NATURAL CONVECTIVE HEAT TRANSFER FROM HORIZONTAL AND INCLINED TWO-SIDED BODIES OF FINITE THICKNESS

by

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Abstract

Natural convective heat transfer from two-sided flat plates that are horizontal or inclined to the horizontal has been numerically and experimentally investigated. The objective of the study was to investigate the influence of changes in the plate thickness, of the boundary condition over the plate side surface, of the plate shape, and of the inclination angle on the heat transfer rate. A further objective was to investigate whether the heat transfer rate from a flat plate can be increased by using a plate having a non-flat surface. The heated elements considered were exposed to air. The heat transfer rate was numerically obtained by using the commercial CFD solver ANSYS FLUENT© and experimentally determined using the lumped capacity method.

In addition, natural convective heat transfer from bottom-heated rectangular enclosures that contain a nanofluid and which have various aspect ratios was numerically studied. The purpose was to determine whether the heat transfer rate from the heated bottom wall of the enclosure can be increased compared to that which would exist with pure water and to study the influence of the nanofluid nanoparticle concentration and of the enclosure aspect ratio on the heat transfer rate.

The results of the studies of natural convective heat transfer from heated plates indicated that the plate thickness and the thermal boundary condition have only a modest influence on the heat transfer rate from the bottom surface of the plate. The influence of the inclination angle on the heat transfer rate from the top surface of the plate is higher than its influence on that from the bottom surface. For the case of the plate having a non-flat surface, the results indicated that the heat transfer rate can be enhanced by using a non-flat surface but this enhancement is relatively small.

The results of the study of heat transfer across a nanofluid filled enclosure showed that the heat transfer rate is significantly increased by replacing pure water with a nanofluid (Cu-water) and by
increasing the nanoparticle concentration. It was also found that the enclosure aspect ratio had only a small influence on the heat transfer rate.
Co-Authorship

All the material contained in the present thesis is the result of my research undertaken under supervision of Prof. Patrick H. Oosthuizen. I hereby acknowledge his strong support in preparing all the publications. However, all the work contained in these publications is my own and my co-author provided guidance and direction in the development of the final product. The present dissertation follows the manuscript format, each chapter is based on the following published, submitted, or to be submitted manuscript articles:

Chapter 4


Chapter 5


Chapter 6
Rafiq Manna and Patrick H. Oosthuizen, A Numerical Study of Natural Convective Heat Transfer from Two-Sided Inclined Square Plates Having a Finite Thickness, submitted to Proceedings of the ASME 2019 International Mechanical Engineering Congress and Exposition (IMECE), Salt Lake City, Utah, USA, 2019. (Accepted for Publication)

Chapter 7

Chapter 8

Chapter 9
Rafiq Manna and Patrick H. Oosthuizen, Natural Convective Heat Transfer in Rectangular Bottom-Heated Enclosures that Contain a Nanofluid and which have Varying Aspect Ratios, to be submitted to Computational Thermal Sciences: An International Journal.
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Latin Symbols

\( A_b \) base area of the upper flat surface of circular plate and of the lower flat surface of circular plate (m\(^2\))

\( A_{plate} \) area of the upper surface of plate and of the lower surface of plate (m\(^2\))

\( A_{side} \) area of the side surface of plate (m\(^2\))

\( A_{total} \) sum of the areas of the heated surfaces of plate (m\(^2\))

\( A_e \) area of the bottom wall of enclosure (m\(^2\))

\( Al \) aluminum

\( AR \) aspect ratio of enclosure

\( Bi \) Biot number

\( C_{1\varepsilon} \) constant which has been arrived as a result of numerous data fitting for a wide range of turbulent flows

\( C_{2\varepsilon} \) constant which has been arrived as a result of numerous data fitting for a wide range of turbulent flows

\( C_{3\varepsilon} \) constant which determines the degree to which turbulent dissipation is affected by the buoyancy force

\( C_{\mu} \) constant which has been arrived as a result of numerous data fitting for a wide range of turbulent flows

\( c \) specific heat of material from which plate is made (J/kg K)

\( c_p \) specific heat at constant pressure (J/kg K)

\( d \) diameter of circular plate (m)

\( g \) gravitational acceleration (m/s\(^2\))

\( G_b \) generation of turbulent kinetic energy due to buoyancy

\( G_k \) generation of turbulent kinetic energy due to mean velocity gradients

\( Gr \) Grashof number

\( h \) thickness of plate (m)

\( h_n \) height of the nonflat surface of circular plate (m)

\( h_t \) overall heat transfer coefficient (W/m\(^2\) K)

\( h_r \) radiant heat transfer coefficient (W/m\(^2\) K)

\( h_c \) convective heat transfer coefficient (W/m\(^2\) K)
\( H \)  
dimensionless thickness of plate

\( k \)  
thermal conductivity of fluid (W/m K)

\( k_s \)  
thermal conductivity of aluminum alloy (Al 6061-T6) (W/m K)

\( l \)  
characteristic length scale of plate (m)

\( l_e \)  
length of enclosure (m)

\( m \)  
mass of plate (kg)

\( n \)  
number of the independent variables

\( Nu \)  
mean Nusselt number

\( Nu_{total} \)  
mean Nusselt number based on the mean heat transfer rate per unit area over heated surfaces of plate

\( Nu_{top} \)  
mean Nusselt number based on the mean heat transfer rate per unit area over upper surface of plate

\( Nu_{bot} \)  
mean Nusselt number based on the mean heat transfer rate per unit area over lower surface of plate

\( Nu_{side} \)  
mean Nusselt number based on the mean heat transfer rate per unit area over side surface of plate

\( P \)  
perimeter of heated surface (m)

\( p \)  
pressure (N/m²)

\( Pr \)  
Prandtl number

\( Pr_T \)  
turbulent Prandtl number

\( \overline{Q} \)  
mean heat transfer rate from the heated surfaces of plate (W)

\( \overline{Q}_{top} \)  
mean heat transfer rate from the upper surface of plate (W)

\( \overline{Q}_{bot} \)  
mean heat transfer rate from the lower surface of plate (W)

\( \overline{Q}_{side} \)  
mean heat transfer rate from the side surface of plate (W)

\( \overline{Q}_e \)  
mean heat transfer rate from the heated wall of enclosure (W)

\( \overline{q} \)  
mean heat transfer rate per unit area (W/m²)

\( R \)  
result which is a function of \( n \) independent variables

\( r \)  
radius of circular plate (m)

\( Re \)  
Reynolds number

\( Ra \)  
Rayleigh number

\( S_k \)  
source terms defined by the user

\( S_e \)  
source terms defined by the user

\( s \)  
arm width of plate (m)
\( t \)    time (s)
\( t_i \)    initial time (s)
\( t_e \)    final time (s)
\( T \)    temperature (K)
\( T_b \)    temperature of bottom surface of enclosure (K)
\( T_t \)    temperature of top surface of enclosure (K)
\( T_w \)    temperature of heated surfaces of plate (K)
\( T_f \)    temperature of fluid far from plate (K)
\( T_{film} \)    film temperature (K)
\( T_i \)    initial temperature of plate (K)
\( T_e \)    final temperature of plate (K)
\( T_m \)    mean temperature of plate (K)
\( T_s \)    temperature of the surroundings to which body is radiating (K)
\( u \)    velocity component in \( x \)-direction (m/s)
\( U_r \)    friction velocity (m/s)
\( V \)    Volume of the heated plate (m\(^3\))
\( v \)    velocity component in \( y \)-direction (m/s)
\( w \)    side length of square plate (m)
\( w_e \)    width of enclosure (m)
\( X \)    independent variable
\( W \)    outside length of I- and +-shaped plates (m)
\( Y_M \)    fluctuating dilatation in compressible turbulence
\( y^+ \)    dimensionless distance from the wall

**Greek Symbols**

\( \alpha \)    thermal diffusivity (m\(^2\)/s)
\( \beta \)    bulk coefficient of expansion (1/K)
\( \sigma \)    Stefan-Boltzmann constant \( =5.67\times10^{-8} \) W/m\(^2\) K\(^4\)
\( \sigma_k \)    turbulent Prandtl number for the turbulent kinetic energy
\( \sigma_e \)    turbulent Prandtl number for the turbulent dissipation rate
\( \delta \)    uncertainty of certain quantity
\( \delta_{ji} \)    Kronecker delta
ρ  mass density (kg/m³)
ε  emissivity of plate
ε  turbulent (eddy) kinematic viscosity (m²/s)
ε_H turbulent (eddy) diffusivity of heat (m²/s)
μ  dynamic viscosity (kg/m s)
μ_t turbulent (eddy) dynamic viscosity (kg/m s)
ν  kinematic viscosity (m²/s)
τ_w shear stress at the wall (N/m²)
φ  inclination angle (angle the side length of plate makes with vertical, degrees)
φ_p particle volume concentration (%)
φ  diagonal inclination angle (angle the diagonal of plate makes with horizontal, degrees)
θ  inclination angle (angle the side length of plate makes with horizontal, degrees)
ω  velocity component in z-direction (m/s)

Acronyms

CFD  computational fluid dynamics
D/A  data acquisition
MS  Microsoft
PRT  precision reference thermometer
RANS Reynolds-averaged Navier-Stokes
TC  thermocouple
Chapter 1

Introduction

1.1 Background and Motivation

In various industrial applications, electrical and electronic components require cooling in order to operate effectively. While cooling of such devices does not pose difficulties as severe as those arising in computer systems, ensuring adequate thermal management can be challenging. Such components are often cooled by natural convective heat transfer due to concerns about long term reliability, cost and noise.

Convective heat transfer as indicated in Figure 1.1 is the process of energy transfer between a surface and a fluid flowing over it as a result of a temperature difference between the surface and the fluid (Oosthuizen and Naylor, 1999).

![Convective heat transfer](image)

**Figure 1.1 Convective heat transfer (Oosthuizen and Naylor, 1999).**

Convection is one of the three well known modes of heat transfer, the other two being conduction and radiation. Convective heat transfer is classified into forced, free (natural) and combined (mixed) convection. In forced convective heat transfer, the fluid motion is caused by external means like a fan or blower, while in natural convective heat transfer the
flow is generated by the body forces resulting from the changes in density which arise from
the temperature change in the flow field. These body forces are generated by pressure
gradients in the flow field which are created by either gravity or centrifugal forces arising
from a rotary motion. These body forces are termed the buoyancy forces. Natural
convective heat transfer is thus often referred to as a buoyancy driven flow. Therefore, the
term forced convective heat transfer is adopted only when the buoyancy force effects are
negligible. However, in some flows where a forced velocity exists, the buoyancy force
effects are not negligible. Such flow is termed mixed free and forced convective flow. All
the preceding types of convective flows can be either laminar or turbulent. The types of
convective heat transfer are illustrated in Figure 1.2 (Oosthuizen and Naylor, 1999).

In natural convective heat transfer, the changes in the fluid properties are usually
relatively small and therefore the fluid properties except for density can be assumed to be
constant. The density change with temperature must be accounted for since it gives the rise
to the buoyancy force. This assumption that all the fluid properties except for density are
assumed to be constant is part of what is known as the Boussinesq approximation which is
widely adopted in the analysis of natural convective heat transfer (Oosthuizen and Naylor,
1999).
As discussed earlier, in many electrical and electronic devices, when the components operate at excessive temperatures above the specified operating temperature, their functionality and operational life can be severely compromised. Operation under such conditions will ultimately lead to a component degradation and failure. To ensure proper operation and reliability, the component must be kept below a prescribed temperature and heat must therefore be effectively dissipated away from the heated element to the ambient surroundings to keep the temperature of the component within the specified operating range. The heat transfer from such components by natural convection occurs in many situations from components which are horizontal or inclined and which have various shapes. Also, the heat transfer can occur from a single-sided plate which is the case where the plate is embedded in an adiabatic material and there is only heat transfer from one side of the plate or the heat transfer can occur from a two-sided plate which is the case where there is heat transfer from all sides of the plate. In addition, the heated elements can be
exposed to air as an ambient fluid or any other fluid like a nanofluid. Figure 1.3 illustrates some examples of components exposed to air and cooled by natural convective heat transfer.

Figure 1.3 Electronic components with different size and orientation cooled by natural convective heat transfer. The pictures were copied from: (a) www.alfredoblogspage.blogspot.com, (b) www.electronicproducts.com, (c) www.eenewspower.com (MOSFET: Metal Oxide Semiconductor Field-Effect Transistor) and (d) www.murata-ps.com.

Adequate thermal management of such components therefore requires a knowledge of the natural convective heat transfer rates that exist in situations that arise with such devices. The results of the studies of natural convective heat transfer from heated plates may therefore be of considerable value to designers of industrial systems involving non-

computer electronic and electrical components. These results provide improved methods for estimating heat transfer rates which will allow the more accurate prediction of operating temperatures of such components. The results will also advance basic knowledge of the type of natural convective flows involved in such situations and therefore potentially indicating how the heat transfer rate in such situations can be enhanced. These considerations are what basically motivated the present study which aims to provide important new basic information for use in the design of such natural convective cooling system components.

Situations considered in the present research are somewhat simplified models of those occurring in real-world electronic and electrical component cooling.

Since the heat transfer rates attainable with natural convective heat transfer are almost always lower than those obtained with forced convective heat transfer, many means of enhancing the heat transfer rates obtained with natural convective heat transfer have therefore been investigated. One potential method is to replace the traditional heat transfer fluids with nanofluids (Minkowycz et al., 2013). Nanofluids are a class of nanotechnology-based heat transfer fluids that are engineered by stably suspending solid nanoparticles which possess an enhanced thermal conductivity compared with those of the traditional base fluids. This results in an enhancement of the heat transfer rate compared to that with a pure fluid (Kim et al., 2004; Sarkar, 2011).

The heat transfer characteristics of the nanofluids depend on the properties of the base liquid and the nanoparticles, on the particle concentration and on the particle size. Many equations for the properties of a nanofluid have been proposed.
Based on a review of the literature of natural convective heat transfer from flat plates, the studies can be classified into the following categories:

1. Investigation of natural convective heat transfer from a heated element using either numerical or experimental method.

2. Investigation of natural convective heat transfer from either a single-sided or two-sided heated element.

3. Investigation of the effect of surface inclination on the natural convective heat transfer rate from a heated element.

4. Investigation of the effect of element shape on the natural convective heat transfer rate from a heated element.

5. Investigation of natural convective heat transfer from a heated element considering either a limited range of Rayleigh numbers which involves laminar flow only or in a few cases, a wider range of Rayleigh numbers which covers conditions involving laminar, transitional, and fully-turbulent flows.

6. Investigation of natural convective heat transfer from a heated element considering the effect of thermal boundary conditions, e.g., isothermal or uniform heat flux conditions existing along the element surface on the heat transfer rate.

7. Investigation of natural convective heat transfer enhancement for a heated element in certain situations.

1.2 Objectives

A review of past studies reveals that natural convective heat transfer from vertical flat plates in air has been extensively studied experimentally as well as numerically. Natural convective heat transfer from horizontal and inclined plates has in the past received less
attention than natural convective heat transfer from vertical plates. The possible reason for this is that the physical nature of the flow over the horizontal and inclined plates is more complex than that over the vertical plates.

Horizontal and inclined plates in situations where there is heat transfer from one side only (single-sided) plate and which are facing either upward or downward have received the most attention. A limited amount of work has been devoted to natural convective heat transfer from horizontal and inclined two-sided plates. Most of this work, however, has focused on investigating natural convective heat transfer under limited conditions such as considering a limited range of Rayleigh numbers which cover only the laminar flow region without obtaining results for transitional and turbulent flow regions, or such as taking into account a single thermal boundary condition existing over the plate surfaces. In addition, the effect of the plate thickness of two-sided plates on the heat transfer rate has not been studied in detail. Moreover, only simple forms of inclination have been considered, i.e., situations in which the angle of inclination is with respect to a side of the plate. No previous work has been found on the natural convective heat transfer from a two-sided plate when it is diagonally inclined (i.e., the inclination angle is between the plate diagonal and the horizontal). Natural convective heat transfer enhancement when the element has a nonflat surface has been studied. The few available studies of this, however, paid attention to the single-sided element only.

Many researchers have given attention to convective heat transfer to nanofluids but the majority of these studies have been concerned with forced convective heat transfer.

Hence, previous literature of natural convective heat transfer shows that further work on heat transfer from two-sided plates of various shapes which are exposed to air and
further work on heat transfer to nanofluids is required. Therefore, the objectives of the present study, which involves both numerical and experimental work, were to:

1. Obtain natural convective heat transfer results for conditions where laminar, transitional and fully-turbulent flow exist for a range of horizontal two-sided element shapes (e.g., square, circular, I-shape and +-shape) elements having a finite thickness and to determine whether by using a certain length scale the results for all element shapes considered can be expressed by means of the same correlation.

2. Investigate the effect of the element thickness and the effect of the thermal boundary conditions (i.e., either all surfaces of the element are isothermal and at the same temperature or the vertical side surface of the element is adiabatic while the top and bottom surfaces are isothermal and at the same temperature) existing over the element surface on the heat transfer rate.

3. Investigate the effect of inclination angle on the heat transfer rate for various inclination situations.

4. Investigate the enhancement of heat transfer rate from a two-sided element having a nonflat surface.

5. Determine whether the natural convective heat transfer rate can be increased when the heat transfer occurs to a nanofluid with various nanoparticle concentrations rather than to a pure liquid in a rectangular enclosure of varying aspect ratios.

1.3 Contribution

The present work has made the following contributions:

- For heated elements exposed to air:
1. Provided the designers of industrial systems involving electronic and electrical components with information for estimating the heat transfer rates which will allow the more accurate prediction of operating temperatures for such components.

2. Provided the designers with the effect of basic parameters on the heat transfer rate from some electronic and electrical components which can be used in the proper selection of such components. These parameters include: the element thickness, the element shape, the thermal boundary conditions over the element surface and the inclination angle at various inclination situations.

3. Provided a comparison of the heat transfer results for surfaces of different shapes when a proper length scale was used.

4. Provided knowledge on the heat transfer enhancement when a nonflat surface is used with a two-sided element.

   - For heated elements exposed to nanofluid:
     1. Compared the heat transfer results from a single-sided element in an enclosure when it occurs to a nanofluid rather than to a pure liquid.
     2. Provided knowledge on how the heat transfer rate could be affected by changing the concentration of nanoparticles in the nanofluid and by changing the enclosure aspect ratio.

1.4 Thesis Outline

This thesis consists of the following chapters:

Chapter 1 provides a background on natural convective heat transfer from flat plates and outlines the motivation, objectives and contribution of the study.
Chapter 2 reviews previous literature concerning numerical and experimental studies of flat plates exposed to air with different shapes, orientations and thermal boundary conditions for various flow conditions as well as the studies when the natural convective heat transfer occurs to a nanofluid rather than to a pure liquid.

Chapter 3 describes the numerical and experimental methodology used in the study.

Chapter 4 describes numerical and experimental investigations of natural convective heat transfer from two-sided circular and square horizontal plates having a finite thickness and discusses the results obtained.

Chapter 5 describes numerical and experimental investigations of natural convective heat transfer from two-sided horizontal plates having a complex shape and a finite thickness and discusses the results obtained.

Chapter 6 describes a numerical investigation of natural convective heat transfer from two-sided inclined square plates having a finite thickness and discusses the results obtained.

Chapter 7 describes numerical and experimental investigations of natural convective heat transfer from two-sided diagonally inclined square plates having a finite thickness and discusses the results obtained.

Chapter 8 describes a numerical investigation of natural convective heat transfer from a two-sided circular horizontal element having a linearly-inclined nonflat surface and discusses the results obtained.

Chapter 9 describes a numerical investigation of natural convective heat transfer in rectangular bottom-heated enclosures with varying aspect ratios that contain a nanofluid and discusses the results obtained.
Chapter 10 presents summary of the studies included in the present thesis and of the conclusions that can be drawn from these studies and presents recommendations for future work.

Appendix A provides a detailed study of the use of the length scale described in the present research.

Appendix B provides typical flow patterns of some of the situations considered in the present research.
Chapter 2

Literature Review

2.1 Overview

In this chapter previous studies which involve experimental and analytical investigations of natural convective heat transfer from a variety of geometries and orientations with various thermal boundary conditions will first be reviewed. Numerical studies of some of these situations will then be considered and the corresponding numerical results will be compared with those obtained in the experimental and analytical investigations. Finally, convective heat transfer to a nanofluid will be discussed to investigate the possibility of enhancing the heat transfer rate by using such a fluid.

2.2 Experimental and Analytical Studies

Natural convective heat transfer from heated flat plates has been widely studied. The heat transfer rate from the plate will depend on the plate orientation, i.e., whether the plate is horizontal or vertical or inclined at a specific angle with respect to the horizontal or vertical. Some experimental and analytical studies that considered horizontal as well as inclined plates will be discussed in the following section.

2.2.1 Horizontal Plates

Natural convective heat transfer from horizontal plates of different shapes and geometries has been studied by a number of researchers. The reason as mentioned earlier is that horizontal plates arise in a number of industrial applications such as cooling of electronic and electrical components. Nevertheless, natural convective heat transfer from horizontal plates has received less attention than natural convective heat transfer from
vertical plates. The possible reason for this is that the physical nature of the flow over horizontal plates is more complex than that for vertical surfaces.

Horizontal plates in which there is only heat transfer from a single surface facing either upward or downward have received the most attention. However, some limited studies of natural convective heat transfer from two-sided plates have also been carried out, most of these studies only considered thin plates. Figure 2.1 shows the surface situations of the heated element with finite thickness. Experimental and analytical studies of all of these situations are discussed in the present section.

![Figure 2.1 Heated element surface situations, (a) facing upward, (b) facing downward and (c) two-sided. (Finite thickness case)](image)

2.2.1.1 Single-Sided Plate

One of the earliest experimental studies that investigated the natural convective heat transfer from single-sided isothermal horizontal plates was conducted by Aihara et al. (1972). Two-dimensional natural convective heat transfer from a downward-facing heated horizontal surface in air was experimentally investigated. The Rayleigh number considered in the study was of the order of $10^7$. Correlations for the local and average Nusselt numbers were proposed.

Al-Arabi and El-Riedy (1976) carried out an experimental study to investigate the natural convective heat transfer from isothermal horizontal plates of different shape facing
upward in air in the range of Rayleigh numbers from $2 \times 10^5$ to $10^9$. Dimensionless equations to predict the Nusselt number over the specified range of Rayleigh numbers were proposed.

Experiments to determine the local and average natural convective heat transfer characteristics over a uniformly heated (uniform surface heat flux) upward-facing horizontal plate in air with and without a shrouded parallel adiabatic surface above the heated surface were performed by Sparrow and Carlson (1986). The size of the plate and the Rayleigh number (in the laminar flow regime) were varied in the experimental work. The authors suggested correlations relating the average Nusselt number with the Rayleigh number.

Clausing and Berton (1989) described an experimental investigation of natural convective heat transfer from a heated upward-facing square isothermal horizontal plate to a surrounding gas medium. The authors reported results which extend the horizontal plate correlation to conditions not covered in earlier work. What distinguished the work is that they used an extended range of wall-to-surrounding temperature ratios ($T_w/T_\infty$) between near 1 and 3.1 rather than being near unity values which were adopted in most previously available studies. Results for heat transfer to Nitrogen ($N_2$) in addition to air were obtained. An improved correlation was developed for natural convective heat transfer for the Rayleigh number values between $2 \times 10^8$ and $2 \times 10^{11}$.

2.2.1.2 Two-Sided Plate

Limited experimental work has been devoted to natural convective heat transfer from two-sided plates. The work however, focused on investigating natural convective heat transfer under a limited set of conditions such as considering a limited range of the Rayleigh numbers covering only the laminar flow region or only taking into account a
single thermal boundary condition existing over the plate. The effect of the plate thickness on the heat transfer rate has also not been studied in detail.

Kobus and Wedekind (2001) carried out an experimental study of natural convective heat transfer from horizontal isothermal two-sided circular disks of different thickness-to-diameter ratios in air. The authors proposed dimensionless correlations for the natural convective heat transfer rate over a Rayleigh number range from $3 \times 10^2$ to $3 \times 10^7$.

### 2.2.2 Inclined Plates

Some previous studies of natural convective heat transfer from inclined plates have been undertaken. Studies dealing with both inclined single-sided and inclined two-sided plates are available.

#### 2.2.2.1 Single-Sided Plate

In an early study of inclined flat surfaces, Hassan and Mohamed (1970) measured natural convective local heat transfer coefficients along an isothermal flat surface in air with inclination angles varying from horizontal facing upward surface through the vertical position and to horizontal facing downward surface. The range of flow conditions considered was in the laminar flow region. Local Nusselt number variations were correlated in terms of the local Grashof number for various values of the inclination angle.

Pera and Gebhart (1973) analytically and experimentally studied the laminar natural convective heat transfer characteristics over horizontal and slightly inclined surfaces. Surface thermal boundary conditions of uniform temperature and uniform heat flux were considered. The similarity solution approach was used to describe the temperature and velocity distributions and the coupled governing equations were integrated numerically. Correlations describing the local Nusselt numbers for Prandtl numbers near unity were
presented for both surface thermal boundary conditions. In addition, the effect of inclination angle on local Nusselt number was shown and compared with the theoretical values for Prandtl number, $Pr=0.7$.

Many studies of natural convective heat transfer from single-sided vertical plates have been undertaken. For example, Churchill and Chu (1975) conducted a theoretical study and developed expressions for Nusselt numbers at different values of Rayleigh and Prandtl numbers (laminar and turbulent flow regimes were covered) for a vertical flat plate with both uniform surface heating and uniform surface temperature conditions being considered. The theoretical solutions were obtained using laminar boundary-layer theory.

2.2.2.2 Two-Sided Plate

Relatively little attention has been given to natural convective heat transfer from inclined two-sided plates. Most of the available studies also only cover laminar flow conditions.

Hassani and Hollands (1989) performed experiments to measure natural convective heat transfer from two-sided isothermal plates of different shapes in air. The plates were oriented in various directions and the range of Rayleigh numbers considered was from $10^2$ to $10^8$.

Kobus and Wedekind (2002) extended their early research (Kobus and Wedekind, 2001) on natural convective heat transfer from horizontal circular disks to include the influence of inclination angle on the heat transfer from thin isothermal two-sided circular disks in air for inclination angles between vertical and horizontal. The authors used their experimental data to develop an empirical correlation for a range of Rayleigh numbers from $2\times10^2$ to $3\times10^7$. 
2.3 Numerical Studies

Numerical studies of situations similar to those considered in previous experimental and analytical studies have been conducted. Some numerical studies that considered horizontal as well as inclined plates will be discussed in the following section.

2.3.1 Horizontal Plates

In this section, numerical studies dealing with natural convective heat transfer from either a single-sided or a two-sided horizontal plate are discussed.

2.3.1.1 Single-Sided Plate

Friedrich and Angirasa (2001) presented a two-dimensional numerical simulation of laminar natural convective flow below a downward-facing heated horizontal surface at different values of Prandtl number. The range of Prandtl numbers considered was \( 0.1 < Pr < 100 \), and Rayleigh numbers of between \( 0.7 \times 10^5 < Ra < 0.7 \times 10^9 \) were considered. The numerical results obtained in this study were compared with the experimental results of Aihara et al. (1972) and good agreement was obtained.

Oosthuizen (2015a) numerically studied the natural convective heat transfer from a horizontal rectangular isothermal element facing upward and downward, the element being embedded in a larger rectangular flat adiabatic surface. The element was exposed to air. Laminar, transitional, and turbulent flows were considered, the governing equations being solved using the k-\( \varepsilon \) turbulence model applied under all conditions. Variations of the Nusselt number with Rayleigh number were obtained for various length to width ratios of the rectangular heated element. It was shown that, as is to be expected, the heat transfer rate for the upward-facing element case is higher than that for the downward-facing
element case. A comparison of results for the laminar range was done with the empirical correlation of Succe (1985) and shown to agree quite closely.

Natural convective heat transfer from horizontal isothermal heated elements in air having a complex shape (I- and +-shaped plates) was studied numerically by Oosthuizen (2015b). The author considered the case where the heated element is embedded in a horizontal adiabatic surrounding surface and facing upward and downward. The range of Rayleigh numbers considered was such that laminar, transitional and turbulent flows can occur. The k-ε turbulence model was used under all conditions. The results obtained indicated that if the Nusselt and Rayleigh numbers based on the length scale \( l = 4A_{plate}/P \) where, \( A_{plate} \) is the heated element surface area and \( P \) is its perimeter were used in laminar and turbulent flow regions, then the variation of the Nusselt number with the Rayleigh number are essentially the same for the element shapes considered in both the upward and downward-facing cases. For transitional flow this conclusion does not apply, the conditions under which transition occurs being dependent on the element shape. Oosthuizen and Kalendar (2016a) studied the same problem of natural convection heat transfer from elements of complex shape (Oosthuizen, 2015b) but considered the case where the heated elements had a uniform surface heat flux rather than being isothermal. The authors gave attention to circular shape with an inner circular adiabatic section (the diameter of this inner section was varied to give different shapes), I- and +-shaped elements. The heated elements considered in this study were embedded in a larger surrounding flat adiabatic surface and were facing upward. The flow conditions considered covered laminar, transitional and turbulent flows. The results were compared with those obtained for a square and a circular element. It was found that for the circular element with an inner circular adiabatic section,
the heat transfer rates are well-correlated in the laminar and fully- turbulent flow regions when results were expressed in terms of Nusselt and Rayleigh numbers based on the length scale $l = 4A_{plate}/P$ defined earlier, while for the case of the square element, circular element with no inner adiabatic section and I- and +-shaped elements, the heat transfer rates are well-correlated in the laminar flow region only if results were expressed in terms of the same length scale.

2.3.1.2 Two-Sided Plate

Chambers and Lee (1997) conducted a numerical simulation to determine the local and average natural convection Nusselt numbers for an upward-facing uniformly heated surface and for the case of downward-facing uniformly heated surface. Both surfaces were exposed to air. The simulation results were compared with the experimental data obtained by Sparrow and Carlson (1986) for the case of upward-facing uniformly heated surface with excellent agreement being found. The complete isothermal horizontal plate with heat being transferred from both upper and lower surfaces simultaneously was also included in this numerical study. The range of Rayleigh numbers used was from 86 to $1.9 \times 10^8$ and it was varied by changing the plate width and heating rate. Correlations for the Nusselt number over the range of Rayleigh numbers considered were proposed.

Wei et al. (2003) numerically investigated the two-dimensional laminar natural convective heat transfer from an upward-facing isothermal surface, from a downward-facing isothermal surface and the simultaneous convective heat transfer above and below a two-sided isothermal horizontal thin plate in an infinite space. The Rayleigh number was varied over the range from $1.0 \times 10^5$ to $1.7 \times 10^7$. The numerical results for the upward-facing horizontal surface were compared with the results given by the analytical solution obtained
by Pera and Gebhart (1973) and with the experimental data of Sparrow and Carlson (1986) for the single-sided plate. For the downward-facing horizontal surface, the numerical results were compared with the numerical calculation of Friedrich and Angirasa (2001) and the experimental data of Aihara et al. (1972) for the single-sided plate. Both comparisons showed quite good agreement.

Fontana (2014) carried out a numerical study to predict the influence of the Prandtl number on natural convective heat transfer from an isothermal horizontal infinite thin strip. The Rayleigh number values between $10^2$ and $10^6$ and Prandtl number values of 0.71, 2.6, 6.7 and 13.5 were considered. The results of this study were compared with the results obtained by Wei and Kawaguchi (2003) for the case of the Prandtl number of 0.71 which is effectively the value for air with good agreement. The results of this study indicated that for $0.71 \leq Pr \leq 13.5$ the average Nusselt number on the upper surface cannot be expressed as a function of the Rayleigh number alone, i.e., is dependent on the value of Prandtl number. Correlations for the Nusselt number in terms of the Rayleigh and Prandtl numbers were proposed.

A numerical study of the simultaneous natural convective heat transfer from the upper and lower surfaces of a thin isothermal horizontal circular plate in air was undertaken by Oosthuizen and Kalendar (2016b). The situation considered in this study was an isothermal circular plate with inner adiabatic section whose dimensionless diameter was varied to investigate its effect on the heat transfer rate. The k-ε turbulence model was adopted in solving the governing equations over a range of conditions that included laminar, transitional and fully-turbulent flows. The results obtained in this study were compared with the experimental results of Hassani and Hollands (1989) and with the correlation of
Kobus and Wedekind (2001) for the laminar flow region and good agreement was obtained. Based on the results obtained in the study, it was concluded that when the Nusselt and Rayleigh numbers are based on the length scale \( l = 4A_{plate}/P \) (defined earlier) then for the case of the upward-facing, the downward-facing and the two-sided heated elements, the variations of the Nusselt number with Rayleigh number are approximately the same in the laminar and fully-turbulent flow regions for all values of the element inner diameter considered.

2.3.2 Inclined Plates

Existing numerical studies dealing with natural convective heat transfer from either single-sided or two-sided inclined plates are discussed in this section.

2.3.2.1 Single-Sided Plate

Oosthuizen (2014) investigated the effect of inclination angle on the natural convection heat transfer from an inclined isothermal upward-facing square flat plate embedded in a flat adiabatic surrounding surface in air. The range of inclination angles with respect to the horizontal and the range of Rayleigh numbers considered were from 0° to 30° and from \(10^5\) to \(10^{13}\), respectively. The k-\(\varepsilon\) turbulence model was used to obtain the numerical solution and the results showing the variation of the Nusselt numbers with the inclination angles were obtained. It was found that the form of variation for the inclination angles less than 3° differed from the form of variation for the inclination angles greater than 3°.

2.3.2.2 Two-Sided Plate

Wei et al. (2002) undertook a numerical study of simultaneous natural convective heat transfer from the two surfaces of a uniformly heated thin plate in air with the plate set at
arbitrary inclination angles from horizontal (angles of 0° to 90° being considered). The values of Rayleigh number used were varied from $4.8\times10^6$ to $1.87\times10^8$ by varying the plate width and heating rate. For validation purposes, the results for the horizontally upward-facing uniformly heated plate were compared with the correlation equation of Chambers and Lee (1997) as well as with the experimental results of Sparrow and Carlson (1986) and good agreement was obtained. It was concluded that for an inclination angle of less than 10°, the heat transfer characteristics of the upper and lower surfaces are different. Therefore, the average Nusselt number for the upper and lower surfaces were expressed in terms of two different equations, whereas for inclination angle greater than or equal to 10°, the average Nusselt number for the two heated upper and lower surfaces could be adequately correlated by a single equation.

Corcione et al. (2011) numerically studied the steady, laminar natural convective heat transfer from an inclined two-sided plate whose sides are simultaneously heated to the same uniform temperature and the situation where one surface was isothermal while the other surface was adiabatic (single-sided case). The range of inclination angles considered was from 0° to 75° (measured from the vertical), the range of the Rayleigh numbers was from 10 to $10^7$ and the range of the Prandtl numbers was from 0.7 to 140. Correlations for the average Nusselt numbers were developed. To validate the numerical procedure, the average Nusselt numbers for the vertical orientation at different values of Rayleigh and Prandtl numbers were compared with the theoretical solution results given by Churchill and Chu (1975) for a single-sided heated plate and by Hassan and Mohamed (1970) for a two-sided heated plate with good agreement.
2.3 Heat Transfer Using Nanofluid

The heat transfer characteristics of nanofluids depend, as mentioned earlier, on the properties of the base liquid and of the nano particles, on the particle concentration, and on the particle size.

Many equations for the properties of a nanofluid have been proposed. For example, Minkowycz et al. (2013) developed expressions for predicting the nanofluid density, specific heat and thermal expansion coefficient. In addition, Minkowycz et al. (2013) proposed two empirical correlating equations to predict the nanofluid dynamic viscosity and thermal conductivity.

Studies of natural convective heat transfer in enclosures that contain a nanofluid have been undertaken. A brief review of some of these available studies is given below.

Putra et al. (2003) experimentally studied the natural convective heat transfer in a horizontal cylinder heated from one end and cooled from another with two nanofluids (aluminum oxide Al₂O₃, 131 nm and copper oxide CuO, 87 nm, both materials being in water with a concentration of up to 4%). Attention was given to predicting the effects of the particle concentration and the nanofluid material on the heat transfer rate. The results obtained in this study indicated that the natural convective heat transfer rate in the nanofluid was lower than that with the base fluid. Additionally, the natural convective heat transfer rate obtained by using the CuO was lower than when using the Al₂O₃.

Experiments on natural convective heat transfer in a bottom-heated horizontal enclosure consisting of two differentially heated discs that was filled with a TiO₂/H₂O nanofluid were carried out by Wen and Ding (2006). The authors observed that an increase in the particle concentration of up to 2.4% leads to a significant decrease in the heat transfer rate and a decrease in the Nusselt number for a wide range of Rayleigh numbers. The results of this
study were similar to the results obtained by Putra et al. (2003) that were discussed above. A numerical study by Khanafer et al. (2003), an analytical study by Kim et al. (2004), and an experimental study by Nanna et al. (2004) showed that unlike Putra et al. (2003), and Wen and Ding (2006) the natural convective heat transfer rate increases as the nano particle concentration increases.

Aminossadati and Ghasemi (2009) numerically studied the natural convection cooling of a heat source embedded on the bottom wall of an enclosure filled with nanofluids. The top and vertical walls of the enclosure were maintained at a relatively low temperature. Attention was given to the influence of the Rayleigh number, the location and geometry of the heat source, the type of nanofluid and the solid volume fraction of nanoparticles on the cooling performance. The results showed that using nanofluids improves the cooling performance compared with the situation when water is used especially at low Rayleigh numbers.

Ho et al. (2010) conducted an experimental study on natural convective heat transfer in side-heated vertical square enclosures of different sizes that were filled with an alumina-water nanofluid. The authors concluded that the natural convective heat transfer across the enclosure was increased due to the increase in thermal conductivity of the nanofluid relative to that of the base fluid, while the relative changes in the remaining properties like viscosity, specific heat and volumetric thermal expansion coefficient contributed detrimentally to the natural convective heat transfer.

Reggio and Vasseur (2012) undertook a numerical study on natural convection from a protruding heater located at the bottom of a square enclosure filled with a copper-water nanofluid. The vertical walls of the enclosure were cooled isothermally while the horizontal
walls were kept adiabatic. The heat source was assumed either to be isothermal or to have a constant heat flux. Attention was given to the effect of the Rayleigh number, the Prandtl number, the geometrical parameters specifying the heater and the nanoparticle concentration. It was found that the heat transfer was enhanced by increasing the nanoparticle concentration for both of the thermal boundary condition applied on the heater.

A numerical study of natural convection in a square enclosure with non-uniform temperature distribution maintained on the bottom wall and filled with different types of nanofluids which had various nanoparticle concentrations was carried out by Ben-Cheikh et al. (2013). The remaining walls of the enclosure are kept at a lower temperature. It was shown that an enhancement in the heat transfer rate is obtained as the nanoparticle concentration increases for the range of Rayleigh numbers considered.
Chapter 3

Methodology

3.1 Introduction

In the present study, natural convective heat transfer from elements of the type being considered has been numerically and experimentally studied. In the numerical part of the study the mass, momentum and energy conservation equations subject to the boundary conditions have been solved numerically in an appropriate solution domain for the case of heated elements in air and for a limited range of conditions for the case of heated elements in a nanofluid. The numerical solutions were obtained using a commercial software code (ANSYS FLUENT©) which is based on the use of the finite volume approach. To generate the meshed computational domain to be solved using FLUENT, a commercial software (GAMBIT) was used.

In this chapter, the numerical methodology, the governing equations and the turbulence model used in the case of heated elements in air are reviewed. In the experimental part of the study, results for heated elements in air were obtained using the “Lumped Capacity” method. An explanation of the experimental setup and procedure will be given later in this chapter.

3.2 Numerical Methodology

3.2.1 Models Considered

The models considered in the numerical study are flat plates of various shapes having a finite thickness and are two-sided which means that the heat is simultaneously transferred from all surfaces of the plate. These models are shown in Figures 3.1 and 3.2. Figure 3.1
shows the plates having a simple shape that were considered and Figure 3.2 shows the plates having a complex shape that were considered. The finite thickness plate situation considered is shown in Figure 3.3 for a square plate and this situation is applied for all other shapes. Natural convective heat transfer from these plates has been numerically investigated for various values of the dimensionless thickness (thickness-to-side length ratio). Some other situations with various vertical side surface thermal boundary conditions and various forms of inclinations existing at these plates have been also investigated.

![Diagram of square and circular plate shapes](image)

**Figure 3.1** Square (left) and circular (right) plate shapes considered showing the definitions of the side length, $w$, of the square plate and the diameter, $d$, and radius, $r$, of the circular plate.

![Diagram of I-shape and +-shape plate shapes](image)

**Figure 3.2** I-shaped (left) and +-shaped (right) plate shapes considered showing the definitions of the side length, $W$, of the plates which is the outside length. As shown $s$ is the arm width of the plates.
Figure 3.3 Finite thickness plate situation considered, a square plate being shown. Also shown are the definitions of the plate thickness, $h$, and of the top (upper) and of the vertical side surface of a two-sided plate. The bottom (lower) surface is below the top surface.

### 3.2.2 Assumptions

The following assumptions have been adopted in the numerical model:

1. The flow is steady.

2. The flow is three dimensional for the square, I-, and +-shaped plates while the flow is two dimensional for the circular shape plate.

3. The flow is single phase.

4. The air properties are constant except for the density variation with temperature which gives the rise to the buoyancy force (i.e., Boussinesq approach was adopted).

5. Radiative heat transfer has been neglected.

### 3.2.3 Governing Equations

The governing equations are as follows (Oosthuizen and Naylor, 1999), the $x$, $y$, and $z$ directions being shown in Figure 3.4:

Conservation of mass (Continuity):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial z} = 0 \quad (3.1)$$

where $u$, $v$ and $\omega$ are the velocity components in the $x$, $y$, and $z$ directions, respectively.

Conservation of Momentum (Navier-Stokes) in the $x$, $y$, and $z$ directions:

$x$-direction:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \beta g (T - T_f) \cos \phi \quad (3.2)$$
\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \beta g (T - T_f) \sin \phi \]  

(3.3)

\[ \frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} + \omega \frac{\partial \omega}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} \right) \]  

(3.4)

where, \( \phi \) is the angle that the \( x \)-axis makes with the vertical as shown in Figure 3.4 and \( \beta \) is the thermal expansion coefficient and is defined as the change of fluid density with temperature at constant pressure. For an ideal gas, it is given by:

\[ \beta = \frac{1}{T} \]  

(3.5)

Conservation of Energy:

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \omega \frac{\partial T}{\partial z} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q'''' \]  

(3.6)

Where, \( q'''' \) is the rate of viscous heat generation. For the situations considered in the present study, \( q'''' \) has been neglected due to the low velocities involved.

Figure 3.4 (a) Coordinate system used in the study (\( z \) is normal to page) and (b) Buoyancy force components (Oosthuizen and Naylor, 1999).

Equations (3.1) to (3.6) describe the flow and temperature distributions. The Boussinesq approximation discussed above was adopted for the case of natural convective
heat transfer to air, i.e., the variation of the fluid properties with temperature is assumed to be sufficiently small to be neglected except for the density variation that gives rise to the buoyancy force. The buoyancy force components terms \( \beta g \rho (T - T_f) \cos \phi \) and \( \beta g \rho (T - T_f) \sin \phi \) which appear in the \( x \) and \( y \)-direction momentum equations (3.2) and (3.3), respectively are the driving force terms in natural convective heat transfer.

### 3.2.4 Boundary Conditions

The following boundary conditions have been considered:

1. All surfaces of the element are isothermal and at the same temperature.
2. Upper and lower surfaces of the element are isothermal and at the same temperature while the vertical side surface is adiabatic (no heat transfer to or from this surface).
3. A uniform pressure condition was assumed on the outer boundaries.

### 3.2.5 Dimensionless Parameters

The following dimensionless parameters will be used in the analysis throughout this study (Oosthuizen and Naylor, 1999):

1. Grashof number, \( Gr \), which is the measure of the ratio of buoyancy forces to viscous force in the flow and is given by:

\[
Gr = \frac{g \beta (T_w - T_f) l^3}{v^2}
\]

(3.7)

where \( l \) is a characteristic length of the heated surface.

2. Prandtl number, \( Pr \), which is the ratio of the momentum to thermal diffusivity of the fluid flow and is given by:

\[
Pr = \frac{v}{\alpha} = \frac{\mu c_p}{k}
\]

(3.8)
In this study, air has been considered as the surrounding fluid of the heated element in the majority of the cases. For air, the value of Prandtl number at standard ambient conditions will be taken as equal to 0.74.

3. Nusselt number, $Nu$, which is a measure of the ratio of the convective heat transfer rate to conductive heat transfer rate that would exist in the absence of fluid motion. The average (mean) Nusselt number based on a certain characteristic length $l$ is defined as:

$$Nu = \frac{\overline{q'} l}{k(T_w - T_f)} \quad (3.9)$$

where $\overline{q'}$ is the mean heat transfer rate per unit area.

4. Rayleigh number, $Ra = Gr \cdot Pr$, which is the product of Grashof and Prandtl numbers is usually used instead of the Grashof number in natural convective heat transfer analysis.

In the present study, conditions have been considered under which laminar, transitional and fully-turbulent flows can occur. Therefore, a turbulence model has been used with full account being taken of buoyancy force effects. The turbulence model is applied under all flow conditions considered and used to predict when turbulence effects develop in the flow. Some work on using this approach in natural convective flows (Albets-Chico et al., 2008; Kalendar et al., 2016; Oosthuizen and Naylor, 2009; Plumb and Kennedy, 1977; Savill, 1993; Schmidt and Patankar, 1991; Xamán et al., 2005, Zheng et al., 1998) have shown relatively good predictions of heat transfer characteristics for situations similar to that being considered in this study for the laminar, transitional and turbulent flow regions. A description of the turbulence model used in the present study is given in the following section. For the natural convective heat transfer investigation using nanofluid, there was no turbulence model used. Laminar flow was assumed to exist in obtaining the heat transfer results.
3.2.6 Turbulence Model

The term “turbulence” denotes a flow in which an irregular fluctuation (mixing, or eddying motion) is superimposed on the main stream flow. Turbulent flow is unsteady, irregular and seemingly random and chaotic. (Pope, 2000; Schlichting, 1979).

The fluctuations in the velocity field cause the transported quantities such as momentum and energy to fluctuate in an unsteady manner. Therefore; a modified set of equations is needed. The RANS (Reynolds-Averaged Navier-Stokes) method is adopted here to predict the effects of turbulence.

The RANS method is based on the development of a set of equations for the mean flow variables. This is done by time-averaging the instantaneous governing equations. To do this the variables in these equations (instantaneous variables) are replaced by the sum of a time-mean quantity plus a fluctuating deviation from the mean quantity (i.e., \( u = \bar{u} + u' \), \( v = \bar{v} + v' \), \( T = \bar{T} + T' \), \( p = \bar{p} + p' \)), where, “\( \bar{\ } \)” denotes the time-mean quantity and “\( \prime \)” denotes the fluctuating deviation from the mean quantity. Substituting these quantities into the governing equations and then time-averaging these equations, the following set of equations is obtained for the case where the mean flow is steady (Cable, 2009).

Continuity:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{\omega}}{\partial z} = 0 \quad (3.10)
\]

Momentum:

x-direction:

\[
\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{\omega} \frac{\partial \bar{u}}{\partial z} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \nu \left( \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2} \right) - \left( \frac{\partial}{\partial x} \bar{u} \bar{v} \right) - \left( \frac{\partial}{\partial y} \bar{u} \bar{\omega} \right) - \left( \frac{\partial}{\partial z} \bar{u} \bar{\omega} \right) \]

\[
\rho g \beta (\bar{T}_f - \bar{T}) \cos \phi \quad (3.11)
\]
\[ y\text{-direction:} \]
\[ \dot{u} \frac{\partial \dot{v}}{\partial x} + \dot{v} \frac{\partial \dot{v}}{\partial y} + \frac{\partial \dot{v}}{\partial z} = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + v \left( \frac{\partial^2 \dot{v}}{\partial x^2} + \frac{\partial^2 \dot{v}}{\partial y^2} + \frac{\partial^2 \dot{v}}{\partial z^2} \right) - \left( \frac{\partial}{\partial x} u'v' + \frac{\partial}{\partial y} v'^2 + \frac{\partial}{\partial z} v'\omega' \right) \]

\[ z\text{-direction:} \]
\[ \dot{u} \frac{\partial \omega}{\partial x} + \dot{v} \frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial z} = -\frac{1}{\rho} \frac{\partial \rho}{\partial z} + v \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} \right) - \left( \frac{\partial}{\partial x} u'\omega' + \frac{\partial}{\partial y} v'^2 + \frac{\partial}{\partial z} v'\omega' \right) \]

Energy:
\[ \dot{u} \frac{\partial \bar{T}}{\partial x} + \dot{v} \frac{\partial \bar{T}}{\partial y} + \frac{\partial \bar{T}}{\partial z} = \frac{k}{\rho c_p} \left( \frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} + \frac{\partial^2 \bar{T}}{\partial z^2} \right) - \left( \frac{\partial}{\partial x} u'\bar{T}' + \frac{\partial}{\partial y} v'\bar{T}' + \frac{\partial}{\partial z} \omega'\bar{T}' \right) \]

As can be seen the time-averaged set of equations contain extra unknowns. To solve for these equations, a “turbulence model” is used.

Several turbulence models have been developed. To select the turbulence model to use in a given situation, some considerations must be taken of the flow situation including the physical nature of the flow, the accuracy level required, and of the computational resources available. In this study, the \( k \)-\( \varepsilon \) turbulence model has been used.

3.2.6.1 \( k \)-\( \varepsilon \) Turbulence Model

The \( k \)-\( \varepsilon \) turbulence model where \( k \) stands for turbulent kinetic energy which determines the energy in the turbulent flow, and \( \varepsilon \) for turbulent dissipation which determines the dissipation rate of the turbulent kinetic energy. The \( k \)-\( \varepsilon \) turbulence model is one of the most widely used models in the analysis of turbulent flows. It provides reasonable accuracy and reasonably economical computational times for a wide range of turbulent flows. The model has three basic versions: standard, RNG, and realizable. The main differences between these versions are the turbulent Prandtl numbers, some terms in
the \( \varepsilon \)-equation and the way in which the turbulent viscosity is calculated. The standard \( k \)-

\( \varepsilon \)-turbulence model has been used in the current studies.

3.2.6.1.1 Standard \( k - \varepsilon \) Turbulence Model

The turbulent kinetic energy \( (k) \) and the turbulent dissipation \( (\varepsilon) \) are determined using

the following equations (ANSYS FLUENT 6.3 user’s guide):

\( k \)-equation:

\[
\frac{\partial}{\partial x}(\rho k \bar{u}) + \frac{\partial}{\partial y}(\rho k \bar{v}) + \frac{\partial}{\partial z}(\rho k \bar{w}) = \frac{\partial}{\partial x}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right) \frac{\partial k}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right) \frac{\partial k}{\partial y}\right] + \frac{\partial}{\partial z}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right) \frac{\partial k}{\partial z}\right] + 
\]

\[
G_k + G_b - \rho \varepsilon - Y_M + S_k \tag{3.15}
\]

\( \varepsilon \)-equation:

\[
\frac{\partial}{\partial x}(\rho \varepsilon \bar{u}) + \frac{\partial}{\partial y}(\rho \varepsilon \bar{v}) + \frac{\partial}{\partial z}(\rho \varepsilon \bar{w}) = \frac{\partial}{\partial x}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right) \frac{\partial \varepsilon}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right) \frac{\partial \varepsilon}{\partial y}\right] + \frac{\partial}{\partial z}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right) \frac{\partial \varepsilon}{\partial z}\right] + 
\]

\[
C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon} G_b) - C_{2\varepsilon} \rho \frac{k^2}{\varepsilon} + \varepsilon \tag{3.16}
\]

where \( G_k \) is the generation of turbulent kinetic energy which arises due to mean velocity

gradients, \( G_b \) is the generation of turbulent kinetic energy which arises due to buoyancy,

\( Y_M \) is the fluctuating dilatation in compressible turbulence which contributes to the overall
dissipation rate, \( S_k \) and \( S_\varepsilon \) are source terms defined by the user, \( \sigma_k \) and \( \sigma_\varepsilon \) are the turbulent

Prandtl numbers for the turbulent kinetic energy and its dissipation rate, respectively, \( C_{3\varepsilon} \)
is a constant which determines the degree to which turbulent dissipation \( (\varepsilon) \) is affected by

the buoyancy force, \( \mu \) is the dynamic viscosity and \( \mu_t \) is the turbulent (eddy) dynamic

viscosity at each point and is defined as follows:

\[
\mu_t = \rho \varepsilon \frac{k^2}{\varepsilon} \tag{3.17}
\]

\( C_{1\varepsilon}, C_{2\varepsilon}, \sigma_k, \sigma_\varepsilon \) and \( \epsilon_\mu \) are constants that have been arrived at as the result of numerous

data fitting for a wide range of turbulent flows, they are:
\( C_{1e} = 1.44, \ C_{2e} = 1.92, \ \sigma_k = 1, \ \sigma_e = 1.3, \ c_\mu = 0.09 \)

\( Y_M, S_k \) and \( S_e \) in the situations considered in this study are equal to zero.

The turbulent velocity terms are assumed to be proportional to the mean velocity gradients and are expressed in the following tensor notation relation (Cable, 2009):

\[
\overline{u'_i u'_j} = \frac{\mu_t}{\rho} \left( \frac{\partial \overline{u_j}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_j} \right) - \frac{2}{3} k \delta_{ij}
\]

where \( i \) and \( j \) represent the components in corresponding directions and \( \delta_{ij} \) is the Kronecker delta.

Also, the turbulent heat fluxes are assumed to be proportional to the mean temperature gradients and are expressed in the following tensor notation relation (Cable, 2009):

\[
-\rho c_p T' \overline{u'_i} = c_p \epsilon_H \frac{\partial T}{\partial x_j}
\]

where \( \epsilon_H \) is the turbulent (eddy) diffusivity of heat.

Introducing the turbulent Prandtl number \( (Pr_T) \) which is defined as (Cable, 2009):

\[
Pr_T = \frac{\epsilon}{\epsilon_H}
\]

where \( \epsilon \) is the turbulent (eddy) kinematic viscosity and is equal to \( \mu_t / \rho \).

Substituting equations (3.18-3.20) into equations (3.10-3.14) leads to the following set of governing equations:

Continuity:

\[
\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0
\]

Momentum:

\( x \)-direction:

\[
\overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} + \overline{w} \frac{\partial \overline{u}}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \left( \nu + \epsilon \right) \left( \frac{\partial^2 \overline{u}}{\partial x^2} + \frac{\partial^2 \overline{u}}{\partial y^2} + \frac{\partial^2 \overline{u}}{\partial z^2} \right) - \rho g \beta (\overline{T_f} - \overline{T}) \cos \phi
\]
\[
\begin{align*}
\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} + \frac{\partial \psi}{\partial z} &= -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + (v + \epsilon) \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) - \rho g \beta (\overline{T}_f - \overline{T}) \sin \phi \\
\frac{\partial \tilde{\omega}}{\partial y} + \frac{\partial \tilde{\omega}}{\partial y} + \frac{\partial \tilde{\omega}}{\partial z} &= -\frac{1}{\rho} \frac{\partial \rho}{\partial z} + (v + \epsilon) \left( \frac{\partial^2 \tilde{\omega}}{\partial x^2} + \frac{\partial^2 \tilde{\omega}}{\partial y^2} + \frac{\partial^2 \tilde{\omega}}{\partial z^2} \right) \\
\frac{\partial \overline{T}}{\partial x} + \frac{\partial \overline{T}}{\partial y} + \frac{\partial \overline{T}}{\partial z} &= (\alpha + \frac{\epsilon}{\rho \tau_f}) \left( \frac{\partial^2 \overline{T}}{\partial x^2} + \frac{\partial^2 \overline{T}}{\partial y^2} + \frac{\partial^2 \overline{T}}{\partial z^2} \right)
\end{align*}
\]

3.2.7 CFD Model Development

To solve the governing equations, computational fluid dynamics (CFD) which uses numerical methods to solve problems that involve fluid flows was used. Numerical results were obtained by using a commercial Finite Volume Method software package (FLUENT).

CFD analysis involves three basic components; the pre-processor, the solver, and the post-processor. Figure 3.5 shows the overall CFD modeling procedure. Firstly, after the model has been developed, the pre-processor is used to generate the mesh that is later used as an input to the solver. In the current study, the preprocessor is a commercial software package (GAMBIT) and this software package was used to generate the mesh in the flow domain.

Secondly, the solver discretizes the differential equations and converts them to algebraic equations which can be solved numerically using the finite volume method. In the present study, the solver is FLUENT. There are two solvers available in FLUENT, segregated solver and coupled solver. The segregated solver which is used in the current study is so called since it solves the governing equations while they are segregated from one another. The converged solution is obtained by performing several iterations of the solution loop. Figure 3.6 illustrates the steps in the segregated solver method. The coupled solver,
however, is more ideal for high-velocity flows which is unnecessary in the current modeling, and it has been revealed that convergence could be reached using the segregated solver with reasonable computational time. The solver settings in Fluent used in this study are as follows:

- Pressure-based, three-dimensional, double-precision (3ddp) segregated steady solver for the Square, I- and +-shaped plates.
- Pressure-based, two-dimensional, double-precision (2ddp) segregated steady solver for the circular shape plate.
- Standard $k - \varepsilon$ turbulence model with enhanced wall treatment.
- SIMPLE pressure-velocity coupling
- PRESTO discretization for pressure.
- Second order upwind discretization for momentum and energy.
- Convergence criteria of between $10^{-5}$ and $10^{-7}$.
- Under-relaxation factors shown in the table 3.1.

Finally, the post-processor is a tool which displays the solution in the form of graphs, plots, and charts. FLUENT and Microsoft Excel are used as post-processors in this study.
**Figure 3.5 Flowchart of CFD modeling procedure (Cable, 2009).**
Figure 3.6 Segregated solver method (Cable, 2009).

Table 3.1 Under-relaxation factors used in Fluent

<table>
<thead>
<tr>
<th>Under-Relaxation Factor</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>0.15</td>
</tr>
<tr>
<td>Density</td>
<td>0.5</td>
</tr>
<tr>
<td>Body force</td>
<td>0.5</td>
</tr>
<tr>
<td>Momentum</td>
<td>0.35</td>
</tr>
<tr>
<td>Turbulent kinetic energy</td>
<td>0.4</td>
</tr>
<tr>
<td>Turbulent dissipation rate</td>
<td>0.4</td>
</tr>
<tr>
<td>Turbulent viscosity</td>
<td>0.5</td>
</tr>
<tr>
<td>Energy</td>
<td>0.5</td>
</tr>
</tbody>
</table>
3.2.7.1 Convergence Criteria

To determine when to terminate the numerical simulation, convergence criteria based on certain tolerance values called the residuals were defined. The residuals are defined by the user and as noted before they were selected to be between $10^{-5}$ and $10^{-7}$. This means that the solution will converge once the residual values for different equations used in the study are being met.

3.2.7.2 Grid Generation

In order to numerically solve the governing equations using FLUENT, the element geometry and mesh were created, and the boundary conditions were applied using a commercial software called GAMBIT. The quality of the mesh plays a significant role in the accuracy of the numerical results as well as the computational time and stability. For example, the high temperature gradient regions require fine mesh enough to resolve the flow properties. Therefore, it is essential to find an optimum mesh size and shape for all cases considered in the present study and this step was done using GAMBIT. The shape of the elements used to generate the mesh and the meshing scheme strongly depend on the model geometry. For the models considered in this study the mesh was generated using quadrilateral elements and the Quad/map scheme was used to mesh the faces while Hex/map scheme was used to mesh the volumes. To ensure that the numerical results obtained are grid independent, an extensive grid-independence testing was undertaken, and this will be discussed later in this chapter.

3.2.7.3 Near Wall Region

In natural convective heat transfer, the boundary layer thickness can be thin, therefore, a fine mesh near the wall is required to resolve the flow properties and the high temperature
and velocity gradients. As explained earlier in this chapter, the $k - \varepsilon$ turbulence model was used in the numerical simulation and to deal with properties near the wall, the enhanced wall treatment function was applied. The enhanced wall treatment is a near-wall modeling method that combines a two-layer model with enhanced wall functions and is applied with a mesh fine enough to resolve the laminar sublayer near the wall (FLUENT). Therefore, the $y^+$ value which represents the dimensionless wall distance is then used to determine the proper size of the cell near the wall and is defined as:

$$y^+ = \frac{yU_{\tau}}{v}$$  \hspace{1cm} (3.26)

where, $y$ is the distance from the wall to the cell center and $U_{\tau}$ is the friction velocity defined as:

$$U_{\tau} = \sqrt{\frac{\tau_w}{\rho}}$$  \hspace{1cm} (3.27)

where the subscript $w$ denotes conditions at the wall.

3.2.7.4 Grid Independence Test

A grid-independence study using a wide range of grid point numbers and convergence-criteria independence testing was done for all cases in the present thesis and the results presented here are grid-independent to better than one per cent. Typical results for the case of a horizontal square plate having a dimensionless thickness, $H$, of 0 are shown in table 3.2 and Figure 3.7. The shaded cell in Table 3.2 represents the optimum number of grid points which has been selected.
Table 3.2 Grid-independence test results for a horizontal square plate having a dimensionless thickness, $H$, of 0

<table>
<thead>
<tr>
<th>Rayleigh Number, Ra</th>
<th>Mesh Size (Total Number of Nodes)</th>
<th>$\text{Nu}_\text{bot}$</th>
<th>$\text{Nu}_\text{top}$</th>
<th>$\text{Nu}_\text{total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.E+07</td>
<td>218660</td>
<td>42.5732</td>
<td>33.9662</td>
<td>38.2697</td>
</tr>
<tr>
<td></td>
<td>485184</td>
<td>41.5571</td>
<td>32.6917</td>
<td>37.1244</td>
</tr>
<tr>
<td></td>
<td>745200</td>
<td>40.9671</td>
<td>32.4667</td>
<td>36.7169</td>
</tr>
<tr>
<td></td>
<td>1160000</td>
<td>39.6397</td>
<td>32.1863</td>
<td>35.9130</td>
</tr>
<tr>
<td></td>
<td>1278900</td>
<td>39.5704</td>
<td>32.2298</td>
<td>35.9001</td>
</tr>
<tr>
<td>1.E+14</td>
<td>218660</td>
<td>2855.6790</td>
<td>17766.0322</td>
<td>10310.8556</td>
</tr>
<tr>
<td></td>
<td>485184</td>
<td>3701.1780</td>
<td>15957.7000</td>
<td>9829.4390</td>
</tr>
<tr>
<td></td>
<td>745200</td>
<td>4014.1630</td>
<td>15247.7810</td>
<td>9630.9720</td>
</tr>
<tr>
<td></td>
<td>1160000</td>
<td>4614.7934</td>
<td>14616.6066</td>
<td>9615.7000</td>
</tr>
<tr>
<td></td>
<td>1278900</td>
<td>4606.3370</td>
<td>14706.9793</td>
<td>9656.6582</td>
</tr>
</tbody>
</table>
Figure 3.7 Grid-independence test results for a horizontal square plate having a dimensionless thickness, $H$, of 0 at (a) $Ra=10^7$ and (b) $Ra=10^{14}$.

3.3 Experimental Methodology

3.3.1 Lumped Capacity Method

In the experimental study, the heat transfer rate from the heated elements considered has been measured using the lumped capacity method. In this method, the models, which were made of high conductivity material, were heated up and then exposed to air allowing them to cool. Temperature-time variations as the model cools were measured and recorded over a selected temperature range. It has been assumed that the model temperature was
uniform at any instant of time during the cooling process. Therefore, the overall heat transfer coefficient from the model can be determined from the measured temperature-time variation by using the following equation:

\[
h_t = \frac{mc}{A_{\text{total}}} \ln \left( \frac{(T_i - T_f)}{(T_e - T_f)} \right)
\]  

(3.28)

where \( h_t \) represents a combination of convective and radiant heat transfer to the surrounding air and conductive heat transfer to the supporting elements.

To check the unsteady effects during an experimental run, plots of \( (T - T_f) \) versus time on a semi-log scale were generated over the cooling interval for all models with different configurations. Figure 3.8 shows a typical plot of \( (T - T_f) \) versus time for the horizontal square plate with dimensionless thickness, \( H \) of 0.1. The variation of \( (T - T_f) \) with time will be seen to be approximately a straight line. The slope of the line is given by the following equation:

\[
\text{Slope} = \frac{\ln \left( \frac{(T_i - T_f)}{(T_e - T_f)} \right)}{t_i - t_e} = -\frac{h_t A_{\text{total}}}{mc}
\]  

(3.29)

The constant slope signified that the quantity \( \frac{h_t A_{\text{total}}}{mc} \) remained essentially constant during the experimental test and since the terms \((A_{\text{total}}, m \text{ and } c)\) remain constants over the experimental temperature range considered, it indicated that \( h_t \) remained constant during the test and therefore the unsteady effects were negligible and the lumped capacity method could be used with good accuracy. This was checked and found to be true in a number of other experimental runs.
3.3.1.1 Justification of the Use of Lumped-Capacity Method

To apply the lumped capacity method, it must be assumed that the model temperature was uniform at any time instant during the cooling process. Now it is usual to assume that if Biot number, \( Bi < 0.1 \) the uniform body temperature assumption can be used. Biot number is a measure of a system’s internal resistance to conduction to the resistance to heat transfer from the surface to the surroundings, therefore it can be defined by the following equation:

\[
Bi = \frac{h_i \left( \frac{V}{\xi_{total}} \right)}{k_s} \tag{3.30}
\]

where \( k_s \) is the thermal conductivity of the aluminum plate at its average temperature. For all tests undertaken in the present study, the maximum value of Biot number during the cooling process for each model was much less than 0.1 (see Table 3.3), which means that the model temperature remained uniform at any time instant while cooling and as a result, the lumped capacity method was justified to use. Figure 3.9 shows the variation of Biot
number during the cooling process for a horizontal square plate having a dimensionless thickness, $H$, of 0.1.

**Table 3.3 Maximum Biot number for different models**

<table>
<thead>
<tr>
<th>Model</th>
<th>Diagonal Inclination Angle, $\varphi$ (degrees)</th>
<th>Dimensionless Thickness, $H$</th>
<th>Maximum Biot Number, $Bi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Plate</td>
<td>0</td>
<td>0.1</td>
<td>0.000178</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.0635</td>
<td>0.000242</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.0635</td>
<td>0.000244</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.0635</td>
<td>0.000252</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0635</td>
<td>0.000259</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>0.0635</td>
<td>0.000260</td>
</tr>
<tr>
<td>Circular Plate</td>
<td>0</td>
<td>0.1</td>
<td>0.000208</td>
</tr>
<tr>
<td>I-Shaped Plate</td>
<td>0</td>
<td>0.1</td>
<td>0.000164</td>
</tr>
<tr>
<td>+-Shaped Plate</td>
<td>0</td>
<td>0.1</td>
<td>0.000169</td>
</tr>
</tbody>
</table>

**Figure 3.9** Biot number during the cooling process for a horizontal square plate having a dimensionless thickness, $H$, of 0.1.
The lumped capacity method has been widely used (e.g., Oosthuizen and Madan, 1971; Oosthuizen, 1976; Oosthuizen and Paul, 1984; Oosthuizen and Bishop, 1987; Oosthuizen and Birk, 1988; Garrett, 2002) because it is easy to apply. The major disadvantage of the method, however, is that it only gives the mean heat transfer coefficient over the entire surface and since it requires that Biot number be small, it can only be applied in situations in which the heat transfer coefficient is relatively low and/or thermal conductivity of the plate is large which is the case in all of the experimental studies of the present thesis.

3.3.2 Apparatus

To study natural convective heat transfer from horizontal and inclined plates support frames on which the models could be mounted were designed and constructed based on the model dimensions, typical forms being shown in Figure 3.10. These support frames were mounted on a leveled horizontal table and were constructed using plexiglas plates on which small diameter, steel elements with sharp points were fixed to make as little contact area as possible with the models. The experimental setup was placed in a large test chamber (shown in Figure 3.11) to avoid the external disturbances in the room air as well as the temperature changes in the room from interfering with the flow over the model while the experiments were taking place.
Figure 3.10 Support frames used to mount the models.

Figure 3.11 The outside view of the test chamber where experiments have been undertaken. The height, width and depth of this chamber were 3.35 m, 2.14 m and 2.44 m, respectively.

3.3.2.1 Flat Plates (Models)

The flat plates used in the test apparatus (shown in Figure 3.12) were machined from aluminum alloy (Al 6061-T6). The dimensions and the mass of the test models used in the
experimental study and the properties of the test models’ material are given in Tables 3.4 and 3.5.

![Figure 3.12 Test models used in the experimental study.](image)

Table 3.4 Dimensions and mass of the models

<table>
<thead>
<tr>
<th>Model</th>
<th>Side Length, $w$ (m)</th>
<th>Thickness, $h$ (m)</th>
<th>Dimensionless Thickness, $H = h/w$</th>
<th>Mass, m (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Plate</td>
<td>0.1</td>
<td>0.01</td>
<td>0.1</td>
<td>0.27071</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.0127</td>
<td>0.0635</td>
<td>1.36192</td>
</tr>
<tr>
<td>Circular Plate</td>
<td>0.12</td>
<td>0.012</td>
<td>0.1</td>
<td>0.36676</td>
</tr>
<tr>
<td>I-Shaped Plate</td>
<td>0.076</td>
<td>0.0076</td>
<td>0.1</td>
<td>0.07495</td>
</tr>
<tr>
<td>+-Shaped Plate</td>
<td>0.076</td>
<td>0.0076</td>
<td>0.1</td>
<td>0.05125</td>
</tr>
</tbody>
</table>
Table 3.5 Properties of the test models material (MatWeb, Material Property Data, www.matweb.com)

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ ($kg/m^3$)</th>
<th>$c$ ($J/kgK$)</th>
<th>$k_s$ ($W/mK$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 6061-T6</td>
<td>2700</td>
<td>896</td>
<td>167</td>
</tr>
</tbody>
</table>

The model temperature variation with time was measured using thermocouples embedded into holes drilled into the models. Depending on the model dimension, seven or five thermocouples were used at various locations to measure the temperature variations.

3.3.2.2 Instruments Used in the Experimental Study

To heat the models to a certain temperature an electric oven, manufactured by Black & Decker shown in Figure 3.13 was used.

![Black & Decker Oven](Figure 3.13 Black & Decker Oven.)

Temperature variation with time of the models as well as the ambient temperature have been measured using thermocouples of type-T. The thermocouple outputs have been monitored by a data acquisition (D/A) system, manufactured by OMEGA
ENGINEERING, model OMB-DAQ-2408, as shown in Figure 3.14. The D/A system has eight temperature ports and a self-calibrating option. This system was connected to a computer through a USB interface cable. The compatible software with this D/A system was TracerDAC version 2.3.3.0 and it was used to display the temperature variation with time.

![Data Acquisition System](image1)

**Figure 3.14 Data Acquisition System.**

The inclination angles which models make with the horizontal have been measured using a protractor, manufactured by Empire, shown in Figure 3.15.

![Protractor](image2)

**Figure 3.15 Protractor used to measure the inclination angle.**
The weights of the models used in the experimental study have been measured using a weight scale, manufactured by Sartorius, model LE6202P, as shown in Figure 3.16. The resolution of this weight scale is 0.01 g.

![Weight scale](image)

**Figure 3.16 Weight scale used to measure the models’ weight.**

### 3.3.3 Thermocouples Calibration

To assess the accuracy and precision of the thermocouple temperature measurements, all thermocouples used to measure the model and ambient temperatures were calibrated by comparison against a precision reference thermometer (PRT), manufactured by Guildline Instruments model 9535/01, 02, 03. The Guildline thermometer was independently calibrated to an accuracy of ±0.01°C (Guideline Certificate of Calibration, 2007). To check the accuracy of the Guildline thermometer, the thermometer has been tested using the ice bath and the boiling water methods as shown in Figure 3.17 and it was found to be accurate to ±0.015°C.
Figure 3.17 Guildline thermometer accuracy check using (a) ice bath method and (b) water boiling method.

For the calibration, the thermocouples and the Guildline thermometer were immersed in a calibration temperature water bath, manufactured by VWR International, model 1166D, as shown in Figure 3.18. The calibration was carried out over a temperature range of from 20°C to 90°C.

Figure 3.18 Calibration temperature water bath.
Figure 3.19 shows the results of an uncalibrated thermocouple (TC) readings versus the PRT readings for a sample thermocouple (the one which has a maximum difference from the PRT reading). Before calibration, the thermocouple readings differed by 0.225°C to 0.529°C from the PRT readings. The uncorrected errors are shown in Figure 3.20. An equation of the linear regression through the data was found to be:

\[ T_{TC(Uncalibrated)} = 0.9977T_{PRT} + 0.521 \]  

(3.31)
The previous results given in Figures 3.19 and 3.20 were used to correct the thermocouple readings. The calibrated thermocouple temperatures versus the PRT temperatures were plotted as shown in Figure 3.21 and an equation of the linear regression through the data was found to be:

\[ T_{TC(\text{Calibrated})} = 0.9977T_{PRT} + 0.1244 \]  

(3.32)

Once calibrated, the accuracy of the thermocouple was improved to be between 0.027°C and 0.171°C from the PRT readings as shown in Figure 3.22.

Figure 3.21 Calibrated thermocouple readings versus PRT readings.

Figure 3.22 Error in thermocouple reading after correction.
The approach described above has been applied to all thermocouples used in the present studies and the peak absolute corrected error among them was found to be 0.171°C. Since the PRT is rated accurate to ±0.015°C, it was assumed that the thermocouples readings were accurate to ±0.186°C.

3.3.4 Air Properties

Air was used as a surrounding fluid in all the experimental studies. The properties of air used to obtain the heat transfer results have been taken at the film temperature (i.e., at \( T_{film} = (T_m + T_f)/2 \)) at each time step where, \( T_m \) is the mean plate temperature during the time interval considered (i.e., \( T_m = (T_i + T_e)/2 \)). To obtain accurate experimental results, accurate air properties are required, therefore, air properties which include density, kinematic viscosity, specific heat and thermal conductivity were defined as a function of temperature using the following equations (Zografos et al., 1987):

\[
\rho = 345.57(T_{film} - 2.6884)^{-1}
\tag{3.33}
\]

\[
\mu = 2.5914 \times 10^{-15} T_{film}^3 - 1.4346 \times 10^{-11} T_{film}^2 + 5.0523 \times 10^{-8} T_{film} \\
+ 4.1130 \times 10^{-6} \tag{3.34}
\]

\[
c = 1.3864 \times 10^{-13} T_{film}^4 - 6.4747 \times 10^{-10} T_{film}^3 + 1.0234 \times 10^{-6} T_{film}^2 \\
- 4.3282 \times 10^{-4} T_{film} + 1.0613 \tag{3.35}
\]

\[
k = 1.5797 \times 10^{-17} T_{film}^5 + 9.4600 \times 10^{-14} T_{film}^4 + 2.2012 \times 10^{-10} T_{film}^2 \\
- 2.3758 \times 10^{-7} T_{film}^2 + 1.7082 \times 10^{-4} T_{film} - 7.488 \times 10^{-3} \tag{3.36}
\]

The above equations are valid for a temperature range of 100-3000 K and at standard atmospheric pressure which are both satisfied in the experimental study in the present thesis.
**3.3.5 Procedure**

1. The thermocouples were inserted and fixed inside holes drilled in the model and connected to the D/A system which was set to measure these temperatures. The D/A system was calibrated before each heating process and was connected to a computer to record the readings.

2. The model with the thermocouples connected was inserted inside an oven and heated to roughly 100°C above ambient.

3. The heated model was then removed from the oven using a heat resistant glove and mounted on the support frame inside the test chamber in accordance with the required test configuration.

4. The test chamber door was closed and the model temperature variation with time and the ambient temperature were recorded by the D/A system.

5. The model was allowed to cool to roughly 15°C above ambient and then the D/A system was stopped.

6. The model was removed from the support frame and the support frame was left to cool before performing another test.

**3.3.6 Analysis**

The measured data when the model’s temperature was between 25°C and 65°C or, in some situations, between 15°C and 65°C above ambient was used in the analysis. The data at higher temperatures were discarded to minimize the errors resulting from the experimenter’s presence near the apparatus during setup and to allow the model taken from the oven and mounted on the support frame to acquire a uniform temperature.
The overall heat transfer coefficient from the model can be determined, as mentioned earlier, from the measured temperature-time variation by using the following equation:

$$h_t = \frac{mc}{A_{total}} \ln \left[ \frac{(T_i-T_f)}{(T_e-T_f)} \right]$$  \hspace{1cm} (3.37)

The data was divided into 5-minute intervals. For each interval, \( \ln \left[ \frac{(T_i-T_f)}{(T_e-T_f)} \right] \) was calculated, where the subscripts \( i \) and \( e \) refer to the beginning and the end of the interval, respectively. Using the known value of \( \frac{mc}{A_{total}} \), \( h_t \) could then be determined.

The value of \( h_t \) found using the above procedure is the result of both convective and radiant heat transfer from the surface to the surrounding air as well as conductive heat transfer from the model to the supporting frame. If the body can be assumed to be small compared with the surroundings to which it is radiating then it can be assumed that the radiant heat transfer coefficient for each interval is given by the following equation:

$$h_r = \sigma \varepsilon (T_m^2 + T_s^2) (T_m + T_s)$$  \hspace{1cm} (3.38)

where \( \varepsilon \) is the emissivity of the aluminum model which was assumed to equal 0.1, based on the surface finish of the models being studied, and \( T_s \) is the temperature of the surroundings to which the body is radiating. In situations considered in the present study, \( T_s = T_f \). Therefore, equation 3.38 could be rewritten as:

$$h_r = \sigma \varepsilon \left( \frac{(T_i+T_e)}{2} \right)^2 + T_f^2 \left( \frac{(T_i+T_e)}{2} + T_f \right)$$  \hspace{1cm} (3.39)

The radiative heat transfer coefficient ranged between 7.1 and 11.7 per cent of the total heat transfer coefficient for the various heated plates considered in the experimental study.

The conductive heat transfer from the model to the supporting element was assumed to be negligible due to very small contact area between the model and the supporting elements.
With the radiant heat transfer coefficient estimated, the convective heat transfer coefficient was calculated using the following equation:

\[ h_c = h_t - h_r \]  \hspace{1cm} (3.40)

Finally, the Nusselt number, \( Nu \) was found for each interval and hence the variation of Nusselt number with Rayleigh number, \( Ra \) could be obtained.

### 3.3.7 Uncertainty Analysis

Uncertainty is the estimated value of the error in the measured results. It refers to the range of values around the measured value within which the true value is believed to lie. The uncertainty analysis refers to the process of estimating how great an effect the uncertainties in the individual measurements have on the calculated result (Moffat, 1988).

#### 3.3.7.1 Propagation of Uncertainty

The propagation of uncertainty is defined as the way in which uncertainties in the measured variables affect the uncertainty in the results (Kline and McClintock, 1953). Let the result, \( R \), be a function of \( n \) independent variables, \( X_1, X_2, X_3, \ldots, X_n \)

i.e.,

\[ R = R(X_1, X_2, X_3, \ldots, X_n) \]

and let \( \partial R \) be the uncertainty in the result, \( R \) and \( \partial X_1, \partial X_2, \partial X_3, \ldots \partial X_n \) be the uncertainties in the independent variables. To estimate the overall uncertainty in the result \( R \) (the uncertainty propagation) the root-sum-square (RSS) method was used and the overall uncertainty in the result \( R \) is given by the following expression (Kline and McClintock, 1953 and Birk, 2009):

\[
\partial R = \left[ \left( \frac{\partial R}{\partial X_1} \partial X_1 \right)^2 + \left( \frac{\partial R}{\partial X_2} \partial X_2 \right)^2 + \left( \frac{\partial R}{\partial X_3} \partial X_3 \right)^2 + \cdots + \left( \frac{\partial R}{\partial X_n} \partial X_n \right)^2 \right]^{1/2}
\]  \hspace{1cm} (3.41)
3.3.7.2 Uncertainty in the Experimental Value of Nusselt Number

The Nusselt number was defined as,

$$Nu = \frac{h_c l}{k}$$  \hspace{1cm} (3.42)

where, $h_c$ is given by equation 3.40. Substituting equations 3.37 and 3.39 into equation 3.40; yields:

$$h_c = \frac{mc}{A_{total}} \ln \left[ \frac{(T_l-T_f)}{(T_e-T_f)} \right] - \sigma \varepsilon \left( \frac{(T_l+T_e)}{2} + T_f \right)^2 \left( \frac{(T_l+T_e)}{2} + T_f \right)$$  \hspace{1cm} (3.43)

thus,

$$Nu = \frac{\frac{mc}{A_{total}} \ln \left[ \frac{(T_l-T_f)}{(T_e-T_f)} \right] - \sigma \varepsilon \left( \frac{(T_l+T_e)}{2} + T_f \right)^2 \left( \frac{(T_l+T_e)}{2} + T_f \right) l}{k}$$  \hspace{1cm} (3.44)

i.e.,

$$Nu = Nu(m, c, A_{total}, t, T_l, T_e, T_f, \sigma, \varepsilon, l, k)$$

By applying the uncertainty expression given by equation 3.41 into equation 3.44, the uncertainty in the Nusselt number will be:

$$\delta Nu = \left[ \left( \frac{\partial Nu}{\partial m} \right)^2 + \left( \frac{\partial Nu}{\partial c} \right)^2 + \left( \frac{\partial Nu}{\partial A_{total}} \right)^2 + \left( \frac{\partial Nu}{\partial t} \right)^2 \right]^{1/2}$$

\hspace{1cm} \left[ + \left( \frac{\partial Nu}{\partial T_l} \right)^2 + \left( \frac{\partial Nu}{\partial T_e} \right)^2 + \left( \frac{\partial Nu}{\partial T_f} \right)^2 \right]^{1/2} \hspace{1cm} (3.45)

where, the partial derivatives are given by:

$$\frac{\partial Nu}{\partial m} = \frac{cl}{A_{total} tk} \ln \left[ \frac{T_l-T_f}{T_e-T_f} \right]$$  \hspace{1cm} (3.46)

$$\frac{\partial Nu}{\partial c} = \frac{ml}{A_{total} tk} \ln \left[ \frac{T_l-T_f}{T_e-T_f} \right]$$  \hspace{1cm} (3.47)

$$\frac{\partial Nu}{\partial A_{total}} = \frac{-mc}{A_{total}^2 tk} \ln \left[ \frac{T_l-T_f}{T_e-T_f} \right]$$  \hspace{1cm} (3.48)

$$\frac{\partial Nu}{\partial t} = \frac{-mc}{A_{total}^2 tk} \ln \left[ \frac{T_l-T_f}{T_e-T_f} \right]$$  \hspace{1cm} (3.49)
\[ \frac{\partial N_u}{\partial T_i} = \frac{mcl}{A_{\text{total} tk}} \left[ \frac{1}{T_i - T_f} \right] - \frac{\varepsilon c_l}{k} \left[ \frac{3}{2} \left( \frac{T_i + T_e}{2} \right)^2 + \frac{T_f^2}{2} + T_f \left( \frac{T_i + T_e}{2} \right) \right] \] (3.50)

\[ \frac{\partial N_u}{\partial T_e} = \frac{-mcl}{A_{\text{total} tk}} \left[ \frac{1}{T_e - T_f} \right] - \frac{\varepsilon c_l}{k} \left[ \frac{3}{2} \left( \frac{T_i + T_e}{2} \right)^2 + \frac{T_f^2}{2} + T_f \left( \frac{T_i + T_e}{2} \right) \right] \] (3.51)

\[ \frac{\partial N_u}{\partial T_f} = \frac{-mcl}{A_{\text{total} tk}} \left[ \frac{-1}{T_i - T_f} + \frac{1}{T_e - T_f} \right] - \frac{\varepsilon c_l}{k} \left[ T_f (T_i + T_e) + \frac{1}{4} (T_i + T_e)^2 + 3T_f^2 \right] \] (3.52)

\[ \frac{\partial N_u}{\partial \sigma} = -\frac{c_l}{k} \left[ \left( \frac{T_i + T_e}{2} \right)^3 + \left( \frac{T_i + T_e}{2} \right) T_f^2 + \left( \frac{T_i + T_e}{2} \right)^2 T_f + T_f^3 \right] \] (3.53)

\[ \frac{\partial N_u}{\partial \varepsilon} = -\frac{c_l}{k} \left[ \left( \frac{T_i + T_e}{2} \right)^3 + \left( \frac{T_i + T_e}{2} \right) T_f^2 + \left( \frac{T_i + T_e}{2} \right)^2 T_f + T_f^3 \right] \] (3.54)

\[ \frac{\partial N_u}{\partial l} = \frac{mc}{A_{\text{total} tk}} \ln \left[ \frac{T_i - T_f}{T_e - T_f} \right] - \frac{\varepsilon c_l}{k} \left[ \left( \frac{T_i + T_e}{2} \right)^3 + \left( \frac{T_i + T_e}{2} \right) T_f^2 + \left( \frac{T_i + T_e}{2} \right)^2 T_f + T_f^3 \right] \] (3.55)

\[ \frac{\partial N_u}{\partial k} = \frac{-mcl}{A_{\text{total} tk}^2} \ln \left[ \frac{T_i - T_f}{T_e - T_f} \right] + \frac{\varepsilon c_l}{k^2} \left[ \left( \frac{T_i + T_e}{2} \right)^3 + \left( \frac{T_i + T_e}{2} \right) T_f^2 + \left( \frac{T_i + T_e}{2} \right)^2 T_f + T_f^3 \right] \] (3.56)

By substituting the partial derivative expressions given by equations 3.46-3.56 into equation 3.45, the uncertainty in the Nusselt number, \( \delta N_u \) could be estimated. The relative uncertainty in the Nusselt number could then be determined by dividing the value of the uncertainty in the Nusselt number by the value of the Nusselt number (i.e., \( \delta N_u / N_u \)).

Table 3.6 gives the relative uncertainties of the terms given in equation 3.45 for a horizontal I-shaped plate with dimensionless thickness of 0.1, while Table 3.7 shows the estimated uncertainties in Nusselt number for the overall cooling period of the model.
Table 3.6 Relative uncertainties of the terms given in equation 3.45 for horizontal I-shaped plate with dimensionless thickness of 0.1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Relative Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>(kg)</td>
<td>0.013</td>
</tr>
<tr>
<td>(c)</td>
<td>(J/kgK)</td>
<td>0.06</td>
</tr>
<tr>
<td>(A_{total})</td>
<td>(m^2)</td>
<td>1.05</td>
</tr>
<tr>
<td>(t)</td>
<td>(s)</td>
<td>0.056</td>
</tr>
<tr>
<td>(T_i)</td>
<td>(^\circ C)</td>
<td>0.21</td>
</tr>
<tr>
<td>(T_e)</td>
<td>(^\circ C)</td>
<td>0.47</td>
</tr>
<tr>
<td>(T_f)</td>
<td>(^\circ C)</td>
<td>0.83</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>--</td>
<td>0.009</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>--</td>
<td>30</td>
</tr>
<tr>
<td>(l)</td>
<td>(m)</td>
<td>1.69</td>
</tr>
<tr>
<td>(k)</td>
<td>(W/mK)</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 3.7 Uncertainty in Nusselt number for horizontal I-shaped plate with dimensionless thickness of 0.1 for the overall cooling period when \(T_i = 90^\circ C\) and \(T_e = 40^\circ C\)

<table>
<thead>
<tr>
<th>(T_m - T_f) (°C)</th>
<th>Rayleigh Number, (Ra)</th>
<th>Nusselt Number, (Nu)</th>
<th>Uncertainty in Nusselt Number, (\delta Nu)</th>
<th>Relative Uncertainty in Nusselt Number, (\frac{\delta Nu}{Nu}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.6</td>
<td>1.745×10^5</td>
<td>13.5</td>
<td>±0.5</td>
<td>±3.7</td>
</tr>
</tbody>
</table>
Chapter 4
A Numerical and Experimental Study of Natural Convective Heat Transfer from Two-Sided Circular and Square Heated Horizontal Plates Having a Finite Thickness

Abstract

Several numerical and experimental studies of natural convective heat transfer from two-sided heated horizontal plates are available. In the past numerical studies it has usually been assumed that the plate is thin, i.e., effectively having no thickness, whereas plates of finite thickness have been used in the experimental studies. The main aim of the present study was to determine whether the plate thickness has a significant influence on the heat transfer rates from the plate surfaces. The effect of the vertical side surface thermal boundary condition and the effect of the plate shape on the heat transfer rates have also been studied. Heat transfer results have, therefore, been numerically obtained for circular and square plates having various thicknesses. The upper and lower plate surfaces have been assumed to be isothermal and at the same temperature. Both the case where the vertical side surface of the plate is isothermal and at the same temperature as the upper and lower plate surfaces and the case where this vertical side surface is adiabatic have been considered. As discussed before the solution has been obtained using ANSYS FLUENT®. Variations of the mean Nusselt number with Rayleigh number for various dimensionless plate thicknesses for the two vertical side surface conditions have been obtained for both plates. To validate the numerical results, a limited range of experiments has been undertaken for both plate shapes. The experimental mean heat transfer rates have been
obtained using the lumped capacity method. Numerical and experimental results have only
been obtained for the case where the plates are exposed to air.

4.1 Introduction

Situations involving natural convective heat transfer from plates where the top, bottom
and side surfaces are heated to the same temperature occur in engineering practice, for
example, in the cooling of electrical and electronic components. A limited number of
numerical and analytical studies (Chambers and Lee, 1997; Wei et al., 2002; Wei et al.,
2003; Corcione et al., 2011; Fontana, 2014; Oosthuizen and Kalendar, 2016b) and
experimental studies (Hassani and Holland, 1989; Kobus and Wedekind, 2001) of natural
convective heat transfer from two-sided horizontal plates are available. All studies consider
air as the fluid except Corcione et al. (2011) who studied other fluids beside air.

Chambers and Lee (1997) conducted a numerical simulation to determine the local and
average natural convection Nusselt numbers for uniformly heated horizontal thin plates
with heat being transferred from both upper and lower surfaces simultaneously. Several
plate widths with a fixed plate thickness were used. The simulation results were compared
with the experimental data obtained by Sparrow and Carlson (1986) for the case of upward-
-facing uniformly heated surface with excellent agreement being obtained. The range of
Rayleigh numbers used was from 86 to $1.9 \times 10^8$. Correlation equations for the Nusselt
number over the range of Rayleigh numbers considered were proposed. Wei et al. (2002)
undertook a numerical study of the simultaneous natural convective heat transfer from the
upper and lower surfaces of a uniformly heated thin plate set at arbitrary inclination angles
($0-90^\circ$) from horizontal. The plate thickness was fixed at $1/51$ of the plate width which was
varied. Average Nusselt number correlation equations for the upper and lower surfaces
were proposed. The values of Rayleigh number used were from $4.8 \times 10^6$ to $1.87 \times 10^8$. For validation purposes, the results for the horizontally upward-facing uniformly heated plate were compared with the correlation equation of Chambers and Lee (1997) as well as with the experimental results of Sparrow and Carlson (1986) and good agreement was obtained. Wei et al. (2003) numerically investigated the simultaneous natural convective heat transfer above and below an isothermal horizontal thin plate in an infinite space. The plate widths were varied while the plate thickness was set at a fixed value which is $1/51$ of the plate width. The range of Rayleigh numbers was from $1.0 \times 10^5$ to $1.7 \times 10^7$. The results for the upward-facing horizontal surface were compared with the results given by the analytical solution obtained by Pera and Gebhart (1973) and with the experimental data of Sparrow and Carlson (1986) and for the downward-facing horizontal surface, the results were compared with the numerical calculation of Friedrich and Angirasa (2001) and the experimental data of Aihara et al. (1972) and good agreement was obtained for both surfaces. Corcione et al. (2011) numerically studied the steady, laminar natural convective heat transfer from an inclined two-sided isothermal plate immersed in different fluids and whose sides are simultaneously heated to the same uniform temperature. For this case, the plate thickness was fixed at $1/50$ of the plate length. The range of inclination angles considered was from $0^\circ$ to $75^\circ$ (measured from the vertical). The range of Rayleigh numbers was from 10 to $10^7$ and the range of Prandtl numbers was from 0.7 to 140. Correlations for the average Nusselt numbers were developed. For validation purposes, the average Nusselt numbers for the vertical orientation at the different values of Rayleigh and Prandtl numbers considered were compared with the theoretical solution given by Churchill and Chu (1975) for a single-sided heated surface and by Hassan and Mohamed.
(1970) for a two-sided heated surface with good agreement being obtained. Fontana (2014) conducted a numerical study to predict the influence of the Prandtl number on natural convective heat transfer from an isothermal horizontal infinite thin strip with zero thickness. Prandtl number values of 0.71, 2.6, 6.7 and 13.5 and the Rayleigh number values between $10^2$ and $10^6$ were considered. The results of this study were compared with the results obtained by Wei et al. (2003) for the case where the Prandtl number is equal to 0.71 which is effectively the value for air and good agreement was obtained. Correlation equations for the Nusselt number in terms of the Rayleigh and Prandtl numbers were developed. More recently, a numerical study of the simultaneous natural convective heat transfer from the upper and lower surfaces of a thin (having no thickness) isothermal horizontal circular plate with an inner adiabatic section whose dimensionless diameter was varied to investigate its effect on the heat transfer rate was undertaken by Oosthuizen and Kalendar (2016b). The range of conditions included laminar, transitional and fully-turbulent flows. The results obtained in this study were compared with the experimental results of Hassani and Hollands (1989) and with the experimental correlation of Kobus and Wedekind (2001) for the laminar flow region with good agreement being obtained.

Hassani and Hollands (1989) performed experiments to measure natural convective heat transfer from two-sided isothermal plates of different shapes oriented in various directions over a range of Rayleigh numbers from 10 to $10^8$. The square and circular plates considered in the study had a dimensionless thickness of only 0.1. A characteristic length was proposed such that the experimental data obtained could be collapsed onto a single curve for all the geometrical shapes considered for a limited range of Rayleigh numbers, thus leading to a universal correlation. Finally, Kobus and Wedekind (2001) reported experimental results
for natural convective heat transfer from horizontal isothermal two-sided circular disks of different thickness-to-diameter ratios ranging from 0.063 to 0.163. Dimensionless correlations for Nusselt number over a Rayleigh numbers range from $3 \times 10^2$ to $3 \times 10^7$ were developed.

The review of the literature has revealed that the existing numerical and experimental studies have mainly dealt with situations in which either the flow over the plate remains laminar, the thickness of the plate remains unchanged (in these studies, either a single value of dimensionless plate thickness was used (Hassani and Hollands, 1989) or various values of dimensionless plate thickness were used by either varying the plate width/length while the plate thickness was kept fixed (Chambers and Lee, 1997; Wei et al., 2002; Wei et al., 2003; Corcione et al., 2011) or by varying both the plate diameter and thickness at each dimensionless thickness considered (Kobus and Wedekind, 2001) or the plate has no thickness (Fontana, 2014; Oosthuizen and Kalendar, 2016b). To the best of the authors’ knowledge, there do not appear any detailed study in the literature for predicting the influence of the plate thickness on the natural convective heat transfer from two-sided horizontal plates when the conditions considered are such that laminar, transitional, and turbulent flow over the plate can occur. This is, therefore, the main objective of the present research where the various dimensionless plate thicknesses used were obtained by varying the plate thickness while the plate side length/diameter was kept fixed. In addition, the effects of the thermal boundary condition of the plate side surface on the heat transfer rates from the plate surfaces for the situations described in the present research have not previously been studied in detail. Also, there is a lack of information in the literature about the effect of the plate shape on the heat transfer rate, a length scale, $l = 4A_{plate}/P$, where
\( A_{plate} \) is the heated surface area and \( P \) is the perimeter of that surface has therefore been proposed to investigate whether if Nusselt and Rayleigh numbers based on this length scale are used the Nusselt-Rayleigh number relation would be the same for all element shapes considered.

This chapter therefore discusses a numerical study of natural convective heat transfer from horizontal two-sided circular and square plates having a finite thickness (see Figure 4.1). A limited range of experiments have also been undertaken for both shapes to validate the numerical results obtained. Attention will first be given to the numerical part and the experimental part will be discussed after.

In the numerical study, the upper and lower surfaces of the plates are assumed to be isothermal and at the same temperature which is higher than that of the surrounding fluid and hence there is heat transfer from both the upper and lower surfaces of both plates. The vertical side surface of the plates considered (Figure 4.2) are either adiabatic or isothermal and at the same temperature as the upper and lower surfaces of the plate. The finite thickness situation considered is shown in Figure 4.2.

![Figure 4.1 Square (left) and circular (right) plate shapes considered showing the definitions of the side length, \( w \), of the square plate and the diameter, \( d \), and radius, \( r \), of the circular plate.](image)
As shown in Figure 4.1 plates having a circular shape and plates having a square shape have been considered in this study. In general, there is an interaction of the flows over the bottom and top surfaces of the plate. The mean heat transfer rates from the top, bottom, and, when the vertical side surface is isothermal, the vertical side surfaces of the plate have been considered as well as the mean heat transfer rate averaged over all the heated surfaces.

![Figure 4.2](image.png)

**Figure 4.2** Finite thickness plate situation considered, a square plate being shown. Also shown are the definitions of the plate thickness, $h$, and of the top (upper) and of the vertical side surface of a two-sided plate. The bottom (lower) surface is below the top surface.

### 4.2 Numerical Solution Procedure

The flow has been assumed to be steady and axisymmetric about the vertical center-line in the case of a circular plate and to be symmetric about the two-vertical center planes in the case of a square plate. The fluid properties have been assumed constant except for the density change with temperature which gives the rise to the buoyancy forces, i.e., the Boussinesq approach has been adopted. The standard $k$-epsilon turbulence model with buoyancy force effects taken into account has been adopted. The solution has been obtained by numerically solving the governing equations using the commercial CFD solver ANSYS FLUENT®. To ensure that the results obtained are grid independent, an extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken, and the heat transfer results presented here are grid and convergence-criteria independent to better than one per cent. Further details of the numerical solution procedure are given in section 3.2.
4.3 Numerical Results and Discussion

The mean heat transfer rates from the various surfaces of the heated plate have been expressed in terms of Nusselt numbers based on the length scale of the plate defined by, \( l = 4A_{\text{plate}} / P \), i.e., in terms of the diameter, \( d \), in the case of the circular plate and in terms of the side length, \( w \), in the case of the square plate, and on the difference between the surface temperature of the plate and the temperature of the undisturbed fluid far from the plate.

The mean Nusselt numbers will be dependent on the Rayleigh number, also based on the characteristic size of the plates and on the difference between the plate temperature and the fluid temperature far from the plate, on the dimensionless thickness of the plate, on the thermal boundary condition existing along the vertical side surface of the plate and on the Prandtl number. It is often assumed, e.g., Oosthuizen (2015b) and Oosthuizen and Kalendae (2016a) that for natural convective heat transfer from horizontal heated elements, if the Nusselt and Rayleigh numbers are expressed in terms of the length scale, \( l \), defined above then the variations of Nusselt number with Rayleigh number will be the same for all element shapes. The Rayleigh number is defined by:

\[
Ra = \frac{\beta g l^3 (T_w - T_f)}{v \alpha}
\]  

(4.1)

Results have only been obtained for a Prandtl number of 0.74, which is effectively the value for air under near standard ambient conditions. The Nusselt numbers will also depend on the dimensionless thickness of the plate, \( H = h/l \), and on the thermal boundary condition existing along the vertical sides of the plate. Mean Nusselt numbers averaged over the upper horizontal surface, over the lower horizontal surface of the plate, averaged over the heated surfaces of the plate, and, in the case of when the plate has isothermal vertical side surface,
the mean Nusselt number averaged over the vertical side surface of the plate have been considered. The following Nusselt numbers have therefore been introduced:

\[ Nu_{total} = \frac{\bar{Q}_l}{A_{total}(T_w - T_f)k}, \quad Nu_{top} = \frac{\bar{Q}_{top} l}{A_{plate}(T_w - T_f)k}, \quad Nu_{bot} = \frac{\bar{Q}_{bot} l}{A_{plate}(T_w - T_f)k}, \quad Nu_{side} = \frac{\bar{Q}_{side} l}{A_{side}(T_w - T_f)k} \]  

(4.2)

where:

\[ \bar{Q}' = \bar{Q}_{top} + \bar{Q}_{bot} + \bar{Q}_{side} \]  

(4.3)

and where \( A_{total}, A_{plate}, \) and \( A_{side} \) are the total area of the heated surfaces of the plate, the areas of the upper and the lower surfaces of the plate, and the area of the vertical side surface of the plate, respectively. Therefore, in the cases where the side surface is adiabatic:

\[ A_{total} = 2A_{plate} \]  

(4.4)

and \( \bar{Q}_{side} = 0 \) while in the situations where the side surface is isothermal:

\[ A_{total} = 2A_{plate} + A_{side} \]  

(4.5)

From the above equations it follows that when the side surfaces are adiabatic:

\[ Nu_{total} = \frac{Nu_{top} + Nu_{bot}}{2} \]  

(4.6)

while when the side surfaces are isothermal:

\[ Nu_{total} = \frac{(Nu_{top} + Nu_{bot})A_{plate} + Nu_{side}A_{side}}{A_{total}} \]  

(4.7)

Results for the case where the vertical side surface of the plate is adiabatic will first be considered. Typical variations of the mean Nusselt numbers averaged over the lower surface of the plate, i.e., \( Nu_{bot} \), over the upper surface of the plate, i.e., \( Nu_{top} \), and over the upper and lower surfaces of the plate, i.e., \( Nu_{total} \), with Rayleigh number for the case of a square
plate for two values of the dimensionless plate thickness, $H$, are shown in Figure 4.3 while values of $Nu_{bot}$, $Nu_{top}$, $Nu_{total}$ for two value of $H$ for the case of a circular plate are shown in Figure 4.4.

It will be seen from these figures that the Nusselt numbers for the square and circular plates have similar values at the lower Rayleigh number values considered but differ somewhat at the higher Rayleigh number values considered mainly as a result of differences in the predicted Rayleigh number values at which transition to turbulent flow starts to occur with the two plate shapes considered. It will also be seen from these figures that in all cases the Nusselt numbers for the top surface are lower than those for the bottom surface at lower Rayleigh number values but are higher than those for the bottom surface at higher Rayleigh number values.
Figure 4.3 Variations of the mean Nusselt number for the bottom and top surfaces and for all heated surfaces with Rayleigh number for a square plate having an adiabatic vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.
Figure 4.4 Variations of the mean Nusselt number for the bottom and top surfaces and for all heated surfaces with Rayleigh number for a circular plate having an adiabatic vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.

To further illustrate the effect of the dimensionless plate thickness on the heat transfer rate, variations of the Nusselt number with $H$ for various values of the Rayleigh number for the top and the bottom surfaces of a square plate are shown in Figure 4.5 and for the top and bottom surfaces of a circular plate are shown in Figure 4.6. It can be seen from these figures that the changes in the value of $H$ have an almost negligible effect on the Nusselt numbers for the bottom surface but that the effect of changes in the value of $H$ on the Nusselt numbers for the
top surface is significant, the nature of this effect being dependent on the Rayleigh number value.

Figure 4.5 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a square plate having an adiabatic vertical side surface with dimensionless plate thickness, $H$, for various values of the Rayleigh number.
Figure 4.6 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a circular plate having an adiabatic vertical side surface with dimensionless plate thickness, $H$, for various values of the Rayleigh number.

Attention will next be turned to the results for the case where the vertical side surface of the plate is isothermal. Typical variations of the mean Nusselt numbers averaged over the lower surface of the plate, i.e., $Nu_{bot}$, over the upper surface of the plate, i.e., $Nu_{top}$, and over the upper, lower, and vertical side surface of the plate, i.e., $Nu_{total}$, with Rayleigh number for the case of a square plate for two values of the dimensionless plate thickness, $H$, are shown.
in Figure 4.7 while values of $Nu_{bot}$, $Nu_{top}$, $Nu_{total}$ for two values of $H$ for the case of a circular plate are shown in Figure 4.8. Comparing these results with those for the adiabatic vertical side surface case given in Figures. 4.3 and 4.4 shows that the thermal vertical side surface boundary condition has only a very small effect on the Nusselt numbers for the bottom surface but that this vertical side surface thermal boundary condition has a significant effect on the Nusselt numbers for the top surface at the lower Rayleigh number values considered. As with the results for the adiabatic vertical side surface case it will be seen from Figures 4.7 and 4.8 that the results for the square and circular plates have similar values at lower Rayleigh number values considered but differ somewhat at the higher Rayleigh number values. Also, as with the results for the adiabatic vertical side surface case, it will also be seen from these figures that in all cases the Nusselt numbers for the top surface are lower than those for the bottom surface at lower Rayleigh number values but are higher than those for the bottom surface at higher Rayleigh number values considered.
Figure 4.7 Variations of the mean Nusselt number for the bottom and top surfaces and for all heated surfaces with Rayleigh number for a square plate having an isothermal vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.
Figure 4.8 Variations of the mean Nusselt number for the bottom and top surfaces and for all heated surfaces with Rayleigh number for a circular plate having an isothermal vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.

Again, to illustrate the effect of the dimensionless plate thickness on the heat transfer rate, variations of the Nusselt numbers with $H$ for various values of the Rayleigh number for the top and the bottom surfaces of a square plate are shown in Figure 4.9 and for the top and bottom surfaces of a circular plate are shown in Figure 4.10.

As was the case with the adiabatic vertical side surface condition it will be seen from these figures that the changes in the value of $H$ have an almost negligible effect on the Nusselt numbers for the bottom surface but that the effect of changes in the value of $H$ on the Nusselt numbers for the top surface is significant, the nature of this effect again being dependent on the Rayleigh number value. Comparing the results for the isothermal vertical side surface case given in Figures 4.9 and 4.10 with the corresponding results for the adiabatic vertical side surface case given in Figures 4.5 and 4.6 shows that the Nusselt numbers for the bottom surface are essentially the same for the two vertical side surface thermal boundary conditions considered whereas at the lower values of Rayleigh number considered (approximately less
than $10^7$) the vertical side surface thermal boundary conditions have a significant effect on the Nusselt numbers for the top surface.

Figure 4.9 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a square plate having an isothermal vertical side surface with dimensionless plate thickness, $H$, for various values of the Rayleigh number.
Figure 4.10 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a circular plate having an isothermal vertical side surface with dimensionless plate thickness, $H$, for various values of the Rayleigh number.

Attention will next be given to the heat transfer from the vertical side surface when it is isothermal. Variations of the mean Nusselt number for this surface with Rayleigh number for the square plate and for the circular plate cases for three values of $H$ are shown in Figures 4.11 and 4.12, respectively. It will be seen from these figures that the variations for the square and circular plates are very similar and that the effect of the $H$ value on the vertical side surface Nusselt number at a particular value of $Ra$ is relatively small except for $H$ values near 0.1.
Figure 4.11 Variations of the mean Nusselt number for the vertical side surface with Rayleigh number for a square plate having an isothermal vertical side surface for dimensionless plate thicknesses, $H$, of 0.1, 0.2, and 0.3.

Figure 4.12 Variations of the mean Nusselt number for the vertical side surface with Rayleigh number for a circular plate having an isothermal vertical side surface for dimensionless plate thicknesses, $H$, of 0.1, 0.2, and 0.3.

To further illustrate the effect of the vertical side surface thermal boundary condition on the heat transfer rate, typical variations of the mean Nusselt number with Rayleigh number for a dimensionless plate thickness, $H$, of 0.1 for the top and bottom surfaces of a square plate are shown in Figure 4.13 and for the top and bottom surfaces of a circular plate are shown in Figure 4.14. It can be seen from these figures that as previously explained the effect of the
vertical side surface thermal boundary condition on the Nusselt numbers for the bottom surface of the square and circular plates is almost negligible whereas this effect is significant for the top surface of the plates at lower values of Rayleigh number as a result of flow pattern changes.

**Figure 4.13** Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for a square plate having a dimensionless thickness, $H$, of 0.1 for the two vertical side surface thermal boundary conditions considered.
Figure 4.14 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for a circular plate having a dimensionless thickness, $H$, of 0.1 for the two vertical side surface thermal boundary conditions considered.

Lastly, the effect of the element shape on the heat transfer rate when the length scale, $l$, is used is shown more clearly by the typical variations of the mean Nusselt number for the top and bottom surfaces with Rayleigh number for the square and circular plates with dimensionless plate thickness, $H$, of 0.1 given in Figures 4.15 and 4.16 for the adiabatic and isothermal vertical side surface boundary conditions, respectively. These figures show that for both vertical side surface thermal boundary conditions and over the range of Rayleigh...
numbers considered the results for the square and circular shapes are in good agreement for the top surface. For the bottom surface, however, the results are in good agreement except at higher values of Rayleigh number.

Figure 4.15 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for square and circular plates having an adiabatic vertical side surface for a dimensionless plate thickness, $H$, of 0.1.
Figure 4.16 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for square and circular plates having an isothermal vertical side surface for a dimensionless plate thickness, $H$, of 0.1.

It should be noted here that when the length scale, $l$, was used the vertical side surface of the plate was not taken into consideration. The plates considered in this study, however, are two-sided, further length scale analysis has, therefore, been performed (see Appendix A). It has been shown that considering the vertical side surface in the length scale used has a negligible effect on the variations of Nusselt number with Rayleigh number when Nusselt and Rayleigh numbers are expressed in terms of the length scale based on the single-sided plate, $l$, for all surface shapes considered.
4.4 Experimental Apparatus and Procedure

The purpose of the experiments was to validate the numerical results used to predict the natural convective heat transfer from the two-sided square and circular plates. In the experimental study, the mean heat transfer rate from the heated element considered was determined using the lumped capacity method.

4.4.1 Apparatus

To experimentally study the natural convective heat transfer from the two-sided horizontal plates considered, flat plates were machined from aluminum alloy (Al 6061-T6). The dimensions and the mass of the plates are given in Table 4.1. A support frame on which the plates are mounted is shown in Figure 4.17. The support frame was constructed using a plexiglas plate on which vertical, small diameter steel elements with sharply pointed tips were fixed to make the contact with the mounted plates and hence the conduction losses as small as possible. Further details of the apparatus used in the experimental study can be found in section 3.3.2.

Table 4.1 Dimensions and mass of the aluminum flat plates

<table>
<thead>
<tr>
<th>Flat Plate</th>
<th>Length Scale, $l$ (m)</th>
<th>Thickness, $h$ (m)</th>
<th>Dimensionless Thickness, $h/l$</th>
<th>Mass, $m$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>0.1</td>
<td>0.01</td>
<td>0.1</td>
<td>0.27071</td>
</tr>
<tr>
<td>Circular</td>
<td>0.12</td>
<td>0.012</td>
<td>0.1</td>
<td>0.36676</td>
</tr>
</tbody>
</table>
4.4.2 Experimental Procedure

The mean heat transfer rate from the square and circular plates was determined using the lumped capacity method. Descriptions of the method and the analysis procedure used in determining the Nusselt numbers are given in sections 3.3.5 and 3.3.6.

4.4.3 Uncertainty Analysis

The overall uncertainty in the experimental values of Nusselt number has then been determined using the root-sum-square (RSS) method described in section 3.3.7. The relative uncertainty obtained for the experimental values of Nusselt number ranged between 6.7% and 9.4% for the square plate and between 6.6% and 9.5% for the circular plate.

4.5 Experimental Results and Comparison with Numerical Results

The experimental results obtained for the square and circular plates were used to examine the validity of the numerical model used. The numerical and experimental heat transfer results have been compared by showing the variations of the mean Nusselt number with Rayleigh number for a dimensionless plate thickness, $H$, of 0.1. These variations are shown in Figures 4.18 and 4.19 for the square and circular plates, respectively. It should
be noted here that the Rayleigh number range is different in these figures because of the difference in the ambient temperature during the experiment for each shape. It can be seen from these figures that the numerical results lie within the experimental error band (the uncertainty of the experimental results) and the numerical and experimental results obtained agree very closely.

Figure 4.18 Comparison between numerical and experimental results for the square plate having a dimensionless thickness, $H$, of 0.1.

Figure 4.19 Comparison between numerical and experimental results for the circular plate having a dimensionless thickness, $H$, of 0.1.
4.6 Conclusions

The results of the present study show that:

1. In all cases the Nusselt numbers for the top surface are lower than those for the bottom surface at the lower Rayleigh number values considered but are higher than those for the bottom surface at the higher Rayleigh number values considered. This seems to be the result of the thickening of the boundary layer on the top surface at lower Rayleigh numbers where the inward flows collide forming the plume along with a thinning of the bottom surface boundary layer where the outward flows divide (Chambers and Lee, 1997). (See appendix B for flow patterns.)

2. Changes in the value of the dimensionless plate thickness, $H$, have an almost negligible effect on the Nusselt numbers for the bottom surfaces of the square and circular plates. However, these changes in the value of $H$ have a significant effect on the Nusselt number for the top surface, the nature of this effect being dependent on the Rayleigh number value being considered.

3. The vertical side surface thermal boundary condition has only a very small effect on the Nusselt numbers for the bottom surface of the square and circular plates but has a significant effect on the Nusselt numbers for the top surface at the lower Rayleigh number values considered.

4. For both vertical side surface thermal boundary conditions, when the length scale, $l$, is used. The variations of Nusselt number with Rayleigh number for the square and circular plates are nearly the same for the top surface of the plates but differ somewhat at the higher Rayleigh number values considered for the bottom surface of the plates mainly as a result of differences in the predicted Rayleigh number values at which transition to turbulent flow starts to occur.
5. In general, the estimated uncertainty in the experimental results was less than 10% and the numerical results lie within the uncertainty of the experimental results. Very good agreement between the numerical and experimental results was obtained over the limited range of the experimental Rayleigh numbers considered.
Chapter 5
A Numerical and Experimental Investigation of Natural Convective Heat Transfer from Two-Sided Horizontal Plates Having a Complex Shape and a Finite Thickness

Abstract

Natural convective heat transfer from two-sided heated horizontal plates has been numerically and experimentally investigated. Most of the existing numerical studies have dealt with the situations where the plate is thin, i.e., has no thickness, whereas in the experimental studies plates having a finite thickness have been used. To determine whether the plate thickness has a significant influence on the heat transfer rates from the surfaces of horizontal plates of various thicknesses having complex shapes results have here been numerically obtained for plates having an I-shape and a +-shape. The upper and lower surfaces of the plates are assumed to be isothermal and at the same temperature. Both the case where the vertical side surface of the plate is isothermal and at the same temperature as the upper and lower plate surfaces and the case where this surface is adiabatic have been considered. Conditions under which laminar, transitional, and turbulent flows exist have been considered. The standard $k$-epsilon turbulence model was used and was applied under all conditions. The numerical results have been obtained using the commercial solver ANSYS FLUENT©. These results have been used to determine whether the plate thickness and the thermal boundary conditions existing on the vertical side surface of the plate has a significant influence on the heat transfer rate. The results have also been used to determine whether the heat transfer rates from the surfaces of the two plate shapes considered can be correlated by using an appropriate length scale based on the ratio of the plate surface area
to its perimeter. Only the case where the heat transfer from the plate is to air has been considered. Variations of the mean Nusselt number with Rayleigh number for a range of dimensionless plate thicknesses (thickness-to-outside length ratios) from 0 to 0.3 for the two vertical side surface thermal boundary conditions considered have been obtained for the I- and +-shaped plates. In order to validate the numerical results, a limited range of experiments has been undertaken for both shapes for a dimensionless plate thickness of 0.1. The results have been obtained using the lumped capacity method. The results of the present study indicate that the plate thickness has a very small effect on the heat transfer rate from the upper surface of the plates. The thermal boundary condition existing along the vertical side surface has a negligible effect on the heat transfer rate from the bottom surface of the plates whereas this effect is significant for the top surface at lower Rayleigh numbers considered. Also, the heat transfer results for the surfaces of the two plate shapes considered were well-correlated at the lower Rayleigh number values when the length scale defined above was used.

5.1 Introduction

In various practical applications, cooling of some electronic devices is effectively accomplished by natural convective heat transfer. Due to the presence of several abutting components in such applications, heat transfer often effectively occurs from surfaces of relatively complex shape. The situation considered here is a simplified model of those occurring in real engineering applications.

The upper and lower surfaces of the plates are assumed to be isothermal and at the same temperature which is higher than that of the surrounding fluid and hence there is heat transfer from both the upper and lower surfaces of the plates. The vertical side surfaces of
the plates are either adiabatic or isothermal at the same temperature as the upper and lower surfaces of the plate. The finite thickness plate situation considered is shown in Figure 5.1. Plates having an I-shape and a + -shape have been considered (see Figure 5.2). Mean heat transfer rates from the top surface, the bottom surface, and, for the case where the vertical side surface is isothermal, from the vertical side surface of the plate have been considered. In addition, the mean heat transfer rate averaged over all of the heated surfaces of the plate has been considered. Results have only been obtained for the geometry where $s/w$ (see Figure 5.2) is equal to 0.25 for both the I-shaped and the + -shaped cases.

Figure 5.1 Finite thickness plate situation considered, an I-shaped plate being shown. Also shown are the definitions of the plate thickness, $h$, and of the top (upper) and of the vertical side surface of a two-sided plate. The bottom (lower) surface is below the top surface.

Figure 5.2 I-shaped (left) and + -shaped (right) plate shapes showing the definitions of the outside length, $W$, and the arm width, $s$, of the plates.

Several numerical and experimental studies of natural convective heat transfer from two-sided heated horizontal plates of relatively simple shape are available, e.g., see (Hassani and Holland, 1989; Chambers and Lee, 1997; Kobus and Wedekind, 2001; Wei
et al., 2002; Wei et al., 2003; Corcione et al., 2011; Fontana, 2014; Oosthuizen and Kalendar, 2016b; Manna and Oosthuizen, 2018).

A few studies have given attention to natural convective heat transfer from horizontal plates of relatively complex shape, Oosthuizen (2015b) numerically investigated the natural convective heat transfer from a horizontal isothermal heated plate having either an I- or a +-shape imbedded in a larger surrounding flat adiabatic surface. The plates considered were single-sided facing either upward or downward. The author proposed a length scale to correlate the heat transfer rate in the laminar, transitional and turbulent flow regions for the surfaces of both shapes considered. Oosthuizen and Kalendar (2016a) undertook a numerical study of natural convective heat transfer from horizontal heated plates of relatively complex shape having a uniform surface heat flux. The plates considered had an I- and a +-shape imbedded in a larger surrounding flat adiabatic surface and are facing upward. The conditions considered were such that laminar, transitional and turbulent flows occur over the plates. A length scale to correlate the results for surfaces with uniform heat flux of the two plate shapes considered was proposed.

The literature review indicated that most available studies dealt with situations where natural convective heat transfer from plates of relatively simple shape have been considered. A very limited attention has been given to natural convective heat transfer from plates of relatively complex shape. These studies, however, dealt with situations where only single-sided plates are considered. Moreover, the effect of thermal boundary conditions existing along the vertical side surface of the plate and the effect of the plate thickness on the heat transfer rate from the plate surfaces has not been studied. This is basically the objective of the present study where the effect of plate thickness, of vertical
side surface thermal boundary condition, and of plate shape on the natural convective heat transfer rate from two-sided plates of complex shape (I and +) has been investigated for conditions where laminar, transitional, and turbulent flow over the plates can occur.

5.2 Numerical Solution Procedure

In the present study it has been assumed that the flow is steady and symmetrical about the two vertical center planes shown in Figure 5.2 for the two shapes considered. The Boussinesq approach has been used in dealing with the buoyancy forces and with the changes in the fluid properties resulting from the temperature changes in the flow. The standard $k$-epsilon turbulence model with buoyancy force effects taken into account has been used in obtaining the results discussed here. This turbulence model has been applied in all calculations and is thus used to determine when transition begins. The commercial CFD solver ANSYS FLUENT© has been used to obtain the numerical solution. An extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken, and the results of this indicated that the heat transfer results presented here are to within approximately one per cent grid and convergence criteria independent. Further details of the numerical solution procedure are given in section 3.2.

5.3 Numerical Results and Discussion

The mean heat transfer rates from the various surfaces of the heated plate have been expressed in terms of Nusselt numbers based on the difference between the surface temperature of the plates and the temperature of the undisturbed fluid far from the plate and on the characteristic length scale of the plate, $l$. This characteristic length scale has here been taken as:
\[ l = \frac{4A_{\text{plate}}}{P} \]  \hspace{1cm} (5.1)

where \( A_{\text{plate}} \) is the heated surface area and \( P \) is the perimeter of that surface. For situations involving natural convective heat transfer from single-sided horizontal surfaces it has been shown that with this definition of \( l \) the variations of mean Nusselt number based on \( l \) with Rayleigh number based on \( l \) are essentially the same for all surface shapes considered for a particular value of the Prandtl number, see Oosthuizen (2015b) and Oosthuizen and Kalendar (2016a). The use of the characteristic length scale for two-sided surfaces has however not been extensively investigated. For the case here being considered where \( s/W = 0.25 \) equation (5.1) gives:

for I-shape: \( l/W = 0.45455 \) and for +-shape: \( l/W = 0.43750 \) \hspace{1cm} (5.2)

It should be noted here that the length scale, \( l \), given in equation (5.2) ignores the existence of the vertical side surface of the plate. However, the plates considered in the present study are two-sided, therefore, further length scale analysis has been undertaken (see Appendix A). It has been shown that considering the vertical side surface in the length scale, \( l \), has a negligible effect on the variation of Nusselt number with Rayleigh number when the length scale based on the single-sided plate, \( l \), is used for all surface shapes considered.

The mean Nusselt numbers considered will be dependent on the Rayleigh number, also based on the characteristic length scale of the plates, on the difference between the plate temperature and the fluid temperature far from the plate, on the dimensionless thickness of the plate, on the thermal boundary condition existing along the vertical side surface of the plate, and on the Prandtl number.
The Rayleigh number is defined by:

\[
Ra = \frac{\beta g l^3(T_w - T_f)}{v \alpha}
\]  

(5.3)

Results have only been obtained for a Prandtl number of 0.74, i.e., effectively for the value for air under near standard ambient conditions. The Nusselt numbers averaged over the upper horizontal surface of the plate, over the lower horizontal surface of the plate, and over the entire heated surfaces of the plate have been considered. In addition, in the case where the plate has an isothermal vertical side surface, the mean Nusselt number averaged over the vertical side surface of the plate has also been considered. The following Nusselt numbers have therefore been introduced:

\[
\begin{align*}
Nu_{top} &= \frac{Q'_{top} l}{A_{plate}(T_w - T_f)k}, \\
Nu_{bot} &= \frac{Q'_{bot} l}{A_{plate}(T_w - T_f)k}, \\
Nu_{side} &= \frac{Q'_{side} l}{A_{side}(T_w - T_f)k} \\
Nu_{total} &= \frac{Q' l}{A_{total}(T_w - T_f)k}
\end{align*}
\]  

(5.4)

where,

\[
Q' = Q'_{top} + Q'_{bot} + Q'_{side}
\]  

(5.5)

and where \(A_{total}\), \(A_{plate}\), and \(A_{side}\) are the total area of the heated surfaces of the plate, the areas of the upper and the lower surfaces of the plate, and the area of the vertical side surface of the plate, respectively.

Therefore, in the situations where the vertical side surface is adiabatic:

\[
A_{total} = 2A_{plate}
\]  

(5.6)

and \(Q'_{side} = 0\) while in the situations where the vertical side surface is isothermal:

\[
A_{total} = 2A_{plate} + A_{side}
\]  

(5.7)
From the above equations it follows that when the side surfaces are adiabatic:

\[ N_{u_{\text{total}}} = \frac{N_{u_{\text{top}}} + N_{u_{\text{bot}}}}{2} \]  \hspace{1cm} (5.8)

while when the side surfaces are isothermal:

\[ N_{u_{\text{total}}} = \frac{(N_{u_{\text{top}}} + N_{u_{\text{bot}}})A_{\text{plate}} + N_{u_{\text{side}}}A_{\text{side}}}{A_{\text{total}}} \]  \hspace{1cm} (5.9)

Attention will first be given to the situation where the vertical side surface of the plate is adiabatic. Typical variations of the mean Nusselt number averaged over the lower surface of the plate, i.e., \( N_{u_{\text{bot}}} \), over the upper surface of the plate, i.e., \( N_{u_{\text{top}}} \), and over the upper and the lower surfaces of the plate, i.e., \( N_{u_{\text{total}}} \), with Rayleigh number for the case of an I-shaped plate for two values of dimensionless plate thickness, \( H \), are shown in Figure 5.3 while variations of \( N_{u_{\text{bot}}} \), \( N_{u_{\text{top}}} \), and \( N_{u_{\text{total}}} \) with Rayleigh number for the case of a +-shaped plate for two values of \( H \) are shown in Figure 5.4. These figures show that the results for the two plate shapes for a given value of \( H \) are close at the lower Rayleigh number values considered but differ somewhat at the higher Rayleigh number values considered, this being mainly the result of differences in the predicted Rayleigh number values at which transition to turbulent flow starts to occur. It will also be seen that the Nusselt numbers for the top surface are lower than those for the bottom surface at lower Rayleigh number values but are higher than those for the bottom surface at higher Rayleigh number values.

To further illustrate the effect of the dimensionless plate thickness on the heat transfer rate variations of the mean Nusselt number with \( H \) for various values of the Rayleigh number for the top and the bottom surfaces of an I-shaped plate are shown in Figure 5.5 and for the top and bottom surfaces of a +-shaped plate are shown in Figure 5.6. It will be
seen from these figures that the changes in the value of $H$ have an almost negligible effect on the Nusselt numbers for the bottom surface but that changes in the value of $H$ have a significant effect on the Nusselt numbers for the top surface, the nature of this effect being dependent on the Rayleigh number value. These changes in $H$ result in changes in the flow pattern changes which are discussed in appendix B.

(a)

![I-Shape, $H=0.1$, Adiabatic Side](image)

(b)

![I-Shape, $H=0.3$, Adiabatic Side](image)

Figure 5.3 Variations of the mean Nusselt number for the bottom and top surfaces and for the total heated surface with Rayleigh number for an I-shaped plate having an adiabatic vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.
Figure 5.4 Variations of the mean Nusselt numbers for the bottom and top surfaces and for the total heated surface with Rayleigh number for a +-shaped plate having an adiabatic vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.
Figure 5.5 Variations of the mean Nusselt number for the (a) top surface and the (b) bottom surface of an I-shaped plate having an adiabatic vertical side surface with dimensionless plate thicknesses, $H$, for various values of the Rayleigh number.
Figure 5.6 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a +-shaped plate having an adiabatic vertical side surface with dimensionless plate thicknesses, $H$, for various values of the Rayleigh number.

Attention will next be turned to the results for the case where the vertical side surface of the plate is isothermal. Typical variations of the mean Nusselt number averaged over the lower surface of the plate, i.e., $Nu_{bot}$, over the upper surface of the plate, i.e., $Nu_{top}$, and over the upper, lower and the vertical side surfaces of the plate, i.e., $Nu_{total}$, with Rayleigh number for the case of an I-shaped plate for two values of the dimensionless plate
thickness, $H$, are shown in Figure 5.7 while variations of $Nu_{bot}$, $Nu_{top}$, and $Nu_{total}$ for two value of $H$ for the case of a +shaped plate are shown in Figure 5.8.

Figure 5.7 Variations of the mean Nusselt number for the bottom and top surfaces and for the total heated surfaces with Rayleigh number for an I-shaped plate having an isothermal vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.
Figure 5.8 Variations of the mean Nusselt number for the bottom and top surfaces and for the total heated surfaces with Rayleigh number for a +\-shaped plate having an isothermal vertical side surface for a dimensionless plate thickness, $H$, of (a) 0.1 and (b) 0.3.

Comparing these results with those for the adiabatic vertical side surface case given in Figures 5.3 and 5.4 shows that the vertical side surface thermal boundary condition has only a very small effect on the Nusselt numbers for the bottom surface but that this side surface thermal boundary condition has a larger effect on the Nusselt numbers for the top surface at the lower Rayleigh number values considered. It will also again be seen from
Figures 5.7 and 5.8 that the results for the I-shaped plate and for the +shaped plate are close at the lower Rayleigh number values considered but differ somewhat at the higher Rayleigh number values. Also, as with the results for the adiabatic vertical side surface case, it will be seen from these figures that with an isothermal vertical side surface in all cases the Nusselt numbers for the top surface are lower than those for the bottom surface at lower Rayleigh number values but are higher than those for the bottom surface at higher Rayleigh number values. In the isothermal vertical side surface case the effect of $H$ on $Nu_{bot}$ is again essentially negligible, the results for the isothermal vertical side surface case being essentially the same as those for the adiabatic vertical side surface. This effect is illustrated in Figure 5.9 which shows the variations of $Nu_{bot}$ with $H$ for various values of Rayleigh number for an I-shaped plate and for a +shaped plate. To illustrate the effect of $H$ on $Nu_{top}$ typical variations of $Nu_{top}$ with $H$ for various values of Rayleigh number for an I-shaped plate and for a +shaped plate are shown in Figure 5.10. Comparing the results given in Figure 5.10 with the corresponding results given in Figures 5.5 and 5.6 shows that at the lower values of Rayleigh number considered (approximately less than $10^7$) the vertical side surface boundary condition has a significant effect on the Nusselt numbers for the top surface of the plates.
Figure 5.9 Variations of the mean Nusselt number for the bottom surface of (a) an I-shaped plate and (b) a +-shaped plate having an isothermal vertical side surface with dimensionless plate thicknesses, $H$, for various values of the Rayleigh number.
Figure 5.10 Variations of the mean Nusselt number for the top surface of (a) an I-shaped plate and (b) a +shaped plate having an isothermal vertical side surface with dimensionless plate thicknesses, $H$, for various values of the Rayleigh number.

Attention will next be given to the heat transfer from the vertical side surface when it is isothermal. Variations of the mean Nusselt number for this surface, $N_{u,\text{side}}$, with Rayleigh number for three values of dimensionless plate thickness, $H$, are shown in Figures 5.11 and 5.12 for the I-shaped plate and for the +shaped plate, respectively. It can be seen from these
figures that the variations for the I-shaped and +-shaped plates are very similar and that the effect of the $H$ value on the vertical side surface Nusselt number at a particular value of $Ra$ is relatively small except when the $H$ values are near 0.1.

Figure 5.11 Variations of the mean Nusselt number for the vertical side surface with Rayleigh number for an I-shaped plate having an isothermal vertical side surface for dimensionless plate thicknesses, $H$, of 0.1, 0.2, and 0.3.

Figure 5.12 Variations of the mean Nusselt number for the vertical side surface with Rayleigh number for a +-shaped plate having an isothermal vertical side surface for dimensionless plate thicknesses, $H$, of 0.1, 0.2, and 0.3.
The effect of the vertical side surface thermal boundary condition on the heat transfer rate from the top and bottom surfaces of the plate is further illustrated by showing typical variations of \( Nu_{top} \) and \( Nu_{bot} \) with \( Ra \) for an \( H \) value of 0.1 for the I-shaped plate and for the +-shaped plate (Figures 5.13 and 5.14, respectively). It will be seen from these figures that for the two plate shapes considered the vertical side surface thermal boundary condition has an almost negligible effect on the Nusselt numbers for the bottom surface while the effect of the vertical side surface thermal boundary condition on the Nusselt numbers for the top surface is significant especially at lower values of Rayleigh number.

![Graph of Nu vs Ra for I-shape plate](image)

**Figure 5.13** Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for an I-shaped plate having a dimensionless thickness, \( H \), of 0.1 for the two vertical side surface thermal boundary conditions considered.
Figure 5.14 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for a +-shaped plate having a dimensionless thickness, $H$, of 0.1 for the two vertical side surface thermal boundary conditions considered.

To further illustrate the effect of the plate shape on the heat transfer rate when the characteristic length scale, $l$, is used typical variations of $Nu_{top}$ and $Nu_{bot}$ with $Ra$ for the two plate shapes considered for the case where the $H$ value is 0.1 are given in Figures 5.15 and 5.16 for the adiabatic and isothermal vertical side surface boundary conditions, respectively. It can be seen from these figures that at lower Rayleigh numbers, i.e., in the
laminar flow region, the results for the two plate shapes are very nearly the same indicating that the use of the characteristic length, \( l \), in defining the Nusselt and Rayleigh numbers is successful. At higher Rayleigh numbers there is some difference between the results for the two plate shapes, this being mainly due to differences in the prediction of the conditions under which transition occurs with the two plate shapes.

![Graphs of mean Nusselt number vs Rayleigh number for I- and +/-shaped plates](image-url)

(a)

(b)

**Figure 5.15** Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for I- and +/-shaped plates having an adiabatic vertical side surface for a dimensionless plate thickness, \( H \), of 0.1.
Figure 5.16 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface with Rayleigh number for I- and +-shaped plates having an isothermal vertical side surface for a dimensionless plate thickness, $H$, of 0.1.

5.4 Experimental Apparatus and Procedure

5.4.1 Apparatus

To experimentally measure the natural convective heat transfer rate from the two-sided plates considered, flat plates were machined from ($Al$ 6061-$T6$) aluminum alloy. The dimensions and the mass of the plates used are given in Table 5.1 A plexiglas plate on
which vertical, small diameter steel elements with sharp pointy tips were fixed was used as a support frame to mount the heated plates as shown in Figure 5.17. The purpose for these steel elements having the sharp pointy tips was to reduce the conduction losses from the mounted plate as much as possible. Further details of the apparatus used in the experimental study are given in section 3.3.2.

**Table 5.1 Dimensions and mass of the aluminum flat plates**

<table>
<thead>
<tr>
<th>Flat Plate</th>
<th>Outside length, ( W ) (m)</th>
<th>Thickness, ( h ) (m)</th>
<th>Dimensionless Thickness, ( h/W )</th>
<th>Mass, ( m ) (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-shaped</td>
<td>0.076</td>
<td>0.0076</td>
<td>0.1</td>
<td>0.07495</td>
</tr>
<tr>
<td>+ shaped</td>
<td>0.076</td>
<td>0.0076</td>
<td>0.1</td>
<td>0.05125</td>
</tr>
</tbody>
</table>

**Figure 5.17** The support frame used to mount the test plates (same as shown in Figure 4.17).

**5.4.2 Experimental Procedure**

The mean heat transfer rate from the I- and +-shaped plates was determined using the lumped capacity method. Descriptions of the method and the analysis procedure used in determining the Nusselt numbers are given in sections 3.3.5 and 3.3.6.
5.4.3 Uncertainty Analysis

The overall uncertainty in the experimentally determined values of Nusselt number has then been determined using the root-sum-square (RSS) method described in section 3.3.7. The relative uncertainty obtained for the experimental values of Nusselt number ranged between 3.7% and 4.8% for the I-shaped plate and between 3.6% and 5.3% for the +-shaped plate.

5.5 Experimental Results and Comparison with Numerical Results

The experimental results obtained for the I- and +-shaped plates were used to validate the numerical results. A comparison between the numerical and experimental heat transfer results obtained has been undertaken and the variations of the mean Nusselt number with Rayleigh number for a dimensionless plate thickness, $H$, of 0.1 are shown in Figures 5.18 and 5.19 for the I-shaped plate and +-shaped plate, respectively. It should be noted here that the Rayleigh number range is different in these figures because of the difference in the ambient temperature during the times the experiments for each of the shapes were undertaken. It will be seen from these figures that for the +-shaped plate the numerical results lie within the experimental error band which is the same as the experimental uncertainty as explained earlier for the range of Rayleigh numbers considered whereas for the I-shaped plate the numerical results lie within the experimental error band for most of the Rayleigh numbers range considered. The numerical and experimental results obtained are generally in very good agreement.
5.6 Conclusions

The results of the present study indicate that:

1. In all cases considered the Nusselt numbers for the top surface are lower than those for the bottom surface at the lower Rayleigh number values but are higher than those for the bottom surface at the higher Rayleigh number values. This seems to
be the result of the thickening of the boundary layer on the top surface at lower Rayleigh numbers where the inward flows collide forming the plume along with a thinning of the bottom surface boundary layer where the outward flows divide (Chambers and Lee, 1997).

2. In all cases considered changes in the value of the dimensionless plate thickness, $H$, have an almost negligible effect on the Nusselt number values for the bottom surface but have a significant effect on the Nusselt numbers for the top surface, the nature of this effect being dependent on the Rayleigh number value considered.

3. The vertical side surface thermal boundary condition has only a very small effect on the Nusselt numbers for the bottom surface but has a significant effect on the Nusselt numbers for the top surface at the lower Rayleigh number values considered.

4. By using the characteristic length scale, $l$, it was found that for both vertical side surface thermal boundary conditions considered, the variations of the Nusselt number with Rayleigh number for the I- and + -shaped plates are almost the same at the lower Rayleigh number values considered but differ somewhat at the higher Rayleigh number values considered mainly as a result of differences in the predicted Rayleigh number values at which transition to turbulence starts to occur.

5. Most of the numerical results lie within the uncertainty range in the experimental results which ranged between 3.7% and 4.8% for the I-shaped plate and between 3.6% and 5.3% for the + -shaped plate and the numerical and experimental results are generally in very good agreement over the limited range of the experimental Rayleigh numbers considered.
Chapter 6
A Numerical Study of Natural Convective Heat Transfer from Two-Sided Inclined Square Plates Having a Finite Thickness

Abstract

Simultaneous natural convective heat transfer from the top, bottom and side surfaces of two-sided inclined square plates having various thicknesses has been numerically investigated. The aim of this work is to determine whether the plate thickness has a significant influence on the heat transfer rates from the plate surfaces when the plate is inclined to the horizontal and to determine how the heat transfer rate varies with this angle of inclination. The upper, lower and side surfaces of the plate have been assumed to be isothermal and at the same temperature which is higher than that of the surrounding fluid. The range of conditions considered is such that laminar, transitional, and turbulent flow occurs over the plate. The numerical solution has been obtained using the commercial CFD solver ANSYS FLUENT©. In this study, results have only been obtained for the case where the plate is exposed to air. Inclination angles of between 0 and 40 degrees from the horizontal and plate dimensionless thicknesses (thickness-to-side length ratios) of between 0 and 0.3 have been considered. Variations of the mean Nusselt number with Rayleigh number for the top surface, the bottom surface, the side surface and that averaged over all heated surfaces of the plate for various inclination angles and for various plate dimensionless thicknesses have been obtained.
6.1 Introduction

Natural convective heat transfer from a two-sided square plate when it is inclined to the horizontal has been numerically investigated in this study. The situation considered is shown in Figure 6.1. The interest in the situation being considered arises from the fact that similar situations do occur in engineering practice. In some such situations a heated plate, similar to that considered in the present study, may be mounted at a relatively small angle to the horizontal. The situation considered in this study is a simplified model of those occurring in real world engineering applications but should indicate the effects of plate inclination and thickness on the heat transfer rate.

![Figure 6.1 Finite thickness two-sided inclined square plate situation considered. Shown are the definitions of the side length, w, of the thickness, h, of the inclination angle, \( \theta \), of the top surface, and of the side surface of the plate. The bottom surface is below the top surface.](image)

Natural convective heat transfer from flat plates has been extensively studied. Most available studies, however, have been concerned with plates that are either vertical or horizontal. Some attention has been given to natural convective heat transfer from inclined plates, e.g., see Hassan and Mohamed, 1970; Pera and Gebhart, 1973; Kobus and Wedekind, 2002, Wei et al., 2002; Corcione et al., 2011; Oosthuizen, 2014, but most of the studies that dealt with inclined plates have only considered a limited range of flow
conditions, i.e., have only considered laminar flow and only considered situations where one surface of the plate is heated, i.e., a single-sided plate. In addition, the effect of the plate thickness on the heat transfer rate has not been studied in detail for such situations. The present study was, therefore, undertaken to determine whether the dimensionless thickness of the square plate and the angle of inclination have a significant effect on the heat transfer rate from the plate surfaces for a wide range of Rayleigh numbers ($10^4$ to $10^{14}$).

6.2 Solution Procedure

The flow has been assumed to be steady and three dimensional and the fluid properties have been assumed constant except for the density change with temperature which gives the rise to the buoyancy forces, i.e., the Boussinesq approach has been adopted. The standard $k$-epsilon turbulence model with full account being taken of buoyancy force effects has been used and applied under all conditions. The solution has been obtained by numerically solving the governing equations using the commercial CFD solver ANSYS FLUENT©. An extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken, and the heat transfer results obtained were grid and convergence-criteria independent to better than one per cent. Further details of the numerical solution procedure are given in section 3.2.

6.3 Results and Discussion

The solution has the following parameters:

1. The Rayleigh number, $Ra$, based on the side length of the square plate and the difference between the plate surface temperature and the temperature of the fluid far from the plate, i.e.,
\[ Ra = \frac{\beta g w^3 (T_w - T_f)}{\nu \alpha} \]  

(6.1)

2. The dimensionless thickness of the plate, \( H \), which is defined as (see Figure 6.1):

\[ H = \frac{h}{w} \]  

(6.2)

3. The inclination angle, \( \theta \).

4. The Prandtl number, \( Pr \).

Because of the applications that motivated this study, results have only been obtained for a Prandtl number of 0.74, which is effectively the value for air at ambient conditions. Inclination angles of between 0° and 40° and Rayleigh numbers between approximately \( 10^4 \) and \( 10^{14} \) have been considered.

The mean heat transfer rates from the various surfaces of the plate have been expressed in terms of the mean Nusselt number based on the side length of the square plate and the difference between the plate surface temperature and the temperature of the fluid far from the plate. The Nusselt number will depend on the Rayleigh number, on the dimensionless thickness of the plate, and on the inclination angle. The mean Nusselt numbers for the top surface, \( Nu_{top} \), for the bottom surface, \( Nu_{bot} \), for the side surface, \( Nu_{side} \), and for the total heated surfaces of the plate, \( Nu_{total} \), have been considered. The following Nusselt numbers have therefore been introduced:

\[ Nu_{top} = \frac{\overline{Q}_w}{A_{plate}(T_w - T_f)k}, \quad Nu_{bot} = \frac{\overline{Q}_w}{A_{plate}(T_w - T_f)k}, \quad Nu_{side} = \frac{\overline{Q}_w}{A_{side}(T_w - T_f)k} \]

\[ Nu_{total} = \frac{\overline{Q}_w}{A_{total}(T_w - T_f)k} \]  

(6.3)

where, \( \overline{Q}_w \), \( \overline{Q}_w \), \( \overline{Q}_w \) and \( \overline{Q}_w \) are the mean heat transfer rates from the total heated surfaces of the plate, from the upper surface of the plate, from the lower surface of the plate.
and from the side surface of the plate, respectively, and where $A_{\text{total}}$, $A_{\text{plate}}$, and $A_{\text{side}}$ are the area of the total heated surfaces of the plate, the areas of the upper and the lower surfaces of the plate, and the area of the side surface of the plate, respectively, therefore:

$$\overline{Q'} = \overline{Q'}_{\text{top}} + \overline{Q'}_{\text{bot}} + \overline{Q'}_{\text{side}}$$  \hspace{1cm} (6.4)$$

$$A_{\text{total}} = 2A_{\text{plate}} + A_{\text{side}}$$  \hspace{1cm} (6.5)$$

$$Nu_{\text{total}} = \frac{(Nu_{\text{top}} + Nu_{\text{bot}})A_{\text{plate}} + Nu_{\text{side}}A_{\text{side}}}{A_{\text{total}}}$$  \hspace{1cm} (6.6)$$

Typical variations of the mean Nusselt number for the top surface, the bottom surface, and for the total heated surfaces of the square plate with Rayleigh number for two values of inclination angles are shown in Figures 6.2 and 6.3 for plate dimensionless thicknesses of 0 and 0.3, respectively. The results given in (a) in each of these figures are for the case where the inclination angle is $0^\circ$, i.e., the case where the plate is horizontal. It can be seen from these figures that the Nusselt number for the bottom surface is higher than that for the top surface at the lower Rayleigh numbers considered but that the Nusselt number for the top surface is higher than that for the bottom surface at the higher Rayleigh numbers considered.

Attention will next be given to the effect of the dimensionless plate thickness on the heat transfer rate for various inclination angles. Typical variations of the mean Nusselt number for the top and bottom surfaces of the square plate with dimensionless plate thickness for various Rayleigh number values are shown in Figures 6.4 and 6.5 for inclination angle values of $0^\circ$ and $30^\circ$, respectively. It will be seen from these figures that for the inclination angles considered the changes in the dimensionless plate thickness have an almost negligible effect on the Nusselt numbers for the bottom surface whereas these changes in the dimensionless plate thickness have a significant effect on the Nusselt
numbers for the top surface, this effect being dependent on the value of Rayleigh number considered. It will also be seen from these figures that for the range of Rayleigh numbers considered the Nusselt numbers for the top surface decrease as the dimensionless plate thickness increases for various inclination angles considered especially at higher values of Rayleigh number.

Figure 6.2 Variations of the mean Nusselt number for the bottom surface, the top surface, and for the total heated surfaces of the square plate with Rayleigh number for angles of inclination of (a) $0^\circ$ and (b) $30^\circ$ for a dimensionless plate thickness, $H$, of 0.
Figure 6.3 Variations of the mean Nusselt number for the bottom surface, the top surface, and for the total heated surfaces of the square plate with Rayleigh number for angles of inclination of (a) 0° and (b) 30° for a dimensionless plate thickness, $H$, of 0.3.
Figure 6.4 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness for various values of Rayleigh number for an angle of inclination of 0°.
Figure 6.5 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness for various values of Rayleigh number for an angle of inclination of 30°.

To illustrate the effect of the dimensionless plate thickness on the heat transfer rate from the side surface of the square plate, typical variations of the mean Nusselt number for the side surface with Rayleigh number for various values of dimensionless plate thickness considered are shown in Figure 6.6 for two values of inclination angle. It will be seen from this figure that for the inclination angle of 0°, i.e., for the case where the plate is horizontal,
the changes in the dimensionless plate thickness have a significant effect on the Nusselt numbers for the side surface for the range of Rayleigh numbers considered except at higher values of Rayleigh number (approximately greater than $10^{10}$). For the inclination angle of $30^\circ$ the effect of changing the dimensionless plate thickness on the Nusselt numbers is significant at lower values of Rayleigh number (approximately less than $10^7$).

Figure 6.6 Variations of the mean Nusselt number for the side surface of the square plate with Rayleigh number for two values of dimensionless plate thickness for angles of inclination of (a) $0^\circ$ and (b) $30^\circ$. 
Attention will next be turned to a more detailed look at the effect of the inclination angle on the heat transfer rate for various dimensionless plate thicknesses. Typical variations of the mean Nusselt number with the inclination angle for the range of Rayleigh numbers considered for a dimensionless plate thickness of 0 are shown in Figures 6.7 and 6.8 for the top and bottom surfaces, respectively, while the typical variations for a dimensionless plate thickness of 0.3 are shown in Figures 6.9 and 6.10 for the top and bottom surfaces, respectively. In these figures, the range of Rayleigh numbers considered was divided into three subranges (lower, intermediate and higher values of Rayleigh number) to show more clearly the effect of the inclination angle on the heat transfer rate. It will be seen from Figure 6.7 that for the top surface of the plate having a dimensionless plate thickness, $H$, of 0 and for Rayleigh number values of $10^4$ and $10^5$ the Nusselt number does not change with inclination angle until an angle of about $10^\circ$ after which the Nusselt number increases as the inclination angle increases. For Rayleigh numbers ranging from $10^5$ to $10^{10}$ the Nusselt number first decreases as the angle increases from $0^\circ$ and starts to increase as the angle increases above about $20^\circ$. For higher values of Rayleigh number, the Nusselt number decreases as the inclination angle increases from $0^\circ$ but starts to increase as the inclination angle increases above about $20^\circ$ and then decreases slightly as the inclination angles increases above about $30^\circ$. For the bottom surface, it can be seen from Figure 6.8 that the effect of the inclination angle on the Nusselt number is less significant than that for the top surface. For the lower values of Rayleigh number, the Nusselt numbers are almost the same for various inclination angles considered except at the Rayleigh number value of $10^4$ where the Nusselt number increases when the angle is between $0^\circ$ and about $10^\circ$. For the intermediate values of Rayleigh numbers, it can be seen that the Nusselt
number increases slightly with the inclination angles. For the higher values of Rayleigh number it will be seen that for Rayleigh number values of $10^{11}$ and $10^{12}$ the Nusselt number decreases with the inclination angle while the Nusselt number variation with the inclination angle is different for the Rayleigh number values of $10^{13}$ and $10^{14}$ such that the Nusselt number first decreases as the inclination angle increases from $0^\circ$ and then starts to increase slightly as the inclination angle is increased above about $10^\circ$. For the plate dimensionless thickness of 0.3 the results given in Figure 6.9 show that for the lower values of Rayleigh number the Nusselt number for the top surface increases as the inclination angle increases from $0^\circ$. For the intermediate and higher values of Rayleigh number, however, the Nusselt number increases as the inclination angle increases from about $10^\circ$ while as the inclination angle increases from $0^\circ$ to about $10^\circ$, it can be seen that the Nusselt number decreases for this range of Rayleigh numbers except at a Rayleigh number value of $10^8$ where the Nusselt number was found to increase. The results given in Figure 6.10 show that the variations of the Nusselt number with the inclination angle for a plate dimensionless thickness of 0.3 is almost the same as that for a plate dimensionless thickness of 0 shown in Figure 6.8. This is due to the negligible effect of the dimensionless plate thickness on the Nusselt numbers for the bottom surface. The form of the variations of Nusselt number with angle of inclination is influenced by the changes in the flow pattern that occur with changes in inclination angle (see appendix B).

To illustrate the effect of the inclination angle on the heat transfer rate from the side surface of the square plate, typical variations of the mean Nusselt number with the inclination angle for the range of Rayleigh numbers considered for a dimensionless plate thickness of 0.3 are shown in Figure 6.11. Again, the range of Rayleigh numbers
considered has been divided into three subranges in this figure. For the lower values of Rayleigh number it can be seen that the Nusselt number does not change with the inclination angle except at Rayleigh numbers of $10^6$ and $10^7$ where a slight increase in the Nusselt number can be noticed above an inclination angle of about $20^\circ$. For the intermediate values of Rayleigh number, it can be seen that the Nusselt number first decreases as the inclination angle increases from $0^\circ$ and then starts to increase when the inclination angle is above about $20^\circ$ for Rayleigh numbers of $10^8$ and $10^{10}$ and above about $10^\circ$ for Rayleigh number of $10^9$. For the higher values of Rayleigh number, the Nusselt number decreases with the inclination angle for Rayleigh numbers of $10^{11}$ and $10^{12}$ while the Nusselt number first increases as the inclination angle increases from $0^\circ$ and then starts to decrease as the inclination angle increases above about $20^\circ$ for Rayleigh numbers of $10^{13}$ and $10^{14}$.
Figure 6.7 Variations of the mean Nusselt number for the top surface of the square plate with inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.
Figure 6.8 Variations of the mean Nusselt number for the bottom surface of the square plate with inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.
Figure 6.9 Variations of the mean Nusselt number for the top surface of the square plate with inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.3.
Figure 6.10 Variations of the mean Nusselt number for the bottom surface of the square plate with inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.3.
Figure 6.11 Variations of the mean Nusselt number for the side surface of the square plate with inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.3.
6.4 Conclusions

A numerical investigation of the natural convective heat transfer from a two-sided inclined square plate has been undertaken to determine whether the plate thickness and the inclination angle have a significant effect on the heat transfer rate from the plate surfaces. It can be concluded from the results obtained in this study that in all cases considered the Nusselt numbers for the top surface are lower than those for the bottom surface at the lower Rayleigh number values considered but are higher than those for the bottom surface at the higher Rayleigh number values. This seems to be the result of the thickening of the boundary layer on the top surface at lower Rayleigh numbers where the inward flows collide forming the plume along with a thinning of the bottom surface boundary layer where the outward flows divide (Chambers and Lee, 1997).

For the inclination angles considered, the changes in the value of the dimensionless plate thickness were found to have an almost negligible effect on the Nusselt numbers for the bottom surface of the square plate but these changes in the dimensionless plate thickness have a significant effect on the Nusselt numbers for the top surface, the nature of this effect being dependent on the Rayleigh number value considered.

For the dimensionless plate thicknesses considered, the effect of the inclination angle on the Nusselt numbers for the top surface of the square plate is higher than that for the bottom and side surfaces for the range of Rayleigh number values considered.

For the range of Rayleigh numbers, inclination angles and dimensionless plate thicknesses considered, it was found that the Nusselt number varies more significantly with Rayleigh number than with the inclination angle, and finally with the plate dimensionless thickness.
Chapter 7

Numerical and Experimental Investigations of Natural Convective Heat Transfer from Two-Sided Diagonally Inclined Square Plates Having a Finite Thickness

Abstract

Natural convective heat transfer from two-sided diagonally inclined square plates having various thicknesses has been numerically and experimentally investigated. The aim of this work is to determine the influence of the plate dimensionless thickness (thickness-to-side length ratio) and the influence of the diagonal inclination angle on the heat transfer rate for the various flow regimes covered (laminar, transitional and turbulent). The mean heat transfer rate was numerically obtained using ANSYS FLUENT© and experimentally determined using the Lumped Capacity Method. In the numerical study, diagonal inclination angles of between 0 and 40 degrees from the horizontal and plate dimensionless thicknesses of between 0 and 0.3 have been considered. In the experimental study, diagonal inclination angles of between 0 and 40 degrees from the horizontal and plate dimensionless thicknesses of only 0.0635 have been considered. The results indicate that the plate thickness does not have a significant influence on the heat transfer rate while the diagonal inclination angle significantly influences the heat transfer rate especially at higher Rayleigh number values considered.

7.1 Introduction

A numerical study of natural convective heat transfer from the top, bottom and side surfaces of two-sided diagonally inclined square plates with various dimensionless
thicknesses (thickness-to-side length ratios) and for various diagonal inclination angles have been conducted for a wide range of Rayleigh number. A limited range of experiments has been undertaken for validation purposes. The upper, lower and side surfaces of the plate are at higher temperature than that of the surrounding fluid. The situation considered is shown in Figure 7.1. Mean heat transfer rates from the upper surface, from the lower surface and from the side surface of the plate as well as the mean heat transfer rate averaged over the heated surfaces of the plate have been considered.

Natural convective heat transfer from plates where all surfaces are heated to the same temperature appear in a variety of engineering applications, for example, in cooling of electrical and electronic components. In such applications the heated plate may in some cases be diagonally inclined to the horizontal with a certain inclination angle. The situation considered in the present study is a simplified model of those arising in such engineering applications. However, the results obtained in this study should give an indication of the effects of the diagonal angle of inclination and of the plate thickness in practical situations.

A limited number of numerical and experimental studies of natural convective heat transfer from inclined plates are available, see (Hassan and Mohamed, 1970; Pera and Gebhart, 1973; Kobus and Wedekind, 2002; Wei et al., 2002; Corcione et al., 2011; Oosthuizen, 2014). All studies undertaken used air as the surrounding fluid. In these studies, however, only simple forms of inclination have been considered, i.e., situations in which the angle of inclination is with respect to a side of the plate. Furthermore, most of these studies have only considered situations with a limited range of flow conditions, i.e., have only considered laminar flow and situations where only one surface of the plate is heated, i.e., a single-sided plate. An early study of the natural convective heat transfer from
the inclined flat surfaces was undertaken by Hassan and Mohamed (1970) who carried out experiments to measure the local heat transfer coefficients along a flat plate for a laminar flow regime. The plate was single-sided and the range of inclination angles considered was from horizontal (plate is facing upward) to the horizontal (plate is facing downward) positions. Pera and Gebhart (1973) later experimentally investigated the natural convective heat transfer above horizontal and slightly inclined surfaces with laminar flow condition being considered. More recently, Kobus and Wedekend (2002) performed experiments measuring the natural convective heat transfer from two-sided thin circular disks of different thickness-to-diameter ratio at arbitrary angles of inclination. The range of Rayleigh numbers considered was from $2 \times 10^2$ to $3 \times 10^7$. Wei et al. (2002) numerically studied the natural convective heat transfer from the upper and lower surfaces of uniformly heated thin plates with various thickness-to-width ratios at arbitrary angles of inclination. The range of Rayleigh numbers considered was from $4.8 \times 10^6$ to $1.87 \times 10^8$. Corcione et al. (2011) undertook a numerical study of laminar natural convective heat transfer from an inclined two-sided isothermal plate immersed in different fluids for various values of inclination angles. Oosthuizen (2014) numerically investigated the natural convective heat transfer from a single-sided inclined isothermal square flat element. The range of flow conditions considered was such that laminar, transitional and turbulent flows can occur.

The above literature review demonstrates that there are no available studies to predict the influence of the plate thickness and the diagonal inclination angle on the heat transfer rate from the surfaces of a square plate when it is diagonally inclined to the horizontal. This is basically the goal of the research described here.
7.2 Numerical Solution Procedure

The flow has been assumed to be steady and three dimensional and the fluid properties have been assumed constant except for the density change with temperature which gives the rise to the buoyancy forces, i.e., the Boussinesq approximation has been adopted. The standard \( k \)-epsilon turbulence model with full account being taken to the buoyancy force effects has been used. This turbulence model has been applied in all calculations and is thus used to determine when transition begins. The solution has been obtained by numerically solving the governing equations using the commercial CFD solver ANSYS FLUENT\(^\circledR\). An extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken, and the heat transfer results presented here are grid and convergence-criteria independent to less than one per cent. Further details of the numerical solution procedure are given in section 3.2.

7.3 Numerical Results and Discussion

The solution has the following parameters:
1. The Rayleigh number, $Ra$, based on the side length, $w$, of the square plate (see Figure 7.1) and the difference between the plate temperature, $T_w$, and the temperature of the fluid far from the plate, $T_f$, i.e.:

$$Ra = \frac{\beta g w^3 (T_w - T_f)}{v \alpha}$$  \hfill (7.1)

2. The diagonal inclination angle, $\varphi$.

3. The Plate dimensionless thickness, $H$, defined by:

$$H = \frac{h}{w}$$  \hfill (7.2)

4. The Prandtl number, $Pr$.

Results have only been obtained for a Prandtl number of 0.74, which is effectively the value for air at ambient conditions. The reason for this is that much of the equipment that is cooled by natural convection uses air as the heat transfer fluid. Diagonal inclination angles of between $0^\circ$ and $40^\circ$ and Rayleigh numbers between approximately $10^4$ and $10^{14}$ have been considered.

The mean heat transfer rate from the top surface, $\bar{Q}_{top}$, from the bottom surface, $\bar{Q}_{bot}$, and from the side surface, $\bar{Q}_{side}$, of the plate as well as the mean heat transfer rate averaged over all heated surfaces, $\bar{Q}$, have been expressed in terms of mean Nusselt numbers based on the side length of the square plate and the difference between the plate temperature and the temperature of the fluid far from the plate. The following Nusselt numbers have, therefore, been introduced:

$$Nu_{top} = \frac{\bar{Q}_{top} w}{A_{plate}(T_w - T_f)k}, Nu_{bot} = \frac{\bar{Q}_{bot} w}{A_{plate}(T_w - T_f)k}, Nu_{side} = \frac{\bar{Q}_{side} w}{A_{side}(T_w - T_f)k}$$  \hfill (7.3)
\[ Nu_{\text{total}} = \frac{\overline{Q}}{A_{\text{total}}(T_w - T_f)k} \]

where,

\[ \overline{Q} = \overline{Q}_{\text{top}} + \overline{Q}_{\text{bot}} + \overline{Q}_{\text{side}} \] (7.4)

and where \( A_{\text{total}} \), \( A_{\text{plate}} \), and \( A_{\text{side}} \) are the total area of the heated surfaces of the plate, the areas of the upper and the lower surfaces of the plate, and the area of the side surface of the plate, respectively. Therefore:

\[ A_{\text{total}} = 2A_{\text{plate}} + A_{\text{side}} \] (7.5)

\[ Nu_{\text{total}} = \frac{(Nu_{\text{top}} + Nu_{\text{bot}})A_{\text{plate}} + Nu_{\text{side}}A_{\text{side}}}{A_{\text{total}}} \] (7.6)

Variations of the mean Nusselt number for the top surface, \( Nu_{\text{top}} \), for the bottom surface, \( Nu_{\text{bot}} \), and for all heated surfaces of the plate, \( Nu_{\text{total}} \), with Rayleigh number for various diagonal inclination angles are shown in Figures 7.2 and 7.3 for plate dimensionless thickness values of 0.0635 and 0.3, respectively. The results given in Figures 7.2-a and 7.3-a are for the case where the diagonal inclination angle is 0°, i.e., for the case where the plate is horizontal. It will be seen from the results given in Figures 7.2 and 7.3 that in all cases, i.e., for all angles of inclination, the Nusselt number for the bottom surface is higher than that for the top surface at the lower Rayleigh numbers considered but that in all cases the Nusselt number for the top surface is higher than that for the bottom surface at the higher Rayleigh numbers considered.
Figure 7.2 Variations of the mean Nusselt number for the bottom surface, the top surface, and for all heated surfaces of the square plate with Rayleigh number for diagonal angles of inclination of (a) $0^\circ$, (b) $10^\circ$, (c) $20^\circ$, (d) $30^\circ$, and (e) $40^\circ$ for a dimensionless plate thickness, $H$, of 0.0635.
Figure 7.3 Variations of the mean Nusselt number for the bottom surface, the top surface, and for all heated surfaces of the square plate with Rayleigh number for diagonal angles of inclination of (a) 0°, (b) 10°, (c) 20°, (d) 30°, and (e) 40° for a dimensionless plate thickness, $H$, of 0.3.

To further illustrate the effect of the plate dimensionless thickness on the heat transfer rate, typical variations of the mean Nusselt number for the top and bottom surfaces with dimensionless plate thickness for various Rayleigh number values and for various diagonal inclination angles considered are shown in Figures 7.4 to 7.8. It can be seen from these figures that for various diagonal inclination angles considered the dimensionless thickness
has an almost negligible effect on the Nusselt number for the bottom surface whereas this
effect is significant for the top surface especially at lower values of dimensionless thickness
considered, the nature of this effect being dependent on the value of the Rayleigh number.
It can be also seen from these figures that for the top surface the Nusselt number decreases
with increasing the dimensionless thickness for various diagonal inclination angles
considered.

Typical variations of the mean Nusselt number with Rayleigh number for the side
surface of the plate for the diagonal inclination angles considered are shown in Figure 7.9
for two values of dimensionless plate thickness. It will be seen from this figure that for the
diagonal inclination angles of 0°, 10° and 20° (Figure 7.9-a, b, c) the dimensionless plate
thickness does affect the Nusselt number for the range of Rayleigh numbers considered
except at higher values of the Rayleigh number (approximately greater than $10^{10}$) where
this effect is almost negligible. For diagonal inclination angles of 30° and 40° (Figure 7.9-
d, e) the effect of the dimensionless plate thickness on the Nusselt number is, however,
significant at lower values of the Rayleigh number (approximately less than $10^{7}$) while this
effect is negligible for Rayleigh numbers between approximately $10^{7}$ and $10^{10}$. Above
Rayleigh number value of approximately $10^{10}$ the dimensionless plate thickness has a
modest effect on the Nusselt number.
Figure 7.4 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness, $H$, for various values of Rayleigh number for a diagonal inclination angle of $0^\circ$. 
Figure 7.5 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness, $H$, for various values of Rayleigh number for a diagonal inclination angle of 10°.
Figure 7.6 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness, $H$, for various values of Rayleigh number for a diagonal inclination angle of 20°.
Figure 7.7 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness, $H$, for various values of Rayleigh number for a diagonal inclination angle of $30^\circ$. 
Figure 7.8 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of the square plate with dimensionless plate thickness, $H$, for various values of Rayleigh number for a diagonal inclination angle of 40°.
Figure 7.9 Variations of the mean Nusselt number for the side surface of the plate with Rayleigh number for two values of dimensionless plate thickness for diagonal inclination angles of (a) 0°, (b) 10°, (c) 20°, (d) 30°, and (e) 40°.

To more clearly illustrate the influence of the diagonal inclination angle on the heat transfer rate, typical variations of the mean Nusselt number with diagonal inclination angle for the range of Rayleigh number considered for the dimensionless plate thickness of 0.0635 are shown in Figures 7.10 and 7.11 for the top and bottom surfaces, respectively while typical variations for a dimensionless plate thickness of 0.3 are shown in Figures 7.12 and 7.13 for the top and bottom surfaces, respectively. In these figures, the range of
Rayleigh numbers considered was divided into three subranges-lower, intermediate and higher values of Rayleigh number. It will be seen from these figures that for the top surface of the plate which has a dimensionless plate thickness, $H$, of 0.0635 (Figure 7.10) the Nusselt number increases as the diagonal inclination angle increases for Rayleigh number values of $10^4$ and $10^5$. After Rayleigh number value of $10^5$ the Nusselt number first decreases as the angle increases from $0^\circ$ and starts to increase as the angle increases above about $10^\circ$ for Rayleigh number value of $10^6$. For the range of Rayleigh numbers between $10^7$ and $10^{11}$ the behavior is almost the same, i.e., the Nusselt number decreases as the diagonal inclination angle increases from $0^\circ$ but starts to increase as the angle increases above about $20^\circ$. For higher values of Rayleigh number, the Nusselt number decreases as the diagonal inclination angle increases from $0^\circ$ until about $20^\circ$ is reached after which the Nusselt number starts to increase with the angle of inclination and then decreases again above an angle of about $30^\circ$. For the bottom surface, it can be seen from Figure 7.11 that the effect of the diagonal inclination angle on the Nusselt number is significantly different from that for the top surface, this effect being significant only at higher values of Rayleigh number (Figure 7.11-c) where the Nusselt number decreases with the diagonal inclination angle for Rayleigh number value of $10^{12}$ whereas for the Rayleigh number values of $10^{13}$ and $10^{14}$ the Nusselt number decreases as the diagonal inclination angle increases from $0^\circ$ to about $10^\circ$ after which the Nusselt number increases with the diagonal inclination angle. For the plate with a dimensionless thickness, $H$, of 0.3, however, it can be noted from the results shown in Figure 7.12 that for the top surface the Nusselt number increases with the diagonal inclination angle for a wider range of Rayleigh number (from $10^4$ to $10^9$). The Nusselt number starts to decrease as the diagonal inclination angle increases from $0^\circ$ and
starts to increase as the angle increases above about $10^\circ$ for the other values of Rayleigh number considered. The effect of the diagonal inclination angle on the Nusselt number for the bottom surface for $H=0.3$ (Figure 7.13) is the same as that for $H=0.0635$ since the dimensionless thickness has an almost negligible effect on the heat transfer rate from the bottom surface.
Figure 7.10 Variations of the mean Nusselt number for the top surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers, and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.0635.
Figure 7.11 Variations of the mean Nusselt number for the bottom surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.0635.
Figure 7.12 Variations of the mean Nusselt number for the top surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.3.
Figure 7.13 Variations of the mean Nusselt number for the bottom surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.3.
The influence of the diagonal inclination angle on the heat transfer rate from the side surface of the plate is shown by the typical variations of the mean Nusselt number for the side surface with the diagonal inclination angle shown in Figures 7.14 and 7.15 for dimensionless thickness values of 0.0635 and 0.3, respectively. Again, the Rayleigh numbers range was divided into three subranges. It can be seen from Figure 7.14 (i.e., for $H=0.0635$) that the influence of diagonal inclination angle starts at Rayleigh number value of about $10^6$ and more specifically when the angle is approximately above $10^\circ$ the Nusselt number then increasing as the diagonal inclination angle increases, this influence being true up until the Rayleigh number value of $10^{10}$ after which the influence of the diagonal inclination angle on the Nusselt number becomes significant after about $20^\circ$ when the Nusselt number starts to increase with angle up to about $30^\circ$ after which the Nusselt number starts to decrease with the angle. For the plate with a dimensionless thickness, $H$, of 0.3, it can be seen from Figure 7.15 that the influence of the diagonal inclination angle on the Nusselt number starts to be significant at Rayleigh number value of about $10^6$ and after about $20^\circ$, this influence being true up to Rayleigh number value of $10^8$. For Rayleigh number values of $10^9$ and $10^{10}$, this influence of the diagonal inclination angle on the Nusselt number is almost negligible. The influence of the diagonal inclination angle on the Nusselt number then becomes significant again at Rayleigh number value of $10^{11}$. It can be seen from Figure 7.15-b, c that the Nusselt number decreases with the diagonal inclination angle for Rayleigh number values of $10^{11}$ and $10^{12}$, whereas for Rayleigh number values of $10^{13}$ and $10^{14}$ it will be noted that the Nusselt number first increases as the angle increases from $0^\circ$ then decreases when the angle is approximately above $30^\circ$. 
Figure 7.14 Variations of the mean Nusselt number for the side surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.0635.
Figure 7.15 Variations of the mean Nusselt number for the side surface of the plate with diagonal inclination angle for (a) lower Rayleigh numbers, (b) intermediate Rayleigh numbers and (c) higher Rayleigh numbers considered for a dimensionless plate thickness, $H$, of 0.3.
7.4 Experimental Apparatus and Procedure

The purpose of the experiments conducted in the present study was to validate the numerical results for heat transfer from a diagonally inclined two-sided square plate. In the experimental study, the mean heat transfer rate from the plate considered was determined using the lumped capacity method.

7.4.1 Apparatus

The heated plate used in the experimental study was machined from aluminum alloy (Al 6061-T6) and had a width, \( w \), of 0.2 m, a dimensionless thickness, \( H \), of 0.0635 and a mass, \( m \), of 1.36192 kg. The support frame used to mount the plate is shown in Figure 7.16. The support frame plates were made from plexiglas and the supporting elements on which the plate was mounted were made from steel and had a small diameter and a sharply pointed tip to make the contact area with the heated plate as small as possible. Further details of the apparatus used in the experimental study can be found in section 3.3.2.

![Figure 7.16 The support frame used in the experimental study.](image)
7.4.2 Experimental Procedure

The mean heat transfer rate from the square plate in the situation being considered was determined using the lumped capacity method. A description of this method and the method of determining the Nusselt number are given in sections 3.3.5 and 3.3.6.

7.4.3 Uncertainty Analysis

The overall uncertainty in Nusselt number has been determined using the root-sum-square (RSS) method described in section 3.3.7. The relative uncertainty range for the experimental values of Nusselt number at every diagonal inclination angle and over the time interval considered was obtained and in general it was less than 11 per cent.

7.5 Experimental Results and Comparison with Numerical Results

As explained earlier, the experimental data were obtained to validate the numerical results. A comparison between the numerical results and the experimental data when the dimensionless plate thickness, \( H \), is 0.0635 for various diagonal inclination angles considered is shown in Figure 7.17. It should be noted here that the Rayleigh numbers range is a bit different in this figure because of the difference in the ambient temperature during the experiment for each diagonal inclination angle. It can be seen from this figure that for the diagonal inclination angles considered the numerical results obtained lie within the experimental uncertainty (error band of the experimental results) which is at most less than 11 per cent and good agreement was obtained except when the diagonal inclination angle is 40° where the numerical results lie outside of the experimental uncertainty range. This is possibly due to inaccurate prediction by the numerical model of when transition to turbulence starts to occur. Nevertheless, the maximum percentage difference between the
experimental and numerical results for this angle (40°) is less than 14 per cent for the range of Rayleigh number considered.
Figure 7.17 Comparison of the numerical results with the experimental data for diagonal inclination angle of (a) 0°, (b) 10°, (c) 20°, (d) 30°, and (e) 40° for dimensionless plate thickness, $H$, of 0.0635. The relative uncertainty range is shown for each diagonal inclination angle.

7.6 Conclusions

Numerical and experimental results have been obtained for natural convective heat transfer from two-sided diagonally inclined square plates having various thicknesses for various diagonal inclination angles. In all cases it was found that the heat transfer rate from the top surface is lower than that from the bottom surface at the lower Rayleigh number values considered but is higher than that for the bottom surface at the higher Rayleigh number values.
The dimensionless plate thickness has an almost negligible effect on the heat transfer rate from the bottom surface but has a significant effect on the heat transfer rate from the top surface especially at lower values of dimensionless plate thickness, the nature of this effect being dependent on the Rayleigh number value considered. The effect of the dimensionless plate thickness on the heat transfer rate is also significant for the side surface of the plate for the lower values of Rayleigh numbers. The diagonal inclination angle has a significant effect on the heat transfer rate from the top and side surfaces for the range of Rayleigh numbers considered while this effect is significant for the bottom surface only at higher values of Rayleigh numbers. For the range of Rayleigh numbers, diagonal inclination angles and dimensionless plate thicknesses considered, it was found that the Nusselt number varies most significantly with Rayleigh number, then with the diagonal inclination angle, and finally with the plate dimensionless thickness.

The numerical results lie within the uncertainty of the experimental data which is at most less than 11 per cent. The only exception to this is at diagonal inclination angle of 40° where the numerical results lie outside of the experimental uncertainty.
Chapter 8

A Numerical Study of Natural Convective Heat Transfer from
a Two-Sided Circular Horizontal Isothermal Element
Having a Linearly-Inclined Nonflat Surface

Abstract

Natural convective heat transfer from the top and bottom surfaces of a two-sided circular horizontal isothermal element with a linearly-inclined nonflat surface of different heights and locations has been numerically investigated. The case where the height of the element surface increases linearly from the outer edge to a maximum at the center of the upper surface, of the lower surface and of both the upper and lower surfaces of the element have been considered. The use of the nonflat surface was to determine if heat transfer rate can be increased compared to that of the plane surface. The top and bottom surfaces of the element are assumed to be isothermal and at the same temperature which is higher than that of the surrounding fluid. The range of conditions considered is such that laminar, transitional, and turbulent flow occurs over the element. The standard $k$-epsilon turbulence model with full account being taken of buoyancy force effects has been used and the solution has been obtained using the commercial CFD solver ANSYS FLUENT©. Results have only been obtained for a Prandtl number of 0.74, which is effectively the value for air. Dimensionless heights of 0.1, 0.2 and 0.3 and Rayleigh numbers of approximately between $10^4$ and $10^{14}$ have been considered. The results show that the use of a nonflat surface of situations considered can produce a heat transfer rate enhancement under many conditions, but this enhancement is relatively small.
8.1 Introduction

The use of the linearly-inclined nonflat surface to increase the natural convective heat transfer rate from an isothermal circular horizontal element has been numerically studied. The main aim of the study was to determine whether the heat transfer rate can be increased by using this approach. The increase in the heat transfer rate by the use of such surface is associated with the increase in the effective area of the surface and with the flow changes produced over the surface. Figure 8.1 shows sectional views of the element for the three situations considered here.

Natural convective heat transfer from horizontal two-sided plane elements has been studied in the past, e.g., see Chambers and Lee, 1997; Kobus and Wedekind, 2001; Kobus and Wedekind, 2002; Wei et al., 2002; Wei et al., 2003; Fontana, 2014; Corcione et al., 2011; Oosthuizen and Kalendar, 2016b; Manna and Oosthuizen, 2018. The use of a wavy surface to increase the heat transfer rate from horizontal elements has been considered, see Oosthuizen, 2016a; 2016b; Oosthuizen, 2017; Prétot et al., 2000; Prétot et al., 2003; Siddiqa and Anwar Hossain, 2013 and Siddiqa et al., 2015. Few studies have however dealt with the two-sided element case, e.g., see Oliveira and Oosthuizen, 2018. These studies have indicated that there is no significant enhancement in the heat transfer rate in many situations when a wavy surface is used. The use of a nonflat surface to increase the convective heat transfer from a horizontal circular element has been discussed in a very limited set of studies, e.g., see Oosthuizen, 2018. Most of these studies, however, dealt with the situation where the element is single-sided. Therefore, the use of a linearly-inclined nonflat surface to increase the heat transfer rate from a two-sided circular horizontal isothermal element has been numerically investigated in the present study.
Figure 8.1 Situations considered, the nonflat surface is on (a) the top side, (b) the bottom side, and (c) both the top and bottom sides of the circular heated element.

8.2 Solution procedure

The flow has been assumed to be steady, two-dimensional and axisymmetric about the vertical center line of the circular heated element, see Figure 8.1. The fluid properties have been assumed constant except for the density change with temperature which gives the rise to the buoyancy forces (i.e., the Boussinesq approach has been adopted). The standard $k$-epsilon turbulence model with full account being taken of buoyancy force effects has been applied under all conditions considered and hence it determines when transition to turbulence occurs. The governing equations subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT. To ensure that the results obtained are grid independent, an extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken and the heat transfer results presented here are grid and convergence-criteria independent to better than one per cent. Further details of the numerical solution procedure can be found in section 3.2.
8.3 Results and Discussion

The solution has the following parameters:

1. The Rayleigh number, \( Ra \), based on the diameter of the circular element and the difference between the element surface temperature and the temperature of the fluid far from the element, i.e.,

\[
Ra = \frac{g \beta d^3 (T_w - T_f)}{v \alpha}
\]  

(8.1)

2. The dimensionless height of the nonflat surface, \( H \), which is defined as (see Figure 8.1):

\[
H = \frac{h_n}{d}
\]

(8.2)

3. The Prandtl number, \( Pr \).

Results have been obtained only for a Prandtl number of 0.74, which is the value for air at ambient conditions. Nonflat surface dimensionless heights of 0.1, 0.2 and 0.3 located on the top side, the bottom side and on both the top and the bottom sides of the circular element and Rayleigh numbers of approximately between \( 10^4 \) and \( 10^{14} \) have been considered.

The mean heat transfer rate from the heated surface has been expressed in terms of the Nusselt number again based on the diameter of the circular element and the difference between the element surface temperature and the temperature of the fluid far from the element. The Nusselt number is dependent on the Rayleigh number and on the dimensionless height of the nonflat surface. The mean Nusselt numbers for the top surface, \( Nu_{\text{top}} \), for the bottom surface, \( Nu_{\text{bot}} \), and for the entire surface of the element, \( Nu_{\text{total}} \), have been considered.
The following Nusselt numbers have therefore been introduced:

\[
Nu_{\text{top}} = \frac{\overline{\dot{Q}'}_{\text{top}} d}{A_b(T_w-T_f)k}, \quad Nu_{\text{bot}} = \frac{\overline{\dot{Q}'}_{\text{bot}} d}{A_b(T_w-T_f)k}, \quad Nu_{\text{total}} = \frac{\overline{\dot{Q}'} d}{A_{\text{total}}(T_w-T_f)k}
\]  \hfill (8.3)

where,

\[
\overline{\dot{Q}'} = \overline{\dot{Q}'}_{\text{top}} + \overline{\dot{Q}'}_{\text{bot}}
\]  \hfill (8.4)

and where, \( A_p \) and \( A_{\text{total}} \), are the base area of the upper and the lower surfaces of the element (i.e., the area of the plane flat surface), and the total area of the heated surfaces of the element, respectively, i.e.,

\[
A_b = \frac{\pi}{4} d^2
\]  \hfill (8.5)

\[
A_{\text{total}} = 2A_b
\]  \hfill (8.6)

The Nusselt number given in equation 8.3 is based on the base area of the surface of the circular element, \( A_b \), because the interest here is in the heat transfer from the circular element with nonflat surface compared to that which would exist with a circular element with plane flat surface.

Variations of the mean Nusselt number with Rayleigh number for the top and bottom surfaces of the circular element for various values of dimensionless height and when the nonflat surface is located on the top side, on the bottom side, and on both the top and bottom sides of the heated element are shown in Figures 8.2, 8.3 and 8.4, respectively.

Attention will first be given to the results for the case where the nonflat surface is on the top side of the circular element, these results being shown in Figure 8.2. It will be seen from this figure that for the top surface of the heated element there is a small enhancement of heat transfer rate for the range of Rayleigh number values considered, this enhancement being more significant at the lower values of Rayleigh number considered (i.e., for the
laminar flow region) where the magnitude of heat transfer rate increases as the dimensionless height of the nonflat surface increases while for the bottom surface of the heated element there is only a very small enhancement in the heat transfer rate for the range of Rayleigh number values considered, this being especially true at the higher values of Rayleigh number considered (i.e., for the turbulent flow region) where this enhancement is more significant and there is no considerable effect of the variation of the dimensionless height on the heat transfer rate.

![Graph](image)

Figure 8.2 Variations of the mean Nusselt number with Rayleigh number for the (a) top surface and (b) bottom surface of a circular element having a nonflat surface on the top side for various values of dimensionless height, H.
Attention will next be turned to the results when the nonflat surface is on the bottom side of the circular element given in Figure 8.3. This figure shows that for the top surface of the heated element the effect of the nonflat surface on the heat transfer rate is almost negligible for the range of dimensionless height and Rayleigh number values considered while for the bottom surface there is an enhancement in the heat transfer rate with the increase of the dimensionless height for all values of Rayleigh number considered except when the dimensionless height is equal to 0.1 where the heat transfer rate decreases at higher values of Rayleigh number.

![Graph](image)

Figure 8.3 Variations of the mean Nusselt number with Rayleigh number for the (a) top surface and (b) bottom surface of a circular element having a nonflat surface on the bottom side for various values of dimensionless height, $H$. 

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Lastly, the results for the case where a nonflat surface is located on both the top and the bottom sides of the circular element as shown in Figure 8.4 will be considered. It will be seen from this figure that for the top surface of the heated element there is a small heat transfer rate enhancement at the lower and intermediate values of Rayleigh number and the heat transfer rate increases as the dimensionless height increases except for the dimensionless height of 0.1 where the heat transfer rate associated with this value is higher than that associated with the other values of dimensionless height at the intermediate values of Rayleigh number. For the bottom surface, it will be seen that the heat transfer variation is similar to that for the bottom surface when the nonflat surface is located on the bottom side of the circular element.
Figure 8.4 Variations of the mean Nusselt number with Rayleigh number for the (a) top surface and (b) bottom surface of a circular element having a nonflat surface on the top and bottom sides for various values of dimensionless height, $H$.

To further illustrate the effect of the dimensionless height of the nonflat surface on the heat transfer rate, variations of the mean Nusselt number with the dimensionless height of the nonflat surface for the top and bottom surfaces of the circular element for the range of Rayleigh number values considered in this study and for the case where the nonflat surface is located on the top side, on the bottom side, and on both the top and bottom sides of the circular element are shown in Figures 8.5, 8.6 and 8.7, respectively. From these figures it can be seen from the results for the case where the nonflat surface is on the top side of the circular element given in Figure 8.5 that for the top surface of the heated element the variation in the dimensionless height has an almost negligible effect on the Nusselt number except at the lower values of Rayleigh number (approximately less than $10^6$) whereas this variation in the dimensionless height has a negligible effect on the Nusselt numbers for the bottom surface except at the higher values of Rayleigh number (approximately higher than $10^{11}$), this effect being significant at lower values of dimensionless height (approximately
less than 0.1). Secondly, when the nonflat surface is on the bottom side of the circular element, it can be seen from the results given in Figure 8.6 that for the top surface of the heated element the variation in the dimensionless height has no significant effect on the Nusselt numbers whereas for the bottom surface the variation in the dimensionless height has a significant effect on the Nusselt numbers, the nature of this effect being dependent on the value of Rayleigh number. Lastly, it will be seen from the results that when the nonflat surface is located on both the top and bottom surfaces of the circular element (see Figure 8.7) that the variation in the dimensionless height has an almost negligible effect on the Nusselt numbers for the top surface of the heated element except at the lower values of Rayleigh number (approximately less than $10^6$) whereas this variation in the dimensionless height has a significant effect on the Nusselt numbers for the bottom surface.
Figure 8.5 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a circular element having a nonflat surface on the top side with the dimensionless height, $H$, for various values of Rayleigh number.

Figure 8.6 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a circular element having a nonflat surface on the bottom side with the dimensionless height, $H$, for various values of Rayleigh number.
Figure 8.7 Variations of the mean Nusselt number for the (a) top surface and (b) bottom surface of a circular element having a nonflat surface on the top and bottom sides with the dimensionless height, \( H \), for various values of Rayleigh number.

To more clearly show the effect of the location of the nonflat surface on the heat transfer rate, typical variations of the mean Nusselt number with Rayleigh number for the top and bottom surfaces of the heated element for the various nonflat surface locations considered are given in Figures 8.8 and 8.9, for dimensionless heights of 0.1 and 0.3, respectively. It will be seen from these figures that the Nusselt numbers are almost the same when the nonflat surface is either located on the bottom side or on both the top and bottom sides of the heated element in all cases considered except at the intermediate values of Rayleigh number.
number for the dimensionless height of 0.1 and at the lower values of Rayleigh number for the dimensionless height of 0.3. Figure 8.8 shows that for a dimensionless height of 0.1, the location of the nonflat surface on the heated element does not play a significant role in the Nusselt number change for the top surface of the heated element while in the case of the bottom surface the Nusselt number is a little bit higher when the nonflat surface is either located on the bottom side or on both the top and bottom sides of the heated element than that obtained when the nonflat surface is located on the top side at lower and intermediate values of Rayleigh number. At higher values of Rayleigh number, the Nusselt number is relatively higher when the nonflat surface is located on the top side of the heated element than that when the nonflat surface is either located on the bottom side or on both the top and bottom sides. For the results obtained for the other dimensionless height of 0.3 given in Figure 8.9, it will be seen that for the top surface of the heated element the Nusselt number is a little bit higher when the nonflat surface is located on the top side of the heated element, this is being especially true at the lower values of Rayleigh number. It can also be seen from this figure that the Nusselt number is higher when the nonflat surface is on both the top and bottom sides of the heated element than that when the nonflat surface is on the bottom side at the lower values of Rayleigh number. From the results for the bottom surface, it can be seen from Figure 8.9 that when the nonflat surface is either on the bottom side or on both the top and bottom sides of the heated element the Nusselt number is higher than that existing when the nonflat surface is on the top side for the Rayleigh number values of up to approximately $10^{11}$ after which this behavior is reversed until at Rayleigh number value of approximately $10^{13}$, the effect of the nonflat surface location being almost negligible at higher values.
Figure 8.8 Variations of the mean Nusselt number with Rayleigh number for the (a) top surface and (b) bottom surface of a circular element having nonflat surface on various sides considered for the dimensionless height, $H$, of 0.1.
Figure 8.9 Variations of the mean Nusselt number with Rayleigh number for the (a) top surface and (b) bottom surface of a circular element having nonflat surface on various sides considered for the dimensionless height, $H$, of 0.3.

8.4 Conclusion

Simultaneous natural convective heat transfer from the top and bottom surfaces of the circular horizontal isothermal heated element having a linearly-increasing nonflat surface located on the top side, or on the bottom side, or on both the top and bottom sides of the heated element has been considered in this study. Various values of nonflat surface
dimensionless heights and a wide range of Rayleigh numbers have been considered. The results of this study show that:

1. In most cases, the existence of the nonflat surface on the heated element with various heights considered can produce heat transfer rate enhancement but this enhancement is relatively small.

2. The Nusselt numbers for the top surface are lower than those for the bottom surface at the lower Rayleigh number values but are higher than those for the bottom surface at the higher Rayleigh number values in all of the cases considered. This seems to be the result of the thickening of the boundary layer on the top surface at lower Rayleigh numbers where the inward flows collide forming the plume along with a thinning of the bottom surface boundary layer where the outward flows divide (Chambers and Lee, 1997).

3. The variation of the dimensionless height of the nonflat surface has a much greater effect on the heat transfer rate from the bottom surface than on the heat transfer rate from the top surface of the heated element, the nature of this effect being dependent on the Rayleigh number value, the only significant effect on the heat transfer rate from the top surface being at the lower values of Rayleigh number.

4. The form of variation of the Nusselt number with the dimensionless height of the nonflat surface for the top and bottom surfaces of the heated element is similar for the range of Rayleigh number values considered except at the higher values of Rayleigh number.

5. The location of the nonflat surface on the heated element, i.e., on the top, on the bottom or on both surfaces affects the heat transfer rate, the nature of this effect
being dependent on the dimensionless height of the nonflat surface and the value of Rayleigh number.
Chapter 9

Natural Convective Heat Transfer in Rectangular Bottom-Heated Enclosures that Contain a Nanofluid and which have Varying Aspect Ratios

Abstract

This chapter discusses a numerical study of natural convective heat transfer from bottom-heated horizontal enclosures of varying aspect ratios filled with a copper-water nanofluid. The top wall of the enclosure is at a lower temperature than the temperature of the bottom wall and the vertical side walls are adiabatic. The aim of this study is to investigate whether the heat transfer rate can be increased when the heat transfer occurs to a nanofluid rather than to a pure fluid and to investigate the influence of the enclosure aspect ratio and the influence of the nanoparticle concentration on the heat transfer rate. The solution was numerically obtained using the commercial CFD solver ANSYS FLUENT©. The results indicate that the heat transfer rate is increased by using a nanofluid compared to that using the base fluid. The enclosure aspect ratio has a modest influence on the heat transfer rate whereas the nanoparticle concentration has a significant influence on the heat transfer rate.

9.1 Introduction

In many engineering applications, natural convective heat transfer is the option for cooling due to concerns about cost, noise and reliability. Therefore, many different ways to enhance the heat transfer rate in such situations have been considered. An innovative way to attempt to enhance the heat transfer rate is by using nanofluids and has been a topic
investigated by many researchers. The potential enhancement of heat transfer by the use of a nanofluid arises because of the enhanced thermal conductivity of the nanofluid compared to that for the base fluid. In addition to its behavior like a pure fluid makes it a prime candidate for heat transfer applications such as microchannel and minichannel heat sinks (Guiet et al., 2012).

Studies of natural convective heat transfer in enclosures that contain a nanofluid have been undertaken. Putra et al. (2003) experimentally studied the natural convective heat transfer in a horizontal cylinder heated from one end and cooled from another with two nanofluids (aluminum oxide Al₂O₃, and copper oxide CuO, both materials being in water). The results obtained in this study indicated that the natural convective heat transfer rate in the nanofluid was lower than that in the base fluid. Also, they found that the natural convective heat transfer rate decreased as the nanoparticle concentration increased.

Experiments on natural convective heat transfer in a bottom-heated horizontal enclosure consisting of two differentially heated discs filled with a TiO₂/H₂O nanofluid have been carried out by Wen and Ding (2006). The results of this study were similar to the results obtained by Putra et al. (2003). However, a numerical study by Khanafer et al. (2003), an analytical study by Kim et al. (2004), and an experimental study by Nanna et al. (2004) showed that unlike Putra et al. (2003), and Wen and Ding (2006) the natural convective heat transfer rate increases as the nanoparticle concentration increases.

Aminossadati and Ghasemi (2009) numerically studied the natural convection cooling of a heat source embedded on the bottom wall of an enclosure filled with nanofluids. The top and vertical walls of the enclosure are maintained at a relatively low temperature. The
results showed that using nanofluids improves the cooling performance compared with the situation when water alone is used especially at low Rayleigh numbers.

Ho et al. (2010) conducted an experimental study on natural convective heat transfer in an alumina–water nanofluid in side-heated vertical square enclosures of different sizes. The authors concluded that the natural convective heat transfer across the enclosure was increased.

Guiet et al. (2012) undertook a numerical study on natural convection from a protruding heater located at the bottom of a square enclosure filled with a copper-water nanofluid. The vertical walls of the enclosure were cooled isothermally while the horizontal walls were kept adiabatic. It was found that the heat transfer was enhanced by increasing the nanoparticle concentration for all of the thermal boundary conditions applied on the heater.

A Numerical study of natural convection in a square enclosure with non-uniform temperature distribution maintained at the bottom wall and filled with different types of nanofluids having various nanoparticle concentrations was carried out by Ben-Cheikh et al. (2013). The remaining walls of the enclosure are kept at a lower temperature. It was shown that an enhancement in the heat transfer rate was obtained, this increasing as the nanoparticle concentration increased.

**9.2 Problem Description**

Figure 9.1 shows a schematic diagram of the enclosure considered in the present study. The bottom wall is heated to a uniform high temperature \(T_b\). The top wall is also isothermal but at a lower temperature \(T_t\) than that of the bottom wall. The vertical side walls are adiabatic. The base fluid considered in this study is water and the solid nanoparticles are copper (Cu). Two concentrations of the nanoparticles have been
considered in the present study, these being 10% and 20% on a volumetric basis. It has been assumed that the base fluid and the solid nanoparticles are in thermal equilibrium. The assumed thermophysical properties of the base fluid and the nanoparticles are given in Table 9.1.

![Schematic diagram of the enclosure](image)

**Figure 9.1** A schematic diagram of the enclosure considered.

**Table 9.1** Thermophysical properties of base fluid and nanoparticles, Abu-Nada, et al. (2008)

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>997.1</td>
<td>8933</td>
</tr>
<tr>
<td>$c_p$ (J/kgK)</td>
<td>4179</td>
<td>385</td>
</tr>
<tr>
<td>$k$ (W/mK)</td>
<td>0.613</td>
<td>401</td>
</tr>
<tr>
<td>$\beta$ (1/K)</td>
<td>0.00021</td>
<td>0.0000167</td>
</tr>
</tbody>
</table>

Since there are three fluids under consideration (water, nanofluid with a 0.1 nanoparticle concentration and nanofluid with a 0.2 nanoparticle concentration), then for each flow situation there will be a specific Rayleigh number corresponding to each fluid. Table 9.2 summarizes the Rayleigh numbers used in this study for the fluids considered.
Table 9.2 Rayleigh numbers used in the present study for various fluids considered in the different flow situations

<table>
<thead>
<tr>
<th>Flow Situation</th>
<th>Rayleigh Number, Ra</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water</td>
</tr>
<tr>
<td>1</td>
<td>1.00E+02</td>
</tr>
<tr>
<td>2</td>
<td>1.00E+03</td>
</tr>
<tr>
<td>3</td>
<td>2.50E+03</td>
</tr>
<tr>
<td>4</td>
<td>5.50E+03</td>
</tr>
<tr>
<td>5</td>
<td>1.00E+04</td>
</tr>
<tr>
<td>6</td>
<td>1.00E+05</td>
</tr>
<tr>
<td>7</td>
<td>1.00E+06</td>
</tr>
<tr>
<td>8</td>
<td>5.50E+06</td>
</tr>
<tr>
<td>9</td>
<td>1.00E+07</td>
</tr>
</tbody>
</table>

9.3 Nanofluid Properties Prediction

Many equations for the properties of a nanofluid have been proposed. The expressions adopted in this study for predicting the nanofluid density, specific heat and thermal expansion coefficient are those developed by Minkowycz et al. (2013). These expressions are:

\[ \rho_{nf} = (1 - \phi_v)\rho_{bf} + \phi_v\rho_p \]  \hspace{1cm} (9.1)

\[ (\rho c_p)_{nf} = (1 - \phi_v)\rho c_p + \phi_v\rho c_{p} \]  \hspace{1cm} (9.2)

\[ (\rho \beta)_{nf} = (1 - \phi_v)(\rho \beta)_{bf} + \phi_v(\rho \beta)_{p} \]  \hspace{1cm} (9.3)

The dynamic viscosity of the nanofluid is assumed to be given by the following equation (Brinkman, 1952):

\[ \frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{(1 - \phi_v)^{2.5}} \]  \hspace{1cm} (9.4)
The thermal conductivity of the nanofluid is given by the following equation (Maxwell, 1904):

\[
\frac{k_{nf}}{k_{bf}} = \frac{(k_p + 2k_{bf}) - 2\phi_v(k_{bf} - k_p)}{(k_p + 2k_{bf}) + \phi_v(k_{bf} - k_p)}
\]  

(9.5)

9.4 Solution Procedure

The flow has been assumed to be steady, two-dimensional and laminar. The fluid properties have been assumed constant except for the density change with temperature which gives the rise to the buoyancy forces (i.e., the Boussinesq approach has been adopted). The governing equations subject to the boundary conditions have been numerically solved using the commercial CFD solver ANSYS FLUENT©. To ensure that the results obtained are grid independent, an extensive grid-independence study using a wide range of numbers of grid points and convergence-criteria independence testing was undertaken, and the heat transfer results presented here are grid and convergence-criteria independent to less than one per cent. Further details of the numerical solution procedure used are given in section 3.2.

9.5 Results and Discussion

The solution has the following parameters:

1. The Rayleigh number, \(Ra\), based on the width of the enclosure and the difference between the temperatures of the bottom and top walls, i.e.,

\[
Ra = \frac{g \beta w_e^3 (T_b - T_l)}{\nu \alpha}
\]

(9.6)
2. The aspect ratio of the enclosure, $AR$, which is defined as (see Figure 9.1):

$$AR = \frac{l_e}{w_e}$$  \hspace{1cm} (9.7)

3. The Prandtl number, $Pr$.

Results have been obtained for 3 values of Prandtl number: 5.83, 3.12 and 2.17, which are effectively the values for water, nanofluid with a 0.1 nanoparticle concentration and nanofluid with a 0.2 nanoparticle concentration, respectively. Three values of the enclosure aspect ratio have been considered: 1, 2, and 3.

The mean heat transfer rate from the heated surface of the enclosure is presented for the three fluids considered (water, nanofluid with a 0.1 nanoparticle concentration and nanofluid with a 0.2 nanoparticle concentration) for the three aspect ratios considered. The heat transfer results are expressed in terms of a mean Nusselt number based on the width of the enclosure and the difference between the temperatures of the bottom and top walls. The mean Nusselt number for a given fluid is dependent on the Rayleigh number, on the aspect ratio of the enclosure and on the Prandtl number. The mean Nusselt numbers for the heated surface, $Nu$ is, therefore, given by:

$$Nu = \frac{\overline{Q_e w_e}}{A_e(T_b - T_t)k}$$  \hspace{1cm} (9.8)

where $A_e$ is the area of the bottom wall of the enclosure.

Attention will first be given to the effect of the enclosure aspect ratio on the heat transfer rate. Variations of the mean Nusselt number with Rayleigh number for water, nanofluid with a 0.1 nanoparticle concentration, and nanofluid with a 0.2 nanoparticle concentration are shown in Figure 9.2. It will be seen from this figure that for the lower values of Rayleigh number the heat transfer is by pure conduction, i.e., there is no flow and the Nusselt number
is equal to one. Above a certain value of Rayleigh number (different for each fluid), flow starts to develop and a natural convective heat transfer mode exists. For water (Figure 9.2-a) the aspect ratio has a significant effect on the Nusselt number at lower Rayleigh numbers, i.e., when the aspect ratio increases above 1, the natural convective heat transfer mode starts at lower Rayleigh number than when the aspect ratio is equal to 1 unlike the case for the two nanofluids considered where the convective heat transfer starts at the same Rayleigh number value for the three aspect ratios considered. When flow starts to take place, i.e., \( \text{Nu} > 1 \) it can be seen from Figure 9.2 that the Nusselt number increases slightly as the aspect ratio increases at a given value of Rayleigh number, this effect existing for the range of Rayleigh numbers considered in the natural convective flow region for all the fluids considered. Figures 9.3-9.5 shows typical isotherms (temperature unit is \( K \)) showing flow patterns for water, for nanofluid with a 0.1 nanoparticle concentration and for nanofluid with a 0.2 nanoparticle concentration respectively for aspect ratios considered at the same flow situation. In these figures, it can be seen how the flow pattern of the fluid varies with the aspect ratio.
Figure 9.2 Variations of the mean Nusselt number with Rayleigh number for (a) water, (b) a nanofluid with a 0.1 nanoparticle concentration and (c) a nanofluid with a 0.2 nanoparticle concentration for various values of enclosure aspect ratio considered.
Figure 9.3 Isotherms showing the flow patterns for water for (a) AR=1, (b) AR=2, and (c) AR=3 at a Rayleigh number value of $10^3$.

Figure 9.4 Isotherms showing the flow patterns for a nanofluid with a 0.1 nanoparticle concentration for (a) AR=1, (b) AR=2, and (c) AR=3 at a Rayleigh number value of $5.52 \times 10^3$. 


Figure 9.5 Isotherms showing the flow patterns for a nanofluid with a 0.2 nanoparticle concentration for (a) AR=1, (b) AR=2, and (c) AR=3 at a Rayleigh number value of $2.98 \times 10^3$.

To more clearly show the effect of replacing the pure water with a nanofluid and the effect of the nanoparticle concentration on the heat transfer rate, variations of the heat transfer rate ratio—which represents the ratio of the mean heat transfer rate of the nanofluid to that of water—with Rayleigh number for water are shown in Figure 9.6 for the three aspect ratios considered. In this figure, the Rayleigh number for water was used as a reference since the interest here is to investigate whether the heat transfer rate can be enhanced by using a nanofluid compared with that existing with pure water. It can be seen from this figure that for all aspect ratios considered, the highest heat transfer enhancement occurs when there is no-flow, i.e., in the pure conduction region, this being due to the increase in the thermal conductivity by replacing the pure water with a nanofluid and by increasing
the concentration of the nanoparticles in this nanofluid. In the natural convective flow region, it can be seen that heat transfer enhancement is relatively lower than that in the conduction region as a result of the effect of viscosity on the flow. In this region, the heat ratio increases as the Rayleigh number increases until the value of approximately $10^6$ after which the heat ratio starts to decrease. This decrease in the heat ratio is mainly due to the significant effect of viscosity on the flow. To illustrate how the flow varies for each fluid considered, typical isotherms showing flow patterns for water, a nanofluid with a 0.1 nanoparticle concentration and a nanofluid with a 0.2 nanoparticle concentration are shown in Figure 9.7 for an enclosure aspect ratio of 2 at the same flow situation.
Figure 9.6 Variations of the heat transfer rate ratio between nanofluid with various nanoparticle concentrations considered and water with Rayleigh number for water for (a) AR=1, (b) AR=2, and (c) AR=3.

Figure 9.7 Isotherms showing the flow patterns for (a) water, (b) a nanofluid with a 0.1 nanoparticle concentration, and (c) a nanofluid with a 0.2 nanoparticle concentration for an enclosure aspect ratio of 2 at the same flow situation.
Variations of the mean Nusselt number with Rayleigh number for the fluids considered are shown in Figure 9.8 for the three aspect ratios considered. It can be seen from this figure that replacing water with a nanofluid with a 0.1 nanoparticle concentration results in the natural convective heat flow to develop at higher Rayleigh number compared with the situation of water, this behavior being true for all aspect ratios considered except when the aspect ratio is equal to 1 where water and a nanofluid with a 0.1 nanoparticle concentration have approximately the same conduction range. The same situation can be noticed by increasing the nanoparticle concentration from 0.1 to 0.2 where the flow in the case of a nanofluid with a 0.2 nanoparticle concentration takes place at higher Rayleigh number compared with the case for a nanofluid with a 0.1 nanoparticle concentration or all aspect ratios considered. It can be also seen from this figures that replacing water with a nanofluid with a 0.1 nanoparticle concentration and increasing the nanoparticle concentration from 0.1 to 0.2 significantly increases the heat transfer rate as a result of the enhanced thermal conductivity for the nanofluid compared with that for water for the whole range of Rayleigh numbers considered except at the transition region between conduction and natural convection, i.e., at Rayleigh numbers between approximately $10^3$ and $10^4$, this being due to the difference in Rayleigh numbers at which natural convective flow of each fluid starts to occur. It will be also seen from this figure that the mean Nusselt number for water is higher than that for a nanofluid with a 0.1 nanoparticle concentration and the mean Nusselt number decreases as the nanoparticle concentration increases from 0.1 to 0.2. This is basically due to the enhanced thermal conductivity for the nanofluid compared with that for water and for the nanofluid with a 0.1 nanoparticle concentration compared to that for the nanofluid with a 0.2 nanoparticle concentration.
Figure 9.8 Variations of Nusselt number for various fluids considered with Rayleigh number for water for (a) AR=1, (b) AR=2, and (c) AR=3.
9.6 Conclusion

The following conclusions can be drawn from this study:

1. The enclosure aspect ratio has a small effect on the heat transfer rate for the fluids considered.

2. A significant enhancement in the heat transfer rate is obtained in the convective heat transfer region by replacing water with a copper-water nanofluid and this enhancement is increased by increasing the nanofluid concentration, this effect being the result of the increase in the thermal conductivity.

3. Increasing the nanofluid concentration results in a delay in the start of natural convective flow.

4. The heat transfer rate ratio between the nanofluid with concentrations considered and water decreases above a certain value of water-based Rayleigh number due to the more significant effect of viscosity on the flow.
Chapter 10

Summary and Conclusions

10.1 Summary

In this thesis natural convective heat transfer from two-sided horizontal and inclined flat plates with various forms of inclination have been numerically and experimentally investigated. The studies were undertaken for the case where the plates are exposed to air. The flow conditions considered were such that laminar, transitional and turbulent flow can occur over the plate. Attention was given to an examination of the influence of the dimensionless plate thickness, of the inclination angle, of the thermal boundary condition over the vertical side surface of the plate, of the plate shape, and of the influence of a nonflat surface on the heat transfer rate from the plate surfaces. In the last part of the thesis attention was given to the natural convective heat transfer from a bottom-heated horizontal enclosure to a nanofluid. The influences of replacing the pure water with the proposed nanofluid and of the enclosure aspect ratio on the heat transfer rate were examined. The conclusions for each of the cases considered are presented in the next section.

10.2 Conclusion

In chapters 4 and 5 a numerical and experimental study of natural convective heat transfer from two-sided heated horizontal plates having a finite thickness were discussed. The plates considered varied from having simple shapes (circular and square) to having complex shapes (I and +). The influence of the dimensionless plate thickness, of the thermal boundary condition over the vertical side surface of the plate and of the plate shape on the heat transfer rate from the plate surfaces was examined. The conclusions from these studies are that in all cases the Nusselt numbers for the top surface are lower than those for
the bottom surface at the lower Rayleigh number values considered but are higher than those for the bottom surface at the higher Rayleigh number values. The changes in the value of the dimensionless plate thickness have an almost negligible influence on the Nusselt numbers for the bottom surface of the plates, however, these changes in the value of the dimensionless plate thickness have a significant influence on the Nusselt number for the top surface. The vertical side surface thermal boundary condition has only a very small influence on the Nusselt numbers for the bottom surface of the plates but has a significant influence on the Nusselt numbers for the top surface of the plates at the lower Rayleigh number values considered. For the various thermal boundary conditions considered, when the length scale, $l$, is used the variations of Nusselt number with Rayleigh number for the square and circular plates are nearly the same for the top surface of the plates but differ somewhat at the higher Rayleigh number values considered for the bottom surface. For the I- and + - shaped plates, however, the variations of Nusselt number with Rayleigh number are almost the same at the lower Rayleigh number values considered but differ slightly at the higher Rayleigh number values. The difference in the variation of Nusselt and Rayleigh number for the circular, square, I-, and + -shaped plates is mainly due to differences in the predicted Rayleigh number values at which transition to turbulence starts to occur.

In general, the estimated uncertainty in the experimental results was less than 10 per cent for the circular and square plates and less than 6 per cent for the I- and + -shaped plates and most of the numerical results lie within this uncertainty. Generally, very good agreement between the numerical and experimental results was obtained.

In chapters 6 and 7 numerical and experimental studies of natural convective heat transfer from two-sided inclined square plates having various thicknesses for two forms of inclination
and at various inclination angles were considered. Chapter 6 dealt with the situation of the square plate having its side length inclined with respect to horizontal while chapter 7 dealt with the situation of a square plate which was diagonally inclined with respect to horizontal. The influence of the dimensionless plate thickness as well as the influence of the various forms of the inclination on the heat transfer rate from the plate surfaces was investigated. It can be concluded from the results presented in these studies that in all cases the Nusselt number for the top surface is lower than that for the bottom surface at the lower Rayleigh number values considered but is higher than that for the bottom surface at the higher Rayleigh number values. For the forms of inclination considered, the changes in the value of the dimensionless plate thickness have an almost negligible influence on the Nusselt numbers for the bottom surface of the plate but these changes in the dimensionless plate thickness have a significant influence on the Nusselt numbers for the top surface. For the dimensionless plate thicknesses considered, the influence of the various forms of inclination on the Nusselt numbers for the top surface of the plate is higher than that for the bottom surface for the range of Rayleigh number values considered. For the diagonally inclined square plate situation, the numerical and experimental results are in very good agreement and the numerical results lie within the uncertainty of the experimental data which is at most 11 per cent except at the diagonal inclination angle of 40° where the numerical results lie outside the experimental uncertainty.

In chapter 8 simultaneous natural convective heat transfer from the top and bottom surfaces of a circular horizontal isothermal heated element having a linearly-increasing nonflat surface located on the top side, on the bottom side and on both the top and bottom sides of the heated element has been numerically studied. Attention was given to the
influence of the dimensionless height and location of the nonflat surface on the heat transfer rate from the plate surfaces. The conclusions from this study are that in most cases, the heat transfer rate could be enhanced by using the nonflat surface but this enhancement is relatively small. In all cases, the Nusselt number for the top surface is lower than that for the bottom surface at the lower Rayleigh number values but is higher than that for the bottom surface at the higher Rayleigh number values. Variations of the dimensionless height of the nonflat surface affects the heat transfer rate especially from the bottom surface of the heated element. The location of the nonflat surface with respect to the heated element affects the heat transfer rate, the nature of this effect being dependent on the dimensionless height of the nonflat surface and the value of Rayleigh number.

In chapter 9 the natural convective heat transfer in rectangular bottom-heated enclosures with varying aspect ratios filled with nanofluids having various nanoparticle concentrations was numerically investigated. Attention was given to whether the heat transfer rate can be increased when the heat transfer occurs to a nanofluid rather than to pure water and to determine the influence of the enclosure aspect ratio and the influence of the nanoparticle concentration on the heat transfer rate. It can be concluded from the results obtained in this study that the heat transfer rate is increased by using a nanofluid compared to that using water. The enclosure aspect ratio has a small influence on the heat transfer rate while the nanoparticle concentration significantly influences the heat transfer rate.

10.3 Future Work

The research presented in this thesis has developed numerical and experimental approach that enables the investigation of natural convective heat transfer from two-sided horizontal and inclined with different forms of inclination flat plates in air. It has also
developed a numerical method to investigate heat transfer enhancement from heated elements in air using a nonflat surface and from heated elements using a nanofluid. There are some ideas that can be considered as a future work to touch more further details for the situations described in this work:

- Evaluate different turbulence models other than k-epsilon to investigate the natural convective heat transfer in the transitional and turbulent flow regime for heated elements in air.

- Study other nonflat surfaces to determine if they can produce a heat transfer enhancement from heated elements in air.

- Use various types of nanofluid with different nanoparticle concentrations to investigate the heat transfer enhancement in other types of enclosures like the partially heated wall enclosures. Also undertake experimental studies of the situations already studied numerically in this area.
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Appendix A

Length Scale Analysis

A.1 Introduction

The proposed length scale used in the present study was that proposed in previous studies for the purpose of correlating the heat transfer results for different plate shapes. This can be done by using Nusselt and Rayleigh numbers based on the length scale proposed. The aim of this analysis is to investigate whether the variations of Nusselt number with Rayleigh number for two-sided plate shapes could be correlated by using the single-sided plate-based length scale. Attention will first be given to the variation of the Nusselt number with Rayleigh number for the two-sided square and circular shape plates and will then be turned to the two-sided I- and +-shaped plates.

A.2 Square and Circular Shape Plates

The two-sided square and circular plates considered in the present study are shown in Figure A.1. A length scale $l$ based on the ratio between the heated surface area and the perimeter of that surface, i.e., $l = 4A_{plate}/P$ will be used. Two cases will be considered, one case in which the existence of the vertical side surface of the plate is ignored, i.e., dealing with the situation that would exist with the single-sided plate and in the other case where consideration is given to the vertical side surface of the plate, i.e., dealing with the situation that would exist with the two-sided plate.
A.2.1 Single-Sided Plate-Based Length Scale

As stated earlier, the length scale used is given by the following equation:

\[ l = 4A_{plate}/P \]  \hspace{1cm} (A.1)

For the case of circular plate:

\[ A_{plate} = \frac{\pi}{4} d^2 \]  \hspace{1cm} (A.2)

\[ P = \pi d \]  \hspace{1cm} (A.3)

and for the case of square plate:

\[ A_{plate} = w^2 \]  \hspace{1cm} (A.4)

\[ P = 4w \]  \hspace{1cm} (A.5)

Therefore, \( l = d \) in the case of the circular plate and \( l = w \) in the case of the square plate.

Typical variations of the mean Nusselt number with Rayleigh number based on the length scale \( l \) for the top, bottom and vertical side surfaces of the circular and square plates are shown in Figures A.2 and A.3 for dimensionless plate thicknesses values of 0.1 and 0.3, respectively.
Figure A.2 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for square and circular plates for a dimensionless plate thickness, $H$, of 0.1 when the single-sided plate-based length scale is used.
Figure A.3 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for square and circular plates for a dimensionless plate thickness, $H$, of 0.3 when the single-sided plate-based length scale is used.
A.2.2 Two-Sided Plate-Based Length Scale

The length scale used is given by equation A.1.

For the case of circular plate and accounting for the vertical side surface:

\[
A_{\text{plate}} = 2 \frac{\pi}{4} d^2 + \pi dh = \pi d \left( \frac{d}{2} + h \right) \quad (A.6)
\]

\[
P = 2\pi d \quad (A.7)
\]

and for the case of square plate and accounting for the vertical side surface:

\[
A_{\text{plate}} = 2w^2 + 4wh \quad (A.8)
\]

\[
P = 8w \quad (A.9)
\]

where, \( h \) is the thickness of the plate.

Therefore, \( l = d + 2h \) in the case of the circular plate and \( l = w + 2h \) in the case of the square plate.

Typical variations of the mean Nusselt number with Rayleigh number based on the length scale \( l \) for the top, bottom and vertical side surfaces of the circular and square plates are shown in Figures A.4 and A.5 for dimensionless plate thicknesses values of 0.1 and 0.3, respectively.
Figure A.4 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for square and circular plates for a dimensionless plate thickness, $H$, of 0.1 when the two-sided plate-based length scale is used.
Figure A.5 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for square and circular plates for a dimensionless plate thickness, $H$, of 0.3 when the two-sided plate-based length scale is used.

Comparing the results given in Figures A.4 and A.5 with those given in Figures A.2 and A.3 shows that the variation of the Nusselt number with Rayleigh number for the circular and square plate shapes when using the two-sided plate-based and the single-sided plate-based length scale is almost the same.
A.3 I- and +- Shaped Plates

The two-sided I- and +- shaped plates considered in the present study are shown in Figure A.6. The same approach as that used in the analysis of circular and square plates will be used in the analysis of the I- and +- shaped plates.

![Figure A.6](image)

Figure A.6 I-shaped (left) and +-shaped (right) plates.

A.3.1 Single-Sided Plate-Based Length Scale

The length scale used is given by equation A.1.

For the case of I-shaped plate:

$$A_{plate} = 0.625W^2$$  (A.10)

$$P = 5.5W$$  (A.11)

and for the case of +-shaped plate:

$$A_{plate} = 0.4375W^2$$  (A.12)

$$P = 4W$$  (A.13)

where $W$ is the outside length of the plate (Figure A.6).

Therefore, $l = 0.45455W$ in the case of the I-shaped plate and $l = 0.43750W$ in the case of the +-shaped plate.
Typical variations of the mean Nusselt number with Rayleigh number based on the length scale $l$ for the top, bottom and vertical side surfaces of the I- and +-shaped plates are shown in Figures A.7 and A.8 for dimensionless plate thicknesses values of 0.1 and 0.3, respectively.
Figure A.7 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for I- and +-shaped plates for a dimensionless plate thickness, $H$, of 0.1 when the single-sided plate-based length scale is used.
Figure A.8 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for I- and +-shaped plates for a dimensionless plate thickness, $H$, of 0.3 when the single-sided plate-based length scale is used.

A.3.2 Two-Sided Plate-Based Length Scale

The length scale used is given by equation A.1.

For the case of I-shaped plate and accounting for the vertical side surface:

$$A_{plate} = 1.25W^2 + 5.5Wh$$  \hspace{1cm} \text{(A.14)}

$$P = 11W$$  \hspace{1cm} \text{(A.15)}

and for the case of +-shaped plate and accounting for the vertical side surface:

$$A_{plate} = 0.875W^2 + 4Wh$$ \hspace{1cm} \text{(A.16)}

$$P = 8W$$ \hspace{1cm} \text{(A.17)}

where, $h$ is again the thickness of the plate.

Therefore, $l = 0.45455W + 2h$ in the case of the I-shaped plate and $l = 0.43750W + 2h$ in the case of the +-shaped plate.

Typical variations of the mean Nusselt number with Rayleigh number based on the length scale $l$ for the top, bottom and vertical side surfaces of the circular and square plates are shown.
in Figures A.9 and A.10 for dimensionless plate thicknesses values of 0.1 and 0.3, respectively.
Figure A.9 Variations of the mean Nusselt number for the (a) top surface, (b) bottom surface, and (c) vertical side surface with Rayleigh number for I- and +-shaped plates for a dimensionless plate thickness, $H$, of 0.1 when the two-sided plate-based length scale is used.
Comparing the results given in Figures A.9 and A.10 with the corresponding results given in Figures A.7 and A.8 shows that the variation of the Nusselt number with Rayleigh number for the I- and +-shaped plates when using the two-sided plate-based and the single-sided plate-based length scale is almost the same.
Appendix B

Flow Patterns

B.1 Introduction

This appendix presents flow patterns for some typical flow situations associated with the various heated elements considered in this study. The changes in the flow patterns are associated with changes in the heat transfer rates from the bodies considered. For example, a good understanding of the reasons for the variations of the Nusselt number for the top surface of the +-shaped isothermal plate with the plate dimensionless thickness can be obtained by considering the flow patterns for various values of thickness at a given Rayleigh number (Figures B.3 and B.4).

B.2 Flow Patterns

The following figures show some typical flow patterns over the heated plates considered in this study. The flow patterns are shown in terms of the isotherms and the unit of the temperature is $K$.
Figure B.1 Horizontal Isothermal Square Plate ($H=0$) at (a) $Ra=10^5$, (b) $Ra=10^6$, (c) $Ra=10^{13}$ and (d) $Ra=10^{14}$.

Figure B.2 Horizontal Square Plate ($H=0.1$) at $Ra=10^4$ (a) isothermal side surface and (b) adiabatic side surface.
Figure B.3 Horizontal Isothermal +-shaped Plate ($H=0$) at (a) Ra=10$^4$, (b) Ra=10$^8$ and (c) Ra=10$^{13}$. 
Figure B.4 Horizontal Isothermal ±-shaped Plate ($H=0.3$) at (a) $Ra=10^4$, (b) $Ra=10^8$ and (c) $Ra=10^{13}$. 
Figure B.5 Inclined square plate (inclination is with respect to side length), \((H=0)\) at \(Ra=10^{10}\)
(a) \(\theta=0^\circ\), (b) \(\theta=20^\circ\), and (c) \(\theta=30^\circ\).