HIGH RESOLUTION SIGNAL PROCESSING TECHNIQUES FOR ENHANCING GPS RECEIVER PERFORMANCE

by

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A thesis submitted to the Department of Electrical and Computer Engineering
In conformity with the requirements for
the degree of Doctor of Philosophy

Queen’s University
Kingston, Ontario, Canada
(March 2015)

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Abstract

Despite the major advances in signal processing methods used nowadays, GPS receivers still face substantial challenges, such as multipath, which remains a dominant source of ranging error. The presence of GPS multipath influences the acquisition and tracking modules inside the receiver leading to biased measurements or loss of lock of the GPS satellite signal. Consequently, GPS receivers cannot provide reliable position, velocity and time solutions. The main objective of this thesis is to introduce high resolution signal processing techniques to enhance the performance of the GPS receiver in harsh navigation environments. This research proposes a two-tier approach. The first introduces a robust spectral estimation method to acquire the carrier frequency accurately after the completion of the coarse acquisition of the GPS signals. The second method targets improving the code delay estimation inside the tracking loops of the GPS receiver to mitigate the challenging closely spaced multipath effect. In this research, a SPIRENT GPS simulator is utilized to examine the performance of the proposed methods against the state of the art techniques. The proposed fine acquisition method uses Gram-Schmidt orthogonalization to provide robust spectral estimation of satellite Doppler frequency. The performance of the proposed fine acquisition method is evaluated against both the computational load and the noise effects. The results show that the proposed method outperforms the FFT-based fine acquisition methods resulting in 70% improvement in the acquisition accuracy at low SNR of -15 dB using only short window size of 1 ms. Moreover, this method speeds up the tracking loops in order to correctly lock the carrier Doppler shift frequency. Furthermore, the proposed multipath mitigation technique, which operates in the tracking module, is based on fast orthogonal search to enhance the code delay estimation for GPS receivers. The proposed tracking module results in an average improvement of 32% in the positioning accuracy when compared to other multipath mitigation techniques. This thesis research resulted in computationally efficient GPS acquisition module that can work in low SNR environment for limited resource GPS receivers. In addition, a robust tracking module capable of mitigating challenging GPS multipath effects is also introduced in this thesis.
Acknowledgements

My full gratitude and appreciation goes to my supervisor, Dr. Aboelmagd Nourelldin, for his professional supervision, patience, flexibility, genuine caring and his continuous support since the very beginning of my studies. Despite all the challenges I faced during the journey of my research studies, Dr. Nourelldin showed unlimited help and support in overcoming these obstacles. His total belief in me was such a great motivation that pushed me to complete my thesis to the fullest. The credit also goes to my co-supervisor Dr. Michael J. Korenberg for his encouragement and guidance throughout my studies. Dr. Korenberg introduced me to the topic of Fast Orthogonal Search and offered me innovative ideas and important guidelines during my studies. His advice was always a source of motivation.

I would like to express my thanks to my sincere friend Mohamed Moussa for his genuine friendship and his support. I would also like to express my appreciation to Dr. Ahmed M. Kamel for his help in several technical aspects of my research. Many thanks to Abdalla Osman, Umar Iqbal, Ali Massoud and my colleagues in Navigation and Instrumentation research group at Royal Military College of Canada. I also wish to thank Dr. Mohamed Youssef for introducing me to Dr. Nourelldin and helping me when I first came to Canada.

I am also thankful to NovAtel Inc. for their help and for facilitating the use of the front-end FireHose GPS receiver.

Finally, I have to mention my father, whom I lost in the midst of this challenging research journey. I owe him everything I am today. I would also like to thank my mother Azza and my sister Dina for being a source of encouragement and love and for bearing my absence for the past years. Special thanks are dedicated to my uncles Tarek Abbas and Adel El-Akkad for caring for my family during my absence. I can’t end this acknowledgment without thanking my beloved
wife Rana for her endless love, support and help. Together we went through this challenging experience and together we made the dream come true.
Dedication

To my father Essam Tamazin

&

My mother Azza Fahmy
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<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BW</td>
<td>Bandwidth</td>
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<tr>
<td>C/A</td>
<td>Coarse/Acquisition</td>
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<tr>
<td>C/N₀</td>
<td>Carrier To Noise Density Ratio</td>
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<tr>
<td>CAF</td>
<td>Cross Ambiguity Function</td>
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<td>CLL</td>
<td>Carrier Lock Loop</td>
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<tr>
<td>CW</td>
<td>Continuous Wave</td>
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<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<td>DLL</td>
<td>Delay Lock Loop</td>
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<tr>
<td>DOA</td>
<td>Directional of Arrival</td>
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<tr>
<td>DOP</td>
<td>Dilution of Precision</td>
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<tr>
<td>ELS</td>
<td>Early-Late-Slope</td>
</tr>
<tr>
<td>EML</td>
<td>Early-Minus-Late</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FLI</td>
<td>Frequency Lock Indicator</td>
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<td>FLL</td>
<td>Frequency Lock Loop</td>
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<td>FOS</td>
<td>Fast Orthogonal Search</td>
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<td>GLONASS</td>
<td>Global Navigation Satellite System</td>
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<td>GNSS</td>
<td>Global Navigation Satellites System</td>
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<td>GPS</td>
<td>Global Positioning System</td>
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<td>Abbreviation</td>
<td>Full Form</td>
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<td>GS</td>
<td>Gram-Schmidt</td>
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<tr>
<td>HDOP</td>
<td>Horizontal Dilution of Precision</td>
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<tr>
<td>HRC</td>
<td>High Resolution Correlator</td>
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<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
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<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
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<td>KF</td>
<td>Kalman Filter</td>
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<tr>
<td>LHCP</td>
<td>Left Hand Circularly Polarized</td>
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<td>LMM</td>
<td>Land Mobile Multipath</td>
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<td>LNA</td>
<td>Low Noise Amplifier</td>
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<td>LOS</td>
<td>Line of Sight</td>
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<td>LS</td>
<td>Least Square</td>
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<td>LUT</td>
<td>Look-up-Table</td>
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<td>MCS</td>
<td>Master Control Station</td>
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<td>Multipath Estimation Delay Lock Loop</td>
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<td>MET</td>
<td>Multipath Elimination Technique</td>
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<td>ML</td>
<td>Maximum Likelihood</td>
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<td>Mean Squared Error</td>
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<td>NavINST</td>
<td>Navigation and Instrumentation</td>
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<td>NCO</td>
<td>Numerical Controlled Oscillator</td>
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<td>Non-Line-Of-Site</td>
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<td>PAC</td>
<td>Pulse Aperture Correlator</td>
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<td>PI</td>
<td>Pre-detection Integration</td>
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<td>PLL</td>
<td>Phase Lock Loop</td>
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<td>Abbreviation</td>
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<tr>
<td>PRN</td>
<td>Pseudo random Noise</td>
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<td>PSD</td>
<td>Power Spectral Density</td>
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<td>PVT</td>
<td>Position, Velocity and Time</td>
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<td>PWV</td>
<td>Perceptible Water Vapor</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<td>RFID</td>
<td>Radio Frequency IDentification</td>
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<td>RHCP</td>
<td>Right Hand Circularly Polarized</td>
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<td>RMS</td>
<td>Root Mean Square</td>
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<td>SC</td>
<td>Strobe Correlator</td>
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<td>SCC</td>
<td>System Control Center</td>
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<td>SF</td>
<td>Scale Factor</td>
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<td>SIS</td>
<td>Signal-in-Space</td>
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<td>SNR</td>
<td>Signal To Noise Ratio</td>
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<td>SV</td>
<td>Satellite Vehicle</td>
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<tr>
<td>TOA</td>
<td>Time-of-Arrival</td>
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<tr>
<td>VDOP</td>
<td>Vertical Dilution of Precision</td>
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<tr>
<td>WSS</td>
<td>Wide Sense Stationary</td>
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<td>ZP</td>
<td>Zero Padding</td>
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Chapter 1

Introduction

1.1 Motivations

Global Positioning System (GPS) technology plays an important role in positioning and navigation applications due to the increasing demands for a positioning system that can provide continuous, accurate and reliable position. However, this may not always be possible due to different environmental effects on GPS signals such as attenuation, fading and multipath [1]. Therefore, efforts need to be exerted to make the civilian part of the system more accurate, reliable and available for challenging navigation environments.

Despite the major advances in signal processing methods and technologies used nowadays inside the GPS receivers, multipath remains a major source of ranging error [2-4]. The multipath phenomena refers to receiving replicas of a reflected and diffracted signal, which causes distortion of the Line-Of-Sight (LOS) GPS signal. The delay of multipath signal depends on the extra time of the travelled path by the reflected signal, which is longer than the direct signal’s path. In deep urban canyons, the high-rise buildings and surrounding objects near the receiver cause the reception of the indirect (reflected) signals in addition to the direct LOS signal. This effect can significantly distort the shape of the correlation function used for Time-Of-Arrival (TOA) estimation inside the GPS receiver [5]. One specific circumstance in multipath propagation is the situation when a LOS component is overlapping with one or several short-delay
multipath components making the TOA estimation process more complicated [6]. This short-delay multipath scenario commonly happens in urban environments. Due to inaccurate LOS delay estimation in multipath environments, receivers may not provide reliable Position, Velocity and Time (PVT) solutions. Moreover, in recent years, GPS receiver chips are found into the most small autonomous devices such as hand-held GPS receivers, smartphones, digital cameras, smart watches and portable tracking devices. These products differ in design and functionality but they all rely on batteries to operate. One way to maximize the operational field life of batteries is to reduce the computational load and complexity of the signal processing algorithms implemented inside the GPS receiver [7]. Users utilize GPS-equipped mobile devices in a diversity of environments such as open sky, forest, rural, suburban and urban canyon environments. When the receiver is challenged to acquire the GPS satellite signal, it consumes larger amounts of power in the acquisition and re-acquisition signal processing modules [8]. Therefore, an accurate, reliable and computationally efficient acquisition algorithm becomes critical for limited resource navigation applications.

The main focus of this thesis is the improvement of the acquisition process of limited resource GPS receivers and the development of a high resolution tracking technique to mitigate multipath effects with a focus on dense multipath environment such as urban areas.
1.2 Limitations of the Previous Work

The GPS signal acquisition is the first signal processing operation performed by the receiver. The purpose of the acquisition stage is to detect the satellite pseudo-random noise (PRN) in view and provide a rough estimation of the code delay and the Doppler frequency of the incoming signals [1]. The acquisition process is performed by correlating the incoming signals with local replica for each PRN code across all possible combinations of code delay and Doppler shift. Once a satellite signal is detected, the necessary parameters can be obtained and passed to the signal tracking process [9]. Thus, the signal acquisition is a time-consuming procedure inside the GPS receiver and its accuracy has direct influence on the tracking performance.

In order to improve the accuracy of the acquisition, two approaches have been widely addressed in the literature [10-13]. The first approach correlates long data in one step [13]. The second approach, known as coarse-to-fine acquisition method, consists of two stages [11]. The first stage uses short data to coarsely estimate the code phase and the carrier frequency of the visible satellites. The second stage strips off the Coarse/Acquisition (C/A) code from the correlated data and uses long C/A code stripped data to accurately estimate the carrier frequency. Since the second approach is faster and more accurate than the one step approach, it is commonly implemented in both hardware and software GPS receivers.

There are several types of the second approach in the literature [10-12]. All types are common in the first coarse estimate stage, which uses short data length to remove the navigation data transition effect. However, the fine acquisition stage differs from one...
type to another. The first type depends on the carrier phase difference but it is subjected to ineffectiveness in case of weak GPS signal [14]. The second type utilizes the Fast Fourier Transform (FFT) nevertheless it has a heavy computational load when using long data records [15]. Another fine method uses the Frequency Locked Loop (FLL), however, the FLL needs time to become steady before the transition of the FLL to the Phase Locked Loop (PLL) [11].

Concerning the multipath mitigation techniques, several multipath mitigation approaches have been introduced in the literature [3, 16-25]. Multipath mitigation techniques can be carried out both spatially and temporally [18]. Spatial processing is related to the adaptive antenna array signal processing for beamforming and Direction-of-Arrival (DoA) estimation. The temporal processing can be performed in code and carrier tracking loops inside the GPS receiver by the use of signal processing techniques for mitigating multipath. The use of spatial multipath mitigation techniques may require extra manufacturing cost and system complexity [26]. Therefore, the focus in this thesis is mainly dedicated to the correlation-based multipath mitigation techniques as being the most commonly used in the commercial GPS receivers.

The most familiar code tracking algorithms used inside the commercial GPS receivers are the classical delay tracking techniques. These techniques include feedback delay estimator loop known as the Delay Lock Loop (DLL) and uses special correlators and discriminator functions. The DLL estimates the code delay by maximizing the correlation between the incoming signal and the local code replica [27]. The structure of the DLL
consists of one or more correlators, a non-linear discriminator for evaluating the code delay error function, a loop filter for improving code delay estimation and a numerically controlled oscillator [1].

In standard DLL discriminator function, two correlators called Early (E) and Late (L) correlators are spaced at a code chip width from each other and they are used to design the discriminator function. The code delay is estimated by balancing the amplitude/power on the E and L correlators. This method is called Early-Minus-Late (EML) technique [1]. The classical EML fails to mitigate the multipath effects. Thus, several enhanced EML-based techniques are introduced in the literature [3] to reduce the multipath effects specially the short delay multipath scenarios.

One method of these enhanced EML techniques is based on reducing the EML spacing, which reduces the multipath impact. This method is known as narrow correlator. The narrow correlator uses spacing in the range of 0.05 to 0.2 code chips [28]. This method requires a wideband front-end (i.e. Bandwidth (BW) > 4 MHz) in order to be able to produce a sharp correlation function.

The Double-Delta technique is another group of discriminator-based DLL proposed to mitigate medium-to-long delay multipath scenarios [29]. This technique typically uses five correlators, which are two correlators for early, two correlators for late and one correlator for prompt [3]. Among the sets of special functions of double-delta technique are the Strobe Correlator (SC), the Pulse Aperture Correlator (PAC) and the High Resolution Correlator (HRC). The discriminator function of the SC is obtained as a
linear combination of two narrow correlator discriminators [30]. The combination of two discriminator functions allows resolving multipath rays with shorter excess delays but it causes noise amplification. The PAC is another implementation of the Double-Delta technique. PAC uses two sets of EML narrow correlators that are spaced at exactly double the width of the first correlator set [31]. The HRC utilizes several correlators to form a linear combination of correlator outputs that give up a net correlation function that is much narrower than the typical C/A code autocorrelation function [32].

The Early-Late-Slope (ELS) technique is another multipath mitigation technique, which is also known as Multipath Elimination Technique (MET). The general idea of ELS method is to determine the slopes of the early and the late sides of the correlation function’s central peak [33]. Based on the slope values, one can calculate the error of code-phase, which goes back to the DLL algorithm to adjust the pairs of correlators. Although the ELS technique has better performance to mitigate the multipath effects than the narrow correlator and HRC technique, it still has limitations in closely spaced multipath scenarios [22].

The Multipath Estimation Delay Lock Loop (MEDLL) is a parametric mitigation technique that attempts to minimize the effect of multipath by estimating the parameters that characterize the received signal. The NovAtel Inc. implements it for GPS receivers [34]. The MEDLL aims at jointly estimating the amplitudes, delays and phases of both the LOS component and multipath components. The MEDLL uses many correlators to determine accurately the shape of the distorted correlation function [24]. The main idea is
that the multipath components are estimated recursively and the effect of each path is removed before the estimation of the next. Several modified MEDLL techniques have been proposed in the literature to reduce the computational cost [35]. There are also several techniques that utilize external sensors or signals to aid the GPS receiver in degraded signal environments. These external systems are inertial sensors, optical systems and wireless networks such as WiFi, Bluetooth and Radio Frequency Identification (RFID). These techniques require their own infrastructure and are normally used for indoor localization.

1.3 Research Objectives

Having studied the limitation of the previous work, the main goal of this thesis is introducing high resolution signal processing techniques to enhance the performance of the GPS receiver in harsh navigation environments. In order to achieve this goal, two objectives are outlined as follows:

1. Proposing a robust fine acquisition method to provide accurate spectral estimation of the GPS satellite Doppler frequency with less computational load than the state-of-the-art fine acquisition techniques for limited resource GPS receiver.

2. Designing, testing and analyzing a high resolution multipath mitigation method which outperforms the state-of-the-art methods while keeping reasonable computational load for navigation receivers.
1.4 Research Contributions

In order to achieve the objectives of this research, the contributions are divided into three main stages. The first stage is analyzing state-of-the-art fine acquisition and multipath mitigation techniques and developing a GPS software receiver, which is a necessary tool to achieve the goal of this thesis. The software receiver is used to process the raw GPS data using different acquisition and tracking algorithms. Moreover, it is essential to examine the performance and confirm the robustness of the proposed methods in comparison to the state-of-the-art techniques. The second stage can be summarized as follows:

- Introducing a robust fine acquisition method based on orthogonal search to improve the acquisition performance of the GPS receivers. This method provides robust spectral estimation of satellite Doppler shift with less computational load and complexity the state-of-the-art techniques. In addition, it adds several spectral estimation capabilities to the limited resource GPS receivers including high frequency resolution, elimination of spectral leakage and operation in challenging environments.

- Proposing an innovative GPS tracking technique to mitigate multipath effects in challenging environments such as in urban canyons where GPS receivers suffer from closely spaced multipath signals. The main contribution here is developing a new approach for multipath mitigation based on Fast Orthogonal Search (FOS),
which offers a higher resolution code tracking performance than that of the conventional techniques in harsh environments.

The third stage is examining the proposed methods using a realistic simulation environment to compare the performance of the proposed methods with the state-of-the-art acquisition and tracking algorithms under certain circumstances such as deep multipath and jamming scenarios. In this research a SPIRENT GSS8000 simulator is utilized to provide a controlled environment to examine the performance of the proposed methods using numerous realistic simulation scenarios.

1.5 Thesis Outline

The structure of this dissertation can be summarized as follows:

Chapter 2 reviews the principles of GPS technology with specific focus on its signal characteristics. The GPS observables and its errors are introduced at the end of this chapter.

Chapter 3 covers the technical background that is related to the GPS receiver designs. It provides an overview of the GPS receiver architecture and covers the basic concepts of the signal processing techniques in a GPS receiver with particular attention to signal acquisition and tracking. Phase and Frequency tracking algorithms are discussed and the benefits of using combined frequency and phase tracking are illustrated. The navigation estimation techniques are briefly reviewed. Finally, a review of current multipath mitigation algorithms is presented to provide a benchmark against which the proposed methods can be assessed.
Chapter 4 introduces a robust fine acquisition method based on orthogonal search to improve the acquisition performance of the GPS receivers. At the beginning of the chapter, several fine acquisition methods are reviewed. Then, the proposed method is discussed in details. Finally, the performance of the proposed method is examined using both simulation and real experiments. Improvements gained using the proposed technique regarding the acquisition accuracy and processing time are addressed and discussed.

Chapter 5 presents a robust correlation-based multipath mitigation technique based on Fast Orthogonal Search (FOS) to mitigate multipath effects in challenging environments such as in urban canyons where GPS receivers suffer from closely spaced multipath signals. The chapter begins with a detailed discussion about the proposed method and the advantages of using the proposed method to mitigate the GPS multipath. In order to assess the performance of the proposed multipath mitigation technique under several operation scenarios, a controlled realistic simulation environmental is required. Thus, the chapter then discusses the capabilities of the simulator used (a Spirent GSS8000) to generate realistic simulation multipath environments through an advanced Land Mobile Multipath (LMM) model implemented in the SPIRENT SimGEN™ software. The experimental setup and the different simulation environments are discussed. A set of static and dynamic realistic simulation data processing results is then presented to compare the performance of the proposed technique with the ones of the conventional and advanced techniques introduced in Chapter 3 in terms of pseudorange and position errors. Finally, the robustness of the proposed methods in this thesis is compared to the
performance of the NovAtel Propak V2 commercial receiver under simulated jamming scenarios.

Chapter 6 concludes the contributions and key findings of the thesis. Recommendations for future work are given.
Chapter 2

Overview of Global Positioning System

2.1 Introduction

This chapter describes the Global Positioning System and its signal characteristics. First, it presents an overview of the GPS and describes its three parts: control, space and user segment. It then addresses the GPS modernization program. The chapter continues by describing the GPS signal characteristics and the GPS measurements errors.

2.2 Overview of Global Positioning System

GPS offers civilian and military users three-dimensional positioning and navigation services [36]. The user can determine his or her position and velocity by using the code pseudorange and carrier phase measurements. The GPS system uses the concept of TOA ranging to determine different parameters such as the user's position and velocity [1]. This concept requires measuring the time interval, referred to as the signal transit time, between the time the signal was transmitted from the satellite and the time it reaches the user's receiver. The transmitter-to-receiver distance can be obtained by multiplying the signal transit time by the speed of light.

For position determination, since distances are measured between the receiver and the position of four or more satellites at a known location, the user’s position may be calculated by trilateration concepts [1]. In actuality, three satellites can determine the
user’s position on the Earth’s surface but at least four satellites are required due to an additional estimation of the receiver clock offset.

Figure 2-1 illustrates the concept of positioning fixing by trilateration using signals from three satellites. There are four satellites available to fix the position and this helps to improve the accuracy of the solution. A single user-to-satellite range measurement defines a sphere centered on the satellite; a second sphere is defined by another range measurement from a different satellite, and the intersection of these two spheres defines a circle of position. A third range measurement defines an ambiguous pair of positions and a fourth range resolves the ambiguity and determines the clock bias. The mathematical solution of the GPS equations is covered in the literature [1, 27, 36-40].

![Figure 2-1: The concept of position fixing by trilateration using signals from three satellites. The user’s position is indicated by red dot [37]](image)
Military signals are less susceptible to interference and spoofing than civilian signals [1], thus, the position determined by using military signals can be more accurate than the position determined by using civilian signals. The GPS signal structure will be discussed in the following sections.

The GPS design includes three segments [39]: space segment which is a constellation of satellites, control segment which is ground-based control facilities and user segment which is user’s equipment. The ground segment consists of a master control station and monitor stations. The user segment consists of all the military and civilian receivers.

2.2.1 The Space Segment
The GPS space segment is made up of a constellation of satellites that continuously broadcasts radio signals to users. The United States of America (USA) is obligated to keep at least twenty-four operational baseline GPS satellites available 95% of the time [39]. The US Air Force has been flying 31 operational GPS satellites for the past few years to maintain coverage whenever the baseline satellites are serviced or decommissioned. GPS satellites fly in medium Earth orbit (MEO) at an altitude of approximately 20,200 km. Each satellite circles the Earth twice a day. The orbital period is about 12 h [39].

The satellites in the GPS constellation are arranged into six equally spaced orbital planes surrounding the Earth as shown in Figure 2-2. The satellites are placed into planes with a target inclination of 55° degrees [40]. This plane scheme ensures GPS users receive the most accurate navigation data at any time, at any place around the world. The satellites
broadcast radio signals that contain coded information and navigation data to enable a receiver to calculate pseudo-ranges and Doppler data to estimate the position and velocity of the user.

Referring to [41], In June 2011, the US Air Force successfully completed a GPS constellation expansion known as the "Expandable 24" configuration. Three of the 24 slots were expanded, and six satellites were repositioned, so that three of the extra satellites became part of the constellation baseline. As a result, GPS now effectively operates as a 27-slot constellation with improved coverage in most parts of the world.

Table 2-1 summarizes features of the current and future generations of GPS satellites, including Block IIA (2nd generation, "Advanced"), Block IIR ("Replenishment"), Block IIR (M) ("Modernized"), Block IIF ("Follow-on"), and GPS III [42].

Figure 2-2: The GPS Satellite Constellation [37]
Table 2-1: Summarizes features of the current and future generations of GPS satellites [42]

<table>
<thead>
<tr>
<th>Legacy Satellites</th>
<th>Modernized Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Legacy Satellites Image" /></td>
<td><img src="image2.png" alt="Modernized Satellites Image" /></td>
</tr>
<tr>
<td>Block IIA</td>
<td>Block IIR</td>
</tr>
<tr>
<td>6 Operational</td>
<td>12 Operational</td>
</tr>
<tr>
<td>- Coarse Acquisition (C/A) code on L1 frequency for civil users</td>
<td>- C/A code on L1</td>
</tr>
<tr>
<td>- Precise P(Y) code on L1 &amp; L2 frequencies for military users</td>
<td>- P(Y) code on L1 &amp; L2</td>
</tr>
<tr>
<td>- On-board clock monitoring</td>
<td>- On-board clock monitoring for enhanced jam resistance</td>
</tr>
<tr>
<td>- 7.5-year design lifespan</td>
<td>- 7.5-year design lifespan</td>
</tr>
<tr>
<td>- Launched in 1990-1997</td>
<td>- Launched in 1997-2004</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.2.2 The Control Segment

The overall control and maintenance of the GPS are the main responsibilities of the GPS control segment [39]. The control segment consists of a global network of ground facilities that are responsible for tracking the GPS satellites, monitoring their signal transmissions, performing analyses and sending commands and data to the GPS constellation.
Referring to [39], the current operational control segment includes a Master Control Station (MCS), an alternate master control station, 12 command and control antennas, and 16 monitoring sites. The locations of these facilities are shown in Figure 2-3.

![Figure 2-3: The Locations of the GPS Master Control Station, an Alternate Master Control Station, 12 Command and Control Antennas, and 16 Monitoring Sites](image)

2.2.3 The User Segment

The user receiver equipment tracks and receives the satellite signals. The purpose of GPS receivers is to process signals transmitted from GPS satellites, estimate the ranges and range rates from these signals and compute a PVT solution [44]. The architecture of the GPS receiver and the related signal processing techniques used will be discussed in details in Chapter 3.
2.3 GPS Signals Characteristics

Each GPS satellite continuously broadcasts radio navigation data on two carrier frequencies, the L1 carrier at 1575.42 MHz and the L2 carrier at 1227.60 MHz [39]. The GPS signals are generated using direct sequence spread spectrum (DSSS) modulation [27]. The GPS signals include the C/A code signal on the L1 carrier frequency and the precision- (P-) code signal on both L1 and L2 carrier frequencies. The C/A code is unencrypted and is associated with the standard positioning service (SPS) set aside for civilian use. The P-code signal is associated with the precise positioning service (PPS) and it is only intended for authorized (military) use. This code is further encrypted to the P(Y)-code for security reasons [45].

As shown in Figure 2-4, three components may be forming the GPS DSSS signal: a spreading waveform, a navigation data and a radiofrequency (RF) carrier. The spreading waveform is an adjoining series of rectangular pulses generated using a deterministic, digital PRN code. The minimum period between transitions in the spreading waveform is referred to as a chip denoted $T_c$ and the reciprocal of this period as the chipping rate, $R_c$, which is the clock rate of the binary PRN code. The PRN code for each C/A code signal is acquired from the family of length-1023 Gold codes [38], which are a family of sequences providing a guaranteed maximum cross-correlation value at the expense of a slight degradation in the auto-correlation function. The PRN codes have good correlation/cross-correlation properties, which are used to spread the navigation message and serve as the basic tool for measuring the transmission time from satellite to receiver [46]. The chipping rates of the P-code and C/A code are 10.23 and 1.023 M/sec,
respectively. Since, the GPS are Direct Sequence Code Division Multiple Access (DS/CDMA) system, therefore, a unique PRN is used for each signal type broadcast by each GPS satellite.

The navigation data consists of information on the satellite’s health, ephemeris (its position and velocity), clock bias parameters, and an almanac that is a less precise version of the ephemeris data but for all the satellites of the constellation [37]. The ephemeris and clock parameters are repeated every 30 s. The navigation waveform is a series of contiguous 20-ms, unit amplitude rectangular pulses generated at 50 Hz. The navigation message is broadcast at the much lower rate of 50 bps and therefore takes 12.5 min to transmit all of the information. The receiver must determine the position of the satellite in order to use the navigation message to convert the range measurements into the position and velocity of the user. For C/A code or P(Y)-code, the RF carrier is simply a pure sinusoid at L1 or L2 frequencies [39].

![Spreading Waveform](image1)

![Navigation Data](image2)

![RF Carrier](image3)

![DSSS Signal](image4)

**Figure 2-4: Direct Sequence Spread Spectrum Modulation [38]**
It should be noted that Figure 2-4 is not drawn to scale. It is difficult to explain the remarkable difference in time scales between the GPS signal components on a common plot. For example, for every single chip in the C/A code there are 1540 cycles of RF carrier and 20460 chips in the spreading waveform for every one data bit [45].

For the purposes of this thesis only L1 C/A code pseudoranges as well as L1 Doppler measurements are considered. A brief discussion of the GPS pseudorange and carrier phase models and their error sources is given in the following sections.

2.4 GPS Observations

2.4.1 Pseudorange Observations

The pseudorange code observations are generated when the satellite specific pseudo-random code (PRN) sequence is correlated with a locally generated replica. The pseudorange code measurement is relatively noisy, with a standard deviation of around 6 cm for high quality receivers [47], and is easily affected by interference caused by signal reflections, otherwise known as multipath. Multipath can result in errors as large as 15 m [48].

The pseudorange model can be written as:

\[ P^j_\alpha = \rho^j_\alpha + d\rho_n + c(d^j - dT_\alpha) + d^j_{ion} + d^j_{trop} + \varepsilon(p_{rx}) + \varepsilon(p_{mult}) \]

where \( P^j_\alpha \) is the pseudorange measurement made from receiver \( \alpha \) to satellite \( j \) at \( t_k \), \( \rho^j_\alpha \) is the geometric range from antenna \( \alpha \) to satellite \( j \) at \( t_k \), \( d\rho_n \) is the nominal broadcast orbital error, \( c \) is the speed of light, \( d^j \) is the satellite clock error, \( dT_\alpha \) is the receiver...
clock error, $d_{ion}^j$ is the ionospheric delay, $d_{trop}^j$ is the tropospheric delay, $\varepsilon(p_{rx})$ is the error due to receiver noise and $\varepsilon(p_{mult})$ is the error in the pseudorange measurement due to multipath.

The geometric range equation expressed in terms of Earth-fixed coordinates is [27]:

$$\rho_\alpha^j = |r^j - r^\alpha| = \left[ (x^j - x_{\alpha})^2 + (y^j - y_{\alpha})^2 + (z^j - z_{\alpha})^2 \right]^{0.5}$$  \hspace{1cm} 2.2

where $r^j$ is the satellite position vector referenced to the Earth-fixed frame computed using the broadcast ephemeris at epoch $t_k$ and $r^\alpha$ is the position vector for antenna $\alpha$ referenced to the Earth-fixed frame at epoch $t_k$.

### 2.4.2 Phase Observations

Carrier phase measurements are calculated by integrating Doppler measurements over time. Given that the phase can be measured to a fraction of a wavelength, carrier phase measurements are much more precise than code measurements, with measurement noise on the order of one millimeter, and maximum multipath effects of one quarter of a wavelength [49].

The carrier phase model for a receiver $\alpha$ and a satellite $j$ at epoch $t_k$ can be written as [50]:

$$\Phi_\alpha^j = \rho_\alpha^j + d\rho_n + d\rho_{sa} + c\left( dt^j - dT_{\alpha} \right) + \lambda N_\alpha^j - d_{ion}^j + d_{trop}^j + \varepsilon(\Phi_{rx})$$  \hspace{1cm} 2.3

$$+ \varepsilon(\Phi_{mult})$$

where $\Phi_\alpha^j$ is the carrier phase measurement made from receiver $\alpha$ to satellite $j$ at epoch $t_k$, $N_\alpha^j$ is the carrier phase ambiguity, $\varepsilon(\Phi_{rx})$ is the error in the carrier phase.
measurement due to receiver noise and \( \epsilon(\Phi_{\text{mult}}) \) is the error in the carrier phase measurement due to multipath.

Unfortunately, the carrier phase measurements are biased by an integer number of carrier wavelength cycles. Therefore, to use the carrier phase measurements most effectively, the ambiguities must be resolved to their integer values. The highest position accuracies are obtained only after the carrier phase ambiguities are estimated as integers. Thus, the pseudorange measurements are the primary observations considered herein.

### 2.4.3 Doppler Observations

Doppler is a measure of the frequency offset of the GPS carrier signal due to clock drift, and relative receiver and satellite motion, and is a measure of the rate of change of the carrier phase measurement. The observation equation in (m/s) can be written as [27]:

\[
\Phi = \rho + d\rho + c(dt - d\hat{t}) - d_{\text{ion}} + d_{\text{trop}} + \epsilon(\Phi_{\text{rx}})
\]

where \( \Phi \) measured Doppler, \( \rho \) geometric range rate, \( d\rho \) orbital error drift, \( dt \) satellite clock error drift, \( d\hat{t} \) receiver clock error drift, \( d_{\text{ion}} \) ionospheric delay drift, \( d_{\text{trop}} \) tropospheric delay drift and \( \epsilon(\Phi_{\text{rx}}) \) noise (1-5 mm/s). Doppler measurements are used primarily for estimating receiver velocity and detecting cycle slips.

### 2.5 GPS Errors

There are three basic sources of error affecting GPS. These include satellite-based errors, signal propagation errors and receivers based errors. Within these basic sources, the major errors are satellite position and clock, ionospheric and tropospheric, multipath and receiver noise [7, 47-49].
2.5.1 Satellite-based Errors

2.5.1.1 Orbit Errors
The GPS satellite positions used for post-mission processing will be generated from the broadcast ephemeris, which is predicted by the GPS control segment using previous measurements of the satellite motion and knowledge of the Earth’s gravity field [1]. As the prediction becomes older, the error in that prediction will increase. The typical error in the satellite positions from broadcast ephemeris is between 2 and 5 m [51].

2.5.1.2 Satellite Clock Error
The satellite’clock behavior is modeled by the GPS control segment using previous measurements of satellite clock errors [52]. This model is used to estimate the individual satellites’ clock offset from a nominal “GPS time” common to all of the satellites. Unaccounted instabilities in the satellite clocks can result in an error in the pseudorange. In single point positioning, the error in the satellite clock prediction can account for up to 4.5 m of error [52]. However, by using differencing methods [37], the satellite clock error on between receiver single difference and the double difference observations is essentially eliminated.

2.5.2 Propagation Errors

2.5.2.1 Ionosphere
The ionosphere is the layer of the atmosphere which contains ionized gases (free electrons and ions) and occupies the region of space approximately from 60 to 1000 km
above the Earth’s surface [27]. The ionosphere affects electromagnetic waves that pass through it by inducing an additional transmission time delay, while at the same time increasing the carrier phase. The magnitude of these effects is determined by the total electron content (TEC), which is a measure of the number of free electrons in a column of the atmosphere [1]. The number of free electrons is largely affected by solar activity. Under normal solar activity, the influence of the ionosphere ranges from several meters to tens of meters. During severe ionosphere storms, the error can reach more than 100 m [53].

Another important characteristic of the ionosphere is that the magnitude of the error is a function of the carrier frequency passing through it (at least for the L-band spectrum). This can be expressed mathematically as follows [53]:

\[ I = \frac{40.3 \times TEC}{f^2} + \text{Higher Order Terms} \quad 2.5 \]

where \( f \) is the carrier frequency. Ignoring the higher order terms does not add considerably to the error effects (a few cm at low satellite elevation angles). Equation 2.5 implies that the L1 and L2 signals are affected differently, which can be exploited to effectively remove most of the ionospheric effect through various linear combinations of the L1 and L2 observations [53].

The ionospheric error is the largest error source in Differential GPS (DGPS) navigation, increasing with baseline length. Therefore, correction of the ionospheric error is the key to high precision DGPS positioning [37].
Typically, ionospheric errors are at the level of 1-3 ppm [54] but can reach larger magnitudes under severe conditions. It is noted that the effect on the pseudo-range and the carrier phase measurements is of the same magnitude, but have opposite sign.

To account for ionospheric errors, several options are available. The simplest approach is to use an ionosphere delay model [53]. This approach is often limited by the spatial and temporal resolution of the model and is therefore not considered ideal. A second alternative is to use the ionosphere-free linear frequency combination to remove the effect of the ionosphere [37]. Unfortunately, this means that the ambiguities cannot be resolved as integers. Finally, the ionosphere errors can be estimated along with the ambiguity and position states in the navigation filter. This approach works well, but is computationally expensive [55].

2.5.2.2 Troposphere
The troposphere consists of dry gases and perceptible water vapor (PWV). Dry gases and PWV cause different delay effects of the GPS signals. The first and larger effect is the dry atmosphere excess delay caused primarily by dry $N_2$ and $O_2$ gases. In theory, the dry atmosphere zenith excess delay can be predicted to the millimeter level [56]. The reason for this predictability is that the dry delay in the zenith direction is a function of the total surface pressure only, which varies in a reasonably predictable manner with altitude and can be measured precisely with barometers. Typical magnitudes for the dry effect are on the order of 2 meters [57].
The PWV effect, on the other hand, is less predictable. Water vapor generally exists only below altitudes of 12 km above sea level, but most of the water vapor is below 4 km. Its density varies widely with position and time and is much more difficult to predict than the dry gases. Fortunately, however, water vapor causes only a relatively small fraction (roughly 10%) of the total tropospheric delay [57].

The above two atmospheric constituents affect the propagation delay of the radio frequency signals quite differently. By assuming that each constituent is both horizontally stratified and azimuthally symmetric, it can be modeled in two parts: the delay experienced in the zenith direction, and the scaling of that zenith delay to the delay experienced along the ray path. The formulation of the total tropospheric delay can then be described as [58]:

\[ d_{trop} = d_{dry}^Z \times m_{dry} + d_{wv}^Z \times m_{wv} \]  

where \( d_{dry}^Z \) is the dry zenith delay caused by the dry gases, \( m_{dry} \) is the function used to map the dry zenith delay to the ray path, \( d_{wv}^Z \) is the wet zenith delay caused by the water vapor and \( m_{wv} \) is the function used to map the wet zenith delay to the ray path.

There are many empirical models to compute the total tropospheric delay. In a series of papers [59] presented one of the first models to estimate tropospheric delay as a function of elevation angle. Many other models have been proposed since then, such as Hopfield’s dual quartic zenith model [60].
2.5.2.3 Multipath

Multipath occurs when the satellite signal reaches the receiver antenna via multiple paths [1]. The first path is usually a direct, unobstructed path from the satellite to the antenna (LOS), and the reflected rays are the result of reflections from nearby objects or the ground. These reflected paths will be delayed relative to the LOS and, usually, will be weaker than the LOS, due to the energy loss from the reflection [61]. However, when the LOS is partially obstructed or when the reflecting surface has a large area (e.g. a building), the LOS may be weaker than some echoes. It is also possible that the LOS signal is completely obstructed so that the receiver processes only the multipath components. Figure 2-5 illustrates a possible multipath scenario, where three rays reach the receiver: LOS, a ray reflected on a building and another ray reflected on the ground. Since the receiver cannot distinguish between the LOS and the Non-line-of-sight (NLOS) signals it will ultimately track the composite signal, which will lead to erroneous measurements [62]. Unlike other error sources, multipath errors are generally uncorrelated between two nearby receivers (even if they are separated by only a few meters) and will not cancel out using differencing process [37]. Therefore, code multipath is still a major issue for positioning applications, because it is one of the largest error sources.
The actual amount of received multipath signals depends on the geometrical conditions. In the context of most existing multipath propagation models, these geometrical conditions are commonly characterized by the amount (or density) and structure of buildings (e.g. buildings’ heights) and natural multipath sources like trees and forest. According to [61], these environments are named “open”, “rural”, “suburban” or “urban”. In urban areas, for example, a mean number of approximately four multipath signals has to be expected most of time.

The power of multipath signal relative to the LOS component is determined mainly by the electromagnetic properties of the reflecting surface. Most reflections cause signal attenuation, so that multipath signals are typically weaker than the LOS component. The
degree of attenuation can be expressed in terms of “multipath relative amplitude” \( \alpha \) (typical range: \( 0 \leq \alpha \leq 1 \)). Due to one or several reflections, the path length of the reflected signal is extended with respect to the length of the LOS path [61]. This path extension is referred to as “multipath delay”. It can be expressed in [meters]. Due to this path delay, the phase of the LOS component and the multipath signal are different. The difference between the phase of the LOS and the multipath signal is referred to as “multipath relative phase” (expressed in [radians] or [degrees]).

In case of multipath, the antenna doesn’t only receive the LOS component but the sum of the LOS plus all existing multipath components with dedicated multipath relative amplitude, path delay and relative phase. More details about the effects of Multipath on GPS receiver performance and Multipath mitigation techniques will be discussed in the next chapter.

2.5.2.4 Interference and Jamming

Since the GPS signals are weak, they are vulnerable to RF interference. The Interference can be intentional or unintentional interference. Any effect that causes degradation in Carrier To Noise Density Ratio \((C/N_0)\) can be considered as interference. Interference can result in poor geometry and large position errors, or in system unavailability [63]. The unintentional interference consists of any unwanted disturbance within a useful frequency band, including distortions of phase and amplitude (e.g. atmospheric effects, multipath), and noise. Intentional (in-band) jamming is the emission of RF energy of sufficient power and characteristics to prevent receivers in the target area from tracking
the GPS signals. According to [1], Table 2-2 shows types of RF interference and typical sources.

A special type of intentional interference is spoofing [64], the intentional transmission of a high power false version of the GPS signal. In case the receiver locks onto this signal, the position process most probably provides a false and useless solution. To avoid this problem with GPS, the public P-code has been encrypted and the resulting P(Y)-code is accessible only via a cryptographic key that is only available for authorized users.

Table 2-2: Types of RF interference and typical sources [1]

<table>
<thead>
<tr>
<th>Jamming Type</th>
<th>Typical Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wideband-Gaussian</td>
<td>Intentional noise jammers</td>
</tr>
<tr>
<td>Wideband phase/frequency modulation</td>
<td>TV transmitter harmonics or near band microwave link transmitters overcoming the front-end filter of GPS receivers</td>
</tr>
<tr>
<td>Wideband spread spectrum</td>
<td>Intentional spread spectrum jammers or near-field of pseudolites</td>
</tr>
<tr>
<td>Wideband pulse</td>
<td>Radar transmitters</td>
</tr>
<tr>
<td>Narrowband phase/frequency modulation</td>
<td>AM station transmitters harmonics or CB radio transmitters’ harmonics</td>
</tr>
<tr>
<td>Narrowband swept continuous wave (CW)</td>
<td>Intentional CW jammers or FM station transmitters’ harmonics</td>
</tr>
<tr>
<td>Narrowband CW</td>
<td>Intentional CW jammers or near-band unmodulated transmitter’s carrier</td>
</tr>
</tbody>
</table>

2.5.3 Receiver-based Errors

2.5.3.1 Antenna Errors
The phase model of the antenna assumes that the antenna acts as a single point source, whereas in reality there are deviations from the ideal due to a non-spherical phase
response of the antenna element and case mounting point offsets [65]. Essentially the measured phase from the same signal coming in at a different elevation or azimuth will be different. The phase center may also vary with the frequency of the GPS signal. For example, the Thales choke ring antenna has a 18.9 mm vertical bias between the L1 and L2 phase centers. A bias in the phase center directly translates to a bias in the final position.

2.5.3.2 Receiver Noise
Measurement noise arises primarily from the thermal noise inside a receiver and the weak GPS signal power. The effect is that GPS measurements are contaminated with a white noise sequence. For pseudorange measurements, the noise is typically at the level of a few decimeters for high quality receivers [1]. The receiver noise term (variance) for a double difference observation is four times that of an observation that has not been differenced.

2.5.4 Other Errors
Other factors can also affect the GPS system accuracy. Listed below are some of the more common errors.

- **Loop filter bandwidth and order** - the loop filter feeds back information from the processed signal to the receiver’s incoming signal carrier and PRN code tracking component. The bandwidth of the loop filter must be wide enough to track under dynamic conditions, however the wider the bandwidth, the noisier the measurements [66]. The order of the filter in addition to the bandwidth determines
the receiver’s response to dynamics such as velocity and acceleration [49]. This error will be discussed in more details in the next chapter.

- **Navigation filter parameters** – the navigation filter parameters and error models within the post-processing software can be a factor, especially where they are not tuned to handle the dynamics of the dynamic receiver.

- **Oscillator effects due to vibration** – the oscillators used in the GPS receiver can be sensitive to vibration such that noise increases [67].

### 2.6 Summary

This chapter reviewed the GPS and its segments. It then addressed the GPS modernization program. It provided an overview of GPS signal characteristics and the GPS measurements errors. In the next chapter, the signal processing techniques in a GPS receiver will be described.
Chapter 3
GPS Receiver Design - Technical Background

3.1 Introduction
This chapter reviews the technical background related to the GPS receiver designs. It also provides an overview of the GPS receiver architecture and covers the basic concepts of the signal processing techniques in a GPS receiver. The outline of this chapter is as follows. General receiver architectures are introduced first, where the signal model, signal acquisition, and theories are discussed. After that, navigation estimation techniques are briefly reviewed. Finally, multipath mitigation techniques are discussed in details. The chapter provides the necessary background for the material covered in the following chapters.

3.2 Overview of GPS Receiver Architecture
The purpose of GPS receivers is to process signals transmitted by GPS satellites, estimate the user-to-satellite ranges and range rates and compute a PVT solution [44]. The high level architecture of a GPS receiver is illustrated in Figure 3-1. As shown in the figure, GPS receivers consist of four blocks: Antenna, Radio frequency (RF) front-end, local oscillator and signal processing block. The antenna is the first element of the receiver architecture. The transmitted signal is Right Hand Circularly Polarized (RHCP), so the antenna must be designed to receive RHCP signals [27]. The antenna gain pattern is an important consideration that indicates how well the antenna performs at different center
frequencies, different polarizations and different elevation angles. Figure 3-2 illustrates
the typical gain patterns of the NovAtel antenna GPS 702GG that can receive GPS L1
and L2, as well as GLONASS L1 and L2 signals [68].

Figure 3-1: High-level architecture of GPS receiver

Figure 3-2: Typical RHCP and LHCP normalized radiation pattern of NovAtel antenna
GPS-702 GG for GPS L1 and GLONASS L1 central frequency [68]
Special applications may need different types of antenna design, technology and location. As shown in Figure 3-3, antennas can range from large rooftop-mounted antennas (e.g. for a static open sky view) to embedded antennas in receiver boards (e.g. for mobile handheld devices).

![Figure 3-3: Examples of GPS antenna types: (a) rooftop antenna, (b) choke ring antenna and (c) patch antenna](image)

The choke ring antenna is commonly used in GPS ground base receivers. The choke ring is constituted of a series of conductive concentric ring surrounding the antenna that efficiently removes creeping signals [69]. It increases the antenna gain at high elevations and reduces it for lower elevations, thus efficiently fighting ground reflected multipath signals. It significantly increases the size of the antenna [70].

The pre-amplifier is the first active component after the antenna. It is often housed in the same enclosure as the antenna element. The antenna may be capable of receiving multiple frequency bands. Thus, there may be one pre-amplifier per frequency band of interest, or
a single pre-amplifier may cover multiple frequency bands. The primary purpose of the preamplifier is to amplify the signal at the output of the antenna for further processing [1]. A typical preamplifier is constituted of (a) Pre-selector filter: It intends to reject out-of-band interference and limit the noise bandwidth, (b) Burnout protection: It is used to protect the electronics components inside the receiver from possible high power interference (in-band or out-of-band) to enter the receiving chain and (c) Low Noise Amplifier (LNA): It is used to amplify the very week received GPS signals (i.e. GPS received signal have a power around -160 dBw or 10-6 W). The LNA gain is usually on the order of 20 to 35 dB [71].

The RF front-end is the second part in the receiver architecture after the antenna and LNA. The goal of the receiver front-end is to provide a sampled signal as clean as possible to the signal-processing block [44]. The RF front-end filters, amplifies, down convert and digitalizes the received signal. Each front-end conditions the narrow band signal in each band and down converts and digitalizes the data.

Filtering in the front-end achieves a number of objectives such as rejecting out of band signals, reducing the noise content in the received signal and reducing the effect of aliasing. Wide bandwidth signals, if appropriately processed, can provide higher resolution measurements in the time domain, but come at the cost of requiring higher sampling rates, and hence result in larger power consumption in the receiver [72].

Down-conversion in the front-end is the process of taking the RF signal down to some lower frequency (either an Intermediate Frequency (IF), or directly to baseband) [1]. The
down-conversion of the incoming signal from RF to IF is necessary to ease the sampling process and the filtering of interference and noise. The common form of the down-conversion is a mixer which multiplies the RF signal by a locally generated sinusoid and filters the output to remove double-frequency terms [1], as shown in Figure 3-4. Typically filtering and down-conversion are achieved in multiple, cascaded stages due to the difficulty in implementing a good high center frequency band-pass filter.

![Figure 3-4: Block Diagram of two cascaded stages down-conversion](image)

The final stage of the RF front-end is conversion of the analogue band-pass or base-band signal to a digital signal. The band-pass sampling achieves both discretization and down-conversion [44]. It is commonly used in the receiver front-end to convert from lower IF to baseband.

The above only describes single RF front-end which amplifies, filters, down-converts and digitalizes a narrow-band signal from a single band in the RF spectrum. With the increasing number of GNSS signals and systems available, more and more receivers have
multi-frequency capability. In order to design a multiple frequency GNSS receiver, the following are some issues which must be taken into consideration as described by [73].

- **Group Delay:** In a multi-frequency receiver, each band processed passes through a different receiver chain, with different effective transfer functions and even different RF path lengths. This results in different delays through the front-end for each band. These delays must either be estimated on the fly or calibrated after manufacturing.

- **Different Sampling Rates:** Different signals have different bandwidths and therefore can be sampled at different rates.

Generally, measurements made by GPS receivers are based on the estimates of the signal TOA and received carrier phase and frequency. There is one local reference oscillator, as shown in Figure 3-1, from which all frequency references in the receiver are derived [74]. The oscillator plays an extremely important role in the receiver performance and, therefore, it must be chosen carefully. Practical considerations include size, power consumption, short-term stability, long-term stability and the effect of temperature and vibration sensitivity [1].

GPS receivers may require multiple frequency references during down-conversion, each mixer stage requires a precise reference frequency and, also, the sampling clock must be generated. Frequency synthesis is the process of generating the desired reference frequencies in the receiver from the local oscillator. This is usually achieved by a combination of integer and rational frequency multiplications [75].
The final stage of a GPS receiver shown in Figure 3-1, and the one that is the focus of this chapter, is the signal processing and navigation-processing block. The output of the RF front-end is a conditioned and down-converted version of the signal received at the antenna, but should contain all the relevant information received at the antenna. The signal and navigation processing stage takes this information, extracts the measurements of range and range rate to all satellites in view and estimates the PVT solution for the antenna.

Generating the PVT solution from the signals at the output of the front-ends is the ultimate goal of most GPS receivers. This processing is usually divided into two stages; the first stage is the estimation of the range and range rates (the measurements or observations) to each satellite using the known signal structure and the second is the estimation of the user’s position, velocity and time information using these observations. In the GPS receiver the signal processing can be divided into the following stages [44]:

1. Signal Acquisition: This involves detection of the signals from satellites in view and provides a rough estimation of the code delay and of the Doppler frequency of the incoming signal.

2. Signal Tracking: This follows acquisition and is a recursive estimation process that maintains continually updated estimates of some critical signal parameters. The parameters from each signal are time varying, the signal tracking stage maintains lock on these parameters, maintaining continuously update estimates.
3. Signal Monitoring: This stage runs in parallel with tracking, and involves estimation of, for example, the C/N$_0$. The receiver uses the signal monitoring to decide when loss of lock occurs, for example.

4. Navigation message extraction: Again, this stage runs in parallel with tracking the navigation message, including satellite ephemerides, is decoded.

5. Measurement Generation: Uses the tracking parameters to estimate ranges and range rates.

6. PVT Solution: Uses the range and range rate estimates to compute a position solution.

3.2.1 GPS Signal Model

The signal model for the ith signal at the output of the GPS receiver antenna is given by [46]:

$$r(t) = \sum_{i=1}^{N} A_i(t) d_i(t - \tau_i^0) c_i(t - \tau_i^0) \cos\{2\pi(f_L + f_{DL,i})t + \phi_i^0(t)\}$$  

$$+ MP(t) + \eta(t)$$  

where $r(t)$ is the received signal that is the sum of $N$ useful signals that are broadcast by $N$ different GPS satellites, $A_i(t)$ is the received carrier power including the effect of atmospheric attenuation and receiver antenna gain pattern, $d_i(t)$ represents the data navigation message, $\tau_i^0$ is the LOS path delay introduced by the channel, $c_i(t)$ represents the product of PRN code, secondary code and sub-carrier and $f_L$ is the carrier frequency which depends on the GPS band under consideration. For GPS L1 band,
\( f_L = 1575.42 \, MHz \) and for GPS L2 band, \( f_L = 1227.60 \, MHz \) and for GPS L5 band, \( f_L = 1176.45 \, MHz \). \( \phi_i^0(t) \) is a time varying phase offset, MP(t) represents the sum total of multipath reflections, \( \eta(t) \) is an Additive White Gaussian Noise (AWGN) with single sided noise Power Spectral Density (PSD) of \( \frac{N_0}{2} \, W/Hz \) and \( f_{D,i}^0 \) is the Doppler frequency affecting the ith useful signal. The Doppler is related to the rate of change of time delay by

\[
\frac{\Delta \tau}{\tau_i} = f_{D,i}^0 \frac{\Delta \tau}{\tau_i}
\]

After the GPS signals reach the receiving antenna, the received signals are filtered, down converted to baseband and digitalized by the radio-frequency (RF) front-end [38]. Neglecting the quantization effect, the GPS signal at the output of the RF front-end can be rewritten as:

\[
\begin{align*}
    r[n] &= \sum_{i=1}^{N} A_i[n] d_i[n - \tau_i^0] c_i[n - \tau_i^0] \cos\left\{2\pi \left(f_{IF} + f_{D,i}^0\right)nT_s + \phi_i^0[n]\right\} \\
    &\quad + MP[n] + \eta[n]
\end{align*}
\]

where \( r[n] = r[nT_s] \) is a discrete-time sequence \( r[n] \), obtained by sampling a continuous-time signal \( r(t) \) with a sampling frequency \( f_s = \frac{1}{T_s} \) and \( f_{IF} \) is the receiver Intermediate Frequency.

According to the above model, the signal is parameterized by the following:

1. the satellite number (SVN) \( : i \)
2. the carrier power \( : A_i \)
3. the code delay \( \tau_i^0 \)
4. the carrier Doppler \( f_{D,i}^0 \)
5. the carrier phase \( \phi_i^0 \)

3.2.2 Signal Acquisition

The first operation performed by a GPS receiver is the signal acquisition. The purpose of the acquisition stage is to detect which signals are present at the output of the front-end and to coarsely estimate sufficient signals parameters to facilitate signal tracking [76]. Since the code delay, \( \tau_i^0 \), and the Doppler frequency, \( f_{D,i}^0 \), of the useful signal in (3.3) are unknown to the receiver, the acquisition process has a two-dimensional search space, one dimension corresponds to the PRN code delay, while the other corresponds to the Doppler frequency [77]. The conventional GPS receiver would search all possible frequencies in the Doppler domain and code delays in the code phase domain in case of no prior information of the incoming signal parameters. In this case, The frequency uncertainty is in a range of 10 kHz to 25 kHz [78], which is mainly contributed by the Doppler effect, caused by the satellite motion and user dynamics, and unknown receiver oscillator frequency offset. The code delay search usually involves all possible chips, which is 1023 chips in the code phase dimension for the GPS L1 C/A code.

All the acquisition techniques used inside the GPS receivers described in literature [46, 79-81] are based on the maximum-likelihood estimation theory. The acquisition operation is based on evaluating and processing the Cross Ambiguity Function (CAF), which evaluates the correlation \( R(.) \) between the received signal and each PRN code.
across all possible combination of local code offset and Doppler shift as shown in Figure 3-5. The CAF can be defined as [46]:

$$\mathcal{R}_c(\tau, f_D) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} r[n] c[n - \bar{\tau}] \exp\{j2\pi(f_{IF} + f_D)nT_3\}$$

where $c[n - \bar{\tau}]$ is the local replica reproducing the C/A code, $\bar{\tau}$ and $f_D$ are the code delay offset and the Doppler shift, respectively, tested by the receiver. The parameter $N_c$ is the number of samples used for computing a single correlation output and $T_c = N_cT_s$ is the coherent integration time in milliseconds for GPS signal. In order to improve the acquisition performance and handle the noise, non-coherent integration is implemented [82] as illustrated in Figure 3-6. The non-coherently average correlation function $\mathcal{R}_{NC}(\bar{\tau}, \bar{f}_D)$ can be written as:

$$\mathcal{R}_{NC}(\bar{\tau}, \bar{f}_D) = \frac{1}{M} \sum_{m=0}^{M-1} \left[ \frac{1}{N_c} \sum_{n=0}^{N_c-1} \mathcal{R}(\tau, f_D) \right]^2$$

where $M$ is the total number of correlation samples that are non-coherently integrated. It should be noted that for $M = 1$ only coherent integration is used.

Figure 3-5: Block diagram of the GPS acquisition [9]
Ideally the output of the acquisition is a sharp peak that corresponds to the values of $\hat{\tau}$ and $\hat{f}_D$ matching the delay and the Doppler frequency of the Signal-in-Space (SIS). The readability of the CAF can be distorted due to the phase of the incoming signal, the noise and other impairments. In this case further processing is required [83].

![Block Diagram of the non-coherent integration](image)

**Figure 3-6: Block Diagram of the non-coherent integration [9]**

Figure 3-7 shows the normalized correlation output for a given GPS satellite. The peak location is related to a C/A code phase and a Doppler frequency of the incoming GPS signal. The acquisition process detects the signal of a visible satellite in case the maximum correlation power exceeds a defined threshold that is defined based on a desired probability of false alarm [81].
Typically there are three modes for acquisition processing: cold, warm and hot modes. The receiver starts in cold mode, since no prior information about the signal parameters is available. In cold mode, the SVs are searched sequentially. The correlator outputs of each satellite are evaluated over a grid of different code delays and Doppler frequencies as shown in Figure 3-7. Once the carrier wave and pseudo-ranging code are properly found, the de-spreading operation reduces the bandwidth of the signal but the noise is effectively unchanged. Therefore, the signal is contained in a narrow bandwidth while the noise power remains spread over a large bandwidth.

The warm start mode functions when the receiver has some knowledge such as almanacs, and last known position and a rough estimate of time [1]. Thus, it can choose most likely
satellites to search. The last mode is the hot start mode. It functions when the receiver has a recent position, time and ephemeris information available, and already knows the satellites in view.

There are mainly three acquisition methods used inside the GPS receivers and described in the literature [9, 38, 46, 79-81, 84]; the serial search, the parallel frequency space search and the parallel code phase search acquisition methods. The parallel code phase search acquisition method is considered in this thesis because it is faster than other methods and commonly used inside GPS receivers [84].

The main drawback of the serial search acquisition method stated in the literature [38, 46, 76, 85, 86] is that it multiplies the input GPS signal with a PRN code with 1023 different code phases. A more suitable way is making a circular cross correlation between the input signal and the PRN code without shifted code phase [85]. Figure 3-8 illustrates the block diagram of the parallel code phase search algorithm. The received signal is multiplied by a locally generated carrier signal. The in-phase (I) signal and the quadrature (Q) signal are obtained by multiplying the received signal with locally generated carrier signal and a 90° phase-shifted version of the signal generates, respectively. The I and Q signals are combined to form a complex input signal

\[ x(n) = I(n) + jQ(n) \]

to the Discrete Fourier Transform (DFT) function [9].

In the parallel code phase search algorithm, the Fourier transform is used to transform the generated PRN code to the frequency domain. The complex conjugate of the Fourier transform of the PRN code is multiplied by the Fourier transform of the incoming signal.
The result of the multiplication is transformed into the time domain by an inverse Fourier transform. The absolute value of the output of the inverse Fourier transform represents the correlation between the incoming GPS signal and the PRN code. If a peak exceeds an established threshold in the correlation domain, the index of this peak defines the PRN code phase and carrier frequency of the incoming signal.

![Diagram](image)

**Figure 3-8: Block diagram of the parallel code phase search algorithm [38]**

### 3.2.3 Signal Tracking

Once the acquisition processing stage has detected the presence of a given satellite (PRN) and provided a coarse estimate of the code phase and carrier frequency, a fine synchronization stage, called signal tracking, is used to estimate the code phase and Doppler frequency, keep track of the changes in the signal parameters and demodulate the navigation data from the particular satellite [1]. The signal tracking consists of two
interoperating feedback loops, a DLL is used to estimate the code delay and the Carrier Lock Loop (CLL) is used to estimate the carrier phase and frequency. The DLL estimates the code phase of the ranging code being tracked while the CLL tracks the incoming carrier phase via a Phase Lock Loop (PLL) or a carrier frequency via a Frequency Lock Loop (FLL) [27]. The two tracking loops are coupled in the sense that the DLL requires a precise estimate of the incoming carrier frequency which the CLL can provide and, similarly, the CLL requires a reasonable estimate of the code phase [1]. Figure 3-9 illustrates a generic block diagram of baseband code and carrier tracking loops.

Referring to Figure 3-9, initially the digital IF incoming signal is stripped of the carrier Doppler by multiplying the incoming signal with the replica carrier signals generated inside the receiver to produce in-phase ($I$) and quadrature ($Q$) sampled data. The $I$ and $Q$ are then correlated with Early (E), Prompt (P) and Late (L) replica codes synthesized by the code generator, a 2-bit shift register, and the code Numerical Control Oscillator (NCO). The E and L are typically separated in phase by 1 chip or less and P is in the middle [38].
Figure 3-9: A Generic block diagram of baseband code and carrier tracking loops [1]

In case the code tracking algorithm tracks the incoming signal code phase correctly, the prompt replica code phase and the incoming signal code phase are aligned and therefore produce maximum correlation. Any misalignment in the replica code phase with respect to the incoming signal code phase produces a difference in the vector magnitudes of the early and late correlated outputs. This difference can help the code tracking loop to detect and correct the amount and direction of the code phase [29].

The three complex pairs of \((I_E, Q_E), (I_P, Q_P)\) and \((I_L, Q_L)\) are passed through “Integration and Dump” accumulators, whose duration establishes the Pre-detection Integration (PI) time for the signal. The PI time usually doesn’t exceed 20 ms for a 50-Hz navigation message data bit period for GPS signals unless special techniques are used to allow for longer integration time [1].
As shown in Figure 3-9, the carrier loop filter output is adjusted by a scale factor and added to the code loop filter output as aiding [1]. This process is called a carrier-aided code loop. This is a typical procedure to remove the LOS dynamic stress from the burden of the latter and hence reduce the code tracking loop bandwidth to minimize the tracking noise. The Scale Factor (SF) is required to map between the carrier frequency and code frequency [1]. Since the Doppler effect on the signal is inversely proportional to the wavelength of the signal, for the same relative velocity between the SV and the GPS receiver, the Doppler shift on the spreading code chip rate is much smaller than the Doppler shift on the L-band carrier. The SF that compensates for this difference in frequency is given by [1]:

\[
\text{Scale factor} = \frac{R_c}{f_L} \text{ (dimensionless)}
\]

where \( R_c \) is the code chipping rate (Hz) and \( f_L \) is the carrier frequency (Hz).

As shown in Figure 3-10, external velocity aiding can be used for the carrier tracking loop using external systems such as INS in the same way as the one described in equation 3.6.

To extract the modulated navigation data from the GPS signals, an accurate estimation of code delay, carrier frequency and phase has to be provided using the tracking loops. Thus, the following sections will be focused on the performance analysis and discussing enhanced techniques for the carrier tracking loop. The carrier tracking loop can be either a PLL, FLL or a FLL-assisted-PLL loops. These tracking loops will be discussed in detail in the following sections.
Figure 3-10 shows a general view of a tracking loop. Discriminator, loop filter, and numerical control oscillator are the basic components of a tracking loop [27]. These three functions regulate the two most important performance characteristics of the receiver carrier loop design, specifically the carrier phase loop thermal noise error and the maximum LOS dynamic stress threshold. Since the carrier tracking loop is always the weak link in a stand-alone GPS receiver, its threshold characterizes the unaided GPS receiver performance.

![Diagram of a tracking loop](image)

**Figure 3-10: A general view of a tracking loop [27]**

3.2.3.1 Non-Linear Discriminator

The non-linear discriminator is responsible for evaluating an error function of the quantity to be tracked. The feedback loop tries to derive the error evaluated by the discriminator to zero. The carrier loop discriminator defines the type of tracking loop as a PLL, a Costas PLL (that tolerates the presence of data modulation on the baseband signal), or an FLL. The objective of the PLL is to minimize the phase difference between incoming signal and local generated signal [1]. The PLL and the Costas loop are the most
accurate, but they are more sensitive to dynamic stress than the FLL. The PLL and Costas loop discriminators produce phase error estimates at their outputs. The Costas loop discriminator is insensitive to navigation data bit transitions compared to the PLL. The FLL discriminator produces a frequency error estimate. Table 3-1 illustrates common types of PLL, Costas PLL and FLL discriminators where $\varphi$ is the phase error and the prompt samples $I_{p1}$ and $Q_{p1}$ are the samples taken at time $t_1$, just prior to the samples $I_{p2}$ and $Q_{p2}$ taken a later time $t_2$.

<table>
<thead>
<tr>
<th>Tracking Loop Type</th>
<th>Discriminator Algorithm</th>
<th>Output Error</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| PLL                | arctan2 ($Q_p, I_p$)     | $\varphi$    | • Four-quadrant arctangent  
|                    |                          |              | • Optimal (maximum likelihood estimator) at high and low SNR |
| Costas PLL         | arctan ($Q_p / I_p$)    | $\varphi$    | • Two-quadrant arctangent  
|                    |                          |              | • Optimal (maximum likelihood estimator) at high and low SNR |
|                    | $\text{sign}(I_p) \times Q_p$ | $\sin(\varphi)$ | • Decision directed Costas  
|                    |                          |              | • Near optimal at high SNR |
|                    | $I_p \times Q_p$         | $\sin(2\varphi)$ | • Classic Costas analog discriminator  
|                    |                          |              | • Near optimal at low SNR |
| FLL Discriminator  | $\frac{\text{arctan2}(\text{dot}, \text{cross})}{(t_2 - t_1)}$ | $\frac{\varphi_2 - \varphi_1}{t_2 - t_1}$ | • Four-quadrant arctangent. Optimal (maximum likelihood estimator) at high and low SNR |

where:
- $\text{dot} = I_{p1} \cdot I_{p2} + Q_{p1} \cdot Q_{p2}$
- $\text{cross} = I_{p1} \cdot Q_{p2} - I_{p2} \cdot Q_{p1}$
Figure 3-11 shows a comparison between the responses corresponding to the different Costas loop discriminators. This figure plots the discriminator output as a function of the input phase, which is commonly named in the literature S-curve. The S-curve defines the lock prosperities of the tracking loop [38]. As shown in Figure 3-11, the discriminator outputs are zero when the phase error is 0 and ±180° and the two-quadrant ARCTAN Costas discriminator is the only Costas PLL discriminator that remains linear over half of input error range ±90°. All other discriminator outputs are only linear near the 0° region.

Figure 3-11: Comparison between the common Costas PLL discriminator responses
3.2.3.2 Loop Filter Design

The loop filter is used in the tracking loop to reduce the noise in order to produce an accurate estimate of the incoming signal by adjusting the Numerical Controlled Oscillator (NCO). As shown in Figure 3-10, the error signal, which is the difference between the loop filter’s output signal and the incoming signal, is feedback into the filter’s input in a closed loop process. Refereeing to [1], the loop filter order and noise bandwidth define the loop filter’s response to signal dynamics. The advantage of a second order loop filter is its unconditional stability. Nevertheless, it suffers from acceleration stress error. Therefore, high dynamic receivers typically use third order PLLs to provide the desirable characteristic of being able to track an accelerating frequency input. Stability becomes a major concern for higher order loops [27]. Table 3-2 summarizes the filter characteristics and provides all of the information required to compute the filter coefficients for first, second and third-order loop filters, where \( \omega_n \) is the loop filter natural radian frequency and \( B_n \) is the loop filter noise bandwidth.

The loop filters shown in Table 3-2 can be presented using analog integrators in the Laplace domain as \( \frac{1}{s} \). The number of the integrators defines the loop filter order and characteristics. For instance the design of a PLL 3\textsuperscript{rd} order loop filter extracted from Table 3-2 is illustrated in Figure 3-12 whereas typical values of multiplier coefficients are presented in Table 3-2.
Table 3-2: Loop Filter Characteristics [1]

<table>
<thead>
<tr>
<th>Loop Order</th>
<th>Noise Bandwidth $B_n$ (Hz)</th>
<th>Typical Filter Values</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| First      | $\frac{\omega_0}{4}$       | $\omega_0$            | • Sensitive to velocity stress  
• Unconditionally stable at all noise bandwidths |
| Second     | $\frac{\omega_0 (1 + a_2^2)}{4a_2}$ | $a_2 \omega_0 = 1.414 \omega_0$; $B_n = 0.53 \omega_0$ | • Sensitive to acceleration stress  
• Unconditionally stable at all noise bandwidths |
| Third      | $\frac{\omega_0 (a_3 b_3^2 + a_2^2 - b_3)}{4(a_3 b_3 - 1)}$ | $a_3 \omega_0^2 = 1.1 \omega_0^2$; $b_3 \omega_0 = 2.4 \omega_0$; $B_n = 0.7845 \omega_0$ | • Sensitive to jerk stress  
• Remains stable at $B_n \leq 18$ Hz |

Figure 3-12: 3rd order PLL analog loop filter [1]

For the filter to be digitally represented, an analog to digital transformation for the integrator has to be performed. The digital bilinear $z$-transform integrator presented in Figure 3-13 linearly interpolates between input samples and more closely approximates
the ideal analog integrator [1]. The time interval between each sample $T$ represents a unit delay $z^{-1}$ in the digital integrator.

![Digital bilinear z-transform integrator](image)

**Figure 3-13: Digital bilinear z-transform integrator [1]**

There is a tradeoff between integration time, loop discriminator, and loop filter bandwidth requirements that have to be taken into consideration by the GPS receiver designer during the design of the carrier tracking loop. In practice, some compromise must be made to resolve the paradox in the tracking loop. The paradox is demonstrated in the fact that to tolerate dynamic stress, the integration time should be short, the carrier loop filter bandwidth should be as wide as possible and the discriminator should be an FLL. However in contrast, for carrier measurements to be accurate and less noisy, the integration time should be long, the carrier loop filter noise bandwidth should be narrow and the discriminator should be a PLL [1]. Figure 3-14 and Figure 3-15 illustrate the tracking performance in terms of Doppler estimation of a known GPS satellite using several loop bandwidths and a zoomed version for better illustration, respectively. It can be seen that while using a wide bandwidth is preferable to tracking high levels of dynamics, it is not advantageous in terms of noise variance.
Figure 3-14: Tracking performance in terms of Doppler shift estimation of a known GPS satellite using several loop filter bandwidths.

Figure 3-15: Tracking performance in terms of Doppler shift estimation of a known GPS satellite using several loop filter bandwidths (zoomed).
Due to the paradox mentioned above, the FLL-assisted-PLL is introduced in literature for very high dynamic conditions with reduced measurement noise. Figure 3-16 shows a 2\textsuperscript{nd} order FLL-assisted 3\textsuperscript{rd} order PLL loop filter design. The filter becomes a pure FLL in case the PLL error input is zeroed in either of these filters. Similarly, the filter becomes a pure PLL in case the FLL error input is zeroed. The FLL-assisted-PLL filter starts by using a pure FLL to speed up the convergence of the tracking loop and to tolerate the dynamic stress. Then the error inputs from both discriminators as an FLL-assisted PLL are applied until phase lock is achieved. Finally, a pure PLL is used to get accurate and less noisy measurements until phase lock is lost. However, if the noise bandwidth parameters are chosen correctly, there is very little loss in the ideal carrier tracking threshold performance when both discriminators are continuously operated \cite{1}. Typical values of multiplier coefficients shown in Figure 3-16 are introduced in Table 3-3, where \( \omega_{dp} \) and \( \omega_{df} \) are the natural radian frequencies of the PLL and FLL filters, respectively.

\[
\begin{align*}
\omega_{dp} & \rightarrow \tau & \omega_{dp} & \rightarrow \Sigma & 1/2 & \rightarrow & \tau & \rightarrow & \Sigma & 1/2 & \rightarrow & \Sigma & \rightarrow & \text{To NCO} \\
\omega_{df} & \rightarrow \tau & \omega_{df} & \rightarrow \Sigma & & & & & & & & & & \\
\omega_{0p} & \rightarrow \tau & \omega_{0f} & \rightarrow \Sigma & & & & & & & & & & \\
\omega_{0f} & \rightarrow \tau & \omega_{0f} & \rightarrow \Sigma & & & & & & & & & & \\
\end{align*}
\]

\textbf{Frequency Discriminator}

\textbf{Phase Discriminator}

\textbf{Acceleration Accumulator}

\textbf{Velocity Accumulator}

\textbf{Figure 3-16: A 2\textsuperscript{nd} order FLL-assisted 3\textsuperscript{rd} order PLL loop filter design using bilinear z-transform integrator} \cite{1}

58
Table 3-3: Typical values of 2nd order FLL-assisted-3rd order PLL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\omega_0f$</th>
<th>$a_2\omega_0f$</th>
<th>$\omega_0p$</th>
<th>$a_3\omega_0^2p$</th>
<th>$b_3\omega_0p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>$\frac{B_{nf}}{0.53}$</td>
<td>$1.414\omega_0f$</td>
<td>$\frac{B_{np}}{0.7845}$</td>
<td>$1.1\omega_0^2p$</td>
<td>$2.4\omega_0p$</td>
</tr>
</tbody>
</table>

Figure 3-17 compares the tracking performance in terms of Doppler shift estimation of a know GPS satellite using pure PLL with BW = 4 Hz and 18 Hz and 2nd order FLL-assisted-3rd order PLL. It can be seen that the time taken for PLL to converge to the correct frequency for the 4 Hz PLL is much larger than the PLL 18 Hz and assisting the tracking scheme with FLL provides better performance in terms of convergence speed.

A well-designed GPS receiver should start tracking with a short PI time using a wide bandwidth FLL, starting from the obtained signal Doppler [44]. Because of the data modulation on the carrier, it should then transfer to a Costas PLL, increasing the PI time while also gradually adjusting the carrier tracking loop bandwidth as narrow as the maximum expected dynamics permits.
3.2.3.3 Code Tracking Loop

The purpose of the code tracking loop is to generate a local replica of the PRN code that is perfectly synchronized with the PRN code in the incoming signal. The code tracking loop estimates the synchronization error between the local replica and the incoming signal using DLL a discriminator [1]. This error is then filtered and used to control the NCO that commands the shift register generating the local replica. Figure 3-18 shows the block diagram of a GPS receiver code tracking loop. As shown in Figure 3-18, the input signal is split into two paths and correlated with three local codes (replicas) referred to as the early, prompt, and late replica, respectively. The three codes are often separated by a
half-chip length [1]. This is done for both the inphase $I$ branch and the quadrature $Q$ branch. The estimation of the ACF in all six signals $I_{ES}, I_{LS}, I_{PS}, Q_{ES}, Q_{LS}$ and $Q_{PS}$, where $E$ denotes early, $L$ denotes late, and $P$ denotes promptly.

There are two types of the code tracking loop can be implemented a coherent DLL or a non-coherent DLL. The coherent DLL requires that the carrier phase and the navigation data bits to be removed from the signal. However, the non-coherent DLL have the ability to track the code phase with the navigation data bit present and the PLL is not necessarily in lock [38]. In this thesis, the non-coherent code tracking loop is implemented.

Refereeing to [1], some common DLL discriminators are listed in Table 3-4. The table shows one coherent and three non-coherent discriminators. The requirement of a DLL discriminator is dependent on the type of application and the noise in the signal [44].

![Figure 3-18: Generic block diagram of GPS receiver code tracking loop [1]](image-url)
discriminator function responses are shown in Figure 3-19. The figure is produced from ideal ACFs, and the space between the early, prompt, and late is $\pm \frac{1}{2}$ chip. The space between the early, prompt, and late codes determines the noise bandwidth in the delay lock loop. The normalized early minus late (EML) envelope discriminator is very common because its output error is linear over a 1-chip range, but the dot product power discriminator slightly outperforms it [73].

<table>
<thead>
<tr>
<th>Discriminator Type</th>
<th>Discriminator Algorithm</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| Coherent           | $I_E - I_L$              | • Simplest of all discriminators  
|                    |                          | • Does not require the Q branch  
|                    |                          | but requires a good carrier tracking loop for optimal functionality |
| EML Power          | $(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)$ | • The discriminator response is nearly the same as the coherent discriminator inside $\pm \frac{1}{2}$ chip |
| Normalized EML Power | $(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)$  
|                    | $(Q_E^2 + Q_E^2) + (Q_L^2 + Q_L^2)$ | • The discriminator has a great property when the chip error is larger than a $\frac{1}{2}$ chip; this will help the DLL to keep track in noisy signals. |
| Dot Product        | $I_F(I_E - I_L) + Q_F(Q_E - Q_L)$ | • This is the only DLL discriminator that uses all six correlator outputs |

Modern GPS receivers often contain many more than three complex correlators because digital correlators are relatively inexpensive [77]. Using more than three complex
correlators improves the performance of GPS receiver especially for multipath mitigation [16].

![Figure 3-19: Comparison between the common DLL discriminator responses](image)

3.3 Overview of Navigation Estimation Techniques

In this section the Least squares and Kalman filter estimation techniques will be reviewed. The least-squares approach will be discussed first and followed by an explanation of the Kalman filtering estimation method.

3.3.1 Least-Squares Approach

The GPS raw data can be post-processed using a parametric least-squares technique with positions fixed to the known test locations. The main concept of the least squares is to
minimize a specific quadratic form, essentially by making the sum of the squares of the weighted residuals as small as possible [37].

The linearized equation that relates GPS pseudorange measurements to the unknown user position and clock offset is

\[ \vec{v} = H \vec{x} \]  

3.7

where \( \vec{v} \) is the vector of pseudorange measurement residuals (the vector offset of the measured and calculated pseudorange measurements), \( H \) is the design matrix containing the linear relationships between the pseudorange measurements and the user position and clock offset and \( \vec{x} \) is the user position and clock offset correction vector.

To estimate the receiver position and clock offset with respect to GPS time, the user state vector is given by

\[ \vec{v} = [\Delta x \quad \Delta y \quad \Delta z \quad \Delta t_{GPS}] \]  

3.8

where \( \Delta x \) is the position deviation in the X direction, \( \Delta y \) is the position deviation in the Y direction, \( \Delta z \) is the position deviation in the Z direction and \( \Delta t_{GPS} \) is the receiver clock offset with respect to GPS.

The design matrix \( H \) is given by

\[
H = \begin{bmatrix}
\alpha_x^{SV1} & \alpha_y^{SV1} & \alpha_z^{SV1} & 1 \\
\alpha_x^{SV2} & \alpha_y^{SV2} & \alpha_z^{SV2} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\alpha_x^{SVn} & \alpha_y^{SVn} & \alpha_z^{SVn} & 0
\end{bmatrix}
\]  

3.9

The term \( \alpha_{x,y,z}^{SVn} \) represents the direction cosines from the receiver to satellite \( n \) in the X, Y and Z directions. The pseudorange residual vector is given by
\[
\tilde{v} = \begin{bmatrix}
PR_{c1} - PR_{m1} \\
PR_{c2} - PR_{m2} \\
\vdots \\
PR_{cn} - PR_{mn}
\end{bmatrix}
\]

where \(PR_{cn}\) is the \(n^{th}\) calculated pseudorange measurement based on the satellite position and the estimated receiver position, \(PR_{mn}\) is the \(n^{th}\) measured pseudorange.

The least-squares solution that minimizes the residuals of the user position is given by

\[
\bar{x} = (H^T W H)^{-1} H^T W \tilde{v}
\]

where \(W\) is the weight matrix.

Assuming that the pseudorange measurements are independent and have a Gaussian distribution, the weight matrix for N GPS is given by

\[
W = \begin{bmatrix}
1/\sigma_{GPS1}^2 & 0 & 0 & 0 \\
0 & 1/\sigma_{GPS2}^2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 1/\sigma_{GPSN}^2
\end{bmatrix}
\]

where \(\sigma_{GPSn}\) is the standard deviation for the \(n^{th}\) GPS satellite

Since the low elevation angle satellites usually contain higher noise levels because the signals travel through more atmosphere than higher elevation satellites, the lower elevation satellites should be weighted less than the higher elevation satellites. The weighting method based on satellite elevation angle is used in the static tests. In this weighting method, the standard deviations for measurements made at the zenith are scaled by \(1/\sin(e)\) where \(e\) is the satellite elevation angle.
The new residual vector $x$ in Equation (3.11) is iteratively generated and added to the estimated position and clock vector. The updated value will be used as a new estimate for the next iteration. The iteration will continue until the norm of the residual vector $x$ converges to a desired value.

According to [36], to relate pseudorange errors to position and clock error, several geometry factors are introduced which are also referred to as Dilution of Precision (DOP) parameters. The DOP parameters are obtained by taking the covariance of both sides of Equation (3.11) as:

$$\text{Cov} \left( \begin{array}{T} x' \\ \end{array} \right) = (H^TWH)^{-1} \text{Cov} \left( \begin{array}{T} v' \\ \end{array} \right)$$  

3.13

Next, $(H^TWH)^{-1}$ is written as a full matrix as:

$$(H^TWH)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}$$  

3.14

The DOP parameters are expressed in terms of the elements of $(H^TWH)^{-1}$ as follows:

$$PDOP = \sqrt{D_{11} + D_{22} + D_{33}}$$  

3.15

$$VDOP = \sqrt{D_{33}}$$  

3.16

$$HDOP = \sqrt{D_{11} + D_{22}}$$  

3.17

where P stands for position, V for vertical and H for horizontal.

### 3.3.2 Kalman Filtering Approach

Kalman filtering is a useful technique for estimating the state of a system given a previous state and external measurements of the state variables. The Kalman filter
extends the concept of least squares to include knowledge of how the state vector behaves in time [87].

The Kalman filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements [87]. Thus, the equations for the Kalman filter divided into two groups: Time update equations and measurement update equations. The time update equations use the current state and error covariance to calculate the a priori estimates for the next time step. The measurement update equations are responsible for improving the posteriori estimate by incorporating a new measurement into the a priori estimate. The time update equations can also be considered as predictor equations, while the measurement update equations can be considered as corrector equations [88]. The Kalman filter uses a predictor-corrector algorithm to solve numerical problems as shown in Figure 3-22.

![Figure 3-20: Algorithm of the Kalman filter](image)
The specific equations for the time and measurement updates are presented as follows [87]:

\[ X_{k+1}^- = A_{k,k+1} X_k^+ + B w_k \]  \hspace{1cm} 3.18

where \( X \) represents the state vector, \( A \) represents the transition matrix, \( B \) is the shaping matrix and \( w_k \) is the random variable representing white noise.

A priori estimate error covariance matrix is

\[ P_{k+1}^- = A_{k,k+1}^+ P_k^+ A_{k,k+1} + Q \]  \hspace{1cm} 3.19

where \( P \) is the error covariance matrix and \( Q \) is the process noise matrix

From equations (3.18) and (3.19), the time update equations represent the state and covariance estimates forward from time step \( k \) to step \( k + 1 \).

The time update equation is as the following

\[ K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \]  \hspace{1cm} 3.20

where \( K \) is the Kalman filter gain and \( Q \) is the process noise matrix, \( H \) is the design matrix (observation matrix) and \( R \) is the measurement noise covariance matrix.

A posterior state estimate is given by:

\[ \hat{x}_k^+ = \hat{x}_k^- + K_k (Z_k - H \hat{x}_k^-) \]  \hspace{1cm} 3.21

The covariance matrix of the updated estimated error is given by

\[ P_k^+ = (I - K_k H) P_k^- \]  \hspace{1cm} 3.22

Where \( P_k^+ \) is the covariance matrix of the updated estimated error, \( I \) is the identity matrix and \( P_k^- \) is the covariance matrix of the previous error.
Figure 3-23 shows a complete Kalman filter operation. The operation starts with the Kalman gain calculation shown in Equation (3.20). The next step is to update the estimate by adding the new measure and to generate an a posteriori state estimate as in Equation (3.21). Then one must obtain an a posteriori error covariance estimate via Equation (3.22). After each time epoch, the process is repeated with the previous a posteriori estimates used to predict the new a priori estimates. This recursive nature of the Kalman filter is an interesting feature. The Kalman filter recursively conditions the current estimate on all of the past measurements [87].

Time Update (Predict)
1. Project the state ahead
   \[ X_{k+1}^- = A_{k,k+1}X_k^+ + BW_k \]
2. Project the error covariance ahead
   \[ P_{k+1}^- = A_{k,k+1}P_k^+ A_{k,k+1}^T + Q \]

Measurement Update (Correction)
1. Compute the Kalman gain
   \[ K_k = P_k^-H^TH_p^-H^T + R)^{-1} \]
2. Update estimate with measurement
   \[ \hat{X}_k^+ = \hat{X}_k^- + K_k(Z_k - H\hat{X}_k^-) \]
3. Update the error covariance
   \[ P_k^+ = (I - K_kH)P_k^- \]

Figure 3-21: Complete picture of the operation of the Kalman filter
3.4 Multipath Mitigation Techniques

In standard DLL discriminator function discussed in section 3.2.3.3, two correlators called Early (E) and Late (L) correlators are spaced at a code chip width from each other and they are used to design the discriminator function. The code delay is estimated by balancing the amplitude/power on the E and L correlators. This method is called EML technique [1]. The classical EML fails to mitigate the multipath effects. Thus, several enhanced EML-based techniques are introduced in the literature [3] to reduce the multipath effects specially the short delay multipath scenarios.

One method of these enhanced EML techniques is based on reducing the EML spacing, which reduces the multipath impact. This method is known as narrow correlator. The narrow correlator uses spacing in the range of 0.05 to 0.2 code chips [28]. This method requires a wideband front-end (i.e. Bandwidth (BW) > 4 MHz) in order to be able to produce a sharp correlation function [29]. In the following sections, several multipath mitigation techniques will be overviewed.

3.4.1 Double Delta Correlator

Double-Delta technique is another group of discriminator-based DLL proposed to mitigate medium-to-long delay multipath scenarios. This technique typically uses five correlators, which are two correlators for early, two correlators for late and one correlator for prompt [3]. Among the sets of special functions of double-delta technique are the SC, the PAC and the HRC. The discriminator function of the SC is obtained as a linear combination of two narrow correlator discriminators [30]. The combination of two
discriminator functions allows resolving multipath rays with shorter excess delays but it causes noise amplification. The PAC is another implementation of the Double-Delta technique. PAC uses two sets of EML narrow correlators that are spaced at exactly double the width of the first correlator set [31]. The HRC utilizes several correlators to form a linear combination of correlator outputs that give up a net correlation function that is much narrower than the typical C/A code autocorrelation function [32].

3.4.1.1 Early-Late-Slope (ELS) technique

The Early-Late-Slope (ELS) technique is another multipath mitigation technique, which is also known as MET. The basic idea of ELS technique is to determine the slopes of the early and the late sides of the correlation function’s central peak [23]. Based on the slope values, the code-phase correction term is calculated, which goes back to the DLL algorithm to shift the pairs of correlators. In the existence of multipath, the slopes at the early and late sides of the autocorrelation function of the received signal are no longer of the same magnitude. Thus, the code-phase correction term is computed in order to compensate the slope difference of the rising and falling edges of the distorted autocorrelation function and it can be expressed as [19]:

$$\Delta \tau = \frac{y_1 - y_2 + d/2 (a_1 + a_2)}{a_1 - a_2}$$  \hspace{1cm} (3.23)

where $\Delta \tau$ is the code tracking error ($\Delta \tau$ will equal zero when the two correlators are located at equal distance on each side of the correlation peak), $a_1 = \frac{2(E_1 - E_2)}{d}$ and $a_2 = \frac{2(L_1 - L_2)}{d}$ are the autocorrelation function slopes at the early and late side,
respectively, \(y_1\) and \(y_2\) are the values of autocorrelation function at \(E_1\) and \(L_1\), and \(d\) is the spacing between \(E_1\) and \(L_1\). The spacing between \(E_2\) and \(L_2\) is equal double the spacing between \(E_1\) and \(L_1\).

3.4.1.2 High Resolution Correlator (HRC)

The High Resolution Correlator (HRC) utilizes the outputs of five correlators \((E_1, E_2, P, L_1, L_2)\) to form a linear combination of correlator outputs that give up a net correlation function that is much narrower than the typical C/A code autocorrelation function [4]. The \(E_2\) and \(L_2\) correlators are spaced twice the space of \(E_1\) and \(L_1\) correlators by \(2d\) chips and the \(P\) (prompt) correlator is in the central. Initially, a created correlator, named HRC Prompt, is modeled as follows [35]:

\[
P_{HRC} = 2P - (E_1 + L_1)
\]

Then, early and late HRC correlators are calculated to form an EML discriminator function for code tracking as follows:

\[
E_{HRC}(\tau) = P_{HRC}(\tau + d) = 2E_1 - (E_2 + P)
\]

\[
L_{HRC}(\tau) = P_{HRC}(\tau - d) = 2L_1 - (L_2 + P)
\]

Finally, the corresponding EML discriminator function can be expressed as

\[
D_{HRC}(\tau) = \frac{(E_{HRC} - L_{HRC})}{2}
\]

\[
D_{HRC}(\tau) = (E_1 - L_1) - (E_2 - L_2)/2
\]

The HRC technique helps only to reduce code multipath. Modified version of HRC was proposed by [89] to solve the issue of the reduction in signal power which degrades the
tracking accuracy at low $C/N_0$ values [32]. This modified implementation uses the following discriminator function:

$$\Delta \tau_{HRC} = \frac{ID_{HRC} \times IP + QP_{HRC} \times QP}{SP}$$  \hspace{1cm} 3.29$$

where $ID_{HRC}$ and $IPHRC$ are the real parts of DHRC and PHRC, $QDHRC$ and $QPHRC$ denote the imaginary parts of DHRC and PHRC and $SP = IP^2 + QP^2$. This implementation of HRC reduces carrier phase multipath as well as providing an enhanced reduction in code multipath.

### 3.4.2 Multipath Estimation Delay Lock Loop (MEDLL)

The MEDLL is a parametric mitigation technique that attempts to minimize the effect of multipath by estimating the parameters that characterize the received signal. The NovAtel Inc. implements it for GPS receivers [34]. The MEDLL aims at jointly estimating the amplitudes, delays and phases of both the LOS component and multipath components. The MEDLL uses many correlators to determine accurately the shape of the distorted correlation function [24]. The main idea is that the multipath components are estimated recursively and the effect of each path is removed before the estimation of the next. After this iteratively process is complete, the remaining is the estimate of the LOS correlation function. Several modified MEDLL techniques have been proposed in the literature to reduce the computational cost [35].

In this research, the non-coherent MEDLL was chosen for comparison analysis in chapter 5. The non-coherent MEDLL algorithm steps can be summarized as follows [19]:

73
1. Find the maximum peak of the correlation function from equation (3.5) and determine the corresponding delay \( \hat{\tau}_1 \), amplitude \( \hat{\alpha}_1 \), and \( \hat{\phi}_1 \).

2. Subtract a reference correlation function at estimated parameters from previous step (i.e. \( \hat{\alpha}_1 \mathcal{R}_{\text{ref}}(t - \hat{\tau}_1)e^{j\hat{\phi}_1} \) from the measured correlation function \( \mathcal{R} \) as follows:

\[
\mathcal{R}^1(\tau) = \mathcal{R} - \hat{\alpha}_1 \mathcal{R}_{\text{ref}}(t - \hat{\tau}_1)e^{j\hat{\phi}_1}
\]

3. Determine the new peak of the remaining function \( \mathcal{R}^1(\tau) \) at its corresponding delay, amplitude and phase.

4. The two previous steps are repeated until the remaining function is below a certain threshold (e.g. set from 0.3 to 0.4 which is used in this research) or until adding a new delay that doesn’t reduce the mean square error between the measured correlation function and the estimated correlation function.

3.5 Summary

Necessary technical background for the GPS receiver architecture and the signal processing techniques inside a GPS receiver was covered in this chapter. Digital designs of PLL, FLL-assisted-PLL and DLL have been presented. The navigation estimation techniques including the Least squares and Kalman filter techniques were overviewed. An introduction to classical and advanced multipath mitigation techniques has been given for later comparisons with new innovative approaches.
Chapter 4

A Robust Fine Acquisition Method for GPS Receivers

4.1 Introduction

As discussed in the previous chapter, the GPS signal acquisition is the first signal processing operation performed by the receiver. The purpose of the acquisition stage is to detect the satellite pseudo-random noise (PRN) in view and to provide coarse estimate of the GPS signal Doppler frequency shift and code delay for use by the tracking loops [44].

The signal acquisition is a time-consuming procedure inside the GPS receiver and its accuracy has direct influence on the tracking performance. The tracking stage regularly includes a preliminary fine frequency estimation method to refine the carrier frequency determination prior to starting the tracking process.

Two approaches have been widely addressed in the literature [10-12, 14, 90-92]. The first approach correlates long data in one step. This approach comprises both the time domain correlation and the circular correlation in the frequency domain. Since the acquisition time significantly increases with increasing data length therefore, the heavy computational load is considered one of the major disadvantages of the one step approach.

The second approach, known as coarse-to-fine acquisition method, consists of two stages [11]. The first stage uses short data to coarsely estimate the code phase and the carrier frequency of the visible satellites. The second stage strips off the C/A code from the correlated data and uses long C/A code stripped data to accurately estimate the carrier
frequency. Since the second approach is faster and more accurate than the one step approach therefore, it is commonly implemented in both hardware and software GPS receivers [91].

There are several types of the second approach in the literature [10-12, 92-94]. All types are common in the first coarse estimate stage, which uses short data length to remove the navigation data transition effect. However, the fine acquisition stage differs from one type to another. The first type depends on the carrier phase difference but it is subjected to ineffectiveness in case of weak GPS signal [14]. The second type utilizes the FFT, nevertheless, it has a heavy computational load when using long data [15]. Generally, the width of the tracking loop is only a few hertz. Thus, using the FFT to fine the carrier frequency is not a suitable approach [9]. The reason is that in order to find 10 Hz resolution, a data record of 100 ms is required. In addition, if the sampling frequency is 5 MHz, then the 100 ms of data contains 500,000 data points, which is very time consuming for FFT operation. Besides, the probability of having phase shift in 100 ms of navigation data is relatively high [9]. Another fine method uses the FLL, however, the FLL needs time to become steady before the transition of the FLL to the PLL [11].

In this chapter, a robust fine acquisition method based on orthogonal search is introduced to improve the acquisition performance of the GPS receivers. This method provides robust spectral estimation of satellite Doppler shift with less computational load and complexity than other state-of-the-art fine acquisition techniques. In addition, it adds several spectral estimation capabilities to the limited resource GPS receivers including
high frequency resolution, elimination of spectral leakage and operation in challenging environments.

4.2 Signal Model

Recalling from the previous chapter, the GPS signal at the output of the GPS receiver antenna can be written as [46]:

\[
r(t) = \sum_{i=1}^{N} A_i d_i(t - \tau_i) c_i(t - \tau_i) \cos\{2\pi(f_L + f_{D,i})t + \varphi_i(t)\} + \eta(t)
\]

where \( r(t) \) is the received signal that is the sum of \( N \) useful signals that are broadcasted by \( N \) different GPS satellites, \( A_i \) is the received carrier power including the effect of atmospheric attenuation and receiver antenna gain pattern, \( d_i(t) \) represents the data navigation message, \( \tau_i \) is the path delay introduced by the channel, \( c_i(t) \) represents the product of PRN code, secondary code and sub-carrier and \( f_L \) is the carrier frequency which depends on the GPS band under consideration. For GPS L1 band, \( f_L = 1575.42 \, MHz \) and for GPS L2 band, \( f_L = 1227.60 \, MHz \) and for GPS L5 band, \( f_L = 1176.45 \, MHz \). \( f_{D,i} \) is the Doppler frequency affecting the \( i^{th} \) useful signal, \( \varphi_i(t) \) is a time varying phase offset and \( \eta(t) \) is an Additive White Gaussian Noise (AWGN) with single sided noise Power Spectral Density (PSD) of \( \frac{N_0}{2} \, W/Hz \).

After the GPS signals reach the receiving antenna, the received signals are filtered, down converted to Intermediate Frequency (IF) and digitalized by the Radio-Frequency (RF) front-end [38]. Neglecting the quantization effect, the GPS signal at the output of the RF front-end can be rewritten as:
\[ r[n] = \sum_{i=1}^{N} A_i d_i [n - \tau_i] c_i [n - \tau_i] \cos \left\{ 2\pi (f_{IF} + f_{Di}) nT_s + \phi_i [n] \right\} + \eta[n] \quad 4.2 \]

where \( r[n] = r[nT_s] \) is a discrete-time sequence \( r[n] \), obtained by sampling a continuous-time signal \( r(t) \) with a sampling frequency \( f_s = \frac{1}{T_s} \) and \( f_{IF} \) is the IF of the carrier.

As mentioned in the previous chapter, the purpose of the acquisition process is to get an initial estimate of the Doppler frequency and code delay of all the satellites in view. All the acquisition techniques used inside the GPS receivers described in literature [46, 79-81] are based on the Maximum Likelihood (ML) estimation theory. As shown in Figure 3.2, the acquisition operation is based on evaluating and processing the Cross Ambiguity Function (CAF), which evaluates the correlation \( \mathcal{R}(\cdot) \) between the received signal and each PRN code across all possible combination of local code offset and Doppler shift.

The CAF can be defined as [46]:

\[ \mathcal{R}_c(\bar{\tau}, \bar{f}_D) = \frac{1}{N_c} \sum_{n=0}^{N_c-1} r[n] c[n - \bar{\tau}] \exp \left\{ j2\pi (f_{IF} + \bar{f}_D) nT_s \right\} \quad 4.3 \]

where \( N_c \) is the coherent integration time in milliseconds for GPS signal, \( c[n - \bar{\tau}] \) is the local replica reproducing the C/A code, \( \bar{\tau} \) and \( \bar{f}_D \) are the code delay offset and the Doppler shift, respectively, tested by the receiver.

The signal acquisition process is basically a two dimensional (2D) search in a grid plane (commonly referred to as search space), as shown in Figure 4-1, where \( \bar{\tau} \in (0, T_c) \), \( \bar{f}_D \in (-f_{D_{max}}, f_{D_{max}}) \) and \( T_c \) is the C/A code period. The variables under test \( \bar{\tau} \) and \( \bar{f}_D \)
are discretized with a step $\tau_s$ for the code delay, and a step $\Delta f$ for the Doppler frequency.

The integration time is $N_c = LT_s$ (where $L$ is the total number of integrated samples).

The number of trial points in the 2D-search space is $N_t = N_c / \tau_s$ and $N_f = 2f_{D_{\text{max}}} / \Delta f$. Therefore the acquisition search grid contains $N_t \times N_f$ cells, and each cell (marked by yellow color in Figure 4-1) corresponds to a parameter pair $\langle \bar{\tau}, \bar{f}_D \rangle$.

Once the CAF is evaluated, the maximum absolute value of the resulting CAF is then compared to a pre-defined threshold to make a decision regarding the presence or absence of the searched satellite [81]. The purpose of a traditional acquisition system is to find the coordinates of the peak cell $\langle \hat{\tau}, \hat{f}_d \rangle$ of the grid plane when the satellite is visible.

To improve the accuracy of the estimates, the steps $\tau_s$ and $\Delta f$ must be small, at the expenses of the computational complexity, since the number of points of the search space increases. The empirical value $\Delta f = 2/(3LT_s)$ is a typical choice [92] for the Doppler frequency step.

![Figure 4-1: Two-dimensional acquisition search space](image)
The effects of Doppler shift and code delay residuals have been studied in the literature [46, 81, 95]. In the presence of a residual of Doppler shift and code delay, the correlator output in the absence of noise, can be shown as [95]:

\[
R_{c,i}(\tau, f_d) = \frac{A_i}{2} R(\delta \tau) e^{-j\phi_i} \text{Sinc}(\pi \delta f N_c) e^{-j\pi \Delta f N_c}
\]

where \(\delta f\) is the carrier frequency residual error (i.e. \(\delta f = f_d - \hat{f}_d\)) and \(\delta \tau\) is the code delay residual error (i.e. \(\delta \tau = \tau - \hat{\tau}\)). As shown in equation (4.4), the negative effects of the frequency offset on the correlation output are twofold [86]. Firstly, the power of the correlation peak is attenuated by increasing the frequency difference between the received carrier and the local replica. Secondly, a constant phase rotation \(\pi \Delta f N_c\) is presented.

### 4.3 Fine Acquisition Methodology

The frequency resolution from the coarse acquisition process may be too rough for the tracking loops [9] and may eventually lead to poor positioning accuracy. Therefore, the fine acquisition stage is required to enhance the estimation of the carrier frequency once the initial code is wiped off from the carrier signal; after the coarse acquisition process is completed. The fine tuning stage can improve the initial estimated Doppler shift within a few tens of Hz [13]. This improvement will speed up the convergence of the FLL in order to correctly lock the carrier frequency [12].

There are several fine acquisition algorithms in the literature [10-12]. One of these methods shows that fine frequency resolution can be resolved by phase relation. This method [9] strips the C/A code from the input signal. As a result, the input becomes a
Continuous Wave (CW) signal. The method assumes that the highest component in 1 ms of data at time $m$ is $X_m(k)$, where $k$ represents the frequency component of the input signal. The initial phase $\theta_m(k)$ of the input signal can, therefore, be found from the Discrete Fourier Transform (DFT) outputs as

$$\theta_m(k) = \tan^{-1}\left\{\frac{l_m[X_m(k)]}{R_e[X_m(k)]}\right\} \quad (4.5)$$

where $l_m$ and $R_e$ represent the imaginary and real parts, respectively. The method assumes that at time $n$, a short time after $m$, the DFT component $X_n(k)$ of 1 ms of data is also the strongest component, because the input frequency will not change that rapidly during a short time. The initial phase angle of the input signal at time $n$ and frequency component $k$ is

$$\theta_n(k) = \tan^{-1}\left\{\frac{l_m[X_n(k)]}{R_e[X_n(k)]}\right\} \quad (4.6)$$

The method states that the two phase angles from equations (4.5) and (4.6) can be used to find the fine frequency as

$$f = \frac{\theta_n(k) - \theta_m(k)}{2\pi(n - m)} \quad (4.7)$$

The frequency obtained from equation (4.7) provides a much finer frequency resolution than DFT. This method requires the phase difference to be less than $2\pi$ [96]. This method is limited in case of weak GPS signals. Zero-Padding FFT-based fine acquisition is another method [38]. Unfortunately, it increases the number of FFT data points to
improve the spectral resolution which in turn contributes to higher computational complexity [97].

4.4 Proposed Fine Acquisition Method
The basic scheme of the acquisition method proposed in this thesis is illustrated in Figure 4-2. The left part of the figure indicates the traditional GPS acquisition process from which a two-dimensional search grid (marked in yellow color) is generally obtained, while the right side shows the presence of a new additional block able to accurately refine the Doppler frequency estimate in a high frequency resolution approach before the tracking loops. The proposed method in this chapter refers to the algorithms used by this additional block to refine the frequency estimation.

![Figure 4-2: Structure of a new Doppler frequency refinement process](image-url)
After the coarse acquisition is accomplished, a 2D search grid is always obtained, and the resulting estimated vector is selected as the location of the peak cell and, at the same time, the other cells in the search grid are abandoned. However, because of the large frequency searching step $\Delta f$, the frequency estimate error is located in the range $[-\Delta f/2, \Delta f/2]$, thus the initial Doppler frequency estimate is usually not accurate enough to pass to the tracking loop directly. Therefore, a new fine acquisition method is proposed here to improve the carrier frequency accuracy further. The proposed method is based on the search grid is already evaluated by the acquisition; that is, no need to compute new correlations, but we only use the neighbor cells of the coarse acquisition peak, already available in the search grid. The flowchart of the proposed algorithm is shown in Figure 4-3.

The code phase and the carrier frequency are determined according to the peak of the correlation function obtained from the coarse acquisition. The length of the received signal fragment used in performing the correlation is restricted typically to 20 ms because the modulation of the received signal fragment by the 50 Hz navigation data corrupts an ordinary correlation using a longer signal fragment.

The proposed method strips off the C/A code phase from the carrier signal. As the code phase accuracy is about 400 ns for the coarse acquisition in a sampling frequency of 2.5 MHz, thus, it has a small effect on the carrier correlation in the proposed fine acquisition algorithm [12].
Figure 4-3: Flowchart of the proposed fine acquisition algorithm

The initial frequency $f_{init}$ is set to be equal to the acquired carrier frequency by the coarse acquisition $\hat{f}_d$. Then, a set of sinusoidal candidate functions is generated, which are pairs of cosine $P_m(n)$ and sine $P_{m+1}(n)$ terms at each of the frequencies range of interest. The candidate functions are given by

$$P_m(n) = \cos (2\pi f_m n)$$

$$P_{m+1}(n) = \sin (2\pi f_m n)$$
where $m = 1, 2, \ldots, K$, $f_m$ is the digital frequency of the candidate pair and $K$ is the number of candidate pairs.

In this research, the Doppler shift range of interest is typically set around the estimated Doppler shift acquired from the coarse acquisition stage as illustrated in Figure 4-4. The estimated Doppler shift is then considered the midpoint of a range that spans from $-\Delta f$ to $\Delta f$ with a total range of two coarse acquisition steps (i.e. $2\times\Delta f$). The proposed method’s Doppler shift range is defined as:

$$\hat{f}_d - \Delta f \leq f_{\text{init}} \leq \hat{f}_d + \Delta f$$  \hspace{1cm} (4.9)

where $\Delta f$ is the coarse acquisition Doppler shift step.

The proposed method’s candidate frequencies are chosen to have a higher resolution than that of the FFT to achieve better de-noising performance. The frequency resolution of the FFT can be given by:

$$\text{fft resolution} = \frac{f_s}{N}$$  \hspace{1cm} (4.10)

where $f_s$ is the sampling frequency and $N$ is the number of points in the record.

For this research, the spacing of proposed method frequencies candidate function is typically set in the order of one tenth the FFT resolution for each segment.
An orthogonal search algorithm based on FOS [98-100] is then utilized to provide both reliable and fast acquisition of GPS signals.

The FOS algorithm uses an arbitrary set of non-orthogonal candidate functions $P_m(n)$ and finds a functional expansion of an input $y(n)$ in order to minimize the mean squared error (MSE) between the input and the functional expansion.

The functional expansion of the input $y(n)$ in terms of the arbitrary candidate functions $P_m(n)$ is given by:

$$y(n) = \sum_{m=0}^{M} a_m P_m(n) + e(n) \quad 4.11$$

where $a_m$ is the weights of the functional expansion, and $e(n)$ is the modeling error.

By choosing non-orthogonal candidate functions, there is no unique solution for (4.11). However, the FOS method may model the input with fewer model terms than an
orthogonal functional expansion. For example, in the conventional fine acquisition methods, which are based on FFT, the FFT uses a basis set of complex sinusoidal functions that have an integral number of periods in the record length [99-101]. For the FFT to model a frequency that does not have an integral number of periods in the record length, energy is spread into all the other frequencies, which is a phenomenon known as spectral leakage. By using candidate functions that are non-orthogonal, the proposed method may be able to model this frequency between two FFT bins with a single term resulting in many fewer weighting terms in the model [99-101].

The FOS method begins by creating a functional expansion using orthogonal basis functions such that:

\[
y(n) = \sum_{m=0}^{M} g_m w_m(n) + e(n)
\]

where \( w_m(n) \) is a set of orthogonal functions derived from the candidate functions \( P_m(n) \), \( g_m \) is the weight, and \( e(n) \) is an error term. The orthogonal function \( w_m(n) \) are derived from the candidate functions \( P_m(n) \) using Gram-Schmidt (GS) orthogonalization algorithm. The orthogonal functions \( w_m(n) \) are implicitly defined by the Gram–Schmidt coefficients \( \alpha_{mr} \) and do not need to be computed point-by-point.

The Gram–Schmidt coefficients \( \alpha_{mr} \) and the orthogonal weights \( g_m \) can be found recursively using the following equations [98]:

\[
w_0 = p_0(n)
\]

\[
D(m, 0) = \frac{p_m(n) p_0(n)}{p_0(n)}
\]
\[ D(m, r) = p_m(n)p_r(n) - \sum_{i=0}^{r-1} \alpha_{ri}D(m, i) \tag{4.15} \]

\[ \alpha_{mr} = \frac{p_m(n)w_r(n)}{w_r^2(n)} = \frac{D(m, r)}{D(r, r)} \tag{4.16} \]

\[ C(0) = y(n)p_0(n) \tag{4.17} \]

\[ C(m) = y(n)p_m(n) - \sum_{r=0}^{m-1} \alpha_{mr}C(r) \tag{4.18} \]

\[ g_m = \frac{C(m)}{D(m, m)} \tag{4.19} \]

In its last stage, the FOS method calculates the weights of the original functional expansion \( a_m \), from the weights of the orthogonal series expansion, \( g_m \). The value of \( a_m \) can be found recursively using

\[ a_m = \sum_{i=m}^{M} g_i v_i \tag{4.20} \]

where \( v_m = 1 \) and

\[ v_i = -\sum_{r=m}^{i-1} \alpha_{ir} v_r, \quad i = m + 1, m + 2, \ldots, M \tag{4.21} \]

From (4.14), (4.15), (4.17) and (4.18) it can be noted that the FOS method requires the calculation of the correlation between the candidate functions and the calculation of the correlation between the input and the candidate functions. The correlation between the input and the candidate function \( y(n)p_m(n) \) are typically calculated point-by-point once at the start of the algorithm and then stored for later quick retrieval.
The MSE of the orthogonal function expansion has been shown to be [99]:

$$\bar{e}^2(n) = \bar{y}^2(n) - \sum_{m=0}^{M} g_m^2 \bar{w}_m^2(n)$$  \hspace{1cm} 4.22

It then follows that the MSE reduction given by the $m^{th}$ candidate function is given by:

$$Q_m = g_m^2 \bar{w}_m^2(n) = g_m^2 D(m, m)$$  \hspace{1cm} 4.23

The FOS method can fit a model with a small number of model terms by fitting terms, which reduce the MSE in order of their significance.

The proposed method is stopped in one of three cases. The first is when a certain maximum number of terms are fitted. The second case is when the ratio of MSE to the mean squared value of the input signal is below a pre-defined threshold. The third case is the FOS algorithm stops fitting model terms when fitting a term reduces the MSE no more than adding white Gaussian noise [98].

The output frequency $f_{out}$ is renewed with the local replica carrier frequency and passed to the signal tracking process.

**4.4.1 Discussion**

There are two significant differences between the proposed method and the conventional fine acquisition method, which is based on Zero-Padding FFT. First, the proposed method yields a parsimonious sinusoidal series representation by selecting the most significant sinusoidal components first. Second, the frequencies of the selected sinusoids should neither be commensurate nor integers multiples of the fundamental frequency.
corresponding to the record length. This translates to better frequency resolution in the spectral model.

The candidate frequencies can be selected so that the candidate functions focus on a particular Doppler shift range of interest. For example, the candidates can be spaced with a high resolution on a range of interest, and outside the range of interest, the candidates can be spaced by the FFT resolution intervals. In general, it is desirable to have an optimal number of candidate frequencies. Too few terms result in a model that does not accurately model the motion dynamics. Too many terms will add noise terms as well as increase the computation time. Therefore, in choosing the number of candidate frequencies, the proposed method tries to maintain a balance between accuracy and computational load.

Thus, a candidate acceptance threshold, requiring a frequency pair to fit a minimum percentage of the overall energy in the signal, is set. These thresholds allow the proposed method to model the motion dynamics and reject frequency terms that model the noise.

### 4.5 Simulation Results

Simulations were carried out to verify the performance of the proposed fine acquisition method. The simulations were conducted using MATLAB to generate IF GPS L1 data. Initially the simulations were performed using the signal model stated by equation (4.2) without data bit modulation. To make the simulation more realistic, the generated signal included five satellites. The format of the data and the simulated Doppler shift values are listed in Table 4-1.
Table 4-1: The format of the data and the simulated Doppler shift values

<table>
<thead>
<tr>
<th>Data Format</th>
<th>Value</th>
<th>Doppler Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Parameter</td>
<td>Value</td>
<td>Satellite PRN</td>
</tr>
<tr>
<td>IF Frequency</td>
<td>21.5 MHz</td>
<td>3</td>
</tr>
<tr>
<td>Sampling Frequency</td>
<td>8 MHz</td>
<td>14</td>
</tr>
<tr>
<td>ADC</td>
<td>2 bits</td>
<td>18</td>
</tr>
<tr>
<td>Data Length</td>
<td>100 ms</td>
<td>21</td>
</tr>
</tbody>
</table>

In the coarse acquisition stage, the frequency band around the IF was set to equal 14 kHz with a step size of 500 Hz. For the fine acquisition stage, the performance of the proposed method was initially compared with conventional FFT-based fine acquisition algorithm in which the radix-2 FFT algorithm was used [97]. The radix-2 FFT can only be performed with sequences of $2^n$ data length. Unfortunately, most sampling rates don’t provide the required power-of-two data length. Therefore, the vector of signal samples is extended by using zero-padding to a length of $L$ samples, where $L = 2^n$ is a power-of-two data size with a frequency resolution of $\frac{f_s}{L}$ Hz.

In addition, the proposed method is compared to zero-padding FFT-based fine acquisition algorithms [38]. These algorithms are based on adding power-of-two zero padding blocks to the data samples by finding the next highest power-of-two of the data length (i.e. $n = \log_2 L$) and multiplying it by a factor of $i$, where $i = 2, 3, 4, ...$ etc. For example, the total number of samples in 1 ms of data sampled at 8 MHz is 8000. Consequently, the next highest power-of-two of the data length is $n = 13$. In order to apply the zero-
padding FFT-based fine acquisition with a factor of 2 (namely FFT-ZP \((2\times n)\)) the total
data length will be \(L = 2^{i\times n} = 2^{2\times13}\) samples.

Since the real Doppler shift is known for the simulation, the errors of the Doppler shift
are the absolute differences between the acquired Doppler frequency and the simulated
value.

Figure 4-5 and Figure 4-6 demonstrate the performance of the proposed method, FFT and
zero-padding FFT-based fine acquisition techniques in estimating the Doppler shift for
satellites PRNs 14 and 18, respectively, for different window sizes. FFT-ZP \((2\times n)\) and
FFT-ZP \((5\times n)\) refer to using the zero-padding FFT-based fine acquisition technique at
factors \(i = 2\) and \(5\), respectively. As shown in Figure 4-5 and Figure 4-6, the proposed
method outperforms the other two methods in estimating the Doppler shift for different
window sizes. In addition, the error values of the estimated Doppler shift produced by the
proposed method are considerably smaller than the ones produced by the other methods,
specially, when using small window size (i.e. 1 ms). For example, in case of PRN 14,
using only 1 ms window size, the proposed method provides an estimate of Doppler shift
with an error of 20 Hz. Otherwise, the FFT, FFT-ZP(2\times n) and FFT-ZP(5\times n) methods
provide an error of 500 kHz, 2 kHz and 700 Hz, respectively.
Figure 4-5: Satellite Doppler shift error versus the window size for PRN 14 using the proposed method, FFT and Zero-Padding FFT-based fine acquisition algorithms.

Figure 4-6: Satellite Doppler shift error versus the window size for PRN 18 using the proposed method, FFT and Zero-Padding FFT-based fine acquisition algorithms.
Figure 4-7 shows the satellite Doppler shift errors for satellites PRNs 03, 14, 18 and 21 computed using the proposed method and the FFT and zero-padding FFT-based fine acquisition algorithms with 5 ms window size of data. The error values of the estimated Doppler shift produced by the proposed method are considerably smaller than the ones produced by the other methods for all of the PRNs. For instance, in case of PRN 14, the proposed method can provide 84% improvement when compared to the FFT-ZP(5×n) method. It is remarkable that the FFT spectral resolution can be improved by zero padding and large window sizes, however, this leads to additional computational load. As showed in this research our proposed method maintains the balance between acquisition accuracy and computational load.

Figure 4-7: Satellite Doppler shift error for all satellites with 5 ms window size using the proposed method, FFT and zero-padding FFT-based fine acquisition algorithms
To examine the performance of the proposed algorithm against noise, the IF GPS data with SNR from -15 to 45 dB were generated using MATLAB. The noise added to the IF GPS data is WGN and 100 Monte Carlo runs were used to calculate the Root-Mean-Square (RMS) value of the Doppler shift error under different SNRs. Figure 4-8 and Figure 4-9 compare the performance of the proposed method, FFT and zero-padding FFT-based fine acquisition techniques in estimating the Doppler shift for satellites PRNs 14 and 18 under different SNRs with 1 ms window size. The RMS values of the Doppler shift errors computed by the proposed method are smaller than the ones produced by the other methods for the different SNRs.

Figure 4-8: RMS of satellite Doppler shift errors versus different SNRs for PRN 14 using the proposed method and the other two methods
Figure 4-9: RMS of satellite Doppler shift errors versus different SNRs for PRN 14 using the proposed method and the other two methods.

Figure 4-10 illustrates the RMS values of the Doppler shift error for the satellites PRNs 03, 14, 18 and 21 computed using the proposed method and the FFT and zero-padding FFT-based fine acquisition algorithms with 1 ms window size at SNR = -10 dB. It can be seen that the proposed method outperforms other methods in estimating the Doppler shift at low SNR and small window size for all of the PRNs.
4.6 Experimental Results

To further verify the performance of the proposed fine acquisition method, real GPS L1 data were generated using SPIRENT GSS6700 simulator and logged using NovAtel FireHose front-end. The tests were performed in static and dynamic modes. The starting point was chosen at a point in Kingston, Ontario at latitude 44° 13.726’, longitude -76° 27.948’ and height 100 m. The real RF data were down converted to the baseband and sampled at a frequency of 5 MHz and quantified with 4 bits. The raw GPS samples were processed using NavINST research group software receiver. Figure 4-11 illustrates a frequency domain, time domain and histogram of the raw GPS L1 I/Q samples,
respectively. In the frequency domain, it clearly shows that the FirHose front-end shifts the received frequencies to the baseband [102]. In the histogram, it is clear that all four bits of the Analog to Digital Converter (ADC) are being produced based on the 16 levels present within the histogram.

Figure 4-11: Frequency domain, time domain and histogram of the raw GPS L1 I/Q samples collected data

There are 10 GPS satellites available above a 5-degree elevation mask at the initial location as shown in Figure 4-12. The number of satellites acquired by the GPS software receiver is shown in Figure 4-13.
Figure 4-12: Skyplot of the GPS satellites in view during the experiment

Acquisition results

Figure 4-13: Acquisition results of the GPS software receiver
Table 4-2 lists the coarse and fine acquisition results of the real experiment. A data length of 8 ms was chosen to fine the coarse acquisition results using the proposed method and the FFT and zero-padding FFT-based fine acquisition algorithms. As mentioned before, the FFT-ZP (2×n) and FFT-ZP (5×n) refer to using the zero-padding FFT-based fine acquisition technique at factors $i = 2$ and 5, respectively.

Since it is difficult to acquire the real Doppler shift in real experiments, the results of FLL are provided and the frequency lock indicator (FLI) was used as a performance metric to evaluate the proposed method with respect to the other methods. Generally, the FLI can be used to evaluate the performance of frequency tracking. FLI [103] is function of frequency error and integration time and is given as:

$$FLI = \cos (4\pi \delta f N_c)$$

where $\delta f$ is the frequency error in the tracking loop and $N_c$ is the integration time.

For example, for a 10 ms integration time, FLI = 0.9 means that the frequency error in frequency tracking loop is 3.6 Hz.
A good carrier frequency tracking performance results in a reliable extraction of the navigation data bits. The criteria for assessing the proposed method in the real experiment in this research is based on the frequency tracking performance indicated by the FLI and the conversing process of the FLL to lock the correct satellite Doppler shift.

Figure 4-14 and Figure 4-15 depict the converging processes of FLL for PRNs 16 and 31, respectively. The converging processes of FLL for other PRNs are similar to that shown in Figure 4-14 and Figure 4-15, which are therefore not provided herein. In Figure 4-14 and Figure 4-15, FLL is just operated during the transient period. After the FLL converges, the PLL starts to track the carrier frequency with higher accuracy. It is clearly shown in Figure 4-14 and Figure 4-15 that the proposed method takes a shorter time to

<table>
<thead>
<tr>
<th>PRN</th>
<th>Frequency by Coarse Acquisition (Hz)</th>
<th>Frequency by Fine Acquisition (Hz)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>FFT</td>
<td>FFT-ZP (2×n)</td>
</tr>
<tr>
<td>16</td>
<td>-3000</td>
<td>-2823</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
<td>76</td>
</tr>
<tr>
<td>20</td>
<td>-1000</td>
<td>-992</td>
</tr>
<tr>
<td>13</td>
<td>-3000</td>
<td>-2670</td>
</tr>
<tr>
<td>31</td>
<td>2000</td>
<td>1755</td>
</tr>
<tr>
<td>14</td>
<td>3000</td>
<td>2670</td>
</tr>
<tr>
<td>11</td>
<td>3000</td>
<td>2747</td>
</tr>
<tr>
<td>30</td>
<td>2500</td>
<td>2060</td>
</tr>
<tr>
<td>23</td>
<td>-2500</td>
<td>-2060</td>
</tr>
<tr>
<td>4</td>
<td>-2500</td>
<td>-2060</td>
</tr>
</tbody>
</table>
Converge than the other methods. Convergence speed doesn’t only affect the tracking of the right frequency but also the tracking of the right signal phase and hence obtaining the required navigation data bits through correlation. Figure 4-16 and Figure 4-18 illustrate the FLI while Figure 4-17 and Figure 4-19 illustrate the in-phase prompt correlator calculated for the four fine acquisition algorithms used for PRNs 16 and 31, respectively. The FLI takes a maximum value of unity when the frequency is perfectly tracked. It can be seen that FLI of the proposed method converges faster than the other two methods. As a result, the tracking loop is able to converge more quickly to the correct Doppler shift and the switching from FLL mode to PLL mode occurs more quickly. It can be determined that navigation bits are obtained much faster when using the proposed method.

**Figure 4-14: The Converging process of FLL for PRN 16**
Figure 4-15: The Converging process of FLL for PRN 31

Figure 4-16: Frequency lock indicator calculated for the four fine acquisition algorithms used for PRN 16
Figure 4-17: Navigation data bits obtained using the four fine acquisition algorithms used for PRN 16

Figure 4-18: Frequency lock indicator calculated for the four fine acquisition algorithms used for PRN 31
4.7 Summary

A high resolution fine acquisition method is proposed to improve the acquisition performance of the GPS receivers. After the coarse acquisition has been accomplished, the proposed method utilizes an orthogonal search algorithm to provide robust spectral estimation of carrier Doppler shift. The proposed method maintains a balance between the acquisition accuracy and computational load. In addition, it is good at detecting the carrier Doppler shift buried in white noise even at poor SNRs. It is important to mention that the MATLAB code of the FOS algorithm used in this research is not optimized to be used for real time applications. It is required to optimize the proposed fine acquisition method in order to make it efficient, reliable, and scalable [104].
Both the simulation and experimental results showed that the proposed method was significantly more accurate than the FFT zero-padding approach. The simulation results demonstrated that the proposed method outperformed other methods in estimating the carrier Doppler shift at SNR = -15 dB and using 1 ms window size. The improvement percentage was more than 70% in the acquisition accuracy. The experimental results indicate that the proposed fine acquisition method speeds up the convergence of the FLL in order to correctly lock the carrier Doppler shift frequency.
Chapter 5
A High Resolution Code Tracking Method to Mitigate Interference Effects

5.1 Introduction
A variety of advanced signal processing techniques have been introduced in the literature [3] to improve the multipath mitigation performance of the GPS receivers. As discussed in Chapter 3, the classical DLL tracking algorithm is one of the most widely used techniques in modern commercial GPS receivers. The DLL tracking algorithm uses different correlators such as double-delta correlator, SC, ELS correlator and HRC to mitigate the code phase multipath and get better TOA estimation of the GPS signals. Although these techniques are easily implemented and effective in weak multipath environments, the performance of these techniques in dense multipath environments is still rather limited. In most urban canyon navigation environments, the LOS component is overlapping one or more NLOS components [19]. The main reason is that the relative delay between the LOS and NLOS signals is less than one C/A code chip duration (i.e. in the order of tens of ns) [62] which distorts the derived correlation peak. So, we cannot rely on that correlation peak to estimate the TOA [5]. Thus, the classical multipath techniques are not capable of accurately following the delay of the LOS signal since it does not consider the bias contributed by the multipath components. Figure 5-1 shows the effect of an in-phase and out-phase reflection on the normalized correlation function. It can be seen from Figure 5-1 that due to the presence of multipath
the measured correlation function doesn’t have the same slope on both of its sides. As a result, even when early and late correlators are balanced, the prompt correlator is still not at the peak of the correlation triangle. This results in error in the pseudorange measurement. Thus, traditional multipath mitigation techniques are unsatisfactory especially for urban canyons.
Figure 5-1: Effect of in-phase and out-phase multipath on normalized correlation function
The MEDLL is another class of code tracking algorithm [34]. The MEDLL is based on ML estimation. The MEDLL techniques outperform the classical multipath mitigation techniques but it does not completely eliminate all multipath errors [19, 34]. Multipath signals with short relative delays specifically are difficult to reject.

This chapter introduces a robust correlation-based multipath mitigation technique based on FOS to mitigate multipath effects in challenging environments such as in urban canyons where GPS receivers suffer from closely spaced multipath signals. The method suggested here provides a higher resolution code tracking performance than conventional techniques in harsh environments.

In order to compare the performance of the proposed method with state-of-the-art multipath mitigation techniques, this research utilized a SPIRENT GSS8000 simulator to provide a controlled environment for different multipath scenarios. The performance of the proposed method has been examined using several realistic simulation scenarios. The performance metrics are based upon pseudorange and positioning errors. The calculated range and position solutions by the proposed method will be compared to the reference solutions from the SPIRENT GNSS simulator that is not affected by multipath in these conditions.

**5.2 Proposed Method**

In this research, the FOS algorithm [98, 105-108] is used to model the distorted correlation function to estimate TOA of the LOS and multipath signals. The motivation behind the development of the multipath mitigation technique based on FOS was to find
such an algorithm that improves the code delay estimation inside the tracking loop of a GPS receiver especially for closely spaced multipath navigation environments. The main idea is utilizing the high-resolution estimation capabilities of FOS to mitigate the mystical multipath signals. The general architecture of the proposed technique inside the delay-tracking loop is shown in Figure 5-2.

**Figure 5-2: General architecture of FOS-based multipath mitigation technique inside the delay tracking loop**

As shown in Figure 5-2, after the essential front-end processing and after the carrier has been wiped off, the received signal is passed to a bank of correlators. The Numerical Control Oscillator (NCO) and PRN generator block produce a group of early and late versions of replicas codes. In case of the classical DLL, the received signal is correlated with each replica in the bank of correlators. Several code tracking techniques (called
discriminator in Figure 5-2) use the output of the correlator bank values as input to generate the estimated LOS delay as output [3]. A loop filter then provides a smooth output of the discriminator.

The proposed algorithm that is highlighted in red in Figure 5-2 uses the corresponding output of the bank of correlators as input to the FOS algorithm to model the measured correlation function (i.e. distorted correlation function due to multipath effects) and estimate the multipath parameters along with the LOS signal.

The FOS algorithm utilizes an arbitrary set of non-orthogonal candidate functions $P_m(n)$ and finds a functional expansion of a measured correlation function $\mathcal{R}(n)$ in order to minimize the mean squared error (MSE) between the measured correlation function and the functional expansion.

The functional expansion of the input $\mathcal{R}(n)$ in terms of the arbitrary candidate functions $P_m(n)$ is given by:

$$\mathcal{R}(n) = \sum_{m=0}^{M} a_m P_m(n) + e(n) \quad \text{5.1}$$

where $a_m$ is the set of weights of the functional expansion, $P_m(n)$ are the model terms selected from the set of candidate functions and $e(n)$ is the modeling error.

These model terms are a set of GPS L1 ideal correlation functions $\mathcal{R}_{\text{ideal}}(n)$ for the middle/prompt correlator of a certain code-delay window range with several multipath delays, phases and amplitudes thereof:

$$P_m(\alpha, \tau, \varphi) = a_m \mathcal{R}_{\text{ideal}}(\tau_m) e^{j\varphi_m} \quad \text{5.2}$$

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where $\alpha$ is the relative amplitude of $l^{th}$ multipath, $\tau$ is the relative delay and $\varphi$ is the relative phase with respect to LOS signal. These candidate functions are generated offline and saved in a lookup table in memory. The set of multipath parameters can be defined as follows:

\[
0 \leq \alpha \leq 1 \\
0 \leq \tau \leq 2T_c \\
0 \leq \varphi \leq 2\pi
\]

where $T_c$ is GPS C/A code chip duration. Since the maximum excessive delay of the channel for near echoes is less than one chip in most closely spaced multipath environments, therefore, the focus here is to search for those components with sub-chip level delays. Thus, we consider the search region to be $\pm 2$ chips around the correlation peak. Herein, only one signal from one of the satellites is considered whereas the contribution of the signals from other satellites is modeled as AWGN due to the weakness of their interference. The code-delay window range is determined based on the estimated correlation peak, theoretically, can be anywhere within the code delay window range of $\pm \tau_w$ chips [3]. The code delay window range essentially depends on the number of correlators (i.e. $M$) and the spacing between the correlators (i.e. $\Delta$) according to

\[
\tau_w = \pm \frac{(M - 1)}{2} \Delta
\]

For example, if 201 correlators are used with a correlator spacing of 0.02 chips, then the resulting code-delay window range will be $\pm 2$ chips with respect to the prompt correlator. Thus, the LOS delay estimate can be anywhere within the $\pm 2$ chips window.
range. Consequently, the FOS pre-designated candidate functions generated by varying all multipath components for the middle correlator (e.g. the 101th correlator for a code-delay window range of ± 2 chips with 0.02 chips correlator spacing) of the code-delay window.

By choosing non-orthogonal candidate functions, there is no unique solution for equation (5.1). However, FOS may model the measured correlation function with fewer model terms than an orthogonal functional expansion [98].

FOS begins by creating a functional expansion using orthogonal basis functions such that:

$$\hat{R}(n) = \sum_{m=0}^{M} g_m w_m(n) + e(n)$$  \hspace{1cm} 5.5

where $w_m(n)$ is a set of orthogonal functions derived from the candidate functions $p_m(n)$, $g_m$ is the weight and $e(n)$ is an error term. The orthogonal functions $w_m(n)$ are derived from the candidate functions $p_m(n)$ using the GS orthogonalization algorithm. The orthogonal functions $w_m(n)$ are implicitly defined by the GS coefficients $\alpha_{mr}$ and do not need to be computed point-by-point [109].

As discussed in Chapter 4, the GS coefficients $\alpha_{mr}$ and the orthogonal weights $g_m$ can be found recursively using the equations [98]:

$$w_0 = p_0(n)$$  \hspace{1cm} 5.6

$$D(m,0) = \frac{p_m(n)p_0(n)}{p_0(n)}$$  \hspace{1cm} 5.7
\[ D(m, r) = \frac{p_m(n)p_r(n)}{w_r^2(n)} - \sum_{i=0}^{r-1} \alpha_{ri}D(m, i) \]  
\[ \alpha_{mr} = \frac{p_m(n)w_r(n)}{w_r^2(n)} = \frac{D(m, r)}{D(r, r)} \]

\[ C(0) = \overline{R}(n)p_0(n) \]

\[ C(m) = \overline{R}(n)p_m(n) - \sum_{r=0}^{m-1} \alpha_{mr}C(r) \]

\[ g_m = \frac{C(m)}{D(m, m)} \]

In its last stage, FOS calculates the weights of the original functional expansion \( a_m \) (equation (5.1)), from the weights of the orthogonal series expansion \( g_m \) and GS coefficients \( \alpha_{mr} \) [109]. The value of \( a_m \) can be found recursively using

\[ a_m = \sum_{i=m}^{M} g_i v_i \]

where \( v_m = 1 \) and

\[ v_i = - \sum_{r=m}^{i-1} \alpha_{ir} v_r, \quad i = m + 1, m + 2, ..., M \]

FOS requires the calculation of the correlation between the candidate functions and the calculation of the correlation between the input and the candidate functions. The correction between the measured correlation function and the candidate function \( \overline{R}(n)p_m(n) \) is typically calculated point-by-point once at the start of the algorithm and then stored for later quick retrieval.
The MSE of the orthogonal function expansion has been shown to be

$$\hat{e}^2(n) = \hat{R}^2(n) - \sum_{m=0}^{M} g_m^2 \overline{w_m^2(n)}$$  \hspace{1cm} 5.15

It then follows that the MSE reduction given by the $m^{th}$ candidate function is given by

$$Q_m = g_m^2 \overline{w_m^2(n)} = g_m^2 D(m,m)$$  \hspace{1cm} 5.16

The candidate with the greatest value for $Q$ is selected as the model term, but optionally its addition to the model may be subject to its $Q$ value exceeding a threshold level [109].

The residual MSE after the addition of each term can be computed by

$$MSE_m = MSE_{m-1} - Q_m$$  \hspace{1cm} 5.17

The search algorithm may be stopped when an acceptably small residual MSE has been achieved (i.e. a ratio of the MSE over the mean squared value of the measured correlation function [107] or an acceptably small percentage of the variance of the time-series being modeled). The search may also stop when a certain number of terms have been fitted. In this research, we assume that the total number of multipath components does not exceed 4. Another stopping criterion is when none of the remaining candidates can yield a sufficient MSE reduction value (this criterion would be representative of not having any candidates that would yield a MSE reduction value greater than the addition of a white Gaussian noise series).

Generally speaking, the developed technique performs a projection of the distorted multipath correlation function into a set of FOS pre-designated candidate functions. The candidate functions are defined as ideal reference correlation functions with certain
amplitude(s), phase(s) and delay(s). The proposed method tries to find a perfect match between the distorted correlation function and the FOS pre-designated candidate functions. The accuracy of the model produced by the proposed method depends on the correlator spacing, the candidate functions and the stopping criteria in the algorithm.

In order to estimate the LOS delay, in this research, it is assumed that the LOS is present and it has the shortest time of arrival from the satellite to the receiver (shortest transit time). In case of the LOS is completely blocked by any obstacle, the proposed algorithm will use the shortest delay estimate. It is also assumed that the Doppler shift $f_D$ is correctly estimated by the carrier-tracking loop and that all-multipath components experience similar Doppler shift.

The performance of the proposed method and the different considered multipath mitigation techniques are compared with 2 path static channels with path amplitude values 1 and 0.8. The performance comparison of the algorithms mainly focuses on the short-delay multipath scenario because this scenario is generally the most challenging situation. Figure 5-3 shows the multipath error envelopes with respect to multipath delay for 2 path static channels with path amplitude values 1 and 0.8. The upper envelope is obtained for in-phase paths and the lower envelope is obtained for 180-degree phase shift between the LOS and NLOS paths.

The EML (with 0.2 chip early-late spacing), HRC (with 0.1 chip early-late spacing and 0.2 chip very early-very late chip spacing) and MEDLL are used for the performance comparison. As shown in Figure 5-3, the proposed method outperforms other methods for
closely spaced multipath scenario. It is also remarkable that the MEDLL algorithm outperforms both EML and HRC multipath mitigation techniques.

Figure 5-3: Multipath error envelop with respect to multipath delay for 2 path static channels with path amplitude values 1 and 0.8

5.3 Simulation Environment

Appropriate multipath mitigation techniques are important in receiver designs for many navigation applications. Besides, there is a crucial need for suitable testing environments. In order to assess the performance of a multipath mitigation technique, a controlled realistic simulation environmental is required [110]. In this research, the SPIRENT GSS8000 simulator, which has the capability to generate realistic simulation multipath environments through an advanced multipath model implemented in the Spirent SimGEN™ software, is used to evaluate the performance of the proposed technique.
under several operation scenarios. Many papers in the literature [111] discussed the matching of the simulated multipath environments using the SPIRENT simulator to real multipath environments.

Two different multipath environments are simulated using SPIRENT GSS8000 simulator using Land Mobile Multipath (LMM) model [112]. The first is a LMM with a rural multipath environment and the second is a LMM with an urban canyon multipath environment.

5.3.1 Land Mobile Multipath (LMM)

The LMM was especially established to simulate the effects of multipath signals on a portable device including GPS positioning (such as a smart mobile phone) [112]. The model allows the user to define the signal conditions through the selection of the kind of surrounding environment, whether it is rural, sub-urban, or urban canyon.

For each environment category, an equivalent category mask editor is defined. The category mask editor is used to define a mask, which is applied over the simulated antenna. The following are four mask categories which can be modified according to the environment category mask set by the user [111]. The signals determined by their arrival angles fall in one of these four categories. According to [112], the categories can be defined as follows:

- **Category A**: Complete Obstruction, that represents a visibility mask where all the satellites with elevation angles below certain degree are excluded.
• **Category B**: LOS only, signals that are unobstructed and not subjected to any reflections. These signals suffer Rician-fading model only.

• **Category C**: LOS and Echoes, signals that are unobstructed but subject to reflections. The LOS signal suffers Rician fading where echoes suffer modified Rayleigh fading.

• **Category D**: Echoes only, these represent obstructed LOS signals that are present as reflections only, and they suffer modified Rayleigh fading model.

Figure 5-4 show the LMM category mask of rural and urban canyon simulation environments, respectively.
Figure 5-4: Land mobile multipath category mask of (a) Rural and (b) Urban Canyon environments (A snapshot of SimGEN™ software)
The Rician fading model describes the fading on LOS signal, however, a Rayleigh model describes the fading on multipath signals [112]. The characteristics of the Rician and Rayleigh models are functions of satellite elevation angle. The details of the Rician and Rayleigh fading models are as follows.

5.3.1.1 Rician LOS Fading Model

The Rician model can be defined as [112]:

\[
f_{Rician}(x) = \begin{cases} 
2Kv \exp\left[-K(v^2 + 1)\right] I_0(2Kv), & x \geq 0 \\
0, & x < 0 
\end{cases} 
\]

where \( v \) is the received signal amplitude relative to the direct path, \( K \) is the ratio of direct to multipath power received and \( I_0 \) is a 0\(^{th}\) order modified Bessel function of the first kind.

5.3.1.2 Rayleigh Multipath Model

According to [112], a modified Rayleigh model describes the fading on multipath channels using three functions. There is a deterministic mean power function, an amplitude noise function following Rayleigh distribution and a delay on multipath channels following exponential distribution. The deterministic mean power reduction in addition to Rayleigh noise can be written as [112]:

\[
P_h(\tau) = P_h(0) - d \times \tau \, dB \]

where \( P_h(0) \) is the echo power for zero echo delay (dB), \( d \) is the power decay (dB.\(\mu\)s\(^{-1}\)) provided by a look-up table and \( \tau \) is the delay of the echo signal.
The amplitude noise on the multipath channel is randomly computed from a Rayleigh
distribution given by:

\[ f_{\text{Rayleigh}}(v) = 2Kv e^{-Kv^2} \quad 5.20 \]

where \( K \) is the ratio of direct to multipath power received and \( v \) is the received voltage
relative to the direct path.

The delay on the multipath channel is calculated at random with the exponential
distribution given by:

\[ f_{\text{exp}}(\tau) = \frac{1}{b} e^{-\frac{\tau}{b}} \quad 5.21 \]

where \( b \) is exponential delay (\( \mu \)s) and it is taken from a look-up table.

The operation of the multipath models is controlled by the contents of a number of Look-
up-Tables (LUTs), determined by satellite elevation angle and iteration rate (equivalent to
simulated user speed of travel). There is a LUT for every environment type. Table 5-1
shows the LUT of rural and urban canyon simulation environments. The SPIRENT
GSS8000 utilizes only the Rayleigh distribution to simulate the multipath leaving us with
no control over the multipath scenarios considered in this thesis.
Table 5-1: Look-up table of rural and urban canyon simulation environment

<table>
<thead>
<tr>
<th>Environment</th>
<th>Rician fading of direct LOS signal</th>
<th>Rayleigh fading, power decay, and exponential delay for reflected signals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Satellite Elevation</td>
<td>$K$</td>
</tr>
<tr>
<td>Rural</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>150</td>
</tr>
<tr>
<td>Urban Canyon</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>46</td>
</tr>
</tbody>
</table>

In order to examine the performance of the proposed multipath mitigation technique and compare its performance to the considered multipath mitigation techniques, a test consists of a set of static and dynamic realistic simulation scenarios was conducted on a SPIRENT GNSS8000 simulator controlled by SimGEN™ software in TECTERRA geomatics laboratory at Calgary, Alberta, Canada. Figure 5-5 shows the hardware experimental setup inside the TECTERRA lab.
The GPS In-phase/Quadrature (I/Q) raw measurements were logged and saved using NovAtel FireHose front-end to a special purpose computer for post processing. The specifications of the digitalized signals collected by the NovAtel FireHose front-end are shown in Table 5-2. The NavINST research group’s modified software receiver processed the raw GPS I/Q samples [38]. The reference location was chosen at a start point in Kingston, Ontario, Canada at latitude 44° 13.726’, longitude -76° 2794’ and height 100m. The experimental duration was 16 minutes.

In the following simulation scenarios, an intermediate frequency (IF) GPS L1 C/A signal with a sampling rate of 20 MHz was used. The coherent integration time $N_c$ was 10 ms. The reason for choosing this value for $N_c$ is that the choice of coherent integration time}
should be larger than 1 ms (one C/A code chip) and smaller than 20 ms (since a non-data-aided scenario was considered). Additionally, the 10 ms coherent integration time is used for high sensitivity receivers operating in urban canyons.

Table 5-2: Settings adopted for data collection

<table>
<thead>
<tr>
<th>Parameters</th>
<th>L1 GPS Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Frequency (MHz)</td>
<td>20</td>
</tr>
<tr>
<td>Sampling</td>
<td>Complex</td>
</tr>
<tr>
<td>Quantization Bits</td>
<td>4</td>
</tr>
</tbody>
</table>

5.4 Results and Discussion

Figure 5-6 illustrates a frequency domain, time domain and histogram of the raw GPS L1 I/Q samples collected data, respectively. There are 9 GPS satellites available above a 5-degree elevation mask at the initialization location at open sky scenario as shown in Figure 5-7. The open sky scenario is built-in scenario inside the Spirent SimGEN™ simulator software. The sky plot shown in Figure 5-7 illustrates the GPS satellite availability before applying the LMM models.
Figure 5-6: Frequency domain, time domain and histogram of the raw GPS L1 I/Q samples collected data

Figure 5-7: GPS satellite availability at the initialization location at open sky scenario
5.4.1 Static Tests

Two different realistic simulation scenarios including rural and urban canyon were considered. The details about these multipath models are discussed in details in section 5.3.

5.4.1.1 Rural Environment

Figure 5-8 shows the sky plot of GPS satellites in the realistic rural simulation environment. There are 7 GPS satellites available. The low elevation satellites such as PRN 4 and 31 (shown in red in Figure 5-8) are completely blocked due to the simulation environmental mask. According to the land mobile multipath category mask of rural environment shown in Figure 5-4(a), the signal received from high elevation GPS satellites such as PRNs 5 and 30 are only LOS signal (shown in green in Figure 5-8). Only the signal received from GPS satellites PRNs 02, 10, 12, 24 and 29 consists of LOS and Multipath components (shown in blue in Figure 5-8). These signals follow the fading models mentioned in sections 5.3.1.2 and 5.3.1.1. The following analysis mainly focuses on the received signals, which is affected by multipath errors. The number of satellites acquired by the GPS software receiver is shown in Figure 5-9.
Figure 5-8: Skyplot of GPS satellites for the rural simulation environment

Figure 5-9: Acquisition results of the GPS software receiver - rural environment
Figure 5-10 shows an example of the received correlation function of satellite PRN 10, the estimated correlation function and LOS and NLOS delays of a GPS satellite signal using the proposed method. In order to estimate the delay and phase of the LOS and NLOS signals, FOS was used to model the measured correlation function by defined pre-designated candidate functions and iteratively adding the term that lowers the MSE of the model by the greatest amount [105]. Figure 5-11 presents the model-fit MSE reduction $Q_m$ versus the number of candidate terms of modeling the measured correlation function of GPS PRN 10. As shown in Figure 5-11, the proposed method successfully modeled the distorted correlation function of satellite PRN 10 by using 4 predefined candidate functions. It is remarkable that none of the remaining candidates can yield a sufficient MSE reduction value.

![Figure 5-10](image-url)
Figure 5-11: Model-fit MSE reduction $Q_m$ versus the number of candidate terms of modeling the measured correlation function of GPS satellite PRN 10

Since the shortest time of arrival from the satellite to the receiver is obtained from the outputs of the proposed method, therefore, the pseudorange error of the corresponding PRN is calculated. Figure 5-12 shows the RMS values of the pseudorange estimation errors computed by the proposed method for the satellites PRNs 02, 10, 12, 24 and 29 which consist of LOS and Multipath components (shown in blue in Figure 5-8). These values are compared to the estimation errors calculated by the considered classical and advanced multipath mitigation techniques discussed in Chapter 3 such as ELS, HRC and MEDLL.

The comparison of the pseudorange estimation errors computed by several techniques is shown in Figure 5-12. The RMS values of the estimated errors produced by the proposed
method are considerably smaller than the ones produced by the other methods for all of the PRNs. This improvement in estimation accuracy is the cost of the additional computation load. For instance, in case of PRN 24, the proposed method can provide 32% improvement when compared to the MEDLL method.

![Pseudorange RMS Errors](image)

**Figure 5-12: RMS values of pseudorange estimation errors - static test**

The estimated pseudoranges for all visible satellites were then used in the computation of the positions of the receiver. Figure 5-13 shows the position errors for the proposed method and other methods. The corresponding RMS values are compared in Figure 5-14. As it was expected from the psudorange errors, the position solution produced by the proposed method is considerably improved when compared to the other conventional methods. The percentage accuracy improvement was more than 33% in the horizontal position.
Figure 5-13: East and north position errors - rural environment - static test

Figure 5-14: RMS values of position errors - rural environment - static test
5.4.1.2 Urban Canyon Environment

The proposed method was also examined through another realistic simulation environment named urban canyon. Figure 5-15 shows the sky plot of GPS satellites in the realistic urban canyon simulation environment. There are 8 GPS satellites available, the low elevation satellites such as PRN 31 (shown in red in Figure 5-15) are completely blocked due to simulation environmental mask. According to the land mobile multipath category mask of urban canyon environment shown in Figure 5-4 (b), the signal received from high elevation GPS satellites such as PRNs 05, 12 and 30 are only LOS signal (shown in green in Figure 5-15). Only the signal received from GPS satellites PRNs 02, 10, 24 and 29 consists of LOS and multipath components (shown in blue in Figure 5-15) and the signal received from GPS satellite PRN 04 consists only of multipath components (i.e. No LOS component, shown in orange in Figure 5-15).

![Skyplot of GPS satellites at urban canyon simulation environment](image)

**Figure 5-15: Skyplot of GPS satellites at urban canyon simulation environment**
The comparison of the pseudorange estimation errors computed by several techniques for urban canyon simulation environment is shown in Figure 5-16. It is clearly shown in Figure 5-16 that the RMS values of the estimated errors produced by the proposed method is considerably smaller than the ones produced by the other methods for all of the PRNs. The RMS values of the estimator error produced by the HRC are slightly smaller than the ones produced by ELS for PRNs 02, 10 and 30. The results show that the proposed method enhances the accuracy of the estimated pseudorange by 22% to 60% when compared to other methods for the urban canyon scenario. Figure 5-17 shows the position errors for the proposed method and the considered methods. The corresponding RMS values are compared in Figure 5-18. The position solution produced by the proposed method is considerably improved compared to the other methods.

![Pseudorange RMS Errors](image)

**Figure 5-16: RMS values of pseudorange estimation errors - urban canyon simulation environment - static test**
Figure 5-17: East and north position errors - urban canyon simulation environment - static test

Figure 5-18: RMS values of position errors - urban canyon simulation environment - static test
5.4.2 Dynamic Test

The third simulation scenario considers a dynamic vehicle. The vehicle trajectory is shown in Figure 5-19. The starting point was chosen at a point in Kingston, Ontario, Canada at latitude of 44° 13.726’, longitude of -76° 27.948’ and altitude of 100 m. First, the scenario comprises a static period of around 4 minutes to initialize the software receiver processing at the beginning of the experiment and then the vehicle velocity is gradually increased until it reaches 60 km/hr. The latter velocity stayed constant for a duration of 1.9 minutes during which the vehicle heading was not changed. A 90° turn was conducted with a turn radius of 10 m leading to a final speed of 50 km/hr. The vehicle maintained its speed of 50 km/hr for 2 minutes, after which it turned 90° to the left with a turn radius of 10 m leading to a final speed of 60 km/hr. Similarly, a 60° right turn with a radius of 10 m was made after 1 minute of constant speed (60 km/hr) and heading. The vehicle speed became 80 km/hr. Then, the vehicle kept its heading and speed for a duration of 2.5 minutes. In order to return to the start point of the trajectory, the vehicle sharply turned right 120° with a turn radius of 5 m and achieved a final speed of 100 km/hr at which it operated for 4 minutes. Finally, the vehicle changed its heading towards the starting point with a turn radius of 10 m and gradually decelerated until it reached rest. A static period of 1.5 minutes was then applied. The experimental total duration was 20 minutes. Figure 5-20 demonstrated the time evolution of the total velocity of the simulated dynamic vehicle. The LMM category mask of urban canyon environment discussed in section 5.3.1 was applied to this scenario. The skyplot of the
satellites visible above a 5-degree elevation mask at the initialization location is shown in Figure 5-15 and explained in section 5.4.2.
Figure 5-20: Dynamic profile of the scenario

Figure 5-21 shows the RMS values of the pseudorange estimation errors computed by the proposed method and the considered classical and advanced multipath mitigation techniques such as ELS, HRC and MEDLL for the satellites PRNs 02, 10, 24 and 29. The received signals of these satellites consist of LOS and Multipath components (shown in blue in Figure 5-15). The RMS values of the estimated errors produced by the proposed method show that it outperforms the other methods for all PRNs in the dynamic test. In case of a low elevation satellite such as PRN 10, the proposed method outperformed MEDLL by 44% improvement. The estimated pseudoranges for all visible satellites were then used in the computation of the positions of the receiver. Figure 5-22 demonstrates the RMS values of the position errors where it can be depicted that the position solution produced by the proposed method is more accurate than ELS, HRC and MEDLL for the simulated dynamic test at urban canyon environment.
Figure 5-21: RMS values of pseudorange estimation errors - urban canyon simulation environment - dynamic test

Figure 5-22: RMS values of position errors - urban canyon simulation environment - dynamic test
5.4.3 Jamming Scenario

The jamming interference can result in degraded navigation accuracy or complete loss of tracking the GPS receiver. The effect of RF interference on code correlation and loop filtering is to reduce the $C/N_0$ for all the incoming signals [63]. The receiver can lose lock if the effective $C/N_0$ is reduced under the tracking threshold. Thus, the final scenario targeted testing the robustness of the methods proposed in this research against the jamming effects.

The SPIRENT GSS8000 hardware simulator was used to generate the jamming scenario for GPS signals. The simulator outputs the RF GPS signal in addition to adjustable additive jamming signals. The starting point was chosen at a point in Kingston, Ontario, Canada at latitude 44° 13.726´, longitude -76° 27.948´ and height 100m. The RF signals are collected through a NovAtel front-end using a sampling frequency of $f_s = 10 MHz$ and quantified in 4 bits. As discussed before, the front-end rotates the received frequencies to the baseband and saves the sampled data for post-processing. The modified NavINST’s software receiver that runs under a MATLAB™ platform is used to process the raw GPS data. The software is capable of performing GPS signal acquisition and tracking using the proposed methods and different tracking algorithms. In this section, the performance of the developed software receiver is compared to the performance of the NovAtel Propak V2 commercial receiver under the simulated jamming scenarios.
There are 10 GPS satellites available above a 5-degree elevation mask at the initial location as shown in Figure 5-23. The number of satellites acquired by the GPS software receiver is shown in Figure 5-24.

**Figure 5-23:** Skyplot of the GPS satellites in view during the experiment-jamming scenario

**Figure 5-24:** Acquisition results of the GPS software receiver - Jamming scenario
Three jamming scenarios are considered at both static and dynamic receiver modes. The trajectory simulated and mentioned in section 5.4.2 is used in this scenario. The signal specifications of these scenarios are given in Table 5-3. In the first scenario, the received signal is corrupted by a swept Continuous Wave (CW) interference for one minute duration when the receiver is static. The frequency range and swept time of the swept-like interference are [1574.42 to 1576.42] MHz and 16 s, respectively. Two different jamming types were added to GPS signals when the receiver was in dynamic mode. The first one is swept CW with the same specifications mentioned before, for a duration of one minute. The second type is noise interference at center frequency $f_{\text{center}} = 1575.42$ MHz and Bandwidth 1 MHz for a duration of two minutes.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Receiver Mode</th>
<th>Interference Type</th>
<th>Frequency (MHz)</th>
<th>Power (dBm)</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Static</td>
<td>Swept Continuous Wave</td>
<td>1574.42 – 1576.42</td>
<td>-80</td>
<td>1 min</td>
</tr>
<tr>
<td>2</td>
<td>Dynamic</td>
<td>Swept Continuous Wave</td>
<td>1574.42 – 1576.42</td>
<td>-80</td>
<td>1 min</td>
</tr>
<tr>
<td>3</td>
<td>Dynamic</td>
<td>Interfere Noise</td>
<td>$f_{\text{center}} = 1575.42$ (BW = 1 MHz)</td>
<td>-80</td>
<td>2 min</td>
</tr>
</tbody>
</table>

To analyze the jamming interference effect, the unjammed $C/N_0$ (db-Hz) has to be first calculated. $C/N_0$ at baseband can be calculated as [63]
\[
\frac{C}{N_0} = P_r + G_d - 10 \log(kT_0) - N_f - L 
\]

where \(P_r\) is the received GPS signal power (dBW), \(G_d\) is the antenna gain (dBic), \(k\) is the Boltzmann’s constant, \(T_0\) is the thermal noise reference temperature (K), \(N_f\) is the noise figure of the receiver including antenna and cable losses (dB), and \(L\) is the implementation loss.

The level to which the unjammed \(C/N_0\) is reduced by RF interference is called the equivalent \(C/N_0\) or \((C/N_0)_{eq}\), which can be calculated as [1]

\[
(C/N_0)_{eq} = -10 \log \left( 10^{-\frac{C}{N_0}} + \frac{10^{\frac{J}{10}}}{QR_c} \right) 
\]

where \(R_c\) is the PRN code chipping rate, \(Q\) is a dimensionless spread spectrum processing gain adjustment factor (different for each interference type), and \(J/S\) is the Jammer-to-Signal power ratio (dB). Equation 5.21 can be rearranged to calculate \(J/S\) as

\[
\frac{J}{S} = 10 \log \left( QR_c \left( 10^{-\frac{(C/N_0)_{eq}}{10}} - 10^{-\frac{C}{N_0}} \right) \right)
\]

Figure 5-25 demonstrates the interference effects on the signal quality, particularly the signal \(C/N_0\) where \(N_0\) refers to the Gaussian noise plus interference. \(C/N_0\) is estimated during FLL-assisted-PLL tracking and computed from the correlator outputs using a Narrow-Band/ Wide-Band power estimator [27]. Lower elevation satellites lose lock at the simulated interference levels as noticed for PRNs 4, 11, 13, 14 and 30. Figure 5-26 shows the estimated Doppler shift of PRN 1. The changes in \(C/N_0\) due to the increasing
interference levels affect the discriminators’ output noise level and hence the estimated Doppler noise. These effects are clearly visible in Figure 5-26. On the contrary, these changes are almost absorbed by the proposed methods, thus the $C/N_0$ changes have a minimum effect on its estimated Doppler noise.

**Figure 5-25: Effect of $J/S$ on $C/N_0$ level changes for PRN 1**
The position solution was calculated using least-squares estimation technique. A single-point epoch-by-epoch least-squares solution was computed at a 1 Hz rate. If there were insufficient satellites in view, at a given epoch, or the solution failed to converge in 10 iterations, no solution would be computed.

Figure 5-27 shows the trajectories obtained using the NovAtel Propak v2 commercial receiver (shown in red) and using the software receiver in which the proposed methods are implemented (shown in green). It is observed that throughout the simulated trajectory, the commercial receiver was unable to fix position solutions during all three jamming events. Moreover, after the jamming was turned off the commercial receiver was still not able to fix a position which means it was not able to re-establish lock.
The advantages of the proposed methods are clearly demonstrated such that after the jamming was turned off; the proposed method provided an accurate code delay and Doppler shift, which in turn enables the tracking loops to re-lock onto the satellite signals. Therefore, the solution availability is increased by 72% compared to the commercial receiver. It is noticed that the commercial receiver reestablished lock and fixed a position solution towards the end of trajectory.

![Test Trajectory (Reference, Commercial Receiver, Proposed) – Jamming Scenario](image)

**Figure 5-27: Test Trajectory (Reference, Commercial Receiver, Proposed) – Jamming Scenario**

### 5.5 Summary

In this chapter, a new high-resolution code tracking method was proposed to mitigate multipath effects for GPS receivers. The new method is based on FOS and it models the
distorted multipath correlation function to get better TOA estimation for both LOS and multipath signals. The proposed method improves the code delay estimation inside the tracking loop of GPS receiver especially for closely spaced multipath navigation environments.

Using the SPIRENT GSS8000 hardware simulator, numerous scenarios were generated involving static and dynamic scenarios under different multipath environments such as rural and urban canyon. It is also used to generate jamming scenario for GPS signals to test the robustness of the methods proposed in this research against the jamming effects.

Four points of assessment were taken into consideration while evaluating the performance of the proposed method. These points are pseudorange and position accuracy, immunity against interference and solution availability.

The results demonstrate the effectiveness of the proposed method in reducing the RMS error values of the estimated pseudorange and position solutions compared to other classical and advanced multipath mitigation techniques. Accuracy improvement ranging from 22% to 60% was achieved in the estimated pseudorange when using the proposed method for the urban canyon scenario at the static mode. The proposed method can provide 44% improvement if compared to the MEDLL method in the dynamic mode. In addition, the proposed method showed superior performance when compared to the NovAtel Propak V2 commercial receiver in terms of signal tracking recovery and solution availability under the jamming scenario.
Chapter 6
Conclusions and Recommendations

This thesis has proposed new methods based on high resolution signal processing to enhance the performance of GPS receivers and analyzed their performance under several realistic simulation scenarios. This chapter summarizes the outcomes of this thesis and provides the conclusions. Recommendations for possible future work in this context are then presented to further extend and enhance the proposed methods.

6.1 Conclusions
This thesis aimed at introducing high resolution signal processing techniques to enhance the performance of the GPS receiver in harsh navigation environments. In order to achieve this goal, the first objective was to introduce a robust fine acquisition method to provide accurate spectral estimation of the GPS satellite Doppler frequency with less computational load. Additionally, the second objective was to design, test and analyze a high resolution multipath mitigation method inside the code tracking loop.

This research started with an overview of the GPS receiver architecture in Chapter 3 with particular attention to the signal acquisition and signal tracking theories. The effects of multipath on code tracking performance were also discussed. The classical and advanced multipath mitigation techniques fail to completely mitigate the severe GPS multipath effects experienced in urban canyons. In deep urban canyons, the closely spaced multipath doesn’t only distort the shape of the correlation function of the received GPS
signal, but they may also result in shifting its peak. Although the advanced multipath mitigation techniques such as MEDLL offer better multipath mitigation performance compared to the classical methods, they don’t completely eliminate all multipath errors especially those resulting from the closely spaced multipath.

This thesis has introduced a robust fine acquisition method in Chapter 4 to improve the acquisition performance. After the coarse acquisition is accomplished, a 2D search grid is obtained, and the resulting estimated vector is selected as the location of the peak cell and, at the same time, the other cells in the search grid are discarded. However, because of the large frequency searching step of the coarse acquisition, the initial Doppler frequency estimate is usually not accurate enough to pass to the tracking loop directly. Therefore, a new fine acquisition method is proposed in this thesis to improve the carrier frequency accuracy further. The proposed method based only on the search grid which is already evaluated by the coarse acquisition; does not need to compute new correlations, but we only use the neighbor cells of the coarse acquisition peak already available in the search grid. The proposed method initially strips off the C/A code phase from the carrier signal. Then, sinusoidal candidate functions are generated at each of the frequencies range of interest, which is typically set around the estimated Doppler shift acquired from the coarse acquisition stage. The proposed method’s candidate frequencies are chosen to have a higher resolution than the FFT in order to achieve better de-noising performance. Finally, an orthogonal search algorithm is utilized to detect the carrier frequency accurately.
The performance of the proposed fine acquisition method was evaluated against the different window sizes and the noise effects. The proposed fine acquisition method is compared to FFT-based fine acquisition algorithms. Simulations were conducted using MATLAB to generate IF GPS L1 data with known Doppler frequency shift. Simulation results showed that the proposed method outperforms the other methods in estimating the Doppler shift at low SNR and small window size, in particularly using 1 ms window size at SNR = -15 dB. The improvement percentage was more than 70% in the acquisition accuracy.

To further verify the performance of the proposed fine acquisition method, real GPS L1 data were generated using SPIRENT GSS8000 simulator and logged using NovAtel FireHose front-end. The experimental results indicated that the proposed fine acquisition method speeds up the convergence of the tracking loop inside the GPS receiver in order to correctly lock the carrier Doppler shift frequency. Convergence speed doesn’t only affect the tracking of the correct frequency but also the tracking of the correct signal phase and hence obtaining the required navigation data bits.

Both the simulation and experimental results showed that the proposed method was significantly more accurate than the FFT zero-padding approach and it maintained a balance between the acquisition accuracy and computational load. In addition, the proposed method is efficient at detecting the carrier Doppler shift buried in white noise even at poor SNRs.
In addition to introducing a high resolution fine acquisition technique, another high resolution signal processing technique has been presented in this thesis inside the tracking loop of a GPS receiver to mitigate multipath effects. The new algorithm is based on FOS, which is used to model the distorted multipath correlation function and get better TOA estimation for both LOS and multipath signals. The performance of the proposed method has been tested using several realistic simulation scenarios using SPIRENT GSS8000 simulator. Both static and dynamic tests were examined under different multipath environments including rural and urban canyon environments. The simulation and data processing results demonstrated the effectiveness of the proposed algorithm in reducing the RMS error values of the estimated pseudorange and positioning solutions compared to other classical and advanced multipath mitigation techniques. In Table 6-1, the RMS of position estimation error of four groups of multipath mitigation techniques obtained from the experiments explained in Chapter 5 is compared. It is important to mention that the MATLAB code of the FOS algorithm used in this research is not optimized to be used for real time applications. It is required to optimize the proposed methods in order to make it efficient, reliable, and scalable.

The final experiment in this thesis targeted testing the robustness of the methods proposed in this research against jamming effects. The SPIRENT GSS8000 hardware simulator was also used to generate the jamming scenario for GPS signals. Three jamming scenarios are considered at both static and dynamic receiver modes. The experimental results showed that the proposed method increased the position solution
availability by 72% compared to position provided by the NovAtel Propak-V2 commercial receiver.

Table 6-1: RMS Position estimation error for different simulation environments within different groups of multipath mitigation methods

<table>
<thead>
<tr>
<th>Mode</th>
<th>Environment</th>
<th>Method</th>
<th>Position RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>East (m)</td>
</tr>
<tr>
<td></td>
<td>Rural</td>
<td>ELS</td>
<td>32</td>
</tr>
<tr>
<td>Static</td>
<td></td>
<td>HRC</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MEDLL</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proposed</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Urban Canyon</td>
<td>ELS</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HRC</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MEDLL</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proposed</td>
<td>10</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Urban Canyon</td>
<td>ELS</td>
<td>74.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HRC</td>
<td>37.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MEDLL</td>
<td>20.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Proposed</td>
<td>10.8</td>
</tr>
</tbody>
</table>

6.2 Recommendations

Considering the presented work and the experimental results in this thesis, the following recommendations for future work are suggested:

1. With the emergence of multiple satellite navigation systems, multi-constellation receivers are becoming widely available. This has been encouraged at the system design level by working towards interoperability and compatibility among all
systems, allowing for seamless combination of the different signal spectra and processing chains into a single, multi-constellation GNSS solution. Although, this thesis limits simulations and experimental tests to GPS L1 signal; the principles based on the proposed methods and general methodologies are the same for other GNSS signals. However, there are different modifications and considerations that may be required for each case. Thus, it is recommended to modify the proposed methods for other GNSS signals, as well as to simulate and perform real data tests for these signals as further developments of the research presented herein.

2. The proposed fine acquisition method is based on estimating the location of the correlation peak that is provided from the coarse acquisition step. Given that the correlation peak is corrupted by noise or not detected by the coarse acquisition, it is recommended to integrate with external sensors such as inertial sensors to assist in the detection of the correlation peak and the definition of the candidate functions of the proposed fine acquisition method.

3. Given the high level of performance achieved by the proposed fine acquisition and tracking methods introduced in this thesis, research focusing on the optimization of the developed methods and realizing them in real-time shall have an important contribution to GPS receivers.
References


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[76] C. Singh and N. Jain, "Signal Acquisition for Software GPS Receiver."


List of Publications


