THE MAGNETIC FIELDS OF AP STARS FROM HIGH RESOLUTION STOKES IQUV SPECTROPOLARIMETRY

by

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Abstract

In this thesis we describe the acquisition of high resolution time resolved spectropolarimetric observations of 7 (bright and well understood) Ap stars in Stokes $IQUV$ using the ESPaDOnS and Narval spectropolarimeters at the Canada-France-Hawaii Telescope and the 2m Télescope Bernard Lyot at Pic du Midi Observatory. We compare these observations with those obtained a decade earlier using the MuSiCoS spectropolarimeter to confirm consistency with the older data and provide evidence that both ESPaDOnS and Narval perform as expected in all Stokes parameters. We demonstrate that our refined longitudinal magnetic field and linear polarisation measurements for these 7 stars are of much greater quality than was previously obtained with MuSiCoS and that the global magnetic properties of these stars are stable over a long timescale. The ultimate aim of these new data is to provide a basis from which mapping of both the magnetic field and abundance structures can be performed on our target stars.

We then describe magnetic field mapping of the Ap star $\alpha^2$ CVn using these data. This mapping is achieved with the use of tomographic inversion of Doppler-broadened Stokes $IQUV$ profiles of a large variety of spectral lines using the INVERS10 Magnetic Doppler imaging code. We show that not only are the new magnetic field maps of $\alpha^2$ CVn consistent with a previous generation of maps of $\alpha^2$ CVn, but that the
same magnetic field topology can be derived from a variety of atomic line sets. This indicates that the magnetic field we derive for $\alpha^2$ CVn is a realistic representation of the star’s true magnetic topology.

Finally we investigate surface abundance structures for $\alpha^2$ CVn for various chemical elements. We investigate the correlation between the location of these abundance features and the magnetic field of $\alpha^2$ CVn. We will demonstrate that whilst the magnetic field plays a role in the formation of abundance structures, the current theoretical framework does not fully explain what we find from our maps. Ultimately this work motivates future mapping of Ap stars by confirming the reliability of both the instrument and associated data and the mapping technique itself.
Statement of Co-Authorship

The research presented in this thesis was done under the supervision of Gregg Wade and David Hanes at Queen’s University (& RMC) and with additional support and supervision from Oleg Kochukhov at Uppsala University, Sweden. All the work presented here was done by the author (James Silvester) except where explicitly stated otherwise.

Chapter 3 contains a version of a paper published in Monthly Notices of the Royal Astronomical Society as: “Stokes IQUV magnetic Doppler imaging of Ap stars - I. ESPaDOnS and NARVAL observations”, Silvester J., Wade G.A., Kochukhov O., Bagnulo S., Folsom C.P., Hanes D., 2012, MNRAS, 426, 1003 (Oxford University Press). I am the lead author of this paper. I wrote the manuscript, acquired the data, performed all the analysis and produced all the figures. Gregg Wade and co-authors edited the manuscript.

Chapter 4 contains a version of a paper published in Monthly Notices of the Royal Astronomical Society as: “Stokes IQUV magnetic Doppler imaging of Ap stars - II. Next generation magnetic Doppler imaging of α² CVn”, Silvester J., Kochukhov O., Wade G.A., 2014, MNRAS, 440, 182 (Oxford University Press). I am the lead author of this paper. I wrote the manuscript, performed all the analysis and produced most of the figures. Gregg Wade and Oleg Kochukhov edited the manuscript.
Chapter 5 contains a version of a paper accepted for publication in Monthly Notices of the Royal Astronomical Society as: “Stokes IQUV magnetic Doppler imaging of Ap stars - III. Next generation abundance mapping of \( \alpha^2 \) CVn”, Silvester J., Kochukhov O., Wade G.A., 2014, MNRAS, Accepted. I am the lead author of this paper. I wrote the manuscript, performed all the analysis and produced all of the figures. Gregg Wade and Oleg Kochukhov edited the manuscript.
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Table of Contents

Abstract i

Statement of Co-Authorship iii

Acknowledgements v

Table of Contents vi

List of Tables ix

List of Figures x

List of Abbreviations xv

List of Symbols xvi

Chapter 1: Introduction ............ 1

1.1 Magnetism and Abundance Anomalies in Ap/Bp Stars .......... 4

1.2 Understanding the Magnetic Field of Ap Stars ............. 9

1.3 Motivation and aim of this thesis ..................... 19

Chapter 2: Magnetic field and Abundance Mapping ....... 20

2.1 Doppler Imaging .................. 21
List of Tables

3.1 Stars discussed in this paper, along with ancillary data . . . . . . . . . 53
3.2 Reduced $\chi^2$s of harmonic fits to the longitudinal magnetic field phase
curves determined from NARVAL and ESPaDOnS data . . . . . . . . . . . 73
3.3 Log of spectropolarmetric observations . . . . . . . . . . . . . . . . . 111
3.4 Longitudinal magnetic field and net linear polarization measurements 114
4.1 Fundamental parameters used/derived for $\alpha^2$ CVn . . . . . . . . . . 128
4.2 Atomic lines used for the $\alpha^2$ CVn mapping . . . . . . . . . . . . . 131
5.1 Fundamental parameters used/derived for the $\alpha^2$ CVn mapping . . 163
5.2 Atomic lines used for the $\alpha^2$ CVn mapping . . . . . . . . . . . . . 164
5.3 Summary of the result of the derived abundance maps for $\alpha^2$ CVn . 175
List of Figures

1.1 The Hertzsprung-Russell diagram ........................................ 2
1.2 Cartoon illustrating the Oblique Rotator Model ..................... 11
1.3 Magnetic field intensity measured from chromium lines versus rota-
tional phase, for the magnetic Ap star HD 125248 .................... 12
1.4 Figure from Mathys et al. (1997) showing a portion of the spectra of
HD 133792 and of HD 94660 and the Zeeman Pattern ................. 13
1.5 Figure from Landstreet & Mathys (2000) showing observations of HD
187474 compared to the best multipole model found .................. 15
1.6 Comparison between the observed linear polarization Q-U diagram for
49 Cam ................................................................. 17
1.7 Variation of Stokes $I, V, Q$ and $U$ LSD profiles of 53 Cam ........ 18

2.1 How a Stokes $I$ line profile changes as a function of rotation if there
are abundance spots on the stellar surface. ............................. 21
2.2 How the Stokes $V$ polarization signature changes as a function of rotation 23
2.3 Cartoon illustration of two different field vector configurations which
lead to the same Stokes $V$ signature. .................................. 25
2.4 The stellar surface adopted for disk integration. ...................... 28
2.5 Stellar and observer coordinate systems as used in INVERS10 ...... 29
2.6 The key parallel computing steps in INVERS10 shown in a schematic form. .......................................................... 30
2.7 Example of an INVERS10 input file, full description is given in text. .......................................................... 32
2.8 Iron and chromium abundance maps for 17 Comae Berenices ............................................... 35
2.9 A comparison between magnetic field models for 53 Camelopardalis ..................................................... 39
2.10 Magnetic field orientation, field intensity and chemical abundance maps of 53 Cam .................................................. 41

3.1 Phase coverage obtained for the target stars. Symbols denote rotational phases at which observations were obtained. ............... 54
3.2 Signal-to-noise ratio as a function of wavelength for two observations of α² CVn obtained with ESPaDOnS and NARVAL .................. 59
3.3 The mean spectral resolving power as a function of spectral order for both ESPaDOnS and NARVAL ............................................ 60
3.4 The total crosstalk of Stokes V into Stokes Q and U of ESPaDOnS as function of time .................................................. 66
3.5 A comparison between the measured crosstalk in Stokes Q and U and the null spectrum for observations of γ Equ taken with ESPaDOnS . 67
3.6 Comparison between Stokes Q and U profiles of HD 32633 in the 5018 Å line for ESPaDOnS/NARVAL and MuSiCoS ..................... 68
3.7 Example abundance fit for HD 32633 .................................................. 72
3.8 Selected regions of the Stokes IVQU and the N spectra of HD 62140, with strongly contributing atomic lines labeled for reference ...... 74
3.9 Two observations of HD 32633 obtained at close phases, one with NARVAL and one with ESPaDOnS ........................................ 77
3.10 Comparison between LSD profiles from ESPaDOnS/NARVAL observations and MuSiCoS observations ............................................. 78
3.11 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 4778 ................................. 81
3.12 Longitudinal field measurements for HD 4778 ............................. 82
3.13 Net linear polarization (Stokes $Q$ and $U$) measurements for HD 4778 ................................................................. 83
3.14 Longitudinal field measurements for HD 32633 ............................. 86
3.15 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 32633 ................................. 87
3.16 Comparison between two phases of the ESPaDOnS/NARVAL observations convolved to a resolution of 35000 and the MuSiCoS data at a similar phase for HD 32633 ......................................................... 88
3.17 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 40312 ................................. 89
3.18 Longitudinal field measurements for HD 40312 ............................. 90
3.19 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 62140 ................................. 92
3.20 Longitudinal field measurements for HD 62140 ............................. 94
3.21 Longitudinal field measurements for HD 62140 for the null spectrum ................................................................. 95
3.22 Net linear polarization measurements for 49 Cam ............................ 96
3.23 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 71866 ................................. 97
3.24 Longitudinal field measurements for HD 71866 ............................. 99
3.25 Net linear polarization measurements for HD 71866 .......................... 100
3.26 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 112413 .............................. 102
3.27 Longitudinal field measurements for HD 112413 ....................... 103
3.28 Net linear polarization measurements for HD 112413 ................... 103
3.29 Comparison between two phases of the ESPaDOnS/NARVAL and the MuSiCoS data at a similar phase for HD 112413 ......................... 104
3.30 Variation of Stokes $I, Q, U$ and $V$ LSD profiles and Variation of Stokes $I, Q, U$ and $V$ profiles for HD 118022 .............................. 105
3.31 Longitudinal field measurements for HD 118022 ......................... 106
3.32 Net linear polarization measurements for HD 118022 ................... 106
3.33 Comparison between the new ESPaDOnS/NARVAL data with the final model profiles adopted in the mapping of HD 112413 .............. 118

4.1 Comparison of magnetic field radial, meridional and azimuthal components derived from MDI maps computed using strong Fe lines, presented in rectangular projection ......................... 129
4.2 Stokes $IQUV$ spectra comparison for strong iron lines; observed, synthetic from ESPaDOnS and Narval data and from the MuSiCoS map 130
4.3 Rectangular maps of three magnetic field vector components .......... 137
4.4 Comparison between observed chromium lines and synthetic profiles based on the chromium lines ................................................. 142
4.5 Comparison between observed (dots) for the weak and strong iron lines used in magnetic mapping and synthetic spectra for a dipolar geometry and for a dipole + quadrupole geometry .......................... 145
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6</td>
<td>Magnetic map computed using all selected iron lines and chromium</td>
<td>146</td>
</tr>
<tr>
<td></td>
<td>lines from the ESPaDOnS/Narval data</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>Comparison between observed and synthetic Stokes $IQUV$ parameter</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td>spectra of $\alpha^2$ CVn for the final magnetic map</td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>Comparison between observed and synthetic Stokes $IQUV$ parameter</td>
<td>149</td>
</tr>
<tr>
<td></td>
<td>spectra of $\alpha^2$ CVn for the final magnetic map</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>Magnetic maps of $\alpha^2$ CVn (top) computed using INVER$S_{10}$</td>
<td>161</td>
</tr>
<tr>
<td>5.2</td>
<td>Chemical abundance distributions for light and iron peak elements</td>
<td>162</td>
</tr>
<tr>
<td>5.3</td>
<td>Chemical abundance distributions for rare earth elements</td>
<td>163</td>
</tr>
<tr>
<td>5.4</td>
<td>Histogram showing the range of abundance values found in the</td>
<td>167</td>
</tr>
<tr>
<td></td>
<td>reconstructed map for chromium</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>Comparison between observed and synthetic Stokes $IV$ spectra for O,</td>
<td>176</td>
</tr>
<tr>
<td></td>
<td>Si and Cl lines.</td>
<td></td>
</tr>
<tr>
<td>5.6</td>
<td>Comparison between observed and synthetic Stokes $IV$ spectra for Ti,</td>
<td>177</td>
</tr>
<tr>
<td></td>
<td>Cr and Fe lines.</td>
<td></td>
</tr>
<tr>
<td>5.7</td>
<td>Comparison between observed and synthetic Stokes $IV$ spectra for Pr,</td>
<td>178</td>
</tr>
<tr>
<td></td>
<td>Nd and Eu lines.</td>
<td></td>
</tr>
</tbody>
</table>
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFHT</td>
<td>Canada-France-Hawaii Telescope</td>
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<tr>
<td>DI</td>
<td>Doppler Imaging</td>
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<tr>
<td>ESPaDOnS</td>
<td>Echelle SpectroPolarimetric Device for the Observation of Stars</td>
</tr>
<tr>
<td>IDL</td>
<td>Interactive Data Language</td>
</tr>
<tr>
<td>ISM</td>
<td>Interstellar medium</td>
</tr>
<tr>
<td>LSD</td>
<td>Least-squares deconvolution</td>
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<tr>
<td>LTE</td>
<td>Local thermodynamic equilibrium</td>
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<td>MDI</td>
<td>Magnetic Doppler Imaging</td>
</tr>
<tr>
<td>NOT</td>
<td>Nordic Optical Telescope</td>
</tr>
<tr>
<td>ORM</td>
<td>Oblique rotator model</td>
</tr>
<tr>
<td>RMS</td>
<td>Root-mean-square</td>
</tr>
<tr>
<td>RT</td>
<td>Radiative Transfer</td>
</tr>
<tr>
<td>RV</td>
<td>Radial velocity</td>
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<tr>
<td>S/N</td>
<td>Signal-to-noise ratio</td>
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<tr>
<td>TBL</td>
<td>Telescope Bernard Lyot</td>
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<tr>
<td>UV</td>
<td>Ultraviolet</td>
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<td>ZDI</td>
<td>Zeeman Doppler Imaging</td>
</tr>
</tbody>
</table>
List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Magnetic obliquity</td>
</tr>
<tr>
<td>$B_l$</td>
<td>Longitudinal magnetic field</td>
</tr>
<tr>
<td>$B_d$</td>
<td>Dipole polar surface magnetic field strength</td>
</tr>
<tr>
<td>$I$</td>
<td>Unpolarized, natural intensity spectrum</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Continuum intensity</td>
</tr>
<tr>
<td>$i$</td>
<td>Inclination angle</td>
</tr>
<tr>
<td>$L_\odot$</td>
<td>Total emitted solar luminosity ($3.989 \times 10^{33}$ ergs s$^{-1}$)</td>
</tr>
<tr>
<td>$L_\star$</td>
<td>Total emitted stellar luminosity</td>
</tr>
<tr>
<td>$\log(g)$</td>
<td>Log of the surface gravity</td>
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<tr>
<td>$M$</td>
<td>Total stellar mass</td>
</tr>
<tr>
<td>$N$</td>
<td>Diagnostic null spectrum</td>
</tr>
<tr>
<td>$P$</td>
<td>Period</td>
</tr>
<tr>
<td>$Q$</td>
<td>Stokes Q spectrum</td>
</tr>
<tr>
<td>$R_\odot$</td>
<td>Solar radius ($6.955 \times 10^5$ km)</td>
</tr>
<tr>
<td>$R_\star$</td>
<td>Stellar radius</td>
</tr>
<tr>
<td>$T_{\text{eff}}$</td>
<td>Effective temperature</td>
</tr>
<tr>
<td>$U$</td>
<td>Stokes U spectrum</td>
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<tr>
<td>$V$</td>
<td>Stokes V spectrum</td>
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<tr>
<td>$v \sin i$</td>
<td>Projected rotational velocity</td>
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</tbody>
</table>
Chapter 1

Introduction

Every star has a place on the Hertzsprung-Russell diagram (the “HR diagram”; Russell 1914), defined by its luminosity and surface (effective) temperature. On the HR diagram (Fig. 1.1), the line of stars running diagonally from lower right to upper left is known as the main sequence. This corresponds to the core hydrogen fusion phase at which most of a star’s life is spent. The Sun is a main sequence star, located near the middle of the main sequence, with a $T_{\text{eff}} = 5780$ K and a luminosity of 1 solar luminosity ($L_\odot$). A star’s position on the HR diagram is primarily indicative of its initial mass, its age and its chemical composition. Stars are classified based on their spectra, which are strongly indicative of their effective temperature. Stellar spectral type is represented using a series of letters, which are ordered in the following manner “O,B,A,F,G,K,M” (Shapley & Cannon 1921), where O type stars are the hottest stars ($T_{\text{eff}} \approx 40,000$ K) and M stars are the coolest ($T_{\text{eff}}$ under 3,500 K).

A great number of stars show similar photospheric chemical abundances to the Sun. However in the middle of the main sequence a fraction of A and B type stars (around 10 to 20 %) show anomalous atmospheric chemical abundances (e.g Folsom
Figure 1.1: The Hertzsprung-Russell diagram. Image courtesy of Ohio State University (http://www.astronomy.ohio-state.edu)
et al. 2012). The distinctive photospheric abundances observed in these chemically peculiar stars (CP stars) cannot be explained in the context of formation in a chemically anomalous region of the galaxy. Nor can they be explained as the product of internal nuclear evolution or of surface nuclear reactions. Instead, it is thought that these stars have undergone chemical fractionation as a result of competition between gravitationally driven diffusion of elements and radiative levitation of some of these elements. This is combined with the interaction of separation processes with mass loss and with mixing processes such as convection and turbulence (Michaud 1970; Babel 1992; Richer et al. 2000). Whilst atomic diffusion itself is somewhat understood, the competing aforementioned physical processes are not. If fully understood, chemical abundances and atomic diffusion theory can provide sensitive diagnostics of a range of physical processes, such as gravitational settling, internal circulation, and advection by winds, which are difficult or impossible to measure by other means.

A subset of these chemically peculiar stars also have strong globally organised magnetic fields (with strengths up to \( \approx 10 \) kG) which can be diagnosed from ground based instruments. These stars (known as Ap/Bp stars, or simply Ap stars) appear to be the only class of middle main-sequence stars for which, in all cases, an observable surface magnetic field is present (Aurière et al. 2007). Historically, the magnetic field geometries of these chemically peculiar Ap stars were modelled in the context of a simple dipole field. However, with the acquisition of increasingly sophisticated diagnostic data, it has become clear that the large-scale field topologies exhibit important departures from this simple model. Recently high-resolution circular and linear polarisation spectroscopy (e.g. Wade et al. 2000) has even hinted at the presence of strong,
small-scale field structures, which were completely unexpected based on earlier modelling. As the origin of these magnetic fields is not well understood, observations of the detailed structure and long term stability of these magnetic fields would provide formidable constraints on the possible underlying physics for the formation of the magnetic field.

Even with recent advances, the details of both the large-scale and small-scale magnetic structures of Ap stars have not been fully investigated. In addition the influence of the magnetic field on the various chemical transport mechanisms (i.e., diffusion, mixing and mass loss) leading to the formation of atmospheric abundance peculiarities and structures is also poorly understood. Recent advances in tomographic imaging techniques and improvements in instrumentation offer the opportunity to improve our understanding of the magnetic field and the effect it has on transport mechanisms. By mapping the magnetic field and chemical surface structure of Ap stars, it is possible to directly investigate the relationship between the local field vector and local surface chemistry.

1.1 Magnetism and Abundance Anomalies in Ap/Bp Stars

Magnetic fields play a fundamental role in the physics of the atmospheres of a significant fraction of stars on the Hertzsprung-Russell diagram. The magnetic fields of early-type stars have quite different characteristics (and probably a different origin) than those of late-type stars like the sun (e.g. Mestel 2003). In early-type stars, the large-scale surface magnetic field is static on timescales of at least many decades,
and appears to be “frozen” into a rigidly rotating atmosphere. The magnetic field is globally organised, permeating the entire stellar surface, with a relatively high field strength (typically of a few hundreds up to a few tens of thousands of gauss).

The favoured theory for the origin of these magnetic fields is the fossil field hypothesis. This proposes that the magnetic field that exists in the interstellar medium (ISM) is pulled in during the collapse of a gas cloud into a star, and forms part of the structure of the star (Braithwaite & Spruit 2004). These field lines then act to provide the magnetic field of the star. Once the star is formed the magnetic field rests in stable equilibrium and the field decays over a slow timescale (longer than the main sequence lifetime) via diffusive processes. Support is given to the fossil field hypothesis by looking at the progenitors of Ap stars, the Herbig Ae/Be stars. The fossil field hypothesis implies that the field should be present during the pre-main sequence phase. Surveys of Herbig Ae/Be stars (Alecian et al. 2013) showed that $\approx 7\%$ are magnetic, which is consistent with incidence of magnetism in main sequence A stars.

During formation in the context of the fossil field theory, Stepién (2000) suggested these stars will lose angular momentum via an interaction between the magnetic field and the circumstellar environment (magnetic braking). This theory is supported observationally, with the projected rotational velocities ($v \sin i$) of Ap stars being on the order of half that of normal (non-magnetic) A stars (Abt & Morrell 1995).

Fossil fields that are theoretically modelled using a simple magnetic field configurations are unstable over time (Goossens et al. 1981). For a stable configuration, both toroidal and poloidal components are required (Braithwaite & Spruit 2004). It has also been suggested that the surface magnetic flux of an Ap star increases with
Magnetohydrodynamics (MHD) simulations of fossil field dynamics by Braithwaite & Spruit (2004) and Braithwaite & Nordlund (2006) have shown that the total magnetic energy should decrease with time, but the surface field strength is expected to increase until the toroidal component emerges at the surface and decays.

The presence of the magnetic field significantly influences energy and mass transport (e.g., diffusion, convection and weak stellar winds) within the atmosphere of a star, and results in the presence of strong chemical abundance nonuniformities in photospheric layers (e.g., Turcotte, 2003), although to what level the magnetic field plays a role with these processes is still to be constrained. Magnetic fields have been shown to modify diffusion in two ways. First, charged particles are strongly constrained to follow field lines. This can result in the magnetic field modifying the diffusion velocity (Alecian & Stift 2006). Secondly, radiative accelerations are also modified by Zeeman desaturation (magnetically induced spectral line desaturation) and splitting of absorption lines (Alecian & Stift 2006).

The abundance anomalies observed in Ap stars are believed to result principally from atomic diffusion in the atmosphere of the star (Michaud 1970). Michaud (1970) showed that if the atmosphere of a star is sufficiently stable, diffusion under the competing influence of gravity and radiation pressure can occur. This is supported by the fact that Ap stars are slow rotators and atmospheric convection is energetically weak in these stars. Another important requirement for diffusion to generate significant peculiarities is that the timescale for the diffusion process to occur must be much shorter than the lifetime of the star. Again, this was shown to be the case by Michaud (1970), with the vertical diffusion timescale in Ap stars being found to be
on the order of 10,000 years.

Therefore, there are credible scenarios where the creation of chemical peculiarities could arise from diffusion. Under gravity alone, elements would sink below the photosphere. However with the addition of a sufficiently large radiation pressure, the upward force could overcome gravity, causing elements to rise within the photosphere. If conditions in the photosphere then reduced this pressure (e.g. by changing ionisation or excitation conditions), both radiation pressure and gravity could balance, allowing elements to accumulate in the photosphere. In the presence of a stellar wind, some elements could in fact be blown out of the star completely leading to under-abundances. This has been the suggested case for certain elements (Babel, 1992) in stars with weak stellar winds.

For an element to be overabundant in the photosphere, the upward force on an ion must be greater than the gravitational force just below the photosphere. Michaud (1970) showed that the radiation force from bound-free (continuous) and bound-bound (line) transitions are frequently larger than the gravitational force in these regions. These conditions are satisfied in all A type stars, but peculiar abundances are not observed in normal A stars, possibly due in part to their high projected rotational velocities and non-magnetic nature allowing for more efficient chemical mixing.

However, not all elements are overabundant in Ap stars. Michaud (1970) explains that continuum absorption is insufficient to overcome gravity, in the region just below the photosphere, for ions with ionization potentials smaller than 10 eV or larger than 18 eV. However, elements heavier than iron (which have rich line spectra) can be supported primarily by line absorption, and these elements are frequently observed
to be overabundant in CP stars. Lighter elements (with more sparse line spectra) must be supported by continuum absorption, resulting in the observed underabundances of these elements in CP stars. Because of the relatively short timescale for diffusion, this allows for the elements which don’t contribute to the opacity to settle below the regions in which the spectrum is formed and to then be diluted by mixing.

Theoretical modelling such as that of Michaud et al. (1981) has attempted to predict how different chemical elements would be distributed and settle in the photosphere as a result of a magnetic field. Most modelling work has considered the idealised case of a purely dipolar structure. Michaud et al. (1981) predicted that in the absence of turbulence, rare earth elements should be concentrated where the magnetic field is horizontal (magnetic equator) in belt-like structures and iron should be enhanced where the field is vertical (magnetic poles). Alecian & Vauclair (1981) modelled the diffusion of silicon and predicted that enhancements will be seen at the magnetic equator. Recent abundance mapping of Ap stars from observations (such as Nesvacil et al. 2012 and Lüftinger et al. 2010, which will be described in the next chapter) have yet to find clear agreement with these theoretical predictions. For example very few belts have been seen in abundance maps. More recently Alecian & Stift (2010) have suggested that if the magnetic field of a star is not purely dipolar, the proposed belt-like enhancements of some elements at the magnetic equator may be too small to be detectable using current Doppler mapping techniques and would result in the appearance of spots in these regions instead.

In summary, it is generally accepted that the observed abundances of CP stars are a result of complex photospheric separation and mixing processes, including diffusion, rotational mixing, weak winds etc. However the detailed physics of these processes,
including their individual contributions to the observed abundances are still not fully understood, with observations of Ap stars yet to show what is predicted by the diffusion theory for Ap stars. However, with the use of high resolution spectropolarimetry and advanced tomographic techniques, these observations provide a detailed set of magnetic field maps combined with self-consistent detailed abundance maps, which can be used to investigate direct correlations between the structure of the magnetic field and the products of atomic diffusion. Ultimately these results will be used to provide critical direction for diffusion theory for Ap stars.

1.2 Understanding the Magnetic Field of Ap Stars

Magnetism in Ap stars is diagnosed via polarization and the Zeeman effect, with the line of sight or longitudinal field component of the magnetic field found by analysing the circularly polarized light from the star, which in most cases is considerably stronger than the stellar plane polarized signal. As a result, circular polarization is the primary source of magnetic field diagnosis. Observations of Ap stars reveal variations in polarization, photometric intensity and spectral line profiles as a function of rotation, these variations are interpreted in the context of a simple model the oblique rotator model (ORM).

The magnetic field of Ap stars can often be approximated as a roughly dipolar field structure. The axis of symmetry of the field is usually oblique to the rotation axis as shown in Fig. 1.2. As the star rotates, the disk-integrated line-of-sight component of the field referred to as the longitudinal magnetic field \( B_\ell \) varies roughly sinusoidally with time.

The oblique rotator model (first discussed by Stibbs 1950) assumes the magnetic
field is a pure dipole which is centred and usually inclined to the axis of stellar rotation. The magnetic field is “frozen” into the star and as the star rotates different aspects of the field are presented to an observer on the visible hemisphere.

This property can be used to infer the rotational period of the star by making multiple observations over time, and searching for periodicity in the measurements of the observable quantities. This phenomenological model was first used to explain the observed periodic variation of the longitudinal magnetic field of HD 125248 as a function of time (illustrated in Fig. 1.3).

Using the ORM principle, Preston (1967) derived a relation between the surface polar field strength $B_d$ and the observed longitudinal magnetic field $B_\ell$:

$$B_d = B_{\ell}^{\max} \left( \frac{15 + u}{20(3 - u)} \left( \cos \beta \cos i + \sin \beta \sin i \right) \right)^{-1}, \quad (1.1)$$

where $u$ denotes the limb darkening parameter, $i$ and $\beta$ are the inclination of the rotational axis and the magnetic dipole obliquity angle respectively. $B_{\ell}^{\max}$ is the maximum longitudinal field value. The rotational axis inclination and obliquity angles $i$ and $\beta$ are related to the ratio of the extrema of the longitudinal magnetic field by:

$$\frac{B_{\ell}^{\min}}{B_{\ell}^{\max}} = \frac{\cos(\beta + i)}{\cos(\beta - i)}. \quad (1.2)$$

These simple equations which are based on easily observable properties provide a lot of information about an Ap star. If the inclination and the extrema of the longitudinal magnetic field are known, the magnetic obliquity angle and the surface polar field strength can be determined. The limitation with this simple model is that it assumes that the magnetic dipole in perfectly centred in the star.
Figure 1.2: Cartoon illustrating the ORM. The inclination $i$ is defined as the angle between the line-of-sight and the rotation axis and the obliquity angle $\beta$ is defined as the angle between the rotation axis and the magnetic axis.
CHAPTER 1. INTRODUCTION

Figure 1.3: Magnetic field intensity measured from chromium lines versus rotational phase, for the magnetic Ap star HD 125248 taken from Babcock (1951). The field strength varies strongly as a function of phase due to the geometry of the magnetic field. The X-axis in this figure is rotational phase. The solid line is the best fit model. © AAS. Reproduced with permission.

Later Preston (1969) was able to model the magnetic field of HD 215441 (Babcock’s star) using a series of Zeeman spectrograms obtained with the coudé spectrograph on the 120-inch reflector at the Lick observatory. Preston discovered that the field geometry of HD 215441 deviated from the centred dipole assumed by Stibbs (1950) in the ORM. The extremely large magnetic field of this star split its sharp spectral lines in their resolved Zeeman components, thereby yielding information about the magnetic field modulus $\langle B \rangle$ (the surface averaged modulus of the magnetic vector). An example of the Zeeman splitting pattern that can be observed using such techniques is shown in Fig. 1.4 from Mathys et al. (1997) for two stars HD 133792 and HD 94660.

Until this point magnetic field measurements had been restricted to the mean longitudinal field. By combining the two measurements, Preston was able to to derive
Figure 1.4: Figure from Mathys et al. (1997) showing a portion of the spectra of HD 133792 and of HD 94660 containing the lines Cr II λ 6147.1, Fe II λ 6147.7 and Fe II λ 6149.2, and Zeeman patterns of those lines. The σ components are shown below the dashed line and the π components above. Figure reproduced with permission from Astronomy & Astrophysics, © ESO.
new constraints on the field structure, showing that the magnetic field of HD 215441 contained an important co-axial quadrupole component.

Further important work was carried out by Landstreet and Mathys (2000), who performed a large survey of the magnetic geometries of slowly rotating Ap stars. This study not only considered the magnetic field modulus and mean longitudinal field, but in addition determined the mean quadratic field ($B_{mq}$) and cross-over field ($B_{xover}$) from higher-order moments of the Stokes V and I profiles. The mean quadratic field manifests itself through the broadening of unpolarized lines, and can be expressed as $\langle \langle B^2 \rangle + \langle B_z^2 \rangle \rangle^{1/2}$ (Mathys, 1993). The crossover field is the difference of the width of lines between the respective states of circular polarization and can be expressed as $v \sin i \langle xB_z \rangle$ (Mathys, 1993). They found that the magnetic fields of all of the stars could be modelled to first approximation by using a simple low-order axisymmetric multipole model (co-axial dipole, quadrupole and octupole). An example of such a model fit to observations is shown in Fig. 1.5 for the star HD 187474.

Leroy et al. (1994, 1996) were the first to systematically study Ap stars using linear polarization measurements, constraining the transverse component of the magnetic field and providing an unambiguous view of the large-scale geometry of the field. The measurements taken were of broadband linear polarization (or net linear polarization), where the total flux from either Stokes $U$ or $Q$ is integrated to give a total percentage level of polarization in Stokes $U$ or $Q$. For one star, 49 Cam, Leroy et al. (1994) found that the phase-resolved linear polarization observations deviated from the predictions of a centred dipole model. Fig 1.6 shows the comparison between the observed levels of Stokes $U$ and $Q$ polarization in the cool Ap star 49 Cam as a function of rotational phase, compared to the result expected by a simple dipole model. Leroy et al. (1996)
Figure 1.5: Figure from Landstreet & Mathys (2000) showing observations of HD 187474 compared to the best multipole model found. The four windows in the figure show (counter-clockwise from the top left) $B_t$, $B_{mq}$, $B_s$, and $B_{xover}$ as functions of rotational phase. Figure reproduced with permission from Astronomy & Astrophysics, © ESO.
then went on to study a selection of Ap stars, characterizing their magnetic field geometries and departures from simple models. They found that whilst some Ap stars did have multipolar fields, others could be modelled in the context of a simple dipole field. Importantly, they also found that the discrepancies between the complex field and the dipole model stars could not be explained by abundance inhomogeneities alone (Leroy et al., 1996). With this work they established a modified dipolar model, where a trend towards an outwards expansion of the field lines over some parts of the magnetic equator is seen, and showed the potential of linear polarization for diagnosing small-scale structure in the magnetic fields. Wade et al. (1996) modelled HD 192678 based on longitudinal field and field modulus measurements, combined with linear polarization measurements obtained by Leroy et al. (1995). They found that the magnetic field was not a pure dipole and by using the modified dipole of Leroy et al. (1996), they obtained an improved fit between the model and the observations.

Although these studies were limited by the fact that they used broadband polarimetry or low resolution spectroscopy, where few (or none in the case of broadband polarimetry) lines are resolved, it was clear that evidence for departures of the field from a centred dipole on both global and local scales, was becoming convincing. Leroy et al. (1996) commented that high-resolution spectropolarimetry represented the next step in furthering the study of the magnetic field geometry of Ap stars.

Five years after the study of Leroy et al. (1995), Wade et al. (2000) published the first ever set of phase-resolved high resolution spectropolarimetric observations of Ap stars in all 4 Stokes parameters. This data set would be the first step in a new generation of abundance and magnetic field mapping of Ap stars. Fig. 1.7 shows Stokes $I, Q, U$ and $V$ averaged, Least-Squares-Deconvolved (LSD) line profiles (as
Figure 1.6: Comparison between the observed linear polarization $Q$-$U$ diagram for 49 Cam (left) and that predicted by a centred dipole model (right). The numbers next to the observed data points indicate the rotational phase. The axes show the respective levels of relative polarization in Stokes $U$ and $Q$. Taken from Leroy et al. (1994). Figure reproduced with permission from Astronomy & Astrophysics, © ESO.

described in chapter 3) obtained for the Ap star 53 Cam by Wade et al. (2000).

These data were used to create ground breaking maps of Ap star 53 Cam (Kochukhov et al. 2004) and $\alpha^2$ CVn (Kochukhov et al. 2002 and Kochukhov and Wade 2010). These maps revealed that the magnetic topologies departed significantly from the commonly-assumed low-order multipolar geometry, and that the surface abundances were not distributed evenly but localised to regions or structures. These maps represented the first time that the magnetic field topology and chemical abundance structures have been derived simultaneously for a star. The analysis techniques involved in this advance will be described in more detail in the next chapter, where we introduce the science of mapping the magnetic field and abundance structures of Ap stars.
Figure 1.7: Variation of Stokes $I,V,Q$ and $U$ LSD (averaged line) profiles of 53 Cam from Wade et al. (2000). Plotted in velocity space, each column is Stokes $I,V,Q$ and $U$ respectively, the phase of rotation is given above each Stokes $I$ profile. Figure reproduced with permission from MNRAS (Oxford University Press).
1.3 Motivation and aim of this thesis

Two main conclusions can be drawn from the above discussion: First, the previous generation of instruments and techniques have established that the magnetic geometry of Ap stars can no longer be considered a pure dipole and contains much more complex substructure. Secondly, theory can explain the formation of abundance structures in the atmosphere of Ap stars in a basic paradigm, but not to a level at which theoretical predictions of abundance structures agree with current observations. Without the use of detailed abundance maps, simultaneously derived with the magnetic field topology, is it is difficult to provide observational results from which theoretical models can be refined.

The aim of this thesis is to obtain improved time-resolved Stokes $IQUV$ spectropolarimetric observations of Ap stars, which will be used to characterize the global magnetic stability in these stars. This data will also be used to evaluate the instrument performance of the current generation of spectropolarimeters as tools for stellar magnetometry. Finally, the data acquired will be used to simultaneously map the magnetic field and abundance distributions of Ap stars. This has never been performed using spectral lines in both circular and linear polarization, acquired at high resolution and high signal-to-noise ratio. Magnetic Doppler imaging will be used to create new state-of-the-art abundance and magnetic field maps for the star $\alpha^2$ CVn to confront current theory - in particular examining the stability of the large scale and small scale magnetic field structures. Also we will investigate the relationships between field and abundance structures, and the origin of the small scale field structures.
Chapter 2

Magnetic field and Abundance Mapping

The spectra of a star contain a great variety of physical information (such as temperature, surface gravity, chemical abundance, projected rotational velocity) which can be diagnosed using various techniques. In the 1980s and 1990s, surface abundance maps emerged using inversion techniques. Inversion uses tomographic mapping to allow us to identify and characterise surface features on stars, which are unresolved in even the largest telescopes (Khokhlova 1976, Goncharsky et al., 1982, Vogt & Penrod, 1983; Piskunov et al., 1990). The general strategy of mapping stellar surfaces using inversion techniques was later termed Doppler Imaging by Vogt et al. (1987), when they mapped active regions in late-type stars.
2.1 Doppler Imaging

Whilst it is not possible to resolve a stellar sphere (other than the sun) using a single optical telescope, by using a technique called Doppler imaging (DI) it is possible to use the morphology and variability of the spectral line profile to create a 2D map of the stellar surface (see Vogt & Penrod, 1983; Piskunov et al. 1990; Kochukhov et al. 2004). This technique uses the fact that a spectral line is broadened by the intrinsic rotation of a star via the Doppler effect. If one observes a star as it rotates, one would observe that one side of the stellar sphere is rotating towards you and the other side is rotating away from you. This results in a redshift on the receding side and a blueshift on the approaching side which ultimately results in the spectral line being broadened about the line centre (central wavelength). If there was a spot on the surface of the star (either a chemical abundance spot or temperature spot), the spot would exhibit itself in a spectral line Stokes $I$ profile as a distortion.

The longitudinal position of a spot on the stellar surface can be determined by the wavelength at which the profile distortion is located, due to the fact that each
wavelength point corresponds to a discrete longitude on the stellar surface. As the spot traverses the stellar sphere from one limb to the other (due to rotation) the distortion will move from one edge of the profile, through the centre of the profile and then eventually to the other edge. The latitude of a spot on the stellar surface can be determined by locating where the distortion first becomes visible (and disappears) in the intensity profile. Spots at high latitudes will only appear near the centre of the profile, whilst spots on the equator will appear as distortions throughout the whole of the profile. An illustration of how the line profile is distorted due to the existence of stellar surface spots is shown in Fig. 2.1.

This technique has been used for many stellar spectral types (e.g Strassmeier & Bartus 2000, Rice et al. 2011, Lindborg et al. 2014, Xiang et al. 2014), usually only providing information about the surface abundance distribution and not the magnetic field geometry. Stars with strong globally ordered magnetic fields will have distortions in the line profile caused by the Zeeman effect. Therefore in these cases it is important to also have information about the magnetic field when reconstructing the stellar surface features. This is where the Zeeman Doppler imaging (ZDI) or magnetic Doppler imaging (MDI) technique is invaluable.

2.2 Magnetic Doppler Imaging

Magnetic Doppler imaging or Zeeman Doppler imaging (e.g Gondoin 1986, Donati et al. 1989, Semel et al. 1993) is a more recent tomographic technique which combines Doppler imaging techniques and spectropolarimetric information (provided by Stokes \textit{IV} or Stokes \textit{IQUV} observations) to reconstruct not only a 2D map of the surface abundance structures, but also a map of the magnetic field topology of a star.
Figure 2.2: How the Stokes $V$ polarization signature changes as a function of rotation and depending on if there is a azimuthal or radial field component. Taken from Hussain (2004). Figure reproduced with permission from Astronomische Nachrichten.
The location of a magnetic field region on the stellar disk is determined by the wavelength (or velocity) position of the Stokes $V$ profile and how the amplitude of the Stokes $V$ profile scales with the longitudinal magnetic field strength. To obtain a more detailed picture of the magnetic field structure on the surface requires three magnetic field vector components to be constrained; radial, meridional and azimuthal components. Because Stokes $V$ is sensitive only to the line of sight component of the field, the observed polarized signatures behave in a certain way depending on which of the three vector components you are observing. This is used to in part to diagnose the three vector components. This behaviour is shown in Fig. 2.2, which shows how both the radial and azimuthal components vary with rotational phase. In the case of the radial (and meridional) field components the resulting Stokes $V$ polarization signatures will vary in amplitude but maintain the same line profile shape (as seen in Fig. 2.2) as the star rotates. This is because the projected magnetic field vectors always point either towards or away from the observer. The azimuthal component on the other hand will result in a flip in the Stokes $V$ profile as the star rotates, this is because the projected direction of the vector will change as the star rotates.

Because both the radial the meridional vector components have the same profile behaviour as a function of rotation, if there is only information from the line of sight component, it is possible to have more than one magnetic field configuration which gives the same set of polarization signatures in Stokes $V$. This is clearly illustrated in Fig. 2.3, where the two configurations will result in a zero circular polarization, but in which the magnetic field vectors point in completely different directions. By including linear polarization which is sensitive to the transverse (or perpendicular) field component, we can use this information to distinguish between
Figure 2.3: Cartoon illustration of two different field vector configurations which lead to the same Stokes $V$ signature, in this case a zero circular polarization. In the top sphere the field vectors are parallel to the rotational axis; in the bottom sphere the field vectors are parallel to the equatorial plane. Red arrows represent the magnetic vector, dashed arrows indicate the line of sight of the observer.
competing configurations, with the case of the magnetic vectors being parallel to the rotational axis showing a Stokes \textit{QU} polarization signatures, whilst in the other case there will be no polarization signature.

2.2.1 INVERS10

In this work we have utilised the INVERS10 MDI code (Piskunov & Kochukhov (2002) and Kochukhov & Piskunov (2002)) which simultaneously reconstructs the magnetic vector field maps and abundance maps using time-resolved medium to high resolution spectropolarimetric observations. INVERS10 calculates a series of synthetic Stokes line profiles and fits them to the same lines in the observed spectrum. The code then iteratively adjusts the surface distributions (both magnetic and abundance) in the model until the synthetic line profiles and observed line profiles (provided by the time-resolved spectropolarimetric observations) meet a given convergence criterion. The calculations can be started based on an initial homogeneous abundance and magnetic field distribution or based on a previously derived distribution.

To reconstruct the surface magnetic field and abundance structures accurately, it is essential that the observations contain sufficient phase coverage of the star which will be mapped. Ideally a minimum of 15 to 20 phases of observations equally spaced, covering a complete rotation are required. It is also important that the spectral lines chosen for inversion are not heavily blended with other atomic species, in addition the code is not able to account for stratification effects (where a given atomic species settles in two or more distinct atmospheric layers in the star) or non-LTE (Local Thermal Equilibrium) effects, so lines which suffer from these effects are best avoided if possible. In the case of magnetic field mapping it is also required that the Stokes
V and Stokes $QU$ profiles (if available) show polarization signatures.

INVERS10 uses a model stellar sphere made up of $X$ discrete surface elements which are adjusted independently (in this study 695 elements were used). An illustration of such a sphere is shown in Fig. 2.4. The local spectra are calculated for each of these 695 elements by solving the polarized radiative transfer equation (details given below) in a given model atmosphere. For each surface element the projected area, limb angle and Doppler shift due to rotation have to be taken into account at each rotational phase.

To solve the polarized radiative transfer requires an evaluation of the magnetic vector of each of the surface elements. To understand how this is performed, it is important to define the observer and stellar coordinate system. As illustrated in Fig. 2.5 there are quite a few important parameters to account for. The stellar reference frame is such that if an observer was looking at the stellar pole at zero longitude, they would observing the stellar sphere at zero inclination. The line CP describes the rotational axis of which the star rotates about counter-clockwise. Phase angle, longitude and latitude on the stellar sphere are defined by $\varphi$, $\eta$ and $\rho$ respectively. The phase angle counted from the $yz$ plane toward the line PA is which at a latitude of zero. The latitude $\rho$ is positive above the stellar rotational equator and negative below. The point M is an example surface point at which the magnetic vector $\mathbf{B}$ is evaluated in terms of radial, meridional and azimuthal components (denoted by $B_r$, $B_\rho$ and $B_\eta$). The observer frame is defined by the axis $xyz$, the tilt angle of the rotational axis is given by $i$ and the azimuth angle is given by $\Theta$. The azimuth angle defines the orientation of the projection $CP'$ of the rotational axis in the observer $xy$ plane. The code performs the required transformations between the two different
Figure 2.4: The stellar surface adopted for disk integration. The surface is divided into 23 latitude belts, and the total number of surface zones is 695 (not all visible in this illustration). Taken from Piskunov & Kochukhov (2002). Figure reproduced with permission from Astronomy & Astrophysics, © ESO.

reference frames (stellar and observer), details of which are described by Piskunov & Kochukhov (2002).

In the code the radiative transfer is performed using a program called RT solver. RT solver constructs a total opacity matrix and solves the radiative transfer equation for 4 Stokes parameters using a Feature Magnetic RT Integrator. The resulting model spectra are then convolved using a Gaussian profile to account for instrumental broadening and macroturbulence. This process is performed consistently and simultaneously for all four Stokes parameters taking into account the abundance and the magnetic field. INVERS10 is a parallel processing code and is typically run on an 8-CPU desktop machine. The key steps taken in code are shown in schematic form in Fig.2.6.

The mathematical technique for convergence to a solution can be thought of as a
Figure 2.5: Stellar and observer coordinate systems as described in the text. Taken from Piskunov & Kochukhov (2002). Figure reproduced with permission from Astronomy & Astrophysics, © ESO.
Figure 2.6: The key parallel computing steps in INVERS10 shown in a schematic form.
CHAPTER 2. MAGNETIC FIELD AND ABUNDANCE MAPPING

least-squares minimisation problem in the following form (Piskunov and Kochukhov, 2002):

\[ \Psi = \sum_{\phi \lambda} \omega_{\phi \lambda}^I \left[ I_{\phi \lambda}^{\text{comp}}(\vec{B}, Z) - I_{\phi \lambda}^{\text{obs}} \right]^2 \]

\[ + \sum_{\phi \lambda} \omega_{\phi \lambda}^Q \left[ Q_{\phi \lambda}^{\text{comp}}(\vec{B}, Z) - Q_{\phi \lambda}^{\text{obs}} \right]^2 \]

\[ + \sum_{\phi \lambda} \omega_{\phi \lambda}^U \left[ U_{\phi \lambda}^{\text{comp}}(\vec{B}, Z) - U_{\phi \lambda}^{\text{obs}} \right]^2 \]

\[ + \sum_{\phi \lambda} \omega_{\phi \lambda}^V \left[ V_{\phi \lambda}^{\text{comp}}(\vec{B}, Z) - V_{\phi \lambda}^{\text{obs}} \right]^2 \]

\[ + \Lambda \cdot R(\vec{B}, Z) \rightarrow \min \]

The total discrepancy is given by \( \Psi \), which finds its optimum when the sum of all the squared terms (in the square brackets) is minimised. The terms in the form of \( X_{\phi \lambda}^{\text{comp}} \) and \( X_{\phi \lambda}^{\text{obs}} \) are the flux/intensity values at a given wavelength for a given Stokes parameter \( X \) for the computed and the observed spectra respectively. The magnetic field is given by \( \vec{B} \) and the abundance distribution is given by \( Z \). Each of the four Stokes parameters are given respective weights which reflect the quality of the data, they are denoted by the \( \omega \) terms. \( R(\vec{B}, Z) \) is the regularization function, with \( \Lambda \) which is the regularization parameter set by the user to scale the regularization function. The regularization function is used to allow the convergence to a smooth continuous solution if the problem is posed is in such a way that there is no unique solution. In the case of the work in this thesis the regularization function used is Tikhonov regularisation (Tikhonov 1977). Tikhonov regularisation function assists the code in converging to a solution by providing a limit on how smooth or patchy the resulting map can be. Providing a reasonable amount of regularisation is a balance of making
CHAPTER 2. MAGNETIC FIELD AND ABUNDANCE MAPPING

Figure 2.7: Example of an INVERS10 input file, full description is given in text.

sure that the map is not so smooth as to ignore the fitting of discrete features in the observed spectra, whilst making sure the map is not allowed to become so patchy that the code has started to fit the noise. It should be noted that with the existence of all four Stokes parameters (Stokes $IQUV$) there should only be one unique solution to the problem (Piskunov 2005).

To illustrate the input parameters required for running an inversion Fig. 2.7 shows the key components of an INVERS10 input file. These components are labeled as
follows: (1) projected rotational velocity ($v \sin i$), inclination of the rotational axis ($i$) with respect to an observer and gamma which is the azimuth angle of rotational axis with respect to the observer, (2) wavelength window to be fitted (in Å), (3) wavelength sampling (in Å) (4) the flux at each wavelength point for each Stokes parameter required (this continues for each phase of rotation), (5) sets the regularization values, (6) defines what is being mapped (in this case the magnetic field and the iron abundance), (7) the atomic data for lines used in the inversion, (8) macro and micro turbulence (if required) and the model atmosphere to be used, Finally (9) specifies the initial values for each of the 695 surface elements for both the abundance and the 3 field components (radial, meridional and azimuthal).

Once an input file has been run by the code successfully and has resulted in a solution, INVERS10 outputs the data in three distinct files, one which contains the observed spectra”DATA1” (which is written as soon as the code starts), and two other files ”DATA2” and ”DATA3” containing the model Stokes spectra and the resulting map for the last two iterations. Both files are rewritten by the code up until convergence and the code always writes the new output to the oldest of the two files, therefore the most recent iteration can be either in ”DATA2” or ”DATA3”. These files are designed to be read by Interactive Data Language (IDL), which has a strong emphasis on the graphical visualisation of scientific data and image processing. IDL is used to extract the Stokes profiles (observed and model) for direct comparison visually and then it is also used to create graphical representations of the reconstructed magnetic topology and the abundance structures.
CHAPTER 2. MAGNETIC FIELD AND ABUNDANCE MAPPING

2.3 Previous Maps of Ap stars

2.3.1 Abundance Mapping

Doppler imaging is a relatively recent development in stellar astrophysics and as such few Ap stars have been mapped in such a way to allow a comparison between the magnetic field topology and chemical abundance structures. Some notable examples of early Doppler mapping of Ap stars include those of Wehlau et al. (1982) with the mapping of Ap star ε Ursae Majoris and Doppler imaging of 17 Comae Berenices (Rice and Wehlau, 1994).

Mapping of ε Ursae Majoris (Wehlau et al. 1982) used observations taken with the coude spectrograph (R ≈ 40,000) on the 1.2m DAO reflector. They created the abundance maps using the following spectral line profiles; Cr I λ 4922, Cr II λ 4588, Cr II λ 4824, Fe I λ 4923, Fe II λ 4919 and Fe II λ 4920 and using the numerical inversion method of Goncharskii et al. (1977). The authors reported that both iron and chromium were concentrated near the rotational equator of the star, but this study reported no direct comparisons to the magnetic field. The study of 17 Comae Berenices (Rice and Wehlau, 1994) was performed using observations taken with the Canada-France-Hawaii telescope (CFHT) and with the coude spectrograph (R ≈ 40,000). The distributions of iron, chromium, barium, cerium and lanthanum were mapped. They derived the latitude and longitude of the magnetic pole from the longitudinal field measurements from the literature and by using the ORM description of Preston (1971). They found that the iron and chromium abundances varied over the surface of the star by a factor of 100 and were localised at one rotational pole of the stellar sphere and they noted that both the abundance distributions of iron and
chromium were sufficiently complex to not allow a clear comparison with the magnetic field. The resulting maps of chromium and iron from this study are shown in Fig. 2.8. Both these studies clearly showed that abundances were localised to regions of the stellar surface, but they were unable to draw any comparisons between the magnetic field and the chemical surface structures.

Later examples of Ap star abundance mapping with comparison to an assumed magnetic geometry include oxygen abundance structure mapping for the star $\theta$ Aur by Rice et al. (2004), multiple abundance mapping of $\varepsilon$ UMa by Lüftinger et al. (2003) and more recently mapping of HD 3980 (Nesvacil et al. 2012).

The study of $\theta$ Aur by Rice et al. (2004) concentrated on the oxygen abundance distribution. Using observations taken with the 1.2m Dominion Astrophysical observatory telescope and the 9681 camera, they achieved a resolution of $R = 55000$ at
7775 Å with a signal-to-noise ratio of \( \approx 500 \). They found that oxygen abundance was highly depleted at the regions of the magnetic pole and that the abundance was only mildly depleted around the magnetic equator. The authors suggested that this particular distribution could be a result of a quadrupolar component to the magnetic field.

The mapping of \( \varepsilon \) UMa (Lüftinger et al. 2003) used observations taken with the AURELIE spectrograph at the Observatorie de Haute Provence (OHP) with a resolution of around \( R=20,000 \) and a signal-to-noise ratio of \( \approx 200 \). The authors mapped the distributions of calcium, chromium, iron, magnesium, manganese, titanium and strontium. The authors reported that chromium, iron and manganese abundances were enhanced at the magnetic poles and avoided the magnetic equator. Strontium was similar but only enhanced on one of the two magnetic poles. Titanium was depleted where iron and chromium accumulated and was enhanced at the magnetic equator. They found no clear correlation with the magnetic field for the elements magnesium and calcium.

The recent study of HD 3890 (Nesvacil et al. 2012) utilised spectra taken from multiple instruments (\( R \approx 48,000 \) to 115,000, \( S/N= 100 \) to 400) to map the abundance distributions of 13 elements (Li, O, Si, Ca, Cr, Mn, Fe, La, Ce, Pr, Nd, Eu and Gd). The magnetic field topology was not derived in this study. They instead used measurements of the longitudinal field from the literature and the ORM relation of Preston (1967) to obtain a dipolar field strength and magnetic obliquity angle (\( \beta \)). Some elements (Li, O, Mg, Pr, Nd) were reported to be concentrated at the magnetic poles and depleted at the magnetic equator. Ca, Cr and Fe were found to be enhanced around the stellar equator and the remaining elements were found in spots located
between the magnetic pole and the magnetic equator. One important conclusion of the paper was that the abundance features for rare earth elements could not be described by theoretical models.

All of the above studies were limited by the fact that the magnetic field was not derived simultaneously and was only modelled in simple terms for comparison. But taken together they do indicate that the current understanding of how abundance structures are related to the magnetic field is somewhat limited, with no clear abundance threads between the respective studies being found.

2.3.2 Combining Abundances and the Magnetic Field

Landstreet (1988) performed one of the first studies that combined modelling of the magnetic field geometry and abundance distributions, for the Ap star 53 Cam. Up until then most studies concentrated on either abundance mapping or magnetic field modelling independently. Landstreet (1988) argued that one cannot readily determine either the magnetic field geometry or abundance distributions of a star, without knowledge of the other. Some spectral lines are strongly affected by magnetic fields and have complex Zeeman patterns. If the field geometry is not known, the resulting abundance analysis will be influenced. Conversely, if the magnetic field has been determined based on elements that are distributed unevenly across the surface of the star the resulting field geometry may be inaccurate.

The study of Landstreet (1988) included spectra from the coulé spectrograph at CFHT \( (R \approx 40,000) \) and H\( \beta \) Zeeman analyzer observations. Landstreet modelled the abundance patterns by performing a forward calculation, one in which assumes a model abundance distribution and then calculates the resulting local mean intensity
profiles which are then compared with observed line profiles. The model abundance distribution is modified and this process is repeated until agreement is reached between the model and observed profiles. The model visible hemisphere is made up of $3i(i+1)$ integration areas, where usually $i = 4$ (Landstreet 1988), which means the surface grid is made of $\approx 60$ areas.

Landstreet found that for the elements chromium and iron the abundances were greater at the magnetic poles than at the magnetic equator. Titanium, on the other hand, was concentrated in a patch near the negative magnetic pole. These findings offer an early clue that the magnetic field may be a strong contributor to the surface inhomogeneities seen in Ap stars. The real limitation of this data set was that it did not model Stokes $V$ and there were no linear polarization measurements.

The very first set of time-series measurements of rotationally-modulated Zeeman circular and linear polarization resolved within stellar line profiles (all 4 Stokes parameters obtained using the MuSiCoS spectropolarimeter on the 2m Telescope Bernard Lyot at Pic du Midi Observatory) were reported by Wade et al. (2000). This data set was later used with the new Magnetic Doppler Imaging technique (MDI) which allowed the reconstruction of both the magnetic vector field and abundance structure at a whole new level of accuracy and resolution. Described by Piskunov & Kochukhov (2002) and Kochukhov & Piskunov (2002) this technique was used to construct high resolution maps of the surface vector magnetic field of the Ap star $\alpha^2$ CVn at first using only Stokes $IV$ and data from the SOFIN spectrograph (Kochukhov et al. 2002) and then using Stokes $IQUV$ with MuSiCoS data (Kochukhov and Wade 2010). The resulting maps showed convincingly that the magnetic field was not a simple dipole, but showed complex sub-structure, which departs from the simple dipole assumption.
Figure 2.9: A comparison between magnetic field models for 53 Camelopardalis derived by a) Landstreet (1988), b) Bagnulo et al. (2001) and c) Kochukhov et al. (2004). The improvement in resolution between successive data sets and procedures can clearly be seen. Kochukhov et al. (2004). Figure reproduced with permission from Astronomy & Astrophysics, © ESO.
Stokes $IQUV$ mapping was also performed for the Ap star 53 Cam (see Fig. 2.10), (Kochukhov et al. 2004). The maps of 53 Cam also revealed that the magnetic topology departed significantly from the commonly-assumed low-order multipolar geometry and that the surface abundances were not distributed evenly but localised to regions or structures, with a calcium overabundance associated with a weak field strength and positive line of sight magnetic field orientation and titanium localised to the negative part of the magnetic field geometry. A comparison between the magnetic map obtained from the data sets of Landstreet (1988), Bagnulo et al. (2001) and Kochukhov et al. (2004) can be seen in Fig. 2.9, showing the improvement in resolution with each successive study as both modelling and observing techniques became more sophisticated. These maps represent the first time that a star has been studied for magnetic intensity and geometry simultaneously with the local abundance of chemical elements within the atmosphere.

The data used in the 53 Cam and the later $\alpha^2$ CVn maps represented the best data set obtained from several years of MuSiCoS observations, Nevertheless with this data set the Stokes $Q$ and $U$ signatures were only really present in a handful of strong lines, with a S/N of 5 or less. The low signal-to-noise ratio and resolving power achievable with the MuSiCoS instrument led to significant ambiguity in the field reconstruction. Because MDI exploits the indirect resolution of the stellar disc due to stellar rotation, this means that those stars best suited to reconstruction (relatively rapidly-rotating stars) were inaccessible using MuSiCoS. Consequently, only an extremely limited range of stellar properties (rotation, mass, temperature, magnetic field, etc.) which may influence the phenomena of interest could be studied using the MuSiCoS data.

Other recent studies mapping both the magnetic field and abundances include
Figure 2.10: Magnetic field orientation, field intensity and chemical abundance maps of 53 Cam, as recovered using the MDI technique, the top rows show the magnetic field strength and orientation. The remaining rows show abundances structures. Each column represents the following phase of rotation: 0.0, 0.2, 0.4, 0.6, 0.8. These maps reveal unexpected complexity of the field in 53 Cam. Kochukhov et al. (2004). Figure reproduced with permission from Astronomy & Astrophysics, © ESO.
the first magnetic Doppler imaging maps of a rapidly oscillating roAp star HD 24712 using Stokes $IV$ (Lüftinger et al. 2010), and more recently using Stokes $IQUV$ (Rusomarov et al. 2013) and mapping of the chemically peculiar star HD 50773 (Lüftinger et al. 2010a). The original mapping of HD 24712 (Lüftinger et al. 2010) was performed using observations from SOFIN ($R \approx 40,000$, $S/N= 500$ to 600 per pixel) on the Nordic Optical Telescope (NOT) in Stokes $IV$. The magnetic field and abundance structures were derived simultaneously. They found a clear dipolar field and that the abundance structures were populated by one of two groups; one group of elements acclimated near or on the positive magnetic pole and the other group around the equatorial region. Work to create updated maps of this star in Stokes $IQUV$ is ongoing (Rusomarov et al. 2013).

The abundance structures of HD 50773 were also mapped using Stokes $IV$ but the magnetic field was mapped using an averaged Stokes $V$ profile (and not based on individual lines). This work used observations from Narval, ESPaDOnS and SemelPol (with data from CoRoT). The key finding was that HD 50773 had a dipolar structure and that the elements chromium, iron and silicon were enhanced around the magnetic poles. The other elements showed varying correlations; Mg showing no correlation to the magnetic field, Ni was found to be the opposite to Cr and Fe by being underabundant at the magnetic poles and overabundant at the magnetic equator. Ca showed a belt like structure around the magnetic equator and a region of overabundance on the positive pole.

This is not an exhaustive list, but a summary of the most significant Ap star mapping research results to date. While it is clear that the number of such studies is few, but it is worth remembering that because of the time-resolved nature of Doppler
or Magnetic Doppler mapping studies they require a large amount of telescope time to obtain complete phase coverage for each target star. For example to obtain sufficient data to map a magnitude $\approx 6$ star will require close to 72 hours of total observing time in Stokes $IQUV$ with an instrument like ESPaDOnS. More importantly the techniques and instrumentation which allow accurate mapping of both the magnetic field and abundance structures in stars have only recently become available.

2.4 General Goals

From the list of previous Ap star mapping results presented above, it is clear that these studies have been limited by one or more of the following: the fact that the abundance structures were diagnosed without simultaneously reconstructing the magnetic field (which prevents detailed comparisons to be drawn between the field and the abundance structures), the magnetic field was diagnosed without linear polarization (which is required for unambiguous reconstruction of the magnetic field geometry) or maps were reliant on lower resolution, lower signal-to-noise data (which limits the number of lines and elements used in mapping). The work in this thesis looks to overcome these limitations by collecting high resolution phase resolved Stokes $IQUV$ spectra of 7 bright Ap stars. These 7 stars have been selected to cover a range of temperatures (7700 to 12800 K) and masses (1.8 to 3.4 $M_\odot$). This allows a probe of how these stellar parameters may influence the magnetic field topology of these stars. Then we will map the magnetic field and chemical abundances of $\alpha^2$ CVn. These maps are an extremely important test for verifying the methodology for mapping Ap stars. These maps will also allow us to search for correlations between the magnetic field and chemical
abundance structures with unprecedented accuracy.

Such correlations would provide important information on the impact of magnetic fields on atomic diffusion. The magnetic map of $\alpha^2$ CVn will be compared to previously published maps, providing an unprecedented diagnostic of the stability of Ap star magnetic fields on small, as well as large scales. To achieve the goals of this thesis the following steps are performed:

1. Obtain Stokes $IQUV$ spectra with ESPaDOnS and Narval and assess the instrumental performance over multiple observing semesters.

2. Map the magnetic topology of Ap star $\alpha^2$ CVn and confirm that the same derived magnetic topology of $\alpha^2$ CVn is found consistently regardless of line choice.

3. Map the chemical abundance structures of $\alpha^2$ CVn and compare with the magnetic field looking for correlations.

Before it is possible to accurately map a star one must obtain multiple observations of that star at phase intervals which fully sample a complete rotation period. We therefore collected a total of 297 Stokes $IQUV$ phase resolved observations of our 7 target stars using the new-generation ESPaDOnS and Narval spectropolarimeters at the Canada-France-Hawaii Telescope and the 2m Telescope Bernard Lyot at the Pic du Midi Observatory. Using these data, we confirmed the suitability of the ESPaDOnS and Narval spectropolarimeters for use in Stokes $IQUV$ time-resolved studies by investigating the stability in resolution and signal-to-noise ratio in the two instruments over multiple observing semesters (spanning almost 4 years). We
then measured the longitudinal magnetic field and net linear polarization of each observation. By comparing the longitudinal magnetic field and linear polarization measurement with data taken a decade earlier with the MuSiCoS spectropolarimeter, we are able to not only confirm consistency, but also probe the stability of the magnetic field over a decade timescale.

We then use observations of one of our target stars, \( \alpha^2 \) CVn, to perform a detailed analysis using tomographic inversion of Doppler-broadened Stokes \( IQUV \) line profiles (or Zeeman split line profiles, but not just Zeeman profiles). This will be done using a variety of spectral lines and the INVERS10 magnetic Doppler imaging code. From this we recover a detailed surface map of the vector magnetic field and chemical abundances for \( \alpha^2 \) CVn. Because \( \alpha^2 \) CVn has been mapped previously using lower resolution data (taken a decade earlier), we can confirm data consistency via direct comparison. This also provides unique information about the global stability of the magnetic fields of Ap stars, as it will be the first time that an Ap star has been mapped twice using spectropolarimetric data. We can use \( \alpha^2 \) CVn to investigate if different lines result in a consistent magnetic field topology, a critically important consistency check if future magnetic maps are going to be considered reliable. Finally we can start to probe for correlations between the magnetic topology and the chemical abundance structures of \( \alpha^2 \) CVn. This study lays the ground work, and helps refine the methodology, for mapping the remaining Ap stars.

This thesis describes all of the above objectives and the respective results separated into 3 distinct works:

(i) Firstly the collection and analysis of Stokes \( IQUV \) spectra for the selected of Ap
stars (Paper 1).

(ii) An investigation into the magnetic field topology of \( \alpha^2 \text{CVn} \) using the INVERS10 MDI code (Paper 2).

(iii) Finally an investigation of the abundance structures of the Ap star \( \alpha^2 \text{CVn} \) using the INVERS10 (Paper 3).

Please note for completeness each paper is presented in its entirety including abstracts, introduction and references. Therefore some introductory material is duplicated between papers. References for the introductory chapters appear at the end of the thesis.
Chapter 3

Paper I: ESPaDOnS and NARVAL observations

Paper Title\textsuperscript{1}: “Stokes $IQUV$ magnetic Doppler imaging of Ap stars - I. ESPaDOnS and NARVAL Observations”

CHAPTER 3. PAPER I: MDI OF AP STARS I. OBSERVATIONS

3.1 Abstract

In this paper we describe and evaluate new spectral linear polarization observations obtained with the goal of mapping the surfaces of magnetic Ap stars in great detail. One hundred complete or partial Stokes $IQUV$ sequences, corresponding to 297 individual polarized spectra, have been obtained for 7 bright Ap stars using the ESPaDOnS and NARVAL high resolution spectropolarimeters. The targets span a range of mass from approximately 1.8 to 3.4 $M_\odot$, a range of rotation period from 2.56 to 6.80 days, and a range of maximum longitudinal magnetic field strength from 0.3 to over 4 kG. For 3 of the 7 stars, we have obtained dense phase coverage sampling the entire rotational cycle. These datasets are suitable for immediate magnetic and chemical abundance surface mapping using Magnetic Doppler Imaging (MDI). For the 4 remaining stars, partial phase coverage has been obtained, and additional observations will be required in order to map the surfaces of these stars. The median signal-to-noise ratio of the reduced observations is over 700 per 1.8 km s$^{-1}$ pixel. Spectra of all stars show Stokes $V$ Zeeman signatures in essentially all individual lines, and most stars show clear Stokes $QU$ signatures in many individual spectral lines. The observations provide a vastly improved data set compared to previous generations of observations in terms of signal-to-noise ratio, resolving power and measurement uncertainties. Measurement of the longitudinal magnetic field demonstrates that the data are internally consistent within computed uncertainties typically at the 50 to 100$\sigma$ level. Data are also shown to be in excellent agreement with published observations and in qualitative agreement with the predictions of published surface structure models. In addition to providing the foundation for the next generation of surface maps of Ap stars, this study establishes the performance and stability of the ESPaDOnS
and NARVAL high-resolution spectropolarimeters during the period 2006-2010.

### 3.2 Introduction

The classification Ap identifies a (main sequence) A or B type star which displays peculiar chemical abundances, usually combined with an observable magnetic field. Although other classes of chemically peculiar stars exist (e.g. Am stars, Hg-Mn stars, He-weak stars), these stars have been demonstrated to lack strong, organised magnetic fields at their surfaces (e.g. Shorlin et al. 2002, Wade et al. 2006, Makaganiuk et al. 2011). Ap stars appear to be the only class of middle main-sequence stars for which, in all cases, an observable magnetic field is present (Aurière et al. 2007).

Since their discovery by Babcock in 1947, the magnetic fields of Ap stars have been established through observation to have important global dipole components with polar strengths ranging from hundreds to tens of thousands of gauss. The symmetry axis of the dipole component is almost always significantly tilted relative the stellar rotation axis. In addition, Ap stars generally spin much more slowly than non-peculiar stars of similar masses (Stepień 2000), and as they spin they exhibit line profile variations attributed to rotational modulation of patchy, non-axisymmetric lateral and vertical distributions of chemical abundance in their photospheres. The distributions of abundance vary significantly from element to element: some are distributed relatively uniformly, while others show strong contrast; some are distributed in relatively simple patterns, while others show complex distributions. While it is generally accepted that the fundamental mechanism responsible for the chemical peculiarities is microscopic chemical diffusion (as described by Michaud 1970), the origin of chemical patchiness, and the relationship to the magnetic field, is poorly understood.
The earliest studies of the magnetic field geometries of Ap stars interpreted the rotational variations of their longitudinal magnetic fields in the context of Stibbs’ Oblique Rotator Model assuming a simple magnetic dipole field (e.g. Babcock 1947, 1951; Stibbs 1950). However, with the acquisition of increasingly sophisticated diagnostic data, the mean surface field (or mean field modulus), and high-resolution line profiles, it became clear that the large-scale field topologies exhibited important departures from the simple dipolar model (e.g. Preston & Sturch 1967, Preston 1969, 1970, Landstreet 1970, 1988, 1989).

Leroy and collaborators (Landolfi et al. 1993, Leroy et al. 1993, 1994, 1995ab, 1996, Bagnulo et al. 1995, Wade et al. 1996) systematically studied Ap stars using broadband linear polarization measurements and models, constraining the transverse component of the magnetic field. Importantly, they found that differences between the observed linear polarization variations and those predicted by the simple dipole model could not be fully explained by abundance inhomogeneities alone (Leroy et al. 1996). With this work, they established a modified dipolar model with a trend toward an outward expansion of the field lines over some parts of the magnetic equator, and showed the potential of linear polarization for diagnosing small-scale structure of the magnetic fields of Ap stars. Thus, the observations and modeling undertaken during the latter half of the 20th century allowed progress from a simple view of the magnetic fields of Ap stars to a relatively sophisticated picture in which fields were known to show both global-scale and local-scale departures from a simple dipole.

Leroy et al. (1996) commented that high-resolution spectropolarimetry represented the next step in furthering the study of the magnetic field geometry of Ap
stars. Four years later, Wade et al. (2000a) published the first compendium of phase-
resolved high-resolution spectropolarimetric observations of Ap stars in both circular
and linear polarization. Using the MuSiCoS spectropolarimeter, \( R = 35,000 \) Stokes
\( IQUV \) spectra with a median S/N of 300 (per 4.6 km s\(^{-1}\) pixel) were obtained for 14
Ap stars. While the quality of the spectra was sufficiently good to show the shape
and phase variation of all Stokes parameters in mean Least-Squares Deconvolved
(LSD) line profiles, measurement in individual spectral lines was restricted to a few
particularly strong lines, principally those of Fe \( \text{II} \) multiplet 42. Nevertheless, the
Stokes profiles of 53 Cam were used by Bagnulo et al. (2001) and Kochukhov et al.
(2004) to evaluate published magnetic models developed based on less sophisticated
data. Those authors found that models based on so-called ”magnetic observables”
(e.g Bagnulo 2000) led to derivation of surface magnetic field characteristics that were
not consistent with the detailed Stokes profiles, and that both circular and especially
linear polarization profiles were required for realistic reconstruction of the field.

Following these conclusions, Kochukhov et al. (2002) for \( \alpha \) \( 2 \) CVn and Lueftinger
et al. (2010) for HD 24712 employed the new Magnetic Doppler Imaging technique
(MDI), described by Piskunov & Kochukhov (2002) and Kochukhov & Piskunov
(2002), to construct high resolution maps of the surface vector magnetic field maps
using Stokes \( IV \) observations and by preferring a global low-order multipolar field
structure. Maps using linear polarization profiles (Stokes \( Q \) and \( U \)) were made by
Kochukhov et al. (2004) and Kochukhov & Wade (2010) for the Ap stars 53 Cam and
\( \alpha \) \( 2 \) CVn. These maps were distinguished from earlier models in that they were com-
puted directly from the observed polarized line profiles, making no \textit{a priori} assumptions
regarding the large-scale or small-scale topology of the field. The MDI surface
magnetic field maps of both stars revealed that their magnetic topologies depart significantly from low-order multipoles. In particular, both studies concluded that while the global topology of the magnetic field was reasonably smooth, the strength of the field was quite patchy, indicating complex structure on relatively small scales. Simultaneous mapping of the distributions of the surface chemical abundances of several elements was also performed, allowing a comparison between the local field properties and local photospheric chemistry.

It is important to note that the observational material used in the MDI studies of 53 Cam and α² CVn represented the best data sets obtained from several years of MuSiCoS observations. In those spectra the uniquely valuable Stokes $Q$ and $U$ Zeeman signatures were only clearly detectable in 3 strong lines, with a significance (i.e. amplitude divided by error bar) of 5 or less. The relatively low signal-to-noise ratio and resolving power achievable with the MuSiCoS instrument led to some ambiguity in the field reconstruction, and limited the useful sample of stars to those with bright apparent magnitudes, strong fields and sharp lines. Because MDI exploits the indirect resolution of the stellar disc due to stellar rotation, this means that those stars best suited to reconstruction (relatively rapidly-rotating stars, with consequentially weaker Stokes profiles) were inaccessible to MuSiCoS. As a result, only an extremely limited range of stellar properties (rotation, mass, temperature, magnetic field, etc.) which may influence the phenomena of interest could be studied using the MuSiCoS data.

To address outstanding questions surrounding the detailed magnetic structure of Ap stars and the effect of the magnetic field on atmospheric chemical transport processes, we have acquired new higher-resolution and signal-to-noise Stokes $IQUV$
Table 3.1: Stars discussed in this paper, along with ancillary data: HD designation, other name, V magnitude, spectral type, projected rotational velocity, rotational period, radius, inclination, the number of longitudinal field measurements obtained, the median longitudinal field uncertainty obtained, the number of net linear polarization measurements obtained. Projected rotational velocities ($v \sin i$) have been measured in this study, as discussed in the analysis section. The periods were obtained from various sources as described in the Sect 6. Masses are those reported by Kochukhov and Bagnulo (2006) and the average longitudinal magnetic fields are taken from the catalogue of stellar effective magnetic fields (Bychkov et al. 2003), using values obtained from Least-Squares Deconvolved profiles if available. Temperatures are those reported by Kochukhov and Bagnulo (2006) and stellar radii as reported in Leone et al. (2000) for HD 32633, Pasinetti Fracassini et al. (2001) for HD 40312, Kochukhov and Wade (2010) for HD 112413 and Wade (1997) for the remaining stars. Inclinations are taken from Stepien (1989) for HD 32633, Rice, Holmgren and Bohlender (2004) for HD 40312 and Kochukhov and Wade (2010) for HD 112413 and from Leroy et al. (1996) for the remaining stars.

spectra of a small sample of well-studied magnetic Ap stars using the new generation of high-resolution spectropolarimeters. In this paper we describe the observations obtained. We demonstrate the stability of the instrumentation during the 5 years of observation by evaluating the internal and external agreement of the data. We illustrate the quality of the observed Stokes profiles, comparing with MuSiCoS results and demonstrating that they represent a qualitative step forward in our ability to diagnose the magnetic structure of Ap stars.
Figure 3.1: Phase coverage obtained for the target stars. Symbols denote rotational phases at which observations were obtained.
3.3 Targets

Targets selected for this study are bright Ap stars demonstrated to exhibit strong Stokes profiles by Wade et al. (2000a). We attempted to select targets spanning a large range of stellar physical properties as well as field strengths and geometries. Target stars were generally required to have the following characteristics to be suitable for this study:

- Well-determined rotation periods - This study requires phase-resolved time-series observations. All targets must therefore have well-determined, unambiguous rotation periods so that each phase is observed correctly.

- Suitable projected rotational velocity - The projected rotational velocity must be neither too rapid (rapid rotators $>50$ km s$^{-1}$ typically have shallow Stokes profiles which are challenging to detect and interpret) nor too slow (<2 km s$^{-1}$ whilst MDI can be applied to stars with such small rotational velocities, the advantage of Doppler tomography is lost).

- Strong magnetic fields and variability - Because MDI relies on both the shape and variation of line profiles to determine the geographic location of magnetic and chemical features, target stars should display strong and variable Stokes profiles.

Ultimately, 7 targets were selected for monitoring. The target list is shown in Table 3.1.
3.4 Observations obtained with ESPaDOnS & NARVAL

Both the ESPaDOnS and NARVAL instruments consist of a bench mounted cross-dispersed échelle spectrograph, fibre-fed from a Cassegrain-mounted polarimeter unit. These instruments are designed to overcome the limitations encountered with MuSiCoS, with improved resolution ($R = \lambda/\Delta\lambda \simeq 65000$), sensitivity (approximately 15-20% throughput) and wavelength coverage from 369-1048 nm (with gaps at 922.4 to 923.4 nm, 960.8 to 963.6 nm and 1002.6 to 1007.4 nm). The ESPaDOnS (Échelle SpectroPolarimetric Device for the Observation of Stars) spectropolarimeter is installed at the 3.6 m Canada-France-Hawaii Telescope (CFHT), and the NARVAL spectropolarimeter is installed at the 2 m Bernard Lyot telescope at Pic du Midi observatory. ESPaDOnS and NARVAL are essentially identical instruments, with NARVAL constructed based on the experience of ESPaDOnS.

The polarimetric unit (at the Cassegrain focus) allows two orthogonal states of a given polarization (circular or linear) to be recorded throughout the entire spectral range. The polarimeters of ESPaDOnS and NARVAL are of a similar design to the ”Sempol” visitor polarimeter (Donati et al. 2003) used on the AAT (Anglo-Australian Telescope). The polarimeter is split into two parts, the upper part is for guiding and calibration, containing the guiding camera (a commercial FLI MaxCam series CCD camera is used in NARVAL and as of semester 2011A a QSI Imaging CCD camera is used in ESPaDOnS), an atmospheric dispersion corrector and calibration wheel. The lower part contains the Fresnel rhomb retarders which are used to perform the polarimetric analysis. These optics are in 4 different drawers which contain the
following (in order): a half-wave rhomb (consisting of a pair of quarter-wave rhombs), a quarter-wave rhomb, a second half-wave rhomb, and finally a Fabry-Perot wheel on one side and a Wollaston/wedge plate slide on the other side. To enhance the achromaticity of the phase delay, and to keep it at $90 \pm 0.5^\circ$ across the whole optical domain, the rhombs are coated with a thin layer of MgF$_2$, yielding a performance significantly better than achromatic crystalline plates which can vary by about $20^\circ$ from the required quarter-wave retardance. Another advantage of Fresnel rhombs is that they do not produce detectable spectral ripples due to the fact that the fringe spacing is on the order of the pixel size. The half-wave rhombs can rotate about the optical axis by a specified angle. The final component, the Wollaston prism, consisting of two orthogonal calcite prisms that are cemented together, acts as a polarizing beamsplitter. The two beams of light from the beamsplitter are transmitted by some 30 m of optical fibre to the spectrograph. ESPaDOnS includes a fiber agitator, which shakes the optical fiber to remove modal noise that may be present.

The ESPaDOnS spectrograph unit consists of a double set of high-reflectance collimators cut from a single 680 mm parabolic mirror, with a focal length of 1500 mm. The grating is a 79 gr/mm monolithic grating with a dimensions of 200 by 400 mm. The camera lens is a fully dioptic $f/2$ 388 mm focal length lens, with a 210 mm free diameter (7 lenses in 4 blocks, one block being a 220 mm quadruplet). For cross-dispersing, a high dispersion prism made of a train of 2 identical PBL25Y prisms with an $35^\circ$ apex and 220 mm cross section is used. Up until the 2011A semester the detector used with ESPaDOnS was a grade 1 EEV detector with $2K \times 4.5K$ 0.0135 mm square pixels (known as EEV1 at CFHT). This was replaced in 2011A with a new E2V detector (named Olapa at CFHT).
The spectrograph unit is mounted on an optical bench, which is housed in a thermal enclosure found in the inner Coudé room at CFHT.

This configuration yields full spectral coverage of the optical domain (from grating order 61 centred at 372 nm to grating order 22 centred at 1029 nm) in a single exposure. In polarimetric mode this should, in principle, achieve a resolution in excess of 65,000, but due to a charge transfer efficiency issue with the EEV1 CCD detector, the true resolution varies from approximately 68,000 in the blue to 61,000 in the red. The peak throughput of the spectrograph (with CCD detector) is about 40% to 45%, bringing the total instrument peak efficiency to a level of about 15% to 20%.

The configuration of NARVAL is much the same with the exception of an 2.8 arcsec aperture pupil (versus 1.6 arcsec for ESPaDOnS). NARVAL does however benefit from better spectrograph thermal stability (by approximately a factor of 10) than ESPaDOnS, due to the use of a double thermal layer enclosure.

The resolving power and signal-to-noise ratio of both instruments vary with wavelength in a predictable manner, as illustrated in Fig. 3.2, which shows the signal-to-noise ratio as a function of wavelength for four observations of α² CVn (two acquired with ESPaDOnS, and two with NARVAL). The variation in the spectral resolving power $R$ as a function of spectral order for a selection of observing nights is shown in Fig. 3.3. This figure illustrates that the characteristics of the instruments do not vary significantly on the timescales relevant for this project (nights to years). Variability in atmospheric conditions (e.g. seeing) may well be a dominant contributor to the scatter in resolving power.

In terms of observations the following steps occur; the two output beams from
Figure 3.2: Signal-to-noise ratio (per 1.8 km s$^{-1}$ spectral pixel) as a function of wavelength for two observations of $\alpha^2$ CVn obtained with ESPaDOnS (top curves, 02 March 07 (airmass = 1.104) and 13 Jan 09 (airmass = 1.058) both total exposures of 120s) and two obtained with NARVAL (lower curves, 20 Dec 06 (airmass = 1.250) and 11 Jan 09 (airmass = 1.005) both total exposures of 240s). Solid vs dashed lines indicate observations taken on respective nights.
Figure 3.3: The mean spectral resolving power as a function of spectral order for both ESPaDOnS and NARVAL for a selection of observing nights (3 observations from each instrument spanning 12 months). Error bars represent the standard deviation of the resolving power of the various observations within each spectral order.
the Wollaston prism, which have been analysed into the two components of circular polarization (using the quarter-wave rhomb) or linear polarization (using the half-wave rhomb, are then carried by the pair of optical fibres to the spectrograph where two interleaved spectra are formed. The $I$ component of the stellar Stokes vector is formed by adding the two corresponding spectra, while the $V$, $Q$ or $U$ polarization component is obtained from the ratio method as discussed by Bagnulo et al. (2009).

To minimise systematic errors due to small misalignments, differences in transmission, effects of seeing, etc., one complete observation of a star consists of four successive sub-exposures; for the second and third, the waveplate settings are changed so as to exchange the positions of the two analysed spectra on the CCD. This same procedure is used for all polarization spectra, Stokes $V$, $Q$ and $U$.

Calibration of the instrument uses a combination of thorium/argon and thorium/neon lamps, with the lamp calibrations taken at the beginning and at the end of each night for a primary wavelength calibration. Telluric lines are then later used to perform a second wavelength calibration during the reduction process using libre-ESpRIT. Filters are used to minimise the blooming on the chip at the red end of the spectrum. Two tungsten lamps are utilised for the flat fields frames, with one low intensity lamp being used with a red filter and the other lamp being higher intensity and used with a blue filter.

The reduction of observations is carried out at the observatories using the dedicated software package libre-ESpRIT, which yields both the $I$ spectrum and the $V$ circular polarization spectrum and/or $QU$ linear polarization spectra of each star observed. It is important to note that libre-ESpRIT automatically finds and removes
continuum polarization. In this work each reduced spectrum is normalised order-by-order using a FORTRAN code specifically optimised to fit the continuum of these stars.

A diagnostic null spectrum called the $N$ spectrum, computed by combining the four sub-exposures in such a way as to have real polarization cancel out, is also calculated by libre-ESpRIT (again, see Bagnulo et al. 2009 for the definition of the null spectrum). The $N$ spectrum tests the system for spurious polarization signals. The result of the reduction and normalisation procedure are continuum-normalised, one-dimensional spectra in the form of wavelength, $I/I_c$, $V/I_c$, $Q/I_c$, $U/I_c$, two independent normalised $N$ spectra $N_1/I_c$ and $N_2/I_c$, and an error bar (computed by propagating photon uncertainties through the reduction procedure), tabulated pixel-by-pixel.

Weak signatures are visible in LSD $N$ profiles associated with some Stokes $Q$ and $U$ spectra. Experiments conducted in order to diagnose and mitigate polarization crosstalk (see Barrick et al., in preparation) indicate that these signatures are related to this phenomenon. However, data acquired during early NARVAL runs in 2006 exhibit substantially stronger signatures. These signatures are reported by TBL staff to result from problems with the coatings on the $\lambda/2$ rhombs in use in 2006. The rhombs were subsequently replaced, and no similar strong signatures are detected in later observations.

Based on our comparisons of Stokes $Q$ and $U$ profiles obtained with the two instruments during the course of our observing program, it is clear that these $N$ signatures are not diagnostic of any detectable contamination of the associated $Q$ and $U$ profiles.
In this study we have obtained 100 polarimetric sequences corresponding to 297 individual polarized spectra. The observations were initially obtained in classical observing mode, and later in service mode at both telescopes. In total 48 sequences were obtained with ESPaDOnS and 52 with NARVAL. The log of observations is reported in Table 3.3, and the achieved phase coverage of the stellar targets is illustrated in Fig. 3.1. The resulting reduced spectra are illustrated in Fig. 3.8 which demonstrates the quality of the data, with Stokes $VQU$ signatures seen in many individual lines.

3.5 Crosstalk

During the commissioning of ESPaDOnS in 2004 it was found that the instrument exhibited crosstalk between linear polarization and circular polarization (and vice versa). Due to the relatively strong circular polarization in the spectral lines of our targets, contamination of the significantly weaker linearly polarized profiles is potentially a serious problem.

The contributing optical component to this crosstalk identified initially was the collimating triplet within the polarimeter unit. This component was replaced in June 2006, resulting in a reduction of the crosstalk to the 5% level (from an initial level of 10-15 %). After replacing the triplet lens again in October 2008, the crosstalk appeared to have been reduced to 2-3%, but still exhibited strong temporal changes. It was discovered in October 2009 that the atmospheric dispersion corrector (ADC) was an important and previously unrecognised source of crosstalk within the polarimetric unit. The ADC was replaced in the fall of 2009, and since that time the crosstalk has been small and stable, with crosstalk from Stokes $V$ into Stokes $U$ at the 0.5% level, and no measurable crosstalk from Stokes $V$ into Stokes $Q$ (i.e. below $\sim 0.1 - 0.2\%$).
The procedure and results of the investigations of the crosstalk are reported by Barrick et al. (in preparation). The evolution of the ESPaDOnS crosstalk with time, based on the results of Barrick et al., is illustrated in Fig. 3.4.

Less extensive crosstalk monitoring has been performed with NARVAL. Results from September 2009 indicated that with the ADC in place, the crosstalk was 3.1% from Stokes $V$ to Stokes $Q$, and below 0.1-0.2% from Stokes $V$ to Stokes $U$. Without the ADC in place, the crosstalk was reduced to 2.1% in Stokes $Q$, but increased to 1% in Stokes $U$ (illustrating that the NARVAL ADC also introduces crosstalk, but again that it is not the sole source). It is important to note that at the current time, no tests of how this crosstalk changes with time have been made with NARVAL.

Although the crosstalk is now below 1% in ESPaDOnS and probably around 2% in NARVAL, observations for this project have been acquired over the last 4 years and during some of this time the crosstalk may have been higher (and almost certainly was for ESPaDOnS). It is important to understand how crosstalk could affect the Stokes $Q$ and $U$ signatures measured in spectral lines. A series of test was therefore performed to evaluate the importance of this crosstalk.

The first test was to examine the Stokes profiles of the Fe $\text{II} \lambda 6149$ line. This line has a relatively large Landé factor, but in the linear regime of the Zeeman effect it is predicted to be purely circularly polarized as a consequence of the $\sigma$ and $\pi$ components of this line having the same strength and identical splitting. In this case we would interpret any signal in Stokes $Q$ and $U$ as due to crosstalk from Stokes $V$. We have examined $\lambda 6149$ in our spectra, as well as spectra discussed by Barrick et al. (in preparation). In no case do we observe any significant signal in this line.

Then, we carefully examined one of the crosstalk diagnostic observations of the
cool magnetic star $\gamma$ Equ obtained during CFHT engineering time. As described by Barrick et al. (in preparation), on-sky crosstalk diagnosis employs observations of slowly-rotating magnetic Ap stars observed in all Stokes parameters at two positions of the CFHT’s Cassegrain bonnette to unambiguously measure the crosstalk into Stokes $Q$ and Stokes $U$. The diagnostic observation of $\gamma$ Equ (obtained in July 2009, with a peak S/N of over 1000) yielded relatively high crosstalk levels of 2.3% in Stokes $Q$ and 5.1% in Stokes $U$. By comparing the observations with and without crosstalk, it was found that the crosstalk contributions to Stokes $Q$ is within the noise, whilst the crosstalk contributions to Stokes $U$ appears to be slightly above the noise, as illustrated by Fig. 3.5. It is important to note that the removal of the crosstalk using techniques as described in Barrick et al. (in preparation) requires the acquisition of a second series of observations following a rotation of the Cassegrain bonnette, a procedure which is not standard practice during regular observations.

$\gamma$ Equ is an extremely sharp lined star, which means the crosstalk contribution will have a greater effect than it would in the broader lined stars studied in this paper. Nevertheless, the potential influence of the crosstalk on MDI will be investigated and discussed in a future paper (Silvester et al. in preparation).

These comparisons demonstrate that the contribution of crosstalk to the Stokes profiles is below, or at most just above, the level of the noise of the best-quality observations of Ap stars acquired with these instruments.
Figure 3.4: The total crosstalk of Stokes $V$ into Stokes $Q$ and $U$ of ESPaDOnS as function of time, as reported by Barrick et al. (in preparation). The dashed boxes indicate periods during which observations were obtained as part of this investigation.
Figure 3.5: A comparison between the measured crosstalk in Stokes $Q$ and $U$ and the null spectrum for observations of $\gamma$ Equ taken with ESPaDOnS (16th July 2009) (top row), with the actual observations shown below (lower frame). In the strong, magnetically sensitive line Fe II 5018 line, the crosstalk in Stokes $Q$ is below the noise, where as the crosstalk in Stokes $U$ is slightly above the noise.
Figure 3.6: Comparison between Stokes $Q$ and $U$ profiles of HD 32633 in the 5018 Å line for ESPaDOnS/NARVAL on the left and MuSiCoS on the right. Rotation phase for each observation are indicated.
3.6 Longitudinal magnetic field and net linear polarization

To examine the self-consistency of the new polarization spectra, as well as to evaluate their consistency with published magnetic data for our targets, we have measured the mean longitudinal magnetic field and net linear polarization using Least-Squares Deconvolution (LSD). LSD (Donati et al. 1997 and Kochukhov et al. 2010) is a multiline analysis method that produces mean Stokes $I$ and $V$ profiles using essentially all metallic lines in the stellar spectrum. It assumes that the observed spectrum can be represented as the convolution of a single mean line profile with an underlying spectrum of unbroadened metal and helium lines of appropriate wavelength, depth and Landé factor (the “line mask” computed using spectrum synthesis; e.g. Wade et al. 2000a).

The LSD model allows the computation of single, average Stokes $I$ and $V$ line profiles, usually characterised by a signal-to-noise ratio significantly higher than that of individual spectral lines, scaling roughly as the square root of the number of lines used.

The projected rotational velocity $v \sin i$ of each star was derived by fitting a selection of spectral lines (typically in the 4500 Å region) with a synthesised spectrum created using the Synth3 spectrum synthesis code (Kochukhov, 2007). Each line in the selection had $v \sin i$ determined by using a $\chi^2$ fitting function which is included as part of the Binmag spectral visualization tool. The resulting mean (and standard deviation) values are reported in Table 3.1. Line masks for this study were compiled...
using Vienna Atomic Line Database (VALD, Kupka et al. 1999) “extract stellar” requests, with effective temperatures (adopted based on the literature) of each target. It should be noted when creating each line mask, a constant log g of 4.0 was used for all stars. Input chemical abundances were determined by a rough abundance analysis, based typically on lines around the 4500 Å region. The abundance analysis was performed by using the Synth3 spectrum synthesis code (Kochukhov 2007) with ATLAS9 solar abundance model atmospheres used to create synthetic spectra that were compared directly to the observations. The input abundances were then adjusted in the synthetic spectrum until a reasonable agreement was found with the observed spectrum. A comparison between of two synthetic spectra, one computed using solar abundances and one using the final determined abundances, is shown in Fig 3.7 for HD 32633. The final abundances were then adopted in the line mask creation. By doing so we ensure that any uncertainties caused by the mask when performing the LSD analysis are minimal.

As discussed by Shorlin et al. (2002), the LSD S/N is only weakly sensitive to the line-depth cutoff employed to populate the mask. Following their results, we have chosen to employ a line-depth cutoff equal to 10% of the continuum. Imposing such a cutoff has a related advantage: because weaker lines are less likely to have published experimental Landé factors (and to generally have more poorly-determined atomic data), we pre-filter our line list to (statistically) exclude those lines with the poorest data. In addition Balmer lines are removed from the mask and the mask is restricted to the ESPaDOnS/NARVAL spectral range. Application of LSD to the data yields a set of mean profiles (Stokes I, Stokes V and N) for each reduced spectrum.

The mean longitudinal magnetic field \( B_\ell \) was computed from each LSD profile set.
This quantity was evaluated by computing the first-order moment of the Stokes $V$ profile in velocity according to:

$$B_t = -2.14 \times 10^{11} \frac{\int (v - v_O) V(v) dv}{\lambda g e \int [1 - I(v)] dv} \quad (3.1)$$

(Mathys 1989, Donati et al. 1997, Wade et al. 2000a) where $v_O$ is the centre-of-gravity of the Stokes $V$ profile, $g$ is the integrated mean Landé factor and $\lambda$ is the weighted mean wavelength of all the lines included in the mask. LSD profiles were locally re-normalised to a continuum level of 1.0 before evaluation of Eq. (3.1). Uncertainties associated with $B_t$ were computed by propagating the formal uncertainties of each LSD spectral pixel through Eq. (3.1). LSD profiles are extracted for each star uniformly weighted to the same landé factor, line depth and wavelength.

To determine the net linear polarization (see e.g. Wade et al 2000a) the LSD Stokes $Q$ or $U$ profile was integrated to compute the normalized equivalent width of the line polarization using:

$$\frac{Q}{I} = \frac{\int Q(v) dv}{\int [1 - I(v)] dv}. \quad (3.2)$$

It was found that both net linear polarization and longitudinal field measurements were sensitive to the integration range used to calculate Eqs. (3.1) and (3.2). Integration ranges were carefully chosen to include the entire line profile, while avoiding including excess continuum outside of the profile (which contributes only noise). This was accomplished initially by selecting limits based on the apparent extent of the wings of the Stokes $I$ profile. We then evaluated visually if any significant polarized flux was located outside of the limits, adjusting the integration limits as necessary.

As was the case with Stokes $V$, LSD Stokes $I$ profiles associated with Stokes $Q$
Figure 3.7: Example abundance fit for HD 32633, with a synthetic spectrum calculation of solar abundance show with a dashed line, and the spectrum computed with the inferred abundances used in the LSD mask shown with a dot-dashed line.

and $U$ profiles were locally re-normalised to a continuum level of 1.0 before evaluation of Eq. (3.2).

Longitudinal field measurements are phased according to the ephemerides given in the respective sections. We verified that all adopted periods were sufficiently precise that no significant relative phase uncertainties exist. A harmonic curve was fitted by least-squares to the phased longitudinal field data. The degree of the harmonic function which yielded the lowest reduced $\chi^2$, while still providing a significant and systematic improvement to the fit, was chose as the “best” fit. The results of the fit are shown in Table 3.2. It should be noted that whilst the fits to the longitudinal field
Table 3.2: Reduced $\chi^2$s of harmonic fits to the longitudinal magnetic field phase curves determined from NARVAL and ESPaDOnS data

<table>
<thead>
<tr>
<th>Star</th>
<th>Degree</th>
<th>Stokes V Fit $\chi^2$</th>
<th>Null N $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 4778</td>
<td>2</td>
<td>1.41</td>
<td>2.68</td>
</tr>
<tr>
<td>HD 32633</td>
<td>3</td>
<td>1.95</td>
<td>5.42</td>
</tr>
<tr>
<td>HD 40312</td>
<td>2</td>
<td>1.96</td>
<td>0.51</td>
</tr>
<tr>
<td>HD 62140</td>
<td>3</td>
<td>1.29</td>
<td>4.78</td>
</tr>
<tr>
<td>HD 71866</td>
<td>4</td>
<td>1.02</td>
<td>3.70</td>
</tr>
<tr>
<td>HD 112413</td>
<td>3</td>
<td>0.83</td>
<td>1.06</td>
</tr>
<tr>
<td>HD 118022</td>
<td>1</td>
<td>1.92</td>
<td>1.12</td>
</tr>
</tbody>
</table>

variations help compare different data sets, quantify the dispersion of the data and verify the accuracy of the error bars, their parameters and degree do not necessarily have any easily-interpretable physical meaning.

To allow a consistent comparison between the MuSiCoS and ESPaDOnS/NARVAL data, the normalisation of the ESPaDOnS/NARVAL data had to be performed carefully. Care also had to be taken with the integration limits when determining both the longitudinal field and net-linear polarization.

When we compared the new longitudinal field measurements with those computed from MuSiCoS spectra, immediate agreement was found between the new measurements and those of Wade et al. (2000b), except in the case of HD 71866 and $\alpha^2$ CVn which showed a slight discrepancy. By re-computing the Stokes V MuSiCoS LSD profiles for these stars with the new masks, the observations were brought into agreement. It is possible that if both HD 71866 and $\alpha^2$ CVn have sufficiently peculiar abundances, a general Ap star line mask (with arbitrarily enhanced and depleted abundances of particular elements, as used in the original analysis of Wade et al.
Figure 3.8: Selected regions of the Stokes $IVQU$ and the N spectra of HD 62140, with strongly contributing atomic lines labeled for reference.
2000b) may result in an underestimation of the longitudinal field. Because each mask used in this study is “tailor-made” to the average chemical abundances of the respective star, we are less susceptible to this problem and thus measure slightly increased longitudinal field values.

An example of the comparison between the new measurements, and those corresponding to the two sets of MuSiCoS LSD profiles, are shown in Section 3.6 (Fig. 27) for $\alpha^2$ CVn. This figure demonstrates that overall the agreement between the new measurements and the re-computed MuSiCoS measurements are acceptable. Using the new masks for the other stars makes little difference to the longitudinal field measurements, so the original measurements were kept for comparison. In addition good agreement can be seen between LSD profiles obtained with MuSiCoS and ESPaDOnS/NARVAL observations. Fig. 3.10 shows that LSD profiles both from MuSiCoS and the current observations are consistent for an identical phase of observation in the case of HD 32633 and HD 112413, with the new data showing much reduced noise levels.

It was quickly seen that both the ESPaDOnS and NARVAL data were consistent with one another. An excellent example of this can be seen in Fig. 3.9, where two observations of HD 32633 are shown, one at a phase of 0.689 (taken with NARVAL) and one with a phase of 0.700 (taken with ESPaDOnS). The detailed agreement between the Stokes profiles indicates that both instruments appear to be performing extremely consistently with one another. Another test of consistency is described in Section 6.2 and 6.6, where for certain phases, the ESPaDOnS/NARVAL data for $\alpha^2$ CVn and HD 32633 have been convolved to the same approximate resolution as MuSiCoS (R=35000) and then compared to MuSiCoS observations of a similar phase.
Very good agreement can be seen in individual lines.

As proposed by Landi Degl’Innocenti et al. (1982), assumed by Landolfi et al. (1993) and Leroy et al. (1993, 1995), and finally confirmed by Wade et al. (2000b), the phenomenon of broadband or net linear polarization results from differential saturation of $\pi$ and $\sigma$ Zeeman components of individual spectral lines. Broadband polarization measurements, such as those reported by Leroy (1995), effectively average the signal from the net polarized spectral lines with regions of continuum (which may be unpolarized, or polarized due to e.g. interstellar polarization). In contrast, net polarization measurements obtained using LSD measure only the net polarization in lines. Therefore the measurements are not equivalent. In particular, Wade et al. (2000b) found that it was necessary to arbitrarily scale and shift the LSD measurements relative to the broadband measurements in order to bring them into agreement. Here we adopt a similar procedure.

Table 3.2 also shows the reduced $\chi^2$ of longitudinal field measurements taken from the null spectra. A value close to 1 indicates that the null spectra and their error bars are consistent with zero longitudinal field and that the instrument is performing as expected. We see for HD 4778, HD 32633, 49 Cam and HD71866, this is not the case. These high values are a result of a handful of non-zero null measurements obtained during the December 2006 observations with NARVAL and very small number with obtained ESPaDOnS, in which the null LSD profiles contained weak signatures (as discussed in Section 3).

In addition, for these stars, we find that the error bars on the Stokes $V$ longitudinal field measurements are larger than the null longitudinal field error bars, typically by a factor of 3-4. This is significant, because it tells us that we begin to see that the Stokes
Figure 3.9: Two observations of HD 32633 obtained at close phases, one with NARVAL (Phase 0.689, taken 17 Dec 2006) and one with ESPaDOnS (Phase 0.700, taken 26 Jan 2008). The signal-to-noise ratio is $\approx 400$ and $\approx 700$ respectively. Excellent agreement between the signatures can been seen, illustrating the consistency between the two instruments.
Figure 3.10: Comparison between LSD profiles from ESPaDOnS/NARVAL observations and MuSiCoS observations taken at near identical phases for HD 32633 and HD 112413 (MuSiCoS data dashed lines, ESPaDOnS/NARVAL solid lines). Good agreement can be seen.
V longitudinal field measurements are no longer limited by photon noise. Rather, the larger error bars for the longitudinal field in Stokes V are telling us that the LSD model is failing to fit the V spectrum within the Libre-ESpRIT computed error bars, likely due to blends and the limitation of the weak-field approximation. This only happens when a significant number of individual lines show very strong Stokes V signatures, and (as demonstrated by our ability to fit the longitudinal field curves within the error bars) is fully compensated for in the error bar calculation for Stokes V. As discussed by Wade et al. (2000b) this compensation is achieved by scaling the photon noise statistical error bars of LSD profiles by a factor equal to the square root of the reduced $\chi^2$. Wade et al. (2000b) describes that this scaling is almost always required for Stokes V in Ap stars. If we recompute the reduced $\chi^2$ calculation again for the null spectra, but use the Stokes V error bars instead of the null error bars, the resulting reduced $\chi^2$ is consistent with a zero longitudinal field (for example in the case of HD 32633, the reduced $\chi^2$ goes from 5.42 to 0.36).

### 3.7 Results for Individual Stars

In the following section, the results for each star will be discussed, including figures showing both the Stokes profile variation in selected individual lines and in the LSD profiles. Also the variation of the longitudinal field and in some cases the variation in net linear polarization as a function of phase will be shown for each target. Results are compared with those of Wade et al. (2000a) and Leroy (1995) where possible. In the cases where the net linear polarization was close to or consistent with zero at all phases (such as HD 32633 and HD 40312), these plots have been omitted.
3.7.1 HD 4778

HD 4778 is the only star in our sample that was not studied by Wade et al. (2000a & 2000b). It was classified by Renson and Manfroid (2009) as an A1CrSrEu star. We obtained Stokes $V$, $Q$ and $U$ spectra for 7 rotational phases (although no Stokes $Q$ observation was obtained at phase 0.366). Using the method described in Sect. 5, the projected rotational velocity ($v \sin i$) was determined to be $36 \pm 2$ km s$^{-1}$ which is a little larger than the Abt & Morrell (1995) value of $33$ km s$^{-1}$. The data have been phased according to the ephemeris reported by Leone et al. (2000):

$$\text{JD} = 2446674.006 + 2.56171 \cdot E.$$  \hspace{1cm} (3.3)

The individual spectral lines show strong signatures in Stokes $V$, and although variation in Stokes $Q$ and $U$ can be seen in individual lines in this star (an example is shown for the Fe $\text{II}$ $\lambda$ 5018 in Fig. 3.11) the signatures in Stokes $Q$ and $U$ are relatively weak. Signatures are more prominent in the LSD profiles shown in Fig. 3.11. Stokes $I$ shows variability in the Fe $\text{II}$ 4923, 5169 and 5018 Å lines (as shown in Fig. 3.11).

Both $B_\ell$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. The average uncertainty on the longitudinal field measurement is 20 gauss. The LSD profiles are illustrated in Fig. 3.11, and the longitudinal field curve shown in Fig. 3.12. The 2nd order Fourier fit to this curve, using only the new measurements gives a reduced $\chi^2$ of 1.41. Also included in Fig. 3.12 are measurements reported by Bohlender (1989) for comparison. The slight apparent phase shift could be due to a cumulative error in phase over an almost 30 year span caused by the period uncertainty.
Figure 3.11: Top: Variation of Stokes $I, Q, U$ and $V$ LSD profiles for HD 4778. Rotational phase increases from top to bottom, represented on the vertical axis. A scaling factor of 25 was used for both the Stokes $Q$ and $U$ observations and a factor of 5 was used for Stokes $V$. Bottom: Variation of Stokes $I, Q, U$ and $V$ profiles for HD 4778 in the Fe II $\lambda 5018$ line (scaled by a factor of 20 for Stokes $Q$ and $U$ and a factor of 4 for Stokes $V$).
Figure 3.12: Longitudinal field measurements for HD 4778 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Bohlender (1989). A 2nd order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.

Leroy et al. (1995) obtained broadband linear polarization measurements of HD 4778; their values are compared with the measurements obtained in this work in Fig. 3.13. Reasonable agreement in Stokes $Q$ and $U$ can be seen between the two epochs of data. The ESPaDOnS/NARVAL data had the mean subtracted and then were scaled by 0.04 to bring them into agreement with those of Leroy et al. (1995).
Figure 3.13: Net linear polarization (Stokes $Q$ and $U$) measurements for HD 4778 obtained with ESPaDOnS/NARVAL (filled circles, scaled 0.04), and those obtained by Leroy (1995) (filled diamonds). The improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements.

### 3.7.2 HD 32633

HD 32633 is a fairly broad-lined B9p star with a strong, non-sinusoidal longitudinal magnetic field variation. We obtained Stokes $V$, $Q$ and $U$ profiles for 20 rotational phases (Stokes $Q$ and $U$ observations are missing at phase 0.086). Using the method described in Sect. 5, the projected rotational velocity ($v \sin i$) was determined to be $19 \pm 2$ km s$^{-1}$ which agrees within uncertainty with the value adopted by Wade et al. (2000b).

The data have been phased according to the ephemeris of Adelman (1997b):

$$\text{JD} = 2437635.200 + 6.43000 \cdot E.$$  \hfill (3.4)

Very strong and variable Stokes $Q$ and $U$ polarization signatures can be seen in
individual lines in the spectra of this star, an example of which is shown for the Fe II 5018 Å line in Fig. 3.15. Interestingly, the shape of the Stokes $Q$ and $U$ profiles change relatively little as a function of phase (in individual lines, as well as LSD profiles of Fig. 3.15). This could be an indication that we are "seeing" a limited part of the transverse geometry of the field due to the overall geometry of the star. It should be noted in Fig. 15 that the shapes of the Stokes $Q$ and $U$ profiles of individual Fe lines look quite different from the LSD profiles. This may be a result of differences between the Stokes profiles of weak versus strong lines, or possibly differences in the Stokes profiles of lines of different chemical elements. As with all the stars with strong linear polarization signatures in this study, HD 32633 has many more individual lines showing signatures in the new observations than in the MuSiCoS observations. Also Stokes $I$ shows variability in lines such as in the Cr II line 4588 Å and Fe II 4824 and 5018 Å lines (as shown in Fig. 3.15).

Both $B_\ell$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. The longitudinal field values obtained by Borra & Landstreet (1980) and Wade et al. (2000b) are compared to the new measurements in Fig. 3.14. Very good agreement can be seen between the three epochs of data, with the clear reduction in uncertainties shown by the smaller error bars associated with the new data. The reduced $\chi^2$ of the Fourier fit to the field curve is 1.95, with an order of fit of 3. The average uncertainty of the longitudinal field measurements is 26 gauss. The net linear polarization plot is not shown for HD 32633, due to the fact the measurements show little or no variation and are consistently null at all phases. In addition for certain phases, the ESPaDOnS/NARVAL data for HD 32633 have been convolved to the same approximate resolution as MuSiCoS (R=35000) and then
compared to MuSiCoS observations of a similar phase. Within the limits imposed by noise, very good agreement can be seen in individual lines between the two data sets in Fig. 3.16.

### 3.7.3 HD 40312 - $\theta$ Aur

$\theta$ Aur is a broader-lined A0p star with a weak magnetic field. We obtained Stokes $V$, $Q$ and $U$ profiles for 7 rotational phases. Using the method as described above, the projected rotational velocity ($v \sin i$) was determined to be $53 \pm 1$ km s$^{-1}$ which agrees within uncertainty with the value adopted by Wade et al. (2000b)

All magnetic measurements have been phased according to the ephemeris of Wade et al. (2000a):

$$JD = 2450001.881 + 3.61860 \cdot E.$$ (3.5)

The individual line profiles show weak signatures in Stokes $V$; marginal Stokes $Q$ and $U$ signatures are visible at phase 0.7-0.8 (an example for the Fe $\text{II}$ 5018 Å line is shown in Fig. 3.17). Signatures are slightly more prominent in the LSD profiles. Stokes $I$ exhibits small variations in the Fe $\text{II}$ 5018 Å and very subtle variation in the Fe $\text{II}$ line 4923 Å. Because of the weak Stokes $Q$ and $U$ signatures in individual lines in the spectra of this star, it would be a challenging candidate for $IQUV$ mapping, but would be very suitable for $IV$ mapping.

Both $B_\ell$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. The longitudinal field values obtained by Wade et al. (2000b) are compared to this work in Fig. 3.18. Agreement can be seen between the two epochs of data. The average uncertainty on the new longitudinal
Figure 3.14: Longitudinal field measurements for HD 32633 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds) and measurements by Borra & Landstreet (1980) (shown as triangles). A good agreement can be seen, confirming consistency between the instruments and the improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements. A 3rd order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.
Figure 3.15: Top: Variation of Stokes $I, Q, U$ and $V$ LSD profiles for HD 32633. Rotational phase is represented on the vertical axis. With a scaling factor of 20 was used both the Stokes $Q$ and $U$ observations. Bottom: Variation of Stokes $I, Q, U$ and $V$ profiles for HD 32633 in the 5018 Fe $\text{II}$ line (scaled by a factor of 10 for Stokes $Q$ and $U$ and 2 for Stokes $V$).
Figure 3.16: Comparison between two phases of the ESPaDOnS/NARVAL observations convolved to a resolution of 35000 and the MuSiCoS data at a similar phase for HD 32633 (MuSiCoS data dashed lines, ESPaDOnS/NARVAL solid lines). Good overall agreement can be seen between the two epochs of data.
Figure 3.17: Top: Variation of Stokes $I$, $Q$, $U$ and $V$ LSD profiles for HD 40312. Rotational phase is represented on the vertical axis. A scaling factor of 200 was used for both the Stokes $Q$ and $U$ observations, while a factor of 50 was used for Stokes $V$. Bottom: Variation of Stokes $I$, $Q$, $U$ and $V$ profiles for HD 40312 in the Fe II 5018 line (scaled by a factor of 80 for Stokes $Q$ and $U$ and 40 for Stokes $V$).
Figure 3.18: Longitudinal field measurements for HD 40312 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds). A 2nd order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.

Field measurement is 14 gauss. A Fourier fit to the longitudinal field curve yields a reduced $\chi^2$ of 1.96 with an order of fit of 2. Because of the limited phase coverage and the small net linear polarization measurements, a plot of net linear polarization is not shown for HD 40312.
3.7.4 HD 62140 - 49 Cam

49 Cam is a fairly broad-lined F0p star with a moderately strong magnetic field. We obtained Stokes $V$, $Q$ and $U$ profiles for 19 rotational phases. The projected rotational velocity ($v \sin i$) was determined to be $24 \pm 2$ km s$^{-1}$ which agrees within uncertainty with the value adopted by Wade et al. (2000b).

All measurements have been phased according to the ephemeris of Adelman (1997a):

$$JD = 2441257.300 + 4.28679 \cdot E. \quad (3.6)$$

Variable signatures can be seen in Stokes $Q$ and $U$ in individual lines in this star, an example of which is shown for the Fe II 5018 Å line in Fig. 3.19. The signatures are even clearer in the LSD profiles (Fig. 3.19). The amplitudes of the Stokes $Q$ and $U$ profiles are small compared to HD 32633 (about 50 % smaller), in both the individual lines and LSD profiles. In Stokes $I$, 49 Cam shows subtle variability in most lines, but as with all the stars with strong linear polarization signatures in this study, many individual lines show signatures, as shown in Fig. 3.8.

Both $B_\ell$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. The longitudinal field values obtained by Wade et al. (2000b) are compared to those derived in this work in Fig. 3.20. Very good agreement can be seen between the two epochs of data. The average uncertainty on the new longitudinal field measurement is 14 gauss, and the Fourier fit to the new measurements gives a reduced $\chi^2$ of 1.29 with an order of fit of 3. To further illustrate the quality of data, Fig. 3.21 shows the longitudinal measurements obtained from the null spectrum. In the case of data free from any spurious signals (e.g. caused by instrumental polarization effects), the longitudinal field in the null spectrum should
CHAPTER 3. PAPER I: MDI OF AP STARS I. OBSERVATIONS

Figure 3.19: Top: Variation of Stokes $I, Q, U$ and $V$ LSD profiles for HD 62140. Rotational phase represented on the vertical axis. With a scaling factor of 12 used for both the Stokes $Q$ and $U$ observations and a scaling factor of 2 for Stokes $V$.

Bottom: Variation of Stokes $I, Q, U$ and $V$ profiles for HD 62140 in the 5018 Fe II line (scaled by a factor of 12 for Stokes $Q$ and $U$ and 4 for Stokes $V$).
be consistent with zero and this is clearly the case with HD 62140 and is representative of most observations in this sample.

The net linear polarization as a function of phase is shown in Fig 3.22. For both Stokes $Q$ and $U$ measurements there is reasonable agreement at most phases with the measurements reported by Wade et al. (2000b) and in most cases very good agreement with values reported by Leroy et al. (1995). The net linear polarization was scaled by the same factor (0.28), double the scaling factor employed by Wade et al. (2000b). In addition the mean was subtracted from all measurements, as prescribed by Wade et al. (2000b).

3.7.5 HD 71866

HD 71866 is a sharp-lined A2p star with a moderately strong magnetic field. We have obtained Stokes $V$, $Q$ and $U$ profiles for 14 rotational phases. The projected rotational velocity ($v \sin i$) we determine is $15 \pm 2$ km s$^{-1}$ which agrees within uncertainty with the value adopted by Wade et al. (2000b).

All measurements have been phased according to the ephemeris of Bagnulo et al. (1995):

$$JD = 2438297.5 + 6.80022 \cdot E.$$  (3.7)

Clear signatures and strong variability can be seen in Stokes $Q$ and $U$ in the individual lines of this star, an example of which is shown for the Fe $\text{II}$ 5018 Å line in Fig. 3.23. The amplitudes of the Stokes $Q$ and $U$ profiles are large compared to many of the other targets in this study (in both the individual line and LSD profiles, see Fig. 3.23). Stokes $I$ appears to vary very slightly with phase, with only small changes in line shape in the Fe $\text{II}$ lines at 4923, 5018 and 5169 Å.
Figure 3.20: Longitudinal field measurements for HD 62140 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds). A good agreement can be seen, confirming consistency between the instruments and the improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements. A 3rd order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.
Figure 3.21: Longitudinal field measurements for HD 62140 obtained with ESPaDOnS/NARVAL for the null spectrum.
Figure 3.22: Net linear polarization measurements (Stokes $Q$ and $U$) for 49 Cam obtained with ESPaDOnS/NARVAL (shown by filled circles, scaled by a factor of 0.28), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds) and those obtained by Leroy (1995) (shown by open squares). A good agreement can be seen, confirming consistency between the instruments and the improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements.
CHAPTER 3. PAPER I: MDI OF AP STARS I. OBSERVATIONS

Figure 3.23: Top: Variation of Stokes $I, Q, U$ and $V$ LSD profiles for HD 71866. Rotational phase represented on the vertical axis. With a scaling factor of 15 used for both the Stokes $Q$ and $U$ observations and a scaling factor of 1 for Stokes $V$. Bottom: Variation of Stokes $I, Q, U$ and $V$ profiles for HD 71866 in the 5018 Fe II line (scaled by a factor of 16 for Stokes $Q$, 16 for Stokes $U$ and 4 for Stokes $V$).
Both $B_\ell$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. To get a good agreement between the two epochs of data required a re-analysis of the Wade et al. (2000b) data, using the updated and abundance specific line mask (as used for the ESPaDOnS/NARVAL data). It is likely that this star is in a temperature regime where measurements are very sensitive to the line mask chosen or because of peculiar abundances. The longitudinal field variation is illustrated in Fig. 3.24. Good agreement can be seen between the two epochs of data. The average uncertainty of the longitudinal field measured from the ESPaDOnS and NARVAL spectra was 27 gauss. A Fourier fit to the longitudinal field curve with a $\chi^2$ of 1.02, with order of fit of 4.

The net linear polarization as a function of phase is shown in Fig 3.25. The Stokes $Q$ and $U$ measurements obtained in this study have been scaled by a factor of 0.10 to bring them into agreement with the measurements of Leroy et al. (1995) and this scaling is consistent with that of Wade et al. (2000b). There is an approximate agreement at most phases.

HD 71866 is an ideal candidate for Magnetic Doppler Imaging. However, more observations will be required to complete phase coverage. As with all the stars with strong linear polarization in this study many individual lines show clear signatures.

3.7.6 HD 112413 - $\alpha^2$ CVn

$\alpha^2$ CVn is a fairly sharp-lined A0p star with a moderately strong magnetic field. We obtained Stokes $V$, $Q$ and $U$ profiles at 24 rotational phases. The projected rotational velocity ($v \sin i$) was determined to be $17 \pm 1$ km s$^{-1}$ which agrees within uncertainty with the values adopted by Wade et al. (2000b) and Kochukhov and Wade (2010).
Figure 3.24: Longitudinal field measurements for HD 71866 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds). A good agreement can be seen (with a re-analysis of the MuSiCoS observations), confirming consistency between the instruments and the improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements. A 4th order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.
All measurements have been phased according to the ephemeris of Farnsworth (1932):

$$\text{JD} = 2419869.720 + 5.46939 \cdot E.$$  \hfill (3.8)

Clear signatures and strong variability can be seen in Stokes $Q$ and $U$ in individual spectral lines of this star. Profile variations of the Fe II $\lambda$ 5018 line and of LSD profiles are shown in Fig. 3.26. Very strong variation in Stokes $I$ can be seen in the Fe II lines at 4923, 5018, 5169 Å: Both $B_\ell$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. The longitudinal field values are compared to the results of Wade et al. (2000b) in Fig. 3.27. To get a good agreement between the two epochs of data required a re-analysis of the Wade et al. (2000b) data, using the updated and abundance specific line mask (as used for
the ESPaDOnS/NARVAL data). It is likely that this star is in a temperature regime where measurements are more sensitive to the line mask chosen or that the abundances are sufficiently peculiar to cause a difference. In addition the new measurements are much more precise. The average uncertainty on the longitudinal field measurement was 27 gauss, with a Fourier fit to the longitudinal field curve yielding a reduced \( \chi^2 \) of 0.83, with an order of fit of 3.

The net linear polarization as a function of phase is shown in Fig 3.28. Both Stokes \( Q \) and \( U \) measurements were compared with the measurements of Wade et al. (2000b), showing very good agreement at most phases. For certain phases, the ESPaDOnS/NARVAL data for \( \alpha^2 \) CVn have been convolved to the same approximate resolution as MuSiCoS (R=35000) and then compared to MuSiCoS as described for HD 32633. Again good agreement can be seen in individual lines between the two data sets in Fig. 3.29.

**3.7.7 HD 118022 - 78 Vir**

78 Vir is a sharp-lined A1p star with a moderately strong magnetic field. We obtained Stokes \( V, Q \) and \( U \) profiles for 5 rotational phases. The projected rotational velocity \( (v \sin i) \) was determined to be \( 13 \pm 1 \) km s\(^{-1} \) which agrees within uncertainty with the value adopted by Wade et al. (2000b). All measurements have been phased according to the ephemeris of Preston (1969):

\[
\text{JD} = 2434816.90 + 3.7220 \cdot E.,
\]

Clear signatures and strong variability can be seen in Stokes \( Q \) and \( U \) in the individual lines of this star, as shown for the Fe \( \Pi \) \( \lambda \) 5018 line and LSD profiles in Fig. 3.30). Stokes \( I \) appears to vary only slightly between phases, with only small
Figure 3.26: Top: Variation of Stokes $I, Q, U$ and $V$ LSD profiles for HD 112413. Rotational phase is represented on the vertical axis. A scaling factor of 25 was used for both the Stokes $Q$ and $U$ observations and a scaling factor of 2 was used for Stokes $V$. Bottom: Variation of Stokes $I, Q, U$ and $V$ profiles of HD 112413 in the Fe II 5018 line (scaled by a factor of 30 for Stokes $Q$ and $U$ and 7 for Stokes $V$).
Figure 3.27: Longitudinal field measurements for HD 112413 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds). The measurements shown without symbols are the result of a re-analysis of the original Wade et al. (2000a) observations with the new line mask. The improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements. A 4th order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.

Figure 3.28: Net linear polarization (Stokes $Q$ and $U$) measurements for HD 112413 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds). A good agreement can be seen, confirming consistency between the instruments and the improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements.
Figure 3.29: Comparison between two phases of the ESPaDOnS/NARVAL convolved to a resolution of R=35000 and the MuSiCoS data at a similar phase for HD 112413 (MuSiCoS data dashed lines, ESPaDOnS/NARVAL solid lines).
Figure 3.30: Top: Variation of Stokes $I, Q, U$ and $V$ LSD profiles for HD 118022. Rotational phase represented on the vertical axis. With a scaling factor of 25 used for both the Stokes $Q$ and $U$ observations and no scaling on Stokes $V$. Bottom: Variation of Stokes $I, Q, U$ and $V$ profiles for HD 118022 in the 5018 Fe II line (scaled by a factor of 10 for Stokes $Q$ and $U$ and 3 for Stokes $V$).
Figure 3.31: Longitudinal field measurements for HD 118022 obtained with ESPaDOnS/NARVAL (shown by filled circles), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds). A 1st order Fourier fit to the ESPaDOnS/NARVAL data is given by the solid curve.

Figure 3.32: Net linear polarization (Stokes $Q$ and $U$) measurements for HD 118022 obtained with ESPaDOnS/NARVAL (shown by filled circles, scaled by 0.10 consistent with Wade et al. (2000b)), compared with those obtained by Wade et al. (2000b) with MuSiCoS (shown by filled diamonds) and Leroy et al. (1995) broadband linear polarization measurements (shown by open squares). Good agreement can be seen, confirming consistency between the instruments. The improvement in data quality is evidenced by the smaller error bars associated with the ESPaDOnS/NARVAL measurements.
changes in the Fe \textsc{ii} lines at 4923, 5018 and 5169 Å.

HD 118022 was studied by Khalack and Wade (2006) who constrained the global magnetic field of the star and determined the abundance distributions of titanium and chromium. This was performed using the magnetic charge distribution method (MCD) (Gerth et al. 1997; Khalack et al. 2001) and lower resolution MuSiCoS spectra, which limits the number of lines that could be modelled. This star is an excellent target for MDI, and further observations are warranted to supplement the currently rather sparse phase coverage.

Both $B_{\ell}$ and net linear polarization measurements have been obtained for each phase and are reported in Table 3.4. The longitudinal field values obtained by Wade et al. (2000b) are compared to this work in Fig. 3.31. The average uncertainty on the longitudinal field measurements was 12 gauss. The net linear polarization as a function of phase is shown in Fig 3.32 and compared with the measurements of Wade et al. (2000b) and Leroy (1995), with good agreement achieved at most phases. The net linear polarization measurements were scaled by the same factor (0.10) as reported by Wade et al. (2000b)

3.8 Discussion and Conclusions

The goal of this project was to obtain a new data set in all four Stokes parameters for a selection of well studied Ap stars, with the ultimate aim to map these stars using Magnetic Doppler Imaging. The target list contained stars which span a large part of the parameter space of interest, with sufficient signal-to-noise ratio to not only greatly improve on the previous observations, but to also be suitable for MDI mapping. The final selection was based primarily on stars already identified by Wade
et al. (2000a) as promising candidates for such study.

Early on in the project it was clear that both ESPaDOnS and NARVAL have greatly improved the level of detail at which Ap stars can be studied. The resulting dataset obtained for this study is far superior to that obtained previously with MuSiCoS, and represents some of the highest resolution phase-resolved observations of Ap stars acquired to date. This data set has more individual lines showing variation, much improved signal-to-noise and smaller error bars associated with measurements of the longitudinal field and net linear polarization. The new data have been shown to be consistent with the previous observations of Wade et al. (2000a) and also those of Leroy et al. (1995), with most targets agreeing well between the different epochs.

Surprisingly we found that even when the data are of such high signal-to-noise and when the magnetic fields are strong, the LSD analysis is sensitive to the normalisation and the measured magnetic field is rather sensitive to the integration ranges chosen, with variations of sometimes on the order of 100 gauss with very small changes of the integration range. A key conclusion of this work is that even with such high-quality data, extreme care must still be taken with all stages of analysis to ensure consistent results at this level of precision.

Although crosstalk was originally a concern, through a series of experiments we have shown that it is at a level which should not have a significant impact on the results. By using observations of γ Equ, we have seen that the highest level of crosstalk in Stokes $Q$ is still within the noise and slightly above the noise in Stokes $U$ (around the 5 % level.). It should be noted that even at these levels, this effect will be less significant in the broader-lined stars studied here.

Considering this, we believe the other uncertainties associated with the analysis
techniques have a greater effect: normalisation, blending, line masks used for LSD and the choice of integration ranges used for longitudinal field measurements. But as regards the final impact of the crosstalk on MDI mapping, this will be discussed and addressed in a future paper (Silvester et al. in preparation). With these high quality observations, we suspect that the limitations for mapping will in fact come not from the data (with strong Stokes $Q$ and $U$ signatures seen in many individual lines), but the ability to deal with line blends within the MDI code.

An important result of this study is the confirmed stability of the global properties of the magnetic fields of these Ap stars. Over multiple epochs of observations the fields have remained constant, with little variation in both longitudinal field and linear polarization measurements. In some cases measurements separated by over a decade still agree with each other within the uncertainties. By comparing MDI maps produced from the new observations of $\alpha^2$ CVn with those of Kochukhov and Wade (2010), we can potentially test for evolution of the field geometry which may occur on small spatial scales.

Considering the longitudinal magnetic field measurements, agreement was found between the new measurements and those of Wade et al. (2000b), with the exception of HD 71866 and $\alpha^2$ CVn which showed a slight discrepancy. By re-reducing the MuSiCoS data with the new masks, the observations were brought into agreement. Whilst the general shapes of the net linear polarization variations were in agreement, one needed to invoke free parameters such as scaling which is consistent with what was described in Wade et al. (2000b).

For any study which requires observations over multiple semesters, it is imperative that the instrument is stable and consistent throughout the campaign. Both NARVAL
and ESPaDOnS proved to be very stable instruments, with resolution and signal-to-noise being constant over the 4 years of data. Indeed we have also shown the two instruments are consistent with one another with close to identical result from similar phases. These facts demonstrate that ESPaDOnS and NARVAL are both very capable instruments, well suited to high-resolution four Stokes measurements of magnetic stars over multiple year timescales.

One of the targets (α² CVn) has already been mapped with MDI using MuSiCoS data by Kochukhov & Wade (2010). HD 112413 is an ideal star for determining how much of an improvement the new polarimetric data could give to MDI mapping. To quantify this improvement and to further confirm consistency, the new observations of α² CVn were compared with the profiles predicted by the model of Kochukhov & Wade (2010). As shown in Fig. 3.33, good general agreement between the new observations and the MuSiCoS-derived model is observed. However, the new profiles show more complexity than was present in the MuSiCoS data, which is not fully reproduced in the current model and would likely require a more complex magnetic field distribution. In addition, the Stokes V profile amplitude is significantly underestimated by the model at a number of phases.

The complete phase coverage of both 49 Cam (Silvester et al. in preparation), α² CVn and HD 32633 will allow the completion of 4 Stokes parameter MDI maps for these stars, doubling the number of Ap stars studied using this technique. Out of the remaining targets HD 4778, HD 71866 and HD 118022 would also be a worthwhile candidates for MDI; conversely HD 40312 has small linear polarization signatures, relative to the noise in their spectra, making them less suitable for MDI analysis. The mapping of 49 Cam is well underway and the results will be presented in Paper
3.9 Measurements

Table 3.3: Log of spectropolarimetric observations, where in the 2nd to last column, an observation is denoted by a star and a missing observation denoted by a dash. In the final column E = ESPaDOnS and N = NARVAL.

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CHAPTER 3. PAPER I: MDI OF AP STARS I. OBSERVATIONS

Table 3.3: continued.

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Table 3.4: Longitudinal magnetic field and net linear polarization measurements of magnetic A and B stars, measured from LSD Stokes $V$ profiles and Stokes $Q$ and $U$ profiles.

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<td>0.980</td>
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<td>−0.000206 ± 0.000341</td>
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$\theta$ Aur (HD 40132)

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<td>4475.268</td>
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49 Cam (HD 62140)

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Acknowledgments

OK is a Royal Swedish Academy of Sciences Research Fellow supported by grants from the Knut and Alice Wallenberg Foundation and the Swedish Research Council. GAW and DAH acknowledge support from the Natural Science and Engineering Research Council of Canada in the form of Discovery Grants.
Figure 3.33: Comparison between the new ESPaDOnS/NARVAL data for the Fe II 5018 line (shown by black points) with the final model profiles adopted in the mapping of HD 112413 (by solid blue lines) by Kochukhov & Wade (2010).
3.10 References


REFERENCES


REFERENCES


REFERENCES


Chapter 4

Paper II: Next generation magnetic Doppler imaging of $\alpha^2$ CVn

Paper Title$^1$: “Stokes $IQUV$ magnetic Doppler imaging of Ap stars - II. Next Generation Magnetic Doppler Imaging of $\alpha^2$ CVn”

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4.1 Abstract

We present updated magnetic field maps of the chemically peculiar star $\alpha^2$ CVn created using a series of time resolved observations obtained using the high-resolution spectropolarimeters ESPaDOnS and Narval. We compare these new magnetic field maps with the original magnetic Doppler imaging maps based on spectra recorded with the MuSiCoS spectropolarimeter and taken a decade earlier. These new maps are inferred from line profiles in all four Stokes parameters using the magnetic Doppler imaging code INVERS10. With the addition of new lines exhibiting Stokes $IQUV$ signatures we have a unique insight into how the derived magnetic surface structure may be affected by the atomic lines chosen for inversion. We report new magnetic maps of $\alpha^2$ CVn created using strong iron lines (directly comparable to the published MuSiCoS maps), weak iron lines and chromium lines, all of which yield a magnetic field structure roughly consistent with that obtained previously.

We then derive an updated magnetic structure map for $\alpha^2$ CVn based on the complete sample of Fe and Cr lines, which we believe to produce a more representative model of the magnetic topology of $\alpha^2$ CVn. In agreement with the previous mapping, this new updated magnetic map shows a dipolar-like field which has complex sub-structure which cannot be explained by a simple low order multipolar geometry. Our new maps show that regardless of the atomic line or species choice, the reconstructed magnetic field is consistent with that published previously, suggesting that the reconstructed field is a realistic representation of the magnetic field of $\alpha^2$ CVn. $\alpha^2$ CVn is the first Ap star for which multiple, high resolution magnetic maps have been derived, providing important observational evidence for the stability of both the large and small-scale magnetic field.
4.2 Introduction

The bright Ap star $\alpha^2$ CVn has been the subject of many observations over the past century, with the first period determination as early as Markov (1930), followed by Farnsworth (1932). There have been many studies of the magnetic and spectral variability of $\alpha^2$ CVn (Babcock & Burd (1952); Pyper (1969); Borra & Landstreet (1977)).

It was not until Kochukhov et al. (2002) employed a new magnetic Doppler imaging technique (MDI) (described by Piskunov & Kochukhov (2002) and Kochukhov & Piskunov (2002)) that the first high resolution maps of the surface vector magnetic field using Stokes $IV$ observations were made for $\alpha^2$ CVn. These maps were later refined by using linear polarization profiles (Stokes $Q$ and $U$) in combination with Stokes $IV$ (Kochukhov & Wade 2010) acquired with the MuSiCoS spectropolarimeter. These maps (along with those of 53 Cam, Kochukhov et al. 2004) were distinguished from earlier models in that they were computed directly from the observed polarized line profiles, making no $a$ priori assumptions regarding the large-scale or small-scale topology of the field. The MDI surface magnetic field maps of both stars revealed that their magnetic topologies depart significantly from low-order multipoles.

These original maps were limited by the quality of the observational data. With MuSiCoS being a relatively inefficient medium resolution spectropolarimeter, only a very small number of lines exhibited Stokes $QU$ profiles of sufficient quality for modelling. With the new observations of Ap stars in all four Stokes $IQUV$ parameters obtained using the new ESPaDOOnS and Narval instruments as described by Silvester et al. (2012), it is now possible to not only study more spectral lines, these new data also allow the study of $\alpha^2$ CVn at a resolution not previously possible and allow a more detailed probe of subtle spectral features which have been unresolved or buried.
in the noise in the MuSiCoS observations.

The mapping performed by Kochukhov & Wade (2010) (herein referred to as K&W) used the inversion code INVERS10, which is also used in this study. INVERS10 employs a single mean metallicity model atmosphere when performing the inversions. It was suggested by Stift et al. (2012) that using a single mean model atmosphere, which did not account for horizontal (local) atmospheric variations would lead to the derivation of incorrect abundance distributions and incorrect magnetic field geometries. To address these concerns, Kochukhov et al. (2012) compared magnetic field maps of $\alpha^2$ CVn reconstructed with INVERS10 with those reconstructed using a version of the INVERS code which incorporated horizontal variation of the model atmosphere. Kochukhov et al. (2012) found no significant differences between the mapping results from the two codes, confirming the suitability of INVERS10 for mapping both the magnetic field and chemical abundance features in Ap stars such as $\alpha^2$ CVn. This work aims to further explore the results of Kochukhov at al. (2012), by investigating whether the same unique magnetic field topology can be obtained from various sets of suitable Stokes $IQUV$ lines, taken with the higher spectral resolution data from ESPaDOnS and Narval.

The remainder of the paper is organised as follows: section 4.2 briefly describes the observations, section 4.3 discusses the procedure for selecting lines suitable for inversion. In section 4.4 we discuss the various magnetic maps of $\alpha^2$ CVn, with the results and implications of these maps. Finally we summarise our findings and the implications in the conclusion.
Table 4.1: Fundamental parameters used/derived for the $\alpha^2$ CVn mapping. References: (1) Kochukhov et al. (2002), (2) Farnsworth (1932), (3) Kochukhov and Wade (2010)

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<td>$120^\circ \pm 5$</td>
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4.3 Spectropolarimetric observations

Observations of $\alpha^2$ CVn were obtained between 2006 and 2010 with both ESPaDOnS and Narval spectropolarimeters during the observing campaign as described by Silvester et al. (2012). In total 28 Stokes $IQUV$ observations of $\alpha^2$ CVn were obtained. The reduction of observations was carried out at the observatories using the dedicated software package Libre-ESpRIT which yields both the $I$ spectrum and the $V$ circular polarization spectrum and/or $QU$ polarization spectra of each star observed. In this work each reduced spectrum is normalised order-by-order using an IDL code specifically optimised to fit the continuum of these stars. The full details of the observations and reduction are reported by Silvester et al. (2012), along with the log of observations for $\alpha^2$ CVn. Importantly Silvester et al. (2012) showed that the resulting longitudinal magnetic field and net linear polarization measurements obtained with ESPaDOnS and Narval were consistent with those measured from with MuSiCoS spectra obtained by Wade et al. (2000), making them suitable for direct comparison.
Figure 4.1: Comparison of magnetic field radial, meridional and azimuthal components derived from MDI maps computed using strong Fe lines, presented in rectangular projection. Upper row: results from the MuSiCoS dataset of K&W. Middle row: results from the ESPaDOnS/Narval dataset of Silvester et al. (2012). Lower row: difference maps corresponding to the middle row minus the upper row. Dashed line indicates the highest possible visible latitude based on the adopted inclination angle $i = 120^\circ$. A contour stepping of 0.5 kG has been used for a range of [-3.5,+3.5] kG.
Figure 4.2: Stokes $IQUV$ spectra comparison for strong iron lines; observed (dots), synthetic from ESPaDOnS and Narval data (solid curve, blue) and from the MuSiCoS map (solid curve, red) for $\alpha^2$ CVn.
Table 4.2: Atomic lines used for the $\alpha^2$ CVn mapping. The log $gf$ values are those as provided by the Vienna Atomic Line Database (VALD, Kupka et al. 1999)

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4.4 Inversion

The MDI mapping is performed using the INVERS10 code (as described by Piskunov & Kochukhov (2002), Kochukhov & Piskunov (2002) and K&W). INVERS10 constructs model line profiles based on an assumed initial spherical surface distribution of free parameters (element abundance and magnetic field geometry) and then iteratively adjusts the parameters until the computed line profiles are in agreement with the observations. Mathematically MDI is a least-squares minimization problem where the polarized radiative transfer depends on the magnetic field and abundance distributions, with a regularization parameter which facilitates convergence and forces the code to choose the simplest of a possible multitude of solutions providing a good fit to the observations.

The code is written in FORTRAN, with the graphical output and file processing
performed under IDL. This code is fully parallelized and was run on a 8 CPU Mac Pro. The time required for the code to converge to a solution for a set of 4 Stokes parameter data (containing 28 phases), with fitting to one or two spectral lines, is of the order of a few hours. During the reconstruction of the magnetic field using INVERS10 we adopted the same value of temperature and effective gravity as used by K&W. We did however adopt a $v \sin i$ of 18.0 km s$^{-1}$, which differs slightly from K&W, who adopted a value of 18.4 km s$^{-1}$. We found that during our mapping runs a value of 18.0 km s$^{-1}$ gave slightly better agreement between the observation and model fit. This adopted value still agrees with K&W within the error bars. Table 4.1 summarises the key stellar parameters used in this study.

Another important parameter in the reconstruction of the magnetic field topology is the choice of the regularisation value within INVERS10. As is described by K&W the Tikhonov regularisation function assists the code in converging to a solution by providing a limit on how smooth or patchy the resulting map can be. Providing a reasonable amount of regularisation is a balance of making sure that the map is not so smooth as to ignore the fitting of discrete features in the observed spectra, whilst making sure the map is not allowed to become so patchy that the code has started to fit to the noise. For the purpose of this work, a value of regularisation was chosen which gave the lowest total discrepancy, whilst still reproducing the Stokes profiles without fitting to a significant amount of noise. When performing inversions to obtain a magnetic field map from a given element, it is also required that we constrain the abundance map for that element. While abundance maps for both iron and chromium are derived in our analysis, we decided that the discussion of the interplay between the magnetic field and chemical atmospheric structure is beyond the scope of this paper.
Therefore, in this paper we will concentrate on the investigation of the robustness of the magnetic field diagnosed using MDI. A future paper will describe the abundance maps and their relationships to the magnetic field.

It is important to note that the MuSiCoS observations employed by K&W were based on 15 complete (full Stokes $IQUV$) and 5 partial (one or more Stokes parameters missing) phases of observation, whilst the new dataset is based on 28 complete phases. The phase coverage of the MuSiCoS time-series is generally good, but because of the lower number of complete phases there are some gaps (e.g. between phase 0.499 and 0.582 and between 0.594 and 0.706). With the new data there are fewer gaps although there is one between phase 0.053 and 0.141. Overall the phase sampling is very similar between the two data sets, making it reasonable to perform a comparison between magnetic field maps reconstructed using the same atomic lines between the two data sets.

For magnetic mapping only lines which show clear signatures in Stokes $Q$ and $U$ were selected. The basis of line selection was to start with the lines used by Kochukhov et al. (2002) and then to expand the list using a visual inspection to pick lines in which signatures were clearly present compared to the noise. Lines which did not show variability or were heavily blended with other lines, in addition to lines which suffered from significant non-LTE effects were avoided. As described in section 3 of Silvester et al. (2012), the ESPaDOnS and Narval spectra offered a much larger resolution than the MuSiCoS spectra with $R = \lambda/\Delta \lambda \simeq 65000$, and wavelength coverage from 3690-10480 Å (with small gaps at 9224 to 9234 Å, 9608 to 9636 Å and 10026 to 10074 Å). Even with this improved data specification, the number of suitable lines based on the aforementioned criteria was only somewhat larger than
the MuSiCoS set, although the new data were still of superior quality (The median
signal-to-noise ratio of the reduced observations is 1000 per 1.8 km s$^{-1}$ pixel vs 580
per 2.6 km s$^{-1}$ pixel for the MuSiCoS data set).

4.5 Results - Magnetic maps

The magnetic maps from the new data set are based on various line sets; first for
direct comparison with the MuSiCoS data the magnetic field was mapped using only
the strong iron lines Fe II $\lambda$ 4923 and $\lambda$ 5018 to determine if by mapping the same
lines using this newer data consistent results were obtained. This will be described
in section 4.5.1. We then identified additional lines which could be used to create
magnetic Stokes $IQUV$ maps. These additional lines allow us to determine if the
reconstructed magnetic field topology depends on factors such as the line choice, the
atomic species and line strength. To investigate this we created a Stokes $IQUV$
magnetic map based on weak iron lines (lines not used in previous mappings of $\alpha^2$
CVn), then for all the iron lines combined and finally for chromium lines. These
experiments are described in sections 4.5.2, 4.5.3 and 4.5.4 respectively. The ultimate
culmination of this search for additional lines was a magnetic field map computed
using both iron and chromium lines of which the results are described in section
4.5.6. In section 4.5.5 we compare a sample of observed profiles with synthetic profiles
obtained from pure dipolar and dipolar + quadrupolar geometries. A list of the lines
used is presented in Table 5.2, along with the the log $gf$ used which was taken from
the Vienna Atomic Line Database (VALD, Kupka et al. 1999).
4.5.1 Strong iron line maps

The first step in mapping with the new data was to produce a magnetic field map obtained from the strong iron lines of Fe II λ 4923 and 5018 as used in the original MuSiCoS maps of K&W. This was to test if the new data produced a field topology consistent with that found by K&W. To create the new map we used the same atomic data and model atmosphere used by K&W. The only differences in the input file were the choice of $v \sin i$, as described in section 3, and the value of regularisation, which was adopted independently of the value used by K&W. Because these new maps are created using a new data set, it is natural to have to independently adopt a value of regularisation. It should be noted that K&W also used Cr II λ 4824 in the mapping of $\alpha^2$ CVn which we have not included in this reconstruction. This line will be included in later reconstructions (sections 4.5.4 and 4.5.6).

A direct comparison of the radial, meridional and azimuthal fields derived from the two the data sets is shown in Fig. 4.1 in a rectangular representation. Good agreement can be seen from the two data sets for all three field components, with the difference plot showing very little in the way of structure. The most significant discrepancies are in the magnitude of the meridional field, with differences on the order of 500 G. We consider this to be a result of the new data giving us a more precise measure of the magnetic field structure in these regions. Given that the code uses Tikhonov regularization when performing magnetic inversions, it is reasonable to expect that the map derived from lower resolution data will have smaller peak magnetic field amplitudes to avoid fitting to the noise. It has been shown in studies such as Brown et al. (1991), Kochukhov & Piskunov (2002) and Rosén & Kochukhov (2012) that the meridional field is the most difficult to constrain by inversions. Therefore it is not
It is interesting to note that the iron abundance map that we derive from the strong iron lines is in very good agreement with that found by K&W. This abundance map will be described in the next paper.

The agreement between the synthetic profiles corresponding to the map and the observations is illustrated in Fig. 4.2. The agreement between the observed and synthetic profiles is good, with most of the features reproduced in all Stokes parameters without fitting to the noise. Within Fig. 4.2 we have also included the synthetic profiles from the MuSiCoS map (shown in red). Whilst the general shape is comparable to the new data fit, it is clear that the MuSiCoS fit does not fully account for the detailed structure of the new Stokes $Q$ and $U$ profiles. These discrepancies may reflect the differences observed in the maps of the meridional field component. It should be noted that whilst the MuSiCoS derived model does provide a good fit to the observed MuSiCoS Stokes $Q$ and $U$ profiles, by reducing the resolution of the new observations to the resolution of MuSiCoS and performing a direct comparison, we find that new observations do indeed lead to a smaller discrepancy between the observations and model in Stokes $Q$ and $U$ compared with the fit obtained in K&W, this suggests the new data contains new information.

Even with these differences, there is good agreement between the two data sets obtained with a separation of over a decade. Thus this new map (herein referred to as the strong line map) is suitable as a basis for comparison to see how the magnetic map may differ depending on the choice of atomic lines used in the mapping by comparison with new magnetic maps created using different line sets.
Figure 4.3: Rectangular maps of three magnetic field vector components for: (from top to bottom) strong Fe lines, weak Fe lines, combined Fe line set, Cr lines and all lines. For information on figure details (dashed line, contours) refer to Fig. 4.1.
4.5.2 Weak iron line maps

The next step was to investigate the effect on the reconstructed magnetic field topology by only using weak iron lines in the inversion. This is the first time $\alpha^2$ CVn has been mapped using weak intensity iron lines, because the previous mapping using MuSiCoS spectra did not present any weak iron lines with clear linear polarization signatures (Kochukhov et al. 2002, K&W). Whilst weak iron lines have reduced Stokes $VQU$ signature amplitudes compared to the strong iron lines, the weak lines are more sensitive to horizontal abundance variations and therefore may be more suitable for reconstruction of chemical abundance maps. The question is can such weak lines still be used for accurate reconstruction of the magnetic topology.

The weak iron lines used were Fe II $\lambda\lambda\lambda$ 4273, 4520 and 4666. These lines were selected on the basis that they exhibited Stokes $QU$ profiles which had amplitudes clearly above the noise, that the intensity profile showed significant variability (which is more important for abundance mapping) and that these lines had predicted line depths smaller than the strong iron line set. Whilst the spectrum of $\alpha^2$ CVn contains many iron lines, most of the lines were strongly blended with other elements or did not show clear Stokes $QU$ profiles, which is why we are limited to three weak iron lines. The log $gf$ values adopted in the inversion calculations are indicated in Table 5.2.

A comparison between the resulting magnetic topology map (herein referred to as the weak line map) and the strong iron line map is shown (with other maps to be described in the following sections) in rectangular form in Fig. 4.3. By comparing the strong and weak iron line maps (first and second row respectively) it can be said that overall, the geometry of the derived field is quite similar, particularly in the radial
field component, but there are some differences in the azimuthal structure and in particular there is a difference at large negative latitudes in the meridional field. This could well be a result of the difference in sensitivity to horizontal structure between the two maps. Strong saturated lines change due to abundance spots over the stellar surface to a lesser extent than weak lines, with strong lines forming over a wider range of abundances than the weaker lines. This means the strong iron line map potentially represents a smoothed version of the field topology. In addition, as has already been mentioned the meridional field is the least constrained of the three field components, so it is not surprising that this component differs between the two maps.

It is important to note that the weak line Stokes \(Q\) and \(U\) signatures are often of very different shape than the strong line signatures, probably due to their different sensitivities to the horizontal abundance structure, but also due to the different relative strengths of the \(\pi\) and \(\sigma\) components as a result of different levels of saturation. If we compute profiles of the weak lines using the strong line magnetic field map, the agreement is quite poor. The same is also found if the reverse comparison is performed. In addition we also checked that these differences were not a consequence of differences in the abundance distribution maps.

This shows that relatively small (but still non-negligible) differences in the magnetic maps can produce quite important differences in the resultant Stokes profiles. This in itself suggests that combining lines of different characteristics will probably result in a map which is more representative of the average field distribution. Also, the validity of the derived field map is tested through this process, since a single field model should be capable of reproducing the profiles and variations of all lines.
4.5.3 Combined iron line maps

The next logical step was to see the effect on the inferred magnetic field by combining the two iron line sets (herein referred to as the combined iron line map). We combined the strong iron lines of Fe II λ 4923 and 5018, in addition to weaker iron lines of Fe II λ 4273, 4520 and 4666. Considering the non negligible differences between the two maps computed using the strong and weak lines, we were curious to investigate if a single magnetic field model was able to reproduce all lines simultaneously. In this map the two sets of lines are combined without applying any relative weights. The resulting magnetic map is shown in rectangular form compared to the strong iron line magnetic map in Fig. 4.3. By comparing the two maps (row one and row 3), once again we see that the overall field structure is consistent with the strong iron line magnetic field map, with the meridional field showing the largest differences at negative latitude. The differences are a result of differences in sensitivity to horizontal structure, as described in the previous section.

For an indication of how well the inversion code has simultaneously fit both the strong and weak iron lines together, Fig. 4.8 illustrates the fit between observed profiles and model for the final map which includes the combined iron lines and chromium. The fit of the iron lines in this figure is representative of the fit obtained for the combined iron line map, with good agreement in all Stokes parameters.

4.5.4 Chromium line map

Next we wanted to investigate the results when using chromium lines to produce Stokes $IQUV$ magnetic maps, again to see if the choice of line and in this case using a different element, modifies the resulting map when compared to the original strong
iron line map. Using the same criteria as used for the weak iron lines, the chromium lines of Cr II $\lambda\lambda 4824$, 5246 and 5280 were chosen. These lines have predicted depths much lower than the weak and strong iron lines. The log $gf$ adopted in the inversion calculations are indicated in Table 5.2. The magnetic map calculated from the chromium lines is compared to the strong iron line magnetic map in Fig. 4.3. The general structure between the two maps is in agreement. There are some differences in particular the meridional component at negative latitude structure. This is similar to what is seen in the weak iron line map (section 4.5.2).

The source of this difference can be seen in Fig. 4.4 which shows the observed chromium Stokes $IQUV$ profiles compared with synthetic profiles obtained from the chromium lines and the equivalent profiles obtained from the strong iron line magnetic map. It is important to note that these synthetic iron line profiles were computed using the chromium abundance map as found from the aforementioned chromium inversion. This is required to eliminate any line profile differences caused by the differing iron and chromium chemical abundance structures. As was mentioned in section 3, the abundance maps will be presented in a future paper.

Within Fig. 4.4 the observed and computed Stokes $IV$ profiles are in good agreement, however there are differences in Stokes $QU$ with most profile features not being reproduced correctly by the model. This difference would lead to a difference in structure between the two maps. Again this difference we believe is the result of the different horizontal structure sensitivities between the strong iron lines and the chromium lines. We believe that we are seeing a more representative model of the field by choosing weak iron lines or chromium lines, in which we are seeing the magnetic field as it is without the smoothing effects of the strong iron lines.
Figure 4.4: Comparison between observed chromium lines (dots) and synthetic profiles based on the chromium lines (solid curve, blue). Over plotted is the strong iron line magnetic field map with the chromium abundance synthetic profiles shown for comparison (solid curve, red).
4.5.5 Comparison to multipolar geometries

As has been described in previous mapping of $\alpha^2$ CVn (K&W, Kochukhov et al. 2012), the magnetic field topology shows complex substructure which could not be described by a low-order multipolar geometry. With the new data it is important to examine whether or not this is still the case and evaluate if the observed profiles cannot be satisfactorily fitted with a more simple field topology. To investigate this, the profiles for the combined iron line set were compared to model profiles one would obtain for an optimal dipolar and dipolar plus quadrupolar geometry. To accomplish this comparison we have fitted four Stokes parameter observations with a modified version of INVERS10 in which a direct model description of the three field components was substituted with a multipolar parameterization similar to the one described by Donati et al. (2006). In this comparison the chemical abundance distribution has been allowed to vary as is the case with the other inversions performed. Further details about our implementation of MDI with multipolar expansion are provided by Kochukhov et al. (2013). In the present study we performed inversions using only poloidal field components and limiting multipolar expansion to angular degrees $\ell = 1$ and $\ell = 2$ for the dipolar and dipolar+quadrupolar models, respectively. The latter field parameterization is mathematically equivalent to the non-axisymmetric dipolar plus quadrupolar model geometry employed by Baguhl et al. (2002) in their statistical study of Ap star magnetic fields.

As illustrated in Fig. 4.5, it can be seen that neither model provides good agreement with the profiles. The dipole model fails to fully reproduce Stokes $V$ and does not reproduce Stokes $Q$ and $U$. The dipolar+quadrupolar model does a better job of Stokes $V$, in the case of both Stokes $Q$ and $U$ it is clear the structure of the model
generally fails to reproduce the high-contrast wavelength variation of the observed profile and it fails most dramatically at those phases where we infer the most significant complex structure to be visible. We can therefore conclude that a simple field topology cannot describe the field structure of $\alpha^2$ CVn, a result in agreement with the findings of K&W.

4.5.6 Iron and Chromium map - The final map

By combining all the lines found to be suitable for Stokes $IQUV$ mapping, we produced a magnetic map using iron lines ($\text{Fe} \, \lambda \, 4923, 5018, 4273, 4520$ and $4666$), combined with chromium lines ($\text{Cr} \, \lambda \, 4824, 5246$ and $5280$). Because these lines all differ in depth and also magnetic sensitivity, this will require the code to find a balance between all the competing lines and still produce a consistent map. It is interesting to test if such a mixture of lines can produce a reasonable result that does not alter the general structure of the field seen in the previous maps. At the same time because of the difference in line depths and respective surface structure sensitivities, this should be considered more representative of the magnetic field topology of $\alpha^2$ CVn.

The fit to the spectra is shown in Fig. 4.8. Considering the variety of line depths included in this inversion, very good agreement between the model and the observations can be seen at all phases and in all Stokes parameters. The magnetic map created with the chromium and iron lines (herein referred to as the final map) is shown compared with the strong iron line magnetic map along with other line maps in Fig. 3 (map bottom row). There is a general agreement in the overall structure, but there is also a difference between the meridional component, with the final map
Figure 4.5: Comparison between observed (dots) for the weak and strong iron lines used in magnetic mapping and synthetic spectra for a dipolar geometry (solid curve, blue) and for a dipole + quadrupole geometry (solid curve, red). Upper frame: Stokes $I$ and $V$ profiles. Lower frame: Stokes $Q$ and $U$ profiles.
Figure 4.6: Magnetic map computed using all selected iron lines and chromium lines from the ESPaDOnS/Narval data. The spherical plots show distributions of the field modulus (a), radial field (b) and field orientation (c).
CHAPTER 4. PAPER II: MDI OF AP STARS II: MDI OF $\alpha^2$ CVN

showing the main positive field structure concentrated at a higher latitude compared to the strong iron line map. This difference is reduced by comparing the final map with the combined iron line map (which includes weak and strong lines) as described in section 4.5.3. This indicates the inclusion of weak iron lines makes a contribution to the overall meridional field at negative latitudes, again combating the potential smoothing effect of the strong iron line map. Whilst still being reasonably consistent with the strong field iron map, we consider this final map to be more representative of the true field, with the smoothing effects of the strong iron lines somewhat limited. This final map is shown in spherical form in Fig. 4.6, with field modulus (a), radial field (b) and field orientation (c) shown.

Figs. 4.1, 4.3 and 4.6 show that the inversion of the Stokes profile consistently recovers structures on a variety of scales. We can roughly estimate the formal resolution of the maps provided by rotational broadening by dividing twice $v\sin i$ by the spectral resolution. For a resolving power of 65000 and taking the relatively low $v\sin i$ of $\alpha^2$ CVn, we obtain approximately 15 resolution elements along the equator, corresponding to about 24°. However, at low $v\sin i$ values (such as that of $\alpha^2$ CVn) information from rotational modulation becomes progressively more important than information coming from the Doppler broadening. Ultimately, there is no standard method to estimate resolution from rotational modulation. A rough lower limit to the achievable spatial resolution can be calculated using average phase sampling (in this case $360° / 28$ phases $= 13°$). It is worth noting that our maps don’t appear to show any consistent structures smaller than 20°.
Figure 4.7: Comparison between observed (dots) and synthetic (solid curve, blue) Stokes $IQUV$ parameter spectra of $\alpha^2$ CVn for the final magnetic map (iron and chromium combined). The solid curves show the best fit to all Cr and Fe lines combined.
Figure 4.8: Comparison between observed (dots) and synthetic (solid curve, blue) Stokes $IQUV$ parameter spectra of $\alpha^2$ CVn for the final magnetic map (iron and chromium combined). The solid curves show the best fit to all Cr and Fe lines combined.
4.6 Conclusion

With the results presented in section 4.5.1 we are able to show that the magnetic maps from K&W and magnetic maps created using the new observations of Silvester et al. (2012) are indeed consistent when considering the iron lines Fe II λ 4923 and 5018, with only small differences likely attributable to the difference in data quality (spectral resolution and signal-to-noise) between the two data sets. This also illustrates that the small scale structure is stable over the timescale between the two epochs of observation.

In section 4.5.2 we investigated the potential impact of line selection on the resulting magnetic map by using only weak iron lines in the inversion. This produced a magnetic map that was broadly consistent with the strong line map and only differed in the meridional field at low latitudes and the azimuthal field at high latitudes. The largest difference in the azimuthal field is on the order of 1 kG and located around a latitude of 25°. The difference in the meridional field is on the order of 1.5 kG and located around a latitude of −60°. Both differences are located around a longitude of 180°. This difference is believed to be a result of the difference in horizontal structure sensitivities between the weak and strong line sets, with stronger saturated lines altered less by abundance spots. It is important to note that profiles computed from weak lines did not agree well with profiles computed from strong iron lines. In addition the meridional field is the most difficult to constrain by inversion. This suggests the strong iron map represents a smoothed version of the magnetic field map.

It should be noted that a potential source of the discrepancy seen between the weak and the strong iron magnetic maps is vertical stratification. Theoretical modelling of abundance stratification by Alecian and Stift (2010) showed that in an Ap star,
vertical stratification leads to a change in chemical abundance as a function of optical depth in the atmosphere. In such a framework, strong lines which sample a larger range of optical depths when compared to weak lines, would be probing different parts of this “abundance vs optical depth” variation. This effect is more significant in cooler Ap stars, however for hotter stars like \( \alpha^2 \text{CVn} \), Stift and Alecian (2012) showed it could potentially lead to variations in abundances on the order of 1 dex over the range of optical depths.

By combining strong and weak iron lines the resulting “combined iron line” map was slightly more consistent with the strong iron line map, than when weak iron lines alone were used in the mapping. The contribution of the weak lines into the combined iron line map still results in a difference in the meridional field at low latitudes. It was possible to reproduce both the weak and strong iron line profiles with this single map. In section 4.5.4 we produced a magnetic map based on chromium lines. The results were similar to those found by mapping weak iron lines, with an overall agreement with the strong iron line map but still with differences in the meridional fit at low latitudes. Interestingly both the weak iron line map and the chromium lines independently give very similar magnetic field maps. This illustrates that very different line sets can give consistent mapping results.

Finally in section 4.5.6 we combined all the above line sets into a final map of the magnetic field of \( \alpha^2 \text{CVn} \). We consider this map to be the best existing representation of the magnetic field of \( \alpha^2 \text{CVn} \). The overall structure of the final map is in agreement with K&W: we find a dipole-like structure with complex sub-structure. As was found by K&W, we find that the magnetic field is strongest at the positive pole (seen clearly at phase 0.6), with an asymmetry compared to the negative pole.
There is clear agreement between the maps of K&W and our new maps (produced from a completely new set of Stokes IQUV observations). When this is combined with the fact that we are able to reproduce the same general magnetic field topology from a variety of lines sets, of varying intensities and from two different atomic species, it provides compelling support to the findings of Kochukhov et al. (2012), and suggests the magnetic field structure we reconstruct for $\alpha^2$ CVn is accurate and not the result of limitations in the inversion technique as has been suggested by Stift et al. (2012). In addition we have also illustrated that the observed profiles cannot be fit with a simple dipolar or dipole + quadrupole geometry and can only be fit with direct magnetic mapping using only local regularization to constrain the map. One other important result from this work is that we have the first confirmation via magnetic Doppler imaging that the global magnetic field of $\alpha^2$ CVn is stable over the period of a decade, which adds further evidence to the current theoretical understanding of the stability of the magnetic fields in Ap stars.

It should be noted that regardless of the line-set choice, the large-scale structure of all the maps is consistent with the original maps of K&W and strong iron line maps. Line selection may have an subtle effect on the resulting magnetic field map, with small differences arising as a result of horizontal structure sensitivity of the atomic line used in the inversion. With this in mind, it is clearly of value to map a variety of lines with different formation heights when possible, such as weak and strong iron lines or strong iron lines and chromium lines, etc. Even when it is not possible to have such a variety, provided that the line profiles used are of sufficient quality and show clear linear polarization amplitudes, the reconstructed field should be reliable. It could be argued that data quality is a far bigger factor potentially affecting the
REFERENCES

reconstructed magnetic topology.

The next stage in the project is to produce a series of abundance maps for $\alpha^2$CVn, looking for any interplay between the magnetic field and chemical atmospheric structures. We will then investigate the magnetic field geometry and chemical abundance structures of other Ap stars using data from Silvester et al. (2012) and the MiMeS project. By increasing the sample of Ap stars studied using MDI, we can probe what influences other stellar parameters (such as mass, temperature, rotation etc) have on the resulting magnetic field geometry.

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4.7 References


REFERENCES


Chapter 5

Paper III: Next generation chemical abundance mapping of $\alpha^2$ CVn

Paper Title$^1$: “Stokes IQUV magnetic Doppler imaging of Ap stars - III. Next generation chemical abundance mapping of $\alpha^2$ CVn”
CHAPTER 5. PAPER III: MDI OF AP STARS III: MAPPING OF $\alpha^2$ CVN

5.1 Abstract

In a previous paper we presented an updated magnetic field map for the chemically peculiar star $\alpha^2$ CVn using ESPaDOnS and Narval time-resolved high resolution Stokes IQUV spectra. In this paper we focus on mapping various chemical element distributions on the surface of $\alpha^2$ CVn. With the new magnetic field map and new chemical abundance distributions we can investigate the interplay between the chemical abundance structures and the magnetic field topology on the surface of $\alpha^2$ CVn.

Previous attempts at chemical abundance mapping of $\alpha^2$ CVn relied on lower resolution data. With our high resolution (R=65,000) dataset we present nine chemical abundance maps for the elements O, Si, Cl, Ti, Cr, Fe, Pr, Nd and Eu. We also derive an updated magnetic field map from Fe and Cr lines in Stokes IQUV and O and Cl in Stokes IV. These new maps are inferred from line profiles in Stokes IV using the magnetic Doppler imaging code INVERS10. We examine these new chemical maps and investigate correlations with the magnetic topology of $\alpha^2$ CVn. We show that chemical abundance distributions vary between elements, with two distinct groups of elements; one accumulates close to the negative part of the radial field, whilst the other group shows higher abundances located where the radial magnetic field is on the order of 2 kG regardless of the polarity of the radial field component. We compare our results with previous works which have mapped chemical abundance structures of Ap stars. With the exception of Cr and Fe, we find no clear trend between what we reconstruct and other mapping results. We also find a lack of agreement with theoretical predictions. This suggests that there is a gap in our theoretical understanding of the formation of horizontal chemical abundance structures and the connection to the magnetic field in Ap stars.
5.2 Introduction

The bright Ap star $\alpha^2$ CVn is a member of the class of chemically peculiar stars which also exhibit a strong globally ordered magnetic field. These chemical peculiarities are exhibited as global over and underabundances relative to the sun and as lateral abundance nonuniformities that have been described in the literature as spots or rings of over/under abundance, but can also be more complex (e.g. Lüftinger et al. 2003, Kochukhov et al. 2004 and Rice et al. 2004). Abundances are also reported to vary vertically through the atmosphere in a significant way. These abundance anomalies are believed to result principally from atomic diffusion in the atmosphere of the star (Michaud 1970). Michaud showed that if the atmosphere of a star is sufficiently stable, diffusion under the competing influence of gravity and radiation pressure can occur. The additional presence of the magnetic field strongly influences energy and mass transport (e.g., diffusion, convection and weak stellar winds) within the atmosphere of a star, and results in strong chemical abundance non-uniformities in photospheric layers (e.g., Turcotte 2003). Magnetic fields have been shown to modify diffusion in two ways. First, charged particles are strongly constrained to follow field lines. This can result in the magnetic field modifying the diffusion velocity (Alecian & Stift 2006). Secondly, radiative accelerations are also modified by Zeeman desaturation (magnetically induced spectral line desaturation) and splitting of absorption lines (Alecian & Stift 2006).

Even with a theoretical framework for the formation of abundance anomalies, very few observational studies have been made of Ap stars using chemical abundance mapping combined with magnetic field topology analysis from the same data (from medium to high resolution observations). With the exception of previous work
on $\alpha^2$ CVn (the subject of the present paper), notable examples include magnetic Doppler imaging of 53 Cam (Kochukhov et al. 2004) and mapping of the roAp star HD 24712 (Lüftinger et al. 2010). Examples of abundance Doppler imaging with a comparison to independent model of the magnetic field geometry include oxygen abundance structures mapped for the star $\theta$ Aur by Rice et al. (2004), multiple element mapping of $\varepsilon$ UMa by Lüftinger et al. (2003) and mapping of the roAp star HD 83368 by Kochukhov et al. (2004). A more recent example of abundance mapping performed (without simultaneously deriving the magnetic field) is HD 3980 (Nesvacil et al. 2012). In these studies, with the exception of a handful of cases, no clear correlations could be found between the magnetic field topology and the horizontal structures of most chemical elements. This was interpreted as a lack of up to date theoretical models predicting the formation of horizontal abundance structures. With such a small sample size there is still limited understanding of the interplay between specific chemical species in the photosphere and the magnetic field of Ap stars.

A new magnetic map of the bright Ap star $\alpha^2$ CVn was reconstructed by Silvester et al. (2014) using Stokes $IQUV$ observations obtained with ESPaDOnS and Narval spectropolarimeters described by Silvester et al. (2012). We demonstrated that the magnetic topology we derived agreed with that of Kochukhov & Wade (2010) which used data taken a decade earlier. Importantly, we showed that the a similar magnetic field topology could be obtained by mapping different atomic line sets, with the only differences seen between the meridional field components. Mapping of the distributions of the surface chemical abundances of several elements for $\alpha^2$ CVn using Stokes $IV$ was performed by Kochukhov et al. (2002), allowing a comparison between the local field properties and local photospheric chemistry. These original
maps were limited by the small wavelength coverage of the SOFIN spectrograph and a lack of a detailed model of the field topology, which generally cannot be derived from Stokes IV alone (Kochukhov et al. 2002, Kochukhov and Wade 2010). With the new spectropolarimetric (Stokes IQUV) observations described by Silvester et al. (2012), we will now investigate the chemical abundance structures of $\alpha^2$ CVn at a level of detail not previously possible, in particular due the increased wavelength coverage. By mapping the chemical surface structures of $\alpha^2$ CVn and comparing them to our updated magnetic map, we hope to further our understanding of how different chemical species are affected by the characteristics of the magnetic field.

The paper is organised as follows: Section 5.2 briefly describes the observations, Section 5.3 discusses the procedure for selecting lines suitable for chemical abundance mapping. In Section 5.4 we discuss the chemical abundance maps and the implications of these maps. Finally we summarise our findings in the conclusion.

### 5.3 Spectropolarimetric observations

Observations of $\alpha^2$ CVn were obtained between 2006 and 2010, with both ESPaDOnS and Narval spectropolarimeters. The full details of the observations and reduction are reported by Silvester et al. (2012), along with the log of observations. For this study only Stokes IV profiles were used for abundance mapping due to the fact that the linear polarization signatures were generally weak for most of the studied elements to be useful for mapping.
Figure 5.1: Magnetic maps of $\alpha^2$ CVn (top) computed using Invers10 for all selected chromium lines and iron lines from Silvester et al. (2014) and (bottom) the new updated magnetic field map for which we included the same lines as Silvester et al. (2014), plus the addition of oxygen and chlorine lines in Stokes IV. The spherical plots show distributions of the field modulus (a), radial field (b) and field orientation (c) and each column is a different phase of rotation (0.0, 0.2, 0.4, 0.6 and 0.8).
Figure 5.2: Chemical abundance distributions based on the observations of Silvester et al. (2012) using INVERS10 for light and iron peak elements: O, Si, Cl, Ti, Cr, and Fe. Each column indicates a different rotational phase (0.0, 0.2, 0.4, 0.6 and 0.8) and the solid line shows the location of the stellar equator. The visible rotational pole is also indicated.
Table 5.1: Fundamental parameters used/derived for the α² CVn mapping. References: (1) Kochukhov et al. (2002), (2) Farnsworth (1932), (3) Kochukhov and Wade (2010).

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<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
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<tr>
<td>$T_{\text{eff}}$</td>
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<td>(1)</td>
</tr>
<tr>
<td>log $g$</td>
<td>3.9 ± 0.1</td>
<td>(1)</td>
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<tr>
<td>$P_{\text{rot}}$</td>
<td>5.46939 days</td>
<td>(2)</td>
</tr>
<tr>
<td>$v \sin i$</td>
<td>18.0 ± 0.5 km/s</td>
<td>(3)</td>
</tr>
<tr>
<td>$i$</td>
<td>120° ± 5</td>
<td>(3)</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>115° ± 5</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Figure 5.3: Chemical abundance distributions based on the observations of Silvester et al. (2012) using INVERS10 for rare earth elements: Pr, Nd and Eu. Each column indicates a different rotational phase (0.0, 0.2, 0.4, 0.6 and 0.8) and the solid line shows the location of the stellar equator. The visible rotational pole is also indicated.
Table 5.2: Atomic lines used for the $\alpha^2$ CVn mapping. The log $gf$ values are those provided by the Vienna Atomic Line Database (VALD, Kupka et al. 1999), the primary references are given where possible and are as follows: (1) NIST10 (Ralchenk et al. 2010) (2) Schulz-Gulde (1969), (3) Blanco et al. (1995), (4) Matheron et al. (2001), (5) Kurucz (2012), (6) Wiese et al. (1969), (7) Wood et al. (2013), (8) Raassen & Uylings (1998), (9) Ryabchikova (2006), (10) DREAM database Biemont et al. (1999), (11) Lawler et al. (2001).

<table>
<thead>
<tr>
<th>Ion</th>
<th>Wavelength (Å)</th>
<th>log $gf$</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>O i</td>
<td>7771.941</td>
<td>0.369</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>7775.388</td>
<td>0.001</td>
<td>(1)</td>
</tr>
<tr>
<td>Si ii</td>
<td>5055.984</td>
<td>0.593</td>
<td>(2,3,4)</td>
</tr>
<tr>
<td></td>
<td>5056.317</td>
<td>-0.359</td>
<td>(2,3)</td>
</tr>
<tr>
<td></td>
<td>5978.930</td>
<td>0.040</td>
<td>(2,3)</td>
</tr>
<tr>
<td></td>
<td>6347.109</td>
<td>0.297</td>
<td>(2,3,4)</td>
</tr>
<tr>
<td></td>
<td>6347.133</td>
<td>-1.200</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td>6347.197</td>
<td>-2.350</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td>6371.371</td>
<td>-0.040</td>
<td>(2,3,4)</td>
</tr>
<tr>
<td>Cl ii</td>
<td>4794.556</td>
<td>0.455</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>4819.480</td>
<td>0.064</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>4819.756</td>
<td>-0.790</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>4904.776</td>
<td>0.310</td>
<td>(6)</td>
</tr>
<tr>
<td>Ti ii</td>
<td>4163.644</td>
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<td>(7)</td>
</tr>
<tr>
<td></td>
<td>4468.507</td>
<td>-0.600</td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td>4571.960</td>
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<td>(7)</td>
</tr>
<tr>
<td>Cr ii</td>
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<td>(8)</td>
</tr>
<tr>
<td></td>
<td>5246.768</td>
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<td>(8)</td>
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<tr>
<td></td>
<td>5279.876</td>
<td>-2.112</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>5280.054</td>
<td>-2.316</td>
<td>(8)</td>
</tr>
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<td>Fe ii</td>
<td>4555.893</td>
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<td>(8)</td>
</tr>
<tr>
<td></td>
<td>5030.630</td>
<td>0.431</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>5032.712</td>
<td>0.077</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>5035.708</td>
<td>0.632</td>
<td>(8)</td>
</tr>
<tr>
<td>Nd iii</td>
<td>4927.420</td>
<td>-0.800</td>
<td>(9)</td>
</tr>
<tr>
<td></td>
<td>5050.695</td>
<td>-1.060</td>
<td>(9)</td>
</tr>
<tr>
<td></td>
<td>5677.120</td>
<td>-1.450</td>
<td>(9)</td>
</tr>
<tr>
<td></td>
<td>6145.068</td>
<td>-1.330</td>
<td>(9)</td>
</tr>
<tr>
<td>Pr iii</td>
<td>5299.993</td>
<td>-0.530</td>
<td>(10)</td>
</tr>
<tr>
<td></td>
<td>5765.243</td>
<td>-1.100</td>
<td>(10)</td>
</tr>
<tr>
<td></td>
<td>7030.390</td>
<td>-0.780</td>
<td>(10)</td>
</tr>
<tr>
<td>Eu ii</td>
<td>6437.640</td>
<td>-0.320</td>
<td>(11)</td>
</tr>
<tr>
<td></td>
<td>6645.100</td>
<td>0.120</td>
<td>(11)</td>
</tr>
</tbody>
</table>
5.4 Chemical and Magnetic Map Inversion

The methodology used to derive magnetic field maps is described by Silvester et al. (2014). In this paper we will concentrate on the abundance mapping of $\alpha^2$ CVn. The abundance mapping was performed using the INVERS10 magnetic Doppler imaging (MDI) code (Piskunov & Kochukhov 2002; and Kochukhov & Piskunov 2002). As was described by Silvester et al. (2014), INVERS10 is a stellar surface mapping code written in FORTRAN that constructs model line profiles based on an assumed initial spherical surface distribution of free parameters (in this case the element abundance) and then iteratively adjusts the parameters until the computed line profiles are in agreement with the observations. The code is parallelized and was run on an 8 CPU Mac Pro. Graphical output and file processing was performed using IDL. The time required for the code to converge to a solution for a set of IV Stokes data (containing 28 phases), with fitting to one or two spectral lines, is of the order of an hour.

As described by Silvester et al. (2014) the fundamental parameters needed for inversion have to be well defined. Table 5.1 shows the key parameters used in this study. The basis of line selection for abundance mapping combined a visual inspection of the spectra looking for candidate lines which were unblended and also a review of the literature looking for unblended lines that have been used in previous studies for stars of a similar spectral type. The selection drew on the line lists of of Pyper (1969), Cohen (1970) and to a lesser extent Roby and Lambert (1990) and those used by Bailey et al. (2014). The next stage was then to eliminate lines from the selection list which did not show strong line depths. Because in this paper we only focus on chemical abundance maps, only Stokes IV profiles were used in the mapping and unlike the requirement by Silvester et al. (2014) candidate lines did not have to
exhibit linear polarization signatures. To ensure that the resulting abundance maps were of sufficient reliability, only elements which had a minimum of two lines suitable for inversion were considered.

An important parameter in the reconstruction of the abundance maps is the choice of the regularisation. As is described by Piskunov & Kochukhov (2002) INVERS10 uses a Tikhonov regularisation function, which assists the code in converging to a solution by providing a limit on how smooth or patchy the resulting map can be. For all the maps a value of regularisation was chosen which gave the lowest total discrepancy, whilst still reproducing the Stokes profiles without fitting to a significant amount of noise. However a lower limit was set such that the total regularisation scaling factor must be at the minimum only a factor of 10 times smaller than the total discrepancy (between the observations and the model). After this limit the improvement to the discrepancy becomes increasingly smaller, but the map will continue to become increasingly patchy.

When producing the final spherical plots for each derived chemical map, the choice of abundance scale range has to be taken with care. In particular it is best to avoid using an abundance scale with an extremely large range if this range is based on a handful of outlying abundance values, otherwise the resulting map may overemphasise unrealistic abundance features. INVERS10 outputs map data in the form of an ascii file which contains a numerical abundance value for each of the surface elements (in this case 695) which make up the complete spherical map. These 695 abundance values were plotted in histogram form to identify any extreme outliers, allowing the abundance scale to be limited to a more realistic/representative range. An example histogram is shown for chromium in Fig. 5.4, with red dashed lines indicating the
CHAPTER 5. PAPER III: MDI OF AP STARS III: MAPPING OF $\alpha^2$ CVN

Figure 5.4: Histogram showing the range of abundance values found in the reconstructed map for chromium. The red dashed lines indicate the abundance range cut off, after which values are not included in the abundance range scale. In this case an abundance range of -3.5 to -7.0 dex was used as the scale for chromium.

abundance range cut off. This process was performed for all the chemical abundance maps presented and in some cases on the order of $\approx$20 points were ignored. In addition because of the inclination of the star, abundance values at latitudes higher than $+60^\circ$ are harder to constrain, so abundance values corresponding to these latitudes were given less weight in the scale selection process. In fact in most cases these values contributed considerably to the outlier values and thus were typically already discarded from the abundance range selection.
CHAPTER 5. PAPER III: MDI OF AP STARS III: MAPPING OF $\alpha^2$ CVN

5.5 Refined Magnetic Field Map

For all of the abundance map inversions the magnetic field was set as a fixed parameter: the magnetic field reconstructed from both chromium and iron lines and referred to as the “final map” by Silvester et al. (2014) was initially used for this fixed magnetic field. But when independently mapping the abundances of oxygen and chlorine it was noticed that whilst the fit for Stokes $I$ was very good, the fit for Stokes $V$ was quite poor, with clear disagreement of the amplitude of Stokes $V$ at various phases between the model and the observations.

To investigate if this poor fit in Stokes $V$ was due to oxygen and chlorine lines emphasising a different region of the magnetic field map to which iron and chromium lines are less sensitive, we recalculated the magnetic field map. We used the same Stokes $IQUV$ iron lines ($\text{Fe} \text{ II} \lambda$ 4923, 5018, 4273, 4520 and 4666) and chromium lines ($\text{Cr} \text{ II} \lambda$ 4824, 5246 and 5280) from Silvester et al. (2014), but also included Stokes $IV$ lines for oxygen ($\text{O} \text{ I} \lambda$ 7771 and 7775) and chlorine ($\text{Cl} \text{ I} \lambda$ 4794, 4819 and 4904). It should be noted that the Stokes $QU$ profiles for oxygen and chlorine showed no significant polarization detected and therefore were not included in the inversion. The resulting fit in Stokes $V$ for oxygen and chlorine was greatly improved with the updated magnetic map and at the same time the fit to Stokes $IQUV$ for iron and chromium was consistent with what was seen when using the magnetic map of Silvester et al. (2014). We therefore chose to adopt the updated magnetic map ($\text{Fe} & \text{Cr}$ Stokes $IQUV$ and $\text{O} & \text{C}$ Stokes $IV$) as the fixed magnetic map for all the inversions in this paper. Whilst the fits to Stokes $V$ were greatly improved for certain elements, the resulting abundance maps for these elements differed little from the maps reconstructed using the magnetic field map by Silvester et al. (2014).
The resulting map is shown with the “final map” of Silvester et al. (2014) in Fig. 5.1. This figure illustrates that by including Stokes \( IV \) for oxygen and chlorine in the reconstruction, the resulting magnetic field is very similar to what is seen by Silvester et al. (2014), but with an additional small spot-like region seen at phase 0.00 below the stellar rotational equator. To retain the quality of the fit to Stokes \( QU \) profiles required that the regularisation of the magnetic field to be reduced by a factor of 3 compared to what was used by Silvester et al. (2014). This reduction in regularisation should result in a slightly more “patchy” map, which is potentially the reason for the new structures seen at phase 0.00. This updated map does not change the findings of Silvester et al. (2014) as the new map is still broadly consistent with the “final map” by Silvester et al. (2014). In fact, at the phases where the small spot does not appear the field distributions are effectively identical. It should be noted that the magnetic mapping of Silvester et al. (2014) concentrated purely on lines which exhibited strong Stokes \( QU \) signatures, which is why lines such as oxygen and chlorine were not included in this original magnetic mapping.

5.6 Chemical Abundance Maps

5.6.1 Oxygen, Silicon and Chlorine Maps

Oxygen chemical maps were produced using the Stokes \( IV \) line profiles of \( \text{O I} \) \( \lambda 7771 \) and 7775. The fit between observations and the model can be seen in Fig. 5.5 and the resulting map is shown in Fig. 5.2. In Fig. 5.5, both lines show clear variability in the Stokes \( I \) profile. The agreement between observations and the model is generally good for the Stokes \( IV \), although there are some phases where the amplitude is
not in agreement. This difference is likely due to non-LTE effects in these oxygen lines. If this small discrepancy seen in Stokes \( V \) is due to non-LTE effects and/or stratification, these effects can also influence the Stokes \( I \) profile, however the code will still find good agreement in Stokes \( I \) by adjusting the abundance accordingly, but such adjustments would have minimal effect on the Stokes \( V \) fit. The end result of this is that whilst the abundance contrast will be realistic, the absolute abundance scale may not, but because it is the contrast that is of greatest importance, comparison with the magnetic field is still possible even if the absolute abundance is not fully realistic.

The oxygen abundance ranges from -2.5 dex to -5.5 dex (solar value = -3.21). A high abundance feature is clearly seen between phases of 0.80 to 0.20, comparing it to the magnetic topology from Silvester et al. (2014) (shown in Fig. 5.1), the highest concentration of oxygen is found at latitudes between -45\(^\circ\) to +45\(^\circ\) and at longitudes between 300\(^\circ\) and 60\(^\circ\). This area aligns with the negative component of the radial magnetic field which is found at a similar latitude and longitude.

Silicon chemical maps were produced using the Stokes \( IV \) line profiles of Si II \( \lambda \) 5055, 5978, 6347 and 6371. The fit between observations and the model can be seen in Fig. 5.5 and the map is shown in Fig. 5.2. In Fig. 5.5, the lines show very slight variability in the Stokes \( I \) profile and there is good agreement between the observations and the model in both Stokes \( I \) and \( V \). Looking at the distribution of silicon, the abundance values vary from -2.0 dex to -6.0 dex (solar value = -4.49), the higher concentration areas are found at upper latitudes between -20\(^\circ\) and 60\(^\circ\) distributed over all longitudes. Comparing this to the magnetic map, we see the higher abundance areas correlate somewhat to the areas where the radial magnetic
field (regardless of sign) has a field modulus of approximately 2 kG, close to the stellar equator between a range of latitudes of -45° to +45°. The lowest abundances are seen in locations where the magnetic field is weakest.

A chlorine map was computed using line profiles of Cl i λ 4794, 4819 and 4904. The fit between observations and the model can be seen in Fig. 5.5 with strong variability in Stokes I with the line profile almost disappearing at a phase of 0.3. There is good general agreement between the observations and model for Stokes I and V. The reconstructed map is shown in Fig. 5.2. Looking at the chlorine map, the abundance ranges from -3.0 to -7.0 dex (solar value = -6.54), and somewhat similar to what was seen in oxygen, the high abundance structure is located at latitudes between -45° to +25° and longitudes between 320° and 40°. Comparing this to the magnetic field we find a similar pattern to that seen for oxygen, with the higher concentration areas aligning with the negative part of the radial magnetic field.

5.6.2 Iron Peak Elements - Titanium, Chromium and Iron Maps

Titanium chemical maps were produced using Stokes IV line profiles of Ti II λ 4163, 4468 and 4571. The fit between observations and the model can be seen in Fig. 5.6 and the map is shown in Fig. 5.2. In Fig. 5.6, all lines show very strong variability in the Stokes I profile, with a clear change in the line shape between phases. There is reasonable agreement between observations and the model, but at some phases the wings in Stokes I are not well fit. Because we are fitting multiple line profiles simultaneously the final fit is a compromise between the different line profiles, therefore a perfect fit is not always expected. If a similar discrepancy is seen
systemically in all lines, then this would be of concern. The titanium abundance ranges from -4.0 dex to -8.0 dex (solar value = -7.02). The abundance structure of titanium is somewhat similar to that of the silicon map, but with the higher abundance areas limited to latitudes of 0° to 60° and spread over all longitudes. When comparing this distribution with the magnetic field map, the higher abundance areas appear to correlate with areas where the radial field modulus is approximately 2 kG, regardless if it is the positive or negative component on the radial sphere which is seen between latitudes of -45° to 45°. There are small discrete low abundance areas seen at longitudes 120° and 260° extending to just above the stellar equator. These are areas where the radial field is small in the transition region between negative and positive components.

Chromium chemical maps using Stokes IV were produced using the line profiles of Cr II λ 4588, 4592, 5246 and 5279. The fit between observations and the model can be seen in Fig. 5.6 and the map is shown in Fig. 5.2. In Fig. 5.6, all lines show very strong variability in the Stokes I profile, with a clear change in the line shape between phases. Like the titanium fits, there is good agreement between observations and the model at most phases, some wings in Stokes I are not well fit, but as described for titanium, this situation arises when fitting multiple lines. The chromium abundance ranges from -3.5 dex to -7.0 dex (solar value = -6.37). The abundance structure is distinct from the light element abundance patterns, with a very large low abundance feature located below the stellar equator, seen at latitudes -80° to -10° and at longitudes between 300° and 60°. On the reverse side there is a slightly smaller low abundance structure seen at latitudes -60° to -10° and at longitudes between 140° and 220°. The larger of these two structures appears to be located at an area where
the radial magnetic field is negligible, this area is found at latitudes -80° to -10° and a longitude between 300° and 60°. Higher abundance areas are seen at latitudes -30° to 30° and at longitudes between 60° and 120° and 220° and 300°. These trace the magnetic field in the areas where the radial field modulus is on the order of 2 kG.

Iron chemical maps were constructed using the Stokes IV line profiles of Fe II λ 4555, 5030 5032 and 5035. The fit between observations and the model can be seen in Fig. 5.6 and the map is shown in Fig. 5.2. In Fig. 5.6, all lines show variability in the Stokes I profile, although the profile change is not as clear as with titanium and chromium. There is good agreement between observations and the model at most phases, there are phases where the core of the Stokes I profiles are perfectly fit. As was described in the case of titanium, these small discrepancies can occur as a result of the compromises the inversion code has to make to fit all lines simultaneously. The iron abundance ranges from -2.0 dex to -5.5 dex (solar value = -4.54), with the abundance structure being very similar to that found for chromium. This similarity extends to a large low abundance structure seen at latitudes -80° to -10° and at longitudes between 320° and 50°, which again corresponds to the location where the radial magnetic field is minimum. One difference is the larger abundance area extends to a slightly lower latitude than is seen in the case of chromium, seen at latitudes -40° to 30° and at most longitudes. Comparing to the magnetic field map, again this is located in areas where the radial field modulus is on the order of 2 kG.
5.6.3 Rare Earth Elements - Praseodymium, Neodymium, Europium

Praseodymium chemical maps were produced using the Stokes $IV$ line profiles of $\text{Pr} \, \lambda 5299, 5765$ and $7030$. The fit between observations and the model can be seen in Fig. 5.7 and the map is shown in Fig. 5.3. In Fig. 5.7, the lines show strong variability in the Stokes $I$ profile, with a clear change in the line shape between phases. There is agreement between observations and model at most phases, with the $\lambda 5299$ wing not fully fit. The praseodymium abundance ranges from -5.0 dex to -8.0 dex (solar value = -11.33), the abundance distribution shows two areas of higher abundance at latitudes between $-30^\circ$ to $+30^\circ$, and at longitudes between $20^\circ$ to $100^\circ$ and $300^\circ$ to $340^\circ$. When comparing to the magnetic field a direct correlation is not clear, but the location of the high abundance areas overlap with areas where the negative radial field is greatest and the depleted areas appear to trace the magnetic equator.

Neodymium chemical maps were produced using the Stokes $IV$ line profiles of $\text{Nd} \, \lambda 4927, 5050, 5677$ and $6145$. The fit between observations and the model can be seen in Fig. 5.7 and the map is shown in Fig. 5.3. In Fig. 5.7, the lines show some variability in the Stokes $I$ profile, with subtle line shape changes between phases. There is good agreement between observations and model at most phases. The neodymium abundance ranges from -5.0 dex to -9.0 dex (solar value = -10.54). Similar to titanium, the higher abundance areas are found at latitudes of $15^\circ$ to $60^\circ$ and spread over all latitudes. When comparing this distribution with the magnetic field map, the higher abundance areas appear to correlate with areas where the radial field modulus is $2 \, \text{kG}$, regardless if it is the positive or negative component. As with titanium there are also small discrete low abundance areas seen at longitudes $120^\circ$
Table 5.3: Summary of the result of the derived abundance maps for $\alpha^2$ CVn

<table>
<thead>
<tr>
<th>Element</th>
<th>Location of enhancement</th>
<th>Location where depleted</th>
<th>General structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>At negative pole</td>
<td>Where field is weak and positive pole</td>
<td>Large single spot / structure</td>
</tr>
<tr>
<td>Si</td>
<td>Where field $\approx 2$ kG</td>
<td>Where field is weak</td>
<td>Distributed at high latitudes</td>
</tr>
<tr>
<td>Ti</td>
<td>Where field $\approx 2$ kG</td>
<td>Where field is weak and positive pole</td>
<td>Distributed at high latitudes</td>
</tr>
<tr>
<td>Cr</td>
<td>Where field $\approx 2$ kG</td>
<td>Where field is weak</td>
<td>Distributed at high latitudes</td>
</tr>
<tr>
<td>Fe</td>
<td>Where field $\approx 2$ kG</td>
<td>Where field is weak</td>
<td>Distributed with two depleted spot regions</td>
</tr>
<tr>
<td>Pr</td>
<td>At negative pole</td>
<td>Magnetic equator and positive pole</td>
<td>Two spots / structures</td>
</tr>
<tr>
<td>Nd</td>
<td>Where field $\approx 2$ kG</td>
<td>Where field is weak</td>
<td>Distributed at high latitudes</td>
</tr>
<tr>
<td>Eu</td>
<td>At negative pole</td>
<td>Magnetic equator and positive pole</td>
<td>Large single spot / structure</td>
</tr>
</tbody>
</table>

and 260° extending to just above the stellar equator, areas where the radial field is small, in the transition region between negative and positive components.

Europium chemical maps were produced using the Stokes $IV$ line profiles of Eu II $\lambda$ 6437 and 6645. The fit between observations and the model can be seen in Fig. 5.7 and the map is shown in Fig. 5.3. In Fig. 5.7, both lines show strong variability in the Stokes $I$ profile, with generally good agreement between observations and model at most phases. It should be noted that these europium lines have hyperfine and isotopic structure (which is not included in the inversions). An experiment was performed to see if by including these structures in the inversion, an improved fit was found. The result was that the fit to the Stokes $IV$ profiles was no better than when hyperfine and isotopic structure were ignored and the resulting abundance map was very similar. The europium abundance ranges from -5.5 dex to -8.0 dex (solar value = -11.53), with the structure very similar to that seen for oxygen. The highest concentrations are found at latitudes between -45° to +45° and a longitude between 320° and 40° which aligns with the negative part of the radial magnetic field. There is an extended low abundance area (seen at longitudes between 100° and 300°) which is seen at the location where the radial magnetic field is negligible and it traces the boundary around the magnetic equator.
Figure 5.5: Comparison between observed (dots) and synthetic (solid curve, blue) Stokes $IV$ parameter spectra of $\alpha^2$ using INVERS10. The solid curves shows the best fit to O, Si and Cl lines.
Figure 5.6: Comparison between observed (dots) and synthetic (solid curve, blue) Stokes \(IV\) parameter spectra of \(\alpha^2\) using INVERS10. The solid curves shows the best fit to Ti, Cr and Fe lines.
Figure 5.7: Comparison between observed (dots) and synthetic (solid curve, blue) Stokes IV parameter spectra of $\alpha^2$ using INVERS10. The solid curves shows the best fit to Pr, Nd and Eu lines.
5.7 Conclusion

We presented a slightly modified and updated magnetic field map of $\alpha^2$ CVn which was required to improve the fit between the model line profiles and observed line profiles in Stokes $V$ for the elements oxygen and chlorine. We adopted this magnetic map as our fixed field map in all our inversions. This new magnetic map which was reconstructed using a lower value of magnetic field regularisation than used by Silvester et al. (2014), is similar to what is seen by Silvester et al. (2014), with an additional small magnetic spot-like region seen at phase 0.00. The inclusion of oxygen and chlorine resulted in this new structures seen at phase 0.00. Overall we find that this updated map does not change the findings of Silvester et al. (2014).

Silvester et al. (2014) used the constraint that we only included lines with clear Stokes $QU$ signatures in the inversions, thus lines such as oxygen and chlorine were not considered. It should be noted that the map that derived by Silvester et al. (2014) is not what we would consider the ultimate magnetic field map of $\alpha^2$ CVn, rather the most realistic representation of the magnetic field taking into account the aforementioned constraint. One consideration we did not previously take into account, was to include lines which exhibited strong abundance structures at opposing phases, where elements sample the stellar surface in a different way.

The resulting chemical abundance maps can be classified into distinct groups; those that accumulate close to the negative pole or the negative part of the radial field and those that are located near the stellar equator where the field is close to a field modulus value of around 2 kG. The results are summarised in Table 5.3.

The elements oxygen, chlorine and europium exhibit one large enhanced abundance structure seen between phases 0.80 to 0.20. This location correlates to the
negative radial field. The praseodymium abundance structure is similar to that of oxygen, chlorine and europium, but with an 0.20 phase offset and with a second enhanced structure. These praseodymium structures appear to only align with the negative radial magnetic field. Unlike the other five elements studied, these elements show enhanced abundances only near the negative radial field and not in the areas of the field which have a strong positive radial component. Furthermore both europium and praseodymium have low abundance areas which trace the magnetic equator.

Rice et al. (2004) studied the distribution of oxygen on the surface of the Ap star θ Aur and found that oxygen was lower in abundance around the magnetic equator and enhanced in bands around the magnetic poles. For the Ap star HD 3980 Nesvacil et al. (2012) found that in the case of oxygen there were circular areas of high abundance around both magnetic poles and that the abundance was depleted around the magnetic equator. In the case of the roAp HD 83368 (Kochukhov et al. 2004) they found oxygen enhanced at the magnetic equator. None of these studies found oxygen located only on one magnetic pole. Chlorine has rarely been mapped in an Ap star. Kochukhov at al. (2002) mapped chlorine on α² CVn and found a very similar distribution for chlorine as we find. For HD 3980 Nesvacil et al. (2012) report that the europium enhancement is located in spots between the magnetic poles and the magnetic equator. In the case of praseodymium they found high abundance regions in the area of the magnetic poles, in our case we find the areas of high abundance are located on the negative radial component and not the positive component of the field.

Iron and chromium were found to have depleted abundance at areas where the magnetic field is weakest and the higher abundance features trace the areas of the
magnetic field where the magnetic field modulus is on the order of 2 kG. This is somewhat consistent with what was found for Cr and Fe in HD 3980 where enhancements were found around the poles and the stellar equator (Nesvacil at al. 2012) and for ε UMa (Lüftinger et al. 2003) where both Cr and Fe were enhanced at the poles.

In the case of the elements silicon, titanium and neodymium the higher abundance areas are broadly distributed around the region where the field modulus is around 2.0 kG, but unlike chromium and iron the depleted abundance regions are larger and cover the lower half of the stellar sphere at most phases, where the field is consistently weak. Looking to previous studies, we find no clear correlation for these elements. Nesvacil et al. (2012) found for silicon on HD 3980 that areas of overabundance were located at various spots along the rotational equator between the magnetic poles and magnetic equator. In the case of titanium, Lüftinger et al. (2003) found that titanium in ε UMa was accumulated around the magnetic equator. Neodymium on HD 3980 was found to be concentrated at the magnetic poles and depleted at the magnetic equator (Nesvacil et al. 2012).

It is clear from our derived abundance maps that the magnetic field has an influence on the formation of horizontal abundance structure for all elements, but by looking at previous mapping result with the exception of Fe and Cr, there appears no clear correlation between our result and results for other Ap stars. In particular given elements don’t correlate to the same magnetic field regions between respective studies.

Michaud et al. (1981) predicted that in the absence of turbulence, rare earth elements should be concentrated where the magnetic field is horizontal (magnetic equator) and iron should be enhanced where the field is vertical (magnetic poles).
This is not clearly seen in our maps. In the case of the elements europium and praseodymium they are depleted at the magnetic equator and not enhanced. Whilst iron and chromium are enhanced near the poles, the region of enhancement is much larger, almost to the magnetic equator in some cases. For silicon Alecian & Vauclair (1981) predicted that enhancements will be seen at the magnetic equator. Again this is not seen in our maps, with silicon being distributed over a large area. Alecian & Stift (2010) comment that if the magnetic field of a star is not purely dipolar, the proposed belt-like enhancements of some elements at the magnetic equator may be too small to be detectable using current Doppler mapping techniques and would result in the appearance of spots in these regions instead. Such spots are not seen in our maps at the magnetic equator. It is also interesting to note that we don’t find any abundance structures for any of the elements studied which we can directly relate to the small-scale structures of the magnetic field modulus.

This all suggests that important details are missing from the theory relating to the formation of horizontal abundance structures and the magnetic field and that a better understanding of the vertical abundance structure and the impact the magnetic field has on this structure, is ultimately required. To obtain further observational constraints, it would be useful to map additional Ap stars. This will be investigated in future papers using the Stokes $IQUV$ data of other Ap stars, in part collected by Silvester et al. (2012). In addition mapping the roAp star HD 24712 in Stokes $IQUV$ (by Rusomarov et al. 2013) is ongoing, which could potentially give additional constraints.
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REFERENCES


Chapter 6

Conclusions

6.1 Summary of Results

In chapter 3 we described the data set obtained in all four Stokes parameters for a sample of Ap stars. We were able to show that the ESPaDOnS and Narval data set greatly improved the level of detail in which Ap stars could be studied and that the obtained spectra were of much higher resolution and signal-to-noise than the previous generation of spectropolarimetric data obtained with MuSiCoS. These data were shown at the same time to be consistent with the previous works. One important finding of chapter 3 was that we confirmed the stability of the global properties in the magnetic fields of these Ap stars. Over a large time span the magnetic fields of these stars have remained constant with very little variation in the longitudinal magnetic field and the linear polarisation. This work showed that both ESPaDOnS and Narval provided a sufficient level of data quality for use in tomographical studies using magnetic Doppler imaging.

In chapter 4 we investigated the role different atomic lines have on the diagnosed
magnetic field topology, we used the bright Ap star $\alpha^2$ CVn for this investigation using the inversion code INVERS10. The first step was to confirm that the magnetic field we derive using strong iron lines agreed with the magnetic field previously derived using MuSiCoS data. Indeed we were able to show that was the case. We then diagnosed a series of magnetic maps from different groups of iron and chromium lines. We showed that regardless of which lines were chosen to be included in the inversion, the resulting magnetic field was generally consistent with the magnetic field found from strong iron lines. This indicates that the magnetic field topology derived for $\alpha^2$ CVn is a realistic representation of the field and not a spurious result caused by the technique.

In chapter 5 we mapped 9 different chemical elements looking for correlations between the formation of abundance structures and the magnetic field topology. We presented a slightly modified magnetic field map which was required for the lines of oxygen and chlorine to be correctly fit in Stokes $V$. With the chemical abundances we found two distinct groups, ones with high abundance structures covering both radial components and those covering just the negative radial field. This result shows that the magnetic field plays an important role in structure formation, but by comparison with previous mapping studies we found no clear correlation between our work and that of other studies. In addition we also looked at the literature describing the theoretical frameworks which lead to the formation of these structures and we found no concrete agreements with what is seen in our maps. These differences suggest that the current theoretical framework for the formation of abundance structure is insufficient to explain what is observed. This has also been the conclusion of other recent mapping studies.
6.2 Discussion

The principal results of the investigations undertaken in this thesis and their consequences are as follows; firstly the evaluation of the instrument performance of ESPaDOnS and Narval for all Stokes parameter observations which was performed in chapter 3, is one of the most detailed instrument monitoring studies of the both ESPaDOnS and Narval spectropolarimeters to date. Because the observational data were taken over a 4 year period, this afforded a relatively long time base to assess instrument changes and improvements in performance. This also provided vital evidence that both instruments are performing to specification in all four Stokes parameters and that the level of stability of both ESPaDOnS and Narval make them suitable for long timescale projects, something that is of great importance to current and future time-series observations. The resulting paper from this work has already been cited favourably as a source for a detailed description of ESPaDOnS and NARVAL, including a comparison of their performances and recent instrumental upgrades (See Petit et al. 2014).

In chapter 3 we were then able to confirm the global magnetic stability of the 7 target Ap stars using both circular and (in most cases) linear polarization. With our new data being consistent with data taken 10 years previously (and in some cases 30 years previously), this provided a unique and important direct observation of this magnetic stability.

Very important information arose from measuring the longitudinal magnetic field from the new data set. We found that even though the new data were of very good quality, high signal-to-noise and that the magnetic fields in the target stars were strong, the LSD analysis was still quite sensitive to the normalisation, and the
measured magnetic field was rather sensitive to the integration ranges chosen, with variations of sometimes on the order of 100 gauss with very small changes of the integration range. This kind of sensitivity was to be expected for stars with faint magnetic fields or polarization signatures, but not as much for stars with such strong polarization signatures. This was a key finding, telling us that extreme care must still be taken with all stages of analysis to ensure consistent results at this level of precision, even when the polarization signatures are strong.

In summary, the results of chapter 3 were of great importance to the next part of the project; the mapping of an Ap star. If the instruments had not been stable or the measurements had been inconsistent, that would have proved problematic for the other stages of the project. In essence the observations part of the project was a “proof of concept” for using ESPaDOnS and Narval for Doppler imaging, magnetic Doppler imaging and other detailed studies.

In the same way as the observations proved the instrument reliability, the results presented in the magnetic field mapping of α² CVn paper (chapter 4) proved the reliability of the magnetic Doppler imaging process using INVERS10. We found that consistent magnetic field maps could be derived from completely different data sets taken from different instruments, something that had not been done before using the same inversion code. We were also able to show that different lines could be used to create very similar magnetic field maps, this sort of investigation had not been previously attempted in such detail. These findings help counter concerns (such as those of Stift et al. 2012) that the field topology derived using such techniques could be spurious and not a real representation of the magnetic field. This has greater implications for future studies using MDI codes similar to INVERS10, as it reaffirms
the suitability of such techniques for diagnosing the field topology and abundance structures in Ap stars. We were also able to present for the first time a comparison between two MDI maps taken of the same star in Stokes $IQUV$ with data taken over a decade apart, this provides a unique confirmation of magnetic stability of both small and larger field structures in Ap stars.

In chapter 5 we derived abundance maps for 9 elements in Stokes $IV$, although the ESPaDOnS / Narval spectra had a large wavelength coverage, finding lines that were not blended or not too shallow for MDI was not a simple task, from the proverbial forest of lines we found just a handful that were suitable. Regardless of this complication all of our maps were derived using at least two different spectral lines (of the same element), and the result showed convincingly that our understanding of the processes that lead to the formation of abundance structures needs to be refined and theoretical models which offer predictions also need to be improved. In presenting an updated magnetic map, whilst this does not change our result presented in chapter 4, we clearly demonstrate that the mapping of a star such as $\alpha^2$ CVn is an ongoing process, with refinements being made all the time. Importantly we learnt that it is useful when mapping the field topology to include lines which present abundance structures at opposing phases from one another, to get a more refined picture of the field topology.

The summation of this work could be described as a verification that current instrumentation and imaging techniques, even when considering their limitations, are sufficiently sophisticated to allow detailed surface mapping of chemically peculiar stars. This indicates that it is now the theory used to explain these phenomena that needs to become more sophisticated. This provides a solid foundation for future
projects, described below.

6.3 Future Work

Through the work in this thesis we have proved that the spectropolarimetric data from ESPaDOnS and Narval is of sufficient quality and constancy for magnetic field and abundance mapping. We have shown that our current mapping techniques are sound and allow us to reconstruct consistent magnetic maps regardless of the spectral lines used. We have also shown that our abundance structure maps indicate that the current theories behind their formation and their relation to the magnetic field needs refining. These facts combined provide an exciting and stable foundation to continue with magnetic field and abundance mapping of other Ap stars.

The dataset in this project contains 6 remaining stars, 3 of which already have complete phase coverage or near complete phase coverage: 49 Cam, HD 32633, HD 71866. To map these stars only requires an investigation of which lines in each star are suitable for MDI mapping. Because each star is different due to the different stellar parameters (such as temperature and gravity), line choice has to be considered on a star by star basis. Future mapping could lead to some very interesting results:

**Differing field complexities of Ap Stars** - Magnetic mapping of 49 Cam, HD 32633, HD 71866 will allow us to expand the sample of Ap stars which have had detailed magnetic field topology analyses. Mapping of the roAp star HD 24712 revealed a magnetic field which appears to be a near pure dipole (Lüstinger et al. 2010 and Rusomarov et al. 2013). This is different to what was found for $\alpha^2$ CVn and 53 Cam which showed detailed substructure. It would be of great interest to see if the
remaining Ap stars in our target list had magnetic field topologies closer to that of HD 24712 or \( \alpha^2 \) CVn and 53 Cam.

**The importance of stellar parameters on the magnetic field** - By mapping stars of different temperatures and masses, we can also probe how these parameters affect the formation or topology of the magnetic field and the atmospheric abundance structures. This may also give an insight into the differing field complexities.

**Differing abundance structures** - To help refine the current theory of abundance structure formation, it would be useful to obtain more data relating to how abundance structures are distributed with respect to the magnetic field. By mapping the abundance structures of more stars, we can expand the amount of data available and potentially give more clues as to why there is currently no clear correlation between a given chemical element and the magnetic field topology between stars.
6.4 References


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