TOWARDS AUTOMATED UNDERGROUND MINE SURVEYING: INTEGRATION OF MOBILE MAPPING WITH GEOREFERENCED GEOMETRIC BEACONS

by

JULIAN VUSUMUZI SIMELA

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Abstract

Within the surveying community, more specifically in underground mining, the challenge for surveying and mapping is affordable, fast and accurate positioning that works well within constraints of underground mining environments. Current surveying methodology for underground environments has not changed much in the last half century. This thesis proposes use of new tools and procedures employed by the robotics community to provide opportunity to improve upon some methods of underground surveying and mapping.

Underground mining environments comprised of ramps, shafts and tunnel type corridors link areas of mineral extraction to the surface environment. Since these environments are GPS-deprived, automatic positioning in a global context providing centimetre level positioning and single-digit, degree orientation is challenging, especially to mark the start and end of mobile mapping traverses.

Current mobile mapping strategies use 2D and 3D laser scanners in conjunction with simultaneous localization and mapping (SLAM) techniques to map and model underground environments from point cloud data. These strategies currently operate mostly within local reference frameworks that can later be transformed to a global mine geodetic reference framework. They utilize beacon technology such as RFID tags, barcodes and other methods that include using natural features as beacons,
providing positioning results outside the positioning accuracies required for surveying and mapping, in the mine surveying context.

This thesis studies and makes contributions in several areas. First it compares and contrasts between conventional surveying and mapping, and mobile mapping. Second, it applies tools and techniques from mobile robotics to bear on problems and tasks in conventional surveying. It importantly focuses on fast, accurate and reliable registration of a mobile mapping tool to a global geodetic reference system, with minimal human input, whilst generating rigorously adjusted position estimates with associated uncertainties. Third, it demonstrates through experiments, strengths, weaknesses, advantages and disadvantages of the automated underground positioning method. Last, it highlights potential applications beyond surveying and mapping where this positioning method may be applicable. Within this list of contributions is the design, construction, calibration and testing of an equivalent mine reference system that uses geometric reference beacon technology to facilitate mobile positioning and mapping.
Acknowledgments

‘Who am I, O Lord God? And what is my house, that You have brought me this far?’ (2 Samuel 8:13)

‘... and honor come from You, .... In Your hand it is to make great and to give strength to all’ (2 Chronicles 29:12)

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My thanks go to Peck Tech Consulting Ltd, for the loan of their uGPS Rapid Mapper™equipment, to Lindsay Vanderbeck for helping me setup my experiments and gather data, and to Oscar Rielo and the Robert M. Buchan Department of Mining technicians, for logistical support in this research. I give thanks to my sons Mongezi...
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<td>Position and orientation of Scan 1176 in mine reference frame</td>
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<td>Scan 1176 convergence criteria at traverse end</td>
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<td>File # 42 raw traverse overlay onto as-built survey</td>
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<td>File # 42 calculated check point locations overlay onto as-built survey</td>
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<td>File # 44 raw traverse overlay onto as-built survey</td>
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<td>Overlay of File # 44 Automated map and as-built survey</td>
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D.112 File # 44 calculated check point locations overlay onto as-built survey 351
Nomenclature

λ  Geodetic longitude
φ  Geodetic latitude
a  Semi-major axis of reference ellipsoid
E  Easting
f  Flattening
H  Height or elevation
h  Geodetic height or elevation
N  Northing
$S_{ij}^{\circ}$  Adjusted distance between two points
X  First coordinate of an orthogonal geocentric coordinate system
x  First coordinate of an orthogonal topocentric coordinate system pointing in east direction
Y  Second coordinate of an orthogonal geocentric coordinate system
y  Second coordinate of an orthogonal topocentric coordinate system pointing in north direction
Z  Third coordinate of an orthogonal geocentric coordinate system
z  Third coordinate of an orthogonal topocentric coordinate system in zenith direction
Chapter 1

Introduction

1.1 Background

The ability for a robot or an automatic mapping system to automatically determine its position and orientation in a known or global reference system is a challenging task for surface based mobile mapping where there is availability of Global Positioning Systems (GPS). It is even more so a challenge in underground mining environments, where GPS positioning is unavailable, that automatic mapping systems be able to determine their location and orientation. While immense efforts by the robotics community have gone into developing systems that speed up the underground mapping process, there have not been concerted efforts to incorporate surveying philosophy into the mapping process, yet, mobile mapping systems have begun to operate in the domain of surveying.

Surveying practice has the components of measurement, mapping and setting out. Surveying involves the determination of relative spatial location of points through measurement of slope, horizontal and vertical distances, and measurement of horizontal and vertical angles between lines and determination of directions of lines [4, 62].
1.1. BACKGROUND

The determinants of a survey process through accompanying mathematical calculations include distances, angles, directions, location elevations, areas and volumes, with much of this information portrayed graphically as maps, profiles, cross-sections, diagrams or by other media viewable by computer.

On surface, satellite-based Global Positioning Systems (GPS) have become an integral tool for positioning, as-built mapping and setting out in surveying practice. However, GPS technology is unavailable in underground mining environments for the same type of work that it is employed in surface mining operations. Hence, positioning, mapping and setting out in conventional underground surveying use methods that have not advanced in the last half century despite technological advancements in instrumentation. Therefore conventional underground surveying processes remain tedious and laborious [7, 63, 30]. While it is true that conventional surveying and mapping methods are lengthy and tedious, new developments in the surveying and mapping process that use LiDAR\(^1\) mapping of the underground environment, by coupling tripod mounted terrestrial laser scanners with conventional traversing techniques to accurately estimate scanner location, are still inefficient and labour intensive [73]. Other technologies such as those highlighted in [37, 42] are used to survey and map the underground environment, and employing off-the-shelf laser scanners mounted on a mobile robotic vehicle in a stop-and-scan process also experience similar problems. While mobile robotics tools for underground mapping such as in [7, 63, 30, 55, 67, 11, 56] promise faster delivery of product, they have yet to satisfy the constraints and accuracies of established systems for conventional underground surveying and mapping.

\(^1\)LiDAR (Light Detection and Radar) is a remote sensing technology with the capability of collecting 3D point cloud data and with applicability in mapping and surface modelling.
As a result of this, other applications that build upon mobile mapping and require analysis of high density spatially related data, such as navigation and tracking of mining equipment in a global geodetic mine reference framework, are tasks that still are not easily and accurately accomplished.

It is apparent from surveying and mapping literature how advancements in surveying and mapping instrumentation and technology have drastically changed the surface surveying and mapping landscape. Such changes have largely come about through use of, as examples, robotic total stations, automatic levels, laser scanners, GPS, LiDAR and even satellite-based mapping systems. While surveying and mapping techniques for surface operations have rapidly advanced, this has not been the case for underground mining operations where advancement in surveying instrumentation have surpassed the surveying and mapping techniques used. A possible cause for this could be the spatial constraints within which underground mine surveying and mapping operates, leading to the restricted techniques that can be employed. While current surveying and mapping technology and mobile mapping improves data capture and data quality, current underground mapping research has focused more on 3D modelling rather than 2D mapping of the underground environment.

Recent developments in mobile robotics have seen aspects of mapping of the underground mining environment undertaken by remotely controlled robots in areas deemed unsafe for mine personnel. The process of mobile mapping of the underground mining environment by the robotics community has the potential to revolutionize how current surveying and mapping is carried out. The products of conventional and mobile mapping in underground mining are maps that are used for a variety of tasks. Conventional survey maps are the basis for design of underground and surface structures,
1.2. MOTIVATION

Compilation of ventilation, power, drainage and haulage plans, tracking of development progress and ore extraction [63] and are also used to solve problems relating to mine safety and protection of underground and surface structures [17]. However, these maps have been found inadequate and not accurate enough for applications such as determining the structural soundness of a mine especially where the mine was abandoned [55, 67].

Map generation from a mobile robotics perspective is currently a secondary product of the Simultaneous Localization and Mapping (SLAM) process [55, 67]. Here a robot determines its position in an environment while at the same time mapping the environment. Map generation can also be the primary product of an offline map generation process whose function is mapping for navigation purposes as described in [7, 6, 51]. In both these cases the resultant mappings provide little or minimal use in terms of positioning for geodetic purposes. These processes generate occupancy grid maps which assign probabilities to cells that indicate if a cell is occupied or not. Occupancy grid maps have the property of being locally consistent [7] and have been used as a basis for vehicle navigation, however, exhibit signs of multi-directional distortion arising possibly due to portrayal of a 3D world on a 2D plane without the process that generates these occupancy grid maps adequately accounting for the projection of scan data onto a defined mapping surface.

1.2 Motivation

Conventional underground surveying and mapping, and mobile mapping by the robotics community, appear to be two distinct fields of research when applied to the underground mining environment and it is this thesis’ contention that these fields are
currently disjoint and both exhibit limited success. While both attempt to solve the same problem, conventional surveying and mapping is regulated through mining law, whilst mobile mapping is currently not. Conventional mapping methods are relatively less accurate, whilst mobile mapping provide high point density to facilitate accurate mapping and a variety of other tasks that are not necessarily map products. This thesis is motivated by the following questions:

- Can the surveying and mapping process be improved in terms of production speed and positional accuracy?
- Can mobile mapping be adapted so as to provide geometric maps that can be a suitable replacement for conventional underground maps?
- Can conventional surveying and mobile mapping be combined into an efficient tool that can be used by the mining industry?

The surveying and mapping aspect is just the tip of the iceberg of the potential that exists for a merger in philosophy of the fields of surveying and mobile robotics. From an underground mining perspective, auto-tramming is a recent phenomenon that enables unmanned load-haul-dump operations in mine tunnels that is made possible by teaching, route profiling and playing back the route a vehicle has been taught [51]. The ability for a vehicle to playback a route is a precursor to a more dynamic system where multi-level tramming will be possible on the basis of maps generated with reference to a global coordinate system.

Automated drilling is a technology that is now a reality in some of the large mines in the world. Here drill patterns are an input into a computer system and these are set-out based on the scan of the working face. This automated process of scan
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registration is described in [50] for the purpose of automating the tunnel profiling process, however one of the issues raised is that in automating the drilling, blasting and removal process, the venting process that requires people to be out of the area for safety reasons is an impediment to work quickly moving forward. If a drill vehicle was automated, its ability to move to the desired place of operation and set itself up for directional drilling based on a model whose basis is a mine coordinate system would be a great asset.

In remote surface exploration environments such as outer space, where GPS is unavailable, automated exploration, navigation, positioning whose basis would be the outcome of this thesis research, even in a local reference system should be possible.

With an ever increasing possibility of underground applications that may employ use of autonomous robotic platforms, up until [38] there did not exist a method by which an autonomous robot could determine its location and orientation in an underground mine within a defined coordinate reference frame to accuracies close to that are required for surveying and mapping. This thesis expounds on and furthers the work first described in [38]. The ability for an autonomous vehicle to determine where it is and how it is oriented in a pre-defined coordinate system opens up other applications such as automated guidance that may require the vehicle to determine its own path to pre-determined coordinate locations, automated mine rescue etc.. The ability to harness large volumes of digital data eventually will lead to support real-time tracking, surveillance, traffic management, environmental monitoring, navigation, auto-tramming asset management and improved production monitoring and reporting [13]. These are but a few of many industry challenges that make this area also challenging and interesting to investigate.
The uGPS Rapid Mapper™ is a self contained tool that attempts to generate a 3D point cloud from a mobile platform in an underground environment with functionality for tunnel mapping, mapping for geotechnical use, volumetric mapping for planning purposes, shaft inspection and scanning for refinement of ventilation calculations [56]. Use of the uGPS Rapid Mapper™ or similar system is integral to the merger of surveying and mobile mapping in such a way as to minimize impact on mine development and production cycles, while at the same time significantly improving the underground surveying and mapping process.

1.3 Research Scope, Objectives and Goals

1.3.1 Scope

This thesis investigates a merger of two bodies of knowledge; surveying and mapping and mobile mapping, in order to optimize how an automatic mapping system or mobile platform can determine its position and orientation in an underground mining environment within a defined mine reference system. This research also applies automatic positioning to the ‘loop closure’ process and automatic alignment of surveys and mappings to a defined mine reference system and mobile mapping for geodetic purposes.

To the best of our knowledge, there is no automated 2D/3D underground mapping and modelling process for geodetic purposes that takes into account mapping accuracy and that also registers important reference marks or landmarks and works well with the underground mining process. This thesis therefore introduces a spatial relational model within which a mobile platform is enabled to connect to a global coordinate reference system and determine its position and orientation with quantifiable
1.3. RESEARCH SCOPE, OBJECTIVES AND GOALS

accuracy.

In developing this spatial relational model, this research develops a low cost, simple to construct, robust beacons used to tie mobile mapping to a mine coordinate system. Associated algorithms and techniques are also developed for beacon calibration and beacon coordinate determination. Algorithms are also developed that utilize the beacons to determine the position and orientation of a laser scanner in an underground environment. Finally tests are carried out to assess the accuracy of this positioning.

1.3.2 Objectives

This research situates and moves underground mine surveying and mapping forward into the robotic age by determining the starting point for any spatial robotic investigation in an underground environment. This research provides a framework for the mobile mapping process that enables quantifying map accuracy of geometric maps used for mining purposes. While current robotic localization and mapping addresses aspects of the basic principles of comparison and repeatability of a mapping process [3], these principles are core to a conventional underground surveying and mapping process. While there has been significant research involving mobile mapping of the underground environment [7, 63, 30, 55, 67, 11, 56, 37, 42, 73], none has been able to quantify the accuracy of mappings. In this research current surveying methods of quantifying accuracy of a mapping are applied to automatically generated maps.

This thesis uses the ability to explicitly compare mobile mapping to as-built survey results and maps, as the basis to evaluate the consistency and repeatability of the of mobile mapping process. This thesis outlines the process required to quantify the accuracy of mobile maps.
The authors in [3] state that current experimental practice and methodology in mobile robotics is not well developed and defined, therefore, this research establishes automated routines for some of the basic principles of conventional surveying and mapping through use and application of tools from the mobile robotics and state estimation communities. Despite the introduction of modern laser scanners, current surveying and mapping methodologies and practice remaining inefficient and cumbersome point based/stop-and-go processes. This research shows that robotic tools can be applied to conventional surveying and mapping techniques with quantifiable results to help improve efficiency.

1.3.3 Goals

The goals of this thesis include:

1. A new positioning concept for mobile robotics in underground environments that utilizes matching of a measured point cloud to a geometric beacon model while at the same time ensuring centimetre level uncertainty in position accuracy and single digit uncertainty in orientation of a mobile platform.

2. Design and development of equivalent mine references through the concept of surveying beacon technology.

3. Conceptualization, design and development of geometric surveying beacon technology with the express purpose of linking coordinate reference systems, specifically transferring a global reference system for use by a mobile robot operating within a local reference framework.

4. A method for automated survey alignment from an arbitrary reference frame to
1.4. Thesis Outline

The remainder of this thesis is as follows:

Chapter 2 provides a literature survey of positioning and orientation methods in conventional underground surveying and mobile robotics.

Chapter 3 provides an in-depth analysis of underground surveying and mapping methods and standards.

Chapter 4 provides an in-depth analysis of a specific mobile mapping system, the uGPS Rapid Mapper™, used in this thesis research.

Chapter 5 presents the 2D automated positioning Mobile Automated Scanner Triangulation (MAST) concept, including beacon design, calibration and associated algorithms.

Chapter 6 presents field tests and experimental procedures undertaken in the ILC complex at Queen’s University.
Chapter 7 presents results and discussions from experiments undertaken in Chapter 6.

Chapter 8 provides a summary of outcomes, primary contributions, recommendations and directions for future work.
Chapter 2

Literature Review

2.1 Introduction

The functions of underground surveying include connecting surface reference networks to establishing underground networks, providing underground horizontal and vertical control networks, surveys enabling preparatory and stope workings and mapping of underground workings. Positioning is the integral part that enables all these functions as from known positions all these are enabled. In conventional surveying the process of positioning has been well studied, and specific to underground environments, the process is restrictive due to the nature of these environments and has remained unchanged for the past half century despite improvements in instrumentation and technology, hence, the need for new methods and techniques of positioning to advance current knowledge. Recently, developments in robotics, and the ability to map the underground environment using robotic tools is making it possible to advance positioning methods now using robotic tools.

As the functions of underground surveying are diverse, this chapter confines itself to conventional and automated instruments and techniques that enable realization of
2.2. UNDERGROUND MINE SURVEYING AND MAPPING

underground surveying and mapping in the context of underground mine passageways.

2.2 Underground Mine Surveying and Mapping

2.2.1 Horizontal and Vertical Surveys

The basis for positioning in conventional surveying stems from the ability to measure and combine different quantities such as angles, distances etc. In three dimensions, position is defined as distances along three orthogonal axes from a point of origin of the axis system. The position of a point in surveying can be determined by various methods that include traversing, intersection, resection, trilateration, triangulation, levelling, etc. These techniques of positioning in surveying have been widely researched and are also the subject of many texts such as [62, 4, 70] and many others. While current positioning techniques in surveying might be slow and cumbersome processes as has been stated in [30, 63, 6, 73] the advent of robotic techniques promises improved data gathering capabilities and improved efficiency in the execution of surveying functions. However, current robotic techniques and methods especially have not been able to sufficiently articulate accuracy/precision of the surveying and mapping process, and normalization of the 2D mapping process with current mine mapping standards. That, coupled with the fact that current experimental practice and methodology in mobile robotics is not as well developed and defined as in conventional surveying and mapping [3], could be significant factors behind difficulties in gaining acceptance by the mining industry.

Advancements in positioning methods and techniques and instrumentation has been more prominent for surface surveying and mapping operations than has been
2.2. UNDERGROUND MINE SURVEYING AND MAPPING

experienced for underground operations due to the advent of GPS, LiDAR and satellite based mapping systems. In addition, the geometry of underground pathways or tunnels continues to restrict the surveying and mapping techniques that can be applied to the underground environment. Traversing is the primary method for positioning in underground surveying and uses similar procedures to surface surveying. Closed rather than open traverses are preferred as they allow for calculation of positioning statistics.

The premise for positioning by traversing is the ability to accurately measure and combine angle and distance measurements obtained in the mapping plane and to further adjust them by various methods such as transit and compass rules, Crandall method and now with the advent of computers, least squares methods. Deakin [26], provides a review of least squares theory as relating to traverse adjustment. Least squares methods provide rigorously adjusted results to network problems where traverse misclosure is within acceptable limits. The author outlines the least squares method of variation of coordinates and also discusses constraining of bearings, distances, angles and precision of adjusted coordinates and traverse bearings and distances in traverse network adjustment. In [4] and [70], the authors give an elaborate analysis of traversing, traverse adjustment methods and the error analysis associated with the different methods.

Done (1982)[27] discusses advantages and disadvantages of the different methods of adjusting traverses. Traverse adjustment methods include least squares methods, Bowditch, Transit, Crandall and Smirnoff methods. Closed traverses always suffer linear closing error despite adjustment for total angular misclosure. This is due to observational and instrumental error. The author argues that the method of traverse
adjustment used should fit the type of traverse undertaken and one or few methods should not be used to solve all types of traverses. Least squares methods are employed for precise traverses and have a basis in formulated condition equations whose solution is found by minimizing the sum of the squared residuals. The Bowditch method was originally developed for compass surveys but is now used mostly in conventional surveys. This method is not suitable for precise surveys or low order traverses where angular measurement is more accurate than distance measurement. It is more suitable where the traverse is almost linear. The transit method has the advantage of smaller changes to previously adjusted bearings than the Bowditch method and is suitable where angular measurement is more accurate than linear measurement. The Crandall method leaves preliminarily adjusted bearing unchanged and since it is now possible to measure distance more accurately than angle measurement this method has now become obsolete. The Smirnoff method quantifies the contribution to the total closing error by each linear and angular measurement combination and does not produce the same linear movement of the station regardless of its bearing. The author contends that adjustment by this method seems more correct than the commonly adopted Bowditch method.

Chrzanowski [22], gives an elaborate analysis and optimization of breakthrough accuracy in tunnelling traverse surveys. Breakthrough accuracy, assuming a project where two tunnels are excavated to meet at some midway point $P$ is calculated as the error or distance between the end points of two open ended traverses emanating from either end of the tunnel. Given that the tasks for a survey engineer in a tunnelling project include setting out grade and tunnel orientation, checking profiles,
guiding drilling for blasting operations and measuring tunnel deformation, achieving optimized breakthrough accuracy is a function of underground control design, intermediate checks using outside control and independent checks using different order level traverses. Lower order level traverses are used to guide construction whilst

![Diagram](image.png)

Figure 2.1: Breakthrough error (Chrzansowski, 1981).

higher order level traversing checks and corrects the position of the last construction traverse point. The author also deals with sources and mitigation of errors that influence breakthrough accuracy and introduces an optimized method for error analysis. Essentially, atmospheric refraction is cited as a critical source of error that comes about due to establishing lines of sight close to tunnel walls. Heat transfer from tunnel walls produces considerable temperature gradients closer to the tunnel walls than at the tunnel centre and this has adverse effects on angular and distance measurements. The author shows the effect of correcting angular observations using gyroscopic data on breakthrough accuracy and also makes recommendations of methods to mitigate or improve breakthrough accuracy.

Stiros(2009) [65] discusses the cause of alignment breakthrough errors associated with open-ended traverses in tunnelling. The author derives simple formulae from error propagation techniques and numerical approximation to quantify the magnitude and alignment of breakthrough error in three dimensions. His work is limited to simple horizontal tunnels with observations from one end of the tunnel.
2.2. UNDERGROUND MINE SURVEYING AND MAPPING

To correctly model the underground environment, in addition to the horizontal distances from the origin of the local mapping plane, heights are transferred from surface geodetic networks to the underground environment. In [62, 4] and other texts, basic levelling is discussed. Vanicek et al. (1980)[69] focus on four aspects of levelling namely measurement procedures and precision, height concepts, use of mean sea level as a datum and utilization of levelling results as a method for detecting vertical displacement in their review of geodetic levelling and its applications. The authors state that spirit levelling is a simple yet precise measurement system that has remained unchanged for over a century. They discuss the levelling process using a level and rod to obtain height differences, the different types of errors (random, systematic and blunders) associated with levelling and how elevation accuracy is a function of individual setups. The authors state that heights may be defined in a variety of ways (gravity potential, orthometric heights and normal heights) and point out the advantages and disadvantages of using these methods to observe elevation differences. The generally adopted reference datum for heights is the geoid which is a gravity equipotential surface closely approximating mean sea level. The authors discuss other vertical datums previously used and their associated problems.

New and faster methods to transfer elevation to the underground environment are discussed in Bahuguna(2005) [8] and Neuhierl et al. (2006)[52]. Bahuguna(2005) [8] describes a new method of transferring heights and orientations to underground workings that is less cumbersome and that takes a much shorter time. The author’s method is based on the use of an optical laser plummet set up at a point of known coordinates at the centre of the shaft on surface to project XY coordinates to all levels below. At the bottom of the shaft a theodolite is set up at the projected
2.2. UNDERGROUND MINE SURVEYING AND MAPPING

Point and vertical distance measurements are taken to targets placed on buntons at each of the levels above to surface. Elevations are subsequently transferred from the target position on each of the buntons on each level by normal levelling methods to underground stations. Figure 2.2 illustrates part of the author’s method. Directions are transferred at each level by observing a target setup on the projected point at each level from a reference point on the level using a gyro-theodolite. The authors also mention that their method takes less than half the time to execute than conventional methods with a much better accuracy and with less disruption to the normal work cycle of a mine.

Figure 2.2: Shaft depth measurement with a total station (Bahuguna, 2005).

Neuhierl et al. (2006)[52] outlined a new method of transferring orientation into
underground workings (see Figure refNeuhierl2006) that makes use of a combination of autocollimation and high-precision inertial navigation systems (INS). Autocollimation requires that an observer at an instrument station observe the reflection of the instrument’s illuminated crosshairs on a mirror that is rigidly fixed to a platform on a mine cage whilst the instrument is focused to infinity. If the reflection of the crosshairs is superimposed or coincident with the instrument crosshairs, it means that incident rays from the instrument and reflected rays from the mirror are coincident and therefore that the mirror is orthogonal to the incident ray. Given that the mirror would be rigidly fixed to a platform on a mine cage and the azimuth of the incident ray to the mirror is known, if the platform onto which the mirror is placed is moved vertically down a shaft, the azimuth can be recovered at any other level underground by autocollimation again. The INS comprises of a set of three accelerometers and three gyros mounted in an orthogonal pattern coincident with the three orthogonal
axes of an inertial reference framework. In their method, the authors augment the autocollimation method with the use of an INS system to measure the accelerations and the spin rates in the directions of motion of the mine cage. A double integration of the accelerometer readings results in the translation parameters of the cage in the local coordinate frame of the INS. A single integration of the spin rate results in the rotations about each of the orthogonal axes to indicate the ‘roll’, ‘pitch’ and ‘yaw’ of the cage as it moves down the mineshaft. On surface, the position of the platform is determined from angular and distance measurements to two targets mounted on the platform and in addition, by an angular measurement to a reference mirror off the platform. On a different level underground, angular observations are made to the mirror on the platform, a control mirror underground and the targets on the platform. Distance measurements are also made to the targets on the platform. The orientation underground is determined as a summation of the difference between the orientation and angle determined on surface and the rotation determined by the INS and the orientation and angle determined underground.

2.2.2 Surveying Instrumentation

Surveying instrumentation has advanced much faster than surveying methods and techniques especially for the underground environment. As a result not as much research has gone into instrumentation for underground surveying as it has for surface surveying.

Goldberg and Ream (1990)[30] discuss the capabilities of total station systems as opposed to traditional surveying instruments and the decision making process leading to purchase of a total station system on the reopening of the Bingham Canyon Mine,
2.2. UNDERGROUND MINE SURVEYING AND MAPPING

Utah in 1986. The authors mention that traditional survey work is slow, repetitive and manual in nature. They also state that purchase of a total station system would improve on the accuracy and speed of surveying work while at the same time reducing manpower needs of the surveying department. In assessing the type of system to purchase, they compare and evaluate different total station systems on the basis of their daily and monthly field operations, ease of use, angular and distance accuracies, measurement time and range, precision, battery life and data collection capability. The authors conclude that while implementation of use of total station has increased productivity and data collection accuracy, some work has continued to be undertaken using older instruments because it was quicker, their initial survey crew was drastically reduced and the total station system is operational 98% of the time.

Lambrou et al. (2006)[41] use a function of a robotic reflectorless total station that enables it to accurately survey and map inaccessible surfaces that include mine walls, cliffs and rocky declivities. The ability for the total station to automatically sweep and scan over a survey area given a user defined window, vertical and horizontal scan rates, enables the instrument to operate in a similar fashion as a laser scanner thereby enabling the total station to automatically gather large amounts of data. Figure 2.4 illustrates the author’s method. The authors indicate that the ability to obtain measurements in a manner that requires no contact with an object offers increased safety to workers and would be ideal for underground applications.

Lewen (2006) [46] explores the use of gyrotheodolites in developing underground geodetic networks. The author’s thesis investigates different gyroscopes in use, their operation when connected to theodolites, geodetic corrections applied, errors and
accuracies associated with gyroscopic observations. The author also discusses different geodetic control networks in tunnelling, breakthrough accuracy, how gyroscopic observations improve network accuracy and network analysis for different types of traverses. The results of the thesis indicates that gyroscopic observations improve or reduce breakthrough error for open-ended traverses, and redundancies in observations improve network accuracies making it easy to detect observational and instrument errors and also that network configuration and traverse lengths have a high influence on traverse or network accuracy.

Langdon (2009) [42] describes one of the latest surveying and mapping tools, a remote surveying vehicle that comprises a Velodyne laser scanner mounted on a Lego Mindstorms kit. This system has a self levelling mechanism originally designed for gun stabilisation to level the scanner and is operated as a stop-and-go system. Four
2.3. AUTOMATED SURVEYING AND MAPPING

 targets are coordinated using GPS or theodolite and a scan to survey these in is undertaken. This scan is then tied to the mine reference network then the system is moved approximately 5-10 m for the next overlapping scan. At the end of the survey a similar tie-in to the mine reference system is undertaken. An image of the system is shown in Figure 2.5.

Figure 2.5: Remote surveying vehicle (Langdon, 2009).

2.3 Automated Surveying and Mapping

The advent of robotics has brought about an opportunity to change how underground mapping is undertaken. The use and fusion of different sensor data to determine the position and orientation of a robot in an underground environment to collect point cloud data of the environment at the same time building a map of the environment underscores the possibility of advancing surveying and mapping methods through
robotic means. SLAM based localization processes enable robots through sophisticated algorithms to determine their location in an environment whilst creating a map of the environment.

2.3.1 Mapping Methods

Online Mapping Methods

Madhavan et al. (1998)[49] discuss map-building and map-based localization of a Load-Haul-Dump (LHD) machine in an underground mine. Their method utilizes statistical pattern matching techniques using a laser scanner to collect data. Map building uses an Extended Kalman Filter (EKF) and the resulting map is composed of polylines. The authors also discuss map-based localization and they propose three methods namely, Iterative Closest Point (ICP) approach, a reflective beacon based approach and an ICP-EKF combined approach. Here the last two methods take into consideration uncertainty in the observation data and employ the EKF. A trial survey of a 150m section of tunnel in an underground Queensland mine was undertaken to validate their method. Figure 2.6 shows the type of map generated and the approximate path of the robot.

Bakambu et al. (2004)[11] describe an autonomous system for exploring and navigating tunnel networks similar to those found in underground mining. In exploration mode, a robotic vehicle is instructed to move through sections of a network gathering range data to produce 2D/3D maps of the environment for use in navigation. Data is gathered by forward-facing and transverse laser range finders corrected by readings from gyroscope, odometry and inclinometer systems. On-line mapping is available for the remote operator to enable mission execution. In navigation mode,
the autonomous vehicle uses acquired maps for motion planning and mission execution. Functions of mission execution include landmark detection, localization using acquired maps and performing planned navigational tasks. The authors validate the functionality of the system through several tests made at an underground mine.

Bakambu et. al (2000)[10] propose an enhanced heading and position determination method that fuses odometry data with inclinometer and gyro data. Inclinometer data is used to compensate for gyro drift that occurs due to roll/pitch perturbation. This data is then used to correct differential odometry that is adversely affected by wheel slippage. Fusion of this enhanced odometry data with range finder measurements of the environment in a Kalman filter improves positioning. The authors give an in-depth discussion on velocity, heading, range and position sensors and support
their choice of sensor for this project. The authors outline gyro drift as a major source of error in inertial navigation, factors that affect the gyro such as earth rotation and latitude, then come up with a system to compensate for these errors for an integrated gyro-clinometer system. Range sensory data is integrated with the odometry data using an extended Kalman filter to significantly reduce lateral positioning error estimation while improving the general positioning estimate.

Bonnifait et al (1998)[15] present a 6DOF localization technique for civil engineering machines based on a fusion of odometry, inclinometer and SIREM, an exteroceptive sensor composed of standard light sources and a rotating CCD camera in a Kalman filter. SIREM measures azimuth and elevation angles to known beacons (light sources) and provides asynchronous measurements. Two inclinometers are used to measure gradient (in direction of travel) and cross-fall angles. Odometry speed measurements are used as the control input during the prediction phase of the extended Kalman filter and the estimation phase uses inclinometer measurements with intermittent correction done when SIREM observables become available. The authors indicate a precision of better than 0.1 degrees is obtained for attitude angles and this is important for the accuracy of pose estimation.

Hoppen et al (1990)[35] discuss laser-radar based mapping and navigation for an autonomous mobile robot MOBOT-III. The authors discuss the robot’s architecture whose components include a sensor data processing capability, and navigation algorithms which comprise course or route planning, execution and operation subroutines. Data processing has five components. The first performs feature extraction, the second, called a correllator, verifies robot position. The third component called a composer, is used to compare laser range data and ultrasonic data to get a consistent
view of the environment, and produce and manage local environment maps. The fourth called a geographer, collects, extracts and manages geometrical environment data by producing and extracting local geometric maps from a maintained global geometrical map. The last, called the topographer contains the knowledge base about objects in the environment. It processes data used in navigation algorithms that enable the robot to successfully navigate an unknown environment. The author’s concept is mostly implemented in a simulated environment and some of the lower level processing algorithms have been tested under realtime conditions.

Shaffer and Stentz (1993)[63] state that surveying is a necessary step in mining from which maps are generated. These maps are used to track development progress and extraction of ore. Conventional maps are generated from transit/theodolite angle and distance observations obtained by a human surveyor. Maps are a coarse approximation of the actual mine structure because while individual observations may be accurate, few measurements are taken making the data collected insufficient to provide adequate resolution to the mapping process. The authors indicate further that the mapping process is slow, the data too sparse and the lag between measurement and map generation too long. They advocate for a new and automated mobile mapping process that will be faster, and produce more detailed maps that will be used to plan and execute navigation and cutting operations in a coal mine. The authors indicate that mapping is rarely a primary end-product in most research undertaken by the robotics community. Generated local maps are used for navigation, obstacle avoidance and planning of a safe path a robot would take to a goal location. The authors approach to an automated or mobile mapping system follows the steps of acquiring range scans, and matching scans to a hand or conventionally measured global
map to determine scanner position. They create a local map from the scanning and append their local map to the global map. They also outline some problems with their method which include deterioration of the global map as the robot moves further from its initial position, and map representation problems in that too much data results in for example multiple representation of single walls and other correspondence problems. The authors outline some solutions to these problems and indicate that this is a useful starting point for a proposed second generation automated mine surveyor.

**Offline Mapping Methods**

Huber et.al (2006)[36] address the problem of sensing and generating high-resolution 3D models of an active mine. They use a high resolution Zoller and Frohlich LARA 25200 3D scanner mounted on a manually moved cart (see Figure 2.7). The scanner is mounted in an inclined position to enable capture of floor and ceiling data. Their system is operated in a stop and go mode with stationary scans every three to five meters. To model collected data, their system automatically registers multiple 3D scans in a common coordinate reference frame without need for knowledge of the individual viewpoints. Their pair-wise scan-matching algorithm requires the conversion of each scan into a surface for matching. The quality of a registration process is evaluated from a network of surface matches that comprise a node for each input viewpoint and edge indicating the pair-wise match in a model graph. Correct scan matches show positional displacements of no more that 10% the scene size from their correct location optimized using a combinatorial search over the space of connected sub-graphs of the model graph. The optimization results in a set of rigid body transforms that
align views in a common coordinate system. The authors determine the accuracy of their modelling as a function of the distances between overlapping surfaces, ensuring the correctness of sensor position by comparing model cross sections with reality.

Thrun et.al (2003)[66] describe a system for volumetric mobile mapping for use in underground mines (see Figure 2.8). The system employs two 2D laser range finders, one pointed forward for horizontal 2D mapping and the other pointed vertical and upwards to collect a vertical 2D profile from which changes in slope can be recovered. It is assumed that a 3D model is generated from additional two 2D laser range finders placed facing up and down and orthogonal to the direction of vehicle travel in conjunction with the SLAM approach.

Lu and Milios (1997)[48] study the problem of consistent registration of laser range
scans so that they can be formed into a global model. The authors indicate that the
general approach taken to building a world model is to incrementally integrate new
data by averaging the data or using a Kalman filter. They indicate that this process
becomes inconsistent as different parts of the model get updated independently and
it becomes difficult to resolve these inconsistencies if data frames have already been
integrated. The author’s approach maintains all data frames as well as a network of
spatial relationships among the data frames. Robot poses in a global reference frame
are treated as variables and spatial relationships between data frames are derived
from scan matching or odometry. An estimate of all robot poses is solved as an opti-
mization problem by taking into account all spatial relationships simultaneously. In
the optimization an objective function is formulated with the network pose coordi-
nates as variables and every link in the network a term in the objective function. The
link is conceived as a spring connecting two nodes which achieves minimum energy
when its value equals that which has been obtained from odometry or scan matching. Pose variables are estimated by minimizing the total energy function. Figures 2.9 a) and b) show misaligned data owing to accumulating error from odometry and aligned data after a global adjustment of the objective function.

![Figure 2.9: Aligning scans from relative pose contraints. (Lu and Milios, 1997).](image)

Jarosz and Langdon (2007)[39] describe a surveying and inspection tool for vertical mining openings and shafts (see Figure 2.10). The components of this tool include a motion sensing Inertial Measuring Unit (IMU), a gyroscope for stabilization, a camera for video and a laser scanning device for distance sensing. This tool collects data as it is lowered into the mine opening by way of a winch and a four-conductor steel armoured wire cable. An inertial spinning-mass gyro is used to stabilize the tool along the vertical axis whilst its orientation is determined by IMU. Cameras collect video images of the walls of the shaft or mine opening and a SICK LD-OEM scanner is used to collect distance measurements from the tool to the walls. The authors have successfully used this tool to inspect ventilation shafts, analyze ore pass wear and conduct an ore pass survey.

Marshall et.al (2008)[51] describe the design, implementation and field testing of
an infrastructureless system for autonomous tramming of a centre-articulated mining vehicle. The system allows for a-priori knowledge of a path, tunnel conditions and driving maneuvers to be implicitly defined for a predefined map. The control architecture and implementation allows for a robust and practical method for handling vehicle steering and driveline dynamics thereby permitting high speed tramming. During the teaching phase sensor data is collected for generation of a map. At this time a path, pause and speed profiles are generated for a sequence of locally consistent metric maps. A path profile defines a sequence of path points and associated vehicle attitudes, a pause profile indicates locations where a vehicle should stop, a speed profile defines for a section of tunnel the speed at which a vehicle should travel, all this occurring within the confines of a manifold of locally consistent metric maps. The authors successfully implemented this on ST1010c LHD and ST14 LHD machines in
2.3. AUTOMATED SURVEYING AND MAPPING

Artan et al. (2009) [6] bring together the ideas of mobile robotics and mapping of underground workings for the purpose of developing globally consistent maps for underground navigation. The authors developed a mobile mapping algorithm that uses a combination of odometry and scan matching to estimate the pose of a vehicle. The authors treat each pose in a sequence of contiguous poses as a node. Edges connect nodes with overlapping scan data. Scan matching estimates the rotation and translation associated with each pose. A globally consistent representation of the vehicle poses is generated using least squares by minimizing a cost function that incorporates all the edge measurements used to generate a map. This map can then be used for navigation purposes. In Artan et al. (2010), the authors successfully applied their initial work and generated globally consistent maps from data collected at two underground mining operations using an Atlas Copco ST14 LHD machine.

Lavigne et al. (2010) [45] further the work by [6] by introducing radio frequency identification (RFID) technology as uniquely identifiable landmarks that can be cheaply added to an underground environment for use as aids to navigation and to automate the map building process. The RFID beacons are randomly placed with no a priori knowledge of their location. A beacon estimate that comprises beacon location, identity number and effective range is obtained from a cloud of RFID measurements whenever a reader-equipped vehicle passes a beacon location. One problem encountered in [6] was a robot’s inability to recognize a previously traversed location. This meant that initiating the ‘closing the loop’ process required human input every time the robot crossed a previously traversed location. The ability to detect RFID beacons enables automatic initialization of this process. The system then generates occupancy
grid maps by dividing an environment into equally sized cells that can be assigned as either occupied or not based on range measurements. The results from the author’s investigations indicate that overlapping beacon estimates initiate the process of forcing position estimates into place thereby enabling generation of a globally aligned map. Also available as a by-product of the alignment process is an estimate of the RFID beacon position.

### 2.3.2 Process Development / Algorithms

Magnusson et al. (2007)[50] discuss scan registration for autonomous mining vehicles using 3D-NDT. In building maps using a range finder, the authors indicate scan registration is an important subtask. They use the three dimensional Normal Distributions Transform (NDT) for registration of 3-D surfaces and apply their work to actual underground mine data. In their conclusions they state that compared to Iterative Closest Point (ICP) methods, the NDT method consistently leads to accurate registration of difficult scan data in a much shorter time because it avoids the computationally challenging nearest-neighbor search that ICP algorithms use.

Borenstein and Feng(1996)[16] discuss a new, very simple and effective method of combining gyro data with measurements obtained from wheel encoders. This method, gyrodometry, is based on a study of the interaction between the ground and the wheel and makes use of the fact that non-systematic odometry errors such as a bump, are momentary and only during such episodes are gyro and odometry readings very different. During such episodes, gyrodometry replaces odometry readings with gyro data thus minimizing the effects of gyro drift. Gyrodometry works on the assumption that the discrepancy between the odometry and gyro curves persist only momentarily
and therefore applies a corrective measure only if this difference exceeds a given threshold.

Nguyen et.al. (2005)[53] discuss and compare line extraction algorithms using a 2D laser range finder for indoor mobile robotics. They list six algorithms namely, split and merge, line regression, incremental, RANSAC, Hough transform and EM algorithm. These algorithms are compared on the basis of complexity, speed, number of lines, correctness and precision. The results of their work indicate the split and merge and incremental methods to best suit mobile robotics application.

Bolstad et.al (1990)[14] discuss the positional uncertainty in digitized maps. The authors mention that one way of assessing accuracy and precision of digital maps is by comparison of field survey data with corresponding map-derived digital coordinates. In this case, the field data is required to be obtained at a much higher accuracy than the digital data. The accuracy of the digital data is obtained from the average difference between the surveyed and digital data whilst the precision is established from a theoretical error probability density function through use of a frequency distribution.

2.3.3 Beacon Based Localization

In determining position in a GPS deprived environment, research has gone into the use of various types of beacons. Wu and Tsai (2008)[71] develop an omni-directional camera that is used in conjunction with a hyperbolic mirror to obtain an image of a circular shaped landmark placed on the ceiling. An ellipse detection algorithm is used to search for and extract circular or elliptical landmarks from images taken by the camera and determines the distance and orientation of the landmark with respect to the camera. Distances are derived from comparing image size to true size on image
plane and orientation from the line joining the image plane centre to the image centre.

Lin and Chen (2011)[47] describe a QR code type barcode landmark beacon that is mounted on the ceiling and photographed using a camera mounted on a robot. This barcode is encoded with its position coordinates and scanned. The authors also describe how position is obtained through use of the beacon.

Durrant-Whyte (1994)[28] describes a method of triangulation that uses distance or bearing measurements observed to fixed beacons placed in the environment. Reflective strips or individually marked barcode beacons placed around the environment are then used to determine the position of the vehicle. If beacons can be uniquely identified by barcode, the vehicle can uniquely locate itself on startup otherwise the vehicle needs to start from initial known position.

Pierlot and Droogenbroeck (2009)[58] describe an active beacon system that continually sends an IR signal and a receiver mounted on a rotating turret such that if the receiver is facing a beacon a binary result of 1 is noted else 0. The authors also describe how the angle measurement is obtained for use in a triangulation process for positioning.

Chao et.al. (2010) [21] describe the design and implementation of a barcode beacon for precise positioning of a vehicle underground. The barcode beacon information comprises ‘bars’ and ‘spaces’ of varying width and reflectance. A laser sensor is used to capture a signal from a series of barcode beacons mounted on the wall. The authors outline a method of determining the vehicles location based on the alignment of the underground passageway with a local axis system.

Hlophe et.al. (2010)[34] describe a localization or posture estimation system that utilizes the method of trilateration to determine the position of a mobile platform from
distances measured from time of flight signals and RF signals emitted by beacons. The system uses two receivers that are mounted on the axis of travel of a vehicle and that independently determine their location and determine the vehicle orientation from coordinates obtained from position calculations. Figure 2.11 from [33] shows the beacon and its deployment. The results from this method indicate better location estimates for locations closer to the centre of the beacon cluster than locations closer to individual beacons.

![Artificial beacon positioning system](image)

**Figure 2.11**: Artificial beacon positioning system and its deployment. (Hlophe and du Plessis, 2013)

Simela et.al. (2013)[38] describe a method of geo-referencing a mobile laser scanner
to a mine reference system for the purposes of underground mine surveying that is able to quantify position and orientation accuracy and minimizes human input. This method utilizes beacons of known geometry that are surveyed in or geo-referenced and scanned during a surveying process. The position of the scanner is determined by a triangulation process that also incorporates matching laser scanner data to the profile of the beacon.

2.4 Current Issues and Limitations

Research in underground network design and mapping has been sporadic and has not garnered as much interest from the research community as its surface counterpart. Most research that has been found has dealt with improving geodetic networks in tunnelling [46, 22] using different strategies such as traversing in a zig-zagging pattern or using braced quadrilaterals in tunnels to minimize the effect of refraction and use of gyrotheodolites. Refraction occurs from observing traverse lines close to tunnel walls as the center of the tunnel is for the most part occupied by vehicles and other equipment. Gyrotheodolites make use of the fact that the Earth is in constant rotational motion and seek at each position, a north facing meridian [46]. Gyrotheodolite observations are then used to continually adjust open-ended underground traverse azimuths in tunnelling to achieve high breakthrough accuracies [46, 22, 65]. Neuhierl et al. (2006) [52] discuss a new method to transfer geodetic orientation to underground workings that makes use of inertial navigation systems.

The advent of robotic and reflectorless total stations and laser scanning has revolutionized data collection capability in conventional surveying. From the safety aspect,
the need to hold target reflectors in hazardous areas is minimized and from the efficiency aspect, the amount of data collected from a specified area for any given time period now far supersedes that which traditional total stations are able to provide for similar areas and time periods [41, 30]. However, the point based nature of conventional surveying which includes the use of total stations and off-the-shelf 3-D laser scanners in a static/semi static sense is in itself limiting. In narrow underground mining corridors, many stations are established to minimize occlusion effects that are common to current conventional 3-D laser scanner practice, total station surveys and off-the-shelf laser scanner mapping operated in stop-and-go mode from remotely controlled vehicles in robotic surveying as in [42]. Mobile mapping that can provide a consistent point density equivalent to that of an off-the-shelf laser scanner whilst addressing the issues of occlusion, lack of efficiency, that affect static or stop-and-go laser scanning or total station surveying, would be faster and might achieve acceptable or better results for underground mine surveying and mapping.

The issue of automated tramming in an underground setting was the subject in [51], the outstanding issues of globally consistent maps and atlases of locally consistent maps have been dealt with in [6, 45]. The outstanding issues in [6] included quantifying the uncertainty associated with globally aligned poses and automating the ‘closing the loop’ process. The ‘closing the loop’ problem was dealt with in [45] by use of RFID beacons. The subject of automated geometric mapping was alluded to in [63, 35, 9] however, there is no indication of further research in this direction in the literature. Quantifying map accuracy is a subject that is routine in conventional surveying. This is discussed in [14] as pertaining to digital maps and is a standard in conventional surveying and mapping practice according to [29]. While underground
mobile mapping produces maps, these have predominantly been for navigation, positioning in the navigation frame and not for geodetic purposes. Also the ability to quantify accuracy of mapping has not been addressed within the robotic community. This issue stems largely from the inability to compare mapping with physical ground truth as is carried out in conventional surveying.

The issue of global position estimation within a global reference network is a current and ongoing problem for the robotics community and this has been highlighted in [61]. The use of artificial beacons in underground global positioning has been widely researched to determine a robot’s position and orientation in GPS deprived environments. Vision based methods using cameras have been used in [47], [71] however these assume good lighting and an ability to extract the beacon from camera images which might not always be available in a mining scenario. Laser and infrared based methods have been implemented in [58] and [28] using triangulation and trilateration concepts. Also in this case, an assumption is made on the visibility of the beacons or ability to easily extract barcodes that have been placed in the underground environment which might not be the case unless each beacon can be uniquely identified. Some issues relating to applicability and accuracy of RFID, radio signals and other technologies for accurate positioning in underground environments are highlighted in [7]. In [38], a method for automated geo-referencing of a laser scanner to a defined reference system is developed that uses geometric beacons. An algorithm is developed and simulated for the underground environment which recovers scanner position and orientation from perturbed point cloud data with accuracies less than 1cm in position and less than 1 degrees in orientation.
2.5 Conclusions

The ability to quantify mapping accuracy from mobile mapping processes requires the ability for an automated survey to be accurately aligned or the capability to determine the error between the automated survey and ground truth. Combining accurate positioning at the beginning and end of a survey with the ability to map an underground area quickly and also compare the automated survey to known checkpoints is the missing link to enabling map accuracy quantification.
Chapter 3

Positioning Methods for Underground Surveying and Mapping

3.1 Introduction

The basic principles and instruments used in underground mine surveying are not that different to those used for surface surveying [19]. Underground surveying and mapping environments are usually spatially constrained or confined, dark, wet, dusty and sometimes unstable, hence, restrict the use of some surveying and mapping techniques and surveying instruments available for surface surveying.

In an underground mining operation, functions of surveying include connection surveys between surface and underground vertical and horizontal geodetic networks, as well as surveys enabling preparatory and stope workings [64, 17]. This chapter limits itself to those techniques, applications and instruments used in underground surveying and mapping of mine passageways. However, the reader is referred to [4, 62] and other texts for other techniques, applications and instruments used.
3.2 Datums and Coordinate Reference Frames

Datums are necessary and common platforms by which relative positions of locations, directions, elevations or spatial data in general can be related in a global reference frame. It is all the more important in an underground mining environment where there are shared boundaries between mineral claims that mines use a common datum as surveying operations have a direct effect on the safety of personnel who work in those environments, as well as the structures above and within an underground mining operation. Datums are characterized as either horizontal or vertical.

The Earth is approximated as an imaginary oblate spheroid of revolution generated by rotating an ellipse about its semi-minor axis [4]. This spheroid represents a surface or geodetic datum upon which a coordinate reference system can be based. In order to relate on a global scale, positions of objects on or near the surface of the earth that have been derived from field measurements, these horizontal or geodetic and vertical datums are used.

The current horizontal or geodetic datums in use include the North American datum of 1983 (NAD 83) and the World Geodetic System of 1984 (WGS 84). These use a reference ellipsoid with a semi-major axis $a = 6378137.0$ m and a flattening, $f = 1/298.257223563$ [4, 62] for surveying computations. On this spheroidal surface the force of gravity is assumed to act in a normal direction at every point on this imaginary surface.

The current vertical datum to which elevations or depths in topographic, geodetic or engineering surveys are referred is the North American Vertical Datum of 1988 (NAVD 88). Other vertical datums such as the Mean Sea Level and at times arbitrary local level datums have been known to be used.
3.2. DATUMS AND COORDINATE REFERENCE FRAMES

The three coordinate reference frames commonly used in conventional mapping include a geocentric cartesian system, a geodetic curvilinear system and a local space rectangular coordinate system. The geocentric cartesian and geodetic curvilinear systems are earth centred and earth fixed, meaning their origins lie at the centre of mass of the earth, rotate with the earth and are defined around three orthogonal axes. In both cases the Z axis lies parallel to the mean spin axis of the earth, the X axis points towards the mean Greenwich meridian and lies on the plane of the equator and the Y axis is right-handed and orthogonal to the X axis and the XZ plane. The cartesian geocentric coordinate system is linear and positive from the origin in all the axis directions defined above. The geocentric XY plane coincides with the plane of the equator. Figures 3.1 illustrate geocentric, geodetic and local space coordinate frames.

The geodetic curvilinear coordinate system comprises a geodetic longitude, $\lambda$, defined as the angle between the XZ plane of the meridian passing through a position of interest and the mean Greenwich meridian measured on the plane of the equator. Longitudes increase east or west from 0° to 180° or alternatively east from 0° to 360°. The geodetic latitude or parallel of latitude, $\phi$, is defined as the angle between the normal to the ellipsoid at a position of interest and the geocentric XY plane whose limits on $\phi$ are $-90^\circ \leq \phi \leq +90^\circ$. Lines of latitude and longitude represent a geographical coordinate system. The geodetic height or ellipsoidal height $h$, is defined as the distance from a position of interest to the reference ellipsoid in a direction normal to the reference ellipsoid.

The local space rectangular coordinate system is a topocentric cartesian coordinate system whose origin is a selected point on or near the surface of the earth’s surface.
The $z$ axis in this case is normal to the reference ellipsoid and is considered positive away from the ellipsoid, up. The $x$ axis is perpendicular to the $z$ axis and the meridian plane through the selected point of origin and is considered increasing positively in a direction of increasing longitude, $\lambda$ or east (Eastings). The $y$ axis forms a right-handed system with the $x$ and $z$ axes and increases positively in a direction of increasing latitude, $\phi$ or north (Northings). This $xy$ plane also called the horizontal plane defines the mapping plane generally used in surveying for topographic or mine mapping. Within this framework, a geodetic azimuth, $\alpha$, is measured in a clockwise direction from the local $y$ or north axis in the plane of the local geodetic horizon and the limits on $\alpha$ are $0^\circ \leq \alpha \leq 360^\circ$. Vertical angles are measured in the plane that contains the geodetic vertical and the limits on vertical angles are $-90^\circ$ to $+90^\circ$ with
the negative values indicating measurements below the horizon or mapping plane and
the positive values above the mapping plane.

Map projections are used to project ellipsoidal coordinate values \((\lambda, \phi, h)\) into
gEOcentric coordinates \((X,Y,Z)\) or eastings and northings \((E,N,H)\) for map representa-
tion or vice-versa. Examples of these projections include Lambert conformal conic,
transverse Mercator, universal transverse Mercator (UTM) and state plane coordi-
nate system (SPCS). The reader is referred to [4] and [62] for an in-depth discussion
on these projections as they fall beyond the scope of this thesis. It is within any
of these map projections in use in a region that stations that comprise the primary
network of a national geodetic reference system derive their coordinate values and it
is from this national geodetic reference framework that a mine reference network is
tied.

A map is a representation of the Earth’s curved surface on a plane and cannot
be made without a degree of distortion. For small areas (approx. 10 km), the earths
curved surface is considered planar and maps constructed by orthographic projection
can represent relative earth surface locations with minimal distortion and as such are
considered conformal. Conformal maps preserve angular measurements made on a
curved surface hence make the shape of small areas appear correct on a plane.

The value of a common geodetic reference framework cannot be underestimated in
cases where there might be abutting mining operations or for surface and subsurface
networks within the same mine more especially as there are legal, health and safety
requirements and other implications for a mining operation.
3.3 Conventional Horizontal Positioning Methods

3.3.1 Traversing

Traversing is currently the method by which horizontal control is predominantly established in an underground mining environment [17]. This method is used where other methods of providing horizontal control such as triangulation, trilateration resection or GPS are unavailable or unsuitable for use. A traverse combines horizontal angular measurements with horizontal distance measurements to relate the spatial location of different observed positions in a common reference frame.

Two types of traverses namely, open and closed traverses, are used to bring control into an area of interest. Open traverses originate from a known position and end at an unknown position. These traverses however provide no computational checks that make it possible to detect errors or blunders in the angular and distance measurements observed [4]. As a result, open traverse results are uncertain and as such traverses of this nature are normally discouraged.

There are two categories of closed traverses. The first category is called a closed loop traverse which starts at a known position and ends at the same known positions. A known position is that which can be referenced to some coordinate system i.e. it has known coordinates (XYZ, Latitude, Longitude, Ht., Easting, Northing, Ht.) in a defined global or local coordinate reference frame. The second category of closed traverse is called a closed link traverse, and this starts at one known position and terminates at a different but also known position. These types of traverses are preferable in that they allow for computational checks that enable detection of systematic errors in distance and angle measurements as well as allow for error analysis of computations by advanced least squares methods. Figures 3.2 and 3.3 show examples of
3.3. CONVENTIONAL HORIZONTAL POSITIONING METHODS

Figure 3.2: Closed loop traverse (Anderson and Mikhail, 1998).

Figure 3.3: Closed link traverse (Anderson and Mikhail, 1998).
3.3. CONVENTIONAL HORIZONTAL POSITIONING METHODS

closed loop and link traverses.

Typically in an underground mining environment, stations or fixed points of known coordinates are established on the roof or sidewalls of an excavation. This is primarily because those are the areas least likely to be disturbed by underground traffic. Examples of survey instruments set up at underground stations are illustrated in Figures 3.4 and 3.5. It is from these physical positions that all surveying and mapping occurs.

![Figure 3.4: Instrument setup below roof station (Image courtesy of Prof. G. Blackwell).](image)

A traverse can be described according to the method by which angles are turned. Typical methods include use of deflection angles, interior angles, angles turned to the right and azimuths. All these are measured in the mapping or horizontal plane orthogonal to the local vertical. An azimuth of a line is defined as the direction given by an angle between the meridian and line passing through a traverse station on the line and measured in a clockwise direction. Any slope distances that are measured
3.3. CONVENTIONAL HORIZONTAL POSITIONING METHODS

Figure 3.5: Instrument setup at sidewall station (Image courtesy of Prof. G. Blackwell).

along any traverse line are projected onto the horizontal or mapping plane.

Figure 3.6: Traverse by deflection angles (Burtch 2011).

In the method of deflection angles, a deflection angle left $\alpha_L$, or right $\alpha_R$ of the current direction is measured. Figure 3.6 from [20] is an example of such a traverse.
3.3. CONVENTIONAL HORIZONTAL POSITIONING METHODS

The deflection angle azimuth condition for a traverse is calculated according to:

\[ Az_1 + \sum_{i=1}^{n} \alpha_{R_i} - \sum_{i=1}^{n} \alpha_{L_i} - Az_2 - 360^\circ = 0 \]  \hspace{1cm} (3.1)

The method of interior angles is used with closed loop traverses. It requires that the interior angles between intersecting lines of a polygon traverse are measured. Typically, this polygon is referenced to the local meridian by an angular observation from a line of known azimuth. The angular error of closure of the polygon is calculated according to:

\[ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \ldots + \alpha_n = (n - 2)(180^\circ) \]  \hspace{1cm} (3.2)

where \( n \) represents the number of sides of the polygon. The azimuth of each succeeding traverse line is calculated by summing the back azimuth of the preceding line and the clockwise angle to the next line.

\[ Az_{2-3} + \alpha_3 = Az_{3-4} \]  \hspace{1cm} (3.3)

The method of traversing using angles to the right works with open or closed loop or link traverses. The start azimuth is calculated from a line defined by the coordinates of two known positions. The forward azimuth of the next line is calculated by adding the back azimuth of the previous line to the measured clockwise angle to the right of the next line. The angle of closure condition for the traverse is expressed as:

\[ Az_1 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \ldots + \alpha_n - (n - 1)(180^\circ) - Az_2 = 0 \]  \hspace{1cm} (3.4)

Traversing by the azimuth method requires the ability to measure azimuths at each
3.3. CONVENTIONAL HORIZONTAL POSITIONING METHODS

traverse station. Here the azimuth of a forward traverse line is directly measured as a clockwise angle from the local meridian.

Appendices A and B provide an in-depth discussion on how traverse computations in conventional surveying are carried out. These discuss formulation of the distance and angle condition equations for each angle and distance which are an input into a traverse and also setting up to solve by the technique of adjustment of indirect observations and the technique of adjustment of observations only [4, 31]. In addition to this, a review of least squares methods for traverse adjustment is outlined in [26] and [20].

3.3.2 Triangulation

In triangulation, position is established from measurement and analysis of mainly angular measurements and the length of a few lines. This method was the preferred method for extending horizontal control over long distances prior to electronic distance measurement and GPS. This method is still used in guiding tunnelling operations as it minimizes interference with traffic. In the case of [59], this was the preferred observation method to minimize influences of random and systematic errors in azimuth, direction and distance measurements. It is common place today that a triangulation survey be combined with a trilateration survey [70].

The basis of a triangulation is a triangle whose angles are measured and a baseline distance between two triangle vertices. Chains of single triangles, polygons or quadrilaterals can be constructed joining network points whose positions are solved by least squares methods. Least squares methods for solving triangulations use condition or observation equations written in terms of azimuths or angles. The parameters
adjusted are approximated coordinates of the network in a plane rectangular system. Special cases of a triangulation include: intersections\(^1\) and resections\(^2\). Triangulation is not a focus of this thesis, hence, the reader is referred to [70],[4] and [31] for further details.

### 3.3.3 Trilateration

The method of trilateration is a common and effective method used to determine the position of an unknown station through measurement of distances from at least two locations of known position or control points to the unknown position. Given known control points \(A\) and \(B\), the measured distances \(bc\) and \(ac\), the coordinates of \(C\) are calculated by first determining the angles \(\beta\) and \(\alpha\) subtended at \(A\) and \(B\) respectively by trigonometry (see Figure 3.7). The calculated angles are applied to the baseline azimuth \(Az_{AB}\) to determine the azimuths of unknown position \(C\) from \(A\) and \(B\). The coordinates of \(C\) are then calculated taking as an example from \(B\) according to:

\[
\begin{bmatrix}
X_C \\
Y_C
\end{bmatrix}
= \begin{bmatrix}
X_B \\
Y_B
\end{bmatrix}
+ bc \begin{bmatrix}
\sin \alpha_1 \\
\cos \alpha_1
\end{bmatrix}
\tag{3.5}
\]

Whenever more than two control points contribute to a trilateration solution, the method of least squares is used. Each control point contributes a distance measurement to the unknown point. The least squares approach described here to solve the trilateration problem is the technique of Indirect Observations [31, 4].

---

\(^1\)Intersection is a method of determining the position of an unknown station from azimuth directions measured at known positions along a baseline or from angles measured from the end points of a baseline of known position and orientation.

\(^2\)Resection is a term used to describe a method of determining an occupied unknown horizontal position by measurement of horizontal angles/directions to at least three stations of known coordinates.
Let $S_{ij}^o$ in equation (3.6) represent the adjusted distance between two points $i$ and $j$. Equation (3.6) is called the distance condition equation.

$$S_{ij}^o = \left[(X_j - X_i^o)^2 + (Y_j - Y_i^o)^2\right]^{1/2}$$

(3.6)

Here, $(X_j, Y_j)$ are the known coordinates of the control point $j$ and $(X_i^o, Y_i^o)$ are the approximate coordinates of the unknown point $i$. For each measured distance, the following form of the linearized distance condition equation is constructed.

$$v_{ij} + b_1 \Delta X_i + b_2 \Delta Y_i = f_{ij}$$

(3.7)

Here:

$$b_1 = \frac{(X_j - X_i^o)}{S_{ij}^o} \quad b_2 = \frac{(Y_j - Y_i^o)}{S_{ij}^o}$$

are elements of the $B$ matrix in equation (3.8), $f_{ij} = S_{ij}^o - S_{ij}$ where $S_{ij}$ is the measured
distance. The linearized distance equations are formulated into matrix form as:

\[ v + B\Delta = f \quad (3.8) \]

The normal equations are constructed and solved in the steps listed in Appendix A. The adjusted observations \( S_{ij}^0 \), are calculated as:

\[ S_{ij}^0 = S_{ij} + v \quad (3.9) \]

As there is redundancy in the observations, the least squares process requires an initial approximation to the solution to which corrections are applied and the process is iterated until convergence. The criteria used to stop the adjustment process here is, if the difference between the current adjustment and the previous, falls below a given threshold the iterative process is stopped. Wolf and Ghilani [70] suggest a good method would be to monitor and stop when the reference variance increases. An increasing reference variance suggests a diverging solution which suggests either a blunder in the observations or no solution is possible or the maximum correction size exceeds the precision of the measurements.

The coordinates of the adjusted position \((X_P,Y_P)\) are calculated as:

\[ X_P = X_{P0} + \Delta X_P \quad (3.10) \]

\[ Y_P = Y_{P0} + \Delta Y_P \quad (3.11) \]

where \(X_{P0} \) and \(Y_{P0} \) are the initial approximations to the unknown position and \(\Delta X_P \) and \(\Delta Y_P \) are the applied corrections.
3.3. CONVENTIONAL HORIZONTAL POSITIONING METHODS

Associated with the adjusted position is a covariance matrix, $\Sigma_{\Delta\Delta}$, that is used in error analysis.

**Error Analysis in Traverse and Trilateration Surveys**

The products of a trilateration adjustment and the traverse technique of indirect observations include an adjusted position $(X_P, Y_P)$ that minimizes the sum of the squares of the errors and a covariance matrix $\Sigma_{\Delta\Delta}$, that signifies the precision of positioning. For traverses, multiple point coordinates are determined during an adjustment whilst in a trilateration, usually a single points coordinates are the required quantity. It follows that the calculated $\Sigma_{\Delta\Delta}$ will have a direct correspondence to the number of point positions determined.

$\Sigma_{\Delta\Delta}$ is a square symmetric matrix whose diagonal values are covariances of the adjusted positions and the off-diagonal values the cross covariances between each of the positions. Error analysis on a position requires that its variance-covariance matrix be extracted from $\Sigma_{\Delta\Delta}$ to which further analysis is applied.

The process of error analysis starts by firstly developing a test statistic, $T_S$, and an a-posteriori estimate of the reference variance, $\hat{\sigma}^2_0$. This is followed by hypothesis testing to evaluate the equality of the true variance, $\sigma^2_0$ and the a-posteriori estimate of the reference variance $\hat{\sigma}^2_0$ at a chosen level of significance, $\alpha$. The null hypothesis assumes equality whilst the alternate assumes that the true variance is much greater than the a-posteriori estimate. A confidence region or error ellipse for the position is then developed depending on the outcome of this hypothesis testing. Typically, if the null hypothesis is rejected, the confidence region generated is generally larger than if it were accepted. A detailed procedure showing the steps taken in investigating
positional accuracy is outlined in Appendix C.

Accuracy for traverses is defined as the inverse ratio of the misclosure distance to the total length of the traverse. The assumption made is that error of positioning accumulates and is directly proportional and related to the number of observations made or distances/angles measured. The tables in Figures E.2 and E.3 in Appendix E are an extract of horizontal control accuracy standards for traversing that are provided by the Federal Geodetic Control Committee (FGCC).

3.3.4 Intersections

Location by intersection refers to the process where direction observations emanating from two positions of known coordinates meet or intersect at the location of a third and unknown point for the purposes of determining its coordinates. Two methods of intersection are used in surveying namely intersection using a baseline method and intersection using azimuth angles. The derivations of these can be found in [4].

Intersection using the Baseline Method

Let A, B and C represent vertices of a triangle in the horizontal plane. Let vertices A and B have known coordinates such that a line between them represents a baseline of known length. Angle $\alpha_1$ and $\alpha_2$ are measured angles from the baseline to Point A. Figure 3.8(a) is an illustration of this scenario.

The coordinates of A are determined according to:

$$ Y_A = \frac{(X_C - X_B) + Y_B \cot \alpha_2 + Y_C \cot \alpha_1}{\cot \alpha_1 + \cot \alpha_2} $$

(3.12)
Intersection using Azimuth Angles

Where intersecting lines using azimuth angles, the vertices A and B have known coordinates and the angles $\alpha_1$ and $\alpha_2$ to Point A are measured in a clockwise angle from local meridian lines that pass through them. Figure 3.8(b) is an illustration of this scenario.

The coordinates of A are calculated according to:

$$X_A = \frac{(Y_B - Y_C) + X_B \cot \alpha_2 + X_C \cot \alpha_1}{\cot \alpha_1 + \cot \alpha_2} \quad (3.13)$$

$$Y_A = Y_C + (X_A - X_C) \cot \alpha_2 \quad (3.15)$$

3.4 Conventional Vertical Positioning Methods

3.4.1 Levelling

Conventional levelling or spirit levelling is a simple yet precise measurement system. It requires that a closed loop traverse of observations onto points of interest emanate from a benchmark of known elevation and terminate on the same or a different benchmark, also of known elevation. Typically a calibrated level and levelling rod are used. Unlike in traversing for horizontal control, where the observations are conducted at traverse stations, in levelling observations are made to points of interest occupied by the level rod. The levelling instrument is adjusted such that all observations are taken
Figure 3.8: (a) Setup for 2D intersection using baseline angles $\alpha_1, \alpha_2$ and baseline BC; (b) Setup for 2D intersection using azimuth angles $\alpha_1, \alpha_2$. 
while its horizontal sweep is parallel to the horizontal plane and the level rod is held in the direction of the local vertical.

A level run of observations comprises backsight and foresight readings. A backsight reading is that which is taken or read from a levelling rod held vertically on a point of known elevation (usually in the direction one is coming from) and a foresight reading is read from a rod held on a point whose elevation needs to be determined (in the direction one is going towards).

The two methods used to reduce or calculate levelling observations include the rise and fall and height of plane of collimation (HPC) methods. The former provides a complete arithmetic check on all observations and is widely used in underground and surface surveying situations whilst the latter is more useful where multiple levels are to be set out on a site.

The height difference $\delta H_{tAB}$, between two positions $A$ and $B$ onto which vertically held levelling rods are placed is:

$$\delta H_{tAB} = A_{rod\text{reading}} - B_{rod\text{reading}}$$  \hspace{1cm} (3.16)

Figure 3.9 shows a typical levelling setup with some observations to points of interest that are placed on the roof of an excavation. In this case the levelling rod is held inverted and vertical on points on the roof.

The height of a second benchmark given a series of backsight $BS$, and foresight observations $FS$, from $n$ instrument setups would be calculated as follows:

$$Elev._B = Elev._A + \left( \sum_{i=1}^{n} BS - \sum_{i=1}^{n} FS \right)$$  \hspace{1cm} (3.17)
Errors in levelling accumulate proportionally to the distance levelled or to the number of instrument setups used during the levelling process. If the progressive distance of each turning point is known from the initial benchmark, each intermediate elevation $E_{\text{lev}}^i_{\text{initial}}$ calculated can be corrected according to:

$$E_{\text{lev}}^i_{\text{final}} = E_{\text{lev}}^i_{\text{initial}} - \frac{d_i}{L} E_c$$

where $E_{\text{lev}}^i_{\text{initial}}$ is the $i^{th}$ unadjusted intermediate elevation corresponding to foresight of the $i^{th}$ setup, $d_i$, the distance of the foresight position from the initial benchmark, $L$, the total level traverse distance and $E_c$, the total level misclosure or error.

Acceptable levelling accuracies depend on the purpose of the levelling exercise. Table E.1 in Appendix E illustrates acceptable level closures and the field procedures that are used to attain these. Where the distance involved might be short but the number of setups high, such as in engineering surveys, an alternate method by which
the allowable error of closure $E_{allow}$ for a loop traverse can be calculated is:

$$E_{allow} = m(n)^{\frac{1}{2}}$$  \hspace{1cm} (3.19)

where $m$ represents the allowable closure limit and $n$, the number of setups otherwise $n$ represents the total distance in kilometres.

Least squares methods can also be used to calculate levels more especially where a network of points of interest are to be tied to several benchmarks. Using the least squares technique of indirect observations, the condition equation of the following form is formulated:

$$v_{ij} + x_i - x_j = -l_{ij}$$  \hspace{1cm} (3.20)

where $v_{ij}$ are the residuals from a loop involving points of interest $i$ and $j$, $x_i$ and $x_j$ are the calculated elevations and $l_{ij}$ on the right hand side is the measured height difference between $i$ and $j$. The condition equation above can be written in the matrix form:

$$v + B\Delta = f$$  \hspace{1cm} (3.21)

where $B$ is a matrix of partial derivatives of the condition equations with respect to the parameters. $\Delta$ is a parameter matrix of the desired elevations of the points of interest, $v$ a residual matrix and $f$ a constant term matrix. This is solved by the usual method outlined in Appendix A. However, in this case the weight matrix for a height difference is a function of the number of setups or distance between any two points that constitute that height difference.

Likewise, the least squares technique of adjustment of observations only can also be used to calculate a set of levels. Here a condition equation is formulated for each
set of points that compose a closed loop. For loops that emanate from one point and terminate at the same point the following condition equation is applied:

\[ v_{1i} + v_{ij} + v_{jk} + v_{k1} = -l_{1i} - l_{ij} - l_{jk} - l_{k1} \]  

where \( v \) are the height differences between two consecutive points. The condition equations above can be written in matrix form:

\[ Av = f \]  

where the matrix \( A \) is a coefficient matrix of the observations, \( v \), a residual vector matrix and \( f \) is a condition equation constant term vector. The least squares solution follows the method outlined in Appendix B on page 231.

### 3.4.2 Trigonometric Levelling

In trigonometrical levelling a theodolite or total station is set up above or below a station and the vertical angle \( \alpha \), and slope distance \( S \) to the another station or position of interest are measured. Of interest is the determination of feature elevations, however this process is usually combined with the determination of the unknown points location coordinates as well which is discussed in the next section.
3.5. CONVENTIONAL MAPPING AND MAP GENERATION

Typically, in an underground mining environment stations are placed on the roof or sidewall of an excavation as illustrated in Figures 3.4 and 3.5. The height of instrument $h_i$, above or below station and the height of target $h_t$, above or below the next station are also measured. Using these observations, the height of a station B from station A, as in the setup in Figure 3.10 is found using:

$$H_B = H_A + h_i + S \sin(\alpha) - h_t$$

(3.25)

Figure 3.10: Trigonometrical levelling (Schoefield and Breach, 2007).

3.5 Conventional Mapping and Map Generation

3.5.1 Underground Mapping

Accurate surveys, reliable maps and confidence in the surveying and mapping process is essential for successful mining [19]. Unlike surface topographic maps, the interest in compiling underground maps is to determine or enable the location of a tunnel or underground passage (xy) and not the elevation of feature points. Usually elevation
3.5. CONVENTIONAL MAPPING AND MAP GENERATION

is depicted at station coordinates.

Underground maps are compiled from a set of observations that include offset distances from a traverse line or angle and distance measurements observed from an underground traverse station in a process called detailing. The term detailing refers to mapping the irregularities or bulges observed along the walls or surface of an excavation and they generally arise from the nature of the rock penetrated or driller experience, following promising indicators of ore [64] or effects of blasting. Two methods of detailing are used and these are the angle-right and offset methods.

The angle-right method or radial positioning method requires a theodolite instrument to be set out below a roof station or at a sidewall traverse station. Angles right are turned from a backsight observation to a previous traverse station onto points of interest along the walls of an excavation and horizontal distance measurements to these positions made by measuring tape. These positions are later transferred onto a map either as coordinates that would have been calculated based on the set-up station coordinates or using a protractor and scale ruler.

With the advent of computers and other electronic surveying instruments, radial positioning is the preferred method for collecting topographic detail. While it uses the same principles as the angle-right method, the difference is that mapping is done from coordinate data that would have been calculated from angular and distance observations that are collected from and referenced to known traverse stations. The typical method to obtain the coordinate of station $B$ from observations made at station $A$ is as follows:
where $\theta$ is the azimuth, $d_{AB}$, the horizontal distance, and $H_i$ and $H_o$, the height of instrument and target respectively.

In the offset method, horizontal distance measurements are made using a measuring tape in an orthogonal direction either side of a line marking two consecutive traverse station positions to interest points along the walls. Offset positions are noted as chainages$^3$ or incremental distances along the traverse lines. Figures 3.11(a) and 3.12 illustrate the methods discussed above.

### 3.6 Surveying and Mapping Standards

Information used for mapping is derived from various sources of data such as GPS, terrestrial, acoustic, satellite surveys or aerial mapping [2]. This data is brought into a single mapping plane through various transformations of the data. Specifications of a project determine the standards used for different data types that are merged together. In the west, several professional associations such as the American Society of Photogrammetry and Remote sensing (ASPRS), American Society of Civil Engineers (ASCE), American Congress on Surveying and Mapping (ACSM) and the FGDC are a major sources of accuracy standards and specifications used in surveying and mapping.

---

$^3$Chainage is a surveying term used to denote cumulative horizontal distances measured along connected line segments having a defined start point. Usually this is applied to road, pipeline, powerline etc. longitudinal sections where the surveys are in the form of corridors.
3.6. SURVEYING AND MAPPING STANDARDS

Figure 3.11: (a) Illustration of plan-view of angle-right method for underground mapping (Staley, 1964); (b) Vertical section showing measurements observed for a radial positioning setup.

Figure 3.12: Offset method of underground mapping (Staley, 1964).
3.6. SURVEYING AND MAPPING STANDARDS

3.6.1 Surveying Standards

Accuracy standards (see Table 3.1) in surveying are specified and classified based on a linear horizontal closure ratio or vertical difference closure standard [4][68][29] and are applicable to traverse, GPS and levelling surveys. Closure checks are used to classify, standardize and evaluate survey work and are also used to determine the relative accuracy of a survey. The basis for relative survey accuracy is the ratio of the traverse loop closure to the total traverse distance.

Typically accepted traverse accuracy ratios for range from 1:5,000 to 1:20,000 or better [25] for tunnel environments surveyed using current equipment. Table 3.1 extracted from [29] shows the minimum closure standards for engineering and construction surveys.

Table 3.1: Minimum closure standards for engineering and construction Surveys (FGDC Part 4, 2002)

<table>
<thead>
<tr>
<th>Classification Order</th>
<th>Closure Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eng. &amp; Constr. Control</td>
<td>Distance (Ratio)</td>
</tr>
<tr>
<td>Second-Order, Class I</td>
<td>1: 50,000</td>
</tr>
<tr>
<td>Second-Order, Class II</td>
<td>1: 20,000</td>
</tr>
<tr>
<td>Third-Order, Class I</td>
<td>1: 10,000</td>
</tr>
<tr>
<td>Third-Order, Class II</td>
<td>1: 5,000</td>
</tr>
<tr>
<td>Construction (Fourth-Order)</td>
<td>1: 2,500</td>
</tr>
</tbody>
</table>

$^1$ Number of angle stations
$^2$ Table 3.1 extracted from [29]

Appendix E shows closure standards specific to traversing and levelling surveys extracted from [4]. These standards state that to achieve a certain traverse accuracy classification, what type of instrumentation should be used, what procedures should
3.6. SURVEYING AND MAPPING STANDARDS

be followed and how many measurements should be obtained.

Geologic field note taking is done on a range of map scales with the mapping scale dependent on the needs of a specific project [25] (p157). Typically the base maps upon which all geologic mapping is done are provided by the mine surveyors. Detailed mapping of the underground environment is done on large scale maps. Table 3.2 compiled from [25] summarizes the map scales and map uses as pertaining to the underground mining environment.

Table 3.2: Mine map classifications (SME 2011)

<table>
<thead>
<tr>
<th>Map Scale Size</th>
<th>Scale (metric)</th>
<th>Scale (in-ft)</th>
<th>Map Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Scale</td>
<td>1: 240</td>
<td>1 in. : 20 ft.</td>
<td>Detailed mapping</td>
</tr>
<tr>
<td></td>
<td>1: 600</td>
<td>1 in. : 50 ft.</td>
<td></td>
</tr>
<tr>
<td>Small Scale</td>
<td>1: 1,200</td>
<td>1 in. : 100 ft.</td>
<td>Simplification of</td>
</tr>
<tr>
<td></td>
<td>1: 2,400</td>
<td>1 in. : 200 ft.</td>
<td>detailed mapping.</td>
</tr>
<tr>
<td>Regional Map</td>
<td>1: 12,000</td>
<td>1 in. : 1,000 ft.</td>
<td>Collect overall resource</td>
</tr>
<tr>
<td></td>
<td>1: 24,000</td>
<td>1 in. : 2,000 ft.</td>
<td>setting data.</td>
</tr>
</tbody>
</table>

1 Table 3.2 compiled from [25]

3.6.2 Mapping Standards

Mapping standards classify maps as statistically meeting a defined level of accuracy [68]. Accuracies on maps are defined based on specific feature positional accuracies. Typically when a desired accuracy specification on a project is stated, it is on the basis of a final or target map scale and contour interval.

The ASPRS standard defines map accuracy through a comparison of well defined points to their true location defined by a more accurate survey. The horizontal planimetric accuracy criteria compares the root mean square error (RMSE) of the
mean of the squared differences in the map and ground survey values. The root mean square errors in the x and y directions are calculated according to:

\[ RMSE_x = \sqrt{\frac{\sum_{i=1}^{n} (X_{Map} - X_{True})^2}{n}} \]  
(3.27)

\[ RMSE_y = \sqrt{\frac{\sum_{i=1}^{n} (Y_{Map} - Y_{True})^2}{n}} \]  
(3.28)

The horizontal radial RMSE is calculated according to:

\[ RMSE_r = \sqrt{RMSE_x^2 + RMSE_y^2} \]  
(3.29)

The 95% CI map accuracy is finally calculated according to:

\[ Accuracy_r = 2.4477(RMSE_x) = 2.4477(RMSE_y) = 1.7308(RMSE_r) \]  
(3.30)

Table 3.3 extracted from [1] shows current ASPRS horizontal accuracy standards for digital planimetric data. The limiting \( RMSE_r \) is defined in terms of the ground scale rather than the target map scale. This results in a linear relationship between the \( RMSE_r \) and the target map scale and therefore establishes the maximum permissible value. These ASPRS limits apply to specific and well-defined check points at a defined map scale. For each 95% CI horizontal accuracy map scale is an associated horizontal data accuracy class. Note that a final map can be plotted at various scales, however, at different data accuracy class.
### Table 3.3: Horizontal accuracy/quality for digital planimetric data (ASPRS 2013)

<table>
<thead>
<tr>
<th>Map Scale</th>
<th>Horizontal Data Accuracy Class</th>
<th>RMSEx (cm)</th>
<th>RMSEy (cm)</th>
<th>Horizontal Accuracy at 95% Confidence Level (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:100</td>
<td>I</td>
<td>1.3</td>
<td>1.8</td>
<td><strong>3.1</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>2.5</td>
<td>3.5</td>
<td><strong>6.1</strong></td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>3.8</td>
<td>5.3</td>
<td><strong>9.2</strong></td>
</tr>
<tr>
<td>1:200</td>
<td>I</td>
<td>2.5</td>
<td>3.5</td>
<td><strong>6.1</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>5.0</td>
<td>7.1</td>
<td><strong>12.2</strong></td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>7.5</td>
<td>10.6</td>
<td><strong>18.4</strong></td>
</tr>
<tr>
<td>1:250</td>
<td>I</td>
<td>3.1</td>
<td>4.4</td>
<td><strong>7.6</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>6.3</td>
<td>8.8</td>
<td><strong>15.3</strong></td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>9.4</td>
<td>13.3</td>
<td><strong>22.9</strong></td>
</tr>
<tr>
<td>1:500</td>
<td>I</td>
<td>6.3</td>
<td>8.8</td>
<td><strong>15.3</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>12.5</td>
<td>17.7</td>
<td><strong>30.6</strong></td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>18.8</td>
<td>26.5</td>
<td><strong>45.9</strong></td>
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<tr>
<td>1:1000</td>
<td>I</td>
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<td>17.7</td>
<td><strong>30.6</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>25.0</td>
<td>35.4</td>
<td><strong>61.2</strong></td>
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<tr>
<td></td>
<td>III</td>
<td>37.5</td>
<td>53.0</td>
<td><strong>91.9</strong></td>
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<td>I</td>
<td>25.0</td>
<td>35.4</td>
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</tr>
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<td></td>
<td>III</td>
<td>75.0</td>
<td>106.1</td>
<td><strong>183.6</strong></td>
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<td>1:2500</td>
<td>I</td>
<td>31.3</td>
<td>44.2</td>
<td><strong>76.5</strong></td>
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<td></td>
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<td>88.4</td>
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<td></td>
<td>III</td>
<td>93.8</td>
<td>132.6</td>
<td><strong>229.5</strong></td>
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<td>1:5000</td>
<td>I</td>
<td>62.5</td>
<td>88.4</td>
<td><strong>153.0</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>125.0</td>
<td>176.8</td>
<td><strong>306.0</strong></td>
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<tr>
<td></td>
<td>III</td>
<td>187.5</td>
<td>265.2</td>
<td><strong>458.9</strong></td>
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<tr>
<td>1:10000</td>
<td>I</td>
<td>125.0</td>
<td>176.8</td>
<td><strong>306.0</strong></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>250.0</td>
<td>353.6</td>
<td><strong>611.9</strong></td>
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<td></td>
<td>III</td>
<td>375.0</td>
<td>530.3</td>
<td><strong>917.9</strong></td>
</tr>
<tr>
<td>1:25000</td>
<td>I</td>
<td>312.5</td>
<td>441.9</td>
<td><strong>764.9</strong></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>III</td>
<td>937.5</td>
<td>1325.8</td>
<td><strong>2294.7</strong></td>
</tr>
</tbody>
</table>

** Table 3.3 compiled from [1].
3.7 Conclusion

In this chapter different facets of mine surveying are developed. The basis for mine coordinates is a global reference frame based on horizontal and vertical datums. Different datums are discussed, and the various methods of conventional horizontal and vertical positioning in the underground environment. Traversing and levelling are the current methods of propagating $x$-$y$ positions and elevations underground.

Also discussed in this chapter is how underground mapping is undertaken. Important to note is that in all these surveying and mapping processes, positional and mapping accuracy is quantifiable and the application of standards used is also discussed.
Chapter 4

Automated Positioning Methods for Underground Mapping

4.1 Introduction

In this chapter different and known or existent mapping systems that have the potential for underground mapping are reviewed. As various researchers use different sensor configurations depending on the research problem, the focus of this chapter are the components of the uGPS Rapid Mapper™system that makes it possible to map the underground mining environment.

4.2 Current products for underground mapping

4.2.1 ZEB1

The GeoSLAM ZEB1 (Zebedee) (see Figure 4.1) is a hand held mobile mapping system developed by CSIRO, Australia. The system is designed to survey and map challenging indoor and underground spaces, document accident and crime scenes and property for real estate valuation. The system is designed to be easy to operate,
4.2. CURRENT PRODUCTS FOR UNDERGROUND MAPPING

lightweight, providing rapid data capture with \( \pm 0.1\% \) accuracy. Data capture is made through an operator walking around the mine tunnels and collecting the data with the hand held device. A fully registered point cloud is obtained through transformation of the field data using simultaneous localization and mapping (SLAM) techniques. The company provides online data processing of the survey data to minimize software costs and maintenance fees.

The system uses a Class 1 eye safe laser scanner with a 270\(^\circ\) field of view. The scanner has a maximum distance measurement range of 30 m and the system has a data acquisition speed of 43,200 measurement points per second. The system uses the scanner and an industrial-grade MEMS inertial measurement unit (IMU) mounted on a simple spring mechanism. The reader is referred to [18] for more details on this system.

Figure 4.1: GeoSlam Zeb1 (Zebedee) hand held mobile mapping system (Bosse et al. 2012).
4.2. CURRENT PRODUCTS FOR UNDERGROUND MAPPING

4.2.2 Trimble Indoor Mobile Mapping Solution (TIMMS)

The Trimble Indoor Mobile Mapping Solution (TIMMS) from the Applanix Corporation, a Trimble company, is the optimal fusion of technologies for capturing spatial data of indoor and other GNSS denied areas [24]. TIMMS output includes geo-located maps, LiDAR point cloud data and spherical video of the mapped area. TIMMS according to the company website is an efficient tool for indoor mapping including underground mines and tunnels with the benefits of high accuracy, low cost, high productivity through a simple workflow and providing a quick data turnaround. Figure 4.2 shows the TIMMS mapping tool.

The company cites a range of applications on their website where their technology is used, however none of these applications include underground mine mapping. The literature cited supporting the technology relates more to GPS guided vehicles and paths with short GPS denied segments, however nothing on underground mapping.

Figure 4.2: Trimble indoor mobile mapping solution(www.applanix.com).
4.2. CURRENT PRODUCTS FOR UNDERGROUND MAPPING

4.2.3 Minefly

MineFly is a drone-based system by Clickmox Solutions designed for accurate scanning of the GPS denied underground environment [23]. The DJI Matrice 100 drone is equipped with a Hokuyo UST-20LX Laser Rangefinder 2D laser scanner and provides accurate 3D point clouds through offline map generation. According to the company website their system is best suited for indoor mapping and profiling, 3D visualization and has been applied to underground drift, stope and ore-pass mapping, convergence monitoring and underground surveying. Their system (see Figure 4.3) employs simultaneous localization and mapping (SLAM) techniques to generate 3D point clouds from 2D laser scan data.

In this case there is no literature that has been found supporting the operation of this system.

Figure 4.3: MineFly (www.clickmox.com).
4.2. CURRENT PRODUCTS FOR UNDERGROUND MAPPING

4.2.4 uGPS Rapid Mapper™

The focus in this research is on the uGPS Rapid Mapper™ (see Figure 4.4), its onboard sensors, its mapping and post processing methods as it is the tool used. The uGPS system is well researched [44, 43, 7, 5, 60] and is a dedicated underground mine mapping system.

Selection to use this system is based on its availability for use in this research, the knowledge that it is well researched and knowledge of how it works, that Dr. J. Marshall, now with Queens University Mining Systems Laboratory was instrumental in its design and the fact that its output data can be easily combined with the Mobile Automated Scanner Triangulation (MAST) system.

Figure 4.4: uGPS Rapid Mapper™ (http://ugpsrapidmapper.com)
4.3 Sensors

4.3.1 Laser Scanners

Laser scanners are a popular sensor for use in robotics applications and are used for distance measurement and proximity detection. In laser based range measurement applications time of flight and phase shift techniques are dominant.

**Time of Flight (TOF)**

TOF techniques send a pulses of light out and measure time until a return signal is recorded. The ranging principle is:

\[ D = \frac{1}{2} c T \]  \hspace{1cm} (4.1)

where \( D \) represents twice the scanner-feature distance and \( c \), the speed of light \( (3 \times 10^8 \text{ ms}^{-1}) \) and \( T \), the time taken to send light pulses from the scanner until a return signal is received. Such a sensor is referred to as a laser or LiDAR.

**Phase Shift**

In phase shift systems, a continuous light wave is transmitted and a comparison in phase is made between the return signal and a reference signal generated by the same source. The distance from the source to the feature target is measured using Doppler shift with the velocity to the target as an additional product of the process.

The issue with this type of technique is the inability to easily differentiate between phase shifts greater than one wavelength from those less than one wavelength.
Scanners

The ability for a laser scanner to scan comes from mounting a laser sensor on a rotating body and having the ability to regulate its rotation or by having a fixed and non-rotational laser sensor and utilize a rotating mirror to enable covering a large field of view.

The SICK LMS range of laser scanners are used with the uGPS Rapid Mapper™ shown in Figure 4.4. These are 2D laser scanners with the following advantages:

1. They provide fast, near instantaneous measurements with a response time of approximately 20 ms, eliminating the need for motion compensation for mobile robotics applications. The ability to rotate the scanning device fast and accurate enough is the only drawback to the system.

2. They provide accurate range measurements within an operating range of 0.5 m to 20 m. The standard deviation of distance measurement for the SICK range is 3 cm with angular resolution ranges of 0.25°, 0.5° and 1° [72].

3. The ability to provide direct outputs as distance and direction.

Disadvantages of using 2D laser scanners include limitation to a single plane making it difficult to sense the 3D world and inability to sense transparent materials.

The SICK LMS 111 according to [72] uses an infrared laser and has a limited distance range of 20 m. The ability to set the angular resolution enables a densification of data collection and in the case of the uGPS system the angular resolution is set at 0.5° providing 541 measurements over a 270° field of view at 50 Hz. Figure 4.5 shows an image of the SICK LMS 111 laser scanner. Table 4.1 from [44] lists the specifications as set for use by the uGPS Rapid Mapper™ system.
4.3.2 Inertial Sensors

Inertial sensors (see Table 4.1 and Figure 4.5) have applicability in navigation. In aviation they are coupled with GPS and radio beacons that facilitate radio navigation. Inertial sensors can also be used for dead reckoning navigation. The following inertial sensors are an input into an inertial measurement unit (IMU) or inertial navigation system (INS):

- Accelerometers
- Gyroscopes
- Tilt sensors

A text such as [54] and many others describe the process by which velocity, displacement and position estimates are obtained.
4.3. SENSORS

Table 4.1: SICK laser scanner specifications for uGPS Rapid Mapper™ (Lavigne 2010)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser wavelength</td>
<td>905 nm (infrared)</td>
</tr>
<tr>
<td>Minimum range</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Maximum range</td>
<td>20 m</td>
</tr>
<tr>
<td>Field of view</td>
<td>270° Start: -45°, End: 225°</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>0.5°</td>
</tr>
<tr>
<td>Scanning frequency</td>
<td>50 Hz.</td>
</tr>
<tr>
<td>Statistical range error (typical)</td>
<td>12 mm</td>
</tr>
<tr>
<td>Systematic range error (typical)</td>
<td>12 mm</td>
</tr>
</tbody>
</table>

**Table extracted from [44]**

Accelerometers

Accelerometers measure acceleration. The INS design requires mounting and aligning the sensors along three orthogonal axes. Integration of accelerations measured in an inertial reference frame and then transforming them into a navigation frame and integrating the results with respect to time produces velocity and displacement estimates.

Gyroscopes

Gyroscopes measure angular motion. These are also placed along the same tri-axis system as the accelerometers. Gyroscopes are categorized as rate gyros or rate integration gyros. The former measures angular velocity whereas the latter measures position. A further classification of gyroscopes is mechanical and optical.

Mechanical gyroscopes measure change in angular motion and apply the law of conservation of momentum. Optical gyroscopes use light to monitor movement.
4.3. SENSORS

Tilt sensors

Inclinometers are tilt sensors that measure angles directly without integration as the gyroscopes and accelerometers do. The elimination of the integration process reduces the possibility of errors that arise due to integration. However this type of sensor works best in stationary situation. While this is a method of determining the attitude of a moving platform, other methods exist such as mentioned in [54].

4.3.3 Radio Frequency IDentification (RFID) Tags

RFID is a technology that wirelessly transmits unique identification numbers to sensors or readers. They are classified as either active or passive. Active RFIDs are based on powered transmitters whilst passive do not have a power source but have excitable integrated circuits. RFID technology is relatively cheap and has a wide range of uses in industry such as in vehicle, human or inventory tracking and many other applications. The uGPS application uses passive RFID technology. Figures 4.6 and 4.7 show the RFID tags used in the ILC experiments and Figure 4.9(c) shows the RFID tag reader on top of the uGPS unit that is mounted on the Taylor Dunn vehicle.

The application of RFID technology to underground mapping with the uGPS Rapid Mapper™ is to locate the position of a known scan previously registered to either ‘close the loop’, start or end a mapping traverse. The application of RFID technology with the Mobile Automated Scanner Triangulation (MAST) beacons is to start or end the MAST positioning process discussed in the next chapter when the mapping vehicle comes within proximity of the beacons.
Figure 4.6: (a) Alien ALN-9654 Passive RFID; (b) Alien ALN-964X Passive RFID;
4.4 MAPPING WITH UGPS RAPID MAPPER™

Figure 4.4 shows an image of the uGPS Rapid Mapper™ unit used in the mapping process. It uses a fusion of sensor inputs, including laser scanner technology to enable rapid and accurate 3D point cloud data acquisition from a mobile platform [56, 57]. The uGPS Rapid Mapper™ allows for multidimensional data acquisition capability for the mining industry.

According to the product website, the uGPS Rapid Mapper™ does not require specialized knowledge of scanning to operate. Useful scans can then be obtained through simple installation and operating procedures by anyone including underground equipment operators. Appendix F is a collection of slide notes shown during a typical uGPS deployment process and is the reference for all slides mentioned in subsequent sections. These show the specifications, procedures and other information pertaining to
4.4. MAPPING WITH UGPS RAPID MAPPER™

the operation of the uGPS instrument.

4.4.1 Automated Machine Routines for the uGPS Rapid Mapper™

The survey process using the uGPS Rapid Mapper™ (see Figure 4.4) starts with the installation of RFID tags at feature rich places such as corridor or tunnel intersections or other places with unique features as shown on Slide 62. In the case of experiments done in the first floor of the Beamish Munroe building or Integrated Learning Centre (ILC) (see Figure 4.8), the locations chosen were STN 120, STN 170, STN 100 and STN 130. At the first two stations is where the MAST beacons were located.

The uGPS Rapid Mapper™ is mounted appropriately according to Slides 10 and 13. This is followed by calibrating and setting the system up for use according to Slides 17 to 25. Once the uGPS Rapid Mapper™ has been set up, it can then be used to register scans required to ‘close the loop’ or perform a mapping.

The coordinates of the RFID tag are determined using conventional surveying methods from stations of known coordinates and these coordinates are entered into the Tag Database as shown on Slides 46 to Slide 58 after visiting and aligning the uGPS Rapid Mapper™ to each tag as shown in Slide 47. This process registers the start, intermediate or finish positions or locations for the survey where the uGPS Rapid Mapper™ software ‘closes the loop’ and corrects for errors due to drift. Figure 4.9 and Slide 47 show the RFID tag illuminated by the laser pointer during the process of linking the scan to a known coordinate location.
Figure 4.8: ILC map showing as-built survey, conventional traverse and stations and beacon positions.
4.5. CONCLUSION

Mapping using the uGPS Rapid Mapper™ starts by ensuring the calibration of the instrument is current. The vehicle or platform upon which the uGPS Rapid Mapper™ is mounted is driven along the path that the mapping is required, passing through or along previously scanned ‘loop closure’ registration positions at the required speed at the same time recording data to be analyzed at a later stage.

4.4.2 uGPS Data Processing

At the end of a mapping exercise, the data is downloaded using a USB drive and uploaded to a computer and can be accessed by the UGPS Rapid Mapper™ post-processing software. The data analysis process starts by selecting a survey input .DTZ file, then selecting constraining parameters from menus on a series of dialogue boxes and if necessary associating the data with relevant tag databases and pressing a button to generate or calculate the solution.

The output of this process are data files that can be accessed by other software packages such as excel etc.. The three output file formats are .XYZ, .PLY and LAS and these enable viewing the point cloud data using other vendor software. This process is illustrated on Slides 28 to 33.

4.5 Conclusion

In this chapter known and existent systems for underground mapping were discussed. The uGPS Rapid Mapper™ is the focus of this chapter as it is a well researched system and its operational principles are known and because of the the easy integration with the MAST system discussed in the next chapter, it is the choice system used in this research.
Figure 4.9: (a) and (b) uGPS laser pointer sighting on RFID tag at ST170; (c) uGPS Rapid Mapper™ mounted on Taylor Dunn SS-534 electric vehicle showing location of laser pointer and flat metal plate on back.
Chapter 5

2D Automated Positioning and Design

5.1 Introduction to MAST

The Mobile Automated Scanner Triangulation\(^1\) (MAST) positioning system is a technique that has been developed to determine the position and orientation of a laser scanner in GPS deprived environments from measurements made by the scanner to geo-referenced beacons that have been placed in the environment. This technique can be used to position any LiDAR-based system as in this case it is used with the GPS Rapid Mapper\(^{TM}\).

MAST is comprised of two parts, namely the hardware and the algorithms. The hardware part takes into consideration the MAST beacons which are integral to the whole positioning process and the laser scanner. The algorithms take the scanner output of the underground environment in the scanner arbitrary frame, using the beacons as reference, and transform the scanner data from the arbitrary reference frame to a mine coordinate frame. MATLAB\(^{TM}\)R2010a computing software was the

\(^1\)Triangulation is a term that has been introduced since [38] to define the process of combining measurement of distances to features in the environment at constant angular intervals specifically using a laser scanner such as the SICK LMS111 in order to determine the position of the laser scanner in 2D space.
basis for executing all MAST simulations and algorithms. This chapter discusses the processes involved and expands upon the earlier work described in [38].

This thesis is limited to 2D positioning where positioning takes place in the $x$-$y$ or horizontal plane.

5.2 Beacon Design, Parameters, Size and Mounting

5.2.1 Beacon Design and Parameters

The MAST beacon is a passive geometric figure of known shape and design comprising interconnected panels of known length $P_i$, and known panel deflection angle $\theta_j$. Figure 5.1(a) shows a schematic of the MAST beacon and Figure 5.1(b) the constructed beacon. The three panels have distinct lengths that enable distinguishing one panel from the other, and in this case, are kept at a constant ratio $P_1 : P_2 : P_3$ of 2 : 3 : 1, here $P_1$ and $P_3$ being the outer panels and $P_2$, the centre panel. The beacon parameters therefore are defined as the panel lengths $P_1$, $P_2$ and $P_3$, and the deflection angles between the panels, $\theta_1$ and $\theta_2$ are reflected in the beacon design.

The MAST beacons are used as pairs mounted side by side or at approximately 90° to each other depending on the underground situation. We define a beacon traverse station (BTS) as the vicinity occupied by a pair of MAST beacons where a laser scanner would have visibility of a pair of beacons. The BTS is defined with respect to the existing main traverse point (not line point) of a mine reference network, in this case the original network points set out for the as-built survey of the ILC. A pair of beacons mounted at a beacon station here is defined such that one beacon is a mirror reflection of the other. In the experiments undertaken in the ILC, both beacon configurations were used, although most results pertain to the second beacon
Figure 5.1: (a) Schematic of the MAST beacon and (b) Image of MAST beacon mounted for experiments at BST170 in the ILC
configuration.

Typical characteristics of the beacons include:

- They can be small, cheap, easy to make, mount and can be surveyed in by conventional surveying methods using a theodolite or total station.
- The geometry of the beacon allows for independent panel segmentation.
- The beacon design allows for transference of the mine coordinate system to an equivalent mine reference system that can be used and manipulated by a robot.
- The horizontal and vertical cross-section of the beacon are the same to facilitate the same algorithm used to determine position to be used to determine elevation.
  
  In this thesis, focus is placed on the $x$-$y$ position although a similar concept can be readily used to determine initial elevation.

- The placement or mounting of the beacons as male and female is to enable unique position determination and also to enable detection of travel direction.

The beacon design is a crucial element of the MAST positioning process. Its simplicity and the way it is mounted and used are one of the major reasons behind the success of the MAST positioning process providing centimetre level uncertainty in positioning accuracy and single digit degree uncertainty in orientation accuracy.

### 5.2.2 Beacon Sizing and Mounting

The basis for MAST positioning is the ability to segment and uniquely define the different panel parameters of the beacon and the interrelationship of these panels.
Therefore, in trying to determine the size of beacon to use for positioning, consideration must be made of the angular resolution of the scanner used and the tunnel width.

Let $\gamma$ represent the angular resolution of the laser scanner and $n$, the minimum number of points required to fall on panel $P_3$. Therefore, at any point in time, the angle $\alpha$ subtending $n$ cloud points on $P_3$ is:

$$\alpha = (n - 1) \times \gamma$$  \hspace{1cm} (5.1)

Let the width of a tunnel be defined as $W$. It is assumed that a mobile surveying and mapping platform scans the beacons from a position that is midway between the beacons, $\frac{W}{2}$. As there is physically no guarantee on the $n$ number of cloud points falling on $P_3$ due to the random nature of the positioning of the laser scanner and the resulting scatter of the point cloud, $n$ cloud points falling on $P_3$ can be guaranteed by using a factor greater than $\frac{W}{2}$, such as $\frac{3W}{4}$. Therefore $P_3$ can be defined in terms of the tunnel width by:

$$P_3 = \frac{3W}{2} \times \tan\left(\frac{\alpha}{2}\right)$$  \hspace{1cm} (5.2)

It follows then that at an optimal beacon depth $h$, $\theta_1$ and $\theta_2$, the outer panel deflection angles with respect to the centre panel, are defined as:

$$\theta_1 = \sin^{-1}\left(\frac{h}{P_3}\right)$$  \hspace{1cm} (5.3)

$$\theta_2 = \sin^{-1}\left(\frac{h}{2P_3}\right)$$  \hspace{1cm} (5.4)

As the vertical and horizontal cross-sections of the beacon are the same, the minimum
beacon length $l$ is defined from:

$$l = 3P_3 + 2P_3 \cos(\theta_2) + P_3 \cos(\theta_1)$$  \hspace{1cm} (5.5)$$

or

$$l = 3P_3 + \sqrt{(2P_3)^2 - h^2} + \sqrt{(P_3)^2 - h^2}$$

Some critical issues that arise are, if $\theta_1$ is too small, then it becomes difficult to differentiate between beacon panel $P_3$ and the surrounding rock, hence difficulty in segmentation. If $\theta_1$ is too big, there is the risk of not being able to segment $P_3$ due to its length being too short and the resulting beacon depth too big and an impediment to traffic. There is also need to balance angular resolution with tunnel width in order to maximize the number of points falling on $P_3$.

In this research, the corridors of the ILC are narrow and there was the need for a beacon whose length was less than a metre and had depth less than 0.2 m and it was decided to use arbitrarily a 0.85 m beacon length design (see Figure 5.3). This is obviously an area that requires optimization and further research.

Figure 5.2 shows the levelling of a MAST beacon using an ordinary level. This process ensures conformity to the mine coordinate system in that the beacon $x$-$y$ coordinates are represented by the locations of the vertical edges/intersection lines and the beacon reference elevations by the horizontal planes corresponding to the horizontal edges/intersection lines.

As mentioned in the previous section, the MAST beacons are used in pairs. Figure 5.3 illustrates the different beacon configurations for beacon traverse stations BTS120 and BTS170. The beacons are mounted as shown in Figure 5.3 such that horizontal
5.2. BEACON DESIGN, PARAMETERS, SIZE AND MOUNTING

edges/intersection lines are horizontal and likewise the vertical edges/intersection lines are vertical.

A local axis system of a properly mounted beacon is defined as an orthogonal three-axis coordinate system that is aligned to the mine coordinate system. The z-axis of the beacon follows the vertical intersection lines/edges of the beacon and is aligned to the z-axis of the mine coordinate system. The local x-axis of the beacon follows the horizontal intersection lines/edges of the beacon and also aligned parallel to the x-y plane of the mine reference system. It follows then that the y-axis of the beacon is orthogonal to the local x-z plane of the beacon.

Figure 5.4 is a schematic that shows the beacon alignment relative to the global axis system. Beacon edge marked 5-6 marked by the red line lies parallel to the xy plane and likewise beacon edge 8-5 lies parallel to the zenith line or z axis.

The mounted beacon should have two degrees of freedom of rotation i.e., the ability to be rotated/tilted backward or forward about the local x axis and rotated left or right about the local y axis. This allows for the adjustment of the beacons horizontal and vertical intersection lines to be made horizontal and vertical respectively.

Figure 5.5 is a schematic that shows how this is achieved.
Figure 5.2: Levelling of the MAST beacon: (b) horizontal ; (b) vertical.
Figure 5.3: Beacon Configurations at MAST Beacon Locations: (a) BTS120 Configuration 1; (b) BTS120 Configuration 2; (c) BTS170 Configuration 1; and, (d) BTS170 Configuration 2.
Figure 5.4: Beacon alignment in relation to global axis system or mine reference frame.
5.3.1 Calibration Flow Process

The beacon calibration and positioning process illustrated in Figure 5.6 is a seven (7) step process. Beacon direction data is collected in the field using a theodolite from traverse points that would have been previously set out and whose coordinates would be known. This direction data is then fed into an algorithm that automatically processes it to determine the beacon calibration, beacon adjustment data and coordinates.

In Step 1, the beacon is mounted, levelled and three control points are set out and
5.3. BEACON CALIBRATION AND POSITIONING

Start Calibration

Step 1: Mount and level beacons, Set out control points

Step 2: Observe/collect raw data for beacon vertex positions

Step 3: Adjust raw data

Step 4: Determine which beacon to calibrate

Step 5: Solve intersection problem and calculate initial beacon coordinates

Step 6: Determine beacon adjustment parameters

Step 7: Determine final beacon coordinates and calibration parameters

End Calibration

Figure 5.6: Flow chart of the calibration process.

used from which observations to the vertices of the beacon will be made. Typically one of these will be an existing mine traverse point and the other two, line points to two nearby traverse points.

In Step 2, the raw direct and reverse telescope position\textsuperscript{2} or face left (FL) and face right (FR) observations or double face observation [62] to the beacon vertices are collected using a theodolite/total station set up at each of these traverse stations. During a face left observation to an object, the vertical circle of the instrument is on

\textsuperscript{2}Direct and reverse telescope position sometimes referred to as face left and face right refer to the relative position of the telescope to the vertical circle as one is doing observations. If the vertical circle is on the left, then this position is called face left or the direct telescope position and observations are collected in the anticlockwise direction. Face right is the opposite of face left.
5.3. BEACON CALIBRATION AND POSITIONING

the left hand side of the observer, likewise face right. The observation cycle for each face begins with an observation to a reference station and cycles through all beacon vertex points and ends at the same reference station. With each vertex observation there is an associated horizontal circle and vertical circle direction reading using the theodolite. The typical observation process has face left readings observed in the anticlockwise direction and the face right readings observed in the clockwise direction and in the reverse order. The reason for this is the assumption that error increments the further away an observation is from the starting position. The telescope is transited and the same process is repeated for the second face. The mean result of double face observation eliminates any systematic errors due to imperfection of the horizontal and vertical circle scales [4, 62].

In Step 3, the raw data is automatically adjusted to provide the angular value inputs for each beacon vertex in the horizontal and vertical planes from each instrument set up position. In Step 4, it is determined which beacon is to be calibrated first.

In Step 5, the method of intersection is used to determine the beacon vertex coordinates. As this is a 3 dimensional problem with redundant information collected, least squares methods are employed.

In Step 6 the beacon adjustment properties tilt and rotation are determined, and in Step 7, the final beacon parameters and coordinates are determined.

Pseudocode Algorithm 5.3.1 summarizes the automated beacon calibration and positioning algorithm.
Algorithm 5.3.1: BCNCalbPostn(FL/FR)

comment: Obtain \( XYZ_{Unadjusted} \)

comment: Obtain Calibration Parameters \( \theta_1, \theta_2, P_1, P_2, P_3 \)

for \( i \leftarrow 1 \) to 2

while \( i \)

\[
\begin{align*}
&\text{AdjustTheodData(FL/FR)} \\
&\text{Determine horizontal/vertical angles(HA/VA)} \\
&\text{return (HA/VA)}
\end{align*}
\]

for \( j \leftarrow 1 \) to 8

3D Intersection(HA/VA)

Position Statistics(XYZ)

return \( (XYZ_{Unadjusted}) \)

for \( j \leftarrow 1 \) to 2

while \( j \)

Beacon Coordinate Adjustment\((XYZ_{Unadjusted})\)

return \( (Tilt/Rotation) \)

if Tilt and Rotation within tolerance

do \[
\begin{align*}
&\text{then } \\
&\begin{align*}
&\text{Beacon Coordinates}(XYZ) \\
&\text{Beacon Parameters}(XYZ)
\end{align*}
\end{align*}
\]

return \( (XYZ_{Adjusted}, \theta_1, \theta_2, P_1, P_2, P_3) \)

else

\[
\begin{align*}
&\text{Adjust Beacon Mounting} \\
&\text{Re-observe Beacon}
\end{align*}
\]

return \( (XYZ_{Adjusted}, \theta_1, \theta_2, P_1, P_2, P_3) \)
5.3.2 Data Collection and Adjustment

Data Collection

In Step 2 a theodolite that measures to 1” (arcsecond) or 5” (arcsecond) can be used to collect data (see Figure 5.7). In this case the former was used. The horizontal and vertical face left and right data for each station setup are compiled into files that are an input into an automated data adjustment algorithm. Care is taken to ensure minimization of systematic and human errors during observations through appropriate observation methods. This data is adjusted according to the data adjustment process outlined below to give the most likely face left/face right observations for each beacon vertex. This step is amenable to use of a data logging system enabling instantaneous detection of observer errors thereby allowing for immediate minimization/correction through re-observation.

Data Adjustment

The automated data adjustment algorithm consists of four steps, namely FL/FR misclosure compensation, FL/FR difference compensation, averaging FL/FR readings and correcting data to reflect original start value. Each control point setup generates an independent observation file. For each file, the input horizontal and vertical FL/FR readings are adjusted to obtain a final adjusted FL dataset. This dataset is used to determine the required horizontal and vertical angles that are input into the least squares intersection algorithm.

Pseudocode Algorithm 5.3.2 summarizes the adjustment process for the three station setups required for the process.
Figure 5.7: Total Station targeting a beacon vertex - collecting horizontal and vertical directional data of beacon vertices
Algorithm 5.3.2: \texttt{AdjustTheodData}(FL/FR)

\begin{algorithm}
\textbf{for} \( i \leftarrow 1 \text{ to } 3 \) \\
\hspace{1em}\textbf{do} Compensate FL/FR for misclosure \\
\hspace{1em}\textbf{do} Compensate for 180° FL/FR difference \\
\hspace{1em}\textbf{do} Mean FL/FR \\
\hspace{1em}\textbf{do} Correct to reflect FL start value \\
\textbf{return} (\textit{FL_{Adjusted}})
\end{algorithm}

5.3.3 3D Intersection Geometry and Solving the Intersection Problem

Determining the Initial Approximation Coordinates of a Beacon Vertex

The initial step to solving the intersection problem is determining each vertex initial coordinate approximation. As only angular data is collected, the minimum number of observations required to obtain initial coordinate approximation of \( A \) in Figure 5.8 is 3 (i.e. two horizontal and one vertical angles).

Figure 5.8 shows a schematic of the 3D intersection problem. Let \( \alpha_1 \) and \( \alpha_2 \) be horizontal angles to position \( A \) measured from baseline \( BC \) at positions \( B \) and \( C \) respectively. Let \( \beta_1 \) and \( \beta_2 \) be angles measured in the vertical plane to position \( A \) from the horizontal plane at \( B \) and \( C \) respectively. Lets define also the horizontal distances from \( B \) and \( C \) to \( A \) as \( b_A \) and \( c_A \) respectively and the baseline distance from \( B \) to \( C \) as \( bc \).

Therefore the initial approximate coordinates of \( A \) by method of intersection by base solution [4] are:
Geometry of the 3D Intersection Problem

The MAST beacon parameters and coordinates are determined from direction observations to eight panel intersection points. Each of the panel intersection points or vertices is observed from three control points/station set-up positions $D$, $C$ and $B$. Figure 5.9 illustrates the geometry of this intersection problem with the directions observed to one beacon vertex on each beacon. The arrows indicate the traverse points $A$, $B$, and $C$. The equations for the intersection points are:

\[
Y_{A_{init}} = \frac{(X_C - X_B) + Y_B \cot \alpha_2 + Y_C \cot \alpha_1}{\cot \alpha_1 + \cot \alpha_2} \tag{5.6}
\]

\[
X_{A_{init}} = \frac{(Y_B - Y_C) + X_B \cot \alpha_2 + X_C \cot \alpha_1}{\cot \alpha_1 + \cot \alpha_2} \tag{5.7}
\]

\[
Z_{A_{init}} = Z_B + \frac{bc \sin(\alpha_1) \tan \beta_1}{\sin(\alpha_1 + \alpha_2)} \tag{5.8}
\]
Angles $\alpha_1$ through $\alpha_4$ are angles measured in the horizontal plane from baseline $DC$ and $CB$ respectively to position $A$. The subscripts $L$ and $R$ indicate a left or right hand side beacon angle. Angles $\beta_1$ through $\beta_3$ are angles measured in the vertical plane from the horizontal line respectively to position $A$. These angles are an input into Pseudocode Algorithm 5.3.3 on page 115.

**Derivation, Formulation and Linearization of the Azimuth Condition Equations**

For each intersection calculation as shown in Figure 5.9, the model, $n$, requires observation of 4 horizontal and 3 vertical angles, a total of 7 observations. The minimum number of observations, $n_0$, required to solve the model is 3. Therefore a redundancy, $r = n - n_0$, equivalent to 4 exists. In accordance with the Least Squares Adjustment of Indirect Observations method [31], $n$ condition equations are written. The condition equations include both parameters and observations. Each condition equation contains only one observation, a unit coefficient and is written in terms of the point coordinates.

In surveying, point coordinates are associated with a defined coordinate reference frame. The azimuth of a line connecting two sets of point coordinates is defined as the clockwise angle measured from a north-south meridian passing through one of these points to the line connecting the two sets of point coordinates. Therefore in developing the angle condition, angles are also defined in the clockwise direction.

As the output of the field observation calculations are angles, taking $\alpha_1$ in Figure 5.10 as an example, lets define lines $BA$ and $BC$ as the directions from $B$ to $A$ and $B$
Figure 5.9: 3D Geometry of the intersection problem. (a) for vertex $A_R$ on right beacon and (b) for vertex $A_R$ on left beacon.
to C respectively. Therefore an adjusted horizontal angle \( \hat{\alpha}_1 \) at B as shown in Figure 5.10, is a clockwise angle measured from line BA to BC is:

\[
\hat{\alpha}_1 = \hat{\theta}_{BC} - \hat{\theta}_{BA}
\]  

where \( \hat{\theta}_{BC} \) and \( \hat{\theta}_{BA} \) are corrected azimuths. In terms of point coordinates, the horizontal angle condition is then written as:

\[
\hat{\alpha}_1 = \tan^{-1} \frac{X_C - X_B}{Y_C - Y_B} - \tan^{-1} \frac{X_C - X_A}{Y_C - Y_A}
\]

The generalized linearized form of this condition equation is written as:

\[
v_{\alpha_1} + b_1 \Delta X_B + b_2 \Delta Y_B + b_3 \Delta X_A + b_4 \Delta Y_A + b_5 \Delta X_C + b_6 \Delta Y_C = f_{\alpha_1}
\]  

where \( \Delta X_i \) represents the unknown parameter and \( b_i \) the numerical coefficient of the
unknown parameter. Therefore, $\hat{\alpha}_1$ written based on an observed value $\alpha_1$ is:

$$\hat{\alpha}_1 = \alpha_1 + v_{\alpha_1} \tag{5.12}$$

where $v_{\alpha_1}$ is the residual of $\alpha_1$. $f_{\alpha_1}$, the difference between the measured angle and its approximation is written as:

$$f_{\alpha_1} = \alpha_1^o - \alpha_1 \tag{5.13}$$

where $\alpha_1^o$ is the initial approximation of $\alpha_1$ based on:

$$\alpha_1^o = \tan^{-1} \frac{X_C - X_B^o}{Y_C^o - Y_B^o} - \tan^{-1} \frac{X_C^o - X_A^o}{Y_C^o - Y_A^o} \tag{5.14}$$

This generalized derivation assumes variable coordinates of $A$, $B$ and $C$, whereas, for the current problem, the only unknown is $A$. Therefore in this case since all triangles have as a known base, that is part of a traverse line (DC, CB), with two known endpoint coordinates, the equation above simplifies to:

$$\alpha_1^o = \tan^{-1} \frac{X_C - X_B}{Y_C - Y_B} - \tan^{-1} \frac{X_C^o - X_A^o}{Y_C^o - Y_A^o} \tag{5.15}$$

Substituting equation 5.15 into 5.13, it follows then that $f_{\alpha_1}$ is written as:

$$f_{\alpha_1} = \tan^{-1} \frac{X_C - X_B}{Y_C - Y_B} - \tan^{-1} \frac{X_C - X_A^o}{Y_C - Y_A^o} - \alpha_1 \tag{5.16}$$

In surveying, angles in the vertical plane are either referenced to the zenith or the horizontal plane. Zenith angles vary from $0^\circ$ to $360^\circ$, are always positive with $0^\circ$
pointing to zenith. Vertical angles referenced to the horizontal plane are considered positive above the horizontal plane and likewise, negative below the horizontal plane. The vertical angle condition equation process used here adopts the latter definition. Using angle $\beta_1$ in Figure 5.8 as an example its adjusted value $\hat{\beta}_1$ is:

$$\hat{\beta}_1 = \tan^{-1} \left( \frac{Z_A - Z_B}{\sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}} \right)$$ \hspace{1cm} (5.17)

The generalized linearized form of this condition equation can be written as:

$$v_{\beta_1} + b_1 \Delta X_B + b_2 \Delta Y_B + b_3 \Delta Z_B + b_4 \Delta X_A + b_5 \Delta Y_A + b_6 \Delta Z_A = f_{\beta_1}$$ \hspace{1cm} (5.18)

Therefore, $\hat{\beta}_1$ written based on an observed value $\beta_1$ is:

$$\hat{\beta}_1 = \beta_1 + v_{\beta_1}$$ \hspace{1cm} (5.19)

where $v_{\beta_1}$ is the residual of $\beta_1$. $f_{\beta_1}$ is written as:

$$f_{\beta_1} = \beta_1^o - \beta_1$$ \hspace{1cm} (5.20)

where $\beta_1^o$ is the initial approximation of $\beta_1$ based on:

$$\hat{\beta}_1^o = \tan^{-1} \left( \frac{Z_A^o - Z_B^o}{\sqrt{(X_A^o - X_B^o)^2 + (Y_A^o - Y_B^o)^2}} \right)$$ \hspace{1cm} (5.21)

As the only unknown is $A$, therefore the equation above simplifies to:

$$\hat{\beta}_1^o = \tan^{-1} \left( \frac{Z_A^o - Z_B^o}{\sqrt{(X_A^o - X_B^o)^2 + (Y_A^o - Y_B^o)^2}} \right)$$ \hspace{1cm} (5.22)
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It follows then that by substituting equation 5.22 into equation 5.20, \( f_{\beta_1} \) be written as follows:

\[
f_{\beta_1} = \tan^{-1} \frac{Z_A^o - Z_B}{\sqrt{(X_A^o - X_B)^2 + (Y_A^o - Y_B)^2}} - \beta_1 \tag{5.23}
\]

As there are seven observations, the condition equations are written as:

\[
F_1 = \alpha_1 - \tan^{-1} \frac{X_C - X_B}{Y_C - Y_B} - \tan^{-1} \frac{X_C - X_A^o}{Y_C - Y_A^o} = 0 \tag{5.24}
\]
\[
F_2 = \alpha_2 - \tan^{-1} \frac{X_C^o - X_C}{Y_C^o - Y_C} - \tan^{-1} \frac{X_B - X_C}{Y_B - Y_C} = 0
\]
\[
F_3 = \alpha_3 - \tan^{-1} \frac{X_D - X_C}{Y_D - Y_C} - \tan^{-1} \frac{X_A^o - X_C}{Y_A^o - Y_C} = 0
\]
\[
F_4 = \alpha_4 - \tan^{-1} \frac{X_A^o - X_D}{Y_A^o - Y_D} - \tan^{-1} \frac{X_C - X_D}{Y_C - Y_D} = 0
\]
\[
F_5 = \beta_1 - \tan^{-1} \frac{Z_A^o - Z_B}{\sqrt{(X_A^o - X_B)^2 + (Y_A^o - Y_B)^2}} = 0
\]
\[
F_6 = \beta_2 - \tan^{-1} \frac{Z_A^o - Z_C}{\sqrt{(X_A^o - X_C)^2 + (Y_A^o - Y_C)^2}} = 0
\]
\[
F_7 = \beta_3 - \tan^{-1} \frac{Z_A^o - Z_D}{\sqrt{(X_A^o - X_D)^2 + (Y_A^o - Y_D)^2}} = 0
\]

The condition equations are linearized with respect to the approximated coordinate position of \( A, (X_A^o, Y_A^o, Z_A^o) \) to the Least Square Adjustment of Indirect Observations matrix form:

\[
v + B\Delta = f \tag{5.25}
\]

where, \( v \) is a residual matrix, \( B \) the numerical coefficient matrix for the unknown parameters, \( \Delta \), the unknown parameter matrix, and \( f \), the right hand side numerical
constant term matrix. The components of the $B$ matrix are:

\[ b_{11} = \frac{\partial F_1}{\partial X_A} = \frac{Y_A^o - Y_B}{D_{AB}} \]
\[ b_{12} = \frac{\partial F_1}{\partial Y_A} = \frac{X_A^o - X_B}{D_{AB}} \]
\[ b_{13} = \frac{\partial F_1}{\partial Z_A} = 0 \quad (5.26) \]

\[ b_{21} = \frac{\partial F_2}{\partial X_A} = \frac{Y_A^o - Y_C}{D_{AC}} \]
\[ b_{22} = \frac{\partial F_2}{\partial Y_A} = \frac{X_A^o - X_C}{D_{AC}} \]
\[ b_{23} = \frac{\partial F_2}{\partial Z_A} = 0 \]

\[ b_{31} = \frac{\partial F_3}{\partial X_A} = \frac{Y_A^o - Y_C}{D_{AC}} \]
\[ b_{32} = \frac{\partial F_3}{\partial Y_A} = \frac{X_A^o - X_B}{D_{AC}} \]
\[ b_{33} = \frac{\partial F_3}{\partial Z_A} = 0 \]

\[ b_{41} = \frac{\partial F_4}{\partial X_A} = \frac{Y_A^o - Y_D}{D_{AD}} \]
\[ b_{42} = \frac{\partial F_4}{\partial Y_A} = \frac{X_A^o - X_D}{D_{AD}} \]
\[ b_{43} = \frac{\partial F_4}{\partial Z_A} = 0 \]

\[ b_{51} = \frac{\partial F_5}{\partial X_A} = \frac{(Z_A - Z_B)(2X_A - 2X_B)}{2((X_A - X_B)^2 + (Y_A - Y_B)^2)^{3/2}} \]
\[ b_{52} = \frac{\partial F_5}{\partial Y_A} = \frac{(Z_A - Z_B)(2Y_A - 2Y_B)}{2((X_A - X_B)^2 + (Y_A - Y_B)^2)^{3/2}} \]
\[ b_{53} = \frac{\partial F_5}{\partial Z_A} = \frac{-1}{((X_A - X_B)^2 + (Y_A - Y_B)^2)^{1/2}} \]

\[ b_{61} = \frac{\partial F_6}{\partial X_A} = \frac{(Z_A - Z_C)(2X_A - 2X_C)}{2((X_A - X_C)^2 + (Y_A - Y_C)^2)^{3/2}} \]
\[ b_{62} = \frac{\partial F_6}{\partial Y_A} = \frac{(Z_A - Z_C)(2Y_A - 2Y_C)}{2((X_A - X_C)^2 + (Y_A - Y_C)^2)^{3/2}} \]
\[ b_{63} = \frac{\partial F_6}{\partial Z_A} = \frac{-1}{((X_A - X_C)^2 + (Y_A - Y_C)^2)^{1/2}} \]

\[ b_{71} = \frac{\partial F_7}{\partial X_A} = \frac{(Z_A - Z_D)(2X_A - 2X_D)}{2((X_A - X_D)^2 + (Y_A - Y_D)^2)^{3/2}} \]
\[ b_{72} = \frac{\partial F_7}{\partial Y_A} = \frac{(Z_A - Z_D)(2Y_A - 2Y_D)}{2((X_A - X_D)^2 + (Y_A - Y_D)^2)^{3/2}} \]
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\[ b_{73} = \frac{\partial F_7}{\partial Z_A} = \frac{-1}{(X_A - X_D)^2 + (Y_A - Y_D)^2 + (Z_A - Z_D)^2} \]

where:

\[ D_{AB}^o = (X_A^o - X_B)^2 + (Y_A^o - Y_B)^2 \quad (5.27) \]
\[ D_{AC}^o = (X_A^o - X_C)^2 + (Y_A^o - Y_C)^2 \]
\[ D_{AD}^o = (X_A^o - X_D)^2 + (Y_A^o - Y_D)^2 \]

represent the updated distances from the traverse points to point A.

The components of the right hand side numerical constant matrix \( f \), are:

\[ f_1 = -\alpha_1 + \tan^{-1} \frac{X_C - X_B}{Y_C - Y_B} + \tan^{-1} \frac{X_C - X_A^o}{Y_C - Y_A^o} \quad (5.28) \]
\[ f_2 = -\alpha_2 + \tan^{-1} \frac{X_A^o - X_C}{Y_A^o - Y_C} + \tan^{-1} \frac{X_B - X_C}{Y_B - Y_C} \quad (5.29) \]
\[ f_3 = -\alpha_3 + \tan^{-1} \frac{X_D - X_C}{Y_D - Y_C} + \tan^{-1} \frac{X_A^o - X_C}{Y_A^o - Y_C} \quad (5.30) \]
\[ f_4 = -\alpha_4 + \tan^{-1} \frac{X_A^o - X_D}{Y_A^o - Y_D} + \tan^{-1} \frac{X_C - X_D}{Y_C - Y_D} \quad (5.31) \]
\[ f_5 = -\beta_1 + \tan^{-1} \frac{Z_A^o - Z_B}{\sqrt{(X_A^o - X_B)^2 + (Y_A^o - Y_B)^2}} \quad (5.32) \]
\[ f_6 = -\beta_2 + \tan^{-1} \frac{Z_A^o - Z_C}{\sqrt{(X_A^o - X_C)^2 + (Y_A^o - Y_C)^2}} \quad (5.33) \]
\[ f_7 = -\beta_3 + \tan^{-1} \frac{Z_A^o - Z_D}{\sqrt{(X_A^o - X_D)^2 + (Y_A^o - Y_D)^2}} \quad (5.34) \]

The Least Squares Adjustment of Indirect Observations solution is outlined in Appendix A and the calculation of relevant position statistics is outlined in Appendix C. Pseudocode Algorithm 5.3.3 below summarizes the 3D intersection solution.
5.3. BEACON CALIBRATION AND POSITIONING

Algorithm 5.3.3: 3DINTERSECT(HA/VA)

for $i \leftarrow 1$ to 8
    do Calculate Initial Approximations($XYZ_{init}$)

while $\delta' \delta > \text{threshold}$
    do
        Update measured distances from traverse points to A
        Populate unknown parameter numerical coeff. matrix - B
        Populate right hand side numerical constant matrix - f
        Calculate normal equations coefficient matrix - N
        Calculate normal equations constant term vector - t
        Calculate correction factor to initial approximations - $\delta$
        Adjust initial approximations - $XYZ_{init} = XYZ_{init} + \delta$
    return ($XYZ_{final}$)

Position Statistics($XYZ_{final}$)

return ($XYX_{final}$, $XYZ_{statistics}$)

5.3.4 Determining Beacon Adjustment Parameters (Tilt and Rotation)

Mounting and levelling the beacon onto the wall may not necessarily ensure the vertical edges are vertical and the horizontal edges horizontal. Therefore the mounting mechanism should allow for two degrees of motion to the beacon, namely, the ability to tilt forward and backward and also rotate about the pinning axis. The tilt action $\beta$, aligns $P_2$ plane to the vertical plane, however, the vertical edges might not be vertical. The rotation action $\alpha$, aligns the vertical edges to zenith and the horizontal edges to the horizontal plane. Figure 5.5 shows a schematic of the mounting mechanism and illustrates the tilt and rotation adjustment.
\( P_{2C} \) is defined as the centroid of an almost vertical plane \( P_2 \) and \( \vec{U}_Z \), a zenith vector emanating from the centroid. Let also \( \vec{a} \) and \( \vec{b} \) be two vectors emanating from \( P_{2C} \) on plane \( P_2 \) to the upper corners of \( P_2 \). These are illustrated in Figure 5.11.

The normal to the plane of panel \( P_2 \) at its centroid is determined by averaging the normals using different combinations of the corner coordinates of the panel. A vector \( \vec{c} \), from \( P_{2C} \) that is orthogonal to the other two vectors \( \vec{a} \) and \( \vec{b} \) is found from:

\[
\vec{c} = \vec{a} \times \vec{b}
\]

A negative sign in the \( Z \) component of \( \vec{c} \) denotes a positive/forward tilt and likewise, a positive sign in the \( Z \) component of \( \vec{c} \) denotes a negative/backward tilt.

A matrix \( A \) can be defined as containing vectors \( \vec{a} \) and \( \vec{b} \).

\[
A = \begin{bmatrix}
a_x & b_x \\
a_y & b_y \\
a_z & b_z
\end{bmatrix}
\]

As the vertical axes of the coordinate system points toward zenith, the tilt angle \( \beta \) is defined as the projection of the vertical axis onto plane \( P_2 \). The projection of \( U_Z \) onto the plane defined by vectors \( \vec{a} \) and \( \vec{b} \), \( P_Z \) is:

\[
P_Z = A(A^T A)^{-1} A^T U_Z
\]
Figure 5.11: Beacon tilt and rotation adjustment
The tilt angle $\beta$ is found from:

$$\beta = \cos^{-1} \frac{P_Z \cdot U_Z}{|P_Z||U_Z|}$$

(5.38)

The beacon rotation $\alpha$, is a correction that follows the tilt adjustment. Let vector $\vec{ab}$ denote a vector on the plane $P_2$ and aligned to the near-vertical edge. The beacon rotation angle is defined between the zenith direction vector $\vec{Z}$ and $\vec{ab}$.

The rotation angle parameter $\alpha$ is found from:

$$\alpha = \cos^{-1} \frac{\vec{ab}^T \cdot \vec{Z}}{|\vec{ab}||\vec{Z}|}$$

(5.39)

A rotation to the left is characterized by a positive $x$ component value in the vector $\vec{ab}$, and likewise, a rotation to the right is characterized by a negative $x$ component value in the vector $\vec{ab}$.

5.3.5 Determining Beacon Calibration Parameters and Beacon Coordinates

Beacon Coordinates

Each vertical edge of the beacon is characterized by a single $XY$ coordinate pair. Taking plane $P_2$ as an example, the top and bottom vertices of a vertical edge share the same $XY$ coordinate but different $Z$ elevation coordinate. Likewise, the bottom two vertices of $P_2$ share the same $Z$ elevation but different $XY$ coordinates. Therefore, if $\beta$ and $\alpha$ are sufficiently small, respective $XY$ and $Z$ coordinates should be averaged to conform to the single $XY$ coordinate pair for a vertical edge and likewise same $Z$ coordinate for horizontal edges.
Beacon Calibration Parameters

The beacon calibration parameters are calculated from the adjusted coordinates of the beacon in both the horizontal and vertical directions. The expectation is that very similar results will be obtained for the two sections as they are symmetric.

Let vector \( \vec{f} \) comprise a vector connecting the mean coordinates of panel \( P_2 \). Let vector \( \vec{g} \) be the extension of this line to and orthogonal to the outer edge of either panel \( P_1 \) or \( P_3 \). The panel deflection angles \( \theta_1 \) and \( \theta_2 \) are found from:

\[
\theta = \cos^{-1} \left( \frac{\vec{f}^T \cdot \vec{g}}{|\vec{f}| |\vec{g}|} \right)
\]  

(5.40)

The lengths of the panels are calculated along the same sections. The calibration values are subsequently used in the MAST algorithms.

Issues with Beacon Calibration

Whilst the calibration process produces accurate coordinate data, there is need for global adjustment of all the observed data at the same time instituting constraints to maintain the design geometry. Some of these constraints are keeping the outer and inner beacon planes parallel as well as ensuring the vertical and horizontal edges on one plane parallel to counterparts on the other plane. Including this is bound to result in an improved and more realistic parameter value or minimize errors in the calculated tilt and rotation parameters.
5.4 Simulated Data Collection

In [38] it was shown how the MAST process operated to recover the known scanner coordinate using perturbed simulated data. In this section elaboration on the purpose and mechanics of the simulation process; i.e., how the original data which includes the wall positions and wall orientations, the original beacon positions, and scanner positions and orientations, were determined.

5.4.1 Simulator Architecture

5.4.2 Variables

Variables that constitute input into the simulator environment include those which define the beacon structure or configuration (panel lengths and angles between panels), and those which define the relationship between two beacons deployed in an underground tunnel environment (distance between two beacons, the wall direction/azimuth at the point where each beacon is mounted and the angular orientation or normal direction/azimuth of one beacon to another).

Beacon Structure

The horizontal cross-section of a beacon for 2D/3D positioning has three planar panels $P_{1:3}$, characterized by different lengths which are held at a constant ratio $R_t$ (e.g., $R_t = [1 3 2]$). It is assumed that the centre panel of the beacon is square and all other trapezoids. The centre panel is mounted such that two of its sides are oriented parallel to the horizontal plane and the other two, parallel to the vertical line or line of gravity.
The inclination angle $\theta_1$, of the first panel to the middle or second panel is regulated between some minimum and maximum value using a small but constant increment. The angle of inclination of the third panel $\theta_2$, to the second, is calculated based on a range of proposed beacon depths $h$. The edge of two vertical intersection panels is a line of constant $x$-$y$ coordinates. Beacon parameters are calculated according to the formulae listed below.

\[
P_1 = \frac{h}{\sin(\theta_1)} \tag{5.41}
\]

\[
P_{[2,3]} = R_{[3,2]} \ast P_1 \tag{5.42}
\]

\[
\theta_2 = \sin^{-1}(h/P_2) \tag{5.43}
\]

\[
l = P_1 \cos(\theta_1) + P_2 \cos(\theta_2) + P_3 \tag{5.44}
\]

$l$ defines the beacon length and a subset of beacon parameters is defined as $[P_1, P_2, P_3, \theta_1, \theta_2, l]$ values concatenation to form a matrix parameters. These parameters are used in the next section.

Beacons are mounted on the walls such that in whatever direction of travel the beacons a scanned using a mobile scanner, the point cloud beacon profile or beacon horizontal cross-section of one beacon on one wall is a mirror image of another mounted on an opposite or adjacent wall.

**Scanner Data Generating Algorithm**

The data collection simulator is an algorithm that simulates MAST beacons in an underground environment and randomly places a scanner within vicinity to scan these beacons and generate a point cloud. As this is a simulation, the random location of the scanner is known as well as its orientation in the mine reference frame. From
the derived point cloud, data coordinates and distance data is calculated to feature positions at known azimuths in the scanner reference frame. The distance data to features in the environment and the scanner field of view directions are the input into the MAST algorithm. Within the MAST process it is assumed the angular resolution of the scanner or direction to features is stable, however, the distance input is perturbed before the arbitrary point cloud coordinates are calculated. The scanner positions and orientations calculated from the scanner generating algorithm are assumed errorless. The goal of the MAST process is to evaluate whether it is possible to recover these calculated positions and orientations and the uncertainty associated with the recovery process.

Algorithm 5.4.1 outlines the steps for the data collection process. The data generating algorithm starts by determining all possible beacon parameter subsets as outlined in the previous section. For the simulator, beacon and scanner positions and orientations are known a-priori in the mine reference system. Each vertical beacon edge has a defined \((x-y)\) coordinate in the mine reference system.

In this simulated scenario, observations are made to the environment and also to each of these beacon positions from a laser scanner placed at a known position, with a known orientation. The known orientation here is defined as the azimuth defined by the middle observation of the laser scanner field of view, where the middle observation symmetrically divides the observations either side of the direction of travel of the mobile platform onto which the scanner is mounted.

Each set of the beacon parameters is used at a defined or known scanner position with a range of known distances between scanner to beacon and also known angular orientation between beacons. At each defined position there is also a known and
associated platform direction/azimuth all of which should be recovered through the MAST algorithm.

**Algorithm 5.4.1: (ScanDataCollection)**

**comment:** Initialize algorithm inputs

\[
\begin{align*}
\text{Wall 1 coordinates} \\
\text{Beacon min./max separation distance} \\
\text{Beacon azimuth orientation range, deflection rate} \\
\text{Laser scanner parameters: FOV, max. dist., dist. accuracy}
\end{align*}
\]

for \( i \leftarrow 1 \) to max. beacon separation

for \( j \leftarrow \text{min} \) to max. beacon orientation

    Determine Wall 2 alignment

    Determine beacon positions

    Determine trajectory between beacon positions

    Determine beacon coordinates

for \( k \leftarrow 1\text{st} \) to last scanner position

    Determine scanner position/orientation along trajectory

    Collect scan data - distance, scan angle

return

\[
\begin{align*}
\text{Scan data} \\
\text{XYZ}_{\text{Scanner}}, \text{Azimuth}_{\text{Scanner}} \\
\text{Beacon parameters, Beacon coordinates} \\
\text{Wall coordinates, directions}
\end{align*}
\]
5.4.3 Generating Simulated Laser Scanner Data

The laser scanner mounted as indicated in Figure 5.12 scans in an anticlockwise direction at an angular resolution of 0.25° - 1°. The field of view of the SICK LMS range of scanners is 270° and is symmetric about the platform direction. The platform direction signified by the X axis in Figure 5.12 is defined as the midway direction in the field of view of the scanner. In generating the scan data, the midway direction or platform direction has a known azimuth and hence the azimuths throughout the field of view are known.

![Scanner orientation for obtaining near to horizontal scanner data.](image)
Simulated Data Collection

For simplicity, it is assumed that the wall in the vicinity of each beacon has the same direction as the middle beacon panel and is continuous on both sides past the ends of the beacon. Azimuths are calculated from the scanner position to each known vertical beacon edge. Given the known azimuths of the walls, beacon planes, intersection locations between the wall and beacon panels as well as between beacon panels, azimuths to the environment as defined from the scanner position, the underground environment coordinates can then be determined by intersection method. The coordinates of a point of interest \((X_3, Y_3)\), are determined from the following condition equations:

\[
x_1 + R_1 \sin(Az_1) = X_3 = x_2 + R_2 \sin(Az_2)
\]

\[
y_1 + R_1 \cos(Az_1) = Y_3 = y_2 + R_2 \cos(Az_2)
\]

where \((X_3, Y_3)\) is the point to be determined from known positions \((x_1, y_1)\) using azimuth \(Az_1\) and \((x_2, y_2)\) using azimuth \(Az_2\). Here \((x_1, y_1)\) is the scanner location and \((x_2, y_2)\), the known feature. \(\hat{R}_1\) and \(\hat{R}_2\) are distance quantities from the scanner and the known feature respectively, to be determined to enable solving either half of the above equations. A least squares solution to determine \(\hat{R} = [\hat{R}_1 \hat{R}_2]^T\) is formulated as follows:

\[
A = \begin{bmatrix}
-\sin(Az_1) & \sin(Az_2) \\
-\cos(Az_1) & \cos(Az_2)
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
x_1 - x_2 \\
y_1 - y_2
\end{bmatrix}
\]

\[
A \hat{R} = B
\]
and

\[
\hat{R} = (A^T A)^{-1} A^T B \tag{5.49}
\]

\[
\begin{bmatrix}
X_3 \\
Y_3
\end{bmatrix} = \begin{bmatrix} x_1 \\
y_1 \end{bmatrix} + \hat{R}_1 \begin{bmatrix} \sin(Az_1) \\
\cos(Az_1) \end{bmatrix} \tag{5.50}
\]

As the azimuths from the scanner position \((x_1, y_1)\) are advanced in the anticlockwise direction or the azimuth values decreasing, based on prior knowledge of the azimuth from the scanner position to the critical intersections between beacon and terrain and beacon panels, the position from which subsequent intersections occur, \((x_2, y_2)\), is also advanced and the appropriate intersection azimuths used to determine \((X_3, Y_3)\).

Where the rank of matrix \(A\) is less than 2 or its condition number less that 0.0005 or any component result of \(\hat{R}\) greater that the maximum measurable distance by the laser scanner or less than 0, the default values for \(\hat{R}\) are the maximum measurable distance by the laser scanner. The use of the parameters of the second beacon is toggled when the azimuth from the scanner equals that of the tunnel direction calculated as the mean of the two walls onto which the beacons are mounted.

The set of calculated \(\hat{R}_1\) distances are compiled and represent those which a laser scanner under normal operating procedure in a surveying or mapping exercise would provide. This data is used in the MAST algorithm described in the next section to recover the original scanner \((x-y)\) coordinate position and platform orientation which were the original basis for the initial data simulation. In addition, to each line of scanner data are augmented associated parameter data that include beacon coordinates, wall directions, scanner coordinates and platform direction, beacon parameters, distance and angular orientation between the two beacons. As these vary with each
scan line, these could be used as a basis for grouping data for analysis based on the different characteristics of the parameters.

Figure 5.13 illustrates on the top sub-plot, the walls (red line) and wall directions upon which the beacons (black star) are mounted, and some of the scanner positions (black dots). The bottom sub-plot shows data collected as a result of intersecting azimuths from the scanner position and those from select beacon/wall or beacon panel intersections. The bottom sub-plot of Figure 5.14 illustrates a blow-up of the laser scanner position (intersection of all red lines and the blue line) and one of the beacons. The beacon reference points are also shown as points 1 through 4.

5.4.4 Relating Beacon size to Tunnel Width

In this thesis the relationship between beacon size to tunnel width was not fully developed. A simulation similar to that which was developed in this chapter can be used to develop the minimum beacon sizes required for use with a LiDAR system in tunnels of a particular width.

5.4.5 Azimuth-Azimuth Intersection

The basis for simulated data collection is the survey method of azimuth-azimuth intersection. The features that the laser scanner measures are the wall positions and the beacons.

In creating the simulated data, the initial assumption made is that all coordinates for the beacons, walls and scanner position and orientations are known. For every scanner position and orientation, its field of view of the environment is unique. Therefore the resultant point cloud will be unique.
Figure 5.13: Scanner data generation outputs.
Figure 5.14: Laser scanner data outline of a beacon.
Given the known scanner position and orientation in the world frame, its field of view is known in the same system. Also, given the known positions of the features in the environment, i.e. the position and orientation of the walls and the attached beacon, and the position of a cloud point on any feature is found by intersecting a ray of known azimuth from the laser scanner and a ray along the plane of the feature. The azimuth-azimuth intersection problem is discussed in Section 5.3.3 and is also outlined in [4], [70] and [31] and possibly other sources in surveying.

5.5 MAST Positioning Algorithm

A laser scanner placed in proximity of two mounted beacons is used to scan and collect data about the underground environment including the beacons. In this thesis, the focus is obtaining the 2D horizontal $XY$ coordinates and orientation of the laser scanner in the mine reference frame.

The automatic scanner positioning process begins by calculating the coordinates of the scan in the arbitrary scanner reference frame. This is followed by segmenting the point cloud into connected linear features that equate to possible panels of the beacon, $P_1$, $P_2$ and $P_3$, that are also connected at pre-defined angles $\theta_1$ and $\theta_2$ to each other. This scenario is illustrated in the Top View diagram of Figure 5.1(a). Each panel is identified by a start and end point number as illustrated in Figure 5.17(b).

Identification of the start and end point numbers of the linear features from the segmented point cloud enables the arbitrary scanner frame to be approximately aligned or associated with the mine reference framework through use of a four parameter transformation. The point numbers relating to each segmented beacon point cloud cluster in the arbitrary scanner reference frame are related to the corresponding
beacon coordinate number in the mine reference frame hence each segmented beacon coordinate \(XY\) pair can be related to its corresponding beacon coordinate \(XY\) pair in the mine reference system. Following the four parameter transformation, a point to point iterative closest point algorithm (ICP) refines this alignment of the scanner point cloud to a point cloud representation of the beacon in the mine reference system. Once the alignment is complete, the coordinates and azimuth orientation of the scanner in the mine reference system are determined. Figure 5.15 illustrates the flow for the MAST positioning process.

![Flowchart of MAST positioning process](image)

Figure 5.15: Flowchart of MAST positioning process

### 5.5.1 Segmentation

The segmentation process comprises two algorithms. The primary algorithm does a simple segmentation of the point cloud into lines using the incremental method described in [53] and a table is compiled of line segment start and end point numbers, coordinates, lengths and line directions.
As the iterative line segment seed cluster growing proceeds using the incremental method, fitting of the line to the data is accomplished using Principal Component Analysis (PCA) at the same time observing the sum of squared residuals do not exceed defined thresholds. In this case two thresholds set for the first and last point of the line are 0.010 m and 0.015 m respectively. When these thresholds are exceeded, the current or last point included into the seed cluster is deemed to belong to the next line.

Principal Component Analysis (PCA) is a quantitative rigorous method for simplifying data. PCA enables easy description of the variance of multi-dimensional data sets. The principal components are a set of new variables that are linear combinations of the original variables and are orthogonal to each other. The first principal component axis is a single axis in space that when observations are projected onto this axis maximize the variance of the new variable that is generated. Likewise, the second principal component is an axis orthogonal to the first and observations that are projected onto this axis generate a new variable of maximum variance.

Consider a dataset $P_{(x,y)} = \{p_1, p_2, p_3, ..., p_n\}$, $p \in \mathbb{R}$ whose mean is $\bar{p}$. The variance of $P$ is obtained from:

$$\sigma^2_{P_x} = \frac{\sum_{i=1}^{n}(p_{x_i} - \bar{p}_x)^2}{n-1}$$  \hspace{1cm} (5.51)

$$\sigma^2_{P_y} = \frac{\sum_{i=1}^{n}(p_{y_i} - \bar{p}_y)^2}{n-1}$$

The covariance of $S$ is found from:

$$\sigma^2_{P_{xy}} = \frac{\sum_{i=1}^{n}(p_{x_i} - \bar{p}_x)(p_{y_i} - \bar{p}_y)}{n-1}$$  \hspace{1cm} (5.52)
The principal component or fitted line to the data lies in the direction of the eigenvector corresponding to the largest eigenvalue of $\sigma_{P_{xy}}^2$. The length of the line is determined as the distance between the extreme points found from projecting the original data points onto the fitted line.

A secondary algorithm, owing to the random nature of data, uses beacon design criteria to extract the different panels from the data. Three or more consecutive line segments are evaluated at a time based on closeness of values to design parameters, panel deflection angles, panel distances and the beacon length.

When three or more line segments are evaluated, tests are made to determine the length of beacon, if two or more line segments belong together and whether parameters fall within threshold limits. Where three line segments are evaluated and they meet the required criteria, a final segmentation file is updated which will contain the start and end point number for each line, coordinates, lengths and line directions.

When more than three lines need to be analyzed together, the main criteria is that the sum of their lengths based on the first and last point coordinates in the point set do not exceed the length of the beacon $l$, within some threshold value. If $l$ is exceeded, an assumption is made that the set of lines do not comprise or denote the beacon location. The first line in the set is discarded and the next three are cumulated and analyzed together as before.

Where more than three line are analyzed together and the beacon length is within limits, the midway point in the point set is taken as a seed point. Knowing the limits or end points of the dataset, line fitting is done in reverse by obtaining a seed point midway the dataset. Typically this would fall on the large middle panel $P_2$, of the beacon. Region growing is now applied in both directions of the seed point, dividing
the dataset into three panel portions. To each of these portions, a line is fitted, its length determined and the deflection angles of the lines to each other determined (see Figure 5.16). Comparisons are made to design data and if these compare within a given threshold, the final segmentation file is updated with the required information.

In testing the calculated lengths of $P_1$ and $P_2$ as well as the deflection angle between them $\theta$ against their design or calibrated values during the growing process, if the calculated length $P_1$ and the angle $\theta$ fall within a threshold value of the design or calibrated value, the growing process in that direction is terminated. Alternately, if the length $P_2$ reaches its threshold distance, the growing process in both directions is terminated and the segmentation process stopped. The reason for the thresholding is that with no guarantee that a data point will fall on the intersection or edge of a panel, the likelihood is that all calculated panel distances will be shorter than their design/calibration values.

The points corresponding to the beacon panel limits based on the segmented lines identified corresponding to the beacon coordinates become the input by which correspondence of the beacon in the arbitrary reference frame of the scanner is related to the beacon coordinate points in the mine reference system.

**Increasing Efficiency of Segmentation**

Where there is much data to work through, it becomes necessary to eliminate the segmentation process in areas where there is likely to be no beacon present. When the beacons are set up, their relative proximity or position to a scanner mounted on a moving platform is a guide to where they can be found on a scan of the environment. The ability to dictate within an algorithm where and where not to look makes it
possible to improve on the efficiency of the segmentation process. This is achieved by dividing the scanner field of view into search zones knowing the scanner resolution and the field of view size.
Figure 5.17: (a) Line segmentation of point cloud and (b) Final segmented beacon points in scanner reference frame.
The segmentation process starts by identifying search zones in operation for each beacon station according to Figure 5.18 and these search zones are illustrated overlayed onto the SICK LMS field of view. Knowing the beacon configuration as indicated in Figure 5.3, the algorithm search can then be limited to the zones defined for that beacon configuration. Figure 5.17(a) is an example of this in operation. In this case, Beacon Configuration 2 requires search in only Zones 1 and 3, hence the missing data in the figure.

![Diagram of Scanner zones for efficient segmentation](image)

**Figure 5.18: Scanner zones for efficient segmentation**

**Point Cloud Data Alignment to Beacons**

Typically, beacons are used in pairs with similar sides of a beacon pair facing the same direction. As indicated in Figure 5.19, the beacons are male and female. The coordinate pairs for each beacon edge or intersection ($XY_{1,R}$, $XY_{1,L}$, e.t.c.) are also
indicated with the $L$ and $R$ subscripts denoting the beacon location left or right of the direction of travel.

As two beacons are used to define the location of the laser scanner we link the laser scanner and the beacon coordinate pairs by the direction the scanner sweeps. Defining a beacon station as two, four-coordinate $XY$ pairs from each beacon stacked together to form a coordinate set, the travel direction is defined according to how they are stacked. The scanner scans in an anticlockwise direction, therefore if we align the stacking of the coordinate pairs with the scanner direction, taking as an example direction A of the mobile platform, the stacking follows the list indicated in the left column of Table 5.1.

![Figure 5.19: Association of travel direction to scan direction](image_url)

**Figure 5.19:** Association of travel direction to scan direction
The reverse direction B, is achieved by simply stacking the two, four-coordinate $XY$ pairs in the reverse order as indicated in the second column of the same table. To obtain the correct segmentation, in the correct direction, the segmented panel distances are compared to the known beacon panel distances based on the forward/reverse stacking order of the coordinate pairs.

### 5.5.2 4-Parameter Transformation

The four parameter similarity transformation is a two dimensional coordinate transformation that relates coordinates measured in two different systems together [4, 70, 31]. Typically, in a survey setting, coordinates may be obtained in a local or arbitrary reference system, $G_{Arb}$, then at a later stage transformed into a mine reference system, $G_{Mine}$. The parameters of this transformation include a scaling factor $u$, a rotation $\beta$, and two orthogonal translations, $T_x$ and $T_y$.

The scaling factor creates equal dimensions in the two coordinate systems. The rotation makes the axes of the two systems parallel and the translations create a common origin for the two systems. A minimum of two common or control points in
both systems are required to enable transforming a set of points in one system onto another.

Let $x_1$ and $x_2$ represent a coordinate set in an arbitrary reference system, $y_1$ and $y_2$ a coordinate set in the mine reference system. The transformation of coordinates in the arbitrary system to the mine reference system is as follows:

$$
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}_{\text{Mine}} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x_1 \\
x_2
\end{bmatrix}_{\text{Arb}} + \begin{bmatrix} c \\
d
\end{bmatrix}
$$

(5.53)

Here

$$
\begin{bmatrix} a & b \\ -b & a \end{bmatrix} = \begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix} \begin{bmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} c \\
d
\end{bmatrix} = \begin{bmatrix} T_x \\
T_y
\end{bmatrix}
$$

and $u$ is a uniform scale change, and $T_x$ and $T_y$ are translations in the $x$ and $y$ directions. $\beta$ is an unknown rotation angle given by:

$$
\beta = \tan^{-1} \left( \frac{b}{a} \right)
$$

(5.54)

An approximation of the orientation of $G_{\text{Arb}}$ to $G_{\text{Mine}}$ in the mine reference system assumes $u$ at unity and is obtained by

$$
G_{\text{Mine}} = \begin{bmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{bmatrix} G_{\text{Arb}} + \begin{bmatrix} T_x \\
T_y
\end{bmatrix}
$$

(5.55)

In this case, due to the detected beacon coordinate points in the arbitrary scanner system not in actual correspondence with the beacon coordinate points in the mine
reference system, the four parameter transformation brings the alignment of the arbitrary system to close alignment with the mine system and the arbitrary coordinates into the mine reference system requiring further alignment by the Iterative Closest Point (ICP) Transformation. Figure 5.20 illustrates results after application of the four-paremeter transformation. The true beacon positions are shown as red lines, the point cloud is shown as the dots in black and the beacon points from segmentation, the red star points with the associated point number.

Figure 5.20: Cloud Position after 4 Parameter Transformation also showing the beacon outline.
5.5.3 Iterative Closest Point (ICP) Transformation

A variation of the point to point ICP algorithm in [40] is used in this research. The point to point ICP algorithm takes as input the rotated and translated point cloud $q$, according to the four parameter transformation, the segmentation locations and a fine and equi-angular reference point cloud $p$, encompassing for each beacon, the line joining the beacon coordinates and arbitrarily calculated base on the centroid of the known beacon coordinates.

As indicated in the previous section, the segmentation algorithm approximates the possible locations of the beacon coordinate positions dividing each beacon into three panel segments and the four parameter transformation rotates and translates the data to close proximity of the beacon coordinates. The ICP process is a four stage process comprising matching, weighting, minimization of error metrics and transformation of minimized data. Figure 5.21 show the flow chart of the ICP process.

In the first stage each measured point in the scanned point cloud $q = [x, y, z]^T$, is matched with a corresponding point in the reference point cloud $p = [x, y, z]^T$ resident in a defined panel on the basis of shortest distance and orthogonality of point to the panel. A weighting factor $W_i$, for each measured point is generated based on the reciprocal of its distance $d_i$ to its corresponding reference point $w_i$.

\[
w_i = \frac{1}{d_i} \quad (5.56)
\]

\[
W_i = \frac{w_i}{\sum_{i=1}^{n} w} \quad (5.57)
\]

The point to point ICP algorithm is applied to match the measured point cloud to the reference point cloud. The flow chart in Figure 5.21 illustrates the different stages
of the ICP process. First the centroids \((\bar{q}, \bar{p})\) and deviation from centroids \((q_d, p_d)\) of the two point clouds are determined.

\[
\bar{p} = \frac{1}{n} \sum_{i=1}^{n} p * W
\]  
\[
(5.58)
\]

\[
\bar{q} = \frac{1}{n} \sum_{i=1}^{n} q * W
\]  
\[
(5.59)
\]

\[
p_d = p - \bar{p}
\]  
\[
(5.60)
\]

\[
q_d = q - \bar{q}
\]  
\[
(5.61)
\]

The data sets are centred at the origin and the covariance matrix \(H\) is computed from:

\[
H = \sum_{i=1}^{n} (p_d * W * q_d^T)
\]  
\[
(5.62)
\]

and the optimal rotation of the measured data onto the reference data is obtained.
using the MATLAB™ singular value decomposition \textit{svd} function from:

\[
[U, S, V] = \text{svd}(H) \tag{5.63}
\]

\[
R = V \ast U^T \tag{5.64}
\]

The translation of the measured data is calculated as:

\[
T = \overline{q} - \overline{p} \tag{5.65}
\]

At each iteration it is checked whether a minimization of the error metric has been achieved, if not, further iterations occur. The iteration stopping criteria is a minimum RMSE calculated from differences in the distance between individual points in the measured point cloud and the corresponding points in the reference point cloud. Figure 5.22 shows the aligned datasets after the ICP adjustment, in red, the line segmentation for each panel, in blue, the beacon outline and the black dots representing the point cloud.

5.5.4 2D Position and Orientation

At the beginning of the MAST process, the arbitrary scanner coordinates are computed. Note that the origin of the calculation is the scanner centre. Therefore by augmenting the origin coordinates to the dataset and manipulating this data accordingly using the MAST process automatically calculates the scanner coordinates. The orientation azimuth is then calculated from the line joining the scanner centre and the mid-scan point in the data set.
5.6. 2D MAPPING

As shown in Figure 4.4, the uGPS Rapid Mapper™ has two scanners, one vertical and the other horizontal. The vertical scanner is used to collect data that is used to determine the position of the scanner during the traversing process and the horizontal scanner for volumetric mapping.

After the position adjustment process, each horizontal scan location and orientation is known, therefore, using the original scan data, for each position, the feature
coordinates are calculated. As the scanner has a 270° field of view that is symmetric about the direction of travel of the moving platform, within the ILC, as the floors are horizontal in a direction orthogonal to the travel direction of the platform, a 2D map is compiled from coordinate data that is orthogonal to the direction of travel of the platform.

In an ideal mining situation where the ground is uneven a different approach is necessary as the current method used is likely to introduce error into the position of the wall features. This will require use of the horizontal scanner used for volumetric mapping.

The position of the horizontal scanner can be determined through calibration in relation to that of the vertical scanner. Based on this relationship a constant approximation of the mapping plane has to be determined throughout the course of the traverse by monitoring the ‘roll’ angle of the uGPS Rapid Mapper™ system. Based on the ‘roll’ angles, corresponding horizontal scanner point cloud coordinates within a given threshold of the mapping plane are averaged to obtain approximations of the wall feature positions.

Figure 5.23 illustrates this concept and shows a vertical scan section of a drift or tunnel and how 2D mapping with the horizontal laser scanner can be accomplished. The ovals indicate the location of the points averaged that are a projection onto the mapping plane.
5.7. Conclusion

In this chapter the MAST process was introduced and developed. MAST consists of hardware and algorithms that enable any LiDAR based scanner position and orientation to be determined in underground GPS deprived environments. The hardware includes a pair of geometric geo-referenced beacons and the algorithms, that which segment beacon laser scanner data from the environment point cloud, transformations that rotate and translate arbitrary beacon point cloud data to approximate the mine reference beacon position. This is followed by refinement of this beacon point cloud to its correct position using the ICP algorithm.

Also in this chapter was discussed the simulation process used to determine initial data input into the MAST algorithm. The simulation took as input known scanner...
position and orientation, known beacon and wall positions to determine distances and azimuths from the laser scanner to feature positions. From this information was only retained the distance data over the scanner field of view which is normal output for the SICK LMS111 laser scanner. This was the input data for the MAST algorithm.
Chapter 6

Field Experimental Procedures

6.1 Introduction

Conventional methods of underground surveying use tripod-based theodolites or total stations and 3D laser scanners to obtain information about the underground environment. Within underground mining environments, which are Global Positioning System (GPS) deprived, instantaneous positioning which is available for surface surveying with GPS is unavailable for underground surveying and mapping. Despite technological advancements in surveying instrumentation, current underground surveying procedures have remained slow, laborious and relatively unchanged leaving the status quo unchanged.

Recent innovations by the robotics community [73, 6, 66, 11, 10, 36] have shown that mobile mapping of underground mining tunnels can be accomplished using 2D and 3D laser scanners. However, surveying was never an intended application.

In order to advance positioning methods for surveying, it is necessary to adopt and use tools from the robotics community to automate the current surveying process. This chapter outlines experiments using the MAST beacons in conjunction
with the uGPS Rapid Mapper™. These experiments are designed to take the reader through the automated surveying and map development process ensuring quantifiable results throughout the whole process. These experiments include determining comparability/compatibility of calculated distances through displacement tests, viability of MAST beacons as a ‘closing the loop’ tool and creation of a method of aligning the mapping to a mine coordinate system. Other experiments include determining and evaluating the MAST positioning process as a means for positioning and orienting a mobile platform within a mine reference system.

In order to achieve this, a baseline survey using conventional methods was required to provide a comparison of results with automated methods.

6.2 Establishing Ground Truth

The process of surveying and mapping establishes from data collected and manipulated a model of ground truth. The most common method of starting this model uses the method of traversing to establish known control points from which the bulk of the detailed as-built survey is undertaken. Typically, closed traverses are established and these start from a traverse point of known coordinates in the mine reference system and end at another traverse station of known coordinates in the same reference system.

While conventional surveying and mapping methods might be slow and cumbersome [7, 30, 63, 73], the processes used ensure accurate positioning with quantifiable accuracy statistics. While there has been significant research within the robotics community pertaining to mapping of the underground environment, the author is yet to find research that unequivocally compares mobile mapping to conventional mapping.
by surveying methods with quantifiable mapping accuracy. Therefore establishing
ground truth serves two purposes: first to know the layout of the experiment site
and second, to enable understanding the limitations of newer surveying and mapping
systems.

6.2.1 Conventional Traverse Survey

The first floor of the Integrated Learning Centre (ILC) at Queen's University was used
to simulate the underground environment and to perform the traverse experiments.
For conventional traversing, poor visibility, short sights and cold conditions were ex-
perienced (as also the experiments were done during the winter) which is a somewhat
similar experience to underground surveying although the difficulties of setting up
the surveying instruments below a roof or at a sidewall station and the ruggedness
of the underground environment were not. The ILC environment is characterized by
flat featureless walls and glass panels and is illustrated in Figure 6.1. This is different
from the underground mining environment where the rough texture of the broken
rock or the smooth but undulating nature of shotcreted wall are features that can be
used by scan matching algorithms in the determination of a robots travelled distance.

Traversing is a popular method for establishing horizontal control in areas where
line of sight is short and where other methods of establishing control such as GPS,
resection, triangulation or trilateration fail. A traverse was established within the ILC
corridors as would occur in a typical mining scenario with traverse stations located
to maximize the amount of detail that could be observed from each traverse station
location. Typically in an underground environment the traverse stations are located
in the roof or sidewall of a tunnel. Figures 3.4 and 3.5 illustrate this. However, in this
Figure 6.1: ILC environment showing featureless straight and smooth walls.
6.2. ESTABLISHING GROUND TRUTH

In this case, all traverse stations were established on the floor rather than the roof or side wall as would be expected in a mining scenario. Angular and distance observations were observed from the established traverse stations to points of interest as would enable creating a planimetric map of the area. In underground mapping, enough data is collected to produce a planimetric map showing a rough outline of the underground environment and this is undertaken from accurately surveyed positions.

The main purposes for traversing include: boundary surveys, supplementary horizontal control, as-built surveys, establishment of ground control for photogrammetric mapping and densification of control networks [17]. In this case the requirement to establish ground truth or an as-built survey of the ILC is multi-fold:

- To establish MAST beacon coordinates in a global reference frame.
- To establish ground truth for simulation of an automated traverse using the MAST beacons and data obtained using the uGPS Rapid Mapper™.
- To link and make comparisons between mapping using conventional surveying methods and that using the uGPS Rapid Mapper system™.

The collection of data from each traverse station is an independent process. Several methods are used in surveying to connect these independent processes namely the transit rule, compass rule, the Crandall method and lastly least squares methods. With the advent of computers, least squares methods are preferred as they provide rigorous adjustment of observations allowing for variation in the precision in the observations, minimizing random variations in the observations and also providing the best estimates in the positions of all traverse stations [4, 70].
6.2. ESTABLISHING GROUND TRUTH

Least squares methods are implemented here to determine traverse coordinates and from these traverse stations an as-built survey for planimetric mapping is undertaken to enable comparison with the uGPS Rapid Mapper™ point cloud output. Figure 6.2 is the map of the ILC resulting from the as-built survey showing all the room allocations to reference the reader. Figure 6.3 shows the main traverse and traverse station locations. Figure 6.4 shows the Futura single second theodolite and the Topcon MZ6628 automatic level used to collect angle, distance and height difference data. Figure 6.5 shows surveying in of a line point, instrument and target setups at different stations. Figure 6.6 shows the traverse stations, line points and MAST beacon locations.

It was necessary that the as-built survey be established at a much higher standard than normal, as the traverse is the basis for determining the MAST beacon coordinates and the UGPS Rapid Mapper™ is designed to provide a high density and high resolution point cloud whilst minimizing data collection times relative to normal surveying and mapping methods [57].

Therefore, the quality of mapping by conventional methods should be high to compare with the quality of mapping and high resolution data provided by the uGPS system. Guidelines establishing comparison, positional accuracy, relative closure ratio accuracy, accuracy testing, verification and reporting which are normal procedures in conventional surveying are listed in [29, 2].

Two independent reference stations (STN110 and STN170) were established as true and assumed without error and the baseline between them was used to orient the traverse survey. A Futura TS-100 single arc-second Total Station was used to collect angular data between lines connecting traverse stations and distance data between
traverse points. The standard deviation of angular and distance measurement with this instrument are 2 arc seconds and 3 mm respectively. Figure 6.4(a) on page 158 shows an image of the Futura TS-100 Total Station.

A set of three tripods or a set including a single tripod and two target rods on rod stands were used to enforce the force centring condition\(^1\) to minimize angular and distance errors that come about due to inability to re-set or re-level on the same position as before. As is standard practice in surveying, observing on both the direct and reverse telescope position, measuring distances in both directions, forced centring, avoiding acute angles and others were employed to minimize observation errors in the survey. On-line points were established to densify the traverse control network from which the as-built survey was performed.

6.2.2 Establishing Line Points

Densification of the ILC coordinate network was done through establishment of line points placed in between the surveyed traverse points. These line points were used for detailed surveying during the as-built survey having been placed to maximize visibility for the survey, and to establish the beacon coordinates. Figure 6.6 shows the location of the traverse and line-points and Figure 6.5 shows some of the setups during traversing and line point determination.

\(^1\)A survey station on the ground or on the hanging wall is defined usually as a point within a defined area. Using optical methods of setting up above/below this station during a traversing process does not ensure that the very same point will be occupied at a later stage. The ability for modern equipment to be de-coupled such that the tripod legs and tribrach can be left set up at a station whilst the theodolite instrument is moved to a different set up ensures the very same point is occupied every time. This is forced centring. The reader is referred to [4] for more detail on the subject.
Figure 6.2: As-built survey of the Integrated Learning Centre (ILC) first floor (with room numbers for reference).
Figure 6.3: Main ILC traverse
Figure 6.4: Futura total station and Topcon automatic level: (a) Futura TS-100, single arc-second total station and (b) Topcon MZ6028 automatic level
Figure 6.5: Traverse Setups: (b) Surveying in a line point; (b) Total station setup at STN170; (d) Target set up at STN100; and, (d) Target set up at STN160.
Figure 6.6: ILC Traverse, Line Points and MAST Beacon Locations
6.2. ESTABLISHING GROUND TRUTH

6.2.3 Level Survey

Determination of elevations for the site was accomplished through spirit levelling. A TOPCON MZ6628 automatic level was used to collect raw level readings which were later reduced to obtain relative elevations and height differences between stations relative to reference station Station 110.

The levelling was accomplished in a leap frog manner. The level was set up in between two stations, and a backsight reading was observed onto a levelling rod held vertically on the first traverse station. From the same level position, a foresight reading was observed to a levelling rod held vertically on a second and forward traverse station. Both readings were recorded. With the second rod position held constant, the level was moved forward to a position in between the second rod position and a rod held on the next traverse station. The same observational procedure was repeated until the terminal point of the levelling traverse was reached. Typically the first and the last readings of a levelling exercise are observed to positions of known elevation.

6.2.4 Topographic/As-built Survey

As-built surveys typically have as a product a planimetric map showing position and elevation of features in an environment and are conducted from positions of known coordinates and elevation. Typically, angular and distance data in the horizontal and vertical planes are measured from known traverse stations to locations of interest and are referenced to other traverse points by method of radial positioning as shown in Section 3.5.1, and in Figure 3.11(a). Figure 6.7 illustrates wall feature locating in the ILC.

Data capture of the environment from each station is an independent process. To
establish the \( xyz \) coordinates of each feature, the station coordinates and elevation were used in conjunction with feature direction and distance measurements measured in the horizontal and vertical planes and also referenced to nearby traverse or control point locations. The as-built survey was established at a higher standard of accuracy to enable comparison of maps with that which would be constructed using the uGPS Rapid Mapper™ data.

As the ILC first floor is multi-level, the survey also established the areas of different elevation and slope. Figure 6.8 on page 164 shows the zones of different slope within the ILC. Floors in zones 1, 3, 5, 6, 7, 9, 10, 12 have a slope of 0\(^\circ\) and floors in zones 2, 4, 8 and 11 are inclined at approximately 2-3\(^\circ\). These slope angles are used to correct the distances obtained from the uGPS data before the loop closure adjustment is implemented.

6.3 Field Experiments

Various tests were undertaken using the uGPS Rapid Mapper™ and the MAST beacons. Within the ILC test environment, scanner position and orientation, loop closure and automatic survey alignment experiments were undertaken using a combination of the uGPS Rapid Mapper™ and the MAST beacons. Displacement experiments were undertaken to evaluate the accuracy of distances obtained from the uGPS Rapid Mapper™ compared to those obtained from conventional surveying methods.
Figure 6.7: Illustration of wall feature positioning in the ILC using the radial positioning method.
Figure 6.8: Different levels of the ILC
6.3.1 Determining Beacon Calibration and Coordinates

The final calculated position and orientation of the laser scanner largely depends on the precision of beacon calibration, how the beacon coordinates are determined and laser measurements to the beacon. The basis for beacon related experiments emanates from questions listed below.

1. What is the allowable tilt and rotation angular error when setting up a pair of beacons?

2. To what precision can the coordinates of the beacon edges be obtained?

3. What amount of error in beacon positioning would allow or enable constancy of the calculated scanner position and orientation?

4. What precision of beacon setting up is required to maintain constancy of the calculated scanner position and orientation? What sensitivity of levelling mechanism/process is required?

5. Does the method of beacon calibration have an effect on positioning accuracy?

Calibration

The beacon calibration process starts by determining beacon vertex coordinates. The beacon vertex coordinates are then used to confirm the design parameters $\theta_1$ and $\theta_2$, the angles of inclination of the outer panels to the centre panel and panel $P_1$, $P_2$ and $P_3$ dimensions. These are illustrated in Figure 5.1. As the beacon design is a square, its vertical and horizontal cross-sections are the same and therefore the similar calibration results should be obtained.
The assumption made is that the beacons are mounted vertically i.e., with the vertical edges vertical within required tolerances. The steps for this experiment include:

1. Set up total station at reference point between mounted beacons and record height above/below station.

2. Set out two line point stations on either side of station, in the vicinity of beacons and in full view of all vertices from which to repeat exercise.

3. On the first theodolite face, observe and record direction orientation to a previous/second reference point. Then turn, observe and record horizontal and vertical angles to each of the eight beacon vertex points. This exercise is repeated on the second theodolite face.

4. The process is repeated from two line points previously set out.

5. The 3D reference coordinates are calculated from which beacon parameters will be determined.

The experimental setup is illustrated in Figure 5.7 and Figure 6.9. Figures 6.10 and 6.11 show a plot of the different beacon configurations at BTS120 and BTS170 after application of the calibration process and determination of the beacon coordinates.

**Beacon Setup and Levelling**

The MAST algorithm assumes verticality of all vertical panels (i.e. each vertical edge points toward the centre of the earth) of mounted beacons. This in turn means vertical edges are vertical and parallel to the vertical axis of the earth at that location.
Figure 6.9: Beacon Calibration: (a) Observing to a beacon vertex; (b) Observing onto a line point; and (c) Observing target set up on STN170
6.3. FIELD EXPERIMENTS

The MAST algorithm also assumes that horizontal position is determined in a plane orthogonal to the local zenith line and the scanner horizontal orientation is calculated on this plane.

The beacon is mounted on the wall and levelled using some levelling mechanism (an ordinary level) in two orthogonal directions to ensure that the panels are vertical. The steps for this experiment include:

1. The beacon is set up such that vertical edges are vertical.
2. The beacon is levelled in two orthogonal directions as shown in Figures 5.2.
3. Adjustments are made to the levelling as necessary to ensure verticality of vertical edges.

As these experiments were undertaken inside the ILC and not in an underground environment, the full spectrum of tests regarding setting up and levelling of the beacon could not be executed. However, the same procedures as would have been taken underground were followed.

**Beacon Coordinates**

In order to determine beacon coordinates, the assumption is that the beacon vertical edges are as near vertical as possible (within some tolerance). Taking as an example a vertical beacon edge, its top vertex $x$-$y$ coordinate should be very similar to the bottom vertex $x$-$y$ coordinate. The assumption that is further made is that if these two coordinate pairs are close then they can be averaged out to obtain a final value. It should be noted that $x$-$y$ coordinates are measured or calculated in the mapping plane.
Figure 6.10: Beacon configuration 1 with beacons adjacent to each other: (a) BTS120 and; (b) BTS170
Figure 6.11: Combined beacon configurations 1 and 2, beacon configuration 2 - beacons opposite each other: (a) BTS120 and; (b) BTS170
6.3.2 Scanner Position and Orientation Tests

Scanner position and orientation tests were conducted to determine the reliability of using the beacons as a tool to close the 'loop' of an automated traverse and for automatic orientation of a survey. These tests were conducted in conjunction with the displacement tests described in the next section.

The position and orientation test requires that the uGPS Rapid Mapper™ be situated within visibility or field of view of a set of MAST beacons. A scan was undertaken of the environment to include the MAST beacons and the coordinates and orientation of the scanner determined using the MAST algorithm. To independently verify the results from the MAST algorithm, the coordinates and orientation of the scanner were also determined using conventional surveying methods.

As the uGPS Rapid Mapper™ is a self contained piece of equipment, several assumptions were made regarding defining position and orientation. First, the cover plate of the vertical scanner has a hole in its centre that is assumed coincident with the scanner axis of revolution, therefore this was assumed as the centre to determine the position of the scanner. Second, with no physical access to the scanner field of view, the only way to determine what direction the scanner was facing was to assume the field of view of the scanner was symmetric to the back panel of the scanner and therefore the direction would be assumed orthogonal to the back panel of the scanner.

To achieve this, a rigid and flat metal plate was attached to the back panel of the scanner. The idea here was, if points marked along this plate could be surveyed in and their coordinates and orientation determined, the direction faced by the scanner should be orthogonal to that of the plate. The position of the laser scanner and the metal plate was determined from surveying in the centre position on the cover plate.
and the marked points on the metal plate from stations of known coordinates and elevation. As no prism target could be placed on these positions, directions to these positions were observed and referenced to nearby traverse stations or line points. Figure 6.12 shows the uGPS Rapid Mapper™ metal plate setup and Figure 6.13(a) shows the actual observation onto a position on the metal plate.

Figure 6.12: uGPS mounted on Taylor Dunn vehicle showing rigid metal plate affixed to the back of the vertical scanner.
6.3.3 Scanner Displacement Tests

The scanner displacement tests were accomplished through a two step process. The first step was the determination of the scanner position and orientation in the vicinity of the MAST beacons using conventional surveying methods. The second step was for the MAST process to determine the location and orientation of the scanner at the same position. The purpose for these tests is to determine and compare distances determined from the uGPS Rapid Mapper™ with those determined using conventional surveying methods. An area that was flat or horizontal was selected to run a batch of displacement tests. The tests included three short and two long runs from which comparisons were made with measurements obtained from conventional surveying methods.

A rigid, flat and straight metal plate was rigidly mounted on the back flat panel of the uGPS Rapid Mapper™. Figure 6.12 shows the uGPS metal plate setup. Several positions were marked on the edge of the metal plate such as to be visible for observation using a theodolite or total station. These points, once their positions were known would enable testing the accuracy of the derived direction. The assumption made here is that the mid-point direction of the uGPS Rapid Mapper™ field of view lies orthogonal to the plate mounted behind the uGPS Rapid Mapper™. On the metal cover above the vertical scanner is a hole that is assumed to line up with the axis of the scanner. Survey observation to this point were also made. This is the primary point used to determine the scanner position at the beginning and end of the experiment and also the distance between the two positions.

At the beginning of the test, the test vehicle was driven onto the test area. A series of directional observations were made to the marks on the plate and onto the
scanner centre from known stations. Coordinates of these points were determined by the method of intersection described in Section 5.3.3. The test vehicle was then driven to a second position also visible from known stations and the positions on the plate and scanner centre were re-observed. As the vehicle is moved from one position to another, a series of scans are collected that will be used to determine displacements from one pose to the next by scan matching and subsequently the coordinates along the vehicle trajectory. Distances using the uGPS were derived from coordinate output in a `dbg_q.csv` file.

A linear distance was calculated between the start and end positions of the scans and this was compared with that which was calculated through conventional surveying methods. Figures 6.14 and 6.15 show the path taken by the uGPS Rapid Mapper™ and the raw scan data superimposed on the drawing.

6.3.4 Closing the Loop and Automatic Survey Alignment using MAST Beacons

In conventional surveying, closed loop or link traverses are preferred as they allow for error analysis computation by advanced least squares methods. As mentioned in Chapter 3, these traverses start and end at locations of known coordinates. Figure 6.16 shows the image of the field of view of the uGPS on its controller. Figure 6.17 shows the automated traverse and mapping exercise for a path running through STN170 - STN100 - STN120 - STN130 - STN160 - STN170. Likewise, the MAST process operating within the vicinity of the beacons establishes known points within a known reference frame.

The purpose of the loop closure experiments were two-fold: first to determine the
Figure 6.13: Displacement tests - (a) Test 1 and (b) Test 4.
Figure 6.14: Raw displacement test data - (a) Tests 1 and 2; (b) Test 3.
6.3. FIELD EXPERIMENTS

Figure 6.15: Raw displacement test data - (a) Tests 4 and 5; (b) Test 5.
6.3. FIELD EXPERIMENTS

Figure 6.16: uGPS field of view on controller at the beginning of traverse.

linear mis-closure and close the loop of a uGPS traverse in the ILC environment. The experiments to ‘close the loop’, required driving the survey platform through the corridors of the ILC and collecting data starting at one set of beacons and ending either at the same set or a different set of beacons. The situations investigated included where the MAST beacons were the only tool used to close the traverse and no RFID tags are used to improve the solution.
Figure 6.17: Automated loop traverse and mapping: (a) Start point at BTS170; (b) Negotiating curve at BTS120; (c) Negotiating ramp near STN160; and (d) Approaching traverse end point at BTS170.
Second, as the result of a uGPS mapping is a 3D point cloud, we generate a 2D plan view of the ILC in the mapping plane of the reference system as would be done by conventional methods, however, this time automatically using the uGPS point cloud data. The goals for experiments to automatically align the uGPS survey, were to rotate and translate the data, and fit and compare the resultant survey to an as-built survey obtained by conventional surveying methods. The derived map would be tested against the as-built map to evaluate and quantify its accuracy according to standards set out by the ASPRS and FGCC.

6.4 Conclusion

In this chapter ground truth in the form of an as-built survey of the ILC was determined from a conventional surveying traverse established to an accuracy higher than required in a typical mine survey. Also from this traverse, calibration and coordinates of the geometric beacons were established. Field experiments to determine scanner position and orientation, automatic survey alignment and displacement tests to compare distances obtained from conventional surveying methods with distances obtained through uGPS derived coordinates were undertaken. A mapping exercise of the ILC was also undertaken using a combination of the georeferenced geometric beacons and the uGPS with the result of map data of quantifiable accuracy.
Chapter 7

Experiment Results

7.1 Introduction

In this chapter are presented the results and analysis from the experiments undertaken. It must be noted here that as the internal algorithms of the uGPS are proprietary, therefore some information critical to especially the automated traverse was lost or not available and extrapolation using other data to obtain distances, orientation of scanner side rays used for mapping, start and end point scanner orientations, hence, a quantitative analysis based on the obtained results may not necessarily be justified. Hence, a more comparative approach to analysis is entertained.

7.2 Control Network Results

Table 7.1 lists the results of the traverse survey. In accordance to [29], this traverse is classified as Second Order, Class II (1:20 000). The typical relative closure ratio accuracy for the underground mining environment is a Third Order, Class II (1:5000) [32]. Figure 6.3 shows this traverse.

Only stations STN 120 and STN 160 were calculated during the traverse. The
reference standard deviation for the traverse adjustment was found to be 0.004 m and the standard error $\sigma_x$, $\sigma_y$ of the individual station’s x and y coordinates surveyed were less than 1 mm. The covariance matrix for the adjustment is listed as well as other information in Table 7.1.

Linear misclosure is the ratio of the closure distance to the overall traverse length. In this case the linear misclosure was found to be 0.0034 m with an overall traverse accuracy of 1: 26 345, well exceeding the requirement for underground mine surveying.

All other station coordinates used as the basis for the detailed as-built survey and determination of the beacon coordinates were calculated from line points placed between existing traverse stations. Table 7.2 is a complete list of these.

7.3 Field Test Results

7.3.1 Beacon Calibration and Coordinates

The beacon design panel lengths $P_1$, $P_2$, $P_3$ for fabrication were given as 0.150 m, 0.450 m and 0.300 m respectively. The panel deviation design angles $\theta_1$ and $\theta_2$ were $39.500^\circ$ and $18.544^\circ$ respectively. The beacon design was such that the horizontal and vertical cross-sections of the beacon were to be the same. This would enable the same positioning algorithm to be used to determine the elevation and the roll angles of the mobile platform.

The final calibration values were generally noted to be within a $2^\circ$ range of the design values however, for the vertical cross section $\theta_2$ was within a $6^\circ$ range. As the beacon calibration was undertaken after the beacons were set up, the tilt and rotation of the beacons were found to be within $1^\circ$ of the zenith and horizontal lines respectively.
Table 7.1: ILC traverse survey results.

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<td>72.074</td>
<td>71.806</td>
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Reference Standard Deviation : +/- 0.004 m
Angular Misclosure : -0deg 0min 17.5sec
Linear Misclosure : 0.0032 metres
Traverse Accuracy : 1 : 26345.04

Covariance Matrix of the Final Coordinates

\[
\begin{bmatrix}
0.0003 & 0.0000 & -0.0000 & -0.0000 \\
0.0000 & 0.0000 & 0.0000 & -0.0000 \\
-0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & -0.0000 & 0.0000
\end{bmatrix}
\]
Table 7.2: Listing of final coordinates from traverse and level surveys

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<th>Y (m)</th>
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The individual intersection calculations for the vertices produced positional errors within 1 cm in the x and y directions with the exception of observations from the first beacon where positional errors of up to 2.3 cm in the x and 1.5 cm in the y directions were recorded. Elevation errors consistently remained less than 0.5 cm. The panel lengths through calibration were found to be within 3 cm, 0.5 cm and 1.5 cm for $P_1$, $P_2$ and $P_3$ respectively. The averaging of relevant coordinate results produced the calibration results in Tables 7.3 through 7.5.

### 7.3.2 Scanner Position and Orientation

The scanner position and orientation tests were undertaken in conjunction with the displacement tests. Essentially the endpoints of the displacement tests were the locations for the scanner position and orientation tests when these positions were in the vicinity of the MAST beacons. Table 7.6 compares the coordinates of the laser
### 7.3. FIELD TEST RESULTS

Table 7.3: Beacon calibration for STN120 - Beacon configuration 1: Left hand side and Right hand side beacons

#### Beacon Calibraion for STN120 - Beacon configuration 1: Left hand side and Right hand side beacons

<table>
<thead>
<tr>
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<th>X(m)</th>
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<th>Y(m)</th>
<th>±t/-(-m)</th>
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#### Final Adjusted Beacon Coordinates

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#### Beacon Parameters

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#### Final Adjusted Beacon Coordinates

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<th>X(m)</th>
<th>±t/-(-m)</th>
<th>Y(m)</th>
<th>±t/-(-m)</th>
<th>Z(m)</th>
<th>±t/-(-m)</th>
<th>Position Err</th>
</tr>
</thead>
<tbody>
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<td>±0.002312</td>
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<td>±0.004548</td>
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<td>±0.01325</td>
<td>0.001766</td>
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<td>101.5997</td>
<td>±0.004118</td>
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<td>±0.00566</td>
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<td>101.5526</td>
<td>±0.004485</td>
<td>10.9635</td>
<td>±0.000547</td>
<td>0.001748</td>
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<tr>
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<td>±0.003386</td>
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<td>±0.004485</td>
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<tr>
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<td>101.5445</td>
<td>±0.005216</td>
<td>10.8189</td>
<td>±0.00553</td>
<td>0.002844</td>
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<tr>
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<td>71.4688</td>
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#### Beacon Parameters

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### Table 7.4: Beacon calibration for STN170 - Beacon configuration 1: Left hand side and Right hand side beacons

#### Left of traverse direction option

<table>
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<tr>
<th>Pt.</th>
<th>[Beacon]</th>
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<th>+/-(-m)</th>
<th>Y(m)</th>
<th>+/-(-m)</th>
<th>Z(m)</th>
<th>+/-(-m)</th>
<th>Positn Err</th>
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<tbody>
<tr>
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<td>0.001402</td>
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Tilt back: 0 deg 35 min 31.03 sec
Rotates left: 0 deg 25 min 5.34 sec

#### Right of traverse direction option

<table>
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<th>Pt.</th>
<th>[Beacon]</th>
<th>X(m)</th>
<th>+/-(-m)</th>
<th>Y(m)</th>
<th>+/-(-m)</th>
<th>Z(m)</th>
<th>+/-(-m)</th>
<th>Positn Err</th>
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</thead>
<tbody>
<tr>
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<td>0.002009</td>
<td>70.6620</td>
<td>0.001603</td>
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<td>0.002009</td>
<td>70.6620</td>
<td>0.001603</td>
<td>11.8290</td>
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<td>0.002489</td>
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</table>

Tilt back: 0 deg 38 min 43.77 sec
Rotates right: 0 deg 3 min 12.18 sec

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**Beacon Parameters**

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<th>Horizontal E-section</th>
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<tbody>
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<td>Deg</td>
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<tr>
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**Beacon Parameters**

<table>
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<th>Vertical E-section</th>
</tr>
</thead>
<tbody>
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<td>Deg</td>
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<td>17</td>
</tr>
<tr>
<td>2</td>
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</tr>
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</table>
Table 7.5: Beacon calibration for STN120 and STN170 - Beacon configuration 2: Left hand side beacons

### Beacon TO LEFT of traverse direction OPTION
#### Calculated Point Coordinate Values

<table>
<thead>
<tr>
<th>Pt. #</th>
<th>Beacon</th>
<th>X(m)</th>
<th>+/- (m)</th>
<th>Y(m)</th>
<th>+/- (m)</th>
<th>Z(m)</th>
<th>+/- (m)</th>
<th>Posn Err</th>
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<tbody>
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<td>98.8010</td>
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<td>0.000330</td>
<td>0.063143</td>
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<td>0.003847</td>
<td>98.8010</td>
<td>0.001597</td>
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<td>0.000330</td>
<td>0.063143</td>
</tr>
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<td>0.000793</td>
<td>0.064954</td>
</tr>
<tr>
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<td>99.1213</td>
<td>0.002129</td>
<td>10.3959</td>
<td>0.000793</td>
<td>0.064954</td>
</tr>
</tbody>
</table>

#### Final Adjusted Beacon Coordinates
#### Calculated Point Coordinate Values

<table>
<thead>
<tr>
<th>Pt. #</th>
<th>Beacon</th>
<th>X(m)</th>
<th>+/- (m)</th>
<th>Y(m)</th>
<th>+/- (m)</th>
<th>Z(m)</th>
<th>+/- (m)</th>
<th>Posn Err</th>
</tr>
</thead>
<tbody>
<tr>
<td>71</td>
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<td>0.003847</td>
<td>98.8010</td>
<td>0.001597</td>
<td>10.2983</td>
<td>0.000330</td>
<td>0.063143</td>
</tr>
<tr>
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<td>0.003847</td>
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<td>0.001597</td>
<td>10.2983</td>
<td>0.000330</td>
<td>0.063143</td>
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<tr>
<td>75</td>
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<td>72.8477</td>
<td>0.004290</td>
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<td>0.002129</td>
<td>10.3959</td>
<td>0.000793</td>
<td>0.064954</td>
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<td>0.004290</td>
<td>99.1213</td>
<td>0.002129</td>
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<td>0.000793</td>
<td>0.064954</td>
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### Beacon Parameters

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### BEACON TO LEFT of traverse direction OPTION
#### Calculated Point Coordinate Values

<table>
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<th>Beacon</th>
<th>X(m)</th>
<th>+/- (m)</th>
<th>Y(m)</th>
<th>+/- (m)</th>
<th>Z(m)</th>
<th>+/- (m)</th>
<th>Posn Err</th>
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</thead>
<tbody>
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<td>71.5729</td>
<td>0.000090</td>
<td>0.002507</td>
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<td>71.5729</td>
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</tr>
<tr>
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<td>0.000365</td>
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#### Final Adjusted Beacon Coordinates
#### Calculated Point Coordinate Values

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<th>Y(m)</th>
<th>+/- (m)</th>
<th>Z(m)</th>
<th>+/- (m)</th>
<th>Posn Err</th>
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<tr>
<td>31</td>
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<td>0.000080</td>
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### Beacon Parameters

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<table>
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</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
scanner as determined using the MAST algorithm and by conventional surveying methods.

From this table it is observed that the two methods of positioning and orientation provide similar results and are consistent with each other. The differences in the x and y values are within 1.6 cm with exception of File 20, Scan #1 where the x difference in 5.5 cm. All azimuths fall within a maximum of $2^\circ$ of each other. The accuracy of positioning is also consistent with the simulated data in [38].

### 7.3.3 Displacement Test Results

A straight line distance was calculated based on the coordinates calculated from the scanner positioning and orientation experiments mentioned in the preceding section and shown in Table 7.6. The displacement test results shown in Table 7.7 indicate that in all cases the distances calculated from the uGPS derived coordinate data are shorter than those derived from conventional surveying. The scaling factor in the last column is the ratio of the uGPS Rapid Mapper™ distance to the conventional surveying distance. The mean scale factor for the distance relationship is the average of the calculated scale factors and this was found to be 0.9671. The ILC is a difficult environment that has very few features, hence this may be the cause for the disparity in the distances. In an underground situation, the jaggedness of the wall or the undulating feature of a smooth shortcreted wall or the presence of ventilation pipes or other ducts are features that aid in obtaining accurate displacement values.
Table 7.6: Comparison of MAST and conventional survey derived coordinates and azimuths

<table>
<thead>
<tr>
<th>File No.</th>
<th>Scan No.</th>
<th>Laser Scanner Positioning</th>
<th>Conventional Surveying</th>
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<tbody>
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</tr>
<tr>
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<td>89</td>
<td>99.711±0.078</td>
<td>71.388±0.078</td>
</tr>
<tr>
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<td>71.911±0.105</td>
</tr>
<tr>
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<tr>
<td>21</td>
</tr>
<tr>
<td>22</td>
</tr>
<tr>
<td>22</td>
</tr>
</tbody>
</table>
Table 7.7: Comparison of uGPS Rapid Mapper™ and conventional survey derived displacements

<table>
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<tr>
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<th>Conventional Surveying Distance</th>
<th>Horizontal Difference</th>
<th>Scaling Factor</th>
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<td>0.9764</td>
</tr>
</tbody>
</table>

7.3.4 Closing the Loop, Automated Checkpoints and Automatic Survey Alignment

Closing the Loop

The initial alignment of the traverse survey or the raw uGPS data to the mine reference system is accomplished by determining the translation and rotation that takes two consecutive or near consecutive scans centres whose positions have been determined using the MAST positioning system then applying the same transformation to the rest of the data. ‘Closing the loop’ therefore pertains to making the physical location of the uGPS at its closing point and its calculated position similar.

The ‘closing of the loop’ experiments emanate from the uGPS mapping physically ending at the desired location, however, the data was not concurring. Here the same concept of starting and ending a traverse at known coordinate locations was applied to the uGPS data.
Table 7.8: ILC automated traverse survey and mapping results.

| Summary of Traverse and Mapping Results Using MAST Beacons NOT Taking into Account uGPS Closure onto RFID Tags |
|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| [File] | Xclos [m] | Yclos [m] | [Distance] | Accuracy | Chkpts | RMSE x | RMSE y | RMSE [m] | Map [cm] | ASPRS Map Class | 95% CI | ASPRS Map Accuracy [cm] |
| 23 | 0.458 | 0.787 | 83.780 | 91.9 | 9 | 73.3 | 67.3 | 99.5 | 172.190 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 183.6 |
| 24 | -0.598 | 0.408 | 128.197 | 177.2 | 20 | 72.0 | 70.2 | 100.6 | 174.090 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 183.6 |
| 25 | 1.515 | 1.388 | 59.655 | 29.0 | 13 | 73.1 | 34.6 | 80.9 | 140.020 | 5000 | 2500 | 2000 | 153.0 | 153.0 | 183.6 |
| 26 | -0.698 | -0.596 | 90.572 | 98.7 | 21 | 35.0 | 49.9 | 60.9 | 105.470 | 2500 | 2000 | 1000 | 153.0 | 122.4 | 183.6 |
| 27 | 0.149 | 0.303 | 96.195 | 12.8 | 7 | 74.6 | 96.6 | 122.1 | 211.290 | 5000 | 2500 | 2500 | 306.0 | 306.0 | 229.5 |
| 28 | 0.223 | 0.132 | 60.749 | 17.7 | 13 | 87.1 | 75.7 | 115.5 | 199.830 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 229.5 |
| 29 | 0.129 | 0.152 | 84.624 | 434.1 | 9 | 70.1 | 38.5 | 80.8 | 330.410 | 5000 | 2500 | 2000 | 153.0 | 153.0 | 183.6 |
| 30 | -2.199 | -3.405 | 129.669 | 32.0 | 18 | 94.7 | 79.8 | 123.8 | 234.290 | 5000 | 2500 | 2500 | 306.0 | 306.0 | 229.5 |
| 31 | -1.459 | 1.069 | 80.425 | 33.4 | 13 | 63.9 | 64.1 | 96.5 | 156.630 | 5000 | 2500 | 1000 | 306.0 | 306.0 | 183.6 |
| 32 | -0.100 | -0.121 | 67.392 | 292.7 | 9 | 46.1 | 30.5 | 55.3 | 95.671 | 2500 | 2000 | 1000 | 153.0 | 122.4 | 183.6 |
| 33 | 1.136 | 0.980 | 84.625 | 56.5 | 9 | 15.9 | 16.0 | 22.6 | 39.072 | 1000 | 500 | 2000 | 61.2 | 61.2 | 45.9 |
| 34 | -3.204 | -2.208 | 54.791 | 14.1 | 14 | 43.5 | 55.6 | 70.6 | 122.250 | 5000 | 2000 | 1000 | 153.0 | 122.4 | 183.6 |
| 35 | 2.569 | 0.763 | 122.173 | 40.0 | 19 | 36.5 | 66.0 | 75.4 | 130.520 | 5000 | 2500 | 2000 | 153.0 | 153.0 | 183.6 |
| 36 | 0.318 | 0.423 | 125.063 | 187.1 | 9 | 68.0 | 35.7 | 117.8 | 203.970 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 229.5 |
| 37 | 1.408 | 0.249 | 95.934 | 67.1 | 21 | 63.8 | 73.7 | 97.4 | 168.660 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 183.6 |
| 38 | 0.211 | 1.280 | 123.165 | 96.2 | 16 | 82.2 | 50.4 | 96.5 | 166.950 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 183.6 |

| Summary Statistics of Traverse and Mapping Results Using MAST Beacons NOT Taking into Account uGPS Closure onto RFID Tags |
|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| [Statistic] | Xclos [m] | Yclos [m] | Distance [Distance] | Accuracy | Chkpts | RMSE x | RMSE y | RMSE [m] | Accuracy | Chkpts | RMSE x | RMSE y | RMSE [m] | Accuracy | Chkpts | RMSE x | RMSE y | RMSE [m] |
| Maximum | 7.026 | 3.035 | 128.669 | 434.1 | 21 | 94.7 | 66.6 | 123.8 | 214.290 | 5000 | 2500 | 2000 | 306.0 | 306.0 | 229.5 |
| Minimum | -3.204 | -3.405 | 54.791 | 12.8 | 7 | 15.9 | 16.0 | 22.6 | 39.072 | 1000 | 500 | 2000 | 61.2 | 61.2 | 45.9 |
| Mean | 0.490 | 0.216 | 93.894 | 104.4 | 15 | 61.1 | 59.7 | 86.6 | 149.887 | 4324 | 2235 | 1676 | 228.6 | 221.4 | 186.3 |
| Std.Dev. | 2.737 | 1.466 | 27.589 | 113.3 | 5 | 21.5 | 22.2 | 27.2 | 47.031 | 1298 | 504 | 611 | 87.3 | 94.6 | 41.3 |
7.3. FIELD TEST RESULTS

Due to the proprietary nature of the uGPS software and algorithms, and the uGPS being a self contained tool, data access was only limited to the final positional coordinate data of the laser scanners along the traverse. It is from this data that the traverse was implemented. The top table of Table 7.8 summarizes the relevant information in the process of the development of an automated map and the bottom table on the same page are its summary statistics.

Columns 1 through 5 show the traverse misclosures, distances and traverse accuracies as would be applied from a conventional surveying perspective. Columns 6 through 10 show the number of checkpoints for each survey, the linear and radial RMSE based on the known and calculated checkpoint locations as well as the derived map accuracy based on ground distance. Columns 11 through 13 show the ASPRS map classifications based on the scale that the map can be plotted at and columns 14 through 16 show the ASPRS map accuracy classifications, also based on ground distance.

Higher traverse accuracies emanate from very low linear misclosures related to the traverse lengths also indicating a good relationship between angular and distance observations. Low RMSE values indicate good correlation between the true values of the checkpoints and their calculated counterparts.

Taking as an example File #23, Figures 7.1 and 7.2 show the traverse start and finish scan locations in both the arbitrary and mine reference frames. The points marked with red stars indicate the panel segmentation from the point cloud data. While this might not be always correct due to the arbitrary nature of the point cloud, it is not critical as the final alignment is adjusted by the ICP algorithm. The red star with the leading line indicates the position and orientation of the laser scanner.
Figure 7.1: Position and orientation of Scan 73 in mine reference frame
Figure 7.2: Position and orientation of Scan 843 in mine reference frame
Figure 7.3: Scan 73 convergence criteria at beginning of traverse
Figure 7.4: Scan 843 convergence criteria at traverse end
For each of these locations, the convergence criteria for the ICP algorithm, which are the RMSEr, translations in x and y and the rotations plotted against the iteration are shown in Figures 7.3 and 7.4 plotted according to [12]. These track the RMSE Error, the translations in the x and y directions and rotations at every iteration of the ICP algorithm.

The raw traverse for File #23 overlayed onto the as-built survey is shown on Figure 7.5. The raw traverse is the line marked in black. The expectation is that the raw traverse should lie central within the passages of the ILC and along the ramps without encroaching onto wall or other features marked in red. In this case a loop traverse was run starting from geometric beacons placed in vicinity of STN170 and ending at the same location. It can be noted that the ending point is significantly different from the start point where these should be relatively close.

The automated map for File #23 is obtained from plotting the scan points either side of the vertical scan for each location point along the traverse. This is also overlayed onto the as-built survey for comparison on Figure 7.6. Shown on this map is the adjusted traverse line in blue as well as the raw traverse line in black as well as the map point cloud in black based on the adjusted traverse. The lack of coincidence between the map point cloud and the as-built map can be attributed to a rotation in the overall traverse as a result of the adjustment or error in the displacement distances between poses because of accuracy of RFID positioning.
Figure 7.5: File # 23 raw traverse overlay onto as-built survey
Figure 7.6: Overlay of File # 23 automated map and as-built survey
Automated Checkpoint Determination

The checkpoints (see Figure 7.7) are distinct corner or wall intersection locations that are easily identifiable and allow for automatic determination of their coordinates from point cloud data in their vicinity. The basis for automated determination of locations to check against a checkpoint location is a radius within which measured point cloud data is captured. In most cases the anticipated deviation or distance of the checkpoint location relative to its calculated counterpart is less than half the tunnel or corridor width. Therefore the adopted distance is half the minimum tunnel/corridor width.

The captured point cloud is subjected to line fitting and when the most probable lines are determined, an intersection algorithm is used to determine the most likely position of the corner from the point cloud data. The last column on Table 7.8 compares points incorrectly identified (IID) by this automated process to the number of checkpoints identified (NCI) along a traverse to the number of possible checkpoints (PCT) per traverse route. From this table, $\frac{9}{14}$ of those converged traverses had an incorrect to tested checkpoint (IID:NCI) ratio less than 25% and likewise $\frac{9}{14}$ had an incorrect to possible (IID:PCT) ratio less than 25%. These values are likely to decrease with improved traverse accuracy and alignment. Also, on only two occasions are less than 50% of the total checkpoints available along a traverse identified. These checkpoints in relation to the traverse lines lie on the sidewalls that the uGPS was turning away from creating a data sparsing situation making it impossible to fit lines. Point cloud densification will increase the likelihood that more of the checkpoints being identified by the algorithm.
Figure 7.7: File # 23 calculated check point locations overlay onto as-built survey
The identified calculated checkpoint locations using the point cloud data for the various locations on File #23 are shown on Figure 7.7. For each traverse survey undertaken, similar data sets can be found on Figures D.1 to D.112 in Appendix D. It is this information that has been captured, summarized and displayed in Table 7.8.

The maximum closure errors in the x and y directions are 7.029 m and 3.405 m. The traverse accuracy ratios range from 1:12.8 to 1:434.1. The radial $RMSE_r$ range from 22.6 cm to 123.8 cm and the calculated map accuracies range from 39.072 cm to 214.290 cm at the ground distance. Based on the ASPRS Map Class, or scale at which the map can be plotted, the ASPRS Map Accuracy range for the maps produced varies between 45.9 cm to 306 cm at ground scale.

**Automatic Survey Alignment**

Survey alignment is a byproduct of the traverse survey starting and ending at known positions in the mine reference system.

The initial alignment of the traverse survey or the raw uGPS data to the mine reference system is accomplished by determining the translation and rotation that takes two consecutive or near consecutive scans centres whose positions have been determined using the MAST positioning system then applying the same transformation to the rest of the data. This in conjunction with the ‘closing the loop’ process, rotates the traverse survey or raw uGPS data further aligning it to the mine reference system.
7.4 Analysis

The control network was performed to a high degree of accuracy, with a traverse accuracy of 1:26 345, exceeding the accuracy standards for underground mine surveying of 1:5000 [32]. This was to enable comparison of the as-built survey with the calculated uGPS map which was obtained from data which was measured accurately. The assumption made here is that if the uGPS traverse is accurate, based on the calculated distances and angles and traverse closures, then the resultant mapping should match the as-built survey. However, it must be noted also that survey grade mapping accuracy was never an intended purpose for the system and therefore the ability to quantify mapping accuracy with this system is currently non-existent.

Surveying methods are proven to be highly accurate despite them being slow. Therefore, in trying to emulate the surveying process through automated means, the expectation was that the automated process would produce similar results within the limits of mine surveying. Fourteen experiments are featured in this thesis all using the second beacon configuration shown in Figures 6.7(b) and 6.1(d). In this case several factors lead to the uGPS traverse survey results obtained:

1. The survey is as accurate as its input data. It is assumed that the distances and angles derived using the uGPS coordinate data to be a true representation of reality, therefore, the results reflect the accuracy of the uGPS system in this environment.

2. The effect of the beacon calibration process on calibration results.

3. The impact of the MAST positioning process.
4. The accuracy of the mapping has its basis on the difference between surveyed points and calculated points, which is also related to the accuracy of the survey alignment. What factors, if any, pertaining to survey alignment contribute to the results?

7.4.1 Constraints Due to Proprietary Nature of the uGPS Process

The basis of the uGPS traverse calculation are the results of a \texttt{dbg.q.csv} file comprising \(x\)-\(y\)-\(z\) coordinates of the vertical scanner produced by the uGPS Rapid Mapper software. If is from this data that the distances and angles required in the adjustment process are computed. This data is also taken as the initial approximation into the MAST least squares adjustment. Typically, knowledge of how the coordinate file was derived is necessary to establish where errors might have occurred, if any, in the uGPS data processing process or whether it emanates from environmental issues.

A general observance of the uGPS survey results suggests an inability to fully converge in the ILC environment. From the uGPS procedures, the start and endpoints of a traverse are established by initial scans made at positions where RFID tags are placed and their coordinates determined. Full convergence here refers to the uGPS system recognizing the beginning or end of the traverse as locations scanned previously and adjust the traverse results to fit to the locations of these scanned positions. However this is not the case as there is a visible, sometimes significant difference, between the traverse endpoint locations and the true positions of the RFID tags. This could possibly be due to a calibration issue that results in a higher distance or scale error in the east-west direction than in the north-south direction resulting in a non-uniform scale in the \(x\)-\(y\) directions. These results can be observed in Table 7.9.
Another possible reason is that the ILC type of environment, comprising smooth and straight walls, and glass walls was not a target environment for the uGPS system, therefore making it a very challenging scenario to tackle. However, this environment with several bends and corridors was the first step towards experiments to be done in a mining environment, which is the target environment for the uGPS Rapid Mapper™.

7.4.2 Assumptions from Data and Data manipulation

The assumption that is made first is that there is a one to one relationship between the scanner position `dbg_q.csv` and the scan data in the `Laser0_range.csv` files i.e., the first point in one file corresponds to the same in the other. The start and end positions of the traverse are calculated based on the `Laser0_range.csv` scan data. Two subsequent or near subsequent position calculations establish the start or end traverse position. The criteria used in establishing position are consistent azimuth values within a threshold value i.e. sequential or near sequential azimuth results will not be divergent from each other. With regards to the start and end positions, the initial and terminal azimuths or poses are assumed aligned with the next or previous point in the `dbg_q.csv` file and this is likely not to be the case. In actuality, there is no link between a pose direction and the direction between two pose positions. In surveying these would be the same. This as a result has the potential to bring error into the angular adjustment portion of the calculation.

Without knowing whether the uGPS data accounted for slope change and how the as-built survey delineated between zones with slope greater than zero and those with zero slope. These are shown in Figure 6.8. As there is no relationship between the initially calculated distances, the position from which they were determined, the
initial input coordinates were overlayed on the slope zone map and corresponding
distances corrected accordingly for slope. In addition to this correction, the mean
distance scale factor calculated in Section 7.3.3, relating survey distances and uGPS
distances was applied. These corrections were applied during the first iteration of the
adjustment and it is assumed these uGPS derived distances and angles reflect a true
representation of reality and therefore the accuracy of uGPS. For each experiment
undertaken, there is shown a raw traverse map that is overlayed onto the as-built
survey. For File #23, this is Figure 7.5.

Based on the alignment method applied, a general pre-adjustment comparative
analysis with the as-built map shows thirteen of fifteen experiment paths encroaching
onto wall features. After the traverse analysis, nine of the fifteen still remained
encroaching onto other features. It was also observed that most raw traverses exhibit
length distortions in either the x or y directions or in both directions. 66.7% of these
length distortions are in the x direction, 33.3% in the y direction and 20% in both
directions.

7.4.3 Impact of Calibration Process on Calibration and Positioning Re-
sults

The calibration process as undertaken in this research has the shortfall that while
beacon vertices were independently observed, there needs to be a bundle adjustment
of the observations with associated constraining factors such as to enforce the design
criteria during the adjustment process. The effect of using an average of independ-
dently adjusted coordinates is a twisting effect of the beacon, thereby distorting and
deviating from the design values.
The averaging process as observed from the results may not be the correct approach to determining the beacon coordinates as it does not take into account or enforce the rigid nature, relationship and orientation of panels to each other. This results in a distortion of the individual panel dimensions. A constraining mechanism is required to enforce that parallel lines of the middle panel be parallel with corresponding lines of the outer panels and also that the normal of the middle panel be similar to that of a plane defined by the outer edges of the beacon.

It has to be noted though that from the results obtained and displayed in Table 7.6, that the closeness of the automated positioning results to conventional survey results are an indicator that the calibration adjustment process may have very little impact on the final coordinate results. This also may be due to the effect of close proximity of the laser scanner to the beacons, however, the calibration process requires further investigation to ensure continued consistency of results with conventional surveying methods.

7.4.4 Impact of MAST Positioning Process

This research has shown that the MAST positioning process is an accurate, fast, efficient and robust method of determining a mobile platform’s position and orientations in a GPS deprived environment. It has been also shown from experimental results how small the deviation of the results are in comparison to conventional survey results and also the similarity with the accuracy of the simulated results obtained in [38].
7.4.5 Impact of Survey Alignment Errors on Map Accuracy

As the alignment is based on start and endpoint positioning using the MAST process, errors in the positioning process have the likelihood of making the alignment slightly incorrect. The effect of this error after adjustment will tend to rotate the overall traverse rather than scale the traverse. It is the author’s belief that as shown from Table 7.6 the MAST process and its derived positions are least likely to contribute adversely to traverse alignment errors and subsequently mapping accuracy. This is also shown from Table 7.9 by the number of issues found relating to positioning.

As the mapping is tied to the traverse alignment, similar feature position errors are experienced which magnify the RMSE errors when calculating the map accuracy. Each traverse alignment and subsequent traverse adjustment is based on four calculated scanner positions. From the fifteen uGPS traverses undertaken, only one calculated position or 3.3% of the overall calculated positions had an issue resulting from incorrect segmentation (File #44, Scan 1430, page 346).

In two experiments, less than 50% of the total check points were available along a traverse route. These check points in relation to the traverse fell on sidewalls where the survey/mapping vehicle tended to turn away creating a data sparsing effect.

7.5 Conclusion

The ILC is a difficult environment for mobile mapping as it has very few features. As a result all data prior to ‘closing the loop’ using the MAST algorithm exhibited linear closing errors ranging from 0.195 m to 7.656 m. While a small closing error is good, its relation to the length of traverse is the basis for traverse accuracy. Analysis of the traverses showed that all traverses fell below the expected traverse accuracy (1:5 000)
as a result of the data input from the uGPS. At this moment, in the absence of data pertaining to a different type of environment, it is impossible to exclusively indicate that the problem is environmental.

The fact that positioning issues exist due to segmentation indicates that further robustness needs to be built into the algorithm as it is a crucial component of the positioning system.

As the mapping process is directly connected to the traverse, improvement in the traverse, especially the inputs into the adjustment will have a huge impact on improving the map accuracy.

Conventional surveying is an indispensable tool to the generation of maps for the underground environment as checkpoints are surveyed and used to determine map accuracy.
### Table 7.9: Comparison of Map Traverses to As-built Survey

<table>
<thead>
<tr>
<th>File No.</th>
<th>Wall/Ramp Feature Encroachment Pre-adj</th>
<th>Post-adj</th>
<th>Distance Direction Error(^2)</th>
<th>MAST Position Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>X</td>
<td></td>
<td>X</td>
<td>4-7-17</td>
</tr>
<tr>
<td>24</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>6-22-24</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>2-12-15</td>
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<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td>2-19-24</td>
</tr>
<tr>
<td>27</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>No Converge</td>
</tr>
<tr>
<td>28</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>5-13-14</td>
</tr>
<tr>
<td>29</td>
<td>X</td>
<td></td>
<td>X</td>
<td>2-9-17</td>
</tr>
<tr>
<td>33</td>
<td>X</td>
<td></td>
<td>X</td>
<td>1-12-15</td>
</tr>
<tr>
<td>34</td>
<td>X</td>
<td></td>
<td></td>
<td>2-9-12</td>
</tr>
<tr>
<td>36</td>
<td>X</td>
<td></td>
<td></td>
<td>1-7-17</td>
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<td></td>
<td>X</td>
<td>2-17-24</td>
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</tr>
<tr>
<td>44</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>5-14-19</td>
</tr>
</tbody>
</table>

| Ratio    | 13/15 | 9/15 | 10/15 | 5/15 | 1/30 |

\(^1\) Checkpoint Mapping Criteria: \#Incorrect - \#Tested - Total traverse checkpoints

\(^2\) Distance Direction Errors in the East - West and North - South directions
Chapter 8

Conclusions and Future Work

In this final chapter a summary of the research outcomes and contributions, conclusions, recommendations and future work are presented.

8.1 Research Outcomes

The following lists outcomes of this thesis research.

1. A geometric reference beacon was designed and implemented.

2. A beacon calibration process and associated algorithms were developed and implemented.

3. Geometric reference beacons were integrated with a mobile mapping system to collect data for analysis.

4. A technique to reference LiDAR-based measurements to a beacon-based global reference system (MAST), that is LiDAR system independent, was developed and implemented.
5. The MAST process was tested and validated using the uGPS Rapid Mapper™, by determining start and end of traverse scanner position and orientation automatically within 3 cm and 2° respectively of true value as well as determination of mapping accuracy.

In the context of research questions posed at the beginning of this thesis, namely:

1. Can the surveying and mapping process be improved in terms of production speed and positional accuracy?
   
   This research has shown that the surveying and mapping process can be improved in terms of production speed through the integration of conventional surveying and mobile mapping, however, in the current test environment, positional accuracy of the mapping did not meet current mapping standards most of the time. It is anticipated that in a feature rich underground environment positional accuracy and mapping standards are more likely to be met as this is the type of environment that the uGPS was designed to operate in.

2. Can mobile mapping be adapted so as to provide geometric maps that can be a suitable replacement for conventional underground maps?

   This research has provided a framework for geometric mapping of the underground environment with quantifiable accuracy and shown that achieving current mapping standards is possible in some cases, therefore extending this research to a mining environment to validate this research is a necessary step.

3. Can conventional surveying and mobile mapping be combined into an efficient tool that can be used by the mining industry?
In this case the proof of concept points toward an increased efficiency of the mapping process due to integration of conventional surveying methods and mobile mapping. Also it has been established in this thesis that this proof of concept is worth extending to an actual underground mine experiment where use in a mining environment can be established.

### 8.2 Primary Contributions

The primary contributions from this thesis research include:

1. A novel concept and design of passive geometric referencing beacons used to provide robust positioning and orienting of LiDAR-based mobile scanners operating in the underground environment and calibration of these beacons.

2. A novel framework (MAST) that is LiDAR system independent, for referencing mobile mapping data to a global mine reference system using geometric referencing beacons.

3. Proof of concept that the geometric reference beacons in conjunction with the MAST positioning process can provide positioning to within 3 cm of true position and within 2° of the true azimuth for traverse start and end points.

4. A framework for automated mobile mapping with quantified accuracy from an automated mapping process.

### 8.3 Recommendations to uGPS Rapid Mapper™ Procedures

Below are some of the recommendations for the uGPS process that come about as a result of this research:
1. Further research is required into calibration of the functionality of the uGPS system and specifically how distance between consecutive poses is calculated especially for difficult environments such as the ILC.

2. It was noted from Table 7.9 through visual inspection of the produced maps that there is a greater scale change or error in distance in the map east-west direction than in the north-south direction. Further research is necessary to investigate the cause of this issue to ensure that scale change is uniform.

3. The traverse is supposed to close at the end regardless of input data. A wide range of traverse misclosure distances were observed that indicated failure to ‘close the loop’ in this environment. Further research should investigate the ‘loop closure’ convergence failure problem.

4. Improvement to the uGPS tag profiling process should consider the concept from conventional GPS, that a minimum of three distance rays are required for a position solution, therefore is the uGPS solution not missing one degree of freedom i.e. a direction to enable a unique solution?

5. A fusion with odometry data will improve the distance solution for difficult environments such as the ILC where there are straight smooth and featureless walls.

8.4 Conclusions and Future Work

As stated at the beginning of this thesis, the ability for a robot or mobile mapping system to automatically determine its position and orientation in a known or global reference system especially in GPS deprived environments is challenging and
important. This research has provided a high level outline showing how an automated surveying and mapping process with quantifiable results can be systematically obtained.

The MAST process for determining the initial position and orientation of a mobile platform gives rise to the possibility of a new generation of mobile robotic algorithms whose basis is the accurate knowledge of the mobile robot location and orientation. An example given at the beginning of this thesis was that of automated drilling, where automated drill face mapping, drill pattern setting out and drilling is a reality, however, the rig still needs to be driven to the face, and during the blasting and venting process people are required to be out of the operation area. The ability for automated tramming which is also a reality, combined with automated drilling and automated positioning using MAST should now make it possible that the whole process can be automated.

Continuing with the automated drilling example, the MAST process would provide the initial position \( xyz \), and orientation of the mobile drill platform in the mine reference system. Automated tramming would guide the mobile platform to its intended location based on a coordinate value. Placement of intermediate MAST beacons would enable re-alignment and positioning, allowing the mobile drill platform algorithms to correct for drift. On getting to the prescribed location, automated drillrig setup and drilling occurs with minimal human input and at its end the drill rig automatically moves out to a safe zone for the blasting, venting, roof bolting and mucking processes, whilst keeping constant note of its position and orientation in the mine reference frame. As soon as these other operations are completed, the drill rig resumes its tramming, positioning, orientation, drill setup, face mapping, drill pattern
setting out and drilling cycle once more.

In a mapping system, MAST beacons would serve as start and end locations for mapping traverses allowing for opening and closing of a mapping exercise that will be automatically oriented, corrected and adjusted to the mine reference system. The ability to obtain start and closing positions and elevations makes it possible to adjust the surveys and mappings according to conventional surveying principles. In a planetary exploration example where there is no GPS, the position and orientation of a rover’s dock may be accurately known in a local reference frame. The rover can then go out to explore and map the area and come back on the basis of the position and reference frame of its dock.

The drawbacks in all these examples is the ability of the mobile platform algorithms to minimize drift in their systems, determine accurately their relative position and orientation such as to ensure convergence at the end of the traverse and accurately map in a defined reference frame. Therefore, in the context of the uGPS Rapid Mapper™, in addition to the recommendations in the previous section, an ability to ensure automatic recognition of the environment in order to apply different algorithms that will ensure accurate relative positioning is also required. The ability to define a mapping reference frame and orient scans to it would make oriented volumetric maps or point clouds in a defined mine reference system more meaningful.

In the context of the MAST positioning system, there is still the need for further investigation in the following areas:

1. This proof of concept should be repeated in a feature rich environment, ideally in an underground mine.
2. There is need to further optimize the calibration, segmentation and ICP processes.

3. Determining the best beacon size and design to use in a tunnel of a given radius as mentioned in Chapter 5 through simulation.

4. Standards for underground mobile mapping should be developed and modelled according to conventional surveying.

5. Since we live in a 3D world, 2D positioning and orienting is not ideal for a 3D environment, how 3D position, orientation and elevations can be obtained.
Bibliography


[13] 


[14] 


[15] 


[16] 

**Borenstein, J., and Feng, L., Eds.** *Gyrodometry: A New Method for Combining Data from Gyros and Odometry in Mobile Robots* (April 1996), IEEE.

[17] 


[18] 


[19] 


[20] 

**Burtch, R.** Traverse and traverse adjustment. Tech. rep., Surveying Engineering Department, Ferris State University, 2011.

[21] 


Appendix A

Solution for Least Squares Adjustment of Indirect Observations

The matrix form of the model for the least squares technique of Adjustment of Indirect Observations is:

\[ v_{n,1} + B_{n,u} \Delta u_{1} = f_{n,1} \]  \hspace{1cm} (A.1)

The subscripts denote the matrix sizes where \( n \) denotes the number of condition equations to be written, \( u \), the number of unknown parameters (which is equal to \( n_0 \), the minimum number of observations required to uniquely determine the model). The condition equations include both observations and parameters and each condition equation contains only one observation with a unit coefficient.

The following steps are taken to obtain a solution by the least squares technique of adjustment of indirect observations. First, for uncorrelated measurements \( \sigma_1...\sigma_n \),
a variance matrix of the following structure is formulated:

$$
\Sigma = \begin{bmatrix}
\sigma_1^2 & 0 & \ldots & 0 \\
0 & \sigma_2^2 & 0 & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \vdots & 0 & \sigma_{n-1}^2 \\
0 & \vdots & \vdots & 0 & \sigma_n^2
\end{bmatrix}
$$

(A.2)

A proportionality constant or reference variance $\sigma_0^2$, usually of unit weight is used to formulate a weight $w$, for each of the measurements such that:

$$
w = \frac{\sigma_0^2}{\sigma^2}
$$

(A.3)

The corresponding weights for the measurements are formulated into a weight matrix $W$ as follows:

$$
W = \begin{bmatrix}
w_1 & 0 & \ldots & 0 \\
0 & w_2 & 0 & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \vdots & 0 & w_{n-1} \\
0 & \vdots & \vdots & 0 & w_n
\end{bmatrix}
= \sigma_0^2 \begin{bmatrix}
1/\sigma_1^2 & 0 & \ldots & 0 \\
0 & 1/\sigma_2^2 & 0 & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \vdots & 0 & 1/\sigma_{n-1}^2 \\
0 & \vdots & \vdots & 0 & 1/\sigma_n^2
\end{bmatrix}
$$

(A.4)

The weight matrix $W$ is an indicator of the precision of the individual observations and is the product of an a-priori reference variance $\sigma_0^2$ and the inverse of the variance matrix $\Sigma$. The coefficient matrix of the normal equations $N$, is calculated from:

$$
N = B^TWB
$$

(A.5)
The vector of constant terms $t$, is calculated from:

$$ t = B^T W f $$  \hspace{1cm} (A.6)

$\Delta$, a vector of the parameter estimates is calculated from:

$$ \Delta = N^{-1} t = (B^T W B)^{-1} B^T W f $$  \hspace{1cm} (A.7)

The residuals from the calculation $v$, are obtained from:

$$ v = f - B \Delta $$  \hspace{1cm} (A.8)

$Q_{\Delta\Delta}$ a cofactor matrix of $\Delta$ and $\Sigma_{\Delta\Delta}$, a covariance matrix of adjusted position are obtained as:

$$ Q_{\Delta\Delta} = (B^T W B)^{-1} $$  \hspace{1cm} (A.9)

$$ \Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta} $$  \hspace{1cm} (A.10)
Appendix B

Solution for Least Squares Adjustment of Observations Only

The matrix form of the model for the least squares technique of Adjustment of Observations Only is:

\[ A_{r,n}v_{n,1} = f_{n,1} \]  \hspace{1cm} (B.1)

Here the subscripts denote the matrix sizes where \( r \) denotes the redundancy or statistical degrees of freedom and \( n \), the number of given observations. \( r \) equations are written in terms of the \( n \) unknown residuals where \( r \) is found from \( r = n - n_o \) and \( n_o \) represents the minimum number of elements required to uniquely determine the model. \( A \) is a coefficient matrix of the observations, \( v \), a matrix of residuals and \( f \), the condition equations constant terms vector. No parameters are included in the condition equations.

\[ Q = W^{-1} \]  \hspace{1cm} (B.2)
The normal equations for this technique of adjustment are written as:

\[ Q_e = AQA^T \quad (B.3) \]

and \( Q_e \) is a cofactor matrix of equivalent observations. A weight matrix of the equivalent observations \( W_e \), is found from:

\[ W_e = Q_e^{-1} \quad (B.4) \]

The vector of Lagrange multipliers \( k \), is found from:

\[ k = W_ef \quad (B.5) \]

where \( f \), represents the condition equation constant term vector. The residuals \( v \), are calculated from:

\[ v = QA^Tk \quad (B.6) \]

The adjusted observations \( \hat{l} \) are obtained from

\[ \hat{l} = l + v \quad (B.7) \]

Applying the law of propagation of cofactors the cofactor matrix of the adjusted observations \( Q_{\hat{l}\hat{l}} \) is obtained from

\[ Q_{\hat{l}\hat{l}} = Q - QA^TW_eAQ \quad (B.8) \]

For the derivation of \( Q_{\hat{l}\hat{l}} \), the reader is referred to [31]. The covariance matrix of the
adjusted observations is obtained from

\[
\Sigma_{\tilde{l}} = \sigma_0^2 Q_{\tilde{l}}
\]

(B.9)
Appendix C

Error Analysis Derivation for Traverse, Intersection and Trilateration Surveys

For both traverse and trilateration surveys, the first step defines a test statistic, $T_S$, as:

$$T_S = \frac{v^T W v}{\sigma_0^2}$$  \hspace{1cm} (C.1)

Let us also define $\hat{\sigma}_0^2$, the a-posteriori estimate of the reference variance as:

$$\hat{\sigma}_0^2 = \frac{v^T W v}{r}$$  \hspace{1cm} (C.2)

where $r$, is the redundancy (number of degrees of freedom). The post adjustment statistical analysis process starts by making a global test on $T_S$, followed by hypothesis testing at a chosen level of significance to determine whether the true variance of unit weight, $\sigma_0^2$, is equal to the assumed variance of unit weight, $\hat{\sigma}_0^2$. A confidence interval and a confidence region are generated for the adjusted position.
Let $H_0$ and $H_1$ represent the null and alternate hypothesis. Specifically:

$$H_0 : \sigma^2 = \sigma_0^2$$

$$H_1 : \sigma^2 > \sigma_0^2$$

A level of significance $\alpha$ is chosen under which the hypothesis statements above are examined. The assumption made is $\sigma_0^2$ follows a chi-distribution with $n-1$ degrees of freedom. Therefore, $H_0$ is accepted if the following probability statement is satisfied:

$$P \left[ \frac{\chi^2_{(a,r)} \sigma_0^2}{r} < \frac{\nu^TW \nu}{\sigma_0^2} < \frac{\chi^2_{(1-a,r)} \sigma_0^2}{r} \right] = 1 - \alpha$$  \hspace{1cm} (C.3)

Under the premise that $H_0$ is accepted, the confidence interval for each parameter of the adjusted position must satisfy the following probability statements:

$$P[\hat{X}_P - z_\frac{\alpha}{2} \sigma_{\hat{X}_p} > \mu_X > \hat{X}_P + z_\frac{\alpha}{2} \sigma_{\hat{X}_p}] \quad P[\hat{Y}_P - z_\frac{\alpha}{2} \sigma_{\hat{Y}_p} > \mu_Y > \hat{Y}_P + z_\frac{\alpha}{2} \sigma_{\hat{Y}_p}]$$  \hspace{1cm} (C.4)

leading to the following confidence intervals:

$$\hat{X}_P \pm z_\frac{\alpha}{2} \sigma_{\hat{X}_p} \quad \hat{Y}_P \pm z_\frac{\alpha}{2} \sigma_{\hat{Y}_p}$$  \hspace{1cm} (C.5)

Here, $z_\frac{\alpha}{2}$ is a standard normal critical value and $\sigma_{\hat{X}_p}$ and $\sigma_{\hat{Y}_p}$ are the relevant values from the covariance matrix. Where the null hypothesis is rejected in favour of the alternate, suggesting that the true variance of unit weight is greater than the reference
variance, the covariance matrix associated with the adjusted position, $\hat{\Sigma}_{\Delta \Delta}$ is:

$$\hat{\Sigma}_{\Delta \Delta} = \hat{\sigma}_0^2 Q_{\Delta \Delta} = \hat{\sigma}_0^2 (B^T W B)^{-1}$$  \hspace{1cm} (C.6)

where $\hat{\sigma}_0^2$ is derived from equation Under this premise, the confidence interval for each parameter of the adjusted position must satisfy the following probability statements:

$$P[\hat{X}_P - t \hat{\sigma}_{\hat{X}_P} > \mu_X > \hat{X}_P + t \hat{\sigma}_{\hat{X}_P}] \quad P[\hat{Y}_P - t \hat{\sigma}_{\hat{Y}_P} > \mu_Y > \hat{Y}_P + t \hat{\sigma}_{\hat{Y}_P}]$$  \hspace{1cm} (C.7)

leading to the following confidence intervals:

$$\hat{X}_P \pm t_r \hat{\sigma}_{\hat{X}_P} \quad \hat{Y}_P \pm t_r \hat{\sigma}_{\hat{Y}_P}$$  \hspace{1cm} (C.8)

Here, $t_r$ is a $t$ critical value based on the chosen level of significance, with $r$ degrees of freedom.

Determining the confidence region takes as input the covariance matrix. First the eigenvalues of the covariance matrix are calculated, these determine the components of the error region, specifically, the larger of the two eigenvalues, the length of the semi-major axis $\sigma_{x'}$, and the smaller, the semi-minor axis length $\sigma_{y'}$ according to:

$$\sigma_{x'} = \sqrt{\lambda_1}$$  \hspace{1cm} (C.9)

$$\sigma_{y'} = \sqrt{\lambda_2}$$  \hspace{1cm} (C.10)

The orientation of the ellipse, $\theta$, is determined from the components of the covariance
matrix itself as the angle between the $x$ axis and the semi-major axis from:

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$  \hfill (C.11)

Figure C.1 shows the components of an error ellipse.

Given a confidence level, $P$ or $1-\alpha$, if $H_0$ were accepted, a scale factor, $C$, is used to correct the ellipse parameters. $C$ is defined as

$$C = \sqrt{\chi^2_{(P,n)}}$$  \hfill (C.12)

Alternatively, if $H_1$ were accepted, $C$ would be defined as

$$C = \sqrt{2F_{(P,n,r)}}$$  \hfill (C.13)

where $F$ is the F-distribution and $n$ is the number of degrees of freedom in your solution $(x,y)$ which is 2 and $r$ is the redundancy.
For the *technique of adjustment of observations only*, the propagated covariance matrix is obtained from using equation (B.8).

For any horizontal positioning method there are specifications to which a survey must adhere. For trilateration, the length measurement standard error $\sigma_m$, is calculated as

$$\sigma_m = \sqrt{\frac{\sum y^2}{n(n-1)}} \quad (C.14)$$

where $y$ represents the residuals from the calculation and $n$ the number of observations. Positioning accuracy would be written as 1 part in $\frac{1}{\sigma_m}$. This equation is obtained from [19].
Appendix D

Results Figures

This Appendix lists all the other figures pertaining to experiments undertaken.
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Figure D.7: File # 24 calculated check point locations overlay onto as-built survey
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Figure D.9: Position and orientation of Scan 609 in mine reference frame
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Figure D.13: Overlay of File # 25 Automated map and as-built survey
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Figure D.16: Position and orientation of Scan 882 in mine reference frame
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Figure D.21: File # 26 calculated check point locations overlay onto as-built survey
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Figure D.23: Position and orientation of Scan 1038 in mine reference frame
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Figure D.25: Scan 1038 convergence criteria at traverse end
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Figure D.43: Position and orientation of Scan 26 in mine reference frame
Scan # 1301, File: 31
Correct Scan Orientation in Mine Reference Frame

Scan # 1301  File: 31
Arbitrary Scan Orientation in Scanner Reference Frame

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Figure D.58: Position and orientation of Scan 1030 in mine reference frame
Figure D.59: Scan 61 convergence criteria at beginning of traverse
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Figure D.60: Scan 1030 convergence criteria at traverse end
Figure D.61: File # 34 raw traverse overlay onto as-built survey
Figure D.62: Overlay of File # 34 Automated map and as-built survey
Figure D.63: File # 34 calculated check point locations overlay onto as-built survey
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Figure D.72: Position and orientation of Scan 1508 in mine reference frame
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Scan # 706, File: 38
Correct Scan Orientation in Mine Reference Frame

Figure D.79: Position and orientation of Scan 706 in mine reference frame
Figure D.80: Scan 107 convergence criteria at beginning of traverse
Figure D.81: Scan 706 convergence criteria at traverse end
Figure D.82: File # 38 raw traverse overlay onto as-built survey
Figure D.83: Overlay of File # 38 Automated map and as-built survey
Figure D.84: File # 38 calculated check point locations overlay onto as-built survey
Figure D.85: Position and orientation of Scan 93 in mine reference frame
Figure D.86: Position and orientation of Scan 1537 in mine reference frame
## Convergence Results Scan Position # 93 File #: 39

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**Figure D.87:** Scan 93 convergence criteria at beginning of traverse
Figure D.88: Scan 1537 convergence criteria at traverse end
Figure D.89: File # 39 raw traverse overlay onto as-built survey
Figure D.90: Overlay of File # 39 Automated map and as-built survey
Figure D.91: File # 39 calculated check point locations overlay onto as-built survey
Figure D.92: Position and orientation of Scan 77 in mine reference frame
Correct Scan Orientation in Mine Reference Frame

Arbitrary Scan Orientation in Scanner Reference Frame

Figure D.93: Position and orientation of Scan 1543 in mine reference frame
Figure D.94: Scan 77 convergence criteria at beginning of traverse
Figure D.95: Scan 1543 convergence criteria at traverse end
Figure D.96: File # 41 raw traverse overlay onto as-built survey
Figure D.97: Overlay of File # 41 Automated map and as-built survey
Figure D.98: File # 41 calculated check point locations overlay onto as-built survey
Figure D.99: Position and orientation of Scan 65 in mine reference frame
Figure D.100: Position and orientation of Scan 1176 in mine reference frame
Figure D.101: Scan 65 convergence criteria at beginning of traverse
Figure D.102: Scan 1176 convergence criteria at traverse end

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Figure D.103: File # 42 raw traverse overlay onto as-built survey
Figure D.104: Overlay of File # 42 Automated map and as-built survey
Figure D.105: File # 42 calculated check point locations overlay onto as-built survey
Figure D.106: Position and orientation of Scan 47 in mine reference frame
Figure D.107: Position and orientation of Scan 1430 in mine reference frame
#### Convergence Results: Scan 93

**File #**: 44  
**Position**: 47

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**Figure D.108**: Scan 93 convergence criteria at beginning of traverse
Figure D.109: Scan 1430 convergence criteria at traverse end
Figure D.110: File # 44 raw traverse overlay onto as-built survey
Figure D.111: Overlay of File # 44 Automated map and as-built survey
Figure D.112: File # 44 calculated check point locations overlay onto as-built survey
Appendix E

Horizontal and Vertical Accuracy Standards
Table E.1: Specifications for vertical control surveys (Anderson and Mikhail, 1998).

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<thead>
<tr>
<th>Classification</th>
<th>First Order, Class 1 and Class 2</th>
<th>Second Order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Specifications</td>
<td>Class 1</td>
</tr>
<tr>
<td>Instrument standards</td>
<td>automatic or tilting levels</td>
<td>geodetic levels and invar</td>
</tr>
<tr>
<td></td>
<td>with parallel plate micrometers;</td>
<td>scale rods</td>
</tr>
<tr>
<td></td>
<td>invar scale rods</td>
<td></td>
</tr>
<tr>
<td>Field procedures</td>
<td>double run; forward and</td>
<td>double</td>
</tr>
<tr>
<td></td>
<td>backward, each section</td>
<td>or single run</td>
</tr>
<tr>
<td>Section length</td>
<td>1–2 km</td>
<td>1–3 km for</td>
</tr>
<tr>
<td>Maximum length of</td>
<td>50 m class 1; 60 m class 2</td>
<td>double run</td>
</tr>
<tr>
<td>sight</td>
<td></td>
<td>70 m</td>
</tr>
<tr>
<td>Field procedures</td>
<td>Maximum difference in -------------</td>
<td>5 m</td>
</tr>
<tr>
<td></td>
<td>lengths forward and backward sights per</td>
<td></td>
</tr>
<tr>
<td>Per section (cumulative)</td>
<td>2 m class 1; 5 m class 2</td>
<td>4 m class 1; 10 m class 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section, forward and</td>
<td>3 mm $K^{1/2}$ class 1;</td>
<td>6 mm $K^{1/2}$</td>
</tr>
<tr>
<td>backward</td>
<td>4 mm $K^{1/2}$ class 2</td>
<td>6 mm $K^{1/2}$</td>
</tr>
<tr>
<td>Loop or line</td>
<td>4 mm $K^{1/2}$ class 1;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 mm $K^{1/2}$ class 2</td>
<td></td>
</tr>
<tr>
<td>Accuracy of height</td>
<td>0.5 mm $K^{1/2}$ class 1;</td>
<td>1.0 mm $K^{1/2}$</td>
</tr>
<tr>
<td>difference, between</td>
<td>0.7 mm $K^{1/2}$ class 2</td>
<td></td>
</tr>
<tr>
<td>directly connected</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bench marks</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Check between forward and backward runnings where $K$ is the distance in kilometers.
Table E.2: Horizontal control accuracy standards for traversing (Anderson and Mikhail, 1998).

<table>
<thead>
<tr>
<th>Class</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Azimuth closure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>At azimuth check point</td>
<td>1.7√N</td>
<td>3.0√N</td>
<td>4.5√N</td>
</tr>
<tr>
<td>(seconds of arc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Position closure</td>
<td>0.04√K</td>
<td>0.08√K</td>
<td>0.20√K</td>
</tr>
<tr>
<td>After azimuth adjustment*</td>
<td>or</td>
<td>or</td>
<td>or</td>
</tr>
<tr>
<td></td>
<td>1:100,000</td>
<td>1:50,000</td>
<td>1:20,000</td>
</tr>
</tbody>
</table>

Note: N is number of segments, K is route distance in km.

*The expression containing the square root is designed for longer lines, where higher proportional accuracy is required. Use the formula that gives the smallest permissible closure. The closure (e.g., 1:100,000) is obtained by computing the difference between the computed and fixed values, and dividing this difference by K. Do not confuse closure with distance accuracy of the survey.
<table>
<thead>
<tr>
<th>Order</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Station spacing not less than (km)</td>
<td>10</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Maximum deviation of main traverse from straight line</td>
<td>20°</td>
<td>20°</td>
<td>25°</td>
</tr>
<tr>
<td>Minimum number of benchmark ties</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Benchmark tie spacing not more than (segments)</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Astronomic azimuth spacing not more than (segments)</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>Minimum number of network control points</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Instrumentation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theodolite, least count</td>
<td>0.2°</td>
<td>1.0°</td>
<td>1.0°</td>
</tr>
<tr>
<td>Field procedures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Directions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of positions</td>
<td>16</td>
<td>8 or 12**</td>
<td>6 or 8***</td>
</tr>
<tr>
<td>Standard deviation of mean not to exceed</td>
<td>0.4°</td>
<td>0.5°</td>
<td>0.8°</td>
</tr>
<tr>
<td>Rejection limit from the mean</td>
<td>4°</td>
<td>5°</td>
<td>5°</td>
</tr>
<tr>
<td>Reciprocal vertical angles (along distance sight path)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of independent observations direct/reverse</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Maximum spread</td>
<td>10°</td>
<td>10°</td>
<td>10°</td>
</tr>
<tr>
<td>Maximum time interval between reciprocal angles (hr)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Astronomic azimuths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations per night</td>
<td>16</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>Number of nights</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Standard deviation of mean not to exceed</td>
<td>0.45°</td>
<td>0.45°</td>
<td>0.6°</td>
</tr>
<tr>
<td>Rejection limit from the mean</td>
<td>5°</td>
<td>5°</td>
<td>5°</td>
</tr>
<tr>
<td>Infrared distances</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum number of measurements</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum number of concentric observations/observation</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum number of offset observations/observation</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Maximum difference from mean of observations (mm)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Minimum number of readings/observation</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Maximum difference from mean of readings (mm)</td>
<td>—</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Microwave distances</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum number of measurements</td>
<td>—</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum number of concentric observations/observation</td>
<td>—</td>
<td>2*</td>
<td>1*</td>
</tr>
<tr>
<td>Minimum number of readings/observation</td>
<td>—</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Maximum difference from mean of readings (mm)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

* 8 if 0.2°, 12 if 1.0° resolution.
** 6 if 0.2°, 8 if 1.0° resolution.
† Only if decimal reading near 0 or high 9s.
‡ As specified by manufacturer.
§ Carried out at both ends of the line.

Table E.3: Horizontal control specifications for traversing (Anderson and Mikhail, 1998).
Appendix F

uGPS Deployment Manual
This Presentation

- I will leave this presentation with you. It contains many useful details.
- For further information, consult the user’s manual or contact me
  - Email: jlavigne@pecktech.ca
  - Phone: (647) 770-5139

What is uGPS?

- uGPS – Underground Positioning System
- It is a technology platform that supports various functional elements:
  - Rapid underground mapping (Rapid Mapper™)
  - Underground positioning sensor

What is uGPS Rapid Mapper™?

- Rapid 3D point cloud acquisition for underground
  - Map data is collected at normal vehicle operating speeds (typically 3 – 10 mph)
  - Can be open loop (zero infrastructure) or georeferenced using RFIDs
  - 3D model can be imported into third party applications
  - Can be used for both horizontal and vertical mapping (shafts / raises)

Mapping Example 1

- In this example 2.2 miles of data collected in 45 minutes @ 3 mph
- +15 minutes post-processing time

Mapping Example 2

Small area acquired in higher density by reducing speed to ~0.5 mph
Slower capture = higher point density
Shaft Scanning Example

~5000 ft scanned at the Sanford Underground Laboratory (previously Homestake Mine)

Technical Specifications

- Dimensions: 8 x 12 x 12 in
- Weight (incl. cables): 26 lb
- Weight as shipped (incl. case): 46 lb
- All core system components are sealed against dust and moisture (IP67)
- Wide input range power supply (10 to 30 V DC)
- Wide operating temperature (-30 to +60 °C)
- Onboard WiFi optional
- Only ONE cable required (power)

Components

- RFID antenna
- WiFi antenna
- USB
- Power
- Ethernet
- Volume laser scanner
- Primary laser scanner

Mounting

- Trailer hitch receiver
- Rail locomotive
- Shaft conveyance

Mounting Considerations

- The scanner will work almost anywhere you can fit it
- Considerations:
  - Scanner must travel in the correct direction
  - Try not to block the laser’s field of view
  - Needs power

Mounting – Power

- 10-30 V DC input
- 25 W power consumption
- Up to 100 W in very cold conditions
- Recommend powering after starting vehicle
  - Voltage drop while cranking can cause the unit to reset

The supplied cable has fused cigarette lighter end. If you modify the cable, add a 10 A fuse inline.
Mounting – Direction of Travel

- Correct axis of travel
  - YES
  - NO

Mounting – Field of View

- Two planar laser scans with 270° field of view
- Try not to block either

uGPS Rapid Mapper™ Operating Principle

- Vertical plane of view from laser scanner
- Volume swept by sensor motion
- Direction of movement
  - Note: blind spot

WiFi

- Connect wirelessly!
- SSID: uGPS Rapid Mapper
- Password: 12345678
- Also, 100 mbps ethernet:
  - 192.168.20.* subnet

uGPS Rapid Mapper™ Interface

- IP address 192.168.0.1
- Top-down view of laser profile
- Scale
- Serial number & hour meter
- Drift cross-section
- Speedometer
- Start & stop scan buttons
- Calibration (click to expand)
- List of saved scans
- ID & firmware version

uGPS Rapid Mapper™ Configuration

- Identifier for this uGPS unit
- Sets a password here to enable security
- Auto-starts recording when powered
- Sets target acquisition speed
- Sets system time & date
- Sets metric or US customary units
- Sava changes
- Selects mounting: forward or backward
- Selects metric or US customary units
The scanner needs to be periodically calibrated. Here the calibration has expired (12 hours), so the calibration tool demands attention.

Park the vehicle on a level surface and press calibrate. The scanner must remain stationary for 10 seconds.

Movement or vibration during this period will cause the calibration to fail, and you will need to restart.

Press Start to end the scan.

Drive! The speedometer turns green when traveling near the configured target speed, or red for "too fast".
You are prompted to describe what was just done. The description is for your own benefit later when sorting through files.

It’s useful to note start & end locations, any conditions that may impact scan quality (e.g., vehicle or personnel traffic).

You can re-open and edit the descriptions at any time later by clicking the file name.

Two ways:
- Download from web interface (previous slide)
- USB sync:
  - Create a folder called "ugps_data" in the root of a USB drive
  - Insert it into this USB port
  - All scan data is automatically sync’ed to the USB drive
  - Use a drive with a blinking activity light: when it stops blinking, it’s finished

Raw scan files cannot be directly used by other software packages (yet)
- A conversion step is necessary to generate point cloud formats
  - We will send improved versions of the SW as they become available

* Look for it in the next release of Vulcan

Default options will work 99% of the time
- Selects a raw scan file
- Restricts the laser field of view
- Laser range filtering
- Point downsampling
- Selects output type & units

Selects a raw scan file
Restricts volume laser field of view
Selects output type & units

Default options will work 99% of the time
### Post-Processing Software

Overrides tab selected

Can be used to override the calibration (e.g., if it wasn’t done before the scan)

Not used much

### Example workflow

1. Install uGPS platform via mounting bracket and energize
2. Energize tablet and connect to uGPS platform via on-board WiFi
3. Ready for data collection - record & drive
4. Stop recording - offload data via USB stick
5. Post-processing -> XYZ or .LAS file
6. Import .XYZ or .LAS file -> GMP software (Maptek, Micromine, etc.)

### Raw data

XYZ or .LAS 3D point cloud format

Import to GMP for manipulation

For Vulcan, use .LAS

### Watch out for... Dirty Lenses

"Spikes" can mean the lens is dirty.
(\Can also just be hitting a non-reflective surface.)

If the lens is scratched, it needs to be replaced (cheap & easy)

### Watch out for... Airborne Dust

- Severe airborne dust can impact scan quality
- One of the reasons it’s better to mount on the front of vehicle

Demo Time
Watch out for... Loose Hitch Receivers

- Hitch receivers are convenient, but wobble and sag will impact results
- Unit should be installed level
  - Laser scan should see the ribs, not the ground
- No-wobble hitch bolts are useful

Open Loop Accuracy

- Recently completed an experimental comparison against survey
- Drift error <2% typical
- Ring or “rib to rib” error is constant ±3 cm

Volumetric Accuracy

- Because the “rib to rib” or “ring” accuracy is constant, the uGPS Rapid Mapper™ will generate volumes as accurate as any tripod-mounted scanner
- But much, much faster

Application: Shotcrete Thickness

- One way to measure shotcrete is to choose easily identified start & end points
- Import to GMP & align by hand
- Or use volumes

RFID Tags & Georeferencing

- Scans shown so far are open loop (unreferenced)
- RFID tags can be used to “close the loop” and tie the scan to survey
- Some setup is required but there are benefits: georeferencing and improved accuracy
RFID Tags & Georeferencing

Tags installed & surveyed (one-time setup)

Recommend the Omni-ID Dura 3000 tag. Order from Omni-ID at around $10 each.

Application: Breakthrough Headings

For this application, can calculate the "last tag" locations using the 2% drift error rule.

RFID Tag Installation

• Tags installed on the back
  – Mesh is convenient for attachment
  – Alternately, can use spads

RFID Tag Profiling

• With tags installed, need to visit each tag with the unit to register into a “tag database”
• Use the laser pointer to align the scanner with the surveyed point
• The web interface has a Tag Install Tool to help in this process

Tag Install Tool
Create a new .tagdb file

Click to select the file. This one has no tags (yet)

Snapshot button turns orange when a tag is within range. Box should be parked directly under the tag at this point.

Click it to take a snapshot of the local area

This information is used to precisely localize w.r.t. the surveyed point when passing near this area later.

Tag height measured

Unique Tag ID

Scan of surrounding area

Tag label

Surveyed tag coordinates (usually convenient to add these later)
Note: tags without coordinates will be ignored.

Save the tag.
Updated list of tags in the file.

Rinse & repeat for all tags.

Download the completed tag file, or sync to USB.
DON'T delete it yet! Leave it here to update it later.

Data processing – open loop: discussed earlier.

The tag database contains the information needed for georeferencing. Save & reuse it.
Post-Processing Software

Georeferencing Tips

- Need to match at least two tags for alignment to be possible
  - Otherwise: tag file error
- Ensure the coordinates entered in the tag file are correct
  - Otherwise: alignment incorrect (no error message)
- Tags should be installed in visually “interesting” locations
  - Otherwise: alignment may be incorrect (no error message)

Georeferencing Tips

- The area within sight of the tag is assumed to be static
- If the area changes, need to update the tag file
  - Just repeat the tag registration process for this tag

Application: Convergence

- Convergence is hard to detect due to the small scale of movement
- It is a problem we are actively working on solving with this scanner
- We are interested in your data

Safety Considerations

- Infrared LIDAR (invisible) is eye-safe (class I)
- The green (visible) laser pointer is not (class IIIa)
  - Use it as any other laser pointer
  - OK to look at the dot but don’t point it in your face
- RFID & WiFi radios are both active while unit is powered
  - Obey site requirements for radio transmitters

Jamie Lavigne, technical project lead
jlavigne@pecktech.ca

Andrew Chapman, technical sales lead
achapman@pecktech.ca