A NUMERICAL INVESTIGATION OF STRESS PATH AND ROCK MASS DAMAGE IN OPEN PITS

by

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Abstract

The importance of establishing reliable slope designs becomes critical as the depth of open pits increases. Defining an optimum slope design that maximizes financial return and ore recovery without compromising safety conditions requires a proper evaluation of the stability conditions around the pit, and of the effects of instability on mining operations. Performing this analysis requires a good understanding of the behavior and strength of the rock mass at the slope scale.

From a conceptual point of view, the behavior of a rock mass is a result of the complex interaction between discontinuities and intact rock. However, our ability to determine rock mass behavior at large scales with a high degree of certainty remains limited. The use of simplified constitutive models, which overlook the effects of damage accumulation on the mechanical behavior of rock masses, adds to the degree of uncertainty of the results of stability analyses of large rock slopes.

In this research, the relationship between damage and stress path is examined by using advanced numerical methods that allow the explicit representation of rock damage. The bonded particle method (BPM) has been used to evaluate the influence of stress path on damage accumulation, and on the strength of intact rock. The analysis of the influence of stress path on damage and strength at rock mass scale has been performed using the synthetic rock mass (SRM).

From this analysis, it was possible to correlate the extension strain with the onset of yielding in the SRM sample. This limit was found to be independent of the confinement stress and the stress path followed to load the sample. A series of 3D numerical models were created to explore the correlation between in situ stress, stress path and rock mass damage. The relationship between the onset of yielding and extension strain from SRM modelling was used to define the areas in the pit models that might suffer relaxation induced damage. The extent of the zone where damage can develop has a direct relationship with the magnitude of the in situ horizontal stress. The geometry of the pit has an influence on the distribution of extensional strain around the pit, as the increased confinement generated by slope curvature reduces the extent of the damage zone in the curved areas of the pit.
Co-Authorship

I hereby certify that all of the work described within this thesis is the original work of the author.
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<tr>
<td>$\mu$</td>
<td>Contact friction coefficient</td>
</tr>
<tr>
<td>$a$</td>
<td>Hoek-Brown dimensionless material constant</td>
</tr>
<tr>
<td>AE</td>
<td>Acoustic Emission</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
</tr>
<tr>
<td>BPM</td>
<td>Bonded Particle Method</td>
</tr>
<tr>
<td>C/T</td>
<td>Contact Shear to Normal Strength Ratio</td>
</tr>
<tr>
<td>CB</td>
<td>Contact Bond</td>
</tr>
<tr>
<td>CI</td>
<td>Crack initiation stress</td>
</tr>
<tr>
<td>CSR</td>
<td>Confining stress reduction</td>
</tr>
<tr>
<td>D</td>
<td>Hoek-Brown Disturbance Factor</td>
</tr>
<tr>
<td>DEM</td>
<td>Distinct Element Method</td>
</tr>
<tr>
<td>DFN</td>
<td>Discrete Fracture Network</td>
</tr>
<tr>
<td>DOE</td>
<td>Design of experiment</td>
</tr>
<tr>
<td>DR</td>
<td>Deconfinement ratio</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>Ec</td>
<td>Young's modulus of the particle-particle contacts</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite Difference Method</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>GSI</td>
<td>Geotechnical Strength Index</td>
</tr>
<tr>
<td>GSIr</td>
<td>Residual value of GSI</td>
</tr>
<tr>
<td>HB</td>
<td>Hoek-Brown</td>
</tr>
<tr>
<td>IRMR</td>
<td>In Situ Rock Mass Rating</td>
</tr>
<tr>
<td>Ja</td>
<td>Joint Alteration</td>
</tr>
<tr>
<td>Jc</td>
<td>Joint Condition</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>Jc89</td>
<td>Joint Condition (Bieniawski 1989)</td>
</tr>
<tr>
<td>JP</td>
<td>Jointing Parameter</td>
</tr>
<tr>
<td>Jr</td>
<td>Joint Roughness</td>
</tr>
<tr>
<td>Js</td>
<td>Joint Smoothness</td>
</tr>
<tr>
<td>Jv</td>
<td>Volumetric Joint Count</td>
</tr>
<tr>
<td>Jw</td>
<td>Joint waviness</td>
</tr>
<tr>
<td>kn/ks</td>
<td>normal to shear stiffness</td>
</tr>
<tr>
<td>L/d</td>
<td>Relation between Length/Diameter of a sample</td>
</tr>
<tr>
<td>m</td>
<td>Hoek-Brown dimensionless material constant for rock</td>
</tr>
<tr>
<td>m_b</td>
<td>Hoek-Brown dimensionless material constant for broken rock</td>
</tr>
<tr>
<td>m_i</td>
<td>Hoek-Brown dimensionless material constant for intact rock</td>
</tr>
<tr>
<td>m_r</td>
<td>m_b residual value (Ribacchi 2000)</td>
</tr>
<tr>
<td>MRMR</td>
<td>Mining Rock Mass Rating</td>
</tr>
<tr>
<td>PB</td>
<td>Parallel Bond</td>
</tr>
<tr>
<td>PFC</td>
<td>Particle Flow Code</td>
</tr>
<tr>
<td>PFC3D</td>
<td>Particle Flow Code in 3 Dimensions</td>
</tr>
<tr>
<td>Q</td>
<td>Rock Tunneling Quality Index</td>
</tr>
<tr>
<td>RBS</td>
<td>Rock Block Strength</td>
</tr>
<tr>
<td>REV</td>
<td>Representative elementary volume</td>
</tr>
<tr>
<td>RMi</td>
<td>Rock Mass Index</td>
</tr>
<tr>
<td>RMR</td>
<td>Rock Mass Rating</td>
</tr>
<tr>
<td>RMR_{76}</td>
<td>Rock Mass Rating (Bieniawski 1976)</td>
</tr>
<tr>
<td>RMR_{89}</td>
<td>Rock Mass Rating (Bieniawski 1989)</td>
</tr>
<tr>
<td>RQD</td>
<td>Rock Quality Designation</td>
</tr>
<tr>
<td>s</td>
<td>Hoek-Brown dimensionless material constant</td>
</tr>
</tbody>
</table>
SCR  Surface Condition Rating
SJCM  Smooth joint contact model
SR  Structure Rating
$s_r$  $s$ residual value (Ribacchi 2000)
SRM  Synthetic Rock Mass
TD  Triaxial deconfinement
TX  Triaxial Compressive Strength
UCS  Uniaxial Compressive Strength
UDEC  Universal Distinct Element Code
$\Delta t$  Numerical timestep
$\delta \varepsilon_a$  Incremental axial strain
$\delta \varepsilon_r$  Incremental radial strain
$\Delta \sigma_1 / \Delta \sigma_3$  Ratio of axial stress increment to confinement stress reduction
$\lambda$  Fracture frequency
$\nu$  Poisson’s ratio
$\sigma_1$  Major Principal Stress
$\sigma_1^{'}$  Major Principal Effective Stress
$\sigma_2$  Intermediate Principal Stress
$\sigma_3$  Minor Principal Stress
$\sigma_3^{'}$  Minor Principal Effective Stress
$\sigma_a$  Axial Stress
$\sigma_c$  Intact Rock Strength
$\sigma_{ci}$  Uniaxial compressive Strength
$\sigma_{cm}$  Rock Mass Strength
$\sigma_{cr}$  $\sigma_c$ residual value (Ribacchi 2000)
$\sigma_r$  Radial Stress
Chapter 1

Introduction

1.1 Project Background
Several open pit mines around the world have reached or are planning to reach depths of 1,000 meters (Robotham, 2011). At this scale, a small increase in the slope angle can significantly reduce the amount of waste rock that needs to be extracted, a condition that, in turn, improves the financial return. In the design of large rock slopes, engineers need to balance the economic incentive of steeper slope angles with the reduction in the degree of stability of the slope caused by the change in the slope angle. Defining the optimum slope design requires an accurate evaluation of the stability of the slopes and an assessment of instability effects on the mining operation. One of the major challenges in the analysis and design of large rock slopes is obtaining a reliable estimate of rock mass strength (Read and Stacey 2009).

The behavior of jointed rock masses is well understood to be controlled by the interaction of intact rock and structural features, such as faults, joints, and veins. Physical tests on samples of intact rock allow us to determine the strength and understand the failure process of intact rock with a certain degree of certainty. For the rock mass system, however, this is not normally practical or even possible. Standard practice in geotechnical design relies on the use of rock mass classification or characterization systems to determine stability or strength at the rock mass scale (Hoek and Brown 1980, Dinc et al. 2011). Most analysis techniques assume elastic rock mass behavior until the peak strength is reached. This assumption overlooks the effect of stress path and damage accumulation and how this damage affects the ultimate rock mass strength in situ.

The development of numerical techniques, such as the Synthetic Rock Mass (SRM) approach, offers the potential to simulate the mechanical behavior of rock mass at a large scale and to model the development of damage in the pre- and post-peak stages. This thesis will use state-of-the-art numerical techniques to analyze the effects of stress path and damage on rock mass behaviour.
1.2 Thesis Objective and Scope

The main objective of this thesis is to determine the influence of stress path on the development of damage at the rock mass scale and the effects that this might have on the assessment of the stability of large open pit slopes. This objective is met through the following:

- To evaluate the influence of stress path on the behaviour of intact rock using numerical methods.
- To quantify the influence of damage on the strength of intact rock.
- To evaluate the use of the Synthetic Rock Mass approach for the evaluation of rock mass behaviour under different lading conditions.
- To quantify the effect of damage on the strength of SRM samples.
- To evaluate the influence of pit geometry and in-situ stress conditions on the stress path and the distribution of extension strains around the slope with 3D continuum numerical models.

1.3 Thesis Outline

This thesis has been prepared in accordance with the requirements outlined by the School of Graduate Studies at Queen’s University in Kingston, Ontario. The thesis is divided into seven chapters, as follows.

Chapter Two presents a literature review of the estimation of rock mass strength and the techniques used for the evaluation of slope stability.

In the first half of this chapter, the most commonly used rock mass classification systems in mining are described, with a focus on the geotechnical strength index (GSI) and different methodologies developed in recent years to obtain this index in a quantitative manner. The Hoek–Brown (HB) criterion and its applicability for different rock mass conditions are discussed. The second half of this chapter deals with slope failure modes and the techniques available for slope stability analysis.

Chapter 3 consists of a review of the numerical methods used in this thesis. The theory and applications of the bonded particle model (BPM), SRM and the finite difference method are described.

Chapter 4 presents the analysis results of the effect of stress path and damage at intact rock scale. BPM samples have been tested with the use of different loading paths to evaluate the effect of loading path on
the accumulation of damage. Finally, the damaged samples have been subjected to numerical UCS and triaxial tests to assess the effect of damage on strength.

Chapter 5 describes the analysis results of the effect of stress path and damage at rock mass scale. The SRM approach was used to conduct numerical tests that allow the identification of the relationship between damage and strain. The influence of stress path on strength was evaluated by testing SRM samples under different loading paths. Finally, the rock mass residual strength obtained from the SRM tests is compared with the empirical estimates of residual strength.

Chapter 6 presents the analysis results of stress path and distribution of extension strain for different pit geometries and in-situ stress conditions.

Chapter 7 discusses the major findings of this research and provides recommendations for future work.

A summary of the structure of this thesis is presented in Figure 1-1.

Figure 1-1. Summarized outline of this thesis describing topics addressed in each chapter.
1.4 Summary of Key Findings
The key findings from this work are summarized in the following sections.

1.4.1 Effect of stress path on intact rock strength and damage accumulation
The strength of intact rock is evaluated with standardized laboratory tests, in which the samples are loaded following prescribed stress paths. The stress path that the rock mass will experience during the construction of underground or surface excavations will be more complex than those used to determine the strength of the rock in these simulations. BPM has been used to evaluate the effect of stress path on damage accumulation and strength. With this objective, a 3D BPM sample has been tested following different loading paths. The results showed that the samples tested under non-standard stress paths (triaxial deconfinement and confining stress reduction) develop less damage than those tested under standardized loading paths (unconfined and triaxial compression).

1.4.2 Influence of pre and post peak damage on rock strength
The effect of damage on intact rock strength has been evaluated by testing the damaged BPM samples. Different methods were used to create the damaged BPM samples, including randomly oriented damage, damage inserted at specific orientations, and damage caused by the previous loading of the sample. The case with a randomly inserted damage showed the greatest reduction in strength. When damage is inserted at random locations and orientations, cracks might be inserted at high-strength contacts that would not fail under loading and that are a part of the internal force chains that carry the load in the sample. Artificially breaking these contacts results in an increase in the impact of a certain amount of damage in strength compared with that of the damage generated by other means.

The results obtained from the tests performed on the previously damaged samples showed that the damage produced in the first tests conducted on the sample has a minor effect on the strength of the sample measured in subsequent tests. During loading, weaker contacts fail first, whereas stronger contacts remain intact and chains of force transfer through the contact network form in a trellis-like structure; these force chains sustain the applied load. Because the orientation of the loading on the sample has not been
changed, the force chains that sustained the load during the first test can carry the load, and, therefore, the effect of broken contacts is minimal.

1.4.3 Evaluation of rock mass strength with the SRM approach
An SRM sample was created by inserting a Discrete Fracture Network (DFN) into an intact BPM sample. The SRM sample was subjected to unconfined and triaxial compression tests with confinements between 1 and 30 MPa. The results captured the brittle–ductile transition and presented a good agreement with the brittle–ductile limit proposed by Mogi (1966). Identifying the onset of yielding was possible through analysis of the stress–strain curves. The onset of yielding occurs at 1.6% of damage, which is independent of the confinement used in the test.

The estimated GSI values for the SRM sample were obtained by relating the geometrical and strength characteristics of the DFN to the parameters used in the GSI quantification methods proposed by Cai et al. (2004) and Hoek et al. (2013). A good agreement was obtained between the UCS obtained from the SRM tests and the UCS obtained with the estimated GSI values and the HB criterion. The confined strength of the SRM sample was lower than the strength obtained from the HB. This result is caused by a limitation of the parallel bond contact model in the Particle Flow Code (PFC) that was used to simulate the intact rock in the BPM.

1.4.4 Evaluation of the effect of intact rock and discontinuity strength on rock mass strength with SRM
The impact of the strength of the intact rock, as well as that of the strength and stiffness characteristics of the discontinuities, on the strength of the SRM samples was evaluated and compared with empirical estimates of rock mass strength obtained with the HB criterion. The SRM was capable of reproducing the same ratio of rock mass UCS to intact rock UCS for materials with different intact rock strengths. Under confined conditions, the sensitivity of SRM to changes in the UCS of intact rock is smaller than that obtained from performing similar changes to the input parameters in the HB criterion.
The evaluation of the impact of the strength and stiffness properties of the discontinuities in the SRM specimen showed that under unconfined conditions, the impact of the increase in the strength of the discontinuities is smaller than the HB estimate. Under confined conditions, the increase in strength in the SRM sample matched the increase in strength obtained from the HB criterion for an increase in GSI of five points.

1.4.5 Effect of Stress Path on Damage Development at Rock Mass Scale
The SRM samples were tested with the use of two different loading paths to evaluate the influence of the loading conditions on the damage development and strength of the SRM specimen. Observing the differences between the level of damage caused by the two loading paths was possible only at lower damage levels.

The samples tested with a triaxial deconfinement test showed an irregular behavior once the stress state passed the onset of yielding limit identified in the UCS and triaxial tests. From these results, it is possible to conclude that the onset of yielding of the SRM samples is independent of the loading path.

1.4.6 Effect of Damage on Rock Mass Strength
A series of unconfined and triaxial tests has been performed on the damaged SRM samples to evaluate the effect of damage on strength. One sample with randomly inserted damage and seven samples with damage caused by the previous loading have been included in the analysis. The sample with randomly inserted damage showed a smaller reduction in strength, than the results obtained from the damaged BPM samples. The difference is caused by the presence of discontinuities and their effect on damage development.

1.4.7 Estimating Rock Mass Residual Strength with SRM
The residual strength values obtained from the SRM tests were compared with the empirical estimates obtained following the recommendations proposed by different researchers. In general, the empirical estimates give lower strengths than what is obtained from the SRM tests. The brittle–ductile transition
observed in the SRM samples is not captured by the empirical estimates, which generate larger
differences between the empirical estimates of residual strength and the results obtained from the SRM
specimens that exhibit ductile post peak behaviour.

1.4.8 Effect of geometry and in-situ stress on the stress path and damage

3D numerical models of large open pits have been analyzed under varying in-situ stress conditions to evaluate the influence of these parameters on the change in stress and on the distribution of extension strains around the pit. The results from the unconfined and triaxial compression tests performed on the SRM samples showed a direct relationship between the damage and the confining stress at which the test was conducted, with increasing levels of damage for increasing confinement. This relationship has been used for the qualitative evaluation of the different levels of damage that the rock mass can develop in different areas of the pit.

In general, the acting stresses follow a particular stress path, with a reduction in the confining stress and a slight increase or a relative constant level in the major principal stress. Therefore, the stress state at which the strength envelope is reached depends on the in-situ stress conditions. On the basis of this finding, larger amounts of damage can be expected in lower parts of the slope under high values of horizontal in-situ stress.
Chapter 2

Rock Mass Strength and Slope Stability Analysis in Open Pit Mining

2.1 Introduction

Several mines around the world have reached or will reach depths of 1,000 meters in their planned mine life. The importance of slope design in the economic profit of an open pit operation increases with the scale of the mine. For a large open pit, even a small increment in the slope angle could reduce the removal of waste rock significantly. On the other hand, the consequences of a large-scale failure could be catastrophic to the operation (Hoek et al. 2000).

Estimating the strength of the rock mass is one of the most important aspects, if not the most important, for slope stability analyses. This is also one of the major challenges engineers and consultants face when working on the design of any structure built in rock. Performing tests on a scale that is representative of the behavior of the rock mass is not feasible, and, therefore, it is impossible to measure the strength at the rock mass scale. Empirical methods are commonly used to obtain rock mass strength estimates; although, given the scale of the slopes, there is some uncertainty about the applicability and reliability of these methods when used in the analysis of large rock slopes. The development of state-of-the-art numerical techniques could provide an alternative for the estimation of rock mass strength.

2.2 Rock Mass Classification Systems

Although originally developed for civil engineering applications, rock mass classification systems have found widespread use in the mining industry. Classification systems were initially used in tunneling to evaluate the quality of a rock mass and to establish a correlation between rock mass quality and rock support. In the process of evaluating the quality of a rock mass, ratings are assigned to those factors that are likely to determine or influence rock mass behavior (Hack 2002, Brady and Brown 2004, Karzulovic and Read 2009). Individual ratings are then combined to obtain a general rating that describes rock mass
quality. A correlation between rock quality and observed rock behavior is determined by using ratings obtained from case histories (Brady and Brown 2004). It is expected that the use of classification systems will produce reliable results if used in rock masses and applications that are similar to those used in their development (Brady and Brown 2004).

Due to the initial success of rock mass classification systems in underground applications, their use was extended to rock slopes. In the evaluation of slopes, rock mass classification systems could be used directly as an indicator of the stability of the slope or as an input to obtain estimates of rock mass strength for slope stability analysis with analytical or numerical methods. Some of the most widely used rock mass classification systems in the assessment of slope stability are described here.

### 2.2.1 Rock Quality Designation (RQD)

The rock quality designation (RQD) index was developed by Deere in 1964 as a tool to obtain a quantitative assessment of rock mass quality from drill core logs. RQD was developed with the objective of assisting in the design of tunnels and large caverns. RQD is a modified core recovery index defined as the percentage length of an intact core longer than 10 cm per core run (Deere et al. 1967) (Figure 2-1). According to Deere and Deere (1988), a core with a diameter greater than 47.5 mm (NX size core) should be used for the evaluation of RQD. The use of diameters smaller than 47.5 mm favors the occurrence of mechanical breaks and core loss, resulting in lower values of RQD.

For cases where a core is not available, it is possible to estimate RQD indirectly through correlation with other parameters. Priest and Hudson (1976) developed a correlation between joint spacing and RQD (Equation 2-1), assuming that joint spacing can be described using a negative exponential distribution.

\[
RQD = 100e^{0.1\lambda} (1+0.1\lambda)
\]  

2-1
Figure 2-1. Procedure for calculation of RQD (Hoek 2013).

As an alternative, RQD may be obtained from the number of fractures per unit volume or volumetric joint count (Jv). Palmström (1974) derived an equation that relates Jv to RQD (Equation 2-2).

\[
RQD = 115 - 3.3 \text{Jv}
\]

2-2

In 2005, Palmström introduced a new equation to calculate RQD (Equation 2-3).

\[
RQD = 110 - 2.5\text{Jv}
\]

2-3

This new equation likely gives better results for prismatic blocks, while Equation 2-3 is more appropriate for bar type blocks (Palmström, 2005). Caution should be used when applying the equations proposed by Priest and Hudson (1976) and Palmström (1974, 2005), as the range of possible RQD values for a single value of spacing or Jv is fairly large, Figure 2-2.
Despite its relatively simple definition, RQD presents limitations and sources of error in its assessment. These include the following (Hack 2002):

- The value of 10 cm for unbroken rock is arbitrary and abrupt. According to this, a rock mass with joint spacing of 90 mm perpendicular to the borehole would give an RQD value of 0%, while a joint spacing of 110 mm would give an RQD of 100%.
- There is a bias caused by the orientation in which RQD is measured with respect to the orientation of the joints.
- RQD can be influenced by the drilling and handling of the core. It is possible to reduce operational influence on RQD by using good-quality drilling techniques and core logging on site.
- RQD is insensitive when the rock mass is moderately fractured (Martin and Grenon 2003). This is of particular importance for the analysis of large rock slopes given the scale of the slopes and the wide range of joint spacing values that are relevant for the stability of the slopes.

In open pit mining, RDQ is regularly used in core logging and surface mapping. Its main use is as input to other classification systems, such as RMR.

### 2.2.2 Rock Mass Rating (RMR)

The geomechanics classification or rock mass rating (RMR) scheme was developed by Bieniawski in 1973. That original version of the rating system was based on 49 unpublished cases of shallow tunnels in sedimentary rocks (Edelbro 2003, Kaiser et al. 1986). The classification system has experienced significant changes as more cases studies have been included. In its 1989 version, a total of 351 case studies were incorporated into the update of the classification system, including different rock types and applications (tunnels, caverns, mines, foundations, and slopes) (Bieniawski 1989). In 2014, RMR was updated with the addition of 2,298 tunnel face case studies (Celada et al. 2014). The 2014 version of the geomechanics classification was specifically developed for tunneling applications. Therefore, the following discussion is based upon the 1989 version of RMR.

The rock mass rating scheme uses six parameters to classify a rock mass: the uniaxial compressive strength of the rock, rock quality designation, joint spacing, joint condition, groundwater condition, and joint orientation. Individual ratings are assigned to each parameter following the guidelines outlined in Table 2-1. The rock mass rating is obtained by adding individual ratings. In the case of rock slopes, the effect of ground water is usually included explicitly in stability analysis, and therefore the rock mass should be considered dry to avoid double counting the effect of pore pressure in slope stability. The adjustment for joint orientation is intended for tunnel applications and is usually not incorporated in slope applications.
2.2.3 Mining Rock Mass Rating (MRMR)

The mining rock mass rating (MRMR) was introduced by Laubscher (1975) as a modification to the rock mass rating developed by Bieniawski (1973). Some adjustments were included in MRMR to add flexibility to the system for its application in different mining conditions. In its original version, RMR was calculated using the different ratings defined by Bieniawski (1973), and then adjustment factors were applied to take into account the effects of mining-induced stresses, weathering, joint orientation, and
blasting effects. Since its introduction in 1975, the MRMR has been updated and modified several times (Laubscher 1977, 1984, 1990, Laubscher and Taylor 1976, Laubscher and Jakubec 2001).

In the 2001 version of MRMR, the concept of in situ rock mass rating (IRMR) was introduced to avoid confusion with Bieniawski’s RMR. IRMR is calculated as the sum of the rock block strength (RBS), joint spacing, and joint condition ratings. The concept of scale is embedded in IRMR by including the effect of heterogeneity and the presence of small-scale defects in the strength of a rock block (Jakubec and Esterhuizen 2002). Once the IRMR value has been determined, MRMR is obtained by multiplying IRMR by adjustment factors related to mining-induced stresses, weathering, joint orientation, blasting, and water. A flowchart outlining the steps involved in the evaluation of IRMR/MRMR is presented in Figure 2-3. A detailed description of the IRMR/MRMR system can be found in Laubscher and Jakubec (2001) and in Karzulovic and Read (2009).

Figure 2-3. Procedure for the evaluation of the in situ rock mass rating (IRMR) and the mining rock mass rating (MRMR) (Laubscher and Jakubec 2001).
2.2.4 Geological Strength Index (Hoek 1994, Hoek et al. 2002, Hoek et al. 2013)

The development of the geological strength index (GSI) is directly linked to the Hoek-Brown failure criterion. During the first years of application of the Hoek-Brown failure criterion, it was found that the RMR system had some problems when applied to poor-quality rock masses (Hoek 1994, Marinos et al. 2005). The use of RMR to calculate the constants in the Hoek-Brown criterion was abandoned in 1992, and the concept of GSI was introduced by Hoek (1994) and Hoek et al. (1995) to overcome the limitations associated with RMR in the characterization of poor quality rock masses. The GSI system differs from other rock mass classification systems, as its objective is to correlate intact to rock mass strength based on rock mass structure and the condition of the discontinuities, instead of being used to determine a certain rock support design. The first GSI chart included four classes of rock mass structure and five classes of surface conditions (Hoek 1994, Hoek et al. 1995, Hoek and Brown 1997). In 1998, the GSI system was expanded to accommodate laminated and sheared rock with non-blocky structure found in tunnels excavated in the Athens Schist formation (Hoek et al. 1998). The GSI chart was further refined in 2000 with the addition of the intact or massive category in rock mass structure (Marinos and Hoek 2000a) and the creation of an alternative chart specifically developed to characterize heterogeneous rock masses (Figure 2-4).

In practice, GSI values are determined through the qualitative visual inspection of rock mass structure and the conditions of the discontinuities following the descriptions provided in the GSI chart. It has been recommended that the correct approach to estimate GSI values is to assign a range instead of a single value. When the GSI concept was introduced, correlations with RMR (Equation 2-4) and Q’ (Equation 2-5) were provided to estimate GSI values.

\[
GSI = RMR_{76} \text{ or } GSI = RMR_{89} - 5 \
\]

\[
GSI = 9 \times \ln(Q') + 44. 
\]

where \( RMR_{76} \) is the rock mass rating calculated according to Bieniawski (1976), and \( RMR_{89} \) is the rock mass rating calculated following recommendations by Bieniawski (1989). Q’ is the rock tunneling quality.
index (Barton et al. 1974) calculated without including the effects of the reduction factors associated to groundwater or stress.

To obtain a good correlation between GSI and RMR, the rock mass should be considered to be in a dry condition and with no adjustment for orientation. Experience has shown that when applied to weak rock masses (GSI<35), the results are not adequate, and therefore the use of equations 2-4 and 2-5 is not recommended (Marinos et al. 2005).

![GSI chart for average rock masses](image)

**Figure 2-4. GSI chart for average rock masses (Hoek and Marinos 2000).**
As with any rock mass classification system, more reliable GSI estimates are obtained when observations are performed by experienced professionals. However, it is often found that data collection tasks are performed by junior staff or by professionals without a strong geological background, who might struggle in assigning a particular value using the qualitative descriptions provided in the GSI chart (Hoek et al. 2013). The qualitative nature of GSI evaluation has been mentioned as one of the limitations of this system, making it possible that different people, with different professional backgrounds and experience, would estimate different GSI values for the same rock mass (Sonmez and Ulusay 1999, Cai et al. 2004, Russo 2009). Obtaining different rock mass evaluations by different observers is not a problem particular to the GSI system—this issue can also be found in other classification systems with a more “quantitative” nature, such as RMR and Q, as shown by Carter (2010).

Different authors have proposed quantitative methods to obtain GSI values in an attempt to facilitate the use of the GSI system by professionals who do not have the experience or background to evaluate GSI by visual inspection, and at the same time to reduce subjectivity and increase the reliability of the GSI values estimated in the field, (Sonmez and Ulusay 1999, Cai et al. 2004, Russo 2009, Hoek et al. 2013). As mentioned previously, GSI is obtained by evaluating the discontinuity surface conditions and the interlocking of the blocks defined by the discontinuities. In general, all quantification methods use scales to correlate structure and surface conditions to the quantitative parameters used in other rock mass classification schemes. A brief description of the most relevant GSI quantification methods is presented in the following section.

2.2.4.1 GSI quantification—Sonmez and Ulusay (1999, 2002)

The first quantitative approach to determine GSI was developed by Sonmez and Ulusay (1999, 2002). In this approach, the structure of the rock mass is evaluated using a structure rating (SR), which is based on the volumetric joint count ($J_v$). The structure rating ranges from 0 to 100 and is equally divided among the upper five categories of structure in the GSI chart (disintegrated to massive). The relationship between the
structure rating and $J_v$ was obtained by linking descriptions provided in the GSI chart with those included in ISRM (1978) (Table 2-2).

Table 2-2. Relationship between the volumetric joint count and structure categories in GSI (modified after Sonmez and Ulusay 1999).

<table>
<thead>
<tr>
<th>Descriptions by ISRM</th>
<th>$J_v$ (joint/m$^3$)</th>
<th>Descriptions for GSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very large blocks</td>
<td>&lt; 1</td>
<td>Intact or massive</td>
</tr>
<tr>
<td>Large blocks</td>
<td>1 - 3</td>
<td>Blocky</td>
</tr>
<tr>
<td>Medium sized blocks</td>
<td>3 - 10</td>
<td>Very blocky</td>
</tr>
<tr>
<td>Small blocks</td>
<td>10 - 30</td>
<td>Blocky/Disturbed/Seamy</td>
</tr>
<tr>
<td>Very small blocks</td>
<td>30 - 60</td>
<td>Disintegrated</td>
</tr>
<tr>
<td>Crushed</td>
<td>&gt; 60</td>
<td>Disintegrated</td>
</tr>
</tbody>
</table>

The surface condition used in the GSI chart is quantified using the surface condition rating, (SCR), which is obtained from the roughness, weathering, and infilling ratings from the RMR$_{89}$ system. Once the structure and the structure condition ratings, SR and SCR, are determined, a GSI value can be obtained from the quantified GSI chart by Sonmez and Ulusay (2002) (Figure 2-5).

Figure 2-5. Quantified GSI chart (Sonmez and Ulusay 2002).
2.2.4.2 GSI quantification—Cai et al. (2004)

The approach developed by Cai et al. (2004) to quantify GSI correlates block volume with the structure description in GSI and a joint condition factor to describe surface condition. When the rock mass is intersected by three joint sets, the block volume can be determined by Equation 2-6.

\[
V_b = \frac{S_1 \times S_2 \times S_3}{\sin \gamma_1 \times \sin \gamma_2 \times \sin \gamma_3}
\]  

2-6

where \( s_i \) and \( \gamma_i \) are representative values of spacing and the angle between the joint sets. Other methods available to calculate block volume can be found in Palmström (1996), Kim et al. (2007, 2015), and Morelli (2016). The joint condition factor, \( J_c \), used by Cai et al. (2004) takes into account the roughness, weathering, and infilling of the joint. The joint condition factor (\( J_c \)) is defined as

\[
J_C = \frac{J_W J_S}{J_A}
\]  

2-7

where \( J_W \), \( J_S \), and \( J_A \) are the joint waviness (large scale), smoothness (small scale), and alteration factor, respectively, as defined by Palmström (1996).

The relationship between block volume and the joint condition factor with their respective axes in the GSI chart was obtained through a calibration process using published data (Cai et al. 2004). The quantified GSI Chart is presented in Figure 2-6.
2.2.4.3 GSI quantification—*Russo (2009)*

Russo (2009) used a different approach for the development of a GSI quantification method, using rock mass strength equations from Palmström (1996) (Equation 2-8) and Hoek et al. (2002) (Equation 2-9) to establish a correlation between GSI and the jointing parameter (JP) in RMi (Equation 2-10).
The jointing parameter (JP) is evaluated using the block volume and the joint condition factor. In this case, the joint condition factor corresponds to the original version defined by Palmström (1996). A detailed description of the jointing parameter and its calculation can be found in Palmström (1996). The quantified GSI chart proposed by Russo (2009) is presented in Figure 2-7.

![Figure 2-7. Quantified GSI chart (Russo 2009).](image)

2.2.4.4 GSI quantification—Hoek et al. (2013)

As in the GSI quantification methods developed by Sonmez and Ulusay (1999, 2002) and Cai et al. (2004), the approach followed by Hoek et al. (2013) defines scales to quantify the structure and structure surface conditions in the GSI chart. In this approach, a GSI value is obtained by adding the values obtained for each scale. Minor modifications to the original chart made the GSI lines parallel and equally
spaced, which allowed the attainment of a better correlation between qualitative and quantitative GSI values (Hoek et al. 2013). The joint condition rating (Bieniawski 1989) and RQD (Deere 1964) are used to quantify the surface condition and structure axes. These parameters were selected due to their simplicity and extended application in the field. The structure scale is defined as RQD/2, and the surface condition scale is defined as $1.5 \cdot J_{c89}$ (Hoek et al. 2013). The quantified GSI chart is presented in Figure 2-8. The LAMINATED/SHEARED and the INTACT OR MASSIVE categories are removed from the GSI chart, as their behavior does not follow fundamental assumptions in the Hoek-Brown criterion.

![Quantified GSI chart (Hoek et al. 2013).](image)

Figure 2-8. Quantified GSI chart (Hoek et al. 2013).
2.3 Rock Mass Strength—The Hoek-Brown Failure Criterion

The Hoek-Brown failure criterion was developed in 1980 with the objective of providing a suitable method for obtaining rock mass strength estimates for the design of underground excavations. Evidence from tests on intact rock and a rather limited suite of in situ tests and laboratory tests on models simulating jointed rock masses showed that the strength envelopes of intact rock and rock masses were not linear (Brown 2008). Hoek’s (1968) experience using the Griffith theory in the analysis of brittle failure served as the conceptual starting point for the development of the Hoek-Brown criterion (Hoek 1983, Eberhardt 2012). The original criterion was obtained through a process of trial and error by fitting curves that would match the Griffith theory for tensile normal stresses and the observed failure conditions of brittle rock under compressive normal stresses (Hoek 1983). The relation between the major and minor principal stresses at failure in the original Hoek-Brown criterion is defined by Equation 2-11

\[
\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( \sqrt{\frac{m \sigma'_3}{\sigma_{ci}}} + s \right)
\]

2-11

where \(\sigma'_1\) and \(\sigma'_3\) are the major and minor principal effective stresses, respectively, at failure, \(\sigma_{ci}\) is the uniaxial compressive strength of intact rock, and \(m\) and \(s\) are material constants related to the characteristics of the rock mass. The Hoek-Brown criterion allowed the attainment of an estimate of the rock mass strength based on the strength of intact rock, determined through laboratory testing, and geological observations performed during data collection in the field. The simplicity of the Hoek-Brown criterion is one of the main reasons for the widespread acceptance this criterion found in the rock mechanics community after its introduction in 1980. Applications of the criterion to problems not considered in its original conception, and the experience gained in its use, motivated several updates over the years. A summary of the modifications of the Hoek-Brown criterion between its introduction, and 1995, is presented in Table 2-3.

<table>
<thead>
<tr>
<th>Publication</th>
<th>Coverage</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoek &amp; Brown [1]</td>
<td>Original criterion for heavily jointed rock masses with no fines. Mohr envelope was obtained by statistical curve fitting to a number of ((\sigma_n', \tau)) pairs calculated by the method published by Balmer [28]. (\sigma_1, \sigma_3) are major and minor effective principal stresses at failure, respectively. (\sigma_n, \tau) are effective normal and shear stresses, respectively.</td>
<td>(\sigma_1' = \sigma_3' + \sigma_c \sqrt{m \sigma_3'/\sigma_c} + s)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sigma_2 = \frac{\sigma_c}{2} \left( m - \sqrt{m^2 + 4s} \right) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\tau = A \sigma_c \left( \left( \frac{\sigma_n - \sigma_1'}{\sigma_c} \right) \right) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sigma_n' = \sigma_3' + \left( (\sigma_1' - \sigma_3') / (1 + \partial \sigma_1 / \partial \sigma_3) \right) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\tau = (\sigma_n' - \sigma_3') \sqrt{\partial \sigma_1 / \partial \sigma_3} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\partial \sigma_1 / \partial \sigma_3 = m \sigma_c / 2 (\sigma_1 - \sigma_3') )</td>
</tr>
<tr>
<td>Hoek [17]</td>
<td>Original criterion for heavily jointed rock masses with no fines with a discussion on anisotropic failure and an exact solution for the Mohr envelope by Dr J.W. Bray.</td>
<td>(\sigma_1' = \sigma_3' + \sigma_c \sqrt{m \sigma_3'/\sigma_c} + s)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\tau = (\cos \phi_1' - \cos \phi_1) m \sigma_c / 8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\phi_1' = \arctan\left( \frac{1}{\sqrt{4h \cos^2 \theta - 1}} \right))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\theta = \left( 90 + \arctan\left( \frac{1}{\sqrt{h^2 - 1}} \right) \right) / 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(h = 1 + \left( 16 (m \sigma_n' + s \sigma_c) / (3m^2 \sigma_c) \right) )</td>
</tr>
<tr>
<td>Hoek &amp; Brown [29]</td>
<td>As for Hoek [17] but with the addition of relationships between constants (m) and (s) and a modified form of RMR (Bieniawski [15]) in which the Groundwater rating was assigned a fixed value of 10 and the Adjustment for Joint Orientation was set at 0. Also a distinction between disturbed and undisturbed rock masses was introduced together with means of estimating deformation modulus (E) (after Serafim and Pereira [18]).</td>
<td>Disturbed rock masses: (m_b / m_i = \exp((RMR - 100) / 14))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(s = \exp((RMR - 100) / 6))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Undisturbed or interlocking rock masses (m_u / m_i = \exp((RMR - 100) / 28))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(s = \exp((RMR - 100) / 9))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(E = 10^{(RMR-10)/40})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(m_b, m_i) are for broken and intact rock, respectively.</td>
</tr>
<tr>
<td>Hoek, Wood &amp; Shah [14]</td>
<td>Modified criterion to account for the fact the heavily jointed rock masses have zero tensile strength. Balmer's technique for calculating shear and normal stress pairs was utilised.</td>
<td>(\sigma_1' = \sigma_3' + \sigma_c \left( m_b \sigma_3'/\sigma_c \right)^{\alpha})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sigma_n' = \sigma_3' + \left( (\sigma_1' - \sigma_3') / (1 + \partial \sigma_1 / \partial \sigma_3) \right) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\tau = (\sigma_n' - \sigma_3') \sqrt{\partial \sigma_1 / \partial \sigma_3} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\partial \sigma_1 / \partial \sigma_3 = 1 + \tan^2 \theta \left( \sigma_3' / \sigma_c \right)^{(\alpha-1)} )</td>
</tr>
<tr>
<td>Hoek [11]</td>
<td>Introduction of the Generalised Hoek-Brown criterion, incorporating both the original criterion for fair to very poor quality rock masses and the modified criterion for very poor quality rock masses with increasing fines content. The Geological Strength Index GSI was introduced to overcome the deficiencies in Bieniawski's RMR for very poor quality rock masses. The distinction between disturbed and undisturbed rock masses was dropped on the basis that disturbance is generally induced by engineering activities and should be allowed for by downgrading the value of GSI.</td>
<td>for (GSI &gt; 25) (m_b / m_i = \exp((GSI - 100) / 28))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(s = \exp((GSI - 100) / 9))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a = 0.5)</td>
</tr>
<tr>
<td>Hoek, Kaiser &amp; Bawden [12]</td>
<td></td>
<td>for (GSI &lt; 25) (s = 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a = 0.65 - GSI / 200)</td>
</tr>
</tbody>
</table>
The current version of the Hoek-Brown criterion was introduced in 2002 (Hoek et al. 2002) and is defined by Equation 2-12.

\[ \sigma'_1 = \sigma'_3 + \sigma'_{ci} \left( m_b \frac{\sigma'_3}{\sigma'_{ci}} + s \right)^a \]  

**2-12**

New equations, valid for the entire range of GSI values, were introduced for the calculation of the constants \( a, m, \) and \( s, \) in equations 2-11 to 2-13. A disturbance factor, \( D, \) was introduced to take into account the effect of blasting and stress relaxation on rock mass strength. A brief discussion on the guidelines for the application of the disturbance factor and its effect in rock mass strength is presented later in this section.

\[ m_b = m_i \exp \left( \frac{GSI - 100}{28 - 14D} \right) \]  

**2-13**

\[ s = \exp \left( \frac{GSI - 100}{9 - 3D} \right) \]  

**2-14**

\[ a = \frac{1}{2} + \frac{1}{6} \left( e^{-GSI/15} - e^{-20/3} \right) \]  

**2-15**

### 2.3.1 Applicability of the Hoek-Brown Criterion

The fundamental assumption behind the Hoek-Brown criterion is that both the intact rock and the rock mass behave isotropically. A jointed rock mass can be considered to be isotropic when there is a large number of joints, to the point where the rock mass has no preferred failure direction. Figure 2-9a shows a typical example used to illustrate the situations where the use of the Hoek-Brown has traditionally been considered to be appropriate. The accelerated development of numerical analysis tools capable of explicitly modelling discontinuities has made it possible to tackle situations where the Hoek-Brown criterion is not recommended (Figure 2-9b).
2.3.2 Scale Effects and the Hoek-Brown Criterion

The concept of scale effect applied to rock mass strength is introduced in Figure 2-9. The scale dependence of rock strength occurs at both the intact rock and rock mass scale (Cundall et al. 2008). Figure 2-10 shows an example of the scale effect at the intact rock scale. Hoek and Brown (1980) found that the uniaxial strength ($\sigma_{cd}$) of a sample of any size is related to the strength of a 50-mm-diameter sample through an exponential function that takes into account the ratio between the size of the samples. The decrease in the strength with increased sample size is explained by the larger number of micro-defects contained in bigger samples. Yoshinaka et al. (2008) analyzed a series of tests on different rock types with samples of different size. The results from this analysis showed that the relation between size and strength proposed by Hoek and Brown (1980) is valid for a wider range of rock types than originally considered with a slight modification allowing for a different exponent depending on the rock type.
Figure 2-10. Decrease in uniaxial compressive strength with size (modified after Hoek and Brown 1980).

The issue of incorporating the effect of micro defects in the strength of intact rock into the Hoek-Brown criterion remains an open question. This is of particular importance in mining environments due to the complex arrangement of veinlets and stockworks generated by the geological processes associated with orebody formation (Figure 2-11). An alternative to the relation between sample size and strength proposed by Yoshinaka et al. (2008) is to estimate the rock block strength included in MRMR following the procedure outlined by Laubscher and Jakubec (2001). The use of numerical methods like the synthetic rock mass could also serve as a means to obtain strength estimates of blocks of rock larger than the samples tested in the laboratory (Pierce et al. 2009).
Day et al. (2012) presented a different approach to address this issue, which deals with micro defects from a rock mass classification perspective. This method uses the GSI quantification chart developed by Cai et al. (2004) to calculate two distinct values of GSI. For the purpose of simplicity, block volume and joint condition scales are replaced with integer ratings on scales BB and JB, respectively (Figure 2-12). The first GSI (GSI$_1$) value is calculated following a standard procedure, which is based on joint spacing and joint condition. A second GSI (GSI$_2$) value is estimated by looking at the spacing and joint condition characteristics of micro defects. A composite GSI is then obtained from Equation 2-16.

$$GSI^* = \min[ GSI_1, GSI_2, \{100 \times (JB^* - 1)/5 \times (BB^* + 4)/11\} + 37 \times ((6-JB^*)/5 \times (BB^* + 1)/8)]$$  \hspace{2cm} 2-16$$

with

$$BB^* = \log_{10}(10^{-BB1/3} + 10^{-BB2/3})^{1/3}$$  \hspace{2cm} 2-17$$

$$JB^* = (JB_1 / BB_1 + JB_2/BB_2) / (1/BB_1 + 1/BB_2)$$  \hspace{2cm} 2-18$$

where BB* and JB* are the composite block volume rating and the composite joint condition rating, respectively.
Figure 2-12. Modification to block volume and joint condition scales for use in calculation of composite GSI (Day et al. 2012). The blue box indicates the range of GSI values based on joint spacing and joint condition, the yellow box indicates the range of GSI values based on spacing and joint condition of the veins, and the green box indicates the range of composite GSI values.

At a larger scale, the reduction in strength from the intact, or rock block, to the rock mass strength is caused by the presence of systematic jointing following a defined structural pattern, not by the presence of randomly located microdefects (Cundall et al. 2008). Figure 2-9 illustrates, conceptually, the relationship between strength and different scales encountered in typical underground and open pit situations. The rate of decrease in strength, from intact to rock mass scale, is determined by the ratio between the size of blocks and the volume of rock being evaluated and reaches a constant value of strength once the volume of rock is relatively large compared to the block size.
Schultz (1996) concluded that to estimate the appropriate strength properties of rock, it was necessary to compare the scale of a particular problem to the scale of the fracture network. This concept is implied in the original Hoek-Brown criterion (Hoek and Brown 1980, 1988, Hoek 1983); however, selecting different material parameters for two different scales was not possible until the introduction of the GSI concept.

In the field, the evaluation of GSI should be performed taking into consideration the scale of the problem at hand as well as the characteristics of the rock mass in terms of block size or joint spacing. This means that the same rock mass may be given two different GSI values depending on the scale of the problem — for example, a tunnel with a diameter of 10 meters versus a 500-meter-high slope in an open pit (see text in upper left corner in Figure 2-8). When a quantitative approach is taken to evaluate GSI, rock mass structure is ranked based on an absolute measure, for example RQD or Jv. With this, the possibility of evaluating GSI depending on the ratio of the rock mass structure and tunnel diameter or slope height is lost. Because of this, the use of quantitative GSI charts should be applied only to problems of a scale similar to those used in their development.

2.3.3 Rock Mass Post Failure Behaviour

The Hoek-Brown criterion was developed with the objective of obtaining estimates of the peak strength of jointed to heavily jointed rock masses. It is known that a rock mass will exhibit a loss of strength after failure. Hoek and Brown (1997) described the expected post peak behavior of rock masses of different quality (Figure 2-13). For good quality rock masses, with GSI values greater than 65–75, it is expected that post peak behavior will be brittle, with almost no plastic strain needed for the rock mass to reach residual strength (Figure 2-13a). Average quality rock masses, with GSI values between 25–30 to 65–75, have a strain softening post peak behavior, reaching residual strength after a significant amount of plastic deformation has occurred (Figure 2-13b). Cai et al. (2007) suggest that residual strength is reached after strain has reached about 5 to 10 times the strain needed to reach peak strength. Hoek and Brown (1997) noted that for average rock masses, the residual strength could be estimated by reducing the GSI value.
from in situ to broken rock mass. Poor quality rock masses, with GSI values less than 25–30, have a perfectly plastic post peak behavior, with no loss in strength after failure (Figure 2-13c).

Figure 2-13. Suggested post peak behavior for different quality rock masses (Hoek and Brown 1997).

Despite the importance of understanding and predicting rock mass post peak behavior for rock engineering and design, the amount of attention this topic has received in the specialized literature is significantly less compared to the efforts devoted to estimating rock mass peak strength. Russo et al. (1998) argue that the residual strength of a rock mass could be obtained with the Hoek-Brown criterion and a residual GSI equal to 36% of the in situ GSI. Ribacchi (2000) proposed the use of residual values of the Hoek and Brown constants $m_b$ and $s$ and also a residual value of the intact rock uniaxial compressive strength (Equation 2-19).
Cai et al. (2007) noted that, given the types of rocks used in Ribacchi’s study, these recommendations would be applicable only for rock masses with joints characterized by a thin infilling or unweathered to slightly weathered surfaces.

Cai et al. (2007) presented a similar approach, where the quantitative method they had developed previously is used to obtain a residual GSI value. As described previously, their method for calculating GSI uses the block volume and a joint condition factor. By determining the volume of blocks and the joint condition factor after failure, it is possible to calculate a residual value of GSI that can be used to calculate the rock mass residual strength. Through the use of numerical modelling with the software ELFEN (Rockfield 2010) and in situ block shear tests performed in Japan, Cai et al. (2007) determined that blocks in failed areas had a size between 1–5 cm. According to their quantified GSI chart (Figure 2-6), this block size would be equivalent to a disintegrated rock mass. For the evaluation of the residual joint factor, Cai et al. (2007) suggest reducing by half the ratings assigned to the joint waviness and joint smoothness.

### 2.4 Open pit slope design

A reliable and robust slope design is fundamental to the successful operation of an open pit mine. The economic cost of a large-scale instability could be catastrophic to mining operations. An example of a large-scale slope failure is presented in Figure 2-14. This instability occurred at a mine in North America in April 2013. This slope failure is the largest ever recorded in an open pit mine and likely the largest non-volcanic one that occurred in North America in recent times (Pankow et al. 2014). The cost of such a failure in terms of loss of production and the cost associated with the implementation of remedial works is estimated to be several hundred million dollars.
Figure 2-14. Large-scale slope failure occurred in April 2013 at Bingham Canyon mine (photo by Kennecott Utah Copper, via earthsky.org)

2.4.1 Failure mechanisms

The expected failure mode in a rock slope is largely dependent on the stress state, structure, rock mass quality, and the scale of the slope. For small-scale slopes, e.g., a bench in an open pit mine, where the stresses are low compared to the strength of the rock, failures are likely controlled by the structure. As the scale of the slope increases, the possibility of finding a fully persistent structure that completely defines the failure surface decreases significantly, and other failure modes control instability. An analysis of the expected modes of failure is an essential step in pit slope design, as the mode of failure dictates the type of analysis required to evaluate the stability of the slopes (Read and Stacey 2009).

A brief review of the most common modes of failures observed in open pit slopes of different scales is presented in the following section. A more detailed review of rock slope failure modes can be found in the work of Sjöberg (1999).
2.4.1.1 Structurally controlled failure

Figure 2-15 shows the typical structurally controlled failure modes found in open pit slopes. In these slopes, planar and wedge failures are commonly found in benches and, with less frequency, at the interramp scale. The scale of these types of instabilities is limited by the persistence of discontinuities that form the instability. It is improbable that a natural discontinuity will have the persistence required to form a failure surface that enables kinematic release (Eberhardt et al. 2004).

![Figure 2-15: Structurally controlled slope failure modes, planar failure with tension crack (left) and wedge failure (right) (modified after Sjöberg 1999).]

2.4.1.2 Circular failure

Circular failures are a common mode of failure of slopes in soil; however, rock slopes can also present this failure mode, particularly in heavily fractured rock masses where the size of the blocks is small compared to the scale of the slope, and when the degree of particle interlocking is low, as shown in Figure 2-16a (Hoek and Brown 1981). In open pit mines, circular failures can also occur in heavily altered/weathered rock masses, where the strength of the intact rock is significantly reduced (Duncan and Wyllie 2004). In large-scale slopes, a circular failure would likely involve failure along pre-existing discontinuities along with the failure of intact rock, as shown in Figure 2-16b (Sjöberg 1999).
2.4.1.3 Toppling failure

Toppling failure occurs in slopes where discontinuities dip steeply into the slope. Toppling is characterized by a rotation of columns of rock formed by discontinuities that dip into the slope at high angles (Sjoberg 1999). This type of failure has been observed in large-scale open pit slopes, as shown in Figure 2-17 (Board et al. 1996, Lorig and Calderón 2002).

Figure 2-16. Circular failures, a) small-scale failure in homogeneous material, b) large-scale circular failure in rock (modified after Sjöberg 1999)

Figure 2-17. Rotation of blocks caused by large-scale toppling failure.
2.4.1.4 Complex failure modes

Structurally controlled failures require the presence of fully persistent structures. For large-scale slopes, the presence of fully continuous structures that could completely define the failure surface is highly unlikely (Eberhardt et al. 2004). At a large scale, more complex failure modes can be expected, including a combination of sliding through pre-existing discontinuities and brittle failure of intact rock (Brideau et al. 2009, Havaej et al. 2013), as shown in Figure 2-18. Through fracturing of intact rock, an interconnection of non-persistent discontinuities is generated, which can result in the formation of a failure surface and, ultimately, in slope failure (Herrero 2015).

![Complex failure mode](image)

**Figure 2-18.** Complex failure mode for large rock slopes involving structurally controlled and rock mass failure (redrawn after Baczynski 2000).

2.4.2 Slope stability analysis

Empirical, analytical, and numerical methods with varying degrees of complexity are used in the mining industry to evaluate the stability of rock slopes. The available methods range from simple empirical charts to sophisticated numerical methods capable of simulating the complex interaction between discontinuities.
and the intact rock. Typically, the selection of the method to be used for stability analysis depends on the stage of the project, the quality of available data, and, when numerical methods are used, the familiarity of the engineer with the software to be used. More sophisticated methods are required with more complex modes of failure. Figure 2-19 illustrates different failure mechanisms and the recommended level of sophistication for the analysis of a particular type of failure.

![Diagram showing different failure mechanisms and their complexity]

**Figure 2-19. Relationship between the complexity of the mode of failure and the recommended method for the analysis (after Stead et al. 2006).**

A brief description of each method is provided in the following section.

2.4.2.1 Empirical methods

Empirical methods could be used in the early stages of a project, when the available data are limited. Slope angle versus slope height charts are the most widely known empirical method. Available charts
include those in the work of Hoek (1970) and Sjöberg (1999), as shown in Figure 2-20. In these charts, the geometry of the slope and the observed behaviour (stable/unstable) are represented. Using this information, both authors have included curves that define the factor of safety for different combinations of slope angle and height (Read and Stacey, 2009). In Figure 2-20, the stable slopes can be identified in areas where the chart indicates an unstable behaviour. This observation is proof of the high level of uncertainty associated with the use of these methods.

Figure 2-20. Slope angle versus slope height charts (redrawn after Stacey and Read 2009). Open symbols show stable slopes and filled symbols correspond to unstable slopes.

2.4.2.2 Limit equilibrium

Despite the tremendous advances in sophisticated methods of analysis, limit equilibrium is still the preferred method to evaluate the stability of open pit slopes. Its popularity is probably related to the simplicity of limit equilibrium software.

In limit equilibrium techniques, a factor of safety is obtained by comparing the resisting forces with the driving forces. This method assumes that the failed slope moves as a rigid block; therefore, information on the deformation characteristics of the rock is not needed. The analysis of circular failures requires knowledge of the distribution of effective normal stress acting on the failure surface (Read and Stacey
This distribution is obtained by subdividing the mobilized section of the slope in slices, as shown in Figure 2-21. Each slice needs to satisfy the conditions of static equilibrium. The equation system formed from the application of equilibrium conditions to each slice is indeterminate because more unknowns are present than equations (Abramson et al. 1996). Varying assumptions are made by different analysis methods to solve the equation system. As a result of these assumptions, some of the methods do not satisfy all conditions of static equilibrium, Table 2-4.

Figure 2-21. Limit equilibrium analysis with the method of slices.

Table 2-4. Static equilibrium conditions satisfied by different slice methods (after Read and Stacey 2009)

<table>
<thead>
<tr>
<th>Method</th>
<th>Force equilibrium</th>
<th>Moment equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fellenius OMS</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Bishop’s simplified</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Janbu’s simplifies</td>
<td>Yes</td>
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<td>US Corps of Engineers</td>
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<td>Lowe and Karafiath</td>
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<td>Morgenstern and Price</td>
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<td>Spencer</td>
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The main limitations of the limit equilibrium method are associated with its inability to incorporate the effect of stress, strain and localized failure in the stability analysis. The limit equilibrium method should not be used when factors, such as stress and deformation of the rock, affect the behaviour of the slope.

2.4.2.3 Numerical methods

The expected failure modes of large rock slopes present many complexities that can be properly addressed with the use of limit equilibrium techniques. The use of numerical methods provides a powerful tool for the analysis of complex failure modes. Based on the characteristics of the different numerical methods they can be grouped into three categories: continuum, discontinuum and hybrid modelling (Stead et al. 2011).

2.4.2.3.1 Continuum methods

In their general form, continuum methods consider the rock mass as a single body with a continuum displacement field. Continuum methods solve the governing differential equations by discretizing the domain into a finite number of elements and thus reducing the number of degrees of freedom of the problem from an infinite to a finite number (Jing 2003). The most commonly used continuum methods are the finite element method (FEM) and the finite difference method (FDM). Despite the requirement of a continuous displacement field, the development of joint or interface elements allows the simulation of the effect of discontinuities on rock mass behaviour. An example of the application of FEM to problems including the presence of discontinuities is presented in Figure 2-22.
Figure 2-22. Application of FEM for the analysis of slope stability in the transition between open pit and block cave mining (courtesy D. Beck).

A large number of applications of continuum methods on slope stability problems can be found in the literature (e.g., Lorig and Varona 2000, Stacey et al. 2013, Noorani et al. 2011). The author has experience in the application of continuum methods to a wide variety of geotechnical problems, including slope stability analysis that incorporates the effect of hydro mechanical coupling, geotechnical design of waste dumps, and general slope stability applications.

2.4.2.3.2 Discontinuum methods
With discontinuum methods, the rock mass is treated as an assembly of rigid or deformable blocks that can move or interact with other blocks at the contact points between them. The distinct element method (DEM) is the most widely used discontinuum numerical approach used in the stability analysis of open pit slopes. UDEC (Itasca Consulting Group 2014) is a commercially available DEM software. Numerous references can be found in the literature regarding the use of DEM in slope stability analysis (e.g., Benko and Stead 1988, Alejano et al. 2011, Bashin and Kaynia 2004, Kveldsvik et al. 2009). The author has
used DEM to back the analysis of large-scale failure with partial structural control, and for the stability analysis of large-scale rock slopes, as shown in Figure 2-23.

Figure 2-23. Stability analysis of a large-scale rock slope with the failure mechanism partially controlled by a major fault. Stability analysis conducted using UDEC. The green area represents the failed area of the slope.

2.4.2.3.3 FEM/DEM

With the combined finite element discrete element method (FEM/DEM), simulating the formation and propagation of fractures in a continuum or in jointed media is possible. ELFEN (Rockfield 2004) is a commercially available FEM/DEM modelling software. In ELFEN, the simulation of crack formation is accomplished by using a fracture energy approach (Stead et al. 2006). The continuum sections of the model are simulated with the Mohr-Coulomb model and the Rankine tensile smeared crack (Lisjak and Grasselli 2014).

Eberhardt et al. (2004) used FEM/DEM to simulate the evolution of failure in massive rock slopes with respect to the development of the slide plane and internal strength degradation. Vyazmensky et al. (2010) applied FEM/DEM to the analysis of large-scale slope failures induced by cave mining at the toe of the slope.
2.4.3 Influence of in situ stress on slope stability

Prior to the excavation of an open pit, the rock mass is subjected to in situ stresses caused by the weight of the overlying rock and locked in tectonic stresses (Hoek et al. 2009). The excavation of the open pit will generate a redistribution of the in situ stress around the pit, with some areas experiencing stress concentrations and other areas experiencing in situ stress relief (Sjöberg 1999, Hoek et al. 2009). Using 2D elastic models, Hustrulid and Kutcha (1995) showed that the influence of in situ stress on the induced stress state is restricted to the lower areas of the slope, whereas the stress state is controlled by gravitational loading in the upper zone. As the open pits become deeper, the magnitude of the deviatoric stress in lower areas of the slope increases, as shown in Figure 2-25, and this condition can lead to the development of rock mass damage (Herero 2015). The new fractures can affect the stability of the slope by providing kinematic release to structurally controlled failures or by weakening the rock, which ultimately reduces the stability of the slope.
Stacey et al. (2003) applied the extension strain criterion (Stacey 1981) to elastic numerical models to evaluate the impact of different in situ stress conditions on the area of the slope where damage caused by stress relaxation might develop. A direct relationship between the magnitude of the horizontal in situ stress and the horizontal extent of the extension strain zone was found. Noorani et al. (2011) analyzed the effect of different magnitudes of horizontal in situ stress on the onset and propagation of failure. The results of their analysis showed a strong correlation between the in situ stress and the location at which the failure initiates. Herrero (2015) used a lattice model to analyze the impact of in situ stress conditions on the development and extension of fractures. The results of the analysis by Herrero (2015) also confirmed the relationship between the magnitude of the horizontal in situ stress and the level of damage caused by stress relaxation.

Most of the studies conducted to evaluate the influence of in situ stress on rock mass damage include simplifications that can affect the conclusions drawn from these studies. Such simplifications include the following:

**Figure 2-25. Variation of principal stresses with depth (after Herrero 2015).**
Use of elastic material, which generates extremely high stress concentrations and does not capture the effect of stress redistribution that occurs after the overstressed rock yields.

Use of dry conditions. The behaviour of rock is controlled by effective stress. For very deep open pit mines, the hydrogeological conditions are expected to affect the distribution of effective stress in the slopes.

Despite the relationship between in situ stress and rock mass damage that has been identified by many authors, the effect of in situ stress is commonly not included in the evaluation of the stability of open pit slopes. Assuming that slopes fail under gravitational loading might be valid for shallow open pits; however, a careful evaluation of the effects of in situ stress on stability needs to be conducted for large open pits in environments with high horizontal stress (Hoek et al. 2009).
Chapter 3

Numerical Methods for the Simulation of Rock Mass Behaviour

3.1 Introduction

Characterizing the mechanical behaviour of jointed rock masses has been a long-standing issue for engineers involved in the design of excavations in rock. The behaviour of jointed rock masses is controlled by a complex interplay of the intact rock and the discontinuities that form the rock mass. The presence of discontinuities introduces non-linearities and anisotropy in the behavior of rock (Lisjak and Grasselli 2014) and creates scale effects, which reduce the strength of the rock mass to a fraction of the strength of the intact rock.

Determining the strength of intact rock with a high degree of certainty is possible through laboratory testing of representative samples. However, for a rock mass, conducting a test at a scale that is representative of the behavior of the rock mass in the field is not possible. Because of the absence of a direct method to measure the strength of rock masses, engineers have relied on the combined use of rock mass classifications and empirical methods to obtain estimates of rock mass strength. Currently, open pit mines are reaching depths that are far beyond the limits at which empirical methods have been derived. This situation introduces an additional level of uncertainty when determining rock mass strength for the stability analysis of large rock slopes.

Recent advances in numerical methods, such as the synthetic rock mass (SRM) model (Pierce et al. 2007, Mas Ivars et al. 2011), provide the means to simulate the behavior of jointed rock masses at scales close to the rock mass representative elementary volume. In SRM, the behavior of intact rock is simulated with the bonded particle method (BPM) (Potyondy and Cundall 2004), which has the capability to explicitly simulate the fracturing process of intact rock. The discontinuities are explicitly represented by inserting a
discrete fracture network into the intact BPM sample. The mechanical behavior of discontinuities is simulated with the use of the smooth joint contact model (SJCM).

The BPM and SRM are used in this study to investigate the relationship between stress path and damage, as well as the effect of damage on strength degradation. In this chapter, the fundamental principles of the BPM and the SRM are reviewed.

3.1 Bonded Particle Model (BPM)

The BPM is implemented in Particle Flow Code (PFC). In such a model, the rock is represented as an assembly of rigid particles bonded together at their contacts. The BPM has been extensively used in the analysis of intact rock (Diederichs 1999, Potyondy and Cundall 2004, Cho et al. 2007, Ghazvinian 2010), and it is an integral part of the Synthetic Rock Mass model used in this thesis.

The basic concepts of PFC are described in the following section¹.

3.1.1 Calculation cycle

In the calculation cycle of PFC, the laws of motion are applied to the particles, the force displacement law is applied to the contacts, and the position of the walls is updated. This process is repeated at each step. At the beginning of each calculation step, the information regarding the set of contacts is updated on the basis of the known positions of the particles and walls. Then, the contact forces are obtained on the basis of the relative displacement between particles and the application of the force displacement law. The law of motion is then applied to the particles to update their position and velocity according to the resultant forces and moments acting on the particle. These resultant moments and forces are obtained from the contact and body forces.

¹ The description of the basic concepts regarding the numerical formulation of the BPM is based on the theory section of PFC’s manual.
3.1.2 Force displacement law

The forces acting at the contacts are obtained from the force displacement law with the use of the relative displacement between particles. The contact force can be represented as the sum of normal ($F_i^n$) and shear ($F_i^s$) components:

$$ F_i = F_i^n + F_i^s. \quad (3-1) $$

The normal and shear components can be obtained from

$$ F_i^n = K^n U^n n_i, \quad (3-2) $$

$$ \Delta F^s = -k^s \Delta U^s, \quad (3-3) $$

where $K^n$ and $k^s$ are the normal and shear stiffness of the contact, $U^n$ is the overlap between particles, and $\Delta U^s$ is the relative shear displacement increment.

3.1.3 Contact models

The mechanical behaviour of the BPM samples is controlled by the contact models used at the contacts (Itasca Consulting Group 2008). The available contact models in PFC3D can be categorized into two groups: linear and Hertz. Two particles could be bonded by a contact or a parallel bond, as shown in Figure 3-1. Calculation cycle in PFC3D (Itasca Consulting Group, 2008)
Figure 3-2. The contact bond acts at the point of contact between two particles. This contact does not resist moment and breaks if the normal or shear force is exceeded. The parallel bond, shown in Figure 3-2b, is the most commonly used contact for intact rock simulation (Potyondy and Cundall 2004). The parallel bond could be considered an analog for finite-sized cement. The parallel bond has the capacity to resist and transfer moment.

Figure 3-2. Illustration of a non-bonded and a parallel-bonded material (Turichshev and Hadjigeorgiou 2016).

The mechanical behaviour of a parallel bond could be described as two particles connected by springs, as shown in Figure 3-3a. Two series of springs act in the normal and shear directions—one set for the stiffness of the linear contact and the other for the parallel bond. The parallel bond can break in tension or in shear, as shown in Figure 3-3b. Once the parallel bond is broken, the springs representing the stiffness of the parallel bond are removed.
3.1.4 Limitations

The BPM presents some limitations for the simulation of the mechanical behavior of intact rock.

According to Turichshev and Hadjigeorgiou (2016), the most important limitations are as follows:

- Particle size dependency. The strength and elasticity of BPM are related to the particle size. Changing the particle size in a calibrated model will affect the macroscopic properties of the model.

- Low compressive to tensile strength ratio. A BPM sample calibrated to the UCS of the rock will overestimate the strength of the rock (Diederichs 1999). With a parallel-bond BPM, matching either the tensile strength or the UCS of the rock is possible, but matching both is not. In the confined region, this is reflected as a low friction angle compared to the friction angles measured in the laboratory (Cho et al. 2007, Potyondy and Cundall 2004).

- Linear failure envelope: Failure envelopes from parallel-bond BPM are almost linear, so they do not match the relationship observed in hard rocks in terms of strength and confinement (Cho et al. 2007).
Attempts to overcome these limitations include the use of clumps (Cho et al. 2007) and clusters (Ghazvinian 2010), as well as the development of new contact modes (Potyondy 2011, Potyondy 2012). Other researchers have opted to replace BPM with other numerical approaches that do not have the problems inherent in the use of spherical particles (e.g., Ghazvinian 2015, Lan et al. 2010, Poulsen et al. 2015, among others).

3.2 Synthetic Rock Mass (SRM)

The SRM approach has been developed to simulate the behaviour of jointed rock masses at large scales. This numerical approach integrates the BPM to model the intact rock, and a DFN to represent the in situ joint fabric. The basics of BPM have been described in the preceding section. The aspects of the numerical representation of the discontinuities and the DFN modelling are presented in the next section.

3.2.1 Smooth joint contact model (SJCM)

Prior to the introduction of the SJCM, the simulation of rock joints in PFC was performed by removing the bonds over a finite area around the joint being modelled, as shown in Figure 3-4 (Bahaaddini et al. 2015). The bond removal approach presents serious difficulties in reproducing actual joint behavior. This situation is caused by the inherent roughness of joints created by the shape of the particles; this roughness prevents the simulation of the sliding of planar surfaces.

![Figure 3-4. Bond removal method for the simulation of rock joints in PFC (after Bahaaddini et al. 2015)](image-url)
The SJCM was developed with the objective of overcoming the limitations of the bond removal method (Mas Ivars et al. 2011). The SJCM simulates the behaviour of a joint at a specified orientation, independent of the contact local orientation, as shown in Figure 3-5. With this contact model, particles can overlap and pass through each other. The problems associated with the use of the bond removal method are thus avoided.

Figure 3-5. Schematic representation of the SJCM (after Mas Ivars et al. 2011)

### 3.2.2 DFN modelling

In SRM modelling, structures are represented explicitly through a Discrete Fracture Network. These networks are constructed with the use of information on fracture orientation, fracture intensity, and joint size distribution within the rock mass. Joint termination and joint hierarchy are important factors in the behaviour of jointed rock masses, and they should be considered during the DFN modelling process. The hierarchy of the joints can be incorporated in SRM by inserting the joints in a specified order (Mas Ivars et al. 2011); joints inserted first will be more continuous, whereas joints inserted at a later stage might present asperities at the point of intersection with joints previously inserted.

A more detailed description of DFN modelling is presented in Section 5.2.2, Chapter 5.
3.2.3 Application of SRM to rock mechanics problems

The first applications of SRM were aimed at evaluating scale effects. Cundall et al. (2008) used SRM to evaluate scale effects and rock mass strength for various rock types at Palabora mine. The results of this analysis were then used by Sainsbury et al. (2008) as an input for mine scale continuum modelling. Pierce et al. (2009) used the SRM approach to investigate scale effects at rock block scale caused by the presence of microdefects. Esmaieli et al. (2010) utilized SRM to determine the representative elementary volume for some rock types present at Brunswick mine in Canada.

![Image](image_url)

**Figure 3-6. Evaluation of scale effect with the SRM approach (after Cundall et al. 2008).**

Other applications of the SRM approach include the following:

- Comparison with empirical estimates or rock mass strength. Poulsen et al. (2015) used SRM to determine rock mass strength and compared the results from numerical models with rock mass strength predictions obtained from the application of the Hoek–Brown criterion. An agreement was found between the strength predicted by the two methods.
Figure 3-7. Comparison between empirical and numerical strength estimates (Poulsen et al. 2015)

- Determining the stability of pillars in underground mining. The SRM has been used for the study of crack propagation in hard rock pillars (Yang et al. 2014) to evaluate the strength and damage of pillars in hard rock mines (Yang et al. 2015) and to analyze the impact of confinement in the behaviour of rock pillars (Zhang et al. 2016).

Figure 3-8. Evolution of damage in hard rock pillar during loading (Yang et al. 2014).
• Analysis of fragmentation in block caving mines. By using a spherical SRM sample, Mas Ivars et al. (2011) were able to subject the SRM sample to a stress path characteristic of the cave back. Size distribution curves were obtained from the SRM tests and then compared to the block size distribution measured at the drawpoints. A good agreement between the two was found.

• Analysis of intact veined rock. Turichshev and Hadjigeorgiou (2015) and Vallejos et al. (2016) have applied the concept of SRM to the analysis of core-sized samples of veined rock under UCS and triaxial test conditions. In both cases, SRM models proved to be successful in replicating results from laboratory tests.

Figure 3-9. Comparison of the fragmentation results obtained from the SRM tests to the block size distribution at drawpoints, (Mas Ivars et al. 2011).
Figure 3-10. Use of SRM for the analysis of intact veined rock (After Turichshev and Hadjigeorgiou 2015)
Chapter 4

Numerical Investigation on the Effect of Stress Path on Intact Rock Damage and the Associated Strength Degradation

4.1 Introduction
The laboratory strength of intact rock is determined through standardized tests, where rock samples are loaded following a specified stress path. During the construction of underground or surface excavations, rock will be subjected to more complex stress paths (e.g. increase of the major principal stress, reduction of the minor principal stress and stress rotation) than those used to determine its strength in the laboratory. An understanding of the effect of different loading scenarios on the behavior of rock can be of assistance in engineering design.

The mechanical behaviour of rocks is largely controlled by the formation, evolution and coalescence of cracks (Potyondy and Cundall 2004). The Bonded Particle Method (BPM) is able to explicitly represent damage as bond-breakage between particles (Potyondy and Cundall 2004), and therefore provides a means to study the development and evolution of damage in rocks as a function of loading. Due to the flexibility of BPM it is possible to examine the behaviour of rock under different loading conditions, which are not possible to reproduce in the laboratory. While the final stress applied to the rock specimen can be identical for the laboratory experiment and the BPM model, the powerful modelling framework provided by BPM permits the complex stress path to be replicated for rock in the proximity of an underground or surface excavation. The associated damage in this case can be extremely difficult or in most cases impossible to reproduce in the laboratory.

This chapter presents the results of exploratory modelling work performed to study the influence of stress path in the development of damage at the intact rock scale. A sensitivity analysis was conducted to evaluate the influence of contact microparameters on the macroscopic response of the sample. Finally, an
investigation of previously damaged samples was performed to study the relationship between damage and strength by comparing the results to those obtained from intact samples.

4.2 SRMLab Description

The modelling work described in this chapter was performed in SRMLab version 1.7 (Itasca Consulting Group 2012). The SRMLab was developed to simulate laboratory tests on Synthetic Rock Mass (SRM) samples to characterize the mechanical behaviour of rock at large scales. SRM models are created by introducing discrete fracture networks (DFN) to the intact rock matrix (represented by BPM). SRMLab is a graphical user interface for PFC3D version 4.0 (Itasca Consulting Group 2008), described in Chapter 3, which takes advantage of PFC3D modelling capabilities, while adding a separate collection of functions for systematic creation and testing of BPM-based SRM samples.

Even though SRMLab was created for the analysis of jointed rock, it can be used to simulate tests on synthetic samples of intact rock. With SRMLab it is possible to create intact BPM samples, insert DFN to the intact sample, therefore generating SRM samples, followed by performing tests on those samples without the need of scripting input data files, as is usually done in PFC3D. This facilitates the use of SRMLab for inexperienced modellers. The FISH scripting language included in PFC3D provides the more advanced users with the capability of modifying the functions included in SRMLab or the development of new functions depending on the requirements of each case being analyzed.

4.3 Sample Genesis Procedure

In BPM, rock is represented as a densely packed group of non-uniform sized balls that are bonded together at their contact points. One of the differences between PFC3D (or SRMLab) or any BPM-based code is that models first need to be synthetized by creating an array of particles and defining how the particles interact with each other. The sample genesis procedure has been described in detail by Potyondy and Cundall (2004) and can be divided into the following five steps:

Compact initial assembly: The sample is created in a material vessel, which is formed by planar frictionless walls. In general, the material vessel can be a parallelepiped, a cylinder or a sphere. In
SRMLab, a parallelepiped is always used to create the sample, and a cylindrical shape is obtained by deleting balls located outside the specified geometry. Particles following a uniform size distribution, bounded by $R_{\text{min}}$ and $R_{\text{max}}$, are placed inside the vessel. In order to obtain a tight particle packing, the number of balls is determined based on a target porosity of 35%. Particles are placed inside the vessel at random locations at half of their final size, so that particles will not overlap. After this process is completed, the size of the balls is doubled reaching their final size (Figure 4-1a), and the model is cycled until static equilibrium is reached.

*Install specified isotropic stress:* The diameter of the balls is uniformly scaled until a specified isotropic stress is achieved (Figure 4-1b). In this study, the target isotropic stress is set to 0.1 MPa.

*Reduce the number of floating particles:* Samples created with particles of different sizes, placed at random locations inside the material vessel and then compacted mechanically, can have a large number of balls with less than three contact points with other balls; these are called floating balls. In order to obtain a dense packing of balls that represents the well-connected and highly interlocked grains of rock, the number of floating particles needs to be reduced. This is achieved by identifying floating balls and increasing their size by a specified factor until floating balls are eliminated from the sample (Figure 4-1c).

*Install parallel bonds:* Parallel bonds will be installed between all particles that are within a certain proximity of each other (Figure 4-1d). The friction coefficient between balls is also assigned.

*Remove from material vessel:* The final step of the material genesis procedure is to remove the sample from the material vessel. After the walls are deleted, the material is allowed to relax by cycling until static equilibrium is reached.
Figure 4-1. Description of the sample genesis procedure in PFC: a) initial assembly with balls at their final size but before rearrangement, b) contact force distributions after balls are scaled to obtain the desired isotropic stress, c) identification of particles with less than three contacts for the floating ball reduction process, d) installation of parallel bonds at the contact points and removal of the material vessel (after Potyondy and Cundall 2004).
4.4 Numerical Testing Procedure

SRMLab includes a collection of FISH functions to simulate laboratory tests on BPM and SRM samples. Supported tests in SRMLab include direct tension, uniaxial compression, triaxial compression and triaxial deconfinement. The numerical procedures used to conduct laboratory tests on BPM samples are presented in the next section.

4.4.1 Unconfined Compression Strength (UCS) Test

There are two approaches to load BPM and SRM samples under unconfined compression; by replicating the action of platens with walls or with a specified velocity acting on the top and lower boundaries of the model, or by assigning internal strains. The use of walls as platens require small strain rates to ensure that the sample remains in quasi-static equilibrium throughout the test. This insures avoiding the inertial effects that would result from stress waves passing through the sample (Itasca Consulting Group, 2008). When testing large samples, the use of low strain rates to load the specimen can result in very long solution times, making this approach unpractical (Pierce et al., 2009). The internal-based strain-application presents a more efficient alternative for loading a large sample. This approach consists of two steps; strain application and sample relaxation. During strain application, particles are assigned a velocity gradient depending on the position of the particle with respect to the center of the sample. The velocity is applied during a specified number of cycles, the amount of strain imposed to the sample in each load step is obtained from:

\[ \text{strain step} = v \times \text{number of cycles} \times \Delta t \]

where \( v \) is the applied velocity at the sample ends and \( \Delta t \) is the numerical timestep used during computation in PFC3D. After each load step the sample is allowed to relax, the top and bottom of the sample are fixed and the velocity applied to each particle is removed. The model then is cycled until static equilibrium is reached. The loading/relax process is repeated until the test is completed (Pierce et al.,
Comparisons between the two loading schemes, platens and internal based strain, show that solution run times can be reduced by an order of magnitude when using the internal-based strain-application loading scheme without affecting material behavior (Pierce et al., 2009). SRMLab is intended as a tool to simulate laboratory tests on SRM samples of large dimensions, and therefore it uses the internal based strain application as the loading scheme.

The termination of the test can be evaluated using a stress or a strain criterion. With the stress drop criterion, the test is terminated once the stress measured in the sample falls below a specified percentage of the peak stress recorded during the test (Figure 4-2a). There are two options when a strain criterion is used; the test can either end when the total strain reaches a certain value, as shown in Figure 4-2b, or by specifying the total strain as a factor of the strain at which the peak stress was recorded (Figure 4-2c).

4.4.1 Triaxial Compression Test

In the triaxial test, axial loading is performed using the same procedure used in the unconfined compression test. The confining stress is set by applying forces to particles on the outer layer of the sample. The magnitude of force to be applied to each particle is determined based on the confining stress magnitude, the lateral area of the sample, and the number of particles in the outer layer.

The first stage in the execution of a triaxial test is the confining stress installation. Here, the sample is loaded isotropically to prevent damage during the stress installation stage. Once the target lateral stress is reached, the sample is loaded in the axial direction while the confinement stress is kept constant. Test completion is evaluated by using one of the criteria described in Figure 4-2.
4.4.2 Triaxial Deconfinement Test

In the triaxial deconfinement test, the lateral stress is reduced during axial loading according to a prescribed axial to lateral stress ratio ($\frac{\sigma_1}{\sigma_3}$ ratio). This test is intended to simulate the loading path that the rock mass experiences on the proximity of an excavation, where the increase in the major principal stress is accompanied by a reduction of the minor principal stress. The initial stress state should match the in situ stress conditions, and the target stress state should be equivalent to the induced stress state at the location of interest.
The stress installation procedure and axial loading are as described for the unconfined and triaxial compression tests, although in this case it is not a requirement for the initial stress state to be isotropic. The lateral stress is modified by adjusting the forces applied on the circumferential surface of the specimen during each loading step according to the stress increment in each load step and the defined $\sigma_1/\sigma_3$ ratio.

An important difference between the UCS and triaxial tests with the triaxial deconfinement test, is the procedure used to perform the unloading after the stress state in the sample reaches the strength envelope. At this point the stress path should follow the strength envelope, however this is not possible when using forces to apply the lateral stress. If the target lateral stress is above the strength envelope, the model will not equilibrate and the sample will suffer an excessive amount of damage. For this reason, once the sample reaches the strength envelope it is unloaded following a user defined strength envelope specified by a prescribed friction angle.

**4.4.3 Stress and Strain Measurement**

In PFC (or SRMLab), particles are rigid and interact with other particles at their contact points. Forces (as opposed to stresses) are calculated at each contact based on the stiffness properties and the relative displacement of each contact. In order to obtain the equivalent continuum stress, it is necessary to average the contact forces over a specified volume (Itasca Consulting Group, 2008). For this purpose, PFC (or SRMLab) provides measurement spheres for which the average stress, among other parameters like porosity or coordination number, is calculated. In this study, three measurement spheres are used in each sample as shown in Figure 4-3. The reported stress values correspond to the average of the three measurement spheres.
Figure 4-3. Measurement spheres used to calculate average stresses during loading.
4.5 Ball and Sample Size Selection

The ball size, size distribution and sample size have an effect on the macroscopic response of BPM samples. The model resolution (Length/diameter: $L/d$), defined as the number of particles across the characteristic length of the sample, needs to be sufficient to reproduce the characteristic failure mechanisms that have an influence on strength properties (Potyondy and Cundall, 2004). In many of the early studies that used particle methods to analyze rock behavior, model resolution was defined by computer limitations and the scale of the problem being studied, without a detailed analysis of the influence of particle size on the physical behavior of the rock (Koyama and Ling, 2007).

Many researchers have performed sensitivity analysis on mechanical and geometrical parameters that influence BPM behaviour in 2D and 3D by analyzing samples with different characteristic length to average particle size ratio ($L/d$). Potyondy and Cundall (2004) found that for 2D BPM with $L/d$ between 10 and 90, the elastic constants and the unconfined compressive strength are relatively insensitive to particle size, but the coefficient of variation reduces with increasing $L/d$ ratio. In PFC, balls are rigid and therefore cracks can only form and propagate between the balls and not through them. In bonded particle models with low resolution, the development of damage and propagation of fractures will be highly influenced by the geometric arrangement of the balls, which explains the higher coefficient of variation observed in samples with lower values of $L/d$.

The results obtained from 3D models on the other hand demonstrate differences between measured properties with model resolution. Poisson’s ratio was found to be independent of particle size, while Young’s modulus exhibits a clear dependence on particle size, increasing with increasing $L/d$ ratio (Potyondy and Cundall 2004). The unconfined compressive strength of the rock also shows a dependence upon the particle size, with increasing trend for models with larger resolution. Similar relationships between the $L/d$ ratio, Young’s modulus and the unconfined compressive strength have been reported by Zhang et al. (2011) and Ding et al. (2014).
Ding et al. (2014) performed a parametric study to analyze the influence of L/d ratio and particle size distribution on UCS in BPM. Their results showed that an important percentage of the strength increment observed with increasing L/d values is related to changes in the porosity of the sample. The default sample genesis procedure used in PFC3D, described earlier in this chapter, produces models with different porosity for samples at different scales. Ding et al. (2014) developed a modified sample genesis procedure that permits the generation of samples of different scale with the same porosity. By testing samples with the same porosity, it was possible to isolate the effect of model resolution on the physical behavior of the sample (Figure 4-4).

![Figure 4-4](image.png)

**Figure 4-4. Influence of sample porosity on the unconfined compressive strength of BPM samples (after Ding et al. 2014).**

### 4.6 Intact Rock Calibration

#### 4.6.1 The Methodology

The macroscopic behaviour of BPM samples is defined by model (i.e. ball and contact) microparameters. Except for very simple packing arrangements, macroscopic properties of BPM samples and rock properties cannot be obtained from direct analysis of contact microparameters (Potyondy and Cundall,
The selection of appropriate microparameters is achieved through a calibration process, where contact properties are adjusted through an iterative process until the relevant properties of rock are matched with an acceptable level of deviation.

The approach used in most studies involving intact rock simulation with BPM is to calibrate the model microparameters in an iterative process until the unconfined compressive strength, Young’s modulus and Poisson’s ratio match the target properties of the rock (Tawadrous et al. 2009, Ghazvinian 2010, Bahrani et al. 2013). In addition to strength and elastic properties, the crack initiation stress (CI) can also be included as a target parameter during the calibration process.

The microparameters available for adjustment during the calibration process are determined by the contact model used to define the mechanics of interaction between particles. In this study, BPM samples have been created using the parallel bond contact model.

The calibration procedure is essentially a trial and error approach and highly dependent on the user’s experience. The following procedure has been described by Potyondy and Cundall (2004) in order to reduce the number of iterations and the time required to obtain a calibrated sample:

1. **Match the Young’s modulus and Poisson’s ratio**: The strength-related parameters of the contacts are set to a large value to prevent bond breakage during loading. The sample is then loaded under unconfined compression and the elastic modulus is obtained from the stress-strain curve. The Young’s modulus of the sample can be adjusted by changing the Young’s modulus of the particle-particle contacts (Ec), and the Young’s modulus of the parallel bonds if they are present. The Poisson’s ratio is controlled by the normal to shear stiffness ratio of the particles (kn/ks) and parallel bonds (\(\overline{kn}/\overline{ks}\)). The Poisson’s ratio increases for increasing values of the normal to shear stiffness ratio.

2. **Peak strength**: After the elastic constants have been calibrated, the unconfined compressive strength of the model is obtained by setting the standard deviation of strength-related contact properties to zero and adjusting the mean contact strength properties. The normal to shear
strength ratio of contacts affects the behaviour of the sample and therefore should be decided upon and kept constant during the calibration process.

3. **Crack initiation stress**: This property is controlled by the standard deviation of the contact strength parameters. Changes in these parameters might affect the sample’s peak strength and therefore additional iterations between steps 2 and 3 might be needed.

Alternative methodologies to obtain appropriate microparameters have been developed in recent years. Yoon (2007) combined a design of experiment (DOE) approach with optimization techniques to determine the microparameter values that would match the macroscopic properties of a wide range of rock types. In this approach, the Placket Burman (PB) design method was used to determine the two microparameters that had the biggest influence on the unconfined compressive strength, Young’s modulus and Poisson’s ratio. Then, the nonlinear relationship between those microparameters and the macroscopic property was obtained with the Central Composite Design Method. Finally, optimization techniques are used to find the optimum set of microparameters to match the target properties (Yoon 2007).

In another effort, Tawadrous et al. (2009) developed an artificial neural network (ANN) approach to determine the appropriate set of microparameters for a given set of rock properties. Neural nets were designed to accept rock properties (Young’s modulus, Poisson’s ratio and unconfined compressive strength) and model characteristics (particle radius ratio, minimum radius and model resolution) as inputs. The output parameters obtained from the neural network are the contact normal stiffness, stiffness ratio and parallel bond strength. Results obtained by Tawadrous et al. (2009) showed a relatively good agreement between target and model properties when the ANN approach was applied to cases with input values close to the center of the range used in the neural network training. An error in excess of 50% was obtained in some cases with inputs close to the limit values used in ANN training.

The estimation procedures proposed by Yoon (2007) and Tawadrous et al. (2009) for BPM microparameters present some advantages over the trial and error approach outlined by Potyondy and
Cundall (2004) in terms of the time required to obtain microparameters that match the rock properties. However, these methodologies do not include the crack initiation stress in their development. This property is important when studying the development of rock damage under different loading conditions and therefore the trial and error approach for model calibration is used in this study.

4.6.2 Calibration Results
The target calibration properties (i.e. unconfined compressive strength, Young’s modulus and Poisson’s ratio) in this study have been selected to match those of typical rocks found in open pit mines in Northern Chile. Determination of the crack initiation stress is not commonly observed in engineering practice when dealing with open pit geomechanics problems. For this reason, typical values obtained by Ghazvinian (2015) will be used for the calibration of this parameter. The list of target properties used for calibration of the intact rock sample is presented in Table 4-1.

Table 4-1. Target properties of the idealized intact rock used in the calibration process

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined Compressive Strength</td>
<td>MPa</td>
<td>75</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>GPa</td>
<td>40</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Crack Initiation Stress*</td>
<td>MPa</td>
<td>33 to 40</td>
</tr>
</tbody>
</table>

* Estimated as percentage of UCS from tests by Ghazvinian (2015)

As it was mentioned earlier in this chapter, the particle size and model resolution have an influence on the macroscopic behaviour of three-dimensional BPM samples. The intact rock calibrated in this chapter will be used to create Synthetic Rock Mass samples in Chapter 5. Thus, the sample size is selected based on constraints imposed by SRM modelling. A detailed discussion of the selection of the sample size used for this study is presented in the next chapter. The sample geometry used for the calibration of microparameters is shown in Figure 4-5. The green layer of particles located at the top and bottom of the sample are called grips, which are used to constrain displacement of the top and bottom of the sample.
during the relaxation stage. The list of calibrated microparameters, obtained by following the calibration steps described previously, is presented in Table 4-2.

Figure 4-5. Geometry of the sample used in the calibration process. The blue particles represent the sample being tested and the green sections at the top and bottom of the sample are the grips.
Table 4-2. The microparameters calibrated to the target properties listed in Table 4-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. Particle radius, Rmin</td>
<td>cm</td>
<td>5.63</td>
</tr>
<tr>
<td>Max./min. Particle radius ratio, Rmax/Rmin</td>
<td></td>
<td>1.66</td>
</tr>
<tr>
<td>Initial number of particles</td>
<td></td>
<td>20,600</td>
</tr>
<tr>
<td>Bulk density, ρ</td>
<td>kg/m³</td>
<td>3150</td>
</tr>
<tr>
<td>Modulus, Ec</td>
<td>GPa</td>
<td>40</td>
</tr>
<tr>
<td>Normal/shear stiffness ratio, kₙ/kₛ</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Friction coefficient, μ</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Modulus, Ec</td>
<td>GPa</td>
<td>40</td>
</tr>
<tr>
<td>Normal/shear stiffness ratio, Kₙ/Kₛ</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Radius multiplier, λ</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Tensile strength (mean), ñn</td>
<td>MPa</td>
<td>57</td>
</tr>
<tr>
<td>Tensile strength (std. dev.), ñ</td>
<td>MPa</td>
<td>15</td>
</tr>
<tr>
<td>Shear strength (mean), c</td>
<td>MPa</td>
<td>114</td>
</tr>
<tr>
<td>Shear Strength (std. dev.), c</td>
<td>MPa</td>
<td>28.5</td>
</tr>
</tbody>
</table>

Several methods have been presented over the years to determine crack damage thresholds from the laboratory tests using the recorded strains and acoustic emission data. Brace et al. (1967) suggested that the crack initiation stress in compression tests could be determined by analyzing volumetric strain to establish the onset of dilatant behaviour in the rock. The crack initiation stress is defined as the point where the “axial stress-volumetric strain” curve deviates from linear behaviour (limit between regions II and III in Figure 4-6).
Figure 4-6. Definition of the onset of damage during axial loading based on the onset of nonlinear behaviour of the volumetric strain (redrawn after Brace et al. 1966).

One of the main problems encountered when using volumetric strain to determine the crack initiation stress is the difficulty in identifying the exact point where the volumetric strain departs from linearity, especially when the sample contains a large number of initial cracks (Martin and Chandler 1994). To overcome this issue, Martina and Chandler (1994) propose the use of crack volumetric strain to define the onset of systematic cracking as shown in Figure 4-7.
The crack volumetric strain for a cylindrical sample is calculated as the difference between the elastic volumetric strain and the volumetric strain. The method proposed by Martin and Chandler (1994) uses the elastic constants of the rock to calculate the elastic volumetric strain. Eberhardt et al. (1998) highlighted the uncertainty in the estimation of the crack initiation stress introduced by the nonlinearity of the “axial stress-lateral strain” response of the rock and the associated implications on the calculated Poisson’s ratio. Based on the testing performed on the pink Lac du Bonnet granite from AECL’s URL in Pinawa, Manitoba, it was found that a difference of ±0.05 in Poisson’s ratio results in ±40% change in the crack initiation stress.

Diederichs et al. (2004) and Eberhardt et al. (1998) have used acoustic emission (AE) data to determine damage thresholds. Figure 4-8 shows the application of this approach to determine different stages of damage from synthetic acoustic emissions events recorded during numerical simulation of compressive tests by Diederichs (2004). The crack initiation stress is determined by the sudden change in the acoustic emission event rate. Diederichs (1999) and Ghazvinian (2010) have successfully applied AE techniques to determine damage thresholds from discrete element simulation results in 2D and 3D, respectively.
The volumetric strain method proposed by Brace et al. (1967) and the AE technique for crack initiation threshold detection method developed by Eberhardt et al. (1998) and Diederichs et al. (2004) have been used during the calibration process. The crack initiation stress obtained from the calibrated case, ranges between 33 MPa (43% of UCS) and 40 MPa (53% of UCS) for the volumetric strain and AE technique, respectively (Figure 4-9). These values are in good agreement with the target values listed in Table 4-1. As mentioned previously, one of the problems with the volumetric strain reversal method is the difficulty in identifying the point where the volumetric strain deviates from linear behaviour. This issue was also encountered during the interpretation of results shown in Figure 4-9a, leading to some uncertainty in the value obtained from this method. The application of the AE threshold detection method to the calibration results also presented some difficulties in determining CI, especially in fitting straight lines to the irregular shape of the number of cracks vs stress curve, Figure 4-9b.
Figure 4-9. Detection of the crack initiation stress from the calibrated model by using: a) volumetric strain, b) AE technique.
A series of triaxial tests were performed on the calibrated sample to define the strength envelope of the synthetic rock (Figure 4-10). An $m_l$ value of 4.5 is obtained when fitting a Hoek-Brown curve to the model results. Typical $m_l$ values for igneous rocks are in the range of 10 to 30. The $m_l$ obtained in this study value is characteristic of BPM samples constructed using the contact bond or parallel bond contact model (Diederichs 1999, Potyondy and Cundall 2004). When a sample is calibrated to the unconfined compressive strength of the rock, a low $m_l$ value translates in an overprediction of the tensile strength and an underprediction of the confined strength.

![Hoek-Brown curve fit](image)

**Figure 4-10. Hoek – Brown strength envelope obtained from the calibrated sample.**

Several approaches have been proposed to overcome (or reduce) the limitation of BPM in reproducing realistic $m_l$ or internal friction angles. These include the use of clusters (Potyondy and Cundall 2004, Cho et al. 2007), clumps (Ghazvinian 2010), and new contact models (Potyondy 2011, 2012).


4.7 BPM Sensitivity Analysis

A series of sensitivity analyses have been performed with the objective of identifying the role of ball and contact microparameters in the macroscopic response of the sample. The sensitivity analysis conducted as part of this study also includes global model aspects such as sample size and loading rate.

4.7.1 Contact Bond Stiffness Ratio \( (k_n/k_s) \)

Previous work has demonstrated that the contact stiffness ratio is the main factor controlling the Poisson’s ratio for BPMs (Diederichs 1999, Potyondy and Cundall 2004, Cho et al. 2007). In this study, the influence of \( k_n/k_s \) on the macroscopic response of BPMs has been investigated by testing samples created with four different \( k_n/k_s \) values. The normal contact bond stiffness has been kept constant at 40 GPa and the contact shear stiffness has been varied obtaining \( k_n/k_s \) values of 1, 2, 4 and 6. The stress-strain response of the samples under unconfined compression conditions are shown in Figure 4-11. It can be observed in this figure that with increasing values of \( k_n/k_s \), the amount of lateral and axial strains at a given stress level increases. It can also be seen that when all other microparameters are kept constant, the peak strength of the sample decreases with increasing values of \( k_n/k_s \). Results obtained from grain based modelling by Ghazvinian (2015) show that with increasing \( k_n/k_s \) the sample becomes more ductile; this effect has not observed in 3D BPM samples tested in this study, although this is probably caused by differences in the range of \( k_n/k_s \) used in this study.

Figure 4-12 shows the variation in Poisson’s ratio for different values of \( k_n/k_s \). It can be seen that the relationship between Poisson’s ratio and \( k_n/k_s \) is not linear, with increasing values of Poisson’s ratio for increasing \( k_n/k_s \) values. This is consistent with previous two-dimensional studies by using contact bond models (Diederichs 1999) as well as a parallel bond model (Yang et al. 2006).
Figure 4-11. Influence of the contact stiffness ratio on the deformation response of the sample.

Figure 4-12. Variation of Poisson’s ratio for different contact stiffness ratios; results from 2D models using the parallel bond (PB) contact model (Yang et al. 2006) and contact bond (CB) model (Diederichs 1999) are included for comparison.
The Young’s modulus can be observed in Figure 4-13 to decrease with increasing $k_n/k_s$ value. The relationship between Young’s modulus and $k_n/k_s$ is not linear and it is expected that the influence of this microparameter becomes less significant for larger $k_n/k_s$ values and ultimately controlled mainly by the $k_n$ value only (Yang et al. 2006). Similar results regarding the relationship between $k_n/k_s$ and Young’s modulus have been reported in 2D BPMs by Diederichs (1999) and Yang et al. (2006).

![Figure 4-13](image)

**Figure 4-13.** Variation of Young’s modulus for different contact stiffness ratios, results from 2D models from Yang et al. (2006) are included for comparison.

The development of damage during loading is also affected by the contact stiffness ratio. Figure 4-14 shows the evolution of damage during a UCS test for the four $k_n/k_s$ cases analyzed. As a result of the difference between the normal and shear stiffness, the tensile forces at the contacts increase (Potyondy and Cundall 2004, Cho et al. 2007). Consequently, an increased number of cracks will develop at a given stress level for samples with increasing $k_n/k_s$ values. When the x-axis in Figure 4-14 is normalized by the
peak stress recorded during testing, it can be observed that the influence of $k_n/k_s$ on damage accumulation only becomes evident at around 70% of the peak stress.

Figure 4-14. Influence of the contact stiffness ratios in damage development up to peak stress during a UCS test.

Figure 4-15 shows the strength envelopes obtained for each case by performing a series of UCS and triaxial tests. The slope of the strength envelope remains nearly constant for the four cases analyzed, which means that $k_n/k_s$ does not influence the friction angle of the material. Cho et al. (2007) and Bahrani et al. (2013) have reported similar results from 2D and 3D BPM simulations, respectively.
Figure 4-15. Strength envelopes obtained from samples with different contact stiffness ratios.

4.7.2 Contact Shear to Normal Strength Ratio (C/T)

The contact tensile and shear strengths are determined through the calibration process as described earlier in this chapter. The contact shear to tensile strength ratio (C/T) affects the damage accumulation process and therefore the macroscopic behaviour of the sample. Despite the major influence of this parameter on the material behaviour, there are no clear guidelines on how to determine this value. Diederichs (1999) combined concepts from fracture mechanics with results from laboratory testing and numerical experimentation to conclude the logical values of C/T for brittle rocks are within a range of 2 to 4. Potyondy and Autio (2001) used BPM to perform a back analysis of breakout notches in underground excavations. By modifying C/T, they analyzed the impact of suppressing the formation of shear cracks in the results. The results showed that when the ratio of shear to tensile strength is equal to 1, the formation
of shear cracks was allowed and the notch geometry could be reproduced with a higher degree of accuracy compared to what was obtained for higher values of C/T that did not allow shear cracks to form. For the sensitivity analysis, the contact tensile strength was kept constant and the shear strength was modified to obtain different C/T values. Figure 4-16 shows the stress-strain response of the UCS samples with different C/T ratios. The C/T ratio has no significance on the elastic portion of the stress strain curves (axial strain < 0.001). The sample with C/T = 1 shows a minor deviation with respect to the behaviour of the samples with larger C/T starting at an axial strain of 0.001. A reduction in peak strength is observed for samples with C/T values of 1 and 2, compared to samples with C/T of 3 and 4.

Figure 4-16. “Axial stress-axial strain” curves from UCS tests performed on samples with different shear to tensile strength ratios.

Figure 4-17 shows the number of shear and tensile cracks that form in the sample during the execution of the test. When the shear strength is equal to the tensile strength (C/T = 1), the number of shear cracks at peak stress is around 20% of the total number of cracks. The number of shear cracks is reduced significantly for C/T of 2, and almost completely suppressed for C/T greater than 2. The differences
observed in the stress-strain curves, and in the peak strength of the samples, are associated with the occurrence of shear cracks obtained in each case.

Figure 4-17. Effect of the shear to tensile strength ratio on the accumulation of shear and tensile cracks during a UCS test.
The relationship between the number of tensile and shear cracks that form during loading is highly dependent on the shape of grains and the kinematic constraints imposed by their geometry. Results from 2D BPM simulations performed by Diederichs (1999) showed that the formation of shear cracks was prevented when the ratio of shear to tensile strengths was greater than 4. For C/T of 2, results from 2D models suggest approximately 15% of the cracks forming under shear at the residual stage. The drastic reduction in shear cracks between 3D BPM (in this study) and 2D models (Diederichs 1999) observed for models with C/T of 2, is largely explained by the extra degree of freedom in 3D models and to a lesser degree by the use of a different contact model. The distribution of tensile and shear cracks will change if polygonal geometries or blocks are used to simulate grains. Ghazvinian (2015) compared results from models constructed using 2D trigon and 3D Voronoi tessellation and demonstrated that the predominance of shear cracks in trigon models is related to geometry of the grains. The edges of triangles form smooth surfaces promoting the occurrence of shear cracks while possible failure surfaces in Voronoi models are irregular, which leads to the simultaneous formation of shear and tensile cracks (Ghazvinian 2015).

4.7.3 Friction Coefficient

The influence of the contact friction coefficient (\(\mu\)) on the model behaviour was examined by testing six different specimens under unconfined and triaxial compression tests. The specimens were created using contact friction coefficient values ranging between 0.5 (~26.5\(^\circ\)) and 2.5 (~68.2\(^\circ\)). The stress-strain curves obtained from the UCS and triaxial tests are presented in Figure 4-18. It is seen in this figure that the friction coefficient has a small effect on the stiffness of the samples.
Figure 4-18. Influence of the friction coefficient on the peak strength and post peak behaviour of the model under different confinement conditions.

The influence of the friction coefficient on the confined strength of the sample is limited, showing a small increase associated with the increasing increments of $\mu$. Damage accumulation during loading is slightly affected by $\mu$, showing an increase in the total number of cracks with decreasing friction coefficients. Results by Fakhimi and Villegas (2007) and Plassiard et al. (2009) show that the influence of the friction coefficient on the residual strength of the sample is negligible. The frictional response observed in the residual state is largely affected by the displacement along an irregular surfaced defined by the disks or spheres and only slightly affected by the friction coefficient (Fakhimi and Villegas 2007).
4.7.4 Model Resolution (L/d)

The ratio between the characteristic length of the sample and the average ball size influences a series of macroscopic properties of the synthetic material. To investigate the influence of model resolution on the behaviour of the simulated material, a series of UCS and triaxial tests were conducted on samples with different resolutions. A uniform distribution was assigned to the ball diameter, with an average value of 15 cm and ratio between the maximum and minimum ball diameter of 1.66. Seven samples were created with diameters ranging from 1.5 m to 12 m as shown in Figure 4-19, resulting in L/d values ranging between 10 to 80.

![Figure 4-19. The diameter of samples tested to evaluate the influence of model resolution (L/d) on macroscopic properties, the height of the samples are 2.5 times the diameter.](image)

Considering the extended time required for the simulation of large BPM samples, only UCS tests were performed as part of the analysis. Figure 4-20 shows the variation of strength with the characteristic length to average particle size ratio. The strengths have been normalized to the peak strength obtained at
an L/d of 20 to make it possible to compare with published results. The results from the numerical tests show an increase of the unconfined compressive strength of the samples with increasing size (resolution or L/d). Results from laboratory tests show a scale effect with reductions in strength with increasing sample size, which is contrary to the results obtained from 3D BPM. Zhang et al. (2011) showed that small scale discontinuities must be included in BPM samples to reproduce the scale effect observed in the laboratory.

Figure 4-20. Influence of model resolution on measured UCS, the results have been normalized to the UCS of the L/d = 20 sample to allow for comparison with published data.

Figure 4-21 shows the variation in Young’s modulus and Poisson’s ratio for different L/d values. Young’s modulus increases in models with higher resolution. For values of L/d over 40, the Young’s modulus remains relatively constant, which means that sample size has reached the representative elementary volume (REV).

The Poisson’s ratio does not show a clear trend for different sample sizes. The tangent Poisson’s ratio values obtained from previous studies (e.g. Ding et al. 2014, Yang et al. 2006) shows decreasing values with increasing resolution. The values of the tangent Poisson’s ratio obtained from this study show some
variation depending on the %UCS where it is calculated. The standard procedure used in laboratory
testing indicates that the elastic constants should be calculated at 50% of the UCS. At this stress level, the
sample might have developed some damage, therefore accelerating increase in lateral strain and thus
affecting the value of the Poisson’s ratio obtained from the test (Diederichs 1999). In order to avoid the
influence of damage, the value of Poison’s ratio was recalculated at 20% of the UCS, however no
significant difference in the relation between Poisson’s ration and model resolution was found with
respect to the values obtained at 50% of UCS. The cause of the absence of a defined trend for the
Poisson’s ratio with sample size, as has been observed in other studies, could not be determined.

Figure 4-21. Effect of model resolution on Young’s modulus and Poisson’s ratio.

A comparison of the evolution of cracks with loading is presented in Table 4-3. Given the difference in
the number of contacts between samples of different sizes, the number of cracks is presented as the
percentage of the total number of contacts at the beginning of the test. The results do not show a clear
relationship between the damage at certain axial strain levels and the size of the sample.
Table 4-3. Comparison of damage at certain strain levels for samples with different size.

<table>
<thead>
<tr>
<th>Axial Strain (%)</th>
<th>Sample size</th>
<th>1.5 m</th>
<th>3.0 m</th>
<th>4.5 m</th>
<th>6.0 m</th>
<th>7.5 m</th>
<th>9.0 m</th>
<th>12.0 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.09</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>0.125</td>
<td>0.18</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.10</td>
<td>0.11</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td>0.44</td>
<td>0.43</td>
<td>0.44</td>
<td>0.40</td>
<td>0.40</td>
<td>0.29</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>0.175</td>
<td>1.08</td>
<td>0.90</td>
<td>0.93</td>
<td>0.88</td>
<td>0.89</td>
<td>0.89</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>2.98</td>
<td>1.96</td>
<td>1.92</td>
<td>1.75</td>
<td>1.76</td>
<td>1.75</td>
<td>2.11</td>
<td></td>
</tr>
<tr>
<td>0.240</td>
<td>6.36</td>
<td>5.88</td>
<td>4.97</td>
<td>6.79</td>
<td>6.39</td>
<td>6.09</td>
<td>7.44</td>
<td></td>
</tr>
</tbody>
</table>

4.7.5 Loading Strain Rate

Different schemes can be used to load BPM samples when simulating laboratory tests. In most cases, loading is performed by fictitious platens that have an assigned velocity with a low magnitude to ensure a quasi-static response of the model. Previous studies that investigated the influence of loading rate when the moving platens approach is used, have shown that the post peak response of the sample is greatly affected by the loading rate (Diederichs 1999, Zhang and Wong 2014, Zhang et al. 2015). When samples are loaded using low loading rates, the post peak behavior is brittle with a steep reduction in stress associated with a small increase in axial strain. Diederichs (1999) and Zhang and Wong (2014) have found that the use of lower loading rates produce a more localized rupture zone. At lower loading rates, less cracks occur during each step, which allows for load redistribution and therefore allows for damage evolution around existing cracks (Diederichs 1999). A more ductile post peak behavior is obtained as the loading rate increases (Diederichs 1999, Zhang and Wong 2014, Zhang et al. 2015). The influence of the loading rate on the peak strength is not significant, showing a slight increase when faster loading rates are used.

A brittle post peak behaviour is obtained when the internal based strain application scheme is used. However, it is not known whether the amount of strain applied during each load step has any influence on the test result. A sensitivity analysis has been conducted to evaluate the influence of loading rate on the model response when the internal-based strain-application loading scheme is used. With the internal
based strain application scheme, a velocity gradient is applied to all the particles of the sample over a given number of steps, after which the velocities are zeroed and the model is cycled until it reaches equilibrium. Different loading rates have been obtained by modifying the number of steps where the velocity gradient is applied. Figure 4-22 shows the “axial stress-axial strain” curves obtained for the four cases analyzed. The value of the stress increment (SI) obtained during each loading step varied from 0.7 MPa to 11.0 MPa. There are no observable changes on the elastic response of the model under different loading rates. The peak strength recorded during the UCS test varied between 75.8 and 77.3 MPa, which is within the acceptable proximity tolerance for bonded particle models. Although the complete post peak response of the models was not obtained, it is confirmed that the increase in ductility observed with other loading schemes is not reproduced when the internal-based strain-application is used.

![Stress strain curves obtained from UCS tests with different stress increment (SI) applied in each load step.](image)

**Figure 4-22.** Stress strain curves obtained from UCS tests with different stress increment (SI) applied in each load step.
The accumulation trend of cracks during loading shows a negligible variation between different loading rates until the last loading step before the peak strength is reached (Figure 4-23). In the loading step where the sample reaches peak strength, samples loaded with higher rates exhibit a significant increase in the number of cracks. This is caused by an excessive straining of the sample at a stage where the current stress is close to the strength of the sample.

![Figure 4-23. Effect of the loading strain rate on the accumulation of crack during a UCS test.](image)

### 4.8 Influence of Stress Path on Intact Rock Strength and Damage Accumulation

One of the objectives of this research is to investigate the influence of stress path on the mechanical response of the rock. All types of excavations in rock, underground or at the surface, induce a complex loading history within the surrounding rock mass, including phases of loading, unloading and stress rotation. In order to evaluate the influence of stress path on the mechanical behavior of rock, a series of numerical samples have been tested using different loading paths. In these models, the effect of stress path on the development of damage during loading and the associated strength of the rock is investigated.
The first stress path corresponds to standardized unconfined and triaxial compression tests (Figure 4-24a). The second set of samples are tested under triaxial deconfinement (Figure 4-24b), with simultaneous increase of the axial stress ($\sigma_1$) and decrease of the confinement stress ($\sigma_2 = \sigma_3$). In the third case, the samples are tested under unloading conditions by reducing the confining stress while maintaining the axial load on the sample (Figure 4-24c). All cases include a stage of isotropic loading until the desired confining stress is reached ($\sigma_1 = \sigma_2 = \sigma_3$).

The calibrated BPM sample described in Section 4.6 is used for the numerical analyses. The geometry of the samples used for these tests is identical to the sample used in the sensitivity analysis (Section 4.7) with the height of 7.5 m and diameter of 3 m as shown in Figure 4-5. The calibrated microparameters defining ball size distribution and the stiffness and strength characteristics of the contacts are listed in Table 4-2.

**Figure 4-24.** Stress paths used to evaluate the influence of loading conditions on strength and damage development, a) standardized tests, b) triaxial deconfinement tests, c) confining stress reduction.
4.8.1 Standardized Loading Tests

A total of seven compression tests, including one unconfined and six confined (i.e. confining pressures of 5, 10, 15, 20, 30, 40 MPa), were simulated using standardized loading paths. The axial stress-strain response of the samples are presented in Figure 4-25. The stress-strain curves for some of the samples show bi-linear slope characteristics at the beginning of the test. This slope change corresponds to the isotropic loading stage and it is caused by the increase in the confining stress during the initial load steps of the test. As expected, an increase of the peak axial stress is observed with increasing confining stress. Although the complete post peak response has not been captured, the results show that brittle post peak behavior is obtained in all tests.

![Axial stress-strain curves from UCS and triaxial tests.](image)

Figure 4-25. Axial stress-strain curves from UCS and triaxial tests.

Figure 4-26 presents the iso-damage contours (percent of broken contacts/total number of contacts) obtained from the results of the UCS and triaxial tests. Iso-damage damage contours are created by connecting points with the same level of damage obtained from different tests. Several interpolation and
regression techniques available in SURFER (Golden Software 2012) were applied to the test data in an attempt to obtain iso-damage contours. Detailed inspection of the results obtained with SURFER showed two important issues with the result of the interpolation. Due to the spatial distribution of the data points, the contours obtained with most of the interpolation algorithms available in Surfer presented a very irregular shape. Some of the interpolation methods produced iso-damage contours with a more regular shape, but after performing a review of the interpolation, it was found that the contours did not match the amount of damage in the input data points. Therefore, a manual interpolation procedure was applied to avoid issues encountered when using SURFER. The relationship between a certain level of damage and confining stress is shown in Figure 4-26 to be non-linear. This becomes more evident at lower levels of damage.

![Figure 4-26. Strength envelope and iso-damage contours (in percent) from the standardized tests.](image)
Samples tested under higher confinement stresses develop higher levels of damage before reaching peak strength envelope. The increasing trend of damage at peak stress in samples tested at higher confinements is an evidence to the additional damage that is required to accumulate for fractures to propagate under confined conditions (Diederichs 1999).

**4.8.2 Triaxial Deconfinement Tests**

Four samples were tested under a triaxial deconfinement loading path. The modelling procedure used to perform this type of test was described earlier in this chapter. A ratio of axial stress increment to confinement stress reduction ($\Delta \sigma_1 / \Delta \sigma_3$) equal to 6 was used in all tests. A comparison between the strength envelopes obtained from this series of tests with the strength envelopes determined using standard loading paths is presented in Figure 4-27. It can be observed in this figure that the strength envelope obtained from triaxial deconfinement tests is slightly higher (~5 MPa across the confining pressures examine) in comparison to the strength envelope determined from standardized tests. A series of points at which the same stress state has been reached under two different loading paths have been selected to evaluate the impact of the loading path on the accumulated damage up to that point.
Figure 4-27. Comparison between the strength envelopes obtained from standardized and triaxial deconfinement tests. Six points have been selected to compare damage development under different loading paths.

Table 4-4 shows the degree of damage (percent of broken contacts/total number of contacts) at each of the selected stress states. The only point that shows a significantly different level of damage between the two stress paths is point 5, which is located close to the strength envelope. The number of cracks that develop at each load step increases significantly when the stress state in the sample is close to the peak strength of the material. Although the stress magnitude is the same in both samples, the strength envelope associated with the standardized tests is lower and therefore more damage is expected in the sample tested with this stress path.
Table 4-4. Difference in damage development at specific points under stress paths from standardized and triaxial deconfinement tests.

<table>
<thead>
<tr>
<th>Point</th>
<th>$\sigma_3$ (MPa)</th>
<th>$\sigma_1$ (MPa)</th>
<th>Damage (%)</th>
<th>Difference in Damage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.25</td>
<td>31.12</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>5.37</td>
<td>82.45</td>
<td>3.13</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>10.13</td>
<td>61.73</td>
<td>0.25</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>15.90</td>
<td>84.48</td>
<td>0.72</td>
<td>-0.12</td>
</tr>
<tr>
<td>5</td>
<td>20.12</td>
<td>129.79</td>
<td>9.04</td>
<td>3.14</td>
</tr>
<tr>
<td>6</td>
<td>31.08</td>
<td>83.47</td>
<td>0.10</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Figure 4-28 shows the iso-damage contours obtained from the triaxial deconfinement tests. As in the case of samples tested under standardized loading paths, the iso-damage contours exhibit a nonlinear relationship with confining stress.

Figure 4-28. Strength envelope and iso-damage contours from the triaxial deconfinement tests.
4.8.3 Confining Stress Reduction Tests

This type of test is not inherently included in SRMLab or in the testing support functions included in PFC Fishtank. Therefore, some modifications to the existing support functions were made to test samples under a confining pressure unloading stress path. With the internal-based strain loading method, samples are loaded by applying a velocity gradient over a period of time, after which the sample ends are fixed and the model is cycled until static equilibrium is reached. Assuming that the sample behaves elastically, the relationship between stresses and strains in a cylindrical sample under triaxial compression is given by equations 4-2 and 4-3:

\[
\delta \varepsilon_a = \frac{1}{E} (\delta \sigma_a - 2v \delta \sigma_r) \tag{4-2}
\]

\[
\delta \varepsilon_r = \frac{1}{E} (\frac{-v \delta \sigma_a - (1 - v) \delta \sigma_r)} \tag{4-3}
\]

where \(\delta \varepsilon_a\) and \(\delta \varepsilon_r\) are the axial and radial strains, \(\delta \sigma_a\) and \(\delta \sigma_r\) are the axial and radial (confining) stresses and \(v\) and \(E\) are the material’s Young’s modulus and Poisson’s ratio, respectively. From equations 4-2 and 4-3, it can be inferred that reducing the confining stress, with no change in strain in the axial direction, will cause a reduction of the axial and radial stress acting on the sample. Therefore, to keep the axial load at a relatively constant level during unloading increment of the test, it is necessary to apply some strain in the axial direction to compensate for the effect of the reduction in confining stress.

A modified loading procedure has been developed to perform the confining stress reduction tests. Following a similar loading procedure of a triaxial test, the samples are first loaded isotropically. Once the target axial stress is reached, the elastic constants are obtained using stress and strain data from the last load step. The confinement reduction is conducted in 0.5 MPa intervals. Using Equations 4-2 and 4-3 it is possible to determine the amount of axial strain required to maintain the axial stress at a constant value. After each confining pressure reduction step, the axial stress is compared with the target value. Axial stress variations under a user defined threshold, 0.25 MPa in this study, are considered to be acceptable. If the difference between the target axial stress and the current axial stress exceeds the defined
threshold, the axial strain is multiplied by a correction factor to increase or decrease the amount of axial strain applied. The stress difference is calculated as the current axial stress minus the target axial stress. For cases where the stress difference is negative, the correction factor increases the applied axial stress and vice versa.

A total of six samples were tested under a confining stress reduction loading path. The target axial stress varied from 85 MPa to 150 MPa. The strength envelope and the stress paths of the six tests are presented in Figure 4-29. A higher strength envelope is obtained from the confining stress reduction tests compared to the envelope obtained from standardized testing. It can be seen that at the beginning of the confinement relaxation in the tests, the magnitude of the axial stress remains relatively constant, however, after a certain point the axial stress starts a decreasing trend. The rate of axial stress reduction increases significantly as the stress path reaches the strength envelope. As expected, the increase of accumulated damage in the sample (as a function of confining pressure relaxation) results in a reduction of the Young’s modulus and an increase in the Poisson’s ratio. The difficulty in maintaining a constant axial stress level during unloading is associated with the variations of the elastic constants with increasing damage. The use of the correction factor to increase the axial strain applied in each load step was not completely effective in correcting the deviation from the target axial stress, particularly at higher levels of damage.
Figure 4-29. Comparison of the strength envelopes obtained from the standardized and confining stress reduction tests.

A series of model states, at which (green markers in Figure 4-30) the same stress state has been reached under a standardized compression test and an unloading stress path, have been selected to evaluate the possible differences in accumulated damage under the two loading paths (Figure 4-30). The amount of damage (percent of broken contacts/total number of contacts) at each of the evaluation points are listed in Table 4-5. A difference in accumulated damage greater than 1% between the two loading paths is observed at stress states close to the strength envelope. The difference in the amount of damage obtained under the two loading paths is explained by the relative difference in stress with respect to the strength envelope associated with each stress path. The strength envelope obtained from standardized tests is lower than the envelope obtained from confining stress reduction tests, and therefore samples tested under standardized UCS or triaxial compression tests develop more damage at a certain stress level compared to the confining stress reduction tests.
Figure 4-30. Points selected for comparison of damage development under standardized and confining stress reduction test loading paths.
Table 4-5. Difference in damage development at specific stress state points in the model for stress paths from standardized and triaxial deconfinement tests.

<table>
<thead>
<tr>
<th>Point</th>
<th>Standardized Tests</th>
<th>Confining Stress Reduction</th>
<th>Difference in Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_3$</td>
<td>$\sigma_1$</td>
<td>Damage (%)</td>
</tr>
<tr>
<td>1</td>
<td>5.36 MPa</td>
<td>84.61 MPa</td>
<td>3.78%</td>
</tr>
<tr>
<td>2</td>
<td>5.21 MPa</td>
<td>89.64 MPa</td>
<td>6.47%</td>
</tr>
<tr>
<td>3</td>
<td>10.17 MPa</td>
<td>72.59 MPa</td>
<td>0.70%</td>
</tr>
<tr>
<td>4</td>
<td>10.19 MPa</td>
<td>94.47 MPa</td>
<td>3.37%</td>
</tr>
<tr>
<td>5</td>
<td>10.12 MPa</td>
<td>101.23 MPa</td>
<td>5.82%</td>
</tr>
<tr>
<td>6</td>
<td>15.92 MPa</td>
<td>112.79 MPa</td>
<td>4.69%</td>
</tr>
<tr>
<td>7</td>
<td>15.98 MPa</td>
<td>107.14 MPa</td>
<td>3.03%</td>
</tr>
<tr>
<td>8</td>
<td>20.13 MPa</td>
<td>123.97 MPa</td>
<td>5.20%</td>
</tr>
<tr>
<td>9</td>
<td>20.20 MPa</td>
<td>127.85 MPa</td>
<td>7.46%</td>
</tr>
<tr>
<td>10</td>
<td>31.17 MPa</td>
<td>105.86 MPa</td>
<td>0.47%</td>
</tr>
<tr>
<td>11</td>
<td>40.83 MPa</td>
<td>125.49 MPa</td>
<td>0.63%</td>
</tr>
</tbody>
</table>

Figure 4-31 shows the strength envelope obtained under the three loading paths investigated in this section. The iso-damage contours of 0.5%, 1%, 3% and 5% are also displayed to compare the effect of the loading path on damage accumulation. The 0.5% damage contour is first reached under the unloading stress path, followed by the damage contours associated with the standardized compression and triaxial deconfinement tests, respectively. The influence of stress path on damage accumulation is quantified by measuring the distance between two consecutive iso-damage contours for each loading path. A greater distance between two contours means that the amount of damage accumulated following that stress path is smaller than the damage that would develop under a stress path with contours located at a closer distance. By analyzing the distance between the contours shown in Figure 4-31, it can be concluded that samples tested under a deconfinement stress path develop less damage compared to the other stress paths. This might seem contradictory given the relative position of damage contours in Figure 4-31, where some points associated with the unloading stress path show more damage than points from other loading paths.
at the same stress state. The reason for the larger amount of damage at these points is related to the large difference in the stress state at the starting point of the stress path after the isotropic stress has been installed in the sample. The influence of accumulated damage on the strength of the sample will be examined in the next section.

![Figure 4-31. Strength envelope and iso-damage contours obtained from standardized, triaxial deconfinement and confining stress reduction tests.](image)

**4.9 Influence of Pre-Peak and Post-Peak Damage on Rock Strength**

The analysis of damage accumulation conducted in the previous sections has been focused on describing the evolution of damage and how this is affected by different BPM aspects such as model resolution, changes in contact behavior and the stress path used to load the sample. In this section, the effect of damage on strength and elastic behavior of BPM is investigated.
4.9.1 Effect of Randomly Induced Damage

A series of samples with varying levels of pre-existing damage have been created to evaluate the impact of randomly induced damage on the sample behaviour. First, an intact sample is created following the sample generation procedure described in Section 4.3. Induced damage is introduced to the model by systematically removing the parallel bond at random locations in the sample until the target level of damage is reached.

From the tests conducted in previous sections, it was determined that at peak stress, the amount of damage in the sample ranges between 5% to 10% for unconfined and confined compression tests, respectively. After peak stress is reached, damage increases significantly and can reach values between 10% to 30% depending on the confining stress used for the test. Samples with different levels of pre-existing damage ranging between 1% to 30% were generated to cover the entire range of pre- and post-peak damage observed in previous tests.

The results of the unconfined compression tests performed on samples with pre-existing damage is presented in Figure 4-32. The linear relationship between UCS and the level of damage is characteristic of BPM samples with high porosity (Schöpfer et al. 2009).

![Figure 4-32. Effect of randomly generated damage on the unconfined compressive strength of the sample.](image-url)
The results of the confined compression tests are shown in Figure 4-33. It can be concluded that the pre-existing damage in rock reduces the cohesive strength of the sample without affecting the friction angle of the material. Similar results were found by Diederichs (1999) using 2D BPM models to analyze the influence of pre-existing damage. Martin and Chandler (1994) proposed that the cohesional component of rock strength starts to decrease at low levels of damage without affecting the instantaneous strength of the sample. This phenomenon is explained by the mobilization of the frictional strength of the rock, in parallel with the loss of cohesion caused by the development of damage. The process of cohesion loss and friction mobilization has been demonstrated experimentally by Zhang et al. (2015b) by testing intact and damaged samples of rock under confined and unconfined compression. The numerical approach adopted in this study is not capable of simulating the mobilization of friction that is associated with the development of damage. The process of friction mobilization occurs due to internal shearing at the crack surfaces that have been created as a consequence of loading. The spherical geometry of particles used in BPMs inhibits the ability to simulate the sliding between polygonal grains that form in damaged rock, therefore limiting the capability of this numerical approach to simulate the mobilization of friction.
Figure 4-33. Effect of randomly generated damage on sample strength.

4.9.2 Crack Orientation at Different Stages of Testing

In the previous section, broken bonds were inserted at random locations within the sample without analyzing the crack orientation with respect to the loading axis. During loading, development of cracks does not occur at random orientations. The distribution of crack orientation is influenced by the geometrical arrangement of particles, the distribution of interparticle strength and loading conditions. In this section, a series of samples with different crack orientation distribution will be tested to evaluate the influence of the orientation of damage on rock strength. Subsequently, the results from samples with preferred orientation of induced damage will be compared with the results obtained from the sample with random damage orientation similar to the specimens investigated in the previous section. This will be called case 1 hereafter in this section.

The orientation of cracks that develop during loading was recorded during the execution of a UCS and a triaxial test. The orientation distribution of cracks was obtained at peak stress and at 75% of the peak...
stress for each of the tests. The comparison of the crack orientation distribution obtained from the UCS test, and the triaxial test with confinement stress of 5 MPa, are presented in Figure 4-34a and Figure 4-34b, respectively. In both tests, it is seen that there is a small increase in the proportion of cracks with a dip angle greater than 40° at peak stress stage compared to the crack orientation distribution at 75% of the peak stress. Based on the UCS and triaxial simulations a crack orientation distribution was decided upon (Figure 4-34c) that will be used to represent damage developed during testing. The model with this crack orientation distribution will be called case 2 hereafter in this section.

Two additional crack orientation distributions (cases 3 and 4) will be used to compare the influence of crack orientation on the strength of damaged samples. In case 3, damage will only be inserted at contacts with a dip angle greater than 40°, with a higher percentage of cracks inserted at contacts with steeper orientations. A uniform distribution of crack orientation is used for case 4. In this case, the same number of cracks have been inserted at each orientation, which is different from the randomly inserted damage, where the number of cracks at each orientation resembles the crack orientation distribution of the sample. The summary of crack orientation for the four cases analyzed is presented in Table 4-6. Samples are created by looping through the contact list and inserting damage at contacts as described in the previous section. The process is continued until the desired distribution of cracks and the specified level of damage are achieved. Samples with 10% and 30% of broken contacts have been created for each case described in Table 4-6.

The results obtained from samples with 10% of damage are presented in Figure 4-35. The strength envelope from the intact sample is included for reference. The strength envelope from cases 2, 3 and 4 represent a very similar threshold. The largest reduction in strength is obtained for the case with randomly inserted damage, case 1.

Figure 4-36 shows the results obtained from the sample with 30% of broken bonds. Similar results to the sample with 10% broken bonds are observed, with the strength of case 2 following the lower-bound
threshold amongst cases 2, 3 and 4 and greater reduction in strength for case 1, where damage is inserted at random orientations.

Figure 4-34. Crack orientation distribution, a) UCS test, b) triaxial test and c) UCS, triaxial and distribution used to represent damage from previously tested samples.
Table 4-6. Crack orientation distribution used to evaluate the influence of crack orientation on sample strength.

<table>
<thead>
<tr>
<th>Crack Orientation</th>
<th>Percentage of Total Cracks at Each Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dip (°)</td>
<td>Case 1</td>
</tr>
<tr>
<td>From</td>
<td>To</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>70</td>
<td>80</td>
</tr>
<tr>
<td>80</td>
<td>90</td>
</tr>
</tbody>
</table>

Case 1: Random orientation  
Case 2: Orientation from UCS and TX tests  
Case 3: High angle cracks  
Case 4: Uniform distribution

Figure 4-35. Effect of damage inserted at specified orientations on the confined strength of the sample, 10% damage.
4.9.3 Testing of Previously Loaded Samples

In the previous section, the orientation of inserted cracks was determined according to a specified orientation distribution. However, the spatial location of the cracks inside the sample was still random.

When rock is loaded at stresses higher than the crack initiation stress, cracks will occur in a random and distributed manner as a consequence of heterogeneity in stresses and micro-scale strength (Diederichs, 1999).

With BPM, external loading results in an internal distribution of compressive and tensile forces acting upon each contact (Figure 4-37a). The tensile strength of the contacts is determined by the tensile strength of the parallel bond and the diameter of the contact. As a result of this, the tensile strength of the contacts has an inherent variability (Figure 4-37b). Parallel bonds break in response to local tensile forces that exceed their tensile strength (Figure 4-37c). When damage is inserted, either randomly or following a specified orientation distribution, cracks are inserted (bonds are broken) independent of the strength of the
contact. Therefore, contacts with high strength, or with an orientation unfavorable for the development of tensile forces, can be broken because of the procedure used to damage the sample, resulting in cracks being inserted at locations than it would have not formed under a realistic external loading.

Figure 4-37. Example of contact geometry and properties in a 2D BPM, a) chain force geometry with contacts sustaining compression and tension forces, b) contact tensile strength distribution, c) contact geometry at peak stress in a UCS test, broken contacts can be observed in upper right corner.

To capture the effect of rock damage induced by real loading conditions, a series of intact samples damaged during the execution of UCS and triaxial tests have been selected for retesting. Two samples have been obtained from each test, the first when the sample reaches 80% of the peak stress and the second at peak stress. A total of ten samples have been selected from the UCS test and triaxial tests conducted at confining stresses of 1, 5, 15 and 30 MPa. The list of cases selected for retesting is presented in Table 4-7. It is seen that in all samples the amount of initial damage obtained at 80% of the peak stress is 1.5%. At peak stress, the amount of damage increases with confining stress at which the sample was tested, starting at 5.7% of broken bonds for the UCS test and increasing up to 10% of broken bonds for the triaxial test with a confinement stress of 30 MPa.
Table 4-7. List of previously tested samples selected for retesting.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Test</th>
<th>Stage</th>
<th>Initial Damage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UCS</td>
<td>Peak stress</td>
<td>5.7</td>
</tr>
<tr>
<td>2</td>
<td>UCS</td>
<td>80% Peak stress</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>TX - 1.0 MPa</td>
<td>Peak stress</td>
<td>6.1</td>
</tr>
<tr>
<td>4</td>
<td>TX - 1.0 MPa</td>
<td>80% Peak stress</td>
<td>1.5</td>
</tr>
<tr>
<td>5</td>
<td>TX - 5.0 MPa</td>
<td>Peak stress</td>
<td>7.3</td>
</tr>
<tr>
<td>6</td>
<td>TX - 5.0 MPa</td>
<td>80% Peak stress</td>
<td>1.6</td>
</tr>
<tr>
<td>7</td>
<td>TX - 15.0 MPa</td>
<td>Peak stress</td>
<td>8.1</td>
</tr>
<tr>
<td>8</td>
<td>TX - 15.0 MPa</td>
<td>80% Peak stress</td>
<td>1.5</td>
</tr>
<tr>
<td>9</td>
<td>TX - 30.0 MPa</td>
<td>Peak stress</td>
<td>10.8</td>
</tr>
<tr>
<td>10</td>
<td>TX - 30.0 MPa</td>
<td>80% Peak stress</td>
<td>1.5</td>
</tr>
</tbody>
</table>

To unload the sample before retesting, the axial stress is gradually reduced using a procedure similar to the one used for loading but with an inverse velocity gradient. To prevent the development of additional damage during the unloading stage, the axial extensional strain applied in each unloading step has been limited to half of the axial strain applied during the compression stage of the test. For the case of samples taken from triaxial tests, the reduction of the confining stress is applied simultaneously with the reduction of the axial stress. The reduction of the confining stress performed at each unloading step is calculated on the basis of the reduction rate of the axial stress in the previous step as shown in Figure 4-38. The unloading process is continued until the axial and radial stresses in the sample are reduced to zero. After the samples are unloaded, one UCS and seven triaxial tests are conducted on each sample. Figure 4-39 illustrates a typical stress path for these tests.
Figure 4-38. Loading and unloading process applied to samples before retesting.

Figure 4-39. Typical stress path from loading-unloading-reloading process.
The strength envelopes obtained from the tests performed on samples with 1.5% of damage do not present significant differences. The results from the tests performed on previously tested samples are compared with results obtained from samples with random induced damage in Figures 4-40 to 4-42. It is possible to observe that in all cases, the strength of samples damaged by previous loading is higher than the strength of samples with random damage inserted. As was described previously, when the sample is loaded, the weakest contacts will fail first while stronger contacts will be able to continue carrying load during external compression. When damage is inserted at random locations, some of the cracks will be created at contacts with high strength, which will modify the force chain in the particle assembly from that which would have developed naturally in an intact sample under loading. This causes a strength reduction greater than what is obtained from damage caused by previous loading.

![Strength envelopes](image)

**Figure 4-40.** Strength envelopes obtained from sample 2 (1.5% damage) in Table 4-7, strength envelopes from the intact sample and samples with similar level of randomly induced damage are included for comparison.
Figure 4-41. Strength envelopes obtained from samples 1, 3 and 5 (5.7%, 6.1% and 7.3% of damage, respectively) in Table 4-7, strength envelopes from the intact sample and samples with similar level of randomly induced damage are included for comparison.

Figure 4-42. Strength envelope obtained from samples 7 and 9 (8.1% and 10.8% of damage, respectively) in Table 4-7, strength envelopes from the intact sample and samples with similar level of randomly induced damage are included for comparison.
Two cases have been selected to evaluate the influence of different stress paths on the development of damage during the test-unload-retest process. In the first case, the confining stress used during the initial test is lower (5 MPa) than the confining stress used for retesting the sample (30 MPa). The second case represents the opposite, with a confining of 30 MPa during the first test and 5 MPa for the second test. Figures 4-43 and 4-44 present the evolution of stress and damage development during the tests for the two cases being analyzed. When the confining stress is set to a higher value for the second test, it is seen that the development of new cracks does not begin until the axial stress is greater than the maximum stress reached during the first loading process (Figure 4-43). A similar effect occurs in the laboratory for rocks under cyclic or loading/unloading conditions, where it has been observed that a significant increase in acoustic emission activity occurs when rock is loaded beyond the maximum stress reached in previous loading cycles. At loads below the previous maximum stress, no acoustic emission activity is recorded.

![Graph showing stress, damage, and crack rate evolution](image)

**Figure 4-43.** Evolution of stress, damage and crack rate during the loading-unloading-reloading process for the model with lower confining stress during the first loading stage.
In BPM, the force chains that form during the first loading stage are able to carry the load during the second loading stage as well. Therefore no new cracks are formed when the sample is loaded below the maximum stress recorded in the first loading stage. This can be seen in Figure 4-43, where during the second loading stage cracks begin to form when the sample is loaded beyond 89.3 MPa, which is the maximum stress recorded during the first loading stage. The rate at which cracks start to form beyond this point is lower than the crack rate recorded in the load steps before unloading the sample. This is related to the increase in the strength of the sample due to the greater confinement stress applied for the second test. Higher crack rates are recorded as the axial stress approaches the strength of the sample. The peak stress recorded during the second test is 153.9 MPa, which is close to the 157.4 MPa recorded in the triaxial test performed on the intact sample at the same confinement.

Figure 4-44. Evolution of stress, damage and crack rate during the loading-unloading-reloading process for the model with higher confining stress during the first loading stage.
In the case where the confinement stress is higher during the first loading stage, Figure 4-44, cracks form at stresses below the maximum stress applied during the first loading stage. The level of damage that the sample developed during the first loading stage is higher than the amount of damage that an intact sample develops at peak stress when it is tested at confinement of 5 MPa. Thus, bonded contacts that would exist in the intact sample are not present, and therefore different force chains form when compared to an intact sample, following different paths and breaking new contacts during this process. The crack rate at peak stress is close to the rate recorded when the load was close to the peak stress during the first loading process. In the second test, the sample reached a peak stress of 80.4 MPa, which represents 90% of the peak stress of an intact sample tested with a confinement stress of 5 MPa.

4.10 Summary

The bonded particle model (BPM) has been used to analyze the behaviour of intact rock. The numerical procedures used to create and performed different types of test on BPM samples have been described. A systematic calibration process was followed to determine the microparameters that replicate the macroscopic properties of the intact rock being modelled.

A sensitivity analysis has been performed to obtain a better understanding of the role of different parameters on the macroscopic response of the sample. The results of the analysis have been compared with results obtained from similar analyses by other researchers.

A series of UCS, triaxial, triaxial deconfinement and confining stress reduction tests were conducted to evaluate the effect of stress path on the behaviour of the BPM specimen. Results showed that tests conducted using a triaxial deconfinement (TD) or a confining stress reduction (CSR) tests accumulate less damage than tests that follow standardized loading paths (UCS and triaxial tests). By accumulating less damage during loading, samples tested under TD or CSR loading paths exhibit a higher strength envelope than the strength determined with UCS and triaxial test. A detailed review of the numerical models was performed to discard that the increase in strength observed in TD and CSR test was not caused by numerical issues. Based on the review of the models it was concluded that the differences in strength and
damage development correspond to a physical behaviour of the sample and is not related to numerical issues. The influence of stress path has received considerable attention from researchers since Jaeger (1966) highlighted the possible influence of stress path on the mode of failure of rock. Some of the early studies conducted to evaluate the effect of stress path showed that the strength of the rock was independent of the stress path followed by tests (Swanson and Brown 1971, Crouch 1972). Xu and Geng (1986) (as cited in Yang et al. 2011), tested soft and hard rocks and found that the strength of hard rocks is slightly lower when is tested in CSR tests, on the contrary, soft rocks showed a higher strength when loaded with a CSR loading path. Chen (1979) used acoustic emission (AE) to evaluate the effect of loading path in the development of damage. It was found that the number of AE events increased gradually when the rock was tested in triaxial compression. Under a CSR loading path, the number of cracks showed a marked increase before reaching peak strength, but the total number of AE events was lower in CSR tests. The findings of Chen (1979) reflect the brittle nature of the failure of rock when subjected to unloading stress path. Successive studies performed over the years confirm that a brittle mode of failure is obtained under unloading conditions (e.g. He et al. 2015, Jinli et al. 2011). Although there seems to be a lack of consensus in the academic community regarding the effects of stress path on the strength of rock, the relation between unloading conditions and brittle failure of rock is accepted and supported by experimental data (e.g. Jinli et al. 2011, Yang et al. 2011).

The influence of damage in the strength of intact rock was analyzed by testing damaged BPM samples. The analysis included testing of samples with randomly inserted damage, samples with randomly inserted damage at specified orientations, and samples damaged by the previous loading. Results showed that samples with randomly inserted damage presented the greatest reduction in strength as a result of damage. External forces applied to BPM samples create internal force chains in a trellis-like structure, Figure 4-37. Weaker contacts fail first, while stronger contacts will remain intact forming the force chains that carry the external load applied to the sample. When damage is inserted at random, some of the cracks will be
inserted at stronger contacts and thus will reduce the strength to a greater magnitude when compared to other forms of damage insertion, such as load cycling.

The analysis of samples damaged by the previous loading showed that the damage caused by the first test has a minor effect on the strength of the sample when compared to the results of subsequent tests. Two factors explain the limited effect of damage caused by the previous loading in the strength of the sample. As previously explained, weaker contacts fail first and stronger contacts remain intact forming the force chains that carry the load. During the second tests, the force chains that formed during the first loading stage are still able to carry the load and therefore the effect of the broken contacts is minimal. Diederichs et al. (2004) demonstrated that stress rotation creates additional damage, which can have a significant impact on rock strength. Due to limitations in the numerical approach selected for this study, it was not possible to simulate the effect of stress rotation that occurs around excavations in rock. A greater reduction in the strength of previously loaded samples is expected if the effects of stress rotation are included in the analysis.
Chapter 5

Modelling the Effect of Stress Path and Damage on Rock Mass Strength with the Synthetic Rock Mass Approach

5.1 Introduction

The major problem engineers face when designing large excavations in rock is obtaining reliable estimates of rock mass strength. A complex mechanical behavior arises from the interplay between the strength of the intact rock, the mechanical and geometrical characteristics of the discontinuities, and the acting stress state. Discontinuities introduce non-linearities and anisotropy in the behavior of the rock mass (Lisjak and Grasselli 2014) and reduce its strength to a fraction of the intact rock strength. The main limitation in obtaining a good understanding of the mechanical behaviour of rock masses is the inability to perform tests at scales that are representative of the rock mass.

The continuous development and refinement of numerical methods, in combination with incredibly fast growth of computing power in recent years, has enhanced our capability to numerically investigate rock masses with a tremendous level of detail. The Synthetic Rock Mass (SRM) approach (Pierce et al. 2007, Mas Ivars et al. 2011) is a state of the art numerical technique that explicitly simulates the interaction between intact rock and discontinuities. In this approach, the intact rock is represented by the Bonded Particle Method (BPM), and an explicit representation of the discontinuities is defined for the model by inserting a Discrete Fracture Network (DFN). By explicitly simulating the interaction between the intact rock and the discontinuities and benefitting from the BPM capabilities, the SRM is capable of reproducing many aspects of rock mass behaviour including the development of new fractures, scale effects, anisotropy, brittle-ductile transition and post peak response.

In this chapter, the SRM technique will be used to investigate the mechanics of rock mass strength, including the influence of intact rock and discontinuity strength on the overall behaviour of SRM samples when subjected to unconfined and triaxial compression conditions. The sensitivity of the SRM to the
changes in the strength of the intact rock and discontinuities is compared to variation in rock mass strength predicted by the Hoek-Brown criterion (Hoek et al. 2002). Furthermore, an analysis of the influence of stress path on damage development and the resultant strength of SRM samples is conducted by performing numerical tests by using standardized and triaxial deconfinement loading paths. Finally, tests are performed on SRM samples to determine its residual strength under confined and unconfined conditions. The results obtained from the SRM tests are compared with empirical estimates of rock mass residual strength

5.2 Synthetic Rock Mass Modelling

5.2.1 Intact Rock

In the previous chapter, it was shown that the model resolution influences the strength of BPM samples. This means that for an identical ball size distribution, samples of varying size will have different strengths, with increasing strength for larger samples. Model resolution effects on the strength of the simulated intact rock need to be considered when using SRM to evaluate rock mass behaviour. By introducing a Discrete Fracture Network to BPM to create an SRM sample, the original intact material is subdivided into smaller blocks, with a size distribution defined by the geometrical characteristics of the DFN. Ideally, the sample size used for the intact rock calibration process should match the average block size defined by the DFN. The blocks that form by inserting a DFN in the BPM require a minimum resolution to capture the mechanics of rock blocks bounded by discontinuities. To correctly capture the behaviour of intact rock blocks, it is recommended that a minimum of five particles should fit across the edge of the average block size (Mas Ivars 2010).

The damage development analysis that was conducted in Chapter 4 required a higher model resolution than is obtained from applying the recommendation of a minimum number of balls across the rock block. The unconfined strength of created blocks in the SRM sample will be lower than 75 MPa as obtained through the calibration process (Chapter 4), since the blocks in the SRM sample will be smaller than the sample size used to calibrate the contact strength and deformation properties. Based on the model size
sensitivity results obtained in the previous chapter, the difference between the calibrated sample strength and the strength of the average block size (from the SRM) is estimated to be less than 5 MPa. This difference is believed to fall within the natural variation of rock mass block strengths, and therefore does not make significant contributions to the strength and damage analysis conducted in this chapter. The calibrated microparameters defined in Chapter 4 will be used for creation of the intact rock matrix of the SRM modelling described in the following sections. The list of BPM microparameters determined in Chapter 4 is repeated in Table 5-1 for reference.

Table 5-1. Calibrated microparameters used to create intact rock for the SRM sample.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material-genesis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min. Particle radius, Rmin</td>
<td>cm</td>
<td>5.63</td>
</tr>
<tr>
<td>Max./min. Particle radius ratio, Rmax/Rmin</td>
<td></td>
<td>1.66</td>
</tr>
<tr>
<td>Initial number of particles</td>
<td></td>
<td>20,600</td>
</tr>
<tr>
<td>Particle micro-properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bulk density, ρ</td>
<td>kg/m³</td>
<td>3150</td>
</tr>
<tr>
<td>Modulus, Ec</td>
<td>GPa</td>
<td>40</td>
</tr>
<tr>
<td>Normal/shear stiffness ratio, k_n/k_s</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Friction coefficient, μ</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Parallel-bond micro-properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulus, E_c</td>
<td>GPa</td>
<td>40</td>
</tr>
<tr>
<td>Normal/shear stiffness ratio, E_n/E_s</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Radius mutiplier, λ</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Tensile strength (mean), σ_t</td>
<td>MPa</td>
<td>57</td>
</tr>
<tr>
<td>Tensile strength (std. dev.), σ_t</td>
<td>MPa</td>
<td>15</td>
</tr>
<tr>
<td>Shear strength (mean), σ_s</td>
<td>MPa</td>
<td>114</td>
</tr>
<tr>
<td>Shear Strength (std. dev.), σ_s</td>
<td>MPa</td>
<td>28.5</td>
</tr>
</tbody>
</table>

5.2.2 Discrete Fracture Network (DFN) Modelling

In SRM modelling, structures are represented explicitly through a Discrete Fracture Network. In theory, a DFN could be constructed deterministically or stochastically. Constructing a deterministic DFN requires detailed information about the geometry of the structure network that is practically impossible to obtain with the current state of practice for fracture mapping. For this reason, in almost all applications of the
SRM approach found in literature, DFN’s are constructed as a statistical representation of the structure network.

The minimum information required to build a DFN includes fracture orientation, fracture intensity, and joint size distribution within the rock mass. Determining the orientation distribution of different sets forming the structure network is a relatively straightforward task. In mining operations, this information is usually gathered from oriented core and/or bench or drift mapping. In the definition of the structural sets, it is important to account for, and correct, any sampling bias introduced by the orientation of the drill holes or benches/drifts with respect to the orientation of structures.

The fracture intensity is a measure of the degree of blockiness of the rock mass, which combined with the fracture size distribution, defines the number of fractures that are created for each set in the DFN. The fracture intensity is quantified through a spatial measure of fracturing such as the linear fracture intensity ($P_{10}$), areal fracture intensity ($P_{21}$), and volume fracture intensity ($P_{32}$). $P_{10}$ corresponds to the number of fractures per unit length, which is equivalent to the fracture frequency used in core logging or surface mapping. The areal fracture intensity ($P_{21}$) is the trace length per unit area. Although this parameter is not routinely evaluated in mining operations, it can be easily determined during mapping of benches or drifts. The volumetric fracture intensity ($P_{32}$) corresponds to the fracture surface area per unit volume. $P_{10}$ and $P_{21}$ are sensitive to the orientation in which they are measured, and therefore $P_{32}$ is usually preferred to define joint intensity (Rogers et al. 2015). The main problem associated with the use of $P_{32}$ to define joint intensity is the limitation in terms of direct evaluation of this parameter. $P_{32}$ can be approximated either by analytical or simulation methods (Rogers et al. 2015).

Determining the fracture size distribution is a complex task as it is almost impossible to see the full extent of a structure in the field. The fracture size distribution is usually determined from trace length data gathered from surface mapping, which only provides a relative measure of the actual size of the structure. In the author’s experience in dealing with structurally controlled failure in open pit slopes, the trace length recorded in window mapping significantly underestimates the real size of the structure. Trace
length data usually follows a lognormal distribution, independent of the 3D fracture size distribution (Baecher 1983). A power law distribution with a negative exponent is often used to define the fracture size for DFN modelling within the context of SRM applications (Pierce and Fairhurst 2011).

For this study, a DFN with three joints sets was modelled. To avoid intersections at low angles between joints of the same set, a fixed orientation was given to each set. All sets were given the same volumetric fracture intensity and size distribution. Synthetic boreholes were used to evaluate the resultant joint spacing. Nine vertical and nine horizontal boreholes were used to obtain the fracture frequency of each set as shown in Figure 5-1. Apparent joint spacing was calculated as the inverse of the fracture frequency. The correct joint spacing was obtained by applying Terzaghi weighting (Terzaghi 1965). The volumetric joint density and the size distribution were adjusted in an iterative process until the target joint spacing of 0.75 – 1.0 m was achieved. The parameters that define the fracture orientation, fracture intensity, and fracture size distribution are listed in Table 5-2. The generated DFN is shown in Figure 5-2.

![Figure 5-1. Synthetic boreholes used to calculate the fracture frequency for DFN modelling.](image)
Table 5-2. Joint set parameters for DFN modelling.

<table>
<thead>
<tr>
<th>Set</th>
<th>Fracture Intensity</th>
<th>Fracture Size Distribution</th>
<th>Joint Orientation</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{32}$ $(m^2/m^3)$</td>
<td>Distribution Exponent $R_{min}$ $(m)$</td>
<td>$R_{max}$ $(m)$</td>
<td>$Dip$ $(^\circ)$</td>
</tr>
<tr>
<td>1</td>
<td>1.05</td>
<td>Power Law 2.5</td>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>Power Law 2.5</td>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>Power Law 2.5</td>
<td>0.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Figure 5-2. Discrete Fracture Network used in SRM modelling.

5.2.3 SRM Simulation Results

5.2.3.1 Sample Description

The SRM sample was constructed by inserting the DFN shown in Figure 5-2 into a cylindrical BPM sample with a diameter of 7.5 m and a height of 18.75 m (Figure 5-3). The BPM contact microparameters are listed in Table 5-1 and the properties that define the strength and deformational characteristics of the
discontinuities are documented in Table 5-3. Identical mechanical properties have been assigned to all joint sets.

![Discrete Fracture Network](image1.jpg) ![Intact BPM sample](image2.jpg) ![SRM sample](image3.jpg)

**Figure 5-3. Components and dimension of the SRM sample.**

**Table 5-3. Mechanical properties of the DFN discontinuities in the SRM sample.**

<table>
<thead>
<tr>
<th>Mechanical parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal stiffness (GPa/m)</td>
<td>50.0</td>
</tr>
<tr>
<td>Shear stiffness (GPa/m)</td>
<td>5.0</td>
</tr>
<tr>
<td>Cohesion (MPa)</td>
<td>0.0</td>
</tr>
<tr>
<td>Friction Angle (°)</td>
<td>30.0</td>
</tr>
</tbody>
</table>

5.2.3.2 Results from UCS and Triaxial Tests

One UCS test and seven triaxial compression tests with confinement stresses ranging between 1 MPa to 30 MPa were conducted to examine the behavior of the jointed specimen. Figure 5-4 shows the “axial stress – axial strain” response of the sample from the UCS and triaxial tests with confinement stresses up
to 5 MPa. Samples tested with confinements of 10 MPa and higher displayed a ductile behaviour. The stress–strain response of these samples are shown in Figure 5-5. The data from the triaxial test with 5 MPa confinement is included in this figure for comparison.

**Figure 5-4.** Stress-strain response of the UCS and triaxial tests with confinement stresses up to 5 MPa.

**Figure 5-5.** Stress-strain response of triaxial tests with confinement stresses greater than 5 MPa.

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The brittle to ductile transition for the SRM sample with increasing confinement is evident from the results shown in Figures 5-4 and 5-5. Many authors have indicated that the brittle-ductile transition is explained by the characteristics of shear failure (Orowan 1960, Mogi 1966, Byerlee 1968). According to Orowan (1960), the frictional strength of a fracture can increase to a point where it is equal or greater than the stress required to create the fracture, and therefore no stress drop is observed after failure. Mogi (1966) analyzed a large set of published data and found that the brittle-ductile transition in silicate rocks is defined by:

$$\sigma_a = 3.4 \times \sigma_c$$  \hspace{1cm} 5-1

where $\sigma_a$ is the axial stress and $\sigma_c$ is the confinement stress. According to this limit, samples with a peak strength lower than 3.4 times the associated confinement stress will fail in a ductile manner.

The strength envelope obtained from the numerical tests with confining stresses up to 10 MPa is presented in Figure 5-6. Mogi’s line has been included in Figure 5-6 to evaluate the observed brittle-ductile transition of the SRM sample across different confining stresses according to the limit defined by Mogi (1966). Based on Mogi’s brittle-ductile transition limit, samples tested at confinement stresses higher than 8 MPa should fail in a ductile fashion. This correlates well with the results from the SRM tests.
Figure 5-6. Strength envelope from SRM tests with confining stresses up to 10 MPa.

The triaxial tests on intact rock conducted in Chapter 4, presented brittle post peak behaviour, independent of the confinement stress applied during the test. This relates to the inability of spherical particles to reproduce sliding mechanisms under applied or resolved shear stresses (rotational moments are generated when using contact bond or parallel bond). In SRM samples, discontinuities are represented by using the smooth joint contact model (SJCM). As described in Chapter 3, the SJCM can simulate sliding along a plane by allowing particles to overlap and slide past each other. With rising confining stress, the resolved normal stress and therefore the frictional strength of the joints also increase, making it possible to numerically reproduce the brittle–ductile transitional behaviour of the rock mass.

5.2.3.3 Comparison of SRM results with an empirical estimate of Rock Mass Strength

The Hoek-Brown criterion is used to obtain an empirical estimate of the strength of a rock mass with similar structure and intact rock properties as the SRM sample used in this study. The Hoek-Brown criterion evaluates rock mass strength based on the strength of intact rock and the quality of the rock mass. The properties of the intact rock have been determined from the tests of intact samples in Chapter 4.
The quality of the rock mass is evaluated using the Geotechnical Strength Index (GSI) (Hoek et al. 1995, Marinos and Hoek 2000), which takes into account the blockiness and the condition of the discontinuity surface. As specified in Table 5-1, the friction angle assigned to the discontinuities is equal to 30°. This friction angle can be considered a typical value of moderately weathered planar joints, which corresponds to the “Fair” category in the GSI chart. The structure category in the GSI chart will be evaluated with the aid of the GSI quantification methods proposed by Cai et al (2004) and Hoek et al. (2013). In the method developed by Cai et al. (2004), the structure category in the GSI chart is defined either by the average joint spacing or from the rock block volume. Based on the DFN with parameters described in Table 5-2, the average block volume in the SRM sample is equal to 700,000 cm³, which combined with a “Fair” surface condition results in a GSI value of approximately 57 (Figure 5-7).
Figure 5-7. Estimated GSI for the defined DFN using the GSI quantification method proposed by Cai et al. (2004), the red dot indicates the GSI value that corresponds to characteristics of the DFN.

The GSI quantification method proposed by Hoek et al. (2013) defines GSI as

\[ GSI = 1.5 \times J_{cond_{B89}} + \frac{RQD}{2} \]  

where \( J_{cond_{B89}} \) is the joint condition rating as defined by Bieniawski (1989). A joint condition rating of 15 could be considered typical for a “Fair” surface condition. RQD has been evaluated using Palmström’s
(2005) relationship between RQD and the volumetric joint count, $J_v$, which is defined as the number of joints per cubic meter. The relationship between RQD and $J_v$ is defined by:

$$RQD = 110 - 2.5 J_v$$  \hspace{1cm} \text{(5-3)}

The number of joints inside the SRM sample was counted and divided by the volume of the sample obtaining a $J_v$ of 3.6, which results in an RQD of 100. A GSI value of 72 is obtained from equation 5-2.

The difference in the GSI values obtained from the application of the two quantification methods (i.e. Cai et al (2004) and Hoek et al. (2013)) resides in the different parameters used to describe the structure of the rock mass and the scale of the excavations each method should be applied to that scale. For comparison purposes, the RQD values that define the rock mass structure categories in the GSI chart for the two quantification methods are presented in Table 5-4. An equivalent RQD was obtained for the joint spacing values used to define rock mass structure in Cai et al. (2004) by applying the relationship between RQD and the fracture frequency ($\lambda$) proposed by Priest and Hudson (1976).

$$RQD = 100 * e^{-0.1\lambda} (0.1\lambda + 1)$$  \hspace{1cm} \text{(5-4)}

The fracture frequency is obtained as the inverse of the joint spacing. There are significant differences in the value of RQD that define rock mass structure in each method. The method proposed by Hoek et al. (2013) was developed for small scale problems at which RQD is able to differentiate between varying degrees of fracturing in the rock mass. For large scale applications, RQD loses sensitivity and consequently is unable to differentiate between degrees of fracturing that might be of interest for design purposes, and therefore other parameters such as block volume, are better suited to describe rock mass structure.
Table 5-4. Comparison of RQD values that define the rock mass structure for the GSI quantification methods proposed by Cai et al. (2004) and Hoek et al. (2013).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact or Massive</td>
<td>100</td>
<td>&gt; 80</td>
</tr>
<tr>
<td>Blocky</td>
<td>95 - 100</td>
<td>60 - 80</td>
</tr>
<tr>
<td>Very Blocky</td>
<td>74 - 95</td>
<td>40 - 60</td>
</tr>
<tr>
<td>Blocky/Disturbed</td>
<td>16 - 74</td>
<td>20 - 40</td>
</tr>
<tr>
<td>Disintegrated</td>
<td>0 - 16</td>
<td>0 - 20</td>
</tr>
<tr>
<td>Laminated/Foliated</td>
<td>1</td>
<td>Not defined</td>
</tr>
</tbody>
</table>

Figure 5-8 shows the comparison between the Hoek-Brown strength envelopes obtained from the two GSI values previously derived, as well as an envelope fitted to the results from the SRM tests.

Figure 5-8. Rock mass strength envelopes derived from empirical approximations and SRM tests.
The SRM strength envelope shows a smaller increase for the peak strength with increasing confinement when compared to the empirical strength envelopes estimated with the Hoek-Brown criterion. This could be caused by a combination of the inability of the parallel bond used in the BPM to reproduce high $m_i$ values with the effect of the discontinuities inserted in the SRM sample.

5.3 Evaluation of the Effect of Intact Rock Strength on Rock Mass Strength using SRM

In this section, the effect of the intact material strength on the overall strength of the SRM sample is evaluated by testing samples with different intact rock strengths. Two BPM samples with intact rock strength of 37.2 MPa and 152.8 MPa were created for the analysis. An identical DFN to that used in the previous section has been used to create the two new SRM samples in addition to the SRM sample investigated in the previous section (Intact BPM strength of 75 MPa). The DFN properties are described in Table 5-3. One UCS and three triaxial tests with confinement stresses up to 5 MPa were simulated for each SRM sample. Figure 5-9 shows the stress-strain curves obtained from the tests. It is observed that the SRM approach captures the direct contribution of the intact rock strength to the overall rock mass strength. It is also interesting to notice that the sample with the lower intact rock strength shows an earlier onset of ductile behavior with the increasing confining stress when compared to the samples with higher intact rock strength, reaching a completely ductile behavior when tested at a 5 MPa confinement. By lowering the strength of the intact material, the strength of the SRM sample is reduced. Consequently, the confinement stress at which the sample starts to exhibit ductile post peak behaviour is also reduced.

In the Hoek-Brown criterion, GSI is used to scale the strength of the intact rock to the strength that is representative of the rock mass scale. For a given GSI value, the percentage of reduction in strength from intact to rock mass strength can be quantified by dividing the rock mass strength by the corresponding intact rock strength. Figure 5-10 shows the Hoek-Brown strength envelopes for three different rock masses with similar GSI and $m_i$ values, but different values of UCS. A GSI of 57 and $m_i$ of 4.5 have been used for the three cases. These values are representative of the $m_i$ for the intact BPM and the properties of the DFN used to construct the SRM samples. The UCS of the three rock masses is 37.2, 75.9 and 152.8
MPa, respectively. The strength envelopes have been normalized by dividing the rock mass strength by the strength of the intact material. In the Hoek-Brown criterion, the relationship between the UCS of the intact rock and the rock mass unconfined strength is a function of GSI. For a rock mass with a GSI of 57 the unconfined strength of the rock mass is approximately 6% of the intact UCS, independent of the UCS for the intact rock. Under confined conditions, the strength of the rock masses with higher intact rock strength is affected by the discontinuities (strength reduction) to a greater extent in comparison to the rock masses with weaker intact rock.

The results of the SRM tests are included in Figure 5-10 to compare the reduction in strength from intact to rock mass scale between the Synthetic Rock Mass and the Hoek-Brown criterion. For unconfined conditions, the normalized strength of the SRM samples is 15% of the intact rock UCS, independent of the intact rock UCS. Under confined conditions, the SRM is able to replicate the increase in strength with increasing confinement stress, although the magnitude of the increase in strength with confinement is different from what the Hoek-Brown criterion predicts.
Figure 5.9. Effect of intact rock strength on the behavior of SRM samples.
Figure 5-10. Comparison of the intact rock strength effect on the overall rock mass strength between the SRM approach and Hoek-Brown criterion.

At confinement stresses close to 1 MPa, the influence of different intact rock UCS values on the strength of the SRM samples is minimal. For the triaxial tests with confinement stresses of 2.5 MPa and 5 MPa, the difference in strength between the samples with intact rock UCS of 37 MPa and 75 MPa is greater than the strength difference between the SRM samples with an intact rock strength of 75 MPa and 152 MPa. Although the SRM method can capture the expected relationship between the strength reduction caused by discontinuities and the strength of the intact material for the range of confining stresses investigated in this study, the magnitude of that reduction differs from what is predicted by the Hoek-Brown criterion.
5.4 Evaluation of the Effect of Structure Properties on Rock Mass Strength using SRM

In the previous section, the influence of the intact rock strength on the strength of SRM samples was analyzed and compared with empirical estimates of rock mass strength obtained with the Hoek-Brown criterion. In this section, the influence of the strength of discontinuities on the overall response of SRM samples is investigated. The strength of the discontinuities in the SRM samples has been modified by increasing the friction angle of the discontinuities to 45° (from 30°). Higher friction angles are characteristic of rough joints with minor or no weathering. These types of joints also present higher normal and shear stiffness than moderately weathered joints. The stiffness properties of the discontinuities in the SRM sample have also been increased to reflect the physical characteristics of the joint type being modeled. A normal contact stiffness of 100 GPa/m and tangent contact stiffness of 10 GPa/m are used in this case.

Three additional SRM samples have been created with the new set of properties for the discontinuities by using intact rock strengths of 37, 75 and 152 MPa. The orientation and size of the discontinuities included in the DFN remain identical to what have been used in the previous sections. One UCS and three triaxial tests with confinements of 1, 2.5 and 5 MPa have been conducted on each sample. Figures 5-11 to 5-13 show the comparison between the stress-strain response for each of the tests conducted on the SRM samples with a friction angle of 30° (DFN1) and 45° (DFN2) for the discontinuities. In general, the increase in the strength of the discontinuities improves the strength of the SRM sample.
Figure 5-11. Effect of joint strength and stiffness on the strength of SRM samples with intact rock UCS of 37 MPa.
Figure 5-12. Effect of joint strength and stiffness on the strength of SRM samples with intact rock UCS of 75 MPa.

Figure 5-13. Effect of joint strength and stiffness on the strength of SRM samples with intact rock UCS of 152 MPa.
The strength of discontinuities is governed by the roughness and degree of weathering of the joints. In the Hoek-Brown criterion, the strength of the joints and its effect on rock mass strength is incorporated by selecting an appropriate surface condition category on the GSI chart. When evaluating the quality of the rock mass with the GSI system, an improvement of the surface condition of the joints results in a higher value for GSI. In the previous section, it was determined that the friction angle of 30° used for the joints would be the equivalent of a “Fair” surface condition, which in combination with the average block volume defined by the DFN, resulted in a GSI of 57. The increase in the strength of the discontinuities in the SRM sample should result in an increase of the estimated GSI for the DFN. Hoek et al. (2013) proposed that as an alternative to the joint condition factor, the ratio of joint alteration to joint roughness ($J_a/J_r$), as defined by Barton et al. (1974), could be used to quantify the surface condition axis in the GSI chart. A $J_a/J_r$ of 1 could be estimated for discontinuities with a friction angle of 45°. In the surface condition axis in the GSI chart, this value of $J_a/J_r$ corresponds to the limit between the “Fair” and “Good” category. This represents an increase of 5 points with respect to the GSI estimated for the DFN with a friction angle of 30°.

In Figure 5-14, the effect of the increase in the strength of the discontinuities in the SRM sample is compared to the strength improvement determined by the Hoek-Brown criterion when the GSI has increased 5 and 10 points. In this figure, the rock mass strength is expressed as a fraction of the corresponding intact rock strength. It is observed that under unconfined compression, the increase in rock mass strength as a result of increasing the discontinuity strength for the SRM samples is limited when compared to the strength improvement calculated from the Hoek-Brown criterion for the GSI increase equal to 5 points. A larger increase in rock mass strength is observed under confined conditions for the SRM samples. The magnitude of the observed strength increase for the SRM samples in confined conditions when improving the joint strength and stiffness properties is comparable to the increase in Hoek-Brown strength when the GSI is increased by 5 points.
Figure 5-14. Comparison between Hoek-Brown strength envelopes and the SRM results to evaluate the effect of increasing the strength of the discontinuities on the rock mass behaviour with different intact rock UCS (the Y axis represents rock mass strength as a fraction of the intact rock strength).
5.5 Effect of Stress Path on Damage Development at Rock Mass Scale

The results from the numerical tests conducted in Chapter 4 indicated that intact BPM samples develop less damage when tested under triaxial deconfinement and unloading paths compared to samples tested using standardized (UCS and triaxial) loading paths. At the rock mass scale, the presence of discontinuities could influence the rock mass damage development and failure process.

In this section, the response of SRM samples tested under different loading paths are analyzed to evaluate the effect of stress path on strength and damage development in jointed rocks. The results from the UCS and triaxial tests described in Section 5.2.3.2 will be compared to results obtained from three triaxial deconfinement tests.

Figure 5-15 shows the “axial stress-axial strain” response and evolution of damage obtained from the UCS and triaxial tests with confinement stresses of 1, 2.5 and 5 MPa. The accumulated damage is evaluated as the percentage of broken bonds in the intact blocks of the SRM samples. Table 5-5 compares the level of damage between equivalent tests performed on intact BPM and SRM specimens. For the UCS and the triaxial test performed at 1MPa confinement, the level of damage in the SRM sample is much lower than the damage accumulated in the BPM sample. When discontinuities are present, the failure surface is partially defined by these discontinuities and therefore a reduced amount of failure of intact rock is required to bring the sample to failure. Results from the triaxial test conducted at the confinement stress of 5 MPa showed a greater amount of damage in the SRM sample than in the intact BPM. This is mainly attributed to the more ductile behavior of the SRM sample compared to the intact BPM sample.

Table 5-5. Comparison of accumulated damage during tests performed on intact BPM and SRM samples.

<table>
<thead>
<tr>
<th>Test</th>
<th>Damage at Peak Stress (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intact BPM</td>
</tr>
<tr>
<td>UCS</td>
<td>5.7</td>
</tr>
<tr>
<td>TX - 1 MPa</td>
<td>6.1</td>
</tr>
<tr>
<td>TX - 5 Mpa</td>
<td>6.8</td>
</tr>
</tbody>
</table>
Figure 5-15. Stress-strain response and evolution of damage during UCS and triaxial tests.
On the stress-strain curve of the triaxial tests in Figure 5-15, it is possible to observe a drop in axial stress before reaching peak strength. This drop in axial stress during loading is attributed to the slips occurring on the surface of discontinuities and could be considered to be the onset of sample failure or yielding. The difference in axial stress, between the point where the first major drop in axial stress is observed and where peak stress is recorded, increases with increasing confinement. This emphasizes the frictional nature of the sample behaviour after the first drop in axial stress.

Figure 5-16 illustrates the iso-damage contours obtained from the UCS and triaxial tests. The amount of damage recorded at peak stress increases with confinement, ranging from 2% for the UCS test to over 7% for 5 MPa triaxial test. The envelope that defines the onset of yielding, identified in Figure 5-15, shows a good agreement with the 1.6% iso-damage contour. Based on this, it is possible to conclude that within the range of confinement stresses investigated, the onset of yielding is governed by the amount of damage accumulated in the sample and not as much by the confinement stress at which the sample is tested.
Table 5-6 presents a summary of the results obtained from the UCS and triaxial tests including peak strength and the associated percentage of damage, and the amount of radial strain in the sample. Results from tests that failed in a ductile manner present some differences depending on when the test was interrupted.

Table 5-6. Summary of the results from UCS and triaxial tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>Target Confinement Stress (MPa)</th>
<th>Peak Stress</th>
<th>Damage (%)</th>
<th>Strain at Peak microstrain</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td>0.0</td>
<td>0.0</td>
<td>11.4</td>
<td>1.7</td>
<td>-780</td>
</tr>
<tr>
<td>Triaxial</td>
<td>1.0</td>
<td>1.1</td>
<td>13.9</td>
<td>1.9</td>
<td>-476</td>
</tr>
<tr>
<td>Triaxial</td>
<td>2.5</td>
<td>2.5</td>
<td>17.0</td>
<td>3.8</td>
<td>-530</td>
</tr>
<tr>
<td>Triaxial</td>
<td>5.0</td>
<td>4.2</td>
<td>20.0</td>
<td>7.2</td>
<td>-644</td>
</tr>
<tr>
<td>Triaxial</td>
<td>10.0</td>
<td>10.9</td>
<td>32.2</td>
<td>51.5</td>
<td>-</td>
</tr>
<tr>
<td>Triaxial</td>
<td>15.0</td>
<td>15.2</td>
<td>30.1</td>
<td>11.7</td>
<td>-</td>
</tr>
<tr>
<td>Triaxial</td>
<td>20.0</td>
<td>21.0</td>
<td>53.7</td>
<td>59.9</td>
<td>-</td>
</tr>
<tr>
<td>Triaxial</td>
<td>25.0</td>
<td>26.2</td>
<td>60.9</td>
<td>60.1</td>
<td>-</td>
</tr>
<tr>
<td>Triaxial</td>
<td>30.0</td>
<td>30.5</td>
<td>49.1</td>
<td>31.9</td>
<td>-</td>
</tr>
</tbody>
</table>

The stress paths followed by the samples using a triaxial deconfinement test procedure (TD) are presented in Figure 5-17. Samples have been loaded isotropically to 4, 6 and 8.5 MPa and then tested following a deconfinement ratio (DR) of 3.5. The strength envelope and the line that defines the onset of yielding, obtained from the UCS and triaxial tests, are included in Figure 5-17 for comparison. The difference in the strength envelopes obtained from the two loading paths (UCS and triaxial versus TD procedure) is minimal and could be considered identical. At some point before reaching peak stress, the stress in the samples deviates from the prescribed loading path and follows an irregular stress path, which indicates the onset of yielding under this loading path. The points at which the stress paths become irregular present a good agreement with the failure envelope identified from the UCS and triaxial tests.
Figure 5-17. Strength envelopes obtained from UCS-triaxial tests and triaxial deconfinement tests, the onset of yielding for the samples as identified from the UCS and triaxial tests is included for reference.

The iso-damage contours obtained from the TD tests and those from the UCS and triaxial tests are presented in Figure 5-18. Given the irregularity of the stress path after the onset of yielding for TD tests, it was not possible to obtain contours for levels of damage greater than 2%, and therefore, a comparison could not be drawn for damage levels beyond 2%. At damage levels below 1.6%, it is observed that at a given stress state, less damage is accumulated under the triaxial deconfinement test. Similar results were obtained from tests performed on intact BPM in Chapter 4.
5.6 Effect of Damage on Rock Mass Strength

At the rock mass scale, failure can include sliding on discontinuities, extension of existing discontinuities, fracturing of intact rock blocks or a combination of these factors. The actual mode of failure will depend on the quality of the rock mass and the in-situ stress conditions. The intact rock damage has an influence on the overall rock mass quality and therefore its effect on rock mass strength is investigated in this section. For this purpose, a series of UCS and triaxial tests were conducted on SRM samples with damaged intact rock. Cases with damage inserted at random locations and orientations were analyzed first. Then, samples damaged during the execution of some of the tests described in this earlier sections of this chapter were unloaded and retested to evaluate the effect of damage caused by the previous loading.
5.6.1 Effect of Randomly Induced Damage

Following the procedure outlined in Chapter 4, four SRM samples with different levels of inserted damage within their intact matrix were created. Damage was generated by setting the parallel bond strength (shear and normal) to zero. The results from the UCS and triaxial tests from Section 5.3.2 with confinement stresses ranging between 1 to 5 MPa, indicated levels of damage between 2% to 8% at peak strength. Based on this, four samples with levels of inserted damage between 3.5% to 8% have been created as listed in Table 5-7.

Table 5-7. List of SRM samples with randomly induced damage.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Inserted Damage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>3.5</td>
</tr>
<tr>
<td>Case 2</td>
<td>5.0</td>
</tr>
<tr>
<td>Case 3</td>
<td>6.5</td>
</tr>
<tr>
<td>Case 4</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Figures 5-18 and 5-19 show the stress-strain curves and the damage evolution for SRM cases 1 and 2 and cases 3 and 4, respectively. As expected, damaged samples exhibit a reduction in the stiffness of the sample, although no significant differences are observed between the two cases with different levels of damage. A minor reduction in UCS of the SRM sample is seen in all cases (1 to 4) compared to the SRM sample with intact rock, Figure 5-19a and Figure 5-20a, although the magnitude of the strength drop can be considered not relevant for practical purposes. Under confined conditions, samples corresponding to cases 1 and 2 do not show a reduction in strength compared to the undamaged specimen, although damaged samples require a larger amount of axial strain to reach peak strength (Figure 5-19b). For cases 3 and 4, a minor reduction in the peak strength is recorded in the triaxial tests in comparison to the undamaged SRM sample (Figure 5-20b), although the strength of the three cases (cases 3 and 4 and intact rock SRM) could be considered to be the same for practical purposes.
Figure 5-19. Stress-strain curves and damage evolution of samples with randomly inserted damage for cases 1 and 2, a) UCS test, b) triaxial test with confinement stress of 5 MPa.
Figure 5-20. Stress-strain curves and damage evolution of samples with randomly inserted damage for cases 3 and 4, a) UCS test, b) triaxial test with confinement stress of 5 MPa.

It is possible to notice a major change in the slope of the damage curves in Figure 5-19 and Figure 5-20 shortly after the peak strength is reached for the samples. This change in the slope of the damage curves represents a reduction of the rate at which new cracks are formed. If the sample is able to maintain a
stress level after failure that is close to its peak strength, the crack rate will not change significantly. In the UCS tests, it is observed that the drop in axial stress after failure occurs more rapidly in damaged samples than in the intact SRM samples, therefore developing less damage in the post-peak region. In the triaxial tests, the damaged samples exhibit more ductile behavior than the intact SRM sample, and thus can maintain a higher level of stress after reaching the peak, and therefore develop more damage at high levels of strain.

Figure 5-21 shows the relation between UCS and the degree of initial damage for the intact BPM samples tested in Chapter 4 and the SRM tests conducted in this chapter. To facilitate the comparison of results, in Figure 5-21, the UCS is expressed as a fraction of the UCS of the undamaged sample. For the same level of inserted damage, a greater reduction in strength is observed for the intact BPM sample. In jointed rock with low confinement stress, the behavior of the rock mass and the development of damage will be largely controlled by the orientation and location of the discontinuities. When introducing random damage to the sample, it is possible that the inserted crack is isolated at a spatial location where it does not contribute to the interaction of the existing discontinuities that form part of the failure surface. This explains the greater influence of randomly inserted damage on intact BPM compared to SRM samples.

![Figure 5-21. Effect of randomly inserted damage on the UCS of intact BPM and SRM samples.](image-url)
5.6.2 Testing of Previously Tested SRM Samples
The effect of damage caused by previous loading on the strength of SRM samples is evaluated by selecting specimens that have developed damage during the execution of UCS and triaxial tests conducted in previous sections of this chapter. The samples have been taken at different stages of the tests to capture the effect of various levels of initial damage. The procedure employed to unload the sample has been described in Section 4.9.3 and will not be repeated here. The list of samples, the test from which the sample was taken, and the level of initial damage present in the sample are detailed in Table 5-8. The intensity of initial damage for the selected samples ranges between 3.4% to 12%. Each of the selected samples is reloaded for one UCS test and two triaxial tests with confinement stresses of 2.5 and 5.0 MPa. Results obtained from the numerical tests conducted on samples 3 and 5 (from Table 5-8) show a non-similar behaviour when compared to the other tests performed in this chapter, and therefore are excluded from further analyses. The numerical models corresponding to these tests were reviewed in detail, and it was not possible to determine if the irregular behavior exhibited by these samples was a real phenomenon or a numerical artifact.

Table 5-8. List of samples selected for retesting.

<table>
<thead>
<tr>
<th>Case</th>
<th>Test</th>
<th>Confinement Stress (MPa)</th>
<th>Initial Damage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UCS</td>
<td>-</td>
<td>3.4</td>
</tr>
<tr>
<td>2</td>
<td>Triaxial</td>
<td>2.5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>UCS</td>
<td>-</td>
<td>6.3</td>
</tr>
<tr>
<td>4</td>
<td>Triaxial</td>
<td>2.5</td>
<td>6.6</td>
</tr>
<tr>
<td>5</td>
<td>Triaxial</td>
<td>2.5</td>
<td>7.5</td>
</tr>
<tr>
<td>6</td>
<td>Triaxial</td>
<td>5.0</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>Triaxial</td>
<td>5.0</td>
<td>12</td>
</tr>
</tbody>
</table>

The stress-strain curves and the evolution of damage during the tests for cases 1, 2, 4, 6 and 7 are presented in Figures 5-22 to 5-24. The level of damage presented in the graphs only considers the additional damage accumulated during the second test conducted on the sample.
For cases 1 and 2 in Figure 5-22, the damaged samples present a softer behaviour than the intact sample, although the difference between the behaviour (peak strength and rock mass modulus) of the intact and damaged material is reduced as the confinement stress increases. In terms of the peak stress recorded in each case (1 and 2), the difference between the strength of the intact and damaged samples is also reduced when the confining stress is increased. In the triaxial test with confinement stress of 5 MPa, the peak axial stress in the intact and damaged samples appears to be nearly identical. This results from a slightly higher confinement stress in the triaxial tests performed on damaged samples. Due to the numerical procedure used in the model for the isotropic stress installation stage of triaxial tests, minor differences can occur between the target confinement stress and the actual confinement stress at which triaxial tests are conducted.

A similar behavior is observed for cases 4 and 6 in Figure 5-23, although the significantly softer behaviour of damaged samples (in comparison to the intact samples) is still detectable for 5 MPa triaxial tests. The amount of new damage that the pre-damaged samples develop during the retesting process is significantly smaller than the damage initially developed by the intact SRM specimen. This might be caused by the presence of a completely developed failure surface during the first test conducted on samples corresponding to cases 4 and 6.

In case 7 (Figure 5-24), major differences in the stress-strain response and the recorded peak strength are observed between the intact and damaged samples for all tests (UCS and triaxial). Again, this is probably caused by the presence of a fully developed failure surface formed during the first test conducted on the sample.
Figure 5-22. Stress-strain curves and damage evolution for samples damaged by previous loading, cases 1 and 2.
Figure 5-23. Stress-strain curves and damage evolution for samples damaged by previous loading, cases 4 and 6.
Figure 5-24. Stress-strain curves and damage evolution for samples damaged by previous loading, case 7.
Figure 5-25 shows the effect of pre-existing damage intensity on the UCS of the rock. The trend between damage intensity and UCS of the intact BPM and SRM samples with random damage is linear. The irregular trend between UCS and damage intensity of the retested samples is related to the influence of the failure surface generated during the first test conducted on the sample. Cases 1 and 2, with initial damage of 3.5% and 5%, respectively, could be considered representative of the damaged rock mass, whereas cases 4, 6 and 7 can be regarded as representative of the strength of the formed failure surface in the samples.

Figure 5-25. Influence of the pre-existing damage intensity on UCS of previously tested SRM samples.

The strength envelopes obtained for the five cases analyzed in this section are compared with the strength envelope obtained for the undamaged SRM sample in Figure 5-26. The strength envelope obtained for the sample with 8% of initial random damage is included for reference as well. The strength envelopes from tests on previously loaded specimens are below the strength envelope for sample with 8% initial random damage. As mentioned earlier in this chapter, the effect of randomly induced damage on the sample
strength is minor compared to the effect of damage caused by the previous loading of the sample. This is caused by the influence of discontinuities on damage development.

The friction angle for each envelope is included in Figure 5-26. It is seen that damaged samples exhibit a higher friction angle compared to the undamaged SRM model. This was not observed in the tests conducted on intact BPM samples in Chapter 4. For the cases where a fully developed failure surface was present at the beginning of the test, the reported friction angle possibly corresponds to the friction on that surface.

![Figure 5-26. Strength envelopes for undamaged and damaged SRM samples.](image)

The obtained friction angles are considered low for a rock mass. The small increase in strength with confinement is caused by the use of the contact bond or parallel bond contact models in PFC. After the bonds break, the particles are free to rotate and fail to capture the behavior of highly interlocked grains at...
intact rock scale (Diederichs 1999, Lisjak and Grasselli 2014), or rock blocks when simulating SRMs. This represents one of the main limitations of BPM and SRM for the analysis of intact and jointed rock under confined conditions.

5.7 Estimating Rock Mass Residual Strength with SRM

The rock mass surrounding underground or large surface excavations develops a plastic zone where the rock mass has been loaded beyond its peak strength. The extent of the plastic zone around an excavation depends on factors such as rock mass strength, excavation geometry and in situ stress conditions. Obtaining a reliable estimate of the strength of the rock mass in the plastic zone is an extremely challenging task. In contrast to the rock mass peak strength for which empirical relationships such as the Hoek-Brown criterion are available for estimation, there is no standard approach defined for determining the rock mass residual strength properties. Conceptually, it is understood that a good quality rock mass will experience a significant and sudden reduction in strength after failure (Figure 5-27a). As the quality of the rock mass degrades, it is expected that its post-peak behaviour is governed by the strain softening model, with a smaller reduction from peak to residual strength than is the case for good quality rock masses. In this type of rock mass, the strength drop from peak to residual occurs over a certain strain increment, reducing gradually until the residual strength is reached (Figure 5-27b). A perfectly plastic post-peak behaviour is expected for poor quality rock masses, with no drop in strength after failure (Figure 5-27c).

Different approaches have been proposed over the years by researchers and practitioners to estimate the residual strength of rock masses (Russo et al. 1998, Ribacchi 2000, Cai et al. 2007, Lorig and Varona 2013). However, the predictions obtained from various methods results in a significantly wide range of values for the rock mass residual strength. In this section, the rock mass residual strength obtained from empirical estimates is reviewed and compared to the values obtained from testing of SRM samples.
Figure 5-27. Expected post-peak behavior for rock masses with different quality (after Hoek and Brown 1997).
5.7.1 Empirical Estimates of Rock Mass Residual Strength

The available methods to estimate the rock mass residual strength establish different relationships between the parameters that define the Hoek-Brown strength envelope for peak and residual conditions. Hoek and Brown (1997) suggested that for an average quality rock mass, the residual strength could be estimated by selecting a residual value of GSI ($GSI_r$) that would match the characteristics of the failed rock mass. Based on the suggestions of Hoek and Brown (1997) and the results of back analysis of actual cases, Russo et al. (1998) proposed that $GSI_r$ could be estimated as $0.36 \times GSI$. Ribacchi (2000) proposed to use the following relationships between peak and residual Hoek-Brown criterion parameters:

$$m_r = 0.65 m_b \quad s_r = 0.04 s \quad (\sigma_c)_r = 0.2 \sigma_c$$

Cai et al. (2007) suggested that the residual strength of rock masses could be estimated by determining the residual values of the block volume and joint condition factor ($J_c$) and by using those values in conjunction with the GSI quantification method proposed by Cai et al. (2004) to obtain $GSI_r$. Based on observations from in situ block shear tests and results from numerical modelling, Cai et al. (2007) suggested that a residual block volume of $10 \ cm^3$ could be used for a wide range of rock mass quality. The residual value of $J_c$ is estimated by reducing the values of the large and small scale roughness of the joints by half. Lorig and Varona (2013) proposed the use of the disturbance factor ($D$) to estimate the residual strength. The value of $D$ to be used depends on GSI and is defined by:

$$D = 0 \quad \text{for} \quad GSI < 20$$

$$D = (GSI - 20)/40 \quad \text{for} \quad 20 \leq GSI \leq 60$$

$$D = 1 \quad \text{for} \quad GSI > 60$$

With this approach, a perfectly plastic post-peak behaviour is obtained for rock masses with GSI less than 20. For rock masses with GSI greater than 20, the difference between peak and residual strengths increases with GSI, which follows the concept of post-peak behaviour presented in Figure 5-27a & b.
The rock mass residual strength calculated from the described approaches are compared to evaluate the resultant range of values obtained from different techniques. Figure 5-28 shows the peak and residual strength envelopes obtained for poor to average quality rock masses, GSI between 30 and 50. A UCS of 75 MPa and m, of 15 are used for the three cases. The strength envelopes obtained for a disturbed rock mass condition are also included for reference. For these cases, the value of D has been set to 0.7 and 1.0. The strength envelopes are presented in the principal stress space normalized to the peak strength to quantify the strength reduction at different confinements for each case. For the given range of GSI values, the higher values of residual strength are obtained with the approach proposed by Lorig and Varona (2013).

The lower-bound residual strength is obtained by using the residual GSI values recommended by Ribacchi (2000). At confinement stresses greater than 0.5 MPa, the residual strengths predicted by the different methods fall within a range bouned by 65% to 90% of the intact rock strength. The four approaches included in the comparison predict a smaller reduction in residual strength as the quality of the rock mass is reduced.

The peak and residual strength envelopes obtained for good quality rock masses, GSI values between 60 and 80, are presented in Figure 5-29. For a GSI value of 60, at a confinement of 5 MPa, all estimates of residual strength range between 48% to 58% of the peak strength. At higher values of GSI, the estimates of residual strength obtained following the recommendations by Lorig and Varona (L-V) exhibit a significant increase compared to the residual strengths determined using any of the other procedures. For a GSI value of 70, the L-V residual strength is equivalent to 62% of the peak strength, while the other methods show a residual strength ranging between 42% to 50% of the peak strength. At a GSI value of 80, the L-V residual strength is equivalent to 70% of the peak strength, while the other methods predict a residual strength between 30% to 40% of the peak strength.
Figure 5-28. Residual and peak strength envelopes for poor to average quality rock masses, a) GSI = 30, b) GSI = 40, c) GSI = 50
Figure 5-29. Residual and peak strength envelopes for good quality rock masses, a) GSI = 60, b) GSI = 70, c) GSI = 80
In the empirical methods proposed by Russo et al. (1998), Cai et al. (2007) and Lorig and Varona (2013), the residual strength envelope is adjusted either by using GSI or an appropriate D value. Since the UCS of the rock is not included, the normalized residual strength of the rock mass is not influenced by the properties of the intact rock. In the approach proposed by Ribacchi (2000), a residual value for the UCS of the intact rock is one of the parameters that defines the rock mass residual strength. Figure 5-30 shows the comparison of the normalized residual strengths for a rock with a GSI of 50, a m, of 15 and UCS values of 75 MPa (left) and 200 MPa (right). The influence of intact rock UCS on the residual strength following Ribacchi’s recommendations is evident. For a UCS of 75 MPa, this approach estimates the lowest residual strength of the four methods used in the comparison. In contrast, when the UCS of the intact rock is elevated to 200 MPa, Ribacchi’s method predicts the highest value for the rock mass residual strength amongst all the approaches. From a conceptual point of view, the strength of the intact rock should not have an influence on the normalized rock mass residual strength. The reduction in rock mass strength after failure is caused by an increase in the degree of fracturing of the rock mass, and by a change in the surface condition of the discontinuities.

Figure 5-30. Effect of intact rock UCS on the residual strength envelope obtained from different approaches.
5.7.2 Rock Mass Residual Strength from SRM samples

As previously described, with the Synthetic Rock Mass approach it is possible to capture the complete stress-strain response of the sample under unconfined and triaxial compression conditions. Due to the extended computation time, in excess of one week in some cases, that is required to completely capture the post-peak behaviour of the SRM sample, most of the tests presented in previous sections of this chapter were terminated shortly after the peak strength is recorded. In order to evaluate the residual strength at rock mass scale, one UCS test and one triaxial test with a confinement of 2.5 MPa have been conducted on SRM samples with intact rock strength of 37 and 75 MPa, SRM-37 and SRM-75 respectively.

Figure 5-31 shows the stress strain response of the tests conducted on the SRM sample with intact UCS of 37 MPa. The triaxial shows test accumulates a significant amount of damage compared to the UCS test. The high level of damage accumulated during the triaxial test is related to the increase in ductility associated with the confinement applied. The peak strength obtained from the UCS and triaxial test are 5.77 MPa and 10.7 MPa respectively. The residual strength measured during the tests are 1.6 MPa and 8.0 MPa, which corresponds to 28% and 75% of the peak strength respectively.

Figure 5-31. Stress strain and damage evolution during UCS and triaxial test, 37 MPa sample.
Figure 5-32 shows the stress strain response of the tests conducted on the SRM sample with intact UCS of 75 MPa. Both tests exhibit a brittle post peak behaviour, with peak strengths of 11.4 MPa for the UCS test and 15.4 MPa for the triaxial test. The residual strengths measured during the tests are 3.4 MPa and 7.5 MPa respectively, which corresponds to 30% and 49% of the peak strength respectively. The percentage of reduction observed in the UCS tests matches the results obtained from the SRM-37 sample. In the triaxial test, the percentage of reduction in SRM-75 is greater than in SRM-37. This is explained by the difference observed in the post peak behaviour (brittle vs brittle/ductile).

![Stress strain and damage evolution during UCS and triaxial test, 75 MPa sample.](image)

**Figure 5-32. Stress strain and damage evolution during UCS and triaxial test, 75 MPa sample.**

The peak and residual strength envelopes for the SRM-37 and SRM 75 samples are presented in Figure 5-33. An increase in the friction angle of the residual strength envelope is observed in the SRM-37 sample, which is associated with the almost ductile post peak behavior observed in the triaxial test. As was shown in Figure 5-9, this sample presents ductile behavior when tested at confinements of 5MPa or
higher. In the case of the SRM-75 sample, a brittle post peak behavior was observed in both tests and therefore no increment in the residual friction was observed. If greater confinements would have been used for the SRM-75 sample, the brittle ductile transition would have been captured. The results shown in Figure 5-5 indicate that the SRM-75 sample exhibits a ductile behavior when tested at confinements of 10 MPa and higher.

Figure 5-33. Peak and residual strength envelopes from SRM samples.

The results obtained from SRM tests performed on the SRM-37 and the SRM-75 sample are compared with the estimates of residual rock mass strength obtained from the methods proposed by Russo et al. (1998), Ribacchi (2000), Cai et al. (2007) and Lorig and Varona (2013), Table 5-9. In order to solve the issue of the difference in the confined strength between the results of the SRM sample and the Hoek
Brown strength envelope, the results are presented as a percentage of reduction over the peak strength at each confinement stress. The empirical estimates have been obtained using a GSI of 57, obtained from the GSI quantification method proposed by Cai et al. (2004), an $m_b$ of 4.5, and UCS of 37 MPa and 75 MPa respectively.

Table 5-9. Comparison of residual strength between results from SRM tests and empirical estimates.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>SRM - 37</td>
<td>UCS</td>
<td>-</td>
<td>-</td>
<td>28.0%</td>
<td>9%</td>
<td>10%</td>
<td>4%</td>
<td>33%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TX</td>
<td>2.5</td>
<td>2.5</td>
<td>75.0%</td>
<td>53%</td>
<td>54%</td>
<td>47%</td>
<td>52%</td>
<td></td>
</tr>
<tr>
<td>SRM - 75</td>
<td>UCS</td>
<td>-</td>
<td>-</td>
<td>30.0%</td>
<td>9%</td>
<td>10%</td>
<td>4%</td>
<td>43%</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td>TX</td>
<td>2.5</td>
<td>1.6</td>
<td>49.0%</td>
<td>43%</td>
<td>44%</td>
<td>38%</td>
<td>52%</td>
<td></td>
</tr>
</tbody>
</table>

For the UCS test, only the empirical estimate of residual strength obtained from Lorig and Varona (2013) shows a good correlation with the results from the SRM tests. A lower value of residual strength for unconfined conditions is obtained with other empirical methods. None of the empirical methods used in this study provided a good match with the results from the triaxial test performed on SRM-37. The empirical methods used to estimate the residual strengths do not replicate the transition from brittle to ductile behavior that occurs as the confining stress is increased, which explains the large difference between empirical predictions and the results from the SRM tests. Although a very limited amount of cases have been analyzed, it can be concluded that at confinements where brittle post peak behavior is anticipated, empirical estimates can provide reasonable estimates of the rock mass residual strength. However, for levels of confining stress where ductile behavior is anticipated, the strength obtained from empirical estimates can underestimate the in situ residual strength.
5.8 Summary
The Synthetic Rock Mass approach was used to study the behaviour of jointed rock and the effect of intact rock and discontinuity strength on the overall behavior of SRM samples under unconfined and confined compression conditions. The transition between brittle to ductile behaviour for the rock mass is captured by the SRM and the results from the numerical tests present a good agreement with the brittle-ductile limit proposed by Mogi (1966). The ductile behavior observed for tests conducted under relatively high confinement stresses (e.g. 5 MPa for SRM-37 or 10 MPa for SRM-75) is associated with the capability of the smooth joint contact model to reproduce shear between particles, and therefore to mobilize a larger frictional strength as a function of increasing confinement.

Using the GSI quantification methods proposed by Cai et al. (2004) and Hoek et al. (2013), it was possible to estimate a range of possible GSI values for the employed DFN. The unconfined peak strength of the SRM sample falls within the range of strengths estimated by Hoek-Brown envelopes (by using the estimated range of GSI). However, the confined strength predicted by the SRM is lower than the strength obtained from the Hoek-Brown criterion. This is caused by intrinsic limitations of the BPM with either contact bond or parallel bond contact model. The flat joint contact model (Potyondy 2012) has been developed that overcome this limitation. However, this contact model is not implemented in the version of the software used for this study.

The SRM was shown to be capable of capturing the effect of changes in the strength of the intact rock and the strength of the discontinuities on the overall response of the SRM material. With respect to the UCS, the SRM is capable of reproducing the same ratio of rock mass UCS to intact rock UCS for materials with different intact rock strengths. Under confined conditions, the sensitivity of the strength of SRM samples to changes in the strength of the intact rock is smaller than what is obtained by performing similar changes to the Hoek-Brown strength envelopes. The results obtained from the increase in the strength of the discontinuities match the predicted increase with the Hoek-Brown criterion.
From the UCS and triaxial tests, it was possible to identify the onset of yielding of the SRM sample before reaching the peak stress. This limit can be used to delineate areas surrounding an excavation that are prone to damage and yielding.

SRM samples were tested under different loading paths to evaluate the influence of this factor on the development of damage, and on the strength of the sample. Only at low levels of damage, approximately 1.6%, was it possible to observe differences in the damage accumulated under the two stress paths (UCS and triaxial versus triaxial deconfinement). The 1.6% of damage contour line corresponds to the onset of yielding identified from the UCS and triaxial tests. Samples tested with a triaxial deconfinement loading path display an irregular behaviour once loaded beyond the line that defines the onset of failure. The irregularity in the stress path observed in the TD tests has also been interpreted as the beginning of failure, which leads to the conclusion that this limit is not affected by the stress path followed, but is only dependent upon the level of accumulated damage in the sample.

The residual strength of SRM samples with intact UCS of 37 MPa and 75 MPa was analyzed by performing one UCS and one triaxial test, loading the sample until the full residual state was captured. The results from the SRM tests were compared to empirical estimates of residual strength obtained from different methods. The comparison shows different results for the UCS and the triaxial tests. Only the method proposed by Lorig and Varona (2013) methods matched the prediction of residual state under unconfined loading. The residual strength obtained from other methods for unconfined conditions is significantly lower than the prediction of residual strength obtained from the SRM tests. Empirical estimates of residual strength failed to predict the residual strength obtained in SRM samples when the post peak approximates ductile behaviour.
Chapter 6

Evaluation of the Influence of Pit Geometry and In Situ Stress on Stress Path and Rock Mass Damage in Rock Slopes

6.1 Introduction

The effect of in situ stress is commonly disregarded in the evaluation of the stability of open pit slopes. This could be considered a valid assumption for the analysis of shallow pits, however, a careful evaluation of the effect of in situ stress on the development of rock mass damage, and ultimately on the stability of the slopes, needs to be performed for large open pits (Hoek et al. 2009). A few open pit mines around the world have reached depths of 1000 m, or are planning to reach those depths in the future (Robotham 2011). Due to the magnitude of the stresses acting at these depths, it is highly likely that the rock mass near the surface of the excavation will experience some level of damage as a result of the stress changes caused by the mining activity. The effects of stress changes are likely not homogeneous through the slope as the rock mass at different areas of the slope will undergo a different stress path due to material contrasts, local pit geometry, mine sequencing, and initial stress gradients with depth. Stress concentrations can develop at the toe of the slope, causing development, propagation and coalescence of fractures, which can affect the stability of the slope either by instigating localized instabilities or acting as the trigger for larger scale failures. In other areas of the pit, the development of new discontinuities will not necessarily cause immediate instability problems, but the increase in the level of fracturing will affect the characteristics of the rock mass. Determining the extent of the damage zone, and the mechanical properties associated with the damaged rock mass, is important to perform a rigorous assessment of the stability of the slope. Stacey (1981) proposed that a correlation can be established between the area around an excavation that is affected by the development of new fractures or extension of existing discontinuities and the amount of extension strain that the rock experiences as a result of the
stress relaxation caused by the excavation. According to this criterion, fracturing initiates when the extension strain in the rock exceeds a critical extension strain threshold, which is dependent upon the characteristics of the rock. Stacey et al. (2003) and Herrero (2015) have applied the extension strain technique to open pit slope stability problems. In these cases, the analyses have been conducted using plane strain and axisymmetric numerical models, and therefore a thorough evaluation of the impact of pit geometry on the extension of the damage zone has not been performed. In this chapter, a series of three dimensional models have been created to assess the effect of geometry on the induced damage zone under different in situ stress regimes. Furthermore, the results from the numerical tests performed on Synthetic Rock Mass (SRM) samples will be used to determine the critical extension strain threshold that will be used in the analysis of the damage zone in the 3D slope models.

6.2 Rock Mass Damage and Slope Stability

The excavation of a large open pit will induce rock mass damage near the surface of the slope, affecting the mechanical properties of the rock mass. The most recognizable form of damage caused by the mining activity is associated with the use of explosives to fragment the rock. Blasting affects the characteristics of the rock mass by inducing new fractures and by disturbing the interlocking of the blocks. When using the Hoek-Brown criterion, the effect of blasting induced damage (BID) is included by adjusting the value of the disturbance factor (D) (Hoek et al. 2002), which can assume any values between 0 for an undisturbed rock mass and 1 for a completely disturbed rock mass. The extension of the BID zone is influenced by the blast design and can range between 0.3 and 2.5 times the height of the slope (Hoek and Karzulovic 2000) as shown in Figure 6-1. In the author’s experience, the use of controlled blasting techniques has become common in most open pit mines, and a standard practice in large open pit mines in Chile. The use of controlled blasting techniques reduces the rock mass disturbance as well as the extension of the BID. The properties of the rock mass in the BID will have a direct influence on the stability of benches, and the stability of interramp slopes (depending on the extent of the BID). Given the
limited extension of the BID zone, the effect of rock mass properties altered by blast damage on the overall slope stability might be limited. However, a small scale failure can act as the trigger for a larger scale instability.

Figure 6-1. Recommendations for estimation of the blast induced damage zone based on the blasting technique (after Hoek and Karzulovic 2000).

Another form of damage caused by the mining activity is related to the stress changes associated with the progress of open pit excavation. The continuous removal of rock from open pits causes a stress relaxation phenomenon, where the rock mass in slopes experiences a reduction of the confining stresses (Hoek and Karzulovic 2000). Figure 6-2, illustrates the reduction in the major, intermediate and minor principal stresses from the initial in situ stress conditions to the stress state after the pit has been excavated. As was
previously mentioned, the rock mass will expand as a result of these stress changes (relaxation effect), causing extensional strains within the rock mass, which can ultimately damage the rock. Selecting an appropriate value of the disturbance factor (D) to incorporate the effect of damage induced by stress relaxation is a challenging task and remains an open question among practitioners (Lorig 2009).

Figure 6-2. Changes in principal stresses caused by the excavation of the open pit (only changes in stress greater than 10 MPa are displayed). The irregular contours observed in the right hand image are caused by an increase in the mesh element size away from the pit boundary.

Figure 6-3 illustrates the concept of rock mass damage in the zone affected by blasting or stress relaxation. The influence of rock mass disturbance on the mechanical properties is controlled by the increase in the intensity of fracturing (Figure 6-3a), either by extension of the existing discontinuities or by the development of new fractures, and through degradation of the interlocking of the rock mass blocks (Figure 6-3c). A significant amount of strain is required to disrupt the interlocking of the rock mass. Hoek and Karzulovic (2000) indicate that although is not possible to establish a direct relationship between
damage and the deformation caused by stress relaxation, it is probable that a rock mass that has deformed\(^2\) over 1% or 2% has reached its residual strength.

Depending on the geometry of the pit, in situ stress conditions and the mechanical properties of the rock mass, the extent of the slope volume that experiences extensional strains might extend past the plastic zone that develops near surface (Figure 6-4). According to Stacey (1981), fracturing will initiate once the extension strain exceeds a critical strain limit. This criterion can be used to delineate the area of the slope where fractures can develop as a result of stress relaxation. This type of damage can occur under small strains before the peak strength of the rock has been reached, and therefore its effect on rock mass strength is primarily associated with block size reduction caused by the development of new fractures (Figure 6-3b). This needs to be taken into account when using the D factor in the Hoek-Brown criterion to incorporate the effect of damage in this zone as the D factor also incorporates the reduction in strength caused by the alteration of the rock mass interlocking condition.

The SRM tests conducted in Chapter 5 suggested that the amount of damage that the sample can develop, between the onset of yielding and peak strength, increases with increasing confining stress. In the next section, an analysis will be performed to evaluate the influence of pit geometry and in situ stress conditions on the stress path in different areas of the slope. Additionally, Stacey’s (1981) extension strain criterion will be used to compare differences in the extent of the slope where extension strains exceed a critical value (determined from the SRM studies) for the simulated cases.

\(^2\) In this case, deformation is defined as the ratio between slope displacement and the height of the slope.
Figure 6-3. Conceptual illustration of the effect of rock mass disturbance on strength.

Figure 6-4. Schematic illustration of the plastic zone and the zone affected by extension strains associated with mining induced stress relaxation.
6.3 Effect of In Situ Stress and Pit Geometry on Stress Path and Rock Mass Damage

The influence of in situ stress and pit geometry on the stress path at different locations in the slope is evaluated using three dimensional numerical models representing open pits with different slope and pit geometries. An extension strain criterion is used to determine the areas of the slope prone to pre-peak damage due to stress relaxation caused by removal of rock as part of the mining operation.

6.3.1 Numerical Model Setup

Four different pit geometries are used for the evaluation of the effect of pit geometry and in situ stress on the stress path at different points in the slope and the development of extension strain associated with stress relaxation. The geometries used include: a circular pit, two oval shaped pits with different major to minor axis ratios, and an irregular shaped pit as shown in Figure 6-5. A pit depth of 1000 m was used for all cases, with 200 m vertical height for interramp slopes (Figure 6-6). Interramp angles of 35° and 55° were considered for the circular, and oval shaped pits. The pit with irregular geometry was constructed using an interramp angle of 55°.

The numerical models have been constructed in FLAC3D (Itasca Consulting Group 2014) using an octree mesh type. This type of mesh permits refinement of the mesh in the areas of interest, while leaving a coarse mesh in other areas of the model for computational efficiency. This helps with the optimization of the run time required to solve the model. The mesh includes a fine zone with element edges of 20 m that extends 350 meters behind the final pit slope.
Figure 6-5. Pit geometries used in the analyses.

Figure 6-6. Slope geometry used in the analysis; Interramp angles of 35° and 55° have been used for the circular and oval shaped pits.
The sequential excavation of the pit is simulated in the numerical models to capture the effect of local stress concentrations that develop as the excavation of the pit progresses. The excavation sequence used in the numerical models for interramp angles of 35° and 55° is presented in Figure 6-7.

Figure 6-7. Cross section with excavation sequence used to simulate the excavation of the pit, a) pits with 35° interramp angle, b) pits with 55° interramp angle. The numbers in the figure indicates the order in which each volume is excavated.
The in situ stress field used in this study is defined by the vertical ($\sigma_z$) and horizontal stress ($\sigma_x$ and $\sigma_y$) components. The vertical stress is calculated as the overburden from the weight of the overlying rock from Equation 6-1. The horizontal stress components are defined by a stress ratio between the vertical stress and horizontal stresses from Equations 6-2 and 6-3. Stress ratios of 1.0, 1.5 and 2.0 were used to simulate different pre-mining stress states. Changes in the in situ stress conditions are only performed in one direction in each case:

\[
\begin{align*}
\sigma_z &= \gamma \cdot z \\
\sigma_x &= k_x \cdot \sigma_z \\
\sigma_y &= k_y \cdot \sigma_z
\end{align*}
\]

Where

- $\gamma = \text{unit weight of the rock}$
- $k_x, k_y = \text{stress ratio in the x and y directions}$

**6.3.2 Effect of Pit Geometry on Stress Path for Different In Situ Stress Conditions**

The evolution of damage in the rock mass surrounding an open pit is governed by the stress changes that the rock mass experiences in different areas. The stress states in different areas and different depths into the walls of the slope are defined by the in situ stress conditions, pit geometry and rock mass properties. In this section, a series of cases are analyzed to compare the influence of in situ stress and geometry of the pit on the stress changes that different zones of the slope experience. This comparison is performed at discrete points around the pit, with points located at 40, 120 and 320 m behind the slope and at elevations of 0 (H0), 200 (H200), 400 (H400), 600 (H600) and 800 (H800) m above the bottom of the pit.
From the SRM tests conducted in Chapter 5 it was found that the amount of damage that develops in the rock mass is primarily related to the confinement at which the test is conducted (see Table 6-1). The damage intensities listed in Table 6-1 can be correlated to the behaviour of the rock mass in different areas of the slope, based on the confining stress at which the strength envelope is reached. In the following sections, the relationship between stress path, pit and slope geometry, and in situ stress is explored.
Table 6-1. Level of damage recorded at peak strength in SRM tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>Target Confinement Stress (MPa)</th>
<th>Damage at Peak Stress (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td>0.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Triaxial</td>
<td>1.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Triaxial</td>
<td>2.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Triaxial</td>
<td>5.0</td>
<td>7.2</td>
</tr>
</tbody>
</table>

6.3.2.1 Effect of Slope Geometry on Stress Path

The influence of the stress path at different points on the slopes with interramp angles of 35° and 55° are presented for the big oval-shaped pit in Figure 6-9. In general, the stress path reflects the reduction in the minor principal stress ($\sigma_3$), with constant or relatively small increasing trend in the magnitude of the major principal stress ($\sigma_1$). It can be seen that for points located at 400, 600 and 800 m elevation, the change in the angle of the slope does not cause a significant change for the stress path. At locations closer to the bottom of the pit (H0 and H200), a slight increase in $\sigma_1$ occurs as $\sigma_3$ decreases. The increase in $\sigma_1$ is greater for the slope with an interramp angle of 55°. This is caused by the stress concentrations that are generated at the toe of the slope as the slope angle increases. A difference in the magnitude of $\sigma_3$ is observed at the final stage of the excavation between the 35° and 55° slopes. In the case of the 35° slope, the points located at 320 m behind the slope exhibit a similar $\sigma_3$, independent of the elevation. This is valid for all points except for H0, where the particular stress state that develops around the toe of the slope creates a different condition.

Regardless of the slope angle, the results show the difference in the stress changes that the rock mass experiences at different elevations as well as different confining stresses at which each point reaches the strength envelope, which can be correlated to the amount of damage that develops at each location. Based on this and the numerical investigations in Chapter 5 (Table 6-1), it is expected that a larger fracture intensity will develop at lower elevations in the slope.
Figure 6-9. Effect of slope angle on the stress path at different points in the slope, a) 35° interramp angle, b) 55° interramp angle. The color indicates the elevation at which the point is located. The red line in the open pit inset indicates line of section as per Figure 6-8.
6.3.2.2 Effect of Pit Geometry on Stress Path

The big oval pit shape with interramp angles of 35° and 55° is used to evaluate the influence of pit geometry on stress path at different locations in the slope. Figure 6-10 shows the stress path at two different sections (P1 and P7) in the pit for the case with an interramp angle of 35°. At elevations above 400 m, the stress paths at all monitored distances behind the slope suggest that the magnitude of $\sigma_1$ remains relatively constant as $\sigma_3$ decreases. At points located close to the bottom of the pit (H0 and H200), $\sigma_1$ has an inverse increasing trend with $\sigma_3$ decreasing, with a larger $\sigma_1$ increase rate for the slope with smaller radius of curvature (section P7). The effect of slope curvature on the confining stress acting on the slopes can be clearly seen in the stress paths of the points located at 320 meters behind the slope (bottom graphs in Figure 6-10). In section P1, the confining stress at the final stage is relatively similar for points with equal distances behind the slope at different elevations, while in section P7 the magnitude of the confining stress increases for points located closer to the bottom of the pit. This is caused by the reduction in the radii of curvature in lower sections of the slope.

Figure 6-11 shows the results obtained for the pit with an interramp angle of 55°. The increase in the slope angle produces a different response in all areas of the pit compared to results obtained for the pit with a slope angle of 35°. In section P7, the magnitude of the reduction of $\sigma_3$ due to excavation is smaller than in the case of the 35° slope. The combination of slope curvature and a higher slope angle creates conditions of higher confinement in this area of the pit (section P7). The increase in the slope angle also contributes to the rate at which $\sigma_1$ increases with $\sigma_3$ reduction.

In section P1, the major change caused by the increase in the angle of the slope is observed at the point located at 320 m behind the slope, where it can be observed that the change in the geometry allows for higher levels of confinement to develop in the lower area of the slope.
Figure 6-10. Influence of pit geometry on stress path at different points around the slope for a big-oval shaped pit with 35° interramp angle, a) section P1, b) section P7. The red line in the open pit inset indicates line of section as per Figure 6-8.
Figure 6-11. Influence of pit geometry on stress path at different points around the slope for a big-oval shaped pit with 55° interramp angle, left) section P1, right) section P7. The red line in the open pit inset indicates line of section as per Figure 6-8.
6.3.2.3 Effect of In Situ Stress Conditions on Stress Path

The mid-oval shaped pit with an interramp angle of 55° is used to examine the effect of in situ stress conditions on stress path in open pit slopes. The change in in situ stress condition is replicated in the models by modifying the horizontal to vertical stress ratio ($k_x$ or $k_y$ depending on the case).

Figure 6-12 shows the results of the case where the major principal stress is aligned with the long axis of the pit. The comparison includes one case with isotropic in situ stress and two cases with horizontal to vertical stress ratios in the y direction ($k_y$) of 1.5 and 2.0. In section P1, it can be seen that the stress path associated with points located at 40 and 120 m behind the slope reach the strength envelope at different stress states. For example, the point located at 40 m on section P1 will reach the strength envelope approximately at $\sigma_1 = 50$ MPa and $\sigma_3 = 9$ MPa when $k_y=2.0$, while for $k_y=1.0$, the strength envelope is reached at $\sigma_1 = 25$ MPa and $\sigma_3 = 2.5$ MPa.

In Chapter 5, the results of the SRM tests showed a direct relationship between confinement stress and the amount of damage that the samples could develop before reaching peak strength. Due to the frictional nature of the strength of discontinuities, an increase in the mobilised strength occurs with increasing confinements. This allows the rock mass to sustain a certain level of load at stress states past its damage threshold. The stress paths also show the difference in the extent of the plastic zone. As an example, at a distance of 320 m behind the slope, only the stress path for $k_y=2.0$ reaches the strength envelope.

Although most of the points in section P5 remain in an elastic condition, it can be seen that the initial difference in $\sigma_1$ caused by the in-situ stress conditions is only maintained nearly constant at the bottom of the pit. In higher areas of the slope, the stress path converges to a similar stress state at the final state of the excavation.
Figure 6-12. Effect of in situ stress conditions on the stress path at different locations in the pit, case with major principal stress aligned with the long axis of the pit (the color of the line indicates the in situ stress conditions, the line type indicates the distance from bottom of the pit), left) section P1, right) section P5.

Figure 6-13 exhibits the results for the case where the major horizontal stress is aligned with the short axis of the pit. In this case, the effect of the larger horizontal stress is observed for both sections (P1 and P5) for H0 and H400. At H800, the effect of the increase in in situ stress in the magnitude of $\sigma_1$ is only
maintained in section P5. A significant increase in $\sigma_1$ with almost no change in $\sigma_3$ is observed in section P5 for cases with high horizontal stress. A stress concentration is created in the curved area of the pit, around section P5, by the combination of the high stress in the out of plane direction and the slope curvature which results in the stress paths shown in the right hand side of Figure 6-13.

Figure 6-13. Effect of in situ stress conditions on the stress path at different locations in the pit, case with major principal stress aligned with the short axis of the pit (the color of the line indicates the in-situ stress conditions, the line type indicates the distance from bottom of the pit), left) section P1, right) section P5.
6.3.3 Influence of In Situ Stress and Pit Geometry on the Distribution of Extension Strains around the Pit

In this section, Stacey’s (1981) extension strain criterion is used to evaluate areas of the pit that can experience pre-peak damage as a result of the stress relaxation caused by mining. This criterion indicates that damage can initiate in moderately jointed rock masses once the extension strain exceeds a certain threshold. Stacey (1981) analyzed results from a series of laboratory tests performed on cylindrical rock specimens to establish extension strain thresholds for different rock types (Table 6-2). These values correspond to crack initiation stress, which is applied to cases where the ultra-long term strength of the rock is relevant. However, for open pit applications this is of little interest and only the development of macroscopic fractures is relevant to the stability of the slopes. Louchnikov (2011) analyzed data from borehole observations and established a correlation between the magnitude of the extension strains and the level of damage as listed Table 6-3. The values proposed by Louchnikov (2011) are site dependent and highly influenced by the rock type (Wesseloo and Stacey 2016).
Table 6-2. Extension strain thresholds determined from laboratory tests for different rock types (after Stacey 1981).

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Core size</th>
<th>Specimen length / diameter ratio</th>
<th>Critical extension strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartzite A</td>
<td>BX</td>
<td>2</td>
<td>0.000120</td>
</tr>
<tr>
<td>Quartzite B</td>
<td>BX</td>
<td>2</td>
<td>0.000109</td>
</tr>
<tr>
<td>Quartzite C</td>
<td>AX</td>
<td>2</td>
<td>0.00081</td>
</tr>
<tr>
<td>Quartzite D</td>
<td>BX</td>
<td>2</td>
<td>0.000107</td>
</tr>
<tr>
<td>Quartzite E</td>
<td>BX</td>
<td>2</td>
<td>0.000130</td>
</tr>
<tr>
<td>Lava A</td>
<td>BX</td>
<td>2</td>
<td>0.000152</td>
</tr>
<tr>
<td>Lava B</td>
<td>BX</td>
<td>2</td>
<td>0.000138</td>
</tr>
<tr>
<td>Diabase</td>
<td>BX</td>
<td>2</td>
<td>0.000175</td>
</tr>
<tr>
<td>Norite</td>
<td>NX</td>
<td>2.5</td>
<td>0.000173</td>
</tr>
<tr>
<td>Conglomerate Reef A</td>
<td>BX</td>
<td>2</td>
<td>0.000086</td>
</tr>
<tr>
<td>Conglomerate Reef B</td>
<td>BX</td>
<td>2</td>
<td>0.000073</td>
</tr>
<tr>
<td>Conglomerate Reef C</td>
<td>BX</td>
<td>2</td>
<td>0.000083</td>
</tr>
<tr>
<td>Sandstone</td>
<td>BX</td>
<td>2</td>
<td>0.000090</td>
</tr>
<tr>
<td>Shale A</td>
<td>BX</td>
<td>2</td>
<td>0.000116</td>
</tr>
<tr>
<td>Shale B</td>
<td>BX</td>
<td>2</td>
<td>0.000150</td>
</tr>
<tr>
<td>Shale C</td>
<td>AX</td>
<td>2</td>
<td>0.000095</td>
</tr>
</tbody>
</table>

Table 6-3. Extension strain thresholds determined from borehole observations (modified after Louchnikov 2011).

<table>
<thead>
<tr>
<th>Rock mass conditions</th>
<th>Extension strain</th>
<th>( \varepsilon_3 ) (( \mu \varepsilon ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushed</td>
<td>&gt;500</td>
<td></td>
</tr>
<tr>
<td>Heavily broken</td>
<td>450-500</td>
<td></td>
</tr>
<tr>
<td>Broken</td>
<td>350-450</td>
<td></td>
</tr>
<tr>
<td>Fractured</td>
<td>250-350</td>
<td></td>
</tr>
<tr>
<td>Fracture initiation = critical extension strain</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

As an alternative to the use of published data, a relationship between extension strain and damage can be derived from the SRM test results. This permits an accurate and reliable correlation to be established by incorporating site specific conditions in the analysis. The magnitude of the extension strains at the onset of yielding and at peak strength identified from the SRM tests, described in Chapter 5, are presented in Table 6-4. The magnitude of the extension strain measured at the onset of yielding decreases with increasing confinement. As a simplification measure, a critical extension strain of 0.0003 will be used to
delineate the areas of the pit that can develop damage associated with the changes in the stress state caused by mining.

**Table 6-4.** Extension strains for the onset of yielding and peak strength obtained from the SRM tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>Target Confinement Stress (MPa)</th>
<th>Extension Strain at the Onset of Yielding</th>
<th>Extension Strain at Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS</td>
<td>0.0</td>
<td>-</td>
<td>-7.8E-04</td>
</tr>
<tr>
<td>Triaxial</td>
<td>1.0</td>
<td>-3.6E-04</td>
<td>-4.8E-04</td>
</tr>
<tr>
<td>Triaxial</td>
<td>2.5</td>
<td>-2.7E-04</td>
<td>-5.3E-04</td>
</tr>
<tr>
<td>Triaxial</td>
<td>5.0</td>
<td>-2.5E-04</td>
<td>-6.4E-04</td>
</tr>
</tbody>
</table>

For linearly elastic material, the extension strain can be calculated from Equation 6-4:

\[ e_3 = \frac{1}{E} [\sigma_3 - \nu(\sigma_1 + \sigma_2)] \]  

where \( E \) and \( \nu \) are the Young’s modulus and Poisson’s ratio of the intact rock, respectively and \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses. For the analysis, a value of 40 GPa was used for the Young’s modulus and 0.25 for Poisson’s ratio.

6.3.3.1 Effect of Slope Angle and Pit Geometry on the Distribution of Extension Strains

The influence of slope geometry on the distribution of extension strains is evaluated for the mid-oval and big-oval pits by comparing the development of extension strains for an identical pit plan geometry with different interramp angles (35° and 55°). An isotropic in situ stress state \((k_x=1.0, k_y=1.0)\) was used for this analysis. Figure 6-14 shows the results obtained for the mid-oval (MO) pit. As expected, the depth of the zone associated with the extension strains varies with the curvature of the pit. This is related to the increased confining stress acting on the curved (concave) areas of the pit. The pit with interramp angle of 55° shows a slight increase in the height of the slope affected by extension strains compared to the pit with 35°. In the lower areas of the slope, a higher confining stress develops for higher interramp angles, which explains the difference observed between the two cases.
Figure 6-14. Effect of interramp angle on the distribution of extension strains for the mid-oval shaped pit.

Figure 6-15 displays the results of the comparison performed for the big-oval (BO) shaped pits. The results show similar trends to what was observed for the mid-oval pit. A slight increase in the extent of the area where extension strains develop is observed in the section located on the side of the pit (first row in Figure 6-15), reflecting the small change in the curvature between the MO and BO geometry.
Figure 6-15. Effect of interramp angle on the distribution of extension strains for the mid-oval shaped pit.
6.3.3.2 Effect of In Situ Stress on the Distribution of Extension Strains

The influence of in situ stress conditions on the development and distribution of extension strains is evaluated for the four pit geometries presented in Figure 6-5. The list of cases being analyzed is presented in Table 6-5. For the mid-oval and the big-oval, the analyses include cases where the major principal stress is aligned with the minor as well as the major axis of the pit.

Table 6-5. Cases analyzed for the evaluation of the effect of in situ stress on the distribution of extension strains in the slopes.

<table>
<thead>
<tr>
<th>Pit Geometry</th>
<th>Interramp Angle (°)</th>
<th>Kx</th>
<th>Ky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>35</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Mid Oval</td>
<td>35</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Big Oval</td>
<td>35</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Irregular</td>
<td>55</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
6.3.3.2.1 Circular Pit

The results of the evaluation of the effect of in situ stress on the plastic and extensional strain zones for the circular pit with interramp angle of 35° is presented in Figure 6-16. In this case, the major principal stress ($\sigma_1$) is aligned with the Y-axis. In section P1, where $\sigma_1$ is perpendicular to the section, the increase in $\sigma_1$ creates a larger plastic zone behind the slope. The area of the slope with extension strains greater than 0.0003 is minimal, and for $k_y$=2 is located inside the plastic zone.

In section P4, the increase in the horizontal stress creates a larger plastic zone. The irregularities observed in the geometry of the plastic zone for $k_y$=2.0 are caused by high stress concentrations that developed at the toe of the slope during intermediate excavation steps. The zone with extension strains greater than 0.0003 also increases significantly with the change in the in situ stress conditions, both in depth and in the height of the slope.

Figure 6-17 presents the results of the comparison performed for the pit with 55° interramp angle. The results present some similarities to the case with the 35° interramp angle. The trend between the extent of the plastic zone and the area with extension strains in section P4 is in agreement with the observations made for the case with lower (35°) interramp angle. A minor difference is observed in section P1 between the two cases, where a small area with extension strains greater than the strain threshold is observed for the case of isotropic stress conditions ($k_x = 1.0, k_y = 1.0$) for a higher slope angle.
Figure 6-16. Influence of in situ stress on the development of plastic zone and distribution of extension strains for a circular pit with an interramp angle of 35°.
Figure 6-17. Influence of in situ stress on the development of plastic zone and distribution of extension strains for a circular pit with an interramp angle of 55°.
6.3.3.2.2 Mid-Oval Pit

Figure 6-18 shows the results of the evaluation of the influence of the in situ stress conditions on the development of the plastic zone and the distribution of extension strains for the mid oval pit with interramp angle of 35°. It is observed in this figure that when $\sigma_1$ is aligned with the major axis of the pit (Figure 6-18a), the plastic zone on section P1 extends in both depth and height. The area with extension strains over the strain threshold in section P1 is only significant for the case of isotropic stress. Under other stress conditions, the area with high values of extension strains is located within the plastic zone. In section P5, the increase in the magnitude of the horizontal stress creates a larger zone affected by extension strains, reaching up to 700 m above the floor of the pit. As it was observed in the circular pit, the geometry of the plastic zone that develops for high horizontal stresses ($k_y=2.0$) is highly irregular. This is caused by stress concentrations that occur at the toe of the slope during intermediate excavation steps.

Similar results are obtained when the major principal stress is aligned with the short axis of the pit (Figure 6-18b). In the section parallel to $\sigma_1$, extension of the area associated with high values of extensional strains is observed as $\sigma_1$ increases. An increase in the extent of the plastic zone is observed in the section perpendicular to the in situ major principal stress as the magnitude of the in situ stress increases.

Figure 6-19 shows the results obtained for the case with an interramp angle of 55°. The results suggest similar trends as the mid oval pit with interramp angle of 35°, with a direct relationship between the magnitude of the in situ horizontal stress and the dimensions of the area with extension strains greater than the defined threshold.

6.3.3.2.3 Big-Oval Pit

The results of the analysis for the big oval pit are presented in Figure 6-20 and Figure 6-21. In general, the results obtained from the analyses of the big-oval pit follow the same trends observed for the mid-oval pit.
Figure 6-18. Influence of in situ stress on the development of plastic zone and distribution of extension strains for the mid-oval pit with an interramp angle of 35°, a) major principal stress aligned with the pit major axis (y), b) major principal stress aligned with the pit minor axis (x).
Figure 6-19. Influence of in situ stress on the development of plastic zone and distribution of extension strains for the mid-oval pit with an interramp angle of 55°, a) major principal stress aligned with the pit major axis (y), b) major principal stress aligned with the pit minor axis (x).
Figure 6-20. Influence of in situ stress on the development of plastic zone and distribution of extension strains for the big-oval pit with an interramp angle of 35°, a) major principal stress aligned with the pit major axis (y), b) major principal stress aligned with the pit minor axis (x).
Figure 6-21. Influence of in situ stress on the development of plastic zone and distribution of extension strains for the big-oval pit with an interramp angle of 55°, a) major principal stress aligned with the pit major axis (y), b) major principal stress aligned with the pit minor axis (x).
6.3.3.2.4 Irregular Pit

Three sections around the pit slope were investigated in detail when examining the irregular pit. This presents an interesting case of study as two of the sections, sections P1 and P2 are located within concave slopes with different curvatures, while the third section is located within the convex slope.

The comparison of the effect of in situ stress on the rock mass response for the irregular pit is shown in Figure 6-22. For an isotropic in situ stress condition (first row in Figure 6-22), the difference in the curvature of the slope has a clear effect on the extent of the plastic zone behind the slope surface. A larger plastic zone is observed in the convex slope, section P3, with respect to the concave slopes (sections P1 and P2). This is expected as one of the problems associated with convex slopes is the reduction in the confinement stress and the consequent reduction in rock mass strength. In the concave slopes, the extent of the zone with extension strains greater than 0.0003 is larger for the slope with lower curvature (section P1) in comparison to what is observed for the slope with smaller radius of curvature (section P2). In this case, the difference in the curvature of the slope does not seem to have an effect on the height of the slope where high extension strains develop. In the convex slope (section P3), the depth of the zone affected by extension strains is similar to what is observed in section P1. However, the height of this zone is reduced with respect to what is observed for concave slopes. In the convex slope, the radius of curvature is decreasing with moving up in the slope. The smaller radius of curvature limits the development of extension strains at higher elevations in the slope.

The increase in the in situ horizontal stress creates larger plastic zones and areas with high extension strains. These results are in agreement with the development of similar zones under comparable in situ stress conditions for other pit geometries. The greater effect on the extent of the area with high values of extension strains is observed in sections aligned with the major
horizontal stress. The increase in the dimensions of this zone is significant, extending several hundreds of meters behind the crest of the slope.

Figure 6-22. Influence of in situ stress on the development of plastic zone and distribution of extension strains for the irregular pit with an interramp angle of 55°.
6.4 Summary

6.4.1 Effect of Pit Geometry and In Situ Stress on Stress Path

The excavation of a large open pit results in redistribution of the stresses acting on the slopes. As the result of the stress changes caused by mining, some areas of the pit will experience stress relaxation, an increase of the major principal stress or a combination of both. These changes in the acting stress state can induce rock mass damage in the form of development of new fractures or extension of existing ones. The intensity of damage that a rock mass can develop by the time its peak strength is reached depends on the corresponding confinement stress. A smaller amount of damage is required to bring the rock to failure under low levels of confinement compared to the accumulated damage in the rock mass at higher levels of confining stress. This is related to the frictional nature of the discontinuities and the strength component that can be mobilised in these features as the confining stress is increased. The results of the Synthetic Rock Mass (SRM) tests conducted in Chapter 5 show the relationship between confining stress and the level of damage the rock can develop at its peak strength. Based on the results of the analysis in this chapter, it is possible to make the following conclusions:

- Under isotropic in situ stress conditions, the slope angle is not a major contributing factor to the stress path. The only noticeable exception was observed far behind the slope face (320 m), where the confinement increases for lower elevations. This is expected as stress concentrations develop at the bottom of the pit.

- The influence of pit geometry on stress path was investigated by using the big-oval shaped pit with interramp angles of 35° and 55°. For points located close to the slope, a minor increase of the major principal stress was observed in the slope with smaller radius of curvature compared to other areas of the pit. The results of the pit with an interramp angle of 55° showed significant differences associated with the change in slope curvature,
with a larger increase in the major principal stress and smaller reductions of the confining stress.

- The effect of in situ stress on the stress path is significant. In slopes with large radius of curvature, close to plane strain conditions, the stress analyses shows that the value of $\sigma_1$ remains constant or has a slight increase as $\sigma_3$ is reduced with the excavation. This means that the stress state at which certain areas of the slope reach peak strength is controlled by the initial stress state. Therefore, higher values of in situ horizontal stress will result in a higher level of damage in areas where the stresses reach or exceed the strength of the rock mass. In the curved areas of the pit, and when $\sigma_1$ is acting in the in-plane direction, the effect of different in situ stress regimes is only considerable at the bottom of the pit. At higher elevations of the slope, the stress state at the end of the excavation converges to similar values. This indicates that in higher areas of the slope, the final stress state is controlled by the geometry of the pit and gravity. When the major principal stress is acting in the out of plane direction, the effect of stress path is observed in the curved area of the slope at all elevations. Therefore, it is expected that different levels of damage will develop under higher levels of in situ stress.

- The increased levels of damage that can develop under varying in situ stress conditions can affect the stability of the slopes in two ways: 1) Fractures that from as a result of the process of stress concentration/stress relaxation can develop at orientations that will be detrimental to the stability of the slope. 2) The increased levels of damage do not necessarily pose a problem for the stability of the slope if the confining stress at which that damage developed is maintained. However, if as a result of the mining activity, the confining stress is reduced, the strength of the damaged rock mass will have a significant drop.
6.4.2 Effect of Pit Geometry and In Situ Stress on the distribution of extensional strains around the pit.

The relationship between the stress state and damage observed in the SRM tests was used to evaluate the differences in the level of damage that can develop in different areas of an open pit. The influence of slope angle and pit geometry, as well as the effect of different in situ stress regimes were evaluated.

The influence that the in situ stress and pit geometry have on the development of a plastic zone behind the slope surface, and on the distribution of extension strains, were evaluated using Stacey’s (1981) extension strain criterion. This criterion indicates that fractures initiate once the extension strain exceeds a certain threshold. The value of the extension strain threshold used in the analysis was defined using the results of the SRM tests conducted in Chapter 5, where the magnitude of the extension strain at the onset of yielding was identified for the investigated rock mass.

The geometry of the plastic zone is highly dependent upon slope geometry and in situ stress conditions. In areas of the pit where the in situ major principal stress is aligned with the sections used in the analysis, the plastic zone grows in size developing an irregular geometry, which is caused by stress concentrations that develop at the toe of the slope during intermediate excavation steps. The increase in the size of the plastic zone is controlled by the curvature of the slope, where larger plastic zones develop for slopes with larger radii of curvature. This corresponds to the inverse relationship between the induced confining stress and the slope curvature radius (function of slope geometry).

The distribution of extension strains around the pit is also dependent upon in situ stress conditions. Larger zones with extension strains above the damage thresholds is observed for higher in situ horizontal stresses, and decreasing depths behind the surface for slopes with smaller radii of curvature.
For a given stress change, the extent of the zone with extension strains is governed by the elastic properties of the rock. In this study, values close to the properties used in the SRM tests conducted in Chapter 5 were used for consistency. However, these values are considered low compared to other applications of the extension strain criterion in open pits published in the literature (e.g. Stacey et al 2013, Herrero 2015). Figure 6-23 compares the zones with extension strains greater than 0.0003 for different material properties. It is seen that the size of the zone with high values of extension strains decreases significantly as the Young’s modulus increases.

As indicated by Louchnikov (2011), the level of damage depends on the magnitude of the extension strain at different locations in the slope. The effect of different levels of damage on rock mass properties can be evaluated by testing damaged SRM samples following a procedure similar to the one used in Chapter 5.
Figure 6-23. Zone with extension strains greater than 0.0003 for different values of Young’s modulus (E) and Poisson’s ratio (v), a) $E=40$ GPa, $v=0.25$ (this study), b) $E=50$ GPa, $v=0.2$, c) $E=80$ GPa, $v=0.17$ (Stacey et al. 2003) and d) $E=78$ GPa, $v=0.20$ (Herrero 2015).
Chapter 7

Conclusions

7.1 Thesis summary
This thesis was conducted to evaluate the influence of stress path and damage on intact rock mass behavior and the possible implications for the stability of large rock slopes. This work consisted of three main phases:

- A numerical analysis of the effect of stress path and damage on the strength of intact rock.
- An evaluation of the effects of stress path and damage at rock mass scale using the Synthetic Rock Mass (SRM) approach.
- A study of the effects of pit geometry and in-situ stress conditions on the stress path and distribution of extension strains around an open pit.

The main findings of this work and recommendations for future research are discussed in the following sections.

7.2 Effects of stress path on intact rock damage accumulation and associated strength degradation
The bonded particle method (BPM) was used to evaluate the influence of stress path on damage accumulation at the intact rock scale. BPM samples were tested under standardized (unconfined and triaxial compression), triaxial deconfinement (TD) and confining stress reduction (CSR) loading paths. The results showed that tests that followed a TD or CSR stress path accumulated less damage than samples tested under standardized loading paths. Consequently, the samples which were tested following unconventional loading paths which accumulated less damage during loading could achieve a higher strength envelope.
A review of the relevant literature showed that the academic community lacks consensus on the relationship between stress path and intact rock strength. The brittle nature of failure under unloading or stress relaxation conditions, though, has been widely accepted and supported by experimental data (e.g. Jinli et al 2011, Yang et al 2011).

The strength degradation associated with accumulated damage was analyzed by testing damaged BPM samples. Different procedures were used to generate damaged BPM samples, including inserting cracks at random orientations and at specified orientation distributions. As well, BPM samples damaged by previous loading in a UCS or triaxial test were tested. The main findings from the analysis of the damaged BPM samples were:

- The samples with damage inserted at random orientations showed the greatest strength reduction. Inserting damage at random orientations could have inserted cracks at high-strength contacts that would otherwise have remained intact under loading and that are a part of the internal force chains that carry the load in the sample.
- A linear relationship was found between the unconfined compressive strength of the damaged samples and the percentage of broken bonds in the samples. The linear relationship between damage and UCS is characteristic of BPM samples with high porosity (Schöpfer et al. 2009).
- Pre-existing damage reduced the cohesive strength of BPM samples without affecting the material’s friction angle. The friction mobilization that occurs in damaged rock is not captured with the BPM samples when the parallel bond contact model is used. The spherical geometry of the particles does not allow for an accurate representation of the sliding and interlocking between polygonal grains that occurs in damaged rock, limiting the BPM’s capability to simulate the mobilization of friction. This is a known limitation of models constructed using spherical particles (Diederichs 1999, Schöpfer et al. 2009).
Damage created by previous loading of the sample did not have significant influence on the strength of the BPM sample. During the first loading stage, weaker contacts broke, while stronger contacts remained intact, forming the force chains that carried the applied load. During the second loading stage, the same force chains could carry the load, so the strength of the sample was not affected by the damage caused during the first loading stage. The results from cyclic loading of Lac du Bonnet specimens in the laboratory by Ghazvinian (2015) support this finding.

### 7.3 Evaluation of the effect of stress path and damage on rock mass strength with SRM
The SRM approach was used to evaluate rock mass behaviour under different loading paths. In addition, the response of the SRM samples to changes in the strength of the intact rock and the discontinuities was determined and compared to the empirical estimates of rock mass strength.

The main findings from this analysis were as follows:

- The SRM tests could replicate the transition from brittle to ductile post-peak behaviour.
  
  The level of confinement at which the transition from brittle to ductile occurred accorded with the empirical brittle–ductile transition limit proposed by Mogi (1966).

- The GSI values of the SRM samples were estimated by relating the geometrical and strength characteristics of the discrete fracture network (DFN) to the parameters used in the GSI quantification methods proposed by Cai et al. (2004) and Hoek et al. (2013).

  Good agreement was found between the UCS of the SRM samples and the UCS predicted by the Hoek-Brown criterion using the estimated GSI values and properties of the intact BPM. Under these confined conditions, the SRM had lower strength than the Hoek-Brown criterion. The reduced increase in the strength of the SRM specimen under confinement was caused by the limited ability of the parallel bond contact model to reproduce high friction angles.
The SRM captured the effects on strength from changing the strength of the intact material and the discontinuities. The SRM could reproduce the same ratio of intact UCS to rock-mass UCS for materials with different intact rock strength. Under confined conditions, the magnitude of the changes in SRM strength caused by modifying the strength of the intact material was lower than the variations obtained using the Hoek-Brown criterion. The change in SRM strength generated by the increased strength of the discontinuities matched the variation predicted by the Hoek Brown criterion.

The results of the triaxial tests made it possible to identify the onset of yielding in the SRM samples. The results showed that yielding began at the same damage level independent of confinement stress. However, the level of damage at peak strength was directly related to the confinement under which the test was conducted. The samples tested under TD loading paths showed irregular behaviour once the stress in the samples passed the onset of yielding identified in the UCS and triaxial tests. The irregular behaviour was interpreted as the onset of yielding in the TD tests.

The influence of the stress path in the development of damage was observed only at low levels of damage. No differences in the damage caused by different stress paths could be observed once yielding of the SRM sample began.

The impact of randomly induced damage in the strength of SRM samples was lower than observed in the damaged BPM samples. In jointed rock, failure was partially controlled by discontinuities. When inserting damage at random locations, it was possible that cracks might have been inserted at locations where they did not contribute to the interactions with the existing discontinuities that contributed to the failure surface.

The rock mass residual strength predicted by SRM was greater than the predictions obtained from empirical estimates. The transition towards ductile behaviour observed in triaxial tests conducted at higher confinement levels, is not included in the empirical
methods, creating larger differences between the empirical estimates of residual strength and the results from SRM samples with ductile behaviour.

7.4 Effects of geometry and in-situ stress on stress path and damage

The amount of damage a rock mass could sustain before reaching peak strength is directly related to the confinement stress at which the sample reached that peak strength. The stress state action on the slope is a function of the pit geometry, in-situ stress conditions and mechanical properties of the rock. The following conclusions regarding the effect of pit geometry and in-situ stress on the stress changes experienced in different areas of the pit were drawn.

- The level of damage caused by stress relaxation in different areas of the pit could be qualitatively differentiated by analyzing the stress path that the rock mass experienced and the level of confinement at which the strength envelope was reached.

- In general, the stress path in slopes was characterized by reduced confinement stress and slightly increased or relatively constant major principal stress. Consequently, the initial stress condition directly influenced the stress state at which the strength envelope was reached. An increased amount of rock mass damage could be expected in slopes excavated in areas with high horizontal in-situ stresses.

- The results of the SRM tests were used to define the areas of slopes where the development of pre-peak damage caused by stress relaxation could be expected. The magnitude of the extension strain recorded at the onset of yielding identified in the SRM tests, along with Stacey’s extension strain criterion, was used to define the areas of slopes where relaxation induced damage could be expected. The large dimensions of the damage zone obtained in this analysis were related to the elastic properties used in this study. A significant reduction of the extent of the damage zone could be expected for stiffer rock types.
The extent of the zone with high-magnitude extension strains increased significantly as the magnitude of the horizontal in-situ stress is increased. The increase in confinement generated by slope curvature reduced the extent of the damage zone in the curved areas of the pit.

Increased amounts of pre-peak damage in different areas of the pit does not necessarily have detrimental effects on the stability of the slopes. As long as the stress state at which that level of damage developed is maintained, the capacity of the rock mass to sustain the acting stress is not affected. Due to the frictional nature of damaged rock, a significant reduction in strength will occur if the magnitude of the confining stresses is reduced as a result of mining activities. This finding has important implications for slope stability analysis as the influence of damage cannot be evaluated by analyzing the final stress state. To accurately quantify the effect of stress relaxation on induced damage, it is necessary to analyze the stress path followed by the rock mass before reaching the final state.

7.5 Future studies
During this research, areas that require further research were identified. Although the numerical methods used in this research present considerable advances with respect to methods used in engineering practice, they still have limited capability to fully capture all aspects of rock behaviour. The following recommendations address areas that require further work and provide alternatives to overcome the limitations of the numerical methods used in this thesis.

- The intact rock analysis was conducted using the parallel bond contact model, which has limitations in reproducing high friction angles. Consequently, the confined strength of BPM samples calibrated to the UCS of the intact rock will underestimate the confined strength. New contact models were added to the most recent versions of the software used in this thesis to overcome the limitations of the BPM in reproducing high friction
angles. It is recommended that additional analyses be performed to investigate the effect of higher friction angles on the results. This could also help improve the confined response of SRM samples. Alternatively, the recently developed Bonded Block Model (BBM) (Turichshev and Hadjigeorgiou 2016) can be used as a replacement for BPM in the construction of SRM samples. The BBM does not have the BPM’s limitations in reproducing high friction angles.

- The SRM samples were constructed using an idealized DFN that did not include variations in the orientation of the joints. As well, joint termination is an important factor in the behaviour of jointed rock masses. The DFN generator in Itasca’s software does not have the capability to model the different types of joint termination observed in the field. Given the importance of structure in the behavior of jointed rock, it is recommended that a more realistic DFN incorporating changes in joint orientation, termination and strength be used to construct SRM samples. More advanced DFN modelling codes, such as Fracman (Golder Associates 2016), have the capability to incorporate joint termination as an input parameter. Palleske (2014) provides additional recommendations regarding the generation of DFNs for rock mechanics applications.

- The influence of higher model resolution on the response of blocks of intact rocks in SRM samples has not been evaluated. Using higher model resolution could help in simulating the behaviour of blocks in the SRM sample with a higher degree of accuracy.

- The effect of stress path on the development of damage was analyzed using cylindrical samples; consequently, the minor and intermediate principal stress in the tests had the same magnitude. In the field, differences between minor and intermediate principal stress exist. Their effect on rock mass behaviour could be evaluated by simulating true triaxial tests on prismatic SRM samples.
In the field, stress rotation accompanies the stress relaxation caused by mining, and the orientation of the principal stresses can result in additional damage. This study did not consider the effects of additional damage and changes in damage orientation. Mas Ivars et al. (2011) describe the use of spherical SRM samples that permit stress rotation to be simulated. This method could be used to evaluate the effect of additional damage caused by stress rotation on the strength of the rock.

The application of Stacey’s extension strain criterion to the prediction of the extent of the zone with damage induced by stress relaxation needs to be validated in the field. This could be done by comparing the extent of the damage zone predicted by the strain criterion with the location of recorded local seismic events.

Some of the processes described in this thesis may not be instantaneous during construction. The impact of time dependant damage process in open pits could be a area of future studies.
References


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