THE ROTATIONAL EVOLUTION AND MAGNETOSPHERIC EMISSION OF THE MAGNETIC EARLY B-TYPE STARS

by

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Abstract

How do the magnetic fields of massive stars evolve over time? Are their gyrochronological ages consistent with ages inferred from evolutionary tracks? Why do most stars predicted to host Centrifugal Magnetospheres (CMs) display no Hα emission? Does plasma escape from CMs via centrifugal breakout events, or by a steady-state leakage mechanism? This thesis investigates these questions via a population study with a sample of 51 magnetic early B-type stars. The longitudinal magnetic field $\langle B_z \rangle$ was measured from Least Squares Deconvolution profiles extracted from high-resolution spectropolarimetric data. New rotational periods $P_{\text{rot}}$ were determined for 15 stars from $\langle B_z \rangle$, leaving only 3 stars for which $P_{\text{rot}}$ is unknown. Projected rotational velocities $v \sin i$ were measured from multiple spectral lines. Effective temperatures and surface gravities were measured via ionization balances and line profile fitting of H Balmer lines. Fundamental physical parameters, $\langle B_z \rangle$, $v \sin i$, and $P_{\text{rot}}$ were then used to determine radii, masses, ages, dipole oblique rotator model, stellar wind, magnetospheric, and spindown parameters using a Monte Carlo approach that self-consistently calculates all parameters while accounting for all available constraints on stellar properties. Dipole magnetic field strengths $B_d$ follow a log-normal distribution similar to that of Ap stars, and decline over time in a fashion consistent with the expected conservation of fossil magnetic flux. $P_{\text{rot}}$ increases with fractional
main sequence age, mass, and $B_d$, as expected from magnetospheric braking. However, comparison of evolutionary track ages to maximum spindown ages $t_{S,\text{max}}$ shows that initial rotation fractions may be far below critical for stars with $M_* > 10M_\odot$. Computing $t_{S,\text{max}}$ with different mass-loss prescriptions indicates that the mass-loss rates of B-type stars are likely much lower than expected from extrapolation from O-type stars. Stars with Hα in emission and absorption occupy distinct regions in the updated rotation-magnetic confinement diagram: Hα-bright stars are found to be younger, more rapidly rotating, and more strongly magnetized than the general population. Emission strength is sensitive both to the volume of the CM and to the mass-loss rate, favouring leakage over centrifugal breakout.
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Statement of Originality

I hereby certify that all of the work described within this thesis is the original work of the author. Any published (or unpublished) ideas and/or techniques from the work of others are fully acknowledged in accordance with the standard referencing practices.

Matthew Eric Shultz
April, 2016
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List of Abbreviations

**Ap**: A-type star with chemical peculiarities.

**ALS**: the Alma Luminous Stars catalogue

**aRRM**: arbitrary Rigidly Rotating Magnetosphere, an RRM model which uses a ZDI map of the surface magnetic field, rather than a dipolar model or a linear superposition of multipolar models, in order to compute the accumulation surface.

**AU**: Astronomical Unit, the distance between the Earth and the Sun, equal to 1.496\times10^8 km.

**BinaMics**: Binarity and Magnetic Interactions in various classes of Stars, an international network of collaborators formed for the study of both hot and cool magnetic stars in close binary systems.

**Bp**: B-type star with chemical peculiarities.

**CFHT**: Canada-France-Hawaii Telescope, a 3.6 m Telescope on the summit of Mauna Kea, Hawai’i

**CM**: Centrifugal Magnetosphere, a region of star’s magnetosphere in which centrifugal forces are stronger than gravity, leading to plasma accumulation within potential minima.

**CP**: Chemically Peculiar

**DI**: Doppler Imaging, a tomographic technique which maps the brightness anisotropies
on the surface of a rotating star, with which temperature or chemical abundance spots may be detected.

**DAO**: Dominion Astrophysical Observatory, an observatory on Observatory Hill near Victoria, B.C., home of the 1.8 Plaskett Telescope

**DM**: Dynamical Magnetosphere, a region of a star’s magnetosphere (for slow rotators the entire magnetosphere) in which centrifugal forces are negligible, and plasma returns to the stellar surface on dynamical timescales

**ESO**: European Southern Observatory, an organization which administers the La Silla Observatory, with multiple large telescopes, and the Paranal Observatory, home of the Very Large Telescope.

**EW**: Equivalent Width, a line strength measurement defined as the width a spectral absorption or emission line would have if it were saturated.

**FAP**: False Alarm Probability, a statistical measure of the significance of a signal. If the FAP is close to 1, the signal is likely to be spurious.

**HD**: Henry Draper, compiler of the HD Catalogue.

**HeBe**: Herbig Be star, a pre-main sequence B-type star, typically showing Balmer line emission from an accretion disk

**He-s**: Helium-strong star, a CP Bp star showing strong overabundances of He

**He-w**: Helium-weak star, a CP Bp star showing strong underabundances of He

**HRD**: Hertzsprung-Russell Diagram

**IUE**: International Ultraviolet Explorer, an ultraviolet spectroscopy space telescope operating from 1978 to 1996.

**LP**: Large Program, an observing program given several semesters of guaranteed time

**LSD**: Least Squares Deconvolution, a multiline analysis technique that deconvolves
a high-SNR mean line profile from an observed spectrum and a list of spectral lines

**LTE**: Local Thermodynamic Equilibrium.

**MiMeS**: Magnetism in Massive Stars, a large international collaboration formed for the purpose of studying magnetism in hot stars

**MS**: Main Sequence, the region in the HRD where stars spend the majority of their life cycle

**NLTE**: Non-Local Thermodynamic Equilibrium

**ORM**: Oblique Rotator Model, a model of a star’s magnetic field consisting at minimum of an inclination of the rotational axis from the line of sight, an tilt angle or obliquity of the magnetic axis with respect to the rotational axis, and a surface strength of the magnetic dipole.

**PCC**: Pearson’s Correlation Coefficient, a measure of the statistical significance of the correlation between two variables.

**PMS**: Pre-Main Sequence, the period in stellar evolution before H fusion ignition.

**SB1/2/3**: Spectroscopic Binary, a star with 1, 2, or 3 stars detectable in the spectrum

**SC**: Survey Component

**SPB**: Slowly Pulsating B-type star

**RRM**: Rigidly Rotating Magnetosphere, a static, analytic model that determines the circumstellar plasma distribution in a CM via locating the potential minima along each magnetic field line, with the distribution of plasma above and below the resulting accumulation surface determined via hydrostatic equilibrium with the stellar wind.

**RV**: Radial Velocity, the line of sight component of a spectral line’s Doppler shift

**TAMS**: Terminal-Age Main Sequence, the end of the MS.
**TBL**: Telescope Bernard Lyot, a 2.2 m Telescope in the French Pyranees

**TC**: Targeted Component

**VLT**: Very Large Telescope, an ESO observatory consisting of 4 8-m Unit Telescopes at the Paranal Observator

**ZAMS**: Zero-Age Main Sequence, the beginning of the MS

**ZDI**: Zeeman Doppler Imaging, similar to DI, ZDI utilizes circularly and, sometimes, linearly polarized spectropolarimetry to map the strength and topology of a star’s magnetic field.
List of Symbols

β: the obliquity angle between the rotational and magnetic axes.

η,: the equatorial wind magnetic confinement parameter: the ratio between the kinetic energy density of the stellar wind and the energy density of the magnetic field.

χ²: the goodness of fit statistic, which evaluates how well a model reproduces the observations. Smaller values indicate a better fit.

χ²/ν: the reduced χ², i.e. χ² normalized to the number of degrees of freedom ν = N − f − 1, where N is the number of observations and f is the number of degrees of freedom in the model. If χ²/ν is close to unity, the model is considered to be a good fit; larger values indicate a worse fit; values significantly below unity indicate over-fitting of the data, i.e. noise as well as signal is being fit.

τJ: the characteristic spindown timescale of a star, given in units of Myr.

τMS: the fractional main sequence age.

a sin i: the projected semi-major axis, generally given in units of AU.

Bd: the surface strength of the dipolar component of a magnetic field.

⟨Bz⟩: the line-of-sight or longitudinal magnetic field, integrated across the stellar disk.

Hα/β/γ: the 1st, 2nd, and 3rd energy level transitions in the H Balmer series, i.e. the transitions to quantum number 2 from a higher quantum number n.
$i$: the *inclination* angle between the rotational axis and the line of sight.

$log g$: logarithmic surface gravity, typically given in cm$^{-1}$.

$log L$: the logarithmic luminosity, typically given in units of solar luminosity $L_\odot$.

$L_\odot$: the solar luminosity, approximately $3.828 \times 10^{26}$ W.

$M_*$: the stellar mass, typically given in units of solar mass $M_\odot$.

$M_\odot$: the solar mass, approximately $1.998435 \times 10^{33}$ g.

$\dot{M}$: the mass-loss rate of a star, given in units of $M_\odot\text{yr}^{-1}$.

$R_*$: the stellar radius, typically given in units of solar radii $R_\odot$.

$R_\odot$: the solar radius, approximately $6.955 \times 10^{10}$ cm.

$R_{\text{eq}}$: the stellar equatorial radius, which can be up to $1.5 \times$ larger than the polar radius due to centrifugal force arising from rotation.

$R_{\text{p}}$: the stellar polar radius.

$R_A$: Alfvén radius: the maximum extent of magnetic confinement and the boundary of a stellar magnetosphere. Typically given in units of stellar radii $R_*$.

$R_K$: the Kepler co-rotation radius, at which the gravitational force acting on a plasma is exactly balanced by the centrifugal force arising due to stellar rotation and magnetically enforced corotation of the plasma. Typically given in units of stellar radii $R_*$.

$T_{\text{eff}}$: the effective temperature, defined as the temperature determined assuming a body emits as a blackbody. Typically given in units of K.

$t_{\text{evol}}$: the evolutionary age of a star, as determined from its location on the HRD and evolutionary models.

$t_{S,\text{max}}$: the maximum spindown time, defined as the time necessary for a star to spin down from initially critical rotation to its present rotational velocity. Given in units
of Myr.

$v_\infty$: the terminal velocity of the stellar wind. Given in units of $\text{km s}^{-1}$.

$v_{\text{mac}}$: the macroturbulent velocity. Given in units of $\text{km s}^{-1}$.

$v \sin i$: projected rotational velocity. Given in units of $\text{km s}^{-1}$.

$W$: the rotation parameter, defined as the ratio between the equatorial rotational velocity and the velocity required to maintain a circular Keplerian orbit at the stellar surface.
Chapter 1

Introduction

This thesis presents a population-level investigation of the rotational, magnetic, and optical emission properties of the magnetic B-type stars. The motivations driving this work are, first, to investigate the consequences of magnetic fields for the stars’ rotational evolution; second, to evaluate the link between optical emission lines and physical, magnetic, and rotational properties; and, third, to determine what insight this can give into the nature of the mechanism by which the stellar wind plasma ultimately escapes magnetic confinement.

The first part of the introduction presents the basic background necessary to understand the remainder of the thesis: the characteristics of early-type stars; the magnetic fields of massive stars; the techniques used to detect stellar magnetic fields; hot star magnetospheres; the known properties and origin of their variable optical emission; angular momentum loss via their magnetized winds; and the results obtained by previous research into the properties of the magnetic A-type stars. The motivation for the thesis is described in the second section. The third section briefly details the specific contributions I have made to this work. The overall organization is outlined in the final section.
1.1 Background

1.1.1 Hot, massive stars

Hot, massive stars are the most luminous objects in the sky. In this work, hot stars are considered to have effective temperatures $T_{\text{eff}} > 15$ kK, and massive stars are defined as stars with stellar masses $M_\star > 5M_\odot$, where the solar mass $M_\odot = 1.998435 \times 10^{30}$ kg. Such stars have spectral types of O and B, and as such are referred to interchangeably as early-type stars. Their spectral energy distributions peak in the ultraviolet, giving them their characteristic blue-white colour. Their high luminosities, ranging from $10^2$ to $10^6$ solar luminosities (where the solar luminosity $L_\odot = 3.828 \times 10^{26}$ W), drive so-called ‘stellar winds’ from their outer atmospheres (e.g. Castor et al. 1975): photon momentum is transferred to metallic ions, which then carry along the bulk of H and He wind via Coulomb interaction. These supersonic outflows (with terminal velocities of order $10^3$ km s$^{-1}$) carry material and momentum into the circumstellar environment which, together with the ionizing radiation that drives the winds in the first place, play a dominant role in the Galactic ecology, both by initiating star-formation (via compression of molecular clouds), and by quenching it (by opening bubbles within star-forming regions, as illustrated in Fig. 1.1).

Despite their large masses, hot stars are short-lived, with main sequence lifetimes on the order of 1 to 10 Myr, as compared to the Gyr evolutionary timescales of cool, solar-type stars. Following the end of their main sequence lifetimes, defined as the point at which core H has been exhausted, core fusion of progressively heavier elements commences, beginning with He and ending with Fe. As Fe fusion consumes more energy than it produces, the star is no longer able to support itself against gravity, thus triggering a catastrophic collapse that leads to a Type II supernova, amongst
1.1. BACKGROUND

Figure 1.1: A Hubble space telescope image of the Bubble Nebula NGC 7635, showing a bubble blown by the wind of a massive star. Image credit: NASA, ESA, and the Hubble Heritage Team (STScI/AURA), F. Summers, G. Bacon, Z. Levay, and L. Frattare (Viz 3D Team, STScI).

the brightest events in the universe. All elements heavier than Fe are produced by these explosions, which then distribute this material to the surrounding environment. Thus, in life as well as death, massive stars play a key part in the chemical evolution of the universe.

1.1.2 Stellar winds

While in some cases the consequences of hot star winds can be seen in the bubbles excavated within star-forming regions, as illustrated in Fig. 1.1, the most common means of detecting and diagnosing a stellar wind is via P Cygni profiles in UV resonance lines and, for O-type stars, Hα. A P Cygni profile is an asymmetric line profile with a blue-shifted absorption trough and a red-shifted emission peak. They arise
due to isotropic radial outflows, where material projected around the star scatters excess photons in the line of sight, leading to emission; the outflow projected in front of the star scatters photons out of the line of sight, leading to absorption; and the material behind the star is eclipsed and, hence invisible. The wind projected around the star is isotropic in velocity space, while the material in front is blue-shifted: the superposition of these two components leads to the canonical P Cygni profile.

P Cygni profiles contain valuable information about the two key properties of the wind: the mass-loss rate, $\dot{M}$ and the wind terminal velocity $v_\infty$. The latter can in general be measured directly from the blue-shifted edge of the absorption trough, with typical values of 1000 to 2000 km s$^{-1}$. Measurements of the former rely upon fitting models to line profiles. For O-type stars, $\dot{M}$ generally ranges from $10^{-7}$ to $10^{-5}$ $M_\odot$ yr$^{-1}$, with $\dot{M}$ increasing with stellar mass and, hence, luminosity. As a typical main sequence lifetime of an O-type star is $\sim 1-10$ Myr, it is immediately apparent that mass-loss plays a decisive role in stellar evolution, growing more important for increasingly massive stars.

As explored in greater detail below, magnetic wind confinement, which breaks the spherical symmetry of the wind, leads to important changes to the morphologies of wind-sensitive spectral lines, making attempts to measure wind parameters directly difficult (Sundqvist et al., 2012; Grunhut et al., 2012c). Magnetic wind confinement, as the term implies, leads to a reduction in the mass-loss rate, however the magnetohydrodynamic simulations used to explore magnetic wind confinement utilize the mass-loss rate that a star would have with no magnetic field (ud-Doula and Owocki, 2002). Therefore, the accepted practice is to use theoretical mass-loss rates, rather than those determined empirically.
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The recipe in the most general use is that given by Vink et al. (1999, 2000, 2001), which was calibrated empirically using $v_\infty$ and $\dot{M}$ from O-type stars. The mass-loss rates of B-type stars are extrapolated downwards from those of the O-type stars, as the much lower $\dot{M}$ of the cooler and less luminous B-type stars (predicted by the Vink recipe to lie between about $10^{-11}$ and $10^{-7.5} \, M_\odot \, \text{yr}^{-1}$) do not in general lead to detectable emission in wind-sensitive lines. The main exception to this rule is amongst the B-type supergiant (SG) stars, which have luminosities comparable to those of O-type stars. Examination of B-type SGs (Markova and Puls, 2008) have found mass-loss rates lower by up to a factor of 3 as compared to the Vink predictions.

While the Vink recipe obtains the mass-loss rates of B-type stars via extrapolation from those of O-type stars, Krtička (2014) calculated the mass-loss rates of main-sequence B-type stars directly. Krtička predicted $\dot{M}$ to be lower by about 1 dex than predicted by the Vink recipe, and to be much more sensitive to the effective temperature, diminishing strongly towards lower temperatures and disappearing entirely near $T_{\text{eff}} = 15 \, \text{kK}$ due to the inability to launch the wind.

Distinguishing between these very different predictions is inherently challenging due to the absence of emission lines in most main-sequence B-type stars. Since some magnetic early B-type stars show emission lines, these rare objects may provide an important probe of the winds of B-type stars in general.

1.1.3 The importance of multiplicity

The majority of the work that has been done on the evolution of massive stars has focused on the consequences of mass-loss and rotation for the evolution of single stars. In binary systems, interactions between stellar components can lead to substantial
modifications to their respective evolution (Podsiadlowski et al., 1992; Wellstein and Langer, 1999; Claeys et al., 2011). In particular, mass exchange can lead to the less massive star accreting both material and angular momentum from the primary component, while the more massive primary may be stripped of much of its outer envelope. If the orbital separation between the two stars is especially close, the stars can merge, leading to strong mixing and rejuvenation of the merger product’s core. Sana et al. (2012) showed that 70% of massive stars will exchange some mass with a binary companion, indicating that far from being a special case, binary interactions likely dominate the evolution of hot stars.

From an observational perspective, there are two primary methods by which binarity can be investigated. The first is direct imaging of individual stellar components, known as a visual binary. Visual binaries are detectable only when the separation between the components is relatively large, and are of no interest from the standpoint of stellar evolution since the components are unlikely to interact in a significant fashion. The second method is via the Doppler shifting of the spectral lines of individual components, known as a Spectroscopic Binary (SB), denoted SB1, SB2, or SB3 depending on the number of detectable components. As the maximum radial velocity (RV) of an orbital system increases with decreasing distance, SBs are the systems of interest for studying the influence of binary interactions on stellar evolution.

The periodicity of an SB’s RVs corresponds to the orbital period. For SB2 stars, the orbital period, together with the detailed shape of the RV curve when phased with the orbital period, enables the semi-major and semi-minor axes, and the mass ratio, of the system to be determined via Kepler’s laws, potentially providing valuable constraints on the component stars’ physical properties.
1.1.4 The magnetic fields of massive stars

Approximately 10% of early-type stars possess strong, globally organized, approximately dipolar magnetic fields (Grunhut et al., 2012b). This is in sharp contrast to late-type stars, in which magnetic fields are practically ubiquitous. This dichotomy is thought to be a consequence of structural differences between hot and cool stars. Low-mass stars have convective envelopes and radiative cores, that is, energy is transported via radiation in the innermost part of a cool star, and by convection in the outer atmosphere. In massive stars the opposite is the case: while their cores are convective, their envelopes are radiative. A rotating, convective plasma drives a dynamo, which sustains the magnetic fields of cool stars (Parker, 1955). The dynamo origin of the solar magnetic field is well-established both observationally and theoretically, and there is a wealth of evidence that the magnetic fields of other cool, low-mass stars are also produced by dynamos (e.g. Donati and Landstreet 2009 and the numerous references therein).

As there is no significant convection in the outer envelope of a massive star, a dynamo cannot be sustained. The theoretical implausibility of dynamo action within a radiative envelope, combined with the absence of typical dynamo behaviour such as e.g. magnetic activity cycles\footnote{In fact, the magnetic fields of stars with radiative envelopes are notable for their stability over timescales of decades.}, has led to the characterization of the magnetic fields of massive stars as so-called ‘fossil fields’: remnant magnetic flux from an earlier stage in the star’s evolution (Cowling, 1945; Mestel and Strittmatter, 1967). The long-term stability of magnetic fields within the radiative envelopes of high-mass stars was conclusively demonstrated in 3D magnetohydrodynamic (MHD) simulations by Braithwaite and Spruit (2004), who found that fields could evolve into a ‘twisted torus’...
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Figure 1.2: Schematic diagram of the Oblique Rotator Model (ORM). The black circle indicates the stellar surface. The blue arrow shows the rotational axis, and the red arrow the magnetic axis. The inclination angle between the rotational axis and the line of sight is denoted by $i$, and the obliquity angle between the magnetic and rotational axes by $\beta$.

geometry in which the poloidal flux (the component of the magnetic field detectable at the stellar surface) is stabilized by a significant internal toroidal field.

Since fossil fields are unrelated to dynamos, they do not exhibit magnetic activity cycles analogous to those of the Sun. As a result of this, all variability can be ascribed to rotational modulation. Fig. 1.2 shows a schematic of the Oblique Rotator Model (ORM) which is used to model longitudinal (or line-of-sight) magnetic field curves (denoted as $\langle B_z \rangle$, where the $z$ axis is aligned with the line of sight, and the $x$ and $y$ axes are in the plane of the sky). In its simplest form, the magnetic field is described as a tilted dipole with an obliquity angle $\beta$ between the magnetic and rotational axes. If the rotational axis is not aligned with the line of sight (i.e. if the inclination angle
1.1. BACKGROUND

$i$ is not zero), then as the star rotates the magnetic pole will be traced out a circle on the sky, producing a rotational modulation of the longitudinal magnetic field. If the magnetic field is predominantly dipolar, as is the case for the majority of magnetic early-type stars, this will lead to a sinusoidal variation of $\langle B_z \rangle$.

1.1.5 Detecting stellar magnetic fields

The first discovery of an astrophysical magnetic field was made by Hale (1908), who detected Zeeman splitting (Zeeman, 1897) in the spectrum of a sunspot and inferred a magnetic field within the sunspot of several kG. The Zeeman effect involves the splitting of magnetically sensitive atomic energy level transitions into three or more components, with the degree of splitting proportional to the strength of the magnetic field and the magnetic sensitivity of the energy level transition. Zeeman splitting arises due to the coupling of orbital and spin angular momentum, as can be seen in the Hamiltonian for an isolated atom in a magnetic field (Schiff, 1955):

$$H = -\frac{\hbar}{2m} \nabla^2 + V(r) + \xi(r) \mathbf{L} \cdot \mathbf{S} + \left[ -\frac{e}{2mc} \mathbf{B} \cdot (\mathbf{L} + 2\mathbf{S}) + \frac{e^2}{8mc^2} B^2 r^2 \sin^2 \theta \right], \quad (1.1)$$

where $m$ and $e$ are the mass and charge of the electron, $c$ is the speed of light, $\mathbf{B}$ is the vector magnetic field strength, and $\mathbf{L}$ and $\mathbf{S}$ are the orbital and spin angular momentum operators. The first three terms in the Hamiltonian are, respectively, the kinetic, potential, and spin-orbit coupling energies, with the spin-orbit interaction given by $\xi(r) = (m^2 c^2 / 2)(1/r) dV dr$. The fourth and fifth terms give the magnetic energy. The magnetic terms are generally small compared to the Coulomb potential $V(r)$ if $B < 10$ MG, in which case the magnetic splitting can be described using
perturbation theory. If $B < 50$ kG, the quadratic field term $\ll$ the linear field term $\ll$ the spin-orbit term, and the splitting is in the regime of the linear Zeeman effect. For reference, the surface magnetic field strength of a magnetic early-type star is typically on the order of 1 kG, and only rarely as high as 30 kG.

In the regime of the linear Zeeman effect, also known as the weak-field regime, for each energy level $i$ in zero magnetic field, for a given total angular momentum quantum number $J = |L \pm S|$ there will be $2J + 1$ magnetic sublevels with energy $E_{i0}$ at $B = 0$, with energies

$$E_i = E_{i0} + g_i \left( \frac{e}{2mc} \right) B m_J \hbar,$$

where $m_J = -J \ldots J$, and $g_i$ is the dimensionless Landé factor given by

$$g_i = 1 + \left( \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)} \right).$$

The splitting in wavelength is then

$$\lambda_{ij} = \lambda_0 + \left( \frac{e\lambda_0^2 B}{4\pi mc^2} \right) (g_j m_j - g_i m_i),$$

with the selection rule $m_j = m_i \pm 0, 1$, and the rest wavelength $\lambda_0$ corresponding to the wavelength of the transition unperturbed by a magnetic field.

For a typical magnetic star with $B \sim 1$ kG, at a typical wavelength of 500 nm, the amount of splitting is only about $\pm 0.0012$ nm, corresponding to about 1 km s$^{-1}$ in Doppler velocity. Since this is less than the typical Doppler widths of stellar spectral lines (around 5–10 km s$^{-1}$), which are furthermore subject to rotational and turbulent broadening (typically tens to hundreds of km s$^{-1}$), the Zeeman effect is inherently
difficult to detect in stellar spectra. As a result of this, the first discovery of a magnetic field in a star other than the Sun did not occur until 1947, when Babcock (1947) detected a 1.5 kG magnetic field in the sharp-lined chemically peculiar Ap star 78 Vir, a relatively cool star (in comparison to the stars examined in this work) with very little Doppler broadening, and no broadening due to either rotation or turbulence.

Hale and Babcock utilized the key property of the Zeeman effect that some of the magnetically sensitive components possess opposite circular polarizations. Thus, by placing a polarization analyzer in front of the spectrograph and measuring the small shift in wavelength between spectra obtained with opposite circular polarizations, a star’s magnetic field can be detected.

For several decades stellar magnetic fields were only detected in Ap stars. In the 1970s, knowledge of the phenomenon was extended to chemically peculiar Bp stars. The first unambiguous detection was in the He-weak star HD 175362 (Wolff’s Star, Wolff and Wolff 1976), and the first detection in a He-strong star was in HD 37479 (σ Ori E, Landstreet and Borra 1978). Magnetic fields were soon shown to be a general phenomenon in the He-peculiar stars (Borra and Landstreet, 1979; Borra et al., 1983).

The 1990s saw the development of two related capabilities that led to a revolution in the detection and measurement of stellar magnetic fields. The first of these was the advent of high-dispersion echelle spectropolarimeters on large telescopes, which were able to obtain polarized spectra at high signal-to-noise (SNR) over a large spectral range whilst simultaneously resolving individual metallic lines. The second was the development of multi-line analysis techniques, principally Least-Squares Deconvolution (Donati et al., 1997), which enables information from multiple spectral lines to
be combined in a single mean line profile with a much higher per-pixel SNR than is practical to obtain in individual spectral lines. Since the degree of polarization in the spectrum of a magnetized star is often quite small ($< 10^{-4}$), achieving a high SNR is a crucial enabling factor in stellar magnetometry. These capabilities enabled the first detection of a magnetic field in a non-chemically peculiar B-type star, the β Cepheid pulsator β Cep (Donati et al., 2001; Henrichs et al., 2013). In short order the first magnetic O-type star, θ¹ Ori C, was discovered by Donati et al. (2002), confirming earlier speculation that the star’s extremely strong and hard X-ray emission might be a consequence of magnetically confined wind shocks (Stahl et al., 1993, 1996a; Babel and Montmerle, 1997).

1.1.6 Massive star magnetospheres

The winds of massive stars are highly ionized and, as such, responsive to magnetic fields. The combination of a stellar wind with a magnetic field can thus lead to the formation of a circumstellar magnetosphere (Babel and Montmerle, 1997; ud-Doula and Owocki, 2002). In recent years the magnetospheres of magnetic hot, massive stars have been the subject of intensive observational and theoretical attention. The observational consequences include: hard, strong X-ray emission (Oskinova et al., 2011; Nazé et al., 2014); ultraviolet, optical, and infrared line emission (Barker et al., 1982; Smith and Groote, 2001; Walborn, 1974; Townsend et al., 2005; Oksala et al., 2015a); photometric variations due to eclipsing of the star by the magnetospheric plasma (Hesser et al., 1976; Townsend, 2008); and variable radio emission (Drake et al., 1987; Chandra et al., 2015).

There has been considerable success in modelling the magnetospheric variability
of magnetic OB stars. Magnetohydrodynamic (MHD) simulations performed in 2 and 3 dimensions are able to reproduce both the periodic and stochastic ultraviolet and optical variability of the magnetic O-type stars (ud-Doula and Owocki, 2002; Sundqvist et al., 2012; ud-Doula et al., 2013), while a semi-analytic formalism predicts the X-ray luminosity of most magnetic, massive stars to within 10% of their observed values (ud-Doula et al., 2014; Nazé et al., 2014).

The simplest case, that of a non-rotating star with aligned magnetic and rotational axes, was investigated by ud-Doula and Owocki (2002) using 2D MHD simulations. The simulations showed that magnetically channeled flows originating from colatitudes in opposite hemispheres collide at the magnetic equator, leading to cooling shocks that produce the strong, hard X-ray emission observed that emanates from magnetic, massive stars. The cooled plasma then collects in a cool (∼10 kK), relatively high-density torus, which falls back to the photosphere under the influence of gravity. 3D MHD simulations presented by ud-Doula et al. (2013) showed that this turbulent infall is the likely origin of the stochastic variations observed in the emission lines of the well-known magnetic O-type star θ¹ Ori C (Stahl et al., 1996b).

If the magnetic axis is not aligned with the rotational axis there will be periodic variations resulting from rotational modulation of the sky-projected geometry of the magnetosphere, a conclusion reached both from the identical periodicities of magnetic and emission-line measurements (e.g. Wade et al. 2011, 2012a, 2015) and from MHD simulations of magnetospheres around stars with tilted dipoles (e.g. Sundqvist et al. 2012).

Magnetic B-type stars with emission are beyond the numerical reach of MHD simulations due both to the much stronger magnetic confinement of their winds, and
1.1. BACKGROUND

Figure 1.3: The Hα line of σ Ori E at maximum emission (solid black) and minimum emission/maximum absorption (dashed blue).

to the prominent role played by rapid rotation in shaping the circumstellar environment (Townsend and Owocki, 2005). However, most of the qualitative features of the optical and photometric variability can be reproduced with the analytical Rigidly Rotating Magnetosphere model (Townsend et al., 2005, 2013; Oksala et al., 2015b).

1.1.7 Variable emission in magnetic B-type stars

The first detection of variable optical emission from a magnetic B-type star was made by Walborn (1974) in the spectrum of σ Ori E (HD 37479, B2 Vp). Walborn also noted emission in the spectra of two other rapidly rotating He-strong stars. σ Ori E’s Hα line is shown in Fig. 1.3, at phases of maximum and minimum emission. Originally interpreted as a mass-transferring binary due to the combination of periodic eclipses and periodic emission peaks at large Doppler velocities (Hesser et al., 1976), it was
1.1. BACKGROUND

later shown by Groote and Hunger (1982) and Nakajima (1981, 1985) that these phenomena can be explained as a consequence of the star’s magnetosphere. This picture emerged from insights gleaned from the Jovian magnetosphere (Michel and Sturrock, 1974). Magnetospheric plasma is locked to Jupiter’s magnetic field and, due to the planet’s rapid rotation and the consequent importance of centrifugal force in shaping the effective potential, plasma travelling along a given flux tube accumulates at the potential minima. The principle difference is that, in the Jovian magnetosphere, ions are supplied externally by Io’s volcanic activity and, to a lesser degree, the solar wind, while in the magnetosphere of a B-type star ions are supplied internally by the star’s radiative wind.

The current state-of-the-art in modelling the magnetospheres of rapidly rotating, strongly magnetized B-type stars is the Rigidly Rotating Magnetosphere (RRM) model developed by Townsend and Owocki (2005), which takes its name from the simplifying assumption that the magnetic field dominates the circumstellar environment and, hence, time-dependent MHD processes can be neglected. An illustration of the RRM model is provided in Fig. 1.4. The model calculates the shape of the accumulation surface within the gravitocentrifugal potential via location of potential minima along magnetic field lines, and then determines the scale height and density of the magnetically confined plasma under the assumption of hydrostatic equilibrium along each field line. For a tilted dipole, the RRM model predicts an accumulation surface in the shape of a warped disk, with the majority of the plasma located in two clouds at the intersections of the rotational and magnetic equators. This naturally explains the double-horned shape of emission lines (e.g. Fig. 1.3; see also Townsend et al. 2005; Townsend and Owocki 2005), the variability of these lines (as illustrated
Figure 1.4: The Rigidly Rotating Magnetosphere (RRM) model (Townsend and Owocki, 2005). The four panels show, clockwise from top left, the sky-projected model in rotational phase increments of 0.25 beginning from phase 0. The yellow arrow indicates the rotational axis, the pink arrow the magnetic axis. Magnetic field lines are shown by green lines. Brightness indicates the optical depth of the circumstellar plasma, while colour is mapped to Doppler velocity (blue corresponding the blue-shift, red to red-shift). Note the cavity between the magnetosphere and the star due to insufficient centrifugal support in the inner region of the magnetosphere. Images obtained from an on-line grid of movies provided by Richard Townsend (available at http://www.astro.wisc.edu/townsend/static.php?ref=rrm-movies).
for σ Ori E’s Hα line in Fig. 1.5), and the depth and duration of photometric eclipses (Townsend, 2008).

The original RRM model considers only simple magnetic geometries, i.e. tilted dipoles (Townsend and Owocki, 2005) or offset dipoles (Townsend et al., 2005). The underlying magnetic geometry is referred to as a dipolar Oblique Rotator Model (ORM). ORM models consist of 3 parameters: the inclination angle $i$ of the rotational axis from the line of sight, the obliquity angle $\beta$ between the rotational and magnetic axes, and the surface strength of the magnetic dipole at the magnetic pole $B_d$. In Fig. 1.4, the rotational axis is shown by the yellow arrow, and the magnetic axis by the purple arrow. Note that while the rotational axis is always oriented the same way relative to the observer, the orientation of the magnetic axis is a function of the rotational phase. This gives rise to the rotationally modulated variability of the star’s magnetic field, as well as the circumstellar structures associated with it.

When projected beside the stellar disk, as in the top left and bottom right panels of Fig. 1.4, there is excess light scattered into the line of sight, along with line emission due to recombination and bound-bound transitions. Line-of-sight or radial velocity is mapped to colour in Fig. 1.4. Because the plasma is rotating as a solid body, its line-of-sight or radial velocity $v$ is directly proportional to its distance $r$ from the star: $v/v\sin i = r/R_*$, where $v\sin i$ is the star’s projected rotational velocity ($\pm v\sin i$ is indicated in Fig. 1.5 by solid vertical lines). This is reflected in the top horizontal axis of Fig. 1.5. As a result peak emission occurs at high Doppler velocities ($v > v\sin i$). When the plasma occults the star, as in the top right and bottom left panels of Fig. 1.4, the star is dimmed, and there is excess line absorption at low Doppler velocities ($v < v\sin i$), as is apparent at phases 0 and 0.4 in Fig. 1.5. This is also the source of
Figure 1.5: Dynamic spectra of σ Ori E's Hα line, adapted from Townsend et al. (2005). One-dimensional residual spectra with respect to a photospheric model (bottom panel) are mapped to the colour-bar (right) and phased with a nonlinear rotational ephemeris accounting for the star’s spindown (top) (Townsend et al., 2010). Vertical lines indicate ±v sin i, dotted lines the Kepler corotation radius ±R_K, both shifted by 20 km s^{-1} into the stellar rest-frame. Dashed lines trace the two magnetospheric clouds. Emission maxima occur at quadrature phases (0.15, 0.75), when the projection angle of the magnetic dipole is closest to the centre of the stellar disk, and the magnetic equator is thus closest to parallel to the plane of the sky. Note that the emission is concentrated above R_K. Approximately 0.25 of a rotational cycle later, the clouds eclipse the star, leading to enhanced absorption in the rotationally broadened core of the line. The slight phase offset between the clouds, along with the asymmetry in emission strengths, are both consequences of the complexity of the star’s surface magnetic field.
photometric dimming.

For a centred dipole, RRM predicts emission that of the same strength in the red and blue line wings, with emission maxima in the red and blue wings occurring at the same rotational phase, and with eclipses occurring 0.25 of a rotational cycle after emission maxima. This does not provide a good match to \( \sigma \) Ori E’s dynamic spectrum, as its magnetic field is more complex than a centred dipole (Bohlender et al., 1987). While an offset dipole can reproduce both the line-of-sight or longitudinal magnetic field \( \langle B_z \rangle \) variation of the star and the qualitative behaviour of H\( \alpha \) (Townsend et al., 2005), this simple modification cannot reproduce the circular polarization profile (Oksala et al., 2012). Oksala et al. (2015b) presented an arbitrary RRM (aRRM) model, which improves on the basic RRM model by modelling the magnetosphere of a star with an arbitrarily complex surface magnetic field. aRRM utilizes a map of the surface magnetic field obtained via Zeeman Doppler Imaging (ZDI, Semel 1989; Donati et al. 1989, 1990; Piskunov and Kochukhov 2002), and then determines the shape of the accumulation surface using potential field extrapolation. The model has shown some success in reproducing the emission and absorption behaviour of \( \sigma \) Ori E. However, Oksala et al. noted that aRRM was unable to reproduce all features of the H line emission or the photometric variability. As their photometric model included the contributions of He, Fe, C, and Si surface chemical abundance spots, obtained via Doppler Imaging (Vogt et al., 1987) of these elements, the disagreement is unlikely to be a consequence of photospheric contributions to the light curve.

A further extension of the RRM model is the Rigid-Field Hydrodynamics (RFHD) formalism, in which the assumption of a rigid magnetic field is retained, but the assumption of hydrostatic equilibrium along each field line is relaxed (Townsend et al.,
In addition to reproducing the cool, dense plasma modelled by RRM, RFHD simulations are able to predict the locations of the post-shock X-ray emitting plasma, as well as near and far-ultraviolet emission, thus giving a window into the high-energy regime.

1.1.8 Emission properties of the population

So far, detailed RRM modelling has only been performed for σ Ori E. However, several other magnetic B-type stars with variable emission characteristics similar to those of σ Ori E have been reported in the literature, e.g.: HD 37017 (B2 Vp: Leone 1993), δ Ori C (HD 36485, B3 Vp: Leone et al. 2010), HD 176582 (B5 IV: Bohlender and Monin 2011), HR 5907 (HD 142184, B2 V: Grunhut et al. 2012a), and HR 7355 (HD 182180, B2 V: Rivinius et al. 2013). In the case of HD 23478 (B3 IV), the discovery of its magnetospheric H line emission (Eikenberry et al., 2014) preceded and indeed, motivated the spectropolarimetric observations that detected its magnetic field (Sikora et al., 2015; Hubrig et al., 2015).

Petit et al. (2013) (hereafter P13) collected available data regarding all known magnetic stars earlier than spectral type B5 with the aim of systematically examining their emission properties. They developed a taxonomy in which magnetic early-type stars were classified as possessing either Dynamical Magnetospheres (DMs) only, or also Centrifugal Magnetospheres (CMs). This distinction is illustrated in Fig. 1.6. In a DM the plasma falls back to the star under the influence of gravity. While all magnetic, massive stars with magnetically confined winds possess a DM, a CM occurs only in those stars which combine rapid rotation with sufficiently strong magnetic fields that the wind remains confined to a radius greater than that at which centrifugal
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Figure 1.6: Illustration of a star with a Dynamical Magnetosphere (DM), and a star with both a DM and a Centrifugal Magnetosphere (CM). Within the DM, the magnetically confined plasma falls back to the stellar surface under the influence of gravity. The magnetic field enforces corotation on the plasma out to the Alfvén radius $R_A$, leading to a centrifugal force on the plasma. The Kepler corotation radius $R_K$ is defined as the point at which centrifugal and gravitational forces balance. If the star’s rotation is slow ($R_A<R_K$), the centrifugal force is negligible, and the star possesses a DM only. Rapidly rotating stars, with $R_A>R_K$, possess a CM. Within the CM, infall is blocked by centrifugal force, leading to the formation of a compressed, high density disk. Reproduced from Fig. 2 from Petit et al. (2013).
and gravitational forces balance.

P13 quantified the distinction between DMs and CMs via the maximum radius of magnetic confinement, given by the *Alfvén radius* $R_A$ (Babel and Montmerle, 1997; ud-Doula and Owocki, 2002), and by the radius of gravitational and centrifugal equilibrium given by the corotation or *Kepler radius* $R_K$ (Townsend and Owocki, 2005; ud-Doula et al., 2008).

The Alfvén radius is defined from the ratio of the magnetic energy density and the kinetic energy density of the wind. This ratio is known as the *wind magnetic confinement parameter* $\eta_*$ (ud-Doula and Owocki, 2002), where if $\eta_* > 1$ the wind is considered to be magnetically confined. Above $R_A$ the magnetic field is opened by the stellar wind, and plasma is carried away from the star. Below $R_A$, the plasma is constrained to follow the magnetic field. This leads flows originating at colatitudes in opposite hemispheres to collide at the magnetic equator, leading to X-ray-producing shocks (Babel and Montmerle, 1997; ud-Doula and Owocki, 2002; ud-Doula et al., 2014) and to the formation of a torus of cool plasma at the magnetic equator.

In analogy to $\eta_*$, $R_K$ is related to the *rotation parameter* $W$, which is the ratio of the surface rotational velocity to the velocity required to maintain a Keplerian orbit just above the surface (ud-Doula et al., 2008). If $W = 0$, the star is non-rotating. If $W = 1$, the star is rotating at its critical or breakup velocity: $W > 1$ is physically impossible as the star’s gravity would no longer be able to maintain it against centrifugal disruption. In Fig. 1.4, the empty region close to the star is at $r < R_K$. In Fig. 1.5 $R_K$ is indicated by vertical dotted lines: note that, at the phases of strongest emission, the emission is located at $r > R_K$.

As shown in Fig. 1.6, a star with $R_A > R_K$ possesses a CM in the outermost region.
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Figure 1.7: The rotation-magnetic confinement diagram in Hα, originally introduced by P13. The diagonal $R_A=R_K$ line divides CMs (above) from DMs (below). Arrows indicate lower limits on $R_A$ and upper limits on $R_K$ for stars for which the magnetic and/or rotational properties are poorly constrained. Herbig Be (HeBe) stars are indicated in purple.

of its magnetosphere. Stars with very small $R_K$ and large $R_A$ are in the regime of applicability of the RRM model described above, in which the magnetic field is so strong that the plasma is effectively locked to the field lines.

Fig. 1.7 shows the rotation-magnetic confinement diagram introduced by P13, where $R_K$ is plotted as a function of $R_A$ (or equivalently, $W$ vs. $\eta_\ast$). The solid diagonal $R_A=R_K$ line in Fig. 1.7 divides stars possessing only a DM (below the line)
from those that also possess a CM (above the line). The status of a star’s Hα line, in emission or absorption, is indicated by colour, while shape corresponds to one of 4 temperature bins. Note that pre-main sequence Herbig Be stars typically show Hα emission due to their accretion disks, thus these are indicated separately.

Only stars with mass-loss rates high enough to replenish the magnetospheric plasma on dynamical timescales possess optically detectable DMs: in other words, the magnetic O-type stars. Virtually all of these are fairly slow rotators (i.e., large \( R_K \) or equivalently, small \( W \)), with relatively small \( R_A \) due to the large kinetic energy densities of their powerful winds. All of the magnetic O-type stars display Hα emission. By contrast, Hα is in absorption for the majority of the magnetic B-type stars. Most of the Hα-bright stars have \( R_A > R_K \), and indeed the majority have \( R_A \gg R_K \). This is more or less in line with the fundamental assumptions of the RRM model: the magnetic field should be strong enough to dominate the circumstellar environment, while the rotational velocity should be high enough to achieve centrifugal support close to the stellar surface.

1.1.9 Rotational evolution

An important consequence of magnetic wind confinement is rotational spindown. A star’s moment arm is extended by the corotating plasma, producing a torque that efficiently sheds angular momentum when material is lost from the system (Weber and Davis, 1967). This is essential to star formation, as otherwise the angular momentum of the contracting protostellar cloud inhibits the final stages of collapse. In contrast to early expectations that protostars should rotate close to their critical or breakup velocity, they rotate at only about 10% of this velocity (Vogel and Kuhi, 1981).
This is easily explained by magnetospheric braking (Bouvier et al., 2014). Models of rotational evolution incorporating magnetic angular momentum loss are highly successful at predicting the surface rotational velocities of cool stars, to the point that gyrochronology has emerged as a powerful dating technique for stellar clusters in which the main-sequence turnoff is in the regime of slowly evolving low-mass stars, in which isochrone-fitting is insensitive to relatively large differences in age (Barnes, 2003, 2007, 2010). Gyrochronology has enabled accurate dating of clusters up to 4 Gyr (Barnes et al., 2016).

In contrast to cool stars, in which magnetic braking is ubiquitous and, in consequence, surface rotational velocities tend to be quite low, massive stars are in general much more rapidly rotating. However, magnetic massive stars are much more slowly rotating than their unmagnetized kin, with rotational periods as long as decades having been reported for some magnetic A-type stars (Landstreet and Mathys, 2000a). As OB stars have much stronger winds than A-type stars, spindown occurs much more rapidly in magnetic, massive stars (ud-Doula et al., 2009). Rotational spindown has actually been measured for two stars (σ Ori E: Townsend et al. 2010; HD 37776: Mikulášek et al. 2008). For these stars, spectroscopic, photometric, and magnetic datasets with long temporal baselines are available, and nonlinear rotational ephemerides are necessary to phase data acquired in different epochs. The dynamic spectrum of σ Ori E in Fig. 1.5 has been phased with the nonlinear ephemeris determined by Townsend et al. (2010): as the spectroscopic data in Fig. 1.5 was acquired over a period of ~10 yr, utilizing a linear ephemeris yields noticeable imperfections in the phasing of the data.
1.1.10 Population studies of Ap/Bp stars

Ap/Bp stars are chemically peculiar A- and B-type stars, characterized by strong over- and under-abundances of various chemical elements. Such stars invariably host strong magnetic fields. Indeed, Babcock (1947) detected the first magnetic field in a star other than the Sun in an Ap star. While some non-chemically peculiar magnetic B-type stars have been discovered, the majority of the known magnetic B-type stars are Bp stars.

As so many magnetic, massive stars are recent discoveries, no systematic examination of the stellar, rotational, and magnetic properties of this population has yet been carried out. Thus, it is not yet known in what ways the properties of this population are similar to, or different from, the properties of Ap stars. However, several population studies of the cooler Ap stars have been performed. As these stars are similar in many respects to Bp stars, it is worthwhile to briefly review the properties of the Ap star population as revealed by these studies.

Mathys et al. presented spectropolarimetric measurements and rotation periods for 44 Ap/Bp stars, noting rotational periods ranging from a few days to several decades (Mathys et al., 1997; Mathys and Hubrig, 1997). Landstreet and Mathys (2000b) compared slowly to rapidly rotating Ap stars, finding $\beta$ to be generally small for slow rotators, which Stępień and Landstreet (2002) interpreted as a consequence of more rapid disk dissipation during the pre-main sequence phase amongst stars with aligned dipoles.

Using low-resolution FORS1 spectropolarimetry presented by Hubrig et al. (2006b), Hubrig et al. (2007) studied a sample of 90 Ap/Bp stars, divided into stars with masses above and below $3 M_\odot$, and comparing Oblique Rotator Model (ORM) parameters
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to stellar parameters and evolutionary statuses. They also found systematically small \( \beta \) angles amongst slow rotators, although the overall sample was dominated by stars with \( \beta > 80^\circ \), in agreement with previous results (Landstreet and Mathys, 2000b; Stępień and Landstreet, 2002). Hubrig et al. found long periods only for stars that had already exhausted at least 40% of their main-sequence lifetimes. They also concluded that \( \beta \) itself may change, moving closer to either 0° or 90° as a star evolves away from the zero-age main sequence. However, for stars with \( M_\ast > 3M_\odot \), they found \( \beta \) to increase with age, in contrast to the results presented by Landstreet and Mathys (2000b).

Power (2007) conducted a volume-limited study of Ap stars and determined ORM parameters for 34 stars, observing 26 stars with the high-resolution spectropolarimeter MuSiCoS and using literature data for the remainder. They found for most of the sample that \( P_{\text{rot}} < 2 \text{ d} \) and \( v \sin i < 70 \text{ km s}^{-1} \); that \( B_d \) peaks near 2.5 kG. They also reported a bimodal distribution of \( \beta \), with more small and large angles than predicted by a random distribution.

Aurière et al. (2007) used high-resolution spectropolarimetric observations to obtain ORM parameters for 24 Ap stars, concentrating their efforts on stars with weak magnetic fields. They found a lower limit of 300 G for \( B_d \), which they interpreted as a cutoff at the critical field \( B_c \), below which magnetic fields are unstable against rotation. This led Lignières et al. (2014) to propose the concept of a ‘magnetic desert’, in which the bimodal distribution for massive star magnetic fields, with strong dipoles present in only 10% of the total population, is a consequence of magnetic fields below \( B_c \) being unstable against rotational perturbations (Jouve et al., 2015). An apparent increase in the minimum magnetic field strength with decreasing rotation period
could be consistent with this (Lignières et al., 2014), although it could also simply indicate observational bias, as the difficulty of detecting a magnetic field increases with $v \sin i$. The discovery of magnetic B-type stars with $B_d \leq 100$ G has further called the existence of a magnetic desert into question (Fossati et al., 2015a), although it must be noted both of the stars reported to have very weak magnetic fields, $\beta$ CMa and $\epsilon$ CMa, are evolved and are likely to have seen substantial reduction in the surface strengths of their magnetic fields due to flux conservation as the stars expanded.

The intrinsic evolution of fossil magnetic fields remains the subject of some controversy. Studies using field stars, in which ages were derived from positions on the Hertzsprung-Russell Diagram (HRD), found evidence that surface magnetic field strengths decrease over time due to flux conservation, but also that the total unsigned magnetic flux increased, particularly for the more massive stars (Kochukhov and Bagnulo, 2006). This result was contradicted by observations of Ap stars in clusters, which were consistent with either conservation of magnetic flux, or decay due to Ohmic dissipation (Landstreet et al., 2007, 2008). Cluster Ap stars have the advantage that the uncertainty in the age of young stars is greatly reduced: evolution close to the Zero-Age Main Sequence (ZAMS) is relatively slow, so typical uncertainties in stellar effective temperatures and luminosities lead to large uncertainties in age (Bagnulo et al., 2006). With star clusters, the age is determined via isochrone fitting to the main-sequence turnoff, leading to a much higher precision in the ages of younger stars. However, ORM parameters were not available for most or any of the program stars used by Kochukhov and Bagnulo (2006), Bagnulo et al. (2006), Landstreet et al. (2007), or Landstreet et al. (2008), so the root-mean-square magnetic field was used as a proxy for precise values of $B_d$. Furthermore, the sample sizes of the cluster Ap
stars was much smaller than the sample of field stars.

1.2 Motivation

1.2.1 The rapid pace of discovery

Over the past several years magnetic hot stars have been the subject of a substantial investment in observational time at large telescopes around the world. Cluster surveys with the low-resolution FORS1 and FORS2 spectropolarimeters at the European Southern Observatory’s (ESO) 4×8 m Very Large Telescope (VLT) have discovered many new magnetic stars (Bagnulo et al., 2006; Landstreet et al., 2007). The data presented in this thesis were largely collected by the Targeted Components (TCs) of the Magnetism in Massive Stars (MiMeS) Collaboration, with Large Programs (LPs) utilizing the high spectral-resolution spectropolarimeters ESPaDOnS at the 3.6 m Canada-France-Hawaii Telescope (CFHT), Narval at the 2 m Telescope Bernard Lyot (TBL) in France, and HARPSpol at ESO’s 3.6 m La Silla telescope (Wade et al., 2016). A large focus of the MiMeS LPs were magnetic O-type stars, and in particular the Of?p stars, which MiMeS data conclusively established as invariably hosting magnetic fields (Wade et al., 2011, 2012a,b, 2015).

The MiMeS LPs were followed by the Binarity and Magnetic Interactions in various classes of Stars (BinaMiCs) LPs at CFHT and TBL, which adopted a similar strategy to the MiMeS LPs, but applied to both hot and cool binary stars. This thesis also makes use of the BinaMiCs hot star data. The high quality and large size of these datasets is a consequence of the surprising rarity of magnetic hot stars in close binary systems: despite observing 200 systems, the BinaMiCs survey detected no new magnetic hot binaries. This leads to a very low fraction (≤ 3%) of close
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Binary systems which contain a magnetic hot star (Alecian et al., 2015). While no new magnetic binaries have been discovered, BinaMics data were instrumental in detecting a magnetic field in the secondary component of $\epsilon$ Lupi (Shultz et al., 2015c), in which the primary was already well-known to be magnetic (Hubrig et al., 2009; Shultz et al., 2012), thus making $\epsilon$ Lupi the first doubly magnetic hot-star binary.

Recently, the B-fields in OB stars (BOB) collaboration has been using HARPSpol and FORS2 to search for weak magnetic fields in OB stars. BOB has discovered a magnetic B-type star in the young Trifid nebula, HD 164492C (Hubrig et al., 2014), and have verified the presence of extremely weak magnetic fields in $\beta$ CMa and $\epsilon$ CMa (Fossati et al., 2015a).

While the rotation-confinement diagram published by P13 and reproduced in Fig. 1.7 is highly suggestive, there is much that remains ambiguous. The magnetic fields of many of the stars included by P13 were only recently discovered, with only 1 or 2 observations available, and their rotational periods and/or surface magnetic field strengths were unknown. For these stars only upper limits for $R_K$ and lower limits for $R_A$ could be established (arrows in Fig. 1.7). In some cases, these limits locate Hα-bright B-type stars well away from the top right of the diagram. High-resolution magnetic data obtained using modern instrumentation were unavailable for many other stars; since different magnetic measurement systems are liable to return systematically different results, even for many of the stars for which rotation periods and ORM parameters are known, a self-consistent comparison of their properties to MiMeS targets could not be performed. Additional ambiguity arises due to the apparent presence in the RRM regime of stars with Hα in absorption. For other stars, Hα data were not available.
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With several magnetic stars discovered subsequent to the first publication of the rotation-magnetic confinement diagram by P13, and much larger datasets available for many of the stars included in the P13 sample, it is necessary to revisit their results in light of these new data.

1.2.2 Mass balance in CMs: violent eruption or leakage?

It is well-understood how material enters a CM. How it gets out is not. This question was first addressed by Havnes and Goertz (1984), who considered two scenarios: diffusion across field lines, and sporadic ejection. The diffusive mechanism they examined was ambipolar diffusion, in which temporarily neutral atoms are able to move across field lines and thus escape the magnetosphere. As the ionization fraction should be quite high, and collisional ionization timescales are expected to be much shorter than the time required for neutrals to escape the magnetosphere, they came to the conclusion that ambipolar diffusion could not efficiently evacuate the plasma. Thus, they suggested that the most likely scenario is that the density increases until the magnetic field can no longer confine the plasma, at which point a catastrophic rupture occurs and the plasma is ejected outwards from the star in a violent reconnection event. 2D MHD simulations of rotating magnetically confined winds supported this basic picture, further predicting that these ejections, termed Centrifugal Breakout (CB) events, should be accompanied by X-ray flaring (ud-Doula et al., 2006, 2008).

While there are good theoretical reasons to expect CB, the observational evidence is far from convincing. X-ray flaring was reported for σ Ori E (Sanz-Forcada et al., 2004), however it was later shown that there is a nearby M-type star which is almost certainly the X-ray source (Bouy et al., 2009). Townsend et al. (2013) analyzed three
weeks of continuous high-precision space photometry obtained with the MOST space telescope, covering 18 rotational cycles, and failed to detect any changes in \( \sigma \) Ori E’s periodic eclipses. While this result is somewhat ambiguous, as typical CB timescales for this star are estimated at \( \sim 200 \) years, Townsend et al. also noted that the upper mass limit inferred from an RRM model capable of reproducing the eclipses was 2 orders of magnitude below the limiting mass necessary for a significant CB event to occur.

There are also puzzling inconsistencies between the distribution of circumstellar material predicted by the RRM model, and inferred both from optical emission lines and from broadband linear polarization scattering measurements. Linear polarimetry especially has found that the material must be far more tightly concentrated in the two clouds than expected, such that a simple two-cloud ‘dumb-bell’ model more accurately reproduces the broadband polarimetry than does a full RRM treatment (Carciofi et al., 2013). Notably, the detailed aRRM analysis presented by Oksala et al. (2015b) found that the light curve could not be fully reproduced even with inclusion of contributions from surface abundance maps obtained via Doppler Imaging. Oksala et al. concluded that there must be missing physics in the RRM model.

Fig. 1.8 shows the P13 sample of magnetic OB stars on the \( \log (R_A/R_K) \) vs. \( \log L \) plane. P13 argued that the positions of stars on this diagram could be used to infer the mechanisms regulating the magnetospheric mass budget, advancing two scenarios: a ‘capacity’ scenario in which only the size of the magnetosphere is of importance, and a ‘leakage’ scenario in which mass-loading via the stellar wind competes with an unidentified mechanism that depletes the CM plasma on comparable timescales. As \( \log (R_A/R_K) \) is a proxy for the volume of the CM, in the capacity scenario the distance
Figure 1.8: \( \log \left( \frac{R_A}{R_K} \right) \), a measure of the volume of the CM, as a function of \( \log L \). The solid line divides stars with CMs (above) from stars with DMs only (below). The dashed line divides O-type stars (to the left) from B-type stars (to the right). Symbols and colors are as in Fig. 1.7, while double-headed arrows indicate stars for which only upper/lower limits are available for \( R_A \) and \( R_K \). The dotted and dot-dashed lines illustrate possible mass-leakage mechanisms from the CM. Adapted from P13.
above the solid log \( R_A/R_K \)=0 line should determine the strength of emission, with the onset of emission occurring once some threshold in log \( R_A/R_K \) is passed (P13 suggested log \( R_A/R_K \)=0.8, indicated by the dotted line). In a leakage scenario, in contrast, stars with higher luminosities should show emission at lower log \( R_A/R_K \), as their higher mass-loss rates are able to better compete with mass-leakage. The leakage scenario is indicated by the diagonal dot-dashed line in Fig. 1.8: in this case emission should increase essentially in the diagonal direction from the lower right to the upper left.

1.3 Objectives and hypotheses

This thesis has three primary objectives:

1. Clarify the positions of magnetic early B-type stars on the rotation-confinement diagram.

2. Compare the ages inferred from evolutionary models with the ages determined from magnetic braking timescales.

3. Investigate the optical emission of H\( \alpha \)-bright stars in the context of their magnetospheric and rotational properties.

The first objective amounts to replacing the limiting values of \( R_A \), \( R_K \), and log \( R_A/R_K \) in Figs. 1.7 and 1.8 with definite values. Thus, for those stars for which rotational periods are unknown, these must be determined. A primary means of establishing rotational periods is via high-resolution magnetometry, which is in turn a key input, along with the rotational periods themselves, in establishing ORM parameters and, finally, magnetospheric parameters. Once the magnetospheric parameters have been determined, the final objective follows in a straightforward fashion.
via standard spectroscopic analysis techniques. As constraining the population’s magnetic and rotational characteristics is a necessary intermediate step, this information can be compared and contrasted to results obtained from previous magnetic surveys of Ap stars.

It is expected that all $\text{H}\alpha$-bright stars with limiting values of $R_\Lambda$ and/or $R_K$ which place them outside the main locus of emission-line stars in Fig. 1.7 will prove to have much smaller $R_K$ and higher $R_\Lambda$. Spindown timescales should be roughly comparable to evolutionary ages. Finally, emission strength is expected to scale with either $\log (R_\Lambda/R_K)$, or $\log (R_\Lambda/R_K)$ and luminosity together, with the latter hypothesis judged as more likely in light of the results already obtained for $\sigma$ Ori E by Townsend et al. (2013) and Carciofi et al. (2013).

1.4 Contributions

As magnetic data were scarce for many of the sample stars, I applied for, and was granted, observing time over two semesters at the CFHT in order to acquire sufficient high-resolution spectropolarimetry to determine the rotation periods and/or surface dipolar magnetic field strengths of those stars for which these were unknown. Before this I was responsible for managing the CFHT Phase 2 observation scheduling for the MiMeS CFHT/ESPaDOnS Targeted Component. Unless otherwise stated, all magnetic measurements are my own.

For binary stars, I developed a line profile disentangling code based upon other versions in the literature, and determined orbital periods for HD 149277 and HD 37061. I measured radial velocities using an IDL tool written by Dr. Jason Grunhut.
1.4. CONTRIBUTIONS

I made the discovery that HD 136504 is doubly magnetic (Shultz et al., 2015c). I developed a line-profile fitting technique based on $\chi^2$-minimization of synthetic spectra, with or without rotational oblateness and gravity darkening, for determination of the surface gravities of component stars.

I applied for and was granted time at the Max Planck Society (MPG) La Silla 2.2 m Telescope with the FEROS spectrograph in order to acquire additional data on emission-line stars. The first of these runs I conducted myself. The second was covered by my ESO supervisor Dr. Rivinius, as I had already left Chile by the time of the observing run. During this final run, using these data, Dr. Rivinius determined the rotational and orbital periods of the HD 156324 SB3 system, as well as the rotational period of ALS 3694, which I later refined using the full spectroscopic datasets for both stars. I had earlier identified ALS 3694 as an H$\alpha$-bright CM host star using ESPaDOnS data from the second of my two CFHT PI programs (Shultz et al., 2014). All FEROS data were reduced by Dr. Rivinius.

I also applied for and obtained time with the ESO VLT/XSHOOTER instrument, and reduced these data myself using the ESO/REFLEX tool.

All other new rotational periods, with the exceptions of HD 189775 (determined by Dr. David Bohlender), HD 136504 (determined by Prof. Ernst Paunzen from BRITE space photometry), and HD 163472 (determined by Prof. Gregg Wade using ESPaDOnS data acquired in the context of the BRITE spectropolarimetry LP at CFHT), are my own.

While $v \sin i$ measurements were already available for many of the program stars, I re-measured these using programs of my own in order to have an internally consistent dataset. Stellar effective temperature measurements using EW ratios, and surface
gravity measurements using H Balmer line wings, are my own. Finally, measurements of emission strength, spectroscopic eclipse durations and depths, and plasma densities, are my own.

Stellar, magnetic, rotational, and magnetospheric parameters were determined using a self-consistent Monte Carlo method utilizing evolutionary tracks and explicitly incorporating rotational oblateness, a method I developed and implemented myself. All discoveries regarding the population-level magnetic and rotational properties of early B-type stars are an original result of this work.

1.5 Organization of thesis

Chapter 2 presents an overview of the observational data.

In Chapter 3 the basic properties of the spectroscopic binary stars in the sample are presented: radial velocity measurements, disentangling results for LSD Stokes I profiles, and orbital periods and parameters. Full disentangling results are given in Appendix A, while radial velocity curves and orbital models are shown in Appendix B.

Magnetometry is presented in Chapter 4, including the extraction of LSD profiles, results for single-element line masks, longitudinal magnetic field measurements, and the prevalence in the sample of longitudinal magnetic field curves indicative of surface magnetic fields substantially more complex than simple dipoles. Longitudinal magnetic field curves for individual stars are given in Appendix C.

Projected surface rotational velocities are presented with period analyses in Chapter 5. In those cases for which photometric and/or spectroscopic data were used to determine the rotation period, these data are included in Appendix C together with
the longitudinal magnetic field curves.

Stellar parameters are presented in Chapter 6. High-resolution spectroscopic data are used to determine both the $T_{\text{eff}}$, via EW ratios of temperature-sensitive ions, and the surface gravity, via the pressure-broadened wings of H Balmer lines. These values replace the photometric determinations utilized by P13 for many of the sample stars. Luminosities are then re-derived on the basis of the new effective temperatures. This section is motivated by achieving the maximum possible precision in ORM, rotational, and magnetospheric properties.

In Chapter 7 the fundamental physical parameters from Chapter 6 are used to determine the stellar radii, masses, and ages of the sample stars. These are combined with $\langle B_z \rangle$ measurements, $v \sin i$, and $P_{\text{rot}}$ to determine ORM parameters in Chapter 8, which are in turn used to calculate magnetospheric parameters and spindown timescales in Chapter 9. While the results of Chapters 7-9 are presented sequentially, they were obtained simultaneously by populating the $T_{\text{eff}}$-$\log L$ and $T_{\text{eff}}$-$\log g$ diagrams with Monte Carlo grids that were then pruned in order to account for all available constraints from rotation, binarity, and cluster ages. In each of these chapters the calculations are illustrated for individual stars, following which the results for the population are analyzed.

Chapter 10 presents a spectroscopic analysis of the variability and emission strength of H$\alpha$-bright stars in the context of the results of the previous chapters. Plasma densities are analyzed using two complementary observational diagnostics, and upper limits on the masses of the magnetospheres of stars with eclipsing CMs are found.

The primary results of this work are summarized, their implications are discussed, and future steps for research are outlined in Chapter 11.
Chapter 2

Observations

2.1 Sample Selection

The sample consists of all magnetic main-sequence B-type stars earlier than B5 identified by P13, for which sufficient spectroscopic and spectropolarimetric data are available to evaluate their rotational and magnetic properties. Three stars with spectral types later than B5 (HD 36526, HD 105382, and HD 125823) are also included, as their effective temperatures are above 15 kK. This is a consequence of surface chemical abundance peculiarities affecting spectral type designations, with He-weak stars in particular having, as their name implies, weaker He lines than expected for their effective temperatures. Since He lines are the primary diagnostic for spectral typing amongst hot stars, the spectral types assigned to He-weak stars are systematically later than would be implied for their effective temperatures. We include 5 additional stars, discovered to be magnetic since the P13 sample was published: HD 23478 (B3 IV, Sikora et al. 2015); the secondary star of the HD 136504 system (€ Lupi, B2 IV/V, Shultz et al. 2015c), in which the primary was already known to be magnetic (Hubrig et al., 2009; Shultz et al., 2012); and HD 164492C (B1.5 V), HD 44743 (β CMa, B1
II/III) and HD 52089 ($\epsilon$ CMa, B1.5 II), discovered by the B-fields in OB stars (BOB) collaboration (Hubrig et al., 2014; Fossati et al., 2015a). In total the initial sample consists of 52 stars reported to host magnetic fields.

The sample is summarized in Table 2.1. Stars are listed in order of their HD number; ALS 3694, which does not appear in the Henry Draper catalogue, is listed last. The 2nd column gives alternate designations. The 3rd column gives the spectral type and luminosity class. Remarks as to chemical peculiarity (He-weak or -strong), binarity (SB1/2/3), and/or pulsation ($\beta$ Cep or Slowly Pulsating B-type star, SPB) are made in the 4th column. The remaining columns provide the number of spectropolarimetric, spectroscopic, and photometric observations available for each target.

As a comprehensive sample drawn from the literature, this study is neither volume nor magnitude limited. The statistical properties of the MiMeS survey were summarized by Wade et al. (2016). The MiMeS survey was complete up to $V \sim 1$, 50% complete up to $V \sim 3$, and overall observed 7% of the OB stars with $V < 8$. In spectral type, completeness was highest for the earliest stars (70% of O4 stars with $V < 8$), declining towards later spectral types. For the B-type stars, the sample is 30% complete at B0, diminishing to about 15% at B5: thus, the sample is most complete for the least common spectral types.

### 2.2 Observing programs

The majority of the observations used in this thesis were acquired under the auspices of the Magnetism in Massive Stars (MiMeS) Large Programs (LPs) at the 3.6 m Canada-France-Hawaii Telescope (CFHT) using ESPaDOnS, the 2 m Bernard Lyot Telescope (TBL) using Narval, and the ESO La Silla Observatory 3.6 m Telescope.
using HARPSpol. Five spectroscopic binaries in the sample (HD 35502, HD 136504, HD 149277, HD 156324, and HD 164492C) have also been observed by the Bina-
ernity and Magnetic Interactions in various classes of Stars (BinaMiS) LPs at CFHT
and TBL. The remainder of the data were acquired by various PI programs using
ESPaDOnS, Narval, and HARPSPol at CFHT, TBL, and the ESO La Silla 3.6 m
telescope. The total number of ESPaDonS, Narval, and HARPSpol observations is
given in the 5th to 7th columns of Table 2.1.

The methodology, observing strategy, instrumentation, and scope of the MiMeS
LPs were described in detail by Wade et al. (2016). The BinaMiS LPs have largely
adopted the strategies and instruments of MiMeS LPs. While the MiMeS LPs ended
in 2012, data remained scarce on several magnetic stars identified by the Survey Com-
ponent. Therefore the Targeted Component was extended in two CFHT/ESPaDOOnS
PI programs in 2013 and 2014. In total 973 spectropolarimetric observations were
obtained (606 ESPaDOnS, 233 Narval, and 134 HARPSpol), with a mean of 15 ob-
servations per star.

2.3 Spectropolarimetry

ESPaDOOnS is a high-spectral resolution spectropolarimeter with a resolving power of
$\lambda/\Delta \lambda \sim 65,000$ at 500 nm and a spectral range of 370-1050 nm across 40 spectral
orders. Narval is identical in all respects. The excellent agreement between observ-
ations obtained with the two instruments was demonstrated by Wade et al. (2016).
Each spectropolarimetric sequence consists of 4 sub-exposures acquired at 4 ortho-
go nal polarizations, yielding 4 unpolarized intensity (Stokes $I$) spectra and 1 polarized
spectrum in either circular (Stokes $V$) or linear (Stokes $Q$ and $U$) polarization. By
2.3. SPECTROPOLARIMETRY

combining the subexposures in such a way as to cancel out intrinsic polarization from the source, a diagnostic null ($N$) spectrum is created, which allows characterization of the photon noise as well as spurious signals due to pulsation or binary motion of the target star. The reduction package, Libre-ESPRIT, was described by Donati et al. (1997). HARPSpol has a greater spectral resolving power than ESPaDOnS and Narval ($\lambda/\Delta\lambda \sim 100,000$), and a narrower spectral range, 378–691 nm, with a gap between 524 and 536 nm, across 71 spectral orders.

The detection and measurement of stellar magnetic fields requires data with a very high signal-to-noise ratio (SNR). Therefore, in some cases spectra were removed from the analysis when the SNR was insufficient to obtain a meaningful measurement (typically, when the maximum SNR per spectral pixel $\text{SNR}_{\text{max}} \leq 100$, below which the uncertainty in the longitudinal magnetic field is generally on the order of several kG). In other cases, if the time difference between observations could reasonably be expected to be small compared to either the known rotational period or the minimum rotational period inferred from the projected rotational velocity, spectra acquired on the same night were binned in order to increase the SNR (e.g. HD 136504, for which each measurement consists of 4 spectropolarimetric sequences). The final number of measurements used for magnetic analysis is listed in the 8th column of Table 2.1. The 9th column gives the median peak SNR per spectral pixel in the final dataset, $\langle \text{SNR}_{\text{max}} \rangle$. The data quality is in general high, with a median SNR across all spectropolarimetric sequences of 612. The log of all spectropolarimetric observations is provided online.

Wade et al. (2016) noted that data collected with Narval during the late summers of 2011 and 2012 were affected by occasional, random loss of control of a Fresnel
rhomb. While this issue cannot produce spurious magnetic signatures, the accuracy of magnetic measurements conducted with these spectra cannot be trusted. Therefore, spectra from the time windows given by Wade et al. were excluded from magnetic analysis. Only two observations within these epochs, of HD 176582 acquired on 24 Aug 2011 and HD 35502 on 21 Sept 2012, show obvious inconsistencies with the expected \( \langle B_z \rangle \) variations. Another outlier, outside the windows given by Wade et al., was found for \( \beta \) Cep on 04 Oct 2009; this measurement was also discarded.

In addition to the high spectral resolution spectropolarimetry, low spectral resolution measurements collected with the dimaPol spectropolarimeter mounted on the 1.8 m Dominion Astrophysical Observatory (DAO) Plaskett Telescope are available for 5 stars. The number of DAO observations are given in the 10th column of Table 2.1. dimaPol has a spectral resolution of approximately 10,000, covering a 25 nm region centred on the rest wavelength of the H\( \beta \) line. The instrument and reduction pipeline are described in detail by Monin et al. (2012).

2.4 Spectroscopy

2.4.1 FEROS

Nine stars with optical emission originating in their magnetospheres were observed using the FEROS spectrograph at the MPG La Silla 2.2 m telescope. FEROS is a high-dispersion echelle spectrograph, with \( \lambda/\Delta \lambda \sim 48,000 \) and a spectral range of 375–890 nm (Kaufer and Pasquini, 1998). The data were reduced using the standard FEROS Data Reduction System MIDAS scripts\(^1\). The number of FEROS spectra are summarized in the 11th column of Table 2.1.

\(^1\)Available at https://www.eso.org/sci/facilities/lasilla/instruments/feros/tools/DRS.html
2.4.2 XSHOOTER

XSHOOTER is a medium resolution spectrograph mounted on the 8-m Unit Telescope 2 (UT2) of the Very Large Telescope (VLT) at ESO’s Paranal Observatory. The instrument covers the near ultraviolet (NUV) to the near infrared (NIR) with three spectral arms: UVB from 300-559.5 nm, VIS from 559.5-1024 nm, and NIR from 1024-2480 nm (Vernet et al., 2011). Observations of 11 stars, principally those with detectable magnetospheric emission, were acquired. The number of XSHOOTER observations for each star is provided in column 12 of Table 2.1.

Data were obtained at four rotation phases, corresponding to emission strength maxima (when the magnetosphere is closest to face-on), and absorption maxima (phases when the magnetosphere may be partially eclipsing the star). The objective of the NIR data was to probe the NIR region out to the Brackett-γ line amongst the emission-line stars, and to search for the presence of emission in some stars with magnetic and rotational parameters close to those of the emission-line stars, yet with no detectable optical emission. The goal for the NUV data was to evaluate the general utility of the density-sensitive Ingliss-Teller effect near the H Balmer jump (Inglis and Teller, 1939).

The spectra were reduced with a developer version of ESO’s REFLEX pipeline (Freudling et al., 2013). This is a Java-based environment built as a general reduction tool for ESO data. For each instrument, reduction steps (calculating bias frames, performing wavelength calibrations, etc.) are loaded as modules into a pipeline, represented to the user as a workflow environment. Each reduction module is fully automated, however the user has the capability to inspect calibration data along with intermediate data products, and to modify the order of modules or the settings of
individual modules so as to optimize the reduction process if necessary. REFLEX
also offers the option of saving intermediate data products, thus greatly speeding up
subsequent reductions should they be necessary.

The NIR is heavily contaminated by telluric features, that is, spectral lines formed
in the Earth’s atmosphere, primarily O$_2$ and H$_2$O. Telluric correction was performed
using the MOLECFIT tool, which fits a spectral model of the local atmosphere cal-
culated via radiative transfer to user-selected spectral windows dominated by telluric
lines; subtracting this model from other spectral windows then removes many of the
contaminating lines (Smette et al., 2015).

2.5 Photometry

Hipparcos (High precision parallax collecting satellite) was an astrometric space tele-
scope, whose mission lasted from 1989 to 1993. While the primary aim was to obtain
high-precision trigonometric parallaxes, it also obtained photometry for a large num-
ber of stars. These data are available for 36 of the sample stars. As chemically peculiar
stars exhibit photometric variability due to surface chemical abundance spots, in some
cases rotational periods can be determined using Hipparcos photometry. Photometric
variability may arise due to other physical mechanisms, e.g. pulsation, however in
such cases the photometric and magnetic data should not phase coherently. These
data were acquired from the online archive (Perryman et al., 1997; van Leeuwen,
2007). The number of Hipparcos measurements is given in the final column of Table
2.1.
Table 2.1: Summary of the sample stars and available spectropolarimetric, spectroscopic, and photometric observations. Stars are listed in order of HD number, with the exception of ALS 3694 which is not in the Henry Draper catalogue. Remarks indicate the type of chemical peculiarity (He-strong or He-weak), the star’s binary status (SB1/2/3), and/or pulsational variability ($\beta$ Cep or SPB). The next four columns give the number of high-dispersion spectropolarimetric observations obtained with ESPaDOnS (E), Narval (N), HARPSpol (H), and the total number (T) of magnetic measurements for each star, once low-SNR observations have been removed and spectral binning has been performed. The following column gives the median peak SNR of the dataset. The next column gives the number of DAO (D) observations obtained with the low-dispersion dimaPol spectropolarimeter. Spectroscopic observations obtained with FEROS (F) and XSHOOTER (X) are given in the next two columns. The final column gives the number of archival Hipparcos (Hip) photometric measurements.

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<td>B1 V</td>
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Chapter 3

Multiplicity

Ten of the stars in the sample are known spectroscopic binaries with 2 or 3 stars (SB2/3) contributing to their line profiles. These are listed in Table 2.1. The unpolarized (Stokes I) line profiles of SB2/3 systems can be strongly affected by the contributions of companion stars. Since the line-of-sight strength of a star’s magnetic field is measured by normalizing the first-order moment of the circular polarization profile to the equivalent width of the Stokes I profile, it is of potential importance to correct the Stokes I profiles for the contributions of non-magnetic stars before making magnetic measurements. Radial velocities (RVs) can also be used to determine orbital periods, for systems for which these are unknown, and to compare against the published ephemerides of systems for which they are. When the system’s components are physically associated, orbital periods and parameters can then be used to infer physical properties of the component stars, which will help to constrain the magnetic and magnetospheric models of the binaries’ magnetic stars.

In addition to the SB2/3 stars, there are three candidate SB1 stars in the sample. RV variability with an amplitude of \( \sim 6 \, \text{km s}^{-1} \) in HD 130807 was reported by Alecian et al. (2011). We have also detected RV variability in HD 63245 and HD 156424, both
with amplitudes of \( \sim 10 \text{ km s}^{-1} \). Both HD 63425 and HD 156424 are potentially within the \( \beta \) Cep instability strip (Moravveji, 2016), and thus may be either pulsators or SB1 stars; HD 130807 is not expected to be within the instability strip. As magnetic diagnostics should not be affected by the presence of faint companion stars, these stars are not considered further in this chapter.

3.1 Radial velocity measurements and line profile disentangling

There are two means of correcting for the contributions of multiple stars to spectral line profiles: either by using model line profiles, or by disentangling the line profiles using an iterative algorithm. In practice, both methods were adopted in a complementary procedure.

First, model fits were found using fit\_LSD\_BINARY, an IDL program written by Jason Grunhut (priv. comm.). The user first selects the full-width at half-maximum (FWHM) of the line profiles. The program then determines the best-fitting solution to each line for a synthetic profile convolved with rotational (\( v \sin i \)) and macroturbulent (\( v_{\text{mac}} \)) broadening, with the radial velocity (RV) and relative equivalent width (EW) of each component as additional free parameters. Example fits are shown in Fig. 3.1.

RVs for HD 25558 and HD 35502 were obtained from the literature (Sógor et al., 2014; Sikora et al., 2016). For HD 122451, HD 136504, and HD 149277, RVs were measured directly from LSD profiles, as the spectral types of the components are quite similar. In other cases there is a more substantial difference in the spectra of the components, in which case RVs were obtained from individual spectral lines selected to maximize the contribution of one or the other component, or in which the components are clearly distinguishable. For HD 36485, the primary’s RVs were measured from the
3.1. RADIAL VELOCITY MEASUREMENTS AND LINE PROFILE

Figure 3.1: Representative model fits for spectroscopic binaries. The observed LSD profiles are shown by black circles. Individual components are indicated by dotted (green), dashed (blue), and dot-dashed (purple) lines, the combined flux by solid (red) lines.
3.1. RADIAL VELOCITY MEASUREMENTS AND LINE PROFILE DISENTANGLING

Mg $\Pi$ 448.1 nm line, while the A-type secondary’s RVs were obtained from the Ti $\Pi$ 456.4 nm line. For HD 37017 and HD 156324, the Mg $\Pi$ 448.1 nm line was used, as all components contribute approximately equally to this line. LSD profiles extracted using only phosphorous lines were also used for the RVs of the tertiary component of HD 156324, as this star is a chemically peculiar PGa star (Alecian et al., 2014), and the P LSD Stokes I profiles are dominated by this component. For HD 37061, the primary’s RVs were obtained from the He $\Pi$ 468.6 nm line, as the much cooler secondary does not contribute at all to this line; the secondary’s RVs were obtained from Si $\Pi$ 637.1 nm, which the primary does not contribute to. Uncertainties were in general estimated based upon the scatter between observations obtained on a given night, as intrinsic line profile variability caused by chemical spots can lead to spurious RV variations larger than the formal error bars.

Model fitting to individual profiles becomes more uncertain at orbital phases at which the components are strongly blended. Therefore, once radial velocities were measured, the second process, iterative disentangling, could proceed. The method starts by generating disk-integrated synthetic line profiles using the estimated values of $v \sin i$, $v_{\text{mac}}$, and the EW ratio. The synthetic flux of each companion star is subtracted from the observed flux at the appropriate RVs; the residual flux of the remaining star is then adjusted to its rest frame, and a mean line profile is calculated. This process is performed in turn for each star. The resulting mean line profiles are then used as input for the next iteration. Iterations continue until successive mean line profiles converge, or until a maximum number of iterations is reached (González and Levato, 2006). Fig. 3.2 shows typical results, using the same LSD profiles as shown in Fig. 3.1 for ease of comparison. The full disentangling results
Figure 3.2: As Fig. 3.1, with mean profiles obtained via disentangling.
for each star are provided in Appendix A. For some stars closer agreement with the observed line profiles is obtained using model fits. Iterative disentangling does not work as well when the radial velocity variation is small. Line profile variability due to e.g. rotation (HD 37017, HD 156324) or pulsation (HD 25558, HD 122451) can also reduce the agreement between the mean line profile and the observed line profile. This is especially the case when more than one star in the system is intrinsically variable. However, disentangled profiles have the advantage that, since information on the line profiles is utilized from all observations, the contribution of each star to the combined flux is more accurately reproduced at phases of strong blending. For this reason, unless stated otherwise disentangled profiles were used for the magnetic measurements presented in Chapter 4.

3.2 Orbital properties

With the radial velocities of individual stellar components measured, it is possible to determine the orbital properties of the binary systems, starting with orbital periods and, from these and Kepler’s laws of motion, parameters such as eccentricities and semi-major axes. These orbital properties can then be used to determine the total masses of the orbital systems and the inclination angles of the orbits from the line of sight.
Table 3.1: Orbital periods and parameters of SB2 stars. The uncertainty in the least significant digit of $P_{\text{orb}}$ is written in brackets.

<table>
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<th>HD No.</th>
<th>$P_{\text{orb}}$ (d)</th>
<th>JD0 -2400000</th>
<th>$v_0$ (km s$^{-1}$)</th>
<th>$K_1$ (km s$^{-1}$)</th>
<th>$K_2$ (km s$^{-1}$)</th>
<th>$e$</th>
<th>$\omega$ (°)</th>
<th>$M_1/M_2$</th>
<th>$M \sin^3 i$ (M$_\odot$)</th>
<th>$a \sin i$ (AU)</th>
<th>$i$ (°)</th>
<th>Reference</th>
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<td>25558</td>
<td>8.9±0.5 yr</td>
<td>-2400000</td>
<td>11.2±0.5</td>
<td>8.0±0.5</td>
<td>12±0.5</td>
<td>&lt;0.1</td>
<td>-</td>
<td>1.5±0.2</td>
<td>2.661±0.008</td>
<td>6.0±0.3</td>
<td>39±4</td>
<td>Sódro et al. (2014)</td>
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<tr>
<td>36485</td>
<td>29.968(2) yr</td>
<td>53774.880</td>
<td>21.3±0.5</td>
<td>10.1±0.8</td>
<td>26±2</td>
<td>0.32±0.07</td>
<td>150±4</td>
<td>2.6±0.4</td>
<td>0.12±0.02</td>
<td>0.094±0.005</td>
<td>14±1</td>
<td>Leone et al. (2010), This work</td>
</tr>
<tr>
<td>37017</td>
<td>18.6561(2) yr</td>
<td>46010.434</td>
<td>46±2</td>
<td>34.4±1.1</td>
<td>86±6</td>
<td>0.468±0.016</td>
<td>116±2</td>
<td>2.5±0.3</td>
<td>2.3±0.4</td>
<td>0.182±0.009</td>
<td>37±3</td>
<td>Bolton et al. (1998)</td>
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<td>37061</td>
<td>14.31(4) yr</td>
<td>42778.941</td>
<td>18±17</td>
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<td>158±30</td>
<td>≤0.07</td>
<td>-</td>
<td>3.1±2.6</td>
<td>13.5±0.5</td>
<td>0.27±0.08</td>
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<td>122451</td>
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<td>51600.030</td>
<td>9.3±0.3</td>
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<td>72.1±0.6</td>
<td>0.825±0.002</td>
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<td>16.5±0.3</td>
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3.2. ORBITAL PROPERTIES

3.2.1 Orbital periods

Orbital ephemerides for eight of the SB2/3 stars in the sample are listed in Table 3.1 together with their orbital elements. The epoch of JD0 is set at periastron. HD 35502 and HD 164492C are not included as both of these systems are hierarchical triples, with the RV variable components orbiting the magnetic primary star (Sikora et al. 2016, Wade et al., in prep.); thus, no information on the magnetic star’s physical properties can be determined from the orbital parameters. In 4 cases we have acquired ephemerides and parameters from the literature. In 4 cases new orbital periods have been determined using the RVs measured above. Table 3.1 provides the references for the orbital analyses for those stars with pre-existing orbital solutions. This information is unavailable only for HD 25558: with the very low level of RV variability, the large number (∼2000) and long temporal baseline (15 yrs) of observations, $P_{\text{orb}}$ is estimated to be ∼9 yrs (Sódor et al., 2014).

Orbital periods were determined using Lomb-Scargle statistics (Lomb, 1976; Scargle, 1982) as implemented in the IDL program PERIODOGRAM.PRO. RVs for each component were analyzed separately. An example periodogram is shown in the top panel of Fig. 3.3, for HD 149277’s secondary component (which has the highest RV semi-amplitude). The orbital period was determined from amplitude peaks in the period spectra appearing at the same period for both components. Spurious periods were checked for by generating synthetic noise data with the same temporal spacing as the real data (dashed blue line in the top panel of Fig. 3.3): amplitude peaks appearing in the periodograms generated for synthetic noise were assumed to be a consequence of the window function and ignored. HD 149277’s RV curve, phased with the highest-amplitude peak from the periodogram, is shown in the bottom panel of
Figure 3.3: *Top:* periodogram for HD 149277’s RV measurements (solid black line) and for synthetic noise with the same window function (dashed blue line). The maximum-amplitude period is indicated by a red circle. *Bottom:* RV curves for HD 149277. Squares indicate primary, triangles secondary. The solid (dashed) lines correspond to the orbital model for the primary (secondary) in Table 3.1.
3.2. ORBITAL PROPERTIES

Fig. 3.3. RV curves for individual stars are provided in Appendix B, together with periodograms for those systems with newly determined orbital periods.

### 3.2.2 Orbital elements

Orbital elements are available in the literature for HD 36485, HD 37017, HD 122451, and HD 136504, and the values in Table 3.1 were obtained from the references therein. For stars with newly determined $P_{\text{orb}}$, orbital elements were constrained via $\chi^2$ minimization of synthetic RV curves using a genetic algorithm. An initial population of RV curves, generated with random parameters, were compared to the observations. The parameters of the curve with the lowest $\chi^2$ were used as the seed for a subsequent generation, with individual parameters varying from the parent parameters by a small, random amount. At random intervals the worst solution, rather than the best, was picked, in order to avoid becoming trapped in local minima in the fitness landscape. The process continued until either $\chi^2 < 1$, or the $\chi^2$ had not improved for three successive iterations. Uncertainties in individual parameters were then determined by the variation within which $\chi^2$ is within $1\sigma$ of the minimum $\chi^2$ solution. Numerical experiments with different starting parameters showed that this method converges to solutions that are unique within the uncertainties of the derived parameters. An example fit is shown in the bottom panel of Fig. 3.3. Fits for other stars are shown by the curved lines in Figs. B.1–B.6 in Appendix B. The synthetic RV curves were calculated as

\[ RV_i = v_0 + K_i \cos(\omega + \nu) + e \cos \omega, \]  

where $v_0$ is the line-of-sight component of the centre-of-mass velocity with respect to
the observer, $K_i$ is the semi-amplitude of the $i^{\text{th}}$ star, $\omega$ is the argument of periapsis, $e$ is the eccentricity, and $\nu$ is the true anomaly. $\nu$ was calculated as

$$\cos \nu = \frac{\cos E - e}{1 - e \cos E},$$ (3.2)

where the eccentricity anomaly $E$ was determined at each orbital phase $\phi$ using root-finding, via

$$E = \frac{\phi - e(E \cos E - \sin E)}{1 - e \cos E}.$$ (3.3)

Table 3.1 lists $V_0$, $K_1$, $K_2$, $e$, and $\omega$ for all stars for which these are known or can be determined.

### 3.2.3 Physical properties of orbits

Orbital parameters contain valuable information which can be used to constrain stellar parameters, in particular stellar masses. The ratio of primary to secondary masses is simply $M_1/M_2 = K_2/K_1$. The projected total mass $M \sin^3 i$ is given by

$$M \sin^3 i = \frac{P_{\text{orb}}}{2\pi G} (K_1 + K_2)^3 (1 - e)^{3/2},$$ (3.4)

in cgs units. $M_1/M_2$ and $M \sin^3 i$ are given in Table 3.1. In the case of HD 25558, an approximate mass ratio was determined by Sódor et al. (2014) on the basis of EW ratios of the star’s spectral lines.

The projected semi-major axis $a$ in AU is given by (Heintz, 1978):

$$a \sin i = 9.1919 \times 10^{-5} (K_1 + K_2) P_{\text{orb}} \sqrt{1 - e^2}.$$ (3.5)
Figure 3.4: Determination of $i_{\text{orb}}$ for HD 149277. The solid line indicates $M_1 + M_2$ as a function of $i$; the dotted lines indicate the $\pm 1\sigma$ uncertainties from the uncertainties in $e$, $K_1$, and $K_2$. Dashed lines show the physically plausible range of masses, given the spectral type of the primary, approximately 15 to 20 $M_\odot$. Dot-dashed lines indicate the limits on $i_{\text{orb}}$ inferred from $M_1$ and $M_2$: in this case, due to the large RV semi-amplitudes, if $i_{\text{orb}} < 60^\circ$, the mass would be much larger than is consistent with the spectral type.

Values for $a \sin i$ are given in Table 3.1.

$i$ can be independently constrained via interferometry or eclipses, in which case $a$, $M_1$, and $M_2$ can be constrained with high precision. Unfortunately, none of the
systems are eclipsing binaries. Interferometric data are available for HD 122451, leading to precise masses for the two components of $M_1 = 10.7 \pm 0.1 M_\odot$ and $M_2 = 10.3 \pm 0.1 M_\odot$ (Davis et al., 2005; Ausseloos et al., 2006). In the remaining cases, $i$ can be constrained by considering the range within which the total mass is physically plausible given the spectral types of the primary components. For example, in the case of HD 149277, a B2 IV/V star, the primary’s mass should be between 8 and 10 $M_\odot$. As illustrated in Fig. 3.4, if $i_{\text{orb}} < 60^\circ$, the high RV semi-amplitudes would imply a much higher mass; below 50°, both stars would be in the mass-range of O-type stars. Limiting the primary’s mass to the range inferred from its spectral type implies $i_{\text{orb}} = 68 \pm 8^\circ$. The masses determined in Chapter 7, on the basis of the stars’ luminosities, effective temperatures, and surface gravities, were used to refine the orbital inclinations given in Table 3.1. For HD 149277, $M_1 = 9.3 \pm 0.5 M_\odot$, leading to $i_{\text{orb}} = 67 \pm 4^\circ$.

3.3 Summary

This chapter has presented disentangled spectra of the SB2/3 stars in the sample, obtained using synthetic line profiles and RV measurements. The disentangled spectra will be used in Chapter 4 to measure the longitudinal magnetic fields of the magnetic components of these systems, thus obtaining a more accurate characterization of the magnetic field strengths of these stars. RV measurements were also used to determine orbital periods for 4 of the systems, such that these are now known for all of the magnetic SB2/3 stars in the sample. Modelling the RV curves then provided information on the stellar masses, which will help to constrain the physical properties of stars (Chapters 6 and 7). These constraints will then assist in determining more
accurate magnetic models (Chapter 8) and magnetospheric parameters (Chapter 9).
Chapter 4

Magnetometry

This chapter presents the new magnetic field measurements, which are later used in Chapter 5 to determine and refine rotation periods, and in Chapter 8 to obtain ORM parameters. As such the magnetic data presented here are of crucial importance to the investigation of the magnetospheric properties and rotational evolution explored in Chapter 9. For two stars, HD 44743 and HD 52089, new LSD profiles were not extracted as these data are not yet public; instead longitudinal magnetic field measurements published by Fossati et al. (2015a) were used.

4.1 Least squares deconvolution

We utilized the least-squares deconvolution (LSD; Donati et al. 1997) procedure in order to maximize the per-pixel SNR; in particular, LSD was performed using the iLSD package (Kochukhov et al., 2010). Line lists were obtained from the Vienna Atomic Line Database\(^1\) (VALD3; Piskunov et al. 1995; Ryabchikova et al. 1997; Kupka et al. 1999, 2000) using ‘extract stellar’ requests, for effective temperatures from 15 kK to 30 kK in increments of 1 kK. A line mask is a list of delta functions at wavelengths

\(^1\)Available at http://vald.astro.uu.se/
4.1. LEAST SQUARES DECONVOLUTION

corresponding to the wavelengths of spectral lines, amplitudes corresponding to the expected line strengths, and Landé factors either measured directly or calculated via spin-orbit coupling. The Landé factor is a measure of the magnetic sensitivity of a line, and ranges from 0 (for a magnetically insensitive line) up to a maximum in the VALD3 line lists used here of 2.5. To ensure a homogeneous analysis, the line lists were automatically cleaned by removing: H Balmer and Paschen lines; He lines with strongly pressure-broadened wings; lines blended with H or He lines; lines in spectral regions that are often significantly contaminated by telluric features, depending on the observing conditions; and lines in spectral regions affected by instrumental ripples. When the spectropolarimetric dataset included both HARPSpol and ESPaDOnS/Narval data, the line lists were further truncated by removing all lines outside of the smaller HARPSpol spectral range. Line masks were then customized to each star by adjusting the depths of the remaining lines (typically 200–300 lines) to the observed line depths (e.g., Grunhut et al., submitted). While inclusion of He lines can distort Stokes $I$, when the magnetic field is weak, and/or the line width is large, and/or the spectral SNR is relatively low, it was sometimes necessary to retain these lines in order to obtain a sufficiently high SNR to detect magnetic signatures in the LSD Stokes $V$ profiles. Table 4.1 gives the number of lines used to extract LSD profiles for each star, for masks including only metallic lines (Z), and masks including both metallic and He lines (YZ). Measurements using H lines (X) are also provided; these are described in further detail in Section 4.2.2. For HD 44743 and HD 52089, the number of lines used for each mask was obtained from Fossati et al. (2015a), who adopted a similar strategy.
4.1. LEAST SQUARES DECONVOLUTION

Figure 4.1: LSD profiles of individual stars. Stokes $I$ in black, Stokes $V$ in red, $N$ in blue. The amplification factor applied to $N$ and $V$ is indicated in the top left. Vertical dashed lines indicate the integration ranges for determining the FAP and $\langle B_z \rangle$. The mean error bar in $N$ and Stokes $V$, scaled to the amplification factor, is displayed in the lower right. The legend on the lower left gives the name of the star, and $\langle B_z \rangle$: the LSD profiles shown here are those yielding the highest $\langle B_z \rangle$ significance. Stokes $I$ profiles of binary stars corrected via disentangling are indicated by red circles in the upper right corner.
4.1. LEAST SQUARES DECONVOLUTION

Figure 4.2: As Fig. 4.1.
Figure 4.3: As Fig. 4.1.
4.1. LEAST SQUARES DECONVOLUTION

LSD profiles were extracted using wavelength, Landé factor, and line depth normalization constants of 500 nm, 1.2, and 1.0, respectively. These normalization constants correspond to the scaling values held by the fictitious spectral line which LSD assumes can reproduce all Stokes I and V profiles in the spectrum when scaled by the actual wavelength, Landé factor, and line depth of a given line (e.g., Donati et al. 1997). Velocity pixel widths were set according to $v\sin i/40$ rounded to the nearest 1.8 km s$^{-1}$ spectral pixel, with a minimum pixel width of 1.8 km s$^{-1}$ adopted for all ESPaDOnS, Narval, and HARPSpol spectra, in order to increase the SNR in Stokes V in those stars with especially broad spectral lines. This pixel width was chosen based on the average velocity pixel width of ESPaDOnS and Narval data. The velocity range used for deconvolution was $\pm 2v\sin i + v_{\text{sys}}$, ensuring inclusion of the full spectral line while minimizing contamination by nearby spectral lines in stars with narrow spectral features. For spectroscopic binary stars velocity ranges were set by line width and radial velocity (RV) semi-amplitudes. Profiles were extracted for
Stokes $I$, Stokes $V$, and both diagnostic null $N$ spectra. $N$ is obtained by combining the polarized subexposures in such a way that intrinsic source polarization cancels out, thus providing a check for spurious signatures introduced by problems with the instrumentation (Donati et al., 1997). Representative LSD profiles for all stars except HD 44743 and HD 52089 (for which we do not possess LSD profiles, but rely on the data published by Fossati et al. 2015a), and HD 35912 (in which a magnetic field is not detected, see below) are shown in Figs. 4.1 to 4.4.
Table 4.1: Summary of magnetometry. For LSD profiles extracted with metallic lines (Z), metallic and He lines (YZ), and for measurements made using H lines (X), we provide the number of lines used in the mask (or the number of H lines used), the maximum $\langle B_z \rangle$ value obtained, and $\Sigma_B = \langle|\langle B_z \rangle|/\sigma_B \rangle$ (Eqn. 4.2), the mean significance level of the $\langle B_z \rangle$ measurements made using that set of lines. The final 3 columns give: the number $N_3$ of single-element masks yielding $\Sigma_B > 3$ as a fraction of the total number of single-element masks $N$ with which LSD profiles were extracted; the significance $A_e/\sigma_e$ of the elemental anomaly index (see section 4.2.1); and the set of measurements selected for further analysis.

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<td>141</td>
<td>0.9 ± 0.7</td>
<td>1.1</td>
<td>154</td>
<td>-1.1 ± 0.1</td>
<td>4.6</td>
<td>2</td>
<td>-3.7 ± 1.0</td>
<td>4.2</td>
<td>1/12</td>
<td>3.0</td>
<td>X</td>
</tr>
</tbody>
</table>
As a first evaluation of the statistical significance of the magnetic signatures in the LSD Stokes $V$ profiles a statistical test was utilized, calculating the False Alarm Probabilities (FAPs) by comparing the signal inside the Stokes $V$ line profile to the signal in the wings (Donati et al., 1992, 1997). This test calculates the reduced $\chi^2/\nu$ of Stokes $V$ inside the line profile, where $\nu$ is in this case the number of velocity pixels spanned by the line profile, and compares this to the value that would be expected for a flat profile (i.e., no magnetic field), in which case $\chi^2/\nu$ should be close to unity. This is then converted into a FAP via the $\chi^2$ probability function (Donati et al., 1992). Integration ranges for calculation of FAPs are indicated by dashed black lines in Figs. 4.1 to 4.4. Fig. 4.5 shows the cumulative distribution function of FAPs for all Stokes $V$ profiles in the dataset, where for each star we used the LSD profiles extracted with the mask yielding the highest SNR. 78% of the LSD profiles have FAP $< 10^{-5}$, the upper boundary for a formal definite detection (DD) (Donati et al., 1997). The remaining $\sim$20% registering marginal detections (MD) with $10^{-5} < \text{FAP} < 10^{-3}$ or non-detections (ND) with FAP $< 10^{-3}$ are accounted for by the presence in the dataset of stars with weak magnetic fields, high $v \sin i$, or typically some combination of the two.

Approximately 4% of $N$ LSD profiles also register a DD (Fig. 4.5, blue line). This is due to stars exhibiting rapid radial velocity variations over short time-spans compared to the exposure times. Neiner et al. (2012b) noted the strong signal in the $N$ profile of the $\beta$ Cep star HD 96446, and we confirm its presence in all spectra in the dataset (Fig. 4.2). Also showing DDs in $N$ in some spectra are HD 46328 (Fig. 4.1) and HD 205021 (Fig. 4.4), both $\beta$ Cep stars; HD 156324, a short-period binary (Fig. 4.3); and HD 156424, which is not listed as either a binary or a $\beta$ Cep variable, but does exhibit radial velocity variations (Fig. 4.3). Such $N$ signatures can be reproduced by including RV variations between sub-exposures (Neiner et al., 2012b), however, correcting for rapid RV variations in the reduction stage does not change $\langle B_z \rangle$ outside error bars (Neiner et al., 2012b). Therefore we do not expect
4.1. LEAST SQUARES DECONVOLUTION

Figure 4.5: Cumulative distribution of False Alarm Probabilities. The dashed (dotted) vertical line indicates the lower cutoff for definite (marginal) detections. \( \sim 80\% \) of Stokes \( V \) measurements are definite detections (DDs), with \( \text{FAP} \leq 10^{-5} \), while \( \sim 4\% \) of \( N \) measurements are also DDs.
4.1. LEAST SQUARES DECONVOLUTION

Figure 4.6: The highest SNR LSD profile of HD 35912. Dotted lines show the integration range used for calculation of FAP and measurement of $\langle B_z \rangle = -13 \pm 36$ G. The Stokes $V$ profile (top) is indistinguishable from the $N$ profile (middle).

that $\langle B_z \rangle$ measurements made using these spectra are unreliable due to RV variability.

ESPaDOnS measurements did not confirm a magnetic field in HD 35912, catalogued as a magnetic star by Bychkov et al. (2005). The LSD profile deconvolved from the spectrum with the highest SNR is shown in Fig. 4.6. HD 35912 has sharp spectral lines, with $v \sin i = 12$ km s$^{-1}$. All 6 of the validated observations, with a median peak SNR per spectral pixel of 340, yielded non-detections in the LSD profile. The median error bar in the longitudinal magnetic field measurements of this star is $\sim 35$ G, more than sufficient to detect the previously reported $\geq 6$ kG magnetic dipole. We therefore removed this star
4.2 Longitudinal magnetic field

To evaluate the strength of the magnetic field, we measured the line-of-sight or longitudinal magnetic field $\langle B_z \rangle$ in G, averaged over the stellar disk, by measuring the first-order moment of Stokes $V$ normalized to the equivalent width of Stokes $I$ (e.g. Mathys 1989):

$$\langle B_z \rangle = -2.14 \times 10^{11} \frac{\int vV(v)dv}{\lambda_0 g_0 c \int [I_C - I(v)]dv},$$

(4.1)

where $v$ is the Doppler velocity in km s$^{-1}$, $I_C$ is the Stokes $I$ continuum, $c$ is the speed of light in km s$^{-1}$, and $\lambda_0$ and $g_0$ are the normalization values of the wavelength and Landé factor used to scale the Stokes $V$ profile. Integration ranges were set by $\pm v\sin i + v_{sys}$. The same measurement can be applied to the diagnostic $N$ profile, yielding an equivalent ‘null longitudinal magnetic field’ $\langle N_z \rangle$, with which $\langle B_z \rangle$ can be compared (Wade et al., 2000).

The error bars $\sigma_B$ and $\sigma_N$ were obtained via error propagation of single-pixel photon noise in the LSD profiles through Eqn. 4.1.

To evaluate the data quality, we calculated the mean significance $\Sigma_B$ of the $\langle B_z \rangle$ measurements, which we define as:

$$\Sigma_B = \frac{1}{n} \sum_{i=1}^{n} \frac{|\langle B_z \rangle_i|}{\sigma_{B,i}},$$

(4.2)

where $i$ denotes an individual measurement out of the total number $n$ of observations for a given star. For an individual measurement, $\langle B_z \rangle/\sigma_B$ is a measurement of the significance of the $\langle B_z \rangle$ measurement, under the assumption that $\sigma_B$ describes the gaussian scatter expected from noise in the data due to photon statistics. Eqn. 4.2 then gives the mean of this significance across all measurements, with $\Sigma_B \leq 1$ indicating that the dataset is dominated by noise. $\Sigma_B$ is given in Table 4.1 for LSD profiles extracted using only metallic
Figure 4.7: Histograms of $\Sigma_B$ and $\Sigma_N$, the mean absolute $\langle B_z \rangle$ and $\langle N_z \rangle$ measurements divided by their error bars. The vertical solid line indicates $3\sigma$. The $\langle B_z \rangle$ distribution peaks at $10\sigma$, while the $\langle N_z \rangle$ distribution peaks below $1\sigma$. There are no $\langle N_z \rangle$ measurements above $3\sigma$ significance, while the majority of $\langle B_z \rangle$ measurements are above the $3\sigma$ threshold.

For HD 44743 and HD 52089 $\Sigma_B$ was evaluated from the $\langle B_z \rangle$ measurements presented by Fossati et al. (2015a). Fig. 4.7 shows a histogram of these values, along with the corresponding histogram for $\Sigma_N$, the equivalent of $\Sigma_B$ for diagnostic $N$ measurements $\langle N_z \rangle$. The significance of $\langle B_z \rangle$ peaks at $\sim 10\sigma$, with a tail extending to $70\sigma$. Approximately 10% of the sample has $\Sigma_B \leq 3$: these are all stars with weak magnetic fields ($\langle B_z \rangle_{\text{max}} \leq 300$ G). $\langle N_z \rangle$ is tightly clustered around $0.5\sigma$, and is below $3\sigma$ for all stars. The DDs obtained from $N$ profiles, examined in the previous sub-section, do not yield spurious signals in $\langle N_z \rangle$.

For DAO data, the longitudinal magnetic field is measured via the Zeeman shift between
two spectra of opposite circular polarizations in the core of the Hβ line, as well as nearby lines such as He i 492.2 nm or Fe ii 492.3 nm. ⟨Bz⟩ is proportional to the Zeeman shift with a per-pixel scaling factor of 6.8 kG in Hβ, 6.6 kG in He i, and 14.2 kG in Fe ii. The DAO ⟨Bz⟩ measurements are summarized in Table 4.2.

As discussed in Chapter 3, ten of the sample stars are multi-lined spectroscopic binaries. The LSD profiles of these stars are indicated in Figs. 4.1 and 4.2 with red circles in the top right corner. Since ⟨Bz⟩ is calculated using the centre-of-gravity of the Stokes I and V profiles, and normalized via the Stokes I profile EW, it can be affected by the contribution of binary companions to the Stokes I spectrum. As described in Chapter 3, in order to remove this influence disentangled LSD profiles were obtained via an iterative algorithm (González and Levato, 2006), and ⟨Bz⟩ was measured from the resulting Stokes I profiles of the magnetic component. Representative fits are shown in Fig. 3.2, while the full disentangling results for each star are given in Appendix A. This correction is negligible for HD 36485 and HD 37061, due to the minimal contribution of the non-magnetic star to Stokes I. For HD 35502, ⟨Bz⟩ is increased by ∼25%. For HD 149277, the correction is minimal for most observations, as the RV amplitude is much larger than the line widths and thus the line profiles are blended in only a few observations. However, for HD 122451 (in which the secondary is the magnetic star) and HD 156324, the correction is important. In the former case, ⟨Bz⟩ is approximately twice as high when measured using disentangled line profiles,
since the components are blended in all observations and contribute approximately equally to the line profile. In the latter case, while the contributions of the non-magnetic stars are not large compared to the magnetic component, scatter in $\langle B_z \rangle$ is greatly reduced due to the strongly variable blending.

Iterative disentangling assumes that the Zeeman signatures in Stokes $V$ are entirely due to one star. This assumption does not hold for HD 136504, in which both components are magnetic (Shultz et al., 2015c), therefore for this star the only measurements used were those obtained when the separation of the stellar line profiles in velocity space was greater than the summed $v \sin i$ of the components. Iterative disentangling was also not adopted for HD 25558: as both stars are Slowly Pulsating B-type (SPB) stars, line-profile variability in both components makes spectral disentangling unreliable. Therefore we limited the HD 25558 dataset to only those observations in which the line profiles are separately distinguishable, and used model fits to remove the flux of the non-magnetic primary as described in Chapter 3 and illustrated in Fig. 3.1. As the EW ratio is a free parameter in model fits, this method is able to compensate for the changing EWs of the two components due to pulsations.

### 4.2.1 Measurements with different elements

While the relative precision of $\langle B_z \rangle$ is quite high, as evaluated by $\Sigma_B$, there is also the question of accuracy. Borra and Landstreet (1977) showed that $\langle B_z \rangle$ measurements of Ap stars displayed systematic differences when measured using spectral lines from different elements. This phenomenon has subsequently been reported for some Bp stars (Bychkov et al., 2005; Yakunin et al., 2011, 2015; Shultz et al., 2015a). The physical origin of these discrepancies is thought to be the same as that leading to the photometric variability of Ap/Bp stars, namely, surface chemical abundance spots. Spots lead to horizontal surface brightness inhomogeneities, causing the polarized flux to be enhanced in some regions relative to others and, hence, warping the Stokes $V$ profile (Yakunin et al., 2015).
Figure 4.8: Top: Single-element $\langle B_z \rangle$ measurements for HD 37776. Bottom: elemental anomaly index $A_e$ (Eqn. 4.3). There is a substantial variance in $\langle B_z \rangle$ between different elements, as well as a systematic pattern in $A_e$ with rotation phase. The dashed line indicates a 3rd-order sinusoidal fit, used to determine the integrated value of $A_e$ across all rotational phases (see text).
To explore the prevalence and influence of this effect on other stars in the sample, we extracted LSD profiles using single-element line masks, i.e. line masks in which lines of only a single chemical element were included. These were obtained from the cleaned and tweaked masks described in Section 4.1, with the criterion that a given element have at least 3 isolated lines in the analysis region. Table 4.1 gives the number of masks $N_3$ compared to the total number of masks $N$ for which LSD profiles could be extracted for each star, where the subscript 3 indicates $\Sigma_B > 3$. $\langle B_z \rangle$ measurements using different elements are shown in Figs. 4.8, 4.9, and 4.10, for the examples of HD 37776, HD 130807, and HD 175362, all of
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Figure 4.10: As Fig. 4.8 for HD 175362. While there is variance in $\langle B_z \rangle$ between different elements, there is no systematic pattern in $A_e$ with rotation phase.

which show a large variance in $\langle B_z \rangle$.

To quantify the degree to which $\langle B_z \rangle$ differs when measured using the spectral lines of different elements, we calculated the elemental anomaly $A_e$. This is the weighted standard deviation across all $n$ single-element measurements obtained from a given spectrum $i$, normalized to the maximum absolute value of $\langle B_z \rangle$: 
\[ A_e = \frac{1}{|\langle B_z \rangle_{\text{max}}|} \sqrt{\frac{1}{\sum_{i=1}^{n} (\sigma_{B,i} - \overline{\sigma_B})^{-2}} \sum_{i=1}^{n} \frac{(\langle B_{z,i} \rangle - \langle B_z \rangle)^2}{(\sigma_{B,i} - \overline{\sigma_B})^2}} \] (4.3)

where \( \sigma_{B,i} \) is the uncertainty in \( \langle B_z \rangle \) for element \( i \), \( \overline{\sigma_B} \) is the mean uncertainty across all \( n \) elements, and \( \langle B_z \rangle \) is likewise the mean \( \langle B_z \rangle \) over all \( n \) elements. An error-bar weighted standard deviation is used because measurements obtained from different single-element line masks have systematic differences in uncertainty, due to the large differences in the number of lines available for a given element. Normalization to \( \langle B_z \rangle_{\text{max}} \) is performed in order to determine the fractional variation in \( \langle B_z \rangle \) across different elements, rather than the absolute variation, which will be higher for stars with intrinsically stronger magnetic fields. The uncertainty \( \sigma_e \) in \( A_e \) was calculated from the weighted mean error bar across all elements.

Examples of \( A_e \) curves are shown in the bottom panels of Figs. 4.8, 4.9, and 4.10. The behaviour of \( A_e \) is different with each star. In the cases of HD 37776 and HD130807, \( A_e \) changes very significantly with rotation phase, peaking near one or more of the magnetic extrema, and reaching a minimum near \( \langle B_z \rangle = 0 \). By contrast, HD 175362 shows little variation of \( A_e \) with rotation. The peak values are also quite different: 150\% for HD 37776, 60\% for HD 130807, and only 6\% for HD 175362. Stars with highly variable \( A_e \) likely exhibit greater surface abundance anisotropies than stars in which \( A_e \) is less variable.

By integrating the best-fit 2\textsuperscript{nd}- or 3\textsuperscript{rd}-order sinusoids to \( A_e \) over all phases, we obtain a single number with which to characterize the average strength of deviations between different elements for a given star. The significance of these deviations is then \( A_e/\sigma_e \), which is given for each star in the 2\textsuperscript{nd}-last column of Table 4.1. If \( A_e/\sigma_e < 1 \), it can be concluded that differences in \( \langle B_z \rangle \) between different elements are a consequence of noise rather than real variations. It is this significance in which we are interested, as it will help to decide whether accurate measurements are available using metallic lines, or whether H lines must
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Figure 4.11: Significance histogram for $A_e$ for (top – bottom) all stars in the sample for which $A_e$ could be measured, Bp stars, pulsating stars, and chemically normal, non-pulsating stars. The dashed line marks $3\sigma$ significance. Approximately half the sample shows at least a $3\sigma$ spread in $\langle B_z \rangle$ measurements conducted with different elements. All of these are Bp stars: the only pulsator with $A_e/\sigma_e \geq 3$ is HD 96446, which is also a He-strong star.
be used instead. Fig. 4.11 shows the histogram of $A_e/\sigma_e$ for all stars. For the full sample (top panel) the median significance is $2.7\sigma$. For the sub-sample of He-weak and He-strong Bp stars, the median of the distribution is at $2.9\sigma$ ($2^{nd}$ panel from the top). Amongst pulsating $\beta$ Cep and SPB stars ($3^{rd}$ panel from the top) and the remainder of the sample with no apparent peculiarities (bottom panel), only one star, HD 96446, has $A_e/\sigma_e \geq 3$: as HD 96446 is also a He-strong star, this likely reflects the star’s chemical peculiarities. The medians of these distributions are $1.4\sigma$ and $1\sigma$, respectively. While there are only 8 non-chemically peculiar stars for which $A_e$ could be measured, these results are consistent with an origin of the effect in distortion due to chemical spots, and confirm that when chemical peculiarities are not present there is no difference in $\langle B_z \rangle$ when measured using different chemical elements. Even amongst the Bp stars, although there are several stars for which $A_e/\sigma_e \gg 3$, differences in $\langle B_z \rangle$ measured from different elements are negligible for many of the stars.

It should be noted that while the mathematical treatment of $A_e$ is based upon random uncertainties, these variations are in fact systematic. Unfortunately, it is not obvious what simple mathematical tools might be utilized in order to capture the systematic variations between different sets of measurements. A rigorous exploration of this effect will require Zeeman Doppler Imaging (ZDI) of the surface magnetic fields together with Doppler Imaging (DI) of the chemical abundance patterns. As ZDI and DI cartography is outside of the scope of this work, we have limited ourselves to determining $A_e$ as a somewhat crude indicator for comparing the magnitude of the effect between different stars. However, it should be kept in mind that as of yet no pattern in the distribution of chemical elements relative to the surface magnetic field has yet been detected, hence, a treatment of these variations in terms of random error might not be entirely unwarranted.
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Figure 4.12: Comparisons of $\langle B_z \rangle$ measurements to historical data. *Top:* HD 175362. H$\beta$ wing measurements reported by Borra et al. (1983) and Bohlender et al. (1987), compared to ESPaDOnS H line measurements, where normalization was performed using either the ‘true continuum’ (tc) or the ‘line continuum’ (lc) (see text for explanation). Only the latter agrees with the historical data. *Bottom:* HD 37776. H$\beta$ wing measurements reported by Thompson and Landstreet (1985) compared to ESPaDOnS and Narval data. In this case only $\langle B_z \rangle$ measured using line continuum normalization is shown. The data are phased using the non-linear ephemeris determined by Mikulášek et al. (2008).
4.2.2 H line $\langle B_z \rangle$ measurements

In addition to yielding different $\langle B_z \rangle$ measurements, chemical spots can also lead to anharmonic $\langle B_z \rangle$ variations that can be mistaken for contributions from higher-order multipoles to the photospheric magnetic field. Borra and Landstreet (1977) showed that measurements performed using H lines avoid this problem, as H is in general distributed relatively uniformly over the photosphere. Historically, such measurements were performed with photopolarimeters using the wings of the H$\beta$ line, however, Landstreet et al. (2015) have shown that, when high-resolution spectropolarimetry is available, $\langle B_z \rangle$ can be measured using the non-LTE rotationally broadened core of H$\alpha$ rather than using the line wings.

Examples of H line $\langle B_z \rangle$ measurements are shown in Fig. 4.12 for HD 175362 (top) and HD 37776 (bottom). Both stars are known to have anharmonic $\langle B_z \rangle$ curves indicative of surface magnetic field topologies more complex than simple dipoles. In both cases we compare to H$\beta$ wing measurements reported in the literature (Borra et al., 1983; Thompson and Landstreet, 1985; Bohlender et al., 1987). The HD 37776 data have been phased with the non-linear ephemeris calculated by Mikulášek et al. (2008), which accounts for the spindown of the star. The modern data are more precise, but the general features of the $\langle B_z \rangle$ curves are essentially identical in both cases. Further comparisons are shown in Appendix C for HD 36485, HD 37017, HD 37058, HD 64740, HD 96446, HD 125823, and HD 142990: in all cases the agreement between modern and historical data is very good.

We measured $\langle B_z \rangle$ using H$\alpha$ through $H\gamma$, except for stars with significant emission, in which case H$\alpha$ measurements were not used. Lines at shorter wavelengths than H$\gamma$ were not used as the SNR of ESPaDOnS/Narval spectra is typically much lower in this region. The final $\langle B_z \rangle$ was calculated as the error bar-weighted mean across all Balmer lines used.

Table 4.1 gives the number of H lines used for each star, as well as $\Sigma B$ for the H line measurements. We used the laboratory wavelengths of the lines for $\lambda_0$, and $g_0 = 1$. $I_c$ in Eqn. 4.1 was taken as the boundary of the rotationally broadened line core, rather than
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Figure 4.13: Measuring $\langle B_z \rangle$ from the H$\alpha$ line of HD 175362. Stokes $I$ (bottom) is renormalized to the continuum. Renormalization regions are shown by vertical dashed lines, with the ‘true continuum’ indicated by the horizontal dotted line. The integration limits are indicated by vertical solid lines. Note that the Stokes $V$ (top) signature is entirely within the rotationally broadened non-LTE core of the line. Obtaining $\langle B_z \rangle$ using the EW of Stokes $I$ calculated from the true continuum yields $\langle B_z \rangle$ much smaller than that obtained from either metallic lines or measurements obtained from the wings of H$\beta$ (Fig. 4.12). Using the ‘line continuum’ (dot-dashed line) brings these measurements into much closer agreement.

The ‘true’ continuum bounding the pressure-broadened wings: this ensures the centroid of the line, and thus the separation of the circularly polarized components, is evaluated at the highest possible SNR and thus the maximum precision (Landstreet et al., 2015). An example of this is shown in Fig. 4.13 for HD 175362. If H-line $\langle B_z \rangle$ measurements are evaluated with $I_c$ at the continuum rather than the boundaries of the non-LTE core, the amplitude of the $\langle B_z \rangle$ curve is greatly reduced such that the measurements no longer agree.
with literature values, as demonstrated in Fig. 4.12 (top) for HD 175362. We have also found that $I_c$ should be evaluated in the same fashion for He lines with strong pressure-broadened wings. This introduces some ambiguity into $\langle B_z \rangle$ measurement for LSD profiles extracted using line masks dominated by He lines: to minimize this problem, He lines with broad wings were excluded from the line masks.

A further consideration relates to the nature of echelle spectra. Before extracting LSD profiles from ESPaDOnS or Narval data, the spectra are in general normalized using a polynomial spline fit to the continuum of each echelle order. However, the broad wings of the H Balmer lines, especially Hβ and Hγ, overlap with the edges of their respective echelle orders: thus, polynomial normalization may distort the line profiles. This will then change the EW, and lead to an incorrect measurement of $\langle B_z \rangle$. To avoid this we used spectra that had not been normalized using polynomial splines, instead renormalizing using a linear fit between the edges of the line cores. Experimentation with different stars indicated that this strategy minimizes scatter in the measurements. As ESO reduction pipelines perform global normalization after merging the subexposures, HARPSpol data should not be affected by this issue.

4.2.3 Selection of $\langle B_z \rangle$ datasets for modelling

The final column of Table 4.1 gives the type of measurement selected for further modelling in Chapter 8: ‘Z’ (LSD profiles extracted using line masks with metallic lines), ‘YZ’ (LSD profiles extracted using line masks with metallic and He lines), or ‘X’ (H lines).

With an ideal dataset in which SNR is not a limitation, H line measurements would be used in all cases in order to avoid distortions due to chemical spots. However, for most stars $\Sigma_B$ is lower in H lines than for LSD profiles, i.e. there is a significant trade-off in precision when H lines are used. Furthermore, in the majority of cases differences in $\langle B_z \rangle$ measured from different elements are negligible (Fig. 4.11). H line measurements were thus selected
only when $A_e/\sigma_e \geq 2$.

For stars with $A_e/\sigma_e \leq 2$, measurements obtained from LSD profiles extracted using metallic line masks are in general preferred, as these are unaffected by the extra broadening introduced by He lines. However, for stars with $\Sigma_B \sim 1$, $\langle B_z \rangle$ measurements using all available spectral lines were selected, as in these cases meaningful measurements are only possible when the maximum possible precision is achieved.

### 4.2.4 Curve shape

Chapter 8 will present dipole oblique rotator model parameters for this sample, thus it is of interest to explore to what degree dipolar models are in fact appropriate. As a pure centred dipole should produce a sinusoidal $\langle B_z \rangle$ variation, the most straightforward way to do this is to evaluate the goodness-of-fit $\chi^2/\nu$ of the best-fitting 1st-order sinusoidal curves, where $\nu$ is the number of degrees of freedom. We utilize the same $\langle B_z \rangle$ measurements chosen for modelling in Section 4.2.3. Since this measure is sensitive to the precision of the dataset, we compare this to the significance of the amplitude of the $\langle B_z \rangle$ curve, $\Sigma_{\text{Amp}} = |\langle B_z \rangle_{\text{max}} - \langle B_z \rangle_{\text{min}}|/\sigma_B$. It should be recalled that $\Sigma_N < 1$ for all stars, indicating that the $\langle B_z \rangle$ error bars are consistent with the scatter in the data, hence large values of $\chi^2/\nu$ should reflect real departures from sinusoidal variations.

Fig. 4.14 shows $\chi^2/\nu$ as a function of $\Sigma_{\text{Amp}}$. The comparison is performed for 42 stars, with the remainder left out due to insufficient high-resolution data and/or the absence of a firmly established $P_{\text{rot}}$. There is an approximate correlation between the two measures, suggesting that, as expected, departures from purely sinusoidal behaviour are more easily detected at higher precision. Only 4 stars have $\Sigma_{\text{Amp}} \leq 3$: for these stars, departures from sinusoidal behaviour cannot be evaluated. Of the 38 stars with $\Sigma_{\text{Amp}} \geq 3$, 26 have $\chi^2/\nu \sim 1$, indicating that a single-order sinusoid provides a reasonably good fit to the data.

Only 12 stars have $\chi^2/\nu \geq 3$, suggesting a dipolar model may not be appropriate in
Figure 4.14: Goodness-of-fit $\chi^2/\nu$ of 1st-order fits to $\langle B_z \rangle$ as a function of the significance of $\langle B_z \rangle$ variation $\Sigma_{\text{Amp}}$. Points to the right of the dotted line possess a $\langle B_z \rangle$ variation of at least 3$\sigma$ significance. The dashed line indicates $\chi^2/\nu = 1$, the formal definition of a ‘good’ fit. The dot-dashed line indicates $\chi^2/\nu = 3$. Points above the dot-dashed line and to the right of the dotted line show the strongest evidence for higher-order multipole contributions to $\langle B_z \rangle$.

these cases. In all 12 cases H line measurements were used to evaluate $\chi^2/\nu$, therefore it is unlikely that this result is merely a consequence of distortion of $\langle B_z \rangle$ by chemical spots. Of these, 4 stars are already well known to exhibit significant departures from the sinusoidal behaviour expected of a centred dipole: HD 175362 (Wolff and Wolff, 1976; Borra et al., 1983), HD 37776 (Thompson and Landstreet, 1985; Kochukhov et al., 2011), HD 37479 (Townsend et al., 2005; Oksala et al., 2015b), and HD 149438 Donati et al. (2006). The remainder, in increasing order of $\chi^2/\nu$, are: HD 130807; HD 142184 (for which the strongly anharmonic $\langle B_z \rangle$ curve was reported by Grunhut et al. 2012a); HD 35502; HD 189775; HD
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Figure 4.15: Calculation of the ‘dipolar anomaly index’ $A_d$, for the cases of HD 37776 (top) and HD 175362 (bottom). The dashed (blue) curve indicates the best-fit 1st-order sinusoid; the solid (black) curve the best-fit 3rd-order sinusoid; the dot-dashed (red) curve the absolute difference between the 1st- and 3rd-order fits; the horizontal dotted line indicates the mean $\sigma_B$; the horizontal solid line indicates $\langle B_z \rangle = 0$ kG.

Examination of $\chi^2/\nu$ can identify poor fits, however it does not necessarily indicate that the departure from purely sinusoidal behaviour is large. To quantify the difference between 1st-order and higher-order sinusoids, we calculate the ‘dipolar anomaly index’ $A_d$, which is the integrated absolute difference between 1st- and 2nd- or 3rd-order least-squares fits to $\langle B_z \rangle$, normalized to $\langle B_z \rangle_{\text{max}}$, where the order of the higher-order fit was chosen depending
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Figure 4.16: Goodness-of-fit $\chi^2/\nu$ of 1st-order fits to $\langle B_z \rangle$ as a function of $A_d$. The dotted line indicates 3σ significance for $A_d$: only 4 stars showing evidence of higher-order multipoles exhibit departures from sinusoidal behaviour at this level.

on the size of the dataset. Fig. 4.15 illustrates the calculation of $A_d$ for two cases: HD 37776, which exhibits the strongest double-wave $\langle B_z \rangle$ variation; and HD 175362, which has the highest $\chi^2/\nu$ for a 1st-order fit to $\langle B_z \rangle$.

Fig. 4.16 shows $\chi^2/\nu$ as a function of $A_d$ for those stars with $\chi^2/\nu \geq 3$, excepting HD 189775, for which there are insufficient observations to constrain a higher-order fit. Only 4 stars exhibit differences between 1st-order and higher-order fits to $\langle B_z \rangle$ that are significant above 10%: HD 175362, HD 37776, and HD 37479, and HD 149438. From this test, we conclude that, in the majority of cases, contributions from higher-order multipoles to the $\langle B_z \rangle$ curve are negligible.
4.3 Summary

Approximately 78% of the highest SNR LSD profiles register definite detections in Stokes $V$, and 98% register at least a marginal detection. No magnetic field was detected in HD 35912 in any of the 6 validated observations, with a median $\langle B_z \rangle$ error bar of 35 G: this star was thus removed from the analysis. Several stars, all of which show rapid RV variations, register definite detections in some diagnostic null profiles, however it was concluded that this should not affect $\langle B_z \rangle$. $\langle B_z \rangle$ was measured from LSD profiles extracted using single-element masks, masks including metallic lines only, and masks including metallic plus He lines, as well as from individual H lines. For SB2/3 stars, $\langle B_z \rangle$ was measured using either disentangled line profiles or, where disentangling was unsuccessful, Stokes $I$ was corrected by subtracting model fits for the non-magnetic stars in the system. The median significance $\Sigma_B$ of $\langle B_z \rangle$ is 7.9, while the median of $\Sigma_N$ is 0.6; there are furthermore no $\langle N_z \rangle$ measurements significant above $3\sigma$, while only 12% of the $\langle B_z \rangle$ measurements are below this threshold.

Several of the stars show statistically significant evidence of systematic deviations in $\langle B_z \rangle$ when measured with spectral lines from different chemical elements, a phenomenon which was quantified with the elemental anomaly index $A_e$. This is most widespread amongst the Bp stars, and is not seen at all in stars showing no evidence of chemical peculiarity, suggesting that it arises due to photospheric anisotropies originating in chemical abundance spots. For stars with $A_e$ significant above the $2\sigma$ level, H line $\langle B_z \rangle$ measurements were selected for modelling in Chapter 8 (indicated in the final column of Table 4.1 as X). In other cases, LSD profile $\langle B_z \rangle$ measurements obtained using either metallic lines (Z in Table 4.1) or metallic plus He lines (YZ in Table 4.1) were selected, with the latter chosen only when the significance of $\langle B_z \rangle$ from metallic lines is low, making the higher SNR worth the tradeoff in the distortion of Stokes $I$ by inclusion of lines with very different shapes in the line mask.
Of the 46 stars for which rotational periods are known (see Chapter 5) and sufficient high-resolution data have been obtained to fit a 1st-order sinusoid, 12 show evidence of a poor fit, potentially indicating contributions from a higher-order multipole to the surface magnetic field. Of these, sufficient data to evaluate higher-order fits has been obtained for 11/12 stars. Only four of these show relatively large deviations from purely sinusoidal behaviour; in the remaining cases, the differences in curve area between 1st- and 2nd- or 3rd-order sinusoidal fits is below about 10%, suggesting that while the surface magnetic fields of these stars may exhibit some degree of complexity, they are still primarily dipolar. This suggests that dipolar ORMs should provide a reasonably accurate characterization of the Alfvén radii calculated in Chapter 9.

The magnetospheric parameters determined in Chapter 9 have three key inputs: the magnetic data presented in this chapter, rotational periods, and stellar parameters. In the following Chapter, the $\langle B_z \rangle$ measurements are used to determine rotational periods for the program stars, thus establishing two of the three legs with which to characterize the stars' magnetospheres.
Chapter 5

Rotation

The rotational properties of the magnetic B-type stars are of key importance to this thesis. First, their current rotational properties tell us the strength of rotational support within the magnetosphere. Second, investigating the rotational evolution of these stars requires that we know how their current rotational velocities.

There are two primary observables associated with stellar rotation: the projected rotational velocity $v \sin i$, and the rotational period $P_{\text{rot}}$. We obtained $v \sin i$ from the broadening of spectral absorption lines. Rotational periods were determined from periodic variability arising from rotational modulation of photospheric or circumstellar features producing either spectroscopic, photometric, or spectropolarimetric variations. While $v \sin i$ can be measured for essentially any star, amongst hot stars $P_{\text{rot}}$ can generally be determined only for the magnetic stars. In this Chapter, both $v \sin i$ and $P_{\text{rot}}$ are determined for the sample stars. In Section 5.1 $v \sin i$ is determined, with special attention paid to other line-broadening mechanisms, in particular macroturbulence $v_{\text{mac}}$, Zeeman splitting, and pulsation. $P_{\text{rot}}$ is primarily determined via the $\langle B_z \rangle$ measurements described in Section 4.2, supplemented in some cases by photometric or spectroscopic data.
5.1 VELOCITY BROADENING

5.1 Velocity broadening

Line-profile fitting was utilized to measure $v \sin i$. ESPaDOnS, Narval, and HARPSpol spectra combine a high spectral resolution with a large spectral range, and offer numerous resolved metallic absorption lines with which to measure line broadening. In order to identify an optimal set of spectral lines, we first searched the VALD3 line lists described in Section 4.1 for isolated metallic lines. The final list for all stars includes: C II 426.7 nm and 658.2 nm; N II 404.4 nm and 568.0 nm; O II 418.5 nm and 445.2 nm; Ne I 640.2 nm; Ne II 439.2 nm; Si II 412.8 nm, 504.1 nm, 637.1 nm, and 567.0 nm; Si III 455.3 nm and 457.5 nm; Si IV 411.6 nm; S II 543.3 nm and 566.5 nm; S III 425.4 nm; and Fe II 526.0 nm and 538.7 nm. For each star, the list was curated to remove lines that were absent (due to chemical peculiarities or effective temperature), or blended with other lines (due to high $v \sin i$). For HD 37061, for which none of the given lines were detected, we selected He I 501.6 nm (pressure broadening being minimal in this line), He II 468.6 nm, and Si III 456.8 nm. We used mean spectra created from all available spectra for each star, so as to minimize the impact of line profile variability; for binary stars, we first decomposed the spectra into their stellar components, using the cascading algorithm described in Section 3.1. The final values of $v \sin i$ and $v_{\text{mac}}$ were taken as the mean of the best-fit values across all analyzed spectral lines. Uncertainties were determined based on the standard deviation of the best-fit parameters across all lines, and are typically on the order of 5 km s$^{-1}$. $v \sin i$ and $v_{\text{mac}}$ are given in Table 5.1.

Line broadening was modelled using a $\chi^2$ goodness of fit test, comparing each line to a grid of synthetic line profiles covering a range of $v \sin i$ and $v_{\text{mac}}$ values. Examples of the $\chi^2$ landscapes are shown in Figs. 5.1-5.3 for 3 stars: the sharp-lined star HD 63425; the He-weak star HD 36526, with an intermediate rotational period; and HD 37479, a rapidly rotating star.
5.1. VELOCITY BROADENING

Figure 5.1: Left: $v \sin i$ vs. $v_{\text{mac}} \chi^2$ landscapes for HD 63425. Shading is proportional to $\log(\chi^2)$, with lighter colours indicating lower $\chi^2$ values. The map reflects the total $\chi^2$ across all lines tested (O $\text{ii}$ 418.5 nm and 445.2 nm; N $\text{ii}$ 567.9 nm; Ne $\text{ii}$ 439.2 nm; Al $\text{iii}$ 451.2 nm; Si $\text{iii}$ 455.3 nm and 457.5 nm; Si $\text{iv}$ 411.6 nm). The best-fit models for individual lines are indicated by red circles. Right: representative model fit. Observed and synthetic line profiles are indicated by black circles and the red line.

Figure 5.2: As Fig. 5.1 for HD 36526, an intermediate rotator. Due to strong blending in this He-weak Bp star, only 4 suitable lines could be found: C $\text{ii}$ 658.3 nm, Ne $\text{i}$ 640.2 nm, Si $\text{ii}$ 637.1 nm, and Mg $\text{ii}$ 448.1 nm.
5.1. VELOCITY BROADENING

Figure 5.3: As Fig. 5.1 for HD 37479, a rapidly rotating star. The lines used were C II 426.7 nm, N II 568.0 nm, O II 466.2 nm, Ne I 640.2 nm, Si II 637.1 nm, and Si III 455.3 nm and 456.8 nm.

Disk integration was performed with an engine similar to that described by Petit and Wade (2012), with some modifications. First, local profile widths were calculated using Maxwellian velocity distributions appropriate to the stellar $T_{\text{eff}}$ and the atomic weight. Second, radial-tangential macroturbulence rather than isotropic turbulence was implemented (Gray, 1975). This was motivated by the inclusion of higher-mass stars in the sample, especially the pulsating $\beta$ Cep stars. For Bp stars $v_{\text{mac}}$ values are likely fictitious in that they do not likely reflect actual velocity fields within the stellar atmosphere, but can be taken as standing in for distortions to the line profile introduced by chemical spots or Zeeman splitting (see below).

For slow rotators (e.g., HD 63425, Fig. 5.1), solutions with high $v_{\text{mac}}$ and $v\sin i \sim 0$ km s$^{-1}$ produce much better fits. For intermediate rotators such as HD 36526 (Fig. 5.2), the quality of the fit is improved by inclusion of non-zero $v_{\text{mac}}$, although the lowest $\chi^2$ solutions with $v_{\text{mac}} = 0$ km s$^{-1}$ yield essentially the same $v\sin i$ as the best-fit solution with higher values of $v_{\text{mac}}$. For rapid rotators, inclusion of $v_{\text{mac}}$ makes very little difference, as
shown for the example of HD 37479 in Fig. 5.3.

Sundqvist et al. (2013) examined $v \sin i$ diagnostics for magnetic O-type stars known to have extremely long rotation periods, such that the true $v \sin i$ should be essentially zero, and found that in such cases $v \sin i$ was often drastically over-estimated. They concluded that macroturbulence was likely contaminating the measurements. Aerts et al. (2014) found that measurements of $v \sin i$ could, at least in the case of the Fast Fourier Transform (FFT) method, be significantly affected by pulsation. Both of these studies suggest that uncertainties in $v \sin i$ may in some cases be underestimated. Therefore, in the case of stars for which we determine extremely long (> 1 yr) rotation periods, we consider our $v \sin i$ measurements to be upper limits based upon the spectral resolution of the data, as the true projected rotational velocities are necessarily much lower.

Pulsations can have a strong impact on line shape, which can affect goodness-of-fit results for narrow-lined stars. HD 46328 and HD 205021 are both $\beta$ Cep pulsators with relatively narrow spectral lines. HD 46328 is a monoperiodic pulsator whose pulsations are likely radial (Saesen et al., 2006). While HD 205021 is a multiperiodic pulsator exhibiting non-radial pulsations, its highest-amplitude pulsation is its fundamental radial mode (Shibahashi and Aerts, 2000), so a simple radial pulsation model should be appropriate to first order. Radial pulsations were included by adding an isotropic velocity field normal to the stellar surface, with the disk centre radial velocity taken to be 1.45 times the radial velocity measured using the centre-of-gravity method, a typical value of the projection factor for $\beta$ Cep stars (Nardetto et al., 2013). In both cases the effect is to reduce $v \sin i$ by $\sim$5–10 km s$^{-1}$. An example of the resulting fits is shown in Fig. 5.4 for HD 46328. The result is in an equivalently good fit with a line profile broadened only by radial pulsation and macroturbulence, as compared to one also including $v \sin i$. This is consistent with the extremely long rotational period of about 30 years inferred from $\langle B_z \rangle$ (Fig. C.9). For HD 205021, this results in a $v \sin i$ of $27 \pm 3$ km s$^{-1}$, very close to the inferred equatorial rotational velocity.
5.1. VELOCITY BROADENING

Figure 5.4: Line profile fits for HD 46328 (ξ¹ CMa) at 3 different pulsation phases. Line profiles are broadened with radial pulsations and macroturbulence only: while a non-zero $v \sin i$ cannot be ruled out, rotational broadening is unnecessary.
5.1. VELOCITY BROADENING

$v_{eq}$ from its rotational period and radius, $29 \pm 1$ km s\(^{-1}\) (Table 8.3). The $v \sin i$ found from its mean line profile without taking pulsation into account is $37 \pm 2$ km s\(^{-1}\), too high to be consistent with $v_{eq}$.

Another mechanism that may affect line broadening is gravity darkening. Townsend et al. (2004) showed that, for stars with surface equatorial rotational velocities above 80% of their critical velocities, line broadening ceases to be a sensitive measure of the projected rotational velocity. This is because gravity darkening reduces the contribution of equatorial regions to the integrated flux. Accounting for this requires detailed spectral modelling including meridional temperature and surface gravity variations, together with knowledge of the inclination of the rotational axis from the line of sight. Only two stars in this sample are rotating in this regime, HD 142184 (Grunhut et al., 2012a), and HD 182180 (Rivinius et al., 2013). In both cases careful spectral modelling accounting for oblateness as well as the meridional spectral differences arising from gravity darkening has been performed, and the $v \sin i$ values so obtained are adopted here.

The line profiles of magnetic stars are subject to additional line broadening due to Zeeman splitting. The shift in wavelength in nm of a magnetically sensitive Zeeman component is given by

$$\Delta \lambda = 4.67 \times 10^{-11} g \lambda^2 B,$$

where $\lambda$ and $g$ are the rest wavelength and the Landé factor of the line, and $B$ is the line-of-sight strength of the magnetic field. For the majority of stars in the sample, this does not significantly affect line broadening: a 1 kG field will cause a line with $g = 1$ to split by $\Delta \lambda \approx 0.0012$ nm at 500 nm, which is less than 1 km s\(^{-1}\). However, for strongly magnetized stars with sharp spectral lines, Zeeman splitting can be a significant source of additional broadening. Fig. 5.5 demonstrates this for four stars which have minimum
5.1. VELOCITY BROADENING

Figure 5.5: $v \sin i$ (black circles) and $v_{\text{mac}}$ (dark blue squares) as a function of Landé $g$ factor for four different stars. Cyan triangles indicate the Zeeman splitting for each line assuming a surface magnetic field strength of $3.4 \langle B_z \rangle$. HD 58260, HD 66522, and HD 96446 all have moderately strong $\langle B_z \rangle$ measurements and sharp spectral lines. In all three cases there is a trend of increasing $v \sin i$ with increasing $g$; in the case of HD 58260, it seems that Zeeman splitting can account for all line broadening. For comparison, results for HD 175362 are shown. This star has the highest $\langle B_z \rangle$ measurements in the sample; while there is essentially no trend in $v \sin i$ with $g$, there is some suggestion that $v_{\text{mac}}$ increases with $g$.

Surface magnetic field strengths of at least $\sim 2.5$ kG. Since the magnetic field strength is unknown ($v \sin i$ itself being a crucial input parameter in magnetic modelling), Eqn. 5.1 was solved with a magnetic field strength $3.4 \langle B_z \rangle_{\text{max}}$, based on the minimum dipolar magnetic field strength. The cyan triangles in Fig. 5.5 show the splitting predicted by Eqn. 5.1 for the Landé factors of the lines used to obtain $v \sin i$ and $v_{\text{mac}}$; $v \sin i$ and $v_{\text{mac}}$ measurements obtained from these lines are indicated by black circles and blue squares. Zeeman splitting can account for all line broadening observed in HD 58260, which has very sharp spectral
5.1. VELOCITY BROADENING

lines and a strong surface magnetic field (∼8 kG). For HD 96446 and HD 66522, both of which have \( v \sin i < 10 \text{ km s}^{-1} \) but weaker magnetic fields (6 kG and 2.5 kG, respectively), the degree of Zeeman splitting is on the order of the uncertainty in \( v \sin i \) and \( v_{\text{mac}} \), although in both cases there is a suggestion of decreasing \( v \sin i \) with decreasing \( g \). Thus, for these three stars the \( v \sin i \) measurements obtained from the line with the lowest Landé factor (the S II 566.5 nm line, \( g = 0.5 \)) were used. The fourth star in Fig. 5.5, HD 175362, has the strongest longitudinal magnetic field of the four (indeed, the highest \( \langle B_z \rangle_{\text{max}} \) in the sample), but a higher \( v \sin i = 34 \pm 4 \text{ km s}^{-1} \), which is apparently insensitive to Zeeman splitting. Given the very strong surface magnetic field of HD 175362, it is surprising that \( v_{\text{mac}} > 0 \text{ km s}^{-1} \) when measured from some lines, as turbulent velocity fields should be suppressed by strong magnetic fields. Notably, \( v_{\text{mac}} \) increases with \( g \), suggesting that the nonzero \( v_{\text{mac}} \) values inferred from some lines may be a consequence of Zeeman splitting.

An important conclusion of this analysis is that Zeeman splitting is important only when \( v \sin i < 10 \text{ km s}^{-1} \) and the magnetic field is simultaneously quite strong, a combination of circumstances which is true only for a very few stars in the sample.

Fig. 5.6 compares these values of \( v \sin i \) to those from Petit et al. (2013). There are two outliers: HD 35298 and HD 37017. These are highlighted in Fig. 5.6. In the former case the value given by P13, 260 km s\(^{-1}\), is much higher than that found here, 60±2 km s\(^{-1}\). In the latter case, we find \( v \sin i = 134 \pm 15 \text{ km s}^{-1} \), while Bolton et al. (1998) settled on 90 km s\(^{-1}\). However, Bolton et al. noted that there was a considerable spread of values in the literature, between 45 and 170 km s\(^{-1}\), and that their own results using the stronger He lines indicated \( v \sin i \sim 140 \text{ km s}^{-1} \). We tested He I 443.7 nm, Ne I 640.2 nm, and Si II 637.1 nm, and found in all cases \( v \sin i \sim 130 \text{ km s}^{-1} \).

Fig. 5.7 compares the \( v \sin i \) and \( v_{\text{mac}} \) measurements of the magnetic B-type stars in the present sample to those reported for B-type stars in the literature, where for the literature stars \( v \sin i \) measurements were taken from Dufton et al. (2013), Simón-Díaz and
Figure 5.6: Comparison of $v \sin i$ measurements with literature values. Note that the axes are logarithmic. The solid red circle and blue square indicate HD 35298 and HD 37017, discussed further in the text.

Herrero (2014), and Garmany et al. (2015), while $v_{\text{mac}}$ measurements were provided only by Simón-Díaz and Herrero (2014). Bin sizes were determined using the Freedman-Diaconis rule, which optimizes the bin size for the variance and size of the data set (Freedman and Diaconis, 1981). Error bars arise from the $1\sigma$ uncertainties in the individual measurements: 10000 simulated datasets were created with values adjusted by addition of gaussian randomly generated numbers to each data point normalized to its $1\sigma$ uncertainty, with the
error bar in each bin corresponding to the standard deviation across the histograms for all synthetic datasets (note that this same method was used to determine histogram error bars throughout this work).

The literature $v\sin i$ measurements combine extragalactic stars (the VLT-FLAMES Tarantula Survey, Dufton et al. 2013) and Galactic stars (Simón-Díaz and Herrero, 2014; Garmany et al., 2015). The sample presented by Simón-Díaz and Herrero (2014) overlaps with the present sample; stars appearing in both were removed from the Simón-Díaz
5.1. VELOCITY BROADENING

and Herrero 2014 data. The two other samples presumably contain some number of magnetic stars, however, the visual magnitudes of these stars are too high for high-resolution magnetometry to distinguish magnetic from non-magnetic stars; in any case, the fraction of magnetic stars should be no higher than 10%. Comparing $v \sin i$ values, there is a clear difference between magnetic and non-magnetic B-type stars, with the distribution of the former peaking at under 100 km s$^{-1}$ while the latter peaks at about 200 km s$^{-1}$. A two-sample Kolmogorov-Smirnov test yields a probability of $8 \times 10^{-11}$ that the two samples belong to the same distribution. This is in line with expectations that magnetic stars should be systematically more slowly rotating than non-magnetic stars due to braking by the magnetized stellar wind. It should be noted that this difference may be even more pronounced than suggested by Fig. 5.7: Dufton et al. (2013) noted the distribution of $v \sin i$ in the Large Magellanic Cloud B-type stars is bimodal, with a broad-lined component ($\sim$75% of the sample) with $v \sin i \sim 250$ km s$^{-1}$, and a narrow-lined component with $v \sin i < 100$ km s$^{-1}$. While Dufton et al. speculated that the narrow-lined population might be a consequence of magnetic braking, this sub-sample has been included here as the most conservative option.

Comparison of $v_{\text{mac}}$ values determined here to those presented for non-magnetic B-type stars by Simón-Díaz and Herrero (2014) (right panels of Fig. 5.7) yields a somewhat more ambiguous result. The two samples peak at a similar value ($\sim 20$ km s$^{-1}$), but there is a systematic offset between the cumulative distributions, with the non-magnetic stars having generally higher values of $v_{\text{mac}}$. Strong magnetic fields are expected to suppress turbulence in the photosphere. However, the much smaller difference in $v_{\text{mac}}$ distributions as compared to $v \sin i$ could also be a consequence of subtle differences in the line profile fitting routines used here and employed by Simón-Díaz and Herrero. Verifying whether or not this difference in turbulent velocities is real will require a self-consistent analysis of the combined sample of magnetic and non-magnetic main sequence B-type stars.
Table 5.1: Projected rotational velocity and rotational periods. The 1\textsuperscript{st} column gives the HD number, the 2\textsuperscript{nd} column the projected rotational velocity $v \sin i$, the 3\textsuperscript{rd} column the macroturbulent velocity $v_{\text{mac}}$, the 4\textsuperscript{th} column the rotational period $P_{\text{rot}}$, the 5\textsuperscript{th} column the epoch $JD0$, the 6\textsuperscript{th} column the reference for the period, and the 7\textsuperscript{th} column the method by which the period was determined ($m$: magnetic; $u$ ultraviolet spectroscopy; $s$: optical spectroscopy; $p$: photometry; $sp$: a non-linear ephemeris accounting for the spindown).

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5.2 Rotation periods

To determine rotation periods, in most cases we have relied on the $\langle B_z \rangle$ measurements described in Chapter 4. In two cases, ALS 3694 and HD 156324, we obtained periods using Hα EWs: both of these stars display Hα emission originating in their magnetospheres, which is modulated by rotation. For HD 66765 we refined $P_{\text{rot}}$ using He EWs, where we assumed the line profile variability in this He-strong star to be due to chemical spots. In one case, HD 66522, $P_{\text{rot}}$ was determined using archival Hipparcos photometry, where once again we assume the origin of the photometric variability to be rotational modulation (the very long period determined for this star, $\sim$900 d, is too long to be compatible with pulsation, while orbital modulation is unlikely as there is no evidence of RV variability). Hipparcos photometry was also used to refine $P_{\text{rot}}$ in two further cases, HD 142990 and HD 35298.

Period analysis was performed using Lomb-Scargle statistics (Lomb, 1976; Scargle, 1982) as implemented in the IDL program PERIODOGRAM.PRO, which normalizes the periodogram to the total variance as described by Horne and Baliunas (1986). Uncertainties in $P_{\text{rot}}$ were determined based upon the full-width at half-maximum of the peak in the periodogram.

An example periodogram is shown in the top panel of Fig. 5.8 for HD 37017; in this case, it is based upon $\langle B_z \rangle$ measurements (bottom panel). HD 37017’s period was originally determined by Bohlender et al. (1987): the periodogram was constructed by combining the ESPaDOnS $\langle B_z \rangle$ measurements with the $\langle B_z \rangle$ measurements published by Bohlender et al., along with the earlier data published by Borra and Landstreet (1979). The long temporal baseline leads to a narrow peak in the periodogram at 0.901186(1) d, where the number in brackets gives the uncertainty in the least significant digit. This is consistent within uncertainty, but more precise than, with the period published by Bohlender et al., 0.90119(5) d. The uncertainty in phase is indicated with horizontal error bars, where the epoch JD0 is the same as that given by Bohlender et al.. Phase uncertainty was computed
Figure 5.8: Top: periodogram for $\langle B_z \rangle$ measurements of HD 37017. The abscissa is limited to the period window. Solid (black) line indicates the periodogram for $\langle B_z \rangle$; dashed (blue) line the periodogram for $\langle N_z \rangle$. The red circle indicates the adopted rotation period. Bottom: ESPaDOnS $\langle B_z \rangle$ measurements and $\langle B_z \rangle$ measurements from the literature (BL79: Borra and Landstreet 1979; B87: Bohlender et al. 1987) phased with $P_{\text{rot}}$. Solid lines indicate the best-fit sinusoidal curves; dashed lines, the $1 \sigma$ uncertainty in the fit arising from the uncertainty in the amplitude and mean of $\langle B_z \rangle$. 
from the uncertainty in $P_{\text{rot}}$, multiplied by the number of elapsed rotational cycles from JD0.

In order to check that the rotation periods are physically plausible, period windows were bounded from above by

$$P_{\text{rot}} \leq \frac{2\pi R_{\text{eq}}}{v \sin i},$$

and from below by the breakup velocity $v_{\text{br}}$ (Jeans, 1928),

$$v_{\text{br}} = \sqrt{\frac{M_* G}{R_{\text{eq}}}},$$

where the equatorial radius $R_{\text{eq}}$ is determined via the oblateness, or ratio between $R_{\text{eq}}$ and the polar radius $R_p$ (Jeans, 1928):

$$\frac{R_p}{R_{\text{eq}}} = \sqrt{1 - \frac{3\Omega^2}{4G\pi\rho}},$$

where $\Omega$ is the angular frequency and $\rho = M_*/(4\pi R_p^3/3)$ is the mean stellar density.

Within a given period window, it is frequently the case that there are multiple peaks in the periodogram that phase the data more or less equally well. This can be due to the relatively small size and uneven temporal sampling of spectropolarimetric time series, typically $\sim 10$ measurements collected over a few years. It can also be a consequence of small semiamplitudes as compared to the mean error bars. In order to check for spurious peaks, we used the $N_Z$ measurements discussed in Section 4.2. Peaks which appear in the $N_Z$ period spectrum are likely a consequence of the window function, and can be ignored. When $\langle N_z \rangle$ is not available (for historical $\langle B_z \rangle$ measurements as well as spectroscopic and photometric data), we used synthetic null measurements obtained via random gaussian noise normalized to the mean uncertainty in order to derive the window contribution to the periodogram within the sampling window.

The statistical significance of a given peak in the periodogram can be quantified by
means of the false alarm probability (FAP), where we use Eqn. 22 from Horne and Baliunas (1986) which gives the FAP as a function of the number of data points and the amplitude of the period spectrum. Similarly to the FAPs used in Chapter 4 to evaluate the statistical significance of the signal within Stokes $V$, smaller FAPs indicate that a signal is less likely to be a consequence of white noise. In Fig. 5.8, the maximum amplitude period at 0.901186(1) d has a FAP of about $10^{-7}$, while the FAP of the highest-amplitude peak in the null spectrum is 0.12.

In the end, new rotation periods have been determined for 15 stars. For a further 15 stars, comparison of the new magnetic data to $\langle B_z \rangle$ measurements in the literature has enabled refinement of the rotation periods. Rotation periods are given in Table 5.1. Unless stated otherwise in the text, JD0 is taken to be date of the observation at which $\langle B_z \rangle = |\langle B_{Z,\text{max}} \rangle|$. Also given in Table 5.1 are references for rotational periods obtained from the literature, and the method by which the period was obtained: via ultraviolet spectroscopy, optical spectroscopy, photometry, or magnetometry. When comparison of our measurements to historical data has allowed refinement of $P_{\text{rot}}$, this work is also given as a reference.

As a sanity check, Fig. 5.9 shows $P_{\text{rot}}$ as a function of $v \sin i$. For clarity, the vertical axis has been truncated at 100 d. As expected, $P_{\text{rot}}$ decreases with increasing $v \sin i$. There is a degree of scatter in the relationship, however this makes sense assuming rotational axis inclinations are random. The equatorial rotational velocity $v_{\text{eq}}$ (obtained by solving Eqn. 5.2 for $\sin i = 1$) is shown via slightly curved diagonal lines for 3 representative sets of physical parameters. Each line terminates in a vertical line corresponding to $v_{\text{br}}$. All stars have $v \sin i \leq v_{\text{eq}}$ and $v \sin i \leq v_{\text{br}}$.

Rotation periods could not be determined for three stars: HD 52089, HD 58260, and HD 136504B. In the cases of HD 52089 and HD 136504B, period analysis is hampered by the small number of high-resolution $\langle B_z \rangle$ measurements, all of which are extremely weak.
Figure 5.9: Rotation periods as a function of $v\sin i$. Vertical lines indicate the breakup velocity (Eqn. 5.3), and the slightly curved diagonal lines indicate $v_{eq}$ (calculated using Eqn. 5.4) as a function of $P_{rot}$, for representative stellar parameters.

(HD 52089: Fossati et al. 2015a; HD 136504B, Fig. C.23). In the case of HD 58260, despite the mean $\langle B_z \rangle$ being significant at the 30$\sigma$ level, the peak-to-peak variation of $\langle B_z \rangle$ is quite small compared to the mean 1$\sigma$ error bar. Bohlender et al. (1987) were unable to determine $P_{rot}$ for the same reasons: our results confirm those of Bohlender et al., but at much higher precision, and in fact fail to detect any statistically significant variation over the entire 35 year period for which magnetic data is available (Fig. C.11). This suggests either a rotational axis aligned with the line-of-sight, a magnetic axis aligned with the rotational axis,
extremely slow rotation, or any combination of these possibilities, all compatible with HD 58260’s extremely narrow spectral lines. Pedersen (1979) suggested \( P_{\text{rot}} = 1.657 \, \text{d} \) based on the star’s photometric variation. While this period is compatible with the period window of 1.42 to 164 d, we cannot confirm this period using Hipparcos photometry. Furthermore, this period requires a very small inclination \( (i \leq 3^\circ) \) and extremely rapid rotation \( (\omega \geq 0.89) \), which is \textit{a priori} less likely than a larger inclination and/or slow rotation.

In two cases, HD 37479 and HD 37776, non-linear ephemerides are available that account for the observed spin-down of the star (Townsend et al., 2010; Mikulášek et al., 2008). Oksala et al. (2012) demonstrated the agreement achieved by this ephemeris between historical and modern \( \langle B_z \rangle \) measurements of HD 37479. On their own, ESPaDOnS and Narval data are unable to distinguish between the spin-down ephemeris provided by Mikulášek et al. (2008) for HD 37776 and the newer ephemeris, in which spin-up of the star is reported (Mikulášek et al., 2011). We therefore adopt the earlier ephemeris as being the more conservative option, as shown in Fig. 4.12. This small ambiguity in ephemeris has no impact on the magnetic modelling.

Comparison of the rotation periods collected and newly determined here to those obtained from earlier studies of Ap/Bp stars indicates no difference between the various populations. Fig. 5.10 shows the histogram of rotation periods for the early B-type stars (red diagonal hatching), together with the combined histogram of periods for Ap stars and cooler (spectral type later than B5) Bp stars (solid black), where the Ap/Bp stars have been compiled from the catalogues published by Landstreet and Mathys (2000a), Bychkov et al. (2005), Aurière et al. (2007), and Power (2007), for a total comparison sample of 172 stars. As there is some overlap between the samples, particularly with the comprehensive Bychkov et al. catalogue, each sample was cleaned by HD number to ensure no duplication. The overall distributions are quite similar: the rotation periods of both the present sample and the cooler stars peak at periods of a few days or less, with tails extending to \( 10^4 \, \text{d} \).
However, there appear to be a higher percentage of rapidly rotating stars amongst the stars earlier than B5.

A two-sample Kolmogorov-Smirnov (K-S) test is shown in Fig. 5.11. The K-S significance is 0.0009, which is quite small and suggests the two distributions may indeed be different.

Figure 5.10: Comparison of rotation periods in this sample to previous studies of Ap stars and cool (spectral type later than B5) Bp stars (Landstreet and Mathys, 2000a; Bychkov et al., 2005; Aurière et al., 2007; Power, 2007).
5.3. Summary

Projected rotational velocities were measured, taking into account radial-tangential macro-turbulence and, in some cases, Zeeman splitting and pulsation as additional line-broadening mechanisms. In some cases line broadening appears to be affected by Zeeman splitting: for these stars, $v \sin i$ was obtained from spectral lines with the lowest Landé factors. Results are largely consistent with previous measurements, although in two cases $v \sin i$ had to be revised, in one case (HD 35298) by about 200 km s$^{-1}$ downward. As expected for strongly
magnetized stars, \(v\sin i\) is quite low, typically below 50 km s\(^{-1}\), as compared to a typical value of \(\sim 200\ \text{km}\ \text{s}\^{-1}\) for non-magnetic main sequence B-type stars. There is some suggestion that \(v_{\text{mac}}\) is systematically lower in magnetic B-type stars as compared to their non-magnetic counterparts, which could be indicative of suppression of turbulent motion by strong magnetic fields: this will need to be verified by a more careful, self-consistent analysis.

Rotation periods were determined using \(\langle B_z \rangle\) measurements, Hipparcos photometry, and line profile variability. New rotation periods were obtained for 15 stars, and rotational periods were refined for another 15 stars via comparison to magnetic data from the literature. \(v\sin i\) and \(P_{\text{rot}}\) are consistent for all stars, insofar as no stars have \(v\sin i\) greater than the equatorial rotational velocity expected from the rotation period. Comparison of the rotation periods obtained for this sample of early B-type stars to published data for Ap/Bp stars suggests a possible preponderance of rapid rotators amongst the hotter population.

This Chapter yielded surprising results for two stars. The first is the extremely slow rotational period of HD 46328, which the magnetic data suggest must have a rotational period of decades. This is in contrast to the periods of 2-4 d inferred on the basis of FORS1/2 data by Hubrig et al. (2011) and an earlier, smaller ESPaDOnS dataset by Fourtune-Ravard et al. (2011). HD 46328 thus has by far the longest rotational period of any star in the sample. The second is that the rotational period of HD 156324, as determined both using magnetic and H\(\alpha\) EWs, is identical to the orbital period obtained from RV measurements in Chapter 3. This indicates that HD 156324 is tidally locked, making it the only known magnetic B-type binary star with such an orbital configuration.

With the projected rotational velocities and rotational periods, the true equatorial rotational velocities can be determined if we also know the stellar radii. In the following Chapter, we examine the observables (effective temperature, luminosities, and surface gravities) with which stellar radii will be determined (Chapter 7). In Chapter 8, \(v\sin i\) and \(P_{\text{rot}}\)
will be used together with the stellar radii to constrain oblique rotator model parameters. The rotational constraints determined here will also be key input parameters in determining the degree of rotational support of the circumstellar plasma, along with magnetic braking timescales, in Chapter 9.
Chapter 6

Stellar Parameters

Previous chapters have concerned themselves with the observed magnetic and rotational properties of the population. The goal of this chapter is to constrain the luminosities, effective temperatures, and surface gravities that will be used, in conjunction with the results from Chapters 4 and 5, to assess the population’s intrinsic rotational, magnetic, and magnetospheric properties.

As a starting point we adopt the values provided by P13, who collected physical parameters from the literature determined via spectral modelling using NLTE fastwind or TLUSTY model atmospheres (Lanz and Hubeny, 2003) for hotter stars, or LTE models such as ATLAS (Kurucz, 1979) for the cooler stars. In these cases, literature values are adopted without modification. More detailed spectral modelling has since been performed for several of the stars for which P13 used photometrically determined parameters. Table 6.1 collects these physical parameters, together with their references.

When spectral modelling was unavailable, P13 derived $T_{\text{eff}}$ and $\log g$ photometrically, with appropriate spectral type calibrations, and obtained the luminosity from a distance estimate using either Hipparcos parallaxes (Perryman et al., 1997) or, for more distant stars, cluster distances. Johnson $UBV$ magnitudes were collected from online catalogues (Reed, 2005; Mermilliod, 2006), and the luminosity determined based on bolometric corrections
(BCs) obtained via interpolation through the TLUSTY BSTAR2006 grid (Lanz and Hubeny, 2007), and extinctions determined as $A_V = R_V E(B - V)$ assuming $R_V = 3.1$. When $RJHK$ photometry was also available in the NOMAD catalogue (Zacharias et al., 2005), P13 refined the BC luminosities using the Bayesian spectrophotometric SED fitting code CHORIZOS (Maíz-Apellániz, 2004). For 9 of the 22 stars for which $RJHK$ magnitudes are available P13 noted a poor fit to the SED, possibly due to near-IR excess, and so reverted to the BC results for these stars.

In many cases, high-resolution spectroscopy has since been obtained that was not available to P13. As spectral analysis is frequently able to achieve a higher precision than photometric determinations, this chapter presents a reanalysis of the fundamental parameters of those stars for which P13 utilized photometric parameters.
Table 6.1: Fundamental physical parameters. The 2nd column indicates, for the spectroscopic binaries, which component the parameters relate to; the magnetic component is indicated with a superscript $m$. In the 3rd column, the superscripts refer to the method by which the luminosity was obtained ($s$: spectral modelling; $p$: photometry; $c$: CHORIZOS SED fitting). References for spectroscopic analyses are provided in the final column, where superscript $l$ indicates log $L$, $t$ indicates $T_{\text{eff}}$, and $g$ indicates log $g$.

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6.1 Effective temperature

A primary weakness of photometric measurements is that they can be affected by reddening. If the degree of reddening is unknown, as is often the case, an educated guess must be made based upon the star’s distance, which is itself often highly uncertain; these uncertainties then propagate through the modelling to significant uncertainties in $T_{\text{eff}}$ and $\log g$. As a sanity check on these determinations we used EW ratios of $T_{\text{eff}}$-sensitive spectral lines of different ionizations but the same atomic species, and compared these to EW ratios measured from a grid of model spectra. While this is not as precise as detailed spectral modelling, it is based on the same physics, yields similar results (Shultz et al., 2015a), and is computationally cheaper; thus, it provides a quick way to check that the photometric $T_{\text{eff}}$ values provided by P13 are consistent with the spectroscopic data.

EWs were measured using mean spectra created from all available ESPaDOnS, Narval, and HARPSpol observations for each star, thus maximizing the SNR. For the hotter stars ($T_{\text{eff}} \geq 25$ kK), EWs of He i 587.6 nm and He i 667.8 nm vs. He ii 468.6 nm, and Si iii 455.3 nm and 456.8 nm vs. Si iv 411.6 nm were compared. For cooler stars the ratios used were Si ii 413.1 nm, 505.6 nm, 634.7 nm, and 637.1 nm vs. Si iii 455.3 nm and 456.8 nm; P ii 604.3 nm vs. P iii 422.2 nm; S ii 564.0 nm vs S iii 425.4 nm; and Fe ii 516.9 nm vs. Fe iii 507.4 nm and 512.7 nm. These lines were selected by searching Vienna Atomic Line Database (VALD3: Piskunov et al. 1995; Ryabchikova et al. 1997; Kupka et al. 1999, 2000) line lists with the criteria that the lines be relatively strong within the $T_{\text{eff}}$ range under consideration, as well as being isolated. Many of the sample stars possess strong chemical abundance peculiarities, raising the possibility of contamination of these lines by spectral lines that would not be expected in a solar metallicity star. Furthermore, some stars are rapid rotators, leading to the possibility of blending with other lines. Therefore the line lists were tailored for each star by excluding any lines strongly blended with other lines, as
well as lines of chemical species for which one of the ionizations does not appear at all in the spectrum.

The measured EW ratios were compared to ratios determined from the BSTAR2006 grid of non-LTE solar abundance synthetic spectra (Lanz and Hubeny, 2007). As described by Shultz et al. (2015a), the grid was first limited to the range appropriate to a given star’s surface gravity log $g$, and the uncertainty in log $g$ propagated through to the uncertainty in $T_{\text{eff}}$. In most cases, there is some scatter in $T_{\text{eff}}$ determined using different elements, in particular Si vs. He; $T_{\text{eff}}$ was thus determined from the mean across all elements, with the standard deviation adopted as the uncertainty. One of the larger changes is in ALS 3694, which shows no Si $\text{II}$ or Si $\text{IV}$ lines apparent in its spectrum, but does possess fairly prominent Si $\text{III}$ lines. Despite the low SNR, a weak He $\text{II}$ 468.6 nm line can also be discerned. These indicate $T_{\text{eff}} = 23 \pm 2$ kK, 3 kK hotter than (although formally consistent with) the photometric determination of $20 \pm 3$ kK.

For spectroscopic binaries, EW ratios were measured from individual spectra when the components are separated in velocity space, whenever possible. In practice this technique was only appropriate for HD 136504 and HD 149277: for the remaining SB2/3 systems, lines are blended at all phases. For HD 136504, EW ratios indicate $T_{\text{eff,P}} = 20.5 \pm 0.5$ kK and $T_{\text{eff,S}} = 18.5 \pm 0.5$ kK, where the subscripts P and S indicate primary and secondary, respectively. For the primary star, Si $\text{III}$ lines are stronger than Si $\text{II}$ lines, while the opposite is the case for the secondary. The difference between the HD 149277 components is less pronounced, with $T_{\text{eff,P}} = 20 \pm 2$ kK and $T_{\text{eff,S}} = 19 \pm 2$ kK.

Fig. 6.1 compares these measurements to those used by P13, where photometrically determined $T_{\text{eff}}$ values are shown by blue (solid) circles, and values determined via spectroscopic modelling by black (open) circles. EW ratio measurements are consistent with those from spectral modelling, suggesting they are reliable, and are moreover consistent with those from photometry. As the error bars are somewhat smaller than measurements
obtained from photometry, and as they are independent of reddening, we adopt these values in preference to the photometric values used by P13.

6.2 Surface gravity

The most sensitive indicators of surface gravity are the pressure-broadened wings of H Balmer lines. In most cases Hβ was used, as this line has a relatively high SNR in ESPaDOnS and Narval spectra when compared to the higher-numbered Balmer lines, and is
6.2. SURFACE GRAVITY

Figure 6.2: Comparison between linear (solid black) and polynomial (dotted blue) normalizations of ESPaDOnS Hβ spectra, for HD 66765. The dashed red line shows a FEROS observation for the same star. The polynomial normalization was performed on individual orders, which were then merged. The linear normalization was performed after merging spectral orders, and yields line wings much closer to those seen in the FEROS data.

in general relatively free of blending with strong metallic or He lines. For stars for which magnetospheric emission is particularly strong, Hγ was used instead, being the best compromise between contamination due to emission, and blending with other lines.

Both Hβ and Hγ are close to the edges of their respective spectral orders in ESPaDOnS/Narval spectra. When preparing the spectra for magnetic analysis they are normalized using an order-by-order polynomial normalization procedure. This has a tendency to warp the line wings of H Balmer lines in overlapping orders, as can be seen for the example of HD 66765 in Fig. 6.2. Therefore, as with the H line ⟨Bz⟩ measurements described in Section 4.2.2, the two overlapping orders were first merged, and then normalized using a linear fit between continuum regions. FEROS spectra are normalized using a polynomial spline, as with ESPaDOnS and Narval data, however this procedure is performed after the
spectral orders are merged, thus, they do not suffer from the same degree of warping. Comparison to FEROS data for HD 66765 in Fig. 6.2 shows that this procedure achieves a much closer match than continuum normalization. In order to maximize the SNR, initially unnormalized spectra were first co-added to improve the SNR, with merging and normalization performed after co-addition.

A goodness-of-fit approach was used to determine log \( g \). Synthetic BSTAR2006 spectra were convolved with the \( v \sin i \) values determined in Chapter 5, and the reduced \( \chi^2 \) was calculated for each fit, with an integration range extending from 483 nm to 489 nm. In most cases a range of 3.5 to 4.5 in log \( g \) was used. As Balmer lines are also weakly sensitive to \( T_{\text{eff}} \), fits were tested at the minimum, mean, and maximum \( T_{\text{eff}} \) according to the values and uncertainties determined above. Since the rotationally broadened cores of H Balmer lines are subject to NLTE effects, fitting was performed only in the region outside \( \pm v \sin i \).

For emission line stars, the range of velocities containing the majority of the H\( \alpha \) emission was also excluded. \( \log g \) was then determined from the minima of polynomial fits to the reduced \( \chi^2 \) curves, with the final value being the mean of the values determined at minimum, mean, and maximum \( T_{\text{eff}} \), and the uncertainty one-half the range of these values.

Figs. 6.3-6.13 show fits for several stars for which only photometric determinations were previously available. The best-fit models for each \( T_{\text{eff}} \) are shown in the top panel, and the residual flux in the bottom, with the mean flux error indicated by horizontal lines. In almost all cases the residual flux is much larger than the mean flux uncertainty. In most cases this can be attributed to chemical peculiarities (i.e., lines not included in the synthetic spectra): residual flux outside of spectral lines is typically below 1% of the the continuum. In the following results for individual stars are briefly discussed.

**HD 35298**: the best fit model (Fig. 6.3) is not a particularly good fit to the details of the spectrum, likely due to strong chemical peculiarities in the atmosphere of this star. The best-fit \( \log g \) at the EW ratio \( T_{\text{eff}} = 15\pm1 \) kK is \( 4.26\pm0.13 \), much higher than the
Figure 6.3: Top: Surface gravity from Hβ for HD 35298. The mean line profile is shown in black. The dark blue line shows the best-fit model for the $T_{\text{eff}}$; the best-fit model for $T_{\text{eff}}-\sigma_T$ is shown in light blue; the best-fit model for $T_{\text{eff}}+\sigma_T$ in purple. Model parameters are indicated in the legend. Bottom: residual flux. The mean flux error bar is indicated by the two horizontal lines above and below 0 (in this case they appear as a single line, as the uncertainty is quite small).

log $g = 3.78 \pm 0.2$ inferred from its temperature and CHORIZOS luminosity. If the slightly higher photometric $T_{\text{eff}}$ (16±2 kK, Landstreet et al. 2007) is used instead, log $g$ would need to be even higher to match the spectrum.

**HD 36526:** P13 gave log $g = 4.0 \pm 0.3$, consistent with, albeit less precise than the value found here, log $g = 4.1 \pm 0.15$. Model fits are shown in Fig. 6.4.

**HD 37058:** the EW ratio $T_{\text{eff}}$, 18.5±0.5 kK, is between that found by spectral fitting, 17 kK (Glagolevskij et al., 2007), and the photometric determination of 20 kK (Landstreet et al., 2007), however we find a much higher log $g = 4.17 \pm 0.07$ than the value given by P13, 3.8±0.2 (Fig. 6.5).

**HD 58260:** the value of log $g$ found here, 3.43±0.15 (Fig. 6.6), is in good agreement with that from the literature (Bohlender, 1989). This is amongst the lowest in the sample, which would imply that HD 58260 is also one of the most evolved magnetic B-type stars.
6.2. SURFACE GRAVITY

Figure 6.4: As Fig. 6.3 for HD 36526.

Figure 6.5: As Fig. 6.3 for HD 37058.
6.2. SURFACE GRAVITY

Figure 6.6: As Fig. 6.3 for HD 58260.

Figure 6.7: As Fig. 6.3 for HD 66522.
6.2. SURFACE GRAVITY

HD 66522: literature values for log \( g \) range widely, from 3.5 to 4.5 (Zboril et al., 1997; Leone et al., 1997). The best fit to H\( \beta \) found here is for \( \log g = 3.72 \pm 0.23 \). The model fit is shown in Fig. 6.7.

HD 142990: the value found from Balmer line fitting, 4.15 \( \pm \) 0.11 (Fig. 6.9), is in good agreement with the value determined from the Balmer discontinuity, 4.27 \( \pm \) 0.2 (Cidale et al., 2007).

HD 105382: Briquet et al. (2001) found \( T_{\text{eff}} = 17.4 \pm 0.4 \) kK and \( \log g = 4.18 \pm 0.15 \) photometrically. The EW ratio \( T_{\text{eff}} = 18.0 \pm 0.5 \) kK, slightly higher than but compatible with the photometric \( T_{\text{eff}} \) within uncertainty. This yields \( \log g = 4.03 \pm 0.07 \), again compatible with the photometric value albeit somewhat lower.

HD 175362: using the EW ratio \( T_{\text{eff}} = 17.6 \pm 0.4 \) kK yields \( \log g = 4.21 \pm 0.06 \) (see Fig. 6.10). This is a substantially higher surface gravity than that adopted by P13, \( \log g = 3.67 \pm 0.16 \), which was obtained from Leone and Manfre (1997) from a simultaneous abundance, \( T_{\text{eff}} \), and \( \log g \) analysis of H\( \beta \) and nearby spectral lines, where \( T_{\text{eff}} \) was determined from H\( \beta \). The discrepancy is due to the much lower \( T_{\text{eff}} \) adopted by Leone and Manfre, 14.6 kK. The
6.2. SURFACE GRAVITY

Figure 6.9: As Fig. 6.3 for HD 105382.

Figure 6.10: As Fig. 6.3 for HD 175362.
EW ratio $T_{\text{eff}}$ is consistent with that determined by Cidale et al. (2007) using the Balmer discontinuity, 17.5 kK, and with the mean of photometric $T_{\text{eff}}$ determinations compiled from the literature by Netopil et al. (2008), 16.8 ± 0.6 kK.

**HD 186205**: we find $\log g = 3.84 ± 0.17$, consistent with the photometric determination of $4.0 ± 0.2$. While the residuals are within 1% of the continuum, there is a systematic bowing in the inner wings of the line (see Fig. 6.11).

**HD 189775**: the EW $T_{\text{eff}} = 17.5 ± 0.6$ kK is in reasonable agreement with the photometric $T_{\text{eff}} = 16.2 ± 0.6$ kK (Lyubimkov et al., 2002). Using the EW $T_{\text{eff}}$ yields $\log g = 4.12 ± 0.08$ (Fig. 6.12), which overlaps within uncertainty with the photometric $\log g = 3.97 ± 0.15$.

**ALS 3694**: this is an emission-line star (Shultz et al., 2014), so $\log g$ was measured using H$\gamma$ as well as H$\beta$. Fig. 6.13 shows the best-fit models to H$\beta$; the weak emission is visible in the residual flux on either side of the line core, and does not exceed 2-4% of the continuum, depending on the model. H$\gamma$ yields the same results, albeit at a lower precision than H$\beta$. $\log g = 3.67 ± 0.19$ is lower than the photometric value, $4.0 ± 0.4$, although the two are consistent within the large error bars of the latter. The relatively large uncertainty is due
6.2. SURFACE GRAVITY

Figure 6.12: As Fig. 6.3 for HD 189775.

Figure 6.13: As Fig. 6.3 for ALS 3694.
6.2. SURFACE GRAVITY

to the particularly noisy spectra of this dim ($V = 10.35$ mag) star.

6.2.1 Binary stars

Determining the surface gravities of binary stars requires special care, since all stellar components will contribute to the Balmer line wings. This leads to a degeneracy in their parameters, therefore the uncertainties are typically greater. This can be partly overcome if enough observations are available, and the variability of the components sufficient, that the contributions of the individual stars can be discerned. Spectral modelling carefully accounting for all components has already been performed for HD 25558 (Sódor et al., 2014), HD 35502 (Sikora et al., 2016), HD 36485 (Leone et al., 2010), HD 37061 (Simón-Díaz et al., 2011), and HD 122451 (Ausseloos et al., 2006).

No analysis of $\log g$ is available for HD 149277, HD 136504, or HD 37017; for HD 156324, an attempt was made to constrain $\log g$, but this was before the availability of orbital periods with which to constrain the masses. For each of these 4 systems a grid of at least 25 synthetic spectra was prepared for each star, covering the approximate range in $T_{\text{eff}}$ and $\log g$ expected for the components, using either TLUSTY BSTAR2006 models (when both components are likely above $T_{\text{eff}}=15$ kK) or ATLAS models (when one of the stars is below 15 kK). The radius and mass of the primary $R_P$ and $M_P$ were determined by interpolating through evolutionary tracks (Ekström et al., 2012) according to the $T_{\text{eff}}$ and $\log g$ of the model. The mass $M_S$ of the secondary was then determined from the mass ratio obtained from the system’s orbital parameters (Table 3.1), from which the radius was obtained directly as $R_S = \sqrt{(GM_S/g)}$. Synthetic spectra were then moved to the measured radial velocities of each component, added together with the contributions of each component scaled by the relative stellar radii, normalized to the synthetic continua calculated in the same fashion, and the reduced $\chi^2$ calculated. The overall fit for each pair of models was taken as the weighted mean $\chi^2$ across all observations, with the weights taken
Figure 6.14: Surface gravity for HD 149277. The left panels show reduced $\chi^2$ landscapes for $\log g_P$, $\log g_S$, $T_{\text{eff},P}$, and $T_{\text{eff},S}$, where darker shades correspond to higher $\chi^2$. The location of the best-fit model is indicated by a circle. The right-hand panels show, from top to bottom, the corresponding fits (combined flux in solid red lines, primary in dashed blue, secondary in dot-dashed green) to observations (black dots) at quadrature (top and bottom) and conjunction (middle).

from the mean flux error bars of each spectrum in order to keep noisier spectra from biasing the results towards overall worse fits.

**HD 149277**: the best results from this method were obtained for this star, given the large dataset, large radial velocity amplitude, and the sharp spectral lines of both components. The reduced $\chi^2$ landscape (Fig. 6.14) shows a sharply defined valley around $\log g_P = 3.75 \pm 0.15$ and $\log g_S = 3.85 \pm 0.3$, suggesting the system to be somewhat evolved. The effective temperatures preferred by this method are additionally in line with those inferred from ionization balances, albeit less precise. These constraints on the surface gravities of the two stars represent a substantial improvement over the estimate of $4.0 \pm 0.4$ used by P13. An
interesting aspect of this result is that, when the rotation velocity inferred from $P_{\text{rot}}$ and $v \sin i$ is taken into consideration, log $g_P > 4.0$ is ruled out as the smaller radius implied by the higher surface gravity would mean $v_{\text{eq}} < v \sin i$.

**HD 136504:** when $T_{\text{eff}}$ and log $g$ are allowed to vary freely, the minimum $\chi^2$ is found with a higher temperature for the secondary (Fig. 6.15). Since ionization balances unambiguously imply a lower temperature for the secondary, the grid is restricted to only those models for which $T_{\text{eff},S} \leq T_{\text{eff},P}$. The result is again a somewhat higher surface gravity for the secondary, log $g_S = 4.13 \pm 0.1$ as compared to log $g_P = 3.97 \pm 0.1$. This makes sense assuming that the primary is somewhat more massive and, hence, more evolved than the secondary. If this restriction is not applied, log $g_P$ remains unchanged, while log $g_S = 4.2$.

**HD 37017:** the primary is a rapidly rotating star, $P_{\text{rot}} = 0.901186(1)$ d. Given this, it is potentially important to take rotational distortion of the star into account when constraining
Figure 6.16: Grid parameters for HD 37017. $T_{\text{eff}}$ and $\log g$ correspond to the value at the rotational pole. At high $T_{\text{eff}}$, lower surface gravities imply super-critical rotation ($\omega > 1$), and so are not included in the grid: these grid points are indicated by closed circles. Models for which $\omega > 0.6$ are synthesized including the effects of oblateness and gravity darkening.

the surface gravity, as $\log g$ may vary by up to 0.5 dex from pole to equator if the star is rotating near critical. For each model, the oblateness was determined via equation 5.4 along with the critical rotational frequency $\Omega_{\text{crit}}$:

$$\Omega_{\text{crit}} = \sqrt{\frac{8GM_*}{27R_p^3}},$$  \hspace{1cm} (6.1)
where $M_*$ was determined via interpolation through evolutionary models in the same manner as before. Models with $\omega = \Omega/\Omega_{\text{crit}} > 1$ were automatically discarded from the analysis. For HD 37017, this is the case for most of the models with $\log g \leq 3.95$ (Fig. 6.16, bottom panels). Models for which $\omega < 0.6$ were convolved with $v \sin i$ according to the usual method, as in this regime the effects of rotational distortion are mild enough that they can be discarded. Gravity darkening was accounted for when $0.6 < \omega < 1$: as is clear from Fig. 6.16, this is the case for about 2/3 of the grid points under consideration for this star.

To account for rotational distortion, the stellar radius $r$ as a function of the meridional colatitude $\theta$ was determined via $x = x(\omega, \theta) \equiv r(\theta)/R_p$ (Cranmer, 1996):

$$x = \frac{3 \cos \left[ \frac{1}{3}(\pi + \cos^{-1} \omega \sin \theta) \right]}{\omega \sin \theta}, \quad (6.2)$$

the local surface gravity $g(\theta)$ is then:

$$g(\theta) = \sqrt{g_r^2 + g_\theta^2}, \quad (6.3)$$

where the ratio of the radial component of the surface gravity at colatitude $\theta$ to the polar surface gravity $g_p$ is

$$\frac{g_r(\theta)}{g_p} = \frac{8}{27} x \omega^2 \sin^2 \theta - \frac{1}{x^2}, \quad (6.4)$$

and the ratio of the meridional component of $g(\theta)$ to $g_p$ is

$$\frac{g_\theta(\theta)}{g_p} = \frac{8}{27} x \omega^2 \sin \theta \cos \theta. \quad (6.5)$$

The gravity darkened flux is then obtained via the von Zeipel theorem (von Zeipel, 1924),
Figure 6.17: As Fig. 6.14 for HD 37017. In this case, due to the primary’s rapid rotation it is necessary to account for rotational distortion and gravity darkening. At higher $T_{\text{eff}}$, log surface gravities near 3.9 imply super-critical rotation, and therefore models with these parameters were not included. Note also the weak emission apparent in the red wing of the top line profile.

\[ T_{\text{eff}}^4(\theta) \propto g_{\text{eff}}(\theta), \]  

which is valid for the radiative envelopes of OB stars.

Since gravity darkening breaks the spherical symmetry of the star, it is also necessary to know the inclination $i$ of the rotational axis, which was determined from $v\sin i$ and $v_{\text{eq}}$. Fig. 6.16 shows the variation of $i$ with log $g$ and $\omega$ in the top two panels: $i$ decreases with increasing $\omega$.

Fig. 6.17 shows the resulting best-fit model. Note that the star displays weak emission in its Balmer wings, originating in its magnetosphere. This emission is weak compared
to other emission-line stars, and is fairly localized, appearing in H\(\alpha\) only between about 200 and 600 km s\(^{-1}\). Thus, in contrast to the strong emission stars such as \(\sigma\) Ori E, HR 5907, or HD 23478, for which emission contaminates essentially the entire line profile, the influence of HD 37017’s emission could be dealt with by the simple expedient of masking out these velocities. As the secondary is much dimmer than the primary, its surface gravity is unsurprisingly almost unconstrained, although it is likely higher than the primary’s, as expected. Constraints are somewhat better for the primary, \(\log g_P = 4.0 \pm 0.1\), although the best-fit model is at the lower limit near \(\log g_P = 3.94\). Referring to Fig. 6.16, this suggests HD 37017 is likely rotating near to the critical limit.

**HD 156324**: the primary of this system is a relatively rapid rotator with a 1.58 d rotation period, however this implies \(\omega = 0.32 \pm 0.01\), well below the threshold at which rotational distortion is important. The analysis is complicated first by emission, which is confined to a
6.3. BOLOMETRIC LUMINOSITIES

narrow cloud that is easily masked out of the spectrum, and second by the presence of a third
stellar component in its spectrum, with a contribution unconstrained by its mass as it does
not appear to be closely associated with the two central stars in the system. Holding the
tertiary’s temperature and surface gravity constant at 14 kK and 4.0 (the values suggested
by Alecian et al. 2014), and the ratio of its radius relative to the secondary’s $R_T/R_S$ constant
at unity, the results do not look unreasonable (Fig. 6.18), however using a slightly larger or
smaller radius ($R_T = 1.5R_S$ or $0.75R_S$) or varying $T_{\text{eff}}$ or $\log g$ by 1 kK or 0.25, the fit is
similarly good with very different stellar parameters for the other two components. Thus
the primary’s surface gravity cannot be constrained better than $\log g_P = 4.0 \pm 0.3$. This was
the same conclusion as reached in the earlier analysis by Alecian et al. (2014), at which
point the orbital period was unknown.

6.3 Bolometric luminosities

P13 utilized luminosities determined via spectral modelling, photometrically via bolometric
corrections ($BC$s), and spectrophotometrically via SED fitting using CHORIZOS. Lumin-
osities from modern spectral modelling are adopted without modification. However, in
most cases $BC$ luminosities are adopted here in preference to the CHORIZOS luminosities.
Distance can be set as a free parameter in CHORIZOS, and in consequence many of the
luminosities determined in this fashion require distances substantially greater than those
inferred from Hipparcos parallaxes, diverging in some cases by up to 1.5 dex in distance
modulus. This in turn leads to $\log L$ being systematically much higher than implied by the
star’s $T_{\text{eff}}$ and parallax or cluster distance. In many cases, these luminosities also imply $\log g$
to be substantially lower than is observed. Thus, only those CHORIZOS luminosities which
obtained distance estimates similar to those obtained from parallax or cluster distances, as
well as a higher precision in $\log L$, were retained. In practice, of the 12 stars for which P13
used CHORIZOS luminosities, all but 4 were discarded in favour of $BC$ luminosities: HD
6.3. BOLOMETRIC LUMINOSITIES

66765, HD 136504, HD 186205, and HD 130807.

The photometric parameters used to determine \( \log L \) are provided in Table 6.2. Visual magnitudes \( V \) were obtained from SIMBAD. Distance moduli \( DM \) were obtained from Hipparcos parallaxes (Perryman et al., 1997) or, when these were unavailable, from cluster distances; in these cases, the references for cluster membership and distance are provided in the final column of Table 6.2. Extinctions \( A_V \) were taken from P13. Absolute visual magnitudes were then determined from \( M_V = V - A_V - DM \). Bolometric corrections were obtained in the same way as those used by P13, i.e. via linear interpolation between the theoretical TLUSTY BSTAR2006 grid (Lanz and Hubeny, 2007), but using the values of \( T_{\text{eff}} \) and \( \log g \) found above. The bolometric luminosity is then \( M_{\text{bol}} = M_V + BC \), and the luminosity is \( \log (L_*/L_\odot) = (M_{\text{bol,}\odot} - M_{\text{bol}})/2.5 \), where \( M_{\text{bol,}\odot} = 4.74 \).

In some cases, it is necessary to adjust the luminosities of binary systems to account for the presence of multiple stars. This was already done by P13 for HD 122451. The total system luminosity \( \log L_{\text{sys}} = \log L_1 + L_2 \) was determined in the usual way from \( V \), \( d \), and \( BC \). While \( BC \) is a function of \( T_{\text{eff}} \), and it is not strictly speaking accurate to use the same \( BC \) for both stars, in practice the luminosity of the primary is only significantly different from the total system luminosity when the two components are close enough in mass, hence also in \( T_{\text{eff}} \) and \( \log L \), for both components to contribute comparable amounts to the system brightness. As close binaries are expected to be primordial (Bonnell and Bate, 1994), the assumption was made that the components are coeval. The individual stellar luminosities were then constrained using isochrones (Ekström et al., 2012). Along each isochrone, \( L_1 \) and \( L_2 \) were determined via \( M_1/M_2 \), with values for which \( \log L_{\text{sys}} \) fell outside the range determined from photometry discarded. The remaining values then constrain the luminosities of the individual components. This is illustrated for the case of HD 149277 in Fig. 6.19. In this case, \( M_1/M_2 = 1.1 \), and \( \log L_{\text{sys}} = 3.9 \pm 0.4 \), thus indicating \( \log L_1 = 3.7 \pm 0.4 \) and \( \log L_2 = 3.6 \pm 0.4 \). Similarly, for HD 136504 with \( \log L_{\text{sys}} = 3.8 \pm 0.2 \).
6.3. BOLOMETRIC LUMINOSITIES

Figure 6.19: Estimating luminosities of individual binary components of HD 149277. The mass ratio inferred from the radial velocity curve is used to locate the two stars on isochrones (primary, dashed blue; secondary, dotted red). Values for which the total luminosity (solid black) lies within the range determined from photometry (horizontal dot-dashed lines) are then used to constrain the luminosities of the individual components. Note that this same process constrains the orbital inclination as it also constrains the total mass of the system.

and $M_1/M_2 = 1.19$, $\log L_1 = 3.6 \pm 0.2$ and $\log L_2 = 3.3 \pm 0.2$. For the remaining systems, all of which have $M_1/M_2 > 2$, this correction is negligible compared to the uncertainties in $\log L$. 
Table 6.2: Photometric data for luminosity determination: $V$ magnitude, Distance Modulus $DM$, extinction $A_V$, Bolometric Correction $BC$, and bolometric magnitude $M_{bol}$. Distance moduli obtained from cluster distances are indicated with a superscript $cl$, the remainder are from Hipparcos parallaxes. References for cluster membership are given in Table 6.3.

<table>
<thead>
<tr>
<th>Star Name</th>
<th>$V$ (mag)</th>
<th>$DM$ (mag)</th>
<th>$A_V$ (mag)</th>
<th>$BC$ (mag)</th>
<th>$M_{bol}$ (mag)</th>
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Continued on next page.
6.4. CLUSTER AGES

Continued from previous page.

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6.4 Cluster ages

In Chapter 7, the absolute and fractional main sequence ages of the sample stars will be derived via their placement on the $T_{eff}$-$\log L$ and $T_{eff}$-$\log g$ diagrams. As discussed by Bagnulo et al. (2006), the evolutionary ages, i.e. the ages inferred from evolutionary tracks and the positions of individual stars on the Hertzsprung-Russell Diagram (HRD), are fairly imprecise for stars close to the Zero-Age Main Sequence (ZAMS). The movement of stars across the $T_{eff}$-$\log L$ diagram is initially relatively slow, speeding up in later evolutionary stages. Thus, a star’s position on the $T_{eff}$-$\log L$ diagram can in practice distinguish only between the first and second half of its main sequence lifetime. Conversely, cluster ages, which are principally determined by the location of the main-sequence turnoff, are much more precise for younger stars. This has motivated a careful search for magnetic stars that are members of stellar clusters.
Table 6.3: Cluster membership and cluster ages.

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<th>Cluster</th>
<th>log (t/yr)</th>
<th>Reference</th>
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<td>Landstreet et al. (2007)</td>
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<td>Landstreet et al. (2007)</td>
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(Bagnulo et al., 2006; Landstreet et al., 2007, 2008). In the present sample, 15 stars have been identified as probable cluster members. These stars are listed with their cluster memberships and cluster ages in Table 6.3.

6.5 Summary

$T_{\text{eff}}$ measurements using EW ratios generally yield very similar results to those obtained via both spectral modelling and photometry. The largest changes are in the surface gravities of those stars for which only photometric determinations were previously available. This is not surprising, as photometry is a much less precise diagnostic of log $g$ than H Balmer line wings.

Luminosities are mostly unchanged from the values used by P13, but in some cases have been revised sharply downwards. For two stars in particular, HD 35298 and HD
175362, a substantially lower luminosity is adopted. In the case of HD 175362, the Hipparcos parallax $\pi = 7.58 \pm 0.27$ mas is quite precise, leading to a distance of $131 \pm 4$ pc, which in turn implies a much lower luminosity of $\log L = 2.65 \pm 0.10$ than that obtained from CHORIZOS, 3.2$\pm$0.1. The parallax measurement for HD 35298 is much more uncertain ($\pi = 1.88 \pm 1.09$ mas), however this star was identified as a likely member of the Ori OB1a association (Landstreet et al., 2007), which would place it at a distance of approximately 330 pc, much closer than the 870 pc assumed by CHORIZOS (the parallax distance, $531^{+733}_{-195}$ pc, is compatible with either distance within the large uncertainty). In both cases, the surface gravities implied by the CHORIZOS luminosities and the stars’ effective temperatures are much lower ($\sim$3.8) than is compatible with their H$\beta$ lines ($\log g \sim 4.2$). Adopting closer distances and lower luminosities, however, brings the stellar temperatures, surface gravities, and luminosities into alignment.

The sample stars are located on the HRD in the top panel of Fig. 6.20. The majority of the sample lie on the main sequence, although some stars are apparently somewhat below the Zero-Age Main Sequence (ZAMS), and two are above the Terminal Age Main Sequence (TAMS). The bottom panel shows the sample on the $T_{\text{eff}}$-$\log g$ plane, confirming the stars as largely main-sequence objects on purely spectroscopic grounds. Importantly, the positions of individual stars on the two diagrams are generally consistent. Notably, in many cases the stellar age and mass are better constrained via surface gravity than luminosity. This will be used to obtain maximal precision in rotational, magnetic, and magnetospheric parameters in the following chapter.

It should be emphasized that the analyses presented here are not intended as
Figure 6.20: Top: the sample stars on the $T_{\text{eff}}$-log $L$ diagram. Evolutionary tracks are indicated by dashed lines, the ZAMS by a solid line, and the TAMS by a dotted line. Bottom: the sample stars on the $T_{\text{eff}}$-log $g$ plane.
substitutes for detailed spectral modelling. While EW ratios utilizing Si II, III, and IV, and He I and II ionization balances are sensitive diagnostics of effective temperature, they are subject to systematic errors if, for instance, the abundance distributions of these elements are not only horizontally but also vertically stratified as has been reported for cooler Bp stars (Bailey and Landstreet, 2013). Furthermore, the Stark-broadened wings of H lines can be affected by over- or under-abundances of He as well as metallic lines (Leone and Manfre, 1997). At least three stars, HD 61556, HD 125823 and HD 184927, all of which show strong He abundance variations, show variability in the Balmer line wings that is closely associated with He variations (Shultz et al., 2015a; Yakunin et al., 2015). Thus, surface gravities should ultimately be derived together with, at the very least, mean surface abundances, and ideally with Doppler imaging in order to determine the horizontal abundance distributions.

With the observable stellar parameters of the stars constrained, we turn in the following chapter to the derivation of their radii, masses, and ages. These will be combined with the magnetic data from Chapter 4 and the rotational observables in Chapter 5 to determine the surface strengths of the magnetic dipoles in Chapter 8. In Chapter 9 these results will also provide the basis for determining stellar wind properties with which to constrain Alfvén radii, Kepler radii, and magnetic braking timescales, as well as to provide a direct comparison between spindown timescales and ages.
Chapter 7

Stellar Radii, Masses, and Ages

A primary goal of this work is to determine the placements of the sample stars on the rotation-magnetic confinement diagram introduced in Chapter 1 (Fig. 1.7), and to use these new parameters to interpret the stars’ emission properties and to evaluate their spindown timescales. Together with $P_{\text{rot}}$ and $v_{\text{sin} \ i}$, a star’s precise rotational parameters are sensitive to the stellar radius and to, due to oblateness, the stellar mass. Thus, rotational axis inclinations are also sensitive to radius and mass. Inclinations are necessary to break the inclination-obliquity degeneracy that is a feature of the dipole oblique rotator model utilizing $\langle B_z \rangle$ measurements, and which are thus essential for constraining the dipole magnetic field strength $B_d$ of each star (the degeneracy can also be broken using magnetic field modulus measurements or linear polarization data, which is sensitive to the transverse magnetic field; such data is unavailable for the majority of stars in this sample). Magnetic wind confinement parameters require stellar mass-loss rates $\dot{M}$ and wind terminal velocities $v_{\text{inf}}$, which act against the magnetic field confining the wind. Wind parameters are also sensitively dependent on the stellar mass and radius, which vary together across the HRD.

Thus, uncertainties in rotational and magnetic wind confinement parameters may
not be entirely uncorrelated. To account for this, in this work all parameters are calculated simultaneously using a Monte Carlo approach in conjunction with interpolation through evolutionary tracks and isochrones to obtain correlated values of mass, radius, and age. The derived properties are then obtained from the peaks of the resulting posterior probability density functions (PDFs), with asymmetrical error bars determined from their areas. In the remainder of this section the steps of the calculation are presented for representative stars, and at each step, the results for the population are examined.

7.1 Calculations

Stellar radii and masses were obtained in two ways. The first and most straightforward is to utilize the $T_{\text{eff}} - \log L$ diagram, while the second is to use the $T_{\text{eff}} - \log g$ diagram. The advantage of the first method is that $R_*$ is calculated directly from $T_{\text{eff}}$ and $\log L$; since, given $v \sin i$ and $P_{\text{rot}}$, $R_*$ is the primary determinant of $i$, this method should in principle yield the most reliable results. The advantage of the second method is that, as it depends only on spectroscopically determined quantities, it is independent of distance. Thus when the distance is highly uncertain, better precision may be available from $\log g$ than from $\log L$. Furthermore, for rapidly rotating stars with significant gravity darkening, the apparent luminosity is a function of $i$: thus, the position on the HRD does not necessarily provide an accurate determination of radius and mass. In practice, both methods are utilized, with the $T_{\text{eff}} - \log g$ results serving as a check on those obtained from $T_{\text{eff}}$ and $\log L$, and used in preference for rapid rotators and stars with highly uncertain distances.
Figure 7.1: *Top:* HD 67621’s position on the HRD. Filled contours indicate point density, with colours corresponding to 1, 2, and 3σ uncertainties in $T_{\text{eff}}$ and $\log L$. The rotating evolutionary models calculated by Ekström et al. (2012) are indicated by solid (evolutionary tracks) and dotted lines (isochrones); ZAMS and TAMS are indicated by dashed and dot-dashed lines. The evolutionary tracks are for 7 $M_\odot$ and 9 $M_\odot$ models. Isochrones are in increments of log ($t$/Myr) = 0.1, beginning from log ($t$/Myr) = 6.5. *Bottom:* The mass-radius relationship inferred from the parameters above via interpolation between models.

When using the $T_{\text{eff}} - \log L$ diagram, the calculation begins from randomly generated normal distributions of effective temperature $T_{\text{eff}}$ and luminosity $\log L/\log \mathcal{L}_\odot$, with standard deviations equal to the 1σ error bars of the stars’ photometrically and spectroscopically determined luminosities and effective temperatures and centered on these values (top panel of Fig. 7.1). At each point, the stellar radius $R_*$ is calculated via $R_*/R_\odot = \sqrt{(L/\mathcal{L}_\odot)/(T_{\text{eff}}/T_\odot)^4}$, where $T_\odot = 5680$ K. The mass $M_*$ is determined via interpolation between evolutionary tracks and isochrones, where rotating Geneva
evolutionary models are used (Ekström et al., 2012). Points below the ZAMS, where interpolation is not possible, are discarded. This is illustrated in the bottom panel of Fig. 7.1. Absolute and fractional main sequence ages $t_*$ and $\tau_{\text{MS}}$ are obtained in the same fashion as mass, while surface gravities are obtained from $R_*$ and $M_*$ as $g = GM_*/R_*^2$.

Using $T_{\text{eff}}$ and $\log g$, the calculation begins in the same fashion, only using a similar randomly generated array of values for $\log g$ instead of $\log L$. $M_*$, $t_*$, and $\tau_{\text{MS}}$ are obtained as before. $\log L$ is also determined via interpolation between evolutionary models, and $R_*$ finally calculated from $T_{\text{eff}}$ and $\log L$. Fig. 7.2 compares the results obtained via this method to those from the first method for the case of the rapidly rotating star HD 182180. Due to gravity darkening, the star’s apparent luminosity is reduced, in fact placing it below the ZAMS (discarding these points leads to a non-elliptical distribution on the HRD). As a consequence of the reduced apparent luminosity, the star’s position on the $T_{\text{eff}}$-$\log L$ diagram yields a smaller mass and radius than that obtained from the $T_{\text{eff}}$-$\log g$ diagram. The uncertainty in $R_*$ is similar in both cases, about $0.2R_\odot$. The uncertainty in $M_*$ is somewhat higher with the $T_{\text{eff}}$-$\log g$ diagram, as $M_*$ is only weakly sensitive to $\log g$.

In one case, HD 122451, an extremely precise stellar mass of $M_*=10.3\pm 0.1M_\odot$ is available thanks to interferometric determination of the inclination of the system’s orbital plane (Ausseloos et al., 2006). Incorporating this additional constraint essentially confines the star to a single evolutionary track, as shown in Fig. 7.3.

In section 6.4 it was noted that cluster ages are available for 15 of the sample stars (listed in Table 6.3). As cluster ages are much more precise than ages determined from the physical parameters of stars in isolation, they offer an additional constraint
Figure 7.2: Top panels: HD 182180’s position on the $T_{\text{eff}}$-$\log L$ (upper left) and $T_{\text{eff}}$-$\log g$ (upper right) diagrams. Filled contours indicate point density, with red, blue, and purple corresponding to 1, 2, and 3$\sigma$ uncertainties in $T_{\text{eff}}$, $\log L$, and $\log g$. Geneva evolutionary models with rotation (Ekström et al., 2012) are indicated by solid (evolutionary tracks) and dotted lines (isochrones); ZAMS and TAMS are indicated by dashed and dot-dashed lines. Bottom panels: The mass-radius relationships inferred from the above parameters via interpolation between models. Using HD 182180’s luminosity yields a mass and radius much lower than obtained from $\log g$. Furthermore, the star’s position on the $T_{\text{eff}}$-$\log L$ diagram indicates it to be below the ZAMS.
Evolutionary tracks are 9, 12, and 15 \( M_\odot \). An extremely precise stellar mass \( M_\star = 10.3 \pm 0.1 \, M_\odot \) is available via interferometric data, essentially confining the star to a single narrow evolutionary track.

by requiring that grid points fall within the isochrones delineated by the cluster ages. Fig. 7.4 shows an example for HD 37479. The left-hand panels show the \( T_{\text{eff}} \) vs. \( \log L \) and \( R_\star \) vs. \( M_\star \) grids across the full range of \( T_{\text{eff}} \) and \( \log L \) possible from the star’s spectroscopically and photometrically determined physical parameters. Parameters constrained by the star’s cluster age are shown on the right: these result in a much narrower range of permitted radii.

Radii, masses, ages, fractional ages, and the ratio of the current stellar radius
Figure 7.4: Top panels: HD 37479’s position on the $T_{\text{eff}}$-$\log L$ diagram. Filled contours indicate point density, with red, blue, and purple corresponding to 1, 2, and 3$\sigma$ uncertainties in $T_{\text{eff}}$ and $\log L$. Geneva evolutionary models with rotation (Ekström et al., 2012) are indicated by solid lines (evolutionary tracks) and dotted lines (isochrones); ZAMS and TAMS are indicated by dashed and dot-dashed lines. Bottom panels: The mass-radius relationships inferred from the above parameters via interpolation between models. Left: as before, but with $T_{\text{eff}}$ and $\log L$ limited to the isochrones determined from the star’s cluster age.
to the ZAMS radius $R_* / R_{ZAMS}$ derived using both the $T_{\text{eff}}$-$\log L$ and the $T_{\text{eff}}$-$\log g$ diagrams are given in Table 7.1, with the set of parameters chosen for further magnetic and magnetospheric modelling indicated by a bullet point. It should be noted that the stellar radii $R_*$ given in Table 7.1 correspond to the polar radii $R_p$. In the non-rotating case, $R_*$ and $R_p$ are equivalent, and these terms are used interchangeably except where rotation is a significant factor.

Table 7.1: Derived physical parameters. For each star, parameters derived using $T_{\text{eff}}$ and $\log L$ are shown first, followed by those derived from $T_{\text{eff}}$ and $\log g$. The final column indicates, with a black bullet point, which of the two strategies was selected for determining the star’s rotational, magnetic, and magnetospheric properties.

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<th>Star Name</th>
<th>$R_*$ $(R_\odot)$</th>
<th>$M_*$ $(M_\odot)$</th>
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### 7.1. CALCULATIONS

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### 7.1. CALCULATIONS

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7.2. DISTRIBUTIONS OF STELLAR MASSES AND AGES

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7.2 Distributions of stellar masses and ages

Histograms of \( \tau_{\text{MS}} \) and \( M_* \) are shown in Fig. 7.5. An inhomogeneous age distribution of low-mass \( (M_* \leq 2M_\odot) \) Ap stars, with such stars tending to concentrate in the middle of the main sequence, was reported by Kochukhov and Bagnulo (2006), although the same study found that more massive Ap stars \( (M_* > 3M_\odot) \) were homogeneously distributed along the main sequence. No pattern is evident in the distribution of \( \tau_{\text{MS}} \) in the present sample. The distribution of \( M_* \) peaks around \( 7M_\odot \), with a tail towards higher masses. The MiMeS Survey Component preferentially observed stars
7.2. DISTRIBUTIONS OF STELLAR MASSES AND AGES

Figure 7.5: Left: histogram of fractional main sequence ages. Right: histogram of stellar masses.

with spectral types earlier than B3 and $V$ magnitudes below 8. Preference was given to the brightest stars and the earliest spectral types. Thus, amongst the B-type stars, completeness is highest for B0 stars at about 30%, declining to $\sim$10% for B5 stars (Wade et al., 2016). As spectral type is a proxy for stellar mass, the distribution of $M_*$ in Fig. 7.5 is unlikely to be due to selection bias, as the completeness of the MiMeS Survey increases towards higher masses.

Fig. 7.6 shows the histogram of stellar masses divided into the sub-samples with and without H$\alpha$ emission. Both samples follow a more or less similar pattern to that seen in Fig. 7.5. The K-S significance of the maximum difference between the two cumulative distributions is 0.50, suggesting that they are drawn from the same population. This is as expected, given that a CM should in principle be able to form in a magnetic hot star of any mass, and one has been reported in a rapidly rotating O-type star (HD 47129; Grunhut et al. 2013).

Dividing the sample into stars with and without H$\alpha$ emission, it is apparent in
Figure 7.6: Top: Cumulative mass distributions of sample stars for the sub-samples with and without H$\alpha$ emission. Bottom: histograms for the same sub-samples.
Figure 7.7: As Fig. 7.6 for fractional main sequence age.
Fig. 7.7 that the age distributions of these two sub-samples differ: the fraction of Hα emission stars decreases with age, while the fraction of stars with Hα in absorption increases with age. The K-S significance of this difference is 0.03, much lower than the K-S significance for the masses of the two samples, suggesting that the Hα-bright stars are indeed systematically younger. This makes sense in the context of the general theory of stellar magnetospheres, which requires both a strong magnetic field and rapid rotation (Townsend and Owocki, 2005), and magnetic braking, which predicts that stars with stronger magnetic fields will lose angular momentum more quickly (Weber and Davis, 1967; ud-Doula et al., 2009).

In Section 4.2.4 the prevalence of complex magnetic topologies in the sample was examined using two simple empirical measures: the goodness of fit $\chi^2/\nu$ of the best-fit 1st-order sinusoid to the $\langle B_z \rangle$ curve, and the dipolar anomaly index $A_d$ (see Figs. 4.14–4.16). While it was concluded that multipolar magnetic fields should not have a strong influence on the magnetic and, especially, magnetospheric parameters derived in this chapter, it is of some interest to see if these diagnostics of multipolarity might change with age. The left-hand panels of Fig. 7.8 show histograms of $\tau_{MS}$, divided into stars showing high vs. low $A_d$ (top) and high vs. low $\chi^2/\nu$ (bottom). Cumulative fractions are shown on the right. By either measure, stars with multipolar fields are most prevalent at young ages, essentially disappearing entirely above $\tau_{MS} = 0.5$. The K-S significance is only 0.20 using $A_d$, due to the small number of stars with $A_d > 0.1$, but falls to 0.03 using $\chi^2/\nu$, indicating that, at least by the latter diagnostic, the difference in ages between stars with simple vs. complex surface magnetic fields is likely real. As stars with primarily dipolar magnetic fields appear at all ages, the observation that stars with complex surface fields are generally young could be
7.3. **Summary**

Stellar radii, masses, and ages were derived from evolutionary models using the fundamental stellar parameters determined in Chapter 6. Parameters were derived using interpreted as evidence that a complex fossil field gradually relaxes into its lowest-energy configuration, i.e. that of a dipole, as the star ages.

Figure 7.8: Age distributions as histograms (left) and cumulative fractions (right) of sample stars for which topological field complexity could be examined, divided into stars with simple dipolar fields vs. stars with significant multipolar contributions, as determined using either the dipolar anomaly index $A_d$ (top) or the $\chi^2/\nu$ obtained from the best-fit sinusoid to $\langle B_z \rangle$ (bottom).
either the $T_{\text{eff}}$-log $L$ or the $T_{\text{eff}}$-log $g$ diagrams, depending on which gave the highest precision. Where available, cluster ages were used as an additional constraint. The sample stars are homogeneously distributed across the main sequence. The distribution of stellar masses does not appear to have been affected by selection effects.

The mass distributions of stars with and without H$\alpha$ emission are quite similar. However, the age distributions exhibit a clear difference, with emission-line stars systematically younger than stars with H$\alpha$ in absorption. There is also some evidence that topological complexity of the surface magnetic field, as evaluated using the empirical diagnostics presented in Section 4.2.4, is predominantly a feature of younger stars.

In the following chapter, the radii and masses found here are used together with the projected rotational velocities and rotational periods from Chapter 5 in order to determine equatorial rotational velocities, and inclination angles between the rotational axes and the lines of sight. These are then combined with the magnetic data from Chapter 4 to obtain the two other dipolar oblique rotator model (ORM) parameters: the angles between the magnetic and rotational axes, and the surface strengths of the magnetic dipoles. Chapter 9 uses the ORM parameters, together with the stellar parameters from Chapter 6, to place the stars on the rotation-magnetic confinement diagram, and to constrain magnetic braking timescales with which to compare to the stellar ages found in the present chapter.
Placing the stars on the rotation-magnetic confinement diagram requires that we know, first, their true (i.e. deprojected) rotational velocities, and the extent of magnetic confinement of their stellar winds. Observable magnetic, rotational, and stellar parameters were examined in Chapters 4–6. In Chapter 7 stellar radii, masses, and ages were determined from the stars’ positions on the $T_{\text{eff}}$-$\log L$ and $T_{\text{eff}}$-$\log g$ diagrams. In this chapter stellar radii and ages are used together with $v \sin i$ and $P_{\text{rot}}$ from Chapter 5 to constrain the inclinations $i$ of the rotational axes from the line of sight, together with rotational oblatenesses and equatorial rotational velocities. The inclinations $i$ are then used together with the $\langle B_z \rangle$ curves from Chapter 4 to calculate dipole oblique rotator model parameters $\beta$ (the obliquity angle between the magnetic and rotational axes) and $B_d$ (the surface magnetic field strength at the magnetic pole). The distributions of $\beta$ and $B_d$ are compared to previous results for Ap stars, and the evolution of $B_d$ examined via comparison with the ages determined in Chapter 7.
8.1 Rotational axis inclinations

A dipolar ORM consists of three parameters: $B_d$, $i$, and $\beta$. When determined using only $\langle B_z \rangle$ measurements, there is a degeneracy between the angles $i$ and $\beta$. However, using rotational constraints $i$ can be determined independently from the magnetic data. Inverting equation 5.2 for $\sin i$ we obtain:

$$\sin i = \frac{P_{\text{rot}} v \sin i}{2\pi R_{\text{eq}}},$$

(8.1)

where $R_{\text{eq}}$ is defined via Eqn. 5.4. As $R_p/R_{\text{eq}}$ is substantially less than 1 for only a few stars in the sample (Table 8.3), in general this correction is negligible, however it is important for the rapid rotators. Since $R_{\text{eq}}$ is related to $M_*$, and since $M_*$ can take on a range of values for a given $R_p$, the parameter space is larger than if $R_*$ is considered in isolation. $R_{\text{eq}}$ was calculated on the grid of $(R_p, M_*)$ values determined in Chapter 7. Points yielding solutions for which the equatorial rotational velocity $v_{\text{eq}} < v \sin i$, or for which $v_{\text{eq}} > v_{\text{br}}$ (Eqn. 5.3), were discarded as unphysical. Inclinations are given together with $\omega$, $R_p/R_{\text{eq}}$, and $v_{\text{eq}}$ in Table 8.1.

The bottom left panel of Fig. 8.1 shows the $M_*$, $R_p$ density plot for the rapid rotator HD 142184, the star with the shortest rotation period in the sample ($P_{\text{rot}} = 0.508276(13)$ d, Grunhut et al. 2012b). Radii below about 2.6 $R_*$ are excluded as $v_{\text{eq}} < v \sin i$, while $R_* > 3.2R_\odot$ are ruled out as $v_{\text{eq}} > v_{\text{br}}$, leading to somewhat tighter constraints on $R_p$ than are possible from $T_{\text{eff}}$ and $\log L$ alone. The stellar parameters in Table 7.1 already include such constraints for HD 142184, as well as for other stars for which parts of the grid are ruled out by rotational properties. $i$ is shown as functions of $M_*$ and $R_p$ in the top left and bottom right panels of Fig. 8.1.
8.1. ROTATIONAL AXIS INCLINATIONS

Figure 8.1: Relationship between mass, radius, and inclination for the rapid rotator HD 182180. Bottom left: $R_p$ vs. $M_*$; solid/dashed/dotted lines indicate 1, 2, and 3σ density contours for the grid inferred from the $T_{\text{eff}}$-$\log L$ diagram, while filled red, blue, and purple contours indicate the same intervals for regions for which $v_{\text{eq}} \leq v \sin i \leq v_{\text{br}}$. Top left: $i$ vs. $M_*$. Bottom right $R_p$ vs. $i$. Top right: cumulative distribution of $i$ (solid black), with the peak and 1σ limits of the probability distribution indicated by solid and dotted vertical blue lines.
Figure 8.2: As Fig. 8.1 for the slowly rotating star HD 66522. The star’s very slow rotation \( P_{\text{rot}} = 909.339(3) \text{ d} \) means that \( v_{\text{eq}} \ll v \sin i \). Since \( i \) is thus unconstrained by rotation, a random distribution across the grid is assumed.

As expected, there is a strong correlation with \( R_p \), and a weaker correlation of \( i \) with \( M_* \).

If \( v_{\text{eq}} \) is much less than the minimum measureable \( v \sin i \) (\( \sim 2 \text{ km s}^{-1} \) in ESPaDOnS spectra), as is the case for several stars with very long \( P_{\text{rot}} \), \( i \) is unconstrained by rotation. These are HD 63425 \( (P_{\text{rot}} = 163.19(1) \text{ d}) \), HD 66522 \( (P_{\text{rot}} = 909.339(3) \text{ d}) \), and likely HD 46328 \( (P_{\text{rot}} \geq 30 \text{ yr}) \). Overall we expect \( i \) to be randomly distributed (e.g, Abt 2001; Jackson and Jeffries 2010), with a cumulative probability following...
8.1. ROTATIONAL AXIS INCLINATIONS

\[ P(i) = 1 - \cos i, \]  \hspace{1cm} (8.2)

therefore for these stars we assume a random distribution of \( i \). An example is shown in Fig. 8.2 for HD 66522.

For the two stars without periods, HD 52089 and HD 58260, two strategies were adopted. The first, and simplest, is to force a random distribution as with the slow rotators. The second is to calculate the minimum and maximum period at each given grid point, and use a random period within these bounds, where in this case a flat distribution of periods was used. A flat distribution does not reflect the distribution determined in Section 5.2 (Fig. 5.10), but was chosen for three reasons. The first is simplicity of implementation. The second is that the observed distribution is strongly biased towards relatively rapidly rotating stars. While it is possible that these stars may be rapid rotators seen close to pole-on, their very sharp spectral lines can also be explained by slow rotation. Furthermore, both stars are of relatively advanced age (Table 7.1), and are likely to have lost significant angular momentum via magnetic braking. The final reason is that a flat distribution of periods tends to bias the grid towards small \( i \): ORM and magnetospheric parameters for these stars were taken from the mean values determined by both methods, thus a flat distribution of periods compensates for the bias towards large \( i \) introduced by a random distribution.
### Table 8.1: Derived rotational properties.

<table>
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<tr>
<th>HD No.</th>
<th>$\omega$ (km s$^{-1}$)</th>
<th>$R_p/R_e$</th>
<th>$v_{eq}$ (km s$^{-1}$)</th>
<th>$i$ (°)</th>
</tr>
</thead>
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<td>$0.990_{-0.001}^{+0.001}$</td>
<td>$54_{-6}^{+5}$</td>
<td>$19_{-3}^{+2}$</td>
</tr>
<tr>
<td>23478</td>
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<td>$0.97_{-0.01}^{+0.00}$</td>
<td>$137_{-32}^{+12}$</td>
<td>$62_{-8}^{+1}$</td>
</tr>
<tr>
<td>25558</td>
<td>$0.3_{-0.1}^{+0.1}$</td>
<td>$0.99_{-0.001}^{+0.001}$</td>
<td>$88_{-32}^{+27}$</td>
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<tr>
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<td>$66_{-3}^{+3}$</td>
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</tr>
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<tr>
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</tr>
<tr>
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<td>$0.99_{-0.01}^{+0.00}$</td>
<td>$85_{-11}^{+11}$</td>
<td>$40_{-5}^{+5}$</td>
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### 8.1. ROTATIONAL AXIS INCLINATIONS

Continued from previous page.

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8.1.1 Additional constraints on $i$

Inclinations and obliquities are available in the literature for some stars. Values found in the literature are summarized in Table 8.2, with the inclinations from Table 8.1 included for comparison. Some studies utilized information beyond $v \sin i$ and $P_{\text{rot}}$ to constrain $i$. The behaviour of emission lines originating in the stellar magnetosphere, and in particular whether eclipses are seen, is an indicator that $i$ must be fairly large (e.g., HD 37479: Townsend et al. 2005; Oksala et al. 2015b; HD 176582:...
### Table 8.2: Inclination angles from the literature. The 2\(^{nd}\) column gives the value of \(i\) derived here. The 3\(^{rd}\) column gives the value of \(i\) from the literature; Literature values adopted in preference to those derived here are given in bold face. The 3\(^{rd}\) column gives the method by which \(i\) was constrained: \(r\): the rotational method used in this work; \(a\): asteroseismology; \(e\): emission properties of the magnetosphere; \(g\): gravity darkening; \(d\): Doppler imaging; \(l\): LSD profile modelling.

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<td>(60^{+4}_{-4}) Donati et al. (2001)</td>
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<td>(60^{+10}_{-10})</td>
<td>(46) Neiner et al. (2015)</td>
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Bohlender and Monin 2011). Amongst pulsators, mode-splitting of non-radial pulsations provides an avenue to determine \( i \) (e.g., HD 25558: Sódor et al. 2014; HD 44743: Fossati et al. 2015a). As the effects of rotational distortion on the stellar spectrum and luminosity are sensitive to \( i \), rapidly rotating stars whose SEDs have been analyzed using synthetic spectra incorporating gravity darkening and oblateness yield independent determinations of \( i \) (HD 142814 Grunhut et al. 2012a, HD 182180 Rivinius et al. 2013). Finally, some stars have been mapped using Doppler and/or
Zeeman Doppler Imaging, which is also sensitive to \( i \) (e.g., HD 37776: Kochukhov et al. 2011; HD 125823; Bohlender et al. 2010). When superior constraints on \( i \) are available via alternate routes, these were adopted in preference to values obtained via rotation by excluding grid points yielding \( i \) outside the given range. These stars are shown in boldface in Table 8.2.

In some cases these constraints in turn place tight limits on \( R_\star \). An example of this is the emission-line star HD 176582, shown in Fig. 8.3. \( R_\star = 3.5 \pm 0.5 R_\odot \) based on the star’s \( T_{\text{eff}} \) and log \( L \). However, the fact that the star is periodically eclipsed by its magnetosphere, combined with the distance of the magnetospheric plasma from the star, indicate that \( i > 85^\circ \) (Bohlender and Monin, 2011), thus requiring \( R_\star = 3.18 \pm 0.05 R_\odot \).

Another instructive case is HD 37479. The inclination inferred from the \( T_{\text{eff}} \) vs. log \( L \) diagram is \( 48^{+10}_{-12}^\circ \). This is much smaller than the \( i = 77 \pm 7^\circ \) determined by Oksala et al. (2015b) via detailed aRRM modelling of the star’s circumstellar emission and absorption properties, via which smaller inclinations were quite explicitly ruled out. Notably, when the grid points are constrained by the star’s cluster age (Fig. 7.4), \( i = 64^{+5}_{-6}^\circ \), much closer to the inclination determined from the star’s emission properties. Pruning the grid to include only those points yielding \( i \) in the range determined by Oksala et al. yields an age consistent with the cluster age, but necessitates even tighter constraints on the radius: from the cluster age, \( R_\star = 3.6 \pm 0.3 R_\odot \), while the emission inclination requires \( R_\star = 3.45 \pm 0.05 R_\odot \).

The first column in Table 8.2 gives the angle \( i \) derived here using the stars’ rotational properties alone. In most cases these are in good agreement with the values
found by previous studies. Literature values adopted in preference to those determined here are indicated in boldface. There are 3 stars for which $i$ differs significantly from the literature value, but we have elected to retain the value determined here.

The first is HD 3360, for which Briquet et al. (2016) found $i = 31 \pm 1^\circ$ using Doppler imaging, differing by more than $3\sigma$ from the value found here, $i = 19^{\circ+2}_{-3}$. Since the inclination found by Briquet et al. cannot be reconciled with the star’s rotational and physical properties, we retain the smaller angle, which is in agreement with that found by Neiner et al. (2003a).

The second star is HD 105382, for which our value is $20^\circ$ smaller than that found by Briquet et al. (2004). In this case the disagreement is due to the adoption of a slightly larger stellar radius (Briquet et al., $3.0 \pm 0.6 R_\odot$; here, $3.3 \pm 0.3 R_\odot$), arising from a slightly higher $T_{\text{eff}}$ and $\log L$.

The final disagreement is for HD 175362, for which we find $i = 65^{\circ+5}_{-6}$, somewhat larger than the $i = 41 \pm 16^\circ$ adopted by Bohlender et al. (1987). This is partly due to the slightly higher $v \sin i$ ($34 \pm 4$ km s$^{-1}$, as compared to $30$ km s$^{-1}$), but mostly a consequence of the smaller radius, $2.61 \pm 0.04 R_\odot$ vs. $3.9 \pm 1.3 R_\odot$. The very small uncertainty in $R_*$ is due first to the highly precision of the Hipparcos parallax distance, and second to discarding grid points that fall beneath the ZAMS, which the majority of them do.

In the case of HD 156324, its rotational inclination can be inferred directly from its orbital inclination as, since the system is tidally locked, the two should be identical. This is especially useful as tight constraints on the star’s physical parameters cannot be established on $R_*$ due to large uncertainties in both luminosity (due to the large distance modulus) and surface gravity (due to the simultaneous spectral contributions
8.1. ROTATIONAL AXIS INCLINATIONS

Figure 8.4: Comparison of $i$ determined via $\log L$ and $\log g$. The two methods yield very similar results, with virtually all of the scatter being within the 1$\sigma$ uncertainties.

of 3 stars). Using $v\sin i$, $P_{\text{rot}}$, $T_{\text{eff}}$, and $\log L$ alone yields $i_{\text{rot}} = 11^{+5}_{-3}$, somewhat smaller than $i_{\text{orb}} = 26 \pm 3^\circ$. However, HD 156324 is a cluster star (Table 6.3): restricting the grid to the range of possible cluster ages yields $i_{\text{rot}} = 23^{+20}_{-3}$, overlapping within error with $i_{\text{orb}}$. 
8.1.2 Comparison of results using log $L$ and log $g$

A comparison between $i$ determined using log $L$ and log $g$ is shown in Fig. 8.4. Stars for which $i$ was forced to follow a predetermined probability distribution (slow rotators, stars without $P_{\text{rot}}$) were excluded, as were stars for which $i$ was forced to match constraints provided by emission, Doppler Imaging, or asteroseismology. While there is some scatter, the agreement is within 1$\sigma$ for most stars. The only 2$\sigma$ outlier is HD 37776, for which log $g$ is not well constrained due to the star’s complex magnetic topology, and correspondingly complex He surface abundance distribution (Kochukhov et al., 2011), which leads to strong variability in H Balmer line wings which is further complicated by circumstellar emission present at essentially all rotational phases (as discussed further in Chapter 10).

8.1.3 Distribution of $i$

Previous investigations of the distribution of $i$ have without exception concluded that it is compatible with a random distribution on a sphere (Abt, 2001; Jackson and Jeffries, 2010), the assumption that was used above to set the inclinations of slow rotators for which $v\sin i$ does not provide any constraints (Fig. 8.2). It is therefore of interest to check that our inclinations do in fact follow a random distribution. Fig. 8.5 compares the cumulative distribution of $i$ to the expected random distribution. The observed distribution is systematically above the expected distribution: the one-sample KS test significance is 0.06. However, it is consistent with the cumulative distributions of the inclinations given by Landstreet and Mathys (2000b) (LM00), Aurière et al. (2007) (A07), Power (2007) (P07), and Hubrig et al. (2007) (H07) for their populations of Ap/Bp stars. The two-sample KS test significances comparing
Figure 8.5: Cumulative distribution of $i$ values determined in this study (solid black line) as compared to a random distribution (dotted line). For comparison, cumulative distributions of $i$ from several previous studies of cooler Ap/Bp stars are overplotted, abbreviated in the legend as LM00 (Landstreet and Mathys, 2000b), A07 (Aurière et al., 2007), P07 (Power, 2007), and H07 ((Hubrig et al., 2007). The curved solid (red) line indicates the cumulative distribution of $i$ for all stars in this and previous studies. All distributions are systematically to the left of the expected random distribution.
the B star inclinations to the previous studies are, respectively, 0.77, 0.32, 0.49, and 0.57, all compatible with belonging to the same population. Comparison of the distributions from these studies to the expected distribution yields respective one-sample KS test significances of 0.32, 0.001, 0.06, and $5 \times 10^{-6}$, suggesting that only the Landstreet and Mathys sample is consistent with a random distribution. For all stars, the one-sample KS significance is $2 \times 10^{-6}$.

Almost all studies find $i$ systematically larger than predicted from equation 8.2. This may be simply an artifact of small-number statistics (the full combined sample consists of 164 stars). Alternatively, it may point towards one or more systematic errors in stellar parameters, with the net effect of increasing the stellar radius and, thus, requiring a larger $i$ for a given $v \sin i$ and $P_{\text{rot}}$. Referring to Table 8.2, in several cases for which constraints other than $v \sin i$ and $P_{\text{rot}}$ are available (HD 37479, HD 44743, HD 125823, HD 176582), the inclination inferred from emission, pulsation, or Doppler Imaging is substantially larger than that determined from rotation alone.

### 8.1.4 Spin-orbit alignment of magnetic binaries

Fig. 8.6 compares the orbital and rotational inclinations $i_{\text{orb}}$ and $i_{\text{rot}}$ obtained for the sample’s binary stars. Note that the values of $i_{\text{rot}}$ shown here are those computed using Eqns. 5.4 and 8.1, i.e. no alignment is assumed: $i_{\text{orb}}$ and $i_{\text{rot}}$ should be identical for HD 156324, as the rotational and orbital periods are the same, and this fact can be used to constrain the star’s magnetic properties. As demonstrated in Fig. 8.6, $i_{\text{rot}}$ is very close to $i_{\text{orb}}$ for this system, as indeed it is for all systems but HD 122451. There is some ambiguity in $P_{\text{rot}}$ for HD 122451 as the variation in $\langle B_z \rangle$ is fairly small compared to the mean error bar, therefore $i_{\text{rot}}$ may be incorrect.
8.1. ROTATIONAL AXIS INCLINATIONS

Figure 8.6: Left: Comparison of rotational and orbital inclinations. Only one system, HD 122451, shows evidence of spin-orbit misalignment. Right: ratio of orbital to rotational angular momentum $L$ (top) and orbital to rotational periods (bottom) as a function of the difference between $i_{\text{orb}}$ and $i_{\text{rot}}$. In all cases $L_{\text{orb}}/L_{\text{rot}} > 7$, above which the timescale for spin-orbit alignment is expected to be longer than the timescale for pseudo-synchronization of orbital and rotational periods. Consistent with this, $P_{\text{orb}} > P_{\text{rot}}$ in almost every case; the one case in which $P_{\text{orb}} < P_{\text{rot}}$, HD 149277, is also the most evolved binary system in the sample.

The relative time-scales of orbital circularization $t_e$, synchronization of orbital and rotational periods at periastron $t_{sp}$, and spin-orbit alignment $t_i$, were investigated by Hut (1981), who found that in general $t_e \gg t_i \sim t_{sp}$, as $t_e$ depends only on the orbital angular momentum, while $t_i$ and $t_{sp}$ also depend on the rotational angular momentum of the primary. Thus, it is not surprising that some of the stars have non-zero eccentricities, despite showing evidence of spin-orbit alignment. Only two stars have $e = 0$, which is consistent with $t_e \gg t_i$. It is also worth pointing out that HD 122451 has by far the longest orbital period in the sample, 356.92 d (see Table 3.1), so $t_i$ will be longer than for the shorter-period binaries, likely explaining
its status as the only star not showing spin-orbit alignment.

$t_i$ and $t_{sp}$ both depend on the ratio of orbital to rotational angular momentum. For $L_{orb}/L_{rot} \gg 7$, $t_i > t_{sp}$; thus, in this regime, $P_{rot}$ will tend to be less than $P_{orb}$ (Hut, 1981). Indeed, there is only one star with $P_{rot} > P_{orb}$, HD 149277, and in this case they differ only by a factor of $\sim 2$. Notably, HD 149277 is the most evolved binary in the sample.
Table 8.3: ORM parameters. The 2\textsuperscript{nd} column gives the limb darkening coefficients $\mu$, the 3\textsuperscript{rd} and 4\textsuperscript{th} columns the sinusoidal fitting parameters $B_0$ and $B_1$ to the $\langle B_z \rangle$ curves, the 5\textsuperscript{th} column the Preston $r$ parameter, the 6\textsuperscript{th} column the obliquity angle $\beta$, and the final column the surface magnetic dipole strength $B_d$.

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### 8.1. ROTATIONAL AXIS INCLINATIONS

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<table>
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<tr>
<th>HD No.</th>
<th>$\mu$ (kG)</th>
<th>$B_0$ (kG)</th>
<th>$B_1$ (kG)</th>
<th>$r$ (°)</th>
<th>$\beta$ (°)</th>
<th>$B_3$ (kG)</th>
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8.2 Dipole oblique rotator models

The periodic magnetic variability of stars is generally understood in the context of the Oblique Rotator Model (ORM; Stibbs 1950; Schwarzschild 1950), first shown by Borra and Vaughan (1976) to be the model most compatible not just with the circular but also with the linear polarization variations of Ap stars. In its simplest dipolar form, an ORM describes a stellar magnetic field via three parameters: the inclination of the rotation axis $i$ from the line of sight, the angle of obliquity $\beta$ between the magnetic axis and the axis of rotation, and the surface strength of the centred magnetic dipole at the magnetic pole $B_d$. A dipolar model predicts a sinusoidal $\langle B_z \rangle$ variation, and the ORM parameters can hence be inferred from $\langle B_z \rangle$ when phased with the rotational period $P_{\text{rot}}$. As was shown in Section 4.2.4, for the majority of the stars in this sample $\langle B_z \rangle$ is well fit ($\chi^2/\nu \sim 1$) by a sinusoid; of those that are not, only 4 show more than minor deviations from a sinusoid.

As discussed in Sections 4.2.1–4.2.3, the $\langle B_z \rangle$ measurements of Bp stars can be strongly affected by chemical spots. This is accounted for by adopting H line $\langle B_z \rangle$ measurements in those cases when a statistically significant dispersion between $\langle B_z \rangle$ measured using spectral lines of different chemical elements was detected. When no such dispersion was found, the most precise set of measurements, typically metallic or metallic+He, were adopted. The $\langle B_z \rangle$ measurements adopted for analysis are listed in Table 4.1.

In the simplest case of a magnetic dipole, there is an analytical solution for ORM parameters developed by Preston (1967). Beginning with a sinusoidal fit to $\langle B_z \rangle$:

$$B_z = B_0 + B_1 \sin 2\pi(\phi - \phi_0),$$  \hspace{1cm} (8.3)
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the obliquity angle $\beta$ of the magnetic axis from the rotational axis is related to $i$ via

$$\tan \beta = \left( \frac{1 - r}{1 + r} \right) \cot i, \quad (8.4)$$

where $r$ is the ratio (Stibbs, 1950)

$$r = \frac{|B_0| - B_1}{|B_0| + B_1} = \frac{\cos (i + \beta)}{\cos (i - \beta)}, \quad (8.5)$$

where $B_0$ and $B_1$ are the sinusoidal fitting parameters determined via Eqn. 8.3. The values of $r$, $B_0$, and $B_1$ determined from $\langle B_z \rangle$ curves are given in Table 8.3.

As is clear from equations 8.4 and 8.5, there is a degeneracy between $i$ and $\beta$. For the majority of the stars, this degeneracy is broken using the values of $i$ determined in Section 8.1. The middle panel of Fig. 8.7 illustrates this degeneracy in the $i - \beta$ plane for HD 52089, a star for which $P_{\text{rot}}$ is unknown and $i$ cannot be determined. As a sinusoid cannot be fit to $\langle B_z \rangle$, $B_0$ and $B_1$ were respectively estimated from the mean and difference between maximum and mean of the 4 $\langle B_z \rangle$ measurements published by Fossati et al. (2015a), while their uncertainties were both taken to be the standard deviation of $\langle B_z \rangle$, yielding $r = -0.2 \pm 0.6$, i.e. essentially unconstrained. $i$ was forced to follow a random distribution (Eqn. 8.2), which weights $i$ towards large angles, as demonstrated by the probability density function in the bottom right panel of Fig. 8.7. Using a flat distribution or calculating $i$ on the basis of random periods would weight $i$ towards moderate and small angles, respectively. This would in turn push $\beta$ towards moderate or high values, in preference to the low $\beta$ that emerges for the random distribution (bottom middle panel of Fig. 8.7). It is worth noting that these angles overlap within error bars between all three methods.
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Figure 8.7: \( i, \beta, \) and \( B_d \) for HD 52089. Red, blue, and purple show 1, 2, and 3\( \sigma \) contours. Left panels, top–bottom: \( B_d \) relative to \( \beta, i, \) and PDF. Middle panel: the \( i − \beta \) plane. Bottom middle and right: PDFs of \( \beta \) and \( i. \)

The degeneracy can in general be broken by determining \( i \) via the star’s rotational properties, as discussed in Section 8.1. Fig. 8.8 shows the same plot as Fig. 8.7, for the moderate rotator HD 36526 (\( P_{\text{rot}} = 1.5415(5) \) d). HD 36526 was chosen as \( i \) is relatively well constrained, but possesses low probability tails covering most of the region between \( 0^\circ \) and \( 90^\circ \). The distribution for \( \beta \) is essentially the reverse of that for \( i \). Their relationship in the \( i − \beta \) plane is much tighter than is the case for HD 52089, due to the small uncertainty in \( r = -0.245 \pm 0.002. \)

The strength of the magnetic dipole at the stellar surface, \( B_d, \) is
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Figure 8.8: As Fig. 8.7 for HD 36526.

\[
B_d = B_z^{\text{max}} \left( \frac{15 + \mu}{20(3 - \mu)}(\cos \beta \cos i + \sin \beta \sin i) \right)^{-1}, \tag{8.6}
\]

where \( \mu \) is the limb darkening coefficient. \( \mu \) was obtained from the tables calculated by Díaz-Cordovés et al. (1995), interpolating according to \( T_{\text{eff}} \) and \( \log g \), and adopting the Johnson \( B \) band values as being the closest to the spectral range of the ESPaDOnS, Narval, and HARPSpol spectropolarimetry. \( \langle B_z \rangle_{\text{max}} \) was given in Table 4.1. The values of \( B_0, B_1, r, \) and \( \mu \) used to solve equations 8.4-8.6 are given in Table 8.3, together with the values of \( \beta \) and \( B_d \) determined using \( i \) from Table 8.1. In Figs. 8.7 and 8.8 \( B_d \) is shown in the left panels in (top–bottom) the \( \beta - B_d \) plane, the
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The $i - B_d$ plane, and vs. probability. For HD 52089, it is clear from inspection that the assumed distribution of $i$ actually has little impact on $B_d$: so long as $10^\circ \leq i \leq 80^\circ$, $B_d \leq 100 \text{ G}$. Thus, although the period is not known for this star, it is very likely that it has the weakest magnetic field in the sample.

For both HD 52089 and HD 36526, the peak of the $B_d$ PDF is near the minimum possible on the full $i - \beta$ plane, however, there is a $3\sigma$ probability tail extending about 1 dex above the peak. Such a tail is not atypical. There are, however, some cases for which the uncertainties in $B_d$ are essentially gaussian, e.g. ALS 3694, as shown in Fig. 8.9, accounting for the occasionally large differences in upper and lower error bars in Table 8.3.

As explored in Section 4.2.4, 4 stars (HD 37479, HD 37776, HD 149438, and HD 175362) have $\langle B_z \rangle$ curves with significant departures from sinusoidal behaviour, indicative of surface magnetic fields with important contributions from non-dipolar multipoles. In these cases, values of $B_d$ and $\beta$ derived via Preston’s method must be treated with care. The surface magnetic field has been mapped using Zeeman Doppler Imaging (ZDI) for HD 37479 (Oksala et al., 2015b), HD 37776 (Kochukhov et al., 2011), and HD 149438 (Donati et al., 2006). The maximum surface magnetic fields determined via ZDI for these stars are $\sim 9.5 \text{ kG}$, $\sim 30 \text{ kG}$, and $\sim 0.5 \text{ kG}$, respectively. The ZDI-derived field strengths for HD 37479 and HD 149438 compare reasonably with the $B_d$ values found via Preston’s method, $9.7^{+1.0}_{-0.3} \text{ kG}$ and $0.31^{+0.02}_{-0.01} \text{ kG}$, however there is a sharp disagreement for HD 37776, for which Preston’s method yields $5.7^{+0.5}_{-0.5} \text{ kG}$. This is not surprising as HD 37776 has a very complex surface topology, with numerous poles of approximately equal strength. Conversely, departures from dipolarity are much less marked for HD 37479, while HD 149438 has a
similarly complex topology to HD 37776, but is dominated by a particularly strong positive pole. As the primary aim of this section is to determine the surface dipolar magnetic field strengths from which to evaluate magnetic confinement in the circumstellar environment, the low value of $B_d$ found for HD 37776, while not reflective of the true surface magnetic field strength, nevertheless likely gives a more or less accurate idea of the physical extent of its magnetosphere as the Alfvén radius is dominated by the dipolar component at large distances (ud-Doula and Owocki, 2002).

No ZDI map has been made available for HD 175362. In this case, $\langle B_z \rangle$ is approximately sinusoidal. The reasonable agreement for HD 37479 and HD 149438 between
magnetic field strengths determined from ZDI and via Preston’s method gives some confidence that the value of $B_d$ so derived is unlikely to differ greatly from the value that would be inferred from the a ZDI map. ZDI analysis of HD 184927 (Yakunin et al., 2015), which has an essentially dipolar surface magnetic field with only small quadrupolar contributions, found a surface magnetic field strength of $7.5\pm1.5$ kG, in good agreement with the value determined here, $7.9\pm1.7$ kG.
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8.2.1 Distribution of $\beta$

With the caveat that there is a slight bias towards smaller angles than expected, $i$ follows an essentially random distribution (Section 8.1.3, Fig. 8.5). It is not clear that this is also true for $\beta$. The cumulative distribution of $\beta$ shown in Fig. 8.10 suggests an excess of small ($\leq 20^\circ$) and large ($\geq 60^\circ$) $\beta$. Comparing to the same studies to which the cumulative distribution of $i$ was compared it is apparent that the A07, P07, and H07 distributions all show a similar pattern. It should be noted that in all four cases, the one-sample K-S test yields a higher significance level than is achieved for $i$: 0.41 for the B-type stars, 0.12 for the A07 sample, 0.12 for the P07 sample, and $3 \times 10^{-4}$ for the H07 sample. The LM00 distribution is clearly not random (the K-S significance is $2 \times 10^{-7}$, the only sample achieving a smaller significance level for $\beta$ than for $i$) but is different from the rest, with a very strong excess below about $\beta \leq 30^\circ$. The LM00 distribution is, however, composed largely of very slowly rotating Ap stars, containing more stars with long $P_{\text{rot}}$ ($\sim$years) than the remaining samples combined, and a principle conclusion of this study is that very slowly rotating stars tend to have aligned magnetic and rotational axes. Comparing the previous studies to the B-type stars with the two-sample K-S test yields significances of 0.44 for the A07 sample, 0.67 for the P07 sample, 0.06 for the H07 sample, and $10^{-4}$ for the LM00 sample, i.e. only the LM00 sample is clearly drawn from a different population.

With the methodology employed here, $\beta$ is highly sensitive to $i$, thus systematic errors in $i$ will also bias $\beta$. Another way to investigate the question of whether or not $\beta$ is random is to examine $r$ directly. While there is a degeneracy between $i$ and $\beta$ for any given value of $r$, the advantage is that $r$ is a directly measured quantity. Fig. 8.11 shows the cumulative distribution of $r$. We computed two expected distributions
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Figure 8.11: Cumulative distribution for \( r \) (solid black line), as compared to the expected distributions assuming random \( i \) and either random \( \beta \) (dashed red line) or flat \( \beta \) (dot-dashed blue line).

by generating 1000 synthetic samples consisting of 50 observations each, in which \( i \) and \( \beta \) were randomly paired. In both cases \( i \) was taken from a random distribution, as per equation 8.2. In the first distribution, \( \beta \) was taken from the same random distribution, i.e. over a solid angle of \( 4\pi \). In the second the distribution of \( \beta \) is flat, i.e. random in 1 dimension. This is the same assumption made by Petit and Wade (2012) in their Bayesian modelling of LSD profiles. While neither assumption fits the observed distribution perfectly, a random flat \( \beta \) distribution matches \( r \) much better.
than a random spherical $\beta$ distribution.

We conclude that the distribution of $\beta$ is not random, and that the overabundance of small $\beta$ angles is likely to be robust against more accurate determination of $i$. Indeed, based upon our examination of the likely effects of systematic biasing of $i$ towards lower values, the overabundance of small $\beta$ angles may well be more pronounced in the population than our results suggest.

### 8.2.2 Distribution of $B_d$

As $B_d$ ranges over 3 decades of strength, it is natural to plot the distribution of the population’s $B_d$ values in logarithmic units, as shown in Fig. 8.12, together with the distributions of of the combined sample of Ap/Bp stars from previous studies (LM00, A07, and P07), and the combined distribution for all 4 studies. Comparing the B-type stars to the Ap/Bp stars with a two-sample K-S test (top panel of Fig. 8.12) yields a significance of 0.39, suggesting that they are drawn from the same distribution. The histograms (bottom panel of Fig. 8.12) are binned using the Freedman-Diaconis rule, and gaussian fits performed using the IDL GAUSSFIT function. The appropriateness of log-normal fits was evaluated via $\chi^2/\nu$, with uncertainties combining the histogram error bars determined from the $1\sigma$ uncertainties in $B_d$ with the fitting errors returned by GAUSSFIT. For the B-type stars $\chi^2/\nu < 3$, and for the LM00 sample $\sim 6$, but $\sim 1$ for the A07, P07, and combined samples. Thus, a log-normal distribution is a reasonable fit for the B-stars and a good fit for all other samples save that of LM00, which is biased towards stars with strong magnetic fields.

The absence of magnetic fields above about 30 kG is likely to be real, as stronger magnetic fields are much easier to detect. The distribution in Fig. 8.12 could be
affected by a bias related to the ease of detection of weak magnetic fields, since magnetic fields become more difficult to detect when they are weaker. This is particularly the case for rapidly rotating stars, as the large line widths spread Stokes $V$ over a larger number of pixels, bringing the amplitude of a magnetic signature closer to the noise level. However, in their analysis of the MiMeS Survey, Wade et al. (2016) found that the median sensitivity of the survey was around $B_d = 300$ G. This is about an order of magnitude below the median value of $B_d$ found here for the magnetic B-type
Figure 8.13: Cumulative distribution (top) and histograms (bottom) of $B_d$ amongst fast ($P_{\text{rot}} \leq 2$ d, solid blue) and slow ($P_{\text{rot}} \geq 2$ d, dashed red) rotators.

stars, suggesting that, if a significant number of massive stars have $B_d$ on the order of 100 G, they should have been detected by the MiMeS Survey.

The standard deviation of the B-type stars is substantially higher ($\log (B_d/kG) = 0.72 \pm 0.15$) than in the other three samples ($\log (B_d/kG) = 0.2$ to 0.4). Again, this is to be expected given that these other studies concentrated on more specific sub-populations of stars than this work. The standard deviation of the combined sample is between these ranges, $\log (B_d/kG) = 0.54 \pm 0.06$, compatible with that inferred from the early B-stars.
Dividing the sample into slow vs. fast rotators, with the division at $P_{\text{rot}} = 2$ d (yielding close to the same number of stars in each bin, 24 slow rotators and 25 fast rotators), yields the interesting result that the slow rotators have systematically lower $B_d$ than the fast rotators (Fig. 8.13). The two-sample K-S test significance is 0.013, suggesting the difference between the two sub-samples is real. This might seem to be contrary to expectation, as more strongly magnetized stars should lose angular momentum more rapidly than stars with weaker magnetic fields.

The answer to this apparent anomaly may lie in the long-term evolution of the star and its magnetic field. As fossil fields are not renewed by convective dynamos, they should weaken over time as a star expands, with the surface magnetic field declining as $\sim R_*^{-2}$ due to conservation of magnetic flux. While such an effect is not expected to be detectable in individual stars, over a large enough sample it would be expected that $B_d$ should be systematically lower amongst older stars. $B_d$ is shown as a function of $\tau_{\text{MS}}$ in the top panel of Fig. 8.14. A linear regression to log $B_d$ as a function of $\tau_{\text{MS}}$ yields a slope of $-1.36 \pm 0.12$, where the uncertainty was determined from the standard deviation of all slopes obtained by iteratively removing one of the stars from the regression.

The second panel of Fig. 8.14 shows the ZAMS $B_d$, i.e. the expected value of $B_d$ at $R_* = R_{\text{ZAMS}}$ assuming $B_d \propto R_*^2$. A linear regression to this data yield a slope of $-0.83 \pm 0.12$, shallower than that obtained for $B_d$ but still apparently significant at $\sim 7\sigma$. This could be interpreted as evidence that magnetic flux declines with time. However, it is possible that the ZAMS radius is poorly estimated, which could lead to an over-correction for the older stars. An alternate means of exploring this is to examine the total unsigned magnetic flux, $B_d R_*^2$, shown as a function of $\tau_{\text{MS}}$, shown
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Figure 8.14: $B_d$ (top), $B_d$ normalized to $R_{ZAMS}$ (middle), and the total unsigned magnetic flux (bottom) as functions of $\tau_{MS}$. Colour is mapped to mass, and demonstrates no correlation. Solid lines are linear regressions of the three quantities with $\tau_{MS}$. 
in the bottom panel of Fig. 8.14. In this case, the slope of the linear regression is 
$-0.68 \pm 0.06$, which again is consistent with flux decay.

Two stars of apparently advanced age but with strong magnetic fields stand out against the trend of declining surface magnetic field strengths and, possibly, magnetic flux with age: HD 149277 ($\tau_{\text{MS}} = 0.85, B_d = 10$ kG) and HD 58260 ($\tau_{\text{MS}} = 1.0, B_d > 6$ kG). HD 58260’s age is inferred from its low surface gravity, $\log g = 3.43 \pm 0.10$. As $P_{\text{rot}}$ could not be determined for this star, only a lower limit can be determined for $B_d$. Its age could turn out to be incorrect if its surface gravity is substantially higher than determined here, due to e.g. He abundance anomalies that were not accounted for in the analysis of the star’s physical parameters. For HD 149277, $P_{\text{rot}}$ is very well-constrained. The age inferred from its surface gravity and effective temperature, $24 \pm 2$ Myr, is in agreement with the cluster age, $14^{+9}_{-5}$ Myr. While a younger fractional age could be consistent with the cluster age if the star is slightly less massive ($\sim 8M_\odot$) than determined above ($9.3 \pm 0.5M_\odot$), this would require $R_* < 5R_\odot$, which is below the lower limit established by $v\sin i$ and $P_{\text{rot}}$.

To explore this question further, the sample was divided into two bins, with $\tau_{\text{MS}} < 0.5$ and $\tau_{\text{MS}} > 0.5$. Only stars that are definitely in one or the other bin were included, i.e. stars which could be in either bin given the uncertainty in $\tau_{\text{MS}}$ were excluded. Fig. 8.15 compares, from left to right, the cumulative distributions and histograms of $B_d$, $B_d(R_*/R_{\text{ZAMS}})^2$, and $B_d R_*^2$ of these two sub-samples. There is a shift towards lower $B_d$ in the older stars, and two-sample K-S test gives a significance of 0.08, as compared to 0.35 for the ZAMS-normalized $B_d$ and 0.66 for the unsigned magnetic flux. These results are compatible with magnetic flux being conserved during the stars’ evolution.
8.3 Summary

In this chapter the derived physical parameters from Chapter 7 were combined with the \( \langle B_z \rangle \) measurements presented in Chapter 4 and the projected rotational velocities and rotational periods from Chapter 5 to derive ORM parameters for the sample stars.

Inclinations \( i \) were determined from stellar parameters and rotational periods, in some cases with additional constraints from emission properties or asteroseismology. Inclinations are generally consistent when derived using radii found from the \( T_{\text{eff}} \)-log \( L \) or \( T_{\text{eff}} \)-log \( g \) diagrams. The cumulative distribution of \( i \) exhibits a small, systematic bias towards smaller angles than expected for a random distribution. A similar bias
is apparent in the cumulative distributions of $i$ published by essentially all previous studies of Ap stars, suggesting that radii may be systematically over-estimated. Comparing orbital and rotational inclinations for close binary systems yields evidence for spin-orbit alignment.

In contrast to $i$, $\beta$ does not appear to be randomly distributed, with an over-abundance of small and large angles as compared to a random distribution. This confirms the results of most previous studies of Ap stars, with the only notable difference being with the slowly-rotating sample presented by Landstreet and Mathys (2000b), which is strongly biased towards small $\beta$. As only 2 or 3 stars in the present sample have $P_{rot}$ in the range of the sample presented by Landstreet and Mathys, a direct comparison between these samples cannot be made.

The surface dipole magnetic field strength $B_d$ is essentially log-normal, with a similar distribution to the range of values seen in cooler Ap stars. There is some evidence for a systematic weakening of $B_d$ which age, consistent with flux conservation of the frozen-in fossil field during expansion of the star. Evidence for a change in flux with age is ambiguous, as the data could interpreted as showing either no increase, or a weak decrease in flux over time. This result is more or less compatible with that found for field Ap stars, which detected a possible increase in flux over time (Kochukhov and Bagnulo, 2006). There is no evidence for the strong decrease in magnetic flux seen in cluster Ap stars (Landstreet et al., 2008). This ambiguity could be due to either incorrect stellar parameters or, potentially, models which make incorrect assumptions. Unfortunately, there is no homogeneous library of stellar evolutionary models which accounts for the simultaneous influences of rotation and magnetic fields on stellar evolution in a consistent way. Thus, the models assuming moderate rotation and no
magnetic field used here (Ekström et al., 2012) do not account for the potentially important role of rotation amongst the younger stars with short rotation periods, whereas they are simultaneously potentially a poor fit for older stars with essentially no rotation. Another important limitation of this study is that many of the stars in the current sample are field stars, and the fractional main-sequence ages of the youngest field stars cannot be determined with the same precision as is possible with cluster stars. On the other hand, previous studies of the evolution of fossil magnetic fields utilized the root-mean-square longitudinal magnetic field as a proxy to $B_d$, as ORM models were unavailable for the majority of stars included in their samples. This result highlights the importance of determining ORM models for cluster stars with fossil magnetic fields.

Now that we have determined $v_{eq}$ and $B_d$ for the sample stars, in the following chapter we can finally turn to the consequences of these properties for the circumstellar environments of the sample stars.
Chapter 9

Magnetospheric Parameters and Spindown

In the Introduction, three primary objectives for this thesis were enumerated: 1) to place the sample stars on the rotation-magnetic confinement diagram; 2) to compare evolutionary and spindown timecales; 3) to investigate the Hα emission properties of the sample stars in the context of their magnetospheric properties. In this chapter, the results of the previous chapters are combined to address the first two of the goals.

In Chapter 7, stellar radii, masses, and ages were derived from the fundamental parameters presented in Chapter 6. The derived parameters were obtained by populating the $T_{\text{eff}}$-$\log L$ and $T_{\text{eff}}$-$\log g$ diagrams with a Monte Carlo grid, and then interpolating between evolutionary models. In Chapter 8, the Monte Carlo grids were pruned by discarding those points yielding unphysical rotational velocities (i.e. $v_{\text{eq}} < v \sin i < v_{\text{br}}$), and by taking into account constraints on $i$ from emission, asteroseismology, and Doppler Imaging.

In this chapter, magnetospheric and rotational parameters are derived on the same Monte Carlo grids, with the aim of revising the rotation-magnetic confinement diagram first presented by P13. Spindown timescales are then compared to the stellar
ages from Chapter 7. Throughout the chapter, comparisons are made between calculations performed using the mass loss prescription adopted by P13 (Vink et al., 2001), and the new Krtička (2014) mass loss rates.

9.1 Rotation parameters and Kepler radii

In the context of the circumstellar environment, the most physically meaningful parameterization of the stellar rotation is in terms of the rotation parameter $W$, defined as (ud-Doula et al., 2008)

$$W \equiv \frac{v_{\text{eq}}}{v_{\text{orb}}}$$

(9.1)

where $v_{\text{orb}} = \sqrt{\frac{GM}{R_{\text{eq}}}}$ is the circular orbital velocity at the surface, i.e. the velocity that must be attained by material orbiting at the surface in order to maintain a circular Keplerian orbit. Assuming that the magnetic field enforces rigid body corotation of the plasma confined within the magnetosphere, the azimuthal velocity increases as $v_\phi = v_{\text{eq}} r / R_{\text{eq}}$. At a certain point, designated the Kepler radius $R_K$, the centrifugal force balances the gravitational force:

$$\frac{R_K}{R_{\text{eq}}} = \left( \frac{GM}{\omega^2 R_{\text{eq}}^3} \right)^{1/3} = W^{-2/3},$$

(9.2)

where $\omega$ is the angular frequency of rotation.
9.2 Wind magnetic confinement parameters and Alfvén radii

If the energy density of the magnetic field in the circumstellar environment is greater than the kinetic energy density of the stellar wind, the wind will tend to be magnetically confined (ud-Doula and Owocki, 2002). To quantify this, we start with the variation of a magnetic dipole $B_d$ with the colatitude $\theta$, given by

$$B^2(\theta) = B_d^2 \left( \cos^2 \theta + \frac{\sin^2 \theta}{4} \right), \quad (9.3)$$

while the radial variation of the magnetic field with distance from the stellar surface $r$ is given by

$$B(r) = B_d \left( \frac{R_*}{r} \right)^3. \quad (9.4)$$

Assuming spherical symmetry, the mass-loss rate $\dot{M}$ is

$$\dot{M} = 4\pi r^2 \rho v, \quad (9.5)$$

where $r$ is the distance from the stellar surface, $\rho$ is the mass density of the wind, and $v$ is the wind velocity. The radial variation of the wind velocity is given by the standard velocity law

$$v(r) = v_\infty \left( 1 - \frac{R_*}{r} \right), \quad (9.6)$$

where $v_\infty$ is the terminal velocity of the wind. With the energy density of the magnetic field $B^2/8\pi$ and the kinetic energy density of the wind $\rho v^2/2$, their ratio at a given colatitude and distance from the star is given by
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\[ \eta(r, \theta) \equiv \frac{2B^2}{8\pi\rho v^2} = \frac{B^2 r^2}{Mv}. \] (9.7)

Rearranging Eqn. 9.7 to isolate the spatial and angular variations in \( \eta \) and making use of Eqns. 9.3-9.6, we then have

\[ \eta(r, \theta) = \left[ \frac{B_d R_e^2}{M v_\infty} \right] \left[ (r/R_\ast)^{-4} \right], \] (9.8)

The left square bracket in Eqn. 9.8 gives a dimensionless constant characterizing the energy density balance between the magnetic field and the wind. Evaluating the left bracket of Eqn. 9.8 at the magnetic equator, where the radial wind outflow is most directly opposed by the magnetic field, yields the equatorial wind magnetic confinement parameter \( \eta_e \):

\[ \eta_e \equiv \frac{B_{eq} R_e^2}{M_{B=0} v_\infty}. \] (9.9)

\( B_{eq} \) is the magnetic field strength at the magnetic equator. \( M_{B=0} \) is the mass-loss rate in the absence of a magnetic field, as magnetic confinement will tend to reduce the net mass-loss rate, but will leave the amount of material entering the wind at the base of the star unchanged. If \( \eta_e > 1 \), the wind is considered to be magnetically confined.

While it is in general possible to measure \( \dot{M} \) and \( v_\infty \) directly from the P Cygni profiles in UV resonance lines, magnetic B stars do not in general show P Cygni profiles as the formation regions of these lines are not entirely within expanding spherical winds, but largely within magnetospheres. This makes empirical measurements of wind parameters difficult to obtain (Sundqvist et al., 2012; Grunhut et al., 2012c).
We therefore use theoretical wind parameters, calculated either using the recipe developed by Vink et al. (2001), or by interpolating through the tables calculated by Krtička (2014) with Bp stars explicitly in mind. Vink mass-loss rates were calculated using the IDL program cal_tot.pro, which takes as input the stellar metallicity $Z$, $T_{\text{eff}}$, log $L$, $M_*$, and optionally $v_\infty$. For simplicity, and as chemical abundances are available for only a few stars, $Z$ was taken as unity (i.e., solar metallicity) for all stars.

$v_\infty$ was left as a free parameter, in which case cal_tot.pro calculates it from the escape velocity $v_{\text{esc}}$:

$$v_\infty = f v_{\text{esc}} = f \left( \frac{2GM_*(1 - \Gamma_e)}{R_*} \right)^{1/2}$$

(9.10)

where $\Gamma_e \equiv \kappa_e L/4\pi GM_*c$ is the Eddington parameter for electron opacity $\kappa_e$ and speed of light $c$, and $f$ is a scaling factor that depends on $T_{\text{eff}}$. There is an abrupt decline from $f = 1.3$ to $f = 2.6$ from the cool to the hot side of the bi-stability jump at $T_{\text{eff}} \sim 25$ kK. The Vink recipe also predicts a decrease of approximately 1 dex in $\dot{M}$ at the bi-stability jump, as a consequence of an increase in the line force due to recombination of iron lines. While investigation of the UV P Cygni profiles of non-magnetic OB stars has established good evidence that $v_\infty$ does indeed change abruptly at the bistability jump (e.g., Lamers et al. 1995), the bi-stability jump in $\dot{M}$ is not without controversy (e.g., Markova and Puls 2008).

Krtička (2014) calculated $\dot{M}$ and $v_\infty$ for main-sequence B-type stars using the NLTE wind models developed by Krtička and Kubát (2010), which utilize a co-moving frame line force that self-consistently predicts wind parameters from stellar physical parameters. Krtička tabulated their predicted wind parameters for models with $T_{\text{eff}}$ between 14 and 30 kK, in 2 kK increments. We determined $\dot{M}$ and $v_\infty$ for
individual stars via linear interpolation according to $T_{\text{eff}}$ within Table 2 in Krtička (2014), scaling $\dot{M}$ as $L^2$ as recommended by Krtička and Kubát (2012).

The Krtička mass-loss rates are generally lower than those predicted from the Vink recipe, leading overall to higher values of $\eta_*$. They are also extremely $T_{\text{eff}}$ sensitive, with rapidly declining $\dot{M}$ and $v_\infty$ with decreasing $T_{\text{eff}}$. Krtička (2014) also provided predictions for specific chemical abundance peculiarities, a key finding being that, in the absence of enhanced Si, no expanding wind can be launched from the photosphere below 15 kK. As chemical abundances are not widely available for these stars, solar abundance tables were used. Some of the cooler stars in the sample have uncertainties in $T_{\text{eff}}$ that extend below 15 kK: at these temperatures, a star has no magnetosphere because there is no wind. Grid values in this range were accordingly discarded in order to avoid divide-by-zero errors in Eqn. 9.9.

Using either the Vink or the Krtička mass-loss rates, we find that $\eta_* \geq 1$ for all stars in the sample, and ranges up to $\sim 10^7$ for cooler stars with especially strong magnetic fields. Thus, all the stars are predicted to have magnetically confined winds.

The physical extent of magnetic confinement is given by the Alfvén radius $R_A$, which is obtained from $\eta_*$ as (ud-Doula and Owocki, 2002; ud-Doula et al., 2008)

$$\frac{R_A}{R_*} \sim 0.3 + (\eta_* + 0.25)^{1/4}. \quad (9.11)$$

The bottom left panel of Fig. 9.1 shows ALS 3694 in the $R_A$–$R_K$ plane (equivalently, the $\eta_* – W$ plane, indicated by secondary axes). This is a rapidly rotating ($P_{\text{rot}} = 1.6779(4)$ d), hot ($T_{\text{eff}} = 22 \pm 1$ kK) star with a strong magnetic field. Since $R_A > R_K$ (solid line), the star has a CM. The star’s magnetospheric parameters show up as two distinct clusters, with the second, less-probable cluster entirely above the
9.2. WIND MAGNETIC CONFINEMENT PARAMETERS AND ALFVÉN RADII

Figure 9.1: Bottom left: ALS 3694’s position in the $R_A$-$R_K$ plane. Top left: PDF for $R_A$. Bottom right: PDF for $R_K$. Top right PDF for log ($R_A/R_K$). While the star’s magnetic parameters are fairly well-defined (Fig. 8.9), the bistability jump the stellar wind leads to additional uncertainty in $R_A$.

$R_A = 10R_K$ threshold (dashed line), suggested by P13 as a typical value for B-type stars with optical emission lines, which ALS 3694 indeed possesses (Shultz et al., 2014). The two clusters are a consequence of the star’s physical parameters straddling one of the bistability jumps at which the Vink recipe predicts an abrupt transition in $\dot{M}$ and $v_\infty$.

The bistability jump is less pronounced in the Krtička simulations, such that calculating $\eta_*$ using the Krtička tables yields a single asymmetric cluster almost entirely...
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Figure 9.2: As Fig. 9.1, using Krtička mass-loss rates.

above $R_A = 10R_K$ (Fig. 9.2). The sense of $R_A$’s variation with $R_K$ is also different: using Vink mass loss, there is a slight decreasing trend of $R_A$ with diminishing $R_K$, whereas using Krtička mass-loss, $R_A$ increases with decreasing $R_K$.

9.2.1 Alfvén radii with different mass-loss prescriptions

Fig. 9.3 compares the $\dot{M}$ and $R_A$ determined using Vink and Krtička mass loss prescriptions. The bistability jump near 22 kK is much more pronounced using the Vink recipe. $\dot{M}$ is generally systematically lower using the Krtička tables, with differences up to 2 dex seen in the cooler stars, while for hotter stars, $\dot{M}$ is quite similar. This
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Figure 9.3: Bottom left: $\dot{M}$ calculated with Krtička vs. Vink prescriptions. The former are generally lower, and the difference increases for cooler stars (legend in top right). Top left: $R_A$ determined from Vink mass loss vs. Vink $\dot{M}$. Bottom right: Krtička $\dot{M}$ vs. $R_A$ determined from Krtička mass loss. Top right: $R_A$ from Vink vs. $R_A$ from Krtička. Differences are negligible for hotter stars, and remain within less than a dex even for the coolest stars.

results in generally larger values of $R_A$ with Krtička mass loss, up to a factor of about 4 for the cooler stars, which is somewhat larger than the systematic differences that exist between Vink and Castor et al. (1975) (CAK) rates explored by P13. Thus, the uncertainty in a star’s position on the rotation-confinement diagram is dominated by systematic uncertainties in the mass-loss rate. However, the primary effect of adopting Krtička mass loss rates is simply to shift the cooler stars towards larger $R_A$, thus
9.2. WIND MAGNETIC CONFINEMENT PARAMETERS AND ALFVÉN RADIi

spreading the stars out: the overall pattern should be unchanged.

The top rows of Table 9.1 provides wind, magnetospheric, and rotational parameters for all stars in the sample, calculated using the Vink mass loss recipe.
Table 9.1: Magnetospheric parameters calculated using Vink mass-loss rates. Data for each star is provided in two rows. The first row provides data on the physical extent of the stellar magnetosphere, the second row information on the star’s angular momentum loss.

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## 9.2. WIND MAGNETIC CONFINEMENT PARAMETERS AND ALFVEN RADII

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9.3. THE UPDATED ROTATION-CONFINEMENT DIAGRAM

A primary motivation for using correlated values of $M_\ast$, $R_\ast$, $\dot{M}$, and $B_d$ was to determine the magnetospheric and rotational parameters of the population as accurately and precisely as possible. Fig. 9.4 compares the fractional errors in $R_A$ and $R_K$ achieved here to those obtained by P13 using error propagation, where for the latter values we show only those for the magnetic B-type stars. Our uncertainties are typically a factor of $\sim2$ smaller than previous results.

Changes in the stars’ positions on the rotation-confinement diagram are shown in Fig. 9.5. Stars with new periods are marked in red. The blue crossed out square indicates HD 35912, which was not detected as magnetic (Fig. 4.6) and therefore discarded. The values of $R_A$ and $R_K$ found here are indicated by filled symbols, old values by open symbols, with lines connecting the two; filled symbols unconnected to open symbols represent magnetic discoveries made after compilation of the P13 catalogue. $R_A$ was calculated using Vink
9.3. THE UPDATED ROTATION-CONFINEMENT DIAGRAM

Figure 9.5: Comparison between the original P13 rotation-confinement diagram (open symbols) and the updated rotation-confinement diagram (filled symbols). The cross indicates HD 35912, in which a magnetic field was not confirmed with ESPaDOnS data. Note the reduced frequency of upper and lower limits on $R_K$ and $R_A$ in the newer data.
mass-loss, as this was used by P13. The largest movements across the rotation confinement
diagram are for stars with new periods, indicated in red. In the majority of these cases, $R_A$ and $R_K$ are respectively above and below the previous lower and upper limits. In some cases the upper limit of $R_K$ derived from $v \sin i$ is higher than the value found with a rotation period. These stars are all slow rotators with low $v \sin i$ values, comparable to or below $v_{\text{mac}}$, a regime in which it becomes difficult to distinguish $v \sin i$ from other line broadening mechanisms such as turbulence, pulsation, and Zeeman splitting. This is also the case for HD 46328, which has made the largest move across the diagram: in this case a previously reported short period was incompatible with the ESPaDOnS data, which favours a period on the order of decades.

The new rotation-confinement diagram is shown in H$\alpha$ in the left panel of Fig. 9.6. In comparison with the previous parameters, there are fewer limiting values on $R_A$ and $R_K$. Furthermore, with the exception of HD 46328, emission-line stars are now concentrated entirely in the top right of the diagram, and absorption-line stars are largely removed from that region. Those that remain are relatively cool stars with very strong magnetic fields (HD 36526, HD 175362, HD 35298), with much weaker and potentially no stellar winds. The absence of emission in these stars may simply be a consequence of their low temperatures. Eclipses from a corotating magnetosphere have been detected around the B8 IIIp star 36 Lyn, although there is no sign of emission at quadrature phases (Smith et al., 2006).

HD 46328 is the only emission line star in the DM regime. Its emission characteristics more closely resemble those of DM than CM stars, rendering it not just the most slowly rotating star in the sample, but also the coolest star with a detectable DM (Shultz et al., in prep.). With the stars showing CM emission now clustered in the upper right of the diagram, much of the ambiguity about the conditions under which CMs become H$\alpha$ bright is removed.
Figure 9.6: The new rotation-confinement diagram in Hα, showing only the magnetic B-type stars. The solid and dashed lines correspond to $R_A = R_K$ (dividing CMs from DMs) and $R_A = 8R_K$, while the dotted line indicates $3R_K$. The latter two lines appear to form a boundary for Hα bright CMs: in contrast to the previous rotation-confinement diagram, all emission-line stars with the exception of HD 46328 are consolidated in the upper right, while the only absorption-line stars sharing this region of the diagram are quite cool.
9.4 Spindown timescales

The solid body rotation of stellar magnetospheres leads to extended moment arms, leading to rapid angular momentum loss (Weber and Davis, 1967; ud-Doula et al., 2009). Even in the absence of a magnetic field, there is a small angular momentum loss $\dot{J}_w$ due to the stellar wind:

$$\dot{J}_w \approx \frac{2}{3} \dot{M} \omega R_{\text{eq}}^2.$$  \hspace{1cm} (9.12)

The star’s angular momentum $J$ is

$$J = f M_* R_{\text{eq}}^2 \omega = I \omega,$$ \hspace{1cm} (9.13)

where $f = r_{\text{gyr}}^2$ is a parameter evaluated from the radius of gyration $r_{\text{gyr}}$, which we take to be $f = 0.1$ for simplicity (Claret, 2004; Petit et al., 2013). In the second equality, $I$ is the moment of inertia. The angular momentum loss timescale $\tau_{Jw}$ is then

$$\tau_{Jw} = \frac{J}{\dot{J}_w} = \frac{3}{2} \tau_M$$ \hspace{1cm} (9.14)

where $\tau_M \equiv M_*/\dot{M}$ is the mass-loss timescale. Extending the moment arm via the magnetic field increases the angular momentum loss rate by $R_A^2$:

$$\dot{J} = \dot{J}_w R_A^2,$$ \hspace{1cm} (9.15)

such that the angular momentum loss timescale becomes

$$\tau_J = \frac{J}{\dot{J}} = \tau_{Jw} \left( \frac{R_{\text{eq}}}{R_A} \right)^2.$$ \hspace{1cm} (9.16)

Making the assumption that all relevant parameters ($\dot{M}$, $r_{\text{gyr}}$, $R_*$) are fixed, the star’s
period $P$ will then increase exponentially from its initial period $P_0$:

$$\begin{equation}
P = P_0 e^{t/\tau_J}
\end{equation}$$

(9.17)

Since $W = P_{\text{crit}}/P$, where $P_{\text{crit}}$ is the critical rotation period, from an initial rotation parameter $W_0$ the spindown age, defined as the time necessary for the star’s rotation to slow from $W_0$ to the present-day value of $W$, is then

$$t_S = \tau_J \ln \left( \frac{W_0}{W} \right),$$

(9.18)

yielding a maximum spindown age $t_{S,\text{max}}$ if $W_0 = 1$, i.e. initially critical rotation is assumed. $t_{S,\text{max}}$ can be directly compared to the stellar age inferred from isochrones $t_{\text{evol}}$. Figs. 9.7–9.8 show this comparison for 2 stars, HD 61556 (a B3V star with $P_{\text{rot}} = 1.90871(7)$ d and $B_d = 2.7$ kG) and HD 205021 (a B1 IV $\beta$ Cep star with $P_{\text{rot}} = 12.00075(1)$ d and $B_d = 0.27$ kG).

For HD 61556, $t_{S,\text{max}}$ is compatible with $t_{\text{evol}}$, although the shape of the PDF on the $t_{S,\text{max}}$-$t_{\text{evol}}$ plane is essentially perpendicular to the $t_{S,\text{max}} = t_{\text{evol}}$ line, i.e. there is an approximately inverse relationship between $t_{S,\text{max}}$ and $t_{\text{evol}}$. This shape on the $t_{S,\text{max}}$-$t_{\text{evol}}$ plane is fairly typical of many of the sample stars. However, $\log \left( t_{\text{evol}}/t_{S,\text{max}} \right) \approx 0$ for this star, indicating the two timescales are basically in agreement.

In contrast, for HD 205021 $t_{S,\text{max}} \gg t_{\text{evol}}$: $198 \pm 14$ Myr, as compared to $13.2 \pm 0.6$ Myr. This substantial disagreement in magnetic braking and evolutionary timescales, on the order of 1 dex, is typical of the more massive stars in the sample: $\log \left( t_{\text{evol}}/t_{S,\text{max}} \right) > 1$ for HD 149438, HD 63425, HD 66665, and HD 44743, all of which have reasonably well-defined periods.

The bottom rows of Table 9.1 gives $\tau_J$, $t_{S,\text{max}}$, and the logarithmic ratio $\log \left( t_{S,\text{max}}/t \right)$ for individual stars, where $t$ is the stellar age from Table 7.1.
9.4. SPINDOWN TIMESCALES

9.4.1 Comparing evolutionary and spindown ages

Since magnetic fields are expected to spin down the star, rotation periods should increase with age, mass-loss rate, and the Alfvén radius. Fig. 9.9 demonstrates that this is qualitatively the case, where $M_*$ is used as a proxy for $\dot{M}$ since the $\dot{M}$ increases with $M_*$, but $M_*$ is a more certain quantity. Regardless of mass, stars with $\tau_{\text{MS}} < 0.2$ have $P_{\text{rot}}$ of a few days or less, while longer rotation periods are seen only in older stars. Furthermore, for a given range in ages, the stars with the longest periods tend to be those with the highest masses (which have the strongest winds) or the strongest surface magnetic fields (hence
larger magnetospheres). HD 66522, with $P_{\text{rot}} = 909$ d, is the oldest star in the sample with a known rotation period. HD 46328, which appears to have $P_{\text{rot}}$ of decades, is both the oldest star and the most strongly magnetized star in its mass bin.

A more direct comparison between stellar ages inferred from evolutionary tracks and maximum spindown ages is shown in Fig. 9.10. The distributions are centred on approximately the same age, $\sim 20$ Myr, but the distribution of $t_{S,\text{max}}$ is much wider than that in $t_{\text{evol}}$, leading to disagreements ranging up to 2 dex. The differences are most pronounced for more massive stars: stars with stronger magnetic fields have $t_{S,\text{max}}$ much less than $t_{\text{evol}}$, while those with weaker $B_d$ have $t_{S,\text{max}}$ much greater than $t_{\text{evol}}$. This is especially the case for the more massive stars in the sample.
Figure 9.9: Fractional main-sequence age as a function of rotation period. Symbol size is proportional to $B_d$, colour to mass. Younger stars tend to have much shorter periods. The distribution of $P_{\text{rot}}$ spreads out with age, with more massive and/or more strongly magnetized stars tending to rotate more slowly than coeval stars with lower masses and/or weaker magnetic dipoles.

The rapidly rotating outliers are all cool stars, with much weaker winds inferred using the Krtička mass-loss rates. Spindown times calculated using these rates are shown in Fig. 9.11. $t_{S_{\text{max}}}$ is systematically shifted towards higher ages, which resolves much of the discrepancy amongst the strongly magnetized stars with short spindown times. However, this increases the disagreement amongst the slow rotators. Note that nearly all of these stars are weakly magnetized ($B_d \leq 0.3$ kG), massive, and in many cases, such as HD 163472, τ Sco, and β Cep, have well-defined rotational periods ranging from days to weeks established.
9.4. SPINDOWN TIMESCALES

Figure 9.10: log $t_{\text{evol}}$ as a function of log $t_{S,\text{max}}$ for all stars, using Vink mass-loss rates. The dashed line indicates $t_{\text{evol}} = t_{S,\text{max}}$: note the much higher range in log $t_{S,\text{max}}$ as compared to log $t_{\text{evol}}$. Arrows indicate stars for which the age is formally an upper limit, i.e. $T_{\text{eff}}$, log $L$, and/or log $g$ are below the ZAMS within uncertainty. Filled symbols indicate stars that are definitely above the ZAMS. Colour is proportional to $M_\star$, symbol size to $B_d$. The histogram on the right shows the logarithmic ratio: the distribution is centred on log ($t_{S,\text{max}}/t_{\text{evol}}$) = 0, but extends out to ±2 dex. Strongly magnetized stars have spindown ages much less than their evolutionary ages. Stars with weaker fields have much longer spindown than evolutionary ages, especially the more massive stars.

from very large datasets (Neiner et al., 2003b; Donati et al., 2006; Henrichs et al., 2013). This appears to be a general problem affecting all stars in this mass range.

Up until now it has been assumed that $W_0 = 1$, i.e. that all stars are rotating at critical velocity on the ZAMS. This is not necessarily the case. If $t_{S,\text{max}}$ is replaced with the stellar age $t$ in equation 9.18, and solving for $W_0$, we have:

$$W_0 = We^{t/\tau_1}.$$  \hspace{1cm} (9.19)

Fig. 9.12 shows $W_0$ as a function of $M_\star$ for Vink mass-loss rates. In several cases, namely when $\tau_1 > t$, Eqn. 9.19 implies $W_0 > 1$. These stars are indicated as having upper limits of
Figure 9.11: As Fig. 9.10, but with Krťka mass loss rates. The longer spindown times bring the spindown ages of strongly-magnetized stars closer to their evolutionary ages, however, the massive stars with spindown ages much longer than their evolutionary ages remain unexplained.

$W_0 = 1$. All such stars are below 11 $M_\odot$ and have strong ($\sim 10$ kG) magnetic fields. With one exception, HD 37061, all stars with $M_\ast > 11M_\odot$ have $W_0 < 0.1$. HD 37061 is the only SB2 star amongst the massive sub-sample, and might have experienced binary interactions that have increased its surface rotational velocity. However, it is also the youngest star in this mass range. Amongst the less massive stars with $W_0 < 1$, there is some suggestion that $W_0$ increases towards smaller masses.

Fig. 9.13 replicates Fig. 9.12 using Krťka mass-loss rates. There are fewer stars with $W_0 \geq 1$ as compared to the Vink mass-loss rates. The overall pattern is similar to that seen in Fig. 9.12.

For many of the less massive stars, $W_0 \gg 1$ using either mass-loss prescription. Unsurprisingly, this problem is much worse using Vink mass-loss rates, which yield $W_0 > 10^2$ in some cases. The highest value obtained using Krťka mass-loss is $W_0 = 35$ for HD 156424. This counts as additional evidence in favour of Krťka mass-loss rates being more
Figure 9.12: $W_0$ as a function of $M_*$ using Vink mass-loss.

appropriate for the cooler stars in the sample.

Equation 9.18 assumes that a star has had its current properties since birth, which is obviously not the case. As stars age, their mass-loss rates either increase (using Vink mass-loss) or decrease (using Krtička mass-loss). At the same time, their radii increase, and as massive star magnetic fields are thought to be fossil fields, flux conservation should lead to a $1/r^2$ decrease in $B_0$ over time, and indeed this sample shows some evidence of this effect (Figs. 8.14 and 8.15). Thus, relatively evolved stars with weak magnetic fields may have had much stronger surface fields near the ZAMS. Finally, the gyration radius $r_{\text{gyr}}$ decreases over time as the H-burning core shrinks.
9.4. SPINDOWN TIMESCALES

To investigate the importance of accounting for evolutionary changes in wind and rotational parameters in obtaining spindown ages, a grid of stellar evolutionary models was utilized in which the tidal evolution constants were explicitly calculated (Claret, 2004). Spindown ages were determined using wind and magnetic confinement parameters calculated from the model parameters at each point in the star’s evolution. This is illustrated in Fig. 9.14 for an 8 $M_\odot$ model, which shows the effect of allowing individual parameters to vary, while holding other parameters constant to the value held at the end of the main sequence. Assuming initially critical rotation, spindown was calculated as function of time by inverting Eqn. 9.18 for $W$:

![Krtiška mass-loss](image)

Figure 9.13: As Fig. 9.12, using Krtiška mass loss.
Figure 9.14: Evolution of the rotation parameter $W$ for an $8 \, M_\odot$ model. The panels on the left show (top–bottom) $\dot{M}$, $B_d$, and $r_{\text{gyr}}$ as a function of fractional main-sequence age. The panel on the right shows the effects of allowing individual parameters to vary. When one parameter is allowed to vary, the remaining parameters are fixed to the value at the main sequence.

$$W(t) = e^{\ln[W(t-1)] - \Delta t/r_{\text{gyr}}},$$  \hspace{1cm} (9.20)

where $W(t-1)$ is $W$ at the previous time step in the model and $\Delta t$ is the length of the time step. Assuming constant vs. varying parameters leads to disagreements of 1 to 2 dex in $W$.

Several spindown tracks were computed with initial $B_d$ values determined by holding $R_A$ constant at 10, 20 and 50 $R_*$, for models with ZAMS masses of 5, 9, and 15 $M_\odot$. The 15 $M_\odot$ model requires $B_{d,ZAMS} \sim 400$ kG, which is unphysically high; all other models are within the observed range of $B_d \leq 40$ kG.

Fig. 9.15 shows the predicted evolution across the rotation-confinement diagram using Vink mass-loss rates. These predict a relatively steady evolution in $R_A$ and $R_K$ due to the continually increasing $\dot{M}$ and decreasing $B_d$. For both models and the observed population,
9.4. SPINDOWN TIMESCALES

Figure 9.15: Spindown tracks computed using Vink mass-loss rates and the evolutionary tracks calculated by Claret (2004), for ZAMS masses of 5 $M_\odot$ (thin solid lines), 10 $M_\odot$ (thick solid lines), and 15 $M_\odot$ (dotted lines). $R_K$ was terminated at 4000 $R_\star$. Colour corresponds to fractional main sequence age $\tau$. Note the different horizontal scales. Three representative initial masses were each evolved from an initial $R_A$ of 10, 20, and 50 $R_\star$. Vink mass-loss rates predict a monotonic decrease in $R_A$ due to the steadily strengthening winds combined with the weakening surface magnetic field. The 10 $M_\odot$ model shows a pronounced change in $R_A$ due to the 24 kK bistability jump around $\tau = 0.7$. 
\( \tau_{\text{MS}} \) increases from top right to bottom left. Spindown is quite abrupt for highly magnetized low-mass stars, and indeed the stars in the top right of the diagram are all, as predicted, fairly young. However, the lower right of the diagram is practically empty, i.e. there are apparently no slowly rotating, low-mass stars with strong magnetic fields. The stars in the lower left are in general older, and are in approximately the location the spindown tracks predict them to be. However, none of the spindown tracks extend all the way into the region they occupy, i.e. the problem found above (Fig. 9.10) using spindown ages calculated with the star’s current stellar parameters persists.

Using Krtička mass-loss rates in Fig. 9.16, the absence of old, highly magnetized, low-mass slow rotators is easily explained: \( \dot{M} \) weakens rapidly with decreasing \( T_{\text{eff}} \), and ceases entirely at 15 kK (\( \sim \tau = 0.5 \)), thus for cooler stars the rate of spindown slows throughout the main sequence and halts entirely partway through. It should also be noted that there is some ambiguity in the age of the two least massive stars in the upper right of the diagram, HD 35298 and HD 175362: while here the luminosity inferred from their apparent magnitudes and distances is used, P13 found higher luminosities using CHORIZOS, which would imply that they are much older than assumed here. If this is the case, their rotation periods could only be explained by Krtička mass-loss rates.

The Krtička mass-loss rates do not, however, successfully reproduce the lower left of the rotation-confinement diagram. Since Krtička mass-loss rates imply a near-constant \( R_A \), the disagreement between theory and model in this region is even greater.

Neither of the mass-loss prescriptions under examination are able to reproduce the distribution of stars on the rotation confinement diagram. The disagreement is at its starkest for the most massive, slowly rotating, and weakly-magnetized stars, as is clear from Fig. 9.10, for which \( \log t_{S,\text{max}}/t_{\text{evol}} > 2 \) for essentially all the stars. Since accounting for stellar evolutionary processes is unable to resolve this discrepancy, we must now ask if changing one or more of the parameters in eqns. 9.12-9.16 might account for this, or what other
Figure 9.16: As Fig. 9.15, using Krtička mass-loss rates. Note the different horizontal scale. Krtička mass-loss is highly sensitive to $T_{\text{eff}}$, and $\dot{M}$ decreases strongly over time. Thus, $R_A$ does not evolve monotonically towards smaller values. For the 5 $M_\odot$ model, the weakening $\dot{M}$ leads to a monotonic increase in $R_A$; mass loss also effectively ceases $\sim 2/3$ of the way through the main sequence, bringing spindown to an end.
9.4. SPINDOWN TIMESCALES

assumptions should be questioned. To do so, HD 205021 is used as an example, as its physical, rotational, and magnetic properties are extremely well-constrained (Henrichs et al., 2013).

**Initially sub-critical rotation**: obviously, if $W_0 < 1$, $t_{s,\text{max}}$ will be less than if $W_0 = 1$. There is some evidence that significant magnetic braking has already occurred during the Pre-Main Sequence (PMS) for both Ap stars (Kochukhov and Baghulo, 2006), and magnetic Herbing Ae/Be stars (Alecian et al., 2013b). As explored in Figs. 9.12 and 9.13, there is an apparently inverse relationship between $W_0$ and $M_\ast$: while relatively low-mass stars can have $W_0 < 1$, with only one exception all of the most massive stars have $W_0 < 0.1$. This would imply that more massive stars lose virtually all of their angular momentum on the PMS. At the same time, the existence of young, highly magnetized, yet rapidly rotating stars indicates that PMS braking is not a universal phenomenon. Stars with $\tau_{\text{MS}} < 0.3$ all have $P_{\text{rot}} < 5$ d (Fig. 9.9). Indeed, the second most massive star in the sample, HD 37061, is, of those stars with $M_\ast > 10M_\odot$, the youngest and the most rapidly rotating: this could argue against significant PMS braking, however it should be kept in mind that HD 37061 is also an SB2 and may have been spun up by tidal interaction. Thus, while it seems very likely that $W_0$ is generally less than unity, it does not necessarily follow that this can explain the slow rotation of the most massive B-type stars.

**Mass-loss**: increasing $\dot{M}$ would lead to shorter spindown timescales. However, reconciling gyrochronological with evolutionary ages would require $\dot{M}$ to be a factor of $\sim 100$ higher than assumed. While there is some ambiguity in the mass-loss rates of early B-type stars, models typically disagree by a factor of $\sim 3$, much less than required. Indeed, mass-loss rates 100 times higher than assumed would put these stars into the same range as O-type stars. Furthermore, as shown above via comparison of evolutionary and spindown ages (Figs. 9.10, 9.11, 9.12, and 9.13), the evidence seems to favour lower mass-loss rates.

**Gyration radius**: if $r_{\text{gyr}}$ is less than assumed from stellar structure models, $\tau_J$ decreases,
thus reducing the spindown time-scale. To resolve discrepancies in the typical magnetic B1-B0 star would, however, require much smaller gyration radii: $\sim 0.05$, as compared to a typical value of $\sim 0.2$. Asteroseismology may shed light on this question, and indeed Briquet et al. (2012) have found that the magnetic $\beta$ Cep star HD 163472 exhibits very little core overshooting, which could be indicative of a more concentrated core.

**Magnetic flux decay:** an initially stronger-than-assumed magnetic field should decrease spindown times. If the surface magnetic field is subject not just to attenuation due to flux conservation, as assumed above, but also to decay of the total magnetic flux due to Ohmic or super-Ohmic dissipation, this would imply magnetic fields that were much stronger than assumed in the past. If this is modelled as an exponential decay, where the flux decreases as $e^{-D_{\text{rms}}}$, where $D$ is a time constant of decay, HD 205021, with present-day $B_d = 0.26 \pm 0.03$ kG, might have had a 30 kG magnetic field on the ZAMS if $D = 6$. Even this extremely strong initial magnetic field, and extremely rapid decay, would result in a present-day $W = 0.16$, three times higher than the observed value of 0.05.

While none of these factors, individually, can plausibly account for the discrepancy, it remains possible that some combination of them might, e.g., a gentler rate of magnetic flux decay ($D = 2$, $B_{d, \text{ZAMS}} = 2$ kG), a mildly higher mass-loss rate (by a factor of 3), a gyration radius smaller by a factor of 2, and $W_0 = 0.5$, would together result in the observed rotational period.

Another possibility is that magnetospheric braking is not the only factor contributing to angular momentum loss. Some simulations have shown that magnetic fields can impose solid-body rotation (Maeder and Meynet, 2005). In the absence of magnetic braking, this has the effect of increasing the surface rotational velocity (Maeder and Meynet, 2005), since solid-body rotation transports angular momentum from the core to the envelope. However when magnetic braking is included in conjunction with solid-body rotation, the angular momentum transported from the interior is then lost via the wind, and spin-down
is enhanced compared to the case of internal differential rotation (Meynet et al., 2011). Published models indicate equatorial velocities of \( \sim 100 \text{ km s}^{-1} \) for stars with \( B_\text{sd} \), \( \log g \), and \( M_* \) in the range of HD 205021 (Meynet et al., 2011), which is still much higher than is observed. However, these models assume a constant surface magnetic field strength throughout the stellar life-cycle. A high priority for future models of massive star evolution with magnetic fields should be to explore the impacts of different assumptions regarding the evolution of the surface magnetic field, in order to see which assumptions provide the best fit with observations.

9.5 Summary

In this chapter stellar wind, magnetospheric, rotational, and spindown parameters were determined from stellar rotational properties (Chapter 5), fundamental and derived physical parameters (Chapters 6 and 7), and ORM parameters (Chapter 8). These parameters were determined self-consistently with stellar radii, masses, ages, and ORM parameters, such that their uncertainties reflect correlations across the \( T_{\text{eff}} - \log L \) and \( T_{\text{eff}} - \log g \) diagrams. This method obtained relative uncertainties substantially smaller than were previously available.

The Alfvén and Kepler radii determined using new rotation periods have improved our view of the division between stars with and without emission on the rotation-confinement diagram: with one exception, all B-type emission-line stars have \( R_K \leq 3.5 R_* \) and \( \log (R_A/R_K) \geq 0.8 \). This result is essentially identical to that found earlier by P13, with the exception that these parameters have proved predictive of the rotational periods and surface magnetic field strengths of all stars in which emission has been detected, including two stars in which emission was unknown at the time the P13 catalogue was compiled. One of these stars, HD 23478, was identified as a possible CM star on the basis of its emission line morphology (Eikenberry et al., 2014), and later confirmed as a magnetic rapid rotator (Sikora et al., 2015; Hubrig et al., 2015). While the systematic uncertainty in \( R_A \) is considerable when
different mass-loss prescriptions are used, the fundamental conclusion is that all magnetic emission-line B-type stars are rapid rotators with strong magnetic fields.

A clear relationship between a star’s fractional age, mass, surface magnetic field strength, and rotation period is apparent in Fig. 9.9: at a given age, the slowest rotators are either the most massive, the most strongly magnetized, or both. This is qualitatively in agreement with the expectations from magnetic braking, which predict angular momentum loss to be more rapid in stars with stronger winds. However, comparison of evolutionary to spindown ages yields large discrepancies in many cases. Using the highly temperature sensitive Krtićka mass-loss rates, the discrepancy between the spindown and evolutionary ages of strongly magnetized stars is largely reduced. Krtićka mass-loss rates also correctly predict that there should be no low-mass, strongly magnetized stars populating the lower right of the rotation-confinement diagram, as is indeed the case. Thus the evidence appears to favour Krtićka mass-loss rates, at least for the cooler Bp stars for which they were calculated.

Neither the Krtićka nor the Vink mass-loss rates can account for the massive, weakly magnetized, slow rotators on the far left of the rotation-confinement diagram. It is important to note that this discrepancy cannot be accounted for by possible systematic errors in fractional ages arising from incorrect evolutionary models, as the spindown ages are typically longer by a factor of 100 than the evolutionary ages. Numerical experiments tweaking various parameters relevant to braking timescales, such as the initial rotation rate, the mass-loss rate, the gyration radius, or the decay rate of the magnetic field, indicate that some combination of these might be able to explain the observed rotation periods. One possible interpretation of this anomaly is that the most massive B-type stars lose almost all of their angular momentum on the PMS. However, the number of free parameters in such an exploration is too large at present to draw meaningful conclusions. Models accounting for the simultaneous impact of rotation and magnetic fields on stellar structure and evolution will be essential to reducing the number of degrees of freedom in this problem. Since it
seems that magnetic braking alone has difficulty in reproducing the rotational periods of the most massive stars using physically plausible assumptions, the possibility that other consequences of a stellar magnetic field, such as solid-body rotation, might significantly impact rotational evolution should certainly be explored.

With the magnetospheric and rotational properties of the sample stars determined, we have established a context within which to investigate the emission properties of the subset of magnetic B-type stars with detectable Hα emission originating in their magnetospheres. In particular, we can now address the question posed by P13: whether the emission properties are wholly determined by $R_A$ and $R_K$, or whether there is some dependence on the mass-loss rate as well. The answer will have implications for the nature of mass-balancing mechanism operating within CMs. This line of investigation is taken up in the following, penultimate chapter of the thesis.
Chapter 10

Magnetospheric Emission

With the exception of Chapter 3, which focused on the small subset of SB2/3 stars in the sample, the bulk of this thesis has dealt with the full population of magnetic early B-type stars. In this chapter we examine the subset of these stars with detectable H\(\alpha\) emission, in the context of the magnetospheric and rotational parameters derived in the previous chapter.

10.1 An overview of emission properties

Of the 51 magnetic stars in the sample, 15 are H\(\alpha\)-bright, i.e. displaying emission in H\(\alpha\). Fig. 10.1 shows the region of the rotation-magnetic confinement diagram containing these stars. In contrast to the diagram presented by P13, all of these stars are in the upper right of the diagram, with \(R_K < 3R_\star\) and \(\log (R_A/R_K) > 0.8\). Furthermore, there is only one star in this region, HD 36526, with H\(\alpha\) in absorption, and this star is much cooler (~15 kK) than the H\(\alpha\)-bright stars. Magnetospheric eclipses have been observed around the even cooler B8 IIIp star 36 Lyn (Smith et al., 2006), with \(T_{\text{eff}} = 13.3 \pm 0.3\) kK (Wade et al., 2006). However, evaluation by eye of the H\(\alpha\) lines of HD 36526, HD 35298, and HD 175362 (the most strongly magnetized of the sample’s cool stars) yields no evidence of variability that might be ascribed to eclipses.
10.1. AN OVERVIEW OF EMISSION PROPERTIES

Figure 10.1: The rotation confinement diagram, as in Fig. 9.6, zoomed in to the region containing stars with Hα emission. The dashed line indicates $R_A = 10R_K$. The dotted line indicates $R_K = 3R_*$. 

This chapter presents an examination of the magnetospheres of these stars. Detailed studies have already been made of $\sigma$ Ori E (Townsend et al., 2005; Oksala et al., 2012, 2015b), $\delta$ Ori C (Leone et al., 2010), HR 5907 (Grunhut et al., 2012a), HR 7355 (Rivinius et al., 2013), and HD 23478 (Sikora et al., 2015). The magnetospheres of the remaining stars have received little attention in the literature, either due to an absence of data, or their very recent discovery. Thus, this is the first time these objects have been studied as a group.
Figure 10.2: Hα profiles at phases of maximum and minimum emission. The horizontal scale is in units of $v/v \sin i$ or equivalently, $R_*$, with $\pm 1R_*$ indicated by dotted lines. Vertical dashed lines indicate $R_K$. Red dashed lines indicate the model spectrum used for analysis, with shaded regions corresponding to 1σ model uncertainties.
10.1. AN OVERVIEW OF EMISSION PROPERTIES

Figure 10.3: Radius of maximum Hα emission $r_{\text{max}}$ vs. $R_K$. In several cases $r_{\text{max}} > R_K$.

Fig. 10.2 shows the Hα profiles all 14 stars at phases of maximum and minimum emission. The profiles are arranged from top to bottom in order of decreasing log ($R_A/R_K$), with $R_K$ indicated by vertical dashed lines. The horizontal scale is in units of $v/v \sin i$. Since magnetic confinement enforces rigid rotation on the plasma, velocity is directly proportional to distance $r$ from the star, i.e. $v/v \sin i = r/R_\ast$. With only a few exceptions, the emission follows a common pattern: the emission is localized into two bumps, with the peaks at $r \sim R_K$; there is very little or no emission at $r < R_K$; and the emission falls off very rapidly after the peak, disappearing entirely after 3–6 $R_\ast$. We designate the radius of maximum emission as $r_{\text{max}}$, and the radius beyond which no emission can be discerned as $r_0$. $r_{\text{max}}$ and $r_0$ are given in Table 10.1.

Fig. 10.3 compares the radius of peak emission, $r_{\text{max}}$, to $R_K$, where $r_{\text{max}}$ was determined by eye for individual clouds, with uncertainties estimated based on the uncertainty in $v \sin i$. There are no stars for which $r_{\text{max}} < R_K$. In several cases, however, $r_{\text{max}} > R_K$, with ratios of $r_{\text{max}}/R_K$ up to 2.3. Thus, while the peak emission is never found below $R_K$, in many cases it is located significantly above this distance.
Some stars show only a single emission bump at maximum emission in Fig. 10.2: in particular HD 37776, HD 142990, HD 37017, and HD 156324. Due to the high complexity of its surface magnetic field, HD 37776 is expected to possess a correspondingly complex magnetosphere (Kochukhov et al., 2011), and this does in fact seem to be the case (see below). HD 142990 has the weakest magnetic field amongst all of the emission-line stars, and its emission is also the weakest in the sample. In Fig. 10.2 emission is only apparent in HD 142990’s red wing, however inspection of the residual flux of this star’s Hα line (Fig. 10.4) reveals the possible signature of very weak emission in the blue line wing, within 1% of the continuum. The remaining two stars showing peculiar emission patterns, HD 37017 and HD 156324, are both close binary systems, each with a hot companion fairly close to the magnetic primary.

Several of the stars (HD 37479, HD 37776, HD 176582, and HD 182180) show enhanced absorption in the core of the line during minimum emission (Fig. 10.2, right panels). This
is due to eclipsing of the star by the magnetosphere (e.g., Townsend et al. 2005, 2013). HD 142990 also shows enhanced absorption in the core, which may be due to eclipses; however, this occurs during its maximum emission phase, and there are unfortunately too few spectra to evaluate whether or not this feature is magnetospheric in origin. HD 36485 also shows enhanced absorption, however the geometry is wrong for eclipses (both $i$ and $\beta$ are small, see Tables 8.1 and 8.3), and the analysis is complicated by the presence of its binary companion. Leone et al. (2010) suggested that HD 36485’s emission may be a consequence of an enhanced stellar wind above the star’s magnetic pole, a feature of MHD simulations (ud-Doula and Owocki, 2002). Finally, both ALS 3694 and HD 164492C show emission in the line core. These features are quite narrow and do not vary with $P_{\text{rot}}$; as both stars are in embedded in nebulae these features are almost certainly nebular in origin.

10.2 Emission variability

The RRM model predicts that the circumstellar plasma will be distributed around the star in a warped disk, approximately in the plane of the magnetic equator, with the two strongest concentrations of material at the intersections of the rotational and magnetic planes. This basic picture is not entirely complete, as there is some dependence on the obliquity angle $\beta$ of the magnetic field with respect to the rotational axis. If $\beta = 0^\circ$, then the plasma should be uniformly distributed in the magnetic and rotational equators, since these are now identical and the potential minima along each magnetic field line occur at the same latitude. In this case there will be no emission-line variability. If $\beta = 90^\circ$, the plasma is predicted to be distributed in two cones, dubbed ‘leaves’, above and below the rotational pole due to potential minima that appear here (leaves are actually present at almost all geometries, however it is only in the $\beta = 90^\circ$ configuration that they dominate the accumulation surface; Townsend and Owocki 2005).

If the magnetic field is dipolar, the variability should be symmetric in the red and blue
Table 10.1: Observed Hα emission properties. Clouds are numbered in the order at which they appear in the blue line wing after phase $\phi = 0$. 

<table>
<thead>
<tr>
<th>Star Name</th>
<th>Cloud No.</th>
<th>$\phi_{\text{max}}$</th>
<th>$r_{\text{max}}$ ($R_*$)</th>
<th>$r_0$ ($R_*$)</th>
<th>EW$_{\text{max}}$ (nm)</th>
<th>$D_{34} \log N_e$ (cm$^{-3}$)</th>
<th>$D_{54} \log N_e$ (cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 23478</td>
<td>1</td>
<td>0.85</td>
<td>2.8</td>
<td>6.0</td>
<td>0.144±0.016</td>
<td>12.3±0.3</td>
<td>12.2±0.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.20</td>
<td>3.2</td>
<td>5.5</td>
<td>0.137±0.013</td>
<td>12.1±0.4</td>
<td>11.8±0.5</td>
</tr>
<tr>
<td>HD 35502</td>
<td>1</td>
<td>0.00</td>
<td>3.5</td>
<td>5.5</td>
<td>0.083±0.008</td>
<td>12.1±0.6</td>
<td>12.1±1.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.95</td>
<td>3.5</td>
<td>5.5</td>
<td>0.099±0.008</td>
<td>12.2±0.5</td>
<td>11.9±0.9</td>
</tr>
<tr>
<td>HD 36485</td>
<td>1</td>
<td>0.90</td>
<td>2.5</td>
<td>4.0</td>
<td>0.039±0.002</td>
<td>12.3±0.2</td>
<td>11.8±0.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.75</td>
<td>2.0</td>
<td>3.5</td>
<td>0.026±0.002</td>
<td>11.5±0.5</td>
<td>11.4±0.4</td>
</tr>
<tr>
<td>HD 37017</td>
<td>1</td>
<td>0.85</td>
<td>3.5</td>
<td>6.0</td>
<td>0.019±0.008</td>
<td>11.09±0.09</td>
<td>12.0±0.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.85</td>
<td>3.5</td>
<td>6.0</td>
<td>0.063±0.008</td>
<td>12.74±0.05</td>
<td>12.99±0.06</td>
</tr>
<tr>
<td>HD 37479</td>
<td>1</td>
<td>0.65</td>
<td>3.2</td>
<td>6.0</td>
<td>0.164±0.015</td>
<td>13.1±0.1</td>
<td>13.4±0.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.75</td>
<td>3.5</td>
<td>6.0</td>
<td>0.239±0.020</td>
<td>12.95±0.04</td>
<td>13.2±0.1</td>
</tr>
<tr>
<td>HD 37776</td>
<td>1</td>
<td>0.40</td>
<td>3.5</td>
<td>6.0</td>
<td>0.027±0.007</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.60</td>
<td>3.5</td>
<td>6.0</td>
<td>0.018±0.007</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>HD64740</td>
<td>1</td>
<td>0.00</td>
<td>3.0</td>
<td>4.0</td>
<td>0.005±0.008</td>
<td>11.8±0.8</td>
<td>12.9±0.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.00</td>
<td>2.5</td>
<td>4.0</td>
<td>0.008±0.007</td>
<td>12.7±0.6</td>
<td>13.3±0.2</td>
</tr>
<tr>
<td>HD 142184</td>
<td>1</td>
<td>0.00</td>
<td>2.0</td>
<td>5.0</td>
<td>0.167±0.019</td>
<td>13.1±0.1</td>
<td>13.4±0.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.80</td>
<td>2.5</td>
<td>4.5</td>
<td>0.104±0.011</td>
<td>13.3±0.2</td>
<td>13.1±0.2</td>
</tr>
<tr>
<td>HD 142990</td>
<td>1</td>
<td>0.15</td>
<td>3.6</td>
<td>4.9</td>
<td>0.004±0.003</td>
<td>12.9±0.6</td>
<td>13.2±0.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.15</td>
<td>3.6</td>
<td>5.3</td>
<td>0.006±0.003</td>
<td>12.0±0.7</td>
<td>12.7±0.5</td>
</tr>
<tr>
<td>HD156324</td>
<td>1</td>
<td>0.95</td>
<td>3.5</td>
<td>6.0</td>
<td>0.074±0.030</td>
<td>12.8±0.2</td>
<td>12.8±0.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>–</td>
<td>3.5</td>
<td>6.0</td>
<td>0.015±0.025</td>
<td>12.9±0.6</td>
<td>11.8±0.8</td>
</tr>
<tr>
<td>HD 156424</td>
<td>1</td>
<td>0.40</td>
<td>4.5</td>
<td>6.5</td>
<td>0.060±0.023</td>
<td>12.2±1.2</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.40</td>
<td>4.5</td>
<td>6.5</td>
<td>0.049±0.013</td>
<td>12.2±1.2</td>
<td>–</td>
</tr>
<tr>
<td>HD164492C</td>
<td>1</td>
<td>0.00</td>
<td>1.7</td>
<td>4.6</td>
<td>0.124±0.004</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.90</td>
<td>1.9</td>
<td>4.5</td>
<td>0.098±0.003</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>HD 176582</td>
<td>1</td>
<td>0.05</td>
<td>4.2</td>
<td>5.5</td>
<td>0.023±0.007</td>
<td>12.1±0.4</td>
<td>12.6±0.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.10</td>
<td>4.5</td>
<td>5.2</td>
<td>0.017±0.006</td>
<td>12.0±0.5</td>
<td>12.3±0.4</td>
</tr>
<tr>
<td>HD 182180</td>
<td>1</td>
<td>0.50</td>
<td>2.5</td>
<td>4.5</td>
<td>0.151±0.017</td>
<td>12.88±0.04</td>
<td>12.92±0.08</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.55</td>
<td>2.0</td>
<td>4.5</td>
<td>0.135±0.009</td>
<td>13.2±0.2</td>
<td>12.8±0.1</td>
</tr>
<tr>
<td>ALS 3694</td>
<td>1</td>
<td>0.00</td>
<td>2.8</td>
<td>5.0</td>
<td>0.039±0.041</td>
<td>12.2±1.2</td>
<td>12.2±1.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.00</td>
<td>3.2</td>
<td>4.5</td>
<td>0.042±0.010</td>
<td>12.2±1.2</td>
<td>12.2±1.2</td>
</tr>
</tbody>
</table>
halves of the line. Emission maxima in the two wings should occur at the same phase, corresponding to the positive or negative extremum of the \( \langle B_z \rangle \) curve, and the emission strength of each cloud should be equivalent. Eclipses, if present, should occur 0.5 rotational cycles apart. If the plasma density is low, such that the CM is optically thin at all projection angles, the second emission maximum (occurring at phase 0.5, when \( \langle B_z \rangle \) is closest to zero) should be as strong as the first; indeed, the EW should not change except during eclipses, when the CM is in absorption.

Fig. 10.5 shows H\( \alpha \) emission EW curves (left panels) for all stars for which sufficient data exists to discern a coherent variation. Variability is ubiquitous, indicating that the optical depth of the CM does indeed change with projection angle, i.e. the plasma is optically thick at least at some rotational phases. The EW curves are arranged from bottom to top in order of \( \Delta \cos \alpha = \cos \alpha(\phi = 0) - \cos \alpha(\phi = 0.5) \), where \( \alpha \) is the angle between the magnetic pole and the line of sight at phase \( \phi \) given by (Preston, 1967)

\[
\cos \alpha = \sin \beta \sin i \cos \phi + \cos \beta \cos i. \tag{10.1}
\]

Stars with \( \Delta \cos \alpha \leq 1 \) all show single-wave variations, while the three stars with \( \Delta \cos \alpha \geq 1 \) all have double-wave variations. This is easily understood as a consequence of projected geometry: when \( \Delta \cos \alpha \leq 1 \), only a single magnetic pole is visible during a rotational cycle, while when \( \Delta \cos \alpha \geq 1 \) two poles are visible, thus giving two opportunities to see the magnetic equator close to pole-on. To get an idea of how geometry affects the amount of variability, the right panels of Fig. 10.5 shows, on the bottom, the difference between the maximum and minimum H\( \alpha \) emission EWs normalized to the mean EW. This index increases from close to zero at low \( \Delta \cos \alpha \) to around 1.5 to 2 at \( \Delta \cos \alpha \sim 0.5 \), plateauing after this value. The top panel shows the difference between maxima or, for single-wave variations, phase 0 to phase 0.5. This index increases from \( \Delta \cos \alpha = 0 \), reaches
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Figure 10.5: Left: Hα emission EWs as a function of rotational phase. EW curves are organized in order of increasing $\Delta \cos \alpha$ (written beside each curve) from bottom to top. Note that double-wave variations occur only for stars with $\Delta \cos \alpha \geq 1$. Right: difference in emission strength between local maxima at phases 0.0 and 0.5 (top) and between maximum and minimum EW (bottom) as a function of $\Delta \cos \alpha$. The amplitude of the curve increases from $\Delta \cos \alpha = 0$ to $\Delta \cos \alpha = 0.25$, levelling off for greater values. The difference between phases 0.0 and 0.5 reaches a maximum at $\Delta \cos \alpha = 1$, and diminishes for greater or lesser values.
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a maximum at Δcosα = 1, and declines thereafter. This demonstrates that intrinsic and projected geometry is broadly predictive of the pattern and strength of variability seen in a given star.

For a more detailed view, Figs. 10.6–10.9 show dynamic spectra for all stars except HD 142990 and HD 156424, neither of which show a coherent variation due to especially weak emission (and a paucity of data) in the first case, and possibly projection on the sky in the second (i = 3 ± 3°, so variability is minimal). All dynamic spectra show residual flux with respect to the models shown in Fig. 10.2. For SB2/3 stars, individual models were used for each observation, consisting of synthetic spectra for each component moved to the appropriate radial velocity. Dynamic spectra for HD 37479, HD 142184, HD 182180, and HD 35502 have been shown previously in the literature and are included here for purposes of comparison (Oksala et al., 2015b; Grunhut et al., 2012a; Rivinius et al., 2013; Bohlender and Monin, 2011; Sikora et al., 2016). Dynamic spectra of HD 36485 and HD 176582 are also available in the literature (Leone et al., 2010; Bohlender and Monin, 2011); those presented here are based upon new data. The data used for HD 23478’s dynamic spectrum has been previously published, although a dynamic spectrum was not shown (Sikora et al., 2015). Dynamic spectra for HD 37776, HD 64740, ALS 3694, HD 37017, HD 156324, and HD 164492C are shown here for the first time (although it should be noted that the Hα emission of HD 37776 and HD 64740 was first noted by Jason Grunhut and Evelyne Alecian, respectively).

In cases of particularly weak emission, the minimum colour-bar flux has been set just below the continuum in order to emphasize emission. Individual clouds are traced by curved white lines. When eclipses are present, these curves have been aligned with the eclipses; otherwise, they are aligned with emission maxima. Symmetric variations in the red and blue line wings, with essentially similar strengths, are seen in only HD 176582, ALS 3694, and HD 35502: that is, perfectly symmetrical variations are actually in the minority. In every
Figure 10.6: Hα dynamic spectra for HD 37479, HD 176582, HD 142184, and HD 182180. The bottom panel shows residual flux as compared to the synthetic spectra in Fig. 10.2. Vertical solid (dotted) lines indicate \( \pm v \sin i \) (\( \pm R_K \)). The top panels show residual flux mapped to colour and plotted as a function of rotational phase. Curved dashed lines trace individual clouds.
Figure 10.7: As Fig. 10.6, for HD 37776, HD 64740, ALS 3694, and HD 23478. Note that 3 clouds are traced for HD 37776.
Figure 10.8: As Fig. 10.6, for the binary stars HD 36485 (SB2), HD 37017 (SB2), HD 156324 (SB3), and HD 35502 (SB3). In this case the reference synthetic spectra are customized to each observation to account for the RV variability of individual components. Note that only 1 cloud is traced for HD 156324.
10.2. EMISSION VARIABILITY

Figure 10.9: As Fig. 10.6, for the binary star HD 164492C (SB3). In this case the reference synthetic spectra are customized to each observation to account for the RV variability of individual components.

other case there is an asymmetry in either peak emission strength, the phase of maximum emission, or both. In the case of HD 37479 it is well understood that this is a consequence of contributions to the surface magnetic field from a non-axisymmetric quadrupolar component (Sikora et al., 2015). The only other star for which a ZDI map is available is HD 37776, which clearly indicates a highly complex surface topology with essentially no dipolar component (Kochukhov et al., 2011); the correspondingly complex emission structure, with at least two and probably 3 clouds, was predicted by an early aRRM model and only later detected in the data (Grunhut et al., in prep.). While a ZDI map is not available for HD 142184, its $\langle B_z \rangle$ curve is strongly anharmonic, likely indicating a surface topology more complex than a simple dipole (Grunhut et al., 2012a). Similarly, while the peak-to-peak variation of HD 36485’s $\langle B_z \rangle$ curve is quite small, making it difficult to draw firm conclusions, its $\langle B_z \rangle$ curve is better fit by a second-order sinusoid, suggesting a surface magnetic field with significant quadrupolar contributions; this possibility was suggested in the discovery paper for HD 36485’s magnetosphere (Leone et al., 2010), and appears to be borne out by the
There is no evidence for a phase offset between HD 64740's clouds, although one cloud is slightly stronger than the other one. This is the opposite of the situation in HD 182180, for which there is a slight phase offset between the emission maxima and eclipses of the clouds, although they are of very nearly even strength. In both cases \( \langle B_z \rangle \) is compatible with an essentially dipolar variation, in keeping with these very small blue/red asymmetries.

For HD 23478, a two-cloud model does not appear to produce a satisfying fit to the data. This may simply be a consequence of the star's very small obliquity \( (\beta = 4 \pm 2^\circ) \), which should lead to a more or less continuous distribution of plasma in the rotational equator, thus, there should be high-velocity emission at all rotational phases. A two-cloud model is a similarly poor fit for HD 142184, which also has a small obliquity.

The most striking asymmetries are present in HD 37017 and HD 156324. In the former case, emission in the red wing is much stronger than in the blue wing. In the latter, only a single cloud is visible. Notably, both of these stars are in close binary systems.

### 10.2.1 Magnetospheres of binary systems

Three of the emission-line stars are in close binary systems: HD 36485, HD 37017, and HD 156324. The orbital properties of the first two systems were provided by Leone et al. (2010) and Bolton et al. (1998); the multiplicity of HD 156324 was reported by Alecian et al. (2014), while its orbital properties were derived in the present work \( P_{\text{orb}} = 1.5805(1) \) d, \( e \leq 0.03 \), \( a = 0.06 \pm 0.01 \) AU; Chapter 3, Table 3.1). HD 37017 and HD 156324 possess anomalous emission lines, each possessing a single strong emission peak in contrast to the double-horned structure that is essentially ubiquitous amongst the single Hα-bright stars.

Hα dynamic spectra for the three stars are shown in Fig. 10.8. HD 36485 displays a more or less typical two-peaked emission structure indicative of two magnetospheric clouds, although there is a small difference in intensity between the two clouds, and a phase offset
between their emission peaks: the blue-shifted peak of the stronger cloud occurs at phase 0.45, while the blue-shifted peak of the weaker cloud occurs near phase 0.75. This is consistent with the data reported by Leone et al. (2010). As the topology of the CM’s accumulation surface is a direct consequence of the surface magnetic field, the most likely explanation for this asymmetry is contribution from higher-order multipoles, and indeed there is some support for this in the star’s $\langle B_z \rangle$ curve, which is best-fit by a 2nd-order sinusoid (Fig. C.3). Similar asymmetries in emission strength and offsets in phase are present in HD 37479 and HD 37776, whose surface magnetic topologies exhibit strong departures from a purely dipolar field (Townsend and Owocki, 2005; Oksala et al., 2015b; Kochukhov et al., 2011).

While the signature of a second magnetospheric cloud is not readily apparent in HD 37017’s intensity spectra when examined in isolation, the dynamic spectrum shows evidence for a much weaker cloud opposite the main emission peak (Fig. 10.8, top right panel). Peak emission in the two clouds occurs at the same phase, suggesting that the asymmetry is unlikely to be the result of a complex surface topology. The $\langle B_z \rangle$ curve is furthermore entirely consistent with a simple dipole (Fig. C.6). HD 156324 exhibits an even stronger asymmetry, with essentially no sign of a second cloud at any phase (Fig. 10.8, bottom left). Its $\langle B_z \rangle$ curve is also consistent with a dipole (Fig. C.27). While the LSD profiles of both HD 37017 and HD 156324 do possess a complex structure at some phases (Figs. 4.1 and 4.2), these correlate closely with irregularities in their Stokes I profiles, and are thus likely to be due to chemical spots.

Thus, for neither HD 37017 nor HD 156324 is there any evidence that multipolar magnetic field topologies can explain the strong asymmetries in cloud structure. Since both of these stars are more compact binary systems than HD 36485 (indeed, HD 156324 is the most compact magnetic binary in the sample), this suggests that the differences in their emission structures, as compared to those of other stars, may be consequences of binarity.
Figure 10.10: The HD 36485, HD 37017, and HD 156324 orbital systems, in the rest-frames of the primaries, projected into the orbital planes. The primary is shown by a red circle, the secondary by a blue circle. The secondary’s orbit is indicated by a solid blue line. The primary’s $R_K$ and $R_A$ are indicated by dotted and dashed lines, while the dash-dotted line indicates $r_0$, the maximum extent of emission. All distances are to scale. For HD 156324, green crosses indicate, from top to bottom, $L_2$, $L_1$, and $L_3$. Note that the HD 156324 secondary is at the Kepler radius.
Fig. 10.10 shows schematics of the orbital systems of HD 36485, HD 37017, and HD 156324. In each case the system is shown in the rest frame of the primary, indicated by the central red circle, with the secondary shown by a blue circle and its orbit by a solid blue line. The primary’s Kepler radius is indicated by a dotted red line, the Alfvén radius by a dashed red line, and $r_0$, the maximum extent of emission, is shown by a dot-dashed red line. Stellar radii and orbital distances are to scale.

HD 36485’s secondary is the smallest and most distant of the three companion stars. Its orbit takes it within the primary’s magnetosphere, but the closest approach is well beyond $r_0$, suggesting that the cool plasma of the CM that is detected in Hα should not be significantly affected by the orbital interaction, as indeed does not seem to be the case.

HD 37017’s companion similarly passes within $R_A$ during its orbit, however, at periastron it is much closer to $r_0$. Whether or not this is close enough to affect the primary’s Hα emission will require detailed RRM modelling accounting for changes in the gravitocentrifugal potential due to the shifting centre of mass. There is also the issue that the rotational and orbital periods do not seem to be synchronized ($P_{\text{rot}} = 0.901186(1)$ d vs. $P_{\text{orb}} = 18.6561(1)$ d), making it unclear why only one cloud should be affected.

The situation for HD 156324, the star showing the most extreme emission anomaly, is also perhaps the easiest to understand. The identical orbital and rotational periods (1.5805(1) d), and the circular orbit, indicate that the system is tidally locked, thus the orientation of the primary with respect to the secondary is static. RV and Hα EW maxima occur at the same phase, indicating that the magnetospheric cloud is directly opposite the secondary. The companion star is furthermore well inside not just $R_A$ but also $r_0$, and is at the same distance from the primary as $R_K$. Since the system is a very close binary, separated by just a few stellar radii, the centre of mass will be significantly offset from the centre of the primary. As a consequence of this, the gravitocentrifugal potential in the direction of the secondary will be much weaker, while opposite the secondary it will be
stronger. Green crosses indicate, from top to bottom, the Lagrange $L_2$, $L_1$, and $L_3$ points. The $L_3$ point is in the middle of the magnetospheric emission region, suggesting that the CM’s accumulation surface is not affected, or possibly even enhanced, by orbital dynamics. The $L_1$ point, however, lies just inside $R_K$, while the secondary star lies just above $R_K$. This likely disrupts the accumulation surface in this region.

It is interesting to note that maxima of HD 156324’s RV and $\langle B_z \rangle$ curves coincide, and that $\beta = 71 \pm 1^\circ$ (Table 8.3). This indicates that the magnetic axis is tangential to the orbit. If the secondary is also magnetized then this would also be a potential minimum. Unfortunately the existing data cannot explore this question with any precision: the mean uncertainty for $\langle B_z \rangle$ measurements of the decomposed secondary’s LSD profiles, using the line mask with metallic + He lines, is 1.1 kG, thus a 3.9 kG magnetic field could easily go unrecognized. There is a contrast however with the one known doubly magnetic binary, HD 136504, in which $\beta$ is clearly small in both cases, and apparently aligned with the orbit, with magnetic axes furthermore apparently oppositely aligned with one another (Shultz et al., 2015c). A potential minimum would imply that the plasma which would ordinarily collect within the accumulation surface could instead accrete directly onto the secondary: thus, the secondary does not so much disrupt the cloud as replace it.

### 10.3 Emission strength

Emission EWs were measured in order to perform a quantitative comparison of emission strengths. To obtain these, for each star a grid of BRUCE-KYLIE spectral models were created (Rivinius et al., 2013). These models take as input synthetic spectra computed using LTE ATLAS9 model atmospheres, with each spectrum stored as a data-cube with axes of wavelength, flux, and incidence angle on the stellar surface. Thus, the code includes wavelength-dependent limb darkening. BRUCE-KYLIE offers the option of incorporating the effects of gravity darkening and rotational oblateness. Since all of these stars are rapid
rotators, rotational distortions can become significant, with pole-to-equator variations in \( T_{\text{eff}} \) and \( \log g \) of several kK and up to 0.4 dex in the critical limit, respectively. Each grid of models was created for the range of \( T_{\text{eff}} \) and \( \log g \) appropriate to the star, and the inclination angle \( i \) and critical rotation fraction \( \omega \) determined based upon \( P_{\text{rot}}, v \sin i \), and the stellar mass and polar radius appropriate to the model. Models yielding \( \omega > 1 \) were discarded. For models with \( \omega < 0.6 \), rotational distortion was turned off, as the effects are negligible below this limit and the computational expense considerable, as a different spectrum must be used for each point on the stellar surface.

The dashed red lines in Fig. 10.2 indicate the mean flux for all models, with the shaded light blue regions indicating the uncertainty in the models. Emission EWs were measured from the residual (observed minus model) flux. Measurements were made in the blue and red wings separately, with integration ranges of \(-r_0\) to \(-v \sin i\) and \(+v \sin i\) to \(+r_0\), with the total emission EW the sum of the red and the blue EWs. The line core was avoided in order to remove any influence of enhanced absorption due to occultations. The uncertainty in these measurements includes the uncertainty in the model flux, which was added in quadrature to the uncertainty obtained by propagation of the flux error bars.

Fig. 10.11 shows the maximum H\( \alpha \) emission EWs as a function of \( R_A, R_K \), and \( \log (R_A/R_K) \).
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with colour corresponding to $T_{\text{eff}}$. Emission strength tends to increase with increasing $R_A$, decreasing $R_K$, and increasing $\log (R_A/R_K)$, with the strongest correlation apparent in the final quantity. Correlations between were evaluated using Pearson’s Correlation Coefficient (PCC), which measures the linear correlation between 2 variables. Values close to unity indicate a perfect correlation, while values close to 0 indicate no correlation. Applying this test yields PCC values of 0.38, -0.31, and 0.68 respectively for $R_A$, $R_K$, and $\log (R_A/R_K)$ vs. logarithmic emission strength. This is more or less to be expected if emission strength is dependent upon both the ability of the magnetic field to confine the stellar wind, and the proximity to the star with which centrifugal support is able to hold the inner edge of the CM, i.e. the strongest correlation should be with $\log (R_A/R_K)$ rather than $R_A$ or $R_K$ individually.

At a given value of $\log (R_A/R_K)$, hotter stars tend to have stronger emission. This is most obviously the case for $\sigma$ Ori E, which, although it does not have the largest $R_A$ nor the smallest $R_K$, has by far the strongest emission in the sample. Dividing the sample into two approximately equally sized sub-samples at $T_{\text{eff}}= 20 \text{ kK}$ and re-evaluating the PCC for $\log (R_A/R_K)$ yields 0.69 for stars with $T_{\text{eff}}< 20 \text{ kK}$ and 0.78 for stars with $T_{\text{eff}}> 20 \text{ kK}$, i.e. a slightly stronger correlation for the hotter stars.

Emission is predicted to be at its strongest when the plane of the CM is seen closest to face-on, that is, when its projected area is the greatest. The EW curves in Fig. 10.5 verify that this is indeed the case. Since $i$ and $\beta$ are different for each star, it might reasonably be expected that there may be some dependence of maximum emission strength on projection. To explore this, symbol size in Fig. 10.11 is proportional to $\cos \alpha_{\text{min}}$, where where the rotation phase $\phi$ at $\alpha_{\text{min}}$ is by convention zero. There does not appear to be any particular dependence of maximum emission strength on $\alpha_{\text{min}}$: two stars for which $\cos \alpha_{\text{min}}$ is quite small, HD 23478 and HD 156424, have emission strengths comparable to hotter stars with larger values of $\cos \alpha_{\text{min}}$. Thus, while geometry is the deciding factor in the variability of stars, it plays a minor role, if any, in the variance between them.
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Figure 10.12: Maximum Hα emission from individual clouds (i.e., in either the red or blue Hα line wings, depending on the star) on the log ($R_A/R_K$) vs. log $L$ plane. The dotted and dash-dotted lines illustrate the two mass-balance scenarios (see text). Filled symbols correspond to emission-line stars, with symbol size proportional to maximum emission strength. Empty symbols denote stars without emission.

As outlined in Chapter 1 and illustrated in Fig. 1.8, P13 suggested that the relationship between log ($R_A/R_K$), log $L$, and emission strength could be indicative of the nature of the mass balancing mechanism within CMs. If emission strength depends only on log ($R_A/R_K$), this would indicate that the capacity of the magnetosphere to confine plasma is the sole deciding factor. If there is also some dependence on luminosity a leakage mechanism would be implied, as in this case a higher mass-loss rate is able to more rapidly fill a CM of lower
Figure 10.13: As Fig. 10.12 on the log \((R_A/R_K)\) vs. \(\dot{M}\) plane. Note that while emission strength increases with log \((R_A/R_K)\), there is a stronger relationship between emission strength, log \((R_A/R_K)\), and \(\dot{M}\): vertical distance from the dashed diagonal lines appears to be more predictive of emission strength than distance from the dotted horizontal line alone. This suggests that it is not only the capacity of the magnetosphere, but the mass-feeding rate from the stellar wind, which determines emission strength.

capacity. Fig. 10.12 shows the sample stars on the log \((R_A/R_K)\) vs. log \(L\) plane, with symbol size proportional to emission strength. As was apparent from Fig. 10.11, emission strength increases with log \((R_A/R_K)\). However, at a given log \((R_A/R_K)\), hotter, more luminous stars tend to have stronger emission.

Another view is provided by Fig. 10.13, where the stars are placed on the log \((R_A/R_K)\) vs. 
10.3. EMISSION STRENGTH

\[ \dot{M} \text{ plane, with symbol size proportional to emission strength.} \]

The horizontal line denotes \( \log \left( \frac{R_A}{R_K} \right) = 0.8 \). While emission does indeed increase with \( \log \left( \frac{R_A}{R_K} \right) \), the relationship is only approximate: stars with the same value of \( \log \left( \frac{R_A}{R_K} \right) \) can have different emission strengths, with the star with the higher mass-loss rate having the higher emission strength.

The diagonal dashed lines are placed on the diagram to help guide the eye. The vertical distance of a star from these lines appears to be more predictive of its emission strength than its distance from the \( \log \left( \frac{R_A}{R_K} \right) = 0.8 \) line alone. This indicates that it is not merely the capacity of the magnetosphere that determines how strong a star’s emission will be, but also the feeding rate from the stellar wind. This in turn implies that the mass-loading is in competition with a continuously operating mass-leakage mechanism.

Figs. 10.11 to 10.13 are not entirely unambiguous. HD 23478 has particularly strong emission, comparable to that of HD 142184 or HD 182180, despite having a substantially lower \( \log \left( \frac{R_A}{R_K} \right) \); conversely, HD 35502 has much weaker emission than HD 23478, despite having a similar \( \dot{M} \).

If Krtička mass loss rates are used instead, the relations seen in Figs. 10.11 to 10.13 disappear almost entirely, with essentially no correlation between emission strength, \( \log \left( \frac{R_A}{R_K} \right) \), and \( \log L \) or \( \dot{M} \). Fig. 10.14 reproduces Fig. 10.11 using Krtička mass loss. Amongst the cooler stars \( (T_{\text{eff}} < 20 \text{ kK}) \) there is a possible trend of increasing EW with \( \log \left( \frac{R_A}{R_K} \right) \),
Figure 10.15: As Fig. 10.12, using Krtička mass loss rates.

however, the hotter stars all have $\log \left( \frac{R_A}{R_K} \right) \sim 0.9$, yet very different emission EWs. The PCC is below 0.3 for both $R_A$ and $\log \left( \frac{R_A}{R_K} \right)$, indicative of no correlation. Fig. 10.15 reproduces Fig. 10.12 with Krtička mass loss. Once again there is no particular obvious relationship between $\log \left( \frac{R_A}{R_K} \right)$, $\log L$, and emission strength. However, the onset of emission seems more consistent with the diagonal dot-dashed line than it does with the horizontal dotted line. Thus, the conclusion that a steady-state leakage mechanism is favoured over centrifugal breakout does not seem to be dependent on the mass loss prescription.
10.4 Eclipses

Stars with \( i + \beta \geq 120^\circ \) are expected to be periodically eclipsed by their magnetospheres (Townsend, 2008), and several stars in the sample show clear signs of this. Eclipse timings can be used to determine the distance of the clouds from the star. The depth of an eclipse contains information on the fraction of the stellar disk occulted by the magnetosphere; together with the Doppler width of the eclipse, which is sensitive the azimuthal extent of the cloud, the depth helps to constrain a cloud’s meridional size.

Only 5 stars possess sufficiently large datasets for eclipses to be studied in any detail: HD 37479, HD 37776, HD 142184, HD 176582, and HD 182180. There is some indication HD 142990 may also possess an eclipsing magnetosphere (see Fig. 10.2), which is not unreasonable based on the star’s moderate inclination \( (i = 46 \pm 7^\circ) \) and large obliquity \( (\beta = 80 \pm 3^\circ) \).

The left panel of Fig. 10.16 shows a dynamic spectrum of HD 37776’s H\( \alpha \) line, zoomed in on the rotationally broadened core, and with the minimum and maximum flux of colour bar range set to highlight the eclipses. These are apparent between near phases 0.2 and 0.4 as narrow increases in absorption strength between \( \pm v \sin i \), each of which crosses the line core within approximately 0.1 rotational cycles.

Since magnetospheres corotate with the stellar surface, the angular velocity of the eclipsing plasma is identical to that of the photosphere. From simple geometry, the duration of an eclipse in rotation phase \( \phi_{ec} \) is thus proportional to the distance of the cloud \( r_{ec} \) from the star:

\[
\frac{r_{ec}}{R_*} = \arctan \left( \frac{\Delta \phi}{2} \right),
\]

thus, an eclipse duration of 0.5 corresponds to a cloud immediately above the photosphere, while shorter durations indicate larger distances. This is illustrated in Fig. 10.17. As the
Figure 10.16: Hα dynamic spectrum of HD 37776, zoomed in on the rotationally broadened core and with the minimum and maximum residual flux set to highlight eclipses by magnetospheric clouds. The bottom panel shows the observed flux (black lines) and, for comparison, the model flux (dashed red).
distance of the cloud from the star increases (dotted lines in Fig. 10.17), the fraction of a rotational cycle during which it eclipses its host star decreases (thick red lines).

EWs were measured in the rotationally broadened core of the Hα line. The resulting EW curves are shown in the left-hand panels of Fig. 10.18, with the approximate start and end phases of each eclipse indicated by vertical dashed lines.

The EWs are of course not only sensitive to the occulting plasma, but also to variable emission (since it is not necessarily the case that all of the cloud is projected in front of the star), as well as to photospheric phenomena such as strong He spots. Therefore EWs were normalized with linear fits to the EWs immediately before and after the eclipse. Since the duration of the eclipses is very brief, this serves as a first-order correction for photospheric effects, which persist for a much longer fraction of the rotational cycle. An example is shown in Fig. 10.19, for HD 37776's He i 667.8 nm line. The variations in absorptions strength in this dynamic spectrum are diagnostic of photospheric abundance spots: note that, in comparison with the eclipse features in Fig. 10.16, they tend to take much longer to cross the line profile than the absorption due to eclipses.

The renormalized EWs of each eclipse are shown in the right-hand panels of Fig. 10.18; note that, due to its small obliquity, HD 142184 shows only 1 eclipse. The peak of the renormalized EWs is directly proportional to the fraction of the disk occulted by the star, ranging from about 8% for HD 176582, to about 60% for HD 37479. Eclipse timings were refined by fitting low-order polynomials to the normalized EWs: duration was found from the points at which the polynomial is equal to 1, while strength was determined from the peak of the fit. Durations range from $\sim 0.09$ cycles (HD 176582) to $\sim 0.3$ cycles (HD 142184).

Fig. 10.20 shows a comparison between $r_{ec}$, $R_K$, and the two characteristic radii inferred from emission maxima, $r_{max}$ and $r_0$. For HD 37479 and HD 142184, $r_{ec}$ is quite close to $R_K$; for the other three stars, $r_{ec}$ is more consistent with $r_{max}$ or $r_0$. This makes sense given that, as already noted, the peak emission is in many cases seen at greater radii than
Figure 10.17: Cartoon illustrating the link between eclipse durations and the distance of the cloud from the star. The photosphere is indicated by the solid line. Dotted lines show the tracks of clouds at 1.5, 2, and 3 $R_\ast$. The cloud eclipses the star when its projected position is between the dashed lines.
Figure 10.18: *Left panels:* Hα EWs measured in the rotationally broadened cores of stars exhibiting magnetospheric eclipses. Eclipses are visible as sharp increases in EW. Eclipse boundaries are indicated by vertical dashed lines; EWs inside these boundaries are indicated by filled symbols. *Right panels:* eclipse EWs normalized via a linear fit between points at the beginning and end of each eclipse. The solid red curves indicate polynomial fits used to determine eclipse strengths and durations, with 1σ uncertainties indicated by dashed curves.
10.4. ECLIPSES

Figure 10.19: He I 667.8 nm dynamic spectrum of HD 37776. In this case a mean spectrum is used to determine residual flux.
Figure 10.20: Comparison of eclipse radii $r_{ec}$, the radii of maximum light $r_{max}$, and the furthest extent of emission $r_0$, to $R_K$. Lower line for each star is for cloud 1, upper line for cloud 2 (see Table 10.1 for numbering).

Since corotation maps the projected velocity of a cloud directly onto units of stellar radius, the azimuthal extent of the cloud can be estimated from the Doppler width during maximum occultation. This was performed by comparing spectra at these phases to both model spectra, and to spectra obtained at phases immediately before and immediately after the occultation. HD 37776, HD 176582, and HD 182180 all have relatively narrow absorption features, so the projected azimuthal extent can be determined relatively unambiguously. In
### Table 10.2: Cloud properties inferred from eclipses.

<table>
<thead>
<tr>
<th>HD No.</th>
<th>Cloud No.</th>
<th>Depth (%)</th>
<th>$\phi_{ec}$</th>
<th>$r_{ec} (R_\ast)$</th>
<th>Width (km s$^{-1}$)</th>
<th>$d_{az} (R_\ast)$</th>
<th>$d_{merid} (R_\ast)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD37479 1</td>
<td>0.53±0.04</td>
<td>0.26±0.03</td>
<td>2.31±0.30</td>
<td>290</td>
<td>2.00±0.27</td>
<td>0.26±0.06</td>
<td></td>
</tr>
<tr>
<td>HD37479 2</td>
<td>0.62±0.04</td>
<td>0.22±0.03</td>
<td>2.78±0.41</td>
<td>290</td>
<td>2.00±0.27</td>
<td>0.31±0.06</td>
<td></td>
</tr>
<tr>
<td>HD37776 1</td>
<td>0.14±0.01</td>
<td>0.12±0.03</td>
<td>5.24±1.34</td>
<td>150</td>
<td>1.49±0.21</td>
<td>0.09±0.02</td>
<td></td>
</tr>
<tr>
<td>HD37776 2</td>
<td>0.11±0.02</td>
<td>0.09±0.02</td>
<td>7.03±1.58</td>
<td>150</td>
<td>1.49±0.21</td>
<td>0.07±0.02</td>
<td></td>
</tr>
<tr>
<td>HD142184 1</td>
<td>0.28±0.01</td>
<td>0.35±0.03</td>
<td>1.63±0.17</td>
<td>500</td>
<td>1.74±0.21</td>
<td>0.16±0.03</td>
<td></td>
</tr>
<tr>
<td>HD176582 1</td>
<td>0.09±0.02</td>
<td>0.13±0.03</td>
<td>4.83±1.15</td>
<td>190</td>
<td>1.84±0.31</td>
<td>0.05±0.02</td>
<td></td>
</tr>
<tr>
<td>HD176582 2</td>
<td>0.08±0.02</td>
<td>0.09±0.02</td>
<td>7.03±1.58</td>
<td>180</td>
<td>1.75±0.29</td>
<td>0.05±0.02</td>
<td></td>
</tr>
<tr>
<td>HD182180 1</td>
<td>0.15±0.02</td>
<td>0.19±0.03</td>
<td>3.25±0.55</td>
<td>450</td>
<td>1.47±0.17</td>
<td>0.10±0.03</td>
<td></td>
</tr>
<tr>
<td>HD182180 2</td>
<td>0.17±0.02</td>
<td>0.15±0.03</td>
<td>4.17±0.86</td>
<td>390</td>
<td>1.27±0.15</td>
<td>0.13±0.03</td>
<td></td>
</tr>
</tbody>
</table>

The case of HD 37479, the absorption feature apparently extends across the full line profile, suggesting the azimuthal extent to be at least 1 $R_\ast$. For the remaining stars, the azimuthal radii are distributed between about 0.6 and 0.9 $R_\ast$.

Under the assumption that during eclipses the cloud is completely opaque, the percent increase in EW in the line core should correspond to the area eclipsed by the cloud. As will be explored in the next section, this assumption is likely correct, as the clouds are very close to the optically thick limit even when the magnetosphere is seen plane-on. To calculate the meridional extent, the simplest assumption in keeping with the geometrically-thin warped disk predicted by an RRM accumulation surface is that the cloud can be described to first order as a simple rectangular slab. This assumption yields meridional extents ranging from about 0.05 $R_\ast$, for HD 176582, to 0.3 $R_\ast$ for HD 37479. In all cases the clouds are more extended in the plane of the stellar rotation (for small $\beta$) or the magnetic equator (for large $\beta$) than they are in the perpendicular direction. Qualitatively this is in basic agreement with the picture presented by the RRM model.
10.5 Density

10.5.1 Balmer decrements

A standard way of measuring the density of an astrophysical plasma is via Balmer decrements, which are simply the ratio of emission strength between two lines. The relation between Balmer decrements and density was determined via non-LTE calculations originally intended for optically thin accretion disks (Williams and Shipman, 1988), and are valid for volume densities between $\log N = 11$ (below which the plasma becomes too optically thin to detect via line emission) and $\log N = 13.5$ (above which the plasma becomes optically thick, and Balmer decrements cease to be sensitive to further increases in density). The relation is only weakly sensitive to temperature in the regime expected for CMs ($\sim 10$ kK, Townsend et al. 2007), and is insensitive to magnetic field strength (Williams and Shipman, 1988).

The method adopted was similar to that used to measure the density of the decretion disk of the classical Be star $\omega$ CMa (Štefl et al., 2003). The emission strength was measured from normalized spectra, and then corrected for the stellar continuum flux, such that the Balmer decrements $D$ are then:

$$
D_{34} = \frac{f_e(\text{H}\alpha)}{f_e(\text{H}\beta) f_c(\text{H}\beta)}
$$

$$
D_{54} = \frac{f_e(\text{H}\gamma)}{f_e(\text{H}\beta) f_c(\text{H}\beta)}
$$

where $f$ refers to the flux, and the subscripts $e$ and $c$ refer to emission and continuum, respectively. Continuum fluxes were calculated from Bruce-Kylie spectral models, with typical ratios of $\sim 0.35$ for $f_c(\text{H}\alpha)/f_c(\text{H}\beta)$ and $1.44$ for $f_c(\text{H}\gamma)/f_c(\text{H}\beta)$.

Emission fluxes were determined from the residual flux after subtraction of synthetic
spectra. Measurements were made at the phase of maximum emission, as at this phase the plane of the magnetosphere is closest to face-on, thus raising the likelihood that the plasma will be optically thin enough to obtain a meaningful measurement. Furthermore, in order to avoid contamination of the measurement by extra absorption due to occultation (which, depending on the geometry, may play a role even at the phase of maximum emission), measurements were performed independently in the blue and red wings of the line, with the outer limit of integration corresponding to the maximum velocity at which emission can be discerned in H\textalpha, and the inner limit set by either the minimum velocity of emission or \( R_K \), depending on the star.

Balmer decrements are generally measured using EWs. Fig. 10.21 shows the results of this technique for EWs measured in the red and blue line wings separately, and for the combined red and blue EWs, as functions of rotational phase. The middle panels show \( D_{34} \) and the bottom panels \( D_{54} \). There is a modulation of the Balmer decrements with rotational phase, most apparent in \( D_{34} \). Since the density is certainly not changing as the star rotates, this must be due to the changing projection angle of the magnetosphere. Curiously, the apparent density increases near the eclipse at phase 0, and decreases near the eclipse at phase 0.4. Measurements obtained at phases of maximum emission (0.2 and 0.75) all yield \( \log N_e \) between 12.5 and 13.0. However, the uncertainties obtained by propagating the EW error bars through Eqn. 10.3 are quite substantial. Since HD 37479 has the strongest H\textalpha emission, and consequently the strongest emission in higher-numbered Balmer lines, this problem is much worse for many of the other stars, in particular those with weaker emission.

An alternative means of determining the density is to use the emission from H\textalpha as a model with which to predict the emission in H\beta and H\gamma, and to then determine the density via a goodness-of-fit test. Fig. 10.22 shows this method applied to HD 37479. The bottom panels show, from left to right, H\textalpha, H\beta, and H\gamma, with the synthetic spectra shown by
Figure 10.21: Balmer decrements from EWs for HD 37479. *Top:* Emission EWs from the red (left) and blue wings (middle), and the combined (right). Hα in black, Hβ in blue, Hγ in red. *Middle and Bottom:* $D_{34}$ and $D_{54}$. The density corresponding to a given Balmer decrement is indicated by the various horizontal lines. Note the modulation with rotational phase, and also the large uncertainty in even the most precise data.

Results for the weak-emission star HD 176582 are shown in Fig. 10.23. Although the fit to observations is quite good, with a difference of 1% of the continuum between model
10.5. DENSITY

Figure 10.22: Balmer decrements from line profiles. The Hα emission profile is obtained by subtracting a spectral model. The flux in Hβ and Hγ is then predicted by scaling the Hα emission via Eqns. 10.3. The density corresponding to a given flux is indicated by the colour bar.

and observation, the residuals in Hβ and Hγ are dominated by uncertainty in the model as the emission flux in these lines is of a similar magnitude. However, in this case the emission is more tightly localized than is generally seen in the strong-emission stars such as HD 37479. As a consequence it is possible to distinguish between residual flux arising from emission, and that arising from model fits, as the latter causes a systematic bowing in the residuals, while emission leads to localized spikes. This general pattern is seen in all of the weak-emission stars (e.g., HD 142990 and HD 64740). Therefore, for these stars the residual fluxes of the red and blue halves of the line were separately renormalized to either side of the integration limits, as illustrated in the bottom panel of Fig. 10.23.

Balmer decrement densities are summarized for each star and cloud in Table 10.1. Fig. 10.24 shows a comparison between densities measured via $D_{34}$ to $D_{54}$. As expected given the weak emission of these stars and, hence, the very weak emission in Hγ, uncertainties are generally greater for $D_{54}$. Despite this, the agreement between densities measured using
10.5. DENSITY

Figure 10.23: As Fig. 10.22 for HD 176582. In the bottom panels, the residual flux of Hβ and Hγ has been separately renormalized in the red and blue halves of the line. The two diagnostics is generally reasonable.

Fig. 10.25 replicates the regression of maximum emission strength vs. $R_A$, $R_K$, and log $(R_A/R_K)$ in Fig. 10.11 for log $N$ as determined via $D_{34}$. In contrast to emission strength, which is weakly correlated to $R_A$ and most strongly correlated with log $(R_A/R_K)$, log $N$ shows the strongest correlation with $R_K$, with a PCC of -0.53, but no correlation with $R_A$ (PCC of 0.14) and only a weak correlation with log $(R_A/R_K)$ (PCC of 0.41). Hotter
10.5. DENSITY

Figure 10.24: Comparison of logarithmic number densities determined via $D_{34}$ to $D_{54}$.

Figure 10.25: $\log N_e$ vs. $R_A$, $R_K$, and $\log (R_A/R_K)$. Symbol size is proportional to $\cos (\alpha_{\phi=0})$. 
stars also show, in general, higher densities at a given $R_K$.

### 10.5.2 Inglis-Teller effect

While absorption due to occultation of the star by its magnetospheres can skew density measurements made using Balmer decrements, occultations can also be exploited to measure the density using the Inglis-Teller effect (Inglis and Teller, 1939). In a diffuse gas, the last visible absorption line in the H Balmer series is sensitive to the density:

$$N_e = 0.027a_0^{-3}n_{\text{max}}^{-7.5},$$  \hspace{1cm} (10.4)

where $n_{\text{max}}$ is the quantum number of the transition, and $a_0 = 0.529 \times 10^{-8}$ cm is the Bohr radius.

In order to evaluate the presence of this effect, several CM stars were observed with XSHOOTER. Observations were made at occultation phases and at phases of maximum emission, with the latter serving as comparison spectra for the former. Since XSHOOTER’s spectral range extends from 300 nm to 2.48 $\mu$m, the emission status of $H\alpha$ could be evaluated in order to ensure that the observations were in fact made at the appropriate phases. Of the stars showing occultations, all were observed except for HD 176582, which is too far north to be observed from the VLT. Only 1 of the 4 planned observations was completed for HD 37776, and this at an emission phase, therefore the Inglis-Teller effect could not be evaluated for this star. However, observations were completed for HD 37479, HD 142184, and HD 182180, the three remaining stars with eclipsing magnetospheres.

Fig. 10.26 shows XSHOOTER spectra in the vicinity of the Balmer jump for these targets. The spectra were normalized to the relatively flat region between 366 and 367 nm, and for HD 37479 and HD 182180 have been offset for clarity. Comparison with spectra at maximum emission phases shows clear excess absorption during occultation phases. At
Figure 10.26: Balmer jump spectra for (top–bottom) $\sigma$ Ori E, HR 7355, and HR 5907. The volume density indicated via the highest-numbered visible Balmer line is compatible with the density determined via Balmer decrements.
emission phases, the highest-numbered visible Balmer line is H16, consistent with a photospheric spectrum for stars with logarithmic surface gravities between 4 and 4.2. During occultation phases, the highest-numbered visible line is H24 for HD 37479 and HD 182180, indicating log \( N \approx 12.9 \); for HD 142184, the highest-numbered line that can be identified is H21, indicating log \( N \approx 13.3 \).

These densities are more properly thought of as upper limits, since it is possible - given the SNR of the data, the absence of detailed time series with which to search for detailed low level variability, and blending due to rotational broadening - that higher-numbered lines might be detectable in the residual flux of large datasets. There is some indication in HD 182180’s spectra that the highest-numbered Balmer lines might differ between the two clouds: during one eclipse, the highest-numbered visible line is H21, while during the second H26 might be visible. Comparing eclipse and quadrature phases, it cannot be ruled out that the possible detection of H26 in HD 182180’s spectra is just noise. Resolving this question would benefit substantially from improved phase coverage. The important conclusion from these data is that the volume densities inferred for all three stars are essentially consistent with those obtained via Balmer decrements (see Table 10.1), giving additional reason for confidence that the latter measurements of at least these three stars are reasonably robust.

### 10.5.3 Magnetospheric masses

For stars that are eclipsed by their magnetospheres, approximate magnetospheric volumes can be determined. Together with the densities determined above, this enables upper limits to be placed on the masses \( M_{\text{mag}} \) confined by the magnetosphere, which can in turn be compared to the limiting masses inferred from a Centrifugal Breakout (CB) analysis. Once again the assumption is made that the cross-sectional area of the clouds \( A_{\text{cl}} \) is directly equivalent to the percent change in EW during the eclipse. Radial extents are found from \( r_{\text{cl}} = r_0 - R_K \) or, for HD 176582 for which the emission does not extend all the way inwards
to $R_K$, $r_{cl} = r_0 - r_{max}$ (Table 10.1). The cross-sectional area of the cloud is assumed constant, such that the volume is simply $V = A_{cl}r_{cl}$. It is furthermore assumed that the cloud is solid, i.e. that the density is constant throughout. This is obviously not the case: the densities found above likely correspond to the central densities, which are expected to decrease vertically with a certain scale height (Townsend and Owocki, 2005). The radial density is furthermore predicted and observed to decline with distance from the star (see below). However, the object here is to establish upper limits for the masses, for which the assumption of constant density is acceptable.

For $\sigma$ Ori E, this calculation yields $M_{mag} \leq 2 \times 10^{-10}M_\odot$. This is the same result obtained for a more sophisticated RRM treatment modelling the depth of the star’s photometric eclipses (Townsend et al., 2013), a remarkable level of agreement given the much simpler analysis. For HR 7355, $M_{mag} \leq 4 \times 10^{-11}M_\odot$. For HR 5907 $M_{mag} \leq 6 \times 10^{-11}M_\odot$. For HD 176582, $M_{mag} \leq 6 \times 2^{-12}M_\odot$.

The limiting mass $M_\infty$ that can be contained in a CM before a CB event occurs is approximately given by Townsend and Owocki (2005):

$$M_\infty \approx 1.5 \times 10^{-8}M_\odot \frac{B_d^2 R_{12}^2 \xi_*^2}{g_4},$$

(10.5)

where $B_d$ is in kG, $R_{12} = R_*/10^{-12}$ cm, $g_4 = g_*/10^4$ cms$^{-2}$, and $\xi_* = R_*/R_K$. For $\sigma$ Ori E, Eqn. 10.5 yields $M_\infty = 1.2 \times 10^{-8}M_\odot$, about 2 orders of magnitude higher than the upper limit on $M_{mag}$. The dichotomies are even greater for HR 7355 ($M_\infty = 2.3 \times 10^{-8}M_\odot$), HR 5907 ($M_\infty = 1.1 \times 10^{-8}M_\odot$) and HD 176582 ($M_\infty = 1.6 \times 10^{-9}M_\odot$): differences of 2.3, 2.8, and 2.9 orders of magnitude, respectively. Noting that the upper limits on $M_{mag}$ assume a constant density, the true difference is likely even greater.
10.5. DENSITY

Figure 10.27: Density spectra from $D_{54}$ (top) and $D_{34}$ (bottom) for HD 23478, obtained by taking the Balmer decrement in individual velocity pixels in the red (dashed lines) and blue (solid lines) wings. The radial density profiles predicted by the RRM model are shown by dash-dotted purple lines. Horizontal dotted lines indicate the optically thin and thick limits at log $N_e = 11$ and 13.5.

10.5.4 Radial density profiles

As the stellar wind loads the magnetosphere with plasma, and assuming the plasma cannot either fall back to the star (due to centrifugal support) or depart outwards (as the ions cannot cross magnetic field lines), the plasma density will increase until the point that the magnetic field can no longer confine it. The approximate conditions under which magnetic confinement will break down can be determined by equating the net outward gravitocentrifugal force with the inward force from the distorted magnetic field, thus yielding a local breakout density $\rho_b$ (Townsend and Owocki, 2005):
Figure 10.28: As Fig. 10.27 for HD 35502.

Figure 10.29: As Fig. 10.27 for HD 36485.
Figure 10.30: As Fig. 10.27 for HD 37017. As emission in the blue wing is negligible, only the red wing is shown.

\[ \rho_b \approx \frac{B(r)^2}{4\pi h_m \Omega^2 R_\ast - GM^* / R_\ast^2}, \]  

(10.6)

where \( B(r) \sim r^{-3} \) is the local magnetic field strength, \( \Omega \) is the angular frequency of rotation, and \( h_m \) is the scale height of the magnetosphere, used as a typical curvature radius within the densest part of the magnetosphere. Under the assumption that plasma along a given field line will be in hydrostatic equilibrium, \( h_m \) is calculated as

\[ h_m = \sqrt[\hat{\psi}]{\frac{2kT/\mu}{GM^*_c / R_K}} \sqrt[\ddot{\psi}]{\frac{1}{\psi}}, \]  

(10.7)

where \( k \) is the Boltzmann constant, \( T \) is the disk temperature, \( \mu \) the mean molecular weight of the H plasma, and \( \ddot{\psi} \) is the second derivative of the gravitocentrifugal potential around the star:
10.5. DENSITY

Figure 10.31: As Fig. 10.27 for HD 37479.

\[
\tilde{\psi} = \frac{1}{R_K^2} \left[ -2 \left( \frac{r}{R_K} \right)^3 + 3 \right]. \tag{10.8}
\]

Since the magnetic field decreases with distance from the star, while the centrifugal force increases, \( \rho_b \) is a smoothly declining function of \( r \). To compare the observed density with the density predicted via Eqn. 10.6, density spectra were calculated by taking the Balmer decrement in each velocity pixel, using spectra obtained at quadrature. These are shown in Figs. 10.27 to 10.38, with density spectra in the blue and red wings plotted in solid blue and dashed red lines, the breakout density plotted with dot-dashed purple lines, and the Balmer decrement validity limits indicated by dotted black lines.

The results are mixed. In most cases, and in particular for the stars with the strongest emission for which density spectra can be most reliably calculated, the observed density decreases with \( r \), although at a shallower grade than predicted. For HD 142184 and HD182180
Figure 10.32: As Fig. 10.27 for HD 64740. As emission in the blue wing is negligible, only the red wing is shown.

(Figs. 10.33 and 10.37), the predicted density is above the optically thick limit over almost the entire range of emission, which could be interpreted as compatible with an almost flat density spectrum which is indeed very close to this limit. For HD 37479 (Fig. 10.31) the observed density is much higher than predicted at large radii in $D_{34}$, although $D_{54}$ agrees somewhat better with model predictions. In many cases, notably HD 176582 (Fig. 10.36), HD 35502 (Fig. 10.28), HD 142990 (Fig. 10.34), HD 36485 (Fig. 10.29), and HD 37017 (Fig. 10.30), there is a reasonable agreement with predictions over most of the range, although in many cases these are the least reliable measurements. HD 156324 (Fig. 10.35) and HD 35502 show good agreement at large radii, however, closer to the stars the density drops, in contrast to model predictions. For ALS 3694, the data are too noisy to perform a useful comparison.
10.5.5 Stars without emission

Having examined the average circumstellar densities and radial density profiles for emission-line stars, we now turn to the question of whether these data can provide insight into the question of why the majority of stars predicted to possess a CM by their positions on the rotation-confinement diagram show no sign of emission. In Fig. 10.39 the predicted radial density profiles (eqns. 10.6-10.8) of all stars with $R_A > R_K$ are plotted as functions of $R_A$, $R_K$, and $\log (R_A/R_K)$, as in Figs. 10.11 and 10.25. In each case the density profile was calculated from $R_K$ to $2.5R_K$, the latter corresponding approximately to the typical value of $r_0$.

Assuming the CM fills until it bursts open in a CB event, the predicted peak densities are in almost all cases above the optically thin limit ($\log N_e = 11$) below which the CM would be undetectable, i.e. the majority of stars should show some emission. However,
the observed densities (filled circles in Fig. 10.39) are in all cases 1-2 dex below the peak density predicted at $R_K$, as is also clear from the radial density profiles in Figs. 10.27-10.38. Presumably a similar offset between predicted and actual densities should affect the CMs of the non-emission line stars. If the mean difference between the predicted density at $R_K$ and the observed density amongst the Hα-bright stars is subtracted from the radial density profiles of the stars with Hα in absorption, many of the stars are now below the optically thin limit.

There remain several cases in which the predicted density remains comparable to that observed in the emission-line stars. Several of these are relatively cool stars with stronger $B$ fields but very low $\dot{M}$. As was demonstrated in Chapter 9, for such cool stars Krtička mass-loss rates are likely to be more appropriate. The very low mass-loss rates of these stars predicted by the Krtička wind models could indicate that mass leakage empties the
Figure 10.35: As Fig. 10.27 for HD 156324. As there is no emission in the red wing, only the blue wing is shown.

CM faster than it can be filled by the wind. Many of the other stars have mass-loss rates comparable to those of the emission-line stars, but much larger $R_K$ and smaller $\log \left( \frac{R_A}{R_K} \right)$. Many of the absorption-line stars predicted to have detectable CMs have comparable mass-loss rates to the emission-line stars, but larger $R_K$ and lower $\log \left( \frac{R_A}{R_K} \right)$: since the volume of the magnetosphere is smaller, in these cases it could again be a matter of mass leakage removing material faster than it can be replenished by the wind. Notably, $r_{max}$ seems to increase with $R_K$, which may suggest that as the inner edge of the CM recedes from the star plasma may leak more efficiently.

10.6 Summary

In Chapter 9, it was shown that the strength of magnetic confinement and rapidity with which a star rotates are broadly predictive of whether a magnetic B-type star will show
Figure 10.36: As Fig. 10.27 for HD 176582.

Figure 10.37: As Fig. 10.27 for HD 182180.
10.6. SUMMARY

Figure 10.38: As Fig. 10.27 for ALS 3694.

Figure 10.39: Predicted radial density profiles for stars with (vertical solid lines) and without (dotted lines) emission. Densities were calculated from $R_K$ to $2.5R_K$ for each star, with the outer limit corresponding approximately to the typical value of $r_0$ in the emission-line stars. Solid circles indicate the observed densities. Open circles correspond to the density predicted for non-emission-line stars by subtracting from the peak density at $R_K$ the mean difference between the predicted log $N_e(R_K)$ and the observed log $N_e$ amongst the emission line stars. The horizontal dashed line indicates log $N_e = 11$, below which the plasma should be optically thin.
emission from its CM, and that the Hα-bright population is typically younger than the overall population. In this Chapter, it has been shown that the Hα variability pattern shown by individual stars is consistent with expectations from the RRM model: the majority of the emission-line stars have emission originating in two clouds; emission strength follows either a single-wave variation (if only one magnetic pole is visible), or a double-wave variation (if both poles are visible); and the amplitude of variability is furthermore closely related to $i$ and $\beta$, where stars with small $i$ or $\beta$ show low levels of variability, while stars with large $\beta$ and/or large $i$ show larger variability amplitudes. Dynamic spectra reveal that the majority of the sample show asymmetries in either or both the relative emission strengths of the two clouds, and the phases of maximum emission. These anomalies are likely consequences of multipolar contributions to the surface magnetic field, a hypothesis that can be explored with aRRM models based on ZDI maps, as has recently been performed for $\sigma$ Ori E by Oksala et al. (2015b). The most striking departures from axial symmetry occur amongst the two closest binaries in the sample, and it seems likely that in these cases orbital interactions play a role in shaping the CM. Finally, determinations of the horizontal and vertical dimensions of the CM clouds obtained via the depths and Doppler width of spectroscopic eclipse absorptions indicate in all cases that the CM plasma is confined to a geometrically thin region that is much more extended horizontally ($\sim 1R_\star$) than vertically ($\sim 0.1R_\star$), which is broadly consistent with the RRM scale height.

Emission strength correlates to some degree with $R_A$, $R_K$, and $\log (R_A/R_K)$, with the latter being the most predictive of the three. However, there is also an apparent correlation with the mass-loss rate: at a given value of $\log (R_A/R_K)$, stars with higher $\dot{M}$ tend to show stronger emission. This strongly suggests the operation of a mass-leakage mechanism within CMs.

For the three stars for which the measurement could be performed, plasma densities measured using both Balmer decrements and the Inglis-Teller effect yield consistent results,
suggesting that densities measured only using Balmer decrements are also reliable. The
densities measured here are also in general consistent with similar measurements of some
stars found in the literature, e.g. δ Ori C, HR 5907, and HR 7355, all measured using
Balmer decrements (Leone et al., 2010; Grunhut et al., 2012a; Rivinius et al., 2013); σ Ori
E, for which shell absorption at the Balmer jump was used (Groote and Hunger, 1982;
Smith and Bohlender, 2007; Townsend et al., 2013). The upper limits on the mass of
the magnetically confined plasma inferred from the observed density and the approximate
volume of the clouds from their radial extents at maximum emission, and areas during
occultations, are consistently 2 to 3 orders of magnitude below the limiting mass required
for significant CB events. While the relative errors of density measurements tend to be too
high to draw firm conclusions from a comparison with magnetospheric parameters, there is
some indication that density increases with $R_K$. This is again consistent with predictions
from the RRM model: plasma held closer to the star is subjected to stronger magnetic
confinement and, hence, can build up to a higher density. Comparisons of RRM radial
density profiles, calculated assuming density is limited by CB, with profiles obtained from
the residual Balmer line flux in individual velocity pixels, indicate reasonable agreement
between the limiting CB densities and the observed densities at large distances. Curiously
however, in many cases the density peaks halfway through the cloud, and drops closer
towards the star, in direct contradiction to RRM expectations.

Thus, on the one hand there is evidence from emission strengths that the mass balance
within the CM is regulated by a leakage mechanism, while at the same time CM masses are
far below the threshold for CB. On the other hand, the densities obtained are consistent
with CB at large distances, but inconsistent at small distances. This may indicate that the
assumption made by Nakajima in his early modelling of CMs, that as the limiting density is
approached an as-yet unidentified diffusion mechanism begins to operate, was correct after
all (Nakajima, 1985). In retrospect this may not be surprising: as density approaches the
critical limit at which magnetic and thermal energy densities approach equipartition, the RRM assumption that plasma is locked to the magnetic field breaks down. Thus, magnetic field lines will instead begin to follow the plasma rather than the reverse. While this could provide a natural qualitative explanation for the observation that the majority of emission-line stars appear to be near the critical breakout density, it is not clear why a diffusive mechanism should apparently operate, thus preventing explosive reconnection events and maintaining the circumstellar plasma distribution in an approximately steady state.
Chapter 11

Summary and Conclusions

This thesis had three primary goals: 1) determining a more accurate rotation-magnetic confinement diagram for the magnetic B-type stars; 2) comparing magnetic braking timescales to stellar ages; and 3) investigating the question of mass leakage in the CMs of magnetic B-type stars, in particular whether it is eruptive or gradual. To this end, a sample of 51 magnetic stars with spectral types between B5 and B0 was examined. Rotational periods, projected rotational velocities, and longitudinal magnetic field curves were evaluated using a large database of modern, high-resolution, high SNR spectropolarimetry. Using these data, oblique rotator models were determined for each star, and the stars' positions on the rotation-magnetic confinement diagram introduced by P13 were refined. These were used to determine braking timescales, which were compared to the ages inferred from stellar parameters and evolutionary models. A spectroscopic analysis of the Hα-bright stars was then performed in the context of these new constraints.

In the remainder of this chapter, the methodologies and primary findings of each chapter are briefly summarized in Section 11.1. The primary conclusions of this thesis are presented in Section 11.2, and an outline of future work presented in Section 11.3.
11.1 Summary

11.1.1 Multiplicity

Using a multi-component line profile fitting code, radial velocities were obtained from the LSD profiles and, in some cases, individual spectral lines of the SB2/3 stars in the sample (Section 3.1, Fig. 3.1). RVs were then refined via residual minimization using spectral disentangling of LSD and spectral line profiles (Section 3.1, Fig. 3.2, and Appendix A). The disentangled line profiles were used to measure $\langle B_z \rangle$ (Section 4.2), while the RVs were used to obtain new orbital periods for 3 spectroscopic binaries: HD 37061, HD 149277, and HD 156324 (Section 3.2, Fig. 3.3, and Appendix B). HD 156324 is actually an SB3 system, and periods for both A and B components were obtained.

The physical parameters of the orbits were determined using a Monte Carlo method that located the minimum in the $\chi^2$ fitness landscape by comparing radial velocity curves calculated using an evolving set of orbital parameters ($e, \omega, K_1, K_2, v_0$) to the observed radial velocities phased with the orbital period (Section 3.2). These were then used to obtain constraints on the projected total mass, mass ratio, semi-major axis, and the inclination angle of the orbital axis from the line of sight. Orbital periods and parameters are summarized in Table 3.1.

11.1.2 Magnetometry

LSD profiles were extracted using multi-element and single-element line masks (Section 4.1, Figs. 4.1-4.4). The flux of non-magnetic companion stars was removed from the unpolarized line profiles of spectroscopic binary stars using either spectral disentangling or model line profiles, as appropriate to the star. A small number of stars exhibiting rapid radial velocity variability display signatures in their diagnostic null profiles, however these have no impact on $\langle B_z \rangle$ (Section 4.2). $\langle B_z \rangle$ curves for individual stars which have not already been published
are presented in Appendix C.

The general occurrence amongst essentially all Bp stars of strong variance in \( \langle B_z \rangle \) when measured using different elements was demonstrated via LSD profiles extracted with single-element line masks (Section 4.2.1, Figs. 4.8-4.10). No such variation was found amongst chemically normal magnetic stars (Fig. 4.11). This phenomenon is likely a consequence of surface chemical abundance spots, and makes \( \langle B_z \rangle \) measurements less reliable for magnetic modelling. To correct for this, \( \langle B_z \rangle \) was measured using the non-LTE cores of H Balmer lines (Section 4.2.2, Fig. 4.13). The results of these measurements are in excellent agreement with older results, albeit at a much higher precision (Fig. 4.12, see also Appendix C).

The \( \langle B_z \rangle \) curves of the vast majority of the sample are well described by centred magnetic dipoles, showing little to no deviation from first-order sinusoids (Section 4.2.4, Fig. 4.14). Only 4 stars show strong evidence of significant contributions from multipolar field moments (Fig. 4.16), all of which are well known in the literature, and for all but one of which ZDI maps are already available. Amongst the handful of remaining stars showing evidence for multipolar magnetic fields, in all cases the deviations from expectations for a dipole are relatively small as determined via comparison of the best-fit 1\(^{st}\) and 2\(^{nd}\) or 3\(^{rd}\)-order sinusoids. Magnetospheric parameters derived using dipolar ORMs are thus unlikely to be strongly affected by more complex magnetic field topologies.

### 11.1.3 Rotation

The projected rotation velocity \( v \sin i \) was measured for all sample stars using a goodness-of-fit approach incorporating both rotational and radial-tangential macroturbulent broadening \( v_{\text{mac}} \) (Section 5.1). These measurements are summarized in Table 5.1. For single stars mean spectra were created by coadding all available data, and multiple spectral lines were examined, with the final value and the uncertainty generally taken from the mean and standard deviation across all lines (Figs. 5.1-5.3). In two cases, \( \beta \) Cep and \( \xi^1 \) CMa, radial
pulsations were also included (Fig. 5.4). Incorporation of these additional line broadening mechanisms results in a superior fit to the line profiles. For strongly magnetized stars with sharp spectral lines, there is evidence that $v \sin i$ and/or $v_{\text{mac}}$ are increased due to the Zeeman effect (Fig. 5.5): thus, for these stars, results from the least magnetically sensitive spectral lines were adopted.

Comparison of $v \sin i$ measurements for the present sample to published measurements for non-magnetic stars of similar spectral types and luminosity classes shows that, as expected, $v \sin i$ is systematically lower for magnetic stars: the distribution of $v \sin i$ peaks at about 100 km s$^{-1}$ for the magnetic stars, and at about 200 km s$^{-1}$ for the non-magnetic stars (Fig. 5.7). The maximum $v \sin i$ measured in non-magnetic stars, about 450 km s$^{-1}$, is furthermore approximately 150 km s$^{-1}$ higher than the maximum $v \sin i$ seen amongst the magnetic B-type stars. A similar comparison for $v_{\text{mac}}$ yields more ambiguous results: magnetic and non-magnetic distributions both peak at about 20 km s$^{-1}$, but there seem to be more magnetic stars with lower values of $v_{\text{mac}}$ (Fig. 5.7).

Using spectropolarimetric, spectroscopic, and photometric data, we have found new rotation periods for 15 stars, and have refined periods for an additional 15 stars, where a refined period is considered to be one in which a small adjustment to $P_{\text{rot}}$ is necessary to coherently phase the data, but does not result in a significant change in the derived rotational parameters (Section 5.2 and Appendix C). The stars’ rotation periods are listed in Table 5.1. Rotation periods and $v \sin i$ values are mutually consistent and physically plausible, in that no stars were found to be rotating faster than the breakup velocity (Fig. 5.9).

This work has resulted in the discovery of the first tidally locked magnetic early B-type star, HD 156324, for which the rotational and orbital periods are identical. Another significant result has been the discovery that $\xi^1$ CMa, formerly thought to have a rotational period of a few days, is in fact the most slowly rotating magnetic B-type star known, with a rotational period of at least 30 years. Comparison of our sample to catalogues of Ap/Bp star
rotational periods shows a similar distribution of $P_{\text{rot}}$ (Fig. 5.10). There is some evidence that the present sample contains more rapid rotators (Fig. 5.11), although this could well be due to a bias introduced by the inclusion of numerous emission-line stars, all of which are rapid rotators.

11.1.4 Physical parameters

As quantitative spectroscopic modelling has not been performed for many of the stars in the sample, an attempt was made to improve determinations of their physical parameters via analysis of the high-resolution spectroscopy acquired by the MiMeS and BinaMIcS Large Programs. Physical parameters are summarized in Table 6.1.

Effective temperatures were measured using EW ratios of temperature sensitive ions, principally Si II and III in cooler stars, and Si III and IV and He I and II in hotter stars (Section 6.1). These measurements are in generally good agreement with results from both detailed spectral modelling and photometric data, although the uncertainties are smaller when compared to the latter (Fig. 6.1).

The surface gravity $\log g$ is difficult to measure precisely using photometry alone, and in many cases no spectroscopic data had previously been available for analysis. Using the EW ratio effective temperatures, surface gravities were measured using the wings of the H$\beta$ line and, in some cases when significant emission is present, H$\gamma$ (Section 6.2, Figs. 6.3-6.13). This resulted in substantial adjustments in the surface gravities of some stars. For binary stars, $\log g$ was constrained using two- or three-star fits, with the relative radii of the components fixed by the mass ratios determined from their orbital parameters (Section 6.2.1). Excellent results were obtained for HD 149277 in particular (Fig. 6.14), for which surface gravities higher than $\log g = 3.9$ are ruled out from both Balmer line fits, as well as from $v \sin i$ and $P_{\text{rot}}$. Results were more ambiguous for other stars, however in general they confirm that the cooler companion stars should have higher surface gravities than the
hotter primaries, in accordance with expectations for coeval close binaries (Figs. 6.15-6.18).

Luminosities were redetermined for some stars, with BCs based on the new effective temperatures and surface gravities (Section 6.3). In some cases these BC luminosities were adopted in preference to the CHORIZOS luminosities used by P13, as the latter seem to yield systematically larger distances and higher luminosities which are in some cases difficult to reconcile with either the star’s parallax distance or surface gravity. Placing the sample on the $T_{\text{eff}}$ vs. log $L$ and the $T_{\text{eff}}$ vs. log $g$ diagrams shows that their locations on these diagrams are mutually consistent (Section 6.5, Fig. 6.20).

11.1.5 Modelling

In Chapters 7-9, the measured magnetic, rotational, and fundamental physical parameters were used to establish stellar, ORM, and magnetospheric parameters. Depending on the relative error bars in log $L$ or log $g$, the $T_{\text{eff}}$ vs. log $L$ or the $T_{\text{eff}}$ vs. log $g$ diagram was populated with gaussian distributions of $T_{\text{eff}}$, log $L$, or log $g$ values, and stellar radii masses, ages determined via interpolation between evolutionary tracks and isochrones (Section 7.1, Figs. 7.1 and 7.2). When cluster memberships were available, the grid was limited to the isochrones corresponding to the age of the cluster age (Fig. 7.4). The grid of stellar parameters was then pruned by removing points yielding equatorial rotational velocities below $v \sin i$ or above the breakup velocity (Section 8.1, Fig. 8.1). In some cases the grid was also pruned in order to ensure that $i$ remained within limits established via asteroseismology, emission properties, or other inclination-sensitive techniques (Section 8.1.1, Fig. 8.3). Each derived parameter was determined from the peak of the resulting probability density function, with asymmetric 1σ error bars arising from the PDF’s area. This method automatically accounts for correlated uncertainties in rotational, magnetic, and wind parameters by ensuring that all values are mutually consistent, and indeed the derived uncertainties in Alfvén and Kepler radii $R_A$ and $R_K$ are much smaller than those adopted in the P13 sample (Fig. 9.4).
Stellar radii, masses, and ages

Comparing the ages of the sub-samples with and without Hα emission yielded strong evidence that the emission-line stars are systematically younger than the absorption-line stars (Section 7.2, Fig. 7.7). Since Hα emission from a CM seems to require both rapid rotation and a strong magnetic field, this is not surprising, as a strongly magnetized star will tend to spin down more rapidly. Stars with topologically complex magnetic fields also appear to be younger than the general population, suggesting that multipolar magnetic field components might decay more rapidly than dipolar components, which are found at all ages (Section 7.2, Fig. 7.8). The stellar parameters derived in this Chapter are summarized in Table 7.1.

Oblique Rotator Models

The majority of the inclinations determined here are in agreement with values found in the literature, where these are available (Section 8.1.1, Table 8.2). In general, the $T_{\text{eff}}$ vs. log $L$ and $T_{\text{eff}}$ vs. log $g$ diagrams return consistent inclinations (Section 8.1.2, Fig. 8.4). The cumulative distribution of $i$ is consistent with the expectation that $i$ is randomly distributed (Section 8.1.3, Fig. 8.5). However, there is some evidence, here and in previous studies, of a small but persistent systematic bias of $i$ towards smaller angles than expected. The rotational and orbital axes of the binary systems appear to be aligned, a finding compatible with the expected timescales for spin-orbit alignment, circularization, and tidal locking (Section 8.1.4, Fig. 8.6). Inclinations are summarized with rotational properties in Table 8.1.

ORM parameters are provided in Table 8.3. Unlike $i$, $\beta$ does not seem to be randomly distributed, but displays an overabundance of very small and very large angles (Section 8.2.1, Figs. 8.10-8.11). Essentially identical distributions of $\beta$ are recovered from the present sample and from published samples of Ap/Bp stars with rapid or moderate rotational periods. The preponderance of small $\beta$ angles amongst extremely slowly rotating Ap stars
cannot be confirmed in the B-type stars. However, since the sample contains only 3 stars with $P_{\text{rot}} > 100$ d, this regime cannot be adequately explored.

The modelling presented here used $\langle B_z \rangle$ measurements, and made the assumption that $\langle B_z \rangle$ is primarily determined by the dipolar component of the surface magnetic field. $B_d$ shows a log-normal distribution essentially identical to that seen for Ap/Bp stars (Section 8.2.2, Fig. 8.12). There is evidence within the sample that the surface magnetic field strength declines with age, and that this is primarily due to conservation of magnetic flux as the fossil field is stretched by the radiative envelope expanding with age (Section 8.2.2, Fig. 8.14). Examination of the total unsigned magnetic flux yields ambiguous results which could be interpreted as evidence for conservation, decay, or growth in the intrinsic magnetic flux. Previous studies have found similarly contradictory results, although the most reliable study, conducted on Ap stars with well-defined cluster ages, has found that flux most likely decays.

**Magnetospheric parameters and spindown**

A primary goal of this thesis was to refine the rotation-confinement diagram. The re-determined magnetospheric parameters are provided in Table 9.1. The qualitative appearance of the rotation confinement diagram is not strongly affected by these results, although it is significantly cleaner (Fig. 9.6). In particular, all Hα bright magnetic B-type stars are now located in the same region of the diagram, while this region now contains no absorption-line stars (Section 9.3). While the formal uncertainties found here are substantially smaller than those found by P13, there is significant systematic uncertainty in $R_A$ due to the great ambiguity in mass-loss rates (Section 9.2.1, Fig. 9.3). However, as differences in $\dot{M}$ tend to be systematic between different prescriptions, the effect of adopting one or the other is simply to shift all stars towards higher or lower $R_A$. Thus, while the positions of individual stars on the rotation-confinement diagram may change well outside of their formal error
bars via adoption of different mass-loss rates, the relative positions of the stars should not be too strongly affected.

The distribution of rotation periods with fractional age, stellar mass, and magnetic field strength is qualitatively similar to expectations from magnetic braking: at a given fractional age, more massive stars and/or stars with stronger magnetic fields are also slower rotators (Section 9.4.1, Fig. 9.9). It is in particular notable that ξ¹ CMa, by far the slowest rotator in the sample, is also amongst the most massive stars and, amongst stars of similar mass, both the most evolved and the most strongly magnetized. Direct comparison of spindown ages calculated using different mass-loss prescriptions to stellar ages inferred from evolutionary tracks favors the mass-loss rates calculated by Krtička (2014) over those found from the Vink recipe (Vink et al., 2001) (Figs. 9.10 and 9.11). The Vink rates predict much shorter spindown timescales for strongly-magnetized, relatively cool stars, in turn predicting that there should be numerous stars in the lower right of the rotation-confinement diagram, where no such stars are observed. The Krtička mass-loss rates, on the other hand, which decline strongly with temperature, predict spindown ages that are much closer to the evolutionary ages for less massive, highly magnetized stars, and furthermore predict that there should be essentially no stars in the lower right of the rotation-confinement diagram (Fig. 9.16).

The most puzzling anomaly is amongst the high-mass stars. Neither the Vink nor the Krtička mass-loss rates are able to reproduce the rotation periods of these stars. Since there is a disagreement of around 2 orders of magnitude between the maximum spindown age and the evolutionary age for these stars, this is unlikely to be resolved through mass-loss rates alone. This result suggests either an additional source of angular momentum loss beyond magnetic braking alone, or that one or more of the assumptions made in the calculation of magnetic braking timescales is very wrong. One possibility is that the most massive magnetic stars lose the majority of their angular momentum before beginning their main-sequence lifetimes (Figs. 9.12 and 9.13).
11.1.6 Magnetospheric emission

An analysis was conducted of the emission strength and variability of all known Hα bright stars, including the newly discovered CM host stars ALS 3694 and HD 164492C, bringing the total number of stars with emission originating in their CMs to 15/51.

While the inner boundary of emission is typically above but quite close to $R_K$, maximum emission typically occurs significantly above $R_K$, and the difference furthermore appears to increase with $R_K$ (Section 10.1, Fig. 10.3). This is in contrast with the RRM prediction that plasma density should peak at the inner edge of the CM, i.e., at $R_K$ itself.

Variability patterns appear to be well explained by the RRM model: the amplitude of variability is sensitive to $i$ and $\beta$, with larger angles resulting in higher levels of variability; furthermore, stars exhibit single- or double-wave variations depending on whether one or two magnetic poles are visible (Section 10.2, Fig. 10.5). There is a widespread prevalence of asymmetries in phase and emission strength between different clouds (Figs. 10.6-10.9). This is most likely explained by the influence of multipolar magnetic field components in shaping the RRM accumulation surface. However, another, not necessarily mutually exclusive possibility that deserves investigation is that surface anisotropies in mass-loss rates, arising due to gravity darkening or chemical spots, may lead to circumstellar asymmetries. The two stars showing the most remarkable asymmetries in emission strength between their clouds, HD 37017 and HD 156324, are both close binaries (Section 10.2.1). In both cases comparison of the orbital and magnetospheric spatial scales suggests an orbital influence on the RRM accumulation surface (Fig. 10.10).

Emission strength appears to be sensitive to both $\log (R_A/R_K)$ and $\dot{M}$, thus, following the conjecture in P13 (see also Fig. 1.8), the evidence supports a mass-leakage scenario over one in which the volume of the CM is the sole determining factor in emission strength (Section 10.3, Figs. 10.11-10.13). The question of what this leakage mechanism might be is considered in further detail below. However, this somewhat coherent relationship is
obtained only using the Vink mass-loss rates: using the much more temperature-sensitive Krtićka prescription the influence of $\dot{M}$ on $R_A$ plays a much stronger role than with the Vink prescription, and there is no correlation between emission strength and $\log (R_A/R_K)$ (Fig. 10.14). Thus, while the overall lower Krtićka mass-loss rates are favoured by the analysis of the stars’ rotational evolution, the Vink mass-loss rates provide more coherent results for the stars’ emission properties.

Plasma densities were measured using Balmer decrements of residual (observed - model) fluxes obtained at maximum emission phases (Section 10.5.1, Figs. 10.21-10.23). For $\sigma$ Ori E, HR 5907, and HR 7355, densities were also measured via the Inglis-Teller effect using shell absorption during eclipses (Section 10.5.2, Fig. 10.26). The Inglis-Teller effect has been used several times for $\sigma$ Ori E (Groote and Hunger, 1982; Smith and Bohlender, 2007), with consistent results including those obtained here; observations of this kind for HR 5907 and HR 7355 are presented here for the first time. Densities are generally consistent between both shell-absorption and Balmer decrements, and using either $D_{34}$ or $D_{54}$ (Fig. 10.24).

Radial density profiles are generally consistent, at large radii, with RRM density profiles that assume CB (Section 10.5.4, Figs. 10.27-10.38). However, close to the star (but still above $R_K$), and in direct contradiction to RRM expectations, in many cases the density appears to drop. This is similar to the observation that maximum emission occurs well above $R_K$, rather than precisely at $R_K$. The radial gradient in density is also typically shallower than expected from the RRM model.

The vertical and horizontal cloud dimensions obtained via examination of eclipses are generally consistent with RRM expectations: the clouds are geometrically thin, with vertical heights comparable to RRM scale heights (about 0.1 $R_*$) and horizontal widths comparable to 1 $R_*$ (Section 10.4, Fig. 10.18). The upper limits on cloud masses inferred from their volumes and densities are however 2-3 orders of magnitude below the limiting mass inferred from a CB scenario (Section 10.5.3). This result was previously available only for $\sigma$ Ori E
(Townsend et al., 2013); this work has thus established that cloud masses are in general much less than required for a significant CB event to occur.

The majority of stars without emission have predicted densities well above the threshold at which optical emission should be detectable (Section 10.5.5, Fig. 10.39). Adjusting these predictions to account for the much lower-than-predicted densities found in the inner magnetospheres does indeed move many of the absorption line stars below the optically thin threshold. Some of the other absorption-line stars predicted to have significant CMs are also fairly cool; if Krtička mass-loss rates are correct, the absence of emission in these stars could simply be a consequence of vanishingly weak winds. There are however still several stars predicted to show emission, which do not. It seems likely that whatever mass-leakage mechanism is in operation in the CMs of Hα-bright stars also operates within the optically invisible CMs of the absorption line stars, i.e. it must prevent accumulation even at low densities. Determining the nature of this mechanism will be crucial to explaining the dichotomy in the emission properties of magnetic B-type stars.

11.2 Conclusions

Determining the magnetospheric parameters of the population required that their rotational and magnetic parameters first be calculated. The longitudinal magnetic field curves of the overwhelming majority of the sample are consistent with primarily dipolar surface magnetic fields, indicating that dipolar Oblique Rotator Models (ORM) should give an adequate description of the population’s magnetic properties. However, it is worth pointing out that close inspection of the variable emission of the Hα-bright stars suggests that there may in many cases be multipolar contributions to the surface field that are not readily apparent in $⟨B_z⟩$. Determining ORM parameters enabled comparison of the population’s magnetic geometries and field strengths to the cooler population of Ap stars. With the exception of very slowly rotating Ap stars, most of which have small obliquity angles $\beta$, the distributions
of $\beta$ for Ap stars and B-type stars are statistically consistent with a single population. The same is true of the distribution of surface magnetic dipole strengths $B_d$, which in both cases is more or less log-normal, peaking around 3-4 kG. There is furthermore good evidence that $B_d$ declines with age, and that this decrease is consistent with the conservation of magnetic flux in an expanding stellar atmosphere.

The positions of the sample stars on the rotation-magnetic confinement diagram have been greatly clarified. In particular, essentially all stars with H$\alpha$ emission are now known to reside in the top right of the diagram, while there are no stars with H$\alpha$ in absorption occupying the same region. Thus, essentially all magnetic early B-type stars with strong magnetic fields ($\sim 10$ kG) and rapid rotation ($P_{\text{rot}} < 1.6$ d) show H$\alpha$ emission.

Emission-line centrifugal magnetospheres are found predominantly amongst the youngest magnetic stars: approximately 2/3 of those stars with fractional main sequence ages below 0.25 host H$\alpha$-bright CMs, while the fraction declines precipitously with advancing age. This is in line with expectations from magnetic braking: strongly magnetized stars should shed angular momentum more rapidly via their magnetically confined stellar winds than less strongly magnetized stars with otherwise similar stellar parameters, thus CM emission should be, and is, a phenomenon limited to stars close to the ZAMS.

$P_{\text{rot}}$ increases with both fractional main sequence age, stellar mass, and $B_d$, in keeping with the expectation that stars with stronger winds and/or magnetic fields should spin down more rapidly. However, direct comparison of stellar ages inferred from evolutionary models $t_{\text{evol}}$ to gyrochronological ages $t_{S,\text{max}}$ revealed two interesting anomalies. First, amongst many of the cooler and more strongly magnetized stars, $t_{S,\text{max}} << t_{\text{evol}}$ when using mass-loss rates extrapolated from O-type stars (Vink et al., 2001), a disagreement that is largely resolved if the (much lower) mass-loss rates calculated explicitly for B-type stars are used instead (Krtička, 2014). Both mass-loss prescriptions yield $t_{S,\text{max}} >> t_{\text{evol}}$ for hotter stars with weaker magnetic fields. The second anomaly cannot be reconciled.
by taking evolutionary effects (i.e. weaker or stronger mass-loss rates, weakening surface magnetic field, and changing gyration radius) into account. One possible resolution is that the initial rotation fractions of the most massive stars in the sample may be substantially less than unity, although a plausible physical explanation for such a discrepancy is not immediately apparent.

The primary goal of this work was to investigate the rotational, magnetic, and emission properties of CM host stars in order to see if this might distinguish between gradual or stochastic mass balancing mechanisms. If emission strength is sensitive to \( \log (R_A/R_K) \) alone, this would indicate that plasma is evacuated from CMs via Centrifugal Breakout (CB). To the contrary, emission seems to be sensitive to both \( \log (R_A/R_K) \) and \( \log L \) or \( \dot{M} \): stars with similar \( \log (R_A/R_K) \), but higher \( \dot{M} \), tend to have stronger emission. The evidence thus favours a leakage mechanism. Furthermore, the lower limits on cloud mass inferred from eclipses are systematically 1-2 dex below the limiting mass implied by CB. The volume density, as determined both using Balmer decrements at quadrature phases and via the Inglis-Teller effect during eclipses, is in many cases well below the limiting density calculated under the assumption of CB.

Comparison of theoretical vs. observed radial density profiles indicates that the greatest disagreements are typically found close to the star, while at large distances the predicted and observed profiles are quite similar. This may be a clue as to the nature of the leakage mechanism. It may also be significant that, while a CB treatment predicts a monotonically increasing density as \( r \) approaches \( R_K \), the observed density profiles often peak at \( r > R_K \), declining towards smaller as well as larger radii. This may be a sign of radial plasma transport within the CM. Along these lines, it is interesting to note that the emission profiles predicted by an aRRM model for HD 37479 extend only to \( \sim \pm 500 \) km s\(^{-1}\), about half the observed range of \( \sim \pm 900 \) km s\(^{-1}\)(Oksala et al., 2015b).
11.3 Future work

Detailed quantitative spectroscopic modelling should be performed for all stars in the sample, in order to determine effective temperatures and surface gravities self-consistently with mean surface abundances, as the latter may influence the former. In addition to enabling more precise physical parameters with which to constrain their rotational and magnetic properties, such modelling will also yield insight into the long-term evolution of the surface abundances of Bp stars. Evolutionary trends in surface abundances have been reported for Ap stars (Bailey et al., 2014), however, no comparable study of Bp stars has yet been performed.

It may also be necessary to revisit the magnetic and magnetospheric modelling when Gaia results become available. Gaia’s extremely high astrometric precision, $\sim 10 \, \mu\text{as}$ in the magnitude range of interest for the present sample, will enable much more accurate distances than are available from Hipparcos parallaxes or cluster distances. The improvement in luminosities and, hence, in radii will enable much tighter constraints on ORM parameters. It will be interesting to see if Gaia parallaxes are able to remove the systematic shift in $i$ observed in this and in previous samples to slightly smaller values than expected from a purely random distribution and, if so, whether some of the conclusions regarding trends in $B_d$ and $\beta$ are robust.

The discrepancy between evolutionary and magnetic braking timescales for massive magnetic stars must be addressed. While there is no doubt that magnetic braking via the wind must play a role, it seems that this alone cannot account for the slow rotation of the most massive stars. Such an investigation will almost certainly require stellar evolutionary models incorporating magnetic phenomena, e.g. solid-body rotation. Is the gyration radius strongly affected by internal magnetic fields? Does the internal topology of the field,
primarily toroidal or poloidal, matter? Must the field permeate the entire star, or is it sufficient that it remain in the radiative zone? Recent simulations have shown that the vigorous convection in the cores of massive stars almost certainly sustains powerful core dynamos (Augustson et al., 2016): do these interact with the fossil magnetic field in the radiative envelope, and if so, what are the consequences for rotational evolution? The possibility that more massive stars stars lose a greater proportion of their angular momentum on the PMS than less massive magnetic stars should also be investigated. A similar population study to that conducted here, but focusing on magnetic Herbig Ae/Be stars, could prove instructive in this regard. Currently, only a few magnetic PMS massive stars are known (Alecian et al., 2013a; Petit et al., 2013), and a rotation period is known for only one of these, HD 36982. A survey using the NIR spectropolarimeter SPIRou will be essential to expanding this sample, and to determining their magnetic and rotational properties.

The rotational evolution of binary magnetic stars deserves much closer attention. Here it has been demonstrated that the spin and orbital inclination angles are in close alignment, and that this is consistent with the expectation that spin-orbit alignment will occur before circularization of the orbit (only 2 close binaries have $e$ consistent with 0), which will in turn occur before synchronization of the orbital and rotational periods (only HD 156324 is tidally locked, and no systems show evidence of obviously harmonic rotational and orbital cycles). All three of the magnetic emission-line binaries are accompanied by non-magnetic companions with lower $v \sin i$ values than the secondary, a pattern which repeats for the rapidly rotating but non-emission line star HD 37061. For HD 37061 and HD 37017 the difference is quite high, over 100 km s$^{-1}$. One would ordinarily expect the magnetic star to be the slower rotator. That it is generally quite the opposite amongst magnetic binaries further suggests the possibility of magnetically mediated spin-orbit coupling. Since the magnetic star’s photosphere is anchored to the circumstellar environment via the magnetic field, does this lead to more efficient transfer of angular momentum from the orbital
reservoir? This should in principle be possible to investigate via MHD simulations and, possibly, semianalytic models based on the RRM formalism.

It is also perhaps significant that, while close binaries containing a magnetic star are vanishingly rare in the overall hot star population (<3%; Alecian et al. 2015), and, excepting HD 25558 and HD 122451 which are both in fairly wide orbits, comprise only 12% of the current sample, half of these systems contain an emission-line star, while emission-line stars themselves make up only about 25% of the overall population. Given the small numbers involved the statistical significance of this result is quite low, however it is suggestive. In all three cases the magnetic star is the primary, and has a higher $v \sin i$ than the secondary star. Amongst the remaining magnetic binaries, HD 37061 continues the pattern of a much more rapidly rotating, as well as more massive magnetic primary; HD 149277 does not continue this pattern; and in the case of HD 136504, both stars are magnetic. Could spin-orbit coupling via the magnetosphere more efficiently spin up the magnetized companion? If so, this could explain the observation that emission appears to be more common amongst close binary systems.

The consequences for orbital evolution may also deserve attention: if orbital angular momentum is drawn down more rapidly in the presence of a magnetic field, and if orbital decay timescales consequently become short in proportion to evolutionary timescales, then rapid decay of the orbits of stars in systems containing a close magnetic binary could explain the remarkable deficit of such systems. The binary fraction for non-magnetic hot stars is close to 100% (Sana et al., 2012), while $\leq 3\%$ of close hot binaries contain a magnetic star (Alecian et al., 2015). This is also much less than the fraction of magnetic stars in the general population (about 10%, Grunhut et al. 2011), indicating that magnetic binaries are exceptionally rare. This could imply a high merger rate for magnetic binaries. Smooth particle hydrodynamics simulations indicate that mixing during a merger process could rejuvenate a star (Schneider et al., 2016), which could in turn be detectable via
incongruencies in stellar ages when compared to nearby stars. The visual binary HD 61556 shows just such an anomaly (Shultz et al., 2015a), however it is not obvious whether its companion is in fact a coeval, physically associated star, or gravitationally captured at a later point in the star’s life, or simply a chance alignment. It will be important to conduct a close comparison of the evolutionary status of both close magnetic binaries to their companions, and single magnetic stars in clusters to other cluster stars, in order to see if magnetic stars appear systematically younger than nonmagnetic stars.

Close magnetic binaries may also provide an excellent laboratory with which to test models of stellar evolution with and without significant surface magnetic fields. HD 149277 in particular is an excellent prospect: the sharp lines of the two components and large radial velocity amplitudes mean that the spectra of the two stars can be unambiguously disentangled, enabling precise physical parameters for each component to be determined.

Virtually all emission stars in this sample show some degree of asymmetry in phase or emission strength between clouds, suggesting pervasive multipolar magnetic fields. This is not so surprising: emission line stars tend to be young, and as demonstrated in Chapter 7 it is precisely amongst the youngest stars that complex field topologies tend to be seen. Thus, modelling their circumstellar structures will require aRRM models, which will in turn require ZDI maps. Kochukhov and Wade (2016) showed that, for broad-lined stars, unique ZDI maps can be obtained with Stokes \( V \) only, but that for sharp-lined stars, Stokes \( Q \) and \( U \) are also required. These data are not available for most stars, and will need to be acquired. These efforts may benefit from magnetospheric tomography under development by Jason Grunhut, which reconstructs the 3D circumstellar structure of the CM. Close comparison of tomographic results to RRM and aRRM models will help to determine what the missing physics in the models might be. It will be important to investigate the effects of both gravity darkening and surface abundance spots, both of which affect the local mass-loss rate and, hence, the magnetosphere. If incorporation of these physics in aRRM models
significantly improves agreement with observations, this will serve as additional evidence that mass-loss rates are generally lower than simple extrapolations from O-type stars would suggest. Remaining inconsistencies may provide clues as to the nature of the mass-leakage mechanism.

Ultraviolet data have not been examined here, and indeed have not received close attention since the formalization of the RRM model. Since UV emission is seen across the rotation-confinement diagram, it is almost certain that it is sensitive to the DM. Whether useful information on CMs can also be obtained from IUE data will require detailed investigation, first by comparing emission variability in CM vs. DM stars with comparable stellar parameters, second by comparing predictions from the Analytical Dynamical Magnetosphere formalism (Owocki et al., in prep.) to the observed emission profiles, and third by determining to what degree incorporation of predictions from Rigid-Field Hydrodynamics simulations can resolve any discrepancies that are noted. The variability in these lines may contain valuable information on the interface between the DM and CM, and hence provide insight into mass leakage.

Other wavelengths, in particular X-ray and radio, will be extremely important to explore. X-ray emission is a well-known consequence of magnetically confined wind shocks (Oskinova et al., 2011; Cassinelli et al., 1994; Babel and Montmerle, 1997; Petit et al., 2013; ud-Doula et al., 2014; Nazé et al., 2014). Ho-bright stars often exhibit even higher X-ray luminosites than can be explained by wind shocks alone (Nazé et al., 2014), which may be a consequence of centrifugal heating of the plasma (Townsend et al., 2007). Unfortunately, X-ray data remain scarce in comparison with optical data, a situation which is being rectified as of the writing of this thesis. As X-rays are sensitive to the hot post-shock plasma, it will be most instructive to see the results of a large survey of the X-ray emission properties of B-type stars.

Radio data are remarkably scarce, with observations available for only a handful of
stars and few with detailed time-series (Drake et al., 1987; Linsky et al., 1992; Leto et al., 2012). Non-thermal radio emission has been detected around some several Bp stars (Drake et al., 1987; Linsky et al., 1992), which has been interpreted as gyrosynchrotron emission originating in the outer magnetosphere. Electron-cyclotron maser emission has also been detected around the cool Bp star CU Vir (Trigilio et al., 2000). A survey of magnetic OB stars at high and low frequencies is currently being conducted. The early results published by Chandra et al. (2015) show some evidence of rapid variability on timescales much shorter than the rotational period. If confirmed with a larger dataset, this could point to radio data as a key diagnostic of time-sensitive plasma dynamics within hot star magnetospheres.

Investigating the physics of CMs is a daunting task from the standpoint of simulations, as the short time steps required to resolve MHD processes in the regime of extremely strong magnetic fields and high centrifugal stresses is much too short for realistic 3D simulations, while even 2D simulations have so far been confined to a regime of moderate magnetic confinement ($\eta_* \sim 100$) and rotation ($W \leq 0.5$) that are far removed from the regimes in which the CMs of real stars become detectable (Townsend and Owocki, 2005; ud-Doula et al., 2006, 2008, 2009). This limitation in numerical simulations motivated the creation of the RRM model, however, due to its static nature RRM is unable to explore the importance of time-dependent processes within CMs. Laboratory astrophysics may provide an answer. The Wisconsin Plasma Astrophysics Laboratory, WiPAL, will have amongst its capabilities the ability to recreate the magnetic confinement and rotational conditions in the circumstellar environment of a rapidly rotating magnetic massive star, for both aligned and oblique dipole geometries (Forest et al., 2015). By investigating CMs in a laboratory setting, with no simplifying assumptions made about the plasma physics, a direct test of the CB hypothesis can be performed.

Evaluating the plausibility of different diffusive mechanisms responsible for maintaining CM plasma in a steady state should be a high priority for theorists. Just as early models of
σ Ori E’s magnetosphere were influenced by then-recent advances in our understanding of the Jovian magnetosphere, it may be useful to return to this relatively well-studied system for inspiration. One leading contender is the interchange instability, in which buoyant convection of magnetic field lines transports the dense, cold plasma of the inner magnetosphere outwards, while magnetic fields from the outer magnetosphere return inwards filled with hot, thin plasma (Blanc et al., 2005; Krupp et al., 2004). Unfortunately, our understanding of plasma transport in the Jovian magnetosphere, in particular the outward transport of cold plasma, remains rudimentary, and it is not obvious how to incorporate such a scenario into RRM models in a physically rigorous fashion.
Bibliography


Appendix A

Disentangling of binary LSD Stokes $I$ profiles

This appendix presents the disentangling results for the spectroscopic multiple systems in the sample.

**HD 25558**: Both of the components of HD 25558 are SPB stars (Sódor et al., 2014), and the resulting line profile variability is clearly apparent in Fig. A.1. As a consequence, disentangling was only moderately successful for this star.

**HD 35502**: HD 35502 is a hierarchical triple (Sikora et al., 2016). The radial velocity of the magnetic B-type primary is not variable, while the two A-type components form a close binary system in a distant orbit around the B-type star. The A-type components do not exhibit line profile variability, while the intrinsic variability of the B-type component is minimal. Disentangling can thus be considered successful for this star (Fig. A.2).

**HD 36485**: The difference in contrast between the early B-type primary and early A-type secondary is quite high (Leone et al., 2010), with the consequence that the secondary contributes minimally to the LSD profile when all lines are included in the mask (Fig. A.3). The secondary’s contribution is much more apparent in LSD profiles extracted using an Fe line mask (Fig. A.4), which were used to determine the radial velocities of the two components. Due to the low level of intrinsic variability in the two components, disentangling yields a good reproduction of the combined line profiles.
Figure A.1: Disentangling for HD 25558. The top panels show the disentangled Stokes I LSD profiles of the two stars (left, the magnetic secondary; right, the non-magnetic primary) in their respective rest frames (black lines), with the mean flux shown by dashed red lines. The bottom panels show (from left to right) the total flux, and the flux of the individual components, for individual observations, arranged in order of acquisition from bottom to top. The combined model flux is shown by solid red lines; the flux of the magnetic component by dotted green lines; the non-magnetic component by dashed blue lines.
Figure A.2: As Fig. A.1 for HD 35502, although for three stars: from left to right, the broad-lined magnetic component, and the two A-type stars.
Figure A.3: As Fig. A.1 for HD 36485. The top panels show the magnetic B-type star (left) and the non-magnetic A-type star (right).
Figure A.4: As Fig. A.3, using only Fe lines.
Figure A.5: As Fig. A.1 for HD 37017. Top panels show the magnetic early B-type star on the left, and the non-magnetic late B-type star on the right.
**HD 37017:** The primary star exhibits strong line profile variations, leading to large differences between the mean disentangled profile and those of individual observations (Fig. A.5). Thus, the reproduction of the observed profiles by the combined model profile is only approximate. However, the sharp-lined secondary is reasonably well-reproduced, so disentangling removes the majority of its contribution from the broad-lined primary’s Stokes $I$ profile.

**HD 37061:** Substantial line profile variability in the magnetic broad-lined primary lead to large residuals in the wings of the narrow-lined secondary, as is clear from Fig. A.6. However, since flux outside of the line profile is discarded, this does not strongly impact the mean line profile, and disentangling removes the majority of the secondary’s contribution to the LSD Stokes $I$ profile.

**HD 122451:** Both components of HD 122451 are $\beta$ Cep stars, contributing approximately equal amounts to the line profile, and the radial velocity variability due to orbital motion in the HARPSpol observations is relatively low as only a small fraction of the orbit is covered. The broad-lined component shows especially strong line-profile variations, as is readily apparent in Fig. A.7, which are clearly due to non-radial pulsations. Despite these limitations a reasonable agreement is achieved for both the combined profiles and the line profile of the narrow-lined magnetic star.

**HD 136504:** The primary star in the HD 136504 system certainly exhibits line profile variability, and the secondary likely does as well (Fig. A.8). In both cases this is likely a consequence of non-radial pulsationUytterhoeven et al. (2005). Nevertheless, intrinsic variability in both cases is relatively small, so disentangling successfully reproduces the combined line profile (Fig. A.8).

**HD 149277:** This SB2 system benefits from a large radial velocity amplitude, a large dataset, and relatively little intrinsic variation in the broad-lined non-magnetic secondary. The narrow-lined magnetic primary exhibits significant line profile variations, which show
Figure A.6: As Fig. A.1 for HD 37061. Top panels show the magnetic early B-type star on the left, and the non-magnetic late B-type star on the right.
Figure A.7: As Fig. A.1 for HD 122451. The narrow-lined magnetic secondary is shown in the top left panel, the broad-lined non-magnetic primary in the top right panel.
Figure A.8: As Fig. A.1 for HD 136504. The top left shows the primary, the the top right the secondary.
Figure A.9: As Fig. A.1 for HD 149277. The top left shows the narrow-lined magnetic primary, the top right the broad-lined non-magnetic secondary.
up as substantial residuals in the continuum of the secondary’s residual flux (Fig. A.9). However, since the components are separated in radial velocity at almost all phases, this does not strongly affect the disentangling process, which is successful in removing the secondary’s contribution to the Stokes $I$ profile in most of the observations in which the two components are blended.

**HD 156324**: This SB3 system is particularly challenging: the magnetic broad-lined primary is blended with the sharp-lined component at almost all phases and exhibits strong line profile variations (Fig. A.11). There is furthermore some indication of line profile variability in the sharp-lined component, which exhibits surface chemical peculiarities of its own Alecian et al. (2014). Thus, the reproduction of the combined profile is only approximate, and in some cases the disentangled Stokes $I$ profile of the primary is either over- or under-corrected for the flux of the sharp-lined component.

**HD 164492C**: while HD 164492C is technically an SB3, the third component, a B5-6 star, is extremely faint compared to the other two components (B2 and B1 stars) and so is not included in the disentangling procedure. Line profile variability is negligible for both components, so disentangling yields a very good fit for both stars.
Figure A.10: As Fig. A.1 for HD 156324. The top row shows the three components, from right to left: the broad-lined magnetic primary, the broad-lined non-magnetic secondary, and the narrow-lined non-magnetic tertiary.
Figure A.11: As Fig. A.1 for HD 164492C. The top row shows, on the left, the broad-lined magnetic star, and on the right, the sharp-lined non-magnetic star.
Appendix B

Radial velocity curves of spectroscopic binaries

**HD 36485**: RVs of the dim secondary star were measured from ESPaDOnS and FEROS data using the Ti ii 456.4 nm line, which is dominated by the contribution from the A-type secondary. Due to the difficulty of measuring the secondary’s RVs, we adopt an error of 4 km s$^{-1}$, twice that adopted for the sharp-lined primary’s RVs. The agreement between the new RVs (filled red and blue symbols in Fig. B.1), and published RVs Leone et al. (2010) (open black symbols in Fig. B.1), is only approximate when phased with the original ephemeris. Period analysis indicates a slightly more precise period of $P_{\text{orb}} = 29.968(2)$ d (Fig. B.1, top). We also find a best-fit orbital model with an argument of periapsis $\omega = 160 \pm 13^\circ$, slightly smaller than the $175 \pm 4^\circ$ originally determined Leone et al. (2010).

**HD 37017**: the orbital period and elements were determined by Bolton et al. (1998). The published measurements Bolton et al. (1998) (open black symbols) are compared to our own (filled red and blue symbols) based upon ESPaDOnS and FEROS data in Fig. B.2. We adopted an uncertainty of 10 km s$^{-1}$ for the RV of primary, due partly to this star’s high $v \sin i$, but also due to the substantial line-profile variability induced by chemical spots. The two datasets are well-phased by the original ephemeris, without modification.

**HD 37061**: the primary has very broad spectral lines and is substantially brighter than the secondary; as the secondary has relatively sharp lines (see Figs. 3.2 and A.6) and a larger RV
Figure B.1: Top: periodogram for RV measurements (solid black line) and for synthetic noise with the same window function (dashed blue line). The maximum-amplitude period is indicated by a red circle. Bottom: RV curve for HD 36485. Open black triangles symbols are measurements from the literature (Leon et al. 2010); filled red circles are from ESPaDOnS data; filled blue squares from FEROS data. The solid (dashed) lines correspond to the orbital model for the primary (secondary) in Table 3.1.
amplitude, period analysis was conducted on the secondary star’s RVs (compare Fig. B.3, top). As with the period analysis for the \( \langle B_z \rangle \) measurements (Fig. C.8), there are numerous peaks in the periodogram; many of these correspond to peaks in the synthetic noise period spectrum. The highest peak is at 14.31(4) d, which we adopt, however, further observations are necessary to confirm this orbital period. Radial velocities for the two components are shown in the bottom panel of Fig. B.3.

**HD 149277**: both components are of approximately equal brightness, and the RV semi-amplitudes are much larger than the profile widths (compare Fig. 3.2 and Table 3.1). Period analyses were therefore conducted separately on both datasets, and yielded the same results (Fig. B.4, top panel): \( P_{\text{orb}} = 11.517(7) \) d. RV measurements, all based upon ESPaDOnS data, are shown phased with \( P_{\text{orb}} \) in the bottom panel of Fig. B.4. It is worth noting that \( P_{\text{orb}} \) was first determined on the basis of the ESPaDOnS data collected in 2014. Subsequent ESPaDONS observations in 2015 confirmed this period with only slight modification.

**HD 156324**: this star is an SB3 system. The semi-amplitudes of the two A components
Figure B.3: *Top:* periodogram for HD 37061’s sharp-lined secondary’s RV measurements (solid black line) and for synthetic noise with the same window function (dashed blue line). The maximum-amplitude period is indicated by a red circle. *Bottom:* RV curve for HD 37061. Squares indicate primary RVs; triangles, secondary. The solid (dashed) lines correspond to the orbital model for the primary (secondary) in Table 3.1.
Figure B.4: Top: periodogram for HD 149277’s RV measurements (solid black line) and for synthetic noise with the same window function (dashed blue line). The maximum-amplitude period is indicated by a red circle. Bottom: RV curves for HD 149277. Squares indicate primary, triangles secondary. The solid (dashed) lines correspond to the orbital model for the primary (secondary) in Table 3.1.

are similar to those of HD 37061 (compare Figs. B.3 and B.5, bottom panels), but the line-width of the primary (∼35 km s⁻¹) is much less than that of HD 37061. Thus, precise RVs could be measured for both A components, and the period analysis was conducted on both stars. This star’s orbital period is the same as the rotational period inferred from both magnetic and spectroscopic data (Table 5.1, and Fig. C.27), implying that the system is tidally locked. Precise RVs could also be measured for the sharp-lined B component,
Figure B.5: Top: periodogram for HD 156324 A’s RV measurements (solid black line) and for synthetic noise with the same window function (dashed blue line). The maximum-amplitude period is indicated by a red circle. Bottom: RV curves for HD 156324. The solid (dashed) lines correspond to the orbital model for the primary (secondary) in Table 3.1.

yielding $P_{\text{orb}} = 6.7(1)$ d (Fig. B.6, top). The RVs of the B component are shown in the bottom panel of Fig. B.6. While a coherent variation is seen in the 2015 RVs (solid circles), RVs measured in 2012 and 2014 (open circles) are systematically higher by $\sim 20$ km s$^{-1}$. This may reflect a change in the centre of gravity of the B component’s orbit, perhaps around the A pair. Further observation of this system is clearly necessary.
Figure B.6: Top: periodogram for the RVs measured for HD 156324 B, using only the 2015 ESPaDOnS and FEROS data. Bottom: RV curve. Open squares indicate RV measurements from 2012; open triangles, 2014; filled circles, 2015.
Appendix C

Rotation periods and longitudinal magnetic field measurements of individual stars

In this appendix we present stars for which we possess new $\langle B_z \rangle$ measurements but do not have improved periods (5 stars), stars with existing periods that we have refined using new data (15 stars), and the stars for which we have determined new rotational periods using magnetic, spectroscopic, or photometric data (15 stars). For stars with new or refined periods we present periodograms and, in some cases, photometric or spectroscopic data together with $\langle B_z \rangle$. The abscissae of the periodograms are restricted to the period windows discussed in Section X. For stars with newly determined rotation periods, we state in the text whether we consider the period to be confidently established (8 stars), or either tentative or only partially constrained (7 stars).

Stars for which the existing magnetic data have already been published, or for which detailed individual studies are in preparation, are not included in this appendix. These are HD 3360 Briquet et al. (2016), HD 23478 Sikora et al. (2015); HD 35502 (Sikora et al., submitted); HD 37479 Oksala et al. (2012); HD 44743 and HD 52089 Fossati et al. (2015a); HD 61556 Shultz et al. (2015a); HD 105382 Alecian et al. (2011); HD 121743 (Briquet et al., in prep); HD 127381 Henrichs et al. (2012); HD 142184 Grunhut et al. (2012a); HD
Measurements from different years are plotted with separate symbols, with the year of observation given in the legends. Measurements from ESPaDOnS, Narval, and HARPSpol are also plotted with separate symbols and abbreviated in the legends as ESP, Nar, and HAR, respectively. The excellent agreement between ESPaDOnS and Narval measurements has been demonstrated by Wade et al. (2016). Those stars for which ESPaDOnS and HARPSpol measurements of similar quality are both available (e.g., HD 67621, HD 130807, HD 156324) demonstrate that longitudinal magnetic field measurements collected with these instruments also agree within 1$\sigma$.

For comparisons to $\langle B_z \rangle$ measurements from the literature, the following abbreviations for the references are used in the legends: BL77 Borra and Landstreet (1977); BL79 Borra and Landstreet (1979); BLT83 Borra et al. (1983); B87 Bohlender et al. (1987); BLT93 Bohlender et al. (1993); M91 Mathys (1991); MH97 Mathys and Hubrig (1997); B04 Briquet et al. (2004); B06 Bagnulo et al. (2006); B07 Briquet et al. (2007); Y11 Yakunin et al. (2011); BM11 Bohlender and Monin (2011); B15 Bagnulo et al. (2015); and F15 Fossati et al. (2015b).

Hubrig et al. (2000) showed that, with H profiles calculated with more modern limb darkening and Stark-broadening, photopolarimetric $\langle B_z \rangle$ measurements made using H$\beta$ lines should be corrected by $\sim$80%. $\langle B_z \rangle$ measurements conducted using this method (BL77, BL79, BLT83, B87) have been corrected accordingly.

**HD 25558:** Sórdor et al. (2014) determined that the rotational period of the magnetic secondary of this SB2 system 1.2±0.6 d. Using the restricted data-set described in Section 4.2, and restricting the period window to the range given by Sórdor et al. (2014), we find maximum amplitude at 1.233(1) d, although there are numerous nearby peaks that provide a comparable phasing of the data (Fig. C.1, top). The FAP of the maximum amplitude
Figure C.1: Top: periodogram for $\langle B_z \rangle$ measurements. The abscissa is limited to the period window. Solid (black) line indicates the periodogram for $\langle B_z \rangle$; dashed (blue) line the periodogram for $\langle N_z \rangle$. The red circle indicates the adopted rotation period. Bottom: $\langle B_z \rangle$ phased with $P_{\text{rot}}$. Solid lines indicate the best-fit sinusoidal curves; dashed lines, the 1σ uncertainty in the fit.
peak in the $\langle B_z \rangle$ period spectrum is 0.12, only slightly lower than the maximum FAP in the $\langle N_z \rangle$ spectrum, 0.17. $\langle B_z \rangle$ is shown phased with this period in the bottom panel of Fig. C.1.

**HD 35298**: North (1984) found a photometric period of $P_{\text{rot}} = 1.85336(1)$ d. The number of ESPaDOnS measurements is insufficient to constrain the period further, although they are not inconsistent with this period. The periodogram constructed from archival Hipparcos photometry shows maximum power at $\sim 0.927$ d, which is almost exactly half of the period reported by North (1984) (Fig. C.2, top). This period does not provide an acceptable phasing of the magnetic data, however. The 2\textsuperscript{nd}-highest peak in the Hipparcos photometry periodogram is at 1.85486(1) d. The FAP of this peak is $\sim 10^{-3}$, much lower than the FAP of the highest peak in the null spectrum, $\sim 0.999$. $\langle B_z \rangle$ and $H_p$ are shown phased with this period in Fig. C.2. $H_p$ shows a double-wave variation, accounting for the appearance of the 0.927 d peak. This period also provides an acceptable phasing of the dimaPol Hβ $\langle B_z \rangle$ measurements.

**HD 36485**: $\langle B_z \rangle$ measurements of HD 36485 ($\delta$ Ori C) have been published by Bohlender et al. (1987), Mathys and Hubrig (1997), Leone et al. (2010), and Yakunin et al. (2011). ESPaDOnS data is unable to improve the period, which was determined spectroscopically using the magnetic primary’s variable Hα emission Leone et al. (2010). They are consistent in magnitude with previous data (see Fig. C.3), but much more precise: in contrast to earlier measurements, a weak variation with a semi-amplitude of 0.06±0.03 kG can be discerned in the high-resolution measurements. The ESPaDOnS data agree in magnitude with the Special Astrophysical Observatory observations reported by Yakunin et al. (2011), however, there is less scatter in the new data. The bottom panel shows the ESPaDOnS measurements only, fit with a second-order sinusoid, which provides a somewhat better fit to the data. The possibility that the star’s magnetic field contains significant quadrupolar contributions was earlier suggested on the basis of a pronounced asymmetry in the Hα emission originating
Figure C.2: Rotational period of HD 35298. *Top*: periodogram constructed for $H_p$ photometry (solid black line) and synthetic null measurements (dashed blue line). The adopted period is indicated by the red circle. *Middle*: $H_p$ phased with $P_{\text{rot}}$. *Bottom*: $\langle B_z \rangle$ phased with $P_{\text{rot}}$. 
Figure C.3: $\langle B_z \rangle$ curves for HD 36485 ($\delta$ Ori C). The top panel shows a comparison to historical data, demonstrating the much higher precision of the ESPaDOnS measurements. The bottom panel shows ESPaDOnS measurements only. Note that the $\langle B_z \rangle$ curve is apparently better fit by a second-order sinusoid.
Figure C.4: As Fig. C.1, for HD 36526.

in its CM (Leone et al. 2010; see also Chapter 10).

HD36526: North (1984) found a photometric period of 1.5405(1) d. This period provides a somewhat imperfect phasing of the ESPaDONS \( \langle B_z \rangle \) measurements; period analysis of the new magnetic data indicates a slightly longer \( P_{\text{rot}} = 1.5415(5) \) d (Fig. C.4). The FAP of this peak is \( 5 \times 10^{-3} \), while the highest peak in the \( \langle N_z \rangle \) periodogram has a FAP of 0.56. \( \langle B_z \rangle \) is shown phased with this period in the bottom panel of Fig. C.4. This period gives
an acceptable phasing between the ESPaDOnS $\langle B_z \rangle$ measurements with the dimaPol H$\beta$ measurements.

**HD 36982:** spectroscopic variability is likely associated with this Herbig Be star’s accretion disk, and therefore cannot be used to constrain $P_{\text{rot}}$. Within the period window, the maximum power in the $\langle B_z \rangle$ periodogram is at 0.4247950(5) d, however, there is a strong
peak in the $\langle N_z \rangle$ periodogram at this period (Fig. C.5, top). We therefore adopt the next-strongest peak, 1.8551(1) d. However, it should be noted that the FAPs of the highest peaks in $\langle B_z \rangle$ and $\langle N_z \rangle$ are both similar, about 0.03. $\langle B_z \rangle$ is shown phased with this period in Fig. C.5 (bottom).

**HD 37017:** Bohlender et al. (1987) combined periodograms for photometric and magnetic data and found $P_{\text{rot}} = 0.901195(5)$ d. This period does not provide a reasonable
phasing of the ESPaDONS data. Combining historical measurements with our own, we find 0.901186(3) d. The maximum amplitude period has a FAP of about $10^{-7}$, while the FAP of the highest-amplitude peak in the $\left<N_z\right>$ spectrum is 0.12. The periodogram and $\left<B_z\right>$ measurements are shown in the top and bottom panels of Fig. C.6.

**HD 37058:** Pedersen (1979) reported a photometric period of $\sim 14$ d, while Bychkov et al. (2005) determined a 1.022 d period. In both cases the available datasets were too small.
to firmly establish $P_{\text{rot}}$. The ESPaDOnS measurements confirm Pedersen (1979)'s period. By combining with $\langle B_z \rangle$ measurements from the literature, we find $P_{\text{rot}} = 14.58095(5)$ d, with a FAP of 0.03, much lower than the FAP of the maximum amplitude peak in the null spectrum, $\sim$0.3. $\langle B_z \rangle$ is shown phased with this period in Fig. C.7.

**HD 37061**: while HD 37061 (NU Ori) displays spectroscopic variability, it is not clear that this is purely due to rotational modulation as periodograms created for these data
yield contradictory results. Therefore we base our period search on the magnetic data. The star’s high $v \sin i$ (225±8 km s$^{-1}$) means the period must be no longer than $\sim$1.4 d. There are two peaks of comparable amplitude within the period window, at $\sim$0.6 d and $\sim$1.1 d (Fig. C.8, top). The shortest period would indicate $v_{eq} \sim v_{br}$, necessitating very tight limits on the stellar parameters. The FAPs of either peak are both about 0.08, only slightly below the $\langle N_z \rangle$ maximum amplitude FAP of about 0.1. We therefore adopt the next-highest peak, 1.09497(4) d, as the more conservative option. $\langle B_z \rangle$ is shown phased with this period in Fig. C.8.

**HD 46328:** The ESPaDOnS $\langle B_z \rangle$ measurements are inconsistent with the 2.179 d period proposed by Hubrig et al. (2011) on the basis of FORS1/2 $\langle B_z \rangle$ measurements. As will be shown in detail by Shultz et al. (in prep), the rotational period is likely to be on the order of decades. While there are several peaks in the periodogram at lower periods (see Fig. C.9), none of these provide a convincing fit to the data; furthermore, the highest amplitude is found at periods greater than 5000 d. The FAP in this region is $\sim 10^{-5}$, much lower than the minimum $\langle N_z \rangle$ FAP of 0.87. The temporal baseline of the magnetic data is not sufficient to constrain the rotation period further. Photometric and spectroscopic variability is dominated by the star’s high-frequency $\beta$ Cep pulsations, and therefore unhelpful for period analysis. It has been suggested that this period is largely a consequence of the inclusion of two lower-quality MuSiCoS measurements (Fossati et al., 2015a), acquired in the year 2000, which show a reversed magnetic polarity Shultz et al. (2015b). The middle panel of Fig. C.9, which shows only the ESPaDOnS $\langle B_z \rangle$ measurements as a function of time, demonstrates that this is not the case: the precision of the ESPaDOnS data is sufficient to detect a modulation on a timescale of years. Comparison to all available magnetic data (bottom panel of Fig. C.9) demonstrates that the the ESPaDOnS and MuSiCoS data can be fit with a sinusoid with a period of 30 years. The FORS1/2 data (Hubrig et al., 2011,
Figure C.9: Top: Periodogram for $\xi^1$ CMa (HD 46328). Middle, bottom: and $\langle B_z \rangle$ as a function of time. The second panel shows only the ESPaDOnS data, both individual measurements and in annual bins. The bottom panel shows all available $\langle B_z \rangle$ measurements.
2006a; Fossati et al., 2015a) are not in contradiction with this fit, being generally within 1-2σ, whereas the limit of reliability of FORS1/2 magnetometry is typically about 5σ (Bagnulo et al., 2012). The sinusoidal fit to the FORS1/2 \langle B_z \rangle measurements obtained by phasing them with the rotation period proposed by Hubrig et al. (2011) implies a \langle B_z \rangle amplitude of only 3.5σ significance when compared to the mean FORS1/2 error bar, suggesting the apparent variation in the FORS1/2 data is reflective of scatter. By contrast, the much more precise ESPaDOnS data detects a long-term modulation in \langle B_z \rangle at a significance of 16σ from maximum to minimum \langle B_z \rangle.

**HD 55522:** Briquet et al. (2004) used spectroscopy and ground-based as well as archival Hipparcos photometry to find \( P_{\text{rot}} = 2.729(1) \) d. This period phases the ESPaDOnS data well when examined in isolation, but comparison of the \langle B_z \rangle measurements presented by Briquet et al. (2007) to our own reveals a small phase offset. While there are numerous closely spaced peaks in the periodogram constructed for the combined magnetic data (Fig. C.10, top), the only peak consistent with the photometric data is at 2.72923(2) d. \langle B_z \rangle is shown phased with this period in the bottom panel of Fig. C.10.

**HD 58260:** Bohlender et al. (1987) were unable to determine a period for this star due to the negligible variation in \langle B_z \rangle. Even at the much higher precision of the ESPaDOnS data, the difference between the maximum and minimum \langle B_z \rangle is only 48 G, with a mean error bar of 27 G. While there are some higher-amplitude peaks at short periods, and very little power at periods longer than \( \sim 10 \) d, this is likely an artifact caused by the relatively short temporal baseline (56 days, with all but 1 observation acquired within 13 days). A higher SNR dataset is therefore necessary to determine \( P_{\text{rot}} \). Comparison to historical data (bottom panel) demonstrates that \langle B_z \rangle is consistent with no variation over a timescale of \( \sim 35 \) years: all measurements are within 1σ of the mean ESPaDOnS \langle B_z \rangle.

**HD 63425:** two periods are compatible with the existing \langle B_z \rangle measurements. Maximum power is at 163.19(1) d, which is compatible with the low \( v \sin i \). The other is very short,
Figure C.10: As Fig. C.2 for HD 55522.
Figure C.11: \( \langle B_z \rangle \) measurements of HD 58260 as a function of time. The top panel shows the ESPaDOnS data, the bottom panel a comparison to historical data. Solid lines indicate the mean ESPaDOnS \( \langle B_z \rangle \), dashed lines the mean ESPaDOnS \( \langle B_z \rangle \) error bar.
∼0.55 d, which would make HD 63425 amongst the most rapidly rotating magnetic massive stars. The FAPs of the two periods are similar, about 0.08, and only slightly lower than the minimum $\langle N_z \rangle$ FAP of 0.1. Given the very small $v \sin i$, this would require the rotational pole to be almost perfectly aligned with the line-of-sight, which is \textit{a priori} less likely. Furthermore, the $\langle B_z \rangle$ measurements from 2010 (the year with the largest number of observations) show statistically significant differences only between those measurements.
separated by $\sim 250$ d, while the variation of those collected within $\sim 10$ d is of the same order as the error bars. We therefore adopt the longer period. $\langle B_z \rangle$ is shown phased with this period in Fig. C.12.

**HD 64740**: combining photometric and magnetic data, Bohlender et al. (1987) found 1.33026(6) d. Combining the magnetic measurements presented by Borra and Landstreet (1979) and Bohlender et al. (1987) with the new ESPaDOnS and HARPSpol data, we find
$P_{\text{rot}} = 1.330205(3) \ \text{d}, \ \text{with a FAP of } 4 \times 10^{-7}, \ \text{much lower than the minimum } \langle N_z \rangle \text{ FAO of 0.28. The periodogram and } \langle B_z \rangle \text{ measurements are shown in the top and bottom panels of Fig. C.13.}

**HD 66522:** a period cannot be determined uniquely from the sparse magnetic data, however, the Hipparcos photometry shows a clear modulation at 909.339(3) d. The FAP is $2 \times 10^{-14}$, while the minimum FAP in the null period spectrum is 0.99. This extremely long period is compatible with the star’s low $v \sin i$. The $H_p$ periodogram is shown in the top panel of Fig. C.14; $H_p$ and $\langle B_z \rangle$ are shown phased with the 909.339 d period in the middle and bottom panels. We chose the date of maximum light as JD0, which seems to correlate well to $| <B_{Z,\text{max}} > |$. The star also shows spectroscopic variation; minimum line strength corresponds to maximum light, and vice versa, suggesting that both spectroscopic and photometric variations are due to the rotational modulation of chemical spots.

**HD 66665:** Petit et al. (2013) reported a period of 21 d based upon a preliminary analysis of ESPaDOnS and Narval magnetic measurements. We find 24.476(2) d using the same data, with a FAP of $2 \times 10^{-4}$. The 21 d period is definitely ruled out. There is very little power in the $\langle N_z \rangle$ periodogram near 25 d, and the minimum FAP of the $\langle N_z \rangle$ period spectrum is 0.08. The periodogram is shown in the top panel of Fig. C.15; $\langle B_z \rangle$ is shown phased with this period in the bottom panel.

**HD 66765:** Alecian et al. (2014) found $P_{\text{rot}} = 1.62 \pm 0.15$ d based on 4 days of observations with HARPSpol. We acquired a single new ESPaDOnS observation approximately 2 years later. The quality of the measurement is not as high as that of the HARPSpol data, and does not enable improvement of the period. However, we also acquired 11 FEROS spectra. As HD 66765 is a He-variable star, we measured the EWs of the He i 447.1 nm, 587.6 nm, and 667.8 nm lines. Within the uncertainty in $P_{\text{rot}}$ determined by Alecian et al. (2011), the highest peak in the periodogram for the combined EWs is at 1.6079(1) d (Fig. C.16, top), with a FAP of $2 \times 10^{-4}$ and a minimum FAP in the null spectrum of 0.58. The combined
Figure C.14: Hipparcos photometry periodogram (top), $H_p$ light curve (middle), and $\langle B_z \rangle$ curve (bottom) of HD 66522. Numbers in brackets in the legends indicate the year of observation.
He I EWs and the $\langle B_z \rangle$ measurements are shown phased with this period in the middle and bottom panels of Fig. C.16.

**HD 67621:** Alecian et al. (2014) determined $P_{\text{rot}} = 3.6(2)$ d. As with HD 66765, we observed the star once more with ESPaDOnS, and refine the period to $3.6191(1)$ d, although we note that the periodogram has numerous nearby peaks within the original range (Fig. C.17, top). While the minimum FAPs of the $\langle B_z \rangle$ and $\langle N_z \rangle$ period spectra are similar (0.27
Figure C.16: Periodogram for combined He i 447.1 nm, 587.6 nm, and 667.8 nm EWs (top); combined He i EWs (middle); \( \langle B_z \rangle \) measurements (bottom).
and 0.31, respectively), there is no power near the 3.6 d period in the $\langle N_2 \rangle$ periodogram. $\langle B_z \rangle$ is shown phased with this period in Fig. C.17 (bottom).

**HD 96446**: Bychkov et al. (2005) adopted a 0.85 d period, as did Neiner et al. (2012b), although Neiner et al. (2012b) noted they were unable to distinguish between periods of 0.85 d and 5.73 d, and commented that longer periods could not be ruled out. With our larger magnetic dataset, there is very little power at periods less than about 10 d, with the
Figure C.18: As Fig. C.1, for HD 96446. Phase errors are larger than the horizontal axis and are not shown.
exception of one peak near 0.93 d; above 10 d there are numerous peaks that provide an acceptable phasing of $\langle B_z \rangle$ (Fig. C.18, top). As there is also a peak in the $\langle N_z \rangle$ periodogram near 0.93 d, this period is likely a consequence of the window function. The highest power is at 35.137(2) d, which we adopt as the rotation period. $\langle B_z \rangle$ is shown phased with this period in Fig. C.18 (bottom). However, we note that this period leaves no $\langle B_z \rangle$ measurements on the descending part of the curve, and does not provide a good phasing of the historical data. Furthermore, the minimum FAPs in the $\langle B_z \rangle$ and $\langle N_z \rangle$ period spectra are similar (0.13 and 0.19 respectively). HD 96446 is a β Cep star, and its LSD $N$ profiles yield definite detections without exception, indicating a rapidly accelerating line profile. While RV variability alone is unlikely to affect $\langle B_z \rangle$ Neiner et al. (2012b), non-radial pulsations may affect Stokes $V$ in a similar fashion to the effect of chemical spots. A full asteroseismological analysis is essential to accurate determination of $P_{\text{rot}}$. As pulsations likely dominate the star’s light curve, the Hipparcos photometry is not useful for determining the rotation period.

**HD 105382**: Briquet et al. (2001) determined the 1.295 d rotation period for HR 4618 using Hipparcos photometry. The star was discovered as magnetic independently by Kochukhov
and Bagnulo (2006) and Hubrig et al. (2006b), and the FORS1 ⟨Bz⟩ measurements were originally published by Briquet et al. (2007). The HARPSpol data were presented by Alecian et al. (2011), who noted that one of the FORS1 measurements was inconsistent with the high-resolution data, having a much higher positive value than expected. In their careful reanalysis of FORS1 ⟨Bz⟩ measurements, Bagnulo et al. (2015) found results consistent with those reported by Briquet et al. (2007), with the exception of the high positive value measurement, which they found instead to be weakly negative. As there are only 3 high-resolution measurements of this star, and the re-analyzed FORS1 data are consistent with the HARPSpol measurements, we include the FORS1 data in the modelling presented in Paper II.

**HD 122451:** There are numerous peaks of comparable amplitude within the period window, however, comparison to the ⟨Nz⟩ periodogram indicates that many of these are likely a result of the window function (Fig. C.20, top). The highest peak with no nearby peak in the ⟨Nz⟩ periodogram is at 2.8981(2) d. ⟨Bz⟩ is shown phased with this period in Fig. C.20 (bottom). It must be noted that the FAP of this peak is 0.33, which is actually slightly higher than the minimum FAP in the ⟨Nz⟩ spectrum, 0.31.

**HD 125823:** Bohlender et al. (2010) found a period of 8.8177(1) d. This period phases the ESPaDOnS data well if examined in isolation, but there is a systematic phase offset when compared to older data. The highest amplitude in the periodogram for the ESPaDOnS data is at 8.8169(3) d. This period phases the ESPaDOnS ⟨Bz⟩ measurements well with the historical data; a periodogram constructed from the combined datasets yields a more precise period, \( P_{\text{rot}} = 8.81694(1) \) d. The FAP of this peak is \( 2.2 \times 10^{-5} \), much lower than the minimum ⟨Nz⟩ FAP of 0.33. This period also provides an acceptable phasing of the FORS2 measurements reported by Fossati et al. (2015b), who noted they were unable to phase their measurements with the historical data using the Bohlender et al. (2010) ephemeris. ⟨Bz⟩ is shown phased with this period in Fig. C.21.
Figure C.20: As Fig. C.1, for HD 122451.

**HD 130807**: Ͽ Lup was reported as magnetic by Alecian et al. (2011). Subsequent observations with HARPSpol in 2012, and ESPaDOnS in 2014, have enabled us to determine \( P_{\text{rot}} = 2.9532(1) \) d, with a FAP of 0.03 as compared to a minimum FAP in the \( \langle N_z \rangle \) periodogram of 0.14. \( \langle B_z \rangle \) is shown phased with \( P_{\text{rot}} \) in the bottom panel of Fig. C.22.

**HD 136504**: this is a B2V/B3V SB2 close binary system in which both stars possess magnetic fields Shultz et al. (2015c). \( v \sin i \) is in both cases moderate (\( \sim 30 \) km s\(^{-1} \)), yielding...
Figure C.21: As Fig. C.1, for HD 125823.

a period window for the primary between 0.2–6.3 d, and for the secondary, 0.25–9.7 d. Both windows contain $P_{\text{orb}} = 4.5597$ d (Table 3.1). As only 5 measurements are available in which the line profiles of the two components are clearly separable, there are not enough $\langle B_z \rangle$ measurements to constraint $P_{\text{rot}}$. Analysis of multi-epoch BRITE photometry indicates two significant frequencies, at 1.85(3) d and 4.6(2) d (Ernst Paunzen, priv. comm.). The 4.6 d period could be due to either rotational modulation of chemical spots, in which case at
least one of the stars would be tidally locked, or it could be a consequence of eclipsing. While the stars are separated by only a few stellar radii Uytterhoeven et al. (2005), the inclination of the orbital axis is relatively small ($i_{\text{orb}} \sim 21^\circ$; Table 3.1), so eclipses are unlikely. On the other hand, if one of the stars is tidally locked, the orbital and rotational inclinations should be similar: however, a 4.6 d period yields $i_{\text{rot}} \sim 80^\circ$ for the primary and 60° for the secondary, much larger than $i_{\text{orb}}$. Since the highest-amplitude modulation in the BRITE
Figure C.23: Top: $\langle B_z \rangle$ for HD 136504 A, phased with the most significant period determined from BRITE photometry. Bottom: $\langle B_z \rangle$ for HD 136504 B (right) phased with the second BRITE period.
photometry is at 1.85(3) d, this is tentatively assigned to the primary. The top panel of Fig. C.23 shows the $\langle B_z \rangle$ measurements of HD 136504 A phased with this period. While the 4.6(2) d period is unlikely to be due to rotational modulation, for display purposes the bottom panel shows the $\langle B_z \rangle$ measurements of HD 136504 B phased with the 4.6(2) d period.

**HD 142990**: Bychkov et al. (2005) found a rotation period of 0.97907 d. The magnetic data, even including the new measurements presented here, is rather sparse and is unable to identify a unique period within the period window. However, a unique period of ~0.979 d is clearly identified in the periodogram obtained using archival Hipparcos photometry, suggesting a period of ~0.979 d is indeed correct (Fig. C.24, top). The FAP of this peak is $2.3 \times 10^{-17}$, much lower than the minimum FAP in the null period spectrum of 0.5. This period provides an adequate phasing of the ESPaDOnS $\langle B_z \rangle$ measurements, but is insufficiently precise to phase the new data with the measurements reported by Borra et al. (1983) and Bohlender et al. (1993). A periodogram constructed for all old and new $\langle B_z \rangle$ measurements, limited to a narrow window around 0.979 d, finds maximum amplitude at 0.978832(1) d. $H_p$ and $\langle B_z \rangle$ are shown phased with this period in the bottom two panels of Fig. C.24.

**HD 149277**: The strongest peak in the periodogram is at 25.3810(5) d (Fig. C.25, top), with a FAP of $9 \times 10^{-5}$, as compared to a minimum FAP in the $\langle N_z \rangle$ periodogram of 0.02. This period was determined at a lower precision using the ESPaDOnS data obtained before 2015, and with only slight modification successfully phases the newer ESPaDOnS data obtained in 2015, as well as the single FORS1 measurement reported by Bagnulo et al. (2006). We therefore consider this to be the correct rotation period with high confidence. $\langle B_z \rangle$ is shown phased with this period in Fig. C.25 (bottom). Note that while the data obtained in 2015 in earlier yield the same period, the FAP is much higher, and is similar in the $\langle B_z \rangle$ and $\langle N_z \rangle$ period spectra, ~0.08 in both cases.
Figure C.24: As Fig. C.2, for HD 142990.
HD149438: $\tau$ Sco is the earliest star in the sample, and is one of the few with a complex surface magnetic topology Donati et al. (2006). Since the first analysis of its magnetic properties, 35 new ESPaDOnS observations were acquired between 2006 and 2012. The periodogram for the new magnetic data does not yield a more precise period than that determined from IUE data by Donati et al. (2006); however, the new $\langle B_z \rangle$ measurements are fully consistent with the original period (see Fig. C.26).
HD 156324: this SB3 star displays emission in the Hα line, likely a consequence of the magnetic primary star’s magnetosphere Alecian et al. (2014). Hα EW measurements from the combined HARPSpol, ESPaDOnS, and FEROS datasets indicate a rotation period of 1.5804(1) d (Fig. C.27, top), with a FAP of $2 \times 10^{-7}$ and a minimum FAP in the null spectrum of 0.28. There is a peak in the $\langle B_z \rangle$ periodogram at the same period, although with a somewhat higher FAP ($3 \times 10^{-4}$), and with 2 other peaks of similar amplitude at 0.6 d and 2.7 d. As the EW period is derived from a larger number of measurements and furthermore shows a clear variation that is coherent across multiple epochs, we adopt 1.5804(1) d as the rotation period. Hα EWs and $\langle B_z \rangle$ are shown phased with this period in the middle and bottom panels of Fig. C.27.

HD 156424: as this star shows weak emission in the high-velocity wings of its Hα line, presumably originating in its magnetosphere, we conducted period analyses using both the magnetic data and Hα EWs, with the EWs supplemented by FEROS spectroscopy. The variation in either case is small compared to the error bar. Almost all the Hα variation is between datasets, suggesting that it is a consequence of systematic differences between

Figure C.26: $\langle B_z \rangle$ curve for HD 149438.
Figure C.27: Top: periodogram for Hα EWs (solid black line) and synthetic null measurements (dashed blue line). The adopted period is indicated by the red circle. Middle: Hα EWs phased with $P_{\text{rot}}$. Bottom: $\langle B_z \rangle$ phased with $P_{\text{rot}}$. 
There is very little power in either periodogram at periods longer than a few days, suggesting that the rotation period must be fairly short despite the star’s low $v \sin i$ (see Fig. C.28, top). The largest-amplitude peak in the $\langle B_z \rangle$ periodogram without a corresponding peak in the $\langle N_z \rangle$ periodogram is at 2.8721(3) d. $\langle B_z \rangle$ is shown phased with this period in the bottom panel of Fig. C.28. It should be noted that the FAP of the highest peaks in the $\langle B_z \rangle$ period spectrum are around 0.06, not much lower than the minimum in

Figure C.28: As Fig. C.1, for HD 156424.
Figure C.29: $\langle B_z \rangle$ curve for HD 176582.

$\langle N_z \rangle$, 0.15.

**HD 176582**: the first magnetic measurements of this star were performed using dimaPol by Bohlender and Monin (2011), who determined $P_{\text{rot}}$ using both magnetic and spectroscopic data. The Narval measurements are more precise than the dimaPol data, and yield a $\langle B_z \rangle$ curve with a somewhat smaller semi-amplitude. However, the two sets of measurements phase well with one another with no change to $P_{\text{rot}}$, as shown in Fig. C.29.

**HD 186205**: the strongest peak in the $\langle B_z \rangle$ periodogram is at 33.395(1) d (Fig. C.30, top), with a FAP of 0.12 (only slightly lower than the minimum FAP in $\langle N_z \rangle$, 0.20). There is very little power at either much higher or much lower frequencies, and the long period is consistent with the low $v \sin i$. Furthermore, there is no power in the $\langle N_z \rangle$ periodogram near this period. However, there are numerous nearby peaks which phase the data approximately as well. $\langle B_z \rangle$ is shown phased with this period in Fig. C.30 (bottom).

**HD 189775**: Petit et al. (2013) reported a period of 2.6048 d for this star, based on unpublished magnetic measurements collected with dimaPol. In addition to Hβ $\langle B_z \rangle$ measurements, there are also $\langle B_z \rangle$ measurements conducted using the Fe II 492.3 nm line. As
Figure C.30: $\langle B_z \rangle$ phased with the newly determined rotational periods. Solid lines indicate the best-fit sinusoidal curves; dashed lines, the $1\sigma$ uncertainty in the fit.

the H$\beta$ $\langle B_z \rangle$ measurements show significantly more scatter than the Fe II measurements, we adopt the latter. ESPaDOnS measurements yield a best-fit period of 2.6067(4) d. For the combined data we find 2.6068(1), as shown in the top panel of Fig. C.31. The FAP of this peak in the periodogram is $3 \times 10^{-4}$, lower than the FAP of the highest peak in $\langle N_z \rangle$, $8 \times 10^{-3}$. The ESPaDOnS and DAO $\langle B_z \rangle$ measurements are shown phased with this period in Fig. C.31.
Figure C.31: As Fig. C.1, for HD 189775.

**HD 208057**: Henrichs et al. (2009) reported $P_{\text{rot}} = 1.441$ d. Using the same data, we find maximum power at 1.3678(5) d (Fig. C.32, top). The FAP of this peak is $7 \times 10^{-4}$, substantially smaller than the minimum FAP in the $\langle N_z \rangle$ period spectrum of 0.12. $\langle B_z \rangle$ is shown phased with this period in Fig. C.32.

**ALS 3694**: no period can be determined from the magnetic data, as $\Sigma_B$ is rather low.
However, this star displays H\(\alpha\) emission consistent with an origin in a centrifugal magnetosphere Shultz et al. (2014). Combining H\(\alpha\) EWs measured from ESPaDOnS and FEROS spectra yields a periodogram with a single strong peak at 1.6779(5) d (Fig. C.33, top). The FAP of this peak is 0.001, much lower than the minimum FAP from the null periodogram, 0.43. H\(\alpha\) EWs and \(\langle B_z \rangle\) measurements are shown phased with this period in the middle and bottom panels of Fig. C.33. While a sinusoidal variation is not distinguishable in \(\langle B_z \rangle\),
Figure C.33: As Fig. C.27, for ALS 3694.
the magnitude of the ESPaDOnS $\langle B_z \rangle$ measurements is compatible with those published by Bagnulo et al. (2006).