Three Essays on Macroeconomic and Financial Stability

by

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Abstract

This thesis studies several issues in the field of macroeconomic and financial stability.

In Chapter 2, I argue that systemic bankruptcy of firms can originate from coordination failure in an economy with investment complementarities. I demonstrate that in such an economy, a very small uncertainty about economic fundamentals can be magnified through the uncertainty about the investment decisions of other firms and can lead to coordination failure, which may be manifested as systemic bankruptcy. Moreover, my model reveals that systemic bankruptcy tends to arise when economic fundamentals are in the middle range where coordination matters. High financial leverage of firms greatly increases the severity of systemic bankruptcy. Optimistic beliefs of firms and banks can alleviate coordination failure, but can also increase the severity of systemic bankruptcy once it happens.

Chapter 3 studies how coordination failure in a country's new technology investment dampens a country's economic growth. I establish a two-sector Overlapping Generation model where capital goods are produced by two different technologies. The first is a conventional technology with constant returns. The second is a new technology exhibiting increasing returns to scale due to technological externalities, about whose returns economic agents have only incomplete information. My model reveals that coordination failure in new technology investment can lead to slower economic growth. More interestingly, the model generates a positive correlation between economic growth and volatility.

In Chapter 4, Frank Milne and I establish a dynamic currency attack model in the presence of a large player. In an attack on a fixed exchange rate regime with a gradually overvalued currency, both the inability of speculators to synchronize their attack and their incentive to time the collapse of the regime lead to the persistent overvaluation of the currency. We find that the presence of a large player can accelerate or delay the collapse of the regime, depending on his incentives to preempt other speculators or to "ride the overvaluation."

Co-Authorship

Frank Milne, for Chapter 4, “The Role of Large Players in a Dynamic Currency Attack Game.”

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Chapter 1

Introduction

Macroeconomic and financial instability (investment booms and subsequent busts, asset bubbles and crashes, and financial crises) can inflict enormous damage on an economy. Global financial markets were full of turbulence in the last three decades of the 20th century. The world witnessed a series of financial crises, such as the collapse of the European Exchange Rate Mechanism in 1992, the Mexican peso crisis in 1994, and the Asian financial crisis started in 1997 and spread to other emerging markets, triggering the Russian default and the devaluation of the Brazilian real. According to the IMF (1998), 158 currency crises and 54 banking crises are identified in over 50 countries during the period 1975-1997. Financial crises can be costly in terms of fiscal and quasi-fiscal costs of restructuring the financial sector and, more broadly, in terms of their effect on economic performance due to the inability of financial markets to function effectively. For example, according to the IMF (1998), resolution costs for banking crises in Chile and Argentina in the early 1980s reached over 40 percent of GDP, and nonperforming loans exceeded 30 percent of total loans in Malaysia during 1988.

The turbulence in global financial markets has spurred the interest of both academic researchers and policy makers in financial stability. The concept of financial stability has been controversial. Here I adopt the definition given by Houben, Kakes and Schinasi (2004):

> ... defines financial stability as a situation in which the financial system is capable of: (1) allocating resources efficiently between activities and across time; (2) assessing and managing financial risks, and (3) absorbing shocks. A stable financial system is thus one that enhances economic performance and wealth accumulation (on account of the first two aspects), while it is also able to prevent adverse disturbances from having an inordinate disruptive impact (the third aspect).

Despite its importance, macroeconomic and financial stability is difficult to study within the traditional neoclassical framework. A few elements that are missing from traditional neoclassical economics are critical to understanding macroeconomic and financial stability. First, economic agents are interdependent, and coordination failure can be an important source of macroeconomic and financial instability. Second, information is incomplete, and the way in which information is diffused to economic agents is critical in determining their expectations and actions. I believe that how interdependent economic agents form their expectations in an uncertain environment, based on their incomplete information, is the key to understanding macroeconomic and financial stability.

Economic theories, such as social learning models and games with strategic complementarities, are extremely useful in studying macroeconomic and financial stability, because they address economic situations where uncertainty, information diffusion (learning), and strategic complementarities are present. Social learning models study how the true state of the world is learned by rational economic agents with private information through the actions of other agents using the Bayesian updating rule. Chamley (2004) gives a comprehensive review of this burgeoning literature. Games with strategic complementarities study the situation where positive payoff externalities exist among economic agents. With perfect information, games with strategic complementarities tend to generate multiple equilibria. This feature is often used to explain the self-fulfilling property of financial crises and business cycles. However, this indeterminacy caused by multiple equilibria makes policy analysis difficult. An excellent survey is given by Vives (2005) on this body of literature.

Global games, first studied by Carlsson and van Damme (1993), introduce incomplete information to games with strategic complementarities, addressing economic situations where both Bayesian learning and strategic complementarities exist. In a typical global game, a finite number or continuum of players plays a binary action game. A player's payoffs from taking an action are increasing in both the level of other players taking the same action and economic fundamentals, which represent the underlying economic states. The players are assumed to have only incomplete information about economic fundamentals: They hold some prior belief about the economic fundamentals. Meanwhile, each player has his own private signal about the fundamentals. It turns out that when the precision of the private signal of each player is high enough relative to that of the prior belief, this seemingly complicated game has a unique Bayesian Nash Equilibrium that survives the iterated elimination of strictly dominated strategies.

Global games are a useful tool for the study of a variety of economic issues related to macroeconomic and financial stability, such as bank runs, currency crises and investment in the presence of macroeconomic complementarities, in which both uncertainty and strategic complementarities are involved. The feature of the unique equilibrium of global games gives them strong predictive power and greatly facilitates policy studies. Morris and Shin (1998) first applied them to currency attacks. Since then they have been widely used in bank runs, currency crises and macroeconomics.

In this thesis I apply these theories to gain a better understanding of macroeconomic and financial stability. I focus on several specific issues in this field. A common feature of my three thesis essays is that they all explore how economic agents form their heterogeneous expectations based on incomplete information, when strategic complementarities exist.

My first essay is a study of systemic bankruptcy of nonfinancial firms originating from firms' coordination failure in investment. Here the meaning of systemic bankruptcy is twofold. First, the bankruptcy studied in my essay occurs in a large number of firms in an economy simultaneously. Second, the bankruptcy originates endogenously in a decentralized economy where coordination matters but is not available. It provides a new mechanism for how the volatility in real investment causes financial crises that are manifested as systemic bankruptcy in the economy. Different from the existing literature which focuses on financial contagion mechanisms, my essay studies systemic bankruptcy that endogenously arises in an economy with investment complementarities. In such an economy, firms face two kinds of uncertainties when investing. First, firms are uncertain about the economic fundamentals of the investment. Second, firms are also uncertain about the investment decisions of other firms. I establish a model in a global game setup to demonstrate that a very small uncertainty about economic fundamentals can be magnified through uncertainty about other firms' investments, and lead to systemic bankruptcy. An important implication of my model is that, due to investment complementarities, the economy will be more vulnerable to financial crises. Systemic bankruptcy can occur even without significant economic shocks. The model also reveals that systemic bankruptcy tends to arise when economic fundamentals are in a middle range where coordination matters, which I call the coordination failure zone. High financial leverage of firms greatly increases the severity of systemic bankruptcy. Optimistic beliefs of firms and banks can alleviate coordination failure, but can also increase the severity of systemic bankruptcy once it happens.

My second essay is a natural extension of my first one. In this essay, investment decisions of nonfinancial firms are studied in a General Equilibrium framework. A global game established by Morris and Shin (2000) is extended to a two-sector Overlapping Generation model where capital goods can be produced by two different technologies. The first is a conventional technology with constant returns, about which economic agents have perfect information. The second is a new technology exhibiting increasing returns to scale due to technological externalities, about whose returns economic agents have only incomplete information. Economic agents have to choose which technology to invest in. In such a setup, a global game is repeatedly played in the capital goods sector each period. My model reveals that under certain circumstances, coordination failure in the capital goods sector will arise and be manifested as under-investment in the new technology. In this way, I demonstrate how coordination failure in a country's technology updating process leads to slower capital accumulation and economic growth. More interestingly, the model generates a positive correlation between economic growth and volatility through a new channel associated with coordination failure. The intuition is as follows: more investment in the new technology will alleviate coordination failure and lead to higher economic growth. Meanwhile, the new technology is riskier by nature and more investment in it leads to higher economic volatility.

My third essay, in collaboration with Frank Milne, studies the role of large players in currency attacks. Different from most existing literature that studies this issue in one-period static models, our study establishes a dynamic currency attack model in the presence of a large player based on Abreu and Brunnermeier (2003). In an attack on a fixed exchange rate regime with a gradually overvalued currency, both the inability of speculators to synchronize their attack and their incentive to time the collapse of the regime lead to the persistent overvaluation of the currency. We find that the presence of a large player, who is defined as a speculator with more wealth and superior information, can accelerate or delay the collapse of the regime, depending on his incentives to preempt other speculators and to "ride the overvaluation." When the incentive of a large player to preempt other speculators is dominant, the presence of a large player will accelerate the collapse of the regime. When the incentive of a large player to "ride the overvaluation" is dominant, the presence of a large player will delay the collapse of the regime. The latter case provides valuable insights into the role that large players play in currency attacks: it differs from the common belief that the presence of large players will alleviate asset mispricing due to their capability and willingness to arbitrage.

Chapter 2

Investment Complementarities, Coordination Failure and Systemic Bankruptcy

2.1 Introduction

Financial stability has gradually become an important topic in economic literature since the 1980's, after the world witnessed a series of financial crises in both developed and developing countries. A large body of literature is devoted to explaining the origin and propagation of financial crises (Krugman (1979, 2000); Diamond and Dybvig (1983); Obstfeld (1996); Cole and Kehoe (1996); Chang and Velasco (2001); Chari and Kehoe (2003)).

Although most financial crises manifest themselves in a seemingly unique manner, a common feature is widespread bankruptcy among both nonfinancial firms and financial institutions. A study on how this kind of bankruptcy arises will help us understand the origin of financial crises and, furthermore, how central banks should tackle them.

Existing literature on systemic bankruptcy of nonfinancial firms and financial institutions primarily focuses on how the bankruptcy of an individual economic agent is spread to other economic agents through different financial contagion mechanisms, such as credit chains and herding behavior (Allen and Gale (2000); Kiyotaki and Moor (2002); Chen (1999)). The origin of systemic bankruptcy, that is, how the first economic agent goes bankrupt, is simply attributed in the literature to some exogenously given shock. By doing so, this literature fails to provide any economic rationale behind the origin of systemic bankruptcy.

This chapter focuses on the origin of systemic bankruptcy of nonfinancial firms associated with volatility in real investment. A formal model is established to demonstrate that systemic bankruptcy of nonfinancial firms can endogenously originate from coordination failure in an economy with investment complementarities. This new explanation promotes better understanding of the origin of financial fragility. Moreover, the model can be used to identify economic situations associated with systemic bankruptcy, and therefore can provide theoretical guidance for central banks to establish an "early warning system" to prevent the occurrence of financial crises.

The model is established in a global game setup, where investment decisions of nonfinancial firms are studied under three conditions. In the first, investment complementarities exist. Therefore, investment returns are determined not only by economic fundamentals, but also by the proportion of the firms investing. With more firms investing, the investment return is higher. In the second, firms only have incomplete information about economic fundamentals. The information structure is specified as follows. At the beginning of the model, firms have a prior belief about the economic fundamentals. After the economic fundamentals are realized, each firm receives a private signal about the economic fundamentals. Each firm will form its posterior belief about the economic fundamentals according to Bayes' rule. In the third, firms have to finance their investment by debts. In such a setup, firms face two kinds of uncertainties when investing: (1) firms are uncertain about the economic fundamentals (they only have noisy private signals about the economic fundamentals), and (2) firms are also uncertain about the investment decisions of other firms (this is a non-cooperative game, and each firm has to form its belief about the actions of other firms based on its noisy private signal). The second kind of uncertainty, ignored in the existing literature, is endogenously generated in an economy with investment complementarities. I demonstrate that even a small uncertainty about economic fundamentals can be magnified through the second kind of uncertainty, causing coordination failure, which may be manifested as systemic bankruptcy. In my model, based on its private signal, each firm will form its own belief about the economic fundamentals (first kind of uncertainty) and about the proportion of other firms investing (second kind of uncertainty). Based on its belief, each firm will make its investment decision. In equilibrium, some firms will invest based on their private signals, even when the investment returns are low enough to lead to bankruptcy. Thus, the meaning of systemic bankruptcy is twofold: (1) it happens to a large number of firms in an economy, instead of to an individual firm, and (2) it is endogenously rooted in a decentralized credit economy, where coordination among firms matters.

One of the key assumptions in this chapter is that investment complementarities exist, and therefore coordination among firms can be critical. Investment complementarities are widely observed in the economy. They can exist in industries with industry-specific externalities, that is, the externalities within an industry. The sources of such externalities can be the benefits of "within-industry specialization, conglomeration, indivisibility and public intermediate inputs such as roads" (Caballero and Lyons (1989)). The industries with network externalities are special examples of such industries. Network externalities are defined as a change in the utility that an agent derives from a good, when the number of other agents consuming the same kind of goods changes. For example, as the Internet is increasingly used as a communication tool, Internet users find it more valuable since they can make greater use of it. So any investment by a firm in this industry attracting more

Internet users will benefit other firms in the industry.

My model can be interpreted as a study of systemic bankruptcy in such an industry. Due to the externalities within the industry, one firm's investment return is higher with higher investment activities by other firms. Therefore, there are investment complementarities among the firms in the industry.

Meanwhile, my model can also be interpreted as a study of systemic bankruptcy in the whole economy, if external economies of scale across industries, or cross-industry externalities, are taken into account. A source of cross-industry externalities can be the demand spillover effect. If other firms in the economy are investing more, they will be spending more too, and this leads to increased demand for the product of an individual firm, which will in general increase the investment return of the firm. In addition, if we drop the unrealistic assumption of a Walrasian auctioneer and admit there are transaction frictions in an economy, a higher level of economic activity will lower trading costs and raise the average investment return due to trading externalities, or the "thick market" effect (Diamond (1982)). The empirical work of Caballero and Lyons (1989, 1990) reveals that cross-industry externalities are significant in the economy. Using the data of two-digit manufacturing industries in Belgium, West Germany, France, the U.K. and the U.S., they test external economies and internal returns to scale in those countries. Strong evidence of external economies is found in all the countries.

Using my model, economic situations where systemic bankruptcy tends to arise can be identified. In this way, my model provides some theoretical guidance for central banks to establish an "early warning system" for financial crises. According to my model, systemic bankruptcy tends to arise when economic fundamentals are neither too strong nor too weak. Coordination failure arises when economic fundamentals fall into this range, which I call the coordination failure zone. More specifically, systemic bankruptcy tends to happen when economic fundamentals take low to medium values in this zone. Comparative statics further reveals that higher financial leverage of firms can greatly increase the severity of systemic bankruptcy. Moreover, optimistic public beliefs of firms and optimistic beliefs of banks about the fundamentals can alleviate coordination failure. The range in which systemic bankruptcy arises is narrowed. However, systemic bankruptcy can be more severe once it occurs.

All of the above results are generally consistent with empirical observations on financial crises. A large body of empirical studies on financial crises finds that financial crises tend to arise at the economic downturn of a business cycle, which is shortly after the economy reaches its peak (Gorton (1988); Kaminsky and Reinhart (1999)). This fact can be interpreted to be consistent with my model's result that systemic bankruptcy tends to arise when economic fundamentals take low to medium value in the coordination failure zone. In my model, coordination failure will not arise when economic fundamentals are extremely high or low. So systemic bankruptcy will not arise at the peak or trough of a business cycle. Only when the fundamentals are in the middle range, especially when the fundamentals are deteriorating from medium to low level, which can be interpreted as the economy being at the downturn from a boom, does systemic bankruptcy arise.

According to my model, systemic bankruptcy tends to be more severe when the financial leverage of firms is high. This result is also consistent with the empirical observation that financial crises tend to happen when the credit/GDP ratio is higher than in the tranquil time (Kaminsky and Reinhart (1999)).

Anecdotal observations on financial crises also reveal that financial crises tend to happen at the end of an economic boom, when both banks and firms are still sanguine about the economy, which is consistent with the result in my model that although optimistic public beliefs of firms and banks can alleviate the coordination failure, they will increase the severity of systemic bankruptcy once it occurs.

This chapter provides a mechanism through which uncertainties in real investment leads to financial fragility, which is manifested as systemic bankruptcy of nonfinancial firms. In this sense this chapter is in favor of the "fundamentalist" opinion that financial crises are caused by real economic factors. However, the mechanism in this chapter can be easily combined with financial contagion theories. As mentioned before, the mechanism that I provide here can be regarded as an alternative explanation about the exogenously given shock in financial contagion theories. Bankruptcy caused by the mechanism in this chapter can be further spread to other economic agents through financial contagion mechanisms, leading to more severe bankruptcy. An important message conveyed in this chapter is that due to investment complementarities, the economy will be more vulnerable to financial crises. Systemic bankruptcy can occur even without significant economic shocks.

The rest of this chapter is organized as follows. Section 2.2 provides a literature survey. In Section 2.3 a basic model without banks is presented. Section 2.4 analyzes how economic fundamentals, financial leverage, and public beliefs of firms affect systemic bankruptcy. Section 2.5 introduces banks without private signals into the basic model and examines the role that banks play in systemic bankruptcy. In Section 2.6 banks with private signals are introduced. In Section 2.7, conclusions and policy implications are given. Future research is also discussed.

2.2 Literature Survey

This chapter is related to three strands of literature. The first strand of literature is on financial crises, especially on systemic bankruptcy in both nonfinancial firms and financial institutions.

Kiyotaki and Moore (2002) study systemic bankruptcy of nonfinancial firms originating from two contagion mechanisms. One is the trade credit chain, and the other is the fall of the price of a collateral asset. Allen and Gale (2000) explore the spread of bank failure from one banking region to another due to the overlapping claims of the banks on each other. Chen (1999) models how the failure of a few banks can cause runs on other banks due to asymmetric information.

The second strand of literature is on macroeconomic complementarities and their implications for the economy. Bryant (1983) uses a special form of production function to study how technological complementarities generate Pareto-ranked multiple equilibria. The business cycle implications of technology complementarities is explored by Baxter and King (1991) in a model whose structure is similar to a standard real business cycle model. In their quantitative work, business cycles generated by a demand shock and propagated through technological complementarities are quantitatively modeled.

Diamond (1982) studies how trading externalities cause "thick market" effects in the presence of trading frictions. He finds that the return of an individual economic agent will be higher due to reduced search costs if more agents are in the market searching for trading partners.

Cooper (1999) comprehensively surveys macroeconomic complementarities and their implications for macroeconomic behavior. He examines a variety of sources of macroeconomic complementarities, such as technological complementarities, demand spillover effects and trading externalities, and studies their implications.

The third strand of literature is about global games and their applications in macroeconomic and financial stability.

Global games were first introduced by Carlsson and van Damme (1993). They incorporate incomplete information into a traditional coordination game with perfect information. In the game each player observes his payoffs with some noise. By iterated elimination of strictly dominated strategies, they prove that when the noise gets infinitely small, there is a unique equilibrium in the game.

Morris and Shin (1998) study currency attacks in a global game setup. They find that when speculators need to coordinate their actions to successfully attack a fixed exchange rate regime, and meanwhile are only able to observe economic fundamentals with some small noise, there is a unique equilibrium in the game, determined by both economic fundamentals and the beliefs of speculators. This result differs from that of a traditional coordination game with perfect information, where a currency attack is solely determined by the self-fulfilling beliefs of speculators. Successfully overcoming the problem of indeterminacy of multiple equilibria models, their model allows for the analysis of policy implications.

Morris and Shin (2000) summarize the applications of global games in macroeconomic modeling by explaining how global games can be used in the context of bank runs, currency crises, and debt pricing. They argue that global games are a useful approach for the analysis of many macroeconomic issues where players' payoffs are interdependent. They reckon that global games provide a more solid ground for policy analysis than multiple equilibria models due to their property of unique equilibrium.

How public information influences equilibrium allocation and social welfare in economies with investment complementarities is studied by Angeletos and Pavan (2004). They demonstrate that when coordination is socially desirable, an increase in the precision of public information will always increase social welfare, given that the complementarities are weak so that the equilibrium is unique. When the complementarities are strong, however, so that multiple equilibria are possible, the increase in public information may facilitate the coordination on both "bad" and "good" equilibria.

Finally, Chamley (2004) gives a survey on coordination games and global games. A detailed summary about the theory and applications of global games is also given by Morris and Shin (2003).

2.3 The Basic Model

This model is based on Morris and Shin (2000). In their model, a continuum of depositors of mass 1 has to decide whether to run a bank or not, based on their beliefs about deposit returns, which are determined by both economic fundamentals and the actions of other depositors. I apply their model to investment decisions of nonfinancial firms in an economy with investment complementarities. The firms are analogous to the depositors, and investment returns are analogous to deposit returns. Technically, my model differs from theirs in that the firms are assumed to finance their investment by debts. Thus the payoff structure of the firms is asymmetric and the firms care about the upside risks only when their capital is positive.[1] This greatly complicates the calculation of the expected payoffs of the firms. Both the investment returns (given realized economic fundamentals levels) and the threshold level of economic fundamentals (above which the firms' capital becomes positive) will change in the firms' strategies. I prove that the main properties of global games still hold in such a situation. Moveover, in Sections 2.5 and 2.6, I introduce banks into the model in two different ways. These two two-stage games, in which banks move at the first stage and firms move at the second stage, further differ from the one-stage game established by Morris and Shin (2000).

There is a continuum of risk-neutral firms with initial wealth w_0 who have to simultaneously decide whether to invest or not.

The gross return rate is 1 if a firm chooses not to invest. The gross return rate from investing is e^{r-l}. Here r denotes economic fundamentals of the investment, and l denotes the proportion of firms not investing. Thus the return of the investment is increasing both

[1]Morris and Shin (2004) study the issue that creditors of a distressed borrower have to decide whether to withdraw their loans or not. The creditors in their model also have an asymmetric payoff structure. But the issue they study is totally different from mine. They intend to explain the liquidation of an *individual project* caused by the actions among *creditors*. While in my model, I try to explain simultaneous bankruptcy among *a large number of firms (projects)* due to the actions among *firms (borrowers)*, and my focus is on how systemic bankruptcy behaves, which is totally irrelevant in their model.

in economic fundamentals and in the proportion of the firms investing, $1 - l$. The latter introduces investment complementarities to the game.

The investment is assumed to have a fixed size of mw_0, where $m > 1$ is exogenously given. So a firm investing has to borrow $(m-1)w_0$. The gross borrowing rate is exogenously given by 1. Later it will be endogenized by introducing banks into the model.

2.3.1 The Case with Perfect Information

With perfect information about r, this game has three possible cases:

When $r > 1$, there is a unique equilibrium in which all firms invest. No bankruptcy occurs.

When $r < 0$, there is also a unique equilibrium in which all firms do not invest. No bankruptcy occurs either.

When $0 < r < 1$, there are two (stable) equilibria. One is that all firms invest, with $l = 0$ and $r - l > 0$. The other is that all firms do not invest, with $l = 1$ and $r - l < 0$. No bankruptcy occurs in both equilibria.

2.3.2 The Case with Incomplete Information

Now I introduce incomplete information into the model. Suppose that at the beginning of the game each firm has an identical prior belief about the fundamentals of the investment, $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. Here α is the precision of $\tilde{r}$ and $1/\alpha$ is its variance. This belief is also called the public belief. In addition, after economic fundamentals are realized, each firm has access to very precise but not perfect information about them before it makes its decision. More specifically, given the realization of $\tilde{r}$, r, firm i observes the realization of signal $x_i = r + \varepsilon_i$, where $\varepsilon_i \sim N(0, 1/\beta)$. ε_i is i.i.d across the firms.

After observing the private signal x_i, firm i updates its belief about the fundamentals according to Bayes' rule. Thus $(\tilde{r}|x_i)$ is also normally distributed with mean

$$\rho_i = \frac{\alpha\bar{r} + \beta x_i}{\alpha + \beta} \tag{2.1}$$

and precision $\alpha + \beta$. Let $\gamma = \frac{\alpha^2}{\beta}$.

Proposition 1. *Provided that $\gamma \leq 2\pi$, there is a unique equilibrium in which each firm follows a symmetric trigger strategy. In this equilibrium, firm i chooses to borrow and invest if and only if $\rho_i > \rho^*$, where ρ^* is the unique solution to*

$$\sqrt{\alpha+\beta}\int_{r^*}^{+\infty}\phi(\sqrt{\alpha+\beta}(r-\rho^*))(me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r}))}-m+1)dr = 1,$$

where $\phi(.)$ and $\Phi(.)$ are respectively the PDF and CDF of a standard normal distribution with mean 0 and variance 1, and r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})) = ln\frac{m-1}{m}.$$

Otherwise, the firm chooses not to invest.

Proof:

There are two ways to prove the equilibrium in this game. One way is to use the iterated elimination of strictly dominated strategies. The other way is to confine attention to symmetric trigger strategy equilibria and to prove that there is such a unique equilibrium.

I will first give the proof that confines attention to symmetric trigger strategies. There are two steps involved. First, I pinpoint the unique value of ρ^* such that in equilibrium each firm i will invest if and only if $\rho_i > \rho^*$. Second, I demonstrate that this strategy is optimal for all the firms.

For ρ^* to be an equilibrium triggering point, a firm with the private signal x^* and updated belief ρ^* must be indifferent between investing and not investing. Recall that the relationship between x^* and ρ^* is given by Equation (2.1).

Let a^{NI} and a^I denote the actions of not investing and investing respectively. We know that

$$R(a^{NI}|x^*) = 1,$$

that is, the gross return of firm x^*(here I abuse the notation of x^*, the firm's private signal, to denote the firm) from not investing is always equal to 1.

The expected gross return rate of firm x^* from investing is given by:

$$ER(a^I|x^*(\rho^*)) = 0 \times \sqrt{\alpha+\beta} \int_{-\infty}^{r^*} \phi(\sqrt{\alpha+\beta}(r-\rho^*))dr + \sqrt{\alpha+\beta}$$
$$\int_{r^*}^{+\infty} \phi(\sqrt{\alpha+\beta}(r-\rho^*))[me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r})))} - m + 1]dr, \quad (2.2)$$

where r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})) = ln\frac{m-1}{m}.$$

Equation (2.2) is the key equation in this chapter, which is derived as follows.

First, the expected gross return rate of firm x^* is based on its belief on the fundamentals, which is $(\tilde{r}|x^*) \sim N(\rho^*, \frac{1}{\alpha+\beta})$. Thus the PDF of $(\tilde{r}|x^*)$ is given by $\sqrt{\alpha+\beta}\phi(\sqrt{\alpha+\beta}(r-\rho^*))$.

Second, given each realized value of $\tilde{r}$, r, and given that all the firms take the trigger strategy ρ^*, the total payoff of firm x^* from investing is a certain number, given by:

$$mw_0e^{r-l(r,\rho^*)} = mw_0e^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r})))},$$

where $\Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})))$ is the proportion of the firms not investing. It is derived as follows:

$$l(r, \rho^*) = Prob(\tilde{x} < x^*|\tilde{r} = r) = Prob(\tilde{x} < \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})) = \Phi(\sqrt{\beta}(\rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}) - r)).$$

That is, it is the proportion of the firms whose private signal $\tilde{x}$ is less than $x^* = \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})$. The CDF of $\tilde{x}$ is $\Phi(\sqrt{\beta}(x-r))$, since given r, $\tilde{x} \sim N(r, \frac{1}{\beta})$.

When $mw_0e^{r-l} < (m-1)w_0$, or $r - l < \ln\frac{m-1}{m}$, or $r < r^*$, the firm loses all of its initial wealth, w_0, and its gross return rate is 0. When $r - l > \ln\frac{m-1}{m}$, or $r > r^*$, the firm earns the gross return rate of $me^{r-l} - m + 1$.

Here r^* is the unique solution to

$$r - l = r - \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r})) = ln\frac{m-1}{m}.$$

Notice that $r - l$ is strictly increasing in r. Thus there is a unique critical level of r, r^*, below which $r - l < \ln\frac{m-1}{m}$ and above which $r - l > \ln\frac{m-1}{m}$.

Until now, I define the PDF of firm x^* over $\tilde{r}$, which is $\sqrt{\alpha+\beta}\phi(\sqrt{\alpha+\beta}(r-\rho^*))$. In addition, I define the gross return rate of firm x^* given each realized value of $\tilde{r}$. It is straightforward to get the expected gross return rate of firm x^*, Equation(2.2). Rearranging the above equation, I get the expected gross return rate of firm x^*:

$$ER(a^I|x^*(\rho^*)) = \sqrt{\alpha+\beta}\int_{r^*}^{+\infty}\phi(\sqrt{\alpha+\beta}(r-\rho^*))(me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r})))} - m + 1)dr \quad (2.3)$$

Firm x^* will be indifferent between investing and not investing if and only if

$$ER(a^I|x^*(\rho^*)) = R(a^{NI}|x^*) = 1. \quad (2.4)$$

I can prove that given $\gamma < 2\pi$, the expected gross return rate of firm x^* is strictly increasing in ρ^*. Therefore, there is a unique solution of ρ^* to Equation (2.4). The proof is given in Appendix A.1.

It is straightforward to demonstrate that the trigger strategy ρ^* is optimal for every firm. For firm x_i, its expected gross return rate from investing is given by:

$$ER(a^I|x_i(\rho_i)) = \sqrt{\alpha+\beta}\int_{r^*}^{+\infty}\phi(\sqrt{\alpha+\beta}(r-\rho_i))(me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r})))} - m + 1)dr.$$

From the above equation we can see that $\Phi(\sqrt{\alpha+\beta}(r-\rho_i))$ first order stochastically dominates $\Phi(\sqrt{\alpha+\beta}(r-\rho^*))$ when $x_i > x^*$, and is first order stochastically dominated when $x_i < x^*$. We also know that $me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r}))}-m+1$ is strictly increasing in r. Since the expected gross return rate is 1 if and only if $\rho_i = \rho^*$, the expected gross return rate is less than 1 when $\rho_i < \rho^*$, and is greater than 1 when $\rho_i > \rho^*$. Thus the trigger strategy ρ^* is optimal for all the firms.

In Appendix A.2, the iterated elimination of strictly dominated strategies is used to prove that this is the unique Bayesian Nash equilibrium.

Q.E.D

Notice that this game can be completely changed by introducing a coordinator, who asks each firm to submit its private signal and makes investment decisions for the firms. The Pareto optimal equilibrium can be at least one possible equilibrium in such a setup. In this equilibrium, the private signals from all firms are collected. Thus the uncertainty about economic fundamentals vanishes. Moreover, since the coordinator can coordinate the investment actions between firms, the uncertainty about other firms' actions vanishes too. But in a decentralized economy without such a coordinator, the uncertainty about both economic fundamentals and other firms' actions leads to inefficiency in the equilibrium, which I will demonstrate later to be manifested as systemic bankruptcy. In this sense I argue that this model provides a new explanation about systemic bankruptcy caused by coordination failure.

2.4 The Analysis of the Basic Model

In this section the basic model is used to study how systemic bankruptcy is influenced by different factors. Section 2.4.1 studies the relationship between realized economic fundamentals r and systemic bankruptcy; how firms' financial leverage influences systemic bankruptcy is analyzed in Section 2.4.2; Section 2.4.3 analyzes the impact of public beliefs about the fundamentals, $\bar{r}$, on systemic bankruptcy.

The numerical examples in this section are not used to quantitatively calibrate systemic bankruptcy in the real economy. Instead, I hope only to give some qualitative insights. I choose $\alpha = 1$ and $\beta = 100$ such that $\gamma < 2\pi$. In addition, the uncertainty about economic fundamentals is assumed to be extremely small ($\beta = 100$) to demonstrate that systemic bankruptcy is mainly caused by the uncertainty about other firms' investment decisions. I choose $m = 2$, or capital/asset ratio= $1/m = 0.5$. This ratio varies from 0.2 to 0.6 in different countries in reality. Finally, $0 < \bar{r} = 0.5 < 1$ since I am interested in the coordination failure zone. Finally, $w_0 = 100$.

2.4.1 Realized Economic Fundamentals and Systemic Bankruptcy

This section studies how the realized economic fundamentals, r influences systemic bankruptcy. I find that *systemic bankruptcy only appears when economic fundamentals are neither too strong nor too weak, which I call the coordination zone. More specifically, systemic bankruptcy begins to arise when r is lower than a threshold level. But the severity of bankruptcy is not monotonically decreasing in r. Instead it reaches its peak when r falls into a low to medium range.*

First, bankruptcy appears only when r is lower than a threshold level, $\underline{r}$. Since $\tilde{x} \sim N(r, 1/\beta)$, the proportion of firms not investing is given by:

$$l(r) = Prob(x \leq x^*(\rho^*)|\tilde{r} = r) = \Phi(\sqrt{\beta}(x^* - r)).$$

A firm will go bankrupt if and only if $mw_0e^{r-l(r)} < (m-1)w_0$, or $r - l(r) < \ln(\frac{m-1}{m})$. Since $r - l(r)$ is strictly increasing in r, there is a unique $\underline{r}$ satisfying

$$\underline{r} - l(\underline{r}) = \ln(\frac{m-1}{m}). \quad (2.5)$$

Second, given $r < \underline{r}$, the bankruptcy rate is $1 - l(r)$, that is, the proportion of the firms investing. Since $l(r)$ is decreasing in r, the bankruptcy rate is increasing in r.

Third, the unpaid debts of an individual firm when $r < \underline{r}$, $(m-1)w_0 - e^{r-l}mw_0$, are decreasing in r.

In order to fully reflect the severity of systemic bankruptcy, I introduce a single creditor, who lends to all the firms. Its total loss from lending, defined by Equation (2.6), is used to measure the severity of bankruptcy:

$$TL(r) = (1 - l(r)) \times max\{(m-1)w_0 - e^{r-l}mw_0, 0\}. \tag{2.6}$$

According to Equation (2.6), TL is not monotonically decreasing in r. Instead, there are two opposite effects on TL when r is decreasing. On the one hand, the unpaid debts of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, are increasing. On the other hand, $1 - l(r)$, the proportion of firms going bankrupt, is decreasing. Due to these two opposite effects, bankruptcy is the most severe when r is at some value lower than $\underline{r}$, where TL reaches its maximum value, which I call the maximum loss.

This result seems counter-intuitive, since we usually expect that bankruptcy is the severest when the fundamentals are the worst. But it is not surprising in this model, since here bankruptcy will happen only when firms invest. A firm can always avoid losses by not investing. So it is not the adverse fundamentals, but the uncertainty about adverse fundamentals and about the actions of other firms, that causes systemic bankruptcy. Later I will show that the latter uncertainty is the main cause of systemic bankruptcy, when the first uncertainty is assumed to be extremely small (firms have very precise private information). The uncertainty about the actions of other firms matters only when $0 < r < 1$, where coordination matters. Lower r reduces the bankruptcy rate, leading to less total loss. When $r < 0$, the economy is out of the coordination failure zone and uncertainty about other firms' actions is vanishingly small. Therefore no systemic bankruptcy arises.

Note here that this specific shape of the total loss is not a special result due to the assumption that private signals x_i are normally distributed. Morris and Shin (1998) prove that under certain circumstances, the feature of a unique symmetric trigger strategy equilibrium still holds when private signals are uniformly distributed. Given this equilibrium strategy, with the decrease in r, the unpaid debts of an individual firm will increase from

0 to some positive number. Meanwhile, with the decrease in r, the proportion of firms investing will decrease from 1 to 0. These two opposite effects still work in this case and lead to some maximum loss when r is in this intermediate to low range. The only difference is that the speed at which the proportion of firms investing is even in the case of a uniform distribution, but uneven in the case of a normal distribution.

Table 2.1: A numerical example with $\alpha = 1$, $\beta = 100$, $m = 2$, $\bar{r} = 0.5$, and $w_0 = 100$

x^*	ρ^*	$\underline{r}$	r at maximum loss	maximum loss
0.4237	0.4244	0.2580	0.2226	0.1334

A numerical example with parameter values given at the beginning of this section reveals the conclusions above. Table 2.1 tells us that given the parameter values above, firm i will invest if and only if its updated belief $\rho_i > 0.4244$, or its private signal $x_i > 0.4237$. Bankruptcy appears when $r < 0.2580$. The total loss reaches its maximum value of 0.1334 when $r = 0.2226$. Notice that r at the maximum loss is pretty high. That is because $1 - l$ rapidly decreases to 0 with the decrease of r, which effectively reduces the total loss.

Figure 2.1 shows that l goes to 1 when the realization of $r > 0.7$ and to 0 when the realization of $r < 0.2$. It is so because β is high and $l(r) = \Phi(\sqrt{\beta}(x^* - r))$ rapidly goes to 1 when r decreases and to 0 when r increases.

From Figure 2.2, we can see that bankruptcy appears only when $0 < r < 1$, where coordination matters. More specifically, it begins to occur after the fundamentals (r) reach 0.2580, and the total loss rapidly increases to its maximum when r decreases to 0.2226. Then it rapidly decreases to 0 when r gets lower. The intuition behind this result is as follows: when r first decreases from the threshold level $\underline{r}$ where bankruptcy begins to occur, the increase from the individual firm's unpaid debts, $(m-1)w_0 - e^{r-l}mw_0$, dominates the decrease in the bankruptcy rate, $1 - l(r)$. However, since β is high, that is, the firms

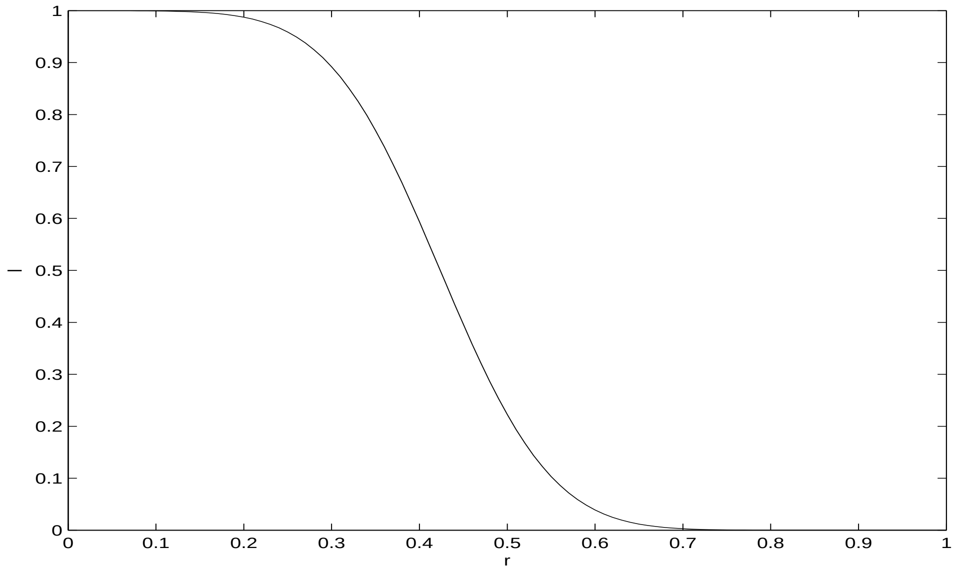

Figure 2.1: How proportion of firms not investing, l, changes with realized economic fundamentals r

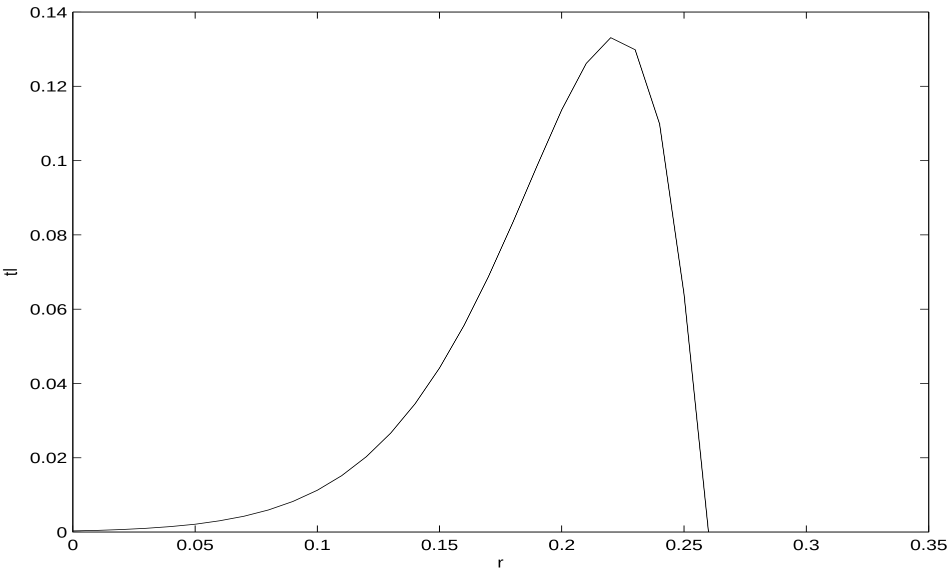

Figure 2.2: How total loss changes with realized economic fundamentals r

have a very precise private signal about r, the proportion of firms not investing, $l(r) = \Phi(\sqrt{\beta}(x^* - r))$, rapidly goes to 1 when r decreases. Therefore, the bankruptcy rate, $1 - l(r)$, rapidly decreases to 0 with r's further decrease, and its effect dominates the increase from the individual firm's unpaid debts, $(m-1)w_0 - e^{r-l}mw_0$.

In order to demonstrate that bankruptcy in this model is mainly caused by the uncertainty about other firms' investment decisions, I give an example without investment complementarities. Keeping all the parameter values unchanged in the above numerical example, I assume that the investment return is only determined by e^r. Table 2.2 gives the results.

Table 2.2: A numerical example with $\alpha = 1$, $\beta = 100$, $m = 2$, $\bar{r} = 0.5$, $w_0 = 100$ and no investment complementarities

x^*	ρ^*	$\underline{r}$	r at maximum loss	maximum loss
-0.01	-0.005	-0.6931	-0.7100	2.1389×10^{-12}

The maximum loss is almost equal to zero in this case, which is far less than that of 0.1334 in the case with investment complementarities. This example clearly reveals that the uncertainty about other firms' investment decisions can be an important source of systemic bankruptcy, even when the uncertainty about the fundamentals is extremely small and causes almost no bankruptcy.

2.4.2 Financial Leverage and Systemic Bankruptcy

This section analyzes how firms' financial leverage influences systemic bankruptcy. I find that *with higher financial leverage, systemic bankruptcy arises for a wider range of economic fundamentals, and is more severe once it happens.*

Observe that m influences bankruptcy in three ways. First, ρ^*, the equilibrium triggering point, is a function of m. But the relationship between ρ^* and m is ambiguous. It depends on the distribution of $(\tilde{r}|x_i)$. With a higher m, the firms can earn more profits when $e^{r-l(r)} > 1$, or $r - l(r) > 0$. But the firms also lose more when $e^{r-l(r)} < 1$, or $r - l(r) < 0$. Second, we know that the threshold fundamentals level for bankruptcy $\underline{r}$ is determined by

Equation (2.5). So when ρ^* is given, $\underline{r}$ is increasing in m. Third, given ρ^* and $r < \underline{r}$, the unpaid debt of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, is increasing in m. Therefore, the second and third effects of a higher m will definitely increase the severity of bankruptcy.

Keeping all the other parameter values unchanged, here I give a numerical example with m varying from 1.5 to 3.0 to help reveal the total effect of m on systemic bankruptcy. I want to see how ρ^*, $\underline{r}$, r at the maximum loss, and the maximum loss itself change in m.

It turns out that m has little impact on ρ^*. ρ^* does not change in m. This is because the precision of the updated belief of firm x^*, $\alpha + \beta = 101$, is so high that its expected payoff is determined only by a small range of values of $me^{r-l} - m + 1$, where e^{r-l} is close to 1 and m has little impact.

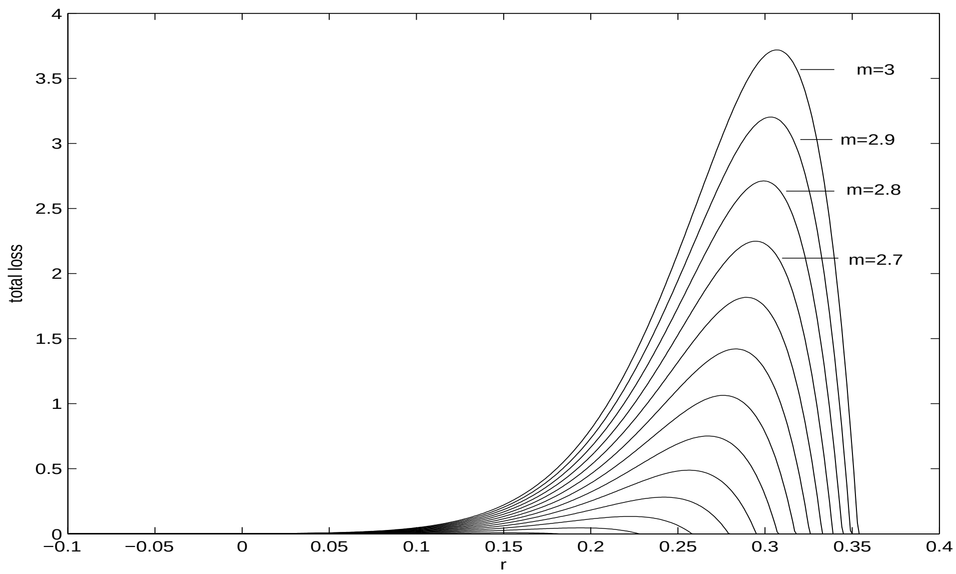

Figure 2.3: How total loss changes with financial leverage m

Figure 2.3 shows that systemic bankruptcy occurs for a wider range of fundamentals when m increases. The threshold level of the fundamentals where systemic bankruptcy begins to appear, $\underline{r}$, is strictly increasing in m. The intuition is straightforward: the more a firm borrows, the more easily it is unable to repay its debts. The realized fundamentals r at the maximum total loss is also strictly increasing in m. In addition, the severity of systemic bankruptcy for a given realized level of economic fundamentals is strictly increasing in m. The maximum total loss is trivial and close to 0 when m is around 1.5. Then it takes off and goes above 3.5 when $m = 3$.

So the impact of m on systemic bankruptcy works mainly through the latter two channels as long as the private signal of firms is highly precise. The severity of systemic bankruptcy increases rapidly with the increase in m.

2.4.3 Public Belief and Systemic Bankruptcy

This section examines the relationship between the mean of the public belief $\bar{r}$ and systemic bankruptcy. It reveals that *a higher $\bar{r}$ leads to more investment and alleviates coordination failure. The range of economic fundamentals where systemic bankruptcy arises is narrowed. However, systemic bankruptcy tends to be more severe once it happens.*

The public belief is given by $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. Since

$$ER(a^I|x^*(\rho^*)) = \sqrt{\alpha+\beta}\int_{r^*}^{+\infty}\phi(\sqrt{\alpha+\beta}(r-\rho^*))(me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r}))}-m+1)dr,$$

is strictly increasing in $\bar{r}$, $x^*(\rho^*)$ is decreasing in $\bar{r}$. Therefore, given each realized r, the proportion of firms investing is increasing in $\bar{r}$, because

$$1 - l(r) = 1 - \Phi(\sqrt{\beta}(x^* - r)),$$

where x^* is decreasing in $\bar{r}$.

Coordination is easier with more optimistic public beliefs about the fundamentals, $\bar{r}$. This is because a firm observing a good public signal not only anticipates that the fundamentals are good, but also anticipates that other firms will believe that the fundamentals are good, and will tend to invest more.

Since $\underline{r} - l(\underline{r}, \bar{r}) = \ln(\frac{m-1}{m})$, a higher $\bar{r}$ will decrease $\underline{r}$. Meanwhile, the unpaid debts of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, decrease for a given realized level of fundamentals

r due to a higher proportion of firms investing. But at the same time, the bankruptcy rate given each $r < \underline{r}$, $1 - l(r, \bar{r})$ increases.

A numerical example is given to show the relationship. Keeping all the other parameter values unchanged, we want to see how ρ^*, $\underline{r}$, r at the maximum loss, and the maximum loss change when $\bar{r}$ changes from 0 to 1.0.

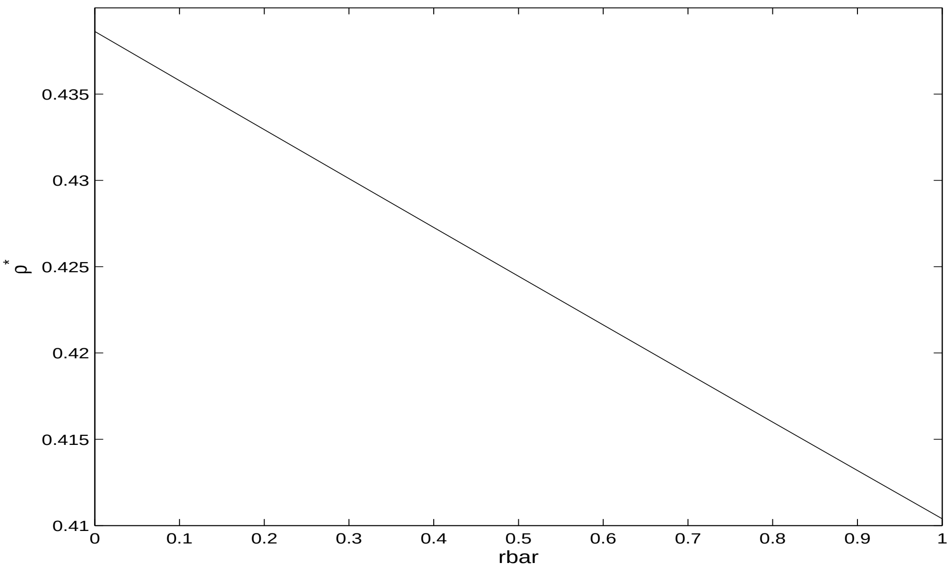

Figure 2.4: How firms' optimal trigger strategy ρ^* changes with public beliefs $\bar{r}$

Figure 2.4 shows that ρ^* is strictly decreasing in $\bar{r}$. That is, optimistic public beliefs about the economic fundamentals encourage more investment and alleviate coordination failure.

Figure 2.5 shows that with higher $\bar{r}$, systemic bankruptcy appears only in a narrower economic fundamentals range. The threshold fundamentals level where bankruptcy begins to arise, $\underline{r}$, is decreasing in $\bar{r}$. This is because with the improvement in coordination, investment return is higher at any given level of economic fundamentals. For the same reason, the level of the fundamentals at which systemic bankruptcy is the severest is also lower with higher $\bar{r}$. It is interesting to see that higher $\bar{r}$ leads to more severe systemic

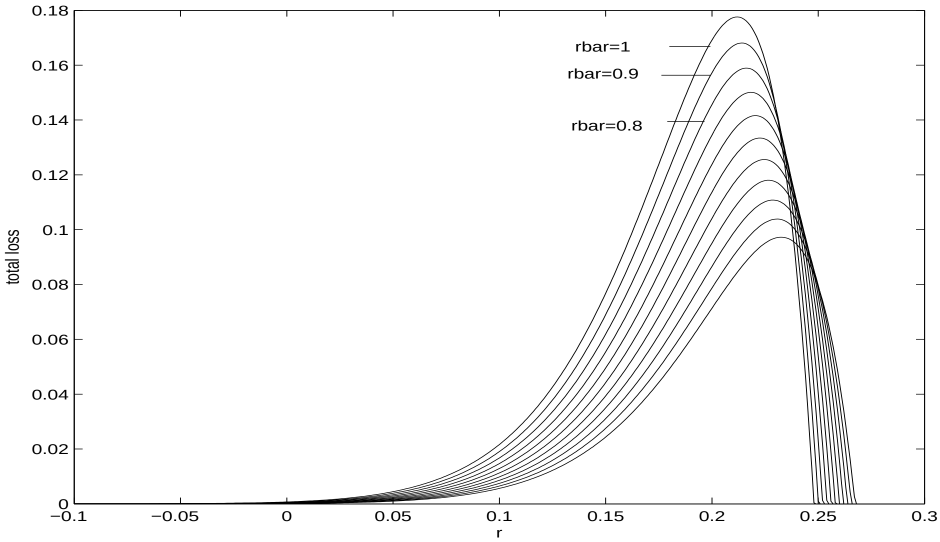

Figure 2.5: How total loss changes with public beliefs $\bar{r}$

bankruptcy once it occurs. The intuition is that optimistic public beliefs induce more firms to invest at each fundamentals level. Thus, when the fundamentals turn out to be weak, the bankruptcy rate is higher, leading to higher total loss.

2.5 A Model with Banks

In this section, banks are introduced into the basic model to endogenize the borrowing rate. Here I do not intend to explain why banks exist in an economy. I simply assume that the transaction costs between investors and firms are prohibitively high, and the firms have to finance their investment via banks.

2.5.1 The Model

N risk neutral banks compete over the borrowing rate to maximize their profits.

The banks are assumed to hold the public belief about economic fundamentals, $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. This public belief is shared by the firms. Later I will introduce banks with their own private signals.

The banks are assumed to have limitless access to funds at the risk-free rate of 1.

The timing of the game is as follows. At the first stage, the banks offer $e^{\bar{r}_b}$, the gross borrowing rate, to the firms. At the second stage, given the borrowing rate and their own private signals, the firms decide whether to invest or not. The game of the firms is the same as before, except that now they face a different borrowing rate. Thus it is a sequential game with the banks as the leaders, and the firms as the followers.

I can prove that there is a unique subgame perfect Bayesian equilibrium in this game. Let $\gamma_0 = \frac{\alpha^2}{\beta}$.

Proposition 2. *Provided that $\gamma_0 \leq 2\pi$, and the public belief $\bar{r}$ is high enough for the banks to make nonnegative profit, there is a unique subgame perfect Bayesian equilibrium. In this equilibrium, the banks offer the borrowing rate of $e^{\bar{r}_b^*}$. Given this borrowing rate, firm i chooses to borrow and invest if and only if $\rho_i > \rho^*$. Otherwise, the firm chooses not to invest. Given $\bar{r}_b = \bar{r}_b^*$, ρ^* is the unique solution to*

$$\int_{r^*}^{+\infty} \sqrt{\alpha+\beta}\phi(\sqrt{\alpha+\beta}(r-\rho^*))(me^{r-l(r,\rho^*)} - (m-1)e^{\bar{r}_b})dr - 1 = 0,$$

and $\bar{r}_b^*$ is the smallest positive solution to

$$\int_{-\infty}^{r^*} \sqrt{\alpha}\phi(\sqrt{\alpha}(r-\bar{r}))(me^{r-l(r,\rho^*)} - (m-1))(1-l(r,\rho^*))dr + \int_{r^*}^{+\infty} \sqrt{\alpha}\phi(\sqrt{\alpha}(r-\bar{r}))(m-1)(e^{\bar{r}_b} - 1)(1-l(r,\rho^*)dr = 0, \tag{2.7}$$

where

$$l(r,\rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))),$$

and r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}))) = ln\frac{m-1}{m} + \bar{r}_b.$$

Proof:

Backward induction is used to find the subgame perfect Bayesian equilibrium in this game. First, I can prove that there is a unique equilibrium in the game of the firms. In equilibrium, a firm will invest if and only if its updated belief $\rho > \rho^*$. The proof is basically the same as that in Section 2.3 with few modifications. Here I only give the proof confined to symmetric trigger strategies. The method of iterated elimination of strictly dominated strategy can also be applied here to prove the unique equilibrium.

To find the unique equilibrium in the game of the firms, I need to pinpoint ρ^* first. Suppose that firm i is at the triggering point, that is, $\rho_i = \rho^*$, then it must be indifferent about investing or not, which means

$$\begin{aligned} &ER(a^I|x^*(\rho^*)) = \\ &\quad \int_{r^*}^{+\infty} \sqrt{\alpha+\beta}\phi(\sqrt{\alpha+\beta}(r-\rho^*))[me^{r-\Phi(\sqrt{\beta}(x^*-r))} - (m-1)e^{\bar{r}_b}]dr \\ &\quad = 1 = R(a^{NI}|x^*(\rho^*)), \end{aligned} \tag{2.8}$$

where r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(x^* - r^*)) = ln\frac{m-1}{m} + \bar{r_b}.$$

By simplifying the above equation and substituting ρ^* for x^*, we get

$$\int_{r^*}^{+\infty} \sqrt{\alpha+\beta}\phi(\sqrt{\alpha+\beta}(r-\rho^*))(me^{r-l(r,\rho^*)} - (m-1)e^{\bar{r}_b})dr = 1, \tag{2.9}$$

where

$$l(r,\rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}))).$$

Using the same method as in Appendix A.1, I can prove that the above equation is strictly increasing in ρ^*, given $\frac{\alpha^2}{\beta} < 2\pi$. Here I omit the proof. Based on the above equation, I find the unique solution of $\rho^*(\bar{r}_b)$. It is easy to show that the symmetric trigger strategy ρ^* is optimal for every firm.

Now let us look at the first mover of this game, the banks. The banks fully understand the game among the firms and the equilibrium strategies of the firms. Taking the equilibrium strategies of the firms into consideration, the banks will set the lowest borrowing rate $\bar{r}_b$ that leads to zero expected profit in the banking sector. This is the unique equilibrium and no bank will deviate. By raising the borrowing rate above the equilibrium rate, a bank will not have any firm to lend to. Lowering the borrowing rate, the bank will make negative profits.

Given that the banks' belief about the fundamentals is $\tilde{r} \sim N(\bar{r}, 1/\alpha)$, the expected profits that the whole banking sector can make are given by:

$$E\Pi_b = \int_{-\infty}^{r^*} \sqrt{\alpha}\phi(\sqrt{\alpha}(r-\bar{r}))(mw_0e^{r-l(r,\rho^*)} - (m-1)w_0)(1-l(r,\rho^*))dr + \int_{r^*}^{+\infty} \sqrt{\alpha}\phi(\sqrt{\alpha}(r-\bar{r}))(m-1)w_0(e^{\bar{r}_b}-1)(1-l(r,\rho^*))dr.$$

The gross borrowing rate, $e^{\bar{r}_b^*}$, that a bank will charge is the smallest positive solution to $E\Pi_b = 0$, which can be simplified as

$$\int_{-\infty}^{r^*} \sqrt{\alpha}\phi(\sqrt{\alpha}(r-\bar{r}))(me^{r-l(r,\rho^*)} - (m-1))(1-l(r,\rho^*))dr + \int_{r^*}^{+\infty} \sqrt{\alpha}\phi(\sqrt{\alpha}(r-\bar{r}))(m-1)(e^{\bar{r}_b}-1)(1-l(r,\rho^*)dr = 0. \quad (2.10)$$

Notice that when $\bar{r}$ is low enough, the expected profits of banks from lending will be always non-positive. Thus the banks will lend if and only if $\bar{r}$ is large enough, such that $max\{E\Pi_b(\rho^*, \bar{r}_b)\} \geq 0$. Or the banks will choose not to lend.

Q.E.D

2.5.2 The Analysis of the Model

In this section, numerical examples are given to examine how systemic bankruptcy will be influenced by the realization of economic fundamentals, firms' financial leverage, and public beliefs when the borrowing rate is endogenously given.

Systemic Bankruptcy and Realized Economic Fundamentals

In this model with banks, firms will still take the optimal trigger strategy ρ^* in equilibrium. Therefore, the relationship between systemic bankruptcy and realized economic fundamentals will be similar to what I found in the basic model without banks except that the level of ρ^* will be different due to different borrowing costs.

Keeping all the parameter values given in Section 2.4 unchanged, the numerical example reveals that banks will charge a borrowing rate of $e^{0.00008}$, which is extremely close to 1. The firms' optimal trigger strategy $\rho^* = 0.4245$ is also very close to 0.4244, the optimal trigger strategy in the case with an exogenously given borrowing rate. Therefore, the results in Section 2.4.1 about the relationship between systemic bankruptcy and economic fundamentals still holds here.

The reason that the gross borrowing rate charged by banks is extremely close to 1 is that β is high and firms have very precise information about economic fundamentals. Thus banks expect that the proportion of firms borrowing, $1 - l(r)$, when bankruptcy arises, that is, $r < r^*$, is extremely low, and that their expected loss from lending is also extremely low. On the other hand, when $r > r^*$, banks expect that the proportion of firms borrowing, $1 - l(r)$, is high, and their expected gain from lending is high too. Since banks only aim at zero profit, they will charge an extremely low borrowing rate. This feature will hold in the rest of the numerical examples. Therefore, in general, banks will not greatly change firms' equilibrium behavior through charging different borrowing rates in this model.

Systemic Bankruptcy and Financial Leverage

Comparative statics reveals that *the equilibrium trigger strategy of firms will slightly increase in m. Meanwhile, the severity of systemic bankruptcy rapidly increases in financial leverage. With higher financial leverage, the range of economic fundamentals where systemic bankruptcy arises is wider, and the total loss at each level of economic fundamentals* *is higher.*

Financial leverage will influence systemic bankruptcy in the following ways: first, the equilibrium borrowing rate charged by the banks, $e^{\bar{r}_b^*}$, and the equilibrium trigger strategy by the firms, ρ^*, are functions of m. Second, given ρ^*, the threshold fundamentals level for bankruptcy, $\underline{r}$, is increasing in m. Third, the unpaid debt of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, is increasing in m. The last two effects are exactly the same as those in the case without banks.

A numerical example with all the other parameter values unchanged and m changing from 1.5 to 3 is given.

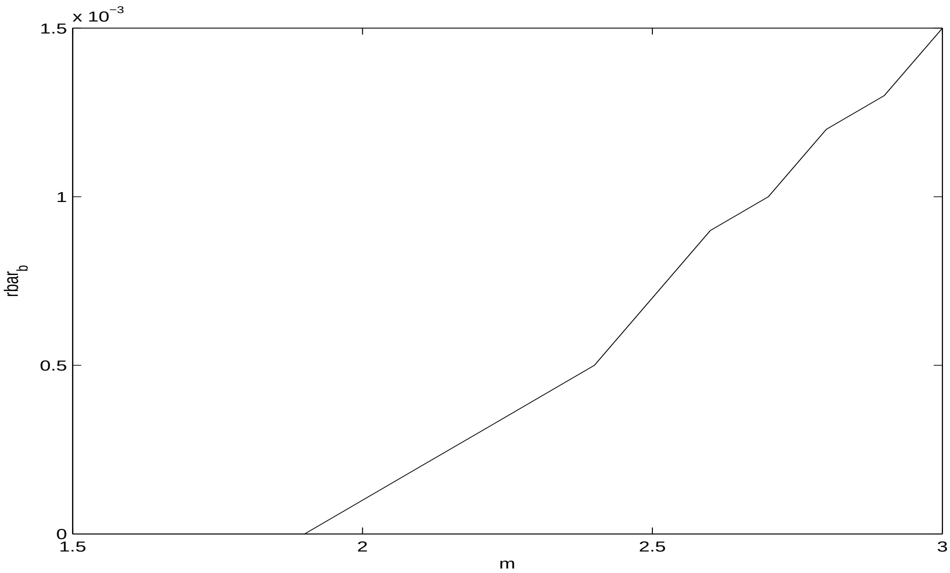

Figure 2.6: How borrowing rate $\bar{r}_b^*$ changes with financial leverage m

Figure 2.6 reveals that the banks will charge higher borrowing rates with higher m. This result is intuitive. With higher m, firms are more easily to go bankrupt and the banks have to charge a higher borrowing rate to gain zero profit.

Figure 2.7 shows that ρ^*, the optimal trigger strategy of the firms, is increasing in m. This is because the higher borrowing rate decreases the expected payoff of the firms from

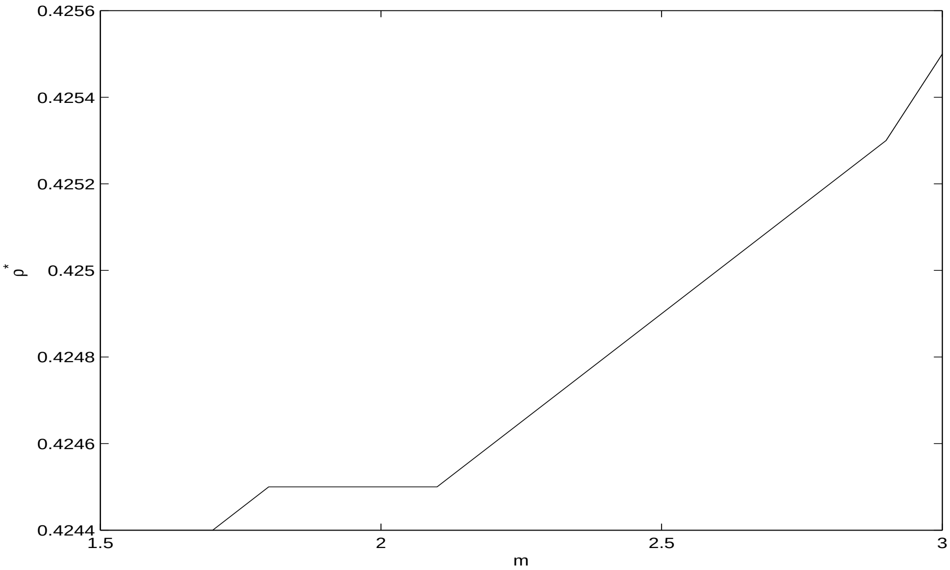

Figure 2.7: How optimal trigger strategy ρ^* changes with financial leverage m

investing. Now firms will invest only when they have higher updated beliefs about the fundamentals.

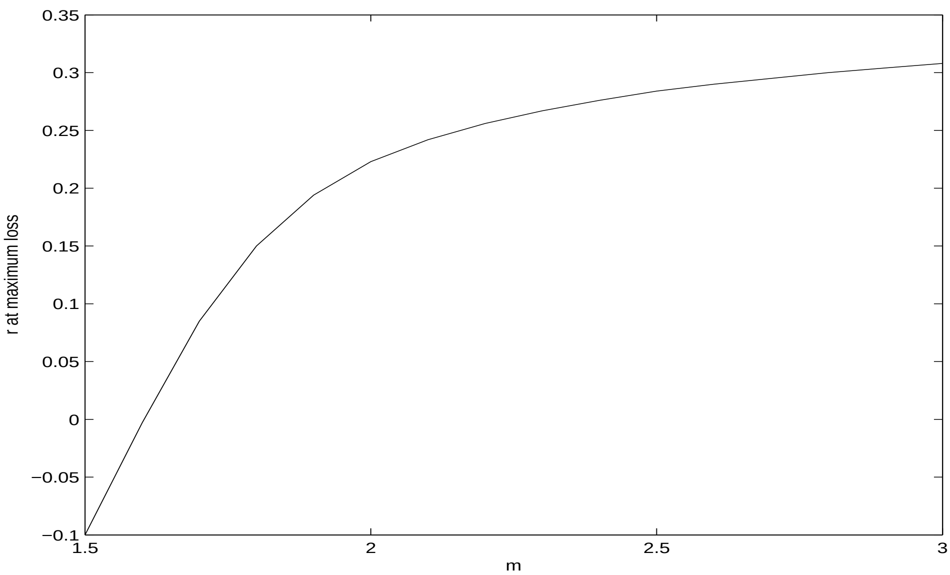

Figure 2.8: How economic fundamentals r at maximum loss changes with financial leverage m

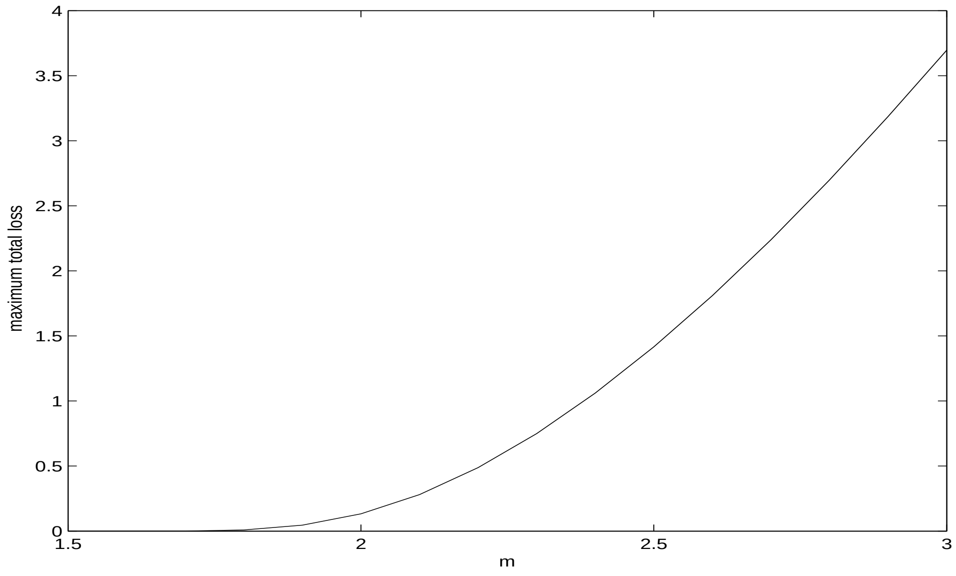

Figure 2.9: How maximum loss changes with financial leverage m

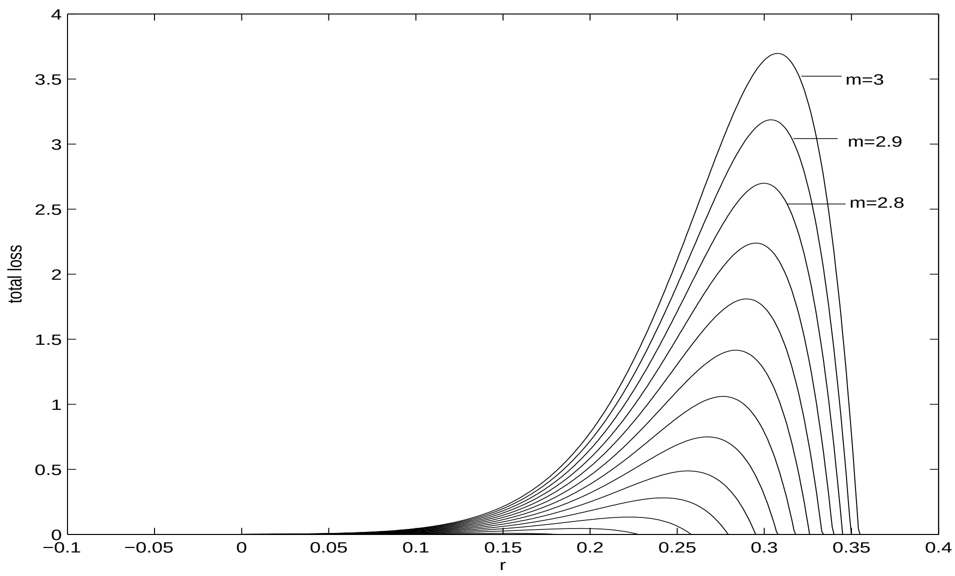

Figure 2.10: How total loss changes with financial leverage m

Figures 2.8, 2.9 and 2.10 illustrate that higher financial leverage m greatly increases the severity of systemic bankruptcy. With higher m, the range of economic fundamentals where systemic bankruptcy arises is wider, and the total loss is higher at any given level of economic fundamentals.

Systemic Bankruptcy and Public Beliefs

Comparative statics reveals that higher public beliefs can alleviate coordination failure. With higher public beliefs, more firms invest at each level of economic fundamentals, leading to higher investment return. *The range of economic fundamentals where systemic bankruptcy arises is narrower with higher public beliefs. But once systemic bankruptcy happens, the total loss is higher with higher public beliefs.*

Analytically, public beliefs will influence systemic bankruptcy in the following way: first, given the borrowing rate $e^{\bar{r}_b}$, public beliefs will influence the equilibrium outcome in the same way as it does in the case without banks. Optimal strategy ρ^* is lower with higher public beliefs $\bar{r}$, inducing more firms to invest at each economic fundamentals level. The threshold economic fundamental level where systemic bankruptcy begins to arise, $\underline{r}$, is lower. Given $r < \underline{r}$, bankruptcy rate $1 - l(r)$ increases, and the unpaid debt of an individual firm, $(m-1)w_0 - e^{r-l}mw_0$, decreases.

Second, with banks in the model, the public belief $\bar{r}$ is also the banks' belief and its change will influence banks' expected profits and consequently the borrowing rate they charge, $e^{\bar{r}_b}$. This will work through two channels: first, higher $\bar{r}$ leads to higher expectations about r, and higher return at each realized economic fundamentals level due to lower expected ρ^*, leading to higher expected profits of the banks. Second, higher $\bar{r}$ leads to higher investment when bankruptcy occurs, leading to lower expected profits for the banks.

It is difficult to find the analytical solution to define the relationship between public beliefs and systemic bankruptcy. Here I will give a numerical example with all the other parameter values unchanged and $\bar{r}$ varying from 0.1 to 1.

The quantitative relationship between public beliefs and systemic bankruptcy is similar to that in the case without banks. However, theoretically, in this model with banks, public beliefs will influence firms' optimal strategy ρ^* through one more channel: changing the borrowing rate $e^{\bar{r}_b}$ that the banks offer. The numerical example reveals that with higher public beliefs, the banks will charge lower borrowing rates, which will further lower the firms' optimal strategy ρ^* compared to the case without banks. Thus optimistic public beliefs have three effects in this case: first, they induce optimistic beliefs of firms about economic fundamentals. Second, they induce optimistic beliefs of firms about other firms' beliefs about economic fundamentals. Third, they induce optimistic beliefs of banks about investment returns. All will lead to lower optimal strategy ρ^* and higher investment returns in equilibrium.

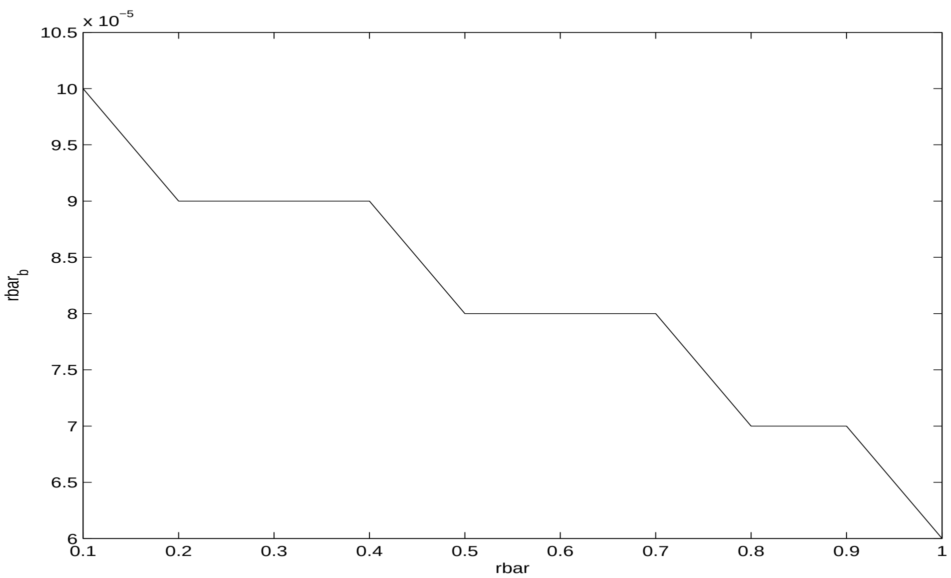

Figure 2.11: How borrowing rate $\bar{r}_b^*$ changes with public beliefs $\bar{r}$

Figure 2.11 shows that the borrowing rate is lower with higher public beliefs. This is because the banks with higher beliefs about economic fundamentals have higher expected payoffs from lending, and, hence, a lower borrowing rate leads to zero profit.

In Figure 2.12, higher public beliefs lead to lower optimal trigger strategy of the firms, ρ^*, for two reasons. First, the banks are charging a lower borrowing rate. Second, higher

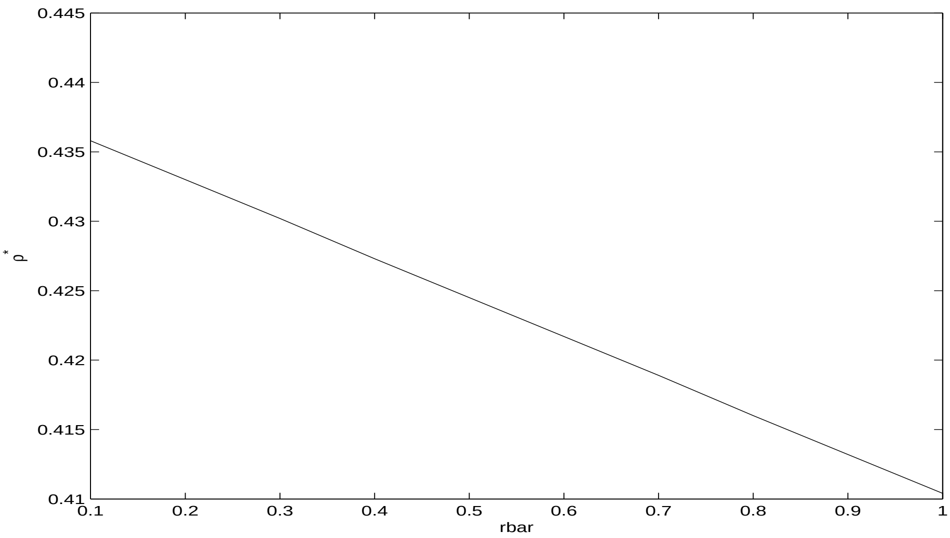

Figure 2.12: How optimal trigger strategy ρ^* changes with public beliefs $\bar{r}$

public beliefs raise the beliefs of the firms about the return from investing. Both will encourage firms to invest.

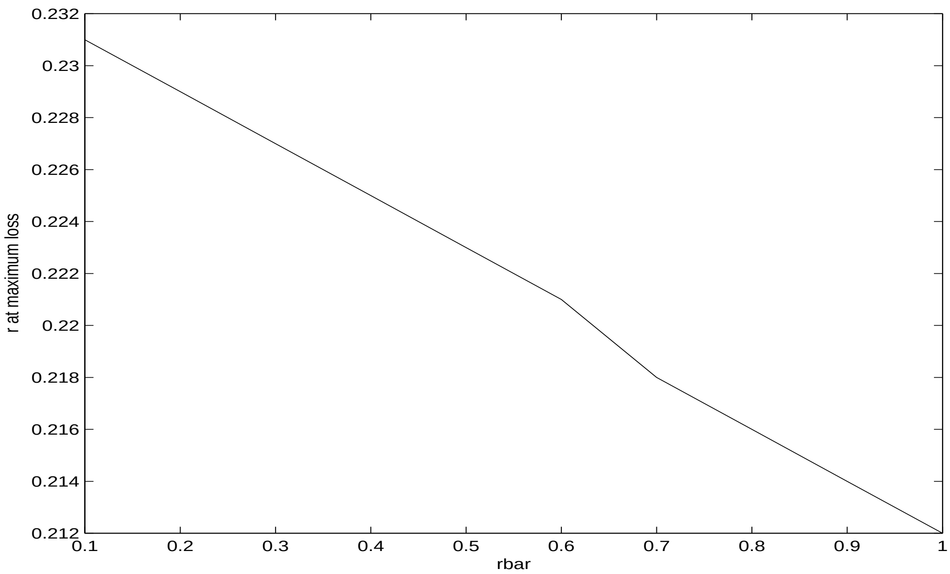

Figure 2.13: How economic fundamentals r at maximum loss changes with public beliefs $\bar{r}$

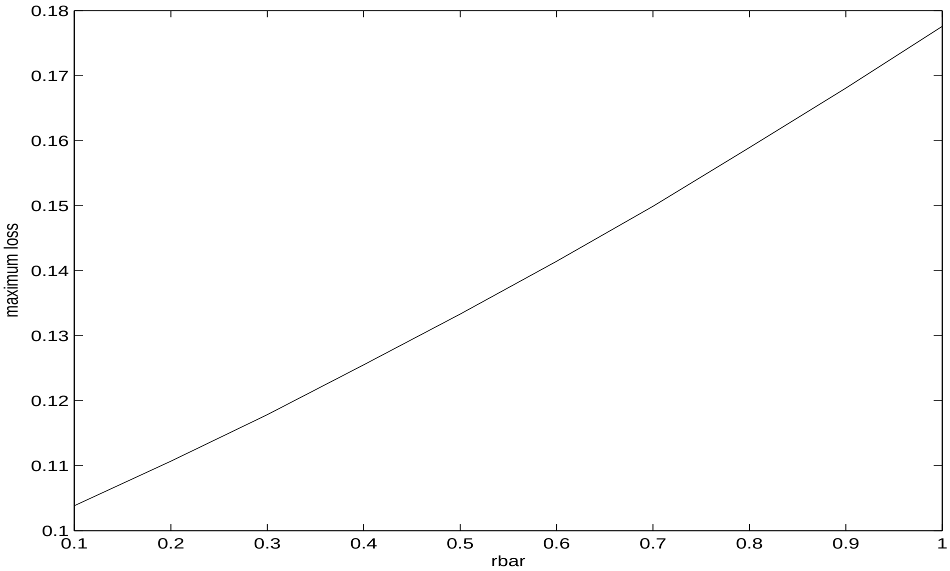

Figure 2.14: How maximum loss changes with public beliefs $\bar{r}$

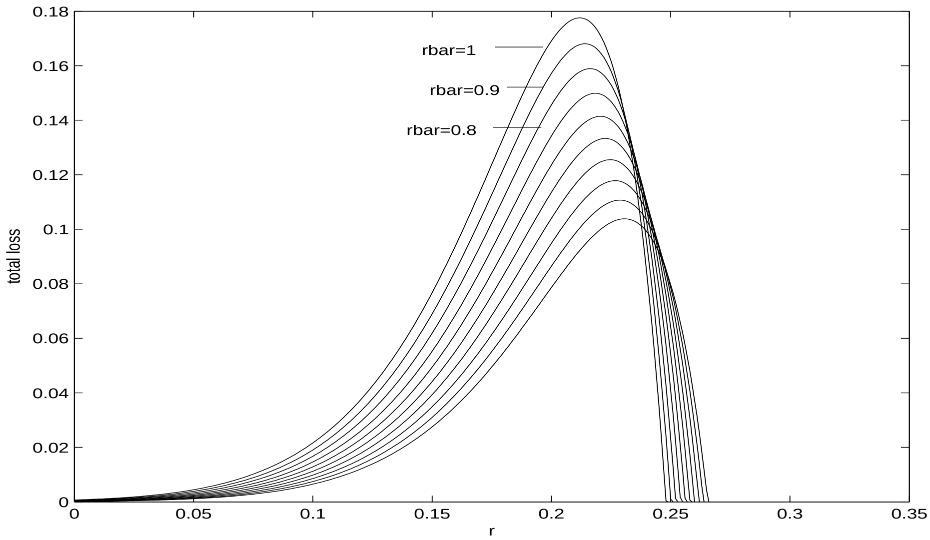

Figure 2.15: How total loss changes with public beliefs $\bar{r}$

Figures 2.13, 2.14 and 2.15 reveal that with higher public beliefs, the range of economic fundamentals where systemic bankruptcy arises is narrower. But the total loss is generally higher once systemic bankruptcy occurs.

2.6 The Model Where Banks Have Private Signals

2.6.1 The Model

This section studies the case when the banks have their own private signals. The prior belief of banks about economic fundamentals is still $\tilde{r} \sim N(\bar{r}, 1/\alpha)$. This public belief is also shared by the firms. In addition, all the banks receive the same realization of private signal about economic fundamentals, $x_b = r + \varepsilon_b$, where ε_b is normally distributed with mean 0 and precision β_b. This signal is also observed by the firms.

I can prove that there is a unique subgame perfect Bayesian equilibrium in this game. Let $\gamma_1 = \frac{(\alpha+\beta_b)^2}{\beta}$.

Proposition 3. *Provided that $\gamma_1 \leq 2\pi$ and the private signal of the banks, x_b, is high enough for them to make zero expected profit from lending, there is a unique subgame perfect equilibrium. In this equilibrium, the banks offer the borrowing rate of $e^{\bar{r}_b^*}$. Given this borrowing rate, firm i chooses to borrow and invest if and only if $\rho_i > \rho^*$. Otherwise, the firm chooses not to invest. Given $\bar{r}_b = \bar{r}_b^*$, ρ^* is the unique solution to*

$$\int_{r^*}^{+\infty} \sqrt{\alpha+\beta_b+\beta}\phi(\sqrt{\alpha+\beta_b+\beta}(r-\rho^*))(me^{r-l(r,\rho^*)} - (m-1)e^{\bar{r}_b})dr = 1,$$

and $\bar{r}_b^*$ is the smallest positive solution to

$$\int_{-\infty}^{r^*} \sqrt{\alpha+\beta_b}\phi(\sqrt{\alpha+\beta_b}(r-\rho_b))(me^{r-l(r,\rho^*)} - (m-1))(1-l(r,\rho^*))dr$$
$$+ \int_{r^*}^{+\infty} \sqrt{\alpha+\beta_b}\phi(\sqrt{\alpha+\beta_b}(r-\rho_b))(m-1)(e^{\bar{r}_b}-1)(1-l(r,\rho^*)dr = 0,$$

where

$$l(r,\rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}) + \frac{\beta_b}{\beta}(\rho^* - x_b))),$$

and r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(\rho^* - r^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}) + \frac{\beta_b}{\beta}(\rho^* - x_b))) = ln\frac{m-1}{m} + \bar{r}_b.$$

Proof: See Appendix A.3.

2.6.2 Banks with Private Signals and Systemic Bankruptcy

This section analyzes the role that banks play in systemic bankruptcy when the banks have their own private signal. Given the setup in the model, systemic bankruptcy is influenced by banks in two ways. First, the belief of the banks about economic fundamentals will determine their lending condition, $e^{\bar{r}_b^*}$, which will influence the expected payoffs of the firms. I call it the payoff effect. Second, the belief of the banks about economic fundamentals will also influence the beliefs of the firms through Bayesian updating. The second effect influences systemic bankruptcy in the same way as public beliefs do, which I analyze in Sections 2.4 and 2.5. I call this effect the information effect.

Now let us look at a numerical example with $\beta_b = 10$, x_b varying from 0.1 to 1, and all the other parameter values unchanged.

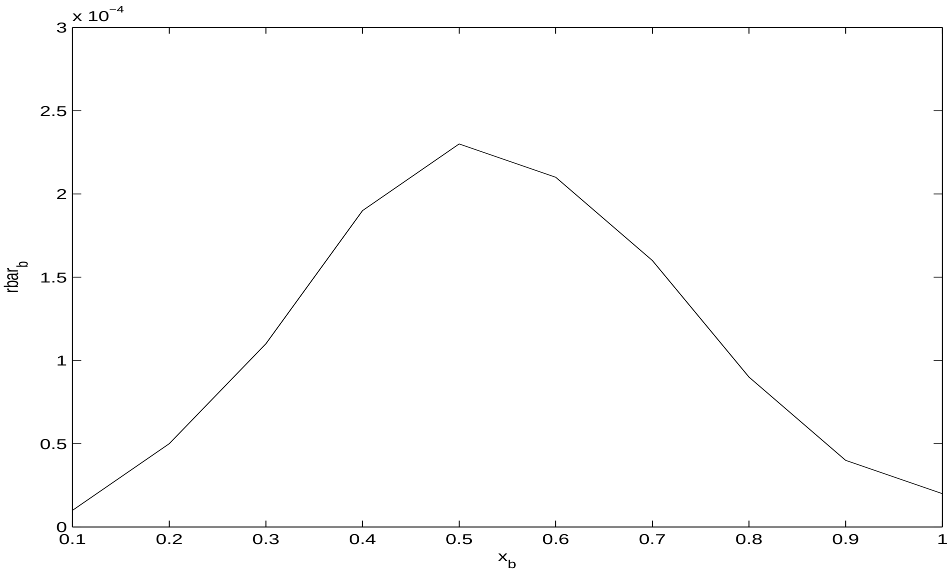

Figure 2.16: How borrowing rate $\bar{r}_b$ changes with the banks' private signal x_b

Figure 2.16 shows a non-monotonic relationship between the banks' beliefs on the fundamentals and the borrowing rates they charge. When x_b is lower than 0.5, the borrowing rate is increasing in x_b, but when x_b is greater than 0.5, the borrowing rate is decreasing in x_b. This is because two opposite effects determine the banks' expected profits. Higher x_b makes the banks put more weight on the higher level of the fundamental r, leading to higher expected profits. But higher x_b also makes the banks expect more firms to borrow when the fundamentals r is low and nonperforming loans arise, inducing lower expected profits.

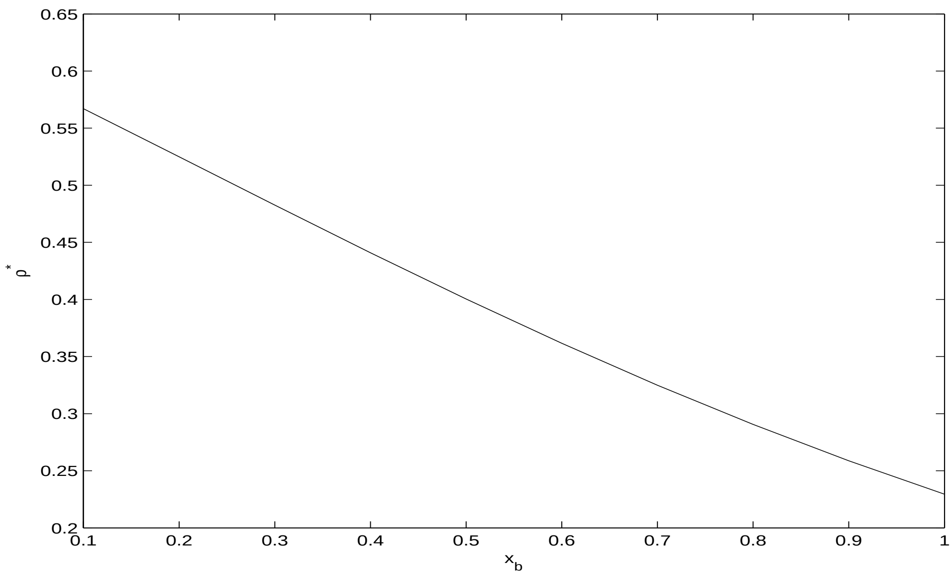

Figure 2.17: How firms' optimal trigger strategy ρ^* changes with the banks' private signal x_b

Figure 2.17 reveals that ρ^* is very sensitive to x_b. When x_b is lower than a certain level, the banks always have negative expected return on the investment, no matter what $\bar{r}_b$ is. No banks will lend, and naturally no bankruptcy will occur. When the banks and the firms hold different beliefs on economic fundamentals, more specifically, when banks hold pessimistic beliefs about economic fundamentals, the demand of the firms for credit may not be satisfied due to the supply side constraint.

When x_b is high enough for banks to lend, the firms' optimal trigger strategy ρ^* rapidly decreases in the banks' private signal x_b. This impact is attained mainly through the information effect, instead of the payoff effect, because we can see that in general the borrowing rate is very close to zero and has little impact on the firms' optimal trigger strategy.

The following figures reveal how systemic bankruptcy is influenced by the banks' private signal x_b.

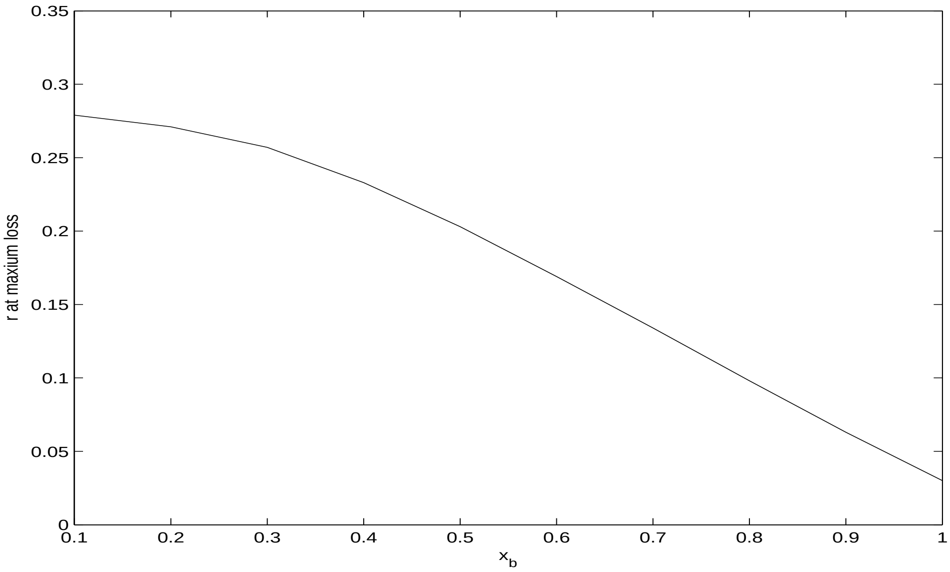

Figure 2.18: How economic fundamentals r at maximum loss changes with banks' private signal x_b

From the figures above we can see that pessimistic banks will make coordination more

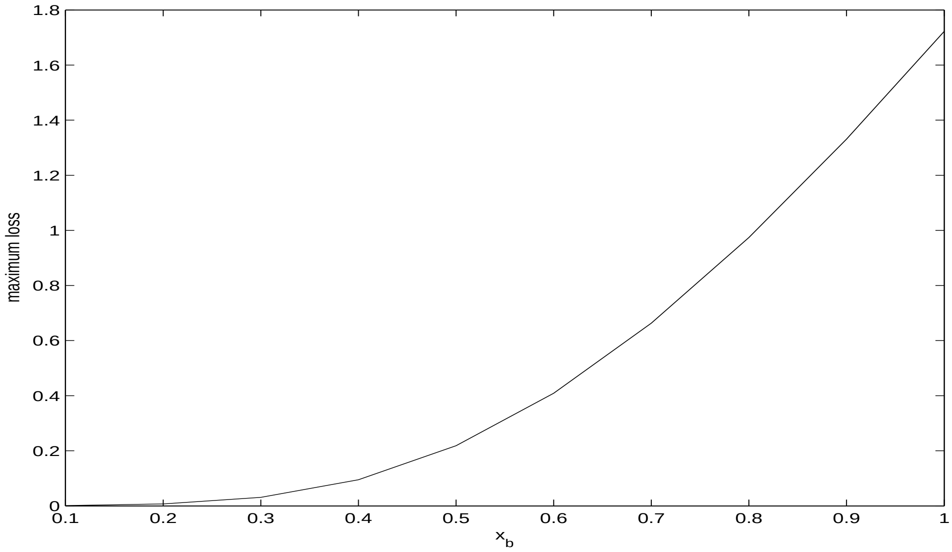

Figure 2.19: How maximum loss changes with the banks' private signal x_b

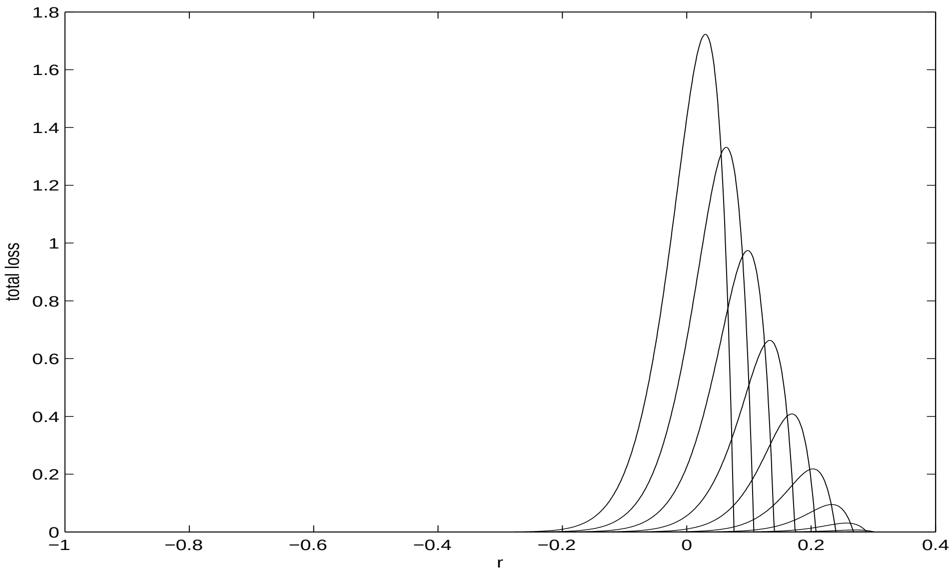

Figure 2.20: How total loss varies with the banks' private signal x_b

difficult, but at the same time will curb systemic bankruptcy. In the extreme case, the banks will not lend at all. Therefore, no bankruptcy occurs. On the other hand, optimism among banks will lead to easier coordination, but systemic bankruptcy also tends to be more severe once it happens.

The policy implication derived from this model is that an optimistic sentiment among banks will be an important indicator of possible severe systemic bankruptcy. This finding is also consistent with the anecdotal observation that severe financial crises usually break out shortly after an economic boom when the banks are still optimistic.

2.7 Conclusions, Policy Implications and Future Research

This chapter explains the origin of systemic bankruptcy of nonfinancial firms by coordination failure in a decentralized credit economy with investment complementarities. By doing so, I provide a new explanation about how uncertainty in real investment can cause financial fragility. I hope that my research can promote a better understanding of the origin of financial fragility, and provide theoretical guidance for central banks to establish an "early warning system" to prevent the occurrence of financial crises.

The main conclusions and policy implications from this chapter are as follows:

1. Systemic bankruptcy can originate from coordination failure in a decentralized credit economy with investment complementarities. Due to investment complementarities, an economy can be more vulnerable to systemic bankruptcy.

2. Systemic bankruptcy becomes possible when economic fundamentals are neither too strong nor too weak. Coordination failure arises when economic fundamentals are in this range. Moreover, systemic bankruptcy tends to break out when the fundamentals are taking low to medium values in this range.

3. Financial leverage of firms is an important indicator revealing the fragility of a financial system. Higher financial leverage of firms greatly increases the possibility and severity of systemic bankruptcy.

4. Optimistic public beliefs of firms and banks can alleviate coordination failure. Systemic bankruptcy will happen with lower economic fundamentals. But once it happens, it tends to be more severe.

In summary, systemic bankruptcy caused by coordination failure will hit an economy most severely when economic fundamentals fall into the coordination failure zone with a lending boom generated by high financial leverage and optimistic public beliefs of firms and optimistic beliefs of banks. Central banks should be highly alert about financial fragility in such economic situations.

A future direction to extend this chapter is to put it in a General Equilibrium framework where uncertainty, investment complementarities and borrowing-lending relationship are present. In the current model, banks are assumed to have limitless access to some funds at an exogenously given cost. In a General Equilibrium model, I will be able to endogenize the size and cost of the funds that banks have. By doing so, I will be able to study investment fluctuations generated by uncertainty, investment complementarities and borrowing-lending relationship. In Chapter 3, I have established an Overlapping Generation model where both uncertainty and investment complementarities exist to study the relationship between economic growth and volatility.

Chapter 3

Coordination Failure in Technological Progress, Economic Growth and Volatility

3.1 Introduction

Technological progress has long been posited to play a key role in economic growth by economists such as Schumpeter dating back to the 1950's. In the 1990's, Romer, Aghion, Howitt, as well as others, formalized this idea by introducing it into the mainstream neo-classical growth models and establishing the so-called endogenous growth theory.

Despite the importance of technological progress in economic growth, it is a topic difficult for formal economic modeling. As Dowrick (1995) summarizes, new technology in general exhibits three characteristics which distinguish it from other ordinary goods: first, new technology contains great uncertainty due to its new and unknown nature. Second, technological externalities exist, leading to investment complementarities in new technology investment. Third, information asymmetry exists. Producers of new products have more information than their creditors or users. All these characteristics are difficult to be modeled by standard economic tools.

Global games, first studied by Carlsson and van Damme (1993), turn out to be a useful tool for the study of new technology. Global games model the situations where both uncertainty and strategic complementarities are present. By doing so, they capture two main characteristics in new technology investment: uncertainty and technological externalities. Hence, global games allow us to formally model new technology investment with these two features to gain valuable insights and policy implications.

This chapter introduces new technology investment modeled by global games into an Overlapping Generation economy with two sectors: the consumption goods sector and capital goods sector. While consumption goods are assumed to be produced by an exogenously given time-invariant technology, I assume that capital goods can be produced by two different technologies in each period. The first is a conventional one with constant returns that are perfectly revealed to economic agents. The other is a new one exhibiting increasing returns to scale due to technological externalities. In addition, economic agents have only incomplete information about the returns of the new technology due to its unknown nature. The returns of the new technology in each period are determined by two parts: the first part is a technology shock, which is random and i.i.d over time. This part is called economic fundamental, because it purely depends on the nature of the technology. The second part is the proportion of economic agents investing in the new technology. The returns are higher with a larger proportion of firms investing. Thus, the second part introduces investment complementarities due to technological externalities. Furthermore, economic agents only have incomplete information about the economic fundamental of the new technology. More specifically, they have a prior belief and receive a noisy private signal about the technology shock before they make their investment decisions. This assumption captures the feature of uncertainty in new technology investment.

The above assumptions are a simple way to model technological progress in an economy. I focus on technological progress in the capital goods sector, because it has more scope for technological innovation than the consumption goods sector. By assuming that in each period capital goods can be produced by both conventional and new technologies, I model the dynamic technological updating process in a simplified way. By assuming that the new technology exhibits externalities and uncertainty, I capture the two main features of a new technology. Technological externalities play an important role in the endogenous growth theory. The theoretical literature emphasizing technological externalities and their implications to economic growth includes Romer (1990), Dowrick (1995), and Aghion and Howitt (1998). Meanwhile, empirical work studying technological externalities is abundant. As Dowrick (1995) surveys, this work reinforces the prevalent view that technological externalities are significant. Moreover, the huge uncertainty in new technology investment is self-evident due to its new and unknown nature: the whole process from inventing a new technology to putting it into production is long and unpredictable.

This chapter does not intend to model the mechanism through which technological externalities originate. Instead, I use the reduced form to assume their existence, and focus on the implications that they have to capital accumulation and economic growth. Due to technological externalities, economic agents face two kinds of uncertainties about the returns of new technology investment: first, they have incomplete information about the technology shock; second, they are uncertain about the actions of other agents. My model reveals that the two uncertainties will lead to inefficiency in equilibrium. Coordination failure can occur, which is manifested as under-investment in the new technology. In such a framework, I study when and how coordination failure leads to slower capital accumulation and economic growth. More interestingly, the model generates a positive correlation between economic growth and volatility through a very peculiar mechanism associated with coordination failure. In my model, more investment in the new technology can alleviate coordination failure and lead to higher economic growth. Meanwhile, the new technology is risker by nature and more investment on it leads to more volatility as well. Policy implications are examined as well.

The rest of the chapter is organized as follows: Section 3.2 gives a survey on related literature. A two-sector Overlapping Generation model is established in Section 3.3 and its equilibrium is characterized. Section 3.4 analyzes when and how coordination failure leads to slower economic growth. The relationship between economic volatility and growth is also examined. Numerical simulation is given in Section 3.5. Section 3.6 explores policy implications, followed by conclusions in Section 3.7.

3.2 Literature Survey

This chapter is related to three strands of economic literature. The first strand is on global games and their applications to macroeconomics. The second strand is on traditional coordination games with perfect information and their applications to macroeconomics. The third strand is on the endogenous growth theory, which emphasizes the importance of technological progress in economic growth.

A survey on the first and second strands of literature has been given in Chapter 2. The third strand of literature is about endogenous economic growth. This literature combines the idea of Schumpeter (1950) that technological progress is crucial in economic growth with standard neoclassical growth models, and studies how technological progress influences a country's economic growth and consequently its policy implications.

Romer (1990) emphasizes the importance of non-rivalry of technology as a main source both of growth and of potential market failure. He argues that the non-rivalry feature of a technology and the consequent increasing returns to scale in the sector that uses the technology make long-run growth possible. Meanwhile, due to non-rivalry of technology, private motives for investing in new technology are usually sub-optimal. Thus an important policy implication is that governments should take steps to stimulate more new technology investment.

Aghion and Howitt (1998) provide a comprehensive survey on the endogenous growth theory literature. They examine how technological progress, influenced by a variety of factors such as organizations, institutions, market structure, market imperfections, trade, government policy and the legal framework, affects long-term economic growth.

3.3 The Model

3.3.1 Environment

The framework of this model follows the two-sector Overlapping Generation model established by Ennis and Keister (2003), who explore the impact of bank runs on capital stock and output. As in their model, I assume that the consumption goods sector exhibits constant capital returns. My model differs from theirs in the capital goods sector, where a global game is applied to the study of new technology investment. The global game used to model the investment in the new technology is based on Morris and Shin (2000).

This is a standard overlapping generation model with infinite time horizon, where each generation lives for two periods. There is also an initial old generation endowed with capital k_0 at the beginning of time.

At the beginning of each period t, a new generation of a continuum of agents with mass 1, denoted by generation t, is born. Each agent is endowed with 1 unit of labor when young, and nothing when old. Labor is supplied inelastically.

Capital goods are produced as follows. At the end of time t, young agents of generation t with wage income can choose from two ways of investing to produce capital goods. One is to invest in a conventional technology, which transforms 1 unit of consumption goods at the end of time t into $R_t^c = r$ units of capital goods at the beginning of time $t+1$. Here r is an exogenously given constant. The other is to invest in a new technology. The return on the new technology, R_t^n, is determined by two factors: the technology shock denoted by θ_t (also called the economic fundamental of the technology) and the proportion of the agents who invest in the new technology, $1-\lambda_t$. So λ_t is the proportion of the agents who invest in the conventional technology. If a young agent invests one unit of the consumption goods in the new technology at the end of time t, he will get $R_t^n = e^{\theta_t - \lambda_t}$ units of capital at the beginning of time $t+1$.

The technology shock θ_t is i.i.d over time and is normally distributed with mean $\bar{\theta}$ and precision α.

In period t, after θ_t is realized, it is observed with noise by the young agents. In particular, each young agent observes his own private signal

$$x_{it} = \theta_t + \varepsilon_{it},$$

where ε_{it} is normally distributed with mean 0 and precision β. It is assumed that $\{\varepsilon_{it}\}$ is i.i.d over agents.

Consumption goods are produced as follows. At the beginning of time $t+1$, old agents of generation t rent their capital produced from the investment at the end of time t to a continuum of perfectly competitive firms. These firms produce a single consumption good in the economy, using labor and capital according to the production function

$$Y_t = \bar{K}_t^{1-\mu} K_t^{\mu} L_t^{1-\mu}, \tag{3.1}$$

where $\bar{K}_t$ is the average capital-labor ratio in the economy at time t. The depreciation rate of capital is assumed to be 1 (In this chapter I use capital and small letters to denote variables at aggregate and individual levels respectively).

The utility function for each agent born at period t is given by

$$u_t = \beta log(c_{2,t}), \tag{3.2}$$

where $c_{2,t}$ denotes the consumption of an old agent born at period t, and *log* represents natural logarithm function. So we can see that an agent will consume nothing when young and consume all he has when old. Figure 3.1 gives the timing of the model.

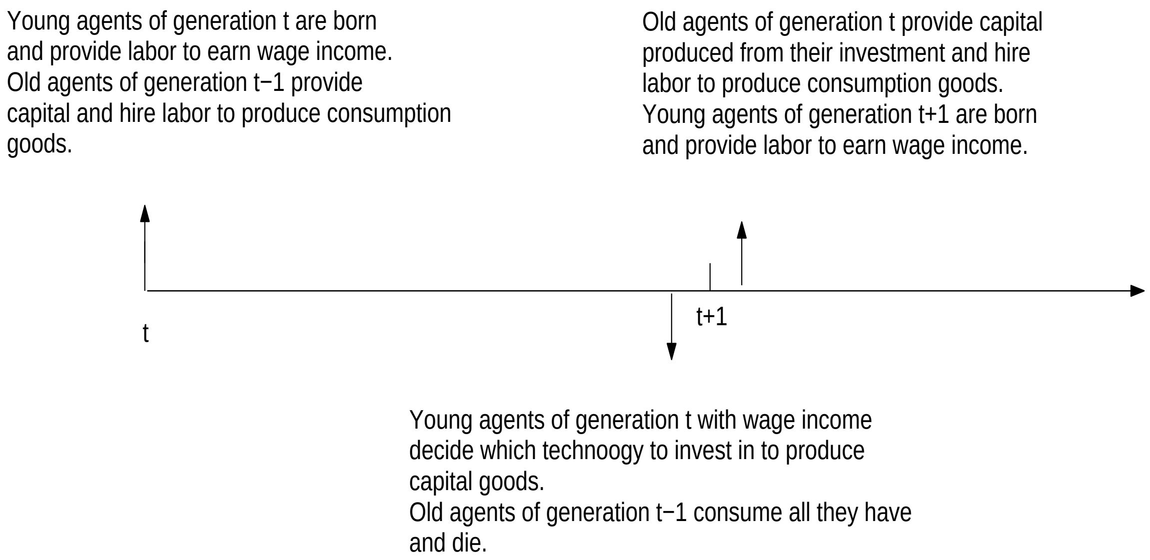

Figure 3.1: The timeline

3.3.2 Market Equilibrium

This section characterizes the market equilibrium in this model.

Consumption Goods Market Equilibrium

The consumption goods market is perfectly competitive. Equilibrium labor supply is given by $L_t = 1$, since labor is supplied inelastically. Also in equilibrium $\bar{K}_t = K_t$. Thus, in this competitive equilibrium, capital rent r_t and wage w_t are respectively

$$r_t = \mu K_t^{1-\mu} K_t^{\mu-1} = \mu;$$

$$w_t = (1-\mu)K_t.$$

Notice that here I follow the assumption of AK models that capital has constant returns. This assumption will greatly simplify the analysis.

Capital Goods Market Equilibrium

In each period t, after observing his own private signal x_{it}, a young agent with wage w_t has to decide to invest in the conventional or new technology at the end of period t.

Given such a setup, I can prove that there is a unique Bayesian Nash equilibrium in this game. In equilibrium, each agent will invest in the new technology if and only if his private signal x_{it} is greater than a threshold level. Otherwise, he will invest in the conventional technology.

After observing the private signal x_{it}, agent it updates his belief about θ_t according to Bayes' rule. Since both θ_t and x_{it} are normally distributed, $(\theta_t|x_{it})$ is also normally distributed.

Moreover, the mean of $(\theta_t|x_{it})$ is

$$\rho_{it} = \frac{\alpha\bar{\theta} + \beta x_{it}}{\alpha + \beta}. \tag{3.3}$$

Its precision is simply $\alpha + \beta$.

Let

$$\gamma = \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}. \tag{3.4}$$

Proposition 4. *Provided that $\gamma \leq 2\pi$, there is a unique symmetric trigger strategy equilibrium in the capitals good market in each period. In this equilibrium, each young agent chooses to invest in the new technology if and only if $\rho > \rho^*$, where ρ^* is the unique solution to*

$$\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})) = log(r).$$

Otherwise, the young agent chooses to invest in the conventional technology.

Proof:

Here I confine attention to symmetric trigger strategy equilibria and prove that there is such a unique equilibrium. In Appendix B.1, I prove that this symmetric trigger strategy equilibrium is the unique equilibrium that survives the iterated elimination of strictly dominated strategies.

Given that attention is confined only to symmetric trigger strategy equilibrium, it takes two steps to prove that there is such a unique equilibrium. First, the unique threshold level ρ^* is pinpointed, given the hypothesis that each young agent follows the strategy of investing in the new technology if and only if his updated belief $\rho_{it} > \rho^*$. Second, it is proved that this strategy is optimal for every agent.

For ρ^* to be an equilibrium triggering point, a young agent with the updated belief ρ^* must be indifferent between investing in the new technology and a conventional one.

Suppose an agent is at the equilibrium triggering point. I will abuse the notation and denote him by ρ^*, his updated belief about the mean of the return from the new technology investment. Then each agent *it* is assumed to invest in the new technology if and only if $\rho_{it} > \rho^*$.

We know that the expected utility of agent ρ^* from investing 1 unit of consumption goods in the new technology is given by

$$E[log(e^{\theta_t - \lambda_t}\mu|x^*)] = E(\theta_t - \lambda_t|x^*) + log(\mu).$$

Recall that the relationship between x^* and ρ^* is given by Equation (3.3).

We already know that

$$E(\theta_t|x^*) = \rho^*.$$

Now I need to find $E(\lambda_t|x^*)$. Given the equilibrium strategy that each firm *it* will invest in the new technology if and only if $\rho_{it} > \rho^*$,

$$E(\lambda_t|x^*) = Prob(\rho_{jt} < \rho^*|x^*).$$

Therefore we get:

$$E(\lambda_t|x^*) = Prob(\rho_{jt} < \rho^*|x^*).$$

From Equation (3.3), we know that

$$\rho_{jt} = \frac{\alpha\bar{\theta} + \beta x_{jt}}{\alpha + \beta}.$$

Therefore, we get

$$Prob(\rho_{jt} < \rho^*|x^*) = Prob(x_{jt} < \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{\theta})|x^*). \tag{3.5}$$

We know that $(x_{jt}|x^*) = (\theta + \varepsilon_{jt}|x^*)$ is normally distributed with mean ρ^* and variance $\frac{1}{\alpha+\beta} + \frac{1}{\beta}$. Therefore, we get

$$Prob(\rho_{jt} < \rho^*|x^*) = \Phi(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}}(\rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{\theta}) - \rho^*)). \tag{3.6}$$

Thus

$$E(\lambda_t|x^*) = \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})), \tag{3.7}$$

where $\Phi(.)$ is the CDF of the standard normal distribution with mean 0 and variance 1.

Then we get

$$E(\theta_t - \lambda_t|x^*) = \rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})). \tag{3.8}$$

The utility of agent ρ^* from investing in the conventional technology is given by $log(r\mu)$.

Since at the triggering point the agent has to be indifferent between investing in the new and conventional technology, we get

$$\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})) = log(r).$$

Given $\gamma < 2\pi$, $\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta}))$ is strictly increasing in ρ^*. So there is a unique solution of ρ^* satisfying the above equation.

Now I need to show that, given ρ^*, the strategy that an agent it will invest in the new technology if and only if $\rho_{it} > \rho^*$ is optimal for every agent.

For agent it with $\rho_{it} > \rho^*$,

$$E(\theta_t - \lambda_t | x_{it}) = \rho_{it} - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta} + \frac{\beta}{\alpha}(\rho^* - \rho_{it}))). \tag{3.9}$$

And its precision is $\alpha + \beta$.

Agent it will invest in the new technology , because

$$E(\theta_t - \lambda_t | x_{it}) - log(r) = \rho_{it} - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta} + \frac{\beta}{\alpha}(\rho^* - \rho_{it}))) - log(r) > 0. \tag{3.10}$$

The reason that the above function is positive is that it is strictly increasing in ρ_{it}, and when $\rho_{it} = \rho^*$, it is equal to 0 by construction. It is easy to show that its first order derivative with respect to ρ_{it} is greater than 0:

$$1 + \sqrt{\gamma}\frac{\beta}{\alpha}\Phi^{'}(\sqrt{\gamma}(\rho^* - \bar{\theta} + \frac{\beta}{\alpha}(\rho^* - \rho_{it}))) > 0.$$

Similarly, I conclude that an agent it with $\rho_{it} < \rho^*$ will invest in the conventional technology, because $E(\theta_t - \lambda_t | x_{it}) - log(r) < 0$. Note that equilibrium strategy ρ^* is time invariant.

Q.E.D

Law of Motion of the Capital

The law of motion of the capital is given as follows:

$$K_{t+1} = (1 - \mu)K_t[\lambda_t r + (1 - \lambda_t)e^{\theta_t - \lambda_t}], \tag{3.11}$$

where

$$\lambda_t = \Phi(\sqrt{\beta}(x^*(\rho^*) - \theta_t)).$$

The consumption profile of a typical generation t is:

$$c_{1,t} = 0; \tag{3.12}$$

$$c_{2,t} = \begin{cases} (1-\mu)k_t r\mu & \lambda_t \text{ of agents investing in conventional technology} \\ (1-\mu)k_t e^{\theta_t - \lambda_t}\mu & 1-\lambda_t \text{ of agents investing in new technology} \end{cases} \quad (3.13)$$

The initial old generation has the consumption profile $\{c_{2,0} = \mu k_0\}$.

3.4 An Analytical Study on the Model

This section applies the above model to the study of how incomplete information and coordination failure influence new technology investment, economic growth, and volatility.

3.4.1 Coordination Failure and Economic Growth

First, let us look at the case where economic agents have perfect information about θ_t. When $\theta_t < log(r)$, no agent will invest in the new technology and it is the first best solution. When $\theta_t > 1 + log(r)$, every agent will invest in the new technology and it is also the first best solution. The interesting case is $log(r) < \theta_t < 1 + log(r)$. With perfect information, this case has two (stable) equilibria: one is that all the agents invest in the conventional technology; the other is that all the agents invest in the new technology. The latter equilibrium is Pareto superior to the former one. Thus, under perfect information, coordination failure is manifested as the randomness about which equilibrium is realized.

In my model, I find that given $\gamma < 2\pi$, there is always a unique equilibrium due to the introduction of incomplete information. Moreover, I find that the first best solution under perfect information can never be achieved in this equilibrium. The inefficiency in equilibrium is caused by two kinds of uncertainties: one is the uncertainty about θ_t, the economic fundamentals of the new technology; the other is the uncertainty about the actions of other agents.

A special case with $\beta \to \infty$ can analytically reveal how the uncertainty about the actions of other agents leads to an inefficient equilibrium outcome. We know that $\beta \to \infty$ means that the private signal of agents almost perfectly reveals the new technology shock. That is, the first kind of uncertainty is vanishingly small. I find that this assumption cannot eliminate the uncertainty about other agents' actions, and the first best equilibrium with perfect information cannot be achieved due to coordination failure among economic agents.

When $\beta \to \infty$, γ, which is strictly decreasing in β, goes to zero. In addition, $\rho^* = \frac{\alpha\bar{\theta}+\beta x^*}{\alpha+\beta} \to x^*$. Thus, the equation pinning down ρ^*,

$$\rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta})) = log(r),$$

is transformed into

$$x^* - \frac{1}{2} = log(r).$$

So we get

$$x^* = \rho^* = log(r) + \frac{1}{2}.$$

The intuition for this result is that when $\beta \to \infty$, each agent believes that his private signal x is exactly the true value of θ so that there is always half of the agents receiving private signals below his, and half of the agents receiving private signals above his.

Now let us examine economic growth, i.e, how outputs Y grow over time. Note that due to the specific production function in the model, $Y = K$ in each period.

We know that

$$K_{t+1} = (1-\mu)K_t[\lambda_t r + (1-\lambda_t)e^{\theta_t - \lambda_t}].$$

So the ex ante expected average gross capital (economic) growth rate is given by

$$g = E(Y_{t+1}/Y_t) = E(K_{t+1}/K_t) = (1-\mu)E[\lambda_t r + (1-\lambda_t)e^{\theta_t - \lambda_t}].$$

We also know that

$$\lambda_t = \Phi(\sqrt{\beta}(x^* - \theta_t)).$$

So $\lambda_t = 1$ when $\theta_t < log(r) + \frac{1}{2}$ since $\sqrt{\beta}(x^* - \theta_t)$ goes to $+\infty$ in this case, and $\lambda_t = 0$ when $\theta_t > log(r) + \frac{1}{2}$ since $\sqrt{\beta}(x^* - \theta_t)$ goes to $-\infty$ in this case.

The ex ante expected gross economic growth rate is given by:

$$g = (1-\mu)[r\Phi(\sqrt{\alpha}(log(r) + \frac{1}{2} - \bar{\theta})) + \int_{log(r)+\frac{1}{2}}^{+\infty} e^{\theta_t} d\Phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))].$$

It is the expected value of K_{t+1}/K_t, given that each period the agents take the equilibrium strategy described above, and that θ_t is normally distributed with mean $\bar{\theta}$ and precision α. Note that in this infinite time horizon model, the actual average gross economic growth rate will converge to the expected average gross economic growth rate when time goes to infinity.

Assume that the Pareto superior equilibrium with perfect information is always realized, that is, the agents will all invest in the new technology when $\theta_t > log(r)$ and all invest in the conventional technology when $\theta_t < log(r)$. The ex ante expected average gross economic growth rate is then given by:

$$g^{FB} = (1-\mu)[r\Phi(\sqrt{\alpha}(log(r) - \bar{\theta})) + \int_{log(r)}^{+\infty} e^{\theta_t} d\Phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))].$$

It is the expected value of K_{t+1}/K_t given the Pareto superior equilibrium is realized each period and θ_t is normally distributed with mean $\bar{\theta}$ and precision α.

We get the above equation because it is optimal for all the agents to invest in the conventional technology if $\theta_t < log(r)$ and to invest in the new technology if $\theta_t > log(r)$.

It is obvious to see that $g^{FB} - g > 0$ and

$$g^{FB} - g = (1-\mu)\int_{log(r)}^{log(r)+\frac{1}{2}} (e^{\theta_t} - r)\sqrt{\alpha}\phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))d\theta_t.$$

The above function indicates that coordination failure will most severely dampen economic growth when $\bar{\theta}$ is around the range $[log(r), log(r) + \frac{1}{2}]$. When $\bar{\theta}$ is in this range, $\phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))$ is at its highest values given $\theta_t \in [log(r), log(r) + \frac{1}{2}]$. Therefore, $g^{FB} - g$ will be higher, indicating more losses from coordination failure.

The policy implication from this result is that when the new technology returns of an economy fall into the range that is close to that of the conventional technology returns, coordination failure turns to be most severe, and the economy should benefit more from encouraging more investment in the new technology.

Note that this solution is Pareto optimal not only because it maximizes the economic growth rate, but also because it maximizes the ex ante expected utility of an agent, which is given by:

$$EU_t = E\beta log(c_{t+1}) = \beta(\Phi(\sqrt{\alpha}(x^* - \bar{\theta}))log(\mu r w_t) + \int_{x^*}^{+\infty} log(\mu e^{\theta_t} w_t) d\Phi(\sqrt{\alpha}(\theta_t - \bar{\theta}))).$$

It is obvious that this expected utility function is maximized when $x^* = log(r)$.

3.4.2 Implications for Economic Volatility

This model can also be used to study economic volatility and its relationship with economic growth. In this model, economic volatility originates from two kinds of uncertainties. One uncertainty is about the economic fundamentals of the new technology. This uncertainty is exogenously given. The other uncertainty is about the actions of other economic agents. This uncertainty is endogenously generated in an economy where investment complementarities exist, but coordination is not available.

As mentioned before, the gross capital (economic) growth rate is given by

$$g_{t+1} = \frac{K_{t+1}}{K_t} = (1-\mu)[\lambda_t r + (1-\lambda_t)e^{\theta_t - \lambda_t}],$$

where

$$\lambda_t = \Phi(\sqrt{\beta}(x^* - \theta_t)).$$

From the above expression, we can see that θ_t, the economic fundamentals of the new technology, represents the first kind of uncertainty. The proportion of agents not investing, λ_t, represents the second kind of uncertainty.

It is difficult to derive an analytical expression for the volatility. But the later numerical simulation shows that the economy is most stable when $\bar{\theta} < log(r)$. The economy is moderately volatile when $\bar{\theta} \in [log(r), log(r) + 1]$, which I call the coordination zone. Finally, the economy is most volatile when $\bar{\theta} > log(r) + 1$. So in general the relationship between economic growth and volatility is positive with the increase in $\bar{\theta}$. A detailed examination will be given in Section 3.5.3.

Another interesting result generated by this model is that economic agents' risk attitude will influence their choices between two technologies, leading to a positive correlation between economic growth and volatility that is associated with coordination failure. The main difference between the conventional technology and the new one is that the returns of the former are constant, while the returns of the latter are uncertain. As mentioned before, the uncertainty embedded in the new technology investment is twofold: first, it stems from the uncertainty about economic fundamentals, θ_t. Second, it stems from the uncertainty about other agents' beliefs about θ_t and their actions based on their beliefs.

My model reveals that the less risk-averse attitude of economic agents can alleviate coordination failure. The intuition is simple: the main consequence of coordination failure is under-investment in the new technology. Since the returns of the new technology are volatile and those of the conventional technology are constant, the less risk-averse attitude will encourage more investment in the new technology, and therefore overcome coordination failure.

Note when an economic agent, say it, decides which technology to choose, his expected utility from investing in the new technology is given by

$$u_{it} = EU(e^{\theta_t - \lambda_t} \mu w_t | x_{it}),$$

where $E(\lambda_t | x_{it}) = \Phi(\sqrt{\gamma}(\rho^* - \bar{\theta}))$.

It is obvious to see that utility function forms will influence economic agents' expected utility derived from the new technology investment. According to Jensen's inequality, a risk neutral agent will more tend to invest in the new technology than a risk averse agent. That is, x^*, the threshold level of private signal above which economic agents will invest in the new technology, is lower for a risk neutral agent than for a risk averse agent. In this way, the less risk-averse attitude of economic agents helps alleviate the coordination failure problem and leads to higher economic growth. Meanwhile, the new technology is more volatile by nature and more investment in the new technology leads to higher economic volatility as well. This mechanism will be most significant when the returns of the new technology are in the coordination failure zone.

In general, my model generates a positive relationship between economic growth and volatility. It is no surprise because the fundamental idea of this model is that economic growth comes from new technology investment, which is volatile by nature. Therefore, the pursuit of higher economic growth is consequently companied by higher economic volatility. Empirical evidence in general finds a negative relationship between economic growth and volatility (Hnatkovska and Loayza (2005)). However, this negative relationship is caused mostly by factors such as institutional and financial development, and fiscal policies, which are missing in my model. Moreover, the negative relationship can be caused by the causal effect of volatility on growth, which is not addressed in my model either. Empirical testing does find a positive relationship between economic growth and volatility (Hnatkovska and Loayza (2005)) among industrious countries, where the factors mentioned above are insignificant.

3.5 A Numerical Study on the Model

This section gives some numerical examples based on my model. These examples will help explain my model more clearly.

3.5.1 Economic Growth Paths with Different Levels of $\bar{\theta}$

Suppose that the capital share of income $\mu = 0.4$. The new technology shock $\theta \sim N(\bar{\theta}, 1/10)$ ($\alpha = 10$) and the precision of the agents' private signal $\beta = 20$. Note that $\gamma = 3 < 2\pi$ such that the condition for a unique equilibrium is held. I also assume $r = 1.7$. Three different levels of $\bar{\theta}$ are given: 0, 1.0 and 1.6. Note that $log(r) = 0.53$, and $log(r) + 1 = 1.53$. So $\bar{\theta} = 0 < log(r) = 0.53$, $\bar{\theta} = 1.0 \in [log(r), log(r) + 1]$, and $\bar{\theta} = 1.6 > log(r) + 1 = 1.53$.

Given the parameter values above, I can calculate the equilibrium ρ^*, time series of the proportion of agents investing in the conventional technology λ_t, and time series of natural logarithm of capital stock (output), $log(K_t)$ ($log(Y_t)$). Each example is simulated for 50 periods. The results are as follows.

It turns out that ρ^* is equal to 0.5675, 1.0981 and 1.5265 respectively, given that $\bar{\theta}$ takes the values of 1.6, 1.0 and 0. The following figures give the time series of realized θ_t, λ_t, and natural logarithm of capital stock (output) $log(K_t)$ ($log(Y_t)$), given that $\bar{\theta}$ is equal to 0, 1 and 1.6.

The Case of $\bar{\theta} = 0$

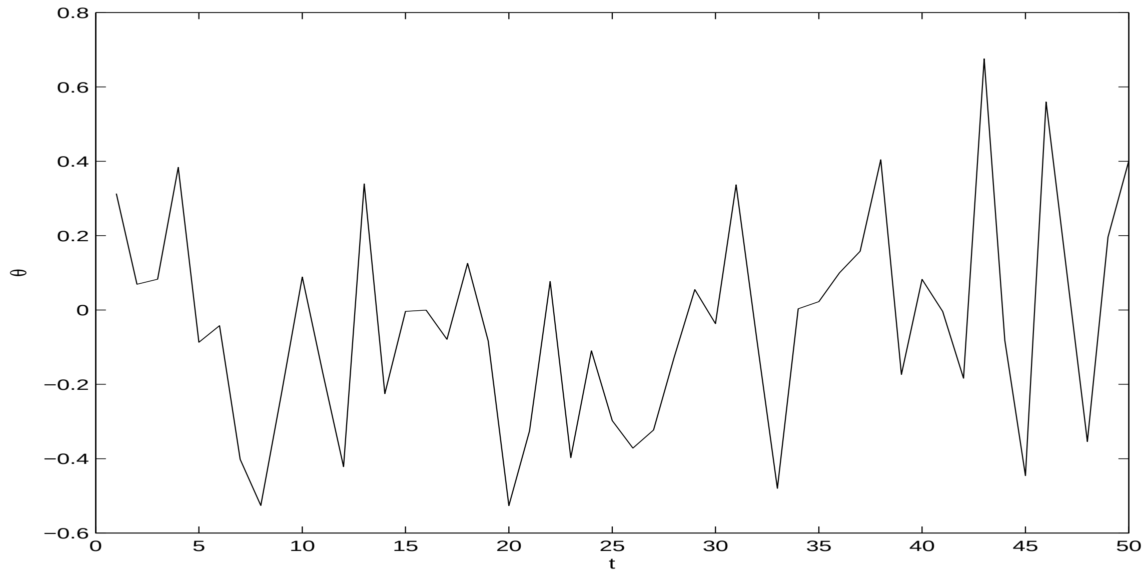

Figure 3.2: Time series of realized θ when $\bar{\theta} = 0$

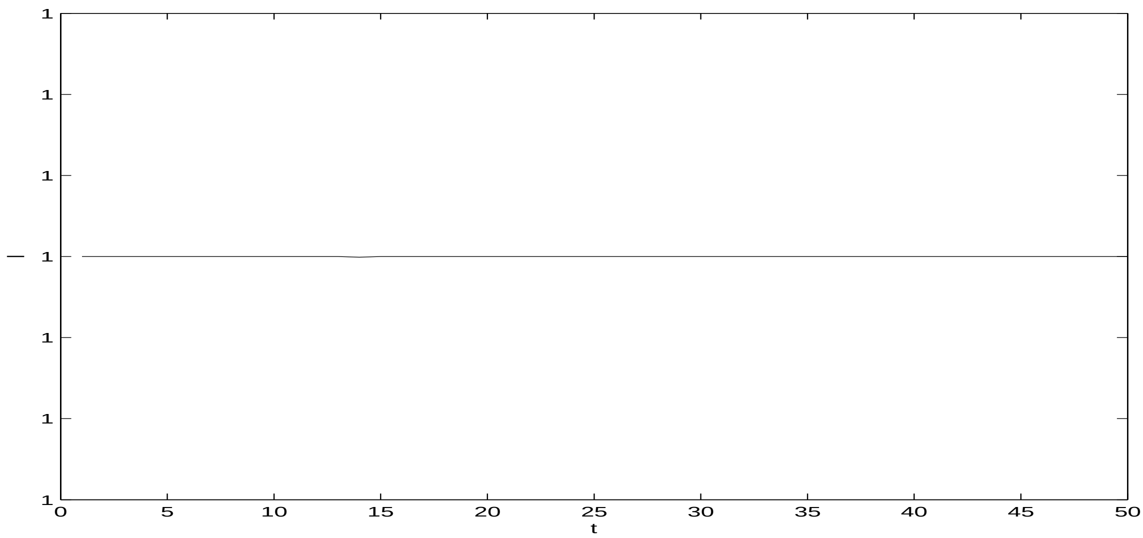

Figure 3.3: Time series of l when $\bar{\theta} = 0$

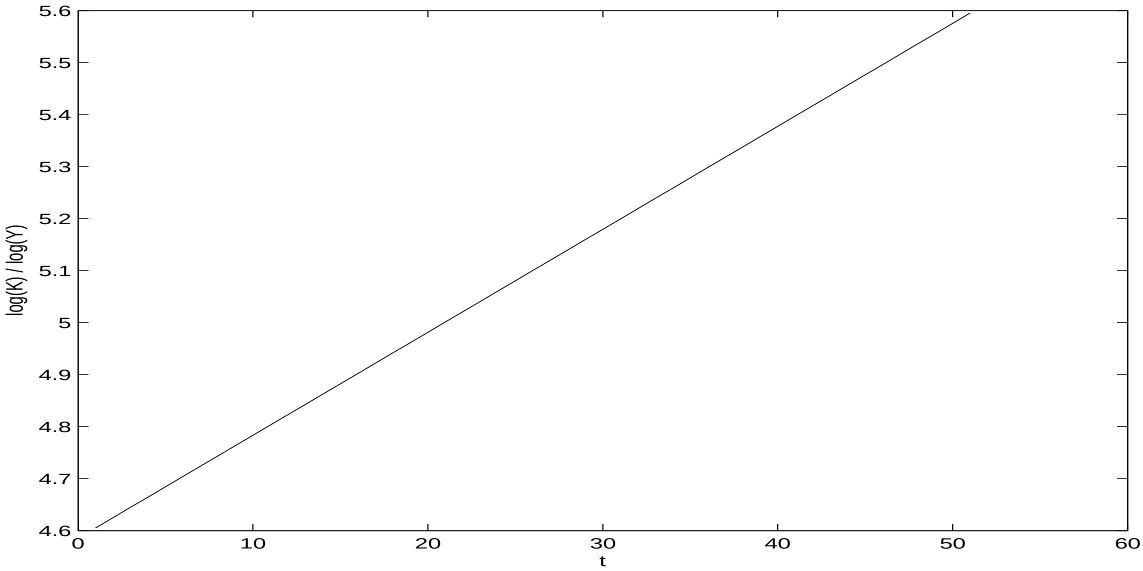

Figure 3.4: Time series of $logK$ $(logY)$ when $\bar{\theta} = 0$

The Case of $\bar{\theta} = 1.0$

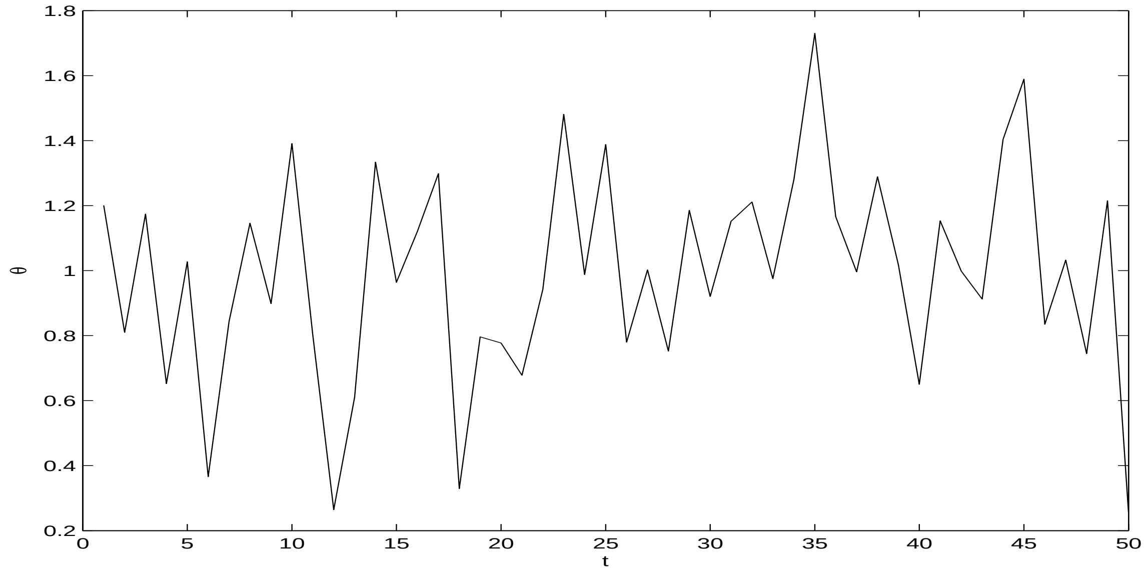

Figure 3.5: Time series of realized θ when $\bar{\theta} = 1$

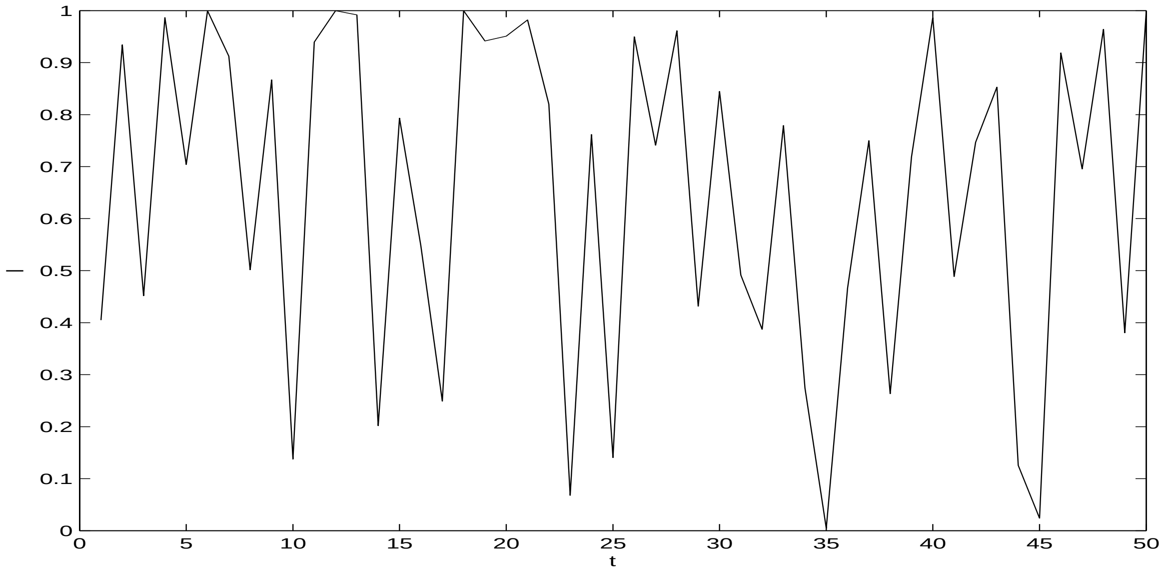

Figure 3.6: Time series of l when $\bar{\theta} = 1$

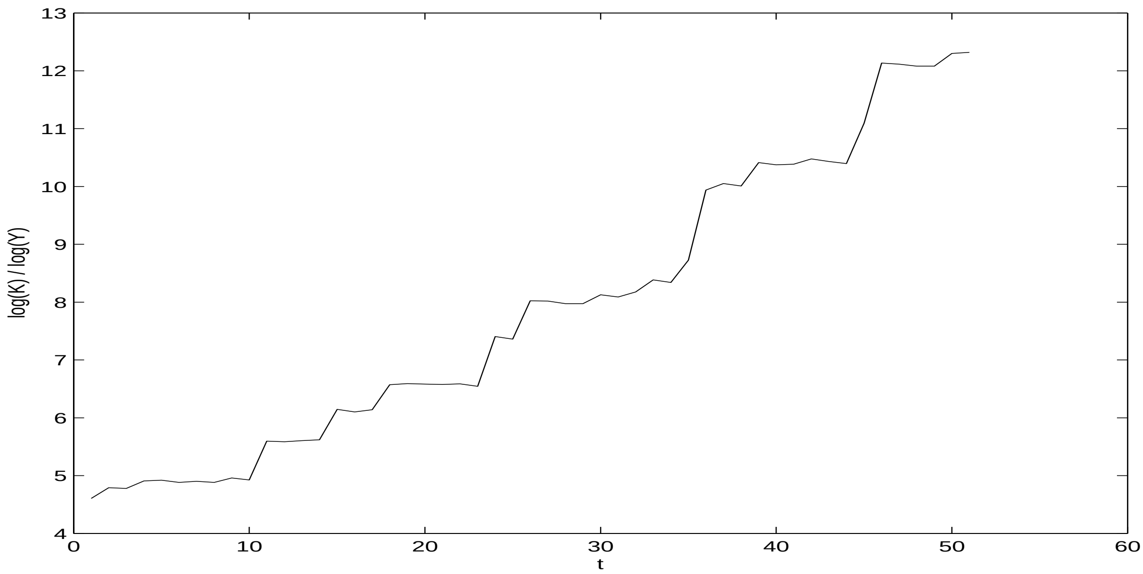

Figure 3.7: Time series of $logK$ $(logY)$ when $\bar{\theta} = 1$

The Case of $\bar{\theta} = 1.6$

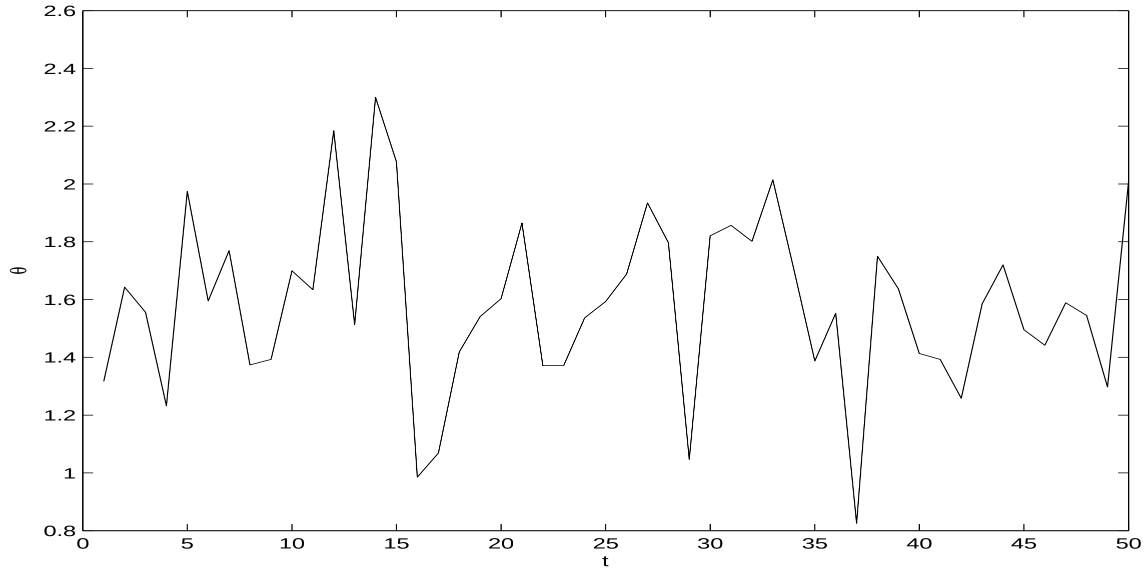

Figure 3.8: Time series of realized θ when $\bar{\theta} = 1.6$

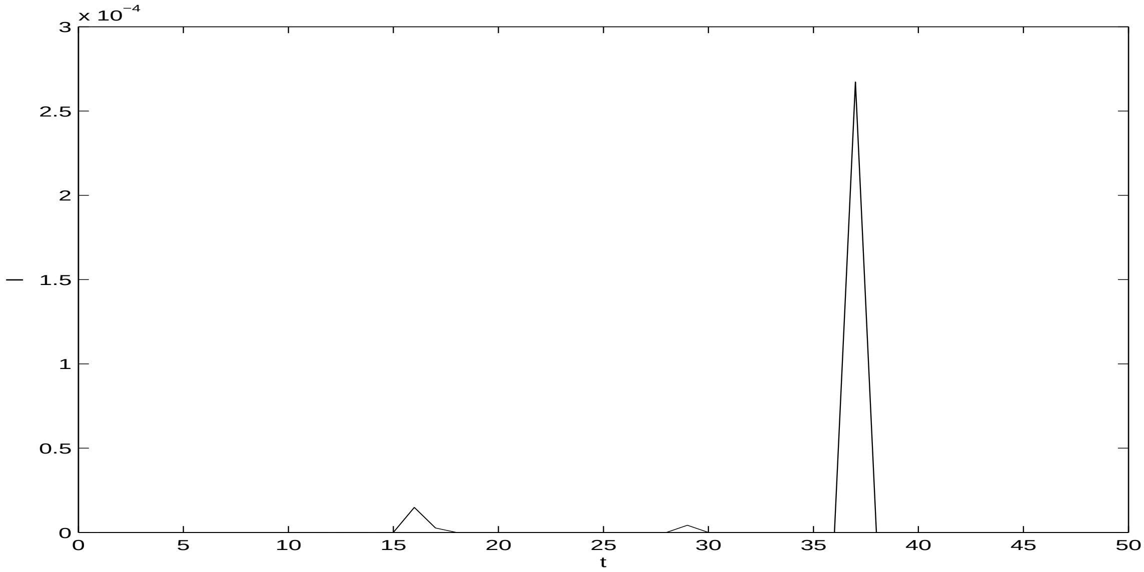

Figure 3.9: Time series of l when $\bar{\theta} = 1.6$

The figures above clearly reveal that coordination failure is closely related to the relative return levels between the conventional technology and new technology.

When $\bar{\theta} < log(r)$, $\theta_t < log(r)$ most of the time. We know that the first best solution

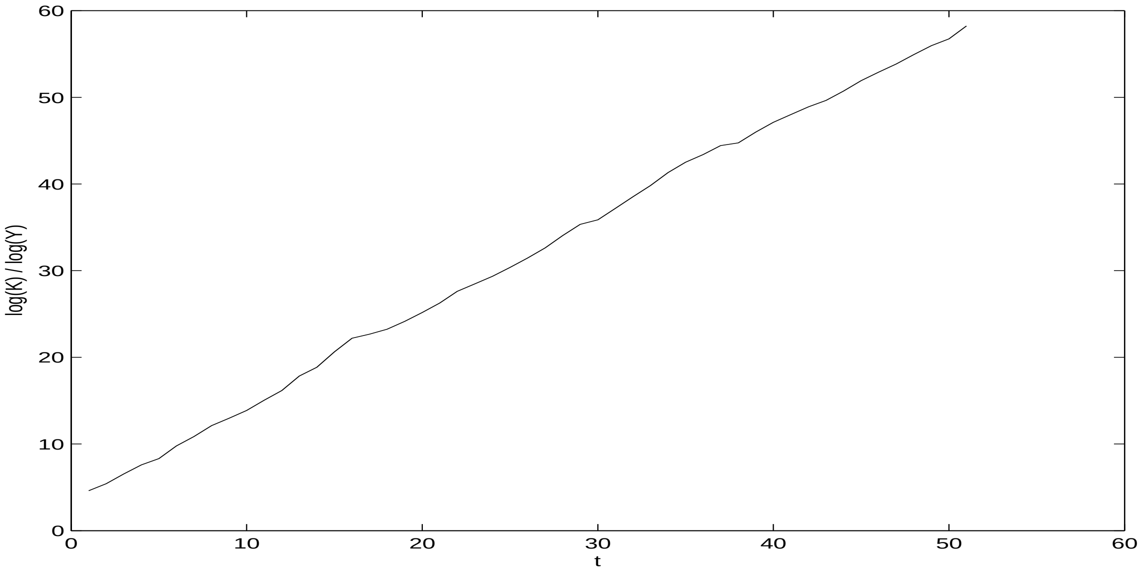

Figure 3.10: Time series of $logK$ ($logY$) when $\bar{\theta} = 1.6$

when $\theta_t < log(r)$ is to invest in the conventional technology, and in equilibrium, λ_t is actually equal to 1 most of the time. Similarly, when $\bar{\theta} > log(r) + 1$, $\theta_t > log(r)$ most of the time. We know that the first best solution when $\theta_t < log(r)$ is to invest in the new technology, and in equilibrium, λ_t is actually equal to 0 most of the time. However, when $log(r) < \bar{\theta} < log(r) + 1$, $log(r) < \theta_t < log(r) + 1$ most of the time. In this case coordination failure becomes severe. The first best solution in this case is that $\lambda_t = 0$ most of the time. But we see that actually λ_t swings between 0 and 1.

3.5.2 Coordination Failure and Economic Growth

Given different $\bar{\theta}$ levels, I am going to compare economic growth rates with and without coordination failure, holding all the other factors the same. In this way, I can explicitly show how coordination failure leads to slower economic growth.

First, given each level of $\bar{\theta}$, I will simulate the model with 500 periods and simulate it 501 times. Then the average economic growth rate will be calculated. All the parameters take the same values as in Section 3.5.1.

Next, given the parameter values unchanged, I assume that the Pareto optimal equilibrium with perfect information is realized in each period. I will simulate this new model with 500 periods and simulate it 501 times. The average net economic growth rates are calculated and compared with those in the case with coordination failure.

Table 3.1 and Figure 3.11 give the results:

Table 3.1: Net economic growth rates with and without coordination

$\bar{\theta}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$g-1$	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0201	0.0204	0.0233
$g^{FB}-1$	0.0262	0.0329	0.0450	0.0647	0.0959	0.1404	0.2010	0.2828	0.3822
$\frac{g-1}{g^{FB}-1}$	0.7626	0.6084	0.4444	0.3089	0.2086	0.1425	0.0998	0.0722	0.0610
$\bar{\theta}$	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	
$g-1$	0.0440	0.1803	0.5840	0.9354	1.1874	1.4328	1.6889	1.9727	
$g^{FB}-1$	0.5041	0.6472	0.8108	0.9960	1.2016	1.4360	1.6896	1.9729	
$\frac{g-1}{g^{FB}-1}$	0.0873	0.2786	0.7203	0.9391	0.9882	0.9978	0.9996	0.9999	

Both Table 3.1 and Figure 3.11 reveal that coordination failure can severely dampen economic growth when $\bar{\theta}$ is in the coordination failure zone. On the other hand, coordination failure will not have significant effects on economic growth when the returns of the new technology are either much lower or higher than those of the conventional technology. We know that the lower $\frac{g-1}{g^{FB}-1}$ is, the more severe the coordination failure is. When $\bar{\theta}$ approaches the coordination zone from below, $\frac{g-1}{g^{FB}-1}$ goes lower and lower, which means that coordination failure is more and more severe. In the extreme case when $\bar{\theta} = 0.8$, the economic growth rate with coordination failure is only 6 percent of that in the Pareto optimal solution. Meanwhile, when $\bar{\theta}$ goes higher and leaves the coordination zone, $\frac{g-1}{g^{FB}-1}$ goes

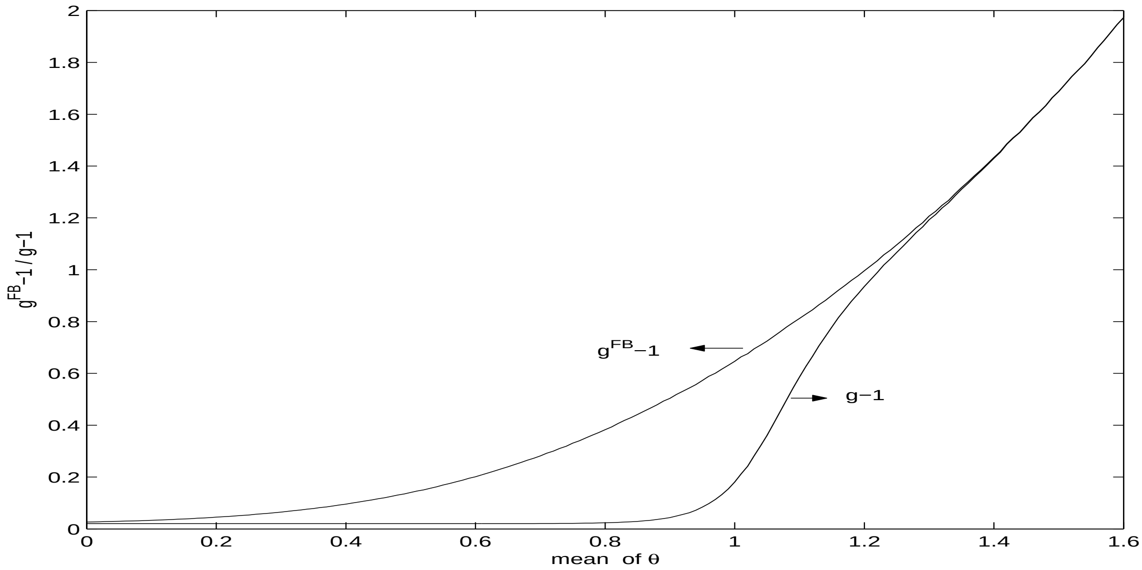

Figure 3.11: Growth rates with and without a coordinator when $\bar{\theta}$ changes

higher and higher, which means that coordination failure is less and less severe.

3.5.3 Economic Growth and Volatility

First, I will use the same method as in Section 3.5.2 to find the average growth rates and variances given different levels of $\bar{\theta}$. Then the relationship between economic growth and variance will be checked.

From the above figures we can see that in general, there is a positive relationship between economic growth and volatility with an increase in $\bar{\theta}$. However, the positive relationship becomes less salient when $\bar{\theta}$ is in some area of the coordination failure zone. We can see that in this range, the economy grows without significant increase in volatility.

The intuition for the above results is as follows. When $\bar{\theta} < log(r)$, λ is relatively stable and equal to 1 most of the time, which means that all the economic agents will choose to invest in the conventional technology most of the time. Since the conventional technology has constant returns, both the uncertainty about the economic fundamentals of the new

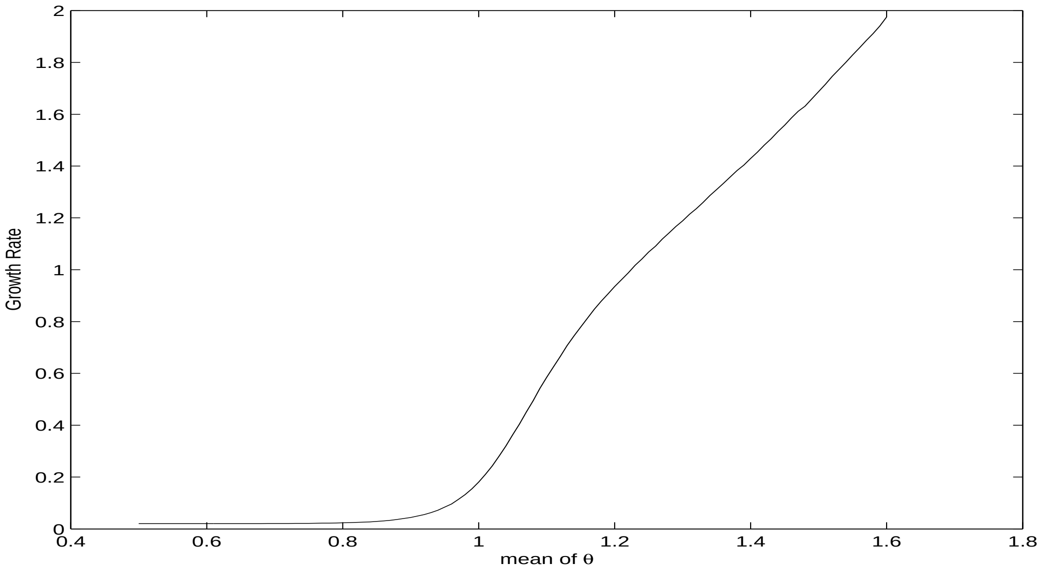

Figure 3.12: How economic growth changes in $\bar{\theta}$

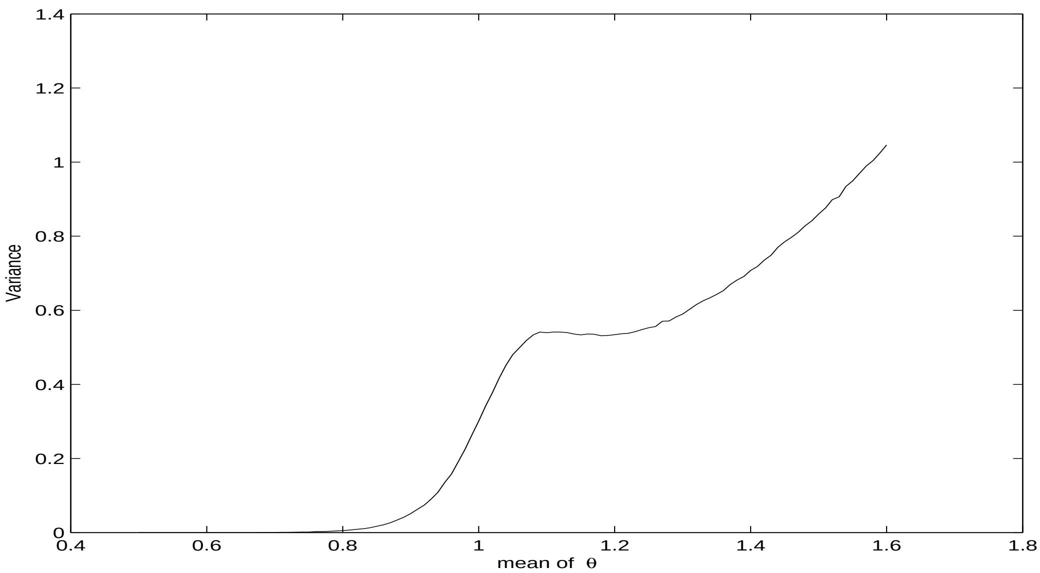

Figure 3.13: How economic volatility changes in $\bar{\theta}$

technology and the uncertainty about the actions of other agents vanish. Thus the economy is the most stable in this case. When $\bar{\theta} \in [log(r), log(r) + 1]$, which is in the coordination zone, λ swings between 0 and 1. Therefore, both kinds of uncertainties contribute to the

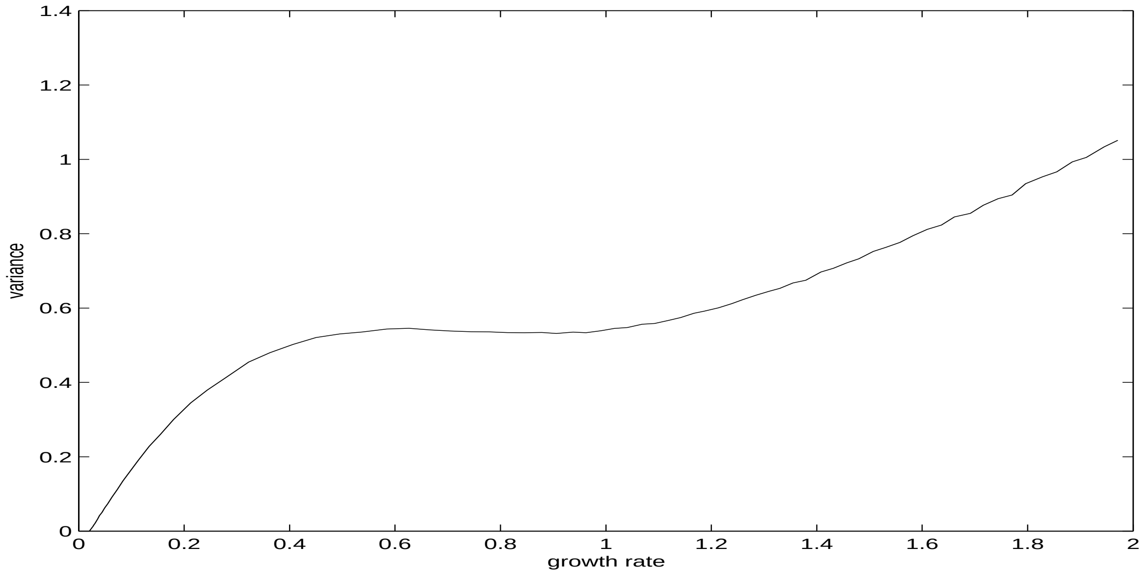

Figure 3.14: Relationship between economic growth and volatility when $\bar{\theta}$ changes

overall economic volatility. The reason for the less salient positive relationship between growth and volatility in some area of this range is that the second kind of uncertainty about the actions of other agents kicks in and prevents agents from investing in the new technology. Consequently, even when θ increases, λ does not increase very significantly, depressing the increase in economic volatility. When $\bar{\theta} > log(r) + 1$, λ is relatively stable and equal to zero most of time. In this case the main uncertainty comes from the economic fundamentals of the new technology. The economy is most volatile in this case due to two reasons. First, all the agents will invest in the new technology, which is riskier by nature compared to the conventional technology, leading to the volatility. Second, the returns of the new technology are assumed to be log-normally distributed. So their variance will increase in the mean. This is because for a log-normally distributed variable $X = e^x$, where x is normally distributed with mean μ and variance σ^2, the variance of X is given by:

$$VAR(X) = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}.$$

In general, economic growth and variance exhibit a positive correlation with the increase in $\bar{\theta}$ because of the shifting of investment to the new technology and and the nature of the new technology modeled in this model.

Next, I will use the same method in Section 3.5.2 to find the average growth rates and variances given different levels of $x^*(\rho^*)$. Here $\bar{\theta} = 1.0$, which is in the coordination failure zone. Then the relationship between economic growth and volatility is examined.

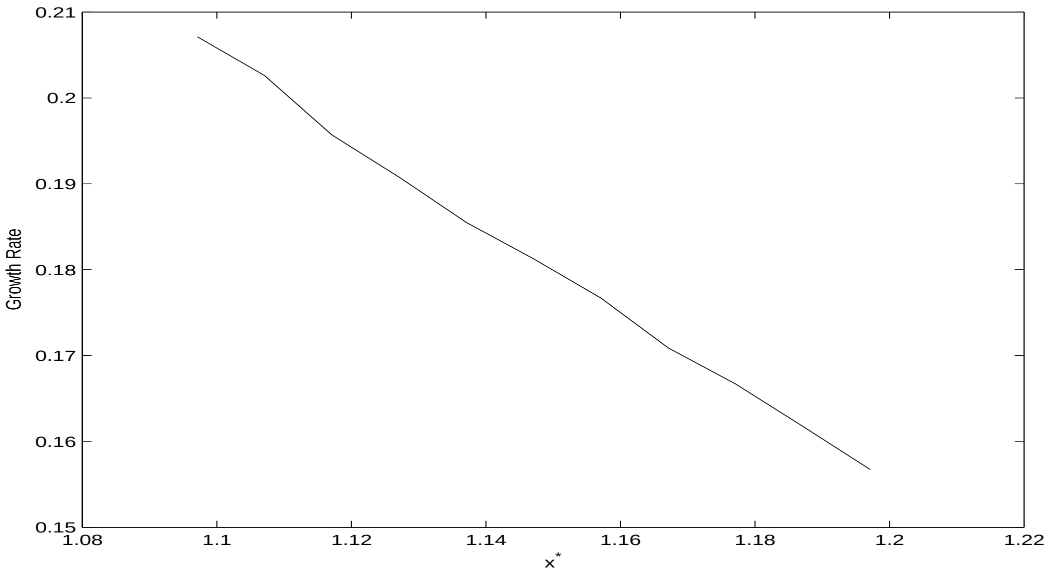

Figure 3.15: How economic growth changes in x^*

The figures above reveal that a small decrease in x^* will greatly increase economic growth and generate a significant positive correlation between economic growth and variance. This is because a decrease in x^* leads to both more investment in the new technology and higher returns of the new technology, both of which contribute to higher growth. Meanwhile, more investment in the new technology results in higher economic volatility. We know that a risk neutral agent will definitely have a lower $x^*(\rho^*)$ than a risk averse agent. So the risk attitude of agents could be a source of a positive correlation between economic growth and volatility associated with coordination failure.

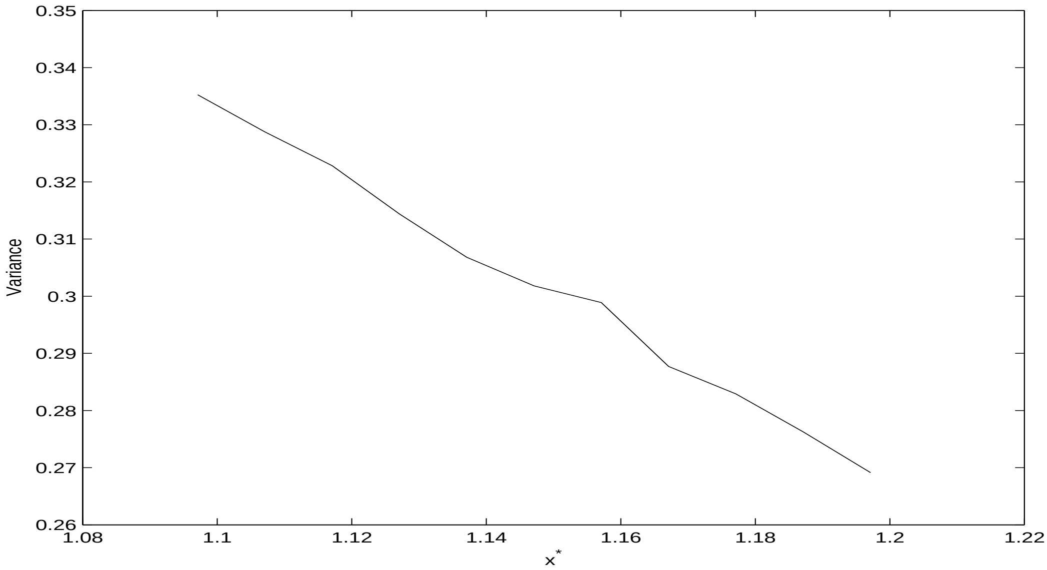

Figure 3.16: How economic volatility changes in x^*

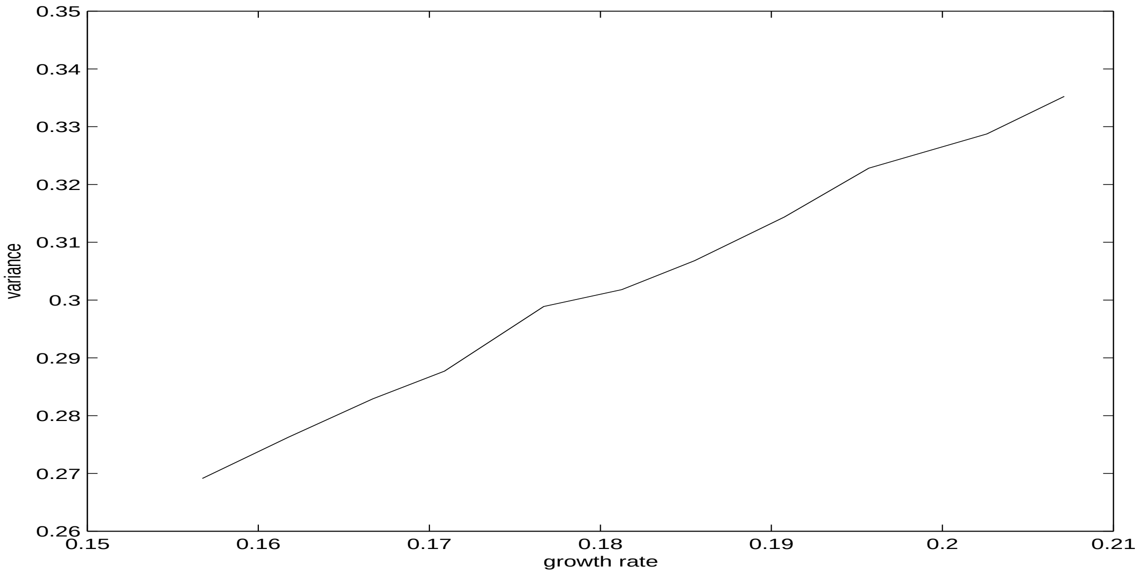

Figure 3.17: Relationship between economic growth and volatility when x^* changes

3.6 Policy Implications

This chapter reveals that coordination failure can hamper a country's new technology investment and economic growth. Therefore, the government needs to play the role of a coordinator to improve social welfare. In summary, there are several implications that I have obtained from the model. First, when the returns of the new technology are close to those of the conventional technology, coordination failure is significant and government intervention is most needed. Second, coordination failure is usually manifested as under-investment in the new technology. Therefore, any government policies stimulating more investment in the new technology will help alleviate coordination failure. Third, public information can play a role in alleviating coordination failure.

Next I will discuss one specific government policy based on my model.

Based on the above example of $\beta \rightarrow \infty$, a simple way to achieve the socially optimal solution is to tax the agents investing in the conventional technology or to subsidize those investing in the new technology. This is a natural result due to my assumption of the increasing returns to scale in the new technology investment. In fact, a lot of research emphasizing the public good property of new technology suggests this policy.

Suppose that the government introduces a penalty τ to the investment in the conventional technology. Now the cutoff level of x^* is determined by:

$$x^* - \frac{1}{2} = log(r - \tau).$$

Then we get $x^* = log(r - \tau) + \frac{1}{2}$. Let $\tau = (1 - e^{-\frac{1}{2}})r$, then $x^* = log(r)$, which is exactly the optimal cutoff level. Suppose this lump-sum tax will be returned to the agents, then the first best solution is achieved. Note that here the government can achieve the first best outcome because this is a very special case with $\beta \rightarrow \infty$. In such a case, the first kind of uncertainty about the economic fundamental of the new technology vanishes. The inefficiency is purely caused by the coordination failure. So the government can step in to play a role as a coordinator and correct the inefficiency.

Note that we have to treat this policy implication with caution. We must be aware that it is derived from a model under some highly simplified assumptions about a real economy. New technology is unknown by nature and involves a severe information asymmetry problem, which increases the difficulty for governments to implement targeted intervention in reality. However, the general message that this analysis sends us is that the presence of externalities in new technology investment demands government intervention to stimulate more new technology investment.

3.7 Conclusions

In this chapter I argue that due to the special features of uncertainty and externalities of new technology, coordination failure can occur in a country's new technology investment and lead to slower capital accumulation and economic growth. Combining a global game into a two-sector Overlapping Generation model, I demonstrate that coordination failure can be manifested as under-investment in the new technology and it is most severe when the returns from the new technology investment are close to those from the conventional technology investment. My model also reveals that the tradeoff between economic growth and volatility can occur because more investment in the new technology will alleviate coordination failure, but meanwhile will lead to higher economic volatility. In addition, policy implications of my model are explored. I find that the government intervention in favor of new technology investment is needed to alleviate coordination failure.

Chapter 4

The Role of Large Players in a Dynamic Currency Attack Game

4.1 Introduction

We often observe that large players such as hedge funds play an active role in currency attacks against fixed exchange rate regimes. They launch currency attacks by employing a large amount of wealth to build large short positions. Afterward they try to influence market sentiment by publicly announcing their short positions and beliefs that devaluation is inevitable. This causes herding among small traders, and/or deters contrarians from taking opposite positions. It seems that the presence of large players facilitates coordination among speculators and increases financial instability in the attacked currencies, specially in small economy currencies. This is sometimes called the "big elephants in small ponds" effect.

We establish a formal model to study the role that large players play in a currency attack, based on the model developed by Abreu and Brunnermeier (2003). In their model, rational arbitrageurs in an asset market become aware of an asset bubble sequentially. Due to the lack of common knowledge about the bubble, and need for coordination to burst the bubble, the bubble will be persistent and its bursting time depends on the incentives of the arbitrageurs to " ride the bubble," as opposed to incentives to preempt other arbitrageurs in selling the asset. Similar to their model, we assume that a currency begins to be overvalued in a fixed exchange rate regime after a certain time. Speculators have dispersed opinions in the sense that they only become aware of the overvaluation sequentially. In addition, we assume that the fixed exchange rate regime will collapse only when attacking pressure reaches a threshold level. This assumption captures the main feature of currency attacks: there is a necessity for coordination among speculators to break a currency peg. This coordination feature is emphasized in Obstfeld (1996) and other currency attack models (see especially Morris and Shin (1998)). In our setup, the speculators try to choose the optimal time to launch their attack, driven by two competing incentives: first, the incentive to "ride the overvaluation;" and second, the incentive to preempt other speculators. The speculators' incentive to "ride the overvaluation" stems from two sources in our model: first, they can reap higher benefits from the devaluation if the overvaluation lasts longer. Second, if they time their attack more precisely, they will save on attacking costs. The speculator attempts to preempt other speculators, because only the speculator attacking early will gain from the collapse of the regime. The late speculators will gain nothing.

Abreu and Brunnermeier (2003) consider a symmetric game with a continuum of atomistic small arbitrageurs. We are more interested in a richer market structure where both a large player and a continuum of small players are present. More specifically, we are interested in studying how the presence of a large player will change equilibrium outcomes. In our model, a large player is defined by two characteristics: first, he has more precise information about the fundamental value of a currency. Here we assume the extreme case where a large player has perfect information about the time when the overvaluation begins. Second, a large player can employ substantially larger amounts of wealth to launch a currency attack. The wealth that a large player employs can come from his own capital, or more importantly, from his accessibility to credit due to his reputation. This is how highly leveraged financial institutions finance their speculation.

Our model differs from most currency attack literature in several aspects. First, it is one of the few papers studying currency attacks in a dynamic setup. Most existing literature uses static models, which miss the complicated market dynamics in currency attacks. Second, while most existing currency attack literature simply assumes exogenously the existence of the overvaluation under a fixed exchange rate regime, we model endogenously the origin of currency overvaluation in the presence of rational arbitrageurs. A key contribution of Abreu and Brunnermeier (2003) lies in that they offer a general explanation of how asset mispricing arises. Even in the presence of rational arbitrageurs, who are capable of correcting the mispricing, they choose not to do it due to their incentive to "ride the bubble." This mechanism can be comfortably applied to explaining how currency overvaluation arises in a fixed exchange rate regime.

Our approach is consistent with the microstructure method for modeling exchange rates. We believe that foreign exchange market participants hold highly dispersed opinions about exchange rates. As argued by Lyons (2001), even if all market participants have the same information about exchange rates, the ways in which they interpret or model the implications of that information can be different. Thus, they can come to different conclusions about exchange rates based on the same information. So it is well justified to assume that speculators do not have perfect information about the time when the overvaluation arises, and only become gradually aware of the overvaluation. In such an opinion-dispersed market where coordination is required for a successful currency attack, the incentive for "riding the overvaluation" naturally arises and leads to the persistent overvaluation of the currency. This explanation is consistent with empirical observations that currency attacks often lead to substantial and sudden devaluations, causing extreme volatility in an economy.

Due to the features of our model, our study of large players in currency attacks focuses on a different aspect compared to the standard, more static models. Most existing literature on large players in currency attacks focuses on the possibility of the collapse of a fixed exchange rate regime, and on whether the presence of large players will increase this possibility or not. In our model, currency devaluation is inevitable, and the issue that we focus on is *when* it will happen. Thus our study focuses on whether the presence of large players will *accelerate* or *delay* a currency attack. Here we do not give a formal welfare analysis to examine whether the presence of a large player is beneficial or harmful to an economy. However, in general, we believe that a currency overvaluation is harmful to an economy, and early correction is always better than a late one if the correction is inevitable. In this sense, a late collapse of the regime will do more harm to an economy than an early one.

Using our model, we find some interesting results. First, we find that the presence of a large player will not necessarily accelerate the collapse of an overvalued fixed exchange rate regime. This result is important because large players are usually believed to facilitate arbitrage in an asset market and reduce asset mispricing. In our model, the presence of a large player will *accelerate* or *delay* the collapse of a fixed exchange rate regime, depending on whether his incentive to "ride the overvaluation" is dominated, or not, by his incentive to preempt the mass of small speculators. If his incentive to preempt is dominant, his presence will accelerate the collapse of the regime. He can do so not only because his wealth facilitates the attack, but also because his presence makes other small speculators attack earlier. Conversely, if a large player's incentive to preempt other speculators is dominated by the incentive to " ride the overvaluation," his presence will delay the collapse of the regime. He can do so not only because he will wait longer, but also because his presence makes other speculators wait longer too. The large player's incentive to "ride the overvaluation" makes the existence of a large player a mechanism for delaying any asset mispricing. Instead, he will use his market power to make greater profits from larger, later asset mispricing.

The rest of the chapter consists of six sections. Section 4.2 provides a literature survey. Section 4.3 discusses the basic model and characterization of a dynamic currency attack. The model is a variation of that established by Abreu and Brunnermeier (2003): the model has a continuum of small arbitrageurs trading in a currency with a fixed exchange rate, that is open to a currency attack. We provide a characterization of the equilibrium and comparative statics. Section 4.4 introduces a large player, proves that there is a unique equilibrium and characterizes that equilibrium. Section 4.5 conducts comparative statics for the model. Section 4.6 observes that our results can be applied to the original Abreu and Brunnermeier (2003) set-up with a stock market. Section 4.7 concludes with observations on further possible extensions.

4.2 Literature Survey

Abreu and Brunnermeier (2003) construct a dynamic coordination game to explain the existence of asset bubbles, even in the presence of rational arbitrageurs who are capable of bursting the bubble. We have already discussed their model in detail. (Our chapter is an application of their model to currency attacks.) In their model, only a continuum of atomistic speculators exists. Since we focus on the study of the role that a large player plays in a currency attack, our model exhibits a richer market structure where both a large player and a continuum of atomistic speculators co-exist.

Both Rochon (2006) and Gara Minguez-Afonso (2007) apply Abreu and Brunnermeier (2003) to currency attacks and try to explain the devaluation that we observe when a fixed exchange rate regime collapses. The most important difference between our model and theirs is that our model focuses on the role of large players in a currency attack with imperfect common knowledge, while they study currency attacks only in a model without large players. In addition, even in our basic model without large players, the way in which we model a currency attack is also slightly different from theirs. We model the payoff structure of speculators who try to gain from the devaluation, while they model the payoff structure of the attackers who try to avoid a capital loss associated with devaluation.

Morris and Shin (1998) study currency attacks in a one-period global game setup. They demonstrate that, although a self-fulfilling currency attack game has multiple equilibria when economic fundamentals are common knowledge, it has a unique equilibrium when speculators can only observe the fundamentals with small noise. Successfully overcoming the problem of indeterminacy of multiple equilibria models, their model allows the analysis of policy implications.

Corsetti, Dasgupta, Morris and Shin (2004) extend the model established by Morris and Shin (1998) to one with a large player. They analyze two cases where the large player has, and has not, a signalling function. They find that in both cases the presence of a large player does increase the possibility of the collapse of a fixed exchange rate regime, and make small speculators more aggressive.

Correstti, Pesenti, and Roubini (2001) give a comprehensive survey on the role that large players play in currency attacks. In the theoretical section of their survey, they apply a traditional coordination game with perfect information, and then a global game established by Corsetti, Dasgupta, Morris and Shin (2004) to the study of the role of a large player in a currency market. In the empirical section, they combine both econometric analysis and case studies to explore examples of currency attacks. Their conclusion is that both theoretical and empirical studies reveal that large players do have a significant role in currency attacks, and more academic research is required to address a number of issues, including the dynamics of currency attacks or crises.

Bannier (2005) modifies the model established by Corsetti, Dasgupta, Morris and Shin (2004) by changing the assumption about a central bank's strategy. Due to that modification, both the large player and small speculators' strategies are symmetric and analytical results are available. She finds that this modification changes the results given by Corsetti, Dasgupta, Morris and Shin (2004). Now a large player can increase the possibility of a regime collapse only when market sentiment is pessimistic. However, the presence of a large player will decrease the possibility of a regime collapse when the market sentiment is optimistic.

4.3 The Benchmark Model without a Large Player

4.3.1 Environment

This model is a simple modification of the Abreu and Brunnermeier (2003) model. We capture the essence of their idea that the difficulty in coordination among arbitrageurs, together with their incentive to time the market, can cause asset mispricing. We modify the model to apply it to foreign exchange markets.

Assume that there is a country with a fixed exchange rate regime where a central bank commits to maintaining the exchange rate at a fixed level until it exhausts all of its foreign reserves, whose level is denoted by $k > 0$.

From time $t_0 > \eta$, the exchange rate becomes overvalued relative to its fundamental value, at a rate of g. Denote the initial exchange rate as E_0. The fundamental exchange rate at t is E_0 when $t < t_0$ and $E_0(1 + g(t - t_0))$ when $t \geq t_0$. Here the exchange rate is denominated in the domestic currency, say wons. So E_0 means that 1 dollar can exchange for E_0 wons.

Without any currency attacks, the fixed exchange rate regime will collapse at some exogenously given time $t_0 + \tau'$. This assumption captures the idea that any asset mispricing is not sustainable in the long run. We follow Abreu and Brunnermeier (2003) in making this simplified assumption to avoid ever greater currency overvaluations. Figure 4.1 shows how the fundamental exchange rate changes with time.

There is a continuum of speculators of mass 1. Each speculator is financially constrained

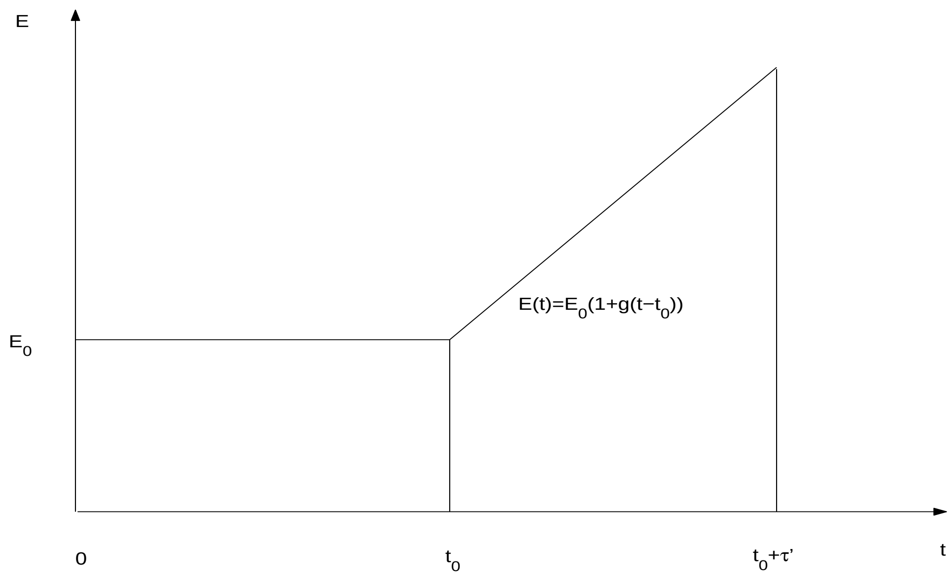

Figure 4.1: How the fundamental exchange rate E changes with time t

and can only access the credit whose worth is normalized to 1 dollar. Each speculator has to choose from two strategies: attacking or refraining. When $t < t_0 + \tau'$, the exchange rate will devalue to the fundamental value if and only if attacking pressure exceeds k. This assumption follows that of Obstfeld (1996) and Morris and Shin (1998) and captures the idea of market liquidity.

We specify the payoff structure of speculators as follows: if they choose refraining, which means that they will do nothing, they will gain zero. If they choose to attack, they will borrow wons from the banks of the attacked country, then exchange them into dollars from the central bank. The costs of attacking consist of two parts. One part is the fixed transaction costs associated with the currency exchanges, which is denoted by c^F. We assume that the fixed transaction costs are not so high that they prevent the speculators from ever attacking, despite the awareness of the overvaluation. The other part is the interest differential between wons and dollars, since we assume that the interest rate of wons is higher than that of dollars. Let c denote the interest differential. Thus, if a speculator keeps attacking during a time interval $\triangle t$, he will incur the cost of $c.\triangle t$. The payoffs of speculators from attacking is as follows. If the regime collapses at instant t, the payoffs of a speculator attacking at instant t with the wealth of 1 dollar will depend on how many other speculators are attacking. If the attacking mass is less than or equal to k, his payoffs are $E_0.g(t-t_0)$. If the attacking mass is greater than k, only the first randomly chosen mass k of attacking speculators will gain the payoffs of $E_0.g(t-t_0)$. So given the attacking pressure $\alpha > k$, the expected payoffs of a speculator are given by $\frac{k}{\alpha}E_0.g(t-t_0)$. For simplicity of the analysis, we assume that no partial attacking is allowed.

The speculators only have imperfect information about t_0, the time at which the overvaluation begins. More specifically, all the speculators have a prior belief about t_0, which is denoted by $\Phi(t_0)$. We assume that the speculators have an improper uniform belief about t_0 over $[0, \infty)$.

From t_0, a new cohort of small speculators with mass $\frac{1}{\eta}$ becomes aware of the overvaluation in each instant from t_0 until $t_0 + \eta$.

Conditional on t_i, speculator t_i's belief about t_0 is given by the CDF

$$\Phi(t_0|t_i) = \frac{t - t_i + \eta}{\eta}, \tag{4.1}$$

where $t \in [t_i - \eta, t_i]$.

Given such a setup, we try to find the equilibrium strategy of a rational speculator t_i.

Let $\sigma(t, t_i)$ denote the strategy of speculator t_i and the function $\sigma : [0, \infty) \times [0, \infty) \mapsto \{0, 1\}$ a strategy profile. Speculator t_i's strategy is given by $\sigma(., t_i) : [0, t_i + \tau'] \mapsto \{0, 1\}$, where 0 means refraining and 1 means attacking. The aggregate attacking pressure of all the speculators at time $t \geq t_0$ is given by

$$s(t, t_0) = \int_{t_0}^{min\{t, t_0+\eta\}} \sigma(t, t_i) dt_i. \tag{4.2}$$

Let

$$T^*(t_0) = inf\{t|s(t,t_0) \geq k \text{ or } t = t_0 + \tau'\} \tag{4.3}$$

denote the collapse time of the fixed exchange rate regime for a given realization of t_0. Recall that $\Phi(.|t_i)$ denotes speculator i's belief about t_0 given that $t_0 \in [t_i - \eta, t_i]$. Hence, his belief about the collapse time is given by

$$\Pi(t|t_i) = \int_{T^*(t_0)<t} d\Phi(t_0|t_i).$$

The time t_i expected payoffs of speculator t_i, who remains refraining until he begins to attack at time t and keeps attacking afterward until the regime collapses, are given by

$$\int_t^{t_i+\tau'} E_0 g(s - T^{*-1}(s)) - c(s-t)d\Pi(s|t_i) - c^F,$$

provided that the attacking pressure at t does not strictly exceed k and that $T^*(.)$ is strictly increasing. Later we will show that in equilibrium all the conditions will hold.

If we normalize the initial exchange rate to 1, we get:

$$\int_t^{t_i+\tau'} g(s - T^{*-1}(s)) - c(s-t)d\Pi(s|t_i) - c^F. \tag{4.4}$$

4.3.2 Equilibrium Characterization

We confine our attention to symmetric trigger strategies. We can prove that there is a unique symmetric trigger strategy equilibrium. In this equilibrium, each speculator t_i will attack at the instant $t_i + \tau^*$ and keep attacking until the regime collapses. Depending on parameter values of η, k, g and c, the regime can collapse exogenously or endogenously. Here we will focus on the endogenous collapse case.

Rochon (2006) proves in a similar setup that this symmetric trigger strategy equilibrium is a strongly rational expectation equilibrium in the set of strategies with the only restriction being that speculators act after being informed.

Proposition 5. *Given* $\tau' > \frac{c}{g}k\eta$ *and* $c \geq g$*, there is a unique symmetric trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each speculator* t_i *begins to attack at the instant* $t_i + \tau^*$ *and keeps attacking until the regime collapses, where* $\tau^* = \frac{c-g}{g}k\eta$*. In equilibrium the regime collapses exactly at the instant* $t_0 + k\eta + \tau^*$*.*

Given $\tau' > k\eta$ *and* $c < g$*, there is a unique symmetric trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each speculator* t_i *begins to attack at the instant* t_i *and keeps attacking until the regime collapses. In equilibrium the regime collapses exactly at the instant* $t_0 + k\eta$*.*

Proof:

Let τ^* define a symmetric trigger equilibrium. That is, all the speculators begin to attack at $t_i + \tau^*$. Given such a strategy, the regime will collapse when speculator $t_0 + k\eta$ attacks, and the collapsing time will be $t_0 + k\eta + \tau^*$.

Now consider the optimal strategy of speculator t_i given that all the other speculators take the strategy τ^*. Thus the regime will collapse at $t_0 + \zeta$, where $\zeta = k\eta + \tau^*$. Speculator t_i believes that $t_0 \in [t_i - \eta, t_i]$, the CDF of his posterior belief about t_0 is given by

$$\Phi(t|t_i) = \frac{t - t_i + \eta}{\eta}. \tag{4.5}$$

Since the collapsing time is $t_0 + \zeta$, he believes that $t_0 + \zeta \in [t_i - \eta + \zeta, t_i + \zeta]$. The CDF of his posterior belief about the collapsing date $t_0 + \zeta$ at time $t_i + \tau$ is given by

$$\Pi(t_i + \tau|t_i) = \frac{t_i + \tau - (t_i - \eta + \zeta)}{\eta} = \frac{\tau + \eta - \zeta}{\eta}. \tag{4.6}$$

Speculator t_i's expected payoff from attacking at t and keeping attacking until the regime collapses is given by:

$$\int_t^{t_i+\zeta} (g(s - T^{*-1}(s)) - c(s - t))d\Pi(s|t_i) - c^F. \tag{4.7}$$

The first order condition gives the optimal τ for him to attack:

$$\frac{\pi(t_i + \tau|t_i)}{1 - \Pi(t_i + \tau|t_i)} = \frac{c}{g(t_i + \tau - T^{*-1}(t_i + \tau))}. \tag{4.8}$$

We also check the second order condition, which turns out that the second order derivative is negative and the second order condition is satisfied.

Taking Equation (4.6) into the left hand side of the first order condition gives us:

$$\frac{\pi(t_i+\tau|t_i)}{1-\Pi(t_i+\tau|t_i)} = \frac{1}{\zeta-\tau}. \tag{4.9}$$

In addition, in this symmetric equilibrium, the duration between the time when the regime collapses and the time when the overvaluation happens is given by: $t_i+\tau-T^{*-1}(t_i+\tau) = \tau^*+k\eta = \zeta$. This is because each speculator will delay a period of τ^* and the regime will collapse exactly at the moment $t_0+k\eta+\tau^*$ when the speculator $t_0+k\eta$ launches his attack.

So we find:

$$\frac{1}{\tau^*+k\eta-\tau} = \frac{c}{g(\tau^*+k\eta)}. \tag{4.10}$$

Since it is a symmetric equilibrium, $\tau=\tau^*$. Solving the above equation, we get

$$\tau^* = \frac{(c-g)k\eta}{g}. \tag{4.11}$$

Given $\frac{c}{g}k\eta < \tau'$, the regime will collapse at $t_0+k\eta+\tau^* < t_0+\tau'$ endogenously.

Note that $\tau^* \geq 0$ if and only if $c \geq g$. When $c < g$, we will get the corner solution of $\tau^* = 0$.

Q.E.D

The intuition of the equilibrium is as follows. Given that all the speculators begin their attack at $t_i+\tau^*$, the instantaneous probability that the regime collapses at $t_i+\tau$ of speculator t_i is given by:

$$\frac{\pi(t_i+\tau|t_i)}{1-\Pi(t_i+\tau|t_i)} = \frac{1}{\tau^*+k\eta-\tau}. \tag{4.12}$$

If the regime exactly collapses at $t_i+\tau$, the gains from attacking will be $g(\tau^*+k\eta)$. Thus, the expected marginal benefits of speculator t_i attacking at $t_i+\tau$ are given by:

$$g(\tau^*+k\eta)\frac{\pi(t_i+\tau|t_i)}{1-\Pi(t_i+\tau|t_i)} = g(\tau^*+k\eta)\frac{1}{\tau^*+k\eta-\tau}.$$

Meanwhile, the marginal costs incurred by attacking at time $t_i + \tau$ are c, which are constant. From the above equations we can see that the expected marginal gains from attacking are strictly increasing in τ, since the speculator t_i's subjective instantaneous probability that the regime collapses at time $t_i + \tau$ is strictly increasing in τ. So there is a unique level of τ, where the expected marginal gains from attacking are exactly equal to the marginal costs incurred by attacking. And it is the optimal time for speculator t_i to attack. Figures 4.2 and 4.3 explain the intuition.

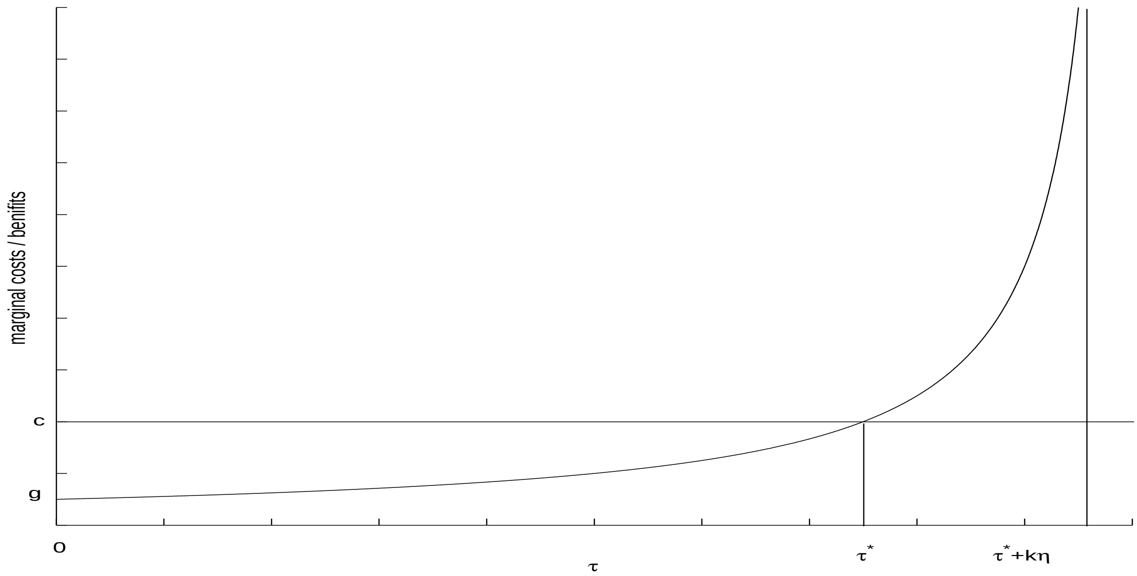

Figure 4.2: How the marginal costs and benefits change in τ in the case of the interior solution of τ^*

4.3.3 Comparative Statics

This section studies how the changes in parameters of the model influence equilibrium results.

We know that in equilibrium

$$\tau^* = \frac{(c-g)k\eta}{g}.$$

First, we can see that the speculators will wait longer with higher c. The intuition is

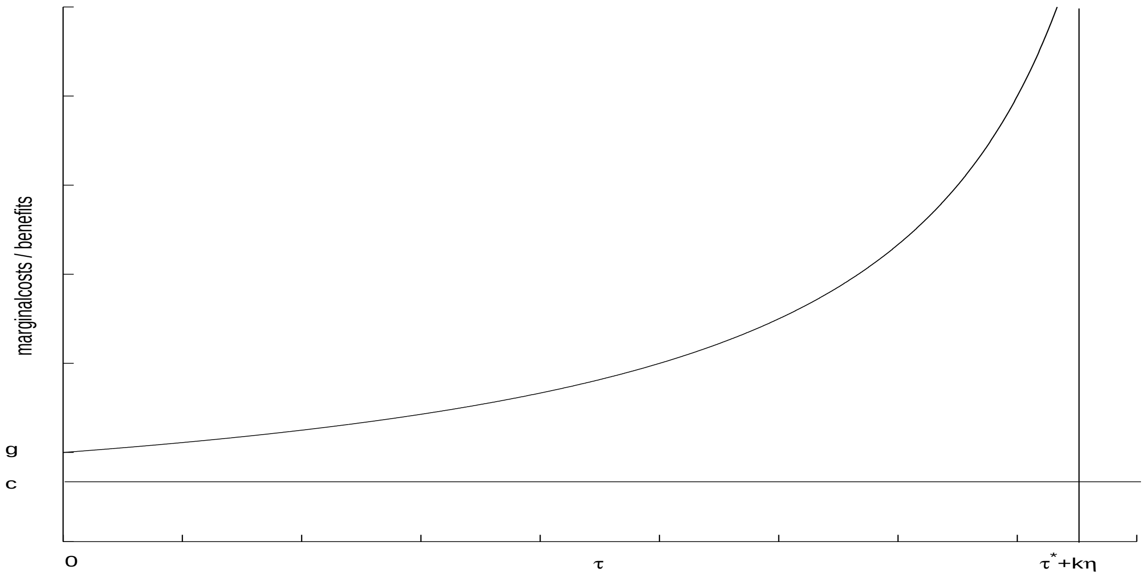

Figure 4.3: How the marginal costs and benefits change in τ in the case of the corner solution of τ^*

simple. Higher c means that it will cost more if a speculator launches an attack early. Hence a speculator would like to wait longer to reduce the costs of attacking.

Second, we find that the speculators will wait longer with both higher k and η. This result is also intuitive. Higher η means more dispersed opinions among the speculators and higher k means a higher requirement for coordination. Both will increase the difficulties in coordination and induce the speculators to wait longer.

We know that c, k and η are all parameters indicating how difficult it is to arbitrage in a foreign exchange market. We find that now the frictions in the market become a blessing for the speculators, since more frictions will induce the speculators to wait longer and make higher profits from the overvaluation.

Third, we find that the speculators will wait longer with lower g, the rate at which the currency is overvalued. In this case, higher g increases the speculators' incentive to preempt other speculators and makes the speculators less patient. In the extreme case when $g > c$, speculators will launch an attack immediately after they become aware of the overvaluation.

Finally, there is an interesting result about the exchange rate level when the regime collapses, which determines the magnitude of the devaluation. It is given by $ck\eta$. We can see that g does not play a role in determining the magnitude of the devaluation. This is because the speed at which the fundamental value of the currency decreases has two opposite effects: First, it affects the optimal delay time of speculators. Second, it affects the fundamental exchange rate at time t. The net result from these two effects is that g will not influence the exchange rate when the regime collapses at all.

4.4 The Model with a Large Player

In this section we introduce a large player into the basic model.

We keep the model as simple as possible, by assuming that the speculators consist of one large player with wealth $\lambda < k$ and a continuum of small speculators of mass 1 with total wealth of 1. Here we assume $\lambda < k$ such that the large player cannot independently break the peg. This assumption is realistic because even a large player like Soros in financial markets cannot single-handedly break a currency peg. Moreover, we assume that the large player has perfect information about t_0; that is, he always becomes aware of the overvaluation at t_0 when the overvaluation happens. In addition, we assume that the action of the large player will not be observed by other speculators.

Now we need to define the equilibrium in such a setup. Given all the assumptions unchanged for small speculators, we will prove that there is a unique trigger strategy equilibrium in this game.

Proposition 6. *Given $\tau' > \frac{(c-g)(k-\lambda)\eta}{g}$ and $\frac{c}{g} > \frac{k}{k-\lambda}$, there is a unique trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each small speculator t_i begins to attack at the instant $t_i + \tau^{SP}$ and keeps attacking until the regime collapses. The large player begins to attack at $t_0 + (k-\lambda)\eta + \tau^{SP}$. Here $\tau^{SP} = \frac{(c-g)(k-\lambda)\eta}{g}$. The regime* *collapses exactly at* $t_0 + \frac{c(k-\lambda)\eta}{g}$*, when the large player launches the attack.*

Given $\tau' > \frac{c(k+\lambda)\eta}{g}$ *and* $\frac{k}{\lambda+k} < \frac{c}{g} < 1$*, there is a unique trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each small speculator begins to attack at the instant* $t_i + \tau^{SP}$ *and keeps attacking until the regime collapses. Here* $\tau^{SP} = \frac{(c-g)k\eta + c\lambda\eta}{g}$*. The large player begins to attack at* $t_0 + k\eta + \tau^{SP}$*. The regime collapses exactly at the time when the large player launches the attack.*

Given $\tau' > k\eta$ *and* $\frac{c}{g} < \frac{k}{\lambda+k}$*, there is a unique trigger strategy equilibrium where the regime collapses endogenously. In this equilibrium, each small speculator begins to attack at the instant* t_i *and keeps attacking until the regime collapses. The large player begins to attack at* $t_0 + k\eta$*. The regime collapses exactly at the time when the large player launches the attack.*

Proof

Since the large player has perfect information about t_0, he will choose the optimal time $t_0 + \tau^{LP}$ to maximize his profits, given the equilibrium strategies taken by small players. Since small players are identical ex ante and atomically small, they will take symmetric strategies. Suppose that each small player plays the symmetric trading strategy $t_i + \tau^{SP}$ in equilibrium. From the moment of $t_0 + (k-\lambda)\eta$ on, the total wealth of the large player and small players exceeds the threshold level k. Thus, the payoffs of the large player from attacking at $t_0 + (k-\lambda)\eta + \tau^{SP} + t$ are given by

$$\lambda g[(k-\lambda)\eta + \tau^{SP} + t]\frac{k}{k + \frac{t}{\eta}} = \lambda g k\eta \frac{(k-\lambda)\eta + \tau^{SP} + t}{k\eta + t},$$

where $0 \le t \le \lambda\eta$.

Notice that $t \le \lambda\eta$, or the regime will collapse solely due to the attacking pressure from small players, and the large player will gain zero. The large player will choose an optimal level of t to maximize his expected payoff. Solving the maximization problem, we get that $t = 0$ given $\lambda\eta - \tau^{SP} < 0$, and $t = \lambda\eta$ given $\lambda\eta - \tau^{SP} > 0$.

Therefore, the optimal strategy for the large player is as follows. Given $\lambda\eta - \tau^{SP} < 0$, the large player will launch the attack at $t_0 + \tau^{LP}$, where $\tau^{LP} = (k - \lambda)\eta + \tau^{SP}$. Given $\lambda\eta - \tau^{SP} > 0$, the large player will launch the attack at $t_0 + \tau^{LP}$, where $\tau^{LP} = k\eta + \tau^{SP}$. (The intuition for the above results is as follows. When the large player delays his attacking, there are two effects on his payoffs. First, he will gain more from the larger devaluation when the regime collapses. Second, he will gain less due to the smaller share in the total attacking wealth. The shorter τ^{SP} is, and the larger λ and η are, the more the large player will gain from delaying.)

Now let us look at the best responses of small players. Our previous proof for the unique symmetric trigger strategy equilibrium still holds in this case. Only now the optimal attacking time $t_i + \tau^{SP}$ is determined by the following conditions.

Given the optimal strategy of the large player, $\tau^{LP} = (k - \lambda)\eta + \tau^{SP}$, in equilibrium the regime collapses at $T^* = t_0 + \zeta = t_0 + (k - \lambda)\eta + \tau^{SP}$. Therefore, the first order condition gives

$$\frac{\pi(t_i + \tau^{SP}|t_i)}{1 - \Pi(t_i + \tau^{SP}|t_i)} = \frac{1}{\zeta - \tau^{SP}} = \frac{1}{(k - \lambda)\eta} = \frac{c}{g\zeta}.$$

In equilibrium, $\zeta = (k - \lambda)\eta + \tau^{SP}$. Thus we get $\tau^{SP} = \frac{(c-g)(k-\lambda)\eta}{g}$. The large player's equilibrium strategy is $\tau^{LP} = (k - \lambda)\eta + \tau^{SP} = \frac{c(k-\lambda)\eta}{g}$. Checking the condition inducing the large player to choose $\tau^{LP} = (k - \lambda)\eta + \tau^{SP}$, we get:

$$\lambda\eta - \tau^{SP} < 0 \Rightarrow \frac{c}{g} > \frac{k}{k - \lambda}.$$

Now let us look at the case in which the large player takes the equilibrium strategy of $\tau^{LP} = k\eta + \tau^{SP}$. In equilibrium $T^* = t_0 + \zeta = t_0 + k\eta + \tau^{SP}$. Given the large player's equilibrium strategy, the first order condition for small players is given by

$$\frac{1}{\zeta - \tau^{SP}} = \frac{c}{\frac{k}{k+\lambda} g(k\eta + \tau^{SP})} = \frac{1}{\eta k} = \frac{c(k + \lambda)}{kg(k\eta + \tau^{SP})}.$$

Solving the above equation, we get $\tau^{SP} = \frac{(c-g)k\eta + c\lambda\eta}{g}$. Therefore, $\tau^{LP} = k\eta + \tau^{SP} = \frac{c(k+\lambda)\eta}{g}$. We need to check the condition inducing the large player to choose $\tau^{LP} = k\eta + \tau^{SP}$, which is

$$\lambda\eta - \tau^{SP} > 0 \Rightarrow \frac{c}{g} < 1.$$

Moreover, notice that in order to ensure τ^{SP} is positive, we have $c(\lambda + k) - gk > 0$, or $\frac{c}{g} > \frac{k}{\lambda+k}$. When $\frac{c}{g} < \frac{k}{\lambda+k}$, we get the corner solution of $\tau^{SP} = 0$. Since $\lambda\eta - \tau^{SP} > 0$ in this case, the condition required for the large player to choose the strategy of $\tau^{LP} = k\eta + \tau^{SP}$ still holds. Thus, the general condition for the large player to take the strategy of $\tau^{LP} = k\eta + \tau^{SP}$ is $\frac{c}{g} < 1$.

Q.E.D.

4.5 The Role of a Large Player

In this section, we analyze the role that a large player plays in a currency attack. Our model reveals that a large player can both accelerate or delay the collapse of a fixed exchange rate regime, depending on the circumstances. This result is important because it differs from the usual perception that the presence of a large player in a foreign exchange market will facilitate arbitrage, therefore helping to reduce the mispricing of exchange rates.

From Proposition 6, we can see that there are two possible equilibria. We will analyze these two cases respectively.

4.5.1 The Case in Which a Large Player Accelerates the Attack

Given $\frac{c}{g} > \frac{k}{k-\lambda}$, the presence of the large player will accelerate the collapse of the regime. The following are some results we find in this case.

1. *The collapse of the regime is accelerated due to two reasons: first, the large player has perfect information about t_0. Thus, more speculators are aware of the mispricing from t_0 on. Second, the presence of a large player makes small players more aggressive and shortens their delay time.*

The regime will collapse at $t_0 + \frac{c(k-\lambda)\eta}{g}$, which is earlier than the regime collapse time without a large player (which is $t_0 + \frac{c}{g}k\eta$). Here the collapse time is earlier, for two reasons: first, the large player will begin to attack exactly when there is enough wealth to correct the overvaluation. Thus, the regime collapses as long as mass of $k - \lambda$ of small players attacks, instead of mass of k in the case without a large player. Second, with the presence of the large player, the small players' equilibrium strategy, which is the waiting time between becoming aware of the overvaluation and before starting an attack, is shorter. In the case without a large player, the small players' strategy is $\tau^* = \frac{c-g}{g}k\eta$. With the large player it becomes $\tau^{SP} = \frac{(c-g)(k-\lambda)}{g}\eta$. In this case, the presence of a large player makes small players take more aggressive strategies and accelerates the arbitrage to correct the overvaluation.

2. *The collapsing time is strictly decreasing in λ, and the devaluation will be also smaller at the collapse time with larger λ. However, λ must be low enough to ensure the existence of this equilibrium.*

   Since $t_0 + \frac{c(k-\lambda)\eta}{g}$, the more wealth a large player has, the faster the fixed exchange rate regime will collapse. The exchange rate at the collapse time is given by $E_0(1 + c(k - \lambda)\eta)$, which is also decreasing in λ. However, in order for this accelerating equilibrium to exist, we must have $\frac{c}{g} > \frac{k}{k-\lambda}$. That is, $\lambda < \frac{(c-g)k}{c}$. Thus, this equilibrium will exist only when the wealth of the large player is low enough.

3. *The large player can make the most profits from the attack when $\lambda = \frac{k}{2}$.*

   The profits of the large player are given by:

$$\lambda g \frac{c(k-\lambda)\eta}{g}.$$

   It is straightforward to see that the optimal λ to maximize the large player's payoffs is $\lambda = \frac{k}{2}$. So there is not a monotonically increasing relationship between the wealth of the large player and the payoffs it reaps from the attack. The intuition is that there is a tradeoff with the increase of the wealth of the large player. On the one hand, more wealth ensures that the large player can claim a higher proportion of the attacking wealth that profits from the collapse of the regime. On the other hand, higher wealth will accelerate the collapse of the regime, leading to less devaluation, and therefore lower profits when the regime collapses.

4.5.2 The Case in Which a Large Player Delays the Attack

Given $\frac{c}{g} < 1$, the presence of a large player delays or causes no acceleration of the regime collapse. The following are some results that we get in this case.

1. *The collapse of the regime is delayed or is not accelerated for two reasons: First, a large player chooses to "ride the overvaluation." Second, the presence of a large player makes small players less aggressive and wait longer before launching the attack.*

   Given $\frac{k}{\lambda+k} < \frac{c}{g} < 1$, the regime will collapse at $t_0 + k\eta + \tau^{SP}$, where $\tau^{SP} = \frac{(c-g)k\eta+c\lambda\eta}{g} > 0$. However, in the case without a large player, we get the corner solution of $\tau^* = 0$, which we can interpret as being that the speculators will launch an attack as soon as they become aware of the overvaluation. There are two reasons to explain why small players delay their attack. First, small players are aware that the large player will "ride the overvaluation." Second, due to the presence of the large player, the gains of small players from the attack will be less, which reduces the incentive of small players to preempt other players. In order to see this, recall that the equilibrium equation to determine τ^{SP} is given by:

$$\frac{1}{\eta k} = \frac{c}{\frac{k}{k+\lambda} g(k\eta + \tau^{SP})},$$

   which is slightly different from that in the case without a large player:

$$\frac{1}{\eta k} = \frac{c}{g(k\eta + \tau^*)}.$$

The only difference between the above two equations is that with the presence of a large player, the expected payoffs of a small player will be the proportion of $\frac{k}{\lambda+k}$ of the total devaluation, instead of the whole devaluation.

Given $\frac{c}{g} < \frac{k}{\lambda+k}$, the presence of a large player will at least cause no acceleration of the collapse of the regime. In this case we get the corner solution of $\tau^{SP} = 0$. Thus the collapse time of the regime will be $t_0 + k\eta$, which is the same as the case without a large player.

This result differs from our common belief that the presence of large players facilitates the arbitrage and alleviates the mispricing in foreign exchange markets. The intuition here is that the market power of the large players, due to their superior information and more wealth, gives them the ability to time the collapse of the regime and " ride the overvaluation." In certain circumstances they prefer to wait longer to reap the most profits from the currency overvaluation.

2. *The collapse time of the regime is strictly increasing in λ. The devaluation will also be greater when the regime collapses with larger λ.*

   Given $\frac{k}{\lambda+k} < \frac{c}{g} < 1$, the collapse time is given by $t_0 + \frac{c(k+\lambda)\eta}{g}$, which is strictly increasing in λ. Moveover, the exchange rate at the collapse of the regime is given by $E_0(1 + c(k + \lambda)\eta)$, which is also strictly increasing in λ. Therefore, the more wealth the large player has, the later the regime will collapse, and the larger the devaluation will be at the time of the collapse. Notice that in order for $\tau^{SP} > 0$, $\lambda > \frac{(g-c)k}{c}$. So in this equilibrium λ has to be large enough to induce small speculators to wait some time after being aware of the overvaluation.

3. *The profits of the large player are strictly increasing in λ.*

   This is straightforward to see from the payoff function of the large player:

$$\lambda g \frac{c(k + \lambda)\eta}{g} = c\lambda(k + \lambda),$$

which is strictly increasing in λ.

Our model reveals that the ratio of $\frac{c}{g}$ is critical to determine whether the presence of a large player will accelerate or delay a currency attack. When $\frac{c}{g} > \frac{k}{k-\lambda} > 1$, the presence of a large player will accelerate the attack. When $\frac{c}{g} < 1$, the presence of a large player will delay or at least will not accelerate the attack.

The intuition is as follows. g and c are key to determining τ^{SP}, that is, how long small players will wait before launching an attack. Moveover, τ^{SP} is critical to determining the gains a large player will get from delay, relative to the losses from delay. The shorter τ^{SP} is, the more are the gains relative to the losses. Thus, higher g and lower c lead to shorter τ^{SP}, inducing the large player to choose the delay equilibrium. Meanwhile, lower g and higher c lead to longer τ^{SP}, inducing the large player to choose the accelerating equilibrium. In summary, only when g is large enough and c is low enough, is the incentive of a large player to "ride the overvaluation" strong enough to dominates the incentive to preempt small speculators, and therefore to make him wait longer. Therefore, his presence delays the regime collapse, and leads to severe currency overvaluation in the attacked country.

Here we argue that the presence of a large player will be harmful to a small economy in the sense that a large player will employ his market power in a small economy to maximize his profits at the expense of the small economy. His presence prevents the small economy from correcting its overvalued currency in time and, we infer, causes more fluctuations in an economy once the devaluation happens.

4.6 A Note on the Application to the Stock Market

The basic results in our model can be extended to the stock market. Suppose that we introduce a large player into the stock market in the Abreu-Brunnermeier (2003) model. In their model, a continuum of small speculators have to decide the optimal time to sell their gradually overvalued stocks. Similar to the arguments we have developed above, we can introduce a large player who has perfect information about the time when the stock becomes overvalued. Given that small players will take a symmetric trigger strategy $t_i + \tau^{SP}$, the large player will choose his optimal strategy $t_0 + \tau^{LP}$ to maximize his payoffs and vice versa. Here the large player also has the incentive to ride the bubble to maximize his payoffs from the burst of the bubble. So in principle, the presence of the large player can also delay the bursting of the bubble. Further analysis of the model is needed to obtain specific conditions under which the incentive of the large player to ride the bubble will lead to the delay from bursting the bubble.

4.7 Conclusions and Future Research

In this chapter we study the role that large players play in currency attacks in a dynamic currency attack game where speculators have to determine when to attack, based on their incentives both to "ride the overvaluation" and to preempt other speculators. Our main finding is that a large player can accelerate or delay the collapse of a fixed exchange rate regime, depending on which incentive is dominant. More specifically, we find that when the incentive of a large player to "ride the currency overvaluation" dominates the incentive to preempt other speculators, the presence of a large player will delay the collapse of a fixed exchange rate regime. This finding is especially interesting because it differs from the common belief that the presence of large players will facilitate arbitrage and reduce asset mispricing.

One direction in which to extend the current model is to introduce multiple large players and to examine how equilibrium outcomes will change. In addition, in the current model with a large player, we assume the extreme case that the large player has perfect information about the time when the currency overvaluation begins. We can relax this assumption to a more general case where the large player has imperfect information about the time when the currency overvaluation begins.

Chapter 5

Summary and Conclusions

This thesis consists of three essays. Each of them focuses on a certain issue in the field of macroeconomic and financial stability, where both strategic complementarities and incomplete information are present.

In the first essay, systemic bankruptcy of nonfinancial firms is studied in an economy where both investment complementarities and incomplete information exist. I demonstrate that in such an economy, very small uncertainty about economic fundamentals can be magnified through the uncertainty about investment decisions of other firms and can lead to systemic bankruptcy. In this way, I show that due to investment complementarities, an economy can be more vulnerable to systemic bankruptcy, and therefore to financial crises associated with systemic bankruptcy in general. Furthermore, the models established in this essay reveal that in an economy with investment complementarities, systemic bankruptcy tends to arise when economic fundamentals are in a middle range where coordination matters. High financial leverage of firms can significantly increase severity of systemic bankruptcy. Optimistic beliefs of firms and banks can alleviate coordination failure. But meanwhile, systemic bankruptcy tends to be more severe once it happens.

The second essay explores how coordination failure in new technology investment leads to slower capital accumulation and economic growth. Here new technology exhibits two main features: first, investment complementarities exist in new technology investment due to technological externalities. Second, the returns of new technology investment are uncertain due to the unknown feature of new technology. I demonstrate that coordination failure in new technology investment, which is manifested as under-investment in new technology, can lead to slower capital accumulation and economic growth. More interestingly, this essay generates a positive correlation between economic growth and volatility associated with coordination failure in new technology investment.

The third essay examines the role of large players in a dynamic currency attack game. A large player and a continuum of small players have to decide when to attack a gradually overvalued currency in a fixed exchange regime, given that small players have only imperfect information about the time at which the overvaluation begins. The model established in this essay shows that the presence of a large player can accelerate or delay the collapse of the regime, depending on his incentives to preempt small players and to "ride the overvaluation." When the incentive of a large player to preempt small players is dominant, the presence of a large player will accelerate the collapse of the regime. However, if the incentive of a large player to "ride the overvaluation" is dominant, the presence of a large player will delay or at least will not accelerate the collapse of the regime.

My future research will involve three strands. The first strand will be the continued study on currency attacks. First, we plan to extend Chapter 4 by introducing multiple large players to examine how the interactions of them will change the equilibrium outcome. Second, I am working on the optimal policy choice of central banks in a currency attack, based on the observation that when a central bank faces an attack on a fixed exchange rate regime, it can either choose to devalue its currency and keep the fixed regime, or to give up the fixed regime and let its currency float. Third, I plan to use a dynamic model combining herding with strategic complementarities to study the role of large players in currency attacks.

The second strand is about the short-term exchange rate determination. This is an extension from the currency attack literature. I find this topic especially challenging, because most existing theories in this field are not satisfying. I am interested in applying the microstructure approach to foreign exchange markets. My research will focus on the information aspect, that is, how information processing and diffusion among different foreign exchange market participants will determine the short term exchange rate. I plan to combine both theoretic and empirical studies on this issue.

The third strand of my future research concerns macroeconomic stability. My focus is to explore how coordination failure, incomplete information and financial frictions cause macroeconomic instability. I am especially interested in explaining why an economic boom is gradually built up and followed by a sudden crash.

Bibliography

[1] Abreu, Dilip and Markus K. Brunnermeier, 2003, "Bubbles and Crushes," *Econometria*, Vol.71, No.1, 173-204.

[2] Aghion, Pilippe and Peter Howitt, 1998, *Endogenous Growth Theory*, Cambridge, MA: MIT Press.

[3] Allen, Franklin, and Douglas Gale, 1998, "Optimal Financial Crises," *Journal of Finance,* 53, 1245-1283.

[4] Allen, Franklin, and Douglas Gale, 2000, "Financial Contagion," *Journal of Political Economy,* 108(1), 1-33.

[5] Angeletos, George-Marios, and Alessandro Pavan, 2004, "Transparency of Information and Coordination in Economies with Investment Complementarities ," *American Economic Review,* 94(2), 91-98

[6] Bannier, Christina, 2005, "Big Elephants in Small Ponds: Do Large Traders Make Financial Markets More Aggressive? " *Journal of Monetary Economics*, 52(2005), 1517-1531.

[7] Baxter, M. and R. King, 1991, "Productive Externalities and Business Cylces," *Institution for Empirical Macroecoomics,* Federal Reserve Bank of Minneapolis, Discussion Paper No.53.

[8] Bernanke, B. S., M. Gertler, and S. Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework, "in J. Taylor, and M. Woodford (eds), *Handbook of Macroeconomics,* Amsterdam.

[9] Blustein, Paul, 2003, *The Chastening: Inside the Crisis That Rocked the Global Financial System and Humbled the IMF,* New York, PublicAffairs Press.

[10] Borio, Claudio, and Philip Lowe, 2004, "Securing Sustainable Price Stability: Should Credit Come Back from the Wilderness," *BIS Working Papers No 157.*

[11] Bryant, J., 1983, "A Simple Rational Expectation Keynes-Type Model," *Quarterly Journal of Economics,* 97, 525-29.

[12] Caballero, Richardo J., and Richard K. Lyons, 1989, "The Role of External Economies in U.S Manufacturing," *Mimeo,* Columbia.

[13] Caballero, Richardo J., and Richard K. Lyons, 1990, "Internal versus External Economics in European Industry," *European Economic Review,* 34, 805-830.

[14] Calomiris, Charles, and Gary Gorton, 1991, "The origins of Banking Panics: Models, Facts and Bank Regulation,"in *Financial Markets and Financial Crises,* edited by R. Glenn Hubbard, The University of Chicago Press, Chicago, US.

[15] Carlsson, H., and van Damme, 1993, "Global Games and Equilibrium Selection," *Econometrica,* 61, 989-1018.

[16] Castro, Rui, Gian Luca Clement, and Glenn MacDonald, 2005, "Legal Institution, Sectoral Heterogeneity, and Economic Development,"*working paper.*

[17] Chamley, Christophe P., 2004, *Rational Herding: Economic Models of Social Learning,* Cambridge University Press.

[18] Chamley, Christophe P., 2003, "Dynamic Speculative Attacks," *American Economic Review,* 93, 603-621.

[19] Chang, Roberto, and Andres Velasco, 2001, "A Model of Financial Crises in Emerging Markets," *Quarterly Journal of Economics,* 116, 489-517.

[20] Chari, V. V., and Patrick J. Kehoe, 2003, "Hot Money," *Journal of Political Economy,* 111, no.6, 1262-1292.

[21] Chen, Nan-Kuang, 2001, "Bank Net Worth, Asset Prices and Economic Activity," *Journal of Monetary Economics*, 48, 415-436.

[22] Chen, Yehning, 1999, "Banking Panics: The Role of the First-Come, First-Served Rule and Information Externalities," *Journal of Political Economy*, vol. 107, no. 5, 946-968.

[23] Cole, Harold L., and Timothy J. Kehoe, 1996, "A Self-fulfilling Model of Mexico's 1994-1995 Debt Crises," *Journal of International Economics,* 41, 309-330.

[24] Cooper, Russell W., 1999, *Coordination Games-complementarities and macroeconomics*, Cambridge University Press.

[25] Corsetti, Giancarlo, Amil Dasgupta, Stephen Morris and Hyun Song Shin,2004, "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders ," *Review of Economic Studies*, 71(1):87-113.

[26] Corsetti, Giancarlo, Paolo Pesenti and Nouriel Roubini, 2001, "The Role of Large Players in Currency Crises, " *NBER Working Paper*, No. 8303.

[27] Demirguc-Kunt, Asli, and Enrica Detragiache, 1998, "The Determinants of Banking Crises in Developing and Developed Countries," *IMF Staff Papers,* Vo. 45, No.1 (March 1998).

[28] Diamond, Douglas W., and Philip H. Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity, " *Journal of Political Economy*, 91(3), 401-419.

[29] Diamond, Peter A., 1982, "Aggregate Demand Management in Search Equilibrium, " *Journal of Political Economy*, 90(5), 881-894.

[30] Dowrick, Steve (eds.), 1995, *Economic Approaches to Innovation*, Edward Elgar Publishing Limited.

[31] Ennis, Huberto M., Todd Keister, 2003, "Economic Growth, Liquidity and Bank Runs," *Federal Reserve Bank of Richmond Working Paper No. 03-01.*

[32] Fourcans, Andre, and Raphael Franck, 2003, *Currency Crises: A Theoretical and Empirical Perspective*, Edward Elgar Publishing Limited.

[33] Goodhart, Charles, and Gerhard Illing (eds.), 2002, *Financial Crises, Contagion, and the Lender of Last Resort*, Oxford University Press.

[34] Gorton, Gary, 1988, "Banking Panics and Business Cycles," *Oxford Economic Papers,* 40, 751-781.

[35] Hnatkovska, Viktoria, and Norman Loayza, 2005, "Volatility and Growth,"in Jushua Aizenman and Brian Pinto (eds), *Managing Economic Volatility and Crises: A Practitioner's Guide*, Combridge University Press, 65-100.

[36] Houben, Aerdt, Jan Kakes, and Carry Schinasi, 2004, "Toward a Framework for Safeguarding Financial Stability, " *IMF Working Paper.*

[37] The IMF, 1998, *IMF World Outlook.*

[38] Jeanne, Olivier, 2000, *Currency Crises : a Perspective on Recent Theoretical Developments*, Princeton University Press.

[39] Kaminsky, Graciela L., and Carmen M. Reinhart, 1999, "The Twin Crises: The Cause of Banking and Balance-of-Payments Problems," *American Economic Review,* Vol.89(3), 473-500.

[40] Kiyotaki, N., and J. Moore, 1997, "Credit Cycle," *Journal of Political Economy,* 105(2), 211-248.

[41] Kiyotaki, N., and J. Moore, 2002, "Balance-Sheet Contagion," *American Economic Review,* Vol. 92(2), 46-50.

[42] Krugman, Paul, 1979, "A Model of Balance of Payment Crises," *Journal of Money, Credit and Banking*, 11(3), 311-325.

[43] Krugman, Paul, 2000, "Balance Sheets, the Transfer Problem, and Financial Crises,"in Peter Isard, Assaf Razin and Andrew K. Rose (eds), *International Finance and Financial Crises, Essays in Honor of Robert P. Flood Jr.*, Boston, MA: Kluwer Academic Publishers, 31-43.

[44] Krugman, Paul, 2001, "Out of the Loop," *New York Times*, March 4, 2001, Sec. 4, 15.

[45] Langlois, Richard N., 1992, "External Economies and Economic Progress: The Case of the Microcomputer Industry," *The Business History Review,* Vol.66 (1), 1-50.

[46] Lyons, Richard K., 2001, *The Microstructure Approach to Exchange Rates*, The MIT Press.

[47] Mehrling, Perry, 1999, "The Vision of Hyman P. Minsky," *Journal of Economic Behavior and Organization*, Vol.39, 129-158.

[48] Minguez-Afonso, Gara, 2007, "Imperfect Common Knowledge in First-Generation Models of Currency Crises, " *International Journal of Central Banking* , Vol. 3 No. 1. 72-81.

[49] Minsky, Hyman P., 1982, *Can It Happen Again?* M. E. Sharpe Inc.

[50] Morris, Stephen, 1995, Co-operation and Timing, *CARESS Working Paper 95-05.*

[51] Morris, Stephen, and Hyun Song Shin, 1998, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *The American Economic Review,* Vol.88, 587-597.

[52] Morris, Stephen, and Hyun Song Shin, 2000, "Rethinking Multiple Equilibria in Macroeconomic Modelling," *NBER Macroeconomics Annual 2000.*

[53] Morris, Stephen, and Hyun Song Shin, 2002, "Social Value of Public Information," *The American Economic Review,* Vol.92, 1521-1533.

[54] Morris, Stephen, and Hyun Song Shin, 2003, "Global Games: Theory and Applications,"Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky (eds.), *Advances in Economics and Econometrics,* Cambridge University Press, 56-114.

[55] Morris, Stephen, and Hyun Song Shin, 2004, "Coordination Risk and the Price of Debt," *European Economic Review,* Vol.48, Issue 1, 133-153.

[56] Obstfeld, M., 1996, "Models of Currency Crises with Self-fulfilling Features," *European Economic Review*, 40(3-5), 1037-1047.

[57] Rochon, Celine, 2006, "Devaluation without Common Knowledge, " *Journal of International Economics*, 70(2), 470-89.

[58] Romer, Paul A., 1990, "Endogenous Technology Change," *Journal of Political Economy*, 98 (5), Part 2: 71-102.

[59] Schumpeter, Joesph A., 1950, *Capitalism, Socialism and Democracy*, 3rd ed., New York: Harper and Row, Publishers, Inc., 1950.

[60] Vives, Xavier, 2005, "Complementarities and Games: New Developments," *Journal of Economic Literature*, Vol. XLIII (June, 2005),437-479.

Appendices

Appendix A

Appendices for Chapter 2

A.1 The Proof of Strictly Increasing Gross Return Rate from Investing

This appendix gives the proof that firm x^*'s expected gross return rate from investing, $ER(a^I|\rho^*)$, is strictly increasing in ρ^*.

We know that:

$$ER(a^I|\rho^*) = \sqrt{\alpha+\beta}\int_{r^*}^{+\infty}\phi(\sqrt{\alpha+\beta}(r-\rho^*))(me^{r-\Phi(\sqrt{\beta}(\rho^*-r+\frac{\alpha}{\beta}(\rho^*-\bar{r}))}-m+1)dr,$$

where r^* is the unique solution to:

$$r^*-\Phi(\sqrt{\beta}(\rho^*-r^*+\frac{\alpha}{\beta}(\rho^*-\bar{r}))=ln\frac{m-1}{m}.$$

Let $r'=r-\rho^*$. The above function can be transformed into:

$$ER(a^I|\rho^*) = \sqrt{\alpha+\beta}\int_{r'^*}^{+\infty}\phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho^*-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho^*-\bar{r}))}-m+1)dr',$$

where $r^{'*}$ is the unique solution to:

$$r^{'*}+\rho^*-\Phi(\sqrt{\beta}(-r^{'*}+\frac{\alpha}{\beta}(\rho^*-\bar{r}))=ln\frac{m-1}{m}.$$

Now pick up any $\rho^* \in R$ and let $\rho' = \rho^* + \triangle\rho$, where $\triangle\rho$ is a small positive number. Then we get:

$$ER(a^I|\rho') = \sqrt{\alpha+\beta}\int_{r''^*}^{+\infty} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho'-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho'-\bar{r}))} - m + 1)dr',$$

where $r^{''*}$ is the unique solution to:

$$r^{''*} + \rho' - \Phi(\sqrt{\beta}(-r^{''*} + \frac{\alpha}{\beta}(\rho' - \bar{r})) = ln\frac{m-1}{m}.$$

Given $\frac{\alpha^2}{\beta} < 2\pi$, we have $r^{''*} < r^{'*}$. Letting $\triangle r = r^{'*} - r^{''*}$, we can rewrite $NR(a^I|\rho')$ as:

$$ER(a^I|\rho') = \sqrt{\alpha+\beta}\int_{r''^*}^{r^{''*}+\triangle r} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho'-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho'-\bar{r}))} - m + 1)dr'$$
$$+ \sqrt{\alpha+\beta}\int_{r'^*}^{+\infty} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho'-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho'-\bar{r}))} - m + 1)dr'. \quad (A.1)$$

Observe that $\sqrt{\alpha+\beta}\int_{r''^*}^{r^{''*}+\triangle r} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho'-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho'-\bar{r}))} - m + 1)dr' > 0$ since it is the integral of a positive function over a normal distribution. This property holds when $\triangle\rho \to 0$.

Now let us look at the item $\sqrt{\alpha+\beta}\int_{r'^*}^{+\infty} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho'-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho'-\bar{r}))} - m + 1)dr'$. I need to compare it with $\sqrt{\alpha+\beta}\int_{r'^*}^{+\infty} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho^*-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho^*-\bar{r}))} - m + 1)dr'$.

Define a function $f(\rho)$, which is given by

$$f(\rho) = \sqrt{\alpha+\beta}\int_{a}^{+\infty} \phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho-\bar{r}))} - m + 1)dr',$$

where $a \in R$ is a constant.

Then I find that:

$$\frac{\partial f}{\partial \rho} = \sqrt{\alpha+\beta}\int_{a}^{+\infty} \phi(\sqrt{\alpha+\beta}r')me^{r'+\rho-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho-\bar{r})))}(1-\phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho-\bar{r})))\frac{\alpha}{\sqrt{\beta}})dr'.$$

Since $\phi(.)$ is the PDF of a standard normal distribution, $0 < \phi(.) \leq \sqrt{\frac{1}{2\pi}}$. So I get $\frac{\partial f}{\partial \rho} > 0$ if $\frac{\alpha^2}{\beta} < 2\pi$. Then it is straightforward to see that:

$$\sqrt{\alpha+\beta}\int_{r'^*}^{+\infty}\phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho'-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho'-\bar{r}))}-m+1)dr'$$
$$> \sqrt{\alpha+\beta}\int_{r'^*}^{+\infty}\phi(\sqrt{\alpha+\beta}r')(me^{r'+\rho^*-\Phi(\sqrt{\beta}(-r'+\frac{\alpha}{\beta}(\rho^*-\bar{r}))}-m+1)dr'$$

if $\rho' > \rho^*$.

Since the first item in $ER(a^I|\rho')$ is greater than 0 and the second item is greater than $ER(a^I|\rho^*)$, I prove that $ER(a^I|\rho') > ER(a^I|\rho^*)$. Let $\triangle\rho \to 0$, I prove that the objective function is strictly increasing in ρ^* given $\frac{\alpha^2}{\beta} < 2\pi$.

A.2 The Proof of Proposition 1

This appendix shows how the unique trigger strategy equilibrium can be obtained by using the iterated elimination of strictly dominated strategies.

The expected gross return rate from investing of a firm receiving a private signal ρ given that all the others follow the trigger strategy $\hat{\rho}$, which is denoted by $ER(\rho,\hat{\rho})$, is given by

$$ER(\rho,\hat{\rho}) = \sqrt{\alpha+\beta}\int_{r^*}^{+\infty}\phi(\sqrt{\alpha+\beta}(r-\rho))(me^{r-\Phi(\sqrt{\beta}(\hat{\rho}-r+\frac{\alpha}{\beta}(\hat{\rho}-\bar{r}))}-m+1)dr,$$

where r^* is the unique solution to:

$$r^* - \Phi(\sqrt{\beta}(\hat{\rho}-r^*+\frac{\alpha}{\beta}(\hat{\rho}-\bar{r}))) = ln\frac{m-1}{m}.$$

Notice that $ER(\rho,\hat{\rho})$ is increasing in ρ, and decreasing in $\hat{\rho}$.

When ρ is sufficiently low, not investing will be the dominant strategy for a firm, no matter what strategies other firms will take. Let us denote it as $\underline{\rho}_0$. All firms realize this and rule out any strategy for firms to invest below $\underline{\rho}_0$. Then investing cannot be optimal for a firm when it receives a private signal below $\underline{\rho}_1$, which solves

$$ER(\underline{\rho}_1,\underline{\rho}_0) = 1$$

This is because the trigger strategy around $\underline{\rho}_1$ is the best response to the trigger strategy around $\underline{\rho}_0$, and all firms believe that other firms will not invest when their private signals are below $\underline{\rho}_1$. Since the firms' expected return is decreasing in the second argument, this rules out any strategy for firms to invest below $\underline{\rho}_1$. Proceeding this way, I get an increasing sequence:

$$\underline{\rho}_0 < \underline{\rho}_1 < \cdots \underline{\rho}_k < \cdots,$$

where any strategy of investing when $\rho < \underline{\rho}_k$ does not survive k rounds of deletion of dominated strategies. The sequence is increasing because $ER(\cdot, \cdot)$ is increasing in the first argument and decreasing in the second one. The smallest solution $\underline{\rho}$ to the equation $ER(\rho, \rho) = 1$ is the least upper bound of this sequence, and hence its limit. Any strategy of investing below $\underline{\rho}$ cannot survive iterated dominance.

Similarly, I can have an analogous argument beginning with the case that ρ is large enough and the strategy to invest is dominant no matter what strategies other firms will take. If ρ is the largest solution to $ER(\rho, \rho) = 1$, any strategy of not investing when the signal is higher than ρ cannot survive the deletion of dominated strategies.

We have proved that given $\gamma < 2\pi$, there is only a unique solution to $ER(\rho, \rho) = 1$. So the smallest solution is equal to the largest solution. There is only one strategy surviving the iterated elimination of dominated strategies, which is the unique equilibrium strategy in this game.

Q.E.D

A.3 The Proof of Proposition 3

Given the borrowing rate $e^{\bar{r}_b}$ offered by the banks, the private signal of the banks, x_b and its own private signal x_i, firm i will update its belief on the fundamental based on Bayes' rule. Thus we get $(\tilde{r}|x_b, x_i) \sim N(\frac{\alpha\bar{r}+\beta_b x_b+\beta x_i}{\alpha+\beta_b+\beta}, \frac{1}{\alpha+\beta_b+\beta})$, where $\rho_i = \frac{\alpha\bar{r}+\beta_b x_b+\beta x_i}{\alpha+\beta_b+\beta}$ is the mean and $\alpha + \beta_b + \beta$ is the precision.

By an analogous arguments to those given in Proposition 1, there is a unique equilibrium in the firms' game. In equilibrium, a firm will invest if and only if its belief ρ_i is greater than some critical value ρ^*. The proof is basically the same as that in Section 2.3 except for some small modifications.

First, I need to pin down ρ^*. Suppose that firm i is at the trigger point, that is, $\rho_i = \rho^*$, then it must be indifferent about investing or not, which means

$$\begin{aligned} &ER(a^I|x^*(\rho^*)) = \\ &\quad \int_{r^*}^{+\infty} \sqrt{\alpha+\beta_b+\beta}\phi(\sqrt{\alpha+\beta_b+\beta}(r-\rho^*))[me^{r-\Phi(\sqrt{\beta}(x^*-r))} - (m-1)e^{\bar{r}_b}]dr \\ &\quad = 1 = R(a^{NI}|x^*(\rho^*)), \end{aligned} \tag{A.2}$$

where r^* is the unique solution to

$$r^* - \Phi(\sqrt{\beta}(x^* - r^*)) = ln\frac{m-1}{m} + \bar{r_b}.$$

By simplifying the above equation and substituting ρ^* for x^*, we get

$$\int_{r^*}^{+\infty} \sqrt{\alpha+\beta_b+\beta}\phi(\sqrt{\alpha+\beta_b+\beta}(r-\rho^*))(me^{r-l(r,\rho^*)} - (m-1)e^{\bar{r}_b})dr = 1, \tag{A.3}$$

where

$$l(r,\rho^*) = \Phi(\sqrt{\beta}(\rho^* - r + \frac{\alpha}{\beta}(\rho^* - \bar{r}) + \frac{\beta_b}{\beta}(\rho^* - x_b))).$$

With the same method I use in Appendix A.1, I can prove that the above equation is strictly increasing in ρ^*, given $\frac{(\alpha+\beta_b)^2}{\beta} < 2\pi$. Here I omit the proof. Based on the above equation, we find the unique solution of $\rho^*(\bar{r}_b, x_b)$.

Now let us look at the first mover of this game, the banks. The banks fully understand the game among the firms and the equilibrium strategies of the firms. Taking the equilibrium strategies of the firms into consideration, the banks will set the lowest borrowing rate $\bar{r}_b$ that makes the zero expected profit in the banking sector. This is the unique equilibrium and no bank will deviate. By raising the borrowing rate, a bank will deter firms from borrowing. While by lowering the borrowing rate, the bank will make negative profits.

After observing x_b, a bank updates its belief about the fundamental, $\tilde{r}$. The mean of $(\tilde{r}|x_b)$ is

$$\rho_b = \frac{\alpha\bar{r} + \beta_b x_b}{\alpha + \beta_b}, \tag{A.4}$$

and the precision is $\alpha + \beta_b$.

Therefore the expected profits of a bank are given by:

$$E\Pi_b = \int_{-\infty}^{r^*} \sqrt{\alpha+\beta_b}\phi(\sqrt{\alpha+\beta_b}(r-\rho_b))(mw_0 e^{r-l(r,\rho^*)} - (m-1)w_0)(1-l(r,\rho^*))dr + \int_{r^*}^{+\infty} \sqrt{\alpha+\beta_b}\phi(\sqrt{\alpha+\beta_b}(r-\rho_b))(m-1)w_0(e^{\bar{r}_b}-1)(1-l(r,\rho^*))dr.$$

The borrowing rate, $e^{\bar{r}_b^*}$, that a bank will charge is the smallest positive solution to $E\Pi_b = 0$, which can be simplified as

$$\int_{-\infty}^{r^*} \phi(\sqrt{\alpha+\beta_b}(r-\rho_b))(me^{r-l(r,\rho^*)} - (m-1))(1-l(r,\rho^*))dr + \int_{r^*}^{+\infty} \phi(\sqrt{\alpha+\beta_b}(r-\rho_b))(m-1)(e^{\bar{r}_b}-1)(1-l(r,\rho^*))dr = 0. \tag{A.5}$$

Notice that when x_b is low enough, the expected profits of banks from lending will always be negative. Thus the banks will lend if and only if x_b is large enough such that $max\{E\Pi_b(\rho^*, \bar{r}_b)\} \geq 0$.

Q.E.D

Appendix B

The Appendix for Chapter 3

B.1 The Proof of Proposition 4

This appendix proves that there is a unique trigger strategy equilibrium in the model by using the iterated elimination of strictly dominated strategies.

In a typical period t, the expected utility of an agent receiving a private signal ρ, given that all the others follow the trigger strategy $\hat{\rho}$, denoted by $u(\rho, \hat{\rho})$, is given by

$$u(\rho, \hat{\rho}) = E(\theta - l|\rho) + log(\mu) = \rho - \Phi(\sqrt{\gamma}(\hat{\rho} - \bar{r} + \frac{\beta}{\alpha}(\hat{\rho} - \rho))) + log(\mu).$$

Note that $u(\rho, \hat{\rho})$ is increasing in ρ, and decreasing in $\hat{\rho}$ (The subscript t is omitted here since the proof can be applied to any period).

When ρ is sufficiently low, investing in the new technology will be the dominant strategy for an agent, no matter what strategies other agents will take. Let us denote it as $\underline{\rho}_0$. All agents realize this and rule out any strategy for agents to invest in the new technology below $\underline{\rho}_0$. Then investing in the new technology cannot be optimal for an agent receiving a private signal below $\underline{\rho}_1$, which solves

$$u(\underline{\rho}_1, \underline{\rho}_0) = log(r) + log(\mu).$$

This is because the trigger strategy around $\underline{\rho}_1$ is the best response to the trigger strategy around $\underline{\rho}_0$, and all agents believe that other agents will invest in the conventional technology when their private signals are below $\underline{\rho}_1$. Since the agents' expected return is decreasing in the second argument, this rules out any strategy for agents to invest in the new technology below $\underline{\rho}_1$. Proceeding this way, we get an increasing sequence:

$$\underline{\rho}_0 < \underline{\rho}_1 < \cdots \underline{\rho}_k < \cdots,$$

where any strategy of investing in the new technology when $\rho < \underline{\rho}_k$ does not survive k rounds of deletion of strictly dominated strategies. The sequence is increasing because $u(\cdot, \cdot)$ is increasing in the first argument and decreasing in the second one. The smallest solution $\underline{\rho}$ to the equation $u(\rho, \rho) = log(r) + log(\mu)$ is the least upper bound of this sequence. Any strategy of investing in the new technology below $\underline{\rho}$ cannot survive iterated dominance.

Similarly we can have an analogous argument beginning with the case that ρ is large enough and the strategy to invest in the new technology is dominant no matter what strategies other agents will take. If ρ is the largest solution to $u(\rho, \rho) = log(r) + log(\mu)$, any strategy of investing in the conventional strategy when the signal is higher than ρ cannot survive the deletion of strictly dominated strategies.

Given $\gamma < 2\pi$, there is only a unique solution to $u(\rho, \rho) = log(r) + log(\mu)$. So the smallest solution is equal to the largest solution. There is only one strategy surviving the iterated elimination of strictly dominated strategies, which is the unique equilibrium strategy in this game.