EXAMINING COGNITIVE PROCESSING IN CHILDREN WITH AN ARITHMETIC DISABILITY

by

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Abstract

Currently there is no consensus as to the specific cognitive impairments that characterize mathematical disabilities (MD) or specific subtypes such as an arithmetic disability (AD). The present study sought to address this concern by examining cognitive processes that might undergird AD in children. The present study utilized archival data to conduct two investigations. The first investigation examined the executive functioning and working memory of children with AD. An age-matched achievement-matched design was employed to explore whether children with AD exhibit developmental lags or deficits in these cognitive domains. While children with AD did not exhibit impairments in verbal working memory or colour word inhibition, they did demonstrate impairments in shifting attention, visual-spatial working memory, and quantity inhibition. As children with AD did not perform more poorly than their younger achievement-matched peers on any of these tasks, impairments in specific areas of executive functioning and working memory appeared to reflect a developmental lag rather than a cognitive deficit. The second study examined the phonological processing performance of children with AD compared to children with comorbid disabilities in arithmetic and word recognition (AD/WRD) and to typically achieving (TA) children. Results indicated that, while children with AD did demonstrate impairments on all isolated naming speed tasks, trail making digits, and memory for digits, they did not demonstrate impairments on measures of phonological awareness, nonword repetition, serial processing speed, or serial naming speed. In contrast, children with AD/WRD demonstrated impairments on measures of phonological awareness, phonological short-term memory, isolated naming speed, serial processing speed, and the alphabet a-z task. Overall, results suggested that phonological processing impairments are more prominent in children with a WRD than children with an AD. Together, these studies further our understanding of the nature
of the cognitive processes that underlie AD by focusing upon rarely used methods (i.e., age-matched achievement-matched design) and under-examined cognitive domains (i.e., phonological processing).
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Mathematical disabilities (MD) are characterized by chronically and significantly low achievement in mathematics, which is related to cognitive deficits rather than intelligence or developmental experiences (Geary, 2011). Those with MD exhibit a range of difficulties such as word problem solving, number sense, retrieval of basic arithmetic facts, and mathematical vocabulary (Bryant & Bryant, 2008; Bryant, Bryant, & Hammill, 2000). Children with MD employ developmentally immature counting strategies and experience difficulty coordinating multi-step mathematics problems (Bryant & Bryant, 2008). One of the most prominent findings across children with MD is difficulty developing calculation competency (Bryant et al., 2000).

The prevalence of MD is approximately 5-7% in the general population (Geary, 2011; Gross-Tsur, Manor, Shalev, 1996). A growing body of evidence suggests that the difficulties faced by those with MD extend beyond the mathematics classroom and general academic environments. A recent national survey found that those with learning disabilities (LD) experience higher rates of unemployment, mental illness, and illiteracy (LDAC, 2007). Students with LD have also been found to experience lower graduation rates and therefore diminished enrolment in post-secondary institutions (Montague & Jitendra, 2012).

Furthermore, early numeracy skills, precursors to a range of mathematics skills, are strong predictors of higher graduation rates, greater financial security, and successful employment (Bynner & Parsons, 2006; Pagani, Fitzpatrick, Archambault, & Janosz, 2010). Conversely, early numeracy difficulties and poor mathematics achievement for MD individuals pose substantial risks for acquiring and maintaining employment, experiencing mental health
challenges (e.g., depression), and incarceration (Bynner & Parsons, 2006). Taken together these findings can paint a potentially dire future for those with MD.

Research on reading disabilities spans a wide variety of disciplines: including cognitive psychology (Hoskyn & Swanson, 2000; Stanovich & Siegel, 1994), curriculum studies (Deno, Fuchs, Marston, & Shin, 2001), and academic interventions (Foorman et al., 1997; Fuchs, Fuchs, & Compton, 2004). In light of the myriad studies examining reading disabilities across a number of contexts, research into mathematical disabilities is in its infancy (Kulak, 1993). Indeed, Murphy, Mazzocco, Hanich, and Early (2007) searched peer-reviewed literature published during 1985 to 2006 and found that research papers on dyslexia outnumbered those on MD by a ratio of nearly 5 to 1. Research into reading disabilities has lead to the classification of three RD subtypes: word recognition disability, fluency disability, and reading comprehension disability (Fletcher, Lyon, Fuchs, & Barnes, 2007). While there is currently no consensus regarding the cognitive underpinnings of MD or the classification of MD subtypes, three prominent theoretical models have emerged.

Geary proposed a model that suggested individuals with MD exhibit cognitive impairments in three domains: semantic memory, procedural knowledge, and visual-spatial representation (Geary, 1993). Visual-spatial impairments are thought to involve difficulty organizing spatial representations in the visual-spatial sketchpad. Visual-spatial impairments may contribute to difficulties with number sense, specifically representing and comparing the magnitude of numbers (Geary & Hoard, 2005). Deficits in procedural knowledge are characterized by the use of inappropriate or developmentally immature strategies and by difficulty coordinating multi-step problems. Furthermore, it has been proposed that procedural knowledge difficulties may be the result of limited conceptual understanding of mathematical
concepts and impairments in cognitive systems, such as working memory or the central executive (Geary & Hoard, 2005). Semantic memory impairments are conceptualized as difficulty retrieving information from long-term memory. Geary proposed that the difficulties with retrieval of arithmetic facts from long-term memory exhibited by children with MD may have a semantic basis and may be associated with the same semantic and phonetic impairments that characterize RD (Geary, 1993; Geary & Hoard, 2005).

A second model, proposed by Fuchs et al. (2008), asserted that individuals with MD belong to one of two MD subtypes: those with problem solving difficulties or those with computation difficulties. There is evidence to suggest that these two groups share some common weaknesses, but also exhibit idiosyncratic impairments (Fuchs et al., 2006; 2008). While inattentive behaviour is associated with both MD subtypes, children with computational difficulties have demonstrated impairments in processing speed and phonological decoding (Fuchs et al., 2006; 2008). Further, children with problem solving difficulties have exhibited impairments in nonverbal problem solving, sight word efficiency, language, and concept formation (Fuchs et al., 2006; 2008).

The third model, the core deficit hypothesis, proposed that numerical representation and processing is a core deficit of MD (Landerl, Bevan, & Butterworth, 2004). While the sources of this deficit are contested, with some researchers proposing it is the product of a defective number module (Butterworth, 1999) and others implicating number sense (Dehaene, 1997), advocates of the latter hypothesis agree that MD children’s underachievement in mathematics is best explained by difficulties with representing and processing numerosity. In contrast, others view MD children’s underachievement as an artifact of impairments in cognitive systems such as working memory or the central executive (Landerl, Fussenegger, Moll, & Willburger, 2009).
In 2013, Szűcs and Goswami discussed the need for researchers to develop profiles of the cognitive impairments exhibited by individuals with MD. Cognitive profiles could assist researchers in classifying MD subtypes and in developing targeted interventions. Despite this mandate, there is currently no consensus as to the particular cognitive domains that underlie MD or proposed subtypes (i.e. arithmetic disability) (Watson & Gable, 2013). The present research sought to address this need by examining cognitive processes that might undergird MD in children. In particular, this research contributes to the field in two ways through two distinct studies. The first study explored two cognitive domains which are commonly regarded as the most promising avenues of research in the field, executive functioning and working memory (Watson & Gable, 2013). The second study explored the contributions of a cognitive domain that has been largely neglected empirically but proposed theoretically, phonological processing.

**Arithmetic versus Mathematics**

It is not uncommon for researchers to apply the term MD to describe individuals experiencing chronic and significant underachievement in one or more mathematical domains (Watson & Gable, 2013). This practice presents a challenge to the field given that mathematics refers to a number of skills and domains of knowledge and there is reason to suppose that individual mathematical skills are likely to be associated with disparate cognitive domains (Geary, 2004). Studies, that examine the cognitive underpinnings of a number of mathematical skills at once, risk committing both type I and type II errors. Indeed, it is possible that this practice could lead researchers to generalize (type I error) or fail to detect (type II error) domain-specific cognitive impairments. Therefore, if researchers wish to examine the cognitive functioning of individuals with MD, they should endeavor to classify participants using measures of multiple mathematical domains. Similarly, if researchers seek to evaluate individuals with a
learning disability specific to a particular mathematical domain, they should employ classification measures that assess the specified domain (Berg & McDonald, 2015). In consonance with this recommendation, the present studies will employ the term arithmetic disability (AD) and classify participants using the numerical operations subtest of the Wechsler Individual Achievement Test-Second Edition (WAIT-II, Wechsler, 2002).

Currently there is no standard method used to define MD (Watson & Gable, 2013). While some researchers employ standardized achievement measures (e.g., Berg, & McDonald, 2015), others classified their subjects by the presence of an achievement delay (van der Sluis, de Jong, & Leij, 2004), and some by subject enrollment in academic remediation programs (Andersson & Lyxell, 2007). Disparate classification procedures are also employed with respect to participant’s IQ scores. While some researchers require those classified with an MD to have an IQ score above a particular cut-off score to ensure participants fall within the normal range of intelligence (Berg, & McDonald, 2015; Swanson, 1993), others do not use IQ scores in their operational definitions of MD (Berg, 2008; McLean & Hitch, 1999). The use of inconsistent classification criteria for MD and possible subtypes make it impossible to generalize results across studies (Berg & McDonald 2015; Watson & Gable, 2013). The present studies’ classification procedures were designed to align with more commonly used definitions of MD and MD subtypes (Swanson & Jerman, 2001). Participants were classified as having an AD if they scored below the 25th percentile on the numerical operations subtest of the WAIT-II, and fell within the normal range of fluid intelligence (i.e., > 90 and < 120) on the Raven’s Colored Progressive Matrices test (RCPM, Raven, 1976).

Evidence is accumulating to suggest that difficulty developing calculation competency and disruptions in arithmetic fact retrieval are defining features of children with MD (Bryant et
al., 2000; Geary, 2004; Jordan, Hanich, & Kaplan, 2003). Some researchers have even proposed that difficulties in these domains may act as a rate limiting step in mathematics achievement (Gersten, Jordan, & Flojo, 2005). In other words, as students with MD have not developed calculation competency, they allocate cognitive resources disproportionately. Without proficiency in basic calculation and arithmetic fact retrieval, these skills become effortful processes that require cognitive resources (e.g., executive functioning, working memory) that could be spent on higher order mathematical concepts and skills (Gersten et al., 2005). A greater understanding of the cognitive processes that contribute to arithmetic disability in children may aid the development of targeted interventions.

**The Present Study**

The present research is comprised of two investigations. Data for both studies came from an archival data set that examined the cognitive processes that undergird children’s mathematics skills. Cognitive processes reflected domains such as memory, processing speed, executive functioning, and phonological processing. Areas of mathematics included skills such as arithmetic calculation, mental math, and mathematical reasoning. The sample included 204 children from Grades 1 to 5, from five schools located in Southern Ontario.

Theoretical and empirical evidence has supported the proposal that working memory and executive functioning play some role in MD (Blair & Razza, 2007; Mazzocco & Kover, 2007). However, the nature of this role remains unclear. In particular, one avenue that has been rarely explored is whether the working memory and executive functioning difficulties exhibited by AD children are more accurately viewed as a developmental lag or as a cognitive deficit. The first study employed an age-matched achievement-matched design to determine whether children with AD exhibited developmental lags or deficits in these cognitive domains.
There is some evidence to suggest the same phonological impairments that define RD may also be impaired in individuals with AD (Geary, 1993; 2013). Some researchers have proposed that difficulties in the retrieval of basic arithmetic facts and computation experienced by those with AD may be associated with interferences in the encoding or retrieval of phonetic and semantic representations of numbers and basic facts (Geary, 1993; 2013; Hecht, Torgesen, Wagner, & Rashotte, 2001). To explore this possibility, the second study examined the phonological processing performance of children with AD. In this investigation, children with AD were compared to children with comorbid disabilities in arithmetic and word recognition (AD/WRD) and to their typically achieving peers (TA). These comparisons provided an understanding of whether phonological processing impairments are exhibited in children with AD or are exhibited only by children with AD/WRD due to the latter group’s reading-related disability. Together, these two studies will further our understanding of the nature of the cognitive processes that underlie AD by focusing upon rarely used methods (i.e., age-matched achievement-matched design) and overlooked cognitive domains (i.e., phonological processing).
Chapter 2
Differentiating the Impairments in Executive Functioning and Working Memory of Children with an Arithmetic Disability

Introduction

Across the last few decades, research on reading disabilities spans a number of disciplines: including cognitive psychology (Hoskyn & Swanson, 2000; Stanovich & Siegel, 1994), curriculum studies (Deno, Fuchs, Marston, & Shin, 2001), and interventions (Foorman et al., 1997; Fuchs, Fuchs, & Compton, 2004). Compared to this multitude of research, the field of mathematical cognition and more specifically mathematical disabilities is in its infancy. Mathematical disabilities (MD) are characterized by chronically and significantly low achievement in mathematics, which is related to cognitive impairments rather than intelligence or developmental experiences (Geary, 2011). Recently, there has been an emphasis within the field to develop cognitive profiles of individuals with MD (Szücs & Goswami, 2013). These profiles could support the classification of MD subtypes and inform the development of targeted interventions. While there is currently no consensus as to the specific cognitive impairments that characterize MD, there is growing evidence to suggest that executive functioning and working memory merit further investigation (e.g., Watson & Gable, 2013).

Arithmetic versus Mathematics

Mathematics is a field that encompasses a wide variety of skills and domains of knowledge, including but not limited to: arithmetic, number sense, algebra, geometry, and problem solving. Thus, the term mathematical disability is often applied by researchers to describe children with a range of mathematical difficulties in a number of areas (Watson & Gable, 2013). As there is evidence to suggest that particular mathematical skills will involve
different cognitive domains (e.g., Geary, 2004), this practice poses an interpretative challenge. That is, assessment of a wide range of mathematical skills could potentially mask or generalize domain-specific cognitive impairments. Furthermore, it is possible that some children classified as having a MD, may only experience chronic and significant underachievement in a single domain of knowledge. Fuchs et al. (2008) proposed that a model of mathematical disability with two distinct subtypes: problem solving difficulties and computational difficulties. When reviewing extant literature, the terminology used by the authors was be used. This paper attempted to highlight those studies that may have employed the term MD while examining arithmetic calculation. The term arithmetic disability (AD) was applied to participants in this study as participants were classified using a test that specifically measures arithmetic calculation.

**Working Memory and Executive Functioning**

Researchers have proposed that both working memory and executive functioning may underlie MD (Blair & Razza, 2007; Mazzocco & Kover, 2007), with both theoretical perspectives and empirical evidence supporting the importance of these cognitive processes in MD. Working memory is a limited capacity cognitive system that is involved in simultaneously maintaining and manipulating relevant information to accomplish a specific task or a general activity (Raghubar, Barnes, & Hecht, 2010). Not only has working memory been found to be a predictor of later mathematical achievement (e.g., Holmes & Adams, 2006), individuals with MD also exhibit impairments in working memory (e.g., Geary, 2011). Some researchers have proposed that limited working memory may constrain the mathematical achievement of individuals with MD by compounding underlying deficits in numeracy and number sense (Watson & Gable, 2013). In other words, the hierarchical nature of mathematics instruction and
the limited storage and processing capacities of individuals with MD may underlie their underachievement.

Baddeley and Hitch’s (1974) multi-component model of working memory has become one of the most prominent and influential frameworks within cognitive psychology (Miyake et al., 2000). In this model, two distinct subsystems known as the phonological loop and visuo-spatial sketchpad, temporarily store and process auditory input and visual and spatial input, respectively. A structure known as the central executive coordinates the activity of the subsystems and the retrieval of relevant information from long-term memory (Baddeley, 2000).

A number of researchers have explored the relationship between the substructures of working memory and mathematics. Recently, it has been proposed that the phonological loop and visuo-spatial sketchpad may affect different mathematical domains (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). The phonological loop has come to be associated with word problem solving and counting (Geary, 2011), while the visuo-spatial sketchpad is associated with number sense (Geary, 2011), mental arithmetic (Rasmussen & Bisanz, 2005), and written calculation (McLean & Hitch, 1999). While studies have examined the contributions of individual components of working memory in mathematics, other researchers have focussed on working memory more generally by examining different forms of working memory (Berg, 2008a; Swanson, 1993). These latter studies combine storage-specific components (e.g., phonological loop) and processing abilities (i.e., central executive) to examine specific forms of working memory such as verbal working memory. There are conflicting findings regarding the connection between MD and impairments in verbal and visual-spatial working memory (Berg, 2008b). While some researchers have found individuals with MD display deficits in verbal working memory alone (Andersson & Lyxell, 2007; Swanson & Sachse-Lee, 2001), others have
found those with MD also exhibit visual-spatial working memory impairments (Szücs, Devine, Soltesz, Nobes, & Gabriel, 2013). Studies that have focused on children with AD or arithmetic difficulties have also found conflicting findings, with some indicating impairments in both verbal and visual-spatial working memory (Berg, 2008b; Swanson, 1993), and others finding only visual-spatial working memory impairments (Berg & McDonald, 2015; McLean & Hitch, 1999). In view of these findings, some researchers have proposed a MD subtype characterized by deficits in visual-spatial working memory (Geary, 1993; Rourke, 1993).

Along with implicating working memory impairments in MD, a number of researchers have proposed that impairments in the central executive might underlie MD (Blair & Razza, 2007; Bull & Scerif, 2001). In particular, it is thought that individuals with MD may experience impairments in the cognitive processes carried out by the central executive, known as executive functioning. Executive functioning monitors and regulates mental processes and behaviours (Blair, Knipe, & Gamson, 2008). The fractioning of the processes that underlay the central executive has led to the proposal of a multitude of constructs and theories (e.g., Jurado & Rosselli, 2007). This fractioning includes attention switching, dual-task performance, selective attention, inhibitory control, and planning (Anderson, 1998; Baddeley, 1996; Baddeley et al., 1996). While identifying the specific dimensions of executive functioning remains a controversial topic, executive functions have been associated with guiding goal-directed behaviour (Jurado & Rosselli, 2007). Using confirmatory factor analysis, Miyake et al. (2000) uncovered three distinct executive functions in a population of undergraduate students: inhibitory control, shifting attention, and updating.

Inhibitory control and shifting attention have both been found to be significant predictors of mathematical achievement (Bull & Scerif, 2001; Clark, Pritchard, & Woodward, 2010).
Inhibitory control refers to an individual’s ability to suppress irrelevant pre-potent knowledge, behaviour, and stimuli (Bull & Scerif, 2001). Within the field of mathematics, inhibitory control is thought to be involved in the suppression of irrelevant problem information and developmentally immature strategies (Szucs et al., 2013). Shifting attention, also referred to as mental flexibility, refers to the ability to switch between mental sets, representations, and tasks (Yeniad et al., 2013). Shifting attention may be involved in alternating between mathematical operations and strategies (Bull & Lee, 2014). Furthermore, shifting attention may play a role in the sequencing of multi-step problems (Bull & Lee, 2014).

To date, few studies have examined the connection between executive dysfunction and MD, the majority of which have reported conflicting findings. Berg and McDonald (2015) found children with AD exhibited impairments in shifting attention and inhibitory control; however, van der Sluis, de Jong, and Leij (2004) only observed shifting impairments. Similarly, while Szucs et al. (2013) found 9 to 10 year-old children with MD demonstrated impairments in inhibitory control, Censabella and Noël (2007) did not observe inhibitory control impairments in 10 year-old children. These disparate findings could be due to a number of methodological limitations within the field. Berg and McDonald (2015) discussed three factors that may contribute to these discrepancies: the use of inconsistent classification procedures for MD, the absence of confirmatory factor analyses, and reliance upon two-group or three-group comparison models. Berg and McDonald (2015), however, did not address the role of achievement-related experiences in cognitive functioning of AD children and typically achieving children.

**Developmental Lag versus Cognitive Deficit**

Studies that compare the cognitive functioning of children with MD or an AD to their typically achieving peers may be confounded by different achievement-related experiences
between these two groups (Stanovich, 1986). That is, any observable differences in cognitive functioning might be attributable to disparities in academic achievement rather than the presence of a specific learning disability. With respect to AD, inclusion of a younger group, matched in achievement with an AD group can control for the potential effects of differences in academic achievement (Vellutino, Pruzek, Steger, & Meshoulam, 1973). If an AD group demonstrates impairments in cognitive functioning relative to their younger achievement-matched peers, this disparity is less likely to be the result of a developmental lag and to academic-related experiences. In this case, differences in cognitive functioning may be better understood as a result of a cognitive deficit. Developmental lags occur when children follow the same developmental path as their peers at a slower rate (Stanovich, 1988; Stanovich & Siegel, 1994). While some researchers have proposed that a deficit occurs when children follow a different developmental path than their typically achieving peers (Stanovich & Siegel, 1994), it is also possible that a cognitive deficit occurs when a child’s development has stagnated and intervention is required to allow them to catch up to their typically achieving peers (Francis, Shaywitz, Stuebing, Shaywitz, & Fletcher, 1996). Age-matched achievement-matched designs have been employed in a number of contexts. These designs have been used to examine the connection between strategy knowledge and working memory in children with MD (Keeler & Swanson, 2001), as well as exploring mathematical problem solving and working memory in LD children (Swanson & Sachse-Lee, 2001). Most relevant to the present investigation are the studies that have utilized this design to examine working memory (Berg, 2008b; Swanson, 1993) and executive functioning (McLean & Hitch, 1999) in children with arithmetic difficulties.

Swanson (1993) explored the working memory performance of children with MD, children with RD, these two groups’ same-age typically achieving peers, and younger children
matched in achievement with both the MD and RD children. Specifically the study had two main goals: (1) to examine whether LD children exhibit domain-general or domain-specific working memory impairments and (2) to determine whether LD children’s poor working memory performance is a product of processing or storage. Though the study referred to children as MD, since they were classified using arithmetic performance they will be referred to as have a disability specific to arithmetic (AD). Children with AD demonstrated impairments in three verbal working memory tasks (backward digit span, backward word span, and semantic categorization) but impairments were not present on an auditory digits sequence task. To determine if LD children’s poor working memory performance is a product of processing or storage demands, tasks were spilt into immediate and delayed recall conditions, as well as non-probe (initial), probe (gain), and post-probe (maintenance) conditions. Swanson found that all groups benefited from the addition of probes. Overall the age-matched and achievement-matched groups experienced the largest performance disparity between the immediate and delayed recall conditions; the LD groups experienced little to no change in performance and were comparable in performance across conditions. For the non-probe condition, AD and RD children experienced impairments on 2 of the 3 verbal working memory immediate recall tasks (story recall and semantic association), and 1 of the 3 verbal working memory delayed recall tasks (phrase sequence). For the same condition AD and RD children experienced impairments on both of the visual-spatial working memory immediate recall tasks (visual matrix and picture sequence) and on 1 of the 3 delayed recall tasks (mapping and directions). Results suggested children with AD and RD experience domain-general working memory impairments.

Berg (2008b) found that 10-year olds with arithmetic difficulties exhibited impairments in both verbal working memory and visual-spatial working memory. Yet, these impairments
were not found to be uniform across working memory tasks. Children with arithmetic difficulties demonstrated impairments in three verbal working memory tasks (backward digit span, backward word span, and semantic categorization) but impairments were not present on an auditory digits sequence task. Similarly for visual-spatial working memory, poor arithmeticians were impaired on three tasks (Corsi Block, visual matrix, and mapping and directions) but not on a picture sequence task. As children with arithmetic difficulties did not perform more poorly than their younger achievement-matched counterparts on any of these tasks, impairments in verbal working memory and visual-spatial working memory impairments appeared to reflect a developmental lag rather than a cognitive deficit.

McLean and Hitch (1999) examined the cognitive functioning of 9 year-olds with arithmetic difficulties using tasks assessing five cognitive domains: attention shifting, inhibition, long-term memory, and two areas of working memory (phonological loop and visuospatial sketchpad). With respect to executive functioning, children with arithmetic difficulties exhibited impairments on all three shifting attention tasks (trails written, trails verbal, and trails color) and semantic long-term memory retrieval (missing item), but not on the only inhibition task (crossing out). Poor arithmeticians did not demonstrate impairments on the working memory measures of the phonological loop (forward digit span, nonword repetition); yet, showed impairments on one (Corsi Block) of the two working memory measures of the visual-spatial sketchpad. Children with arithmetic difficulties only performed more poorly than their younger achievement-matched controls on the missing item task, which led the authors to conclude that those with difficulties in arithmetic may experience a cognitive deficit related to retrieval of information from long-term memory. However, as the missing item task required children to finish incomplete addition problems, their poor performance may be an artefact of the classification scheme employed
rather than the presence of a cognitive deficit. In other words, the poor performance of children with arithmetic difficulties may be a product of their arithmetic competency rather than difficulties with retrieval of information from long-term memory.

The Present Study

The present study will employ an age-matched achievement-matched design to examine whether the working memory and executive functioning difficulties exhibited by AD children are accurately viewed as a developmental lag or as a deficit. This study is the first to examine both executive functioning and working memory in children with a learning disability specific to arithmetic using an age-matched achievement-matched design. As well, this study employs a stricter categorization of AD than other studies. For instance, both Berg (2008b) and McLean and Hitch (1999) examined children with difficulties in arithmetic and not specifically those identified as AD through the use of an IQ measure. Thus, these studies’ results could be confounded by IQ differences between the typically achieving children and children with mathematical difficulties, and by extension, arithmetic disabilities. It is possible that any observable differences in cognitive functioning might be attributable to disparities in IQ rather than the presence of difficulties specific to arithmetic. The term arithmetic disability refers to chronic and significantly low achievement in arithmetic that is biologically based, and not the product of intelligence, developmental characteristics, or instructional experiences (Mazzocco, 2007). The present study’s design will allow for a more accurate identification of the cognitive functioning of children with AD compared to their typically achieving age-matched peers and their younger achievement-matched counterparts.

Method

Participants
The present study included 60 children (29 boys, 31 girls) ranging in age from 73 to 133 months ($M = 109.27, SD = 12.89$). Case-by-case matching was employed to form three groups: 20 children (7 boys, 13 girls) with arithmetic disabilities (AD) ($M = 116.05, SD = 6.69$), 20 children (11 boys, 9 girls) typically achieving in arithmetic and reading (CM) ($M = 117.75, SD = 6.68$) matched in chronological age with the AD children, and 20 younger children (11 boys, 9 girls) typically achieving in arithmetic and reading (AM) ($M = 94.00, SD = 7.55$) matched in achievement with the AD group. Participants were classified using the numerical operations and word reading subtests from the WIAT-II (Wechsler, 2002) and the Raven’s Colored Progressive Matrices test (RCPM, Raven, 1976). Only those participants who scored below the 25th percentile on the numerical operations subtest and above the 30th percentile on the word reading subset of the WIAT-II were classified as AD. To be considered typically achieving, participants must have scored above the 30th percentile on both the numerical operations and word reading subsets of the WIAT-II. The chronologically age-matched group (CM) consisted of typically achieving peers matched in age to each child in the AD group. The achievement-matched group (AM) consisted of younger typically achieving controls who were matched case-by-case to a participant in the AD group using their raw scores from the numerical operations and word reading tests. Case-by-case matching involves matching one participant to another participant on one or more performance measures. This allows researchers to identify accurately whether performance differences are present between two groups (Treiman & Hirsh-Pasek, 1985). Only participants who fell within the normal range of fluid intelligence (i.e., > 90 and < 120) on the RCPM were included in this study. Data were collected as part of a larger study examining the cognitive underpinnings of children’s mathematical performance.

**Instruments**
**Academic Achievement.** The WIAT-II numerical operations and word reading subtests were used to assess participants’ arithmetic and reading performance. The WIAT-II numerical operations subtest consists of tasks that assess an individual’s ability to read numbers, count, and perform written calculations for simple addition, subtraction, multiplication, and division problems. Participants’ score was the number of problems solved correctly. The word reading subtest assesses an individual’s ability to read individual words. Each participant’s score is the number of words read correctly. The Cronbach’s alpha for these measures were .88 for arithmetic and .97 for reading.

**Fluid Intelligence.** The Raven’s Colored Progressive Matrices task was used to assess the participants’ fluid intelligence (RCPM, Raven, 1967). The test requires participants to select one of four figures to complete a pattern. The test consists of 3 sets of 12 items, with each set increasing in difficulty. A participant’s score was the number of items solved correctly across all sets. The Cronbach’s alpha for fluid intelligence measured .60.

**Executive Functioning and Working Memory.** To assess children’s executive functioning across the domains of inhibitory control and shifting attention, four tasks were administered. Four tasks were administered to assess children’s working memory across the domains of visual-spatial and verbal working memory. Berg and McDonald (2015) employed the same executive functioning and working memory battery as the present study to explore cognitive processing in a younger group of AD children. They conducted a confirmatory factor analysis (CFA) to determine the validity of the executive functioning and working memory batteries; with a chi-square test of $\chi^2 = 16.93$, $df = 14$, $p = .260$ suggesting a good fit for the overall model. In addition, the goodness of fit index (GFI) was .97, the root mean square error of approximation (RMSEA) was .04, and the comparative fit index (CFI) was .99. The CFA
supported the theoretical framework of the cognitive batteries. Due to the relatively small sample size of the present study a confirmatory factor analysis (CFA) was not conducted.

**Shifting Attention.** Two versions of the Making Trails task were administered to assess participants’ shifting ability. Both versions of the Making Trails task were derived from Berg and McDonald (2015), who adapted their tasks from van der Sluis et al. (2004). In the Making Trails Letters task children were presented with a sheet of paper with 22 circles. These circles consisted of two colored sets (pink and yellow) of the first eleven letters of the alphabet (i.e., A-K). Participants were instructed to draw lines from the yellow circles to the corresponding letter of the pink circles in ascending alphabetical order as quickly as possible (i.e., yellow A to pink A, yellow B to pink B, continued to pink K). The Making Trails Digits task can be considered the numerical analogue of the letters task, as it required children to follow the same procedure with the numbers 1 to 11 instead of the letters A to K (i.e., yellow 1 to pink 1, yellow 2 to pink 2, continued to pink 11). For both of these tasks the participant’s score was the time required to complete the sequence correctly.

**Inhibitory Control.** Two versions of a stroop-type task were administered to assess children’s inhibitory control. Both versions of the stroop task were derived from Berg and McDonald (2015), who adapted their tasks from Bull and Scerif (2001) and van der Sluis et al. (2004). In the Quantity-Digits Inhibition task, each item consisted of number of digits that did not match the magnitude represented by the digits (i.e., 444). Participants were shown 40 items and asked to name the number of digits in each item as quickly as possible. For example, if the item “44” was presented the correct response would be “2” instead of “forty-four”. In the Colour-Word Inhibition task each item consisted of a colour name that did not match the colour of the word itself. Participants were asked to name the colour in which the word was printed
instead of the word itself for all 40 items. For each of these tasks the participant’s score was the
time required to identify all items.

**Visual-spatial Working Memory.** To assess participants’ visual-spatial working memory
the Visual Matrix Task and the Mapping and Directions tasks from the S-CPT (Swanson, 1995)
were administered. In each trial of Visual Matrix task the participant was shown a matrix that
contained a series of dots and was given 5 seconds to study the matrix. They were then asked a
process question about the matrix, “Were there any dots in the third column?” If the process
question was answered incorrectly the experimenter stopped the task. If the process question was
answered correctly, the participant was asked to reproduce the dot array on a blank matrix of the
same size. The difficulty of each trail varied from a matrix of 4 squares with 2 dots to a matrix of
45 squares and 12 dots. The participant’s score was the number of matrices completed correctly.
Cronbach’s alpha for the visual matrix task measured .62.

In the Mapping and Directions task the participant was given a street map with three
types of symbols: dots (streetlights), lines (route), and arrows (directions). Participants were
given 10 seconds to study the map. They were then asked a process question about the map,
“Were there any streetlights in the second column?” If the process question was answered
incorrectly the experimenter stopped the task. If the process question was answered correctly, the
participant was asked to reproduce all dots, lines, and arrows on a blank map. A participant’s
score was the number of maps labelled correctly. Cronbach’s alpha for the mapping and
directions task measured .55.

**Verbal Working Memory.** To assess participants’ verbal working memory, the Auditory
Digit Sequence and Semantic Categorization subtests from the Swanson Cognitive Processing
Test (S-CPT, Swanson, 1995) were administered. In the Auditory Digit Sequence subtest, the
experimenter read aloud a sentence containing a street address. After each sentence was read the
participant was asked a process question where they had to identify the name of the street in the
previous sentence. If the process question was answered incorrectly the experimenter stopped the
test. If the process question was answered correctly, the participant was asked to recall the
numbers in the street address. The length of each set ranged from two to nine sentences. A
participant’s score was the number of correctly recalled sets. Cronbach’s alpha for Auditory
Digit Sequence measured .50.

The Semantic Categorization subtest asked participants to recall a set of words and their
respective prearranged groups. The difficulty of each set varied from one group of two words to
eight groups with three words in each. After each set was presented the participant was asked a
process question where they had to identify which of two words appeared in the previous set. If
the process question was answered incorrectly the experimenter stopped the task. If the process
question was answered correctly, the participant was asked to recall the groups and words within
each group. A participant’s score was the number of correctly recalled sets. Cronbach’s alpha for
semantic categorization measured .50.

**Procedure**

Each participant completed two test batteries. The first battery assessed arithmetic and
word reading performance as well as fluid intelligence, while the second assessed four cognitive
domains: inhibitory control, shifting attention, verbal working memory, and visual-spatial
working memory. The assessment batteries were each administered in approximately one hour.

**Results**

*Classification Measures*
Table 1 displays the means and standard deviations for raw, standard, and percentile scores of each group in arithmetic, reading, and fluid intelligence. A series of analysis of variance (ANOVA) tests were conducted to identify if children with AD differed significantly from the CM and AM groups in chronological age, academic achievement, and fluid intelligence (see Table 1). The Games Howell procedure was employed to control for non-homogenous variance across groups. Results indicated significant differences in chronological age among the groups, $F(2,57) = 71.91, p < .001$. Post hoc tests indicated that the mean chronological age of the AM group was significantly lower than AD ($p < .001$) and CM ($p < .001$) children. The mean chronological age of the AD and CM children did not differ significantly ($p = .723$).

An ANOVA, exploring differences in arithmetic performance indicated significant differences among groups on raw arithmetic scores $F(2,57) = 64.33, p < .001$, standard arithmetic scores $F(2,57) = 98.90, p < .001$, and percentile arithmetic scores $F(2,57) = 92.34, p < .001$. For participant’s raw arithmetic scores post hoc tests indicated that the CM group performed better than the AD ($p < .001$) and AM groups ($p < .001$). The performance of AD and AM children did not differ significantly ($p = .749$). For participants’ standard arithmetic scores post hoc tests indicated that the AD group performed more poorly than the CM ($p < .001$) and AM groups ($p < .001$). The performance of CM and AM children did not differ significantly ($p = .392$). For participants’ percentile arithmetic scores post hoc tests indicated that the AD group performed more poorly than the CM ($p < .001$) and AM groups ($p < .001$). The performance of CM and AM children did not differ significantly ($p = .328$).

An ANOVA, exploring differences in word recognition performance indicated significant differences among groups on raw word recognition scores $F(2,57) = 25.28, p < .001$ and percentile scores $F(2,57) = 3.59, p = .034$, but failed to find significant differences for standard
word recognition scores $F(2,57) = 2.36, p = .104$. For participants’ raw word recognition scores post hoc tests indicated that the AM group performed more poorly than the AD ($p < .001$) and CM groups ($p < .001$). The performance of AD and CM children did not differ significantly ($p = .102$). For participants’ percentile word recognition scores post hoc tests indicated that the AD group performed more poorly than their CM peers ($p = .026$). The performance of AD and AM children did not differ significantly ($p = .410$). The performance of CM and AM children did not differ significantly ($p = .350$). An ANOVA was conducted to examine potential differences in fluid intelligence among groups. Results indicated no significant differences between groups, $F(2,57) = 2.00, p = .145$. 

The classification framework employed in this study required that participants score below the 25th percentile in arithmetic and above the 30th percentile on word recognition to be identified as having an AD. While members of the CM and AM groups had to score above the 30th percentile in arithmetic and word recognition to be considered TA. Data analyses supported this classification scheme. Significant differences in mean chronological age were observed where expected among groups, with no differences between AD and CM children, and both of these groups being older than the AM children. Significant differences in the mean percentile scores in arithmetic were observed as expected among groups, with AD children scoring lower than the CM and AM groups, and no differences between the CM and AM groups. Unexpectedly, there were significant differences observed between the mean percentile scores in word recognition among groups. Post hoc tests indicated that mean word recognition percentile scores between the CM and AM groups did not differ significantly; however, mean percentile scores of AD children were significantly lower than CM children. While this difference is noteworthy, the mean percentile scores for word recognition of children with AD ($M = 55.95$) was well above the
30\textsuperscript{th} percentile cut-off score and no significant differences in word recognition raw and standard scores were observed between the AD and CM groups.

\textit{Cognitive Processing}

Table 2 displays the mean and standard deviations for each group’s scores on the cognitive processing measures of shifting attention, inhibitory control, verbal working memory, and visual-spatial working memory. To examine differences in working memory and executive functioning, analysis was directed at comparing the performance among the CM, AM, and AD groups. A series of ANOVAs was conducted to identify if children with AD differed significantly from the CM and AM groups on any cognitive measure (see Table 2). In light of the relatively small sample size and considerations for statistical power, effects sizes (Cohen’s $d$) were calculated to determine the magnitude of any differences, significant and nonsignificant. Cohen’s estimates of strength ranges were employed to interpret effect sizes (small effect, around $d = .20$; medium effect, around $d = .50$; large effect, greater than $d = .80$) (Cohen, 1988).

\textit{Shifting Attention}

An ANOVA on the coloured trails digits task revealed significant group differences, $F(2,57) = 7.36, p < .001$. Post hoc tests indicated that the CM group performed better than the AD ($p < .001$, $d = 1.44$) and AM ($p = .009$, $d = 1.04$) groups. The performance of AD and AM children did not differ significantly ($p = .696$, $d = .26$). The same pattern of results was observed with the coloured trails letters task. An ANOVA on the coloured trails letters task revealed significant group differences in performance, $F(2,57) = 7.93, p < .001$. Post hoc tests indicated that the CM group performed better than the AD ($p < .001$, $d = 1.43$) and AM ($p = .005$, $d = 1.12$) groups. The performance of AD and AM children did not differ significantly ($p = .977$, $d = .07$).
Inhibitory Control

An ANOVA on the quantity-digits inhibition task revealed significant group differences, $F(2,57) = 9.47, p < .001$. Post hoc tests indicated that the CM group performed better than the AD ($p < .001, d = 1.53$) and AM ($p = .004, d = 1.15$) groups. The performance of AD and AM children did not differ significantly ($p = .110, d = .67$). A different pattern of results was observed with the colour-word inhibition task. An ANOVA on the colour-word inhibition task revealed significant group differences, $F(2,57) = 6.67, p = .002$. Post hoc tests indicated that the CM group performed better than the AM group ($p = .002, d = 1.02$). The performance of AD children did not differ significantly from their CM ($p = .186, d = 1.10$) or AM ($p = .156, d = 0.49$) peers.

Visual-spatial Working Memory

An ANOVA on the visual matrix task revealed significant group differences, $F(2,57) = 9.28, p < .001$. Post hoc tests indicated that the CM group performed better than the AD ($p = .008, d = 0.96$) and AM ($p < .001, d = 1.48$) groups. The performance of AD and AM children did not differ significantly ($p = .582, d = .29$). The same pattern of results was observed with the mapping and directions task. An ANOVA on the mapping and directions task revealed significant group differences in performance, $F(2,57) = 14.60, p < .001$. Post hoc tests indicated that the CM group performed better than the AD ($p = .001, d = 1.24$) and AM ($p < .001, d = 1.76$) groups. The performance of AD and AM children did not differ significantly ($p = .476, d = .37$).

Verbal Working Memory

An ANOVA on the auditory digit sequence task revealed significant group differences, $F(2,57) = 7.27, p = .002$. Post hoc tests indicated that the AM group performed more poorly than
their AD ($p = .028, d = .84$) and CM ($p = .001, d = 1.25$) peers. The performance of AD and CM children did not differ significantly ($p = .544, d = .31$). An ANOVA on the semantic categorization task did not reveal significant group differences, $F(2,57) = 1.52, p = .229$.

**Discussion**

This study examined the working memory and executive functioning performance of children with AD relative to their age-matched and achievement-matched peers. The purpose of this study was to explore whether the impairments AD children exhibit in these domains are better understood as developmental lags or as cognitive deficits. Results indicated that, while children with AD did not exhibit impairments in verbal working memory and colour-word inhibition, they did demonstrate impairments in shifting attention, visual-spatial working memory, and quantity-digits inhibition. The performance of AD children was comparable to their younger achievement-matched peers on the inhibitory control, shifting attention, visual-spatial working memory, and verbal working memory tasks. This suggests that the executive functioning and working memory impairments of children with AD reflect a developmental lag rather than a cognitive deficit. Furthermore, Cohen’s $d$ estimates of magnitude ranges for significant differences indicated large degrees of impairment in group performances across these domains: shifting attention ($d = 1.43$ for coloured trails letters and $d = 1.44$ for coloured trails digits), visual-spatial working memory ($d = 0.96$ for visual matrix and $d = 1.24$ for mapping and directions), and quantity-digits inhibition ($d = 1.53$) (Cohen, 1988).

The results of this study align with a previous study on executive functioning and working memory impairments of children with AD (Berg & McDonald, 2015). Berg and McDonald (2015) found that children with AD exhibited impairments in inhibition, shifting attention, and visual-spatial working memory relative to their TA peers. As mentioned
previously, Berg and McDonald (2015) explored executive functioning and working memory in a younger group of AD children using the same cognitive battery as the present study.

While both studies found that AD children demonstrated shifting impairments relative to their CM peers, regardless of the nature of the stimuli employed (i.e., numeric or alphabetic), the present study did not witness the same pattern of results on inhibitory control. Berg and McDonald found that children with AD demonstrated impairments in both colour-word inhibition and quantity-digits inhibition, while the present study found AD children only demonstrated impairments on the quantity-digits inhibition task. Two possible explanations might underlie these disparate findings. First, it maybe that the present study failed to detect significant differences in colour-word inhibition performance between the AD and CM groups due to the relatively small size of the present study. The performance of AD children on the colour-word inhibition task did not differ significantly from their CM counterparts ($p = .186, d = 1.10$). However, Cohen’s $d$ estimated that the magnitude of this difference was large (Cohen, 1988); this suggested that the present study may have committed a type II error due to a lack of statistical power. Second, it is possible that these inconsistent findings are a product of the difference in age of the AD groups between the two studies. The younger group of AD children in Berg and McDonald (2015) may be demonstrating a floor effect on both inhibition tasks, because the processing demands of these tasks are too great for children of their age. The older group of AD children in the present study may have demonstrated impairments on the quantity-digits inhibition task due to the nature of the stimuli itself. As older AD children only exhibited an impairment when required to inhibit processing of numerical content, their poor performance may be due to the difficulties with number sense that define MDs (Landerl, Bevan, & Butterworth, 2004).

The present study’s findings largely correspond to those of McLean and Hitch (1999). Both studies found that AD children (and children with arithmetic difficulties) demonstrated
impairments in shifting attention. While the current study found children with AD exhibited impairments in visual-spatial working memory, McLean and Hitch found that children of a similar age group with arithmetic difficulties experienced impairments in measures of the visual-spatial sketchpad. There was only one area in which these studies’ findings significantly differed. McLean and Hitch (1999) found that children with arithmetic difficulties did not exhibit impairments in inhibitory control, while the present study found that children with AD demonstrated impairments in quantity-digit inhibition. It is unlikely that these disparate findings are due to the nature of the stimuli in the tasks themselves, as both tasks utilized numerical stimuli.

Both Berg (2008b) and Swanson (1993) found that those with arithmetic difficulties exhibited impairments in both verbal and visual-spatial working memory. However, it is notable that in both these studies, results were not uniform among verbal or visual-spatial working memory tasks. Therefore it is possible that verbal working memory impairments may have emerged in the AD group if a wider range of tasks were employed.

In keeping with the findings of Berg (2008b), the present study found that AD children (and children with arithmetic difficulties) performed at a level comparable to their younger achievement-matched peers. As children with AD were found to perform at the same level as children roughly two years younger than them, this finding may contribute to researchers developing an understanding of the magnitude of developmental lags experienced by children with AD (Berg, 2008b).

Connections to Theoretical Models

Geary’s model of mathematical disabilities posits that individuals with MD experience impairments in three cognitive domains: semantic memory, procedural knowledge, and visual-spatial representation (Geary, 1993). Visual-spatial representation impairments are characterized by difficulty coordinating spatial representations in the visual-spatial sketchpad (Geary, 1993).
Procedural knowledge impairments are defined by recurrent procedural errors in mathematical processes, that may be a product of limited conceptual understanding or impaired cognitive systems, such as the central executive (Geary & Hoard, 2005). Semantic memory impairments are characterized by difficulty retrieving information from long-term memory. The findings of the present study align with the visual-spatial representation and procedural knowledge impairments outlined by Geary’s model; measures of semantic memory were not included in the present study. Indeed, not only did children with AD exhibit visual-spatial representation impairments in comparison to their CM peers, they also performed at the same level as children approximately two years younger. Children with AD also demonstrated impairments on both shifting attention tasks and the quantity-digit tasks, which may indicate procedural knowledge impairments (Geary & Hoard, 2005). Further research is needed to uncover more clearly the relationship between the central executive and procedural knowledge impairments.

The core deficit hypothesis of mathematical disabilities proposes that deficits in numerical processing and representation are fundamental features of MD. While the present study found that AD children demonstrated shifting impairments regardless of the nature of the stimuli employed (i.e., numeric or alphabetic), inhibitory control impairments appeared to be task-specific. As AD children only exhibited impairments on the quantity-digit inhibition task it is possible that their poor performance was attributable to the numerical content of the task rather than difficulty inhibiting irrelevant information (Landerl et al., 2004). However, as discussed earlier the disparity in performance on the quantity-digit and colour-word inhibition tasks could also be explained by a lack of statistical power. Overall the present study found that the impairments displayed by children with AD align more closely with the cognitive phenotype postulated by Geary’s model, than the profile proposed by the core deficit hypothesis. However, this finding may be an artefact of the cognitive battery employed by this study. Unlike the executive functioning tasks, the working memory battery did not consist of tasks that contained
only numeric or alphabetic stimuli. This means that any conclusions that can be drawn about the role of numerosity processing in verbal and visual-spatial working memory in AD children are limited.

Limitations

One limitation of the first study is the absence of measures of updating. Miyake et al.’s (2000) model of executive functioning proposes three distinct constructs: inhibitory control, shifting attention, and updating. While all three of these constructs have been found to be significant predictors of general mathematical achievement (e.g., Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013), converging evidence indicates that updating is the most robust predictor of the three executive functions (Bull & Lee, 2014). Updating is the ability to monitor and encode new information in working memory (Kroesbergen, van Luit, van Lieshout, van Loosbroek, & van de Rijt, 2009). Updating is also associated with maintaining and manipulating relevant information in working memory while problem solving (Bull & Lee, 2014). Children with AD have been found to exhibit impairments in both shifting attention and inhibitory control (Berg & McDonald, 2015). While updating has been shown to play a role in mental arithmetic (Deschuyteneer, Vandierendonck, & Muylleart, 2006), no study has examined if updating impairments contribute to AD in children. Further, to date, no study has investigated the relative contributions of updating, inhibitory control, and shifting attention to arithmetic disabilities. Future studies should endeavour to explore the relative contributions of these three constructs in children with a specific learning disability in arithmetic.

The performance of AD was not lower than that of their younger achievement-matched peers on the inhibitory control, shifting attention, visual-spatial working memory, and verbal working memory tasks. These findings suggest that the executive functioning and working memory impairments of children with AD reflect a developmental lag rather than a cognitive
deficit. However, this finding may be an artefact of the study’s methodology. While an age-matched achievement-matched design allows for an examination of cognitive differences among groups at a single moment in time, it is possible that this method may mask the presence of a developing deficit or a deficit reflective of a shorter developmental span. Future studies should employ a longitudinal design to examine the developmental trajectories of executive functioning and working memory in children with AD. For instance, Jordan, Hanich, and Kaplan (2003) examined the development of children’s mathematical competencies from the 2nd to 3rd grade, across three domains: problem solving, calculation, and base-ten concepts. All children were classified into 1 of 4 groups: children with MD, children with RD, children comorbid disabilities in math and reading (MD/RD), and TA children. They found that while children with MD/RD experienced a greater degree of difficulty with word problem solving than their MD or RD counterparts, both the MD/RD and MD groups were equally impaired in arithmetic calculation. Furthermore, both MD/RD and MD children exhibited significant difficulties with arithmetic fact retrieval compared to their RD and TA peers.
Table 1. Study 1 Summary of Group Differences in Chronological Age, Arithmetic, Word Recognition, and Fluid Intelligence

| Tasks              | AD  
|                   | (n = 20) | CM  
|                   | (n = 20) | AM  
<p>|                   | (n = 20) |</p>
<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>SD</th>
<th>M</th>
<th>SD</th>
<th>M</th>
<th>SD</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (Months)</td>
<td>116.05&lt;sup&gt;a&lt;/sup&gt;</td>
<td>6.69</td>
<td>117.75&lt;sup&gt;b&lt;/sup&gt;</td>
<td>6.68</td>
<td>94.00&lt;sup&gt;ab&lt;/sup&gt;</td>
<td>7.55</td>
<td>71.91</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Arithmetic</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw</td>
<td>13.90&lt;sup&gt;a&lt;/sup&gt;</td>
<td>2.15</td>
<td>23.00&lt;sup&gt;ab&lt;/sup&gt;</td>
<td>3.81</td>
<td>14.55&lt;sup&gt;b&lt;/sup&gt;</td>
<td>2.21</td>
<td>64.33</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Standard</td>
<td>70.45&lt;sup&gt;ab&lt;/sup&gt;</td>
<td>10.68</td>
<td>105.90&lt;sup&gt;a&lt;/sup&gt;</td>
<td>8.27</td>
<td>102.25&lt;sup&gt;b&lt;/sup&gt;</td>
<td>6.94</td>
<td>98.90</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Percentile</td>
<td>5.03&lt;sup&gt;ab&lt;/sup&gt;</td>
<td>5.13</td>
<td>63.25&lt;sup&gt;a&lt;/sup&gt;</td>
<td>18.41</td>
<td>55.20&lt;sup&gt;b&lt;/sup&gt;</td>
<td>16.78</td>
<td>92.34</td>
<td>&lt;.001</td>
</tr>
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<td>Word Recognition</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw</td>
<td>104.50&lt;sup&gt;a&lt;/sup&gt;</td>
<td>9.44</td>
<td>110.05&lt;sup&gt;b&lt;/sup&gt;</td>
<td>7.00</td>
<td>85.95&lt;sup&gt;ab&lt;/sup&gt;</td>
<td>15.49</td>
<td>25.28</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Standard</td>
<td>103.90</td>
<td>12.04</td>
<td>110.55</td>
<td>8.36</td>
<td>106.15</td>
<td>8.74</td>
<td>2.36</td>
<td>.104</td>
</tr>
<tr>
<td>Percentile</td>
<td>55.95&lt;sup&gt;a&lt;/sup&gt;</td>
<td>22.97</td>
<td>72.65&lt;sup&gt;a&lt;/sup&gt;</td>
<td>15.09</td>
<td>63.95</td>
<td>20.24</td>
<td>3.59</td>
<td>.034</td>
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<td>Fluid Intelligence</td>
<td></td>
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<td></td>
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<tr>
<td>Full Score IQ</td>
<td>103.05</td>
<td>6.35</td>
<td>105.45</td>
<td>6.83</td>
<td>106.95</td>
<td>5.40</td>
<td>2.00</td>
<td>.145</td>
</tr>
</tbody>
</table>

Note: Scores with the same superscript differ significantly. AD = arithmetically disabled; CM = chronological age matched controls; AM = achievement matched controls.
Table 2. Summary of Group Differences in Executive Functioning and Working Memory

| Tasks                        | AD  
|                             | (n = 20) | CM  
|                             | (n = 20) | AM  
|                             | (n = 20) |     |     |     | F   | p    |
|------------------------------|----------|----------|----------|----------|----------|----------|----------|
|                              | M  | SD    | M  | SD    | M  | SD    |     |     |     |     |        |        |
| Shifting                     |        |        |        |        |        |        |        |        |        |        |        |        |
| Coloured Trails Digits       | 37.26  | 11.74  | 22.79b | 7.95   | 42.35b | 25.27  | 7.36 | .001 |
| Coloured Trails Letters      | 35.83a | 14.83  | 20.06ab| 4.74   | 37.01b | 20.86  | 7.93 | .001 |
| Inhibition                   |        |        |        |        |        |        |        |        |        |        |        |        |
| Quantity-Digits Inhibition   | 53.53a | 9.56   | 41.18ab| 6.23   | 71.33b | 36.41  | 9.47 | <.001|
| Visual-Spatial WM            |        |        |        |        |        |        |        |        |        |        |        |        |
| Visual Matrix                | 3.00a  | 1.34   | 4.10ab | 0.91   | 2.65b  | 1.04   | 9.28 | <.001|
| Mapping and Directions       | 0.95a  | 1.00   | 2.05ab | 0.76   | 0.60b  | 0.88   | 14.60| <.001|
| Verbal WM                    |        |        |        |        |        |        |        |        |        |        |        |        |
| Auditory Digit Sequence      | 1.95a  | 1.00   | 2.25b  | 0.91   | 1.20ab | 0.77   | 7.27 | .002 |
| Semantic Categorization      | 2.35   | 0.88   | 2.50   | 0.76   | 2.00   | 1.12   | 1.52 | .229 |

Note: Scores with the same superscript differ significantly. AD = arithmetically disabled; CM = chronological age matched controls; AM = achievement matched controls.
Chapter 3

Examining the Phonological Processing of Children with an Arithmetic Disability

Introduction

Over the past 20 years there has been a significant shift in the conceptualization of the cognitive underpinnings of mathematics. Increasingly researchers have focused their attention on working memory and executive functioning. This undertaking has led to an increased understanding of the roles of these systems in mathematics and provided the framework necessary for the exploration of new avenues of research. One possible avenue of further research is phonological processing, which has been theorized to play a role in computational competency, as it may be involved in the encoding or retrieval of phonetic and semantic representations of numbers and basic facts (Geary, 1993; 2013; Hecht, Torgesen, Wagner, & Rashotte, 2001). Research examining the role of phonological processing in mathematics performance is relatively scarce; rarer still is an examination of phonological processing in children with MD.

Arithmetic and Phonological Processing

The work that has explored the relationship between mathematics and phonological processing suggests a possible connection between arithmetic and phonological processing (Bull & Johnston, 1997; Fuchs et al., 2006; Hecht, Torgesen, Wagner, & Rashotte, 2001). Hecht et al. (2001) examined the contributions of phonological processing to general computation skills across three phonological domains: phonological awareness, phonological memory, and access to phonological codes from long-term memory (serial and isolated). Phonological awareness is “a metacognitive understanding that the words we hear and read have internal structures based on sounds” (Fletcher, Lyon, Fuchs, & Barnes, 2007, p. 87). Phonemes are the fundamental units
of speech; successful decoding is reliant upon an individual’s understanding of phonemes and their role in the phonological framework of a language. A confirmatory factor analysis supported the theoretical underpinnings of the cognitive batteries employed. The study followed 201 students from the 2\textsuperscript{nd} to 5\textsuperscript{th} grade, with cognitive and academic assessments being conducted annually. While this study did find that all three cognitive domains were significant predictors of growth in general computation skills from the 2\textsuperscript{nd} to the 5\textsuperscript{th} grade, the predictive power of phonological memory and rate of access was limited in comparison to that of phonological awareness. Furthermore, while all three domains were significant predictors of growth in computational skills from the 2\textsuperscript{nd} to 3\textsuperscript{rd} grade, only phonological awareness emerged as a significant predictor of annual computational growth from the 3\textsuperscript{rd} to 4\textsuperscript{th} grades and from the 4\textsuperscript{th} to 5\textsuperscript{th} grades. This suggests that the role of specific phonological domains in computational competency may not be static and, therefore, is capable of changing over time.

Bull and Johnston (1997) explored processing speed, retrieval from long-term memory (serial and isolated), short-term memory, speech rate, and single-digit addition in 7 year-old children with arithmetic difficulties. Children were classified into one of two groups: high math ability and low math ability. Bull and Johnston examined the role of children’s reading ability by conducting two separate analyses. When reading ability was not controlled, poor arithmeticians exhibited impairments on two of the three short term memory tasks (digit span and word span), one of three measures of speech rate (1 syllable rate), all three measures of processing speed (cross-out task, visual matching, and pegboard), and all five long-term memory measures (letter naming, number naming, alphabetical order, numerical order, and reciting the alphabet). After controlling for reading ability, poor arithmeticians displayed impairments on one of the three measures of speech rate (1 syllable rate), two of the three measures of processing speed (visual
matching and pegboard), and three of the five measures of long-term memory retrieval (number naming, numerical order, and reciting the alphabet). Poor arithmeticians experienced greater difficulty with arithmetic facts retrieval and were more likely to use developmentally immature strategies (i.e., finger counting) when solving single-digit addition problems than their high math ability peers. Correlational analyses conducted between cognitive functioning measures and math ability revealed that all cognitive measures were associated with math ability when reading ability was not controlled through partial correlational analyses. When reading was controlled, all measures of processing speed and short-term memory, as well as two of the five long-term memory retrieval tasks (number naming and letter naming) were associated with math ability.

The findings of Bull and Johnston (1997) are significant given converging evidence suggesting that calculation competency and retrieval of arithmetic facts from long-term memory are core deficits in children with MD (Bryant, Bryant, & Hammill, 2000; Gersten, Jordan, & Flojo, 2005; Jordan, Hanich, & Kaplan, 2003). Researchers have proposed a number of mechanisms to account for these difficulties. One theory posits that difficulties in computation and arithmetic fact retrieval are linked to disruptions in the retrieval or encoding of phonetic and semantic representations of numbers and basic facts (Geary, 1993; 2013; Hecht et al., 2001). Geary and Hoard (2001) have also proposed that the same cognitive processes that enable word recognition could be involved in the retrieval of phonetic and semantic representations of numbers during counting; which suggests the same phonological impairments that define reading disabilities (RD) may also underlie impairments in individuals with MD.

**The Connection between MD and RD**

It is not uncommon for children with MD to exhibit reading difficulties; it is also not uncommon for RD to co-occur with MD (Geary, 1993; Watson & Gable, 2013). The extent to
which these learning disabilities co-occur is still unclear, as there is a high degree of variance in comorbidity estimates (Moll, Kunze, Neuhoff, Bruder, & Schulte-Körne, 2014). Indeed, researchers have reported comorbidity rates ranging from 17% (Gross-Tsur, Manor, & Shalev, 1996), 23-39% (Landerl & Moll, 2010), 43-65% (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005), to 70% (von Aster, Schweiter, & Zulauf, 2007 as cited in Moll et al. 2014). The high degree of comorbidity coupled with a growing body of evidence that indicates shared cognitive impairments between these learning disabilities has led some researchers to propose that these disorders may be the product of a common origin (Geary, 1993). Though research examining the relationship between MD, RD, and MD/RD is relatively sparse, there is support for this perspective (Swanson & Jerman, 2006; Willcutt et al., 2013).

Willcutt et al. (2013) conducted a twin study to explore the etiology of comorbidity between MD and RD by examining the concurrent psychopathology, functional impairments, and neuropsychological functioning of children and adolescents with MD, RD, MD/RD, in comparison to their TA peers. The assessment battery included measures of 10 neuropsychological domains: phoneme awareness, verbal comprehension, response inhibition, verbal working memory, set shifting, interference control, processing speed, naming speed, vigilance, and response variability. Results of a series of multiple regression analyses indicated that while both mathematics and reading were associated with impairments in verbal comprehension, verbal working memory, and processing speed; impairments in set shifting were only linked to mathematics and difficulties with phonemic awareness and naming speed were only associated with reading. The MD/RD group experienced a greater degree of impairment than their MD and RD counterparts on all neuropsychological domains excluding interference control, inhibition, and set shifting. Results suggested a correlated liabilities model of
comorbidity between RD and MD. This model proposes that these learning disabilities are correlated but distinct disorders, which means that while MD and RD do share some cognitive impairments, they also possess idiosyncratic cognitive deficits.

Swanson and Jerman (2006) performed a selective meta-analysis of 85 studies examining cognitive functioning in children with MD, RD, and MD/RD. The meta-analysis focused on comparing the LD groups’ performance across 10 cognitive domains: literacy (reading comprehension, phonological awareness, vocabulary, & writing), verbal problem solving, visual-spatial problem solving, naming speed, long-term memory, short-term memory for words, short-term memory for numbers, verbal working memory, visual-spatial working memory, and attention. Overall they found that children with MD and children with RD shared so many of the same cognitive impairments, it was difficult to discriminate between MD and RD children. In fact, the only differences that emerged between the two groups were differences in naming speed and visual-spatial working memory, with children with RD outperforming their MD counterparts. It is important to note, however, that effect sizes indicated the magnitude of group differences were negligible. Further research is needed to determine the cognitive processes that are shared between MD and RD children and that discriminate between these groups.

The Present Study

The present study employed a three-group comparison model to explore the phonological processing of AD children by comparing their performance to children with comorbid disabilities in arithmetic and word recognition (AD/WRD) and to their typically achieving (TA) peers. A word recognition disability (WRD), also known as dyslexia, is characterized by significant difficulties in single-word decoding and spelling (Fletcher, 2009; Lyon, Shaywitz, & Shaywitz, 2003). WRD is the most studied RD subtype; this may be partly due to its ability to bottleneck
higher order processes such as word reading fluency and reading comprehension (Fletcher, 2009). WRD is commonly associated with impairments in phonological awareness and rapid automatized naming (RAN) (Catts, Gillispie, Leonard, Kail, & Miller, 2002). RAN is the ability to identify correctly and quickly stimuli such as letters, digits, and objects as quickly as possible (Georgiou, Parrila, Cui, & Papadopoulos, 2013).

By comparing the phonological processing of children with AD to their AD/WRD and TA peers, it is possible to examine whether phonological impairments are characteristic of AD or if they are more characteristic of children with a WRD. If phonological processing impairments are exhibited by children with AD, this study’s design will allow us to investigate if children with AD/WRD experience a greater degree of cognitive impairment than their AD peers. By contrasting the performance of AD and AD/WRD groups, it is possible to uncover whether phonological impairments are attributable to the presence of an AD or WRD.

**Method**

**Participants**

The present study included 72 children (36 boys, 36 girls) ranging in age from 95 to 131 months (\(M = 112.32, SD = 9.57\)). All participants were classified into one of three chronologically aged-matched groups: children with arithmetic disabilities (AD), children with a comorbid arithmetic disability and a word recognition disability (AD/WRD), and a typically achieving group (TA). Participants were classified using the numerical operations and word reading subtests from the WIAT-II (Wechsler, 2002) and the Raven’s Colored Progressive Matrices test (RCPM, Raven, 1976). Participants who scored below the 25th percentile on the numerical operations subtest and above the 30th percentile on the word reading subset of the WIAT-II were classified as AD (\(n = 25; 10\) boys, 15 girls). Participants who scored below the
25th percentile on both the numerical operations subtest and the word reading subsets of the WIAT-II were classified as AD/WRD (n = 20; 11 boys, 9 girls). To be classified as TA (n = 27; 15 boys, 12 girls) participants had to score above the 30th percentile on both the numerical operations and word reading subset of the WIAT-II. The proposed study only included participants who fell within the normal range of fluid intelligence (i.e., > 90 and < 120) on the RCPM. Data were collected as part of a larger study examining the cognitive underpinnings of children’s mathematical performance.

Instruments

**Academic Achievement.** The WIAT-II numerical operations and word reading subtests were used to assess participants’ arithmetic and word recognition performance, respectively. The WIAT-II numerical operations subtest consists of tasks that assess an individual’s ability to read numbers, count, and perform written calculations for simple addition, subtraction, multiplication, and division problems. The participants’ score was the number of problems solved correctly. The word reading subset assesses an individual’s ability to read individual words. Each participant’s score is the number of words read correctly. Cronbach’s alpha for these measures was .87 for arithmetic and .96 for reading.

**Fluid Intelligence.** The Raven’s Colored Progressive Matrices task was used to assess the participants’ fluid intelligence (Raven, 1967). The test required participants to select one of four figures to complete a pattern. The test consists of 3 sets of 12 items, with each set increasing in difficulty. A participant’s score was the number of items solved correctly. Cronbach’s alpha for fluid intelligence measured .46.

**Phonological Awareness.** To assess participants’ phonological awareness, the elision and blending words subtests from the Comprehensive Test of Phonological Processing (CTOPP,
Wagner, Torgesen, & Rashotte, 1999) were administered. In the elision subtest, participants were asked to say a word and then say what the word would be after omitting a particular phoneme. If the participant answered correctly they proceeded to the next item. The experimenter stopped the task if the child gave three incorrect answers consecutively. The task consisted of 20 items and a participant’s score was the number of items named correctly. In the blending words subtest children were asked to combine specific phonemes to create a word. If the participant answered correctly they proceeded to the next item. The experimenter stopped the task if the child gave three incorrect answers consecutively. The task consisted of 20 items and a participant’s score was the number of items named correctly. Cronbach’s alpha for these measures were .89 for the elision and .73 for blending words.

**Phonological Short-term Memory.** The memory for digits and nonword repetition subtests from the CTOPP were administered to assess participants’ phonological short-term memory (Wagner et al., 1999). In the memory for digits task, children were presented with a recorded set of digits and then asked to repeat the digits in the same order they were presented. Each set varied from in length from 2 to 8 digits and each set was presented at a rate of 2 digits per second. The experimenter stopped the task if the child gave three incorrect answers consecutively. The task consisted of 21 sets and a participant’s score was the number of sets recalled correctly. In the nonword repetition task, children were presented with a set of recorded nonwords and then asked to repeat each nonword using the same pronunciation with which the word was presented. Each nonword varied in length from 3 to 15 sounds. The experimenter stopped the task if the child gave three incorrect answers consecutively. The task consisted of 18 sets and a participant’s score was the number of sets recalled correctly. Cronbach’s alpha for these measures were .80 for the memory for digits task and .64 for nonword repetition.
Isolated Naming Speed. Three isolated naming speed tasks were administered to assess participants’ speed at naming individual stimuli. These included the rapid letter naming and rapid digit naming subtests from the CTOPP (Wagner et al., 1999) and a quantity naming task adapted from van der Sluis et al. (2004). In the rapid letter naming task participants were shown two pages which depicted a total of 72 randomly ordered letters. Each page consisted of 36 letters arranged in four rows and nine columns. Participants were asked to name the letters as quickly as possible, starting from left to right at the top row, continue to name the letters starting in the next row, and repeat this procedure until they have named all of the letters. The rapid digit naming task could be considered the numerical analogue of the letters task, as it required children to follow the same procedure with 72 numbers instead of 72 letters. For both of these tasks, a participant’s score was the time taken to name all 72 items. In the quantity naming task, participants were shown 40 items (i.e., triangles) and asked to name the quantity of each item as quickly as possible. For example, if the item “△△” was presented the correct response would be “two”. A participant’s score was the time required to identify all of the items.

Serial Naming Speed. Two serial naming speed tasks were administered to assess participants’ speed to name stimuli presented serially. The tasks included articulating the alphabet from a to z and counting from 1 to 20. In the naming the alphabet task, participants were asked to recite the alphabet as quickly as possible. A participant’s score was the time taken to recite the complete alphabet. In the counting from 1 to 20 task, children were asked to count aloud from 1 to 20 as quickly as possible. A participant’s score was the time taken to complete the number sequence.

Serial Processing Speed. Two versions of the trail making task were administered to assess participants’ serial processing speed. Both versions of the Making Trails task were used in
Berg and McDonald (2015) who adapted their tasks from van der Sluis et al. (2004). In the trail making letters task children were presented with a sheet of paper with 11 circles. These circles consisted of the first eleven letters of the alphabet (i.e., A-K). Participants were instructed to draw lines from the letters in each circle in alphabetical order as quickly as possible. The trail making digit task can be considered the numerical analogue of the letters task, as it required children to follow the same procedure with the numbers from 1 to 11 instead of the letters A to K. For both of these tasks, a participant’s score was the time required to complete the sequence.

**Procedure**

Each participant completed two test batteries. The first battery assessed reading, mathematical performance, and fluid intelligence. The second battery assessed the five cognitive domains under investigation: phonological awareness, phonological short-term memory, isolated naming speed, serial naming speed, and serial processing speed. The batteries were administered in approximately one hour each, with the academic performance and fluid intelligence battery being administered first.

**Results**

*Classification Measures*

Means and standard deviations for raw, standard, and percentile scores of each group in arithmetic, reading, and fluid intelligence are displayed in Table 1. A series of analysis of variance (ANOVA) tests was conducted to identify if children with AD differed significantly from the AD/WRD and TA groups in chronological age, academic achievement, and fluid intelligence (see Table 1). The Games Howell procedure was employed to control for non-homogenous variance across groups.
The ANOVA examining possible differences in chronological age indicated that there were no significant differences in chronological age among the groups, $F(2,69) = .32, p = .729$. An ANOVA, exploring potential differences in arithmetic performance indicated significant differences among groups on arithmetic raw scores $F(2,69) = 38.52, p < .001$, arithmetic standard scores $F(2,69) = 102.00, p < .001$, and arithmetic percentile scores $F(2,69) = 152.71, p < .001$. For participants’ arithmetic raw scores, post hoc tests indicated that the TA group performed better than the AD ($p < .001$) and AD/WRD ($p < .001$) groups. The performance of AD and AD/WRD children did not differ significantly ($p = .946$). For participant’s’ arithmetic standard scores, post hoc tests indicated that the TA group performed better than the AD ($p < .001$) and AD/WRD ($p < .001$) groups. The performance of AD and AD/WRD children did not differ significantly ($p = .435$). For participants’ arithmetic percentile scores, post hoc tests indicated that the TA group performed better than the AD ($p < .001$) and AD/WRD ($p < .001$) groups. The performance of AD and AD/WRD children did not differ significantly ($p = .974$).

An ANOVA exploring potential differences in word recognition performance indicated significant differences among groups on raw scores $F(2,69) = 49.36, p < .001$, standard scores $F(2,69) = 133.41, p < .001$, and percentile scores $F(2,69) = 113.27, p < .001$. For participants’ word recognition raw scores, post hoc tests indicated that the AD/WRD group performed more poorly than the AD ($p < .001$) and TA ($p < .001$) groups. The performance of AD and TA children did not differ significantly ($p = .070$). For participants’ word recognition standard scores, post hoc tests indicated that the AD/WRD group performed more poorly than the AD ($p < .001$) and TA ($p < .001$) groups. The performance of AD and TA children differed significantly ($p = .009$). For participants’ word recognition percentile scores, post hoc tests indicated that the AD/WRD group performed more poorly than the AD ($p < .001$) and TA groups ($p < .001$). The
performance of AD and TA children differed significantly ($p = .004$). An ANOVA was conducted to examine potential differences in fluid intelligence among groups. Results indicated no significant group differences, $F(2,69) = 1.73$, $p = .186$.

Data analyses appeared to support the classification framework employed in this study. Significant differences in the mean arithmetic percentile scores were observed where expected among groups, with no differences among the AD and AD/WRD groups, and both of these groups performing more poorly than the TA group. However, there were significant differences observed between the mean percentile scores in word recognition among groups. Post hoc tests indicated that mean word recognition percentile scores of the AD/WRD children were significantly lower than AD and TA children. Unexpectedly, the mean percentile scores of AD children were found to be significantly lower than TA children. While this difference is noteworthy, the mean percentile word recognition score of children with AD ($M = 51.52$) was above the 30th percentile cutoff score, and no significant differences in word recognition raw scores were observed between the AD and TA groups.

**Cognitive Processing**

Means and standard deviations for each group’s scores on the cognitive processing measures of phonological awareness, phonological short-term memory, isolated naming speed, serial naming speed, and serial processing speed are presented in Table 2. To examine differences in phonological processing, analyses was directed at comparing the performance among the AD, AD/WRD, and TA groups. A series of ANOVAs was conducted to identify if children with AD differ significantly from the AD/WRD and TA groups on any cognitive measure (see Table 2). In light of the relatively small sample size and considerations for statistical power, effects sizes (Cohen’s $d$) were calculated to determine the magnitude of any
differences, significant or nonsignificant. Cohen’s estimates of strength ranges were employed to interpret effect sizes (small effect, around $d = .20$; medium effect, around $d = .50$; large effect, greater than $d = .80$) (Cohen, 1988).

**Phonological Awareness**

An ANOVA on the elision task revealed significant group differences, $F(2,69) = 10.67, p < .001$. Post hoc tests indicated that the AD/WRD group performed more poorly than the AD ($p < .001, d = 1.19$) and TA ($p = .003, d = 1.08$) groups. The performance of AD and TA children did not differ significantly ($p = .992, d = 0.03$). This pattern of results was not observed with the blending words task. An ANOVA on the blending words task revealed significant group differences, $F(2,69) = 6.92, p = .002$. Post hoc tests indicated that the AD/WRD group performed more poorly than the TA group ($p < .001, d = 1.25$). The performance of AD/WRD and AD children did not differ significantly ($p = .249, d = 0.48$). The performance of AD and TA children did not differ significantly ($p = .084, d = 0.51$).

**Phonological Short-term Memory**

An ANOVA on the memory for digits task revealed significant group differences, $F(2,69) = 7.16, p < .001$. Post hoc tests indicated that the TA group performed better than the AD ($p = .031, d = 0.68$) and AD/WRD ($p = .002, d = 1.13$) groups. The performance of AD and AD/WRD children did not differ significantly ($p = .474, d = 0.37$). A different pattern of results was observed with the nonword repetition task. An ANOVA on the nonword repetition task revealed significant group differences, $F(2,69) = 4.06, p = .021$. Post hoc tests indicated that the AD/WRD group performed more poorly than the TA group ($p = .031, d = 0.74$). The performance of AD/WRD and AD children did not differ significantly ($p = .881, d = 0.15$). The performance of AD and TA children did not differ significantly ($p = .073, d = 0.61$).
**Isolated Naming Speed**

An ANOVA on the rapid letter naming task revealed significant group differences, \( F(2,69) = 9.23, p < .001 \). Post hoc tests indicated that the TA group performed better than the AD \((p = .037, d = 0.76)\) and AD/WRD \((p < .001, d = 1.23)\) groups. The performance of AD and AD/WRD children did not differ significantly \((p = .164, d = 0.50)\). The same pattern of results was observed with the rapid digit naming task and the quantity naming task. An ANOVA on the rapid digit naming task revealed significant group differences, \( F(2,69) = 15.73, p < .001 \). Post hoc tests indicated that the TA group performed better than the AD \((p = .002, d = 1.07)\) and AD/WRD \((p < .001, d = 1.53)\) groups. The performance of AD and AD/WRD children did not differ significantly \((p = .098, d = 0.59)\). An ANOVA on the quantity naming task revealed significant group differences, \( F(2,69) = 18.81, p < .001 \). Post hoc tests indicated that the TA group performed better than the AD \((p < .001, d = 1.16)\) and AD/WRD \((p < .001, d = 1.70)\) groups. The performance of AD and AD/WRD children did not differ significantly \((p = .244, d = 0.55)\).

**Serial Naming Speed**

An ANOVA on the counting 1-20 task revealed significant differences between groups, \( F(2,69) = 3.55, p = .034 \). The performance of the TA group did not differ significantly from their AD \((p = .074, d = 0.64)\) and AD/WRD \((p = .060, d = .72)\) peers. The performance of AD and AD/WRD children also did not differ significantly \((p = .973, d = 0.06)\). An ANOVA on the alphabet A-Z task revealed significant group differences, \( F(2,69) = 3.46, p = .037 \). Post hoc tests indicated that the AD/WRD group performed more poorly than the TA group \((p = .037, d = 0.60)\). The performance of AD children did not differ significantly from AD/WRD children \((p = .108, d = 0.50)\) or from TA children \((p = .888, d = 0.21)\).
Serial Processing Speed

An ANOVA on the trail making digits revealed significant group differences, $F(2,69) = 6.31, p = .003$. Post hoc tests indicated that the TA group performed better than the AD ($p = .004, d = 1.01$) and AD/WRD ($p = .024, d = 0.82$) groups. The performance of AD and AD/WRD children did not differ significantly ($p = .915, d = 0.10$). An ANOVA on the trail making letters task did not reveal significant group differences, $F(2,69) = 2.81, p = .067$.

Discussion

This study examined the phonological processing performance of children with AD relative to their AD/WRD and TA peers. The purpose of this study was to explore whether phonological impairments are a distinguishing feature of AD. Results indicated that, while children with AD did demonstrate impairments on all isolated naming speed tasks, trail making digits, and memory for digits, they did not demonstrate impairments on measures of phonological awareness, nonword repetition, serial processing speed, or serial naming speed. Cohen’s estimates of magnitude ranges for significant differences signified a medium to large degree of impairment ranging from ($d = 0.68$ to $d = 1.16$) (Cohen, 1988). In contrast, children with AD/WRD demonstrated impairments on measures of phonological awareness, phonological short-term memory, isolated naming speed, serial processing speed, and the alphabet a-z task. Furthermore, Cohen’s estimates of magnitude ranges for significant differences signified a medium to large degree of impairment ranging from ($d = 0.60$ to $d = 1.70$) (Cohen, 1988).

Notably, while data analysis revealed a significant main effect for the counting 1-20 task, post-hoc tests indicated there were no significant differences in group performance. The most likely explanation for this contradictory finding has to do with the rigour of the post-hoc tests employed. Conservative post-hoc tests decrease the risk of committing a type I, while inflating
the chances of committing a type II error (Davis, 2013). As the present study employed post-hoc tests (Games-Howell and Tukey) that are generally considered to be more conservative (Davis, 2013), it is possible a more liberal post-hoc test, such as the least square difference (LSD) test, may have detected a significant difference among the groups.

While the results of the present study suggest that the phonological processing impairments that define WRD are not characteristic of AD, this finding may be a product of the age of the participants (mean age 9 years 4 months). Evidence suggests that the predictive power of phonological awareness decreases over the course of reading development (Sprugevica, Paunina, & Hoien, 2006). Reading proficiency is the product of the interaction of a number of factors, including but not limited to: phonological awareness, vocabulary, and syntactic knowledge (Hulme, Snowling, Caravolas, & Carroll, 2005). Over the development of reading proficiency the fundamental unit of understanding appears to shift from phonemes to morphemes (Singson, Mahony, & Mann, 2000). Thus it may be possible, that phonological processing impairments in particular domains such as phonemic awareness, may be easier to detect in younger AD children. Conceptually, the blending words subtest can be regarded as the addition of phonemes, while the elision subtest can be thought of as subtraction of phonemes. It is possible that a younger group of AD children may demonstrate impairments on the elision and blending words subtest, just as these children demonstrate significant impairments in basic calculation in addition and subtraction.

**Connections to Theoretical Models**

Geary proposed that individuals with MD exhibit cognitive impairments in three domains: semantic memory, procedural knowledge, and visual-spatial representation (Geary, 1993). In keeping with the semantic memory impairments that characterize Geary's model,
children with AD exhibited impairments on all three isolated naming speed tasks. This suggests that children with AD experience difficulty retrieving information from long-term memory.

While children with AD did demonstrate difficulty with isolated naming tasks regardless of the nature of stimuli employed (i.e., numeric or alphabetic), their difficulties in other domains such as phonological short-term memory and serial processing speed were only observed on tasks that contained numerical content. This suggests that for AD children the source of their impairment is not necessarily due to difficulties with phonological short-term memory and serial processing, but instead, may be a product of the numerical content of the specific tasks used to assess these domains (Landerl, Bevan, & Butterworth, 2004). Furthermore, the present study’s findings partially correspond to those of Landerl et al. (2009).

Landerl et al. (2009) examined the cognitive functioning of 8 to 10 year-old children with learning disabilities across five domains: phonological awareness (phoneme deletion), naming speed (RAN digits, verbal fluency), phonological and visual-spatial short-term memory (nonword span, Corsi blocks, forward and backward digit span), and numerosity processing (symbolic magnitude comparison, physical size comparison, nonsymbolic magnitude comparison, number line). All children were classified into 1 of 4 groups: children with AD, children with RD, children with comorbid disabilities in arithmetic and reading (AD/RD), and TA children. They found that while children with AD experienced impairments on the symbolic magnitude comparison and number line tasks, they did not exhibit impairments in physical size comparison, nonsymbolic magnitude comparison, phonemic awareness, naming speed, and phonological or visual-spatial short-term memory. Children with RD exhibited impairments on phonological awareness, RAN digits, and nonword span, but did not experience impairments on the verbal fluency task, numerosity processing tasks, Corsi blocks, or digit span forward or backward tasks.
Finally, the AD/RD group experienced the same impairments as their RD and AD counterparts, in addition to exhibiting impairments on Corsi blocks, verbal fluency, and the digit span backwards task. These findings led the authors to conclude that children with AD experience a core deficit in numerosity processing, while children with RD experience a fundamental deficit in phonological processing. Both the findings of Landerl et al. (2009) and the present study provide support for the core deficit hypothesis of mathematical disabilities, which proposes that numerical representation and processing is a core deficit of MD (Landerl et al., 2004).

**Limitations**

One limitation of the present study is that it employed a limited number of measures to assess each phonological domain. Additional tests from the CTOPP could have been employed to gain a greater degree of specificity in the nature of the phonological impairments that may be exhibited by children with AD. In particular, two additional isolated speed tasks (rapid color naming and rapid object naming) from the CTOPP may have provided greater insight into the semantic retrieval impairments posited by Geary’s model of mathematical disabilities (Geary, 1993, Wagner et al., 1999). Three additional phonological awareness tasks (blending nonwords, segmenting nonwords, and sound matching) could have been used to gain a better understanding of the possible connections between AD and phonological processing. Future studies should employ phonological measures to assess a more varied range of phonological processing in children with AD.

Another limitation of the present study is the absence of a group of children with WRD. Employing a four-group comparison model would enable us to examine the impairments of each LD subtype with a greater degree of specificity (Berg & McDonald, 2015). By examining the phonological processing of children with AD, WRD, AD/WRD, and their TA peers, it would be
possible to determine the specific impairments associated with AD, WRD, and AD/WRD. It would also be possible to determine the degree of impairment experienced by those with comorbid learning disabilities in comparison to their AD and WRD peers. For instance, Willcutt et al. (2013) found that children and adolescents with MD/RD exhibited a greater degree of impairment than their MD and RD counterparts, on 7 of 10 neuropsychological domains examined. Future studies should explore the phonological processing of AD children using a four-group comparison model. Ideally, these studies would compare the phonological processing of AD children across various age groups and employ a longitudinal measurement model to gain a better understanding of the developmental trajectory of phonological processing in AD children.
Table 3. Study 2 Summary of Group Differences in Chronological Age, Arithmetic, Reading, and Fluid Intelligence

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<td>(n = 20)</td>
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<td>&lt;.001</td>
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<tr>
<td>Standard</td>
<td>102.81&lt;sup&gt;ab&lt;/sup&gt; 7.34</td>
<td>73.68&lt;sup&gt;a&lt;/sup&gt; 7.95</td>
<td>70.40&lt;sup&gt;b&lt;/sup&gt; 11.39</td>
<td>102.00</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Percentile</td>
<td>56.41&lt;sup&gt;ab&lt;/sup&gt; 17.73</td>
<td>5.90&lt;sup&gt;a&lt;/sup&gt; 5.56</td>
<td>5.49&lt;sup&gt;b&lt;/sup&gt; 6.81</td>
<td>152.71</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Reading</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw</td>
<td>103.96&lt;sup&gt;a&lt;/sup&gt; 7.21</td>
<td>99.08&lt;sup&gt;b&lt;/sup&gt; 8.20</td>
<td>77.50&lt;sup&gt;ab&lt;/sup&gt; 12.82</td>
<td>49.36</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Standard</td>
<td>106.63&lt;sup&gt;a&lt;/sup&gt; 5.69</td>
<td>100.76&lt;sup&gt;a&lt;/sup&gt; 6.64</td>
<td>74.65&lt;sup&gt;a&lt;/sup&gt; 8.57</td>
<td>133.41</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Percentile</td>
<td>65.82&lt;sup&gt;a&lt;/sup&gt; 13.83</td>
<td>51.52&lt;sup&gt;a&lt;/sup&gt; 16.58</td>
<td>7.12&lt;sup&gt;a&lt;/sup&gt; 7.42</td>
<td>113.27</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Fluid Intelligence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full Score IQ</td>
<td>103.78 6.034</td>
<td>104.68 6.42</td>
<td>101.40 5.42</td>
<td>1.73</td>
<td>.186</td>
</tr>
</tbody>
</table>

Note: Scores with the same superscript differ significantly. TA = typically achieving; AD = arithmetically disabled; AD/WRD = comorbid arithmetic disabled and word recognition disabled.
Table 4. Summary of Group Differences in Phonological Processing

<table>
<thead>
<tr>
<th>Tasks</th>
<th>TA (n = 27)</th>
<th>AD (n = 25)</th>
<th>AD/WRD (n = 20)</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Phonological Awareness</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elision</td>
<td>15.44^a</td>
<td>3.83</td>
<td>15.56^b</td>
<td>3.04</td>
<td>10.80^ab</td>
</tr>
<tr>
<td>Blending Words</td>
<td>13.41^a</td>
<td>2.31</td>
<td>11.96</td>
<td>2.85</td>
<td>10.80^a</td>
</tr>
<tr>
<td>Phonological Short-term Memory</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Memory for Digits</td>
<td>13.44^ab</td>
<td>2.68</td>
<td>11.60^a</td>
<td>2.74</td>
<td>10.70^b</td>
</tr>
<tr>
<td>Nonword Repetition</td>
<td>12.70^a</td>
<td>2.61</td>
<td>11.20</td>
<td>2.27</td>
<td>10.85^a</td>
</tr>
<tr>
<td>Isolated Naming Speed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rapid Letter Naming</td>
<td>35.41^ab</td>
<td>8.13</td>
<td>42.53^a</td>
<td>10.36</td>
<td>48.15^b</td>
</tr>
<tr>
<td>Rapid Digit Naming</td>
<td>31.80^ab</td>
<td>8.12</td>
<td>40.54^a</td>
<td>8.29</td>
<td>46.11^b</td>
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<tr>
<td>Quantity Naming</td>
<td>25.29^ab</td>
<td>5.76</td>
<td>31.37^a</td>
<td>4.63</td>
<td>33.76^b</td>
</tr>
<tr>
<td>Serial Naming Speed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Counting 1-20</td>
<td>5.08</td>
<td>0.94</td>
<td>5.91</td>
<td>1.56</td>
<td>6.00</td>
</tr>
<tr>
<td>Alphabet A-Z</td>
<td>5.31^a</td>
<td>1.85</td>
<td>5.71</td>
<td>1.88</td>
<td>7.61^a</td>
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<tr>
<td>Serial Processing Speed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trail Making Digits</td>
<td>9.24^ab</td>
<td>2.31</td>
<td>12.87^a</td>
<td>4.51</td>
<td>12.39^b</td>
</tr>
<tr>
<td>Trail Making Letters</td>
<td>8.71^a</td>
<td>1.71</td>
<td>12.17</td>
<td>8.90</td>
<td>11.85^a</td>
</tr>
</tbody>
</table>

Note: Scores with the same superscript differ significantly. TA = typically achieving; AD = arithmetically disabled; AD/WRD = comorbid arithmetic disabled and word recognition disabled.
Chapter 4

Conclusion

The present research sought to further our understanding of the nature of the cognitive processes that underlie arithmetic disabilities (AD) by conducting two investigations. The first investigation employed an age-matched achievement-matched design to examine whether the working memory and executive functioning difficulties exhibited by AD children are better viewed as developmental lags or as cognitive deficits. The primary purpose of the second investigation was to examine whether the same phonological impairments that define a word recognition disability (WRD) also underlie an AD. This investigation compared the phonological processing of AD children to children with comorbid disabilities in arithmetic and word recognition (AD/WRD) and to their typically achieving peers (TA), to examine whether phonological impairments are characteristic of AD or if they are more characteristic of children with a WRD.

Results from the first investigation suggest that the working memory and executive functioning difficulties exhibited by AD children are better understood as a developmental lag rather than a deficit. As the impairments of children with AD were comparable to that of their younger achievement-matched peers on the inhibitory control, shifting attention, and visual-spatial working memory tasks; developmental lags appeared in these areas of executive functioning and working memory.

The results of the second investigation suggested that phonological processing impairments are more characteristic of children with a WRD than children with an AD. Indeed, while children with AD did demonstrate impairments on all isolated naming speed tasks, trail making digits, and memory for digits, they did not demonstrate impairments on measures of
phonological awareness, nonword repetition, serial processing speed, or serial naming speed. In contrast, children with AD/WRD demonstrated impairments on measures of phonological awareness, phonological short-term memory, isolated naming speed, serial processing speed, and the alphabet a-z task.

Overall, results from both investigations suggest that children with AD experience cognitive impairments across a range of domains, including working memory, executive functioning, and phonological processing. However, there is evidence to suggest that the impairments experienced by AD children are not uniform across these domains. Indeed, children with AD demonstrated impairments on visual-spatial but not verbal working memory tasks, on quantity-digits inhibition but not colour-word inhibition, memory for digits but not nonword repetition, and trail making digits but not trail making letters. It is possible that the absence of cognitive impairments on particular tasks within these domains may mean that these tasks are more closely associated with other areas of mathematics, and thus, might also be representative of a disability in another area of mathematics. For instance, children with a learning disability specific to problem solving may exhibit impairments on the trail making letters task due to an association between letter identification and the reading requirements of word-based mathematics problems. In other words, stimuli-specific effects might mask or highlight the presence of impairments depending on the classification criteria employed or the MD subtype being investigated.

The findings of these studies also suggest that given the breadth and magnitude of working memory and executive functioning impairments exhibited by AD children, researchers may want to privilege interventions that target these domains over phonological processing. Furthermore, it is possible that the isolated naming speed impairments demonstrated by children
with AD, may be a product of disruptions in the retrieval of information from long-term memory, a process that is posited to be mediated by the central executive (Baddeley, 1996). As stated earlier, the findings of the first investigation indicated that the working memory and executive functioning difficulties experienced by children with AD are better viewed as developmental lags than cognitive deficits. As the working memory and executive functioning performance of AD children was comparable to that of children approximately two years younger, this suggests that children with AD may experience a developmental lag of roughly two years. While further research is needed to examine the developmental trajectories of working memory and executive functioning in children with AD, and to examine the magnitude of any developmental lags, preliminary results seem to bode well for the remediation of AD. Indeed, a developmental lag may be more responsive to classroom intervention than a cognitive deficit.

While developmental lags occur when children follow the same developmental path as their peers, yet, at a slower rate (Stanovich, 1988; Stanovich & Siegel, 1994), a cognitive deficit indicates that children follow a different developmental path than their typically achieving peers (Stanovich & Siegel, 1994). As the fundamental nature of a developmental lag is dynamic, children are already experiencing incremental change over time, targeted interventions and instructional strategies may encourage a larger degree of developmental growth. Little is known about the nature of cognitive deficits, it is possible that a deficit occurs when developmental growth has stagnated (Francis, Shaywitz, Stuebing, Shaywitz, & Fletcher, 1996); it could be more difficult to intervene successfully when the nature of the impairment children experience is static. Students with AD may benefit from instructional strategies that minimize the cognitive resources required by working memory and executive functioning systems. As well, these
students may benefit from the incorporation of working memory and executive functioning training programs in the classroom.

**Connections to Theoretical Models**

In an attempt to gain insight into the cognitive underpinnings and possible subtypes of mathematical disabilities, the present studies’ findings were examined through the lenses of three theoretical models of MD. First, taken together, both studies provide support for the model posited by Geary (1993). This model proposed that those with MD experience impairments in three cognitive domains: semantic memory, procedural knowledge, and visual-spatial representation. In the first study, the visual-spatial working memory and executive functioning (shifting attention and quantity-digit inhibition) impairments witnessed in children with AD, can be interpreted as evidence of the visual-spatial representation and procedural knowledge impairments outlined by Geary’s model. Visual-spatial representation impairments are thought to involve difficulty processing and representing visual and spatial representations in the visuo-spatial sketchpad (Geary, 1993; Geary & Hoard, 2005). Visual-spatial working memory is a limited capacity cognitive system that is involved in simultaneously maintaining and manipulating relevant visual and spatial information to accomplish a specific task (Raghubar, Barnes, & Hecht, 2010). Given the apparent connection between these two domains, the visual-spatial working memory impairments exhibited by children with AD were taken as evidence of visual-spatial representation impairments. Procedural knowledge difficulties may be the product of poor conceptual understanding of mathematical concepts or impairments in the central executive or working memory (Geary & Hoard, 2005). For instance in arithmetic, procedural knowledge difficulties may be an artifact of the use of developmentally immature counting strategies (i.e., counting-all versus counting-on), disruptions in the retrieval of arithmetic facts
due to weak connections between the central executive and long-term memory (Baddeley, 1996),
or limited working memory capacity (Geary & Hoard, 2005). It is unclear whether procedural
knowledge impairments occur because children lack the relevant mathematical knowledge or
because they experience difficulty applying this knowledge due to constraints on cognitive
systems. For example, do MD children experience difficulty with regrouping or carrying over in
arithmetic due to a lack of basic math facts (knowledge) or difficulty shifting their attention from
one column to the next (executive functioning)? The findings of the second study are in keeping
with the semantic memory impairments proposed by Geary’s model. Children with AD exhibited
impairments on all three of the isolated naming speed tasks, indicating that AD children
experience difficulties storing and retrieving information from long-term memory (e.g., basic
math facts).

Second, both studies provide partial support for the core deficit hypothesis of MD, which
proposed that deficits in numerical processing and numerical representation are core attributes of
MD (Landerl, Bevan, & Butterworth, 2004). In the first study, children with AD were found to
exhibit impairments in quantity-digit inhibition but not in colour-word inhibition. Notably, in the
second study, the performance of children with AD on phonological short-term memory and
serial processing speed measures appeared to be associated with the nature of stimuli employed
in a given task, as AD children only exhibited impairments on tasks with numerical content
(memory for digits and trail making digits). However, both studies also provided evidence that
children with AD exhibited impairments in cognitive domains regardless of the nature of the
stimuli employed (shifting attention and isolated naming speed). Furthermore, in the first
investigation, children with AD performed at a level comparable to their same-age typically
achieving peers on the auditory digit sequence task, indicating that the presence of numerical
stimuli in the task did not impede their performance. Thus, it appears that the core deficit hypothesis does not fully account for the impairments exhibited by children with AD in this study. However, it is also possible that stimuli-specific effects related to numerosity were not detected in some domains because the cognitive demands of particular tasks may have been too great or because numerical representation impairments may only be detectable in tasks that are closely associated with particular mathematical skills.

Third, the present studies were unable to apply fully the model of MD proposed by Fuchs et al. (2008). This model posited that MD is comprised of two subtypes: those with problem solving difficulties or those with computation difficulties. While the children in the present studies did exhibit arithmetic disabilities, it is unclear whether they experienced disabilities in other mathematical domains such as mathematical reasoning. As participants in the present studies were only measured for performance in arithmetic, it is likely that participants within both studies fall under the computation subtype of the Fuchs et al. model. Findings across both studies suggest that computation difficulties may be associated with impairments in shifting attention, quantity-digits inhibition, visual-spatial working memory, isolated naming speed, memory for digits, and trail making digits. More generally, a computational subtype is likely characterized by core impairments in the central executive (i.e., executive functioning and access to long-term memory) and in numerical-based processing found in other cognitive domains (e.g., short-term memory).

**Recommendations for Future Studies**

Future studies should employ a longitudinal design to examine the developmental trajectories of executive functioning, working memory, and phonological processing in children with AD. Not only do longitudinal designs lend themselves to discriminating between
developmental lags and deficits, they can also provide information on the magnitude of any impairment or deficit (Szűcs & Goswami, 2013). The field would also benefit from studies that employ a wider range of measures to assess working memory, executive functioning, and phonological processing, as well as studies that explore additional cognitive domains that may underlie MD (e.g., updating). Research that explores the classification of MD subtypes is needed to inform the development of targeted interventions. Furthermore, studies that investigate subtype-specific cognitive impairments and impairments general to MD are also essential for the remediation of MD.

With respect to Geary’s model of mathematical disabilities, further research is needed to explore the roles of conceptual understanding, executive functioning, and working memory in MD and AD. A mixed-methods study that employs executive functioning and working memory batteries, in addition to interviews exploring conceptual understanding and strategy use by MD children may help us to better understand this mechanism. Further research is also needed to determine the role of numerical representation and processing in MD. In fact, the field may benefit from employing an age-matched achievement-matched design to examine whether children with MD and MD subtypes experience a developmental lag or cognitive deficit in processing and representing numerosity. This may help researchers to understand better the nature of any impairment related to numerical representation and processing.
References


algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, 98(1), 29. doi: 10.1037/0022-0663.98.1.29


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February 23, 2016

Ms. Pamela McDonald
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Faculty of Education
Queen's University
Duncan McArthur Hall
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Kingston, ON, K7M 5R7

GREB Ref #: GEDUC-795-16; Romeo # 6017758
Title: "GEDUC-795-16 Examining Cognitive Processing in Children with an Arithmetic Disability"

Dear Ms. McDonald:

The General Research Ethics Board (GREB), by means of a delegated board review, has cleared your proposal entitled "GEDUC-795-16 Examining Cognitive Processing in Children with an Arithmetic Disability" for ethical compliance with the Tri-Council Guidelines (TCPS 2 (2014)) and Queen's ethics policies. In accordance with the Tri-Council Guidelines (Article 6.14) and Standard Operating Procedures (405.001), your project has been cleared for one year. You are reminded of your obligation to submit an annual renewal form prior to the annual renewal due date (access this form at http://www.queensu.ca/traq/signon.html; click on "Events"; under "Create New Event" click on "General Research Ethics Board Annual Renewal Form for Approved Studies").

You are reminded of your obligation to advise the GREB of any adverse event(s) that occur during this one year period (access this form at http://www.queensu.ca/traq/signon.html; click on "Events"; under "Create New Event" click on "General Research Ethics Board Adverse Event Form"). An adverse event includes, but is not limited to, a complaint, a change or unexpected event that alters the level of risk for the researcher or participants or situation that requires a substantial change in approach to a participant(s). You are also advised that all adverse events must be reported to the GREB within 48 hours.

You are also reminded that all changes that might affect human participants must be cleared by the GREB. For example you must report changes to the level of risk, applicant characteristics, and implementation of new procedures. To submit an amendment form, access the application by at http://www.queensu.ca/traq/signon.html; click on "Events"; under "Create New Event" click on "General Research Ethics Board Request for Amendment of Approved Studies". Once submitted, these changes will automatically be sent to the Ethics Coordinator, Ms. Gail Irving, at the Office of Research Services for further review and clearance by the GREB or GREB Chair.

On behalf of the General Research Ethics Board, I wish you continued success in your research.

Sincerely,

John Freeman, Ph.D.
Chair
General Research Ethics Board

c: Dr. Derek Berg, Faculty Supervisor
   Dr. Liying Cheng, Chair, Unit REB
   Ms. Erin Wicklam, Dept. Admin.