ANALYSIS OF SEISMICITY IN MINES AND DEVELOPMENT OF RE-ENTRY PROTOCOLS

by

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Abstract

Following a large seismic event in hard rock mines, there is a short-term increase in aftershock activity, which over time decays to background levels. During this time of elevated seismic activity the risk of aftershocks with sufficiently high magnitude to cause damage is also high. Workers are therefore restricted from re-entering affected areas for a specified time period. This is the re-entry protocol. The objective of this research has been to produce practical, standardized guidelines for the development of re-entry protocols.

This thesis is divided into two parts. In Part I, current re-entry practices are evaluated based on the results of a survey of 18 seismically active mines, mostly in Ontario. Based on this compilation, a complete set of best practice guidelines are proposed.

Part II of this thesis presents an in-depth study of the temporal evolution and characteristics of aftershocks and their implications for re-entry protocol development. Through this part of the thesis, eight mining and two crustal seismicity catalogues are used to study the statistical properties of aftershock sequences. These include the modified Omori’s law, Båth’s law, the Gutenberg-Richter frequency-magnitude relation, the Reasenberg and Jones model, and the Epidemic Type Aftershock Sequence (ETAS) model. Statistical procedures for identifying the most consistent parameters of these relations were developed and tested.

Despite the site specific nature of mining seismicity, consistent statistics were identified and used to develop guidelines for re-entry protocol development in Ontario mines without the requirement for previous intensive calibration. These simple recommendations are intended for those mines with limited historical seismicity, and to serve as a first approach guide for developing a re-entry protocol. Their applicability is, however, limited to single aftershock sequences.
For mines with a significant seismic database a probabilistic approach for setting a family of decay curves (seismic envelopes) and estimating seismicity rate thresholds for re-entry protocol development is proposed. With this formulation it is possible to interpret the decay behaviour of new aftershock sequences and quantify the degree of confidence of the re-entry protocol decision making process in real-time. This is the recommended method for the standardization of re-entry protocol development.
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Nomenclature

$A$  
Productivity of an aftershock sequence

$AIC$  
Akaike information criterion

$AUC$  
Area under the curve

$a_1, a_2$  
Adjustable parameters

$a, b, c$  
Major, intermediate and minor axis of the ellipsoid

$a, b$  
Parameters of a uniform distribution

$a, b$  
Coefficients in the Gutenberg-Richter relationship

$\alpha$  
Decay constant of the exponential decay function

$B$  
Background rate

$B_i, S_i$  
Observed and predicted cumulative number of events in each magnitude bin

$b_w$  
Histogram bin width

$\beta_k$  
Regression coefficients of the set of explanatory variables in a logistic model

$C, D$  
Adjustable parameters of the seismic work equation

$C_{\%}$  
Percentage of clustering

$CAD$  
Cumulative ascending distribution

$CAV$  
Cumulative apparent volume

$CDD$  
Cumulative descending distribution

$CV$  
Coefficient of variation

$\chi, \beta$  
Adjustable parameters of the correlation between $T_{MC}$ and $N_1$

$D$  
Maximum absolute difference between the cumulative distribution function of the sample and the one specified in the null hypothesis

$D_C$  
Critical value for rejecting the null hypothesis in the Kolmogorov-Smirnov test

$D^*$  
Cluster dimension

$D_{\text{max}}$  
Maximum displacement

$\Delta K, \Delta p, \Delta c$  
Uncertainties of the parameters of the modified Omori’s law

$\Delta M$  
Båth’s law

$\Delta T_{MC}$  
Maximum curvature time window

$\Delta T_{S}, \Delta T_{E}$  
Intervals of possible start and end times of power-law decay

$\Delta b$  
Standard deviation of the Gutenberg-Richter $b$-value

$\Delta i$  
Measure of model $i$ relative to the best model of a set of candidate models

$\Delta r$  
Location error

$\Delta \sigma$  
Static stress drop
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\Delta\sigma_d$</td>
<td>Dynamic stress drop</td>
</tr>
<tr>
<td>$E$</td>
<td>Seismic energy</td>
</tr>
<tr>
<td>$EAD$</td>
<td>Early aftershock deficiency</td>
</tr>
<tr>
<td>$EC$</td>
<td>Event count</td>
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<tr>
<td>$EI$</td>
<td>Energy index</td>
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<tr>
<td>ETAS</td>
<td>Epidemic type aftershock sequence</td>
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<tr>
<td>$E_i$</td>
<td>Entropy of event $i$</td>
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<td>Cumulative entropy</td>
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<td>$\bar{E}$</td>
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<td>Cut-off magnitude bin</td>
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<td>Magnitude of the largest aftershock</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<td>--------</td>
<td>-------------</td>
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<td>$M_{1-12}^{LAM}$</td>
<td>Magnitude of the largest event occurring during the 1-12 hour time interval after the principal event</td>
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<td>$MLE$</td>
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<td>Number of events used to perform the regression</td>
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<td>$N_{1-2}$</td>
<td>Number of events occurring between the first and second unit time after the principal event</td>
</tr>
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<td>$n$</td>
<td>Number of observations in a dataset</td>
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<tr>
<td>$n_i$</td>
<td>Number of neighbours for event $i$</td>
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<td>$n_p$</td>
<td>Number of adjustable parameters of a model</td>
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<td>$n(t)$</td>
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<td>Probability</td>
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<td>$PAP$</td>
<td>Peak acceleration parameter</td>
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</tbody>
</table>
$PVP$  
Peak velocity parameter

$P_{\text{Chi-Sq}}$  
Observed level of significance of the Chi-Square test. As the $P$-value approaches one, there is no basis to reject the hypothesis that the fitted distribution actually generated the data set.

$P(z)$  
Categorical response variable that represents the probability of a particular outcome in the logistic model

$Q$  
Generic seismic quantity

$Q^T$  
Total proportion of normal level of seismicity

$Q^U$  
Upper bound proportion of normal level of seismicity

$\overline{Q}$  
Average normal levels of seismicity for a generic seismic quantity $Q$

$\theta$  
Constant of proportionality between the release of cumulative seismic moment and the volume of mined rock

$R$  
Goodness of fit of the Gutenberg-Richter relationship

$R\text{-J}$  
Reasenberg and Jones stochastic parametric model

$R_{KS}$  
Kolmogorov-Smirnov goodness of fit ratio

$\text{ROC}$  
Receiver operating characteristic curve

$\text{RSS}$  
Residual sum of squares

$R^2$  
Coefficient of determination

$R(t)$  
Cumulative seismic quantity at time $t$

$r_{Nb-Na}$  
Ratio of the times of the $N_b$th earthquake following the principal event $T_{Na}$ and the $N_B$th earthquake preceding the principal event $T_{Na}$

$r_1, r_2, r(t)$  
Seismicity rate

$r^c$  
Critical ratio generated by a random process with a certain probability

$r_a$  
Asperity radius

$r_s$  
Source radius

$\rho$  
Correlation coefficient

$\rho(t)$  
Curvature as a function of time

$\text{SLC}$  
Single-link clustering procedure

$SR$  
Spherical radius representing the exclusion zone

$\overline{S}$  
Average spacing of sensors in a microseismic array

$\overline{S}_V$  
Volumetric spacing of sensors in a microseismic array

$SW$  
Cumulative seismic work

$SW(t)$  
Cumulative seismic work at time $t$

$SW_1$  
Cumulative seismic work at one time unit after the principal event

$sr$  
Strain rate

$sw$  
Seismic work rate
Apparent stress
Total time of continuously bins of seismic activity
Start and end times of power-law decay
The time of the \( N_A \) earthquake following the principal event and the \( N_B \) earthquake preceding the principal event
True negative
True positive
True positive rate
Time of maximum curvature
Average maximum curvature time
Maximum curvature time for the 95% prediction interval
Target time interval for fitting a decay-law formula
Time, time window
Magnitude scale based on the peak vectorial amplitudes of triaxial sensors (meaningless with no triaxial sensors)
Occurrence time of the individual \( i \) event after a principal event
Inter-event median time relative to the principal event, \( \bar{t}_i = (t_i + t_{i-1}) / 2 \)
Division between the possible start \( \Delta T_S \) and end \( \Delta T_E \) time intervals of power-law decay
Occurrence times of the first and last events of a seismic sequence
Estimated time required for a seismic quantity to decay to some predefined level of seismicity rate \( X \)
Long term average decay time per unit of volume removed
Magnitude scale based on the peak amplitudes of uniaxial sensors
Modified Omori’s law cumulative density function
Volume
Variance-account-for
Volume of mined rock
Anderson-Darling statistic
Accumulated sum of event count, seismic work, and seismic moment
Akaike weights
Set of \( k \) explanatory variables in a logistic model
Coefficient that modifies the relative contributions of events in different magnitude ranges
Depth below surface
Chapter 1

Introduction

1.1 Problem definition

A common characteristic of deep mines in hard rock is induced seismicity. This results from stress changes and rock failure around excavations. Some seismic events cause damage to excavations and injury to personnel due to ejected rock, called rockbursts. The single most important characteristic of mining seismicity relevant to this research is that following large seismic events or blasts, there is a short-term increase in levels of seismic activity (both in magnitude and frequency) that over time decays to background levels. Two examples of this decay are shown in Figure 1.1 and Figure 1.2, for a development blast and a rockburst related sequences respectively.

![Craig 53-0 Down Ramp](image)

Figure 1.1: Example of increased levels of seismicity following a development blast at Craig Mine. The spike in event rate decays within 3 hours after the blast (Simser, 2006).
Figure 1.2: Example of increased levels of seismicity following a $M_n=2.4$ rockburst at the Copper Cliff North Mine. Spatial distribution of events in the affected zone (frame a). Event frequency histogram (frame b) and moment magnitude (frame c) as a function of the hour of the day. The rockburst ($M_n=2.4$) was triggered on the 3050 level. A second large magnitude event ($M_n=1.4$) occurred in the same zone 6.2 hours after the initial event.

During this time of elevated seismic activity the risk of aftershocks with sufficiently high magnitude to cause damage is also high; therefore the policy adopted by mines is to restrict access to the affected areas for a time period. This is the re-entry protocol.

In mines, this decay period is generally a matter of hours, but at the crustal scale, aftershock sequences can occur over months or years following the main event. This phenomenon is referred to as modified Omori’s law (Omori, 1894; Utsu, 1961). Restricting access to areas of a mine
affected by large seismic events for sufficient time to allow this decay of aftershock events is the
main approach in re-entry strategies and is central to this thesis.

1.2 Objectives and approach
The overall goal of this research is to develop guidelines for the development of re-entry
protocols that are based on a scientific understanding of the decay patterns of seismicity
following large seismic events and blasts. In this thesis the analysis is restricted to the temporal
evolution of seismic activity without spatial dependence because the main interest is to reflect
current re-entry practices, where the microseismic source parameter data is used for the re-entry
protocol decision making process in a specified volume or zone without any formal consideration
of the spatial distribution of seismicity.

Due to the large number of Ontario mines experiencing seismicity and the need of standards, the
Ground Control Committee of Mines Aggregates Safety and Health Association (MASHA)
selected the problem of re-entry protocol guideline development as its highest research priority.
Through the support of MASHA and its member mines, this research project was funded by the
Workplace Safety and Insurance Board of Ontario (WSIB). For research topics such as this that
are fundamentally complex, the WSIB approach funded two main phases of work:

1. Development of practical guidelines for the development of re-entry protocols at a mine,
   based on a synthesis and analysis of currently used protocols, including advice on instru-
   mentation, analysis of data, and final development of the protocol.
2. Based on the results of the first phase, a more focussed and in-depth research was conducted.
   In this second phase of research, catalogues of seismicity were collected from several mines
   and detailed statistical analyzes of aftershocks were made. This enabled a better
understanding of the characteristics of aftershock sequences to be developed, enabling more reliable re-entry guidelines to be developed and tested.

1.3 Thesis structure

As part of the research sponsorship requirements, much of what is presented has been published in the form of reports and a mining industry MASHA-sponsored workshop for WSIB (Vallejos and McKinnon, 2006), in quarterly update meetings with MASHA members, conferences, symposiums and journal articles (Vallejos and McKinnon, 2008a, 2008b, 2009a, 2009b, and 2010). The information from these manuscripts has been rearranged here to be consistent with the expected thesis format and is divided into two main parts, in connection with the WSIB approach.

1.3.1 Part I

Part I of this thesis including Chapter 1 to 4, is focused on the results of the first phase of the project. A review of existing literature of mining seismicity relevant to workplace re-entry is performed in Chapter 2. The literature does contain a significant number of papers on related topics of mining seismicity, including hazard and risk assessment, mine design to minimize seismicity, rockburst ground control methods, data analysis, and theoretical aspects of mining seismicity. However, the conclusion was reached that there has been no systematic study of re-entry protocols such as outlined in this thesis.

In Chapter 3, current re-entry practices are presented and evaluated based on the results of a survey of 18 seismically active mines, mostly in Ontario. The Appendix A presents the questionnaire developed to collect the data from seismically active mines in a standardized format. Based on this evaluation, improvements, concerns, requirements and limitations of current re-entry protocols were identified. It is clear from the information presented in Chapter 3 that a wide variety of protocols are used, and that seismicity and therefore re-entry protocols are
quite site specific. Re-entry protocols must be compatible with rock mass behaviour in a particular geological/mining environment, as measured by the seismic monitoring system. For this reason, the preliminary guidelines discussed in Chapter 4, emphasize the use of procedures to develop re-entry protocols based on the analysis of locally observed seismicity.

1.3.2 Part II

Part II of this thesis presents an in-depth study of the statistical patterns of aftershocks and their implications for re-entry protocol development. Through this part of the thesis eight mining and two crustal seismicity catalogues are used to study the statistical properties of aftershock sequences. The sources of data and the methods used to identify and prepare these aftershock sequences for the analysis are presented in Chapter 5.

The temporal characteristics of aftershocks expressed in terms of scaling laws are investigated in Chapter 6. These include the modified Omori’s law (Omori, 1894; Utsu, 1961), Båth’s law (Båth, 1965), the Gutenberg-Richter frequency-magnitude relation (Gutenberg and Richter, 1944), the Reasenberg and Jones stochastic model (Reasenberg and Jones, 1989, 1990, 1994), and the Epidemic Type Aftershock Sequence (ETAS) model proposed by Ogata (1988, 1989, 1992). The patterns of aftershock sequences described by these scaling laws and their implications for re-entry assessment are addressed and discussed. The main objective of this chapter is to verify these relations for mining seismicity and to identify how they can be used for the statistical development of re-entry protocols.

Based on the significance of the modified Omori’s law for describing the decay of aftershock sequences, Chapter 7 presents a generic decay-law formula applicable to event count, seismic work and seismic moment. The implications of this new formulation are discussed in terms of current re-entry practices. Using a uniform statistical method the best fit parameters of the event
count and seismic work decay-formulas are estimated and compared. A probabilistic approach for
representing the decay behaviour of aftershock sequences and estimating seismicity rate
thresholds for re-entry protocol development is also presented. Four new concepts are introduced:
seismic envelopes, seismic path, rate histograms and rate diagram. In addition, logistic regression
is considered for estimating the probability of invoking a re-entry protocol based on microseismic
source parameters.

The concepts developed in Chapter 7 are applied to several case histories in Chapter 8 where the
decay of mining-induced aftershock sequences is correlated with mining activities, such as:
volume of extracted rock, and depth. In addition, a database of large magnitude events and
rockbursts is developed and analyzed, resulting in preliminary correlations between the decay
time and size of the exclusion zone as a function of the Nuttli magnitude of the main event.

The conclusions, including the scientific contribution, final guidelines for re-entry protocol
development in Ontario mines without the requirement for previous intensive calibration and
recommendations for future research are provided in Chapter 9.

Each chapter was written to provide a self-contained treatment of one major topic. To facilitate
the reading, the summary and discussions have been included at the end of each chapter. Figure
1.3 presents the interrelation between the different chapters presented in this thesis.
Figure 1.3: Outline of the thesis and main contributions of the study.
1.4 Terminology

To a large extent, this thesis is concerned with mining seismicity relevant to Ontario mines. All of the Ontario mines that facilitated data for this thesis make use of the Engineering Seismology Group (ESG) microseismic monitoring systems. Therefore, the terminology introduced by ESG in mining seismicity is used throughout this thesis.

Hereafter, symbols and terms are briefly defined as they occur in the text. They can also be found in the nomenclature section at the beginning of the thesis. For additional explanation on how the microseismic source parameters are evaluated, the reader is referred to the Hyperion Software User's Guide–Appendixes F, G, H, and I.
Chapter 2
Literature review

Most of the science of mining seismicity is derived from or parallels that of crustal seismology. However, mining seismicity has developed into a recognized sub-field of seismology with its own literature. The most widely known venue for dissemination of research and practice in mining seismicity is a series of conferences held every four years, known as the International Symposia on Rockburst and Seismicity in Mines. The most recent of these (the sixth), prior to the publication of results from the research presented here, was held in Australia in March 2005. A careful review of all past proceedings showed a significant fact: there is not a single paper on seismic re-entry protocols.

In an extensive literature search, only one paper explicitly addressing seismic re-entry protocols was found (Malek and Leslie, 2006), which was a case history of three rockbursts at Inco’s Copper Cliff North Mine as opposed to the outline of general procedures. The re-entry time was determined using a combination of parameters: cumulative seismic work, spatial clustering, and strain rate. However, the seismic work was the only parameter that could be used for all the three rockbursts studied. Time histories of these parameters were used to subjectively establish typical background levels. No methodology was described to statistically determine these thresholds. The authors stressed the necessity to interpret the parameters in conjunction with knowledge of the mining and geological circumstances. This case history illustrates the importance of site specific calibration.

Seismic re-entry protocols are also mentioned briefly in a small number of papers that deal with general aspects of mining seismicity (Turner and Player, 2000; Heal et al., 2005; Kaiser et al., 2005; Simser, 2005; Simser, 2006). Turner and Player (2000) described seismicity at the Big Bell
Mine in Australia. Rockbursts were found to be related to nearby blasting, and re-entry restrictions were placed for standard time periods (12 or 24 hours) and for standard distances around the event locations. Event frequency appeared to be the basis for these restrictions. Simser (2005) reported on how microseismic monitoring was used at the Craig Mine, in Ontario. Of relevance to this review is the emphasis on event rates for proactive workplace closure and re-entry. However, due to the inability to predict the time and location of large seismic events, most effort in research on mining seismicity is placed on strategies to minimize risk, including support design, mine layout, and extraction sequence. Simser also stresses the importance of geology and local ground conditions in understanding rockburst risk. In an update to that paper, Simser (2006) noted the relationship between stope blasting and induced seismicity with an interesting observation that there is an offset between the time of blasting and the peak of high magnitude event activity. It was also mentioned that the mine did not have a formal re-entry protocol due to the variable behaviour of different areas although event rates and return to background levels was the primary tool in making re-entry decisions. Kaiser et al. (2005) takes a completely different approach to analyzing patterns of seismic activity and determining risk. Their approach centers on the use of a virtual reality visualization system and the associated database management software. They state that seismic migration patterns and interaction levels between seismically active blocks can be used to formulate seismic re-entry policies. This is certainly true, but in order to be practical for ground control practitioners at mines, tools must be cost-effective and be capable of being used in real-time data as opposed to being reliant on highly specialized software and hardware that operates on historical data. Another method that has been used for re-entry after blasting involves the cumulative percentage of energy for a given time window following the blast (Heal et al., 2005). Re-entry is defined at the time where 90% of the measured cumulative seismic energy is released for a time window of 24 hours. The limitations of this method are that
it can only be used for back analysis (not for real-time re-entry decision making), and, by definition, small blasts that do not produce a well-defined decay sequence can lead to large re-entry times.

The literature, however, does contain a significant number of papers on related topics including hazard and risk assessment, mine design to minimize seismicity, rockburst ground control methods, data analysis, and theoretical aspects of mining seismicity. Although a deep understanding of the seismicity and rockburst problem does not exist, considerable experience has enabled risk management procedures to be adopted by mines. The area of mining seismicity most relevant to re-entry protocol is that of hazard assessment, since many of these studies focus on identifying parameters relevant to predicting seismic events. Alcott et al. (1998) working on data from the Brunswick Mine, New Brunswick, determined that seismic energy, apparent stress and seismic moment were primary parameters to monitor. Trends in these parameters were correlated with observed damage, establishing critical thresholds that could be used to evaluate whether seismic hazard in an area was increasing or decreasing. However, these correlations and thresholds were site specific. Eneva (1998) used pattern recognition techniques to show that degree of non-randomness, spatial correlation dimension, and time interval for the occurrence of a constant number of events were correlated with changes in patterns of microseismicity, although no reliable precursory trends could be identified. Mansurov (2001) attempted to predict the occurrence of strong seismic events in mines based on variations in space and time clustering of microseismic events, plus density of faults in the rock mass. Spatial and time clustering patterns were also found to be significant indicators of seismic hazard by Kaiser et al. (2005). Based on seismic monitoring at the Mt. Charlotte Mine in Australia, Poplawski (1997) found that hazard assessment required monitoring of multiple parameters. Departure indexes were found to be particularly useful, i.e., the departure of various observed seismic parameters from normal
trends over time. The drawback of the departure index approach is that it requires a history of observations in similar ground conditions. Recognizing that seismicity is associated with mining-induced rock failure, Mercer and Bawden (2005) combined stress analysis with observations of microseismicity. Through extensive multivariate statistical analysis they showed significant correlations between observed seismicity and states of stress, but concluded that none of the relations were reliable enough to be used as routine decision making tools. Milev and Spottiswoode (2001) also noted the effect of rock type on induced seismicity, although they did not carry out detailed stress analysis.

These papers show the large number of parameters that have been associated with mining seismicity. However, based on discussion with mining seismologists, event rate is clearly the single most important parameter that is monitored for re-entry protocol. Immediately following large seismic events in mines, the frequency of events is high, but gradually decays to background levels over a matter of several hours. This pattern follows the modified Omori’s law (Omori, 1894; Utsu, 1961) developed from observed decay rates of large earthquake aftershocks. An important consequence of the Omori’s law is the time dependent nature of rock strength (Scholz, 2003). The increase in aftershock activity in the source region is generally attributed to increases in stress and resulting damage to the rock fabric. This has been studied in various ways, including stress corrosion (Das and Scholz, 1981), damage mechanics (Shcherbakov and Turcotte, 2004a), and statistical branching models (Saichev et al., 2005).

Although there is no direct literature on seismic re-entry protocol, there is a reasonable body of research on related topics, especially on earthquake aftershocks. Most results of these studies are expressed in terms of scaling laws, of which the modified Omori’s law is one and the Gutenberg-Richter (Gutenberg and Richter, 1944) frequency-magnitude relation is another. A third scaling law, Båth’s law (Båth, 1965), is not well known in mining seismology, but is of considerable
interest for re-entry. Báth’s law states that the difference in magnitude between the main event and the largest aftershock is a constant (which for crustal earthquakes is approximately 1.2). This result, if verified for mining seismic events, would be very useful in estimating the magnitude of the largest aftershock after a main event. In general, these laws are statistical fits of empirical functions to observable patterns of aftershock sequences, and regardless of whether they explain the underlying physics of the problem, the patterns that they fit are valuable in terms of applications such as the development of re-entry protocols. By combining the modified Omori’s law and Gutenberg-Richter scaling relations in a stochastic parametric model, the possibility of the occurrence of either significant aftershocks or an even stronger main shock during intervals following the main shock has been evaluated by Reasenberg and Jones (1989, 1990, and 1994). Ogata (1988, 1989, 1992, 1999, and 2001) introduced the Epidemic Type Aftershock Sequence (ETAS) model which is a point process in which every event can produce its offspring of events and can be considered as an extension of a single modified Omori’s law. These topics are central to developing a more reliable understanding and for developing a sound basis for selecting parameters to monitor for re-entry protocol.

As is apparent from the above review, there is no paper that outlines general procedures for the development of re-entry protocols. Many different procedures are used by mines, but not published. Mines that are just starting to experience seismicity and rockbursting are faced with the difficulty of having to develop re-entry protocols without the benefit of significant local experience. The conclusion is that there has been no systematic in-depth research of re-entry protocols such as outlined in this thesis.
Chapter 3
Current re-entry practices

To evaluate current re-entry practices and identify key seismic parameters currently in use for quantifying the decay of seismicity following large seismic events, a questionnaire was developed and distributed to 60 operating mines in seven different countries. Appendix A presents the questionnaire developed to collect data from seismically active mines in a standardized format. A total of 18 seismically active mines replied to the survey. The main mining methods at the surveyed mines are: entry mining (7), open stope (9) and caving (2). Most (75%) of these mines are from Ontario, representing a solid basis of information from Ontario’s operating mines. Following the receipt of the surveys, 13 mines in Ontario were visited. The principal factors assessed for each site surveyed included:

1. Development, description and experience with currently used re-entry protocols.
2. Decision indicators and key parameters used in the re-entry protocol.
3. Physical implementation of the re-entry protocol at the mine.
4. Management, reporting and communication of the re-entry protocol to the workforce.
5. Description of seismic monitoring systems.
6. Main concerns, needs for improvement, identification of limitations.
7. Site conditions (mining methods, geology, depth, rock properties).

In the following sections the main results of the survey are presented. Additional information can be found in Vallejos and McKinnon (2006).
3.1 Triggering of re-entry incidents

Blasting was found to be a significant factor in triggering seismic events. Ninety percent of re-entry incidents were reported to be triggered by blasting, making the re-entry protocol for blast related events the most fundamental one. Since blasting was found to trigger most large seismic events, it would be expected that these events would be relatively close to mining activity. However, it was found that the majority of events triggering re-entry restrictions were located between 50–100 m from mining, which is larger than might be expected on the basis of the influence of stress changes resulting from stope enlargement during blasting. It appears, therefore, that geological structures in the vicinity of mining must account for a significant portion of large magnitude events triggered in addition to fracturing around stopes.

The following guidelines were used at some of the surveyed mines for invoking a re-entry protocol:

1. Any seismic event with a Nuttli magnitude of 3.0 or greater, regardless of location and whether or not there was damage to mine excavations.

2. Any seismic event triggering a rockburst (damage to mine excavations).

3. Any seismic event with a Nuttli magnitude of 1.5 or greater and affecting the main accesses (e.g. ramp, footwall drifts), which could require workers to be confined to underground refuge stations and/or could require the evacuation of workers.

4. Any seismic event with a Nuttli magnitude greater than 1.5 and less than 3.0, located within 30 m from mine excavations and/or main infrastructure (e.g. cross-cuts, ramp, refuge station, electrical sub-station, garage, crusher station).

5. Excessive seismicity in close proximity of mine excavations.
3.2 **Logic involved in re-entry and representation of data**

The decision to re-enter is based on the requirement for the monitored parameter to return to a previously defined seismic background/normal level for a specified time/event window. If the monitored parameter exceeds a pre-set threshold during that time/event window, the re-entry restriction continues. Re-entry is permitted at the time where seismicity is comparable or lower than the threshold representing background level.

### 3.2.1 Time window

This type of analysis is mainly used to estimate the rate of a seismic quantity. Using the cumulative seismic quantity, at each time \( t \) the rate \( r \) is calculated using the last time window \( \Delta t \) of data, the process being repeated at successive points by using a time shift. Figure 3.1 illustrate this concept for the event count parameter after a rockburst in a mine using a 2 hour time window and a 0.1 hour shift.

![Figure 3.1: Time window analysis for re-entry time determination. (a) Cumulative number of events after a rockburst; (b) Mean \( r_1 \) and regression \( r_2 \) seismicity rates using a 2 hour time window with a 0.1 hour shift.](image)
Two approaches can be used to estimate the seismicity rate:

1. Mean window:

\[
r_1(t) = \frac{N(t) - N(t - \Delta t)}{\Delta t}
\]  

(3.1)

2. Regression window: Using the corresponding data set \( \{t_i, N(t_i)\} \) of \( n \) events within the last time window \( \Delta t \) the slope is determined:

\[
r_2(t) = \frac{n \sum_{i=1}^{n} t_i N(t_i) - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} N(t_i)}{n \sum_{i=1}^{n} t_i^2 - (\sum_{i=1}^{n} t_i)^2}
\]  

(3.2)

Both definitions produce similar results in the example presented in Figure 3.1, however, the regression technique make use of the whole sample in the time window, giving a better estimation of the first derivative of the seismic quantity of interest. Note that if Eq. (3.1) is used with a time and shift window of 1 hour the result is coincident with the classical event frequency histogram (see Figure 1.1 and Figure 1.2b).

The reasons for using a time window are twofold: (i) smoothing of the data, and (ii) additionally safety, because the longer the time window the longer the waiting time for a given event to move out of the window. In addition, the time window sets a minimum amount of time for data to be collected for calculating the rate, and therefore, when applied to re-entry, this also sets a minimum re-entry time after the initial event. Based on the most common practices at the surveyed mines a minimum time window of 2 hours is recommended.
With this procedure, and when background levels are reached, it can be stated that the seismicity rate in the last time window is comparable to background levels. Therefore, when used for re-entry is referred to as the background time window.

### 3.2.2 Event window

This type of analysis is necessary to track the time variation of seismicity parameters that are explicitly affected by the size of the sample used in the calculation. In this case it is necessary to calculate the time \( t \) elapsed for the occurrence of a constant number of events, the process being repeated at successive points using an event shift.

### 3.3 Development and description of currently used re-entry protocols

Over 80% of the surveyed mines have seismic re-entry protocols. Most (89%) were developed in-house, by ground control personnel, and over 70% were based on local experience. This has resulted in a large variety of re-entry protocols, many being specific to particular mining activities, such as: mining through sensitive zones and production blasts, as shown in Figure 3.2. The most frequent re-entry protocol is the one created for production blasts, which in the simplest case corresponds to a gas check. Given the variable behaviour of different zones throughout the mine, there is also a group that makes re-entry decisions on a case by case basis with no standards for the re-entry protocol.
Of the mines with re-entry protocols, a direct correlation was found between the number of years that the protocol had been in place and the perceived reliability informed by the surveyed mines defined in a subjectively scale from 1 to 5 (Figure 3.3). This is not surprising, considering that the majority of protocols have been developed on the basis of local experience. The lowest levels of reliability were found in mines that were just starting to experience seismicity and rockbursting and were faced with the difficulty of developing re-entry protocols without the benefit of significant local experience and no standard procedure to follow. In these cases, and also for those mines with more experience with seismicity, there is a need for standardization regarding the data that needs to be collected, and how it should be analyzed and to a lesser extent, advice on suitable seismic system configurations.
Figure 3.3: Perceived reliability and number of triggered re-entry protocols per year at the surveyed mines as a function of time in place of the policy.

### 3.4 Key re-entry seismic parameters

Figure 3.4 presents the key seismic parameters that are currently monitored at the surveyed mines in connection with re-entry decisions.
Since the number of events is easily measured by actual microseismic monitoring systems, over half of the mines surveyed use event count analysis as their primary decision making parameter. The main shortcoming of this approach is that the energy/strength release of the event is not considered into the re-entry policy. It has been observed that large magnitude events can occur after the main event. The question then occurs as to whether the re-entry clock should be reset based on magnitude. The survey showed that only 56% of mines have resetting magnitude thresholds.

Other mines that have purchased proprietary software have additional ways of visualizing the data provided by the seismic monitoring system. Two specialized software packages were found to be in use for re-entry purposes: (1) SeisWatch, and (2) MS-RAP. In the following sections, a
brief description and critical discussion of the procedures used by each of these softwares is provided.

3.4.1 SeisWatch software
SeisWatch is an on-line seismic tool developed as part of a research project based on the data of Creighton Mine (Van Dusen and Shumila, 2001). The purpose of this software is to monitor for a user-specified volume the return to background level of the following five indicators:

1. Seismic work.
2. Spatial clustering.
3. Strain rate.
4. Event decay rate-Omori analysis.
5. Depth-Event frequency histogram.

The definition of how parameters 1 to 4 are calculated by SeisWatch was found in Malek and Leslie (2006) as the Software User’s Guide (SeisWatch, 2007) does not provide a description. Indicator number 5 was found in SeisWatch (2007).

3.4.1.1 Seismic work
This is the principal parameter used by the mines that have purchased SeisWatch. Three of the four mines indicated in Figure 3.4 using Energy/Strength analysis in their re-entry protocol use this parameter as their principal decision-making. The seismic work ($SW$) parameter is defined from the seismic moment $M_o$, which is an estimate of event strength. For re-entry purposes, this is plotted cumulatively at each $N^{th}$ event occurrence since the selected time zero:
\[ SW = \sum_{i=1}^{N} (M_{oi})^{0.5} \]  

(3.3)

It is supposed that this type of parameter shows trends more accurately than event count because the strength of the event is incorporated into the analysis. Figure 3.5 illustrates an example of a response curve for this parameter.

Figure 3.5: SeisWatch response curve for the Seismic Work parameter (Modified from Malek and Leslie, 2006). Red, pink and blue lines are indicated by the rockburst data, calibrated curve and background levels text boxes respectively.

Four different curves are indicated in Figure 3.5. The red line indicates the cumulative seismic work for seismic events after the rockburst, the dashed line indicates the regression line for a time window in order to compare the actual rate to background levels. A steeply sloping regression line indicates ongoing high seismic activity. The two blue lines indicate the slope of maximum
and minimum background levels of seismicity and the pink line indicates a calibrated curve for a previous event, given by:

\[ SW(t) = Ct^D \]  

(3.4)

where \( C \) and \( D \) are adjustable parameters.

For re-entry, the regression line must be comparable or lower than the maximum background level of seismicity.

The following remarks can be made regarding this parameter:

1. “Seismic work” is not a standard seismological term. This definition is similar to the Benioff strain (the square root of seismic energy) given by Benioff (1951) in the crustal literature.

2. The definition of highly unstable, transition and return to stable conditions in Figure 3.5, are somewhat arbitrary.

3. The data of the new rockburst does not necessary follow the previously calibrated typical event in Figure 3.5. This indicates that the equation calibrated from a past rockburst or large magnitude event, does not represent an intrinsic response for new occurrences. In addition, if no rockbursts/large magnitude events are present in the database the equation cannot be calibrated.

4. The re-entry time depends directly on the pre-defined level of background seismicity.

3.4.1.2 Spatial clustering

This parameter measures the spatial dispersion of events throughout the mine. Typically, events will be more tightly clustered following a large magnitude event, and less clustered (more evenly distributed over a larger region) under stable conditions. This parameter quantifies the degree of
clustering by calculation of a normalized entropy. Only one of the surveyed mines used this parameter for re-entry. The algorithm for the calculation of the percentage clustering is as follows:

1. Define the number of events $k$ to be used in the event window and the cluster spatial dimension $D^*$. 

2. Calculate the number of neighbours $n_i$ for each $i^{th}$ event within the cluster dimension $D^*$ and find the total number of neighbours $N_o$ for all $k$ number of events.

3. Calculate the entropy for each event $E_i$ as:

$$E_i = -\frac{n_i}{N_o} \ln \left( \frac{n_i}{N_o} \right) \quad (3.5)$$

4. Calculate the cumulative entropy $E_o$ for all $k$ number of events:

$$E_o = \sum_{i=1}^{k} E_i \quad (3.6)$$

5. Finally, the cumulative entropy is normalized to display as a percentage of clustering $C\%$:

$$C\% = 100 \frac{N_o E_o}{(k+1)k} \quad (3.7)$$

A high clustering value indicates that events are occurring in a spatial region smaller than the cluster spatial dimension $D^*$.

Comments on this procedure are as follow:

1. The most important input parameter is the cluster spatial dimension $D^*$. If a large $D^*$ value is selected, so all the events have the same number of neighbours, the percentage of clustering
will be 1.0 at all times. This indicates that the selected cluster dimension can be appropriate for a specific aftershock sequence but inadequate for a different sequence.

2. The normalization provided in Eq. (3.7) does not correspond to the classical normalization of descriptive statistics given by $\ln(k)$.

3. Maximum and minimum background levels of seismicity also need to be defined for this parameter.

4. It is unclear how the software manages events without neighbours.

### 3.4.1.3 Strain rate

Strain rate is a calculation of seismically induced strain. It is not directly related to strain in engineering units. A high seismic strain, however, is an indication of ongoing seismicity in the monitored region. For example, many events occurring in a small volume over a short period of time will yield a high seismic strain rate. Only one of the surveyed mines used this parameter for re-entry.

The algorithm for the calculation of the strain rate is as follows:

1. Define the number of events $k$ to be used in the event window.

2. Determine the spatial distribution of the $k$ events. The cluster is approximated by an ellipsoid with major, intermediate and minor axis $a$, $b$ and $c$ respectively. The volume of the ellipsoid is given by:

$$V = \frac{4}{3} \pi a b c$$  \hspace{1cm} (3.8)

3. Calculate the time $t$ elapsed during the window of $k$ events.

4. The strain rate parameter is given by:
\[ sr = \log \left( \frac{\sum_{j=1}^{k} M_{o_j}}{V_t} \right) \]  

(3.9)

Comments on this procedure are as follow:

1. The volume estimated using an ellipsoid may not be a good fit to the spatial distribution of the events.

2. Maximum and minimum background levels of seismicity have to be defined for this parameter.

3.4.1.4 Event decay rate-Omori analysis

The event decay rate-Omori analysis chart provided by the SeisWatch software shows the logarithmic decay of number of events per day since the main event (Figure 3.6).

![Figure 3.6: SeisWatch event decay rate chart (SeisWatch, 2007).](image-url)
The Omori typical process plot (smooth curve in Figure 3.6) shows how the event rate can be expected to behave at time $t$ after a large magnitude event and is given by:

$$n(t) = \frac{K}{(1 + t)^p} \quad (3.10)$$

where $n(t)$ is the number of events per day, and $p$ and $K$ are user-supplied constants.

Critical comments on this analysis are as follow:

1. The parameter $K$ and $p$ differs from sequence to sequence.

2. There are no guidelines of how this plot can be used for re-entry.

3. The time unit used, corresponding to a day, is impractical for re-entry purposes where the decision has to be made in hours.

4. Considering that the modified Omori’s law (Omori, 1894; Utsu et al., 1995) formula is given by:

$$n(t) = \frac{K}{(c + t)^p} \quad (3.11)$$

where $n(t)$ is the instantaneous rate/intensity of events and $c$ is an adjustable offset time constant, it is unclear why an offset time equal to one day is set as default in the denominator of Eq. (3.10).

5. The Omori formula is applied mistakenly to the number of events in a time period following the main event rather than to the rate of activity after the initial event. For applying Eq. (3.11) to the number of events at time $t$ occurring during the last time window $\Delta t$ it is necessary to consider its integral form for a time interval $[T_{\Delta}, t]$, given by:
\[ N_{[t,T]} = \int_{t_i}^{t} n(t)dt = \begin{cases} K \ln \frac{(t + c)}{(T_A + c)} & \text{for } p = 1 \\ K \left( t + c \right)^{1-p} - (T_A + c)^{1-p} & \text{for } p \neq 1 \end{cases} \] (3.12)

By fixing the range of the time interval \([T_A, t]\) to a given time period \(\Delta t\):

\[ t - T_A = \Delta t \] (3.13)

and replacing Eq. (3.13) in (3.12):

\[ N_{[t, t-\Delta t]} = \begin{cases} K \ln \frac{(t + c)}{(t - \Delta t + c)} & \text{for } p = 1 \\ K \left( t + c \right)^{1-p} - (t - \Delta t + c)^{1-p} & \text{for } p \neq 1 \end{cases} \] (3.14)

Equation (3.14) represents the number of events at time \(t\) occurring during the previous time window \(\Delta t\).

### 3.4.1.5 Depth-Event frequency histogram

This is a bar graph histogram of the frequency of events against depth (Figure 3.7).

![Event Frequency Histogram](image)

**Figure 3.7:** SeisWatch depth-event frequency histogram (SeisWatch, 2007).
Three types of events are shown in Figure 3.7: those that occurred within one hour after (red bar) and before (green bar) the large magnitude event, and those based on the average hourly activity for a period of two days before the selected time zero (blue bar). These histograms of activity by level can assist in identifying where clustering is occurring relative to the location of the large event and if activity is migrating to other levels (Alexander and Trifu, 2005).

### 3.4.2 MS-RAP software

Another seismic data post processing tool used for re-entry decisions is the MS–RAP (Mine Seismic Risk Analysis Program) software, developed by the Australian Centre for Geomechanics (ACG). This software is not exclusively for re-entry analysis and includes several options for interpreting seismicity in mines. The analysis for re-entry considers all the seismic events occurring within a defined spherical radius around the main event coordinates within a defined time period after the event. The criteria used to assess re-entry at the surveyed mines with this software are:

1. Event decay rate-Omori analysis.
2. Cumulative event/Energy release within a defined time period.

In addition, the following analysis was found to be used as an indicator of stress levels:

3. Energy index ($EI$)/Cumulative apparent volume ($CAV$)

Details of how the analysis is performed and used were obtained from the surveyed mines, the MS-RAP User’s Manual (Hudyma, 2005), and papers (Hudyma et al., 2003; Heal et al., 2005; Potvin, 2008).
3.4.2.1 Event decay rate-Omori analysis

Figure 3.8 presents the type of event decay rate analysis performed with this software.

![Figure 3.8](image_url)

Figure 3.8: MS-RAP event decay rate analysis following a mine blast (Hudyma et al., 2003).

In Figure 3.8 the recorded seismicity is presented as a bar chart, showing the number of events occurring each hour after a blast. As a reference the line corresponding to the Omori formula is also plotted in Figure 3.8. This software seems to use the following formula:

\[ n(t) = \frac{K}{t^p} \]  

(3.15)

where \( n(t) \) is the number of events in a time period \( t \) following the main event (usually in hours), and \( p \) and \( K \) are adjustable parameters. Hudyma et al. (2003) argue that by back analysis useful values for \( K \) and \( p \) can be determined, however, no examples were provided. In Figure 3.8 the number of events occurring during the first hour \( N_{1\text{hour}} \) is coincident with the first point of Eq. (3.15). By imposing \( t = 1 \) in Eq. (3.15) implies that \( K = N_{1\text{hour}} \). In addition, a \( p \) value of 1.5 is
indicated, which based on the author’s visual criterion gives a good fit to the decrease in the number of seismic events. Hudyma (2005), Hudyma et al. (2003), Heal et al. (2005) and Potvin (2008) indicated that a management decision is needed to decide what portion of the seismic decay should occur prior to re-entry. However, no guidelines were provided by the authors.

Additional comments are as follow:

1. In this case the authors have decided to set the parameter \( c \) of the modified Omori’s law (Eq. (3.11)) equal to zero.

2. No guidelines of how the Omori formula can be used for re-entry purposes are provided.

3. The Omori formula is applied mistakenly to the number of events in a time period following the main event rather than to the rate of activity after the initial event. This has already been discussed in Section 3.4.1.4.

### 3.4.2.2 Cumulative event/Energy release within a defined time period

This analysis makes use of the cumulative number of events or cumulative seismic energy normalized by the corresponding total within a defined time period (Figure 3.9). Re-entry is defined at the 90% of the cumulative events/energy dissipated. In the example presented in Figure 3.9, 80% of the seismic events occur within the first 4 hours after the blast, but the majority of the seismic energy release occurs 10 hours after the blast. Hudyma et al. (2003) state that this type of pattern is relatively common for structurally related seismicity, where large magnitude seismic events can occur many hours or days following mine blasts. They argue that in these cases, re-entry procedures based on number of events are not effective for managing workforce exposure and seismic energy is recommended. However, if the 90% rule is applied both to the normalized cumulative number of events and seismic energy both criteria give exactly the same re-entry time as shown in Figure 3.9.
Comments on this procedure are as follow:

1. The results depend on the defined time period of analysis.

2. The 90% re-entry rule is a completely arbitrary rule. The underlying assumption is that once 90% of the total energy has been released, the rock mass can be considered to have readjusted to the new state of stress and is unlikely to produce a significant event (Potvin, 2008).

3. The method can only be used for back analysis (not for real-time re-entry decision making).

4. By definition small events that do not produce a well-defined decay sequence can cause large re-entry times.

5. There is no published statistically comparison between the cumulative events and energy criteria.
3.4.2.3 Energy index/Cumulative apparent volume

Energy index/Cumulative apparent volume analysis, called instability analysis, is a technique used in South African mines to look for precursory patterns of seismicity prior to large magnitude seismic events (Mendecki et al., 1999). The assumption is that energy index represents the stress distribution in the rock mass. The higher the index, the higher is the stress in the rock mass. Figure 3.10 illustrates how energy index is evaluated for a group of seismic events.

As shown in Figure 3.10 energy index is defined as the ratio of the seismic energy $E$ of an event with seismic moment $M_o$ to the average energy $\bar{E}$ derived from the regional log $E$ vs log $M_o$ relationship (van Aswegen and Butler, 1993):

$$EI = \frac{E}{\bar{E}(M_o)}$$

$$\bar{E}(M_o) = 10^{a_1 + a_2 \log M_o}$$

where

- $a_1 = -11.434$
- $a_2 = 1.692$
- $R^2 = 0.877$

Figure 3.10: Energy index definition for a group of seismic events.
\[ EI = \frac{E}{\overline{E}(M_o)} \quad (3.16) \]

with \( \overline{E}(M_o) = 10^{a_1 + a_2 \log M_o} \), where \( a_1 \) and \( a_2 \) are adjustable constants for a given volume and time period. For energy index greater than 1 (above the regression line), there is more energy being released in seismic events than normal, suggesting that stress is accumulating in the rock mass.

Cumulative apparent volume is supposed to be a measure related to rock mass deformation. For the \( N^{th} \) event occurrence since the selected time zero the cumulative apparent volume is given by:

\[ CAV = \frac{1}{2G} \sum_{i=1}^{N} \left( \frac{M_{o_i}}{E_i} \right)^2 \quad (3.17) \]

where \( G \) is the shear modulus of the rock.

The instability analysis assumes that there is a basic model for rock mass failure. This failure model starts with stress accumulating in the rock mass due to nearby mining. As stress accumulates, the seismicity generated shows large fluctuations in energy index (\( EI > 1 \)), but relatively little deformation (cumulative apparent volume). As the rock mass fractures and starts to deform, stress starts to be shed to other locations, resulting in a drop in the energy index below 1. As significant deformation starts to occur, cumulative apparent volume starts to increase significantly and is coincident with a drop in the energy index.

Figure 3.11 shows the energy index/cumulative apparent volume time histories for a cluster of events at Laronde Mine (Heal et al., 2005). The cluster starts by showing a fluctuating increase of energy index with little coseismic deformation. This is followed by a general decrease in energy index in July 2004, corresponding to local stress shedding, and ultimately large shear deformation (large cumulative apparent volume).
Figure 3.11: Energy index/Cumulative apparent volume analysis for a cluster of events at Laronde Mine (Heal et al., 2005).

Comments on this parameter are as follow:

1. If there is more than one rock type or there is one rock type but many large scale features such as faults the $\log E$ vs $\log M_o$ relationship may be scattered.

2. Events with a low pair $(E, M_o)$ can have the same $EI$ as events with large $(E, M_o)$. To filter events with little impact on the workplace, thresholds for the parameters of energy and seismic moment can be established (see for example Alcott et al., 1998).

3. The relationship between stress and $EI$ has never been verified.

4. The instability concept is a theoretical failure model, which can sometimes match rock mass failure observed in mines. If the rock mass failure process occurring at a seismic source does not match the failure model, $EI/CAV$ analysis may shed little or no insight into the rock mass failure. While this model is frequently used to try “forecast” large magnitude seismic events
in South African mines, there are few examples of routinely successful application of the technique in Canadian and Australian mines (Heal et al., 2005).

### 3.5 Exclusion zone

The exclusion zone is that volume around the blast/seismic event for which access is restricted by the re-entry protocol. Three main procedures were identified at the surveyed mines (Figure 3.12).

![Figure 3.12: Re-entry exclusion zones at the surveyed mines. The number of mines for each group is indicated.](image)

The first procedure is based on the spatial extent of the seismic data, and the restricted area is normally established to contain the volume of events. It should be cautioned that the restriction zone determined using this approach depends on the monitoring system coverage and quality of
data. Local experience may lead to closure of additional zones outside of the currently affected area. A second procedure restricts access for a fixed distance in all directions from the main event. Distances less than 50 m are related mostly to strain burst damage and entry mining methods, while distances between 50 and 100 m include open stope mining. Distances larger than 100 m were associated with regional fault slip in certain mines. The advantage of this second alternative is that zones with high and low seismic system coverage are equally protected. The third procedure restricts access only on the basis of nearby excavations, without considering any particular distance.

Generally, Ground Control personnel are responsible for placing the exclusion zone, as they have direct access to the seismic data and experience in interpretation. However, with appropriate training and experience, operators can identify hazardous conditions for active workplaces and place an exclusion zone until reported to and inspected by Ground Control personnel.

3.6 Re-entry procedure flowchart

Figure 3.13 summarizes the most common steps followed at the surveyed mines as part of the procedure prior to re-entering an affected zone. This flowchart can be used either for blast and non-blast triggered restrictions. The only modification is the order of installation of barricades and signs for the case of large magnitude seismic events.
Figure 3.13: Most common steps followed at the surveyed mines prior to re-enter.

The following relevant standard precautions were incorporated into the re-entry flowchart shown in Figure 3.13:
1. Monitor and evaluate ground response for 12 hours after the restricted period.

2. Study in more detail seismicity that appears outside of blast hours.

As a part of the re-entry procedure some of the surveyed mines have developed their own methodologies to evaluate risk to help determine if the task is critical (for example, before a blast). Figure 3.14 present an example of the implemented procedure. The direct application of this methodology without a deep understanding and site evaluation is not recommended.

Figure 3.14: Risk assessment chart used at some of the surveyed mines.

The risk assessment presented in Figure 3.14 is not based on any ground control concerns. The factors affecting the risk assessment, in this case, are the likelihood of occurrence (probability), exposure and possible consequences. Once these factors have been evaluated, a risk score and the associated risk level can be calculated for the specific task by:
Risk Score = Likelihood × Exposure × Possible consequences  

(3.18)

For the example presented in Figure 3.14 the risk score is given by: 
$1 \times 2 \times 7 = 14$, giving a low risk level. However, the possible consequences can be serious with injuries.

If measures are adopted to mitigate those risks that were identified as intolerable, the risk assessment procedure is repeated. Note that these mitigation measures will only reduce the probability of an identified risk occurring and the consequences typically will not change (Brummer and Andrieux, 2008).

3.7 Microseismic monitoring systems

Most Ontario mines have either none or very few triaxial sensors in their microseismic monitoring arrays. It was found that the ratio of triaxial to uniaxial sensors was approximately 1/10, as opposed to 1/4 for overseas mines. The reliance on larger numbers of uniaxial sensors is related to the priority placed on event location and the requirement to cover larger volumes. However, the lack of or low density of triaxial sensors places restrictions on more advanced analyzes that can be carried out on seismic waveforms, such as: moment tensor (e.g., Trifu, 2001).

For each surveyed site, the following factors were asssessed regarding the array of sensors:

1. Total number of channels $N_C$.
2. Approximate volume covered by the array $V$.
3. Average spacing of sensors $\bar{S}$.
Figure 3.15 presents the total number of channels against the approximate volume covered by the array. Note that both axes have been elevated to the cubic root in order to represent the average volumetric spacing $S_{V} = \sqrt[3]{V/N_{C}}$ of the arrays.

Accordingly, the following best-fit equation was determined by the method of least squares (standard errors in square brackets):

$$N_{C} = \left(2.6 \pm 0.2\right) + 1.4\left[\pm 0.3\right]V^{1/3}$$

(3.19)

Despite some scatter and low coefficient of determination $R^2 = 0.54$, Eq. (3.19) is significant at the 5% level.
Using the average volumetric $\bar{S}_V$ and spacing $\bar{S}$ of sensors, a Factor of Coverage ($FC$) is defined by:

$$FC = \frac{\bar{S}_V}{\bar{S}} = \sqrt[3]{\frac{V}{N_c}}$$  \hspace{1cm} (3.20)

Figure 3.16 indicates that $FC$ varies empirically between 0.5 to 2.6, with a mean of 1.3 and standard deviation of 0.6. Using the Chi-Sq test it was found that $FC$ follows a normal probability distribution with an observed significance level of 0.94.

Figure 3.16: Cumulative ascending distribution for the factor of coverage $FC$.

These empirical relationships are satisfactory for preliminary global estimation of the number of channels and average spacing for a given volume. For example, using Eq. (3.19) and the 95% confidence interval for a volume of 0.5 km$^3$ yields $N_c = 39 - 60$ with an average of 49 channels.
The spacing between sensors can be obtained by using the normal distribution for $FC$ and the range of possible number of channels, yielding: $\overline{S} = 78 - 452$ m, with an average of 164 m.

Since the array geometry at specific mines is influenced by a variety of factors, including those of a non-technical nature, scatter in the reported data is inevitable. Therefore this method should be complemented with more sophisticated site specific methods, such as error-space analysis, to provide additional design refinement.

### 3.8 Summary and discussion

As part of a study to provide a complete set of guidelines for the development of re-entry protocols, a survey of re-entry practices at 18 seismically active mines, mostly in Ontario, was analyzed and evaluated. Some results from the survey are: the majority of re-entry incidents are triggered by production blasts, resulting in classical Omori-style decay in seismic event frequency over a number of hours. The distance from active mining that seismic events typically trigger re-entry restrictions is between 50-100 m. Seismic parameters typically evaluated are event frequency and magnitude, energy, and moment.

The variation in the choice of parameters used in the re-entry evaluation process by the surveyed mines indicates that the controlling parameters for the time-dependant decay of seismic aftershocks are still poorly understood, and that there are no recognized formal standards for selecting which parameters to monitor or calculate on a routine basis. It is recommended that at least event count and their associated magnitude/strength release per hour be monitored and used in the determination of re-entry time. This is more reliable than using one parameter alone. This approach, however, requires the determination of background/normal levels of seismicity and resetting thresholds.
Re-entry protocols cannot be generalized or directly adapted from other mines. A policy can have success in one environment but can be of questionable value in others. Re-entry protocols must be compatible with rock mass behaviour in a particular geological/mining environment based on observations of seismicity patterns.

The most commonly used parameters/procedures to assess re-entry at the surveyed mines are:

1. Event decay rate-Omori analysis.

2. Seismic work.

In the case of the Omori analysis there seems to be a misunderstanding of how the modified Omori’s law has to be applied to the data. This was discussed in Section 3.4.1.4 and a theoretical framework of how this equation should be used for representing the event frequency histogram has been proposed. The current approach with the use of this equation is to establish parameters for a previously measured large magnitude event/rockburst and use this set of parameters for the future. This formulation is not completely representative for new aftershock sequences. In addition, there are no guidelines of how the modified Omori’s law can be quantitatively used for re-entry. These topics are reviewed and addressed in-depth later in this thesis.

Due to the possibility of large aftershocks in the event decay sequence, re-entry is generally delayed for a minimum time window once background/normal levels of seismic activity are re-established. This provides an additional safety level and degree of confidence to the policy. Different background time windows may be selected for different zones of a mine depending on the local characteristics of seismicity.

Several controlling parameters were identified during this first phase of the project, such as: blast size, layout geometry, stresses, geology, etc., however, there is insufficient data to rank their
relative importance and provide a methodology of how to account for these factors in the re-entry protocol.

In summary, various needs related to re-entry protocols were identified from mining personnel that would enable them to make more informed decisions about workers re-entry into areas after large magnitude seismic events. The study clearly illustrated the significant differences in patterns of seismicity between various mines and in different regions of individual mines. This showed the need not for a single policy on re-entry, but for guidelines that would outline how ground control personnel at mines could make use of data collected locally in order to develop a re-entry protocol relevant to their particular geological and mining conditions. Preliminary guidelines for re-entry protocol development are proposed in Chapter 4.

There is also a need to develop a better understanding of the patterns of seismicity following large magnitude seismic events and blasts. These items are the priorities in the part II of this thesis (Chapter 6 to 8).
Chapter 4

Preliminary guidelines for re-entry protocol development

Based on the evaluation and analysis of current re-entry practices various needs related to re-entry protocols were identified from mining personnel that would enable them to make more informed decisions about workers re-entry into areas after large seismic events. The study clearly illustrated the significant differences in patterns of seismicity between various mines and in different regions of individual mines. This showed the need not for a single policy on re-entry, but for guidelines that would outline how ground control personnel at mines could make use of data collected locally in order to develop a re-entry protocol relevant to their particular geological and mining conditions. For this reason, the preliminary guidelines proposed in this chapter emphasize the use of procedures to develop re-entry protocols, focusing on the data that need to be collected, presentation and interpretation of the collected data, and how to develop the re-entry protocol following the data analysis.

As noted in the previous chapter (Figure 3.4), over half of the mines surveyed use event count analysis as their primary decision making parameter. The decision to re-enter is based on the requirement for the event count to return to background level for a specified time window. If the event count exceeds a pre-set event count threshold during that time window, the re-entry restriction continues. A major disadvantage of this approach is that magnitude or energy of events is not accounted for. It has been observed that large magnitude events can occur after the main event. The question then occurs as to whether the re-entry clock should be reset based on magnitude. The survey showed that only 56% of mines have resetting magnitude thresholds. The lack of resetting thresholds in many mines appears to be related to the absence of any procedures or guidelines on how to choose an appropriate level, and to the difficulty in accurately
determining magnitude. Therefore two important components of re-entry protocols need to be fulfilled: background/normal levels of seismicity, and resetting thresholds.

The following step-by-step guidelines are proposed for the identification of background levels, resetting thresholds and the development of a site specific re-entry protocol based on the analysis of locally observed seismicity.

4.1 Data collection

The following seismic parameters are considered for the development of a re-entry protocol for a given volume:

1. Event location and origin time.

2. Location error $\Delta r$ and number of triggered sensors $N_s$.

3. Magnitude/strength of the events.

4.2 Filtering

To identify trends, the seismic database must be filtered so seismic events and blasts that are poorly located are removed from the analysis. Filtering the data by limiting the location error and the number of triggered sensors is recommended due to its simplicity. The limit values can be selected by plotting the location error versus the number of triggered sensors as shown in Figure 4.1. After selecting tentative limit values, the percentage of activity and energy retained for the analysis is evaluated. It is recommended to employ more than 95% of the total event activity and energy, as shown in Figure 4.1. If possible, events labelled as blasts should be filtered separately from seismic events.
4.3 Normal levels of seismic activity—Diurnal charts

An important component of a re-entry protocol is the definition of background/normal levels of seismicity. Use of a level that is too high will lead to unnecessary exposure of the workforce to seismicity, while one too low will delay production, decreasing the reliability of the re-entry protocol. The most challenging process is to define when seismicity is at a normal level. Based on the survey, most mines have identified, at least approximately, typical normal levels for the re-entry protocol. Still, there is no straightforward practical method for calculating the background levels to be used in the development of the re-entry protocol. The current approach consists in establishing typical minimum and maximum background levels during a shut-down at the mine.
and during periods of minor mining activities, respectively (Malek and Leslie, 2006). However, no statistical method to determine them was described.

As a simple and practical statistical tool, the use of diurnal charts is recommended, as shown in Figure 4.2 for event count. This figure is based on the superposition of seismicity in a 24-hour chart over a time period that represents relatively constant mining conditions, which could be several months of observations at a particular location. In the example presented in Figure 4.2 a total of 4,164 events were located over a three month time period. To normalize the plot the diurnal chart has been divided by the total value of the particular seismic parameter. The selection of one hour time unit interval to represent the data is arbitrary. However, one hour was the most common time unit used to represent the event frequency histogram for re-entry purposes at the surveyed mines.

Figure 4.2: Event count diurnal chart for a zone of a mine for a three month time period.
These types of plots have been used to show evidence of time-dependent rock failure (Cook, 1976), to identify an offset between the time of blasting and the peak of large magnitude events (Simser, 2006), and to determine the seismic source mechanism causing seismic events (Hudyma, 2005). From the example shown in Figure 4.2, it is clear that there is a typical, approximately constant, normal level of seismic activity plus two daily blasting shifts, each of which triggers an increase in seismicity that decays over a three to four hour time period. Therefore, normal levels of seismicity can be estimated by omitting hours of highest activity and determining an upper bound percentage for the parameter of interest. Visual inspection of Figure 4.2 suggests that the hours of highest activity are: 4, 5, 6, 7, 16, 17, 18, and 19, yielding an upper bound proportion of $EC_{\%}^U = 2.79\%$. By adding all the occurrences $EC_{\%}^i$ below $EC_{\%}^U$ the total proportion of normal level of seismicity is obtained $EC_{\%}^T = \sum_{i \in U} EC_{\%}^i = 26.68\%$.

The average upper bound of normal levels is calculated for a generic seismic quantity $Q$ by:

$$\overline{Q} = Q_{\%}^T \frac{Q}{T}$$  \hspace{1cm} (4.1)

where $Q_{\%}^T$ is the total proportion of normal levels of seismicity below $Q_{\%}^U$, $Q = \sum_i Q_i$ is the total value of the seismic quantity $Q$ for the time period of interest and $T$ is the total of non-omitted time units of seismicity determined from the diurnal chart. Using the data presented in Figure 4.2 the following value is obtained for event count:

$$EC = 26.68\% \frac{4,164 \text{ events}}{466 \text{ hours}} = 2.3 \text{ events/hour}$$  \hspace{1cm} (4.2)
Figure 4.3 presents the event frequency histogram with the determined upper bound of normal
seismicity for a selected time period. The method provides reasonable values for the estimation of
an upper normal level of seismicity.

![Event Frequency Histogram](image)

**Figure 4.3:** Event frequency histogram for a selected time period showing the determined
upper normal level of seismicity for a zone of a mine.

One of the limitations of the proposed method is the selection of the hours of highest activity. In
order to reduce the subjectivity and automate the procedure, the cumulative ascending
distribution (CAD) for the sample data of $Q_\%$ is used to examine if there is a distinct change in
the population that may divide the hours of highest activity. The result (black squares in Figure
4.4) indicate that there is a characteristic point that separates hours of low and high activity,
which is coincident with the previously visually determined upper bound proportion of 2.79% in
Figure 4.2. Therefore, the upper bound proportion of normal levels of seismicity is defined at this point.

Further inspection of Figure 4.4, suggest that all the values below the point that separates hours of low and high activity are in a straight line, i.e., the CAD can be approximated by a uniform distribution. To test the null hypothesis $H_0$ that a range of the CAD is a random sample from a uniform distribution $U(a, b)$ the Kolmogorov-Smirnov test was selected. This non-parametric statistic is based on the maximum absolute difference $D$ between the CAD of the sample $Q_{n}$ and the CAD of the probability function specified in the null hypothesis (Chakravarti et al., 1967). Critical regions for rejecting $H_0$ are of the form $D \geq D_C$, where $D_C$ are tabulated values for a significance level and sample size (Ayyub and McCuen, 2003). The data is considered to follow a uniform distribution if the $D$ value is lower than the critical value $D_C$ for a significance level of...
20%, ensuring no evidence against the null hypothesis. The parameters $a$, $b$ of the uniform distribution are estimated by the maximum likelihood method, and are given by the first and last data points of the sample respectively. Figure 4.4 illustrates the approach for event count, which shows the ratio $R_{K,S}=D/D_C$ as a function of the event count proportion $EC_{\%}$ for the sample data of Figure 4.2. If a tentative $EC_{\%}^{U}$ is higher than the ‘correct’ $EC_{\%}^{C}$, the uniform distribution cannot model the sample data adequately and, consequently, the goodness of fit, measured by the ratio $R_{K,S}$ is higher than 1.0. The goodness of fit ratio $R_{K,S}$ decreases with decreasing $EC_{\%}$ and reaches a value less than 1.0 at $EC_{\%}^{U}=2.79\%$ in this example. Beyond this point $R_{K,S}$ can gradually decrease or increase. The upper bound proportion of normal levels $Q_{\%}^{U}$ is defined at the point at which $R_{K,S}$ reaches a value less than or equal to 1.0. Note that the proposed method is not limited to event count and can be used to estimate the upper bound level of normal levels of seismicity for any seismic quantity of interest. An additional example is shown in Figure 4.5 for the seismic moment parameter.

Figure 4.5. Determination of background/normal levels for the seismic moment parameter. (a) Seismic moment diurnal chart; (b) Cumulative ascending distribution and Kolmogorov-Smirnov goodness of fit ratio for the $SM_{\%}$ sample data.
4.4 Thresholds

Re-entry protocols are invoked, in real-time, when there are seismic events with magnitude larger than a specified threshold. This is defined as a magnitude event protocol. Another important threshold to establish in the re-entry protocol is the resetting threshold, which is used to reset the re-entry clock if there is a subsequent seismic event of significant magnitude. Re-entry thresholds are established from the seismic history recorded at the mine or in zones of the mine, reflecting actual conditions of the rock mass responding to mining activities.

4.4.1 Resetting thresholds

The objective of these thresholds is to introduce the microseismic energy/strength release into the re-entry protocol. They are related to the energy/strength measured at the mine or zone of the mine during background levels. The course of action adopted using this type of thresholds is to reset the re-entry clock if, during the specified background time window, following a re-entry event, there are events larger than the specified resetting thresholds.

These thresholds can be selected using the determined background levels of seismic activity and seismic moment. For the example presented in Figure 4.2 and Figure 4.5 the following background levels were determined:

\[
\begin{align*}
EC &= 2.3 \ \frac{\text{events}}{\text{hour}} \\
SM &= 5.4 \times 10^6 \ \frac{\text{Nm}}{\text{hour}}
\end{align*}
\]

Replacing \( m_{SM} \) in the relationship between moment magnitude and seismic moment (Hanks and Kanamori, 1979):

\[
m_{SM} = \frac{SM}{EC} = 2.4 \times 10^6 \ \frac{\text{Nm}}{\text{event}}
\]
\[ \text{MomMag} = \frac{2}{3} \log M_o - 6.0 \]  

(4.4)

gives a resetting threshold of \( \text{MomMag} \approx -1.75 \). This result suggests that if during the specified background time window there are events with magnitude higher than -1.75 the re-entry clock should be reset independently of the event count rate. This level of magnitude may appear low in absolute value; however, it has to be compared with the population used to estimate the background levels. Only 6.5% of this population has a moment magnitude higher than the selected threshold (Figure 4.6).

![Figure 4.6](image)

Figure 4.6: Comparison of the determined resetting threshold with the rest of the population. (a) Magnitude-time plot; (b) Cumulative moment magnitude distribution.

### 4.4.2 Microseismic magnitude event protocol

These thresholds are related to the large magnitude events recorded at the mine. The course of action adopted with this type of threshold is to invoke the re-entry restriction. From the evaluation
of current re-entry practices it was determined that a re-entry protocol is invoked mainly for large magnitude events, measured in the Nuttli magnitude scale (Section 3.1).

In this section, an attempt is made to use the microseismic scale to set these thresholds by identifying key microseismic source parameters for seismic events that have been labelled as reported. Reportable seismicity may include: Ontario’s reportable occurrences (rockburst damage greater than 5 tonnes), events that cause visible damage to the excavations, events that are felt on surface or underground and are also typically recorded by the on-site strong ground motion seismic system.

The following two examples suggest that a plot of the moment magnitude ($MomMag$) versus the S-wave to P-wave energy ratio ($E_s/E_p$) can be used to identify thresholds for establishing a microseismic magnitude event protocol (Figure 4.7). This combination of microseismic parameters: $E_s/E_p − MomMag$, was selected to represent the mechanism and strength of the events respectively.

![Figure 4.7: Thresholds identification for defining a microseismic magnitude event protocol based on the S-wave to P-wave energy ratio and the moment magnitude of reported events at two different mine sites.](image-url)
Figure 4.7 can be approximately divided into three regions: seismic events, blasts, and reported incidents. Although, there is a natural mix of the three categories a rough boundary can be traced and used to define the thresholds that would invoke the re-entry restriction. When a particular event has a combination of $E_s/E_p$ and $MomMag$ above the defined boundary a microseismic magnitude re-entry protocol should be invoked.

4.5 Real-time application of the preliminary guidelines

Figure 4.8 illustrates an example of the implementation of the proposed guidelines using a two hour background time window and the $MomMag$ resetting threshold after a blast in a zone of a mine.

Figure 4.8: Application example of the proposed preliminary guidelines after a blast in a zone of a mine.
In Figure 4.8 the events per hour satisfy the 2 hour background time window, 9 hours after the blast. However, during this time window an event with magnitude higher than the established resetting threshold occurred, resetting the re-entry clock and extending the re-entry time until hour 14. In this case, the implementation of the re-entry clock reset incorporates an additional level of confidence in the policy, extending the restriction period over the secondary activity that occurred in the zone between hours 10-12 after the blast.

Figure 4.9 shows how the Seismic Visualizer software (SeisVis), available at all the Ontario mines that have purchased Engineering Seismology Group (ESG) products, can be configured to display in real-time resetting thresholds in the event frequency histogram without the need of additional software.

![Figure 4.9: Option in the seismic visualizer software (SeisVis) that can be used to display the resetting threshold in the event frequency histogram for re-entry protocol development. (a) Version 14; (b) Version 11.](image)
4.6 Summary and discussion

The main assumptions of the proposed method for calculating normal levels of seismicity are the use of superposition (diurnal charts), unit time intervals to display the data and the quantity and quality of the available data. Superimposing several days of seismic data has the effect of producing representative average conditions of the zone. However, it also has the effect of hiding new trends in the data. The selected time periods for the diurnal charts should be related to mining conditions to ensure a good understanding of the seismic history of the zone. As an alternative, a moving time window can be selected and used to evaluate and trace changes in the upper normal levels of seismicity. The estimated upper normal levels of seismicity may vary as mine development progresses and the nature of seismicity changes. The analysis and experience with the method shows that for building diurnal charts, at least 250 seismic events should be used. The selected length of the unit time bin for displaying the diurnal chart will affect the results. Lower units will display more details of the sample CAD changing the upper bound of normal levels. It is necessary to be consistent in the analysis with the unit time bin used to display the data. Despite these assumptions, the proposed method provides a practical procedure that enables an evaluation of reasonable normal levels of seismicity for re-entry protocol development.

For the identification of microseismic magnitude thresholds for invoking a re-entry protocol, ongoing maintenance of the database is required. All events should be flagged as to the known mechanism (blast, seismic event, reported, unknown, noise), where reportable seismicity may include: Ontario’s reportable occurrences (rockburst damage greater than 5 tonnes), events that cause visible damage to the excavations, events that are felt on surface or underground and are also typically recorded by the on-site strong ground motion seismic system. The plot of moment magnitude versus the S-wave to P-wave energy ratio suggests that events, blasts and reported occurrences have different mechanisms and that the microseismic source parameters are able to
reflect these mechanisms. A rough boundary was used to identify microseismic thresholds for
invoking a re-entry restriction. This approach will be refined in Chapter 7 by means of a
probabilistic approach. In addition, it is necessary to evaluate which microseismic source
parameters are the most suitable for setting the thresholds.
Chapter 5
Sources of data and methods

The preliminary guidelines proposed in Chapter 4 fulfilled an immediate industry requirement for re-entry protocol development, but it is clear that these guidelines do not provide an understanding of the patterns of seismicity following large seismic events and blasts. This topic is addressed further in this part of the research.

In order to develop representative guidelines it is necessary to include in the analysis a wide range of mining, geology, and seismic settings that can be found in mining operations in Ontario. Therefore, several mining seismicity catalogues had to be included in this part of the thesis.

5.1 Sources of data

Seismic data from the following eight mining operations in Ontario, Canada, are used throughout Part II of this thesis, including:

1. Mine A (name and description of the mine withheld).
2. Copper Cliff North.
3. Craig.
4. Creighton.
5. Kidd Creek.
6. Macassa.
7. McCreedy East.
8. Williams.
In addition, in order to compare mining-induced aftershocks with natural earthquake sequences, two earthquake catalogues are considered from:

9. California and Italy.

In the following, an overview of the monitoring systems, mining methods and geology of each mining site is provided.

5.1.1 Copper Cliff North

North Mine is a nickel-copper deposit, located within the Copper Cliff Offset in Sudbury, Ontario. The study area consists of the 100/900 orebodies, between the 2700 and 4200 levels, from September 2004 to September 2005. The microseismic monitoring system covering this zone is composed of 24 uniaxial and 3 triaxial accelerometers (Figure 5.1).

During the study period a total of 25,046 microseismic events were located within this volume with moment and uniaxial magnitudes between -2.2 to 1.3 and -4.5 to 0.8, with the highest frequency of events occurring at the magnitude bin of -1.6 and -3.1 respectively (Figure 5.2a, b).
Two main blast shifts are observed in Figure 5.2c, with a higher portion of seismic events associated to the day shift.

![Graphs showing frequency-moment and uniaxial magnitude distributions and diurnal chart.](image)

Figure 5.2: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a and b respectively) and diurnal chart (frame c) for the recorded microseismic events and blasts from September 2004 to September 2005 at the 100/900OB Copper Cliff North Mine.

The following description of the geology and mining method was obtained from Malek and Leslie (2006). A variety of mining methods have been used over the years at North Mine but the
primary methods used presently are slot/slash and vertical retreat mining in the main mining block of the mine. The ore deposits of the North Mine environment predominantly occur within the intrusive quartz diorite dyke. The quartz diorite dyke striking north-south, is approximately 50 meters wide and generally dips vertically or steeply west. The nickel-copper sulphides are generally located in the central portion of the quartz diorite dyke, and form elongated steeply plunging pipe-like orebodies. The country rocks west of the dyke are predominantly granite and granodiorite rocks of the Creighton Pluton. The country rock east of the dyke are metavolcanic and metasedimentary rocks of the Elsie Mountain Formation. Sudbury Breccia predominantly occurs east of the dyke and is widespread at breaks in the dyke. Narrow quartz diabase and olivine diabase dykes crosscut the quartz diorite dyke. The North Mine is associated with major faults: number 2 mine fault, number 1 Cross fault, 900 Orebody Cross fault and Creighton fault at the south end of the North Mine. The faulting was the last geological event affecting the North Mine environment: it displaced the quartz diorite dyke and its associated ore deposits, the quartz diabase dykes, and the olivine diabase dykes.

5.1.2 Craig
Craig Mine is a nickel-copper deposit located on the Northwest rim of the Sudbury Basin. Five isolated aftershock sequences occurring from 2007 to 2008 were provided by mine personnel for zones 10 and 11. These zones of Craig Mine have a fault region obliquely traversing the orebody generating high seismic activity and occasional large magnitude events. These zones are currently being extracted via blasthole open stoping. The seismic array has 55 uniaxial sensors and covers a volume of approximately 1000 m x 1300 m x 900 m.

The frequency-moment and uniaxial magnitudes for the year 2007 goes from -2.8 to 1.2 and -4.5 to 1.4, with the highest frequency of events occurring at the magnitude bin of -1.6 and -2.7
respectively (Figure 5.3a, b). Two main shift blasts are identified in Figure 5.3c, with a higher portion of seismic events associated to the night shift.

Figure 5.3: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a and b respectively) and diurnal chart (frame c) for the recorded microseismic events and blasts during 2007 at Craig Mine.
5.1.3 Creighton

Creighton Mine is located within the Creighton embayment on the outer rim of the South Range of the Sudbury Igneous Complex. At depth, the Creighton main ore zone strikes roughly east-west and dips steeply to the north. Creighton Mine comprises 15 orebodies of which the majority of the higher grade mineralization has been depleted. Mineralization is contained within a north-west plunging embayment of norite in the footwall. Creighton Mine is characterized by several late-stage faults, locally termed shears. The structures consist of foliated material. Depending on structure, shear zones vary in thickness from a few centimetres to tens of meters (Malek et al., 2009). The study region corresponds to the Creighton Deep, between the 6600 and 7800 levels (between 1,828 and 2,377 meters below surface). The underground microseismic monitoring system covering this area consists of 24 uniaxial and seven triaxial accelerometers (Figure 5.4).

Figure 5.4: View of the Creighton Mine showing the study volume.
During the study period (January to December 2008) a total of 26,176 microseismic events were located within this volume with moment and uniaxial magnitudes between -2.8 to 0.9 and -4.1 to 1.9, with the highest frequency of events occurring at the magnitude bin of -1.5 and -2.4 respectively (Figure 5.5a, b). Two main shift blasts are identified in Figure 5.5c, with a higher portion of seismic events associated to the night shift.

Figure 5.5: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a and b respectively) and diurnal chart (frame c) for the recorded microseismic events and blasts during 2008 at Creighton Mine.
5.1.4 Kidd Creek

The Kidd Mine orebody is a large scale copper-zinc deposit, located near Timmins, Ontario. The study region corresponds to the complete Mine D, covering a volume of approximately 300 m x 200 m x 500 m, between the 6800 and 8800 levels (between 2,073 and 2,682 meters below surface) from August 2004 to December 2007. In this zone, the underground microseismic monitoring system consists of 15 uniaxial and 4 triaxial accelerometers (Figure 5.6).

![Figure 5.6: View of the seismicity and monitoring system at the Kidd Creek D Mine.](image)

During the study period a total of 23,452 microseismic events were located within this volume with moment and uniaxial magnitudes between -2.5 to 0.9 and -3.0 to 2.0, with the highest frequency of events occurring at the bin of -1.9 and -1.6 respectively (Figure 5.7a, b). Two main shift blasts are identified in Figure 5.7c, with a higher portion of seismic events associated to the night shift.
Figure 5.7: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a and b respectively) and diurnal chart (frame c) for the recorded microseismic events and blasts from August 2004 to December 2007 at the Kidd Creek D Mine.

Blasthole mining with delayed paste backfill is used to extract the ore underground. The following description of the geology of the mine was adapted from Board et al. (2001). Kidd Mine’s main mineralized lenses are called the main (copper stringer and massive sulphides) and south lenses. These orebodies are located near the top of a locally thickened rhyolite, which is
underlayed to the east by ultramafics and overlayed to the west by mafic flows and associated intrusions. The startigraphy trends north-south, is overturned, and dips steeply to the east. All the lithologies in the Kidd Mine, including the ore, have been subjected to complex folding and faulting. The major faults that potentially affect mine-wide stability can be defined in two systems: the Gouge Fault and the south-dipping echelon faults. The south-dipping faults have been associated with the larger seismic events at Kidd, while the Gouge Fault and its splays primarily impact hangingwall dilution.

5.1.5 Macassa
The zone under study is composed of a set of primary, longitudinal, continuous retreat sublevel longhole stopes with delayed paste backfill at a depth below surface of 1500 m (Figure 5.8).

Figure 5.8: 3D view of the 5036LH zone at the Macassa Mine.

The underground microseismic monitoring system surrounding the zone consists of a dense array of 66 uniaxial accelerometers. The volume of interest consists of approximately 150 m x 100 m x 100 m. During the study period (December 2004 to May 2007) a total of 10,351 microseismic
events were located within this volume with moment and uniaxial magnitudes between -1.7 to 0.7 and -4.3 to 0.1, with the highest frequency of events occurring at the magnitude bin of -1.2 and -3.3 respectively (Figure 5.9a, b). Two main shift blasts are identified in Figure 5.9c, with a higher portion of seismic events associated to the night shift.

Figure 5.9: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a and b respectively) and diurnal chart (frame c) for the recorded microseismic events and blasts from December 2004 to May 2007 for the 5036LH zone at the Macassa Mine.
The geotechnical domains include very brittle, massive, high-strength tuff rocks in the hangingwall and massive-to-moderately jointed high strength orebody in a series of sub-parallel faults. These faults strike northeast and dip steeply (~70°) to the south. The footwall consists of a medium-strength basic syenite, which is generally quite blocky. The average stope dimensions are approximately 3 m in thickness and 30 m width. The dominant geological structure of the zone is a single major fault intersecting the stopes, which can be parallel to the hangingwall or footwall contact or be the actual contact itself. Inclusions are infrequent and discontinuous.

5.1.6 McCreedy East

At present, the Main Ore Body (MOB1, MOB2, MOB3), West (WOB2, WOB3) and 153 Ore Bodies are being mined. The study area embeds the 4400 level of the 153 Orebody (Figure 5.10) from January to December 2003.

Figure 5.10: View of the McCreedy East Mine 153 OB showing the location of the 4400 level.
The microseismic monitoring system covering this zone is composed of 24 uniaxial and 4 triaxial accelerometers. During the study period a total of 7,794 microseismic events were located within this volume with moment and uniaxial magnitudes between -2.8 to 0.8 and -4.2 to 1.3, with the highest frequency of events occurring at the magnitude bin of -2.1 and -2.9 respectively (Figure 5.11a, b).

Figure 5.11: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a and b respectively) and diurnal chart (frame c) for the recorded microseismic events and blasts from January to December 2003 for the 4400 level/153 OB at the McCready East Mine.
Two main shift blasts are identified in Figure 5.11c, with a higher portion of seismic events associated to the night shift. The geology consists of a complex system of copper-nickel veins that are contained within an east-west striking, southerly dipping breccia zone. Cut-and-fill production headings with hydraulic fill are used to extract the ore.

5.1.7 Williams

Williams Mine is one of the three operating mines in the Hemlo gold deposit. The deposit is located 40km east of Marathon, along the north shore of Lake Superior. The zone under study corresponds to the B Zone from levels 9190 to 9215-46 stope, from May 2007 to March 2008. The mining method was longhole open stoping with cemented paste filling. The area mined was a diminishing pillar (Figure 5.12).

![Figure 5.12: View of the B Zone 9190 to 9215-46 stope at Williams Mine.](image)

The microseismic monitoring system covering the zone consists of 12 uniaxial sensors. The geology consists of feldspathic ore host, with banded sediments to the north. The orebody dips at 70 degrees and strikes east-west. During the study period a total of 13,927 microseismic events
were located within this volume with moment and uniaxial magnitudes between -2.5 to -0.1 and
-4.0 to -0.3, with the highest frequency of events occurring at the magnitude bin of -1.8 and -3.3
respectively (Figure 5.13a, b). Two main shift blasts are identified in Figure 5.13c, with a higher
portion of seismic events associated to the day shift.

Figure 5.13: Non-cumulative frequency-moment and uniaxial magnitude distributions (frame a
and b respectively) and diurnal chart (frame c) for the recorded microseismic events
and blasts from May 2007 to March 2008 for the B Zone 9190 to 9215-46 stope at
Williams Mine.
5.1.8 Crustal sequences

To compare the characteristics of mining-induced and natural aftershocks a total of 78 previously published natural aftershock sequences were analyzed. These sequences occurred in Southern California (61) from 1933 to 2004, in Italy (15) from 1976 to 2004, the March 9, 1994, deep Tonga earthquake, and the September 28, 2004, Mw6.0 Parkfield, California, earthquake. For Southern California and Italy, the aftershock sequences available from Lolli and Gasperini (2006) (ftp://ibogfs.df.unibo.it/lolli/aft2005) were used. These aftershock sequences were obtained from regional catalogues using a clustering algorithm based on a space window varying as a function of the main shock magnitude and a fixed one year time window. Only sequences including at least 100 events with magnitude not lower than the main shock magnitude minus 3.5 were considered by Lolli and Gasperini (2006). Given the simplicity of the clustering algorithm, these sequences may contain significant secondary aftershock sequences and/or background seismicity.

For the Tonga sequence the data published by Wiens and McGuire (2000) was used. Finally, for the case of the Parkfield earthquake the Northern California Seismic Network (NCSN) catalogue within the spatial box suggested by the Northern California Earthquake Data Center (NCEDC) site (http://www.ncedc.org/2004parkfield.html) with one year time window and no depth restriction was used. Figure 5.14 presents the frequency-magnitude distribution and diurnal charts of all the aftershock sequences isolated by Lolli and Gasperini (2006). The variable cut-off magnitude used by these authors may affect the range and shape of the frequency-magnitude distribution, however, the scale of magnitude ranges from 1.5 to 7.3 and 1.6 to 6.5 for California and Italy respectively (Figure 5.14a). In addition, no characteristic spikes of seismic events are observed from the diurnal chart (Figure 5.14b).
Figure 5.14: Non-cumulative frequency-magnitude distribution (frame a) and diurnal chart (frame b) for all the crustal aftershock sequences isolated by Lolli and Gasperini (2006).

5.1.9 Summary

Table 5.1 presents a summary of some of the magnitude characteristics of mining and crustal seismicity at each site, such as: minimum and maximum magnitude bin recorded ($M_{\min}$, $M_{\max}$), and the magnitude bin with the highest frequency of events in a non-cumulative frequency-magnitude distribution ($M_c$). Assuming the difference $M_{\max} - M_c$ as an indicator of the
effective range of magnitudes recorded at each site, it is absolutely clear that the uniaxial magnitude scale provides a wider range of measured magnitudes than the moment magnitude scale, which is comparable to the one recorded for crustal sequences.

Table 5.1. Moment and uniaxial magnitude characteristics of the selected mining and crustal seismicity catalogues. $M_{\text{min}}$, $M_{\text{max}}$: minimum and maximum recorded magnitude bins respectively. $M_c$: magnitude bin with the highest frequency of events in a non-cumulative frequency-magnitude distribution. Crustal seismicity is included in the uniaxial magnitude scale.

<table>
<thead>
<tr>
<th>Site</th>
<th>$M_{\text{min}}$</th>
<th>$M_{\text{max}}$</th>
<th>$M_c$</th>
<th>$M_{\text{max}} - M_c$</th>
<th>$M_{\text{min}}$</th>
<th>$M_{\text{max}}$</th>
<th>$M_c$</th>
<th>$M_{\text{max}} - M_c$</th>
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<tbody>
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<td>A</td>
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<td>-0.1</td>
<td>-2.0</td>
<td>1.9</td>
<td>-4.1</td>
<td>-0.2</td>
<td>-3.5</td>
<td>3.3</td>
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<td>2.7</td>
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<td>-2.4</td>
<td>4.3</td>
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<td>1.3</td>
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</tr>
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<td>4.2</td>
</tr>
</tbody>
</table>

5.2 Identification of mining-induced seismic sequences

There is no commonly accepted standard, rigorous definition of aftershock events, but several have been used for specific purposes (Isacks et al., 1967; Shlien and Toksoz, 1974; Kagan and Knopoff, 1976; Reasenberg, 1985; Ogata, 1988; Davis and Frohlich, 1991a and 1991b; Kagan and Jackson, 2006). Molchan and Dmitrieva (1992) point out that aftershock identification depends on the research goals. The principal research objective of this thesis is to provide guidelines that reflect current re-entry practices, where the seismic source parameter data, either as calculated automatically by the system or as calculated after manually processing of
waveforms, is used for re-entry protocol decision-making in a specified volume or zone without any formal consideration of the spatial distribution of seismicity. In some mines, the analysis was done for the complete mine, in others the analysis was done level by level, while others tried visually to isolate cluster of events. Therefore, the analysis is relaxed to the temporal identification of aftershock sequences without spatial dependence. If space-time clustering schemes for aftershock detection were used, the results would depend to some extent on the assumptions of the clustering method, restricting the applicability of the guidelines.

For a specified zone or target volume, aftershock sequences are identified in time by the following approaches. The first involves following the mining sequence provided by the mine personnel. When this identification is not available, the start of a sequence is identified by a modified version of the ratios method (Frohlich and Davis, 1985). For a particular event, or principal event, the ratios method evaluates the relative origin times of events in a test sequence of events occurring after and before the principal event. The method determines if the observed time differences would be likely by chance alone. When the observed time differences are very improbable, the method presumes that the test sequence is not generated by a random, or Poisson process, i.e., that some of the events following the principal event are aftershocks. To apply the ratios method (Figure 5.15), consider the time $T_{N_A}$ of the $N_A^{th}$ earthquake following the principal event, the time $T_{N_B}$ of the $N_B^{th}$ event preceding the principal event, and then form the ratio:

$$ r_{N_B-N_A} = \frac{T_{N_A}}{T_{N_B}} \quad (5.1) $$

If the ratio is sufficiently small, it implies that some or all of the $N_A$ events occurring after the principal event are aftershocks.
Figure 5.15: Definition of the ratios method for the temporal identification of aftershocks (redrawn from Frohlich and Davis, 1985).

A following event is defined to be an aftershock if the ratio is smaller than a critical ratio \( r^c \) generated by a random process with a certain probability \( P \). Table 5.2 presents the critical values for the ratio \( r_{N_A-N_A} \) for various combinations of \( N_A \) and \( N_B \) for a one per cent probability. Note that with this definition 1% of all the observed values of \( r_{N_A-N_A} \) will be smaller than the tabulated value even if a Poisson process generated the catalogue. The main objective of the ratios method is to determine a critical time interval that defines which following events are aftershocks with confidence \( 1 - P \) for a given principal event.
Table 5.2: Critical ratios $r^c$ calculated for the temporal identification of aftershocks (Frohlich and Davis, 1985).

<table>
<thead>
<tr>
<th>$N_a$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0101</td>
<td>0.1111</td>
<td>0.2746</td>
<td>0.4625</td>
<td>0.6614</td>
</tr>
<tr>
<td>2</td>
<td>0.0050</td>
<td>0.0626</td>
<td>0.1640</td>
<td>0.2855</td>
<td>0.4171</td>
</tr>
<tr>
<td>3</td>
<td>0.0034</td>
<td>0.0438</td>
<td>0.1181</td>
<td>0.2093</td>
<td>0.3095</td>
</tr>
<tr>
<td>4</td>
<td>0.0025</td>
<td>0.0338</td>
<td>0.0926</td>
<td>0.1659</td>
<td>0.2472</td>
</tr>
<tr>
<td>5</td>
<td>0.0020</td>
<td>0.0275</td>
<td>0.0762</td>
<td>0.1376</td>
<td>0.2062</td>
</tr>
</tbody>
</table>

For the analysis of mining-induced aftershock sequences this method is applied in a different manner. Instead of testing which events are aftershocks for a given principal event, the ratio $r_{N_B-N_a}$ is continuously evaluated for the given seismicity catalogue. A group of consecutive events satisfy the condition $r_{N_B-N_a} < r^c$ is interpreted as the beginning of a seismic sequence. For the analysis $N_a = 1, N_B = 5$ with a probability of one per cent were used, giving a critical value of $r^c_{5-1} = 0.002$ (Table 5.2). The start of the sequence is defined if the ratio $r_{5-1}$ is less than the critical value for a group of at least three consecutive events. The first event of the group is identified as the principal event of the sequence regardless of its magnitude. The events between principal events are considered as aftershocks. Note that the objective of this scheme is to identify in time, within a specified volume, the beginning of a seismic sequence based on spikes of event rate (Figure 5.16) and does not attempt to classify events as foreshocks or aftershocks. Also no restriction is applied to the magnitude of the aftershocks to be less than the first event in the sequence. In the example presented Figure 5.16 most of the sequences, but not all, are initiated by blasts.
Figure 5.16: Example of the seismic sequences identification provided by the ratios method for a time period of two months at the Macassa site (events labelled as blasts are identified by red stars).
5.3 Location error and magnitude filtering

Each identified aftershock sequence was filtered by limiting the location error-number of triggered sensors (see Section 4.2), and by a cut-off magnitude $M_c$, selected at the magnitude bin with the highest frequency of events in a non-cumulative frequency-magnitude distribution (Wiemer and Wyss, 2000; Woessner and Wiemer, 2005). The effect of these two filters was to remove poorly located seismic events from the analysis and to provide some degree of uniformity to the data respectively. The moment magnitude scale was selected as this type of magnitude provides an estimate of the energy release of the source which is model independent and can be used to compare magnitudes from different mining environments (Trifu, 2008). For the California and Italy catalogues, in general, the same cut-off magnitude used by Lolli and Gasperini (2006) was used. However, whenever possible, the cut-off magnitude was selected using the same procedure described above. Only mining sequences with at least 10 events and two hours of duration after filtering were retained for the analysis. Table 5.3 presents the resulting number of sequences at each site.

Table 5.3. Number of sequences retained for the analysis at each site.

<table>
<thead>
<tr>
<th>Site</th>
<th>Number of sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>51</td>
</tr>
<tr>
<td>Craig</td>
<td>5</td>
</tr>
<tr>
<td>Creighton</td>
<td>44</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>70</td>
</tr>
<tr>
<td>Macassa</td>
<td>49</td>
</tr>
<tr>
<td>McCready East</td>
<td>24</td>
</tr>
<tr>
<td>Williams</td>
<td>23</td>
</tr>
<tr>
<td>California</td>
<td>61</td>
</tr>
<tr>
<td>Italy</td>
<td>15</td>
</tr>
<tr>
<td>Parkfield + Tonga</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>372</strong></td>
</tr>
</tbody>
</table>
Table 5.4 presents the cut-off magnitude bin determined from the average of individual aftershock sequences $\bar{M}_c$ and from the total event population $M_c$ recorded during the study period at each site.

Table 5.4: Cut-off magnitude bin determined from the average of individual aftershock sequences $\bar{M}_c$ and from the total event population $M_c$ recorded during the study period at each site. Crustal sequences are included in the uniaxial magnitude scale.

<table>
<thead>
<tr>
<th>Site</th>
<th>Moment magnitude $\bar{M}_c$</th>
<th>$M_c$</th>
<th>Uniaxial magnitude $\bar{M}_c$</th>
<th>$M_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-2.01±0.05</td>
<td>-2.0</td>
<td>-3.51±0.17</td>
<td>-3.50</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>-1.59±0.12</td>
<td>-1.6</td>
<td>-3.05±0.26</td>
<td>-3.10</td>
</tr>
<tr>
<td>Craig</td>
<td>-1.66±0.11</td>
<td>-1.6</td>
<td>-2.74±0.17</td>
<td>-2.70</td>
</tr>
<tr>
<td>Creighton</td>
<td>-1.42±0.12</td>
<td>-1.5</td>
<td>-2.29±0.22</td>
<td>-2.40</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>-1.88±0.11</td>
<td>-1.9</td>
<td>-1.73±0.38</td>
<td>-1.60</td>
</tr>
<tr>
<td>Macassa</td>
<td>-1.18±0.08</td>
<td>-1.2</td>
<td>-3.25±0.26</td>
<td>-3.30</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>-2.06±0.10</td>
<td>-2.1</td>
<td>-2.97±0.29</td>
<td>-2.90</td>
</tr>
<tr>
<td>Williams</td>
<td>-1.83±0.12</td>
<td>-1.8</td>
<td>-3.17±0.20</td>
<td>-3.30</td>
</tr>
<tr>
<td>California</td>
<td>-</td>
<td>-</td>
<td>2.39±0.62</td>
<td>1.60</td>
</tr>
<tr>
<td>Italy</td>
<td>-</td>
<td>-</td>
<td>2.28±0.36</td>
<td>2.30</td>
</tr>
</tbody>
</table>

In general, all mining sites presented an approximately uniform response in terms of $\bar{M}_c$ (small variability) reflecting a consistent sensitivity of the microseismic monitoring systems for locating events above this magnitude during the study period. Smaller standard deviations are obtained for the moment magnitude compared to the uniaxial magnitude. This is an effect of the lower curvature close to $M_c$ of the uniaxial magnitude compared to the moment magnitude (see for example Figure 5.9). Another outcome of the above analysis is that in most of the cases $\bar{M}_c$ is approximately within 0.05 of $M_c$. The practical implication of this result is that the cut-off magnitude can be simply selected at the magnitude bin with the highest frequency using the
frequency-magnitude distribution of the total population of events for the period under study, if and only if the distribution does not present a bimodal shape. In contrast, the crustal sequences (California and Italy) presented large fluctuations on $\bar{M}_c$ determined from individual sequences. The main reason is the variable cut-off magnitude dependent on the main shock magnitude (Section 5.1.8) used by Lolli and Gasperini (2006).
Chapter 6
Scaling laws and aftershock statistics

Prediction of the time and location of seismic events, whether they are crustal-scale earthquakes or mining-induced seismic events, remains an unsolved problem. It has been recognized for some time now that seismicity in the earth’s crust is a complex dynamical problem exhibiting many elements of chaotic behaviour, implying that prediction will not be possible (Bak and Tang, 1989; Grasso, 1998). Despite our limited understanding of the mechanisms involved, certain characteristics of aftershocks have been recognized that enable useful decisions on tasks such as re-entry to be made.

Aftershock characteristics of crustal-scale seismic events have been studied extensively with the objective of understanding the basic physics of the process and for application to hazard assessment. Studies related to spatial and temporal migration of earthquake aftershocks are therefore highly relevant to this research. Aftershock patterns and statistics have been attributed to a number of causes. Stress change induced by the main event clearly has some influence, either in terms of static stress change (Stein, 1999; Casarotti et al., 2001) or dynamic stress change (Kilb et al., 2000; Felzer et al., 2006). Other mechanisms have been proposed including stress corrosion (Das and Scholz, 1981), and damage mechanics (Shcherbakov and Turcotte, 2004a). Purely statistical approaches of assessing spatial and temporal patterns of seismicity include branching models (Saichev et al., 2005), self-organized criticality (Huang et al., 1998), and random walk models (Helmstetter and Sornette, 2002a).

Most results of these studies are expressed in terms of scaling laws, of which the modified Omori’s law (Omori, 1894; Utsu, 1961) is one and the well-known Gutenberg-Richter (Gutenberg and Richter, 1944) frequency-magnitude relation is another. In general, these laws are
statistical fits of empirical functions to observable patterns of aftershock sequences, and regardless of whether they explain the underlying physics of the problem the patterns that they fit are valuable in terms of applications such as the development of re-entry protocols. A third scaling law, Båth’s law (Båth, 1965), is not well known in mining seismology, but is of considerable interest for re-entry. Båth’s law states that the difference in magnitude between the main event and the largest aftershock is a constant (which for crustal earthquakes is approximately 1.2). This result, if verified for mining-induced seismic events, would be very useful in selecting values for parameters such as the magnitude of re-entry resetting events. By combining the modified Omori’s law and Gutenberg-Richter scaling relations in a stochastic parametric model, the possibility of the occurrence of either significant aftershocks or an even stronger main shock during intervals following the main shock has been evaluated by Reasenberg and Jones (1989, 1990, and 1994). Ogata (1988, 1989, 1992, 1999, and 2001) introduced the Epidemic Type Aftershock Sequence (ETAS) model which is a point process in which every event can produce its offspring of events and can be considered as an extension of a single modified Omori’s law. These topics are central to developing a more reliable understanding and for developing a sound basis for selecting parameters to monitor for re-entry protocol.

The primary objective of this chapter is to study in detail the statistical properties of mining-induced aftershock sequences. To accomplish this, the scaling relations (Modified Omori’s law, Gutenberg-Richter and Båth’s law) and two stochastic models (Reasenberg and Jones, and ETAS) are applied to several mining-induced aftershock sequences from different mine sites in Ontario, Canada.
6.1 Modified Omori’s law (MOL)

Following the principal event, the rate of aftershock occurrence \( n(t) \) is considered to decrease with time as a power-law function (Omori, 1894; Utsu, 1961) known as modified Omori’s law (MOL):

\[
n(t) = \frac{K}{(c + t)^p}
\]  

(6.1)

where \( t \) is the time measured from the principal event, \( c \) is an offset time constant, \( p \) is a parameter related to the speed of decay, and \( K \) is a productivity parameter related to the number of events occurring in a time period \([T_a, T_b]\) and the other two parameters:

\[
K = \begin{cases} 
\frac{N_{T_b-T_a}}{\ln\frac{(T_b + c)}{(T_a + c)}} & \text{for } p = 1 \\
\frac{N_{T_b-T_a}(1 - p)}{\left[(T_b + c)^{1 - p} - (T_a + c)^{1 - p}\right]} & \text{for } p \neq 1 
\end{cases}
\]  

(6.2)

The power-law behaviour of Eq. (6.1) is indicative of a physical process generally slower than those typically observed in nature, which are usually described by negative exponentials (Utsu et al., 1995). Although a complete theory of aftershock generation is not available, a number of worldwide crustal studies have shown that the MOL satisfactorily describes the time decay of aftershock rate for most sequences. The parameter \( p \) differs from sequence to sequence, with a typical range of 0.6–1.6 and a median value of 1.1 (Utsu et al., 1995). This variability has been previously related to the fault system heterogeneity, surrounding lithosphere, local stress fields and crustal temperature (e.g., Mogi, 1967; Kisslinger and Jones, 1991; Kisslinger, 1996), but it is still not clear which are the most significant factors controlling the \( p \) value (Utsu et al., 1995).
Using a rate-state dependent fault strength model, Dieterich (1994) indicated that \( p > 1 \) may arise when the stresses on the fault surface decrease with time following the principal event. The existence of a \( c \) value and its physical meaning are still under debate (Enescu and Ito, 2002; Kagan, 2004; Shcherbakov et al., 2004; Kagan and Houston, 2005; Enescu et al., 2007; Enescu et al., 2009). The inclusion of \( c \) is essentially a mathematic artefact to avoid the singularity of the MOL equation for \( t = 0 \) (Gross and Kisslinger, 1994; Narteau et al., 2002). Kagan and Knopoff (1981) justify this divergence by considering the main shock as the superposition of an infinite number of shocks occurring in an infinitesimal time interval. Notwithstanding these uncertainties on its physical meaning, the parameter \( c \) is usually assumed different from zero by most, due to the convenience of eliminating the divergence at \( t = 0 \) (Lolli and Gasperini, 2006).

Figure 6.1 illustrates the effect of the parameters of the MOL on the event rate decay in a \( \log n(t) - \log t \) plot.
Figure 6.1: Effect of the MOL parameters $K$, $c$ and $p$ on the event rate decay.

The majority of published research on aftershock decay refers to crustal as opposed to mining-induced seismicity. McGarr and Green (1978) noted that the cumulative number of aftershocks of two mine tremors could be described by the MOL, with $p = 1$. Scott Phillips et al. (1999) found that the production of aftershocks induced by a controlled mine collapse followed the MOL with $p = 1.3$. Spottiswoode (2000) established, for eleven sequences in four different mines, that blast aftershocks were in agreement with the MOL, with $p$ values ranging from 0.54 to 1.10 with an
average of $0.82 \pm 0.20$. These results were obtained through superposition of the time series and by a least-squares fit in a $\log n(t) - \log t$ plot. Stacking aftershock sequences permits the study of $p$ values for sequences possessing too few events to allow individual analysis. However, this method may introduce artefacts into the aftershock time sequence and, therefore, influence the estimated parameters (Nyffenegger and Frohlich, 1998). Also by using this procedure the estimated parameter are affected by the size of the bin used to represent the event rate.

As noted above, there is no systematic rigorous statistical study of how well the MOL describes the decay of mining-induced aftershock sequences, and there are no well-defined recommendations of how this equation can be used in practice for re-entry protocol development. Thus, the primary goal of this section is to address statistically how well the MOL can describe the decay rate of mining seismicity using a standardized statistical method to fit the equation. Next, practical guidelines for how the MOL can be used for the development of re-entry protocols are proposed. These guidelines are applicable for the range of mining conditions that can be found in some mines in Ontario, Canada.

6.1.1 Maximum likelihood estimate of the MOL parameters

If the occurrence times $t_i$ ($i = 1, \ldots, N$) of the individual events are available for a time interval $[T_A, T_B]$, then the parameters $K$, $p$, $c$, and their uncertainties $\Delta K$, $\Delta p$, $\Delta c$ can be estimated by the maximum likelihood method (Ogata, 1983). Assuming that all the events in an aftershock sequence are conditionally independent and distributed according to a non-stationary Poisson process with intensity function given by Eq. (6.1), the log-likelihood function is given by:

$$\ln L(K, p, c, T_A, T_B) = N \ln K - p \sum_{i=1}^{N} \ln(t_i + c) - KA(p, c, T_A, T_B)$$ (6.3)
where

\[ A(p, c, T_A, T_B) = \begin{cases} \ln(T_B + c) - \ln(T_A + c) & \text{for } p = 1 \\ \left[ (T_B + c)^{1-p} - (T_A + c)^{1-p} \right] (1 - p) & \text{for } p \neq 1 \end{cases} \]  

(6.4)

The maximum likelihood estimates (MLE) of the parameters \( K, p, \) and \( c \) are those that maximize Eq. (6.3).

Ogata (1983) also provides a framework for determining an estimate of the standard error associated with the MLE of each parameter through the use of the Fisher information matrix \( J(K, c, p, T_A, T_B) \) given by:

\[
J(K, p, c, T_A, T_B) = \int_{T_A}^{T_B} \frac{1}{n(t)} \begin{bmatrix}
\frac{\partial n(t)}{\partial K} & \frac{\partial n(t)}{\partial K} & \frac{\partial n(t)}{\partial c} \\
\frac{\partial n(t)}{\partial p} & \frac{\partial n(t)}{\partial p} & \frac{\partial n(t)}{\partial c} \\
- & - & - \\
- & - & - \\
\end{bmatrix} dt
\]

(6.5)

The inverse matrix \( J(K, c, p, T_A, T_B)^{-1} \) is the variance-covariance matrix with respect to the parameters \( K, p, \) and \( c \). Their associated standard errors \( \Delta K, \Delta p, \) and \( \Delta c \) are the square roots of the corresponding diagonal terms respectively.

### 6.1.2 Statistical criteria

One of the applications of the MLE is to the statistical model selection on the basis of the Akaike Information Criterion (AIC), given by (Akaike, 1974):

\[
AIC = -2 \max \{ \ln L(K, p, c, T_A, T_B) \} + 2n_p
\]

(6.6)
where \( n_p \) is the number of adjustable parameters of the model and \( \ln L(K, p, c, T_A, T_B) \) is the log-likelihood function. The AIC can be used to compare different rate models having different number of parameters and has a minimum value for the best fitting model. This criterion is used later to compare different rate models.

A more useful interpretation of the AIC is accomplished by using the Akaike weights \( (w_i) \). If \( AIC_i \) is the AIC value for model \( i \), and \( \min_{m=1}^{M} \{ AIC_m \} \) is the minimum AIC value of a set of \( M \) candidate models, then the Akaike weights are expressed by:

\[
W_i = \frac{\exp(-\Delta_i/2)}{\sum_{m=1}^{M} \exp(-\Delta_m/2)}
\]

where \( \Delta_i \) is a measure of each model relative to the best model of a set of candidate models:

\[
\Delta_i = AIC_i - \min_{m=1}^{M} \{ AIC_m \}
\]

The \( w_i \)'s are interpreted directly as the probability that the model is the best among the set of candidate models.

It should be noted that the AIC only measures the relative fit of competing models and cannot be used as an absolute measure of suitability of fit (Nyffenegger and Frohlich, 1998). Therefore, to evaluate whether the MOL fitted parameters adequately describe the time sequence, the non-parametric Anderson-Darling statistic \( W^2 \) is used (Anderson and Darling, 1954; Lewis, 1961), given by:

\[
W^2 = -N - \sum_{i=1}^{N} \frac{(2i-1)}{N} \left[ \ln(u_i) + \ln(1 - u_{N+1-i}) \right]
\]
where \( u_i \) is the MOL cumulative density function for event \( i \), given by:

\[
\begin{align*}
    u_i &= \begin{cases} 
        \frac{\ln(t_i + c) - \ln(T_A + c)}{\ln(T_B + c) - \ln(T_A + c)} & \text{for } p = 1 \\
        \frac{(t_i + c)^{1-p} - (T_A + c)^{1-p}}{(T_B + c)^{1-p} - (T_A + c)^{1-p}} & \text{for } p \neq 1
    \end{cases}
\end{align*}
\]

(6.10)

\( W^2 = 0 \) indicates a perfect fit to the data. Sequences with \( W^2 \leq 2.0 \) have been previously considered to follow the MOL distribution, and those with \( W^2 \leq 1.0 \) are considered to fit well (Nyffenegger, 1998; Nyffenegger and Frohlich, 1998, 2000).

### 6.1.3 Effect of the location error-filtering on the MOL

To illustrate the effect of the location error-filtering applied to the aftershock sequences (Section 5.3), a plot of the Anderson-Darling statistic \( W^2 \) as a function of the location error for a blast related aftershock sequence is presented in Figure 6.2. In this particular case, a reduction of only 3% of the population with the highest location error produces a dramatic reduction in \( W^2 \), from 18.6 to 1.8, increasing the fit of the MOL to the data.
To quantitatively evaluate the effect of the error-filtering, the MOL parameters were estimated with and without filtering for 50 mining-induced aftershock sequences. In 75% of the cases, the error-filtering did not change the MOL parameters by more than 10%. The remaining 25% of the cases presented an improvement in the fit of the MOL to the data (lower $W^2$). It can be concluded that this filter does not significantly affect the MOL parameters and, in general, improves the fit to the data.

### 6.1.4 Temporal power-law decay limits of the MOL

The following steps are used to estimate the MOL parameters for an aftershock sequence:

1. Specify a target time interval $[T_A, T_B]$.

2. Maximize the log-likelihood function given by Eq. (6.3).
3. Test if the proposed decay relation adequately describes the aftershock sequence for the given time interval with the Anderson-Darling statistic $W^2$ (Eq. (6.9)).

There are, however, numerous time intervals that satisfy $W^2 \leq 1$ for a given aftershock sequence. Despite the importance assigned to the $p$ value in the literature, there seems to be no consensus on the time interval $[T_A, T_B]$ that should be used to fit the equation. If $T_A$ is selected as the time of the principal event (i.e., $T_A = 0$) then $c$ must be different from zero to eliminate the singularity of the MOL equation at $t = 0$. Ogata (1983) suggested that the interplay between $p$ and $c$ in the MOL, combined with the difficulty of detecting early aftershocks following the principal event, may bias $p$ toward higher values. He outlined a procedure for choosing the best start time $T_A$ through the use of the $AIC$. This method relies on the fact that at some $T_A$ the sequence can be best modeled by two MOL’s compared to a single MOL. On the other hand, $T_B$ is influenced by the clustering algorithm used to associate members to an aftershock sequence. For example, the clustering technique of Reasenberg (1985) considers as aftershocks events occurring up to hundred of days after the principal event, whereas the single-link clustering (SLC) procedure (Davis and Frohlich, 1991a and 1991b) emphasizes events occurring within several days of a main event (Nyffenegger, 1998). In both methods, spatial and temporal properties of aftershocks will depend on the input parameters used for the algorithm.

Several studies have subjectively limited the time interval $[T_A, T_B]$ to find a reliable and representative estimate of the MOL parameters for single aftershock sequences (e.g., Nyffenegger, 1998; Bohnenstiehl et al., 2002; Peng et al., 2006) and for stacked sequences (e.g., Nyffenegger and Frohlich, 1998; Ouillon and Sornette, 2005). Nyffenegger (1998) fixed $c$ equal to zero and then eliminated 10% of the events from each end of the sequences in an attempt to
reduce the bias on the estimated $p$ values. Then he used the Anderson-Darling $W^2$ statistic to test if the decay relation was able to describe the sequence.

From a preliminary analysis of aftershock sequences, it was determined that the MLE of the MOL parameters were sensitive to the time interval used to fit the equation, especially to the behaviour at the beginning and end of the sequence. Two examples are presented in Figure 6.3 for a blast-related and a crustal sequence at the Kidd Creek Mine and California respectively. For representing the seismicity rate the data was divided into logarithmic intervals (Utsu, 1962).

![Figure 6.3: Event rate and maximum likelihood estimate of the MOL parameters for the complete time interval $[t_0, t_N]$ for a blast-related aftershock sequence at the Kidd Creek Mine (frame a) and a crustal aftershock sequence Cal37 (frame b).](image)

In Figure 6.3, the reported parameters, their uncertainties and goodness of fit, have been estimated for the complete duration of the sequence, i.e., $[T_A, T_B] = [t_0, t_N]$, where $t_0$ and $t_N$ are the origin times of the principal and last events in the sequence respectively. In the case of the
mining-induced seismic sequences these were detected by the ratios method, while for the crustal sequences it was provided by Lolli and Gasperini (2006).

Inspection of Figure 6.3a provides the following:

1. There is high uncertainty in the MLE of the MOL parameters.

2. The \( p \) value seems high compared to the usual range reported in the crustal literature (0.6-1.6).

3. Based on the Anderson-Darling statistic, the sequence can be considered to follow a MOL distribution \( (W^2 \leq 2.0) \).

4. The fitted MOL does not adequately describe the aftershocks at very short occurrence times \((<-0.1\ \text{hours})\).

A few remarks can also be made about Figure 6.3b:

1. The inclusion of late events, probably not related to the principal event, can produce low \( p \) and \( c \) values.

2. The Anderson-Darling statistic indicates that this sequence does not follow the MOL distribution \( (W^2 > 2.0) \).

This parameter sensitivity motivated an in-depth research of the effects of the time interval selection on the MLE of the MOL parameters. It was concluded that, to estimate consistent decay parameters, it is necessary to exclude some aftershocks from the start and end of the sequence and to consider only the time interval that satisfies power-law behaviour \( [T_s, T_e] \). Ideally, the maximum time interval that satisfies power-law behaviour should be estimated, in order to identify possible transition points inside the time sequences. In addition, if there are any
misidentified events by the clustering algorithm at the beginning or end of the sequence, the selection of power-law parameters will compensate for these errors. Therefore, a uniform algorithm based on formal properties of aftershock time sequences and numerical simulations was proposed. The method is described and justified in detail in Appendix B. The proposed method was applied to the 372 isolated aftershock sequences (Table 5.3) and power-law MOL parameters were estimated. Figure 6.4 shows the same example presented in Figure 6.3, but with the MOL parameters estimated for the time interval of power-law decay.

Figure 6.4: Event rate and maximum likelihood estimate of the MOL parameters for the determined time interval of power-law decay $[T_s, T_e]$ for a blast-related aftershock sequence at the Kidd Creek Mine (frame a) and a crustal aftershock sequence Cal37 (frame b).

Regarding Figure 6.4a, events occurring at very short ($< 0.1$ hours) and long ($> 5$ hours) delay times were automatically excluded. Also, the uncertainties of the MOL parameters are considerably reduced and the $p$ value seems to be in the usual reported range for crustal sequences. In Figure 6.4b the late aftershocks were excluded and the $p$ value seems much more...
reasonable. In both cases the data segment used to fit the equation conforms to the MOL much better than the entire data set (lower $W^2$).

6.1.5 Justification and interpretation of the time interval of power-law decay

By using the AIC (Section 6.1.2), the hypothesis that the start and end times of power-law decay determined by the proposed method represents justifiable conditions for the onset and cessation of power-law behaviour is statistically verified. The problem is reduced to compare the AIC between competing models.

6.1.5.1 End time of power-law decay $T_E$

To demonstrate that there is a justifiable reason to split the sequence at $T_E$ independently of the selected start time of power-law decay $T_s$, an MOL adjusted for the complete time interval $[t_0, t_N]$ is compared to a fixed MOL for the time interval $[t_0, T_E]$ followed by a different rate model from the following six combinations:

\[
\begin{align*}
    n(t) &= \begin{cases} 
        \frac{K_1}{(c_1 + t)^{p_1}} & \text{for } t_0 < t \leq T_E \\
        \frac{K_1}{(c_1 + t)^{p_1}} + \frac{K_2}{(c_2 + t - T_E)^{p_2}} & \\
        \frac{K_1}{(c_1 + t)^{p_1}} + K_2 e^{-\alpha(t - T_E)} & \quad (6.11) \\
        \frac{K_1}{(c_1 + t)^{p_1}} + K_2 & \text{for } T_E < t \leq t_N \\
        \frac{K_2}{(c_2 + t)^{p_2}} & \\
        K_2 e^{-\alpha t} & \\
        K_2 & 
    \end{cases} 
\end{align*}
\]
Equations (6.1) and (6.11)-(6.16) give a total of three different functions and seven combinations of rate models to compare. Equations (6.11)-(6.13) are associated with two superimposed mechanisms after $T_E$, while Eqs. (6.14)-(6.16) are representative of a transition between two independent regimes. The three types of functions are well-known in the literature. Equations (6.11) and (6.15) have been suggested by Ogata (1983) to confirm significant secondary aftershocks and a possible transition from a power-law to an exponential decay respectively. A permanently present exponential and power-law (Eq. (6.12)) was suggested by Narteau et al. (2002). Finally, as an alternative rate model, a transition to background levels (Eqs. (6.13) and (6.16)) is considered. This comparison also makes it possible to identify whether or not a systematic new physical process commences after the detected power-law decay. The decay patterns of each rate model compared to a single MOL are presented in Figure 6.5 and Figure 6.6 respectively.

To ensure stable solutions, only aftershock sequences with at least 10 events in the time interval $[T_E, t_N]$ were considered for the analysis. As a result, 42 of the mining-induced and 54 of the crustal earthquake sequences were retained for the analysis giving a total of 96 cases. The low number of retained cases in the case of mining-induced seismic sequences is a direct consequence of the mining process. Before the power-law ends, a new sequence is generally induced (by blasting).
Figure 6.5: Rate decay patterns of the different superimposed models used to test the efficiency of the end time of power-law decay $T_E$ determined by the proposed method.
Figure 6.6: Rate decay patterns of the different independent models used to test the efficiency of the end time of power-law decay $T_E$ determined by the proposed method.
It was found that the alternative rate formulas (Eqs. (6.11)-(6.16)) gave better results than a single MOL (Eq. (6.1)), in 91 of the 96 cases analyzed with an average probability of 96%. The five cases where Eq. (6.1) gave better results than Eqs. (6.11)-(6.16) were further analyzed. These sequences have a low number of events (less than 20) in the time interval \([T_E, t_N]\), suggesting that the available data in this time interval was insufficient to adequately determine that a new mechanism was established after \(T_E\). Also, the probabilities that Eq. (6.1) is the best model among the set of candidate models are low (less than 50%). The implication is that a single MOL does not hold after \(T_E\), confirming that a new process becomes established, which has been correctly identified by the proposed method.

Another implication of the identified mechanism transition occurring at \(T_E\) is related to the physically unacceptable fact of infinite productivity of aftershocks of Eq. (6.1) as \(t \to \infty\) when \(p \leq 1\) (Utsu et al., 1995; Kisslinger, 1996). This can be demonstrated by using the expression for the cumulative number of events from zero until time \(t\):

\[
N(t) = \begin{cases} 
K\{(\ln(t + c) - \ln c) \} & \text{for } p = 1 \\
K \frac{(t + c)^{1-p} - c^{1-p}}{1-p} & \text{for } p \neq 1 
\end{cases}
\]  

(6.17)

In Eq. (6.17) when \(p > 1\), \(N(t)\) tends to a constant level \(N_\infty = K/\{(p-1)c^{p-1}\}\) as \(t \to \infty\). While, when \(p \leq 1\), \(N(t) \to \infty\) as \(t \to \infty\). The detected mechanism transition occurring at \(T_E\) suggests that the unbounded number of events of the MOL for \(p \leq 1\) is not relevant in practice, since at some finite time \(T_E\) other rate models will be dominating the sequence.
extracting main sequences from the mining seismicity catalogues (no secondary superimposed aftershock sequences).

To identify which one of the rate models is the most effective for justifying the change point of the sequence at $T_E$, each of the alternative rate model (Eqs. (6.11)-(6.16)) is compared with Eq. (6.1) on an individual basis for each aftershock sequence. It is found that Eqs. (6.14), (6.15), (6.11), (6.12), (6.13), (6.16) fit better than Eq. (6.1) in 90%, 82%, 77%, 73%, 72% and 63% of the cases respectively. The implication is that, if $T_E$ is selected based exclusively on statistical model selection, i.e., by minimizing $AIC$, the double non-superimposed MOL (Eq. (6.14)) is the most effective model for identifying the change point of the sequence at $T_E$. The second most effective corresponds to Eq. (6.15), which identifies a transition from a power-law to an exponential decay, and has been interpreted in the literature as a “correlation time”, after which aftershocks cease to occur and healing dominates (Narteau et al., 2002).

Figure 6.8 presents examples of the determined best fit rate models for justifying the change point of the sequence at $T_E$ for four crustal sequences corresponding to: Cal27, Ita09, Cal26 and Cal37 of Lolli and Gasperini (2006). A secondary sequence triggered at the time of the largest magnitude aftershock was found to be superimposed on the initial sequence induced by the principal event in Figure 6.8a. In the case of Figure 6.8b the best fit transition point was initiated at a change in slope in the decay, in which the generation of aftershocks slows down. Figure 6.8c shows a change in the decay nature from a power-law to an exponential decay and Figure 6.8d exhibits a change in the aftershock generation where the power-law ceases and a linear rate starts. In all cases, visual inspection suggests that there is a justified reason for splitting the sequences at the change points selected by the proposed method.
Figure 6.7 presents the distribution of the best fit rate model for justifying the end time of power-law decay $T_E$. The results are presented separately for mining and crustal sequences given the differences in the methods used to isolate the seismic sequences.

![Figure 6.7: Distribution of the best fit rate model for justifying the end time of power-law decay $T_E$ determined by the proposed method. (a) Crustal sequences; (b) Mining-induced seismic sequences. The number of sequences for which each model is the best is included.](image)

Figure 6.7a indicates that in the case of crustal sequences the most repeated best fit rate model corresponds to Eq. (6.11), i.e., a model with a superimposed secondary aftershock sequence. This is a direct consequence of the clustering method used by Lolli and Gasperini (2006) (Section 5.1.8). For the case of mining-induced seismic sequences (Figure 6.7b) the most frequent best fit rate model corresponds to Eq. (6.16) followed by Eq. (6.13), i.e., the models that include a transition to background levels. In addition, there is a low number of cases (only six) for which Eq. (6.11) is found to be the best model, indicating that the ratios method performs well in
Figure 6.8: Examples of the determined best fit rate models for justifying the division of the sequence at $T_E$ for four crustal sequences. (a) Cal27; (b) Ita04; (c) Cal26; (d) Cal37.
6.1.5.2 Start time of power-law decay $T_S$

For the start time of power-law decay, the $AIC$ obtained for the time interval $[t_0, T_E]$ was compared with the composite of $AIC$ obtained for a MOL in the time interval $[T_S, T_E]$ preceded by the following alternative rate models:

$$n(t) = \begin{cases} 
\frac{K_2}{(c_2 + t)^{p_2}} & \text{for } t_0 < t \leq T_S \\
K_2e^{-at} & \text{for } T_S < t \leq T_E \\
\frac{K_2 + K_3t}{K_3} & \text{for } T_s \leq t \leq T_E
\end{cases}$$

(Eq. 6.18)

$\frac{K_1}{(c_1 + t)^{p_1}}$

(Eq. 6.19)

$\frac{K_2}{(c_2 + t)^{p_2}}$

(Eq. 6.20)

$\frac{K_1}{(c_1 + t)^{p_1}}$

(Eq. 6.21)

Equation (6.18) is equivalent to the procedure outlined by Ogata (1983). His method relies on the fact that at some $T_S$ the sequence can be best modeled by two MOL’s instead of a single MOL.

Three alternative rate models are considered for the analysis. The exponential and constant rate models were already mentioned in the previous section, while a linear decay function (Eq. (6.20)) was proposed by Narteau et al. (2002). Only sequences with at least 10 events in the time interval $[t_o, T_S]$ were considered for the comparison. A total of 37 mining-induced and 41 crustal sequences (30 from Southern California, 9 from Italy, the Tonga and the Parkfield earthquake) were retained for the analysis giving a total of 78 cases. It was found that the combined rate formulas (Eqs. (6.18)-(6.21)) gave better results than a single MOL in only 65% of the cases. This result suggests that, based on the $AIC$ criterion, there is no strong motivation for dividing the sequence at $T_S$. Figure 6.9 presents the distribution of the best fir rate model for justifying the change point of the sequence at $T_S$. Figure 6.9 indicates that there is no consistent best-fit rate
model in $[t_0, T_S]$, suggesting that there is no physical property of the aftershock sequences in this time interval. It is left open as a possibility that there is another equation that can fit the data better than those considered here.

![Figure 6.9: Distribution of the best fit rate model for justifying the change point at $T_S$. (a) Crustal sequences; (b) Mining-induced seismic sequences. The number of sequences for which each model is the best is included.](image)

However, in the proposed method $T_S$ has a fundamental statistical and physical meaning as illustrated in Figure 6.10. For the $T_E$ detected by the method, $T_S$ always corresponds to the first inter-event median time that satisfies a power-law process. This is shown in Figure 6.10c where the parameter $c$ and $W^2$ have been evaluated for different inter-event median time at the beginning of the sequence. It is possible to verify that the detected $T_S$ corresponds to the time when the solution constraints of $c = 0$ and $W^2 \leq 1$. Beyond this point, $c$ can gradually increase and/or $W^2$ can continue decreasing or increasing.
Figure 6.10: Statistical interpretation of the start time of power-law decay. (a) Event rate with the determined time interval of power-law decay; (b) Magnitude-time plot; (c) $c$ and $W^2$ as a function of a possible start time of power-law decay $T_S$; (d) Akaike weights for a MOL with $c = 0$ and $c \neq 0$ as a function of a possible start time of power-law decay $T_S$. 
An alternative interpretation of the start time of power-law decay determined by the proposed method is through the use of statistical model selection (Section 6.1.2). This corresponds to determine the $T_s$ for which Eq. (6.1) with $c = 0$ is a better fit than with $c \neq 0$. This is illustrated in Figure 6.10d where the Akaike weights for a MOL with $c = 0$ and $c \neq 0$ are compared for different possible start times of power-law decay. It is found that at $T_s$, selected by the proposed method, the Akaike weight of the MOL with $c = 0$ is higher than the Akaike weight of the MOL with $c \neq 0$ as shown in Figure 6.10d.

### 6.1.6 MOL parameter statistics

In the following the statistical results and interpretation of the start and end times of power-law decay $T_s$, $T_e$, and the power-law MOL parameters determined using the proposed methodology are presented. The objectives are to identify and understand common characteristics of the parameters of the MOL for representing the decay of aftershock sequences and produce guidelines on the use of the MOL for re-entry protocol development. Also, the correlation between the parameters of the MOL is investigated.

#### 6.1.6.1 Start time of power-law decay $T_s$

Figure 6.11 presents the population distributions for the start times of power-law decay for all the seismicity catalogues analyzed. Most (98%) of the sequences analyzed started the power-law decay in less than one time unit after the principal event (long dashed line in Figure 6.11), indicating that, both mining and crustal aftershock sequences display non-power-law behaviour only for short times ($< 1$ hour, 1 day). Median values for mining and crustal sequences range from 0.001 to 0.2 hours and 0.04 to 0.4 days respectively.
Figure 6.11: Cumulative ascending distributions of the start times of power-law decay. Hours are used for mining seismicity, while days for crustal sequences.

The three mining-induced seismic sequences with $T_S > 1$ hour presented an initial decay rate followed by a nearly rate constant period during the first hour before the onset of power-law decay. Two of these sequences are illustrated in Figure 6.12. Seven crustal sequences presented $T_S > 1$ day, corresponding to the following sequences of Lolli and Gasperini (2006): 02, 06, 21, 42 and 43 for Southern California, and 12 and 13 for Italy. These sequences presented a similar behaviour to the ones presented in Figure 6.12.
Figure 6.12: Examples of the event rate behaviour and the determined power-law MOL for two mining-induced aftershock sequences that presented $T_S > 1$ hour. Relevant estimated parameters are listed.

Several factors may be affecting the decay rate before the onset of the power-law decay, such as: overlapping of seismic records that make it difficult to identify and locate the many events (Kagan, 2004), a complex process which the MOL equation is not able to adequately describe, or the sequence may actually begin gradually and build to a higher rate before the onset of smooth decay. This is likely to be due to undetected events missing from the early part of the sequence (Gross and Kisslinger, 1994). In the literature, these complexities have been previously accounted for by using the $c$ parameter in the MOL. Considering that these features can affect the estimated MOL parameters (see Figure 6.3 and Figure 6.12) the proposed power-law interval detection method excludes events occurring at very short and long times and considers only the portion that statistically satisfies power-law behaviour (see Figure 6.4 and Figure 6.12).

Fitting the MOL to the time interval $[t_0, t_E]$ with $c$ as a parameter and obtaining $T_S$ with the proposed method are not equivalent. The first analysis finds the time shift that gives the best fit to
the available data from $t = 0$, while the second, statistically drops all data prior to the start of power-law decay. However, both parameters are positively correlated as shown in Figure 6.13.

![Figure 6.13: Correlation between the start time of power-law decay $T_S$ and the MOL parameter $c$ estimated for the time interval $[t_0, T_E]$ for crustal sequences (frame a), and mining-induced seismic sequences (frame b).](image)

This positive correlation further suggests that the parameter $c$ cannot be treated as a physical parameter as it is only the best fit time that represents the complexities during the beginning of the sequence. A practical implication of the above correlation is that for a given $T_E$, the start time of power-law decay can be estimated using the value $c$, in order to reduce the number of iterations used by the proposed method to find the optimal $T_S$.

A more useful interpretation of $T_S$ is derived by using the concept of Early Aftershock Deficiency ($EAD$), introduced by Peng et al. (2006). The $EAD$ can be quantified by extrapolating the determined power-law MOL curve toward the principal event and calculating the number of
events between the first measured inter-event time $t_i = (t_1 - t_0)/2$ and the start time of power-law decay $T_s$:

$$N_{t_i-T_s}^{calc} = \frac{K}{1-p} \left( T_s^{1-p} - t_i^{1-p} \right)$$

(6.22)

The following ratio is defined for evaluating the degree of $EAD$:

$$EAD = \frac{N_{t_i-T_s}^{calc}}{N_{t_i-T_s}^{meas}}$$

(6.23)

where $N_{t_i-T_s}^{meas}$ is the measured number of events between $t_i$ and $T_s$. Figure 6.14 show that the Copper Cliff North and the Macassa data present lower $EAD$ ratios compared to the Kidd Creek catalogue.

Figure 6.14. Cumulative ascending distribution of the early aftershock deficiency ratio for three mining seismicity catalogues.
Figure 6.14 also suggests that the Copper Cliff North and the Macassa microseismic monitoring systems are more sensitive in detecting events above their corresponding cut-off magnitude closer to the principal event than the Kidd Creek array.

6.1.6.2 End time of power-law decay $T_E$

Figure 6.15 presents the population distributions for the end time of power-law decay for all the seismicity catalogues analyzed.

![Cumulative ascending distributions of the end time of power-law decay $T_E$. Hours are used for mining seismicity, while days for crustal sequences.](image)

For mining-induced aftershock sequences the power-law may continue for over 100 hours after the principal event, however, median values between 7 to 22 hours are found, which is driven by the mining process. Of the mining seismic catalogues the Macassa site presented the largest end
times of power-law decay as a consequence of the large average time between production blasts. The power-law decay of crustal aftershock sequences can be extended for five and three decades with median values of 86.6 and 44.5 days for the Southern California and Italy catalogues, respectively. A total of 13 crustal sequences satisfying $T_E = t_N$ were found, indicating that a fixed time-window of one year, as used by Lolli and Gasperini (2006), may not be always long enough to detect the deviation of the sequence from a power-law regime. From the 372 sequences analyzed, only 11 cases presented a maximum curvature time longer than the end time of power-law decay (i.e., $T_{MC} > T_E$). These sequences correspond to: Cal56 and Ita09 of Lolli and Gasperini (2006), one at Copper Cliff North, one at Creighton, four at Kidd Creek, one at Macassa and two at Williams Mine. In all cases a secondary superimposed MOL starting at $T_E$ fitted the data better than a single MOL, confirming that the condition $T_{MC} \leq T_E$ needed to be relaxed. Therefore, the power-law solution of these cases may not be the best, as the sequences cannot be completely modeled by a single MOL. When developing aftershock statistics these cases are treated with caution.

6.1.6.3 $p$ values

For each sequence, the MOL parameters, with their uncertainties and the corresponding goodness of fit $W^2$ were estimated for the following three time intervals:

1. The complete duration of the sequence $[t_0, t_N]$.

2. The determined time interval of power-law decay $[T_S, T_E]$.

3. From the principal event until the end time of the power-law decay $[t_0, T_E]$. 
These three solutions are compared to draw more conclusions about the MLE parameter sensitivities, and also to emphasize the importance of selecting an adequate time interval for estimating the MOL parameters, focusing on the $p$ value. Figure 6.16 presents the cumulative distributions of the $p$ values estimated for the three time intervals at each site.

Figure 6.16. Cumulative ascending distributions of the determined $p$ values for all the sites analyzed. (a) $[t_0, t_N]$; (b) $[T_S, T_E]$; (c) $[t_0, T_E]$. 

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It is observed in Figure 6.16 that there are some cases with large $p$ values ($p > 2$) for the time intervals $[t_0, t_N]$ and $[t_0, T_E]$. In the case of the power-law time interval $[T_S, T_E]$ in 98% of the cases, the $p$ value range from 0.4 to 1.6, with average values from 0.74 to 1.04 (Figure 6.16b). Each distribution seems to represent specific conditions of the local environment, with higher average power-law decay $p$ values for the Mine A, Macassa and Kidd Creek sites compared to the Sudbury sites.

By comparing the $p$ values determined for $[t_0, t_N]$ and $[t_0, T_E]$ the effect of selecting an adequate end time can be evaluated. In 73% of the cases, the $p$ value determined for $[t_0, t_N]$ is lower than that obtained for $[t_0, T_E]$. In addition, except for the Craig mine, lower average $p$ values are obtained for the time interval $[t_0, t_N]$ compared to $[t_0, T_E]$ (see Figure 6.16a and c). This underestimation of the $p$ values is probably due to the inclusion of background seismicity or secondary aftershock sequences in the $[t_0, t_N]$ time interval (for an example see Figure 6.3b).

Next, the effect of selecting an adequate start time is evaluated. Gasperini and Lolli (2006) evidenced a positive correlation between $c$ and $p$ in sequences from Italy and New Zealand. They interpreted that a high $c$ value has the effect of slowing the predicted sequence decay for $t < c$, which is counterbalanced by an increase of $p$. Similar correlations between $p$ and $c$ are presented in Table 6.1 for the $[t_0, T_E]$ time interval.
Table 6.1: Regression results between the MOL parameters $p_{[t_0, t_E]}$ and $c_{[t_0, t_E]}$. Best fit function for each site is shown in bold.

<table>
<thead>
<tr>
<th>Site</th>
<th>$p_{[t_0, t_E]} = a_1 + a_2 c_{[t_0, t_E]}$</th>
<th>$p_{[t_0, t_E]} = a_1 + a_2 \ln c_{[t_0, t_E]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>A</td>
<td>0.893</td>
<td>1.197</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>0.755</td>
<td>0.243</td>
</tr>
<tr>
<td>Craig</td>
<td>0.874</td>
<td>1.221</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.767</td>
<td>0.587</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>0.888</td>
<td>0.839</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.921</td>
<td>1.805</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>0.850</td>
<td>1.093</td>
</tr>
<tr>
<td>Williams</td>
<td>0.669</td>
<td>2.475</td>
</tr>
<tr>
<td>California</td>
<td>0.939</td>
<td>0.483</td>
</tr>
<tr>
<td>Italy</td>
<td>0.896</td>
<td>0.353</td>
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</table>

Table 6.1 confirms that $p$ and $c$ are positively correlated. In some cases (6/10) a linear correlation fit the data in a better manner compared to a semi-logarithmic relationship. These correlations highlight the interplay between the two parameters, i.e., the reason for the high $p$ values found for the $[t_0, t_N]$ and $[t_0, t_E]$ time intervals is related to the inclusion of the aftermath of the principal event. This trade-off between $p$ and $c$ might indicate a general inadequacy of the MOL describing the real properties of simple aftershock sequences (Gasperini and Lolli, 2006). By combining the time intervals $[t_0, T_E]$ and $[T_S, T_E]$, the following empirical linear relationships between $p_{[t_0, t_E]}$, $c_{[t_0, t_E]}$ and $p_{[t_S, t_E]}$ is established:

$$p_{[t_0, t_E]} = a_1 c_{[t_0, t_E]} + a_2 p_{[t_S, t_E]}$$  \hspace{1cm} (6.24)

Table 6.2 presents the results of the regression of Eq. (6.24).
Table 6.2: Regression results between the MOL parameters $p_{[t_0, t_N]}$, $c_{[t_0, t_N]}$, and $p_{[t_0, T_E]}$.

<table>
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<th>$a_2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
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<tr>
<td>Copper Cliff North</td>
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<td>0.830</td>
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<td>0.873</td>
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</tr>
<tr>
<td>Williams</td>
<td>0.842</td>
<td>1.001</td>
<td>0.968</td>
</tr>
<tr>
<td>California</td>
<td>0.225</td>
<td>1.032</td>
<td>0.818</td>
</tr>
<tr>
<td>Italy</td>
<td>0.212</td>
<td>0.977</td>
<td>0.893</td>
</tr>
</tbody>
</table>

Table 6.2 clarifies that the effect of $c_{[t_0, t_N]}$ on $p_{[t_0, t_N]}$ is a bias toward higher values relative to the power-law MOL solution.

Next, the estimated power-law $p$ values are compared with previously published values for crustal sequences. Figure 6.17 presents the relative frequency histogram of $p$ values determined for the three time intervals $([t_0, t_N], [T_S, T_E], [t_0, T_E])$ and by other authors.
Figure 6.17. Relative frequency histogram of $p$ values determined for different time intervals and by other authors for Southern California (frame a), and Italy (frame b). The vertical bold and dash lines represent the average and one standard deviation from the average. The vertical scale of the panels is indicated.

The average values obtained by the power-law solution for Southern California (0.966±0.252) is lower than the one reported by Kisslinger and Jones (1991) (1.122±0.262), but higher that the one determined by Nyffenegger (1998) (0.886±0.298), however, it is within one standard deviation from both cases. Note that the standard deviation for the power-law solution is the lowest in all cases for both catalogues. Some of the reasons for the discrepancies are, among others:

1. The scheme used to identify members of the aftershock sequences: Kisslinger and Jones (1991) used the clustering algorithm proposed by Reasenberg (1985), while Nyffenegger (1998) used the single-link clustering (SLC) procedure (Davis and Frohlich, 1991a and 1991b). In these methods, both spatial and temporal properties depend on the input parameters used for the algorithm. On a temporal basis, the Reasenberg method considers aftershock events occurring up to hundreds of days after the principal event, whereas the SLC algorithm emphasizes events occurring within several days of a main shock (Nyffenegger, 1998). In the present analysis the sequences isolated by Lolli and Gasperini
(2006) were used for the analysis. In this case the space window depends on the magnitude of the main event, but the duration of the sequence was estimated by the proposed power-law determination method.

2. The method used to determine the \( p \) value: Kisslinger and Jones (1991) allowed \( c \) to vary, which in general, leads to higher \( p \) values as it was demonstrated previously. Nyffenegger (1998) determined the \( p \) value by imposing \( c = 0 \) and removing 10\% of the events from the beginning and end of the sequence. He considered that the sequences conform to a MOL decay if \( W^2 \leq 2 \). The proposed power-law method takes this one step further, for each sequence a representative power-law MOL time interval that is completely free from any interplay between the parameters \( p \) and \( c \) has been systematically identified.

3. Others, such as: number of events used to fit the equation, cut-off magnitude and spatial distribution.

Finally, for the Tonga and Parkfield earthquakes, it was found \( P_{[T_s,T_e]} = 1.027 \pm 0.009 \); \( [T_s,T_e] = [0.0128, 41.8584] \) days, and \( P_{[T_s,T_e]} = 0.85 \pm 0.02 \); \( [T_s,T_e] = [0.0528, 288.6103] \) days respectively. These results are consistent with the estimates published by Nyffenegger and Frohlich (2000): \( p = 1.04 \); \( [0.017, 41.0] \) days and Peng et al. (2006): \( p = 0.86 \pm 0.03 \); \( [0.0417, 335.6481] \) days respectively.

6.1.6.4 \( K \) values

By definition, for a sequence conforming to power-law decay, the parameter \( K \) corresponds to the rate of activity one unit time after the principal event. However, rate estimates can present high fluctuations during the first time unit and any evaluation will depend on the widths of the
bins used to represent the data. This can bias estimates of $K$. In the crustal literature the average productivity $K$ of aftershocks is usually considered to be related to the magnitude of the principal event $M_{PE}$ (e.g., Reasenberg, 1985; Singh and Suarez, 1988; Reasenberg and Jones, 1989; Helmstetter and Sornette, 2002b; Felzer et al., 2004; Helmstetter et al., 2005; Yang and Ben-Zion, 2009) by:

$$K = k_o 10^{\lambda(M_{PE} - M_c)}$$

(6.25)

where $k_o$ and $\lambda$ are parameters and $M_c$ is the cut-off magnitude. Reasenberg (1985) used for his aftershock identification clustering method $k_o = 10^{-2/3}$ and $\lambda = 2/3$ based on personal communication with Stein (1982). However, no statistical support of this relationship was provided. Reasenberg and Jones (1989) set $A = 10^4$, $\lambda = b$, where $A$ is correlated with the $K$ value of the MOL estimated from the time series and $b$ is the coefficient in the Gutenberg-Richter relationship determined by fitting the distribution of aftershock magnitudes independently of time. Once calibrated the average or median values of $A$ and $b$ are considered to be common to all aftershock sequences and used to estimate the probabilities for aftershocks and large magnitude events during time intervals following a main shock. The applicability and performance of this model for mining seismicity is explored and discussed in detail in Section 6.3.1. Figure 6.18 presents the estimated power-law MOL $K$ values as a function of the difference between the magnitude of the principal event and the cut-off for the aftershock sequences analyzed at four different mining sites.
Figure 6.18. Estimated power-law MOL $K$ values as a function of the difference between the moment magnitude of the principal event and the cut-off magnitude for the aftershock sequences analyzed at four different mining sites.

No significant correlation between $K$ and the moment magnitude of the principal event is found (Figure 6.18). There are several reasons why this correlation may not work on a global basis for mining seismicity derived from microseismic measurements. First, the mechanism of the event may play a fundamental role in the aftershock productivity, for example a pillar burst may be different from a regional fault slip event. Second, the microseismic monitoring systems of the mines analyzed have a moment magnitude limitation between -2.0 to 1.0 (see Table 5.1). For
larger magnitude events all sensors of the array will clip (Trifu and Suorineni, 2009). This can be
solved by using the magnitude derived by one or more triaxial geophones located in the periphery
of the mine. Finally, at the crustal scale Eq. (6.25) has been obtained based on the
superimposition of several aftershock sequences to smooth out fluctuations associated with
individual sequences and highlight the common features inside a region.

Based on these observations, an alternative criterion to estimate a real-time $K$ value based on the
number of events was investigated. For a power-law MOL, the calculated number of events at
time $t$ since the first inter-event median time $t_1$ is given by:

$$
N(t) = \int_{n_1}^{t} n(t) dt = \begin{cases}
K \ln \left( \frac{t}{t_1} \right) & \text{for } p = 1 \\
\frac{K}{1-p} \left\{ t^{1-p} - t_1^{1-p} \right\} & \text{for } p \neq 1
\end{cases}
$$

(6.26)

By assuming $t = 1$ in Eq. (6.26) the following productivity ratio can be defined:

$$
\kappa_{\text{calc}} = \frac{K}{N_1} = \begin{cases}
\frac{-1}{\ln t_1} & \text{for } p = 1 \\
\frac{1-p}{1-t_1^{1-p}} & \text{for } p \neq 1
\end{cases}
$$

(6.27)

It can be seen from Eq. (6.27) that $\kappa$ depends on the time occurrence of the first event $t_1$ and the
$p$ value of the sequence. For the eight different mining seismicity and the two crustal catalogues
under study, it was found that $t_1$ can vary from 0.00001 to 0.1 hours, and from 0.0001 to 0.1 days
respectively. Figure 6.19 presents the productivity ratio $\kappa$ given by Eq. (6.27) as a function of $p$, for two different values of $t_1$. 

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Figure 6.19. Productivity ratio $\kappa$ as a function of $p$ for two different values of $T_s$. (a) Crustal sequences; (b) Mining-induced seismic sequences.

Figure 6.19 indicates that for the same $T_s$, a higher $p$ value implies a lower $\kappa$, and that in theory the only way for $\kappa \geq 1$ is when $p = 0$ in the first time unit of data. Measured productivity ratios
\( \kappa_{\text{meas}} = K/N_1 \) are included in Figure 6.19 using the estimated power-law MOL parameters \( K \) and \( p \), and the measured number of events during the first time unit after the principal event \( N_1 \).

Most of the cases are within the boundaries of Eq. (6.27). The cases outside of the boundaries are a direct consequence of the natural variations in the inter-event times, limited by the sensitivity of the monitoring systems to detect events above the cut-off magnitude during the first time unit after the principal event. A large number of mining-induced aftershock sequences (10%) with high \( \kappa \) values correspond to the Kidd Creek site, which also presented high EAD ratios (see Figure 6.14). Figure 6.19 also implies that in theory \( \kappa \leq 0.8 \) for the range of \( p \) and \( t_1 \) values found for the crustal and mining-induced aftershock sequences analyzed.

Next, a normal distribution with parameters \( \mu_\kappa \) and \( \sigma_\kappa \) is evaluated for representing the population of \( \kappa \) values (Table 6.3).

<table>
<thead>
<tr>
<th>Site</th>
<th>( \mu_\kappa )</th>
<th>( \sigma_\kappa )</th>
<th>( P_{\text{Chi-Square}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.310</td>
<td>0.159</td>
<td>0.000</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>0.395</td>
<td>0.324</td>
<td>0.000</td>
</tr>
<tr>
<td>Craig</td>
<td>0.287</td>
<td>0.112</td>
<td>0.655</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.392</td>
<td>0.166</td>
<td>0.055</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>0.412</td>
<td>0.191</td>
<td>0.506</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.260</td>
<td>0.108</td>
<td>0.807</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>0.289</td>
<td>0.100</td>
<td>0.841</td>
</tr>
<tr>
<td>Williams</td>
<td>0.394</td>
<td>0.128</td>
<td>0.889</td>
</tr>
<tr>
<td>California</td>
<td>0.435</td>
<td>0.365</td>
<td>0.000</td>
</tr>
<tr>
<td>Italy</td>
<td>0.630</td>
<td>0.504</td>
<td>0.165</td>
</tr>
</tbody>
</table>

\[ \begin{array}{ccc} 
\text{min} & 0.260 & 0.100 & 0.000 \\
\text{max} & 0.630 & 0.504 & 0.889 \\
\text{average} & 0.380 & 0.216 & 0.392 \\
\text{S.D.} & 0.107 & 0.136 & 0.384 \\
\end{array} \]
In most of the cases (7/10), a normal distribution is an acceptable fit (Craig, Creighton, Kidd Creek, Macassa, McCreedy East, Williams and Italy). The parameters $\mu_\kappa$ and $\sigma_\kappa$ are in a well constrained range, from 0.26 to 0.63 and 0.10 to 0.50, with averages of 0.38±0.11 and 0.22±0.14 respectively.

This analysis provides the following conclusions:

1. The parameter $K$ does not correspond to the number of events occurring during the first time unit, as considered by Liu (1984), Hudyma et al. (2003) and Heal et al. (2005), and previously commented upon by Page (1986), unless $p = 0$ in the first time unit of data.

2. The normal distribution of $\kappa$, found in most of the cases, suggests that some average correlation may exist between $K$ and $N_1$:

$$K = \kappa N_1$$

(6.28)

The significance of Eq. (6.28) depends mainly on the behaviour of the aftershock sequence and the sensitivity of the monitoring system in locating events during the first time unit. This is in general ensured if the power-law process is established during the first time unit, which based on the previous analysis (Figure 6.11) occurred in 98% of the 372 sequences analyzed. Figure 6.20 presents the correlation between the start time of power-law decay $T_s$ and the aftershock productivity ratio $\kappa$. A linear increase in $\kappa$ as a function of $T_s$ is observed, confirming that the initial non-power-law behaviour period is responsible for the high $\kappa$ values. Therefore, cases that satisfy $T_s \geq 1$ and/or $\kappa \geq 1$, are immediately excluded from the regression analysis of Eq. (6.28).
In addition, cases with secondary aftershock sequences close to the principal event, i.e., when \( T_{MC} > T_E \) is satisfied, are individually studied in the regression analysis of Eq. (6.28). These cases were excluded from the analysis when they produced a significant impact on the regression coefficient.

Figure 6.21 and Figure 6.22 presents the regression between \( K \) and \( N_1 \). Cases that satisfy \( T_S \geq 1 \) and/or \( \kappa \geq 1 \), or \( T_{MC} > T_E \) are indicated by black and grey arrows in these figures. The results are summarized in Table 6.4. A surprisingly narrow range, between 0.25 and 0.5 with an average of 0.374±0.071 is obtained for \( \kappa \).
Table 6.4: Determined productivity $\kappa$ ratios and their associated standard errors for each of the seismicity catalogues under study.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\kappa$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.267±0.024</td>
<td>0.607</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>0.451±0.021</td>
<td>0.843</td>
</tr>
<tr>
<td>Craig</td>
<td>0.391±0.019</td>
<td>0.982</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.411±0.021</td>
<td>0.670</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>0.468±0.019</td>
<td>0.753</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.305±0.010</td>
<td>0.886</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>0.291±0.015</td>
<td>0.892</td>
</tr>
<tr>
<td>Williams</td>
<td>0.371±0.027</td>
<td>0.826</td>
</tr>
<tr>
<td>California</td>
<td>0.339±0.017</td>
<td>0.792</td>
</tr>
<tr>
<td>Italy</td>
<td>0.443±0.027</td>
<td>0.841</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\kappa$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>0.267</td>
<td>0.607</td>
</tr>
<tr>
<td>max</td>
<td>0.468</td>
<td>0.982</td>
</tr>
<tr>
<td>average</td>
<td>0.374</td>
<td>0.809</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.071</td>
<td>0.110</td>
</tr>
</tbody>
</table>
Figure 6.21. Correlation between the parameter $K$ and the measured number of events occurring during the first unit time after the principal event $N_1$ for six mining seismicity catalogues.

Mine A

$K = 0.267 \pm 0.024 N_1$ hour

$R^2 = 0.607$

Copper Cliff North

$K = 0.451 \pm 0.021 N_1$ hour

$R^2 = 0.843$

Craig

$K = 0.391 \pm 0.019 N_1$ hour

$R^2 = 0.982$

Creighton

$K = 0.411 \pm 0.021 N_1$ hour

$R^2 = 0.670$

Kidd Creek

$K = 0.468 \pm 0.019 N_1$ hour

$R^2 = 0.753$

Macassa

$K = 0.305 \pm 0.010 N_1$ hour

$R^2 = 0.886$
Figure 6.22. Correlation between the parameter $K$ and the measured number of events occurring during the first unit time after the principal event $N_1$ for two mining and two crustal seismicity catalogues.
6.1.7 Guidelines for the use of the MOL for re-entry protocol development

It was statistically demonstrated that the MOL can be adequately used to describe the event decay rate of mining-induced aftershock sequences. In addition, a rigorous approach for estimating the MOL parameters and delineating an expected decay rate was provided for both mining-induced and crustal aftershock sequences. However, the task remains of developing a criterion for how the MOL can be quantitatively used to make a preliminary estimate of the time at which it may be considered appropriate to re-enter the area. The following criteria are presented and discussed:

1. Preset level in the MOL cumulative density function.

2. MOL and a previously defined rate level.

3. Time maximum curvature $T_{MC}$ of the MOL.

As will be shown, the time of maximum curvature $T_{MC}$ presents some physical attributes of a time series obeying a MOL decay that can be used as a preliminary estimate of the time at which it may be considered appropriate to re-enter the area.

6.1.7.1 Preset level in the MOL cumulative density function

For a power-law MOL, the cumulative density function at time $t$ since $T_A \neq 0$ is given by:

$$u(t) = \left\{ \begin{array}{ll}
\frac{\ln t - \ln T_A}{\ln T_B - \ln T_A} & \text{for } p = 1 \\
\frac{T_B^{1-p} - T_A^{1-p}}{T_B^{1-p} - T_A^{1-p}} & \text{for } p \neq 1
\end{array} \right. \quad (6.29)$$

This criterion consist on presetting some level $L$ in Eq. (6.29), which corresponds to a percentage of the total number of events occurring in a given time window $[T_A, T_B]$ after the principal event. The decay time is obtained directly from Eq. (6.29) by:
\[
T_d = \begin{cases} 
T_A \exp \left( L \ln \left( \frac{T_B}{T_A} \right) \right) & \text{for } p = 1 \\
\left[ L (T_B^{1-p} - T_A^{1-p}) + T_A^{1-p} \right]^{\frac{1}{1-p}} & \text{for } p \neq 1
\end{cases}
\] (6.30)

It is clear from Eq. (6.30) that the re-entry times estimated with this criterion does not depend on the activity parameter \( K \), which seems unrealistic. Figure 6.23 presents the MOL cumulative density function for different values of \( p \), assuming reasonable values for the parameters: \( L = 0.9 \), and \([T_A, T_B] = [0.01, 12]\) hours. It can be seen that the range of re-entry times, obtained with this criterion and given parameters, can range from 0.38 to 10.1 hours, which is an impractically wide range for re-entry purposes.

![Figure 6.23. MOL cumulative density functions for different values of \( p \).](image)

6.1.7.2 MOL and a previously defined rate level

It seems reasonable to define re-entry as the time required for the estimated MOL to decay to some predefined fixed level or background rate \( B \), i.e.:
The definition of $B$ can be a challenging process. When used in the industry, it is mainly defined based on the experience of the mine personnel. Malek and Leslie (2006) established typical maximum and minimum background levels at the Copper Cliff North Mine. For the minimum level, they used a maintenance shutdown with no mining activities. For the maximum level, periods with no anomalous seismic events or large crown blasts occurred were used. Based on a superposition of seismicity in a 24-hour chart, a practical method for estimating an upper level for $B$ was proposed in Section 4.3. Similarly, several methods were identified in the literature of crustal seismology for recognizing background levels. Most define background levels as the independent temporal component that yields the time sequence closest to a Poisson process (McNally, 1976; Habermann and Wyss, 1984). For some sequences, the $B$ value was estimated using the above methods and it was concluded that using a previously defined $B$ value and the power-law MOL parameters can lead to long re-entry times. For example, using the parameters listed in Figure 6.4a and $B = 2$ events/hour produces $t_B^d = 24.1$ hours, which is longer than the actual measured duration of the sequence.

Next, a more rigorous method, proposed by Utsu et al. (1995), which also makes use of background seismicity, is evaluated. The $AIC$ values of Eq. (6.1) are compared with the following alternative rate model:

$$n(t) = \frac{K}{(c + t)^p} + B$$

(6.32)

If $B > 0$ and the $AIC$ of Eq. (6.32) is smaller than that for Eq. (6.1), then the duration of the aftershock sequence can be defined by replacing the estimated parameters of Eq. (6.32) in Eq.
The continuation of background seismicity during a decay sequence would imply that the principal event creates new sites of later slip (Gross and Kisslinger, 1994). Another interpretation (Dieterich, 1986) is that aftershocks are the same events that would have occurred as background seismicity, but they occur sooner because of the stress load imposed by the principal event.

For the analysis, the aftershock sequences for the complete time interval \([t_0, t_N]\) from two different mines, Kidd Creek and Macassa, were considered. These catalogues presented the shortest and longest \(t_N\) times respectively. Gross and Kisslinger (1994) concluded that background rates can be reliably recovered only if the sequence is longer than the time at which the rate of the MOL has decreased to the background rate, i.e., when the time decay estimated using Eq. (6.31) with the parameters of Eq. (6.32) is less than \(t_N\). For the Macassa and Kidd Creek sites, it was found that just 59% and 34% of the cases, respectively, where Eq. (6.32) gave a better fit than Eq. (6.1). In addition, the models with background rate yielded \(p\) values higher than 2.0, for 16% and 47% of the cases at the Macassa and Kidd Creek sites respectively, which seem particularly high. Finally, re-entry times estimated with this criterion in some cases led to impractically long times. An example is presented in Figure 6.24. Using the parameters listed in Figure 6.24, a re-entry time \(t_B^{d} = 164.6\) hours is obtained. Based on these observations this criterion was not further investigated.
6.1.7.3 Time of maximum curvature $T_{MC}$ of the MOL

The power-law form of Eq. (6.1) indicates that there is no characteristic timescale and, for large time ($t > c$), the equation is temporally self-similar (Ito and Matsuzaki, 1990). However, if the curvature of the MOL is traced in time, given by:

$$
\rho(t) = \frac{\dddot{n}(t)}{\left[1 + (\ddot{n}(t))^2\right]^{3/2}}
$$

(6.33)

where $\ddot{n}(t)$ and $\dddot{n}(t)$ are the first and second time derivatives of Eq. (6.1) respectively, a characteristic point emerges at the time of maximum curvature ($\dot{\rho}(t) = 0$):

$$
T_{MC} = \left[ Kp \left(\frac{1 + 2p}{2 + p}\right)^{\frac{1}{1+p}} \right] - c
$$

(6.34)
Figure 6.25 presents an example of the curvature of the MOL for a blast related aftershock sequence. To estimate the first and second time derivatives of the rate a moving regression technique with 100 points was used.

Figure 6.25.  Rate and curvature of the MOL for a blast related aftershock sequence.

It can be seen from Figure 6.25b that initially the aftershock sequence has a low curvature, similar to a straight line, which with time increases and reaches a maximum. After that, the curvature decreases to lower values. Although there is no characteristic time in the log $n(t)$ - log $t$ plot (Figure 6.25a) it will be demonstrated that $T_{MC}$ has a physical meaning suitable for use in the development of a re-entry protocol. Taking the first derivative of Eq. (6.1):

$$\dot{n}(t) = -p \frac{K}{(c + t)^{1-p}}$$

(6.35)
The negative sign of Eq. (6.35) reflects the decrease in the change of event rate as time increases. Substitution of Eq. (6.34) into Eq. (6.35) gives the following expression for the change in rate at \( T_{MC} \):

\[
\dot{n}(t = T_{MC}) = -\sqrt{\frac{2 + p}{1 + 2p}}
\]  

(6.36)

Using the typical range of \( p \) values, between 0.4 and 1.6, it is found that \( \dot{n}(t = T_{MC}) \) can range from -1.15 to -0.93, respectively. The implication is that, at \( T_{MC} \), the event rate starts to decrease at a rate approximately less than one, indicating a transition from high-to-low event rate change. In the case of a power-law MOL, the maximum change in rate is always found before \( T_{MC} \). For example, using the parameters listed in Figure 6.25, it is found that this sequence decayed to a rate of \( n(T_{MC}) = 8.5 \) events/hour during the first \( T_{MC} = 7.98 \) hours, while it took over 27.7 additional hours to decay to 2.0 events/hour. In terms of the mean rate, approximately 40 events/hour is produced in the first \( T_{MC} = 7.98 \) hours while a mean rate of 5.6 events/hour is found in the next 7.98 hours.

The general form of Eq. (6.34) indicates that \( T_{MC} \) is a scale-invariant function of \( K \). By replacing Eq. (6.28) in (6.34) for a power-law process \( (c = 0) \), the following expression is obtained:

\[
T_{MC} = \chi(N_t)^\beta
\]  

(6.37)

where:

\[
\chi = \left[ \kappa p \left( \frac{2 - \beta}{1 + \beta} \right) \right]^\beta ; \beta = \frac{1}{1 + p}
\]  

(6.38)
Using the estimated power-law parameters and performing a nonlinear regression between $T_{MC}$ and $N_1$, a surprisingly narrow range, between 0.2-0.6 and 0.45-0.8 with averages of 0.389±0.120 and 0.625±0.102 (Table 6.5) is obtained for $\chi$ and $\beta$ respectively. The main advantage of these correlations is that $T_{MC}$ can be estimated based only on $N_1$ without the necessity of specifying a $p$ value. Equation (6.37) appeared to be less sensitive to the cases that satisfy $T_s \geq 1$, $\kappa \geq 1$ and/or $T_{MC} > T_E$ compared to Eq. (6.28). The exclusion of these cases from the correlations was studied individually by analyzing the residuals and regression coefficients (black arrows in Figure 6.26 and Figure 6.27).

<table>
<thead>
<tr>
<th>Site</th>
<th>$\chi$</th>
<th>$\beta$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.466±0.087</td>
<td>0.531±0.046</td>
<td>0.814</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>0.199±0.025</td>
<td>0.792±0.030</td>
<td>0.926</td>
</tr>
<tr>
<td>Craig</td>
<td>0.296±0.073</td>
<td>0.661±0.051</td>
<td>0.987</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.403±0.077</td>
<td>0.626±0.064</td>
<td>0.695</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>0.404±0.066</td>
<td>0.646±0.045</td>
<td>0.792</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.395±0.043</td>
<td>0.567±0.025</td>
<td>0.925</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>0.393±0.052</td>
<td>0.575±0.038</td>
<td>0.916</td>
</tr>
<tr>
<td>Williams</td>
<td>0.533±0.128</td>
<td>0.533±0.054</td>
<td>0.823</td>
</tr>
<tr>
<td>California</td>
<td>0.592±0.115</td>
<td>0.489±0.039</td>
<td>0.776</td>
</tr>
<tr>
<td>Italy</td>
<td>0.339±0.131</td>
<td>0.697±0.086</td>
<td>0.844</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th>$\chi$</th>
<th>$\beta$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>0.199</td>
<td>0.489</td>
<td>0.695</td>
</tr>
<tr>
<td>max</td>
<td>0.592</td>
<td>0.792</td>
<td>0.987</td>
</tr>
<tr>
<td>average</td>
<td>0.402</td>
<td>0.612</td>
<td>0.850</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.112</td>
<td>0.091</td>
<td>0.088</td>
</tr>
</tbody>
</table>
Figure 6.26. Correlation between the parameter $T_{MC}$ and the measured number of events occurring during the first unit time after the principal event $N_1$ for six mining-induced seismicity catalogues.
Figure 6.27. Correlation between the parameter $T_{MC}$ and the measured number of events occurring during the first unit time after the principal event $N_1$ for two mining-induced and two crustal seismicity catalogues.
6.1.8 Application of the proposed MOL guidelines

A comprehensive statistical analysis of the MOL has been provided. In terms of the MOL parameters, there are basically only two parameters to specify: $p$ and $K$. Both can be estimated based on the analysis of locally observed historical data using the procedure already described.

The applications of these findings for developing a real-time re-entry protocol are:

1. The average or median power-law $p$ value can be assumed (Figure 6.16b).

2. Following the principal event and after recording the number of seismic events during the first unit time $N_1$, the value of $K$ and $T_{MC}$ can be established with an associated level of confidence.

Given that the correlations for $T_{MC}$ were obtained for the power-law portion of the aftershock sequences, an additional time window after $T_{MC}$ can be considered as a “factor of safety” by using the variability of the data. Given $N_1$ and using the average correlation and the 95% prediction interval shown in Figure 6.26 and Figure 6.27, a maximum curvature time window can be defined by:

$$\Delta T^{T_{MC}} = T_{MC}^{95\% \text{ PI}} - T_{MC}^{\text{avg}}$$  \hspace{1cm} (6.39)

By using this definition the variability of the data indicates that the maximum curvature time window can range from 0.68 to 2.73 hours. However, most of the cases are between 1 and 2 hours (Figure 6.28). Note that the maximum curvature time window is not exactly the background time window as the event rate in this time interval is not necessary at background levels. However, $\Delta T^{T_{MC}}$ is related to the irregularity in the decay of aftershocks and thus represents the inherent variability of the re-entry time estimates. If a strong deviation from the MOL decay curve occurs
during this observation time period extra precautions should be taken, as for example wait until the data plots again below the average MOL curve.

A second application of the correlation between $T_{MC}$ and $N_1$ is for establishing an excessive seismicity protocol. This can be directly defined as the number of events per hour necessary to raise the maximum curvature point above of some time threshold. Considering that when a re-entry protocol is invoked, the recommended minimum re-entry time is two hours (Section 3.2.1), the number of events per hour required to raise $T_{MC}$ above two hours is used to set an excessive seismicity protocol (Table 6.6).
Table 6.6: Number of events per hour $N_1^*$ above the corresponding average cut-off moment magnitude bin $\bar{M}_c$, required to raise $T_{MC}$ more than two hours.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\bar{M}_c$</th>
<th>$N_1^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-2.01±0.05</td>
<td>19</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>-1.59±0.12</td>
<td>19</td>
</tr>
<tr>
<td>Craig</td>
<td>-1.66±0.11</td>
<td>11</td>
</tr>
<tr>
<td>Creighton</td>
<td>-1.42±0.12</td>
<td>13</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>-1.88±0.11</td>
<td>16</td>
</tr>
<tr>
<td>Macassa</td>
<td>-1.18±0.08</td>
<td>14</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>-2.06±0.10</td>
<td>17</td>
</tr>
<tr>
<td>Williams</td>
<td>-1.83±0.12</td>
<td>13</td>
</tr>
</tbody>
</table>

min 11  
max 19  
average 15  
S.D. 3

Each site seems to have its own site specific threshold, however, an average of 15±3 is found. Note that this criterion considers only the events per hour above the average cut-off moment magnitude $\bar{M}_c$ used to fit the equation (Table 5.4). Using the same logic, the number of events per hour required to raise $T_{MC}$ above any time threshold can be established. An alternative will be to evaluate $T_{MC}$ continuously based on the number of events occurring during the last hour and invoke an excessive seismicity protocol when $T_{MC}$ is above the specified time threshold.

Next, five examples are presented to illustrate the concepts and the application of the proposed MOL guidelines. When possible, the determined re-entry time determined is compared to the estimates of other authors. The characteristic parameters used to establish the MOL decay curve, using Eq. (3.14) with $\Delta t = 1$ hour, are presented in each frame.

The first case corresponds to the example already presented in Section 1.1 for a development blast at Craig Mine (Figure 1.1). Figure 6.29 shows the figure used by Simser (2006) to establish
that the spike in event rate decays within 3 hours after the blast. It can be noticed that $T_{MC}^{avg}$ is in excellent agreement with the visual estimate made by Simser (2006). The data follows the average MOL decay curve very well, except for hour number three after the blast where a small deviation occurs.

Figure 6.29. Application of the proposed MOL guidelines for the development blast at Craig Mine presented in Figure 1.1.

In the following four cases a 1 hour time window with a 0.1 hour shift is used to represent the event rate. In addition, the maximum MOL curve using the maximum productivity aftershock ratio $\kappa$ at each site (Figure 6.19b) is included in the plots.

Figure 6.30 presents an aftershock sequence at the Kidd Creek Mine that was triggered by a 1.6 Nuttlí magnitude event. In this case, the data follows the 95% prediction interval MOL during the first 1.7 hours. From 1.7 to 3.5 hours the event rate increases reaching the maximum MOL. After that, the data returns to the 95% prediction interval boundaries where $T_{MC}^{avg}$ is reached. A
secondary aftershock sequence appeared during the maximum curvature time window which resets the re-entry clock and the restriction continues until hour 9.3 where the data joins again the average MOL.

![Graph showing time after principal event and events per hour with overlaying MOL guidelines]

Figure 6.30. Application of the proposed MOL guidelines for a large magnitude event (\(M_n=1.6\)) at the Kidd Creek Mine.

A blast induced aftershock sequence at the Copper Cliff North Mine is presented in Figure 6.31. In this case the data went below the average MOL until hour 4.9, where it starts oscillating within the 95% prediction interval. After \(T_{MC}^{avg}\) the data starts decaying at a rate below than the average MOL. In this particular case hour 10 is appropriate for re-entering.
The third case (Figure 6.32) corresponds to a large rockburst ($M_n=2.4$) at the Copper Cliff North Mine. This sequence was accompanied by a second large magnitude event ($M_n=1.4$) 6.2 hours after the initial event. In this case, the sequence deviates from the average MOL from the very beginning. In this case $T_{MC}^{avg}$ includes the second large magnitude event. Also, after this point the overall decay starts to oscillate at an approximately constant rate. In this case hour 11.9 results appropriate for re-entering, which is similar to the re-entry time period of 12.8 hours determined by Malek and Leslie (2006) based on the visual inspection of the return of cumulative seismic work to background levels.
Figure 6.32. Application of the proposed MOL guidelines for a rockburst sequence ($M_n=2.4$) at the Copper Cliff North Mine. A second large magnitude event ($M_n=1.4$) occurred in the same zone 6.2 hours after the initial event.

The final example (Figure 6.33) presents a blast related sequence at the Macassa Mine. This sequence follows closely the average MOL pattern until hour 2.3. A deviation from the average MOL starts at this time until hour 4.4, where the data goes below the average MOL curve. In this case $T_{MC}^{avg}$ includes all of these features giving a good estimate of the re-entry time.
Figure 6.33. Application of the proposed MOL guidelines for a blast (6,752 lbs) related aftershock sequence at the Macassa Mine.

### 6.2 Frequency-magnitude distribution

In this section, the frequency-magnitude distribution (Gutenberg and Richter, 1944) is fitted to the aftershock magnitudes for the time interval of power-law decay. The objective is to prepare the data for the Reasenberg and Jones (1989) stochastic model (Section 6.3.1). In this model the average parameters of the Gutenberg-Richter (G-R) relation are used to describe some statistical features of seismicity. In addition to the moment magnitude scale, uniaxial magnitude is also considered in the analysis. Therefore, the power-law MOL parameters were re-estimated for the uniaxial magnitude scale following the procedure described in Appendix B.

The frequency-magnitude distribution (FMD) describes the relationship between the frequency of occurrence and magnitude of events by:
where $N(M)$ is the cumulative number of events having magnitude larger than equal to $M$, and $a$ and $b$ are constants that may vary in space and time. The parameter $a$ characterizes the general level of seismicity in a given area during the study period, while the parameter $b$, which is typically close to 1, describes the relative abundance of large to smaller events. The $b$-value is believed to depend on the stress regime and tectonic character of the region (Allen et al., 1965; Mogi, 1967; Scholz, 1968; Hatzidimitriou et al., 1985; Tsapanos, 1990a; Urbancic et al., 1992).

Under the assumption that the magnitude data are random samples from a population obeying the G-R relation, the method of moments (Utsu, 1965) and the method of maximum likelihood (Aki, 1965) both yield the solution:

$$b = \frac{1}{\ln 10[M - M_z]}$$

(6.41)

where $\bar{M}$ is the mean magnitude of events with $M \geq M_z$ and $M_z$ is the threshold magnitude above which the data should be complete, i.e., the lowest magnitude at which 100% of the events in a space-time window are detected (Rydelek and Sacks, 1989). An estimate of the standard deviation of $b$ can be obtained using the equation first derived by Aki (1965), or the improved formulation by Shi and Bolt (1982):

$$\Delta b = 2.3b^2 \sqrt{\frac{\sum (M_i - \bar{M})^2}{n(n-1)}}$$

(6.42)

where $n$ is the sample size. Note that $\Delta b$ does not necessarily indicate the goodness of fit of the G-R relation to the data. However, a large $\Delta b$ may indicate insufficient sample size.
For computing the \( a \) and \( b \) values, knowledge of \( M_z \) is critical. \( M_z \) has to be estimated either for every aftershock sequence or defined assuming a homogeneous recording quality for the entire dataset (Schorlemmer et al., 2004). Different techniques can be used to estimate \( M_z \) (Gomberg, 1991; Kijko and Sellevoll, 1992; Rydelek and Sacks, 1989). A review is given in Wiemer and Wyss (2000). A simple technique used frequently is based on estimating \( M_z \) by visual examination of the cumulative or non-cumulative frequency-magnitude distribution (FMD).

In this thesis, the quantitative criterion of Wiemer and Wyss (2000) was selected for estimating \( M_z \). The following steps are taken to estimate \( M_z \). First, estimate the \( a \) and \( b \) values of the FMD as a function of a minimum magnitude \( M_i \) using the maximum likelihood method (Eq. (6.41)). Next, a synthetic distribution of magnitudes with the same \( a \), \( b \) and \( M_i \) values is computed. The goodness of fit, or percentage of data variability, is computed as the absolute difference of the number of events in each magnitude bin between the observed and synthetic distribution:

\[
R = 1 - \frac{\sum_{i}^{M_{max}} |B_i - S_i|}{\sum_{i} B_i}
\]  

where \( B_i \) and \( S_i \) are the observed and predicted cumulative number of events in each magnitude bin. The distribution is normalized by dividing by total number of observed events. The approach is illustrated in Figure 6.34, which shows \( b \) and \( R \) as a function of \( M_i \).
Figure 6.34. Wiemer and Wyss (2000) method used to estimate the minimum magnitude of completeness $M_z$ applied to the entire Macassa seismicity catalogue.

It can be observed in Figure 6.34, that if $M_i$ is smaller than the optimal $M_z$ the synthetic distribution cannot model the FMD adequately and, consequently, the goodness of fit is low (point 1 in Figure 6.34). The goodness of fit value $R$ increases with increasing $M_i$ and reaches a maximum value of $R \sim 0.92$ at $M_z = -0.7$, in this example. At this $M_z$, a simple power-law with the assumed $a$, $b$, and $M_z$ can explain 92% of the data variability.

Wiemer and Wyss (2000) suggested to define $M_z$ at the 0.9 level, that is, $M_z$ is defined as the point at which a power-law can model 90% or more of the FMD. Wiemer and Wyss (2000) mention that not all FMD’s will reach the 0.9 level, such as: curved or bimodal FMD’s. In these cases a simple power-law cannot be readily applied.
In the example presented in Figure 6.34 it is determined that \( R \geq 0.9 \) for \( M_z = -1.2 \), giving \( b = 3.2 \). This \( b \)-value seems particularly high, however, the value is accepted based on the goodness of fit. Note that this method selects \( M_z \) by solely increasing \( M_i \), until the power-law is able to describe enough of the remaining data. It is not appropriate to use this method to determine an optimal power-law magnitude range by truncating the data. This is because the Aki method depends on the FMD following a power-law with a constant slope in the \( M - \log_{10} N \) plot. The slope of an upper-truncated power-law is not constant in the \( M - \log_{10} N \) invalidating the method. In addition, the level of acceptance of 0.9 was arbitrarily selected by Wiemer and Wyss (2000).

This approach was applied to the event magnitudes in the time interval of power-law decay \([T_s, T_e]\) determined for each aftershock sequence. To ensure stable solutions, only aftershock sequences with at least 10 events in the time interval \([T_s, T_e]\) were considered for the analysis.

First, the benefit of choosing a \( M_z \) higher than the \( M_c \) already used to fit the MOL and selected at the magnitude bin with the highest frequency of events in a non-cumulative FMD is evaluated. Table 6.7 presents the percentage of mining sequences with a goodness of fit \( R \geq 0.9 \), when \( M_z = M_c \) and when \( M_z \) is increased according to the method of Wiemer and Wyss (2000). These cases are represented in Table 6.7 by the columns \( M_z = M_c \) and \( M_z \geq M_c \) respectively.
Table 6.7: Percentage of mining-induced aftershock sequences that satisfy $R \geq 0.9$ when $M_z$ is equal to the cut-off magnitude $M_c$ already used to fit the MOL ($M_z = M_c$), and when $M_z$ is increased ($M_z \geq M_c$) according to the method of Wiemer and Wyss (2000) for the moment and uniaxial magnitude scales.

<table>
<thead>
<tr>
<th>Site</th>
<th>$M_z = M_c$</th>
<th>$M_z \geq M_c$</th>
<th>$M_z = M_c$</th>
<th>$M_z \geq M_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>21%</td>
<td>29%</td>
<td>57%</td>
<td>61%</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>45%</td>
<td>65%</td>
<td>42%</td>
<td>44%</td>
</tr>
<tr>
<td>Craig</td>
<td>40%</td>
<td>40%</td>
<td>60%</td>
<td>60%</td>
</tr>
<tr>
<td>Creighton</td>
<td>59%</td>
<td>76%</td>
<td>56%</td>
<td>67%</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>39%</td>
<td>55%</td>
<td>38%</td>
<td>66%</td>
</tr>
<tr>
<td>Macassa</td>
<td>24%</td>
<td>38%</td>
<td>52%</td>
<td>55%</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>14%</td>
<td>23%</td>
<td>39%</td>
<td>52%</td>
</tr>
<tr>
<td>Williams</td>
<td>59%</td>
<td>64%</td>
<td>61%</td>
<td>78%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>38%</strong></td>
<td><strong>49%</strong></td>
<td><strong>51%</strong></td>
<td><strong>60%</strong></td>
</tr>
</tbody>
</table>

Inspection of Table 6.7 provides the following:

1. If $M_z$ is increased according to the method of Wiemer and Wyss (2000), on average only 49% and 60% of the studied cases satisfied the condition $R \geq 0.9$ for the moment and uniaxial magnitude scales respectively.

2. The average benefit of choosing $M_z$ higher than $M_c$, selected at the bin with the highest frequency of events in a non-cumulative FMD, is 11% and 9% for the moment and uniaxial magnitude scales respectively.

In the case of crustal sequences, when $M_z = M_c$ is used, the percentage of sequences with $R \geq 0.9$ are 72% and 73% for California and Italy respectively. If $M_z$ is increased, the resulting percentages with $R \geq 0.9$ are 82% and 80% for California and Italy respectively. Given the average modest benefit of selecting $M_z$ higher than the $M_c$ already used to fit the MOL it was...
decided to fix $M_z = M_c$ for future analysis. This selection also guarantees that the same number of events in both scaling laws (MOL and G-R) is used.

Figure 6.35 presents the cumulative descending distribution of sequences as a function of the goodness of fit $R$ at each site for the moment and uniaxial magnitude scales.

From Figure 6.35 the cumulative descending distribution of sequences for the uniaxial magnitude scale seems more tight (Figure 6.35b) than the moment magnitude scale (Figure 6.35a). This suggests that, in general, a better fit of the G-R relationship is attained for the uniaxial magnitude scale.

The resulting average $b$-values at each site as a function of the goodness of fit $R$ are presented in Figure 6.36.
The average $b$-values are not strongly affected by the selected level of $R$ (Figure 6.36). There are two sites (A and Macassa) that presented high average $b$-values for the moment magnitude scale with a slight tendency to increase as $R$ increases (Figure 6.36a). The rest of the mining sites presented average $b$-values for the moment magnitude scale from 1.00 to 1.85. The average $b$-values for the aftershock sequences of site A and Macassa are particularly high (>3.0) compared to the usual range reported in the crustal literature:

- $0.95\pm0.19$ for Japan (Guo and Ogata, 1997),
- $1.01\pm0.15$ for New Zealand (Eberhart-Phillips, 1998),
- $0.90\pm0.18$ for California (Reasenberg and Jones, 1989),
- $0.99\pm0.17$ for Italy (Lolli and Gasperini, 2003; Gasperini and Lolli, 2006), and
- $0.88\pm0.21$ for the Nevada Test site (Ford and Walter, 2010).
However, values of 3.0 have been indicated by Okal and Romanowicz (1994) for the portion of largest crustal events of the frequency-moment relationship. Large temporal variations in a range from about 0.5 to 3.0 were found by Nuannin et al. (2002) for mining tremors with moment magnitude between -1.6 to 2.6 in the Zinkgruvan Mine in south-central Sweden. Pockets of anomalously high $b$-values (>2.0) have been found underneath volcanoes (Wiemer and Wyss, 2002). A bimodal frequency-moment magnitude distribution has been identified by Richardson and Jordan (2002) using seismicity of deep gold mines of South Africa. They attribute this behaviour to the small events tightly clustered in time and space that are created by the process of blasting. Wiemer and Wyss (2002) also suggested that explosions may bias the $b$-estimates toward large values because explosions are smaller than earthquakes and of similar sizes. This interpretation seems to fit very well the large average $b$-values found for the moment magnitude scale for the aftershock sequences of mine sites A and Macassa. However, the same effect is not observed when the uniaxial magnitude scale is used (Figure 6.36b). In this scale of magnitudes the average $b$-values ranges from 0.68 to 1.31, this is more consistent with the range of average $b$-values observed at the crustal scale.

Figure 6.37 presents an example for a blast related aftershock sequence with the corresponding determined power-law MOL time interval and the $b$-value for the moment and uniaxial magnitude scales. Similar power-law MOL parameters are obtained for both scales of magnitudes. However, a different situation appears for the FMD. In the case of moment magnitude, the distribution seems to be highly concentrated at low magnitudes following the blast resulting in a high $b$-value (Figure 6.37a), while for the uniaxial magnitude the FMD is more evenly distributed resulting in a $b$-value close to one, which is in the usual range reported for crustal seismicity.

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6.3 Other rate stochastic models

In order to have a better overview of stochastic models that can be used for representing the event rate decay of time sequences two additional models available in the crustal literature are considered:

Figure 6.37. Estimated MOL and G-R parameters for a blast-induced aftershock sequence at the Macassa site for the moment (frame a) and uniaxial (frame b) magnitude scales.

2. Epidemic type aftershock sequences (ETAS) (Ogata, 1988).

These two models have similar characteristics and assumptions, such as: the necessity of setting a-priori representative parameter values for a number of specific features for describing the decay and productivity of the aftershock sequence. However, their approach for aftershock hazard forecasting is completely different. The Reasenberg and Jones model uses exclusively the main shock magnitude to establish the productivity of the sequence and forecast the time decay, while in the ETAS model each shock in the sequence can generate aftershock at a rate that decreases according to the MOL.

In the following, both approaches are explored and discussed in detail. The implications and suitability for re-entry protocol development are identified and guidelines on the use of these models are provided.

6.3.1 Reasenberg and Jones (R-J)

Reasenberg and Jones (1989, 1994) proposed a simple model, which is still in use at the U.S. Geological Survey, to forecast aftershock rates and probabilities in Southern California and in few other areas (Gasperini and Lolli, 2006). This model combines the G-R and the MOL to describe the aftershock decay rate as a function of the time \( t \) elapsed after the main shock by:

\[
n(t) = \frac{10^{A+b(M_{PE}-M_c)}}{(c+t)^p}
\]

(6.44)

where \( A \) is correlated with the \( K \) value of the MOL by:

\[
A = \log K - b(M_{PE} - M_c)
\]

(6.45)
and is referred by the authors as the “productivity” of the sequence. $b$ in Eq. (6.45) is the coefficient in the G-R relationship determined by fitting the distribution of aftershock magnitudes, $M_{PE}$ is the magnitude of the principal event or main shock ($t = 0$) and $M_c$ is the cut-off magnitude used to fit the equation. This model has the merit to allow the forecasting of aftershocks behaviour based only on the main shock magnitude.

Note that, the following expression can be obtained for $A$ using Eq. (6.2) to correlate $K$ and the number of events $N_{T_a-T_s}$ occurring in a time interval $[T_A, T_B]$:

$$A = \begin{cases} 
\log(N_{T_a-T_s}) - \log\left(\frac{\ln(T_B + c)}{\ln(T_a + c)}\right) - b(M_{PE} - M_c) & \text{for } p = 1 \\
\log(N_{T_a-T_s}) - \log\left(\frac{(T_B + c)^{1-p} - (T_a + c)^{1-p}}{(1-p)}\right) - b(M_{PE} - M_c) & \text{for } p \neq 1 
\end{cases} \quad (6.46)$$

For calibrating this model it is necessary to fit Eq. (6.44) to several aftershock sequences. For each aftershock sequence the procedure is as follow:

1. For a cut-off magnitude $M_c$ and target time interval $[T_A, T_B]$ with $N_{T_a-T_s}^T$ events, estimate the parameters $K$, $p$ and $c$ by the maximum likelihood method (Eq. (6.3)).

2. Estimate the $b$-value by fitting the frequency-magnitude distribution in the same time interval $[T_A, T_B]$ considering all the events $N_{T_a-T_s}^z$ with magnitude above the threshold of magnitude of completeness $M_z$.

3. Evaluate the parameter $A$ using Eq. (6.45).
4. The average or median values of $A$ and $b$ are considered to be common to all aftershock sequences and used to estimate the probabilities for aftershocks and larger principal events during intervals following a main shock.

It must be mentioned that the procedure to estimate the parameter $A$ includes a further step that is relevant when the magnitude of completeness $M_z$ used to fit the G-R relationship is higher than the cut-off magnitude $M_c$ used to fit the MOL (Gasperini and Lolli, 2006). In this case, in fact, the number $N^c_{T_a-T_d}$ of events with magnitude above $M_c$ actually recorded in the catalogue may be underestimated and then an ad-hoc correction is applied by extrapolating the G-R law from $M_z$ down to $M_c$. Namely, being $N^z_{T_a-T_d}$ the number of events in the catalogue with $M \geq M_z$ and assuming the completeness in such interval, the corrected number $N^{corr}_{T_a-T_d}$ of events with $M \geq M_c$ to be used in Eq. (6.45) instead of $N^c_{T_a-T_d}$ is given by:

$$\log(N^{corr}_{T_a-T_d}) = \log(N^z_{T_a-T_d}) - b(M_c - M_z)$$ (6.47)

Thus, the corrected values of parameters $K$ and $A$ are computed using only the complete part of the aftershock sequence:

$$\log K^{corr} = \begin{cases} \log(N^z_{T_a-T_d}) - b(M_c - M_z) - \log \left( \frac{(T_a + c)}{(T_A + c)} \right) & \text{for } p = 1 \\ \log(N^z_{T_a-T_d}) - b(M_c - M_z) - \log \left( \frac{(T_a + c)^{1-p} - (T_A + c)^{1-p}}{(1-p)} \right) & \text{for } p \neq 1 \end{cases}$$ (6.48)

$$A^{corr} = \log(K^{corr}) - b(M_{PE} - M_c)$$ (6.49)
This model assumes that the largest magnitude $M_{PE}$ in the time sequence is the event occurring at $t = 0$. This is true if the magnitude of the identified aftershocks is restricted to be lower than the magnitude of the considered event at $t = 0$ (main shock). In the case of mining seismicity the beginning of a sequence was identified by an increase in rate by the ratios method (see Section 5.2) and the event occurring at $t = 0$ is not necessary coincident with the largest magnitude occurring at the very beginning of the sequence. There is also a limitation of current microseismic monitoring systems for quantifying large magnitude events. These considerations may limit the direct applicability of the Reasenberg-Jones (R-J) model to mining seismicity. Figure 6.38 illustrates the situation for a rockburst related seismic sequence. In this example, there is a larger magnitude event ($MomMag = 0.55$, square in Figure 6.38c) occurring shortly (0.00996 hours) after the first event of the sequence ($MomMag_{t=0} = -0.44$). Given the short delay, there is no doubt that this larger event is associated with the rockburst initiation process. In the classical view of main shock/aftershocks, all the events prior and after the 0.55 are considered as foreshocks and aftershock respectively. However, the beginning of the sequence ($t = 0$) was correctly identified by the ratios method as there is an abrupt change in the cumulative number of events (Figure 6.38a). Also a higher rate can be confirmed for the events that occurred between the principal event and 0.00996 hours (Figure 6.38b). Similar time-magnitude behaviour was observed for blast related sequences where the $t = 0$ event is known. It is expected, however, that the delay between the $t = 0$ event and a related larger magnitude event may be small.
Figure 6.38. Example of a rockburst related sequence where the magnitude of the principal event \((t = 0)\) is not necessarily coincident with the largest magnitude occurring at the very beginning of the sequence. (a) Cumulative number of event as a function of time relative to the rockburst. Event rate (frame b) and moment magnitude (frame c) as a function of time after the principal event.

The determined start time of power-law decay \(T_s\) was included in Figure 6.38. It can be observed that the large magnitude event occurring shortly after the first event (red square) also occurred before the start time of power-law decay, i.e., during the time in which a complex rate behaviour dominated the sequence. It was determined that in 28% of all the mining sequences analyzed an event with magnitude larger than the selected at \(t = 0\) occurred before the start of power-law decay \(T_s\). Based on the fact that 99% of the mining-induced aftershock sequences presented
$T_s < 1$ (Figure 6.11) the largest magnitude occurring during the first hour is selected as representative of the magnitude of the main event, i.e., $M_{PE} = M_1$.

Using the above considerations, steps 1 to 4 were applied to the aftershock sequence of each site with $M_z = M_c$ to avoid the correction for missing events (Eq. (6.49)). Also, the choice of $M_z = M_c$ seems more appropriate than using a theoretical correction, in that aftershocks below $M_c$ may not be fully recorded. The average $A$-values are presented as a function of the goodness of fit $R$ for both moment and uniaxial magnitude scales (Figure 6.39).

![Figure 6.39](image)

**Figure 6.39.** Average $A$-values as a function of the goodness of fit $R$. (a) Moment magnitude; (b) Uniaxial magnitude.

Inspection of Figure 6.39 provides the following:

1. In general, the average $A$-values are not strongly affected by the selected level of $R$. At some sites, the most significant change is produced at the $R = 90\%$ level, and is due to the low number of sequences retained for the analysis.
2. Most of the average $A$-values range from -4.6 to -0.7, and from -1.8 to -0.7 for the moment and uniaxial magnitude scales respectively. Average $A$-values at the crustal level calculated using the reported values from other authors are:

- $-1.79 \pm 0.74$ for New Zealand (Eberhart-Phillips, 1998),
- $-1.83 \pm 0.67$ for Italy (Lolli and Gasperini, 2003; Gasperini and Lolli, 2006),
- $-1.81 \pm 0.62$ for California (Reasenberg and Jones, 1989; Gasperini and Lolli, 2006),
- $-1.23 \pm 0.26$ for the Nevada Test Site (Ford and Walter, 2010),
- $-1.83$ for Japan (Yamanaka and Shimazaki, 1990), and
- $-2.18$ for stacked California aftershock sequences (Felzer et al., 2003).

These results indicate that at the crustal scale an expected range for average $A$-values is: -2.18 to -1.23. The implication is that the average $A$-values estimated using the uniaxial magnitudes are in better agreement with those obtained for crustal sequences.

Next, the predictions of the $K$ values using the R-J model are compared to the actual $K$ values obtained from fitting the MOL to the time sequences. Assuming the average values of $A$ and $b$ ($\bar{A}$ and $\bar{b}$) of the site the estimated $K$ value using the R-J model is given by:

$$K_{R-J} = 10^{\frac{A + b(M_{pe} - M_c)}{2}}$$

To conclude about the effectiveness of this model for estimating $K$, the variance-account-for ($VAF$) is used:

$$VAF = 1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)}$$
If the $VAF$ is equal to one, then the predictions of $K_{R-J}$ are in excellent agreement with the actual $K$ values. A negative $VAF$ will indicate that the prediction of $K_{R-J}$ are even worse than a horizontal line.

Table 6.8 presents the $VAF$ between the predictions of the $K$ values using Eq. (6.50) and the actual $K$ values for different levels of the goodness of fit $R$. Almost all sites have a negative $VAF$ (Table 6.8). In some mining cases a weak positive $VAF$ is obtained for some level of the goodness of fit $R$. Similar $VAF$ values are obtained from tabulated values of the R–J model from other authors:

- -1.21 for New Zealand (Eberhart-Phillips, 1998),
- -0.11 for Italy (Lolli and Gasperini, 2003; Gasperini and Lolli, 2006), and
- 0.85 for the Nevada Test site (Ford and Walter, 2010).

There are only two cases with significant positive $VAF$ corresponding to California (Table 6.8b) and to the Nevada Test site. In the case of California, the large $VAF$ is influenced by one aftershock sequence with large $K$ and large $M_{PE} - M_c$. If this occurrence is removed the $VAF$ is reduced to 0.22. In the case of the Nevada Test site, there are two aftershock sequences with large $M_{PE} - M_c$ that may be influencing the correlation. When these two occurrences are removed the $VAF$ reduces to -0.67. The implication is that the correlation between the predictions of the R-J model and the observed $K$ values are low.
Table 6.8: Variance-account-for ($VAF$) values between the R–J model predictions of $K$ using the average $A$ and $b$ values of each site and the actual $K$ values as a function of the goodness of fit $R$. (a) Moment magnitude scale; (b) Uniaxial magnitude scale. Crustal sequences are included in the uniaxial magnitude scale.

(a) $MomMag$ $VAF$

<table>
<thead>
<tr>
<th>Site / R</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-290</td>
<td>-575</td>
<td>-1462</td>
<td>-12255</td>
<td>-28694</td>
<td>-1036266</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>-0.12</td>
<td>-0.12</td>
<td>-0.12</td>
<td>-0.88</td>
<td>-2.70</td>
<td>-4.88</td>
</tr>
<tr>
<td>Craig</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.27</td>
<td>-1.24</td>
<td>-1.24</td>
</tr>
<tr>
<td>Creighton</td>
<td>-10</td>
<td>-10</td>
<td>-12</td>
<td>-13</td>
<td>-13</td>
<td>-5</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-3</td>
<td>-6</td>
<td>-10</td>
</tr>
<tr>
<td>Macassa</td>
<td>-41</td>
<td>-61</td>
<td>-87</td>
<td>-100</td>
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<td>-2203</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>-764</td>
<td>-764</td>
<td>-1466</td>
<td>-1610</td>
<td>-211</td>
<td>-30</td>
</tr>
<tr>
<td>Williams</td>
<td>-0.15</td>
<td>-0.15</td>
<td>-0.15</td>
<td>-0.15</td>
<td>0.03</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

(b) $uMag$ $VAF$

<table>
<thead>
<tr>
<th>Site / R</th>
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<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.85</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>-0.10</td>
<td>-0.10</td>
<td>-0.93</td>
<td>-1.04</td>
<td>-0.94</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>0.13</td>
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<td>0.13</td>
<td>0.11</td>
<td>0.10</td>
<td>0.00</td>
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<tr>
<td>Craig</td>
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<td>-2.24</td>
<td>-2.24</td>
<td>-2.24</td>
<td>0.87</td>
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<td>Creighton</td>
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<td>-226</td>
<td>-251</td>
<td>-89</td>
<td>-29</td>
</tr>
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<td>Kidd Creek</td>
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<td>-8</td>
<td>-8</td>
<td>-10</td>
<td>-19</td>
<td>-58</td>
</tr>
<tr>
<td>Macassa</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.06</td>
<td>-0.22</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>-6</td>
<td>-6</td>
<td>-6</td>
<td>-8</td>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td>Williams</td>
<td>-0.93</td>
<td>-0.93</td>
<td>-0.93</td>
<td>-0.93</td>
<td>-1.13</td>
<td>-2.47</td>
</tr>
<tr>
<td>California</td>
<td>0.78</td>
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<td>0.78</td>
<td>0.77</td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td>Italy</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.18</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Despite the low performance of this model for estimating $K$, it is currently used to estimate the probability of crustal aftershocks in the following day, week, month etc., during an ongoing sequence. By assuming a random process of events, the probability $P$ of one or more events occurring in the magnitude range ($M_a \leq M \leq M_b$) in a time window $\Delta t$ following the time $t$ can be computed by (Reasenberg and Jones, 1989; Lolli and Gasperini, 2003):

\[
P(t, \Delta t, M) = 1 - \exp \left[ -\int_{M_a}^{M_b} \int_{t}^{t+\Delta t} n(t, M)dt dM \right]
\]  (6.52)
where \( n(t, M) \) is given by Eq. (6.44) using \( M_c \) as a variable of integration. The obtained solution is given by:

\[
P(t, \Delta t, M_a, M_b) = 1 - \exp \left\{ \frac{1}{(1-p) b \ln(10)} \left[ 10^{4 + b(M_c - M_a)} - 10^{4 + b(M_c - M_b)} \right] \times \left[ (t + \Delta t + c)^{-p} - (t + c)^{-p} \right] \right\} \quad (6.53)
\]

where \( A, b, p \) and \( c \) are the representative values of the site. By using Eq. (6.53) it is possible to estimate probabilities for strong aftershocks/larger main shocks and for larger main shocks only by setting: \( M_a = M_{PE} - 1 \), \( M_b = \infty \), and \( M_a = M_{PE} \), \( M_b = \infty \) respectively (Reasenberg and Jones, 1989).

Figure 6.40 presents the probabilities for having an event of magnitude \( M_{PE} - 1 \) or larger than \( M_{PE} \) in the next \( \Delta t = 2 \) hours as a function of time \( t \) after a principal event for the different studied mining seismicity catalogues.

![Diagram](image-url)
Each curve seems to represent specific characteristics of decay and productivity of each site. There are two extreme cases: Creighton and Macassa. Creighton Mine presented the largest average $A$ and $b$ values, increasing the productivity and probability, while the Macassa site has the lowest average $A$ value, reducing the productivity and probability.

For using this model for re-entry it is necessary to set a magnitude threshold $M^*_LA$ and a probability level $P^*$ acceptable for re-entry for a given time window $\Delta t$. Figure 6.41 illustrates an example of probability decay curves for the uniaxial magnitude scale for two aftershock sequences at the Copper Cliff North Mine using: $\Delta t = 2$ hours, $M^*_LA = -0.5$, and the parameters listed in Figure 6.40b.

![Figure 6.41](image)

Figure 6.41. Example of probability decay curves derived with the R-J model using the uniaxial magnitude scale for a blast (frame a) and a rockburst (frame b) related sequences at the Copper Cliff North Mine.
In this example, the rockburst (Figure 6.41b) has a higher probability at all \( t \) for an \( M_{LA}^* = -0.5 \) occurrence compared to the blast (Figure 6.41a). The time-magnitude plot included in Figure 6.41 confirms that there are higher magnitude events as a function of time for the rockburst case compared to the blast.

A shortcoming of this approach is that the probabilistic curves depend only on the magnitude of the principal event and not on the ongoing data of the sequence. Reasenberg and Jones (1989) suggested that the model parameters for an ongoing seismic sequence can be estimated with Bayes rule (Bowker and Lieberman, 1972; Hogg and Craig, 1978). Assuming that the a-priori estimates of each parameter \( \theta \) are normally distributed with some mean value \( \theta_o \) and standard deviation \( \sigma_\theta \), and that the a-posteriori estimate of the parameter, determined from a sample of size \( n \), is normally distributed with some mean \( \hat{\theta} \) and standard deviation \( \hat{\sigma} \). Then the Bayesian estimate of \( \theta \), for a mean squared error loss function, is given by:

\[
\hat{\theta}_B = \left( \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma^2/n} \right) \hat{\theta} + \left( \frac{\sigma^2/n}{\sigma_\theta^2 + \sigma^2/n} \right) \theta_o
\]

(6.54)

With this procedure, immediately after the main shock the a-priori mean parameter values would heavily weight, while during the course of the aftershock sequence, the a-posteriori parameter estimates would be increasingly weighted as the current sequence data become more numerous and consequently their estimation error become smaller (Lolli and Gasperini, 2003). As commented by Rydelek (1990) the statistical distribution of the model parameters is not always normal and this would prevent the formal derivation of the above formula from Bayes rule. Although, Eq. (6.54) can still be used for practical purposes (Reasenberg and Jones, 1990) the implementation of this approach for re-entry protocol development will require specialised
trained of personnel to do the proper adjustments to the model as more data becomes available, in a matter of hours. Unfortunately, this may be difficult or maybe impossible at some of the mines.

Regardless of the low performance of the R-J model for estimating the $K$ values, there is, however, a possible application of this model for the selection of magnitude thresholds for invoking a microseismic magnitude event protocol. By replacing the average correlation between $K$ and $M_{pe} - M_c$ (Eq. (6.50)) in the formula of the time of maximum curvature $T_{MC}$ (Eq. (6.34)) for a power-law MOL ($c = 0$) the following expression is obtained:

$$T_{MC} = \left[10^{11+3(M_{pe}-M_c)} \frac{1}{\sqrt[1+\bar{p}]{\bar{p}}} \right]^{1+\bar{p}}$$  (6.55)

where $\bar{p}$ is the average $p$ value of the site. In this manner, the maximum curvature time can be linked to the magnitude of the principal event. By using Eq. (6.55) it is possible to recognize the increasing trend between $T_{MC}$ and $M_{pe} - M_c$ without the necessity of performing a direct fit to the data. Figure 6.42 presents an example of this relationship for all the aftershock sequences at four different mine sites for the moment scale.
Figure 6.42. Time of maximum curvature as a function of the moment magnitude of the principal event minus the cut-off magnitude and the predictions of the average R-J model at four different mining sites.

It is not expected that Eq. (6.55) will be accurate enough for predictions of $T_{MC}$. However, it can be used to discriminate in the highly scattered data a median principal event magnitude threshold necessary to raise the maximum curvature time above some time threshold (e.g., 2 hours) and set a microseismic magnitude event protocol. From Eq. (6.55) the following expression is obtained for $\overline{M}_{pe}$:
where $T_{MC}^*$ is the time threshold and $\bar{M}_c$ is the average cut-off magnitude of the aftershock sequences considered during the study period (Table 5.4). Table 6.9 presents the difference $\bar{M}_{PE} - \bar{M}_c$ and the corresponding event magnitude threshold $\bar{M}_{PE}$ to raise the maximum curvature time above two hours at each mining site.

Table 6.9: Difference $\bar{M}_{PE} - \bar{M}_c$ and event magnitude threshold $\bar{M}_{PE}$ of a principal event with $T_{MC}^* = 2$ hours for the moment and uniaxial magnitude scales.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\bar{M}_{PE} - \bar{M}_c$</th>
<th>$\bar{M}_{PE}$</th>
<th>$\bar{M}_{PE} - \bar{M}_c$</th>
<th>$\bar{M}_{PE}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.15</td>
<td>-0.90</td>
<td>2.13</td>
<td>-1.45</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>1.65</td>
<td>0.00</td>
<td>2.53</td>
<td>-0.55</td>
</tr>
<tr>
<td>Craig</td>
<td>1.35</td>
<td>-0.35</td>
<td>1.74</td>
<td>-1.00</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.81</td>
<td>-0.65</td>
<td>1.40</td>
<td>-0.90</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>1.77</td>
<td>-0.20</td>
<td>1.47</td>
<td>-0.30</td>
</tr>
<tr>
<td>Macassa</td>
<td>1.23</td>
<td>0.00</td>
<td>2.47</td>
<td>-0.85</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>1.36</td>
<td>-0.75</td>
<td>2.08</td>
<td>-0.95</td>
</tr>
<tr>
<td>Williams</td>
<td>1.09</td>
<td>-0.80</td>
<td>1.64</td>
<td>-1.60</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>0.81</td>
<td>-0.90</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>1.77</td>
<td>0.00</td>
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<td>-0.46</td>
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<tr>
<td></td>
<td>S.D.</td>
<td>0.31</td>
<td>0.36</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Although, by definition magnitude thresholds are site specific, the difference $\bar{M}_{PE} - \bar{M}_c$ has a well constraint, normally distributed, range of values with an average of $1.30 \pm 0.31$ and $1.93 \pm 0.44$ for the moment and uniaxial magnitude scales respectively.
6.3.2 Epidemic type aftershock sequence (ETAS)

It was shown in Section 6.1.5.1 that some sequences may contain significant secondary aftershocks and that these cases can be better represented by the superposition of MOL functions. Based on these considerations Ogata (1988, 1989, 1992, 1999, and 2001) proposed the Epidemic Type Aftershock Sequence (ETAS) model which is a point process in which every event can produce its offspring of events and can be considered as an extension of a single MOL. In the ETAS model, an earthquake occurring at time \( t_i \) can generate aftershocks at a rate that decreases according to the MOL:

\[
n_i(t) = \frac{K_i}{(c + t - t_i)^p}, \quad t > t_i
\]  

(6.57)

where \( K_i \) is proportional to the expected number of aftershocks whose magnitude exceed the cut-off magnitude \( M_c \), generated by an event of magnitude \( M_i \). The model fits well if \( K_i \) is of the form:

\[
K_i = K_o e^{\alpha(M_i - M_c)}
\]  

(6.58)

where \( K_o \) and \( \alpha \) are parameters to be estimated. Therefore, the ETAS model is defined by the superposition:

\[
n(t) = \sum_{i < t} \frac{K_o e^{\alpha(M_i - M_c)}}{(c + t - t_i)^p}
\]  

(6.59)

which depends on the history of occurrence times \( t_i \) and magnitudes \( M_i \) before time \( t \). The parameters \( K_o \), \( c \), \( \alpha \), and \( p \) are assumed to take the same values for all events, thus representing regional characteristics of seismicity (Utsu et al., 1995). Maximum likelihood
estimates of the four parameters can be obtained by maximizing the log-likelihood function $L$ which is given in the same form as Eq. (6.3). Most $p$ and $c$ values obtained for various earthquake data sets fall in the range between 0.9 and 1.4, and between 0.003 and 0.3 days, respectively (Ogata, 1988, 1989, 1992). The parameter $\alpha$, which is typically between 0.2 and 3.0, measures an efficiency with which events above the threshold magnitude $M_c$ generate offspring and is useful in characterizing earthquake sequences quantitatively in relation to the classification into seismic types (Ogata, 1992). For example, earthquake swarms have $\alpha$ values less than 1, and clear and simple main shock-aftershock activity has $\alpha > 2$.

When the ETAS model is applied to aftershock sequences, the event occurring at $t = 0$ must be included in the data. This limits the comparison of the ETAS and MOL to the time intervals $[t_0, t_N]$ and $[t_0, T_E]$. From a preliminary analysis, it was determined that some sequences did not fit the ETAS model which was evidenced by very small $K_o$ or $\alpha$ values ($K_o < 10^{-5}, \alpha < 0.1$) or when the solution did not converge within the fixed number of iterations ($10^8$). These limit values were arbitrarily selected, however, $K_o \approx 10^{-5}$ is the smallest value and $\alpha \approx 0.1$ is within the order of magnitudes reported in Ogata (1992), and in Ogata and Zhuang (2006). Using this criterion, from the 294 mining sequences, and when the moment magnitude scale is employed a total of 42 and 57 sequences did not fit the ETAS model for the time intervals $[t_0, t_N]$ and $[t_0, T_E]$ respectively. If the uniaxial magnitude is used the corresponding results changed to 44 and 52. For the case of the 78 crustal sequences, 2 and 10 sequences did not fit the ETAS model for $[t_0, t_N]$ and $[t_0, T_E]$ respectively. When comparing which model (MOL or ETAS) fits better the data, the aftershock sequences that did not fit the ETAS model are included in the analysis using the solution obtained
in the last iteration. However, for the statistical treatment of the resulting parameters only the sequences that fit the ETAS model are considered.

For each aftershock sequence, the Akaike Information Criterion (AIC) of the ETAS model is compared with that of the MOL for the time intervals $[t_0, t_N]$ and $[t_0, T_E]$ as a function of the number of events per sequence as shown in Figure 6.43 and Figure 6.44 respectively. The results are presented in terms of the fraction of sequences that satisfy: $AIC_e < AIC^\alpha$, where $AIC^\alpha$ and $AIC_e$ denotes the AIC values in the MOL and ETAS formulas respectively.

![Comparison of the MOL and ETAS models through the Akaike Information Criterion as a function of the number of events per sequence for the time interval $[t_0, t_N]$. (a) Moment magnitude; (b) Uniaxial magnitude. Crustal sequences are included in the uniaxial magnitude scale.](image)

Figure 6.43. Comparison of the MOL and ETAS models through the Akaike Information Criterion as a function of the number of events per sequence for the time interval $[t_0, t_N]$. (a) Moment magnitude; (b) Uniaxial magnitude. Crustal sequences are included in the uniaxial magnitude scale.
Figure 6.44. Comparison of the MOL and ETAS models through the Akaike Information Criterion as a function of the number of events per sequence for the time interval \([t_0, T_E]\). (a) Moment magnitude; (b) Uniaxial magnitude. Crustal sequences are included in the uniaxial magnitude scale.

When the time interval \([t_0, t_N]\) is considered (Figure 6.43) and sequences with at least 50 events, it is observed that for mining sequences the ETAS model is preferred (more than 70% of the cases) by \(AIC\) to the MOL for both moment and uniaxial magnitude scales. In the case of crustal sequences the ETAS model is always preferred by \(AIC\) independently of the number of events per sequence (Figure 6.43b).

If the time interval \([t_0, T_E]\) is considered then the number of cases where the ETAS fit better than the MOL is reduced (Figure 6.44). In the case of mining sequences, the MOL performs as well as ETAS. This result indicates that most of the mining sequences correspond to typical main event/aftershock sequences if the end time of a single MOL is carefully selected, for example with the proposed methodology for detecting the power-law time interval (Appendix B). However, for crustal sequences, the ETAS model still performs better than MOL in most of the cases (Figure 6.44b).
The better fit of the ETAS for the crustal sequence for the time interval \([t_0, T_e]\) suggests that these sequences still have significant time clustering within the aftershock sequences in the time interval \([t_0, T_s]\). To check this hypothesis, the ETAS model was fitted to the crustal sequences using the time interval \([T_s, T_e]\). It was found that in only 18% of the 78 crustal sequences the ETAS model is preferred by AIC in this time interval. The implication is that in the time interval \([T_s, T_e]\) there is little time clustering within the aftershock sequences making the MOL a better model. This also corroborates that the proposed method for detecting the power-law time interval \([T_s, T_e]\) is able to properly establish the limits of a single MOL. Based on these results it can be concluded that the ETAS model generally performs better that the MOL when the data includes significant time clustering within the aftershock sequence, however, if the time interval for a single MOL is carefully selected, then the MOL performs as well as or better than ETAS.

Next, the ETAS model parameters are analyzed and compared to those of the MOL. Hereafter, the symbols \(c^{\alpha}, p^{\alpha}\) denote the \(c\) and \(p\) values in the MOL, and \(c^e, p^e\) the ones in the ETAS model. First, the interplay between the parameters of the ETAS model is investigated. Table 6.10 presents the regression results between the parameters \(p^e\) and \(c^e\). This table confirms that \(p^e\) and \(c^e\) are positively correlated in the ETAS model. The estimated values of the regression coefficients are quite similar for the different datasets, time intervals and magnitude scales. Although this correlation was expected, as the MOL already presented the same trade-off (Section 6.1.6.3), it has not been mentioned and/or quantified in the literature. An additional correlation, not reported in the crustal literature, was found for the parameters \(K_o\) and \(\alpha\) (Table 6.11). A higher value of \(\alpha\) corresponds to a lower value of \(K_o\).
Table 6.10: Regression results between the ETAS model parameters $p^e$ and $c^e$ for the time intervals $[t_0, t_N]$ and $[t_0, T_E]$. (a) Moment magnitude; (b) Uniaxial magnitude. Crustal sequences are included in the uniaxial magnitude scale.

(a) MomMag

\[ p^e = a_1 + a_2 \ln c^e \]

<table>
<thead>
<tr>
<th>Site</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$R^2$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.421</td>
<td>0.057</td>
<td>0.602</td>
<td>1.540</td>
<td>0.072</td>
<td>0.718</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>1.220</td>
<td>0.040</td>
<td>0.282</td>
<td>1.285</td>
<td>0.047</td>
<td>0.384</td>
</tr>
<tr>
<td>Craig</td>
<td>1.188</td>
<td>0.031</td>
<td>0.150</td>
<td>1.316</td>
<td>0.057</td>
<td>0.336</td>
</tr>
<tr>
<td>Creighton</td>
<td>1.603</td>
<td>0.115</td>
<td>0.549</td>
<td>1.367</td>
<td>0.075</td>
<td>0.422</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>1.361</td>
<td>0.060</td>
<td>0.583</td>
<td>1.431</td>
<td>0.068</td>
<td>0.593</td>
</tr>
<tr>
<td>Macassa</td>
<td>1.402</td>
<td>0.059</td>
<td>0.399</td>
<td>1.265</td>
<td>0.036</td>
<td>0.192</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>1.366</td>
<td>0.059</td>
<td>0.768</td>
<td>1.476</td>
<td>0.071</td>
<td>0.658</td>
</tr>
<tr>
<td>Williams</td>
<td>1.484</td>
<td>0.091</td>
<td>0.759</td>
<td>1.551</td>
<td>0.097</td>
<td>0.713</td>
</tr>
</tbody>
</table>

(b) uMag

\[ p^e = a_1 + a_2 \ln c^e \]

<table>
<thead>
<tr>
<th>Site</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$R^2$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.376</td>
<td>0.052</td>
<td>0.658</td>
<td>1.482</td>
<td>0.070</td>
<td>0.674</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>1.268</td>
<td>0.044</td>
<td>0.333</td>
<td>1.243</td>
<td>0.037</td>
<td>0.197</td>
</tr>
<tr>
<td>Craig</td>
<td>1.469</td>
<td>0.073</td>
<td>0.746</td>
<td>1.358</td>
<td>0.065</td>
<td>0.456</td>
</tr>
<tr>
<td>Creighton</td>
<td>1.297</td>
<td>0.067</td>
<td>0.422</td>
<td>1.238</td>
<td>0.058</td>
<td>0.412</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>1.537</td>
<td>0.081</td>
<td>0.524</td>
<td>1.686</td>
<td>0.104</td>
<td>0.636</td>
</tr>
<tr>
<td>Macassa</td>
<td>1.328</td>
<td>0.050</td>
<td>0.325</td>
<td>1.555</td>
<td>0.081</td>
<td>0.544</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>1.377</td>
<td>0.061</td>
<td>0.795</td>
<td>1.397</td>
<td>0.062</td>
<td>0.772</td>
</tr>
<tr>
<td>Williams</td>
<td>1.297</td>
<td>0.062</td>
<td>0.672</td>
<td>1.452</td>
<td>0.083</td>
<td>0.604</td>
</tr>
<tr>
<td>California</td>
<td>1.312</td>
<td>0.058</td>
<td>0.428</td>
<td>1.456</td>
<td>0.085</td>
<td>0.256</td>
</tr>
<tr>
<td>Italy</td>
<td>1.359</td>
<td>0.077</td>
<td>0.273</td>
<td>1.333</td>
<td>0.076</td>
<td>0.256</td>
</tr>
</tbody>
</table>
Table 6.11: Regression results between the ETAS model parameters $K_o$ and $\alpha$ for the time intervals $[t_0, t_N]$ and $[t_0, T_E]$. (a) Moment magnitude; (b) Uniaxial magnitude. Crustal sequences are included in the uniaxial magnitude scale.

(a) $MomMag$

\[ K_o = a_1 e^{-a_2 \alpha} \]

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Site & $a_1$ & $a_2$ & $R^2$ & $a_1$ & $a_2$ & $R^2$ \\
\hline
A & 0.158 & 0.894 & 0.672 & 0.132 & 0.668 & 0.387 \\
Copper Cliff North & 0.109 & 0.848 & 0.658 & 0.211 & 1.271 & 0.777 \\
Craig & 0.319 & 1.016 & 0.702 & 0.496 & 1.155 & 0.771 \\
Creighton & 0.235 & 0.542 & 0.771 & 0.207 & 0.578 & 0.840 \\
Kidd Creek & 0.222 & 1.036 & 0.633 & 0.267 & 1.183 & 0.667 \\
Macassa & 0.157 & 0.573 & 0.703 & 0.092 & 0.389 & 0.324 \\
McCreedy East & 0.106 & 0.639 & 0.569 & 0.196 & 1.133 & 0.671 \\
Williams & 0.168 & 0.613 & 0.736 & 0.427 & 0.977 & 0.743 \\
\hline
\end{tabular}

(b) $uMag$

\[ K_o = a_1 e^{-a_2 \alpha} \]

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Site & $a_1$ & $a_2$ & $R^2$ & $a_1$ & $a_2$ & $R^2$ \\
\hline
A & 0.152 & 1.803 & 0.886 & 0.217 & 1.948 & 0.819 \\
Copper Cliff North & 0.195 & 1.861 & 0.858 & 0.136 & 1.547 & 0.732 \\
Craig & 1.156 & 2.160 & 0.800 & 0.152 & 1.270 & 0.923 \\
Creighton & 0.181 & 0.682 & 0.920 & 0.302 & 0.920 & 0.735 \\
Kidd Creek & 0.093 & 0.710 & 0.694 & 0.160 & 0.969 & 0.761 \\
Macassa & 0.198 & 1.670 & 0.850 & 0.232 & 1.778 & 0.729 \\
McCreedy East & 0.197 & 1.871 & 0.920 & 0.278 & 2.052 & 0.927 \\
Williams & 0.183 & 0.931 & 0.709 & 0.347 & 0.882 & 0.522 \\
California & 0.362 & 1.742 & 0.861 & 0.435 & 1.870 & 0.871 \\
Italy & 0.385 & 1.618 & 0.698 & 1.375 & 2.345 & 0.908 \\
\hline
\end{tabular}

Figure 6.45 and Figure 6.46 present the scatter plots of $(c^o, c^e)$ and $(p^o, p^e)$ respectively for the time interval $[t_0, T_E]$. In the case of mining sequences moment and uniaxial magnitude are included in the same plot for the sake of brevity. Figure 6.45 and Figure 6.46 shows that $c^e$ is smaller than $c^o$, while $p^e$ is generally larger than $p^o$.
Figure 6.45. Scatter plot of \( c^o \) and \( c^e \) for the time interval \([t_0, T_E]\).

Figure 6.46. Scatter plot of \( p^o \) and \( p^e \) for the time interval \([t_0, T_E]\).

Table 6.12 summarizes the average ETAS model parameters for the \([t_0, t_N]\) time interval. This time interval was selected given the better fit of ETAS over the MOL. Also, the dispersion of the parameters was less for this time interval compared to \([t_0, T_E]\).
Table 6.12: Determined range and average ETAS model parameters for the time interval $[t_0, t_N]$. (a) Moment magnitude; (b) Uniaxial magnitude. Crustal sequences are included in the uniaxial magnitude.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\log c^e$ min-max</th>
<th>average</th>
<th>$\log K_o$ min-max</th>
<th>average</th>
<th>$p^e$ min-max</th>
<th>average</th>
<th>$\alpha$ min-max</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-4.30 ~ -1.00</td>
<td>-2.93±0.84</td>
<td>-2.74 ~ -0.97</td>
<td>-1.40±0.38</td>
<td>0.81~1.03</td>
<td>1.04±0.14</td>
<td>0.31~3.37</td>
<td>1.54±0.81</td>
</tr>
<tr>
<td>CCN</td>
<td>-4.54 ~ -1.22</td>
<td>-3.70±0.62</td>
<td>-2.54 ~ -1.06</td>
<td>-1.49±0.34</td>
<td>0.67~1.23</td>
<td>0.88±0.11</td>
<td>0.31~3.28</td>
<td>1.43±0.74</td>
</tr>
<tr>
<td>CRA</td>
<td>-3.29 ~ -1.59</td>
<td>-2.25±0.76</td>
<td>-2.60 ~ -1.66</td>
<td>-1.98±0.31</td>
<td>0.84~1.17</td>
<td>1.03±0.14</td>
<td>2.77~4.01</td>
<td>3.36±0.60</td>
</tr>
<tr>
<td>CRE</td>
<td>-4.23 ~ 0.14</td>
<td>-2.31±0.99</td>
<td>-1.36 ~ -0.98</td>
<td>-1.49±0.88</td>
<td>0.54~2.51</td>
<td>0.99±0.39</td>
<td>0.25~13.05</td>
<td>3.65±3.29</td>
</tr>
<tr>
<td>KC</td>
<td>-4.82 ~ 0.29</td>
<td>-2.81±1.11</td>
<td>-3.56 ~ -0.91</td>
<td>-1.61±0.58</td>
<td>0.69~1.84</td>
<td>0.97±0.20</td>
<td>0.32~5.05</td>
<td>2.12±1.02</td>
</tr>
<tr>
<td>MA</td>
<td>-4.73 ~ -0.73</td>
<td>-2.54±0.93</td>
<td>-3.36 ~ -0.98</td>
<td>-1.47±0.46</td>
<td>0.76~1.58</td>
<td>1.06±0.20</td>
<td>1.22~9.10</td>
<td>2.67±1.54</td>
</tr>
<tr>
<td>MC</td>
<td>-4.11 ~ -1.75</td>
<td>-3.26±0.85</td>
<td>-1.66 ~ -0.94</td>
<td>-1.31±0.22</td>
<td>0.78~1.24</td>
<td>0.92±0.13</td>
<td>0.12~2.43</td>
<td>1.20±0.61</td>
</tr>
<tr>
<td>W</td>
<td>-3.88 ~ -1.65</td>
<td>-2.77±0.50</td>
<td>-1.91 ~ -1.06</td>
<td>-1.49±0.21</td>
<td>0.62~1.21</td>
<td>0.90±0.12</td>
<td>1.07~3.70</td>
<td>2.68±0.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site</th>
<th>$\log c^e$ min-max</th>
<th>average</th>
<th>$\log K_o$ min-max</th>
<th>average</th>
<th>$p^e$ min-max</th>
<th>average</th>
<th>$\alpha$ min-max</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-4.73 ~ -1.10</td>
<td>-2.88±0.91</td>
<td>-3.40 ~ -0.93</td>
<td>-1.79±0.78</td>
<td>0.75~1.29</td>
<td>1.03±0.13</td>
<td>0.13~3.15</td>
<td>1.25±0.94</td>
</tr>
<tr>
<td>CCN</td>
<td>-4.46 ~ -1.20</td>
<td>-3.74±0.62</td>
<td>-4.86 ~ -1.01</td>
<td>-1.51±0.61</td>
<td>0.67~1.29</td>
<td>0.89±0.11</td>
<td>0.20~4.33</td>
<td>0.99±0.70</td>
</tr>
<tr>
<td>CRA</td>
<td>-3.71 ~ -1.77</td>
<td>-2.63±0.80</td>
<td>-3.45 ~ -1.57</td>
<td>-2.08±0.92</td>
<td>0.86~1.18</td>
<td>1.03±0.16</td>
<td>1.31~3.43</td>
<td>2.28±0.87</td>
</tr>
<tr>
<td>CRE</td>
<td>-4.17 ~ 0.15</td>
<td>-2.50±1.23</td>
<td>-3.73 ~ -0.68</td>
<td>-1.46±0.62</td>
<td>0.60~2.00</td>
<td>0.91±0.29</td>
<td>0.24~10.30</td>
<td>2.43±0.01</td>
</tr>
<tr>
<td>KC</td>
<td>-4.81 ~ -0.90</td>
<td>-2.95±0.85</td>
<td>-4.88 ~ -0.70</td>
<td>-1.82±0.92</td>
<td>0.51~1.93</td>
<td>0.99±0.22</td>
<td>0.26~13.87</td>
<td>2.55±2.45</td>
</tr>
<tr>
<td>MA</td>
<td>-4.28 ~ -0.76</td>
<td>-2.48±0.86</td>
<td>-4.96 ~ -1.04</td>
<td>-2.08±0.83</td>
<td>0.73~1.65</td>
<td>1.04±0.17</td>
<td>0.18~9.42</td>
<td>1.89±1.06</td>
</tr>
<tr>
<td>MC</td>
<td>-4.89 ~ -2.10</td>
<td>-3.50±0.73</td>
<td>-4.60 ~ -0.95</td>
<td>-1.63±0.81</td>
<td>0.71~1.11</td>
<td>0.89±0.12</td>
<td>0.19~4.14</td>
<td>1.13±0.95</td>
</tr>
<tr>
<td>W</td>
<td>-4.95 ~ -1.56</td>
<td>-2.94±0.73</td>
<td>-2.06 ~ -0.87</td>
<td>-1.42±0.29</td>
<td>0.62~1.25</td>
<td>0.88±0.13</td>
<td>0.32~2.78</td>
<td>1.69±0.60</td>
</tr>
<tr>
<td>CAL</td>
<td>-4.37 ~ -0.29</td>
<td>-2.25±0.76</td>
<td>-3.06 ~ -0.97</td>
<td>-1.64±0.43</td>
<td>0.67~1.44</td>
<td>1.01±0.15</td>
<td>0.83~3.64</td>
<td>1.58±0.53</td>
</tr>
<tr>
<td>ITA</td>
<td>-1.87 ~ -0.70</td>
<td>-1.34±0.42</td>
<td>-2.27 ~ -1.04</td>
<td>-1.65±0.38</td>
<td>0.93~1.38</td>
<td>1.12±0.14</td>
<td>1.00~2.65</td>
<td>1.76±0.45</td>
</tr>
</tbody>
</table>

Note: CCN: Copper Cliff North, CRA: Craig, CRE: Creighton, KC: Kidd Creek, MA: Macassa, MC: McCriddy East, W: Williams, CAL: California, ITA: Italy.

Inspection of Table 6.12 provides the following regarding the ETAS model parameters:

1. There is a wide range of possible values for the parameters $c^e$ and $K_o$. The value $c^e$ may affect the response of the model for short $t$ after the principal event, but the proportional constant $K_o$ may affect the overall aftershock productivity of the model.
2. Average $p^*$ values are similar for both magnitude scales.

3. There is a considerable scatter in the $\alpha$ parameter, with standard deviations ranging from 0.45 to 3.29. A narrower range of average values of $\alpha$ is found for the uniaxial magnitude (0.99–2.54) compared to the moment magnitude (1.20–3.65). A main statistical study of the ETAS model parameters correspond to Guo and Ogata (1997) for Japanese crustal aftershock sequences. These authors obtained a range for $\alpha$ from 0.55 to 4.30 with an average of 1.98±0.89, which matches very well with the corresponding values obtained in this study for the crustal seismicity in California (1.58±0.53) and Italy (1.76±0.45). Except for Craig and Creighton mines when using the moment magnitude scale, the average values of $\alpha$ obtained for mining-induced aftershock sequences are in the same range as the crustal values (less than 3).

4. Average values larger than 2.0 are found for Craig, Creighton, Kidd Creek, Macassa and Williams for the moment magnitude scale, and for Craig, Creighton and Kidd Creek for the uniaxial magnitude scale. These high values of $\alpha$, explains why the MOL equation fits better than the ETAS model in most of the mining seismic sequences (Figure 6.43 and Figure 6.44) as they correspond in general to simple main shock-aftershock activity.

There are certain clear advantages of the ETAS model over the MOL:

1. All parameters are fixed a-priori based on site specific averages.

2. The estimated parameters are less sensitive to the selected time interval used to fit the equation.

3. Magnitude is explicitly accounted for into the formulation.
Given the ability of the ETAS model to reproduce time clustering within the sequence the parameters can be established using individual sequences or the complete catalogue. However, for evaluating the range of possible values of the parameters, analysis of individual aftershock sequences is mandatory.

There are also certain limitations:

1. There are more parameters to specify. Some of them with high variability, e.g., \( K_o \), \( c^e \), and \( \alpha \) (Table 6.12).

2. The correlation between the parameters \( c^e \) and \( p^e \), \( K_o \) and \( \alpha \) cannot be avoided.

3. To use the ETAS model for estimating the event rate it is necessary to have as an input the magnitude of each event. In the case of microseismic arrays sensors may clip for large magnitude events.

4. The forecasting and maximum curvature point of the ETAS model depends on the history of occurrences times \( t_i \) and magnitudes \( M_i \) before time \( t \), i.e., the development of the decay curve is an ongoing process that has to be updated after each occurrence of a new event. For the cumulative number of events from the principal event until time \( t \), the following equation is obtained:

\[
N(t) = \frac{K_o}{1 - p^e} \left[ \sum_{t_i < t} e^{\alpha(M_i - M_o)} \left( (t - t_i + c^e)^{1-p^e} - (c^e)^{1-p^e} \right) \right] (6.60)
\]

The number of events occurring during the last time window \( \Delta t \) until time \( t \) is given by an equation similar to Eq. (3.14). The time of maximum curvature has to be evaluated at each time \( t \) after the principal event by finding numerically the time that maximizes the following equation:
\[ \rho(t) = \frac{j(t)}{\left(1 + \left(\dot{j}(t)\right)^2\right)^{3/2}} = \frac{K_o p (p+1) \sum \frac{e^{\alpha(M_c-M_t)}}{(c^s + t - t_i)^p + 2}}{\left[1 + \left(\sum \frac{e^{\alpha(M_c-M_t)}}{(c^s + t - t_i)^p + 1}\right)^{3/2}\right]} \] (6.61)

Overall, the ETAS model can be seen as a complementary tool for evaluating the response of the ground to seismic activity instead of a replacement of the MOL. As with the other models, for using this formulation for re-entry protocol development, it is necessary to set a-priori parameters. Figure 6.47 presents an example of the ETAS guidelines applied to the rockburst aftershock sequence at the Copper Cliff North Mine presented in Figure 6.32 using the determined average parameters listed in Table 6.12 for the moment magnitude scale. It is observed in Figure 6.47a that the ETAS model produces low productivity, in terms of events per hour, in the first 4 hours of the sequence compared to the average MOL and the actual sequence. After hour 4, the average MOL and ETAS are almost coincident, with the exception that the ETAS model is able to reproduce local features of the data. In Figure 6.47b the maximum curvature relative to the actual time \( t \) after the principal event reaches a peak at hour 1.2, and then starts decreasing until a second large magnitude event (\( M_n=1.4 \)) occurring 6.2 hours after the initial event creates a spike in the maximum curvature. It is not until hour 12 that the decay starts to oscillate at an approximately constant rate. This compares very well with the time of maximum curvature calculated with the MOL using \( N_1 \) (Figure 6.32).
Figure 6.47. Proposed ETAS guidelines for the rockburst sequence (Mₖ=2.4) at the Copper Cliff North Mine presented in Figure 6.32. (a) Number of events per hour; (b) Magnitude-time plot; (c) Time of maximum curvature at each time t after the principal event. A second large magnitude event (Mₖ=1.4) occurred in the same zone 6.2 hours after the initial event.
6.4 Båth’s law

This scaling relation states that the difference in magnitude between the main event and its largest aftershock is approximately constant, independent of the magnitude of the main event (Båth, 1965). That is:

$$\Delta M = M_{PE} - M_{LA}$$ (6.62)

where $M_{PE}$ and $M_{LA}$ are the magnitude of the principal event and largest aftershock respectively. The scaling associated with Båth’s law implies that the stress transfer responsible for the occurrence of aftershocks is a self-similar process (Shcherbakov and Turcotte, 2004b). For crustal earthquakes this difference is typically taken to be $\Delta M \approx 1.2$. This law has been examined in detail by several authors (Utsu, 1961; Vere-Jones, 1969; Kisslinger and Jones, 1991; Guo and Ogata, 1997; Tsapanos, 1990b; Drakatos and Latoussakis, 2001; Felzer et al., 2002; Console et al., 2003; Helmstetter and Sornette, 2003). Shcherbakov et al. (2004) and Shcherbakov and Turcotte (2004b and 2005) proposed a modified form of this law by introducing the notion of inferred largest aftershock from an extrapolation of the Gutenberg-Richter frequency-magnitude statistic of the sequence following a given main shock. For 10 large earthquakes that occurred in California they found $\Delta M = 1.16 \pm 0.46$ from the detected data and $1.11 \pm 0.29$ by using the inferred largest aftershock.

Figure 6.48 presents $\Delta M$ for different datasets compiled from the crustal literature. Taking into account the differences in the methods used for the selection of main shock and largest aftershock, these studies agree on average with Båth’s law ($1.35 \pm 0.66$). However, $\Delta M$ differs from sequence to sequence with fluctuation between 0.0 and 3.2. Average values of $\Delta M$ for different sites around the world range from 0.84 to 1.53. Note that there are no negative $\Delta M$ values in accordance with the definition of aftershocks used by these authors.
Figure 6.48. Relative frequency histogram of Båth’s law for different crustal datasets compiled from the literature. A normal distribution (solid line) with the observed Chi-Sq significance level is included in each frame.
In terms of re-entry protocol development Båth’s law provides a potential method to statistically estimate the magnitude of the largest aftershock. From a preliminary analysis of mining-induced aftershock sequences, it was determined that the majority of the larger aftershocks have a tendency to occur close to the principal event in the time sequence. In addition, for some sequences, the largest magnitude event was not always associated with the first event of the sequence (see Figure 6.38). These considerations limited the direct applicability of Båth’s law to mining seismicity. In order to be useful for re-entry purposes, Båth’s law should consider the main event as the largest magnitude induced at the beginning of the sequence and the largest aftershock at a latter stage. The following adjustments are considered:

1. The magnitude of the principal event is defined as the largest magnitude event occurring during the first hour $M_1$ after the event that initiated the sequence. This selection was based on the fact that 98% of the seismic sequences analyzed presented a start time of power-law decay $T_s$ less than one hour (Figure 6.11).

2. The largest aftershock needs to be defined for a time period of interest after the first hour. A natural choice is the period where the time sequence still corresponds to a single aftershock sequence, i.e., before the end of power-law decay $T_E$. The distribution of $T_E$ for mining seismicity (Figure 6.15), indicates values of 7.4, 14.1, 26.7, and 69.4 hours for percentiles 0.25, 0.5, 0.75 and 0.9 respectively. For re-entry purposes it is impractical to have an estimate of the magnitude of the largest aftershock one day after the main event. It was decided that the most practical time period for developing statistics for the largest aftershock is for a time period less than 12 hours after the principal event. This is almost coincident with the median of end times of power-law decay, and with the two working 10.5 hour shifts.
per day used at most of the surveyed mines. This is the period of greatest interest, should there be a large aftershock event.

3. There is one additional consideration for the application of Eq. (6.62) to mining seismicity. The reliability of $M_{PE}$ and $M_{LA}$ has to be considered. There is no simple manner to compensate $M_{PE}$ due to the clipping of amplitudes recorded by the sensors in the microseismic array for large magnitude events. However, the statistical treatment of the data would camouflage or reflect this deficiency, for example, with a low $\Delta M$ value.

Another concern is the reliability of microseismic monitoring system for detecting the largest aftershock magnitude associated with a specific principal event. It is not expected that Båth’s law will scale for a principal event with a largest aftershock magnitude lower than the lowest magnitude that can be reliably detected by the microseismic monitoring system. For example, if a principal event of magnitude -1.5 is considered and the lowest reliable magnitude of the system is -1.9, then the maximum value that Båth’s law can take is $\Delta M = -1.5 - (-1.9) = 0.4$. To avoid this bias it is necessary to include into the analysis only principal events with a certain magnitude higher than the lowest detection magnitude. It is tentative to use all principal events that have a magnitude higher than lowest detection magnitude plus 1.2. However, in this criterion there will be a possible bias in the results as the average Båth’s law from crustal sequences is assumed in advance. It is necessary to set a principal event magnitude threshold based on an independently principle.

Previously, the average parameters of the Reasenberg-Jones model (Section 6.3.1) were used to set a microseismic magnitude event protocol (Table 6.9). This magnitude threshold is used to filter out small principal events and test the validity of Båth’s law for mining seismicity.
With these considerations Båth’s law can be re-written by:

\[
\Delta M = M_1 - M_{LA}^{1-12 \text{ hours}}; \quad M_1 \geq \overline{M}_{PE}
\]  

(6.63)

where \( \overline{M}_{PE} \) is obtained for each site from Table 6.9. This modified form of Båth’s law was evaluated for both the seismic moment and uniaxial magnitude scales (Table 6.13).

Table 6.13: Modified Båth’s law statistics for the analyzed mining seismic sequences. (a) Moment magnitude; (b) Uniaxial magnitude.

<table>
<thead>
<tr>
<th>Site</th>
<th>MomMag M Δ</th>
<th>uMag M Δ</th>
<th>PChi-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site</td>
<td>min</td>
<td>max</td>
<td>average</td>
</tr>
<tr>
<td>A</td>
<td>-0.21</td>
<td>1.82</td>
<td>1.28±0.54</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>-0.35</td>
<td>1.94</td>
<td>0.69±0.48</td>
</tr>
<tr>
<td>Craig</td>
<td>0.10</td>
<td>1.26</td>
<td>0.59±0.50</td>
</tr>
<tr>
<td>Creighton</td>
<td>-0.24</td>
<td>0.77</td>
<td>0.30±0.30</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>0.43</td>
<td>1.98</td>
<td>1.27±0.44</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.04</td>
<td>1.60</td>
<td>1.04±0.33</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>-0.57</td>
<td>1.62</td>
<td>0.78±0.68</td>
</tr>
<tr>
<td>Williams</td>
<td>-0.25</td>
<td>0.95</td>
<td>0.45±0.34</td>
</tr>
<tr>
<td></td>
<td>-0.32</td>
<td>2.63</td>
<td>1.08±0.68</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>-0.48</td>
<td>2.44</td>
<td>1.18±0.73</td>
</tr>
<tr>
<td>Craig</td>
<td>0.56</td>
<td>1.78</td>
<td>0.95±0.57</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.34</td>
<td>2.81</td>
<td>0.91±0.62</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>-0.26</td>
<td>1.74</td>
<td>0.67±0.50</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.19</td>
<td>2.47</td>
<td>1.55±0.57</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>-0.05</td>
<td>2.45</td>
<td>1.07±0.80</td>
</tr>
<tr>
<td>Williams</td>
<td>-0.40</td>
<td>1.75</td>
<td>0.80±0.58</td>
</tr>
</tbody>
</table>

*: Not enough cases for evaluating \( P_{\text{Chi-Square}} \).

Excluding Craig, Creighton and Williams mines for the moment magnitude scale, the average values of \( \Delta M \) range from 0.67 to 1.57, very similar to the ones found for crustal sequences (Figure 6.48). Table 6.13 also provides that \( \Delta M \) evaluated using the uniaxial magnitude scale...
has a wider range compared to the moment magnitude scale. Overall, the uniaxial magnitude is more suitable for evaluating and using Båth’s law for mining-induced seismicity. Note that on average, Båth’s law is still valid, without the necessity of restricting the magnitude of the aftershocks to be lower than the main shock.

In the case of the crustal sequences analyzed, Båth’s law is evaluated in the conventional way by using Eq. (6.62) with the principal event magnitude occurring at \( t = 0 \). However, the largest aftershock was picked for two time intervals: \( (t_0, t_N) \) and \( (t_0, T_E) \). In addition to the largest aftershock, statistics for the second largest aftershock were also developed for both time intervals (Table 6.14).

<table>
<thead>
<tr>
<th></th>
<th>( (t_0, t_N) )</th>
<th>( (t_0, T_E) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta M )</td>
<td>( t_0 ) ( t_N )</td>
<td>( t_0 ) ( T_E )</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>California</td>
<td>0.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Italy</td>
<td>0.10</td>
<td>1.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( (t_0, t_N) )</th>
<th>( (t_0, T_E) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta M )</td>
<td>( t_0 ) ( t_N )</td>
<td>( t_0 ) ( T_E )</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>California</td>
<td>0.10</td>
<td>2.20</td>
</tr>
<tr>
<td>Italy</td>
<td>0.10</td>
<td>2.10</td>
</tr>
</tbody>
</table>

Table 6.14a shows that when the largest aftershock is selected in the time interval \( (t_0, T_E) \), \( \Delta M \) is closer to the value 1.2 originally suggested by Båth’s. However, there is no improvement in the
standard deviation compared to the time interval \( \left( t_0, t_N \right) \). In addition, Båth’s law seems to increase in approximately 0.26±0.03 when the second largest aftershock is used (Table 6.14b).

The developed statistics for Båth’s law can also be used to estimate the probability of occurrence of a seismic event with magnitude higher than a certain threshold \( M^*_L \). Given the maximum magnitude event recorded during the first hour \( M_1 \) and using the fact that \( \Delta M \) is approximately normally distributed (Table 6.13), with some mean value \( \mu_{\Delta M} \) and standard deviation \( \sigma_{\Delta M} \), the probability of occurrence of a seismic event with magnitude higher than \( M^*_L \) during the next 11 hours can be estimated by:

\[
P(M \geq M^*_L) = \text{Normal}(M_1 - M^*_L, \mu_{\Delta M}, \sigma_{\Delta M})
\]  

(6.64)

This approach assumes, however, that the occurrence time of the largest aftershock \( t_{LA} \) measured from the principal event is a uniform process with constant probability and does not decay in time as with the Reasenberg-Jones model (Figure 6.40 and Figure 6.41). This is not formally correct as there is a higher concentration of largest aftershocks closer to the principal event (Figure 6.49).

![Figure 6.49. Frequency distribution of the time of the largest aftershock for crustal (frame a) and mining sequences (frame b).](image)
Båth’s law is significant for re-entry protocol development as the largest magnitude that should be expected will depend (within a certain error) on the measured event magnitude at the beginning of the sequence. Figure 6.50 shows an example for a rockburst related sequence. It can be observed that Båth’s law provides a framework for discriminating which events are significant given the magnitude induced at the beginning of the sequence.

Figure 6.50. Båth’s law applied to a rockburst sequence at the Copper Cliff North Mine.

This law is integrated with the MOL guidelines applied in Section 6.1.8. The course of action is to consider a time window of at least two hours after the maximum curvature point. If an event
with magnitude higher than the one estimated by Båth’s law occurs during these two hours, the re-entry clock is reset and the time window is moved, the procedure being repeated until no significant magnitude events are measured in the time window.

6.5 Summary and discussion
In this chapter the characteristics of aftershock were analyzed to determine the applicability of three empirical scaling relations to mining-induced seismicity: (1) Modified Omori’s law for the temporal decay of aftershocks, (2) Gutenberg-Richter frequency-magnitude scaling, and (3) Båth’s law for the magnitude of the largest aftershock. In addition, two alternative rate stochastic models (Reasenberg-Jones and ETAS) were presented and examined in detail. The discussion will concentrate on the applicability of each of these scaling laws and stochastic models for re-entry protocol development.

6.5.1 Modified Omori’s law (MOL)
It was statistically demonstrated that the event decay rate \( n(t) \) of mining-induced aftershock time sequences can be satisfactorily described by the modified Omori’s law:

\[
n(t) = \frac{K}{(c + t)^p}
\]

where \( t \) is the time measured from the principal event, \( c \) is an offset time constant, \( p \) is a parameter related to the speed of decay, and \( K \) is a productivity parameter related to the number of events occurring in a time period \([T_A, T_B]\). For estimating consistent decay parameters \( K \) and \( p \) it is necessary to exclude aftershocks from the start and end of the sequence and to consider the time interval \([T_S, T_E]\) that satisfies power-law decay. Based on synthetic aftershock sequence simulations and formal properties of aftershock time sequences, a new uniform and systematic
method was developed for estimating the power-law time interval decay (Appendix B). For all the considered aftershock sequences, the proposed method performed well in estimating a proper power-law time interval, which was statistically confirmed by means of the Akaike Information Criterion (Section 6.1.5).

Four main stages were consistently identified for both crustal and mining-induced seismic sequences (Figure 6.51):

1. \([t_0, T_s]\): Initial noisy stage before the onset of the power-law decay. Several factors may affect the decay during this stage, such as: overlapping of seismic records that make it difficult to identify and locate the many events (Kagan, 2004), a complex process which the MOL is not able to adequately describe, or the sequence may actually begin gradually and build to a higher rate before the onset of smooth decay (Gross and Kisslinger, 1994). It was found that both crustal and mining-induced aftershock sequences display a non-power-law decay only for short times (< 1 time unit) and that there is no consistent rate model in the time interval \([t_0, T_s]\). This suggests that the parameter \(c\) does not represent any intrinsic property of the aftershock sequence.

2. \([T_s, T_{MC}]\): Once the power-law decay starts, there is a characteristic point at the maximum curvature time of the MOL. This point defines the transition from high to low event change rate.

3. \([T_{MC}, T_E]\): After \(T_{MC}\) the power-law decay can still continue until a time where the data deviates from power-law behaviour.
Figure 6.51. Example of the main stages identified for aftershock time sequences. (a) Event rate; (b) Cumulative number of events.
4. \([T_E, t_N]\): \(T_E\) defines the end of the power-law regime and a transition to a different process, while \(t_N\) is the last event identified by the (temporal) clustering algorithm. In the example presented in Figure 6.51, \(T_E\) is coincident with the start of a secondary aftershock sequence. It was found that in most of the cases an exponential transition systematically commenced after the determined end time of power-law decay. This time has been previously interpreted in the literature as a “correlation time”, after which aftershocks cease to occur and healing dominates (Narteau et al., 2002).

In the context of developing guidelines for re-entry protocols, there are some practical advantages to determining power-law MOL parameters. Firstly, there is one less parameter (\(c\)) to consider a-priori. This also eliminates the positive interplay between the parameters \(p\) and \(c\). In addition, \(c\) values may be difficult to estimate accurately (Utsu et al., 1995). Secondly, by removing the complex aftermath of the principal event it is possible to eliminate any instrument related factors that can affect the estimated parameters enabling them to be compared for different mining environments.

The \(p\) value varies from sequence to sequence, with most (98%) being within a typical range of 0.4–1.6, with average values ranging from 0.74 to 1.04. Each distribution seems to represent specific conditions of the local environment, with higher decay values for the Mine A, Macassa and Kidd Creek sites than the Sudbury sites. This variability may be related to the properties and characteristics of the fault system, as suggested by Kisslinger (1996) for crustal earthquakes, but it is still not possible to draw conclusions of the significant factors that control the \(p\) value in mining-induced aftershock sequences. The parameter \(K\) can be adequately expressed by:

\[ K = \kappa N_1 \]

where \(\kappa\) is an activity ratio and \(N_1\) is the measured number of events occurring
during the first hour after the principal event. It is found that the $\kappa$ values fall in a well-constrained range. Theoretically, $\kappa \leq 0.8$ and the data analysis of ten different seismic environments suggests that $\kappa \in [0.3 - 0.5]$. Based on the analysis of locally observed historical seismicity, both the $p$ and $\kappa$ values can be estimated. Using the MOL formula and the above guidelines, it is possible to establish a decay event rate that can be used as a frame of reference for comparing the behaviour of the ongoing sequence.

The use of the first time unit of data to establish $K$ may be viewed as a limitation, however, for mining-induced aftershock sequences this is not a concern as, in general, when a re-entry protocol is invoked the minimum re-entry time is two hours. In addition, a minimum time is required after blasts to allow the gases to dissipate before the site is re-entered. If this minimum gas dissipation time is set to 1 hour after the blast it gives enough time to establish the MOL $K$ parameter and forecast the aftershock rate decay.

Three different criteria were presented and discussed for estimating the time at which it may be considered appropriate to re-enter the area by using the MOL. The time of maximum curvature $T_{MC}$ resulted in a practical criterion for developing a defendable re-entry protocol. In the case of a time series obeying a MOL process, this time defines the transition between the highest to lowest event rate change. Based solely on the aftershock decay rate, therefore, it is recommended using $T_{MC}$ as a preliminary estimate of the time at which it may be considered appropriate to re-enter the area. It was found that $T_{MC}$ can be estimated, without the need of specifying a $p$ value, by the expression: $T_{MC} = \chi (N_1)^\beta$ where $\chi$ and $\beta$ are two parameters dependent on local conditions. Both parameters presented remarkably well-defined average empirical ranges for the
sites analyzed: $\chi \in [0.2 - 0.6]$ and $\beta \in [0.45 - 0.8]$. The maximum curvature point was also used to establish:

1. An excessive seismicity protocol.
2. An equivalent to the background time window which in most of the cases was between 1 and 2 hours.

### 6.5.2 Frequency-magnitude distribution

The frequency-magnitude distribution (Gutenberg and Richter, 1944) was fitted to the aftershock magnitudes for the time interval of power-law decay $[T_s, T_e]$ for both the moment and uniaxial magnitude scales. This was necessary to prepare the data for the Reasenberg and Jones (1989) model presented in Section 6.3.1. However, individual conclusions on the Gutenberg-Richter (G-R) scaling law applied to mining-induced aftershock sequences were found.

The criterion of Wiemer and Wyss (2000) was used for estimating the magnitude completeness $M_c$. This method makes use of a goodness of fit $R$ to identify and define $M_c$ at the point at which a power-law can model 90% of more of the frequency-magnitude distribution (FMD). Accordingly, it was found that only 49% and 60% of the mining-induced aftershock sequences analyzed satisfied the condition $R \geq 0.9$ for the moment and uniaxial magnitude scales respectively. These low number of cases for which the G-R law is actually able to describe more than 90% of the FMD is consistent with observations of previous authors. For example, Trifu et al. (1993) mentioned a limited self-similar magnitude domain between moment magnitudes -1.1 and -0.4 for a rockburst related aftershock sequence in Strathcona Mine, and, Trifu and Shumila (2005) recognized that fitting the observed magnitude event recurrence graph to log-linear distributions can be highly subjective if not impossible.
The higher number of sequences for which the uniaxial magnitude scale presented $R \geq 0.9$, suggests that the G-R relationship fits better the uniaxial than the moment FMD.

A small benefit was found from choosing $M_z$ higher than the one selected at the bin with the highest frequency of events in a non-cumulative frequency-magnitude distribution $M_e$.

It was found that the average $b$-values were not affected by the selected level of $R$. Two mining sites presented particular high average $b$-values for the moment magnitude scale. However, the same effect was not observed when the uniaxial magnitude scale was used. In this scale of magnitude the average $b$-values ranges from 0.68 to 1.31 which is more consistent with the range of $b$-values observed at the crustal scale.

It can be concluded that $M_c$ can be used for estimating the G-R parameters and that the uniaxial magnitude scale is more consistent for estimating the FMD parameters and performing aftershock statistics.

### 6.5.3 Other rate stochastic models

In addition to the MOL, two additional rate models available in the crustal literature were presented and examined in detail:

1. Reasenberg and Jones model.
2. Epidemic type aftershock sequences (ETAS).

These models are an attempt to link magnitude and rate in a single model. Therefore, they should bridge the limitation of the MOL, which does not incorporate the magnitude of the event. The Reasenberg and Jones model makes use exclusively of the principal event magnitude to establish
the productivity of the sequence and forecast the time decay, while in the ETAS model each event in the sequence can generate aftershocks at a rate that decreases according to the MOL.

6.5.3.1 Reasenberg and Jones (R-J)
A detailed study of the R-J model was presented. The analysis indicated that it is much more accurate to estimate \( K \) from the number of events occurring during the first time unit after the principal event \( N_1 \) for both crustal and mining sequences, than using the magnitude of the principal event.

There are several reasons for the low performance of this model. Good parameter fitting requires a well-recorded sequence with a magnitude range of about 3 or 4 between the completeness magnitude \( M_c \) and the main shock magnitude (Klein et al., 2006). This is not always satisfied, especially for the microseismic moment magnitude (see Table 5.1). It has been also noted by Helmstetter (2003) that taking \( b \) as the linear coefficient of \( M_{PE} \) is only a hypothesis, although assumed in many other studies (Kagan and Knopoff, 1987; Davis and Frolich, 1991a and 1991b; Console and Murru, 2001 and Felzer et al., 2002), and there is a lack of a clear theoretical and empirical justification. It can be heuristically deduced from the assumption that the main shock and the aftershocks belong to the same population and follow the same G–R law. Notwithstanding that this hypothesis might sound reasonable, no strict physical arguments or unequivocal empirical evidence can be found proving that the productivity scales with magnitude with a coefficient exactly equal to \( b \) (Gasperini and Lolli, 2006). An additional reason, not mentioned in the crustal literature, could be the effect of the mechanism on the productivity of aftershocks, which was not considered in the analysis. It is expected that for mining seismicity, the mechanism of the sequence (e.g., pillar burst, fault slip, strain burst) may play a significant role in the aftershock productivity. This has been suggested in the crustal literature by considering
aftershock statistics separately for intraplate and interplate earthquakes (Yamanaka and Shimazaki, 1990; Guo and Ogata, 1995).

Despite the low performance of the R-J model for estimating the $K$ values, an application of the model for the selection of magnitude thresholds for invoking a microseismic magnitude event protocol was proposed. By using this model it was possible to discriminate, in the highly scattered data, a median principal event magnitude threshold necessary to raise the maximum curvature time above some time threshold and set the microseismic magnitude event protocol.

### 6.5.3.2 Epidemic type aftershock sequence (ETAS)

The parameters of the ETAS model ($K_o$, $c^e$, $\alpha$, $p^e$) were estimated for the time intervals $[t_0, t_N]$ and $[t_0, T_E]$ for both mining and crustal seismic sequences. For each aftershock sequence, ETAS and MOL were compared through the Akaike Information Criterion ($AIC$).

It was determined that for mining sequences with at least 50 events and when the time interval $[t_0, t_N]$ is considered the ETAS model is generally preferred by $AIC$ to the MOL. In the case of crustal sequences the ETAS model was always preferred by $AIC$ independently of the number of events of the sequence.

A different result was obtained when the time interval $[t_0, T_E]$ was considered. The number of cases where the ETAS model fit better than the MOL was reduced. In the case of mining sequences the MOL performs as well as ETAS. For crustal sequences, the ETAS model still performs better than MOL in most of the cases.

When the ETAS model parameters were fitted for the crustal sequences using the determined time interval of power-law decay $[T_s, T_E]$, in only 18% of the 78 sequences analyzed the ETAS
model was preferred by $AIC$ to the MOL. Based on these results it can be concluded that the ETAS model generally performs better that the MOL when the data includes significant time clustering within the aftershock sequence, however, if the time interval for a single MOL is carefully selected, then the MOL performs as well or better than ETAS.

The interplay between the parameters of the ETAS model was investigated. It was found that $p^e$, $c^e$ and $K_\alpha$, $\alpha$ are positively and negative correlated respectively. The use of model parameters representing truly independent physical properties of the aftershock occurrence process is crucial when their estimates are averaged to compute a-priori values to be used for forecasting of future sequences (Gasperini and Lolli, 2006). These correlations indicate that the parameters do not represent independent effects, suggesting a general inadequacy of the ETAS model in describing the real physical properties of simple aftershock sequences.

The average values of $\alpha$ obtained for mining seismicity are in the same range with the crustal ones (less than 3). Average values larger than 2.0 are found for Craig, Creighton, Kidd Creek, Macassa and Williams for the moment magnitude scale, and Craig, Creighton and Kidd Creek for the uniaxial magnitude scale. These results indicate that most of the mining seismic sequences correspond to typical main event/aftershock sequences. It also, explains why the MOL equation fits better than the ETAS model in most of the mining cases.

Guidelines on the use of the ETAS model for re-entry protocol development were proposed. For this model the forecasting and maximum curvature point has to be updated with the occurrence of each new event. This is certainly different from the MOL model where a unique estimate of the forecast and $T_{MC}$ can be made using only the number of events occurring during the first time unit of data $N_1$. It was also observed that the ETAS model with the average parameters of the
zone produced a low number of events per hour at the beginning of the sequence compared to the
average MOL and the actual data.

Based on these considerations, the MOL with the proposed guidelines is selected as the most
suitable model for representing the time decay of mining-induced aftershock sequences.

### 6.5.4 Båth’s Law

The applicability of Båth’s law was evaluated using the moment and uniaxial magnitude scales.
For practical purposes the magnitude of the main event and largest aftershock were defined for
the largest magnitude occurring during the first hour \( M_1 \) after the event that initiated the
sequence and for the largest magnitude recorded during the next 11 hours \( M_{L4}^{1-12 \text{ hours}} \)
respectively.

Except for three mining sites for the moment magnitude scale, and despite the fact that actual
microseismic monitoring systems have limitations for appropriately quantifying large magnitude
events, \( \Delta M \) presented a range of average values from 0.67 to 1.57, similar to crustal
earthquakes.

It was determined that, uniaxial magnitude is more suitable for evaluating and using Båth’s law
after a blast, large magnitude event or rockburst. In terms of re-entry protocol development
Båth’s law provides a method to statistically estimate the magnitude of the largest aftershock
given the induced magnitude at the very beginning of the sequence.

This law is integrated with the MOL guidelines applied in Section 6.1.8. The course of action is
to consider a time window of at least two hours after the maximum curvature point. If an event
with magnitude higher than the one estimated by Båth’s law occurs during these two hours, the
re-entry clock is reset and the time window is moved, the procedure being repeated until no significant magnitude events are measured in the time window.

6.5.5 Concluding remarks
Aftershock time sequences are complex in nature, nevertheless different seismic environments, from crustal earthquakes to mining microseismic events, presented common statistical properties. These statistics are valuable in terms of application for re-entry protocol development.

In order to have an estimate of the decay rate, all of the studied stochastic models are based on the principle of setting a-priori a group of parameters. The productivity of two of them (MOL and R-J) is estimated based on limited input data, such as: \( N_1, M_1 \) or \( M_{PE} \). The only model studied that needs to be updated during the ongoing sequence is the ETAS model, which presented low number of events per hour at the beginning of the sequence compared to the average MOL and actual data.

It can be conclude that the MOL with the guidelines proposed here are the best alternative for establishing an average decay rate and use as a frame of reference for comparing the decay of real-time data. It was also found that the maximum curvature point \( T_{MC} \) represents a physical property of aftershock time sequences that can be used as a rapid estimate of the time at which it may be considered appropriate to re-enter an area.

However, in this approach, only the average parameters of the zone are assumed for future forecasting and some isolated sequences that presented high \( \kappa \) values were excluded from the analysis. It is necessary to develop a technique that incorporates into the analysis the inherent variability of the site, and provides a deep understanding of the behaviour of the seismic
sequences by comparing real-time data with the previous decay behaviour of the seismic environment. This is a priority in the next Chapter.

The two models studied (R-J and ETAS) that attempt to relate event rate with magnitude in one equation presented a low fit to actual sequences when the average parameters were used. In the next chapter, event count and magnitude are addressed as separate seismic quantities in connection with current re-entry practices, and a generic decay-law is formulated. In addition, a method for evaluating background levels of seismicity rate is proposed.
Chapter 7

Generic decay-law formulation and probabilistic framework

Predominantly, two types of seismic quantities were found to be used for re-entry determination on a daily basis at the surveyed mines: event count and seismic work (Figure 3.4). The modified Omori’s law applied to event count was investigated in the previous chapter and guidelines of how to use this scaling law were proposed. These guidelines make use of the first unit time of data to establish an average decay-law curve, providing a preliminary estimate of the re-entry time based on the maximum curvature of the modified Omori’s law. Although, these guidelines are valuable, they do not provide a deep understanding of the real-time behaviour of the seismic sequences. In addition, background levels have not been included in the analysis. This chapter addresses these limitations by:

1. Extending the applicability of decay-law formulas to seismic work.

2. Adapting and including the previously identified characteristics of decay into a real-time process.

3. Developing a method for estimating background levels/thresholds of seismicity rate in connection with the regression window.

4. Considering the inherent variability of the seismic data and microseismic magnitude thresholds by means of a probabilistic methodology.

7.1 Generic decay-law formula

Decay-law formulas provide a framework for representing the decay in time of aftershock sequences of seismic quantities. They can also be used to test if the current data is following a particular pattern. A generic decay-law model can be formulated by:
\begin{equation}
    r(t) = \frac{d\Omega}{dt} = \frac{K}{(c + t)^p} \tag{7.1}
\end{equation}

where \( r(t) \) is the rate at time \( t \) measured from the principal event, \( \Omega \) is the accumulated sum of the seismic quantity of interest until time \( t \), and \( K, c, \) and \( p \) are adjustable parameters. For the particular case of event count, seismic work and seismic moment \( \Omega \) can be conveniently expressed at each \( N \)th event after time zero by:

\begin{equation}
    \Omega = \sum_{i=1}^{N} (M_{o_i})^\xi \tag{7.2}
\end{equation}

where \((M_{o_i})^\xi\) is the \( i \)th seismic moment after time zero for \( \xi = 1 \), event count for \( \xi = 0 \) referred to as the modified Omori’s law (Omori, 1894; Utsu, 1961), and equivalent to Benioff strain for \( \xi = 0.5 \) referred to as seismic work. Figure 7.1 presents an example of \( \sum (M_{o_i})^\xi \) for a rockburst related sequence for different values of \( \xi \).

![Figure 7.1](image_url)

**Figure 7.1.** Example of \( \sum (M_{o_i})^\xi \) for a rockburst related sequence for different values of \( \xi \).
It can be observed from Figure 7.1 that $\xi = 1$ and $\xi = 0$ gives dominating weights to the largest and smallest events within the analysis, respectively, while fractional values of $\xi$ provide filters that modify the relative contributions of events in different magnitude ranges (Ben-Zion and Lyakhovsky, 2002).

From Eq. (7.1) the predicted cumulative seismic quantity at time $t$ since $T_A$ is expressed by:

$$R(t) = \int_{t_A}^{t} r(t) dt = \begin{cases} K \left[ \ln(c + t) - \ln(c + T_A) \right] & \text{for } p = 1 \\
1 - p \left[ \ln(c + t) - \ln(c + T_A) \right] & \text{for } p \neq 1 \end{cases}$$

(7.3)

There are some particular mathematical features of Eq. (7.3). When $p \geq 1$ and $T_A$ is selected as the time of the principal event (i.e., $T_A = 0$) then $c$ must be different from zero to eliminate the singularity in this equation. Alternatively, if $p < 1$ and $T_A = 0$ then $c$ can be set equal to zero. This clarifies the necessity of the parameter $c$ on the equation. It is just an additional parameter that enables the $p$ value to be greater than or equal to one. With this definition, a positive correlation may be expected between $c$ and $p$ which was already revealed for the MOL (Section 6.1.6.3) and ETAS (Section 6.3.2). If $T_A = 0$ and $c = 0$ are forced then Eq. (7.3) becomes:

$$R(t) = \frac{K}{1 - p} t^{1-p} \quad \text{for } p < 1$$

(7.4)

Equation (7.4) is the type of formula currently in use for representing the cumulative seismic work of large magnitude events by the ESG software SeisWatch (Section 3.4.1.1). In this equation the decay parameter $p$ has been arbitrarily constrained to be less than one, which is not necessarily the case.
Note that by normalization, the total measured cumulative seismic quantity $\Omega_{T_A-T_B}$ on a time interval $[T_A, T_B]$ must be coincident with that predicted by Eq. (7.3). Therefore, the parameter $K$ is related to $\Omega_{T_A-T_B}$ and the other two parameters by:

$$K = \begin{cases} 
\frac{1}{\Omega_{T_A-T_B} \ln(c + T_B) - \ln(c + T_A)} & \text{for } p = 1 \\
\frac{(1 - p)}{\Omega_{T_A-T_B} (c + T_B)^{1-p} - (c + T_A)^{1-p}} & \text{for } p \neq 1 
\end{cases}$$

One of the objectives of this section is to investigate the applicability of Eq. (7.3) and (7.4) for representing the time decay of seismic work. More specifically, the following topics are addressed:

1. Using a uniform statistical method compare the decay-law formulas for event count and seismic work.

2. Determine which of Eq. (7.3) or (7.4) is the most appropriate for representing the decay of aftershock sequences from a theoretical and practical point of view.

### 7.1.1 Generic decay-law parameter determination: Maximum likelihood vs least squares

For the case of event count ($\xi = 0$) the decay parameters $K$, $p$, and $c$ can be estimated by maximum likelihood (ML) as shown in Section 6.1.1. An alternative to ML would be to use nonlinear least-squares (LS) fitting to the cumulative seismic quantity, commonly used in the earthquake literature to perform time-to-failure analysis (Varnes, 1989; Bufe and Varnes, 1993; Gross and Rundle, 1998; Robinson, 2000; Zhou et al., 2006). This method involves selecting the parameters $p$ and $c$ that minimize the following residual sum of squares:
The advantage of the LS is that it is not limited to event count, providing a uniform statistical method for estimating and comparing the decay-law parameters for different $\xi$.

In this section the ML and LS parameter estimates are compared for event count using the aftershock sequences filtered by moment magnitude analyzed in Chapter 6 for the $[t_0, T_E]$ time interval. To conclude about the effectiveness of LS for estimating decay-law parameters, the variance-account-for ($VAF$) and the coefficient of determination ($R^2$) are used:

$$VAF = 1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)}$$

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum y^2 - \frac{(\sum y)^2}{n}}$$

where $\text{var}$ denotes the variance, $y$ is the reference value (ML estimates), $\hat{y}$ is the estimated value and $n$ is the number of observations. If the $VAF$ and the $R^2$ are equal to one, then the LS recovered decay parameters are in excellent agreement with ML.

In only 24 from the 372 aftershock sequences considered here, was the LS not able to converge to a reasonable solution. In these cases, the solution obtained was at the limit of the maximum possible value tolerated for $p$, which was set equal to 15. Six of these sequences had less than 20 events in the time interval $[t_0, T_E]$. The rest had $p$ values determined by ML higher than 2.0, with an average of $2.40 \pm 0.86$. The implication is that, the LS method was not able to converge.
only for specific cases. Figure 7.2 presents the comparison of the MOL parameters estimated by ML and LS.

Figure 7.2. Comparison between maximum likelihood (ML) and least squares (LS) methods for estimating the event count decay-law parameters $K$ (frame a), $p$ (frame b) and $c$ (frame c).

Except for the case of the $p$ value for the Copper Cliff North site ($VAF$ and $R^2 \sim 0.7$) ML and LS are in good agreement as indicated by both statistical indices. Considering all the 372 aftershock sequences analyzed, LS is in agreement with ML in 96, 85, and 85% of the cases for the
parameters $K$, $p$, and $c$ respectively. Despite the fact that the LS method has some limitations compared to ML (Vere-Jones et al., 2001) it can be concluded that it provides reasonable results, and for practical purposes can be used to estimate decay-law parameters of aftershock sequences. In the following sections, both statistical techniques are used for estimating the decay-law parameters. When comparing the decay-law formulas for event count and seismic work LS is used. However, when providing guidelines for the use of decay-law formulas for re-entry protocol development, ML and LS are used for event count and seismic work respectively. This is to be consistent with the previous work on the MOL presented in Chapter 6.

### 7.1.2 Comparison of decay-law formulas

The $t_0 - T_E$ time interval was selected to compare the decay-law formulas with $\xi = 0.0$ and $\xi = 0.5$. Note that the end of power-law decay $T_E$ was determined for event count and does not necessarily imply the same physical meaning for seismic work. However, in order to compare the solutions the same time interval must be used. By employing $R^2$ as an indicator of how well the decay-law formulas fit the data, the decay formula with $\xi = 0.0$ was found to fit better than with $\xi = 0.5$ in 85% of the cases. The reason is clear: when more weight is given to the magnitude of the events, occasional departures from the underlying curve take place for large magnitude events during the sequence.

Next, the decay-law parameters $c$ and $p$ obtained for event count and seismic work are compared. Hereafter, the symbols $c^o$, $p^o$ denote the $c$ and $p$ values in the MOL and $c^{sw}$, $p^{sw}$ denote those in the seismic work model. When all the aftershock sequences are considered it is found that the conditions: $c^{sw} < c^o$ and $p^{sw} < p^o$ are satisfied in 64% and 49% of the cases respectively. This indicates that, in general, lower $c$ values are obtained for seismic work.
compared to event count and that there is no consistent trend in the comparison of the $p$ values.

Considering that the number of events in the aftershock sequence is one of the crucial elements in determining accurate decay-law parameters (Nyffenegger and Frolich, 1998), Figure 7.3 presents the fraction of sequences that satisfy $c^{sw} < c^o$ and $p^{sw} < p^o$ as a function of the number of events per sequence.

A steady increase in the fraction of sequences that satisfy $c^{sw} < c^o$ and $p^{sw} < p^o$ as the number of events per sequence increases is found (Figure 7.3). For example, if only sequences with more than 100 events are considered, the conditions $c^{sw} < c^o$ and $p^{sw} < p^o$ are satisfied in 75% and 67% of the cases respectively, corroborating that lower $c$ and $p$ values are obtained for seismic work compared to event count.
Next, the decay-law formulas with and without $c$ values (Eqs. (7.3) and (7.4) respectively) are compared. The difference in the number of parameters between Eq. (7.3) and (7.4) is considered by the use of the Akaike Information Criterion ($AIC$). The $AIC$ is computed for LS by:

$$AIC = n \ln \left( \frac{RSS}{n} \right) + 2n_p,$$

where $n$ is the number of observations, $RSS$ is the estimated residual sum of squares for a candidate model, and $n_p$ is the number of parameters. The preferred model is the one with the lower $AIC$. Table 7.1 presents the percentage of the top ten most active sequences at each site for which the $AIC$ of the decay-law formula with $c$ ($AIC_{c=0}$) is lower than the formula without $c$ ($AIC_{c=0}$).

<table>
<thead>
<tr>
<th>Site</th>
<th>Top ten most active sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Event count</td>
</tr>
<tr>
<td>A</td>
<td>80%</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>80%</td>
</tr>
<tr>
<td>Craig</td>
<td>80%</td>
</tr>
<tr>
<td>Creighton</td>
<td>40%</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>100%</td>
</tr>
<tr>
<td>Macassa</td>
<td>100%</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>80%</td>
</tr>
<tr>
<td>Williams</td>
<td>60%</td>
</tr>
<tr>
<td>California</td>
<td>100%</td>
</tr>
<tr>
<td>Italy</td>
<td>100%</td>
</tr>
</tbody>
</table>

At the following sites the preferred model by $AIC$ (more than 70% of the sequences) is Eq. (7.3) compared to Eq. (7.4): A, Copper Cliff North, Craig, Kidd Creek, Macassa, McCreedy East, Italy and California for event count, and: A, Craig, Kidd Creek and Macassa for seismic work. This
suggests that Eq. (7.4) may provide a reasonable representation of the time sequence at some sites, particularly for the seismic work parameter.

The fulfilment of the condition $AIC_{c \neq 0} < AIC_{c = 0}$ can be explained by the nature of decay of the sequences by identifying the combination of $c$ and $p$ values for the sequences that satisfied $AIC_{c \neq 0} < AIC_{c = 0}$ and $AIC_{c \neq 0} > AIC_{c = 0}$ (Figure 7.4).

Figure 7.4. Combination of $c$ and $p$ values for sequences that satisfied $AIC_{c \neq 0} < AIC_{c = 0}$ and $AIC_{c \neq 0} > AIC_{c = 0}$. (a) Event count; (b) Seismic work. Cases with $c = 10^{-10}$ correspond to sequences where the $c$ value is actually zero. Crustal sequences are included in the event count frame with $c^*$ in days.

A clear boundary can be drawn between the cases for which the sequence is fit better with or without $c$. This result indicates that the combination of parameters $c$ and $p$ have to be above this boundary in order for Eq. (7.3) to fit better than Eq. (7.4). The implication is that, Eq. (7.3) fits better the data compared to Eq. (7.4), depending on the site specific nature of decay and that Eq. (7.4) is better for representing seismic work than event count.
Despite the fact that Eq. (7.4) is not formally correct it has some practical advantages compared to Eq. (7.3):

1. There are only two parameters to specify: $K$ and $p$.
2. Because the $p$ value of Eq. (7.4) is constrained to be less than one, the resulting range of possible values for this parameter will be narrower than that obtained using Eq. (7.3).
3. Ground Control personnel are already familiar with this type of equation for the seismic work parameter.
4. In environments with low average $c$ and $p$ values, such as: Copper Cliff North, Creighton, McCready East and Williams, Eq. (7.4) provides a representation of the seismic work data as good as Eq. (7.3).

Based on these observations it can be concluded that Eq. (7.4) is a practical form of formula for representing the decay of seismic work, however, it is limited to the case of $p < 1$. Considering that this type of formula is currently in use by the ESG software SeisWatch in several mines, and in order to produce guidelines that can be readily used by mine personnel, Eq. (7.4) was selected for further analysis. However, for the case of event count the conventional formula, which allows the $p$ value to take values higher than 1.0, is used. This is consistent with the previous work on the MOL, and with the fact that for event count Eq. (7.3) fits better the data compared to Eq. (7.4) in most of the sites (7/10). The next section presents the decay-law parameter statistics for event count and seismic work.
7.1.3 Decay-law parameter statistics

In this section the statistics of the decay-law parameter are presented. The principal objective is to
determine the probability distribution function that best represents the population of the decay-
law parameters for event count and seismic work.

7.1.3.1 Event count

The cumulative distribution functions of the $p^o$ values estimated for the time interval of power-
law decay were presented in Figure 6.16b. Two probability distribution functions are tested for
representing the power-law $p^o$ values: normal and log-normal (Table 7.2). In most of the cases
(8/10) a normal distribution fits the datasets very well (Copper Cliff North, Craig, Kidd Creek,
Macassa, McCreedy East, California and Italy). However, the most consistent statistical
distribution for representing the population of $p^o$ values is determined by a log-normal
distribution (higher $P_{Chi-Sq}$ in Table 7.2). The two parameters $\mu_{p^o}$ and $\sigma_{p^o}$ are in well
constrained ranges from 0.74 to 1.05 and 0.16 to 0.33, with averages of 0.91±0.1 and 0.23±0.05
respectively.
Table 7.2: Estimated parameters $\mu_{p^o}$, $\sigma_{p^o}$ and Chi-Square observed significance level for a normal and log-normal distributions representing the population of power-law $p^o$ values. Best fit distribution at each site is shown in bold.

<table>
<thead>
<tr>
<th>Site</th>
<th>Normal</th>
<th>Log-normal</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_{p^o}$</td>
<td>$\sigma_{p^o}$</td>
<td>$P_{\text{Chi-Sq}}$</td>
<td>$\mu_{p^o}$</td>
<td>$\sigma_{p^o}$</td>
<td>$P_{\text{Chi-Sq}}$</td>
</tr>
<tr>
<td>A</td>
<td>0.944</td>
<td>0.263</td>
<td>0.051</td>
<td>0.943</td>
<td>0.246</td>
<td>0.366</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>0.743</td>
<td>0.171</td>
<td>0.764</td>
<td>0.744</td>
<td>0.181</td>
<td>0.801</td>
</tr>
<tr>
<td>Craig</td>
<td>0.922</td>
<td>0.222</td>
<td>0.655</td>
<td>0.922</td>
<td>0.202</td>
<td>0.655</td>
</tr>
<tr>
<td>Creighton</td>
<td>0.777</td>
<td>0.307</td>
<td>0.019</td>
<td>0.773</td>
<td>0.262</td>
<td>0.853</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>1.047</td>
<td>0.324</td>
<td>0.306</td>
<td>1.038</td>
<td>0.332</td>
<td>0.563</td>
</tr>
<tr>
<td>Macassa</td>
<td>0.980</td>
<td>0.182</td>
<td>0.540</td>
<td>0.978</td>
<td>0.186</td>
<td>0.801</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>0.897</td>
<td>0.244</td>
<td>0.271</td>
<td>0.895</td>
<td>0.212</td>
<td>0.363</td>
</tr>
<tr>
<td>Williams</td>
<td>0.820</td>
<td>0.298</td>
<td>0.000</td>
<td>0.816</td>
<td>0.257</td>
<td>0.580</td>
</tr>
<tr>
<td>California</td>
<td>0.966</td>
<td>0.252</td>
<td>0.503</td>
<td>0.966</td>
<td>0.257</td>
<td>0.503</td>
</tr>
<tr>
<td>Italy</td>
<td>0.972</td>
<td>0.177</td>
<td>0.819</td>
<td>0.972</td>
<td>0.158</td>
<td>0.819</td>
</tr>
<tr>
<td>min</td>
<td>0.743</td>
<td>0.171</td>
<td>0.000</td>
<td>0.744</td>
<td>0.158</td>
<td>0.351</td>
</tr>
<tr>
<td>max</td>
<td>1.047</td>
<td>0.324</td>
<td>0.819</td>
<td>1.048</td>
<td>0.332</td>
<td>0.903</td>
</tr>
<tr>
<td>average</td>
<td>0.907</td>
<td>0.244</td>
<td>0.393</td>
<td>0.906</td>
<td>0.230</td>
<td>0.619</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.098</td>
<td>0.056</td>
<td>0.308</td>
<td>0.099</td>
<td>0.051</td>
<td>0.218</td>
</tr>
</tbody>
</table>

In Section 6.1.6.4 it was found that $K$ can be satisfactorily expressed by: $K = \kappa N_1$, where $N_1$ is the measured number of events occurring during the first time unit following the principal event, and $\kappa$ is a site specific parameter ranging from 0.25 to 0.50. The estimates of $\kappa$ were obtained by a least squares fit between $K$ and $N_1$, and some sequences with high $\kappa$ values were excluded from the analysis. In this section, those previously excluded cases are included in the analysis by using the entire distribution of $K$ values. At all sites, the entire population of $K$ values are adequately described by a log-normal distribution (Figure 7.5a). To confirm the previous suggestion, that $K$ is correlated with $N_1$, the distributions of $N_1$ with the corresponding log-normal fit parameters $\mu_{N_1}$ and $\sigma_{N_1}$ are included in Figure 7.5b.
It is found that the parameters $\mu_K$ and $\sigma_K$ are in a direct correlation with the parameters $\mu_{N_1}$ and $\sigma_{N_1}$ respectively (Figure 7.6). The implication of this correlation is that the parameters $\mu_K$ and $\sigma_K$ can be estimated in a first approximation from the distribution of $N_1$.
Given the above correlation, an alternative criterion was investigated for estimating $\mu_K$ and $\sigma_K$.

By using Eq. (6.26) the following relationship between $K$ and the number of events occurring between the first and second unit time after the principal event $N_{i-2}$ is obtained:

$$K = \begin{cases} 
\frac{N_{i-2}}{\ln(2)} & \text{for } p^o = 1 \\
\frac{N_{i-2}(1-p^o)}{2^{1-p^o} - 1} & \text{for } p^o \neq 1 
\end{cases} \quad (7.10)$$

It is expected that Eq. (7.10) may be a better approximation for estimating $K$, given that only 3 and 7 crustal sequences presented a start time of power-law decay higher than one unit time. The assumption is that the parameters can be estimated more accurately once regular decay has initiated, which is more likely to have occurred using data from the second time unit after the principal event. Figure 7.7 presents the scatter-plots of $(\mu_K, \mu_{N_{i-2}})$ and $(\sigma_K, \sigma_{N_{i-2}})$. An improved correlation is found. This is highly significant as the parameters for representing the distribution of $K$ values can be estimated just using $N_{i-2}$ from the isolated sequences.
Figure 7.7. Scatter plots of \( (\mu_K, \mu_{N_{1-2}}) \) and \( (\sigma_K, \sigma_{N_{1-2}}) \).

### 7.1.3.2 Seismic work

The selected formula for representing the cumulative seismic work has the following form:

\[
SW(t) = Ct^D \quad \text{with } D < 1
\]  

(7.11)

where \( C = K/(1 - p^{sw}) \) and \( D = 1 - p^{sw} \) are two alternative model parameters. Figure 7.8 presents the cumulative distributions of the \( D \) values estimated for the time interval \([t_0, T_E]\) for all the sites under study.
It is found that in 96% of the cases the $D$ value ranges from 0.05 to 0.55 and that the parameters of the best fit log-normal distribution $\mu_D$ and $\sigma_D$ are in a well constrained range, from 0.18 to 0.36, and 0.11 to 0.19, with averages of $0.25 \pm 0.06$ and $0.14 \pm 0.03$ respectively. At all sites, the entire population of $D$ values are adequately described by a log-normal distribution.

Equation (7.11) shows directly that $C$ is the cumulative seismic work at one time unit after the principal event $SW_1$. However, the exact measured cumulative seismic work released at $t = 1$ is not always known and it has to be determined from the best fit parameters of Eq. (7.11). As expected, a direct correlation between $C$ and $SW_1$ is encountered (Figure 7.9).
Figure 7.9. Correlation between the parameter $C$ and the measured seismic work occurring during the first unit time after the principal event $SW_1$ for all the 294 mining-induced aftershock sequences analyzed.

The distributions of the $C$ and $SW_1$ values with the corresponding log-normal parameters are presented in Figure 7.10a and b respectively. The parameters $\mu_C$ and $\sigma_C$ have an extremely well defined correlation with $\mu_{SW_1}$ and $\sigma_{SW_1}$ respectively (Figure 7.11).
Figure 7.10. Cumulative ascending distributions of the estimated $C$ values (frame a) and $SW_1$ (frame b) at each site.

![Graphs of cumulative ascending distributions of $C$ and $SW_1$ at each site.](image)

Figure 7.11. Scatter plots of $(\mu_C, \mu_{SW_1})$ and $(\sigma_C, \sigma_{SW_1})$.

![Scatter plots of $\mu_C$, $\mu_{SW_1}$, $\sigma_C$, and $\sigma_{SW_1}$.](image)
7.2 Isoprobability decay curves-Seismic envelopes and seismic path

In Section 3.4.1.1, the procedure of monitoring the decay of seismic work was described, including an illustration of how it is used in the commercially available package SeisWatch. In this section, a more rigorous application of decay in event count and seismic work is developed.

Figure 7.12 presents all of the seismic sequences identified and analyzed for the Copper Cliff North Mine, in terms of their cumulative seismic work release in a 24 hour window. It is clear from this figure that a unique reference decay-law curve calibrated from one previous large magnitude event/rockburst, as employed in current re-entry practices, will not be representative of the variability nor the average conditions of the seismic environment. An approach able to represent all of the past aftershock sequences may help to develop a better understanding of the decay pattern of an ongoing sequence.

![Diagram showing cumulative seismic work release over time](image)

Figure 7.12. Identified aftershock sequences at the Copper Cliff North Mine in terms of their cumulative seismic work release in a 24 hour window.
Instead of setting a unique deterministic reference decay-law curve, the concept of seismic envelopes is introduced. In this approach, a series of decay-law curves reflecting both the inherent variability and specific average conditions of the aftershock sequences at each site are employed as a reference. For this purpose the well-defined distributions of the decay-law parameters $p$, $K$ and $D$, $C$ presented in Table 7.2, Figure 7.5a and Figure 7.8, Figure 7.10a respectively, are embed in a Monte Carlo simulation. This is possible as the proposed parameters of the decay-law formulas describe truly independent effects. If two correlated parameters are included in the formula, such as the one found between $c$ and $p$, then the samples of input probability distributions should be correlated. At each time after a principal event, a Monte Carlo analysis is performed with 1,000,000 simulations using the Latin Hypercube technique on Eq. (7.11) and the cumulative version of the power-law MOL:

$$N(t) = \frac{K}{1 - p} \left( t^{1-p} - 1 \right) ; \ t > 1$$  \hspace{1cm} (7.12)

Equation (7.12) was established one hour after the main event. The reason is that the event count decay-law parameters have been estimated using the time interval that follows power-law behaviour and some events from the early portion of the sequence have been excluded from the analysis (see for example Figure 6.51). By establishing the event count envelopes one hour after the main event there is a 98% of confidence that the power-law decay has already started (Figure 6.11) and that the event count seismic envelopes are representative of the average conditions of the site. Note that the seismic envelopes obtained from Eq. (7.12) are valid one hour after the principal event so they must be shifted to match the measured number of events occurring during the first hour. The cumulative version of the decay-law formulas were preferred in the Monte Carlo simulations to avoid the use of bins required for estimating the seismicity rate and representing the data.
Figure 7.13a presents the resulting isoprobability curves for the seismic work parameter at the Copper Cliff North Mine. These curves are interpreted directly as the probability for the decay to be lower than a certain value at a certain time after the principal event and are referred to as the seismic envelopes. The sequences analyzed used to develop these probabilistic decay curves are included as a reference in Figure 7.13b. The framework makes it possible to interpolate and extrapolate decay curves where there are no measured sequences.

Figure 7.13. Seismic envelopes (frame a) and analyzed aftershock sequences (frame b) for the cumulative seismic work parameter at the Copper Cliff North Mine.

After a principal event the real-time data is superimposed on the seismic envelopes, enabling the seismic path of the data to be tracked. It is still not clear which factors are the most significant in controlling the speed of decay. Using a rate-state dependent fault strength model a \( p > 1 \) may arise when the stresses on the fault surface decrease with time following the principal event (Dieterich, 1994). Assuming that this statement is correct, then determining real-time \( p \) values can be particularly useful for re-entry protocol development. However, this has been proven to be
sensitive (Section 6.1.4), especially at the beginning of the sequence where little data is available. Also, in specific seismic environments with a high number of sequences in which \( p < 1 \), the validity of the statement is uncertain. It is considered that a better understanding of the behaviour of aftershock sequences is obtained if the actual sequence is compared with a statistical analysis of previous occurrences. This approach assumes that the seismic environment responds in a particular manner to the mining process and can be represented by statistical parameters. In this context, seismic envelopes are introduced to represent both the inherent variability and the average mechanism of the seismic environment. Different speeds of decay can be identified by comparing the seismic path with the developed seismic envelopes as illustrated in Figure 7.14 for the seismic work parameter. In Figure 7.14 the seismic path is represented by Eq. (7.11) using different values of \( D \) with the same total release of seismic work in a 12 hour window.

![Figure 7.14. Illustration of the seismic path concept relative to the seismic envelopes for the seismic work parameter. Small \( D \) values implies a faster speed of decay.](image-url)
Note that for seismic work $D$ and the actual decay constant $p_{sw}$ are related by: $D = 1 - p_{sw}$, i.e., a small $D$ implies a high $p_{sw}$ and therefore a faster decay. A small $D$ value represents a seismic response where almost all of the seismic work in a sequence is released during the very first hours after the principal event. As the $D$ value is increased the released seismic work is more uniformly distributed through time after the principal event.

The practical implications of the seismic envelopes in terms of the interpretation of the seismic path of the data are in order:

1. They allow a classification to be made of the type of aftershock sequence under analysis based on the envelope that is been followed, i.e., it can be seen and tested in real-time to which level of probability the actual seismicity corresponds.

2. Speeds of decay higher than the average (low $D$ in Figure 7.14) are identified by a seismic path that terminates in a fast manner, crossing the seismic envelopes in a sub-horizontal direction. This type of behaviour indicates a sequence where the seismicity is mainly associated with the principal event.

3. Speeds of decay lower than the average (high $D$ in Figure 7.14) are indicated by a seismic path crossing several envelopes in a sub-vertical direction. This condition may arise mainly for two reasons: (1) the principal event had little influence and the new data is indistinguishable from the normal/background level of seismicity, or (2) large magnitude events or significant time clustering is occurring at long delay times after the principal event.

The seismic envelopes may be used as a reference framework, however, for the determination of the re-entry time the rate of the seismic data still has to be included and compared to a previously defined level of seismicity rate. In addition, as already mentioned, a sequence with high $D$ value
(low speed of decay) may cross several envelopes, indicating a delayed response of the ground. However, this can also be the case for a small magnitude principal event that does not produce a well-defined decay sequence and the new data is indistinguishable from the normal/background level of seismicity. To avoid this misinterpretation of the behaviour of the sequence, it is necessary to compare the actual rate of the sequence with a previously defined normal/background level of seismicity. In the next section a method is developed for estimating thresholds of seismicity rate in connection with the regression window. This will incorporate a rate diagram in addition to the seismic envelopes.

The decay-law formulas can also be used to obtain a characteristic time decay that can be estimated by using the seismic path followed by the data. This characteristic time corresponds to the time of maximum curvature $T_{MC}$ given by Eq. (6.34). This point is suitable for re-entry protocol development in the case of event count, but has no physically sound meaning for seismic work given the corresponding units (Joule$^{0.5}$/hour). Equation (6.34) is incorporated into a Monte Carlo simulation with the distributions already presented for the decay-law parameters $K$ and $p$. The resulting probabilistic $T_{MC}$ is included in the event rate diagram as a type of boundary, i.e., when the event rate crosses this boundary the maximum curvature has been reached.

Figure 7.15 illustrates the above concepts (seismic envelopes, seismic path, rate diagram and maximum curvature boundary) for the event count and seismic work parameters after a rockburst ($M_n=2.4$) at the Copper Cliff North Mine. Characteristic features are indicated by vertical arrows. In the case of event count (Figure 7.15a1) the data initially follows the 97% envelope for approximately 3.0 hours after the first event. At this time, a jump occurs in the data and starts following the 98% envelope. In the case of seismic work (Figure 7.15a2) the data crosses several envelopes during the first six hours. A large magnitude event ($M_n=1.4$) occurs 6.2 hours after the
initial event producing an increase in the event and seismic work rate (Figure 7.15b). The current data starts to deviate from the seismic envelopes around hour 8.6 and 7.9 for event count and seismic work respectively (Figure 7.15a), very close to the time where the maximum curvature boundary is crossed (Figure 7.15b1). After the maximum curvature boundary is crossed there is little change in the event and seismic work rate during the next 12 hours. This suggests that most of the physical readjustment imposed by the rockburst has already occurred. It can be also observed in Figure 7.15b that both event count and seismic gives similar rate behaviour of the sequence, however, in terms of the seismic path the cumulative seismic work (Figure 7.15a2) presents a more clear cross with the seismic envelopes compared to event count. This is a result of giving extra weight to events with large magnitudes.
7.3 Background levels/Thresholds of seismicity rate

As already reviewed in Sections 4.3 and 6.1.7.2 the determination of when the mining seismicity is at background levels can be a challenging process for the development of re-entry protocols. Currently there is no recognized method to decide this and practices vary across the industry.
The technique proposed in this section is justified on using rate histograms (Section 7.3.2). Instead of subjectively selecting individual days for defining background levels, the seismicity rate is continuously evaluated for the time period and volume of interest using a time window regression technique (Section 3.2.1) coincident with the one used for re-entry purposes. Therefore, the first step it to understand the factors that control and affect this method. This will be reviewed in the next section.

### 7.3.1 Seismicity rate estimated by a time window regression technique

There are three main factors affecting the estimated seismicity rate by using a time window regression technique that need to be understood:

1. The minimum number of events required to perform the regression $N_{\text{reg}}$.

2. The shift window.

3. The regression window.

The selection of these three factors is influenced by the quantity of data, the updating rate and the degree of smoothing and additional safety of the re-entry policy respectively. Figure 7.16 presents an example of how these factors influence the estimate of the seismicity rate for an aftershock sequence. The minimum number of events required to perform the regression is two. On the other hand, if $N_{\text{reg}}$ is selected too high data will be lost. A minimum of three events was selected. The shift window affects the representation of the sequence as a continuous process (Figure 7.16b). Differences between 0.05, 0.1, 0.2, and 0.5 hours do not create a big impact. A value of 0.1 hour was selected for further analysis. The biggest effect is created by the regression window (Figure 7.16c). If one hour is used, small fluctuations of the time sequence become apparent, making more difficult to interpret. In this example, three hours seems to merge all the data points of the
sequence. Current re-entry practices indicate that the most common regression window is two hours. Therefore, two hours is initially adopted for building the rate histograms and establishing seismicity rate thresholds for re-entry protocol development.

Figure 7.16. Example of the factors affecting the seismicity rate estimated with a time window regression scheme. (a) Number of events required to perform the regression $N_{\text{reg}}$; (b) Shift window; (c) Regression window.
### 7.3.2 Rate histograms

To illustrate the technique, Figure 7.17a presents the resulting event rate time sequence after a rockburst using a 2 hour regression window and a 0.1 hour shift. An approximately constant level between hours 12 and 18 is observed. This indicates that most of the physical readjustment imposed by the rockburst is released during the first 12 hours. It is suggested the use of this approximately constant level as representative of the normal levels of seismic activity for re-entry protocol development. Note that rigorously speaking the decay still continues, but as seen in Figure 7.17a there is no reason for delaying re-entry as the maximum change in rate has already occurred and it may take several hours to achieve a stable lower level of seismicity rate.

![Event rate time sequence for a rockburst sequence (frame a) and corresponding rate histogram and cumulative descending distribution (CDD) (frame b). A 2 hour regression window with a 0.1 hour shift was used for estimating the seismicity rate.](image)

Using the rate time sequence, the rate histogram and cumulative descending distributions (CDD) are built (Figure 7.17b). For representing the rate histogram the data was divided into logarithmic intervals of 0.05. Note that the rate histogram and the CDD in Figure 7.17b are normalized by the
most frequent level of seismicity rate and by the total population respectively, so they both take 
values from zero to one. For this particular aftershock sequence the seismicity rate distribution 
presents a bimodal feature. However, a categorical most frequent level of occurrence of 
seismicity rate in time is observed, coincident with the one previously identified in Figure 7.17a.

This scheme can be applied to any regressed seismic source parameter of interest and for any 
particular time period or target volume. There is, however, one crucial consideration for the 
systematic application of this scheme, which corresponds to the definition of the width of the bins 
used to represent the data for estimating the mode in the rate histogram. This is important to 
establish for the automation of the technique. To select an appropriate bin size two statistical rules 
are implemented: Scott’s rule and Freedman/Diaconis rule. Scott (1979) proposed that the bin 
width $b_w$ be determined by:

$$ b_w = 3.49 \frac{\sigma}{n^{1/3}} $$

(7.13)

where $\sigma$ is the sample standard deviation of the $n$ data values. The equation is derived from 
attempting to minimize the bias in variance of the histogram compared to the data set. The 
underlying theory requires knowledge of the distribution form of the data, which in general is 
unknown, so the above equation assumes normality. Freedman and Diaconis (1981) proposed that 
$b_w$ should be determined as follows:

$$ b_w = 2 \frac{IQR}{n^{1/3}} $$

(7.14)

where $IQR$ is the sample inter-quartile range of the $n$ data values, i.e. the difference between the 
$75^{th}$ and $25^{th}$ percentile of the data. This rule was based on the goal of minimizing the sum of 
squared errors between the histogram bar height and the probability density of the underlying
distribution which gave the \( n^{1/3} \) part of the equation. The use of \( 2IQR \) as a measure of spread was determined from their empirical experiments. Both rules (Eqs. (7.13) and (7.14)) are applied to the corresponding seismicity rate dataset and a range of possible bin width values is obtained. Next, the mode is estimated for each possible bin width by using increments of 0.01. In this manner, for each seismicity rate dataset, a population of modes is generated, making it possible to estimate the variability of the mode and analyze how the bin width affects the results. A low variability will indicate that the mode can be easily determined, while the opposite will hold for a large variability. As a measure of the spread of the central tendency of the mode the coefficient of variation is used:

\[
CV = \frac{\sigma}{\mu}
\]

(7.15)

where \( \sigma \) and \( \mu \) are the standard deviation and mean estimated from the mode population respectively. As a rule of thumb, coefficients of variation below 0.1 are thought to be low, between 0.15 and 0.3 moderate, and higher than 0.3, high (Harr, 1987).

Figure 7.18-Figure 7.19 and Figure 7.20-Figure 7.21 presents the rate time series, rate histograms and CDD’s obtained for the complete catalogues at each site under study for event count and seismic work respectively. To be consistent with the previous analysis of aftershock sequences, the catalogues were filtered by the cut-off moment magnitude \( M_c \) presented in Table 5.1.

Without exception, a pronounced most frequent level of seismicity rate is observed at all sites. This most frequent level is interpreted as a “global” level of seismicity rate \( \eta^G \) for re-entry protocol development. The resulting average, standard deviation and \( CV \) of the mode are summarized in Table 7.3. Except for mine site A, all the studied catalogues presented a \( CV \) less than 0.2, indicating that the mode can be reliable estimated using the proposed method.
Table 7.3: Global most frequent level of seismicity rate for re-entry protocol development at each of the sites under study.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\eta_{EC}^G$ (events/hour)</th>
<th>$\eta_{SW}^G$ (Joule$^{0.5}$/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average</td>
<td>CV</td>
</tr>
<tr>
<td>A</td>
<td>3.5±1.0</td>
<td>0.29</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>4.0±0.5</td>
<td>0.13</td>
</tr>
<tr>
<td>Craig</td>
<td>3.5±0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Creighton</td>
<td>2.6±0.1</td>
<td>0.04</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>2.4±0.1</td>
<td>0.04</td>
</tr>
<tr>
<td>Macassa</td>
<td>2.9±0.2</td>
<td>0.07</td>
</tr>
<tr>
<td>McCreedy East</td>
<td>2.7±0.4</td>
<td>0.15</td>
</tr>
<tr>
<td>Williams</td>
<td>2.9±0.1</td>
<td>0.03</td>
</tr>
</tbody>
</table>

For the Copper Cliff North Mine (Figure 7.20b) the seismic work re-entry rate level currently used at the mine (4,600 Joule$^{0.5}$/hour) and the one established by Malek and Leslie (2006) (62,000 Joule$^{0.5}$/hour) are included as a reference. The first value is low compared with the rest of the population for this site (0.95 percentile), while the second one seems high (0.23 percentile). This is the result of selecting a small data set for setting the seismicity rate level. The estimated level of seismicity rate with the proposed technique is in between the above two values (16,800 Joule$^{0.5}$/hour) as it is more centered with the rest of the population (0.58 percentile).

As a second benchmarking case, the seismic work re-entry rate level currently used at Creighton Mine (30,000 Joule$^{0.5}$/hour) is included in Figure 7.20d. This level is higher than the one estimated using the mode of the complete population (18,300 Joule$^{0.5}$/hour), however, the difference in percentiles in only 0.13.
Figure 7.18. Event rate time series and rate histograms for sites: (a) A; (b) Copper Cliff North; (c) Craig; (d) Creighton.
Figure 7.19. Event rate time series and rate histograms for sites: (a) Kidd Creek; (b) Macassa; (c) McCreedy East; (d) Williams.
Figure 7.20. Seismic work rate time series and rate histograms for sites: (a) A; (b) Copper Cliff North; (c) Craig; (d) Creighton.
Figure 7.21. Seismic work rate time series and rate histograms for sites: (a) Kidd Creek; (b) Macassa; (c) McCreedy East; (d) Williams.
Inspection of Figure 7.18 to Figure 7.21 shows that there are few isolated spikes in the time rate series, indicating that a time window regression technique with the selected values for: $N_{reg}$, shift window and regression window, is appropriate for continuously evaluating the seismicity rate. The main exception is Kidd Creek (Figure 7.19a and Figure 7.21a). To further understand the rate behaviour at this site, Figure 7.22 presents the event count rate histograms and CCD’s for different regression windows and $N_{reg}$ using a 0.1 hour shift window.

![Figure 7.22](image.png)

**Figure 7.22.** Event rate histograms and CDD’s for different regression windows and $N_{reg}$ at the Kidd Creek Mine. A 0.1 hour shift window was used in all cases.
It can be noticed that the population of spikes is reduced as the regression window and $N_{\text{reg}}$ are increased. However, for a 4 hour regression window and $N_{\text{reg}} = 4$ there is still a population of spikes of approximately 8%. These spikes are a result of isolated group of events with very short inter-event times that enter the regression window. The results presented in Figure 7.22 implies that they can be reduced by increasing the size of the regression window and $N_{\text{reg}}$ but not completely eliminated. This indicates that the time window regression technique may not be the most appropriate for continuously evaluating the seismicity rate at this site. However, the problem is reduced when applied to aftershock sequences where there is a significant source of continuous data close to the principal event.

Next, in order to evaluate the effect of the regression and shift windows on the estimated global level of seismicity rate, different values were tested as shown in Figure 7.23 for three different catalogues. Figure 7.23 confirms that the most frequent level of seismicity rate is insensitive to the selection of the shift window and verifies that this level is not an artefact. In addition, as the regression window increases the most frequent level of seismicity rate decreases. This is an effect of smoothing the data to a higher degree.
Figure 7.23. Global most frequent level of seismicity rate estimated for different values of the regression and shift windows. (a) Copper Cliff North; (b) Kidd Creek; (c) Macassa.
As defined previously, re-entry seismicity rate thresholds can be estimated by using the most frequent level of seismic activity determined from rate histograms for the complete catalogues. However, this only represents the global response of the seismic environment and the variability of this quantity needs to be evaluated. Figure 7.24 presents a portion of the rate time series at the Copper Cliff North Mine for a highly active period. Four seismic sequences were identified by the ratios method. Sequence A, B and C corresponds to blasts while sequence D to a rockburst. The rate histogram and most frequent level of seismicity rate for each sequence are included as separately frames. In this example the locally observed event rate can range from 5.6 to 23.4 events/hour, which are higher than the global estimate of 4.0 events/hour. This suggests that the use of a fixed level of seismicity rate for re-entry purposes may lead to long re-entry times and for some highly active sequences a higher seismicity rate for re-entry may be justifiable. This will be examined in more detail in Section 7.3.3.
Figure 7.24. Event rate time series, rate histograms and CDD’s for a highly active period at the Copper Cliff North Mine.
For applying this technique to isolated sequences, it is necessary to consider two factors: the total duration $t_N$ and the number of events in the sequence. The ratios method, used to isolate mining seismic sequences, requires an abrupt change in the rate to label the beginning of a new sequence, generating large data samples and thus making it appropriated for this application. To ensure that the determined level of seismicity rate is representative, only sequences with $t_N$ higher than 12 hours are considered. In addition, to have a large enough population for estimating the seismicity rate, only sequences with at least $N_1^*$ events during the first hour after the principal event (Table 6.6) were retained for the analysis.

For each sequence that satisfied the above conditions the mode was estimated following the same methodology that was applied to the complete catalogues. Figure 7.25 displays the number of cases retained for the analysis and the fraction of sequences as a function of the coefficient of variation of the mode of seismicity rate for event count and seismic work.

Figure 7.25. Cumulative descending distribution of sequences as a function of the coefficient of variation of the mode of seismicity rate for event count (frame a) and seismic work (frame b).
The value of $CV=0.3$ is included in Figure 7.25 as the limit between moderate to high $CV$. It can be seen that 83% and 76% of the sequences analyzed satisfy $CV\leq 0.3$ for event count and seismic work respectively. After this point, the number of cases with lower $CV$’s start to decreases more abruptly at some sites. This analysis suggests that $CV=0.3$ seems to be the limit in reliability for estimating the mode with the proposed methodology and is considered as the cut-off value. These levels will be referred to as the “local” level of seismicity rate $\eta^L$ for re-entry protocol development. Figure 7.26 presents the resulting distributions of the local level of seismicity rate for event count and seismic work.

![Graph](image)

**Figure 7.26.** Distributions of the local level of seismicity rate for re-entry protocol development. (a) Event count; (b) Seismic work.
Figure 7.26 confirms that there is a local variability of the seismicity rate that can be approximately represented with a log-normal distribution. The distributions presented in Figure 7.26 are used to estimate the probability of having a level of seismicity rate higher than a certain value. The probability ranking system indicated in Table 7.4 was adopted from Davies (1997) and used to define a scale of significance of seismicity rate for re-entry protocol development.

Table 7.4: Probability ranking system for defining a scale of significance of seismicity rate for re-entry protocol development. (a) Event count; (b) Seismic work.

<table>
<thead>
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<th>(a) Event count</th>
<th>Significance scale</th>
<th>Site</th>
<th>Very high 1%</th>
<th>High 10%</th>
<th>Medium 50%</th>
<th>Low 90%</th>
<th>Very low 99%</th>
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<tr>
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<td></td>
<td>A</td>
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<td>2.6</td>
<td>1.7</td>
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<td>3.9</td>
<td>1.9</td>
<td>1.0</td>
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<td>Creighton</td>
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<td>5.2</td>
<td>3.2</td>
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<td>1.6</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Seismic work</th>
<th>Significance scale</th>
<th>Site</th>
<th>Very high 1%</th>
<th>High 10%</th>
<th>Medium 50%</th>
<th>Low 90%</th>
<th>Very low 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>24,400</td>
<td>13,643</td>
<td>6,687</td>
<td>3,277</td>
<td>1,833</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Copper Cliff North</td>
<td>130,184</td>
<td>55,742</td>
<td>19,694</td>
<td>6,958</td>
<td>2,979</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Craig</td>
<td>55,076</td>
<td>29,582</td>
<td>13,802</td>
<td>6,439</td>
<td>3,459</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Creighton</td>
<td>76,106</td>
<td>42,145</td>
<td>20,413</td>
<td>9,887</td>
<td>5,475</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kidd Creek</td>
<td>27,174</td>
<td>13,864</td>
<td>6,073</td>
<td>2,660</td>
<td>1,357</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Macassa</td>
<td>54,103</td>
<td>38,082</td>
<td>24,755</td>
<td>16,092</td>
<td>11,327</td>
</tr>
<tr>
<td></td>
<td></td>
<td>McCreedy East</td>
<td>18,101</td>
<td>9,359</td>
<td>4,167</td>
<td>1,855</td>
<td>959</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Williams</td>
<td>22,804</td>
<td>14,365</td>
<td>8,150</td>
<td>4,624</td>
<td>2,913</td>
</tr>
</tbody>
</table>

In order to identify which seismicity rate level is appropriate for re-entry, Table 7.5 shows a comparison between the global $\eta^G$ (Table 7.3) and the medium local $\eta^{50\%}$ (Table 7.4) levels of seismicity rate.
Table 7.5: Comparison between the global $\eta^G$ and the medium local $\eta^{L,50%}$ levels of seismicity rate at each site.

<table>
<thead>
<tr>
<th>Site</th>
<th>Event count</th>
<th>Seismic work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta^G_{EC}$</td>
<td>$\eta^{L,50%}_{EC}$</td>
</tr>
<tr>
<td>A</td>
<td>3.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Copper Cliff North</td>
<td>4.0</td>
<td>3.9</td>
</tr>
<tr>
<td>Craig</td>
<td>3.5</td>
<td>3.9</td>
</tr>
<tr>
<td>Creighton</td>
<td>2.6</td>
<td>3.2</td>
</tr>
<tr>
<td>Kidd Creek</td>
<td>2.4</td>
<td>3.7</td>
</tr>
<tr>
<td>Macassa</td>
<td>2.9</td>
<td>3.3</td>
</tr>
<tr>
<td>McCready East</td>
<td>2.7</td>
<td>2.3</td>
</tr>
<tr>
<td>Williams</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>min</th>
<th>max</th>
<th>average</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta^{L,50%}$</td>
<td>0.87</td>
<td>1.52</td>
<td>1.15</td>
<td>0.20</td>
</tr>
<tr>
<td>$\eta^G$</td>
<td>0.52</td>
<td>1.19</td>
<td>0.96</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The ratio $\eta^{L,50%}/\eta^G$ included in Table 7.5 indicates that on average $\eta^{L,50%}$ is coincident with $\eta^G$ within 15% and 0.04% for event count and seismic work respectively. A main exception is the Kidd Creek site which presented a higher and lower $\eta^{L,50%}$ compared to $\eta^G$ for event count and seismic work respectively. In the case of event count, the rate histogram for the complete catalogue (Figure 7.19a) presented a slight bimodal feature with a secondary most frequent level at 3.5 events/hour, very close to the $\eta^{L,50%}$ presented in Table 7.5. In the case of seismic work $\eta^G$ seems high compared to $\eta^{L,50%}$. Considering that most of the sites presented similar values of $\eta^{L,50%}$ and $\eta^G$, the $\eta^{L,50%}$ was selected as the most appropriate level for defining re-entry.

The scale of significance of seismicity rate presented in Table 7.4 is incorporated into the rate diagram and used to estimate different re-entry times based on the associated probability. This enables re-entry times to be defined with different degrees of confidence.
Figure 7.27 presents an example for a $M_n=2.1$ rockburst at the Copper Cliff North Mine. The following features are included in Figure 7.27: seismic envelopes, rate diagram with the seismicity rate 6 hours previous to the principal event, the maximum curvature boundary and the scale of significance of seismicity rate. The time relative to the principal event where the seismic path crosses the maximum curvature boundary and reaches different levels of seismicity rate are indicated by a circle and squares respectively.

In this example presented in Figure 7.27, the event count seismic path follows mainly the 80% decay envelope in a kind of activation-deactivation cycle (Figure 7.27b1). In the case of seismic work several envelopes were crossed in a sub-vertical manner during the first 2.0 hours (Figure 7.27b2), indicating a major readjustment. The re-entry time corresponding with the medium level (50%) for event count and seismic work are 8.4 and 5.1 hours respectively. After this point the seismicity rate oscillates between the medium (50%) and high (10%) levels (Figure 7.27c) until it disappears completely at hour 12.

The above result suggests that the re-entry determined by seismic work can be lower than that determined by event count. The difference in the decay times between event count and seismic work will be examined statistically in Section 7.3.4. In addition, several examples of re-entry protocol development are presented in Section 7.5.
Figure 7.27. Re-entry protocol development for event count and seismic work parameters for a rockburst ($M_n=2.1$) at the Copper Cliff North Mine. (a) Spatial distribution of events in the affected zone; (b) Seismic envelopes; (c) Rate diagram.
7.3.3 Observation time window

In this section, the difference in the decay times between the high (10%) and medium (50%) level of seismicity rate defined in Table 7.4 are analyzed. This time period is referred to as the observation time window and is related to two observed behaviours:

1. Irregularity in the decay of aftershock events.

2. Stabilization of the seismicity rate for several hours before reaching the medium level.

These two behaviours are illustrated in Figure 7.28 for a large magnitude event ($M_n=1.6$) and a rockburst ($M_n=2.4$) sequence at the Kidd Creek and Copper Cliff North mines respectively. These sequences presented secondary activity (Figure 7.28I) and a stabilization (Figure 7.28II) of the seismicity rate between the high and medium levels. The statistical analysis of this decay time difference will help to understand in which situations/sites a higher seismicity rate may be justifiable for re-entry.
Figure 7.28. Examples of the decay behaviour between the high and medium level of seismicity rate. (a) Seismic envelopes; (b) Rate diagram.
Figure 7.29 presents the comparison between the decay times estimated for the high $t^d_{\eta_{10\%}}$ and medium $t^d_{\eta_{50\%}}$ levels of seismicity rate. Statistics for the time difference $t^d_{\eta_{10\%}} - t^d_{\eta_{50\%}}$ are included as a reference.

Figure 7.29. Comparison between the decay times estimated for the high and medium levels of seismicity rate for all the 294 mining-induced aftershock sequences analyzed. (a) Event count; (b) Seismic work.

In 87%, 8%, and 2% of the cases the event count decay times (Figure 7.29a) estimated for the high level of seismicity rate is within 2, 4 and 6 hours from the one estimated using the medium level respectively. Similar statistics were found for the seismic work parameter (Figure 7.29b). The implication is that in 97% and 99% of the 294 mining seismic sequences analyzed the decay from the high to the medium level takes no longer than 6 hours for the event count and seismic work parameters respectively.
In addition, most of the sequences that presented $t_{\eta_{10\%}}^d - t_{\eta_{50\%}}^d > 6$ hours belong to the Copper Cliff North Mine. These sequences presented a stabilization of the seismicity rate for several hours before reaching the medium level; similar to the behaviour presented in Figure 7.28II. Based on this result the following preliminary guideline is suggested for the Copper Cliff North Mine:

Once the high level of seismicity rate is reached a minimum 6 hour observation time period is recommended. If during these 6 hours the medium level of seismicity rate is not reached and the seismic path is separating from the seismic envelopes in a sub-horizontal manner, and the seismicity rate remains approximately constant and/or below the high level, as the one shown in Figure 7.28II, then the zone can be released for re-entry. It has to be understood that this recommendation is based on the statistical analysis of several aftershock sequences at different mines and its implementation will result in a less confident re-entry protocol. If the pattern of decay is not clear then the medium level of seismicity rate is recommended for re-entry.

### 7.3.4 Comparison of decay times between event count and seismic work

Rate histograms provide a framework to compare the decay times obtained by using event count and seismic work. This comparison is done to determine if it is necessary to use both parameters as independent measures of the re-entry time. Figure 7.30 presents the comparison of decay times between event count $t_{\eta_{10\%}}^d$ and seismic work $t_{\eta_{50\%}}^d$ using the medium level of seismicity rate presented in Table 7.4.
Figure 7.30. Comparison of the decay times between event count and seismic work by using the medium level of seismicity rate for all the 294 mining-induced aftershock sequences analyzed.

The decay times estimated by event count and seismic work are quite similar for all the 294 mining-induced aftershock sequences analyzed (Figure 7.30). In only 6 of the sequences the decay times differ by more than 4 hours. From these 6 sequences 5 of them presented longer re-entry time for the event count parameter. The reason for this difference is simple. At some stages during the aftershock sequence the production of events corresponds mainly to small magnitude events, and consequently the regressed seismic work has decayed to a rate lower than the specified rate threshold, however, the event count rate can still be high. The implication is that, if
the event count parameter is used properly it provides an estimate as good as that provided by seismic work.

It can be concluded that both parameters have similar rate behaviour, however, for some sequences the seismic path of the cumulative seismic work shows a more clear cross with the seismic envelopes compared to event count (see for example Figure 7.15). This is a result of giving extra weight to events with large magnitude. Therefore, it is recommended to use both parameters for a better understanding of the decay of the seismic sequence.

7.4 Probabilistic microseismic magnitude event protocol-Logistic model
In Section 4.4.2 the plot of moment magnitude versus the S-wave to P-wave energy ratio (Figure 4.7) was used to visually identify thresholds for invoking a re-entry restriction. This plot suggested that the mechanism of events labelled as: events, blasts and reported, are different and that the microseismic source parameters are able to reflect this mechanism. In this section, logistic regression is applied for the identification of events that may trigger an event protocol by using multiple microseismic source parameters. This linkage implicitly assumes that there is a direct correlation between the microseismic source parameters of an individual event and the consequences as observed underground.

The ESG full-waveform systems commonly used in Ontario mines automatically provides 13 microseismic source parameters, namely: uniaxial magnitude (\(uMag\)), triaxial magnitude (\(tMag\)), moment magnitude (\(MomMag\)), seismic energy (\(E\)), S-wave to P-wave energy ratio (\(E_s/E_p\)), source radius (\(r_s\)), asperity radius (\(r_a\)), static stress drop (\(\Delta\sigma\)), apparent stress (\(\sigma_a\)), dynamic stress drop (\(\Delta\sigma_d\)), maximum displacement (\(D_{max}\)), peak velocity parameter (\(PVP\)), and the peak acceleration parameter (\(PAP\)). Therefore, the first step is to identify key
microseismic source parameters that should be used for establishing the microseismic magnitude event protocol. Based on seismic monitoring at the Mt. Charlotte Mine in Australia, Poplawski (1997) found that hazard assessment required monitoring of multiple parameters. No key seismic source parameters to monitor were mentioned. Alcott et al. (1998) working on data from the Brunswick Mine, New Brunswick, suggested that seismic energy, apparent stress and seismic moment were primary parameters to monitor. These source parameters were selected because scalar parameters can be more easily handled and thus lend themselves to routine analysis (Alcott et al., 1998). Trends in these parameters were correlated with site observations, establishing critical thresholds that could be used to evaluate whether seismic hazard in an area was increasing or decreasing. Thresholds were set up to filter out events that had little impact on the workplace. One of the main conclusions of the study was that the selected thresholds employed by the seismic hazard assessment would have to be adjusted for different rock mass and stress conditions, requiring that thresholds be site specific.

Assuming that seismic events producing incidents of concern, such as: damage to the excavations and/or apprehension to the safety of workers, are consistently-time-labelled in the database, the probability of having this type of occurrence can be estimated by fitting the data to a logistic model. The logistic model is defined by the following equations:

\[ z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k \]  \hspace{1cm} (7.16)

\[ P(z) = \frac{1}{1 + e^{-z}} \]  \hspace{1cm} (7.17)

where \( z \) is a measure of the total contribution of all the independent explanatory variables \( x_k \) used in the model, \( \beta_k \) are the regression coefficients of the set of explanatory variables, which are obtained by maximum likelihood in conjunction with their standard errors \( \Delta \beta_k \), and \( P(z) \) is
the categorical response variable that represents the probability of a particular outcome and is assigned a value of 1 or 0. In this particular application \( x_k \) are the independent microseismic source parameters and \( P(z) \) is the probability of having an incident of concern.

The following procedure was developed for calibrating the logistic model:

1. Initially, the model is calibrated using the 13 microseismic source parameters provided by full-waveform system. Parameters that do not contribute significantly are removed from the model. The process continues until all the variables are statistically significant at the 5% level.

2. Once all the variables are statistically significant the regression coefficients are analyzed. These coefficients describe the size of the contribution of each microseismic source parameter. A positive/negative regression coefficient means that the explanatory variable increases/decreases the probability of the outcome respectively. In this application, variables with negative coefficients are removed from the model as they may suggest a possible correlation with other microseismic source parameters.

3. To ensure that the selected significant variables with positive regression coefficients are independent, a correlation analysis of the microseismic source parameter pairs for reported events is performed. The following approximation is used for assessing the presence of correlation (Larose, 2006): \( |\rho| > 0.7 \) the variables are correlated, \( 0.33 \leq |\rho| \leq 0.7 \) the variables are mildly correlated, \( |\rho| < 0.33 \) the variables are not correlated, where \( \rho \) is the correlation coefficient.

This procedure will identify the key microseismic source parameters that best describe the classification of incidents of concern. Once the \( \beta_k \) have been calibrated, the probability of
having an incident of concern for any observation or any hypothetical values of $x_k$ can be estimated.

The quality of the logistic regression is assessed by classifying the predictions of the model compared to the actual values for different probability levels. This information is displayed in a classification matrix as shown in Table 7.6.

<table>
<thead>
<tr>
<th>Predicted outcome</th>
<th>Actual value</th>
<th>Incident of concern</th>
<th>No concern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incident of concern</td>
<td>$TP$</td>
<td>$FP$</td>
<td></td>
</tr>
<tr>
<td>No concern</td>
<td>$FN$</td>
<td>$TN$</td>
<td></td>
</tr>
</tbody>
</table>

There are four possible outcomes from the logistic model (Table 7.6). If the outcome from a prediction is an incident of concern and the actual value is also an incident of concern, then it is called a true positive ($TP$), however, if the actual value is not an incident of concern then it is said to be a false positive ($FP$). Conversely, a true negative ($TN$) occurs when both the prediction outcome and the actual value are not incidents of concern. A false negative ($FN$) is when the prediction outcome is of no concern while the actual value it is. Based on this classification, several metrics have been defined to evaluate the performance of logistic models (Fawcett, 2006), two of them are:

$$TPR = \frac{TP}{TP + FN}; \quad FPR = \frac{FP}{FP + TN}$$  \hspace{1cm} (7.18)

where $TPR$ and $FPR$ are the true positive rate and the false positive rate respectively.
A receiver operating characteristic (ROC) curve is a graph plotting the combination of $FPR$ and $TPR$ as $x$ and $y$ axes respectively across a series of probability levels as shown in Figure 7.31.

![ROC Curve](image)

Figure 7.31. Example of an ROC curve. The circles represent different probability levels.

A common method for evaluating the performance of the model is to calculate the area under the ROC curve, abbreviated $AUC$, which takes values between 0.5 and 1.0 (Fawcett, 2006). A $AUC$ closer to 1.0 indicates a better model. The ROC can also be used to select a probability cut-off value. For example, maximizing the true positive rate $TPR$ corresponds to some large $y$ value on the ROC curve, while minimizing the false positive rate $FPR$ corresponds to a small $x$ value on the ROC curve. Thus a good first choice for a cut-off probability value is that value which corresponds to a point on the ROC curve nearest to the upper left corner of the ROC graph. However, this is not always true, and there is no universal definition of the cut-off value in the literature (SigmaPlot, 2006). For example, if it is vital not to miss detecting abnormal incidents, then it is more important to maximize $TPR$ than minimize $FPR$. In this case the optimal cut-off
point on the ROC curve will move from the vicinity of the upper left corner over toward the upper right corner. In a case where false alarms are undesirable, minimizing $FPR$ is important (moving toward the lower left corner of the ROC curve).

In the case of re-entry protocols it is necessary to have a more flexible classification. If the main objective of the logistic model is to indicate in real-time which events are similar to the ones already classified as reported and draw the attention of mine personnel, then the objective should be to maximize $TPR$, which in turn will produce a large amount of false alarms ($FP$). Conversely, if an automatic detection alarm is the objective, then $FPR$ needs to be minimized. This will make the estimates more reliable (decrease in $FP$) but less accurate (decrease in $TP$).

The following preliminary probability ranking system was adopted from Davies (1997): Low: 0.1, Moderate: 0.5. By implementing this ranking system the following course of action is suggested:

1. Events with probability lower than low are of minor concern.

2. Between low and moderate an evaluation should be carried out to decide whether or not to invoke a re-entry protocol. Further considerations, such as: time of the event relative to blasting hours, distance of the event to the excavations and trigger of the strong ground motion monitor system may be used to finally decide on invoking or not a re-entry protocol.

3. Everything above moderate should be treated as a seismic concern protocol.

This ranking system is preliminary in nature and needs to be reviewed with the actual range of probabilities estimated by the logistic model.

In the following sections, the logistic model is applied to the databases of Kidd Creek and Creighton mines to explore the possibility of establishing a probabilistic microseismic magnitude
event protocol using the reported events as triggers. Events labelled as blasts are not considered in the analysis.

### 7.4.1 Kidd Creek database

The available microseismic database from 08/06/2004 to 12/11/2007 contains a total of 23,012 seismic occurrences with the 13 microseismic source parameters evaluated, from which 17,258 are labelled as seismic events, 5,684 as blasts, and 70 as reported. Figure 7.32 presents the spatial distribution of the reported incidents throughout the mine.

![Figure 7.32. Spatial distribution of reported incidents occurring at Kidd Creek D Mine from 2004 to 2007.](image)

To confirm the uniformity of the seismic records, Figure 7.33 presents the time series of four microseismic source parameters from 2004 to 2007.
Figure 7.33. Time series of four microseismic source parameters at the Kidd Creek Mine from 2004 to 2007. Blasts not included.

It can be observed from Figure 7.33 that there is a vertical shift in the range of values of the microseismic source parameters at approximately March of 2005. To avoid this inconsistency only dates from 06/01/2005 to 12/11/2007 are considered for the calibration of the logistic model,
resulting in: 11,591 events, 63 reported occurrences, and a mean daily rate of reported incidents of 0.07. Table 7.7 presents the calibration of the logistic model.

Table 7.7: Calibration of the logistic model for the Kidd Creek reported incidents from 06/01/2005 to 12/11/2007. Removed parameters at each step are indicated.

<table>
<thead>
<tr>
<th></th>
<th>uMag</th>
<th>tMag</th>
<th>MomMag</th>
<th>log E</th>
<th>log E / Es</th>
<th>r1</th>
<th>r2</th>
<th>log Δσ</th>
<th>log σu</th>
<th>log Δσu</th>
<th>log Δσv</th>
<th>log Dmax</th>
<th>log PVP</th>
<th>log PAP</th>
<th>min.</th>
<th>max.</th>
<th>average</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>β</td>
<td>0.92</td>
<td>1.87</td>
<td>-5.01</td>
<td>1.74</td>
<td>1.70</td>
<td>2.00</td>
<td>0.60</td>
<td>3.79</td>
<td>-2.86</td>
<td>8.11</td>
<td>98.61</td>
<td>-99.07</td>
<td>-24.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Δβ</td>
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<td>0.89</td>
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<td>1.94</td>
<td>0.71</td>
<td>0.86</td>
<td>0.79</td>
<td>3.10</td>
<td>2.24</td>
<td>222.85</td>
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<td>206.76</td>
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<td>0.04</td>
<td>0.37</td>
<td>0.02</td>
<td>0.02</td>
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<td></td>
</tr>
<tr>
<td></td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
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<td>Y</td>
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</tr>
<tr>
<td>2</td>
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<td>1.70</td>
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<td>-</td>
<td>98.55</td>
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<td>Δβ</td>
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<td>0.02</td>
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<tr>
<td></td>
<td>Significant?</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
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<td>Y</td>
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</tr>
<tr>
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<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
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<td>-</td>
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</table>
As observed in Table 7.7 all of the parameters are significant at the 5% level at step 8. However, the regression coefficient for \( \text{MomMag} \) is negative, which may indicate a correlation with other parameters. When \( \text{MomMag} \) is removed from the model (step 8) the static stress drop (\( \Delta \sigma \)) ceases to be significant and is further removed from the model (step 9). The final calibrated logistic model (step 10) for estimating the probability of having a reported occurrence is given by:

\[
z = -6.89 + 0.92u\text{Mag} + 2.44t\text{Mag} + 0.99 \log \frac{E_s}{E_p} + 1.63r_o
\]

\[
AUC = 0.992
\]

Based on the \( AUC \) presented in Eq. (7.19) it can be stated that the ability for making inferences with this model are good. The selected final microseismic source parameters are truly independent as shown by the correlation matrix in Table 7.8. In this case, the removal of \( \text{MomMag} \) is justified given the correlation with \( t\text{Mag} \). It can be concluded that the subset of microseismic source parameters: \( u\text{Mag} \), \( t\text{Mag} \), \( E_s/E_p \), and \( r_o \) can provide the basic information.

Table 7.8: Correlation matrix of the microseismic source parameters labelled as reported at the Kidd Creek D Mine from 06/01/2005 to 12/11/2007. Correlation coefficients higher than 0.7 are indicated.

<table>
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<tr>
<th></th>
<th>( u\text{Mag} )</th>
<th>( t\text{Mag} )</th>
<th>( \text{MomMag} )</th>
<th>( \log E )</th>
<th>( \log \frac{E_s}{E_p} )</th>
<th>( r_s )</th>
<th>( r_o )</th>
<th>( \log \Delta \sigma )</th>
<th>( \log \sigma_a )</th>
<th>( \log \Delta \sigma_d )</th>
<th>( \log D_{max} )</th>
<th>( \log PVP )</th>
<th>( \log PAP )</th>
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<tr>
<td>( t\text{Mag} )</td>
<td>0.37</td>
<td>1.00</td>
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<tr>
<td>( \text{MomMag} )</td>
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<tr>
<td>( \log E )</td>
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<td>0.84</td>
<td>0.91</td>
<td>1.00</td>
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<tr>
<td>( \log \frac{E_s}{E_p} )</td>
<td>0.36</td>
<td>0.54</td>
<td>0.30</td>
<td>0.40</td>
<td>1.00</td>
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<tr>
<td>( r_s )</td>
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<td>0.43</td>
<td>0.20</td>
<td>0.02</td>
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<tr>
<td>( r_o )</td>
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<td>0.01</td>
<td>0.07</td>
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<td>0.05</td>
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<tr>
<td>( \log \Delta \sigma )</td>
<td>0.27</td>
<td>0.78</td>
<td>0.55</td>
<td>0.95</td>
<td>0.29</td>
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<tr>
<td>( \log \sigma_a )</td>
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<td>0.83</td>
<td>0.82</td>
<td>0.97</td>
<td>0.51</td>
<td>0.05</td>
<td>-0.19</td>
<td>0.93</td>
<td>1.00</td>
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<td>( \log \Delta \sigma_d )</td>
<td>0.18</td>
<td>0.75</td>
<td>0.67</td>
<td>0.85</td>
<td>0.32</td>
<td>-0.11</td>
<td>-0.50</td>
<td>0.86</td>
<td>0.84</td>
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<td>( \log D_{max} )</td>
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<td>0.89</td>
<td>0.82</td>
<td>0.93</td>
<td>0.44</td>
<td>0.12</td>
<td>-0.16</td>
<td>0.89</td>
<td>0.91</td>
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<tr>
<td>( \log PVP )</td>
<td>0.56</td>
<td>0.89</td>
<td>0.92</td>
<td>0.91</td>
<td>0.44</td>
<td>0.12</td>
<td>-0.16</td>
<td>0.89</td>
<td>0.91</td>
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<td>1.00</td>
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<tr>
<td>( \log PAP )</td>
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<td>1.00</td>
<td>0.93</td>
<td>0.93</td>
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</table>
Figure 7.34 presents the ROC for the calibrated logistic model. Based on this curve it can be observed that for probability level of 0.1, the model is able to reproduce 52 of 63 (83%) of the reported incidents used to calibrate the model with a total of 82 false alarms. This suggests that the probability level of 0.1 is a good starting point for considering invoking a re-entry protocol.

Next, the classification provided by the logistic model is compared with that resulting from selecting individual thresholds for each of the microseismic source parameters included in Eq. (7.19). These thresholds are selected at the intersection of the cumulative ascending and descending distributions for the event and reported occurrences respectively (Figure 7.35). This represents the thresholds values that separates events and reported occurrences with the least amount of error.
Figure 7.35. Threshold selection based on the intersection of the cumulative ascending and descending distributions of events and reported occurrences respectively at the Kidd Creek Mine from 06/01/2005 to 12/11/2007.

Using these thresholds the classification matrix is obtained and compared to that provided by the calibrated logistic model producing the same number of true positives (Table 7.9).

Table 7.9: Classification matrices using individual thresholds for each of the parameters (matrix a) and the calibrated logistic model producing the same number of true positives (matrix b) at the Kidd Creek Mine from 06/01/2005 to 12/11/2007.

<table>
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<th>Actual value</th>
<th>(a) Thresholds</th>
<th>(b) Logistic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted outcome</td>
<td>r</td>
<td>e</td>
</tr>
<tr>
<td>r</td>
<td>45</td>
<td>100</td>
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<tr>
<td>e</td>
<td>18</td>
<td>11,509</td>
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</table>
By using individual thresholds it is possible to correctly identify 45 of the 63 reported incidents with 100 false alarms. For the same number of true positives the logistic model is able to reduce the number of false alarms to 43. The benefit of using the logistic model is apparent, reducing by 57% the number of false alarms.

Next, the ability of the calibrated logistic model (Eq. (7.19)) to identify reported incidents is tested with additional data from 12/12/2007 to 03/24/2009. A total of 5,348 seismic records are available, where: 1,468, 3,835, and 45 are labelled as blasts, events and reported respectively. No significant shifts were detected in the microseismic source parameters of interest ($uMag$, $tMag$, $E_s/E_p$, and $r_o$) in this time period. Figure 7.36 presents the ROC curve between the predictions of the calibrated model and the actual reported occurrences for this additional dataset.

![ROC curve for the predictions of the calibrated logistic model compared to the actual reported occurrences at Kidd Creek Mine for the time period 12/12/2007 to 03/24/2009. Several probability levels are indicated by circles.](image)

**Figure 7.36.** ROC curve for the predictions of the calibrated logistic model compared to the actual reported occurrences at Kidd Creek Mine for the time period 12/12/2007 to 03/24/2009. Several probability levels are indicated by circles.
The calibrated logistic model generalizes extremely well, as the model achieves similar classification accuracy to both the calibrated and the tested samples. In this new dataset, and by using the suggested probability level of 0.1, the model correctly predicts 41 of 45 reported occurrences with only 22 false alarms in 468 days.

Next, the timing of false alarms relative to the last blast is investigated. Figure 7.37 presents the cumulative ascending distributions of the time interval between the false alarms and the last event labelled as a blast for the 2007-2009 dataset and two probability levels.

Figure 7.37. Cumulative ascending distributions of the time interval between the predicted false alarms (FP) and the last event labelled as a blast for two probability levels at Kidd Creek Mine for the time period 12/12/2007 to 03/24/2009

Figure 7.37 provides the following:

1. The cumulative population distributions are similar for both probability levels.
2. Approximately 35% and 45% of the false alarms (FP) are triggered within 1 and 2 hours after the last event labelled as a blast respectively.

7.4.2 Creighton database

The available microseismic database from 01/01/2002 to 12/31/2006 contains 194,528 seismic records with the 13 microseismic source parameters evaluated, where: 181,872 are labelled as seismic events, 10,998 as blasts, and 1,658 as reported. Figure 7.38 presents the spatial distribution of reported incidents throughout the mine.

![Figure 7.38. Spatial distribution of reported incidents at Creighton Mine from 2002 to 2006.](image)

For the period under consideration the microseismic source parameters presented an approximately consistent response in time so no additional filtering was applied to the database, resulting in a mean daily rate of reported occurrences of 0.91.
The database is separated into two target groups for calibration and testing of the logistic model. The first group corresponds to the time period 01/01/2002-12/31/2004 and contains 128,463 events, 7,866 blasts, and 926 reported incidents. This group is used to calibrate the logistic model (Table 7.10).

Table 7.10: Calibration of the logistic model for the reported incidents occurring at Creighton Mine from 01/01/2002 to 12/31/2004. Removed parameters at each step are indicated.

<table>
<thead>
<tr>
<th></th>
<th>aMag</th>
<th>tMag</th>
<th>MomMag</th>
<th>logE</th>
<th>logEs/Ep</th>
<th>ro</th>
<th>logσr</th>
<th>logΔσ</th>
<th>logDmax</th>
<th>logPVP</th>
<th>logPAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>min.</td>
<td>-3.36</td>
<td>-3.23</td>
<td>-1.44</td>
<td>-0.02</td>
<td>-2.00</td>
<td>1.60</td>
<td>0.33</td>
<td>4.89</td>
<td>3.27</td>
<td>5.09</td>
<td>-3.06</td>
</tr>
<tr>
<td>max.</td>
<td>0.40</td>
<td>2.83</td>
<td>1.32</td>
<td>8.33</td>
<td>3.00</td>
<td>6.49</td>
<td>4.21</td>
<td>8.92</td>
<td>8.15</td>
<td>19.12</td>
<td>-1.09</td>
</tr>
<tr>
<td>average</td>
<td>-1.27</td>
<td>-0.28</td>
<td>0.39</td>
<td>5.46</td>
<td>1.04</td>
<td>4.40</td>
<td>1.70</td>
<td>7.30</td>
<td>6.43</td>
<td>7.72</td>
<td>-2.51</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.50</td>
<td>0.80</td>
<td>0.45</td>
<td>1.49</td>
<td>0.48</td>
<td>0.76</td>
<td>0.65</td>
<td>0.86</td>
<td>0.86</td>
<td>0.70</td>
<td>0.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>β</th>
<th>Δβi</th>
<th>Prob&gt;Chi-Sq</th>
<th>Significant?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.45</td>
<td>0.46</td>
<td>-1.61</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>2.33</td>
<td>0.72</td>
<td>0.94</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>0.94</td>
<td>-0.38</td>
<td>-0.42</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>-1.33</td>
<td>84.46</td>
<td>38.79</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>-38.10</td>
<td>49.92</td>
<td>49.92</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>-86.34</td>
<td>49.92</td>
<td>49.92</td>
<td>N</td>
</tr>
</tbody>
</table>

| 2      | 2.45  | 0.46  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 3      | 2.45  | 0.46  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 4      | 2.46  | 0.47  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 5      | 2.45  | 0.47  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 6      | 2.46  | 0.37  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 7      | 2.40  | 0.38  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 8      | 2.46  | 0.47  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |

| 9      | 2.46  | 0.47  | -1.61        | Y            |
|        | 2.33  | 0.72  | 0.94         | N            |
|        | 0.94  | -0.38 | -0.42        | Y            |
|        | -1.33 | 84.46 | 38.79        | Y            |
|        | -38.10| 49.92 | 49.92        | N            |
|        | -86.34| 49.92 | 49.92        | N            |
In this case, all of the retained parameters are significant at step 5. However, there are four parameters with negative coefficients: $r_a$, $\Delta \sigma$, $\sigma_a$, and $PAP$. Further removal of these parameters $\Delta \sigma$ (step 5), $r_a$ (step 6), $\sigma_a$ (step 7) and $PAP$ (step 8) ensures that all of the coefficients are positive and significant. The resulting logistic model is given by:

$$z = -4.56 + 2.64 uMag + 0.33 tMag + 0.14 \log E + 0.75 \log E_s/E_p + 1.17 r_o$$  \hspace{1em} (7.20)$$

$$AUC = 0.978$$

Based on the $AUC$ presented in Eq. (7.20) it can be stated that the ability for make inferences using this model is good. Table 7.11 confirms that the final selected microseismic source parameters are truly independent, as: $r_a$ is related with $r_o$, while $\Delta \sigma$, $\sigma_a$, and $PAP$ are correlated with $E$.

Table 7.11: Correlation matrix of the microseismic source parameters labelled as reported at Creighton Mine from 01/01/2002 to 12/31/2004. Correlation coefficients higher than 0.7 are indicated.

<table>
<thead>
<tr>
<th></th>
<th>uMag</th>
<th>tMag</th>
<th>MomMag</th>
<th>log E</th>
<th>log E/E_p</th>
<th>r_a</th>
<th>r_o</th>
<th>log $\Delta \sigma$</th>
<th>log $\sigma_a$</th>
<th>log $\Delta \sigma_d$</th>
<th>log PVP</th>
<th>log PAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>uMag</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tMag</td>
<td>0.62</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MomMag</td>
<td>0.61</td>
<td>0.70</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log E</td>
<td>0.57</td>
<td>0.73</td>
<td>0.95</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log E/E_p</td>
<td>0.28</td>
<td>0.39</td>
<td>0.20</td>
<td>0.29</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.19</td>
<td>0.01</td>
<td>0.31</td>
<td>0.14</td>
<td>-0.06</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_o$</td>
<td>0.05</td>
<td>-0.10</td>
<td>0.13</td>
<td>0.02</td>
<td>-0.14</td>
<td>0.72</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log $\Delta \sigma$</td>
<td>0.49</td>
<td>0.75</td>
<td>0.82</td>
<td>0.90</td>
<td>0.19</td>
<td>-0.05</td>
<td>-0.10</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log $\sigma_a$</td>
<td>0.50</td>
<td>0.66</td>
<td>0.85</td>
<td>0.97</td>
<td>0.35</td>
<td>0.00</td>
<td>-0.07</td>
<td>0.90</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log $\Delta \sigma_d$</td>
<td>0.56</td>
<td>0.67</td>
<td>0.84</td>
<td>0.91</td>
<td>0.33</td>
<td>-0.06</td>
<td>-0.24</td>
<td>0.81</td>
<td>0.90</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log PVP</td>
<td>0.61</td>
<td>0.69</td>
<td>0.92</td>
<td>0.95</td>
<td>0.34</td>
<td>0.10</td>
<td>-0.04</td>
<td>0.81</td>
<td>0.92</td>
<td>0.97</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>log PAP</td>
<td>0.56</td>
<td>0.67</td>
<td>0.84</td>
<td>0.91</td>
<td>0.33</td>
<td>-0.06</td>
<td>-0.24</td>
<td>0.81</td>
<td>0.90</td>
<td>1.00</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Figure 7.39 presents the ROC for the calibrated logistic model.
Based on the curve in Figure 7.39 it can be observed that for a probability level of 0.1, the model is able to reproduce 668 of 926 (72%) of the reported incidents used to calibrate the model with a total of 1,284 false alarms. By reducing the probability to 0.05 the number of correctly classified reported occurrences increases to 781 but with an additional 1,043 false alarms. The probability of 0.1 seems to have a reasonable balance between correctly predicted and false alarms and is considered as a reasonable starting point for invoking a re-entry protocol.

Next, the ability of the calibrated logistic model (Eq. (7.20)) to identify reported incidents is tested with additional data from 01/01/2005-12/31/2006, which contains: 53,409 events, 3,132 blasts, and 732 reported incidents. Figure 7.40 presents the ROC curve between the predictions of the calibrated model and the actual reported occurrences for this second database. The calibrated
logistic model generalizes well, as the model achieves similar classification accuracy to both the calibrated and the tested samples.

Figure 7.40. ROC curve for the predictions of the calibrated logistic model compared to the actual reported occurrences at Creighton Mine from 01/01/2005 to 12/31/2006. Several probability levels are indicated by circles.

To gain insight into when false alarms tend to occur, Figure 7.41 presents the cumulative ascending distributions of the time interval between the false alarms and the last event labelled as a blast for the 2005-2006 dataset and two probability levels.
Figure 7.41. Cumulative ascending distributions of the time interval between the predicted false alarms ($FP$) and the last event labelled as a blast for two probability levels at Creighton Mine for the time period 01/01/2005 to 12/31/2006.

Some remarks are:

1. The cumulative population distributions are independent of the probability level.

2. Approximately 20% and 30% of the false alarms ($FP$) are triggered within 1 and 2 hours after the last event labelled as a blast respectively.
7.5 Application examples

In this section several examples are presented to illustrate the application and impact of the following concepts for re-entry protocol development:

1. Seismic envelopes.

2. Rate diagram.

3. Probability of having a reported occurrence-Logistic model (when applicable).

The significance scale of seismicity rate (very high, high, medium, low, very low) presented in Table 7.4 is incorporated into the rate diagram and used to estimate different re-entry times based on the associated probability. In this scale, the medium level of seismicity rate was identified as an appropriate level for defining re-entry. A major benefit of defining a probabilistic distribution of seismicity rate for re-entry is that the confidence of the re-entry protocol can be quantified in real-time. The following indicators are used for the determination of the re-entry time:

1. The pattern of decay of the sequence before the seismicity rate reaches the previously defined very high and high levels of seismicity rate and the maximum curvature boundary.

2. The seismicity rate level 6 to 12 hours before the principal event, depending on the availability of data.

As explained before (Section 7.3.3) for highly active sequences at the Copper Cliff North Mine a higher seismicity rate for re-entry may be justifiable. This guideline can be implemented when a stabilization of the seismicity rate is observed for 6 hours after the high level of seismicity rate is reached.

Given that the decay times estimated by event count and seismic work rate were similar (Section 7.3.4) most of the examples presented here make use of the event count rate and the
corresponding patterns of decay for the determination of the re-entry time. Seismic work was used in only one case as it presented a clearer cross with the seismic envelopes. The following examples are presented:

1. Two $M_n=2.8$ and one $M_n=2.4$ rockburst related sequences at the Copper Cliff North 100/900 orebodies occurring on 2006 and 2008 respectively (Figure 7.42 to Figure 7.44).

2. One rockburst ($M_n=3.8$) related sequence at the Kidd Creek D Mine on 2009 (Figure 7.45).

3. Three blasts and one large magnitude event ($M_n=3.1$) related sequences at the Macassa Mine occurring on 2007 and 2008 respectively (Figure 7.46 to Figure 7.49).

4. A large magnitude event ($M_n=2.0$) at Craig Mine that triggered a $M_n=0.5$ and a rockburst ($M_n=2.2$) approximately 38 and 40 hours after the main event respectively (Figure 7.50).

Note that except for the three blasts presented for the Macassa Mine, all of the examples are outside of the databases used to calibrate the seismic envelopes, rate diagram and logistic model, testing the capacity of the proposed concepts for new aftershock sequences. To facilitate the understating of the examples the observations for each case are included with the figures.
Observations:
1. Rate level before rockburst approximately at the 50% rate level.
2. When the maximum curvature boundary and the 1% rate level are crossed, the seismic path presents a slight sub-horizontal behaviour relative to the seismic envelopes.
3. Between Tmc and the 1% rate level the seismicity rate shows a small increase.
4. Between the 1% and 10% rate levels there is additional decay rate of the sequence. The seismic path increases the sub-horizontal direction relative to the seismic envelopes.
5. When the 10% rate level is reached 6 hours are considered. The stabilization of the seismicity rate and further sub-horizontal behaviour of the seismic path during these 6 hours suggests that the 10% rate level is appropriate for re-entry.
6. Re-entry time = 24.7 hours.

Figure 7.42. Application example for a rockburst (Mn=2.8) related sequence at the Copper Cliff North 100/900 orebodies. (a) Spatial distribution of events in the affected zone; (b) Seismic envelopes; (c) Rate diagram.
Observations:
1. Rate level before rockburst approximately at the 50% rate level.
2. Fast decay between the 1% and 10% rate levels.
3. Maximum curvature crossed between the 10% and 50% rate levels, very close in time to the 10% rate level.
4. Re-entry time = 5 hours.
5. Note that the magnitude of the main event of this sequence is the same one than the previous example (Figure 7.42), however, they have significantly different re-entry times, emphasizing the importance of real-time analysis.

Figure 7.43. Application example for a rockburst ($M_c=2.8$) related sequence at the Copper Cliff North 100/900 orebodies. (a) Spatial distribution of events in the affected zone; (b) Seismic envelopes; (c) Rate diagram.
Observations:
1. Rate level before rockburst approximately at the 90% rate level.
2. Fast decay between the 1% and 10% rate levels.
3. Maximum curvature crossed between the 10% and 50% rate levels.
4. Re-entry time = 3.1 hours.

Figure 7.44. Application example for a rockburst (M_n=2.4) related sequence at the Copper Cliff North 100/900 orebodies. (a) Spatial distribution of events in the affected zone; (b) Seismic envelopes; (c) Rate diagram.
Observations:

1. Almost no data available before main event.
2. Seismic work parameter was used as it presented a clearer cross with the seismic envelopes.
3. The seismic path crosses in a sub-vertical direction the seismic envelopes during the first 6 hours, suggesting a delayed ground response. Another large magnitude event ($M_n=1.1$) was triggered 3.4 hours after the initial event.
4. All the large magnitude events triggered reported incidents with high probabilities as shown by the logistic model. All of them were covered by the return of the seismicity rate to the 50% rate level.
5. Re-entry time = 10.3 hours.

Figure 7.45. Application example for a rockburst ($M_n=3.8$) related sequence at the Kidd Creek Mine. (a) Spatial distribution of events in the affected zone; (b) Seismic envelopes; (c) Rate diagram; (d) Probability of having a reported event.
Observations:
1. No data available before blast.
2. Seismic path shows sub-horizontal behaviour relative to the seismic envelopes at the 1% rate level. However, some recovery of activity occurs after crossing the maximum curvature boundary.
3. Fast decay after crossing the 10% rate level.
4. Re-entry time = 8.5 hours.

Figure 7.46. Application example for a blast (1,656 lbs) related sequence at the Macassa Mine. (a) Seismic envelopes; (b) Rate diagram.
Observations:

1. No data available before blast.
2. Seismic path presents a slight sub-horizontal behaviour relative to the seismic envelopes at the 1% rate level.
3. The seismicity rate shows an almost constant behaviour between the 1% and 10% rate levels. This is reflected by a vertical shift in the seismic envelope being followed.
4. Re-entry time = 13.2 hours.

Figure 7.47. Application example for a blast (3,843 lbs) related sequence at the Macassa Mine. (a) Seismic envelopes; (b) Rate diagram.
Observations:

1. No data available before blast.
2. At the 1% rate level the seismic path already shows a sub-horizontal behaviour relative to the seismic envelopes.
3. Between the 1% and 10% rate levels the seismicity rate shows oscillations.
4. Re-entry time = 12.1 hours.

Figure 7.48. Application example for a blast (6,752 lbs) related sequence at the Macassa Mine. (a) Seismic envelopes; (b) Rate diagram.
Observations:

1. No data available before main event.
2. At the 1% rate level the seismic path already shows a sub-horizontal behaviour relative to the seismic envelopes.
3. Between the 1% and 10% rate levels the seismicity rate shows oscillations.
4. There is a minor recovery of activity after the 50% rate level.
5. Re-entry time = 14.8 hours.

Figure 7.49. Application example for a slot blast that trigger a large magnitude event ($M_n=3.1$) related sequence at the Macassa Mine. (a) Spatial distribution of the first 20 events in the affected zone; (b) Seismic envelopes; (c) Rate diagram.
Observations:
1. Rate before the main event ($M_n=2.0$) is approximately at the 10% rate level.
2. Once the seismicity rate after the main event reaches the 10% rate level (hour 5.3) the seismic path has already crossed in a sub-vertical direction the 70 and 75% seismic envelopes.
3. The seismicity rate locally reaches the 50% rate level 16.1 hours after the main event. Until that time, the seismic path continued crossing seismic envelopes in a sub-vertical direction.
4. Additional seismicity starts to enter at approximately 06/21/2007: 18. This is reflected by a spike of seismicity rate that continues until 06/22/2007: 0. After this spike the seismicity rate oscillates around the 50% rate level. The seismic path continues crossing envelopes in a sub-vertical direction before the $M_n=0.5$ event.
5. After the $M_n=0.5$ event, the seismic path and rate shows an increase that leads to the $M_n=2.2$ event.
6. Although, the formal re-entry time is 16.1 hours, i.e., once the 50% seismicity rate level is reached, the low speed of decay, the additional seismicity measured from 06/21/2007: 18 to 06/22/2007: 0, and the $M_n=0.5$ event, may suggest a continuation of the restriction until the decay starts crossing the seismic envelopes in a sub-horizontal direction.

Figure 7.50. Application example for a large magnitude event ($M_n=2.0$) at Craig Mine that trigger a $M_n=0.5$ and a rockburst ($M_n=2.2$) approximately 38 and 40 hours after the main event respectively. (a) Seismic envelopes; (b) Rate diagram.
7.6 Summary and discussion

In this chapter a generic decay-law model was proposed for representing three seismic quantities: event count, seismic work and seismic moment. The model is given by:

\[ r(t) = \frac{d\Omega}{dt} = \frac{K}{(c + t)^p} \]  

(7.21)

where \( r(t) \) is the rate at time \( t \) measured from the principal event, \( \Omega = \sum_{i=1}^{N} (M_s^i)^{\xi} \) is the accumulated sum of seismic moments until time \( t \), and \( K \), \( c \) and \( p \) are adjustable parameters. Equation (7.21) was fitted to several mining-induced aftershock sequences for the case of event count (\( \xi = 0 \)) and seismic work (\( \xi = 0.5 \)). The specific case of the equation currently used by the ESG software SeisWatch, which sets \( c = 0 \) and \( p < 1 \) in Eq. (7.21) was also considered.

7.6.1 Comparison of decay-law formulas

For comparing the decay-law parameters \( K \), \( c \) and \( p \) of these two seismic quantities (event count and seismic work) it was necessary to use a uniform statistical method. For this the least squares (LS) method was selected. It was determined that LS can be used to estimate decay-law parameters of aftershock sequences.

By employing \( R^2 \) as goodness of fit, the decay-law formula with \( \xi = 0.0 \) was found to fit better than with \( \xi = 0.5 \) in 85% of the cases. The source of this difference was associated with the occasional departures from the underlying curve that take place for large magnitude events occurring within the sequence.

In terms of the parameters \( c \) and \( p \) lower values were obtained for seismic work compared to event count. The implication of this property is that by using seismic work release rather than
event count, the problem of undercounting small events at the beginning of the sequence is reduced, since most of the total moment in aftershock sequences is contained in the largest events at the beginning (Kagan, 2002).

When comparing the decay-law formulas with and without the $c$ value, a clear boundary was identified between the cases for which the sequence is fit better with or without $c$. This result indicated that fitting a decay-law curve (Eq. (7.21)) with $c = 0$ and $p < 1$ may provide a satisfactory representation of the time sequence depending on the site specific nature of decay and that this formula is better for representing seismic work than event count.

### 7.6.2 Decay-law parameter statistics

The statistical analysis of the decay-law parameters revealed that the populations of $K$ and $p$ can be represented by log-normal distributions with parameters $\mu$ and $\sigma$. A well constrained range was found for the speed of decay $p$, with lower average values for seismic work compared to event count (Table 7.12).

| Table 7.12: Statistics of the parameters $\mu$ and $\sigma$ of the log-normal distribution representing the speeds of decay $p$ for event count and seismic work. |
|---|---|---|---|
| Event count | Seismic work |
| $\mu$ | $\sigma$ | $\mu$ | $\sigma$ |
| min | 0.74 | 0.16 | 0.64 | 0.11 |
| max | 1.05 | 0.33 | 0.81 | 0.19 |
| average | 0.91 | 0.23 | 0.75 | 0.14 |
| S.D. | 0.10 | 0.05 | 0.06 | 0.03 |

In the case of the $K$ values for event count there is a direct correlation between $\mu_K$, $\sigma_K$ with $\mu_{N_{t-2}}$, $\sigma_{N_{t-2}}$ respectively, where $\mu_{N_{t-2}}$ and $\sigma_{N_{t-2}}$ are the parameters of a log-normal distribution.
representing the population of number of events occurring between the first and second unit time after the principal event. In a similar manner, $\mu_c$, $\sigma_c$ of the log-normal distribution representing the population of the alternative parameter $C = K/(1 - p^{sw})$ for seismic work, presented an extremely well defined correlation with the parameters $\mu_{sw}$, $\sigma_{sw}$ respectively, where $\mu_{sw}$ and $\sigma_{sw}$ are the parameters of the log-normal distribution representing the cumulative seismic work at one time unit after the principal event $SW_1$.

### 7.6.3 Seismic envelopes

The current approach with the decay-law formulas is not completely representative of new aftershock sequences. To avoid setting a unique deterministic reference decay-law curve, the concept of seismic envelopes was introduced. For this purpose the well-defined distributions of the decay-law parameters were embedded in a Monte Carlo simulation using the cumulative version of the decay-law formulas and the maximum curvature equation.

After a principal event the real-time data is superimposed on the seismic envelopes, enabling the seismic path of the data to be tracked and develop a better understanding of the decay pattern of the ongoing sequence. Using the seismic envelopes as a reference framework, and the maximum curvature boundary, the previously identified characteristics of decay (see Chapter 6) were included into a real-time analysis.

### 7.6.4 Rate histograms

For the determination of the re-entry time the rate of the seismic data has to be included and compared to a previously defined seismicity rate threshold. A method for estimating thresholds of seismicity rate in connection with the regression window was developed and incorporated into a
rate diagram. This approach exploits the condition that the rock mass responds in a particular manner to the mining process which is represented by the most frequent level of seismicity rate.

The factors affecting the time window regression technique (shift window, regression window) in relation with the most frequent level of seismicity rate were investigated. It was found that the most frequent level of seismicity rate is insensitive to the selection of the shift window, but dependent on the time length of the regression window. Therefore, for estimating seismicity rate thresholds with the proposed method, it is necessary to be consistent with the regression window in the analysis used to display the data.

Using this definition, the local variability of the seismicity rate was evaluated. It was found that it can be represented by a log-normal distribution of parameters $\mu_{\eta'}$ and $\sigma_{\eta'}$. Probabilistic seismicity rate thresholds were used to define a scale of significance (very high, high, medium, low, and very low) of seismicity rate for re-entry protocol development. These levels were incorporated into the rate diagram and used to estimate different re-entry times based on the associated probability. This enabled the level of confidence of the re-entry protocol to be quantified in real-time and to introduce observation time windows in the rate diagram for developing a better understanding of the patterns of decay. In addition, if the consequences of allowing re-entry at different levels of seismicity rate can be quantified, then a risk assessment for re-entry can be developed.

Another application of rate histograms was for comparing the decay times obtained by using event count and seismic work. It was found that the re-entry times estimated by event count and seismic work were quite similar. It can be concluded that the event count provides an estimate of the re-entry time as good as the one provided by seismic work. In addition, for evaluating the seismic work it is necessary that the seismic moment is evaluated correctly by the microseismic
monitoring system. If the seismic moment is not uniformly detected then the correct choice of parameter should be event count.

7.6.5 Application examples

Nine examples of the application of the seismic envelopes-rate diagram were presented. Seven of the cases (Figure 7.42 to Figure 7.44 and Figure 7.46 to Figure 7.49) presented seismic paths in a sub-horizontal direction indicating higher speeds of decay compared to the average of the corresponding site. Two cases with a different pattern were presented. The first case (Figure 7.45) corresponds to a $M_n=3.8$ that triggered a $M_n=1.5$ and $M_n=1.1$ events at 0.3 and 3.4 hours after the main event respectively. The second case (Figure 7.50) was initiated by a $M_n=2.0$ event, which triggered a $M_n=0.5$ event and a rockburst ($M_n=2.2$) approximately 38 and 40 hours after the main event respectively. Both sequences presented patterns crossing several envelopes in a sub-vertical direction before triggering additional large magnitude events.

It is still not clear which factors are the most significant in controlling the speed of decay of mining-induced aftershock sequences. However, Dieterich’s (1994) theoretical results offer a possible explanation of $p$ values higher than 1: the shear stress applied to the fault after the main shock may decrease with time.

In the approach considered here, speeds of decay lower than the average of the site are indicated by a seismic path crossing several envelopes in a sub-vertical direction and can be interpreted as cases where stress are increasing with time following the principal event. As a preventative measure, re-entry should be delayed until the decay starts crossing the seismic envelopes in a sub-horizontal direction and the rate approaches the medium level of seismicity rate.

The prediction of potentially damaging events is outside of the scope of a re-entry protocol. However, the seismic envelopes and the rate diagram had correctly identified in these cases the
lower speeds of decay of the sequences. It is worthwhile to examine more field evidence related to this concept.

### 7.6.6 Logistic model

The applicability of logistic regression was considered for establishing the probability of invoking a microseismic magnitude event protocol. The logistic model was applied to the databases of Kidd Creek and Creighton Mines using the reported events as triggers. At both sites $uMag$, $tMag$, $E_s/E_p$ and $r_o$ were identified as key microseismic source parameters for invoking a re-entry protocol. In addition, energy was determined to be significant at Creighton Mine.

Once the model was calibrated, its ability for identifying reported incidents was tested with additional data. In the case of Kidd Creek, from 3,880 events of which 45 were labelled as reported it was possible to correctly predict 41 of these occurrences with only 22 false alarms for a time period of 1 year and 3 months. In the case of Creighton Mine, from 54,141 events of which 732 were labelled as reported the model predict correctly 553 with 602 false alarms for a 2 year time period. In both cases the calibrated logistic model generalized well, as the model achieved similar classification accuracy for both the calibrated and tested samples. The accuracy of the logistic model is higher for the Kidd Creek site and reflects in some degree the quality of the database. At the Kidd Creek Mine manual picking of waveforms is done for almost all events, while at the Creighton Mine is almost none for the considered period.

Some advantages of this approach are:

- The probability of having an incident of concern is the first component of any formal risk assessment.
• For mines without a history of large magnitude events this method will enable them to develop their own in-house classification based on the microseismic source parameters.

Some limitations are:

• It is necessary to have several incidents of concern consistently-time-labelled to calibrate the model, which is currently not done at all the mines. Hopefully, this type of analysis will motivate mines to keep a well maintained database.

• The monitoring system has to be able to evaluate consistently the microseismic source parameters in time. It has been shown that at some mines the microseismic source parameters can undergo vertical shifts. This was commented in Section 7.4.1 when the Kidd Creek data was analyzed. Figure 7.51 presents an additional example for two microseismic source parameters at Creighton Mine for the time period 2007-2008.

Figure 7.51. Time series for moment magnitude (frame a) and source radius (frame b) at Creighton Mine from 01/01/2007 to 12/31/2008.
In this case, two dramatic vertical shifts are observed in the considered time period. The source of this shift seems to be related to a “bug” in the ESG processing software (Leslie, 2009). More important are the consequences of the shift. First, any correlation involving microseismic source parameters will be invalidated. Second, since the seismic work is a function of the seismic moment, the re-entry protocol will need to be re-calibrated. This will not be the case if event count is used.

The successful predictions of the logistic model invite further refinement in the identification of reported incidents. In addition, other parameters can be introduced in the algorithm such as: location error, distance from mining excavations and to the microseismic array, time relative to the last blast, triggering of the strong ground motion system, etc. The natural extension will be to include blasts as an additional category and perform a multinomial logistic regression for the on-line identification of blast, events and reported occurrences. Other types of classification algorithms, such as, neural networks may be used as well, as the logistic regression has a limited complexity on the constructed boundary for classification.
Chapter 8

Case histories and correlation with mining activities

In this section several case histories collected during the project are presented. These cases were selected to illustrate the possible effects of factors, such as: volume of mined rock, depth, and magnitude of large events, on the time decay of mining-induced aftershock sequences. These correlations are not intended to replace real-time data analysis. They were developed to quantify and reflect the site specific influence of diverse mining parameters on the time decay response of the rock mass. However, the methodology should be widely applicable.

8.1 Theoretical framework

The following theoretical development can be made between decay time and seismicity by using the proposed generic decay-law formulation (Section 7.1), which is repeated for convenience:

\[
\frac{d\Omega}{dt} = \frac{K}{(c + t)^p}
\]

(8.1)

\[
\Omega = \sum_{i=1}^{N} (M_{o_i})^p
\]

(8.2)

where the parameter \(K\) is related to the total measured cumulative seismic quantity on a time interval \([T_A, T_B]\) and the other two parameters by:

\[
K = \begin{cases} 
\Omega_{T_A-T_B} \frac{1}{\ln(c + T_B) - \ln(c + T_A)} & \text{for } p = 1 \\
\Omega_{T_A-T_B} \frac{(1 - p)}{(c + T_B)^{1-p} - (c + T_A)^{1-p}} & \text{for } p \neq 1
\end{cases}
\]

(8.3)
Equation (8.1) can also be used to estimate the time required for the seismic sequence to decay to some predefined level of seismic rate $B$:

$$t^d_B = \left( \frac{K}{B} \right)^{\frac{1}{p}} - c \quad (8.4)$$

Equation (8.4) indicates that a physical process that generates a higher $K$ value in a specified time period $[T_A, T_B]$, will produce a longer decay time to reach a certain level of seismicity rate $B$ for given values of $p$, $c$. Note that the correlation between decay time and $K$ is not necessarily linear and will depend on the average $p$ value of the zone under analysis.

### 8.2 Volume of mined rock

The dependency between seismic activity and the volume of extracted rock has long been recognized from theory and observations (McGarr, 1976; Cook, 1976; Glowacka and Kijko, 1989; Srinivasan et al., 1997). These models state that when a volume of rock $V_m$ is removed at time $t_0$ the cumulative seismic moment released in a specific volume and within a given period of time is proportional to $V_m$:

$$\sum_{i=0}^{N} M_{o_i} = \theta V_m \quad (8.5)$$

Note the similarity of Eq. (8.5) with Eq. (8.2) for $\xi = 1$. The only difference is that in the decay-law formulas the main event is not considered as part of the sequence. The reason why the main event is used in Eq. (8.5) is that this equation is evaluated continuously for the whole time period under consideration, while for re-entry purposes the time of the main event is set as zero to start the analysis. It was found that the inclusion of the main event or not in Eq. (8.5) affected only the
constant of proportionality \( \theta \), so in general both definitions are applicable. By replacing Eq. (8.5) in (8.3) the following equation is obtained:

\[
K = \begin{cases} 
\frac{1}{\theta V_m \ln(c + T_g) - \ln(c + T_A)} & \text{for } p = 1 \\
\frac{(1 - p)}{\theta V_m (c + T_g)^{1-p} - (c + T_A)^{1-p}} & \text{for } p \neq 1
\end{cases}
\]  

Equation (8.6) indicates that the parameter \( K \) is proportional to the mined volume and that, in general, the time for a seismic sequence to decay to a certain level of seismic moment rate will increase as the volume of extracted rock increases.

The case history presented in this section corresponds to a series of 5 primary, longitudinal, continuous retreat sublevel longhole stopes with delayed paste backfill at a depth below surface of 1500 m at the Macassa Mine which includes 25 mining steps (Figure 8.1). The geotechnical domains were described previously in Section 5.1.5. The mining sequence, identified aftershock sequences with their associated cumulative seismic moment for a 24 hour period after the principal event and the corresponding volume of mined rock for each mining step are indicated in Figure 8.1. Some comments on the released seismic moment after each mining step are:

1. In stopes D and E there is a visual agreement of increasing seismic moment as the volume of extracted rock increases.

2. There seems to be an effect of induced stresses as mining approached the abutments on stopes A and B. A5 and A6 are lower in volume than A2 but produced higher release of seismic moment. B3 has the lowest volume of stope B but produced the second highest moment release in this stope.
Figure 8.1. Mining sequence and cumulative seismic moment for a 24 hour period after the principal event. The mined volume is included for each mining step.
Given the availability of sufficiently detailed data for this case study, the hypothesis that the sum of released seismic moment is proportional to the volume of mined rock is tested. Figure 8.2a presents the cumulative seismic moment after each mining step versus the volume of extracted rock. An overall linear correlation between the two parameters can be recognized, with the average release of seismic moment in the zone increasing at a rate of approximately $44\pm4\times10^6$ (Nm/extracted ton). Next, the time decay to a predefined rate level is estimated and correlated with the volume of mined rock. The global most frequent level of seismic moment rate $\eta_{SM}$ was established for the complete available catalogue (Figure 8.2b) and used to estimate the decay time after each mining step. Figure 8.2c presents the seismic moment decay times $t_{d_{GSM}}$ as a function of the mined volume in the zone. There is no improvement in the fit by assuming a power-law compared to the linear fit (Figure 8.2c). Thus, as mining progresses a linear increase in the decay time of approximate 0.011 (hours/extracted ton) is observed. Note that the above analysis is simplified. There is a possible interaction of induced stresses between stopes affected by the fault crossing the zone, the mining rate and backfilling sequence, which were not included in the analysis.
This back analysis justifies the use of these correlations for estimating the decay time response of the rock mass to a new mining configuration. However, if the regression between $t^d_{GSM}$ and $V_m$ is performed when few data points are available, for example using only the data from stope A, the correlations may not be significant. Therefore, the applicability of these correlations is checked as if they were tracked and updated continuously in real-time. The following procedure is applied:

**Figure 8.2.** (a) Cumulative seismic moment after each mining step as a function of the volume of mined rock; (b) Most frequent level of seismic moment rate; (c) Seismic moment decay times as a function of the mined volume in the zone.
The most frequent levels of seismic moment rate $\eta_{SM}$ is estimated using the rate histogram of the available data before the first mining step. Once the first volume of rock is removed the decay time is estimated and the first pair of data $V_m, t_{\eta_{SM}}^d$ becomes available. Next, $\eta_{SM}^G$ is updated with and used for the second mining step, obtaining the second data point. The process being repeated until the last mining step. Once data has been collected for mining step $i-1$ a regression between the collected pairs $V_m, t_{\eta_{SM}}^d$ can be performed to estimate the decay time for the next step $i$.

However, the regression may be weak at the beginning of the process. Instead of performing a regression the long term average is evaluated, given by:

$$\tau_{\nu i-1} = \frac{\sum_{k=1}^{i-1} t_{\eta_{SM}^G k} t_{\eta_{SM}}^d}{\sum_{k=1}^{i-1} V_{m k}}$$  

(8.7)

where, $\tau_{\nu i-1}$ is the long term average decay time per unit of volume removed until step $i-1$, evaluated using the past $i-1$ data. Therefore, the estimated decay time $t_{V_{mi}}^d$ for a mined volume $V_{m i}$ is given by:

$$t_{V_{mi}}^d = \tau_{\nu i-1} V_{mi}$$  

(8.8)

Figure 8.3a presents the estimated most frequent level of seismicity rate $\eta_{SM}^G$ as a function of mining step. It can be observed that, in this zone, $\eta_{SM}^G$ starts to stabilize at approximately mining step 12. In Figure 8.3b the mined volume and actual decay times of the time sequences estimated using the corresponding $\eta_{SM}^G$ for each step are provided.
Figure 8.3. Most frequent level of seismic moment rate (frame a), and mined volume/seismic moment decay times (frame b) as a function of the mining step.

From the data presented in Figure 8.3b the long term average decay time per unit of mined volume $\tau_v$ is calculated for each step (Figure 8.4). Using $\tau_v$ of step $i-1$, the decay time $t_{d,im}^o$ for the next mined volume $V_{mi}$ is estimated and compared to the actual $t_{d,sm}^o$ (Figure 8.4).

Figure 8.4. Long term average decay time per unit of mined volume $\tau_v$ and comparison between the actual and estimated decay times as a function of mining step.
From Figure 8.4 the behaviour of the long term average decay time per unit of mined volume in time can be tracked. In this case $\tau_0$ reached a stable behaviour only for a minor period (from step 16 to 22) and is always increasing as new data of larger mined volumes comes in. Except for the estimate of the last step, an acceptable correlation ($R^2 = 0.69$) is observed between the actual and the estimated decay time response of the rock mass to a given new mining step.

8.3 Depth

Although there is considerable scatter in the measured stress databases at any given depth, a linear or exponential model is generally assumed as a best fit trend (Martin et al., 2003). It is expected, therefore, that an increase in mining depth will result in an increase in the magnitude and frequency of mining seismicity (Cai et al., 2005; He et al., 2007). Figure 8.5 presents the distribution of mining seismicity as a function of depth for different moment magnitude ($M_w$) levels at Creighton Deep.

![Figure 8.5. Distribution of mining seismicity as a function of depth for different moment magnitude levels at Creighton Deep.](image)
Under the current mining scale and conditions, there is an exponential increase in the frequency of mining events for almost all magnitude bands (Figure 8.5). Also, larger microseismic events seem to occur in the deepest part of the orebody. As a reference the best fit line is shown for events with $M_w \geq -1.6$. Based on the above observations, the decay time as a function of depth was evaluated. For each of the identified aftershock sequences the depth of the principal event was considered. The decay times to reach the most frequent level of seismic work rate presented in Table 7.3 after each principal event as a function of depth are presented in Figure 8.6.

![Figure 8.6.](image)

(a) Seismic work decay times versus depth; (b) Distribution of seismic work decay times as a function of depth.

The identification of the type of principal event (reported or blast) was obtained from the mine database, where reportable seismicity is defined as seismic events that are felt on surface or underground and are also typically recorded by the on-site strong ground motion seismic system. Inspection of Figure 8.6 provides the following:

1. The longest two decay times correspond to reported events.
2. There are few cases (10%) with decay times longer than 8 hours.

3. Decay times longer than 10 hours only occur at depths greater than 7000 ft.

4. The distribution of the decay times seems to increase exponentially with depth with the same proportional constant of the distribution of microseismicity with depth (Figure 8.5).

This dependence with depth confirms quantitatively that as stresses increase, and for sustained mining conditions, the rock mass will take more time to respond to a new mining configuration.

8.4 Ontario’s large magnitude event database

To provide some guidance on the possible decay times and exclusion zone size for large Nuttli magnitude events, data has been collected from several mine-wide rockbursts and large magnitude events at different sites in Ontario. Table 8.1 presents the cases collected so far together with some of their characteristics. The end time $t_N$ of the sequences listed in Table 8.1 was detected by the ratios method with $N_A = 1$, $N_B = 5$ and $r_{5-1}^c = 0.002$. The apparent mechanism was given by mine personnel.

First, the decay times of the sequences are analyzed. In order to compare the decay times of different sites it is necessary to use a normalized uniform criterion independent of the site specific nature of seismicity. A common most frequent level of seismicity rate may not exist for all the sites. Therefore, individual rate-histogram plots were examined and for each aftershock sequence the most frequent level of event rate was determined. The decay time was defined as the time to decay to this individual most frequent level of seismicity. An example of this application was presented in Figure 7.17. The premise behind this methodology is that aftershock sequences in different seismic environments will have their own delay time of most significant readjustment.
indicated by the beginning of the most frequent level of seismicity rate. Using this criterion the back analysis of the aftershock sequences was performed.

Once the decay time was estimated the spatial extent of the events was analyzed. For simplicity, the exclusion zone is represented by a sphere. In some cases, it was found that the first event in the sequence was not necessarily associated with the cluster of seismicity (Figure 8.7). Considering that this can be attributed to a poor location of the selected first event (Figure 8.7b), the centroid of seismicity occurring during the first hour after the principal event was used as the

Table 8.1: List of analyzed large magnitude events from Ontario mines.

<table>
<thead>
<tr>
<th>#</th>
<th>Site</th>
<th>Date (mm/dd/yyyy)</th>
<th>(M_n)</th>
<th>(t_N) (hours)</th>
<th>(N)</th>
<th>Apparent mechanism*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>10/13/2006</td>
<td>1.1</td>
<td>27.3</td>
<td>48</td>
<td>SB</td>
</tr>
<tr>
<td>2</td>
<td>Copper Cliff North</td>
<td>09/30/2004</td>
<td>1.9</td>
<td>25.6</td>
<td>51</td>
<td>SB</td>
</tr>
<tr>
<td>3</td>
<td>Copper Cliff North</td>
<td>11/30/2004</td>
<td>2.4</td>
<td>29.9</td>
<td>855</td>
<td>FS</td>
</tr>
<tr>
<td>4</td>
<td>Copper Cliff North</td>
<td>06/10/2005</td>
<td>2.1</td>
<td>10.3</td>
<td>172</td>
<td>FS</td>
</tr>
<tr>
<td>5</td>
<td>Copper Cliff North</td>
<td>06/11/2006</td>
<td>2.8</td>
<td>72.1</td>
<td>1880</td>
<td>SB</td>
</tr>
<tr>
<td>6</td>
<td>Copper Cliff North</td>
<td>09/24/2008</td>
<td>2.4</td>
<td>69.4</td>
<td>164</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>Craig</td>
<td>06/22/2007</td>
<td>2.2</td>
<td>37.5</td>
<td>507</td>
<td>FS</td>
</tr>
<tr>
<td>8</td>
<td>Creighton</td>
<td>11/29/2006</td>
<td>4.1</td>
<td>165.5</td>
<td>3742</td>
<td>FS</td>
</tr>
<tr>
<td>9</td>
<td>Creighton</td>
<td>06/15/2007</td>
<td>3.0</td>
<td>27.6</td>
<td>591</td>
<td>FS</td>
</tr>
<tr>
<td>10</td>
<td>Creighton</td>
<td>10/07/2007</td>
<td>3.1</td>
<td>53.6</td>
<td>801</td>
<td>FS</td>
</tr>
<tr>
<td>11</td>
<td>Creighton</td>
<td>10/17/2007</td>
<td>1.0</td>
<td>18.4</td>
<td>86</td>
<td>FS</td>
</tr>
<tr>
<td>12</td>
<td>Creighton</td>
<td>11/20/2007</td>
<td>0.6</td>
<td>9.3</td>
<td>24</td>
<td>FS</td>
</tr>
<tr>
<td>13</td>
<td>Creighton</td>
<td>03/14/2009</td>
<td>2.6</td>
<td>197.7</td>
<td>2933</td>
<td>FS</td>
</tr>
<tr>
<td>14</td>
<td>Creighton</td>
<td>02/07/2008</td>
<td>2.4</td>
<td>45.9</td>
<td>197</td>
<td>?</td>
</tr>
<tr>
<td>15</td>
<td>Creighton</td>
<td>04/17/2008</td>
<td>1.5</td>
<td>7.1</td>
<td>23</td>
<td>?</td>
</tr>
<tr>
<td>16</td>
<td>Creighton</td>
<td>12/06/2008</td>
<td>2.9</td>
<td>25.3</td>
<td>161</td>
<td>?</td>
</tr>
<tr>
<td>17</td>
<td>Fraser</td>
<td>10/16/2008</td>
<td>2.4</td>
<td>19.0</td>
<td>114</td>
<td>?</td>
</tr>
<tr>
<td>18</td>
<td>Garson</td>
<td>12/05/2008</td>
<td>3.3</td>
<td>14.4</td>
<td>117</td>
<td>FS</td>
</tr>
<tr>
<td>19</td>
<td>Kidd</td>
<td>03/02/2006</td>
<td>1.6</td>
<td>23.5</td>
<td>223</td>
<td>PB</td>
</tr>
<tr>
<td>20</td>
<td>Kidd</td>
<td>01/06/2009</td>
<td>3.8</td>
<td>71.6</td>
<td>116</td>
<td>FS</td>
</tr>
<tr>
<td>21</td>
<td>Macassa</td>
<td>07/12/2008</td>
<td>3.1</td>
<td>469.7</td>
<td>583</td>
<td>PB</td>
</tr>
</tbody>
</table>

* Apparent mechanism given by mine personnel: SB: Strain burst, FS: Fault slip, PB: Pillar burst.
center of the sphere. This point leads to a better statistical representation of the size of the affected zone (Figure 8.7c).

Figure 8.7. Spatial distribution of a $M_n=2.8$ rockburst at the Copper Cliff North Mine. Location of the first event in the sequence relative to the seismicity occurring during the first 24 hours (frame a). Distance distributions measured from the coordinates of the event occurring at $t=0$ (frame b) and from the first hour centroid (frame c).
For estimating the spherical exclusion zone, only events that occurred before the decay of the sequence to the most frequent level of seismicity rate are considered. This ensures that the estimated exclusion zone only includes events that are related to the maximum change in rate of the sequence. Figure 8.8 presents the resulting event count decay times, the best fit spherical radius calculated by least squares as a function of the Nuttli magnitude for all the 21 sequences analyzed. As a more conservative estimate of the exclusion zone, the spherical radius containing 90% of the events has also been included in Figure 8.8.

Figure 8.8. Event count decay times (frame a), best fit spherical radius (frame b) and spherical radius containing 90% of the events (frame c) as a function of the Nuttli magnitude for different aftershock sequences in Ontario mines.
Despite some natural dispersion in the data, a remarkable and significant exponential increase in the decay time and the size of the spherical radius representing the exclusion zone can be recognized as the Nuttli magnitude of the event increases. The size of the best fit sphere and the one containing 90% of the events correspond very well with the identified ranges of exclusion zones currently used at the surveyed mines (Section 3.5). The dispersion in Figure 8.8 reflects the site specific nature of seismicity but also suggests that the mechanism of the sequence may play a role in determination of the decay time. It is expected that for mining-induced aftershock sequences, the mechanism involved (e.g., pillar burst, fault slip, strain burst) may play a role in the aftershock productivity. This has been suggested in the crustal literature by Yamanaka and Shimazaki (1990) and Guo and Ogata (1995), which considered aftershock statistics separately for intraplate and interplate earthquakes.

Next, the irregularity in the decay of the aftershock sequences is investigated. The difference between the decay time estimated using the most frequent level of seismicity rate and the time of maximum curvature as a function of the Nuttli magnitude is presented in Figure 8.9. An increase in the time difference is observed as the Nuttli magnitude increases. This can be used as a guideline for considering a time window after the maximum curvature of the actual sequence, estimated with the correlations provided in Section 6.1.7.3 (see Table 6.5).
Figure 8.9.  Difference between the decay time estimated using the most frequent level of seismicity rate and the time of maximum curvature as a function of the Nuttli magnitude for different aftershock sequences in Ontario mines.

The correlations shown in Figure 8.8 and Figure 8.9 should be used with caution and should be interpreted as a guideline. It is recommended that re-entry times should be based on real-time data analysis, considering the behaviour and actual rate of the sequence. The above correlation is intended to assist mines with less seismic history, having their first large magnitude event or for areas with little coverage of the microseismic monitoring system. The Nuttli magnitude can be confirmed with the Geological Survey of Canada (http://earthquakescanada.nrcan.gc.ca/) and/or by a strong ground motion system, and based on the above correlations a space-time restriction for re-entry can be estimated.
8.5 Summary and discussion

Different case histories relating the decay of seismicity and re-entry protocol development with mining activities on a basis of a theoretical framework were presented.

A significant positive linear correlation was established between the volume of mined rock and the decay time of seismic moment. The applicability of these correlations was checked as if they were tracked and updated continuously in time. It was found that by using the data of previous mining steps it is possible to estimate in advance the decay time response of the rock mass to a new mining configuration. This can be particularly usefully to develop a proactive re-entry protocol that can be adjusted as the new data becomes available.

Using data from the Creighton Mine, microseismicity was shown to correlate with depth. An exponential increase in the frequency of mining events for almost all magnitude bands was observed. This resulted also in an exponential increase in the frequency of the decay time of seismic work. As mining progresses to deeper levels and for a sustained mining condition, a higher number of cases with longer decay times is found. This is a direct consequence of the in-situ stresses that increase with depth.

Perhaps, the most significant correlations proposed in this chapter are those that relate the decay time and size of the exclusion zone with the Nuttli magnitude of the main event. This is a major development for mines with little experience developing their first re-entry protocol and for areas with poor coverage of the microseismic monitoring system.
Chapter 9

Contributions, conclusions, final recommendations and future research

To summarize the methodological procedures developed in this thesis and the important set of practical rules resulting from the application of these procedures, this chapter is divided into three parts: The first part documents the proposed methodologies to standardize the development of re-entry protocols in mines where sufficient historical data is available, and summarizes the most significant conclusions resulting from the application of these procedures to eight mining-induced and two crustal seismicity catalogues. The second part presents generic guidelines for the development of re-entry protocols in Ontario mines without the requirement for previous intensive calibration. These are considered as recommendations when limited data is available. Suggestions for future research possibilities are made in the final part.

9.1 Contributions and conclusions

The present study has resulted in both an increase of the general knowledge of re-entry protocols, the decay of aftershock sequences and a methodology in seismicity analysis for the standardization of re-entry protocol development. The major contributions and main conclusions of this work are presented below.

- A uniform statistical procedure for identifying the most consistent parameters of empirical scaling relations was developed and tested. These empirical relations include: (1) Modified Omori’s law for the temporal decay of aftershocks, (2) Gutenberg-Richter frequency-magnitude scaling, (3) Båth’s law for the magnitude of the largest aftershock, and two alternative rate stochastic models (Reasenberg-Jones and ETAS).
The developed procedure for fitting scaling relations was successfully applied to the aftershock sequences identified in eight mining-induced and two crustal seismic catalogues. It can be concluded that: (1) mining-induced seismicity aftershocks follow similar patterns as crustal scale aftershocks, and (2) the proposed methodology can be routinely applied to seismicity catalogues.

Guidelines of how the scaling relations can be applied for the selection of parameters relevant to re-entry protocols were provided, including procedures for selecting triggering magnitude, reference decay-law curves, background time window, and resetting thresholds.

Individual conclusions and recommendations for each scaling relation are listed below.

1. In order to obtain consistent parameter estimates for the modified Omori’s law it is necessary to exclude some events from the beginning and end of the seismic sequence and consider the portion that satisfy power-law behaviour.

2. It is still not possible to conclude which are the most significant factors controlling the $p$ value in mining-induced aftershock sequences.

3. The Gutenberg-Richter relationship can be fitted, without introducing significant errors for magnitude completeness, from the bin with the highest frequency of events in a non-cumulative frequency-magnitude distribution. The uniaxial magnitude resulted in a more consistent magnitude scale for estimating the Gutenberg-Richter parameters and performing aftershock statistics.

4. The Reasenberg-Jones model presented a low performance for estimating $K$ values. However, the model resulted useful for the selection of magnitude thresholds for invoking a microseismic magnitude event protocol.
5. For applying Båth’s law to mining-induced aftershock sequences it is necessary to consider as the principal event the largest magnitude measured during the first hour, with a magnitude higher than the one used for invoking a microseismic magnitude event protocol.

6. Using the ETAS model with the average parameters of a zone produced a low number of events per hour at the beginning of the sequence compared to the average MOL and the actual data. It was concluded that the MOL with the proposed guidelines is the most suitable model for representing the time decay of mining-induced aftershock sequences.

- By applying the developed procedure for fitting scaling relations common statistical properties of aftershock sequences at different seismic environments were recognized that enables useful decisions on tasks such as re-entry to be made. These statistics are valuable for mines without significant seismic history data.

- Using rigorous statistical assessments the time decay patterns after blasts, large magnitude events/rockbursts and earthquakes were studied in detail. The main pattern stages that were revealed for both crustal and mining-induced seismic sequences are:

  1. Initial noisy/complex aftermath before the onset of power-law decay.

  2. High change in rate until the maximum curvature time $T_{MC}$ of the modified Omori’s law.

  3. After $T_{MC}$ the power-law decay can still continue until a time where the data deviates from power-law behaviour and a transition to a different process takes place.

The time of maximum curvature $T_{MC}$ is a physical property related to the change in decay rate of single time sequences. This is significant for re-entry protocol development as it defines the transition from high to low event change rate.
To provide insight into the decay patterns of an ongoing sequence the concept of seismic envelopes was developed. With this family of decay-law curves it is possible to evaluate the path and decay pattern of a new large magnitude event or rockburst. The approach considered here is to analyze several mining seismic sequences in order to identify the natural response of the rock mass to the mining process. This is completely new as the current approach is to fix a unique reference curve from a single past large magnitude event or rockbursts and use it for new occurrences.

A procedure for estimating seismicity rate thresholds for re-entry protocol development was proposed. This technique is based on the physical concept that the most frequent level of seismicity rate is where the rock mass is most likely to be most of the time, and blast and large magnitude events disrupt this equilibrium. This is a major improvement for the standardization and development of re-entry protocols and is not limited to large data samples. It can be applied to a complete catalogue for evaluating the global response of the seismic environment or to isolated aftershock sequences for estimating the local variability of the seismicity rate thresholds. There is also almost no need for interpretation, as the most frequent level and its associated error is selected automatically by the scheme. It can be concluded that they provide a major tool for standardization of re-entry protocol development.

The re-entry times determined by event count and seismic work were compared. It can be concluded that the event count parameter provides an estimate of the re-entry time as good as the one provided by seismic work.

The characteristics of aftershocks decay and seismicity rate thresholds have been included in a probabilistic based design. The end product is reflected in two charts: seismic envelopes
and the rate diagram. These tools lead to an improved understanding of how seismicity returns to background levels and enable to quantify the degree of confidence of the re-entry protocol decision making process. This is the recommended method for the standardization of re-entry protocol development.

- Logistic regression was considered for establishing the probability of invoking a microseismic magnitude event protocol. A procedure was proposed for identifying key microseismic source parameters that should be considered in the logistic model. Applying the procedure to two mining sites it was determined that: \(u\text{Mag}, t\text{Mag}, E_s/E_p\) and \(r_o\) were truly independent parameters to consider for invoking a re-entry protocol.

- This study represents, as far as the author is aware, the first attempt to quantify relations between mining factors, such as: volume of mined rock, depth, and magnitude of large events, to the time decay of mining-induced aftershock sequences.

- Preliminary correlations for estimating the time decay and size of the exclusion zone based on the Nuttli magnitude of the main event have been proposed. These correlations are particularly useful for assisting mines with less seismic history, having their first large magnitude event or for areas with little coverage of the microseismic monitoring system.

- The aim of re-entry protocols is to enhance workplace safety by restricting access or proactively closing areas based on seismic information. By applying the proposed methods to standard seismic databases, the risk after large magnitude events/rockbursts and blasts can be managed more effectively.
9.2 Aftershock statistics for Ontario mines

Despite the site specific nature of mining seismicity consistent statistics have been identified that can be used to develop some generic guidelines for re-entry protocol development in Ontario mines without the requirement for previous intensive calibration. These simple recommendations are intended for those mines with limited historical seismicity, and to serve as a first approach guide for developing a re-entry protocol. Their applicability is, however, limited to single aftershock sequences.

Based on the characteristics of 294 aftershock sequences studied in 8 mining seismicity catalogues, and 21 large magnitude events, with a wide range of mining, geology, and seismic settings that can be found in Ontario mines, a set of guidelines has been derived for triggering of re-entry incidents, establishing the reference decay-law curves (modified Omori’s law and seismic work equation) and resetting thresholds. The guidelines are presented and briefly described in the following sections.

9.2.1 Triggering of re-entry incidents

The magnitude of a principal event $M_{PE}$ relative to the cut-off magnitude $M_c$ required to raise the maximum curvature time above two hours was used to set a microseismic magnitude event protocol (Table 6.9). The difference $M_{PE} - M_c$ was normally distributed with average values of:

- 1.3±0.3 for moment magnitude, and
- 1.9±0.4 for uniaxial magnitude.

To use these guidelines it is necessary to identify in the seismic data of interest the cut-off magnitude $M_c$ selected at the magnitude bin with the highest frequency of events in a non-cumulative frequency-magnitude distribution. To build the frequency-magnitude distribution it is
recommended to filter events labelled as blasts. Then, using the above averages, the magnitude of a principal event $\bar{M}_{PE}$ triggering a re-entry protocol can be estimated and compared with the actual frequency-magnitude distribution of the zone. If necessary, $M_{PE}$ can be adjusted by considering the standard deviation.

In addition to the microseismic magnitude oriented re-entry protocol it is recommended that an excessive seismicity re-entry protocol be set, which is represented by the measured events per hour. An average of $15\pm3$ events per hour is recommended. This guideline was derived in Section 6.1.8 (Table 6.6) and considers only the events above the cut-off magnitude $M_c$.

### 9.2.2 Reference decay-law curves

Two reference decay-law curves are used in practice:

- The modified Omori’s law, and

- The seismic work equation.

In the following, both equations are briefly presented with the corresponding guidelines for the selection of representative parameters. A recommendation is made for setting the reference decay-law curves in connection with current re-entry practices.
9.2.2.1 Modified Omori’s law

The modified Omori’s law for representing the decay rate of aftershocks \( n(t) \) is given by:

\[
n(t) = \frac{K}{(c + t)^p}
\]  

(9.1)

where \( t \) is the time measured from the principal event, \( c \) is set equal to zero for the guidelines, \( p \) is related to the speed of decay, and \( K \) to the number of events in the sequence.

The \( p \) values vary from sequence to sequence with average values ranging from 0.74 to 1.04 for the mining seismicity catalogues analyzed. The average \( p \) value from all the 315 mining-induced aftershock sequences analyzed is given by: \( \bar{p} = 0.90 \pm 0.28 \).

The parameter \( K \) can be adequately expressed by (Figure 9.1a):

\[
K = 0.42N_1
\]  

(9.2)

where \( N_1 \) is the measured number of events occurring during the first hour after the principal event.

Equation (9.1) has a characteristic time that should be considered as a reference for re-entry. This corresponds to the time of maximum curvature and can be expressed by (Figure 9.1b):

\[
T_{MC} = 0.42(N_1)^{0.60}
\]  

(9.3)
Correlations between the parameters $K$ (frame a) and $T_{MC}$ (frame b) with the measured $N_1$ for all the 315 mining-induced aftershock sequences analyzed.

9.2.2.2 Seismic work

The formula for representing the cumulative seismic work is given by:

$$SW(t) = Ct^D \quad \text{with } D < 1$$  \hspace{1cm} (9.4)

where $t$ is the time measured from the principal event, and $C$ and $D$ are adjustable model parameters. It was found that the $D$ values vary from sequence to sequence with average values ranging from 0.18 to 0.36 for the mining seismicity catalogues analyzed. The average $D$ value for all the 315 mining-induced aftershock sequences analyzed is given by: $\bar{D} = 0.26 \pm 0.14$.

The parameter $C$ can be estimated from (Figure 9.2):

$$C = 1.05SW_1$$  \hspace{1cm} (9.5)

where $SW_1$ is the cumulative seismic work measured during the first hour after the principal event.
9.2.2.3 Recommendations for setting the reference decay-law curves

The current approach using the reference decay-law curves is to calibrate the parameters for a past rockburst or large magnitude event and to use it as representative for new occurrences. Typical ranges and guidelines for the selection of parameters of these curves were presented. Using these guidelines it is recommended to establish the following three reference decay-law curves:

- One that represents a previous occurrence, usually a large magnitude event/rockburst or a large blast depending on the availability of data.
- Two curves representing the ongoing sequence by using the average and 95% prediction interval.
To accomplish this, it is only necessary to evaluate $N_i$ and $SW_i$ for the sequence of interest, and replace them in Eq. (9.2) and (9.5) respectively. For the constants $p$ and $D$ the average values can be assumed.

### 9.2.3 Resetting thresholds - Båth’s Law

The objective of these thresholds is to introduce the microseismic energy/strength release in the re-entry protocol. The statistic used here corresponds to Båth’s law and is given by:

$$\Delta M = M_1 - M_{1-12 \text{ hours}}$$

(9.6)

where $M_1$ and $M_{1-12 \text{ hours}}$ are the maximum magnitude recorded during the first hour after the principal event and the magnitude of the largest aftershock for the next 11 hours respectively.

The recommended average $\Delta M$ values, obtained from all the 315 mining-induced aftershock sequences analyzed, are given by:

$$\Delta M_{MomMag} = 0.94 \pm 0.50$$

$$\Delta M_{MomMag} = 1.11 \pm 0.64$$

(9.7)

To establish the resetting magnitude it is necessary to replace $M_1$ in the corresponding Eq. (9.7). Note that this law is only valid for a principal event with magnitude higher than the triggering magnitude identified in Section 9.2.1, i.e., $M_1 \geq M_{PE}$.

This law is incorporated into the re-entry protocol by monitoring the magnitude of the events during a time window after the time of maximum curvature estimated with Eq. (9.3). The course of action is to reset the re-entry clock if, during this time window, there are events larger than the resetting threshold given by Eq. (9.7). A minimum time window of two hours is recommended.
Based on the 95% prediction interval of Figure 9.1b. Alternatively, the time window can be estimated based on the Nuttli magnitude of the main event (see Figure 8.9).

9.3 Recommendations for future research

This research has involved an exploration of the applicability of aftershock statistics and principles governing the time decay of aftershock sequences after rockbursts, large magnitude events, blasts and earthquakes. From an examination of this body of research, it is possible to suggest a number of avenues for further research and development in this area (listed in order of priority).

1. Develop a software tool that incorporates the developed methodology in seismicity analysis and guidelines for re-entry protocol development.

2. Using the procedure developed to estimate the power-law parameters of the modified Omori’s law it would be useful to relate the decay $p$ constant to geotechnical-mining parameters in order to elucidate the most significant factors controlling the decay of mining-induced aftershock sequences. It is also worthwhile to examine more field evidence related to Dieterich’s (1994) theoretical results for $p$ values higher than 1: the shear stress applied to the fault after the main shock may decrease with time.

3. By using rate histograms it is possible to compare the decay times of different regressed microseismic source parameters. A statistical study could be carried out to determine key microseismic source parameters for a general re-entry protocol. In addition, this tool can be used for back analysis of individual aftershock sequences and therefore to correlate the decay time with geotechnical-mining parameters.
4. A grid map could be defined throughout the mine and seismicity rate thresholds could be calculated for each grid volume for a given past time period. This result will be a major development as it will be possible to visualize how the seismicity above the specified threshold evolves in space with time after a large magnitude event, rockburst or blast, showing in real-time which zones are above the rate threshold. This will define immediately the affected zones to be excluded for re-entry without the necessity of using pre-defined parametric shapes, such as: spheres and/or ellipsoids, for representing the exclusion zone. As time passes the zones that are returning to background levels will be indicated. This approach will eliminate the subjectivity in selecting the affected volume and will also reduce the exclusion zone to more specific areas.

5. The developed database of large magnitude events from Ontario mines needs to be extended, in terms of both information of currently included cases and addition of more cases. A more refined classification of the cases with respect to mining factors may help to introduce the mechanism in the proposed correlations.

6. Develop and implement a non-parametric space-time clustering identification scheme. This will enable aftershock sequences to be identified in space and time and reduce the subjectivity in the selection of the volume to monitor.

7. An in-depth study of aftershock migration after large magnitude events in mining seismicity would provide insights into the mechanisms that control the space-time exclusion zone.

8. Apply other classification algorithms to the identification of seismic event, blasts and reported occurrences, such as: neural networks, as the logistic regression model has a limited complexity on the constructed boundary for classification.
References


Appendix A: Survey of re-entry protocols in seismically active mines

**Background**

This survey is being carried out by the Department of Mining Engineering at Queen's University on behalf of the Ground Control Committee of the Mines and Aggregates Safety and Health Association of Ontario (MASHA). It is part of a WSIB sponsored research project whose goal is to produce practical guidelines for the development of re-entry protocols in seismically active mines.

The first phase of the project is to document current re-entry practice following significant seismic events. This will be followed-up by site visits. As some of the information collected may be considered sensitive, it will be kept confidential. The results of the survey will be distributed to participants in aggregate form only.

Your participation in this survey is vital to the successful outcome of the project. When completed, please return by clicking on the "Submit by Email" button at the top of this form. Alternatively, attach a copy of the completed file by e-mail and return to survey@mine.queensu.ca, or print a copy and mail to:

Re-entry Protocol Questionnaire  
Department of Mining Engineering  
Queen's University  
Kingston  
Ontario K7L 3N6  

**Contact Information**

<table>
<thead>
<tr>
<th>Company</th>
<th>Name of Mine</th>
<th>Ground Control Engineer</th>
<th>Name</th>
<th>Title</th>
<th>Telephone</th>
<th>Fax</th>
<th>e-mail</th>
</tr>
</thead>
</table>

**Seismic Re-Entry Protocol Description**

Statement of the protocol (alternatively, please attach a copy of the policy, procedure, or decision flow chart for seismic re-entry)
<table>
<thead>
<tr>
<th>Development</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Does the mine have a re-entry protocol</td>
<td>![Yes] ![No]</td>
<td></td>
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<tr>
<td>If the mine does not have a formal re-entry protocol, how is re-entry controlled following seismic events?</td>
<td></td>
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<tr>
<td>Who designed the protocol?</td>
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<tr>
<td>Is there a re-entry protocol after production blasts, after crown blasts, or in sensitive areas? Please specify and describe the protocol.</td>
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<tr>
<td>How long has the mine had a seismic re-entry protocol (this relates to the degree of experience with using the policy)?</td>
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<tr>
<td>Was the policy based on experience at the mine or was it adapted from re-entry practice at other mines?</td>
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<tr>
<td>How reliable is the policy in practice, e.g. does it provide an adequate (not too short or too long) re-entry period in all situations?</td>
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<table>
<thead>
<tr>
<th>Criteria</th>
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<tr>
<td>What are the key parameters (monitored or calculated, and their threshold) and used in the re-entry protocol?</td>
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<tr>
<td>Is there more than one re-entry protocol for seismic events/rockbursts?</td>
<td>![Yes] ![No]</td>
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<tr>
<td>Does the protocol differentiate between seismic events and rockbursts?</td>
<td>![Yes] ![No]</td>
<td></td>
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<tr>
<td>If there is another seismic event during the re-entry restriction period, is the re-entry clock reset?</td>
<td>![Yes] ![No]</td>
<td></td>
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<tr>
<td>Question</td>
<td>Answer</td>
<td></td>
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<tr>
<td>-------------------------------------------------------------------------</td>
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<td>If it is reset, is there a threshold magnitude for a resetting event?</td>
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<tr>
<td>Does the protocol vary with region of the mine, depth, rock type, proximity to geological structures, or any other factor? Please explain.</td>
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<tr>
<td>Are there any special cases to the policy (i.e. special mining situations, geological conditions) in deciding on re-entry?</td>
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<tr>
<td>Have any limitations of the policy (e.g. should additional parameters or criteria be incorporated?) been identified?</td>
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**Implementation**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
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</thead>
<tbody>
<tr>
<td>How many times per year (on average) is access restricted by re-entry protocol?</td>
<td></td>
</tr>
<tr>
<td>What information is obtained as part of the re-entry procedure and prior to permitting re-entry (examples: appropriate analysis of seismic event location, energy, post-event activity; cause of the event; inspection results-loose, integrity of the support system; rehab requirements; precautions to be taken)?</td>
<td></td>
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<tr>
<td>What standard precautions, if any, are incorporated into the protocol (e.g. re-entry inspection for lifting the restriction to the area for resumption of work)?</td>
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<tr>
<td>Is there a formal re-entry inspection? Who conducts it?</td>
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<tr>
<td>How far is access restricted from the source area (e.g. levels, distance) during re-entry restriction?</td>
<td></td>
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<tr>
<td>Following a catastrophic event, is the protocol a stand-alone procedure or does it link to other mine operating procedures?</td>
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</table>
**Management, Reporting & Communication**

<table>
<thead>
<tr>
<th>Question</th>
<th>Yes</th>
<th>No</th>
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<tbody>
<tr>
<td>Is the seismic re-entry protocol in writing?</td>
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<td>Is it a standard mine operating procedure (SOP)?</td>
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<td>Is it signed by senior management personnel?</td>
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<tr>
<td>Has it been reviewed by the mine Joint Health and Safety Committee?</td>
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<tr>
<td>Is there a formal process in place for the regular review of the re-entry protocols?</td>
<td></td>
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<tr>
<td>If yes to the above question, does the review involve the Joint Health and Safety Committee?</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>How is the re-entry protocol communicated to the workforce?</td>
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<tr>
<td>How is entry restricted (barricade, caution - do not enter tape, fence etc.)?</td>
<td></td>
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<tr>
<td>Who places the restriction?</td>
<td></td>
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<tr>
<td>If there is another seismic event during the re-entry restriction period, is the re-entry clock reset?</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

**Main concerns:**

**Suggested improvements:**

---

**General Aspects of Mining Seismicity**

**Seismic monitoring system (system information, data collected)**

<p>| Manufacturer, model name or number of microseismic monitoring |     |    |</p>
<table>
<thead>
<tr>
<th>system:</th>
<th></th>
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<tbody>
<tr>
<td>Number and types of sensors (uniaxial, triaxial)</td>
<td></td>
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<tr>
<td>What volume (length, width, depth) is covered by the array?</td>
<td></td>
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<tr>
<td>Typical spacing of sensors</td>
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<tr>
<td>What is the lower threshold of event magnitude that can be detected by the monitoring system?</td>
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<tr>
<td>What is the maximum event magnitude recorded at the mine?</td>
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<tr>
<td>Approximate source location accuracy</td>
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<tr>
<td>Parameters routinely monitored</td>
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<tr>
<td>What level of post-processing of microseismic data is routinely carried out at the mine?</td>
<td></td>
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<tr>
<td>Does the mine have a dedicated specialist to operate the microseismic system?</td>
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</table>

**Characteristics of seismic activity related to re-entry:**

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Have typical background levels of seismic activity been identified?</td>
<td>Yes  No</td>
</tr>
<tr>
<td>How far from active mining do events occur that trigger re-entry restrictions?</td>
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</tr>
<tr>
<td>What percentage of re-entry incidences are triggered by blasting vs. those that are apparently unrelated to mining activities?</td>
<td></td>
</tr>
<tr>
<td>At what depth in the mine did seismic activity commence?</td>
<td></td>
</tr>
</tbody>
</table>
Has a trend in seismic activity with depth been identified? If possible, please quantify.

Have distinct seismogenic regions of the mine been identified (could be related to rock type, depth etc.)? Please explain.

Does rockburst damage appear to be related to distance from the seismic event source or is it more random?

Are there any distinct mechanisms that are causing seismicity and rockbursting (mining geometry, pillars, contacts, rock property contrasts, etc.)?

Has a rate of rockbursting been noted (e.g. rockburst/tonnes mined)?

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**Rock Mechanics, Geology and Mining Information**

Information regarding rock properties, geology and mining will be required for the analysis, but in order to keep the questionnaire as brief as possible, we will collect this information from the mine design package. Therefore, **please provide a copy of the mine design package**, or portions that provide this information.
Appendix B: Justification of the criteria used to select the temporal power-law decay limits of the modified Omori’s law

This Appendix describes and justifies the criteria used in Section 6.1.4 for selecting the temporal power-law decay limits of the modified Omori’s law (MOL). The procedure is described in the following seven steps.

Step 1

Given the time occurrence \( t_i \) (\( i = 1, \ldots, N \)) of the individual events of an aftershock sequence calculate the inter-event median times relative to the principal event: \( \bar{t}_i = (t_i + t_{i-1})/2 \). Next, define intervals of possible start and end times of power-law decay: \( \Delta T_S = [\bar{t}_1, \bar{t}_i^*] \), \( \Delta T_E = [\bar{t}_{i+1}, \bar{t}_N] \) as shown in Figure B.1.

![Figure B.1. Definition of the inter-event median times relative to the principal event \( \bar{t}_i \) and intervals of possible start \( \Delta T_S \) and end \( \Delta T_E \) times of power-law decay.](image_url)
Initially, for the benefit of computational efficiency, the division between $\Delta T_S$ and $\Delta T_E$ is set at the event closest to one time unit, i.e., $t_i^* \approx 1$. To ensure stable solutions the beginning of the interval of end times of power-law decay $\bar{t}_{i+1}^*$ is set at the inter-event median time closest to $\bar{t}_i^*$ plus one time unit, i.e., $\bar{t}_{i+1}^* \approx \bar{t}_i^* + 1$.

**Step 2**

For each possible combination of inter-event median times $T_A$ and $T_B$ contained in $\Delta T_S$ and $\Delta T_E$, respectively, the maximum likelihood estimate (MLE) of the MOL parameters $K$, $p$, $c$ with their uncertainties $\Delta K$, $\Delta p$, $\Delta c$ is obtained, corresponding goodness of fit $W^2$ and maximum curvature point $T_{MC}$ given by:

$$T_{MC} = \left[ Kp \left( 1 + \frac{2p}{2 + p} \right)^{\frac{1}{1+p}} \right] - c$$  (B.1)

$T_{MC}$ defines the transition from high to low event change rate. The theoretical derivation of Eq. (B.1) and the demonstration of the physical meaning of $T_{MC}$ can be found in Section 6.1.7.3.

If the time intervals $\Delta T_S$ and $\Delta T_E$ contain, respectively, $N_S$ and $N_E$ events, this procedure will generate a start-end times matrix of $(N_S + 2)(N_E + 2)$ elements, where each cell contains a vector with the estimated parameters $c$, $K$, $p$, $T_{MC}$ respectively uncertainties $\Delta c$, $\Delta K$, $\Delta p$ and goodness of fit $W^2$. 

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Step 3

From the solutions determined in the steps 1 and 2, consider only inter-event median times that conform to power-law behaviour:

\[ W^2 \leq 1 \]
\[ c = 0 \]  \hspace{1cm} (B.2)

Step 4

Next, consider all the inter-event median times that include the maximum curvature point:

\[ T_A \leq T_{MC} \leq T_B \]  \hspace{1cm} (B.3)

This constraint was identified from numerical simulations. Essentially, 100,000 MOL time sequences with a normally distributed \( p - \text{Normal}(p_{\text{avg}}, p_{SD}) \) value, \( c = 0 \) and \( N \) events in a time interval \([T_A, T_B]\) were generated. The variables \( N_{T_a-T_d}, T_A \) and \( T_B \) were varied uniformly over certain ranges \( \Delta N, \Delta T_A \) and \( \Delta T_B \) respectively. Reasonable values for these ranges using as a reference crustal aftershock sequences were selected (Kisslinger and Jones, 1991; Nyffenegger and Frohlich, 1998):

- \( p = 1.1 \pm 0.2 \),
- \( \Delta N = [10, 1000] \),
- \( \Delta T_A = [0.0001, 1.0] \), and
- \( \Delta T_B = [2, 100] \).

From this analysis it was found that the condition \( T_A \leq T_{MC} \) is always satisfied and that there is a small percentage (9.4%) of cases satisfying \( T_{MC} > T_B \).
The condition \( T_{MC} > T_B \) can be further examined by replacing the following expression obtained for \( K \):

\[
K = \begin{cases} 
\frac{N_{T_g-T_s}}{(T_B + c)(T_A + c)} & \text{for } p = 1 \\
\frac{N_{T_g-T_s}(1-p)}{(T_B + c)^{1-p} - (T_A + c)^{1-p}} & \text{for } p \neq 1
\end{cases}
\]  (B.4)

in Eq. (B.3) with \( c = 0 \) and imposing \( T_{MC} = T_B \):

\[
T_B = \begin{cases} 
\frac{N_{T_g-T_s}}{\ln T_B - \ln T_A} p \left( \frac{1+2p}{2+p} \right)^{\frac{1}{\frac{1}{p}}} & \text{for } p = 1 \\
\frac{N_{T_g-T_s}(1-p)}{T_B^{1-p} - T_A^{1-p}} p \left( \frac{1+2p}{2+p} \right)^{\frac{1}{\frac{1}{p}}} & \text{for } p \neq 1
\end{cases}
\]  (B.5)

Given \( T_A, p \) and \( N_{T_g-T_s} \), then Eq. (B.5) can be solved numerically to obtain the corresponding \( T_B \) that satisfies \( T_{MC} = T_B \). Figure B.2 presents the results from the numerical simulations and the curve obtained by solving Eq. (B.5).
Figure B.2. Results of the numerical simulation of 100,000 MOL sequences testing the condition: $T_A \leq T_{MC} \leq T_B$. Only 9.4% of the sequences do not satisfy the tested condition.

A clear boundary in the $T_B - N_{T_B-T_A}$ plane discriminating the cases that satisfy $T_A \leq T_{MC} \leq T_B$ can be identified in Figure B.2. The interpretation is that for a given $T_B$, a minimum number of events $N_{T_B-T_A}$ is required to raise $T_{MC}$ above $T_B$. In real aftershock sequences this situation can appear in two cases:

(a) When $T_B$ is the time of last event in the sequence and $N_{T_B-T_A}$ is high enough to raise $T_{MC}$ above $T_B$.

(b) When a significant secondary aftershock sequence or change in slope has been triggered forcing the power-law to end promptly (for an example see Figure B.6).
If after applying the condition $T_A \leq T_{MC} \leq T_B$ no feasible solution is found, then the condition $T_{MC} \leq T_B$ is relaxed. If after this relaxation no solution is found then the division between the possible start and end times of power-law decay $t^*$ is increased by one time unit and the algorithm is repeated from step 1. If with the relaxed condition feasible solutions are obtained then the algorithm continues to step 5, keeping in memory that the condition $T_{MC} \leq T_B$ was transgressed. In this situation an additional check for change in slope or secondary aftershock sequences is applied to the solution obtained from step 6.

**Step 5**

After applying Eqs. (B3) and (B4), the space of feasible solutions is reduced considerably. However, there are still multiple potential solutions. Several studies of crustal aftershock sequences have suggested that the immediate events are more directly related to the main event than the late-occurring events (Doser and Kanamori, 1986; Wang, 1994). This is also consistent with several physical models of seismicity (e.g., Mikumo and Miyatake, 1979; Dieterich, 1994). Therefore, an optimal solution should try to maximize the time interval that contains these more related events. A characteristic time $T_{MC}$ of sequences that follow a MOL was already introduced in step 2. Considering that $T_{MC}$ defines the transition from high to low change in rate, it seems appropriate to introduce this point as a reference.

For maximizing the time between $T_A$ and $T_{MC}$ two different criterions are tested in the following:

\[
\text{max}\{T_{MC} - T_A\} \quad \text{(B.6)}
\]

\[
\text{max}\{T_{MC} / T_A\} \quad \text{(B.7)}
\]
To evaluate which criteria is the most appropriate, synthetic MOL time sequences were generated using the method described in Nyffenegger and Frohlich (1998). This method makes use of the cumulative MOL density function given by:

\[ u_i = \begin{cases} 
  \frac{\ln(t_i + c) - \ln(T_a + c)}{\ln(T_b + c) - \ln(T_a + c)} & \text{for } p = 1 \\
  \frac{(t_i + c)^{1-p} - (T_a + c)^{1-p}}{(T_b + c)^{1-p} - (T_a + c)^{1-p}} & \text{for } p \neq 1
\end{cases} \]  

(B.8)

By rearranging Eq. (B.8) the following expression is obtained for the delay time of the \(i^{th}\) event after the principal event:

\[ t_i = \begin{cases} 
  \exp\left[\ln(T_b + c) - \ln(T_a + c)\right]u_i + \ln(T_a + c) - c & \text{for } p = 1 \\
  \left[\left((T_b + c)^{1-p} - (T_a + c)^{1-p}\right)u_i + (T_a + c)^{1-p}\right]^{\frac{1}{1-p}} - c & \text{for } p \neq 1
\end{cases} \]  

(B.9)

This equation is used to create sequences having a population of \(N\) aftershocks for cumulative density function positions \(u_i = (u_1, u_2, \ldots, u_N)\) drawn randomly from a uniform distribution in a target time interval \([T_a, T_b]\). This method creates mathematically perfect synthetic sequences in the sense that they do not include events related to background seismicity or spurious aftershocks that in real sequences might be caused by misidentified or mislocated events (Nyffenegger and Frohlich, 1998). However, the natural variations and main features in the inter-event times of real aftershock sequences are well reproduced as shown in Figure B.3 for a mining-induced sequence.
Figure B.3. Generation of synthetic MOL sequences. The parameters shown in the figure were estimated by maximum likelihood for a real mining-induced aftershock sequence and then used to generate several synthetic sequences.

The idea is to generate synthetic MOL sequences with a known transition point $T_c$ at the beginning of the sequence, that are followed by a power-law decay with known constant $p_{syn}$, and evaluate which criteria: $\max\{T_{MC} - T_A\}$ or $\max\{T_{MC} / T_A\}$, is the most effective for recovering $T_c$ and $p_{syn}$.

It is necessary to generate synthetic sequences that represent as close as possible real aftershock sequences. The following synthetic generation was considered appropriate:
• First generate one synthetic MOL with \( c_{\text{syn}1} \neq 0 \), \( p_{\text{syn}} \) and \( N_{\text{syn}1} \) for a time interval \( [T_a, T_c] \).

• Generate a second synthetic MOL with the same \( p_{\text{syn}} \) value but with \( c_{\text{syn}2} = 0 \) and \( N_{\text{syn}2} \) for a time interval \( [T_c, T_b] \).

This type of synthetic generation creates a noisy beginning of known duration \( T_c \), followed by a power-law decay with known constant \( p_{\text{syn}} \) (Figure B.4).

**Figure B.4.** Example of a two phase synthetic sequence generated to test which criterion: \( \max\{T_{MC} - T_A\} \) or \( \max\{T_{MC} / T_A\} \), is the most effective for estimating the power-law MOL decay parameters.
By using of this approach 1,000 synthetic sequences were generated using: \( c_{\text{syn}1} \sim \text{Uniform}(0,1) \), \( p_{\text{syn}} \sim \text{Normal}(1.1, 0.2) \), \( N_{\text{syn}1} \sim \text{Uniform}(0,50) \), \( T_a = 0 \) and \( T_c \sim \text{Uniform}(0.001,1) \) for the first MOL, and \( N_{\text{syn}2} \sim \text{Uniform}(50,450) \) and \( T_h \sim \text{Uniform}(10,100) \) for the second MOL.

For each of the synthetic sequences generated, steps 1 to 4 were followed. Then the transition point \( T_c \) and the decay parameter \( p_{\text{syn}} \) were recovered by applying the criterions: \( \max\{T_{MC} - T_A\} \) and \( \max\{T_{MC}/T_A\} \). To evaluate which of the criterions is the most effective for recovering \( T_c \) and \( p_{\text{syn}} \) two statistical indices are used, the variance-account-for (VAF) and the coefficient of determination \( R^2 \):

\[
VAF = 1 - \frac{\text{var}(y - \hat{y})}{\text{var}(y)}
\]

\[
R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum y^2 - \left(\sum y\right)^2/n}
\]

where var denotes the variance, \( y \) is the reference value, \( \hat{y} \) is the estimated value and \( n \) is the number of cases. If the VAF and the \( R^2 \) are equal to one, then the generated and recovered decay parameters are in excellent agreement.

Figure B.5 presents the probability and cumulative density functions of the differences between the reference values (\( T_c \) and \( p_{\text{syn}} \)) used to generate the synthetic sequences and the ones recovered by each criterion: \( T^\text{max\{MC-TR\}}_c \), \( p^\text{max\{MC-TR\}} \) and \( T^\text{max\{MC/TR\}}_c \), \( p^\text{max\{MC/TR\}} \). The VAF and \( R^2 \) are included in each frame.
Figure B.5. Probability and cumulative density functions of the differences between the reference values ($T_c$ and $p_{\text{syn}}$) used to generate synthetic sequences and the ones estimated by using the criterions: $\max\{T_{\text{MC}} - T_A\}$ (frame a) and $\max\{T_{\text{MC}} / T_A\}$ (frame b). The $VAF$ and $R^2$ are included in each frame.

It can be concluded that $\max\{T_{\text{MC}} - T_A\}$ produces the most reliable results for recovering: $T_c$ and $p_{\text{syn}}$ of the synthetic aftershock sequences (higher $VAF$ and $R^2$). The main reason why $\max\{T_{\text{MC}} - T_A\}$ resulted in a better criterion than $\max\{T_{\text{MC}} / T_A\}$ can be observed in Figure B.5.
The recovered transition point $T_c$ by the criterion $\max\{T_{MC}/T_A\}$ has a tendency to lower values than the actual one, giving a distribution of $T_c - T_c^{\max(T_{MC}/T_A)}$ shifted to the right (Figure B.5b1). The implication is that the criterion $\max\{T_{MC}/T_A\}$ tends to be contaminated by the first phase of the synthetic sequence given higher recovered decay constant (Figure B.5b2).

Based on this analysis, the criterion $\max\{T_{MC} - T_A\}$ is chosen for maximizing the time interval between $T_A$ and $T_{MC}$. Once this criterion is applied to the remaining solutions from steps 3 and 4 a reference time interval $[T_{A_r}, T_{B_r}]$ is selected. This also fixes a reference decay constant $p_r \pm \Delta p_r$ of the MOL.

**Step 6**

The optimal solution is selected as the one that maximizes the time interval relative to the reference solution determined in step 5, and is referred to as start $T_S$ and end $T_E$ times of power-law decay. This condition is defined as the inter-event median time interval that maximizes the ratio $T_B/T_A$ and satisfies $T_A \leq T_{A_r}$, $T_B \geq T_{B_r}$. This criterion was established by using the identified previous result that: $\max\{T_B/T_A\}$ produces lower $T_A$ values and therefore a longer time interval, which is the objective in this case. To avoid selecting an optimal solution too different from the reference one, the optimum $p_{t_5 - t_6}$ value is constrained within a 95% confidence limit of the reference value $p_r$, i.e., $|p_{t_5 - t_6} - p_r| \leq 1.96 \Delta p_r$. 

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Step 7

If the condition $T_{MC} \leq T_B$ was transgressed in step 4, it is necessary to check if a secondary aftershock sequence or a change in slope was actually triggered at the $T_E$ determined from step 6.

To confirm this, the following two rate models are compared through the $AIC$:

$$n(t) = \frac{K_0}{(c_0 + t)^{p_0}} \quad \text{for } T_S \leq t \leq t_N \quad (B.12)$$

$$n(t) = \begin{cases} 
\frac{K_1}{(c_1 + t)^{p_1}} & \text{for } T_S \leq t \leq T_E \\
\frac{K_1}{(c_1 + t)^{p_1}} + \frac{K_2}{(c_2 + t - T_E)^{p_2}} & \text{for } T_E < t \leq t_N \\
\frac{K_2}{(c_2 + t)^{p_2}} & \text{for } T_S \leq t \leq T_E 
\end{cases} \quad (B.13)$$

$$n(t) = \begin{cases} 
\frac{K_0}{(c_0 + t)^{p_0}} & \text{for } T_S \leq t \leq t_N \\
\frac{K_1}{(c_1 + t)^{p_1}} + \frac{K_2}{(c_2 + t - T_E)^{p_2}} & \text{for } T_E < t \leq t_N \\
\frac{K_2}{(c_2 + t)^{p_2}} & \text{for } T_S \leq t \leq T_E 
\end{cases} \quad (B.14)$$

If it is possible to establish that one of the models considered in Eqs. (B.13)-(B.14) is better (smaller $AIC$) than Eq. (B.12), then it can be concluded that a secondary sequence or change in slope was actually triggered at $T_E$ and the solution with $T_{MC} > T_E$ is adopted. Otherwise, $t^*$ is increased by one time unit and the scheme is repeated from step 1.

Figure B.6 presents an example of an aftershock sequence in which the power-law decay has been interrupted at $T_E$ and the condition $T_{MC} \leq T_B$ has been relaxed to find an optimal solution. In Figure B.6 when the condition of step 4 ($T_A \leq T_{MC} \leq T_B$) was applied no solution was found. After relaxing $T_{MC} \leq T_B$ and following steps 5 and 6 the solution presented in Figure B.6a was obtained (solid red line). Then Eq. (B.12) and (B.14) were fitted to the data (dashed black line and solid grey line respectively) and compared through the $AIC$ (Figure B.6b). It can be
concluded that a rate model that includes a change in slope at time $T_E$ fits the data better than a single MOL. Under these considerations the final solution is accepted as the power-law decay.

![Graph showing rate decay over time with parameters](image)

**Figure B.6.** Example of an aftershock sequence in which the power-law decay has been interrupted at $T_E$ and the condition $T_{MC} \leq T_B$ has to be relaxed to find an optimal solution. (a) Aftershock sequence and detected time intervals; (b) Relevant parameters for each time interval justifying the division at $T_E$.

**Figure B.7** presents a flowchart of the proposed method for determining the temporal power-law decay limits of the MOL that was implemented in a Visual Basic code.
Figure B.7. Flowchart of the proposed method for determining the temporal power-law decay limits of the MOL.

\[ t_i (i = 1, ..., N) \rightarrow \bar{t}_i = (t_i + t_{i-1}) / 2 \]
Set \( t^* = t_i \approx 1 \) and \( \bar{t}_i^* \approx \bar{t}_i^* + 1 \)
\[ \rightarrow \Delta T_S = [\bar{t}_1^*, \bar{t}_i^*] \Delta T_E = [\bar{t}_i^* + 1, t_N] \]

For each possible \( T_A \in \Delta T_S \) and \( T_B \in \Delta T_E \)
\[ \rightarrow K \pm \Delta K, \ p \pm \Delta p, \ c \pm \Delta c, \ T_{MC}, \ W^2 \]

Keep solutions that satisfy:
\[ W^2 \leq 1 \]
\[ c = 0 \]

Relax:
\[ T_{MC} \leq T_B \]

Keep solutions that satisfy:
\[ T_A \leq T_{MC} \leq T_B \]

Find: \( \max \{ T_{MC} - T_A \} \)
\[ \rightarrow \text{Reference solution: } [T_{A_r}, T_{B_r}], \ p_r \pm \Delta p_r \]

Find: \( \max \{ T_B / T_A \} \) with \( T_A \leq T_{A_r}, \ T_B \geq T_{B_r}, \ |p_{T_S - T_E} - p_r| \leq 1.96 \Delta p_r \)
\[ \rightarrow \text{Optimal solution: } [T_S, T_E], \ K_{T_S - T_E}, \ p_{T_S - T_E}, \ T_{MC{T}_S - T_E} \]

\[ n_s(t) = \begin{cases} 
K_s / (c_s + t)^r & \text{for } T_s \leq t \leq t_s \\
K_s / (c_s + t)^r + K_s / (c_s + t - T_s)^r & \text{for } T_s \leq t \leq T_e \\
K_s / (c_s + t - T_e)^r & \text{for } T_e < t \leq t_s 
\end{cases} \]

\[ T_{MC{T}_S - T_E} > T_E ? \]

YES

END

NO

AIC_1 < AIC_2 ?

YES

NO