better off under this matching method or under the QCM. For simplicity, we only consider the fallback in which Rome's contribution in the last stage is zero. Such a fallback can occur for one of two reasons. 1) $g = 0$ is *ex post* optimal for Rome in a natural Stackelberg fallback (i.e., $g = 0$ solves Rome's maximization problem in the Stackelberg fallback, given the optimal contributions by Greece); 2) As in extension 1 above, Rome finds that $g = 0$ is *ex ante* optimal as it weakens Greece's fallback position and gives itself higher utility in the QCM, although $g = 0$ may not be *ex post* optimal for Rome if the fallback does occur.

In the next subsection, we briefly characterize the full-commitment outcomes and then compare them with those arising from the QCM.

In the case where only Rome can commit to a matching rate, only Rome chooses m in Stage 1, and direct contributions by both countries g and γ are made in Stage 2. Rome's utility function is $u(G, x) = u(g + \gamma, w - g - m\gamma)$, and that of Greece is $\upsilon(G, \chi) = \upsilon(g+\gamma, \omega-\gamma)$. Note that the total level of the public good is $G = g + \gamma + m\gamma$. We begin our analysis with Stage 2.

Stage 2: Rome and Greece Choose g and γ

Assuming an interior solution, the choice of g and γ by Rome and Greece satisfy the following first-order conditions, respectively,

$$F(g, \gamma, m) \equiv u_G - u_x = 0, \qquad \Phi(g, \gamma, m) \equiv (1 + m)\upsilon_G - \upsilon_x = 0 \tag{25}$$

These yield reaction functions $g(\gamma; m)$ and $\gamma(g; m)$, whose solutions give the Nash Equilibrium (NE for short below) contributions $g(m)$ and $\gamma(m)$. The local properties of the solutions can be obtained by differentiating the first-order conditions to get:

$$\begin{bmatrix} F_g & F_\gamma \\ \Phi_g & \Phi_\gamma \end{bmatrix} \begin{bmatrix} dg \\ d\gamma \end{bmatrix} = \begin{bmatrix} -F_m & -F_\mu \\ -\Phi_m & -\Phi_\mu \end{bmatrix} \begin{bmatrix} dm \\ d\mu \end{bmatrix} \tag{26}$$

Thus, for example,

$$\frac{d\gamma}{dm} = \frac{-F_g\Phi_m + \Phi_g F_m}{D} = -\frac{\upsilon_G[u_{GG} - 2u_{Gx} + u_{xx}]}{D}. \tag{27}$$

Where

$$D \equiv F_g\Phi_\gamma - \Phi_g F_\gamma. \tag{28}$$

Using the expressions for the derivative involved, we get

$$D = [(\upsilon_{\chi\chi} - (1+m)\upsilon_{G\chi})(u_{GG} - u_{Gx}) + ((1+m)\upsilon_{G\chi} - \upsilon_{\chi\chi})(u_{Gx} - u_{xx})$$

$$+(u_{xx} - u_{Gx})(\upsilon_{GG} - \upsilon_{G\chi})] > 0, \tag{29}$$

because terms in the square brackets are positive.[9] Therefore $d\gamma/dm > 0$, since the numerator is negative by the second-order conditions to Rome's Stage 2 problem and $D > 0$.

Thus, increases in m cause γ to increase as long as the solution is interior. However, one can check that $dg/dm \gtreqless 0$ because of the indeterminacy in the sign of Φ_m.

Stage 1: Rome Chooses m

Rome chooses m, anticipating the NE outcomes $g(m)$ and $\gamma(m)$ in Stage 2. Rome's utility is $u(g(m) + (1+m)\gamma(m),\ w - g(m) - m\gamma(m))$. Assuming an interior solution, we have

$$\frac{du}{dm} = \frac{\partial u}{\partial m} + \frac{\partial u}{\partial g}\frac{dg}{dm} + \frac{\partial u}{\partial \gamma}\frac{d\gamma}{dm}.$$

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[9] This comes from the normality of G, x, and χ. For example, for Rome, the normality of G and x requires:

$$\frac{dg}{dw} = -\frac{F_w}{F_g} = -\frac{u_{Gx} - u_{xx}}{F_g} > 0, \quad \frac{dx}{dw} = 1 - \frac{dg}{dw} = \frac{u_{GG} - u_{Gx}}{F_g} > 0$$

and $F_g < 0$ by the second order conditions. Analogous conditions apply for Greece.

Noting $\partial u/\partial g = 0$ by Rome's first-order condition in Stage 2 and substituting for the remaining terms, we have

$$\frac{du}{dm} = \gamma(u_G - u_x) + [(1+m)u_G - mu_x]\frac{\partial\gamma}{\partial m} = u_G\frac{\partial\gamma}{\partial m} > 0 \tag{30}$$

where we have used $u_G = u_x$ from Rome's Stage 2 first-order condition and $\partial\gamma/\partial m > 0$ from above. Therefore, Rome's utility increases in m as long as the NE is interior.

Given that $du/dm > 0$, Rome will continue to increase m until it eventually crowds out its own direct contribution g in Stage 2.[10] At the point where g is just crowded out,

$$\frac{du}{dm} = u_G\frac{\partial\gamma}{\partial m} = -u_G\frac{\Phi_m}{\Phi_\gamma}$$

the sign of which is given by the sign of $\Phi_m = (1+m)\gamma v_{GG} - \gamma v_{Gx} + v_G$, which is ambiguous. Therefore, Rome may or may not increase m further once g is crowded out.

Comparing with the QCM

Assume that, under the QCM, Greece has no bargaining power and its reservation utility is just equal to its utility in the fallback.

We showed above that, in the equilibrium of the rate-matching, $g = 0$ and $m > 0$ in equilibrium and Greece's contribution solves $\max_\gamma\ v((1+m)\gamma, \omega - \gamma)$. We assumed above that Rome contributes nothing in the fallback of the QCM. Then, Greece's maximization problem in the fallback of the QCM will be the same as its maximization problem in the rate-matching game but with $m = 0$.

[10] This is so even though $dg/dm \gtreqless 0$. If $dg/dm < 0$, Rome will increase m until g falls to zero. On the other hand, if $dg/dm > 0$, $m\gamma$ increases with m, so Rome eventually exhausts its budget. In either case, m increases at least to the level where until g falls to zero.

Because of the absence of a strictly positive m, Greece is necessarily worse off under the QCM. The QCM is efficient, so the total surplus from internalizing the free rider problem to be divided between Rome and Greece is larger than under the rate-matching case in this extension. Since Greece's utility in the QCM is just equal to its utility in the fallback situation, which is smaller than that in the rate-matching, and Rome gets all the remaining surplus, Rome must be better off under the QCM.

However, if Greece has some bargaining power and can get in a QCM a utility level that is higher than that in the fallback situation, or if $g \neq 0$ in the fallback, the comparison is likely to be ambiguous.[11]

To summarize, in the case where only Rome can commit to a matching rate m, Rome increases m until it crowds out its direct contribution; Greece's contribution γ rises with m; if $g = 0$ in the fallback of a QCM and the QCM only leaves Greece its reservation utility in the fallback situation, Rome is better off, Greece is worse off in the QCM than in the case in this extension.

2.4.5 **Comparing with Rate-matching Where Both Players Can Commit**

Now, both Rome and Greece can set matching rates in Stage 1 and commit to them. The two-stage process then becomes the Guttman-Danziger-Schnytzer mechanism. Since this structure underlies part of the analysis in the next chapter and is explored in detail there, we only make some qualitative comments here.

In this case, we can show[12] that the equilibrium allocation is efficient; the allocation replicates the Lindahl allocation in that the total contributions by

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[11] In section 4 of Boadway, Song and Tremblay (forthcoming), we also compared the rate-matching case in this subsection with the simple one stage Nash equilibrium in contributions, and with a rate-matching case where both countries can commit to their matching rates.

[12] See the benchmark case in section 3 of Boadway, Song and Tremblay (forthcoming).

Rome and Greece, $c \equiv g + m\gamma$ and $\varsigma \equiv \gamma + \mu g$ respectively, reflect Lindahl prices; the matching rates satisfy $m = c/\varsigma = 1/\mu$, so $m\mu = 1$; direct contributions g and γ are indeterminate, but the total contributions c and ς are determinate; and both countries are indifferent between direct and matching contributions.

Since the QCM allocation is also efficient, it is not clear which matching arrangement makes Rome (and Greece) better off. It seems that the comparison can go either way and may depend on the specifics of the utility functions, the fallback situation of the QCM, and the bargaining powers of the players in the QCM.

2.5 Concluding Remarks

This chapter tries to explore a method for increasing contributions to a public good—the QCM. It is inspired by the "corporate challenge gift" used in real world fund-raising.

In this chapter, we showed that a QCM can bring efficiency in public good contribution, given any fallback condition. The fallback positions could well depend on the players' ability to commit, as shown in subsection 2.4.1.

The strongest assumption we employed was perhaps that preferences are public information. Admittedly, an issue that motivated much of the work in the mechanism design in the public good contribution context was private information about preferences. Exploring the consequences of that for the QCM could be the next task.

However, some other assumption may be strong from a purely theoretical perspective but nonetheless reasonable from a practical point of view. For example, we assumed that one player can credibly commit to preannounced contingent matching plan. In reality, we do see such individuals or institutions. It may also be interesting to explore the consequences of partially incredible commitment.

Chapter III

Matching and Quantity-contingent Methods for Emissions Abatement

3.1 Introduction

Environmental pollution has been studied in economics as a problem involving a negative externality. When players fail to internalize this externality, there is usually more than the socially optimal level of pollution. Various non-cooperative and cooperative mechanisms have been proposed that can help players internalize the negative externality to reduce pollution to an efficient level.

In this chapter, we study the pollution abatement problem in an international context, where the players are countries and there is no "international government" or organization that has coercive power over the national governments. In reality, international pollution abatement usually involves some degree of cooperation among countries. The various international environmental agreements are examples. However, it may still be of interest to study the effectiveness of non-cooperative arrangements in this context. Some international agreement is not fully observed by the signatories; introducing some non-cooperative elements into the cooperative arrangements or into the general cooperative setting, if feasible, might enhance the effectiveness of the latter. Besides, the absence of a supra-national government may also make non-cooperative arrangements relevant.

We wish to examine whether two multi-stage, non-cooperative processes for increasing voluntary public good contributions could be adapted and used to bring efficient pollution abatement in an international context.

The first process studied is the Guttman-type mechanism for voluntary public good contribution. This type of mechanism was first proposed by Guttman

(1978), who used quasi-linear preferences, and later extended to general preferences by Danziger and Schnytzer (1991) and Varian (1994), and to general externality problems by Guttman and Schnytzer (1992). The mechanism involves perfect information about preferences and consists of two stages. Take the model of Danziger and Schnytzer (1991) for example. In the first stage, each party simultaneously announces a subsidy rate at which it will subsidize all other parties' stage-two flat contributions ("purchases") to the public good. In stage two, each party simultaneously contributes (purchases) its flat amount of the public good, at a price that is equal to the constant per unit market price of the public good minus the sum of per unit subsidies from all other parties. Under certain conditions, a subset of the subgame-perfect equilibria of the two-stage game is the Lindahl equilibrium. As Danziger and Schnytzer commented (p. 57), this Lindahl type of voluntary-exchange mechanism somehow "transformed" the public good into a private good, for which individuals can decide the quantity to "purchase", given the price that is decided, not by him/her-self, but by all other individuals in the game. Guttman and Schnytzer (1992) apply this idea of matching the other players' actions to a general model where the agents' actions can cause either positive or negative externalities to others. They view this type of mechanism as a non-cooperative way of implementing the Coasean solution to the externality problem, where arriving at private compensations is the essence. In these models, matching the other players' actions in-kind is akin to direct monetary compensation envisioned by Coase, as Guttman and Schnytzer put it.

The second process is the QCM studied in the previous chapter. As explained in that chapter, the QCM is based on the so-called "corporate challenge gift" way of fund-raising. Sometimes, some donors, especially large donors, may wish to leverage their support for a philanthropic cause or nonprofit organization in public fund-raising. Some of these donors may require that a specific amount be raised from the general public, before they make a sizable donation. The details of this arrangement will be given again in section 3 below.

A common feature of these processes is the commitment ability of the players in the game. In the first process, all players are assumed to be able to commit to the subsidy rates announced in the first stage; in the second, the (large) donors who wish to leverage the donations of the general public are assumed to be able to commit to the contingent matching plan announced at the beginning of the game. Without such commitment ability, these processes will not work. Boadway, Song and Tremblay (forthcoming) examined, among other things, the consequences of partially relaxing or modifying the commitment assumption in the Guttman type mechanism. For example, one of the results was that, when only one of the two players in a Guttman type game of public good contributions can commit to a matching rate, this player is better off than when both players can commit to such matching rates. The expositions in some parts of this chapter are parallel to those in that paper.

In the sections below, we adapt the processes above to a two-country pollution abatement model. Our focus is to examine whether and how the adapted processes could improve upon the outcomes when there is not any such scheme in place.

We find that both schemes generate constrained efficiency: the schemes induce each country to internalize the negative externality it causes to the other country; however, due to the difference in the abatement cost functions, global abatement effort is in general not efficiently distributed across the two countries such that the marginal costs of abatement are equalized. This problem is similar that found in some models of voluntary contributions to public good. Buchholz and Konrad (1995), for example, showed that, when contribution productivities differ, the less productive country can gain by making unconditional transfers to the more productive country. Ihori (1996) analyzed the welfare effect of changes in productivity differentials in a similar context and showed that a country with high productivity does not necessarily enjoy high welfare. In the model of this chapter, government policies seem to lack one degree of freedom, and the inefficient distribution of abatement cost could only be overcome by introducing another instrument. On this point, a comparison may be made to the model of Caplan and Silva (2005). In their model, countries make environmental policies before an international agency implements *ex post* redistributive transfers among them. Beside these, there is also an international market for trading pollution permits among the countries. Fully efficient allocation of pollution can be attained in equilibrium. Thus, the matching mechanisms we consider play a similar role as the international redistributive transfers in the Caplan and Silva (2005) model, but they cannot replace the international permit trading in that model, which ensures that the marginal costs of abatement are equalized.

Since we are adapting some voluntary public good contribution processes to study the environmental pollution abatement problem, it may be interesting to see what happens if the two issues are linked and have to be resolved together. Thus, after the analysis above, we also introduce voluntary contributions to a public good into the pollution abatement models above, and examine how the abatement outcomes are affected under different orderings of the public good contribution issue and the abatement issue. The purpose is to consider some of the consequences of linking the two issues in a simple model. It turned out that, with our special setup, the ordering of issues matters. If the public good contribution decisions follow the abatement decisions and if both countries contribute to the public good, the subgame perfect Nash equilibrium in abatement becomes fully efficient in the sense mentioned above, although the public good contributions remain inefficient. Thus, neither matching process would be useful for abatement any more. If the order is reversed, both processes can still play the same role and to the extent as before for abatement, but the public good contributions are again inefficient.

The rest of the chapter is organized as follows. Section 2 presents the model. In section 3, we analyze the rate-matching method and characterize some of its properties. In section 4, we study the quantity-contingent method. In section 5, we introduce the public good contribution issue into modified versions of the pollution abatement models of sections 3 and 4, and study the outcomes when the two issues are resolved sequentially. Section 6 concludes.

3.2 Model

The setup resembles those of Weitzman (1974), Roberts and Spence (1976), and Kwerel (1977). There are two countries in the world, country 1 and country 2. Whenever possible, we will use Roman letters for quantities and functions associated with country 1, and Greek ones for those associated with country 2. Each country j, $j = 1, 2$, has a polluting sector. In the absence of any government intervention, these sectors emit amounts e and η, respectively, of pollutants into the environment. These emissions are trans-boundary and cause various damages to both countries. The damages in terms of economic resources are summarized by the damage functions, $D(e+\eta)$ and $\mathit{\Delta}(e+\eta)$, for the two countries respectively. Country 1's (2's) government, if it wishes, can reduce e (η) by a desired amount r_0 (ρ_0) through such market-based pollution control instruments as a permit trading system, a tax system, or a mixture of them. These abatement efforts, r_0 and ρ_0, are also costly, and their costs are summarized by the abatement cost functions, $C(e - r_0)$ and $\mathcal{C}(\eta - \rho_0)$, respectively. Note that the arguments of these cost functions are the levels of emissions after abatement, rather than the abatement efforts themselves. The total resource cost to country 1 arising from the damages of the world's emissions and the abatement of its own emissions is $D(e-r_0+\eta-\rho_0)+C(e-r_0)$, and that to country 2 is $\mathit{\Delta}(e-r_0+\eta-\rho_0)+\mathcal{C}(\eta-\rho_0)$.

We assume that the damage functions and the abatement cost functions satisfy the following assumptions

$$\begin{gathered} D'(\cdot)>0,\ D''(\cdot)>0, \quad \mathit{\Delta}'(\cdot)>0,\ \mathit{\Delta}''(\cdot)>0, \\ C(e)=0,\ C'(\cdot)<0,\ C''(\cdot)>0, \quad \mathcal{C}(\eta)=0,\ \mathcal{C}'(\cdot)<0,\ \mathcal{C}''(\cdot)>0, \end{gathered} \tag{1}$$

recall that e and η are the "laissez faire" levels of emissions. Thus, the damages are increasing and strictly convex in the level of global emissions $e-r_0+\eta-\rho_0$; the abatement costs are not incurred when there is no cleaning up, and they are increasing and strictly convex in efforts, r_0 and ρ_0, respectively (or, equivalently, deceasing and strictly convex in the levels of the countries' emissions, $e-r_0$ and $\eta-\rho_0$, respectively).

Each country's government sets abatement effort to minimize its total cost. We assume that, in the absence of any matching mechanism discussed below, the two countries behave as Nash competitors. Our assumptions on the functions then imply that there is a unique Nash equilibrium in abatement efforts r_0 and ρ_0. One can immediately see that, because each country neglects the negative externality it imposes on the other, the Nash equilibrium, denoted as $\{r^{\mathrm{NE}}, \rho^{\mathrm{NE}}\}$, is inefficient. For future reference, note that globally efficient allocation of abatement efforts solves

$$\max_{r,\rho}\ D(e-r+\eta-\rho)+C(e-r)+\mathit{\Delta}(e-r+\eta-\rho)+\mathcal{C}(\eta-\rho),$$

the first-order conditions for which imply that

$$-\frac{D'}{C'}-\frac{\mathit{\Delta}'}{\mathcal{C}'}=1 \quad \text{and} \quad C'=\mathcal{C}'. \tag{2}$$

The first one says that the negative externality is internalized, and the second one says that marginal costs of abatement are equalized across countries.

3.3 A Rate-Matching Method for Emissions Abatement

Suppose now that country 1 and country 2 have individually optimal levels of emissions $e^{\mathrm{NE}} \equiv e - r^{\mathrm{NE}}$ and $\eta^{\mathrm{NE}} \equiv \eta - \rho^{\mathrm{NE}}$ respectively, to start with. Suppose that they are contemplating further reducing emissions in order to have better environment. In this section, we look at the Guttman type rate-matching mechanism. In particular, both can make direct abatements of own pollution levels $r \geqslant 0$ and $\rho \geqslant 0$,[13] respectively, but both can also commit to match the direct abatement of the other country by further abatement of own emissions, at rates $m \geqslant 0$ and $\mu \geqslant 0$, respectively. We analyze a situation involving two stages. In Stage 1, the two countries simultaneously choose matching rates $\{m, \mu\}$. In Stage 2, they simultaneously choose the levels of direct abatements $\{r, \rho\}$. After these two stages, country 1 then actually reduces its pollution level by a total of $R \equiv r + m\rho$, and country 2 by a total of $\Upsilon \equiv \rho + \mu r$. Thus, each country's total abatement effort is the sum of it direct and matching abatements. Both countries are assumed to be able to commit to their matching rates. We focus on the subgame perfect equilibria for this two-stage game.

The following result is demonstrated below.

Result:[14] An equilibrium involving positive abatement efforts r and ρ in the rate-matching game described above is constrained Pareto efficient if there is no international permit trading, in the sense that the externalities are internalized but marginal costs of abatement are not equalized across countries; it is fully efficient otherwise. Furthermore, an equilibrium satisfies the following properties:

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[13] In the Nash equilibrium with which we begin our analysis, each country is making some direct abatement efforts. To simplify, we shall simply call further abatements efforts over and above those in the Nash equilibrium "abatement efforts" below.

[14] Most of the results in this proposition parallel those in the Guttman-Danziger-Schnytzer mechanism in a public good contribution context, which can be found in the related papers cited in the Introduction.

i. Abatements by the two countries, $R \equiv r + m\rho$ and $\varUpsilon \equiv \rho + \mu r$, reflect Lindahl prices;
ii. Matching rates satisfy $m = R/\varUpsilon = 1/\mu$, so $m\mu = 1$;
iii. Direct abatements r and ρ are indeterminate, but total abatements R and $\varUpsilon$ are determinate; and
iv. Both countries are indifferent between direct and matching abatements.

The demonstration involves solving the two stages by backward induction. The exposition parallels that of Boadway, Song and Tremblay (forthcoming).

Stage 2: Choosing Direct Abatements r and ρ

In this stage, the matching rates m and μ chosen in Stage 1 are taken as given, and the countries set direct abatement efforts, r and ρ respectively, taking each other's choice as given. Thus, country 1 solves

$$\min_{\{r\}} D[(e^{\mathrm{NE}} - r - m\rho) + (\eta^{\mathrm{NE}} - \rho - \mu r)] + C(e^{\mathrm{NE}} - r - m\rho)$$

The first-order condition, denoted $F(r, \rho, m, \mu)$, is:

$$F(\cdot) \equiv -(1+\mu)D' - C' = 0, \quad \text{or} \quad \frac{D'}{C'} = -\frac{1}{1+\mu} \tag{3}$$

The solution yields country 1's reaction curve $r(\rho; m, \mu)$, whose slope is given by $\partial r/\partial \rho = -F_\rho/F_r$. Similarly, country 2 solves

$$\min_{\{\rho\}} \varDelta[(e^{\mathrm{NE}} - r - m\rho) + (\eta^{\mathrm{NE}} - \rho - \mu r)] + \mathcal{C}(\eta^{\mathrm{NE}} - \rho - \mu r)$$

The first-order condition, denoted $\varPhi(r, \rho, m, \mu)$, is:

$$\varPhi(\cdot) \equiv -(1+m)\varDelta' - \mathcal{C}' = 0, \quad \text{or} \quad \frac{\varDelta'}{\mathcal{C}'} = -\frac{1}{1+m} \tag{4}$$

The solution yields country 2's reaction curve $\rho(r; m, \mu)$, whose slope is given by $\partial \rho/\partial r = -\varPhi_r/\varPhi_\rho$.

The simultaneous solution to (3) and (4) yields the Stage 2 Nash Equilibrium direct abatements, $r(m, \mu)$ and $\rho(m, \mu)$.[15] The local properties of the solution can be obtained by differentiating $F(\cdot)$ and $\Phi(\cdot)$ to get:

$$\begin{bmatrix} F_r & F_\rho \\ \Phi_r & \Phi_\rho \end{bmatrix} \begin{bmatrix} dr \\ d\rho \end{bmatrix} = \begin{bmatrix} -F_m & -F_\mu \\ -\Phi_m & -\Phi_\mu \end{bmatrix} \begin{bmatrix} dm \\ d\mu \end{bmatrix} \tag{5}$$

So, for example,

$$\left.\frac{dr}{dm}\right|_\mu = \frac{-F_m\Phi_\rho + \Phi_m F_\rho}{det}, \quad \text{and} \quad \left.\frac{d\rho}{dm}\right|_\mu = \frac{-F_r\Phi_m + \Phi_r F_m}{det} \tag{6}$$

where $det \equiv F_r\Phi_\rho - \Phi_r F_\rho$. Stability of the NE requires that the slope of country 1's reaction curve in the $(\rho, r)-$space exceeds that of country 2's, $-F_\rho/F_r > -\Phi_\rho/\Phi_r$, which corresponds to $det > 0$.

Stage 1: Choosing Matching Rates m and μ

Anticipating $r(m, \mu)$ and $\rho(m, \mu)$, country 1 and country 2 choose m and μ simultaneously. Consider first country 1's choice of m, given μ. Country 1's total cost is now given by

$$D[(e^{\text{NE}} - r(m, \mu) - m\rho(m, \mu)) + (\eta^{\text{NE}} - \rho(m, \mu) - \mu r(m, \mu))]$$

$$+C(e^{\text{NE}} - r(m, \mu) - m\rho(m, \mu))$$

Differentiating this with respect to m, we obtain:

$$\left.\frac{d(D+C)}{dm}\right|_\mu = D' \cdot \left[-(1+\mu)\frac{\partial r}{\partial m} - (1+m)\frac{\partial \rho}{\partial m} - \rho\right] + C' \cdot \left[-\frac{\partial r}{\partial m} - m\frac{\partial \rho}{\partial m} - \rho\right] \tag{7}$$

[15] In the derivations near here and those in the appendix, we write direct abatements r and ρ as functions of Stage 1 matching rates m and μ. This may make it clearer where the matching rates enter, although it may make the notations a bit repetitive.

Using (3), (6), and the expressions for F_m, F_ρ and F_r, and assuming an interior solution for Stage 2, we can rewrite this as:[16]

$$\left.\frac{d(D+C)}{dm}\right|_{\mu} = -\frac{1}{det}(1-m\mu)D'\Delta' F_r \tag{8}$$

An analogous condition applies for country 2.

To interpret (8), we rewrite $det = F_r\Phi_\rho - \Phi_r F_\rho$ as follows, using the derivatives of (3) and (4):

$$det = (1-m\mu)[(1+\mu)D''\mathcal{C}'' + (1+m)\Delta''C'' + C''\mathcal{C}''] \tag{9}$$

so det is proportional to $1-m\mu$. By the assumptions on the damage functions and the abatement cost functions, the terms in the square bracket above are positive (thus Stage 2 Nash equilibrium is always stable if $m\mu < 1$); the term $D'\Delta' F_r$ in (8) above is also positive. These imply that

$$\frac{d(D+C)}{dm} < 0 \qquad \text{if } m\mu \neq 1 \tag{10}$$

Similar reasoning applies to country 2. Thus, an interior equilibrium in Stage 1 cannot occur if $m\mu \neq 1$. In particular, when $m\mu < 1$, both countries would want to increase their matching rates to reduce total costs, until $m\mu = 1$, when $d(D+C)/dm$ and $d(\Delta+\mathcal{C})/d\mu$ become indeterminate.[17] When $m\mu > 1$, both countries would want to increase their matching rates indefinitely without reaching an equilibrium. Besides, in reality, doing so will exhaust their budgets, as long as at least one country is making positive direct abatement effort. In that case, such increases in matching abatements would eventually crowd out direct abatements r and ρ, and neither direct nor indirect abatements will be made.

[16] See the Appendix on page 123 for the derivations for equations (8), (9), and (11). $A1$ of the Appendix also contains the expressions for F_r etc.

[17] See the Appendix for some derivations.

Such an allocation is dominated by an allocation involving a positive amount of at least one of r and ρ. Given these considerations, we might reasonably rule out any case involving $m\mu > 1$ as an equilibrium.

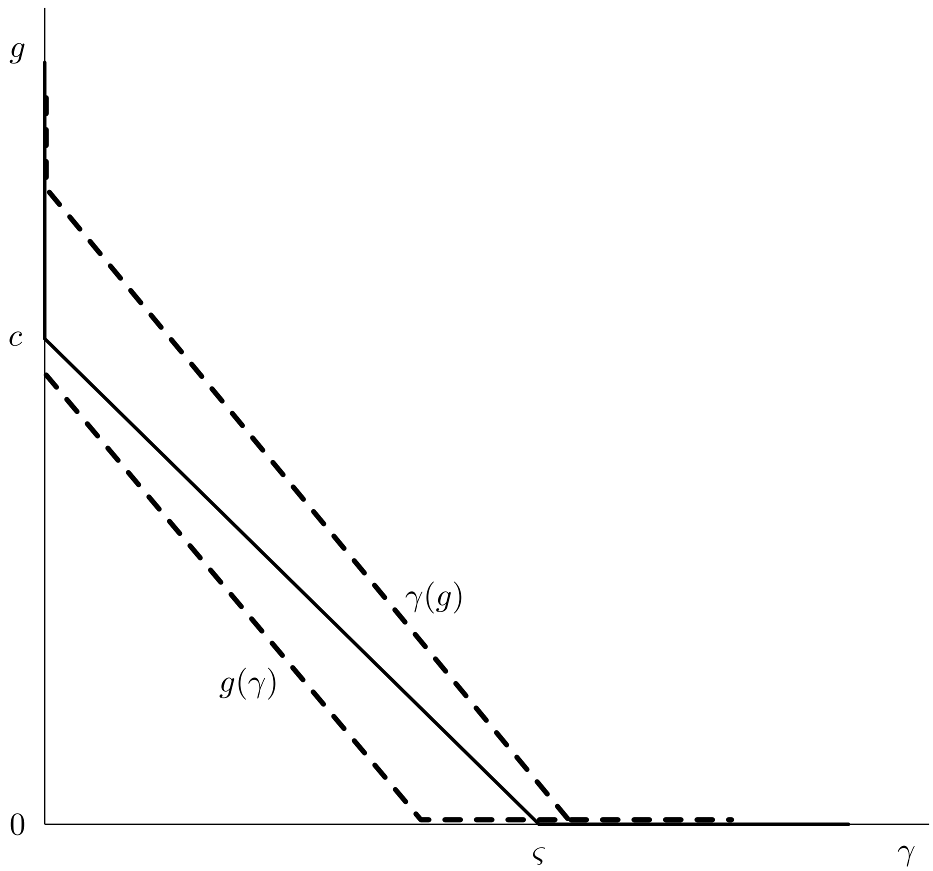

Figure 3.1. Stage 2, Rate-matching Method

Next, we show that the equilibrium values of m and μ are unique. Note first that, using (3) and (4), when $m\mu = 1$ the slopes of the two countries' reaction curves are, respectively:

$$\left.\frac{\partial r}{\partial \rho}\right|_{m\mu=1} = -\frac{F_\rho}{F_r} = -m, \quad \text{and} \quad \left.\frac{\partial \rho}{\partial r}\right|_{m\mu=1} = -\frac{\Phi_r}{\Phi_\rho} = -\mu \tag{11}$$

which implies that they are linear and parallel in the $(\rho, r)-$space, since $m = 1/\mu$.

Furthermore, in equilibrium, the reaction curves will be overlapping. To see this, suppose that the reaction curves were parallel but not overlapping, such as the dashed lines in Figure 3.1. In this case, the Stage 2 NE would be a corner solution with country 2 reducing emissions directly and country 1 making only a matching abatement. The Stage 2 first-order conditions would be

$$-(1+\mu)D' - C' > 0, \quad -(1+m)\mathit{\Delta}' - \mathcal{C}' = 0.$$

The first one can be written

$$D' < -\frac{C'}{1+\mu}.$$

The left hand side is the marginal benefit from further abatement, or equivalently the marginal damage. The right hand side is a measure of the effective marginal cost to country 1 of further abatement; country 1's marginal cost is deflated by a factor of $1/(1+\mu)$ due to the fact that, for each unit of country 1's direct abatement effort, country 2 will match by μ units. Thus the first-order condition simply says that the marginal benefit of further abatement is less than country 1's effective marginal cost. Country 1 then has an incentive to further reduce global abatement efforts by lowering its matching rate m to reduce its total abatement effort $R = m\rho$. Thus, such a pair of m and μ would not be an equilibrium. A similar argument would apply if reaction curves were such that only country 1 makes a direct abatement. Therefore, in Stage 1 equilibrium, m and μ are those for which the reaction curves overlap, as illustrated by the solid line in Figure 3.1.[18]

[18] The preceding analysis assumes interior Stage 2 contributions. A strict corner solution for one country can be ruled out. When $m\mu < 1$, if a country is making a strictly positive direct abatement, there is no cost to increasing its matching rate until the point where the other would just start to reduce emissions directly. At that point, the Stage 2 first-order conditions hold with equality for both countries, and both would increase their matching rates. When $m\mu > 1$, suppose only country

We can note from the above that the ratio $1/(1+\mu)$ can serve as an indicator of country 1's cost of direct abatement. It reflects the ratio at which country 1's marginal cost of abatement is effectively decreased due to country 2's matching. Similarly, $m/(1+m)$, which is what $1/(1+\mu)$ is equal to when $m\mu = 1$, can be interpreted as an indicator of country 1's cost of indirect/matching abatement. This expression is increasing in m. As country 1 applies matching abatement at rate m, the cost to it is effectively reduced by that factor since country 2 exerts a direct abatement effort that is being matched. Similar interpretations would apply for $1/(1+m)$ and $\mu/(1+\mu)$, which are equal when $m\mu = 1$, for country 2.

We can now show that an equilibrium will have the characteristics listed in the Result above. Rearranging $m\mu = 1$ and using conditions (3) and (4) imply:

$$m\mu = 1 \iff \frac{1}{1+m} + \frac{1}{1+\mu} = 1$$

$$\implies -\frac{D'}{C'} - \frac{\Delta'}{\mathcal{C}'} = 1, \quad \text{but} \quad C' \neq \mathcal{C}' \text{ in general}$$

Comparing these conditions with (2) for the fully efficient allocation, we can see that the equilibrium is constrained efficient when there is no mechanism such as permit trading between the two countries. That is, the matching scheme induces each country to internalize the negative externality it causes to the other country; yet, since there is no international permit trading etc., the marginal costs of abatement are not equalized and there is still room for Pareto improvement. Thus, we may clearly see the different effects of matching and instruments such as international permit trading in this context.

2 makes a direct abatement. Then, for country 1, $D' < -C/(1+\mu)$. But if $m\mu > 1$, it must be the case that country 1's cost of reducing directly is smaller than its cost of reducing indirectly, that is, $1/(1+\mu) < m/(1+m)$; this point will be explained in more detail in the next paragraph in text. Thus, $D < -mC/(1+m)$, which implies that country 1 would want to reduce m. A similar argument applies for country 2.

Moreover, the total abatement each country makes replicates their Lindahl abatement efforts (Danziger and Schnytzer 1991). To see this, note that the country 1's Lindahl price is $-D'/C'$, which is equal to $1/(1+\mu)$ by the first-order condition (1), and that country 1's Lindahl abatement effort is:

$$\frac{1}{1+\mu}(R+\Upsilon) = \frac{(1+\mu)r+(1+m)\rho}{1+\mu} = r + \frac{1+m}{1+\mu}\rho = r + m\rho$$

using $\mu = 1/m$. Thus, country 1's total abatement, $R = r + m\rho$, equals its marginal rate of substitution, which is equal to $1/(1+\mu)$, applied to the world's total abatements, $(R+\Upsilon)$. The same applies for country 2.

Next, in equilibrium, country 1 and country 2 are indifferent between making direct abatements and matching abatements. When $m\mu = 1$, $1/(1+\mu) = m/(1+m)$ and $1/(1+m) = \mu/(1+\mu)$. The first of these says that the ratio at which country 2's matching rate μ effectively shrinks the marginal cost to country 1 of reducing emissions directly is equal to the ratio at which country 1's own matching rate m effectively shrinks its marginal cost of reducing emissions indirectly. The second says the same for country 2. Thus, if country 1 were to increase its matching rate, starting from an equilibrium with $m\mu = 1$, it would be reducing emissions indirectly at a cost higher than the cost at which it can reduce emissions directly. The same would apply for country 2. Therefore, neither country would want to increase their matching rate beyond $m\mu = 1$. By the same token, when $m\mu < 1$, $1/(1+\mu) > m/(1+m)$. Therefore, it will be cheaper for country 1 to subsidize country 2 than to reduce emissions directly itself, so it will increase m. The same logic applies to country 2.

Finally, since the slope of country 1's reaction curve is $-m$ when $m\mu = 1$, country 1's reaction curve can be written $r = R - m\rho$, where R is the intercept on the $g-$axis. Thus, country 1's total abatement $R = r + m\rho$ is constant along the reaction curve and is equal to the value of the intercept R. Similarly, country 2's reaction curve can be written $\rho = \Upsilon - \mu r$, where Υ is country 2's total abatement. Furthermore, the values of R and Υ are constant in equilibrium. Consider the borderline case where, for example, $\gamma = 0$ and the first-order conditions (3) and (4) just hold with equality. (3) and (4) will then determine unique values of g and μ.[19] Similarly, the borderline case in the other extreme uniquely determines γ and m. Now recall that the reaction curves overlap off-axis, and that each country is indifferent between direct and matching contributions. Therefore, the unique values of R and Υ would be the same in these two borderline cases, as well as in the infinitely many interior cases, where both r and ρ are strictly positive. This shows the uniqueness of the equilibrium R and Υ. When the reaction curves overlap, $m = 1/\mu = R/\Upsilon$. Direct contributions r and ρ are indeterminate in Stage 2, but total contributions R and Υ are determinate.

This completes our discussion of the rate-matching method, which requires that both countries can commit to the matching rates announced before direct and matching abatements are carried out. We would like to point out that, in principle, the efficiency result and many other results above could be generalized to a setting where there are more than two countries, based on the results in the papers cited above. Besides, as Boadway, Song, and Tremblay (forthcoming) showed, allowing only one country to have the commitment ability in a two country setting can also lead to many interesting results, although in some cases it may be difficulty to tell whether global emissions are higher or lower than in a NE.

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[19] As Bergstrom, Blume, and Varian (1986) pointed out, it can be treacherous to assert uniqueness of solution to a system of nonlinear equations simply by checking if the number of unknowns and the number of equations are the same. For simplicity, we assume that our system is regular enough such that this is the case. Danziger and Schnytzer (1991), under a somewhat different setup for the public good contribution problem, showed uniqueness in another way that avoids the potential problem mentioned above with the uniqueness argument we are using here.

We now turn to the case where only country 1 can commit to a contingent matching plan on direct abatement efforts, and it uses this commitment ability to leverage higher direct abatement effort from country 2.

3.4 A Quantity-contingent Method for Emissions Abatement

The setting involves three stages. In Stage 1, country 1 announces a contingent abatement plan. The plan would state that, if country 2 sets its abatement effort ρ to be no smaller than some (high) threshold level $\tilde{\rho}$ in Stage 2, country 1 will set its own, r, to be (a high) $\tilde{r}$ in Stage 3 and can commit to $\tilde{\rho}$; however, if country 2 sets ρ to be smaller than $\tilde{\rho}$ in Stage 2, country 1 is no longer obliged to setting $\tilde{r}$ in Stage 3, and will instead choose an individually optimal r, given ρ from Stage 2. Then, in Stage 2, country 2 actually sets its ρ. In Stage 3, after observing country 2's action in Stage 2, country 1 actually acts according to the contingent matching plan announced in Stage 1. Country 1 is assumed to be able to commit in Stage 3 to the contingent matching plan announced in Stage 1 and does not renege after country 1 has taken effort to reduce its emissions in Stage 2. As we will see below, *ex post*, it is in Country 1's self-interest to renege and set an abatement effort lower than $\tilde{r}$ announced at the beginning, so the ability to commit is essential.

To induce country 2 to participate in such a quantity-contingent method (QCM) for abatement, country 1's QCM plan has to make country 2 at least as well off as in some fallback situation. A natural fallback situation here would be the one that arises when country 2 sets some ρ lower than $\tilde{\rho}$. Thus, in setting a QCM, country 1 essentially maximizes own utility subject to the participation of country 2. As one could foresee, given the assumption of country 1's commitment ability, the outcome is likely to be efficient, since country 1's problem is a Pareto optimizing one.

In what follows, we first characterize the fallback situation, which determines reservation utilities. Next, we analyze the allocation that will arise under a QCM and show that it is constrained efficient by comparing it to the solution of a the social planner's problem. Then, we show why this allocation can be sustained as a sub-game perfect Nash equilibrium outcome.

3.4.1 Reservation Utilities

In Stage 2 in the fallback situation, country 2 sets some ρ lower than $\tilde{\rho}$. Then, in Stage 3, country 1 sets r to maximize own utility, given ρ. When setting ρ in Stage 2, country 2 anticipates country 1's optimal response in Stage 3. Thus, country 2 is effectively a Stackelberg leader in Stage 2 in the fallback. We solve backwards.

Stage 3

Given ρ, country 1 solves

$$\min_{\{r\}} \; D(e^{\mathrm{NE}} - r + \eta^{\mathrm{NE}} - \rho) - C(e^{\mathrm{NE}} - r)$$

When an interior solution is assumed,[20] the first order condition is

$$F(\cdot) \equiv -D' - C' = 0, \tag{12}$$

from which we can obtain country 1's reaction function $r(\rho)$ with

$$\frac{dr}{d\rho} = -\frac{D''}{D'' + C''} \in (-1, 0), \tag{13}$$

since D'' and C'' are both positive.

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[20] We focus on the interior solution case. Allowing for the possibility of corner solution complicates the analysis somewhat, but it does not seem that it will add much for the main purpose of the analysis here. Interested readers are referred to Varian (1994).

Stage 2

Country 2 chooses ρ, anticipating the response $r(\rho)$ by country 1 in Stage 3. Country 2 solves

$$\min_{\{\rho\}} \ \Delta(e^{\mathrm{NE}} - r(\rho) + \eta^{\mathrm{NE}} - \rho) - \mathcal{C}(\eta^{\mathrm{NE}} - \rho)$$

The first order condition is

$$-\Delta' \cdot (1 + r'(\rho)) - \mathcal{C}' = 0. \tag{14}$$

Evaluating the LHS at ρ^{NE} and noting that $-\Delta' - \mathcal{C}' = 0$ in a Nash equilibrium, we can see that country 2's abatement effort in the Stackelberg equilibrium in the fallback is smaller than that in a simultaneous move NE. Thus, relative to the case where both countries move at the same time, country 2 sets its abatement effort "too low", in order to induce a higher abatement effort by country 1 and free ride on that effort. This tendency comes from country 2's Stackelberg leader advantage. Varian (1994) showed this result for the public good contribution game. In a sequential public good contribution game, when the follower contributes a positive amount, a Stackelberg leader contributes less than in the simultaneous move game, so as to free ride on the follower's contribution.

We denote by P and Π (standing for "payoff") the minimized sums of damage and abatement cost, for country 1 and 2 respectively, resulting from these sequential interactions in the fallback situation. In the QCM analyzed below, these would be the highest possible (worst) levels of sums that the countries require in order to participate. We also allow for reservation payoffs that are lower (thus, better) than these maxima, although in a reduced form way.

3.4.2 Quantity-contingent Method

Taking into account the outcomes in the fallback situation, country 1 sets the quantities, $\tilde{r}$ and $\tilde{\rho}$, for the QCM, and announces the matching plan in Stage 1. In later stages, country 1 can commit to what it announced before.

As mentioned above, country 1 would maximize its own sum of damage and abatement cost by choosing $\tilde{r}$ and $\tilde{\rho}$, subject to country 2's participation. Thus, country 1 solves the following problem

$$\begin{aligned} \min_{\{\tilde{r},\tilde{\rho}\}} \; & D(e^{\mathrm{NE}} - \tilde{r} + \eta^{\mathrm{NE}} - \tilde{\rho}) + C(e^{\mathrm{NE}} - \tilde{r}) \\ \text{s.t. } & \mathit{\Delta}(e^{\mathrm{NE}} - \tilde{r} + \eta^{\mathrm{NE}} - \tilde{\rho}) + \mathcal{C}(\eta^{\mathrm{NE}} - \tilde{\rho}) \leqslant \mathit{\Pi} \end{aligned} \tag{15}$$

Let the Lagrange multiplier associated with the constraint be λ. The Lagrangian function is

$$\mathcal{L} = D(e^{\mathrm{NE}} - \tilde{r} + \eta^{\mathrm{NE}} - \tilde{\rho}) + C(e^{\mathrm{NE}} - \tilde{r}) + \lambda[-\mathit{\Pi} + \mathit{\Delta}(e^{\mathrm{NE}} - \tilde{r} + \eta^{\mathrm{NE}} - \tilde{\rho}) + \mathcal{C}(\eta^{\mathrm{NE}} - \tilde{\rho})] \tag{16}$$

The first order conditions are the binding participation constraint for country 2 and

$$-D' - C' - \lambda \mathit{\Delta}' = 0 \tag{17}$$

$$-D' + \lambda(-\mathit{\Delta}' - \mathcal{C}') = 0 \tag{18}$$

Rearranging, we get

$$-\frac{D'}{C'} - \frac{\mathit{\Delta}'}{\mathcal{C}'} = 1 \tag{19}$$

However, $C'/\mathcal{C}' = \lambda = \partial P/\partial(\mathit{\Pi}) \neq 1$ in general, since in general there is no guarantee that the last partial derivative is equal to 1. Thus, the QCM allocation would be constrained efficient, provided it could be sustained as equilibrium. We now show that it can indeed be sustained as an equilibrium of the QCM game.

In the FOC's above, $D' > 0$ and $\mathit{\Delta}' > 0$ by our assumptions on the damage functions, and $\lambda > 0$ by the construction of the Langrangian function. Thus, under the QCM, $-D' - C' > 0$ and $-\mathit{\Delta}' - \mathcal{C}' > 0$. Referring to the first order condition for individual country's minimization above, we see that $\tilde{r}$ and $\tilde{\rho}$ under the QCM are "too high" from each country's individual point of view. Thus, no country wants to increase its abatement effort above $\tilde{r}$ and $\tilde{\rho}$, respectively.

One the other hand, no country wants to reduce its abatement effort below $\tilde{r}$ or $\tilde{\rho}$, respectively, as doing so would upset the mechanism and bring both countries to the fallback situation. For country 2, this would imply a sum of damage and abatement cost back to the level $\mathit{\Pi}$, which is no smaller than $k\mathit{\Pi}$ that it could get in the QCM. Similar reasoning applies to country 1.

We assumed that country 1 can commit to $\tilde{r}$ once country 2 indeed sets its emissions level to $\tilde{\rho}$. This is an important assumption. We saw above that, from each country's individual point of view, QCM abatement effort, $\tilde{r}$ and $\tilde{\rho}$ respectively, is too high. Thus, each has an incentive to unilaterally deviate and decrease its effort if it had the opportunity to do so. Country 2 does not have such an opportunity: since it moves before country 1, doing so will be observed and "punished" by country 1 in Stage 3. However, country 1 is the last mover, and country 2 cannot effectively punish country 1 if the latter reneges on its promise and free-rides on the former's abatement effort. Thus, it is important that country 1 can commit to the plan it announced in Stage 1. If country 2 cannot be sure that country 1 will commit, it will not choose $\tilde{\rho}$.

Thus, the QCM can bring constrained efficiency when there is no mechanism to equalize marginal costs of abatement. As before, allowing for international trade in permits would imply full efficiency.[21] Therefore, as with rate-matching

[21] Presumably, introducing such trade would require changing the setup, and the (constrained) efficiency condition may have a different form. Besides, in one case in the next section, where a public good contributions issue has to be resolved after the abatement issue is, full efficiency obtains without the use of either rate-matching or QCM when both countries make positive contributions to the public good. Although the formulation of payoff functions in that section is somewhat different from that in this and the previous sections, the full efficiency result is in contrast with the result in this and the previous sections.

in the previous section, a QCM helps each country internalize the negative externality imposed on the other country, while mechanisms such as international permit trading are needed to further equate marginal costs of abatement across countries.

The previous chapter showed that the QCM may be generalized to a multiple player setting, with some modifications of the game. Presumably, similar results would hold here, since the nature of the mechanisms is the same.

3.5 Adding a Public Good

In the previous sections, we looked at the abatement issue in isolation. In the real world, many international issues are interrelated. In this section, we examine a special case of such interrelation. We ask what happens if the matching games in abatement above are either followed or preceded by a public good contribution game, for which there is no matching scheme in place.[22] As we will see below, it suffices to consider the rate-matching only; the outcome for the QCM is straightforward to see.

We assume that the utility functions of the two countries are, respectively,

$$u(G,x) = u[g+\gamma, w-g-D(e^0-r-m\rho+\eta^0-\rho-\mu r)-C(e^0-r-m\rho)]$$

and

$$\nu(G,\chi) = \nu[g+\gamma, \omega-\gamma-\mathit{\Delta}(e^0-r-m\rho+\eta^0-\rho-\mu r)+\mathcal{C}(\eta^0-\rho-\mu r)].$$

We use e^0 and η^0 to denote the initial levels of emission, since now the game is a multi-stage one. These initial levels could be the Nash equilibrium levels or levels determined in some other way. G is the level of public good; g and γ are country

[22] At the end of this section, we discuss how introducing a matching scheme to the public good would change the results.

1 and 2's respective contributions to the public good; the second arguments in the utility functions are the private consumptions of the two countries, which we will sometimes denote as x and χ respectively; w and ω are the two countries' respective endowments. Thus, the damages and the abatement costs enter as resource costs that reduce endowments, which can be used to finance public good contribution and private consumption. In effect, the damages and the abatement costs are perfect substitute for private consumptions. This may be a strong assumption, but it allows us to conveniently introduce a public good into the models covered in the previous sections.[23] Furthermore, we assume that the utility function are quasi-concave and increasing in both arguments, and that both the public good and the private consumption are normal.

Using this setup, we find that different orders of events lead to different outcomes. If the public good contribution decisions follow the abatement decisions and if both countries contribute to the public good, the subgame perfect Nash equilibrium in emissions abatement *without any matching scheme* in place becomes fully efficient. An efficient total amount of emissions abatement will be carried out, and that amount is efficiently allocated across countries such that the marginal costs of abatement are equalized. Public good contributions are not efficient, however. Thus, neither rate-matching nor QCM would be necessary for emissions abatement any more. The reason for these results is the neutrality outcome from the last stage public good contribution game. The neutrality outcome makes the world's total resource costs associated with emission, namely the *world's total* damages and abatement costs, the only relevant variable to *both* countries. Under our assumptions on preferences, this aligns the interests of the

[23] In what follows, we will give one example where the damage is less of a direct resource cost and enters the utility functions as a separate argument. How this affects the results will be discussed there.

two countries completely. Furthermore, with the assumptions on preferences, each country's utility will be maximized when the total resource costs are minimized. Thus, the neutrality outcome in public good contributions leads to full efficiency in abatement and renders both mechanisms unnecessary.

If, instead, the public good contribution decisions precede the abatement decisions, both the rate-matching and the QCM would play the same role and to the same extent as before for abatement, but public good contribution is again inefficient. This is due to the separability of the two types of decisions under our setup. Provided that the abatement effort does not drive private consumption to zero, neither country's contribution to the public good affects any country's choice of abatement effort, because both the damage and the abatement cost enter as resource costs that reduce private consumption in the utility functions. To be more specific, after the public good contribution decisions have been made, the problem of each country becomes one of minimizing the sum of the damage and the cost, which has a unique solution that only depends on the type of matching scheme used.

Let us look at the first case in more detail now. We characterize the subgame perfect Nash equilibrium without any matching and then analyze its implications for the two matching mechanisms. Then, we look at the other case.

3.5.1 Public Good Contributions Follow Emission Abatements

Subgame Perfect Nash Equilibrium

In this subsection, there is no matching scheme of any type. The timing is as follows. In Stage 1, the two countries choose emission abatements simultaneously. In Stage 2, both countries contribute to a usual public good, $G = g + \gamma$, where g and γ are the two countries' respective contributions. Thus, the two countries have the same productivity in producing public good contributions. We work backwards to solve for the subgame perfect Nash equilibrium.

Stage 2

At the beginning of this stage, the two countries' endowments are $w - D(e^0 - r + \eta^0 - \rho) - C(e^0 - r)$ and $\omega - \varDelta(e^0 - r + \eta^0 - \rho) - \mathcal{C}(\eta^0 - \rho)$, respectively. r and ρ are the two countries' respective abatements in Stage 1. Each chooses its contribution to the public good. Assuming that an interior solution always holds, we know from the Neutrality Theorem (Bergstrom, Blume, and Varian, 1986, for example) that the private consumptions of the two countries and the level of public good will depend on the sum of the two countries' endowments only, and not on the distribution of that sum across the two countries. Total endowments can be written $w + \omega - I$, where

$$I \equiv D(e^0 - r + \eta^0 - \rho) + C(e^0 - r) + \varDelta(e^0 - r + \eta^0 - \rho) + \mathcal{C}(\eta^0 - \rho). \quad (20)$$

Then, the two countries' utilities after playing this stage of the game can be written $u[x(I), G(I)]$ and $\nu[\chi(I), G(I)]$, respectively, since $w + \omega$ is constant.

Because G, x, and χ are normal goods, and because utilities are increasing in both arguments, minimizing I will maximize the two countries' utilities *at the same time.* This completely aligns the two countries' objectives.

Stage 1

In this stage, the countries choose their abatement efforts, anticipating the future outcome in Stage 2. Country 1, for example, solves

$$\max_{\{r\}} \; u[x(I), G(I)]$$

the first-order condition of which implies that

$$-D'(e^0 - r + \eta^0 - \rho) - \varDelta'(e^0 - r + \eta^0 - \rho) = C'(e^0 - r). \quad (21)$$

Similarly, country 2 solves

$$\max_{\{\rho\}} \ \nu[\chi(I), G(I)]$$

the first-order condition of which is

$$-D'(e^0 - r + \eta^0 - \rho) - \Delta'(e^0 - r + \eta^0 - \rho) = \mathcal{C}'(\eta^0 - \rho). \tag{22}$$

As we can see, (21) together with (22) coincides with the solution to the social planner's problem characterized by condition (2), where externality is internalized and marginal abatement costs are equalized across countries.

These imply that the matching schemes will not be necessary any more.[24] This result of course depends on our simplifying assumptions of interior public good contributions and assumptions on preferences. Although the result should not be taken too literally, it seems to suggest that linking issues involving externality in a sequential manner can sometimes produce unexpected outcomes.

We would also like to note that, if we adopted some other formulation of preferences, the neutrality theorem may not hold for the last stage public good contribution, and the analysis would be more complicated. For example, suppose country 1's utility function were

$$u[G, x, D(e^0 - r - m\rho + \eta^0 - \rho - \mu r)]$$

where

$$x = w - g - C(e^0 - r).$$

[24] In terms of the rate-matching method, this result here is for the case where both countries can commit to some matching rate. As Boadway, Song, Tremblay (forthcoming) showed in the context of public good contributions, the same result applies if only one of the two countries can commit to a matching rate, as long as the last stage contributions to the public good are interior.

Thus, the abatement cost is a material, resource cost; the damage is less "material", is not a perfect substitute for private consumption, and enters as something that directly reduces utility. The Stage 2 first-order condition for g would be

$$u_G[g+\gamma, w-g-C(e^0-r), D(e^0-r-m\rho+\eta^0-\rho-\mu r)]$$

$$+u_x[g+\gamma, w-g-C(e^0-r), D(e^0-r-m\rho+\eta^0-\rho-\mu r)] = 0.$$

Country 1's income at the beginning of Stage 2 is now $w-C(e^0-r)$, and that of country 2's is $\omega-\mathcal{C}(\eta^0-\rho)$. The world's total income is $w-C(e^0-r)+\omega-\mathcal{C}(\eta^0-\rho)$. We would like to know the effect on each country's public good contribution and private consumption of a small enough change in the composition of the world income, keeping the world's total income constant. Following the logic on page 31 of Bergstrom, Blume, and Varian (1986), we would like to see if the first-order condition above for country 1 (and a similar one for country 2) will still be satisfied, when each country changes its contribution by the amount of its income change. There can be two types of changes in the composition of the world's income. The first type is the redistribution of the initial endowments w and ω. The second type is the change in income caused by changes in abatement costs $C(e^0-r)$ and $\mathcal{C}(\eta^0-\rho)$, which keep the sum of C and $\mathcal{C}$ constant. There is now a third argument $D(\cdot)$ ($\mathit{\Delta}(\cdot)$ for country 2) that depends on e^0-r ($\eta^0-\rho$ for country 2) nonlinearly. Therefore, in general, the first-order conditions will still be satisfied when each country changes its public good contribution by the amount of its income change, only if the change in composition of world's income is of the first type; this will not be the case, if the change in composition is of the second type. The reason is that such compensating changes in public good contributions and incomes leave G, x, and χ unchanged; however, in general, $D(e^0-r-m\rho+\eta^0-\rho-\mu r)$ and $\mathit{\Delta}(e^0-r-m\rho+\eta^0-\rho-\mu r)$ are left unchanged only if the redistribution is of the first type.

From these, we could also see that our formulation of the preferences is an important reason for getting the results above.

Furthermore, even with the original setup, if a rate-matching or a quantity-contingent matching method is applied to the public good contributions too, the neutrality outcome does not obtain for the public good contributions, and the result above will not hold. The neutrality outcome does not obtain for the following reason. Since the matching schemes lead to efficient levels of public good contributions, a redistribution of the two countries' endowment at the beginning of the public good contribution stage moves the two countries from one point on the utility possibility frontier to another and necessarily changes their utilities. This is in contrast to the neutrality result that the consumptions of the public good and the private good and therefore utilities do not change after a redistribution of endowments that do not change the set contributors in the Nash equilibria. Thus, the two countries' overall utilities can be changed by the relative size of their initial endowments when a matching scheme is in place for the public good contributions.

3.5.2 **Public Good Contributions Precede Emission Abatements**

The timing of events is as follows. In Stage 1, both countries decide on their public good contributions. If rate-matching is used, in Stage 2, country 1 and country 2 set matching rates for direct abatement efforts to be decided upon in Stage 3; in Stage 3, direct abatement efforts are actually chosen and the matching abatement efforts are applied. If a QCM is used, in Stage 2, country 1 announces its QCM which specifies a high-for-high and low-for-low type of matching for country 2's abatement effort as we saw before; in Stage 3, country 2 chooses its abatement effort; in Stage 4, country 1 chooses it abatement effort according to the QCM. We only need to analyze the rate-matching case to see the results mentioned in the introduction to this section.

After Stage 1

After the public good contribution decisions have been made in Stage 1, country 1, for example, solves the following problem in Stage 2, anticipating the choices r and ρ in Stage 3

$$\max_{\{m\}} u[g + \gamma, w - g - D(e_1 - r - m\rho + e_2 - \rho - \mu r) - C(e_1 - r - m\rho)]$$

As one immediately sees, this is equivalent to minimizing $D(\cdot) + C(\cdot)$, given g and γ and anticipating the choices r and ρ in Stage 3

$$\min_{\{m\}} D(e_1 - r - m\rho + e_2 - \rho - \mu r) + C(e_1 - r - m\rho)$$

Thus, because $D(\cdot) + C(\cdot)$ enters private consumption *only* and does so in an *additively separable* way, utility is always maximized by minimizing $D(\cdot) + C(\cdot)$. Therefore, the public good contribution decisions and the abatement decisions are essentially separated. From previous sections, we know that, under our assumptions, equilibrium total abatement in each country and the minimized sum of damage and abatement cost of each country are all unique, given the type of matching scheme being considered. Therefore, the choices of g and γ in Stage 1 would not affect the choices in the subsequent abatement game.

Therefore, there is no need for further analysis. The emission abatement choices and the public good contribution choices are separated due to our setup. Abatements would be constrained efficient when either of the two matching schemes is used, and public good contributions would still be in efficient.

3.6 Concluding Remarks

In this chapter, we adapted some multi-stage, non-cooperative matching schemes that can generate efficient outcomes in public good contribution to a simple two-country pollution abatement model. It was shown that the matching schemes can help countries internalize the negative externality they impose on others and can thus be a substitute for the international redistributive transfers as used in the Caplan and Silva (2005) model, for example. However, in equilibrium, the marginal costs of abatement efforts are not equalized across countries, leaving room for further Pareto improvement. This problem seems to arise from the lack of enough policy instruments. If, in addition, pollution permits, for example, can be traded internationally, marginal costs of abatement would probably be equalized across countries, and full efficiency would obtain. When there is a voluntary contributions to public good issue beside the abatement issue and when public good contribution decisions are made after abatement decisions, the subgame perfect equilibrium outcome for abatement is already fully efficient, and none of the matching schemes would be of any use for the pollution abatement issue. When the two issues are resolved in reverse order, they become essentially separated, due to our setup, and the matching schemes remain useful to the same extent as when the issues are isolated.

Throughout the chapter, commitment ability is simply assumed, although it is an open question as to what extent countries could commit on such issues. Besides, in reality, such commitment on the part of all or most countries and their individual citizens to tackle the environmental pollution problems might be what really matters.

Chapter IV

Charitable Giving of Money and Time by the Elderly —an Analysis Using PSID Data

4.1 Introduction

Charitable giving of money and time is pervasive in many countries. There is a sizable empirical literature on this subject, which typically analyzes the various determinants of money donation, volunteering, or both. Among the determinants analyzed are the tax price of monetary giving(one minus marginal income tax rate), income, wage rate, and demographic factors such as education, family size, number of dependent children, age, gender, and marital status. Elasticities of both types of giving with respect to tax price and income have been a focus in many studies. Elasticity and related results can potentially be used to inform policy on income tax deductibility of charitable donation. Another more or less common feature of previous studies is perhaps that the samples mainly consisted of working-age adults. A third aspect worth noting is that, probably in part due to data availability, money giving and time giving have often been studied separately. To our knowledge, only Brown and Lankford (1992) and Andreoni et al (1996) examine the two jointly. The first paper employed a reduced form and essentially probit type of model; the second one was perhaps the first structural estimation in this area, with the likelihood function based on a pair of first-order conditions for consumer maximization.

This chapter attempts to add to the analysis of the determinants of giving. The current version uses a sample of the elderly (age 60 or above) from the year 2003 Panel Study of Income Dynamics (PSID) data and estimates the two forms of giving jointly. Such an inquiry informs our understanding of time and resource allocation, particularly among the elderly, and serves as an input into analyses of economic well-being. More specifically, we focus on the elderly for the following three reasons. First, there are not many studies in the economics literature focusing on the charitable donations of the elderly. Donors of different age may face different resource constraints, and their donation behaviour may systematically differ. For example, if one classify donors into the young, the middle aged, and the elderly. The young may have relatively more time but less money; the middle aged may have the money but less time; the elderly may have relatively more of both money and time. If such differences are true to some extent, conducting an empirical analysis over people of all age and using a dummy variable to represent the elderly might not capture the systematic difference in donation behaviours well. Second, existing survey studies show that the elderly is an increasingly important constituent of the donor community. Third, in some discussions of welfare reforms and some other issues, the elderly was sometimes viewed as passive recipients of resources from society. Their continued contributions of, for example, financial and human resources, together with the adverse impact on their wellbeing of some of the proposed policy measures, were ignored, to greater or lesser extent. In this chapter, we wish to document their contributions to society in this respect.

We only selected from the full sample elderly householders who are not married. We felt that persons in such a sample were more likely to be making giving decisions by themselves only and thus allow us to avoid intra-household decision-making issues.

We take a reduced form approach and look at several determinants traditionally studied in the literature. The estimates give us some idea about the income, wealth, and age profiles of giving, as well as the roles of health and education. Although we also obtain price elasticities of giving, their interpretation and implication should be taken with caution, due to the lack of variation in the tax price variable in our data. We will discuss this in more detail when summarizing the data and when presenting estimation results.

Our data is cross-sectional, and we assume that the system of two equations to be estimated belongs to the class of seemingly unrelated regressions (SUR) models. The dependent variables are the amounts of money and time donated. Because the data contains many zero observations on the dependent variables, our model is formulated as an SUR Tobit model. This is a common phenomenon in such contexts. Since the model contains only two equations, we estimated it using straight maximum likelihood. This can be done particularly conveniently with Stata, because its built-in univariate normal PDF and CDF options and bivariate normal CDF option allow us to program the loglikelihood function easily.

The rest of this chapter is organized as follows. Section 2 summarizes some studies on charitable giving. Section 3 describes the data. Section 4 presents the econometric model and the loglikelihood function. In section 5, we present and discuss the estimation results. Section 6 concludes.

4.2 Related Studies on Charitable Giving

Clotfelter (2002) provides an excellent recent discussion of the evolution and current patterns of charitable money giving, as well as the various issues examined in scholarly studies. We also refer to the literature review sections of Brown and Lankford (1992), Andreoni et al (1996), and Gruber (2004).

Some of the features of money giving summarized by Clotfelter's study are the following. 1) Charitable giving can take many forms. Donors can be individuals (through direct donations or bequests), companies, or other entities such as private foundations. In the US, for example, individuals contribute the lion's share. According to "Giving USA" 1996, individual donations were

80.61% of all donations in 1995 in the US. 2) Donees these days are mainly institutions, as opposed to individuals. According to Hodgkinson and Weitzman (1996), the missions of donee organizations, in descending order of individual contributions received, were religion, human service, education, health, youth development, arts-culture-humanities, international, public/societal benefit, environment, recreation-adults, other areas, and private and community foundations. 57.5% of all individual donations went to religious organizations, followed by 9.4% to human services. 3) Tax systems in many countries provide favourable treatment of many forms of charitable money/asset giving by allowing some form of deductions that reduce a donor's income tax burden. 4) The types of organizations to which donors contribute differ systematically by income level. In the 1973 "National Study of Philanthropy" in the US, the share of religious contributions decreases as income increases. Taking the place of religious giving at higher income levels was giving to higher education, cultural organizations, and other types such as hospitals. 5) As a percentage of income, money giving shows a U-shape pattern.

There is a sizable empirical literature on charitable money giving. In the 1970s and 1980s, most works used cross-sectional data, and the focus was on the responsiveness of charitable money giving with respect to income and to the after-tax price of money giving. Clotfelter (1985) and Steinberg (1990), among others, summarize some of these studies. In most of them, estimates of price elasticity were in the range of -0.5 to -1.75, and estimates of income elasticity were in the range of 0.4 to 0.8. More recent studies, however, tended to produce larger income elasticities and smaller price elasticities. In the 1990s, some argue that previous studies estimated responses of charitable giving to transitory changes in tax rates. It is possible that individuals optimize the timing of their giving in response to transitory tax changes, but do not change the long-run levels of giving very much. Studies by Randolph (1995) and Barrett et al (1997), for example, found that responses to permanent tax changes are lower than previously reported. Randolph's study found a price elasticity of -0.5 and an income elasticity of 1.1. Barrett et al found a price elasticity of -0.471. However, some more recent studies questioned those in the 1990s and produced larger price elasticity estimates again. Auten et al (2002) questioned the parametrization of transitory and permanent effects in previous studies and estimated elasticities for several time periods between 1980 and 1992 using panel data. Auten et al's estimated long-run price elasticities were in the range of -0.79 to -1.26, and their long-run income elasticities were in the range of 0.40 to 0.87. Just as Clotfelter put it, it seems that the issue of the magnitude of responsiveness is still not resolved.

Perhaps mainly due to data limitations, there is significantly less work on charitable time giving, that is, volunteering. Brown and Lankford (1992) provides a relatively recent review. The following short summary follows that of their paper. Mueller (1975) found neither husband's income nor a woman's gross wage a significant predictor in OLS equations explaining hours of volunteer work. Dye (1980) found that volunteering is negatively associated with the tax price of monetary donations, and estimated a cross-price elasticity of -0.83. Clotfelter (1985, p. 167-170) looked at the volunteer behavior of women in models of sequential decision-making, assuming that the labor force participation decision is determined prior to making decisions on volunteering. In a logit model predicting participation in the volunteer labor force, he also found significant negative effects of the price of money donations, and also of hours of market work. Besides, he found that volunteering is positively related to age, education, and the presence of children under age 18 at home. Menchik and Weisbrod (1987)'s Tobit estimates showed a significant, negative effect of the after-tax wage on hours of volunteer work. They find a positive but declining effect of income and a negative effect from the price of money donations, both statistically significant.

Brown and Lankford (1992) was perhaps the first study that estimated a money giving equation and a time giving equation jointly, to take into account the potential implication of censoring in one equation on the estimation of the other. Their data were from the *Florida Consumer Attitude Survey* for the year 1983. Their econometric model is a system of three equations, with the first equation for money giving by both men and women and the other two for volunteering by men and by women, respectively. All equations have the form $\log(y_j - a_j) = X_j\beta_j + u_j$, $j = 1, 2, 3$ where y_j is money/time giving, a_j is an equation-specific constant, some regressors in X_j are in logarithm, and u_j is the error term. $[u_1, u_2]^\top$ and $[u_1, u_3]^\top$ are, respectively, jointly normal and i.i.d. Censoring could occur for any or all of the y_j's. If one takes these equations as corresponding to strict consumer maximization over the amounts of giving, one would be inclined to use Lee and Pitt's (1986) framework that explicitly accounts for the effect of a binding non-negativity constraint for one form of giving on the other form of giving. The loglikelihood function would then be of the Tobit type. As will be explained in more detail below, Brown and Lankford found that that framework and the functional form above do not fit their data well. Instead they adopted a less restrictive approach that preserves the essence of Lee and Pitt's framework but avoids its numerous cross-equation restrictions and other complications. The loglikelihood function is of the probit type for censored observations. They found statistically significant effects of the money tax price in all equations. The estimated elasticities are -1.7 for money giving, -2.1 for women's time giving, and -1.1 for men's time giving. Their specification testing showed that available time, rather than the unit cost of time—the wage rate—is a better explanatory variable and can be treated as exogenous. This is consistent with the sequential decision making model that Clotfelter (1985) used before, or with any model in which hours of paid work is determined before the volunteering decision. As in previous studies, they also found that money giving and time giving are complementary.

Andreoni et al (1996) is perhaps the first to estimate a structural model in the area. They use data from the 1990 Gallop Survey for the Independent Sector. Their economic model is one where agents maximize a quadratic utility function with respect to the amount of money giving and the amount of time giving. A quadratic utility function can be seen as an approximation of an arbitrary utility function and is used for other empirical works. Utility maximization results in two first-order conditions, or expressions for marginal utilities. The utility function parameter that enters additively in the marginal utility expressions are assumed to be joint normal and independently and identically distributed. These then allow one to construct the loglikelihood function. This structural framework allows one to calculate elasticities by simulation, instead of by the conventional ways in previous works. One first derives closed form expressions for desired money giving and time giving. Then one takes repeated random draws from the estimated error term distribution and fit demands for each household. Negative simulated demands are set to zero. The sum over all households gives the aggregate demands. Then one perturbs, for example, the tax price variable a little bit[25] and repeats the above steps to obtain another set of aggregate demands. The percentage change in aggregate demand over the percentage change in the explanatory variable is taken to be the elasticity. The tax price elasticity of money giving they found, -0.35, is considerably smaller than many previous ones, but is similar to that found by Randolph (1995).[26] Their income elasticity

[25] Andreoni et al (1996) did not mention exactly how this was done.

[26] One may want to bear in mind that structural elasticities and reduced-form ones

estimate of 0.28 is also small. For purpose of comparison, Andreoni et al also calculate the tax price elasticity of money giving using a conventional double-log model. The elasticity is -1.845. This shows, as the authors argue, that the difference in elasticity results does not come from the different data set used, but from the new estimation method. Because of the explicit use of the structural framework, they can calculate not only the uncompensated cross-price elasticity of time giving with respect to the money tax price, as the previous studies did, but also the compensated ones. They find that the two forms of giving are gross (uncompensated) complements, as found in most previous studies, but are weak Hicksian (compensated) substitutes, with the compensated cross-price elasticity close to zero. If one agrees with their representation of preferences, their structural framework allows for better policy simulation analysis. In fact, if preferences are as they model them, previous log-log or Tobit specifications would be misspecified. "In particular, previously estimated price effects would reflect a mixture of utility function parameters, budget constraint variables, and their interactions. In this situation the estimated parameters will not provide accurate estimates of the effect of price changes on gifts of money and time."[27] Although their elasticity estimates are small, simulations show that tax policies still have quantitatively significant impacts on giving.

4.3 Data

In 2001 and 2003, the PSID collected information on the two types of giving. In particular, details of money donations by households and volunteering by individuals in households were collected.

We use both the individual file and the family file for the year 2003. Because

may lack comparability in some cases.

[27] Quote from paragraph 1 on p. 5 of their paper.

the econometric model we will use is a seemingly unrelated regression model with censoring in the dependent variables, we leave the task of using the panel data from both years for future work. There are 687 individuals who are householders, are age 60 or above, and are not married. Information on itemization status, before-tax income, wealth, and demographic variables are extracted from the data files. We follow Gruber (2004) and derive the price of money donation using the TAXSIM calculator.[28] In particular, the tax price is calculated as the difference between the combined federal and state income tax liabilities under the actual amount of money donation observed in the data, and that under an amount of money donation that is one thousand dollars more than the actual one, divided by one thousand. According to Gruber (2004), such a large artificial increment is used to avoid some "strange notches" in the income tax system, which could render results using small increments inaccurate. Gruber (2004) had actually tried smaller increments too, and the results were not much different; the same is true for our exercise. Tax price calculated this way captures the effect of donating one more dollar on tax liability.

In arriving at a data set that does not include *Wealth* as an explanatory variable, we first drop a total of 227 observations that have a non-response for some variable, dependent or independent. We then drop one observation with extreme time giving; for this observation, money and time giving are 600 dollars and 5425 hours, respectively, and the next highest time giving in the sample is 3120 hours. These deletions leave us with 418 observations. Whether to

[28] TAXSIM is the NBER's FORTRAN program for calculating tax liabilities and marginal tax rates under US Federal and State income tax laws from individual data. The program itself is not released. Users upload files containing individual data, and output will be returned as a web page or a file in a few seconds. See Feenberg, D. R. and E. Coutts (1993) and the website `http://www.nber.org/ taxsim/taxsim-calc7/`.

include *Wealth* poses a difficult problem: to include it, we have to further drop 105 observations (and are thus left with merely 313 observations) because of non-responses to items used in calculating wealth. Thus, as will be mentioned further when presenting results, we estimate two broad specifications of a similar econometric model, one with wealth dummies as explanatory variables but having a smaller sample size and the other without wealth but having a larger sample.

Table 4.4[29] summarizes the numbers and proportions of positive and zero observations for the dependent variables. Although the incidence percentages in the two estimation samples are quite similar, they are different from those in our initial sample. The difference mainly comes from dropping observations which have both positive money giving and positive time giving, but also non-response in some variables. This may lead to less general conclusions, especially with respect to volunteering.

Table 4.5 contains summary statistics of the variables for the two estimation samples. For ease of reference, we will refer to the sample for the base specification (i.e., not including *Wealth*) as sample 1 and that for the alternative specification (i.e., including *Wealth*) as sample 2. Both money and time giving are for year 2002. 75.60% of all individuals in sample 1 and 75.40% in sample 2 are the only person in the household. 73.68% (70.29%, respectively) are females. One may note the high means and small variances of the tax price. There are actually only 38 (9.09% of 418) observations in sample 1 and 35 (11.18% of 313) in sample 2 for which the price is less than one; all others have a price equal to its maximum, 1. An individual faces a tax price equal to one if either she does not itemize for deduction in filing income tax return, does not pay any tax, or her federal and state income tax liability is the same with and without one thousand dollars more of money giving according to the TAXSIM calculator. This

[29] Tables 4.4 to 4.9 are at the end of this chapter, pages 111 to 116.

lack of variation can certainly affect identification. However, as we mentioned in the introduction, estimating the effect of price is not the main purpose of this chapter.

4.4 Econometric Model and Method

4.4.1 **Empirical Model**

We take a reduced form approach and our empirical model is similar to that of Brown and Lankford (1992). In particular, both money and time giving functions are of the form:

$$y_{ji}^* = X_{ji}\beta_j + u_{ji}, \qquad j = 1, 2,\ i = 1, \ldots, n,$$
$$y_{ji} = \begin{cases} y_{ji}^*, & \text{if } y_{ji}^* > 0, \\ 0, & \text{if } y_{ji}^* \leqslant 0. \end{cases}$$

y_{ji}^* are individual i's desired (latent) giving of money ($j = 1$) or time ($j = 2$), and y_{ji} are individual i's observed giving.[30] y_{ji}^* could take positive or negative values; in the latter case, we do not observe y_{ji}^*, but instead observe $y_{ji} = 0$ in our data. For each observation, one or both of the desired amounts of giving may thus be censored.

u_{ji} are mean zero disturbances. Let $u_i = [u_{1i}\ u_{2i}]^\top$. Since our data is cross-sectional and the two types of giving by an individual are conceivably correlated, it seems reasonable to assume that

$$\begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix} \sim i.i.d. \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \right).$$

[30] More on model specification will be said below and in the Appendix to this chapter. We use j to denote the type of giving and i to denote observation. If a variable has two subscripts, the first subscript indicates the type of giving, and the second indicates observation. Except for the variance/covariance terms ($\sigma_{rs},\ r, s = 1, 2$), the meaning of subscripts is similar for parameters, with the second subscript now indicating explanatory variable rather than observation.

That is, u_i are not correlated across observations, but u_{1i} and u_{2i} may be correlated. Thus, desired giving y_{ji}^* have the following distribution

$$\begin{bmatrix} y_{1i}^* \\ y_{2i}^* \end{bmatrix} \sim \ i.i.d.\, N\left(\begin{bmatrix} X_{1i}\beta_1 \\ X_{2i}\beta_2 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}\right).$$

As Brown and Lankford (1992) explained, these equations for y_{ji}^* can be viewed as consumer demand equations. In other words, we are implicitly assuming a consumption model for giving here. Presumably, other motives for giving, such as social climbing and investment, were not present to a significant extent in our sample of the elderly.

The explanatory variables are:

I_i : individual's after-tax income (before money giving)

$Inc1 \sim Inc4$: dummy variables for I_i in the intervals [0, 20,000], [20,000, 40,000], [40,000, 60,000], and [60,000 $\sim$ above]

Pm_i : tax price of money giving, calculation explained in text above

Ph_i : price of time giving, taken to be hourly after-tax[31] wage rate if working and zero if not

$Wealth1 \sim Wealth5$: wealth is taken to be the net value of all assets and debts, including house, automobile, etc.; these are dummy variables for $Wealth$

[31] This is calculated as pre-tax wage rate times one minus the marginal tax rate with respect to wage income, where the marginal tax rate is obtained from the TAXSIM calculator. Menchik and Weisbrod (1987) found that after-tax wage rate is a statistically significant explanatory variable for hours of volunteer work. Brown and Lankford (1992) found instead that available time does a better job than wage rate. We do not have information on individuals' available time. Another issue is whether before or after tax wage rate should be used. Maybe the rules of the income tax system are relevant here: if wage income is taxed so that the wage earners' take-home wage rate is reduced by the tax system, it would be fine to use the after-tax wage rate as a proxy for the opportunity cost of time giving; otherwise, it may be better to use the gross wage rate. In this chapter, after-tax wage rate is used.

in [below $\sim$ 1,000], [1,000, 50,000], [50,000, 150,000], [150,000, 500,000], and [500,000 $\sim$ above]

W_i : demographic variables

Demographic variables in W_i include:

$Fsize$: number of persons living in household

$Age1 \sim Age4$: dummy variables for Age in [60, 69], [70, 79], [80, 89], and [90 $\sim$ above]

$Health$: subjective rating of own health status from 1 (excellent) to 5 (poor)

$Educ$: years of education

$Times$: number of Church visits in year 2002, used as a proxy for religiosity

$Gender$: gender of householder: 1 (female), 0 (male)

4.4.2 **Loglikelihood Function**

Our empirical model is a seemingly unrelated regressions model where the dependent variables are censored. Huang (1999) called this a SUR Tobit model. As mentioned in the literature review section, Brown and Lankford (1992), however, actually estimated such a model, which has a partially log-linear functional form for the giving equations, though using a probit type of likelihood function. The reason they gave is roughly the following. If one takes the view that money and time giving are results of strict consumer utility maximization with non-negativity constraints on money given and on time given sometimes binding, one would have to either explicitly account for these constraints or use the shadow (instead of the actual) price for money giving. Brown and Lankford referred to Lee and Pitt (1986) for more on these issues. They took the latter approach. However, the results were not to their satisfaction. They concluded that their partially log-linear functional form together with the numerous cross-equation parameter restrictions arising from the shadow price approach were inappropriate for their data. To retain the functional form that is standard in the literature and to have a somewhat less restrictive model which would still be consistent with the virtual price approach, they adopted the probit type likelihood function for those observations for which one of money giving and time giving is zero and the other is positive.

We take a more parsimonious approach here. We view the money giving equation and time giving equation as consumer demand equations, too. The desired level of giving may thus be negative. These imply that the equations are Tobit in nature. However, we felt that, for most people, money giving and time giving decisions are made separately, rather than jointly and through a process of strict consumer maximization. This seems especially true if one considers the fact that "money giving" and "time giving" in our exercise each consists of many types of giving that differ in nature; each type within money giving or time giving may well be made for a different reason and over a different time horizon. Thus, even though they are related and may be affected by some common factors, it seemed hard to say that the money giving and the time giving in the data are results of consumer maximization in the above narrow sense. Furthermore, other types of activities, for example, participation in various social activities, may well interact with money giving and/or time giving; this might render a system of only two equations potentially an inaccurate description of giving activities, if one took a strict consumer maximization view. Given these considerations, we are inclined to think of money giving and time giving as two loosely related activities, which are affected more by individuals' general taste for giving and specific, physical circumstances that may or may not facilitate giving and less by parameters of a strict economic maximization framework. Thus, we maximize a Tobit type of likelihood functions. However, for comparison, we will also give in an appendix results from estimating the Brown and Lankford econometric model, which is log-linear in some variables and uses a probit likelihood function, as well as those from a model which is log-linear in some variable but uses a Tobit likelihood function. We will explain these two econometric models and comment on their results mainly in the appendix.

Next, we briefly describe how we rewrite the loglikelihood function for our SUR Tobit model so that it is easier to code.[32] Consider the four possible cases of censoring of y_i^* in our model:

$$\text{regime 1} \begin{cases} y_{1i} = y_{1i}^* > 0 \\ y_{2i} = y_{2i}^* > 0 \end{cases}, \quad \text{regime 2} \begin{cases} y_{1i} = y_{1i}^* > 0 \\ y_{2i} = 0 \geqslant y_{2i}^* \end{cases},$$

$$\text{regime 3} \begin{cases} y_{1i} = 0 \geqslant y_{1i}^* \\ y_{2i} = y_{2i}^* > 0 \end{cases}, \quad \text{regime 4} \begin{cases} y_{1i} = 0 \geqslant y_{1i}^* \\ y_{2i} = 0 \geqslant y_{2i}^* \end{cases}.$$

Correspondingly, the contribution of observation i to the likelihood function also has four possible forms:

1. Regime 1: Both y_{1i}^* and y_{2i}^* are positive and observed. The contribution is the density of the normal random vector $y_i^* = [y_{1i}^* \; y_{2i}^*]^\top$

$$L_i^1(y_i;\, b, \Omega) = f_i(y_i^*;\, b, \Omega),$$

where $b = [(\beta_1)^\top \; (\beta_2)^\top]^\top$, Ω is the covariance matrix of $u_i = [u_{1i} \; u_{2i}]^\top$, and $f_i(y_i^*;\, b, \Omega)$ is

$$(2\pi)^{-1}|\Omega|^{-\frac{1}{2}} exp\left\{ -\frac{1}{2} \begin{bmatrix} y_{1i}^* - X_{1i}\beta_1 \\ y_{2i}^* - X_{2i}\beta_2 \end{bmatrix}^\top \Omega^{-1} \begin{bmatrix} y_{1i}^* - X_{1i}\beta_1 \\ y_{2i}^* - X_{2i}\beta_2 \end{bmatrix} \right\}.$$

2. Regime 2: y_{1i}^* is positive and observed; y_{2i}^* is non-positive and observed as zero. First note that

$$\int_{-\infty}^{0} f_i(y_i^*;\, b, \Omega)\, dy_{2i}^* = \int_{-\infty}^{0} \frac{f_i(y_i^*;\, b, \Omega)}{\phi_i(y_{1i}^*)} \phi_i(y_{1i}^*)\, dy_{2i}^*$$

[32] We used Intercooled Stata, Version 9.2, to perform all estimations.

$$= \phi_i(y_{1i}^*)\left[\int_{-\infty}^{0} \frac{f_i(y_i^*;\, b, \Omega)}{\phi_i(y_{1i}^*)}\, dy_{2i}^*\right] = \frac{1}{\sqrt{\sigma_{11}}}\phi(\frac{y_{1i}^* - X_{1i}\beta_1}{\sqrt{\sigma_{11}}})\Phi(-\frac{\tilde{\mu}_2}{\sqrt{\tilde{\sigma}_2}})$$

In the expressions above, $\phi_i()$ in the second and the third expressions is the density function of the univariate normal random variable y_{1i}^*, which has mean $X_{1i}\beta_1$ and variance σ_{11}; the integrand in the third expression is the density for the conditional distribution of y_{2i}^*, given the realization of y_{1i}^*; $\Phi()$ and $\phi()$ in the last expression are the CDF and the PDF of the standard normal distribution; $1/\sqrt{\sigma_{11}}$ is the Jacobian of the transformation from u_{1i} to y_{1i}^*. The conditional distribution just mentioned is

$$y_{2i}^*|y_{1i}^* \;\sim\; N(\tilde{\mu}_2, \tilde{\sigma}_2), \qquad \text{with } \tilde{\mu}_2 = X_{2i}\beta_2 + \frac{\sigma_{12}}{\sigma_{11}}(y_{1i}^* - X_{1i}\beta_1),\; \tilde{\sigma}_2 = \sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}.$$

3. Regime 3: y_{1i}^* is non-positive and observed as zero; y_{2i}^* is positive and observed. The expression is symmetric to that for regime 2

$$L_i^3(y_i;\, b, \Omega) = \int_{-\infty}^{0} f_i(y_i^*;\, b, \Omega)\, dy_{1i}^* = \frac{1}{\sqrt{\sigma_{22}}}\phi(\frac{y_{2i}^* - X_{2i}\beta_2}{\sqrt{\sigma_{22}}})\Phi(-\frac{\tilde{\mu}_1}{\sqrt{\tilde{\sigma}_1}}),$$

where the conditional distribution involved is

$$y_{1i}^*|y_{2i}^* \;\sim\; N(\tilde{\mu}_1, \tilde{\sigma}_1), \qquad \text{with } \tilde{\mu}_1 = X_{1i}\beta_1 + \frac{\sigma_{12}}{\sigma_{22}}(y_{2i}^* - X_{2i}\beta_2),\; \tilde{\sigma}_1 = \sigma_{11} - \frac{\sigma_{12}^2}{\sigma_{22}}.$$

4. Regime 4: Both y_{1i}^* and y_{2i}^* are non-positive and observed as zero. The contribution is

$$L_i^4(y_i;\, b, \Omega) = \int_{-\infty}^{0}\int_{-\infty}^{0} f_i(y_i^*;\, b, \Omega)\, dy_{2i}^* dy_{1i}^* = \mathbf{\Phi}(-\frac{X_{1i}\beta_1}{\sqrt{\sigma_{11}}}, -\frac{X_{2i}\beta_2}{\sqrt{\sigma_{22}}}, \rho),$$

where $\mathbf{\Phi}()$ denotes the CDF of the bivariate normal distribution with the variances of both variates equal to one and the correlation coefficient equal to ρ.

In deriving these expressions, we referred to Appendix A1 of Brown and Lankford (1992) and Hogg and Craig (1995, p. 146).

The SUR Tobit model described above is a small, two-equation system and can be estimated in several ways, including Amemiya (1974)'s method, straight maximum likelihood, as well as the expectation-maximization algorithm (Huang 1999, for example). We chose to implement straight ML with Stata. Stata provides built-in options for the CDF's of the univariate and the bivariate normal distributions as well as the PDF of the univariate normal distribution. This makes the loglikelihood function very easy to program.

4.5 Results and Analysis

As mentioned in the data section, we estimate two specifications of a similar econometric model. The base specification does not use information on wealth and uses 418 observations in its estimation sample. The alternative specification controls for wealth but can only uses 313 observations.

For each specification, we select the final model to estimate by the following procedure. First, we include all explanatory variables that we have in both the money equation and the time equation; this is the initial, unrestricted model. Then, we consider dropping those regressors that are not statistically significant and do not have strong economic rationale for inclusion in the particular equation. The resulting smaller, restricted models are then tested against the initial, unrestricted model, using joint likelihood ratio tests. If they are not rejected, we reach a satisfactory restricted model. For each specification, we will present results for both the initial, unrestricted model and the final, restricted model.

4.5.1 **Base Specification**

Tables 4.6 and 4.7 contain results for the unrestricted model and the restricted model, respectively, of this specification that does not use *Wealth*. In each equation of the small model, only own price is included, while the price of the other form of giving is dropped as highly insignificant. Number of church visits is dropped from the time giving equation. Family size and gender are dropped from both equations. As we can see, the estimates and the $z-$ratios are not very different across the two models. The likelihood ratio test of the small model against the initial model does not reject the former. In what follows, we focus on the final, smaller model.

Money Giving Equation

Most variables in the money equation are individually statistically significant at least at the 10% level of significance.

Higher income is correlated with more money giving, with the income level of 40,000~60,000 having the largest positive association.

Although the tax price of money giving lacks much variation in our sample, this variable has statistically significant negative relation with money giving. For itemizers, the higher their marginal income tax rates are (or the lower the after-tax price are), the more they give. This is suggestive of the stimulating effect of government tax expenditures (through more generous deduction allowance) for at least some individuals.

Money giving also seems to increase with age. The age variable usually captures the influence of many factors observable or unobservable in the data. We see that the second age group (70 to 80) and the fourth age group (90 and above) tend to give more on average; simple averages of giving of the four age groups show that this is approximately true even if we do not condition on other covariates, although the gap between the average for the second group and that for the third group appears to be larger than that suggested by the regression estimates.

Subjective rating of one's health has a statistically significant association with money giving: the better one feels about one's health, the higher money giving is. Years in school has a similar association with money giving.

Although our proxy for religiosity, the number of church visits during year 2002, is not significant, it seems to have a small positive relation with money giving. This suggests a possible topic for future work: given the detailed data we have, it might be feasible and interesting to see how donations to religious institutions are related to the number of church visits. Using other data sets, Gruber (2004) found a negative correlation between the two, which implies that some people substitute money donations for religious participation. Then, government policies encouraging money donation may lead to less church participation; however, church participation may be important for the positive externality of religious activities to be realized.

Time Giving Equation

Higher income seems to be correlated with less volunteering hours, although none of the income dummy coefficient estimates is individually significant at the 5% level of significance. In our sample, average age and the proportion of retired decrease over income groups; average wage, subjective health status, and years in school increase over income groups.[33] That is, in our sample, more younger persons are working, earning more and having better subjective health and more education. Besides, the wage rate, which is intended to serve a measure of the opportunity cost of time giving, is not statistically significant and has the

[33] Averages by income groups of some explanatory variables are: age: 73.54, 70,65, 68.96, and 68.90, respectively; health: 3.3680, 2.8538, 2.7143, and 2.2000, respectively; after-tax wage rate: 12.82, 48.08, 63.86, and 110.97 dollars per hour, respectively; proportion of retired: 87.20%, 55.38%, 50%, and 50%, respectively; education: 10.3, 12.58, 14.18, and 14.6 years, respectively. When averaging wages, we included both working and retired individuals in an income group; the same method of averaging is used in several footnotes below.

"wrong" sign. Taken together, these two observations seem to suggest that the overall effect of working and earning income on volunteer hours is perhaps more related to available time or physical capacity, rather than such opportunity cost measures as the wage rate. This point may to some extent be consistent with Brown and Lankford (1992), who found that the wage rate did not have much explanatory power and that available time did.

Age has a joint statistically significant[34] relation with volunteer hours. The "age profile"[35] of volunteering is hump-shaped: other things being equal, the middle age groups, 70 to 80 and 80 to 90, volunteered more than the youngest group, 60 to 70, and the most senior group, 90 and above. Average years of education do not vary systematically across age groups; average subjective health status exhibits slight decrease over age groups; however, the proportion of working individuals, wages, and to a lesser extent income all decrease over age groups.[36] The picture is thus similar to that above. We suspect that middle aged seniors volunteer more because, on the one hand, more of them are retired and thus have more available time, and, on the other hand, they are physically more capable to perform the volunteer work than people in the most senior group.

Finally, better subjective health status and more years in school when young are both significantly positively correlated with volunteer hours.

Error Covariance Matrix

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[34] For the final, restricted model, the LR test statistic for the restriction that all of the coefficients of the age dummies are zero is 6.66, and $P[\chi^2(3) > 6.66] \approx 0.0836$.

[35] Of course, this is not the pattern of change over time for the individuals.

[36] By age group, averages of some explanatory variables are: health: 3.0549, 3.2339, 3.0822, and 3.2000, respectively; years in school: 11.6646, 11.2573, 10.9726, and 11.5000 years, respectively; proportion of retired: 53.66%, 82.46%, 97.26%, and 90%, respectively; wage: 55.24, 17.30, 41.10, and 3.33 dollars per hour, respectively; income: 23,285, 19,637, 17,210, and 19,012 dollars, respectively.

The error variance for money giving is smaller than that for time giving, and the correlation coefficient is positive and of moderate size. All elements of the covariance matrix estimate are highly significant. The positive correlation is consistent with results in many previous studies. It could be interpreted as suggesting that "some people have a taste for giving in general" (Brown and Lankford 1992, p. 332).

Overall, the findings are similar to those in the literature, in particular, those of Brown and Lankford (1992). Although we do not have an available time variable, our reasoning suggests that it seems to play an important role behind time giving.

4.5.2 **Alternative Specification**

Tables 4.8 and 4.9 contain results for the unrestricted model and the restricted model, respectively, for this specification using *Wealth*. Wealth dummies are jointly statistically significant[37] in both equation. *Health* is dropped from the money giving equation now. Again, we can see that the estimates and the $z-$ratios are not very different across the unrestricted and the restricted models, and the likelihood ratio joint test does not reject the restrictions in the smaller model. In what follows, we focus on the smaller model. We shall also skip or only briefly discuss those variables the parameter estimates for which have qualitatively similar features to those in the base specification. (These include almost all variables except the wealth dummies and the price of time giving.)

Money Giving Equation

[37] The following results refer to the final, restricted model. The LR test statistic for the restrictions that all of the coefficients of the wealth dummies in the money giving equation are zero is 16.26, and $\mathrm{P}[\chi^2(4) > 16.26] \approx 0.0027$. For the same test for the wealth dummies in the time giving equation, the test statistic is 9.92, and $\mathrm{P}[\chi^2(4) > 9.92 \approx 0.0558$. Finally, for the system of equations, the test statistic for a similar test is 29.15, and $\mathrm{P}[\chi^2(8) > 29.15] \approx 0.0003$.

More wealth is associated on net with more money giving. Recall that the wealth categories are [negative, 1,000], [1,000, 50,000], [50,000, 150,000], [150,000, 500,000], and [500,000, above]. The estimates show that the positive relation is stronger, the higher the wealth level. This seems intuitive. Besides, the money equation result seems to suggest that both income and wealth have some independent influence on money giving. Also note that the relative magnitudes of income dummies are roughly the same as in the base specification.

Time Giving Equation

Income dummies were dropped from this equation. Estimates for a smaller model that *includes* these dummies showed that the "income profile" is somewhat different from that in the base specification. In particular, the coefficients for $inc2-4$ are .1715, .2105, and -.5790, respectively, implying that, in the alternative specification estimation sample, the "hump" of the profile lasts over a longer range of middle incomes. However, wealth appears to be a more significant determinant of time giving than income when both sets of regressors are included.

Wealth is negatively correlated with volunteer hours. Over the wealth categories in this estimation sample, average income, wage, proportion working, subjective health status, and years in school all increase. Average age is roughly the same across wealth groups.[38] Thus, we again suspect that the negative association between wealth and volunteer hour is mainly due to the implied availability of time besides working.

[38] By wealth category, the averages of some explanatory variables are: income: 111532, 162003, 209195, 302771, and 399091 dollars per year, respectively; wage: 11.307, 36.082, 38.310, 28.161, and 42.231 dollars per hour, respectively; proportion of retired: 87.88%, 64.63%, 64.06%, 76.00%, and 65.38%, respectively; age: 71.97, 70.84, 70.97, 72.83, and 74.42 years, respectively; health: 3.64, 3.44, 2.97, 2.84, and 2.38, respectively; education: 9.24, 10.74, 11.74, 12.97, and 14.62 years, respectively.

The price of time giving, wage rate, now has a negative estimate as expected, although it is still not statistically significant.

Other explanatory variables have qualitatively the same association with time giving as in the base specification.

Error Covariance Matrix

The qualitative features of these estimates are the same as in the base specification, except that the error variance of the time equation is now smaller than its counterpart in the base specification.

Overall, the results for this specification are similar to those for the base specification. Wealth seems to have the expected relation with money and time giving and have some independent explanatory power beside income.

4.5.3 Elasticities and Conditional Probabilities of Giving

Following the literature, we also calculated several elasticities, in order to get an idea of the responsiveness of giving with respect to changes in various explanatory variables. For the limited dependent variables in our models, the unconditional tax price elasticity of observed money giving, for example, is defined as

$$\eta = \frac{\partial E(y_{1i})}{\partial pm} \frac{pm}{E(y_{1i})},$$

where y_{1i} is the observed money giving by a typical individual in our sample[39] and pm and all other explanatory variables used in calculating $E(y_{1i})$ are evaluated at their sample means. The conditional counterpart is defined by replacing the expectations above by those conditional on y_{1i} being positive. We focus on the unconditional ones.

[39] See the explanation of our model on page 91.

Using the expression of Brown and Lankford, we can write

$$E(y_{1i}) = E(y_{1i}|y_{1i} > 0, y_{2i} > 0) \cdot P(y_{1i} > 0, y_{2i} > 0))$$

$$+E(y_{1i}|y_{1i} > 0, y_{2i} = 0) \cdot P(y_{1i} > 0, y_{2i} = 0)) + 0 \cdot P(y_{1i} = 0)$$

The component terms above are somewhat different from those of Brown and Lankford. The explanation of the difference follows the expressions below:

$$E(y_{1i}|y_{1i} > 0, y_{2i} > 0) = E(X_{1i}\beta_1 + u_{1i}|X_{1i}\beta_1 + u_{1i} > 0, X_{2i}\beta_2 + u_{2i} > 0)$$

$$= X_{1i}\beta_1 + E(u_{1i}|u_{1i} > -X_{1i}\beta_1, u_{2i} > -X_{2i}\beta_2)$$

$$= X_{1i}\beta_1 - E(u_{1i}|u_{1i} < X_{1i}\beta_1, u_{2i} < X_{2i}\beta_2)$$

$$= X_{1i}\beta_1 - \sqrt{\sigma_{11}} E\left(\frac{u_{1i}}{\sqrt{\sigma_{11}}}\middle| \frac{u_{1i}}{\sqrt{\sigma_{11}}} < \frac{X_{1i}\beta_1}{\sqrt{\sigma_{11}}}, \frac{u_{2i}}{\sqrt{\sigma_{22}}} < \frac{X_{2i}\beta_2}{\sqrt{\sigma_{22}}}\right) \quad (a)$$

$$P(y_{1i} > 0, y_{2i} > 0) = P(X_{1i}\beta_1 + u_{1i} > 0, X_{2i}\beta_2 + u_{2i} > 0)$$

$$= P(u_{1i} > -X_{1i}\beta_1, u_{2i} > -X_{2i}\beta_2)$$

$$= P(u_{1i} < X_{1i}\beta_1, u_{2i} < X_{2i}\beta_2)$$

$$= P\left(\frac{u_{1i}}{\sqrt{\sigma_{11}}} < \frac{X_{1i}\beta_1}{\sqrt{\sigma_{11}}}, \frac{u_{2i}}{\sqrt{\sigma_{22}}} < \frac{X_{2i}\beta_2}{\sqrt{\sigma_{22}}}\right) \quad (b)$$

$$E(y_{1i}|y_{1i}>0,y_{2i}=0)=E(X_{1i}\beta_1+u_{1i}|X_{1i}\beta_1+u_{1i}>0,X_{2i}\beta_2+u_{2i}\leqslant 0)$$

$$=X_{1i}\beta_1+E(u_{1i}|u_{1i}>-X_{1i}\beta_1,u_{2i}\leqslant -X_{2i}\beta_2)$$

$$=X_{1i}\beta_1-E(\tilde{u}_{1i}|\tilde{u}_{1i}<X_{1i}\beta_1,\tilde{u}_{2i}\leqslant -X_{2i}\beta_2)$$

$$=X_{1i}\beta_1-\sqrt{\sigma_{11}}E\left(\frac{\tilde{u}_{1i}}{\sqrt{\sigma_{11}}}\middle|\frac{\tilde{u}_{1i}}{\sqrt{\sigma_{11}}}<\frac{X_{1i}\beta_1}{\sqrt{\sigma_{11}}},\frac{\tilde{u}_{2i}}{\sqrt{\sigma_{22}}}\leqslant -\frac{X_{2i}\beta_2}{\sqrt{\sigma_{22}}}\right)\quad (c)$$

$$P(y_{1i}>0,y_{2i}=0))=P(X_{1i}\beta_1+u_{1i}>0,X_{2i}\beta_2+u_{2i}\leqslant 0)$$

$$=P(u_{1i}>-X_{1i}\beta_1,u_{2i}\leqslant -X_{2i}\beta_2)$$

$$=P(u_{2i}\leqslant -X_{2i}\beta_2)-P(u_{1i}\leqslant -X_{1i}\beta_1,u_{2i}\leqslant -X_{2i}\beta_2)$$

$$=P\left(\frac{u_{2i}}{\sqrt{\sigma_{22}}}\right)-P\left(\frac{u_{1i}}{\sqrt{\sigma_{11}}}\leqslant -\frac{X_{1i}\beta_1}{\sqrt{\sigma_{11}}},\frac{u_{2i}}{\sqrt{\sigma_{22}}}\leqslant -\frac{X_{2i}\beta_2}{\sqrt{\sigma_{22}}}\right)\quad (d).$$

In the above, $[\tilde{u}_{1i},\tilde{u}_{2i}]^\top$ are bivariate normal random vector that has a correlation coefficient $-\rho$ but is the same as $[u_{1i},u_{2i}]^\top$ in other aspects. Thus,

$$\begin{bmatrix}\tilde{u}_{1i}\\ \tilde{u}_{2i}\end{bmatrix}\sim NID.\left(\begin{bmatrix}0\\0\end{bmatrix},\ \begin{bmatrix}\sigma_{11} & -\sigma_{12}\\ -\sigma_{21} & \sigma_{22}\end{bmatrix}\right).$$

The third equality in (c) uses the symmetry of the distributions of $[u_{1i},u_{2i}]^\top$ and $[\tilde{u}_{1i},\tilde{u}_{2i}]^\top$ according to the u_{2i} axis.

Following Brown and Lankford, we calculated the elasticities using numerical derivatives of the unconditional expectations. All explanatory variables are set at their sample means. Then we perturbed the relevant sample mean by plus and minus 10^{-11}, calculated the difference between the two resulting $E(y_{ji})$'s, and divided the difference by 2×10^{-11} to get an estimate of $\partial E(y_{ji})/\partial(x)$ evaluated at the sample means of all the regressors.

As Brown and Lankford, for example, put it, with limited dependent variable models, an elasticity reflects "changes in both the expected value of the dependent variable conditional on its being positive and in the probability of its being positive." We can see this clearly from the expressions associated with $E(y_{ji})$ above.

Table 4.1 Elasticities, base specification

	pm	*educ*	*wage*	*times*
y_{1i}	-3.5277	0.7957	0.0000	0.0309
y_{2i}	0.0000	2.4224	0.0655	0.0000

Table 4.2 Elasticities, alternative specification

	pm	*educ*	*wage*	*times*
y_{1i}	-3.2875	0.7479	0.0000	0.0238
y_{2i}	0.0000	3.0988	-0.0362	0.0000

y_{1i} : observed money donation

y_{2i} : observed time donation

Since Brown and Lankford used a probit type likelihood function when one dependent variable is censored and the other is not, it is not feasible for them to calculate expressions like $E(y_{1i}|y_{1i} > 0, y_{2i} = 0)$, as the giving function of y_{1i} conditional on $y_{2i} = 0$, for example, is not estimated. Their setup does allow one to calculate elasticities of one dependent variable conditional on the other being positive. The formula for the elasticity of money giving with respect to the tax price conditional on time giving being positive, for example, is

$$\eta = \frac{\partial E(y_{1i}|y_{2i} > 0)}{\partial pm} \frac{pm}{E(y_{1i}|y_{2i} > 0)},$$

where

$$E(y_{1i}|y_{2i} > 0) = E(y_{1i}|y_{1i} > 0, y_{2i} > 0) \cdot P(y_{1i} > 0|y_{2i} > 0)).$$

As we reasoned above, we prefer to use a Tobit type likelihood function and thus the giving functions are estimated for those censoring cases too. Therefore, we can also calculate the unconditional elasticities above. All elasticities reported below are unconditional ones.

Elasticities with respect to other explanatory variables are not reported either because the variables are categorized or the elasticities are very close to zero.

As we can see, the elasticities are broadly similar across specifications. The sign of the elasticity of time giving with respect to wage rate are different in the two specifications, in part due to the opposite signs of the coefficient estimates. The cross price elasticities are close to zero, given our econometric model.

Although the tax price variable lacks much variation, the money giving elasticities associated with it are large in absolute value. As Andreoni et al (1996) stated, previous estimates vary from -0.4 to -3.0, with most estimates falling in the range of -1.0 to -1.3. Ours is certainly very large in absolute value. The reason for this is not entirely clear. It may be due to our focus on the older people. It is not clear whether the lack of variation in some explanatory variable could cause potential bias in coefficient estimates. It may also have to do with the specifics of our way of calculating the elasticities—evaluating every variable at the sample mean and calculating numerical derivatives in the way we did it. It may even in part be the result of the potentially disproportionate influence of the small number of people who do itemize on the expected values of money giving. In the base specification, for example, average money giving is 784.71 dollars for the entire sample of 418 individuals, 603.08 dollars for the 380 who do not itemize, and 2601 dollars for the 38 who do itemize. This fact may have lead to the sizable coefficient estimate and consequently the large elasticity estimate. The model specification also seems to affect the estimates a lot. In the appendix, we show results from estimating the Brown and Lankford model, where the dependent variable and some independent variables are log-transformed and the likelihood function is of the probit type. The tax price elasticities of money giving are only -0.1219 and -0.7725 for the base and the alternative specifications, respectively. We need to examine these and other possible causes in the future.

Years in school has a significant influence on both types of giving, especially time giving. To give some more idea of the meaning of the numbers, let us focus on the base specification. Starting from the sample mean years in school of 11.37 years, an increase in one year of schooling when young would correspond to an increase in expected money giving of 69.99 dollars and an increase in expected time giving of 5.92 hours. (The expected money giving and time giving, when all explanatory variables are evaluated at their sample means, are 796.01 dollars and 67.34 hours, respectively.) These changes are not sizable, but are also not very small.

We also calculated the probability of one form of giving conditional on the other form of giving being positive or being zero.

Table 4.3 Conditional Probabilities of Giving

	base spec.	alter. spec.
$P(y_{1i} > 0 \mid y_{2i} > 0)$	0.7225	0.7225
$P(y_{1i} > 0 \mid y_{2i} = 0)$	0.5175	0.5013
$P(y_{2i} > 0 \mid y_{1i} > 0)$	0.2125	0.2102
$P(y_{2i} > 0 \mid y_{1i} = 0)$	0.1000	0.0932

y_{1i} : observed money donation

y_{2i} : observed time donation

Thus, volunteers are about 40% more likely than non-volunteers to donate money; money donors are about 100% more likely than non-donors to volunteer. These reflect the intuition that some people have a taste to give in general, which was mentioned above and will be mentioned again in the Appendix to this chapter.

Since our model is based on Brown and Lankford (1992), it is of interest to compare our results with their results. In the Appendix, we present the likelihood function and the estimation results of the Brown and Lankford econometric model and those of another model that is in some sense between the Brown and Lankford model and the main model in the text.

4.6 Concluding Remarks

By this exercise, we wish to draw more attention to the contributions to society made by the elderly. Beside the charitable donations of money and time we analyzed here, they also provide many other invaluable resources to their family members (eg., child care for the young), their communities (eg., formal and informal help to community members), and the society in general (eg., their wisdom and experience handed down to the young in many ways). When considering policies that affect the resources they receive from society, we may also want to bear in mind their various contributions to society and, when appropriate, to take into account how such policies would affect these invaluable contributions and the overall wellbeing of these respectable people.

In the econometric analysis, we examined the effects of some determinants of money giving and time giving. In general, income, wealth, subjective health status, and years in school when young are found to have statistically significant impacts. The tax price of monetary donation also has a statistically significant effect on money giving, although the lack of variation in this explanatory variable cautions us against drawing general conclusions. Besides, as in many previous studies, we find that the two types of giving are complementary, reflecting the fact that some people may have a general taste for giving.

In future work, it might be interesting to break down money giving and time giving by categories and analyze them separately. For example, one could separately look at giving to different types of charitable organizations.

Table 4.4 Incidence of Censoring

(initial sample, 687 obs)						
	n.o.b.			in percentages		
	$m > 0$	$m = 0$		$m > 0$	$m = 0$	
$h > 0$	307	10	317	44.69	1.46	46.14
$h = 0$	184	186	370	26.78	27.07	53.86
	491	196		71.47	28.53	

(base specification estimation sample, 418 obs)						
	n.o.b.			in percentages		
	$m > 0$	$m = 0$		$m > 0$	$m = 0$	
$h > 0$	79	10	89	18.90	2.39	21.29
$h = 0$	184	145	329	44.02	34.69	78.71
	263	155		62.91	37.08	

(alternative specification estimation sample, 313 obs)						
	n.o.b.			in percentages		
	$m > 0$	$m = 0$		$m > 0$	$m = 0$	
$h > 0$	57	7	64	18.21	2.24	20.45
$h = 0$	135	114	249	43.13	36.42	79.55
	192	121		61.34	38.66	

m : money donation, h : time donation

Table 4.5 Summary Statistics

base specification, without wealth, 418 obs.						
variable	mean	std. dev.	min	max	%> 0	mean| > 0
money[a]	.7847	1.4299	0	9.250	62.92	1.2472
time[b]	.0676	.2951	0	3.120	21.29	.3175
income[c]	2.0629	1.5040	0^{ℓ}	10.8016		
price	.9799	.0683	0.66	1		
wage[d]	.2955	.6858	0^{m}	8.5409	26.08	1.1331
size[e]	1.4163	.9764	1	8		
age[f]	7.2225	0.8005	6	9.8		
health[g]	3.1364	1.1007	1	5		
education[h]	1.1373	.3246	0^{n}	1.7		
visits[i]	.4586	1.1096	0	6.5	75.84	.6047
gender[j]	.7368	.4409	0	1		

alternative specification, with wealth, 313 obs.						
variable	mean	std. dev.	min	max	%> 0	mean| > 0
money[a]	.7870	1.4668	0	9.250	61.34	1.2830
time[b]	.0599	.2486	0	3120	20.45	0.2929
income[c]	2.1443	1.5870	0^{ℓ}	10.8016		
wealth[k]	16.9052	32.1754	-2.5000	271.5000	84.35	20.0748
price	.9753	.0746	0.66	1		
wage[d]	.2993	.5793	0^{m}	3.2165	27.80	1.0766
size[e]	1.3994	.9042	1	7		
age[f]	7.1712	0.8000	6	9.7		
health[g]	3.1534	1.1275	1	5		
education[h]	1.1514	.3333	0^{n}	1.7		
visits[i]	.4153	.9905	0	5.2	74.44	0.5579
gender[j]	.7029	.4577	0	1		

a: money donation, in 1,000 dollars; b: volunteering, in 1,000 hours;
c: net income, in 10,000 dollars; d: in 10 dollars; e: family size
f: in 10 years; g: subjective rating from excellent(1) to poor(5)
h: years in school, in 10 years; i: church visits in 2002, in 10 times
j: male(0), female(1); k: net wealth, in 10,000 dollars

ℓ: only two observations, these were set to 1 dollar in regressions
m: 0 exactly corresponds to retirement
n: only two observations have no education at all

Table 4.6
Base Specification, initial model

	coef.	s.e.	z	$P > z$
		money equation		
inc2	0.3272	0.1119	2.92	0.003
inc3	0.9876	0.2053	4.81	0.000
inc4	0.3150	0.3295	0.96	0.339
pm	-2.7071	0.7841	-3.45	0.001
ph	-0.0945	0.0888	-1.06	0.288
fsize	-0.0239	0.0492	-0.49	0.627
age2	0.2270	0.1073	2.12	0.034
age3	0.1993	0.1385	1.44	0.150
age4	0.3436	0.3066	1.12	0.262
health	-0.0911	0.0443	-2.06	0.040
educ	0.4862	0.1693	2.87	0.004
times	0.0574	0.0398	1.44	0.150
gender	-0.1294	0.1072	-1.21	0.228
cons	2.3133	0.8393	2.76	0.006
		time equation		
inc2	0.0930	0.2073	0.45	0.653
inc3	-0.0959	0.3869	-0.25	0.804
inc4	-1.1597	0.7755	-1.50	0.135
pm	0.3358	1.5306	0.22	0.826
ph	0.1413	0.1395	1.01	0.311
fsize	-0.1128	0.1129	-1.00	0.318
age2	0.4724	0.2084	2.27	0.023
age3	0.4478	0.2658	1.68	0.092
age4	-0.0617	0.6347	-0.10	0.923
health	-0.1715	0.0861	-1.99	0.046
educ	1.3426	0.3827	3.51	0.000
times	0.0149	0.0781	0.19	0.848
gender	-0.0218	0.2043	-0.11	0.915
cons	-2.6913	1.6867	-1.60	0.111
		error covariance matrix		
σ_{11}	0.7250	0.0663	10.94	0.000
ρ	0.2903	0.0636	4.56	0.000
σ_{22}	1.5503	0.2599	5.97	0.000

number of observations: 418
loglikelihood: -664.60953

Table 4.7
Base Specification, small model

	coef.	s.e.	z	$P > z$
		money equation		
inc2	0.3307	0.1075	3.08	0.002
inc3	1.0054	0.2005	5.01	0.000
inc4	0.3295	0.3206	1.03	0.304
pm	-2.6020	0.7491	-3.47	0.001
age2	0.2600	0.1038	2.50	0.012
age3	0.2537	0.1330	1.91	0.056
age4	0.4029	0.3044	1.32	0.186
health	-0.0866	0.0443	-1.95	0.051
educ	0.5068	0.1678	3.02	0.003
times	0.0487	0.0389	1.25	0.210
cons	1.9930	0.7932	2.51	0.012
		time equation		
inc2	0.0906	0.2014	0.45	0.653
inc3	-0.1378	0.3477	-0.40	0.692
inc4	-1.1975	0.7182	-1.67	0.095
ph	0.1458	0.1307	1.12	0.264
age2	0.4880	0.2065	2.36	0.018
age3	0.4836	0.2646	1.83	0.068
age4	-0.0236	0.6345	-0.04	0.970
health	-0.1701	0.0855	-1.99	0.047
educ	1.4004	0.3772	3.71	0.000
cons	-2.6050	0.6122	-4.25	0.000
		error covariance matrix		
σ_{11}	0.7299	0.0667	10.94	0.000
ρ	0.2924	0.0632	4.62	0.000
σ_{22}	1.5517	0.2598	5.97	0.000

number of observations: 418
loglikelihood: -666.6061
LR test of small model against initial model:
statistic$\approx$3.99, P$[\chi^2(7) > 3.99] \approx 0.7806$

Table 4.8
Alternative Specification, initial model

	coef.	s.e.	z	$P > z$
		money equation		
inc2	0.2181	0.1387	1.57	0.116
inc3	0.8296	0.2318	3.58	0.000
inc4	0.1712	0.3756	0.46	0.648
wealth2	0.0463	0.1754	0.26	0.792
wealth3	0.4276	0.1854	2.31	0.021
wealth4	0.4795	0.1917	2.50	0.012
wealth5	0.5733	0.2515	2.28	0.023
pm	-2.7190	0.8576	-3.17	0.002
ph	-0.1239	0.1109	-1.12	0.264
fsize	-0.0475	0.0645	-0.74	0.461
health	-0.0182	0.0523	-0.35	0.729
educ	0.4359	0.2042	2.14	0.033
times	0.0467	0.0521	0.90	0.370
gender	-0.1281	0.1227	-1.04	0.296
cons	2.0574	0.9365	2.20	0.028
		time equation		
inc2	0.1905	0.2311	0.82	0.410
inc3	0.2773	0.3852	0.72	0.472
inc4	-0.5204	0.7419	-0.70	0.483
wealth2	-0.5150	0.2778	-1.85	0.064
wealth3	-0.7348	0.3074	-2.39	0.017
wealth4	-0.5960	0.3120	-1.91	0.056
wealth5	-1.1880	0.4586	-2.59	0.010
pm	-0.4000	1.4885	-0.27	0.788
ph	-0.2086	0.2066	-1.01	0.312
fsize	-0.0975	0.1103	-0.88	0.377
health	-0.1741	0.0881	-1.98	0.048
educ	1.3179	0.3950	3.34	0.001
times	-0.0140	0.0966	-0.14	0.885
gender	0.1127	0.2021	0.56	0.577
cons	-1.0780	1.6567	-0.65	0.515
		error covariance matrix		
σ_{11}	0.7268	0.0776	9.36	0.000
ρ	0.3035	0.0765	3.97	0.000
σ_{22}	1.1493	0.2291	5.02	0.000

number of observations: 313

loglikelihood: -473.46032

Table 4.9
Alternative Specification, small model

	coef.	s.e.	z	$P > z$
		money equation		
inc2	0.1848	0.1286	1.44	0.151
inc3	0.7884	0.2199	3.59	0.000
inc4	0.2229	0.3528	0.63	0.528
wealth2	0.0448	0.1737	0.26	0.796
wealth3	0.4601	0.1815	2.53	0.011
wealth4	0.5418	0.1878	2.89	0.004
wealth5	0.6501	0.2454	2.65	0.008
pm	-2.4014	0.8018	-3.00	0.003
educ	0.4628	0.1992	2.32	0.020
times	0.0407	0.0508	0.80	0.423
cons	1.4531	0.8456	1.72	0.086
		time equation		
wealth2	-0.4744	0.2745	-1.73	0.084
wealth3	-0.6586	0.2917	-2.26	0.024
wealth4	-0.5058	0.2862	-1.77	0.077
wealth5	-1.1591	0.4166	-2.78	0.005
ph	-0.0669	0.1722	-0.39	0.698
age2	0.2953	0.1997	1.48	0.139
age3	0.2776	0.2603	1.07	0.286
age4	-0.7188	0.7876	-0.91	0.361
health	-0.1621	0.0838	-1.93	0.053
educ	1.4888	0.3896	3.82	0.000
cons	-1.9231	0.6042	-3.18	0.001
		error covariance matrix		
σ_{11}	0.7343	0.0785	9.36	0.000
ρ	0.3111	0.0756	4.12	0.000
σ_{22}	1.1195	0.2225	5.03	0.000

number of observations: 313
loglikelihood: -474.5459
LR test of small model against initial model:
statistic≈2.17, P[$\chi^2(8) > 2.17$] ≈ 0.9753

Chapter V

Summary and Conclusions

In this thesis, we explored the use of matching methods to increase voluntary contributions to public good and to reduce environmental pollution in an international context. We also documented the charitable giving of money and time by the elderly and analyzed some of its determinants.

Focusing on the idea of quantity-contingent matching found in a corporate challenge gift, we built a simple model of this matching method in chapter II and found that, under certain assumptions, it can increase public good contributions to an efficient level.

Extending the analysis of Boadway, Song and Tremblay (forthcoming), chapter III examined whether the rate-matching and quantity-contingent matching methods for public good contribution can be adapted and used to reduce activities that cause a negative externality. In a simple two-country model of international pollution abatement, we found that the methods help internalize negative externalities, but cannot equate the marginal abatement costs across countries, which would be required for full efficiency.

Turning to the kind of public good contribution that many people may do in their everyday life, chapter IV provided an empirical account of the donations of money and time by the elderly in a recent PSID survey in the United States. Such donations only represent a small part of the many contributions by the elderly to the society. In a SUR-Tobit framework, we examined the statistical association between the two types of giving and some economic and demographic determining factors. In general, income, wealth, the tax price of money donation, wage, the subjective rating of health status, and the years of schooling tend to have statistically significant relations with the two forms of giving.

In chapters II and III, we made the important assumption that some or all of the players in the games can commit to matching plans that are *ex ante* optimal to them, but are not so *ex post*, that is, after the other players have taken actions based on the belief that the plans will be carried out as promised. This assumption may need justification. Guttman and Schnytzer (1992) generalize the Guttman-Danziger-Schnytzer type mechanism for public good contributions to both positive and negative externality problems and showed the existence of a subgame perfect equilibrium in the matching game for the two player case. This paper discussed two potential ways by which one may justify the commitment assumption. The first way is to posit the existence of binding, unilateral contracts, in which matching commitments are made. However, as they pointed out, this requires the players to know the socially optimal levels of the externality-generating action before the game starts. The matching mechanisms studied by the three papers above do not require that, however; instead, the optimal levels emerge as the Nash equilibria in the games. Thus, the first way can be overly restrictive. Besides, the binding contracts themselves would require an enforcement mechanism. The second way is to postulate that agents have an incentive to maintain reputation. Reputation can be especially valuable in repeated games. However, due to the complexity of the models, it is hard to introduce repeated interactions to them. Given these considerations, Guttman and Schnytzer (1992) chose to proceed in the first way, treating the matching pre-commitments *as if* they were binding contracts. They pointed out that this treatment reflects their view "that there remain interesting problems to explore in understanding the incentives to cooperate in the presence of externality" (p. 75).

Our views on this assumption in both the joint work and this thesis are similar to that of Guttman and Schnytzer's. Furthermore, in the real world, it is not extremely hard to find examples of such commitment. In fund-raising, individuals, companies, and government agencies do show some degree of commitment. Although the events may not be repeated, these agents seem to care about their reputation at a general level. Their reputation in one event may be important in the future in some other events. Or, losing reputation itself can cause large utility loss, which might not be considered in a usual economic model.

In chapter IV, we were not able to utilize the full panel data available in the PSID, but have to restrict ourselves to one year's cross-sectional portion of the data. Exploring how Tobit estimators for panel data can be developed for a system of seemingly unrelated equations might be a useful but challenging topic for future research.

For good reasons, such as tractability, economists tend to model preferences for public goods in a simple but meaningful way and to take that preferences as given. However, in reality, preferences can be changed by social interactions, by family, school, and religious educations, or even by the very act of contributing to the public good. If such influences make the agents averse to profiting on others' efforts and cause them value the public good a lot, these non-economic channels may be much more effective in overcoming the free rider problem. One work along this line is that of Brown, Rooney, Steinberg and Wilhelm (2006), which studies the intergenerational transmission of generosity. Theoretical and empirical research in this direction might be interesting and worthwhile pursuing.

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Appendix to Chapter III

This appendix shows some derivations that lead to equations (8), (9) and (11) on pages 62 to 63 in the text. At the end, it also displays how we attempted to use the l'Hôpital's rule to determine the magnitude of (8) when $m\mu = 1$, but found that the rule is not applicable here and are left with the conclusion that (8) is indeterminate when $m\mu = 1$.

Recall that country 2's minimization problem is

$$\min_{\{r\}} \; D[(e^{\mathrm{NE}} - r - m\rho) + (\eta^{\mathrm{NE}} - \rho - \mu r)] + C(e^{\mathrm{NE}} - r - m\rho)$$

The first-order condition, denoted $F(r, \rho, m, \mu)$, is:

$$F(\cdot) \equiv -(1+\mu)D' - C' = 0, \quad \text{or} \quad \frac{D'}{C'} = -\frac{1}{1+\mu} \tag{3}$$

Country 2's minimization problem is

$$\min_{\{\rho\}} \; \mathit{\Delta}[(e^{\mathrm{NE}} - r - m\rho) + (\eta^{\mathrm{NE}} - \rho - \mu r)] + \mathcal{C}(\eta^{\mathrm{NE}} - \rho - \mu r)$$

The first-order condition, denoted $\mathit{\Phi}(r, \rho, m, \mu)$, is:

$$\mathit{\Phi}(\cdot) \equiv -(1+m)\mathit{\Delta}' - \mathcal{C}' = 0, \quad \text{or} \quad \frac{\mathit{\Delta}'}{\mathcal{C}'} = -\frac{1}{1+m} \tag{4}$$

Using these, we can write down the derivatives of the first-order conditions (3) and (4):

$$\begin{aligned}
F_r &= (1+\mu)^2 D'' + C'', & F_\rho &= (1+m)(1+\mu)D'' + mC'', \\
F_m &= (1+\mu)\rho D'' + \rho C'', & F_\mu &= -D' + (1+\mu)rD'', \\
\mathit{\Phi}_r &= (1+m)(1+\mu)\mathit{\Delta}'' + \mu\mathcal{C}'', & \mathit{\Phi}_\rho &= (1+m)^2\mathit{\Delta}'' + \mathcal{C}'' \\
\mathit{\Phi}_m &= -\mathit{\Delta}' + (1+m)\rho\mathit{\Delta}'', & \mathit{\Phi}_\mu &= (1+m)r\mathit{\Delta}'' + r\mathcal{C}''
\end{aligned} \tag{A1}$$

Equation (8)

Recall that (7) is

$$\left.\frac{d(D+C)}{dm}\right|_{\mu} = D' \cdot \left[-(1+\mu)\frac{\partial r}{\partial m} - (1+m)\frac{\partial \rho}{\partial m} - \rho\right] + C' \cdot \left[-\frac{\partial r}{\partial m} - m\frac{\partial \rho}{\partial m} - \rho\right]$$

Denote $d(D+C)/dm|_{\mu}$ by y for ease of notation. Dividing (7) through by C' and using (3), we get

$$\begin{aligned}\frac{y}{C'} &= -\frac{1}{1+\mu}\left[-(1+\mu)\frac{\partial r}{\partial m} - (1+m)\frac{\partial \rho}{\partial m} - \rho\right] + \left[-\frac{\partial r}{\partial m} - m\frac{\partial \rho}{\partial m} - \rho\right] \\ &= \frac{1}{1+\mu}\left[(1-m\mu)\frac{\partial \rho}{\partial m} - \mu\rho\right]\end{aligned}$$

By (3) again, this can be written

$$-\frac{y}{D'} = \frac{1}{1+\mu}\frac{-F_r\Phi_m + \Phi_r F_m}{det} - \frac{\mu\rho \; det}{det} \tag{$A2$}$$

Using (6), which is

$$\left.\frac{dr}{dm}\right|_{\mu} = \frac{-F_m\Phi_\rho + \Phi_m F_\rho}{det}, \quad \text{and} \quad \left.\frac{d\rho}{dm}\right|_{\mu} = \frac{-F_r\Phi_m + \Phi_r F_m}{det} \tag{6}$$

and the definition of $det \equiv F_r\Phi_\rho - \Phi_r F_\rho$, we can obtain

$$\begin{aligned}-\frac{det}{D'}y &= (1-m\mu)(\Phi_r F_m - F_r\Phi_m) - \mu\rho(F_r\Phi_\rho - \Phi_r F_\rho) \\ &= -F_r[(1-m\mu)\Phi_m + \mu\rho\Phi_\rho] + \Phi_r[(1-m\mu)F_m + \mu\rho F_\rho]\end{aligned}$$

Using $(A1)$, we can rewrite

$$\begin{aligned}&(1-m\mu)F_m + \mu\rho F_\rho \\ =&(1-m\mu)[(1+\mu)\rho D'' + \rho C''] + \mu\rho[(1+m)(1+\mu)D'' + mC''] \\ =&\rho(1+\mu)[1-m\mu+\mu+m\mu]D'' + \rho[1-m\mu+m\mu]C'' \\ =&\rho[(1+\mu)^2 D'' + C''] = \rho F_r\end{aligned}$$

Then

$$-\frac{det}{D'}y = \rho F_r\mathit{\Phi}_r - F_r[(1-m\mu)\mathit{\Phi}_m + \mu\rho\mathit{\Phi}_\rho]$$

and

$$\begin{aligned}
&-\frac{det}{D'F_r}y = \rho\mathit{\Phi}_r - (1-m\mu)\mathit{\Phi}_m - \mu\rho\mathit{\Phi}_\rho \\
=&\rho(1+m)(1+\mu)\mathit{\Delta}'' + \rho\mu\mathcal{C}'' - (1-m\mu)[-\mathit{\Delta}' + (1+m)\rho\mathit{\Delta}''] \\
&-\mu\rho[(1+m)^2\mathit{\Delta}'' + \mathcal{C}''] \\
=&\rho(1+m)(1+\mu)\mathit{\Delta}'' + \rho\mu\mathcal{C}'' + (1-m\mu)\mathit{\Delta}' - (1-m\mu)(1+m)\rho\mathit{\Delta}'' \\
&-\rho\mu(1+m)^2\mathit{\Delta}'' - \rho\mu\mathcal{C}'' \\
=&\rho(1+m+\mu+m\mu-1-m+m\mu+m^2\mu-\mu-2m\mu-m^2\mu)\mathit{\Delta}'' \\
&+(1-m\mu)\mathit{\Delta}' \\
=&(1-m\mu)\mathit{\Delta}'
\end{aligned}$$

Thus, we can get (8).

Equation (9)

Using $(A1)$, we can get (9) in the following way

$$\begin{aligned}
&det \equiv F_r\mathit{\Phi}_\rho - \mathit{\Phi}_r F_\rho \\
=&[(1+\mu)^2D'' + C''][(1+m)^2\mathit{\Delta}'' + \mathcal{C}''] \\
&-[(1+m)(1+\mu)\mathit{\Delta}'' + \mu\mathcal{C}''][(1+m)(1+\mu)D'' + mC''] \\
=&(1+m)^2(1+\mu)^2D''\mathit{\Delta}'' + (1+\mu)^2D''\mathcal{C}'' + (1+m)^2\mathit{\Delta}''C'' + C''\mathcal{C}'' \\
&-(1+m)^2(1+\mu)^2D''\mathit{\Delta}'' - m(1+m)(1+\mu)\mathit{\Delta}''C'' \\
&-\mu(1+m)(1+\mu)D''\mathcal{C}'' - m\mu C''\mathcal{C}'' \\
=&D''\mathcal{C}''(1+\mu)(1+\mu-\mu-m\mu) + \mathit{\Delta}''C''(1+m)(1+m-m-m\mu) \\
&+C''\mathcal{C}''(1-m\mu) \\
=&(1-m\mu)[(1+\mu)D''\mathcal{C}'' + (1+m)\mathit{\Delta}''C'' + C''\mathcal{C}'']
\end{aligned}$$

Equation (10)

Using $(A1)$ and $m\mu = 1$, we can get

$$\left.\frac{\partial r}{\partial \rho}\right|_{m\mu=1} = -\frac{F_\rho}{F_r} = -\frac{(1+m)(1+\mu)D'' + mC''}{(1+\mu)^2 D'' + C''} = -\frac{1}{\mu} = -m$$

and

$$\left.\frac{\partial \rho}{\partial r}\right|_{m\mu=1} = -\frac{\Phi_r}{\Phi_\rho} = \frac{(1+m)(1+\mu)\Delta'' + \mu\mathcal{C}''}{(1+m)^2 \Delta'' + \mathcal{C}''} = -\frac{1}{m} = -\mu$$

Applying l'Hôpital's Rule to (8)

Substituting (9) into (8), we get

$$y \equiv \left.\frac{d(D+C)}{dm}\right|_{\mu} = -\frac{(1-m\mu)D'\Delta' F_r}{(1-m\mu)[(1+\mu)D''\mathcal{C}'' + (1+m)\Delta'' C'' + C''\mathcal{C}'']} \qquad (A3)$$

To simplify notation, define

$$x_1 \equiv D'\Delta' F_r$$

$$x_2 \equiv (1+\mu)D''\mathcal{C}'' + (1+m)\Delta'' C'' + C''\mathcal{C}''.$$

Because y is country 1's first-order condition with respect to m, where country 1 takes country 2's matching rate μ as given, $m\mu \to 1$ is equivalent to $m \to 1/\mu$. Thus, in applying l'Hôpital's rule, we take the partial derivative of the numerator and the denominator with respect to m.

$$\lim_{m\to\frac{1}{\mu}} y = \lim_{m\to\frac{1}{\mu}} \frac{\frac{\partial}{\partial m}(1-m\mu)x_1}{\frac{\partial}{\partial m}(1-m\mu)x_2} = \lim_{m\to\frac{1}{\mu}} \frac{-\mu x_1 + (1-m\mu)\frac{\partial x_1}{\partial m}}{-\mu x_2 + (1-m\mu)\frac{\partial x_2}{\partial m}}$$

However, if we rearrange this expression in the way we did for equation $(A2)$ above both $(1-m\mu)(\partial x_1/\partial m)$ and $(1-m\mu)(\partial x_1/\partial m)$ can again be written in a form that is similar to that of y, that is, $(1-m\mu)$ times some terms in the numerator and $(1-m\mu)$ times some other terms in the denominator. One may see this by noting that 1) both x_1 and x_2 contain $r(m,\mu)$ and $\rho(m,\mu)$ in their arguments (see $(A1)$ above), so $\partial r/\partial m|_\mu \; \partial\rho/\partial m|_\mu$ will appear when x_1 and x_2 are differentiated with respect to m, and 2) by (6) and (9), both $\partial r/\partial m$ and $\partial\rho/\partial m|_\mu$ have *det* and thus $(1 - m\mu)$ in their denominators.

Therefore, the limits as $m \to 1/\mu$ of the partial derivatives of the denominator of y and of the numerator of y with respect to m would again have terms that are similar to the denominator and the numerator of y themselves as in $(A3)$. Furthermore, one can see that this pattern would occur for all subsequent partial derivatives with respect to m. We know that, if the limit of the ratio of the partial derivatives does not exist, l'Hôpital's rule does not apply.

Another way that is sometimes used to compute the limits of indeterminate forms is to first perform Taylor series expansion of the functions involved and then examine the resulting expression to see how to proceed. For our case, applying this method would encounter the same kind of difficulty as that with applying l'Hôpital's rule. When we expand the functions in the denominator and the numerator around $m = 1/\mu$, we need to take the derivatives of them with respect to m. This means that we need to differentiate $r(m, \mu)$ and $\rho(m, \mu)$ in these function with respect to m again, so we would run into the same kind of situation as above.

Based on these observations, we think that (8) is indeterminate as $m \to 1/\mu$ or as $m\mu \to 1$ holding μ constant. Similar reasoning would apply to country 2's first-order condition with respect to μ.

Appendix to Chapter IV

This appendix refers to section 4.2, "Loglikelihood Function" (p. 93), and Appendix Tables A4.7 to A4.10 (at the end of this Appendix), which contain results from estimating the Brown and Lankford (1992) model and another model.

As mentioned in the text, for the purpose of comparison, we also estimated two other models. In the model in the text, both the dependent variable and the independent variables enter linearly, and the likelihood function is of the Tobit type. It is of interest to compare this model with the one used by Brown and Lankford, where both dependent variables and some of the independent variables are subject to a logarithmic transformation and the likelihood function is of the probit type. We call this *model BL* below. Another model that is in some sense between the two above is one where the dependent and some of the independent variables are log-transformed but the likelihood function is of the Tobit type. We call this *model b* below, where *b* stands for "between".

More specifically, *model BL* can be written as

$$y^*_{ji} = a_j + exp(X_{ji}\beta_j + u_{ji}) \qquad j = 1, 2,\ i = 1, \ldots, n,$$

$$a_j < 0 \text{ are constants}$$

$$y_{ji} = \begin{cases} y^*_{ji}, & \text{if } y^*_{ji} > 0, \\ 0, & \text{if } y^*_{ji} \leqslant 0 \end{cases}$$

where the error structure is the same as before

$$\begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}\right).$$

Rearranging, we can write

$$\log(y^*_{ji} - a_j) = X_{ji}\beta_j + u_{ji} \qquad j = 1, 2.$$

It is in this sense that we say the dependent variable is subject to a log-transformation.

The strictly negative constants a_j are location parameters that only affect the lower bounds of the y^*_{ji}.

Two independent variables, income and the tax price, now enter the model after log-transformation; all other independent variables remain the same as in the main model in the text.

The contributions to the likelihood function for this model has four cases:

1. Regime 1: $y^*_{1i} > 0$ and $y^*_{2i} > 0$

From the expression of the model, we can write

$$u_{ji} = \log(y^*_{ji} - a_j) - X_{ji}\beta_j.$$

Let $u_i \equiv [u_{1i}, u_{2i}]\top$. The contribution for this case can be written as

$$L^1_i(y^*; b, \Omega) = f_i(u_i; b, \Omega)/[(y^*_{1i} - a_1)(y^*_{2i} - a_2)]$$

where f_i is the joint density of u_i and the divisor is the determinant of the Jacobian of the transformation from $[u_{1i}\ u_{2i}]^\top$ to $[y^*_{1i}\ y^*_{2i}]\top$.

2. Regime 2: $y^*_{1i} > 0$ and $y^*_{2i} \leqslant 0$

We may first define

$$\overline{u}_{ji} \equiv \log(-a_j) - X_{ji}\beta_j,\ j = 1, 2,$$

and then define the "standardized censoring points"

$$h_j = \frac{\overline{u}_{ji}}{\sqrt{\sigma_{jj}}}.$$

Using these notations, we can write the contribution to the probit type likelihood function as

$$L_i^2(y^*; b, \Omega) = P(y_{1i} > 0, y_{2i} \leqslant 0)$$

$$= \int_{\overline{u}_{1i}}^{\infty} \int_{-\infty}^{\overline{u}_{2i}} f_i(u_i; b, \Omega)\, du_{2i}\, du_{1i} = \Phi(h_2) - \mathbf{\Phi}(h_1, h_2, \rho).$$

3. Regime 3: $y_{1i}^* \leqslant 0$ and $y_{2i}^* > 0$

$$L_i^3(y^*; b, \Omega) = P(y_{1i} \leqslant 0, y_{2i} > 0)$$

$$= \int_{-\infty}^{\overline{u}_{1i}} \int_{\overline{u}_{2i}}^{\infty} f_i(u_i; b, \Omega)\, du_{2i}\, du_{1i} = \Phi(h_1) - \mathbf{\Phi}(h_1, h_2, \rho).$$

4. Regime 4: $y_{1i}^* \leqslant 0$ and $y_{2i}^* \leqslant 0$

$$L_i^4(y^*; b, \Omega) = P(y_{1i} \leqslant 0, y_{2i} \leqslant 0)$$

$$= \int_{-\infty}^{\overline{u}_{1i}} \int_{-\infty}^{\overline{u}_{2i}} f_i(u_i; b, \Omega)\, du_{2i}\, du_{1i} = \mathbf{\Phi}(h_1, h_2, \rho).$$

Model b has exactly the same expression as *model BL* above, in that the dependent variable and some of the independent variables enter after a log-transformation. However, the likelihood function is of the Tobit type and is the same as that in the text.

Appendix Tables A4.7 and A4.8 contain estimates for *model BL* for the base specification and the alternative specification. Appendix Tables A4.9 and A4.10 are for *model b*.

As we can see, there are some quantitative and qualitative differences in the results of the main model in the text and the two here.

Comparing across the models for the base specification (Table 4.7 (p. 114), Appendix Tables A4.7 and A4.9 (p. 136 and p. 138)), we may note that the signs of almost all parameter estimates are the same, except for those of *Age*4 and *Times*. The differences are associated with statistically insignificant estimates and are not dramatic. The relative magnitudes of some age dummy coefficients and of some error covariance matrix estimates show some variations across the models. In general, the results for the two Tobit likelihood function models (the main model and *model b*) are closer to each other, as compared with those for the probit model (*model BL*).

Much the same can be said about the alternative specification (Tables 4.9 (p. 116), Appendix Table A4.8 and A4.10 (p. 137 and p. 139)). One may also note that many of the wealth dummies are not statistically significant in the probit model, whereas many of them are in the two Tobit models.

We also calculated elasticities and conditional probabilities for these two models. The elasticities for *model BL* are conditional ones as explained in the text (section 4.5.3, p. 106), while those for *model b* are unconditional ones and defined in the same way as those for the main model in the text (section 4.5.3, p. 103).

Table A4.1 Elasticities, base spec., *model BL*

	inc	*pm*	*educ*	*wage*	*times*
y_{1i}	0.0162	-0.1219	4.6880	0.0053	-0.0063
y_{2i}	-0.0305	-0.0411	1.5650	-0.0447	0.0002

Table A4.2 Elasticities, alternative spec., *model BL*

	inc	*pm*	*educ*	*wage*	*times*
y_{1i}	0.4148	-0.7725	5.2974	0.0109	-0.1391
y_{2i}	-0.0112	0.0210	1.8553	-0.0738	0.0038

Table A4.3 Conditional Probabilities of Giving, *model BL*

	base spec.	alter. spec.
$P(y_{1i} > 0 \mid y_{2i} > 0)$	0.2225	0.2209
$P(y_{1i} > 0 \mid y_{2i} = 0)$	0.0914	0.0734
$P(y_{2i} > 0 \mid y_{1i} > 0)$	0.8115	0.8497
$P(y_{2i} > 0 \mid y_{1i} = 0)$	0.6021	0.6123

y_{1i} : observed money donation

y_{2i} : observed time donation

Table A4.4 Elasticities, base spec., *model b*

	ince	*pm*	*educ*	*wage*	*times*
y_{1i}	0.8990	-4.7909	1.3093	0.0028	0.0408
y_{2i}	0.3805	-0.4222	4.0119	0.0496	0.0036

Table A4.5 Elasticities, alternative spec., *model b*

	inc	*pm*	*educ*	*wage*	*times*
y_{1i}	0.0712	-1.5075	1.2327	-0.0020	0.0510
y_{2i}	0.0060	-0.1276	5.1430	-0.0320	0.0043

Table A4.6 Conditional Probabilities of Giving, *model b*

	base spec.	alter. spec.
$P(y_{1i} > 0 \mid y_{2i} > 0)$	0.6843	0.6833
$P(y_{1i} > 0 \mid y_{2i} = 0)$	0.4210	0.3968
$P(y_{2i} > 0 \mid y_{1i} > 0)$	0.5170	0.5013
$P(y_{2i} > 0 \mid y_{1i} = 0)$	0.2642	0.2346

y_{1i} : observed money donation

y_{2i} : observed time donation

Although the conditional probabilities of giving have exactly the same formula and are thus directly comparable across the three models, the elasticities for the Tobit models and those for the probit model use different formulae and therefore lack comparability. As we may note, the conditional probabilities of giving are quite close across the two specifications for each model. However, except for the probabilities of giving some money conditional on doing some volunteer work ($P(y_{1i} > 0|y_{2i} > 0)$) in the two Tobit models, they differ significantly across the three models for each speciication; this is especially the case for the last two conditional probabilities, those of positive volunteer work. This indicates the importance of model selection.

Despite the differences, the conditional probabilities from all three models do show one similarity: the probability of making one form of giving conditional on positive giving of the other form is always greater than conditional on no giving of the latter. Many previous studies have found the same result. This seems to be another piece of evidence that some people have a taste for giving in general.

We may also compare our elasticity estimates of *model BL* above with those of Brown and Lankford (1992), since the two sets of estimates use the same formulae and are comparable. Their estimate of the tax price elasticity is -1.7 for money giving, and -2.1 (-1.1) for men's (women's) time giving. Our *model BL* base specification estimate of the tax price elasticity is -0.1219 for money giving, and -0.0411 for time giving; those for the alternative specification are -0.7725 and 0.0210, respectively. Overall, ours are smaller in absolute value than theirs.

Appendix Table A4.7
Base Specification, probit, log

	coef.	s.e.	z	$P > z$
	money equation			
linc	0.1603	0.1098	1.46	0.144
lpm	-0.1010	0.6539	-0.15	0.877
age2	0.2188	0.1428	1.53	0.125
age3	0.1870	0.1752	1.07	0.286
age4	0.0874	0.3782	0.23	0.817
health	-0.1001	0.0639	-1.57	0.118
educ	1.0326	0.4176	2.47	0.013
times	-0.0033	0.0477	-0.07	0.945
cons	-2.1548	1.2967	-1.66	0.097
a_1	-1.0161	0.5175	-1.96	0.050
	time equation			
linc	0.5869	0.1714	3.42	0.001
ph	-0.1151	0.1518	-0.76	0.448
age2	0.3432	0.2116	1.62	0.105
age3	0.6834	0.2892	2.36	0.018
age4	0.3512	0.6300	0.56	0.577
health	-0.1765	0.0914	-1.93	0.053
educ	1.1387	0.3641	3.13	0.002
cons	-5.3152	0.9363	-5.68	0.000
a_2	-0.0381	0.0162	-2.35	0.019
	error covariance matrix			
σ_{11}	0.6362	0.4278	1.49	0.137
ρ	0.3321	0.0869	3.82	0.000
σ_{22}	2.3090	0.6132	3.77	0.000

number of observations: 418
loglikelihood: -504.8971

Appendix Table A4.8
Alternative Specification, probit, log

	coef.	s.e.	z	$P > z$
	money equation			
linc	0.1319	0.1681	0.78	0.433
lpm	-0.2460	0.9809	-0.25	0.802
wealth2	-0.0116	0.3026	-0.04	0.969
wealth3	0.0471	0.3171	0.15	0.882
wealth4	0.1707	0.3271	0.52	0.602
wealth5	-0.1823	0.4275	-0.43	0.670
educ	1.5469	0.6185	2.5	0.012
times	-0.1067	0.1011	-1.06	0.291
cons	-3.6804	1.6189	-2.27	0.023
a_1	-0.6112	0.3274	-1.87	0.062
	time equation			
wealth2	-0.1890	0.2967	-0.64	0.524
wealth3	0.3848	0.3355	1.15	0.251
wealth4	0.5782	0.3461	1.67	0.095
wealth5	0.8736	0.5047	1.73	0.083
ph	0.1443	0.1916	0.75	0.451
age2	-0.0436	0.2229	-0.20	0.845
age3	0.2576	0.2946	0.87	0.382
age4	0.0058	0.6721	0.01	0.993
health	-0.0887	0.0941	-0.94	0.346
educ	1.3358	0.4299	3.11	0.002
cons	-4.2034	0.9220	-4.56	0.000
a_2	-0.0454	0.0236	-1.92	0.054
	error covariance matrix			
σ_{11}	1.1989	0.7673	1.56	0.118
ρ	0.3883	0.1000	3.88	0.000
σ_{22}	2.0073	0.6837	2.94	0.003

number of observations: 313
loglikelihood: -371.8851

Appendix Table A4.9
Base Specification, Tobit, log

	coef.	s.e.	z	$P > z$
	money equation			
linc	0.1497	0.0311	4.81	0.000
lpm	-0.8133	0.2319	-3.51	0.000
age2	0.0873	0.0422	2.07	0.039
age3	0.0983	0.0542	1.81	0.070
age4	0.1720	0.1246	1.38	0.167
health	-0.0324	0.0181	-1.79	0.073
educ	0.1622	0.0683	2.37	0.018
times	0.0151	0.0154	0.98	0.327
cons	0.0464	0.1687	0.28	0.783
a_1	-1.8242	0.1391	-13.11	0.000
	time equation			
linc	0.1310	0.1310	1.00	0.317
ph	0.0725	0.1210	0.60	0.549
age2	0.4774	0.2005	2.38	0.017
age3	0.6847	0.2633	2.60	0.009
age4	0.1461	0.5880	0.25	0.804
health	-0.1411	0.0828	-1.70	0.088
educ	1.4878	0.3982	3.74	0.000
cons	-4.1737	0.8281	-5.04	0.000
a_2	-0.1460	0.0422	-3.46	0.001
	error covariance matrix			
σ_{11}	0.1178	0.0126	9.36	0.000
ρ	0.4014	0.0656	6.12	0.000
σ_{22}	1.3249	0.3668	3.61	0.000

number of observations: 418
loglikelihood: -399.8794

Appendix Table A4.10
Alternative Specification, Tobit, log

	coef.	s.e.	z	$P > z$
money equation				
linc	0.1235	0.0390	3.16	0.002
lpm	-0.5798	0.2433	-2.38	0.017
wealth2	-0.0099	0.0688	-0.14	0.886
wealth3	0.1755	0.0712	2.47	0.014
wealth4	0.1959	0.0748	2.62	0.009
wealth5	0.2630	0.0963	2.73	0.006
educ	0.1275	0.0790	1.61	0.107
times	0.0195	0.0194	1.01	0.314
cons	0.0426	0.1618	0.26	0.792
a_1	-1.9281	0.1691	-11.40	0.000
time equation				
wealth2	-0.3501	0.2558	-1.37	0.171
wealth3	-0.4586	0.2850	-1.61	0.108
wealth4	-0.2175	0.2733	-0.80	0.426
wealth5	-0.6109	0.4125	-1.48	0.139
ph	-0.0393	0.1796	-0.22	0.827
age2	0.2366	0.1930	1.23	0.220
age3	0.4015	0.2490	1.61	0.107
age4	-0.7142	0.7461	-0.96	0.338
health	-0.1120	0.0805	-1.39	0.164
educ	1.6171	0.4389	3.68	0.000
cons	-3.3069	0.7774	-4.25	0.000
a_2	-0.1972	0.0610	-3.23	0.001
error covariance matrix				
σ_{11}	0.1092	0.0137	7.95	0.000
ρ	0.4307	0.0787	5.48	0.000
σ_{22}	0.9732	0.3267	2.98	0.003

number of observations: 313
loglikelihood: -281.7600