EASTERN ONTARIO PRE-SERVICE INTERMEDIATE-SENIOR MATHEMATICS TEACHERS’ BELIEFS ABOUT PROBLEM SOLVING

by

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Abstract

Problem solving in the Ontario mathematics curricula is described as central to learning mathematics. Scholars of mathematics education similarly stress the importance of problem solving, noting its potential to develop students’ academic ability in mathematics and prepare students to be effective lifelong learners. However, despite the value attributed to problem solving in the learning of mathematics, little is known about how pre-service teachers are thinking about problem solving. Given that beliefs are a fundamental domain of cognition, the first step in addressing this gap in the literature is to explore pre-service teachers’ beliefs about problem solving. Hence, the purpose of this mixed methods study was to explore Eastern Ontario pre-service intermediate-senior mathematics teachers’ beliefs about mathematics problem solving.

Following a pragmatic mixed methods approach, quantitative questionnaire data were collected from a sample of 44 pre-service teachers in an Eastern Ontario teacher education program, and qualitative interview data were collected from a subsample of four pre-service teachers. Both types of data were collected at the beginning and end of the Fall 2016 academic term. Analysis followed a convergent parallel design, where quantitative and qualitative data were analyzed separately then merged to enable the identification of key features being converged upon by the two data sets.

Analysis of the qualitative interview data identified and described the pre-service teachers’ ontological and epistemological beliefs about mathematics problem solving (i.e., beliefs about what mathematics problem solving is and how mathematics problem-solving knowledge is acquired, respectively). Additionally, analysis of the quantitative questionnaire data identified statistically significant differences in the pre-service
teachers’ beliefs about problem solving with respect to experience in the teacher education program and teacher-related variables (i.e., gender and teaching subject). Finally, by comparing and integrating the questionnaire and interview findings, it was possible to characterize the prominent features of problem solving the pre-service teachers emphasized when communicating their thoughts about problem solving. The implications for mathematics educators focus on opportunities for recognizing the complex conceptualization of problem solving espoused by pre-service intermediate-senior mathematics teachers. Recommendations for future research highlight the need to expand the scope of this initial investigation and to investigate pre-service teachers’ knowledge of problem solving.
Acknowledgements

Reaching this point in my academic journey would not have been possible without my supervisor, Dr. Jamie Pyper. Your knowledge, insight, patience, and unwavering encouragement not only made the preparation and completion of this manuscript possible, but also enjoyable. Your contributions extend far beyond this single manuscript, however, and owing to your guidance I have grown as a researcher, a writer, and a teacher. I hope you know the profound influence you have had on me both as a scholar and a friend. I look forward to the projects we will take on in future years.

I would also like to acknowledge the contributions of my thesis committee member, Dr. John Freeman, and my thesis examining committee, Dr. Richard Reeve, and Dr. Peter Taylor. John, although you could not see the final product, the research presented in this thesis is unquestionably of higher quality because of your contributions. Dr. Reeve, thank you stepping in for John during such a difficult time for our faculty. To both Dr. Reeve and Dr. Taylor: your critique and wise suggestions have allowed this manuscript to develop and mature, and for that I am truly grateful.

To the pre-service teachers who participated in this study: thank you for contributing your thoughts and time to this master’s thesis. That you were so generous with your time, all while completing a program that demands a great deal of your attention, shows a depth of character that I know will serve you well in your future careers as teachers of mathematics.

To my friends: I am forever grateful for the countless hours you surrendered to listen to my ruminations about problem solving and teacher education. I hesitate to list names, as there have been so many of you who were instrumental in helping me reach
this point today. It has been an honour and a privilege to learn from each of you, and I look forward to the lessons we will teach one another in the years to come.

To my parents: thank you for your boundless support and understanding throughout my entire academic journey. Your influence cuts so completely across my life that it feels like an understatement to say I would not be at this point today without your love and support. I owe who I am today to both of you, and while I may never find a way to fully communicate my gratitude, I hope you know I love you dearly.

Finally, to Stephanie: throughout my many years of post-secondary study, I have always been able to count on your support and encouragement. Even when I have undoubtedly worked too much and lost valuable time that could have been spent with you, you were never anything but patient and understanding. I love you, and I can think of no better way to spend the rest of my life than exploring this world with you.
Dedication

This thesis is dedicated to the memory of Dr. John Freeman. A graduate student could never hope for someone more caring, inspiring, and wise in a committee member or mentor. His influence on my academic work, my thinking as a scholar, and the Queen’s University Faculty of Education is ineffable. An irreplaceable personality, John’s memory lives on in the excellence that he inspired in every person who had the privilege of knowing him.
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# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>B.Ed.</td>
<td>Bachelor of Education</td>
</tr>
<tr>
<td>BMPSQ</td>
<td>Beliefs About Mathematical Problem Solving Questionnaire</td>
</tr>
<tr>
<td>IS</td>
<td>Intermediate-Senior</td>
</tr>
<tr>
<td>MOE</td>
<td>Ministry of Education</td>
</tr>
<tr>
<td>NCTM</td>
<td>National Council of Teachers of Mathematics</td>
</tr>
<tr>
<td>TBI</td>
<td>Teacher Beliefs Interview</td>
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Chapter 1

Introduction

In the contexts of the Ontario mathematics curricula, problem solving is understood to be an integral aspect to students’ learning of mathematics (e.g., Ministry of Education [MOE], 2005a, 2005b, 2007). In fact, the Ontario mathematics curricula treat problem solving in an analogous way to the scholars of mathematical problem solving (e.g., Schoenfeld, 1992; Wilson, Fernandez, & Hadaway, 1993), considering it to be at the heart of learning mathematics. Given the importance attributed to problem solving by the Ontario MOE, Ontario mathematics teachers must be prepared to educate students on effective problem-solving practices and ways of thinking about problem solving in relation to mathematics.

Ensuring teachers are prepared to effectively embed problem solving in their classroom practice is instrumental for students’ learning of mathematics, as a wealth of literature suggests that purposeful problem-solving experiences increase student achievement, facilitate academic skill development, and promote positive views of the learning environment (Butera et al., 2014; Jitendra, Dupuis, & Zaslofsky, 2014; Swanson, Orosco, & Lussier, 2014). Additionally, problem-solving experiences prepare students to be effective lifelong learners by developing conceptual understandings, analytic abilities, and logical reasoning skills (National Council of Teachers of Mathematics [NCTM], 2010). However, while it is expected that mathematics teachers will treat problem solving as mandated by the Ontario MOE, teachers’ interpretation and translation of the
mathematics curricula is inevitably influenced by their cognitive processes (Ball, Maguire, Braun, & Hoskins, 2011).

Beliefs are a cognitive process of foundational importance for understanding how teachers interpret and translate problem solving as it is outlined in the Ontario mathematics curricula, as beliefs directly influence teachers’ classroom practice (Pajares, 1992; Philipp, 2007; Raths & McAninch, 2003). Teachers’ beliefs guide instructional decisions, influence the form and use of assessments, and act as a filter through which classroom experiences are processed (Luft & Roehrig, 2007). Moreover, examining mathematics teachers’ beliefs has been described as imperative for anyone seeking to improve mathematics education (Barlow & Reddish, 2006). Pre-service teachers’ beliefs are particularly important to understand, as teacher education is an ideal time to positively influence teachers’ beliefs about their classroom practice (Luft & Roehrig, 2007; Pajares, 1992).

Despite the known influence of beliefs on teachers’ classroom practice (Luft & Roehrig, 2007; Wilson et al., 1993), there appears to be no research that has examined Ontario mathematics teachers’ beliefs about problem solving. This gap in the literature is especially relevant for the preparation of intermediate-senior (IS) mathematics teachers (i.e., teachers who teach mathematics in grades 7 through 12 in Ontario), as these teachers must be familiar with all three of the Ontario curriculum documents for mathematics from grades 7 to 12 (MOE, 2005a, 2005b, 2007). Additionally, of the scant literature that does exist for any other pre-service mathematics teachers’ beliefs about problem solving (Bal, 2015; Memnun, Hart, & Akkaya, 2012; Sağlam & Dost, 2014; Xenofontos & Andrews, 2014; Yavuz & Erbay, 2014), teachers in the IS divisions have
received comparatively less attention. The current literature has also yet to reach a consensus about the influence of teacher-related variables on pre-service mathematics teachers’ beliefs about problem solving (e.g., the influence of gender and teaching subject), so additional work is needed to explicate the effects of such variables.

Purpose and Research Questions

The purpose of this mixed methods study was to address the gap in the literature regarding Ontario pre-service IS mathematics teachers’ beliefs about problem solving in the context of teaching Ontario mathematics curricula. Given the complexity of belief structures (Pajares, 1992), addressing this gap in the literature required a multi-faceted examination of ontological beliefs (i.e., beliefs concerned with the perceived reality of problem solving; what it is) and epistemological beliefs (i.e., beliefs concerned with how knowledge about problem solving is acquired and the limits of that knowledge).

Developing an understanding of these beliefs about problem solving will ultimately provide an informative basis for further research and support mathematics teacher educators in preparing pre-service teachers who can effectively develop students’ problem-solving ability. The objectives of this mixed methods study were: (a) to interpret the ontological and epistemological beliefs about problem solving held by Eastern Ontario pre-service IS mathematics teachers, and (b) to determine the features of problem solving that are emphasized in the pre-service teachers’ communication of problem solving, as influenced by their problem-solving belief structures. The following two research questions informed the development, execution, and reporting of this study:
1. What are the ontological and epistemological beliefs about mathematics problem solving held by Eastern Ontario pre-service IS mathematics teachers?
   a. How do experience and gender interact regarding pre-service teachers’ beliefs about mathematics problem solving?
   b. How do experience and teaching subject interact regarding pre-service teachers’ beliefs about mathematics problem solving?

2. What features of problem solving are emphasized in Eastern Ontario pre-service teachers’ communication of mathematics problem solving, as influenced by their problem-solving belief structures?

Rationale

This study of Eastern Ontario pre-service IS mathematics teachers’ beliefs about mathematics problem solving was an important addition to the current literature for several reasons. First, the majority of research that has examined teachers’ beliefs about problem solving has occurred in a Middle Eastern context (Bal, 2015; Memnun et al., 2012; Sağlam & Dost, 2014; Xenofontos & Andrews, 2014; Yavuz & Erbay, 2014), which is culturally distinct from a Canadian context. Teachers in Middle Eastern and Canadian contexts operate under distinctly different cultural expectations for teaching mathematics, and it is well-established that beliefs are a culturally-informed domain of cognition (Pajares, 1992; Raths & McAninch, 2003). As such, it is not reasonable to draw clear inferences about Canadian mathematics teachers’ beliefs about problem solving based on the findings from research that has examined Middle Eastern mathematics teachers’ beliefs about problem solving. Thus, currently we do not have a robust foundation for how Canadian mathematics teachers are thinking about problem solving.
Another reason this study was an important addition to the literature is that the Ontario MOE has placed great emphasis on problem solving as an essential process in mathematics, yet there does not appear to be research into whether Ontario teachers are adopting a similar viewpoint. Additionally, although the Ontario mathematics curricula do provide numerous criteria that highlight the value of problem solving in mathematics, problem solving is not explicitly defined in any of the curriculum documents (MOE, 2005a, 2005b, 2007). Therefore, when teachers are planning instruction related to problem solving they must rely on their espoused beliefs about the essential meaning of problem solving. Considering our current lack of understanding for Ontario mathematics teachers’ beliefs about problem solving, we do not know if there is convergence or divergence between teachers’ beliefs and problem solving as it exists in the Ontario mathematics curricula. Consequently, we cannot be sure that problem solving is being implemented by IS mathematics teachers as mandated by the Ontario MOE, which is an issue that must be remedied by any curricular jurisdiction looking to effectively embed problem solving in mathematics (Wilson et al., 1993).

Finally, this study is personally important because it addresses an issue I experienced in my learning of secondary-school mathematics. Although I was an academically successful student in mathematics, my learning experiences related to problem solving typically treated it as a concept separate from mathematics, and problem-solving concepts were often covered passively en route to factual mathematics content. As a result, it was not until my undergraduate studies that I gained an appreciation for the importance of a robust problem-solving knowledge base, and I began actively addressing my lack of problem-solving knowledge. Later, when completing my
Bachelor of Education, I became interested in how my learning experiences related to problem solving in secondary-school mathematics could have been improved. However, I noticed a distinct gap in the literature when I examined the intersection of problem solving and beginning mathematics teachers, especially for newly certified mathematics teachers in Canada. If mathematics teachers are to be influential actors in students’ learning of problem solving, it is first necessary to understand how newly certified teachers are thinking about problem solving. Recognizing this gap in the literature was the impetus for me to begin the Master of Education program, with the intent of shedding light on how Ontario pre-service IS mathematics teachers are thinking about problem solving—in particular, their beliefs about mathematics problem solving.

**Overview of Thesis**

This thesis is organized into five chapters that describe the design, execution, and interpretation stages of the study. Chapter One—the current chapter—introduces the study by detailing the purpose and research questions, and outlining the rationale for conducting this study. Chapter Two presents a review of the relevant literature that situates this study in the research landscape, elucidates the basis for methodological choices, and provides a foundation for how the research questions were answered. Chapter Three outlines the methodological and method choices that informed the design of this study, the phases of data collection, and the analysis decisions. Additionally, a brief description is included for how personal biases were managed throughout the study. Chapter Four details the quantitative and qualitative findings of this study that developed from data analysis. Finally, Chapter Five discusses answers to the research questions while drawing connections to the problem-solving literature and to the Ontario
mathematics curricula, presents the limitations of this study, and concludes by offering implications for the preparation of Ontario pre-service IS mathematics teachers and suggested directions for future research.
Chapter 2

Literature Review

This study was built upon literature examining the characteristics of meaningful mathematics problem-solving learning experiences, pre-service teachers’ beliefs, teachers’ classroom practice, and an overview of the empirical work that has examined teachers’ beliefs about problem solving. These overarching areas provided an informative basis from which the research questions were fully explored and satisfied. The organizing structure of each theme includes an introduction connecting the theme to the purpose of the study followed by a review of relevant literature that informs how the research questions were approached and provides context for methodological choices. Finally, a summary is provided as a synthesis of the ideas presented in this section, elucidating how this study is situated within the literature.

Problem Solving

Educational literature has produced a notable array of definitions for problem solving in the contexts of mathematics (e.g., see Jonassen, 2004). A well-cited definition proffered by Metallidou (2009) defines problem solving as “a goal-directed behavior [which] requires an appropriate mental representation of the problem and the subsequent application of certain methods or strategies in order to move from an initial, current state to a desired, goal state” (p. 76). Metallidou’s definition suggests problem solving to be a form of cognitive processing; problem solving occurs within the mind of a student. The process of moving from fully conceptualizing a problem to evaluating the appropriateness of a solution holds potential for enhancing intellectual development.
(National Council of Teachers of Mathematics [NCTM], 2010). However, describing problem solving as a whole does little to explicate the complexities of how it enhances intellectual development or fits within teachers’ classroom practice. A more powerful examination that does explicate the complexities of problem solving includes the characteristics of problems, the characteristics of problem solving, and the manifestation of problem-solving instruction in mathematics contexts.

**Characteristics of problems.** Looking more closely at the definition for problem solving provided by Metallidou (2009), a problem can be understood as a situation possessing a solution, a solution that is obscured in its current state. A problem only becomes a *problem* when the means to reach a solution are not immediately obvious or available. In lieu of an immediate solution, students are required to reflect on their previous knowledge to determine what additional information is required and what relationships can be garnered among the problem components (Dewey, 1938). Moreover, variation exists regarding the degree to which any problem is *problematic*.

A simple, yet powerful model of understanding problem characteristics is to situate problems along a continuum from well-defined to ill-defined (Hollingworth & McLoughlin, 2005). Differences between well-defined and ill-defined problems are seen to arise from the data provided, the knowledge domain (i.e., documented knowledge of similar problems), the provision of relevant rules and principles, whether a solution process is available, and the structure of the final answer (see Table 1). Well-defined problems are those where the given state, goal state, and allowable operators are clearly specified, whereas ill-defined problems are those where the given state, goal state, and allowable operators are not clearly specified (Mayer & Wittrock, 2006). Whether a given
problem is well-defined or ill-defined is independent of the student; students with different levels of knowledge and ability will similarly identify a problem as well-defined or ill-defined.

Table 1

*Characteristics of Well-Defined and Ill-Defined Problems*

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Well-defined problem</th>
<th>Ill-defined problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>Complete</td>
<td>Incomplete or not given</td>
</tr>
<tr>
<td>Knowledge Domain</td>
<td>Well-defined</td>
<td>Ill-defined</td>
</tr>
<tr>
<td>Rules and principles</td>
<td>Limited rules and principles in organized arrangement</td>
<td>Uncertainty about concepts and principles necessary for solution</td>
</tr>
<tr>
<td>Solution process</td>
<td>Familiar; knowable, comprehensible method</td>
<td>Unfamiliar; no explicit means for action</td>
</tr>
<tr>
<td>Answer</td>
<td>Clear goal, convergent; possesses a correct answer</td>
<td>Uncertain, multiple or no solution; need to make judgements and evaluation</td>
</tr>
</tbody>
</table>


Ill-defined problems are less common in educational materials, as they pull from interdisciplinary concepts and require epistemic assumptions in place of highly automatized solving procedures (Hollingworth & McLoughlin, 2005; Schraw, Dunkle, & Bendixen, 1995). Hollingworth and McLoughlin describe ill-defined problems as desirable if students are to develop high-level problem-solving skills, as these problems “require students to interpret some of the problem elements…[consider] which rules or
principles are necessary for a solution...[and] think strategically, employ metacognitive skills, and defend his or her solution” (p. 68). In contrast, well-defined problems are those commonly encountered at the end of textbook chapters, typically highlighting only key concepts and requiring highly automatized solving procedures. Mayer and Wittrock (2006) contend well-defined problems to be much more common in educational materials, whereas most real-world problems are ill-defined. Successful solving of ill-defined problems requires additional skills, relying on a student’s propensity for cognitive regulation and attitudes toward the subject matter, which is supplementary to the domain knowledge and reasoning skills required by well-defined problems (Shin, Jonassen, & McGee, 2003). Despite ill-defined problems developing high-order thinking skills and commonly being stressed in the literature, well-defined problems also have benefits as they are necessary in developing well-structured domain knowledge (Hollingworth & McLoughlin, 2005). Consequently, both well-structured problems and ill-structured problems hold potential for enhancing students’ development.

Another useful system of characterizing problems, which adds to the well-defined—ill-defined classification (Hollingworth & McLoughlin, 2005), is to position problems on a continuum from routine to nonroutine (Jonassen, 2011; Mayer & Wittrock, 2006). Routine problems require students to use well-established algorithms (i.e., solving procedures) to find solutions similar to those from questions practiced during instructional periods. Problems classified as routine often form the basis of many educational lessons, present students with familiar data, and require minimal creative thinking throughout solving processes. Despite the emphasis often placed on routine problems by mathematics teachers, few real-world problems reflect simple algorithmic
solving procedures (Mayer & Wittrock, 2006). Conversely, nonroutine problems require students to use complex solving processes combining both creative and critical thinking, rarely relying on algorithmic solving procedures. Nonroutine problems are often interdisciplinary in structure (i.e., drawing content from multiple subjects) and require solving processes akin to those encountered in real-world problems (Mayer & Wittrock, 2006). The extent to which a problem is classified as routine or nonroutine depends primarily on the knowledge of the student. While a problem might pose a genuine challenge to students at a particular grade level and be classified as nonroutine, students at a higher grade level might find the problem routine and merely a component of the more complex problems being solved. If students are to develop competency in problem solving, both routine and nonroutine problems are necessary instructional tools (Santos-Trigo, 2014)

Integrating the Hollingworth and McLoughlin (2005) and Mayer and Wittrock (2006) problem classification systems provides a model for understanding the various types of problems that could potentially be addressed in problem-solving tasks (see Figure 1). Similar to a Cartesian coordinate system, the merged model for classifying problems is comprised of four distinct quadrants, each corresponding to a different type of problem. In the ill-defined, nonroutine quadrant the problems present an uncertain given state, goal state, and allowable operators, and incorporate content that is mostly unfamiliar to the student. For example, assigning students a task where they must draw upon content from multiple subjects (e.g., a science discipline together with mathematics) to conceive a solution to a current societal issue. In the ill-defined, routine quadrant the problems again present an uncertain given state, goal state, and allowable operators, yet
the mathematics content is comparatively familiar. For example, a collaborative design project where students must employ familiar mathematics to address a real-world problem identified by the students.

In the well-defined, non-routine quadrant the problems present a clear given state, goal state, and allowable operators, yet the mathematics is unfamiliar to the students. For example, asking students to plot and describe the characteristics of a quadratic function when their prior experience has solely been with linear functions. Finally, in the well-defined, routine quadrant the problems present a clear given state, goal state, and allowable operators; and the content is mostly familiar to the students. For example, asking students to solve a simple algebraic equation involving mathematics covered in detail during previous instructional periods.

**Figure 1.** Integration of the Hollingworth and McLoughlin (2005) well-defined—ill-defined continuum and the Mayer and Wittrock (2006) routine—nonroutine continuum for problem classification.

**Characteristics of problem solving.** Problem solving embodies the process of transitioning from an initial, unclear problem state to an acceptable solution state through appropriate methods and strategies (Metallidou, 2009). Wilson, Fernandez, & Hadaway (1993) note that the majority of problem-solving research concerning secondary school
students emanates from the seminal work of Polya (1945). In his book, *How to Solve It*, Polya suggests that successful problem-solving experiences ought to follow four phases. First, students must *understand* the problem, recognizing both what is needed and what is already given. Second, students must *plan*, taking care to note connections between aspects of the problem and to their prior knowledge; this stage involves generating a conception of how the solution might take form. Third, following their developed plan, students must *carry out* the solving procedure(s). Lastly, students must *look back* at their solution, reviewing and discussing whether the solution aptly answers the problem, and consolidating any new-found understandings with prior knowledge.

Since Polya (1945) introduced his problem-solving framework, various scholars have added to and altered his framework (e.g., see Resnick & Glaser, 1976), yet all tend to follow a similar structure of component cognitive processes, including: representing, planning/monitoring, executing, and self-regulating (Mayer & Wittrock, 2006). These processes are largely indistinguishable from those proffered by Polya; what has emerged is the appreciation that self-regulation is continuously applied throughout the solving process, rather than solely once the solution is reached. Despite the typical instructional emphasis on the execution stage, literature has consistently shown students to struggle most with representing, planning and monitoring, and self-regulating (Mayer & Wittrock, 2006). To better understand the reason for this discrepancy between instructional emphasis and student learning, Mayer and Wittrock (2006) created a general framework to describe the action of problem-solving.

The framework constructed by Mayer and Wittrock (2006) suggests problem solving to be an action composed of four main characteristics. First, problem solving is a
cognitive endeavour, where the solving of a problem is localized to the cognitive system of a student and only observable indirectly through behaviour measurement. Second, problem solving is a process, as the solving of a problem occurs through an iterative process of mental representation and knowledge manipulation within a problem space in the student’s cognitive system. Third, problem solving represents directed behaviour, as a student’s cognitive processing within the problem space is guided by the student’s goals. Fourth, problem solving is dependent on qualities of the student (e.g., knowledge of mathematics content and heuristic strategies), insofar that the perceived difficulty of a problem will be contingent on the student’s knowledge and skillset. Collectively, these four characteristics provide a general sense of how to think about the action of problem solving.

Buttressing the framework given by Mayer and Wittrock (2006), Schoenfeld (1985, 2013) provided a framework from which to understand the success or failure of a student’s attempts at solving a problem, aligning his framework with the student’s problem-solving behaviours. The framework comprises four categories: the student’s knowledge and resources, the student’s use of heuristic strategies in the solving process, the student’s metacognitive ability to self-monitor and self-regulate throughout the solving process, and the student’s belief systems—specifically those beliefs that have been forged through experience (Schoenfeld, 2013). These four categories alone are necessary and sufficient to determine why any individual problem-solving experience was or was not successful: necessary in that examining all categories is required, and sufficient in that no additional categories are required (Schoenfeld, 2013). The value in the framework proposed by Schoenfeld (1985), which distinguishes his framework from
the Mayer and Wittrock (2006) framework, is that it provides an ability to perform post-hoc analysis on individual problem-solving experiences. Furthermore, Schoenfeld (2013) contends that his framework should apply to all problem-solving disciplines, regardless of content being qualitative or quantitative.

Finally, in defining the general characteristics of problem solving, Mayer and Wittrock (2006) specify commonly misattributed terminology, such as thinking or reasoning. Thinking, unlike problem solving, does not require a directedness condition, for example, contemplating the place of philosophy in mathematics; reasoning, unlike problem solving, emphasizes deduction through logic rather than process-based solving procedures, for example, using simple deduction to assert that uncountable infinities are larger than countable infinities. In place of terms such as thinking or reasoning, Mayer and Wittrock suggest creative thinking and critical thinking to be useful terms that better specify elements of problem solving. Although the use of thinking appears contradictory to their argument against this terminology, by preceding thinking with creative or critical, Mayer and Wittrock are retaining the directedness condition and process-based solving procedures that are necessary elements of problem solving. Specifically, creative thinking represents the component of problem-solving processes aimed at generating ideas that may lead to the solving of a problem (retains directedness condition), and critical thinking represents the evaluation of these ideas (retains process-based solving procedures).

**Problem solving in mathematics teachers’ classroom practice.** Meaningful problem-solving experiences are those with “the potential to provide intellectual challenges that can enhance students’ mathematical development” (NCTM, 2010, p. 1),
and it is the teacher who is ultimately responsible for creating the context of such problem-solving experiences (Wilson et al., 1993). Therefore, if problem solving is to be meaningfully crafted as a component of mathematics, it is necessary to deduce how the characteristics of problems and problem solving translate into teachers’ classroom practice.

The NCTM (2010) defines four problem criteria that are essential to meaningful problem-solving experiences: problems incorporating mathematics that is both academically and personally important for students; problems requiring higher-level thinking, such as self-monitoring and self-regulating; problems promoting conceptual development; and problems providing an opportunity to assess where learning is stagnating. Further problem criteria can be combined to bolster particular learning experiences (e.g., problems possessing various solutions, problems addressing multiple concepts within a curriculum, and problems addressing concepts across subject areas), although all problem-solving experiences should be built upon a foundation of the previous four problem-criteria (NCTM, 2010).

In addition to the four essential problem criteria, teachers need to “develop a problem-solving culture in the classroom” (NCTM, 2010, p. 4). If teachers hope to engage students in meaningful problem solving, they must make problem-solving experiences a regular and consistent aspect of their classroom practice. Such a culture of problem solving primarily manifests through classroom discourse, both teacher-student and among students. Finally, problem solving taught separately from subject concepts and procedures is inconsistent with research evidence on effective problem-solving instruction mounted over the last 30 years (NCTM, 2010). Problem solving must be made
an integral aspect of instruction, directly connected with subject concepts and procedures. In developing this connection, teachers must be conscious to model effective problem-solving processes, yet not take over when students encounter minor struggles in the solving process. Teachers’ manner of interaction with students during problem-solving learning experiences largely emanates from the teachers’ problem-solving belief structures; as such, there is a need to ensure these belief structures are consistent with the literature on effective problem solving (Wilson et al., 1993) and the relevant teaching resources (e.g., mathematics curricula). It is valuable then to consider teachers’ beliefs about problem solving and the corresponding belief structures.

**Pre-Service Teachers’ Beliefs**

The last two decades have seen teachers’ beliefs become an increasingly appreciated construct in understanding teachers’ classroom practice (Mason, 2003). The importance of understanding teachers’ beliefs lies in their capacity to “guide instructional decisions, influence classroom management, and serve as a lens of understanding for classroom events” (Luft & Roehrig, 2007, p. 38). Beliefs and knowledge are often accompanying constructs in attempts to understand the thinking processes underlying teachers’ classroom practice, yet studies have shown that teachers with similar knowledge often teach differently (e.g., see Ernest, 1989). Moreover, various constructs used to understand teachers’ classroom practice can be reduced to belief structures (Pajares, 1992), such as attitudes, values, opinions, perceptions, conceptions, and perspectives. Given the encompassing quality of beliefs, any attempt to grasp the nature of teachers’ pedagogical decisions needs to place teachers’ beliefs at the heart of the investigation. Studies examining pre-service teachers’ beliefs are particularly valuable, as
pre-service education is a critical period during which teachers’ beliefs can be actively monitored and, if effectively targeted, positively influenced (Luft & Roehrig, 2007).

**Defining beliefs.** Systematically reviewing the literature for pre-service teachers’ beliefs, Raths and McAninch (2003) compiled a comprehensive resource on teachers’ beliefs and classroom practice. Early in their edited book, pre-service teachers’ beliefs are defined as “psychologically held understandings, premises, or propositions about the world that are felt to be true” (Richardson, 2003, p. 2). Beliefs are viewed as distinct from other forms of cognition; in particular, teachers’ beliefs are seen as distinct from teachers’ knowledge. This distinction is proposed to emanate from knowledge requiring a form of supporting evidence, or *truth condition*, whereas beliefs can be founded on interpretations and speculation (Richardson, 2003). This separation is primarily understood as a philosophical viewpoint, as the psychological viewpoint does not treat these two constructs separately. However, for the sake of consistency among definitions of knowledge, and to allow for these constructs to be deeply analyzed, the philosophical definition tends to hold greater empirical value (Richardson, 2003).

Similar to Raths and McAninch (2003), Pajares (1992) synthesized a wealth of literature on teachers’ beliefs in an effort to better define the construct and allow it to be studied. Resulting from his efforts, Pajares presented 16 inferences and generalizations about teachers’ beliefs extensively grounded in the literature (see Table 2). These offerings should not be interpreted as incontrovertible truths, but rather as fundamental assumptions that should inform any study examining teachers’ educational beliefs (Pajares, 1992).
Table 2

*Fundamental Assumptions of Teachers’ Educational Beliefs*

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Beliefs form early and, even when presented with contradictory evidence, are not easily changed.</td>
</tr>
<tr>
<td>2</td>
<td>Through a process of cultural transmission, individual beliefs coalesce to form systems of beliefs.</td>
</tr>
<tr>
<td>3</td>
<td>Beliefs serve to help individuals understand and define experiences and themselves.</td>
</tr>
<tr>
<td>4</td>
<td>An inextricable linkage binds knowledge and beliefs, yet beliefs are the filter through which experiences are processed.</td>
</tr>
<tr>
<td>5</td>
<td>Thinking and information processing are guided by beliefs.</td>
</tr>
<tr>
<td>6</td>
<td>Epistemological beliefs have influence on knowledge interpretations and cognitive monitoring.</td>
</tr>
<tr>
<td>7</td>
<td>The structure of beliefs within a system of beliefs suggests a hierarchy.</td>
</tr>
<tr>
<td>8</td>
<td>Understanding particular beliefs necessitates the understanding of connections within the belief system.</td>
</tr>
<tr>
<td>9</td>
<td>Some beliefs are more vulnerable to change than others.</td>
</tr>
<tr>
<td>10</td>
<td>The age of a belief and its vulnerability to change appear negatively correlated.</td>
</tr>
<tr>
<td>11</td>
<td>Beliefs are unlikely to be altered in adulthood, even when presented with contradictory evidence.</td>
</tr>
<tr>
<td>12</td>
<td>Beliefs influence the planning and organizing of tasks.</td>
</tr>
<tr>
<td>13</td>
<td>Beliefs strongly influence perceptions.</td>
</tr>
<tr>
<td>14</td>
<td>Beliefs strongly affect behaviour.</td>
</tr>
<tr>
<td>15</td>
<td>Beliefs can only be inferred, not directly measured.</td>
</tr>
<tr>
<td>16</td>
<td>Beliefs about teaching are well established prior to adulthood.</td>
</tr>
</tbody>
</table>
In the context of the proposed study, the most salient assumption from Pajares (1992) was that “the beliefs teachers hold influence their perceptions and judgements, which, in turn, affect their behaviour in the classroom” (p. 307). Therefore, understanding how teachers teach problem solving first necessitates understanding their beliefs about problem solving. Adding to this claim, factors external to the classroom and teacher are also relevant when examining how teachers’ beliefs influence their classroom practice, for example, professional development and induction programs (Luft & Roehrig, 2007).

Another contribution from Pajares was to suggest that many of the constructs in educational research are beliefs in disguise (e.g., attitudes, values, opinions, perceptions, conceptions, and perspectives). The encompassing nature of beliefs reveals their fundamental importance in understanding teachers’ classroom practice. However, beliefs are a difficult construct to measure, as beliefs can only be inferred from teachers’ speech and actions (Pajares, 1992). Consequently, techniques that are particularly effective at eliciting teachers’ beliefs are interviews, ranking tasks, and constructed response questions (Luft & Roehrig, 2007).

Another important piece of literature is an extensive summary of research on teachers’ beliefs and how teachers’ beliefs influence their practice, compiled by Philipp (2007). While much of Philipp’s work reiterated and reinforced the concepts provided by Raths and McAninch (2003) and Pajares (1992), an extension was made based on how to reconcile apparent inconsistencies between teachers’ beliefs and practice. In researching how different categories of beliefs affect teachers’ practice, many researchers have concluded that once teachers’ thinking was better understood most recorded inconsistencies disappeared (Philipp, 2007). In fact, recorded inconsistencies likely rest
within researchers’ minds, not necessarily within teachers’ minds (Philipp, 2007). As such, when attempting to analyze teachers’ beliefs, researchers must be aware of their preconceptions regarding the relationship between beliefs and affect.

**Beliefs instruments.** In the contexts of mathematics and science education, a beliefs instrument found to be particularly valuable is the Beliefs about Mathematical Problem Solving Questionnaire (BMPSQ) developed by Kloosterman and Stage (1992). A synthesis of the Indiana Mathematics Beliefs Scales (Kloosterman & Stage, 1992) and the Usefulness of Mathematics Scale (Fennema & Sherman, 1976), the validated BMPSQ consists of 36 five-point Likert-type questions equally distributed among six scales. Two scales correspond to beliefs about mathematics as a discipline: *there are word problems that cannot be solved with simple, step-by-step procedures* and *word problems are important in mathematics*; three scales correspond to beliefs about the learning of mathematics: *I can solve time-consuming mathematics problems*, *understanding concepts is important in mathematics*, and *effort can increase mathematical ability*; and one scale is extracted directly from the Fennema and Sherman (1976) scale: *mathematics is useful in daily life*. Excluding the effort-related scale, which possesses six positively worded items, each of the scales possess three items with positive wording and three items with negative wording. Positively worded items are scored with “strongly agree” as the highest-scored response, while negatively worded items are scored with “strongly disagree” as the highest-scored response—both awarding five points as the highest score and one point as the lowest score. Interpretations of the response data are developed by summing the scores for all Likert items in a specific scale.
The BMPSQ functions as a measurement tool for determining “students’ beliefs about mathematics as a subject and about how mathematics is learned” (Kloosterman & Stage, 1992, p. 109). While the instrument was originally designed for students learning mathematics, it also holds value in measuring teachers’ beliefs about problem solving, as demonstrated by Haciömeroğlu (2011). Furthermore, the efficacy of the BSMPQ has been established for a variety of mathematics teaching and learning contexts, including both elementary and secondary mathematics, and both pre-service and in-service teachers (Bal, 2015; Memnun, Hart, & Akkaya, 2012; Sağlam & Dost, 2014; Xenofontos & Andrews, 2014; Yavuz & Erbay, 2014). This validated instrument is proposed to provide a less time-consuming alternative to interview-based and observation-based beliefs measurements, yet interviews and observations still hold the prospect for greater depth and a lesser influence of social desirability bias in response data (Kloosterman & Stage, 1992).

An instrument that provides depth to responses garnered using the BMPSQ (Kloosterman & Stage, 1992) is the interview-based beliefs instrument developed by Luft and Roehrig (2007)—the Teacher Beliefs Interview (TBI). The TBI was originally constructed to elucidate how teacher education experiences impact teachers’ epistemological beliefs, specifically with respect to instruction and assessment. Following a semi-structured design, the TBI is a seven-item interview protocol that examines teachers’ beliefs in seven areas (see Table 3). It is important to note that although Luft and Roehrig developed the TBI using belief data from science teachers, the TBI questions were pertinent for this study of pre-service mathematics teachers’ beliefs about problem solving. The necessary adjustments to the TBI were to incorporate the
term problem solving for general epistemology questions and to replace science with problem solving in question three (see Appendices D & E).

Responses to the TBI questions can be classified as teacher-centered, student-centered, or transitional (i.e., learning experiences beginning to appreciate learning as a subjective student-teacher collaboration). Teacher-centered responses can be partitioned into traditional responses (i.e., learning experiences based on transfer of facts, rules, and methods from teacher to student) and instructive responses (i.e., learning experiences that are decided upon and exemplified by the teacher). Similarly, student-centered responses can be partitioned into responsive responses (i.e., learning experiences based on collaboration and exchange between student and teacher) and reform-based responses (i.e., learning experiences that are guided by students’ understanding and interest). The TBI has been demonstrated as effective at revealing both pre-service and in-service teachers’ epistemological beliefs of teaching and learning (Fletcher & Luft, 2011; Luft & Roehrig, 2007), including teachers with mathematics as a teaching subject (Adams, 2010). Furthermore, the instrument is constructed to provide a comprehensive account of a teacher’s beliefs by examining seven aspects of teaching and learning through beliefs related to: the learning environment, the constructing of student knowledge, the structuring of learning experiences, what should be taught, assessment, student understanding, and attentiveness to student learning.
**Table 3**

*TBI Questions and Associated Belief Area Addressed*

<table>
<thead>
<tr>
<th>Question</th>
<th>Format</th>
<th>Area Addressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 How do (will) you maximize student learning in your classroom?</td>
<td>Environment</td>
<td></td>
</tr>
<tr>
<td>2 How do you describe your role as a teacher?</td>
<td>Student knowledge</td>
<td></td>
</tr>
<tr>
<td>3 How do your students learn science best?</td>
<td>Learning</td>
<td></td>
</tr>
<tr>
<td>4 In the public school setting, how do you decide what to teach and what not to teach?</td>
<td>Student and standards</td>
<td></td>
</tr>
<tr>
<td>5 How do you decide to move on to a new topic?</td>
<td>Assessment</td>
<td></td>
</tr>
<tr>
<td>6 How do you know when students understand?</td>
<td>Understanding</td>
<td></td>
</tr>
<tr>
<td>7 How do you know learning is occurring in the classroom?</td>
<td>Student response</td>
<td></td>
</tr>
</tbody>
</table>


**Teachers’ Classroom Practice**

Professional practice is understood as “appropriate pedagogy, assessment and evaluation, resources and technology in planning for and responding to the needs of individual students and learning communities” (Ontario College of Teachers, p. 13). For the purposes of this study, classroom practice refers to the component of professional practice that deals with teachers’ pedagogical decisions made during implementation of the mathematics curriculum documents. As such, it is necessary to consider both how
curricula are implemented as a component of teachers’ classroom practice and the specific content of the Ontario mathematics curricula relevant to this study.

**Classroom practice and curriculum implementation.** With respect to teachers’ classroom practice, the primary objects of interest in this study were the Ontario mathematics curriculum documents (MOE, 2005a, 2005b, 2007). Given that curriculum documents are a form of educational policy, it is necessary to recognize teachers as actors in the implementation of educational policy. The implementation stage of educational policy is commonly the most challenging, primarily because “implementation of policy occurs in a highly complex social environment with official policy agendas seldom intersecting with local interests” (Delaney, 2002, p. 57), suggesting that policy implementation is contingent upon individual actor’s incentives, beliefs, and capacity. As such, in an effort to fit personal interpretations to a dynamic social climate, teachers inevitably implement curriculum differently based on their cognitive structures.

Teachers’ decisions in policy implementation evolve from two components: how teachers interpret policy and how teachers translate policy (Ball, Maguire, Braun, & Hoskins, 2011). In the context of curriculum, interpretations involve a decoding process wherein teachers attempt to connect smaller curriculum components (e.g., specific concepts, mathematical processes emphasized by the Ontario MOE, and assessment and evaluation criteria stressed by the MOE) to a bigger picture (i.e., students becoming effective learners in mathematics). This process of interpretation embodies the creation of a teacher’s intended curriculum. On the other hand, translation involves the process by which interpretations are transformed into the enacted curriculum, recoding policy through the creation of “materials, practices, concepts, procedures and orientations” (Ball
et al., 2011, p. 620). Transitioning from intended curriculum to enacted curriculum manifests differently for each teacher, with the subversion or acceptance of curriculum content providing clues to teachers’ cognitive structures.

How teachers enact the curriculum can be classified according to four overlapping categories: curriculum use as following or subverting the text, curriculum use as drawing on the text, curriculum use as interpretation of text, and curriculum use as participation with the text (Remillard, 2005). The type of enactment is contingent on the particular teacher, the curriculum being implemented, and the context surrounding the implementation process. Therefore, prior to considering teachers’ beliefs, there is a need to examine the curricular mandates for how Ontario pre-service IS mathematics are expected to teach problem solving.

**Problem solving in Ontario mathematics curricula.** In the contexts of the Ontario mathematics curricula, problem solving is acknowledged as an essential process in the learning of mathematics (MOE, 2005a, 2005b, 2007). In fact, of the seven essential processes identified in the Ontario mathematics curricula, problem solving is recognized as the principal process and as “the motor that drives the development of the other processes” (MOE, 2007, p. 17). Although minor variation exists between the problem-solving content in each of the Ontario mathematics curriculum documents, the principles offered for teachers’ consideration of problem solving are largely consistent: (a) it is the connection of mathematics to the real world, (b) it bolsters students’ confidence with mathematics, (c) it develops students’ conceptual and procedural understandings of mathematics, (d) it provides opportunities for students to enhance their communication with mathematics, (e) it offers diverse opportunities to assess students’ understanding of
mathematics, (f) it promotes idea sharing and collaboration, (g) it supports students in developing an appreciation for mathematics, and (h) it presents opportunities to enact and strengthen critical-thinking skills (MOE, 2005a, 2005b, 2007).

Despite the various criteria presented to describe problem solving, none of the Ontario mathematics curricula provide a specific definition of problem solving. Additionally, although the curricula use the terminology of routine and nonroutine problems, the routine-nonroutine problem continuum is not described or referenced. Inexplicably, the curricula also include *instructional problems* in the routine-nonroutine classification, but a description of how instructional problems are a unique classification is not provided. The curricula do, however, provide detail about the selection of problem-solving strategies, showing a conceptualization of problem solving that emphasizes the use of general and specific problem-solving procedures (e.g., drawing a picture, guessing and checking, and working backwards). Finally, the Ontario Curriculum: Grades 1-8 (MOE, 2005a) includes a detailed description of Polya’s (1945) problem-solving framework, yet the Ontario curricula for secondary school mathematics (MOE, 2005b, 2007) do not similarly include a well-established problem-solving framework.

**Overview of Empirical Studies for Teachers’ Beliefs About Problem Solving**

Only a handful of empirical studies have explored pre-service teachers’ beliefs about problem solving in mathematics contexts (e.g., see Bal, 2015; Memnun et al., 2012; Sağlam & Dost, 2014; Xenofontos & Andrews, 2014; Yavuz & Erbay, 2014), and those that have were primarily within a Middle Eastern setting (ibid.). Of the studies that have examined pre-service teachers’ beliefs about problem solving, the BMPSQ has been utilized as the primary method for collecting quantitative belief data (Memnun et al.,
2012; Sağlam & Dost, 2014; Yavuz & Erbay, 2014), semi-structured interviews have been the primary method for collecting qualitative belief data (Bal, 2015; Xenofontos & Andrews, 2014), and very few studies have combined quantitative and qualitative methods for collecting belief data (e.g., see Bal, 2015). In a Canadian context—or even more specifically, an Ontario context—there does not appear to have been any research examining pre-service teachers’ beliefs about problem solving, so any implications from the literature are grounded in alternate contexts.

Nearly every study that has used the BMPSQ to understand pre-service teachers’ beliefs about problem solving has occurred in Turkey, using the translated version of the instrument produced by Hacıömeroğlu (2011). The BMPSQ studies have documented various findings, including: a conflicting range of scores for the BMPSQ scales (Memnun et al., 2012; Yavuz & Erbay, 2014), contradictory evidence for the influence of gender and teaching subjects on beliefs (Memnun et al., 2012; Sağlam & Dost, 2014), and the need for a qualitative lens to further explicate beliefs associated with the scales (Yavuz & Erbay, 2014). Despite the variation in findings for BMPSQ studies, an inference consistently reported by researchers has been the need for further research in different contexts to help expand upon current findings (Memnun et al., 2012; Yavuz & Erbay, 2014).

Contrasting the varied findings for studies using the BMPSQ, findings for studies that have utilized semi-structured interviews have been mostly consistent for understanding pre-service teachers’ beliefs about problem solving. The core finding from interview-based studies has been that pre-service teachers’ espoused beliefs are largely influenced by the teaching and learning context (Bal, 2015; Xenofontos & Andrews,
For example, a comparison of the beliefs about problem solving held by pre-service teachers in England and Cyprus found that while there was a shared belief of problem solving as applying mathematics to the real world, there were diverging beliefs about the structure of problem solving (Xenofontos & Andrews, 2014). The authors explained that the difference in beliefs for English and Cypriot teachers was consistent with the notion that beliefs are culturally situated, as each group of pre-service teachers’ beliefs were consistent with problem solving as it exists in their curricular systems.

Overall, based on the empirical work that has examined pre-service teachers’ beliefs about problem solving, several gaps in the literature are evident: (a) the need for more empirical work that examines beliefs through the combination of quantitative and qualitative lenses; (b) the need for research in contexts that are distinctly different from the Middle Eastern context where most research has been conducted, as beliefs are influenced by context; and (c) the need for research that examines the influence of various factors on beliefs, such as gender and teaching subjects, as current research has not definitively exposed the influence of these factors.

**Summary**

The literature used in constructing this study was essential in providing an informative basis from which the complexities of the research questions could be considered. The central themes underlying the literature review are the characteristics of meaningful problem-solving learning experiences, pre-service teachers’ beliefs, teachers’ classroom practice, and the empirical work that has examined teachers’ beliefs about problem solving. First, meaningful problem-solving experiences possess several key characteristics supported by established teaching techniques. As such, regardless of what
teachers believe to be effective problem solving, research can provide a guideline for evaluating the efficacy of problem-solving experiences. Furthermore, if problem-solving experiences are to be efficacious, problem solving must intimately be entwined with subject concepts, breaking free from common approaches that teach problem solving solely heuristically. The deep ingraining of problem solving with mathematics concepts requires the building of a problem-solving culture in the classroom.

Second, pre-service teachers’ beliefs influence their classroom practice. Although the context underlying teachers’ enactment of the curriculum is important to consider, beliefs clearly have an observable and prominent impact on teachers’ pedagogical decisions. Moreover, beliefs can be understood as a fundamental domain of cognition; understanding their influence can provide key insight into teachers’ pedagogical decisions. Obtaining this insight requires the use of carefully planned methods and a robust understanding of belief structures.

Third, teachers’ classroom practice is largely dependent on the highly complex social climate of the school environment. In an effort to interpret and translate curriculum policy, teachers inevitably either accept or subvert particular aspects of the curriculum, and thus it is not enough to simply expect that curricula will be implemented as outlined. Regarding the Ontario mathematics curricula, criteria for the value of problem solving in mathematics are clearly articulated, yet an explicit definition of problem solving is not provided. Additionally, the Ontario mathematics curricula treat problem solving as a strategy-based mathematical process and recommend general and specific strategies that are useful for developing students problem-solving knowledge (e.g., the framework proffered by Polya [1945]).
Finally, there is a dearth of empirical studies that have examined Ontario pre-service mathematics teachers’ beliefs about problem solving. In fact, there does not seem to be any research that has examined Canadian mathematics teachers’ beliefs about problem solving. Moreover, the empirical studies that have examined teachers’ beliefs about problem solving have noted the need to extend findings beyond the primarily Middle Eastern context, and the need to examine teachers’ beliefs about problem solving using the combination of quantitative and qualitative lenses.
Chapter 3

Methodology and Methods

The objectives of this mixed methods study were: (a) to interpret the ontological and epistemological beliefs about mathematics problem solving held by Eastern Ontario pre-service intermediate-senior (IS) mathematics teachers, and (b) to determine the features of problem solving that are emphasized in the pre-service teachers’ communication of mathematics problem solving, as influenced by their problem-solving belief structures. Fulfilling these objectives required carefully structuring the research design, sampling procedures, methods for dealing with ethical considerations, and data collection and analysis decisions. In what follows, each of these aspects is documented in detail to fully describe the organizing and conducting of this study.

Research Design

This mixed methods study followed a pragmatic approach (Morgan, 2007), which “sidesteps the contentious issues of truth and reality, accepts, philosophically, that there are singular and multiple realities that are open to empirical inquiry and orients itself toward solving practical problems in the ‘real world’” (Feilzer, 2009, p. 3). The first implication of a pragmatic approach was treating data and theory as interconnected, which meant using language recognized in the problem-solving literature to articulate qualitative findings (Shannon-Baker, 2015). The second implication of a pragmatic approach was treating the relationship between the researcher and the research process as intersubjective. Contrasting the notion of researchers working in strictly objective or subjective frames of reference, an intersubjective lens recognizes that researchers drift
between these mindsets developing a shared meaning of quantitative and qualitative understandings (Morgan, 2007). The final implication of the pragmatic approach was treating inferences developing from the data as transferable rather than purely bound in the research context or as generalizable. Thus, findings from the data are treated as transferable and usable in settings similar to the research setting.

A sequential design underpinned each phase of data collection (Creswell 2014), allowing insights that developed from preliminary statistical analysis of quantitative data to be refined and extended through subsequent collection of qualitative data (Ivankova, Creswell, & Stick, 2006). This sequencing of quantitative and qualitative data was present in each of the two phases of study. Specifically, in each phase, collection of survey data from all participants was followed by in-depth semi-structured interviews with a selected subsample of participants (the same subsample in each phase). Data analysis, however, did not follow a sequential design and instead was framed by a convergent parallel design (Creswell, 2014). The convergent design meant moving through data reduction (i.e., reducing raw data to manageable parts) and transformation (i.e., representing data in a meaningful way) separately for quantitative and qualitative data sets before ultimately merging the two data sets for comparison and integration (Li, Marquart, & Zercher, 2000). A convergent design enhances the quality of the final inferences by allowing the inherent weaknesses of each data source to be compensated for by the strengths of the accompanying data source (Li et al., 2000).

Given the complexity of belief structures (Pajares, 1992), it was determined the sequential design for data collection and convergent parallel design for data analysis would be optimal for addressing the objectives of this study. Moreover, as mixed
methods designs are valuable for studying new questions and initiatives (Creswell, Klassen, Plano Clark, & Smith, 2011), these design choices were pragmatic given the lack of research concerning Ontario pre-service teachers’ beliefs about problem solving and the features of problem solving emphasized by Ontario pre-service teachers.

**Sampling Procedures**

Participants in this study were pre-service IS mathematics teachers enrolled in a Bachelor of Education (B.Ed.) program at an Eastern Ontario university. At the university chosen for study, these participants were the entire population of pre-service IS mathematics teachers. The pre-service teachers at this university are the second cohort in the enhanced B.Ed. program (i.e., the newly structured, extended-year teacher preparation program mandated by the Ontario College of Teachers). Pre-service teachers were chosen as the population of study for several reasons, including: (a) alignment with the recent focus on improving pre-service teacher education in Ontario—exhibited most prominently by the enhanced B.Ed. program, (b) recognition that the development of pre-service teachers is well-established as the crux of improving students’ educational experiences (Barlow & Reddish, 2006; Sheridan, 2016), and (c) the likelihood that focusing on pre-service teachers’ beliefs about problem solving will ultimately have a greater impact on students’ learning of problem solving than a focus on in-service teachers’ beliefs (Luft & Roehrig, 2007; Pajares, 1992).

In total, 45 participants completed the BMPSQ in each phase of the study, yet one participant was unique to each phase (i.e., 44 repeated-measure participants). Data from the participants who were unique to each phase were removed prior to analysis to ensure accuracy in reliability and validity comparisons across phases. The gender and teaching
subject distribution of the remaining 44 participants was as follows: 20 females and eight males with a second teaching subject in a science discipline (i.e., biology, chemistry, physics, geography, or computer science), and 14 females and two males with a second teaching subject in a non-science discipline (i.e., French, English, history, music, or visual arts). In alignment with the research questions, and due to the low number of males with a second teaching subject in a non-science subject, the categories in each phase were collapsed to simple female and male, and science and non-science groupings.

Interviews were conducted with a subsample of four pre-service teachers from the 44 repeated-measure participants who indicated their interest in follow-up interviews when completing the initial BMPSQ during phase one. Interview participants included two males who possessed science teaching subjects and two females who possessed non-science teaching subjects. The imbalance of gender and teaching subject (i.e., males with a science teaching subject and females with a non-science teaching subject) arose because these BMPSQ participants were the only pre-service teachers who agreed to participate in the interviews. Interview participants participated in both phases of the study. When reporting results from interview analysis, male participants are referenced using the pseudonyms Isaac and Albert, and female participants are referenced using the pseudonyms Marie and Judith.

**Dealing with Ethical Considerations**

As a consequence of this study being part of a larger three-phase research project managed by the pre-service IS mathematics teachers’ Mathematics Education instructor, it was necessary to carefully address ethical concerns when collecting data. The BMPSQ and subsequent interviews were implemented by the course instructor as components of
the Mathematics Education course. The instructor having access to these data during the 2016-2017 academic year would have created a power imbalance, so I acted as a research assistant and collected all data for these course components. I presented the BMPSQ and follow-up interviews to the pre-service IS mathematics teachers during an in-person information session in the first Mathematics Education class in September 2016, during which the instructor was not present. While detailing these course components I distributed a Letter of Information and Consent Form requesting access to pre-service mathematics teachers’ data for use in my master’s thesis. As the BMPSQ and interview data were collected by the instructor at two times in the fall term of the 2016-2017 academic year, access to both data sets was requested. Permission was sought as a researcher interested in the data being collected by the course instructor—a sincere explanation. The instructor was not given full access to the data until he was no longer in an evaluative role, and even when access was provided, all BMPSQ data and interview data were anonymized and de-identified, respectively.

It was further necessary to protect pre-service teachers’ identity throughout the process of data analysis. With regard to the BMPSQ analysis, data were anonymized prior to seeking discussion or feedback from my supervisor. Thus, it was never possible to connect raw or analyzed BMPSQ data to any study participants. Conversely, I never presented raw interview data (in the form of full transcripts) to my supervisor. Feedback was only sought on the emergent themes and categorical codes. Interview excerpts did comprise these qualitative elements, yet all data was de-identified immediately following interviews, stripping the data of all participant identifiers.
**Data Collection**

**Types of data collected.** This study collected data in the form of a Likert-type questionnaire; in-depth, semi-structured, one-on-one interviews; and curriculum documents constructed by the Ontario MOE. The BMPSQ (Kloosterman & Stage, 1992) is a Likert-type questionnaire containing six scales. These scales represent six beliefs about mathematical problem solving (see Table 4), with each scale possessing three positive and three negative Likert items. In the original construction of this validated questionnaire by Kloosterman and Stage (1992), reliabilities in the form of Cronbach’s alpha were calculated for all scales except the Usefulness of Mathematics Scale (Fennema & Sherman, 1976), although the authors’ reasoning behind this exclusion was not communicated in the literature. Low reliability scores for the Word Problems scale were suggested to result from confusion regarding what constitutes a *word problem*, and the authors recommended using this scale only when the term word problem has been explained or when word problems are prominent in the curriculum (Kloosterman & Stage, 1992). As the participants in this study were pre-service IS mathematics teachers and word problems are often a prominent feature in the learning of mathematical problem solving (Wilson et al., 1993), this scale was retained to inspect whether a similar reliability would be obtained.
Table 4

*BMPSQ Scales and Corresponding Reliabilities (n = 517) from Kloosterman & Stage (1992)*

<table>
<thead>
<tr>
<th>Scale</th>
<th>Belief Addressed</th>
<th>Cronbach’s Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficult Problems</td>
<td>I can solve time-consuming mathematics problems</td>
<td>0.77</td>
</tr>
<tr>
<td>Steps</td>
<td>There are word problems that cannot be solved with simple, step-by-step procedures</td>
<td>0.67</td>
</tr>
<tr>
<td>Understandinga</td>
<td>Understanding concepts is important</td>
<td>0.76</td>
</tr>
<tr>
<td>Word Problems</td>
<td>Word problems are important in mathematics</td>
<td>0.54</td>
</tr>
<tr>
<td>Effort</td>
<td>Effort can increase mathematical ability</td>
<td>0.84</td>
</tr>
<tr>
<td>Usefulness</td>
<td>Mathematics is useful in daily life</td>
<td>N/A</td>
</tr>
</tbody>
</table>

*aCalculations for the Understanding scale were based on a supplemental sample of 88 college students.*

Data from the BMPSQ were utilized to provide a sense of the beliefs about problem solving before and after pre-service IS mathematics teachers’ Mathematics Education coursework and teaching-based practicum (Fall term, 2016). In particular, questionnaire data were used to generate descriptions of the sample using descriptive statistical analysis, and to make inferences about the population of Ontario pre-service IS mathematics teachers using inferential statistical analysis. Prior to statistical analysis, exploratory factory analysis was used to examine the factor structure of the BMPSQ, and Cronbach’s alpha was calculated to test the internal consistency of the BMPSQ scales. Inferential statistics were generated using several two-way mixed ANOVA (gender x time, and teaching subject x time). Results from inferential statistics were used to develop an understanding of how pre-service teachers’ beliefs about problem solving...
evolve following coursework and the practicum (i.e., how experience affects beliefs), how gender and teaching subject relate to beliefs about problem solving, and how experience interacts with gender and teaching subject for beliefs about problem solving.

In-depth, semi-structured, one-on-one interviews followed the collection of BMPSQ data in both phases, and served to deepen and extend findings obtained from the questionnaire. As interviews are a valuable method of data collection when the phenomenon being studied cannot be directly observed (Patton, 2002), it was determined that interviews would be ideal for eliciting participants’ beliefs about problem solving. In phase one, interviews were used to generate an initial understanding of participants’ ontological and epistemological beliefs about problem solving. In phase two, following participants’ coursework and the practicum, interviews again probed participants’ ontological and epistemological beliefs about problem solving; however, findings from phase one interviews informed several prompts incorporated into phase two interviews.

Interviews followed a standardized open-ended structure, which provided consistency across participants’ interviews and still allowed participants to fully detail their thoughts and experiences (Turner, 2010). A detailed protocol was followed throughout each interview to ensure valuable information was not missed, such as: ethics details included at the beginning and end of each interview, the precise wording of questions, and prompts for clarification and expansion of responses (Jacob & Furgerson, 2012; Turner 2010). Interview questions targeted participants’ ontological beliefs through inquiry into what they considered problem solving to be, and their epistemological beliefs through questions modelled after the TBI (Luft & Roehrig, 2007). Wording of TBI was slightly modified to incorporate the language of problem solving, yet all seven areas of
the original TBI (see Table 3) were retained to fully understand participants’ epistemological beliefs. While largely identical to the phase one protocol, the interview protocol followed in phase two featured additional prompts informed by analysis of the phase one BMPSQ and interview data.

The final type of data collected in this study was curriculum policy documents constructed by the Ontario MOE, which are mandated to be followed by pre-service teachers in the development of course material they use during teaching practica. In Ontario, three documents exist for IS mathematics teachers to consider: the Grades 1-8 Curriculum (MOE, 2005a), the Grades 9 and 10 Curriculum (MOE, 2005b), and the Grades 11 and 12 Curriculum (MOE, 2007). In each of these documents problem solving is identified as one of the seven mathematical processes that supports effective learning in mathematics. As such, each of the Ontario mathematics curriculum documents describe aspects of problem solving that teachers should be considering when planning their instruction. By utilizing this data source, this research sought to explore the relationship between participants’ ontological and epistemological beliefs about problem solving and problem solving as it exists in Ontario mathematics curricula.

**Phases of data collection.** Data collection encompassed two phases: a pre-coursework and practicum phase (i.e., phase one) that occurred between August and October 2016, and a post-coursework and practicum phase (i.e., phase two) that occurred in November and December 2016. Evidently, these dates were chosen to bestride the Fall term of the teacher education program, which began in the last week of August 2016 and ended in the third week of December 2016. At the onset of this study participants had already experienced an observation-based practicum, so a basic understanding of
enacting curriculum in professional practice was present. While data were collected a third time in April 2017 as a part of the course instructor’s study, these data fell outside the scope of this master’s study and were not included.

The objective in the first phase of this study was to generate an initial understanding of participants’ ontological and epistemological perceptions of problem solving. The first step towards fulfilling this objective was to distribute the Letter of Information and Consent Form for the instructor’s study, which occurred during the first class of the Mathematics Education course in August 2016. Immediately following collection of participants’ Consent Forms, access to the BMPSQ was provided. The instrument was created and made electronically available using Fluid Surveys—the survey tool licensed at the institution for data collection. Participants were provided access to faculty-owned laptops for completing the BMPSQ, although participants were also permitted to complete the BMPSQ using a personal electronic device.

Following the collection of phase one BMPSQ data, purposefully selected pre-service teachers were contacted and invited to participate in follow-up interviews. Interviews occurred at a time and place of the participants’ choosing between the first week of September 2016 and the first week of October 2016. Each interview lasted between 30 and 60 minutes. The strategic placement of interviews following the BMPSQ allowed for probing into participants’ thought processes behind their answers to specific BMPSQ questions (e.g., questions with a Likert response of neutral). Once all phase one interviews were complete and participants had left for their practicum, analysis of the full phase one data set commenced. The intent of this initial analysis was to inform interview prompts for phase two interviews.
The objectives in the second phase of this study were to review participants’ ontological and epistemological beliefs about problem solving following their experience teaching, and to begin holistic analysis of all collected data. Collection of the phase two BMPSQ data occurred in November 2016 during participants’ first Mathematics Education class period following the practicum. Replicating the procedure from phase one, participants completed the BMPSQ through Fluid Surveys using either a faculty-owned laptop or a personal electronic device. Following the collection of phase two BMPSQ data, the interview participants from phase one were contacted by email to arrange follow-up interviews. All interview participants from phase one agreed to the follow-up interviews, which occurred at a time and place of the participants’ choosing between the third week of November 2016 and the second week of December 2016. Similar to phase one, each interview lasted between 30 and 60 minutes. Immediately following the final interview, extensive holistic analysis of phase one and phase two data began. Table 5 provides a summary of the timeline for data collection and analysis.

Table 5

Timeline of Data Collection and Analysis

<table>
<thead>
<tr>
<th>Phase</th>
<th>Event</th>
<th>Time of Event</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Distributed Letter of Information and Consent Form to pre-service IS mathematics teachers</td>
<td>August 2016: During the first Mathematics Education class</td>
<td>Consenting pre-service teachers provided their university-assigned email address for contact regarding interviews</td>
</tr>
<tr>
<td></td>
<td>Collection of phase one BMPSQ data</td>
<td>August 2016: Immediately following signing of Consent Form</td>
<td>Conducted using Fluid Surveys</td>
</tr>
<tr>
<td>Event</td>
<td>Time Period</td>
<td>Details</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>-----------------------</td>
<td>-------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Phase one interviews</td>
<td>September—October 2016</td>
<td>Occurred at a time and place of participants’ choosing</td>
<td></td>
</tr>
<tr>
<td>Analysis of phase one data</td>
<td>October—November 2016</td>
<td>Generated initial interpretations that informed prompts in phase two interviews</td>
<td></td>
</tr>
<tr>
<td>(6 weeks: duration of the first teaching practicum)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Collection of phase two BMPSQ data</td>
<td>November 2016</td>
<td>Conducted using Fluid Surveys</td>
<td></td>
</tr>
<tr>
<td>Phase two interviews</td>
<td>November—December 2016</td>
<td>Occurred at a time and place of participants’ choosing</td>
<td></td>
</tr>
<tr>
<td>Holistic analysis of data</td>
<td>January—May 2017</td>
<td>Followed a convergent parallel design</td>
<td></td>
</tr>
</tbody>
</table>

**Data Analysis**

Three distinct actions comprised the convergent parallel design for data analysis in this study: analysis of the BMPSQ data, analysis of the interview data, and the comparing and integrating of the results from these two data sets. Rigour was achieved for each analytic action by the careful following of established procedures, and in what follows the details for each of these procedures is outlined.

**BMPSQ analysis.** Analysis of the BMPSQ data included both descriptive and inferential statistical analysis to fully describe pre-service teachers’ response characteristics and develop inferences about the larger population of Ontario pre-service IS mathematics teachers. Performing these analyses necessitated reverse coding all
negatively-worded Likert items to ensure that all scale items were coded in the same direction. A score was then generated for each BMPSQ scale by summing the Likert items associated with that scale, which yielded six total scores corresponding to the six BMPSQ scales (see Table 4). *Scale scores* is the term used to denote this value, which was used for all descriptive and inferential statistical analysis.

The internal consistency of each BMPSQ scale (i.e., the reliability of each scale) was calculated using Cronbach’s alpha. This measure of reliability provided insight on whether each group of Likert items were indeed measuring the same underlying belief, and the well-established, minimally-acceptable value of .7 was used as the cut-off for scale retention (Lance, Butts, & Michels, 2006; Nunnally, 1978). In addition, Likert items were removed from the scales when specific criteria were met: (a) if removal lead to a modest increase (greater than .01) in reliability, (b) if the removed item possessed a corrected total-item correlation (i.e., Pearson correlation) value lower than .3 (Laerd Statistics, 2015a), and (c) if these item removal criteria were satisfied in both phases of the study.

Following reliability analysis, an exploratory factor analysis (EFA) was performed to determine the overall validity of the BMPSQ, excluding scales that failed reliability analysis. While the original instrument claims to measure six latent constructs (see Table 4), EFA allowed for determination of whether this dimensionality held for the data collected in this study. The number of factors to retain was established using an eigenvalue Monte Carlo simulation (i.e., parallel analysis; O’Connor, 2000)—a process that compares the eigenvalues of the matrix of raw data to averaged eigenvalues from a set of 1000 (chosen by the researcher) similar sized matrices of the raw data randomly
shuffled. Prior to conducting the EFA, a direct oblimin rotation method was chosen to allow for correlation between the scales, as scales are related in that they measure beliefs about mathematical problem solving. Additionally, the presence and influence of outliers in the data were examined prior-to and while conducting the EFA.

Once reliability and validity testing of the data was completed, a two-way mixed ANOVA was conducted for each retained scale to examine the interaction of experience (i.e., initial coursework and the first teaching-based practicum) and gender, and experience and teaching subject. Rather than control for the family-wise error rate as is typically done with multiple comparisons (Glickman, Rao, & Schultz, 2014), the approach of controlling the false discovery rate was implemented for this study. Specifically, this approach corrected significance considerations by controlling the false discover rate at 10% using the Benjamini-Hochberg procedure (Benjamini & Hochberg, 1995; Benjamini & Yekutieli, 2001). A demonstrably more powerful correction for multiple comparisons (Glickman et al., 2014), the Benjamini-Hochberg procedure is also more appropriate for this exploratory study (Benjamini & Hochberg, 1995), as the procedure allows for identification of effects that can be missed by controlling for the more conservative family-wise error rate. Prior to conducting the two-way mixed ANOVA for each retained scale, the data were inspected for adherence to the necessary assumptions: the presence of outliers was assessed by boxplot and studentized residuals; deviations from normality were examined using Normal Q-Q plots and the Shapiro-Wilk test of normality; homogeneity of variances and covariances were explored using Levene's test for equality of variances and Box's test of equality of covariance matrices, respectively; and the variances between groups was tested using Mauchly's test of
sphericity. For all analyses the data were split by gender and teaching subject separately to ensure the necessary assumptions held for each between-subjects factor.

**Interview analysis.** For each phase and interview, participant data from the ontological and epistemological interview sections were analyzed separately. All audio-recorded interview data were transcribed and analyzed in an iterative and recursive process (Merriam, 1998), adding value to interpretations that developed from the data by treating analysis as an active and ongoing procedure. Specifically, analysis of the interview data followed a general inductive approach, wherein coding progressed from text segments, to categorical codes, and finally to emergent themes (Thomas, 2006).

Text segments were formed directly from analysis of raw transcript data and represented a process of “breaking down, examining, comparing, conceptualizing, and categorizing data” (Strauss & Corbin, 1990, p. 61). Ranging from single words to short sentences, text segments were created both directly from participants’ words (*in vivo* codes) and by the researcher. Once all data had been reduced to text segments, categorical codes were formed by identifying relationships among the segments and subsequently grouping segments into categories and sub-categories (Kendall, 1999). These categorical codes were created to assist the researcher in thinking systematically about the data and to develop connections between portions of data. Finally, emergent themes were identified by determining the core meanings encompassing the various categories and sub-categories (i.e., the categorical codes). Identifying emergent themes involved determining the underlying idea pervading the data and then relating that idea to the categories and sub-categories through a process of integration and refinement (Kendall, 1999).
Comparing and integrating quantitative and qualitative data. Analysis procedures culminated in the comparison and integration processes of the convergent parallel design (Creswell, 2014). Specifically, after the BMPSQ and interview findings had been fully represented in meaningful ways, findings were merged to enable the identification of key features being converged upon by the two data sets. In this way, the separate and parallel analysis of quantitative and qualitative data permitted the fulfilling of the first study objective and the merging of findings permitted the fulfilling of the second study objective (see Figure 2).

**Figure 2.** The convergent parallel design underpinning data analysis, with identification of the study objective addressed by each section of analysis. Adapted from Creswell (2014).

The merging of BMPSQ and interview findings was treated as a dynamic process where the researcher drifted between the two data sets identifying parallel features, critically examining whether those features were indeed being converged upon, and building explanations for the meaning of those convergent features. The verisimilitude of identified convergent features was assessed through the critical appraisal of other researchers with experience in mixed methods research designs. Specifically, the
convergent features and the supporting reasoning for how those features were represented in the findings were appraised by the researcher’s supervisor, who has extensive experience in mixed methods research, and by colleagues of the researcher who have experience with mixed methods research designs in their graduate research.

**Managing Personal Biases**

As indicated in the rationale section, this study was personally important because it ultimately had bearing on an issue I experienced in my learning of secondary-school mathematics. Consequently, it was necessary to take measures to ensure that my personal biases for the topic of problem solving in mathematics did not excessively influence interpretations of the data. The primary measure for controlling my bias was to discuss all interpretations that developed from data analysis with my supervisor and colleagues. Whenever soliciting the critical review of these outside examiners, I would provide relevant portions of the data along with my interpretations of the data to allow their appraisal of whether my personal biases had overtly influenced interpretations. If there was misalignment between my interpretations and the outside examiner’s interpretations, I would revise the problematic portion and the critical review process continued. Another measure for controlling my bias was to consistently rely on the literature when developing descriptions of the data; whenever a finding appeared to echo a concept from the literature, I would first ensure the finding indeed fit the established descriptive criteria (e.g., the criteria for an ill-defined problem). If the criteria indeed appeared to be met, I would use language established in the literature to describe my interpretations of the data, thus reducing bias that might be present in my own descriptive language.
Chapter 4

Findings

In this chapter the empirical foundation for answering the research questions is presented. First, I present the results from analysis of the phase one and phase two Beliefs about Mathematical Problem Solving Questionnaire (BMPSQ) data (i.e., the quantitative data). Findings presented for the BMPSQ data include reliability and validity analysis of the BMPSQ scales, descriptive statistics for the scales, and inferential statistics for how scale scores changed between phase one and phase two. Second, I present the results from analysis of the phase one and phase two interview data (i.e., the qualitative data). These results include the ontological and epistemological themes from each phase and the corresponding categorical codes from which these themes emerged.

Quantitative Data

Quantitative data, collected using the BMPSQ, was the first set of data collected in each phase of this study. In total, 44 repeated-measure participants completed the BMPSQ. Prior to analyzing the data, all 36 Likert items were numerically coded following the outline given by Kloosterman and Stage (1992), which is displayed in Appendix B. Subsequently, scale scores for the six subscales (i.e., difficult problems, steps, understanding, word problems, effort, and usefulness) were calculated by summing the corresponding Likert items for each participant and determining the mean across participant groups. Analysis of the BMPSQ scales included three segments: (1) determining the reliability and validity of each scale, (2) reviewing descriptive statistics.
for each scale, and (3) calculating how scale scores changed from phase one to phase two.

**BMPSQ reliability and validity.** Prior to calculating descriptive statistics or determining whether significant differences existed within or between the groupings of females and males or science and non-science teaching subjects, it was necessary to determine whether the BMPSQ scales, as originally constructed, were reliable and whether each scale item was indeed measuring the expected latent construct. Reliability was determined by calculating the internal consistency for each scale using Cronbach’s alpha, whereas validity was determined by conducting an exploratory factor analysis (EFA).

**Reliability analysis.** To determine whether the BMPSQ scales as originally constructed were reliable, the internal consistency of each scale was calculated for both phases using Cronbach’s alpha. Results of the reliability analysis, following removal of poorly correlated Likert items and highly inconsistent cases, are given in Table 6 (see Appendix B for a complete list of the BMPSQ Likert items and corresponding scales).

Table 6

<table>
<thead>
<tr>
<th>Scale</th>
<th>Phase One Reliability</th>
<th>Phase Two Reliability</th>
<th>Revised Phase One Reliability</th>
<th>Revised Phase Two Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficult Problems</td>
<td>0.587</td>
<td>0.822</td>
<td>0.730^1,2</td>
<td>0.844^1,2</td>
</tr>
<tr>
<td>Steps</td>
<td>0.658</td>
<td>0.666</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Understanding</td>
<td>0.804</td>
<td>0.704</td>
<td>0.808^2</td>
<td>0.811^2</td>
</tr>
</tbody>
</table>
As shown in Table 6, four of the six scales in the BMPSQ—difficult problems, word problems, effort, and usefulness—benefitted from removal of a single item. The remaining scales—steps and understanding—did not have agreed-upon item removals between phases. Following item removal, the difficult problems, understanding, and usefulness scales showed a large discrepancy (greater than .1) in reliability values between phases. These large discrepancies can introduce inaccuracies during analysis with two-way mixed ANOVAs, so systematic case removal and recalculation of reliabilities was conducted for each discrepant scale to uncover inconsistent cases. Criteria for case removal was determined to be: (a) removal leads to a modest increase (greater than .01) in reliability for the less internally consistent phase, and (b) removal does not lead to a modest reduction (greater than 0.1) in reliability for the more internally consistent phase. Overall, three cases (two females with a non-science teaching subject and one male with a science teaching subject) were removed from the difficult problems scale data, three cases (one female and two males with a science teaching subject) were removed from the understanding scale data, and one case (a male with a science teaching subject) was removed from the usefulness scale data. In each scenario, the removals improved the reliability of both phases.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Reliability</th>
<th>Reliability</th>
<th>Reliability</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word Problems</td>
<td>0.584</td>
<td>0.468</td>
<td>0.632(^1)</td>
<td>0.583(^1)</td>
</tr>
<tr>
<td>Effort</td>
<td>0.896</td>
<td>0.895</td>
<td>0.919(^1)</td>
<td>0.915(^1)</td>
</tr>
<tr>
<td>Usefulness</td>
<td>0.667</td>
<td>0.789</td>
<td>0.731(^{1,2})</td>
<td>0.836(^{1,2})</td>
</tr>
</tbody>
</table>

\(^1\) Reliability following removal of one Likert item agreed upon at both time points
\(^2\) Reliability following removal of highly inconsistent cases
The steps scale and word problems scale did not attain the cutoff reliability criteria of .7, indicating that items in each scale could not be presumed to be measuring the same latent construct. Accordingly, inferential statistical analysis did not make use of these two scales. Thus, to summarize, inferential analysis included only the difficult problems, effort, usefulness, and understanding scales; the latter being the only scale to retain all six items, whereas the other scales retained five items: 21 Likert items in total.

**Factor analysis.** To determine whether each BMPSQ item was indeed measuring the expected latent construct, the validity of scales remaining after reliability adjustments was investigated using EFA for both research phases. Principle axis factoring was chosen as the factor extraction method due to BMPSQ items deviating from multivariate normality (Fabrigar, Wegener, MacCallum, & Strahan, 1999), and a direct oblimin rotation method was chosen to allow for correlation between the scales. The overall Kaiser-Meyer-Olkin measure was .599 for phase one and .550 for phase two, classifying the interpretability of the EFAs as “miserable” (Kaiser, 1974). However, Bartlett’s Test of Sphericity was statistically significant ($p < .0005$), indicating that the data were likely factorizable. The number of factors to retain in each phase was determined by conducting an eigenvalue Monte Carlo simulation (i.e., parallel analysis; O’Connor, 2000), which suggested three factors in phase one and four factors in phase two. This result was supported by the extracted factors possessing eigenvalues greater than one and by collectively explaining at least 50% of the total variance in both phases. When interpreting the EFA results, only factor loadings in excess of .4 were retained (Field, 2013) and items with cross-loadings were removed when other loadings on the factor were adequate to strong (approximately .5; Costello & Osborne, 2005). Factor loadings
of the rotated solution for phase one are presented in Table 7, and factor loadings of the rotated solution for phase two are presented in Table 8.

Table 7

*Factor Loadings for EFA of Phase One Data*

<table>
<thead>
<tr>
<th></th>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Factor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factor 1 – Effort Scale</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Ability in math increases when one studies hard.</td>
<td>.918</td>
<td></td>
<td></td>
</tr>
<tr>
<td>29. I get smarter in math by trying hard.</td>
<td>.917</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23. Hard work can increase one’s ability to do math.</td>
<td>.871</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35. I can get smarter in math if I try hard.</td>
<td>.763</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. By trying hard, one can become smarter in math.</td>
<td>.687</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Factor 2 – Understanding Scale</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.</td>
<td></td>
<td>.751</td>
<td></td>
</tr>
<tr>
<td>21. Getting a right answer in math is more important than understanding why the answer works.</td>
<td></td>
<td>.738</td>
<td></td>
</tr>
<tr>
<td>15. A person who doesn’t understand why an answer to a math problem is correct hasn’t really solved the problem.</td>
<td></td>
<td>.707</td>
<td></td>
</tr>
<tr>
<td>3. Time used to investigate why a solution to a math problems works is time well spent.</td>
<td></td>
<td>.660</td>
<td></td>
</tr>
<tr>
<td>33. It doesn’t really matter if you understand a math problem if you can get the right answer.</td>
<td></td>
<td>.653</td>
<td></td>
</tr>
<tr>
<td>9. It’s not important to understand why a mathematical procedure works as long as it gives a correct answer.</td>
<td></td>
<td>.535</td>
<td></td>
</tr>
<tr>
<td><strong>Factor 3 – Difficult Problems Scale (without item 25)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19. If I can’t solve a math problem quickly, I quit trying.</td>
<td></td>
<td>.736</td>
<td></td>
</tr>
<tr>
<td>31. I’m not very good at solving math problems that take a while to figure out.</td>
<td></td>
<td>.645</td>
<td></td>
</tr>
<tr>
<td>13. I feel I can do math problems that take a long time to complete.</td>
<td></td>
<td>.634</td>
<td></td>
</tr>
<tr>
<td>1. Math problems that take a long time don’t bother me.</td>
<td></td>
<td>.455</td>
<td></td>
</tr>
</tbody>
</table>

Table 8

Factor Loadings for EFA of Phase Two Data

<table>
<thead>
<tr>
<th>Factor 1 – Effort Scale</th>
<th>Factor 2 – Difficult Problems Scale</th>
<th>Factor 3 – Usefulness Scale</th>
<th>Factor 4 – Understanding Scale (without item 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11. Ability in math increases when one studies hard.</td>
<td>31. I’m not very good at solving math problems that take a while to figure out.</td>
<td>24. Mathematics is of no relevance to my life.</td>
<td>15. A person who doesn’t understand why an answer to a math problem is correct hasn’t really solved the problem.</td>
</tr>
<tr>
<td>23. Hard work can increase one’s ability to do math.</td>
<td>13. I feel I can do math problems that take a long time to complete.</td>
<td>18. Knowing mathematics will help me earn a living.</td>
<td>21. Getting a right answer in math is more important than understanding why the answer works.</td>
</tr>
<tr>
<td>35. I can get smarter in math if I try hard.</td>
<td>25. I find I can do hard math problems if I just hang in there.</td>
<td>36. Studying mathematics is a waste of time.</td>
<td></td>
</tr>
<tr>
<td>5. By trying hard, one can become smarter in math.</td>
<td>19. If I can’t solve a math problem quickly, I quit trying.</td>
<td>12. Mathematics will not be important to me in my life’s work.</td>
<td></td>
</tr>
<tr>
<td>29. I get smarter in math by trying hard.</td>
<td>1. Math problems that take a long time don’t bother me.</td>
<td>30. Mathematics is a worthwhile and necessary subject.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>11. Ability in math increases when one studies hard.</td>
<td>.952</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23. Hard work can increase one’s ability to do math.</td>
<td>.832</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35. I can get smarter in math if I try hard.</td>
<td>.827</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. By trying hard, one can become smarter in math.</td>
<td>.819</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29. I get smarter in math by trying hard.</td>
<td>.486</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
27. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.

33. It doesn’t really matter if you understand a math problem if you can get the right answer.

9. It’s not important to understand why a mathematical procedure works as long as it gives a correct answer.


EFA results for phase one, as displayed in Table 7, exactly returned the understanding and effort scales, and returned the difficult problems scale without Likert item 25 (see Appendix B for a complete list of the BMPSQ Likert items and corresponding scales). Items 12, 18, and 24 from the usefulness scale did not meet the minimum factor loading criteria of .4, and items 30 and 36 from the usefulness scale were found to cross load onto the understanding scale. The cross loadings were suppressed because other items in the factors loaded adequately to strongly within the factor (factor loadings of approximately .5 or greater; Costello & Osborne, 2005). EFA results for phase two, as displayed in Table 8, exactly returned the effort, difficult problems, and usefulness scales. The understanding scale was returned without item 3, as the item did not meet the minimum factor loading criteria. In both phases, factor correlation matrices indicated that the difficult problems scale and effort scale were weakly correlated (i.e., 0.3 < r < 0.4; Mukaka, 2012). All other scale correlations were found to be negligible (|r| < 0.3; Mukaka, 2012).

Descriptive statistics. In line with the research questions, participants’ scale scores were analyzed in groupings of females and males, and science and non-science teaching subjects. Complete details for how these groups responded to each Likert item is
available in Appendix F; however, this study focused on the six scales that evenly distributed and summed these 36 Likert items. Thus, descriptive statistics displayed in Table 9 and Table 10 provide contrasts of BMPSQ scale scores for these groupings. Reported descriptive statistics do not include the removed Likert items or highly discrepant cases identified in reliability analysis. However, while not included in inferential analysis, descriptive statistics are reported for the steps and word problems scales to provide insight into how participants performed on these scales.

Table 9

**Gender-Based Descriptive Statistics of BMPSQ Scales for Phase One and Phase Two**

<table>
<thead>
<tr>
<th>Scale</th>
<th>Phase One (mean; standard deviation)</th>
<th>Phase Two (mean; standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
<td>Male</td>
</tr>
<tr>
<td>Difficult Problems(^1) (maximum score = 25; minimum score = 5)</td>
<td>18.8; 2.4</td>
<td>18.8; 2.5</td>
</tr>
<tr>
<td>Steps(^2) (maximum score = 30; minimum score = 6)</td>
<td>18.6; 2.8</td>
<td>20.8; 4.1</td>
</tr>
<tr>
<td>Understanding(^3) (maximum score = 30; minimum score = 6)</td>
<td>25.9; 2.8</td>
<td>24.0; 1.8</td>
</tr>
<tr>
<td>Word Problems(^2) (maximum score = 25; minimum score = 5)</td>
<td>16.6; 2.8</td>
<td>16.5; 2.2</td>
</tr>
<tr>
<td>Effort(^2) (maximum score = 25; minimum score = 5)</td>
<td>19.9; 3.1</td>
<td>19.9; 3.0</td>
</tr>
<tr>
<td>Usefulness(^4) (maximum score = 25; minimum score = 5)</td>
<td>22.6; 1.7</td>
<td>22.8; 2.2</td>
</tr>
</tbody>
</table>

\(^1\)\(N_{\text{Females}} = 32, N_{\text{Males}} = 9\)
\(^2\)\(N_{\text{Females}} = 34, N_{\text{Males}} = 10\)
\(^3\)\(N_{\text{Females}} = 33, N_{\text{Males}} = 8\)
\(^4\)\(N_{\text{Females}} = 34, N_{\text{Males}} = 9\)
Table 10

*Teaching Subject-Based Descriptive Statistics of BMPSQ Scales for Phase One and Phase Two*

**Phase Two**

<table>
<thead>
<tr>
<th>Scale</th>
<th>Phase One (mean; standard deviation)</th>
<th>Phase Two (mean; standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Science</td>
<td>non-Science</td>
</tr>
<tr>
<td>Difficult Problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(maximum score = 25;</td>
<td>19.0; 2.5</td>
<td>18.4; 2.1</td>
</tr>
<tr>
<td>minimum score = 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steps</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(maximum score = 30;</td>
<td>19.7; 3.1</td>
<td>18.1; 3.2</td>
</tr>
<tr>
<td>minimum score = 6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understanding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(maximum score = 30;</td>
<td>25.6; 3.0</td>
<td>25.4; 2.4</td>
</tr>
<tr>
<td>minimum score = 6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Word Problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(maximum score = 25;</td>
<td>16.6; 2.7</td>
<td>16.6; 2.6</td>
</tr>
<tr>
<td>minimum score = 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effort</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(maximum score = 25;</td>
<td>19.8; 3.0</td>
<td>20.2; 3.3</td>
</tr>
<tr>
<td>minimum score = 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usefulness</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(maximum score = 25;</td>
<td>23.0; 1.6</td>
<td>22.1; 2.0</td>
</tr>
<tr>
<td>minimum score = 5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^1\text{N}_{\text{Science}} = 27, \text{N}_{\text{non-Science}} = 14\)
\(^2\text{N}_{\text{Science}} = 28, \text{N}_{\text{non-Science}} = 16\)
\(^3\text{N}_{\text{Science}} = 25, \text{N}_{\text{non-Science}} = 16\)
\(^4\text{N}_{\text{Science}} = 27, \text{N}_{\text{non-Science}} = 16\)

The six scales indicated in Table 9 and Table 10 are each a summation of Likert items scored on a 5-point scale. As such, scales with a maximum score of 30 possessed the original six Likert items, whereas scales with a maximum score of 25 possessed five Likert items, as one item was removed during reliability analysis. All groups in both phases scored highest on the usefulness scale (mean scores greater than 85% the maximum) and understanding scale (means scores greater than 80% the maximum), and
lowest on the steps and word problems scales (mean scores lower than 70% the maximum). The dispersion of scale scores (i.e., standard deviation) was found to be greatest for the steps and effort scales (standard deviation values greater than 10% of the maximum).

**Two-way mixed ANOVAs.** In order to examine how scale scores changed from phase one to phase two, a two-way mixed ANOVA was conducted for each retained scale to examine the interaction of experience (i.e., initial coursework and the first teaching-based practicum) and gender, and experience and teaching subject. Scales retained and examined were those that passed reliability and validity analysis (i.e., difficult problems, understanding, effort, and usefulness scales). In total, this required conducting eight two-way mixed ANOVAs: four for the gender grouping and four for the teaching subject grouping.

Prior to conducting the two-way mixed ANOVA for each scale, the data were inspected for outliers, deviations from normality, and homogeneity of variances and covariances, taking care to split the analyses by gender and teaching subject separately. As assessed by boxplot, outliers at more than three times the interquartile range were discovered for the effort scale in both the gender and teaching subject groupings. Removal of these outliers did not affect result significance, so all data were retained. Normality of the data was assessed using Shapiro-Wilk’s test of normality, which revealed minor violations for the effort and usefulness scales for both gender and teaching subject groupings. Data transformations—square root, logarithmic, and reciprocal conversions (and the corresponding reflection of each conversion; Laerd Statistics, 2015b)—were not successful at adapting data to pass Shapiro-Wilk’s test of
normality. Normality violations were accepted and analysis continued as it is well-established that ANOVA procedures are robust to violations of normality (Erceg-Hurn & Mirosevich, 2008; Laerd Statistics, 2015c; Schmider, Ziegler, Danay, Beyer, & Bühner, 2010). Finally, there was homogeneity of variances ($p > .05$) and covariances ($p > .05$) for both groupings, as assessed by Levene's test of homogeneity of variances and Box's $M$ test, respectively.

All ANOVA findings are presented using partial $\eta^2$ as a measure of effect size, following Cohen’s (1988) classification system: $\eta^2 = 0.01$ (small), $\eta^2 = 0.06$ (medium), and $\eta^2 = 0.14$ (large). Similarly, all simple main effects are presented using Cohen’s $d$ as a measure of effect size, again following Cohen’s (1988) classification system: $d = 0.2$ (small), $d = 0.5$ (medium), and $d = 0.8$ (large). A summary of all ANOVA findings for the four retained scales (difficult problems, understanding, effort, and usefulness scales) is presented in Table 11.

**Difficult problems scale.** A statistically significant interaction was found between experience and gender, $F(1, 39) = 7.470, p = .017$, partial $\eta^2 = .138$. Scale scores for males were statistically significantly greater in phase two ($M = 1.33, SE = 0.33, p = .004,$ $d = 0.486$) compared to phase one. Scale scores for females were not statistically significantly different between phase one and phase two ($M = 0.13, SE = 0.29, p = .673$). Scale scores between males and females were not statistically significantly different in phase one ($M = 0.03, SE = 0.92, p = .970$) or phase two ($M = 1.42, SE = 1.13, p = .215$). There was no statistically significant interaction between experience and teaching subject, along with no statistically significant main effects for time or teaching subject.
**Usefulness scale.** There was no statistically significant interaction between experience and gender, $F(1, 41) = 1.197, p = .280$, partial $\eta^2 = .028$, along with no statistically significant main effects for time or gender. Similarly, there was no statistically significant interaction between experience and teaching subject, $F(1, 41) = 3.142, p = .084$, partial $\eta^2 = .071$. However, the main effect of teaching subject showed a statistically significant difference in mean scale scores over the two phases $F(1, 41) = 7.198, p = .010$, partial $\eta^2 = .149$. Specifically, in phase two, scale scores for science teaching subject were statistically significantly greater ($M = 1.9, SE = 0.61, p = .004, d = 0.984$) compared to scale scores for non-science teaching subject. The main effect of time did not show a statistically significant difference in mean scores for teaching subject, $F(1, 41) = 1.242, p = .272$, partial $\eta^2 = .029$.

**Understanding scale.** There was no statistically significant interaction between experience and gender, $F(1, 39) = 1.667, p = .203$, partial $\eta^2 = .041$, or between experience and teaching subject, $F(1, 39) = 0.098, p = .756$, partial $\eta^2 = .003$. Similarly, there were no significant main effects for time, gender, or teaching subject.

**Effort scale.** There was no statistically significant interaction between experience and gender, $F(1, 42) = 0.564, p = .457$, partial $\eta^2 = .013$, or between experience and teaching subject, $F(1, 42) = 2.104, p = .421$, partial $\eta^2 = .010$. Similarly, there were no significant main effects for time, gender, or teaching subject.
Table 11

Summary of Significant Findings from Two-Way ANOVAs

<table>
<thead>
<tr>
<th>Scale</th>
<th>Significant Two-Way Interactions</th>
<th>Significant Simple Main Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difficult Problems</td>
<td>Experience and gender (p = .017, \eta^2 = .138)</td>
<td>Males’ scale scores were greater in phase two compared to phase one (p = .004, d = 0.486)</td>
</tr>
<tr>
<td>Usefulness</td>
<td>None</td>
<td>Teachers with science teaching subjects possessed greater scale scores than teachers with non-science teaching subjects (p = .004, d = 0.984)</td>
</tr>
<tr>
<td>Understanding</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Effort</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

Qualitative Data

Two ontological themes and two epistemological themes emerged from the data for both phase one and phase two of this study. All themes are displayed in Table 12. Descriptions for the themes in each phase will be presented using the categorical codes that collectively formed the theme and specific interview excerpts that display how those categorical codes developed. For clarity and continuity, ontological themes from both phases will be presented first and epistemological themes from both phases will be presented second.
Table 12

Ontological and Epistemological Themes for Phases One and Two

<table>
<thead>
<tr>
<th></th>
<th>Phase One</th>
<th>Phase Two</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ontological Themes</strong></td>
<td>1) Problem solving is a collaborative thinking process for solving real-world problems</td>
<td>1) Problem solving is a challenging and collaborative thinking process for solving real-world problems</td>
</tr>
<tr>
<td>(What is mathematics problem solving?)</td>
<td>2) Problem solving is bounded by mathematics education</td>
<td>2) Problem solving is a process for solving academic mathematics problems</td>
</tr>
<tr>
<td><strong>Epistemological Themes</strong></td>
<td>1) Establishing a problem-solving learning environment</td>
<td>1) Exploring problem solving and communicating thinking</td>
</tr>
<tr>
<td>(How is mathematics problem-solving knowledge acquired?)</td>
<td>2) Balanced problem-solving instruction and assessment</td>
<td>2) Demonstrating problem-solving techniques and assessing students’ ability with those techniques</td>
</tr>
</tbody>
</table>

**Ontological themes: Phase one.** Two ontological themes (i.e., what mathematics problem solving is) emerged from participants’ interview data in phase one: (1) problem solving is a collaborative thinking process for solving real-world problems, and (2) problem solving is bounded by mathematics education. The first theme suggests that participants primarily considered problem solving to be a way of addressing real-world problems, and solving such problems requires collaborative thinking. This viewpoint situates problem solving as mathematics with a tangible real-world connection that requires unfamiliar solving processes, and it is the quality of unfamiliarity that requires problem solvers to collaborate. The second theme suggests that participants also considered problem solving to be bounded by the cultural values of mathematics education. Following from this view, how problem solving is perceived requires consideration for what a culture values in mathematics and how that translates to
mathematics curricula. Consequently, how problem solving is perceived in the Canadian context could be distinctly different from other curricular jurisdictions.

**Development of theme one.** The first ontological theme in phase one—problem solving is a collaborative thinking process for solving real-world problems—emerged from three categorical codes: *problem solving is a thinking process*, *problem solving deals with real-world application*, and *problem solving is primarily group based*. The first categorical code—problem solving is a thinking process—represents the idea that problem solving deals with problems requiring unfamiliar solving processes. Due to this unfamiliarity, students are required to think through different ways of approaching a problem and how to evaluate potential strategies. Albert expressed this viewpoint when describing problem solving as “something that is more complex rather than a simple calculation.” For Albert, problem complexity is a distinguishing feature of problem solving that distinguishes it from simple calculation. In making his assertion, Albert used the term *complexity* to describe problems that require deeper thinking than simple rote calculations. Isaac built upon this distinction by noting “you can ‘problem solve’ but it's really just going through the motions rather than understanding what you're doing.” Isaac’s careful choice of wording highlights the distinction between problem solving and rote calculations, which he termed *problem solve*. The idea he was conveying is that problem solving requires the student to think through each aspect of the solving process and to truly understand decisions made in the problem-solving process.

Albert’s and Isaac’s thoughts were taken a step further when Judith described her view of problem solving as “a question with concepts that we haven't touched base on yet . . . they can take whatever they know already, see if they can meddle around with it, and
see if they can solve the problem.” Similar to Albert, by describing the solving process as “meddle around,” Judith was suggesting that working toward a solution is not an obvious task, and effortful thinking is therefore required. Additionally, Judith indicated that problem solving deals with concepts that have not been covered—an opinion again suggesting that problem solving requires thinking beyond simple calculation. A final piece of detail was provided by Marie, who described problem solving as a type of learning for “developing our thinking process, and how we think critically about something, and how we approach something.” Marie directly stated that problem solving develops thinking processes, and she distinguished the type of thinking that is developed. Marie’s categorization of thinking as critical reveals her stance that problem solving is more about the thoughtful evaluation of ideas rather than the generation of novel ideas. All participants were in agreeance that mathematical tasks where a solution state is attainable without effortful thinking did not qualify as problem solving.

The second categorical code—problem solving deals with real-world application—identifies the participants’ views that problem solving is the application of mathematics to address real-world situations. Such real-world situations are those that address problems external to the classroom and that students can find relatable to their experiences (either directly or indirectly). Moreover, the quality of relatability suggests that problems should be authentically embedded in the real world—where the mathematics addresses a real-world problem, not where real-world context is added postscript to a mathematics problem. Marie outlined these views when describing what comes to mind when hearing the term problem solving: “What I think about problem solving is applying it to the real world and all the math that we learn we're able to apply it
to real life.” By repeating herself, Marie was emphasizing how she values real-world application as a key component of problem solving. Additionally, by referring to applying “all the math that we learn,” Marie was communicating that knowledge of and use of mathematics alone is not problem solving—the real-world application must be present. Albert expressed a similar viewpoint for what comes to mind when hearing the term problem solving: “[when you] take things you apply in math—you can use those methods of problem solving to apply them in your real life.” In describing his viewpoint, Albert contrasted application within mathematics and application in the real world. To him, problem solving focuses on the latter, where students apply mathematics to experiences in their “real life.”

Reflecting on his own learning, Isaac supported Marie’s and Albert’s views of problem solving as dealing with real-world application: “I've learned that problem solving in mathematics isn't ‘follow these rules, follow these steps,’ it's a way of analyzing the world around you.” To Isaac, problem solving is a way of analyzing real-world situations using mathematics, which he contrasted with the thoughtless following of rules or steps. Moreover, Isaac’s statement highlighted how he perceived relatability to be an essential quality of problem-solving tasks, evident by his stance that problem solving occurs in “the world around you.” Finally, expanding on when problem solving occurs, Judith stated “it's a real thing that you do on a day-to-day basis.” Although not directly using the terminology of application, Judith was describing problem solving as a real-world activity that we confront every day. Again, Judith’s view showing that participants perceived problem solving as directly tied to real-world experiences.
The third categorical code—problem solving is primarily group based—is the perspective that problem solving should occur in a group setting. Participants’ reasoning for this perspective centered around the value of experiencing how problems can be solved using a variety of strategies. Judith and Albert proposed, respectively, that “I think together would be better, having different perspectives and ideas come together,” and “by seeing what went through someone else's mind, you can draw on that in your future when you're trying to solve.” Their view of problem solving as group-based stemmed from the appreciation that interactions are valuable for sharing novel ways of thinking about mathematics. Furthermore, Albert suggested that collaboration can add to students’ knowledge of problem-solving processes by providing opportunities to see how others use different solving strategies. While neither Judith or Albert claimed problem solving cannot occur individually, their feeling was that individual problem solving was inferior to group-based problem solving. Marie supported this feeling when recalling how her own learning of problem solving looked:

[Instances where] we had group work and we had to figure out “How do you solve this problem, and why did that group use this to solve the problem? Which one is right? Is there one that is right or wrong?”

In Marie’s recollection, the group aspect of problem solving is key for requiring students to coordinate their thinking. Specifically, working in groups provides an opportunity for students to observe how each other approach problem-solving tasks, to collectively evaluate the efficacy of different solving strategies, and to experience thinking that may not have come naturally with independent problem solving.
**Development of theme two.** The second ontological theme in phase one—problem solving is bounded by mathematics education—emerged from two categorical codes: *problem solving is defined by educational culture* and *problem solving is a structured mathematical procedure*. The first categorical code—problem solving is defined by educational culture—represents the opinion that defining problem solving requires consideration for what a culture values in mathematics and how those values transfer into school-based learning and interactions with parents and peers. That said, while participants agreed that perceptions (i.e., personal definitions) of problem solving primarily develop within the school system, participants were divided on the influence of parents and peers. Albert communicated a view that emphasized school-based learning and deemphasized learning from parents:

> [My beliefs] probably originated from what we were taught in elementary school, I would think, because that's where it was applied the most. You were directed to actually use certain steps to solve the problem. It's not like you go home and your mom says "Alright, let's do some problem solving." Parents probably don't directly lay it out, they may imply that but it's more so in the class.

Albert suggested his perception of problem solving largely developed from his experiences in school, which emphasized problem solving as a step-based procedure. In making this point, Albert noted that parental influence on a students’ perception of problem solving is minimal, even joking about this possibility. By making the joke, Albert clearly expressed that he did not consider parents to be natural teachers of problem solving.
Judith described her perceptions of problem solving as having developed in a similar fashion: “I think the problems that they gave us and how they taught students to attempt problem solving. I think that could have been what influenced me.” In reflecting on her learning experiences, Judith considered school-based learning to be what developed her perception of problem solving. Specifically, Judith’s understanding of problem solving developed from assigned problem-solving tasks and from problem-solving instruction, and any influence from parents or peers was not mentioned.

Offering an opinion that partially differs from Albert and Judith, Marie suggested problem solving “has been traditionally taught out of the textbook . . . [and] home environment is a really big factor, because you live at home and you are taught by your parents about the way that they approach problems.” The order that Marie presented her thinking is meaningful: the importance of the school system—expressed by reference to problem solving being taught by textbook—was the first influence that came to mind when considering how her problem-solving perceptions developed. The home environment, while described as a “really big factor,” was not mentioned until after Marie had considered school-based influences. As such, Marie likely viewed school-based learning of problem solving as more influential for how it is ultimately perceived by students. Isaac suggested the learning of problem solving “sort of becomes a cultural fitting in.” What Isaac meant by this statement is that many different cultural elements influence the development of perceptions about problem solving. For example, Isaac suggested considerations should include “adult figures and classmates, or even pop culture,” the latter two not mentioned by the other participants. Thus, the overall
viewpoint expressed by participants was that problem-solving perceptions develop to reflect what the relevant culture values in mathematics.

The second categorical code—problem solving is a structured mathematical procedure—represents the idea that, in mathematics, problem solving is identifiable by the use of procedural methods. Specifically, participants felt the use of known steps and strategies indicates problem solving in mathematics. Isaac expressed this idea when reflecting on how he recognizes learning of problem solving in mathematics: “I haven't come across a lot of other ways that I can identify it, but seeing clear steps and following the procedure we've outlined in class is indicative [of problem solving].” A noteworthy point about Isaac’s statement is that he mentioned not having come across many ways of recognizing problem solving, other than procedural methods. To him, a key aspect of problem solving is following procedures that have been demonstrated by an instructor. Reflecting on her own learning, Marie expressed a similar view of how mathematical problem solving is identified: “I learned the GRASP method—given, required, apply, solve, and then solution or something. It helps to memorize what you have to do, but then it's important to understand the steps as well.” Marie’s view on identifying problem solving, given by the GRASP acronym, represented a highly procedural view. While she mentioned that it is important to understand the steps, Marie prefaced her thinking by mentioning the value of memorizing solving procedures—a point made despite her own inability to recall the full GRASP acronym.

Albert and Judith also viewed problem solving as identifiable through procedural mathematics, yet their views revealed greater flexibility in those procedures:
Using knowledge that you have previously acquired to first identify a plan of what you are going to do and then carry it out in a series of steps to reach a final conclusion . . . It could take only a few steps or it could take multiple steps. (Albert)

I feel like there are different ways to get to an answer. So, if you were to get like “step 1 - do this, step 2 - do this,” you can follow that but there are other ways to get your answers, not necessarily all in the same order. (Judith)

By Albert and Judith mentioning the use and sequencing of steps, they were clearly articulating that problem solving utilizes procedural mathematics. However, a distinction of their view is that the mathematical procedures can take various forms, and there is perhaps no single, correct procedure. This view was evident when Albert indicated that the steps only need to reach a “final conclusion,” and when Judith plainly stated that “other ways” exist to reach an answer. Although Albert’s and Judith’s views on the use of procedures were less rigid than Isaac’s and Marie’s views, all participants agreed that procedural mathematics are an identifier of problem solving.

**Ontological themes: Phase two.** Similar to phase one, two ontological themes emerged from participants’ interview data in phase two: (1) problem solving is a challenging and collaborative thinking process for solving real-world problems, and (2) problem solving is a process for solving academic mathematics problems. The first theme suggests that participants considered problem solving to be a process that is both not immediately obvious and requires collaboration. Similar to phase one, a key aspect of this thinking is that problem solving necessitates the exchange and questioning of ideas—students need to think about solving choices. A distinguishing feature of this theme in
phase two is participants’ use of the word “challenging” when discussing the types of problems addressed by problem solving. The second theme stands as a contrast to the first theme and reveals that participants also considered problem solving to be a process that lives within academic mathematics. Specifically, it was thought that real-world application is not always necessary and problem solving can simply be a procedure for developing and demonstrating mathematical understanding. Participants viewing real-world application as an unnecessary criterion for problem solving is what distinguishes this theme from the problem solving is bounded by mathematics education ontological theme from phase one.

**Development of theme one.** The first ontological theme in phase two—problem solving is a challenging and collaborative thinking process for solving real-world problems—emerged from three categorical codes: problem solving is a challenging thinking process, problem solving deals with real-world application, and problem solving is primarily group based. The first categorical code—problem solving is a challenging thinking process—identifies an opinion that problem solving should involve tasks that are not immediately solvable. Consequently, problem-solving tasks should require students to think through the efficacy of potential strategies for reaching a desired solution. Isaac expressed this focus on thinking when discussing what he perceives to be important facets of problem solving: “what matters to me is whether you are thinking about how to solve the problem and how you want to structure your approach, and the difference between that and recognizing steps you need to follow.” In his statement, Isaac clearly distinguishes between the simple following of steps and thinking-based problem solving. To him, simply knowing the steps to reach a desired conclusion is insufficient, and
instead students need to contemplate how solving choices, and the organization of those choices, will be valuable for reaching a conclusion. Therefore, in Isaac’s opinion, problem solving is necessarily a thinking process.

Isaac’s opinion was expanded on by Albert, who clearly articulated that problem-solving tasks should challenge students’ understanding of mathematics: “It should actually challenge you so that you need to go through the motion of trying to solve the problem and not taking short-cuts to just come up with the solution.” What Albert meant by “challenge” is that the thinking process for identifying and evaluating potential solution strategies should be non-trivial. He reasoned that this element of challenge compels students to progress through problem-solving processes rather than use short-cuts that can exist for more simple problems. Albert felt it is the actual progression through problem-solving processes that strengthens a students’ problem-solving ability. Reflecting on where problem-solving tasks are typically encountered, Judith expressed a similar view on the element of challenge: “So it's like when you have tests its always one of the last questions, so the hardest question would be problem solving.” Although Judith does not directly use the language of “challenging,” she described problem-solving questions as the most difficult problems on mathematics tests. Judith’s description suggests that she viewed thinking to be an essential element of problem-solving tasks, as thinking is a mitigating factor that allows students to work through the inherent difficulty and reach a solution state.

The second categorical code—problem solving deals with real-world application—represents participants’ views that problem solving should focus on the practicality of mathematics and how it can be applied to real-world situations. Similar to
phase one, real-world situations are those that students can find relatable to their experiences and that are not restricted to purely mathematical concepts. This emphasis on application can be noted in Isaac’s personal definition of problem solving: “[it is] looking at something in your life that you don’t understand or you want to change in some way and applying mathematical tools or algorithms or styles of thinking to change those things.” To him, problem solving is dealing with real-world situations using mathematics, and the way mathematics is used can take several forms. Explicitly, Isaac suggested that problem solving is the use of mathematical tools, algorithms, or ways of thinking to change or better understand real-life situations. Isaac’s view was corroborated by Albert, who posited the value of problem solving lies in that “it’s not just random thinking, but actually putting it into a process so that they can use those skills outside of the classroom as well, and in the solving of everyday problems as well, not just math.” The focus of Albert’s statement was that problem solving is the use of mathematical skills to solve everyday problems. To emphasize his point, Albert contrasted problem solving with “random thinking,” by which he meant the use of mathematics without a directedness condition (i.e., a desired solution state in mind).

Marie and Judith also displayed positive feelings about problem-solving tasks related to real-world applications: “I think those are really good because you can make that connection,” (Marie) and “I think it's something that we do every single day, and not knowing how to address those issues we see in the real world sucks” (Judith). What Marie meant by “make that connection” is that problem-solving tasks allow students to observe how mathematics connects with the real world. Marie suggested that developing real-world connections with problem solving is critical for students to perceive the value
of mathematics. Relatedly, Judith’s statement reflects an opinion that problem solving is an aspect of everyday life. Following Judith’s opinion, a lack of experience with problem-solving tasks can lead to an inability to deal with challenging real-world situations later in life. Specifically, she was insinuating that real-world situations requiring problem-solving knowledge are made achievable through experience with problem-solving tasks related to the real world.

The third categorical code—problem solving is primarily group based—identifies a view that problem solving occurs optimally when students are explaining their thinking to peers and collectively building an understanding of the task. While participants did not openly disparage individual problem solving, the general feeling was that group-based problem solving was more genuine. Group-based problem solving emphasizes that students need to engage in their learning of problem solving by challenging each other’s ideas and discussing the efficacy of different solving strategies. This thinking is demonstrated by Albert’s response to what problem-solving looks like:

[Students] would probably be very on-task, conversing back and forth, and not necessarily agreeing either. It could be a little bit of a conflict at some points - students saying: "no I don't think we should do it that way, we should do it this way." Individually, I don't know.

In his response, Albert noted that while problem solving students would be conversing about the task and potentially experiencing conflict with each other’s ideas. However, Albert was not suggesting this conflict is negative; rather, he viewed conflict as a way for students to discover the best approach for dealing with problem-solving tasks. Albert also
noted that he is not sure how individual problem solving would look, which shows how strongly he associated working in a group with problem solving.

Adding details for the value of group-based problem solving, Marie viewed groups as valuable because “in groups you'll have more ideas and it'll open up more different tools to solve a problem because you'll have the help of others.” The key point of her statement is that groups allow a greater number of ideas and solving tools (or solving strategies) to be considered. Marie viewed this as a positive aspect of problem solving because, as described in the first categorical code, problem solving is challenging, and working in groups is a way to mitigate the challenge of problem solving.

Judith described her feelings on the matter as:

I think it's important to do both individually and with a group or with another person. I think individually if you don't have someone there knowing how to problem solve, then doing it on your own is one thing, but knowing how to work with other people to solve a problem is another thing.

To Judith, limitations exist for both individual and group-based problem solving. Specifically, individual problem solving is challenging when unaware of solving processes, and group-based problem solving is challenging because there is a required ability to work effectively in groups. Given these limitations, Judith perceived problem solving as something that needs to be experienced both individually and in groups. Hence, while participants viewed problem solving as primarily group based, they did not entirely discount the value of individual problem solving.

Development of theme two. The second ontological theme in phase two—problem solving is a process for solving academic mathematics problems—emerged from
three categorical codes: problem solving is defined by educational culture, problem solving is a structured mathematical procedure, and problem solving is not constrained to application. The first categorical code—problem solving is defined by educational culture—denotes a similar perspective to that encountered in phase one: problem solving has developed out of what is valued in mathematics education. Specifically, defining problem solving requires consideration for how cultural values of mathematics influence school-based learning and interactions with parents and peers. This perspective was evident when Judith reflected on her own educational experiences:

I think culture and parental figures, obviously, coming from a family of a different culture. I think comparing the Asian culture versus the Western culture there are different approaches to problem solving or what they value in education, so I think that maybe for some students that could be where their ideas and beliefs of problem solving come from.

Speaking directly from her experience with problem solving in Asian and Western cultures, Judith suggested that ideas and beliefs about problem solving are primarily influenced by culture and parents. Her reasoning for this perspective is that culture influences what is valued in education generally, so problem solving in a mathematics classroom will necessarily reflect those values.

Albert and Isaac also presented perspectives that highlight the influence of culture on defining problem solving: “whereas ya your parents can influence it too, I think education is the most heavily weighted [influence] for sure,” (Albert) and “misconceptions and cultural norms guide a lot more than I knew they did until very recently” (Isaac). Albert, while noting that parental influence is considerable, purports
education to be the primary influence for defining problem solving. When Albert said “education,” he was referring to school-based learning experiences that are shaped by what is valued in education. Isaac, conversely, spoke more generally when proposing how problem solving is defined. To him, ideas about problem solving are largely influenced from cultural norms, by which he meant views on problem solving that could be identified as typical or standard within a culture. Moreover, Isaac noted that his viewpoint developed recently—indirectly referencing his recent experiences in the teacher education program.

The second categorical code—problem solving is a structured mathematical procedure—represents the idea that problem solving is identifiable by students using recognized steps and resources. As such, participants suggested the most obvious way to recognize whether a student has successfully performed problem solving is through their use of steps and resources. For example, when asked how she recognizes problem solving, Marie replied: “I would say through steps. If someone just gave me the answer then I can't say they have done any problem solving or showed me any.” In her view, simply providing the solution to a task is insufficient for identifying whether actual problem solving occurred. To identify that problem solving has occurred, Marie considered it necessary to see the steps used to progress from the initial state to the solution state.

This way of thinking was echoed by Albert and Judith when describing qualities of problem solving: “It shouldn't be something that is just easy and that you can just not necessarily actually have to use any setup steps” (Albert), and “using your resources and not having to ask ‘hey miss, help me out’” (Judith). Although they were speaking to the
difficulty of problem solving, Albert and Judith mentioned the use of steps and resources when discussing how problem-solving tasks are resolved. To Albert, not having to use steps would bring into question whether the task is truly indicative of problem solving. And to Judith, the use of available resources is indicative of problem solving because the student is not relying directly on the teacher. Evidently, Judith identified the use of resources as a separating factor between problem solving and simply being lead to a solution.

The third categorical code—problem solving is not constrained to application—identifies the opinion that problem solving is not solely the application of mathematics to the real world. Standing in obvious contrast to the ideas from theme one, this opinion reveals that participants also viewed problem solving as something that can be purely based in mathematics. For example, when asked if problem solving questions should be related to the real-world, Isaac explained “not always related, no, because even though it might not relate to life it still gives you practical skills to be able to use it in other problems that do relate to life.” Isaac viewed an application focus in problem solving as unnecessary to achieve the goal of practicing and instilling transferable mathematics skills. Thus, even though a problem-solving task may lack a real-world application, Isaac would first inquire whether the task develops skills that have bearing in real-world situations.

Marie and Judith expressed a broader view of problem solving that contests the notion of problem solving as solely based in real-world application: “math in general is solving problems—you're given a question and you need to find something. The fact that we have to always be finding something in math is a type of problem solving,” (Marie)
and “I think math is problem solving. I don’t know how else to explain it” (Judith). To them, math is problem solving and there is no need for further definitional elements. Therefore, any focus on application is a way to contextualize problem-solving tasks but not necessary to identify the task as problem solving. Providing further reasoning for why an application focus is unnecessary, Albert suggested “You're not always going to be able to relate it back to things that actually happen in life. It is useful if you can as much as you can, but in practicality you're not always going to be able.” Albert’s point is that while an application focus can be useful it is not always practical. What he meant by this claim is that problem-solving tasks can be constructed as purely mathematical, and attempting to connect those tasks to the real world is contrived. As such, rather than tacking on an unnatural real-world connection, Albert considered it necessary to expand the scope of problem solving to include tasks that are purely mathematical.

**Epistemological themes: Phase one.** Two epistemological themes (i.e., how mathematics problem-solving knowledge is acquired) emerged from participants’ interview data in phase one: (1) establishing a problem-solving learning environment, and (2) balanced problem-solving instruction and assessment. The first theme suggests that participants think problem-solving knowledge is primarily acquired by developing a classroom environment that encourages and supports problem solving. Such an environment is one that guides students through the learning of problem solving, encourages active participation in learning processes, and provides ample thinking opportunities. The second theme treats the acquisition of problem-solving knowledge as occurring when students are directly instructed how to problem solve and then asked to complete similar procedures. Additionally, a key aspect of this view is that problem-
solving instruction and assessment tasks should be developed and implemented with students’ learning needs in mind.

**Development of theme one.** The first epistemological theme in phase one—establishing a problem-solving learning environment—emerged from two categorical codes: *supporting student autonomy* and *emphasizing thinking opportunities*. The first categorical code—supporting student autonomy—captures the view that students acquire problem-solving knowledge when guided through the learning process and when actively involved in their learning. Isaac expressed the view of teacher guidance when describing a teacher’s role in the learning of problem solving: “I would describe the role as more of a guide, where I provide the materials and opportunities and stuff,” In his view, Isaac would simply provide the opportunities for learning problem-solving and any requisite resources. Constructing a teacher’s role in this way supports student autonomy by transferring the responsibility for learning problem solving from the teacher to the students. Judith expressed a similar way of thinking, describing her role as a teacher of problem solving to be “providing them with the opportunities—with questions that allow them to do it on their own instead of me just teaching every single day.” Here, Judith used the same language of *providing opportunities* that was used by Isaac, which suggests both Isaac and Judith saw the learning of problem solving as closely linked to student autonomy. To them, a teacher should establish an environment for learning problem solving but not dictate exactly how the learning looks. In fact, Judith specifically stated that learning opportunities should consist of students working on their own, not simply with instruction by the teacher.
To Albert, an ideal problem-solving learning environment was “An active classroom, not just sitting there receiving a lecture from the teacher . . . the teacher is having a conversation and moving around from person to person having conversations with them.” Albert’s envisioned learning environment focused on active student involvement. In his opinion, students should be conversing and working through problems as the teacher circulates. Marie added to Albert’s opinion with her description of problem-solving learning environments: “[students] exploring by themselves, like discovery is a good way to learn.” By mentioning exploring and discovery, Marie was communicating that student autonomy is an important component of learning problem solving.

The second categorical code—emphasizing thinking opportunities—denotes the opinion that students gain problem-solving knowledge when teachers provide ample opportunities for thinking about problem solving. Participants considered thinking opportunities to be essential for developing students’ understanding of why particular problem-solving techniques are useful, when specific techniques should be utilized, and that multiple techniques can often lead to the same solution state. Albert highlights the importance of thinking opportunities when describing how he envisions problem-solving experiences: “not just wildly attempting to solve it—you have an order to how you’re thinking . . . they can lay out their thinking, identify why this makes sense and how they got to the answer. Explaining their thinking.” In addition to explicitly mentioning thinking frequently during conversation, Albert noted that it is not enough for students to be able to solve a problem—students should be able to explain the why and how underpinning chosen techniques. Isaac communicated a similar opinion when describing
what problem solving should ideally look like: “actually think[ing] about it rather than following prescribed steps . . . critically analyzing problems and thinking about problems and mathematics.” Similar to Albert, Isaac explicitly mentioned thinking multiple times when describing what he perceived to be ideal problem-solving experiences for students. Moreover, by stating that students should be “critically analyzing problems,” Isaac was communicating that students need to be capable of thinking through how particular problem-solving techniques will connect the unclear problem state to an acceptable solution state—evident by his use of critical, which denotes the evaluation of techniques.

Building on Albert’s and Isaac’s ideas, Judith proposed that students’ problem-solving knowledge is strengthened and expanded by “ask[ing] those thinking questions, like: ‘What did you do to get to this?’ ‘Why did you do that?’ ‘How did you do that?’” Whether it is students asking these questions or the teacher, Judith viewed problem-solving experiences as most valuable for students’ learning when students are required to explain and support their thinking. Additionally, Judith noting that she would ask what, why, and how questions to probe students problem-solving choices shows that she assigned considerable value to thinking opportunities for developing robust problem-solving knowledge. Marie further highlighted the importance of thinking by suggesting that one of a teacher’s responsibilities in students’ learning of problem solving is the “encouragement of different ways of thinking, different ways of approaching a question.”

In addition to ensuring that students can support their thinking, Marie felt it was important for teachers to emphasize that problems can be thought through in a variety of ways and solved using a variety of techniques. Marie considered the encouragement of different ways of thinking to be an invaluable factor in extending students’ knowledge of
problem-solving techniques and their perceptions about the flexibility of those techniques.

**Development of theme two.** The second epistemological theme in phase one—balanced problem-solving instruction and assessment—emerged from two categorical codes: *accounting for students’ needs* and *demonstrating problem-solving techniques and requiring practice*. The first categorical code—accounting for students’ needs—represents a perspective that for students to become knowledgeable problem solvers, teachers need to account for student’s unique learning needs. By combining consideration for students’ learning needs with curricular goals, participants felt that students more readily acquire problem-solving knowledge. While all participants recognized the value of teaching problem solving as mandated by the Ontario curricula, there was a common feeling that this would be ineffective if not combined with student-based considerations. A focus on students’ needs can be seen when Marie discussed that she would teach problem solving “based on the needs of the curriculum, but also the needs of my class—the needs of my students. Maybe they're not ready for one part of it so I have to build up to their needs as well.” Marie viewed the teaching of problem solving as optimally situated between curricular expectations and students’ needs. Moreover, by noting that she would “build up to their needs,” Marie was suggesting that she would be continuously revising her teaching of problem solving based on the progress of students’ learning. Judith conveyed a similar way of thinking when discussing how she would pace her teaching of problem solving: “I think that would be based on what my students need. To just go with the flow and see what they need. What do they need the most right now.”

In contrast to Marie, Judith did not mention the curriculum and instead focused entirely
Albert and Isaac also expressed views that highlighted learning needs as an essential consideration for developing students’ problem-solving knowledge:

I think you need to make problems fairly achievable so they don't get discouraged and dislike problem solving, because often times if it's too difficult of a problem, no matter the steps I'm following, I'm not going to be inclined to continue if it's something that's super difficult. I think you need to gradually build it up. (Albert)

It's different for every student. As we discover through being in school and being in teacher's college, people are different, students are different . . . As long as you hit all the expectations and can show me that you learned something from this unit, then make whatever you want for me. Just do something. (Isaac)

Albert’s passage, while not directly using the language of students’ needs, references a need for problem-solving tasks to be achievable, by which he meant that all students should be able to achieve some level of success with problem solving. However, for problem-solving tasks to be achievable, students’ learning needs are a requisite consideration. Additionally, similar to the “building up” described by Marie, Albert mentioned that problem-solving tasks should gradually build in difficulty, which references the learning of problem solving based on students’ progress. Taking an even more open view when discussing problem-solving tasks, Isaac described only looking to see that learning has occurred through the fulfillment of curriculum expectations; how students show their achievement of curriculum expectations, however, is left open to
students’ discretion. Isaac attributed his viewpoint to both personal experiences in school and learning experiences in the teacher education program. Overall, participants felt accounting for students’ needs was essential for students to perceive problem solving as a feasible and valuable tool.

The second categorical code—*demonstrating problem-solving techniques and requiring practice*—represents a perception that problem-solving knowledge is optimally acquired when students are directly shown general and specific problem-solving techniques and subsequently tasked with using those techniques in the completion of problem-solving tasks. In contrast to the other epistemological theme in phase one, this theme emphasizes students gaining experience with valuable problem-solving techniques in lieu of their learning needs. Participants’ reasoning for this perspective was that if students are to develop robust problem-solving knowledge, it is necessary to directly demonstrate problem-solving techniques and require students to practice those techniques in related tasks. This thinking was exemplified in Marie’s response to her perceived role in students’ learning of problem solving: “my role is to teach the skills and the strategies and to facilitate their learning of problem solving . . . true teacher demonstration at first I think.” A key point in Marie’s statement is that teacher demonstration should occur first. Rather than letting students confront problems using unguided strategies, Marie felt it was necessary to teach students how to utilize specific problem-solving skills and strategies.

Supporting Marie’s stance, Albert proposed his responsibility in students’ learning of problem solving to be “giving them the knowledge and the steps that go with problem solving, so first show them what problem solving looks like—what things we
should be thinking about while doing problem solving.” Similar to Marie, a key point of Albert’s statement is that teacher demonstration should be the first step in students’ learning of problem solving. In Albert’s opinion, a teacher needs to ensure that students have knowledge of useful problem-solving techniques and an understanding of the efficacy of those techniques. Additionally, Marie’s and Albert’s condition that teacher demonstration should occur first highlights their perception that teacher demonstration is just one piece of learning problem solving—acquiring problem-solving knowledge is not accomplished exclusively through instruction.

In addition to viewing the demonstration of problem-solving techniques as necessary for developing students problem-solving knowledge, participants highlighted the importance of students practicing demonstrated techniques. Judith, for example, speculated that she would know her students had acquired problem-solving knowledge “if they can solve it, like a general [problem], they understand how to solve a problem.” By “general problem,” Judith was referring to problems requiring problem-solving techniques she would have previously demonstrated during instructional sessions. Therefore, in Judith’s opinion, students’ ability to utilize demonstrated problem-solving techniques was directly linked to their problem-solving knowledge. Isaac communicated a similar opinion when describing the task criteria from which he would evaluate students’ problem-solving knowledge:

Hav[ing] a clear outline of what is expected (e.g., use these words, do this at least once, have at least one picture, have at least three sources, etc.). Have some expectations for that, and whether they follow the expectations or not and whether they understand the material using those expectations.
When Isaac repeatedly uses the language of “expectations,” he is articulating the importance of seeing students use problem-solving techniques that he has previously demonstrated (e.g., expecting that students will draw a diagram). To Isaac, students’ ability to employ demonstrated problem-solving techniques allows for judgement of their understanding of problem solving.

**Epistemological themes: Phase two.** Two themes emerged from participants’ interview data in phase two: (1) exploring problem solving and communicating thinking, (2) demonstrating problem-solving techniques and assessing students’ ability with those techniques. The first theme suggests that participants felt problem-solving knowledge is primarily acquired when students are given the freedom to explore problem solving and ample opportunities to communicate their thinking related to problem solving. A key tenet of this view is that teachers can assess students’ understanding of problem solving through their communication of thinking processes; however, these thinking processes should manifest naturally through investigative learning. The second theme represents an opinion that problem-solving knowledge is acquired when students receive instruction related to problem-solving techniques and are subsequently expected to practice those techniques on related tasks.

**Development of theme one.** The first epistemological theme in phase two—exploring problem solving and communicating thinking—emerged from two categorical codes: *exploring problem solving* and *students demonstrating their thinking*. The first categorical code—exploring problem solving—describes the view that problem-solving knowledge is acquired when students are given the opportunity to explore problem solving through investigative learning. Participants reasoned this investigative approach
allows for students to develop their thinking processes related to problem solving without relying on the teacher. Isaac articulated this viewpoint: “I think it's important to encourage exploration and curiosity and not necessarily staying within the lines.” When mentioning “staying within the lines,” Isaac was referring to traditional instructional practices where the students receive all information about problem solving from the knowledgeable instructor. Contrasting this way of learning, Isaac advocated for encouraging students to explore problem solving without relying solely on the teacher for information. Expressing a similar viewpoint, Albert felt that optimal learning of problem solving occurred in his teaching when “Using investigations where they had to figure it out on their own, where you're not just showing them how to do it.” Here, Albert plainly stated that optimal learning occurred when the learning was investigative. Moreover, when Albert noted that investigative learning needs to be students working on their own, he was suggesting that students should be working (potentially in groups) towards solutions with minimal teacher direction.

In addition, participants felt explorative learning experiences were valuable at any point in the learning process, such as at the introduction of a new topic, as described by Marie: “I like to do investigation and discovery learning in the beginning before I teach them something. That allows them to think on their own and think creatively on solving a problem without actually giving them a formula first.” The way Marie described her investigation-first approach to teaching shows that she perceived the acquisition of problem-solving knowledge to be fostered when students are given explorative freedom. Marie viewed the investigative aspect of problem solving as essential for developing
students’ thinking processes. Judith provided an example of such an approach from her teaching experience:

I did an assignment where I just kind of threw them into it, "sorry guys threw you into the water," and then I was just there . . . I was there, obviously, to guide them and I think that worked.

Although Judith did not exactly articulate this approach as investigation or explorative learning, her description of “I just kind of threw them into it,” and that she acted as a guide conveys an investigative approach to teaching. Judith felt that by providing the opportunity to enact problem solving without teacher direction, students were more likely to truly exercise and grow their problem-solving knowledge.

The second categorical code—students demonstrating their thinking—denotes the opinion that it is essential for teachers to see students’ thinking processes as their problem-solving knowledge develops. Several components form this opinion: students communicating their thinking to the teacher, students communicating their thinking to peers, and students demonstrably organizing their thinking processes (e.g., written planning). By collating these different ways of observing students’ thinking, participants considered it possible to make an accurate conclusion about whether problem-solving knowledge has been acquired. For example, when discussing how she would evaluate students’ understanding of problem solving, Judith proposed:

If they are talking about the task and not going off topic. I think listening for the different ideas and their input . . . If they can tell me ideas, like how they would proceed, whether this is right or not. If they are on the path toward solving it . . .

As a teacher I think it is important to be like "hey, show me how you got to this,"
because, like I said, there are different ways to solve a problem, right. Maybe there is one way that I’m doing but a student has solved the question a different way that I didn't think of, so I could share that.

In this discussion, Judith indicates multiple ways she analyzed students’ thinking: listening to students’ conversations, engaging the students in one-on-one exchanges, and examining how students’ problem-solving organization helped to connect them with the solution state. The final point made by Judith shows that by requiring students to demonstrate their thinking, opportunities can arise where novel thinking is shared among students for analysis and questioning.

Marie and Albert also expressed ways of teaching problem solving that highlighted students’ thinking: “if they didn't have the steps labelled or something it doesn't really matter as long as they show their thought” (Marie), and

Going around the classroom and asking them questions: ‘What's happening here?’ ‘Why did that heat up?’ Or ‘why did the colour change occur there?’ ‘What do you think that means?’ Always asking questions and never straight up telling them the answer, just asking another question. (Albert)

The common emphasis of each statement is getting students to think carefully about their problem solving, which is inextricably tied to the importance of observing students’ thinking related to problem solving. Marie highlighted the importance of observing students’ thinking by noting her lack of concern for minor details like organizing steps; what mattered to Marie is observing students’ ability to communicate how they attained the solution state for a problem-solving task. Similarly, Albert described the posing of
various questions to students, as he is interested in hearing how students are thinking about a problem rather than providing students with direct confirmation.

**Development of theme two.** The second epistemological theme—demonstrating problem-solving techniques and assessing students’ ability with those techniques—emerged from two categorical codes: *demonstrating problem-solving techniques* and *assessing students’ ability with problem-solving techniques*. The first categorical code—demonstrating problem-solving techniques—represents a view that for problem-solving knowledge to be acquired, students should to be directly shown problem-solving techniques. Whereas the explorative approach focused on students investigating problem-solving tasks with minimal teacher direction, the demonstration approach stresses the importance of teachers presenting and explaining problem-solving techniques they deem useful. Albert described how this demonstration approach looked in his teaching: “In the lecture we would do a sample question and say ‘These are the steps we should follow to solve this type of problem,’ and then they would go about that trying to do those same steps as well.” In the description of his teaching, Albert directly referenced the demonstration and provision of problem-solving steps that students were subsequently expected to practice. Albert’s hope was that students would later be able to use the steps he had demonstrated to solve problems of a similar type. Judith’s reflection on her teaching showed a similar approach: “I think we're models, so if students can see how we're going about solving an issue and also we give them the opportunities.” To Judith, teachers need to model proper problem solving, by which she means presenting particular problem-solving techniques and explaining the efficacy of those techniques. By acting as a model of problem solving, Judith thought students would later be able to draw upon the
experience of seeing her use problem-solving techniques to address other problem-solving tasks.

Expanding on the details of a demonstration-based approach, Isaac outlined his view of how problem-solving knowledge is optimally acquired:

So my role as a teacher is to give them tools to problem solve and explaining why you might you want to check that and why it's useful . . . Being given general tools or even specific tools and told the general purpose, like the checking your units, is really effective.

In this excerpt, Isaac described teacher demonstration as a way of imbuing students with the techniques (or tools) of problem solving and helping students understand why those techniques are useful. Additionally, Isaac noted that he would provide both general and specific problem-solving techniques, thus ensuring that students are fully equipped for future problem-solving tasks.

The second categorical code—assessing students’ ability with problem-solving techniques—identifies an opinion that to know whether students have acquired problem-solving knowledge it is necessary to see them using previously demonstrated techniques. Marie conveyed this opinion when reflecting on an assessment task she created:

They were solving by substitution, so I would write out all the steps for them, like prompts and they would just fill in the blanks, and on the next page it was just the problem. Some of them decided to look at the front, which they were supposed to make use of, and some of them didn't and didn't know how to do it. That was kind of a guide for them to look at and then do on their own.
For her assessment task, Marie required students to complete two problems: the first directly led students through the previously-instructed solving techniques, and the second expected students to use those problem-solving techniques without explicit instruction. Evidently, what Marie wanted to see was students’ ability to successfully employ problem-solving techniques that she considered important to know. By first providing students with the expected steps and subsequently taking those steps away, Marie was hoping to see that students were understanding when the problem-solving techniques should be used.

Judith and Albert also expressed the opinion that gauging students’ problem-solving knowledge requires assessing their use of previously demonstrated problem-solving techniques: “Maybe like an unexpected task, not a pop quiz but something along those lines to see—kind of gauge where they are in comparison to something we’ve done in class. So that could have been modeled already” (Judith), and “So if they can tell me and I can see that they have gone through those steps then I would know that they are doing a good job of problem solving” (Albert). For Judith’s proposed task, she comments it would be used to determine students’ current ability in relation to problem-solving content covered during a previous instructional period. What Judith meant by this statement is that she would like to see students using problem-solving techniques previously demonstrated, and how well those solving techniques are emulated would be representative of students’ problem-solving knowledge. Albert, when mentioning “those steps,” was referring to steps previously demonstrated during an instructional session. As such, Albert was viewing students’ use of instructionally-similar techniques (or steps) while progressing to a solution state as representative of their problem-solving
knowledge. Additionally, Albert described his expectation that students should be able to discuss the problem-solving techniques they have used, as he is hoping students have retained an understanding of why those techniques are valuable.

**Findings Summary**

This study of Eastern Ontario pre-service IS mathematics teachers’ beliefs about problem solving produced several intriguing and noteworthy findings. With regard to the quantitative BMPSQ data, a two-way interaction between gender and experience along with several simple main effects were discovered. The two-way interaction revealed that significant, gender-related changes occurred in pre-service teachers’ thinking about difficult problems following initial coursework and the first teaching-based practicum. Specifically, following coursework and the practicum, male pre-service teachers expressed more positive views towards difficult problems in relation to problem solving as compared to before the coursework and the practicum. Another significant finding was the greater perceived usefulness of mathematics by pre-service mathematics teachers with science as an additional teaching subject, as compared to pre-service mathematics teachers without science as an additional teaching subject. When looking at BMPSQ data in each phase separately, this difference was only measured after pre-service teachers returned from their teaching-based practicum.

With regard to the qualitative interview data, several prominent themes were identified for pre-service teachers’ ontological and epistemological expressions of problem solving in phases one and two. The ontological themes were somewhat consistent between the two phases of study, with the first identified theme—problem solving is a collaborative thinking process for solving real-world problems—only
evolving to indicate that the thinking process should be challenging. There was, however, considerable development in the other ontological theme: in phase one pre-service teachers felt problem solving was bounded by mathematics education, while in phase two they felt problem solving could also be purely a process of solving academic mathematics problems. Although these themes are similar, differences between the themes meaningfully influence interpretations of pre-service teachers’ ontological beliefs about problem solving in each phase.

In contrast, there was greater development between phases for the identified epistemological themes. The epistemological themes in phase one suggest that pre-service teachers felt the acquisition of problem-solving knowledge was accomplished through establishing a problem-solving learning environment, and through balanced problem-solving instruction and assessment. While there are some contrasting ideas in these themes, pre-service teachers generally viewed the learning of problem solving as a balance between exploration with problem solving and being shown the mathematical tools of problem solving. In phase two pre-service teachers elaborated on the epistemological themes from phase one by describing the acquisition of problem-solving knowledge as accomplished through exploring problem solving and communicating thinking, and demonstrating problem-solving techniques and assessing students’ ability with those techniques. Evidently, the themes from phase two have similar core meanings to the themes in phase one, with the change being a greater ability to articulate important characteristics of the themes.
Chapter 5

Discussion

The discussion chapter of this study is divided into three sections: the first section presents answers to the research questions outlined in the introduction of this study. The second section identifies the limitations of this study and how those limitations should be considered when interpreting the overall meaning of the findings. The third section, based on the findings from this study, presents recommendations for the preparation of Ontario intermediate-senior (IS) mathematics teachers and for future research.

Answering the Research Questions

The development and execution of this mixed methods study was guided by two research questions:

1) What are the ontological and epistemological beliefs about mathematics problem solving held by Eastern Ontario pre-service IS mathematics teachers?
   a. How do experience and gender interact regarding pre-service teachers’ beliefs about mathematics problem solving?
   b. How do experience and teaching subject interact regarding pre-service teachers’ beliefs about mathematics problem solving?

2) What features of problem solving are emphasized in Eastern Ontario pre-service teachers’ communication of mathematics problem solving, as influenced by their problem-solving belief structures?

In answering the first research question I draw from the qualitative findings of the pre-service IS mathematics teachers’ interview data, incorporating both phases of ontological
and epistemological data. The sub-questions associated with the first research question are answered using the quantitative findings of the Beliefs about Mathematical Problem Solving Questionnaire (BMPSQ) data for phases one and two. To answer the second research question I present a mixing of the qualitative and quantitative data, which followed a convergent parallel design (Creswell, 2014; Li, Marquart, & Zercher, 2000). Where relevant, answers to the research questions are supplemented with connections to the literature.

**Research Question One**

*What are the ontological and epistemological beliefs about mathematics problem solving held by Eastern Ontario pre-service IS mathematics teachers?*

Interpreting pre-service teachers’ ontological and epistemological beliefs about problem solving is an inherently complex task. The complexity of interpreting beliefs primarily derives from three qualities of beliefs: beliefs can only be inferred, not directly measured; understanding particular beliefs necessitates the understanding of connections within the belief structure; and beliefs form intricate structures influenced through a process of cultural transmission (Pajares, 1992). A multi-faceted approach that examines belief structures from various viewpoints is therefore necessary to develop a coherent picture of a sample’s beliefs. Accordingly, participating pre-service IS mathematics teachers’ beliefs about problem solving were analyzed from the ontological and epistemological perspectives at two distinct points in time.

The resultant descriptions of pre-service teachers’ ontological and epistemological beliefs were drawn directly from the qualitative findings of the interview data. It is important to note that each overarching belief contains several component beliefs, which
reflects the well-established principle that beliefs form intricate structures, so understanding an overarching belief structure (e.g., an ontological belief) requires understanding the associated component beliefs (Pajares, 1992). For example, although the pre-service teachers were asked to define problem solving during the interviews, their personal definitions were treated as merely one component of their ontological beliefs about mathematics problem solving. As such, descriptions of pre-service teachers’ ontological and epistemological beliefs are guided by descriptions of the associated component beliefs. Alignment of pre-service teachers’ beliefs with the problem-solving literature and Ontario mathematics curricula is provided to further explicate their beliefs and reveal connections to their teaching context, respectively. The beliefs identified by this study are shown in Table 13, presented in no particular order. It is important to note that the identified beliefs developed from analysis of the ontological and epistemological themes in each phase individually and from the noted changes in themes between each phase. Therefore, the identified beliefs are not taken solely from phase one or phase two, but instead were formulated by drawing from the holistic meanings of pre-service teachers’ perceptions across the entire study.
Table 13

*Pre-Service IS Mathematics Teachers’ Beliefs about Mathematics Problem Solving*

<table>
<thead>
<tr>
<th>Ontological Beliefs</th>
<th>Epistemological Beliefs</th>
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<tbody>
<tr>
<td>1) Problem solving is a challenging and collaborative thinking process for solving real-world problems</td>
<td>1) Problem-solving knowledge is acquired by establishing an environment where students can explore problem solving and communicate their thinking</td>
</tr>
<tr>
<td>2) Problem solving is a process for solving academic mathematics problems</td>
<td>2) Problem-solving knowledge is acquired when problem-solving techniques are demonstrated for students, and students are subsequently given opportunities to enact those techniques</td>
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**Ontological beliefs.** In each phase of this study the first objective of the interviews was to determine the pre-service teachers’ ontological perceptions of problem solving; that is, the intent was to determine what they thought problem solving *is*. Two ontological themes emerged from inductive analysis of the interview data for each phase of this study, and analysis of these themes in each phase and across phases suggested two underlying ontological beliefs.

**Ontological belief one.** The ontological belief most clearly represented in the data was that *problem solving is a challenging and collaborative thinking process for solving real-world problems*. Although there was greater emphasis on problem solving as challenging in phase two as compared to phase one, all components of the first ontological belief were clearly represented in both phases (i.e., the real-world connection, the thinking process attribute, and the need for collaboration).

A main component of the first ontological belief was pre-service teachers’ belief that problem solving is used to solve real-world problems. Pre-service teachers talked
extensively about problem solving as a mathematical process deeply connected to real experiences. Consequently, when discussing the essential characteristics of problem-solving questions, pre-service teachers consistently communicated that problem-solving involves problems with direct or indirect relation to students’ experiences in the world. The connection of problem solving with students’ experiences aligns with one of the four essential problem criteria noted by the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2010): incorporation of mathematics that is both academically and personally important for students. The real-world aspect is also reflected in the Ontario mathematics curricula, which identify problem solving as “the primary focus and goal of mathematics in the real world” (MOE, 2005a, p. 12; MOE, 2005b, p. 13), and in the literature concerning important criteria for problem-solving tasks (Jonassen, 2004, Hollingworth & McLoughlin, 2005).

Another component of the first ontological belief was pre-service teachers’ view of problem solving as a thinking process. Rather than relying on rote procedures that students could simply memorize, pre-service teachers believed that problem solving requires students to enact focused thinking about potential solving strategies, which reflects two of the Mayer and Wittrock (2006) problem-solving characteristics: problem-solving is a cognitive endeavour (i.e., it relies on focused thought) and problem-solving is a process (i.e., it occurs through an iterative process of mental representation and knowledge manipulation). This view of problem solving as a thinking process is also reflected in the Ontario mathematics curricula (MOE, 2005a, 2005b, 2007), which identify problem solving as an opportunity to develop students’ critical thinking skills.
Interestingly, after pre-service teachers gained experience through initial coursework and the first teaching-based practicum, the thinking aspect of problem solving was further articulated to include an aspect of challenge. Prior to coursework and the practicum, pre-service teachers described problem solving as a thinking process without much emphasis on why they possessed this view; however, after coursework and the practicum pre-service teachers specifically articulated that problem solving is non-trivial and should pose a challenge for students because it draws on unfamiliar mathematics and solving techniques. Characterizing problem solving in this way reflects the nonroutine portion of the integrated problem framework shown in Figure 1 (Hollingworth & McLoughlin, 2005; Mayer & Wittrock, 2006), yet this characterization does not clearly identify pre-service teachers’ thoughts on well-defined versus ill-defined problems.

The challenge aspect is connected to the third component of the first ontological belief, which identifies problem solving as a collaborative activity. Pre-service teachers viewed problem solving as a process that rarely occurs as an individual activity in the real world, so problem solving in a mathematics classroom should similarly be collaborative. Through collaboration, pre-service teachers felt that students would be able to share, analyze, and question each other’s thinking, which would enhance their thinking processes for future problem solving—a viewpoint reflected in both the NCTM (2010) problem criteria and the Ontario mathematics curricula (MOE, 2005a, 2005b, 2007). Although the notion of individual problem solving was not outright rejected, pre-service teachers more heavily emphasized the importance of students working collaboratively.
The value of individual problem solving was felt to primarily stem from an ability to assess individual students’ problem-solving ability.

**Ontological belief two.** The second ontological belief about problem solving held by pre-service teachers was that *problem solving is a process for solving academic mathematics problems*. Although this belief might seem to have contrasting elements with ontological belief one, the two ontological beliefs are more parallel in meaning than contradictory. Whereas the first ontological belief represents a largely unfettered way of thinking about problem solving, the second ontological belief shows that pre-service teachers recognized problem solving is also defined by the education system in which they will be employed. The duality of pre-service teachers’ beliefs is to be expected, as it is well-established that individuals often hold belief structures with differing elements for a single topic (Wallace & Kang, 2014; Yoon & Kim, 2016).

Two components form the second ontological belief, with the first being pre-service teachers’ belief that problem solving is defined by cultural values for mathematics within the broad teaching and learning context. Specifically, pre-service teachers felt that identifying the characteristics of problem solving required consideration for how problem solving was reflected in the relevant mathematics curricula and the rhetoric of influential figures in students’ lives (e.g., teachers and parents). Therefore, pre-service teachers were recognizing that problem solving may look distinctly different depending on the curricular jurisdiction, and it is this dependency that led to the use of *academic* as a descriptive term. Specifically, academic mathematics problems are those whose elements students need to become proficient with and knowledgeable about in order to demonstrate competency in problem solving. Viewing problem solving in this culturally-dependent
way is consistent with empirical studies that have shown pre-service teachers’ beliefs about problem solving are similarly influenced by the teaching and learning context (Bal, 2015; Xenofontos & Andrews, 2014).

The other component of the second ontological belief identifies pre-service teachers’ view of problem solving in the context of Ontario mathematics classrooms. Based on the pre-service teachers’ experiences, problem solving has often been treated as a structured mathematical procedure (e.g., drawing a diagram, identifying given information and unknowns, and specific algorithms) and existed without a plainly obvious connection to the real world. As such, being successful with problem solving in Ontario mathematics (i.e., the solving of academic mathematics problems) requires a robust knowledge of problem-solving techniques and the ability to apply those techniques without the conceptual aid of real-world context. An essential detail to note with this component belief is that both elements—problem solving as a structured mathematical procedure and problem solving as not requiring real-world context—were not viewed in a negative sense. On the contrary, pre-service teachers believed that the ability to manage these elements was necessary to be proficient in problem solving as it exists in an Ontario mathematics context.

Pre-service teachers’ conceptualization of problem solving in Ontario mathematics is reflected in the definition of problem solving detailed by Metallidou (2009), who directly referenced the use of methods and strategies; the framework provided by Schoenfeld (1985, 2013), which highlighted the importance of heuristic strategies; the NCTM (2010) essential problem criteria, which note that problem solving should be academically relevant for students; and the Ontario mathematics curricula
which emphasize the importance of problem solving strategies. Additionally, characterizing problem solving as a structured mathematical procedure reflects the routine portion of the integrated problem framework shown in Figure 1 (Hollingworth & McLoughlin, 2005; Mayer & Wittrock, 2006), yet pre-service teachers’ thoughts on whether problem solving deals with well-defined or ill-defined problems was not clearly communicated.

Interestingly, pre-service teachers’ emphasis on real-world context as not compulsory for the characterization as problem solving did not emerge until after the coursework and practicum. However, the emphasis on real-world context as unnecessary is not representative of a distinct change in pre-service teachers’ beliefs about problem solving, as beliefs are resistant to change (Pajares, 1992). Instead, as previously stated, the development is representative of an improved ability to articulate beliefs, owing to cultural transmission of understandings of problem solving through teacher education experiences. Such development in teachers’ ability to articulate their beliefs after teacher education experiences is well-supported in the literature (Borg, 2011).

Epistemological beliefs. The second objective of the interviews in each phase of this study was to determine the pre-service teachers’ epistemological perceptions of problem solving; that is, the intent was to determine how they thought problem-solving knowledge is acquired. Two epistemological themes emerged from inductive analysis of the interview for each phase of this study, and analysis of these themes in each phase and across phases suggested two underlying epistemological beliefs.

**Epistemological belief one.** The first epistemological belief clearly represented in the data was that *problem-solving knowledge is acquired by establishing an environment*
where students can explore problem solving and communicate their thinking. Between the two phases, pre-service teachers again displayed an improved ability to articulate their beliefs. Specifically, following coursework and the first teaching-based practicum, pre-service teachers emphasized the importance of observing students’ thinking processes when exploring problem solving. While the provision of thinking opportunities was discussed prior to coursework and the practicum, it was not until afterwards that pre-service teachers emphasized the need to see the progress of students’ thinking related to problem solving.

Evidently, a main component of the first epistemological belief is that pre-service teachers considered opportunities to explore problem solving as essential for developing students’ problem-solving knowledge. When discussing the exploring of problem solving, pre-service teachers were referring to opportunities where students work through different problem-solving tasks without direct teacher guidance. Pre-service teachers believed the exploration approach to be valuable because it supports student autonomy by strategically removing the teacher as a crutch for students, requiring them to take ownership over their learning—an idea also highlighted by Schoenfeld’s (1985, 2013) problem solving framework. Additionally, because problems with real-world relevance were discussed in relation to the exploration approach, pre-service teachers believed that exploration opportunities with problem solving were valuable for rousing student engagement, which is a sentiment mirrored by the Ontario mathematics curricula (2005a, 2005b, 2007).

The second component of the first epistemological belief is that pre-service teachers felt it was necessary for students to communicate their thinking while working
through problem-solving tasks. By observing how students think through a problem-solving task—communicated verbally or in writing—pre-service teachers believed it was possible to gauge students’ understanding of problem solving, which exactly aligns with the NCTM’s (2010) stance that problem-solving tasks should always provide assessment opportunities. Moreover, pre-service teachers felt it was important to observe students’ thinking through direct communication with the students and through indirect monitoring of communication amongst students. Student-to-student communication was considered valuable for helping students recognize that problem solving can occur in a variety of forms. Pre-service teachers’ emphasis on the value of communication in problem solving is similarly emphasized by the Ontario mathematics curricula (2005a, 2005b, 2007), which identify communication as vital for students learning to share, collaborate on, and question ideas.

In addition to the component beliefs exhibiting connections to the literature and curricular documentation, the overall meaning of the first epistemological belief reflects the NCTM’s (2010) purported foundation to all efficacious problem solving in mathematics—developing a problem-solving culture in the classroom. Pre-service teachers, similar to NCTM’s (2010) statements, believed that students’ learning of problem solving necessarily needs to occur in an environment where it is treated as an integral aspect of mathematics, supported by the communication of problem solving between all members of the classroom.

**Epistemological belief two.** The second epistemological belief held by the pre-service teachers was that *problem-solving knowledge is acquired when problem-solving techniques are demonstrated for students, and students are subsequently given*
opportunities to enact those techniques. Again, while the second epistemological belief might seem to have contrasting elements with epistemological belief one, the two epistemological beliefs are more parallel in meaning than contradictory. Similar to the ordering of ontological beliefs, epistemological belief one is a largely unfettered way of teaching problem solving, and epistemological belief two shows pre-service teachers’ recognition that teaching problem solving in Ontario mathematics includes necessary elements. When articulating the second epistemological belief prior to the coursework and practicum, pre-service teachers more strongly emphasized the need for instruction and assessment to account for students’ learning needs. Although this emphasis was not entirely absent in post-coursework and practicum interviews, there was a marked shift toward emphasizing the importance of ensuring students can utilize common problem-solving processes.

Pre-service teachers’ change in emphasis related to students’ learning needs highlights one of the main components of the second epistemological belief: teachers should demonstrate problem-solving techniques. Pre-service teachers believed students’ inability to employ useful problem-solving techniques—such as the commonly referenced framework developed by Polya (1945)—would jeopardize students’ future success with problem solving. Therefore, pre-service teachers considered it the teachers’ responsibility to impart a foundation of problem-solving techniques. The demonstration approach is thoroughly supported by the problem-solving literature, which has shown the instruction of specific and general problem-solving processes to be instrumental in students’ development as effective problem solvers (Griffin & Jitendra, 2009; Jitendra, Dupuis, & Zaslofsky, 2014). Additionally, the demonstration approach is reflected in
Schoenfeld’s (1985, 2013) framework, which emphasizes heuristic strategies; and the Ontario mathematics curricula (2005a, 2005b, 2007), which underscore the importance of students learning general and specific problem-solving strategies.

The other component of the second epistemological belief is that students should be given opportunities to enact previously demonstrated problem-solving techniques. Pre-service teachers perceived the value of the demonstrate-then-enact structure to be twofold: one, it allows students to exercise newly-learned problem-solving techniques and explore unclear elements; and two, it provides teachers with an avenue to assess students’ understanding of specific problem-solving techniques. Effectively, pre-service teachers believed students’ practicing of previously demonstrated problem-solving techniques to be a necessary element to ensuring that students are equipped with the knowledge and confidence to handle future problem-solving tasks. This belief is supported by the NCTM (2010) essential problem criteria and the Ontario mathematics curricula (MOE, 2005a, 2005b, 2007), both of which highlight the importance of assessment opportunities in students’ learning of problem solving.

**Research Question One: Sub-Question One**

*How do experience and gender interact regarding pre-service teachers’ beliefs about mathematics problem solving?*

The answer to this question stems from quantitative analysis of pre-service teachers’ BMPSQ data. Following from reliability analysis of the BMPSQ scales, this question was examined only for the difficult problems, usefulness, understanding, and effort scales (scales with reliabilities ≥ .7). The difficult problems scale was the sole scale of the BMPSQ to display a significant, gender-related change in pre-service teachers’
beliefs between phase one and phase two. Specifically, male pre-service teachers expressed a more positive view towards difficult problems following initial coursework and the first teaching-based practicum.

The belief addressed by the difficult problems scale is “I can solve time-consuming mathematics problems” (Kloosterman & Stage, 1992, p. 115). Apparently, male pre-service teachers felt more confident in their ability to solve difficult mathematics problems following the coursework and practicum. Changes in beliefs primarily occur through a process of cultural transmission (Pajares, 1992), and relatedly, while completing coursework and the practicum, pre-service teachers garnered first-hand experience teaching and discussing problem solving as expected by the Ontario mathematics curricula (MOE, 2005a, 2005b, 2007). Considering that experience is known to positively influence teachers’ confidence in mathematics (Harper & Daane, 1998), coursework and practicum teaching experience appear to have been a positive influence for male pre-service teachers’ confidence with mathematical problem solving. However, it is important to note that the change in male pre-service teachers’ scores on the difficult problems scale was subtle, which is to be expected. Beliefs tend to resist change (Pajares, 1992), so a subtle increase in scale scores suggests that beliefs about difficult problems are, as expected, not easily changed. The inference that beliefs about difficult problems are not easily changed is supported by the lack of significant change in female pre-service teachers’ scores on the difficult problems scale.

Overall, the data indicated that experience and gender do interact to influence pre-service teachers’ beliefs about difficult problems, with male pre-service teachers becoming more confident in their ability to solve difficult problems following the
coursework and practicum. Conversely, it does not appear that experience and gender interact to influence pre-service teachers’ beliefs about the usefulness of mathematics, the importance of understanding concepts in mathematics, or the effect of effort on mathematical ability.

**Research Question One: Sub-Question Two**

*How to experience and teaching subjects interact regarding pre-service teachers’ beliefs about mathematics problem solving?*

Similar to sub-question one, the answer for sub-question two stems from quantitative analysis of pre-service teachers’ BMPSQ data—specifically the difficult problems, usefulness, understanding, and effort scales. None of these scales were found to display a significant, teaching subject-related change in pre-service teachers’ beliefs between phase one and phase two. However, collapsing the data across the two phases did reveal a significant difference in scores on the usefulness scale for science and non-science teaching subjects. Analyzing each phase separately revealed that this significant difference emanated primarily from data collected following initial coursework and the first teaching-based practicum.

The belief addressed by the usefulness scale is “Mathematics is useful in daily life” (Kloosterman & Stage, 1992, p. 115). As such, pre-service teachers with a science teaching subject had a greater appreciation for the usefulness of mathematics in daily life than pre-service teachers with a non-science teaching subject. Explaining this change required an awareness that individual beliefs coalesce to form intricate systems of beliefs (Pajares, 1992). Pre-service teachers with a science teaching subject hold a belief structure for problem solving that is informed by many academic experiences applying
mathematical problem solving. Pre-service teachers with a non-science teaching subject have relatively fewer academic experiences applying mathematical problem solving. Given that experience has been considered an essential element in the formation of beliefs (Irez, 2006; Philipp, 2007), the difference in academic experience of applying mathematical problem solving can explain the different scale scores for pre-service teachers with science and non-science teaching subjects. The positive influence of applying mathematical problem solving is further supported by literature that has shown experiences related to science, technology, engineering, and mathematics shape the development of an individuals’ self-efficacy and interests related to those disciplines (Wang & Degol, 2013).

Overall, the data indicates that experience and teaching subject do not interact to influence pre-service teachers’ beliefs about their ability to solve difficult problems, the usefulness of mathematics, the importance of understanding concepts in mathematics, or the effect of effort on mathematical ability. However, suppressing the experience factor does reveal a significant difference in beliefs about the usefulness of mathematics between pre-service teachers with a science teaching subject and pre-service teachers with a non-science teaching subject.

Research Question Two

*What features of problem solving are emphasized in Eastern Ontario pre-service teachers’ communication of mathematics problem solving, as influenced by their problem-solving belief structures?*

Moving beyond identifying pre-service IS mathematics teachers’ ontological and epistemological beliefs about problem solving, this research question sought to determine
how those beliefs influenced pre-service teachers’ communication of problem solving. The answer for this question came from the comparison and integration phases of the convergent parallel design for analysis (Creswell, 2014). In total, two overarching features were emphasized in pre-service teachers’ communication of problem solving: (a) usefulness, and (b) complexity. The following is a description of these features that draws upon the findings from each data set.

The first feature of pre-service teachers’ communication of problem solving is that usefulness was acknowledged as a core attribute of problem solving. To pre-service teachers, problem solving was considered a fundamental way of learning and doing mathematics because of its sweeping applicability and the transferability of problem-solving processes across mathematical domains. During the mixing of quantitative and qualitative data sets, the usefulness feature was first noted when examining descriptive statistics for the BMPSQ data. The usefulness scale of the BMPSQ was the highest scored scale both before and after the coursework and practicum, which indicated a perception among pre-service teachers that problem solving is an essential process in mathematics.

Following identification of the usefulness feature from the BMPSQ, the duality of pre-service teachers’ ontological beliefs about problem solving provided the next piece of support for usefulness as a core attribute of problem solving. When considering what mathematics problem solving is, pre-service teachers believed that problem solving is a process for applying mathematics to real-world problems and a process for solving academic mathematics problems. An important aspect of the two perspectives is that neither way of thinking about the use of problem solving limits it to specific
mathematical domains or applications of mathematics; rather, pre-service teachers considered problem solving to be useful in all domains of mathematics and in all applications of mathematics (i.e., real-world and academic mathematics).

Pre-service teachers’ interpretation that usefulness was a core attribute of problem solving was further supported by the low scores obtained for the steps and word problems scales of the BMPSQ. The wording of Likert items in the steps scale presented the following of steps in a negative sense (i.e., following steps is not indicative of problem solving; see Appendix B), yet the interviews revealed pre-service teachers’ belief that learning specific problem-solving techniques was an important part of acquiring problem-solving knowledge. This positive outlook on the learning of specific solving processes is similarly present in the Ontario mathematics curricula (MOE, 2005a, 2005b, 2007). Therefore, pre-service teachers’ interpretation of problem solving was aligned with the curricula, which appreciates the usefulness of common problem-solving processes. The word problems scale presented a similarly limited perspective by treating word problems as representative of problem solving (see Appendix B). On the contrary, pre-service teachers refrained from describing problem solving as limited by any specific type of problem. To them, the notion of world problems as fully representative of problem solving was ignoring the many other mathematical representations of problem solving. Similarly, at no point do the Ontario mathematics curricula mention word problems when describing problem solving (MOE, 2005a, 2005b, 2007)—problem solving is considered more multifaceted than the simple designation of word problems.

Finally, pre-service teachers’ perception that usefulness is a core attribute of problem solving was reinforced by the development of their epistemological belief
related to the demonstration of problem solving. Following the coursework and practicum, pre-service teachers quieted emphasis on students learning needs and increased emphasis on students’ ability to use common problem-solving processes. The change in emphasis reflects an attentiveness to students developing competency in problem-solving processes that pre-service teachers perceived as widely useful in mathematics. Correspondingly, while the Ontario mathematics curricula do acknowledge students learning needs, emphasis in the sections describing problem solving primarily emphasize the learning of commonly used problem-solving processes, such as Polya’s (1945) framework (MOE, 2005a).

The second feature of pre-service teachers’ communication of problem solving is that complexity was considered an inherent attribute of problem solving. Amongst the pre-service teachers, the perception of problem solving as complex was pervasive and found to intensify as they gained experience in the teacher education program. The feature of problem solving as complex was first noted in the development of pre-service teachers’ ontological perceptions of problem solving. Prior to the coursework and practicum, teachers were viewing problem solving as a collaborative thinking process—collaborative because multiple perspectives help to generate solution strategies, and a thinking process because working towards a solution is not obvious. Following the coursework and practicum, pre-service teachers’ communication developed to directly specify an element of challenge. Specifically, they felt that problem solving requires students to identify and evaluate potential solving strategies, and performing these actions was perceived as challenging. The incorporation of challenge into pre-service teachers’
conception of problem solving meant that students would need to exert effort to learn
problem solving, which is reflected in the BMPSQ scales.

The effort scale showed the greatest variance in scores, which in association with
the interview data revealed pre-service teachers’ recognition that effort could be thought
about in two ways: (a) mindful, purposeful effort, and (b) casual, unplanned effort.
Mindful, purposeful effort represents the careful choosing of solution strategies to move
from an initial problem state to a solution state. In contrast, casual, unplanned effort
represents the comparatively haphazard application of different mathematical methods to
move from an initial problem state to a solution state. Based on the interview data, pre-
service teachers were viewing problem solving as requiring mindful, purposeful effort
because it is a complex activity; however, the effort scale does not distinguish between
types of effort. Therefore, the variance in scores for the effort scale can be explained by
whether pre-service teachers were astute to note the lack of distinguishing in the effort
scale. Additionally, the effort and difficult problems scales were the only two scales of
the BMPSQ found to be correlated, meaning the pre-service teachers’ thinking across
these scales was related. The correlation between the effort and difficult problems scales
suggests that the pre-service teachers were viewing problem solving as an activity that
necessitates effort because it is challenging activity.

A final piece of support for pre-service teachers considering complexity to be a
key attribute of problem solving is the development of their epistemological belief about
the value of learning through exploration. Prior to the coursework and practicum, pre-
service teachers were viewing the acquisition of problem-solving knowledge as optimally
stimulated by a learning environment that supports student autonomy and provides
thinking opportunities. However, following the coursework and practicum, the aspect of providing thinking opportunities was further articulated to highlight the importance of students communicating how they think through problem-solving activities. The development in articulating the importance of communication reflected an appreciation for the complexity of problem solving, as pre-service teachers needed to know if students were indeed learning how to use problem-solving techniques. Although examining students’ thinking is a part of any learning experience, pre-service teachers stressed the monitoring of students’ problem solving as especially important. In contrast, when discussing computational mathematics during interviews, the importance of seeing students’ ability to perform computations was not similarly emphasized.

**Limitations**

There are three limitations of this study that bear mentioning. The first limitation of this study came from the quantitative data collection instrument—the BMPSQ. Despite the intriguing findings from the BMPSQ, several issues were noted with the instrument. The first issue, which corroborates results obtained by Kloosterman and Stage (1992), is the paltry reliability of the steps scale and word problems scale. Considering the poor reliabilities obtained for these scales, there is reason to question whether these scales should be revamped or removed altogether. The second issue is whether the usefulness scale borrowed from Fennema & Sherman (1976) truly belongs with the other BMPSQ scales. In phase one, exploratory factor analysis (EFA) did not return the usefulness scale; however, this finding may be a result of the poor sampling adequacy in phase one, as EFA did return the usefulness scale in phase two. The third issue is the apparent redundancy and incongruity of some Likert items in BMPSQ scales. Reliability analysis
of the BMPSQ data revealed that four scales (difficult problems, word problems, effort, and usefulness) benefitted from removal of a Likert item, and this finding was agreed upon in both phases. Given these issues, the significant BMPSQ findings identified by this study need to be examined through additional empirical research before being considered transferable to similar research settings.

The second limitation of this study arose from the sample of Eastern Ontario pre-service IS mathematics teachers who participated in the quantitative and qualitative data collection. With regard to the quantitative data collection, the chosen sample of pre-service teachers introduced two complications: (a) a gender imbalance, with 34 females and only 10 males; and (b) a teaching-subject imbalance, with 28 pre-service teachers possessing a science teaching subject and only 16 pre-service teachers possessing a non-science teaching subject. Although all necessary assumptions were met when conducting inferential statistical analysis, the sample imbalances introduce concern as to whether the data for each group in the sample accurately represents the population of Eastern Ontario pre-service IS mathematics teachers. Similarly, with regard to the qualitative data collection, the sample of pre-service teachers who agreed to participate did not cover an equal distribution of gender and teaching subjects. Specifically, all of the male pre-service teachers who agreed to participate in the interviews possessed science teaching subjects, and all of the females who agreed to participate in the interviews possessed non-science teaching subjects. While it is unlikely the interview imbalances profoundly influenced the beliefs ultimately identified, the imbalance could have influenced minor nuances in data interpretations.
The third limitation of this study resulted from the timeline of data collection only spanning the Fall term of a B.Ed. program. Despite this timeline being purposefully chosen, the lack of data collection in winter term—such as prior to pre-service teachers’ completion of the program—means that the identified beliefs may have further evolved. Similar to the minor changes in emergent themes between phase one and phase two (see Table 12), there may have been additional development in pre-service teachers’ ontological and epistemological perceptions following the other half of Mathematics Education coursework and practicum experience. Therefore, the findings identified in this study need to be interpreted with the consideration that the pre-service teachers still had half of their B.Ed. program left to complete.

Implications and Recommendations

As a conclusion to this study, two implications for practice are provided regarding the preparation of Ontario pre-service IS mathematics teachers and two recommendations are provided for future research. It is imperative that readings of the implications for practice and recommendations for future research maintain an appreciation of this study’s limitations, as each of these final components were constructed with the limitations in mind.

Implications for Practice

Two implications for practice naturally followed from the findings of the study: a primary implication and a secondary implication. The primary implication for practice is to recognize that Ontario pre-service IS mathematics teachers possess a rather complex understanding of mathematics problem solving in the early stages of a teacher education program. As evidenced by their ontological and epistemological beliefs about problem
solving, pre-service teachers were thinking about problem solving in a way mostly consistent with the problem-solving literature and the Ontario mathematics curricula. With regard to the characteristics of problems, pre-service teachers in this study clearly understood that problems could exist on a continuum from routine to nonroutine (Mayer & Wittrock, 2006; MOE, 2005a, 2005b, 2007), yet their conceptualization for problems as ill-defined or well-defined was mostly ambiguous (Hollingworth & McLoughlin, 2005). Therefore, while pre-service teachers had a grasp on student-based problem characteristics (the routine—non-routine continuum; Mayer & Wittrock; 2006), a similar understanding was not present for student-independent problem characteristics (the well-defined—ill-defined continuum; Hollingworth & McLoughlin, 2005).

With regard to the characteristics of problem solving, pre-service teachers’ beliefs were again mostly consistent with the literature and Ontario mathematics curricula. One characteristic of problem solving that did appear to escape pre-service teachers’ thinking was the metacognitive element that more recently became recognized as an inseparable component of problem solving (Mayer & Wittrock, 2006; Metallidou, 2009; NCTM, 2010). Although the pre-service teachers talked extensively about thinking as an essential element of problem solving, the need for students to monitor and self-regulate their problem solving was not discussed. Additionally, the pre-service teachers did not appear to have fully conceptualized the distinct kinds of thinking involved in problem solving (Mayer & Wittrock, 2006)—while critical thinking was a term mildly used in both phases, creative thinking did not enter pre-service teachers’ rhetoric. It should be noted, however, that the Ontario mathematics curricula also focus solely on critical thinking when describing problem solving (MOE, 2005a, 2005b, 2007).
Given pre-service teachers’ complex understanding of mathematics problem solving, teacher educators could save valuable instructional time by focusing their efforts on helping pre-service teachers refine and recognize their problem-solving beliefs. This study clearly revealed that many Eastern Ontario pre-service IS mathematics begin the B.Ed. program with an impressive base understanding of problem solving that further develops in the first half of the B.Ed. program. Therefore, mathematics teacher educators might avoid extensive efforts covering the basic ideas of problem solving and instead focus on developing pre-service teachers’ thinking about more nuanced elements of problem solving (e.g., metacognition and the different kinds of thinking).

The secondary implication for practice is that mathematics teacher educators need to be aware of the influence of teacher-related variables on pre-service mathematics teachers’ beliefs about mathematics problem solving. This study corroborated earlier research by showing that gender and teaching subject significantly influence pre-service mathematics teachers’ beliefs about problem solving (Memnun et al., 2012; Sağlam & Dost, 2014; Yavuz & Erbay, 2014). While more research is needed to confirm the validity of these significant findings, it would be prudent for mathematics teacher educators to recognize that different groups of pre-service mathematics teachers think differently about problem solving. As such, instruction related to problem solving should cover all criteria highlighted in the Ontario mathematics curricula (MOE, 2005a, 2005b, 2007), and pre-service teachers should be given opportunities to examine how their peers are thinking about problem solving.
**Recommendations for Future Research**

The first recommendation for future research is to expand the scope of this initial investigation of Ontario pre-service IS mathematics teachers’ beliefs about problem solving. This study investigated the pre-service teachers’ beliefs under several contextual boundaries: the Eastern Ontario setting of data collection, the focus on pre-service teachers in IS divisions, and the timeline of data collection not including the second half of the B.Ed. program. As such, while the results are treated as transferrable to settings similar to the research setting, the contextual boundaries of this study inherently introduce a restriction for how transferable. Future research could therefore extend the findings from this study by including pre-service mathematics teachers from different geographic regions in Ontario, by expanding the population to include Ontario pre-service teachers who will be teaching problem solving in primary and junior divisions (i.e., kindergarten through grade 6 in Ontario), and by extending the timeline of data collection to follow pre-service teachers throughout the entire B.Ed. program.

The second recommendation for future research is to supplement this initial investigation of Ontario pre-service IS mathematics teachers’ beliefs about problem solving with an investigation into their knowledge about problem solving. Beliefs and knowledge are inextricably linked constructs (Ernest, 1989; Pajares, 1992), and any robust understanding of the thinking processes underlying teachers’ classroom practice will ultimately need to draw from both their beliefs and knowledge. While there has been some work in the general sphere of teachers’ problem-solving knowledge (e.g., see Chapman, 2015), there is a lack of empirical work specifically examining the problem-solving knowledge of pre-service teachers. By addressing both this recommendation and
the recommendation for additional empirical work to expand the scope of this initial research, a foundation for enhancing Ontario teachers’ classroom practice related to problem solving will be established.
References


Appendix A

Letter of Information

Problem-Solving Beliefs and Pre-Service Teachers’ Intentions for Professional Practice in Mathematics

Dear Teacher Candidate,

This research is being conducted by Dr. xxxxx xxxxx, in the Faculty of Education at xxxxxxx University in xxxxx, Ontario. This study has been granted clearance by the General Research Ethics Board according to Canadian research ethics principles (http://www.ethics.gc.ca/default.aspx) and xxxxxx University policies.

What is this study about? The purpose of this research is to develop an understanding of mathematics teacher candidates’ beliefs about problem solving, and to interpret how those problem-solving beliefs affect teacher candidates’ interpretation and communication of the curriculum.

What is involved to participate in this study? During this coming academic year, Dr. xxxxx xxxxx will be collecting information about your problem-solving beliefs and intentions for professional practice through the completion of 15-minute questionnaires during class time on three occasions to increase his understanding of his teaching. In addition, you are invited to take part in three 60-minute follow-up interviews during the academic year (total time: three hours). For the interviews, some of your questionnaire answers may be discussed to further understand your beliefs and knowledge about problem solving, and you are encouraged to bring artifacts (lesson plans, assessment tools, and learning objectives) that show how you were thinking about problem solving in your teaching. Between four and eight teacher candidates will be sought for the interviews. All interviews will occur at a time and location convenient for the participants. We are seeking permission to use all your questionnaire responses and interview data for research purposes. There are no known physical, psychological, economic, or social risks associated with this study.

The benefits to this study are an enhanced understanding of problem solving in mathematics instruction, contributing to the growing body of literature concerned with teacher candidates’ role in developing students’ problem-solving ability, and contributing to teacher education programs that aim to ensure teacher candidates are prepared to educate students on how to navigate societal, environmental, and technological problems they will encounter throughout their lives.

Is participation voluntary? Yes. You should not feel obliged to answer any questions that you find objectionable or that make you feel uncomfortable. You may choose to
withdraw from the study with no effect on your evaluations in the Bachelor of Education program up to three months after the conclusion of the third interview. Dr. xxxxx xxxxx will receive the questionnaire and interview data for research purposes stripped of all identifiers only after he has submitted the final marks for the course. Pseudonyms will have already be applied to the data. He will not know whether or not you participated in the interviews. If you wish to withdraw, contact Stephen MacGregor at 11sm36@queensu.ca. If you withdraw, you may request removal of all or part of your data from the study.

What will happen to your responses? Your responses will be kept confidential. Only Stephen MacGregor will have access to this information during the course. Dr. xxxxx xxxxx will have access to this information at the end of May 2017 when all the course marks have been submitted. Your confidentiality will be maintained to the extent possible. All participants in the interview part of the study will be asked not to discuss their participation with other people or Dr. xxxxx xxxxx, however, it is possible people may learn of your participation indirectly. Results from this study will be published in Stephen MacGregor’s master’s study and may be published in professional journals or presented at scientific conferences, but any such presentations will maintain individual confidentiality. In accordance with the General Research Ethics Board Standard Operating Procedures, data will be securely/password protected for a minimum of five years. If data are used for secondary analysis, they will contain no identifying information. You are entitled to a copy of the findings, if you are interested. If you would like a copy of the findings, please contact: Stephen MacGregor at 11sm36@queensu.ca.

Will you be compensated for your participation? No. However, your name will be entered into a draw to receive a problem-solving teaching resource. One resource will go to a randomly selected questionnaire participant, and one resource will go to a randomly selected interview participant. Each teaching resource will provide information and strategies on how to effectively imbed problem solving within your professional practice.

What if you have concerns? Any questions about study participation may be directed to Stephen MacGregor at 11sm36@queensu.ca or Dr. xxxxx xxxxx at xxx-xxx-xxxx extension xxxxx and xxxxxxxxxxx. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at xxxxxxxxxxx or x-xxx-xxx-xxxx.

Thank you for your interest in participating in this research study.
Consent Form

Problem-Solving Beliefs and Pre-Service Teachers’ Intentions for Professional Practice in Mathematics

Name (please print clearly): ________________________________________

1. I have read the Letter of Information and have had any questions answered to my satisfaction.

2. I understand that my course work (the questionnaires) and the interviews and artifacts will be used for research purposes in the study called Problem-Solving Beliefs and Pre-Service Teachers’ Intentions for Professional Practice in Mathematics.

3. I understand that my participation in this study is voluntary and I may withdraw up to three months after the third interview. I understand that every effort will be made to maintain the confidentiality of the data now and in the future. Only Stephen MacGregor will have access to my data during the course. Dr. xxxxx xxxxx will have access to this information after it has been de-identified, anonymized, and have had pseudonyms applied, at the end of May 2017 when all the course marks have been submitted. The data from this study will be published in Stephen MacGregor’s master’s study and may also be published in professional journals or presented at scientific conferences, but any such presentations will never breach individual confidentiality. I understand that I am entitled to a copy of the findings, if I am interested.

4. I am aware that if I have any questions, concerns, or complaints, I may contact Stephen MacGregor at 11sm36@queensu.ca or Dr. xxxxx xxxxx at xxx-xxx-xxxx extension xxxxx and xxxxxxxxxxxx. Any ethical concerns about the study may be directed to the Chair of the General Research Ethics Board at xxxxxxxxxx or x-xxx-xxx-xxxx.

I have read the above statements and freely consent to:

☐ my questionnaire data being used for the study
☐ being interviewed for the study on three occasions
☐ my artifacts data being used for the study

Signature: ________________________   Date: ______________________________

Section A

I agree to allow Stephen MacGregor to contact me through my xxxxxxx University email address to request participation in the three interviews associated with this study.

Email: _______________________________________

PLEASE RETURN ONE COPY OF THIS CONSENT FORM TO STEPHEN MACGREGOR AND RETAIN A SECOND COPY FOR YOUR RECORDS.
Appendix B

Outline of the Beliefs About Mathematical Problem Solving Questionnaire

The following questions will be randomly distributed throughout a single questionnaire such that no items from the same scale are consecutive.

Each question will have the following options for response: strongly agree, agree, uncertain, disagree, and strongly disagree.

Items are scored on a scale from one to five. Positively scored items (denoted by a ‘+’ symbol) receive a score of five for strongly agree and a score of one for strongly disagree. Negatively scored items (denoted by a ‘-’ symbol) receive a score of five for strongly disagree and a score of one for strongly agree. Following data collection, each scale is scored separately, and no overall score is calculated.

Belief 1: I can solve time-consuming mathematics problems.
   + Math problems that take a long time don’t bother me.
   + I feel I can do math problems that take a long time to complete.
   + I find I can do hard math problems if I just hang in there.
   - If I can’t do a math problem in a few minutes, I probably can’t do it at all.
   - If I can’t solve a math problem quickly, I quit trying.
   - I’m not very good at solving math problems that take a while to figure out.

Belief 2: There are word problems that cannot be solved with simple, step-by-step procedures.
   + There are word problems that just can’t be solved by following a predetermined sequence of steps.
   + Word problems can be solved without remembering formulas.
   + Memorizing steps is not that useful for learning to solve word problems.
   - Any word problem can be solved if you know the right steps to follow.
   - Most word problems can be solved by using the correct step-by-step procedure.
   - Learning to do word problems is mostly a matter of memorizing the right steps to follow.

Belief 3: Understanding concepts is important in mathematics.
   + Time used to investigate why a solution to a math problems works is time well spent.
   + A person who doesn’t understand why an answer to a math problem is correct hasn’t really solved the problem.
In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.\(^1\)
- It’s not important to understand why a mathematical procedure works as long as it gives a correct answer.
- Getting a right answer in math is more important than understanding why the answer works.
- It doesn’t really matter if you understand a math problem if you can get the right answer.

Belief 4: **Word problems are important in mathematics.**
- A person who can’t solve word problems really can’t do math.
- Computational skills are of little value if you can’t use them to solve word problems.
- Computational skills are useless if you can’t apply them to real life situations.
- Learning computational skills is more important than learning to solve word problems.
- Math classes should not emphasize word problems.
- Word problems are not a very important part of mathematics.

Belief 5: **Effort can increase mathematical ability.**
- By trying hard, one can become smarter in math.
- Working can improve one’s ability in mathematics.
- I get smarter in math by trying hard.
- Ability in math increases when one studies hard.
- Hard work can increase one’s ability to do math.
- I can get smarter in math if I try hard.

Belief 6: **Mathematics is useful in daily life.**
- I study mathematics because I know how useful it is.
- Knowing mathematics will help me earn a living.
- Mathematics is a worthwhile and necessary subject.
- Mathematics will not be important to me in my life’s work.
- Mathematics is of no relevance to my life.
- Studying mathematics is a waste of time.

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\(^1\) This item is to be used in administration of the questionnaire, but the item is not included in any statistics reported for the five-item version of the Understanding scale.
Appendix C

Implemented Beliefs About Mathematical Problem Solving Questionnaire

1. Math problems that take a long time don’t bother me.
2. There are word problems that just can’t be solved by following a predetermined sequence of steps.
3. Time used to investigate why a solution to a math problem works is time well spent.
4. A person who can’t solve word problems really can’t do math.
5. By trying hard, one can become smarter in math.
6. I study mathematics because I know how useful it is.
7. If I can’t do a math problem in a few minutes, I probably can’t do it at all.
8. Any word problem can be solved if you know the right steps to follow.
9. It’s not important to understand why a mathematical procedure works as long as it gives a correct answer.
10. Learning computational skills is more important than learning to solve word problems.
11. Ability in math increases when one studies hard.
12. Mathematics will not be important to me in my life’s work.
13. I feel I can do math problems that take a long time to complete.
14. Word problems can be solved without remembering formulas.
15. A person who doesn’t understand why an answer to a math problem is correct hasn’t really solved the problem.
16. Computational skills are of little value if you can’t use them to solve word problems.
17. Working can improve one’s ability in mathematics.
18. Knowing mathematics will help me earn a living.
19. If I can’t solve a math problem quickly, I quit trying.
20. Most word problems can be solved by using the correct step-by-step procedure.
21. Getting a right answer in math is more important than understanding why the answer works.
22. Math classes should not emphasize word problems.
23. Hard work can increase one’s ability to do math.
24. Mathematics is of no relevance to my life.
25. I find I can do hard math problems if I just hang in there.
26. Memorizing steps is not that useful for learning to solve word problems.
27. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
28. Computational skills are useless if you can’t apply them to real life situations.
29. I get smarter in math by trying hard.
30. Mathematics is a worthwhile and necessary subject.
31. I’m not very good at solving math problems that take a while to figure out.
32. Learning to do word problems is mostly a matter of memorizing the right steps to follow.
33. It doesn’t really matter if you understand a math problem if you can get the right answer.
34. Word problems are not a very important part of mathematics.
35. I can get smarter in math if I try hard.
36. Studying mathematics is a waste of time.
Appendix D

Pre-Coursework and Practicum Interview Guide

Participant’s name: ______________________________________
Date: ______________________________________
Time: ______________________________________
Location: ______________________________________
Additional contact information: ______________________________________

Introductory Elements (5 minutes)
• Explain what is being studied and why.
• Explain the format of the interview and the approximate amount of time it will take (60 minutes).
• Explain the notion of informed consent.
• Provide a statement that will alleviate any concerns of confidentiality. This student would include both confidentiality within the university and in published results.
• Build rapport by introducing and telling a little about myself.
• Ask the interviewees if they have any questions prior to beginning the interview. Explain that there are no incorrect answers to any of the interview questions.

Ontological Elements (20 minutes)
1) What comes to mind when I say ‘problem solving’?
   • What definition would be best suited (i.e., what is your personal definition)?
   • How is it recognized or measured?
   • How, if at all, does your definition suggest different levels of problem solving?
   • Where do we find/observe it?
   • What role does it serve?
2) Expand on responses to the questionnaire items where appropriate.
   • In the questionnaire completed during class there was the statement, [insert chosen item]. Can you expand on what you were thinking when you answered this question? Why did you [insert participant response]?
   • What thoughts came to mind while answering the questionnaire? What thoughts related to problem solving?
3) How would you describe the origin of a person’s beliefs about problem solving? What do you suppose might influence beliefs about problem solving, and whether or not these beliefs are subject to change?
Epistemological Elements (30 minutes)

1) Environment: How do (will) you maximize student learning of problem solving in your classroom?
2) Student knowledge: How do you describe your role as a teacher with regard to problem solving?
3) Learning: How do students you teach best learn problem solving?
4) Student and standards: In the public school setting, how do you decide what to teach and what not to teach?
5) Assessment: How do you decide to move on to a new topic or aspect of problem solving?
6) Understanding: How do you know when students understand?
7) Student response: How do you know learning is occurring in the classroom?

Concluding Elements (5 minutes)

- Thank the interviewees for their participation.
- Inquire as to whether they would like to add anything to what was discussed or if they have any questions.
- Reassure confidentiality and secure handling of all data.
- Request permission to follow-up either in-person, by email, or through a phone call regarding data that may seem unclear during analysis.
- Request permission to contact participant regarding the post-practicum interview once they return from the fall 2016 practicum.
Appendix E

Post-Coursework and Practicum Interview Guide

Participant’s name: ______________________________________
Date: ______________________________________
Time: ______________________________________
Location: ______________________________________
Additional contact information: ______________________________________

Introductory Elements (5 minutes)
- Restate what is being studied and why.
- Explain the format of the interview and the approximate amount of time it will take (approximately 60 minutes).
- Explain the notion of informed consent.
- Provide a statement that will alleviate any concerns of confidentiality. This statement would include both confidentiality within the university and in published results.
- Ask the interviewees if they have any questions prior to beginning the interview. Explain that there are no incorrect answers to any of the interview questions.

Ontological Elements (20 minutes)
1) What comes to mind when I say ‘problem solving’?
   - What definition would be best suited (i.e., what is your personal definition)?
   - How is it recognized or measured?
   - How, if at all, does your definition suggest different levels of problem solving?
   - Where do we find/observe it?
   - What role does it serve?
   - With regard to the origins of a student’s beliefs about problem solving, what do you think about the relative influences of culture, the education system, and parental figures and peers.
2) Expand on responses to the questionnaire items where appropriate.
   - In the questionnaire completed during class there was the statement, [insert chosen item]. Can you expand on what you were thinking when you answered this question? Why did you [insert participant response]?
   - What thoughts came to mind while answering the questionnaire? What thoughts related to problem solving?
3) Expand and re-examine responses from the pre-coursework and practicum interviews where appropriate.
Epistemological Elements (30 minutes)

1) Environment: How did you maximize student learning of problem solving in your classroom?
2) Student knowledge: How would you describe your role as a teacher with regard to problem solving?
3) Learning: How did the students you taught best learn problem solving?
4) Student and standards: In the public school setting, how did you decide what to teach and what not to teach with regard to problem solving?
5) Assessment: How did you decide when it was time to move on to a new topic or aspect of your problem solving instructional tasks?
6) Understanding: How did you know when students understood your instructional tasks related to problem solving?
7) Student response: How did you know students were learning how to problem solve in your classroom?

Concluding Elements (5 minutes)

- What influence, if any, do you think the in-class focus on PS and previous interview had on your instructional practice and thinking while on the fall practicum?
- Thank the interviewees for their participation.
- Inquire as to whether they would like to add anything to what was discussed or if they have any questions.
- Reassure confidentiality and secure handling of all data.
- Request permission to follow-up either in-person, by email, or through a phone call regarding data that may seem unclear during analysis.
### Appendix F

**Descriptive Statistics for Likert Items in the Beliefs About Mathematical Problem Solving Questionnaire**

<table>
<thead>
<tr>
<th>Likert Item</th>
<th>Phase One (mean; standard deviation)</th>
<th>Phase Two (mean; standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Math problems that take a long time don’t bother me.</td>
<td>3.5; 1.0</td>
<td>3.6; 0.9</td>
</tr>
<tr>
<td>2. There are word problems that just can’t be solved by following a predetermined sequence of steps.</td>
<td>3.5; 0.9</td>
<td>3.6; 0.9</td>
</tr>
<tr>
<td>3. Time used to investigate why a solution to a math problems works is time well spent.</td>
<td>4.3; 0.6</td>
<td>4.3; 0.6</td>
</tr>
<tr>
<td>4. A person who can’t solve word problems really can’t do math.</td>
<td>2.2; 0.9</td>
<td>2.5; 0.8</td>
</tr>
<tr>
<td>5. By trying hard, one can become smarter in math.</td>
<td>4.0; 0.8</td>
<td>4.1; 0.6</td>
</tr>
<tr>
<td>6. I study mathematics because I know how useful it is.</td>
<td>3.8; 0.7</td>
<td>4.1; 0.7</td>
</tr>
<tr>
<td>7. If I can’t do a math problem in a few minutes, I probably can’t do it at all.</td>
<td>4.4; 0.5</td>
<td>4.3; 0.5</td>
</tr>
<tr>
<td>8. Any word problem can be solved if you know the right steps to follow.</td>
<td>2.7; 0.9</td>
<td>2.9; 0.8</td>
</tr>
<tr>
<td>9. It’s not important to understand why a mathematical procedure works as long as it gives a</td>
<td>4.4; 0.7</td>
<td>4.2; 0.8</td>
</tr>
</tbody>
</table>
correct answer.

10. Learning computational skills is more important than learning to solve word problems.

3.5; 0.8

11. Ability in math increases when one studies hard.

3.8; 0.7

12. Mathematics will not be important to me in my life’s work.

4.6; 0.5

13. I feel I can do math problems that take a long time to complete.

3.9; 0.6

14. Word problems can be solved without remembering formulas.

3.4; 0.9

15. A person who doesn’t understand why an answer to a math problem is correct hasn’t really solved the problem.

3.8; 0.9

16. Computational skills are of little value if you can’t use them to solve word problems.

3.0; 0.9

17. Working can improve one’s ability in mathematics.

4.4; 0.7

18. Knowing mathematics will help me earn a living.

4.1; 0.7

19. If I can’t solve a math problem quickly, I quit trying.

4.0; 0.6

20. Most word problems can be solved by using the correct step-by-step procedure.

2.8; 0.9

21. Getting a right answer

4.3; 0.6
in math is more important than understanding why the answer works.

22. Math classes should not emphasize word problems.  
   4.1; 0.6  4.0; 0.7

23. Hard work can increase one’s ability to do math.  
   4.2; 0.6  4.2; 0.6

24. Mathematics is of no relevance to my life.  
   4.6; 0.5  4.6; 0.5

25. I find I can do hard math problems if I just hang in there.  
   3.9; 0.6  3.8; 0.7

26. Memorizing steps is not that useful for learning to solve word problems.  
   3.0; 0.9  3.1; 0.9

27. In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.  
   4.6; 0.5  4.5; 0.5

28. Computational skills are useless if you can’t apply them to real life situations.  
   3.1; 1.0  3.1; 0.9

29. I get smarter in math by trying hard.  
   3.9; 0.8  4.1; 0.7

30. Mathematics is a worthwhile and necessary subject.  
   4.7; 0.5  4.5; 0.5

31. I’m not very good at solving math problems that take a while to figure out.  
   3.6; 0.8  3.6; 0.9

32. Learning to do word problems is mostly a matter of memorizing the right steps to follow.  
   3.7; 0.8  3.7; 0.7

33. It doesn’t really matter if you understand a math problem if you can get the  
   4.2; 0.6  4.0; 0.6
right answer.

34. Word problems are not a very important part of mathematics.  4.2; 0.6  4.1; 0.7

35. I can get smarter in math if I try hard.  4.0; 0.6  4.1; 0.7

36. Studying mathematics is a waste of time.  4.7; 0.5  4.6; 0.5
Appendix G

General Research Ethics Board Approval

August 08, 2016

GREB Ref #: GEDUC-819-16; Romeo # 6018957
Title: "GEDUC-819-16 Problem-Solving Beliefs and Pre-Service Teachers' Intentions for Professional Practice in Mathematics"

Dear [Name],

The General Research Ethics Board (GREB), by means of a delegated board review, has cleared your proposal entitled "GEDUC-819-16 Problem-Solving Beliefs and Pre-Service Teachers' Intentions for Professional Practice in Mathematics" for ethical compliance with the Tri-Council Guidelines (TCPS 2 (2014)) and Queen's ethics policies. In accordance with the Tri-Council Guidelines (Article 6.14) and Standard Operating Procedures (405.001), your project has been cleared for one year. You are reminded of your obligation to submit an annual renewal form prior to the annual renewal due date (access this form at http://www.queensu.ca/traq/signon.html; click on "Events"; under "Create New Event" click on "General Research Ethics Board Annual Renewal/Closure Form for Cleared Studies"). Please note that when your research project is completed, you need to submit an Annual Renewal/Closure Form in Romeo/traq indicating that the project is 'completed' so that the file can be closed. This should be submitted at the time of completion; there is no need to wait until the annual renewal due date.

You are reminded of your obligation to advise the GREB of any adverse event(s) that occur during this one year period (access this form at http://www.queensu.ca/traq/signon.html; click on "Events"; under "Create New Event" click on "General Research Ethics Board Adverse Event Form"). An adverse event includes, but is not limited to, a complaint, a change or unexpected event that alters the level of risk for the researcher or participants or situation that requires a substantial change in approach to a participant(s). You are also advised that all adverse events must be reported to the GREB within 48 hours.

You are also reminded that all changes that might affect human participants must be cleared by the GREB. For example, you must report changes to the level of risk, applicant characteristics, and implementation of new procedures. To submit an amendment form, access the application by at http://www.queensu.ca/traq/signon.html; click on "Events"; under "Create New Event" click on "General Research Ethics Board Request for the Amendment of Approved Studies". Once submitted, these changes will automatically be sent to the Ethics Coordinator, Ms. Gail Irving, at the Office of Research Services for further review and clearance by the GREB or GREB Chair.

On behalf of the General Research Ethics Board, I wish you continued success in your research.

Sincerely,

[Signature]
John Freeman, Ph.D.
Chair
General Research Ethics Board

c: Dr. John Freeman and Mr. Stephen MacGregor, Co-investigators
Dr. Richard Reeve, Chair, Unit REB
Ms. Erin Wicklum, Dept. Admin.